Topological order in quantum matter

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Talk online: sachdev.physics.harvard.edu

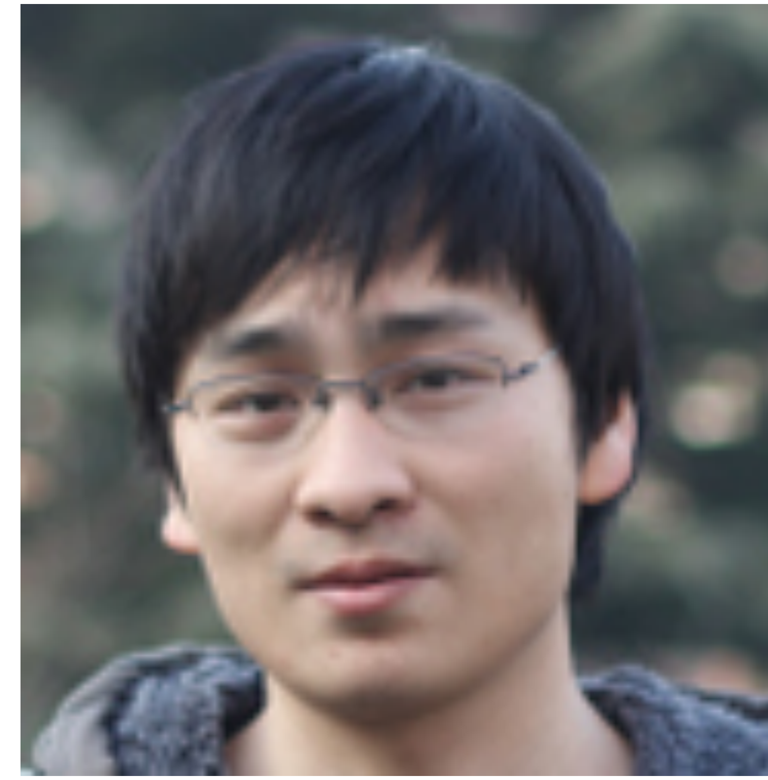
PHYSICS

VE RI TAS

HARVARD

Mathias Scheurer

Shubhayu Chatterjee



Wei Wu

Michel Ferrero

arXiv:1711.09925

Antoine Georges

1. Classical XY model in 2 and 3 dimensions

2. Topological order in the classical XY model in 3 dimensions

3. Topological order in the quantum XY model in 2+1 dimensions

4. Topological order in the Hubbard model

1. Classical XY model in 2 and 3 dimensions

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$$\mathcal{Z}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-H/T\right)$$

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

Describes non-zero *T* phase transitions of superfluids, magnets with `easy-plane' spins, ...

In spatial dimension $d = 3$, in the low T phase, the symmetry $\theta_i \to \theta_i + c$ is "spontaneously broken". There is (off-diagonal) long-range order (LRO) characterized by ($\Psi_i \equiv e^{i\theta_i}$)

$$\lim_{|r_i - r_j| \to \infty} \langle \Psi_i \Psi_j^* \rangle = |\Psi_0|^2 \neq 0 \,.$$

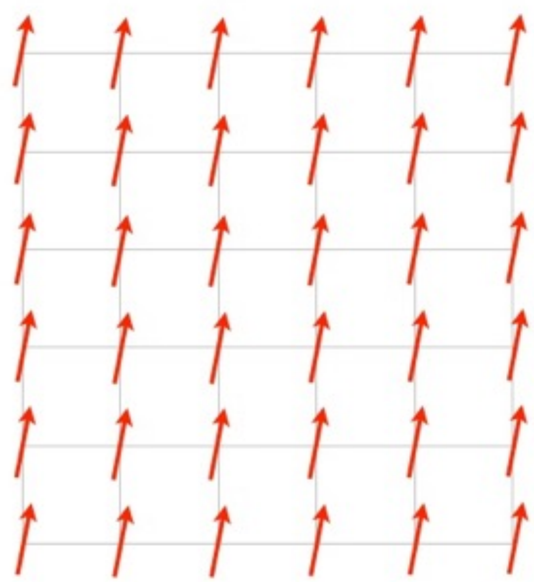
We break the symmetry by choosing an overall phase so that

$$\langle \Psi_i \rangle = \Psi_0 \neq 0$$

Wilson-Fisher theory
(Nobel Prize, 1982)

$\langle \Psi_i \rangle = \Psi_0 \neq 0$

LRO



$\langle \Psi_i \rangle = 0$

$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$

SRO

T_c T

Kosterlitz-Thouless theory in *d*=2

In spatial dimension $d = 2,$ the symmetry $\theta_i \to \theta_i + c$ is preserved at all non-zero T. There is no LRO, and

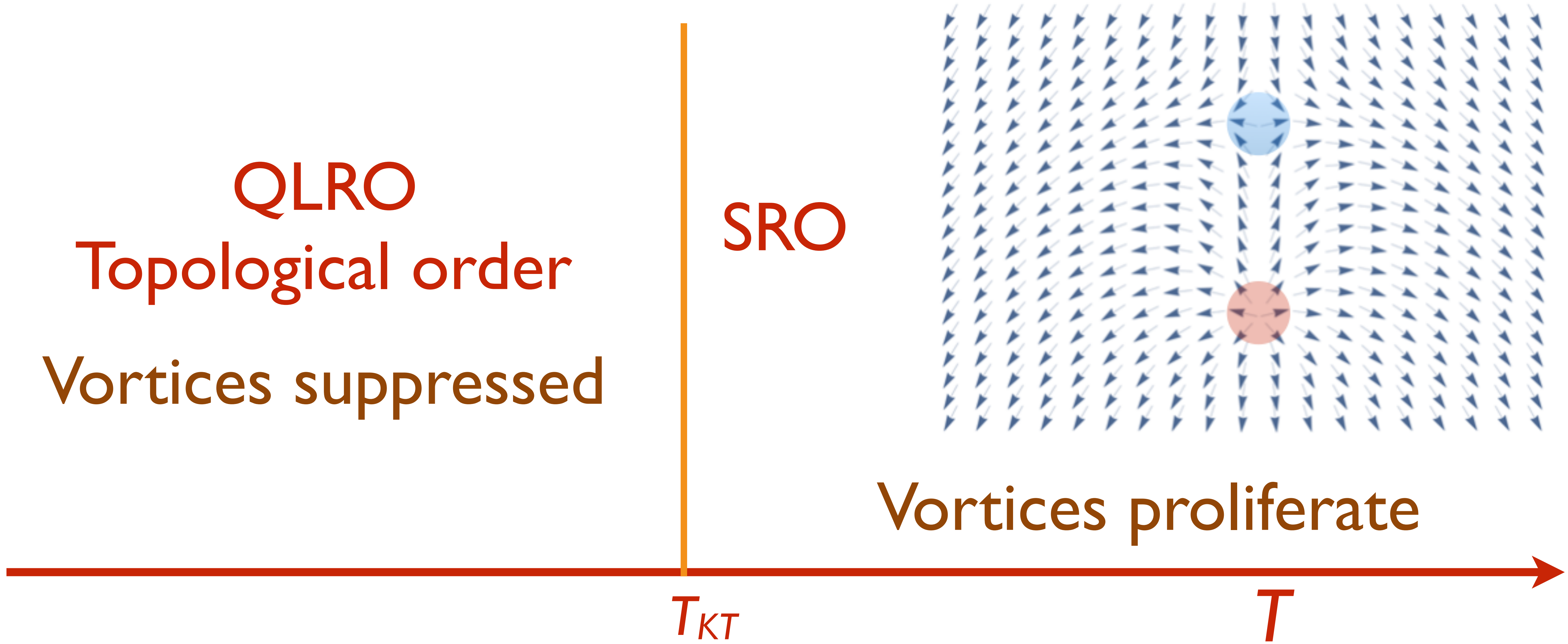
$$\langle \Psi_i \rangle = 0 \text{ for all } T > 0.$$

Nevertheless, there is a phase transition at $T = T_{KT}$, where the nature of the correlations changes

$$\lim_{|r_i - r_j| \to \infty} \langle \Psi_i \Psi_j^* \rangle \sim \begin{cases} |r_i - r_j|^{-\alpha}, & \text{for } T < T_{KT}, \text{ (QLRO)} \\ \exp(-|r_i - r_j|/\xi), & \text{for } T > T_{KT}, \text{ (SRO)} \end{cases}$$

KT theory
(Nobel Prize, 2016)

Kosterlitz-Thouless theory in d=2



The low T phase also has **topological order** associated with the suppression of vortices.

KT theory
(Nobel Prize, 2016)

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Can we modify the XY model Hamiltonian to obtain a phase with "topological order" in d=3 ?

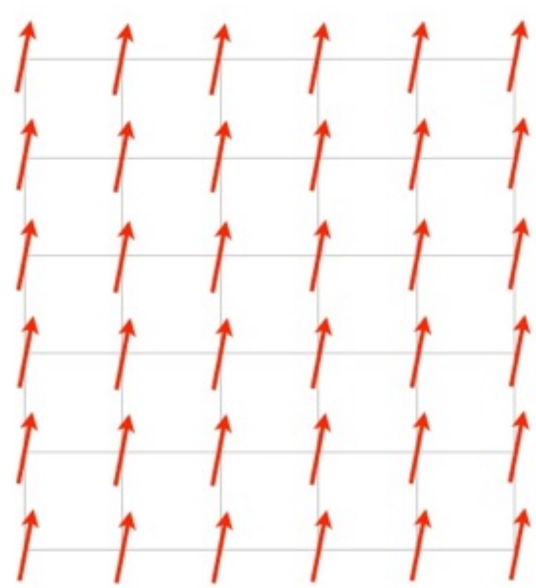
$$\langle \Psi_i \rangle = 0$$

$$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$$

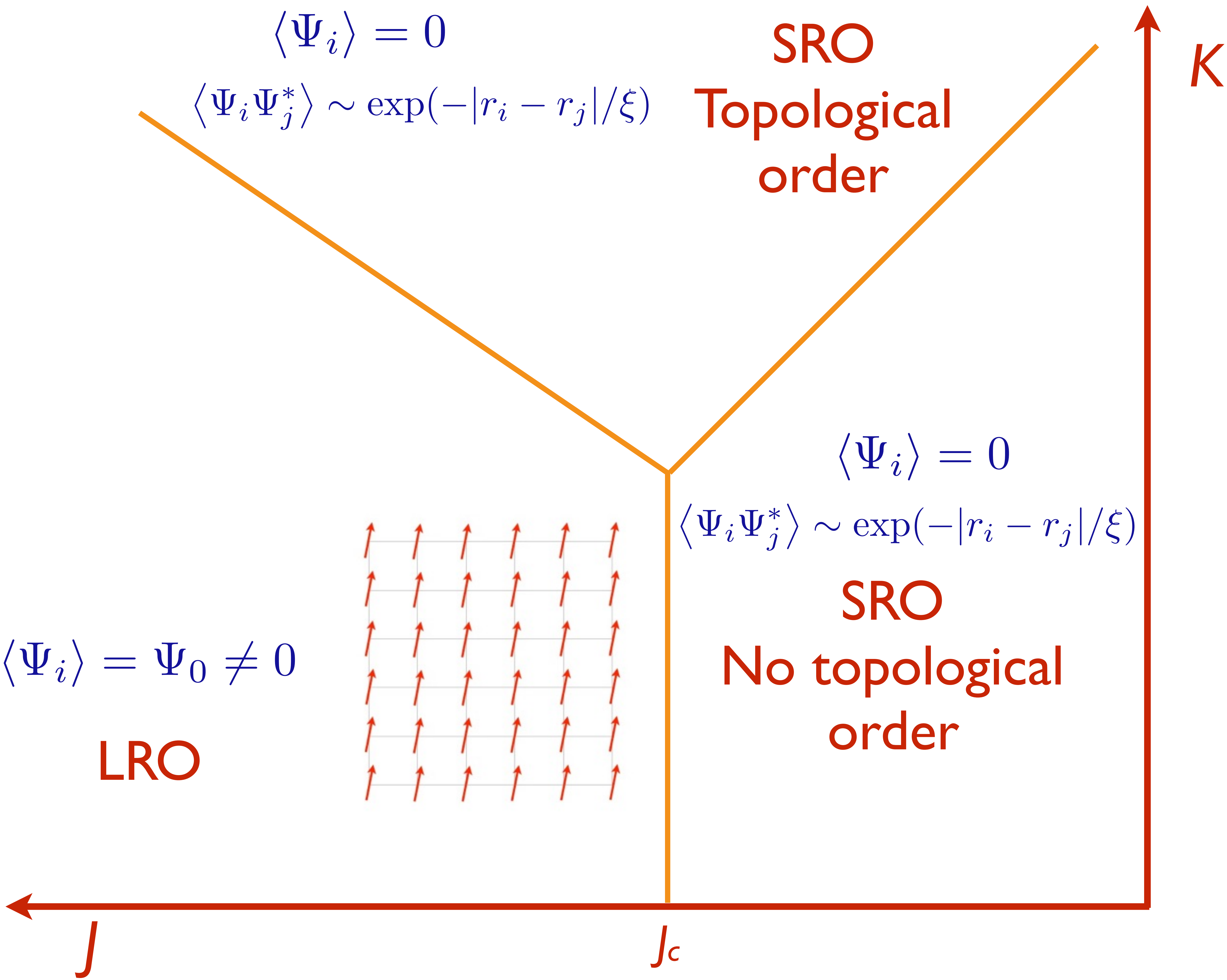
SRO

$$\langle \Psi_i \rangle = \Psi_0 \neq 0$$

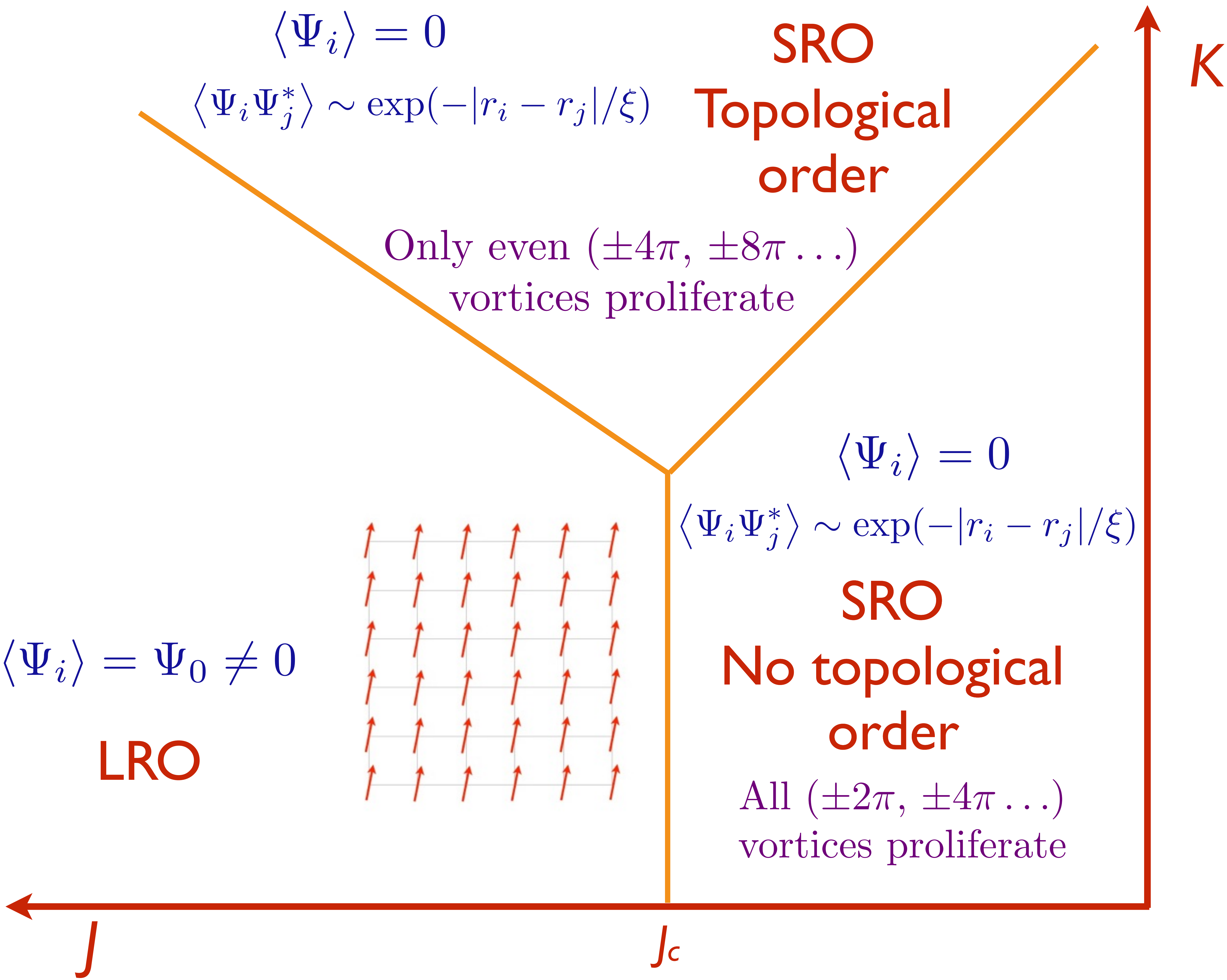
LRO



J J_c



$\langle \Psi_i \rangle = 0$
$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$
SRO
Topological order
K
$\langle \Psi_i \rangle = 0$
$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$
SRO
No topological order
$\langle \Psi_i \rangle = \Psi_0 \neq 0$
LRO
J
J_c



$\langle \Psi_i \rangle = 0$
$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$
SRO
Topological
order
K
Only even $(\pm 4\pi, \pm 8\pi \ldots)$
vortices proliferate
$\langle \Psi_i \rangle = 0$
$\langle \Psi_i \Psi_j^* \rangle \sim \exp(-|r_i - r_j|/\xi)$
SRO
No topological
order
All $(\pm 2\pi, \pm 4\pi \ldots)$
vortices proliferate
$\langle \Psi_i \rangle = \Psi_0 \neq 0$
LRO
J
J_c

$$\widetilde{\mathcal{Z}}_{XY} = \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) + \sum_{ijk\ell} J_{ijk\ell} \cos(\theta_i + \theta_j - \theta_k - \theta_\ell) + \ldots\ldots$$

Add terms which suppress single but not double vortices…..

