

Unveiling the order of the high temperature superconductors

Stanford University
April 15, 2014

Subir Sachdev

Talk online: sachdev.physics.harvard.edu



Theorists at Harvard



Max Metlitski



Rolando
La Placa



Andrea Allais



Johannes
Bauer



Debanjan
Chowdhury



Jay Deep
Sau

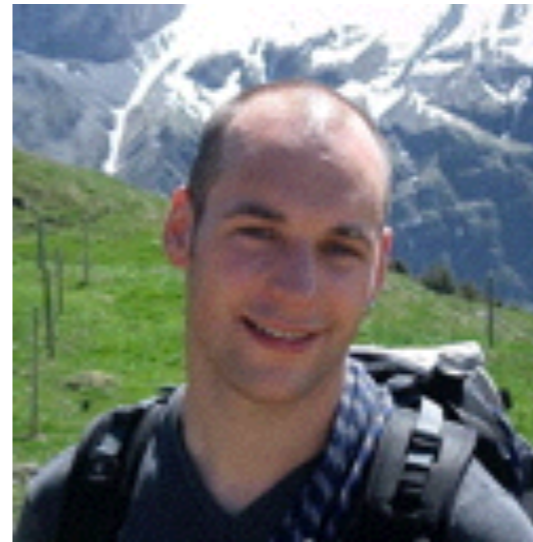
Experimentalists at Harvard



Yang He



Yi Yin



Martin Zech



Anjan
Soumyanarayanan



Ilija Zelkovic



Michael Yee



Jennifer Hoffman

Experimentalists at Cornell



Kazuhiro Fujita
Cornell/ BNL



Mohammad Hamidian
Cornell / BNL

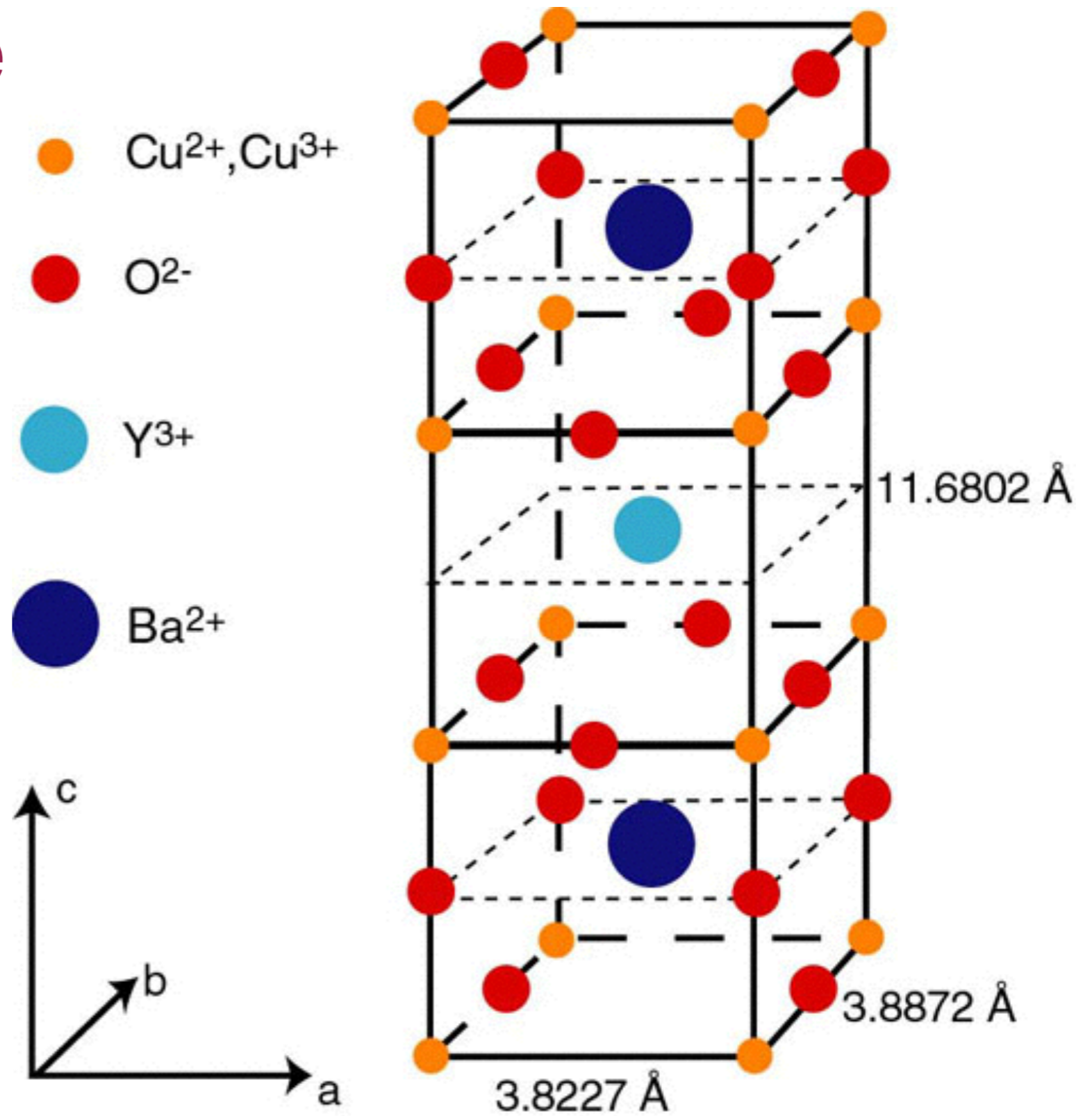


Stephen Edkins
Cornell / St Andrews

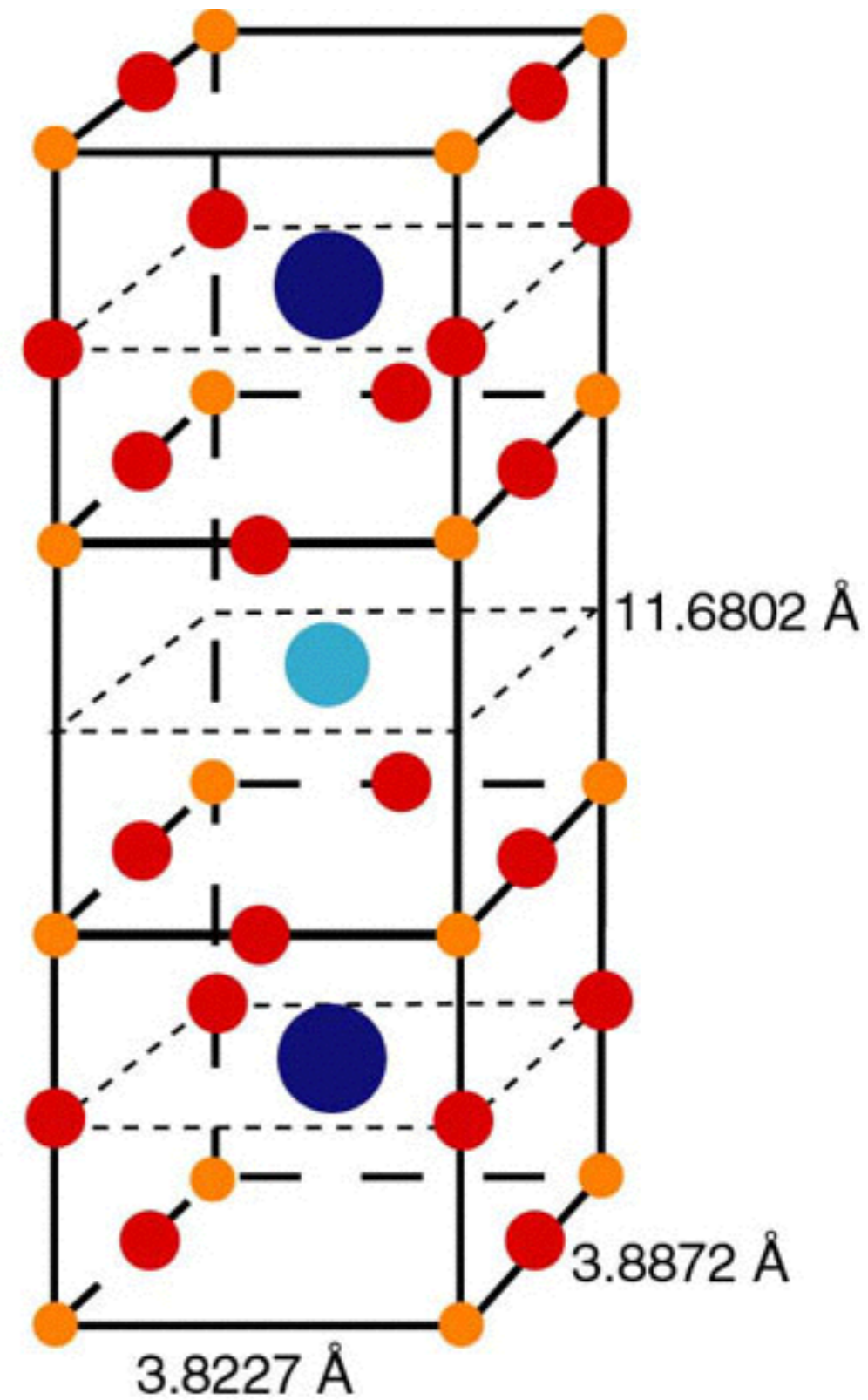
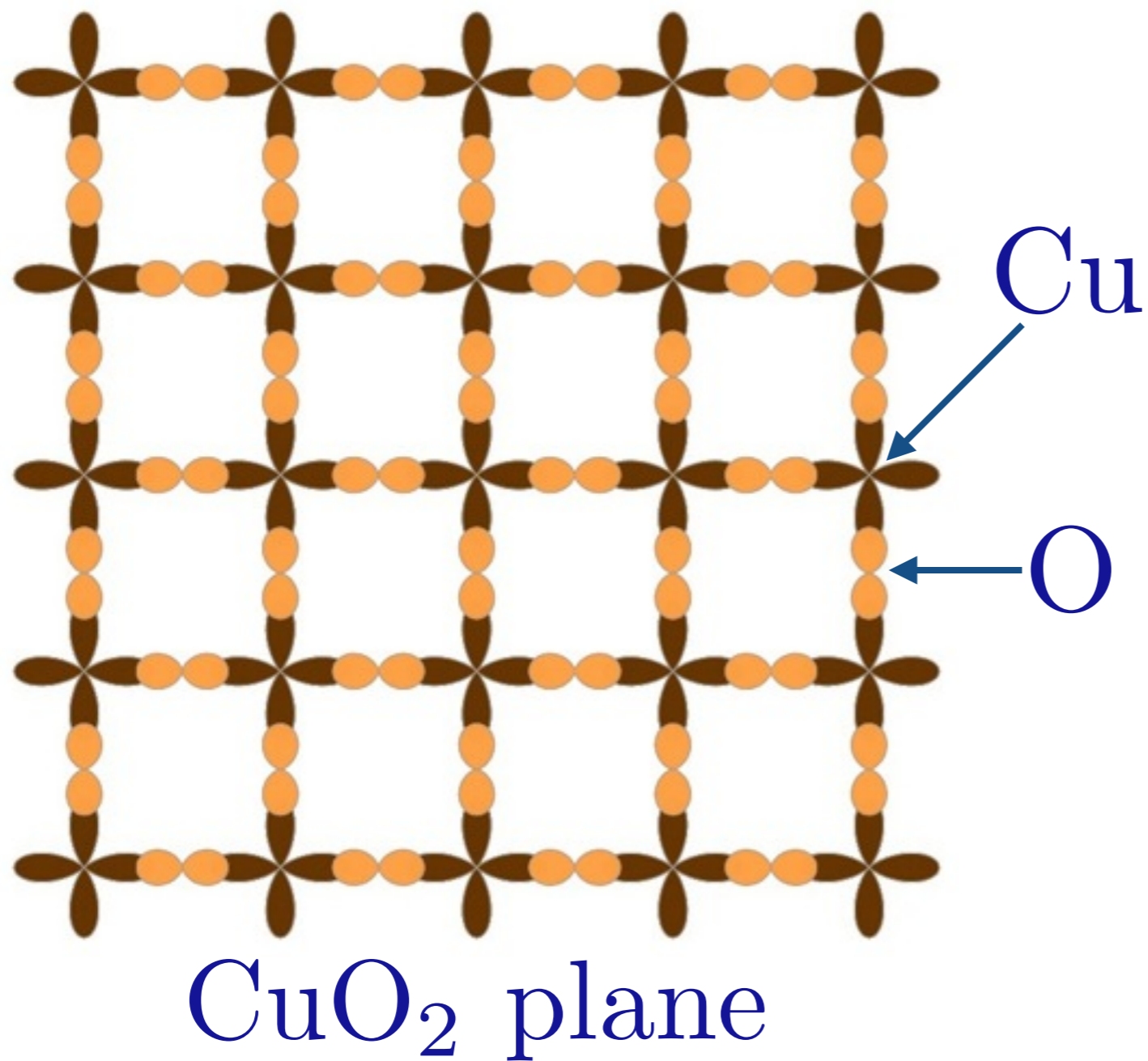


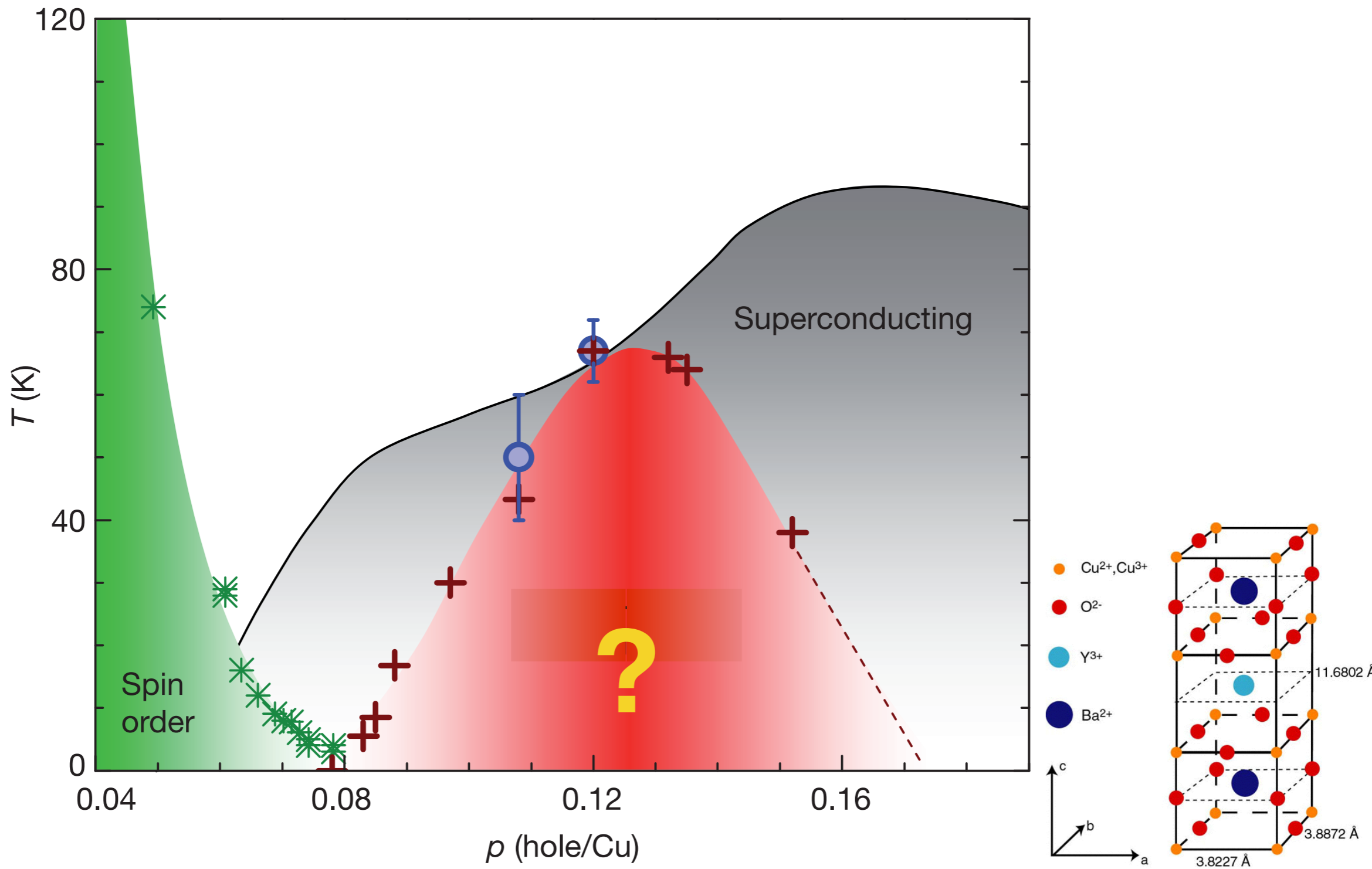
J. C. Seamus Davis

High temperature superconductors

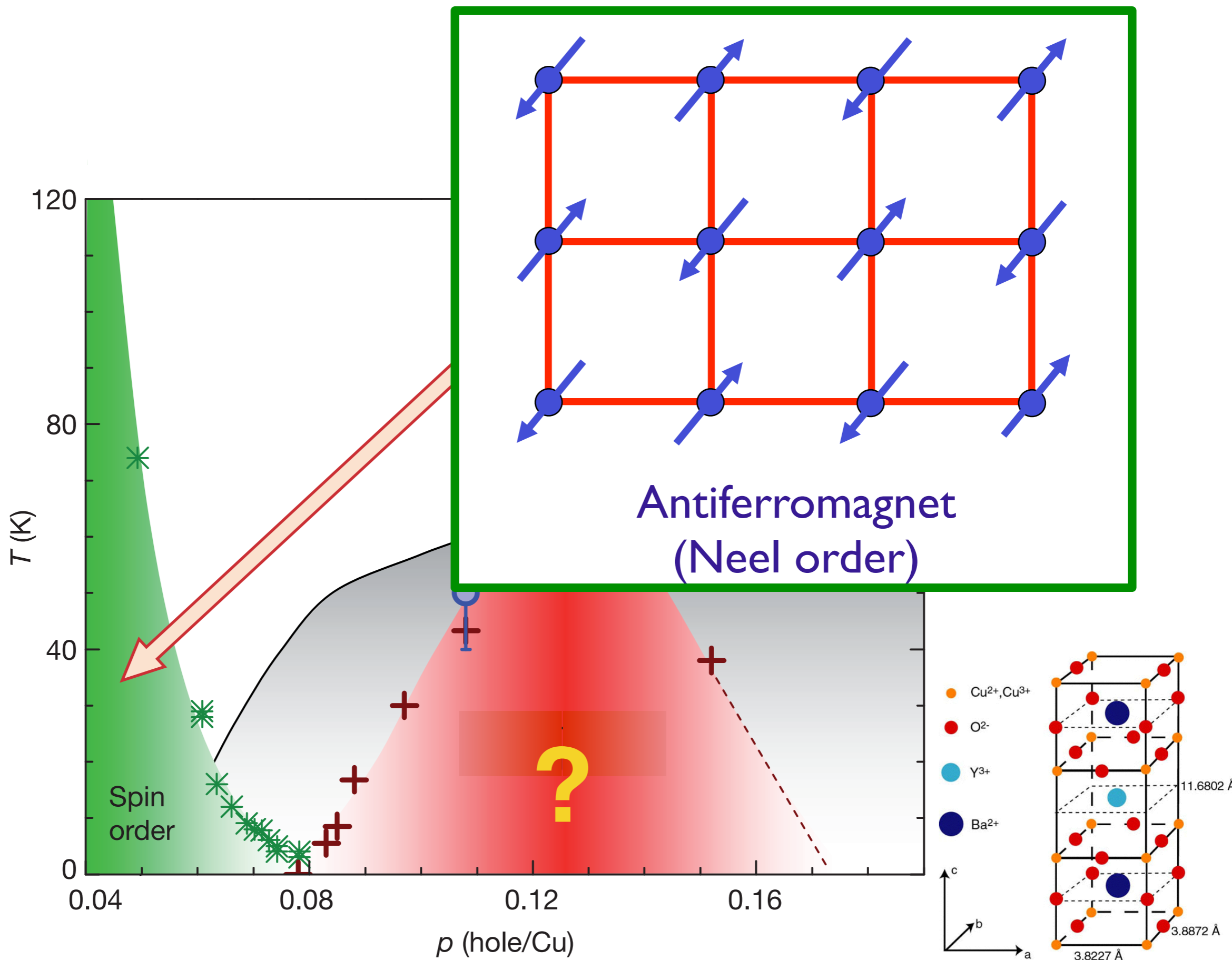


High temperature superconductors

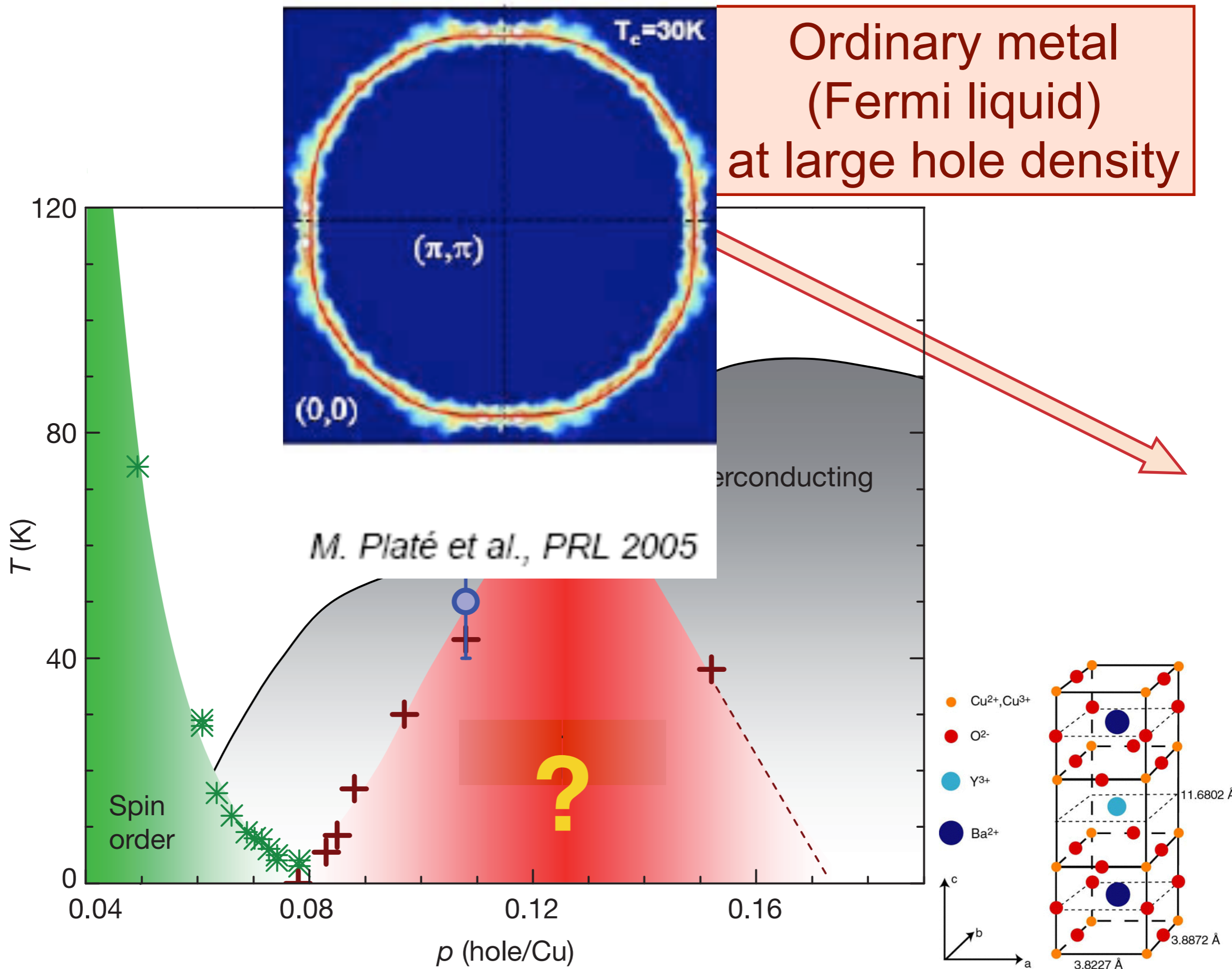




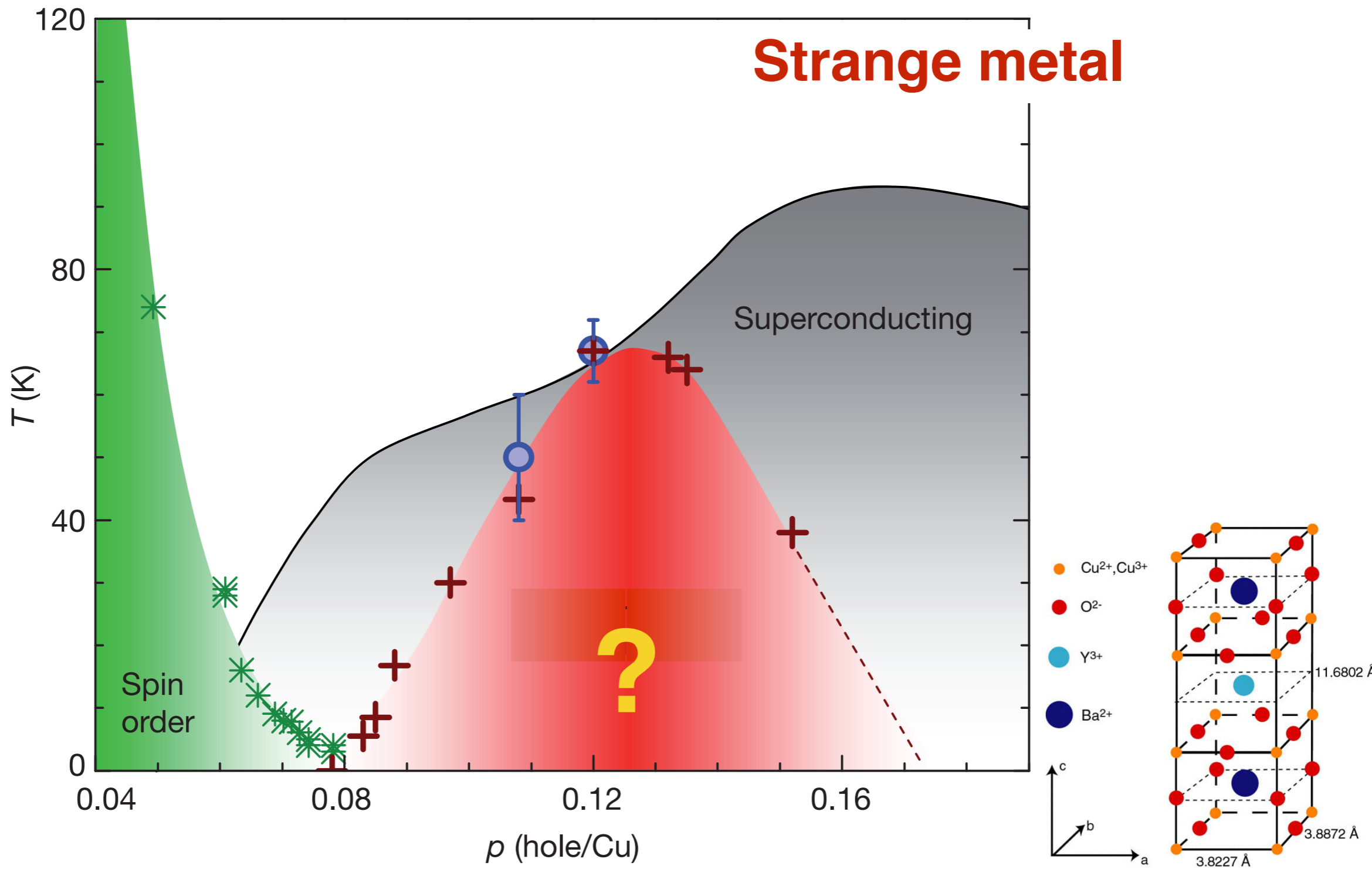
T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



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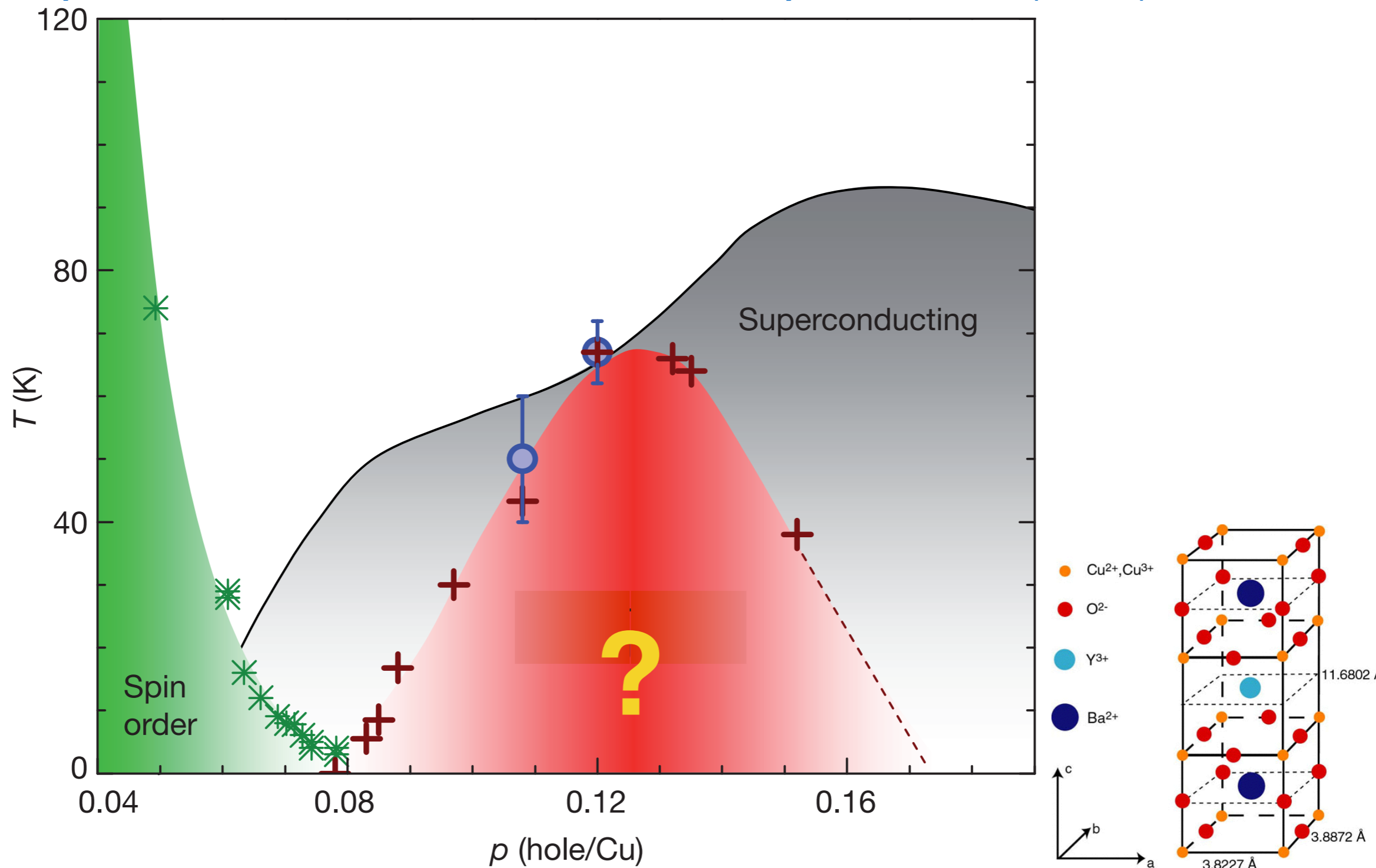


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In the La-based superconductors, the region with magnetic order overlaps with the red region - I will not discuss these materials: see S.A. Kivelson, I.P. Bindloss, E. Fradkin, V. Oganesyan, J. M. Tranquada, A. Kapitulnik, and C. Howald, *Rev. Mod. Phys.* **75**, 1201 (2003)



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Outline

1. *d*-wave superconductivity
2. Low hole density state:
d-wave bond order
3. Theoretical background
4. Evolution of Fermi surface

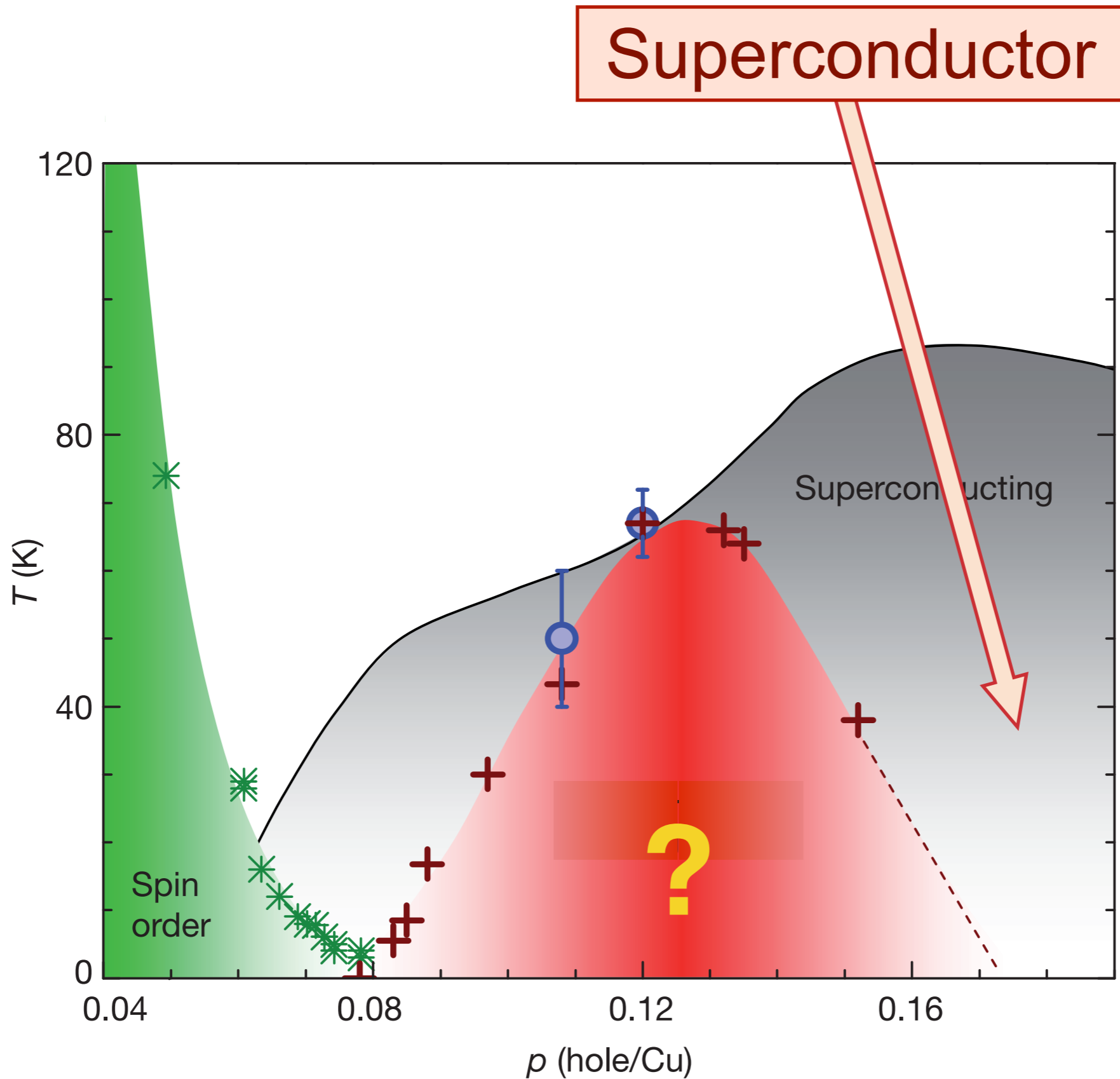
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Superconductivity: Bose condensation of Cooper pairs of electrons

$$\varepsilon^{\alpha\beta} \left\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\beta}^{\dagger}(\mathbf{r}_2) \right\rangle = \left[P(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{SC} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)$$

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

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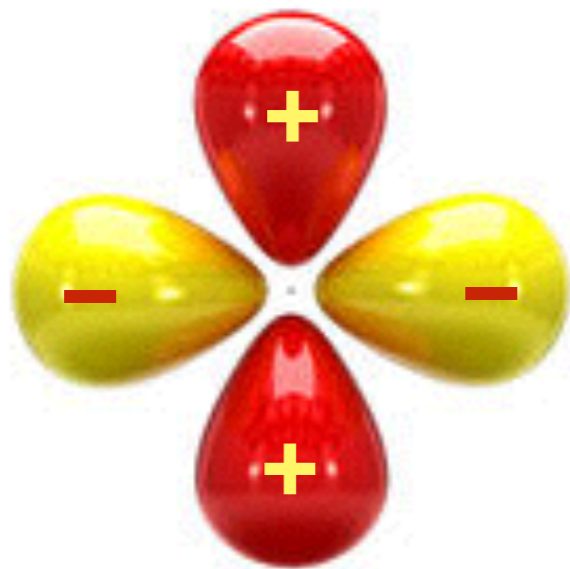
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Nearly constant condensate wavefunction
(superconducting order parameter)

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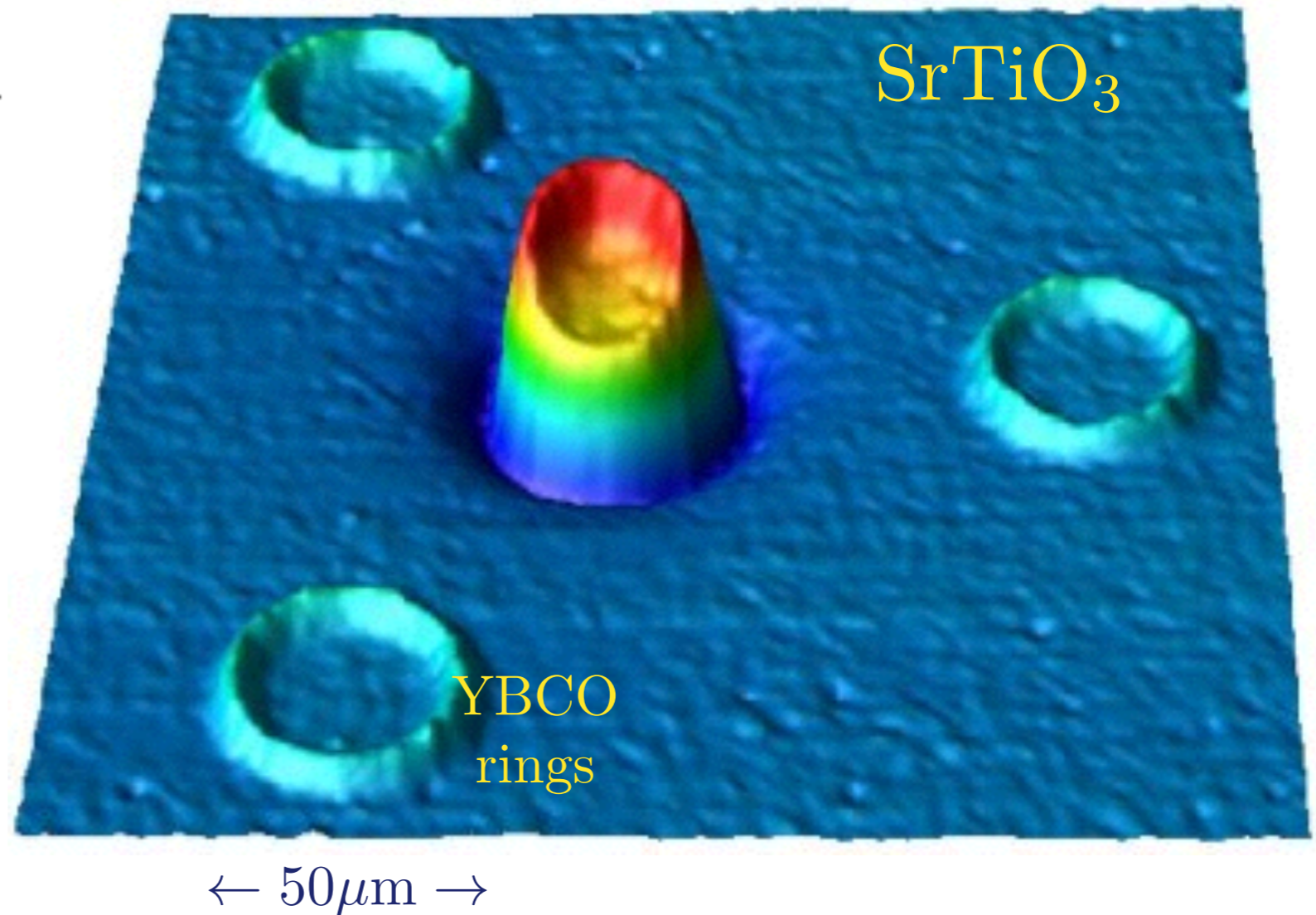
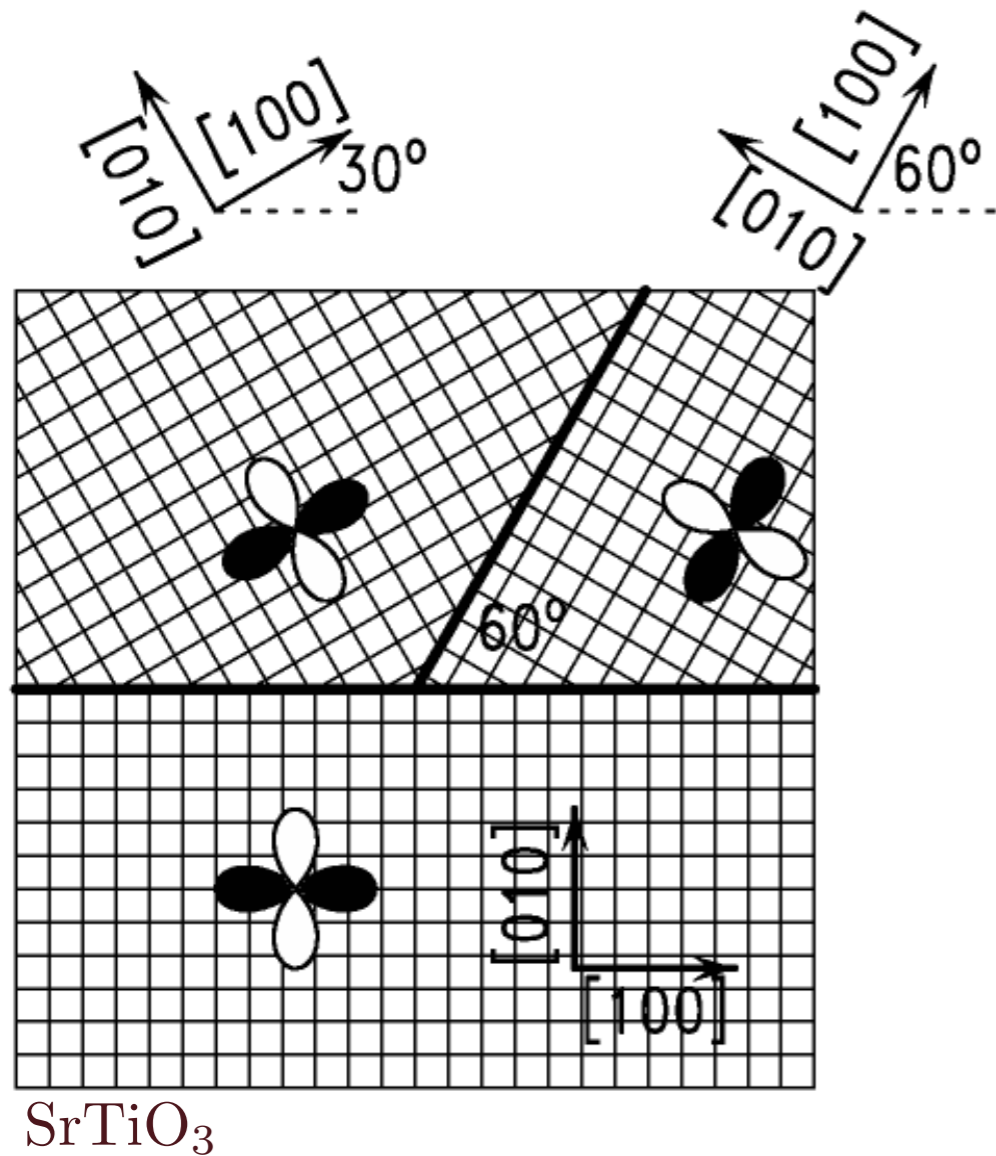
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Internal Cooper-pair wavefunction.
Has *d*-wave form in cuprates

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

Phase-sensitive measurement of the d -wave symmetry of Cooper pairs



Pairing Symmetry and Flux Quantization in a Tricrystal Superconducting Ring of YBa₂Cu₃O_{7-δ}

C. C. Tsuei, J. R. Kirtley, C. C. Chi,* Lock See Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun, and M. B. Ketchen
IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

Phys. Rev. Lett. **73**, 593 (1994)

