

A Unified Theory Based on $SO(5)$ Symmetry of Superconductivity and Antiferromagnetism

Shou-Cheng Zhang

The complex phase diagram of high-critical temperature (T_c) superconductors can be deduced from an $SO(5)$ symmetry principle that unifies antiferromagnetism and d -wave superconductivity. The approximate $SO(5)$ symmetry has been derived from the microscopic Hamiltonian, and it becomes exact under renormalization group flow toward a bicritical point. This symmetry enables the construction of a $SO(5)$ quantum nonlinear σ model that describes the phase diagram and the effective low-energy dynamics of the system. This model naturally explains the basic phenomenology of the high- T_c superconductors from the insulating to the underdoped and the optimally doped region.

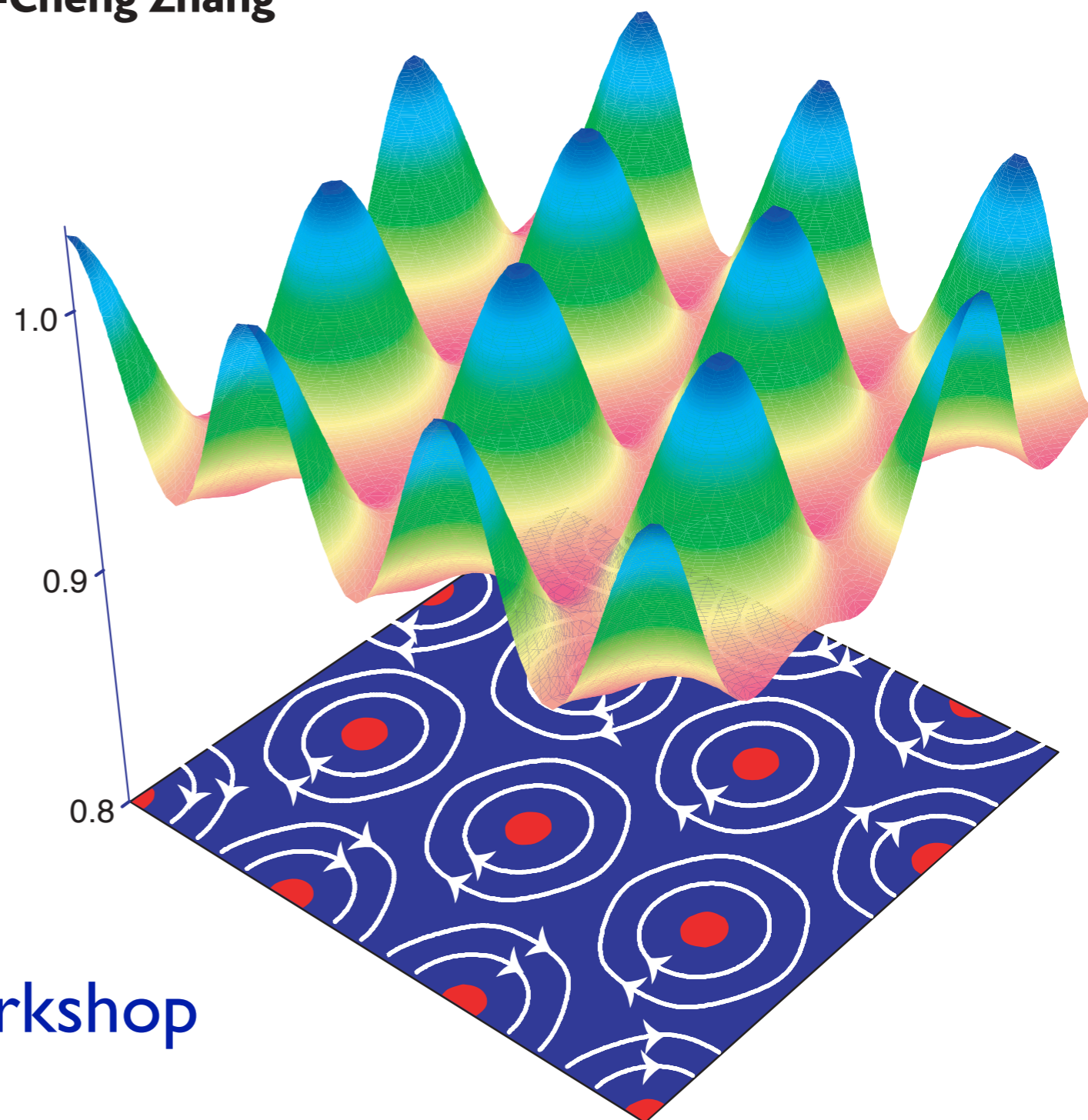
Science **275**, 5303 (1997)

Shoucheng Zhang Memorial Workshop
May 4, 2019

Tuning Order in Cuprate Superconductors

Subir Sachdev and Shou-Cheng Zhang

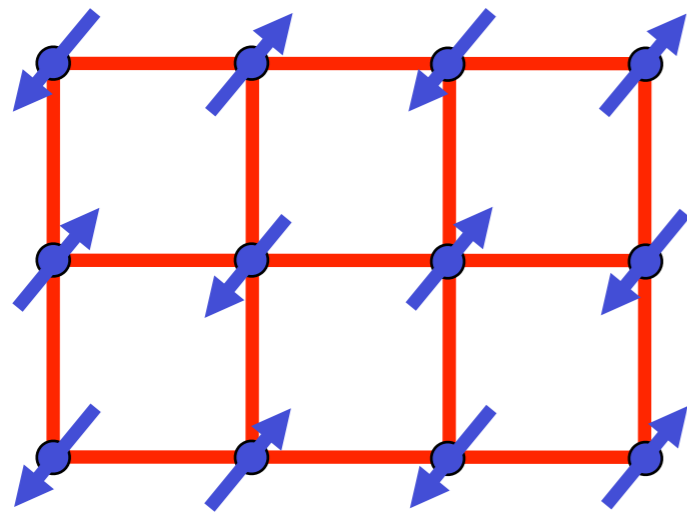
Science **295**, 452 (2002)



Shoucheng Zhang Memorial Workshop
May 4, 2019

Quantum criticality in a frustrated square lattice antiferromagnet

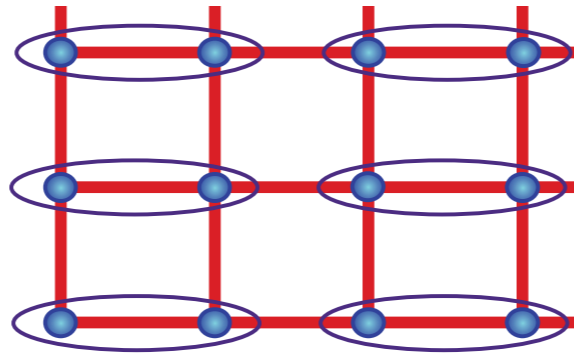
N. Read and S. Sachdev, PRL **62**, 1694 (1989)



$$\langle z_\alpha \rangle \neq 0$$

Néel state

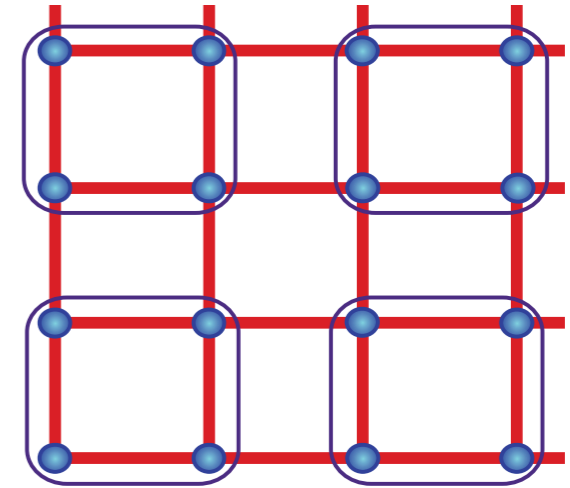
$$\vec{N} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$



$$\langle z_\alpha \rangle = 0$$

Valence bond solid (VBS) state, V_x, V_y
with a nearly gapless, emergent “photon”

or



s_c

s

The critical theory has a $SO(5)$ global symmetry, and the Néel and VBS order parameters transform as a $SO(5)$ fundamental.

$$(N_x, N_y, N_z, V_x, V_y)$$

Theory of a Planckian metal with a remnant large Fermi surface

Shoucheng Zhang Memorial Workshop
May 4, 2019

Aavishkar Patel and Subir Sachdev,
to appear



Talk online: sachdev.physics.harvard.edu



Ordinary metals and quasiparticles

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\text{eq}} \sim \frac{\hbar E_F^3}{U^2 (k_B T)^2} \quad , \quad \text{as } T \rightarrow 0,$$

where U is the strength of interactions, and E_F is the Fermi energy.

- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau} \sim U^2 T^2 \quad \text{with } \tau \sim \tau_i$$



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- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau} \sim U^2 T^2 \quad \text{with } \tau \sim \tau_{\text{eq}}$$

- These times are much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau \sim \tau_{\text{eq}} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \rightarrow 0.$$



Remarkable recent observation of
'Planckian' strange metal transport in cuprates,
pnictides, magic-angle graphene, and
ultracold atoms: the resistivity, ρ , is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

independent of the strength of interactions!



Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity is associated with a universal scattering time $\approx \hbar/(k_B T)$.

Universal T -linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142-147

A. Legros^{1,2}, S. Benhabib³, W. Tabis^{3,4}, F. Laliberté¹, M. Dion¹, M. Lizaire¹, B. Vignolle³, D. Vignolles³, H. Raffy⁵, Z. Z. Li⁵, P. Auban-Senzier⁵, N. Doiron-Leyraud¹, P. Fournier^{1,6}, D. Colson², L. Taillefer^{1,6*} and C. Proust^{3,6*}

arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,^{1,2} Tristin Metz,² Christopher Eckberg,² Kevin Kirshenbaum,² Alex Hughes,² Renxiong Wang,² Limin Wang,² Shanta R. Saha,² I-Lin Liu,^{2,3,4} Nicholas P. Butch,^{2,4} Zhonghao Liu,^{5,6} Sergey V. Borisenko,⁵ Peter Y. Zavalij,⁷ and Johnpierre Paglione^{2,8}

Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,^{1,*} Debanjan Chowdhury,^{1,*} Daniel Rodan-Legrain,¹ Oriol Rubies-Bigordà,¹ Kenji Watanabe,² Takashi Taniguchi,² T. Senthil,^{1,†} and Pablo Jarillo-Herrero^{1,†}

arXiv:1901.03710

Bad metallic transport in a cold atom Fermi-Hubbard system

Science **363**, 379–382 (2019)

Peter T. Brown¹, Debayan Mitra¹, Elmer Guardado-Sanchez¹, Reza Nourafkan², Alexis Reymbaut², Charles-David Hébert², Simon Bergeron², A.-M. S. Tremblay^{2,3}, Jure Kokalj^{4,5}, David A. Huse¹, Peter Schauf^{1*}, Waseem S. Bakr^{1†}

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11 \text{ kbar}$	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of T -linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, *Nature Physics* **15**, 142 (2019)

The complex SYK model

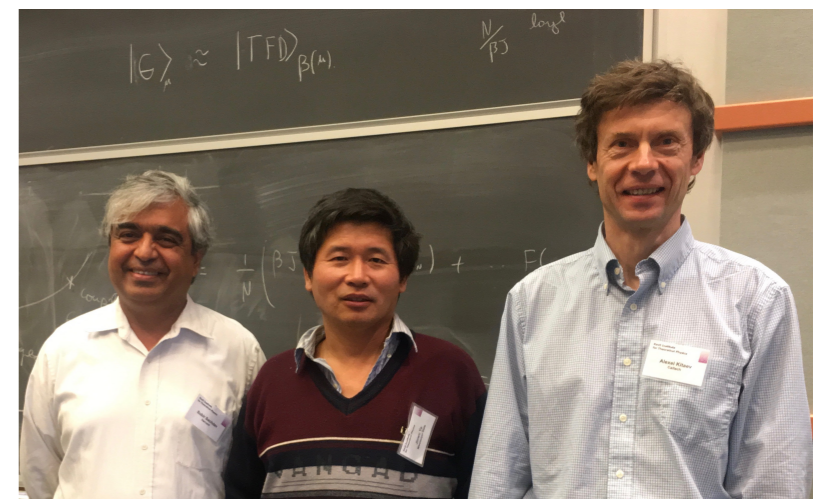
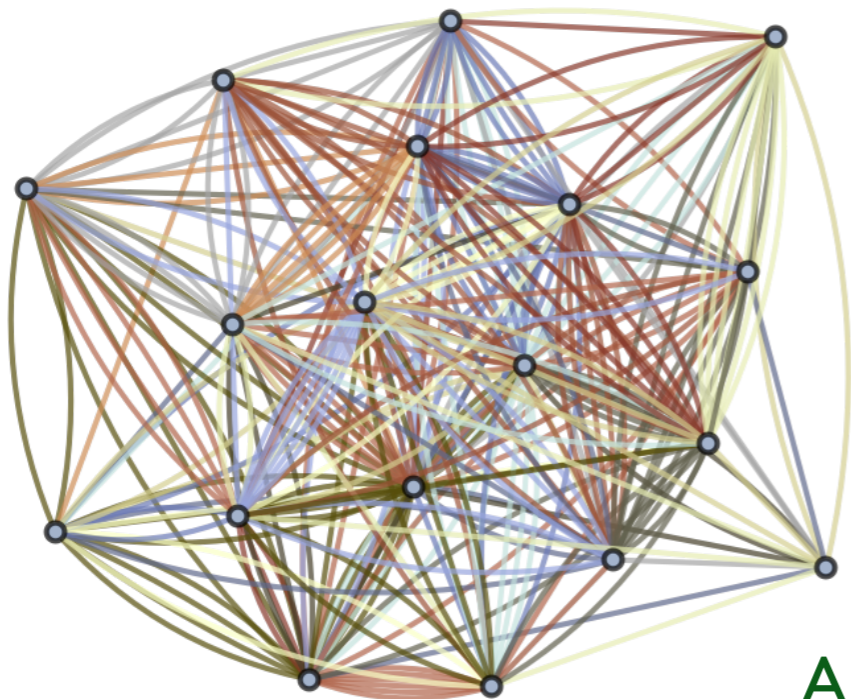
(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} + \epsilon \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$Q = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta; \gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta; \gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta; \gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

The large N limit is given by the sum of “melon” Feynman graphs

For long times $\tau > 0$

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle = e^{\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

$$\langle c_\alpha^\dagger(\tau) c_\alpha(0) \rangle = e^{-\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter $\mathcal{E} = \mathbb{C} \epsilon/U$ determines the particle-hole asymmetry.

