Paramagnon fractionalization in the pseudogap, and implications for the cuprate phase diagram

Stanford University
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Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

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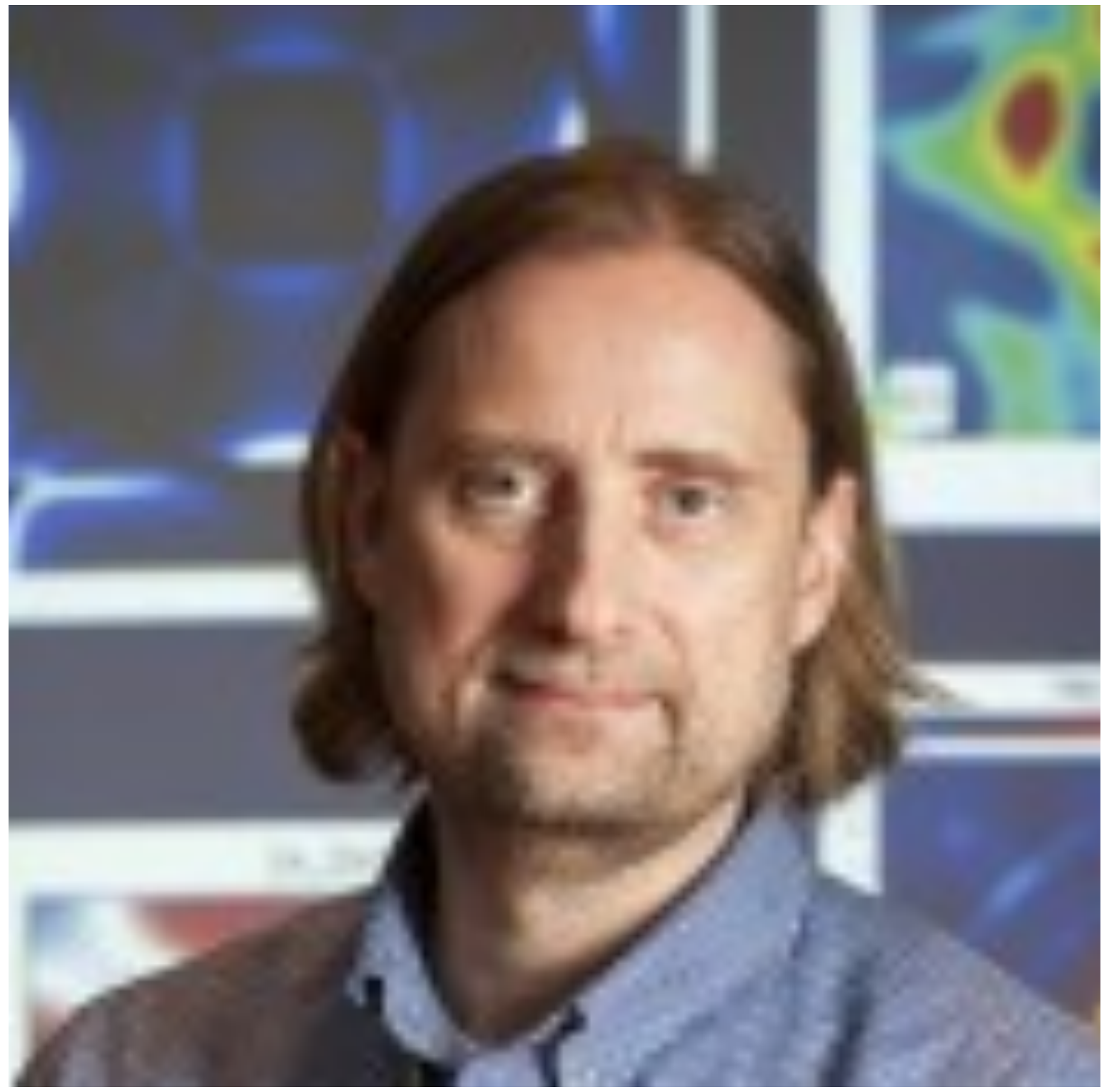
arXiv: 2001.09159
arXiv: 2103.05009

arXiv: 2006.01140
arXiv: 2111.13703

Dirk Morr



1. Luttinger relations - old and new

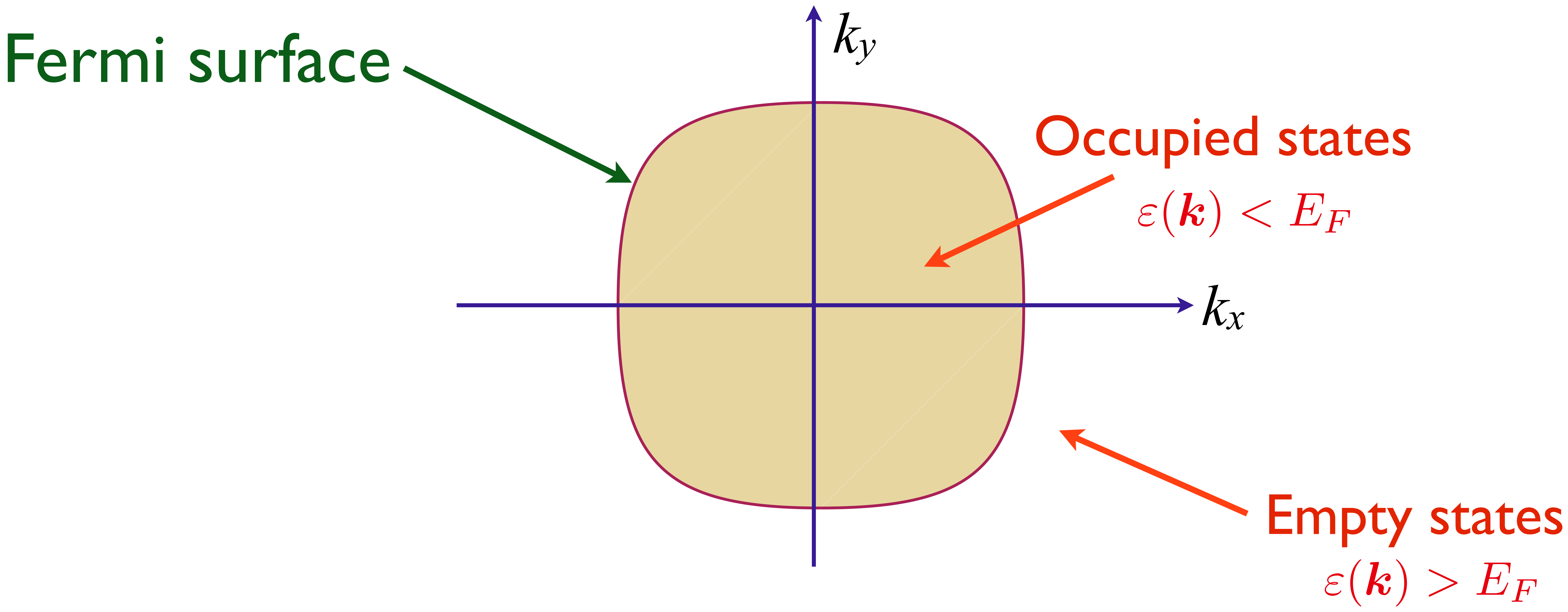
2. Paramagnon fractionalization in the single band Hubbard model

3. Comparison with photoemission

4. Confinement transition in the random *t-J* model

Luttinger relation

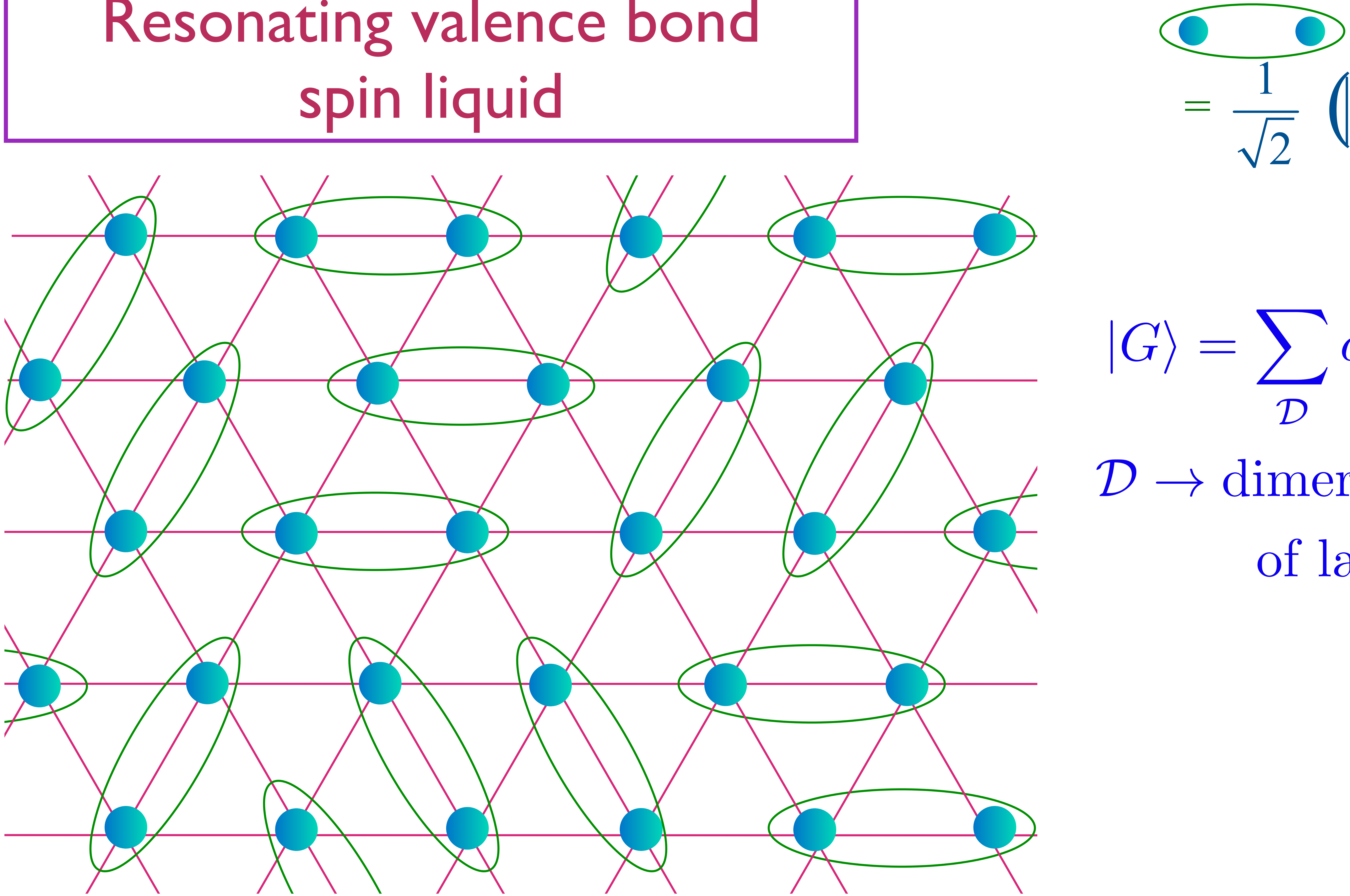
Electrons move with momentum $\mathbf{k}$ through the lattice with dispersion $\varepsilon(\boldsymbol{k})$



$$2 \times \frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons (mod 2)}$$

Mott insulator: Triangular lattice antiferromagnet

Resonating valence bond spin liquid



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

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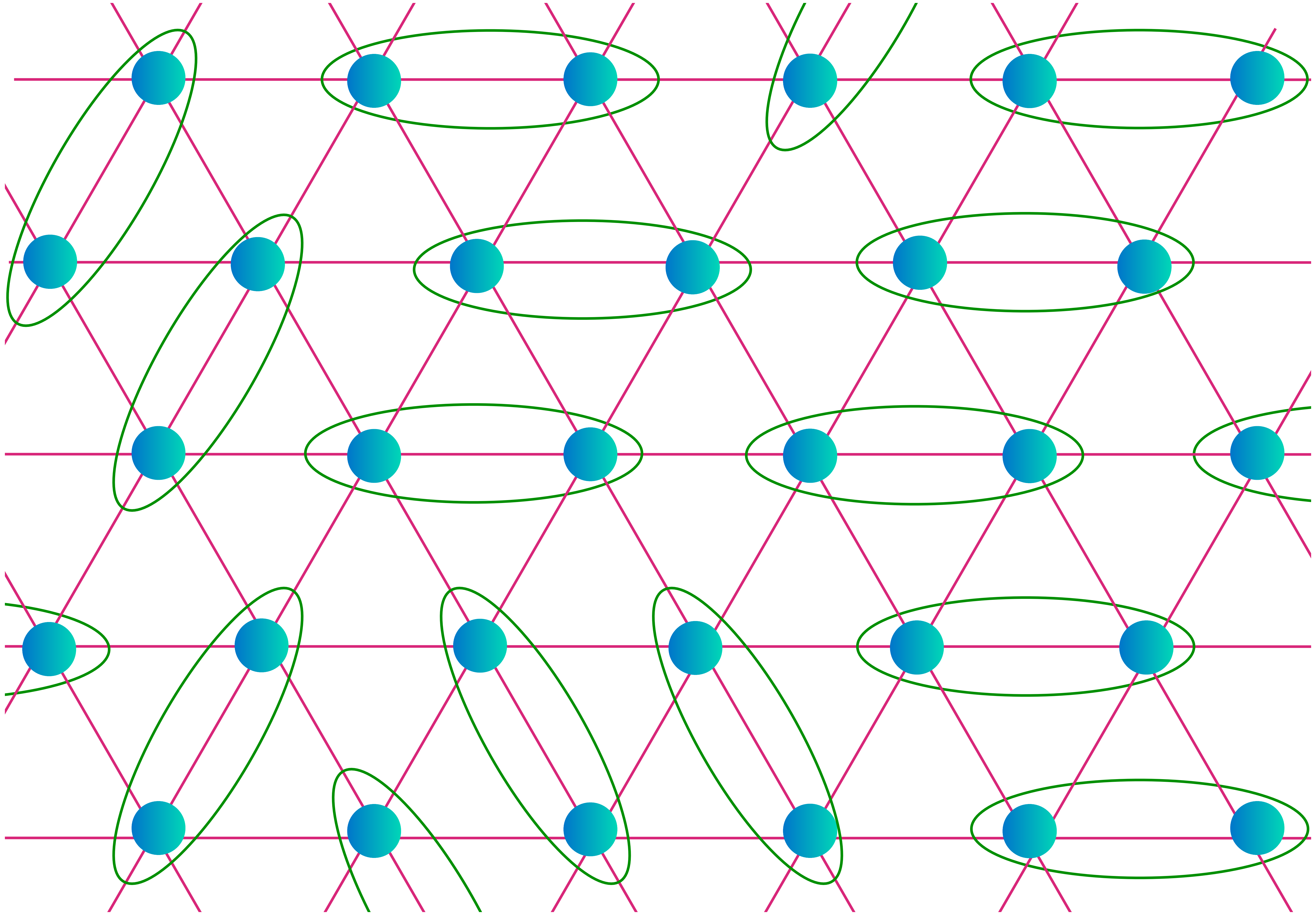

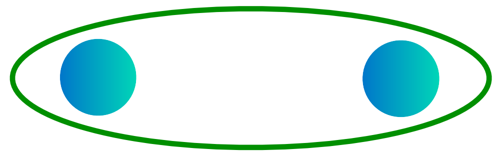

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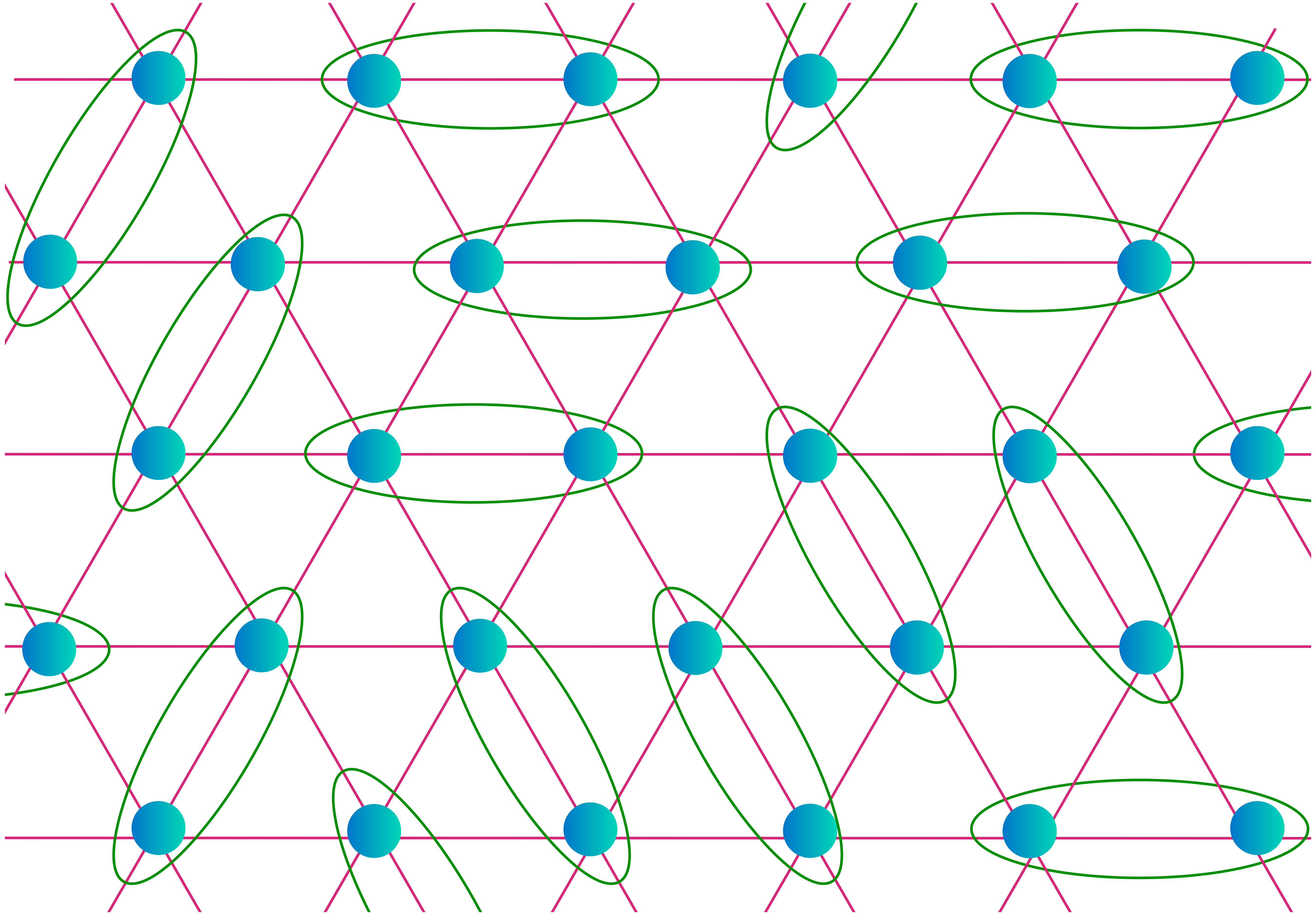

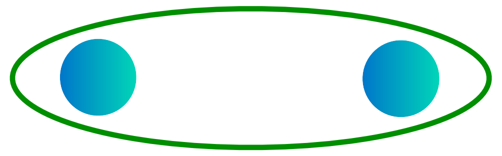

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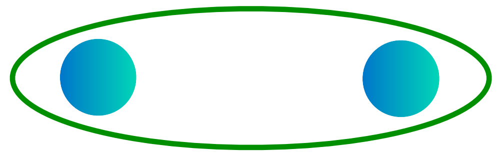

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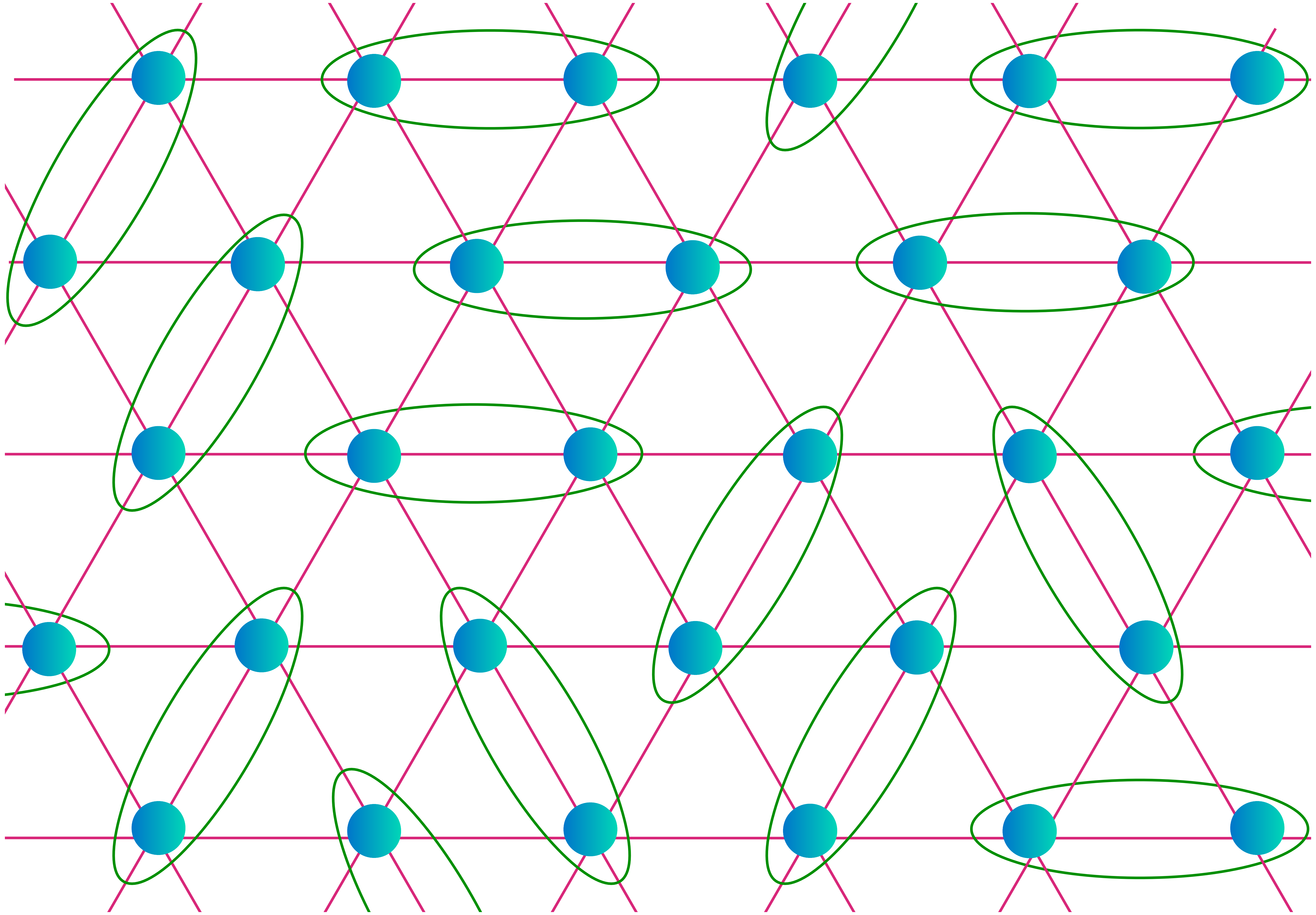

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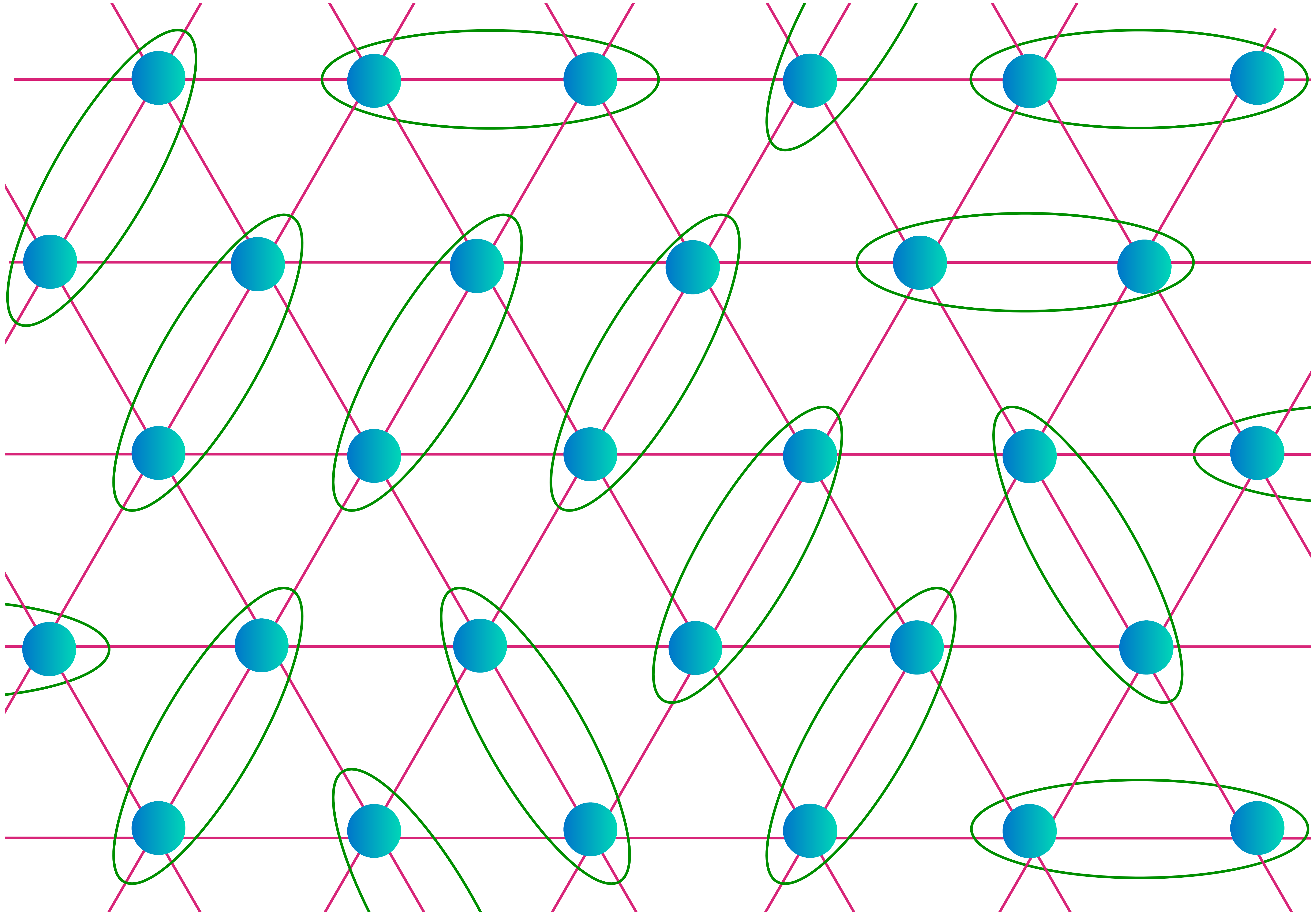

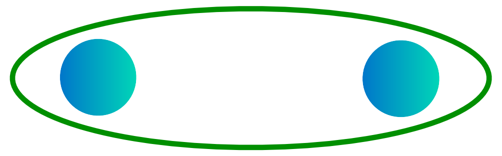

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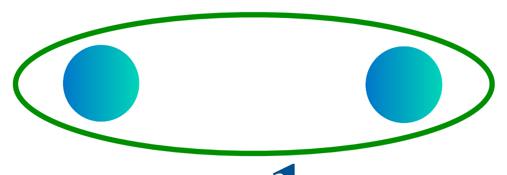