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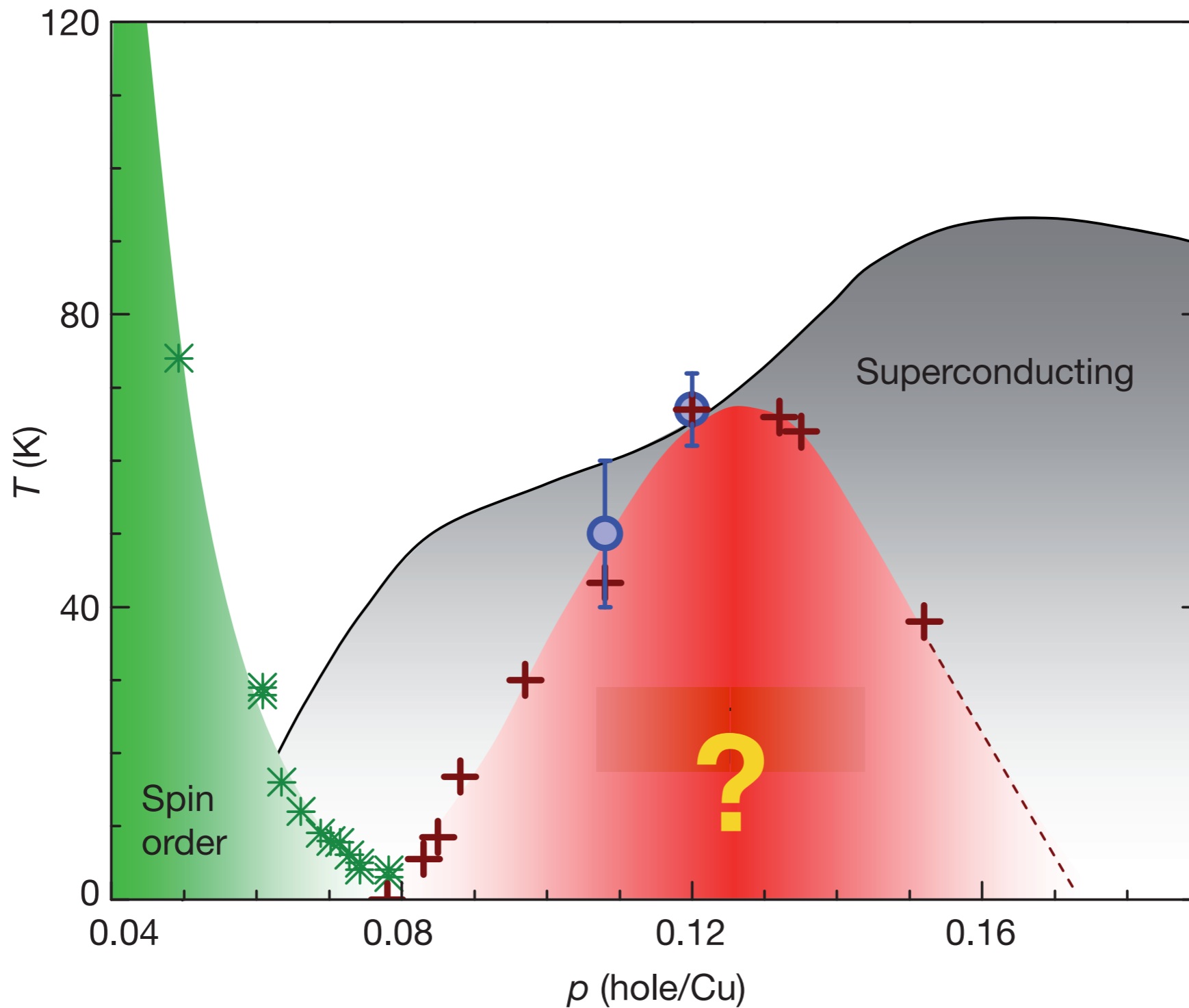
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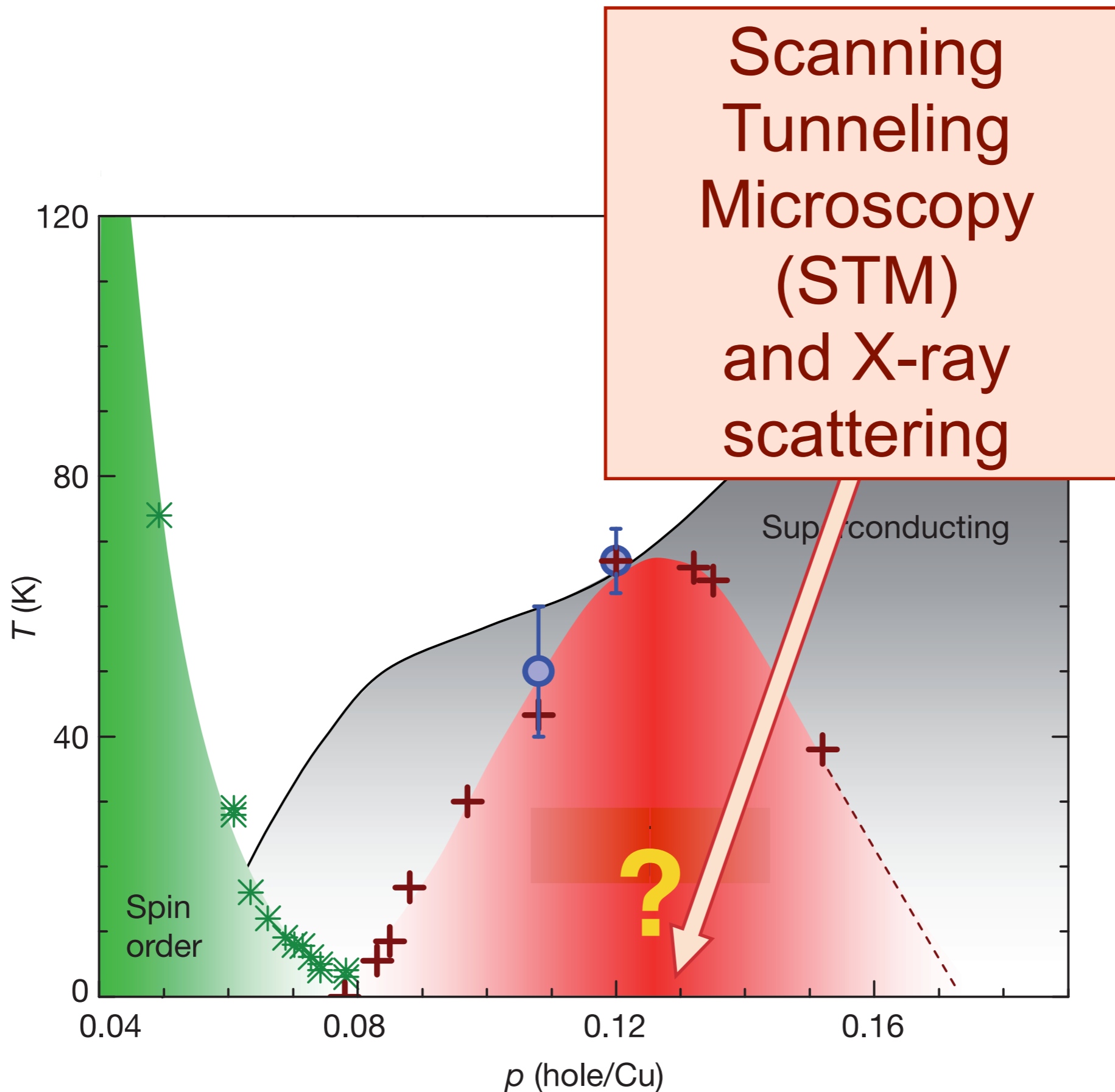
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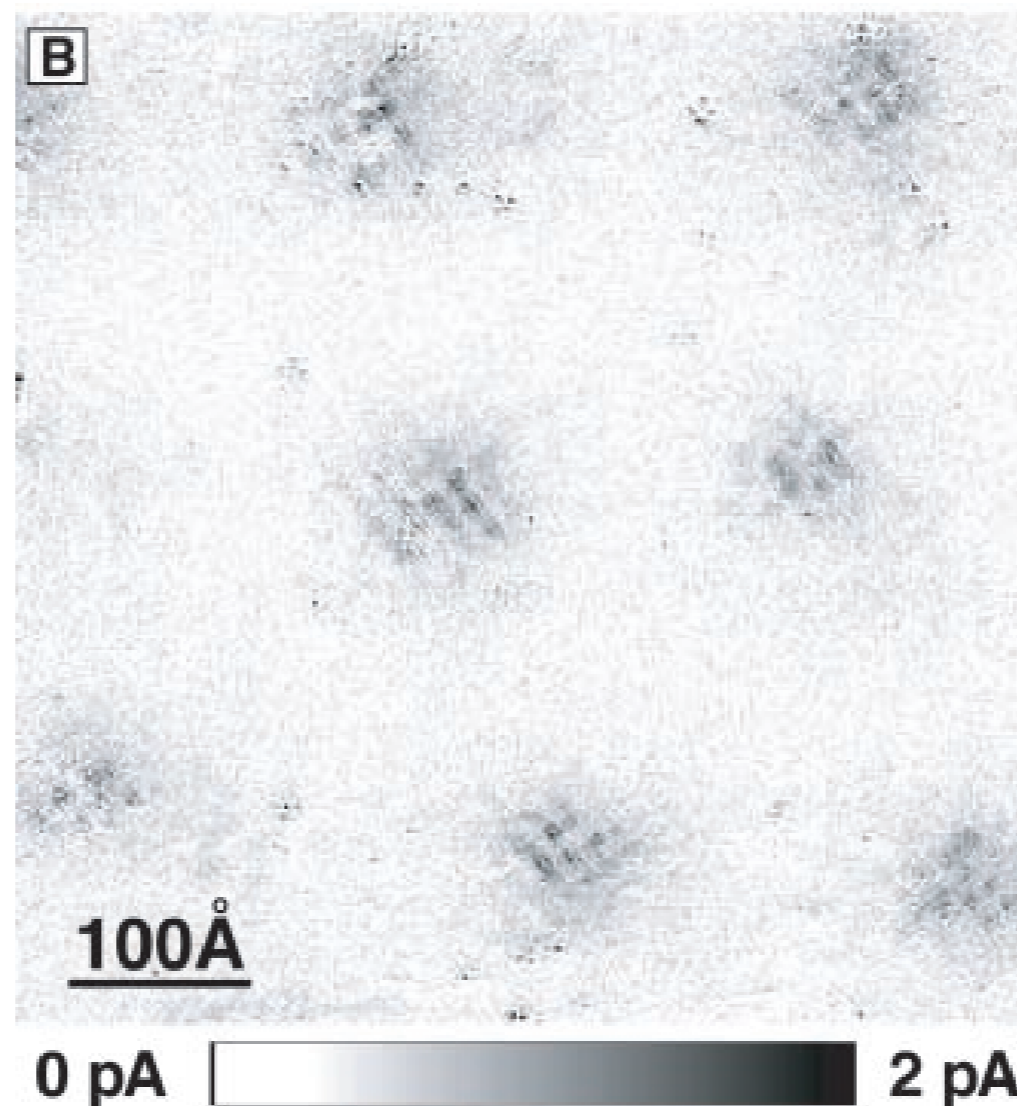
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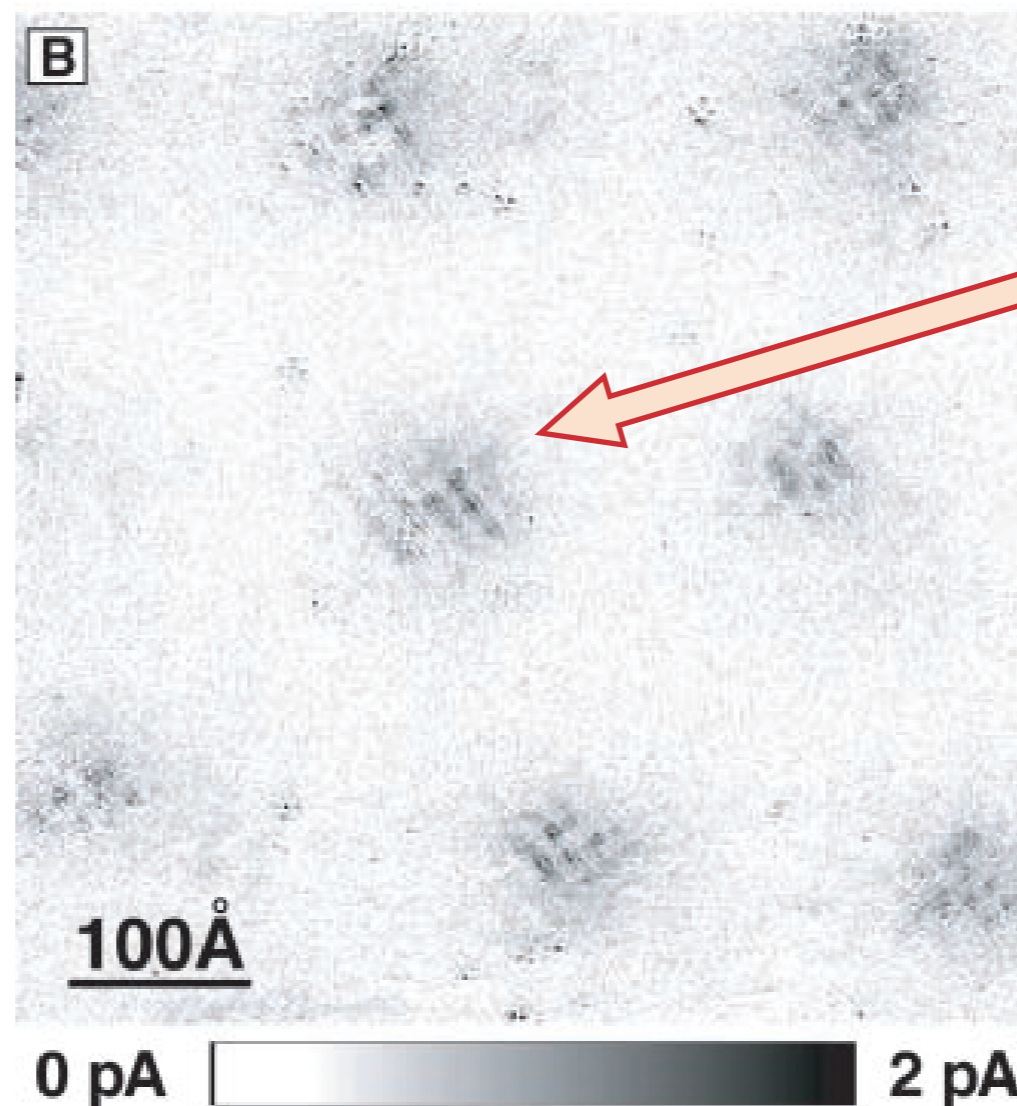
A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan,
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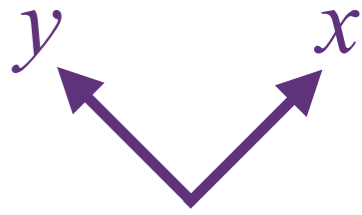
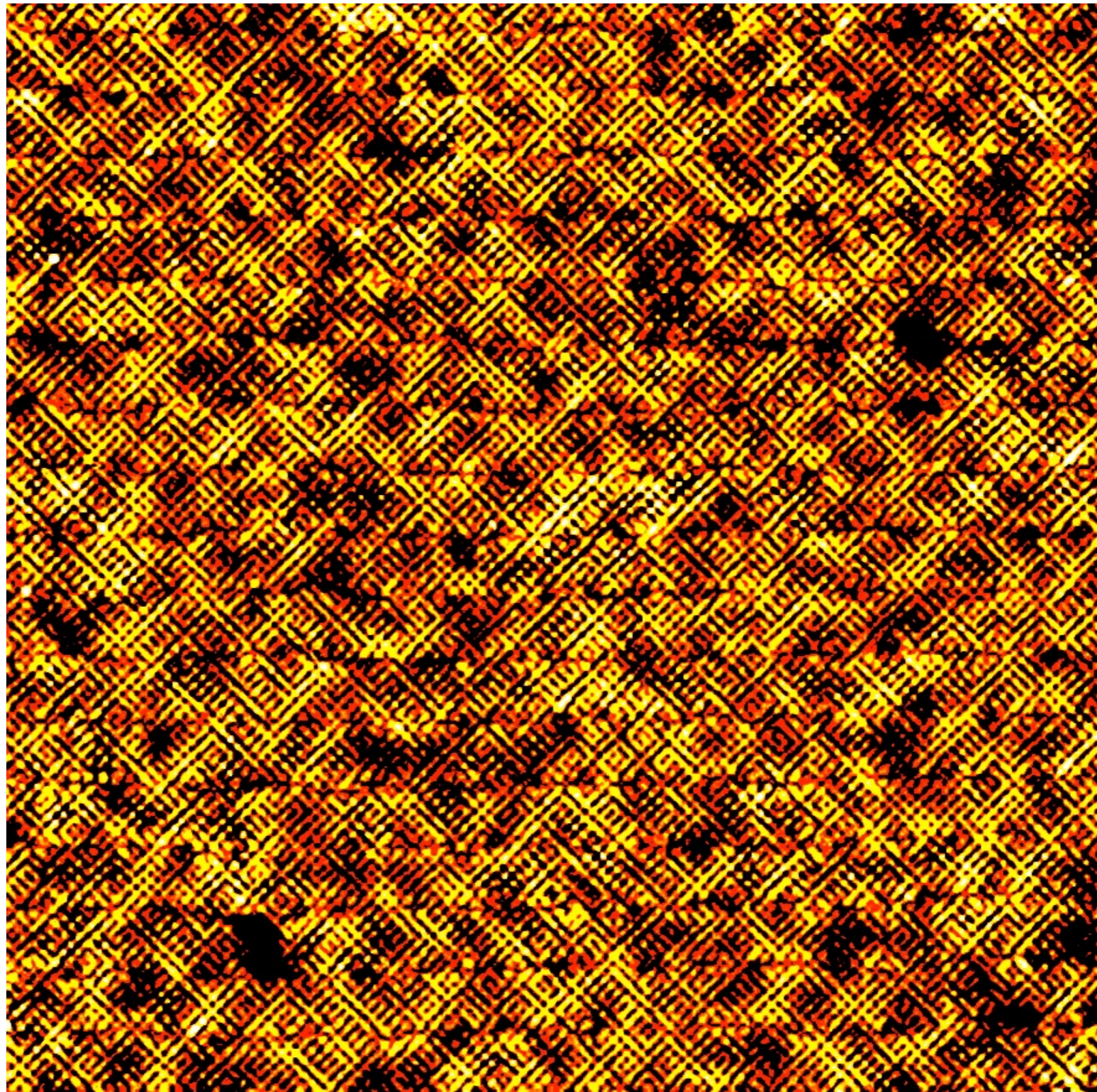
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A charge density wave (CDW) with wavelength ≈ 4 lattice spacings around vortex cores ?

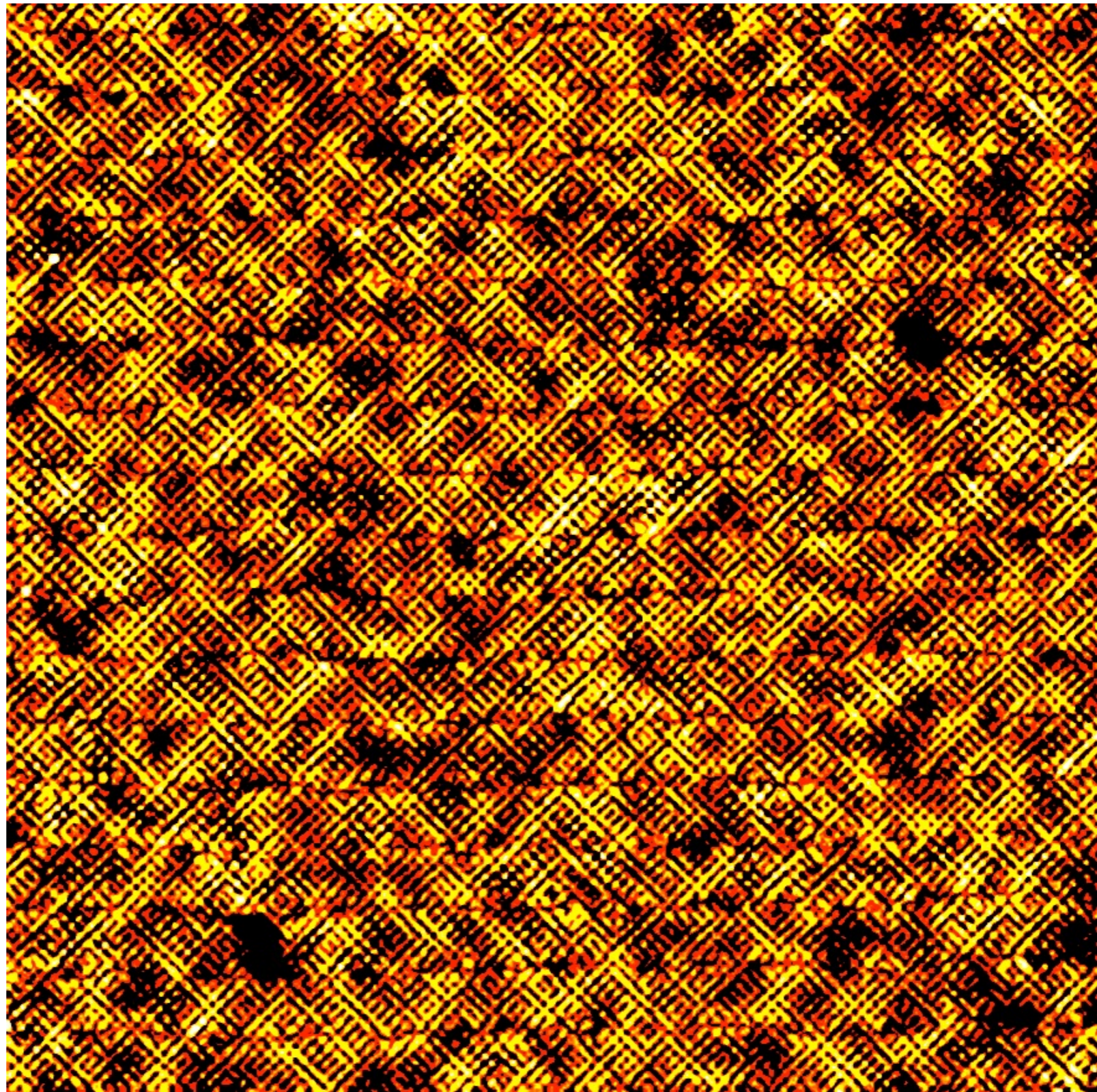
See also C. Howald,
H. Eisaki, N. Kaneko,
M. Greven, and
A. Kapitulnik, *Phys.*
Rev. B **67**, 014533
(2003).



“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007).

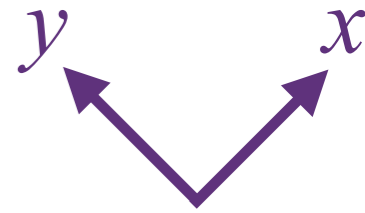
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*“Cosmic
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map of a
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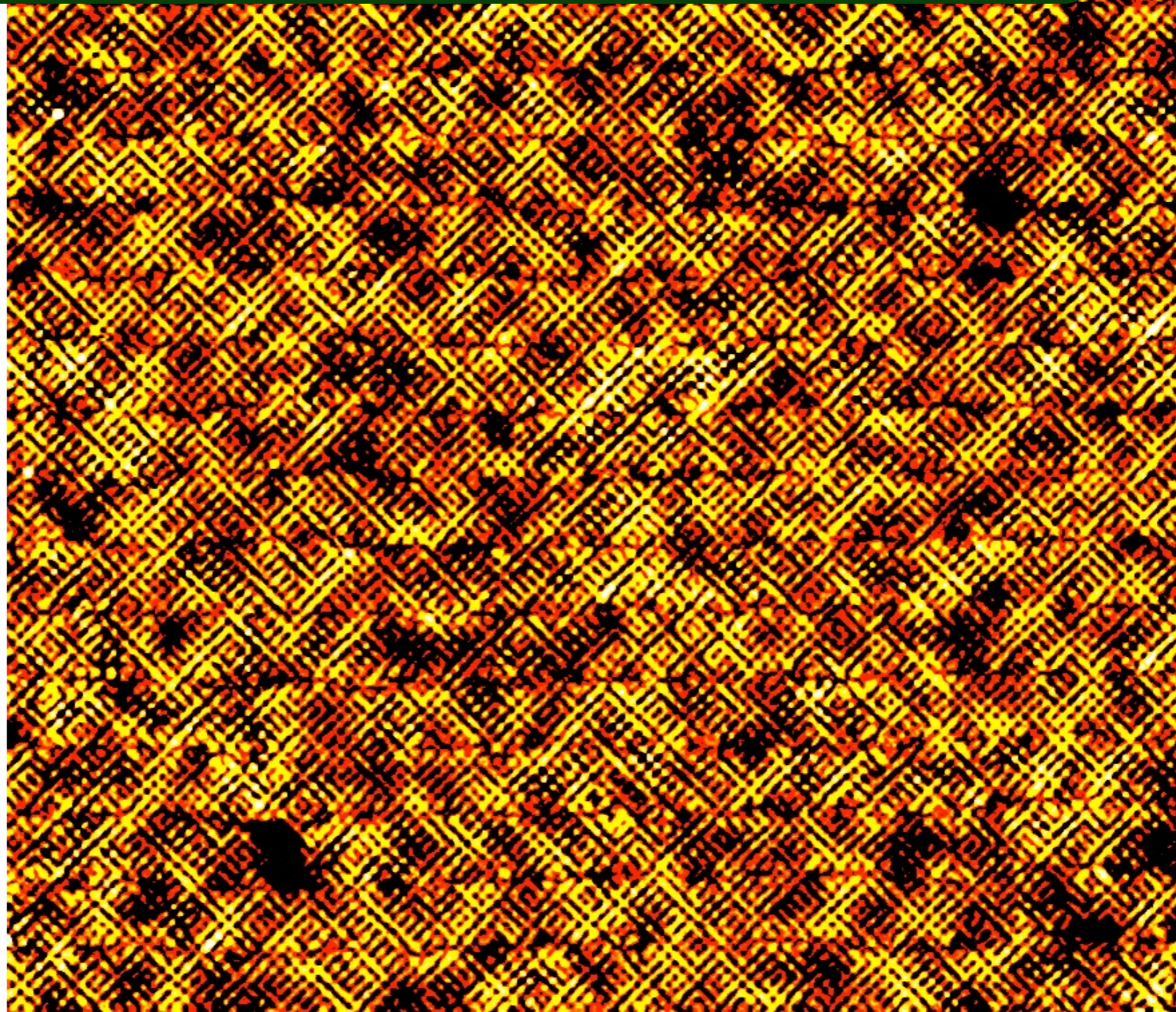
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Fourier transform yields non-dispersing peaks at
 $Q \approx 2\pi(\pm 1/4, 0)$ and $\approx 2\pi(0, \pm 1/4)$



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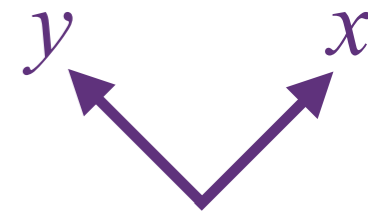
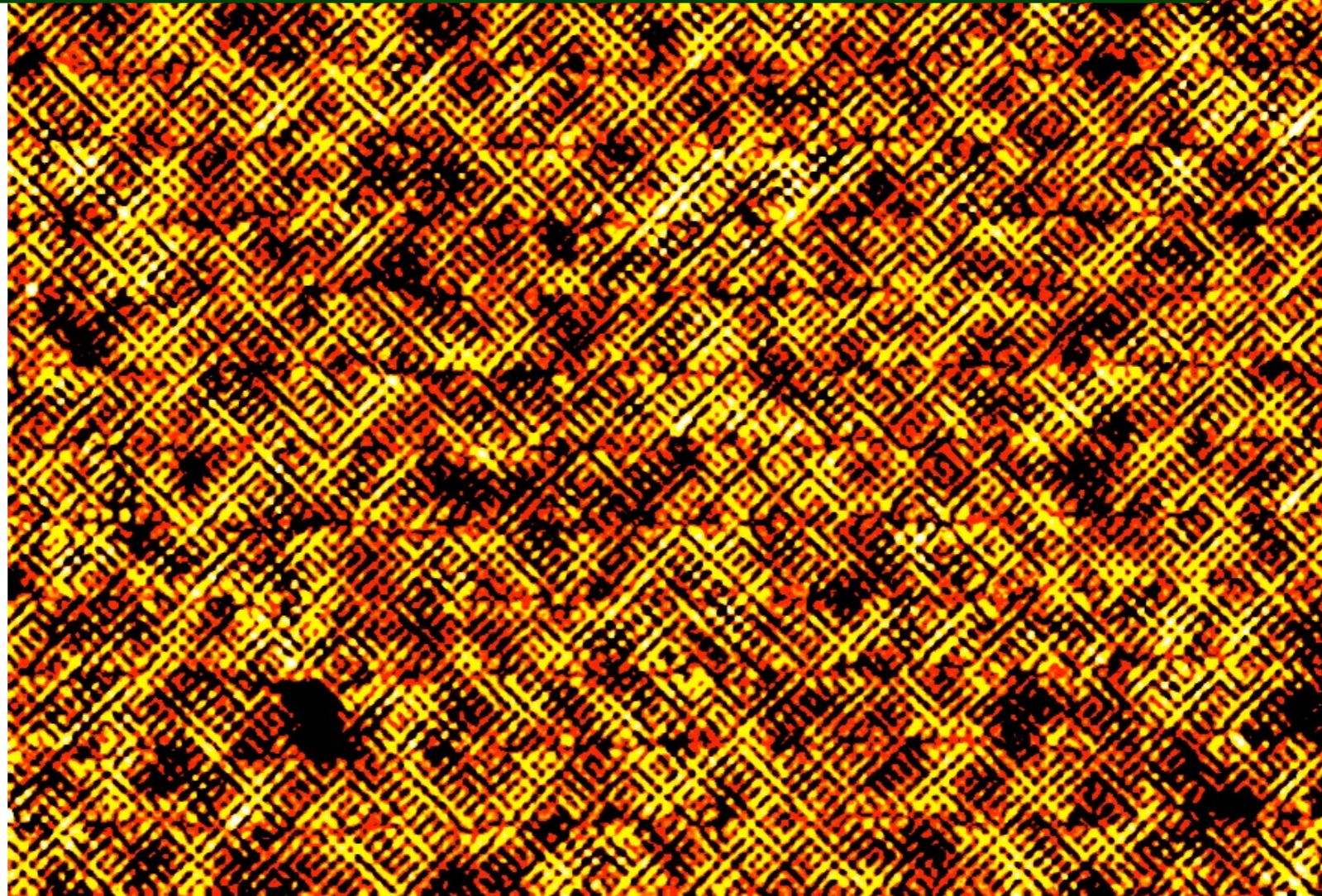
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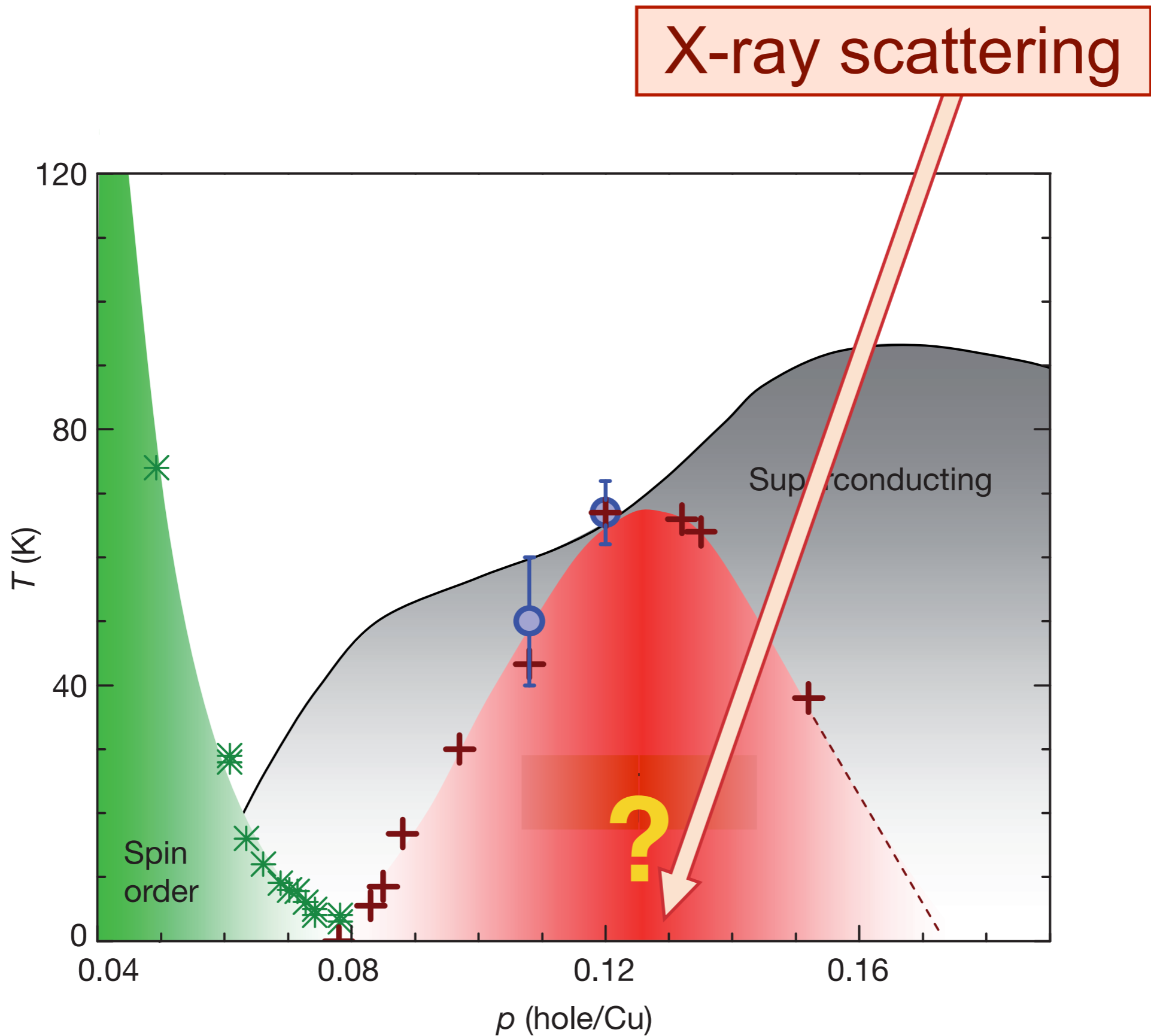
But there seems to be
much more structure in the map ...

See also C. Howald,
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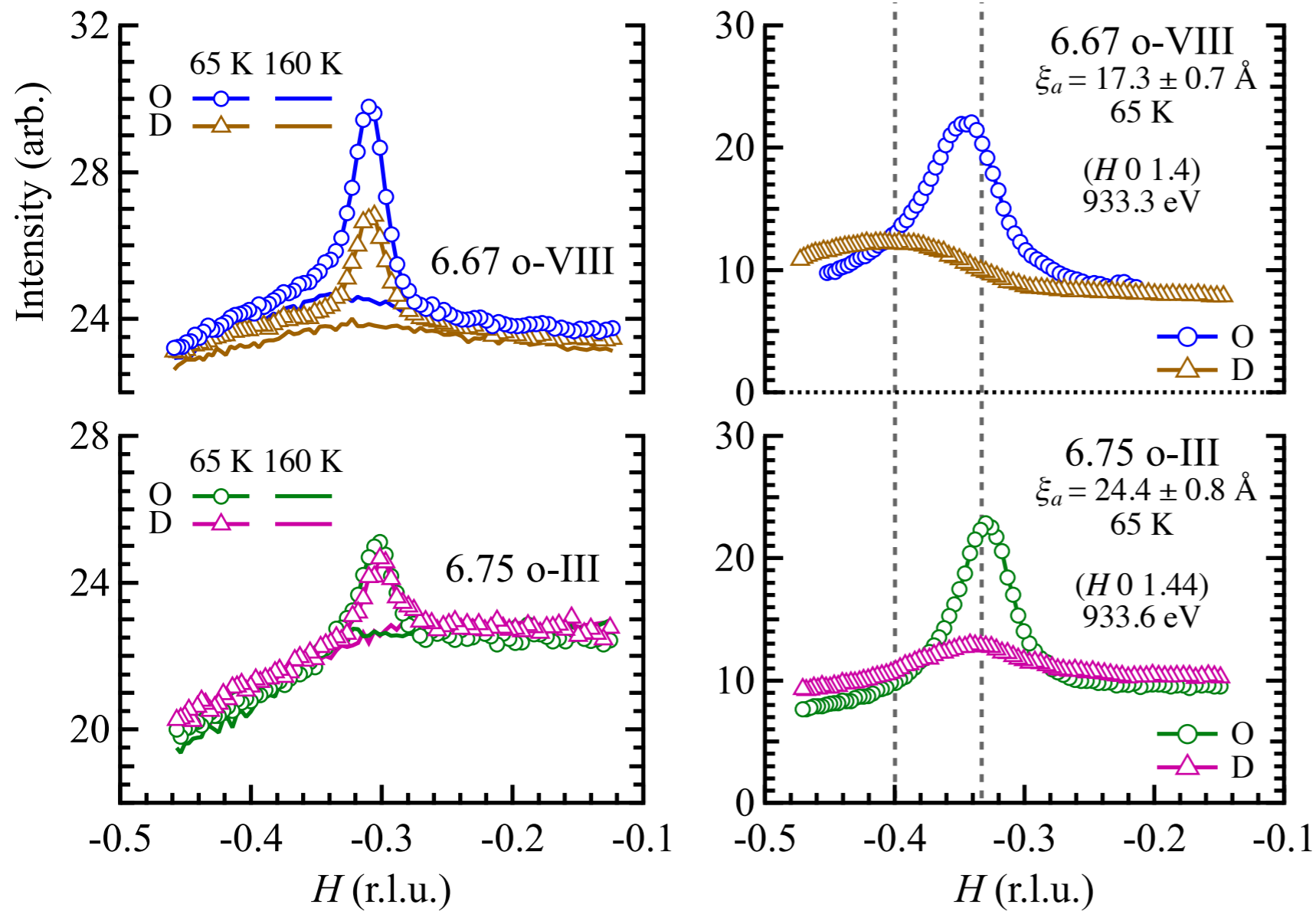
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A. J. Achkar, X. Mao, Christopher McMahon, R. Sutarto, F. He, Ruixing Liang, D. A. Bonn, W. N. Hardy, and D. G. Hawthorn, arXiv:1312.6630

Many groups have observed CDW peaks in X-ray scattering over the past 2 years.

The X-ray wavevector is found to agree with the wavevector obtained from STM. R. Comin, A. Frano, M. M. Yee, Y. Yoshida, H. Eisaki, E. Schierle, E. Weschke, R. Sutarto, F. He, A. Soumyanarayanan, Yang He, M. Le Tacon, I.S. Elfimov, J. E. Hoffman, G.A. Sawatzky, B. Keimer, A. Damascelli, Science **343**, 390 (2014).

Charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r})c_{\alpha}(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

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CDW wavevector \mathbf{Q}

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Nearly constant CDW order parameter

Generalized charge density wave (CDW) : Bose condensation of particle-hole pairs

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{CDW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$



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Internal particle-hole pair wavefunction $\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y) \dots$$

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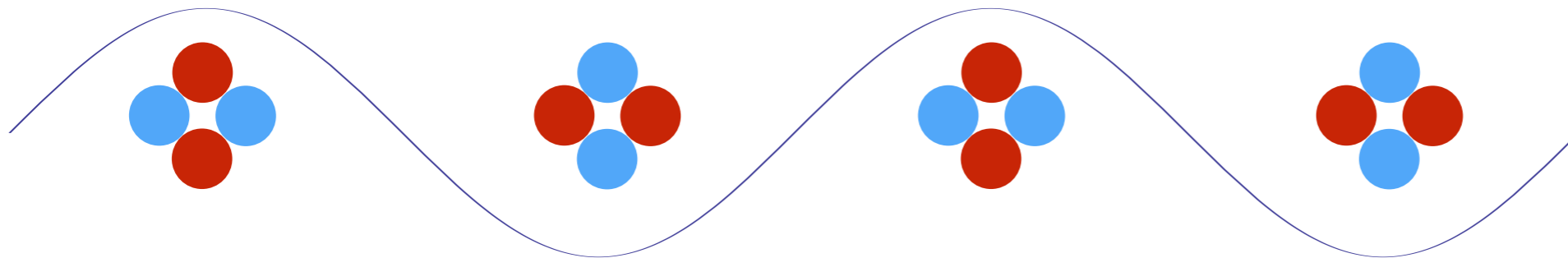
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A CDW with $\mathcal{P}_d \neq 0$ is nearly degenerate with SC near the quantum-critical point for antiferromagnetism in a metal.

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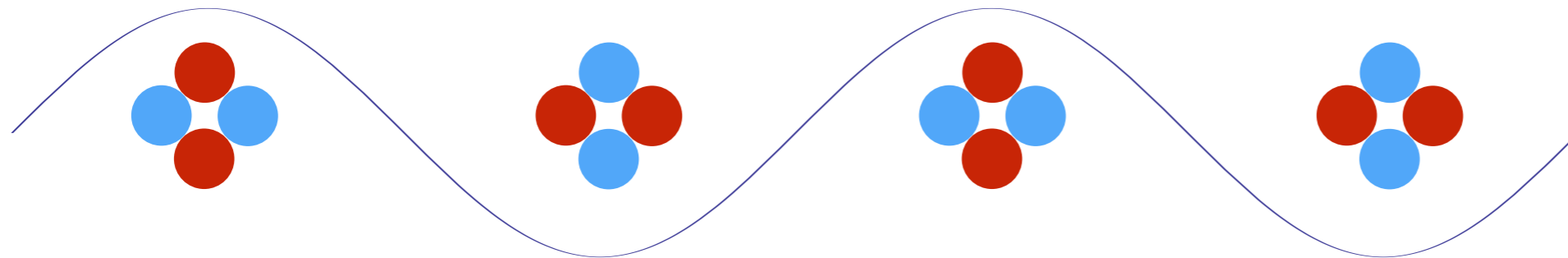


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M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010)

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(Adapted from a picture of a gravity wave)



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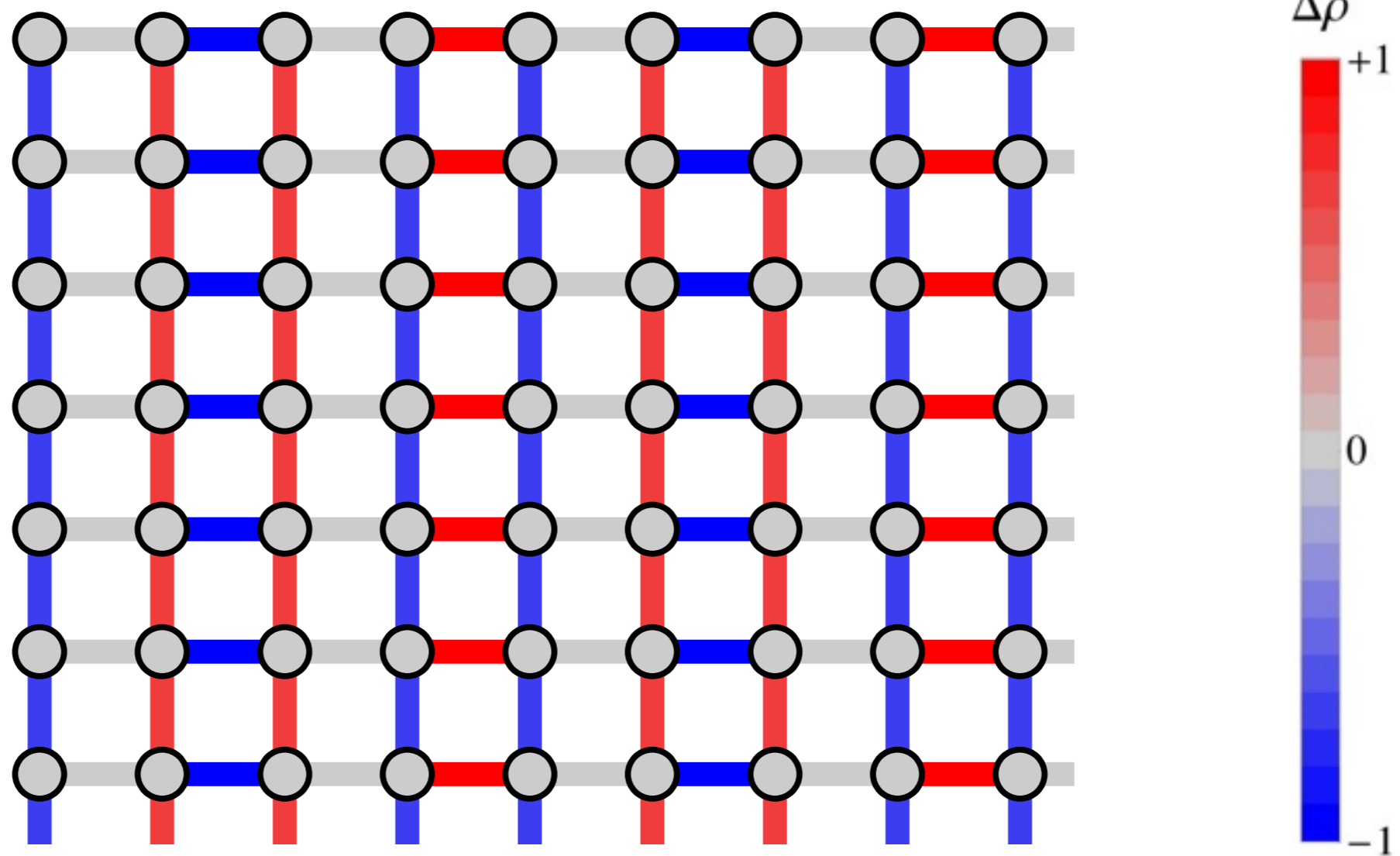
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010)

d -wave bond order.

