Exotic phases of the Kondo lattice, and holography

Stanford, July 15, 2010

Talk online: sachdev.physics.harvard.edu

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Outline

1. The Anderson/Kondo lattice models
   Luttinger's theorem

2. Fractionalized Fermi liquids
   Metallic spin-liquid states

3. A mean field theory of a fractionalized Fermi liquid
   Marginal Fermi liquid physics

4. An AdS/CFT perspective
   Holographic metals as fractionalized Fermi liquids

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Anderson/Kondo lattice models

Anderson model Hamiltonian for intermetallic compound with conduction electrons, $c_{i\sigma}$, and localized orbitals, $f_{i\sigma}$

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \Big(V c_{i\sigma}^\dagger f_{i\sigma} + V f_{i\sigma}^\dagger c_{i\sigma} + \varepsilon_f \left(n_{fi\uparrow} + n_{fi\downarrow}\right) + U n_{fi\uparrow} n_{fi\downarrow} \Big)$$

$$n_{fi\sigma} = f_{i\sigma}^\dagger f_{i\sigma} \quad ; \quad n_{ci\sigma} = c_{i\sigma}^\dagger c_{i\sigma} \quad ; \quad n_T = n_f + n_c$$

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&\qquad + \varepsilon_f \left(n_{fi\uparrow} + n_{fi\downarrow} \right) + U n_{fi\uparrow} n_{fi\downarrow} \Big)
\end{aligned}
$$

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$$

In the limit of large U, this maps onto the Kondo lattice model of conduction electrons, $c_{i\sigma}$, and spins $\vec{S}_{fi} = f^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} f_{i\sigma'}$

$$
\begin{aligned}
H_K &= -\sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_i J_K c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma'} \cdot \vec{S}_{fi} \\
&\qquad + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}
\end{aligned}
$$

Luttinger's theorem on a d-dimensional lattice

For simplicity, we consider systems with SU(2) spin rotation invariance, which is preserved in the ground state.

Let v_0 be the volume of the unit cell of the ground state,
n_T be the total number density of electrons per volume v_0.
(need not be an integer)

Then, in a metallic Fermi liquid state with a sharp electron-like Fermi surface:

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= n_T \left(\text{mod } 2 \right)$$

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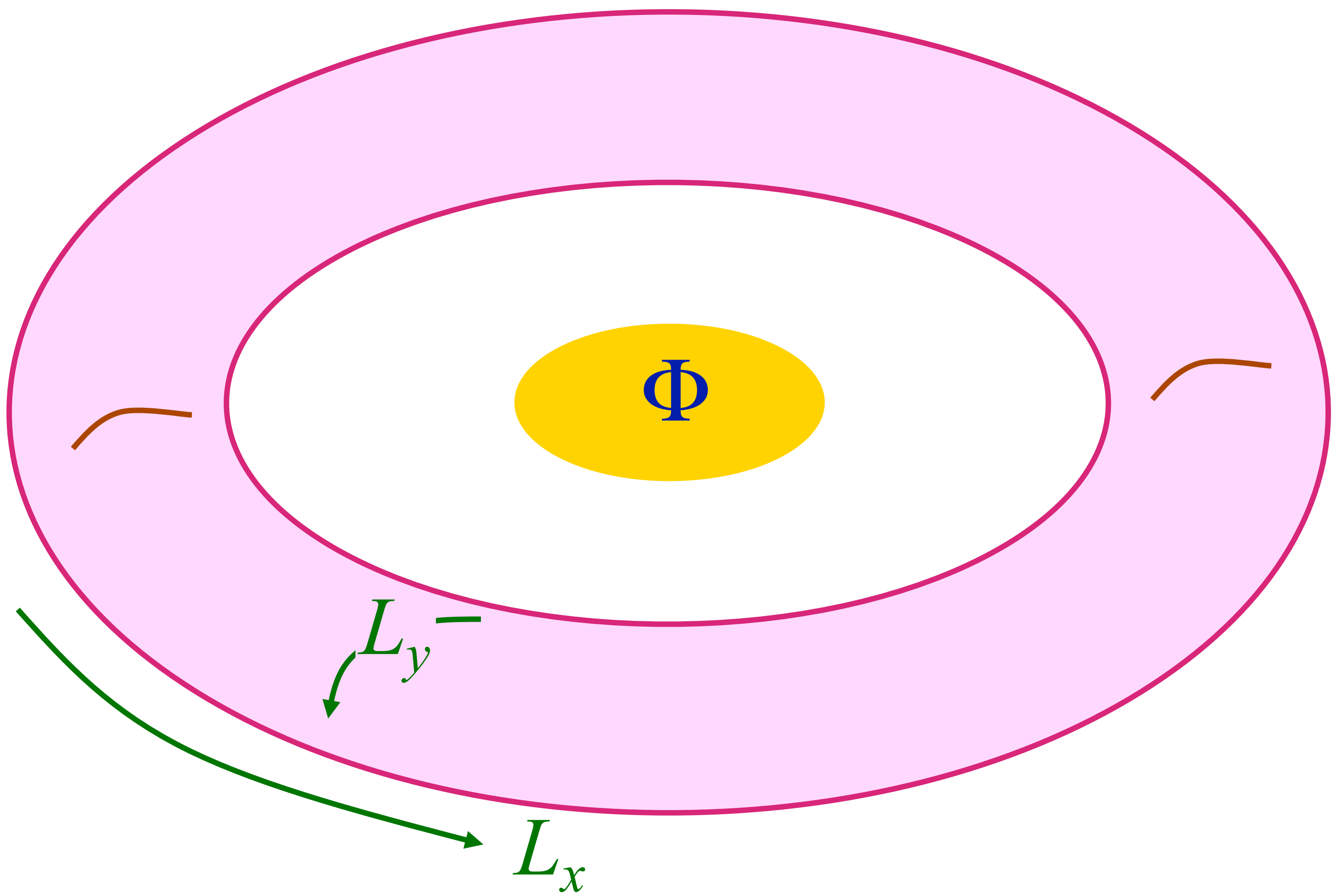
A "large" Fermi surface

Lieb-Schultz-Mattis-Laughlin-Bonesteel-Affleck-Yamanaka-Oshikawa flux-piercing arguments



Unit cell a_x, a_y.
L_x/a_x, L_y/a_y
coprime integers

Adiabatically insert flux $\Phi=2\pi$ (units $\hbar$=c=e=1) acting on $\uparrow$ electrons.