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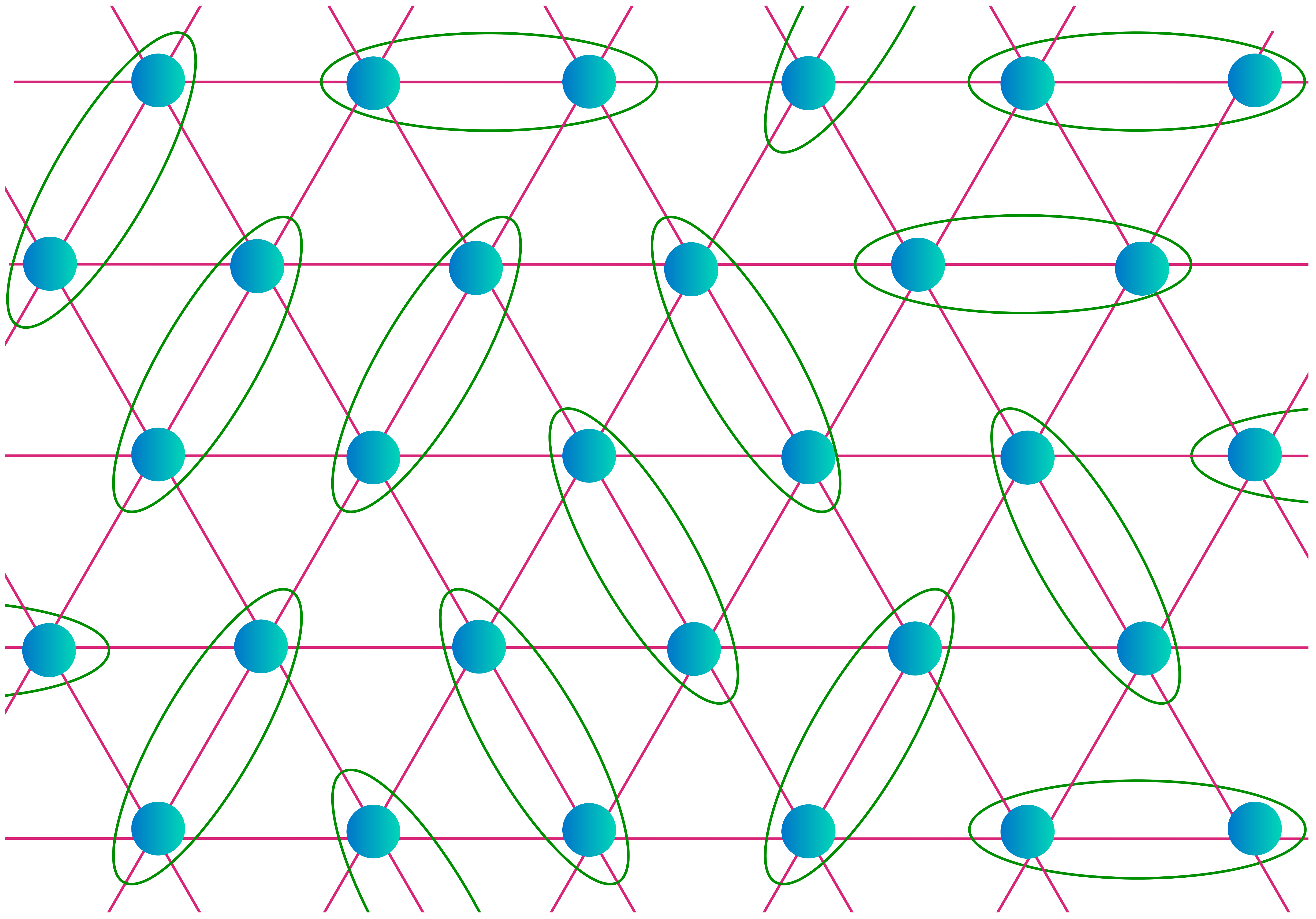
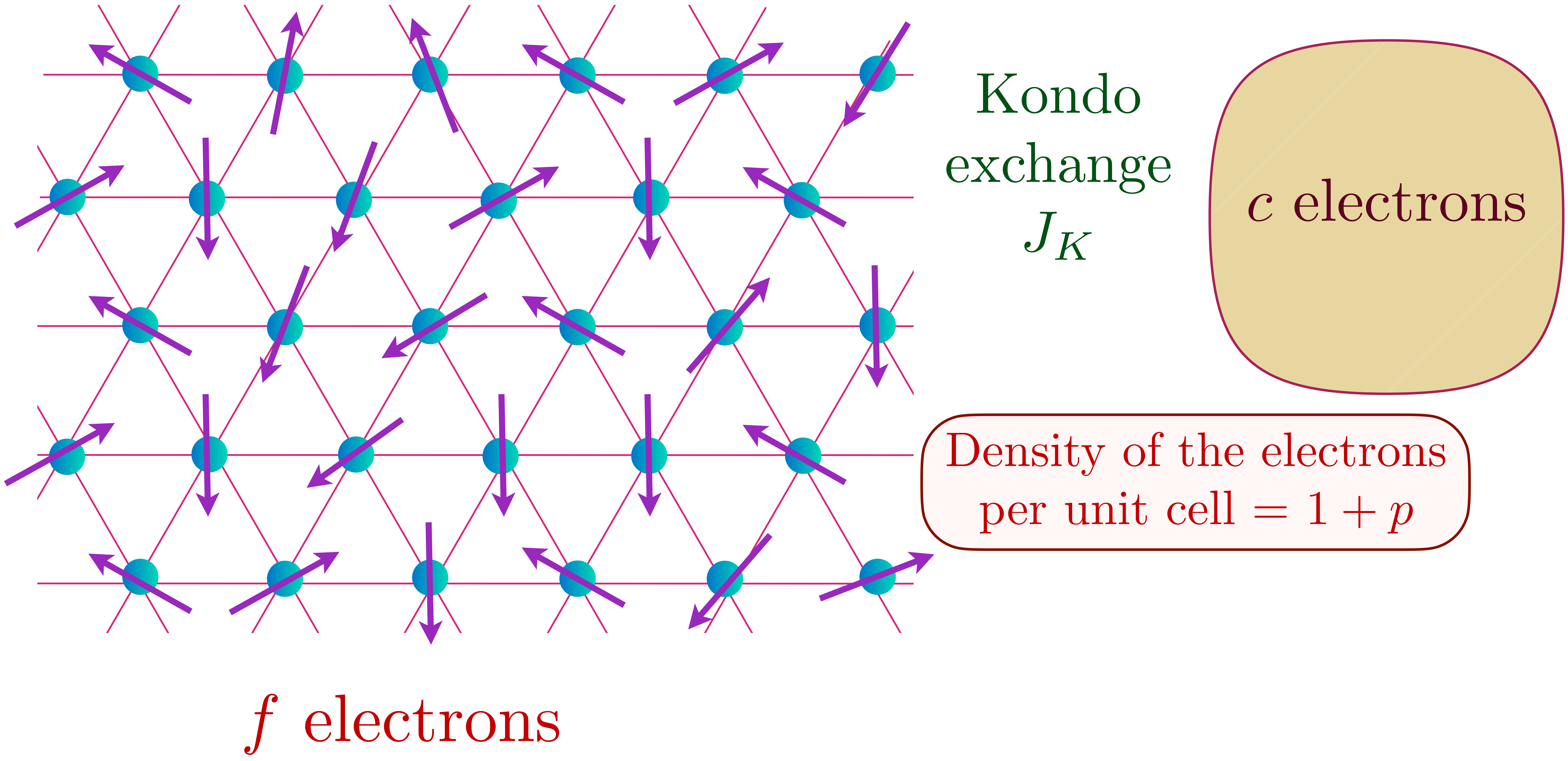


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

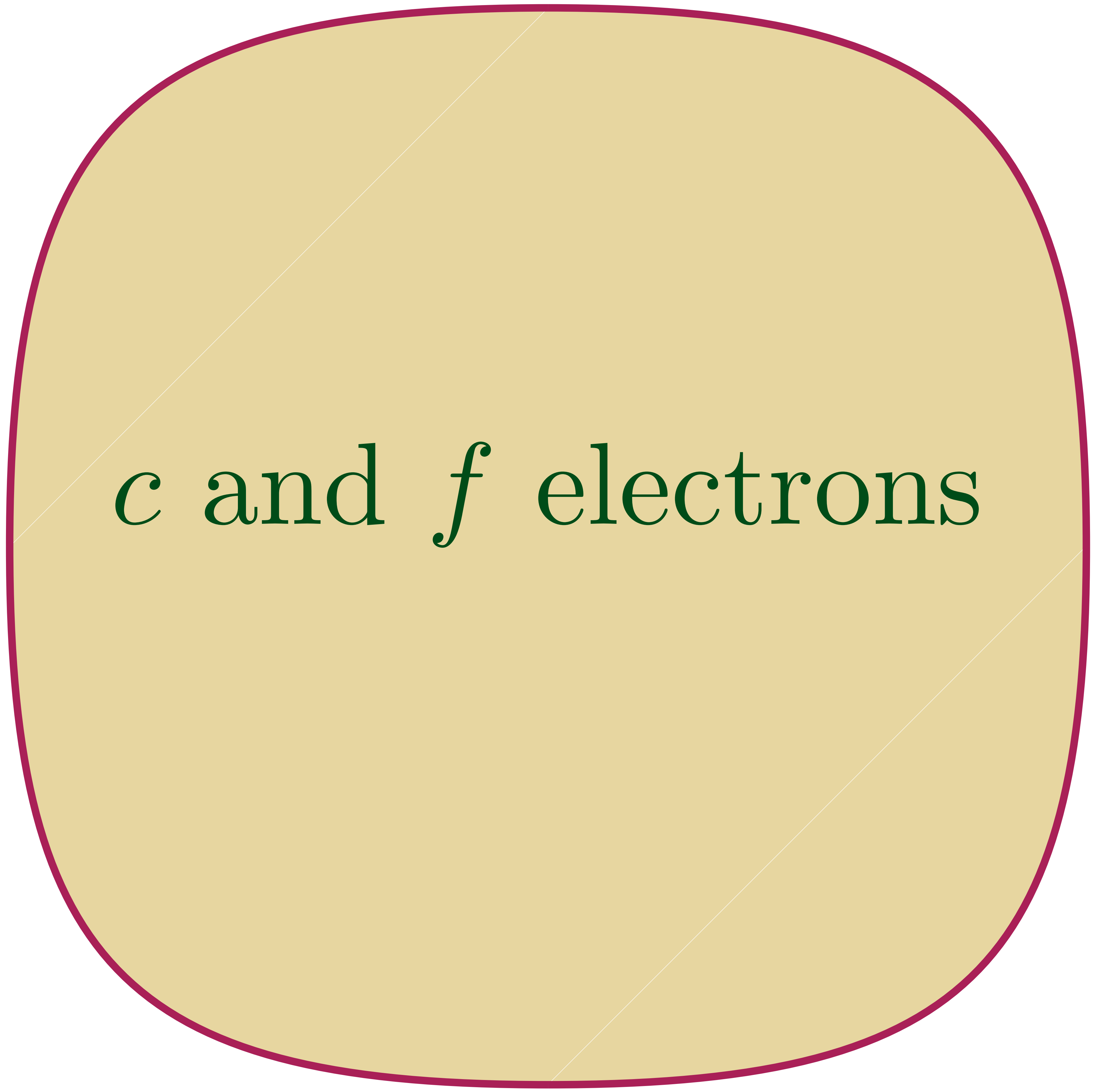
$\mathcal{D} \rightarrow$ dimer covering of lattice

Density of the electrons per unit cell = 1, but no Fermi surface $\Rightarrow$ Violation of Luttinger relation

Kondo lattice



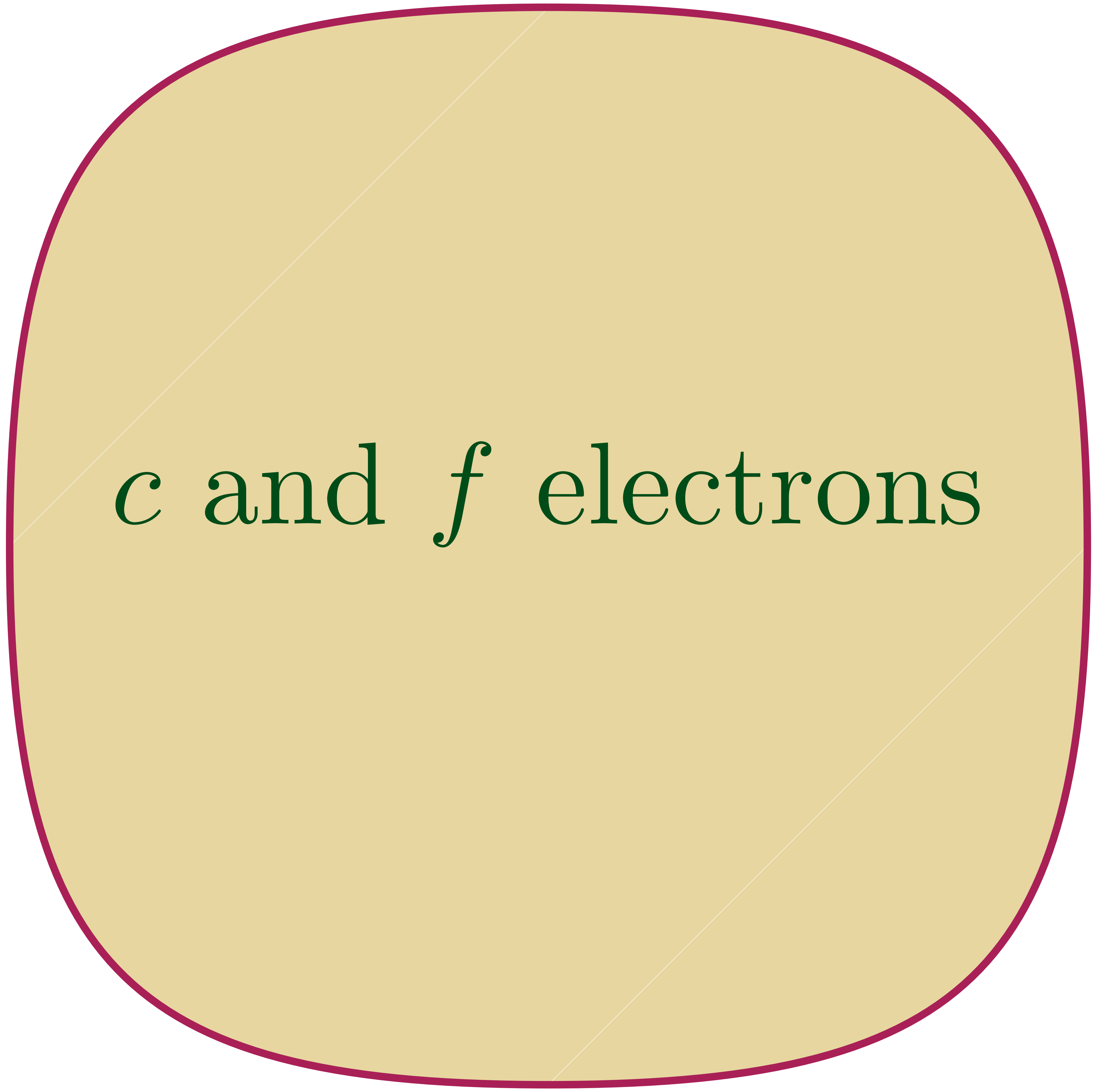
Kondo lattice: HFL phase



Kondo
exchange
J_K

Density of the electrons
per unit cell $= 1 + p$,
Fermi surface size $= 1 + p$.
Luttinger volume "large" Fermi surface.

Kondo lattice: HFL phase



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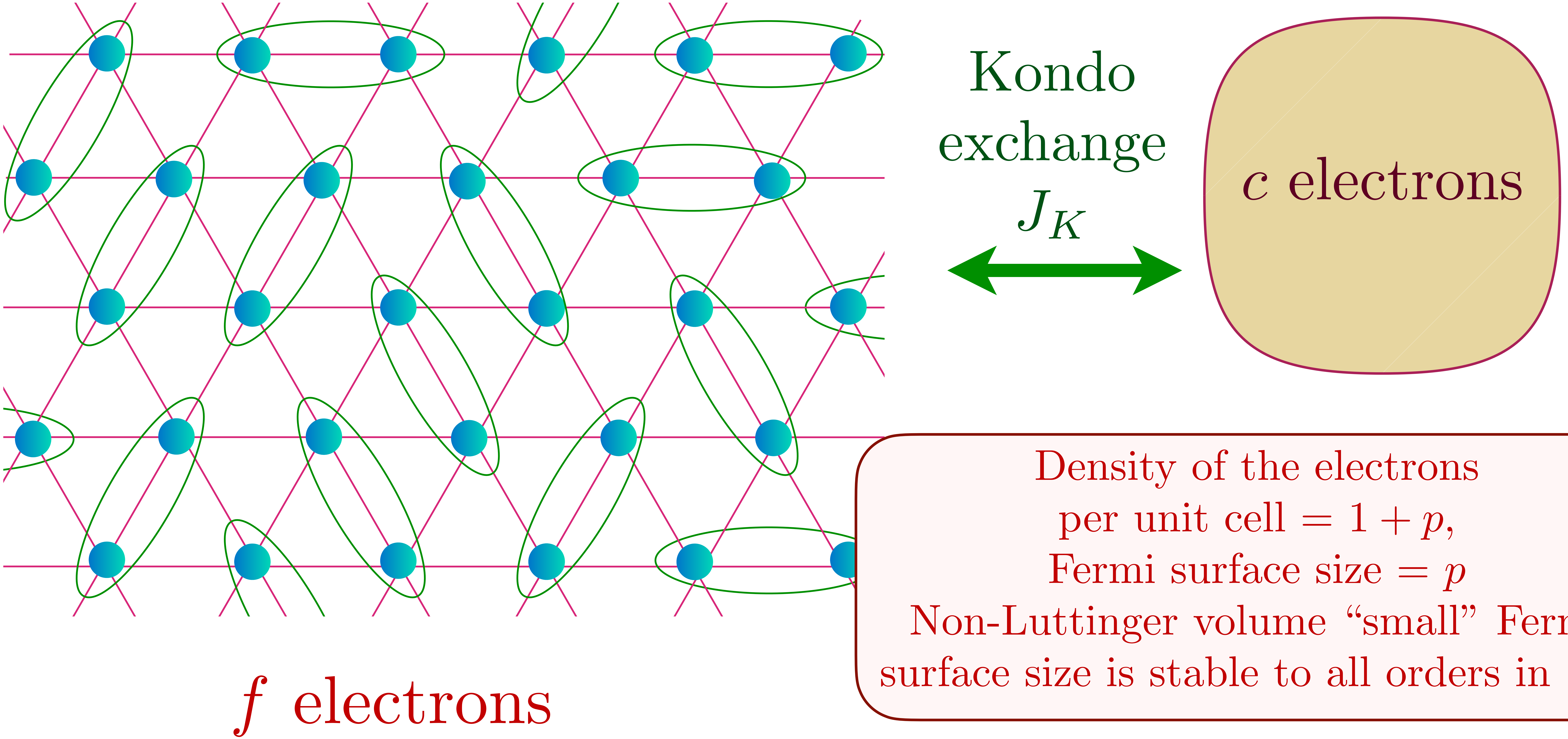
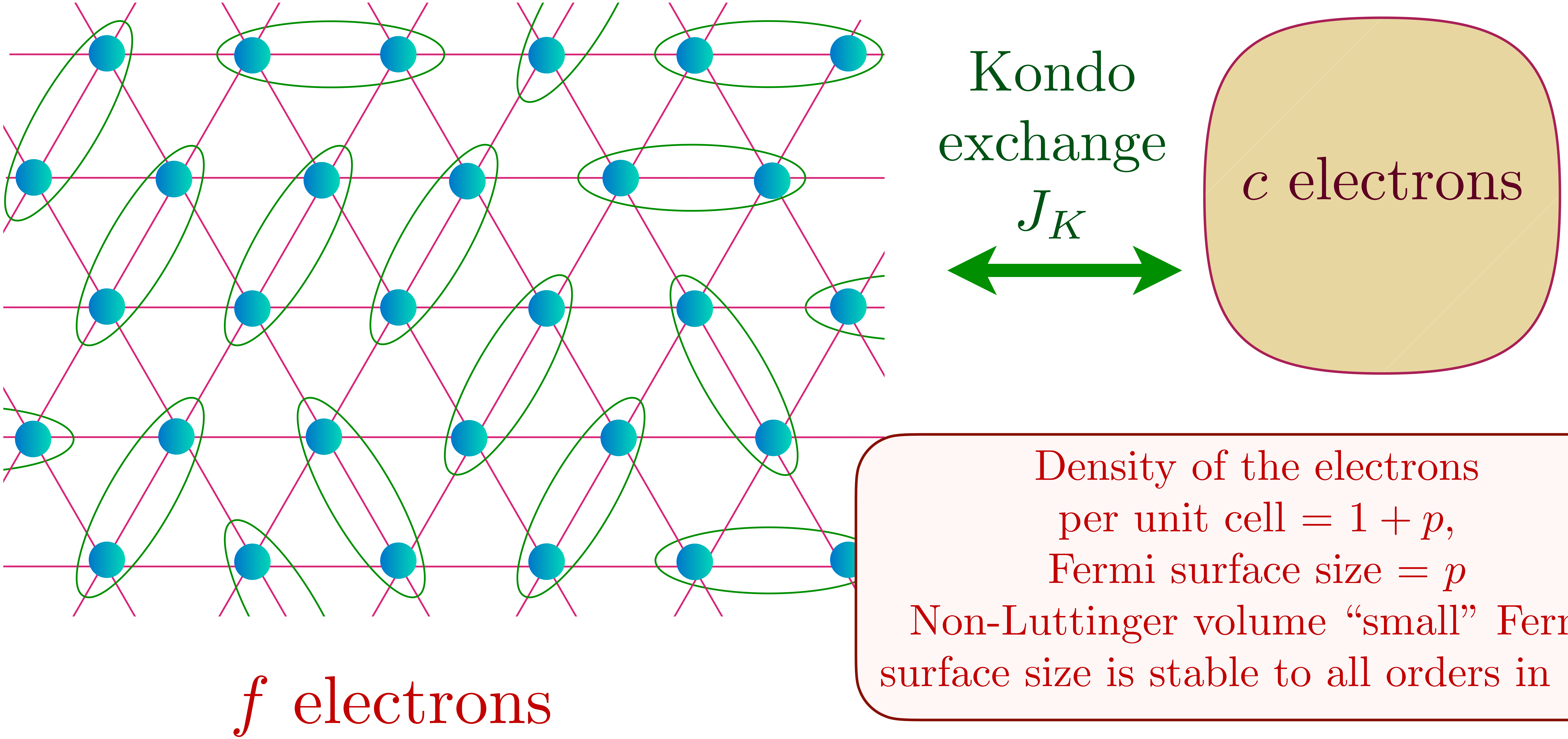
Kondo lattice: FL* phase



Density of the electrons
per unit cell $= 1 + p$,
Fermi surface size $= p$
Non-Luttinger volume "small" Fermi
surface size is stable to all orders in J_K.

T. Senthil, M. Vojta, and S. Sachdev, PRL **90**, 216403 (2003)

Kondo lattice: FL* phase



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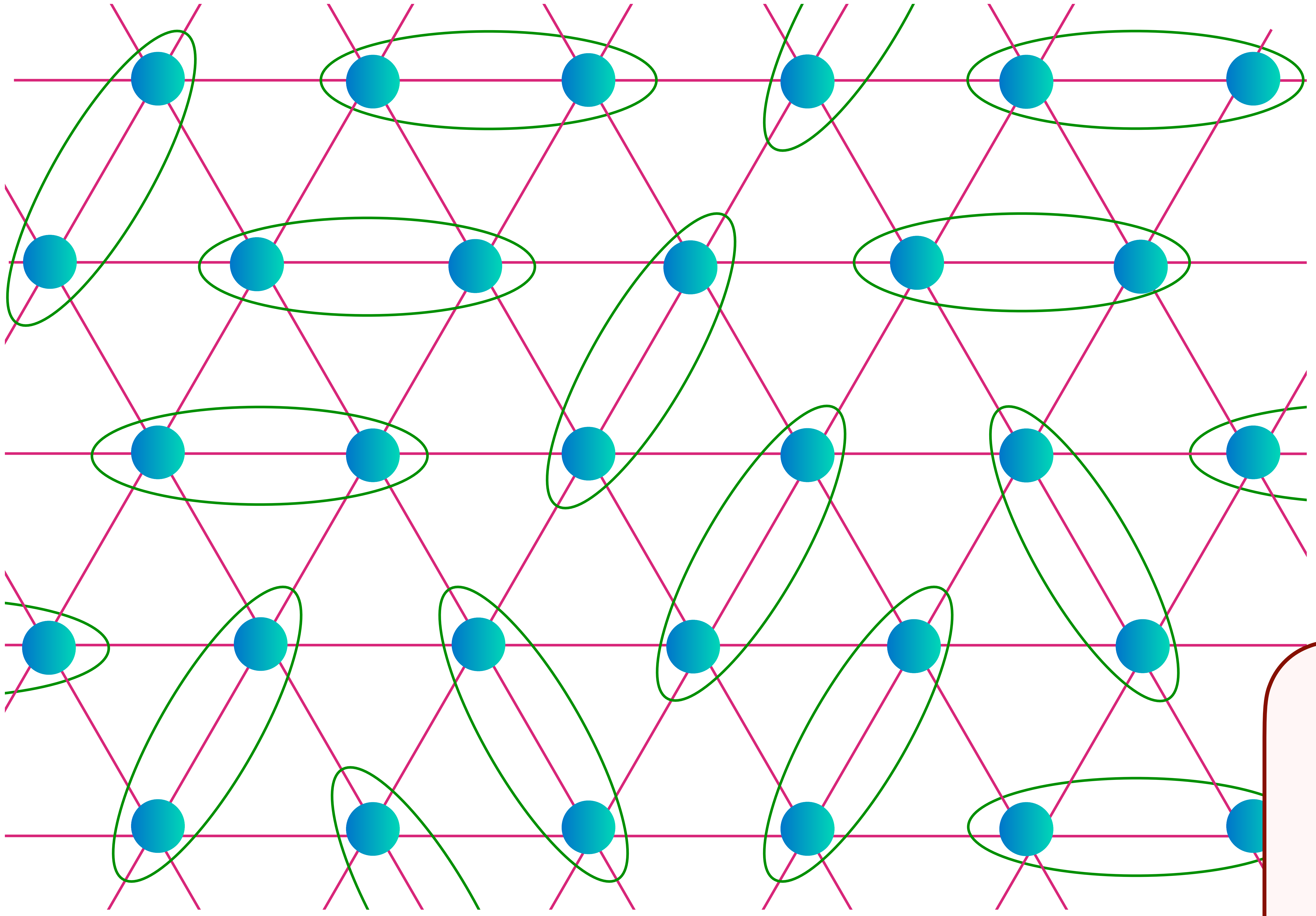
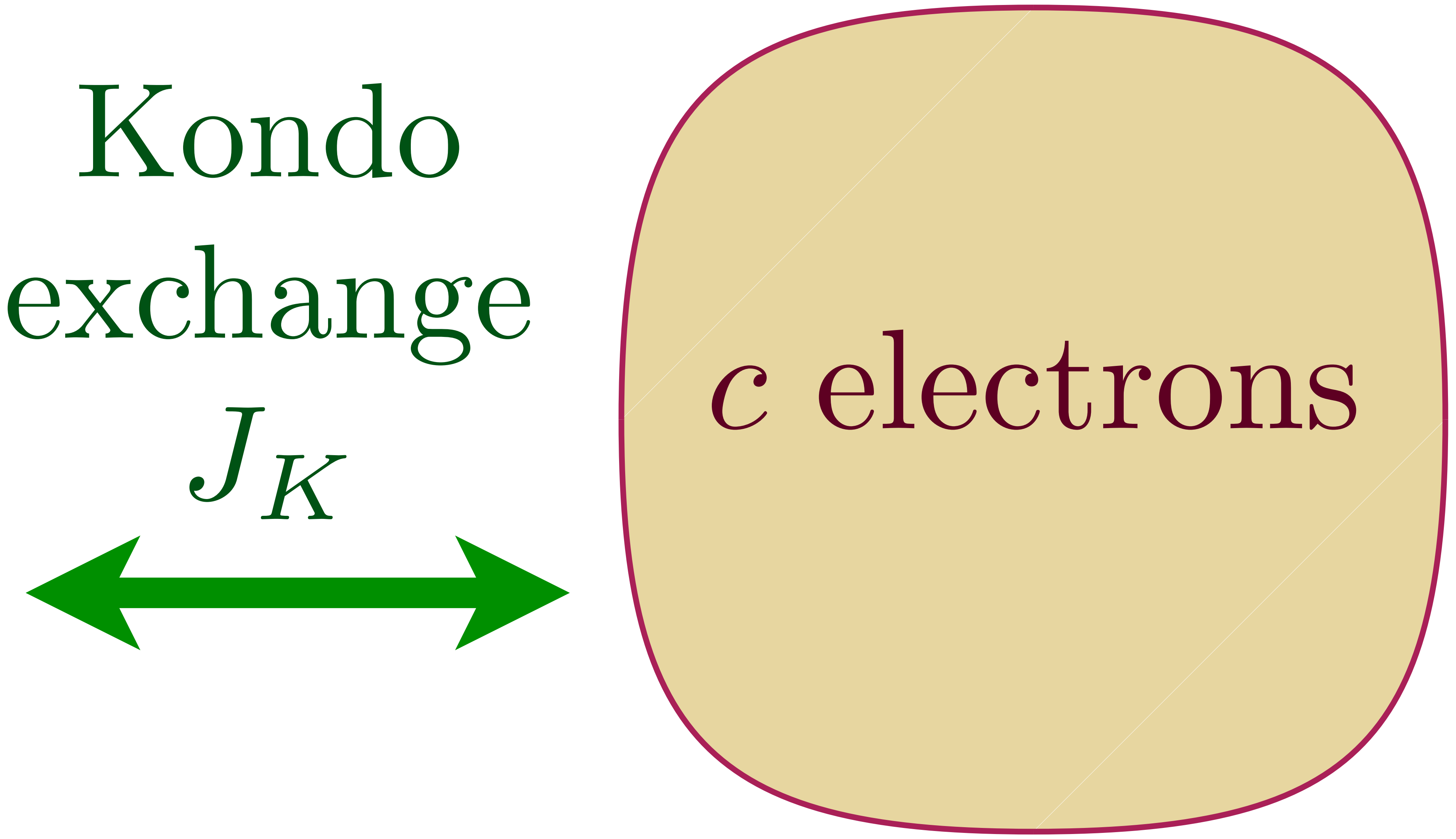
Kondo lattice: FL* phase



