Transport near the quantum critical points of metals

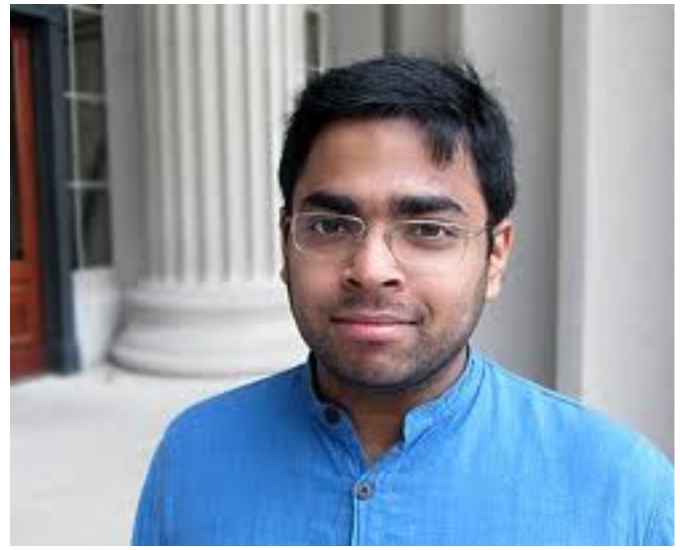
Conference on Non-Fermi Liquids
Stanford Institute for Theoretical Physics
April 18, 2014

Subir Sachdev

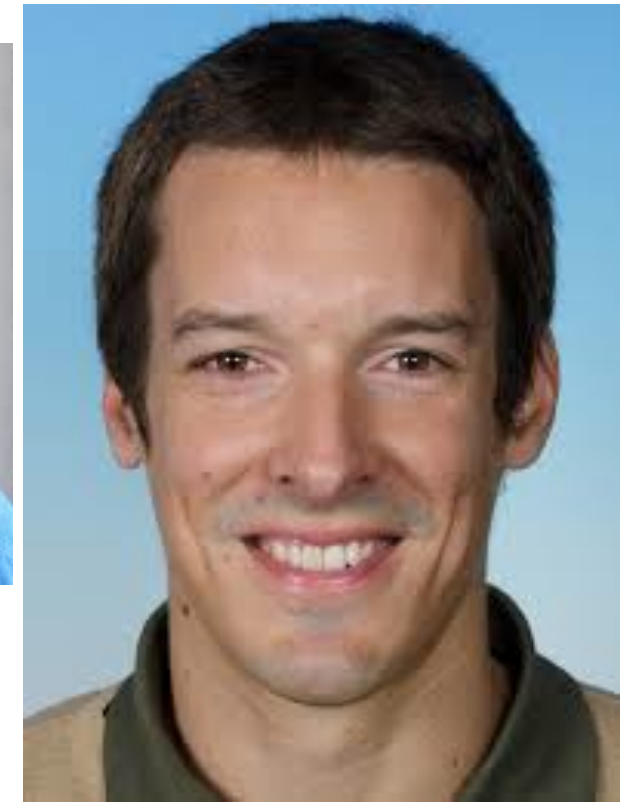
Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD

Sean Hartnoll
Stanford

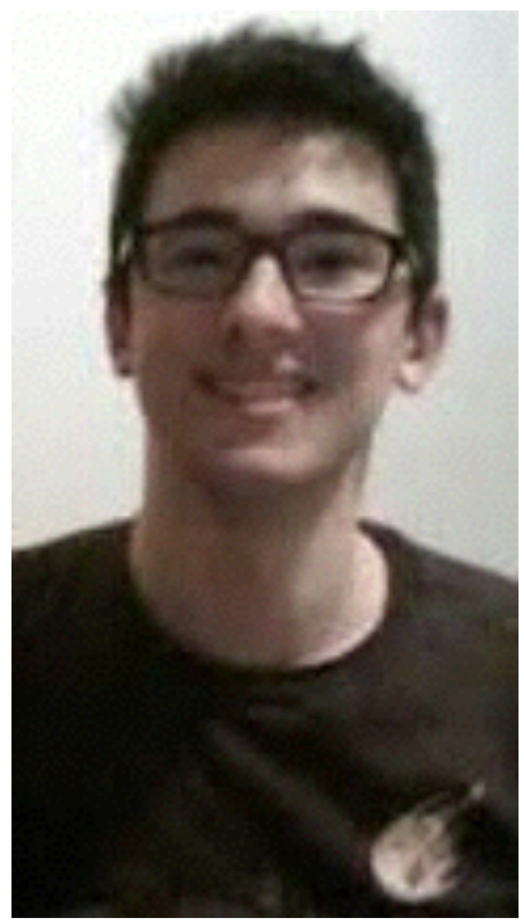


Raghu Mahajan
Stanford



Matthias Punk
Innsbruck

Koenraad Schalm
Leiden



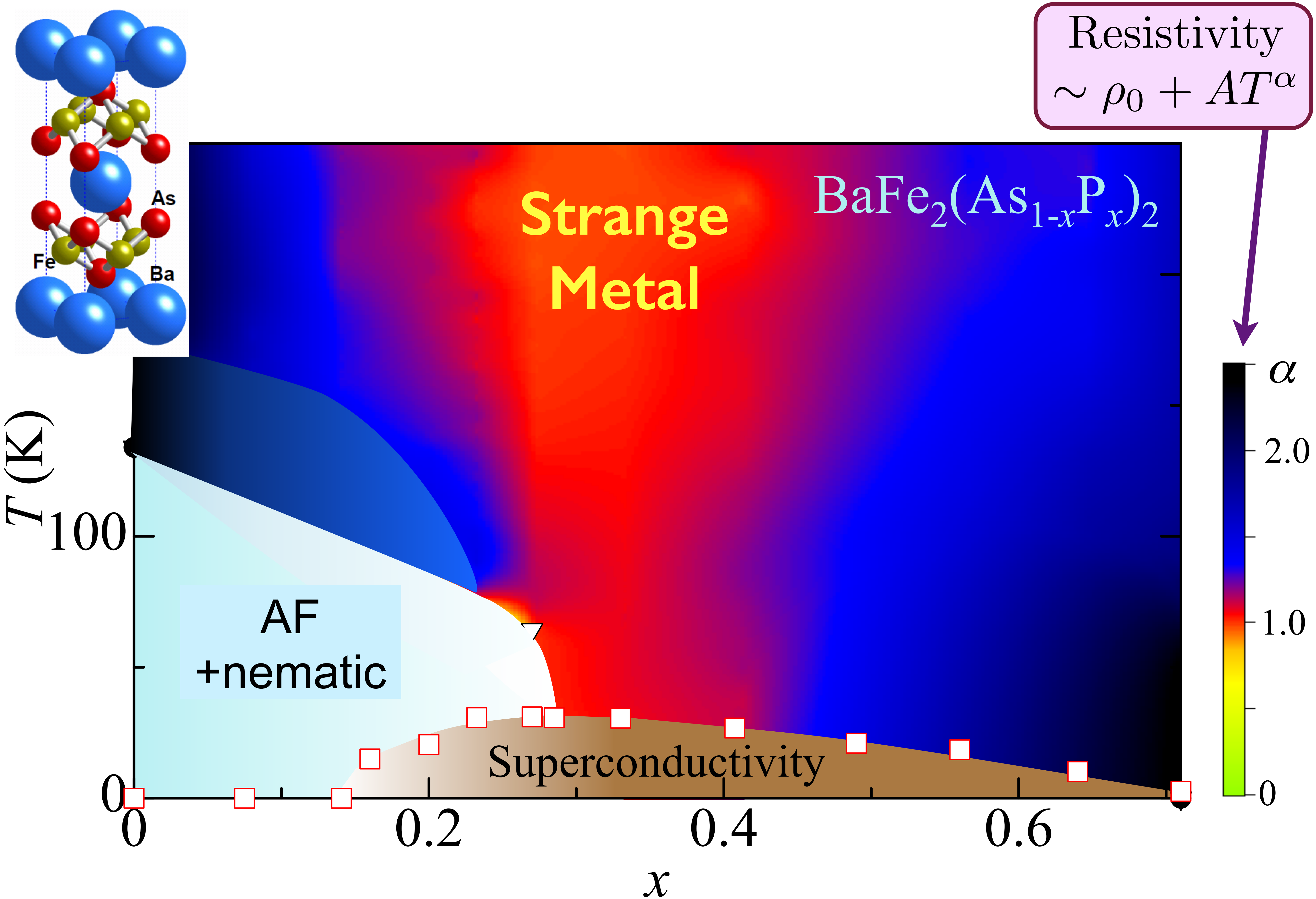
Andrew Lucas
Harvard

Friday April 25th

PROFS GONE WILD

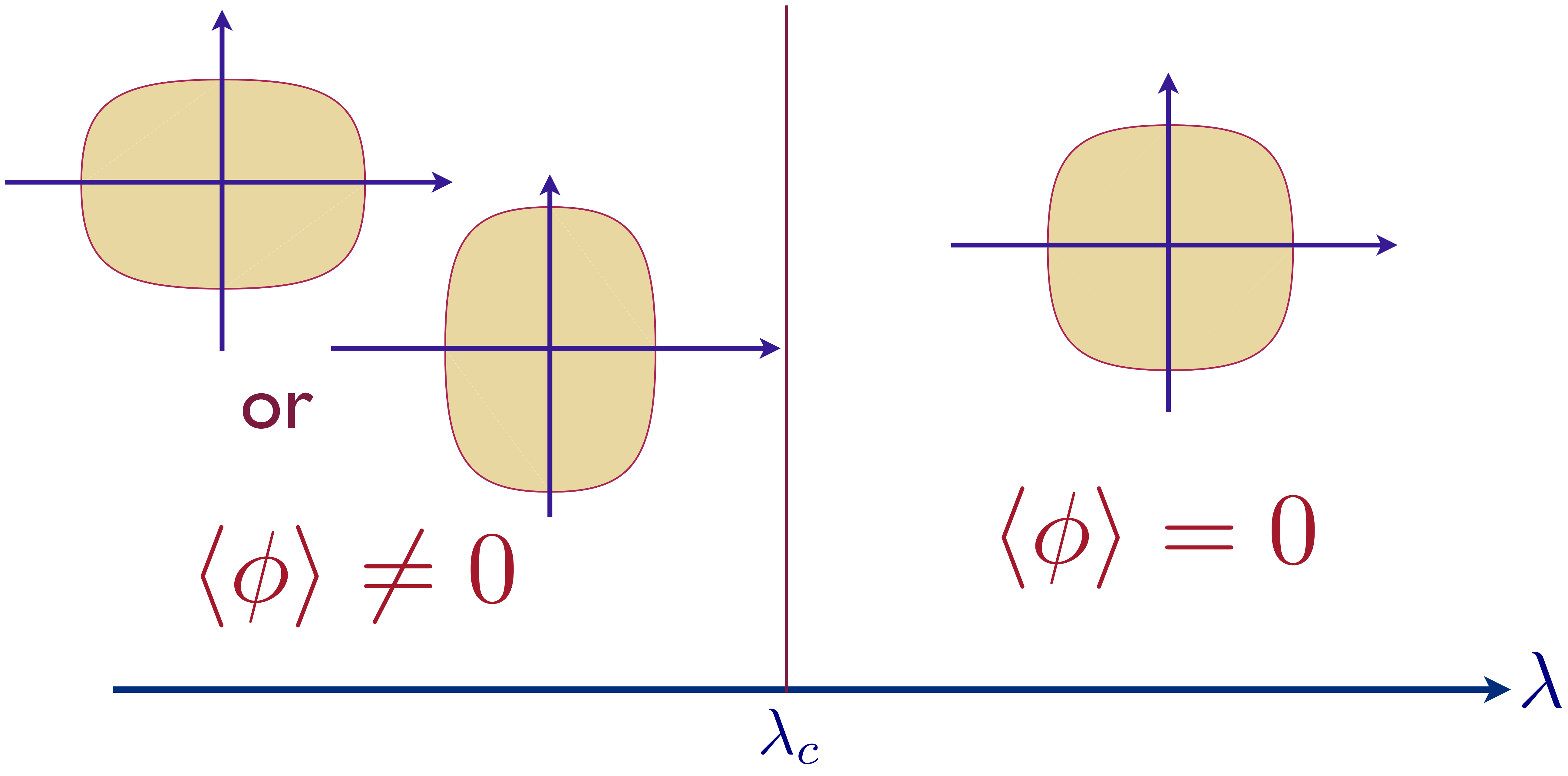
7pm REFRESHMENTS IN THE LIBRARY

8pm PUPPET SHOW IN J250



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c^\dagger_{\mathbf{k}\alpha} \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

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Field theory of bosonic order parameter

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Electrons with a Fermi surface: $\varepsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - \mu \ldots$

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"Yukawa" coupling between bosons and fermions

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Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic ϕ fluctuations.

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- "Bloch's law" for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$.

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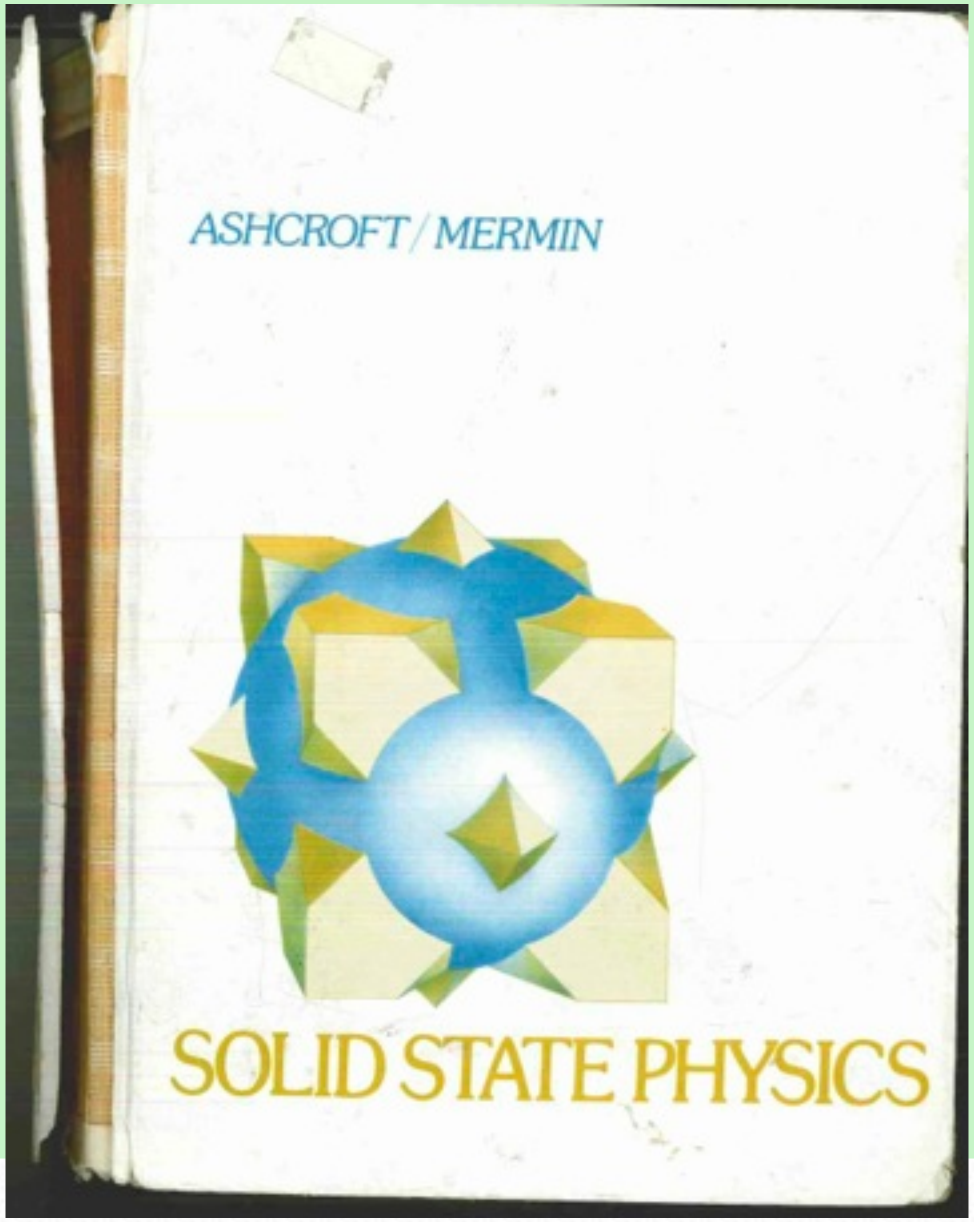

However, this ignores "phonon drag"

PHONON DRAG

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[28] R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).