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J\sum_{\langle ij\rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}$$

Lattice gauge theory precursors (without symmetry broken phases):
F.Wegner, J.Math. Phys. **12**, 2259 (1971).
E. Fradkin and S. H. Shenker, PRD **19**, 3682 (1979).
S. Sachdev and N. Read, Int. J. Mod. Phys. B, **5**, 219 (1991).
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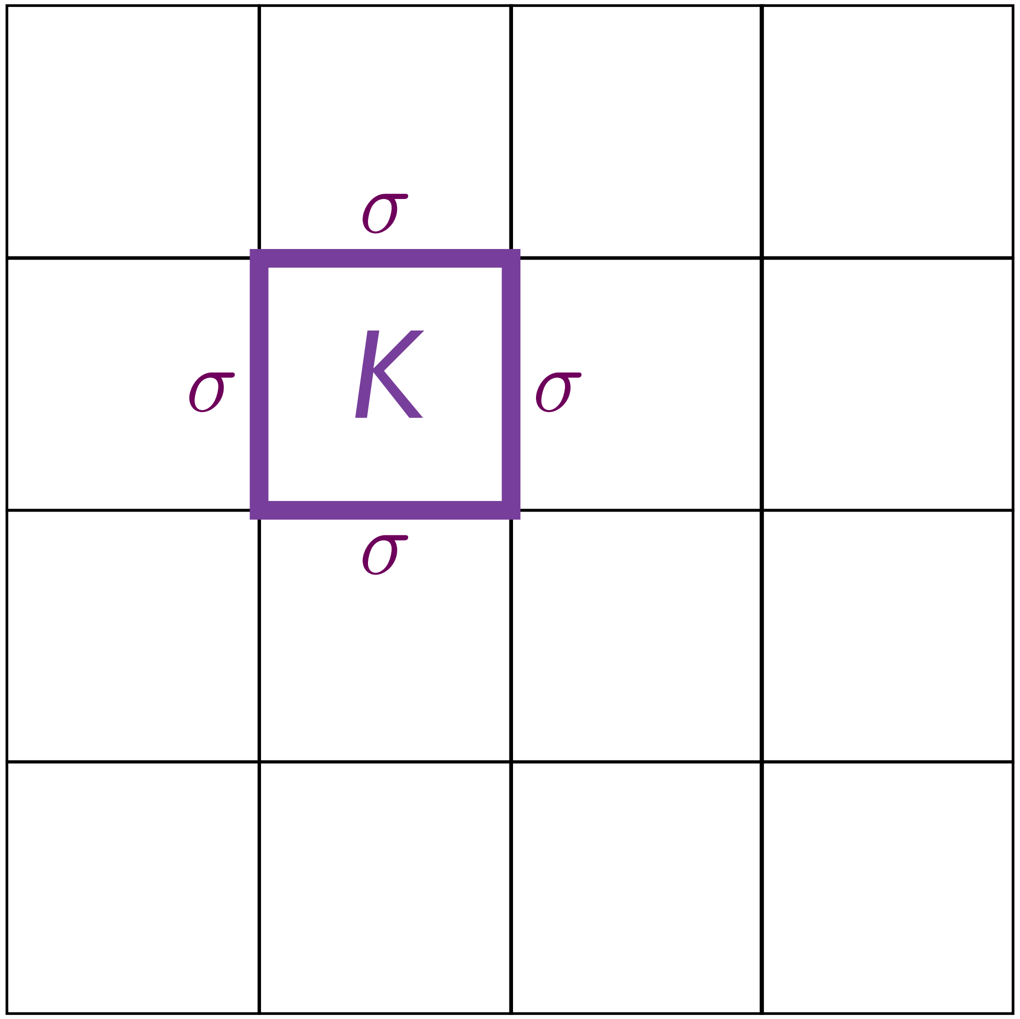


$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\Box} \prod_{(ij)\in\Box} \sigma_{ij}$$

- At small K, we can explicitly sum over σ_{ij}, order-by-order in K, and the theory reduces to an ordinary XY model with multi-site interactions. The resulting effective action of the XY model is periodic in $\theta_i \to \theta_i + 2\pi$ (for any site i), and preserves the symmetry $\theta_i \to \theta_i + c$ (for all sites i).

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\Box} \prod_{(ij)\in\Box} \sigma_{ij}$$

- The theory has a $\mathbb{Z}_2$ gauge invariance: we can change

$$\theta_i \rightarrow \theta_i + \pi(1-\eta_i)$$
$$\sigma_{ij} \rightarrow \eta_i \sigma_{ij} \eta_j \,,$$

  with $\eta_i = \pm 1$, and the energy remains unchanged.

- The XY order parameter $\Psi_i = e^{i\theta_i}$ is gauge invariant, as are all physical observables. So this is an XY model with a modified Hamiltonian, and no additional degrees of freedom have been introduced.

$$\begin{aligned} \widetilde{\mathcal{Z}}_{XY} &= \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right) \\ \widetilde{H} &= -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij} \end{aligned}$$

- A single (odd) 2π vortex in θ_i has $\prod_{(ij)\in\square} \cos\left[(\theta_i - \theta_j)/2\right] < 0$.

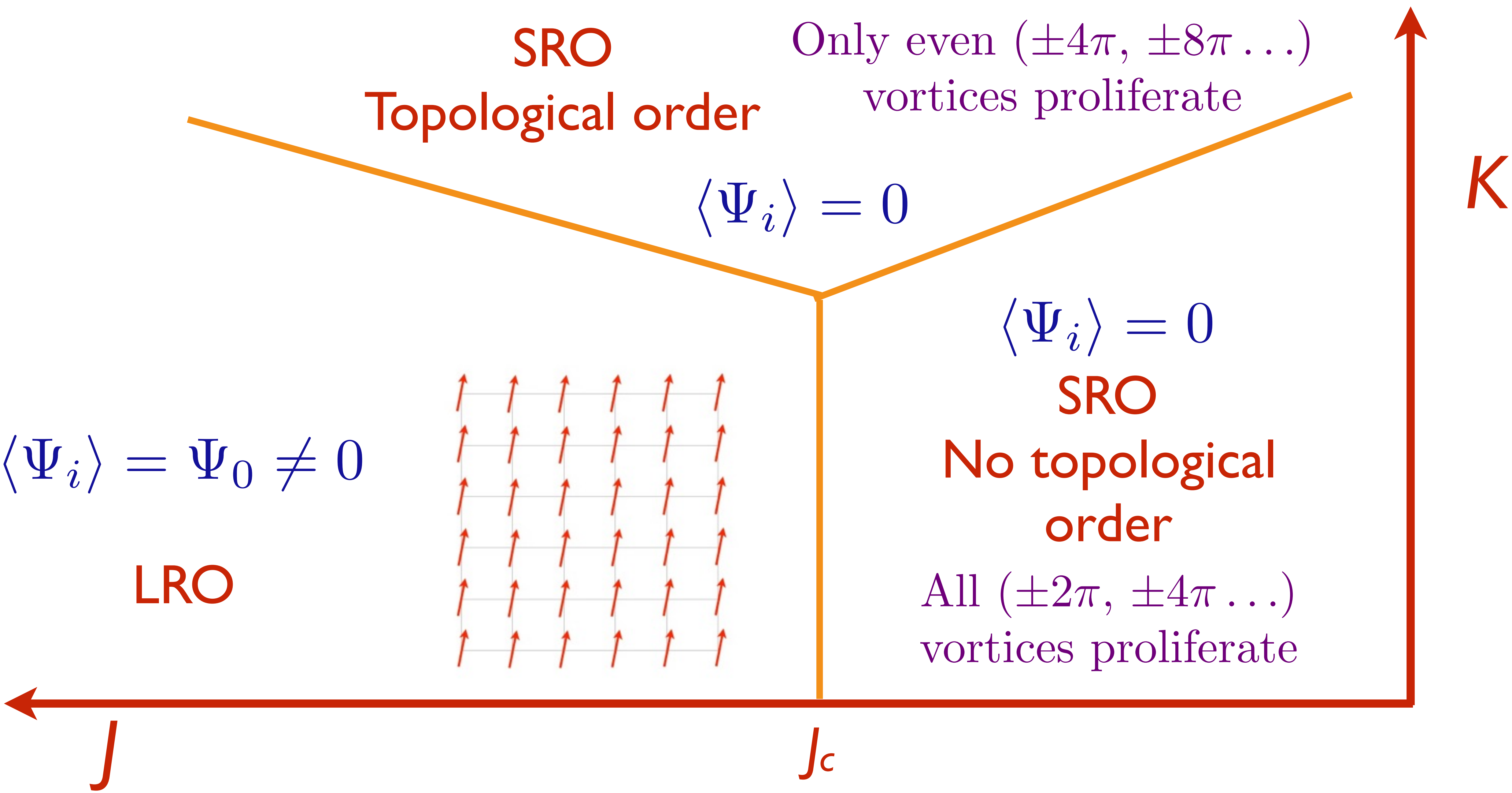
- So for $J > 0$, such a vortex will prefer $\prod_{(ij)\in\square} \sigma_{ij} = -1$, *i.e.* a 2π vortex has $\mathbb{Z}_2$ flux $= -1$ in its core.

- So a large $K > 0$ will suppress (odd) 2π vortices.

- There is no analogous suppression of (even) 4π vortices.

$$\widetilde{\mathcal{Z}}_{XY} = \sum_{\{\sigma_{ij}\}=\pm 1} \prod_i \int_0^{2\pi} \frac{d\theta_i}{2\pi} \exp\left(-\widetilde{H}/T\right)$$

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij} \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}$$



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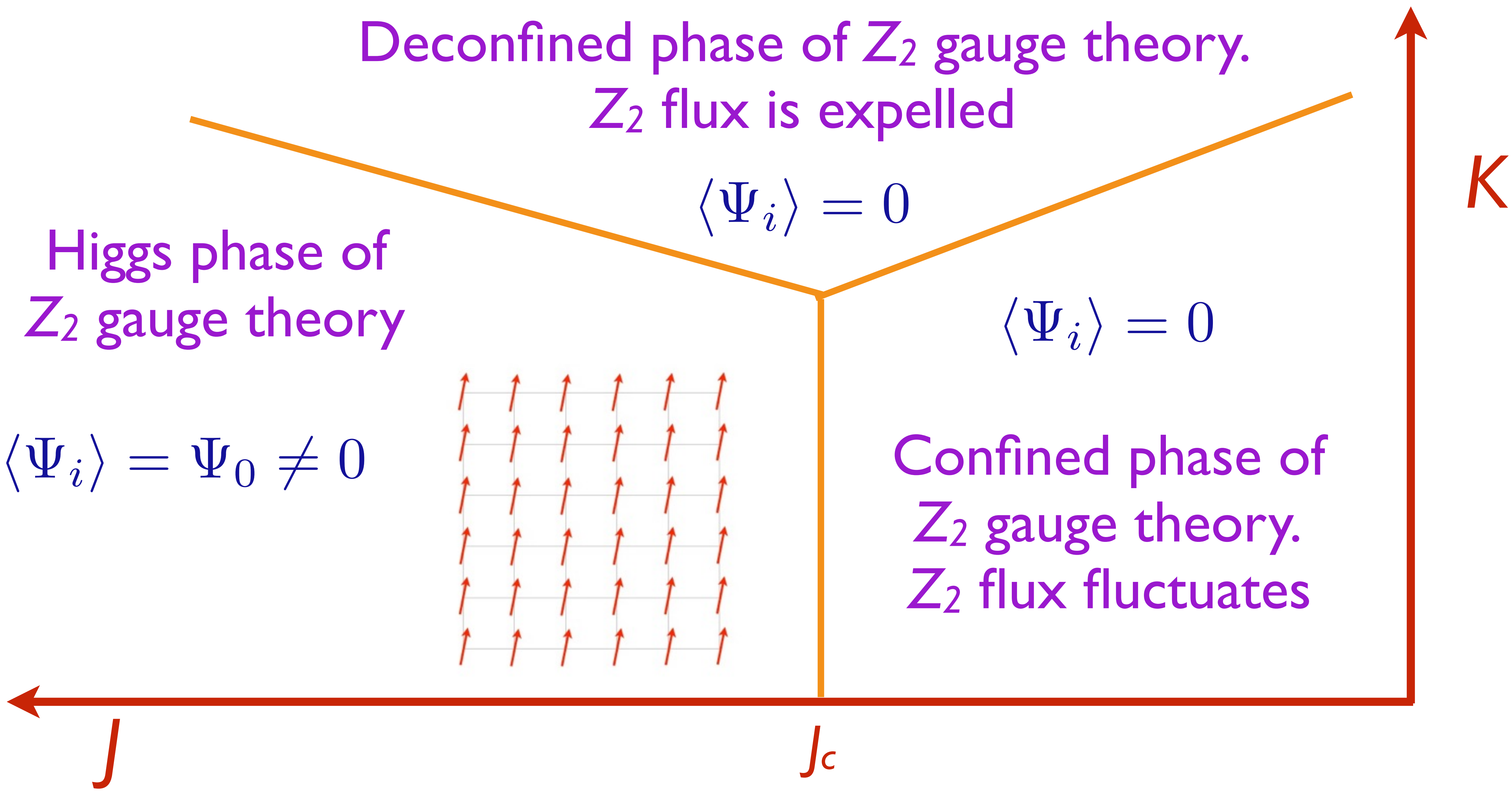

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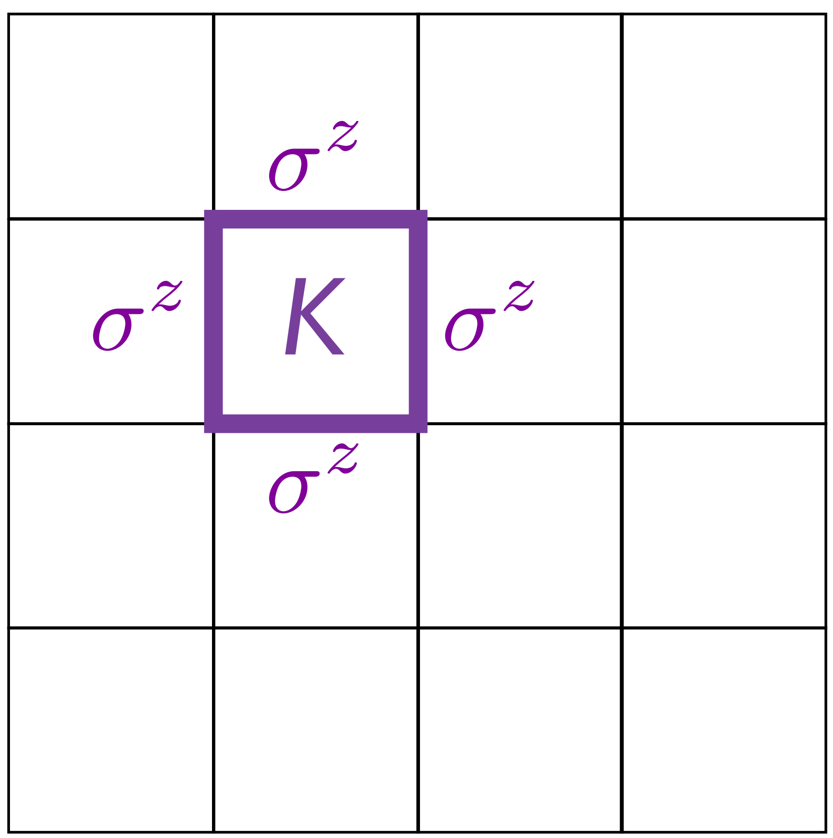
A quantum Hamiltonian in 2+1 dimensions

$$\widetilde{H} = -J \sum_{\langle ij \rangle} \sigma_{ij}^z \cos\left[(\theta_i - \theta_j)/2\right] - K \sum_{\square} \prod_{(ij)\in\square} \sigma_{ij}^z$$

$$+U \sum_i (\hat{n}_i)^2 - g \sum_{\langle ij \rangle} \sigma_{ij}^x \quad ; \quad [\theta_i, \hat{n}_j] = i\delta_{ij}$$



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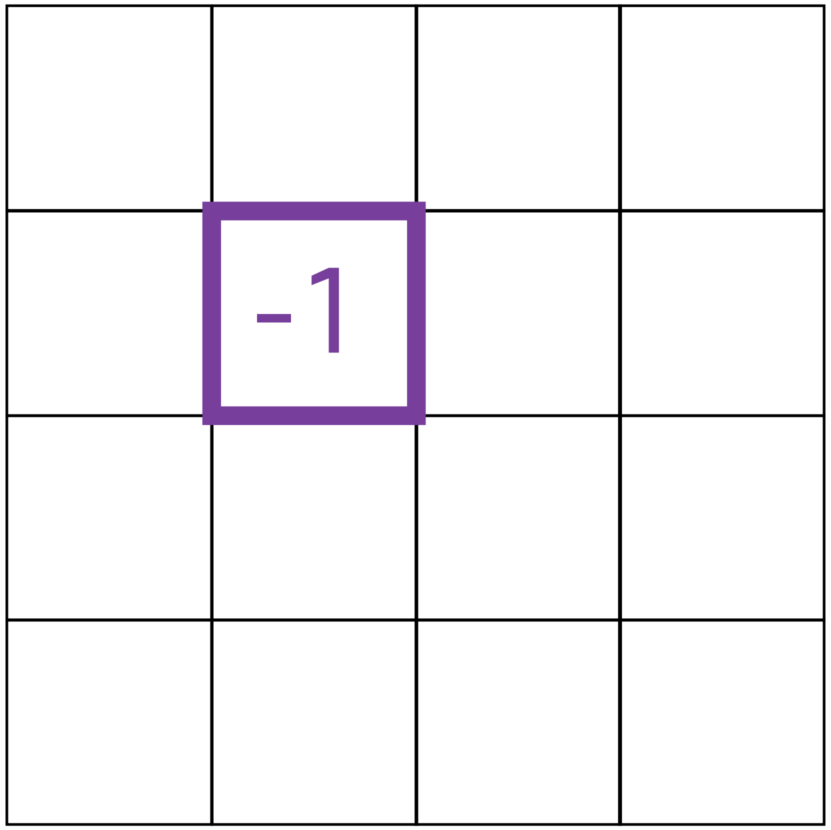
- In the topological phase, the suppressed Z_2 fluxes of -1 become well-defined gapped quasiparticle excitations ('visons') above the ground state.



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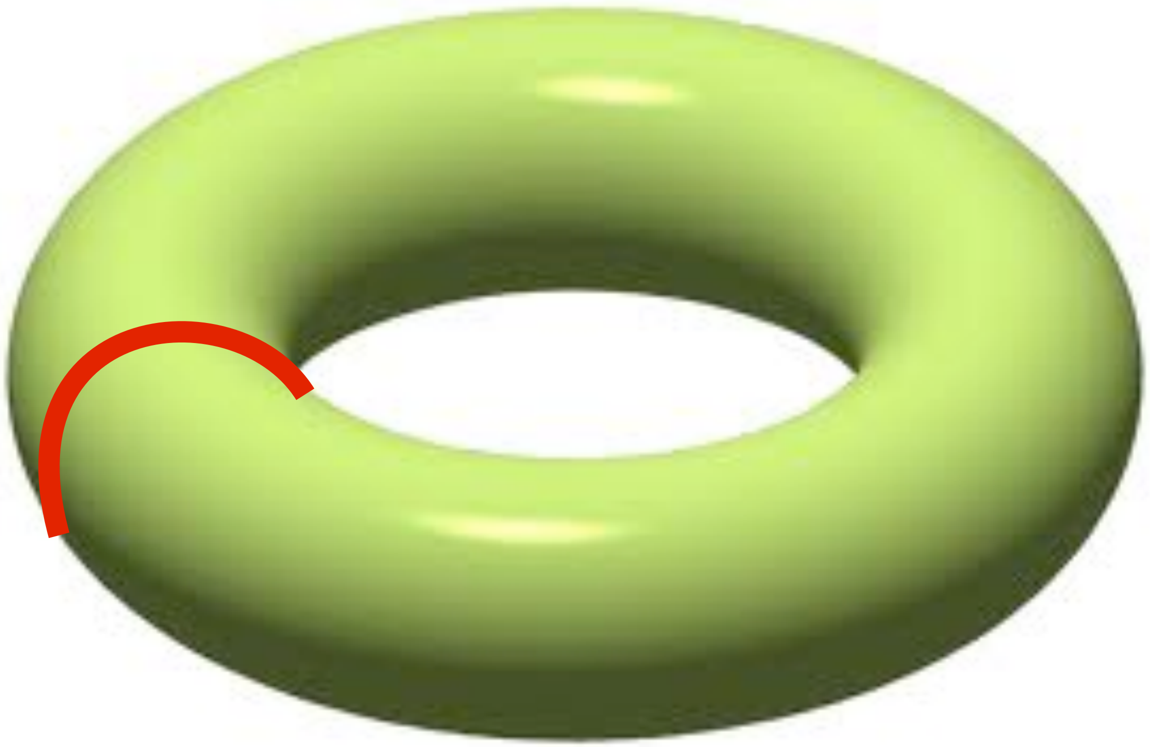
$$+U \sum_i (\hat{n}_i)^2 - g \sum_{\langle ij \rangle} \sigma^x_{ij} \quad ; \quad [\theta_i, \hat{n}_j] = i\delta_{ij}$$

- In the topological phase, on a torus, inserting the Z_2 flux of -1 into one of the cycles of the torus leads to an orthogonal state whose energy cost vanishes exponentially in the size of the torus: there are 4 degenerate ground states on a large torus.



