# Gauge theory of optimal doping criticality in the cuprates

AharonFest
Stanford University
September 15, 2018

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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J. Phys. A: Math. Gen. **17** (1984) L197–L202. Printed in Great Britain

## LETTER TO THE EDITOR

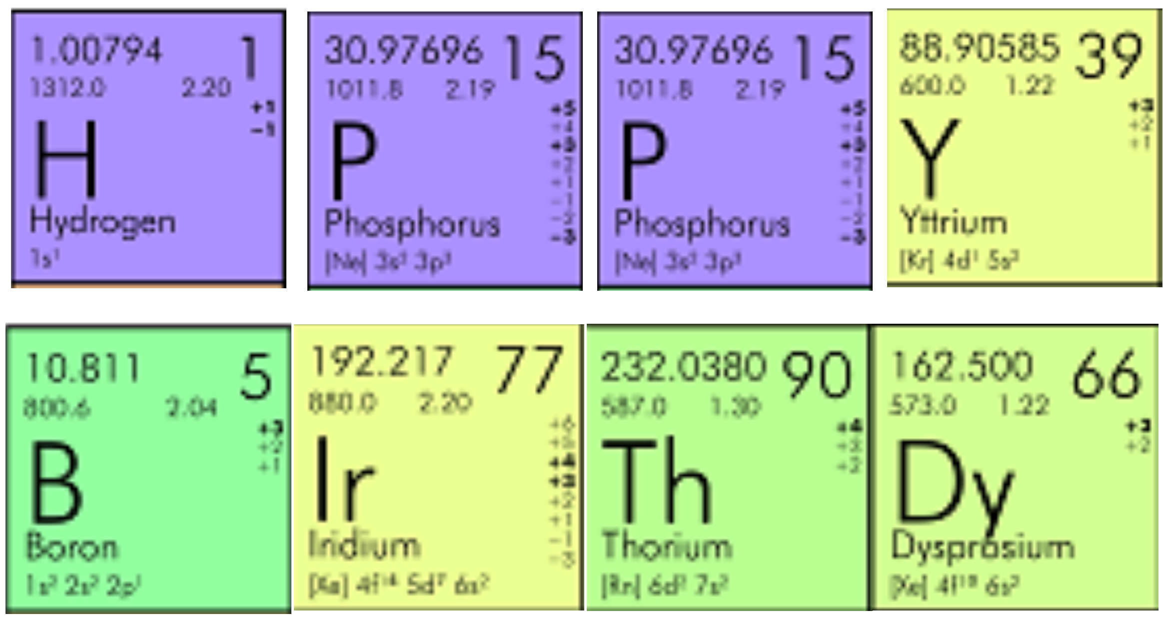
# Scaling at the percolation threshold above six dimensions

Amnon Aharony, Yuval Gefen and Aharon Kapitulnik

Department of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel

Received 20 July 1983

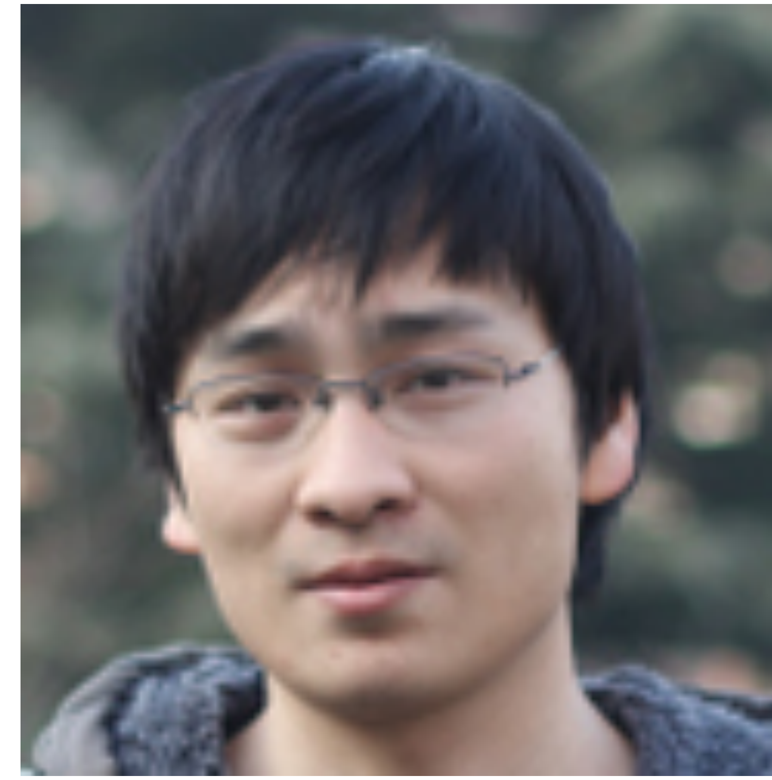
**Abstract.** The fractal dimensionality of the infinite cluster at the percolation threshold for dimensionalities $d>6$ is shown to be $D=4$ (rather than the naive finite size scaling prediction $D=d-2$). Similarly, the conductivity of a sample of size $L$ scales as $L^{-d}$ (rather than $L^{-6}$). This anomalous behaviour is related to a dangerous irrelevant variable, associated with the probability to have vertices of three bonds. The crossover to the 'homogeneous' behaviour occurs at length scales which are short compared with the correlation length. The 'links and blobs' picture is confirmed for $d>6$, and the size of the latter is estimated.


1.00794 1
H
Hydrogen
30.97696 15
P
Phosphorus
30.97696 15
P
Phosphorus
88.90585 39
Y
Yttrium
10.811 5
B
Boron
192.217 77
Ir
Iridium
232.0380 90
Th
Thorium
162.500 66
Dy
Dysprosium


AharonFest

Mathias Scheurer

Shubhayu Chatterjee



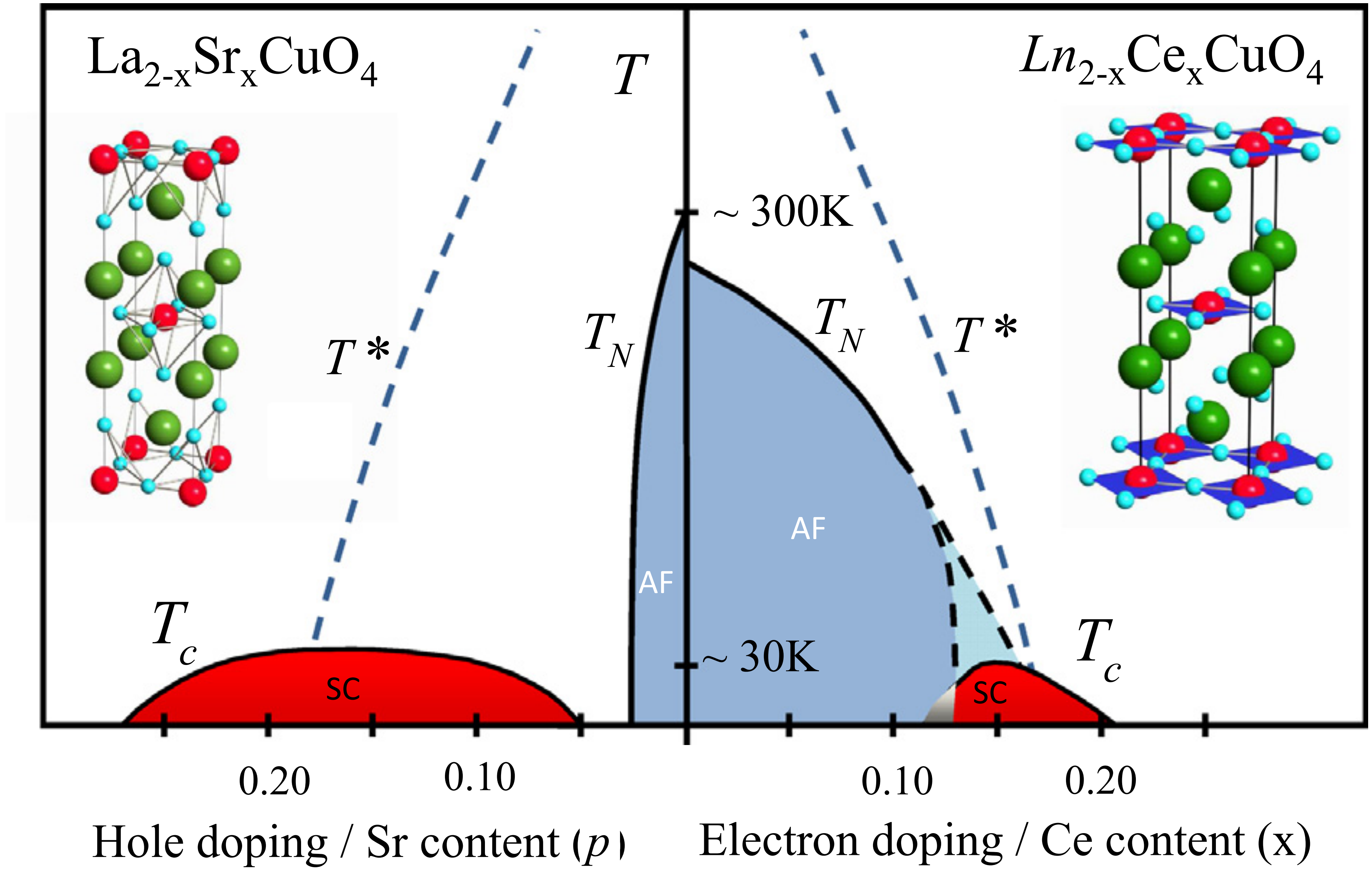
Wei Wu

Michel Ferrero

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero, Physical Review X **8**, 021048 (2018)

Antoine Georges



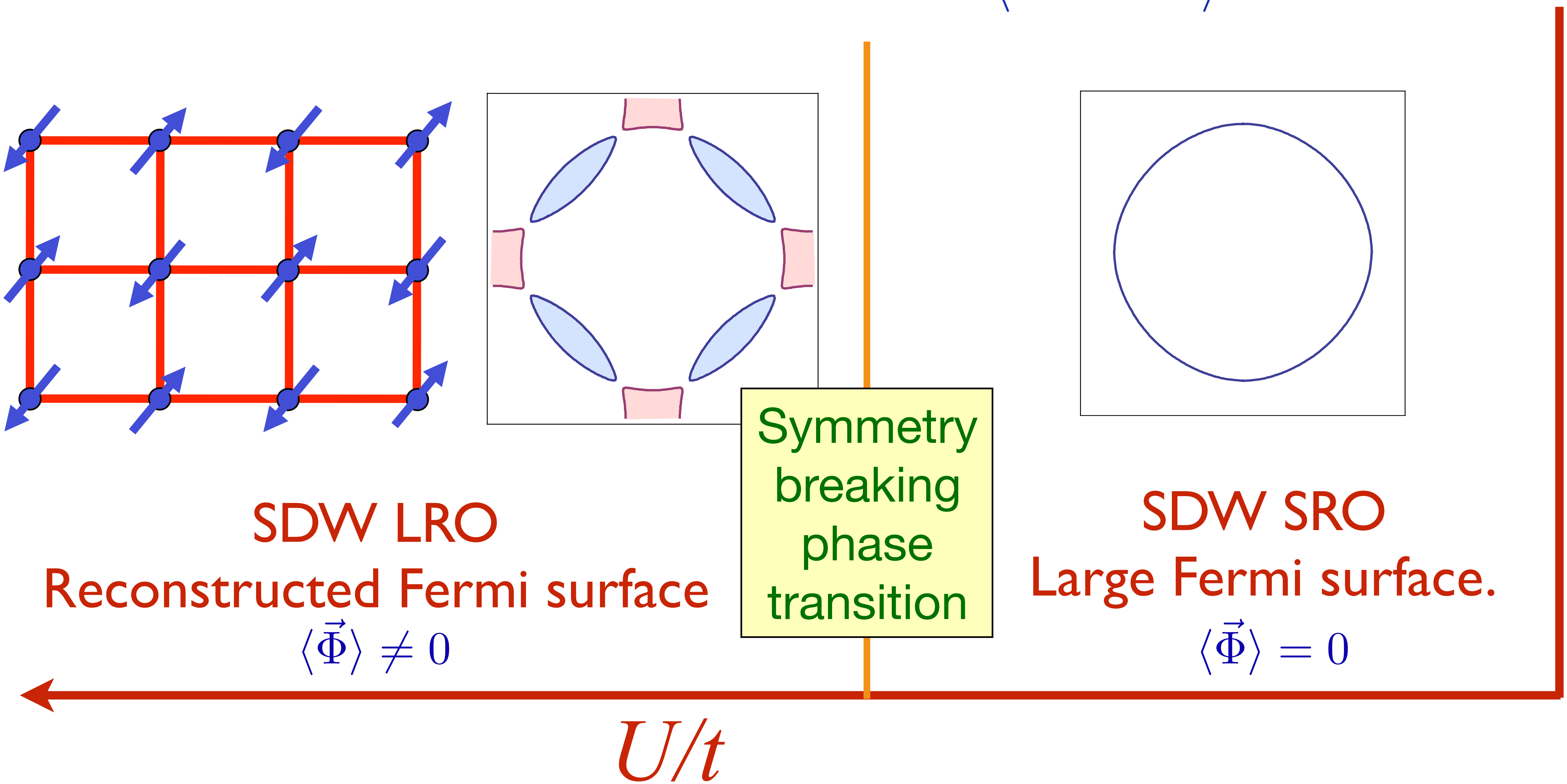
La2-xSr2CuO4
T
Ln2-xCexCuO4
~ 300K
T*
TN
TN
T*
AF
AF
Tc
Tc
~ 30K
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

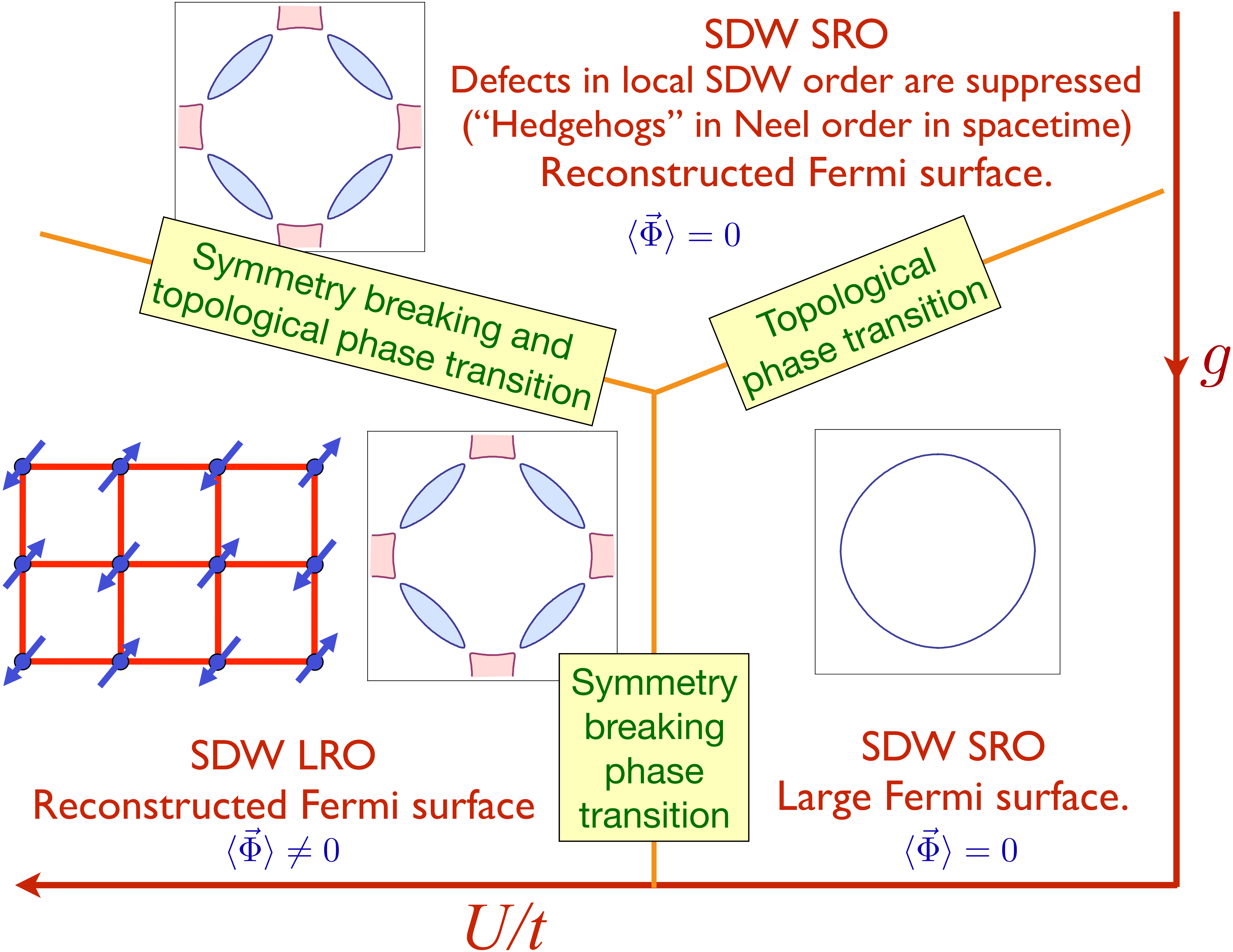
# Antiferromagnetism in the Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_i \left( n_{i\uparrow} - \frac{1}{2} \right) \left( n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^{\dagger} c_{i\alpha}$$

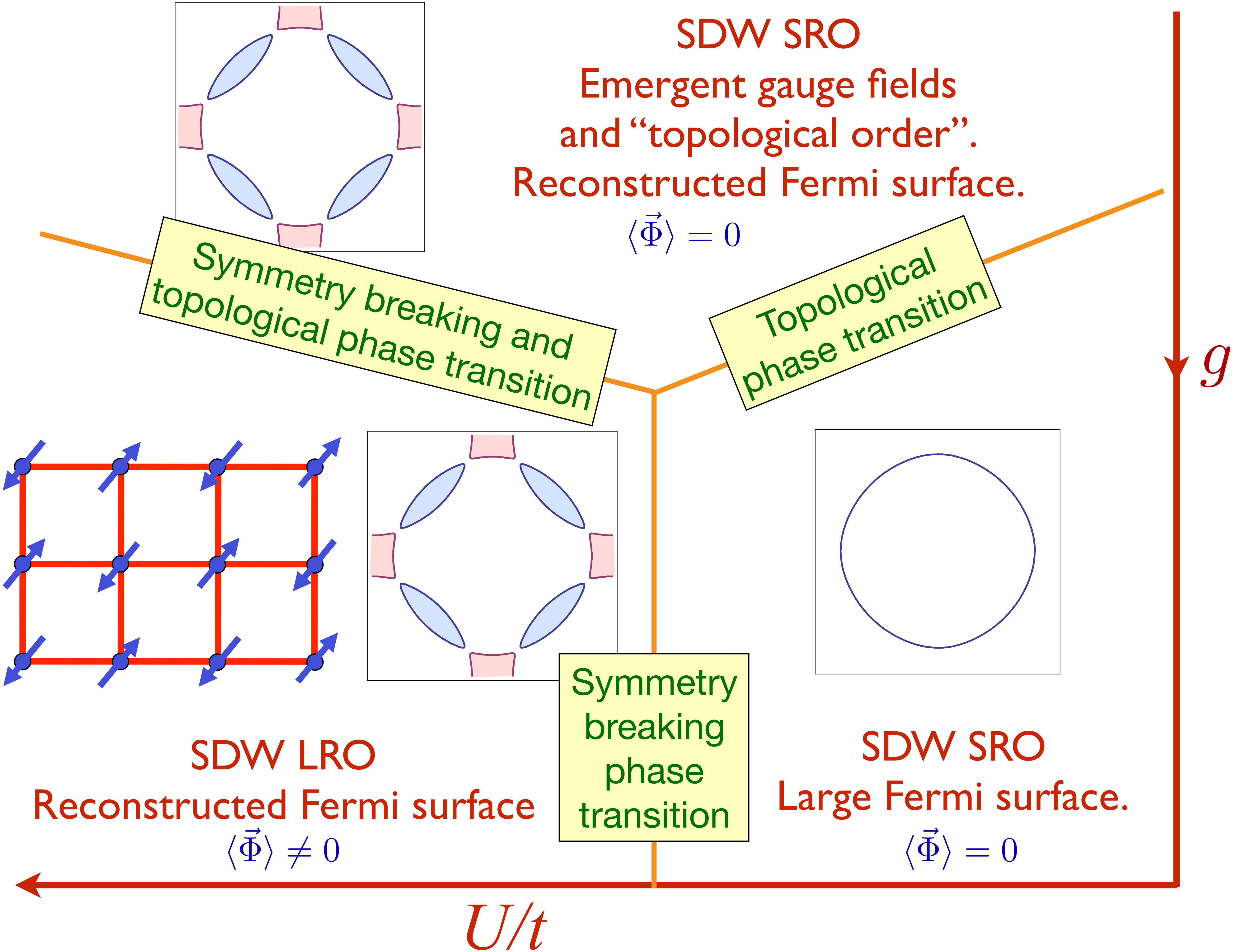
$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Mean-field theory with a spin density wave (SDW) order parameter $\vec{\Phi}_i = (-1)^{i_x + i_y} \left\langle c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta} \right\rangle / 2$





SDW SRO
Defects in local SDW order are suppressed
("Hedgehogs" in Neel order in spacetime)
Reconstructed Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
Symmetry breaking and topological phase transition
Topological phase transition
$g$
Symmetry breaking phase transition
SDW LRO
Reconstructed Fermi surface
$\langle \vec{\Phi} \rangle \neq 0$
SDW SRO
Large Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
$U/t$


SDW SRO
Emergent gauge fields
and "topological order".
Reconstructed Fermi surface.
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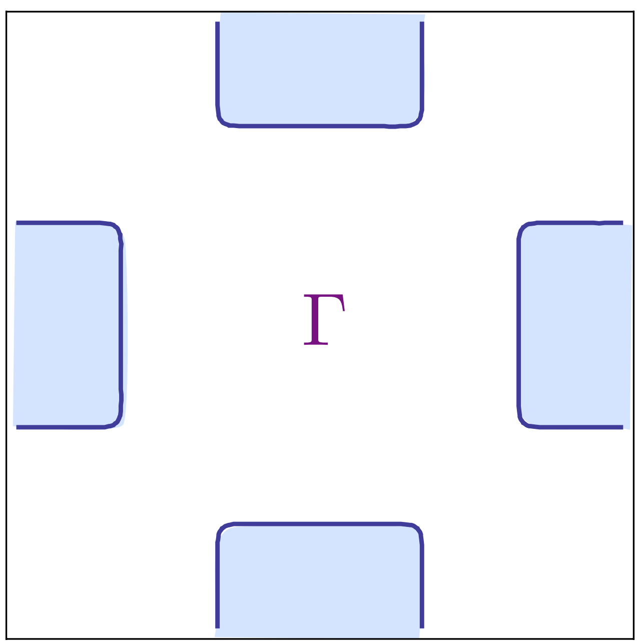
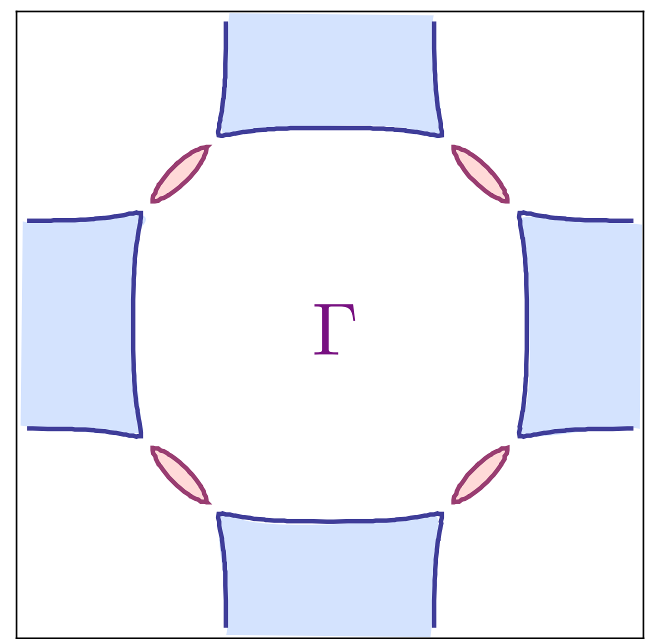
# Square lattice Hubbard model with electron doping