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = (\pi/2, 0)$$



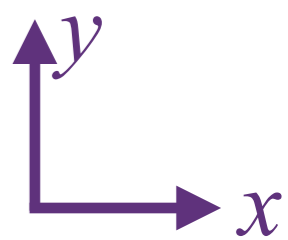
This d -wave bond order was first discussed in
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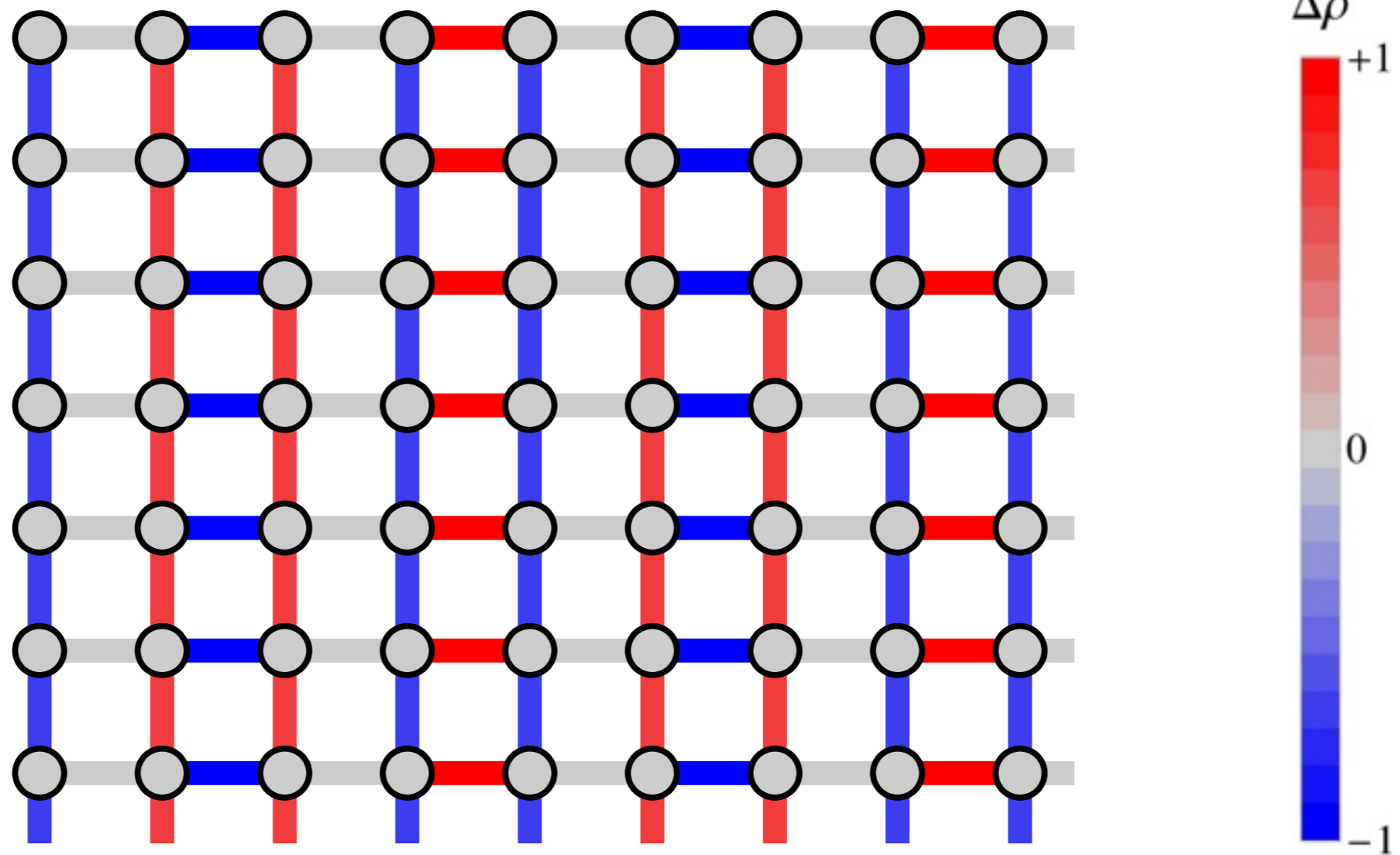
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Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



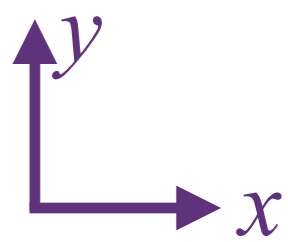
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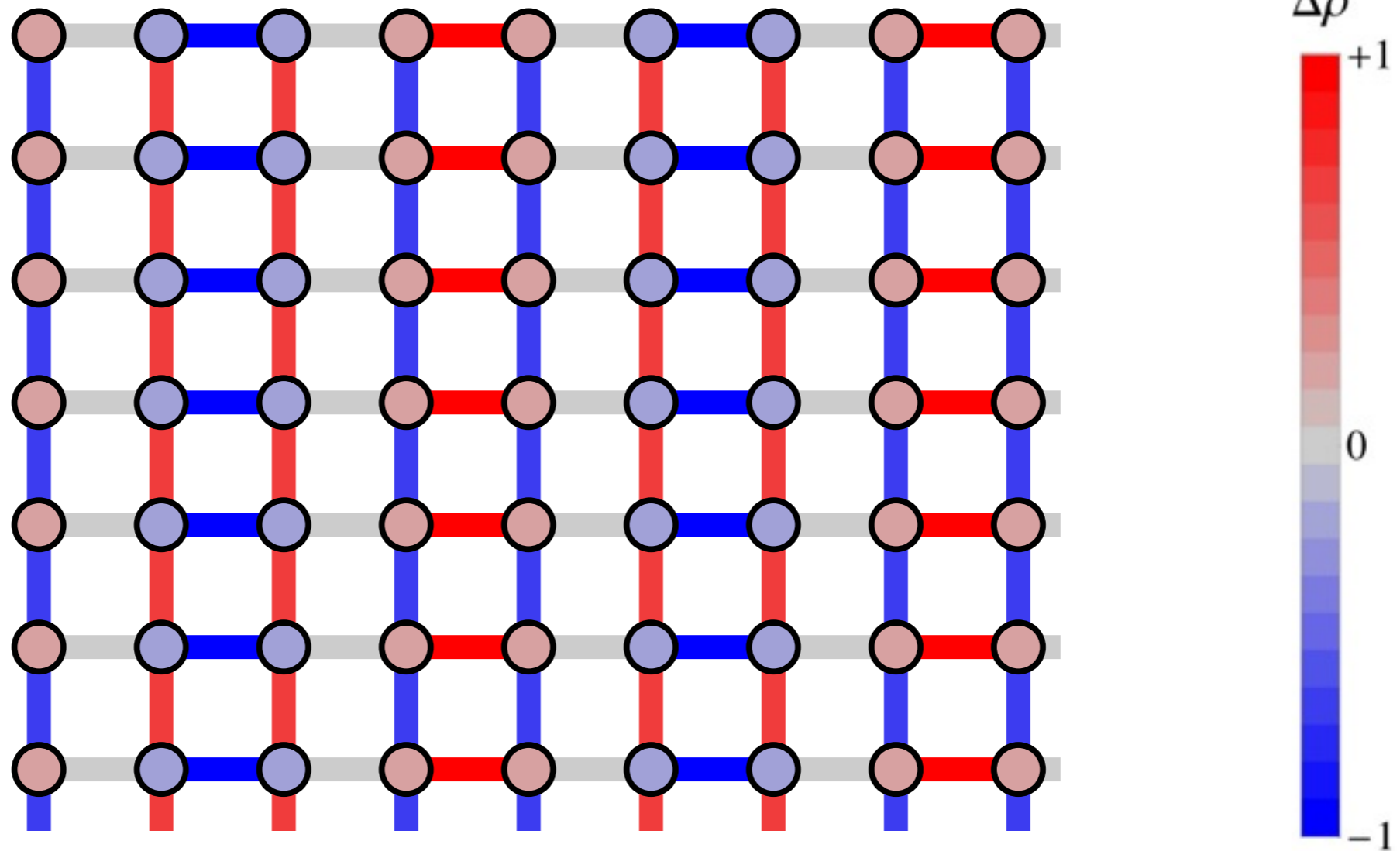
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$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [0.3 + \cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = (\pi/2, 0)$$

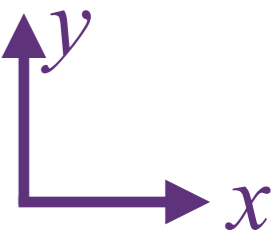


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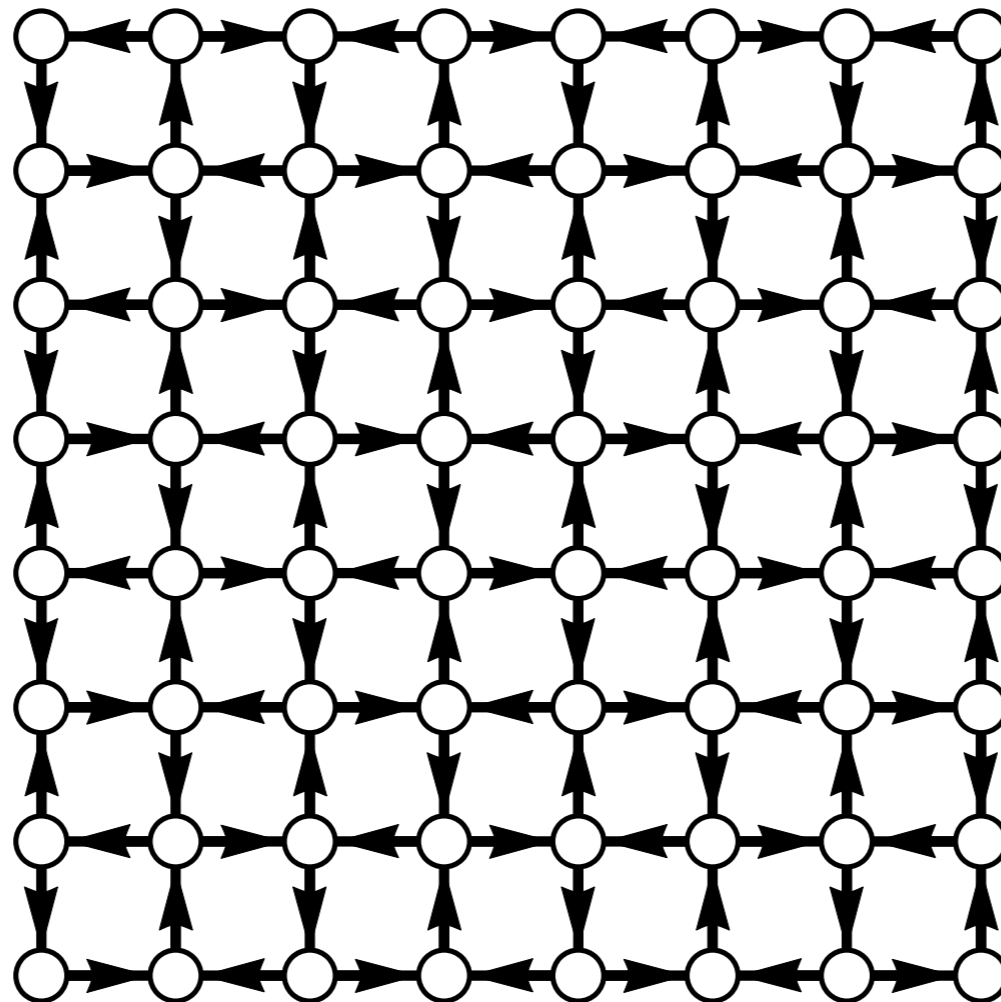
Time-reversal symmetry breaking state



Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\sum_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

$$\mathcal{P}(\mathbf{k}) = \sin(k_x) - \sin(k_y) \quad \text{and} \quad \mathbf{Q} = (\pi, \pi)$$



This state breaks time-reversal and is also known as “*d*-density wave” (*unfortunately*), and “staggered-flux (SF)”.

p-wave current order

S. Chakravarty, R. B. Laughlin, D. K. Morr, and C. Nayak, Phys. Rev. B **63**, 094503 (2001).

P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006).

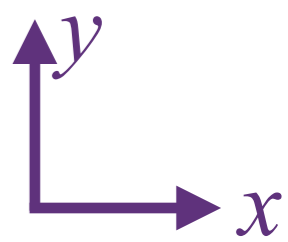
R. B. Laughlin, Phys. Rev. B **89**, 035134 (2014).

d -wave bond order.

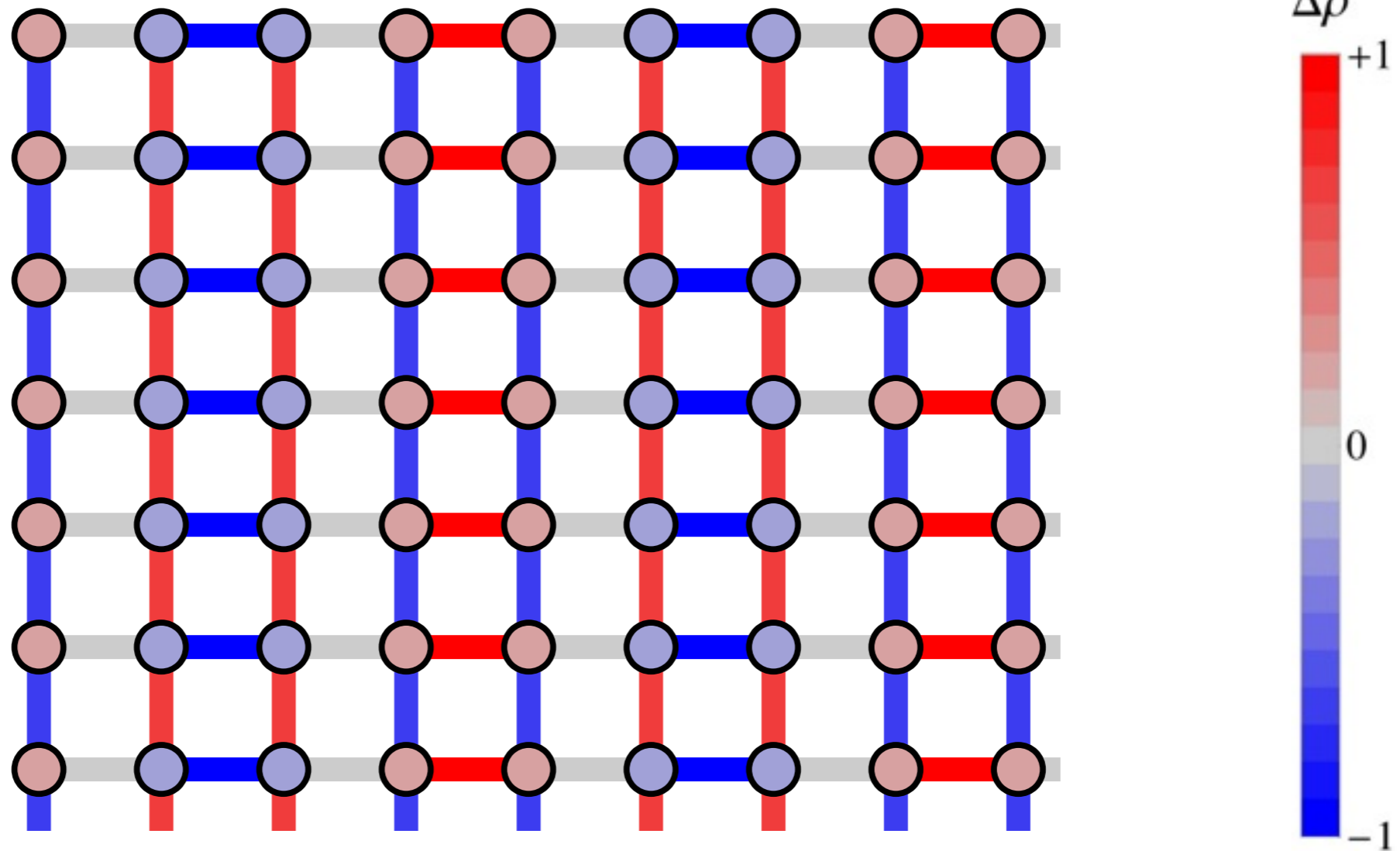
Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [0.3 + \cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = (\pi/2, 0)$$



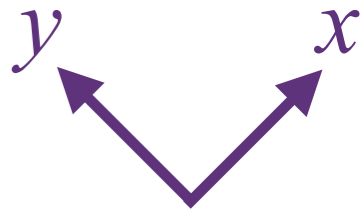
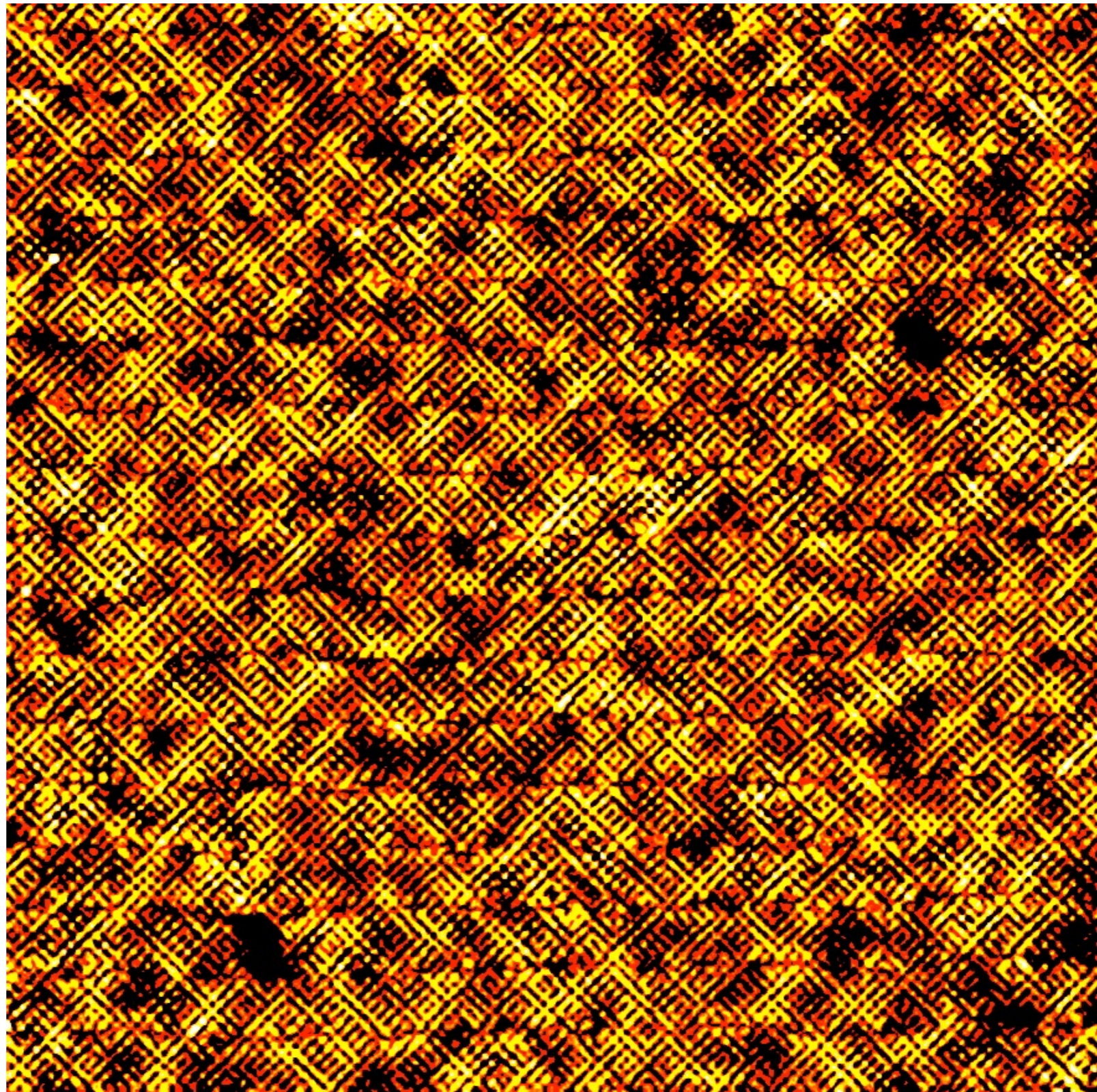
Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



This d -wave bond order was first discussed in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

See also C. Howald,
H. Eisaki, N. Kaneko,
M. Greven, and
A. Kapitulnik, *Phys.
Rev. B* **67**, 014533
(2003).

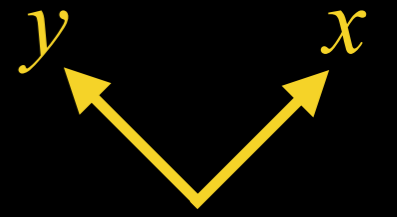
*“Cosmic
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superconductor*



“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007).

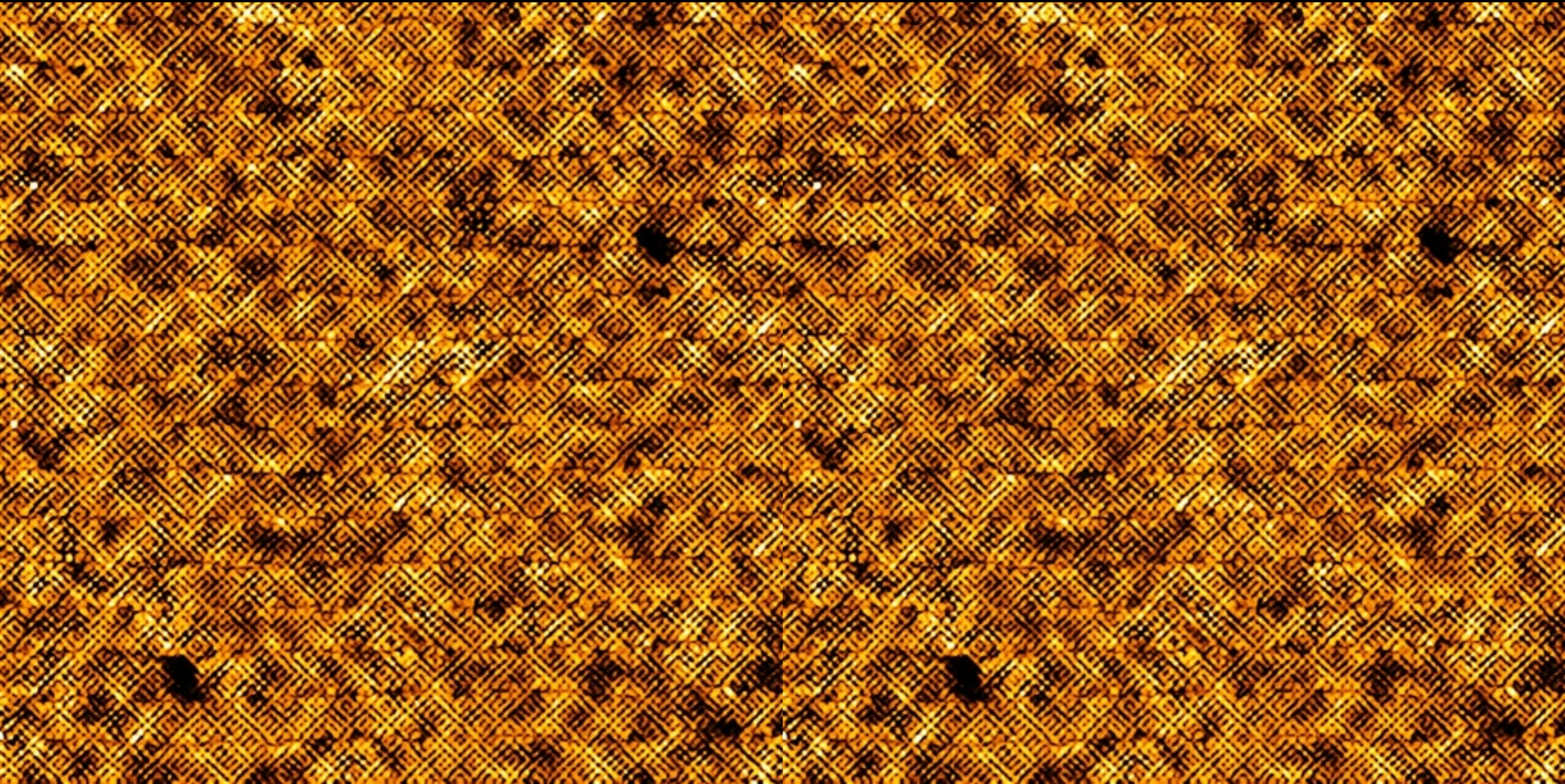
UD45K
BSCCO

$R(r, 150\text{mV})$



$R(r, 150\text{mV})$

$R(r, 150\text{mV})$



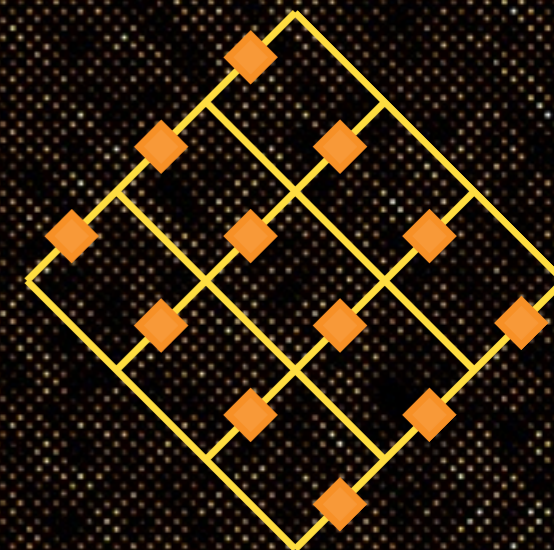
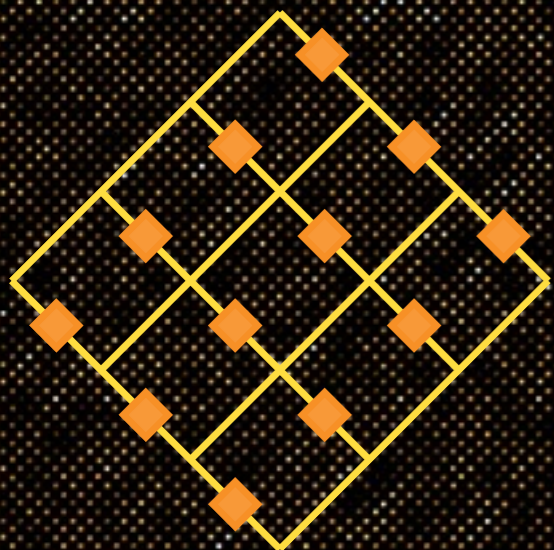
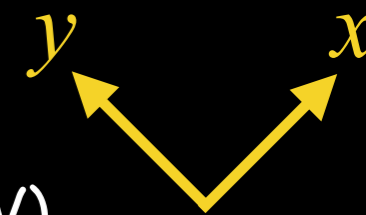
Note that these are identical images.

UD45K

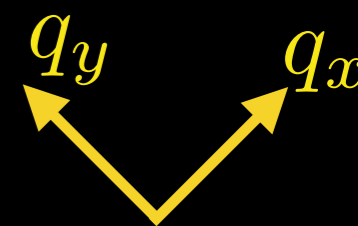
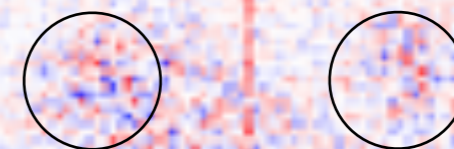
$R(r=0, 150\text{mV})$

$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$



UD45K

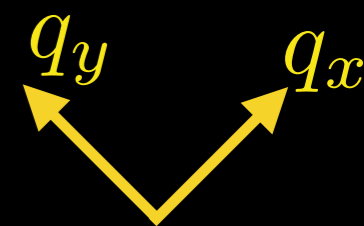
Broad $(0,Q)$ and $(Q,0)$ DW Features $\text{Re}O_x(q)$ $(1,0)$ $\text{Re} O_x(q)$ $\text{Im} O_x(q)$ $\text{Re} O_y(q)$ $\text{Im} O_y(q)$ 

-1



+1

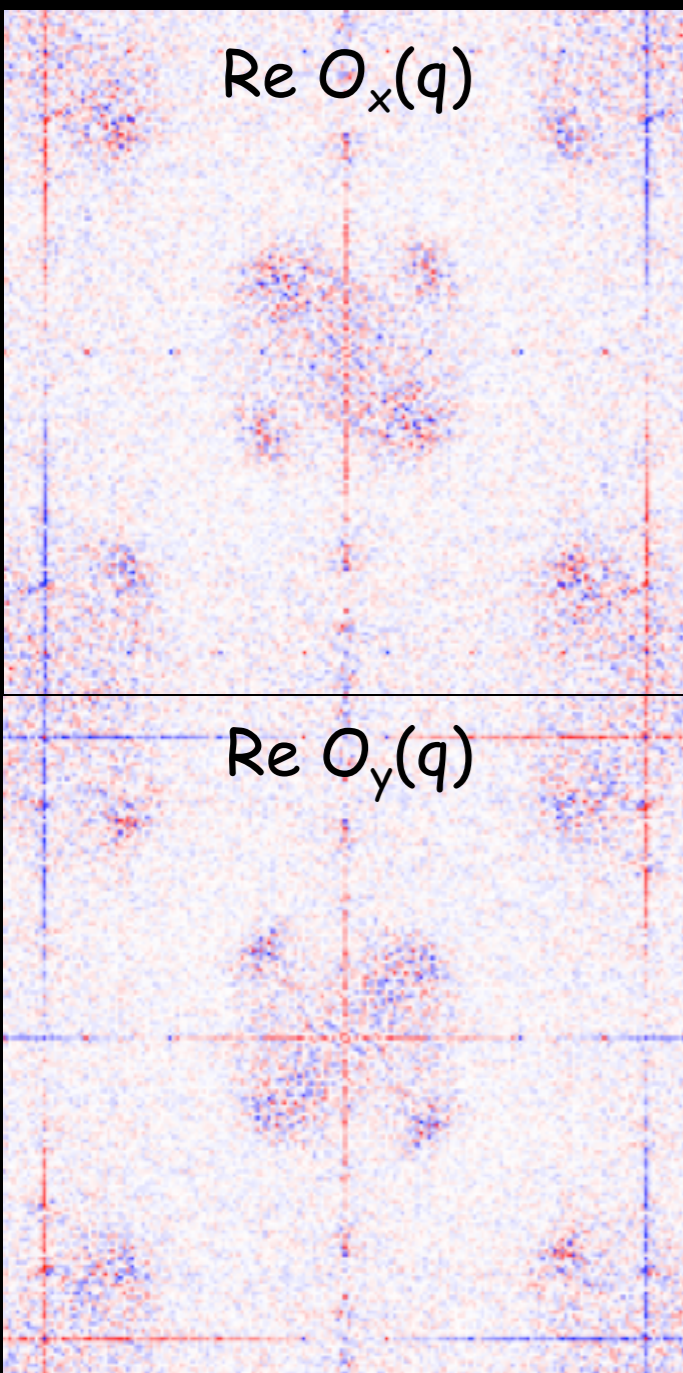
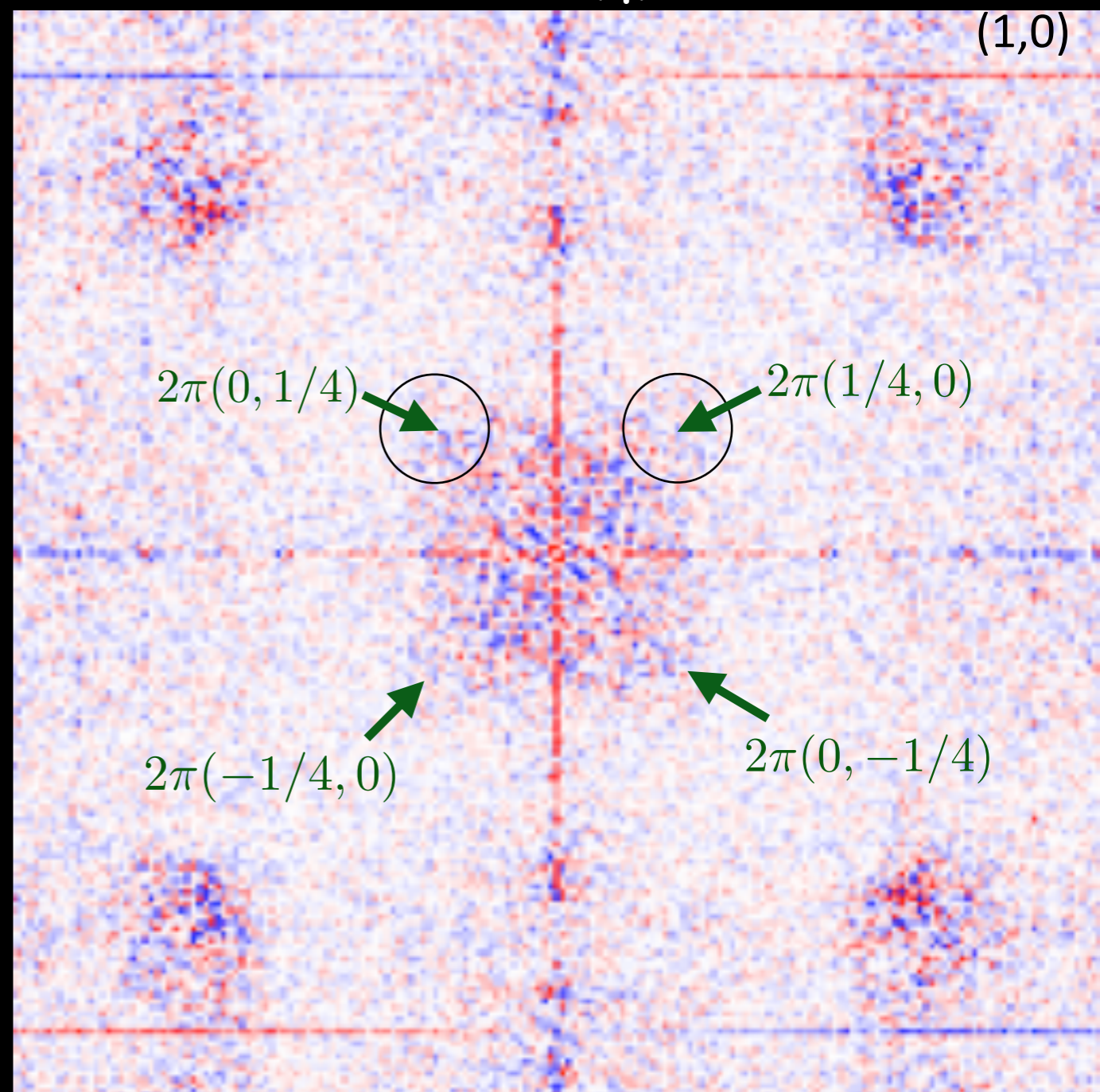
UD45K

Broad $(0,Q)$ and $(Q,0)$ DW Features $\text{Re}O_x(q)$ $(1,0)$ $\text{Re} O_x(q)$ $\text{Im} O_x(q)$ $\text{Re} O_y(q)$ $\text{Im} O_y(q)$ $2\pi(0, 1/4)$ $2\pi(1/4, 0)$ $2\pi(-1/4, 0)$ $2\pi(0, -1/4)$

-1

+1

UD45K

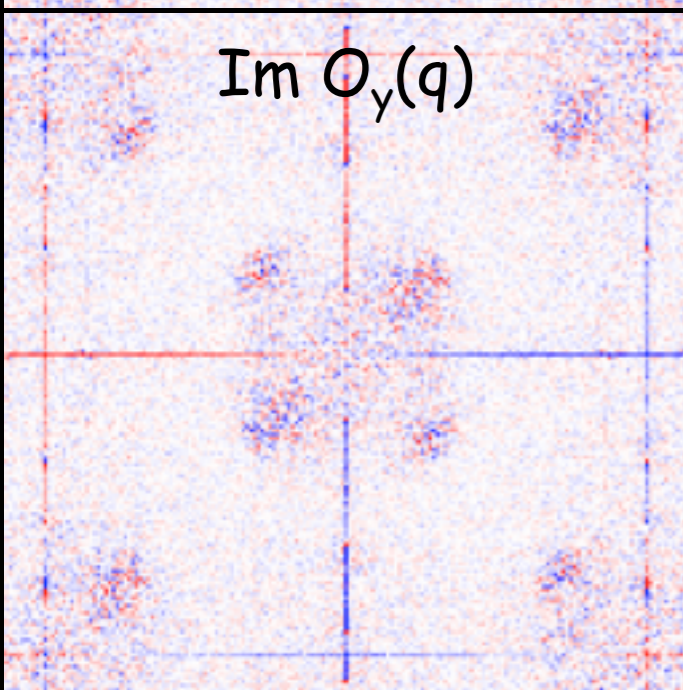
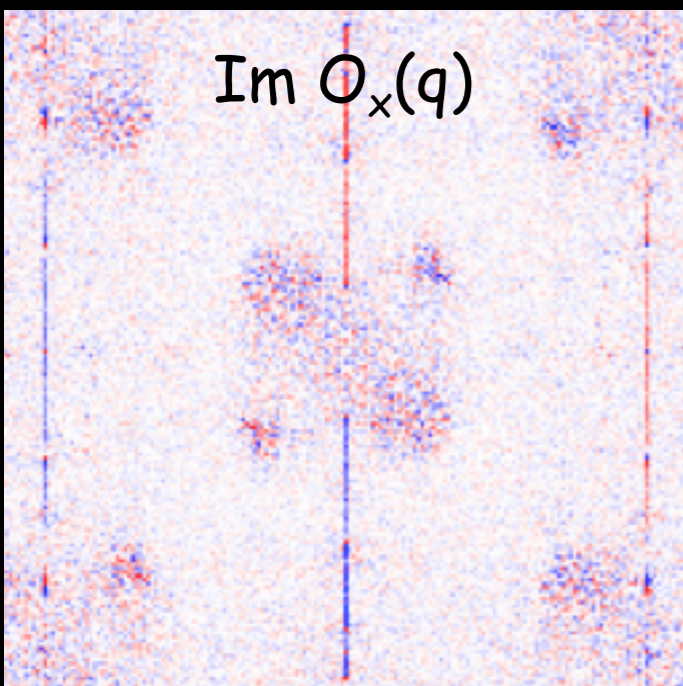
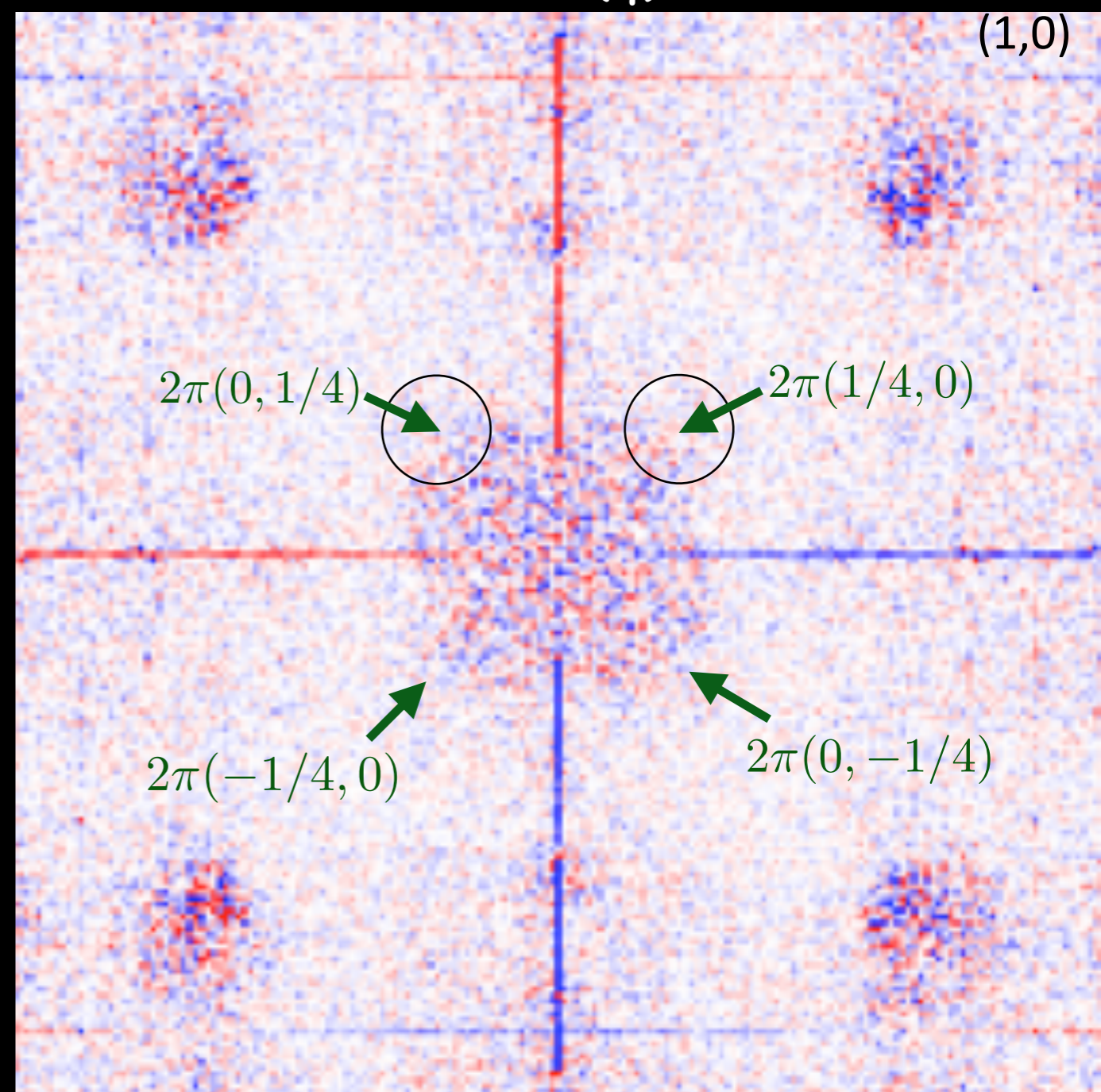
 $\text{Re}Q_x(0,Q)$ Vs $\text{Re}O_y(0,Q)$ q_y
 q_x $\text{Re}O(q)$  $\text{Re} O_x + \text{Re} O_y$ 

$$\text{Re}O_x(Q,0) + \text{Re}O_y(Q,0) \approx 0$$

$$\text{Re}O_x(0,Q) + \text{Re}O_y(0,Q) \approx 0$$

-1  +1

UD45K

 $\text{Im}Q_x(0,Q)$ Vs $\text{Im}O_y(0,Q)$ q_y q_x $\text{Im}O(q)$  $\text{Im} O_x + \text{Im} O_y$ 

$$\text{Im}O_x(0,Q) + \text{Im}O_y(0,Q) \approx 0$$

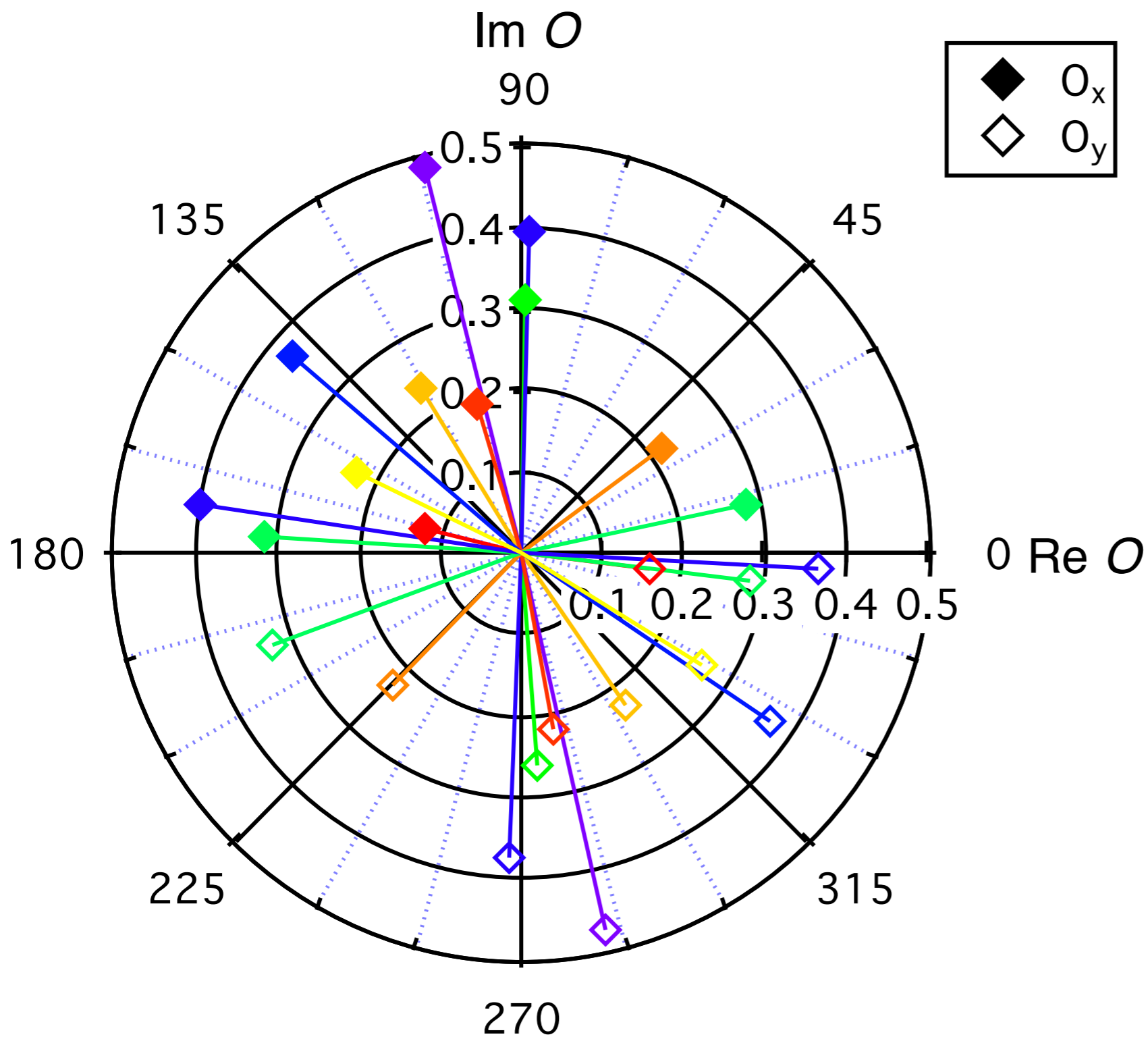
$$\text{Im}O_x(Q,0) + \text{Im}O_y(Q,0) \approx 0$$