A quantum Hamiltonian in 2+1 dimensions

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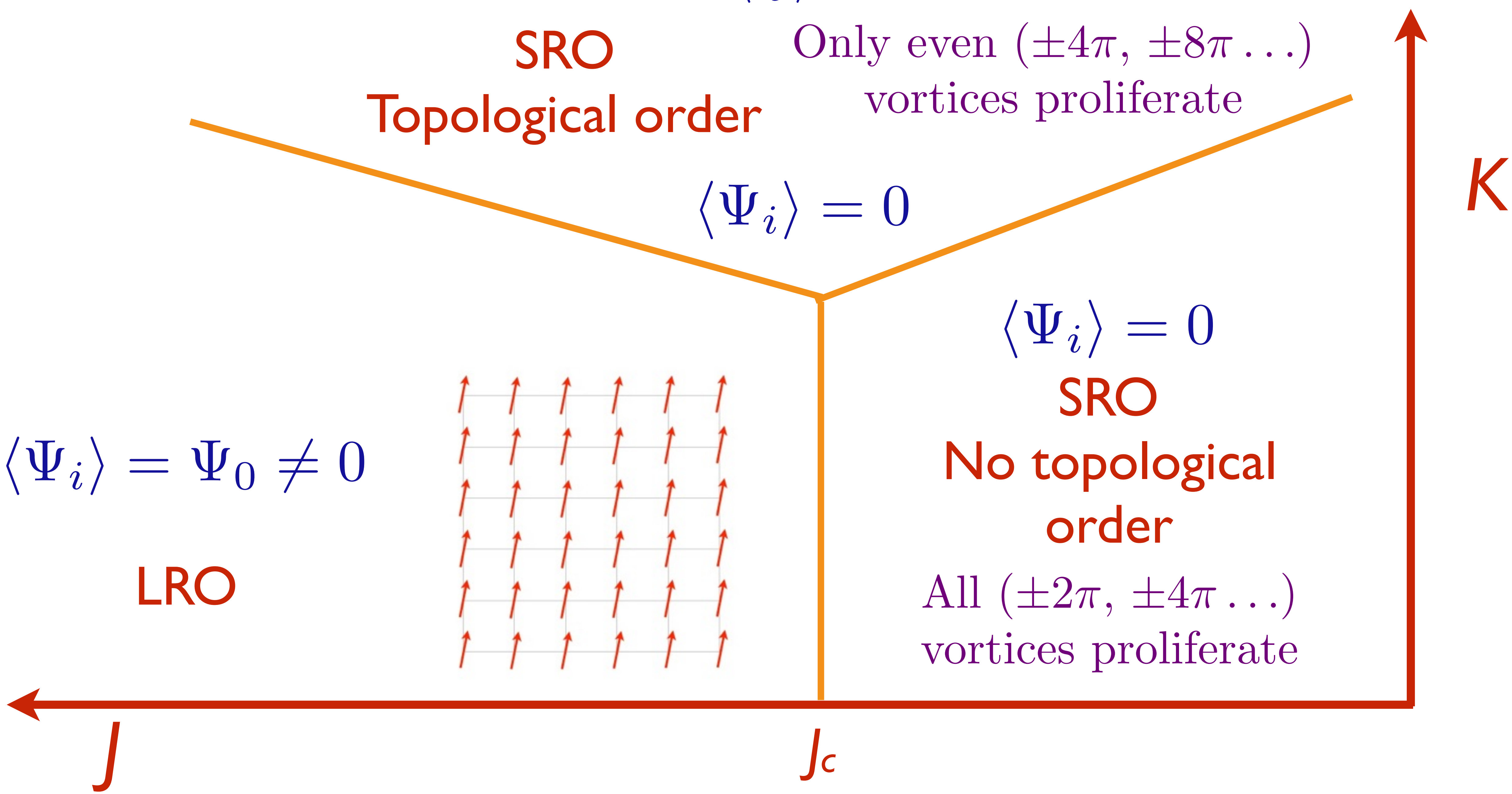


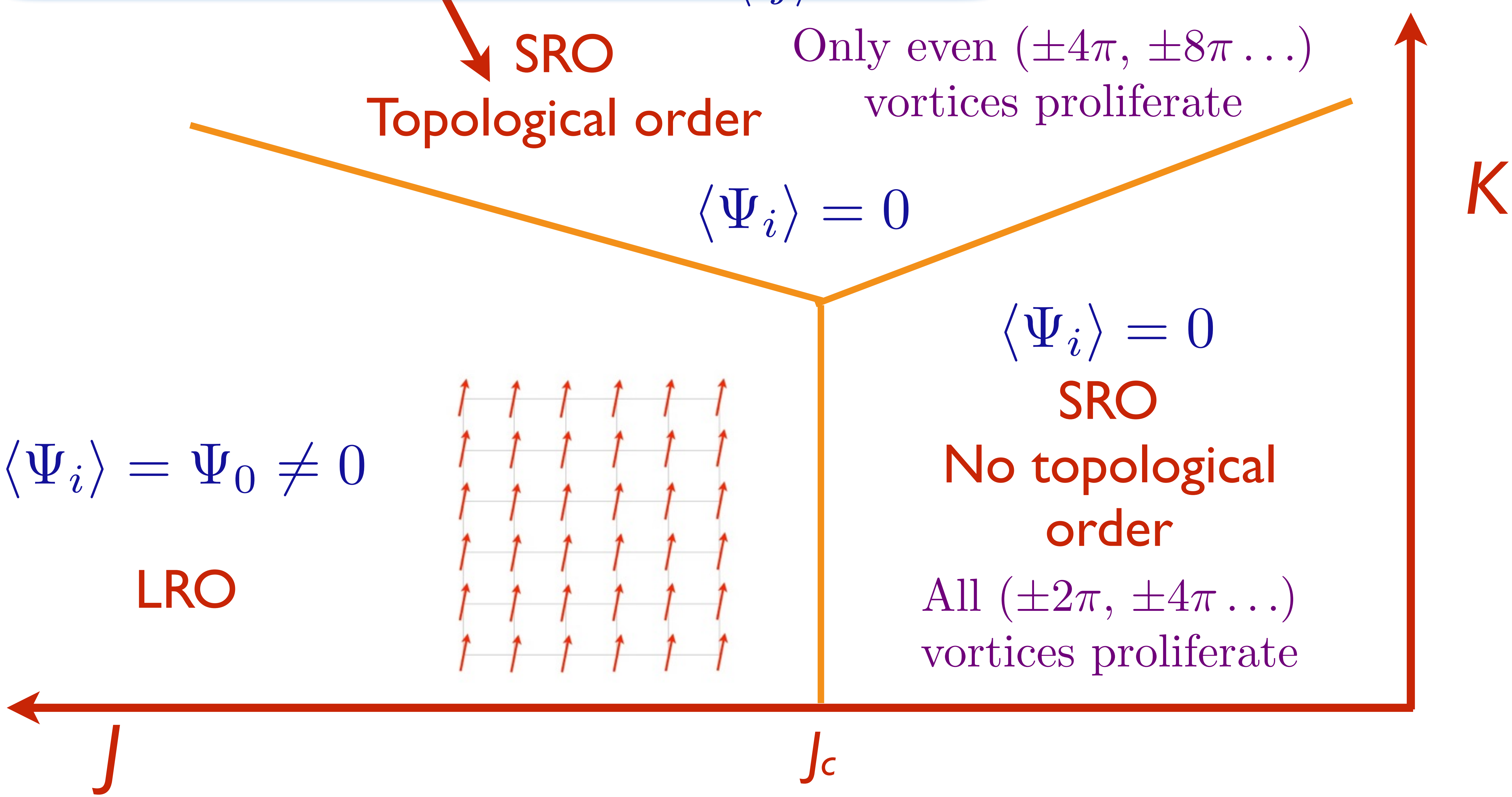
A quantum Hamiltonian in 2+1 dimensions

The topological order is the same
as that of the 'toric code',
or of the U(1)×U(1) Chern-Simons theory

$$\mathcal{L}_{cs} = \frac{1}{\pi}\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda$$

$$- K \sum_{\square} \prod_{(ij)\in\square} \sigma^z_{ij}$$

$$[\theta_i, \hat{n}_j] = i\delta_{ij}$$



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The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

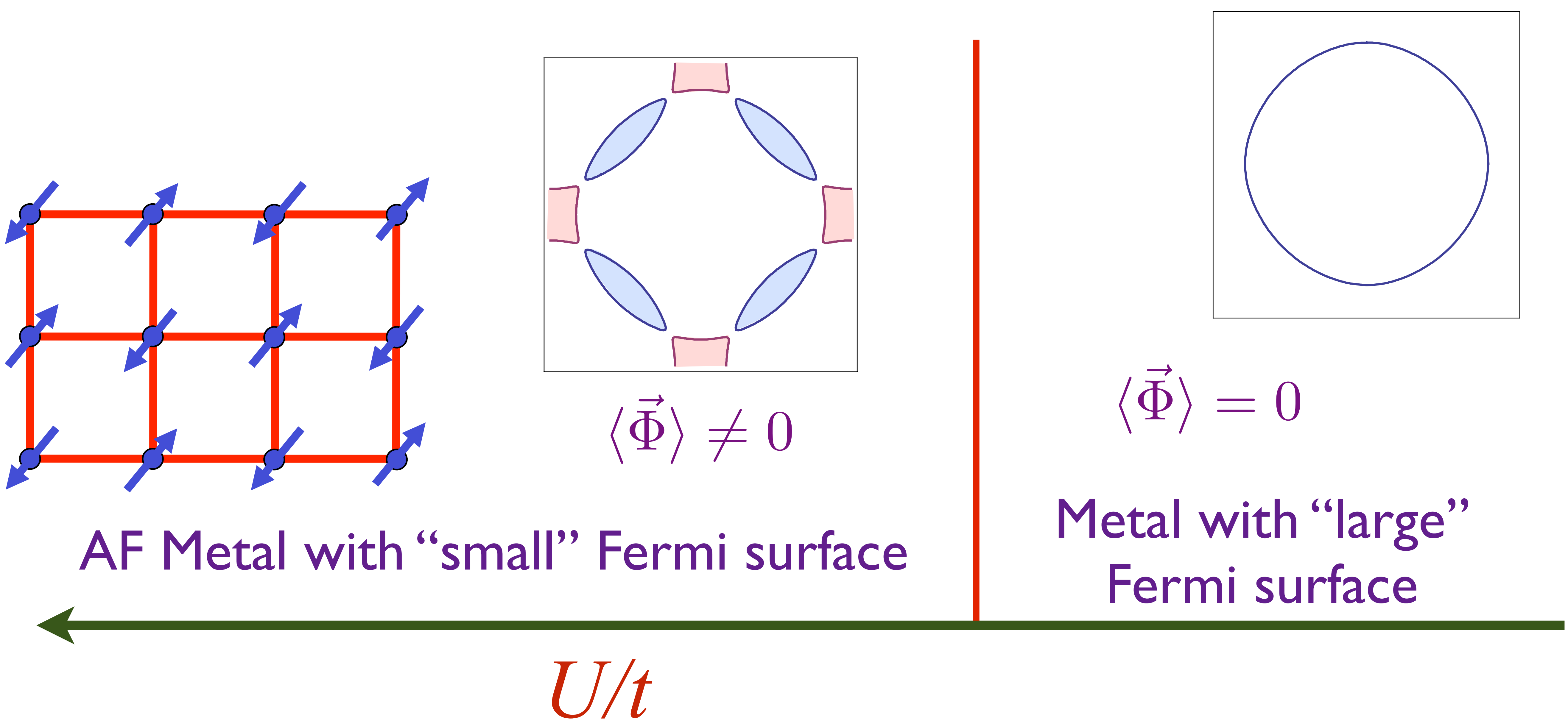
$$n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}$$

$$c_{i\alpha}^\dagger c_{j\beta} + c_{j\beta} c_{i\alpha}^\dagger = \delta_{ij}\delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

Will study on the square lattice

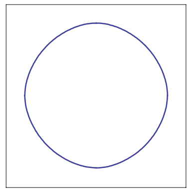
Fermi surface+antiferromagnetism

Mean-field theory with an antiferromagnetic order parameter $\vec{\Phi}_i = (-1)^{i_x+i_y} \left\langle \vec{S}_i \right\rangle$



We can (exactly) transform the Hubbard model to the "spin-fermion" model: **electrons** $c_{i\alpha}$ on the square lattice with dispersion

$$\mathcal{H}_c = -\sum_{i,\rho} t_\rho \left(c^\dagger_{i,\alpha} c_{i+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{i+\boldsymbol{v}_\rho,\alpha} c_{i,\alpha} \right) - \mu \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} + \mathcal{H}_{\text{int}}$$



are coupled to an **antiferromagnetic order parameter** $\Phi^\ell(i)$**,** $\ell = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i \Phi^\ell(i) c^\dagger_{i,\alpha} \sigma^\ell_{\alpha\beta} c_{i,\beta} + V_\Phi$$

where $\eta_i = \pm 1$ on the two sublattices. (For suitable V_Φ, integrating out the Φ yields back the Hubbard model).

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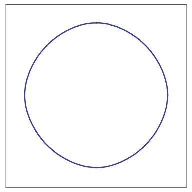

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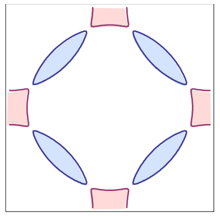
$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i \Phi^\ell(i) c^\dagger_{i,\alpha} \sigma^\ell_{\alpha\beta} c_{i,\beta} + V_\Phi$$

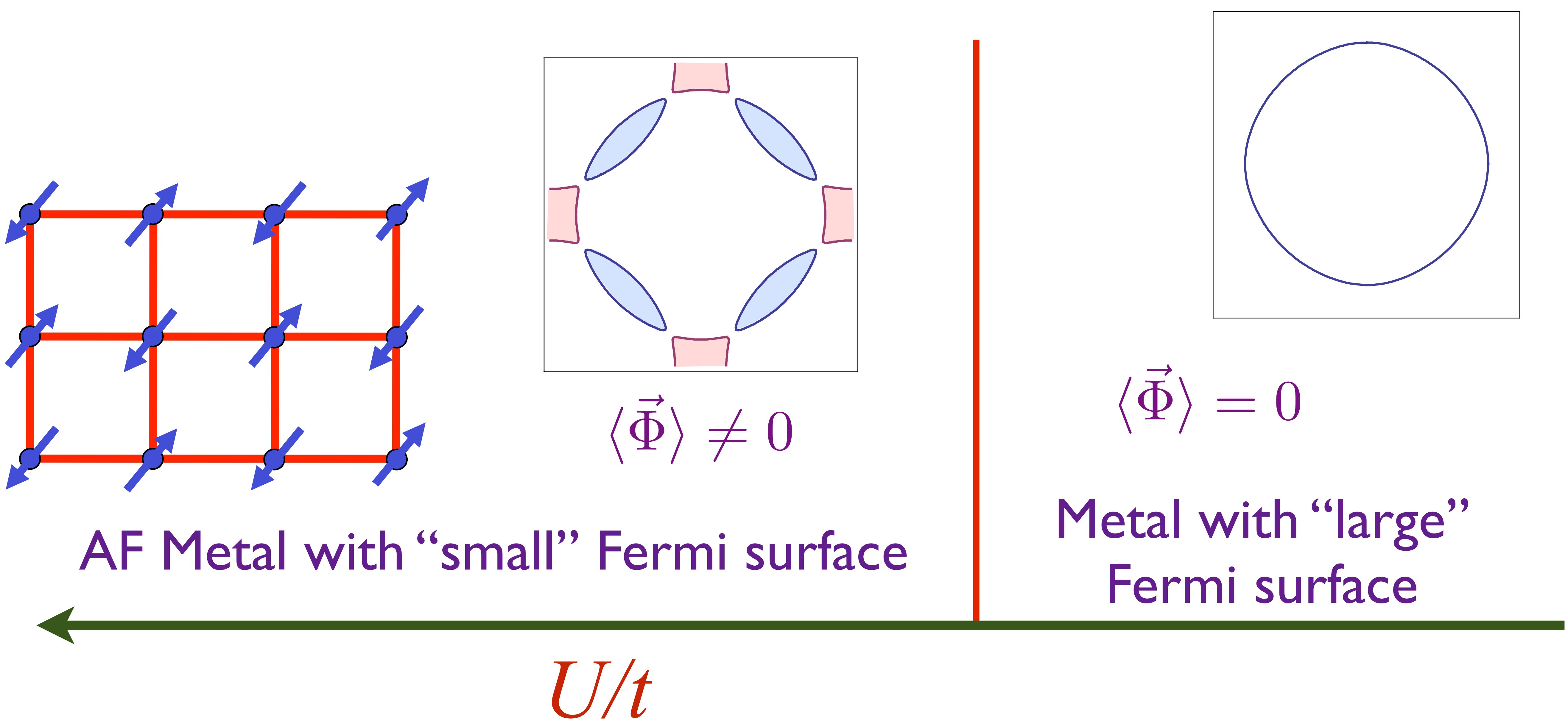
where $\eta_i = \pm 1$ on the two sublattices. (For suitable V_Φ, integrating out the Φ yields back the Hubbard model).

When $\Phi^\ell(i) =$ (non-zero constant) independent of i, we have long-range AF order, which transforms the Fermi surfaces from large to small.

