Detecting boson-vortex duality in the cuprate superconductors

Physical Review B **71**, 144508 and 144509 (2005),
cond-mat/0602429

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Talk online at http://sachdev.physics.harvard.edu

Outline

I. Bose-Einstein condensation and superfluidity

II. The cuprate superconductors, and their proximity to a superfluid-insulator transition

III. The superfluid-insulator quantum phase transition

IV. Duality

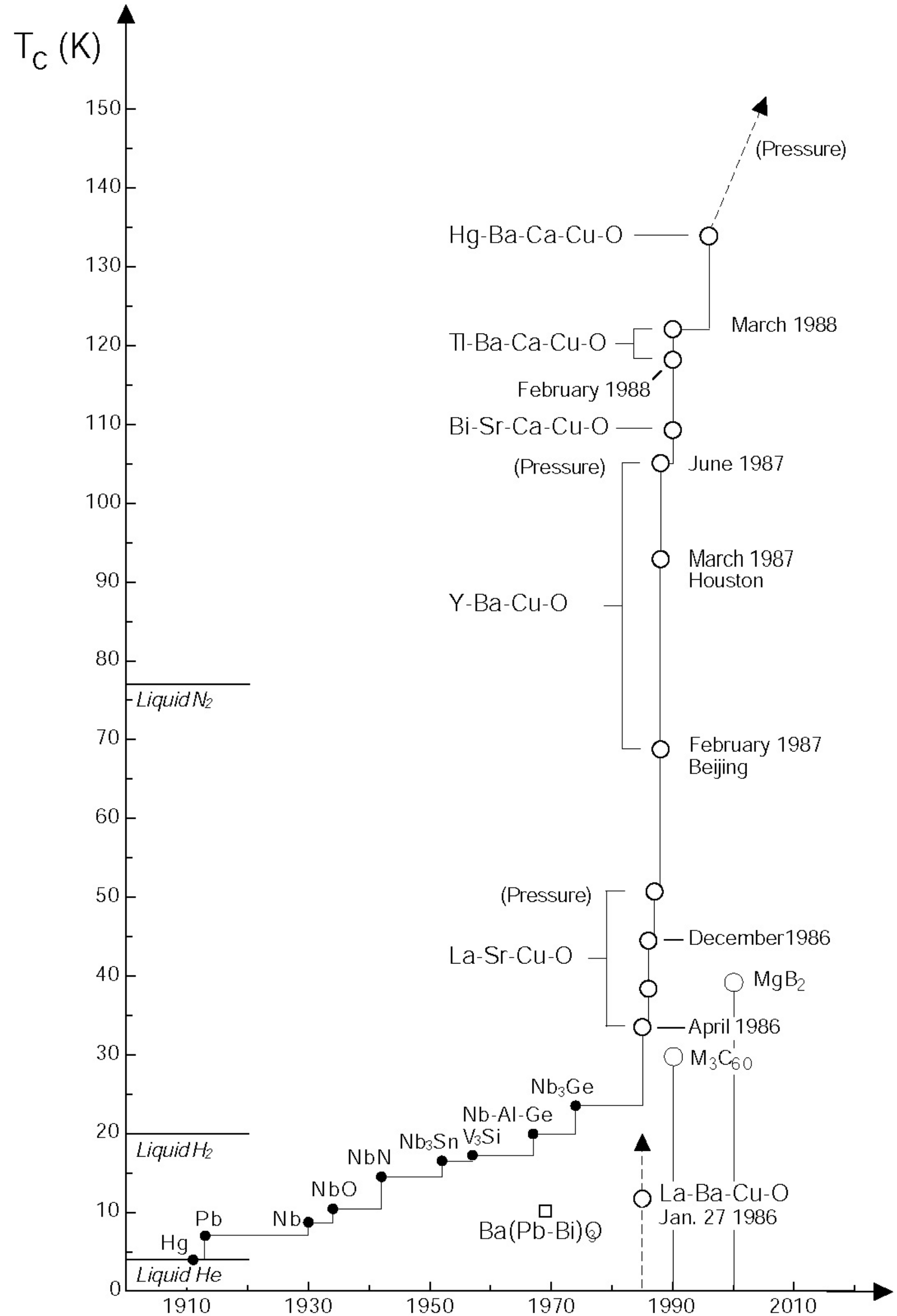
V. The quantum mechanics of vortices near the superfluid-insulator transition

Dual theory of superfluid-insulator transition as the proliferation of vortex-anti-vortex pairs

I. Bose-Einstein condensation and superfluidity

Superfluidity/superconductivity occur in:

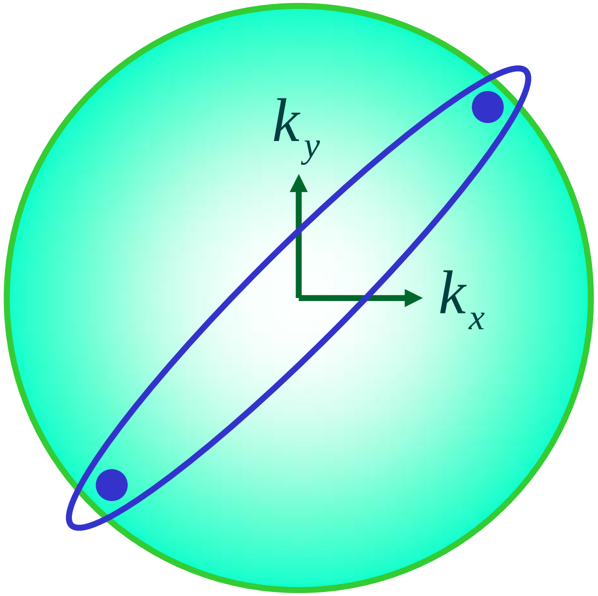
- liquid ^{4}He
- metals Hg, Al, Pb, Nb, Nb_3Sn…..
- liquid ^{3}He
- neutron stars
- cuprates $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{6+y}$….
- M_3C_{60}
- ultracold trapped atoms
- MgB_2



The Bose-Einstein condensate:

A macroscopic number of bosons occupy the lowest energy quantum state

Such a condensate also forms in systems of fermions, where the bosons are Cooper pairs of fermions:

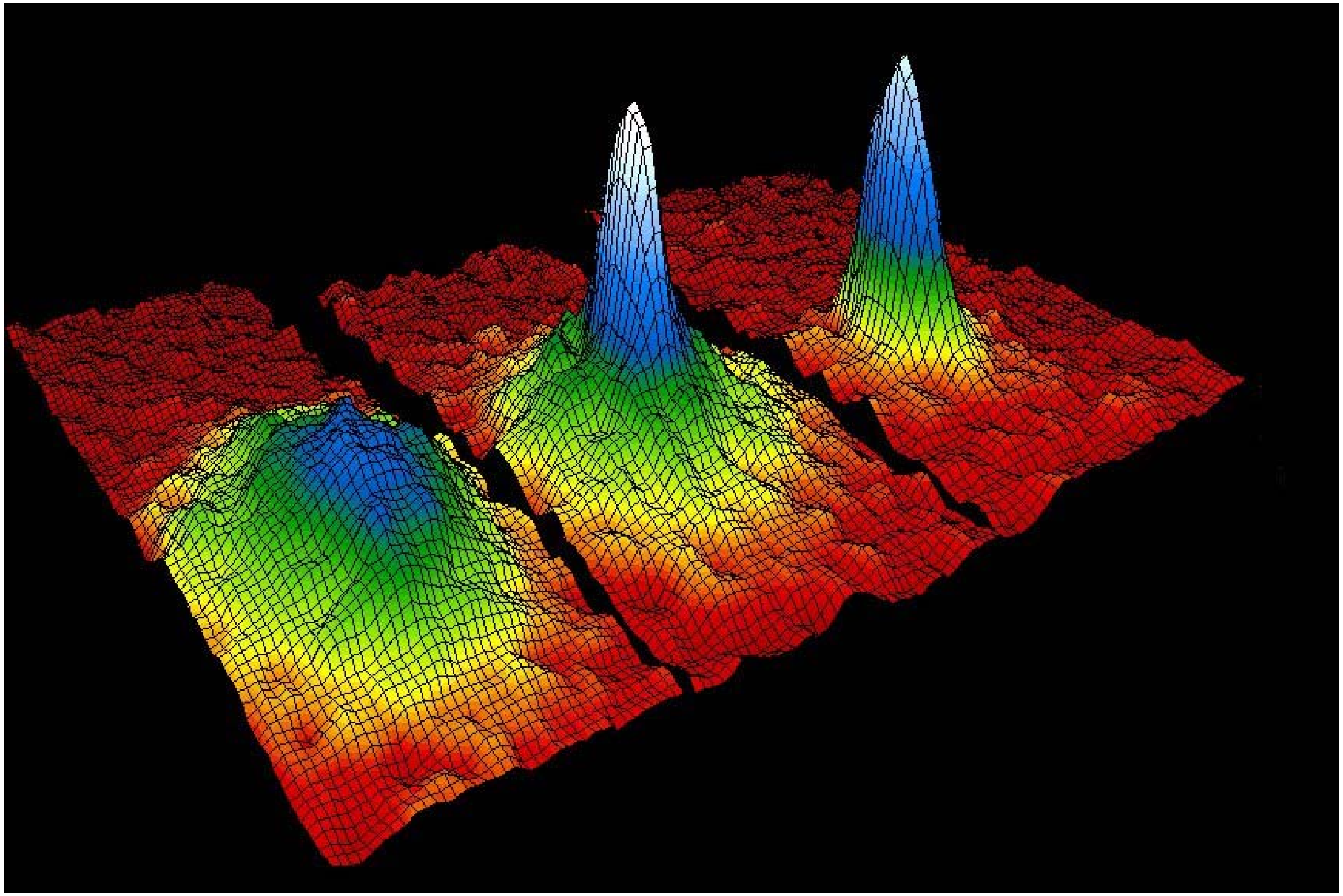


Pair wavefunction in cuprates:

$$\Psi = \left(k_x^2 - k_y^2\right)\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$\left\langle \vec{S} \right\rangle = 0$$

Velocity distribution function of ultracold ^{87}Rb atoms



M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, *Science* **269**, 198 (1995)

Superflow:

The wavefunction of the condensate

$$\Psi \rightarrow \Psi e^{i\theta(\boldsymbol{r})}$$

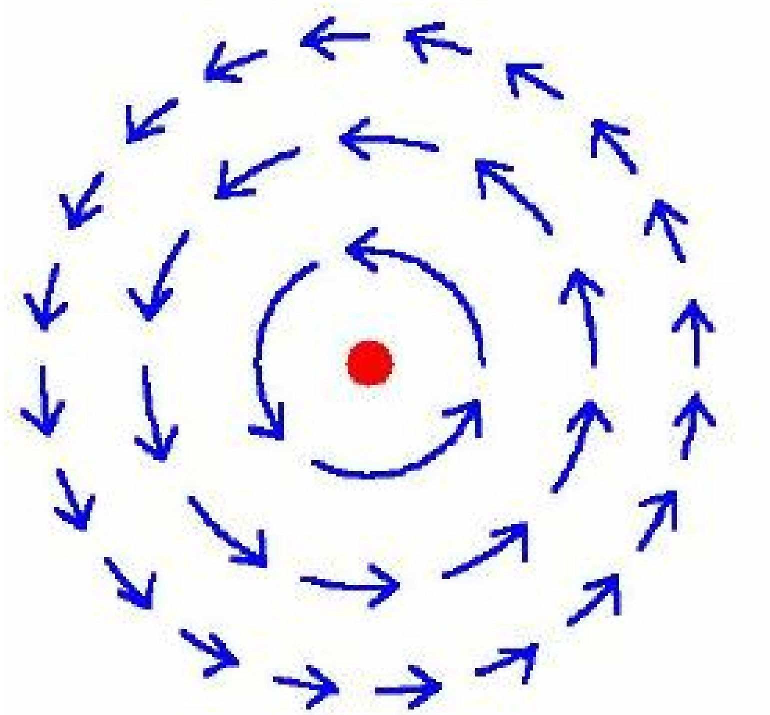
Superfluid velocity

$$\boldsymbol{v}_s = \frac{\mathrm{h}}{m} \nabla \theta$$

(for non-Galilean invariant superfluids,

the co-efficient of $\nabla \theta$ is modified)

Excitations of the superfluid: **Vortices**

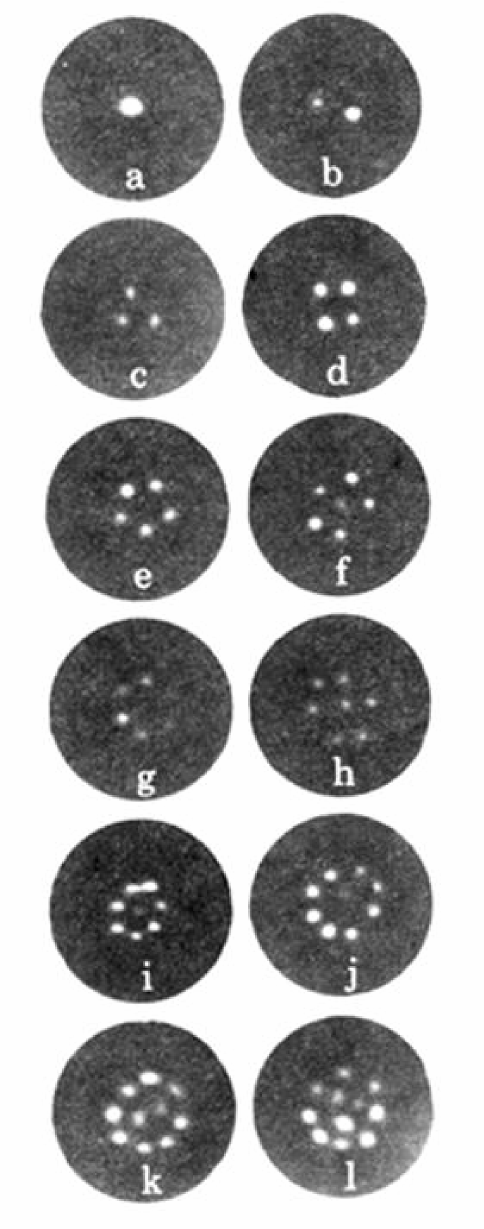


The circulation of a vortex is quantized:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \oint \nabla\theta \cdot d\mathbf{r} = n\frac{h}{m}$$

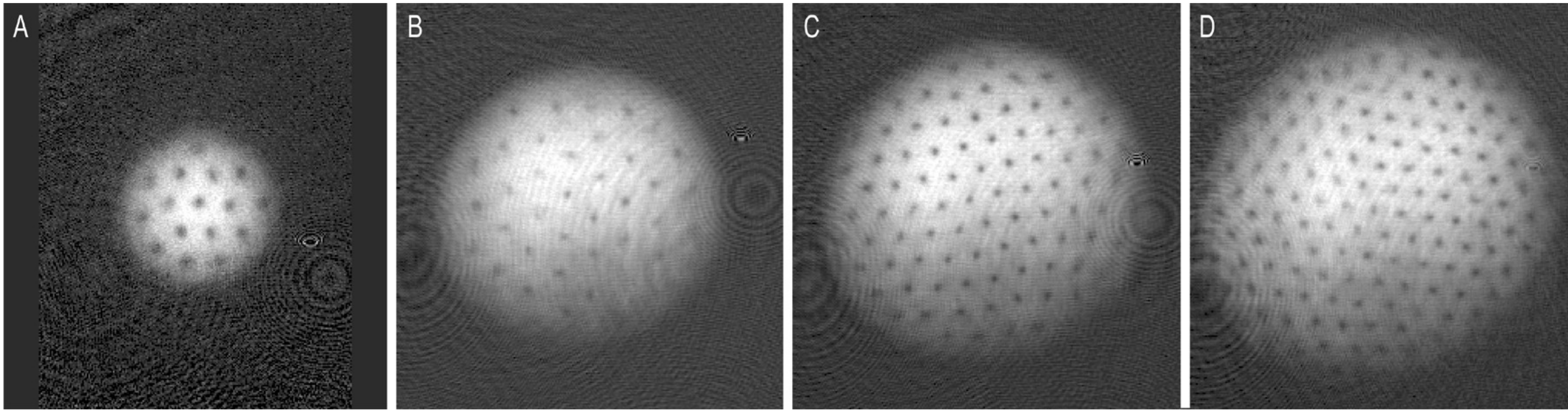
where n is an integer.