-1



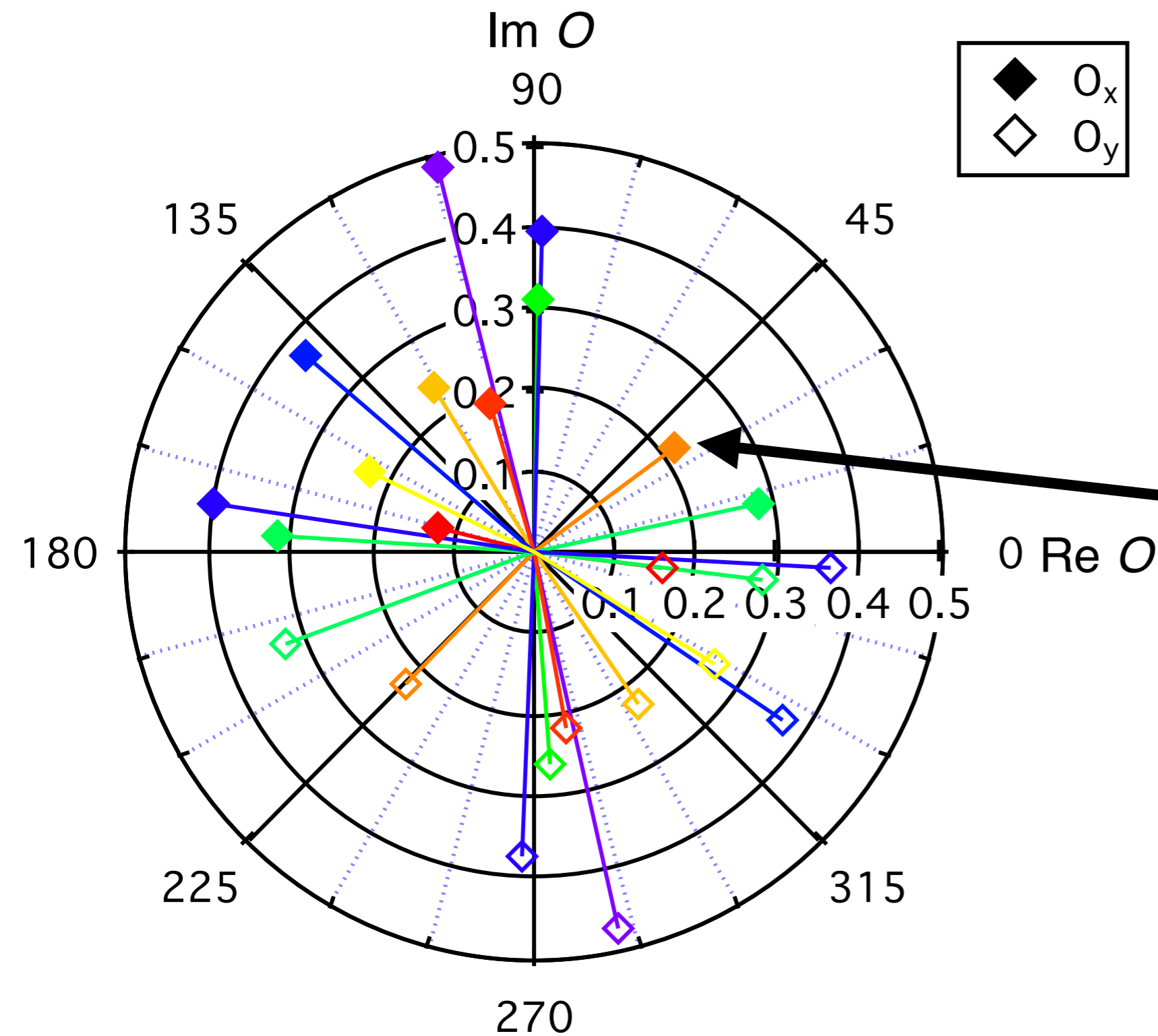
+1

**Phase-sensitive
measurement of
the d symmetry of
charge density
wave order**



**Phase-sensitive
measurement of
the d symmetry of
charge density
wave order**

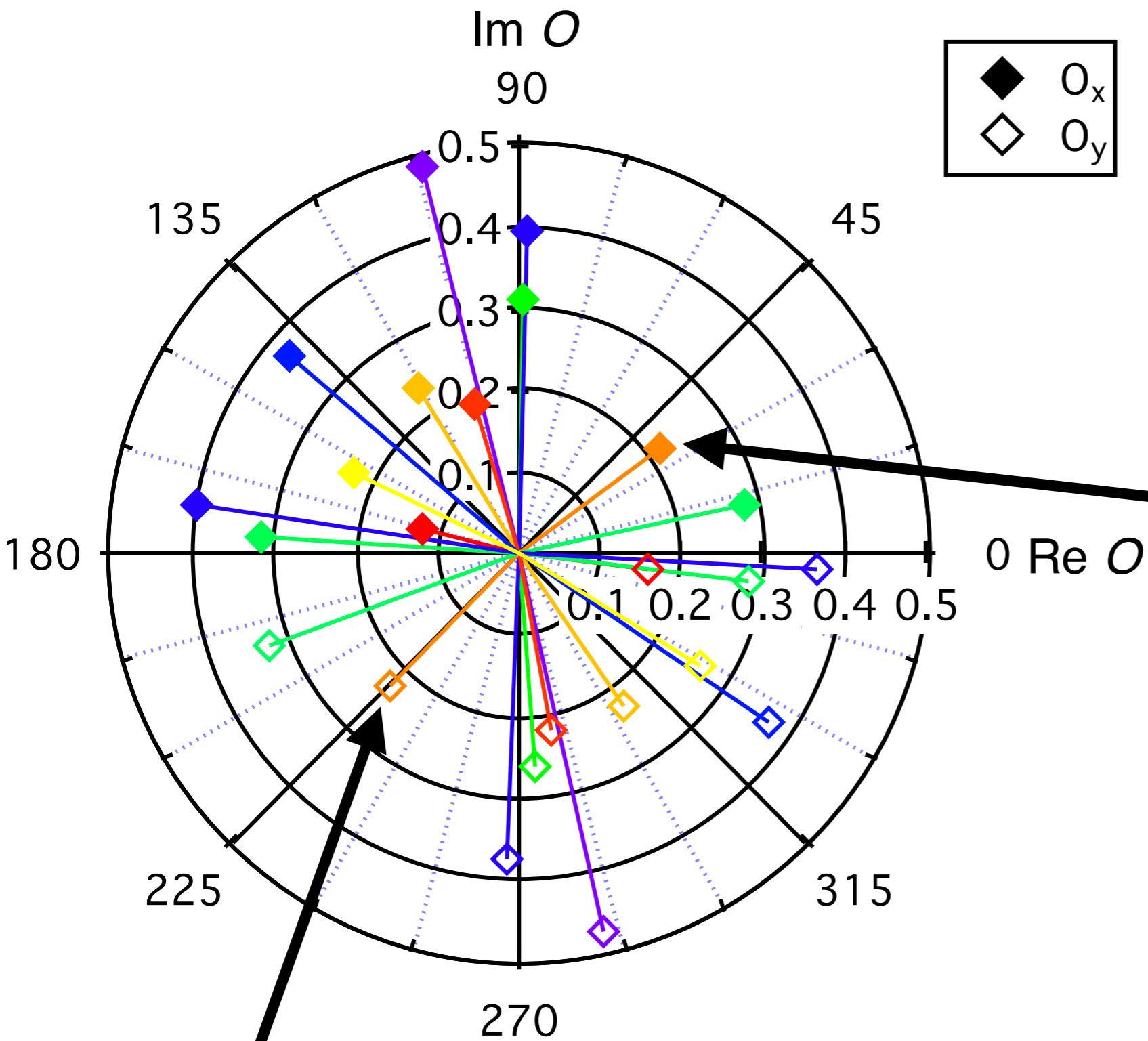
Complex value of
 O_x at a pixel



Phase-sensitive measurement of the d symmetry of charge density wave order

Complex value of O_x at a pixel

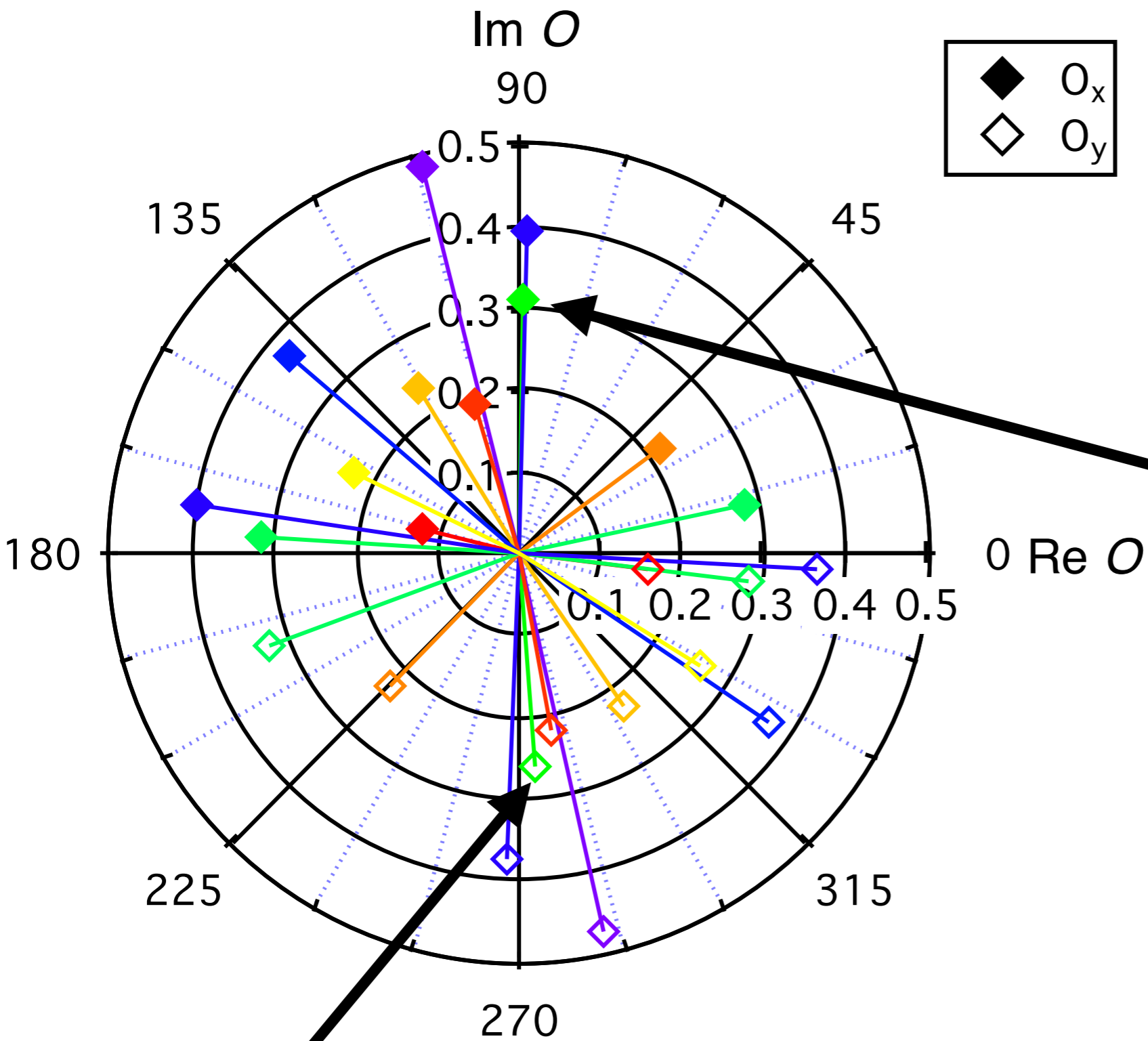
Complex value of O_y at same pixel



Phase-sensitive measurement of the d symmetry of charge density wave order

Complex value of O_x at a pixel

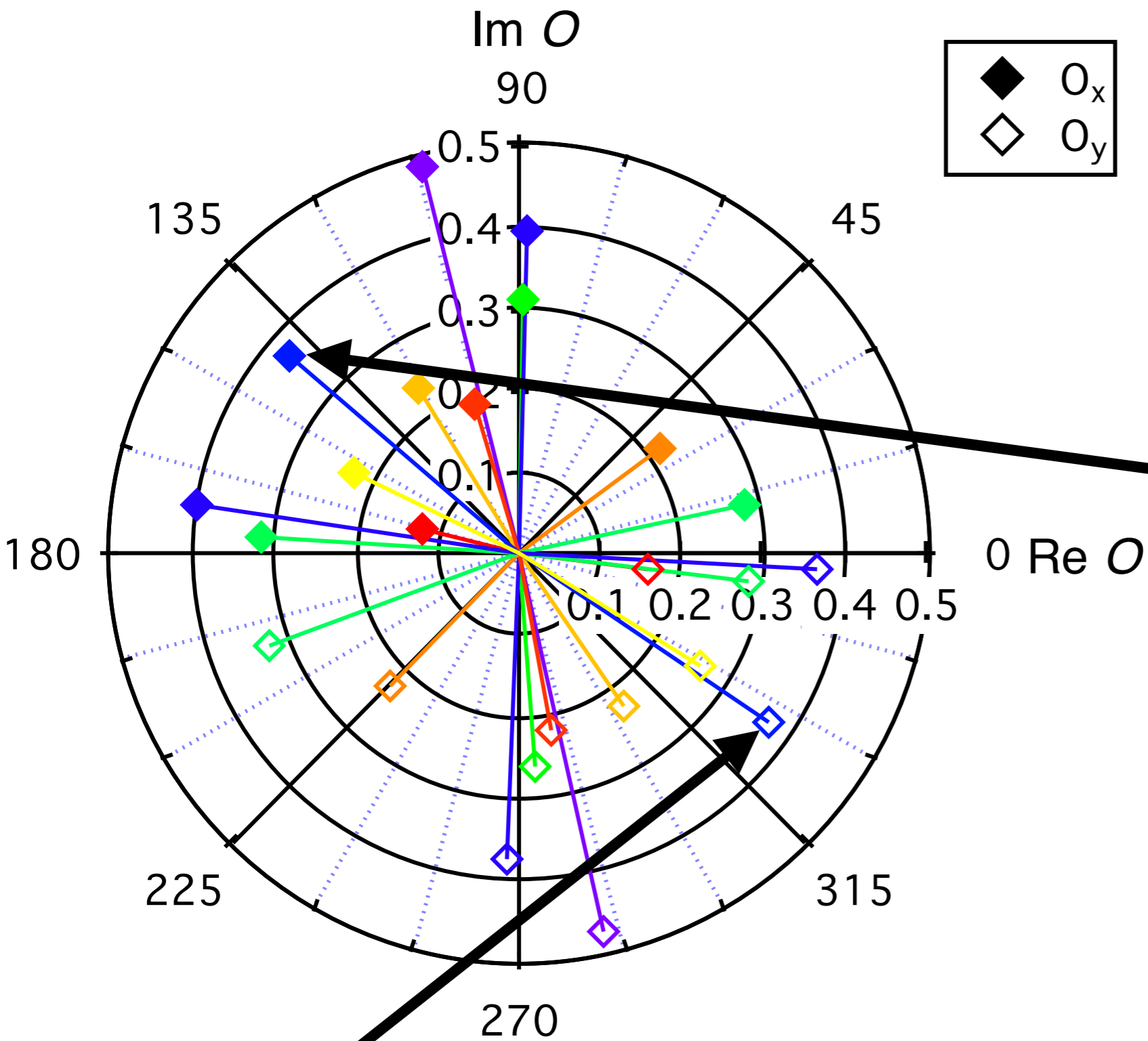
Complex value of O_y at same pixel



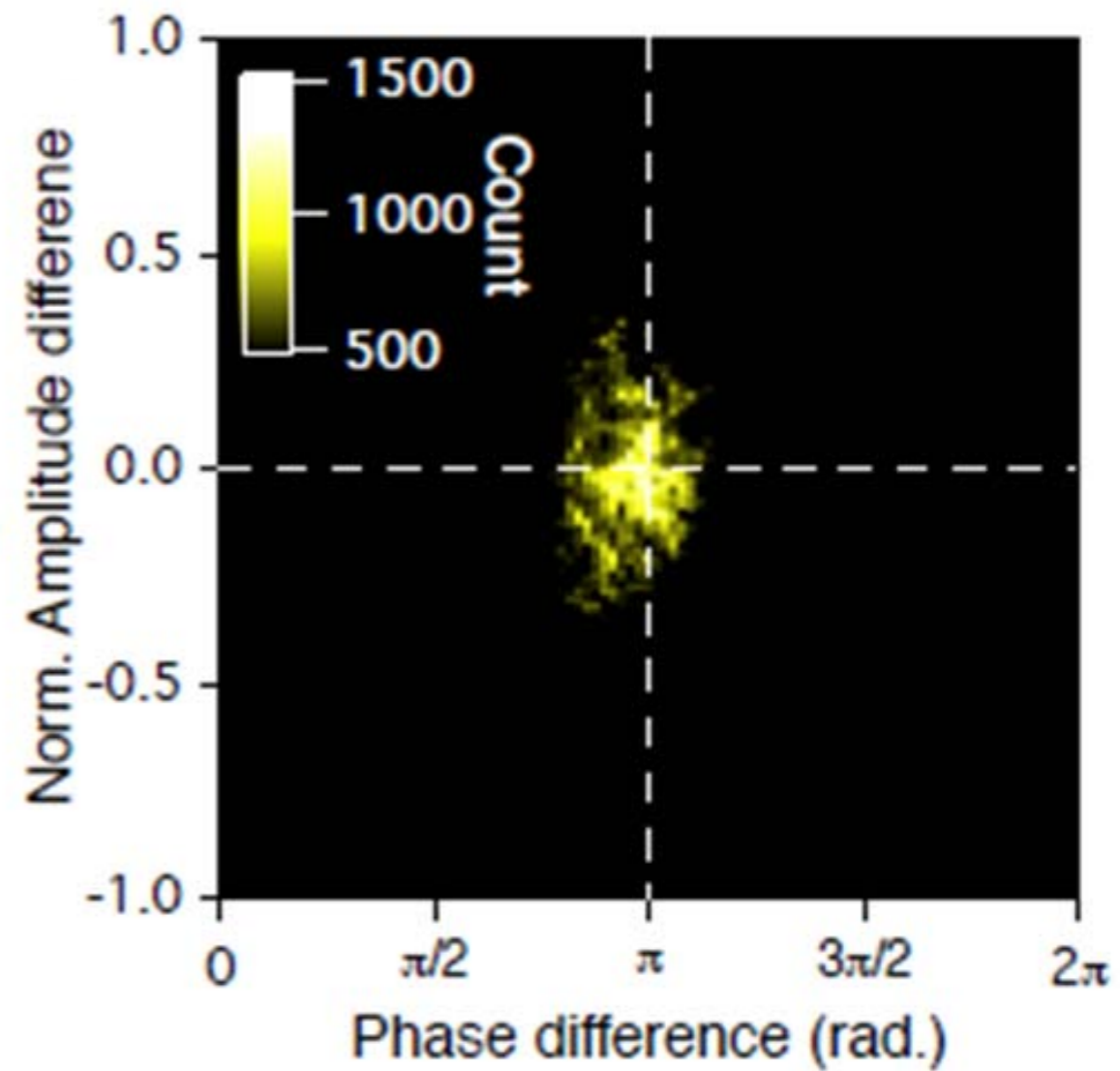
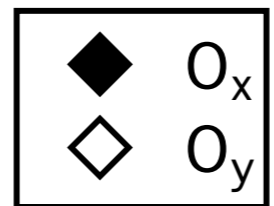
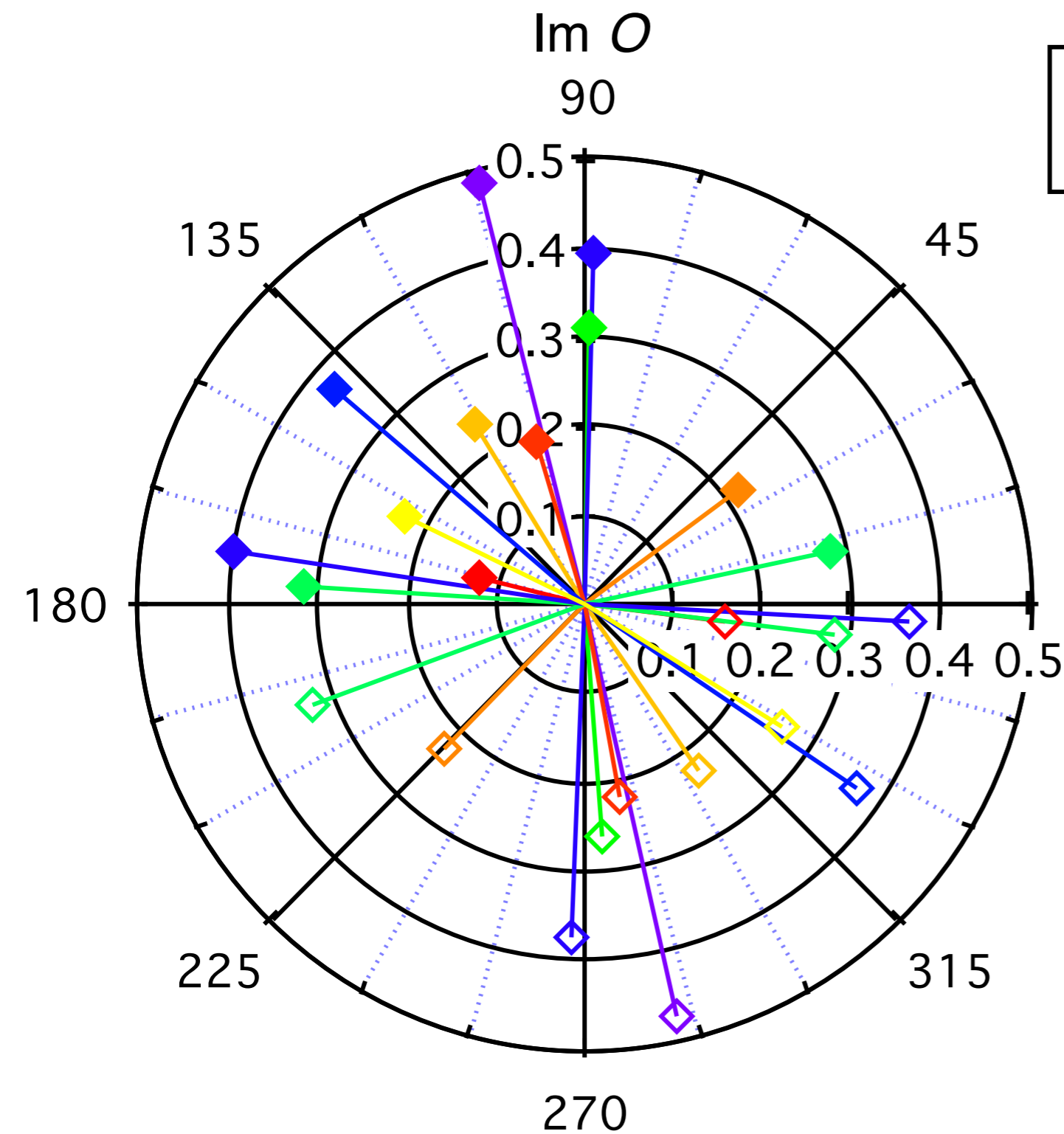
Phase-sensitive measurement of the d symmetry of charge density wave order

Complex value of O_x at a pixel

Complex value of O_y at same pixel

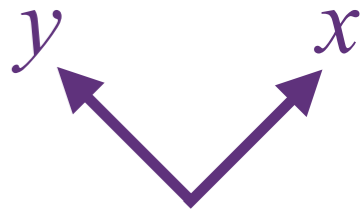
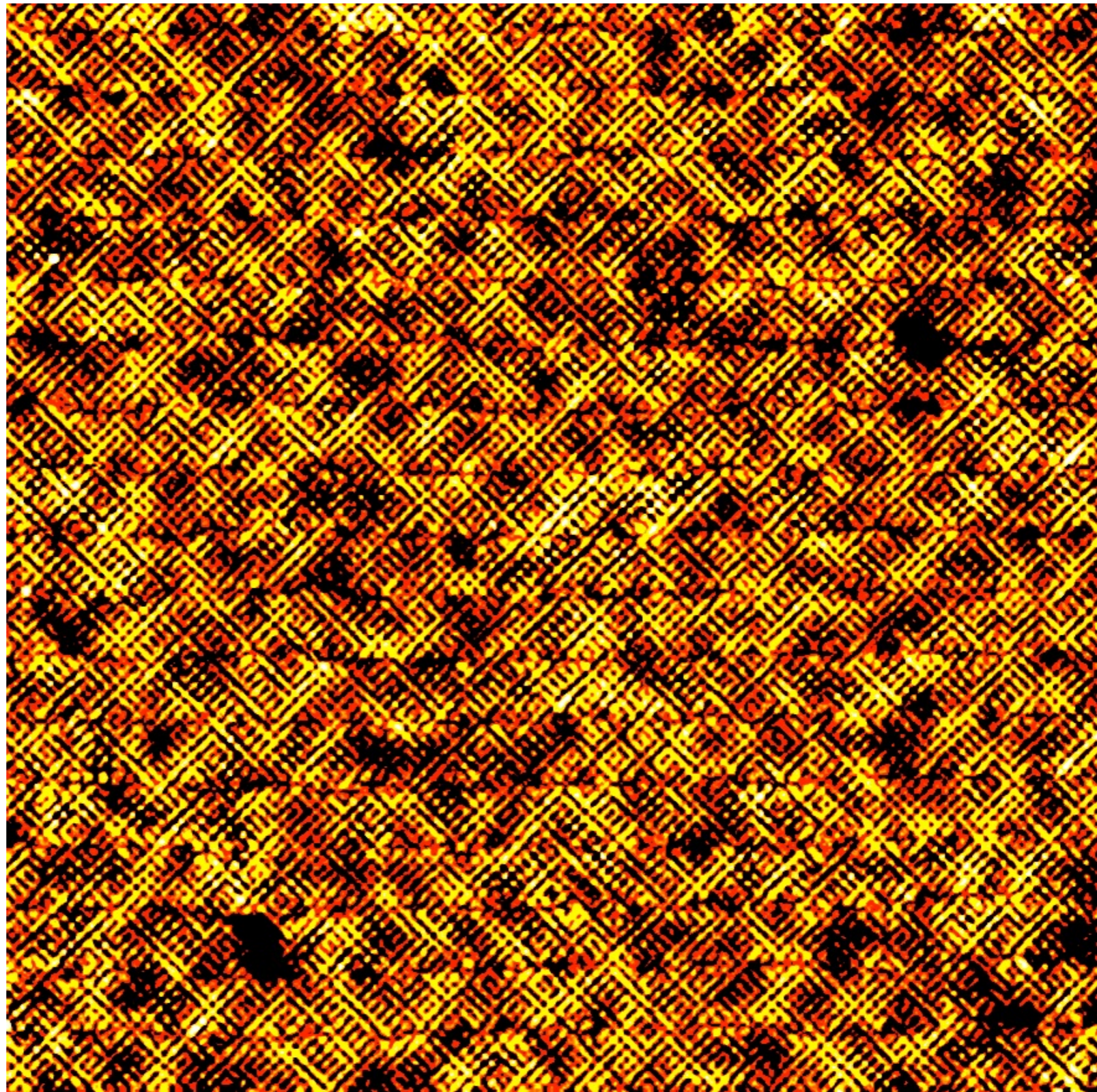


Phase-sensitive measurement of the d symmetry of charge density wave order



See also C. Howald,
H. Eisaki, N. Kaneko,
M. Greven, and
A. Kapitulnik, *Phys.
Rev. B* **67**, 014533
(2003).

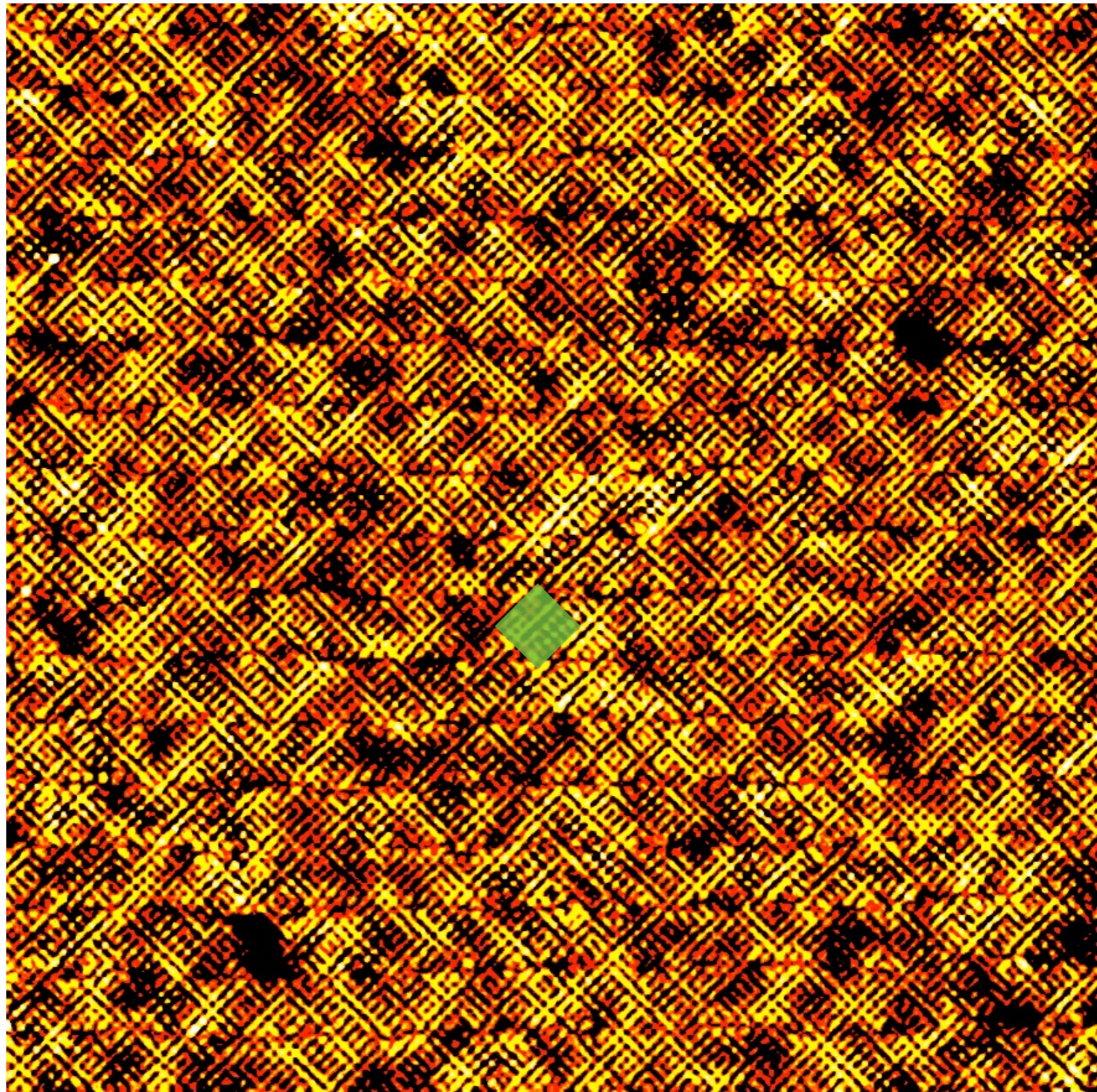
*“Cosmic
background”
map of a
superconductor*



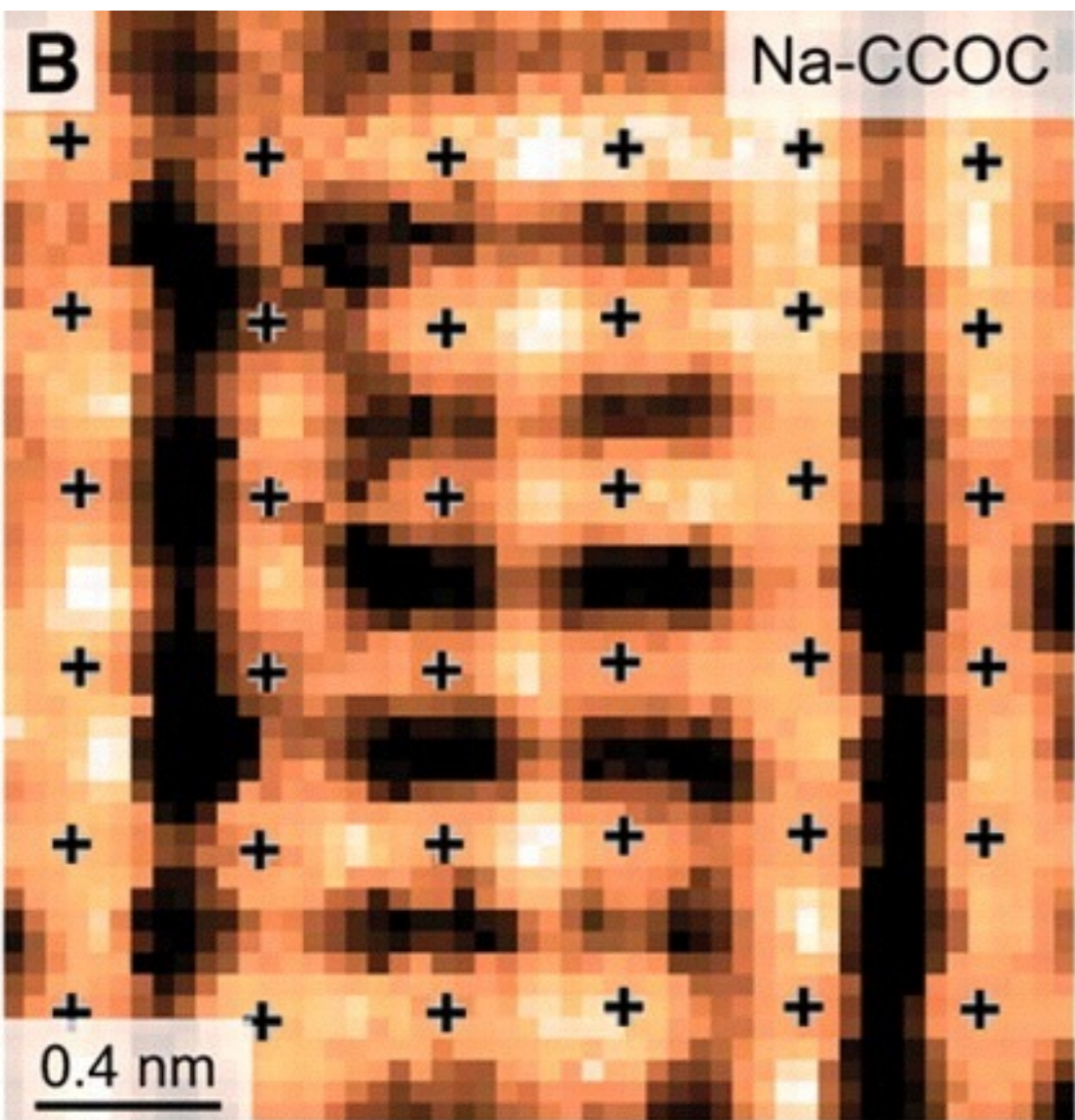
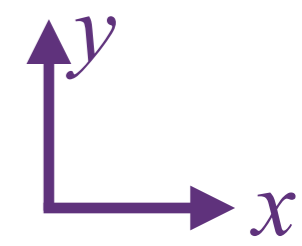
“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007).

See also C. Howald,
H. Eisaki, N. Kaneko,
M. Greven, and
A. Kapitulnik, *Phys. Rev. B* **67**, 014533
(2003).

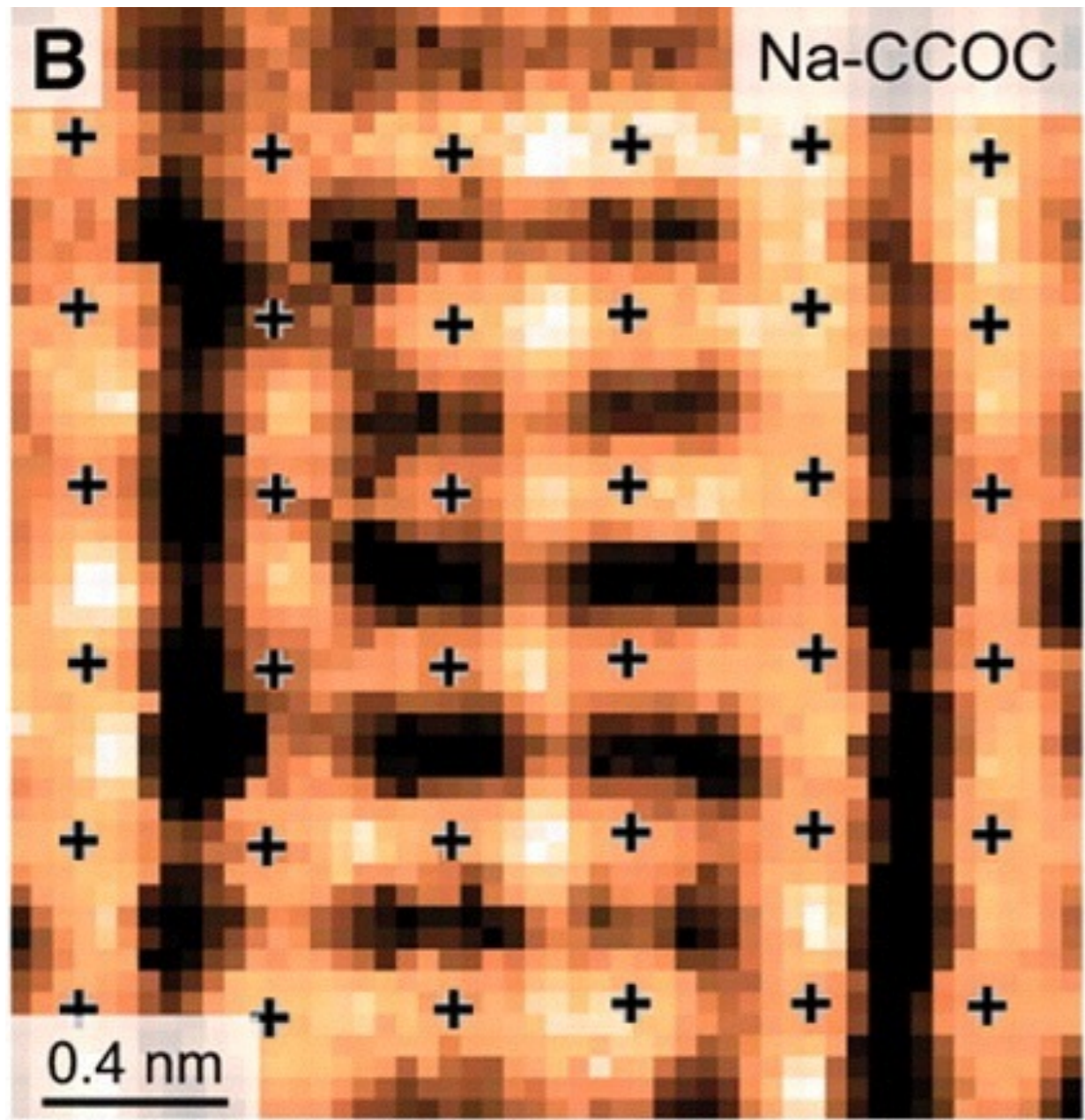
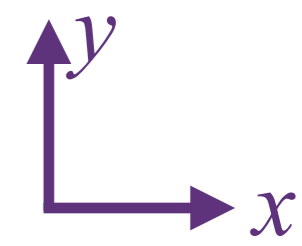
*“Cosmic
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map of a
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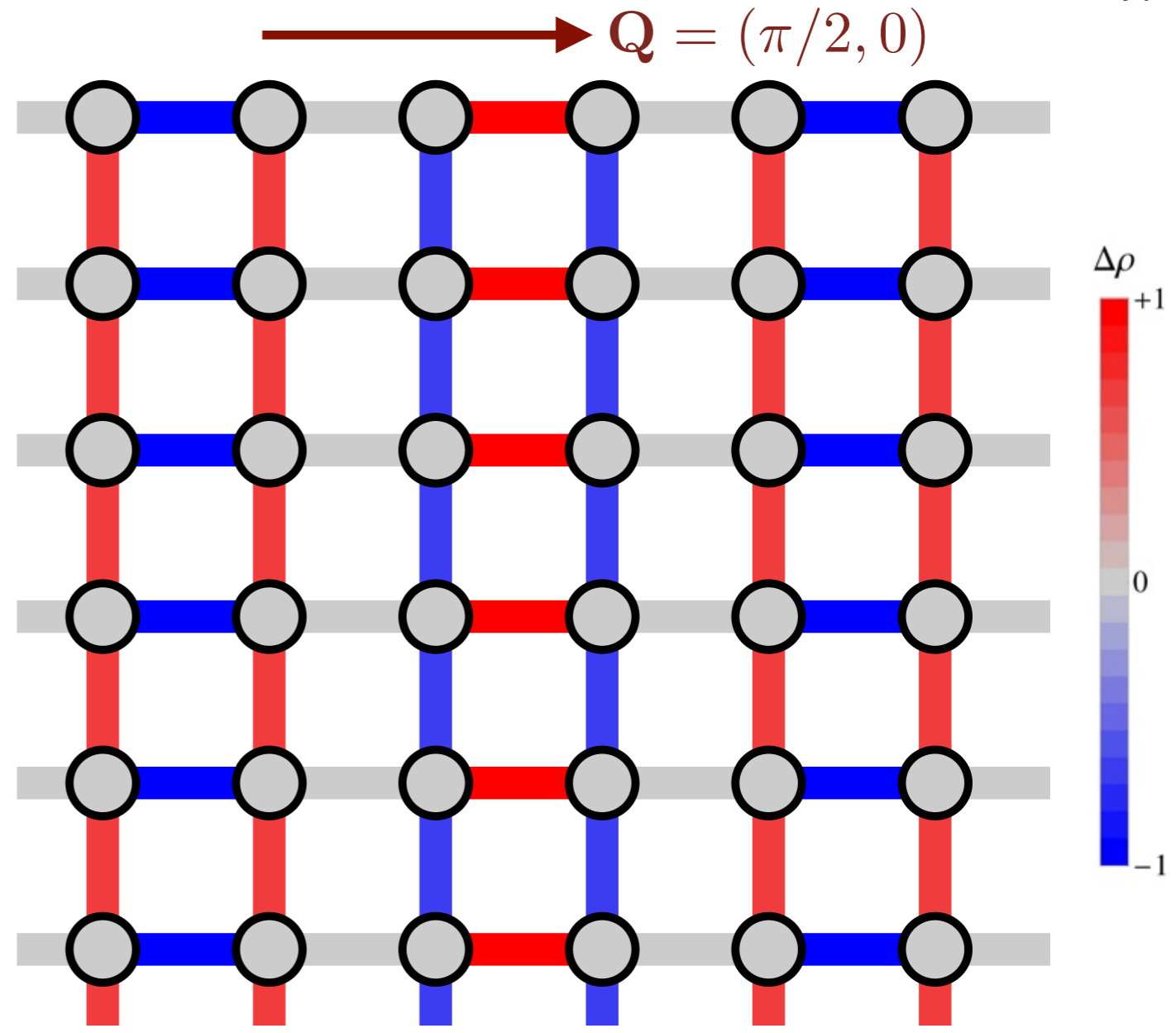
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Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

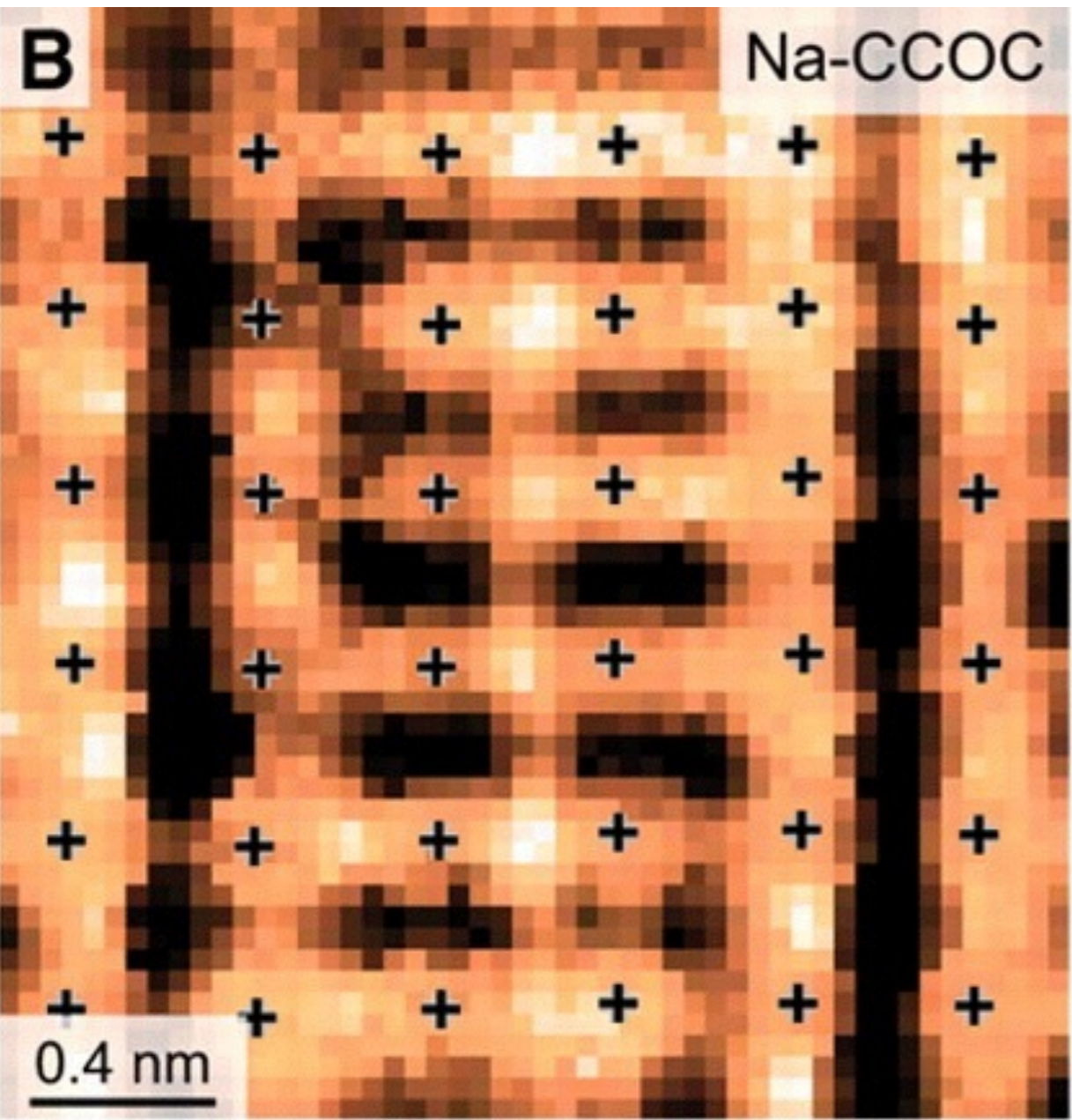
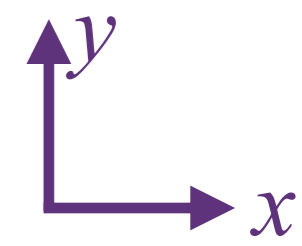


Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

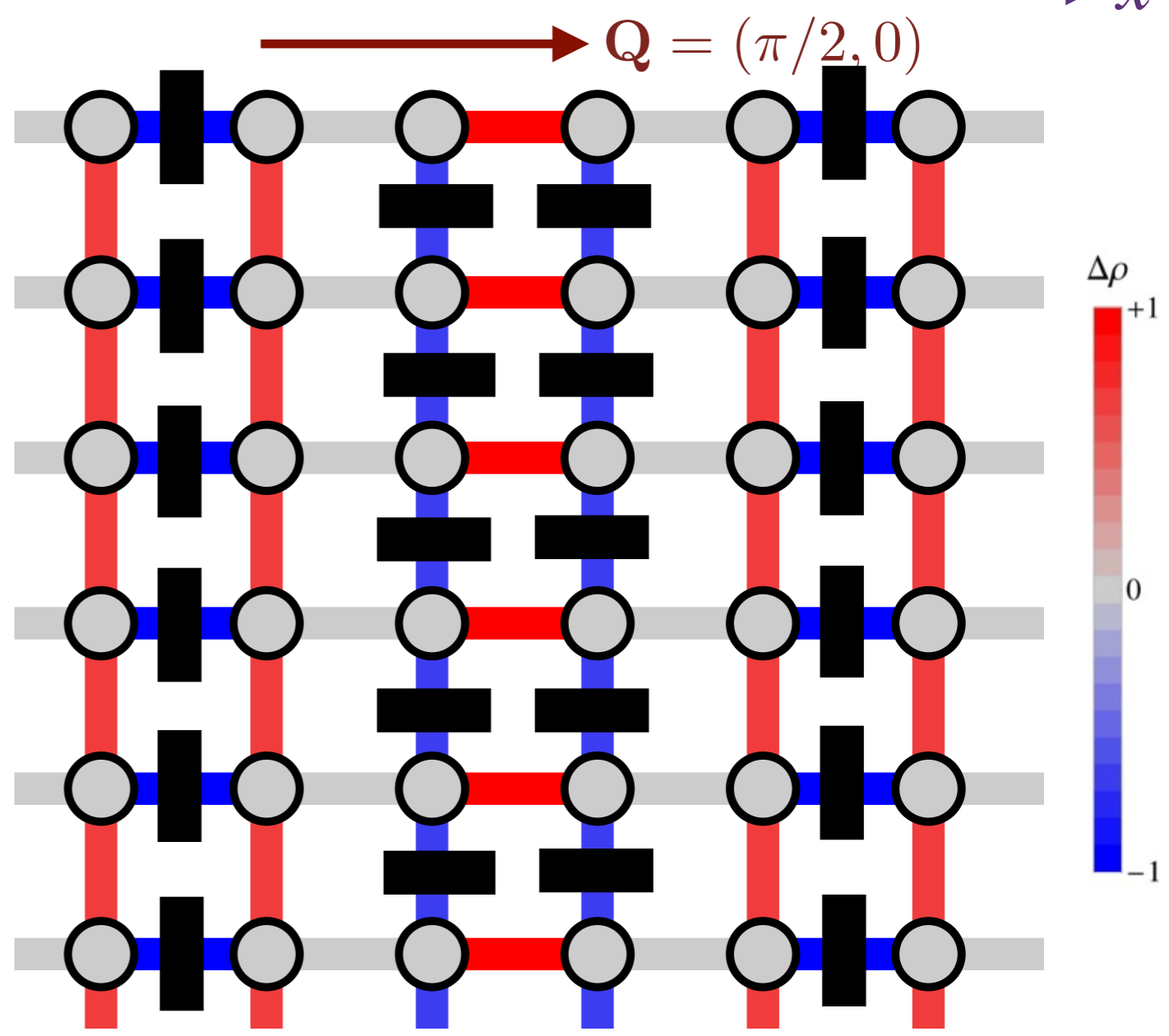


d-wave bond order

This *d*-wave bond order was first discussed in
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