$\langle \vec{\Phi} \rangle \neq 0$ and large



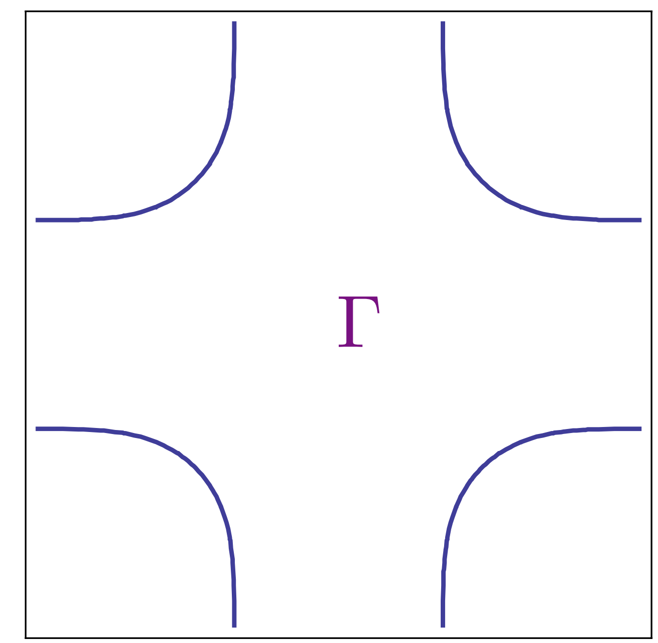

Metal with electron pockets

$\langle \vec{\Phi} \rangle \neq 0$ and small




Metal with electron and hole pockets

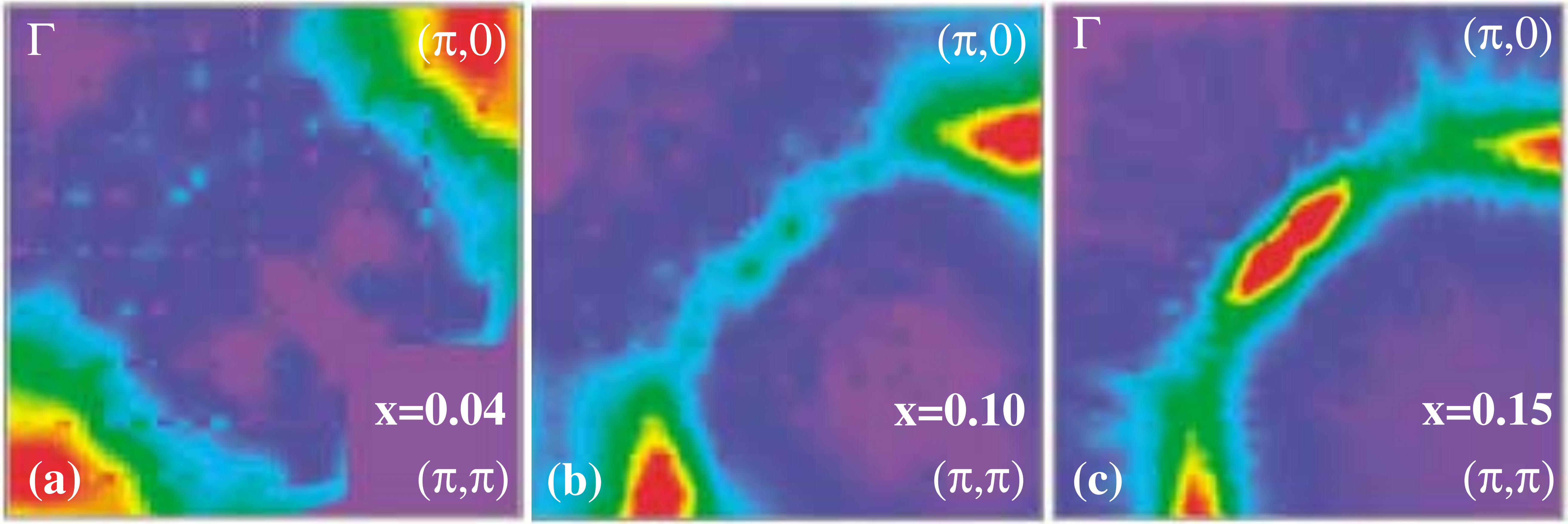
$\langle \vec{\Phi} \rangle = 0$




Metal with "large" Fermi surface

$s$

# $Nd_{2-x}Ce_xCuO_{4\pm\delta}$




Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura

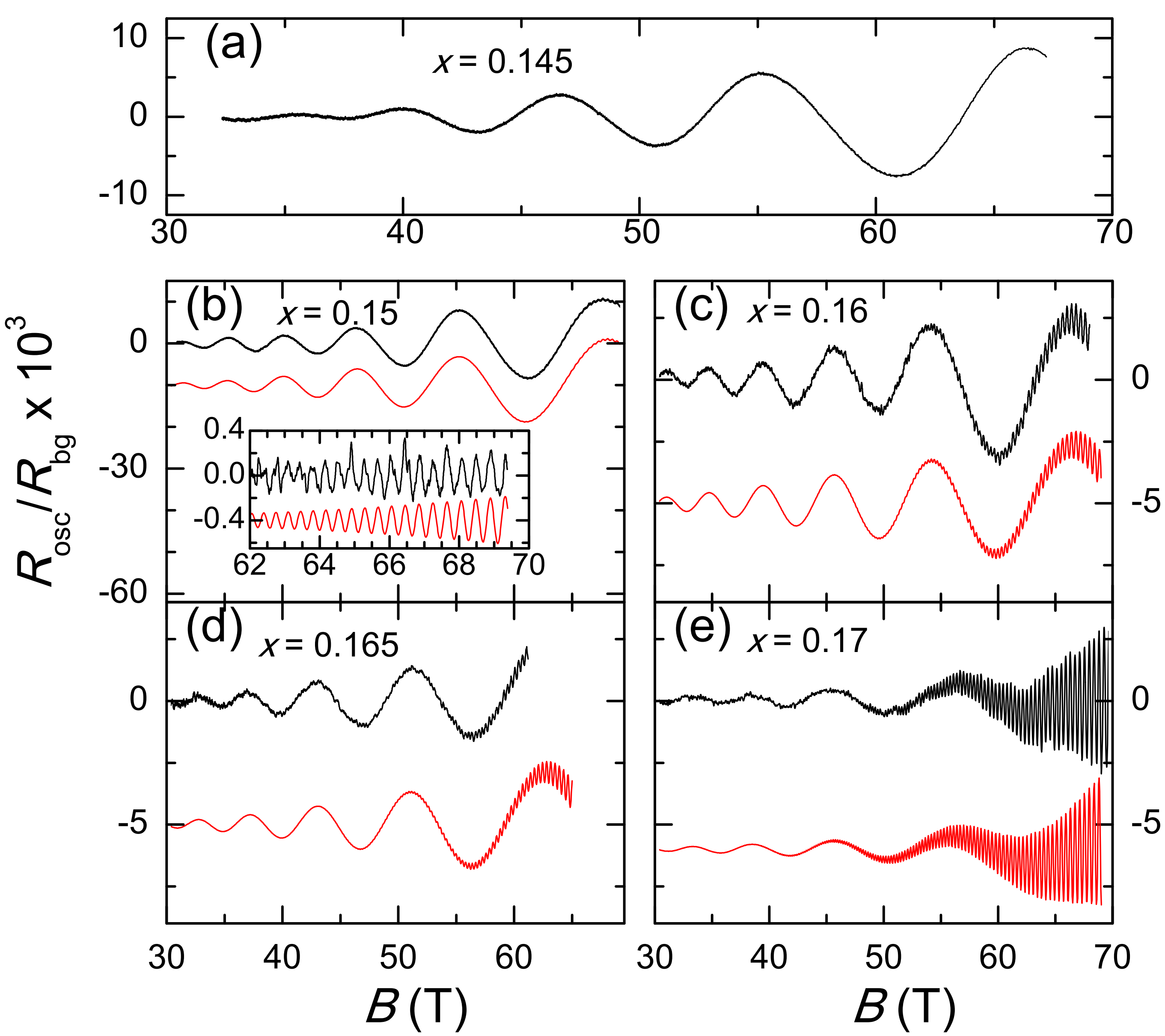
Phys. Rev. Lett. **88**, 257001 (2002)

PHYSICAL REVIEW B **92**, 094501 (2015)

**Correlation between Fermi surface transformations and superconductivity in the electron-doped high-$T_c$ superconductor $Nd_{2-x}Ce_xCuO_4$**

T. Helm,[1,*] M. V. Kartsovnik,[1,†] C. Proust,[2] B. Vignolle,[2] C. Putzke,[3,‡] E. Kampert,[3] I. Sheikin,[4] E.-S. Choi,[5] J. S. Brooks,[5] N. Bittner,[1,§] W. Biberacher,[1] A. Erb,[1,6] J. Wosnitza,[3] and R. Gross[1,6,‖]

- Quantum oscillations show the presence of small hole pockets up to a doping $x = 0.175$



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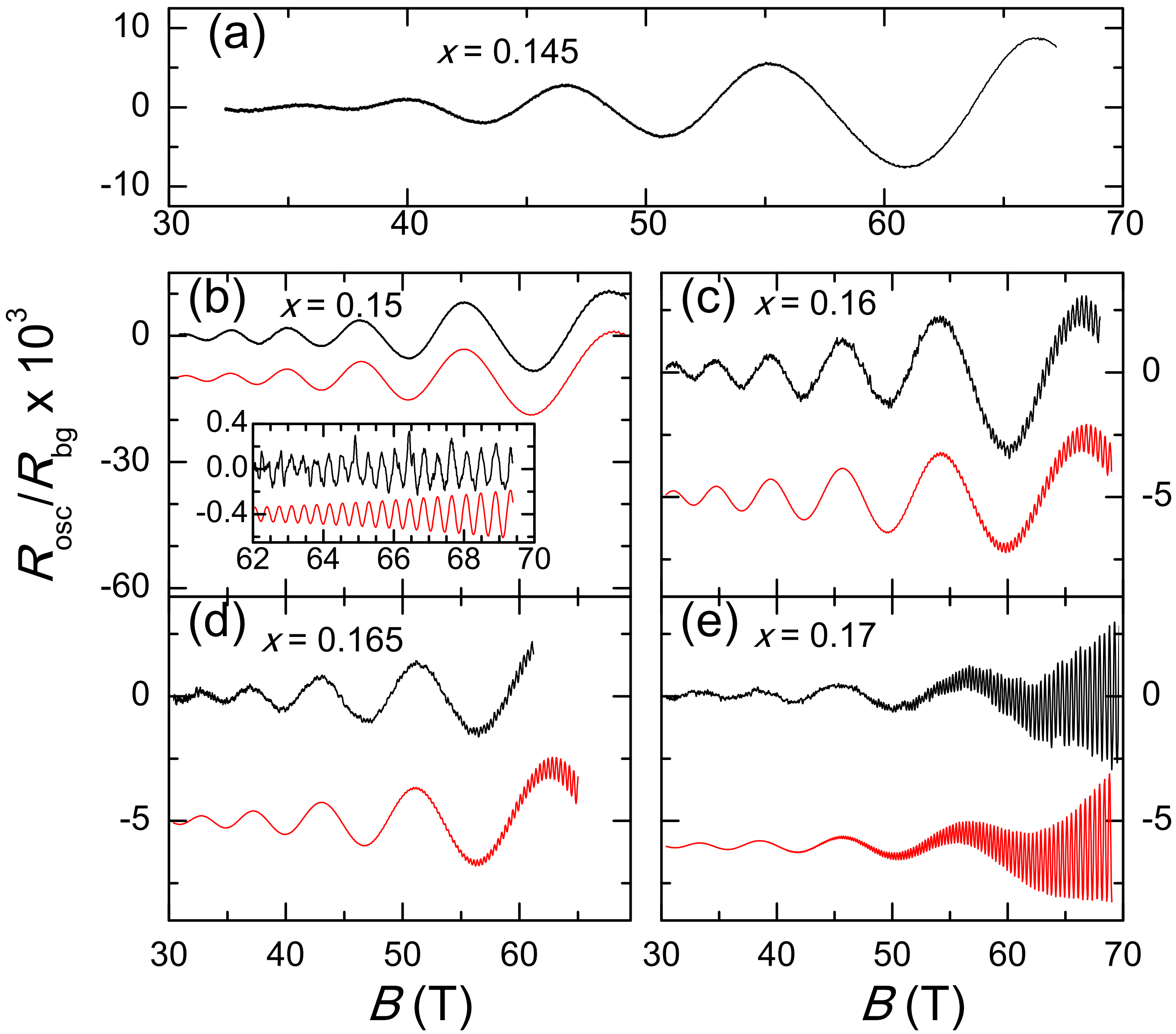


Although antiferromagnetic order disappears near $x = 0.14$, perhaps there is field-induced antiferromagnetic order up to $x = 0.175$ ?

## Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen

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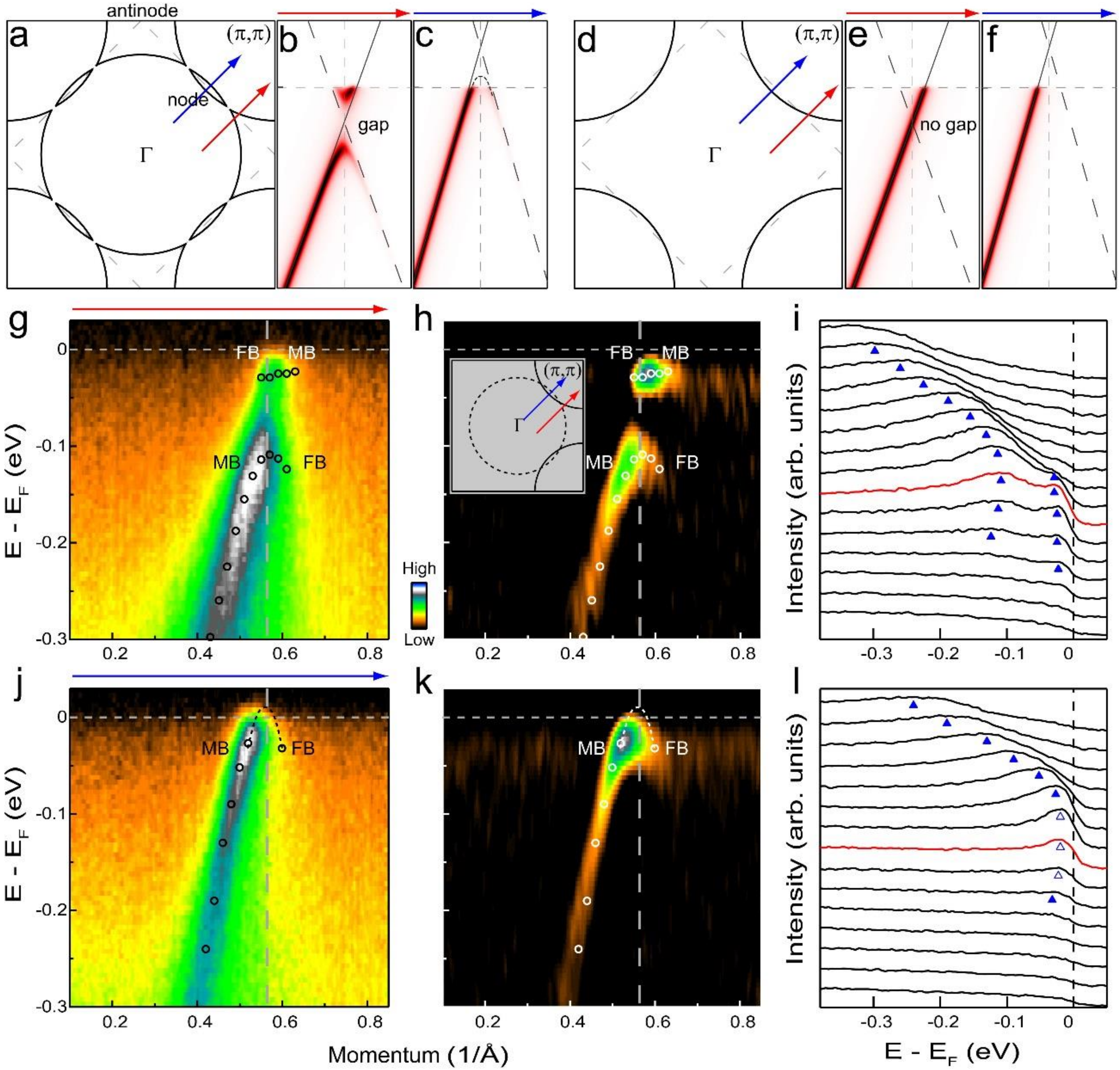
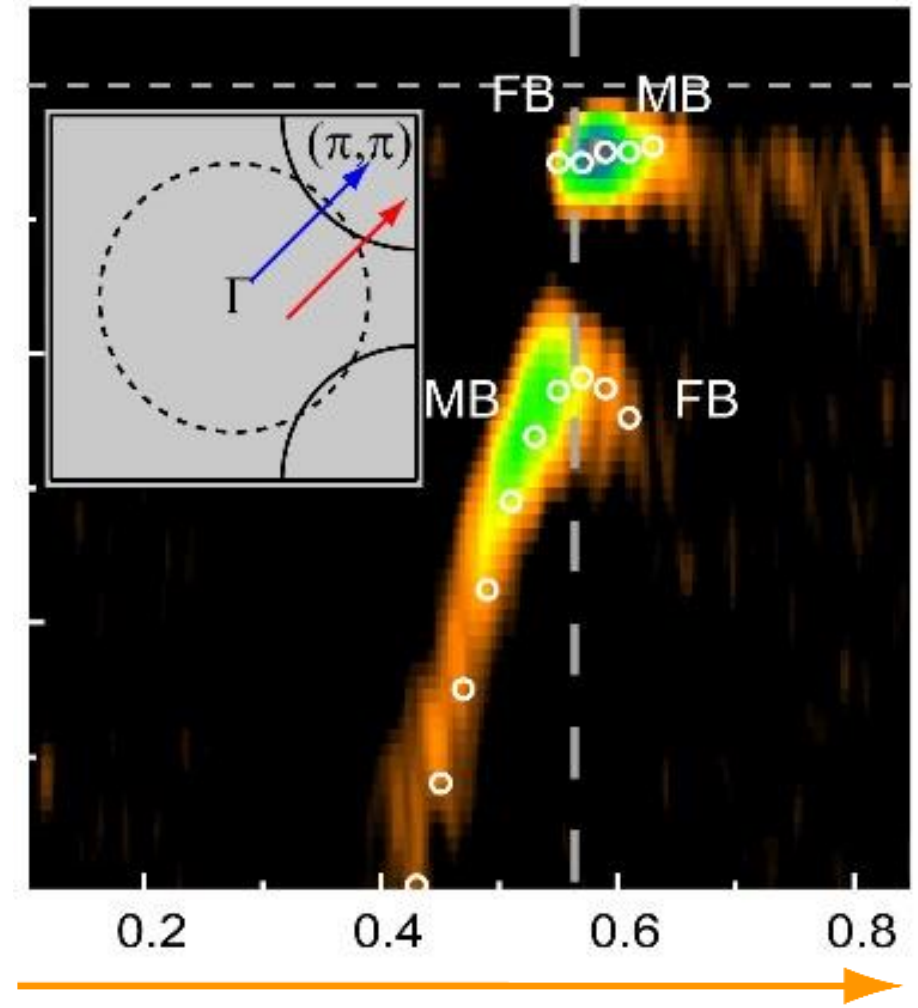
- "The totality of the data points to a mysterious order between $x = 0.14$ and $x = 0.17$, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin."

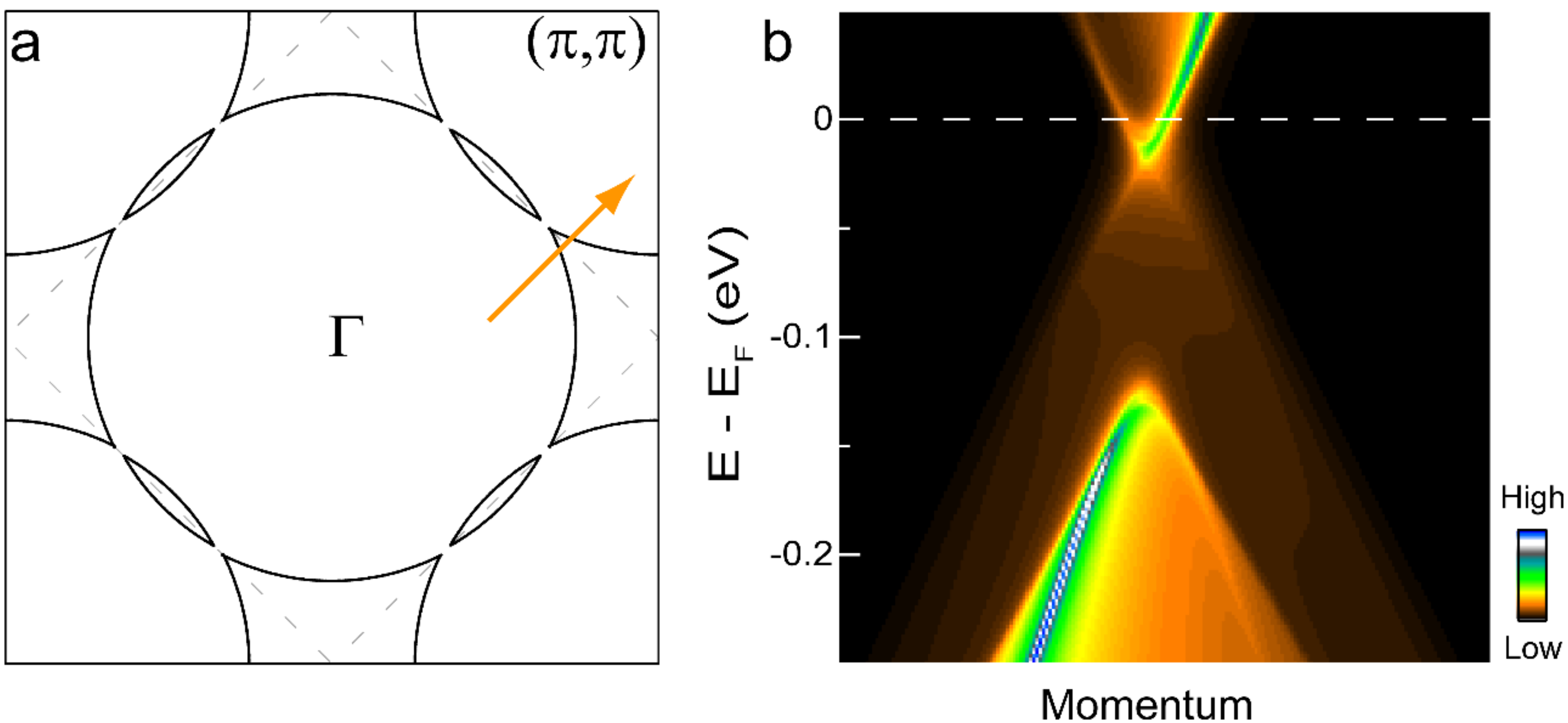


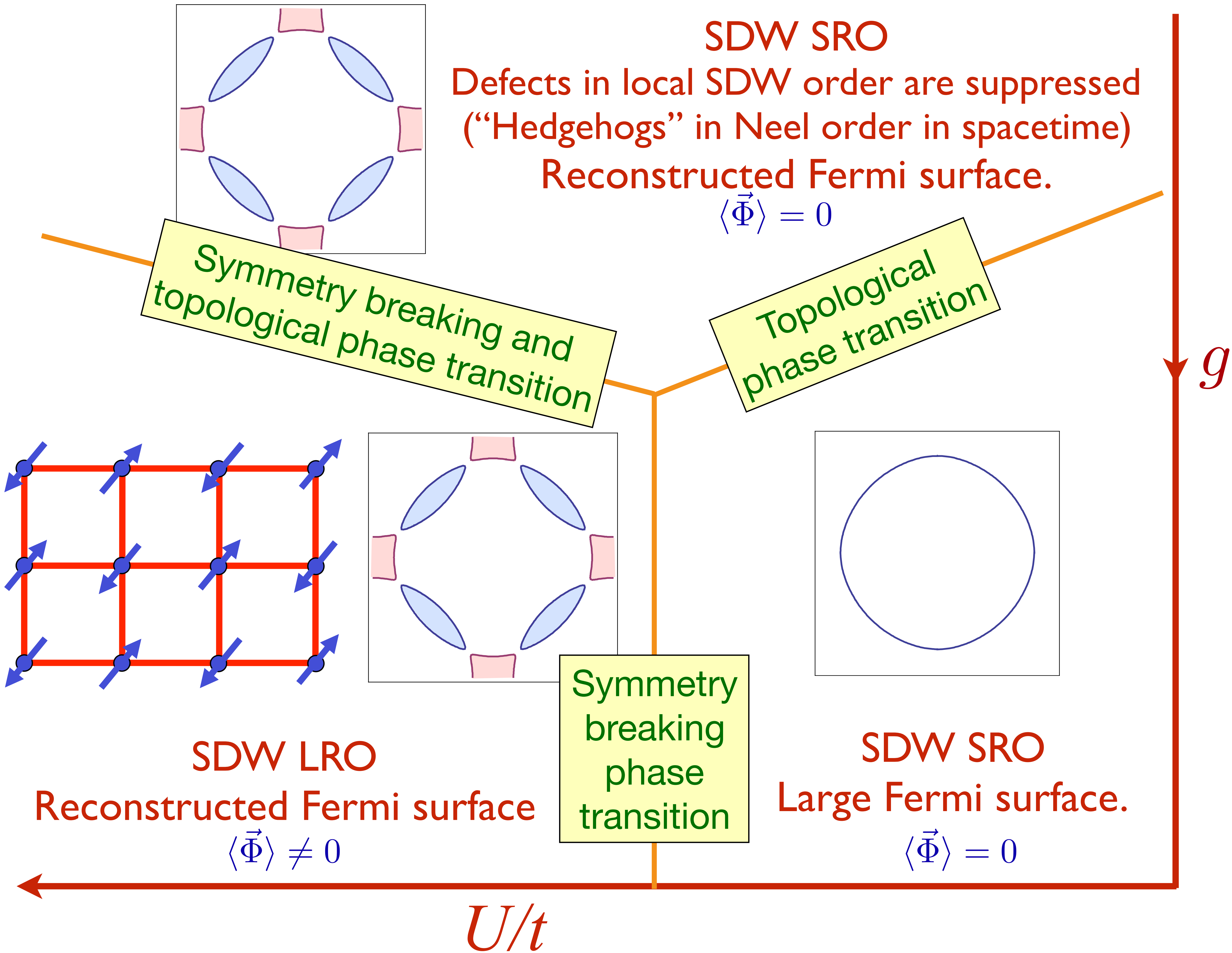
antinode
a
(π,π)
node
Γ
b
gap
c
d
(π,π)
Γ
e
no gap
f
g
h
i
FB
MB
FB
MB
FB
MB
FB
MB
(π,π)
Γ
High
Low
E - E_F (eV)
0
-0.1
-0.2
-0.3
0.2
0.4
0.6
0.8
Intensity (arb. units)
-0.3
-0.2
-0.1
0
j
k
l
MB
FB
MB
FB
Momentum (1/Å)
E - E_F (eV)

