Quantum criticality, the AdS/CFT correspondence and the cuprate superconductors

Statphys 24, Cairns, Australia, July 19 2010

Talk online: sachdev.physics.harvard.edu

PHYSICS

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Outline

1. Coupled dimer antiferromagnets

   Quantum criticality and CFTs

2. The AdS/CFT correspondence

   Diffusion and transport in strongly interacting CFTs

3. Quantum matter at non-zero density

   Holographic strange metals

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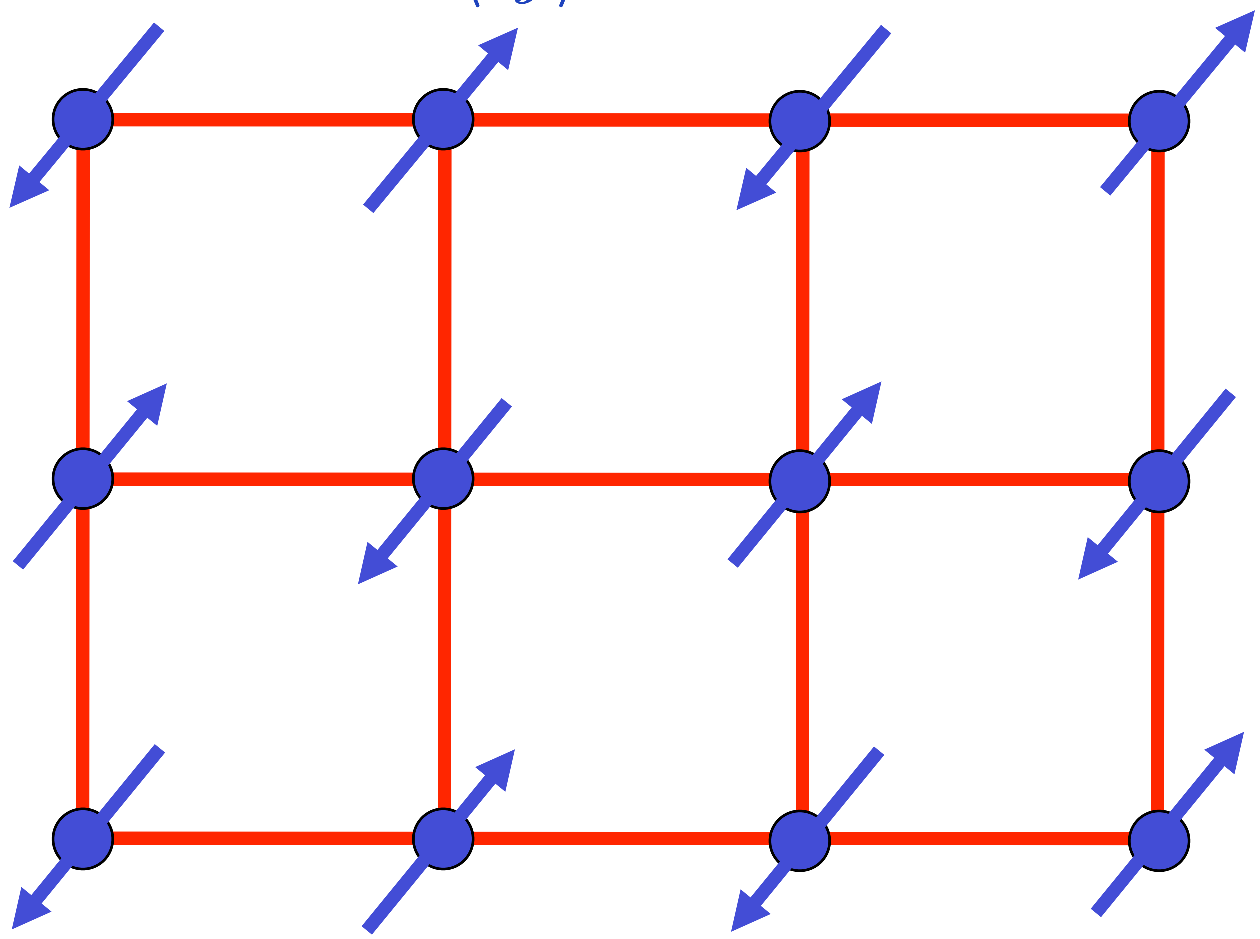
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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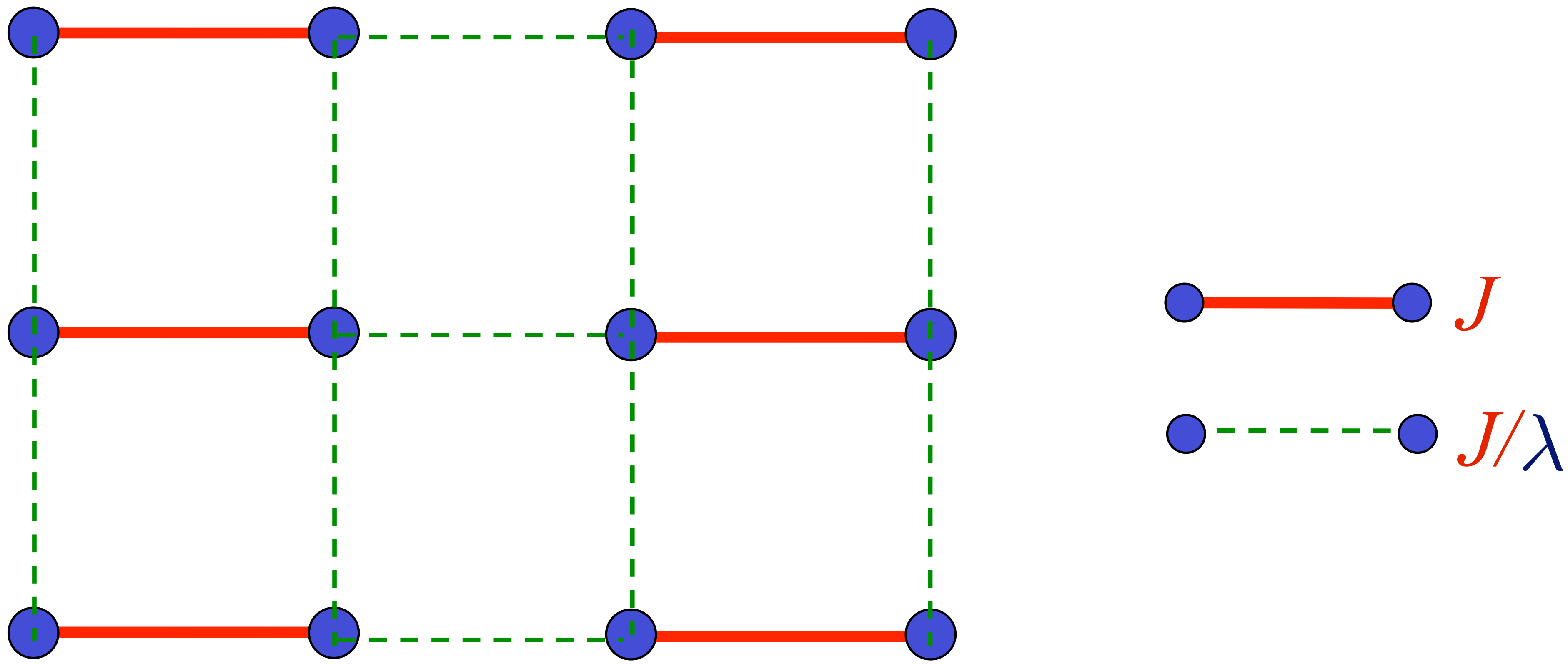

Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

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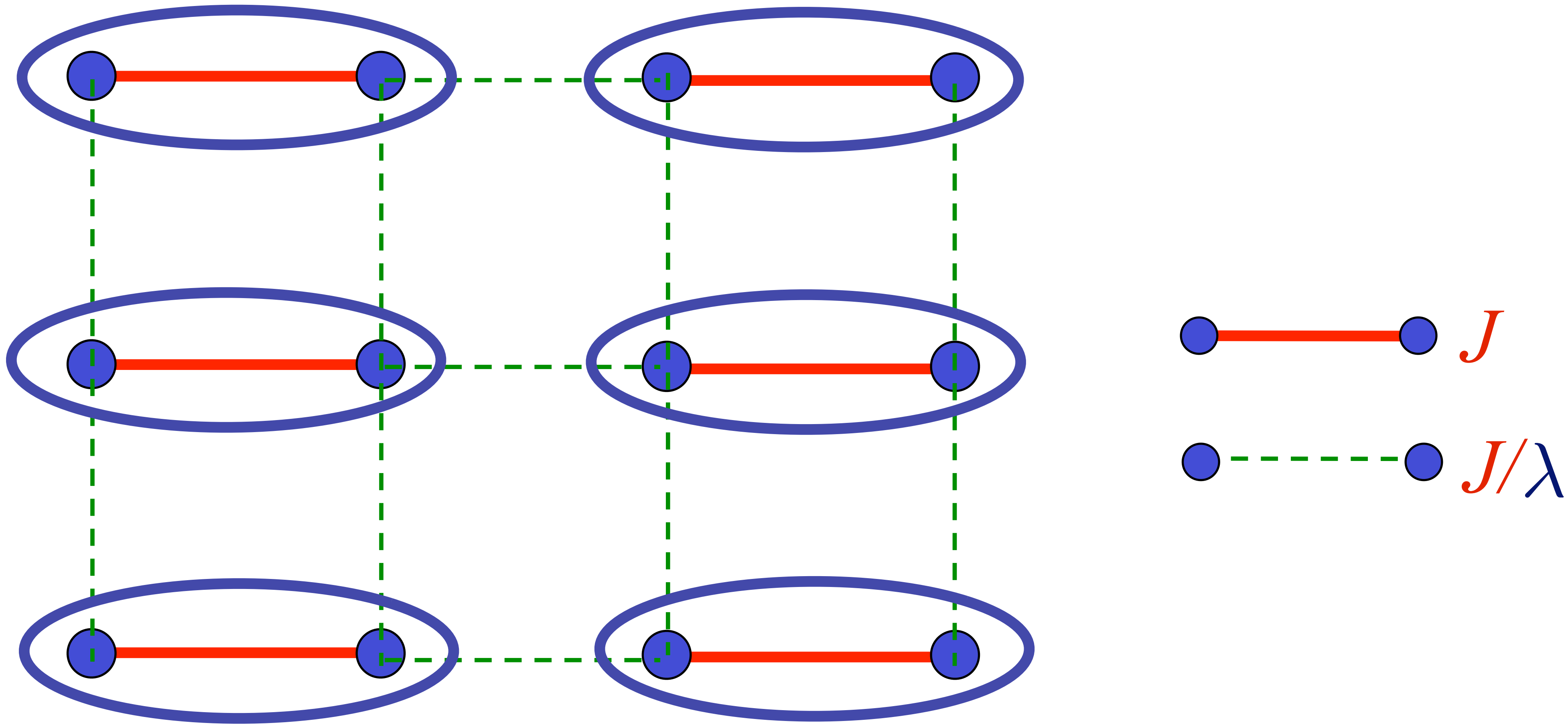
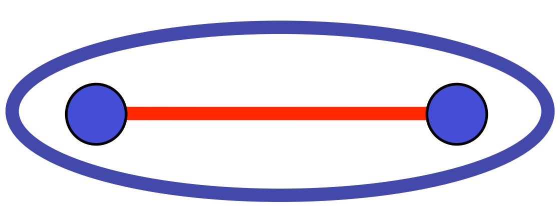
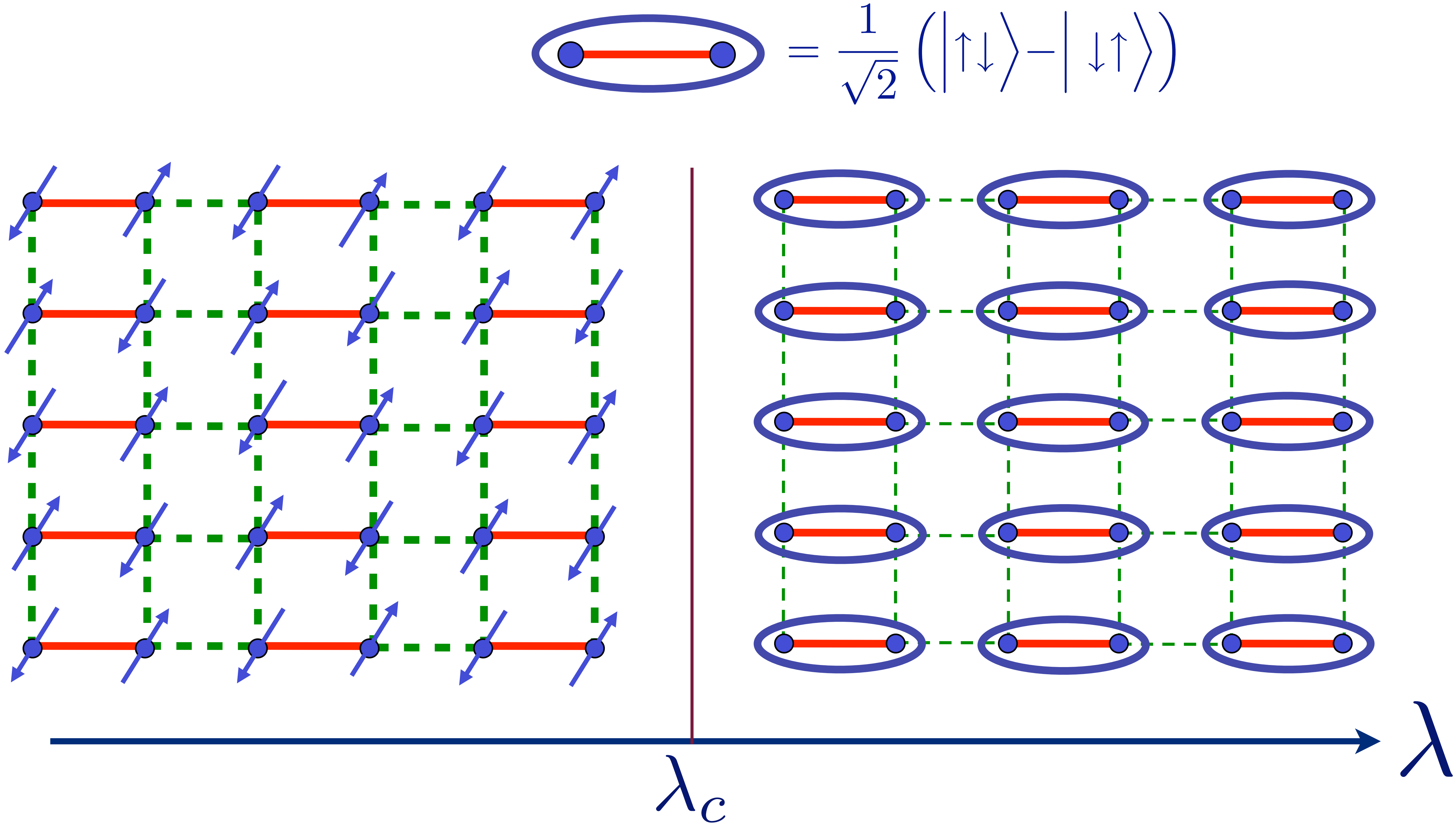
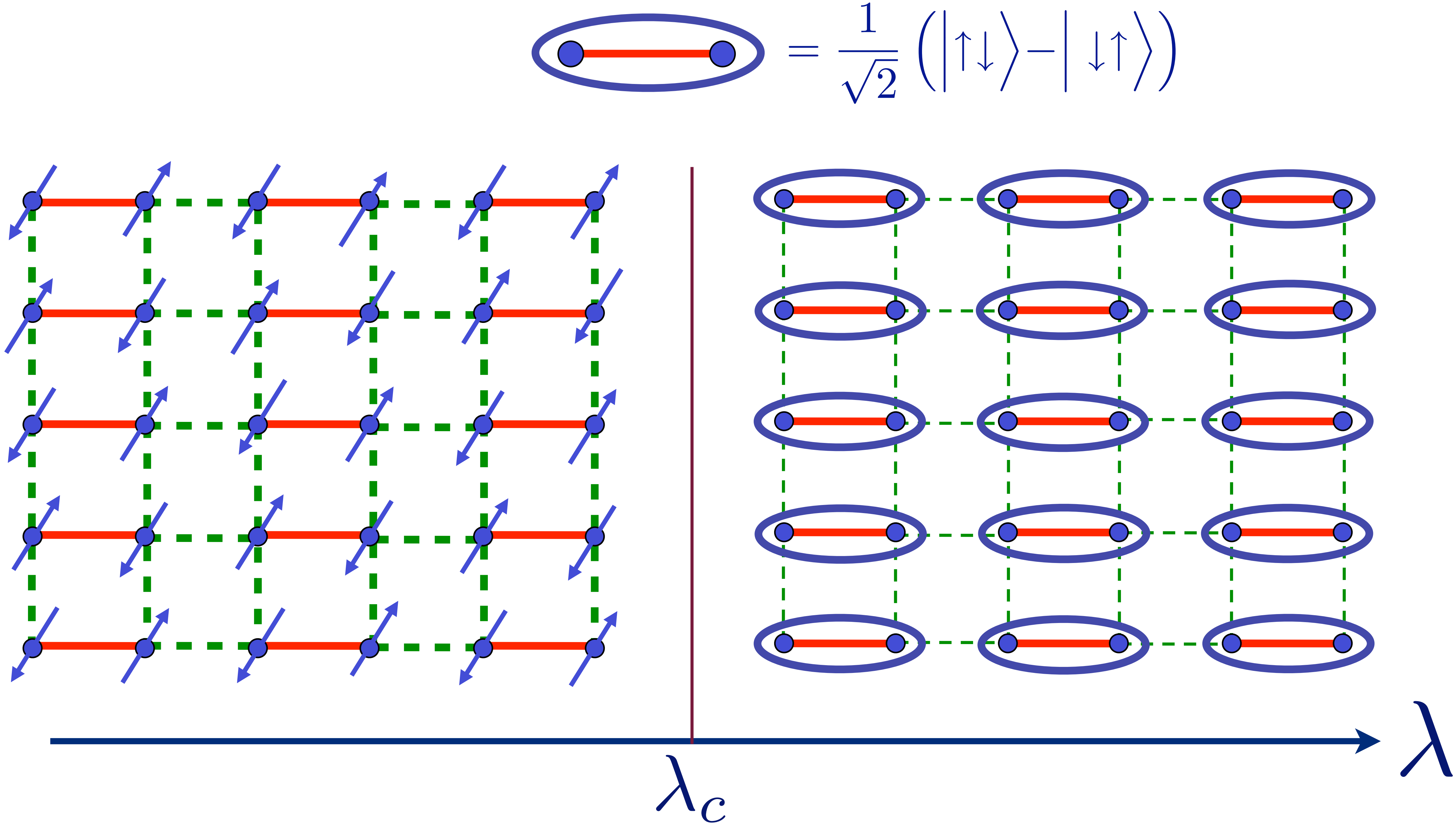


Ground state is a "quantum paramagnet" with spins locked in valence bond singlets



$$= \frac{1}{\sqrt{2}} \Big(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \Big)$$

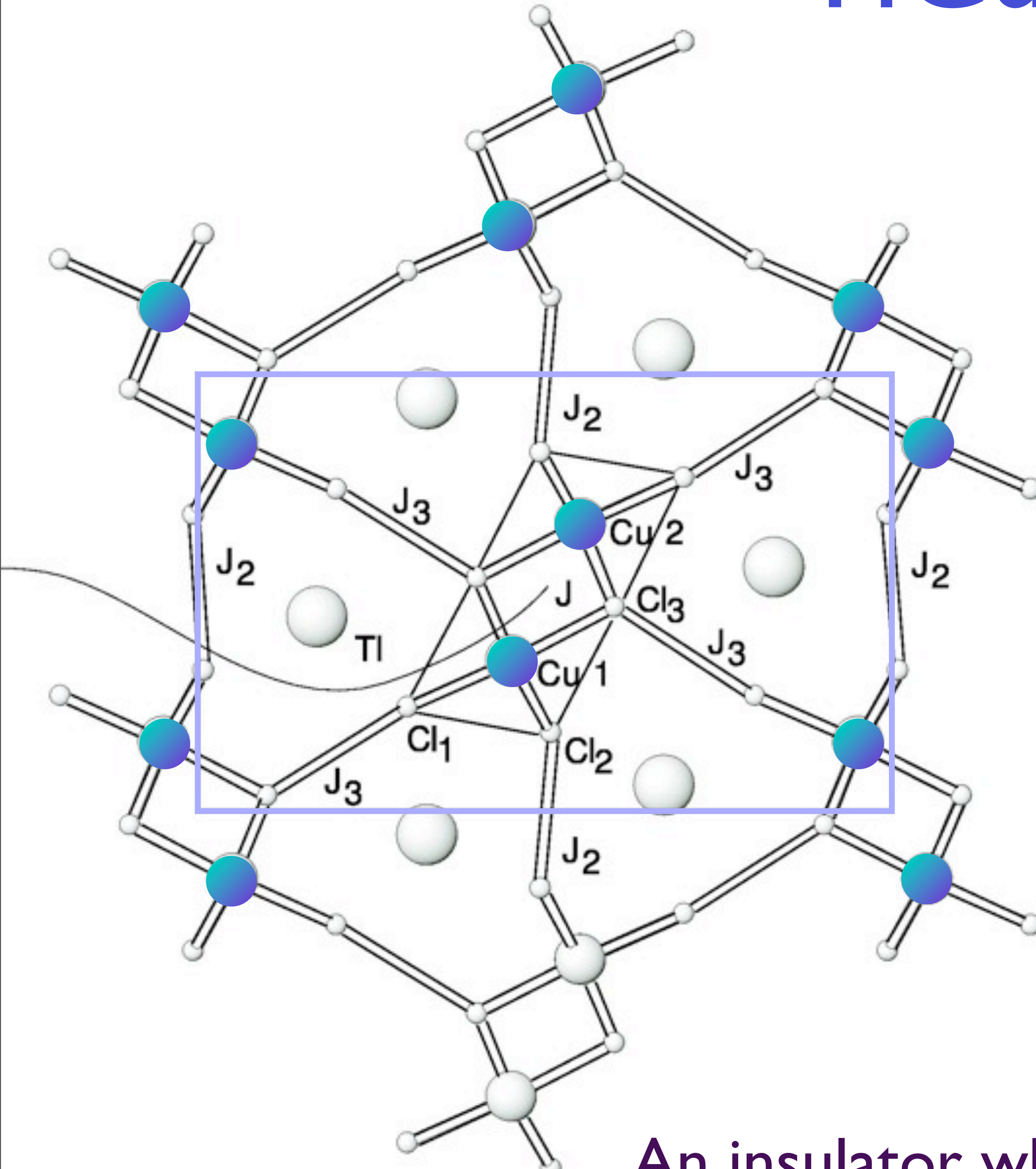

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ

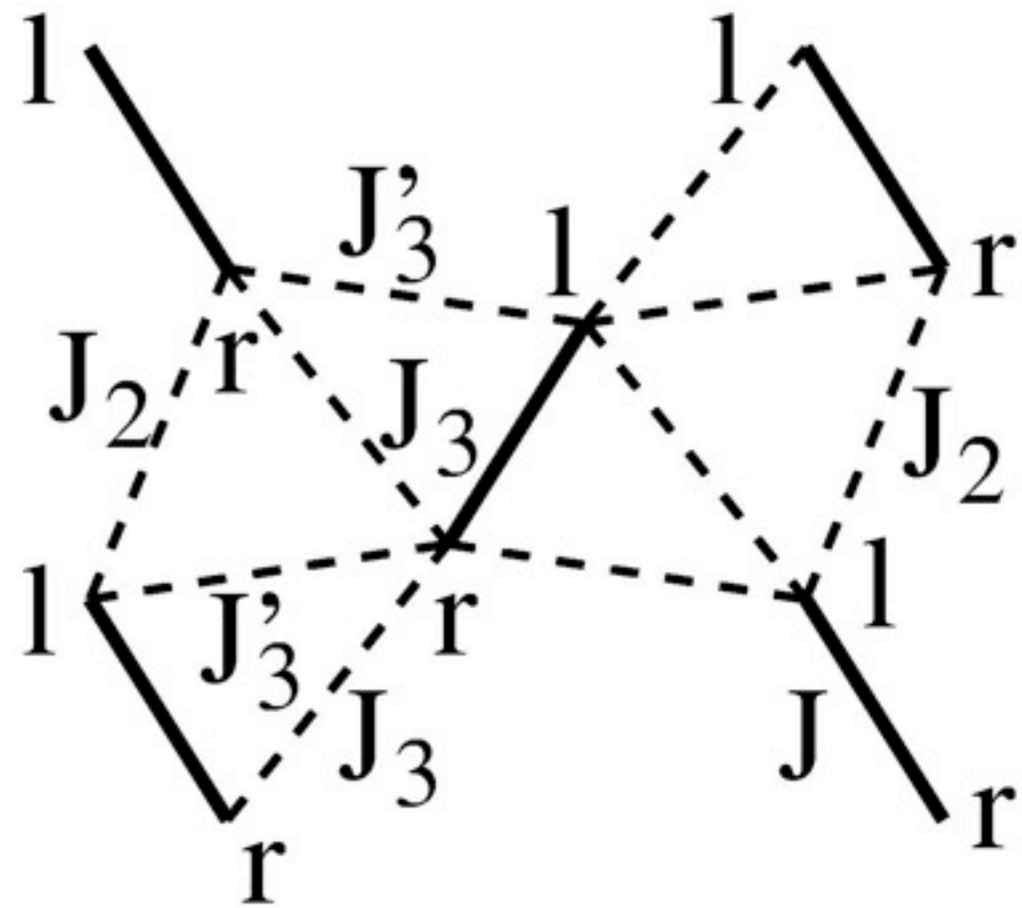


Pressure in $TlCuCl_3$

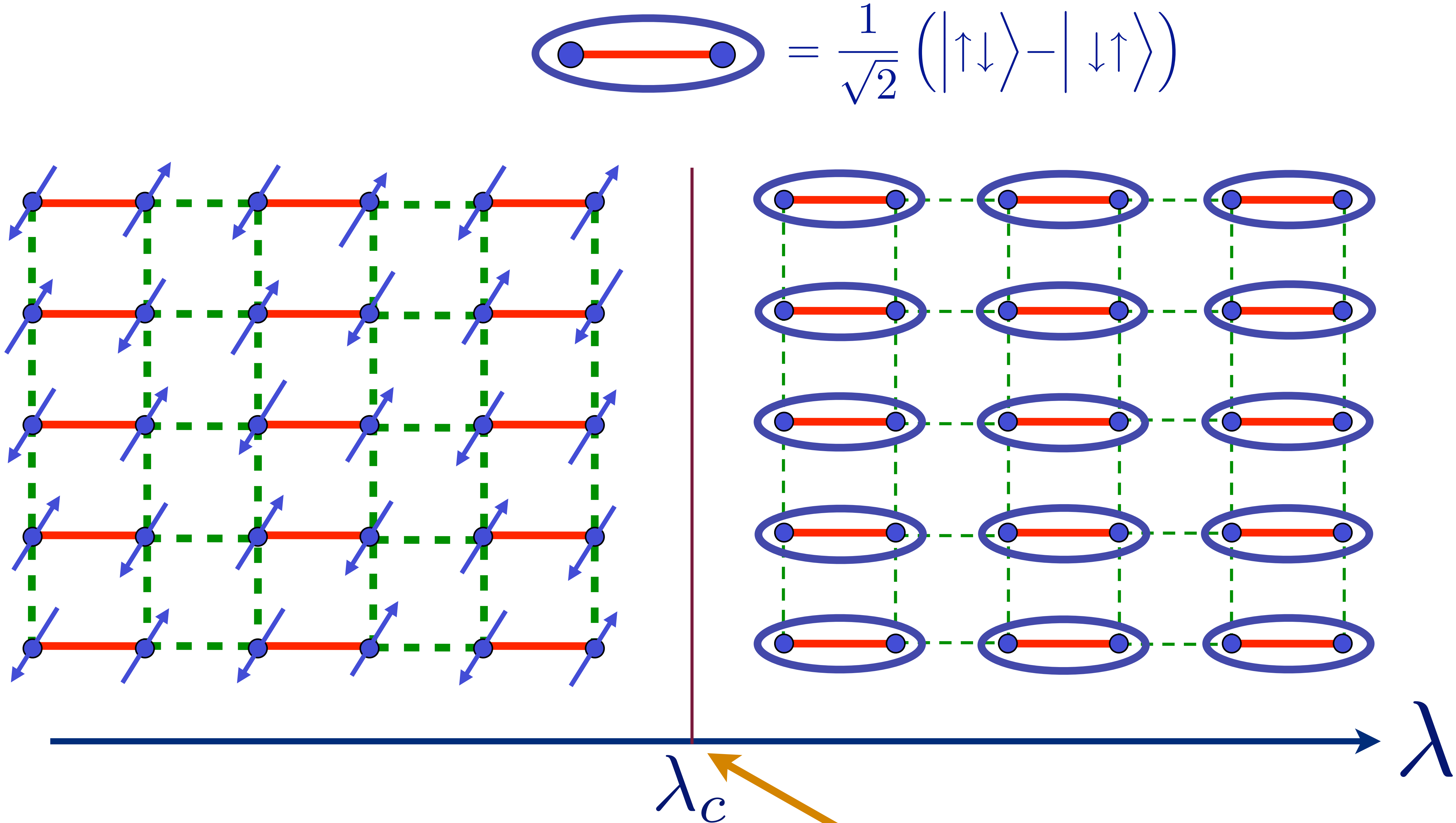
A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

$TlCuCl_3$



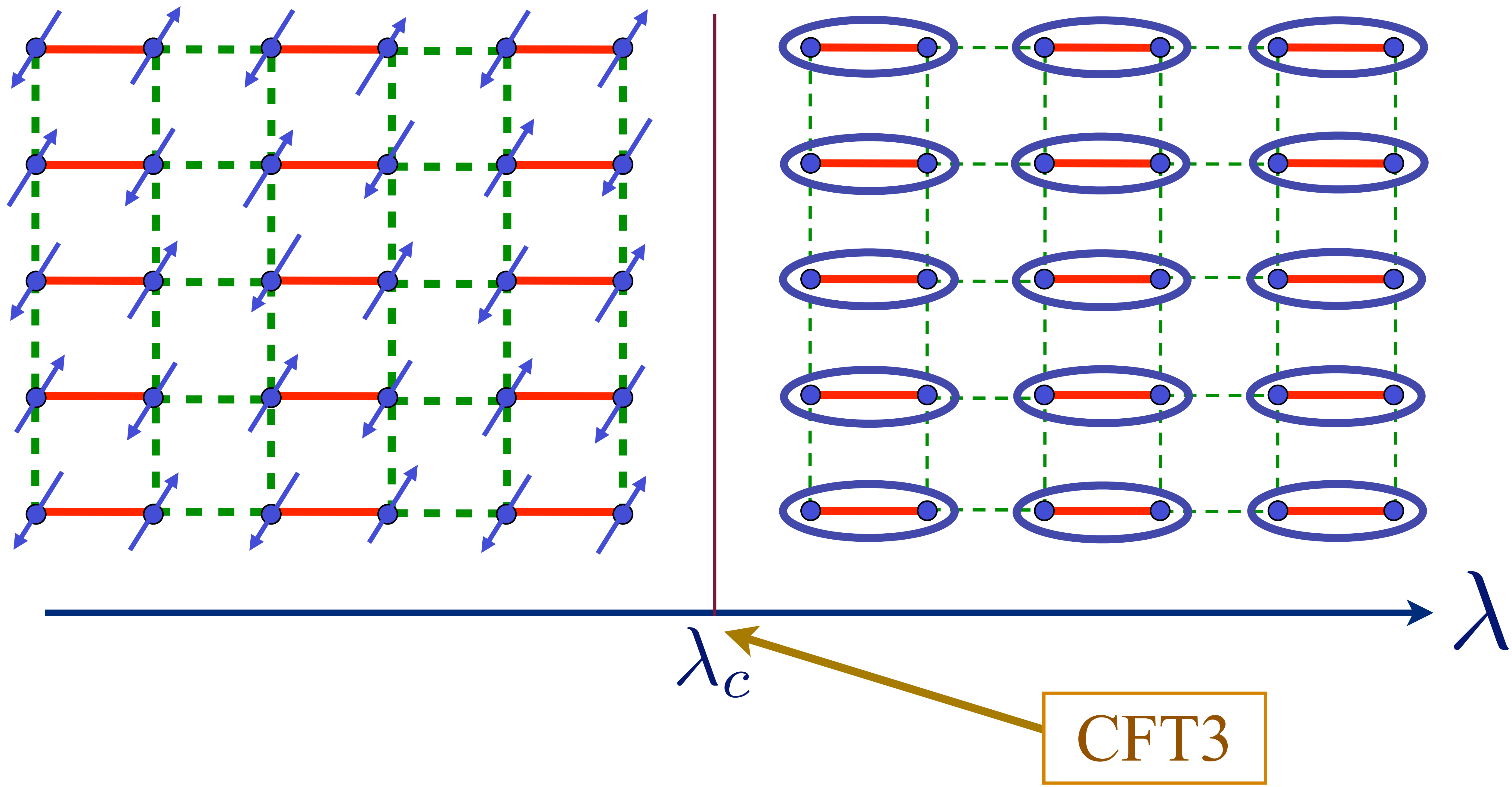


An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.