Observation of quantized vortices in rotating ^{4}He



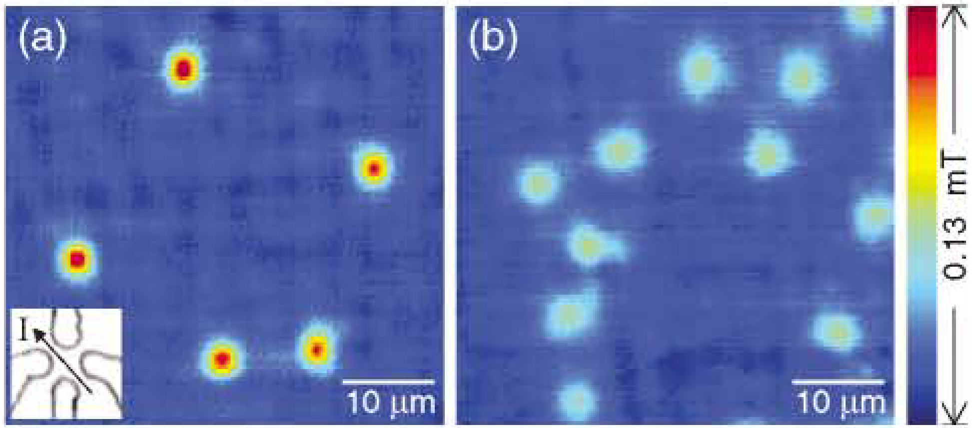
E.J. Yarmchuk, M.J.V. Gordon, and R.E. Packard, *Observation of Stationary Vortex Arrays in Rotating Superfluid Helium*, Phys. Rev. Lett. **43**, 214 (1979).

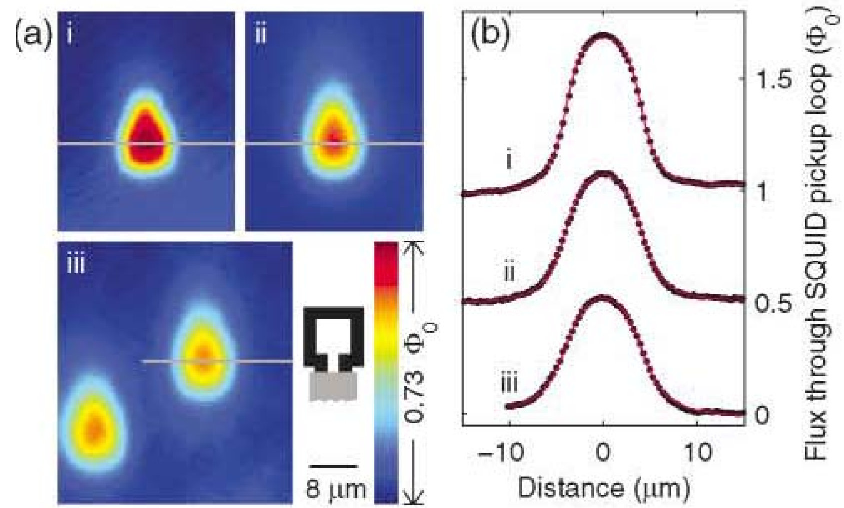
Observation of quantized vortices in rotating ultracold Na



J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle, *Observation of Vortex Lattices in Bose-Einstein Condensates*, Science **292**, 476 (2001).

Quantized fluxoids in $YBa_2Cu_3O_{6+y}$





J. C. Wynn, D. A. Bonn, B.W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett.* **87**, 197002 (2001).

In superconductors, vortices carry quantized magnetic flux:

$$\int \mathbf{B} \cdot d\mathbf{S} = n\frac{hc}{2e}$$

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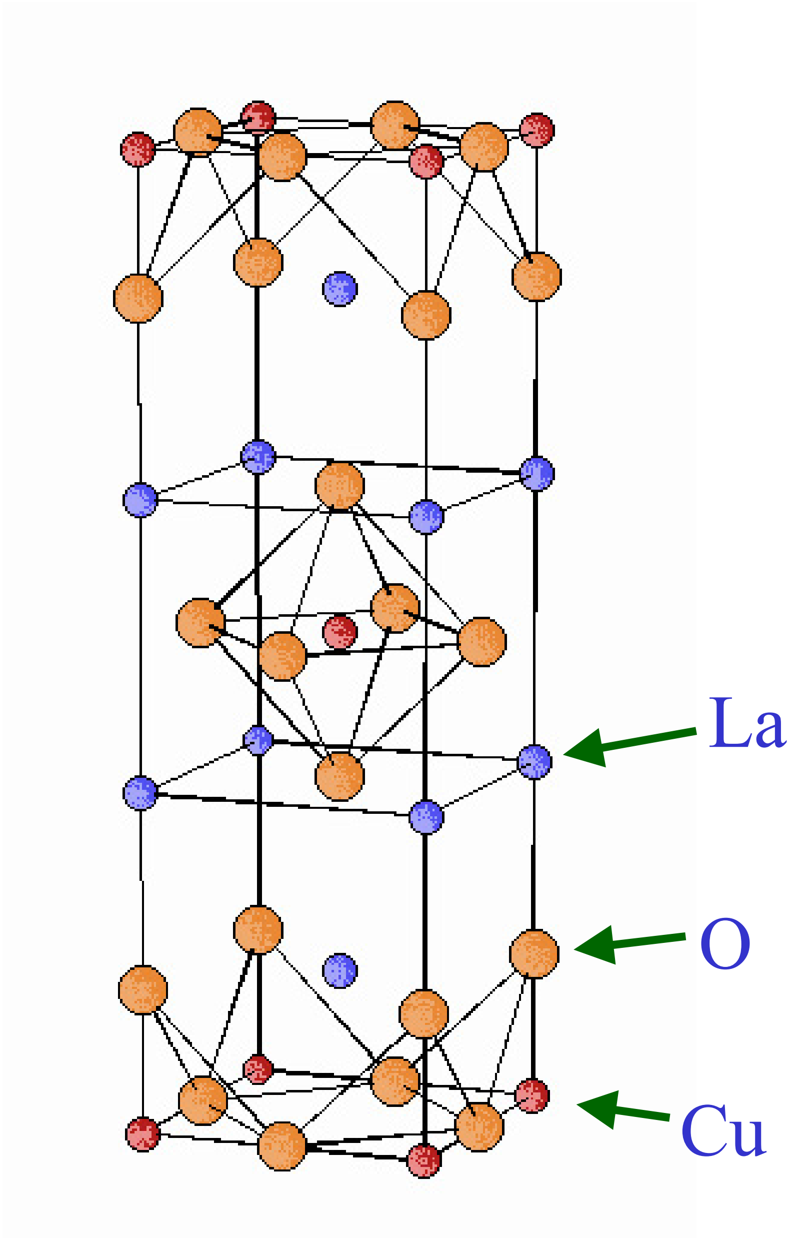
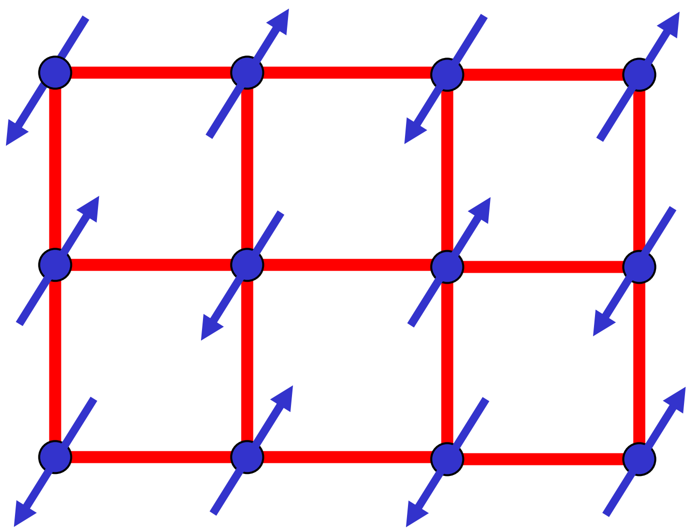