S. Sachdev, Topological order and Fermi surface reconstruction, arXiv:1801.01125

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)





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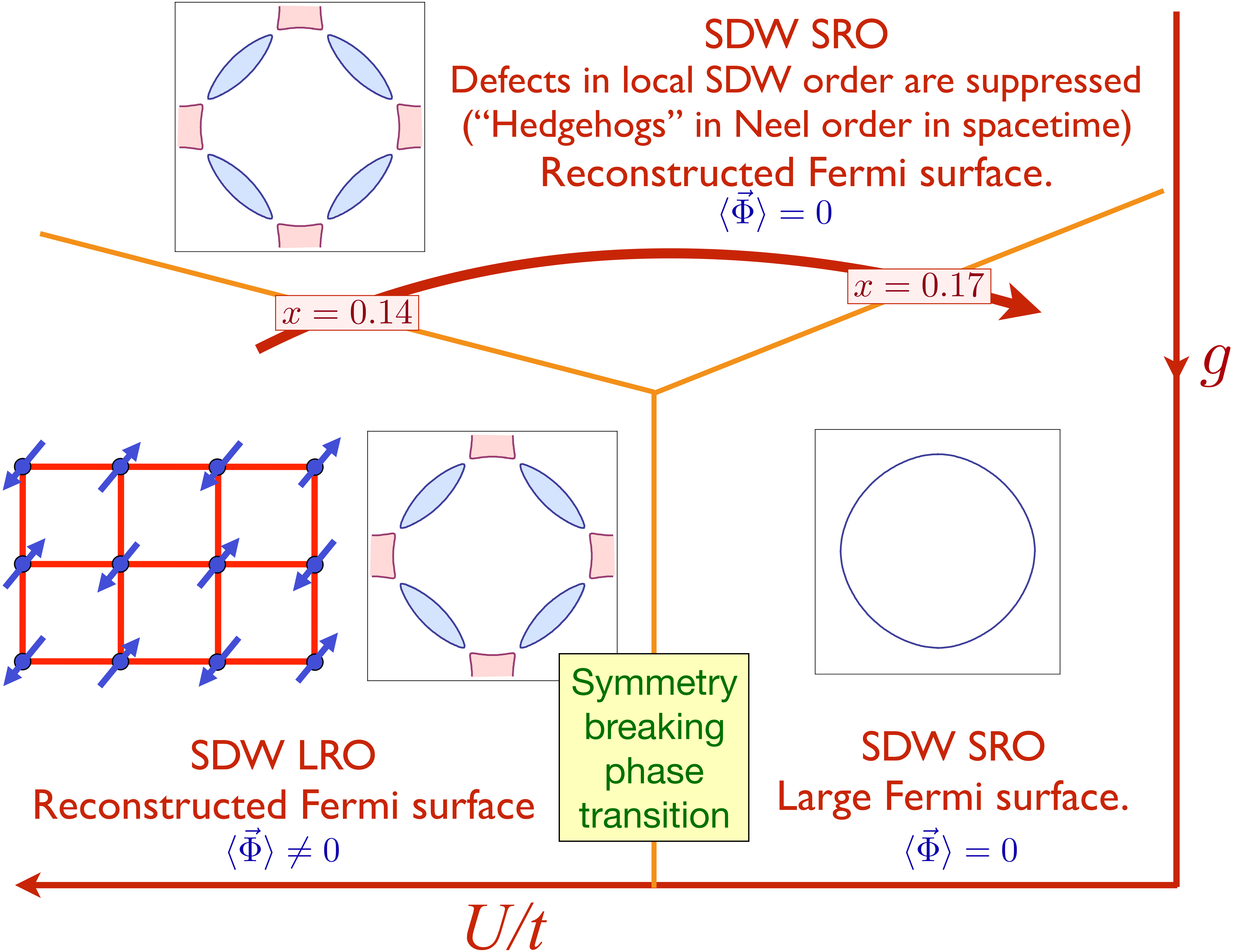

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$U/t$

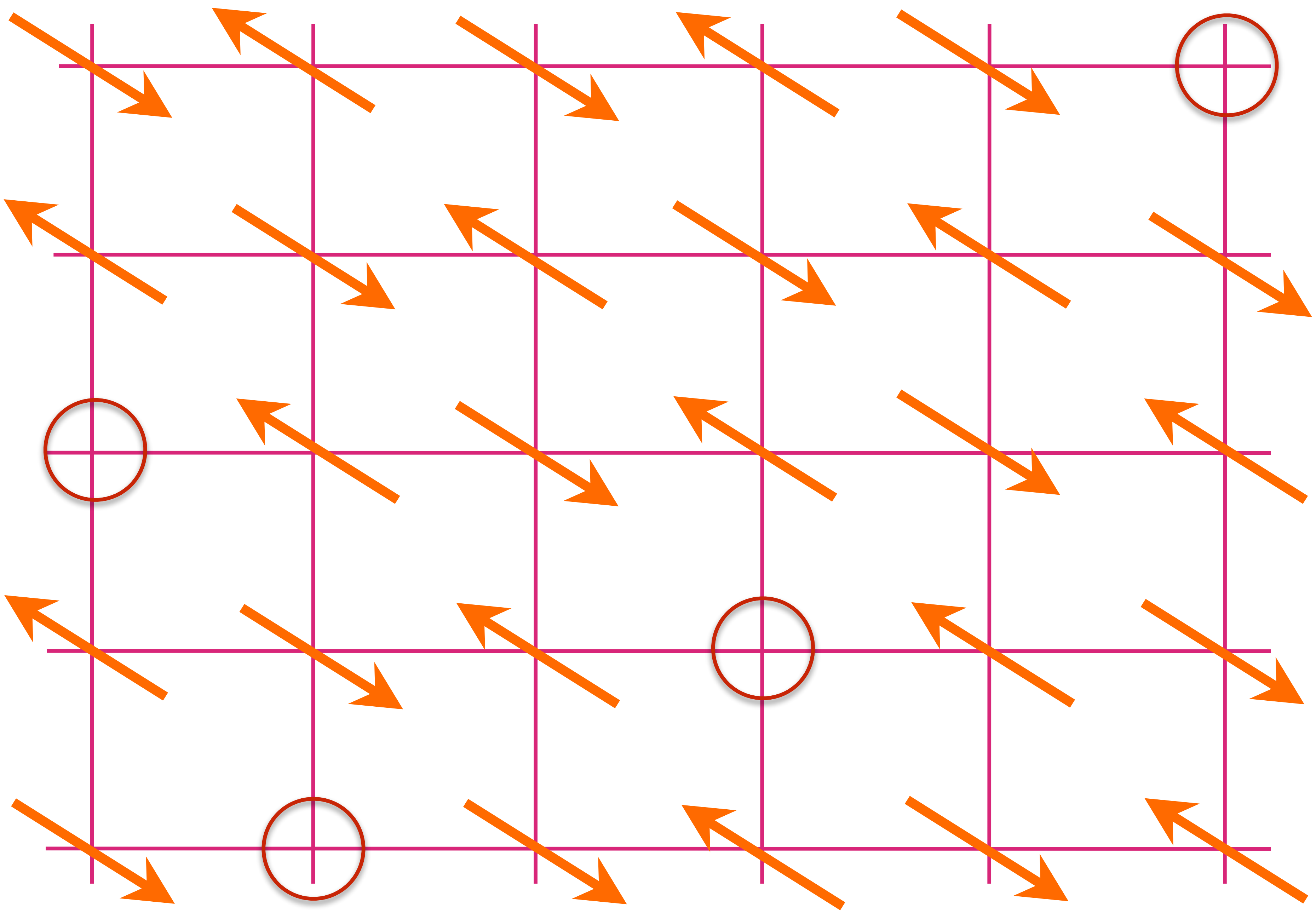
# Constraints on volume enclosed by the Fermi surface

- In a conventional Fermi liquid state, Fermi volume must equal $1+x$ *(mod 2)*.
- When the unit cell is doubled by SDW order, total Fermi volume must equal $x$ *(mod 1)*.

# Constraints on volume enclosed by the Fermi surface

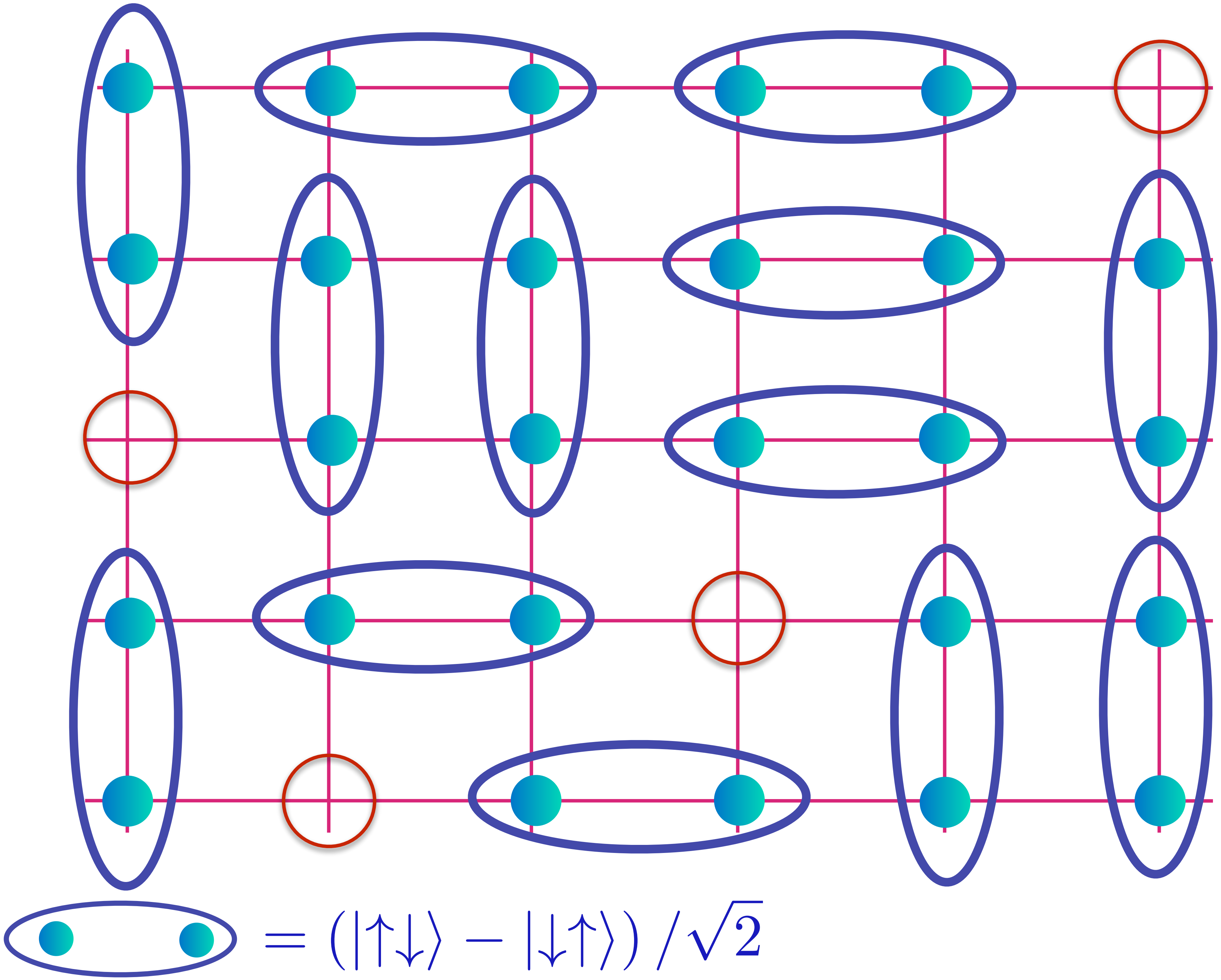
- In a conventional Fermi liquid state, Fermi volume must equal *1+x (mod 2)*.
- When the unit cell is doubled by SDW order, total Fermi volume must equal *x (mod 1)*.
- A state with Fermi volume *x (mod 2)*, but no translational symmetry breaking, must have non-quasiparticle excitations with vanishing energy on a torus *i.e.* emergent gauge fields

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)



Anti-ferromagnet with $p$ holes per square

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid with density $p$ of spinless, charge +e (or -e) “chargons”.

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
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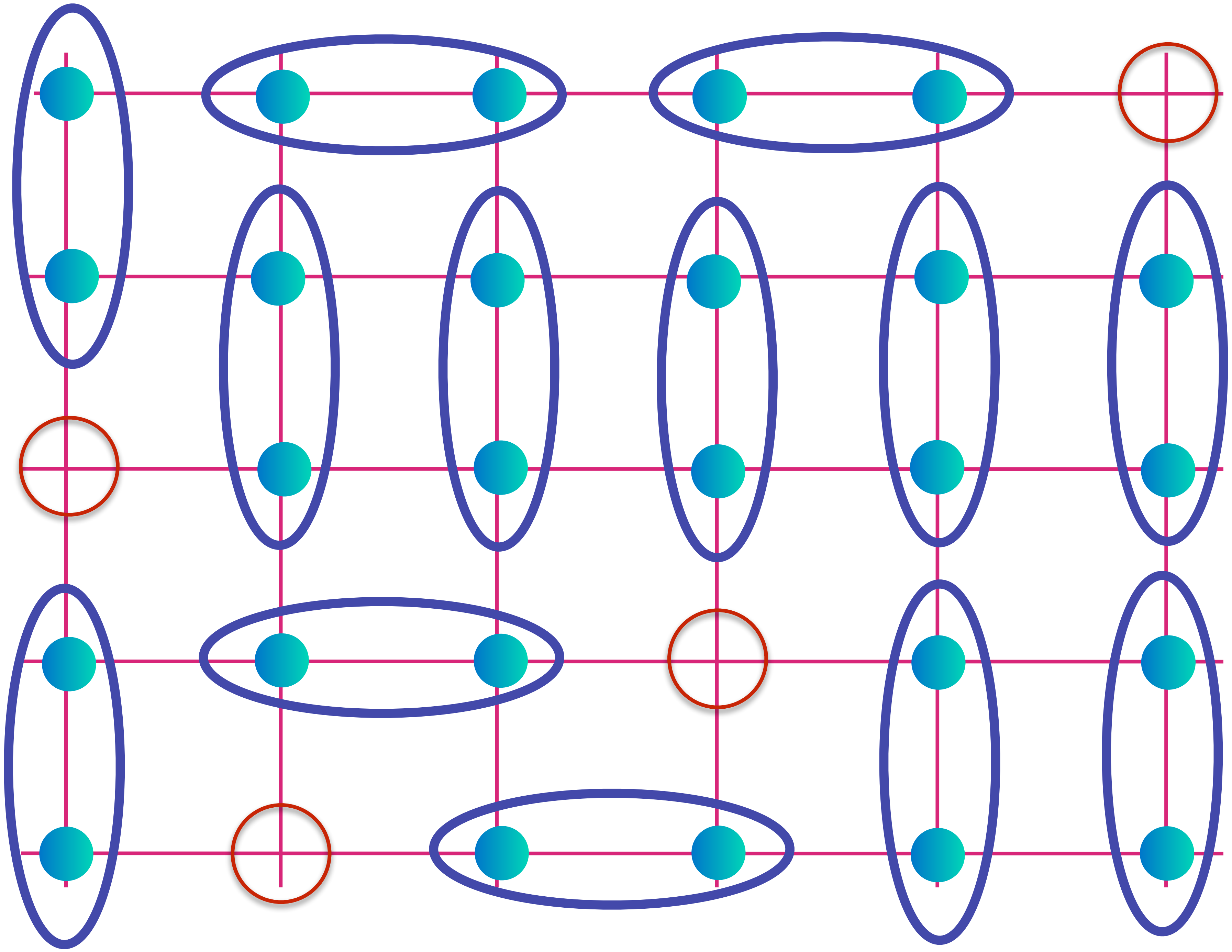

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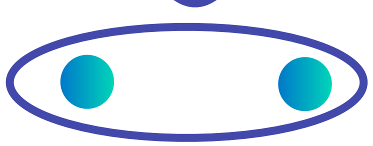

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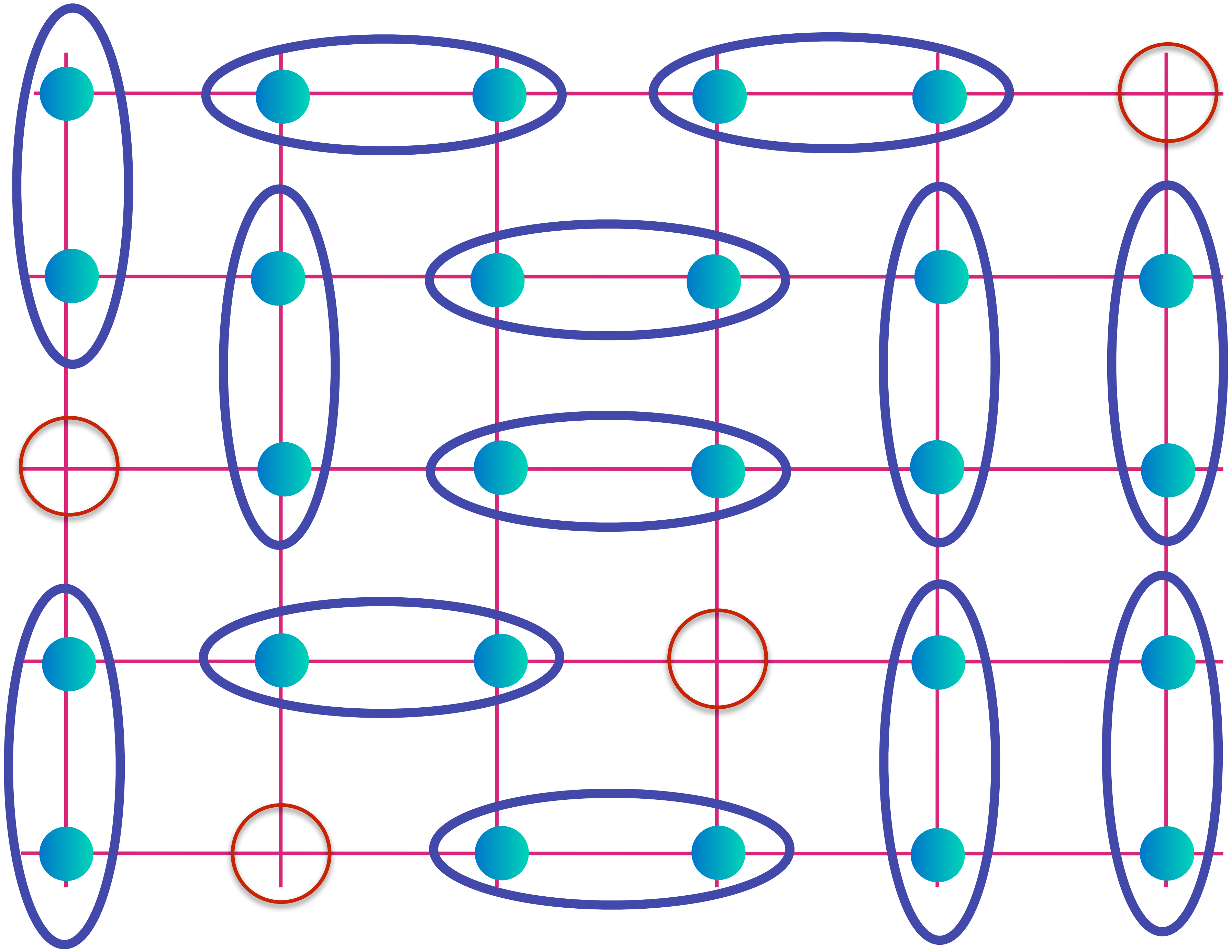

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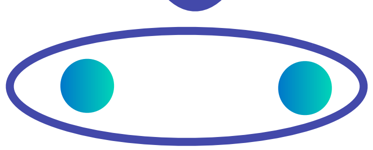 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

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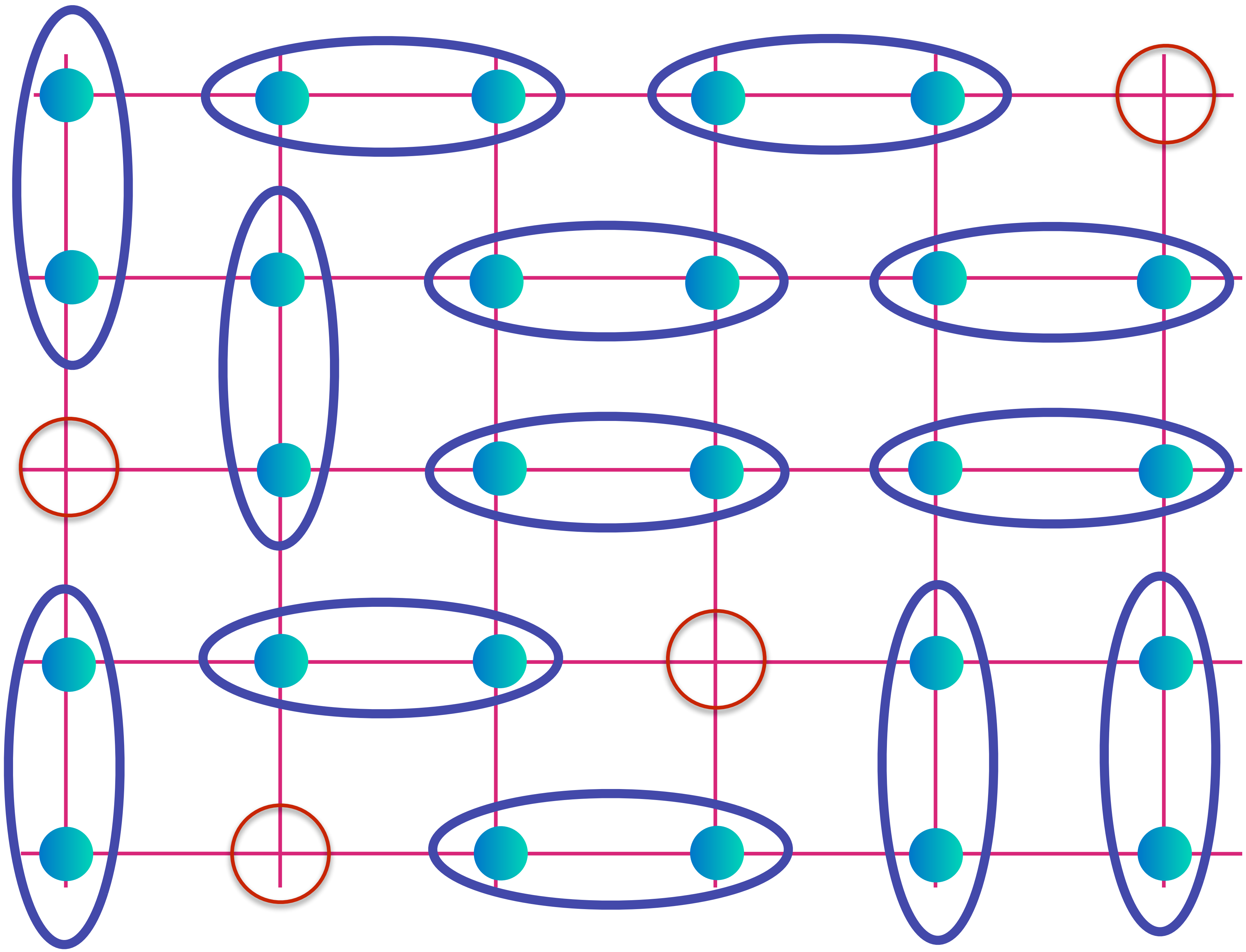