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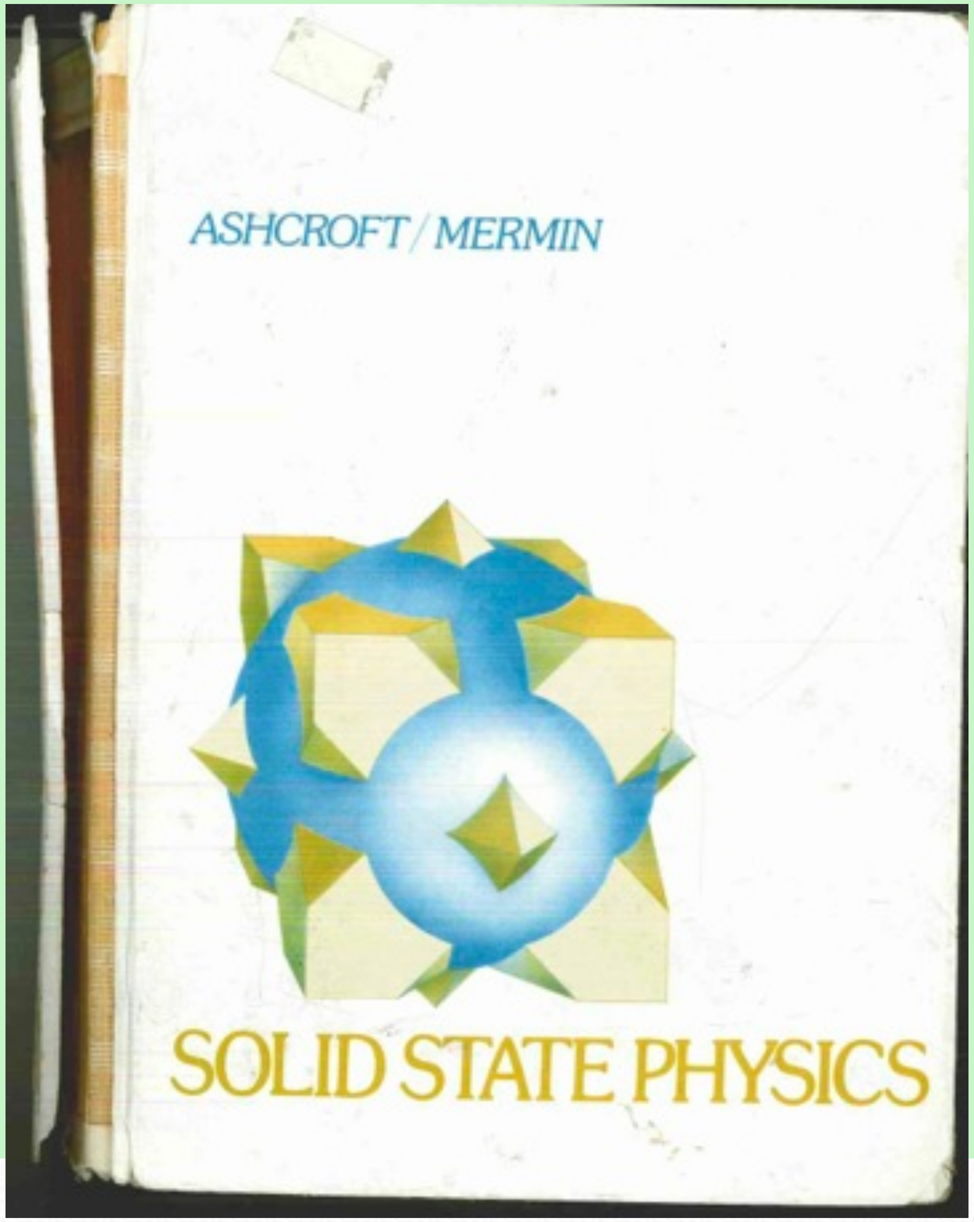

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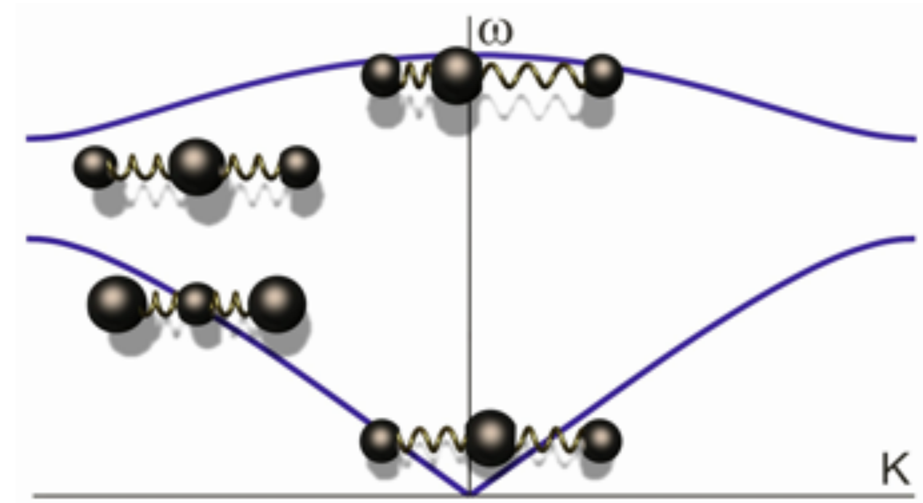
Rates of Momentum Flow

Electrons

SLOW

Phonons



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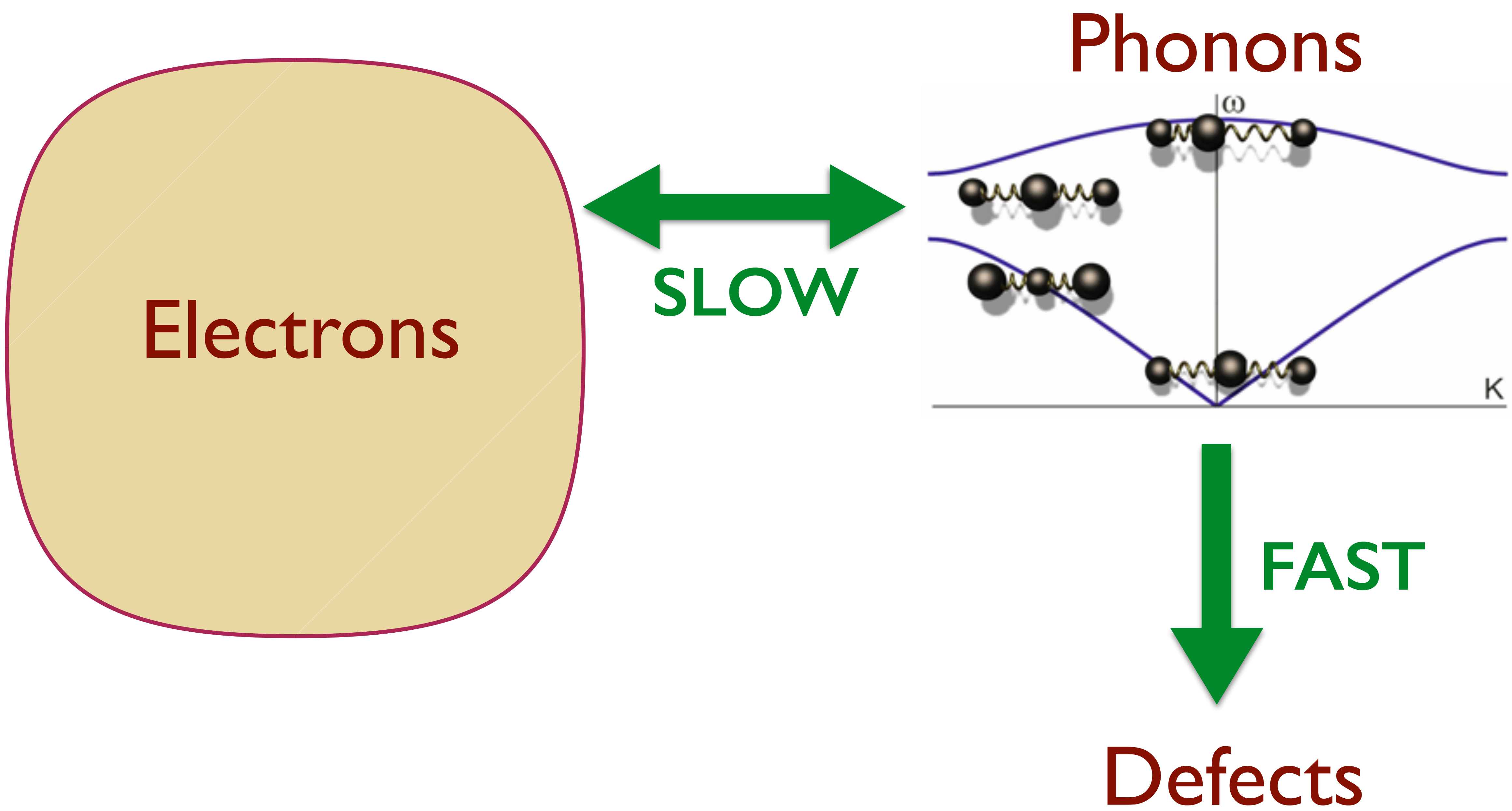

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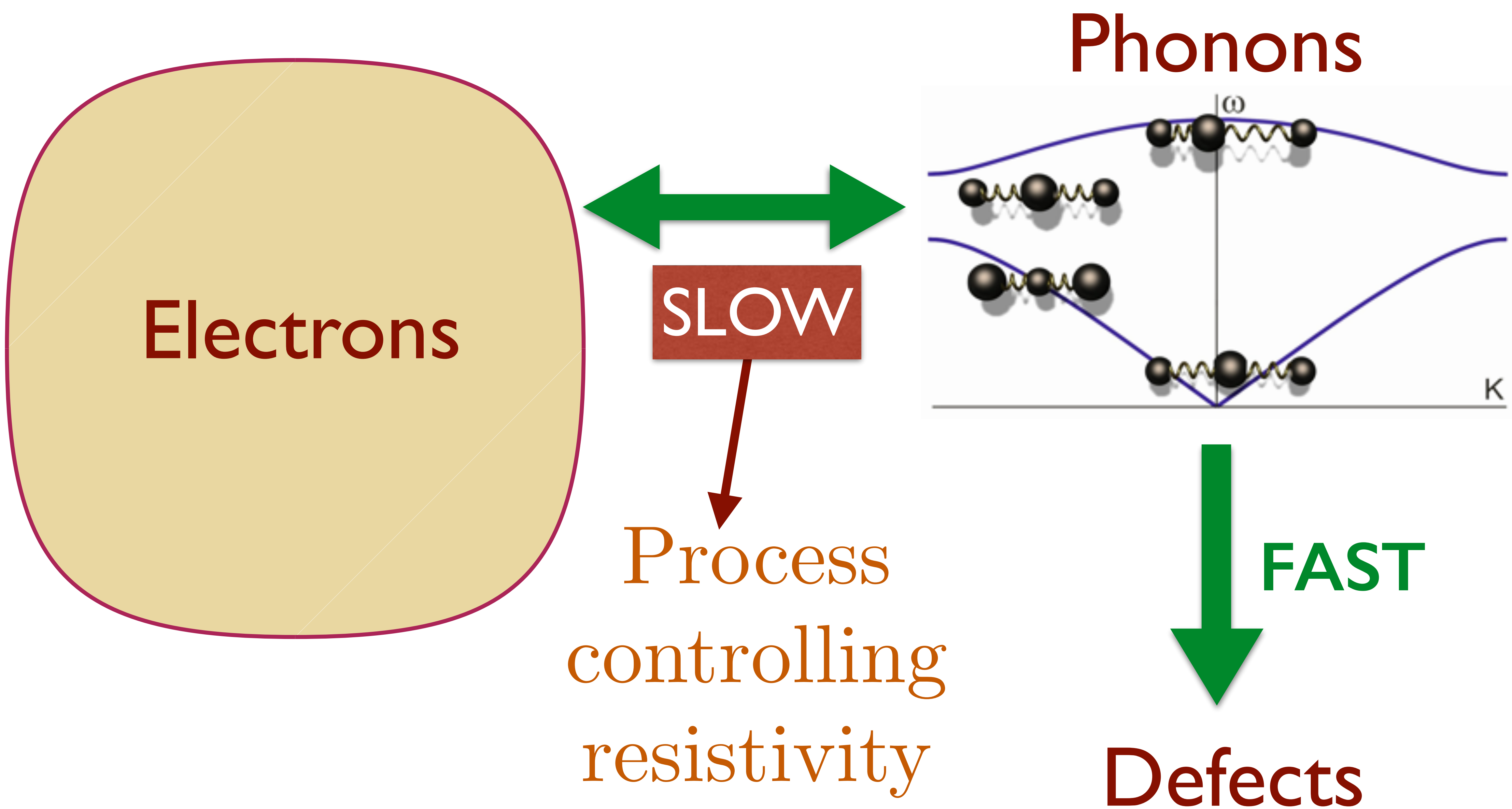