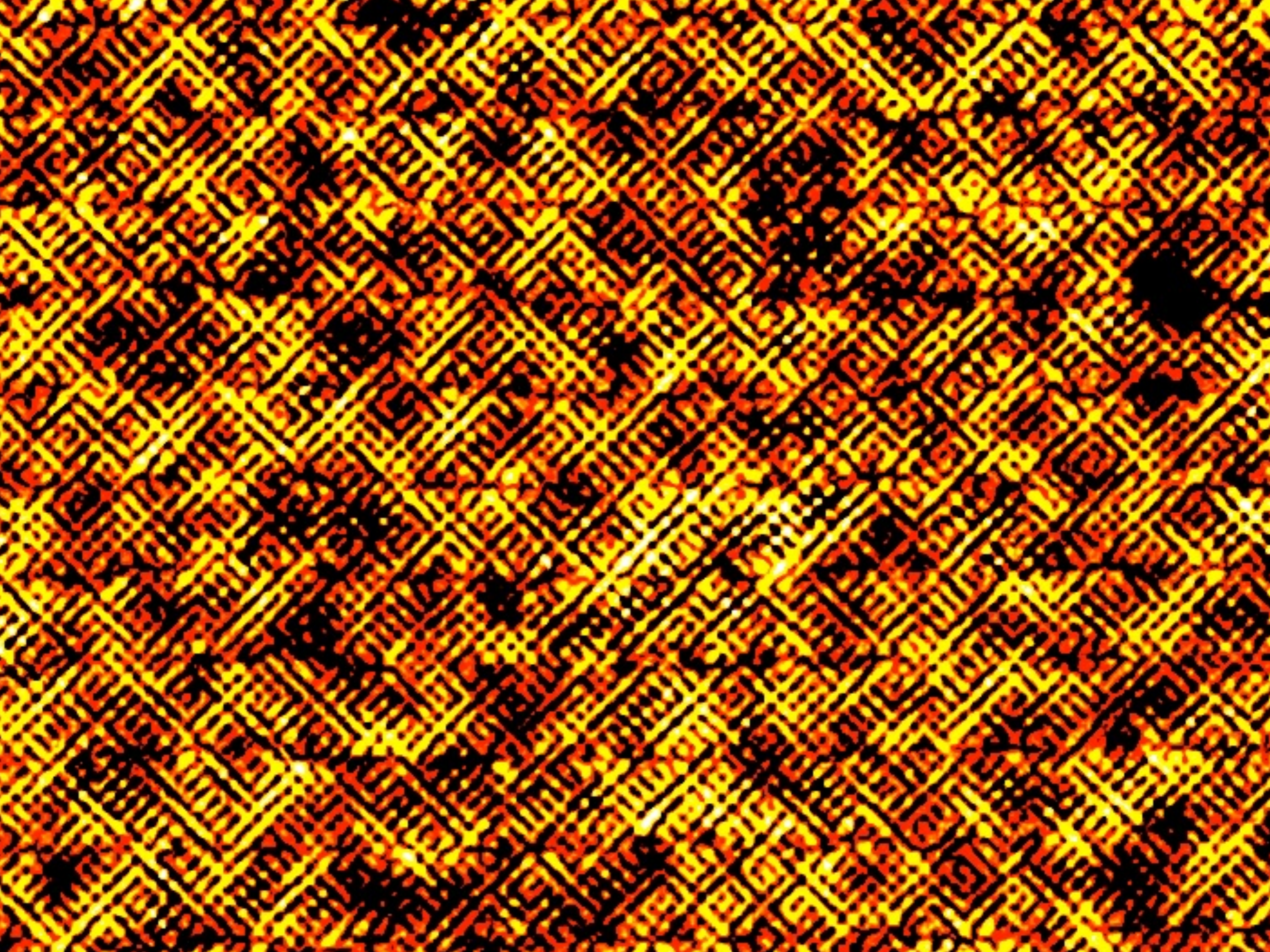


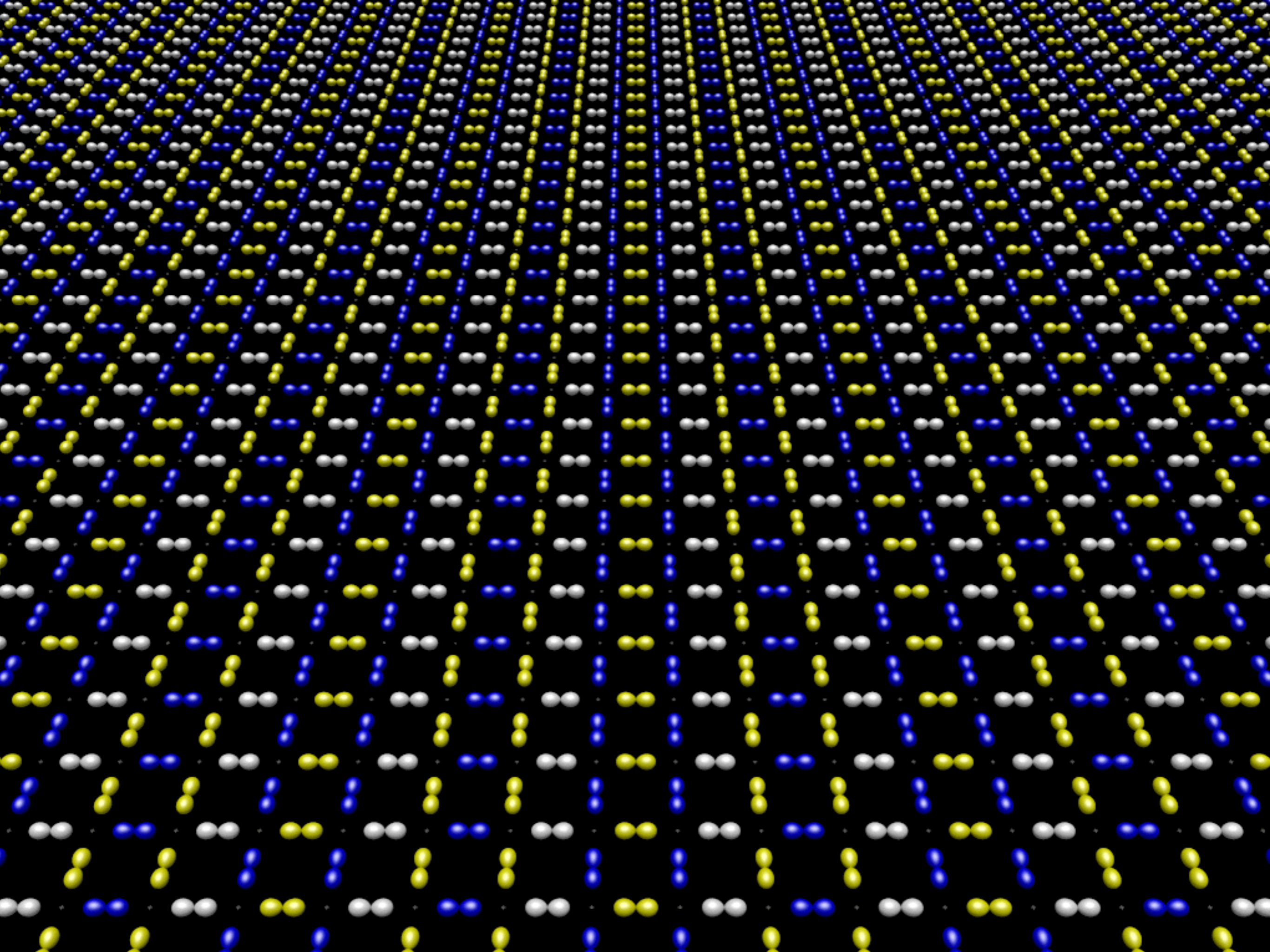
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

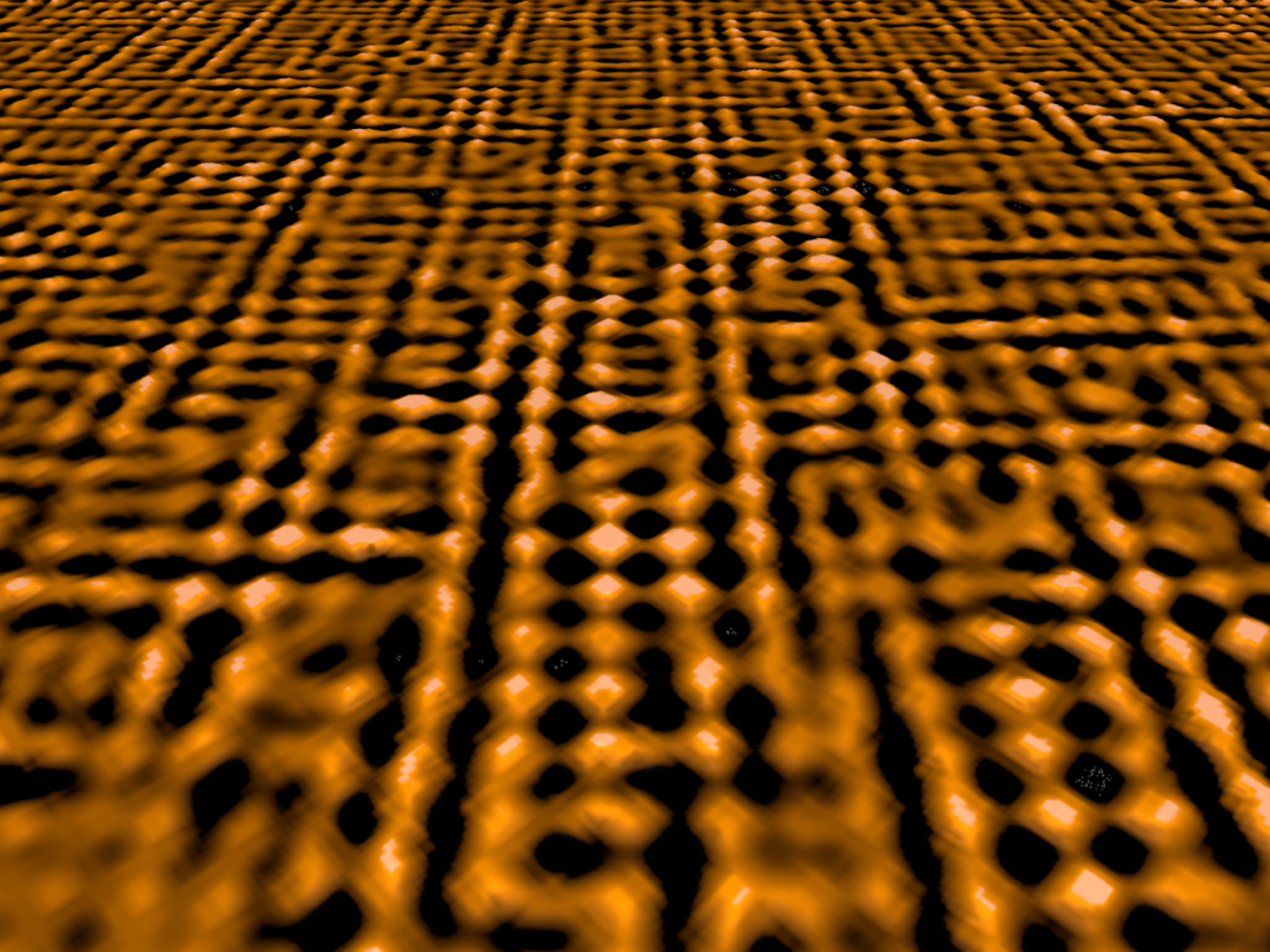


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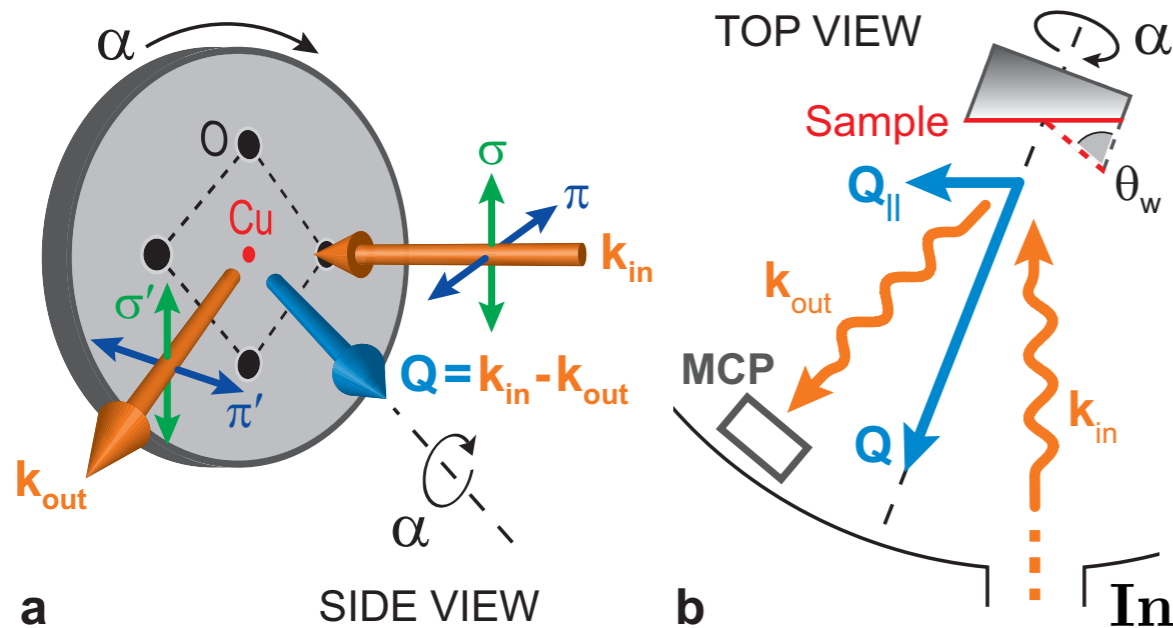






The symmetry of charge order in the cuprates

R. Comin, R. Sutarto, F. He, E. da Silva Neto, L. Chauviere, A. Frano, R. Liang, W.N. Hardy, D.A. Bonn, Y. Yoshida, H. Eisaki, J. E. Hoffman, B. Keimer, G.A. Sawatzky, and A. Damascelli, arXiv:1402.5415

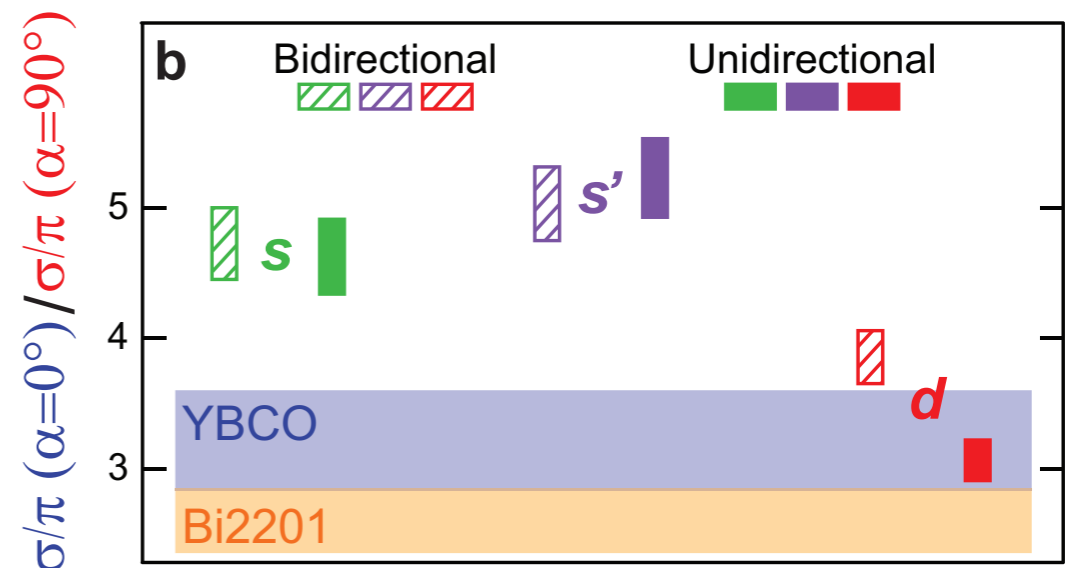
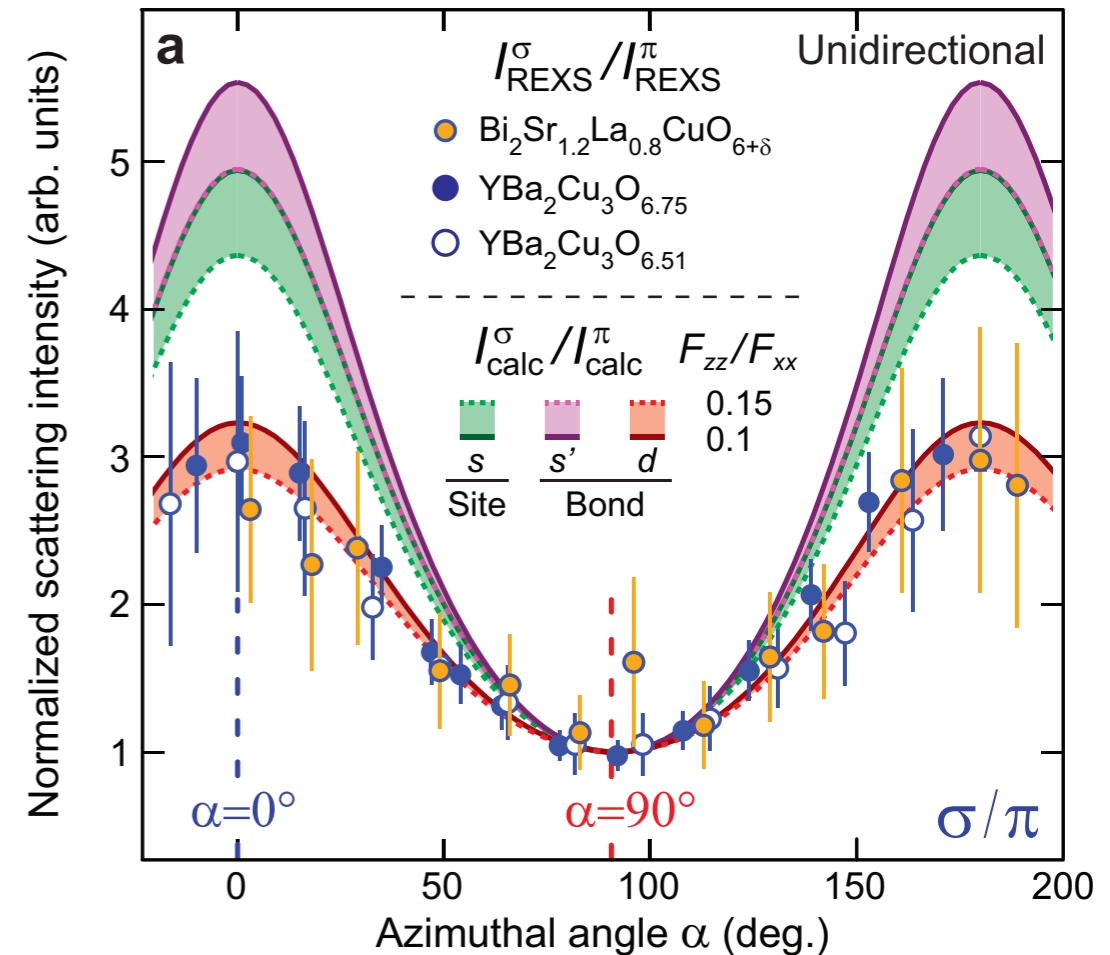
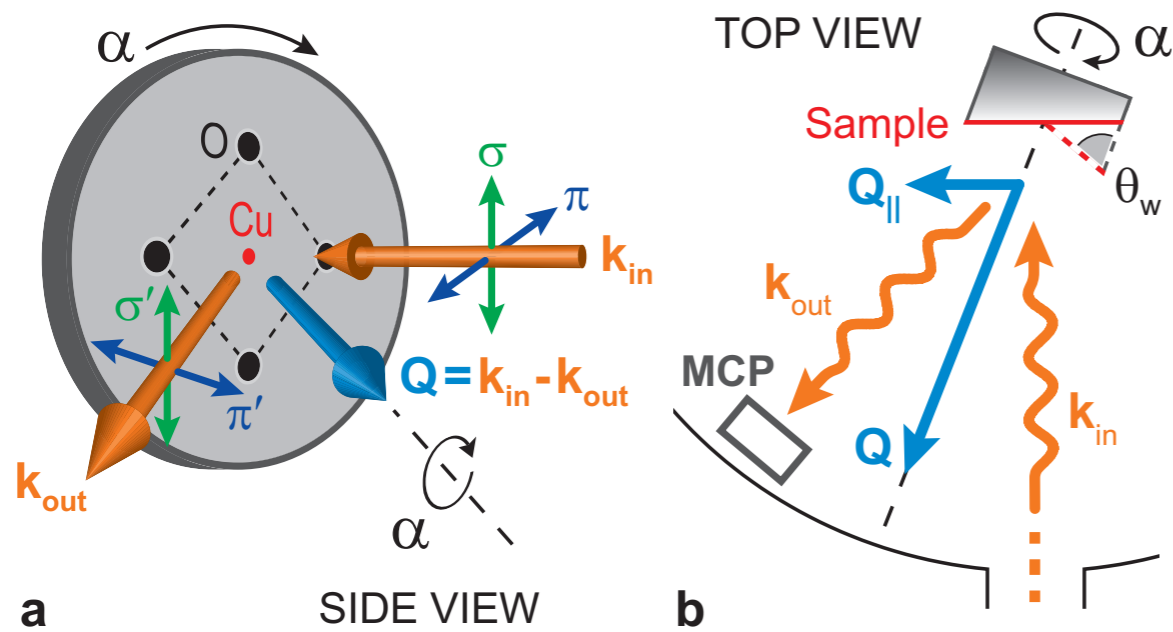


Polarization and angular analysis of X-ray scattering at $\mathbf{Q} = (Q_0, 0)$ in Bi2201 and YBCO

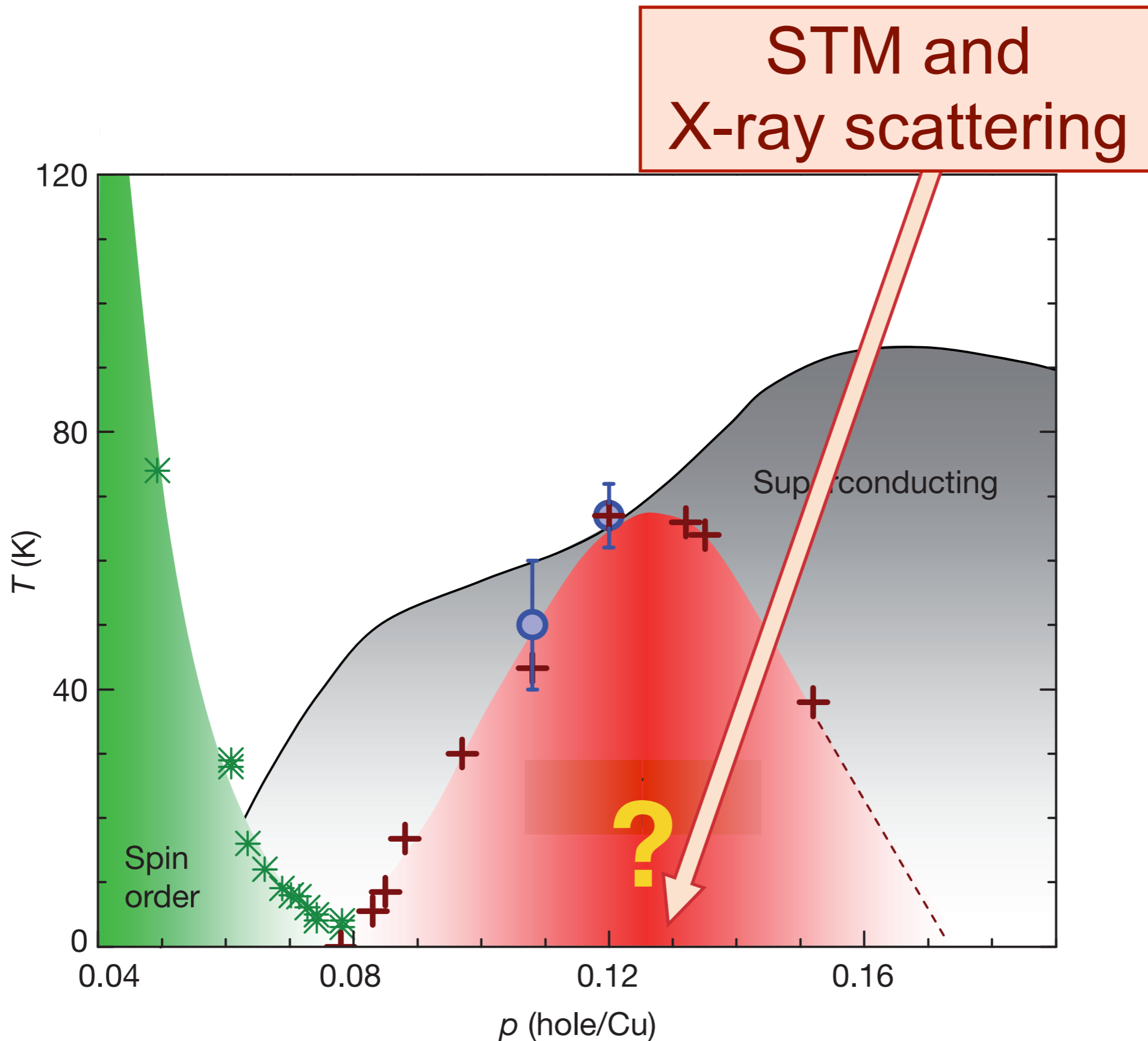
In addition, by adopting a special experimental geometry, we also resolve the *intra-unit-cell* symmetry of the charge ordered state, which is revealed to be a ***d-wave bond-order***. These results represent a fundamental advancement in our microscopic description of charge order in cuprates, and provide crucial insights for the understanding of its origin and interplay with superconductivity and magnetism.

The symmetry of charge order in the cuprates

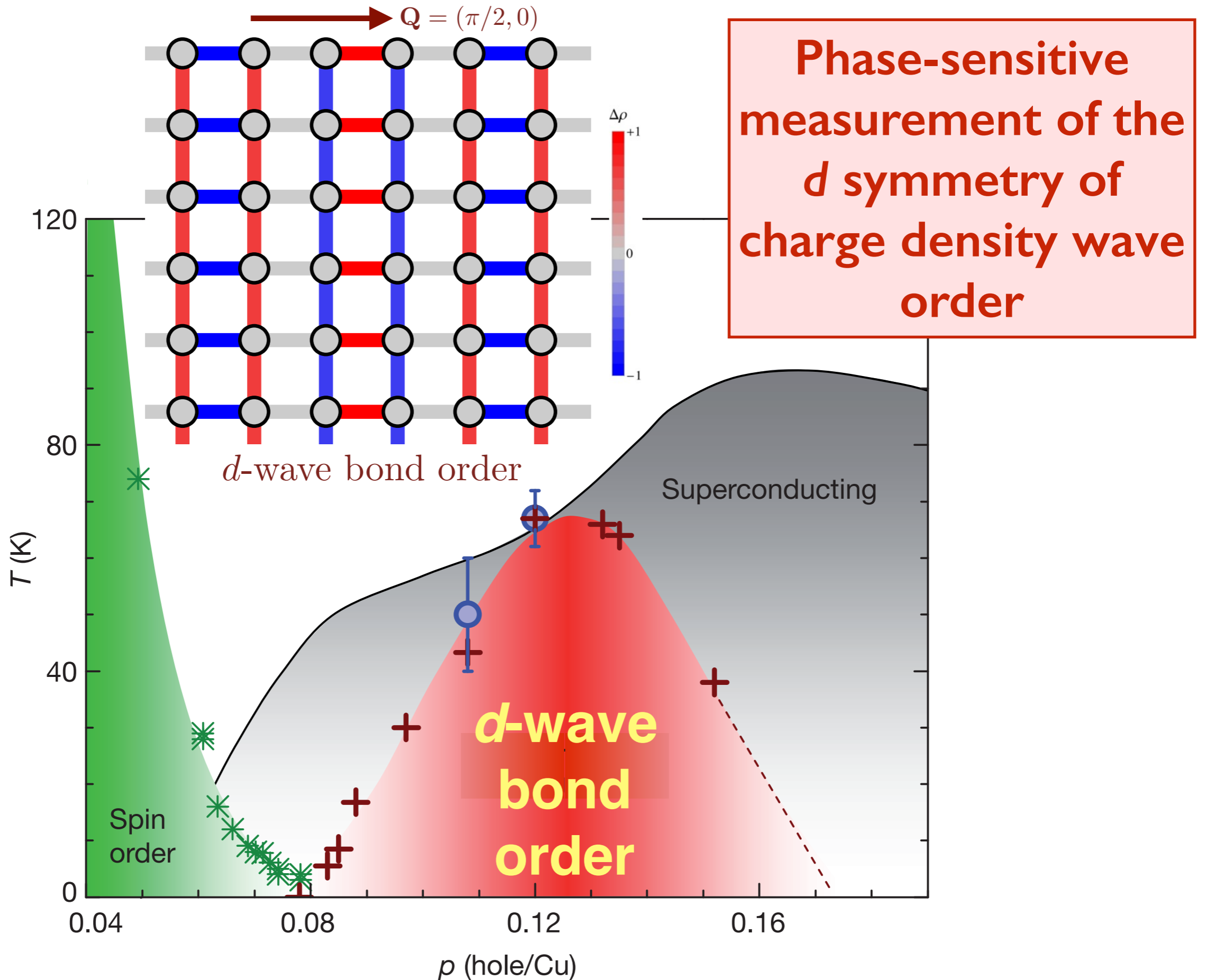
R. Comin, R. Sutarto, F. He, E. da Silva Neto, L. Chauviere, A. Frano, R. Liang, W.N. Hardy, D.A. Bonn, Y. Yoshida, H. Eisaki, J. E. Hoffman, B. Keimer, G.A. Sawatzky, and A. Damascelli, arXiv:1402.5415



Δ_{CDW}	Probability levels P (%)	
	Bidirectional	Unidirectional
Δ_s	30.3	38.8
$\Delta_{s'} (\cos k_x + \cos k_y)$	12.0	6.0
$\Delta_d (\cos k_x - \cos k_y)$	81.8	87.6



T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



Outline

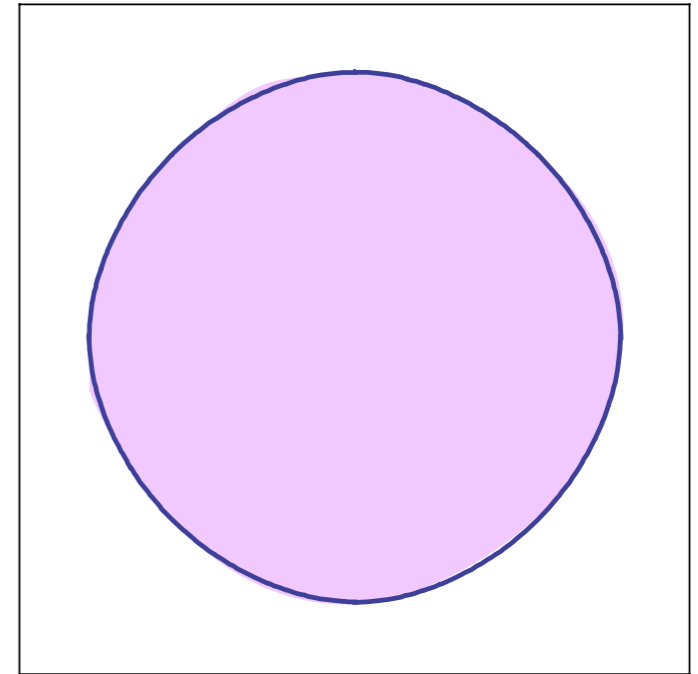
1. *d*-wave superconductivity
2. Low hole density state:
d-wave bond order
3. Theoretical background
4. Evolution of Fermi surface

Outline

1. *d*-wave superconductivity
2. Low hole density state:
d-wave bond order
3. Theoretical background
4. Evolution of Fermi surface

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

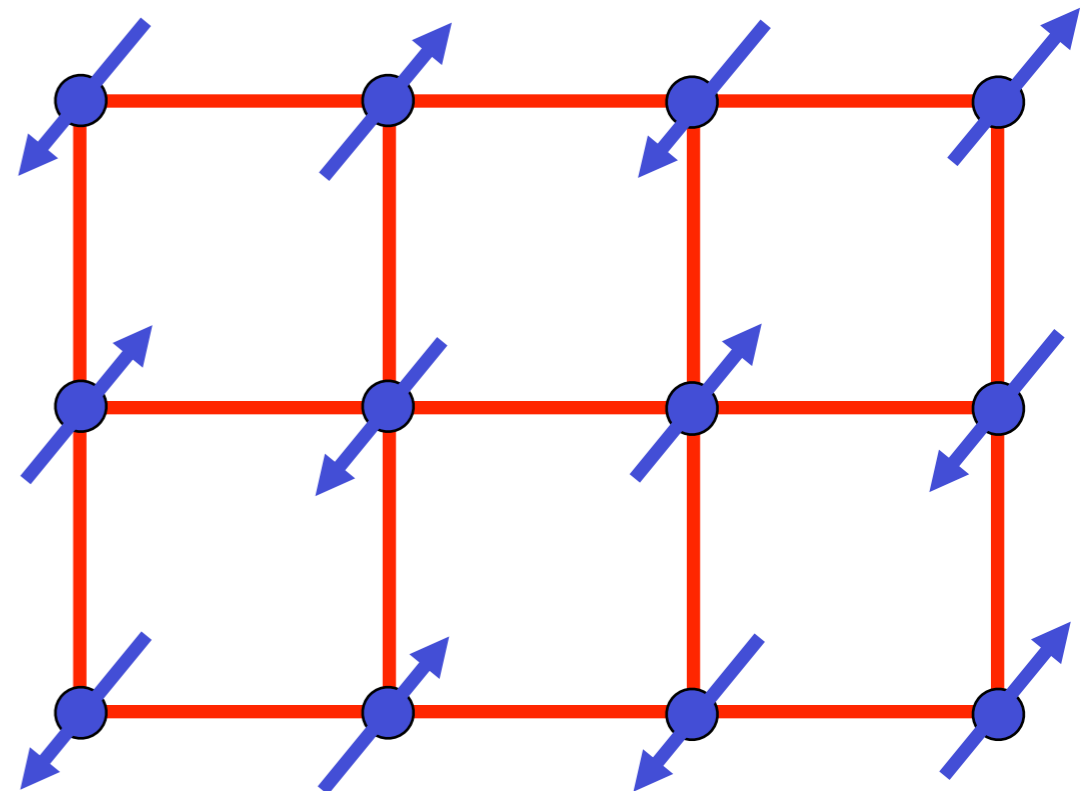


+

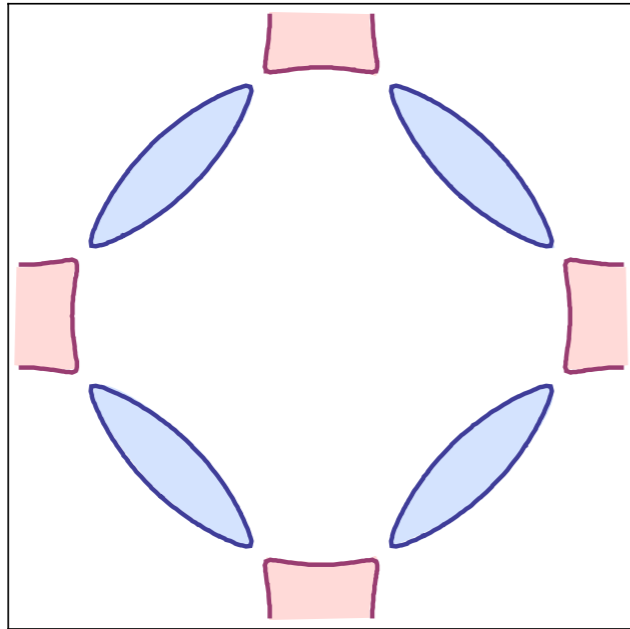
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\mathbf{K} = (\pi, \pi)$ is the ordering
wavevector.

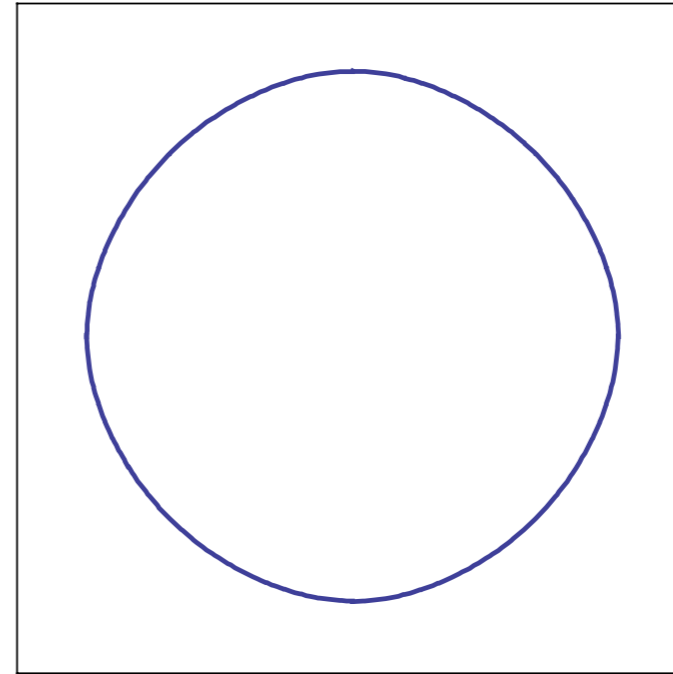


Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

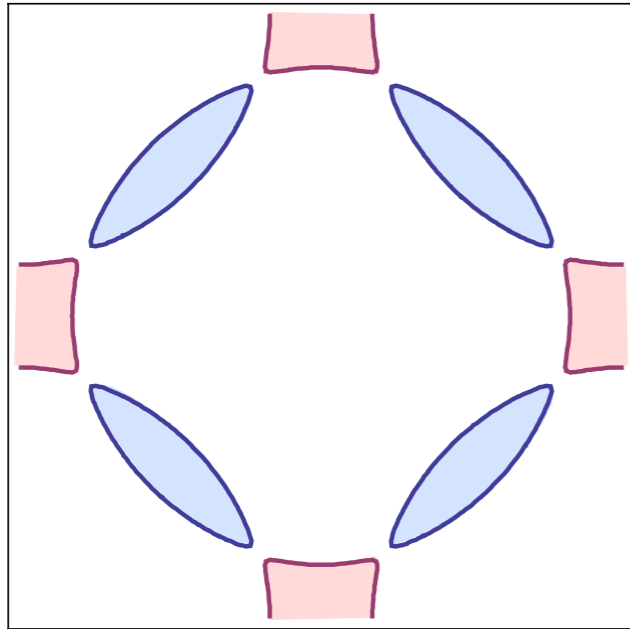


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

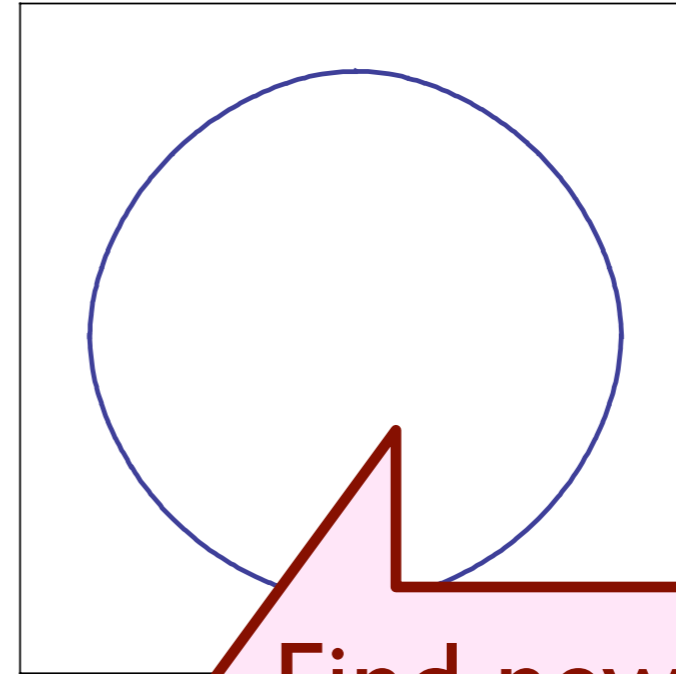
r

Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



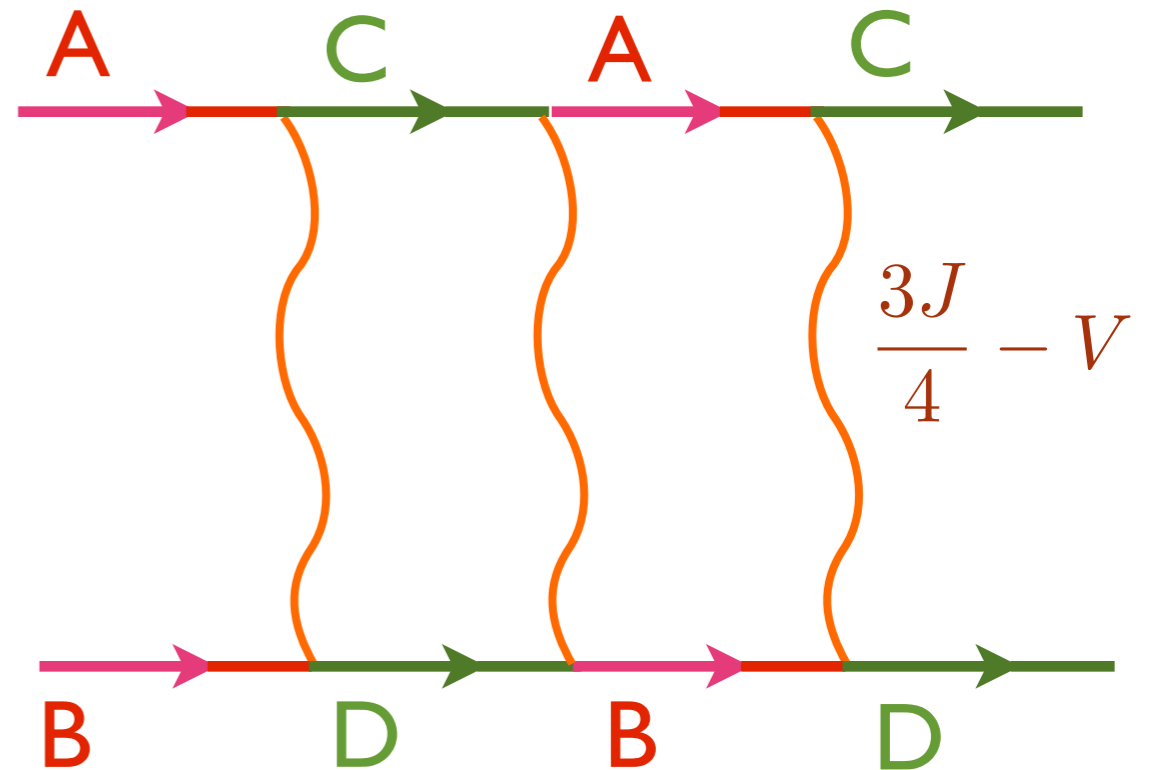
Metal with "large"
Fermi surface

Find new instabilities
upon approaching
critical point

r

Pairing “glue” from antiferromagnetic fluctuations

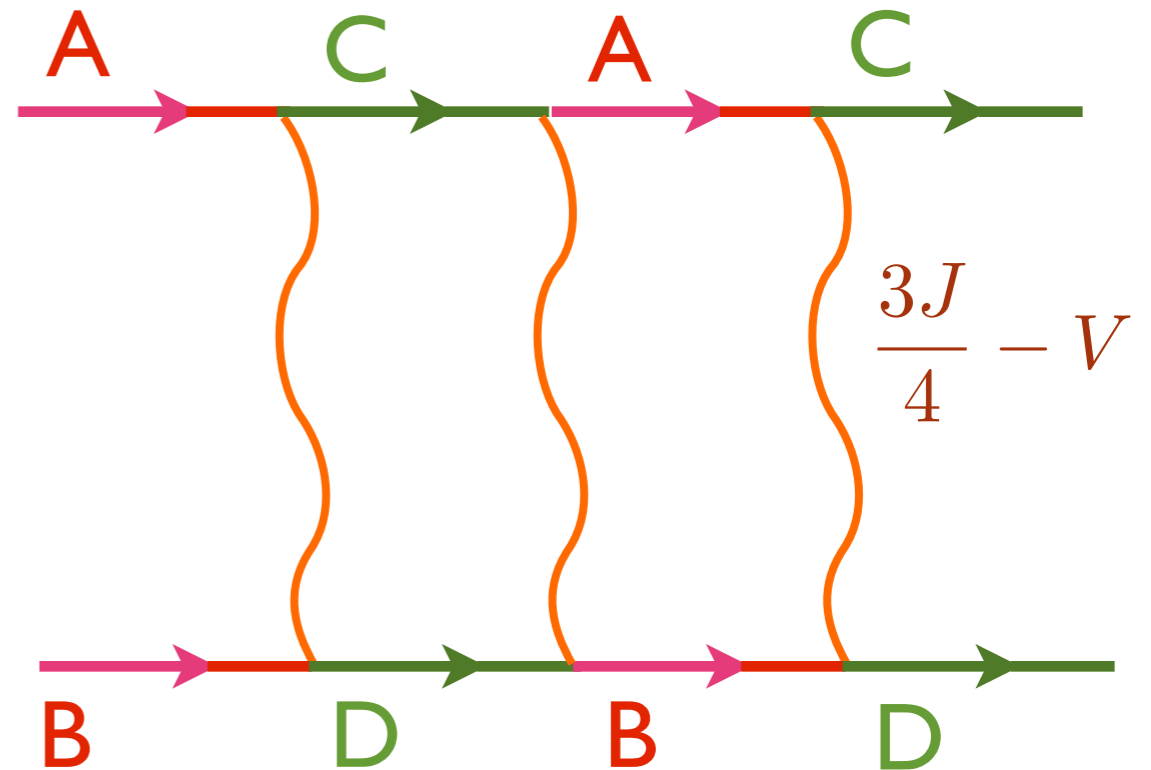
$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



Pairing “glue” from antiferromagnetic fluctuations

Electron hopping

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



V.J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)