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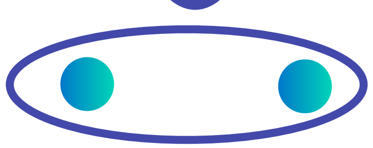
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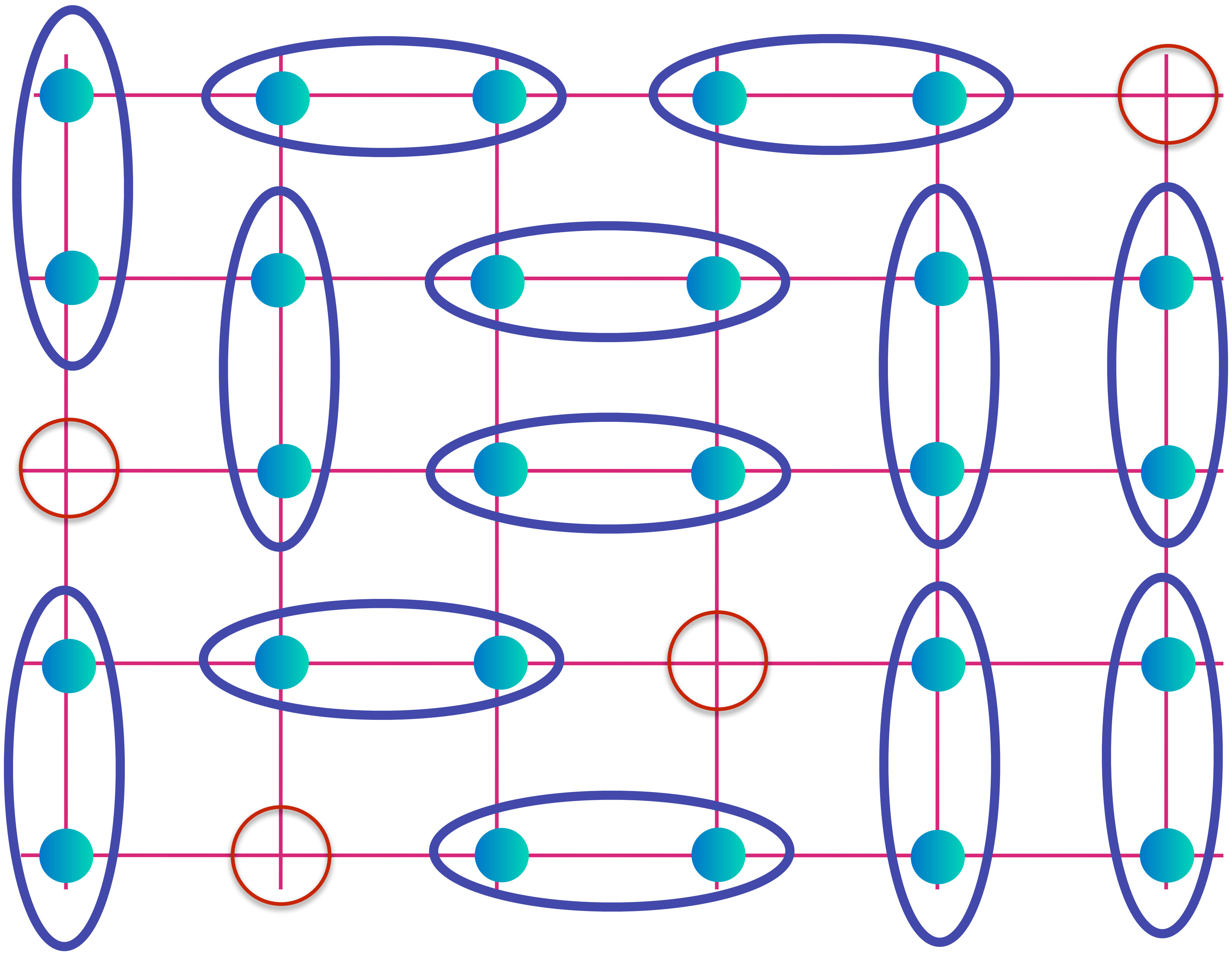

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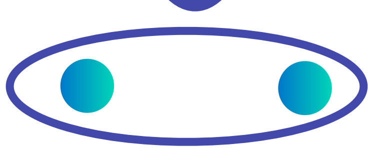

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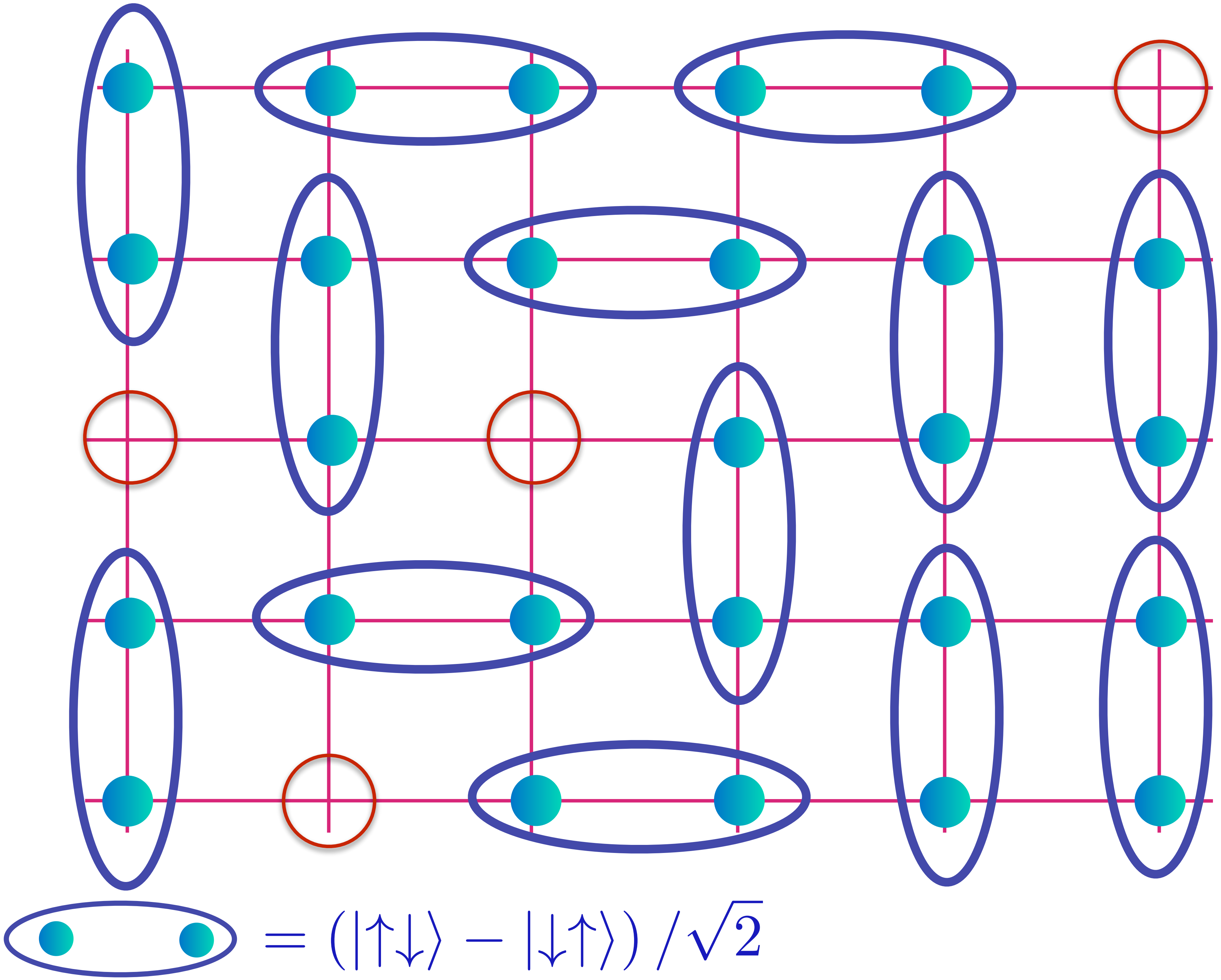

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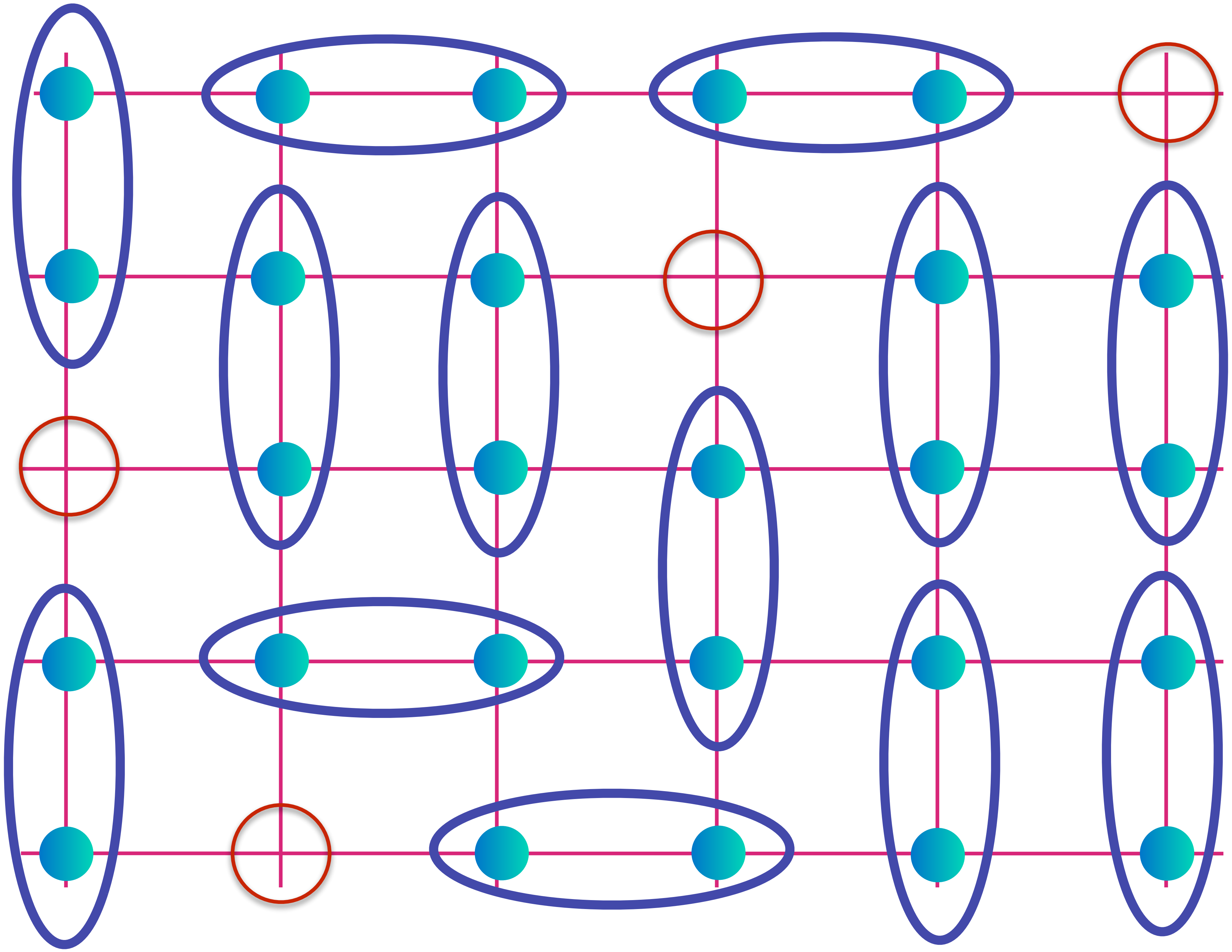

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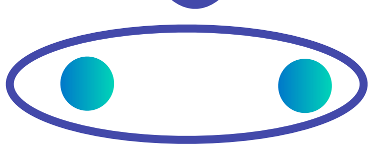 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

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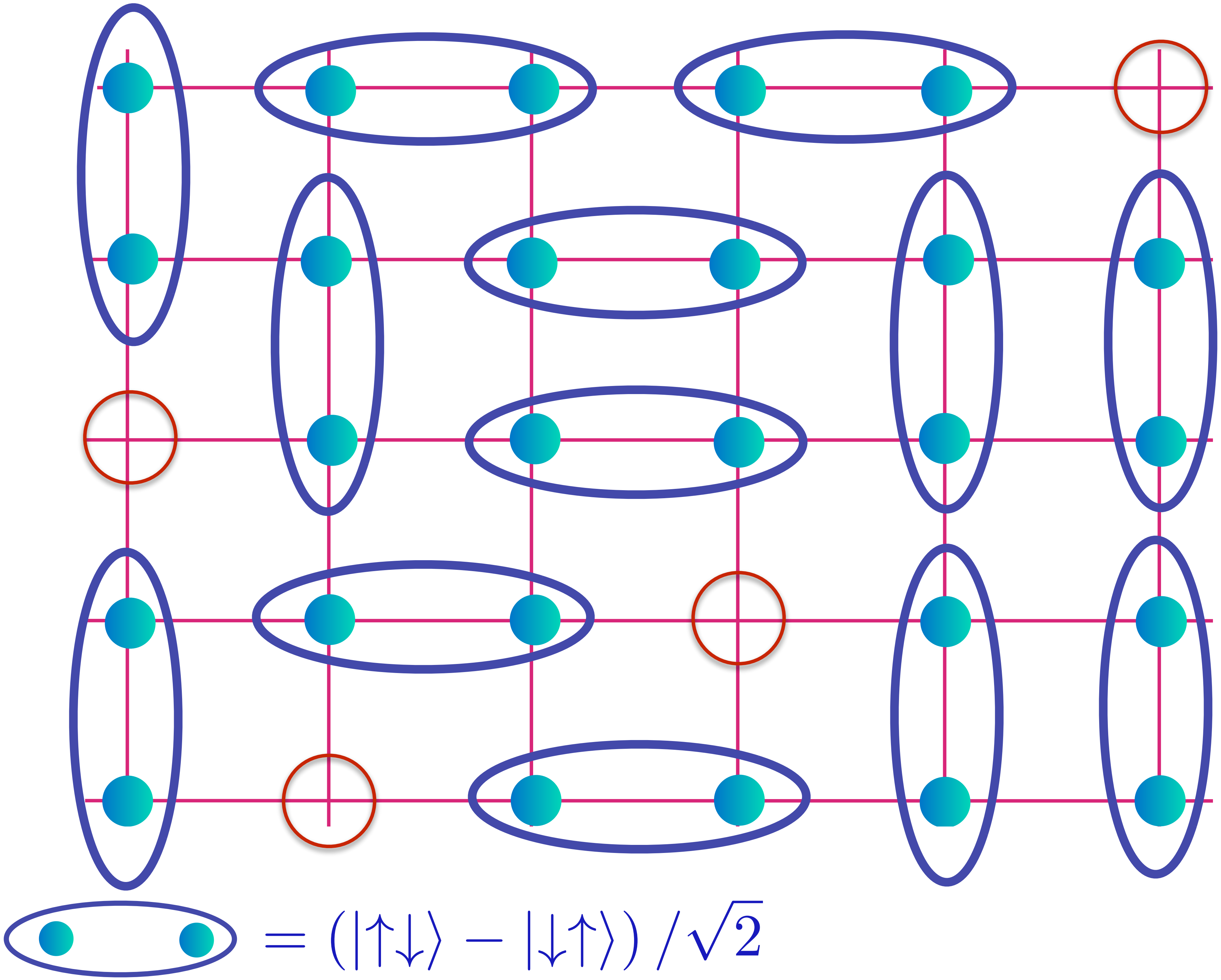

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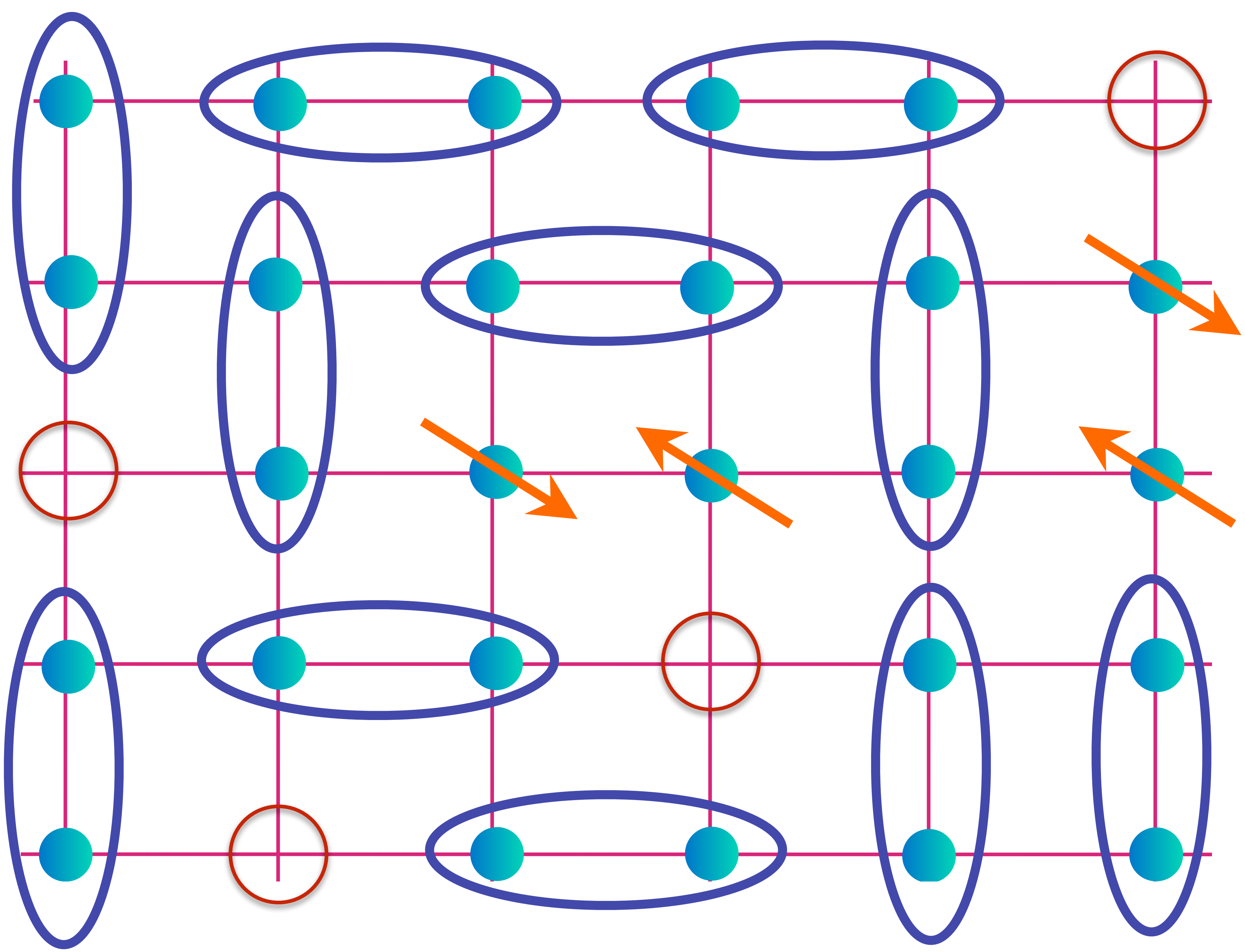
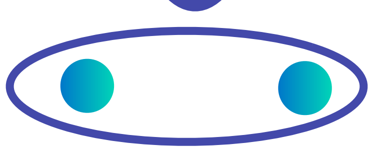


Spin liquid
with density
*p* of spinless,
charge +e (or -e)
"chargons",
and charge 0,
spin 1/2
"spinons"



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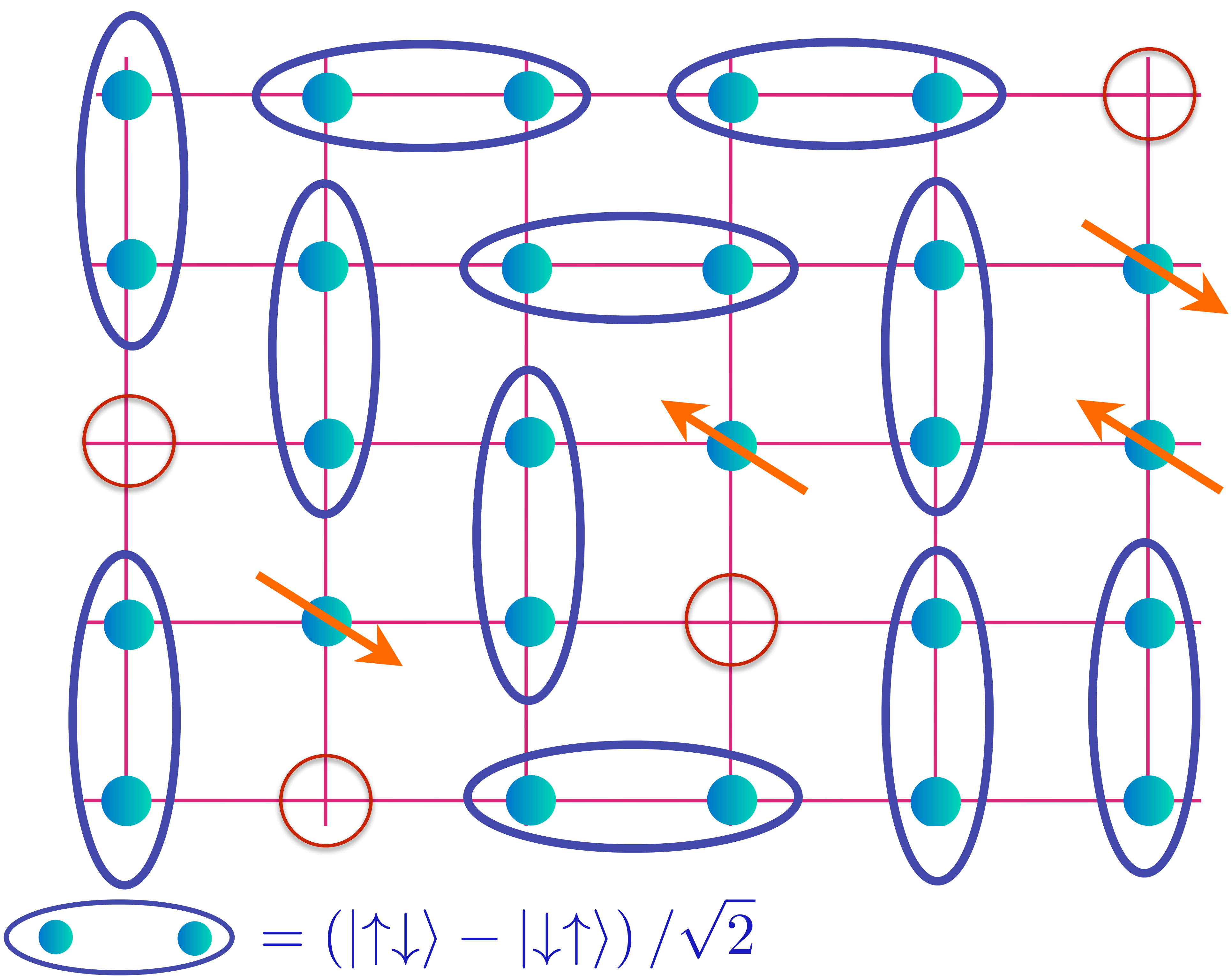

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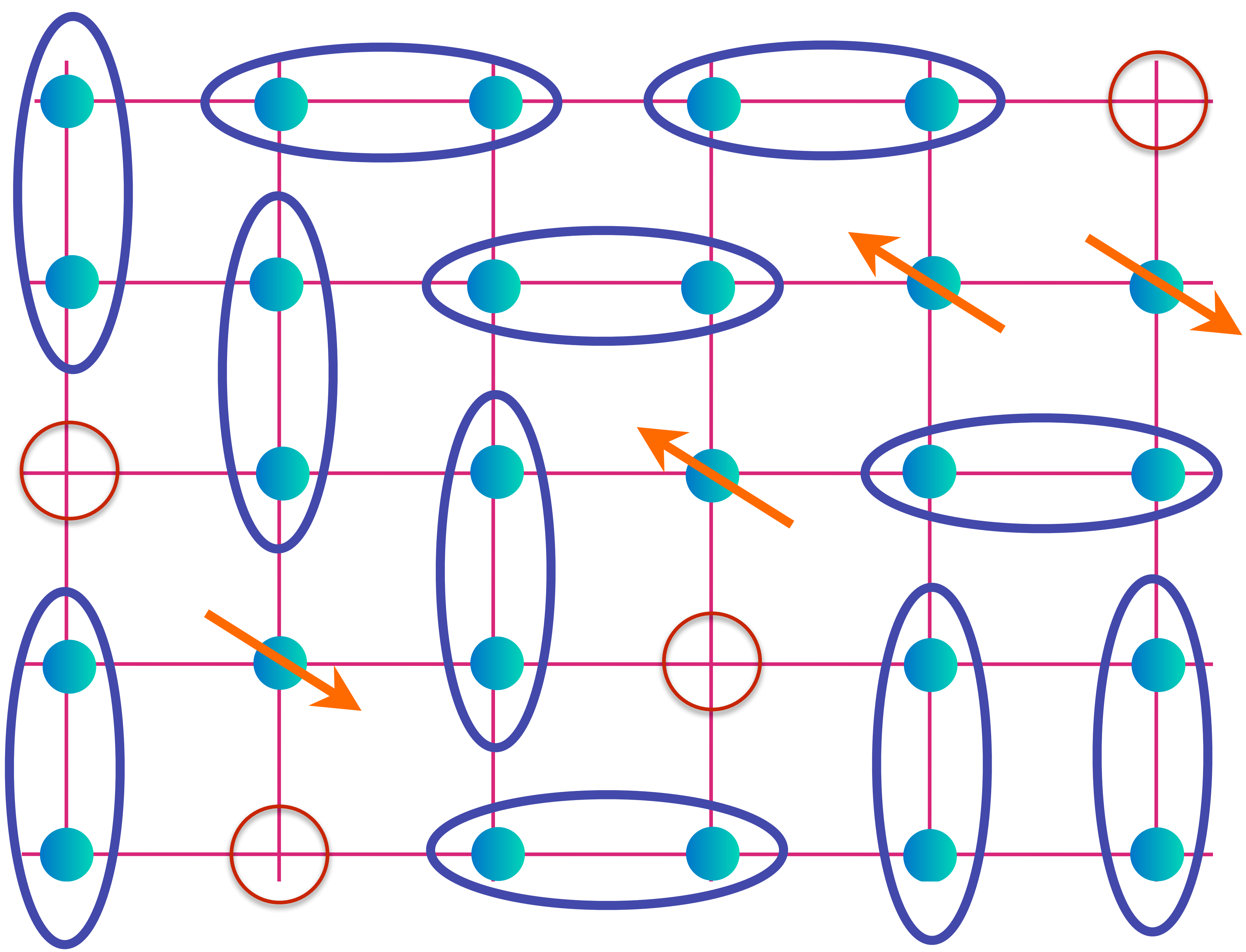