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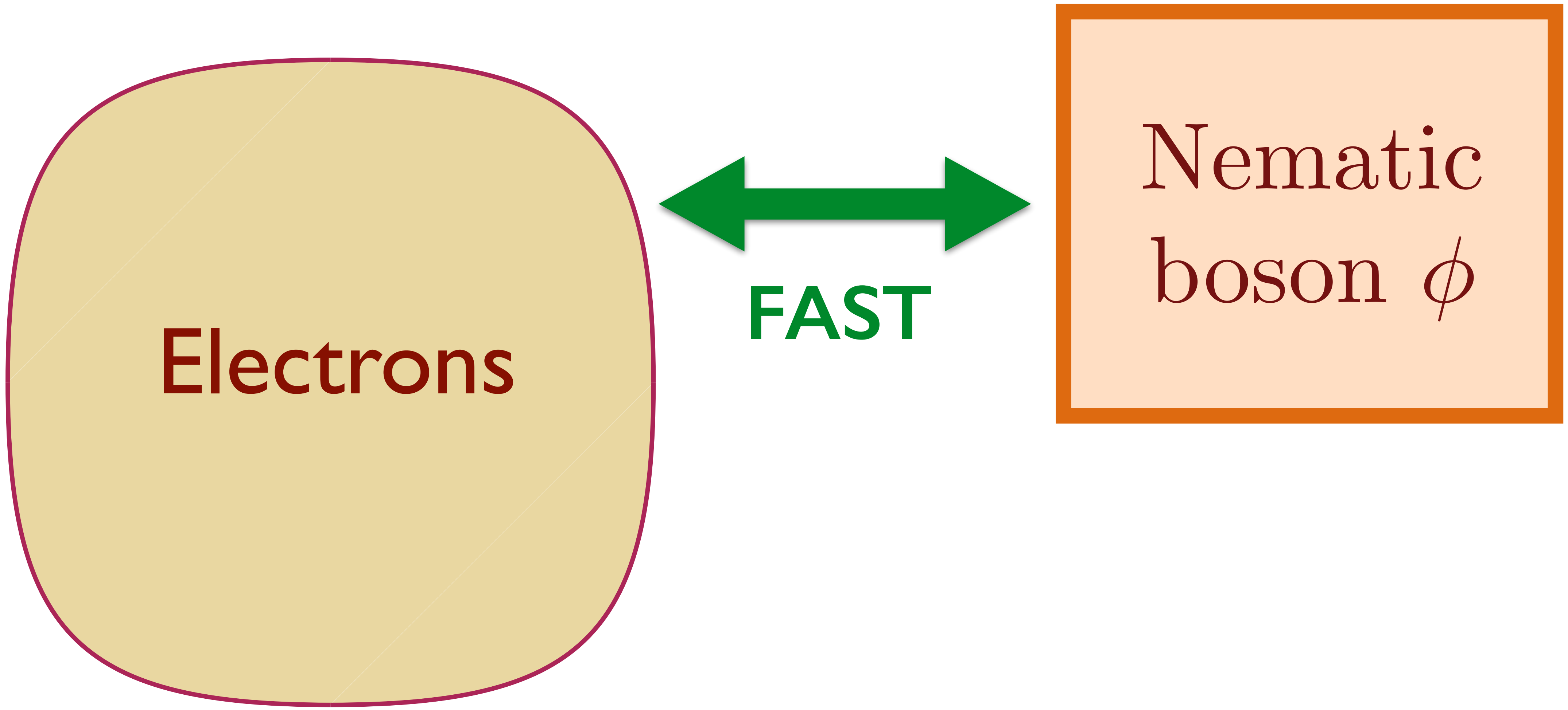

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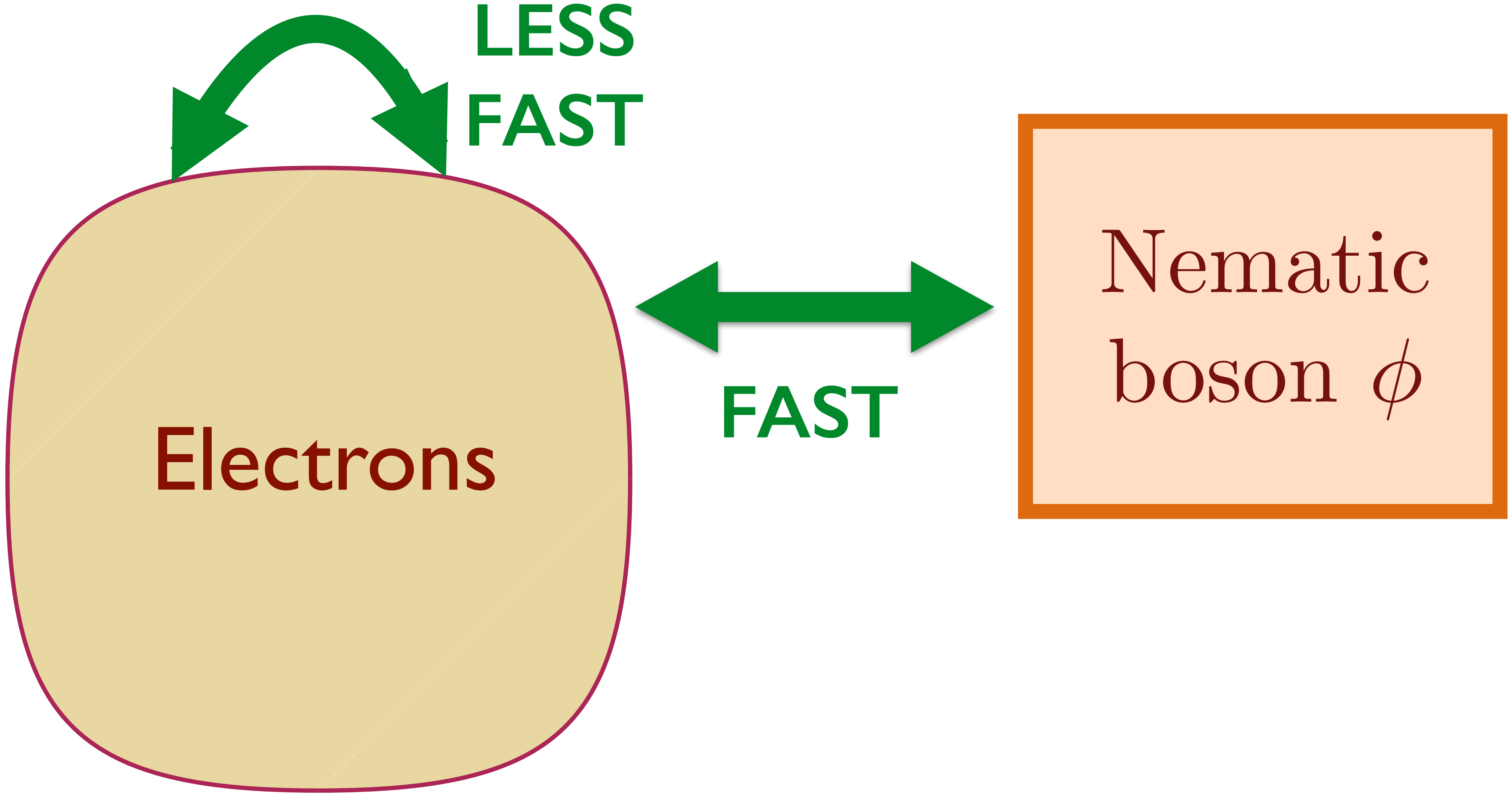

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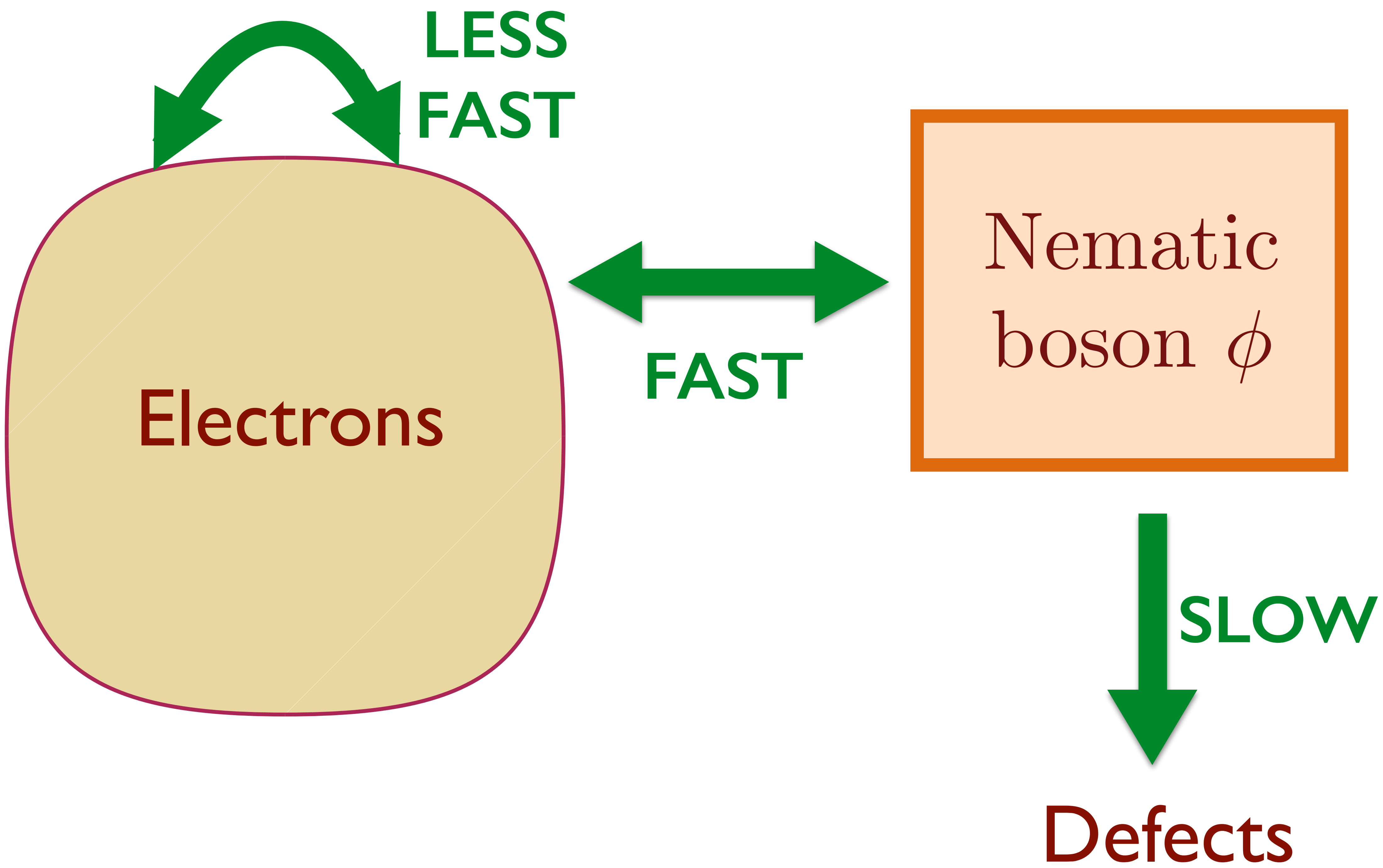

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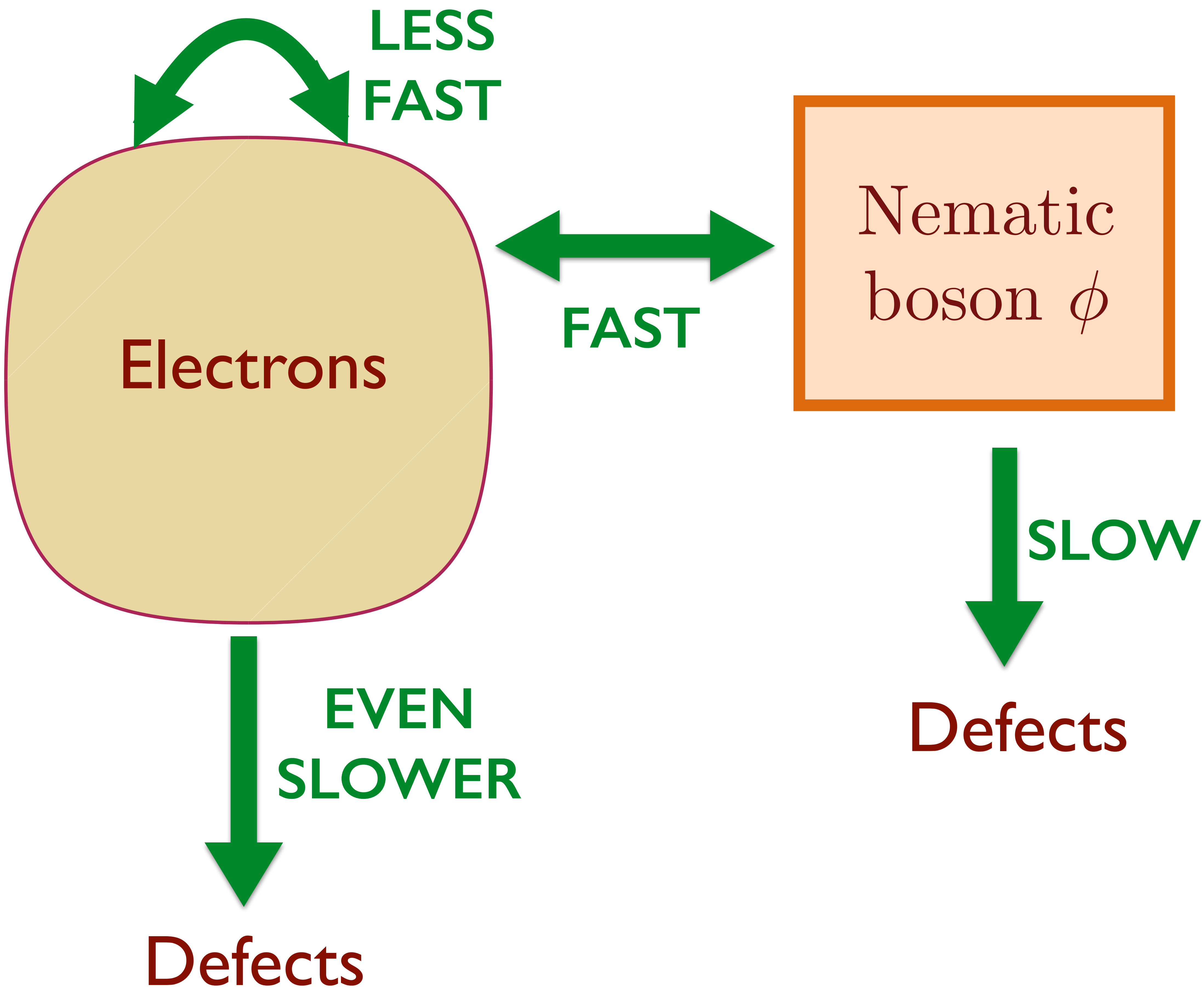