State changes from $|\Psi\rangle$ to $|\Psi'\rangle$, and $UH(0)U^{-1} = H(\Phi)$, where

$$U = \exp\left[\frac{2\pi i}{L_x}\sum_r x\,\hat{n}_{r\uparrow}\right].$$

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000).

Adiabatic process commutes with the translation operator T_x, so momentum P_x is conserved.

$$\text{However } U^{-1} T_x U = T_x \exp\left[\frac{2\pi i}{L_x}\sum_{\boldsymbol{r}} \hat{n}_{T\boldsymbol{r}\uparrow}\right];$$

so shift in momentum ΔP_x between states $U|\Psi'\rangle$ and $|\Psi\rangle$ is

$$\Delta P_x = \frac{\pi L_y}{v_0} n_T \left(\text{mod}\, \frac{2\pi}{a_x}\right) \qquad (1).$$

Alternatively, we can compute ΔP_x by assuming it is absorbed by quasiparticles of a Fermi liquid. Each quasiparticle has its momentum shifted by $2\pi/L_x$, and so

$$\Delta P_x = \frac{2\pi}{L_x}\frac{(\text{Volume enclosed by Fermi surface})}{(2\pi)^2/(L_x L_y)}\left(\text{mod}\, \frac{2\pi}{a_x}\right) \qquad (2).$$

From (1) and (2), same argument in y direction, using coprime $L_x/a_x, L_y/a_y$:

$$2 \times \frac{v_0}{(2\pi)^2}(\text{Volume enclosed by Fermi surface}) = n_T \;(\text{mod } 2)$$

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For small U, we obtain a Fermi liquid ground state, with a "large" Fermi surface volume determined by n_T (mod 2)

This is adiabatically connected to a Fermi liquid ground state at large U, where n_f=1, and whose Fermi surface volume must also be determined by

$$n_T \,(\mathrm{mod}\ 2) = (1 + n_c)\,(\mathrm{mod}\ 2)$$

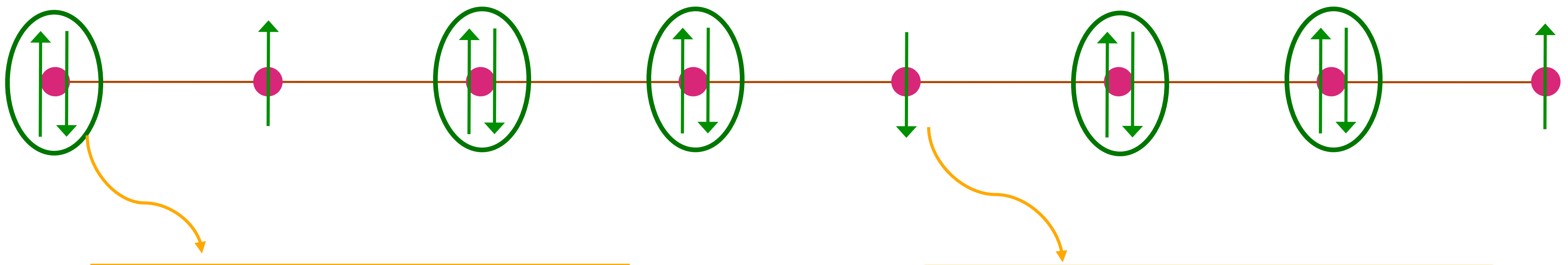
Anderson/Kondo lattice models

$$H_K = -\sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_i J_K c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma'} \cdot \vec{S}_{fi} + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

We can also the Kondo lattice model to argue for a Fermi liquid ground state whose "large" Fermi surface volume is $(1+n_c)(\text{mod } 2)$

Arguments for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies



$$\left(c_{i\uparrow}^\dagger f_{i\downarrow}^\dagger - c_{i\downarrow}^\dagger f_{i\uparrow}^\dagger\right)|0\rangle$$

$$f_{i\downarrow}^\dagger |0\rangle, \; S=1/2 \text{ hole}$$

Fermi liquid of S=1/2 holes with hard-core repulsion

$$\begin{aligned}\text{Fermi surface volume } &= -\left(\text{density of holes}\right) \bmod 2 \\ &= -\left(1-n_c\right) = \left(1+n_c\right) \bmod 2\end{aligned}$$

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There exist "topologically ordered" ground states in dimensions $d > 1$ with a Fermi surface of electron-like quasiparticles for which