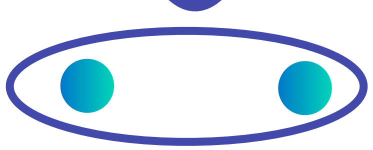 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Spin liquid
with density
*p* of spinless,
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“chargons”,
and charge 0,
spin 1/2
“spinons”

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
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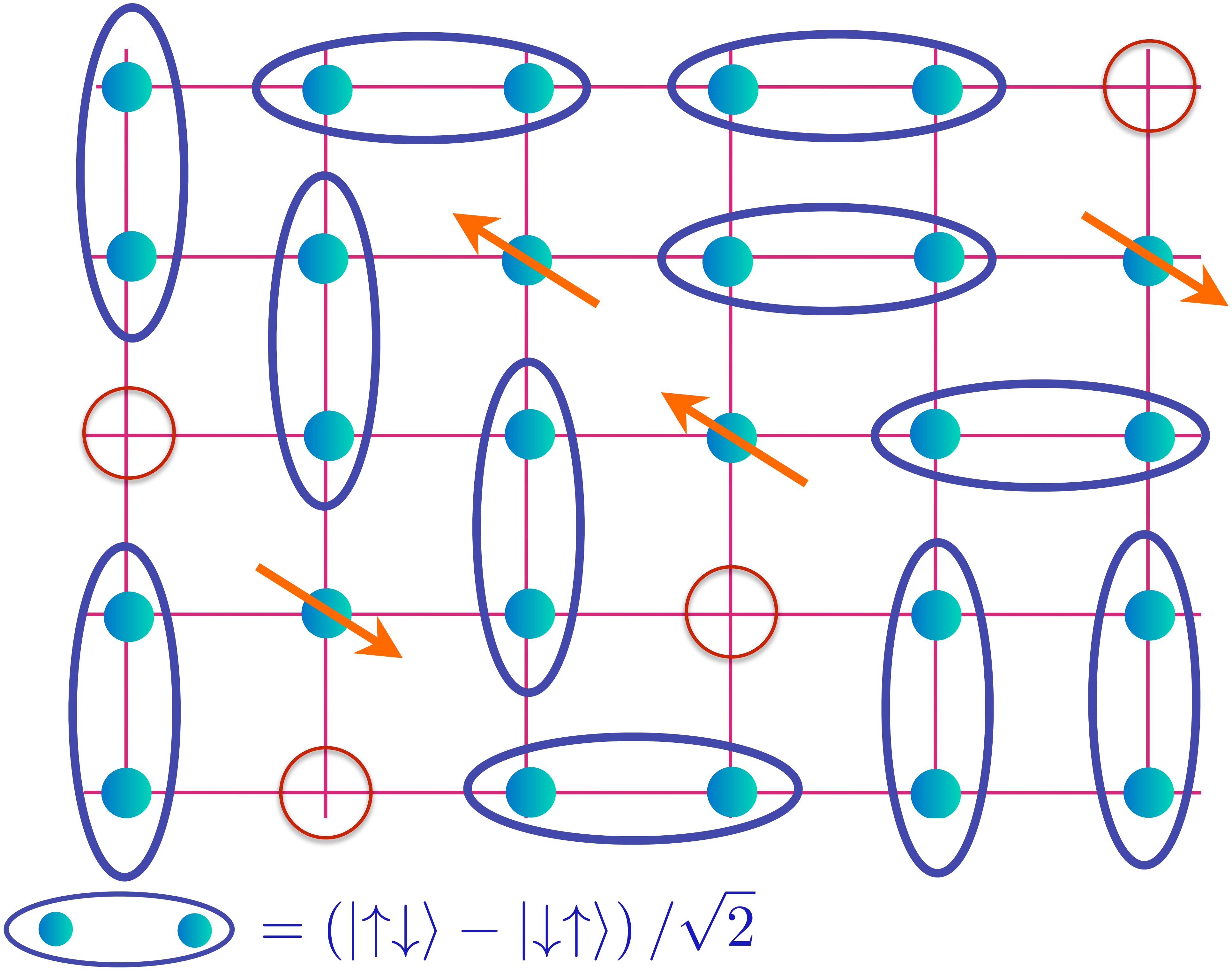

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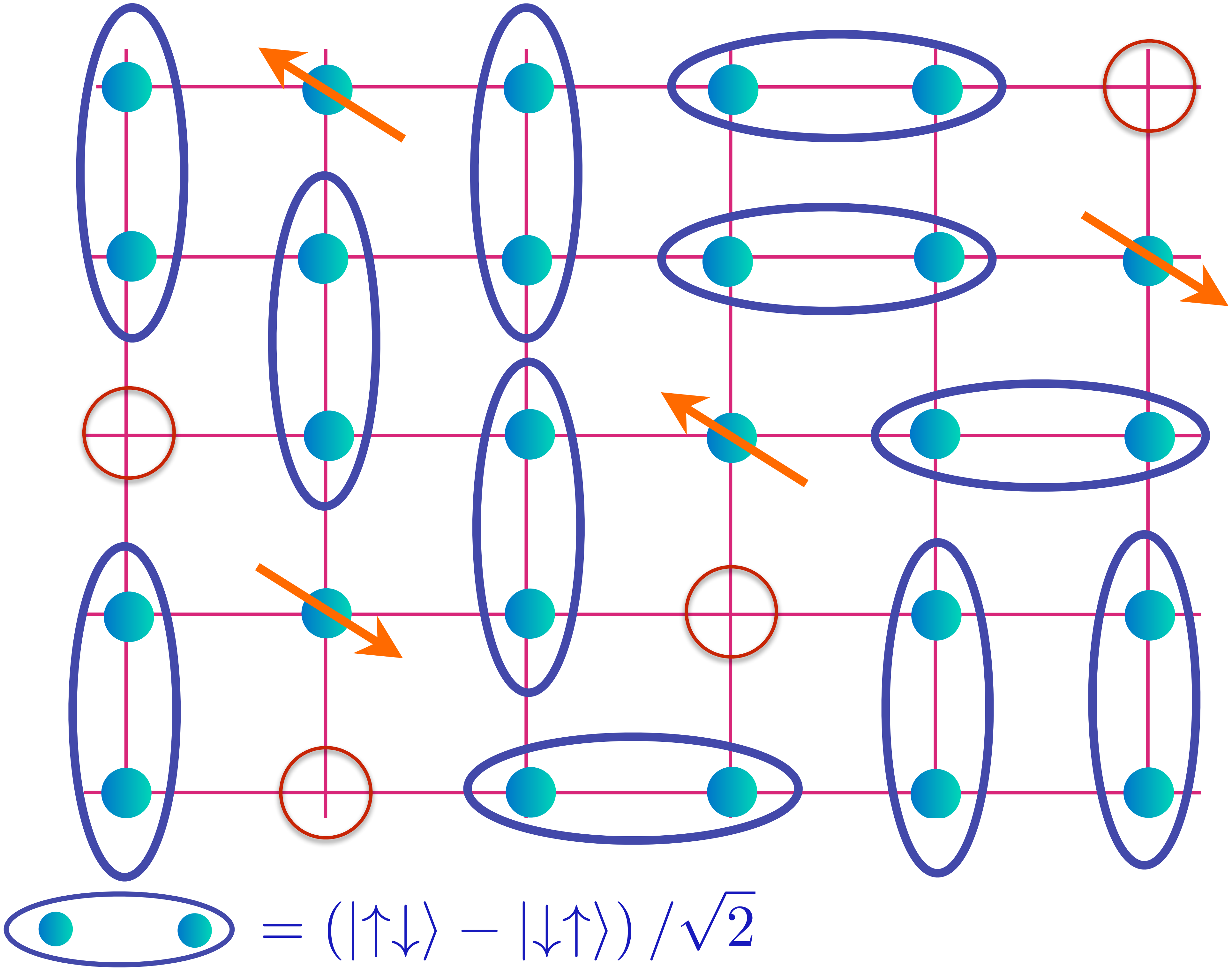

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

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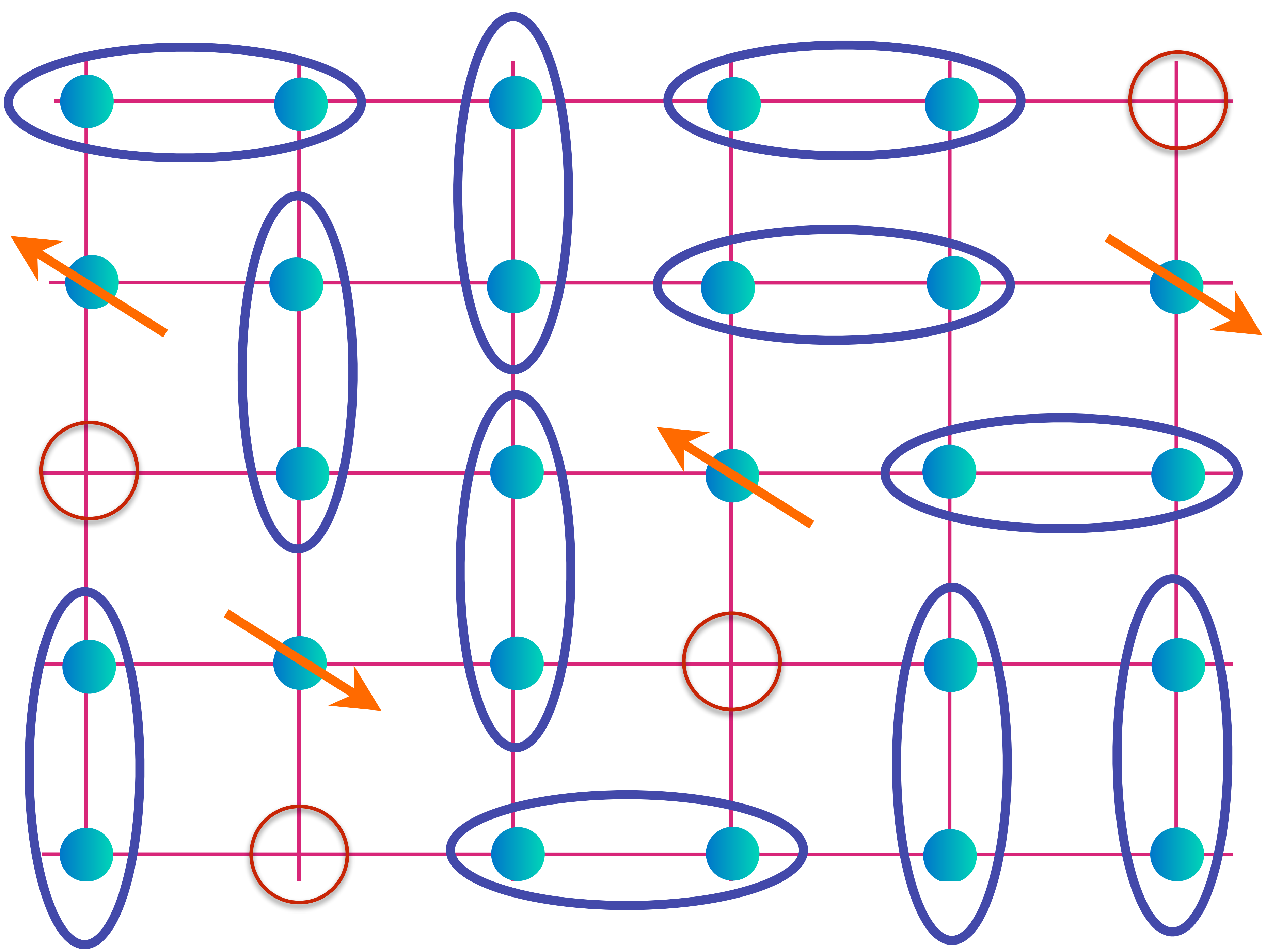

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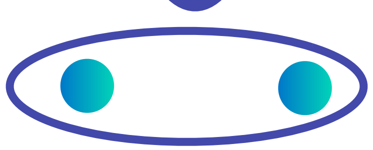

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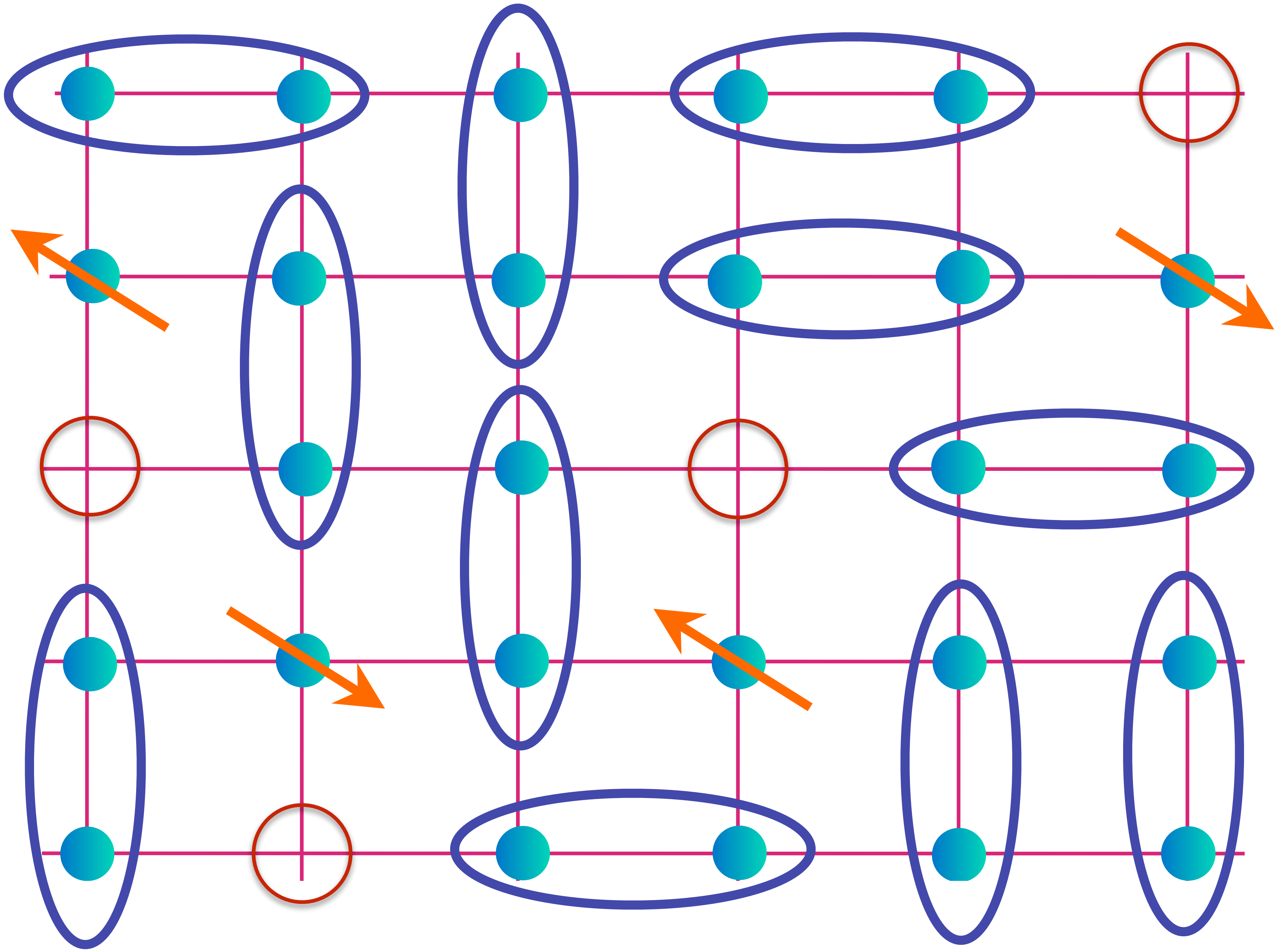

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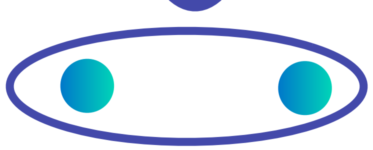

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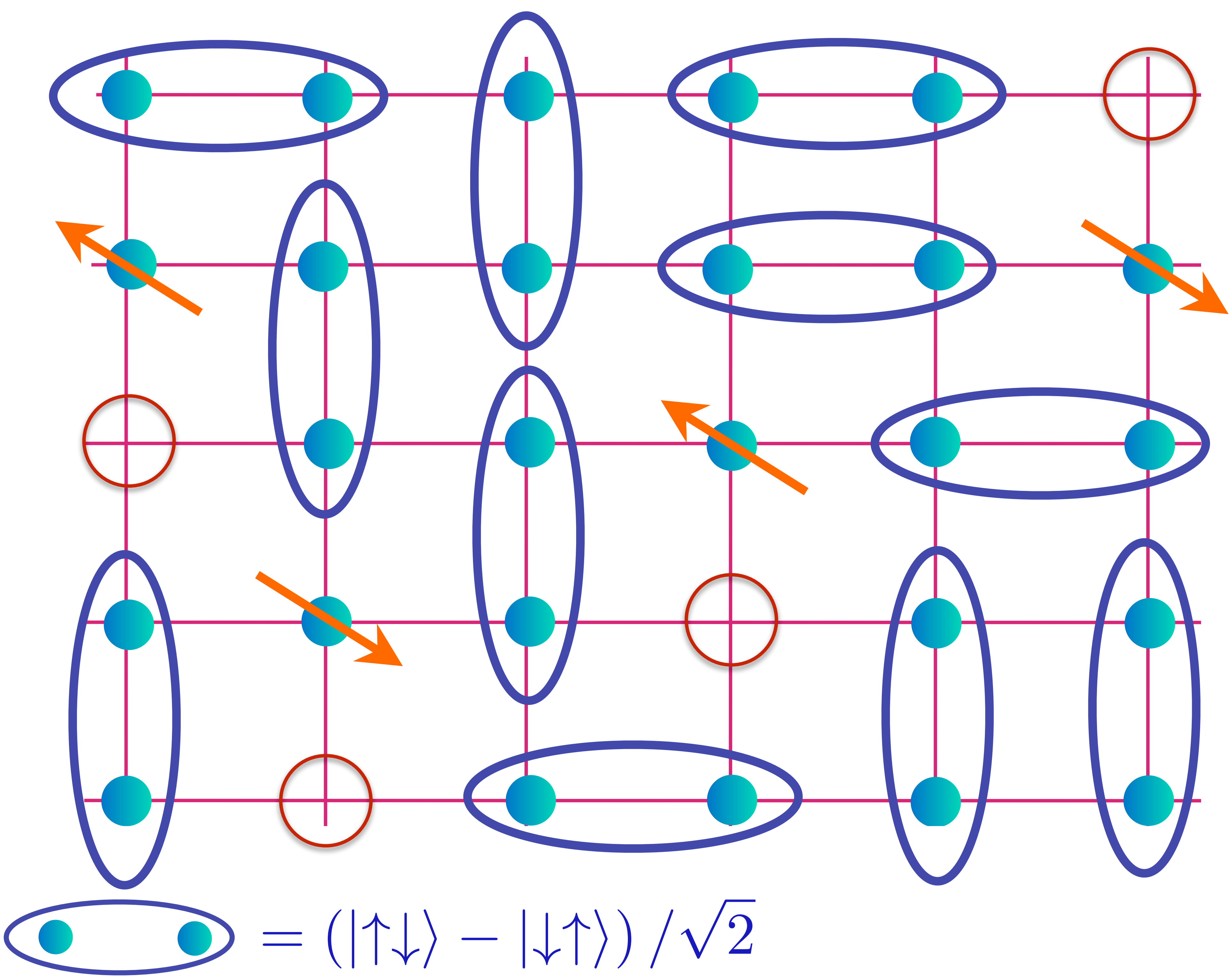

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Spin liquid
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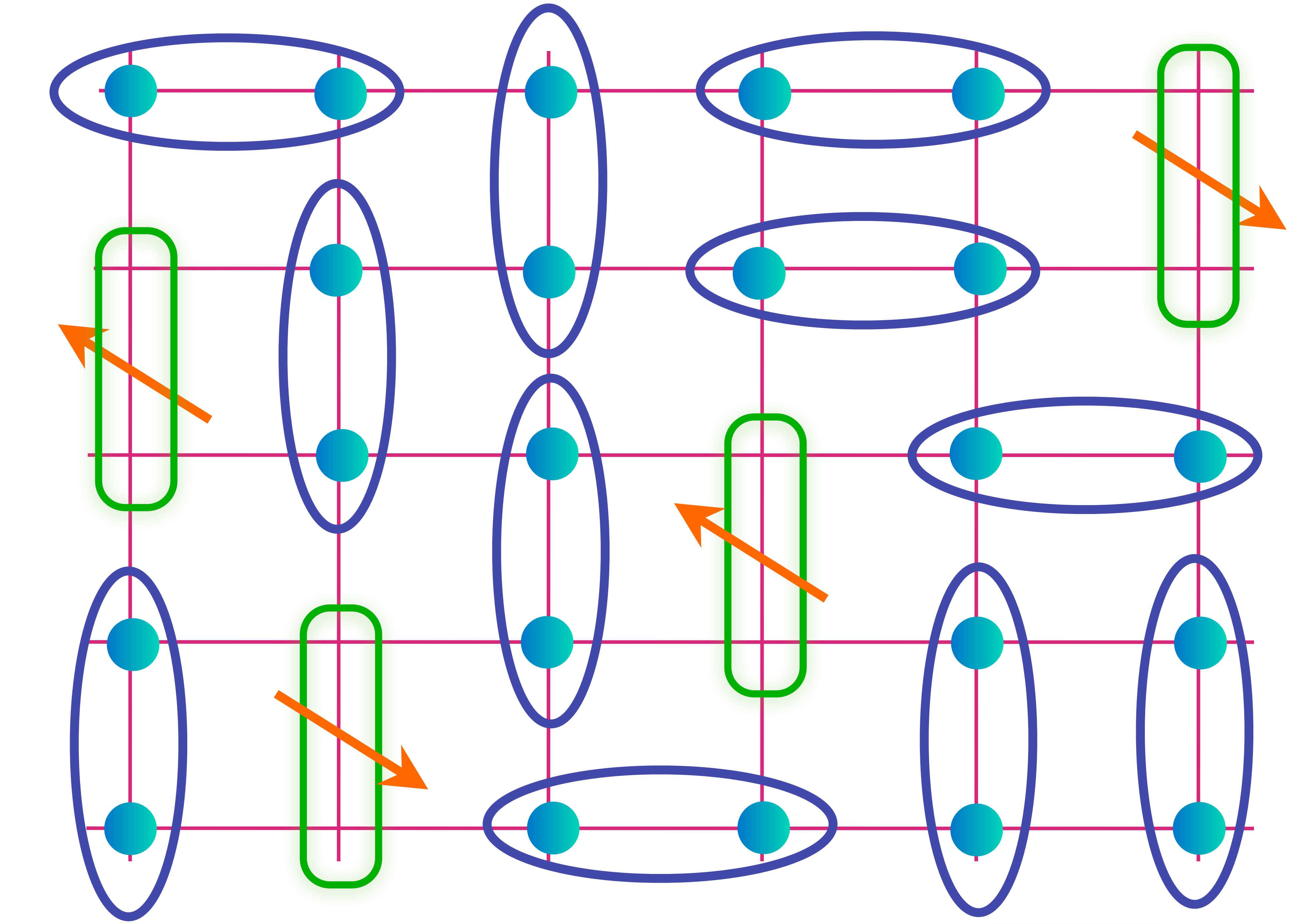
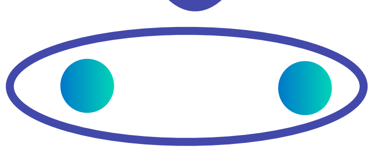
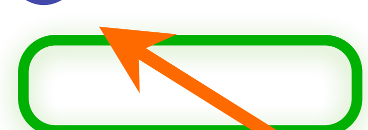