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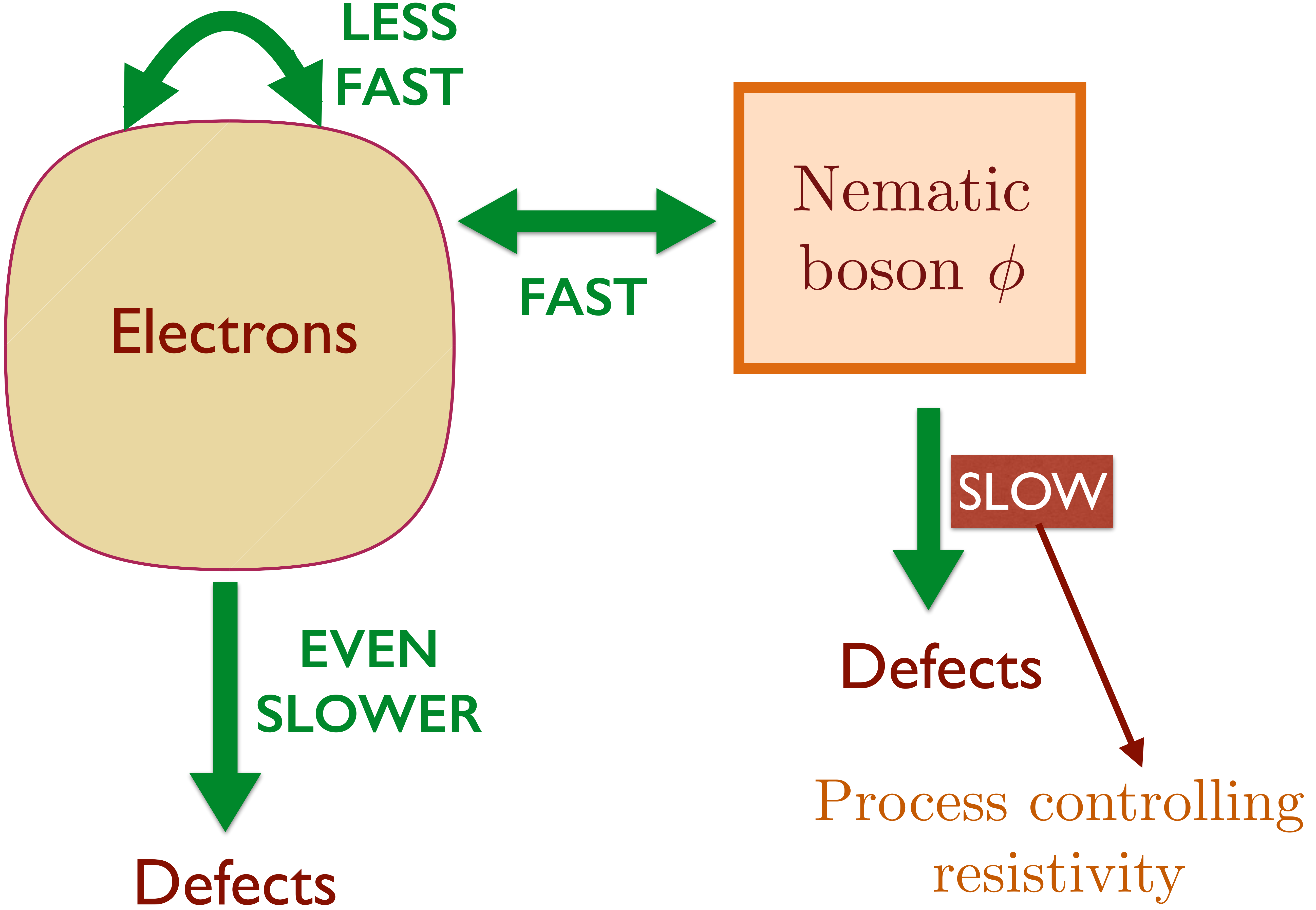

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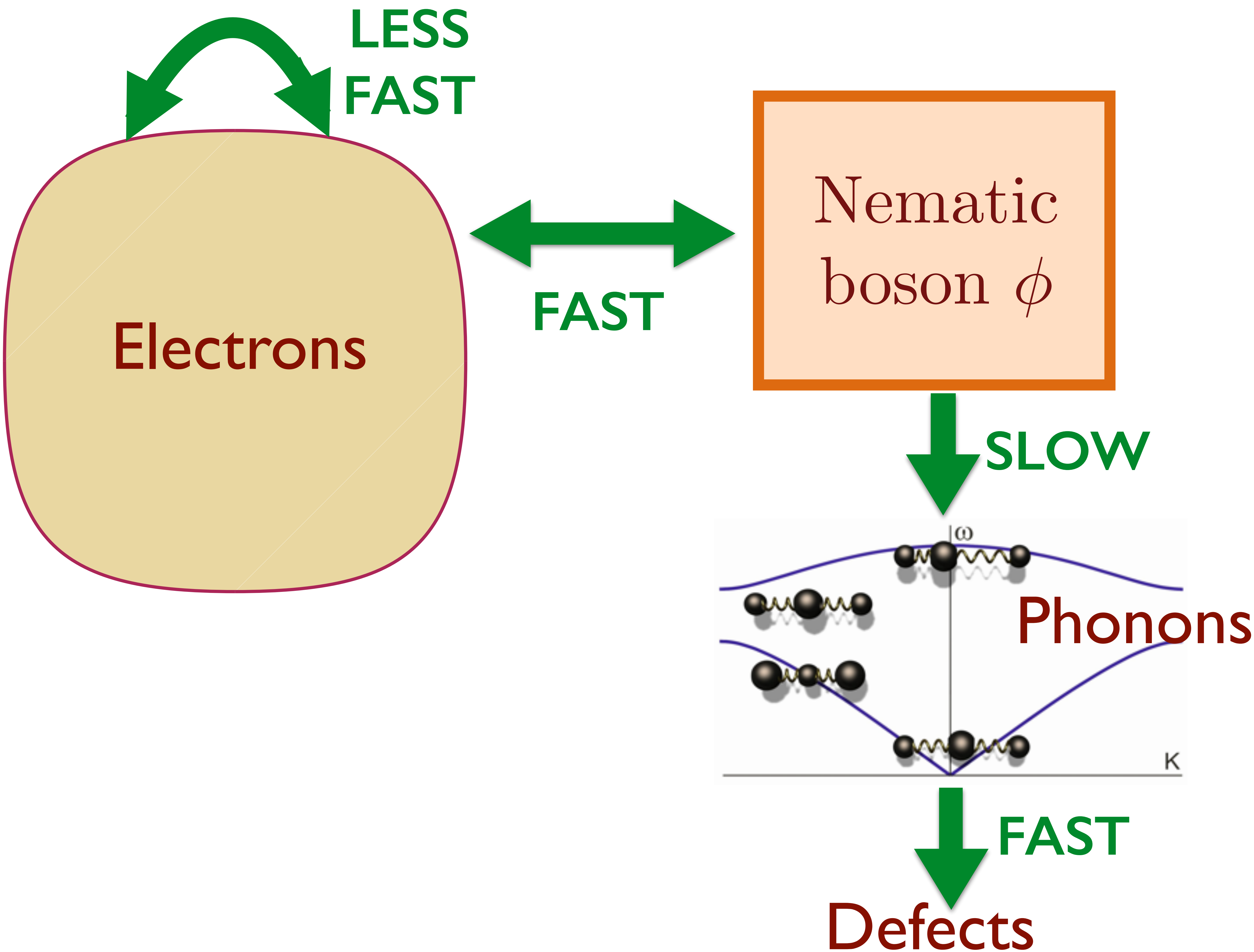

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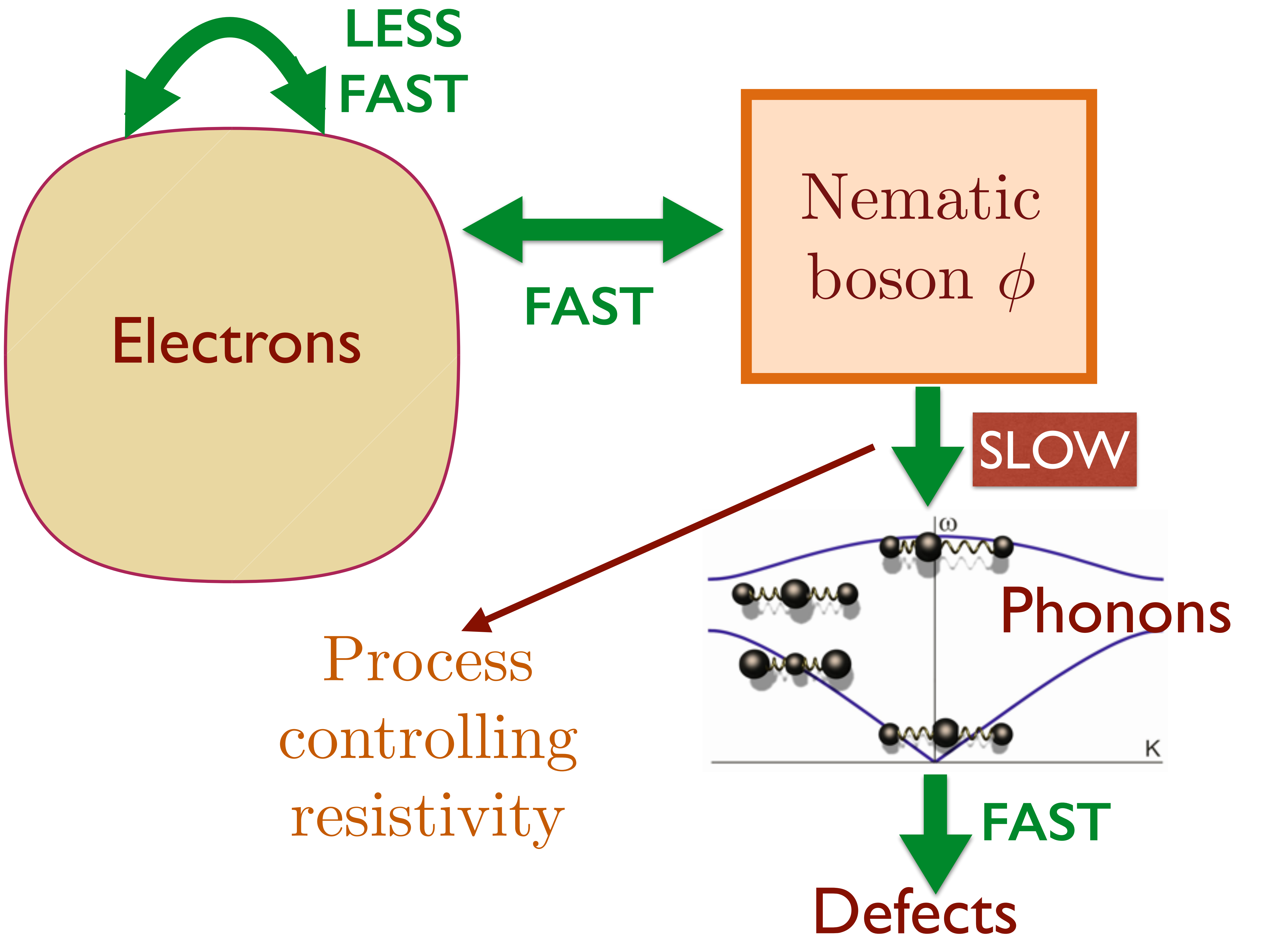

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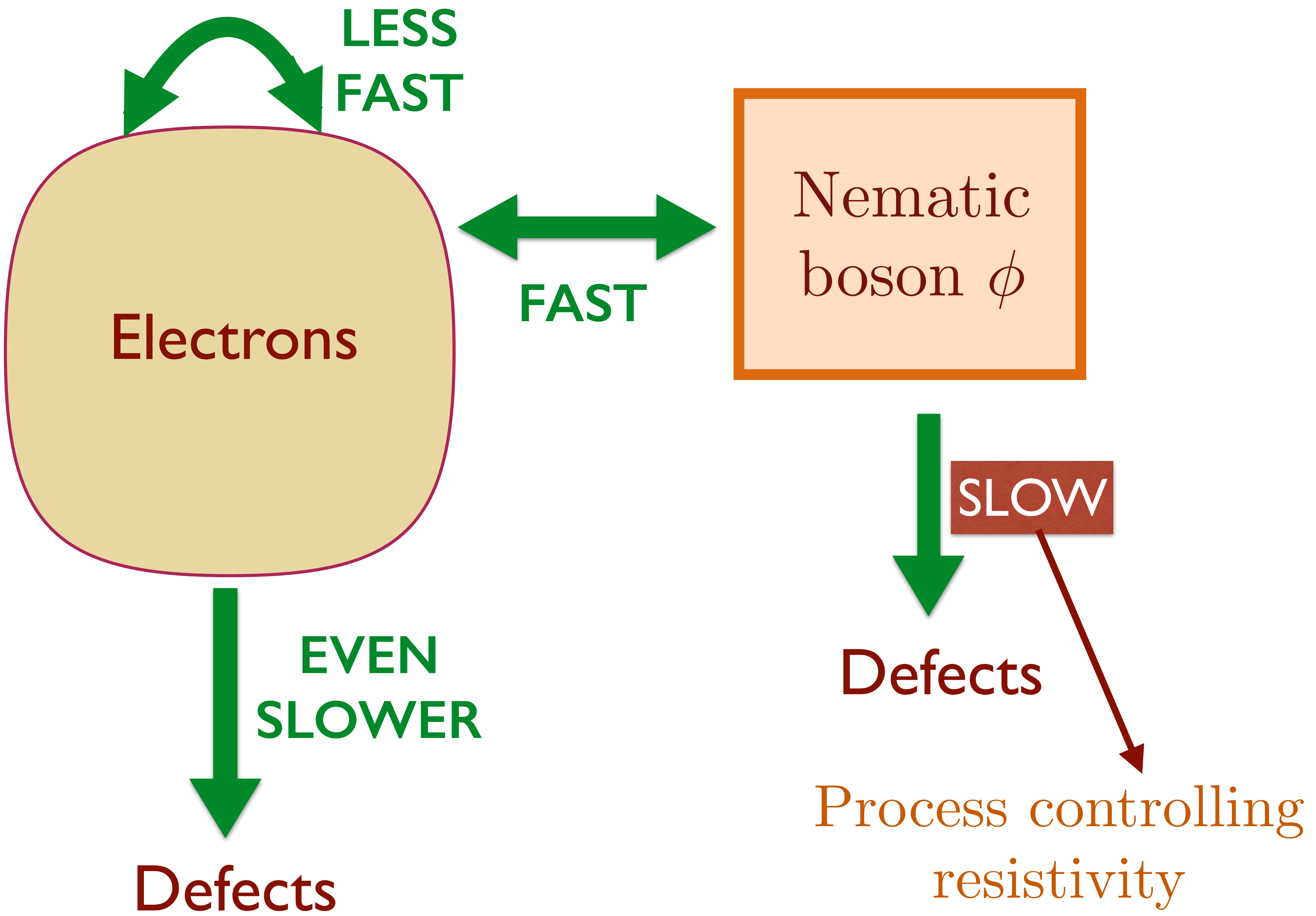