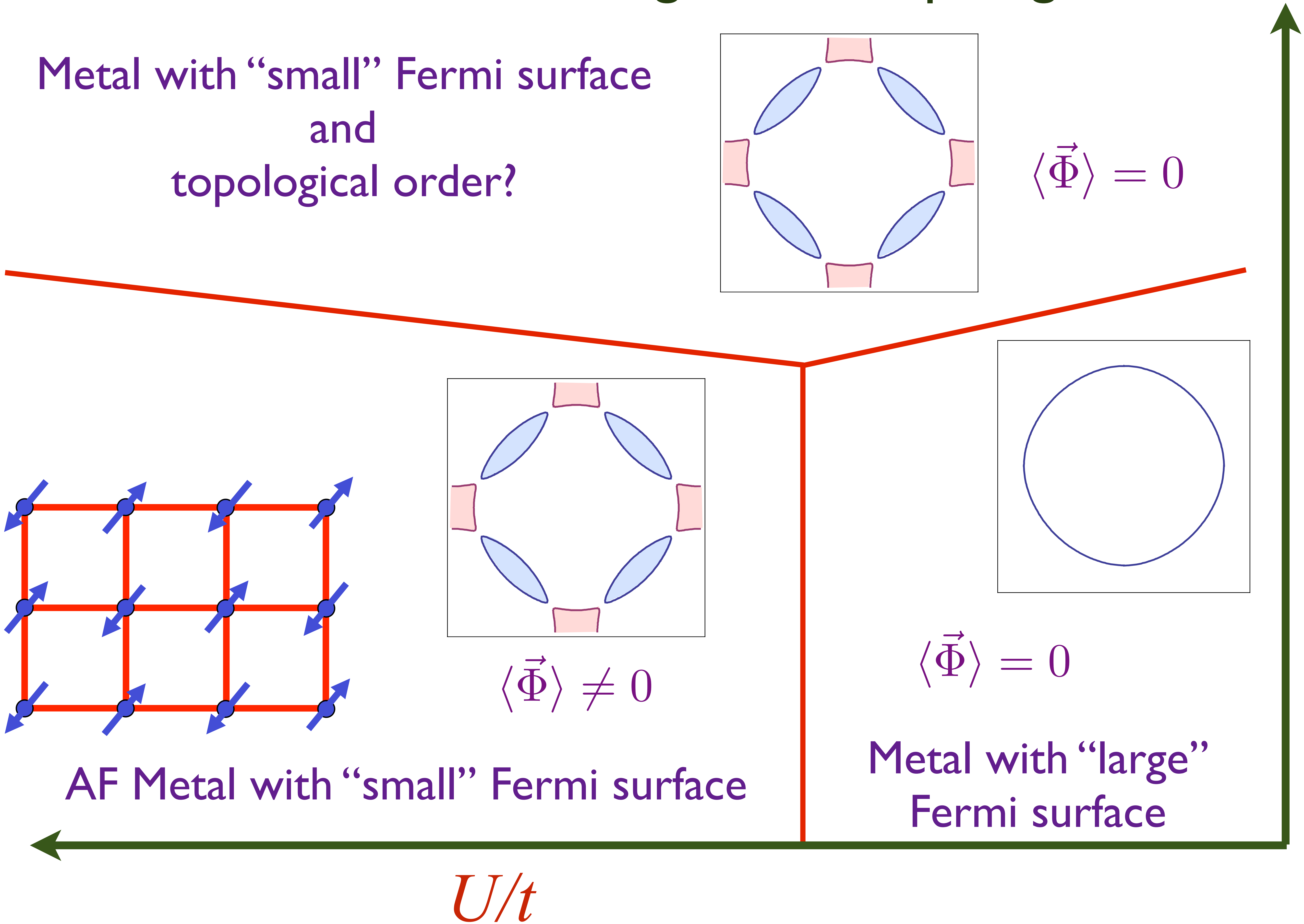


Fermi surface+antiferromagnetism

Mean-field theory with an antiferromagnetic order parameter $\vec{\Phi}_i = (-1)^{i_x + i_y} \left\langle \vec{S}_i \right\rangle$



Fermi surface+antiferromagnetism+topological order



For fluctuating antiferromagnetism, we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the AFM order in the rotating reference frame.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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The Higgs field is the AFM order in the rotating reference frame.
Note that this representation is ambiguous up to a SU(2) gauge transformation, V_i

$$\left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right) \rightarrow V_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right)$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Fluctuating antiferromagnetism

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the AFM order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho,s} + \psi^\dagger_{i+\boldsymbol{v}_\rho,s} \psi_{i,s} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'} + V_H$$

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$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V_H$$

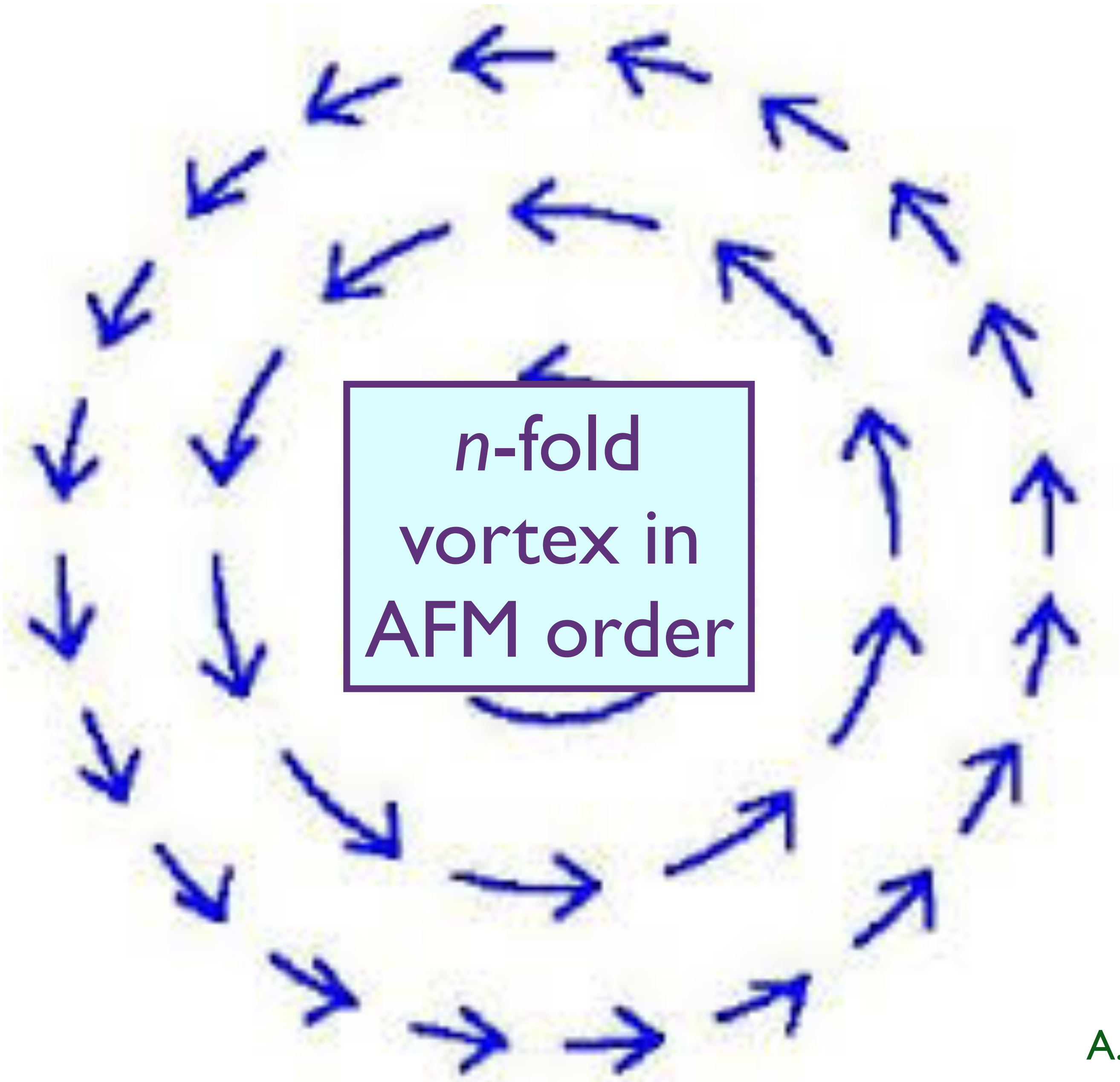
IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of fluctuating AFM will inherit the small Fermi surfaces of the electrons in the presence of static AFM.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Fluctuating antiferromagnetism

We cannot always find a single-valued SU(2) rotation R_i to make the Higgs field $H^a(i)$ a constant !

(assume easy-plane AFM for simplicity)



A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994);
S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

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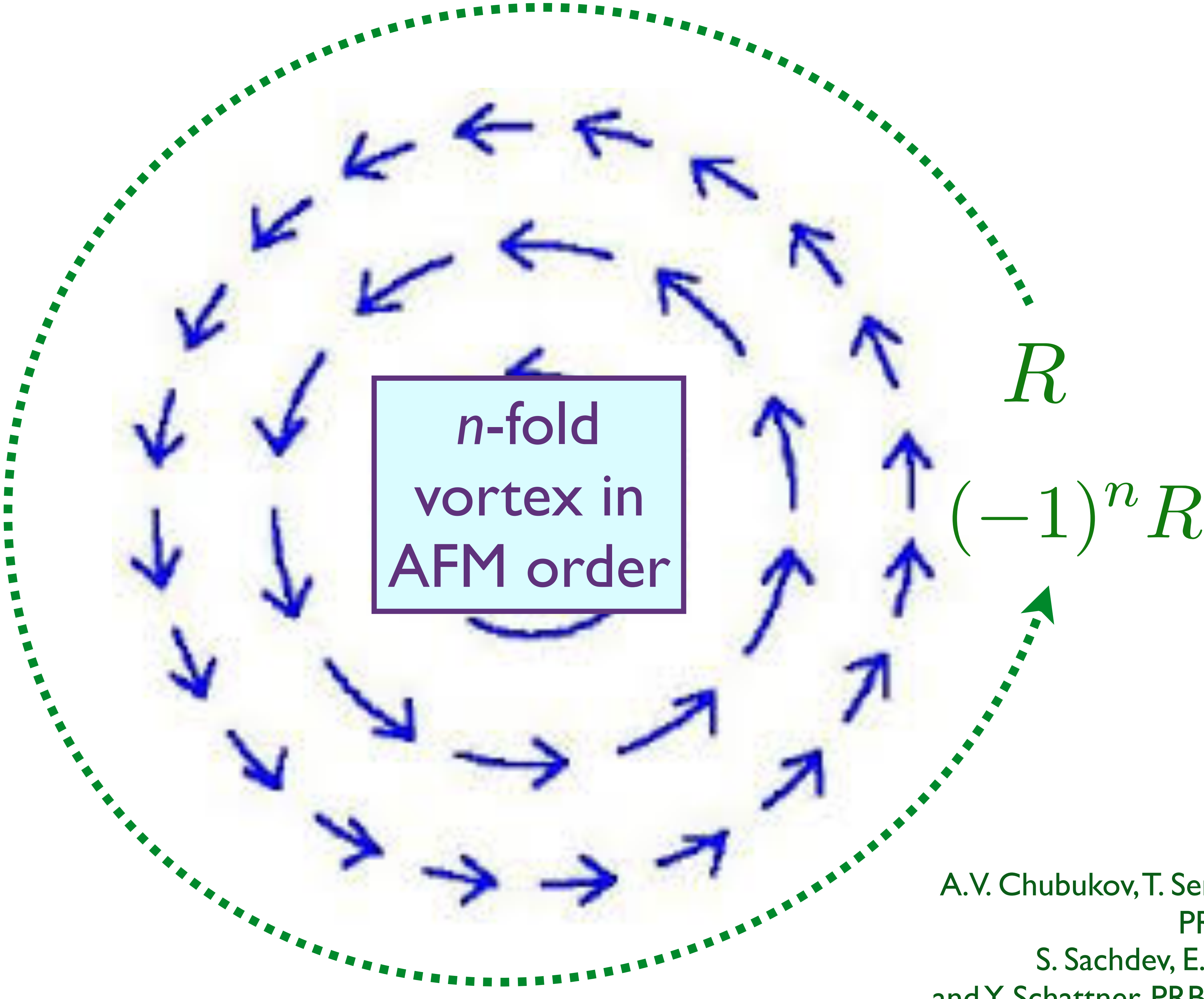

A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994);
S. Sachdev, E. Berg, S. Chatterjee, and Y. Schattner, PRB **94**, 115147 (2016)

Topological order

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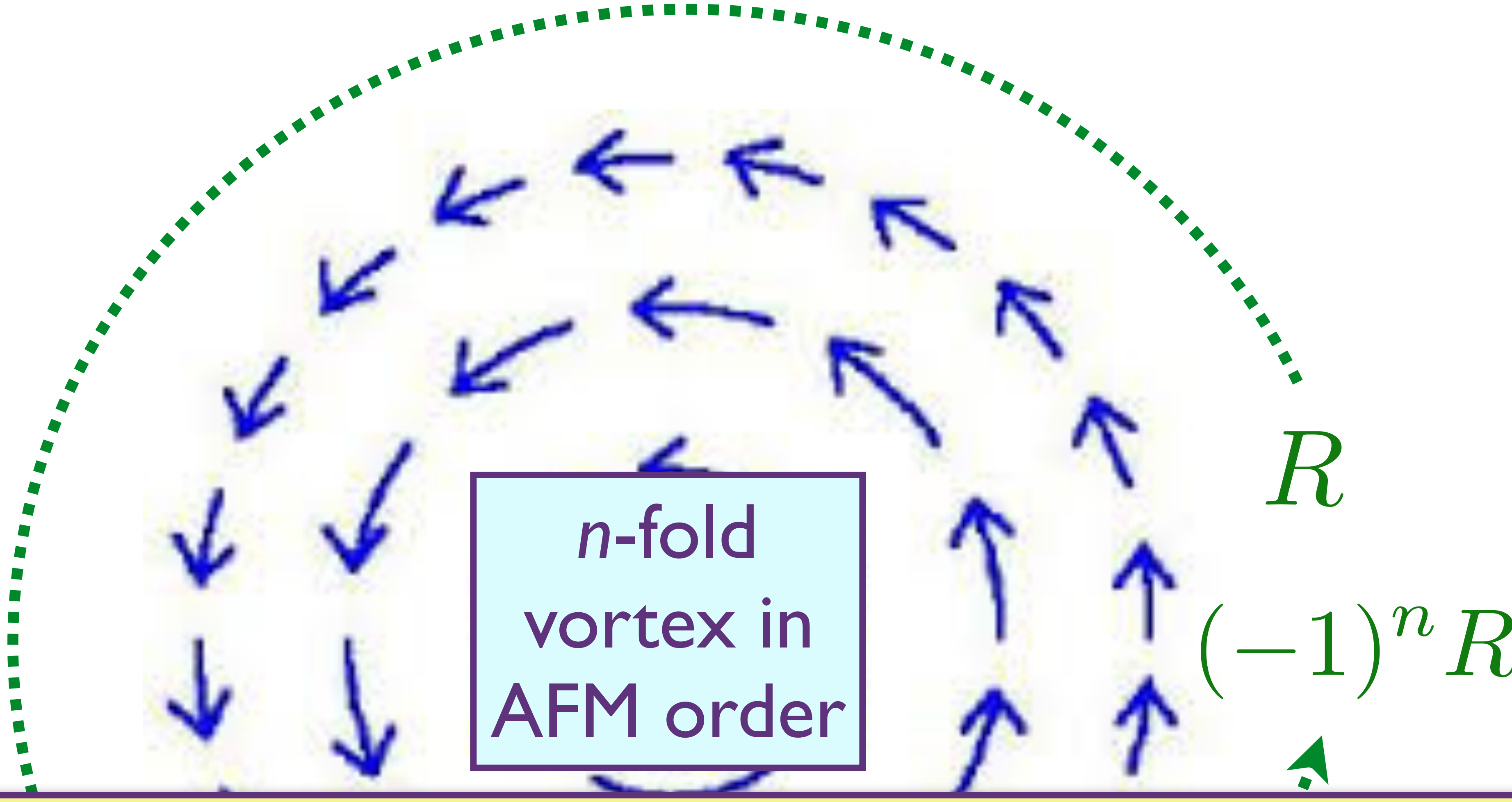
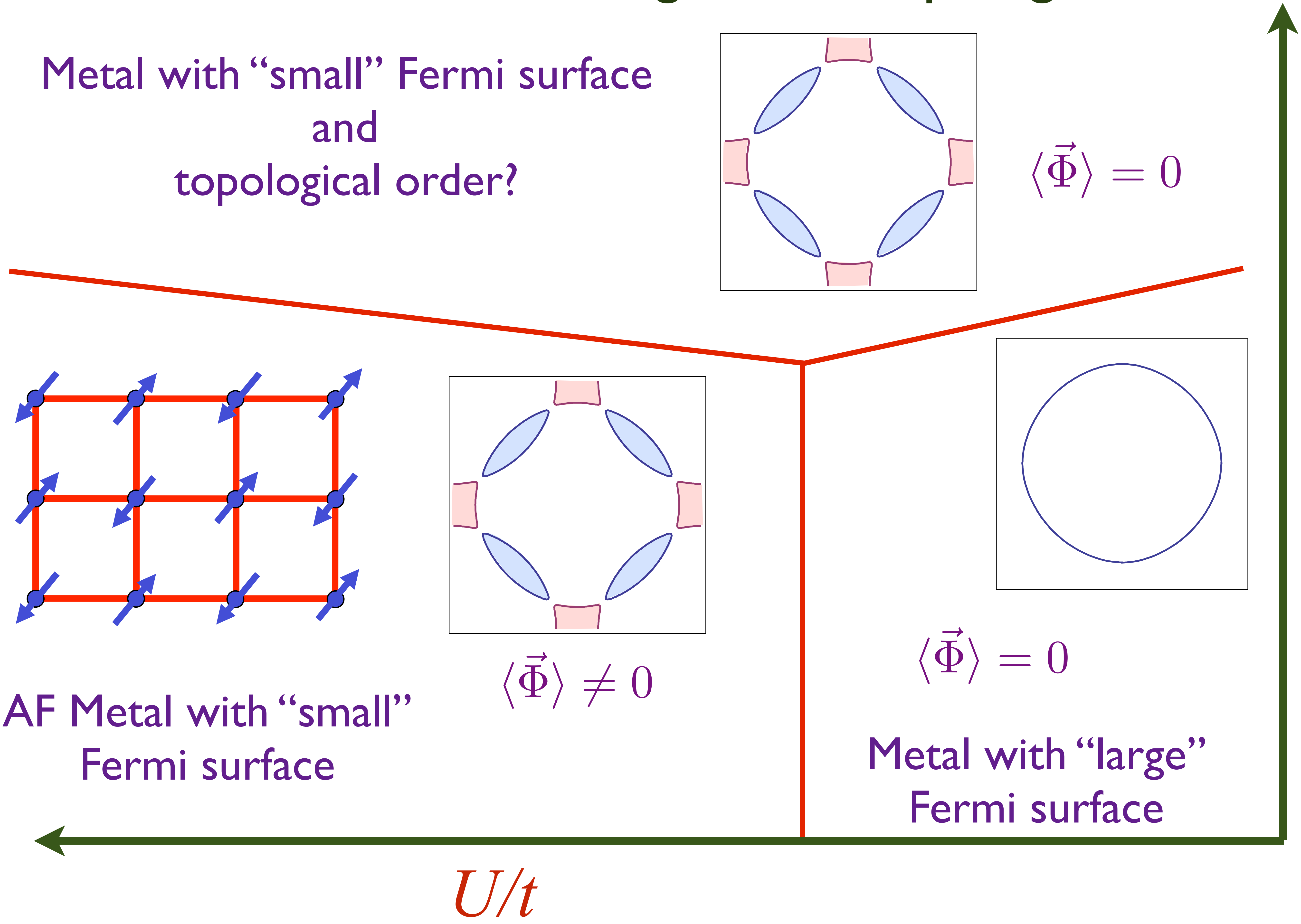
(assume easy-plane AFM for simplicity)



Vortices with n odd must be suppressed: such a metal with "fluctuating antiferromagnetism" has BULK $\mathbb{Z}_2$ TOPOLOGICAL ORDER and fermions which inherit the small Fermi surfaces of the antiferromagnetic metal.