La_2CuO_4

La_2CuO_4

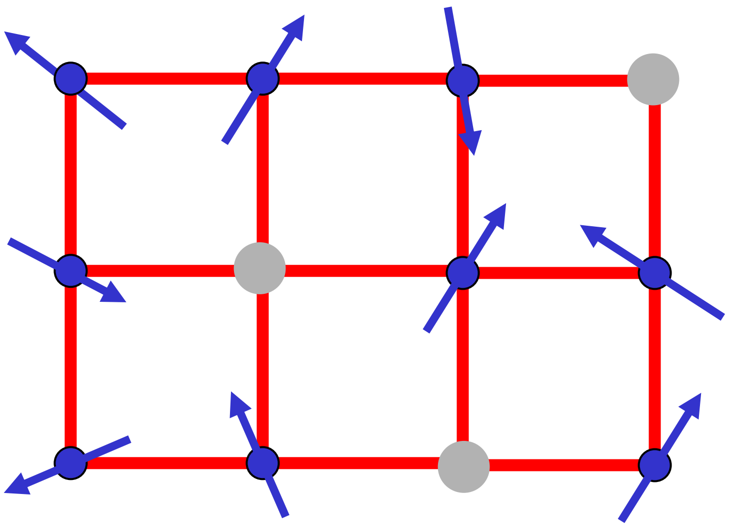
Mott insulator: square lattice antiferromagnet



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

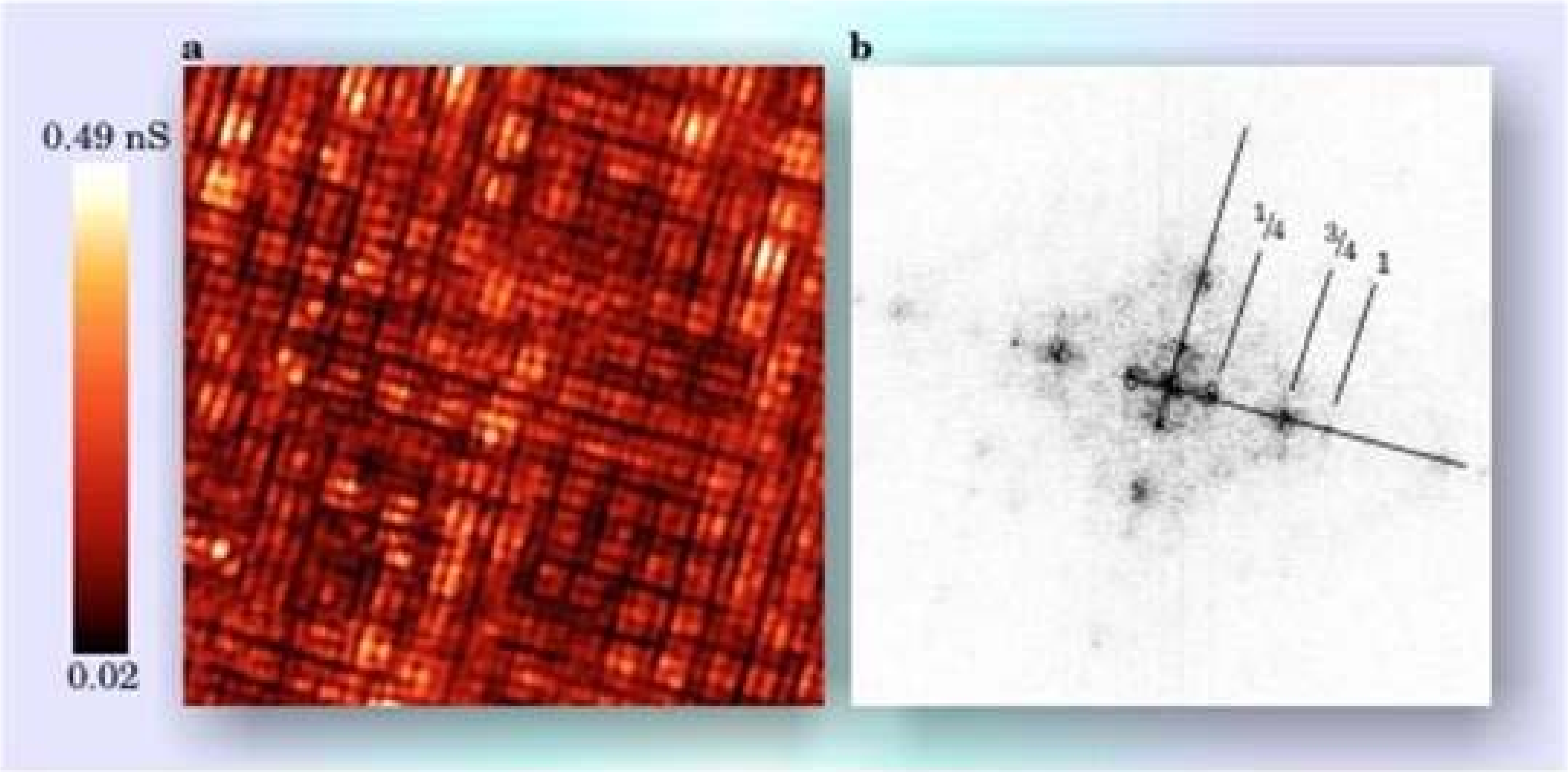
$La_{2-\delta}Sr_{\delta}CuO_4$

Superfluid: condensate of paired holes



$$\langle \vec{S} \rangle = 0$$

The cuprate superconductor $Ca_{2-x}Na_xCuO_2Cl_2$



T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004). Closely related modulations in superconducting $Bi_2Sr_2CaCu_2O_{8+\delta}$ observed first by C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546 and Physical Review B **67**, 014533 (2003).

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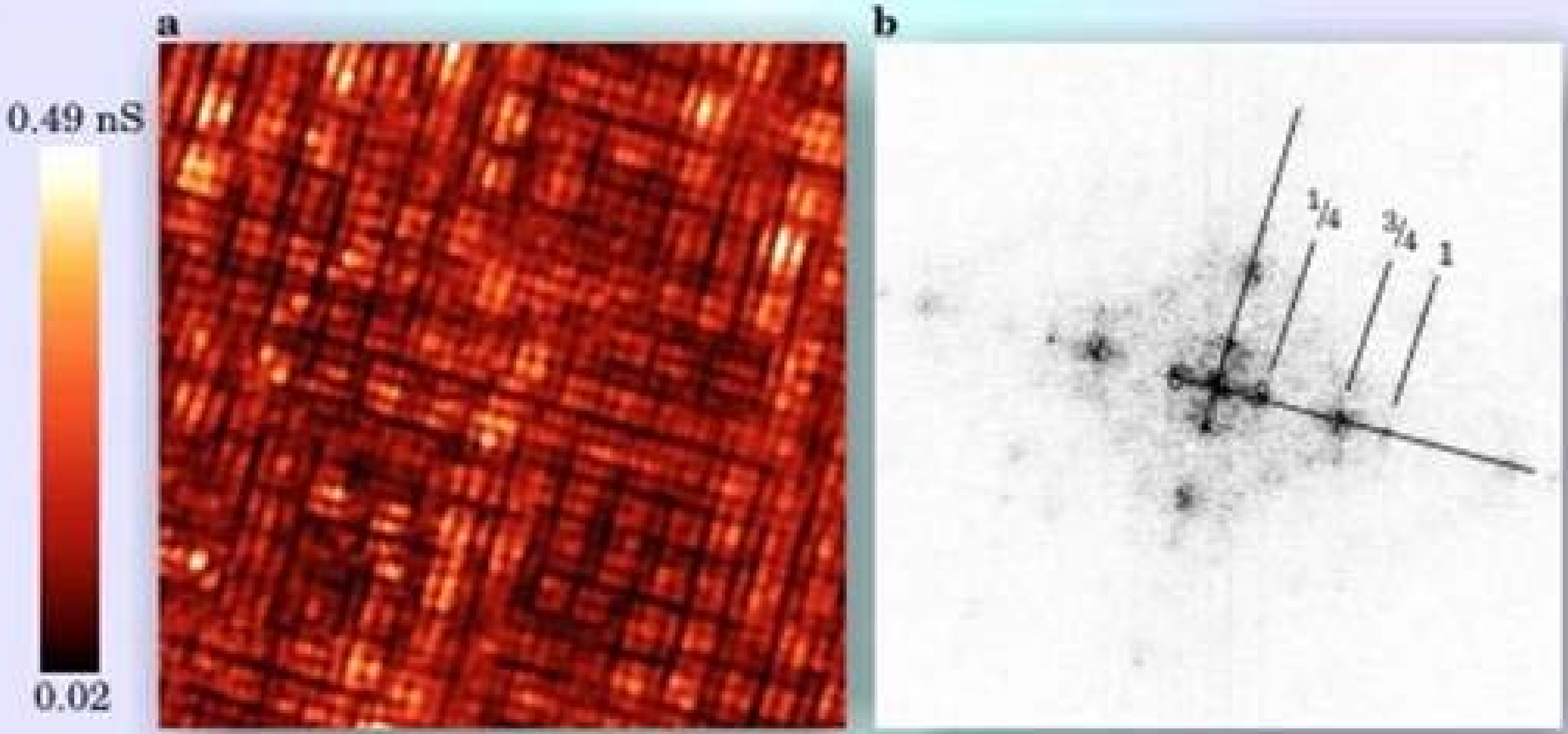