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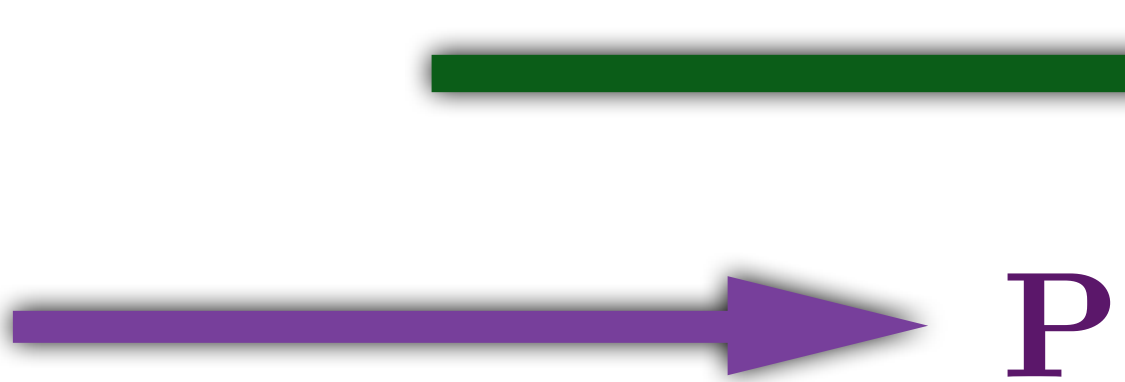
$$\mathcal{S}_{\phi c} = -g \int d^2r d\tau \sum_{\alpha=1}^{N_f} \phi \left[c_\alpha^\dagger \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha \right\} + \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha^\dagger \right\} c_\alpha \right]$$

This continuum theory has has strong electron$-\phi$ scattering, and no quasi-particle excitations. But it has a conserved momentum $\mathbf{P}$, and $\chi_{\mathbf{J},\mathbf{P}} \neq 0$ ("phonon drag"), and so the resistivity $\rho(T) = 0$.

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

- Focus on the interplay between J_μ and $T_{\mu\nu}$!



The most-probable state with a non-zero current $\mathbf{J}$ has a non-zero momentum $\mathbf{P}$ (and vice versa). At non-zero density, $\mathbf{J}$ "drags" $\mathbf{P}$.

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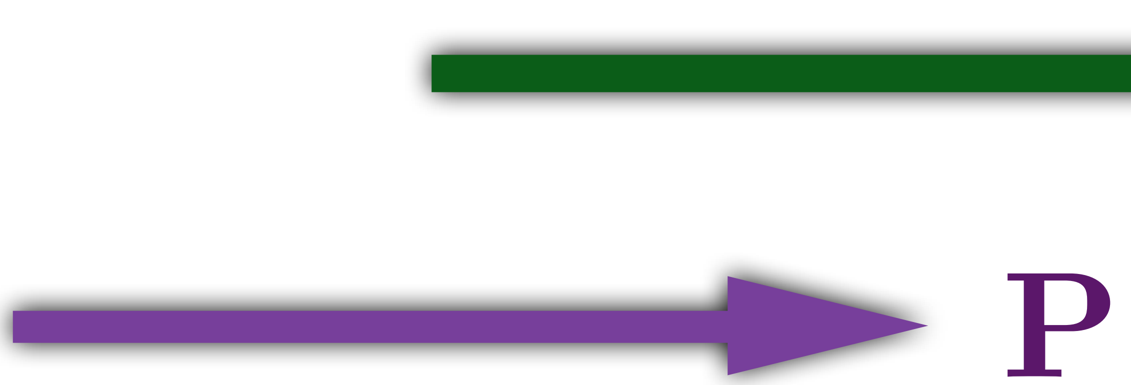

The most-probable state with a non-zero current $\mathbf{J}$ has a non-zero momentum $\mathbf{P}$ (and vice versa). At non-zero density, $\mathbf{J}$ "drags" $\mathbf{P}$.

The resistivity of this metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, arXiv:1401.7012.
A. Lucas, S. Sachdev, and K. Schalm, arXiv:1401.7933

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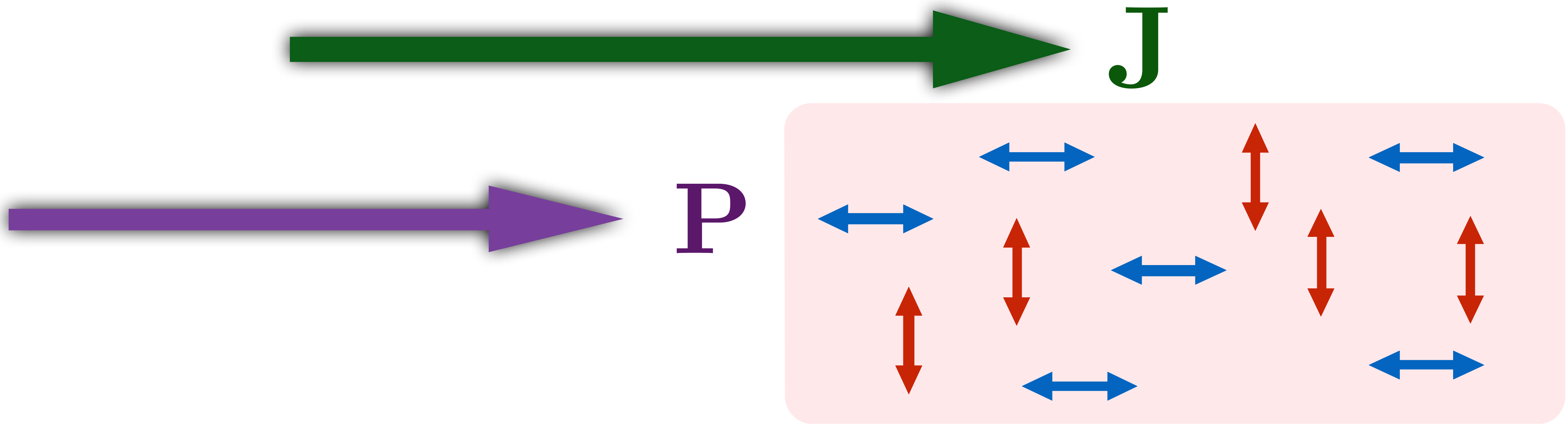
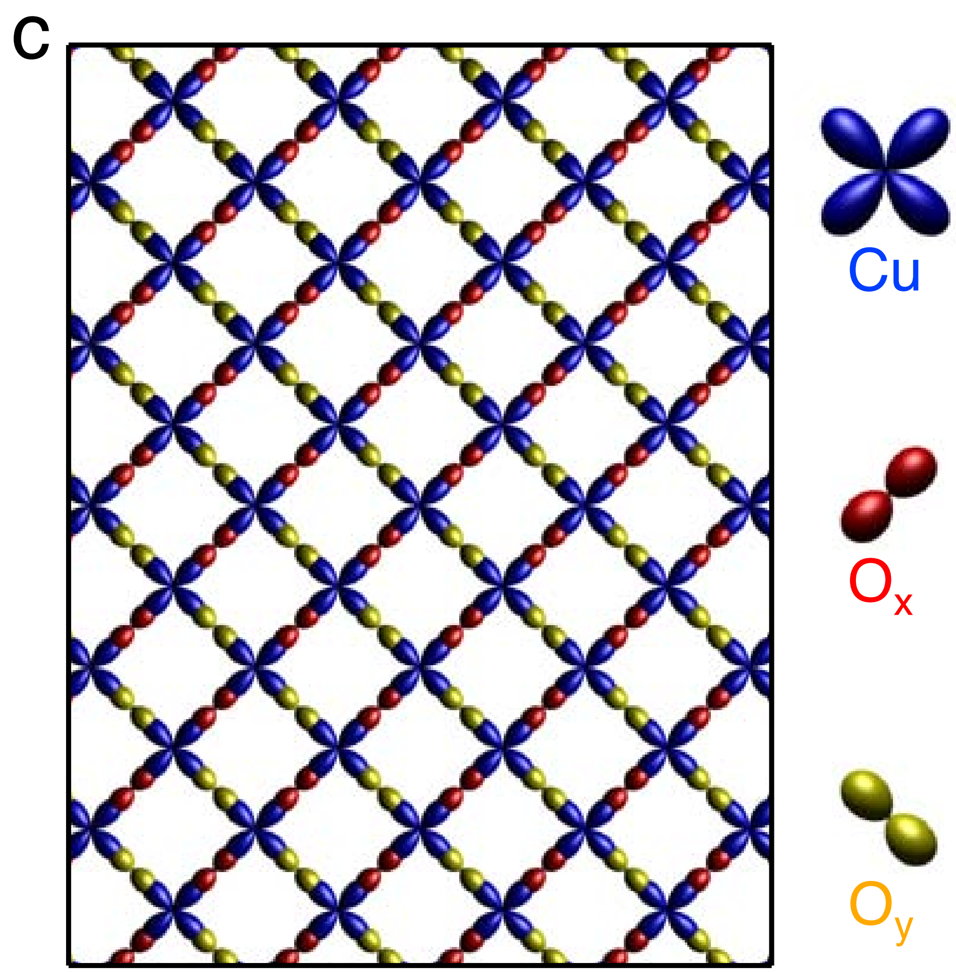


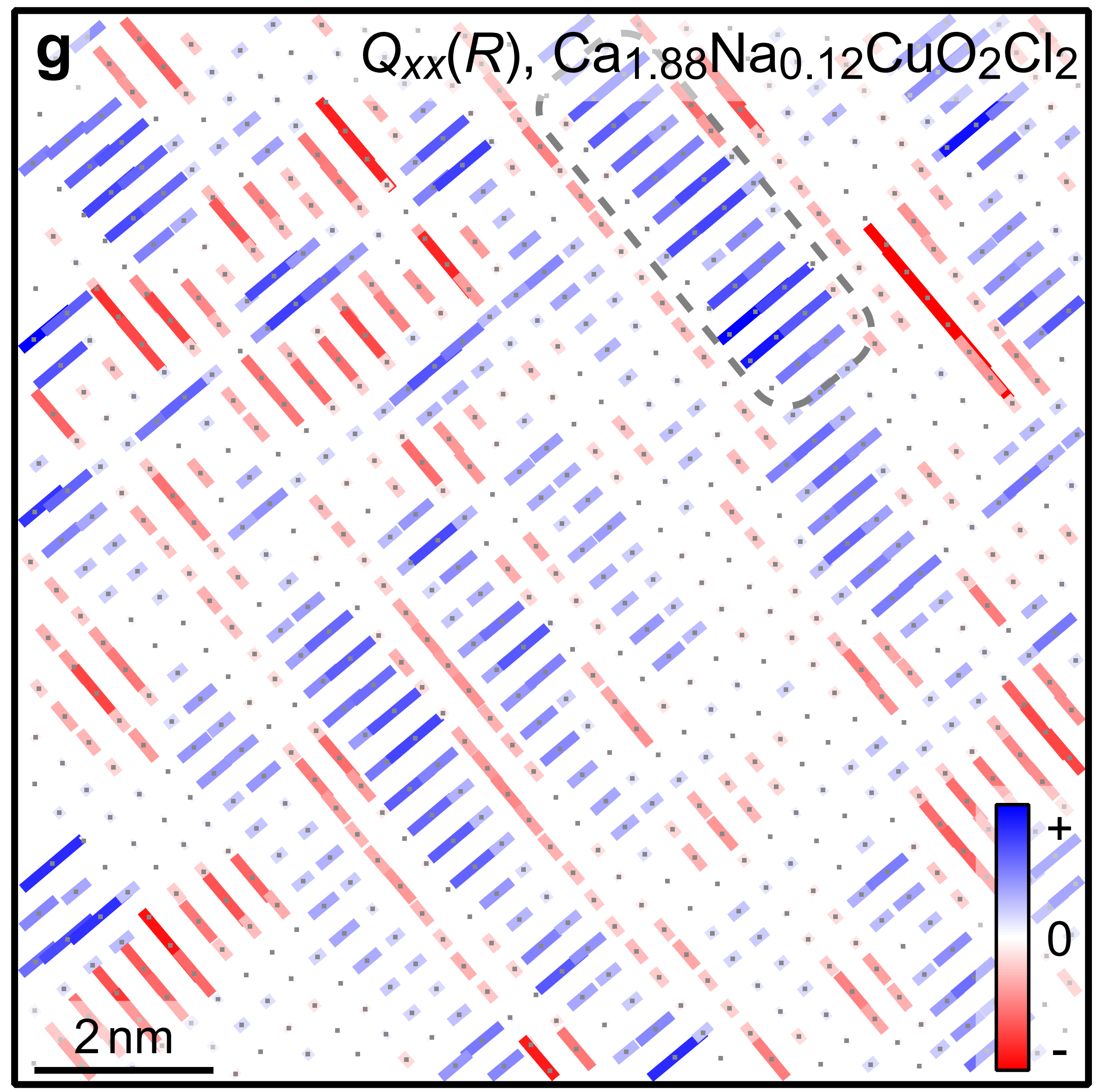
The dominant momentum loss occurs via the scattering of the neutral bosonic ϕ excitations off **random fields**.
This is good news for the AdS/CMT approaches, which do not capture the Fermi surface of most of the charged carriers.

Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi

Nature Physics, **8**, 534 (2012).





Evidence for "nematic" order
(*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$\mathcal{S}_{\rm dis} = \int d^2r d\tau \left[V(\mathbf{r})\, c^\dagger c + h(\mathbf{r})\, \phi \right] \,,$$

$$\overline{V(\mathbf{r})} = 0 \quad ; \quad \overline{V(\mathbf{r}) V(\mathbf{r}')} = V_0^2\, \delta(\mathbf{r} - \mathbf{r}')\,,$$

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we use the memory-function approach to obtain the *resistivity* for current along angle ϑ

$$\rho(T) = \frac{1}{\chi^2_{\mathbf{J},\mathbf{P}}} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2}\, k^2 \cos^2(\theta_{\mathbf{k}} - \vartheta) \left(V_0^2\, \frac{\mathrm{Im}\, \Pi^R_{c^\dagger c}(\omega, \mathbf{k})}{\omega} + h_0^2\, \frac{\mathrm{Im}\, D^R_\phi(\omega, \mathbf{k})}{\omega} \right) .$$

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Fermi surface term: Obtain T-dependent corrections to residual resistivity similar to earlier work

G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001)
I. Paul, C. Pépin, B. N. Narozhny, and D. L. Maslov, Phys. Rev. Lett. **95**, 017206 (2005).