Quantum critical point with non-local entanglement in spin wavefunction

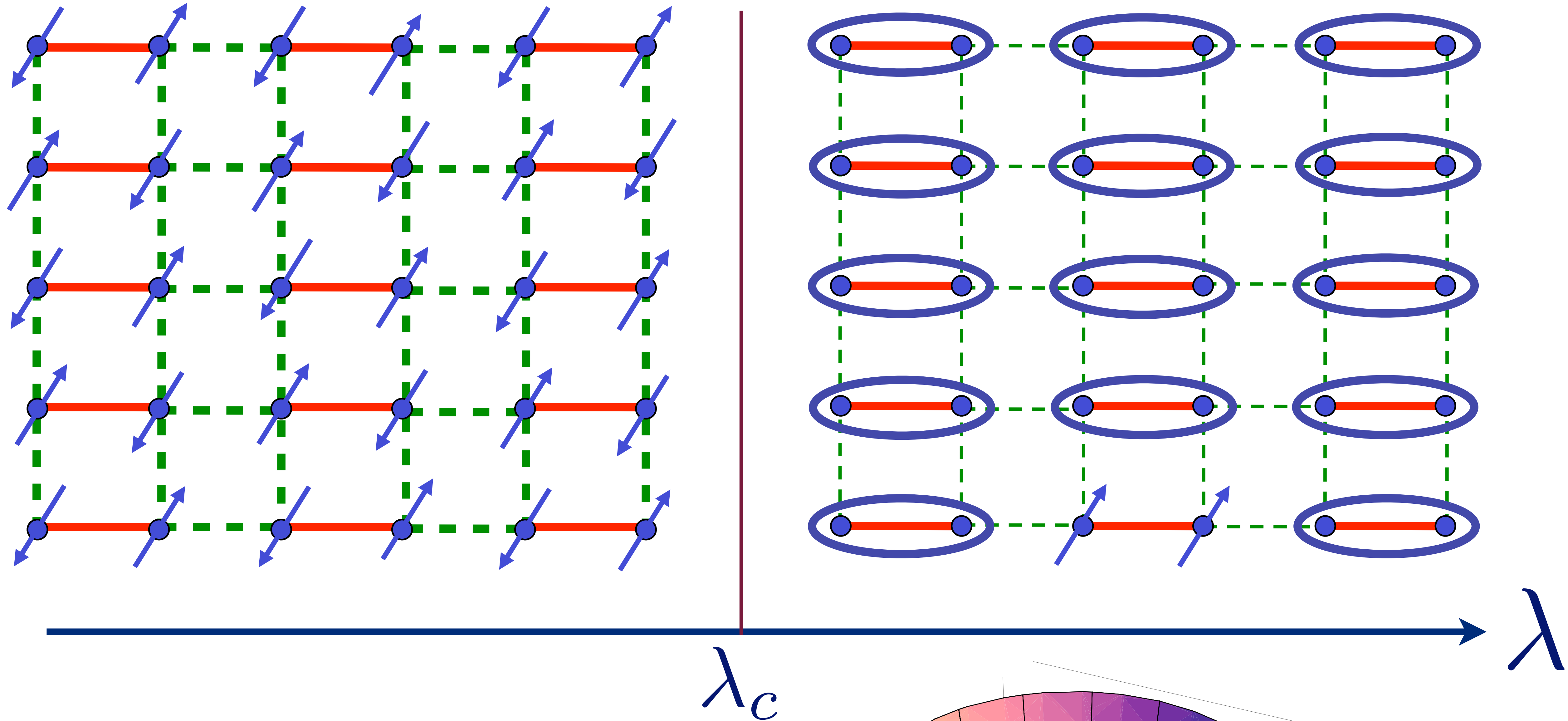
Description using Landau-Ginzburg field theory



O(3) order parameter $\vec{\varphi}$

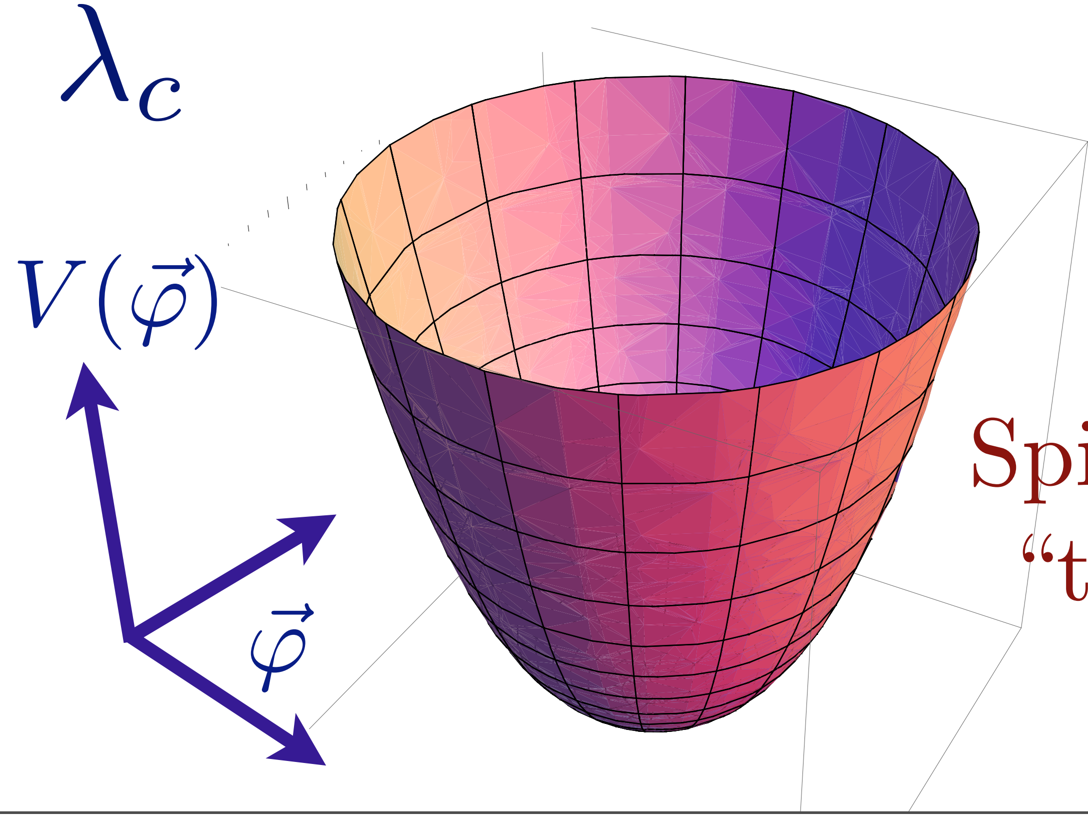
$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

Excitation spectrum in the paramagnetic phase



$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$



Spin $S = 1$
"triplon"

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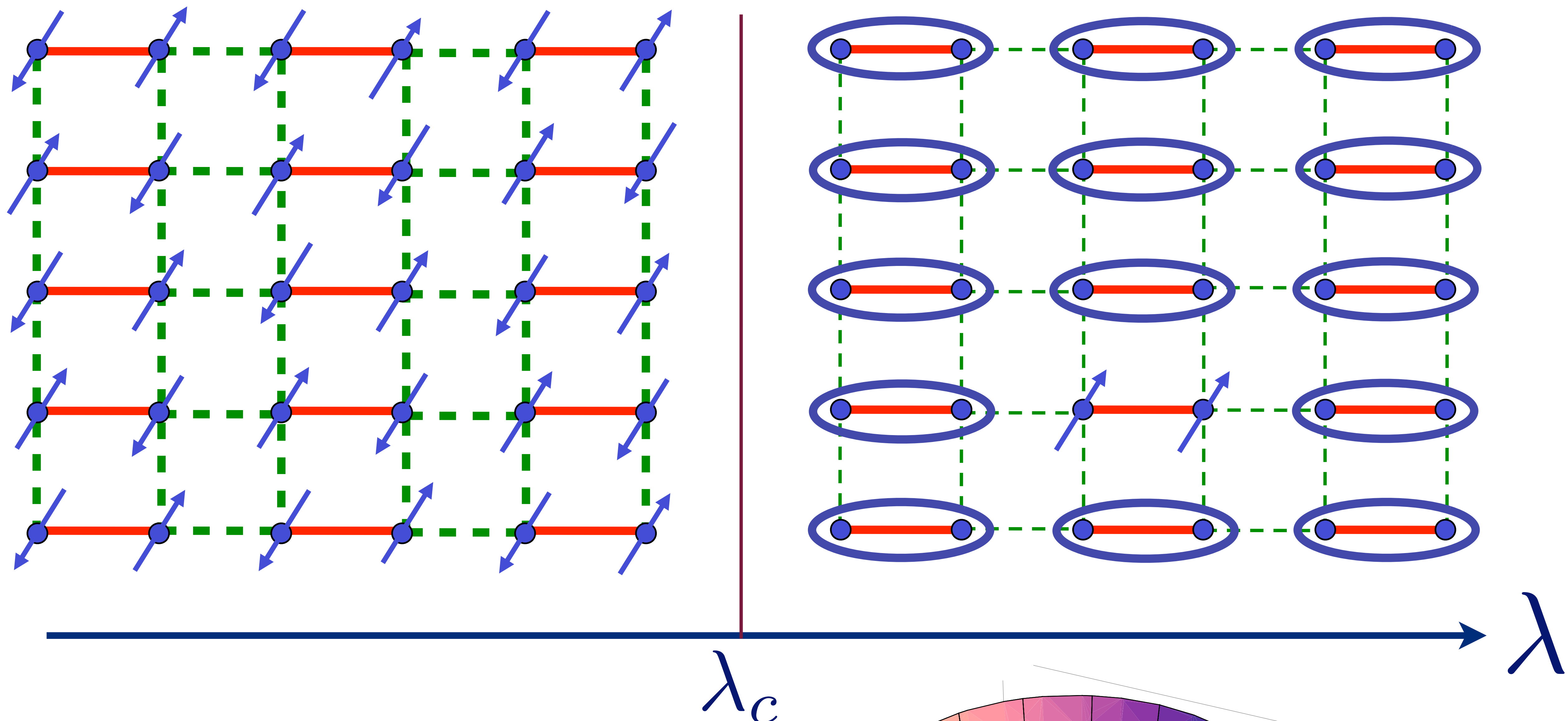

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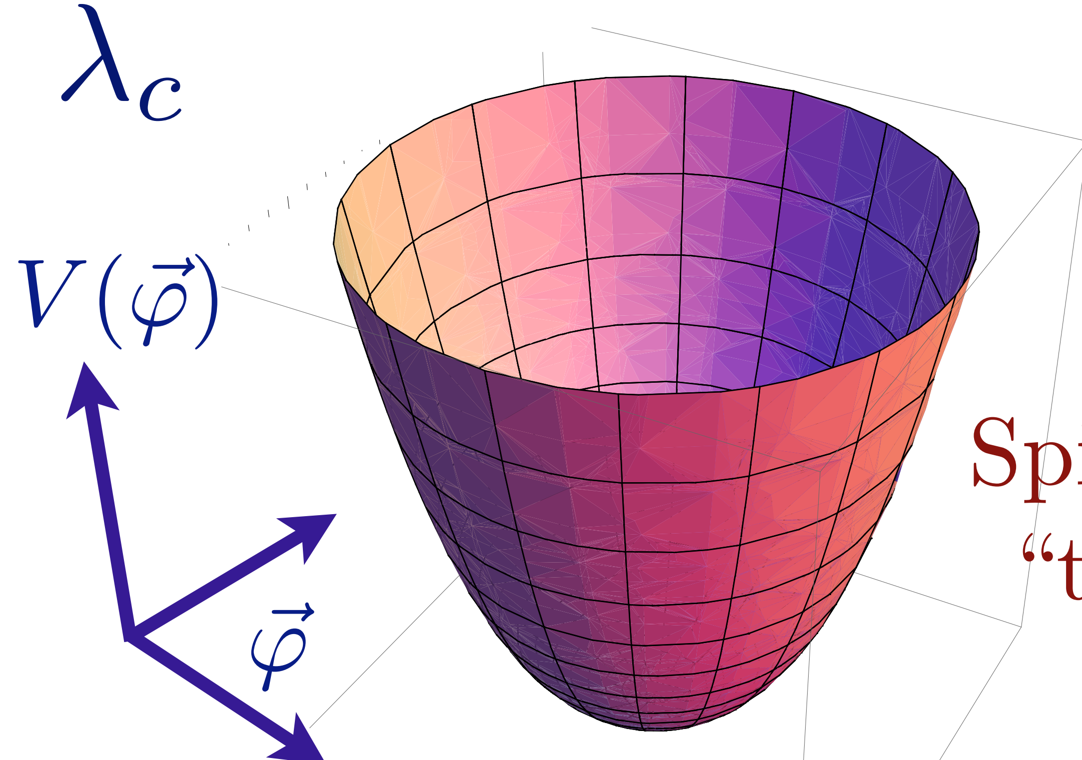

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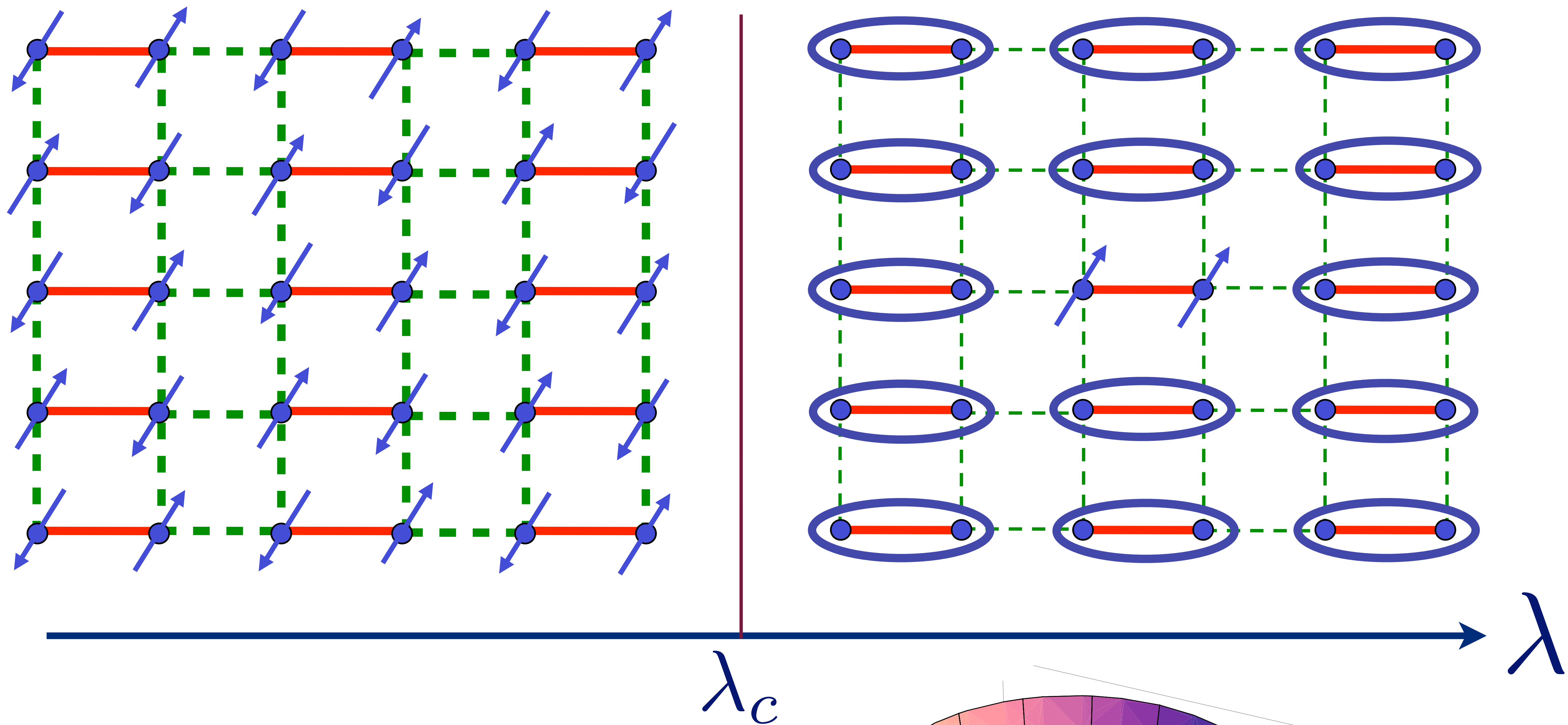

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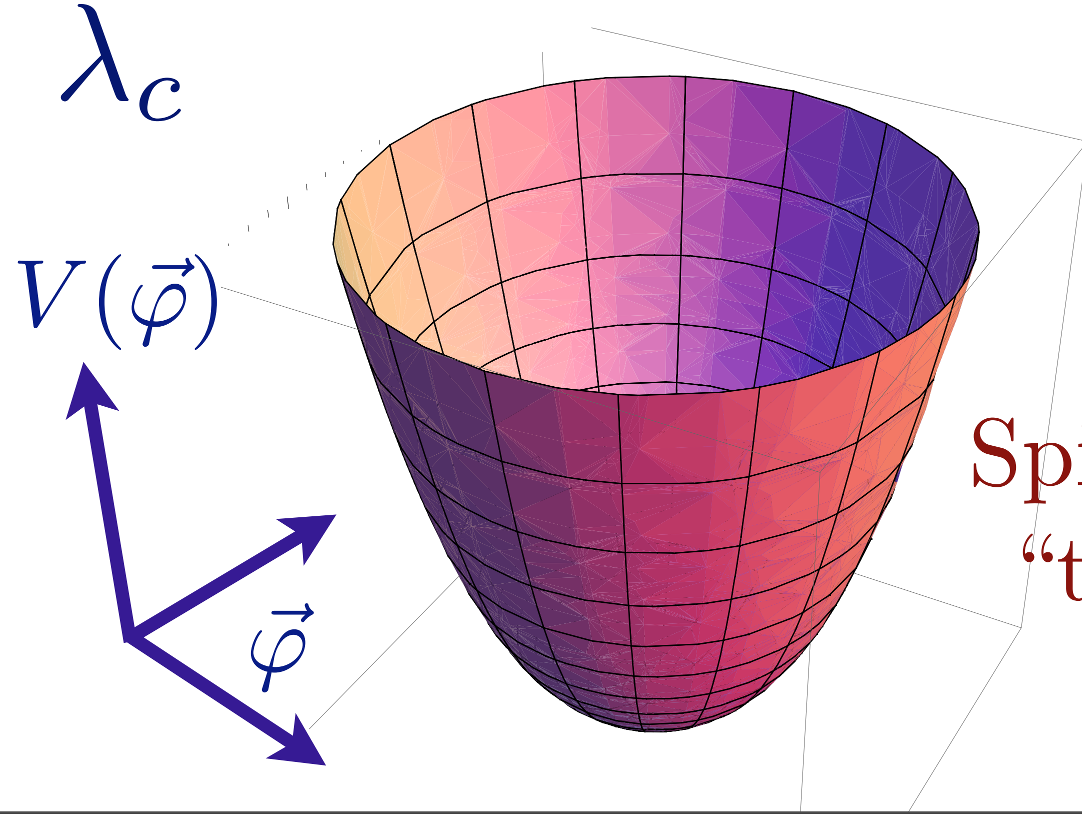

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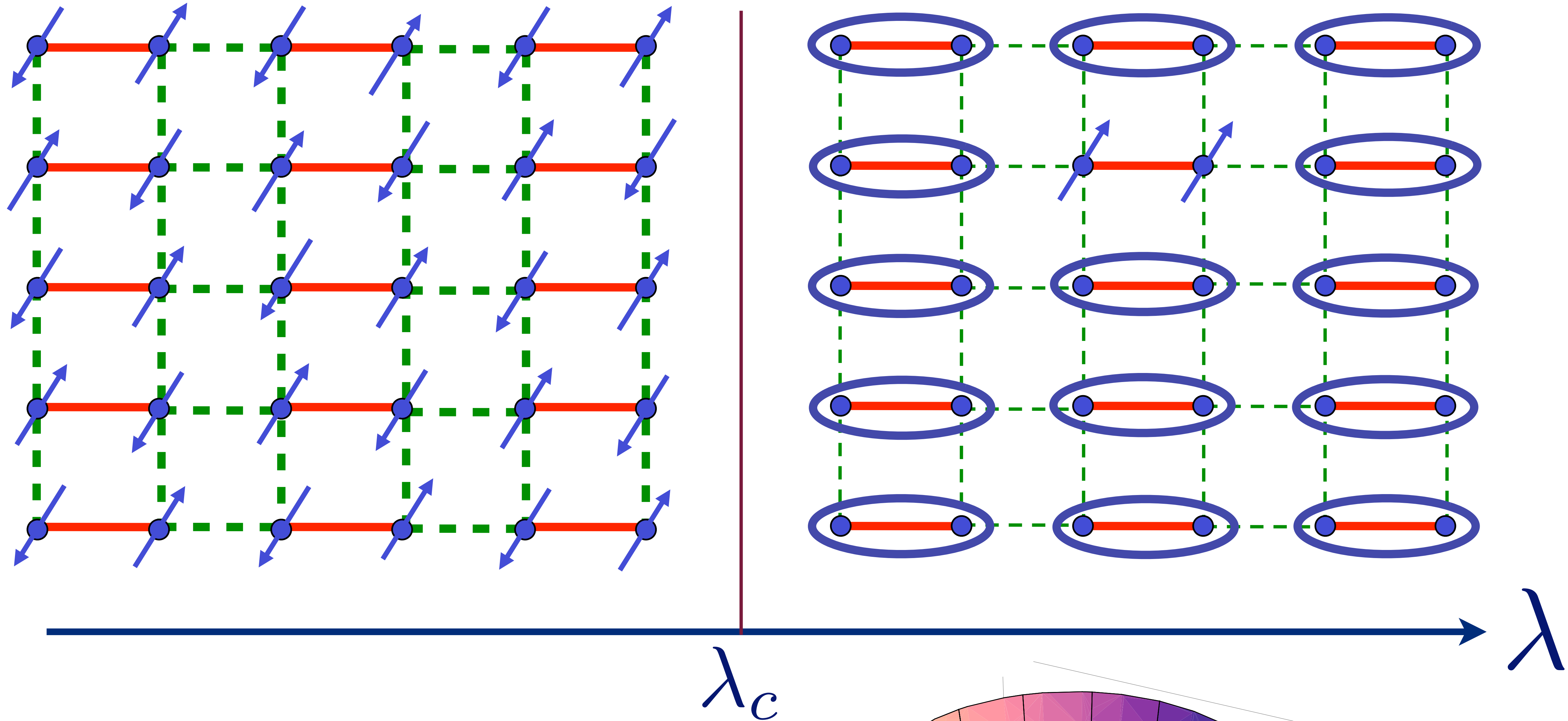

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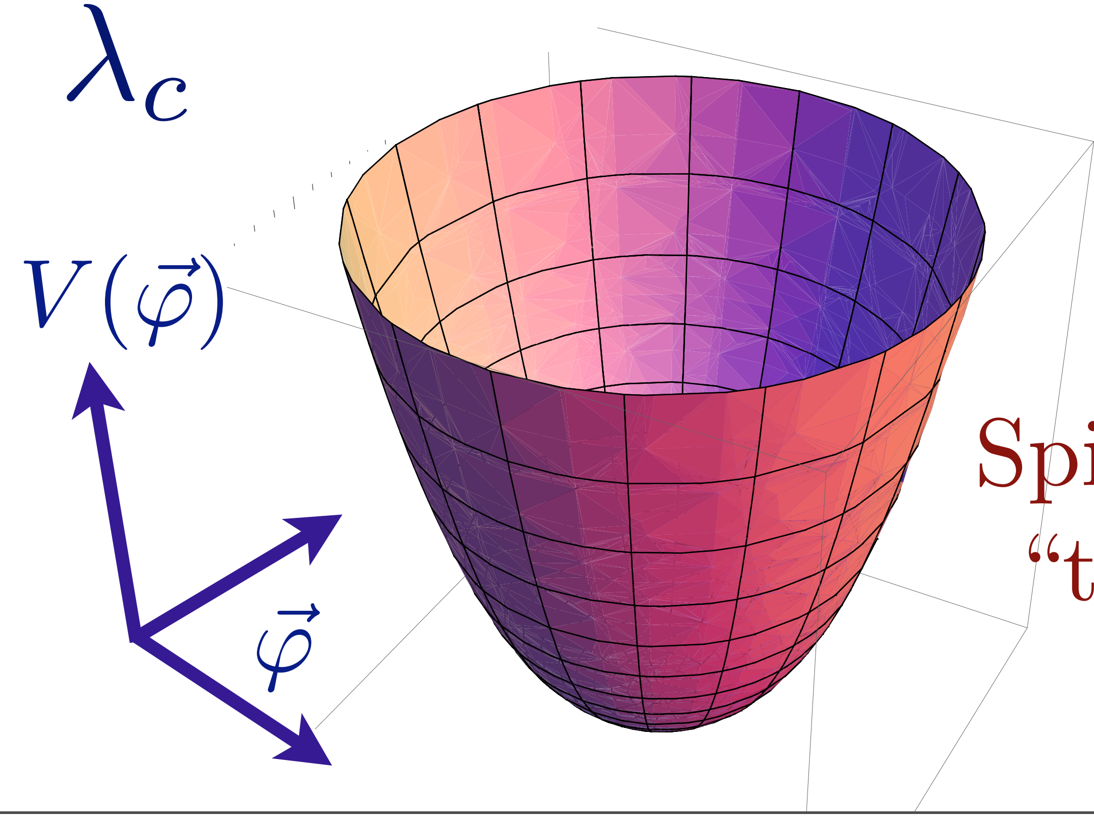