Metal with electron-like quasiparticles on a Fermi surface of size $p$, and emergent gauge fields

 $= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left( |\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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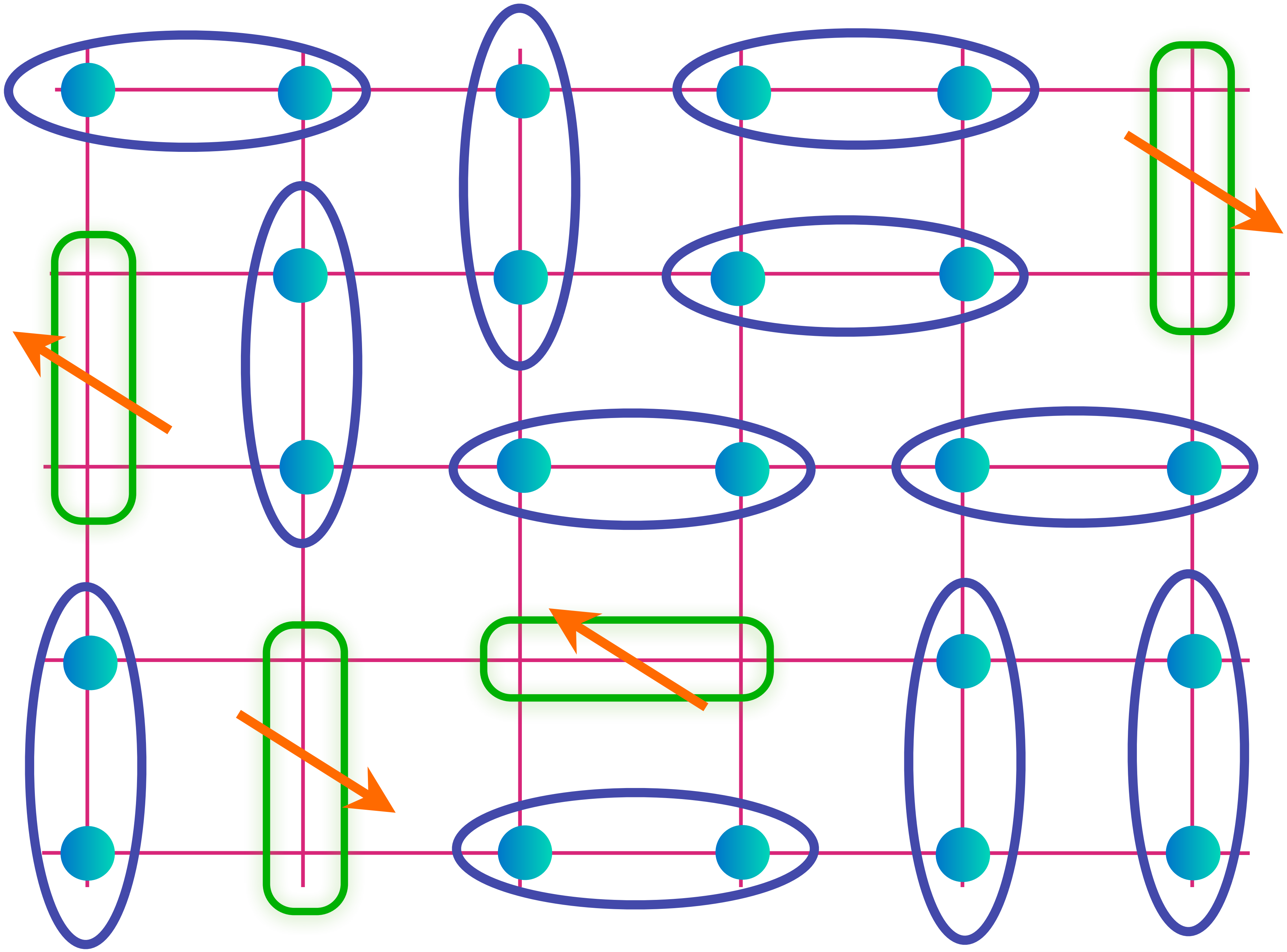

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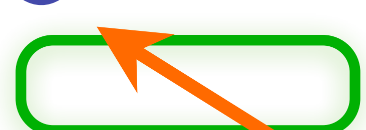 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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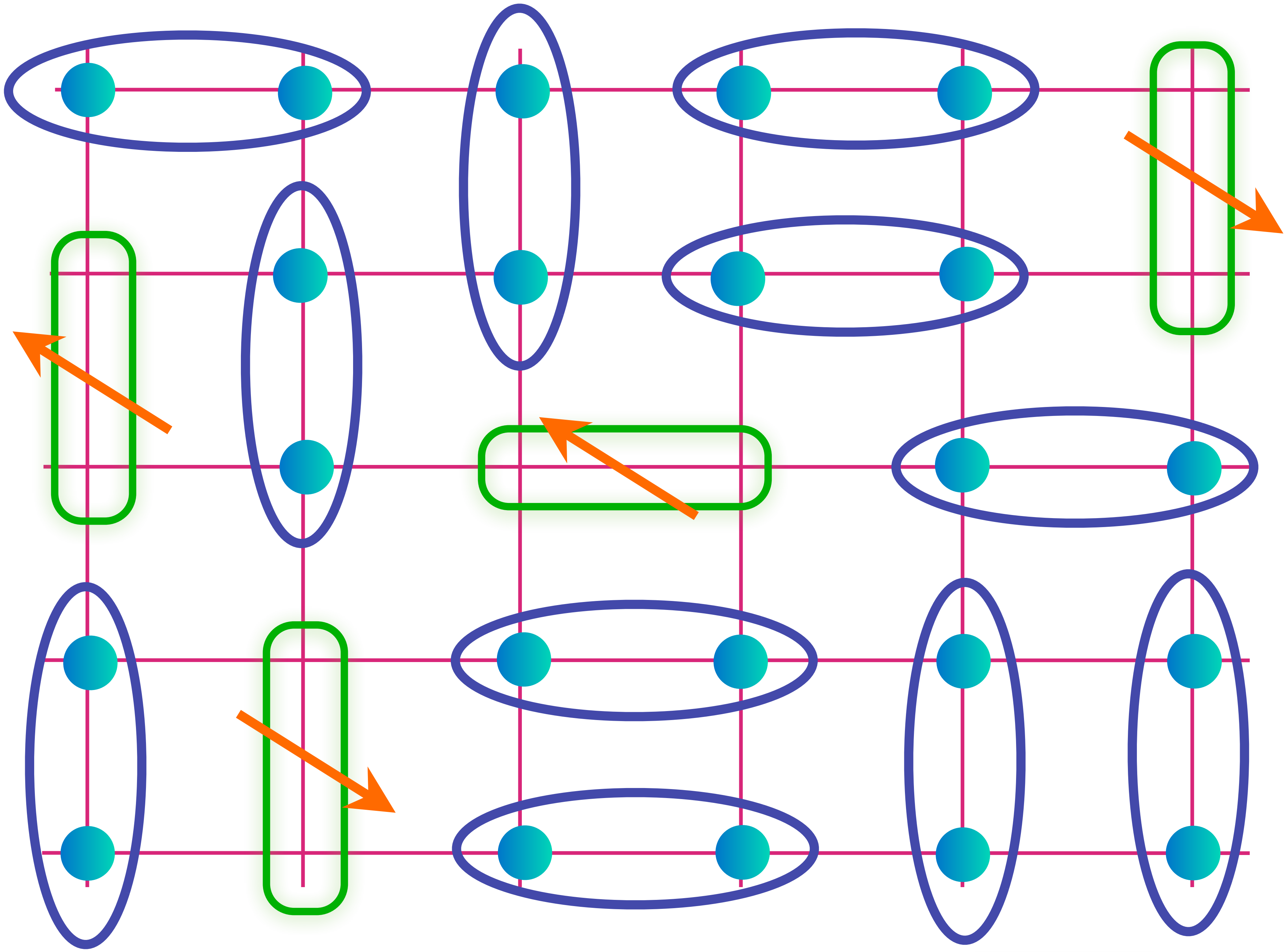

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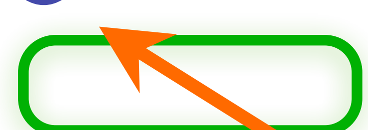 $= \left( |\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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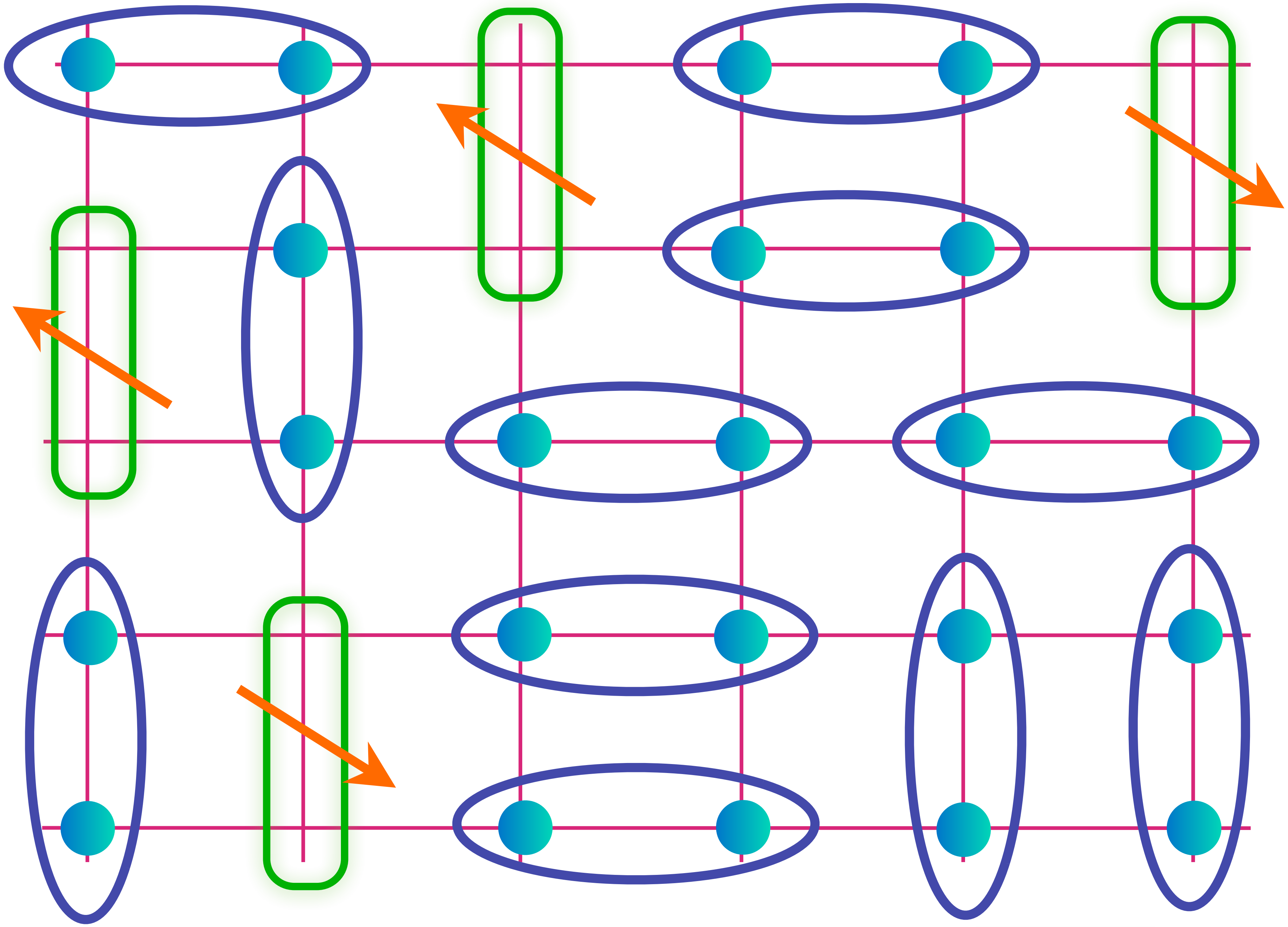

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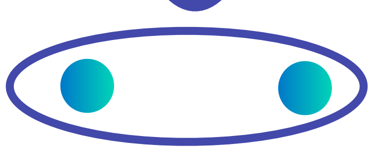
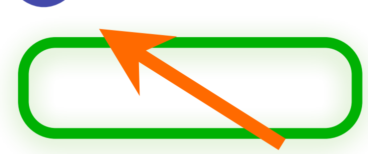
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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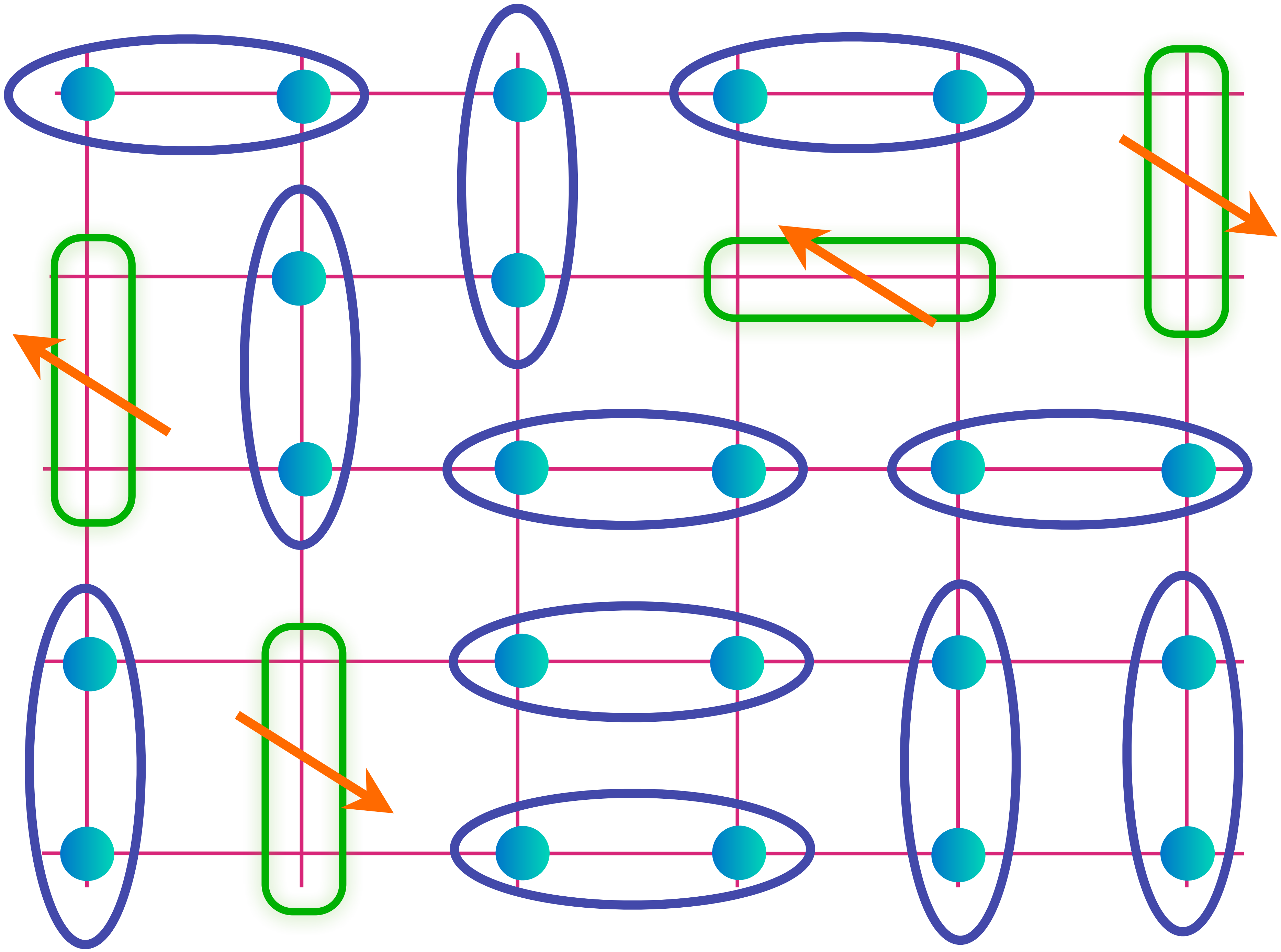

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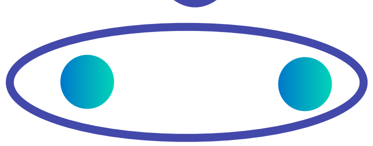
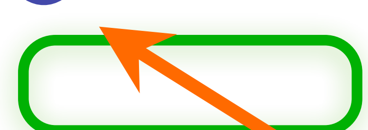
 $= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left( |\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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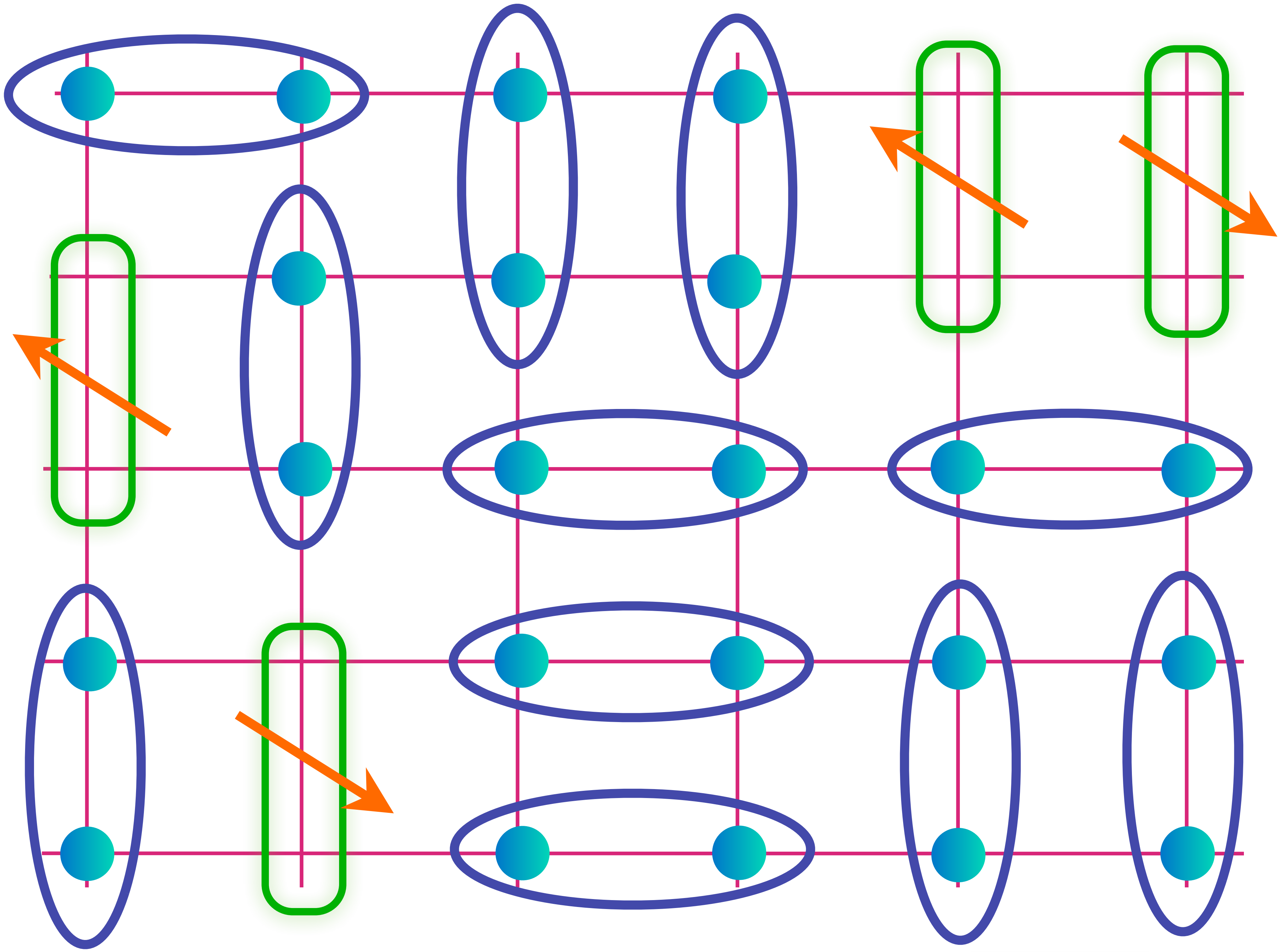

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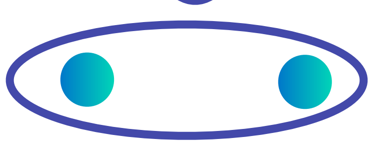
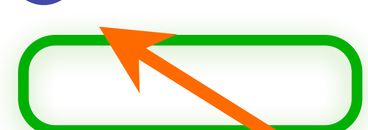
 $= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left( |\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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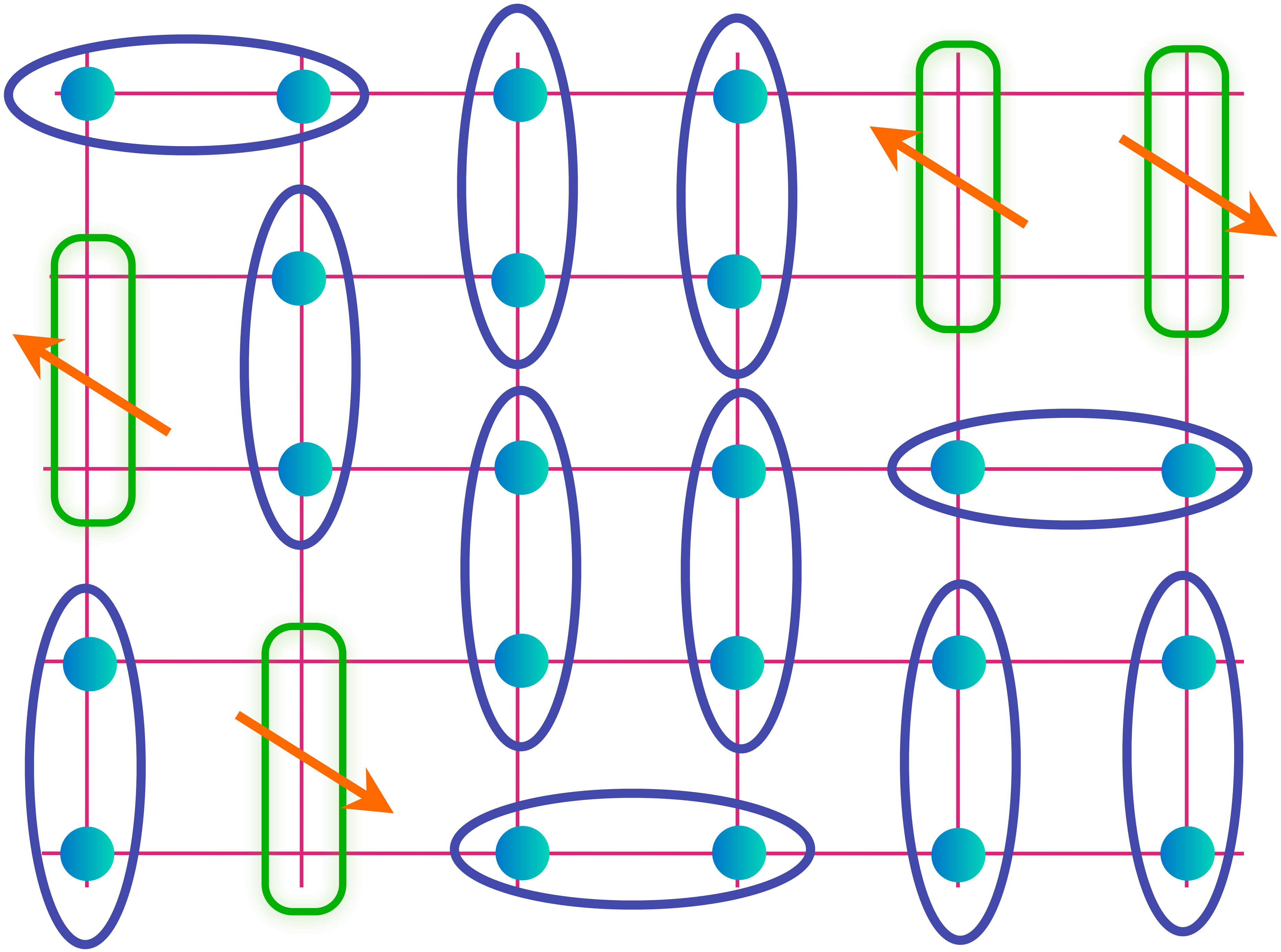