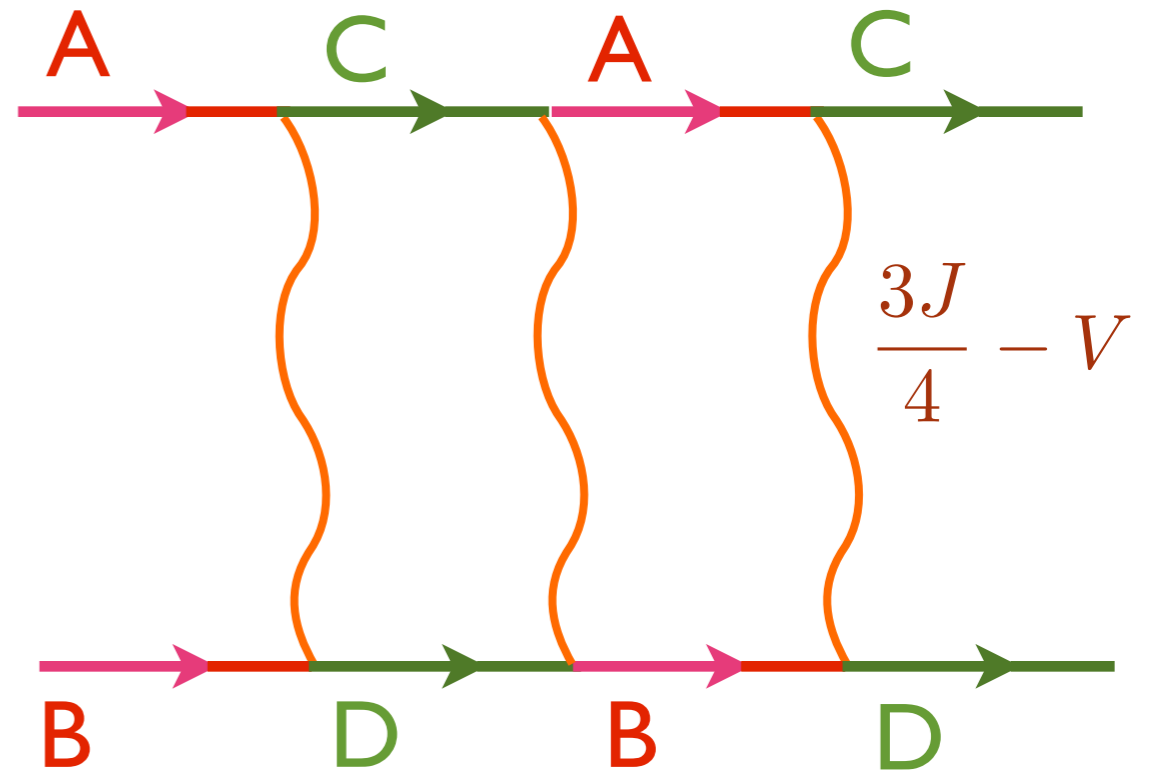
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)

K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

Pairing “glue” from antiferromagnetic fluctuations

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

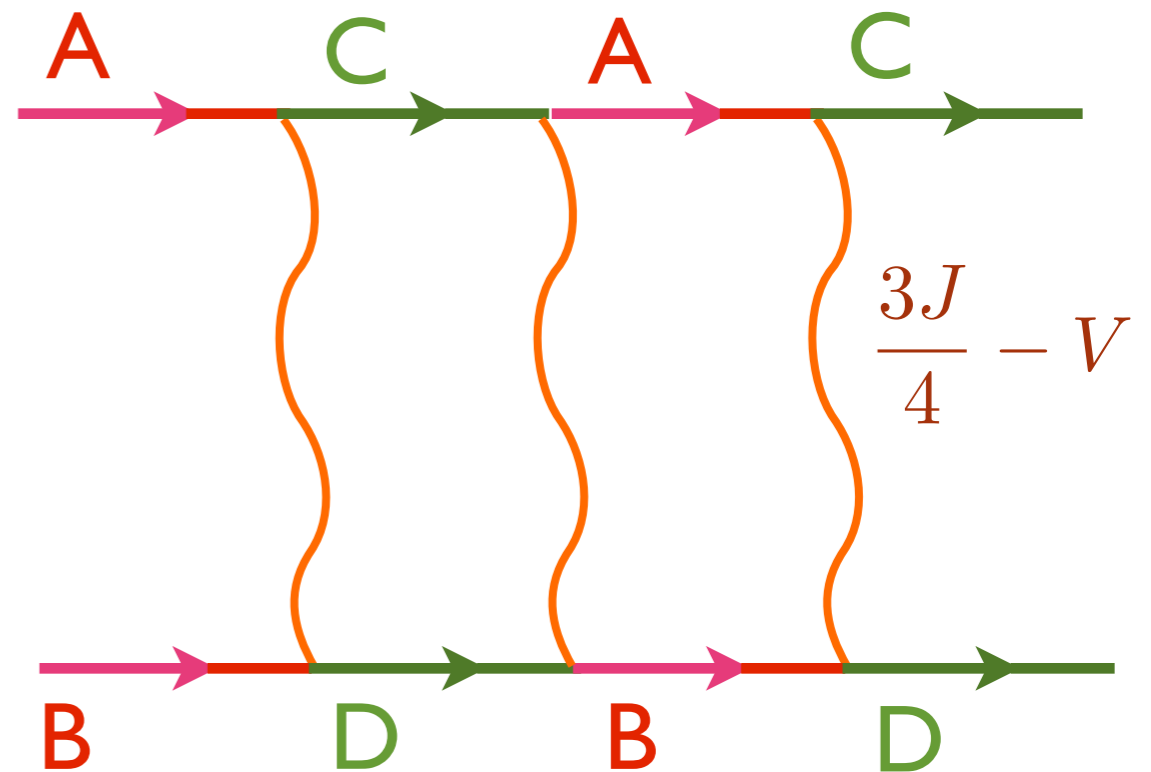
Antiferromagnetic
exchange
interaction



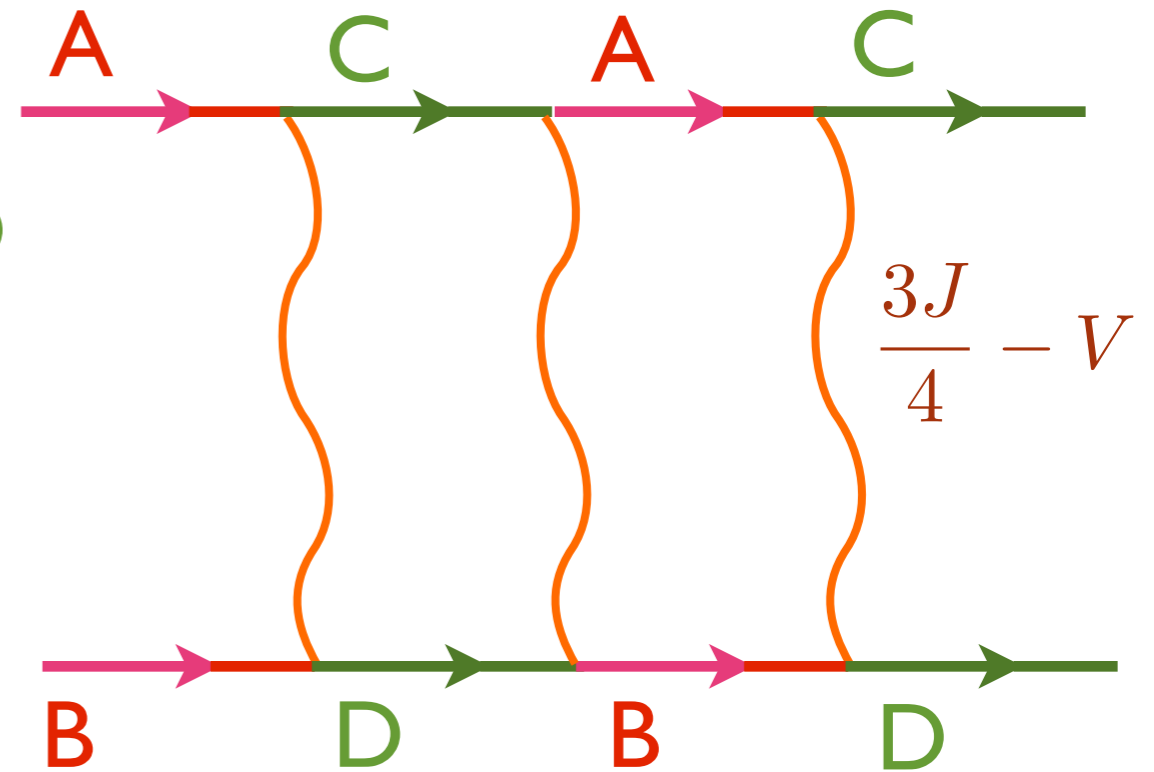
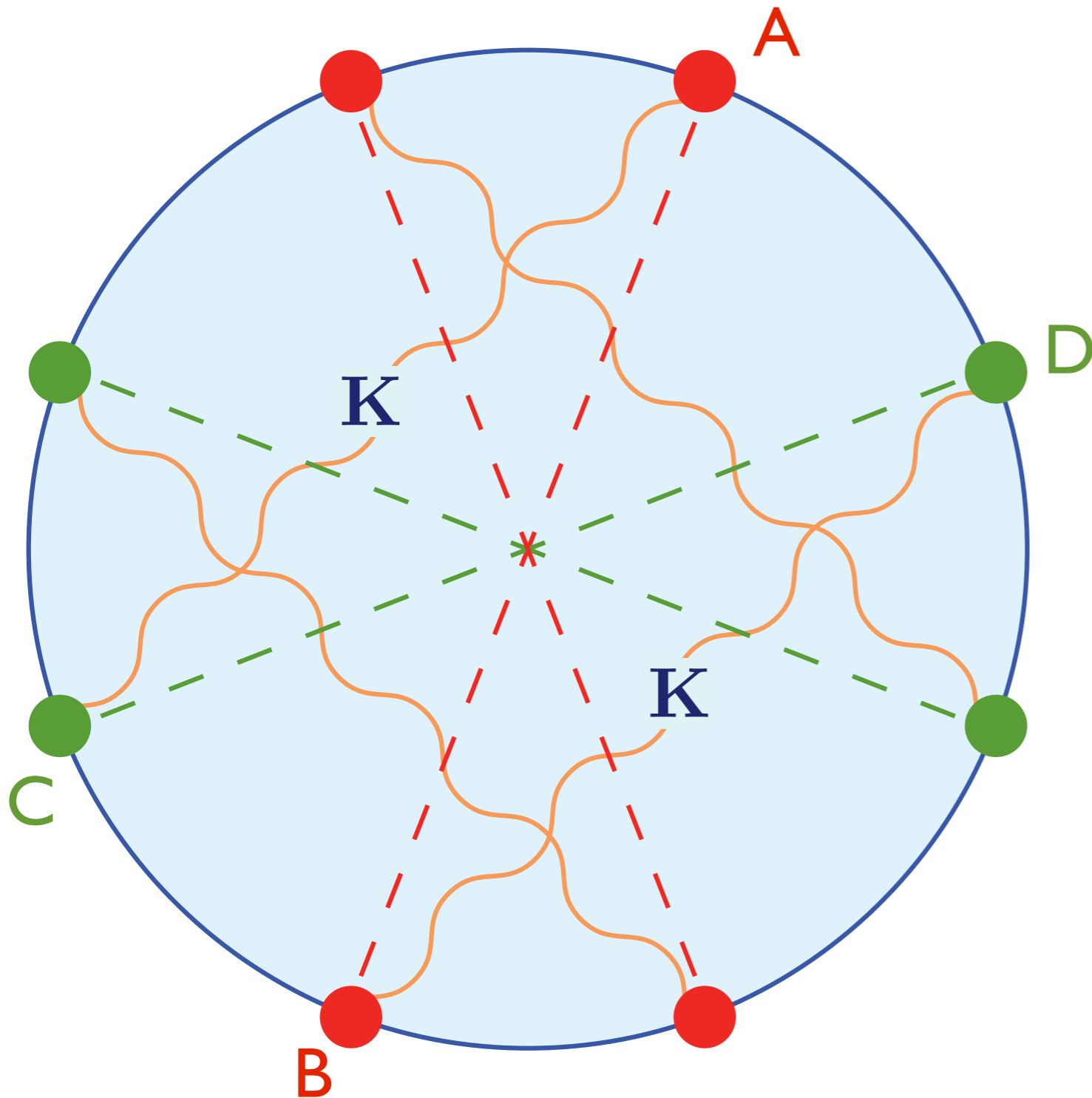
Pairing “glue” from antiferromagnetic fluctuations

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

Coulomb repulsion

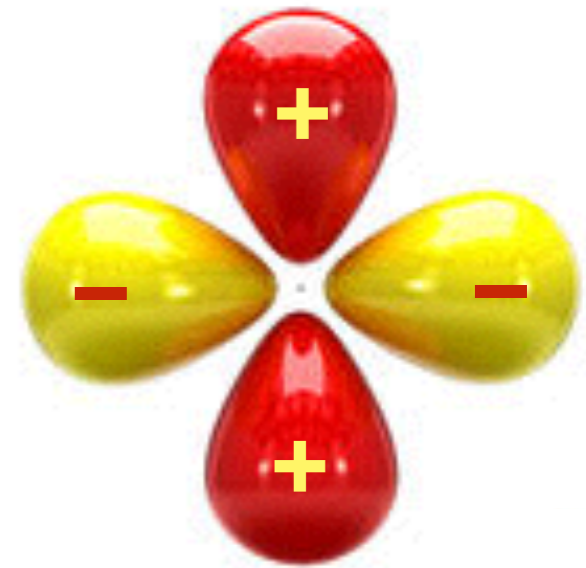
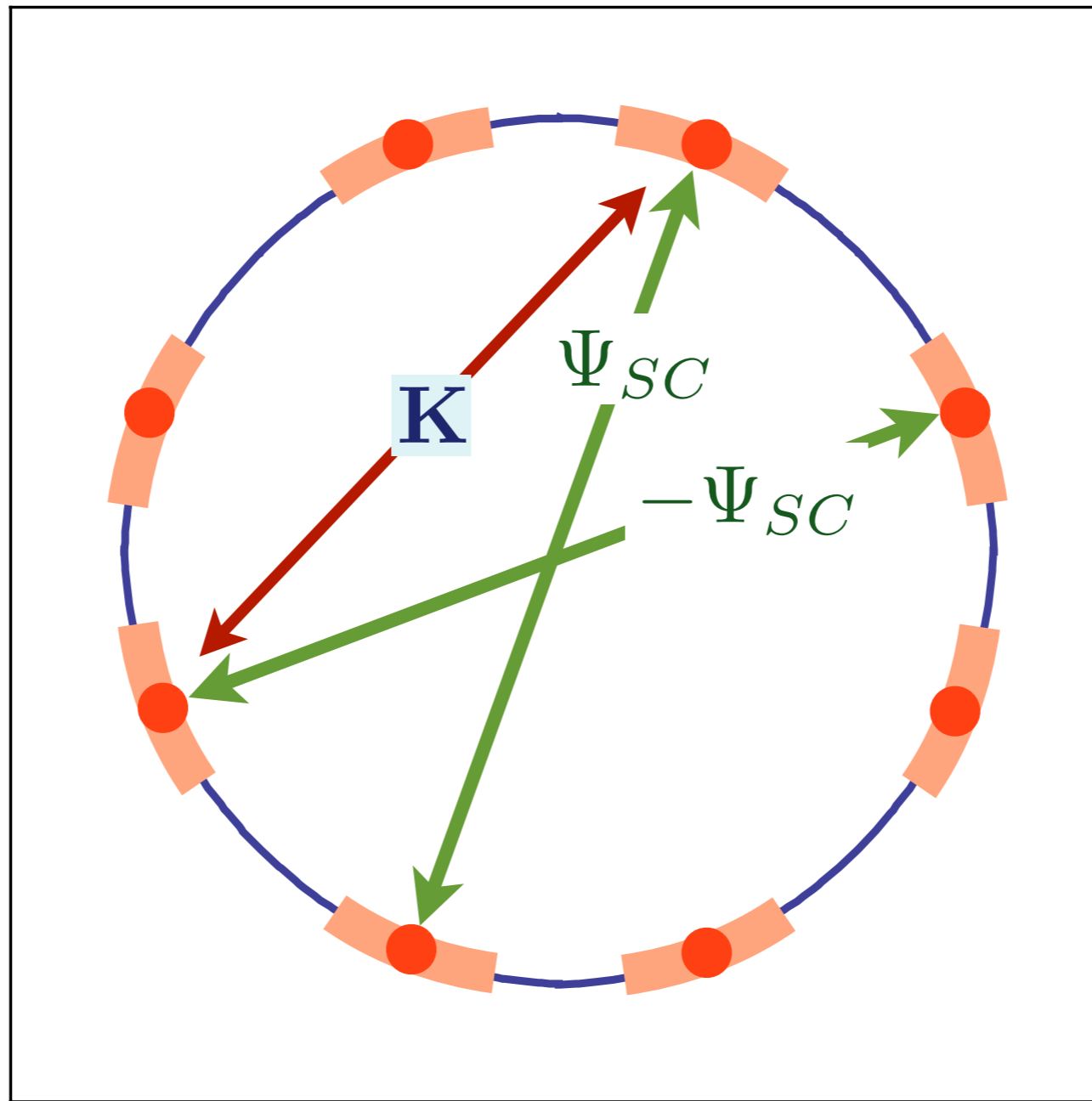


Pairing “glue” from antiferromagnetic fluctuations



V.J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} (\cos k_x - \cos k_y) \Psi_{SC}$$



**d-wave superconductor:
sign-changing pairing amplitude**

V. J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

Pseudospin symmetry of the exchange interaction

$$H_J = J \vec{S}_1 \cdot \vec{S}_2$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} J \left(\Psi_{1\alpha a}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{1\beta a} \right) \cdot \left(\Psi_{2\gamma b}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{2\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site $\Psi_{i\alpha a} \rightarrow U_{i,ab} \Psi_{i\alpha b}$

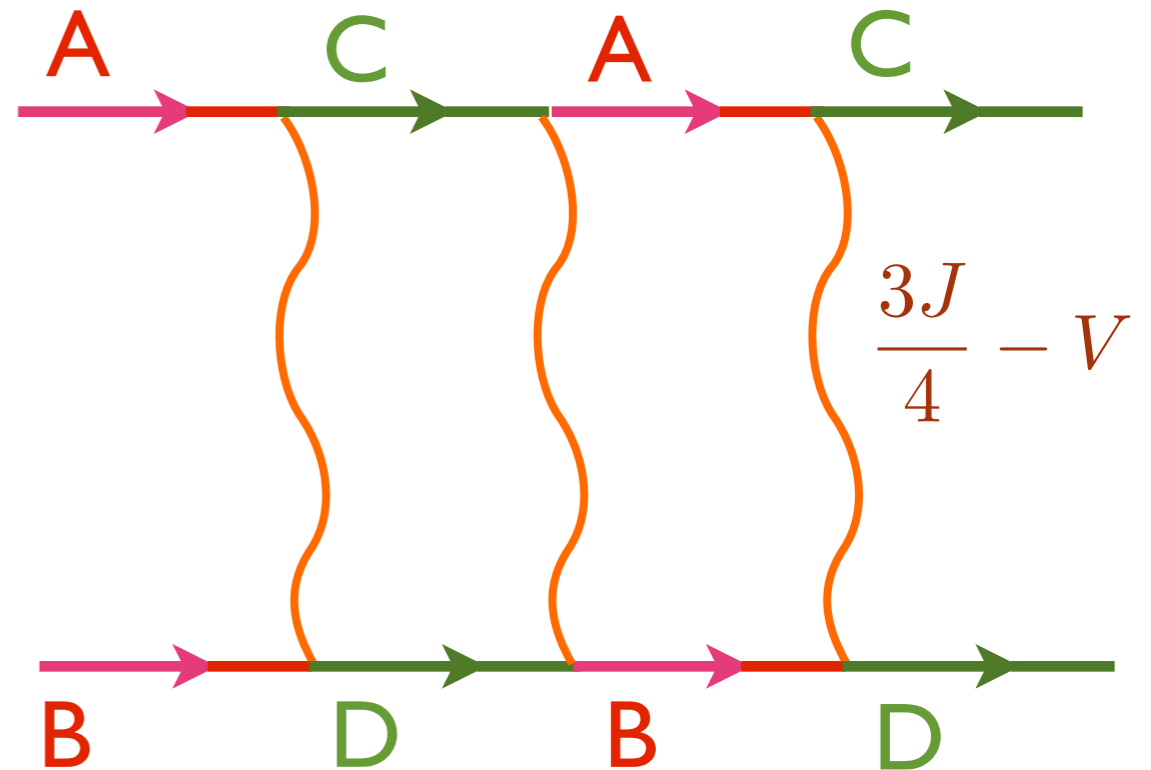
I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)

E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)

P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

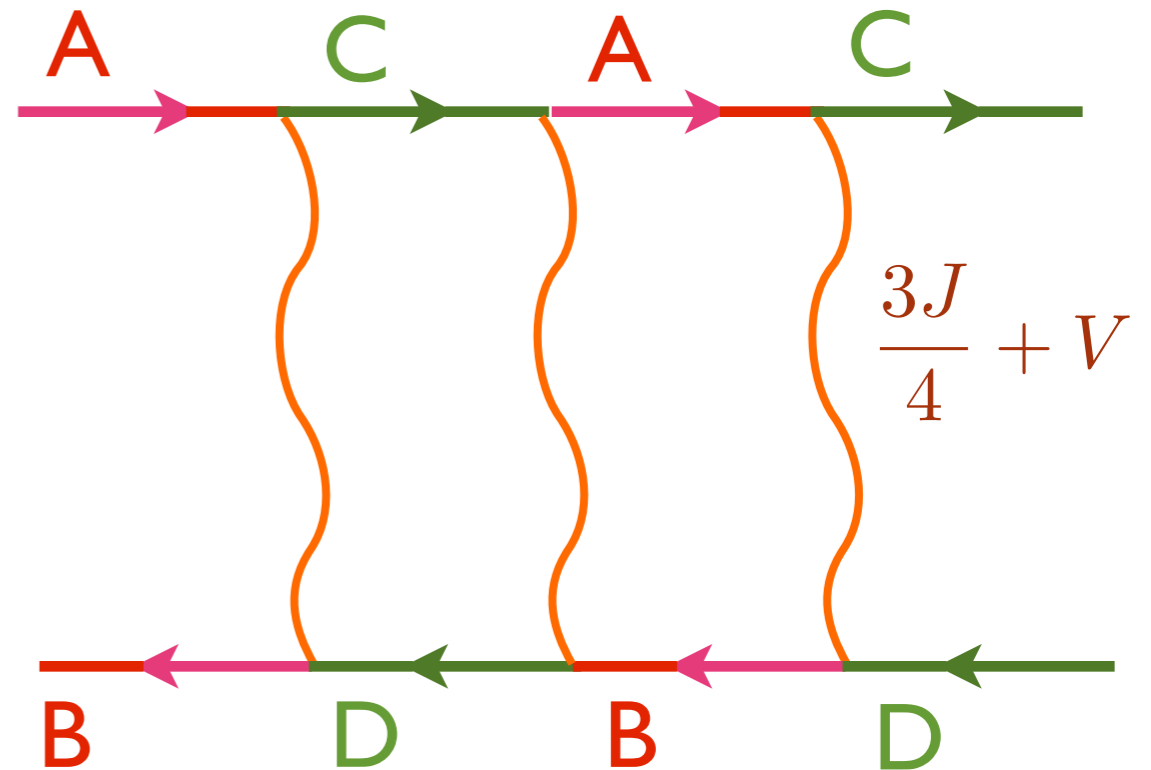
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 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



Same “glue” leads to *d*-wave particle-hole pairing !

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

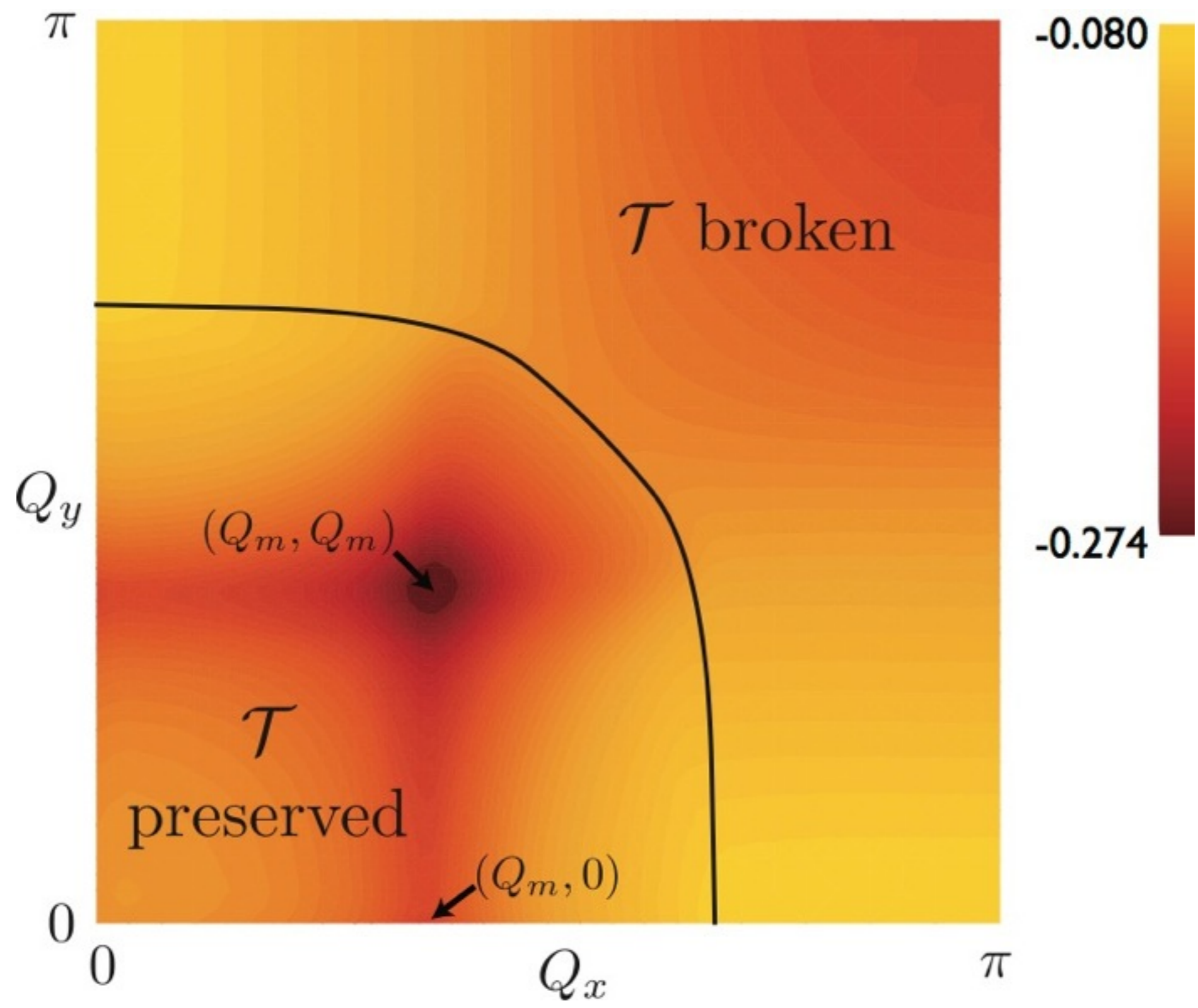


M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

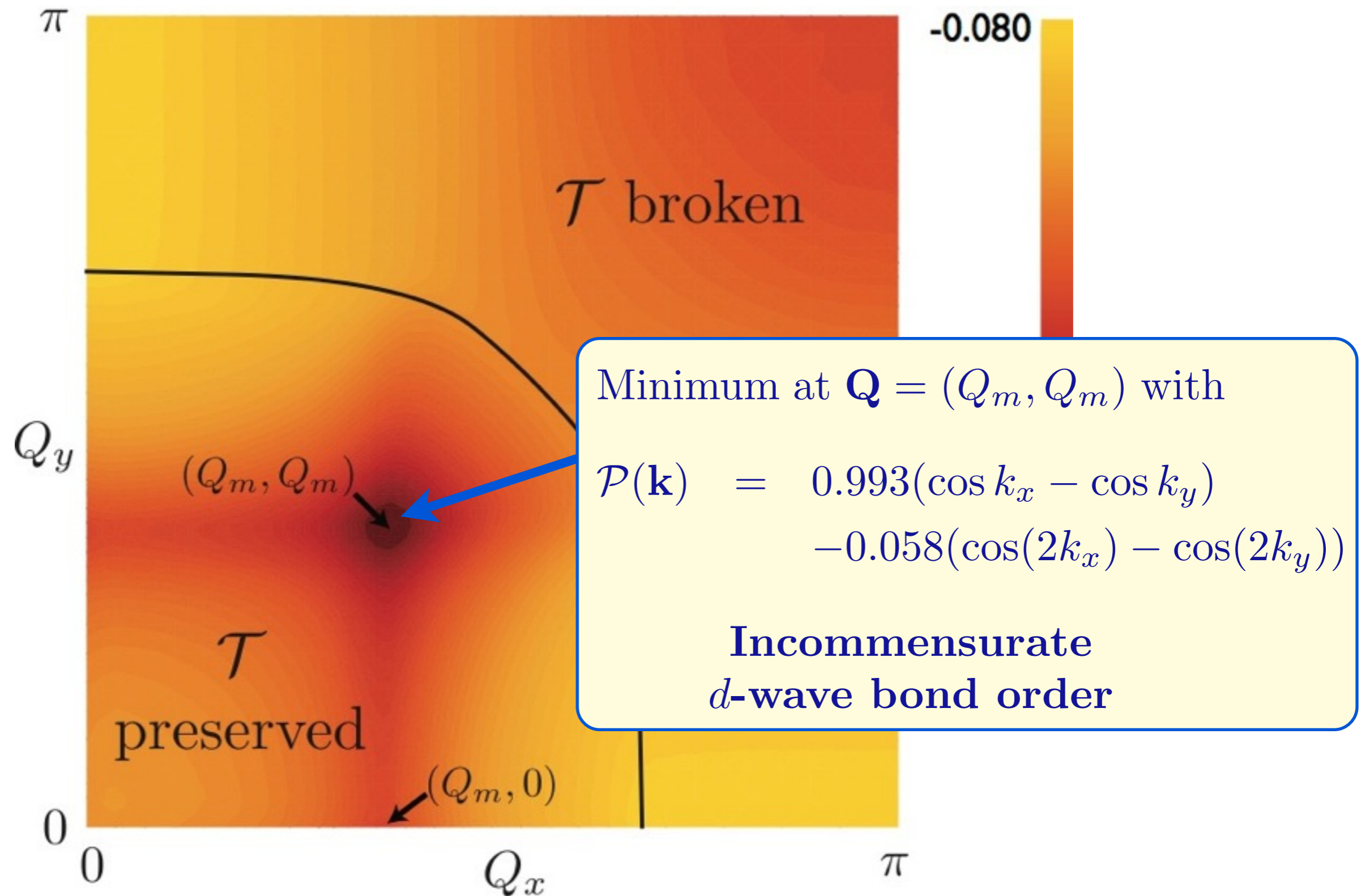
S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)

J. C. Davis and Dung-Hai Lee, *Proc. Natl. Acad. Sci.* **110**, 17623 (2013)

J. D. Sau and S. Sachdev, *Phys. Rev. B* **89**, 075129 (2014)

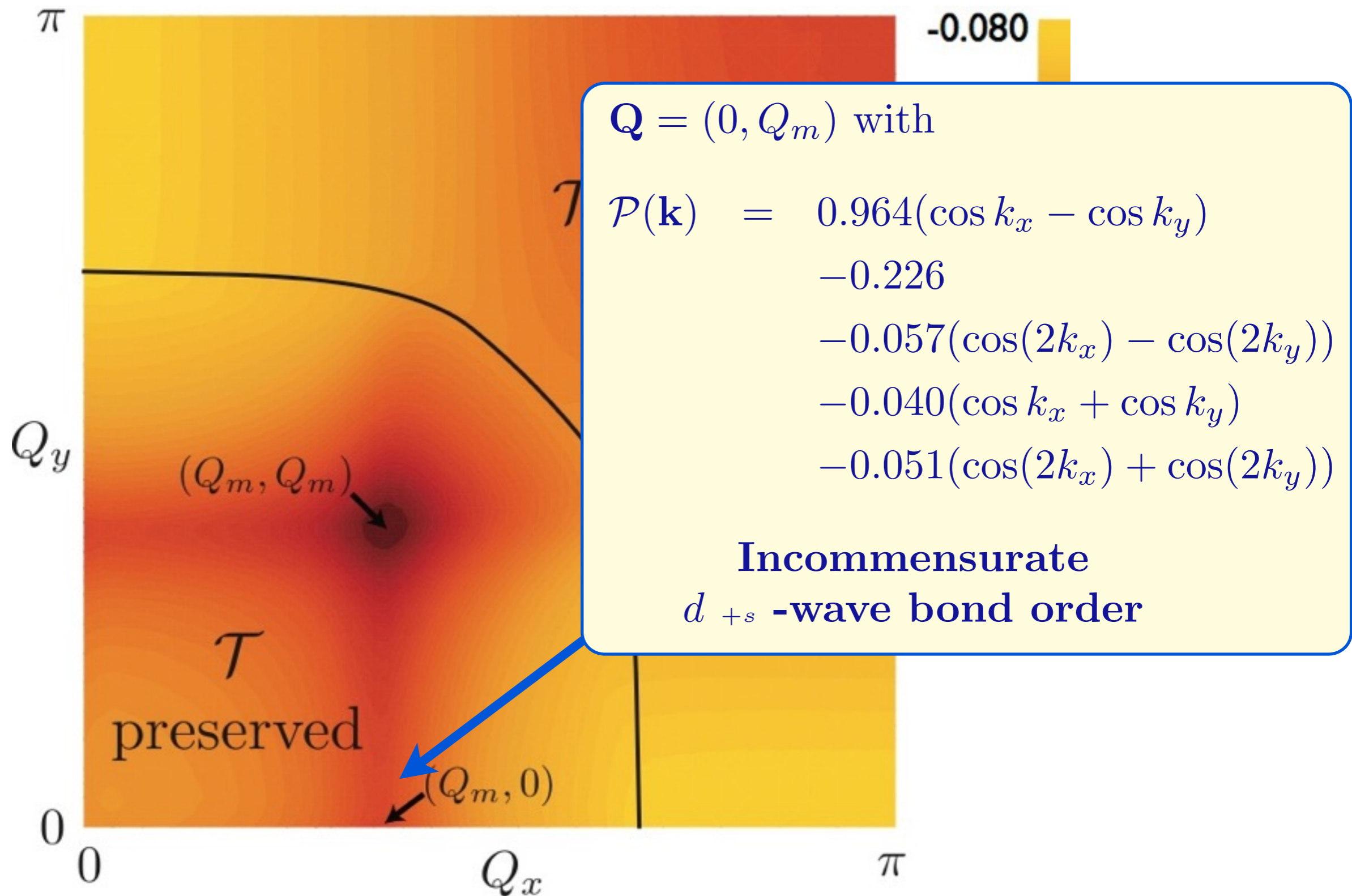


Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order $\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = [\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$



Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

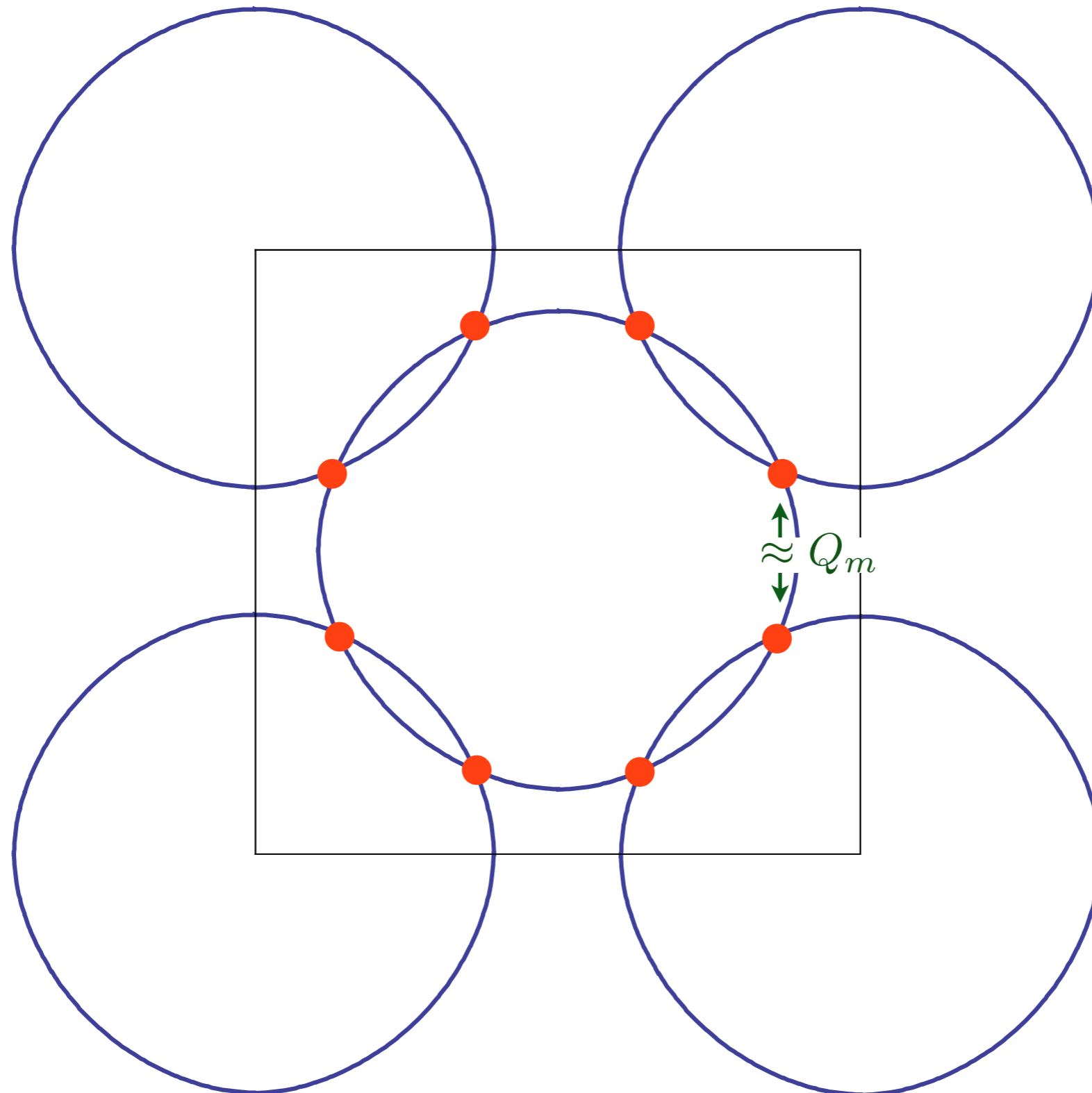
$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$



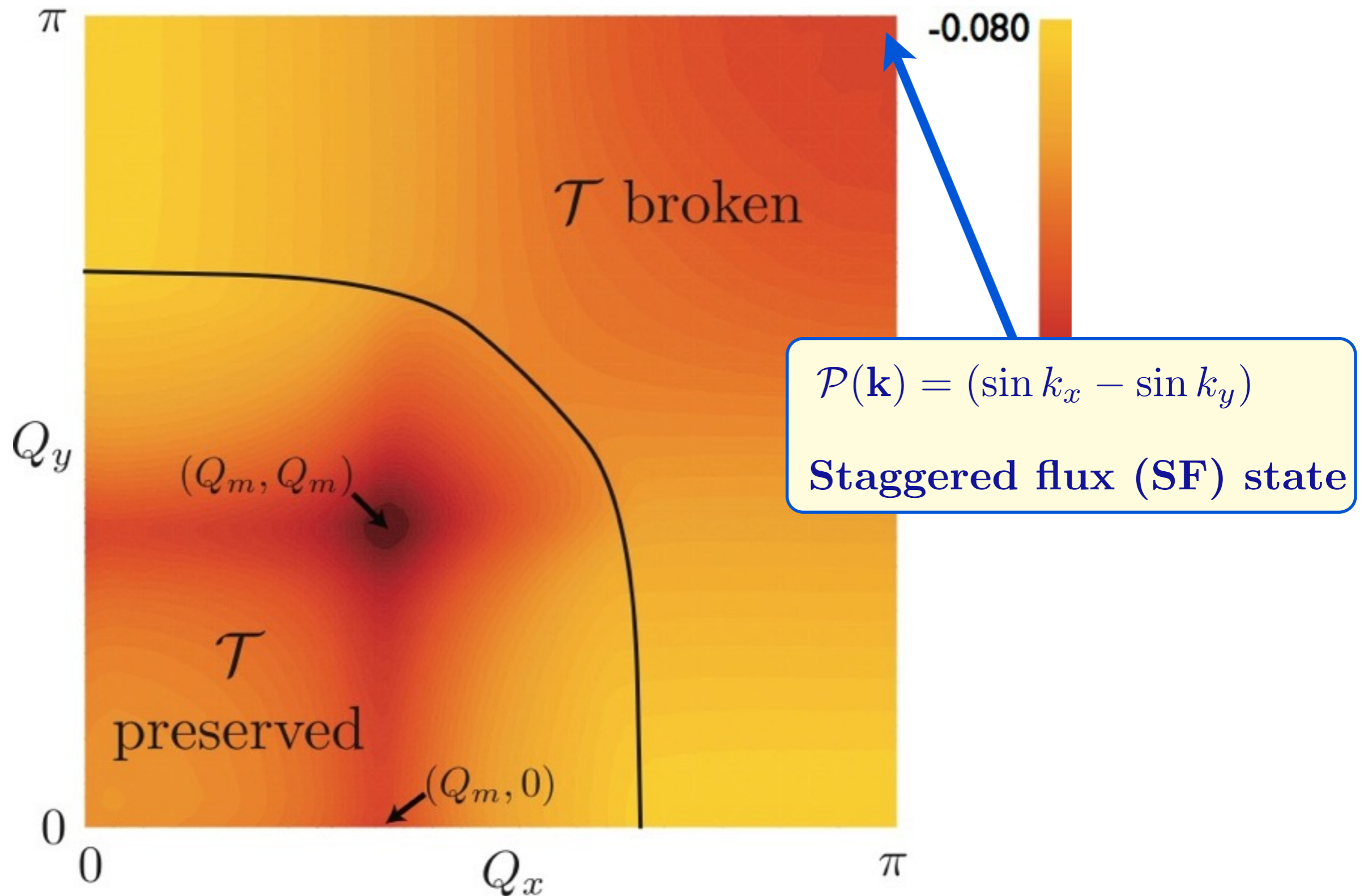
Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

Fermi surface+antiferromagnetism



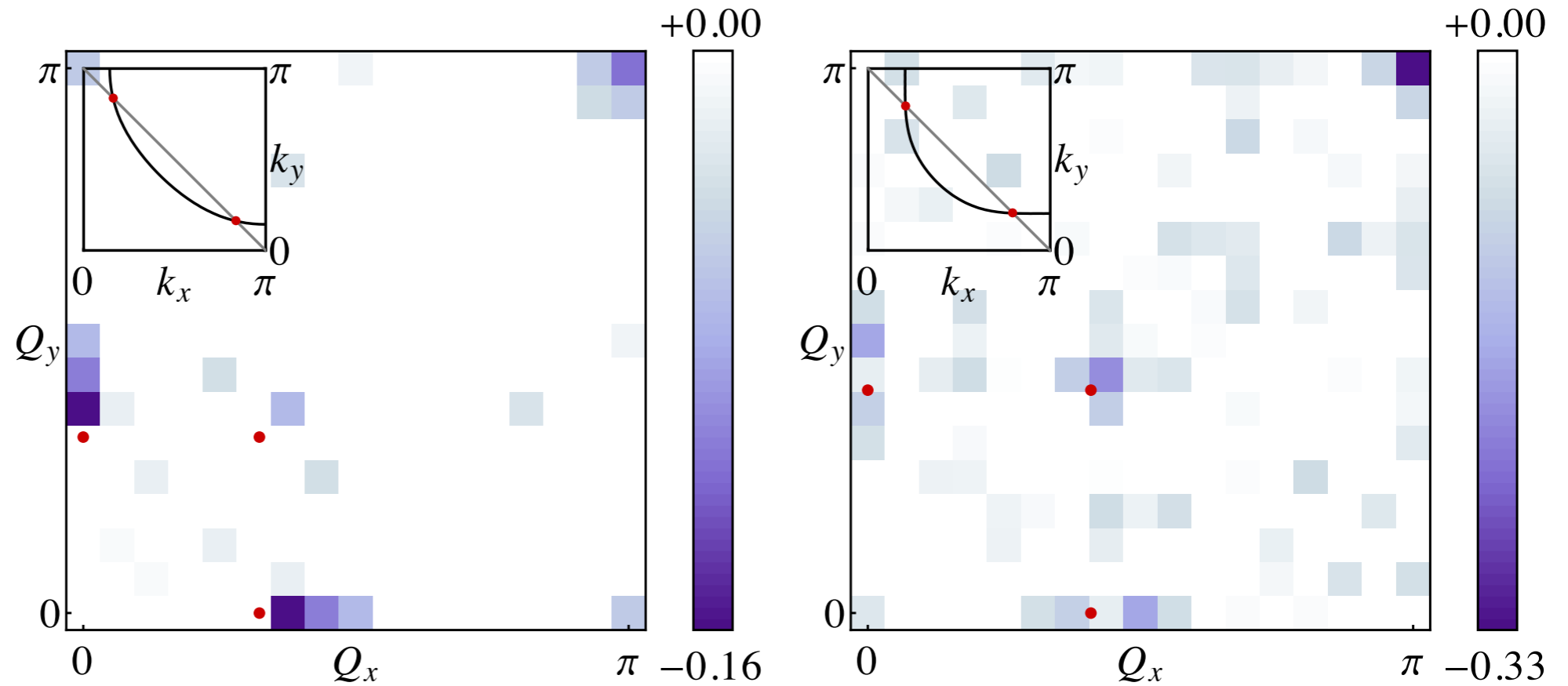
Q_m is approximately the separation between hotspots.