In a Fermi liquid,

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle = \langle c_\alpha^\dagger(\tau) c_\alpha(0) \rangle = \tilde{A}/\tau$$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)



The complex SYK model

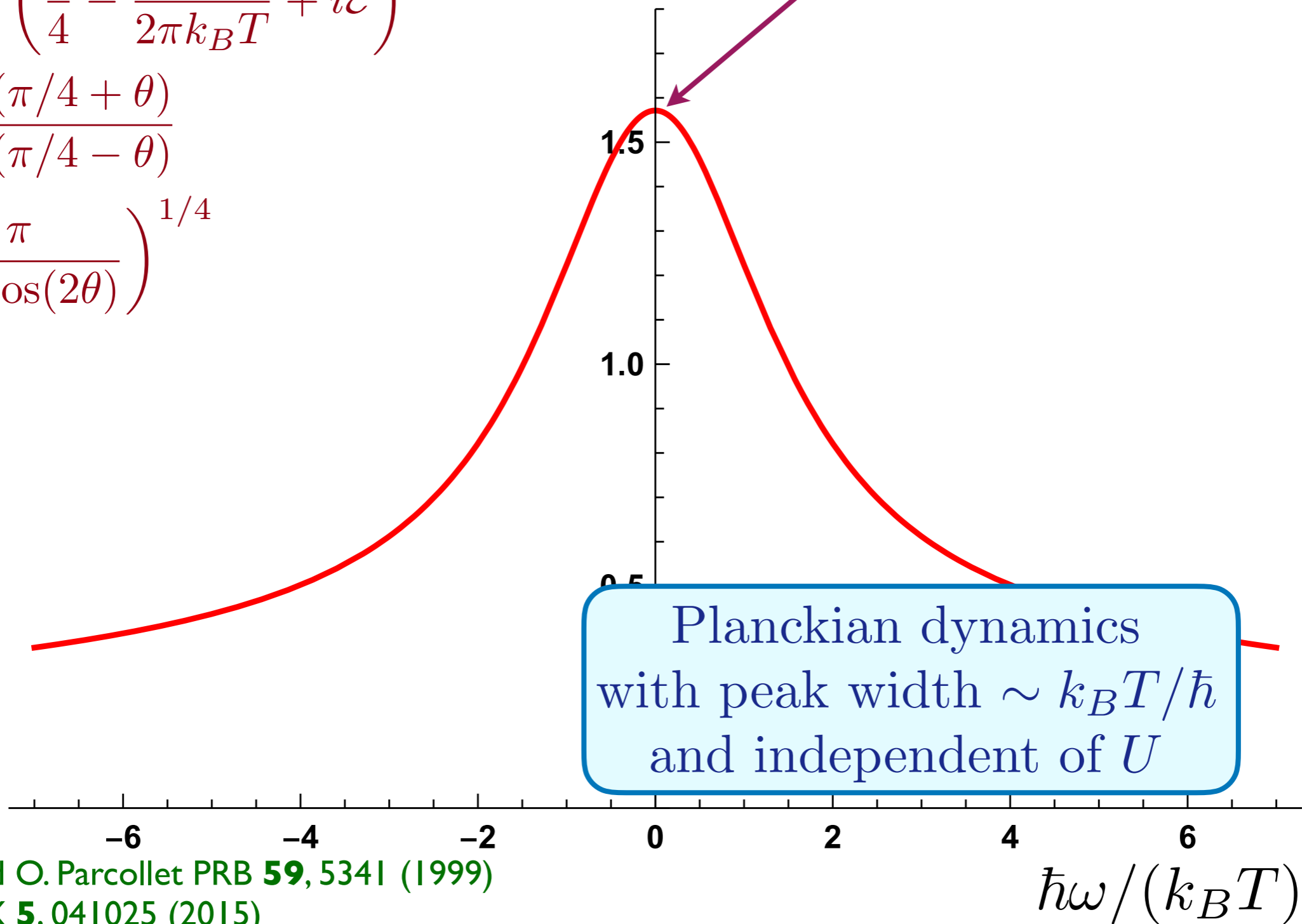
$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$

$$G_{\text{SYK}}^R(\epsilon, \hbar\omega/(k_B T)) = \frac{-iC e^{-i\theta} \Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{(2\pi T)^{1/2} \Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E} = 0$$



The complex SYK model

$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$

$$G_{\text{SYK}}^R(\epsilon, \hbar\omega/(k_B T)) =$$

$$\frac{-iC e^{-i\theta} \Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{(2\pi T)^{1/2} \Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}$$

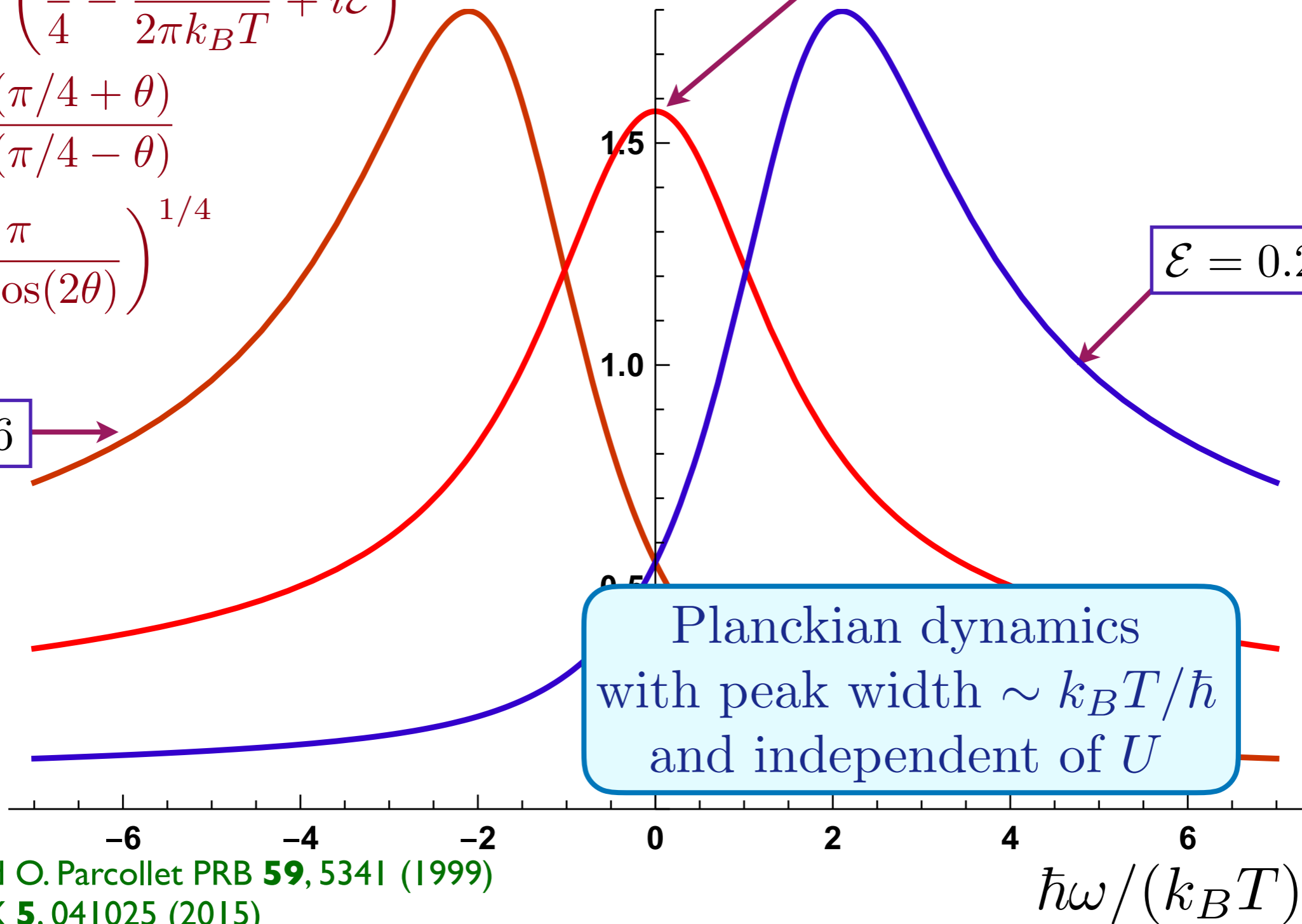
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$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E} = 0$$

$$\mathcal{E} = -0.26$$

$$\mathcal{E} = 0.26$$



Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U

The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} + \epsilon \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$

A generalized SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)
 ϵ_k has a range of values of width W .