f electrons



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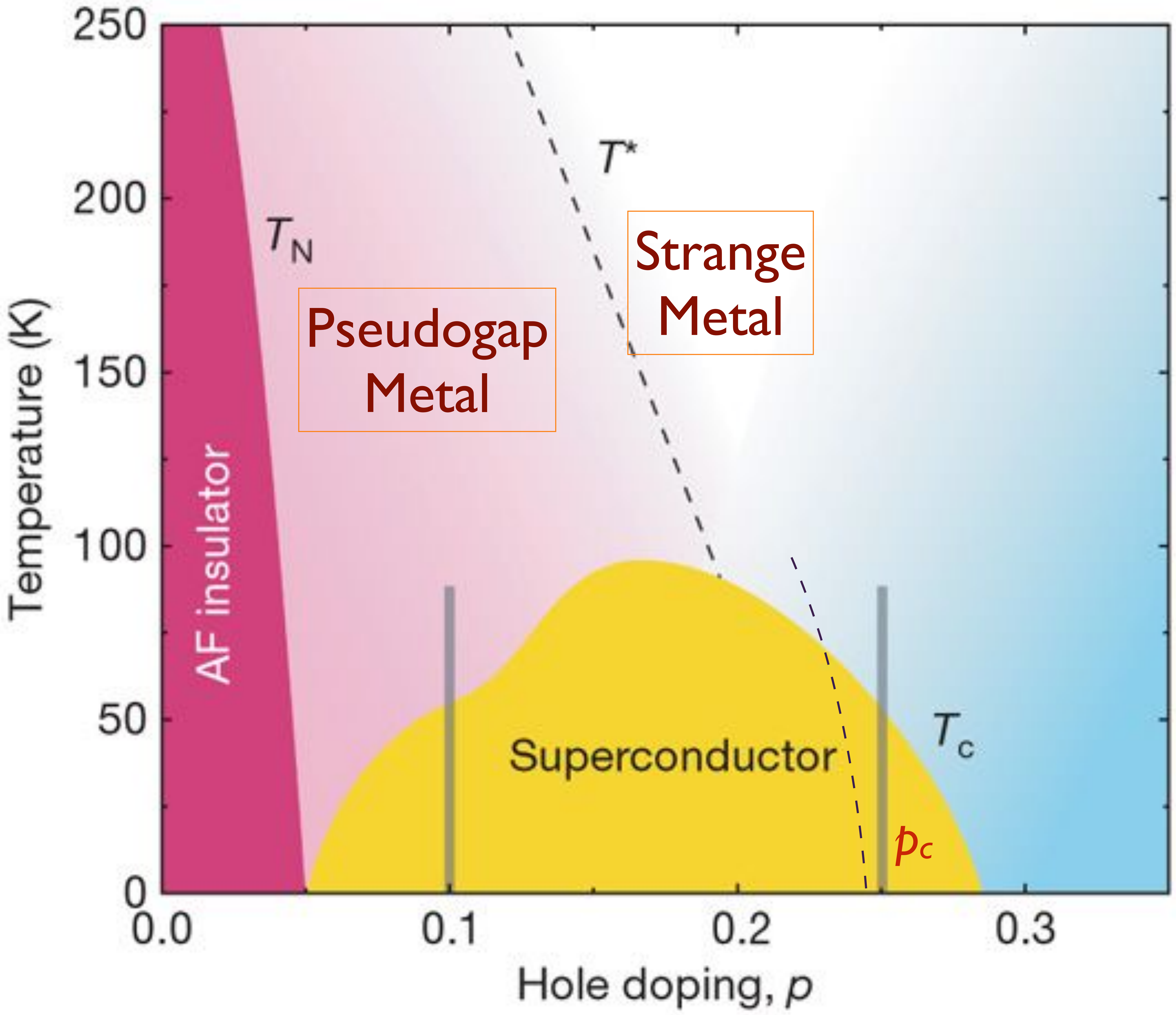
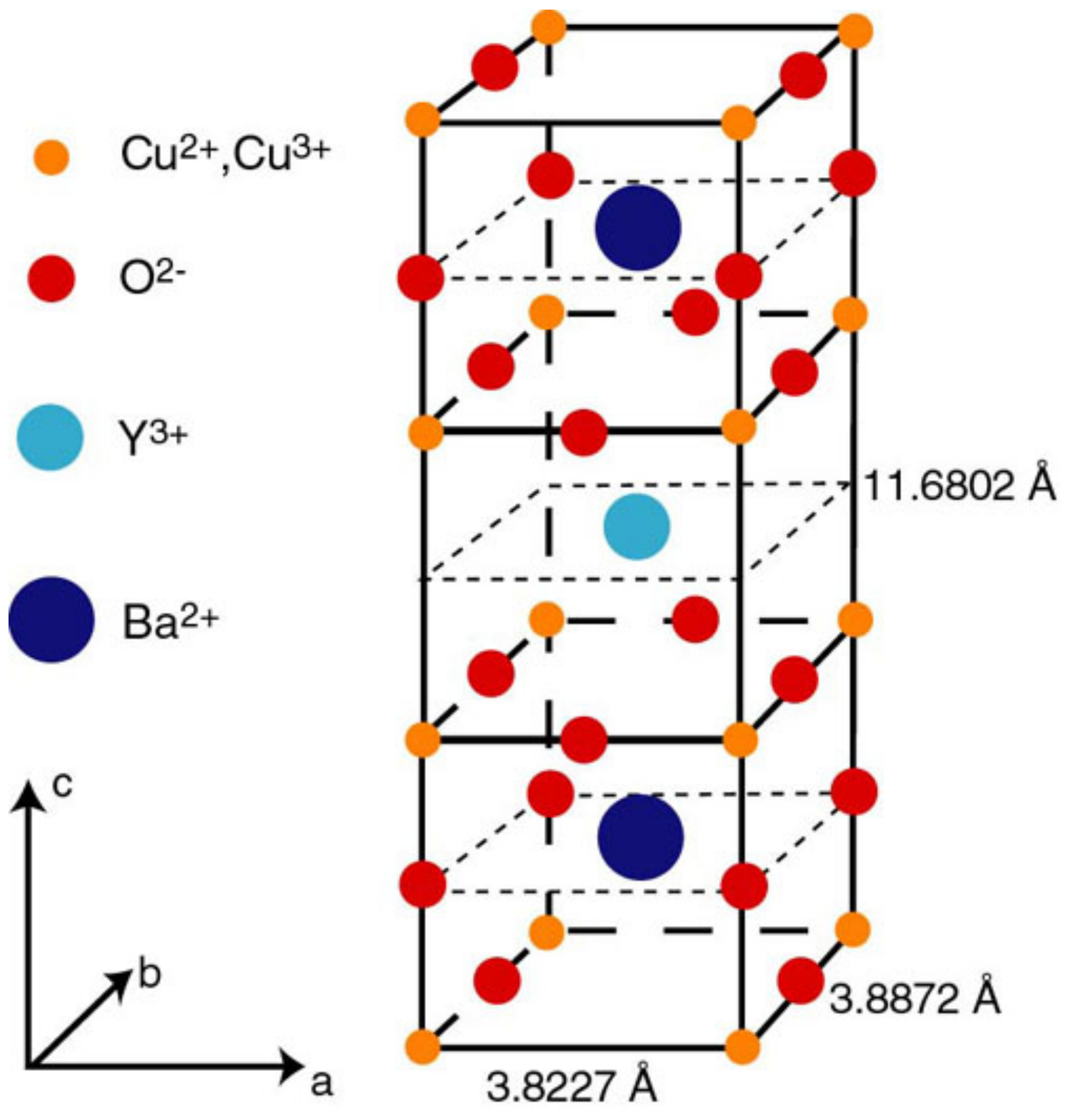
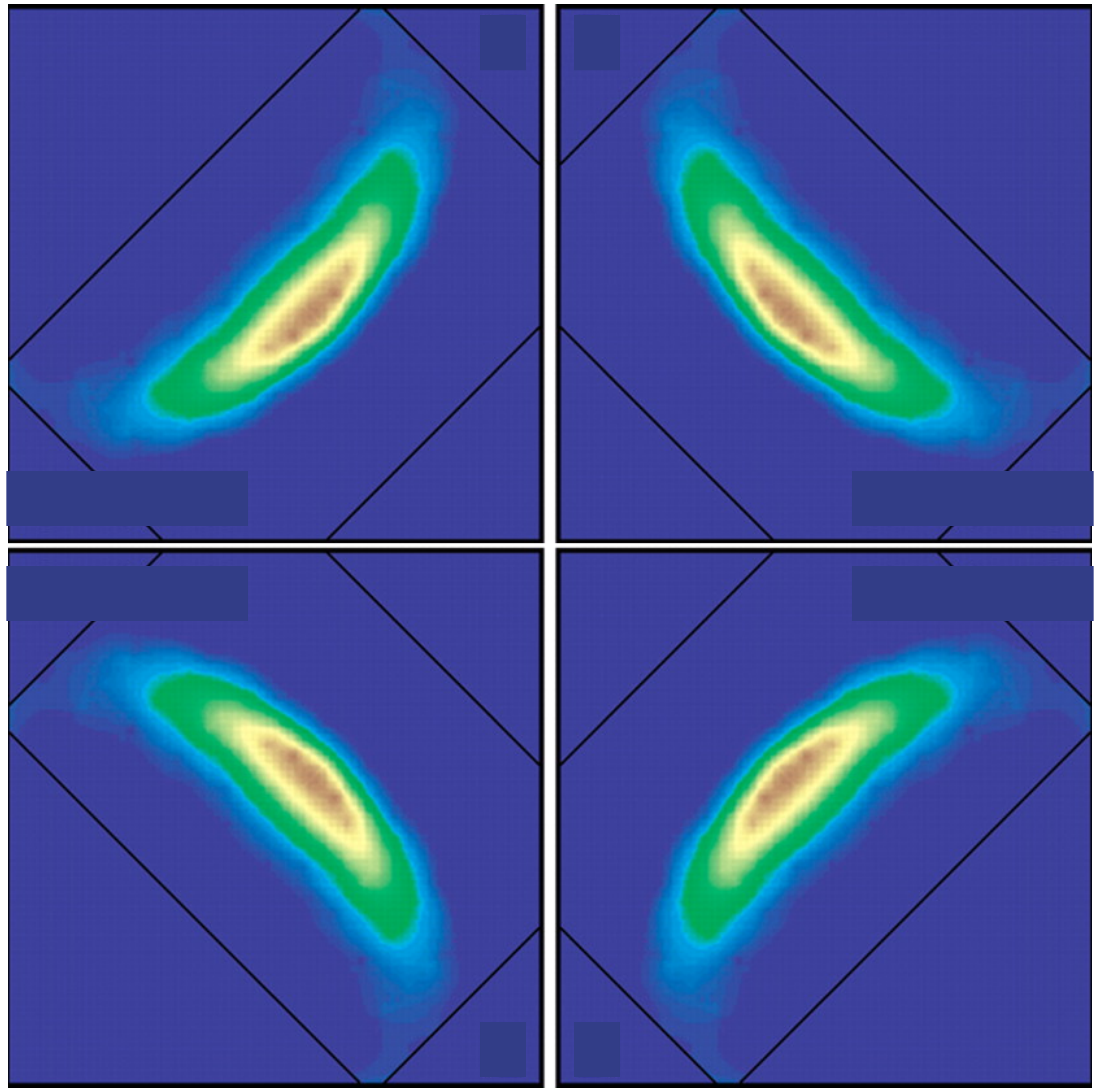


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Strange Metal
Superconductor
T_c
p_c



Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

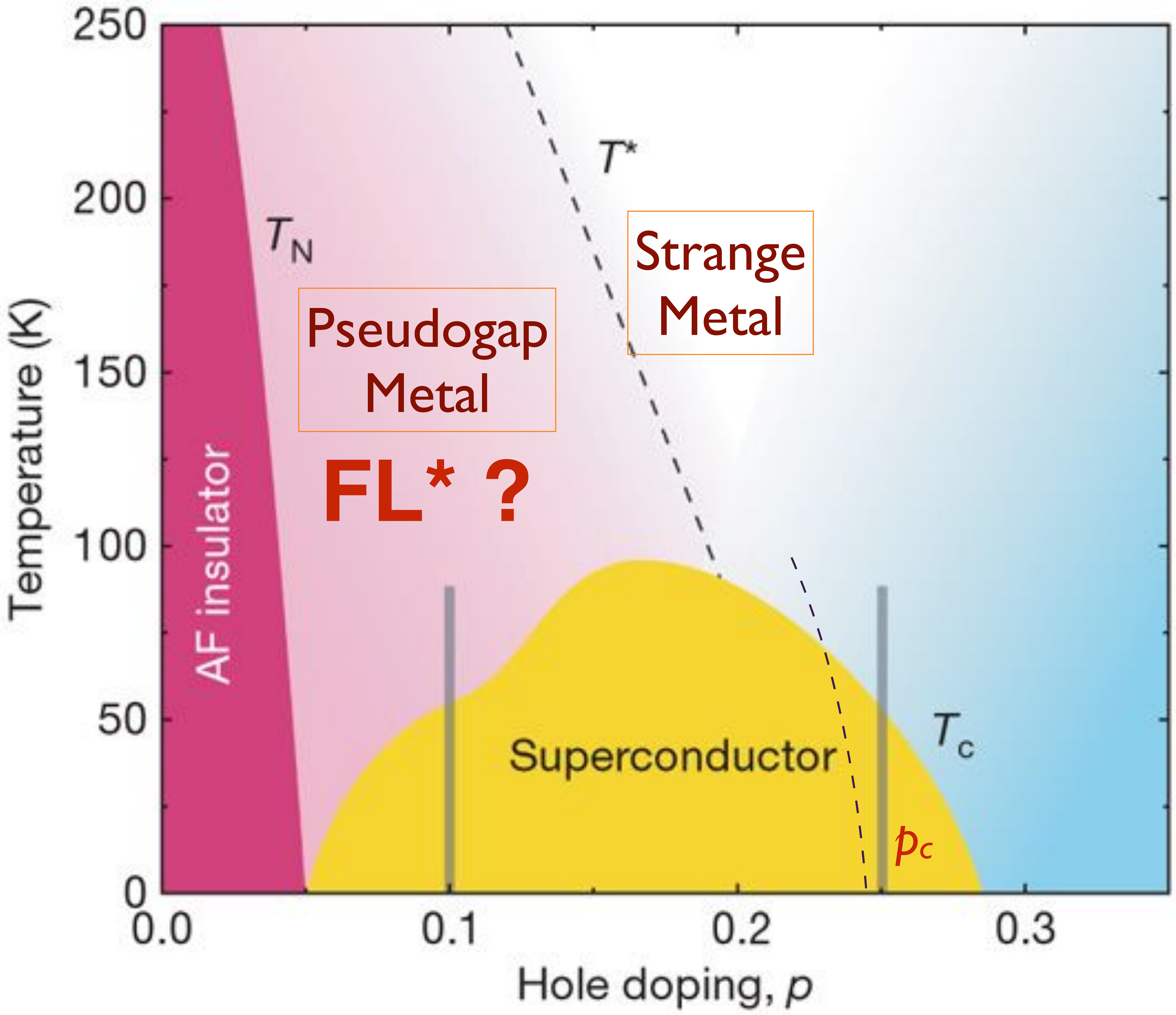
Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)



Can a FL* state
in a
single-band Hubbard model
describe the
pseudogap metal
over an intermediate
temperature range,
along with a
crossover/transition to
confinement at lower
temperatures?

The pseudogap metal = FL*

Main lesson from the Kondo lattice for Hubbard model

For $t \gg J$,

do not fractionalize the mobile electron, $c_{i\sigma} \neq f_{i\sigma} b_i^\dagger$ or $f_i b_{i\sigma}^\dagger$.

Fractionalize the paramagnon instead!

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

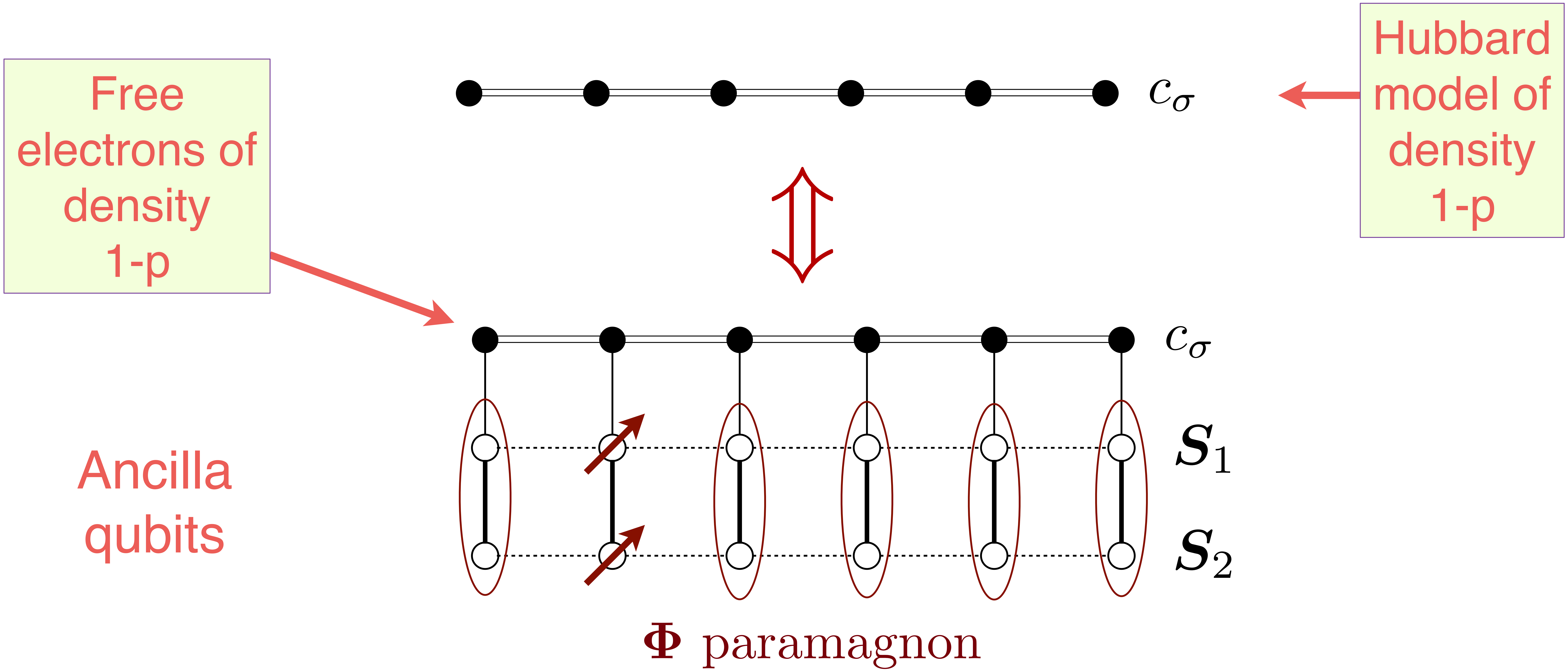
$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

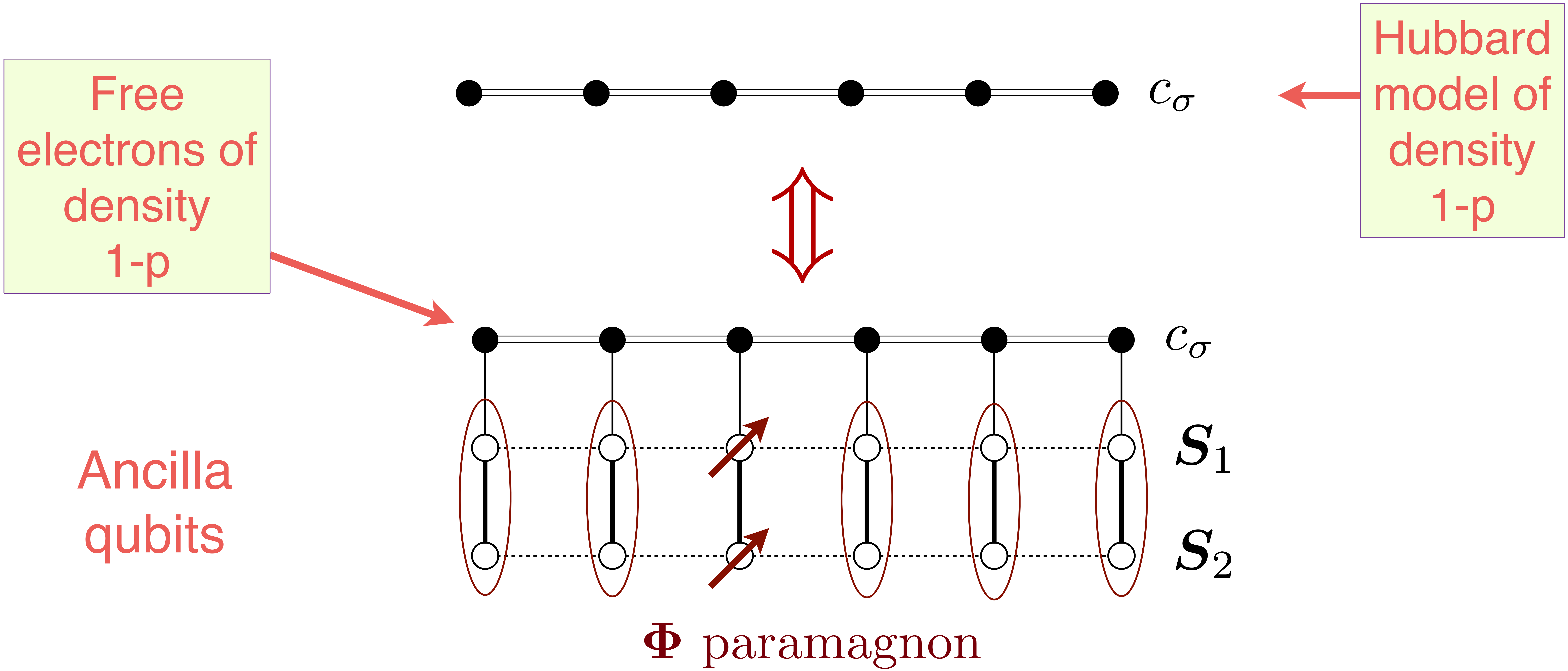
This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model



$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

Paramagnon theory of the Hubbard model



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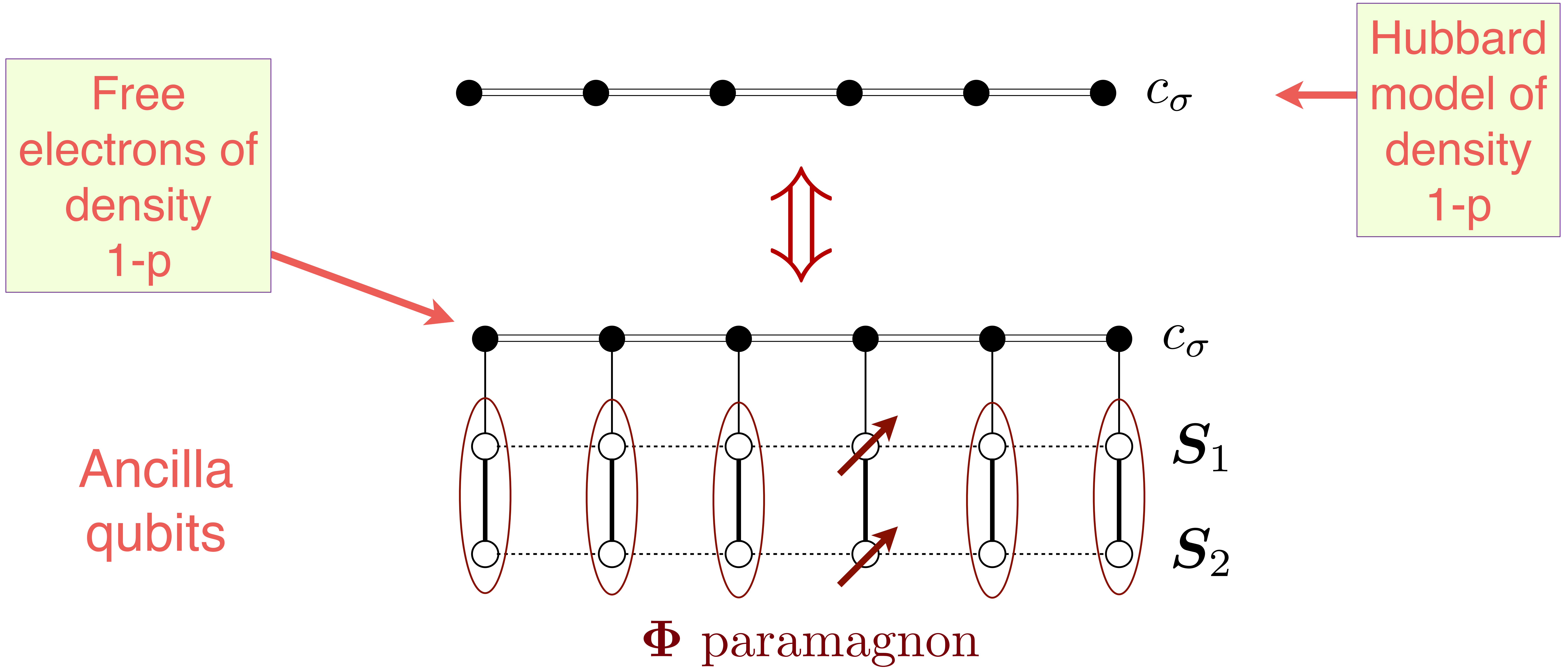