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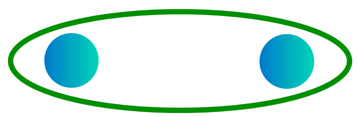
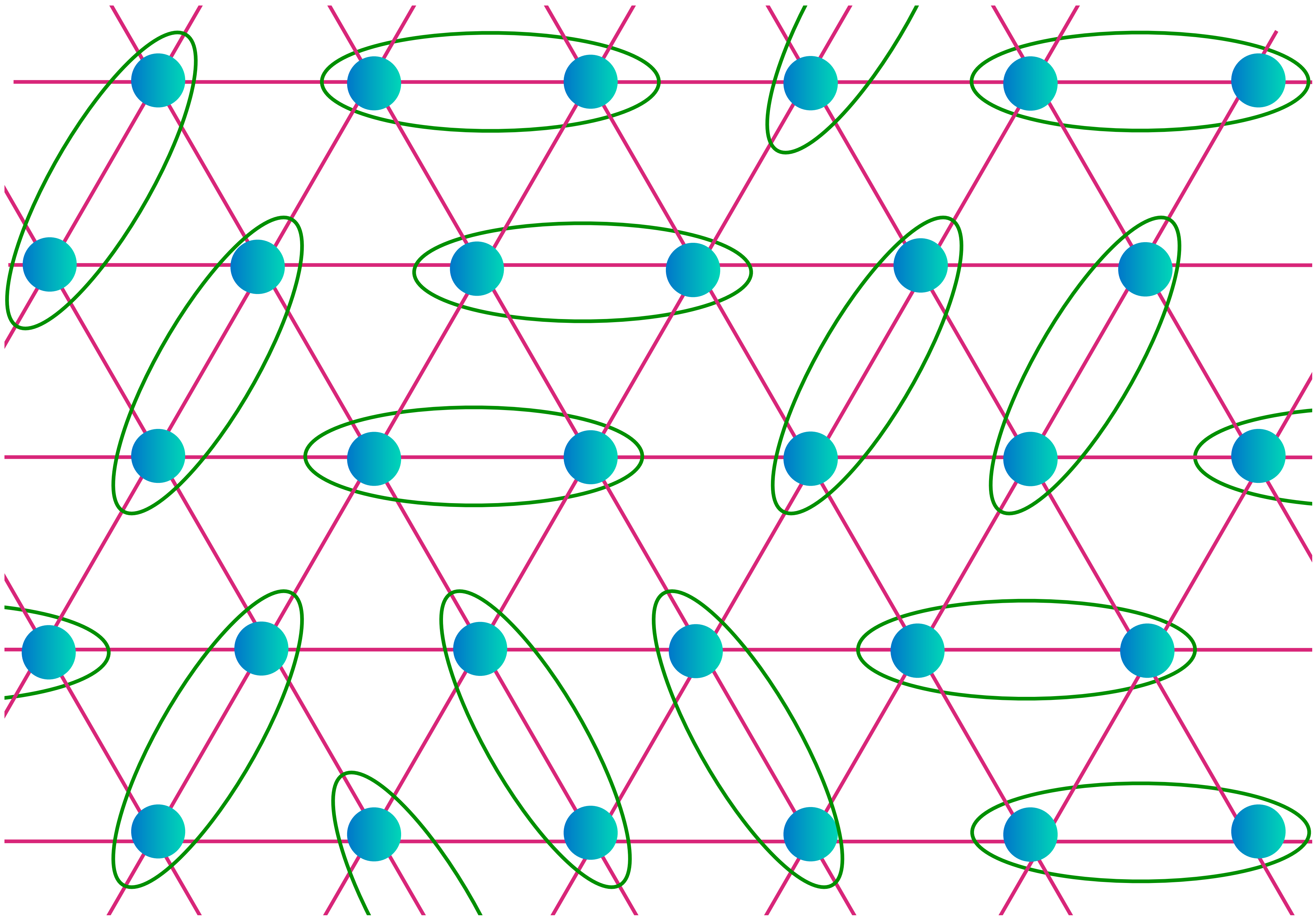
A Fractionalized Fermi Liquid

Start with the Kondo lattice, with $J_H \gg J_K$.
Assume J_H are so that the $\vec{S}_f$ spins form a spin liquid



$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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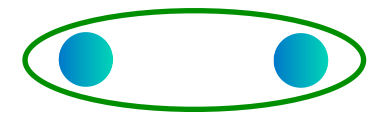

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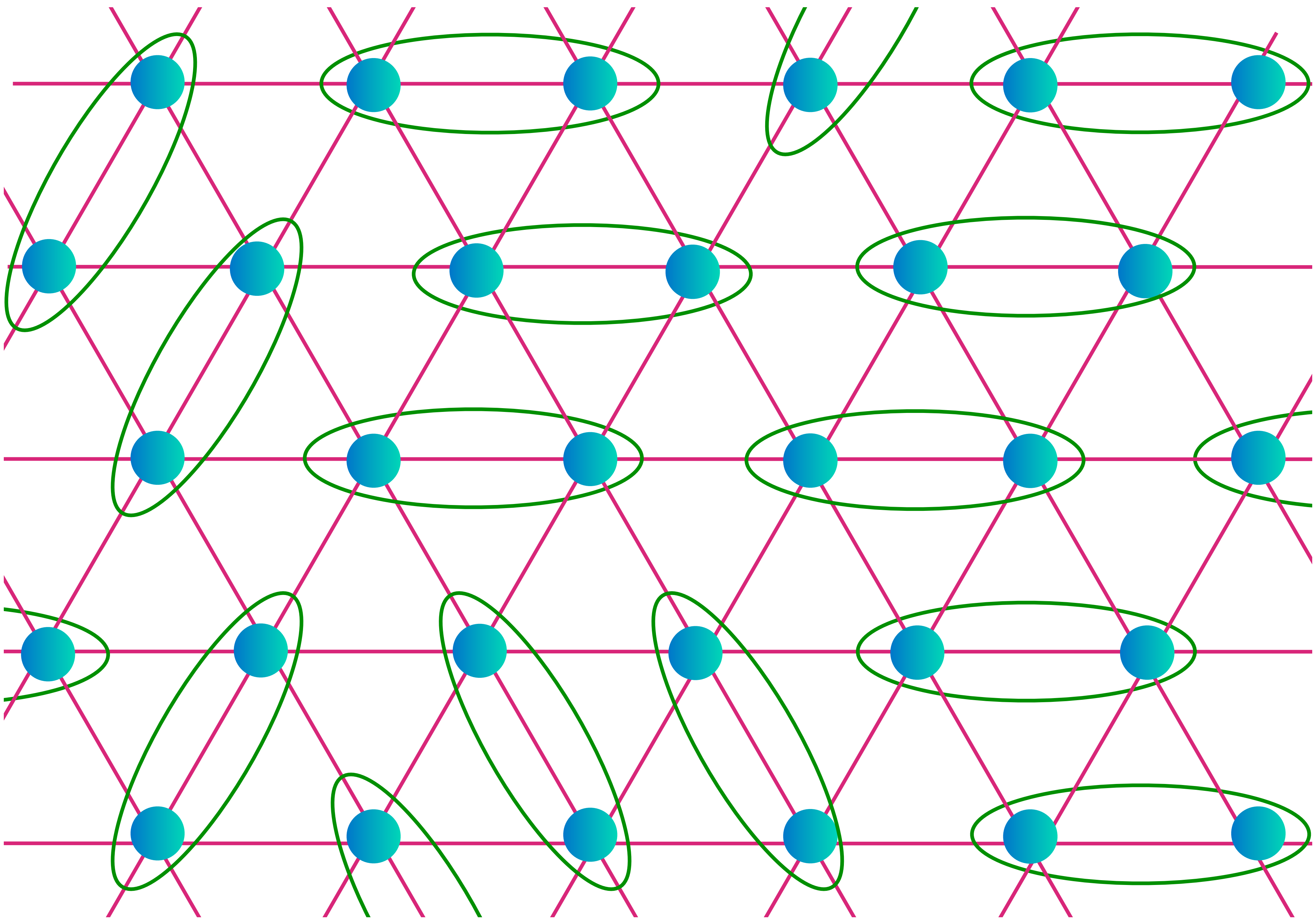