Fermi surface+antiferromagnetism+topological order



Fermi surface+antiferromagnetism+topological order

Metal with “small” Fermi surface;
Higgs phase of a SU(2) gauge theory
with Z_2 or U(1) topological order
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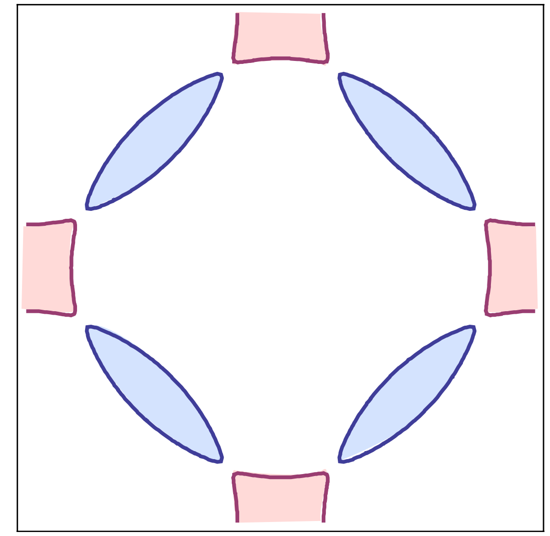
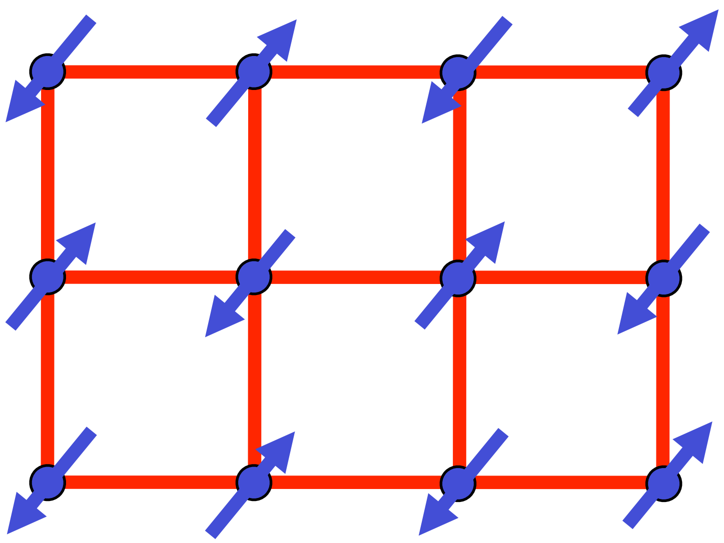
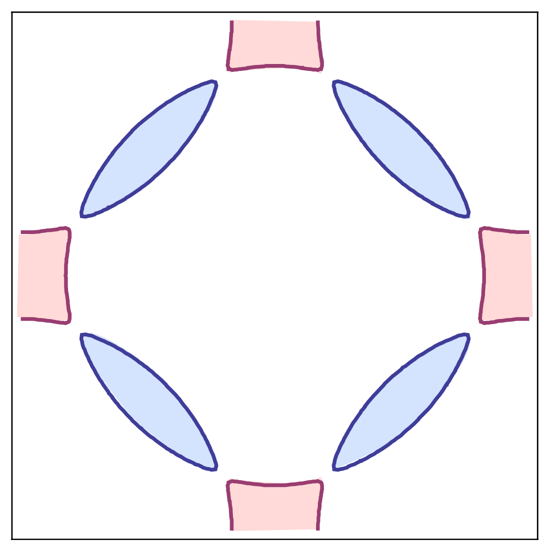
hedgehogs respectively)



$\langle \vec{H} \rangle \neq 0$

$\langle R \rangle = 0$





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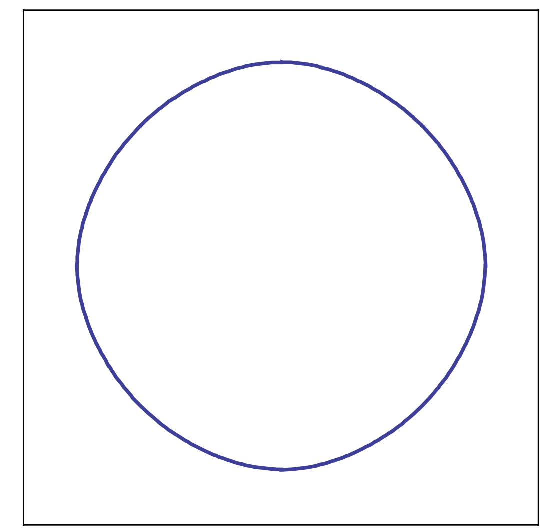

AF Metal with “small”
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Confining phase of
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Metal with “large”
Fermi surface

U/t

S. Sachdev and D. Chowdhury,
Prog. Theor. Exp. Phys. 12C102 (2016)

Topological order in the pseudogap metal

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero,
A. Georges, and S. Sachdev

We compute the electronic Green's function of the topologically ordered Higgs phase of a SU(2) gauge theory of fluctuating antiferromagnetism on the square lattice. The results are compared with cluster extensions of dynamical mean field theory, and quantum Monte Carlo calculations, on the pseudogap phase of the strongly interacting hole-doped Hubbard model. Good agreement is found in the momentum, frequency, hopping, and doping dependencies of the spectral function and electronic self-energy. We show that lines of (approximate) zeros of the zero-frequency electronic Green's function are signs of the underlying topological order of the gauge theory, and describe how these lines of zeros appear in our theory of the Hubbard model. We also derive a modified, non-perturbative version of the Luttinger theorem that holds in the Higgs phase.

arXiv:1711.09925

Common features of many cluster-DMFT computations of pseudogap metal:

- Momentum-space differentiation: electron self-energy is enhanced at low frequencies in the anti-nodal region, and vanishes in the nodal region.
- Gapped spectrum in the anti-nodal region
- Fermi arcs in the nodal region
- Apparent zero of Green's function on a "Luttinger surface".

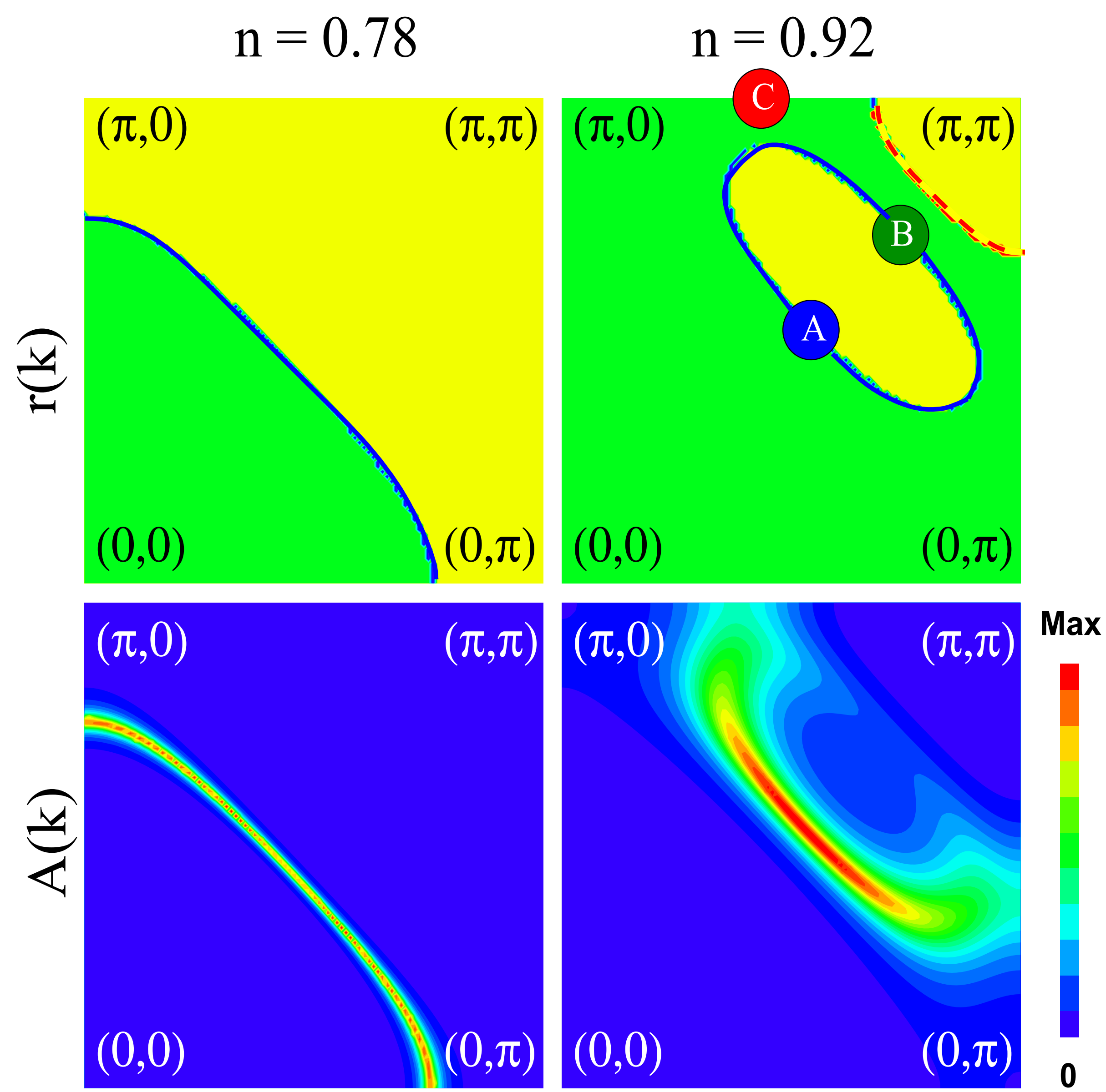


FIG. 4. (Color online) Renormalized energy $r(\mathbf{k})$ (upper panels) and spectral function $A(\mathbf{k})$ (lower panels) for the 2D Hubbard model with $U=8t$ and $T=0$. The color code for the upper panels is green/gray ($r<0$), blue/dark gray line ($r=0$), yellow/light gray ($r>0$), red dashed line ($r\to\infty$).

T.D. Stanescu and G. Kotliar, PRB **74**, 125110 (2006)

Electron Green's function in SU(2) gauge theory higgsed down to U(1)

Electron is fractionalized into bosonic spinons and fermionic chargons:

$$\begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} = R_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix},$$

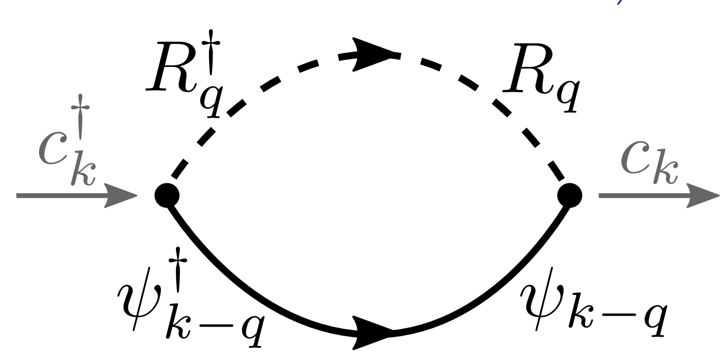
The chargons, ψ, are treated as free fermions in a Higgs background: this reconstructs the chargon Fermi surface into "small pockets", even though translational and spin rotation symmetries remain *unbroken*

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi^\dagger_{i,s} \psi_{i+\boldsymbol{v}_\rho,s} + \psi^\dagger_{i+\boldsymbol{v}_\rho,s} \psi_{i,s} \right) - \mu \sum_i \psi^\dagger_{i,s} \psi_{i,s} - \lambda \sum_i (-1)^{i_x+i_y} H_0^a \psi^\dagger_{i,s} \sigma^a_{ss'} \psi_{i,s'}$$

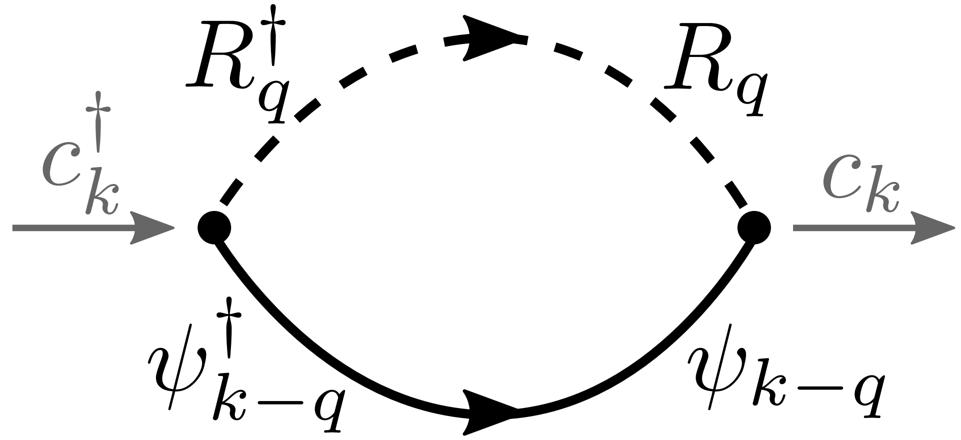
The diagonal chargon Green's function is

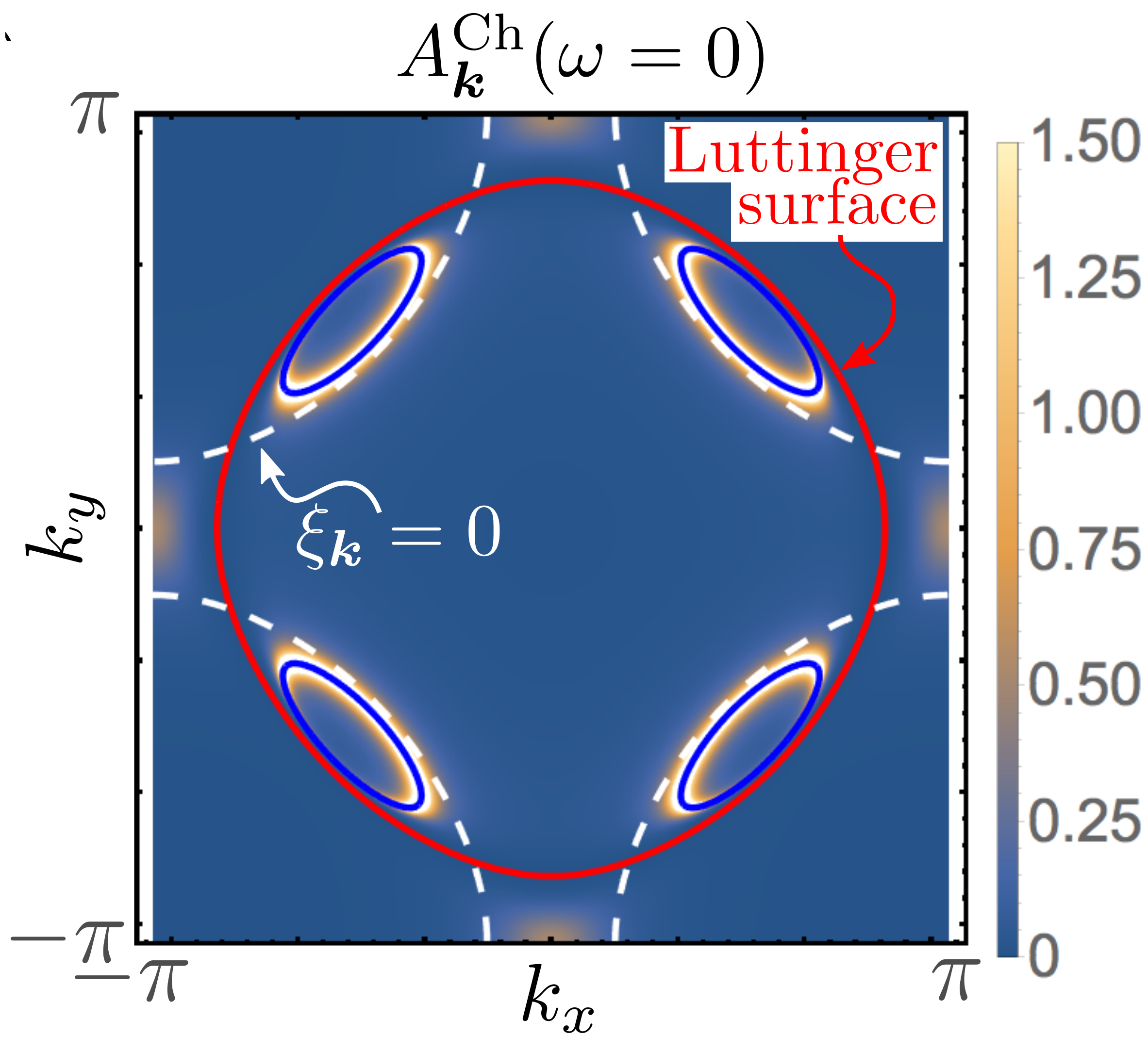
$$G_\Psi(\omega, \vec{k}) = \frac{1}{\omega - \varepsilon_{\vec{k}} - \Sigma_\Psi(\omega, \vec{k})}, \quad \Sigma_\Psi(\omega, \vec{k}) = \frac{H_0^2}{\omega - \varepsilon_{\vec{k}+\vec{Q}}}, \quad \vec{Q} = (\pi, \pi).$$

This has poles at the pocket Fermi surfaces, and zeros at $\varepsilon_{\vec{k}+\vec{Q}}$.
The electron Green's function is computed via a convolution, and then the zeros are smeared to *approximate zeros*.

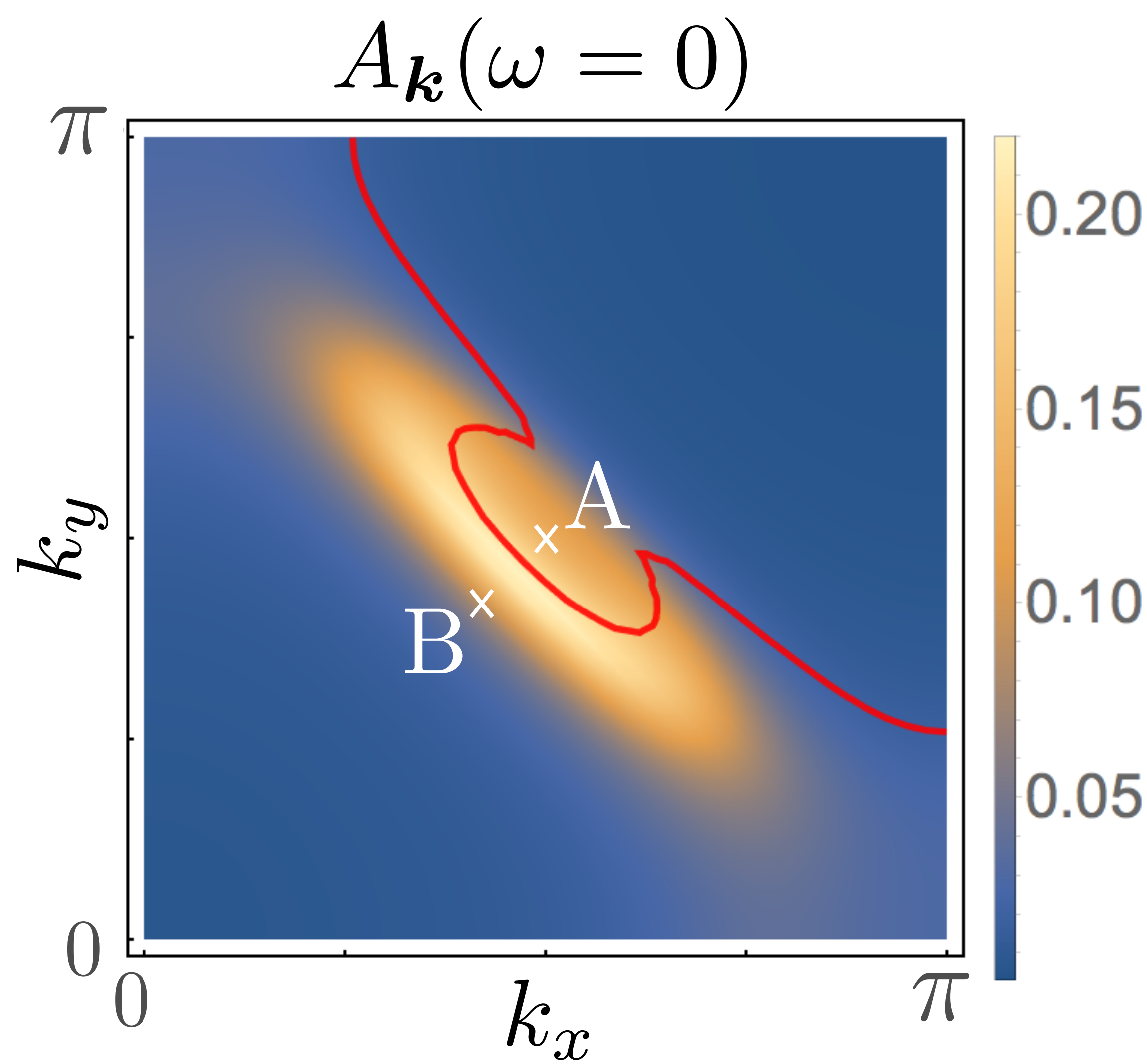


Electron Green's function in SU(2) gauge theory higgsed down to U(1)