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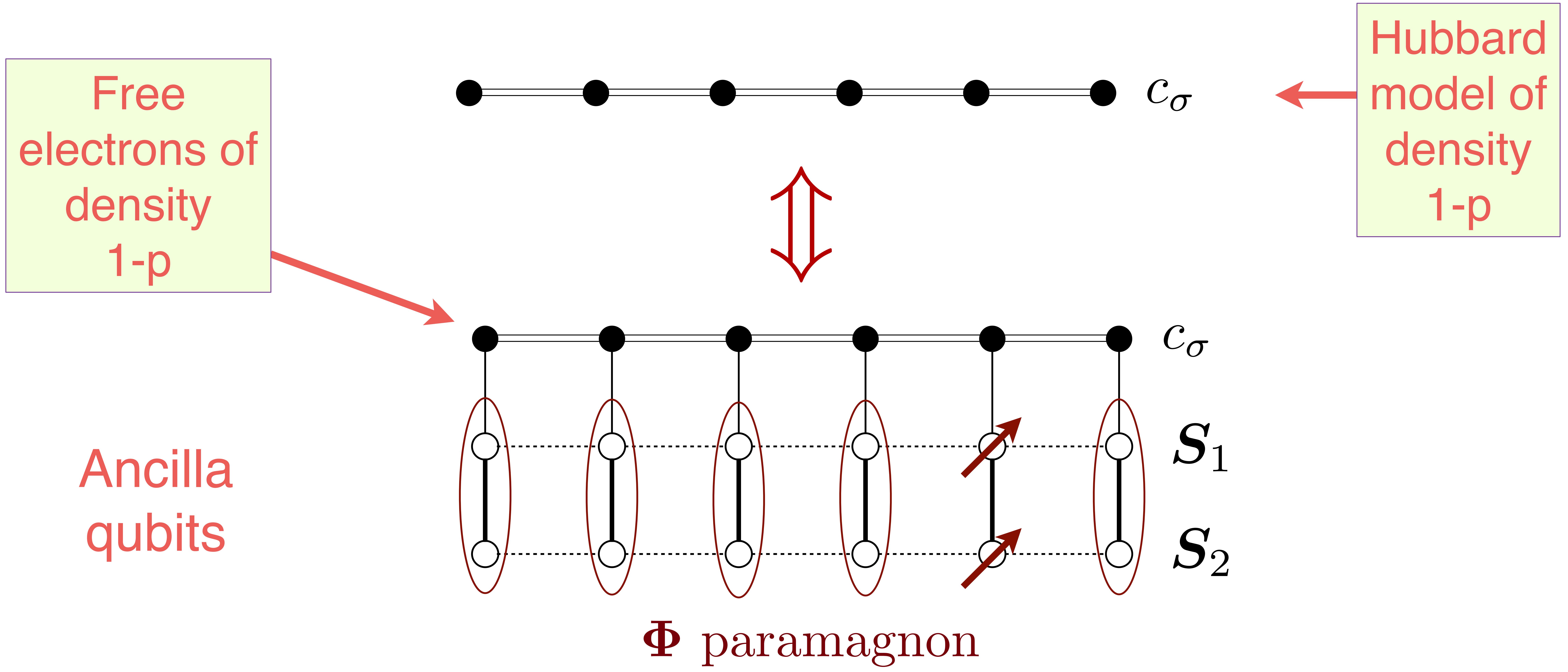

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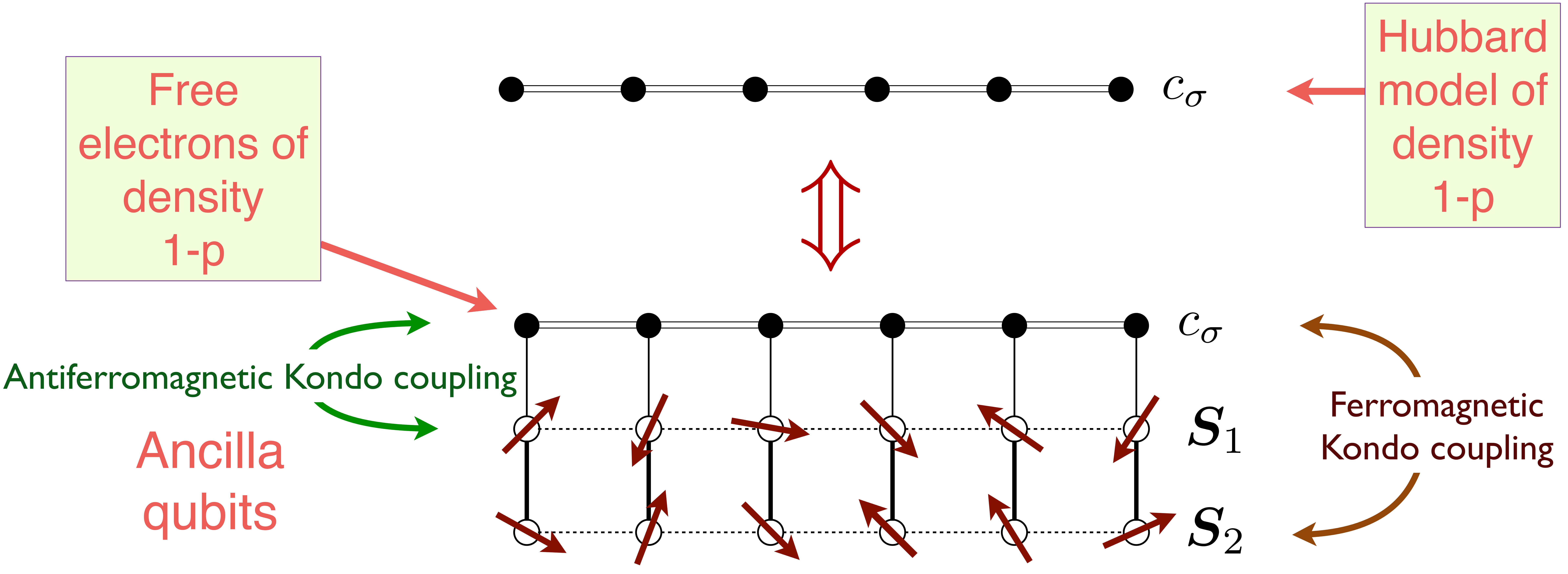
Paramagnon fractionalization theory of the Hubbard model



$$\boldsymbol{\Phi}_i = \frac{1}{\sqrt{3}} \left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i} \right)$$

$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} - \lambda \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

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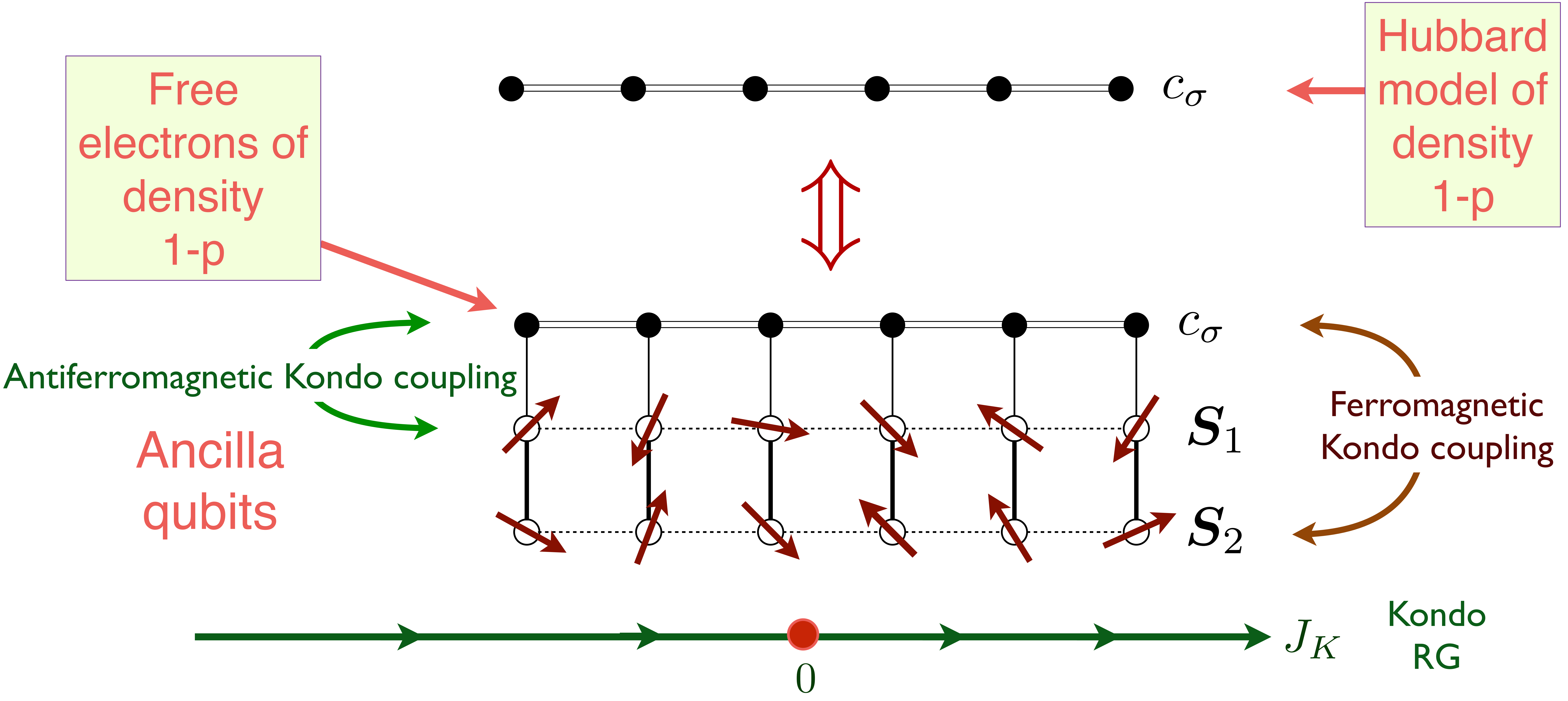
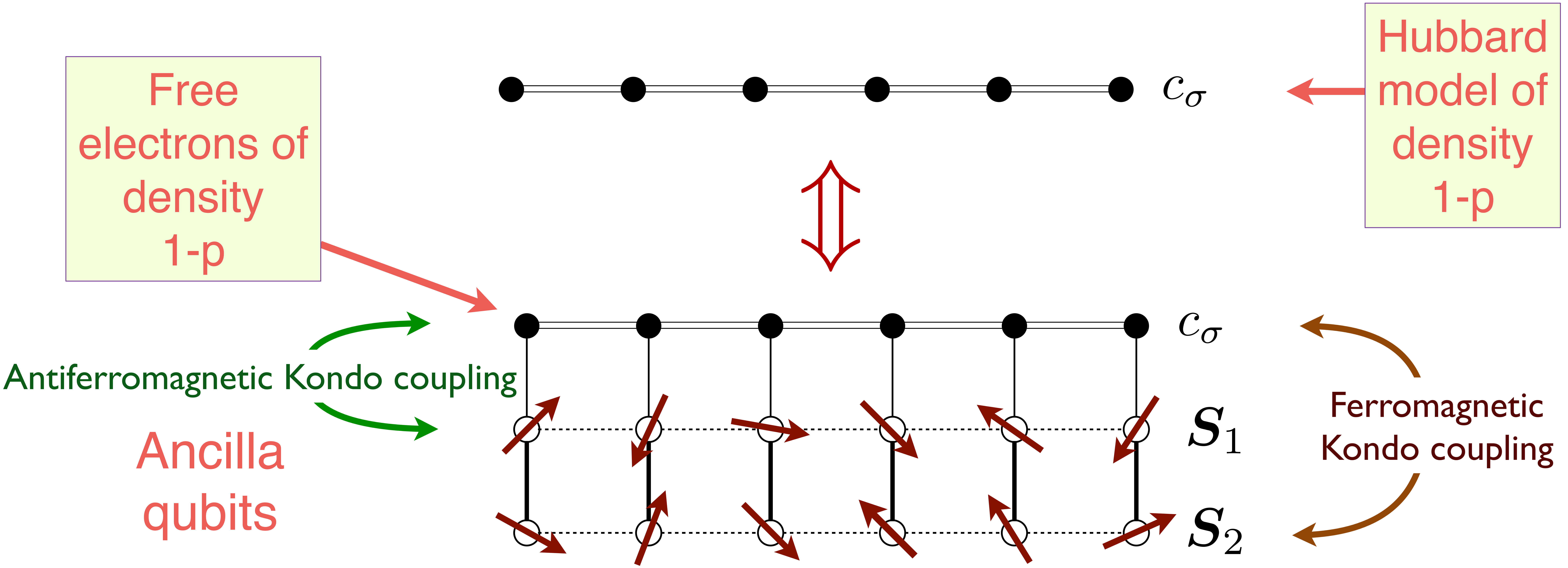


$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + J_K \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + - \widetilde{J}_K \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{2i} + \ldots$$

Paramagnon fractionalization theory of the Hubbard model



A FL* state is realized when the antiferromagnetic Kondo coupling dominates, and the c_σ and $\boldsymbol{S}_1$ form a "large" Fermi surface of hole density

$$(1+p)+1 = 2+p = p \text{ mod } 2!$$

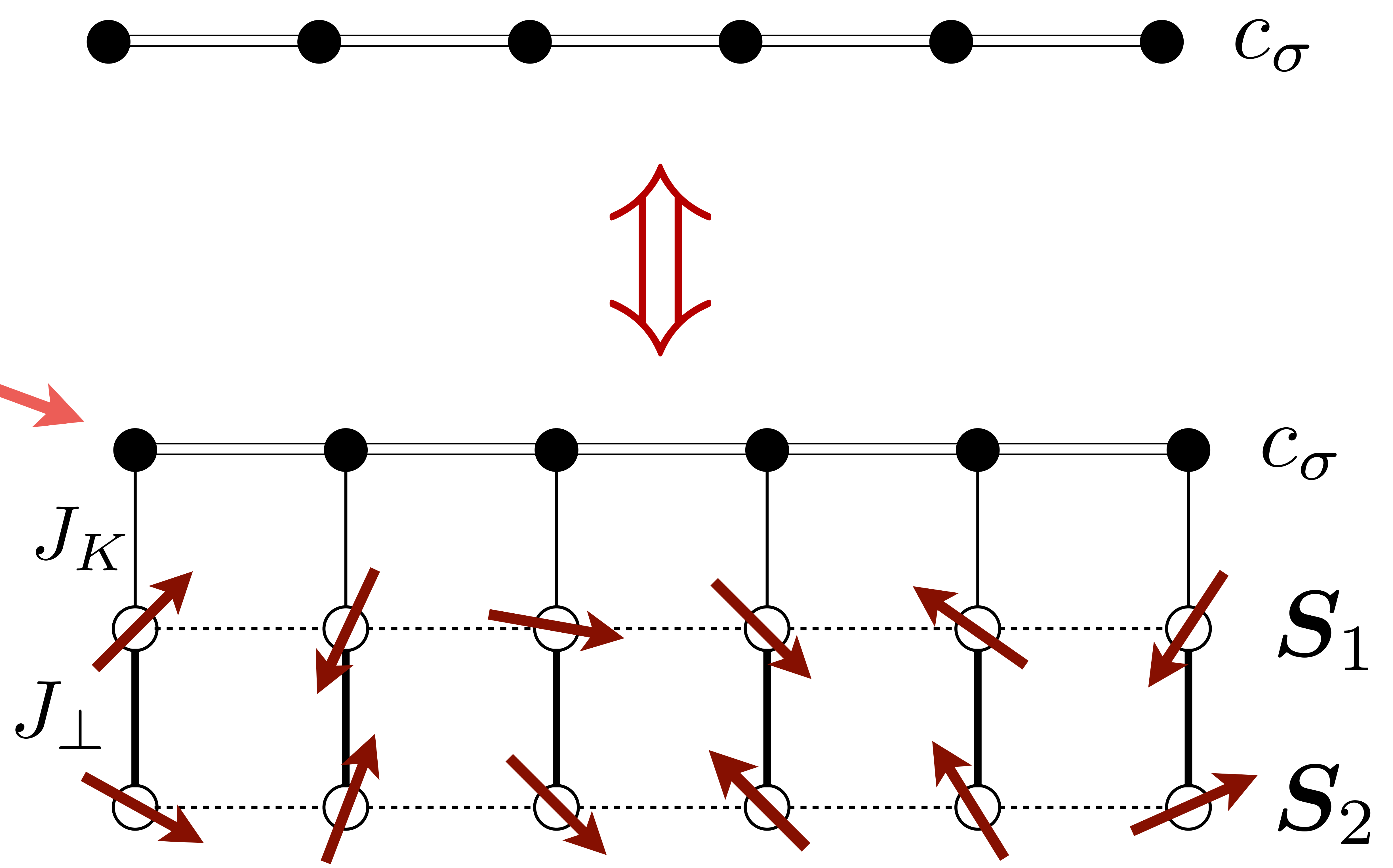
The $\boldsymbol{S}_2$ must form a decoupled spin liquid which does not break translational symmetry, to obtain a metal with a non-Luttinger volume Fermi surface.

Paramagnon fractionalization theory of the Hubbard model

Hubbard model of density 1-p

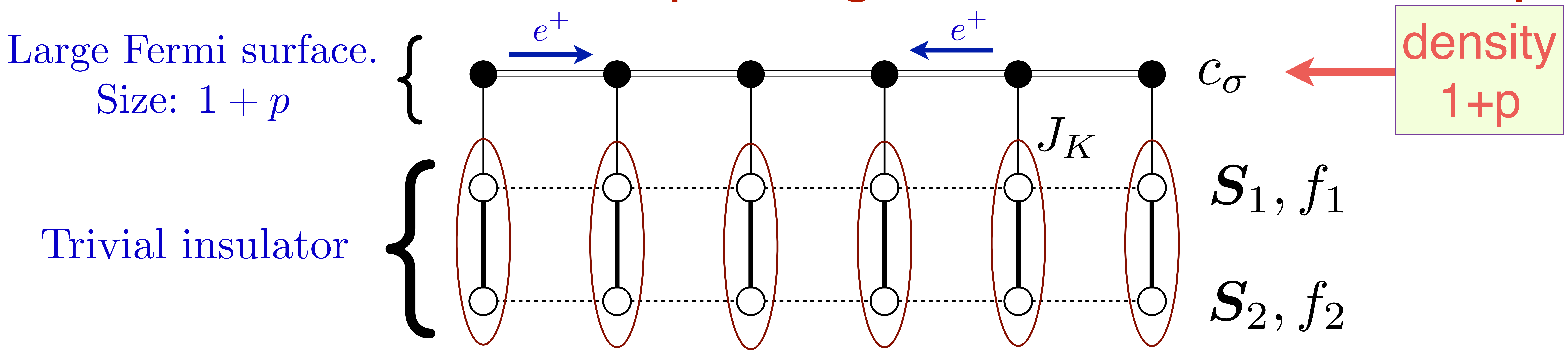
Free electrons of density 1-p

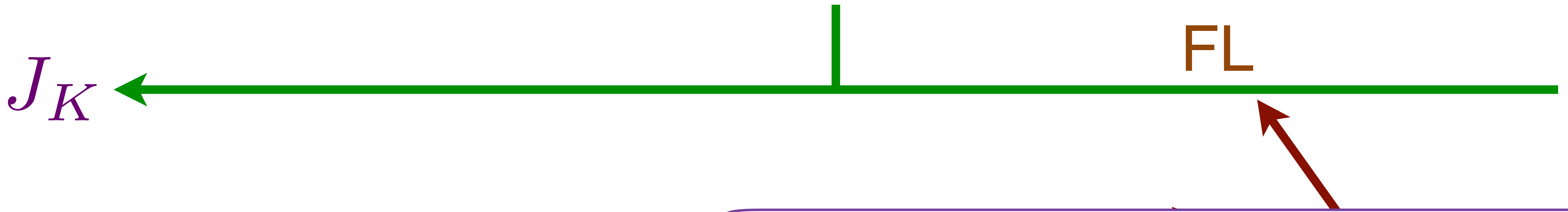
Ancilla qubits



Related by a Schrieffer-Wolff canonical transformation with $U = \dfrac{3J_K^2}{8J_\perp} + \dfrac{3J_K^3}{16J_\perp} + \ldots$

Trial wavefunctions in the paramagnon fractionalization theory

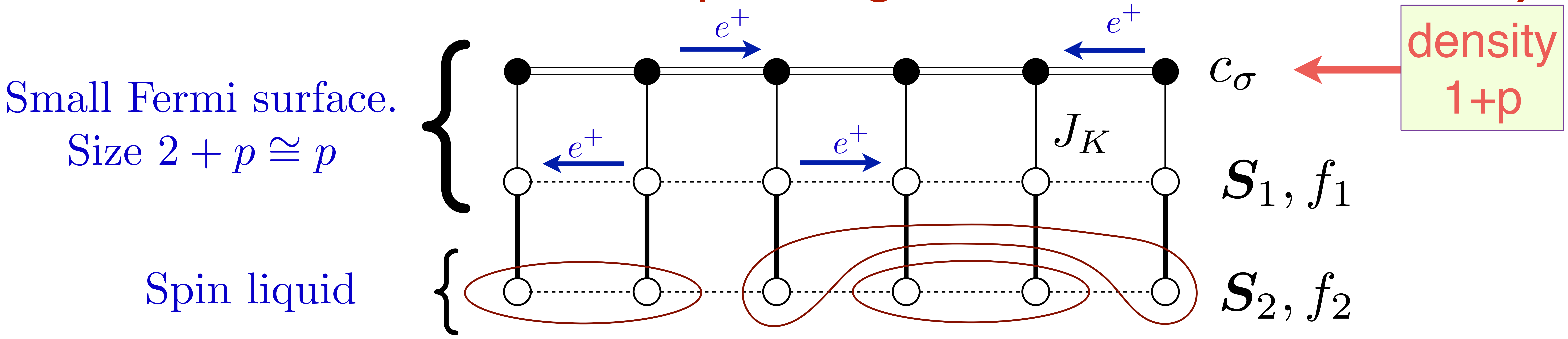


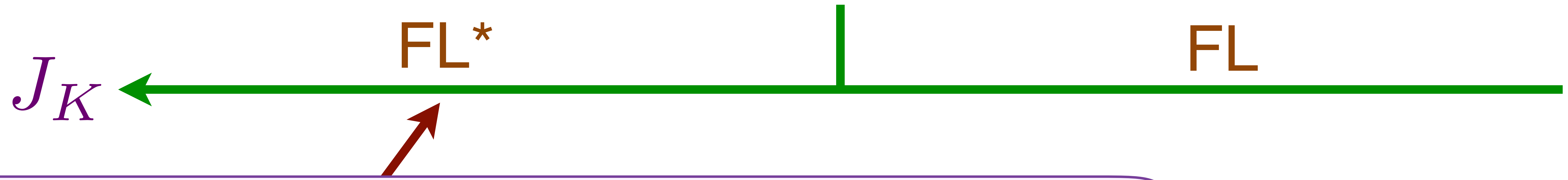


Large Fermi surface of size $1+p$

$$|\mathrm{FL}\rangle = |\text{Rung singlets of } f_1, f_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory

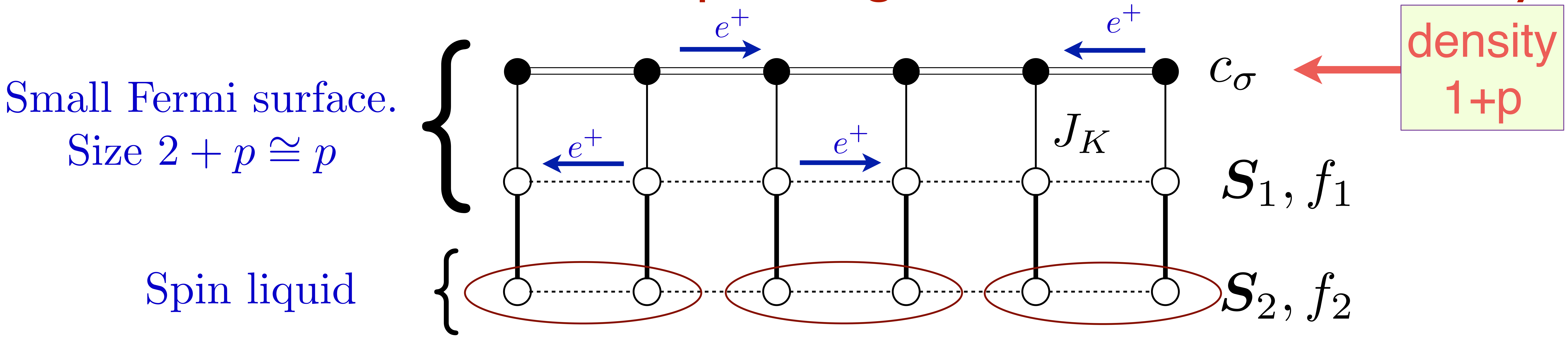


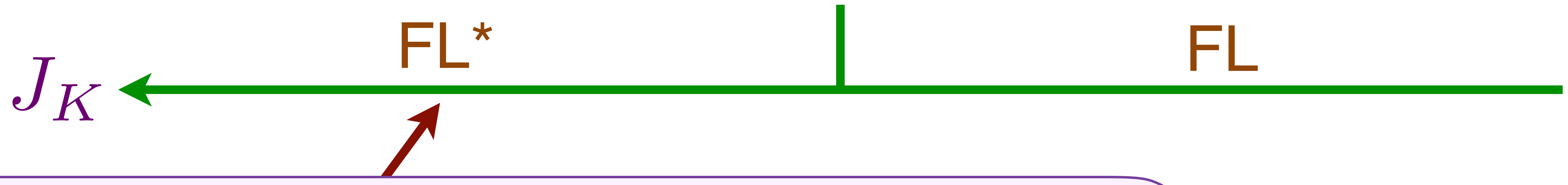


Small Fermi surface of size p

$$|\text{FL}*\rangle = [\text{Projection onto rung singlets of } f_1, f_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f_2\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory





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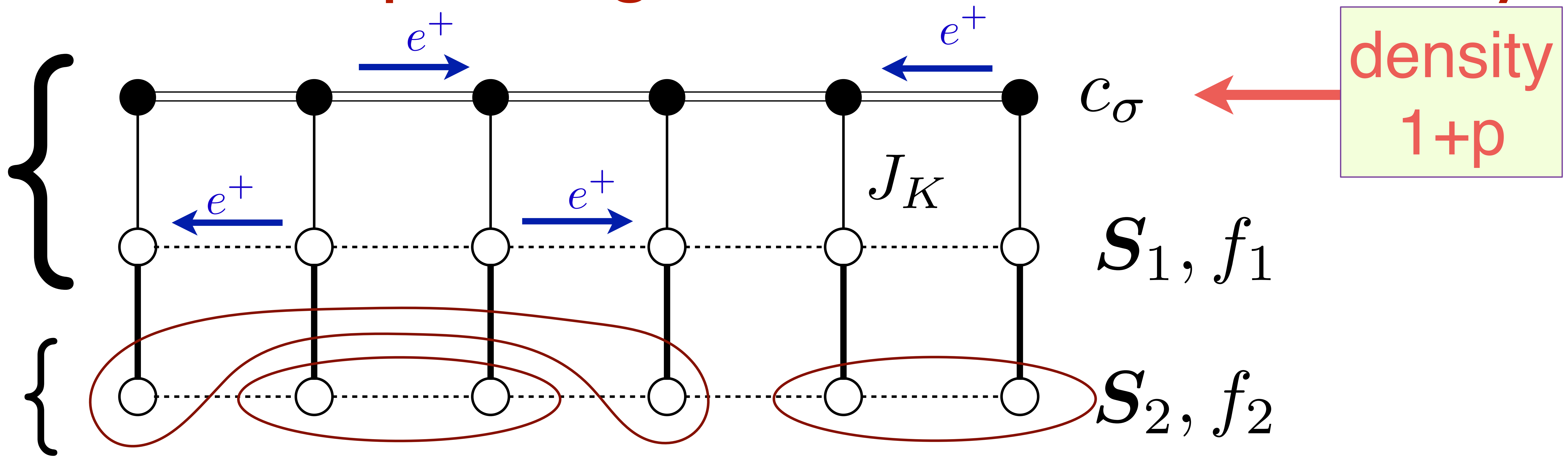
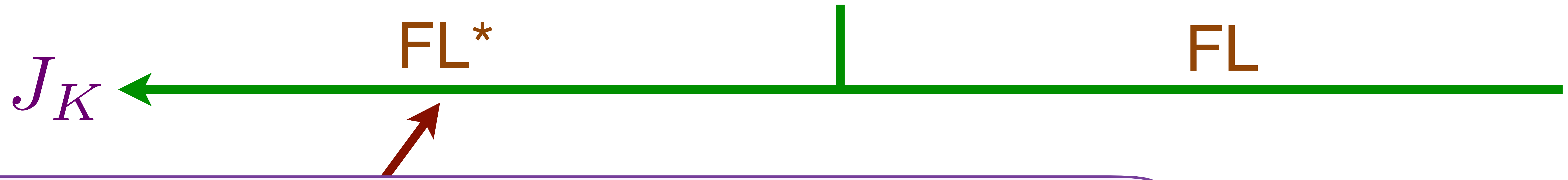
Trial wavefunctions in the paramagnon fractionalization theory

Small Fermi surface.
Size $2+p \cong p$

Spin liquid





Small Fermi surface of size p

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$$\bowtie |\text{Slater determinant of } (c, f_1)\rangle$$
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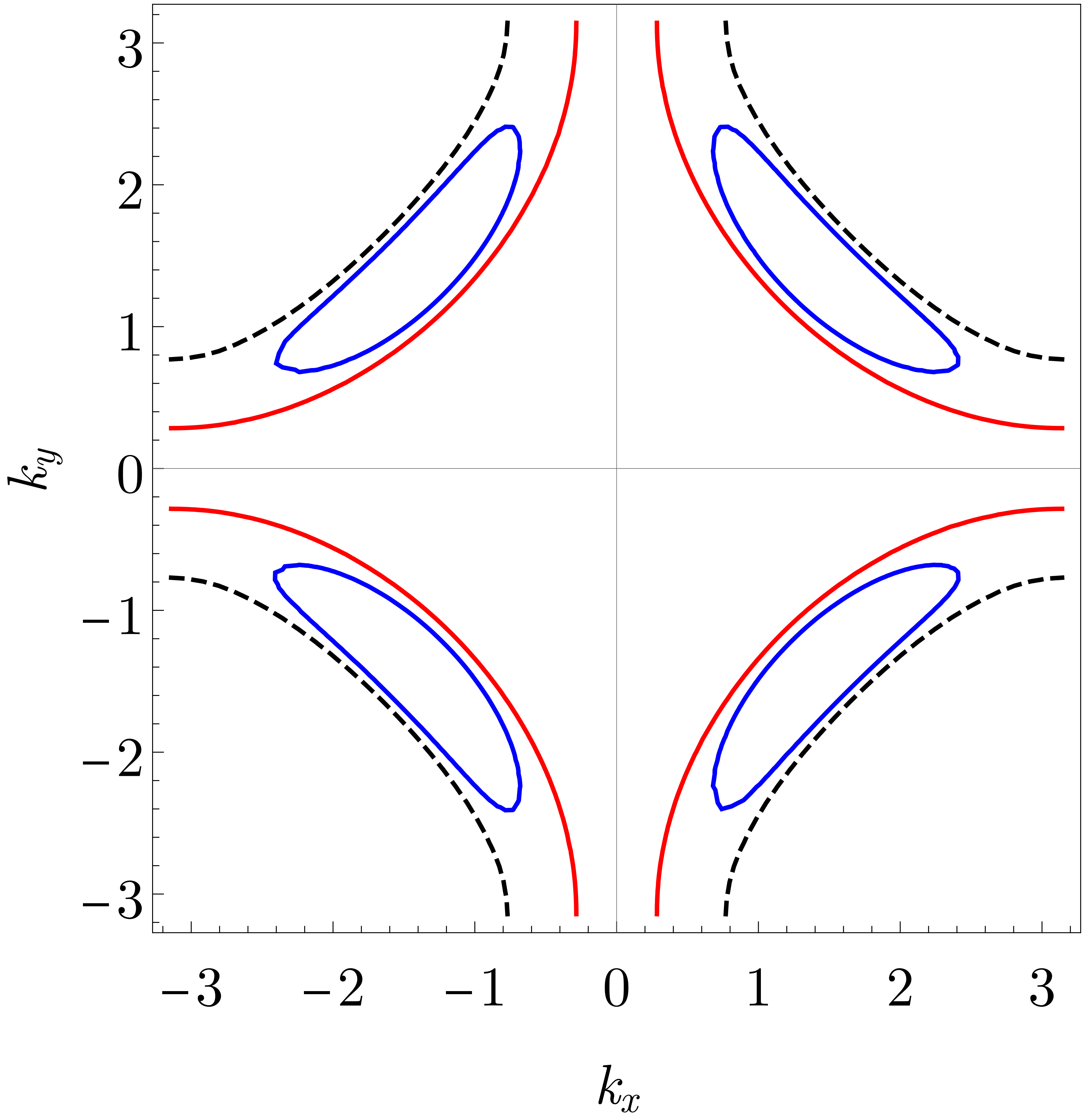
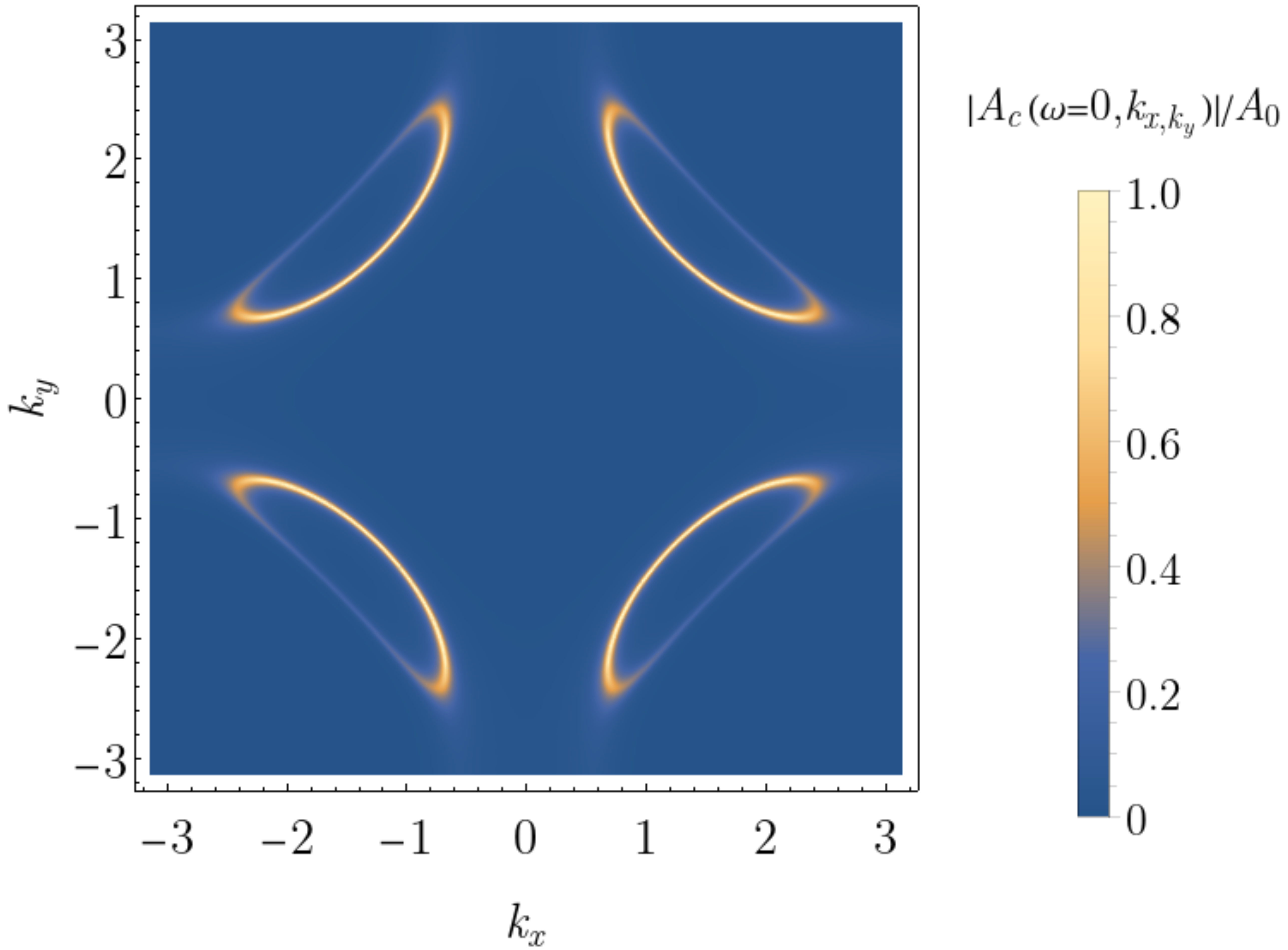
FL* in a ***one-band*** model

"Fermi arc" spectral functions

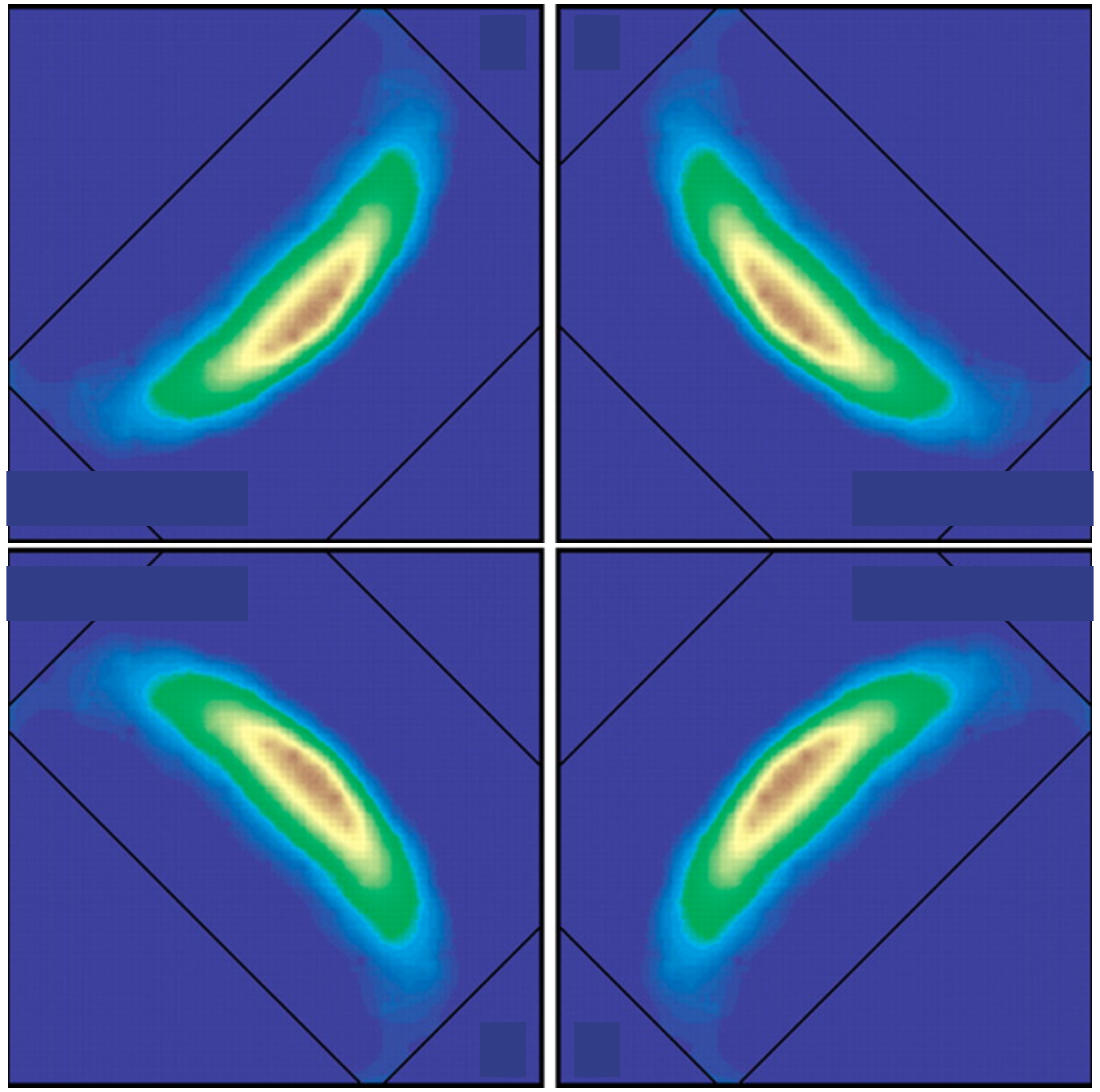




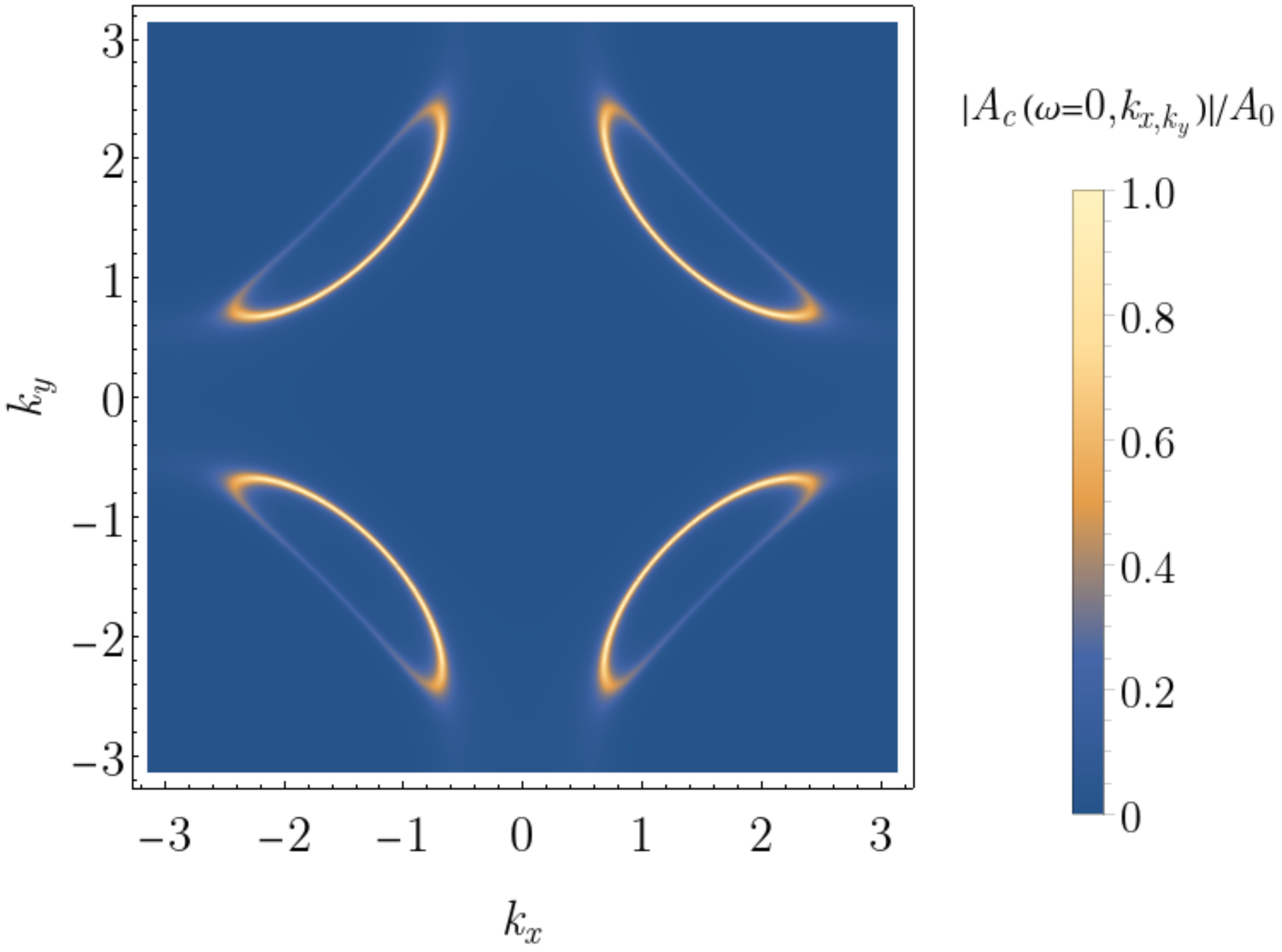
FL* Hamiltonian: $[(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2] \times \mathrm{U}(1)_{\mathrm{em}}$ is broken to $\mathrm{U}(1)_{\mathrm{diag}}$ by Higgs condensate Φ:

$$H = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} + \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_\sigma \right)$$

Photoemission at small p



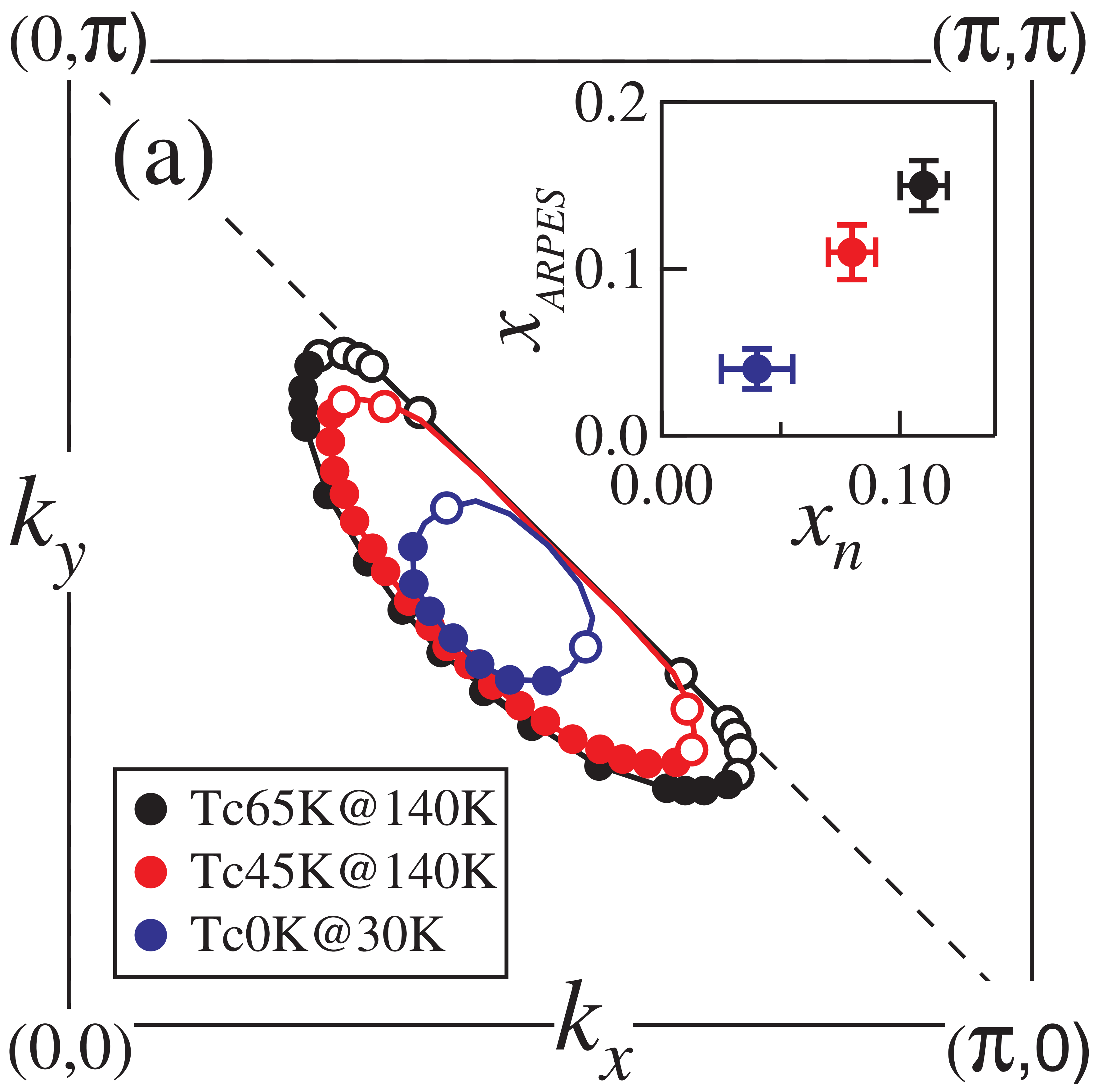
$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

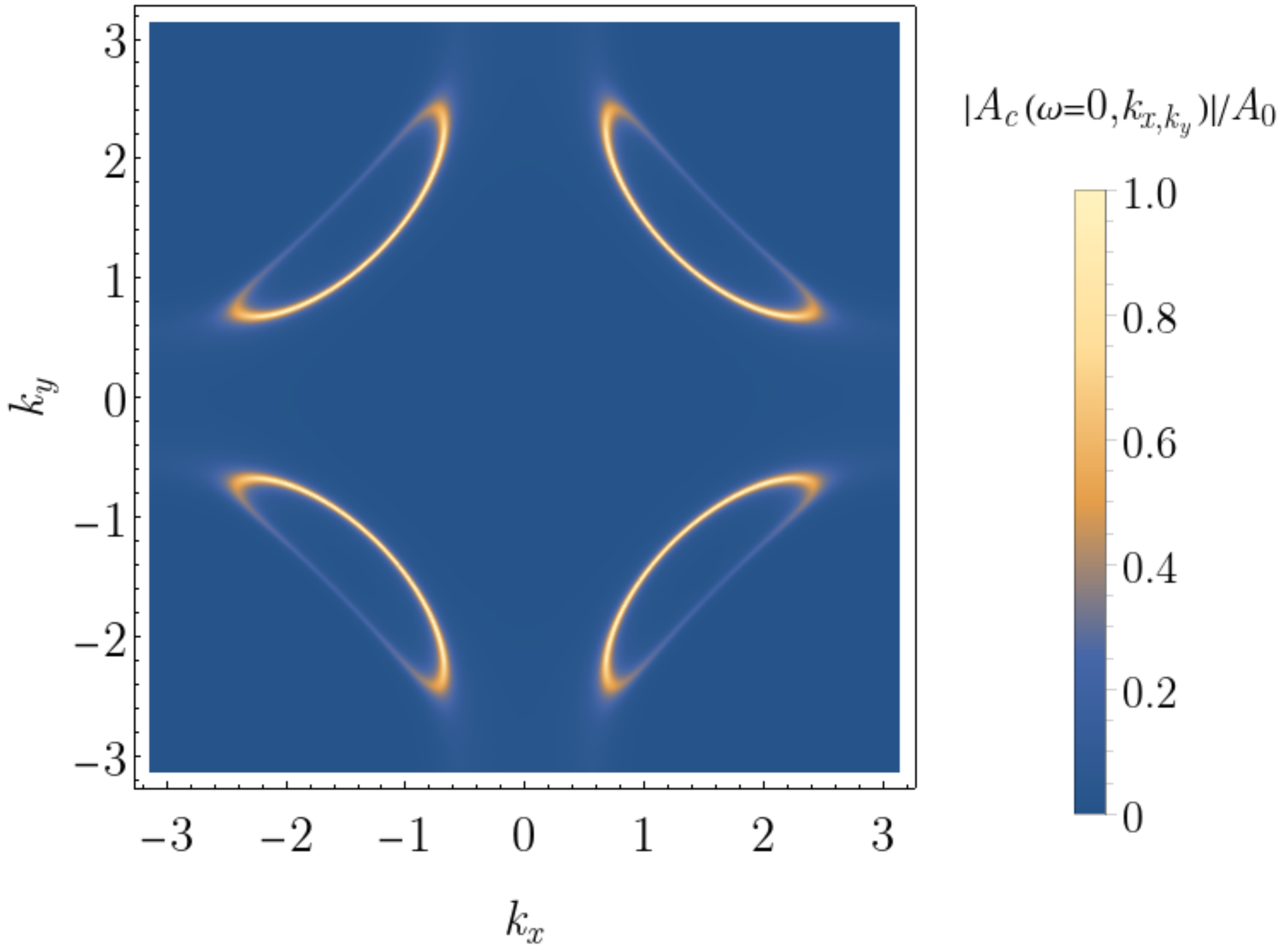


"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission at small p





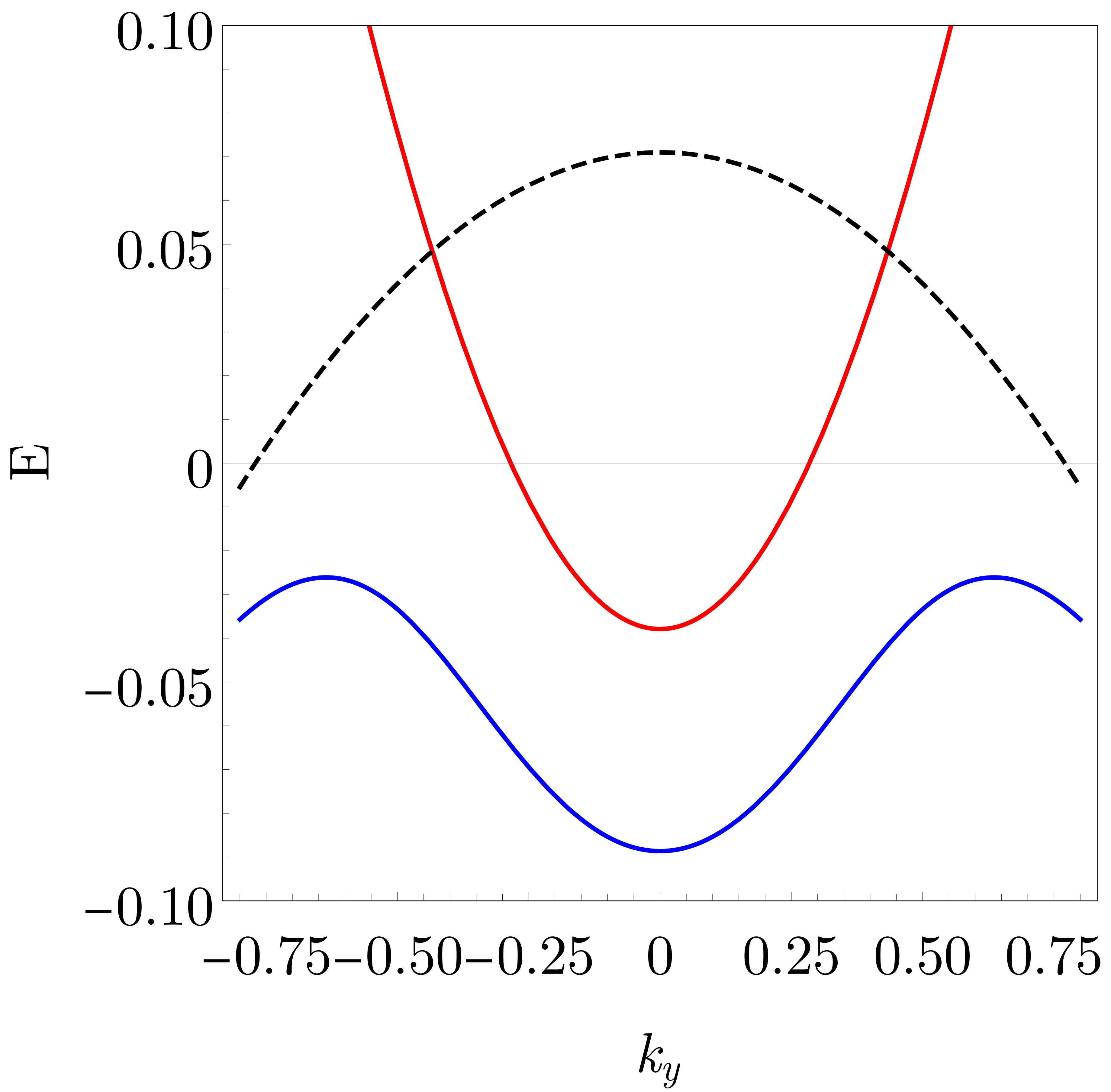
"Fermi pockets"

Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

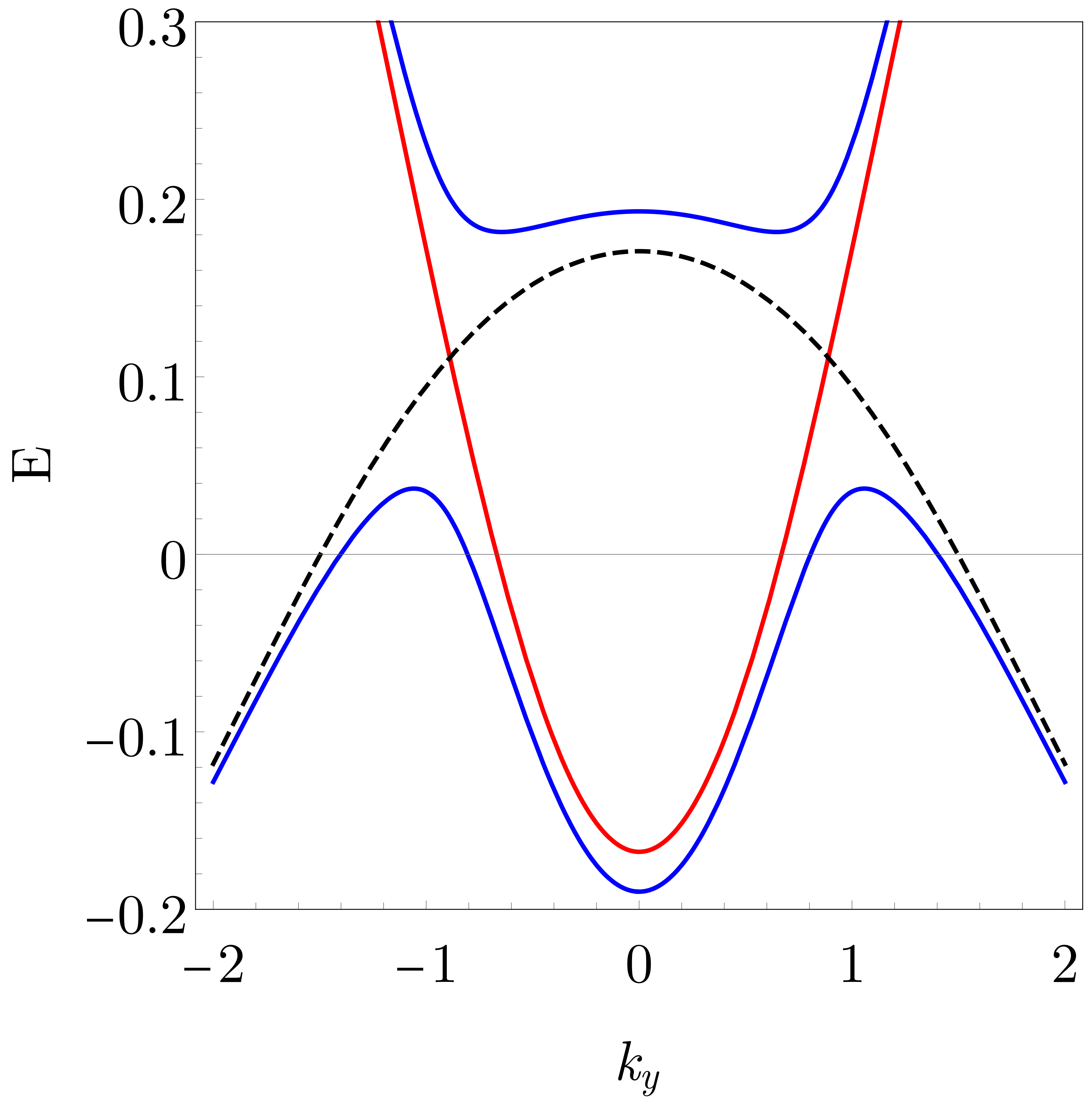
FL* in a ***one-band*** model

Electronic dispersion

Anti-node: $k_x = \pi$

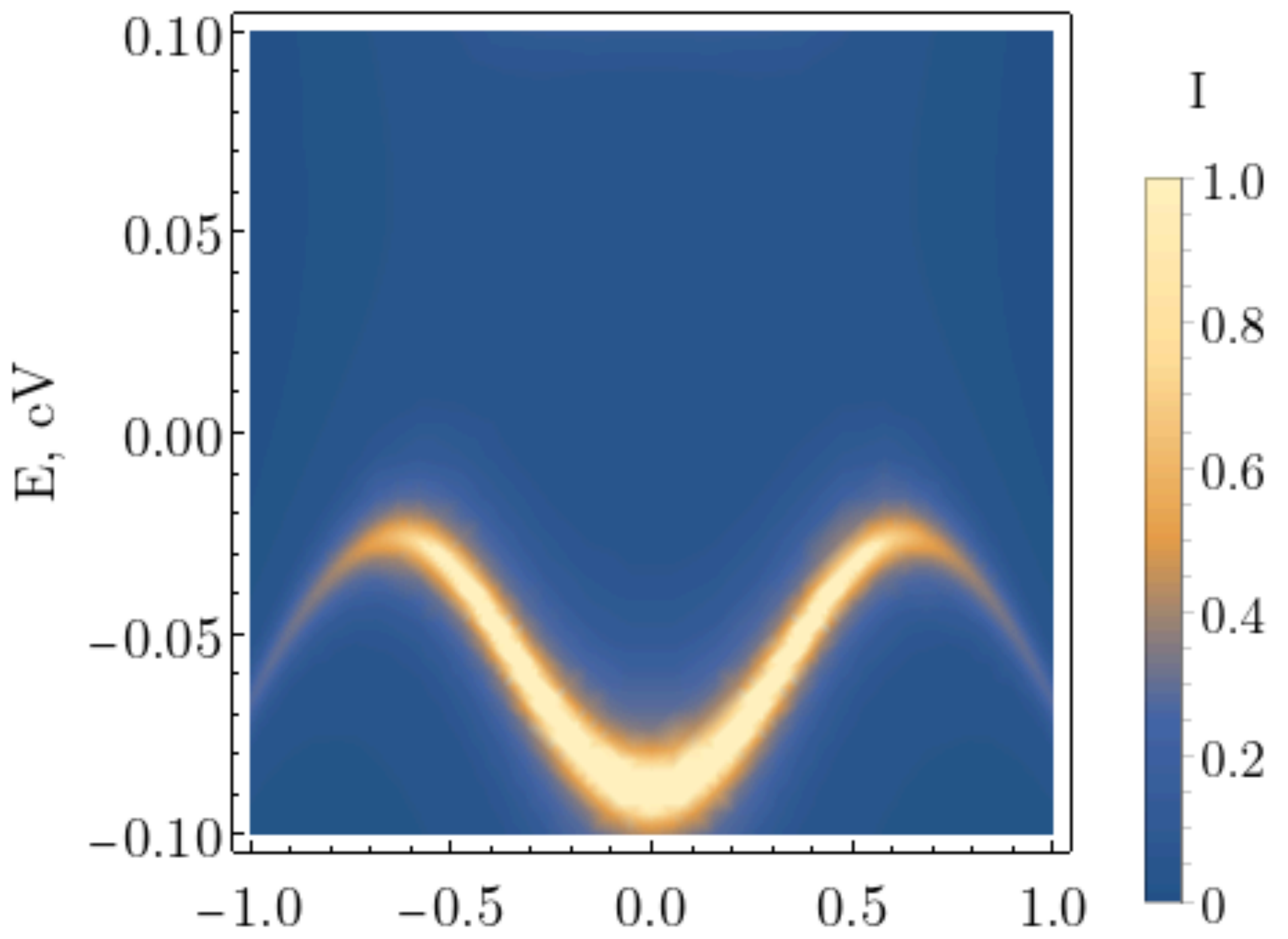


Node: $k_x = 2$

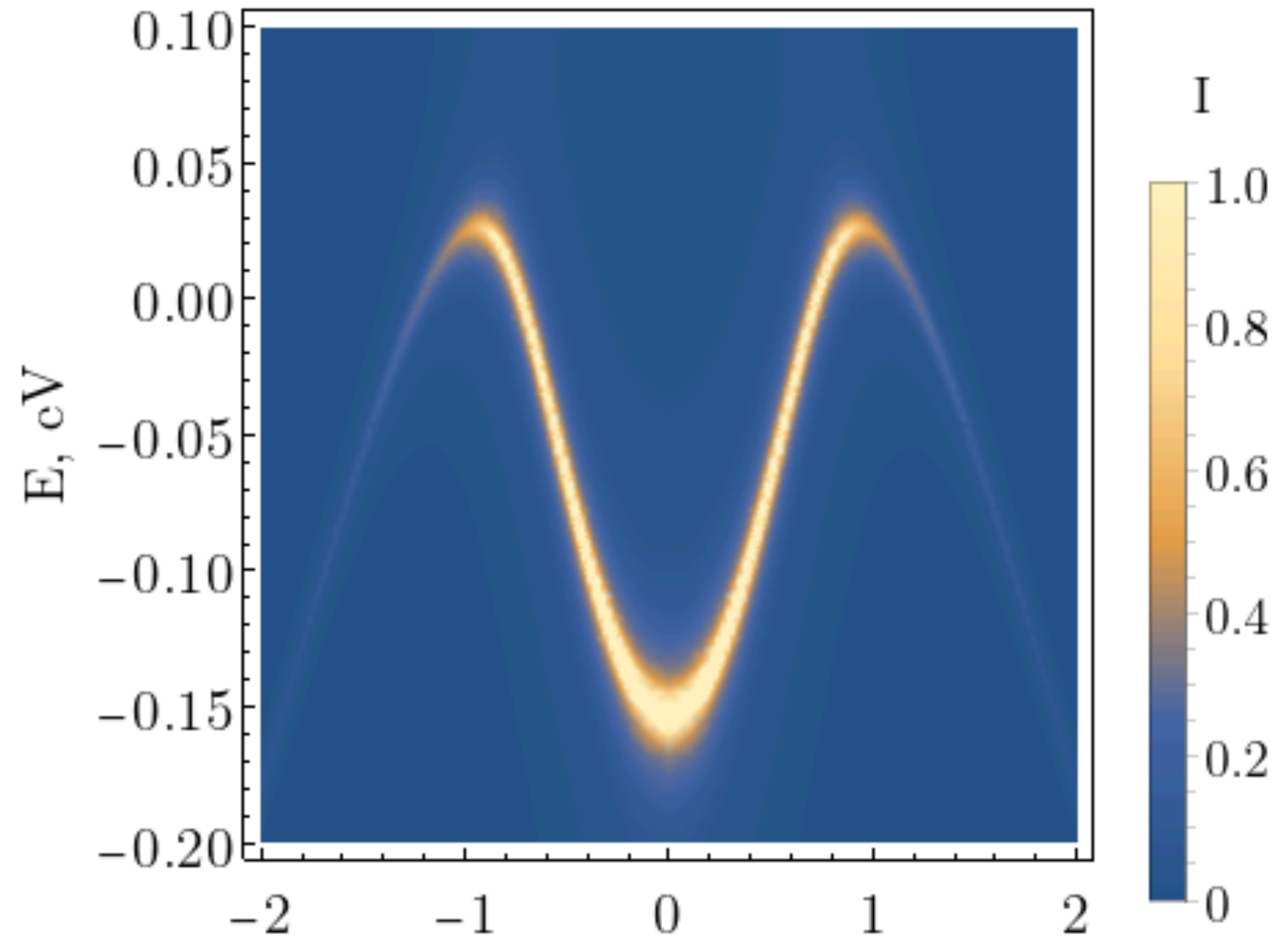


FL* Hamiltonian: $[(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2] \times \mathrm{U}(1)_{\mathrm{em}}$ is broken to $\mathrm{U}(1)_{\mathrm{diag}}$ by Higgs condensate Φ:

$$H = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} + \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_\sigma \right)$$

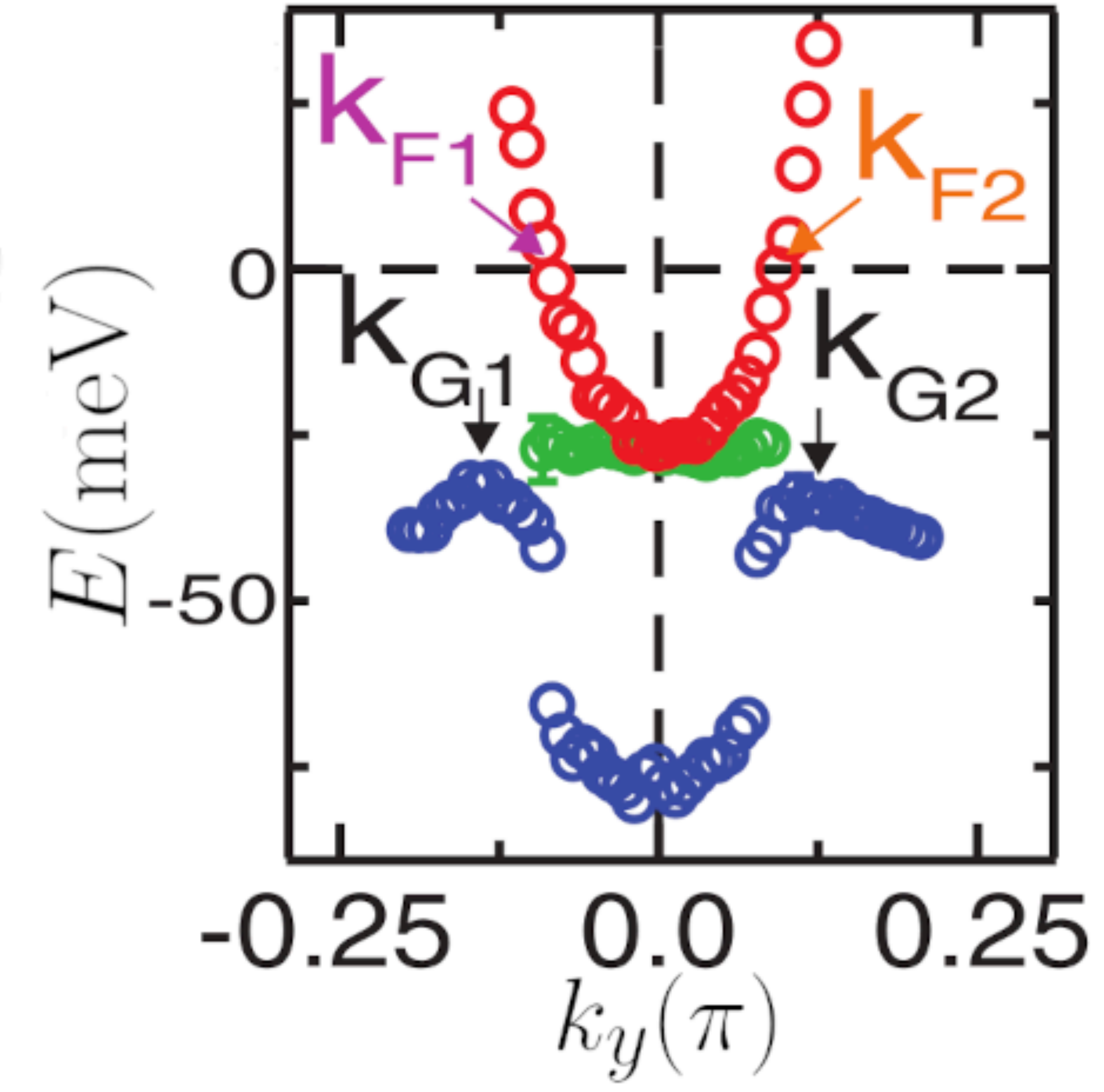


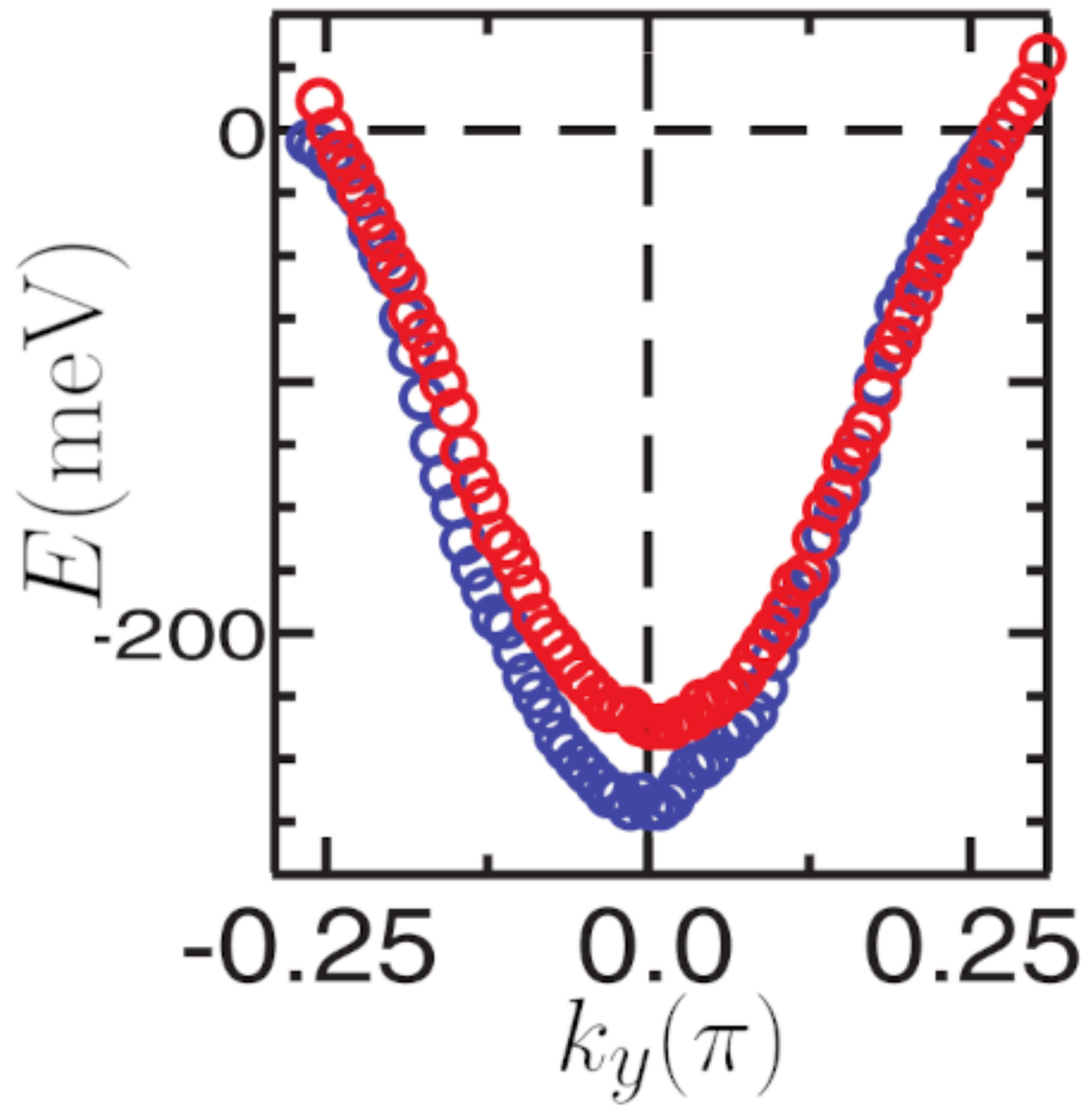
Anti-node



Node

FL* in a *one-band* model

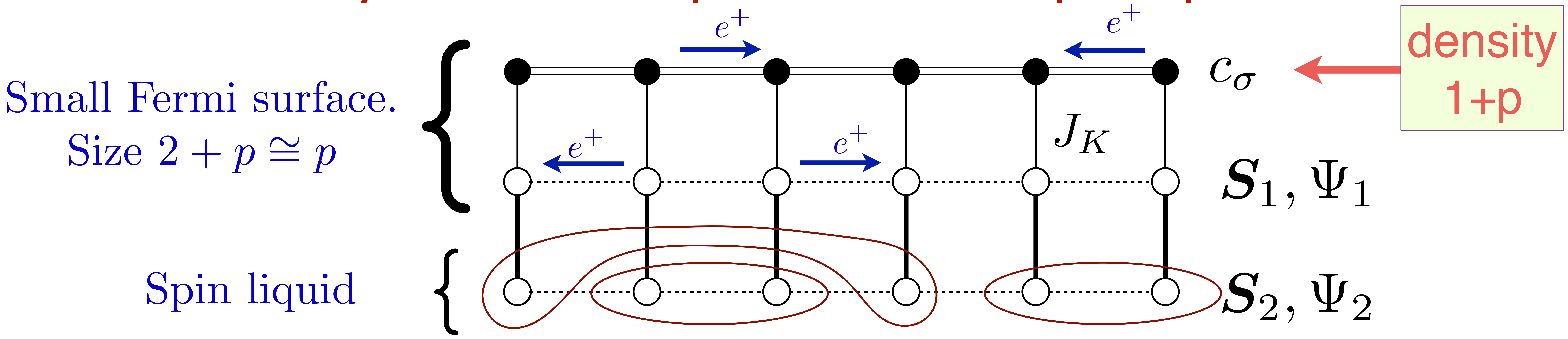




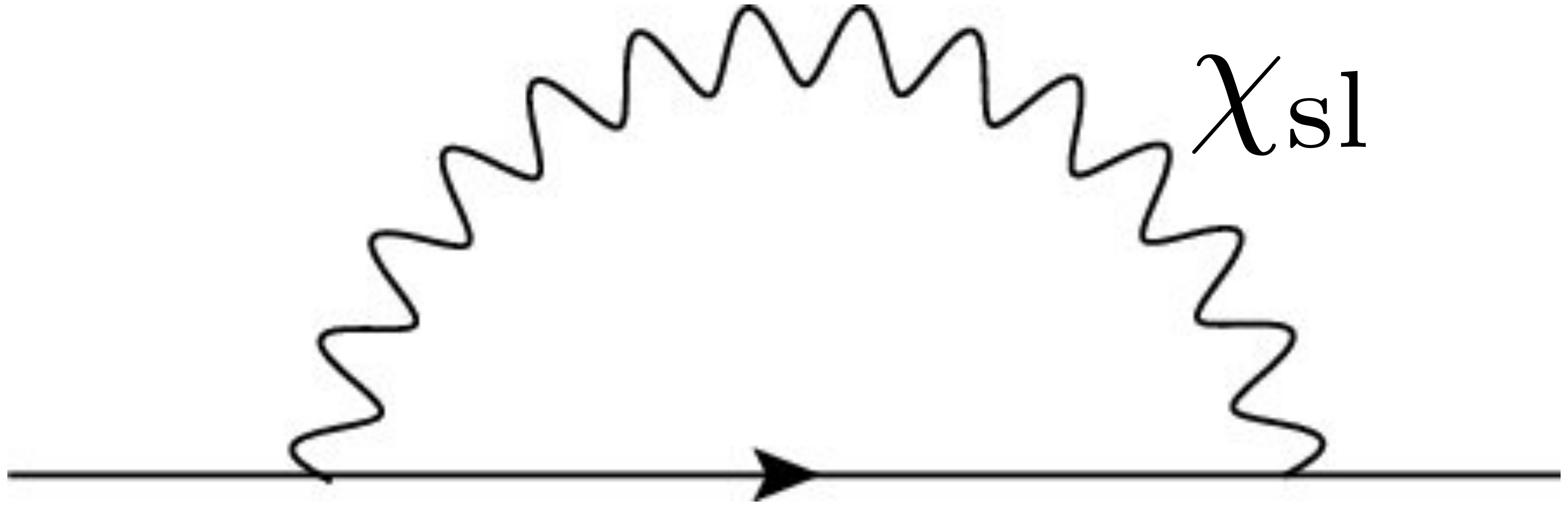
ARPES on Bi2201

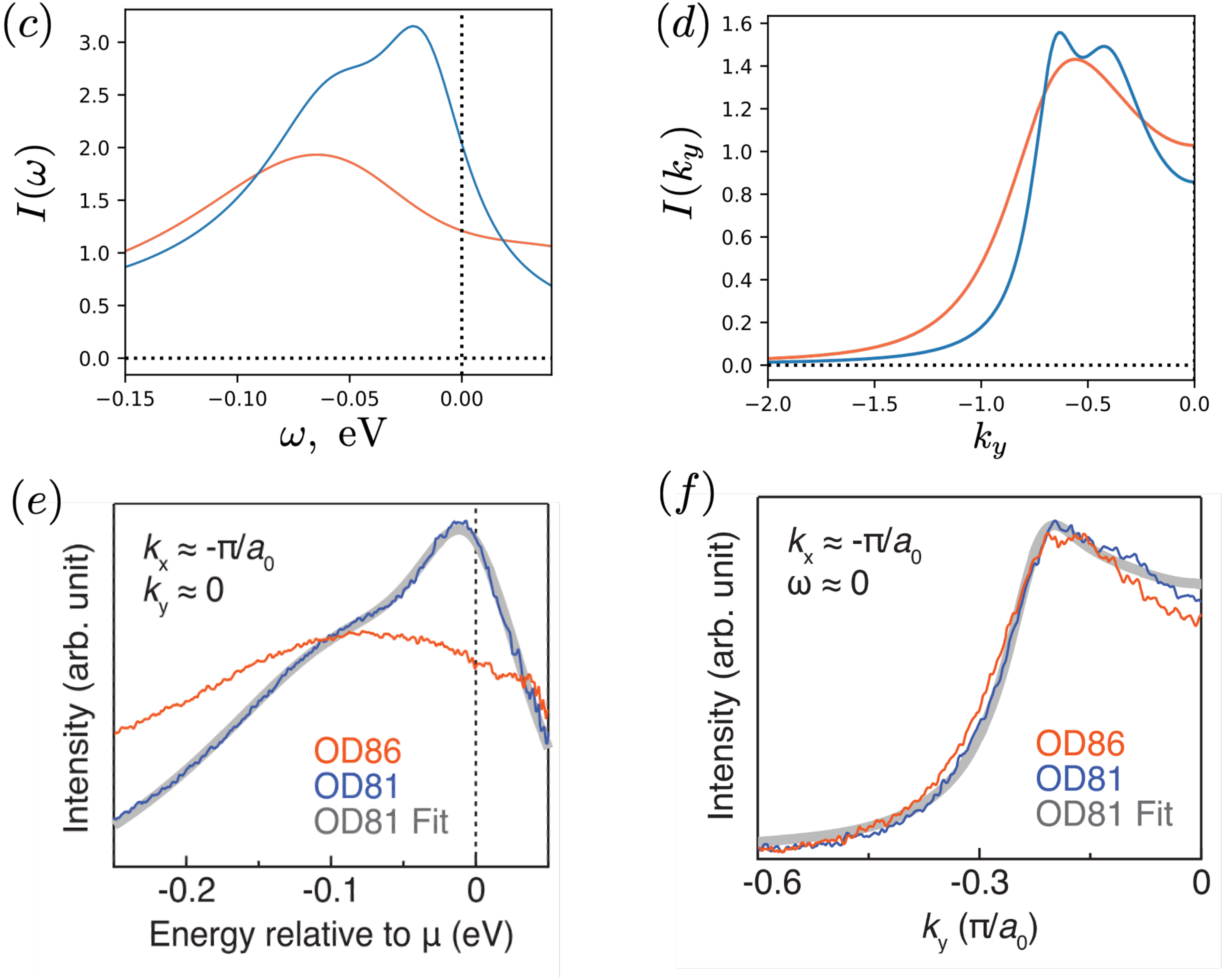
R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

Dynamic consequences of the spin liquid



The only singular gauge fluctuations are those in the spin liquid of the Ψ_2. We can compute their influence on the electronic spectrum perturbatively in the exchange couplings in terms of the dynamic spin susceptibility $\chi_{\rm sl}$.