$\hbar\omega/(k_B T)$ scaling behavior of SYK holds for $W^2/U \ll k_B T \ll U$.

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Pengfei Zhang, PRB **96**, 205138 (2017)

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018)

Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A lattice SYK model

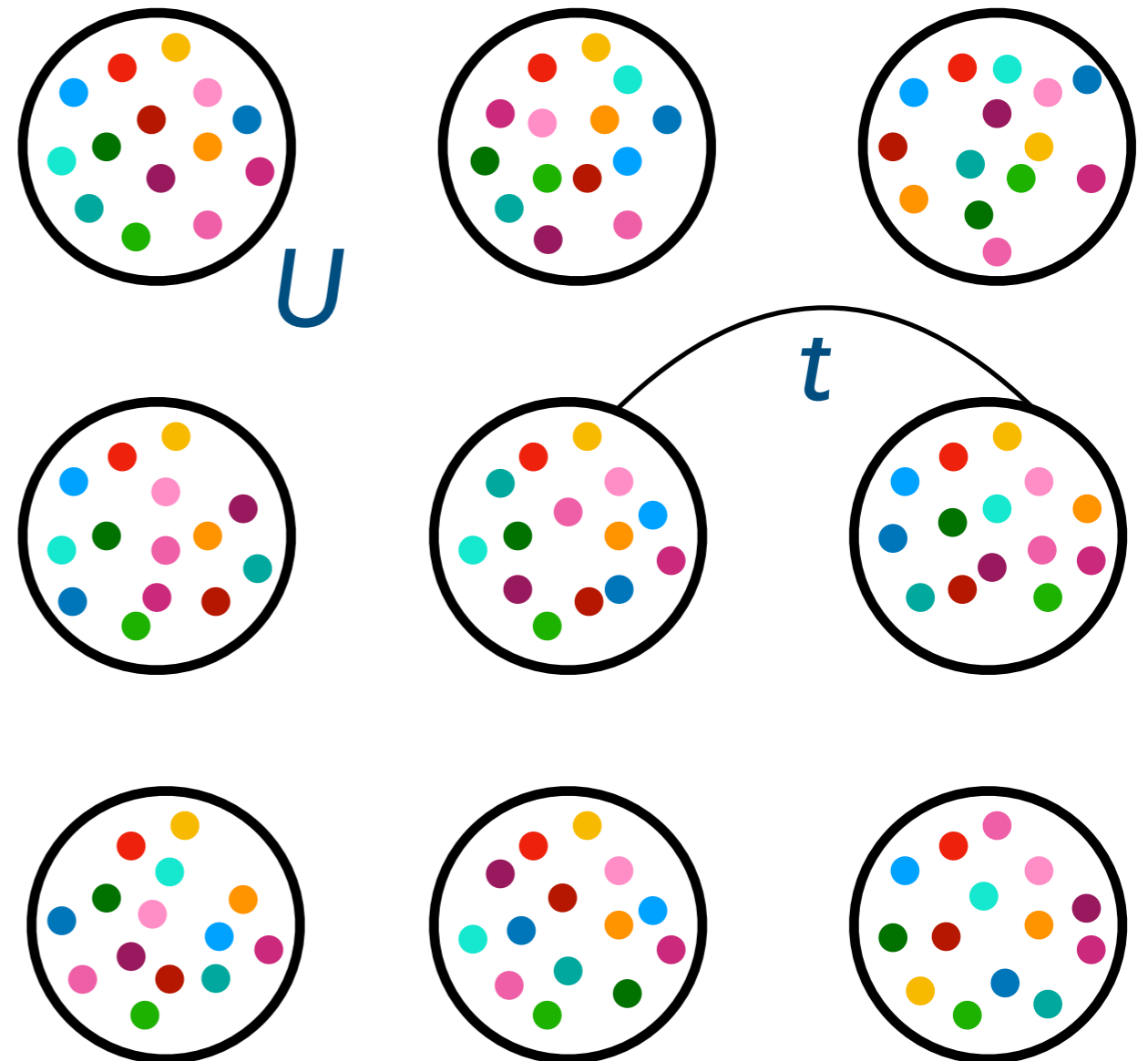
$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

Choose U on-site,
and uncorrelated between sites;
yields ‘incoherent metal’
with no Fermi surface
for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k}, \omega) = G_{\text{SYK}}(\epsilon, \hbar\omega/(k_B T))$$

independent of \mathbf{k} ,
and linear-in- T resistivity $\rho \gg h/e^2$

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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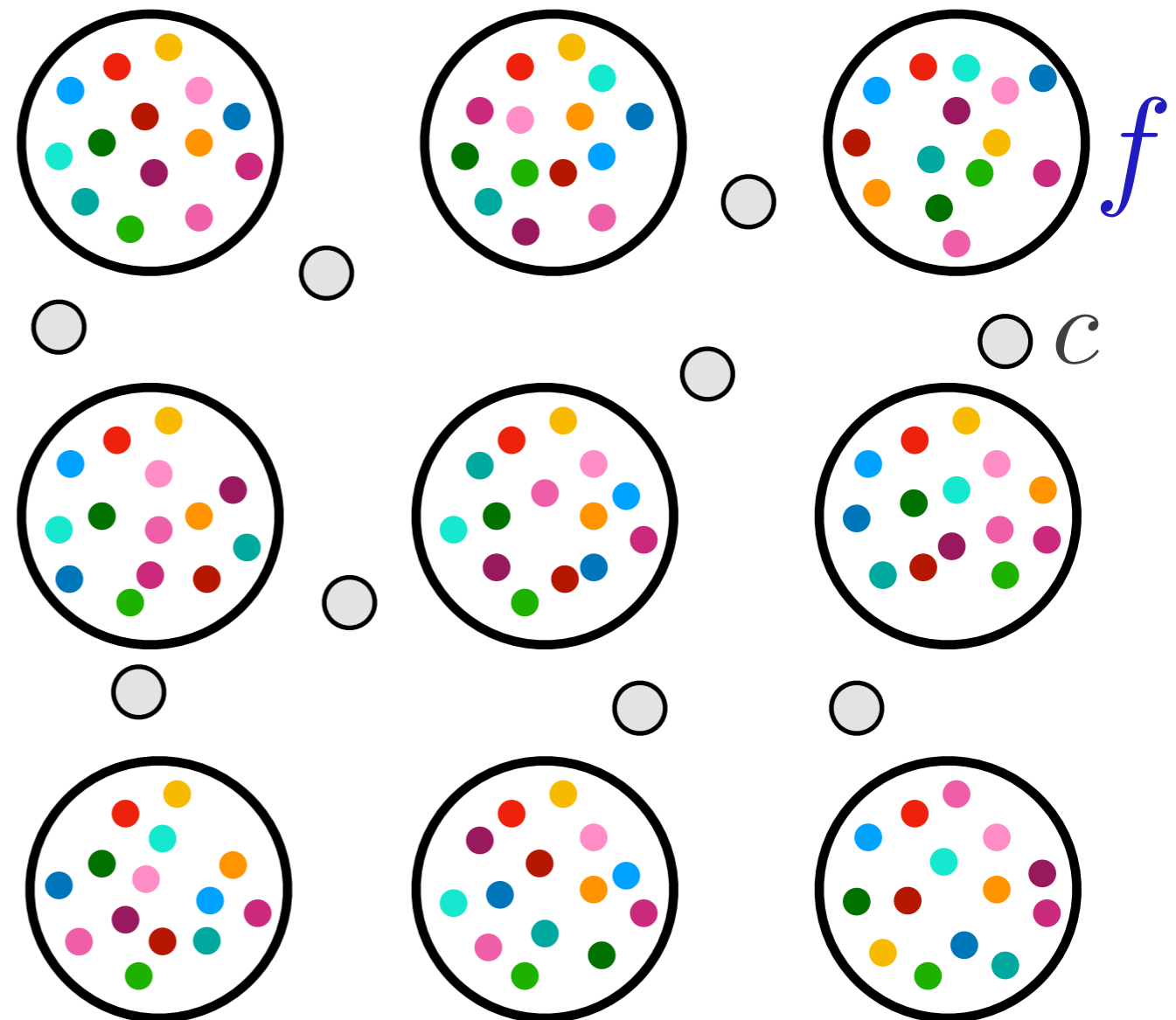
Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A Kondo-SYK model

Mobile electrons (c) coupled to SYK quantum islands (f) with exchange interactions.