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<u>Bosonic term</u>: Dominant contribution:

$$\rho(T) \sim h_0^2\, T^{(d-z+\eta)/z}$$

Crosses over from the "relativistic" form ($z = 1$, $\eta \approx 0$) with $\rho(T) \sim h_0^2\, T$ at higher T, to the "Landau-damped" form ($z = 3$, $\eta = 0$) with $\rho(T) \sim h_0^2\, (T \ln(1/T))^{-1/2}$ at lower T (subtle corrections to scaling specific to this field theory).

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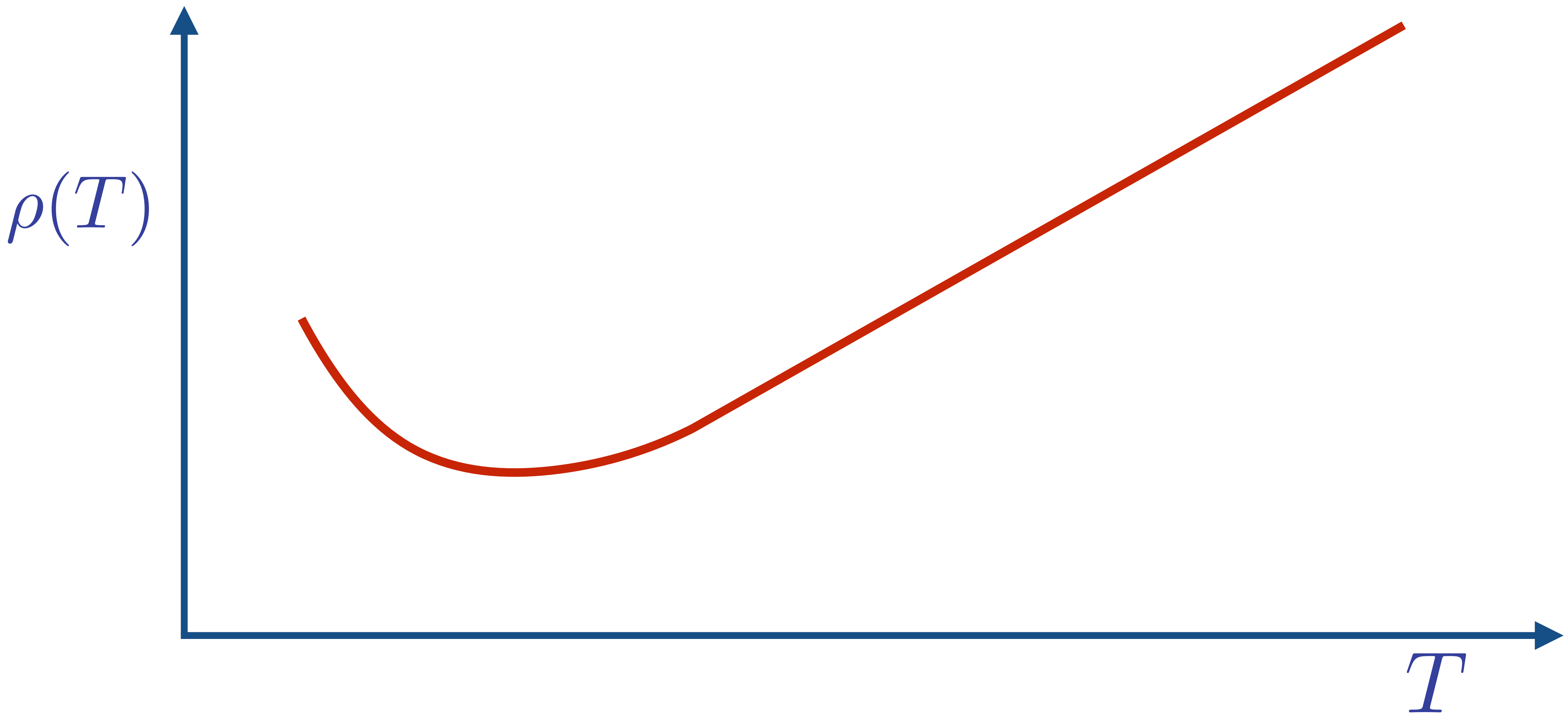
Transport without quasiparticles:

Resistivity from random-field disorder



S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, arXiv:1401.7012.
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Quantum criticality of Ising-nematic ordering in a metal

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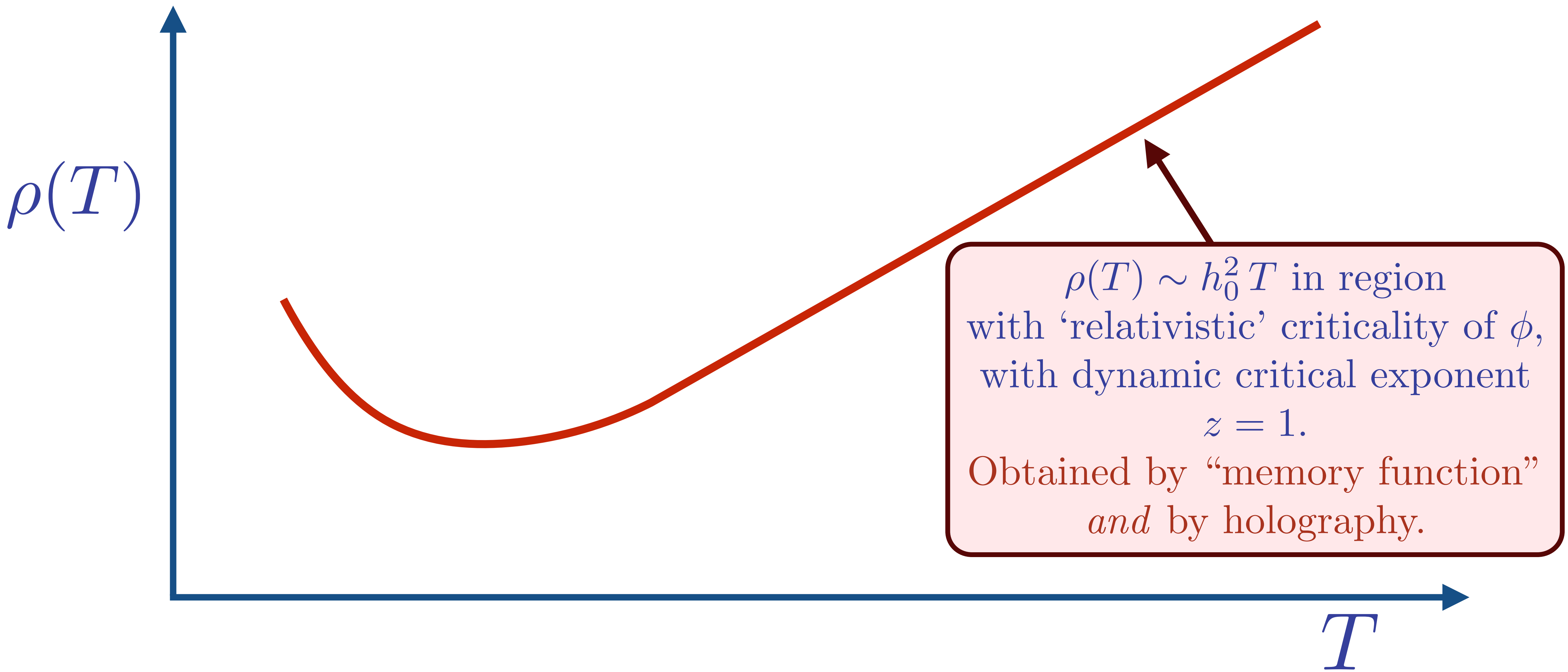

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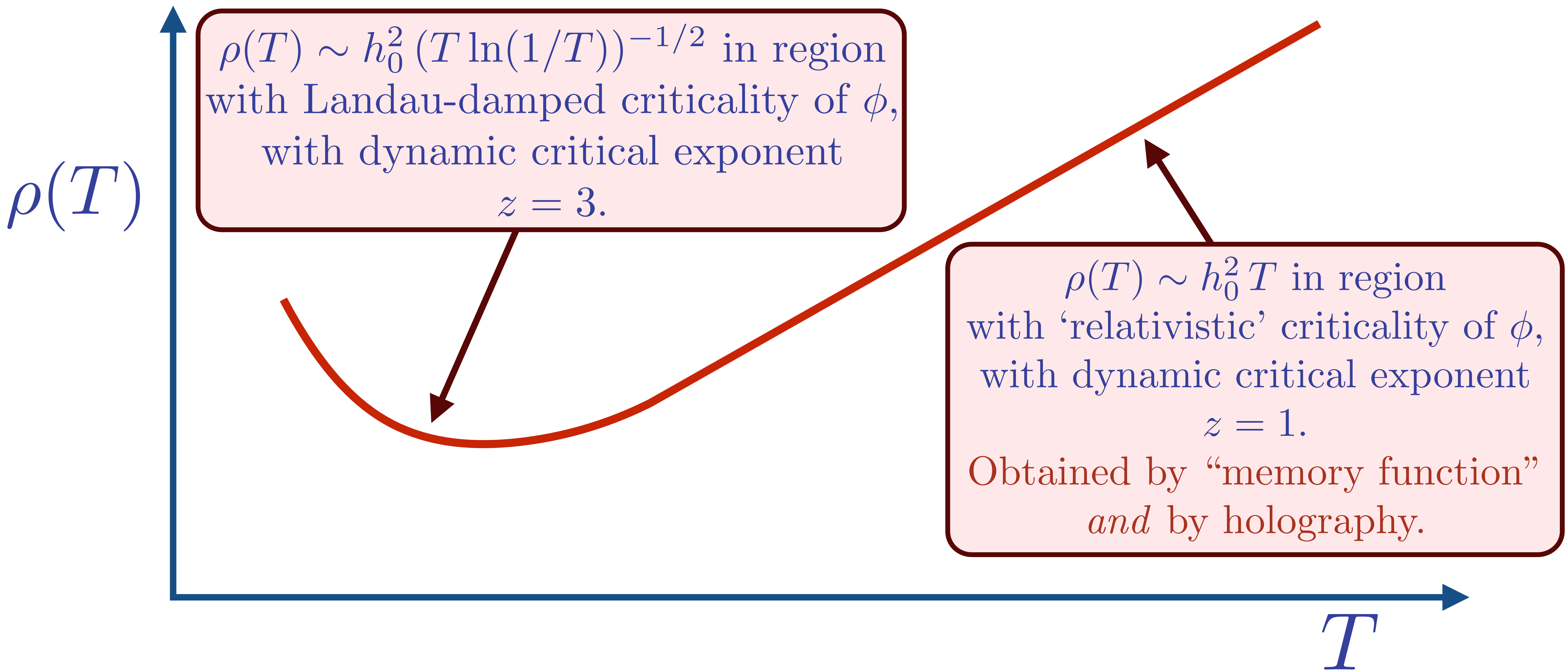

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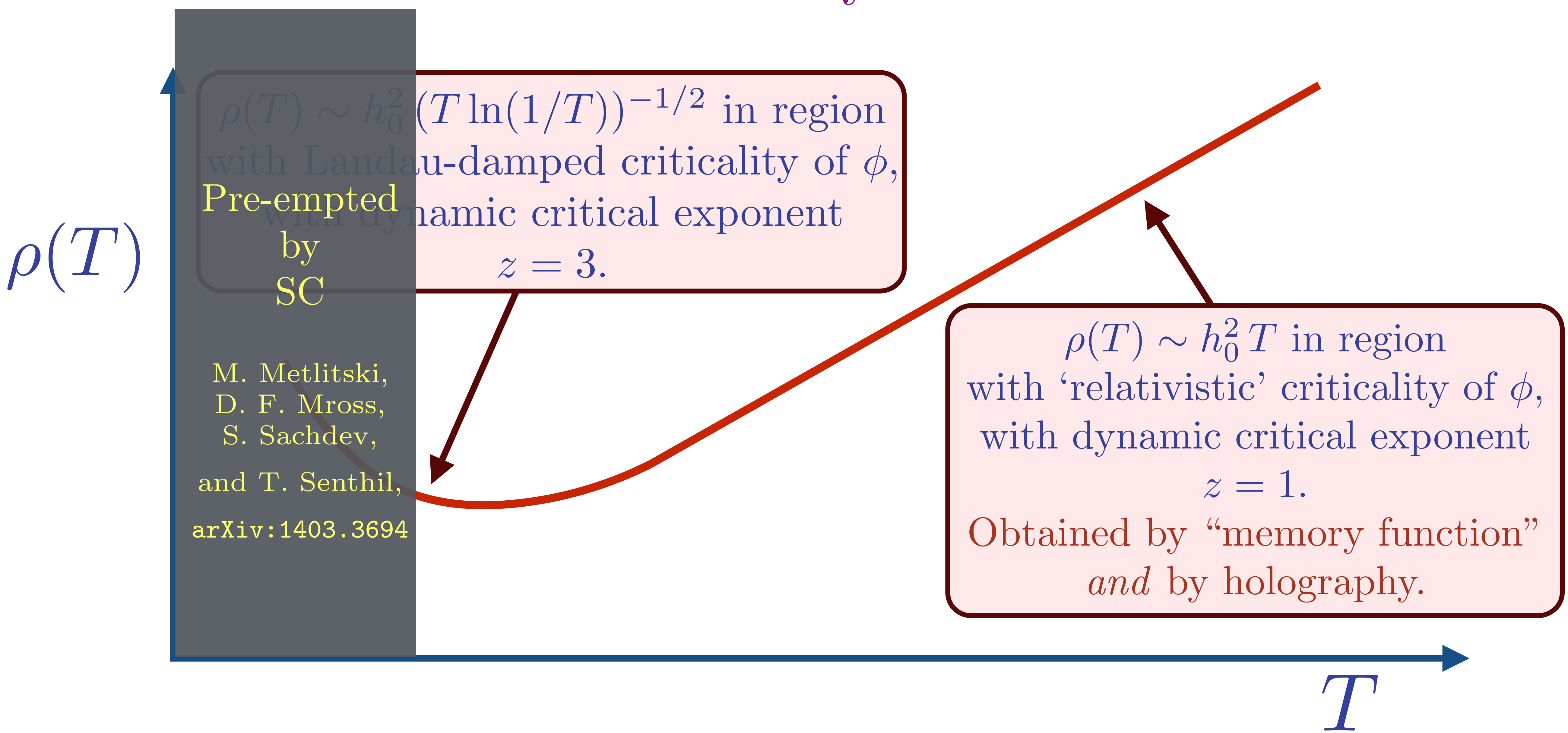