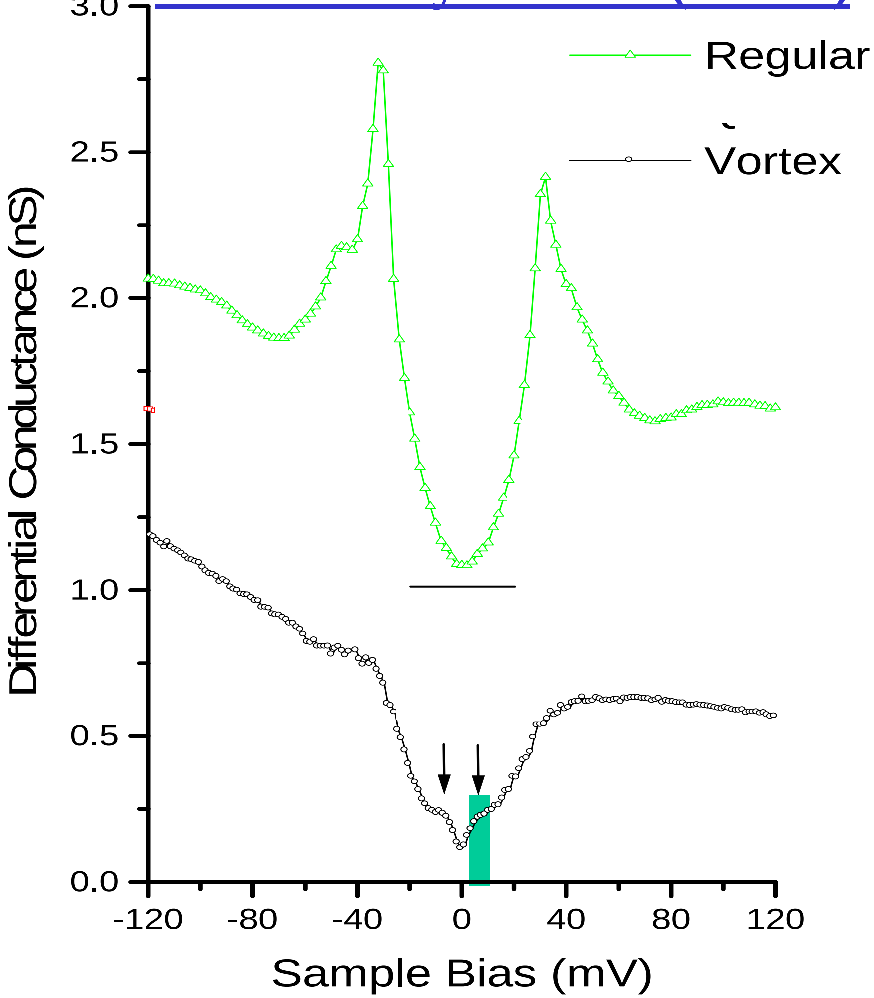
Evidence that holes can form an insulating state with period ≈ 4 modulation in the density

T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004). Closely related modulations in superconducting $Bi_2Sr_2CaCu_2O_{8+\delta}$ observed first by C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546 and Physical Review B **67**, 014533 (2003).

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states (LDOS)



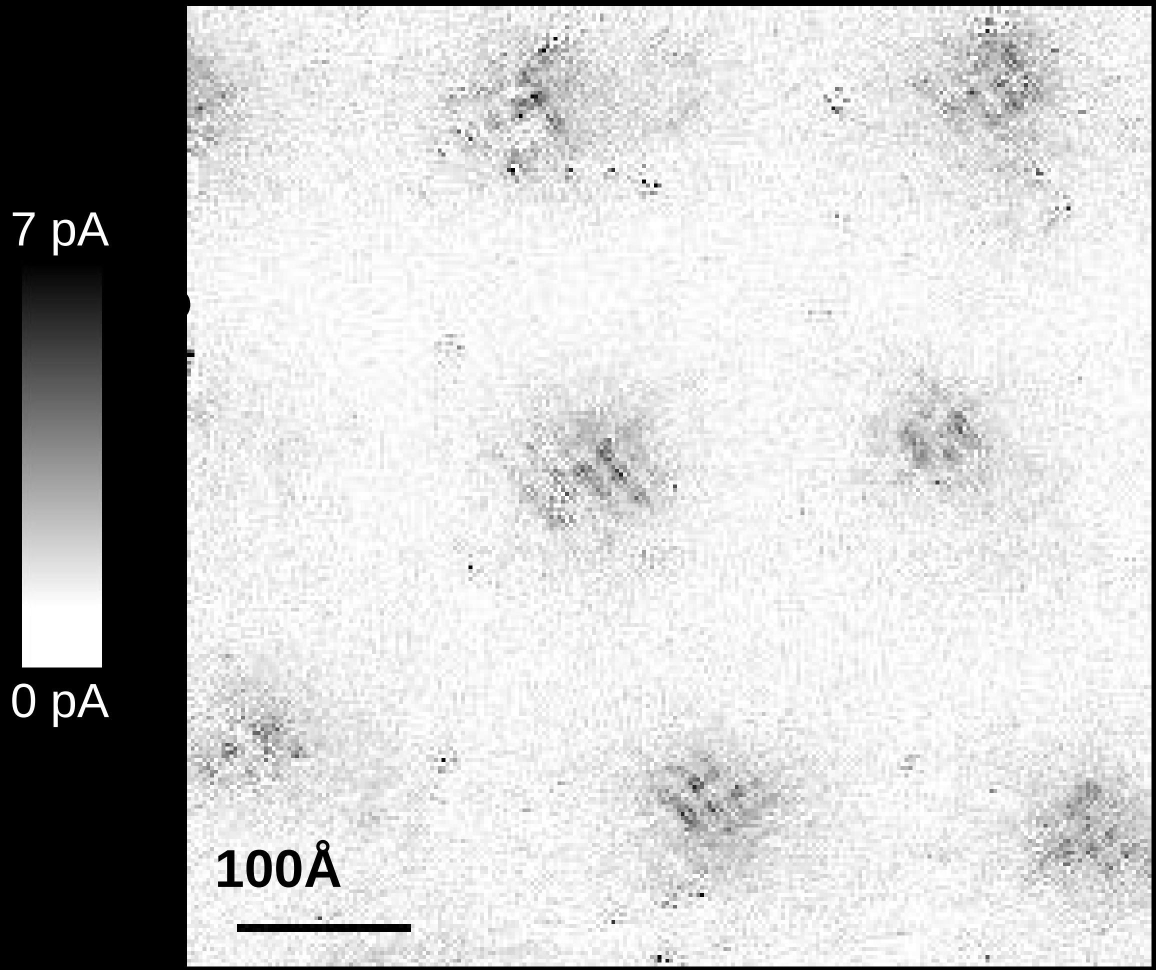
1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

I. Maggio-Aprile *et al. Phys. Rev. Lett.* **75**, 2754 (1995).
S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

Berkeley University of California

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

J. Hoffman et al., *Science* 295, 466 (2002).
G. Levy et al., Phys. Rev. Lett. 95, 257005 (2005).

Prediction of periodic LDOS modulations near vortices:
K. Park and S. Sachdev, Phys. Rev. B 64, 184510 (2001).