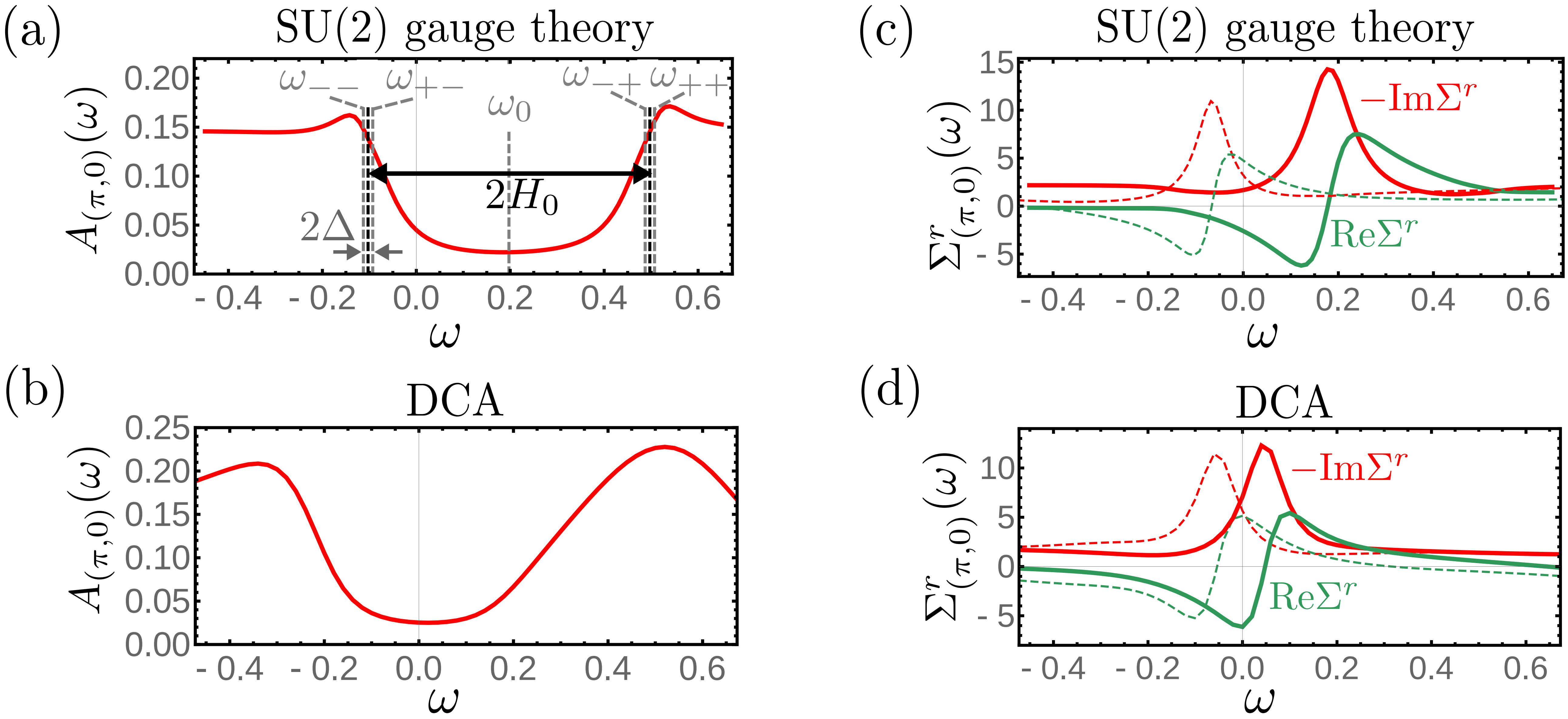
Red line indicates the locus of $G(\mathbf{k}, \omega = 0) = 0$



Red line indicates the locus of $\mathrm{Re}\, G(\mathbf{k}, \omega = 0) = 0$

Full Brillouin zone spectra of chargons (ψ) and electrons (c)

Electron Green's function in SU(2) gauge theory higgsed down to U(1)

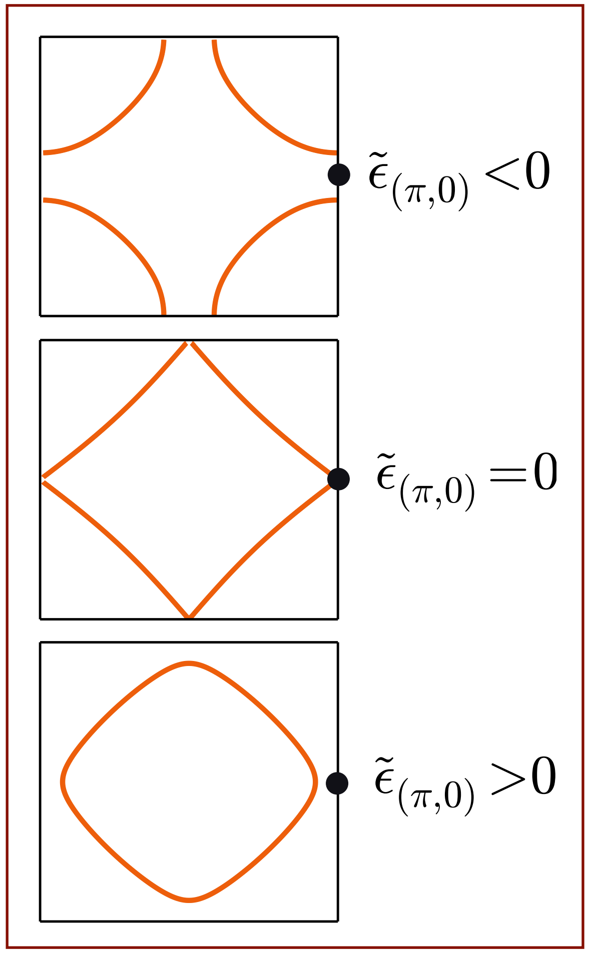


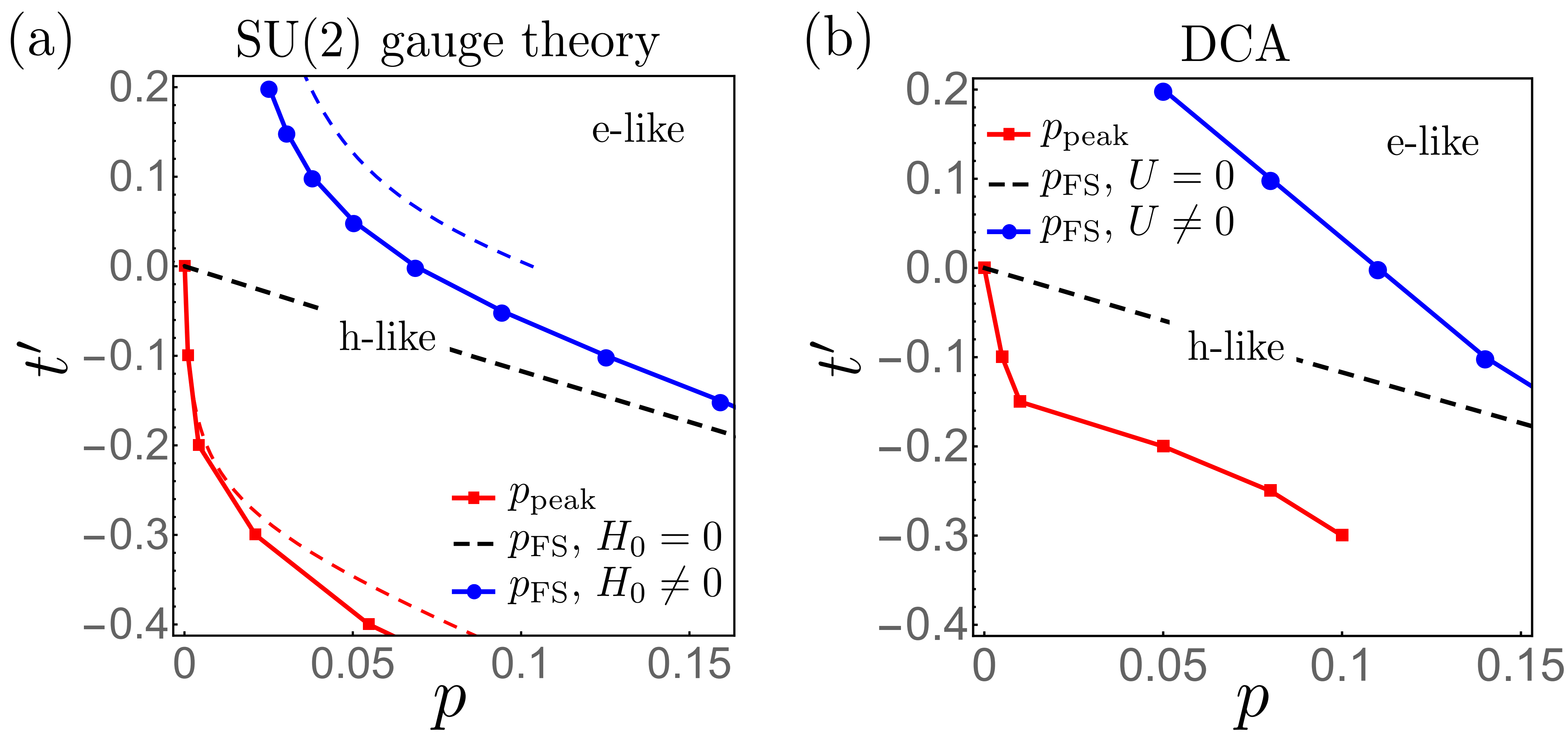
$$T = t/30 \quad , \quad U = 7t \quad , \quad p = 0.05$$

t' takes different negative values

Anti-nodal spectra compared to cluster DMFT

Lifshitz transition compared to cluster DMFT

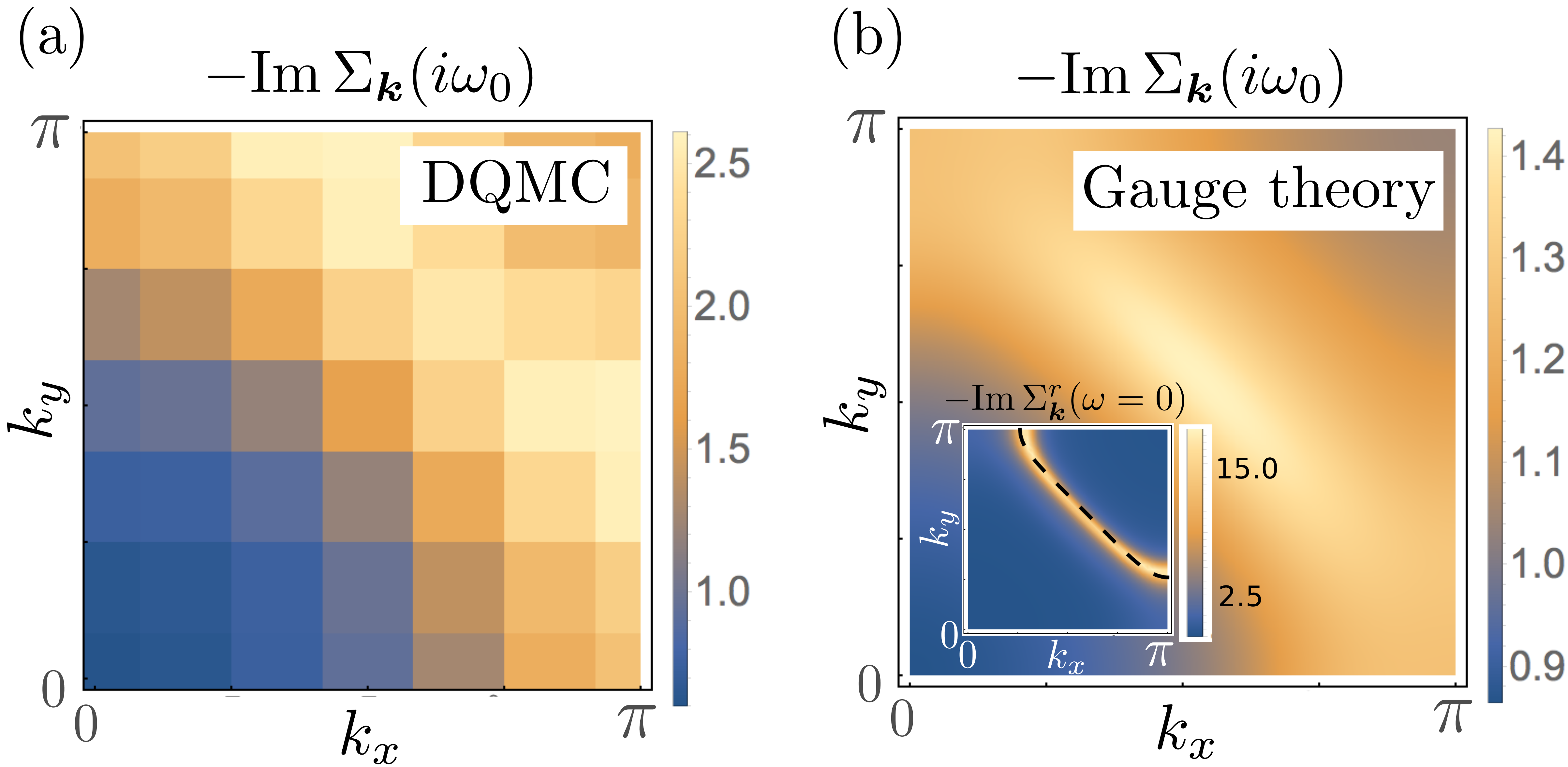




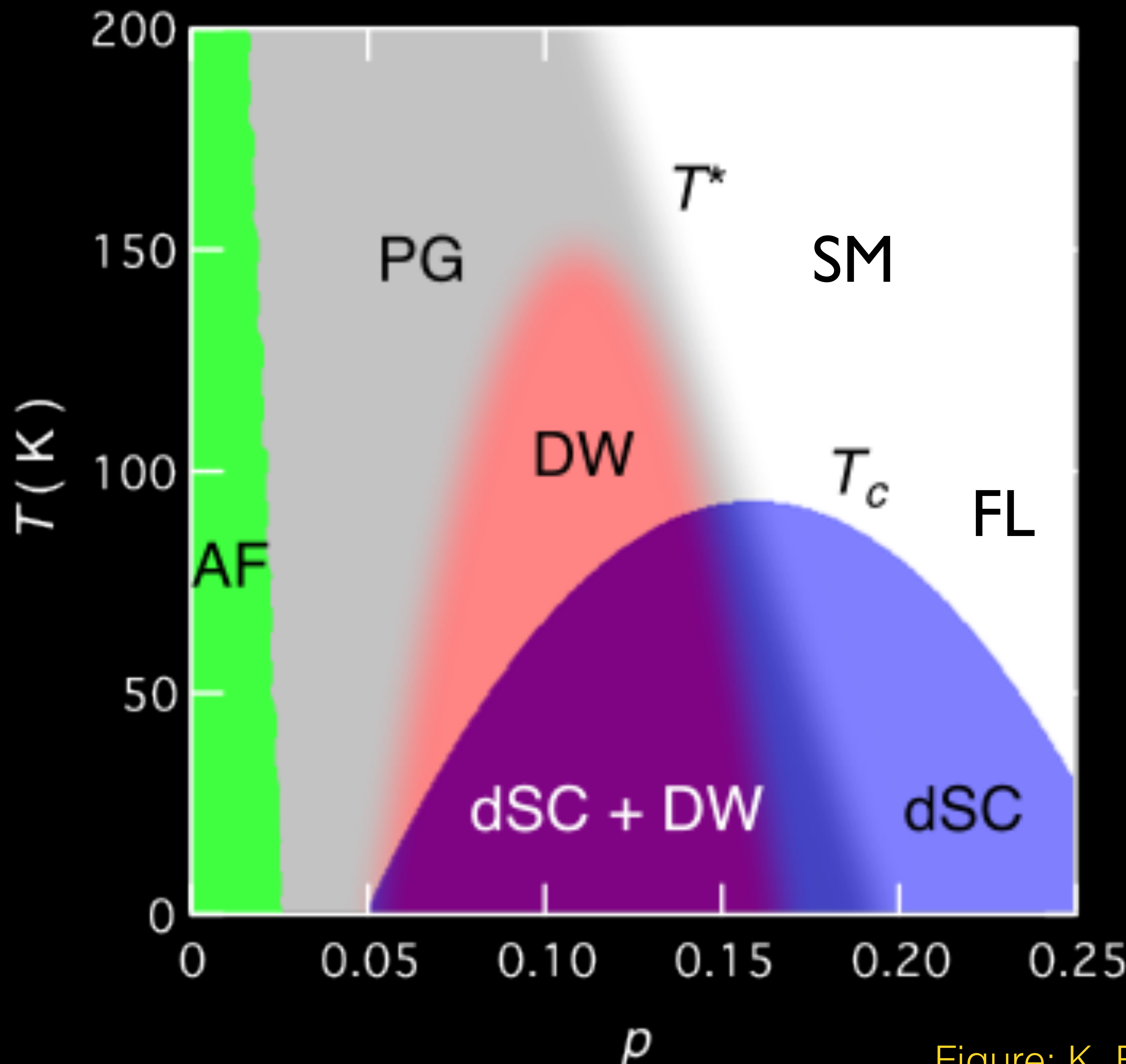
$$\tilde{\epsilon}_{\vec{k}} = \epsilon_{\vec{k}} + \mathrm{Re}\,\Sigma_{\vec{k}}(\omega = 0) = -\mathrm{Re}\left(G_c(\omega = 0, \vec{k})\right)^{-1}$$

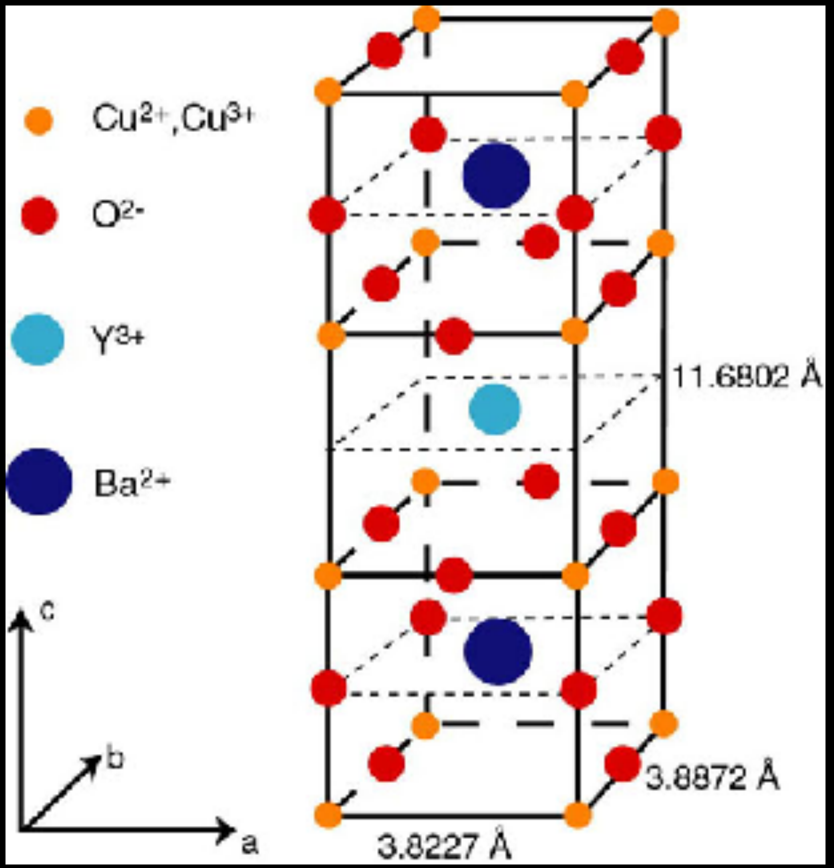
The p-t' dependence of the "interacting Lifshitz transition", defined by the sign change of the renormalized quasiparticle energy $\tilde{\epsilon}_{(\pi,0)}$ at $\omega_{\mathrm{peak}} > 0$, is shown as solid blue lines calculated from the SU(2) gauge theory, part (a), and DCA, part (b). The black dashed lines show the location of the same transition for non-interacting electrons. The red lines indicate where the particle-hole asymmetry of the self-energy changes, *i.e.*, where the peak position ω_{peak} of the anti-nodal Im(self-energy) changes sign.