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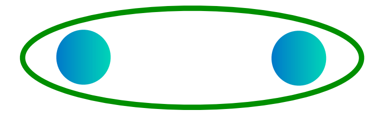

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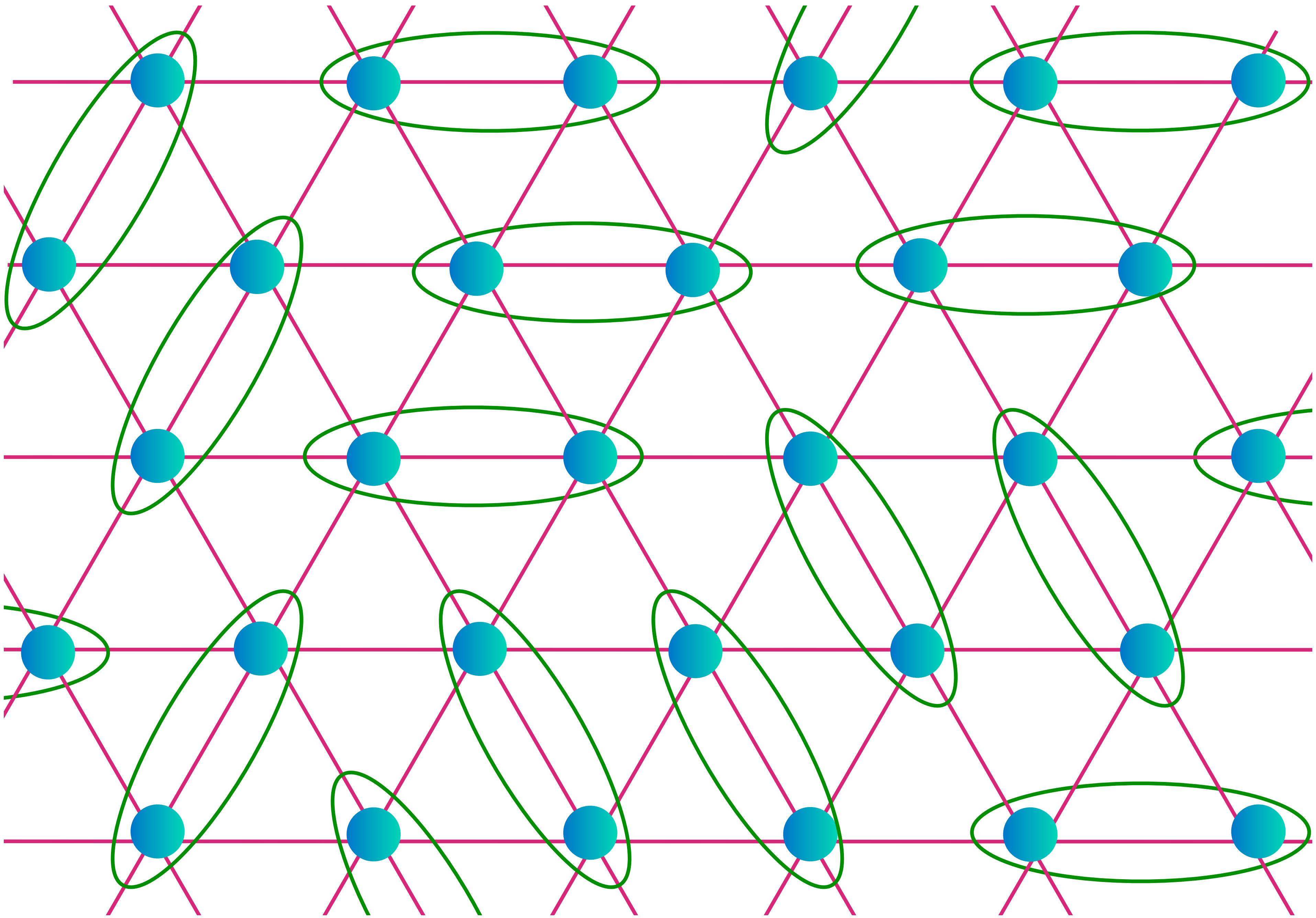

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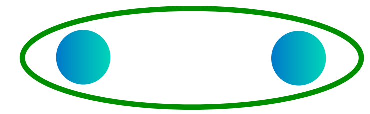