Questions on the cuprate superconductors

- What is the quantum theory of the ground state as it evolves from the superconductor to the modulated insulator ?

- What happens to the vortices near such a quantum transition ?

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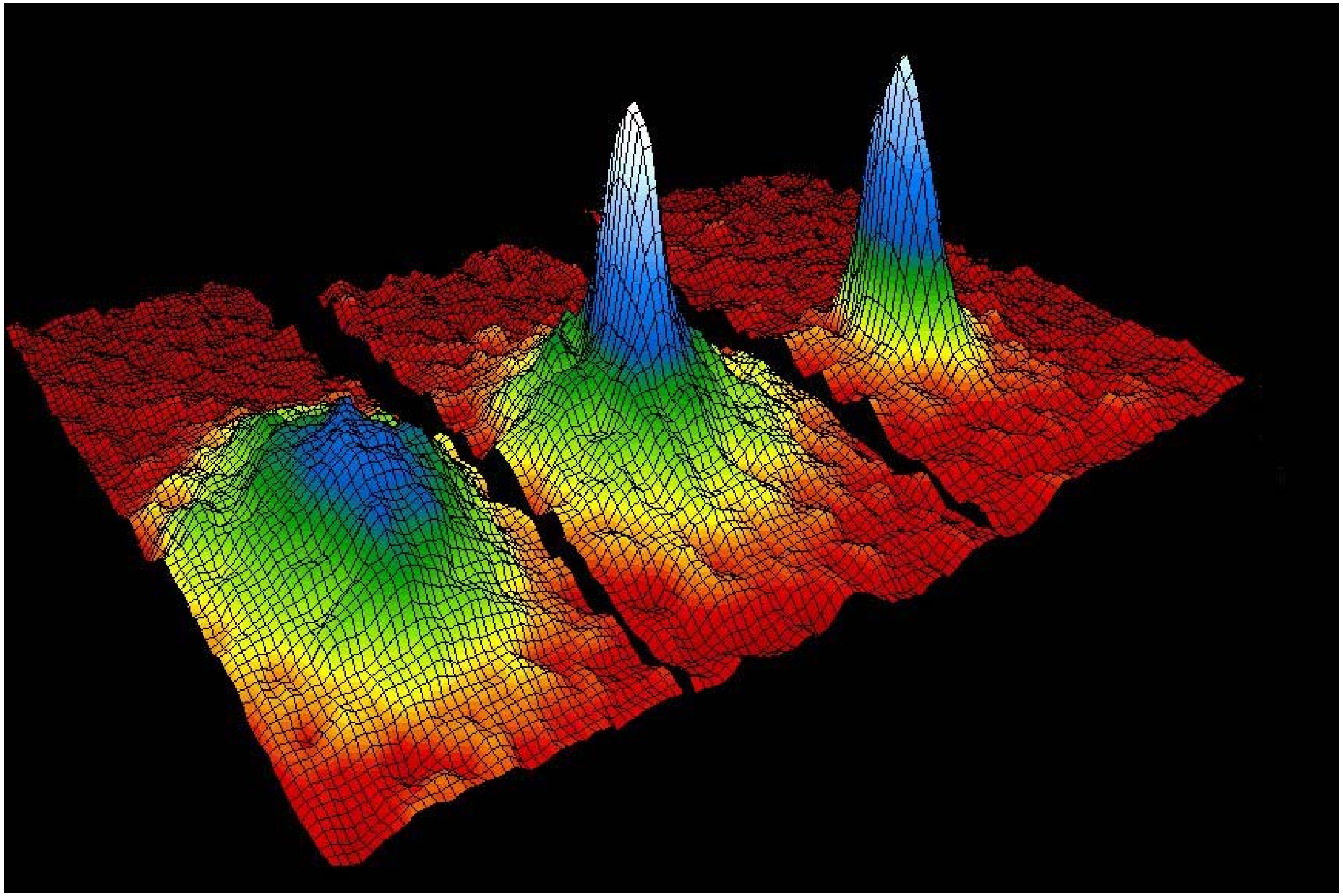
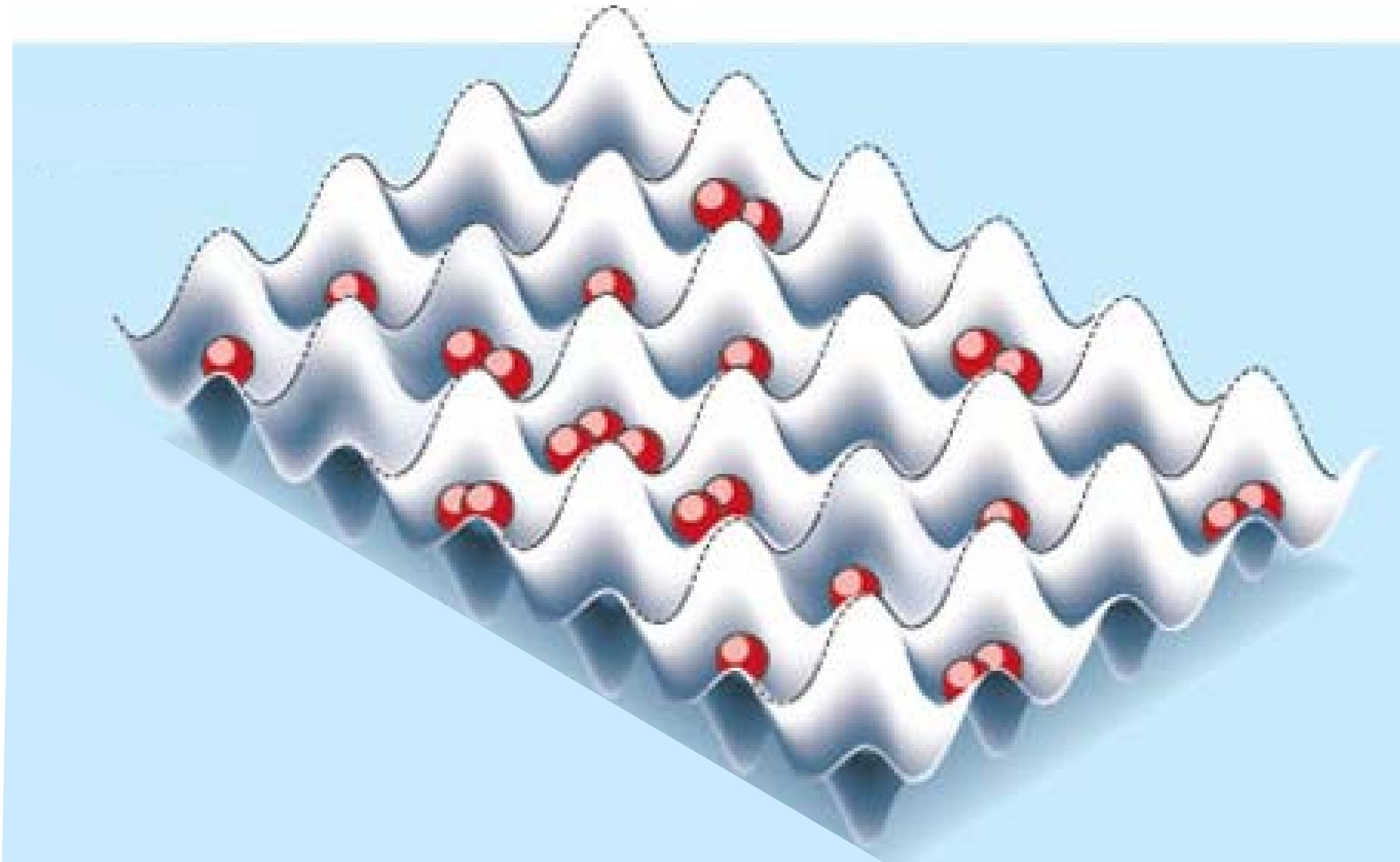
Velocity distribution function of ultracold ^{87}Rb atoms