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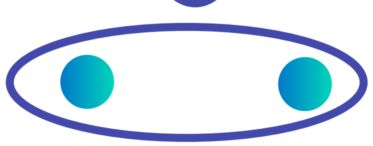
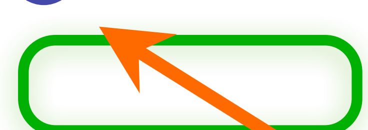
 $= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left( |\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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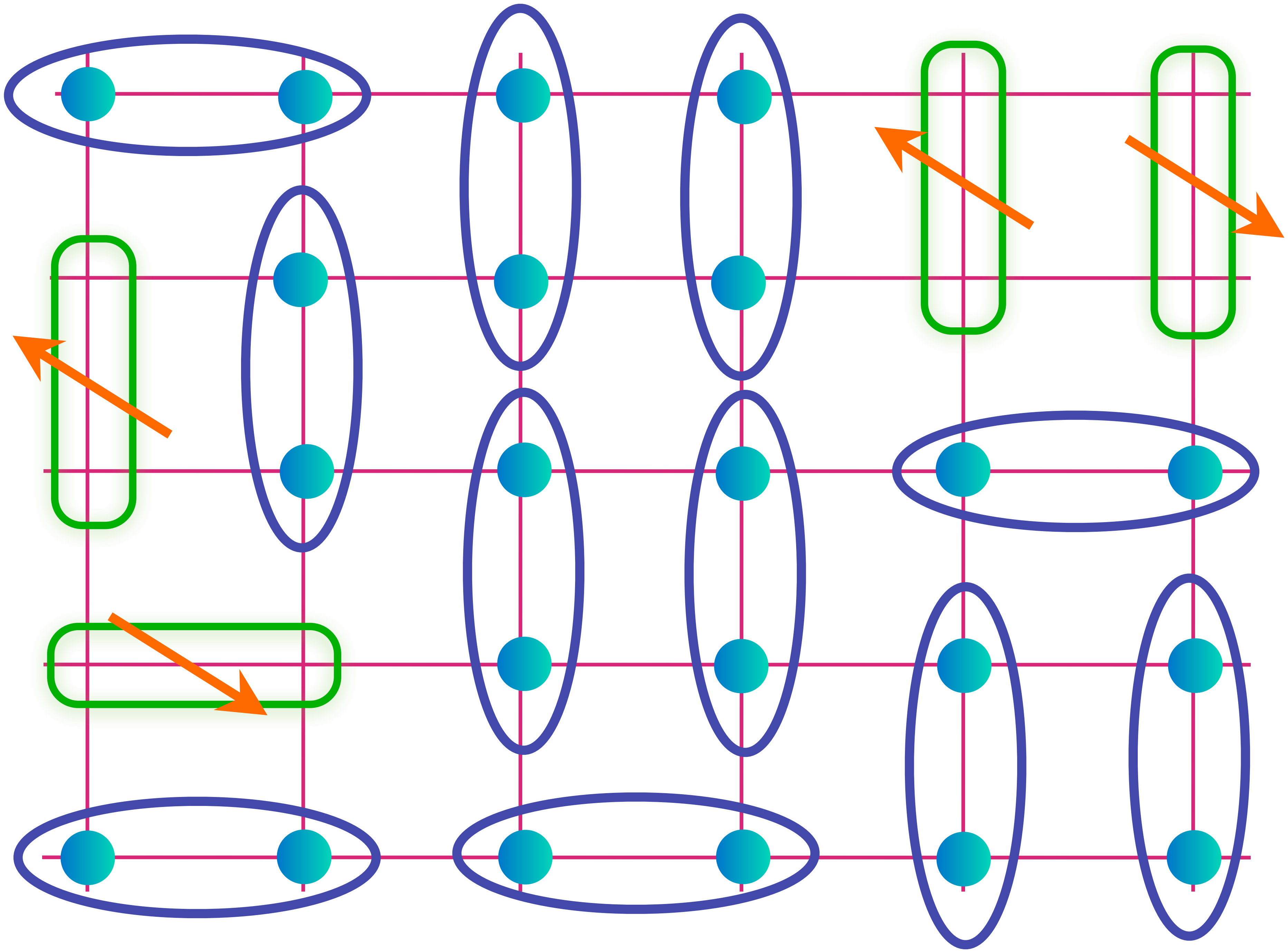

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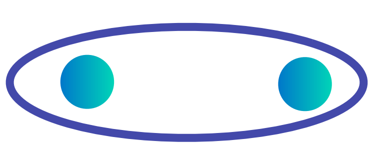
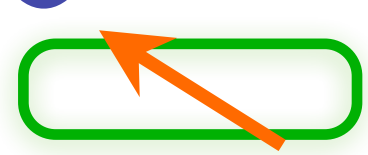
 $= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left( |\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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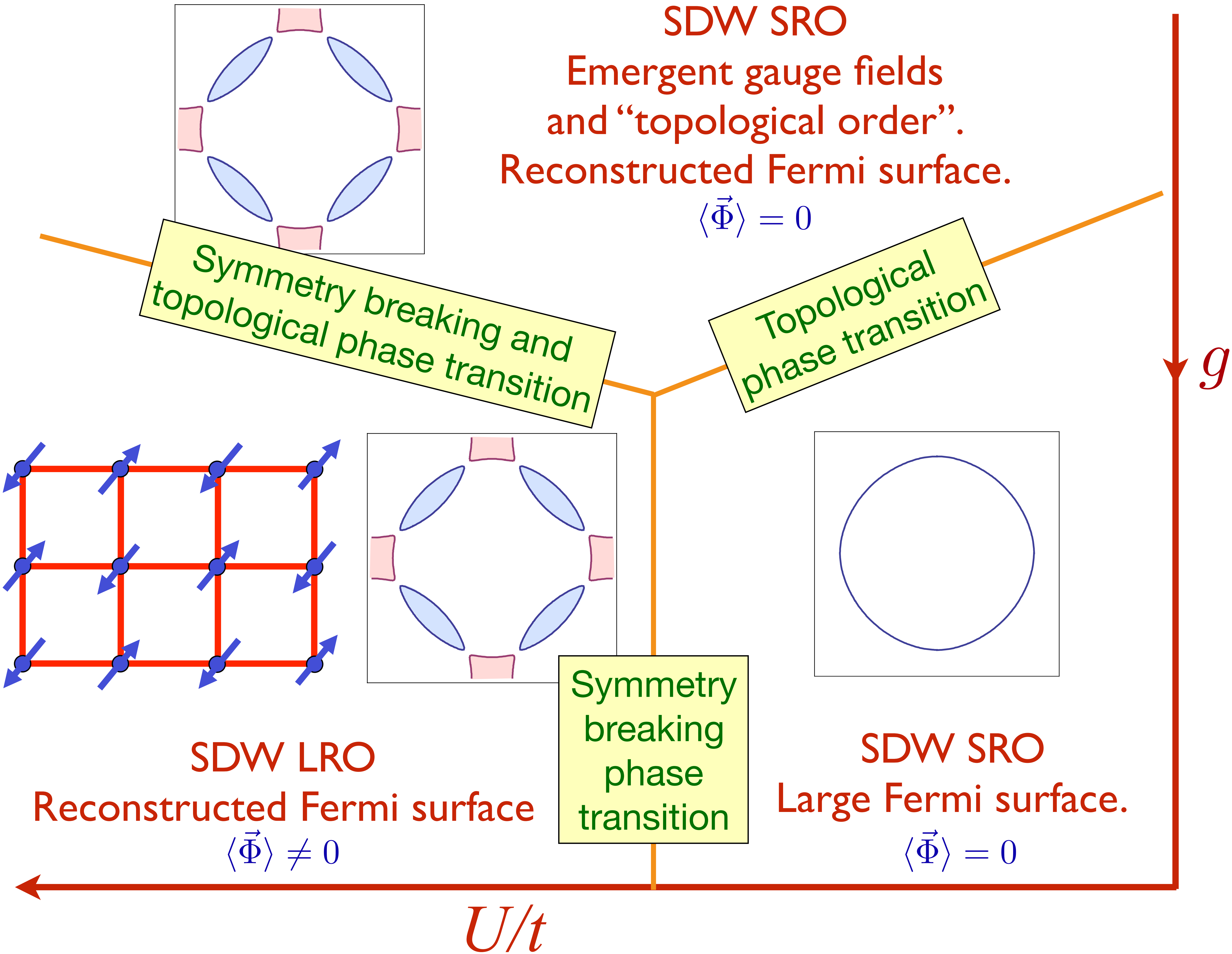
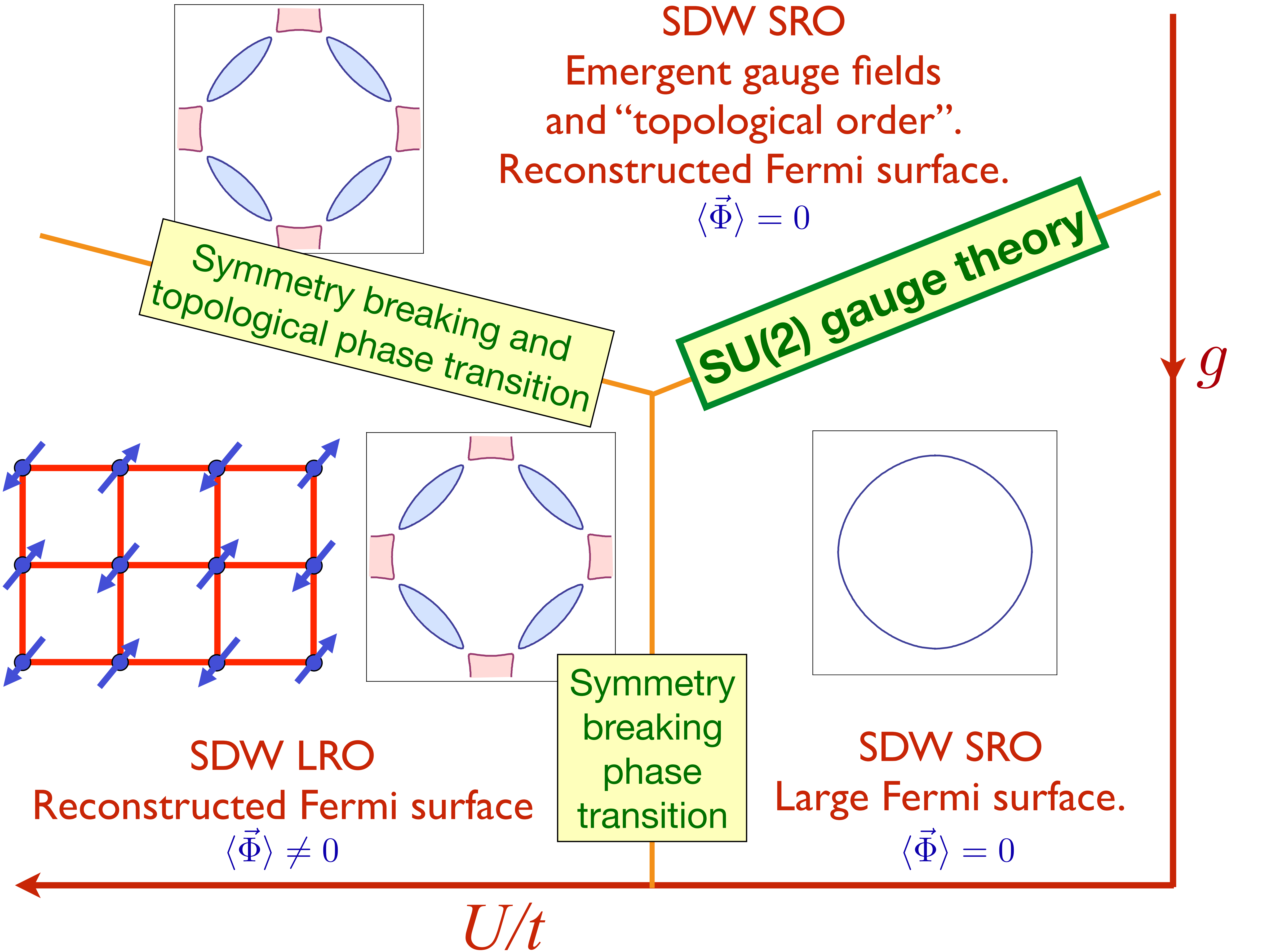


SDW SRO
Emergent gauge fields
and "topological order".
Reconstructed Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
Symmetry breaking and topological phase transition
Topological phase transition
Symmetry breaking phase transition
SDW LRO
Reconstructed Fermi surface
$\langle \vec{\Phi} \rangle \neq 0$
SDW SRO
Large Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
$g$
$U/t$



SDW SRO
Emergent gauge fields
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Reconstructed Fermi surface.
$\langle \vec{\Phi} \rangle = 0$
Symmetry breaking and topological phase transition
SU(2) gauge theory
$g$
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Large Fermi surface.
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# SDW theory

We can (exactly) transform the Hubbard model to the "spin-fermion" model: **electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left( c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}}$$

are coupled to a magnetic moment order parameter $\Phi^p(i)$, $p = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \Phi^p(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^p c_{i,\beta} + V_\Phi$$

For (fluctuating) SDW SRO, we transform to a rotating reference frame using the SU(2) rotation $R_i$

$$\begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} = R_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix},$$

in terms of fermionic "chargons" $\psi_s$ and a **Higgs field** $H^a(i)$

$$\sigma^p \Phi^p(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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The Higgs field is the SDW order in the rotating reference frame. Note that this representation is ambiguous up to a SU(2) gauge transformation, $V_i$

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \rightarrow V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i\, \sigma^b H^b(i)\, V_i^\dagger .$$

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# Particle content

| Field | Symbol | Statistics | $SU(2)_{\text{gauge}}$ | $SU(2)_{\text{spin}}$ | $U(1)_{\text{e.m.charge}}$ |
|---|---|---|---|---|---|
| Electron | $c$ | fermion | $\mathbf{1}$ | $\mathbf{2}$ | -1 |
| AF order | $\Phi$ | boson | $\mathbf{1}$ | $\mathbf{3}$ | 0 |
| Chargon | $\psi$ | fermion | $\mathbf{2}$ | $\mathbf{1}$ | -1 |
| Spinon | $R$ or $z$ | boson | $\bar{\mathbf{2}}$ | $\mathbf{2}$ | 0 |
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Spin density wave theory

# Particle content

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|---|---|---|---|---|---|
| Electron | $c$ | fermion | $\mathbf{1}$ | $\mathbf{2}$ | -1 |
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*SU(2) gauge theory:* fractionalize the SDW order parameter into the Higgs field ($H$) and the spinons ($R$); fractionalize the electron ($c$) into chargons ($\psi$) and spinons. Such a theory was used to model the photoemission and the cluster DMFT results in the intermediate phase.

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)

# Particle content

| Field | Symbol | Statistics | $SU(2)_{\rm gauge}$ | $SU(2)_{\rm spin}$ | $U(1)_{\rm e.m.charge}$ |
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*SU(2) gauge theory for* **quantum criticality***:* fractionalize the SDW order parameter into the Higgs field ($H$) and the spinons ($R$); but do *not* fractionalize the electron ($c$) into chargons ($\psi$) and spinons. The hopping $t$ leads to a strong attraction between $\psi$ and $R$, leading to an electron-like bound state *i.e.* assume all the low energy fermionic excitations have the quantum numbers of an electron

# SDW theory

We can (exactly) transform the Hubbard model to the "spin-fermion" model: **electrons** $c_{i\alpha}$ on the square lattice with dispersion

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# SU(2) gauge theory

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are coupled to a magnetic moment order parameter $\Phi^p(i)$, $p = x, y, z$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \Phi^p(i) c^\dagger_{i,\alpha} \sigma^p_{\alpha\beta} c_{i,\beta} + V_\Phi$$

Express $\Phi^\ell$ in terms the Higgs field

$$\sigma^p \Phi^p(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

Integrate out high energy $c_\alpha$ and spinons $R$, to obtain effective theory for $H^a$ and low energy $c_\alpha$ near the Fermi surface.

# SU(2) gauge theory

We obtain different numbers of adjoint Higgs scalars, $N_f$, depending upon the spatial dependence of the local spin correlations:

Neel correlations: $N_f = 1$,

$\boldsymbol{K} = (\pi, \pi)$,

$H^a(i) = H_1^a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$

Unidirectional incommensurate correlations: $N_f = 2$,

$\boldsymbol{K} = (\pi, \pi - \delta)$,

$H^a(i) = \mathrm{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}\right\}$

Bidirectional incommensurate correlations: $N_f = 4$,

$\boldsymbol{K}_y = (\pi, \pi - \delta)$, $\boldsymbol{K}_x = (\pi - \delta, \pi)$,

$H^a(i) = \mathrm{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H_3^a(\boldsymbol{r}) + iH_4^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right\}$

# SU(2) gauge theory

SU(2) gauge theory with $N_f$ adjoint Higgs fields with potential $V(H^a_\ell)$, $a = 1, 2, 3$, $\ell = 1 \ldots N_f$

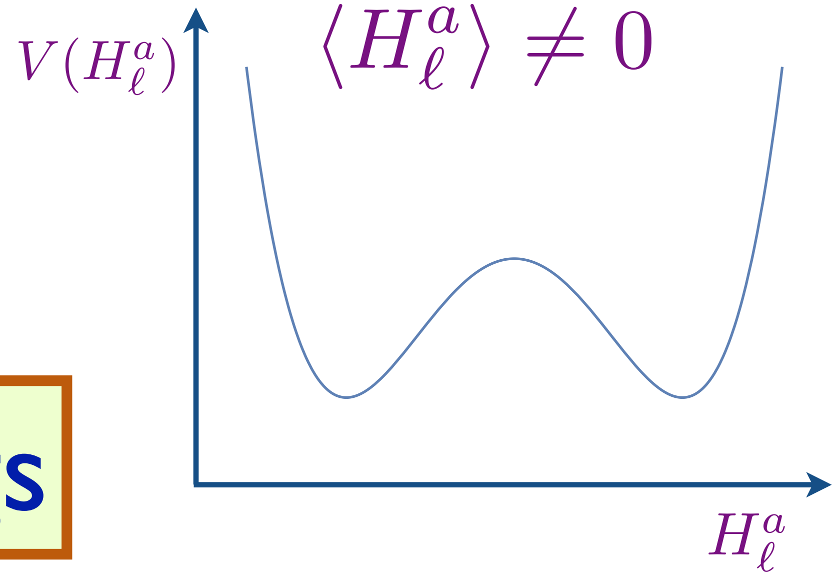
$$-\sum_{i,\rho} t_\rho \left( c^\dagger_{i,\alpha} c_{i+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{i+\boldsymbol{v}_\rho,\alpha} c_{i,\alpha} \right) - \mu \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} + \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} H^a(i) H^a(i)$$

$$V(H^a_\ell) = s\, H^a_\ell H^a_\ell + u_1 (H^a_\ell H^a_\ell)^2 + u_2\, H^a_\ell H^a_m H^b_\ell H^b_m + \ldots$$

$N_f = 1$

# Phase diagrams of SU(2) gauge theory

**Higgs**

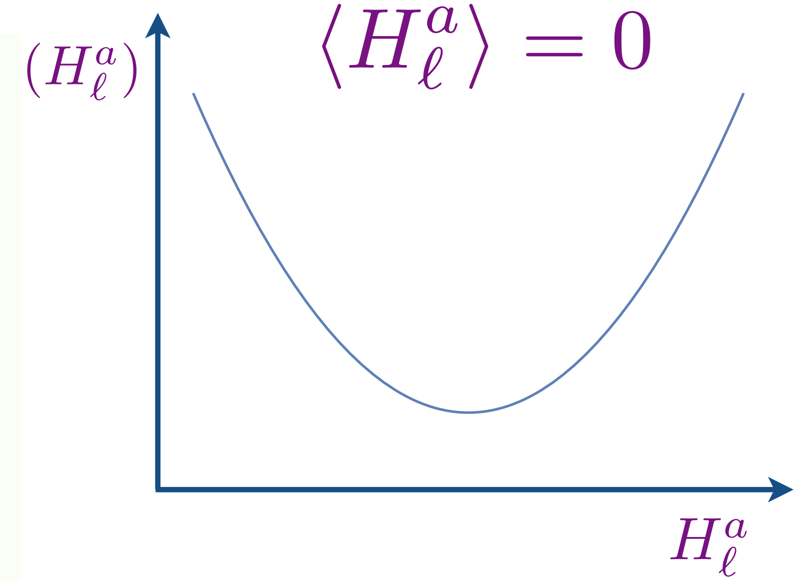



Exponentially large confinement length

Reconstructed Fermi surfaces at distances smaller than the confinement length

Crossover

**Confinement**



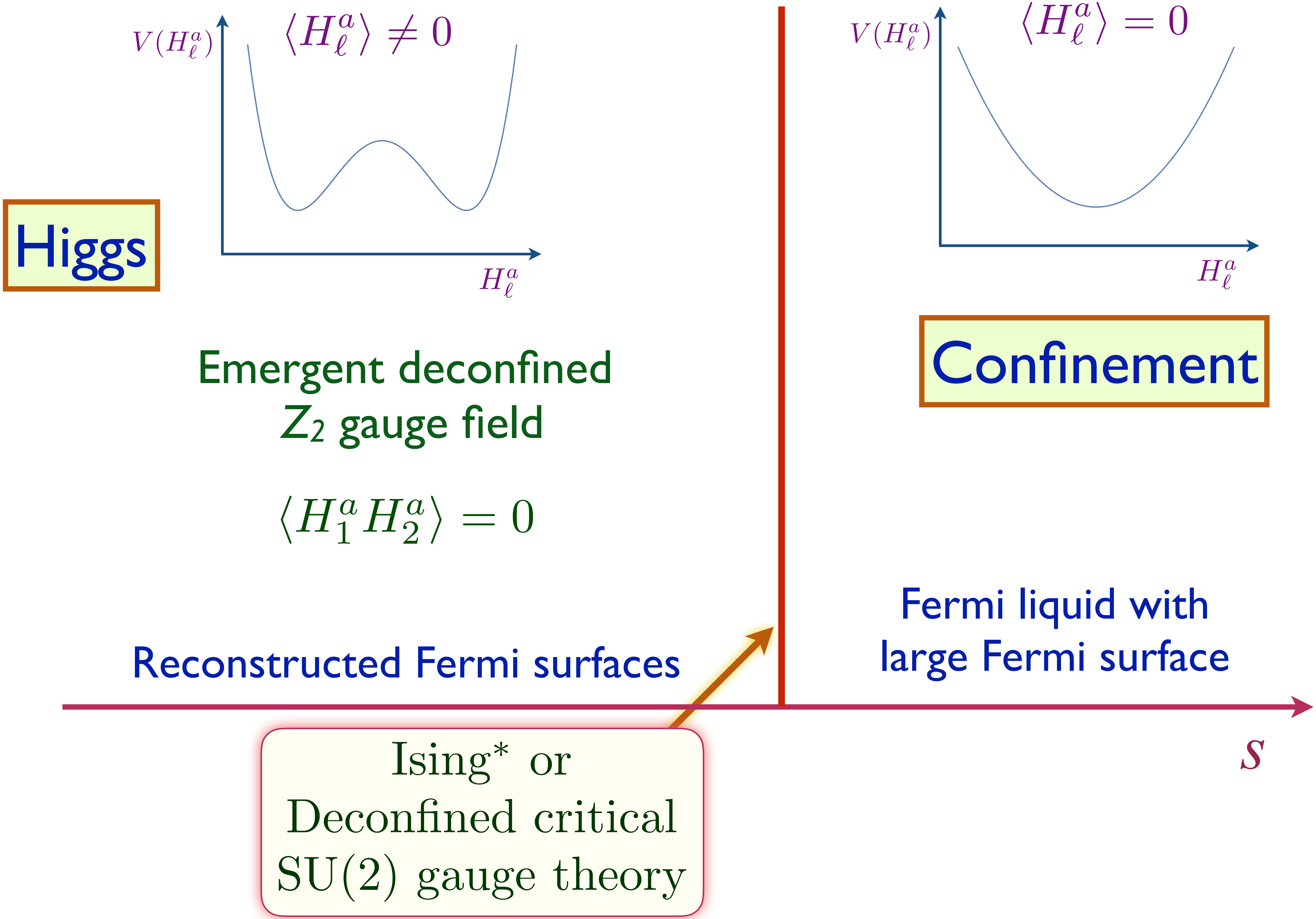

Fermi liquid with large Fermi surface

$s$


A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

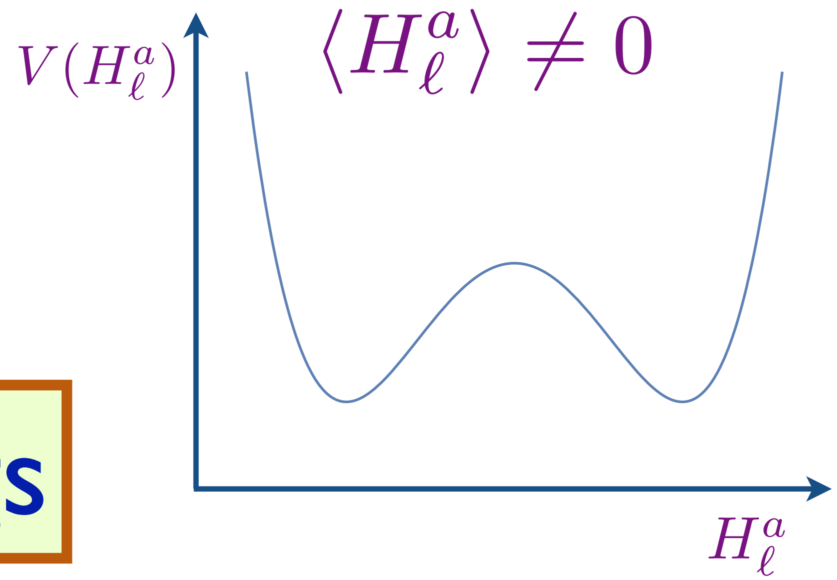
$N_f = 2$

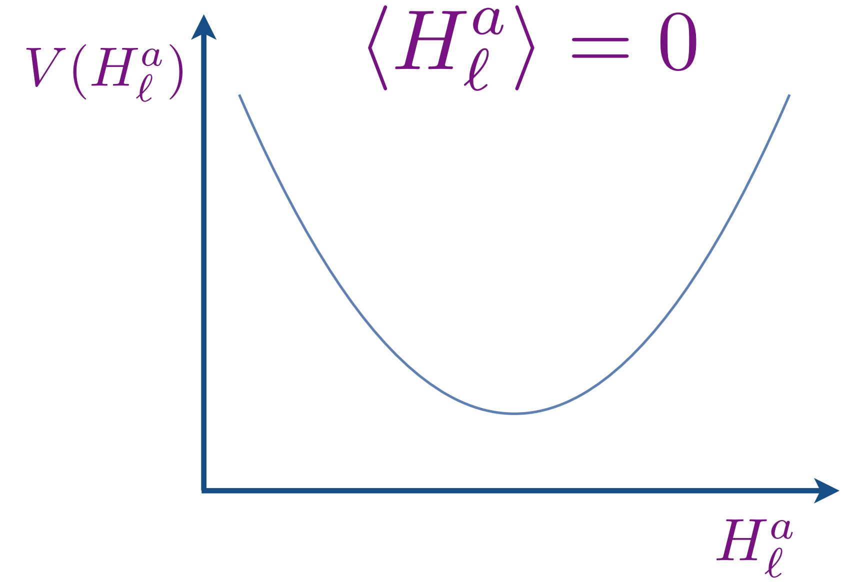
# Phase diagrams of SU(2) gauge theory



$N_f = 2$

# Phase diagrams of SU(2) gauge theory




**Higgs**

Charge density wave order
Exponentially large confinement length

Gauge invariant order parameter
$\langle H_\ell^a H_m^a - \delta_{\ell m}(H_n^a H_n^a)/N_f \rangle \neq 0$
breaks $\mathrm{O}(N_f)$ symmetry