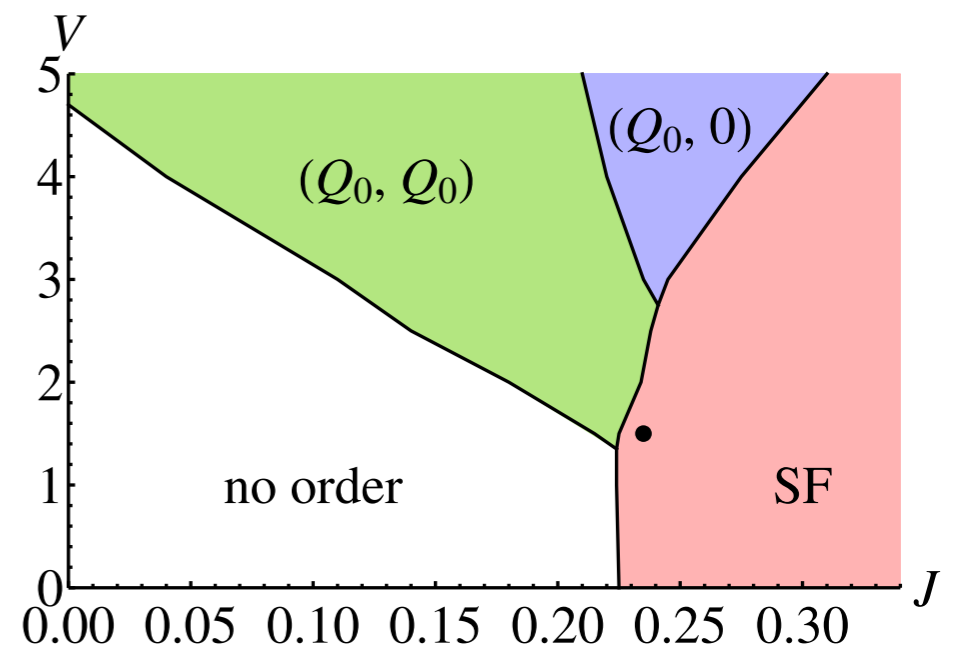
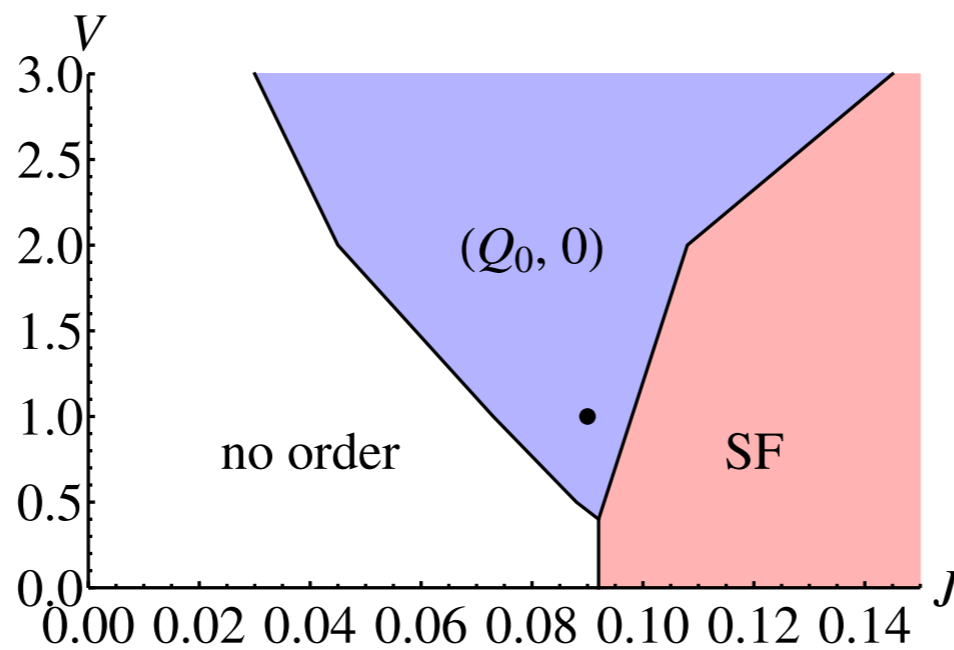
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$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j) / 2}$$

Results of a variational Monte Carlo computation on a wavefunction with double-occupancy projected out.
 A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.4807



Andrea Allais

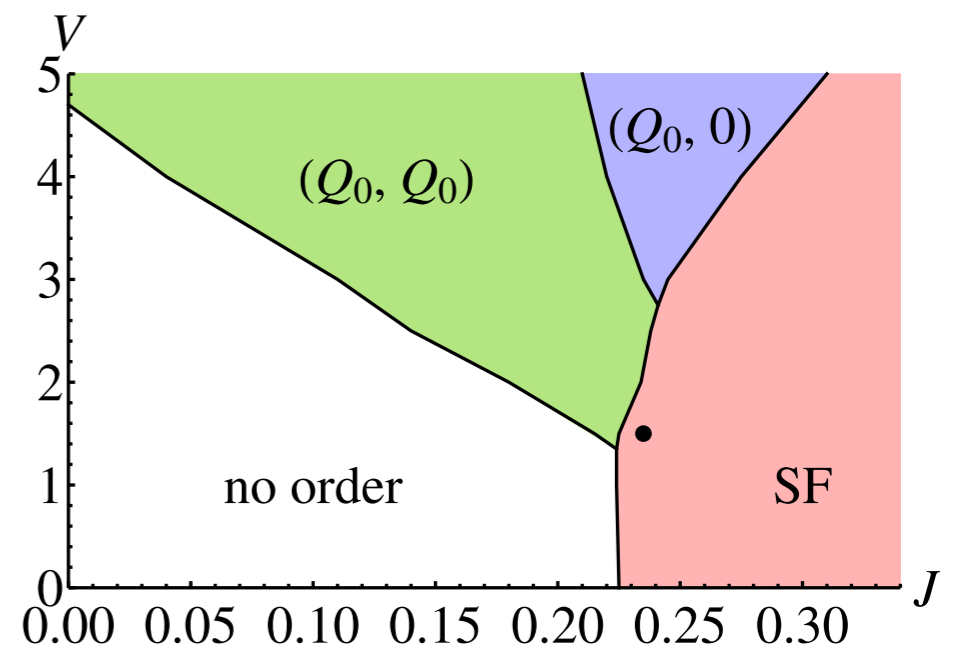
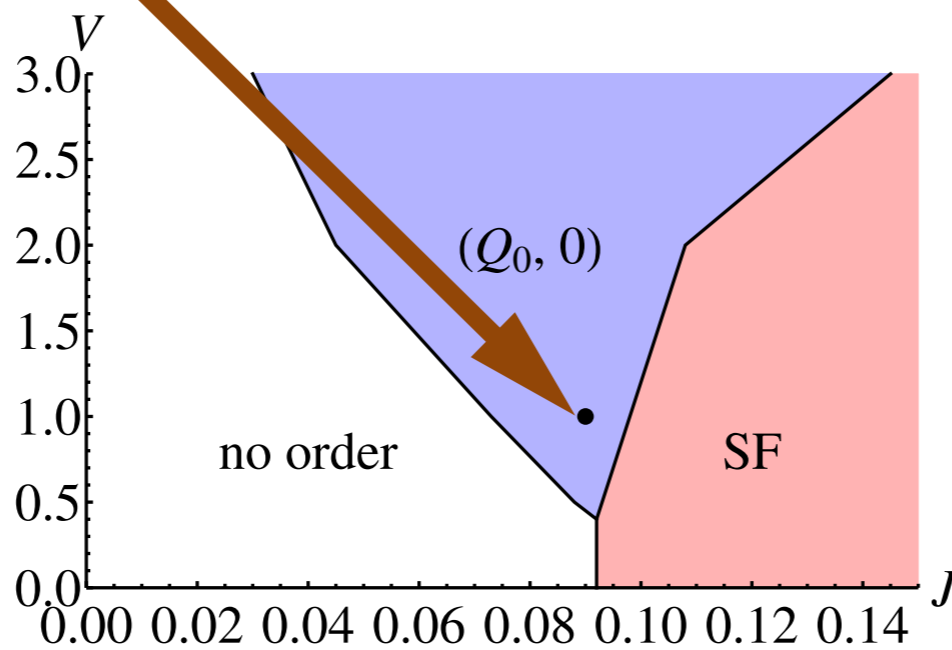
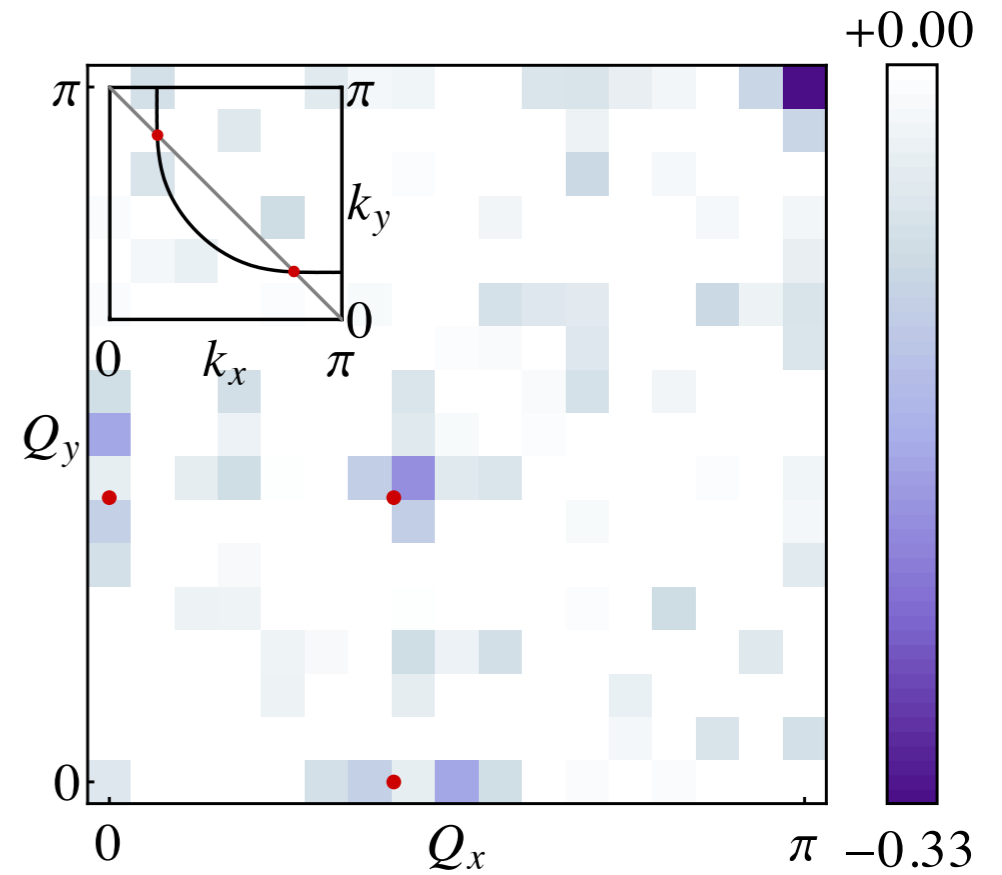
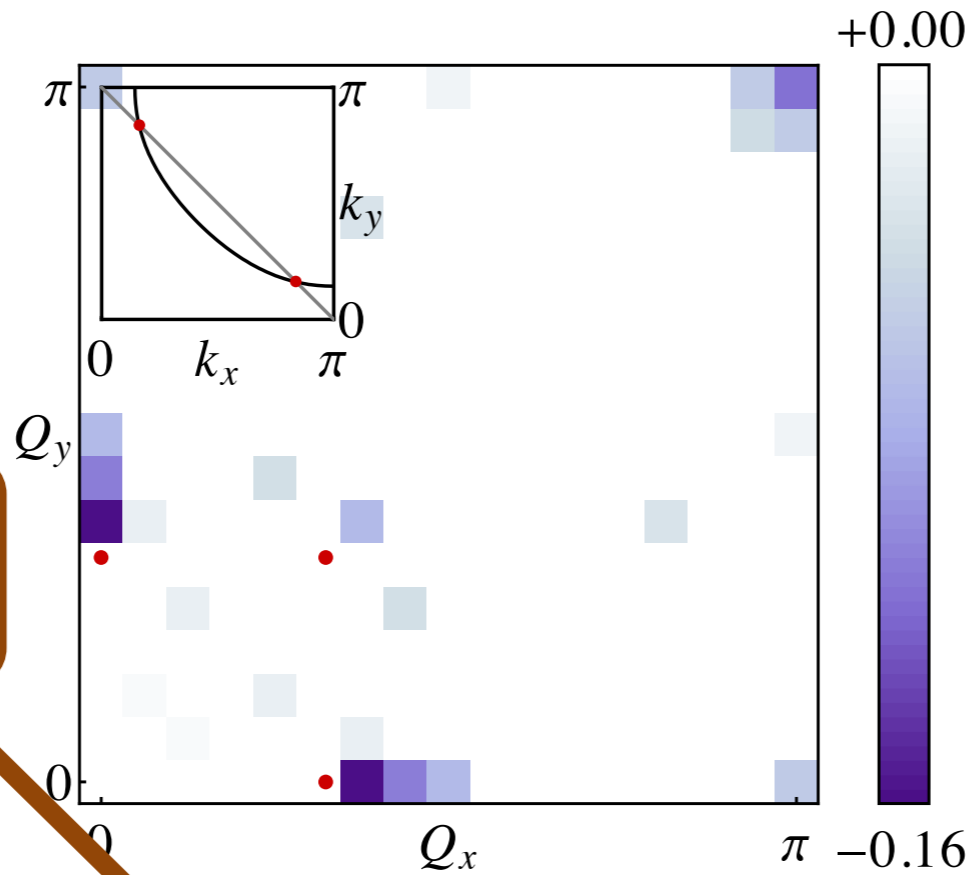


Results of a variational Monte Carlo computation on a wavefunction with double-occupancy projected out.
 A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.4807

Q-plot above at this value of J, V



Andrea Allais



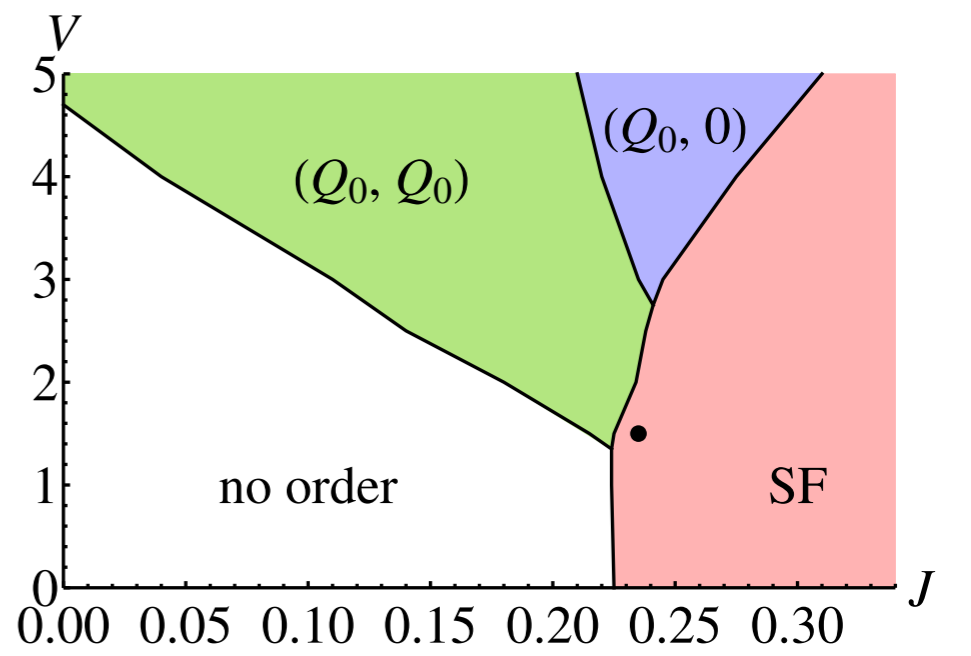
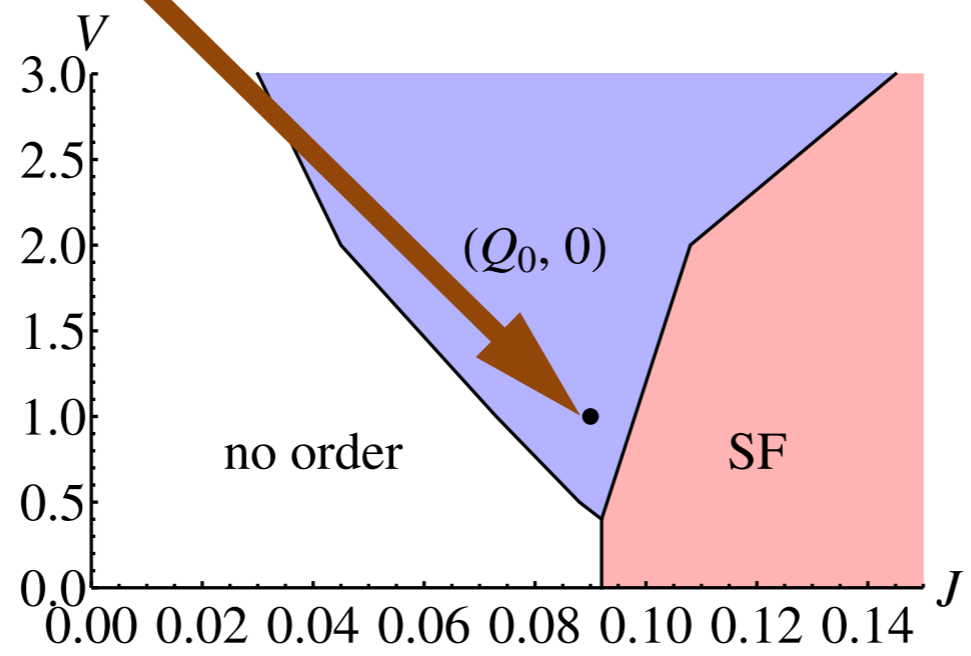
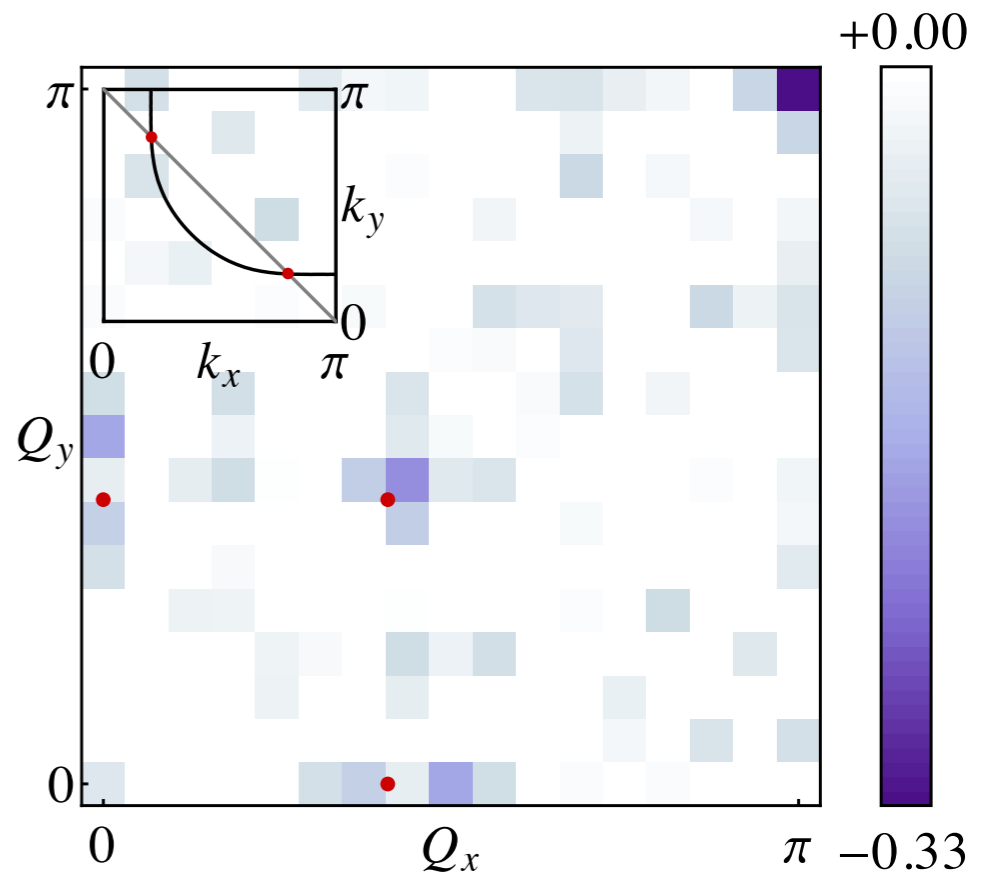
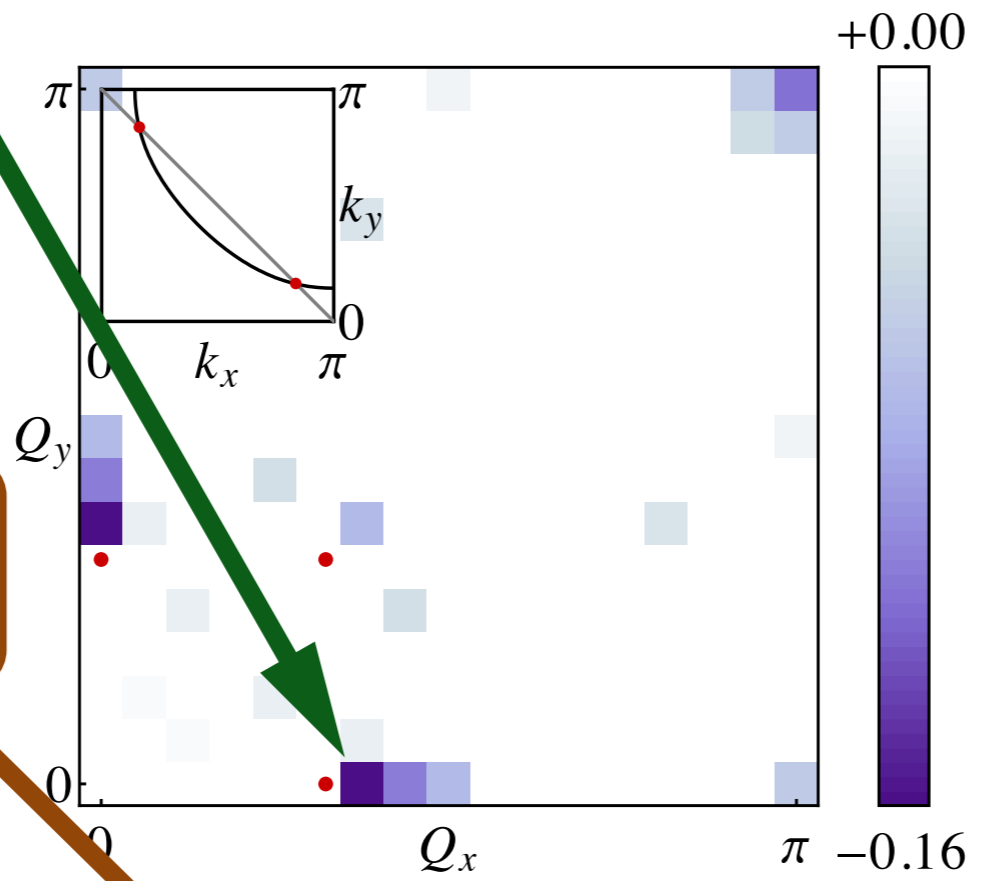
Results of a variational Monte Carlo computation on a wavefunction with double-occupancy projected out.
 A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.4807

Q of the lowest energy state. Predominantly *d* wave

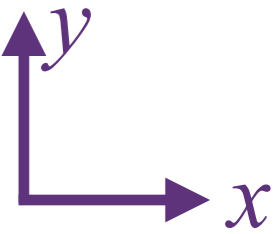
Q-plot above at this value of J, V



Andrea Allais



d -wave bond order.

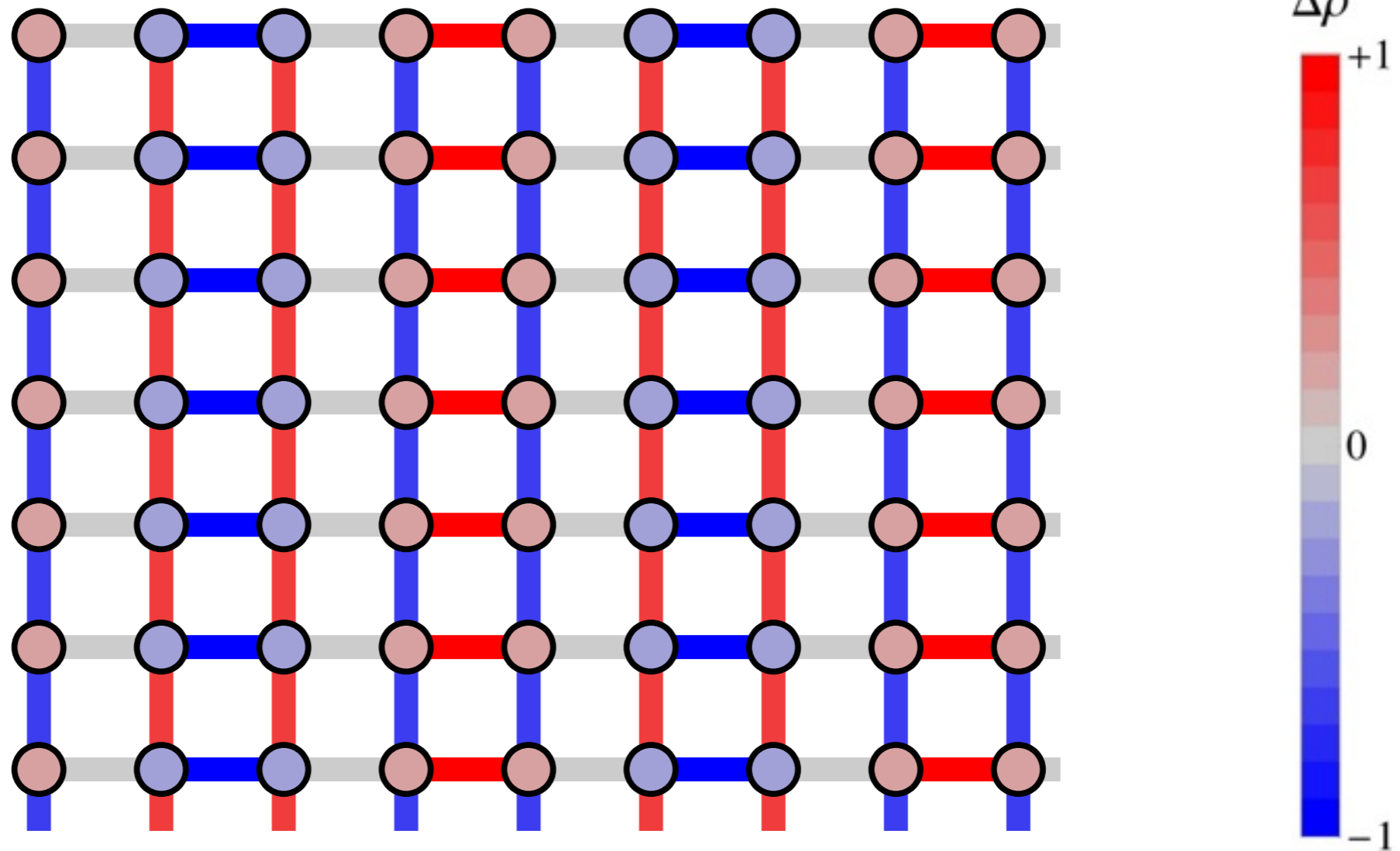


Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [0.3 + \cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = (\pi/2, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



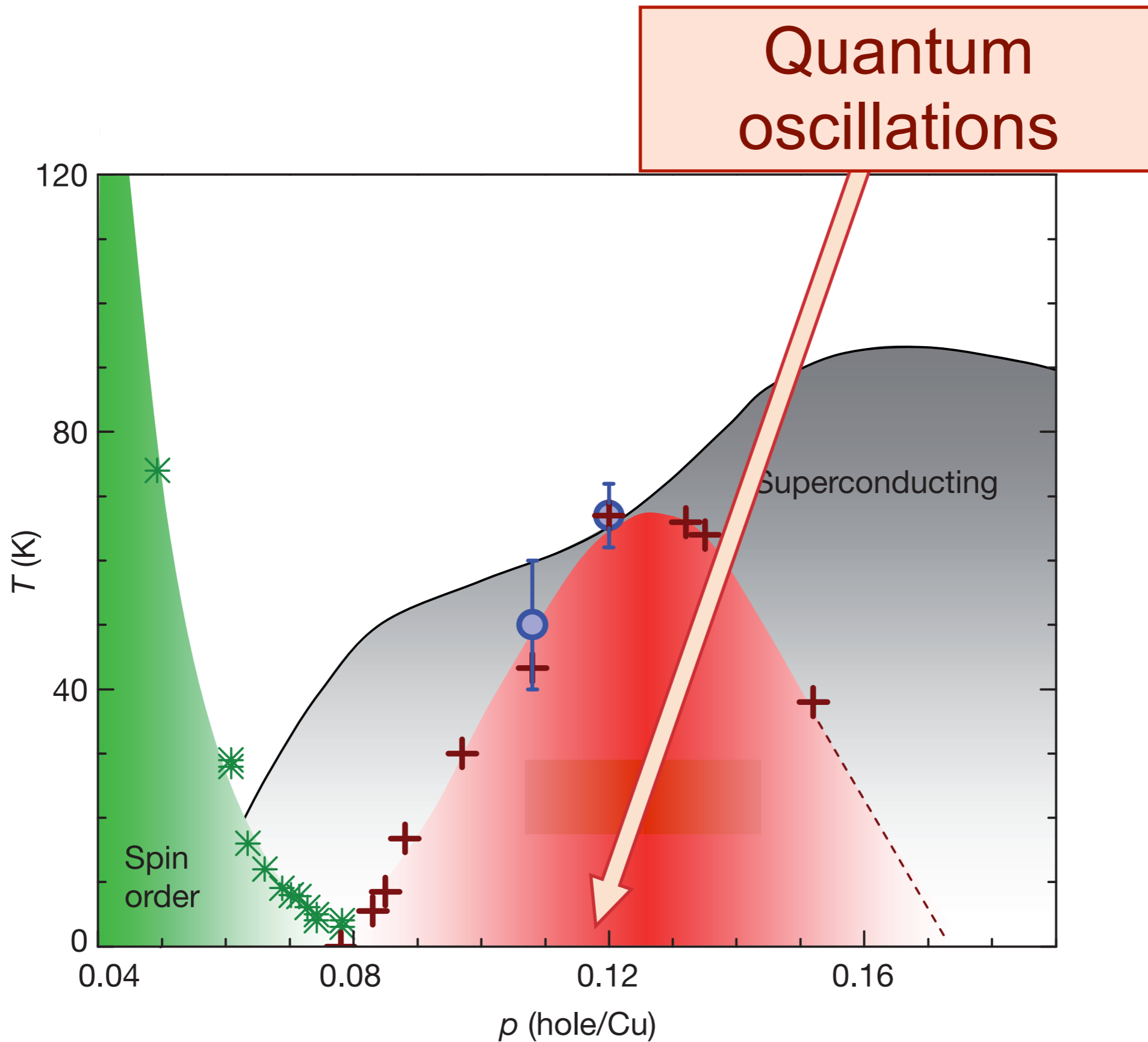
This d -wave bond order was first discussed in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Outline

1. *d*-wave superconductivity
2. Low hole density state:
d-wave bond order
3. Theoretical background
4. Evolution of Fermi surface

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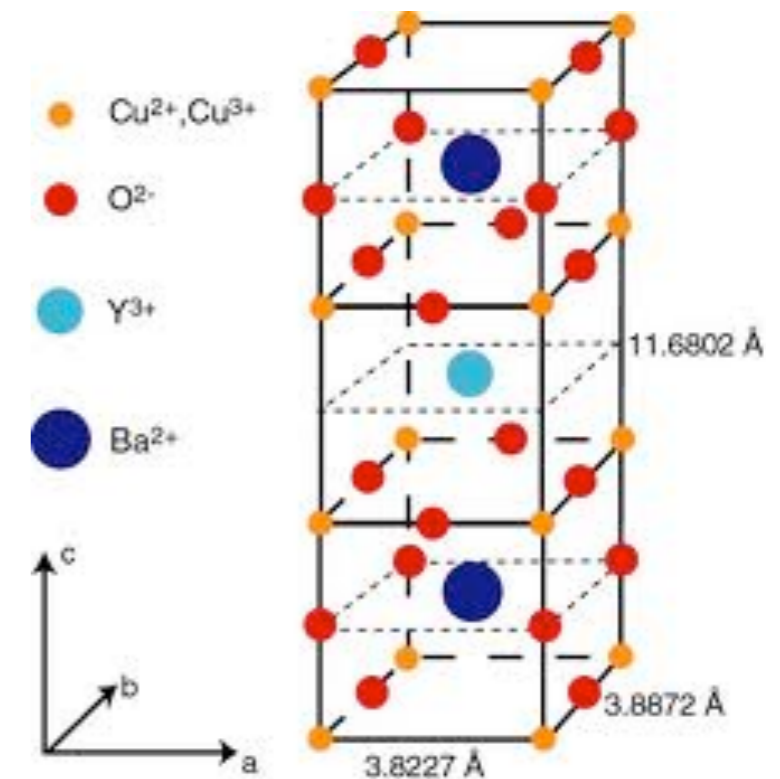
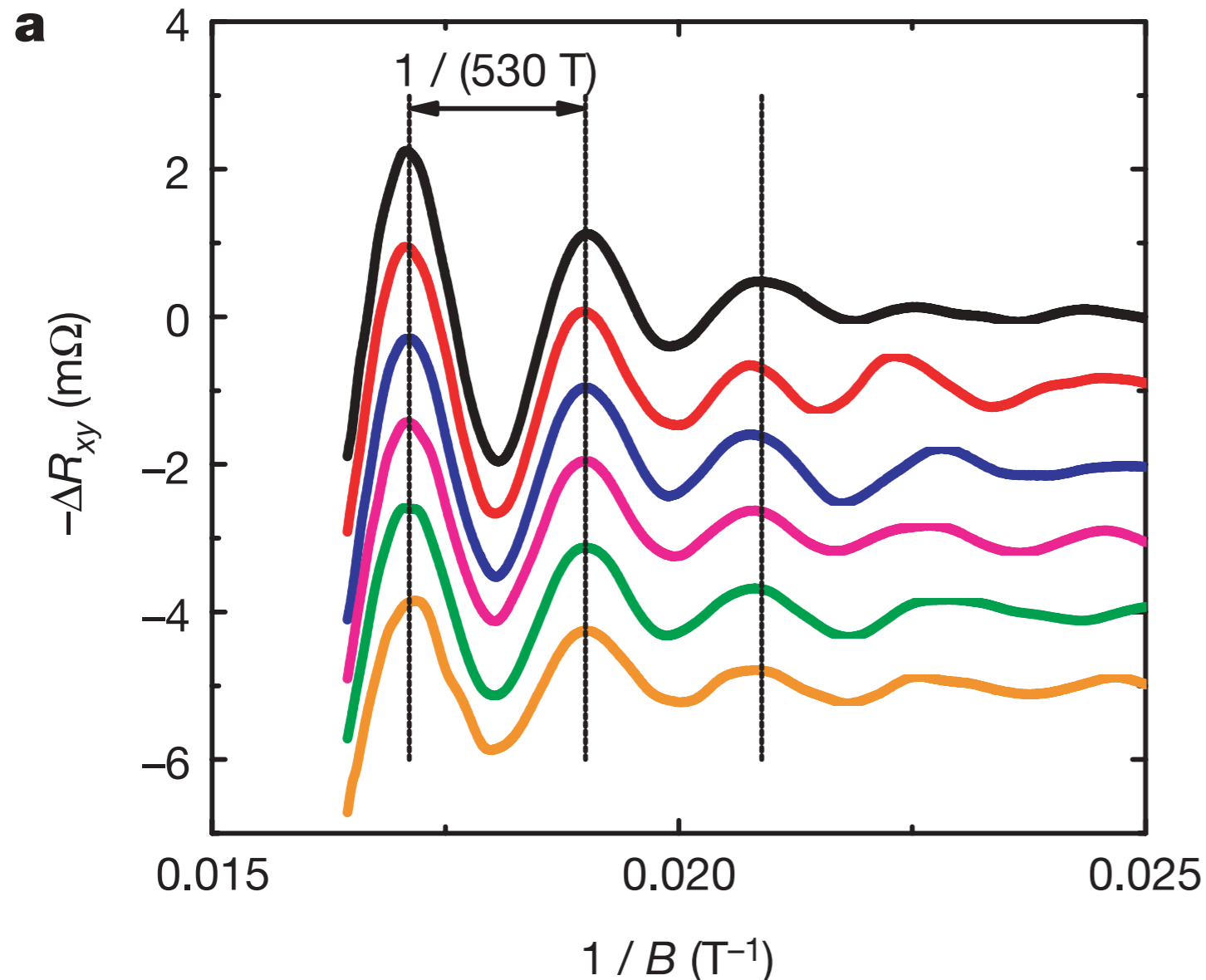


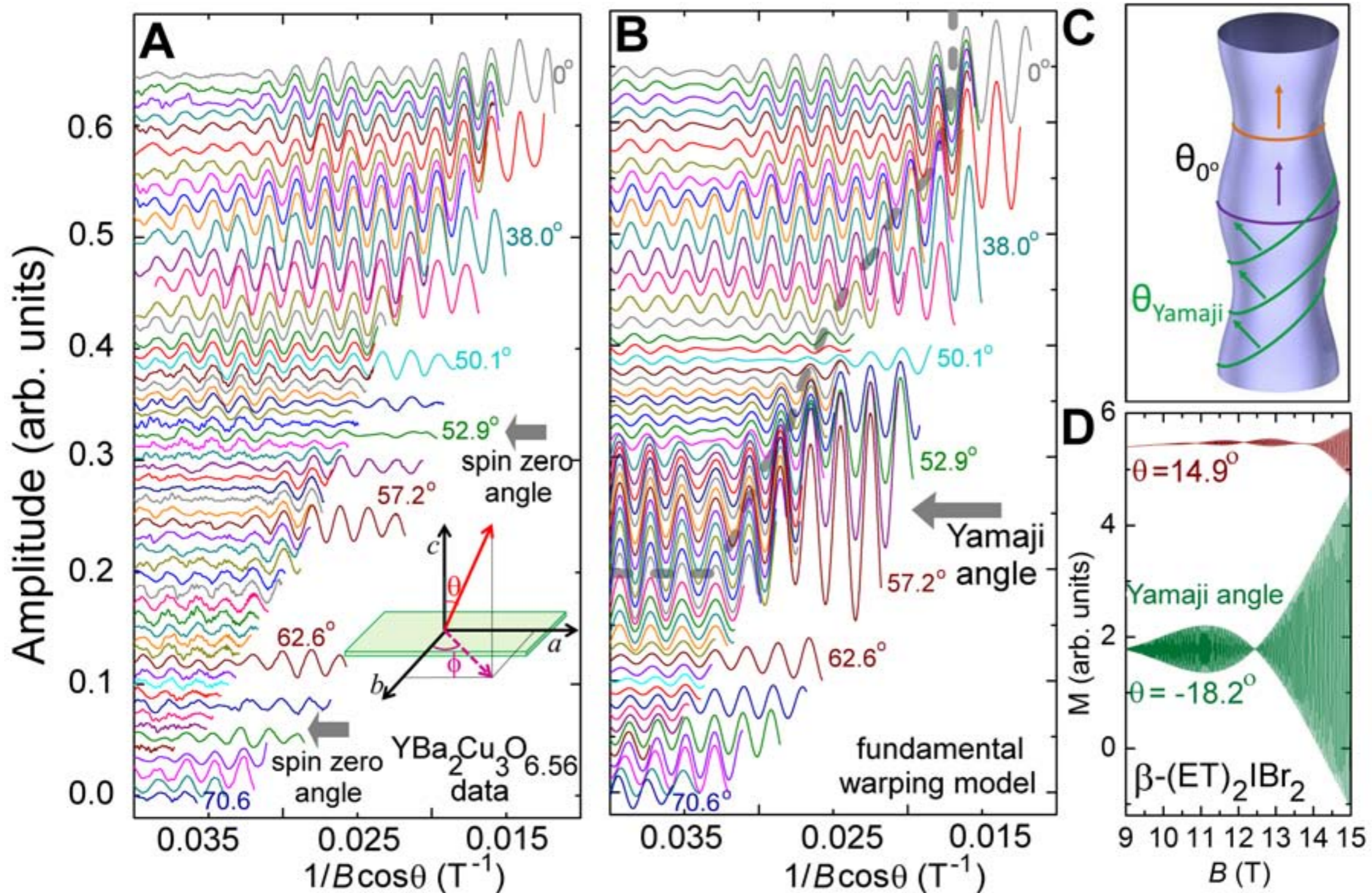
T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).

Quantum oscillations and the Fermi surface in an underdoped high- T_c superconductor

Nicolas Doiron-Leyraud¹, Cyril Proust², David LeBoeuf¹, Julien Levallois², Jean-Baptiste Bonnemaïson¹, Ruixing Liang^{3,4}, D. A. Bonn^{3,4}, W. N. Hardy^{3,4} & Louis Taillefer^{1,4}

Nature 447, 565 (2007)

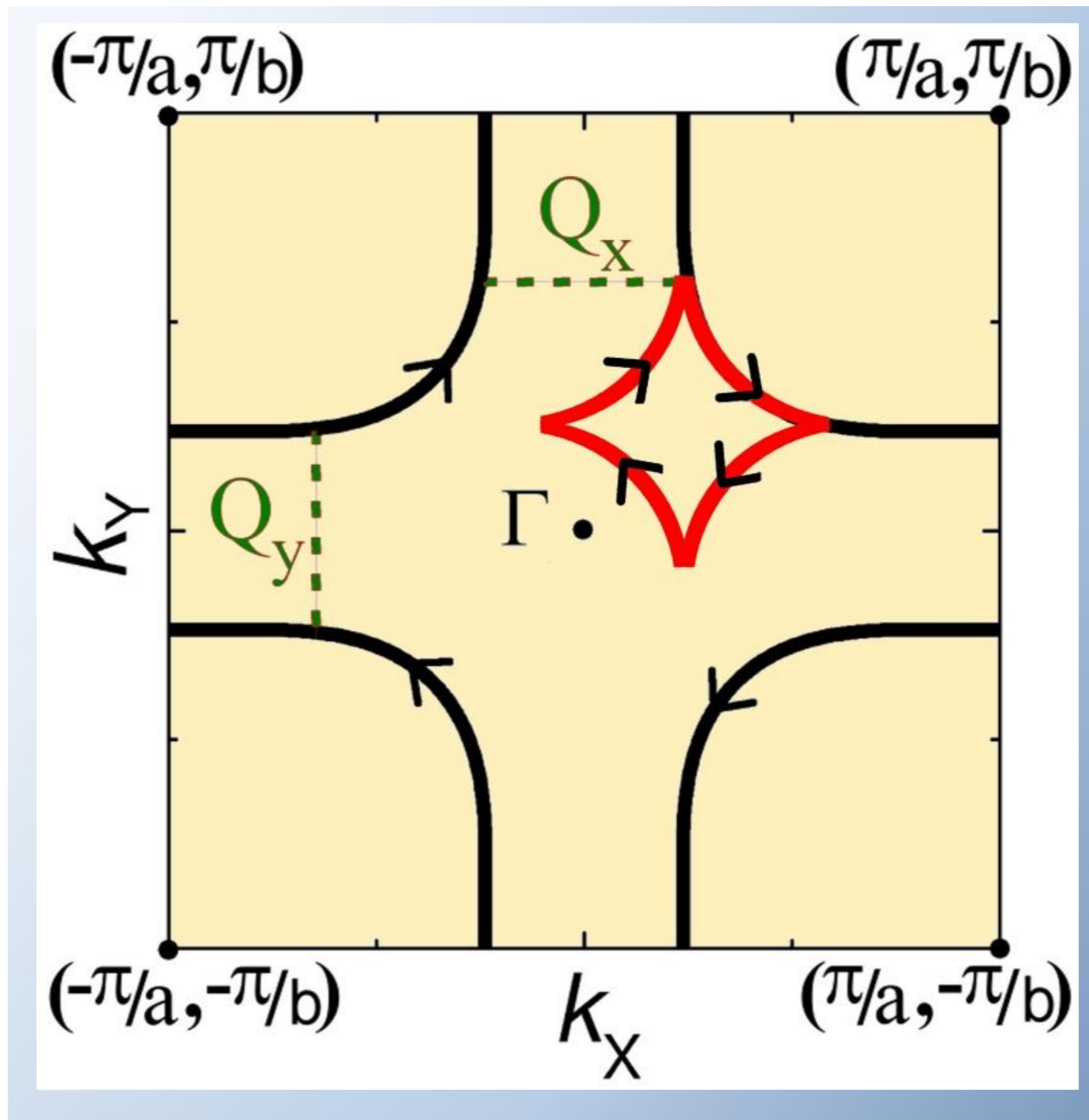




Twofold twisted Fermi surface from staggered order in an underdoped high T_c superconductor

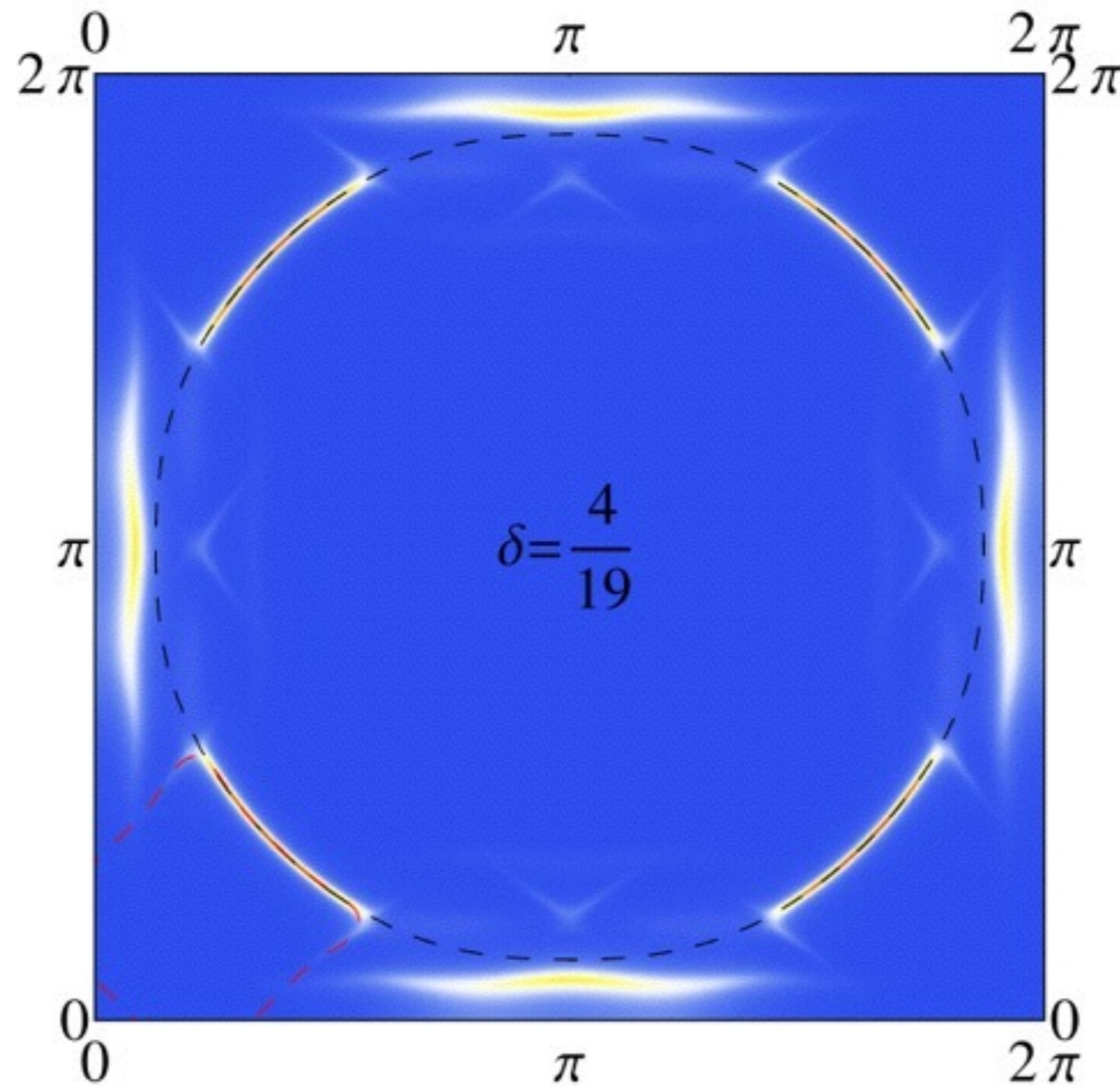
Suchitra E. Sebastian,^{1*} N. Harrison,² F. F. Balakirev,² M. M. Altarawneh,^{2,3}
 Ruixing Liang,^{4,5} D. A. Bonn,^{4,5} W. N. Hardy,^{4,5} G. G. Lonzarich,¹

APS March
 meeting 2013
 B2.00004,
 Nature, to
 appear



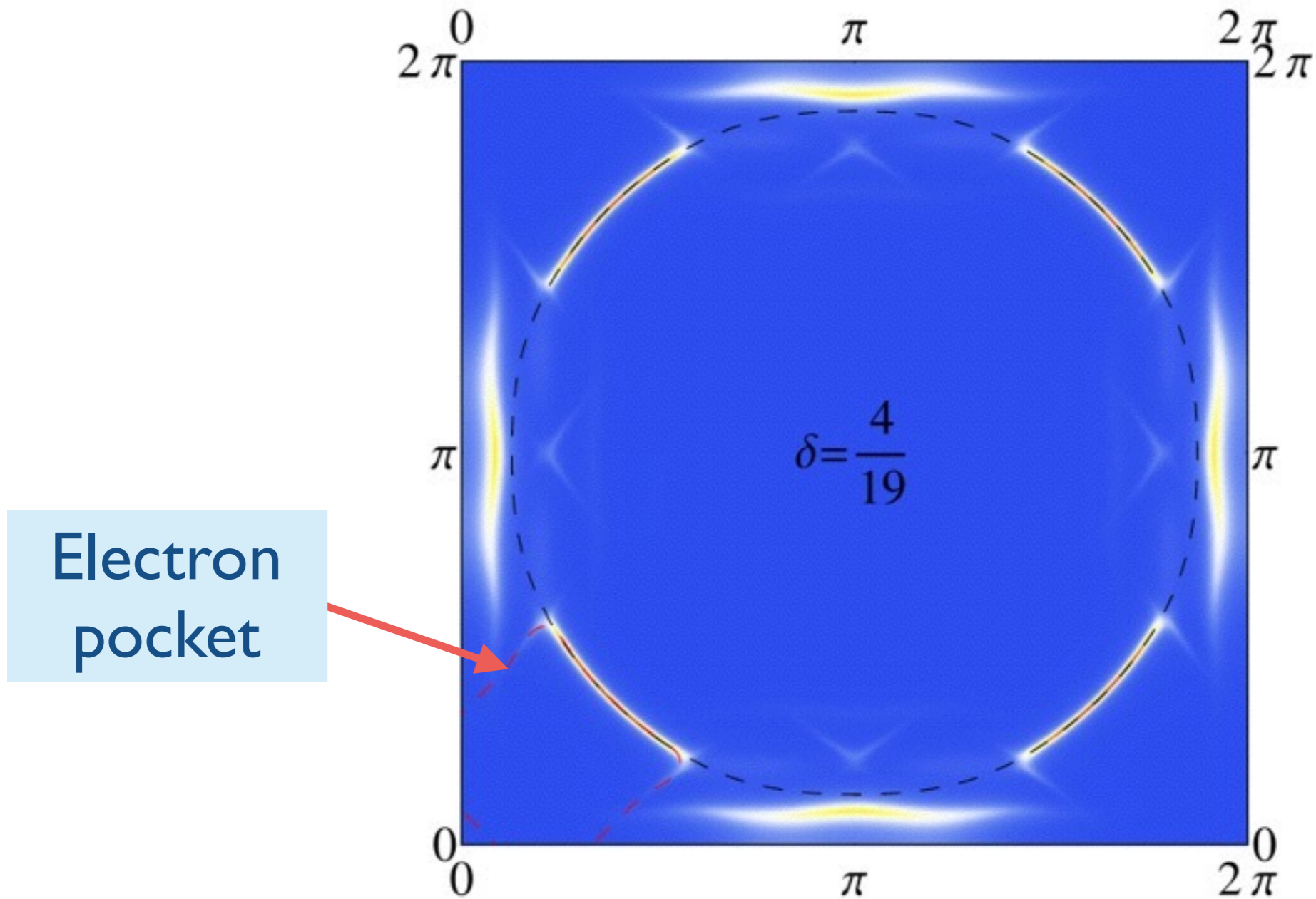
N. Harrison and S. E. Sebastian, Phys. Rev. Lett. **106**, 226402 (2011).