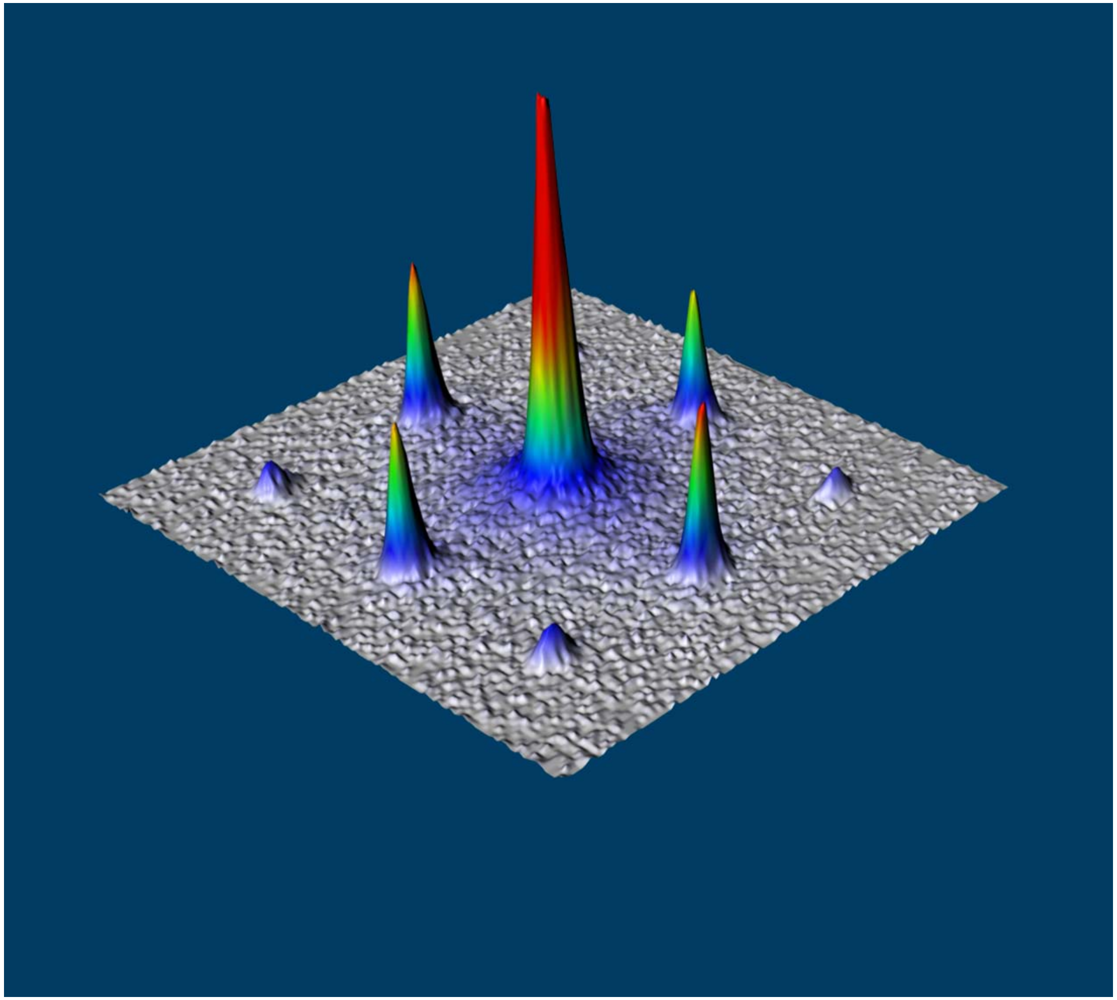
M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, *Science* **269**, 198 (1995)

Apply a periodic potential (standing laser beams) to trapped ultracold bosons (^{87}Rb)



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

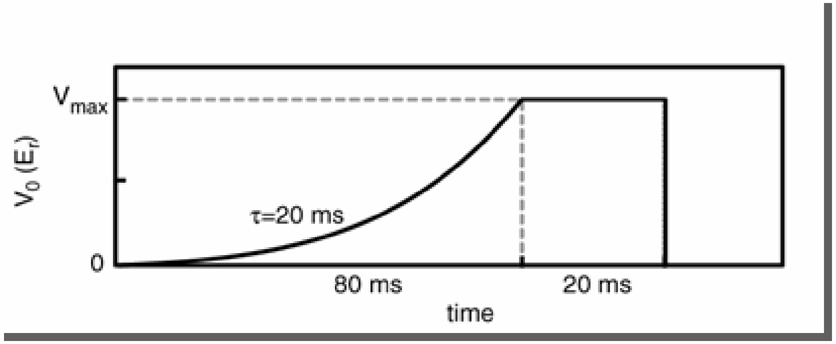
Momentum distribution function of bosons