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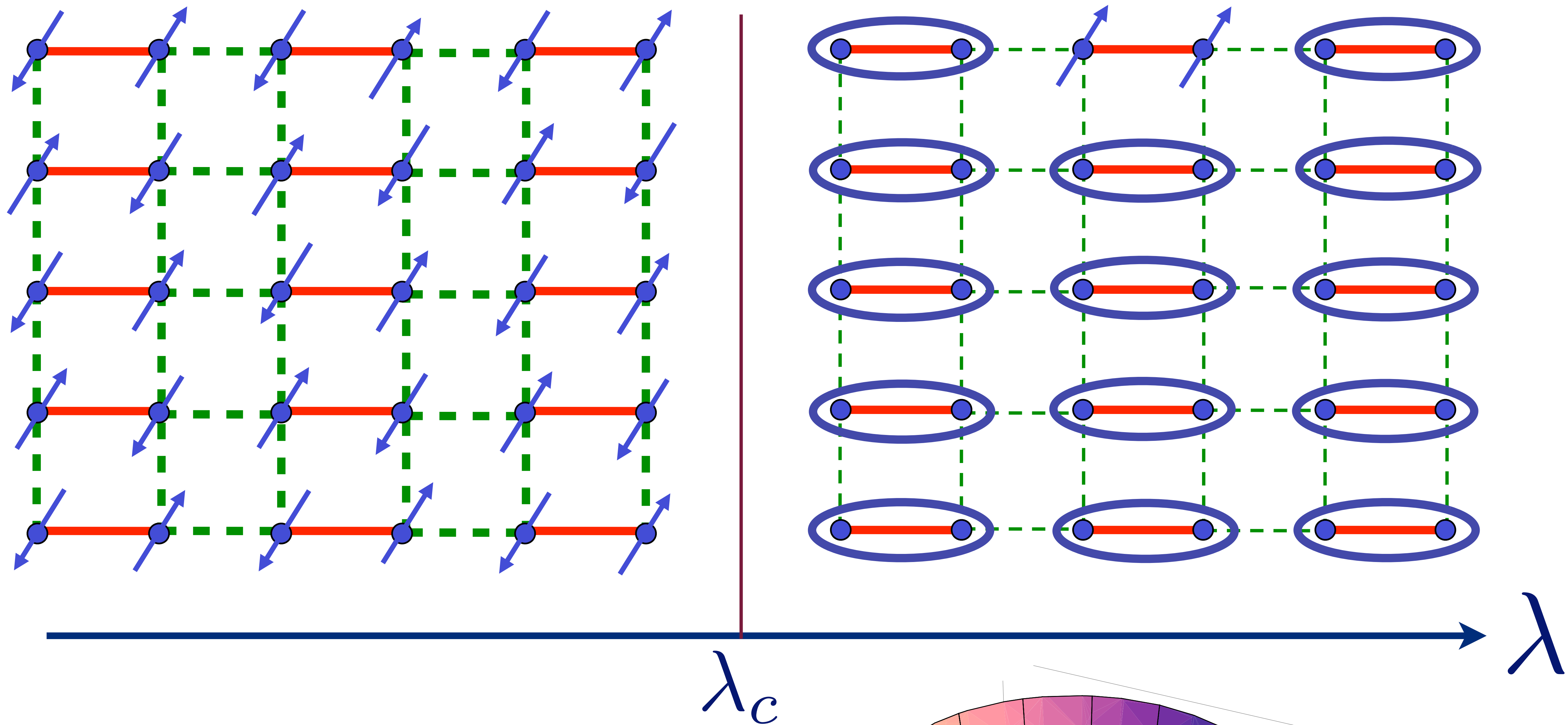

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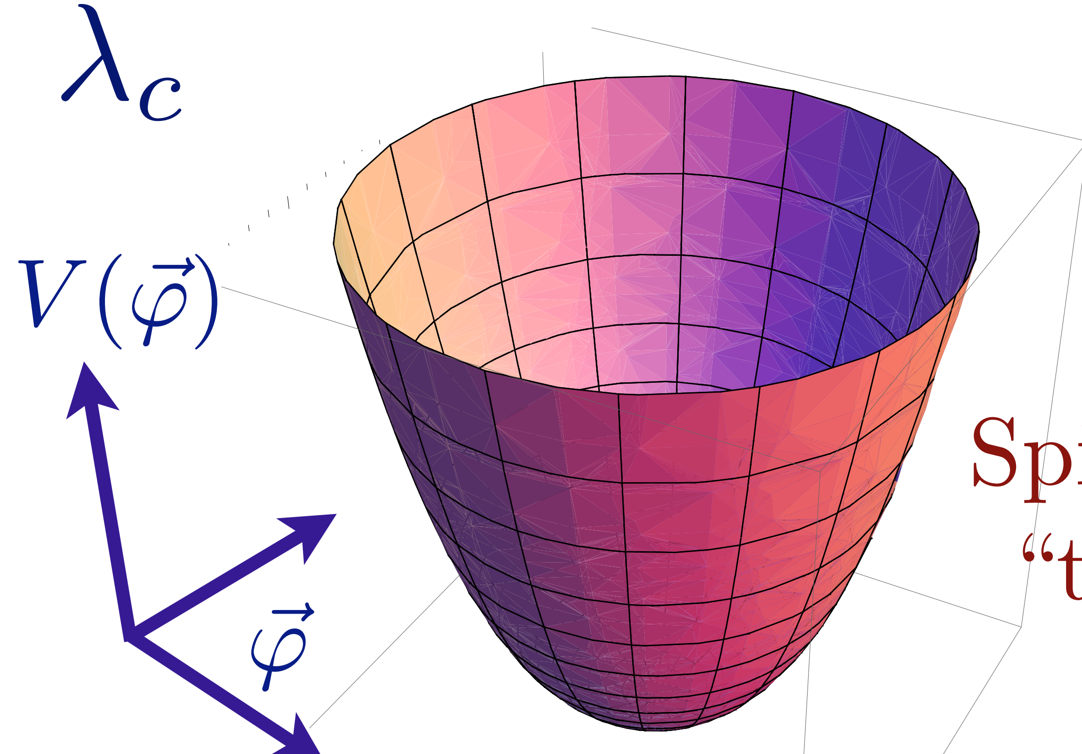

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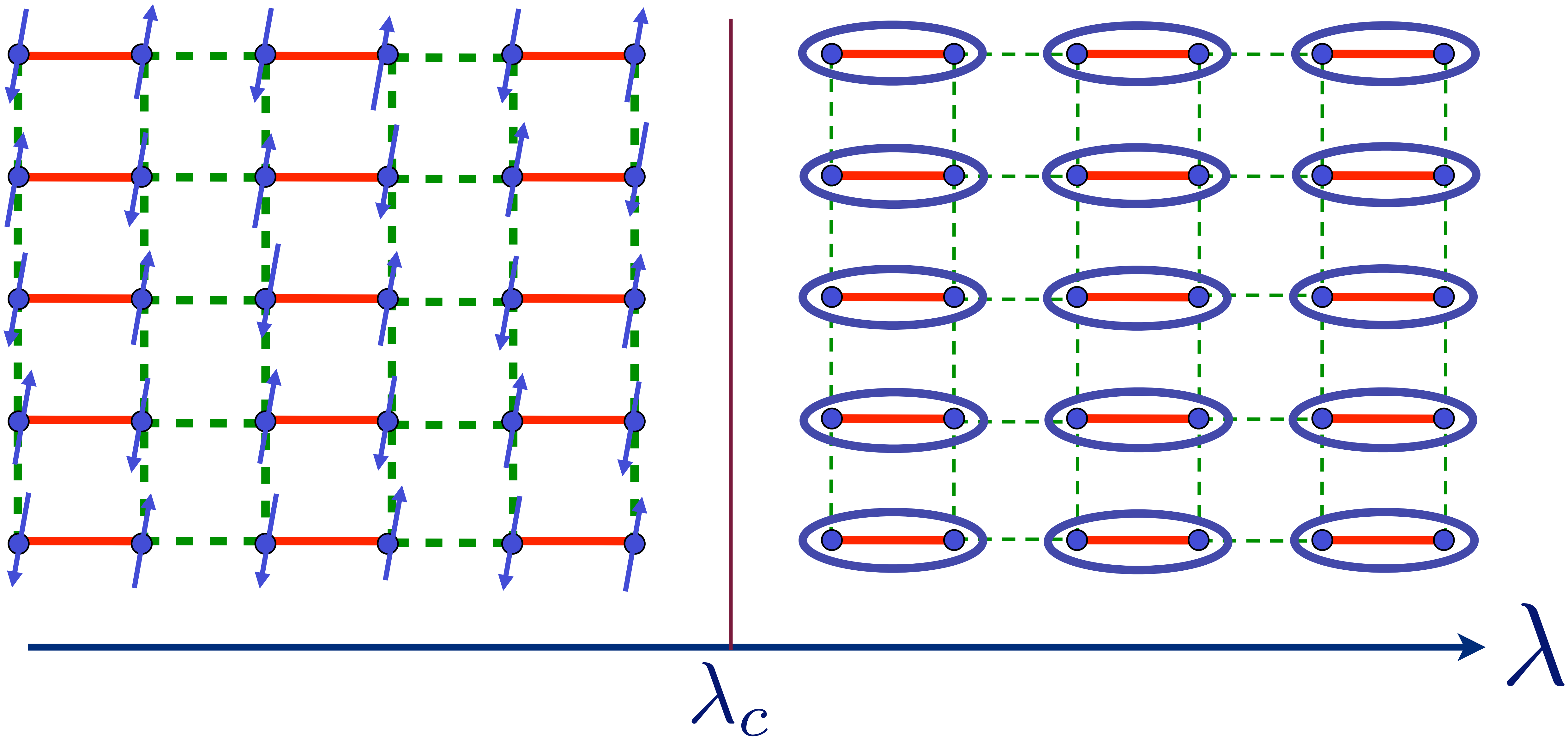
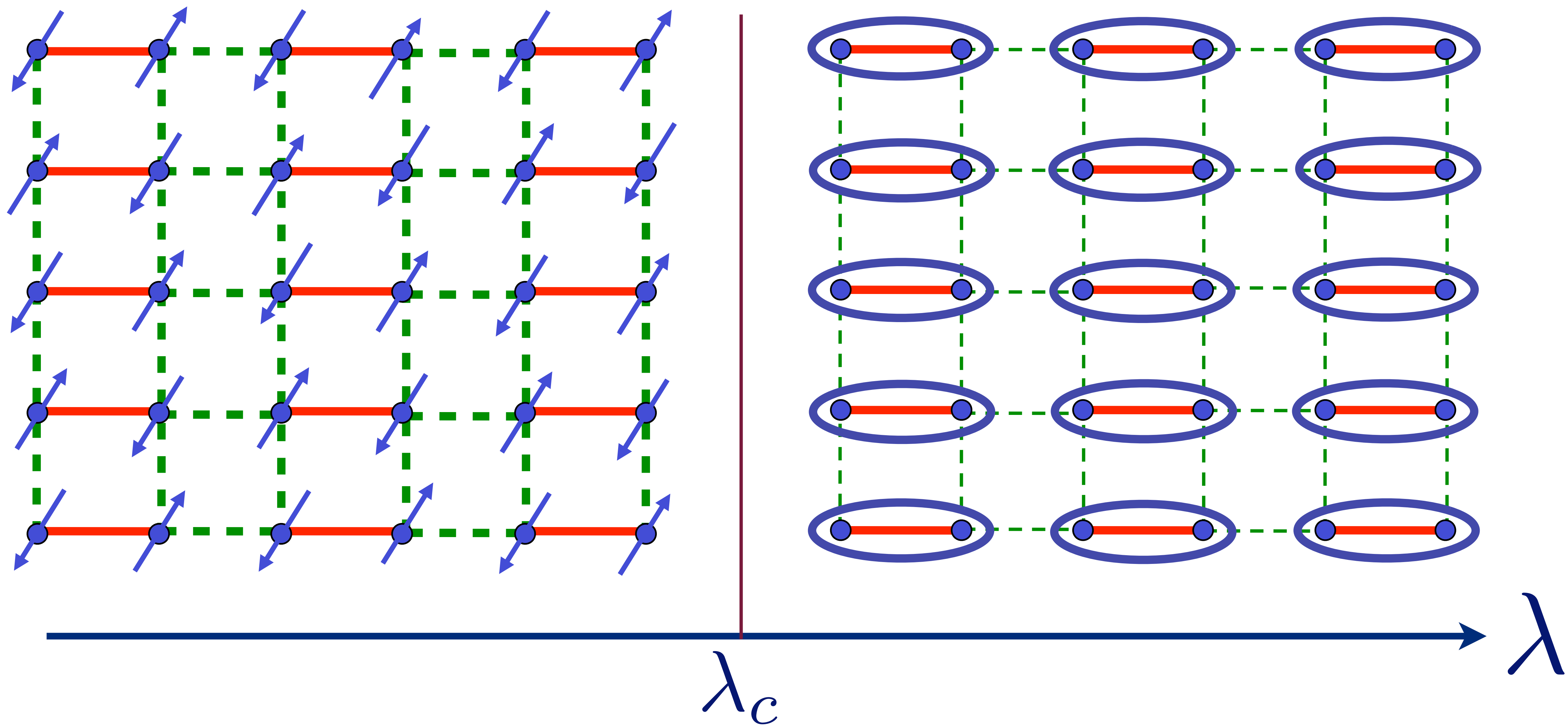
Excitation spectrum in the Néel phase



Spin waves

Excitation spectrum in the Néel phase



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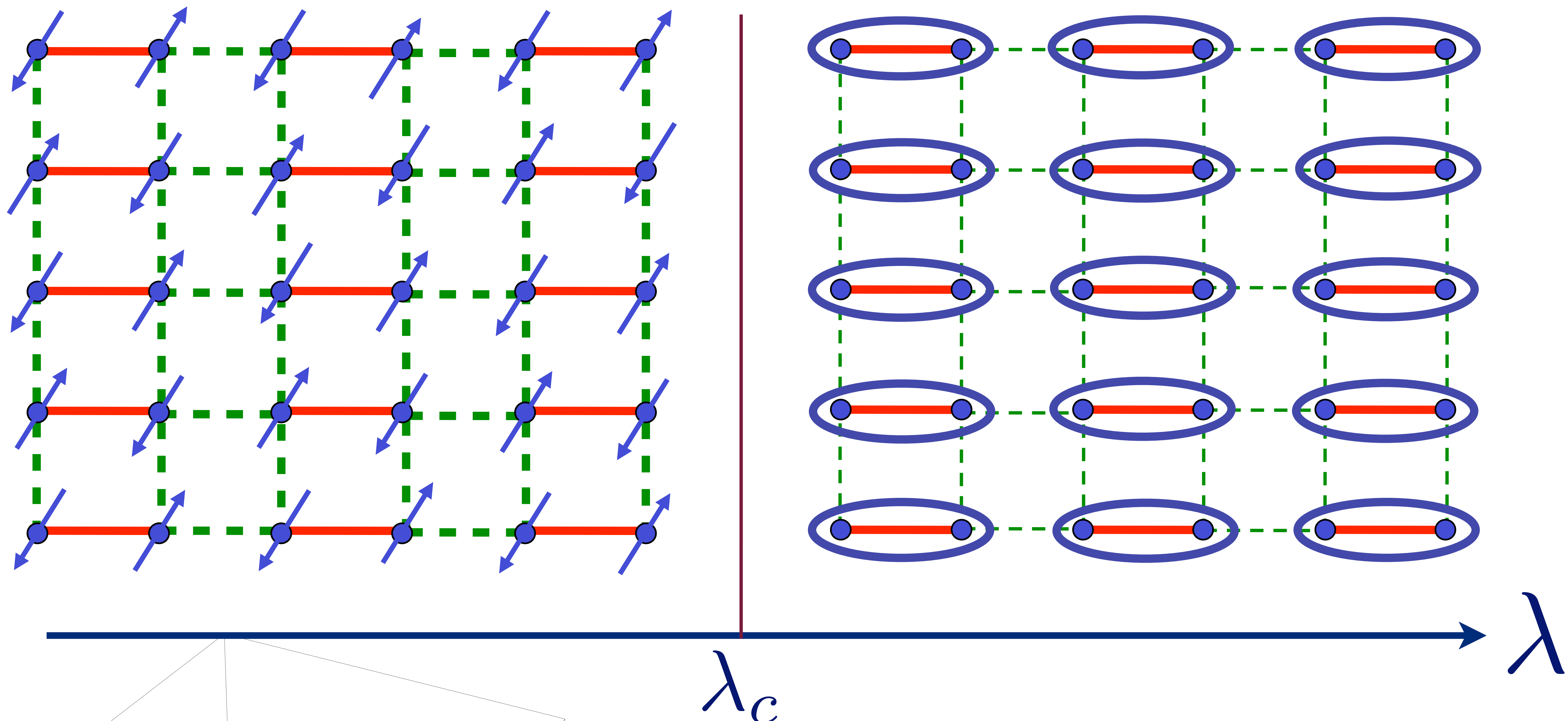

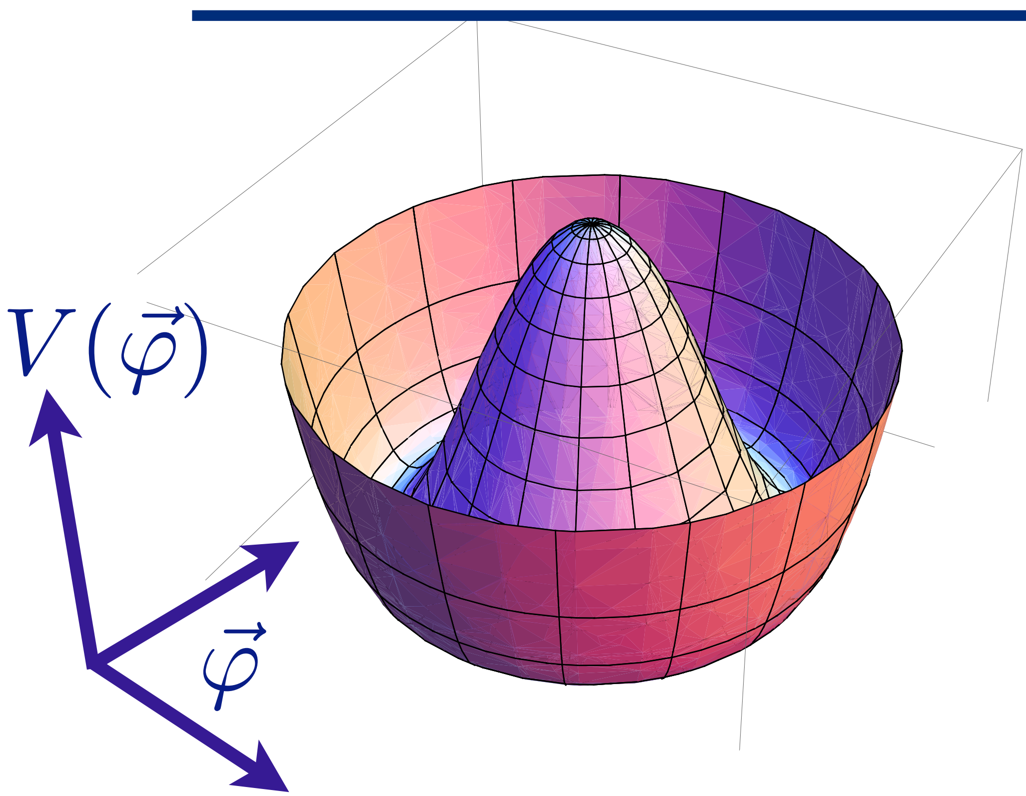

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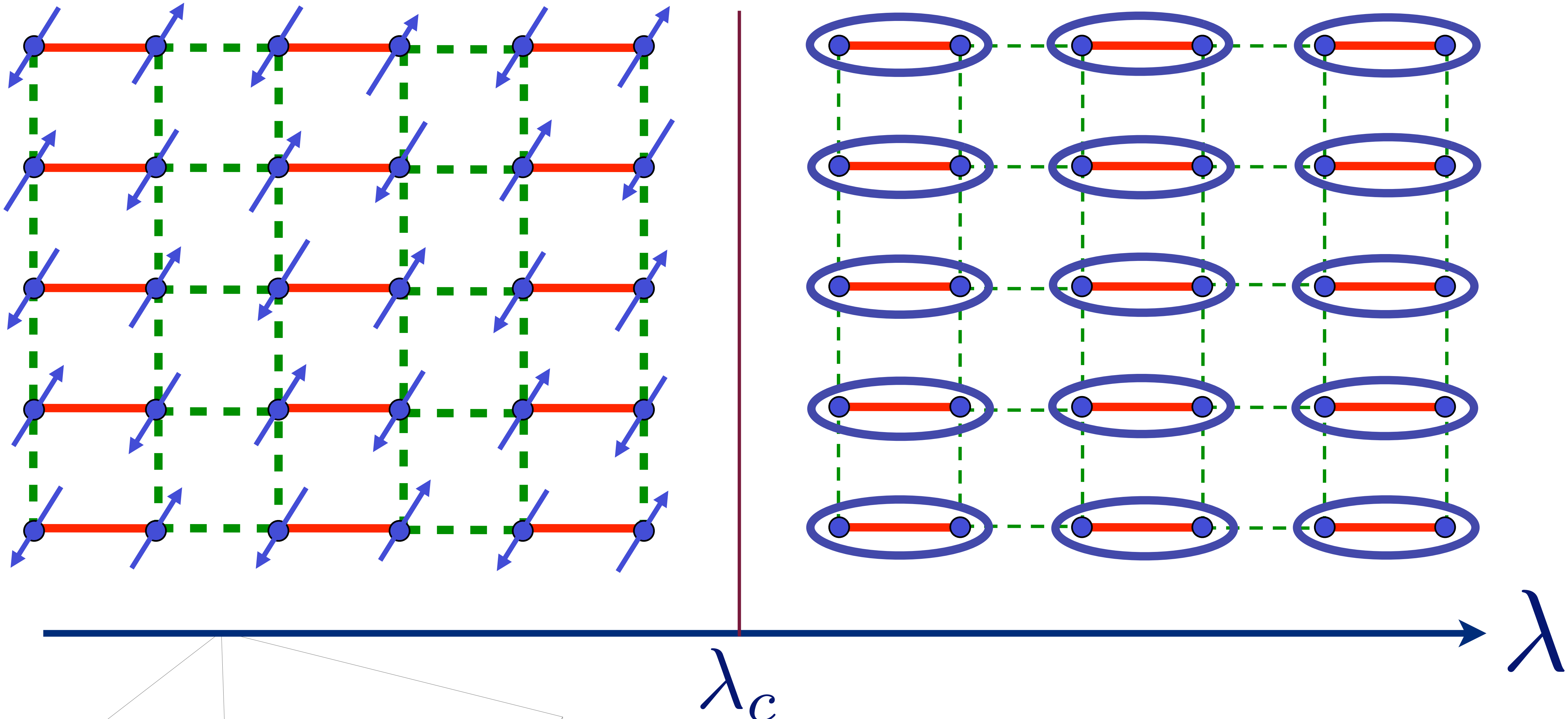

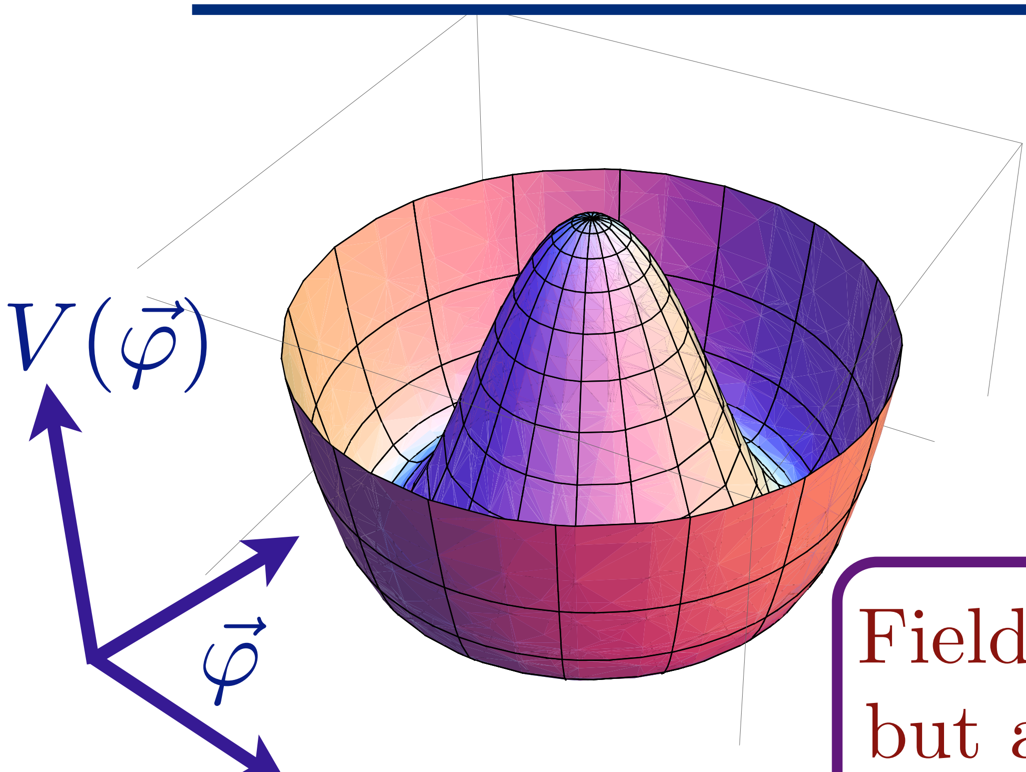

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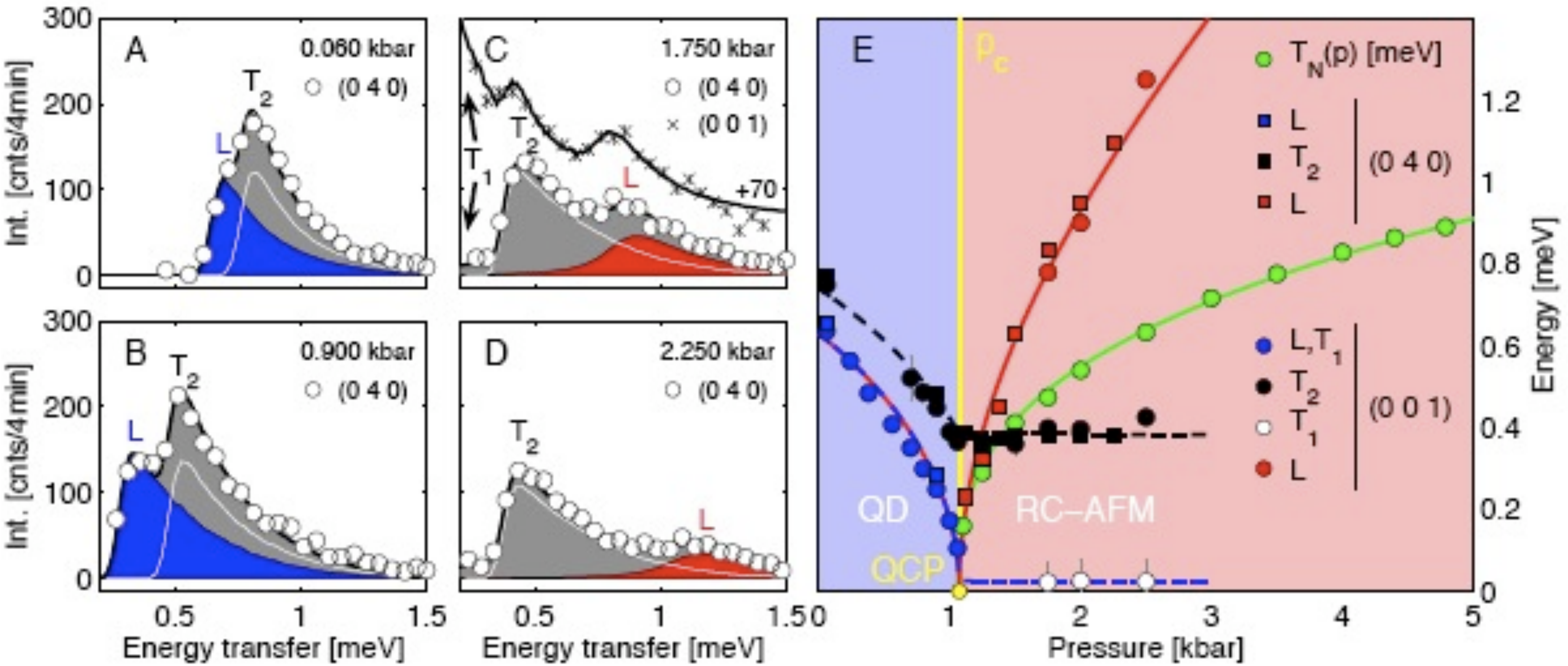
$$\lambda < \lambda_c$$

Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle

$TlCuCl_3$ with varying pressure



Observation of 3 → 2 low energy modes,
emergence of new Higgs particle in the Néel phase,
and vanishing of Néel temperature at the quantum critical point

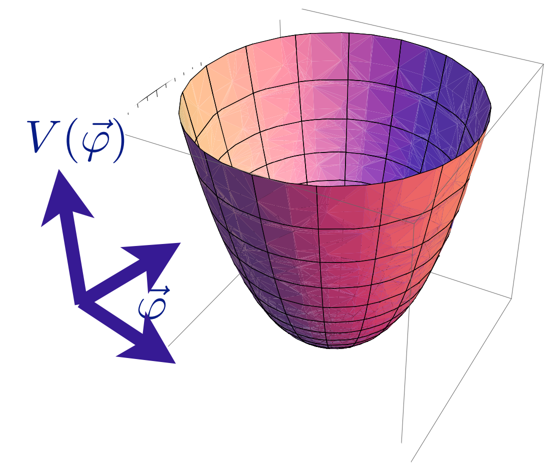
Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer,
Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya,
Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\vec{\varphi} = 0$:



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Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

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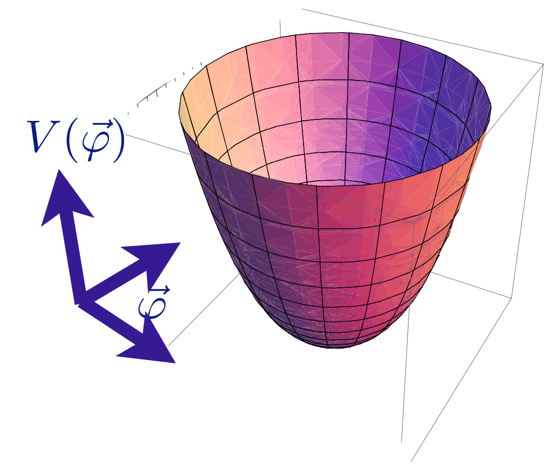

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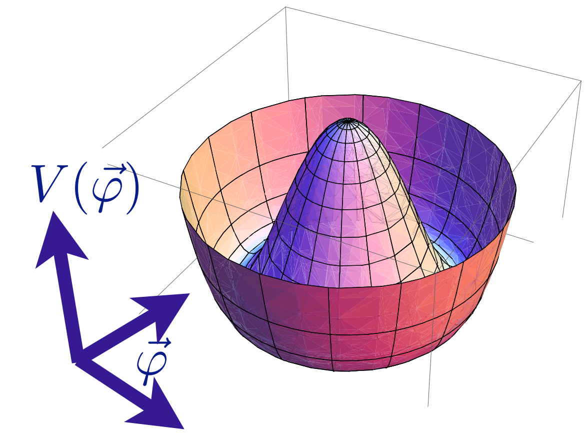
Néel phase, $\lambda < \lambda_c$

Expand $\vec{\varphi} = \left(0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}\right) + \vec{\varphi}_1$:



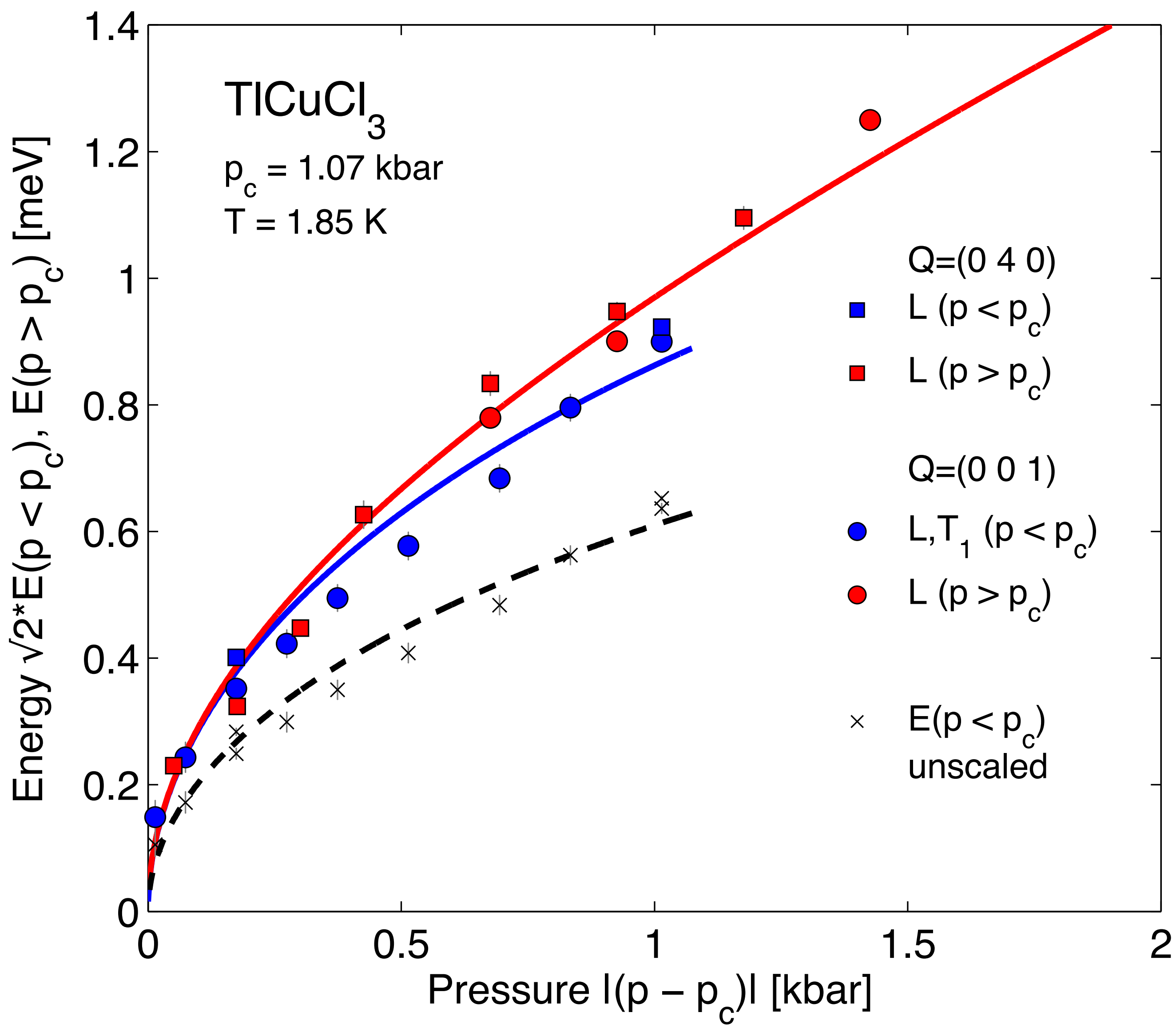
$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

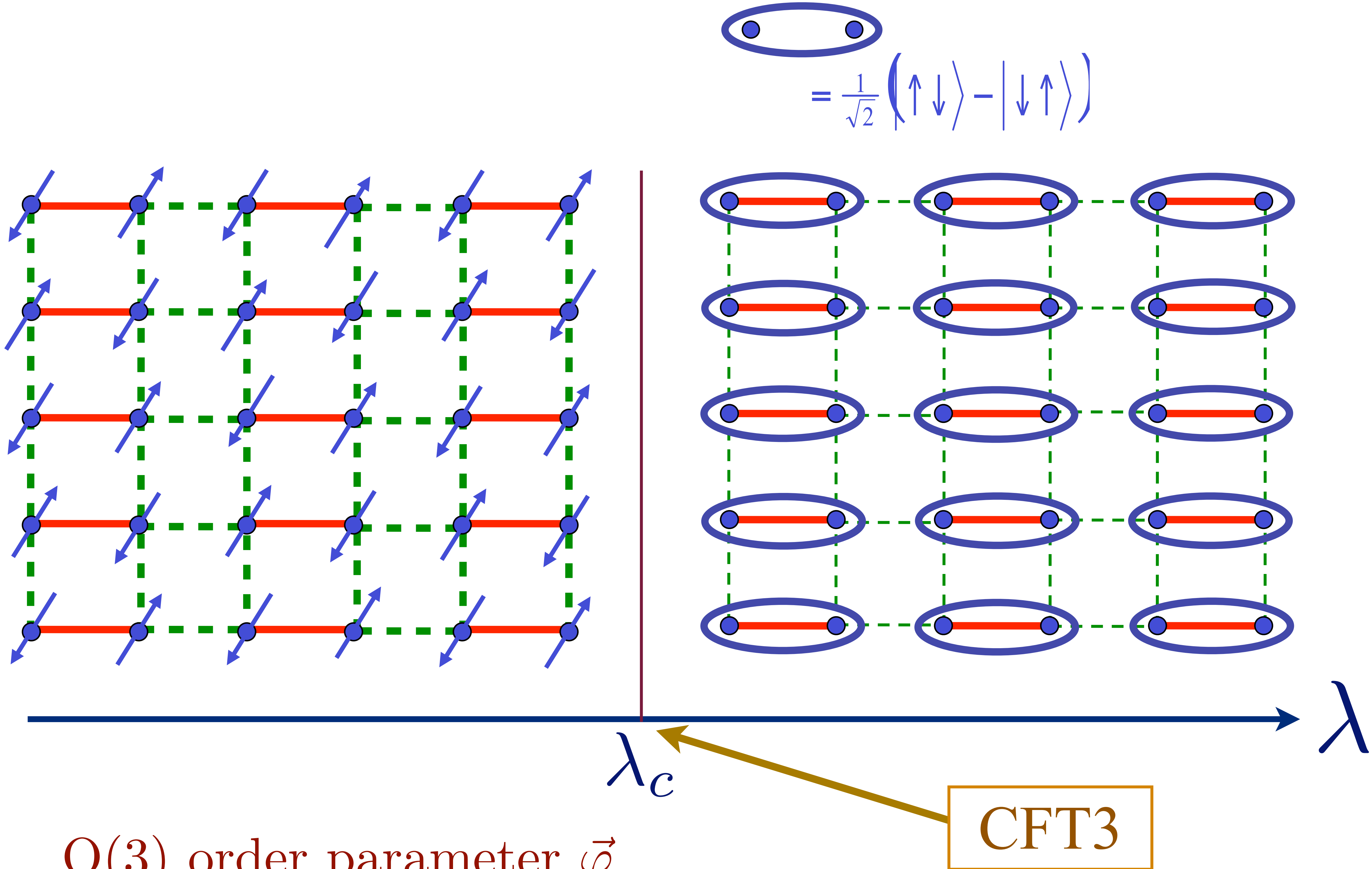
Prediction of quantum field theory

$$\frac{\text{Energy of Higgs particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$



S. Sachdev, arXiv:0901.4103



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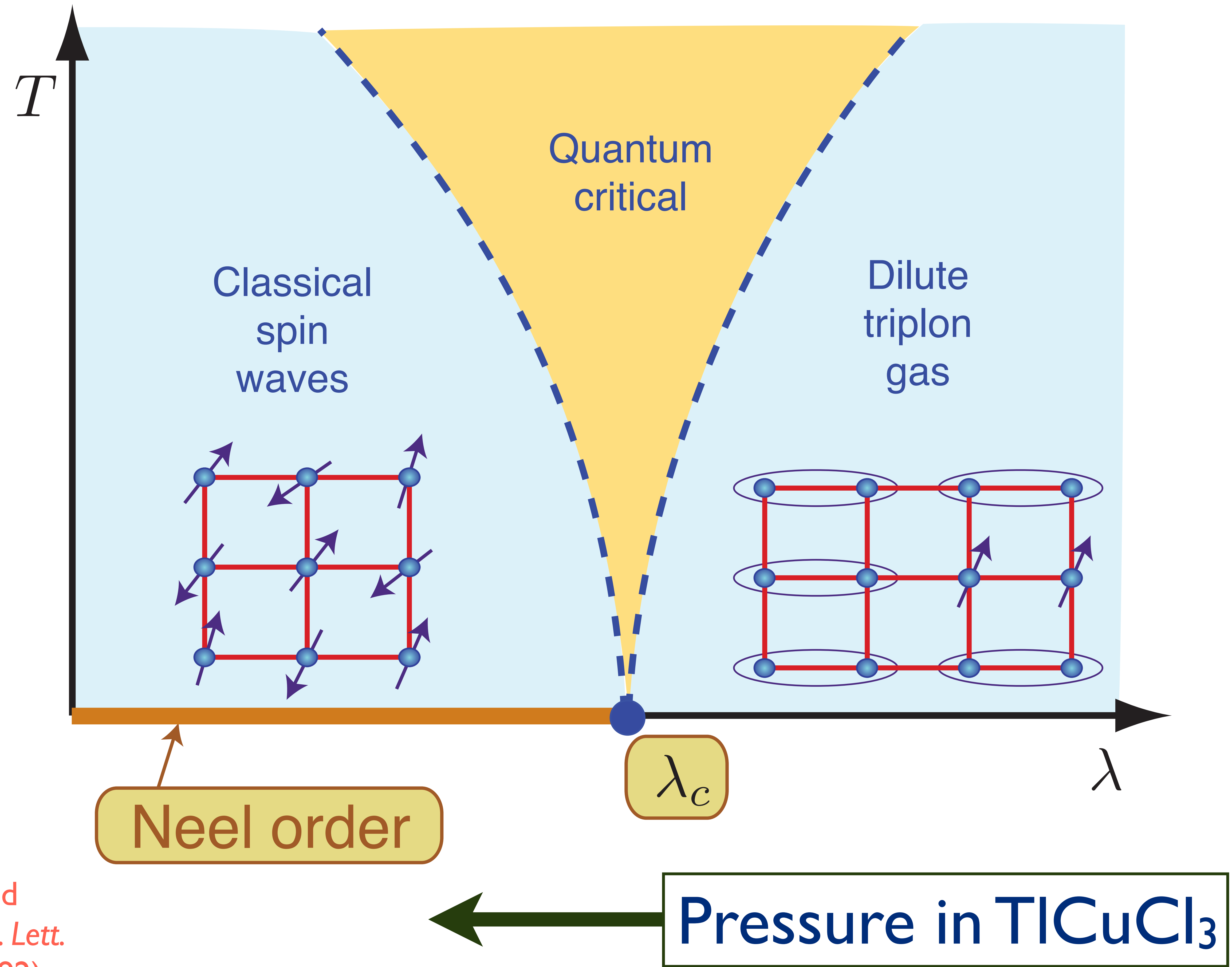

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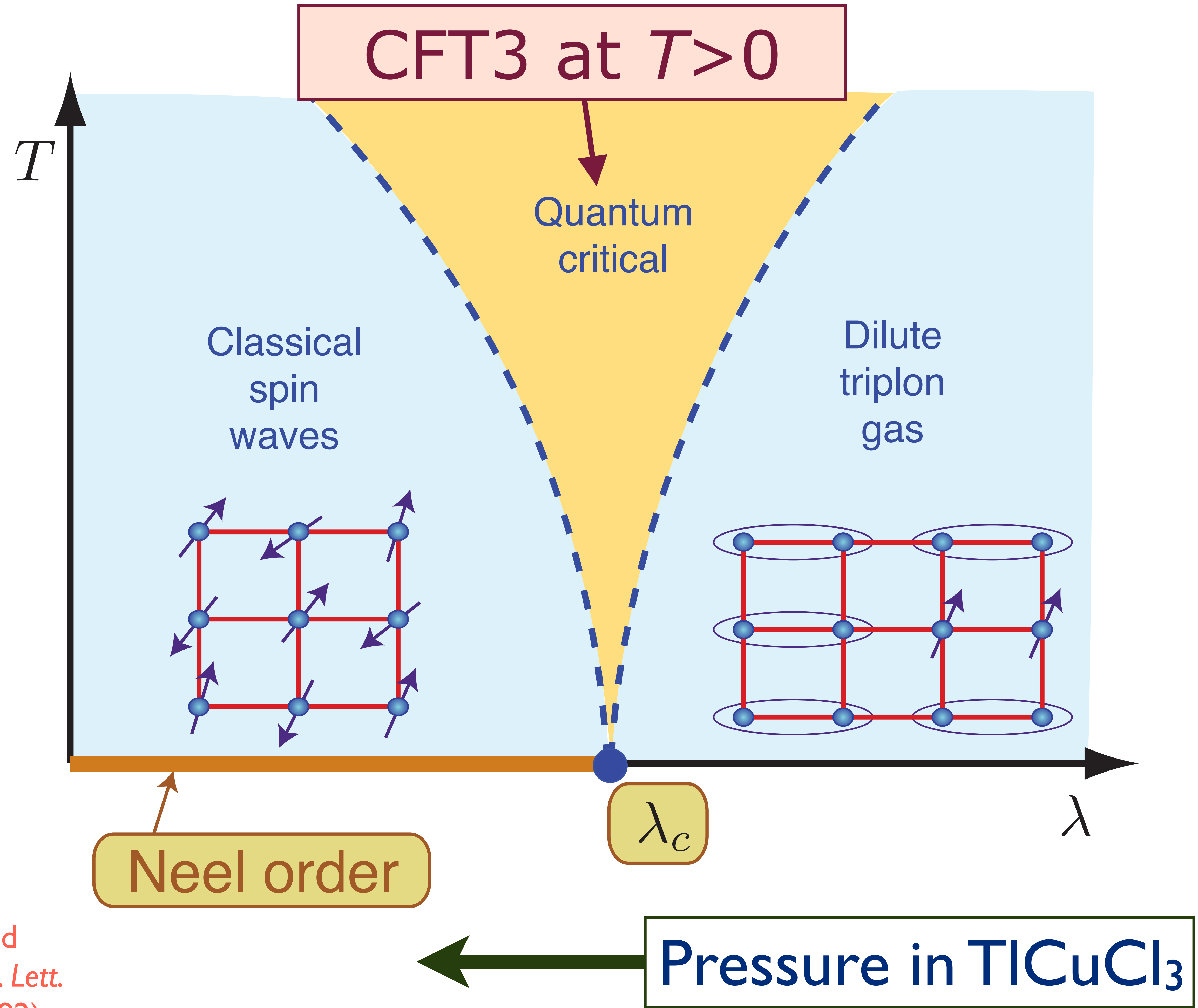

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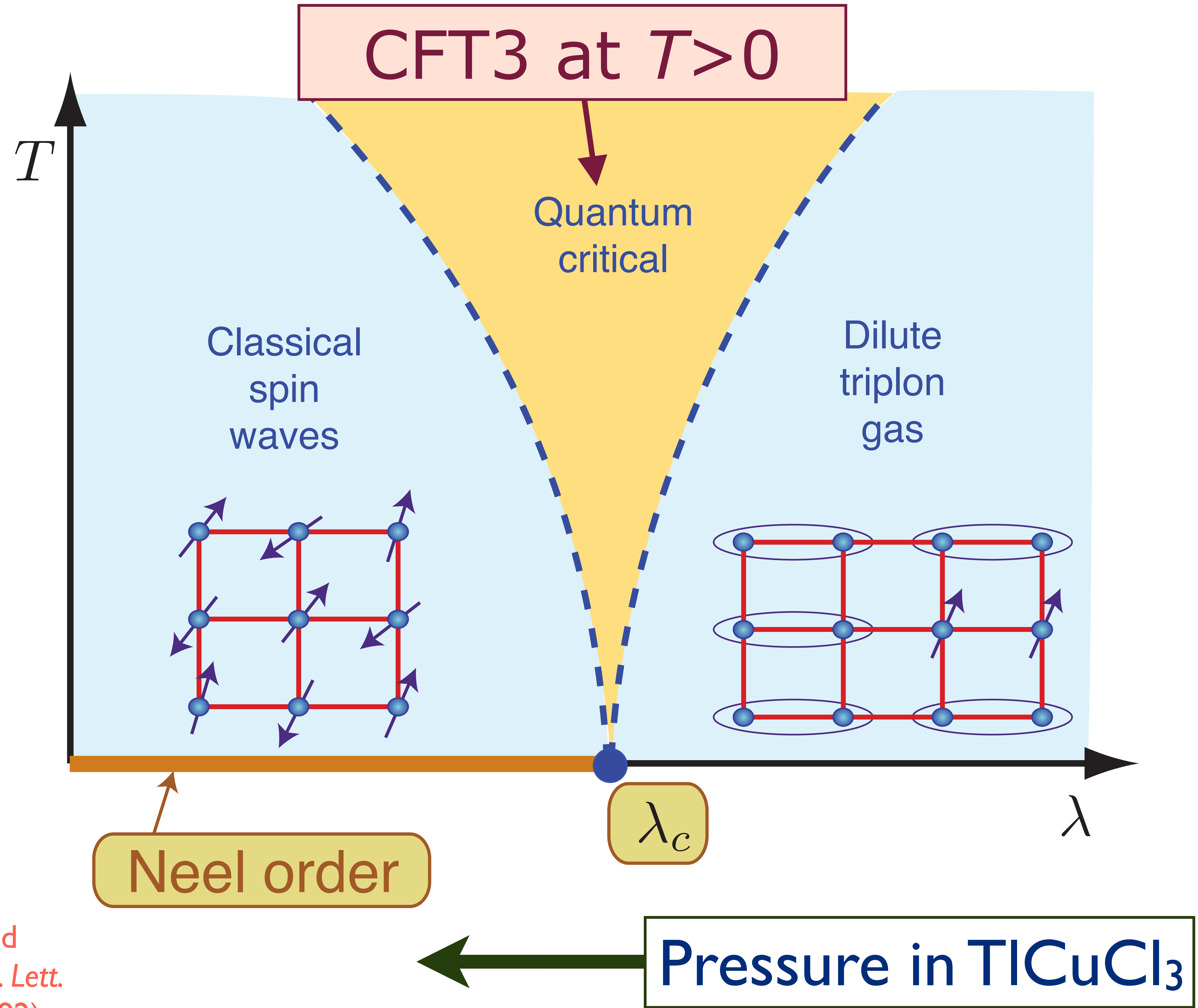

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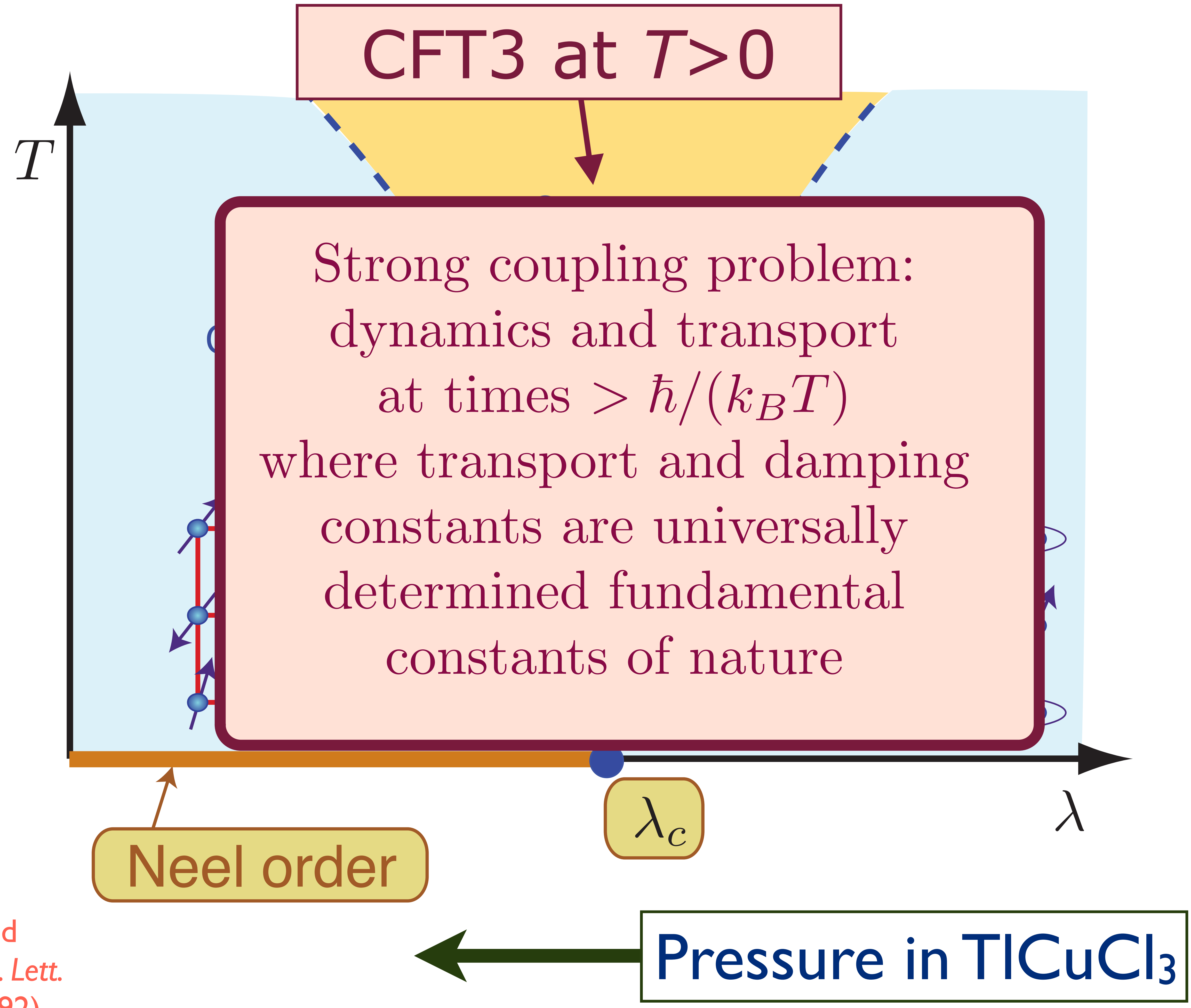

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Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

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Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $D+1$ dimensions.

At the RG fixed point, $\beta(g) = 0$, the D dimensional field theory is invariant under the scale transformation

$$x^\mu \to x^\mu/b \quad , \quad u \to b\,u$$

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This is an invariance of the *metric* of the theory in $D+1$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS_{D+1}, and L is the AdS radius.