Electron Green's function in SU(2) gauge theory higgsed down to U(1)



The imaginary part of the self-energy at the lowest Matsubara frequency $\omega_0 = \pi T$ determined from DQMC on the Hubbard model ($U = 7t$, $t' = -0.1t$, $T = 0.25t$, $p = 0.042$) and from the SU(2) gauge theory is shown in (a) and (b), respectively. To avoid too much broadening, we have applied a slightly smaller temperature of $T = 0.15t$ for the gauge theory. The inset in (b) shows the gauge theory prediction at zero frequency and low temperature (as before $T = t/30$). The black dashed line corresponds to the position of the Luttinger surface of the chargons.





$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

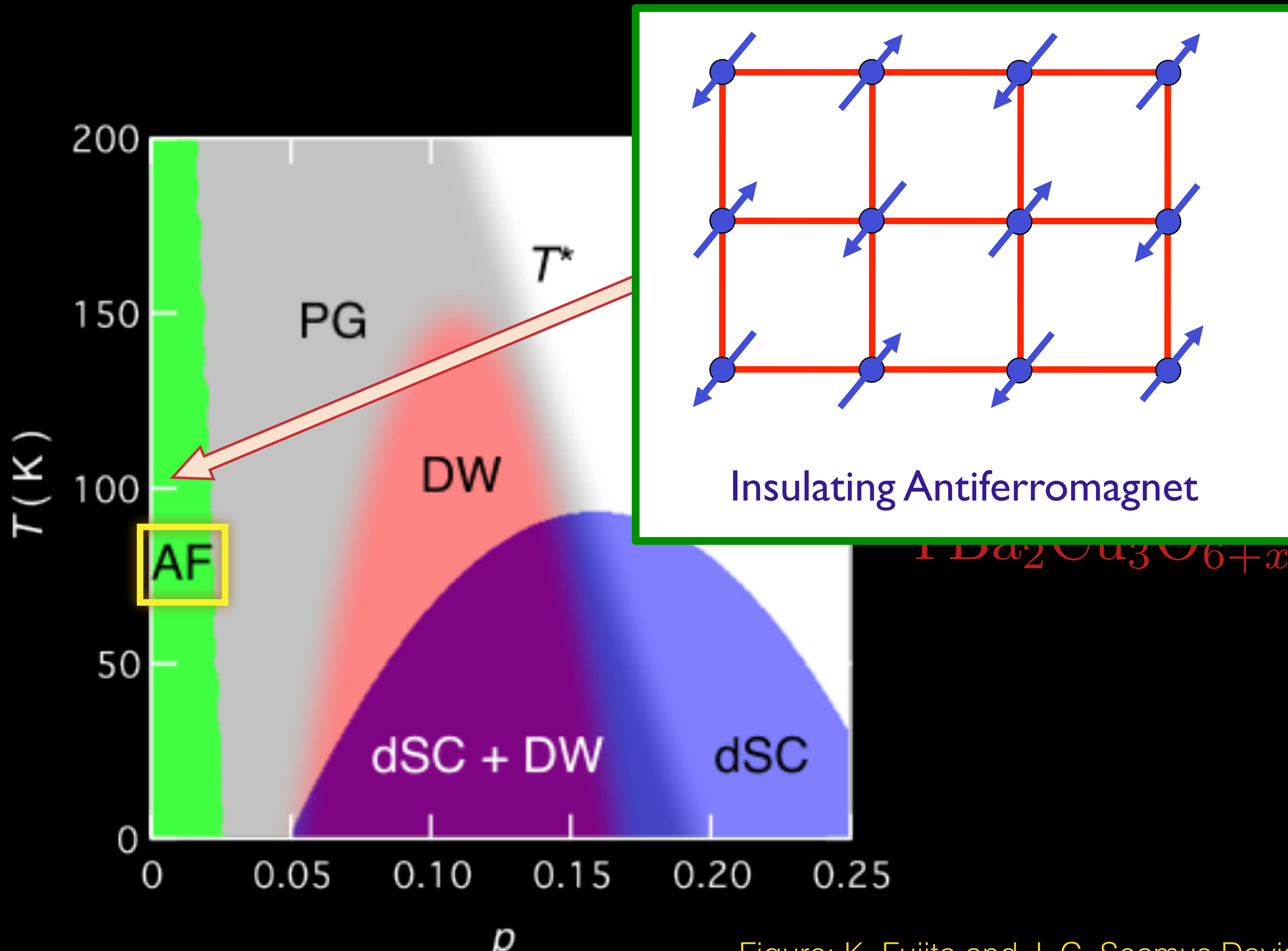
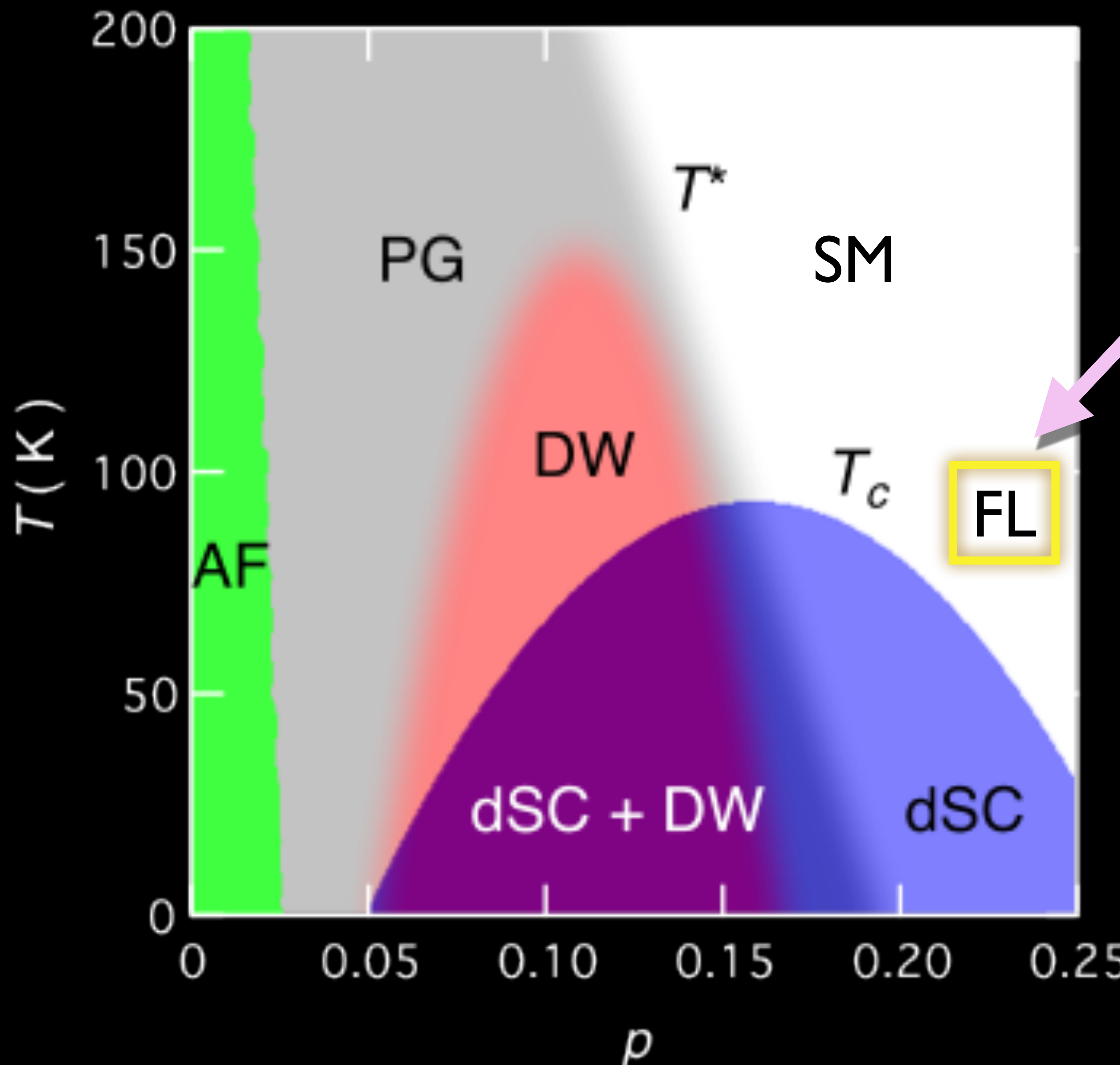
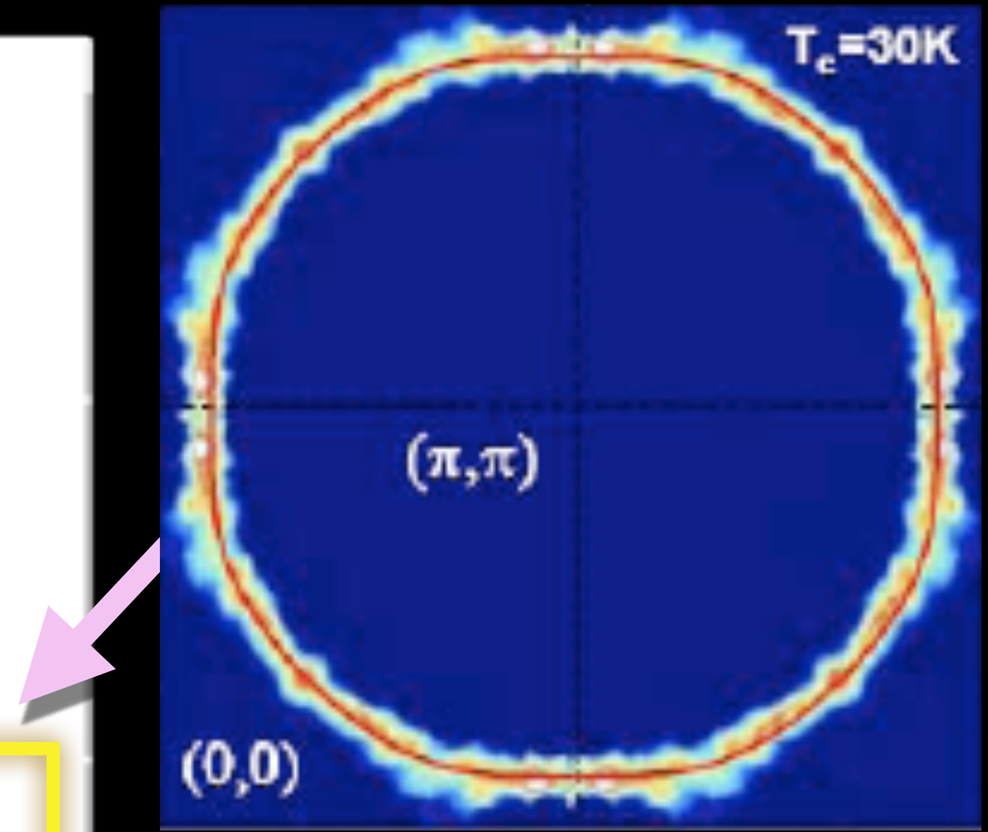


Figure: K. Fujita and J. C. Seamus Davis

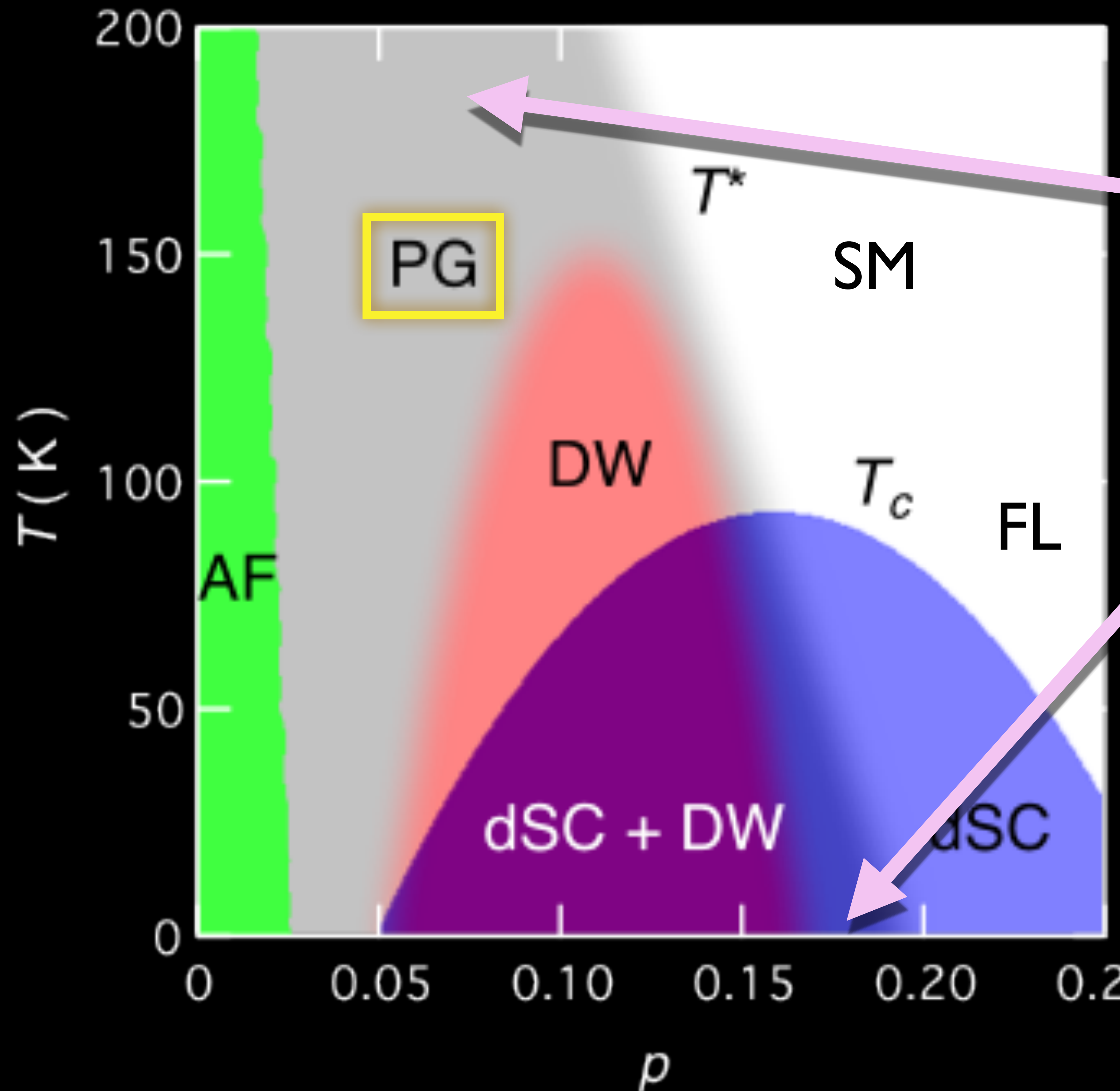
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





A conventional metal: the Fermi liquid with Fermi surface of size $1+p$

T. Senthil, M. Vojta and S. Sachdev, PRB **69**, 035111 (2004)



Pseudogap metal at low p

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

If present at $T=0$, a metal with a size p Fermi surface (and translational symmetry preserved) ***must*** have ***topological order***

Fermi surface+antiferromagnetism+topological order

Metal with "small" Fermi surface;
Higgs phase of a SU(2) gauge theory
with Z_2 or U(1) topological order
(with suppressed Z_2 vortices and
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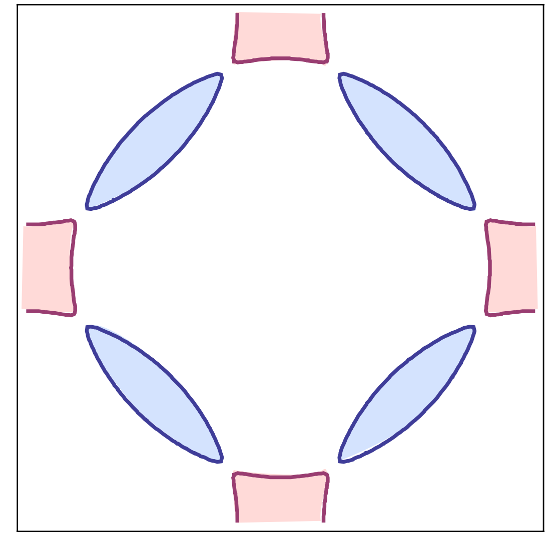
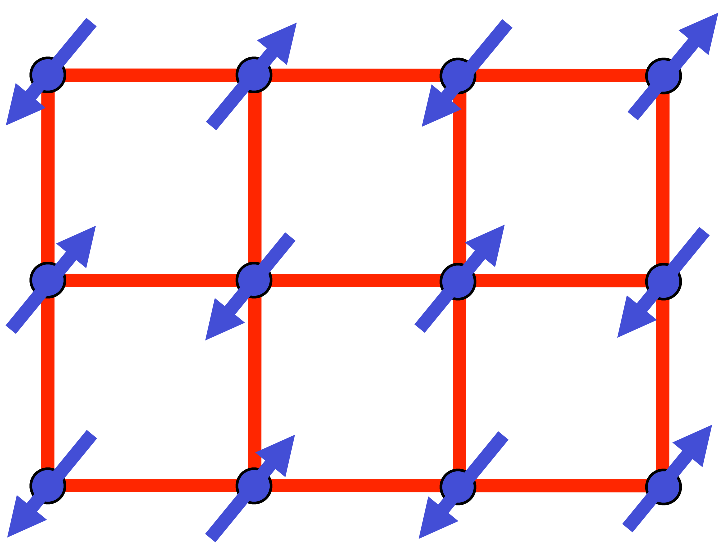
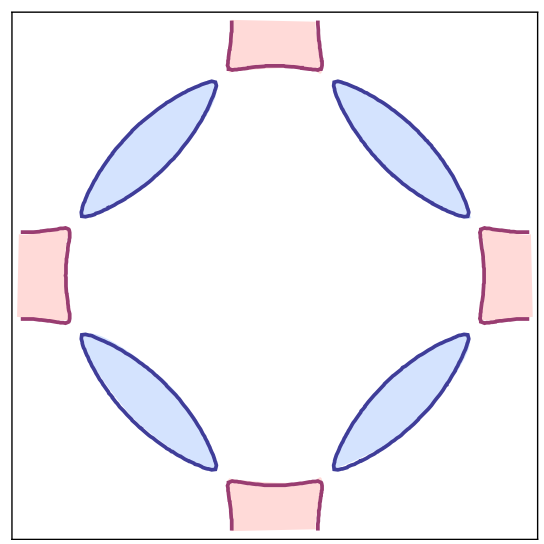


$\langle \vec{H} \rangle \neq 0$
$\langle R \rangle = 0$





AF Metal with "small"
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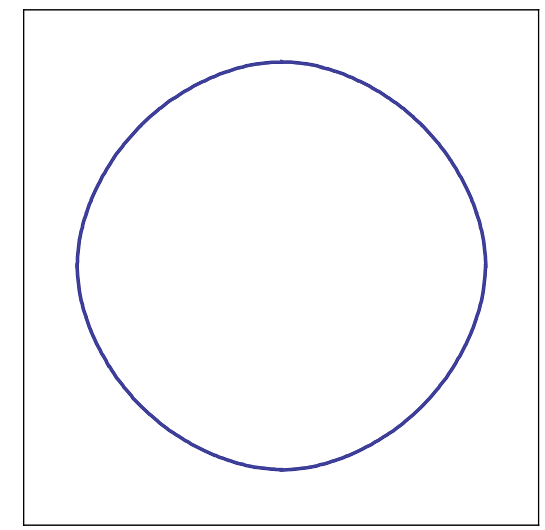