Has a regime where the c electrons form a marginal Fermi liquid with a linear-in- T resistivity dependent upon interaction strength, and a small Fermi surface which does not count the f electrons.



Similar results for many earlier 'marginal Fermi liquid' and holographic models

A lattice SYK model

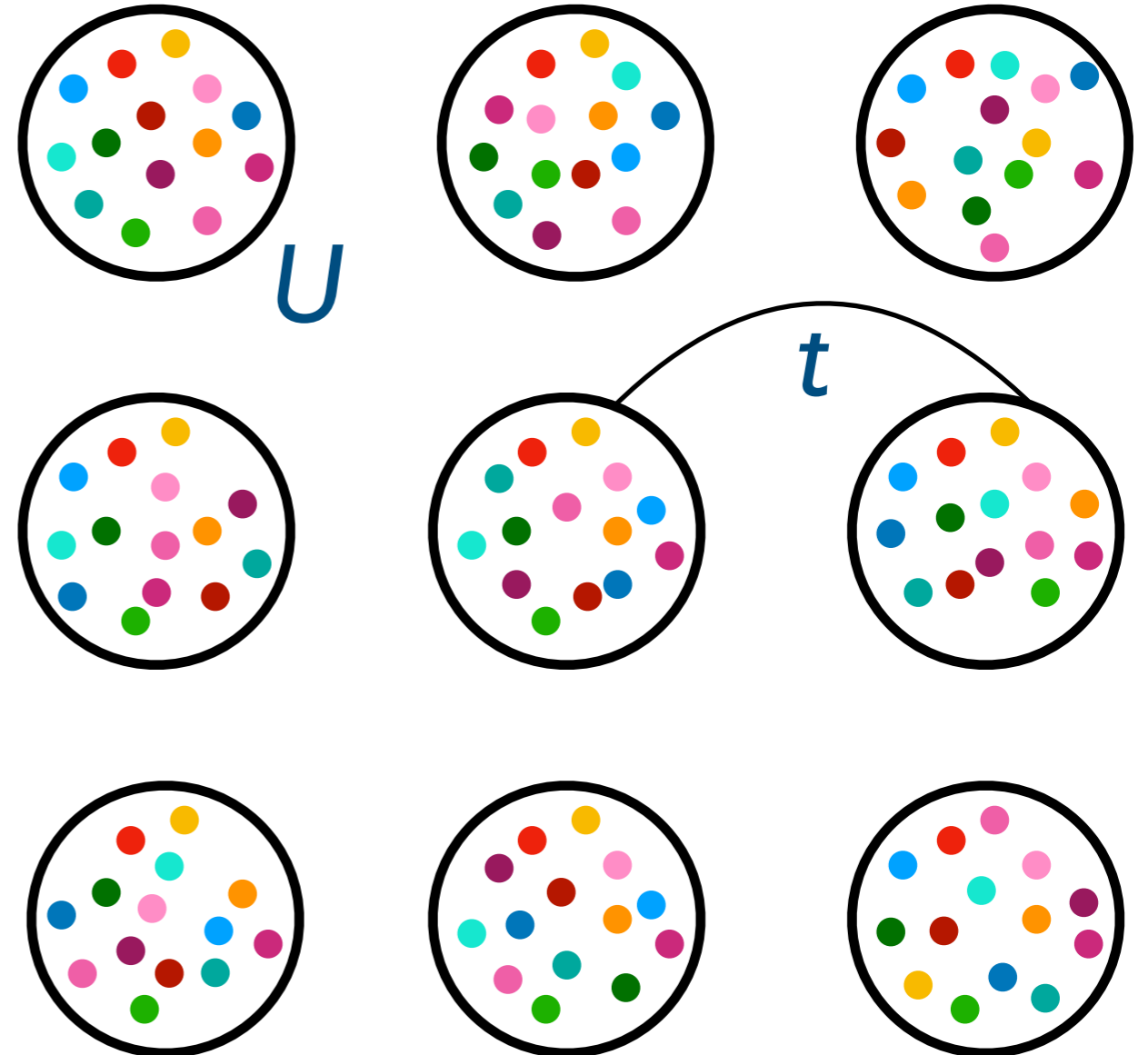
$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

Choose U on-site,
and uncorrelated between sites;
yields ‘incoherent metal’
with no Fermi surface
for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k}, \omega) = G_{\text{SYK}}(\epsilon, \hbar\omega / (k_B T))$$

independent of \mathbf{k} ,
and linear-in- T resistivity $\rho \gg h/e^2$

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Pengfei Zhang, PRB **96**, 205138 (2017)

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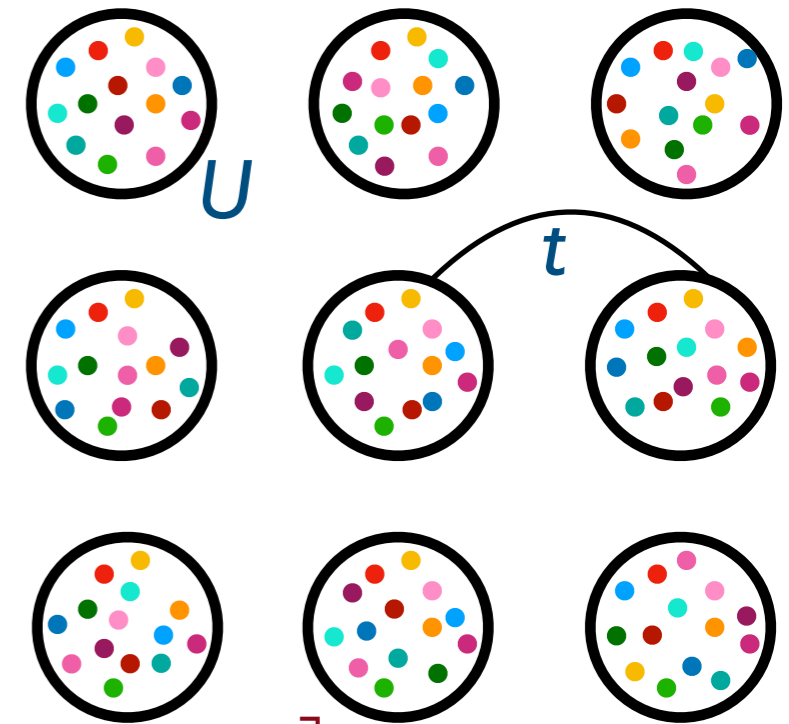
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

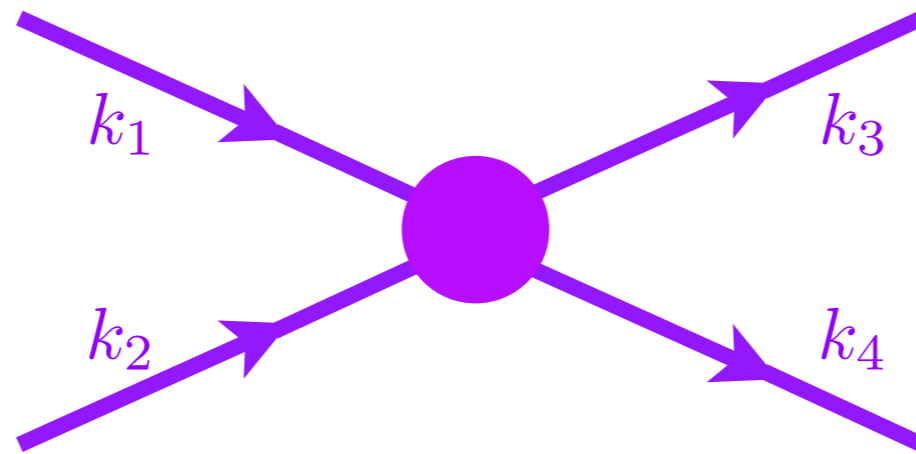
$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$
 ϵ_k has a bandwidth W .