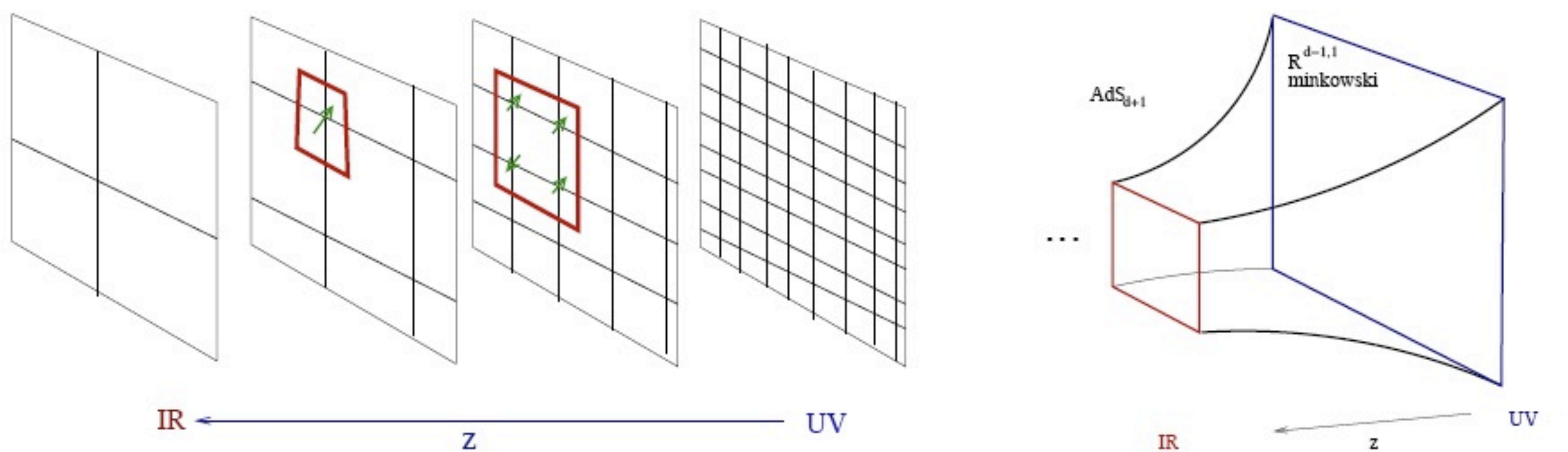
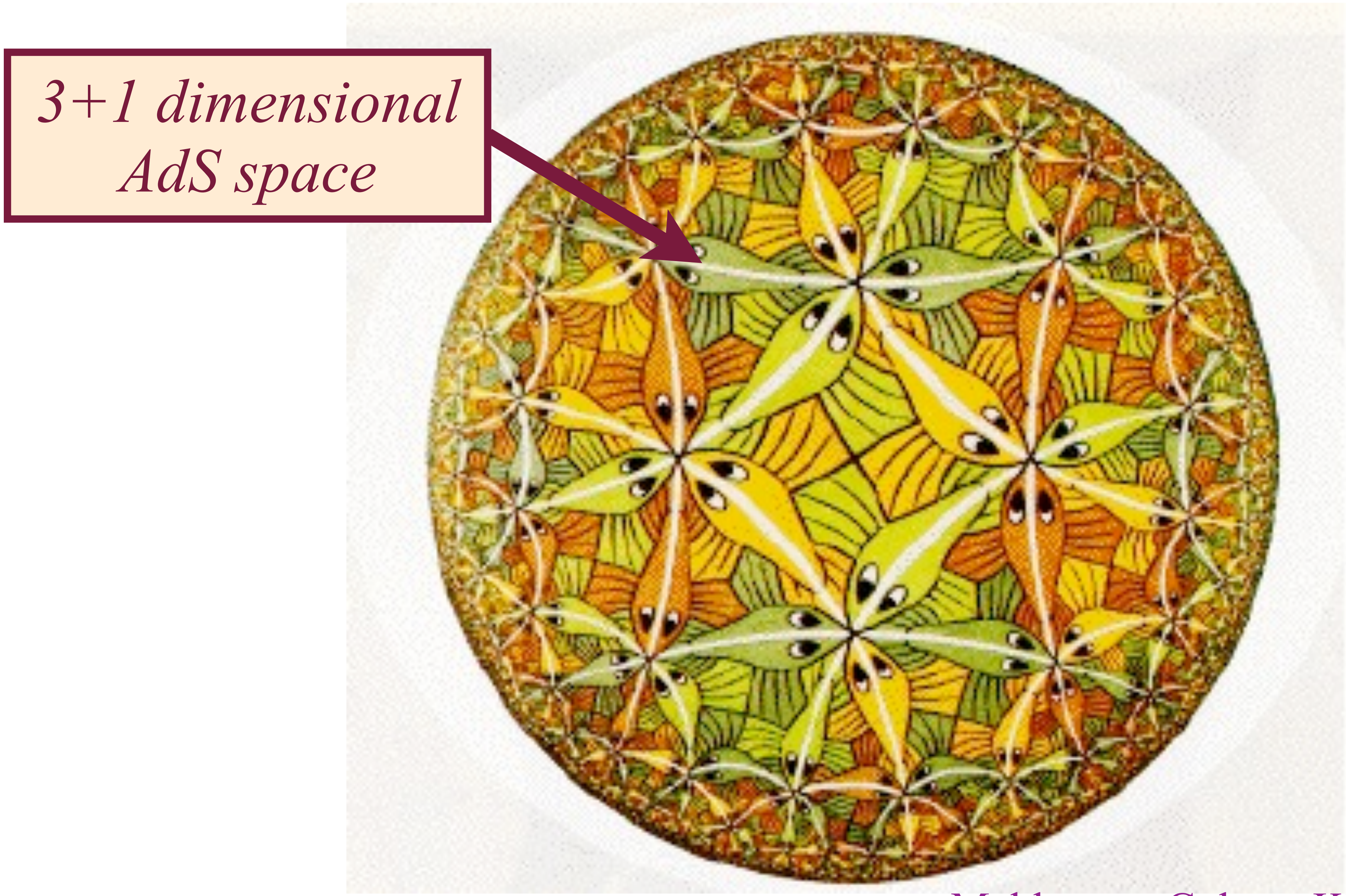


Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

J. McGreevy, arXiv0909.0518

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



Maldacena, Gubser, Klebanov, Polyakov, Witten

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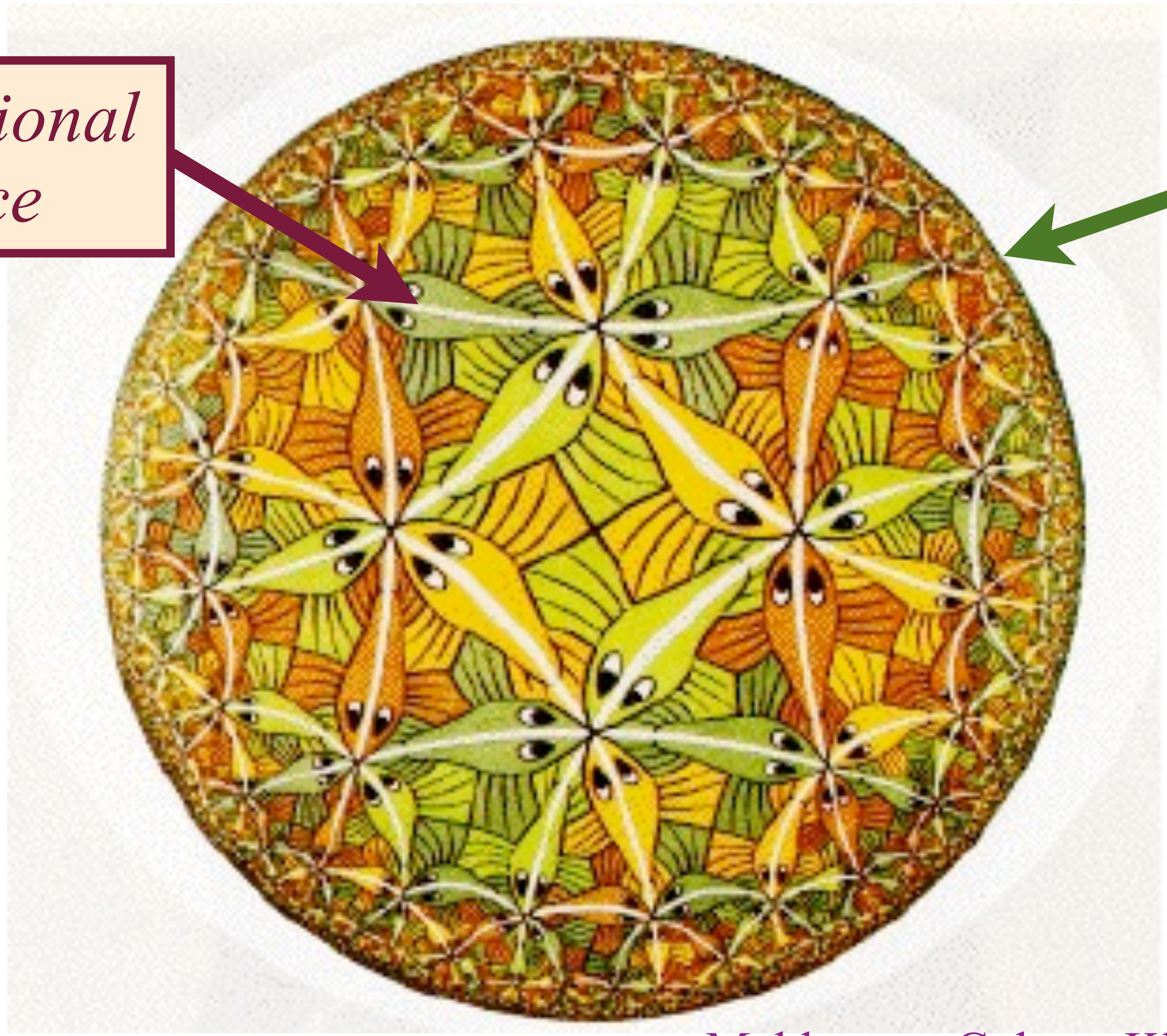
3+1 dimensional AdS space



A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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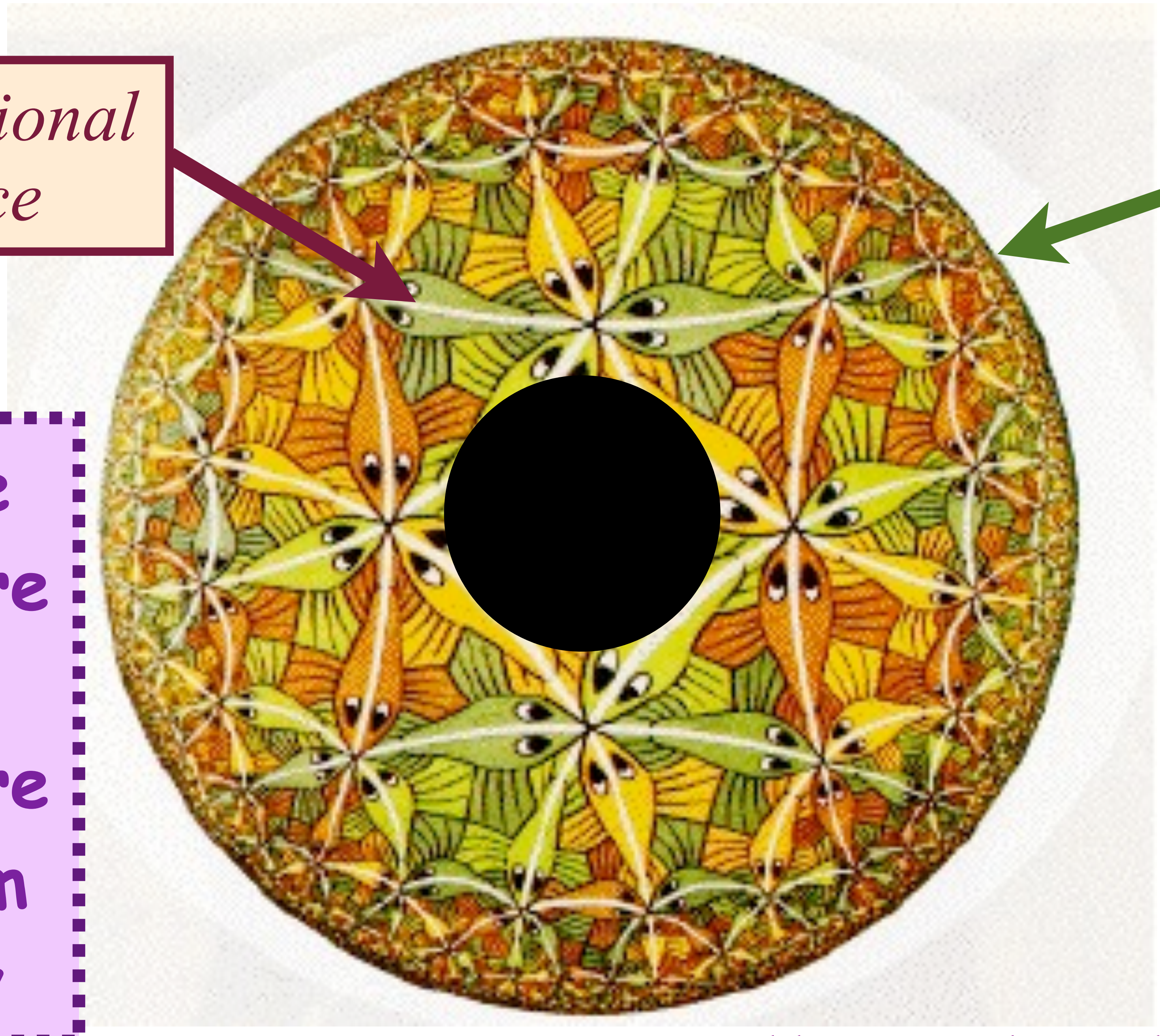

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten

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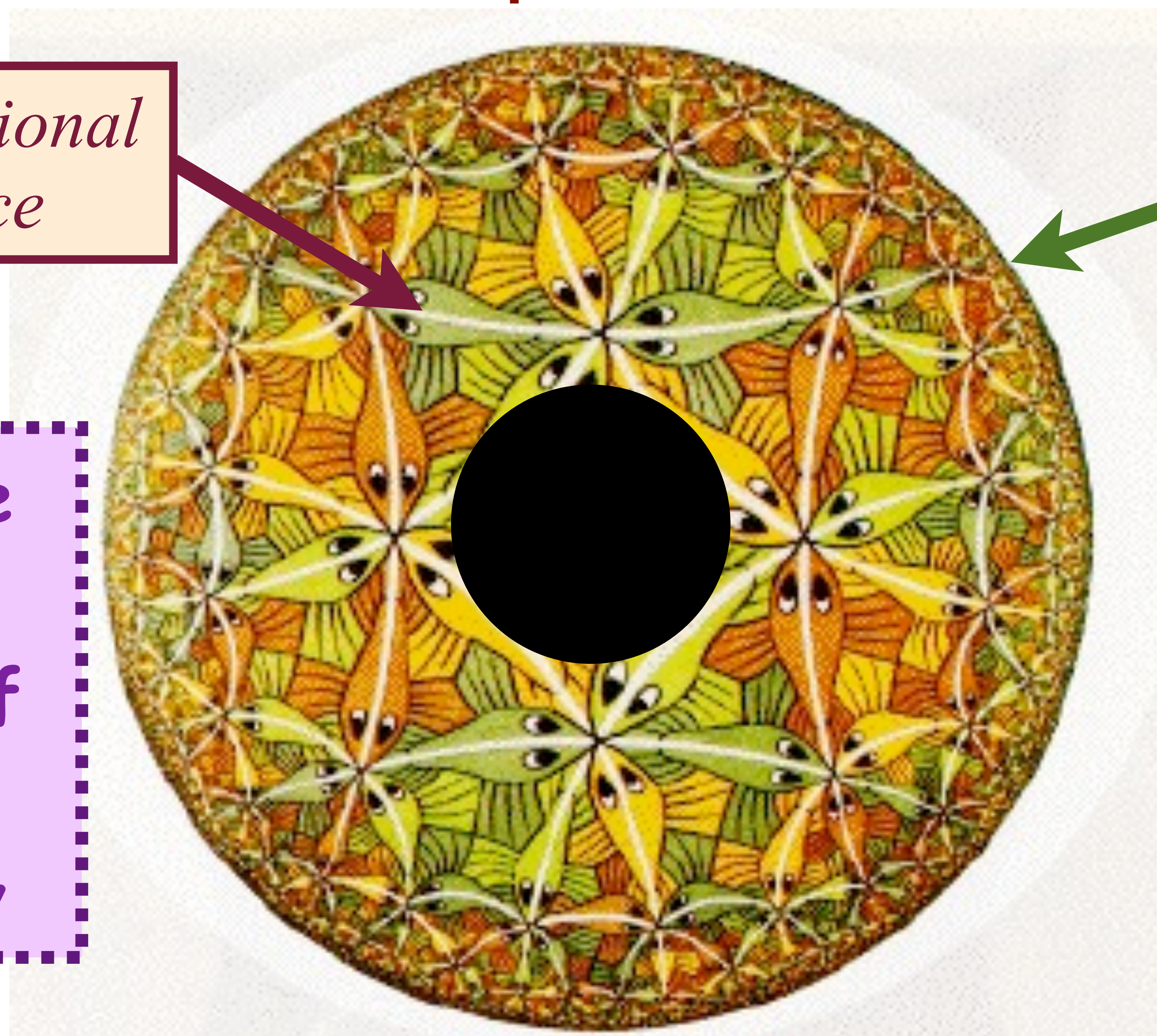

Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

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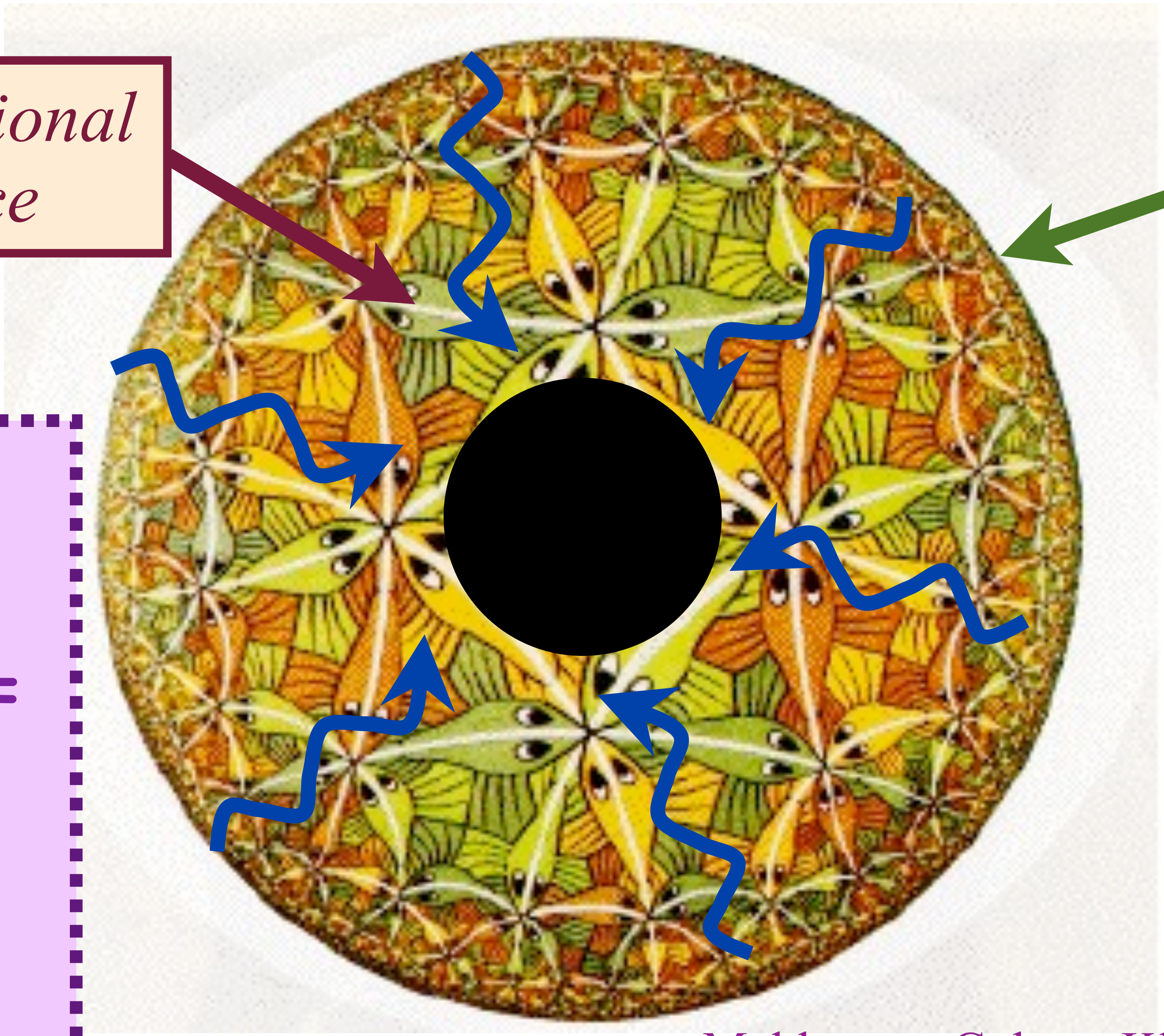
3+1 dimensional AdS space



Quantum criticality in 2+1 dimensions

Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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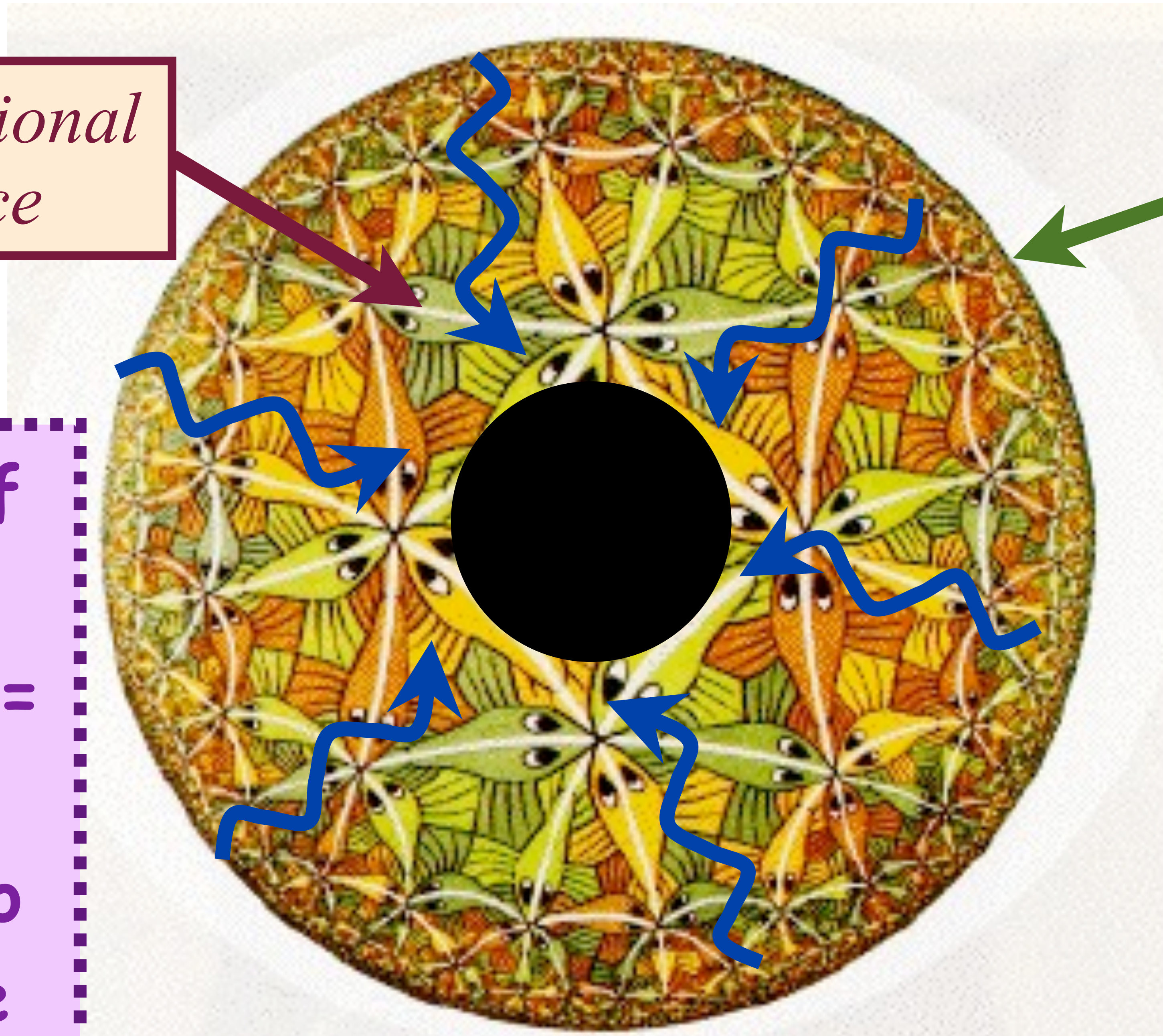

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Friction of quantum criticality = waves falling into black hole

Kovtun, Policastro, Son

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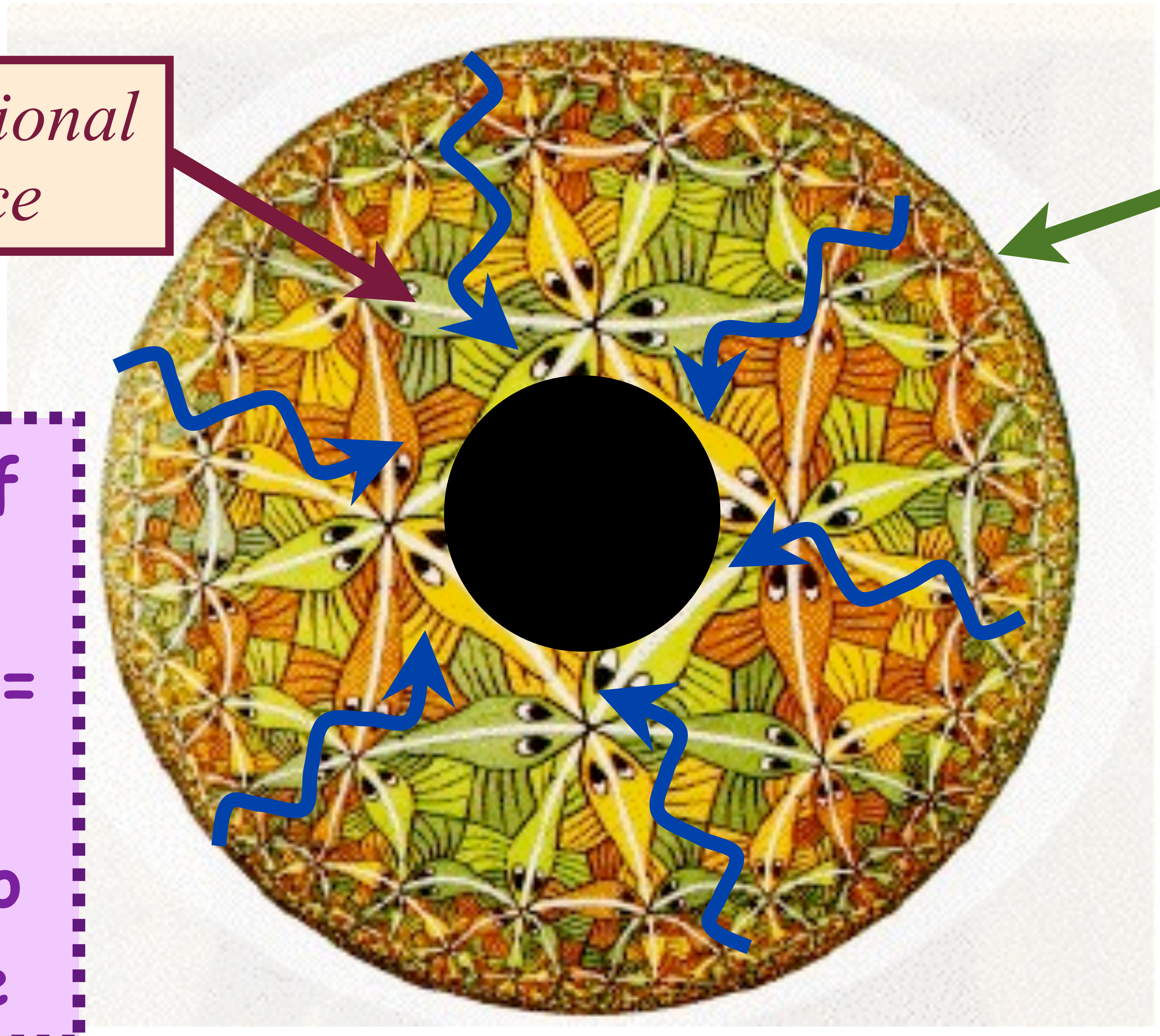

3+1 dimensional AdS space

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AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-

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Strong coupling problem: General solution of spin and magneto-thermo-electric transport in quantum critical region.

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).
S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76**, 144502 (2007).