Reconstructed Fermi surfaces at distances smaller than the confinement length

**Confinement**

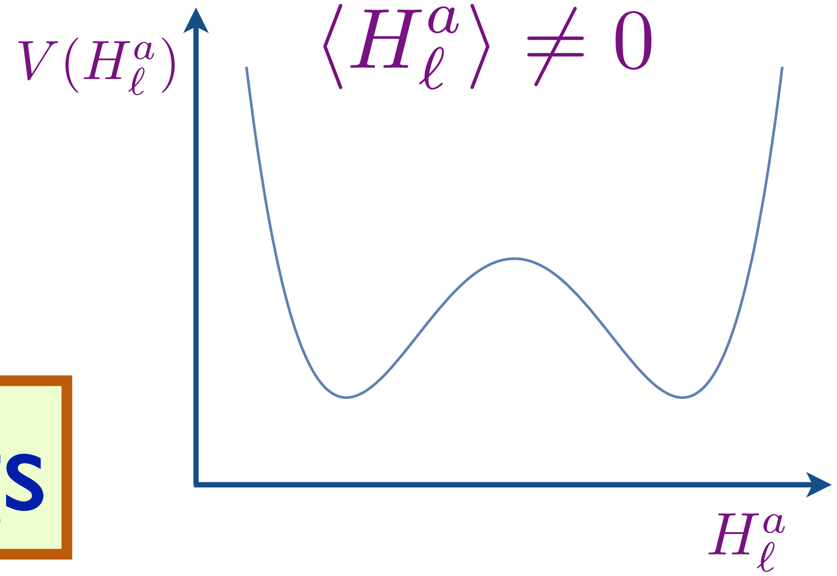
Fermi liquid with large Fermi surface

XY Wilson-Fisher or Deconfined critical SU(2) gauge theory

$s$

$N_f = 2$

# Phase diagrams of SU(2) gauge theory




**Higgs**

Charge density wave order
Exponentially large confinement length

Gauge invariant order parameter
$\langle H_\ell^a H_m^a - \delta_{\ell m}(H_n^a H_n^a)/N_f \rangle \neq 0$
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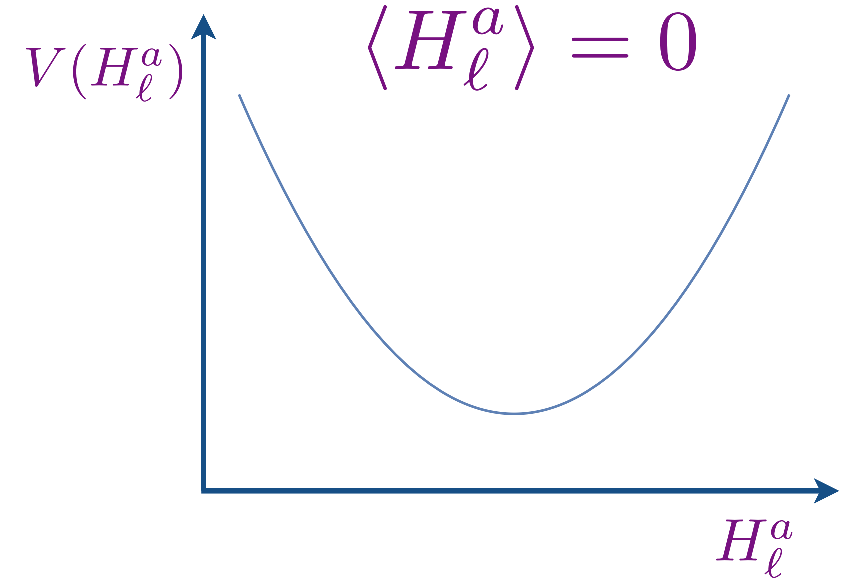
Reconstructed Fermi surfaces at distances smaller than the confinement length




**Confinement**

Fermi liquid with large Fermi surface

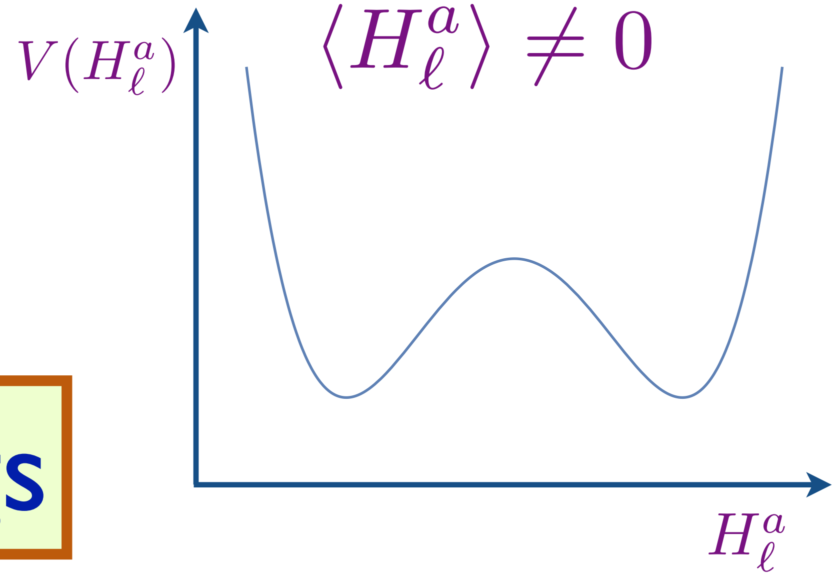
$s$

Transition between conventional phases with and without broken symmetry need not be of the LGW type !

XY Wilson-Fisher or Deconfined critical SU(2) gauge theory

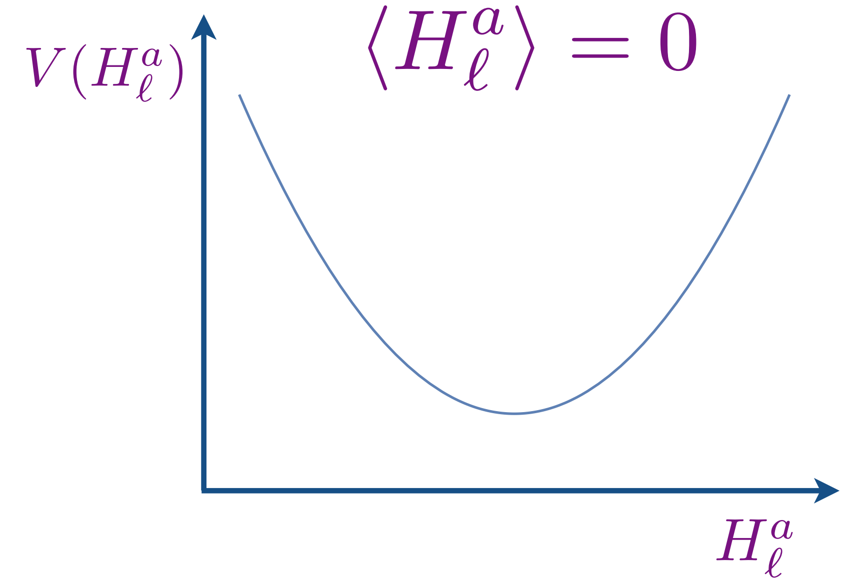
$N_f = 4$

# Phase diagrams of SU(2) gauge theory




**Higgs**

Emergent deconfined
$Z_2$ gauge field
and/or
Charge density wave order
and/or
Ising-nematic order
(Reconstructed Fermi surfaces)



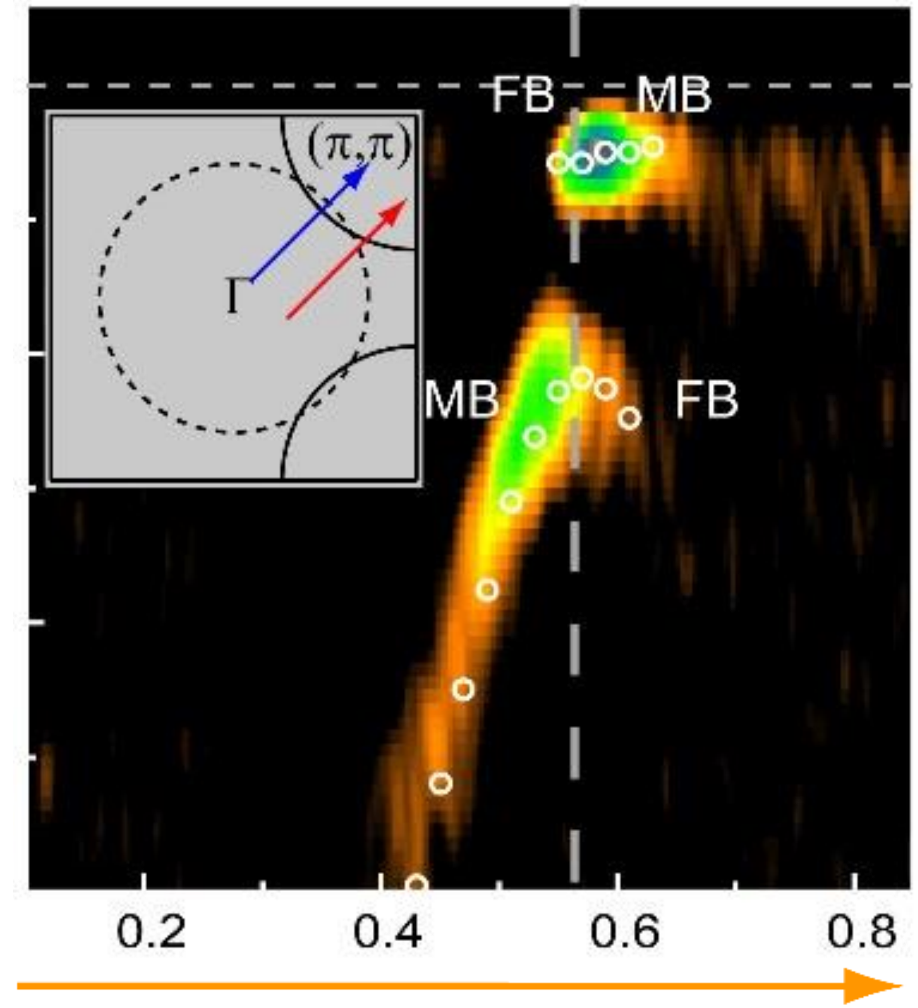

**Confinement**

(Fermi liquid with
large Fermi surface)

$s$

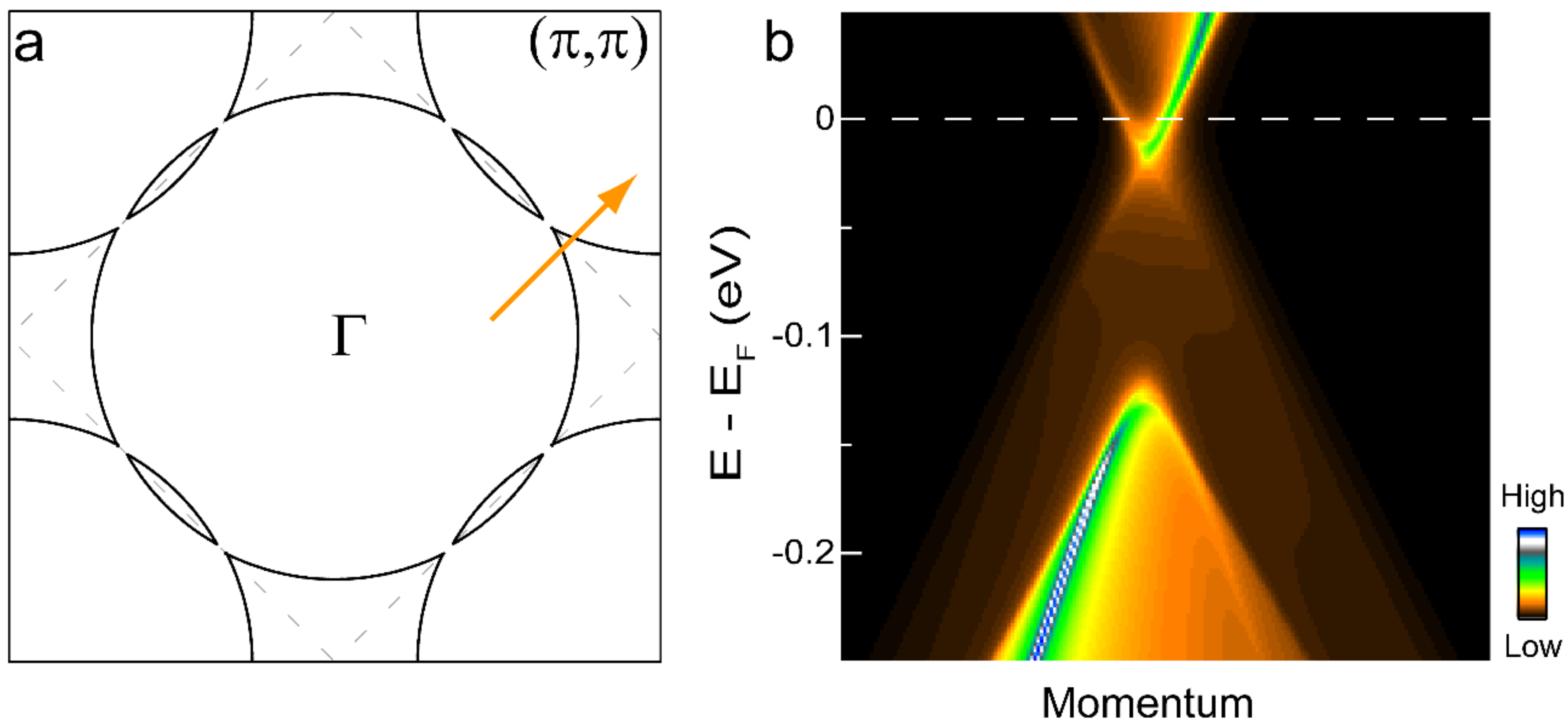
Deconfined critical
SU(2) gauge theory

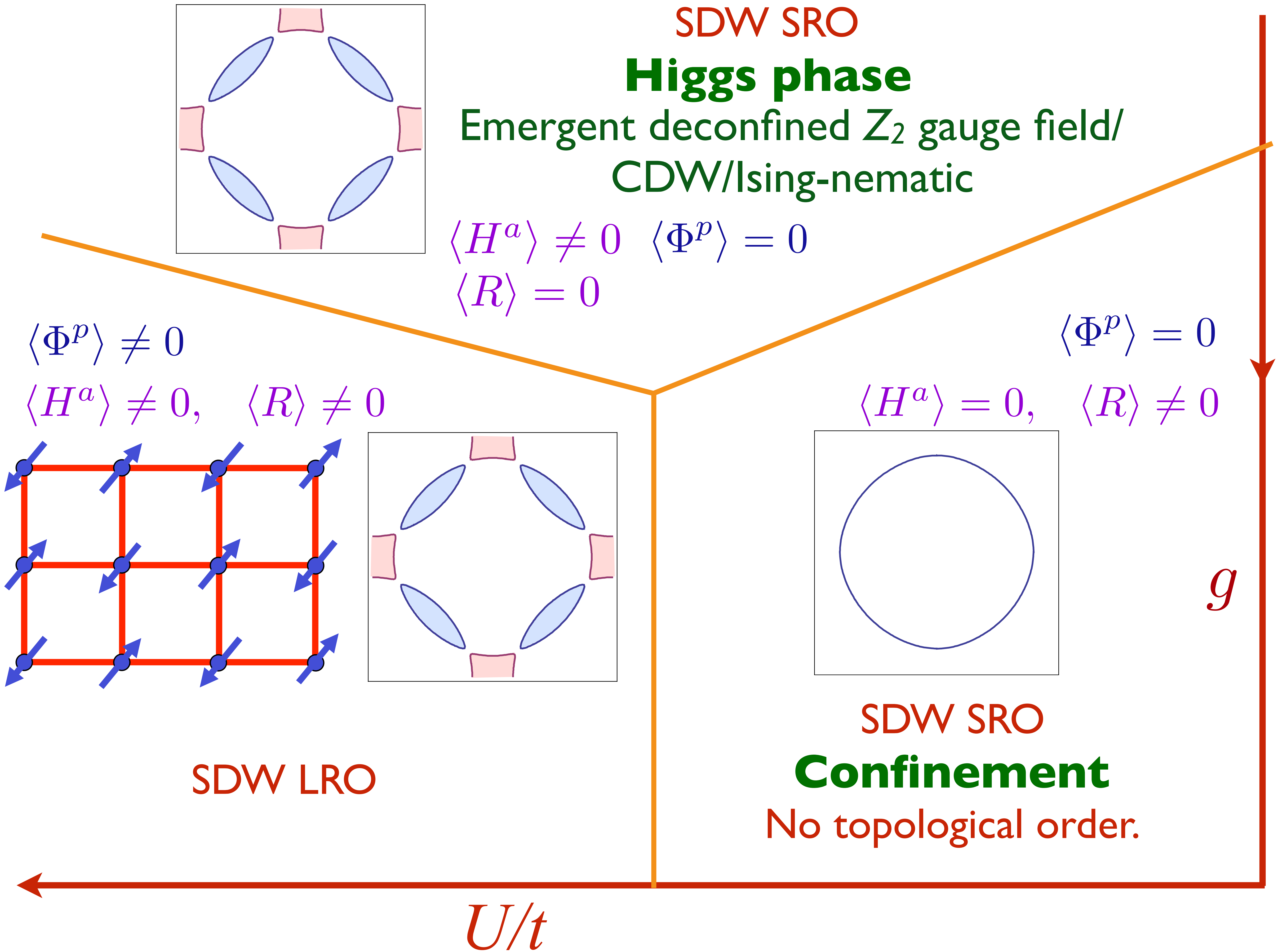
Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen,
Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order



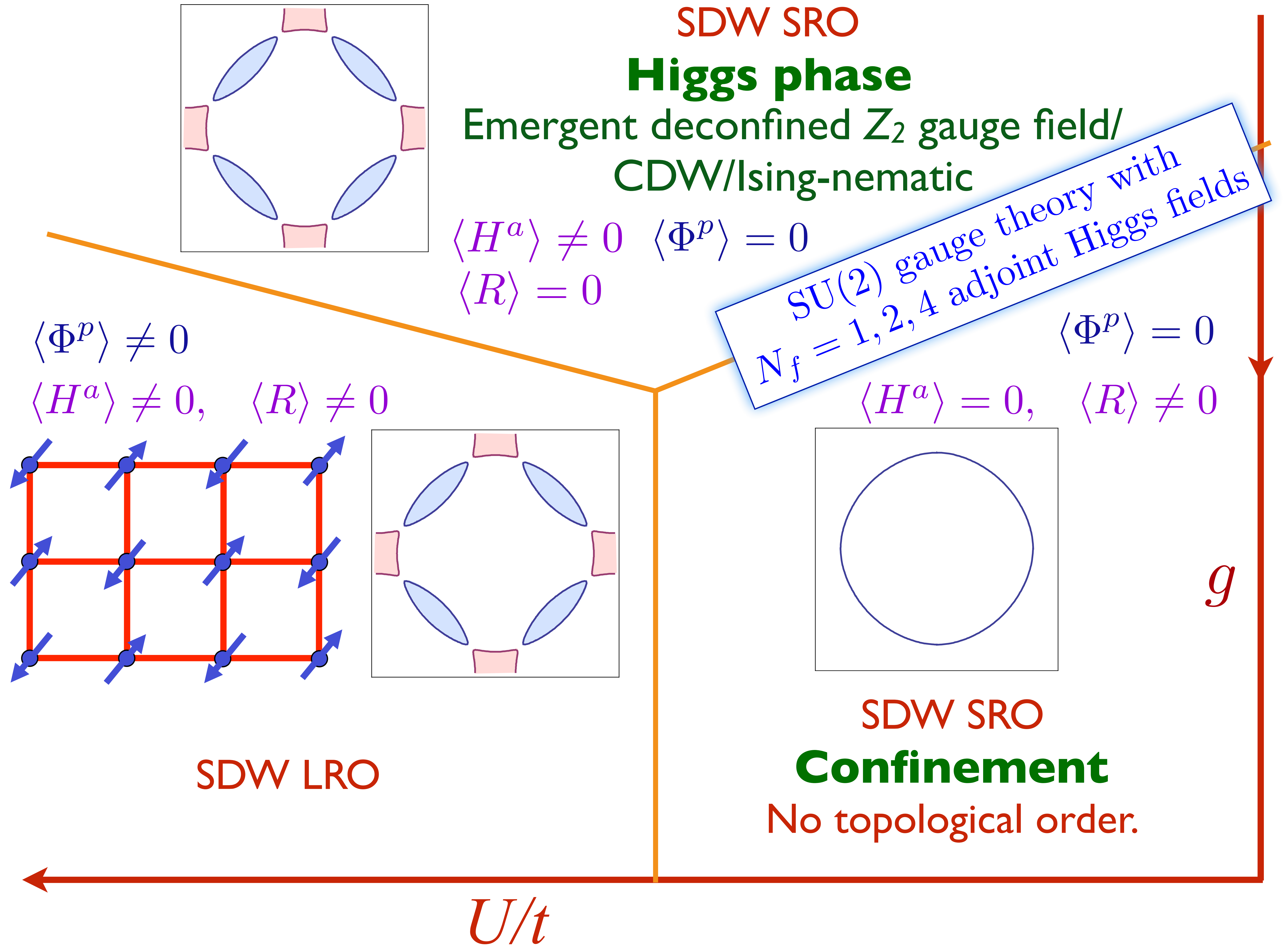

S. Sachdev, Topological order and Fermi surface reconstruction, arXiv:1801.01125

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)





SDW SRO
Higgs phase
Emergent deconfined $Z_2$ gauge field/
CDW/Ising-nematic
$\langle H^a \rangle \neq 0$ $\langle \Phi^p \rangle = 0$
$\langle R \rangle = 0$
$\langle \Phi^p \rangle \neq 0$
$\langle H^a \rangle \neq 0, \quad \langle R \rangle \neq 0$
$\langle \Phi^p \rangle = 0$
$\langle H^a \rangle = 0, \quad \langle R \rangle \neq 0$
$g$
SDW SRO
Confinement
No topological order.
SDW LRO
$U/t$



SDW SRO
Higgs phase
Emergent deconfined $Z_2$ gauge field/
CDW/Ising-nematic
$\langle H^a \rangle \neq 0$ $\langle \Phi^p \rangle = 0$
$\langle R \rangle = 0$
SU(2) gauge theory with
$N_f = 1, 2, 4$ adjoint Higgs fields
$\langle \Phi^p \rangle \neq 0$
$\langle H^a \rangle \neq 0$, $\langle R \rangle \neq 0$
$\langle \Phi^p \rangle = 0$
$\langle H^a \rangle = 0$, $\langle R \rangle \neq 0$
$g$
SDW SRO
Confinement
No topological order.
SDW LRO
$U/t$