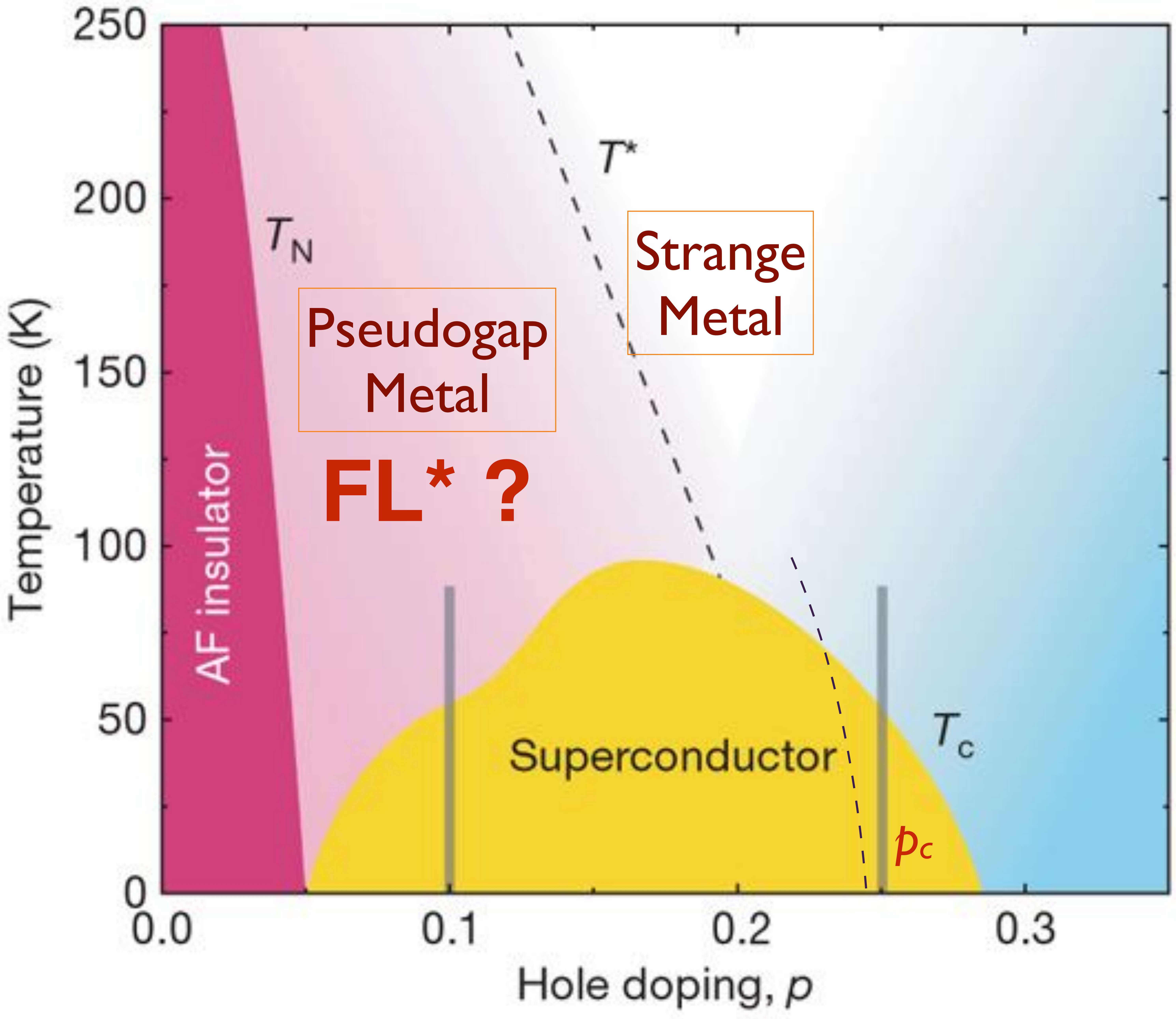


Antinodal EDC and MDC

(c,d) Theory with SYK spin liquid in Ψ_2 layer. Similar EDC obtained by gapless $\mathbb{Z}_2$ spin liquid

(e,f) Experiments on Bi2212 by S.-D. Chen, M. Hashimoto, Y. He, D. Song, K.-J. Xu, J.-F. He, T. P. Devereaux, H. Eisaki, D.-H. Lu, J. Zaanen, and Z.-X. Shen, Science **366**, 1099 (2019).

1. Luttinger relations - old and new
2. Paramagnon fractionalization in the single band Hubbard model
3. Comparison with photoemission
4. Confinement transition in the random t-J model



Can a FL* state
in a
single-band Hubbard model
describe the
pseudogap metal
over an intermediate
temperature range,
along with a
crossover/transition to
confinement at lower
temperatures?

Quantum generalization of the Sherrington-Kirkpatrick model to $S = 1/2$ spins with SU(2) symmetry

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$[S_{i\mu}, S_{j\nu}] = i\delta_{ij}\epsilon_{\mu\nu\lambda}S_{i\lambda} \quad , \quad \boldsymbol{S}_i^2 = 3/4$$

$\overline{J_{ij}} = 0$, $\overline{J_{ij}^2} = J^2$, Different J_{ij} uncorrelated.

Two possible ground states

I. Gapless spin liquid

$$\lim_{\tau \to \infty} \langle \boldsymbol{S}_i(\tau) \cdot \boldsymbol{S}_i(0) \rangle \sim \frac{1}{|\tau|^a}$$

Ground state of the SU($M \to \infty$) model with $a = 1$ (Sachdev, Ye 1993).

II. Spin glass order

$$\lim_{\tau \to \infty} \langle \boldsymbol{S}_i(\tau) \cdot \boldsymbol{S}_i(0) \rangle = q_{EA} > 0$$

where q_{EA} is the spin glass order parameter.

Random *J* model (insulator): SU(*M*) symmetry

Express the spin operator in terms of fermionic spinons $\vec{S} = (1/2) f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1 \ldots M$. The fermions obey the constraint

$$\sum_{\alpha=1}^{M} f_\alpha^\dagger f_\alpha = \frac{M}{2}$$

In the large M limit we obtain the SYK equations for the spinon Green's function G and self energy Σ; similar results apply for bosonic spinons.

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution is

$$G(\tau) \sim \frac{\mathrm{sgn}(\tau)}{\sqrt{\tau}} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

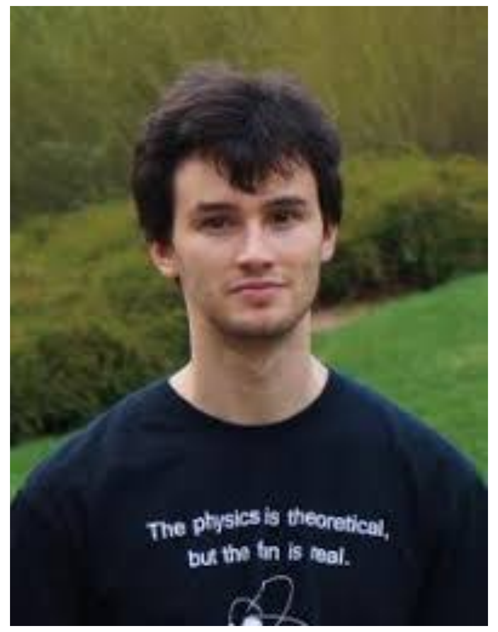
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Dynamic spin susceptibility of the spin liquid at $M = \infty$

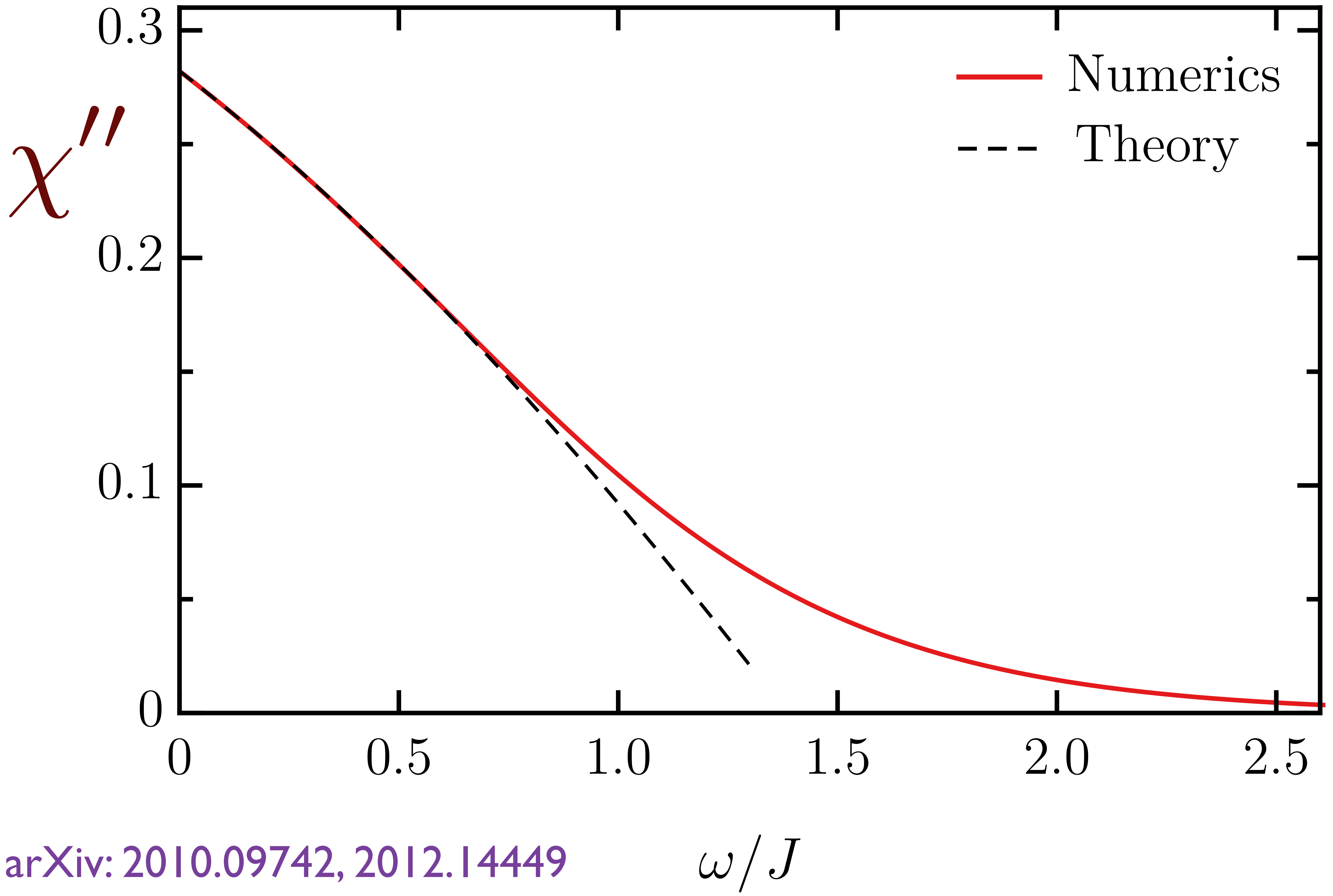
$$Q(\tau) = \int_0^\infty \frac{d\omega}{\pi} \chi''(\omega) e^{-\omega\tau}$$

$$\chi''(\omega) \sim \mathrm{sgn}(\omega)\left[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\ldots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \ldots\right]$$





Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory. $\mathcal{C}$ is a known number, and γ is the co-efficient of the action for the 'boundary graviton' in holographic dual.



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

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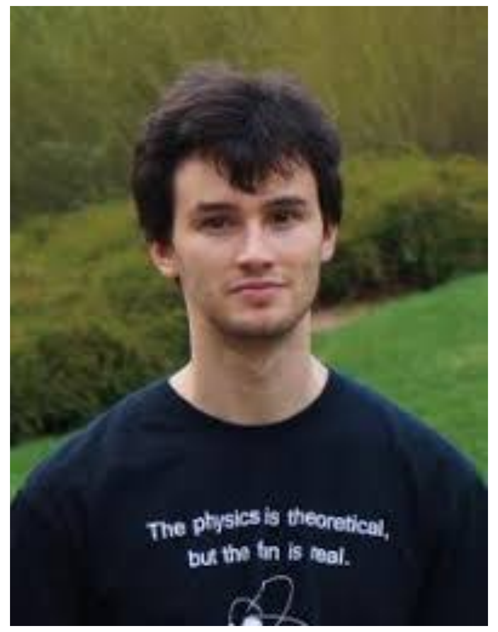

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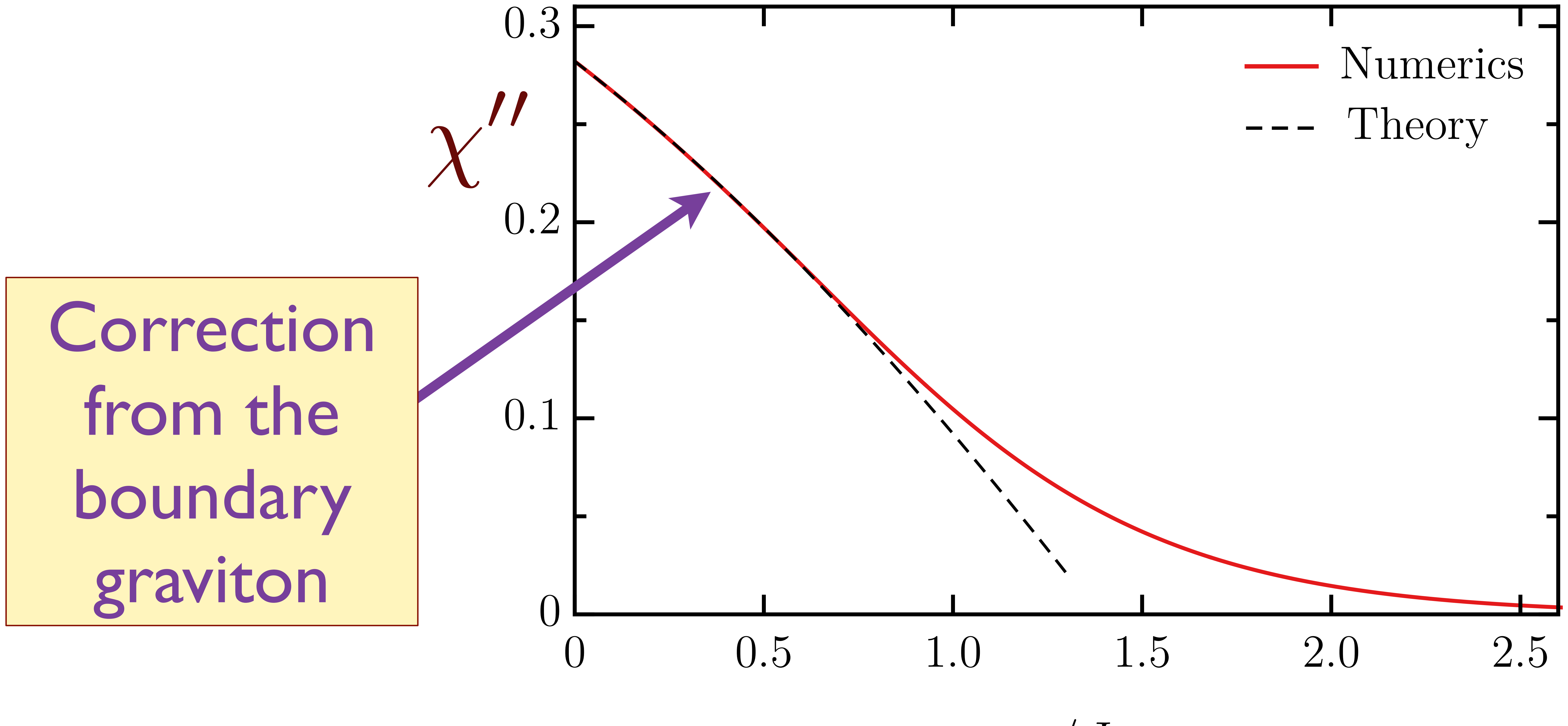
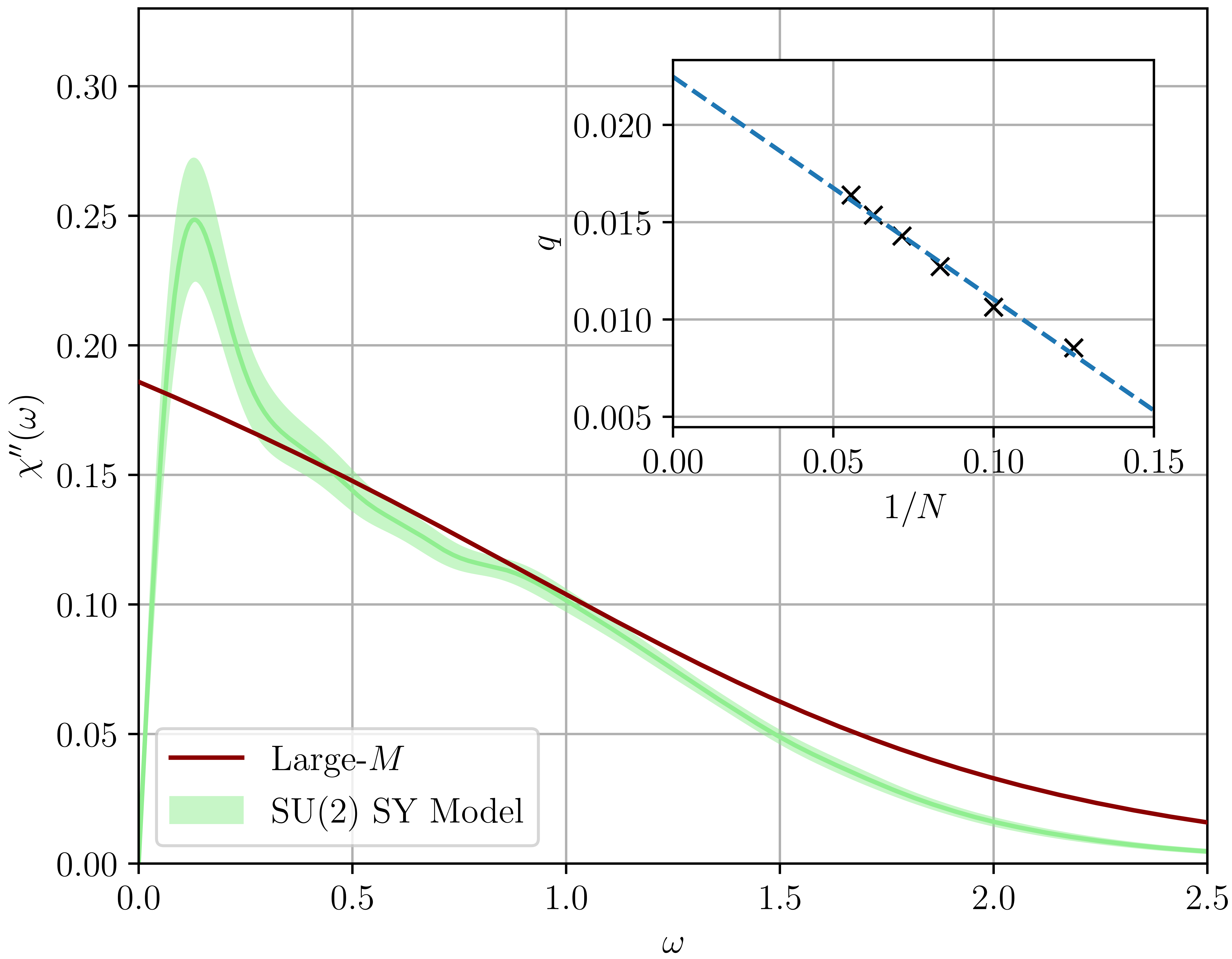


Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742, 2012.14449

Exact diagonalization of clusters of SU(2) spins



H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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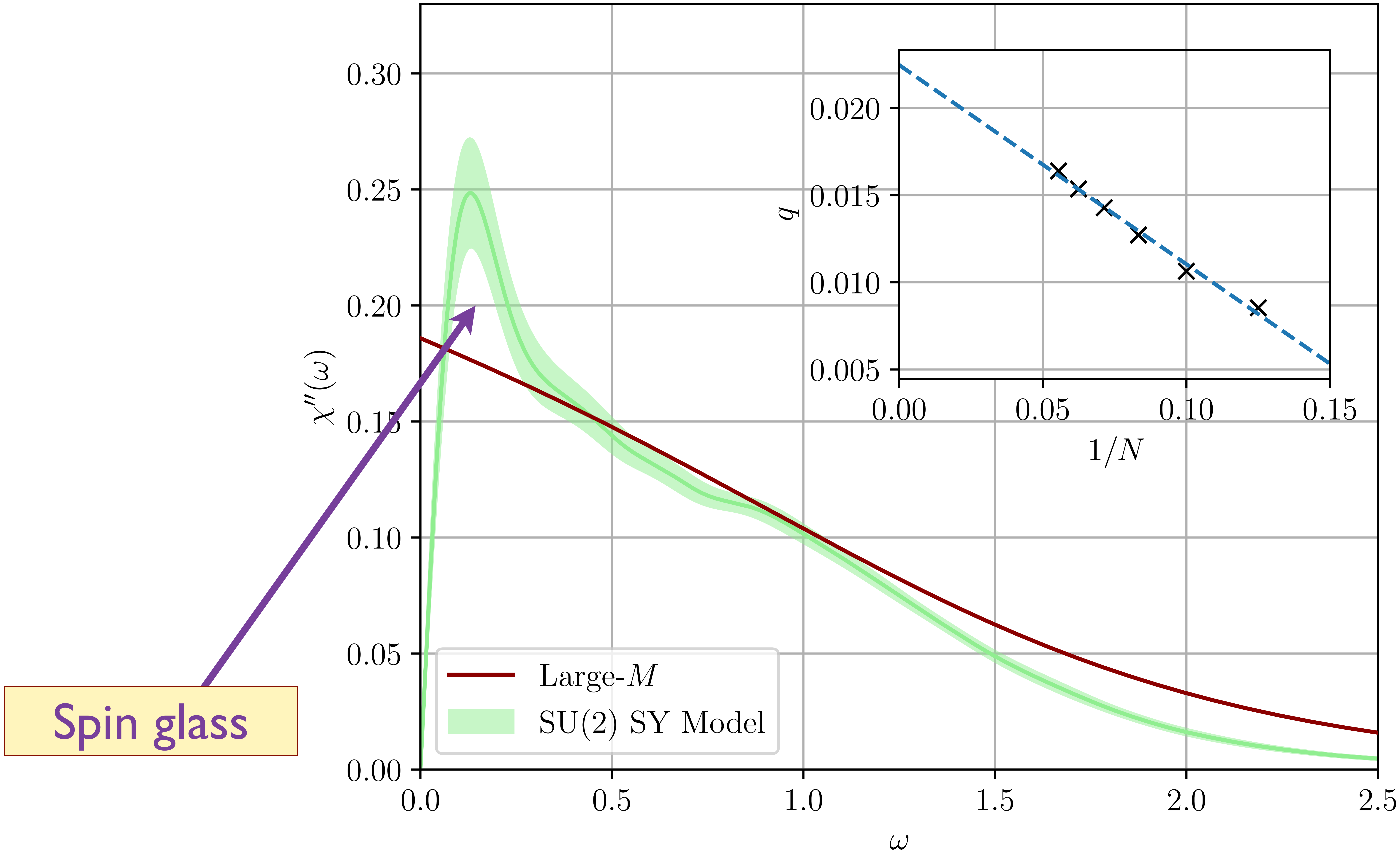

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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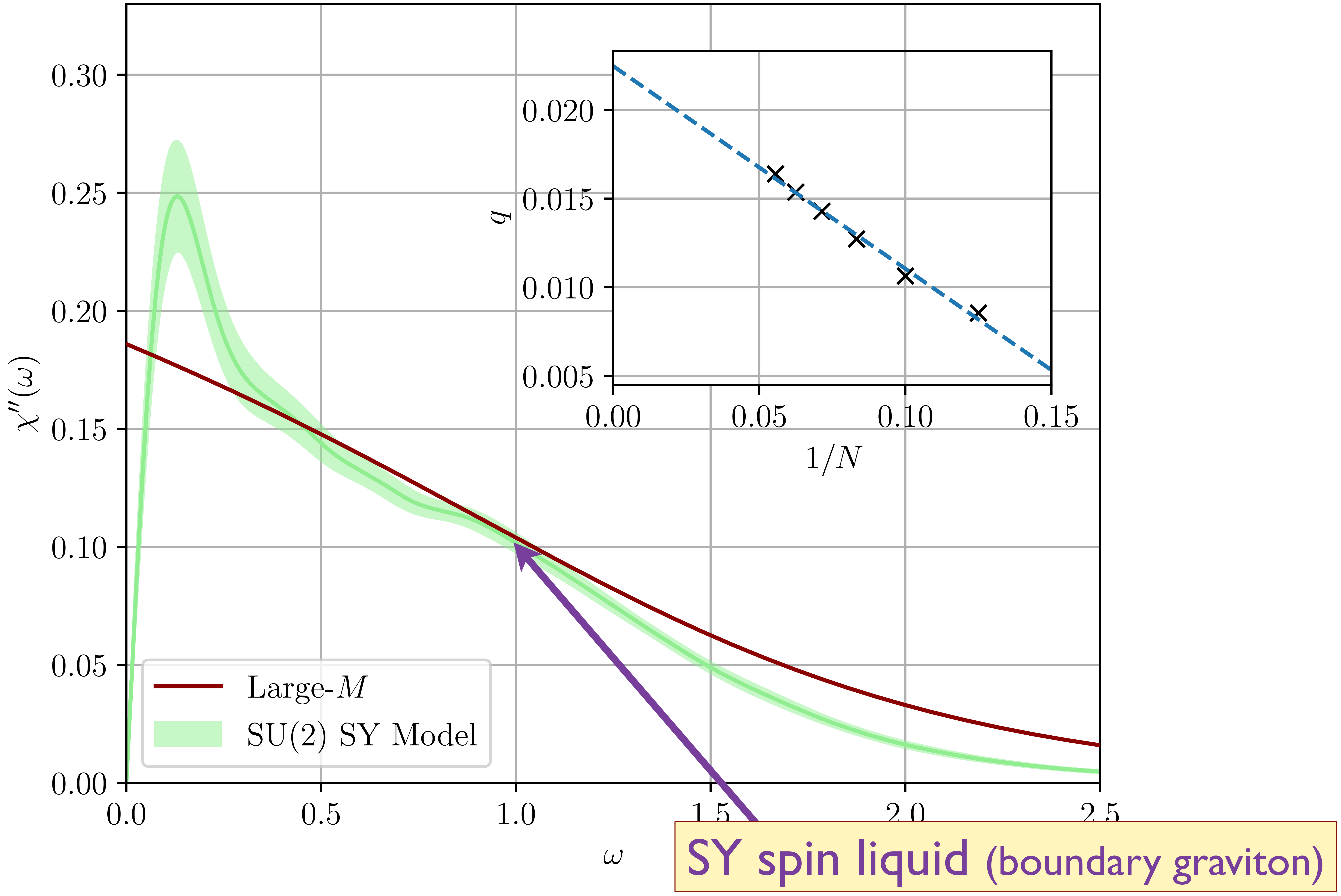
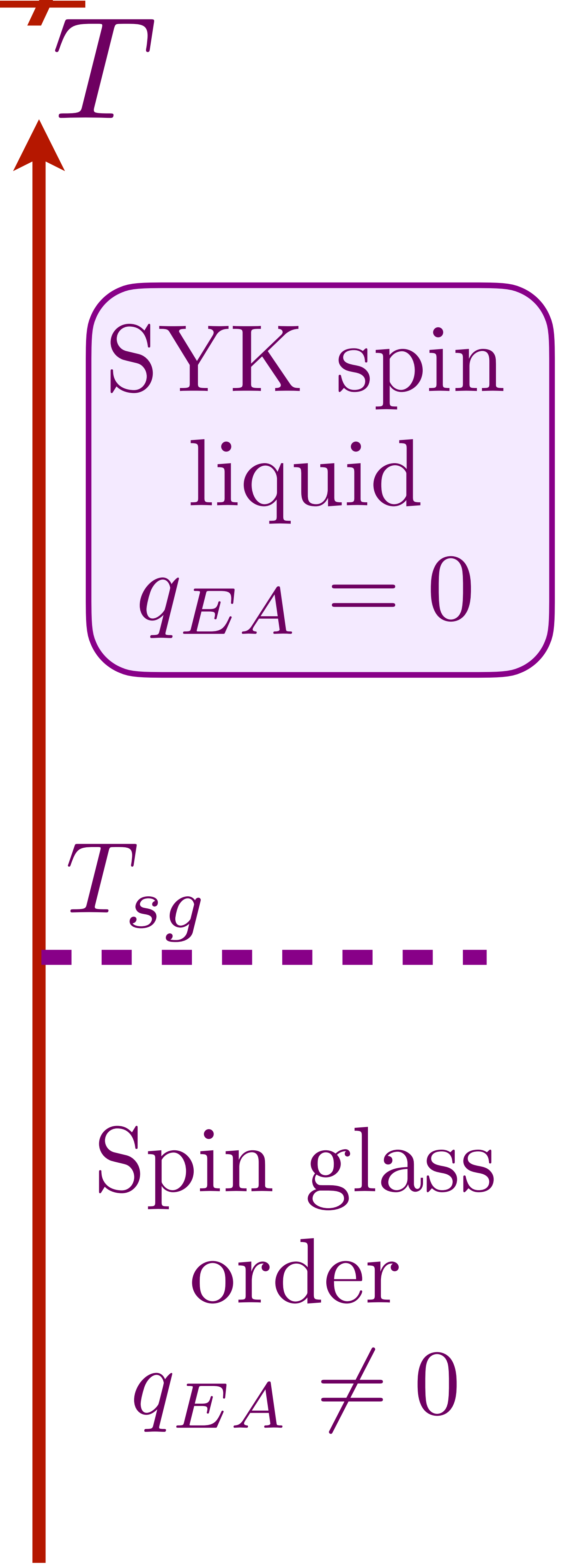