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Fricti
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falling into
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Kovtun, Policastro, Son

Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Spin/charge conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the quantum of spin/charge)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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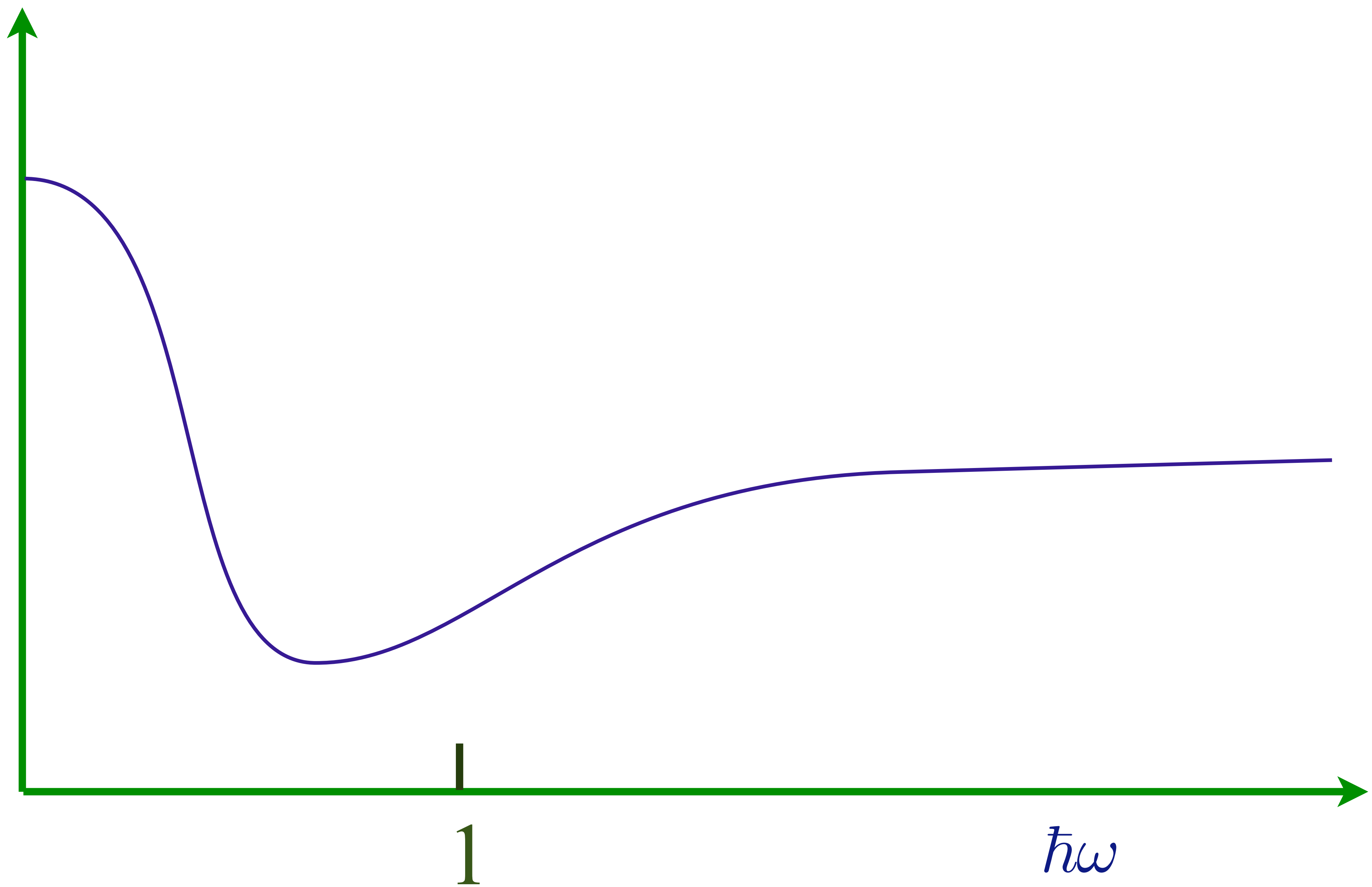
Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

Boltzmann theory of quantum critical transport

$$\sigma = \frac{4e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \;\; ; \quad \Sigma \rightarrow \text{a universal function}$$



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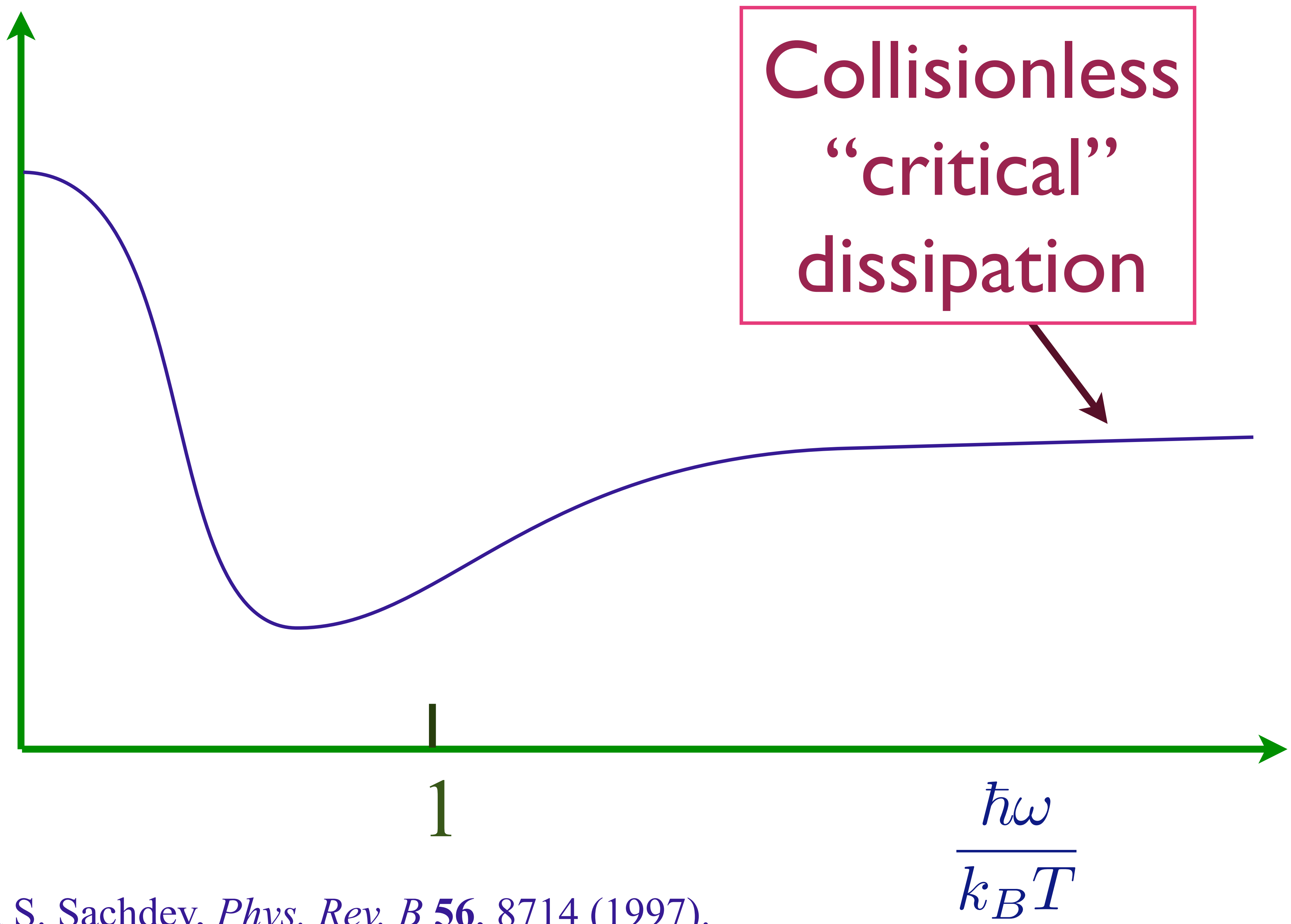

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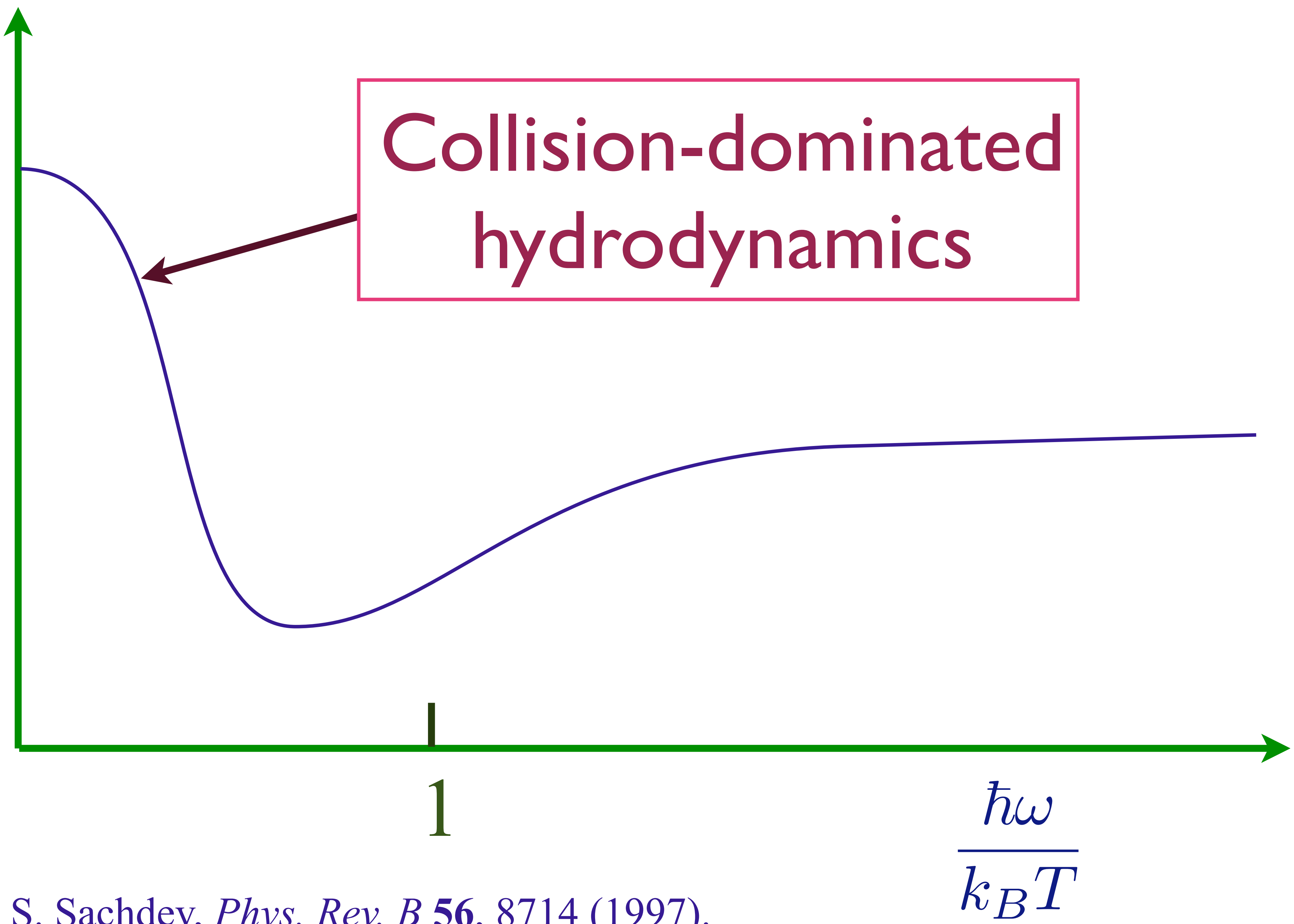

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AdS theory of strongly interacting "perfect fluids"

An infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4 with the following common properties:

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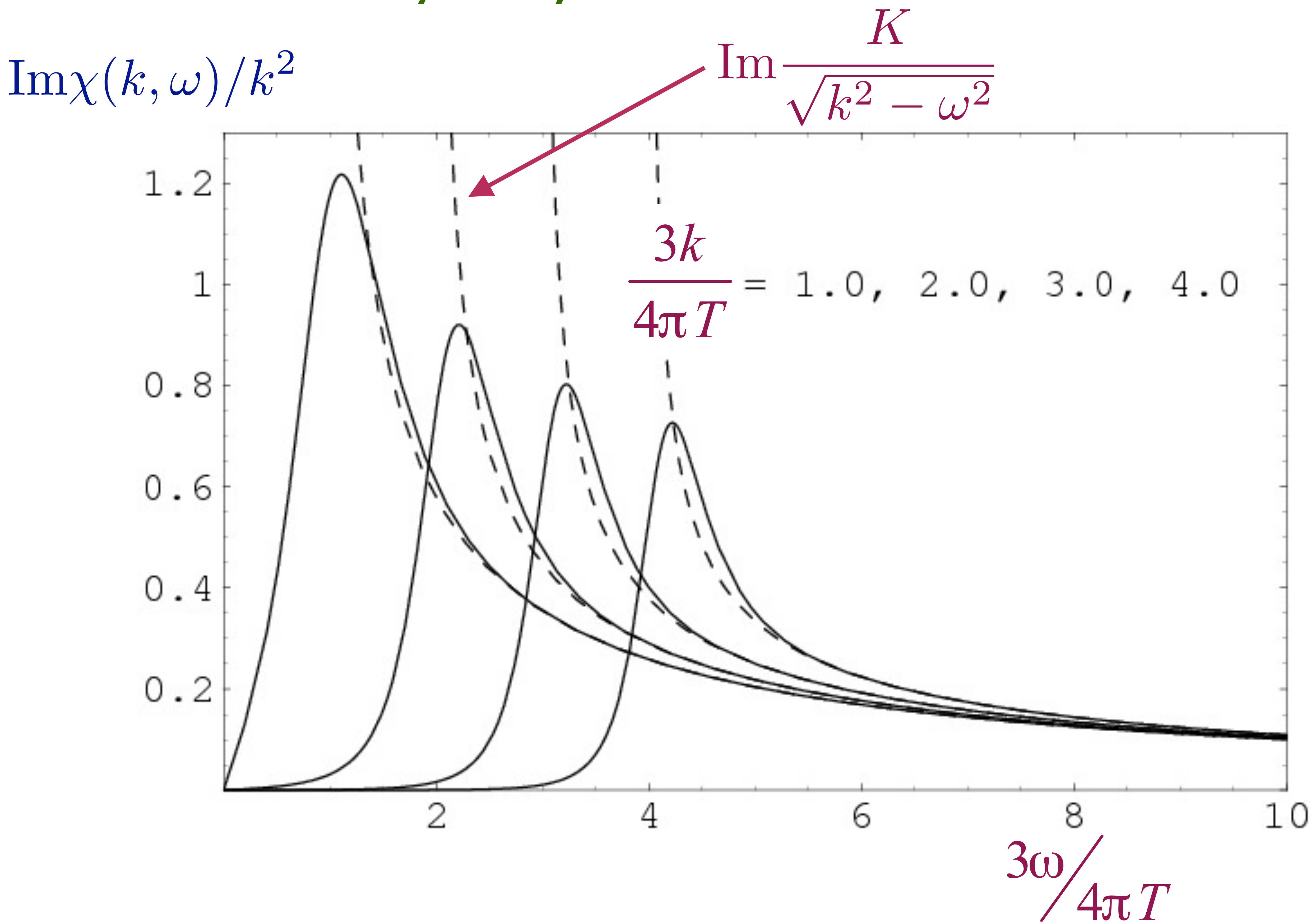
Collisionless to hydrodynamic crossover of SYM3



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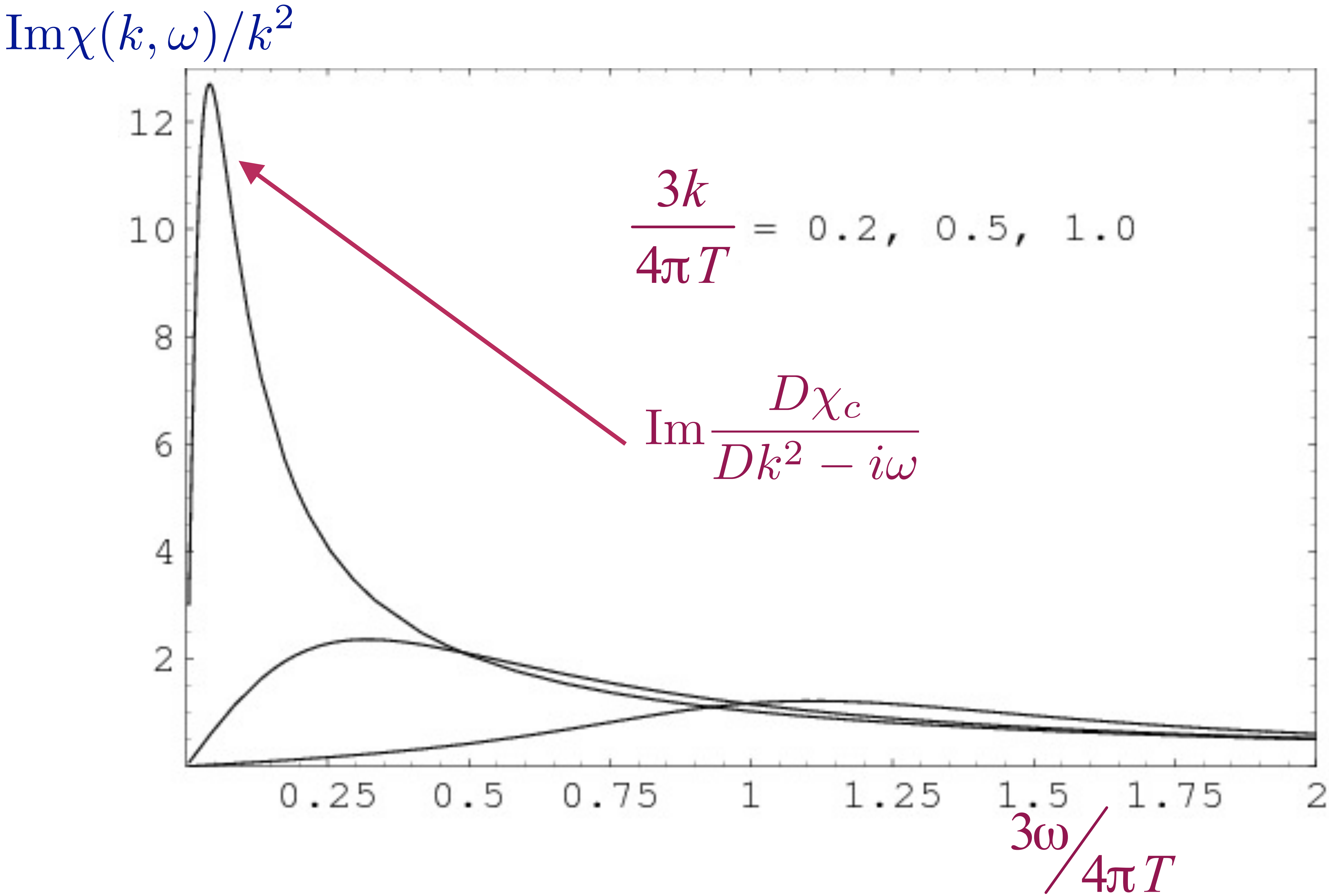

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- The viscosity/entropy-density ratio is

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

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- All continuous global symmetries are self dual.
- The conductivity σ is ω-independent and equal to the self-dual value. Frequency-dependent corrections are obtained from higher-derivative gravity theories.

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$
$$\sigma_{\text{Insulator}}(T \to 0) = 0$$
$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

- Self-dual value $= 4e^2/h$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

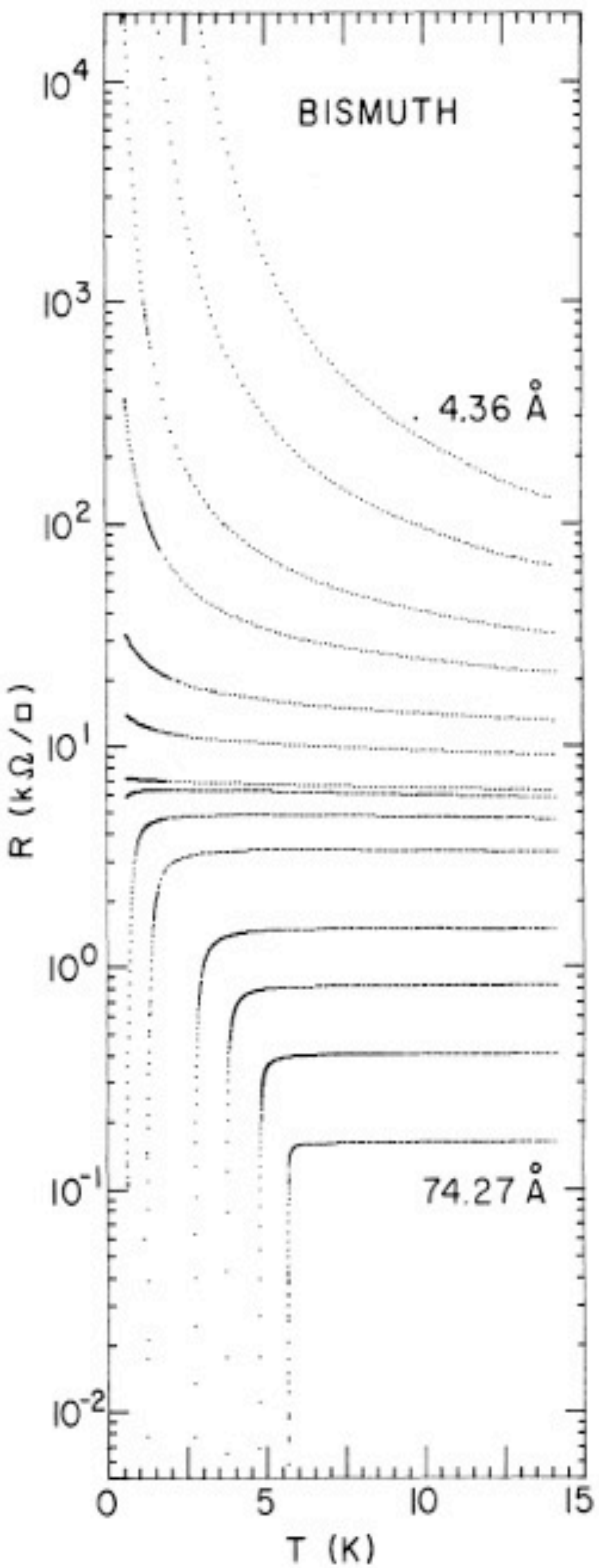

FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Outline

1. Coupled dimer antiferromagnets
   Quantum criticality and CFTs

2. The AdS/CFT correspondence
   Diffusion and transport in strongly interacting CFTs

3. Quantum matter at non-zero density
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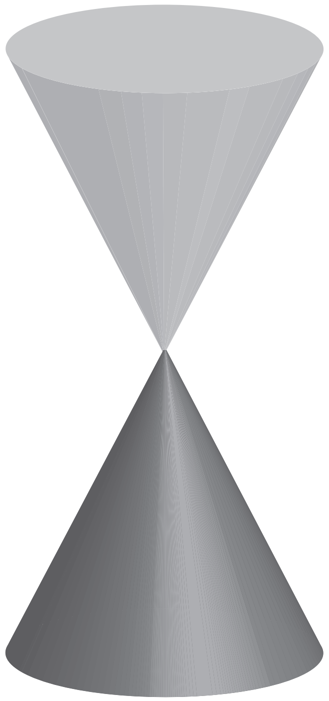
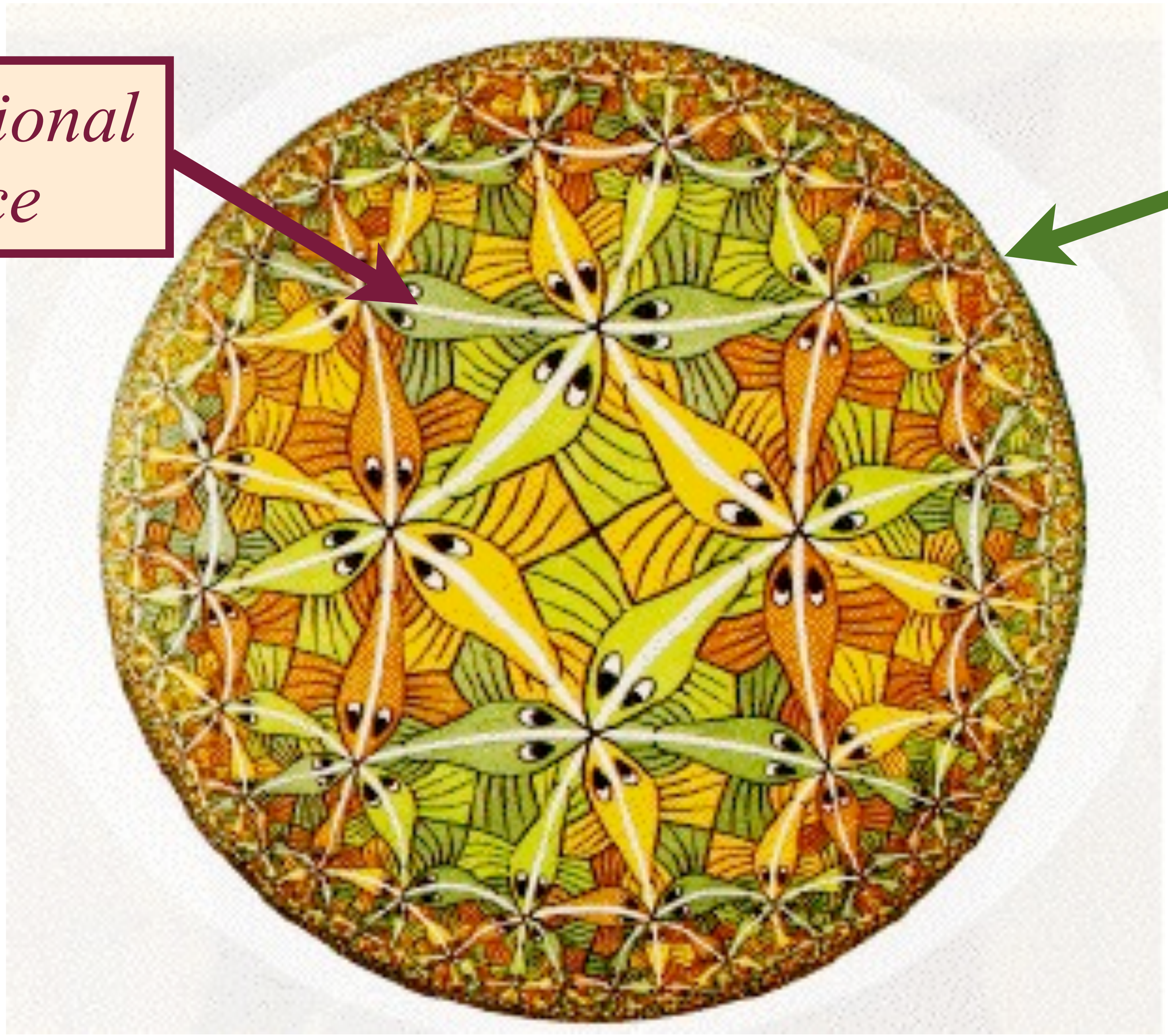
AdS/CFT correspondence

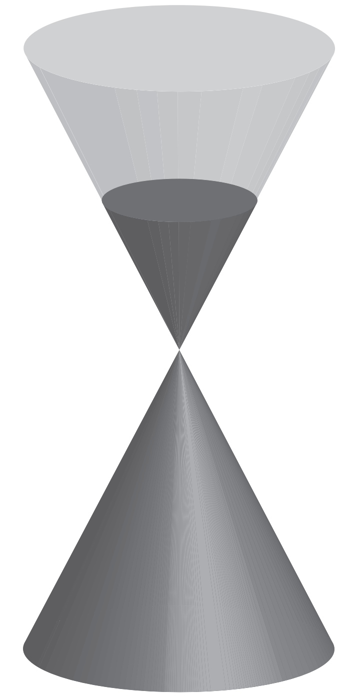




3+1 dimensional AdS space

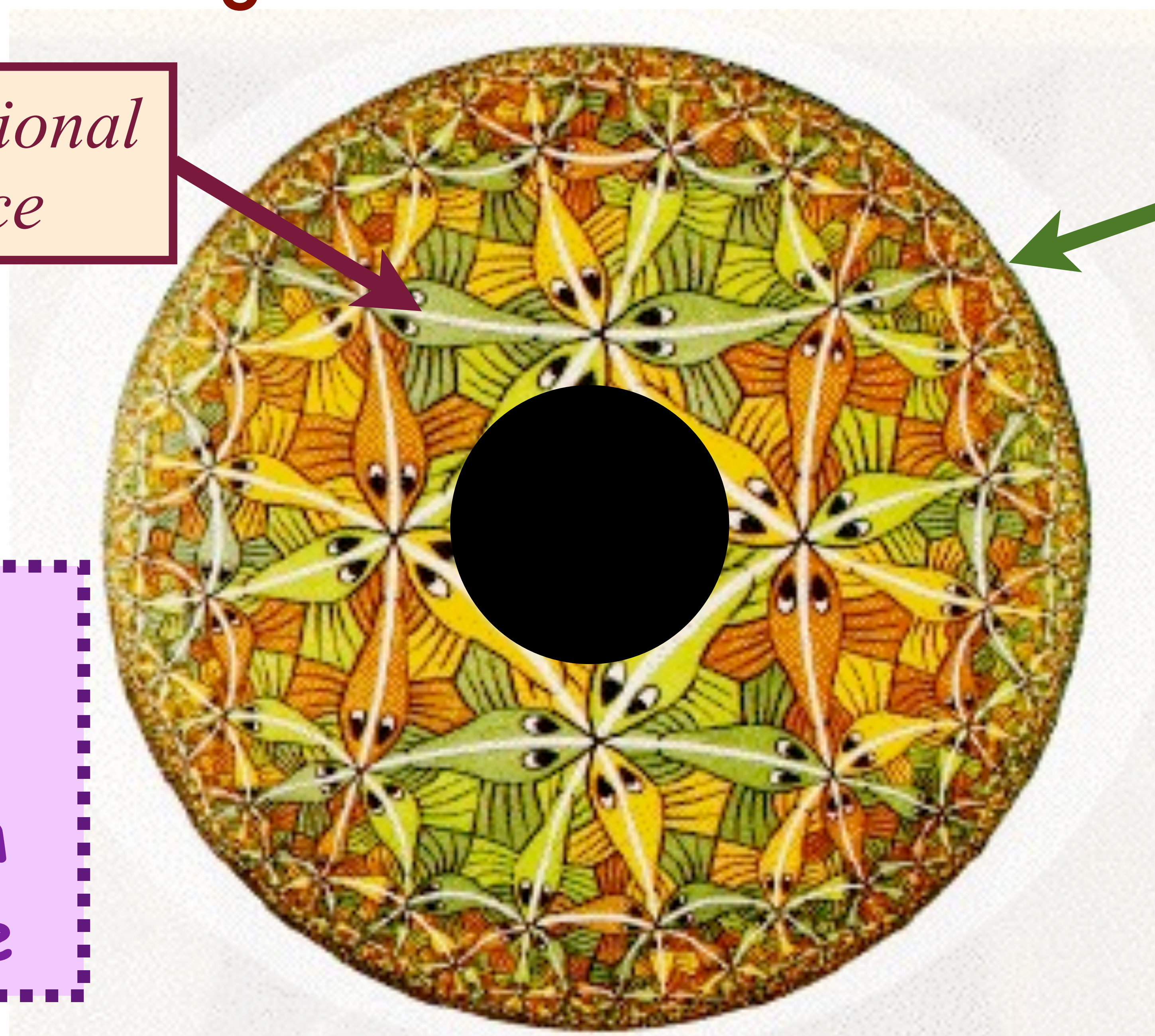
Quantum criticality in 2+1 dimensions

AdS/CFT correspondence



Move away from the quantum critical point to a system of matter at non-zero density: equivalent to adding an electrical charge to the black hole.