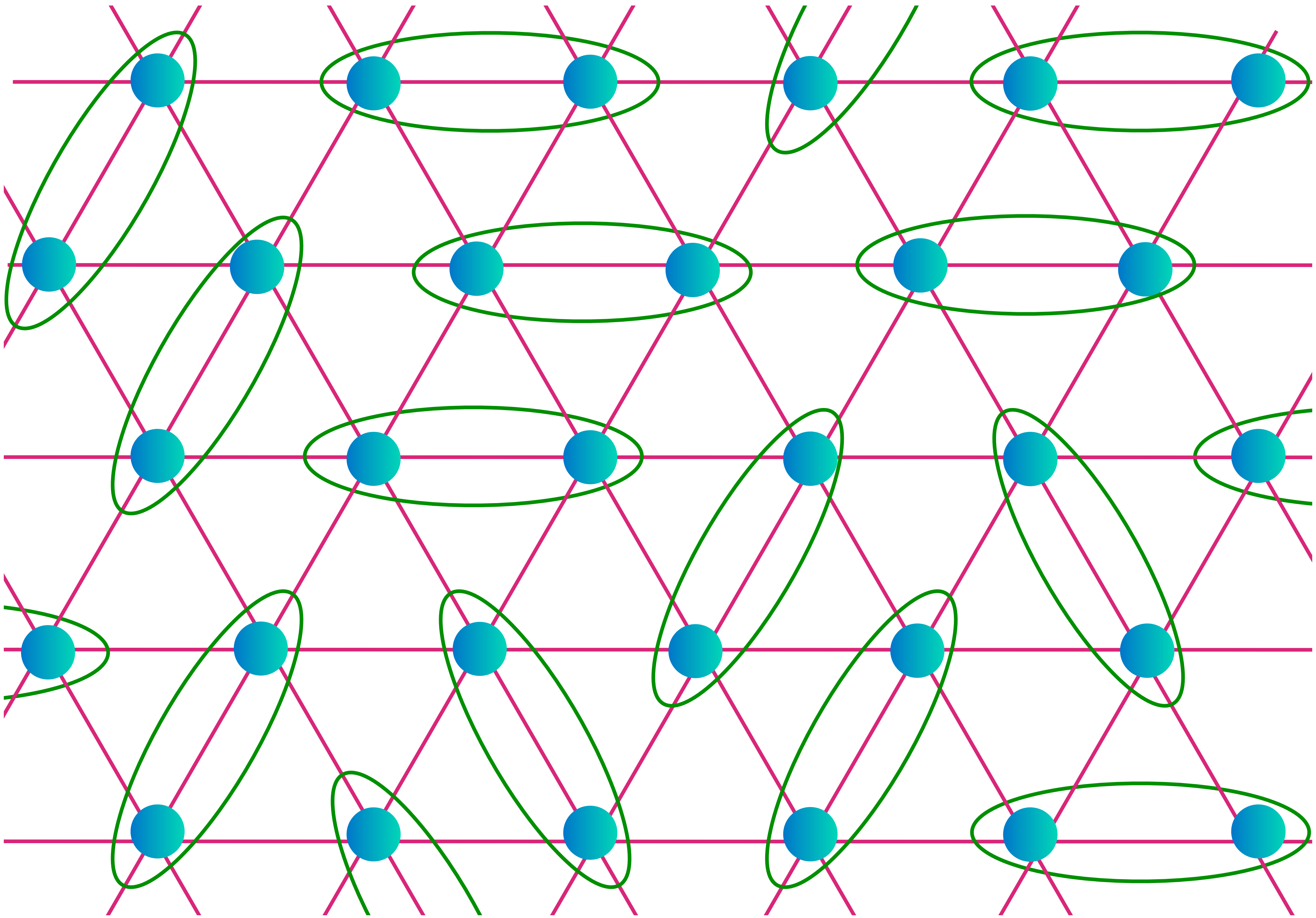

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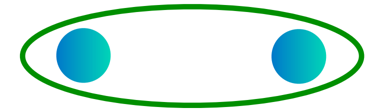

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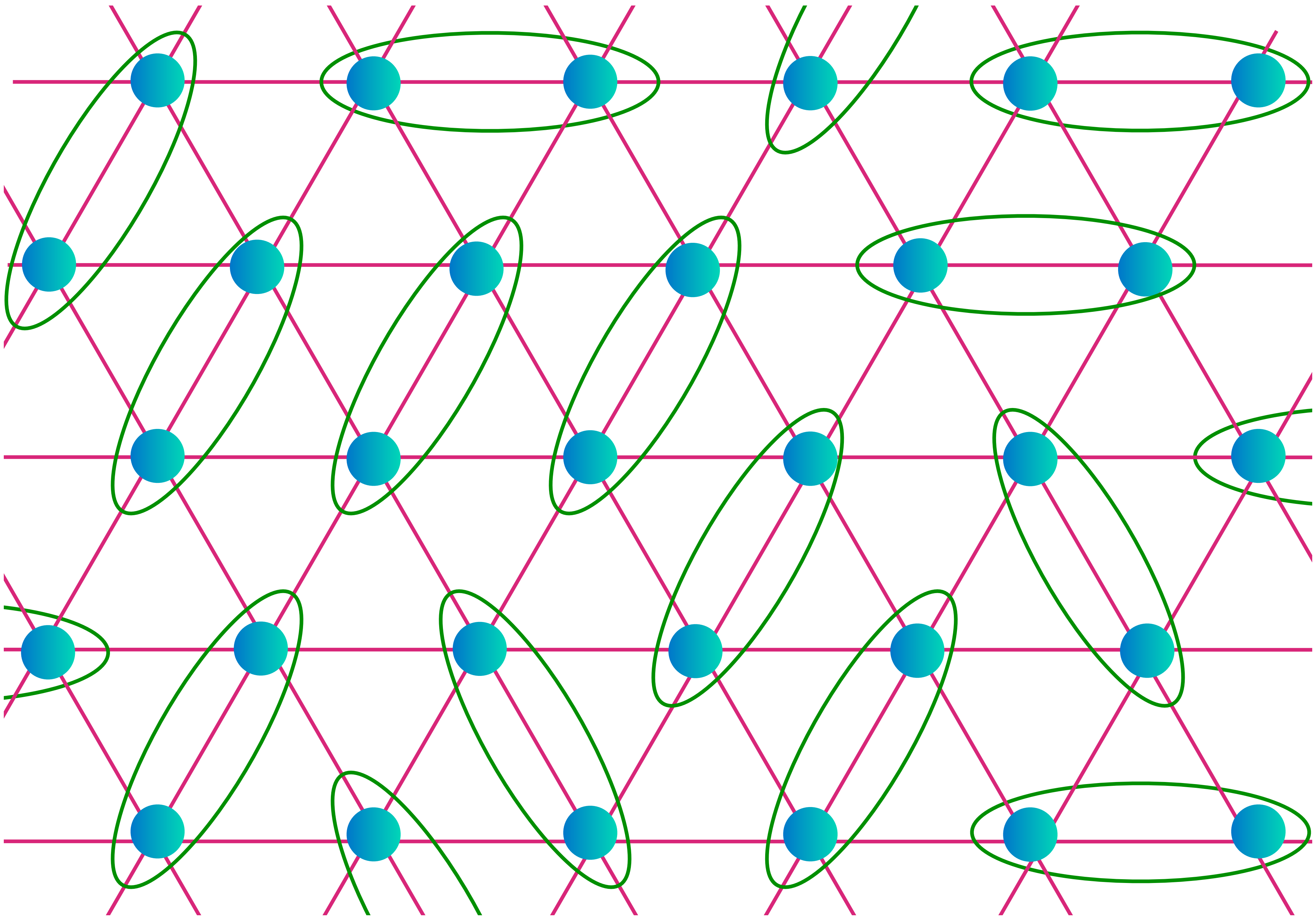