Electron spectral function in the presence of charge order
at $\mathbf{Q} = 2\pi(\delta, 0)$ and $\mathbf{Q} = 2\pi(0, \delta)$



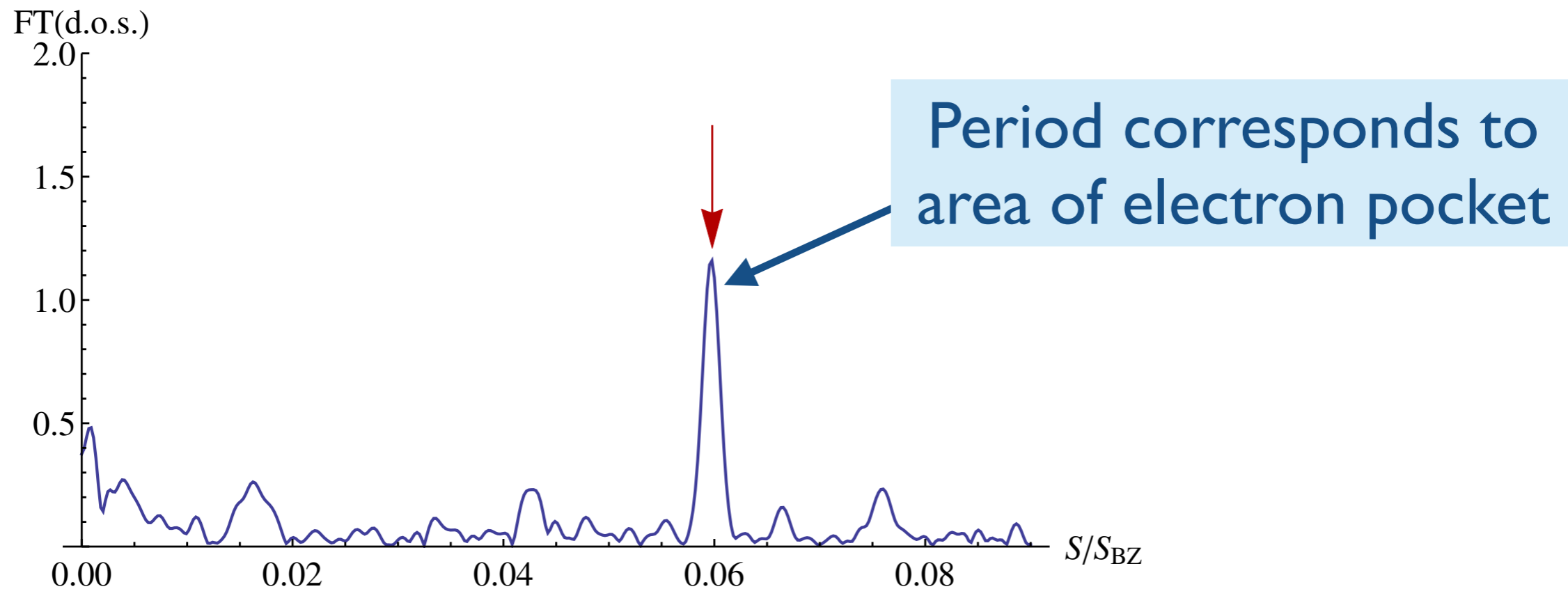
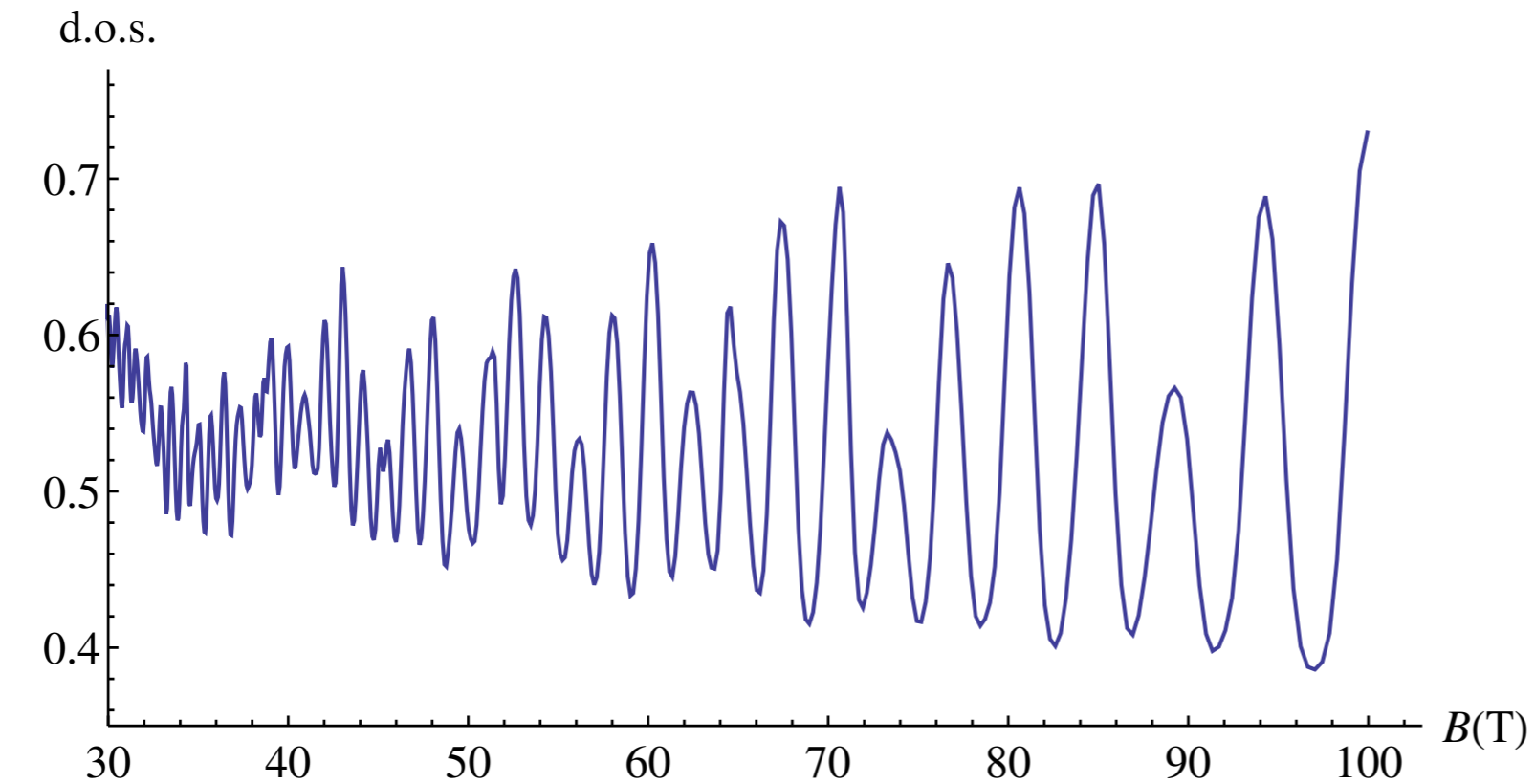
S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)
A. Allais, D. Chowdhury, and S. Sachdev, to appear

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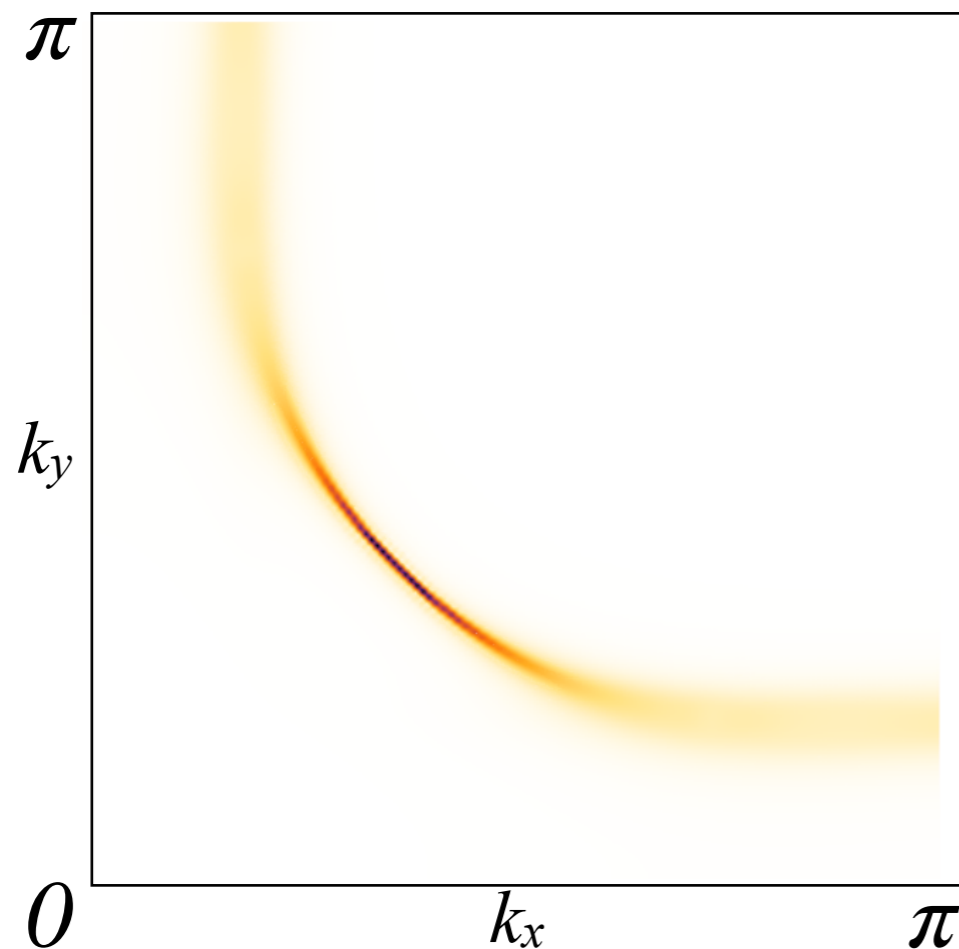
S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)
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Quantum oscillation in the presence of charge order at $\mathbf{Q} = 2\pi(\delta, 0)$ and $\mathbf{Q} = 2\pi(0, \delta)$

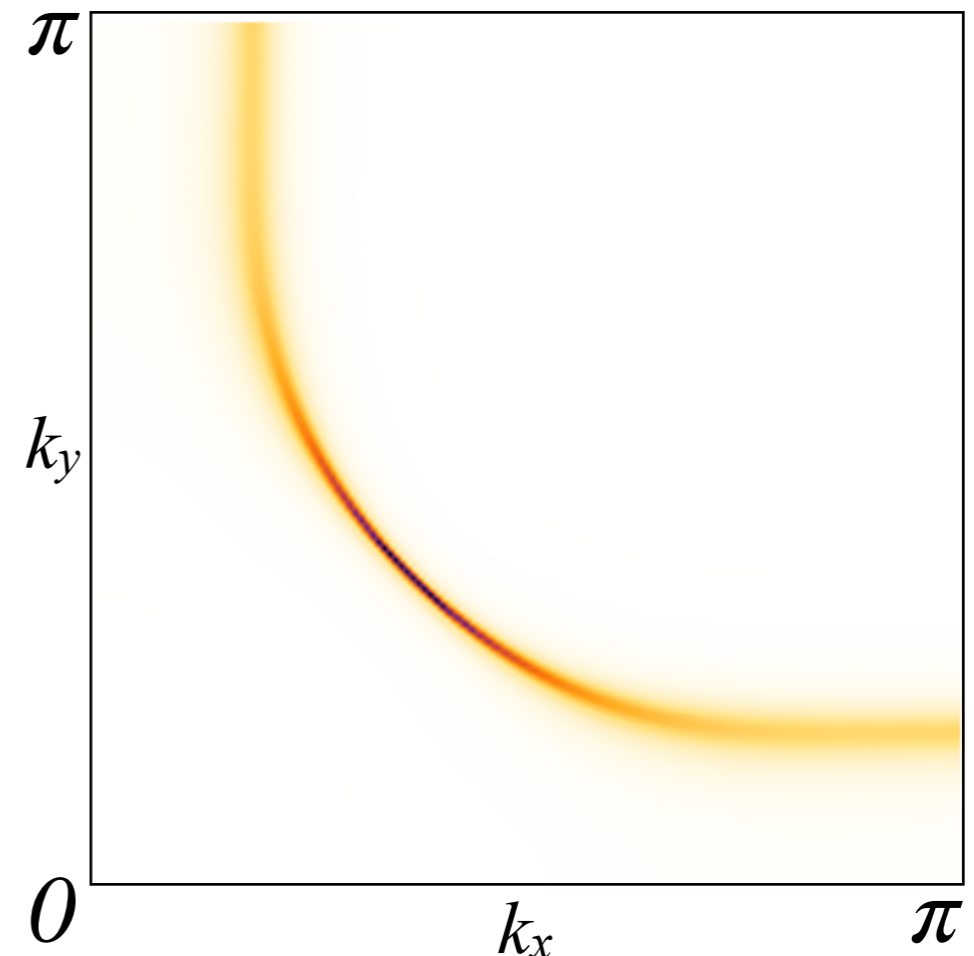


Electron spectral function with fluctuating charge order and superconductivity at temperatures above T_c .

SC+BO
T=0.05

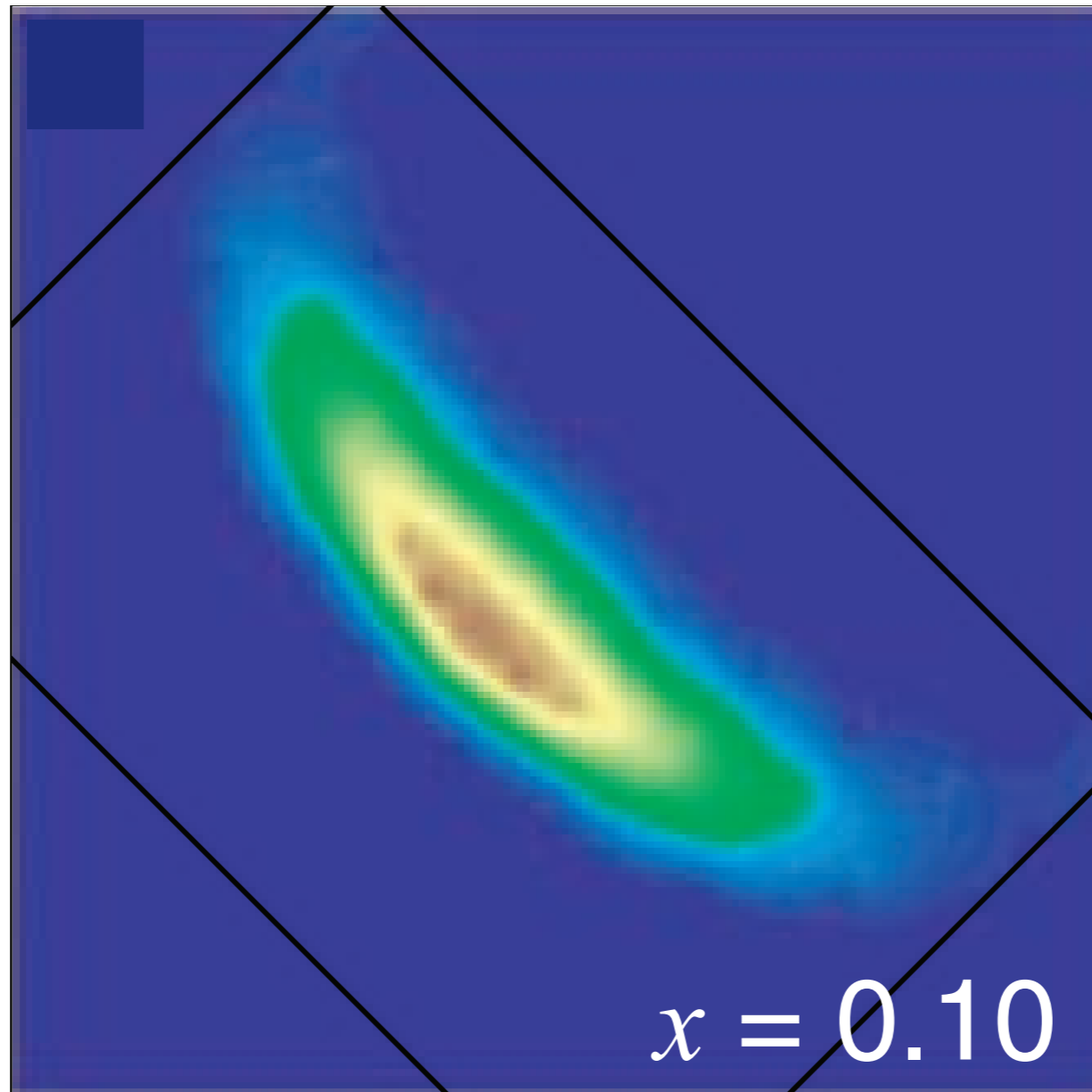


SC+BO
T=0.10



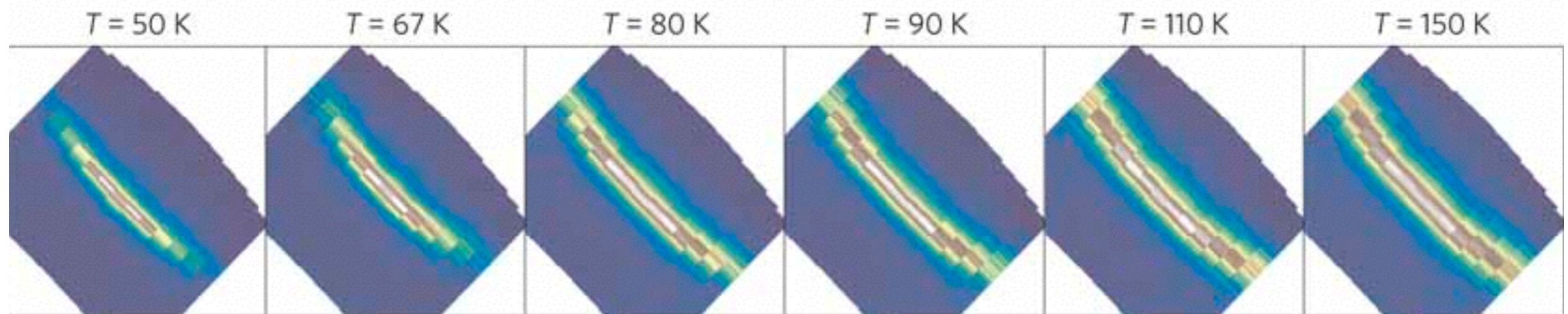
Spectral functions $A(k, \omega=0)$ in the presence of fluctuating SC and BO (with wave vectors $(Q, 0)$, $(0, Q)$) at two different temperatures. Notice how spectral weight fills up in the antinodal regions as a function of increasing temperature.

A. Allais, D. Chowdhury, and S. Sachdev, to appear

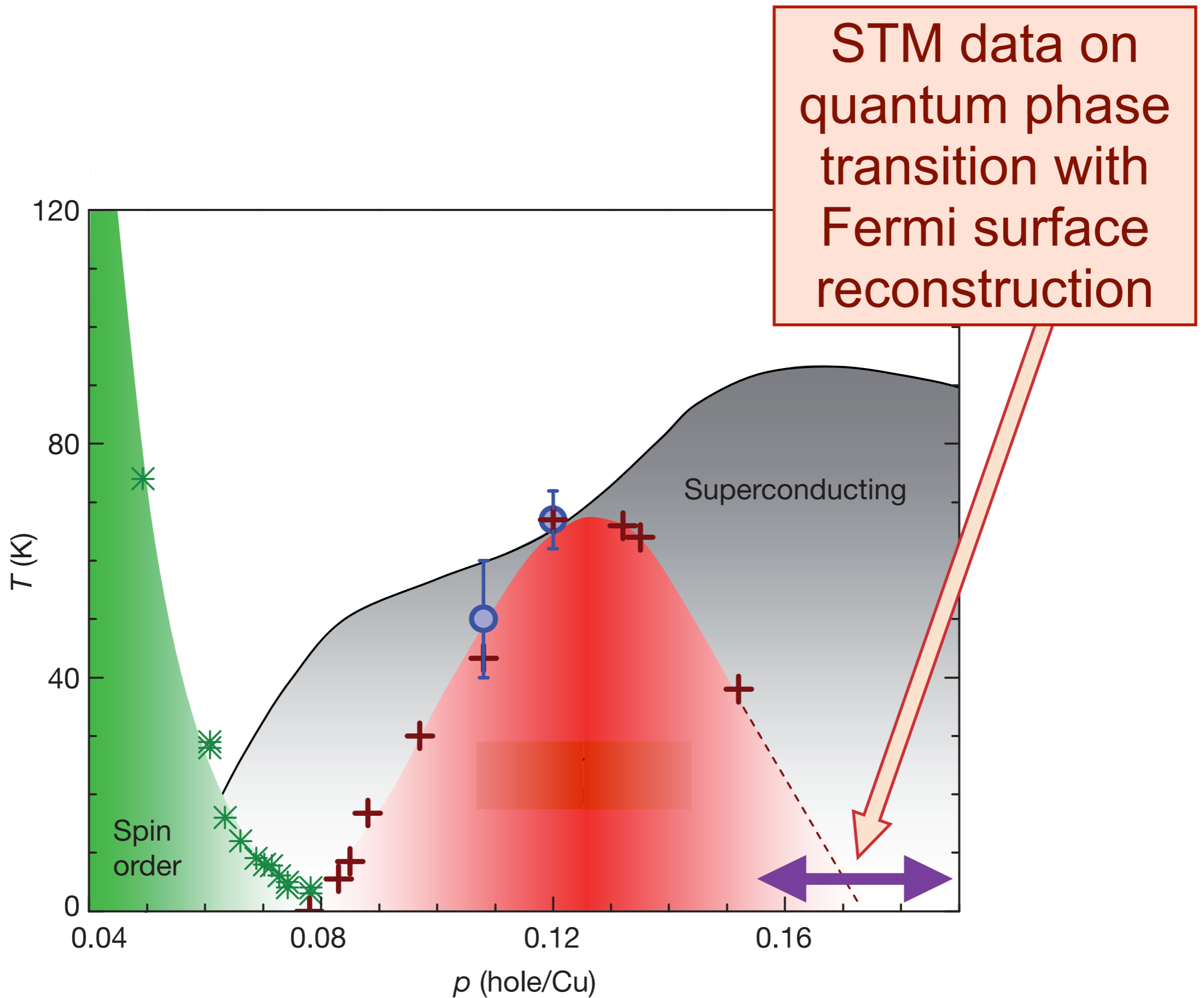


**Nodal Quasiparticles and
Antinodal Charge Ordering in
 $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$**

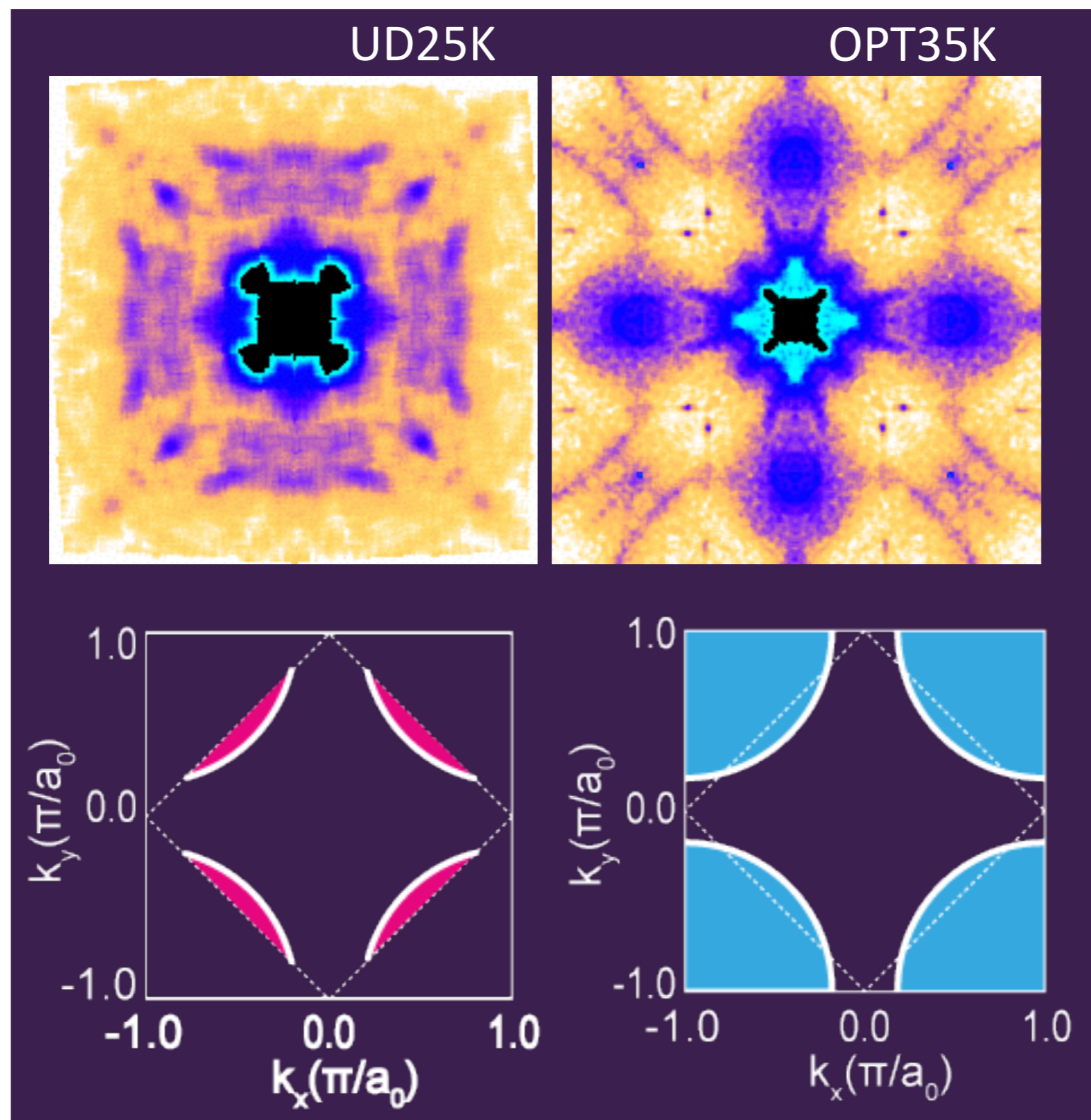
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle,
W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi,
Z.-X. Shen, *Science* **307**, 901 (2005).



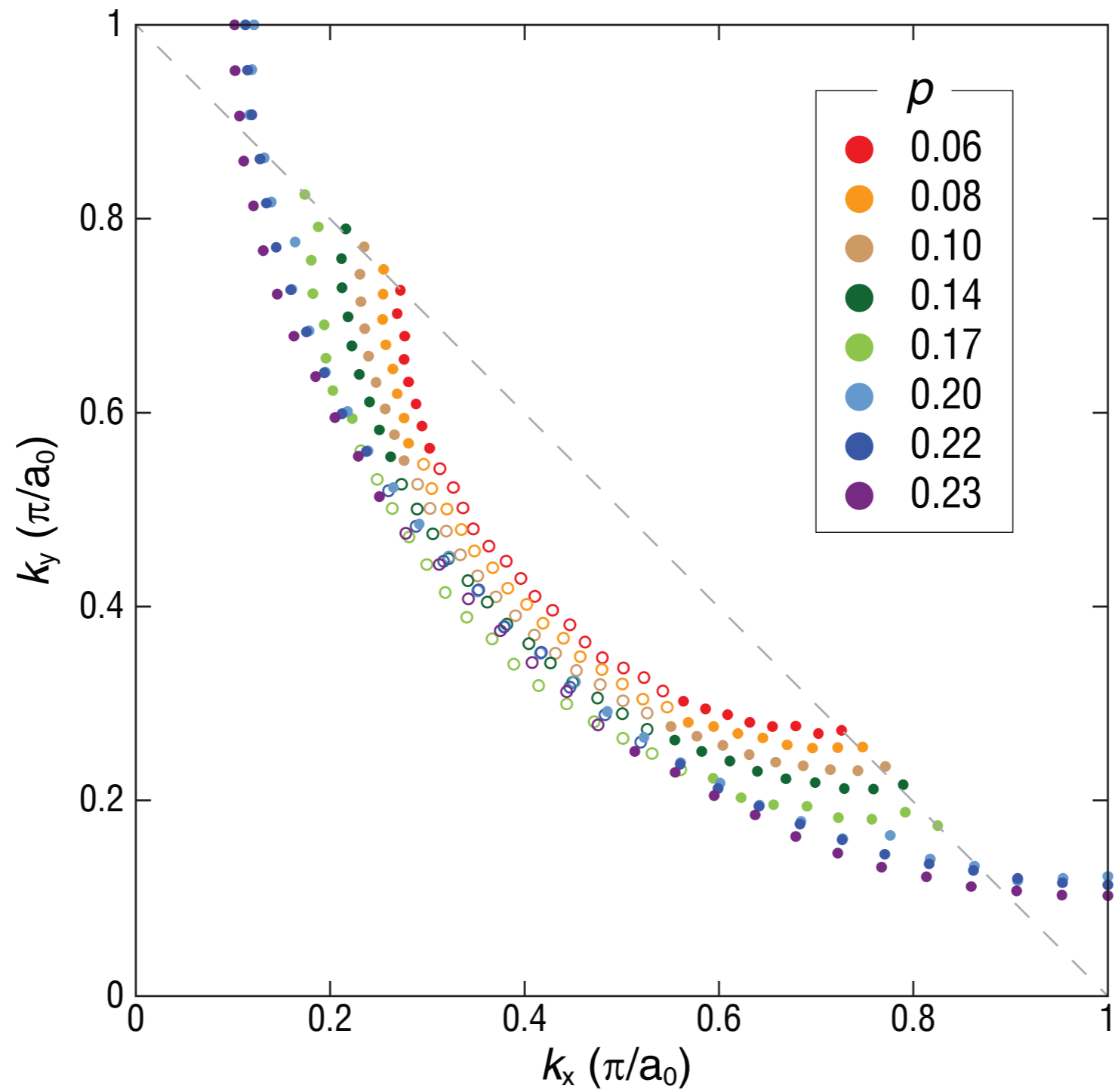
T. J. Reber, N. C. Plumb, Z. Sun, Y. Cao, Q. Wang, K. McElroy, H. Iwasawa, M. Arita, J. S. Wen, Z. J. Xu, G. Gu, Y. Yoshida, H. Eisaki, Y. Aiura, and D. S. Dessau, *Nature Physics* **8**, 606 (2012)



T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



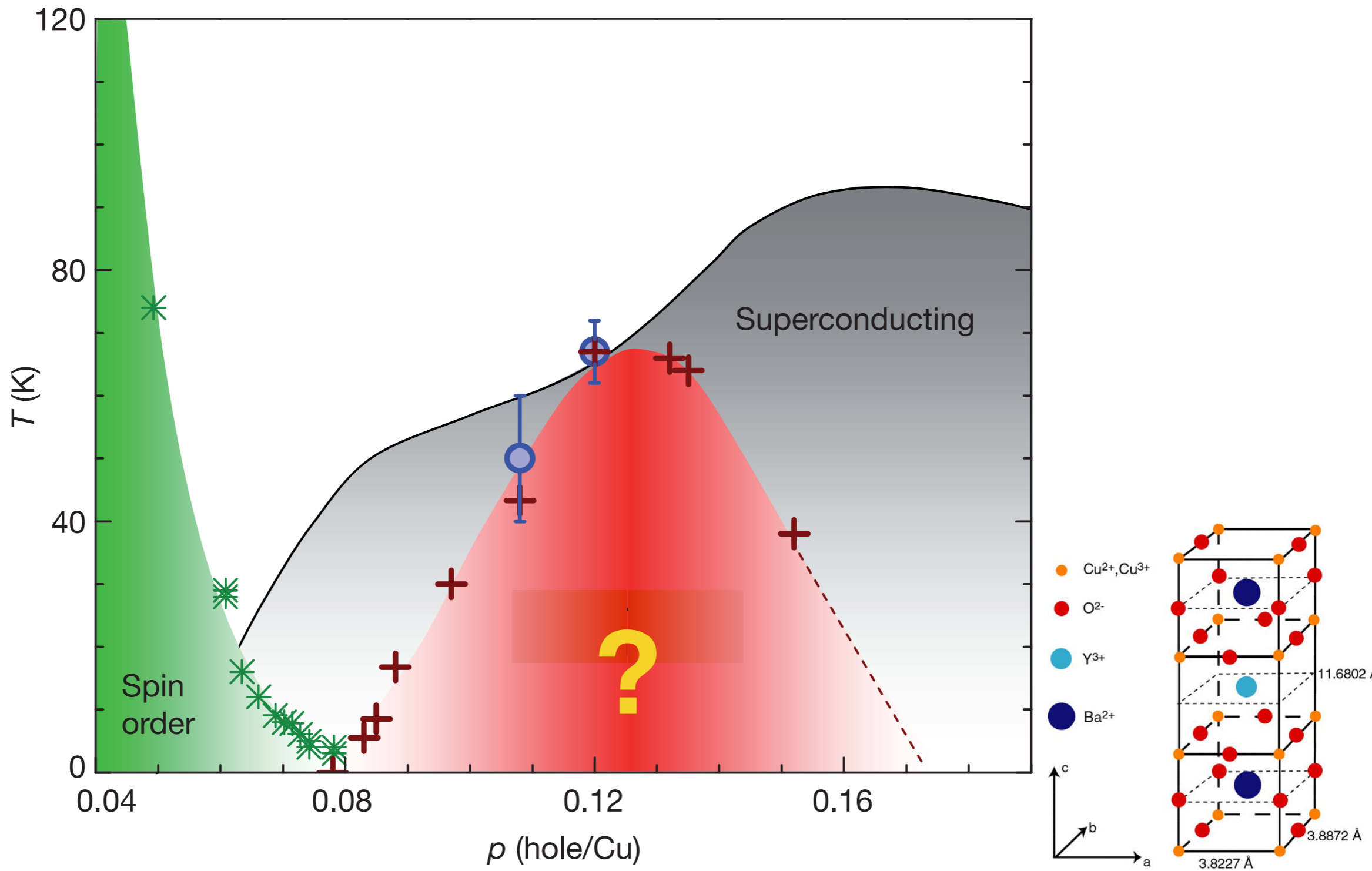
Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, J. E. Hoffman, arXiv:1305.2778, *Science*, to appear.



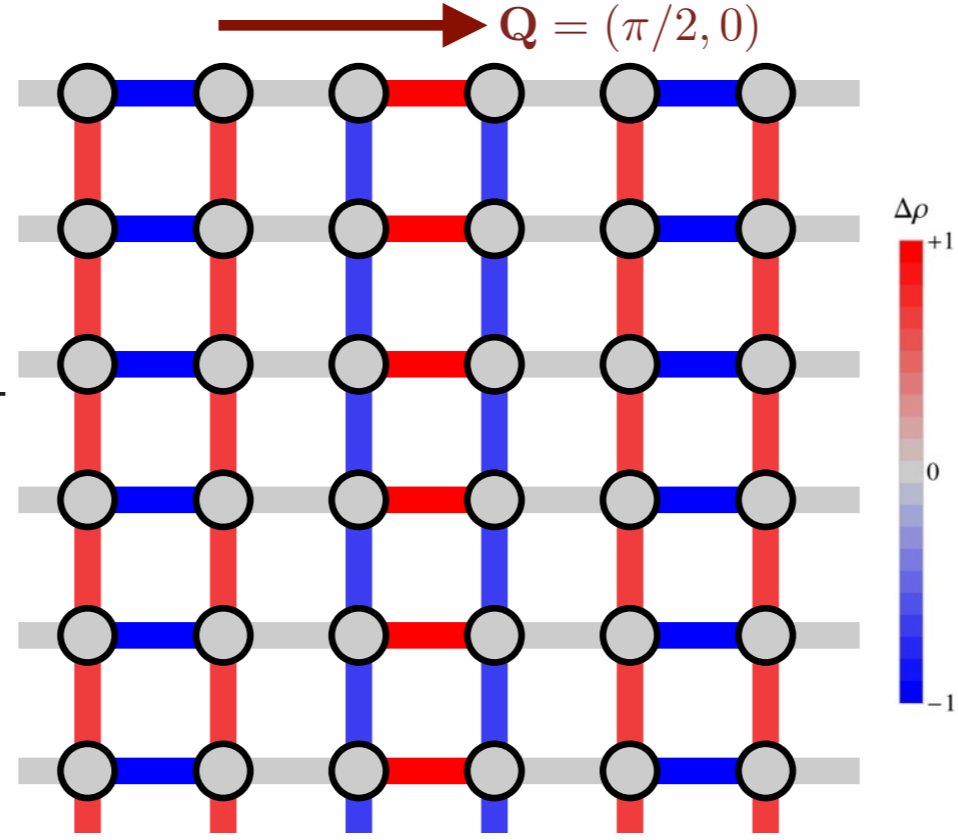
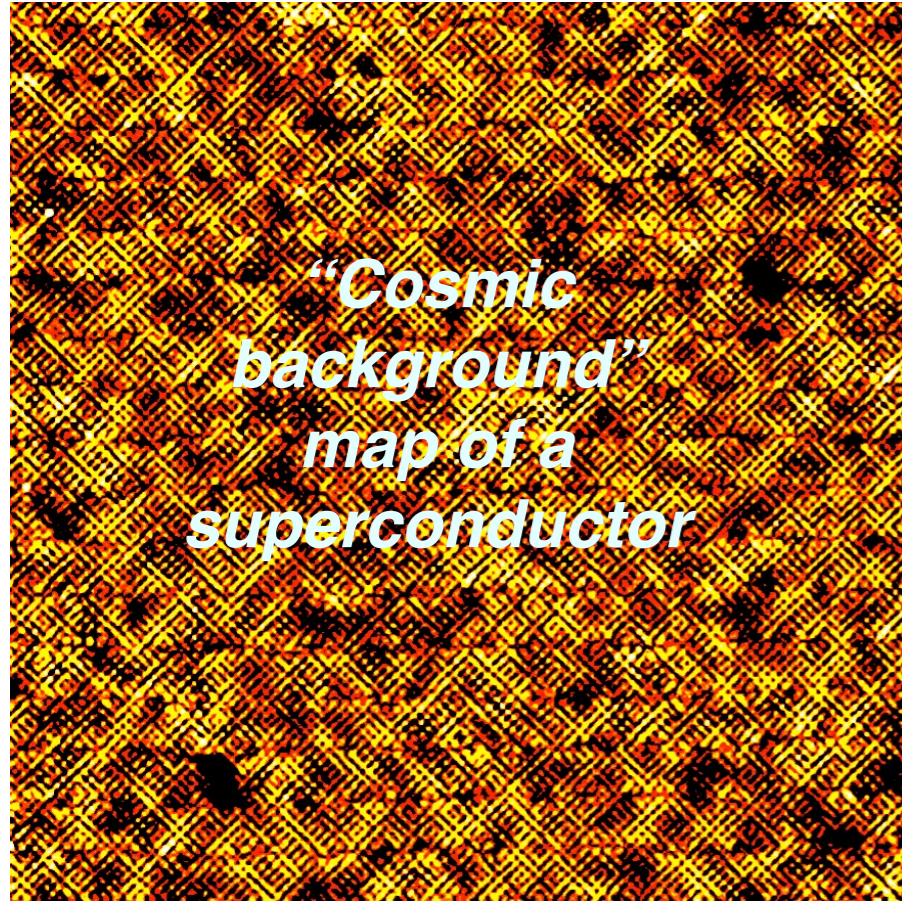
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, and J. C. Davis, *Science*, to appear.

Conclusions

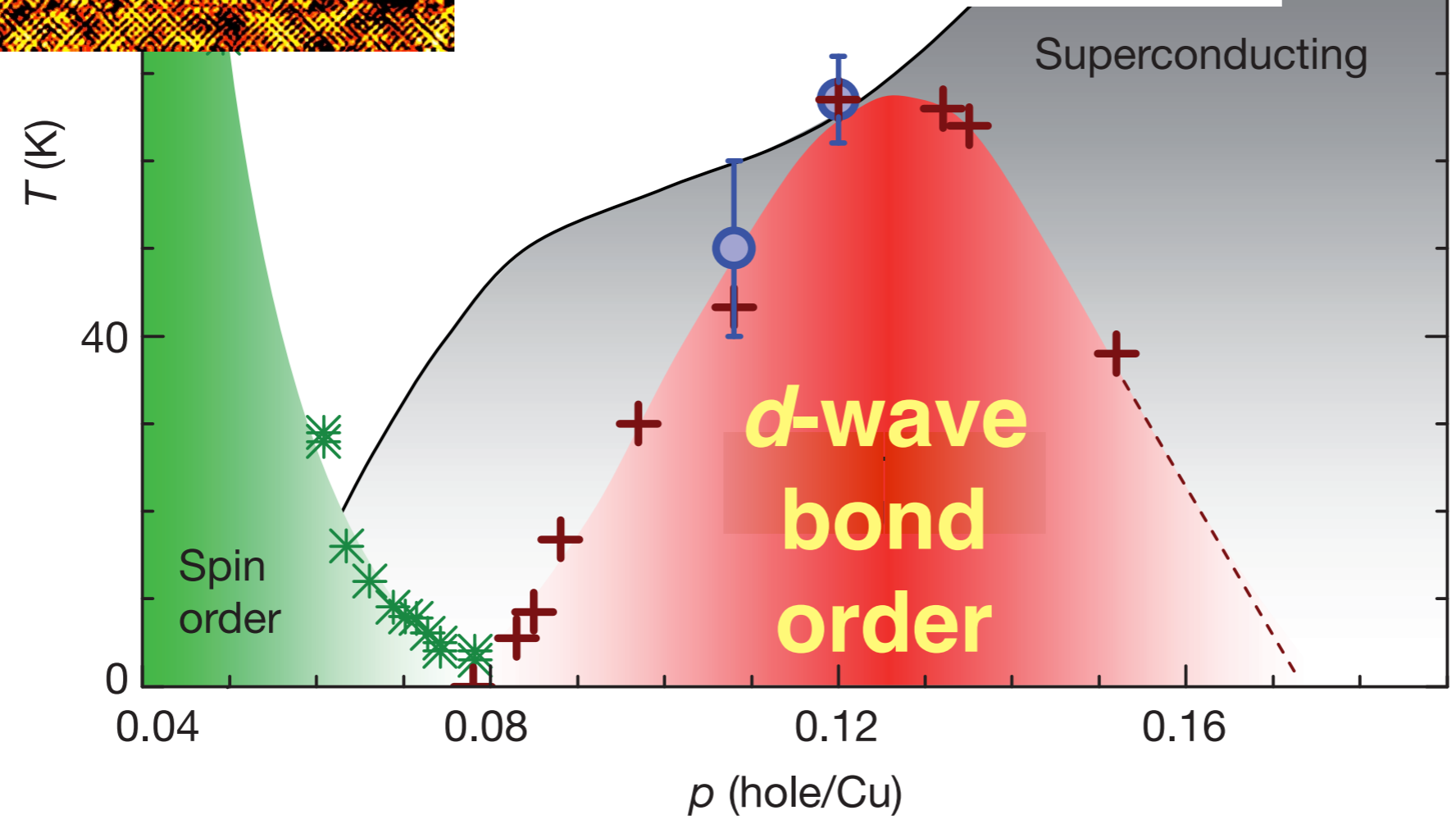
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Phase-sensitive measurement of the *d* symmetry of charge density wave order



K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, arXiv:1404.0362