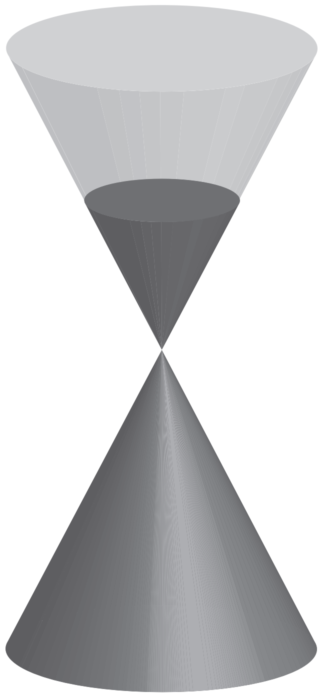
3+1 dimensional AdS space



Finite density matter in 2+1 dimensions

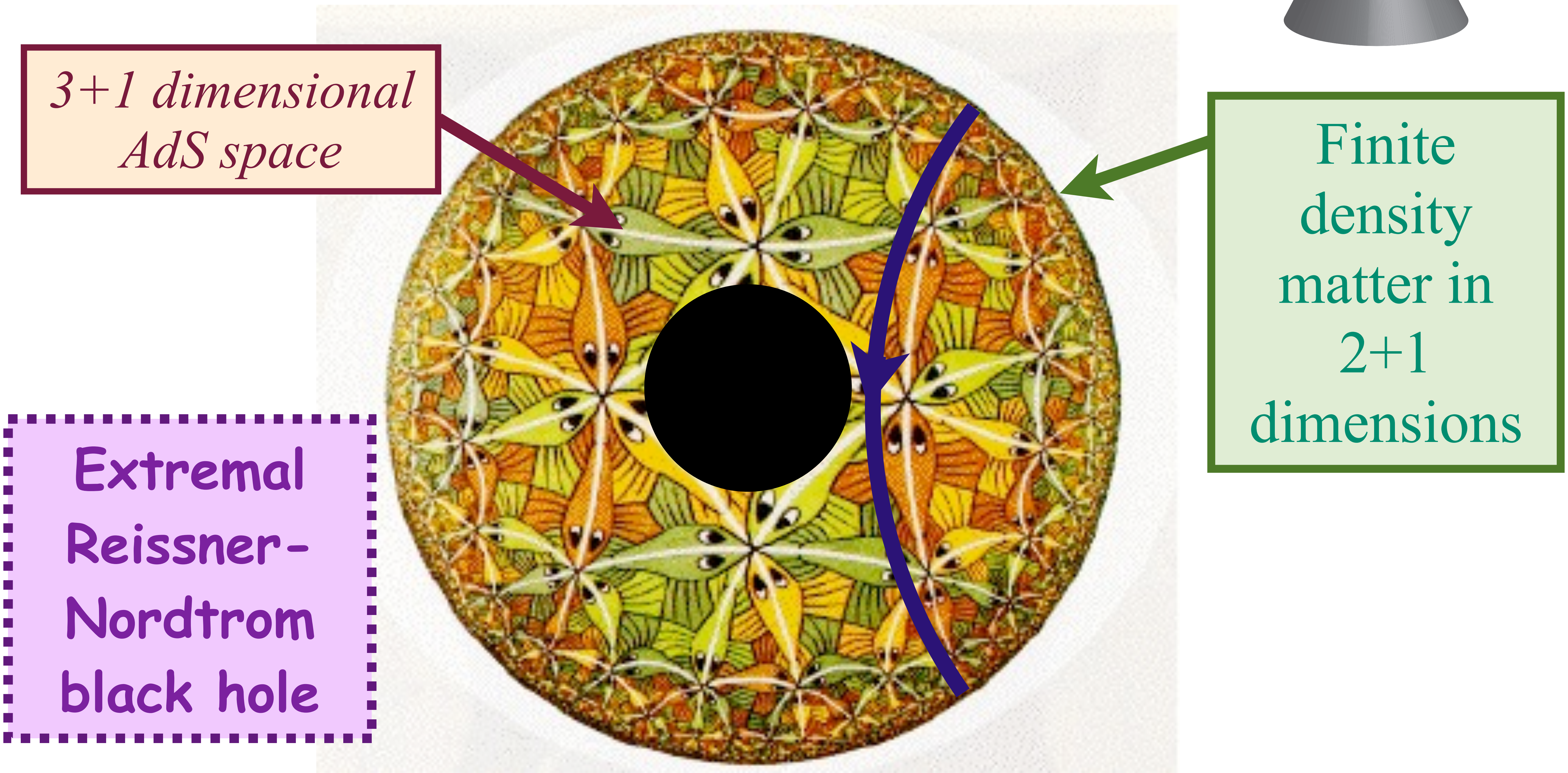
Extremal Reissner-Nordtrom black hole

AdS/CFT correspondence

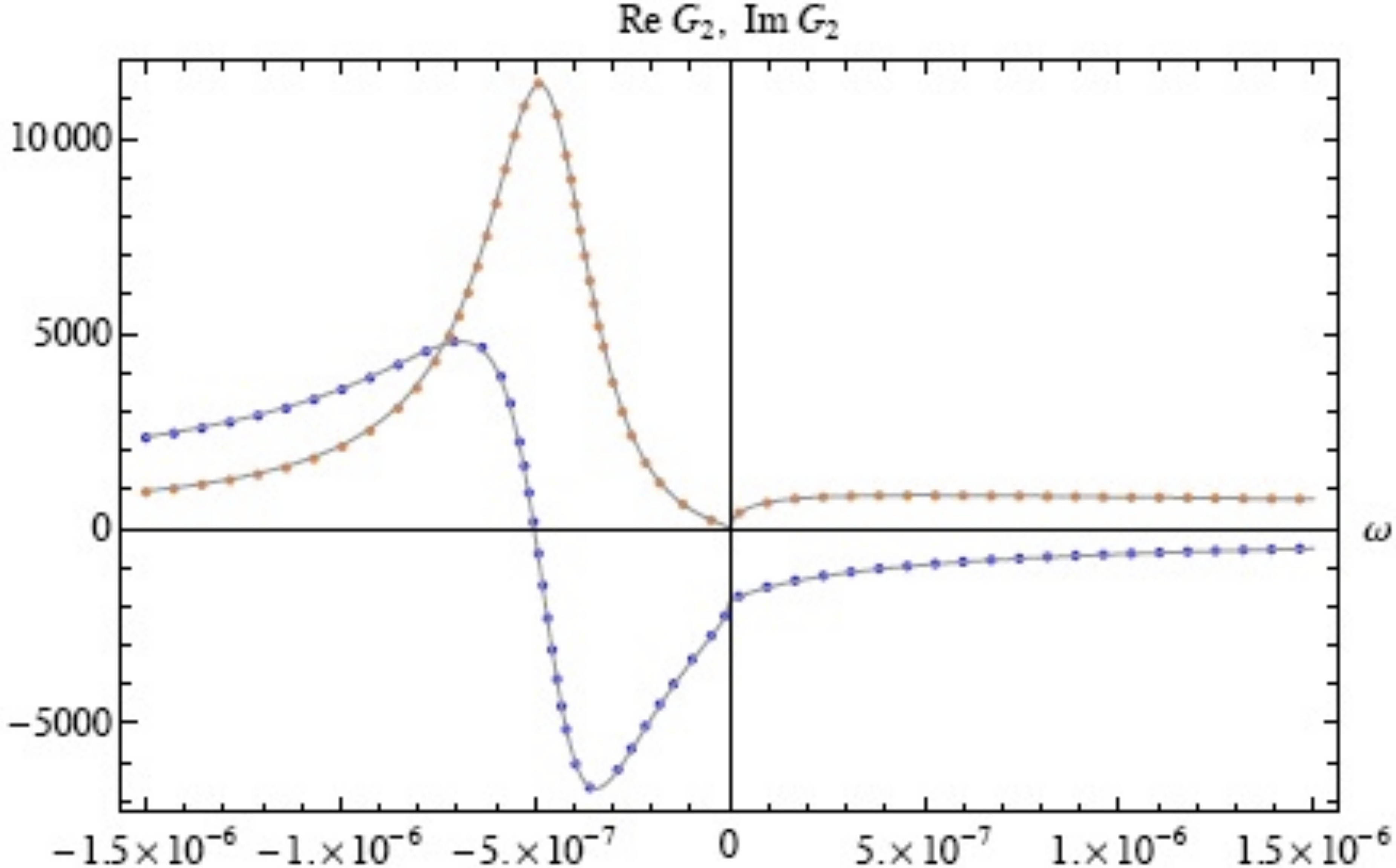


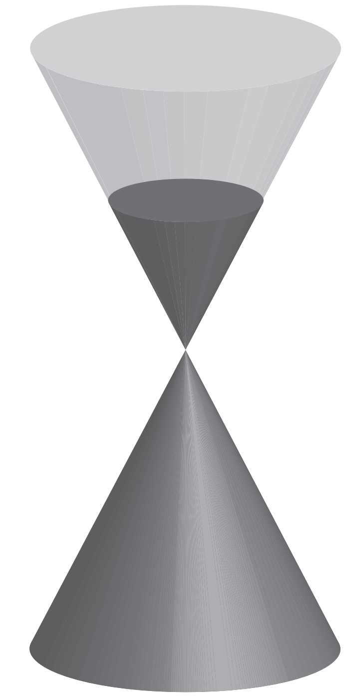
Examine the free energy and Green's function of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



Green's function of a fermion





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$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);
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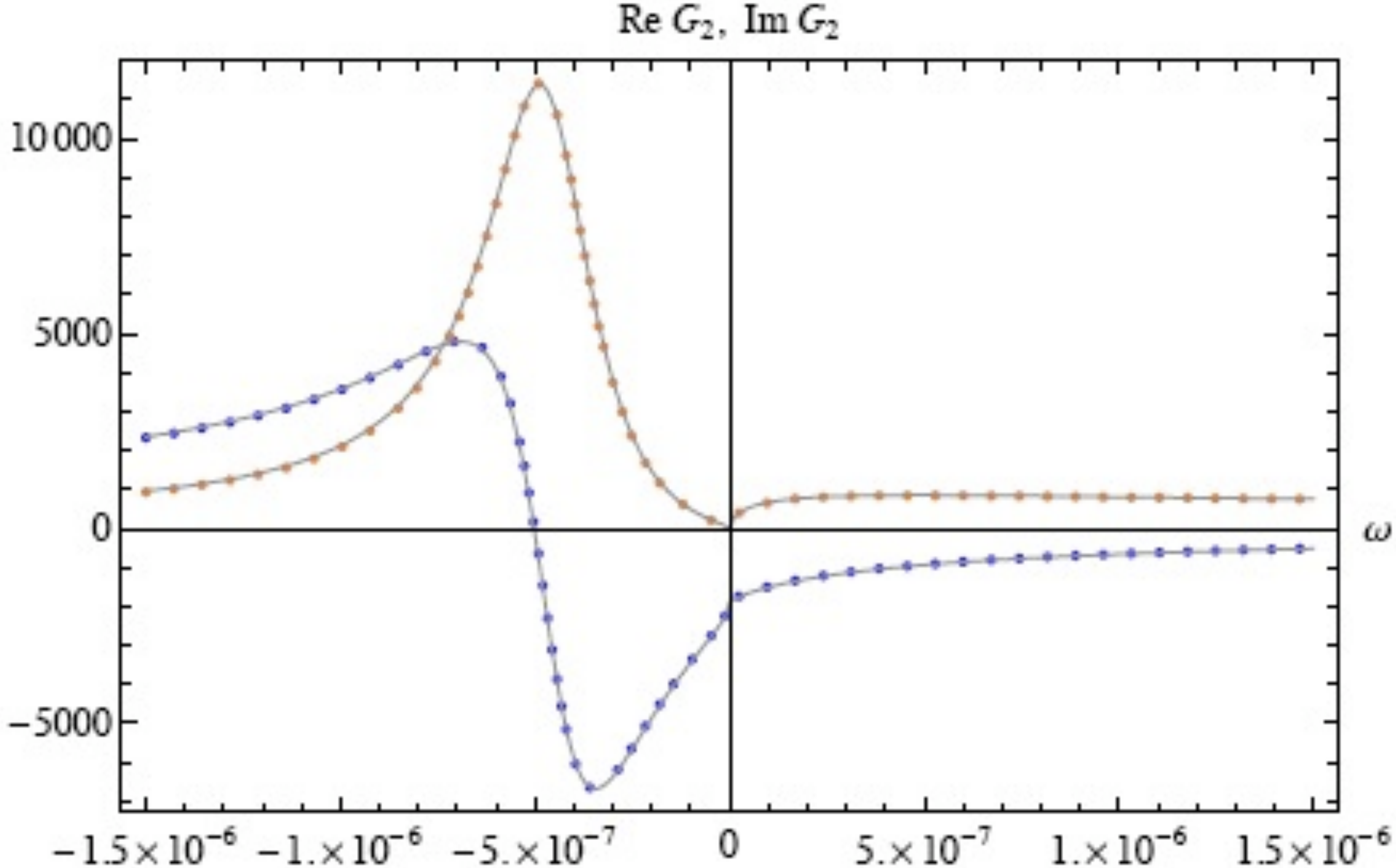

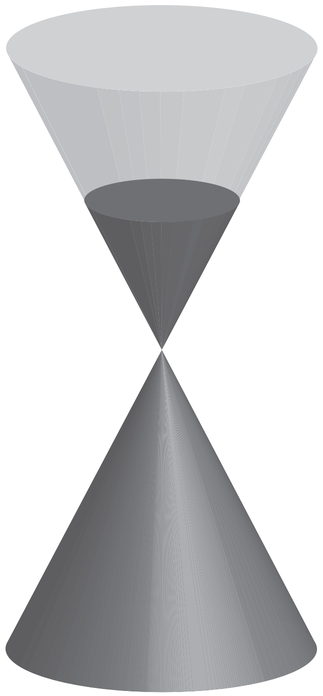

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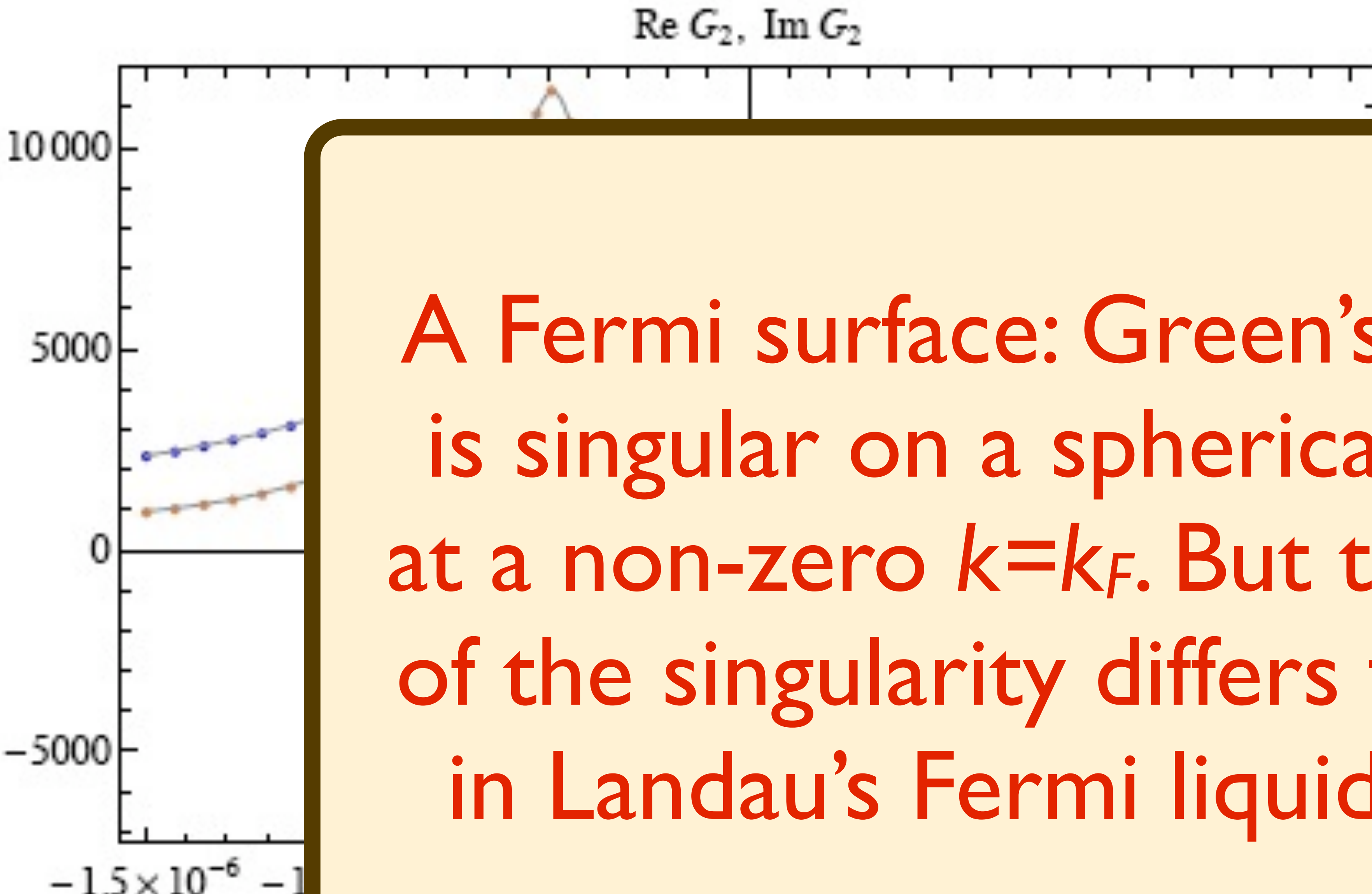

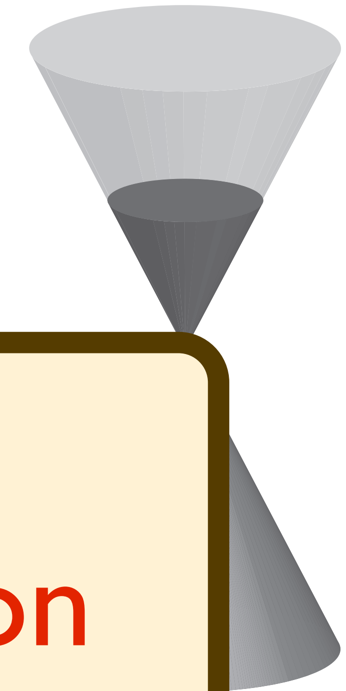

A Fermi surface: Green's function is singular on a spherical surface at a non-zero $k=k_F$. But the nature of the singularity differs from that in Landau's Fermi liquid theory.

H. Liu,
y, and
n,
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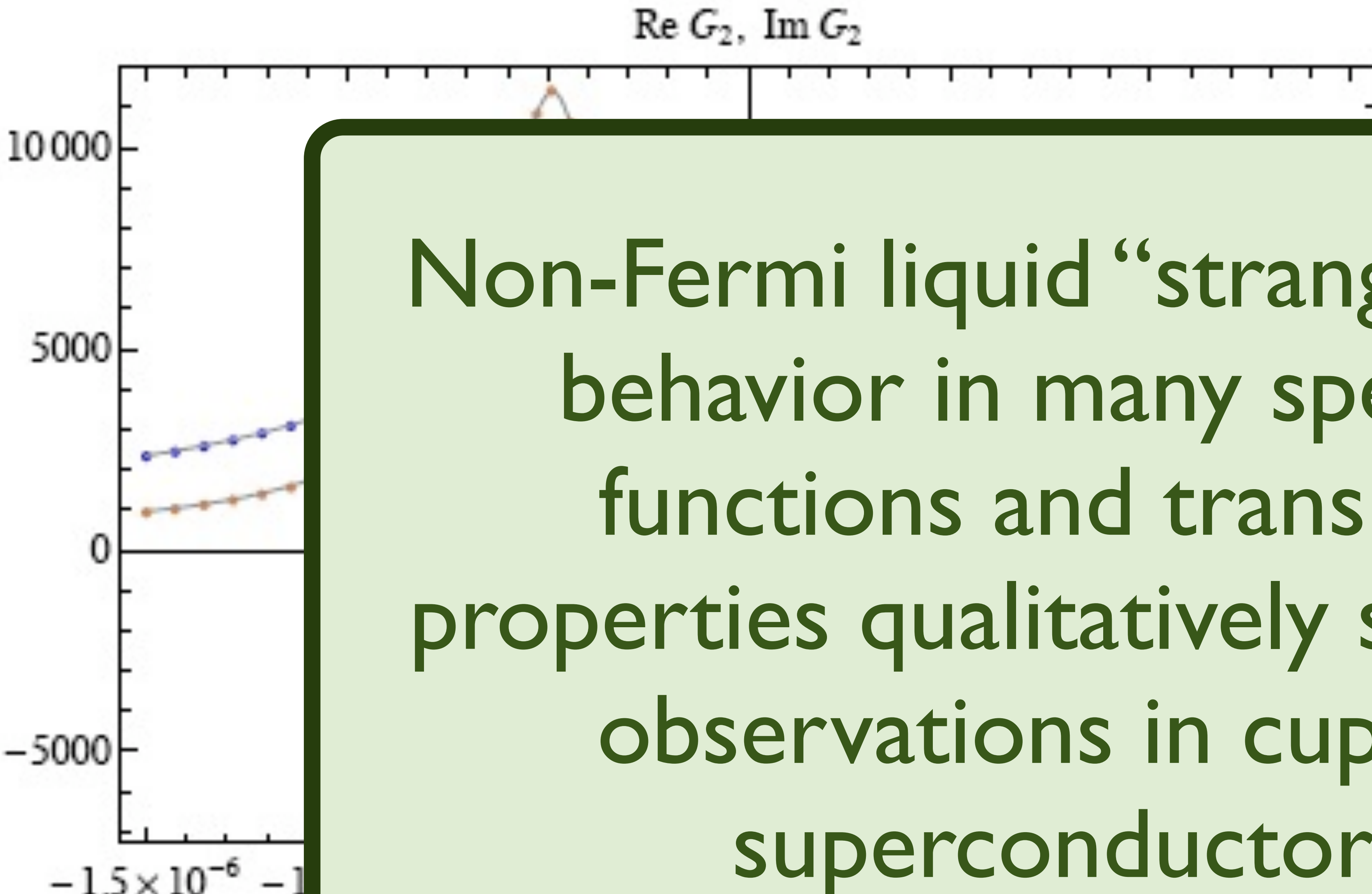

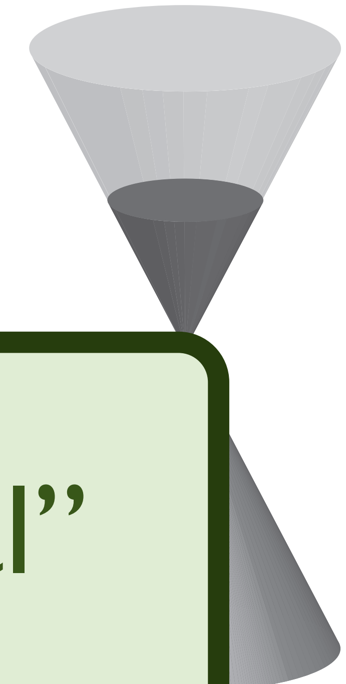

Non-Fermi liquid "strange metal" behavior in many spectral functions and transport properties qualitatively similar to observations in cuprate superconductors

H. Liu,
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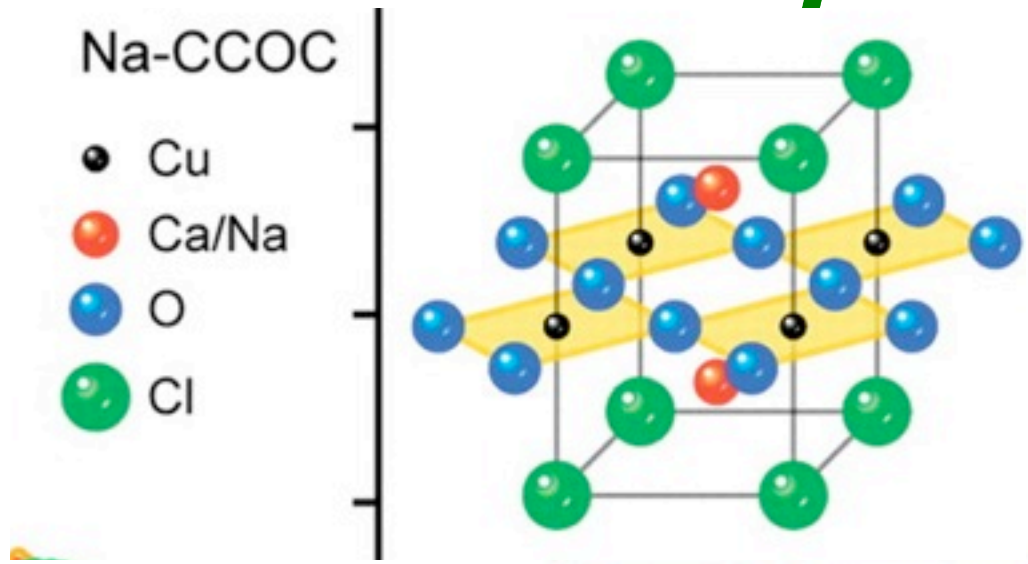
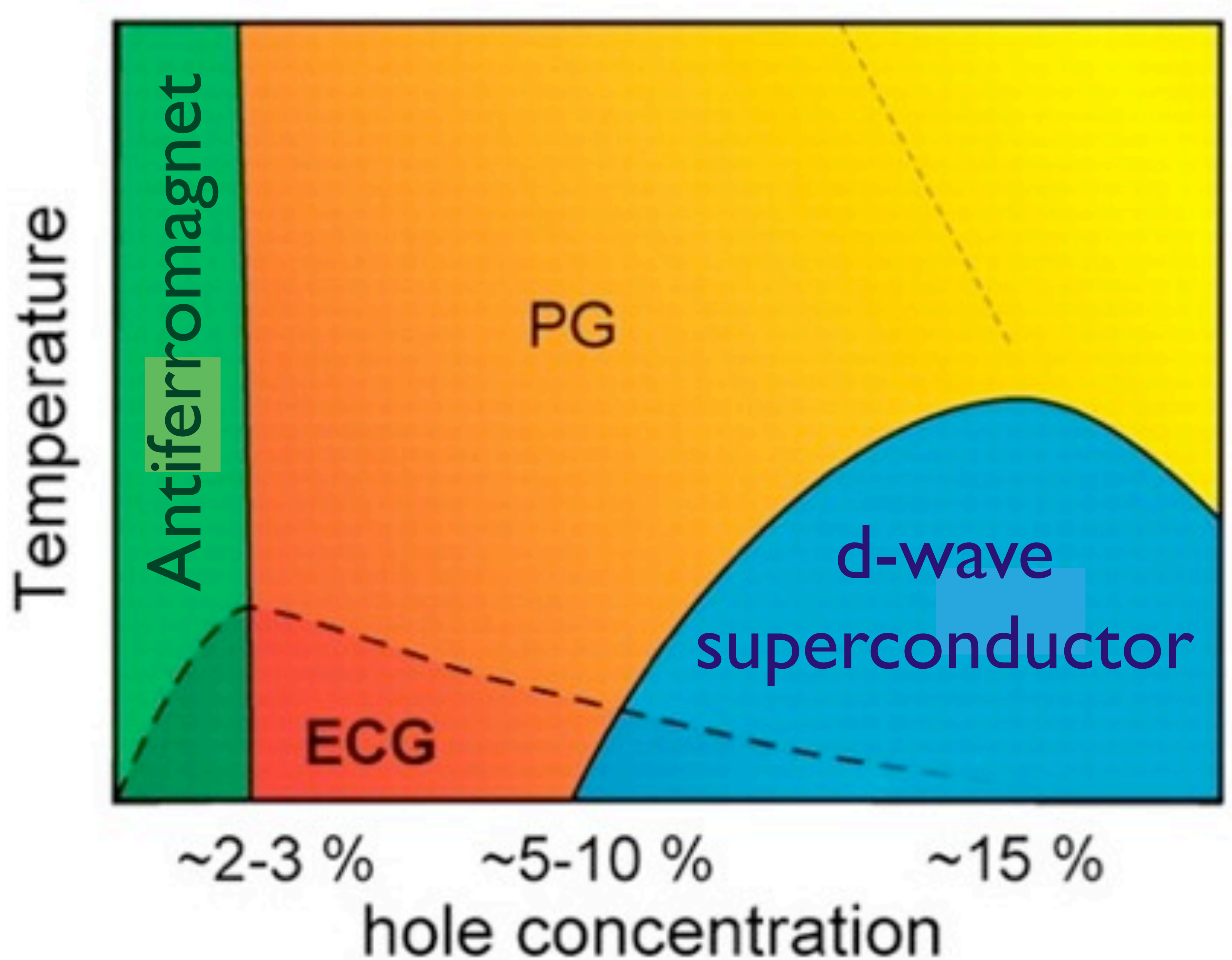
The cuprate superconductors