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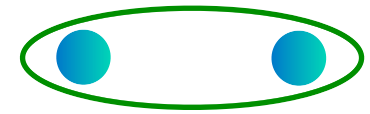

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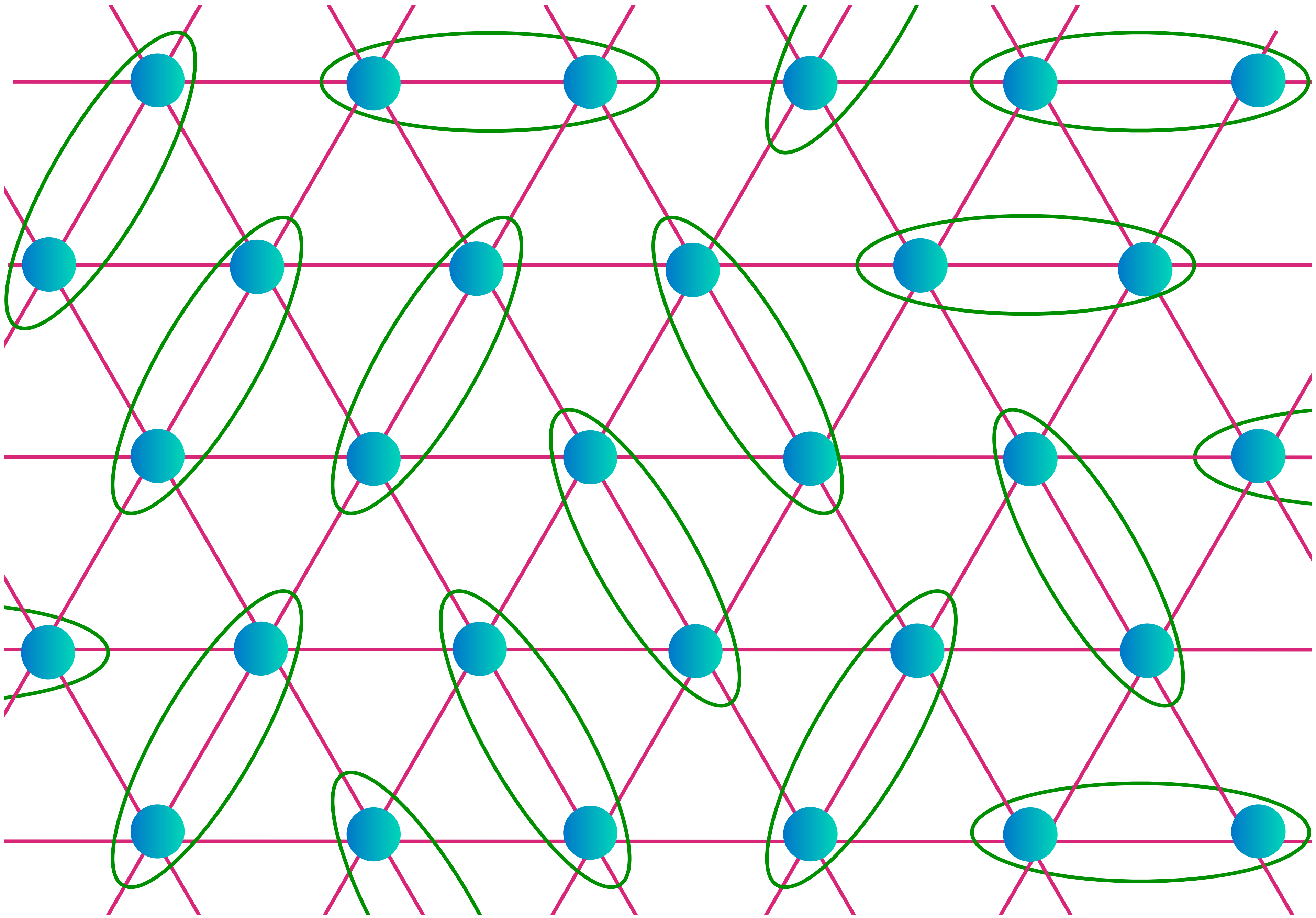

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Effect of flux-piercing on a spin liquid