H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Random J model (insulator): SU(M) symmetry

$$T_{\rm sg} \sim J \exp\left(-\sqrt{M\pi}\right) \quad , \quad e^{-\sqrt{2\pi}} = 0.0815\ldots$$

$$\chi''(\omega) = \frac{\pi\omega}{T} q_{EA}\, \delta(\omega) + \frac{1}{J}\Phi_\chi\left(\frac{\omega}{J q_{EA}}\right) + \ldots, \quad T \to 0$$



$\omega / J q_{EA}$

M. Christos, F. M. Haehl, and S. Sachdev, PRB **105**, 085120 (2022)

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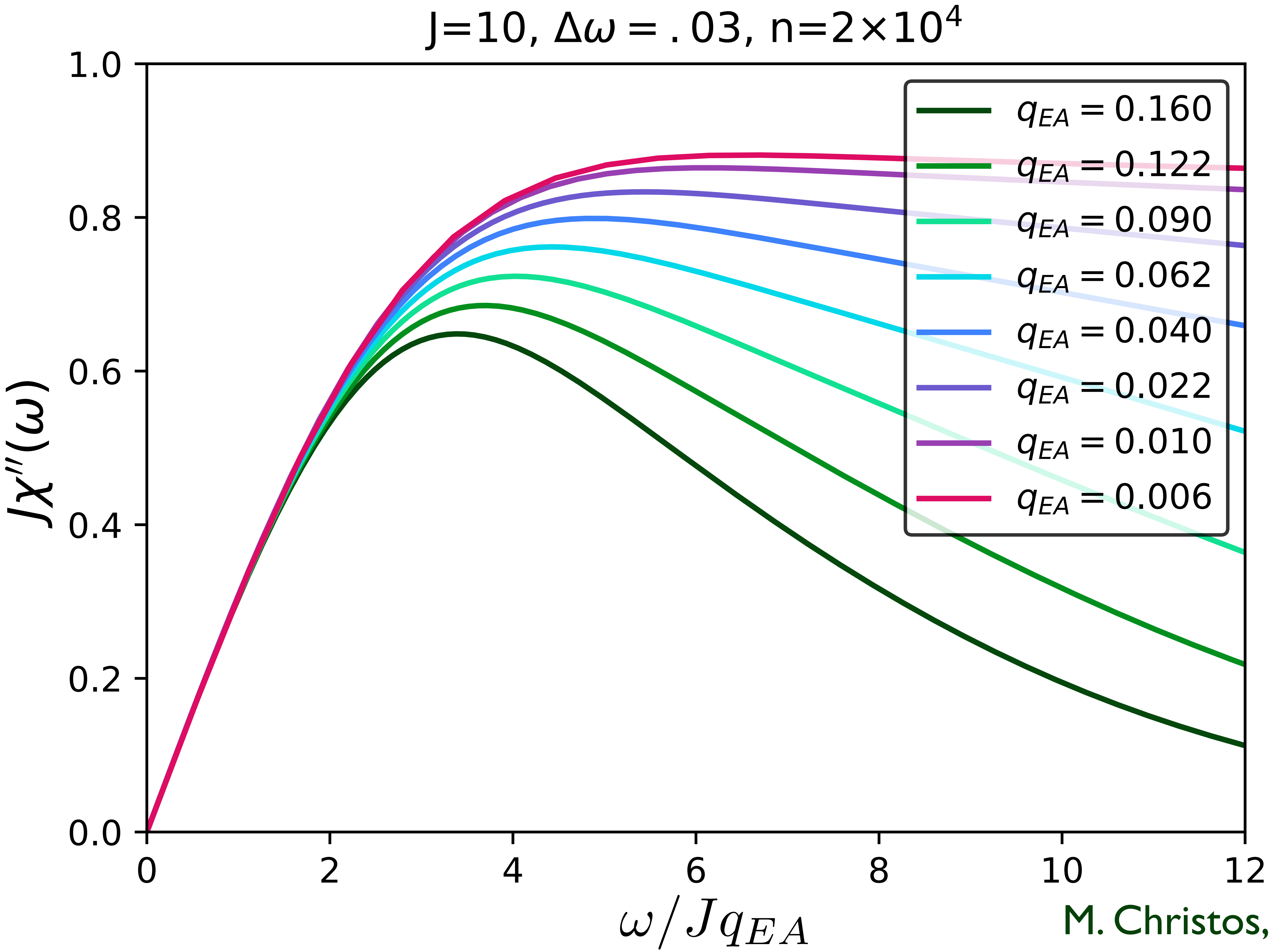
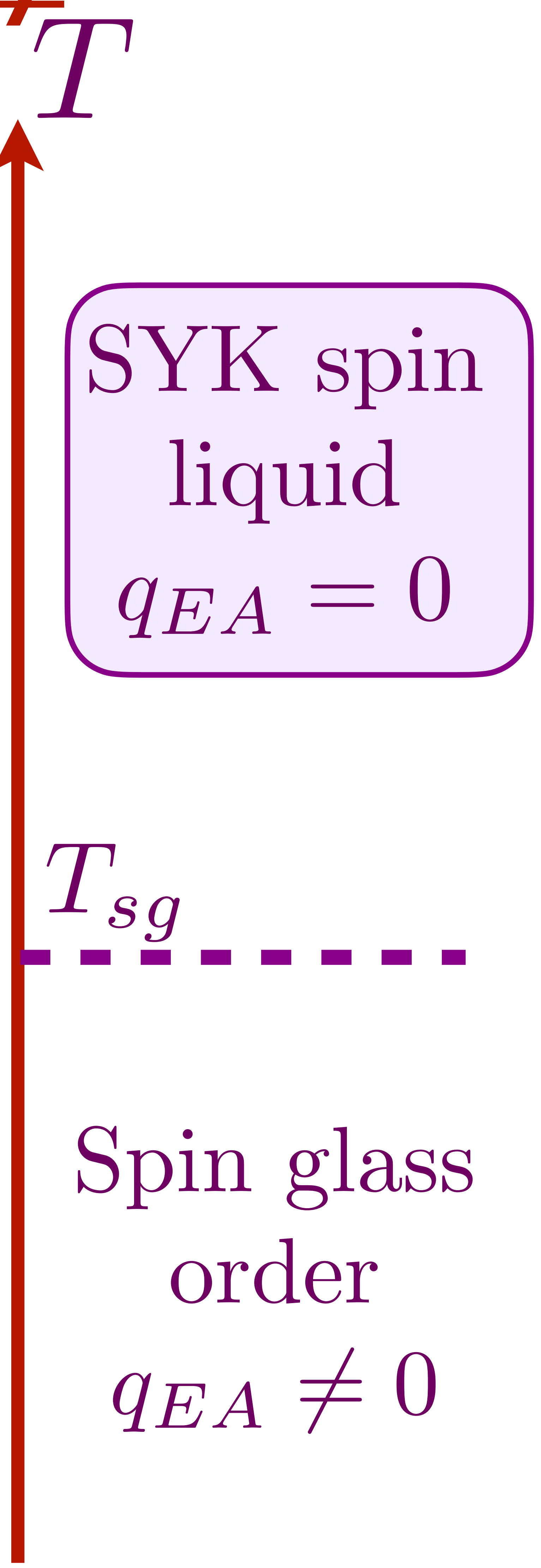




M. Christos, F. M. Haehl, and S. Sachdev, PRB **105**, 085120 (2022)

Dope the quantum Sherrington-Kirkpatrick model with mobile electrons

$$H = \sum_{i<j} \left[-t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \text{H.c.} + J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \right]$$

$$\boldsymbol{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{i\beta}$$

$$[c_{i\alpha}, c_{j\beta}^\dagger]_+ = \delta_{ij}\delta_{\alpha\beta} \quad , \quad \sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} \leq 1$$

$$\frac{1}{N} \sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

$$\overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2, \quad \text{Different } J_{ij} \text{ uncorrelated.}$$

$$\overline{t_{ij}} = 0, \quad \overline{t_{ij}^2} = t^2, \quad \text{Different } t_{ij} \text{ uncorrelated.}$$

Review: D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv:2109.05037

Random t-J model: SU(M) symmetry



Each site has 3 states which we map to the space of a boson b (the holon) and a fermion f_α (the spin):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \quad , \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$
$$c_\alpha = f_\alpha b^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

To obtain a large M limit, let $\alpha = 1 \ldots M$, endow the boson with an 'orbital' index $a = 1 \ldots M'$ and send $M \to \infty$ at fixed $k = M'/M$. Then

$$c_{a\alpha} = f_\alpha b_a^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b_a^\dagger b_a = \frac{M}{2}$$

M. Christos, D. G. Joshi, S. S., and M. Tikhanovskaya, arXiv:2203.16548

Random *t-J* model: SU(*M*) symmetry

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green's functions:

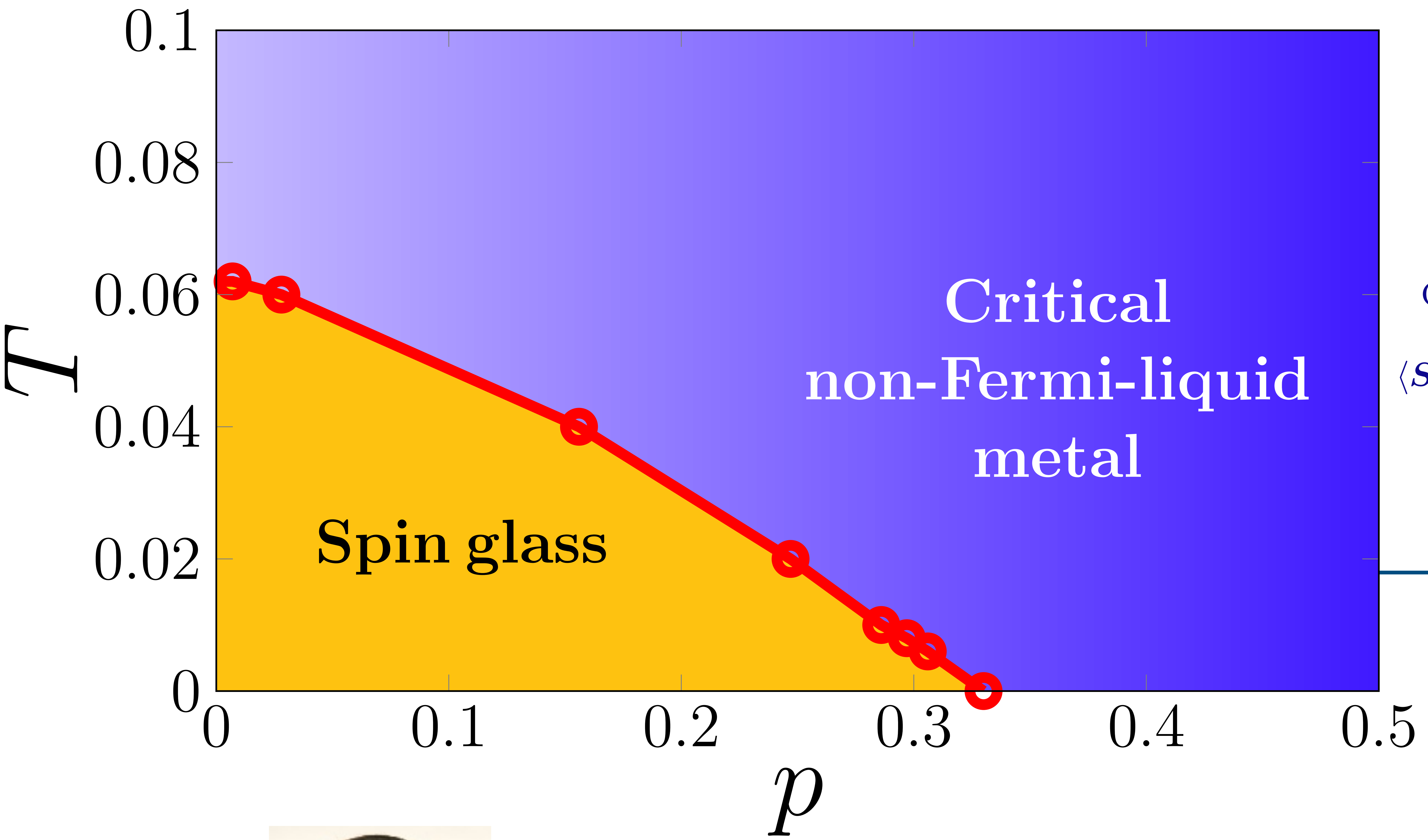
$$
\begin{aligned}
G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\
\Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\
G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\
\Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k\, t^2 G_f(\tau) G_b(\tau) G_b(-\tau)
\end{aligned}
$$

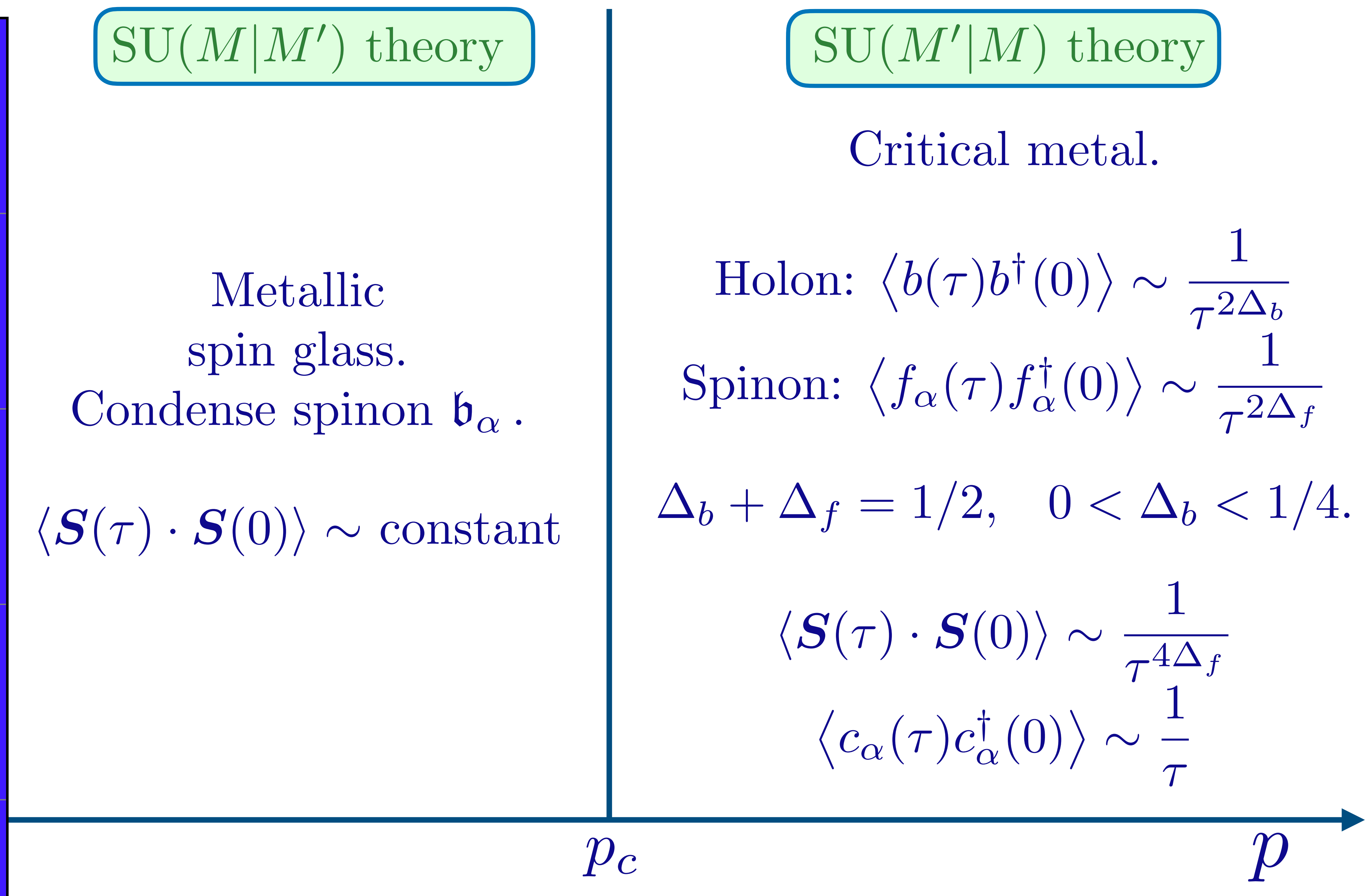
Here μ_f and μ_b are chemical potentials chosen to satisfy

$$
\langle f^\dagger f \rangle = \frac{1}{2} - kp \quad , \quad \langle b^\dagger b \rangle = p \,.
$$

M. Christos, D. G. Joshi, S. S., and M. Tikhanovskaya, arXiv:2203.16548

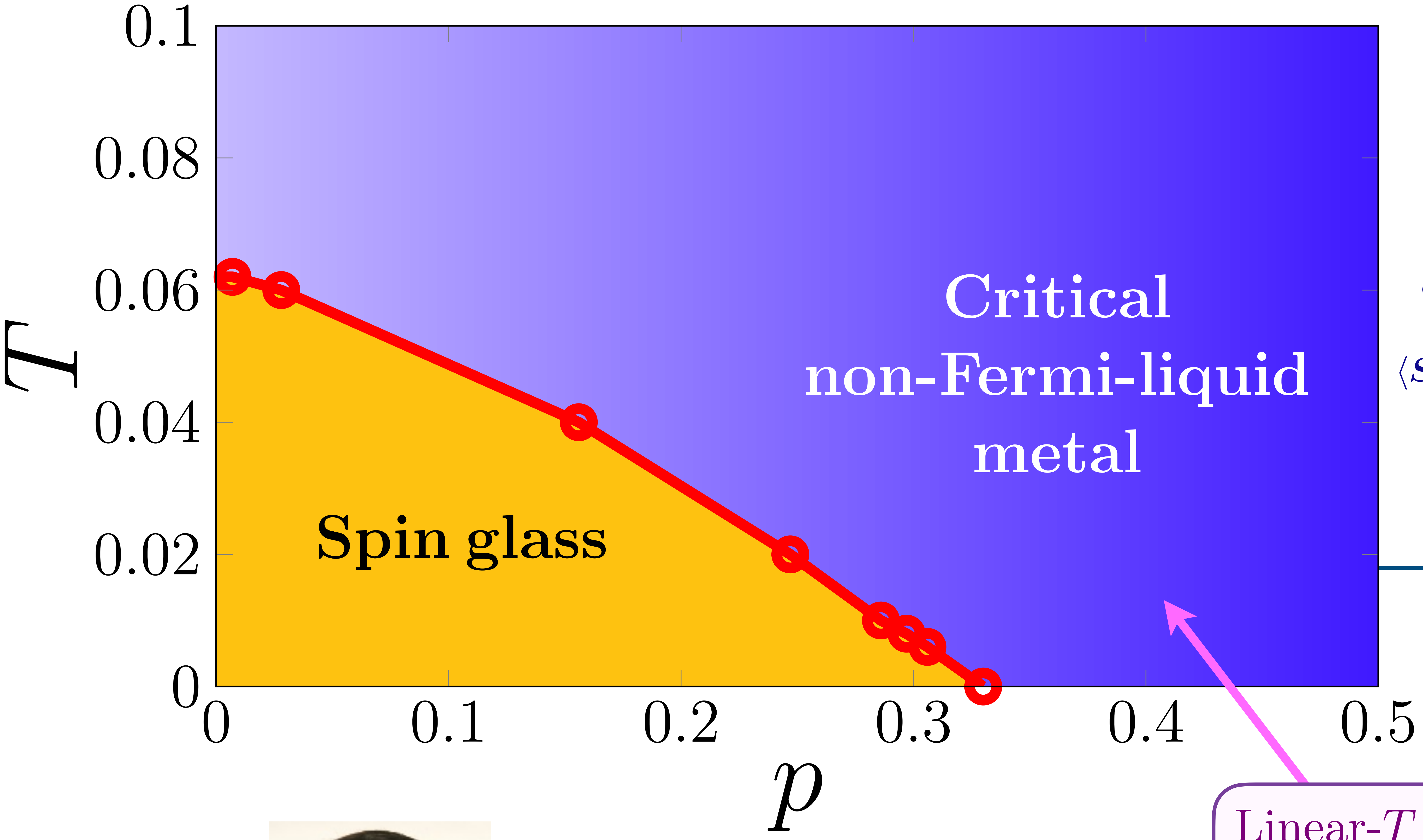
Random *t-J* model: SU(*M*) symmetry

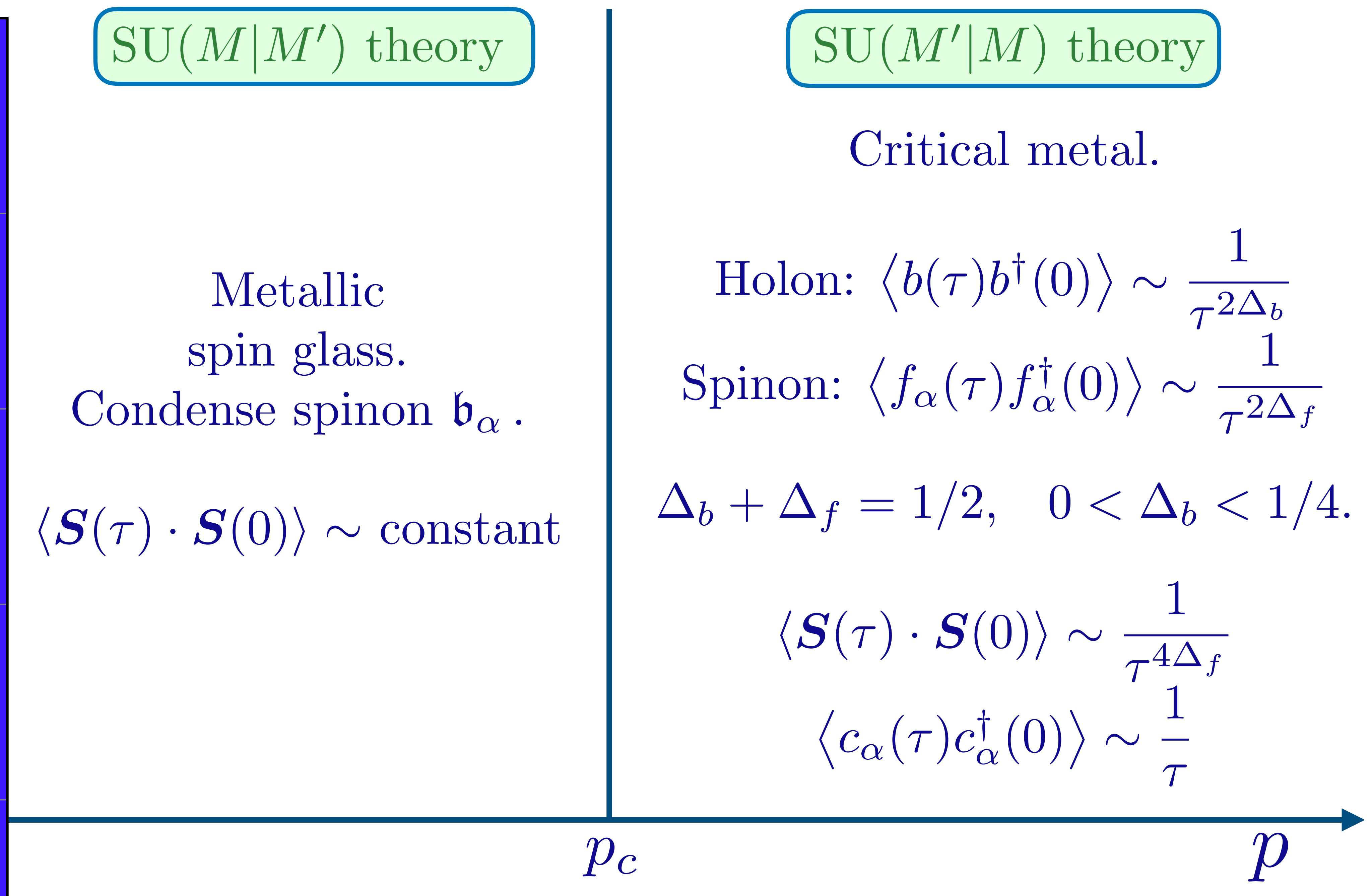




M. Christos, D. G. Joshi, S. S., and M. Tikhanovskaya, arXiv:2203.16548

Random *t-J* model: SU(*M*) symmetry





Linear-T resistivity from boundary graviton mode
Haoyu Guo, Yingfei Gu, and S.S.,
Annals of Physics **418**, 168202 (2020)



M. Christos, D. G. Joshi, S. S., and M. Tikhanovskaya, arXiv:2203.16548

Summary

- Ancilla theory of FL* for the pseudogap metal of the cuprates: Don't fractionalize the mobile electron, but fractionalize the 'paramagnon rotor' into 'ancilla qubits'. Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.

Summary

- Ancilla theory of FL* for the pseudogap metal of the cuprates: Don't fractionalize the mobile electron, but fractionalize the 'paramagnon rotor' into 'ancilla qubits'. Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.

- Theory of spin liquid to confinement transition in the random t-J model: Spin glass order at small p, and a critical non-Fermi liquid metal at larger p.