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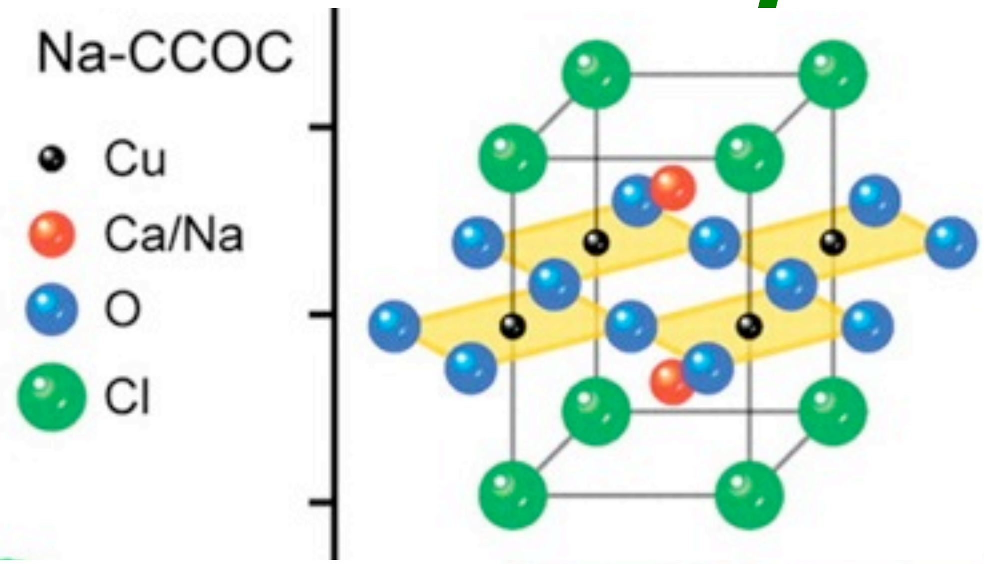

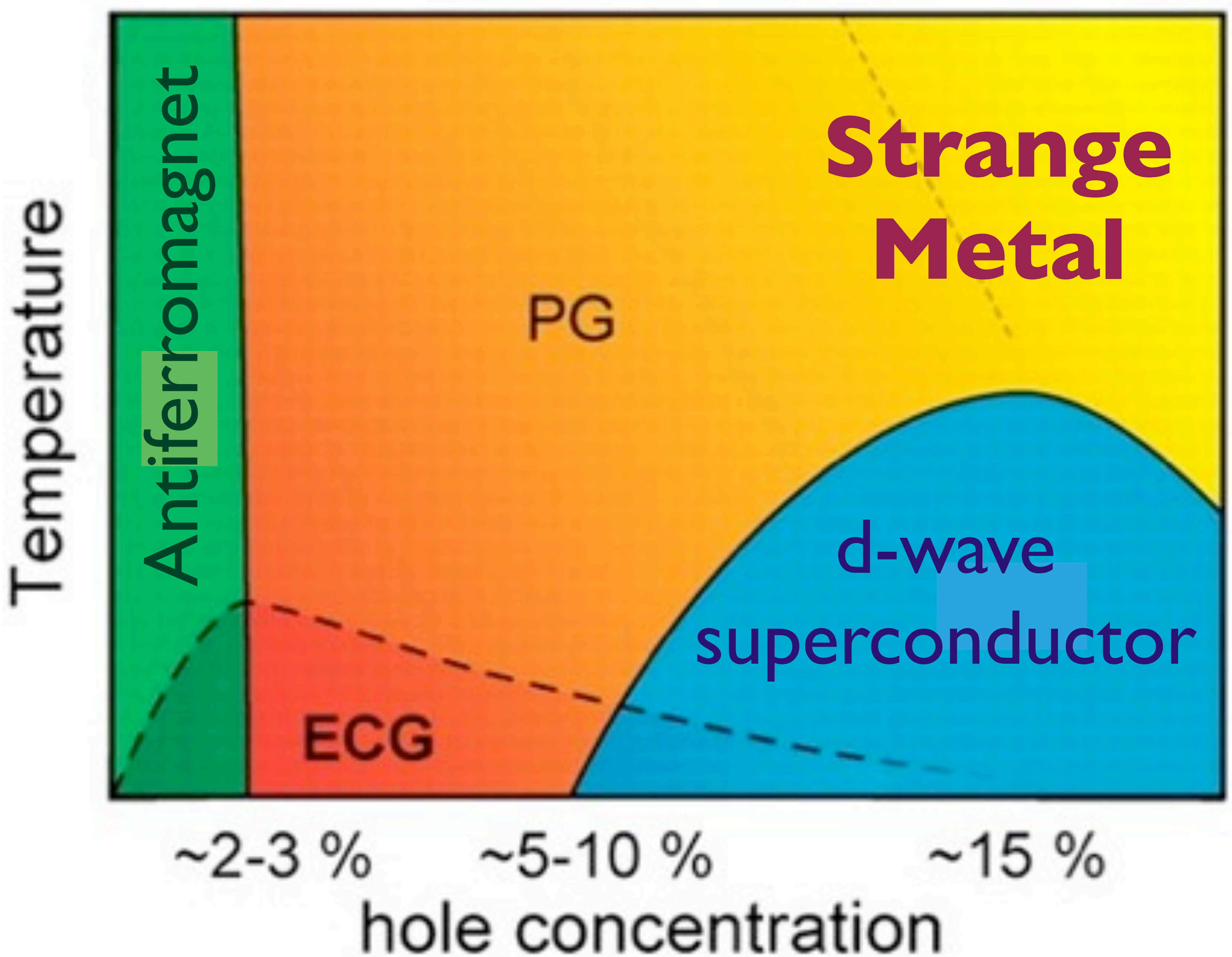

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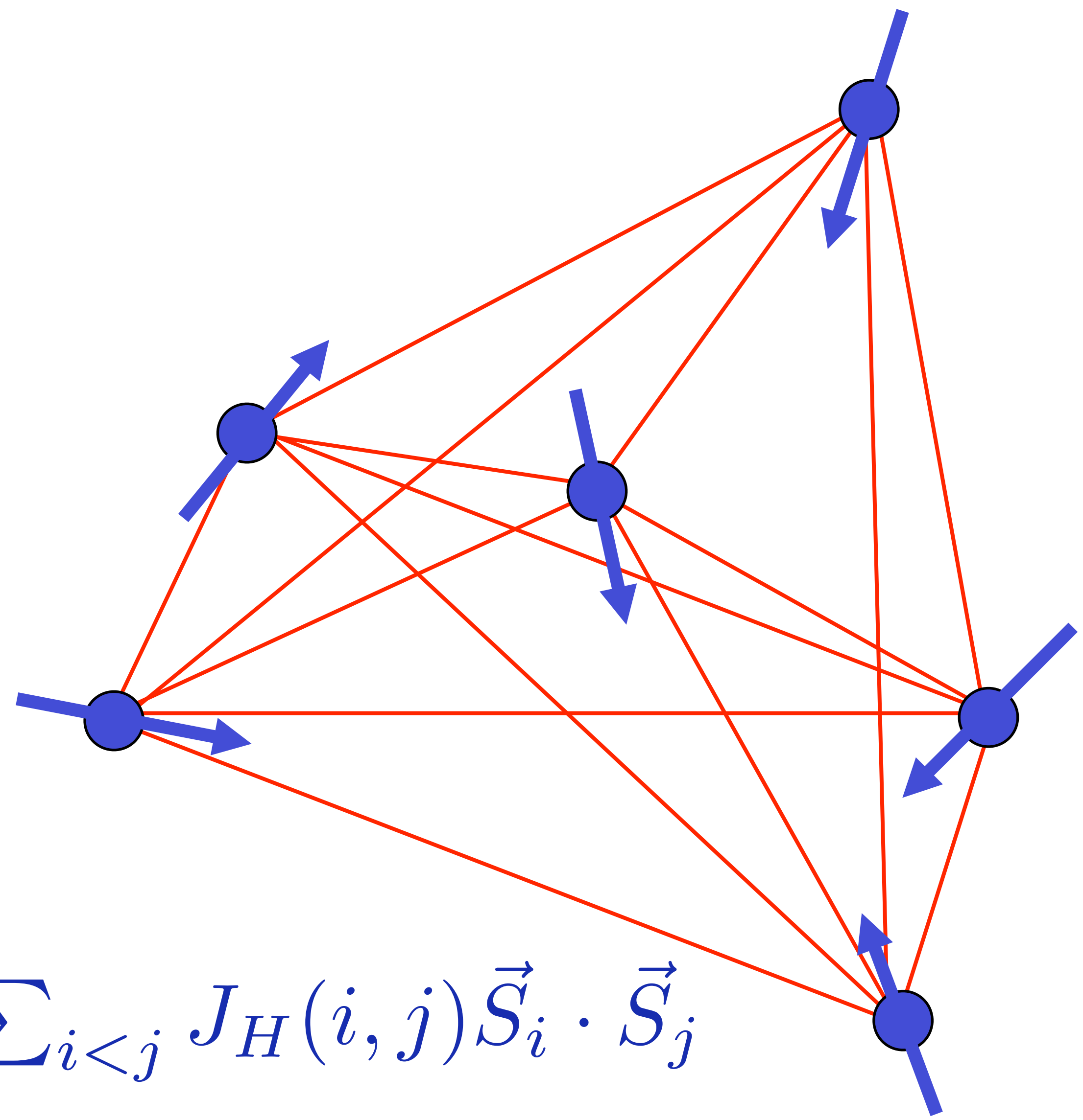
- Single particle self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
- Low energy singularities are described by "conformal quantum mechanics": a 0+1 dimensional defect in a 2+1 dimensional CFT. This is linked to the factorization of the near-horizon metric to $AdS_2 \times R^2$,

A Kondo lattice model for the $AdS_2 \times R^d$ region of an extremal Reissner-Nordstrom black hole



$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$

$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick
model of SU(N) spins.

S. Sachdev, arXiv:1006.3794

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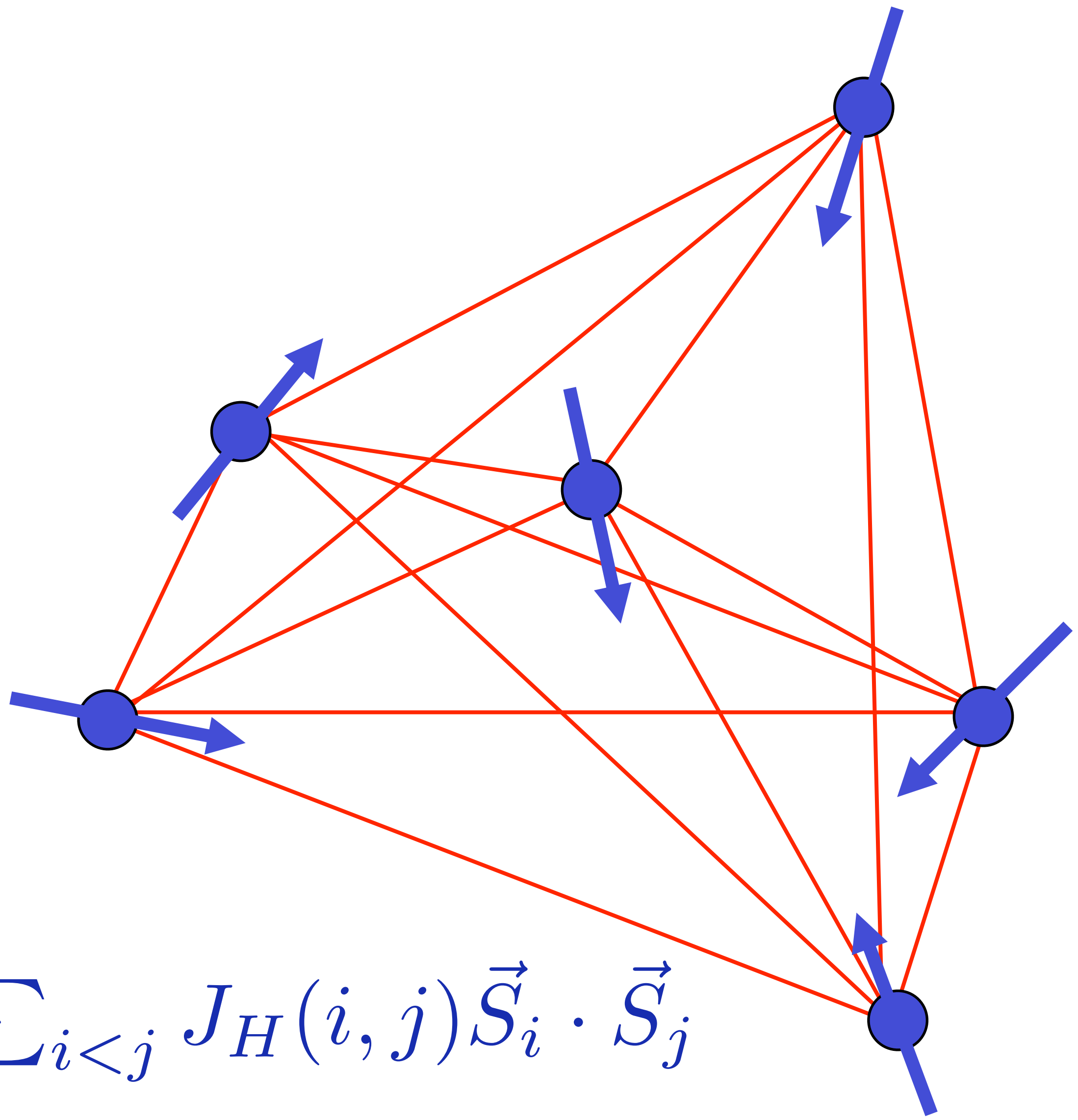

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Described by the conformal quantum mechanics of a quantum spin fluctuating in a self-consistent time-dependent magnetic field: a realization the finite entropy density $\mathrm{AdS}_2 \times R^d$ state

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001).

AdS_2 realization in the quantum SK model

Focus on a single $\vec{S}$ spin, and represent its imaginary time fluctuations by a unit vector $\vec{S} = \vec{n}(\tau)/2$ which is controlled by the partition function

$$\mathcal{Z} = \int \mathcal{D}\vec{n}(\tau)\, \delta(\vec{n}^2(\tau) - 1) \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \frac{i}{2} \int_0^1 du \int_0^{1/T} d\tau\, \vec{n} \cdot \left(\frac{\partial \vec{n}}{\partial u} \times \frac{\partial \vec{n}}{\partial \tau} \right) - \int_0^{1/T} d\tau\, \vec{h}(\tau) \cdot \vec{n}(\tau)$$

The first term is a Wess-Zumino term, with the "extra dimension" u defined so that $\vec{n}(\tau, u = 1) \equiv \vec{n}(\tau)$ and $\vec{n}(\tau, u = 0) = (0, 0, 1)$.

The field $\vec{h}(\tau)$ represents the "environment", which is determined as a Gaussian field obeying the self-consistency condition

$$\left\langle \vec{h}(\tau) \cdot \vec{h}(0) \right\rangle \propto \langle \vec{n}(\tau) \cdot \vec{n}(0) \rangle$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
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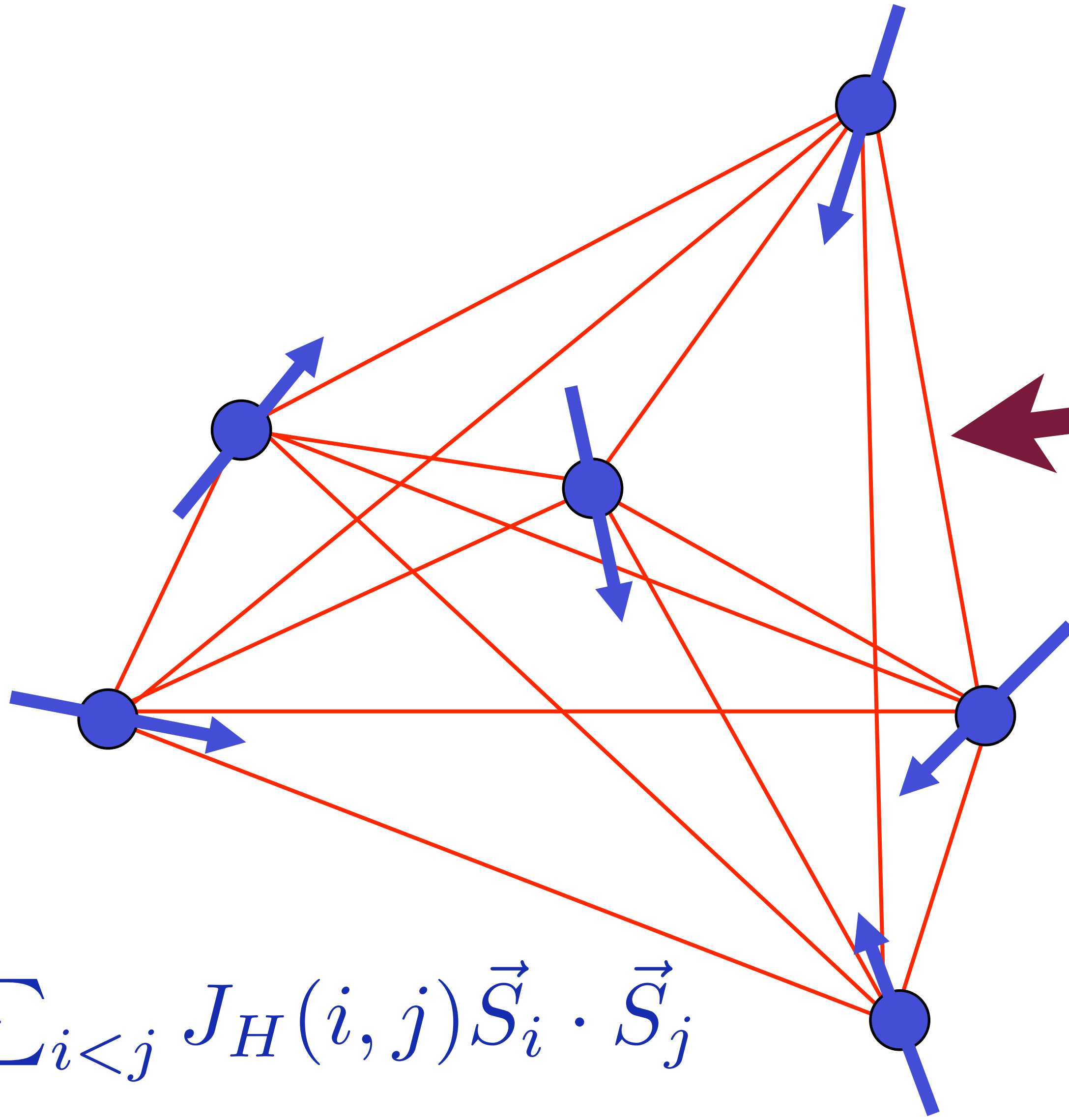

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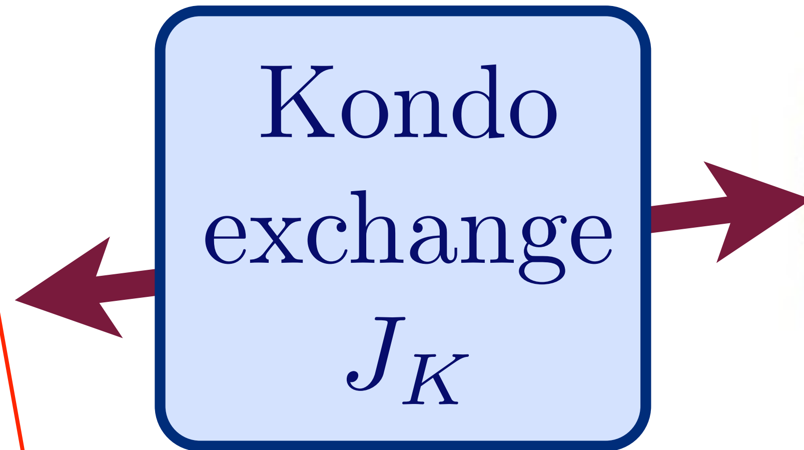

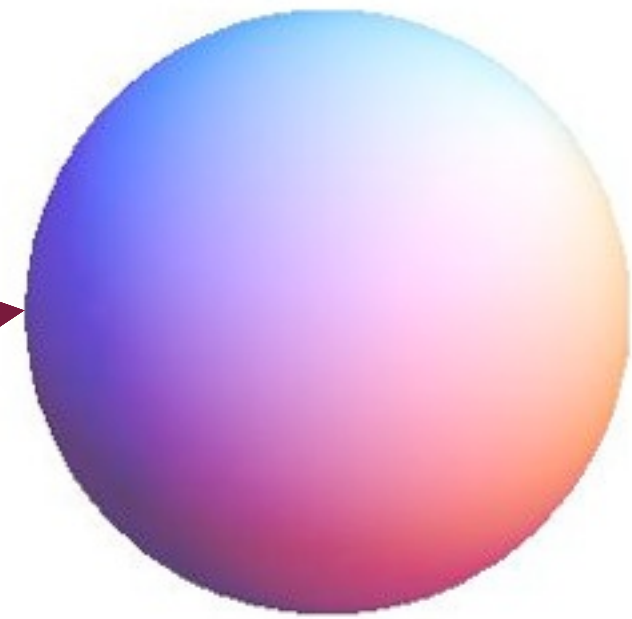

A small Fermi surface of probe fermions

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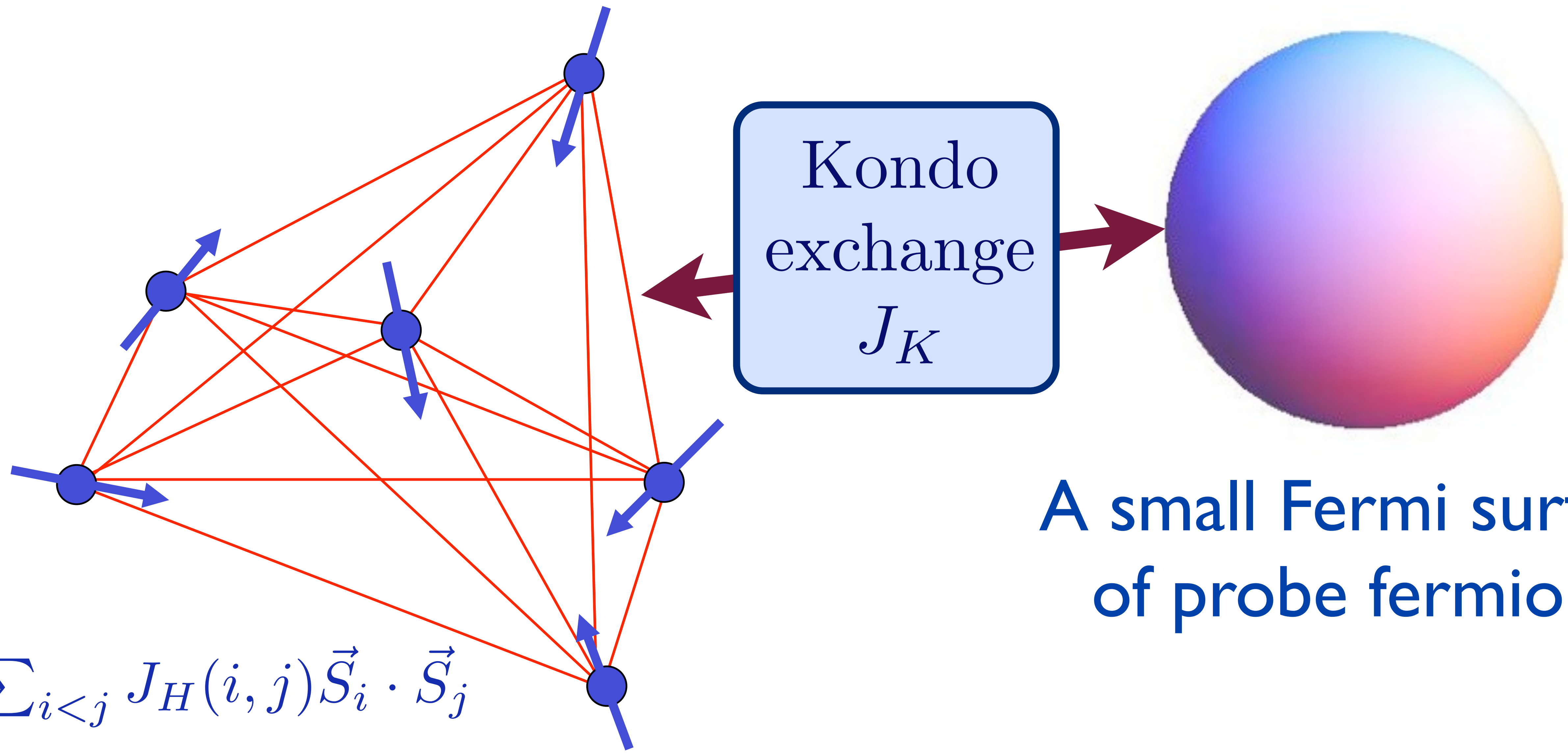

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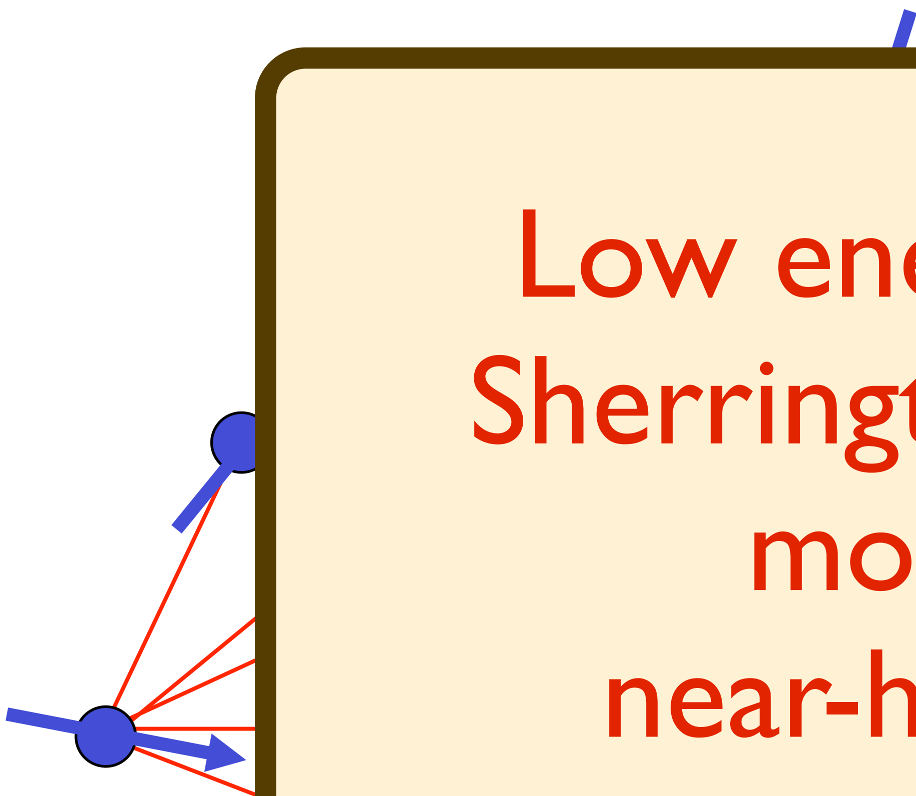

Low energy properties of the Sherrington-Kirkpatrick-Kondo model map onto the near-horizon physics of an extremal Reissner-Nordstrom black hole

surface

ions

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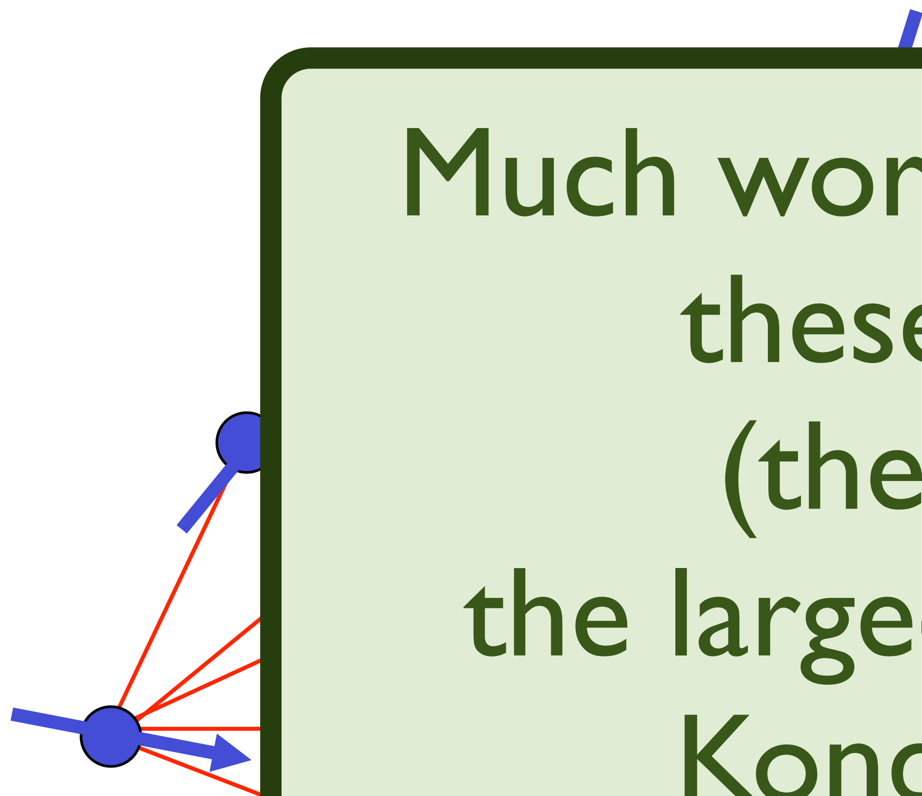

Much work remains in extending these solvable models (the AdS theories or the large-connectivity limits of Kondo lattice models) to a realistic theory of the cuprate superconductors

surface

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Conclusions

New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle picture.

Conclusions

The AdS/CFT correspondence offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density