Rewriting of lattice model of incoherent and bad metal in momentum space



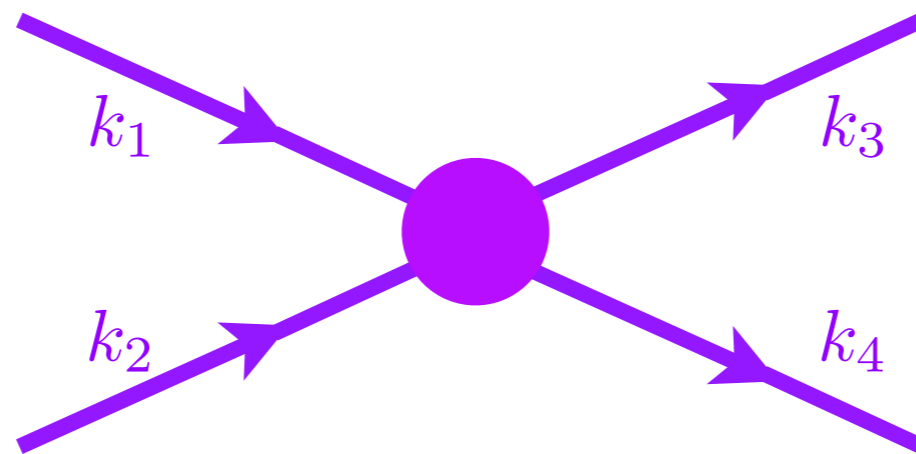
$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right]$$

Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we “integrate these out” in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k , which we have already accounted for.

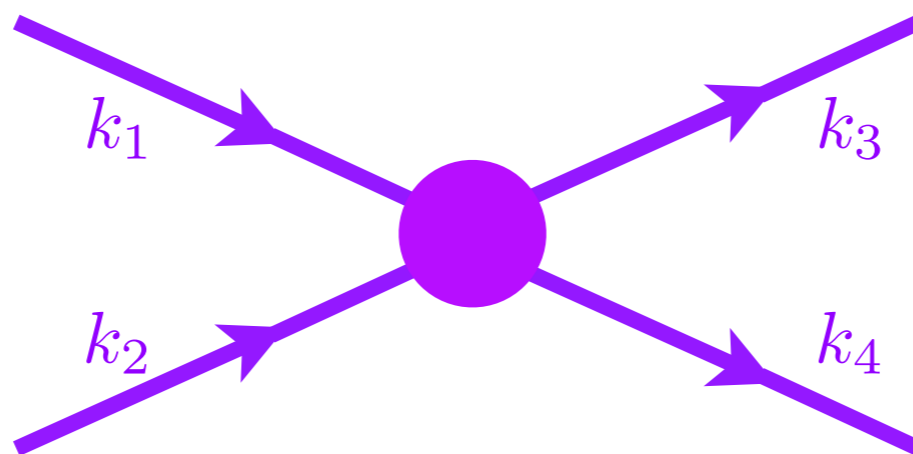
Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we “integrate these out” in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k , which we have already accounted for.

Keep only the interactions resonant in the bare quasiparticle energy with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

Resonant SYK model



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Keep only the interactions resonant in the bare quasiparticle energy with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

This is precisely the effective Hamiltonian method, when low energy states are separated from high energy states by a gap; we are assuming it can also apply in a gapless system.

Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

The random k_i dependence of U allows only interactions resonant in the bare quasiparticle energies

with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

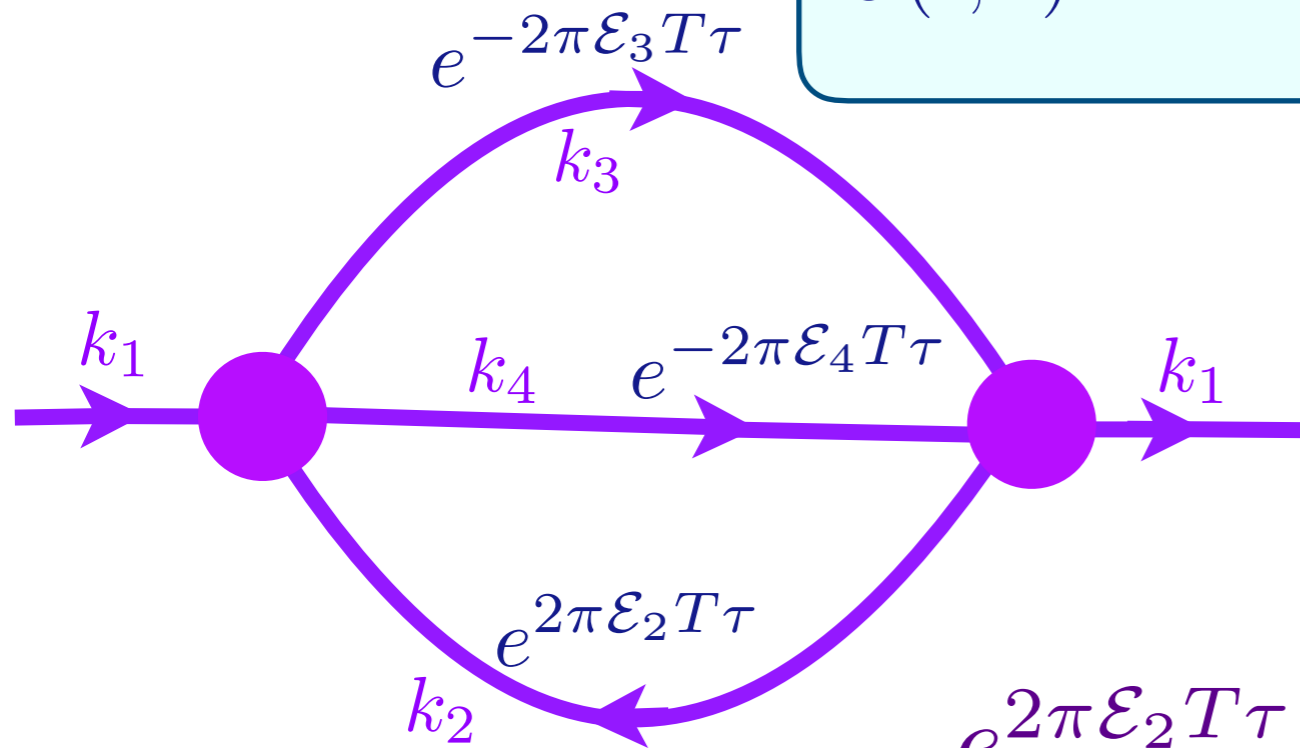
$$\overline{U(k_1, k_2, k_3, k_4)U^*(k_5, k_6, k_7, k_8)} = \\ U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right] \\ \times \left[\delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} + \epsilon_{k_6} - \epsilon_{k_7} - \epsilon_{k_8}) \right]$$

This implies off-site interactions with correlations which decay with a power-law in space.