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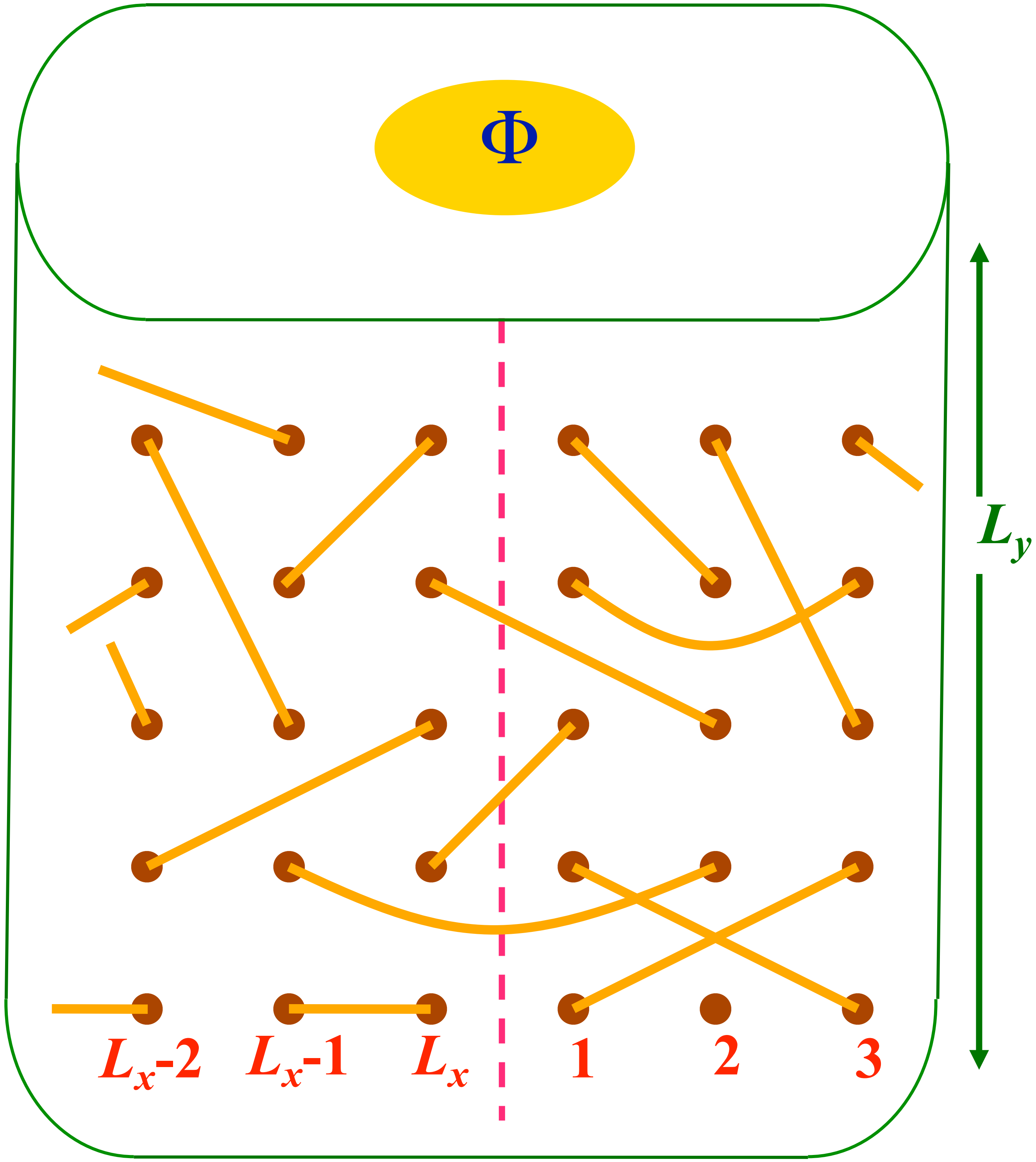

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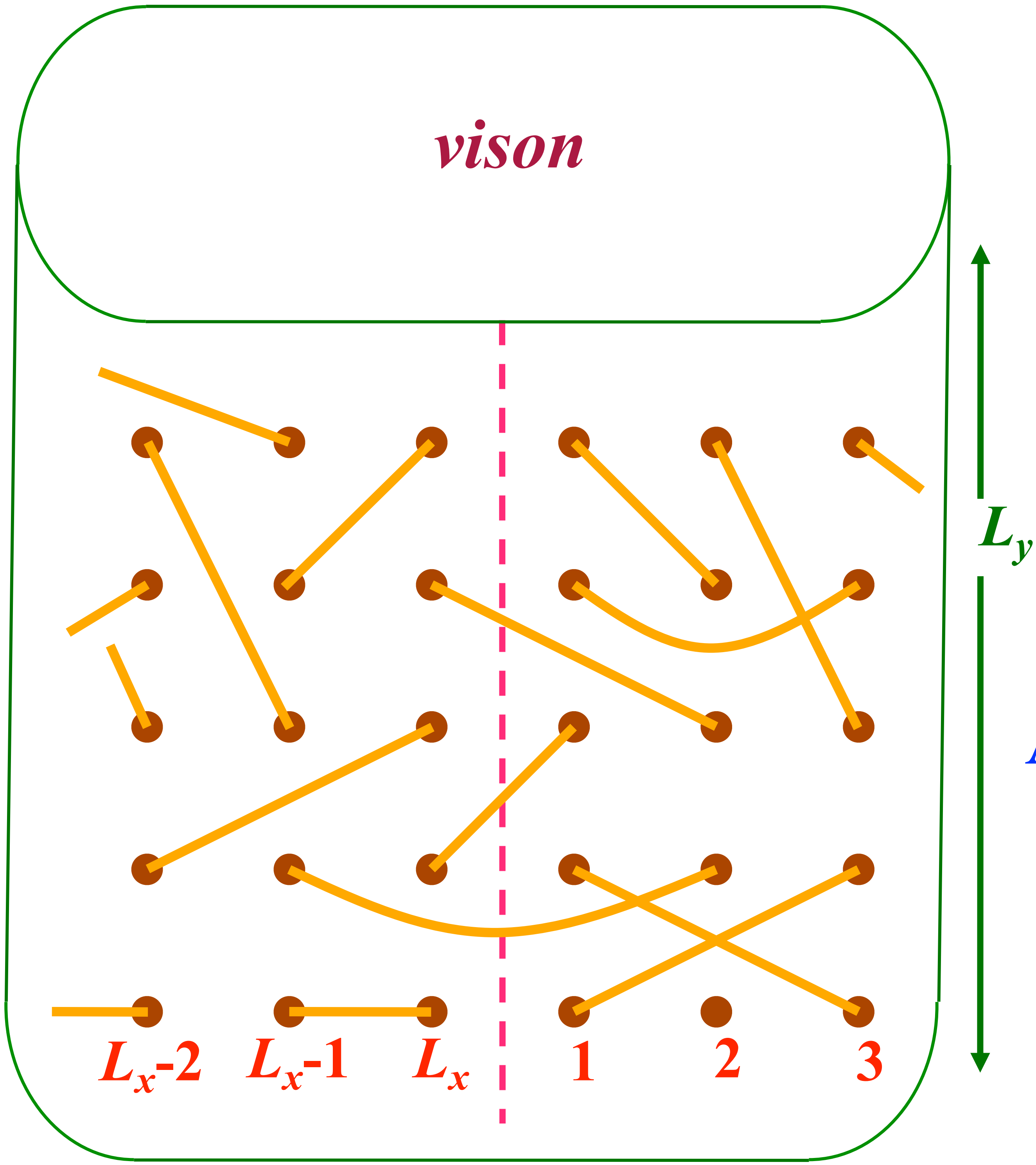

$$|\Psi\rangle = \sum_D a_D |D\rangle$$

After flux insertion $|D\rangle \Rightarrow$

$$(-1)^{\text{Number of bonds cutting dashed line}} |D\rangle;$$

Equivalent to inserting a ***vison*** inside hole of the torus.