Confining phase of
SU(2) gauge theory.
Metal with "large"
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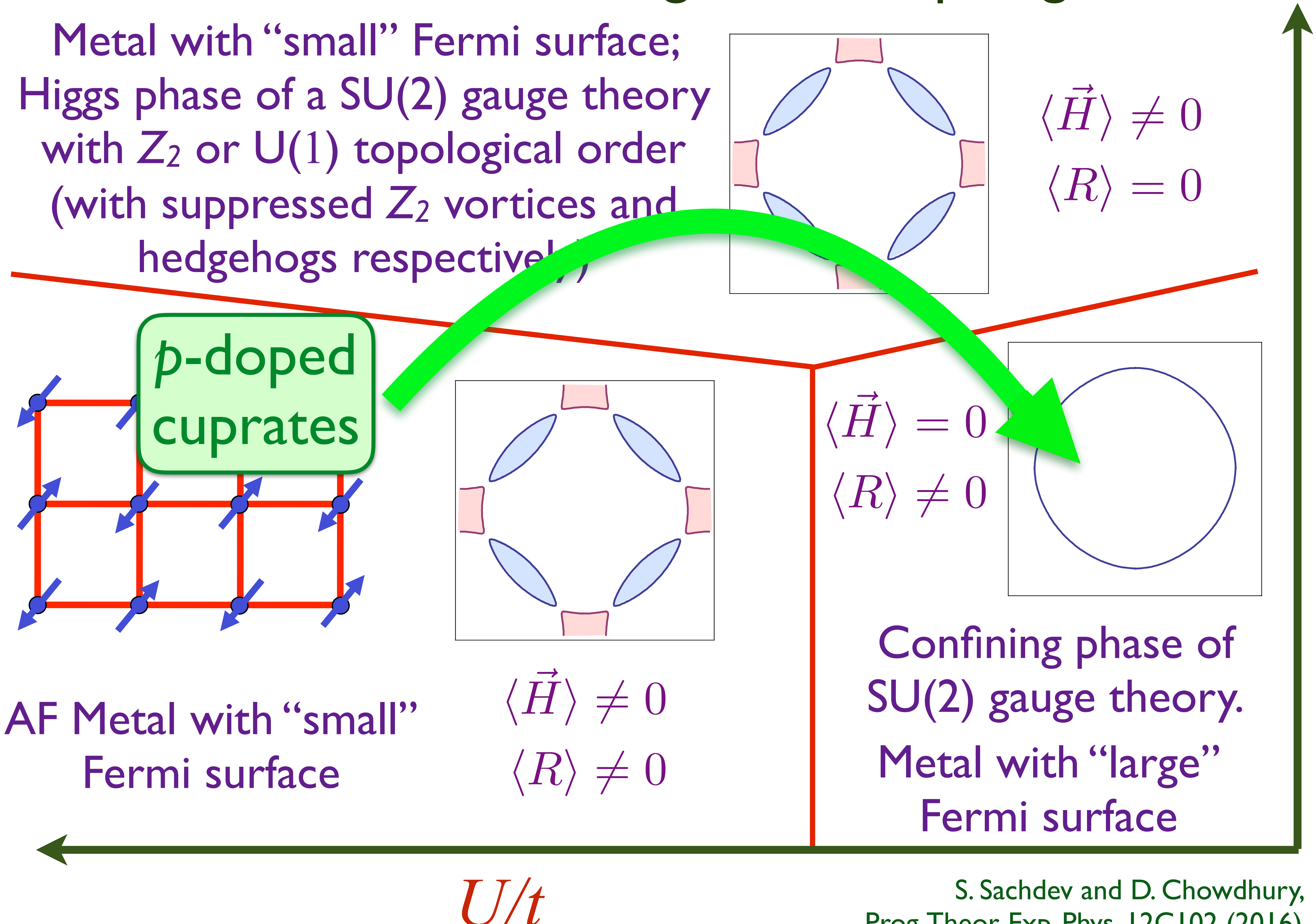
U/t

S. Sachdev and D. Chowdhury,
Prog. Theor. Exp. Phys. 12C102 (2016)

Fermi surface+antiferromagnetism+topological order



S. Sachdev and D. Chowdhury,
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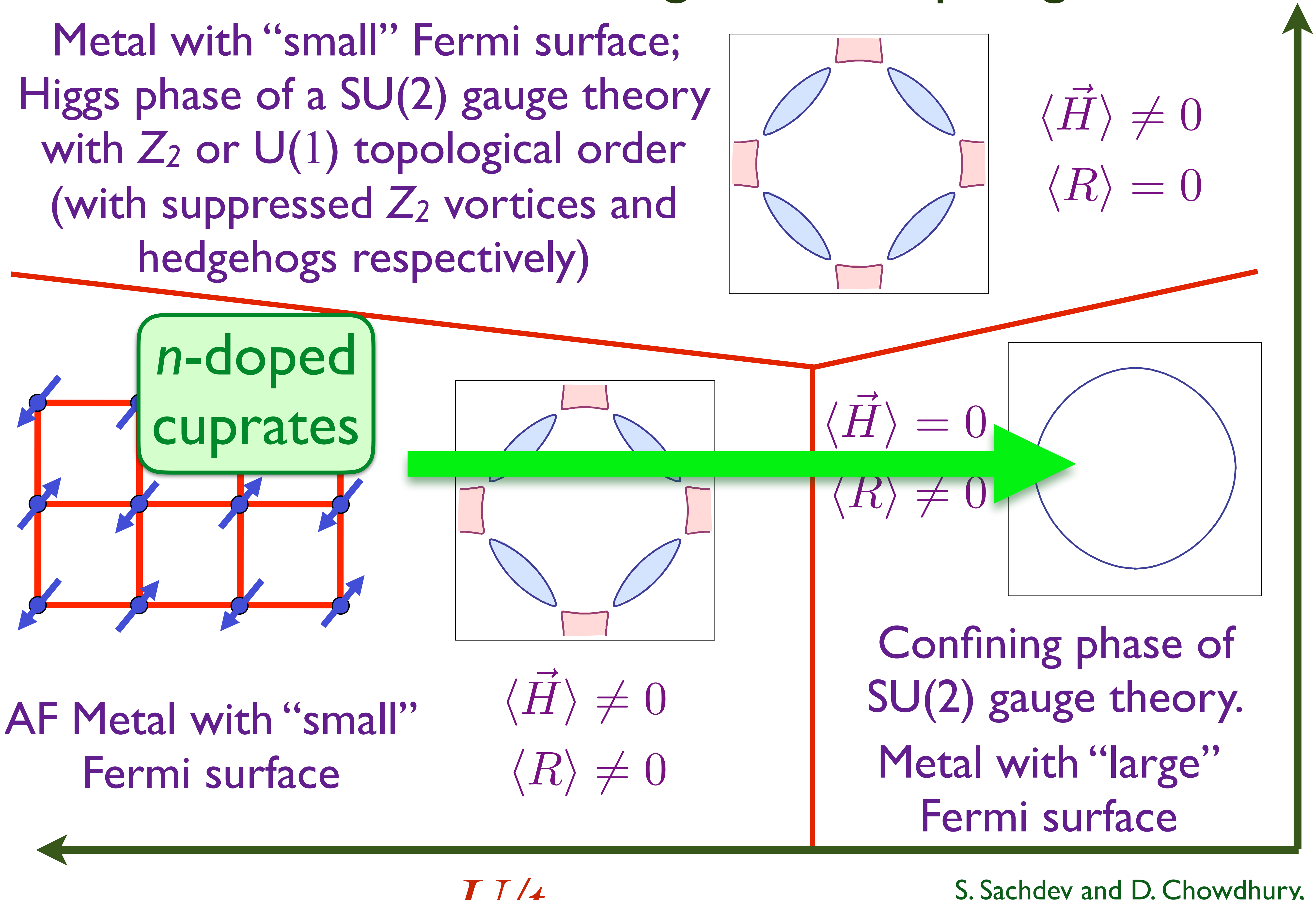
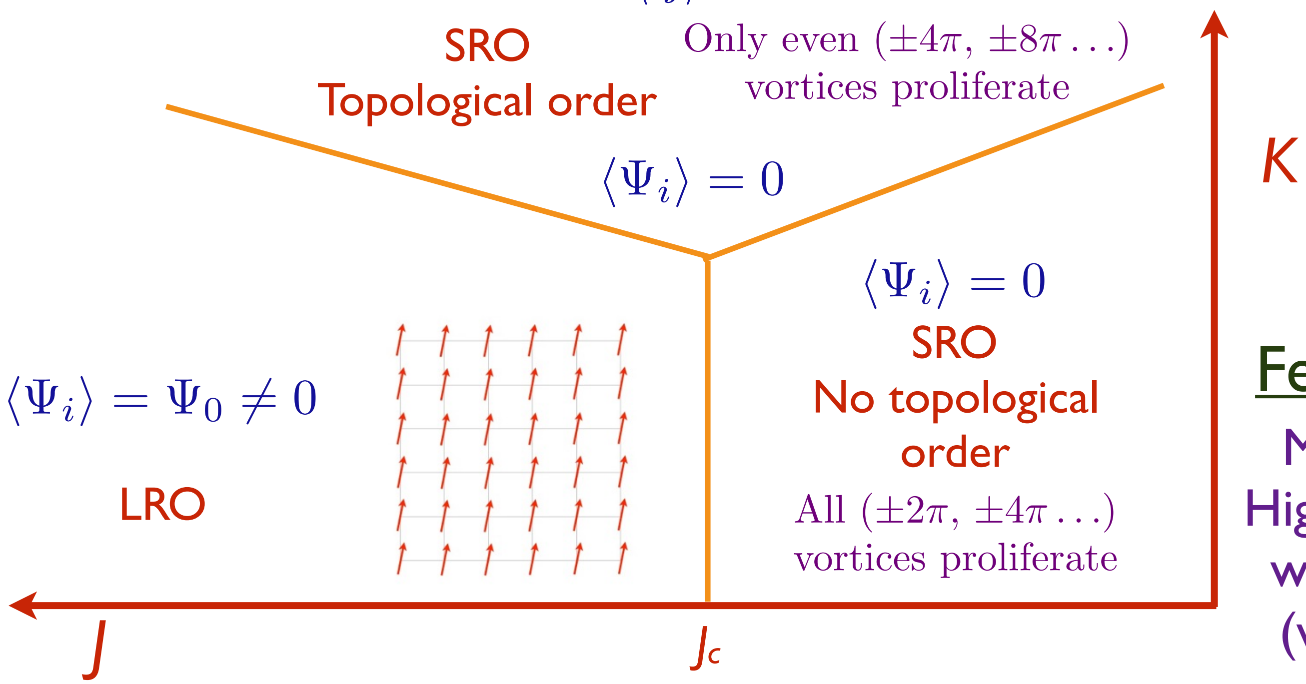
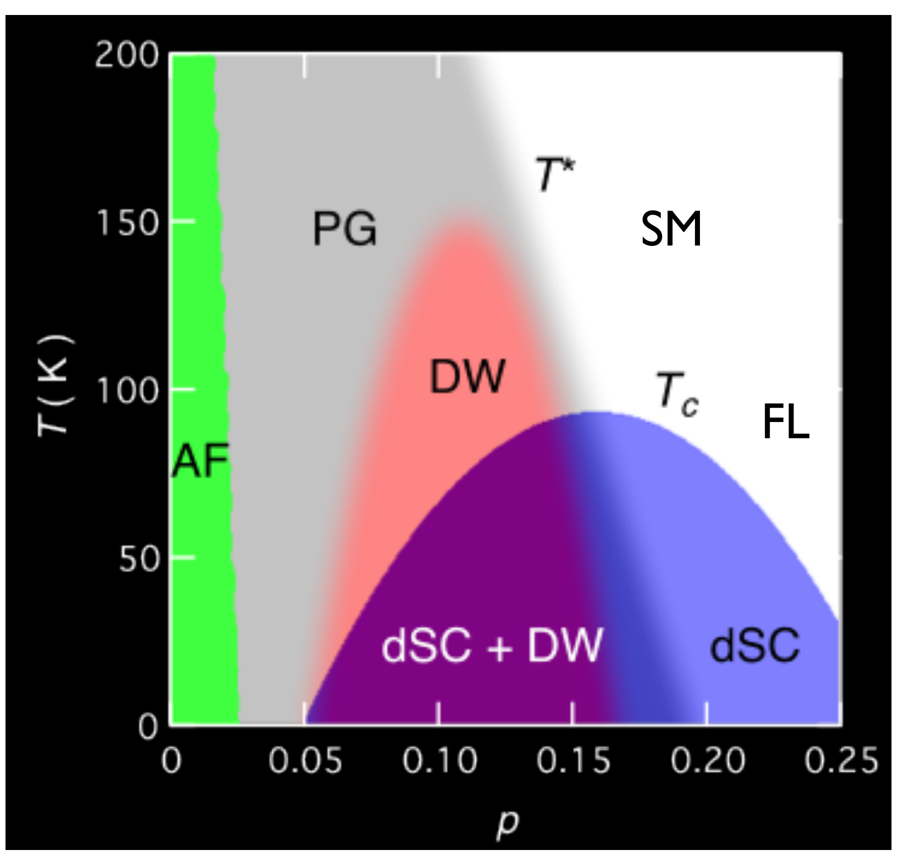


S. Sachdev and D. Chowdhury,
Prog. Theor. Exp. Phys. 12C102 (2016)

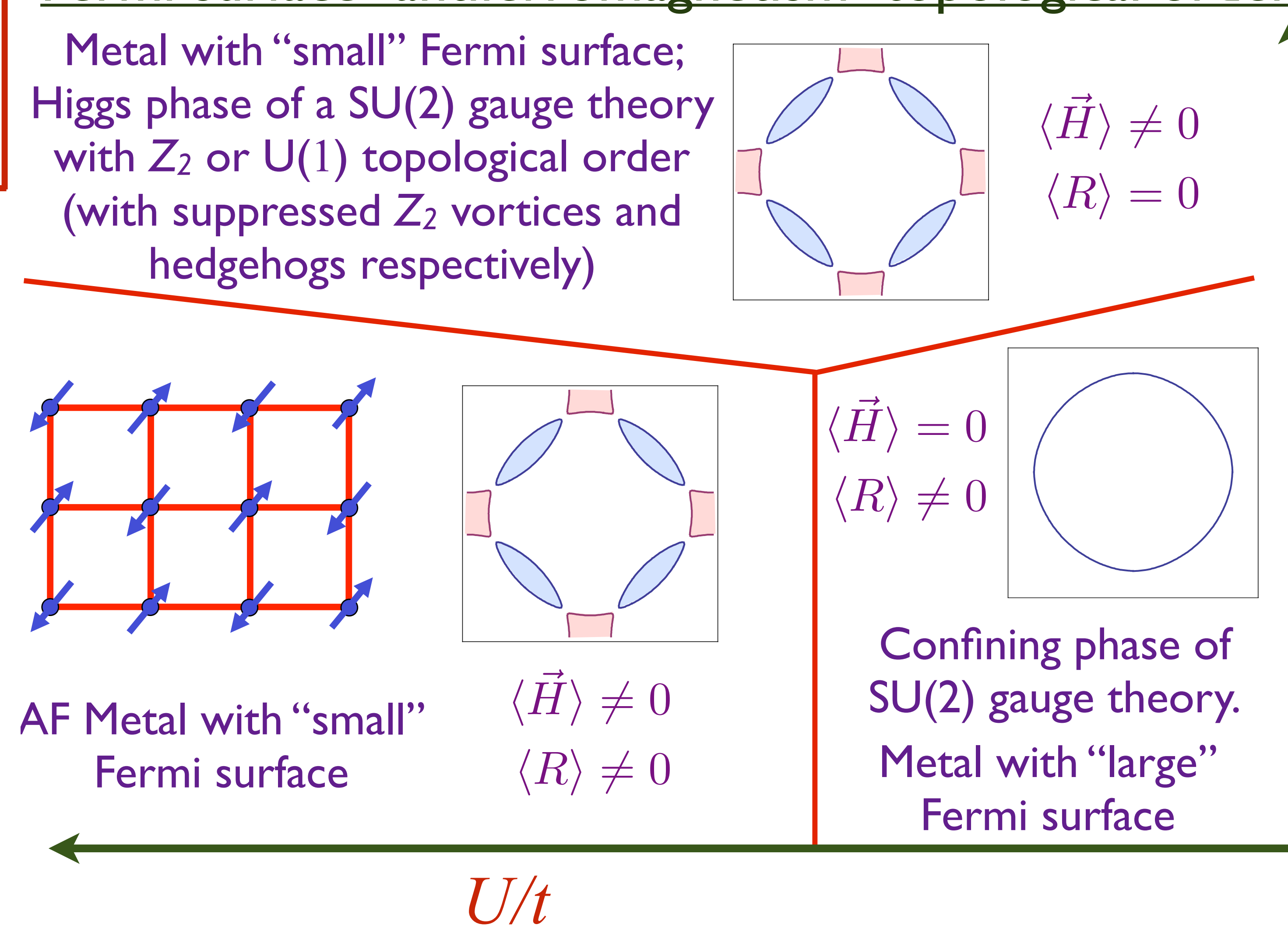
A quantum Hamiltonian in 2+1 dimensions

$$\widetilde{H} = -J\sum_{\langle ij\rangle}\sigma^z_{ij}\cos\left[(\theta_i-\theta_j)/2\right] - K\sum_{\square}\prod_{(ij)\in\square}\sigma^z_{ij}$$
$$+U\sum_i(\hat{n}_i)^2 - g\sum_{\langle ij\rangle}\sigma^x_{ij} \quad ; \quad [\theta_i,\hat{n}_j]=i\delta_{ij}$$





Fermi surface+antiferromagnetism+topological order



- New classes of quantum states with topological order

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- Can be understood as:
(a) defect suppression in states with fluctuating order associated with broken symmetries
(b) Higgs phases of emergent gauge fields

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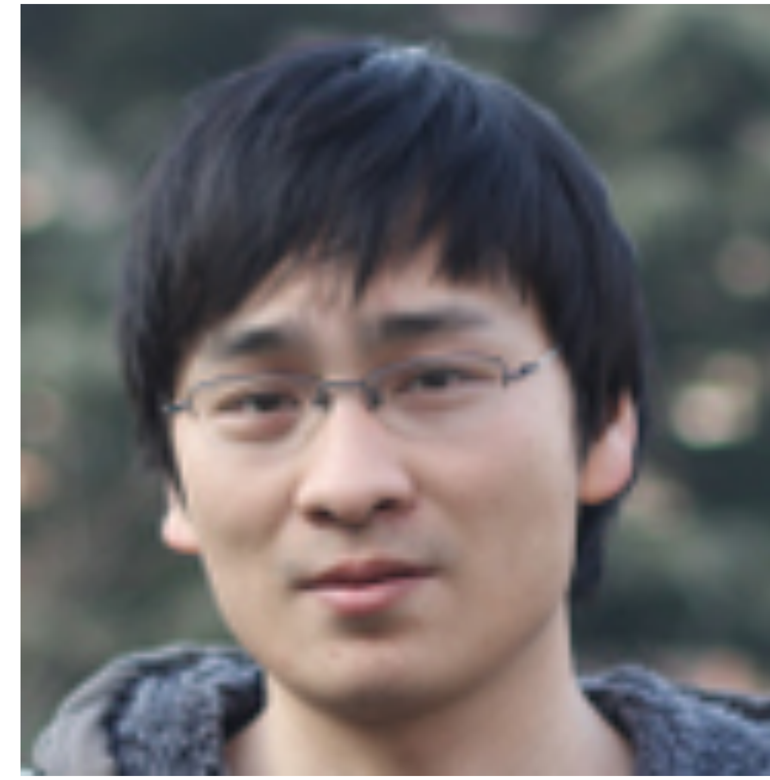
  (a) defect suppression in states with fluctuating order associated with broken symmetries
  (b) Higgs phases of emergent gauge fields

- A metal with bulk topological order (*i.e.* long-range quantum entanglement) can explain existing experiments in cuprates, and agrees well with cluster-DMFT

Mathias Scheurer

Shubhayu Chatterjee



Wei Wu

Michel Ferrero

arXiv:1711.09925

Antoine Georges