Resonant SYK model

Conformal Green's function at $T > 0$ must have the form

$$G(\epsilon, \tau) \sim e^{-2\pi\mathcal{E}T\tau} \left(\frac{T}{\sin(\pi T\tau)} \right)^{1/2}, \quad 0 < \tau < 1/T.$$



$$e^{2\pi\mathcal{E}_2 T\tau} e^{-2\pi\mathcal{E}_3 T\tau} e^{-2\pi\mathcal{E}_4 T\tau} = e^{-2\pi\mathcal{E}_1 T\tau}$$

if

$$\mathcal{E}_a = \mathbb{C}\epsilon_a/U$$

and

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$$

SYK behavior in a Planckian metal as $T \rightarrow 0$ with a remnant Fermi surface:
 $G(k, \omega) = G_{\text{SYK}}(\epsilon_k, \hbar\omega/(k_B T))$,
 with $\mathcal{E}_k = \mathbb{C}\epsilon_k/U$

Incoherent metal

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$
$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi\mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter \mathcal{E} is independent of k ,
and determined by the total density

Planckian metal with remnant Fermi surface

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathbb{C} \epsilon_k / U} \frac{A}{\sqrt{\tau}}$$
$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathbb{C} \epsilon_k / U} \frac{A}{\sqrt{\tau}}$$

The particle-hole asymmetry changes as
we cross the Fermi surface

The complex SYK model

$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$

$$G_{\text{SYK}}^R(\epsilon, \hbar\omega/(k_B T)) = \frac{-iC e^{-i\theta} \Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{(2\pi T)^{1/2} \Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$-\text{Im}G^R(\omega)$ $\mathcal{E} = 0$

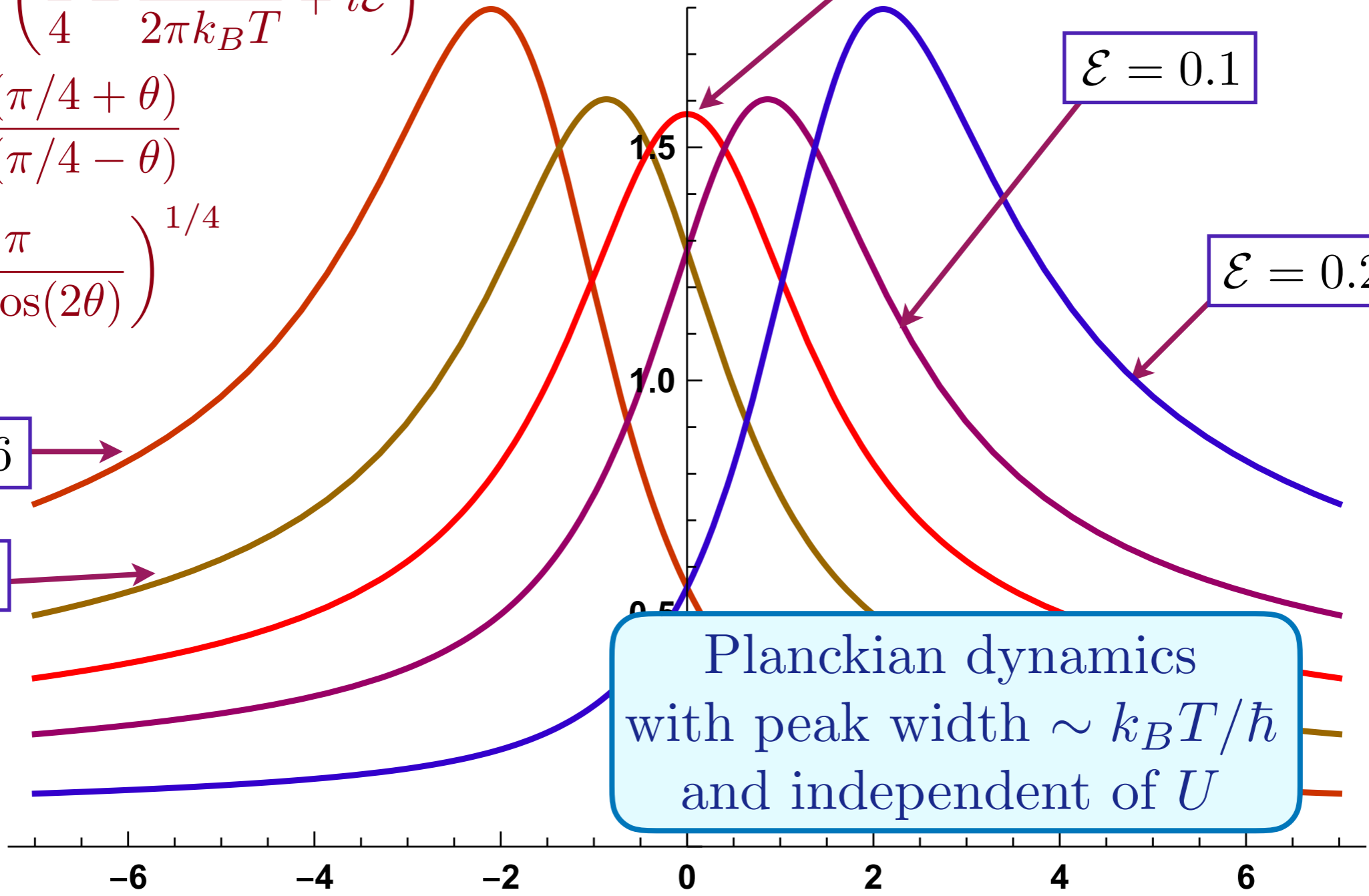
$\mathcal{E} = 0.1$

$\mathcal{E} = 0.26$

$\mathcal{E} = -0.26$

$\mathcal{E} = -0.1$

Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U



The complex SYK model

$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$

$$G_{\text{SYK}}^R(\epsilon, \hbar\omega/(k_B T)) =$$

$$\frac{-iC e^{-i\theta} \Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{(2\pi T)^{1/2} \Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)}\right)^{1/4}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E} = 0$$