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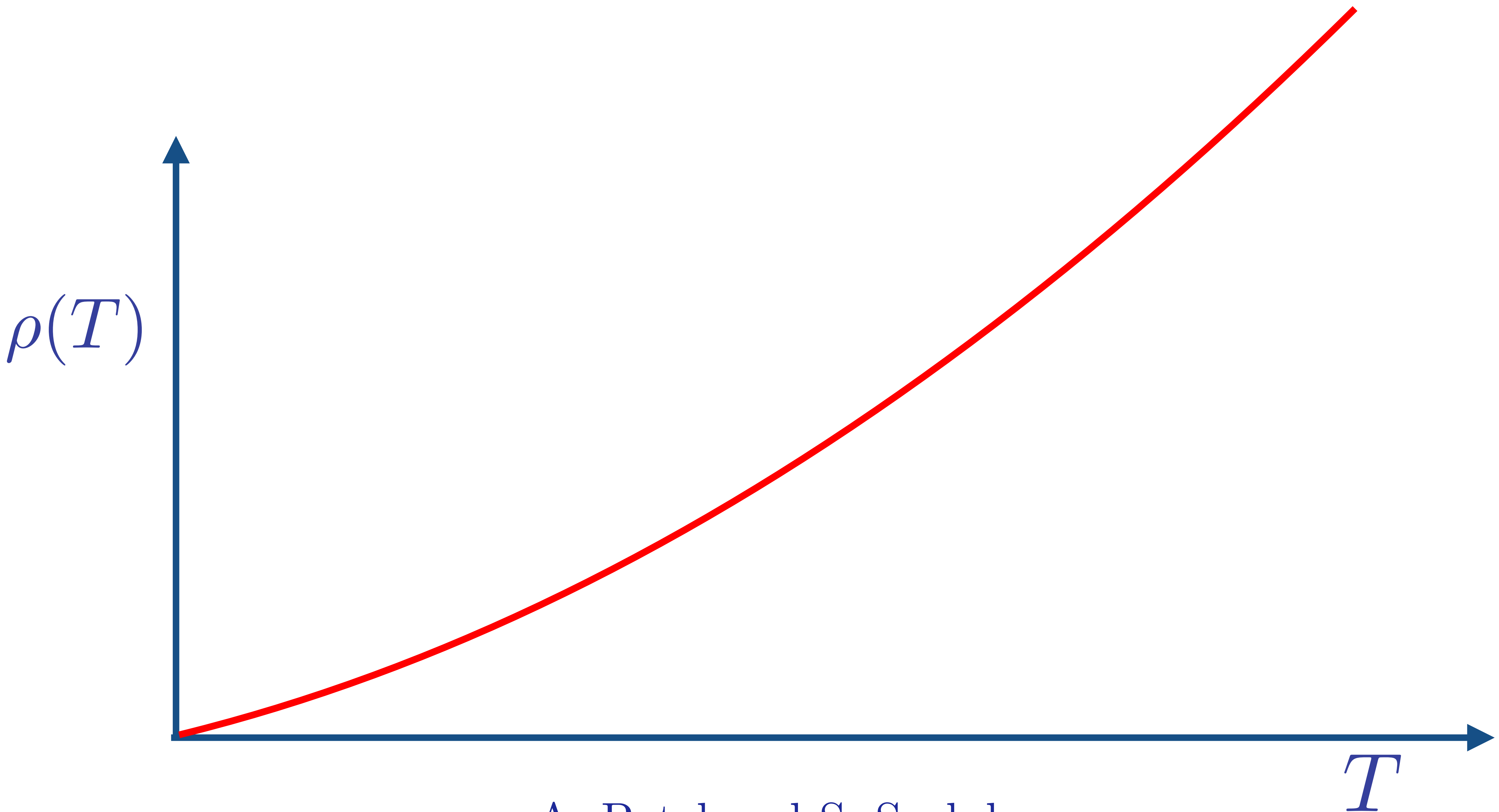

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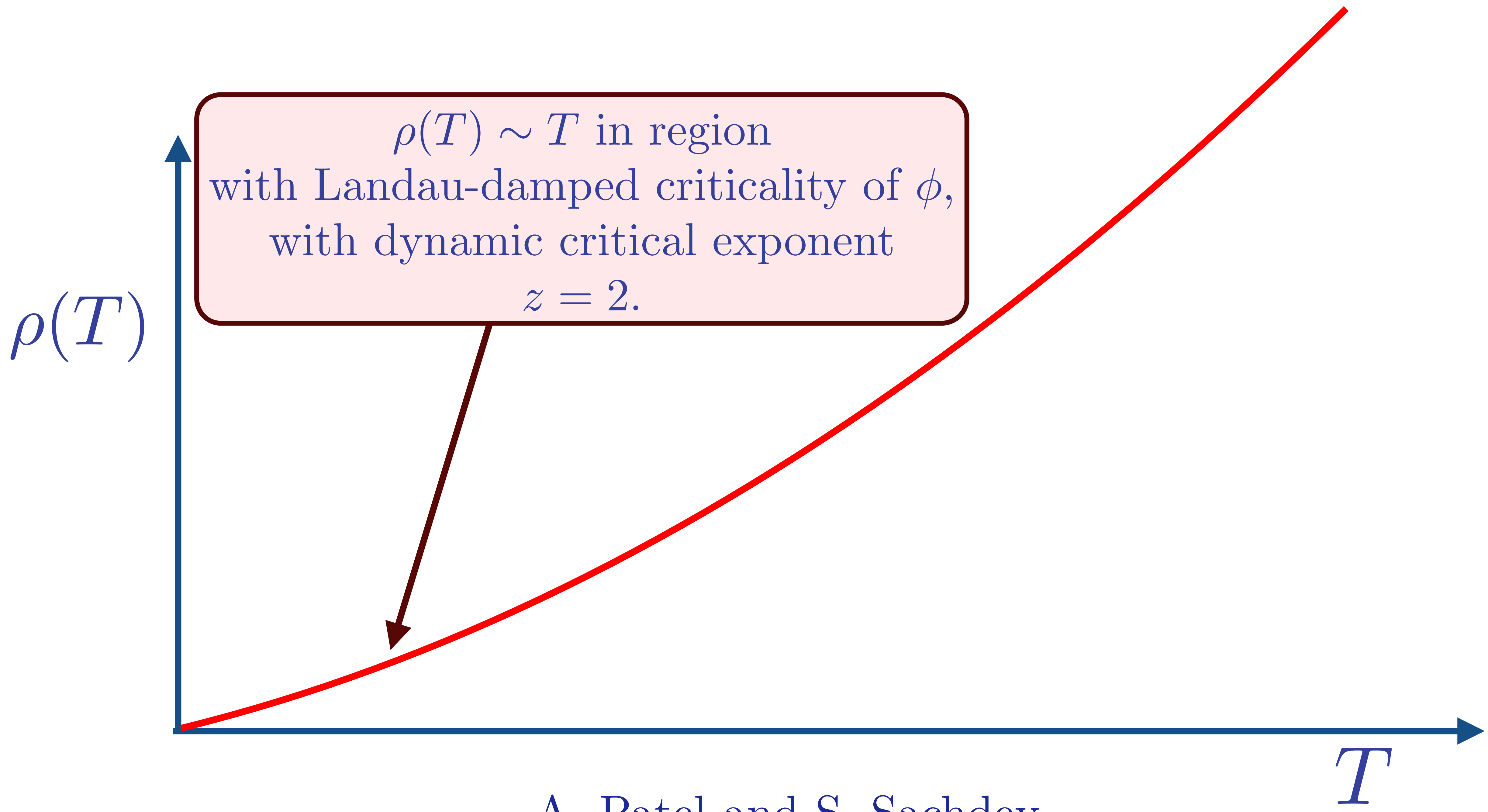

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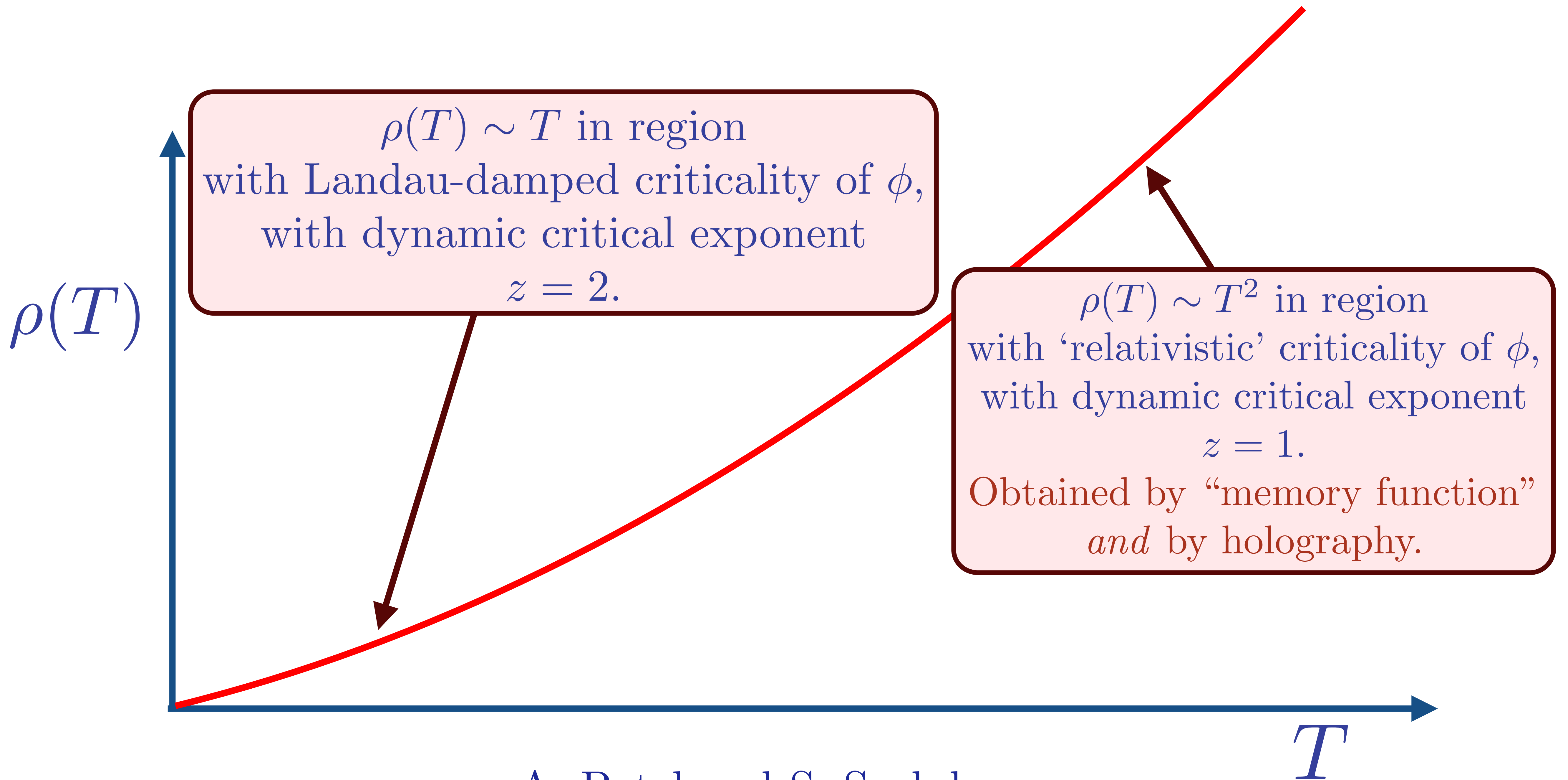

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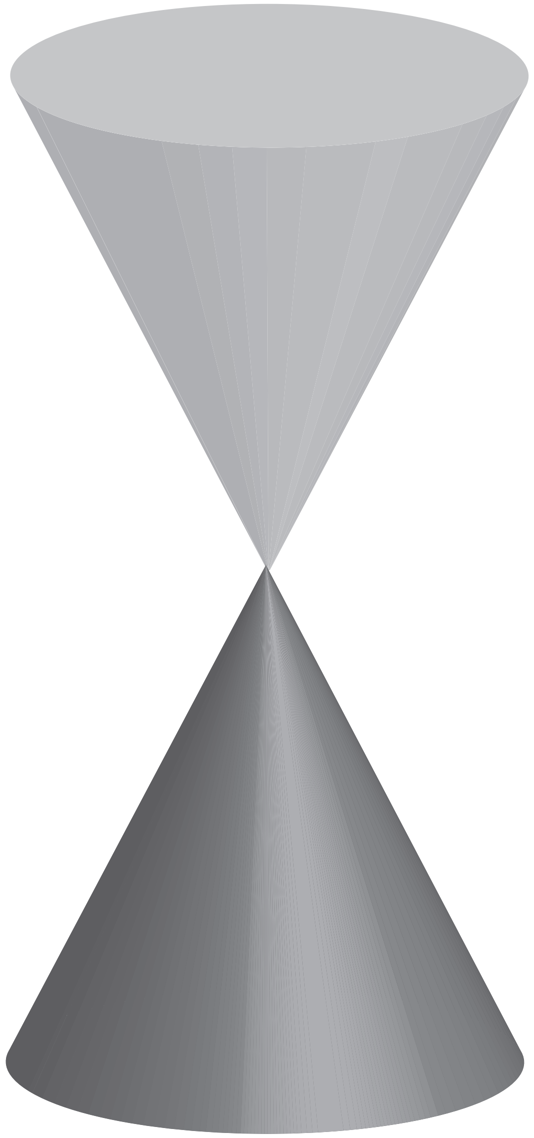
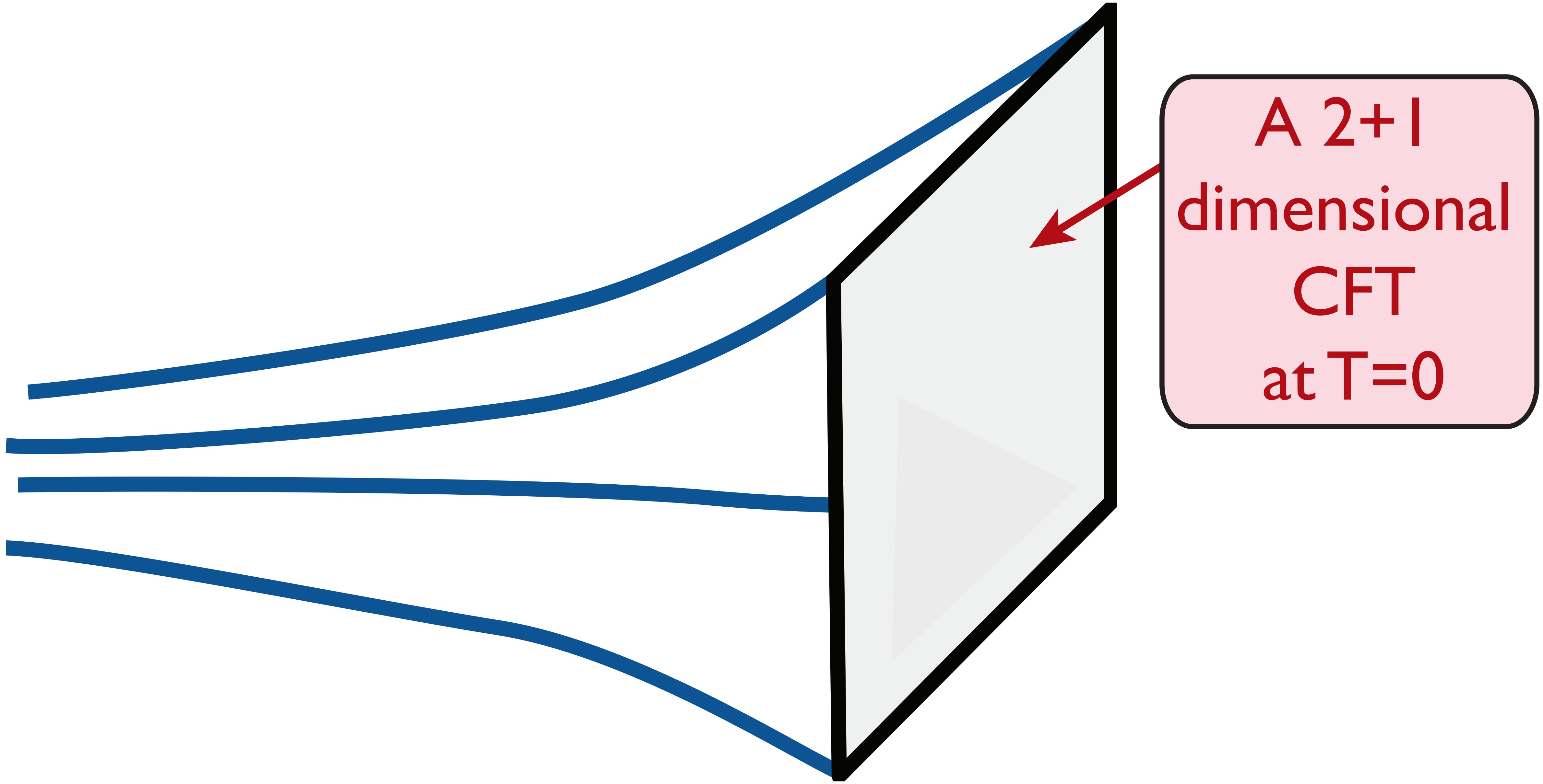
Begin with a CFT