Vison carries momentum $\pi L_y / v_0$

Flux piercing argument in Kondo lattice

Shift in momentum is carried by n_T electrons, where

$$n_T = n_f + n_c$$

Treat the Kondo lattice perturbatively in J_K. In the spin liquid, momentum associated with n_f=1 electron is absorbed by creation of vison. The remaining momentum is absorbed by Fermi surface quasiparticles, which enclose a volume associated with n_c electrons.

There exist "topologically ordered" ground states in dimensions $d > 1$ with a Fermi surface of electron-like quasiparticles for which

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A Fractionalized Fermi Liquid

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003).
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004).

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Focus on a single $\vec{S}_f$ spin, and represent its imaginary time fluctuations by a unit vector $\vec{S}_f = \vec{n}(\tau)/2$ which is controlled by the partition function

$$\begin{aligned}\mathcal{Z} &= \int \mathcal{D}\vec{n}(\tau)\,\delta(\vec{n}^2(\tau)-1)\exp\left(-\mathcal{S}\right) \\ \mathcal{S} &= \frac{i}{2}\int_0^1 du \int_0^{1/T} d\tau\,\vec{n}\cdot\left(\frac{\partial\vec{n}}{\partial u}\times\frac{\partial\vec{n}}{\partial\tau}\right) - \int_0^{1/T} d\tau\,\vec{h}(\tau)\cdot\vec{n}(\tau)\end{aligned}$$

The first term is a Wess-Zumino term, with the "extra dimension" u defined so that $\vec{n}(\tau, u=1) \equiv \vec{n}(\tau)$ and $\vec{n}(\tau, u=0) = (0,0,1)$.

The field $\vec{h}(\tau)$ represents the "environment" which has to be determined self-consistently.

Simplest self-consistency condition:

$\vec{h}(\tau)$ is a Gaussian random variable with two-point correlation

$$\langle \vec{h}(\tau) \cdot \vec{h}(0) \rangle \propto \langle \vec{n}(\tau) \cdot \vec{n}(0) \rangle$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001).

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Solution:

$$\langle \vec{n}(\tau) \cdot \vec{n}(0) \rangle \sim \frac{\pi T}{\sin(\pi T \tau)} \quad \text{at large } \tau.$$

This has the structure of correlations on a conformally-invariant 0+1 dimensional boundary of a CFT2.

There is also a non-zero ground state entropy per spin.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001).

Effective low energy theory for conduction electrons

The operators acting on the low energy subspace are c_i and $\vec{S}_{fi}$. For the c_i we have the effective theory

$$\mathcal{S}_c = \int \frac{d^d k}{(2\pi)^d} \int d\tau \left[c_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\sigma} - V F_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} - V c_{\mathbf{k}\sigma}^\dagger F_{\mathbf{k}\sigma} \right]$$

Here the $F_{i\sigma}$ are strongly renormalized operators on the f orbitals, which project onto the low energy theory as

$$F_{i\sigma} \sim \frac{1}{U} \left(\vec{\tau}_{\sigma\sigma'} \cdot \vec{S}_{fi} \right) c_{i\sigma'}$$

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From this we obtain the marginal Fermi liquid behavior in the conduction electron self energy

$$\Sigma_c(\tau) \sim \left[\frac{\pi T}{\sin(\pi T \tau)} \right]^2$$

S. Burdin, D. R. Grempel, and A. Georges, Phys. Rev. B **66**, 045111 (2002)

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- The CFT3 is dual to a gravity theory on $AdS_4 \times S^7$

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- Add some SO(8) charge by turning on a chemical potential (this will break the SO(8) symmetry)

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- In the Einstein-Maxwell theory, the chemical potential leads to an extremal Reissner-Nordtrom black hole in the AdS_4 spacetime.
- The near-horizon geometry of the RN black hole is $AdS_2 \times R^2$. There has been no clear interpretation of the AdS_2 theory, the R^2 degeneracy, and the finite ground state entropy density

What is the meaning of $\mathrm{AdS}_2 \times \mathrm{R}^2$?

The AdS_2 represents the dynamics of a quantum "spin" with partition function

$$\mathcal{Z} = \int \mathcal{D}\vec{n}(\tau) \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \mathcal{S}_{\mathrm{Wess-Zumino}}[\vec{n}(\tau, u)] - \int_0^{1/T} d\tau \, \vec{h}(\tau) \cdot \vec{n}(\tau)$$

S. Sachdev, arXiv:1006.3794

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- The environment field $\vec{h}(\tau)$ represents the dynamics of the CFT3.

S. Sachdev, arXiv:1006.3794

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- The spin carries a global SO(8) charge.
- The R^2 represents a finite density of such spins.
- In the classical gravity theory, these spins do not interact with each other: this leads to the R^2 degeneracy and the finite ground state entropy density.

S. Sachdev, arXiv:1006.3794

Effective low energy theory for "conduction electrons"

The operators acting on the low energy subspace are the probe fermions c_i, and the F_i with the effective theory

$$\mathcal{S}_c = \int \frac{d^d k}{(2\pi)^d} \int d\tau \left[c^\dagger_{\mathbf{k}\sigma} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\sigma} - V F^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - V c^\dagger_{\mathbf{k}\sigma} F_{\mathbf{k}\sigma} \right]$$

Here the $F_{i\sigma}$ are gauge-invariant operators in the AdS$_2$ with the same quantum numbers as the electron, with the correlator

$$\left\langle F_{\mathbf{k}\sigma}(\tau) F^\dagger_{\mathbf{k}\sigma}(0) \right\rangle \sim \left(\frac{\pi T}{\sin(\pi T \tau)} \right)^{2\Delta_k}$$

T. Faulkner, H. Liu, J. McGreevy and D. Vegh, arXiv:0907.2694.
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From this we obtain the non-Fermi liquid behavior in the conduction electron self energy

$$\Sigma_c(\tau) \sim \left[\frac{\pi T}{\sin(\pi T \tau)} \right]^{2\Delta_k}$$

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S. Kachru, A. Karch, and S. Yaida, Phys. Rev. D **81**, 026007 (2010) have given an explicit construction of such a Kondo lattice in AdS_5.