$$\mathcal{E} = 0.1$$

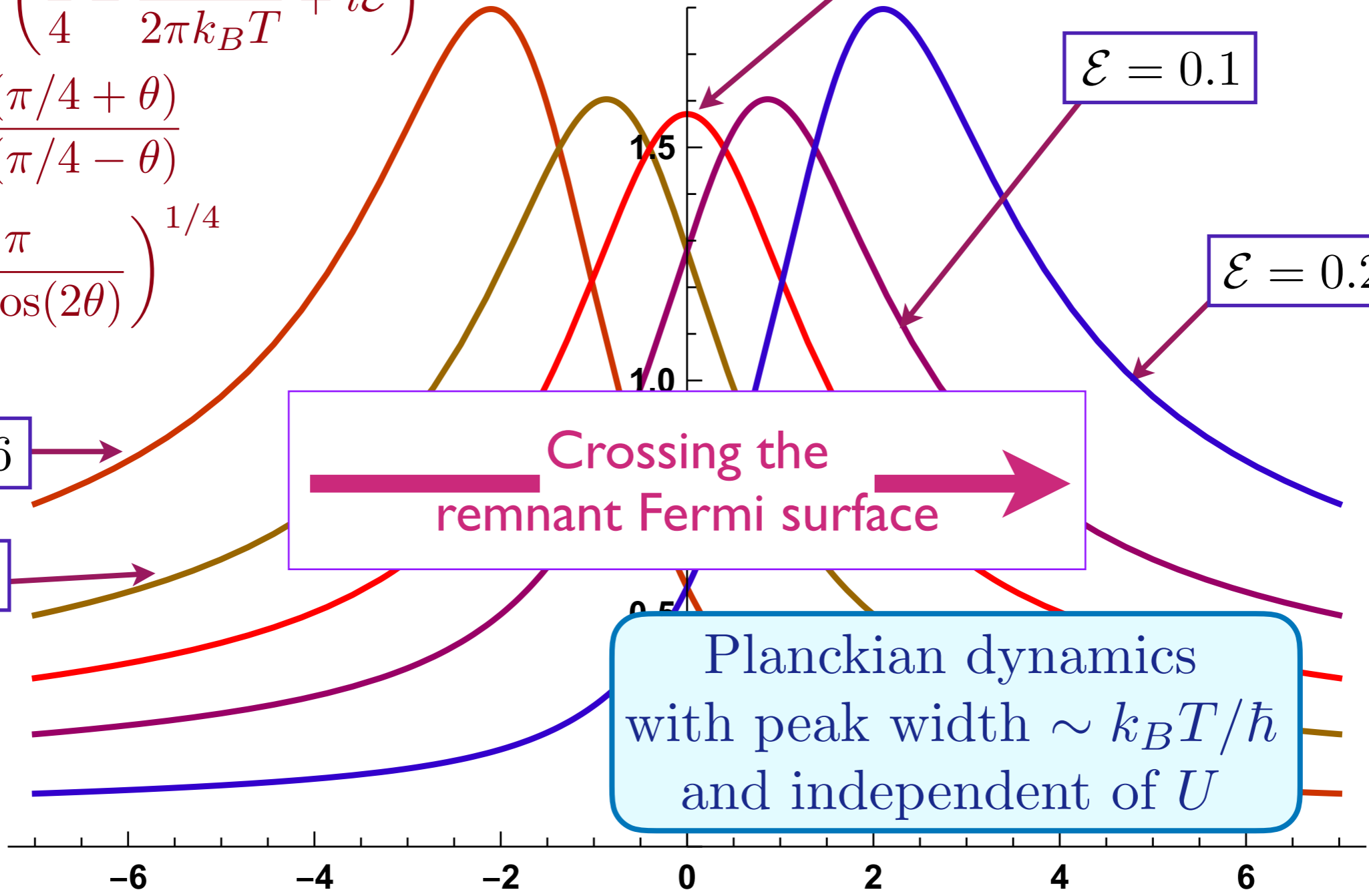
$$\mathcal{E} = 0.26$$

$$\mathcal{E} = -0.26$$

$$\mathcal{E} = -0.1$$

Crossing the remnant Fermi surface

Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U



Resonant SYK model

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

The random k_i dependence of U allows only
interactions resonant in the bare quasiparticle energies
with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

Resistivity of a Planckian metal as $T \rightarrow 0$

From the Kubo formula, in the large N limit

$$\sigma = \frac{Ne^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\text{Im} G_{\text{SYK}}^R \left(\epsilon, \frac{\omega}{T} \right) \right]^2 \text{sech}^2 \left(\frac{\omega}{2T} \right)$$

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$$\rho = \frac{m^*}{ne^2} 2.71\mathbb{C} \frac{k_B T}{\hbar}, \quad \text{using } \mathcal{E} = \mathbb{C}\epsilon/U,$$

where

$$m^* = \frac{d V_{FS}}{\oint_{FS} |\mathbf{v}_F|},$$

where d is spatial dimensionality and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^* .

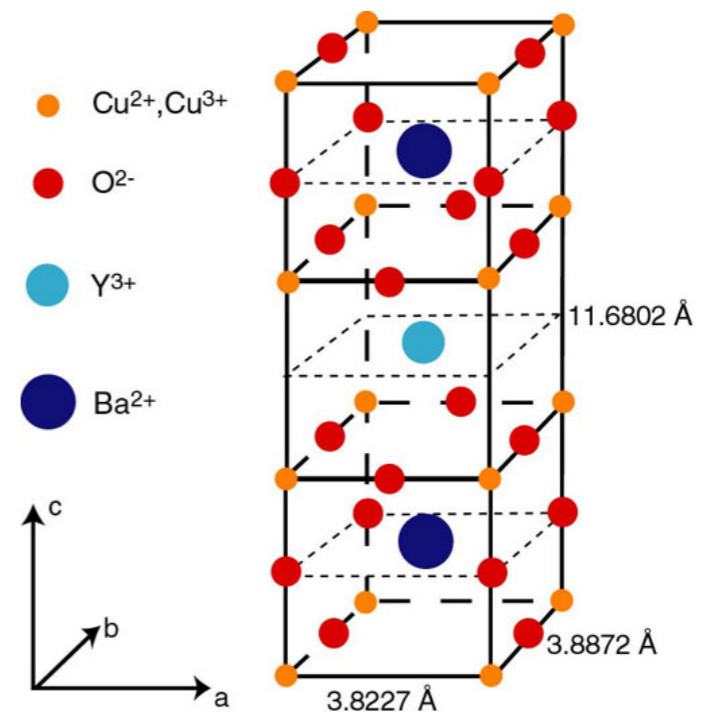
Resonant SYK model

Resistivity of a Planckian metal as $T \rightarrow 0$

$$\rho = \frac{m^*}{ne^2} 2.71\mathbb{C} \frac{k_B T}{\hbar}$$

Note that all explicit dependence on U has cancelled out!

The number \mathbb{C} is defined by $\mathcal{E}_k = \mathbb{C} \epsilon_k / U$ as $|\epsilon_k| \rightarrow 0$. This is determined by UV physics, and is weakly dependent upon the energy width of the interactions W_U / U .



Resonant SYK model

In an independent momentum shell model, $W_U/U \rightarrow 0$,

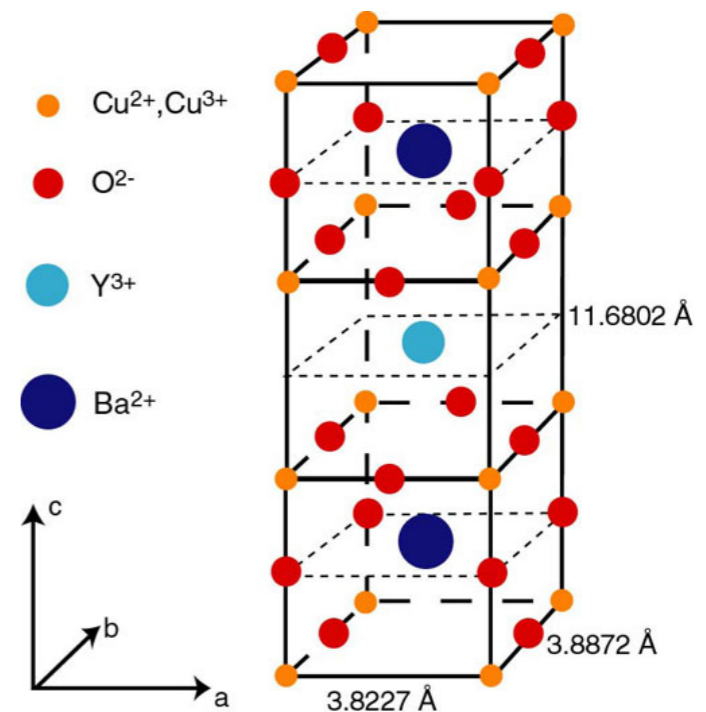
$$\overline{U(k_1, k_2, k_3, k_4)U^*(k_5, k_6, k_7, k_8)} =$$

$$U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right]$$

$$\times \left[\delta(\epsilon_{k_1} - \epsilon_{k_2})\delta(\epsilon_{k_2} - \epsilon_{k_3})\delta(\epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} - \epsilon_{k_6})\delta(\epsilon_{k_6} - \epsilon_{k_7})\delta(\epsilon_{k_7} - \epsilon_{k_8}) \right]$$

$\mathbb{C} = 0.41$ as in a single SYK model,
and we obtain a Planckian metal with

$$\rho = \frac{m^*}{ne^2} 1.11 \frac{k_B T}{\hbar}$$



Planckian metals with a remnant Fermi surface

- Planckian dynamics, *i.e.* $\hbar\omega/(k_B T)$ scaling of response functions and fastest possible local thermalization in a time $\sim \hbar/(k_B T)$, is realized in the ‘solvable’ SYK models.



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- The low energy quantum theory of charged black holes in Einstein-Maxwell theory co-incides with that of complex SYK models as the (Hawking) temperature $\rightarrow 0$.

S. Sachdev, arXiv:1902.04078