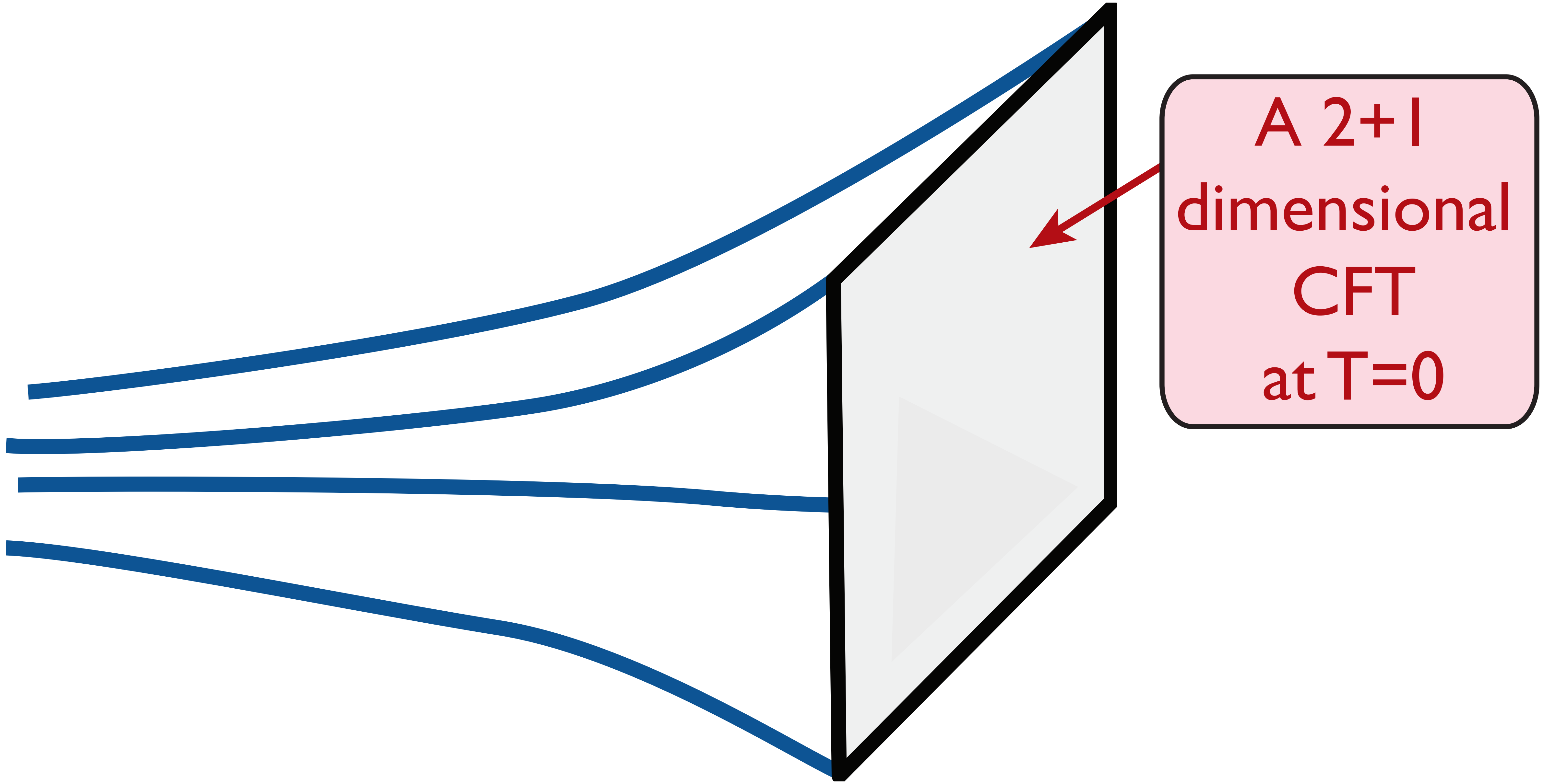


Holographic representation: AdS_4



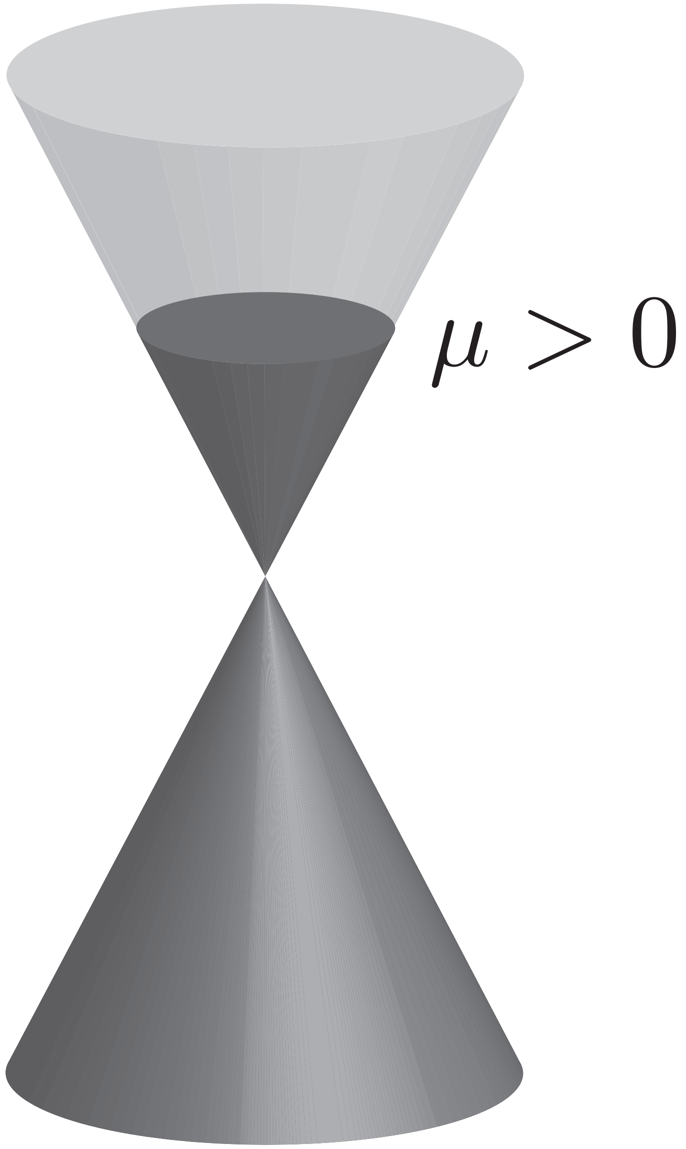
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Holographic representation: AdS$_4$

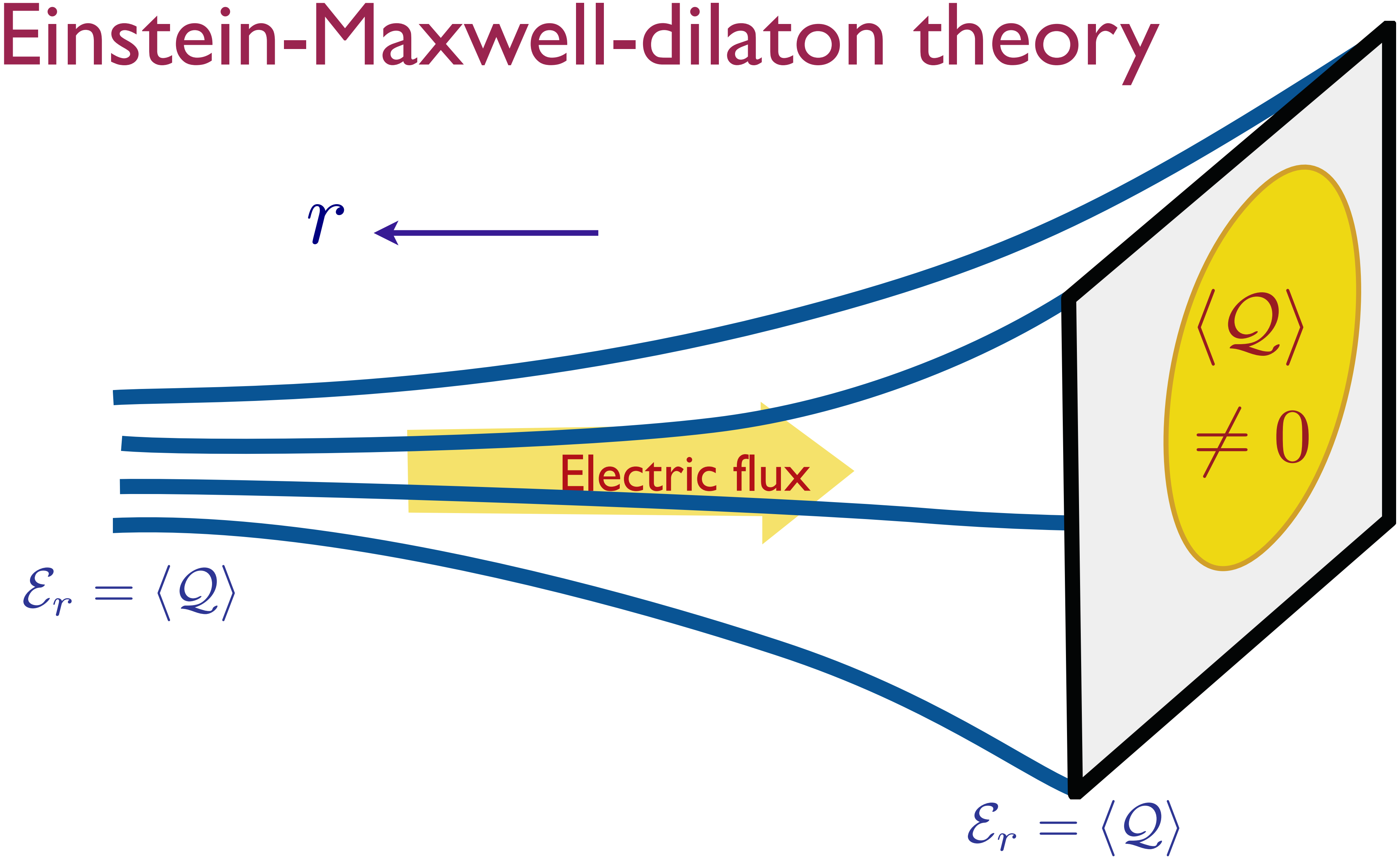


$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$
with $f(r) = 1$

Apply a chemical potential



Einstein-Maxwell-dilaton theory



$$\mathcal{S} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2}\right) - \frac{Z(\Phi)}{4e^2}F_{ab}F^{ab}\right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holography of a non-Fermi liquid

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The $r \to \infty$ limit of the metric of the Einstein-Maxwell-dilaton (EMD) theory has the most general form with

$$\begin{aligned}\theta &= \frac{d^2\beta}{\alpha + (d-1)\beta} \\ z &= 1 + \frac{\theta}{d} + \frac{8(d(d-\theta)+\theta)^2}{d^2(d-\theta)\alpha^2}.\end{aligned}$$

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Computation of the entanglement entropy in the EMD theory via the Ryu-Takayanagi formula for $\theta = d - 1$ yields

$$\mathcal{S}_E = \mathcal{C}_E \mathcal{Q}^{(d-1)/d} P \ln P$$

where $\mathcal{C}_E$ is independent of UV details.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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This is precisely as expected for a Fermi surface, when combined with the Luttinger theorem!

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To relax momentum, add a 'random-field' coupling to the field operator $\mathcal{O}$:

$$\mathcal{S} \to \mathcal{S} + \int d^d r d\tau h(r)\mathcal{O}(r,\tau) \quad \text{with } \overline{h(r)} = 0 \text{ and } \overline{h(r)h(r')} = h_0^2\delta^d(r - r')$$

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Solution of Einstein-Maxwell equations for small h_0 yields the resistivity

$$\rho(T) \sim h_0^2\, T^{(d-z+\eta)/z},$$

where $\dim[\mathcal{O}] = (d + z - 2 + \eta)/2$. This agrees with the *memory function* computation of the bosonic contribution of the "standard model" field theory. The crossover at higher energies to the Wilson-Fisher CFT (with $z = 1$, $\eta \approx 0$) yields $\rho(T) \sim T$.

A.Lucas, S. Sachdev, and K. Schalm `arXiv:1401.7933`

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- Thermodynamic/entanglement/transport (but not Fermi surface) properties of strongly-coupled metals can be modeled by charged black hole holographic duals.