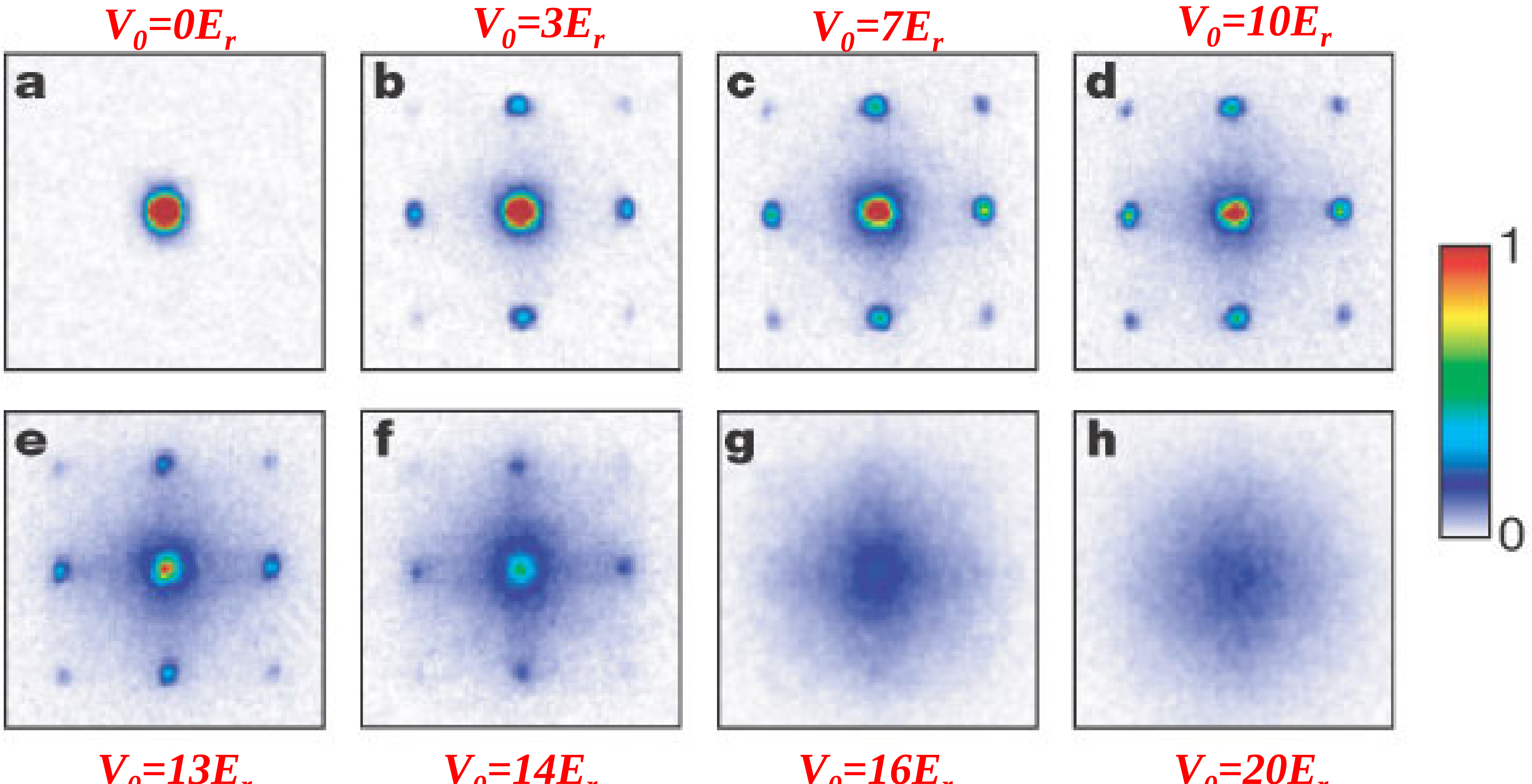


Bragg reflections of condensate at reciprocal lattice vectors

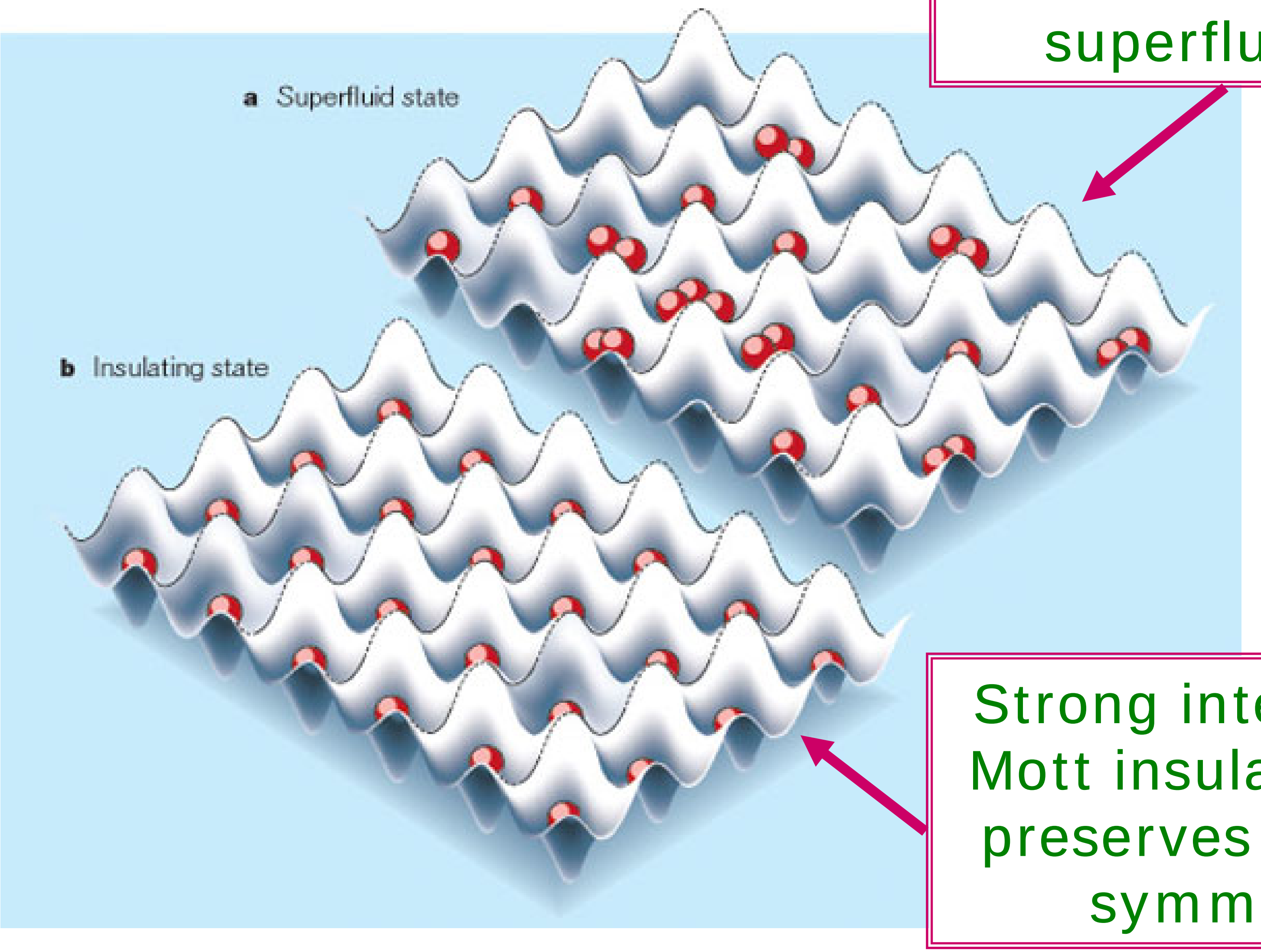
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Superfluid-insulator quantum phase transition at T=0





Bosons at filling fraction f = 1



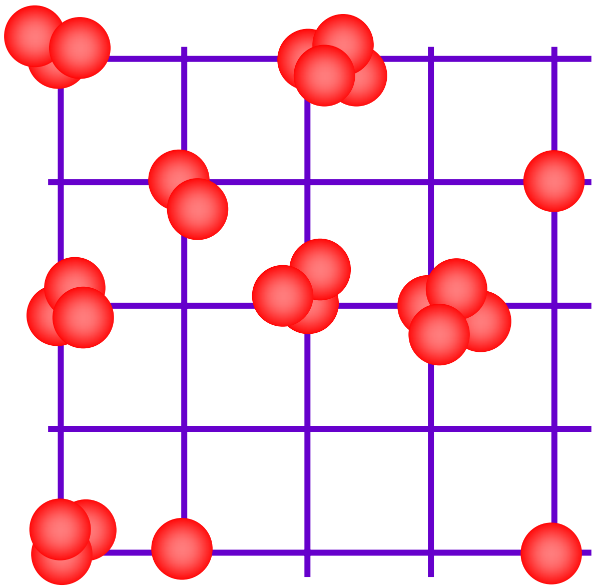
Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Related earlier work by C. Orzel, A.K. Tuchman, M. L. Fenselau, M. Yasuda, and M. A. Kasevich, *Science* **291**, 2386 (2001).

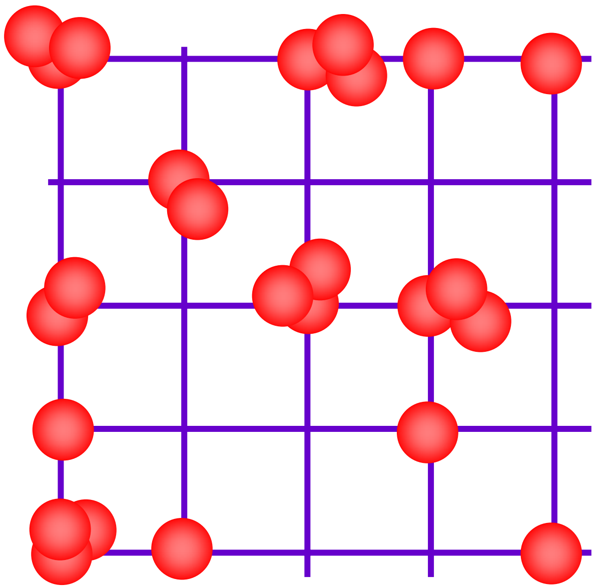
Bosons at filling fraction $f = 1$



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

Bosons at filling fraction $f = 1$



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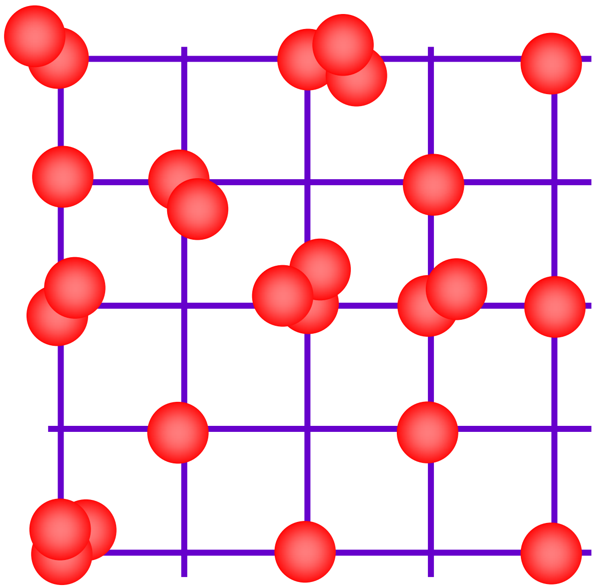

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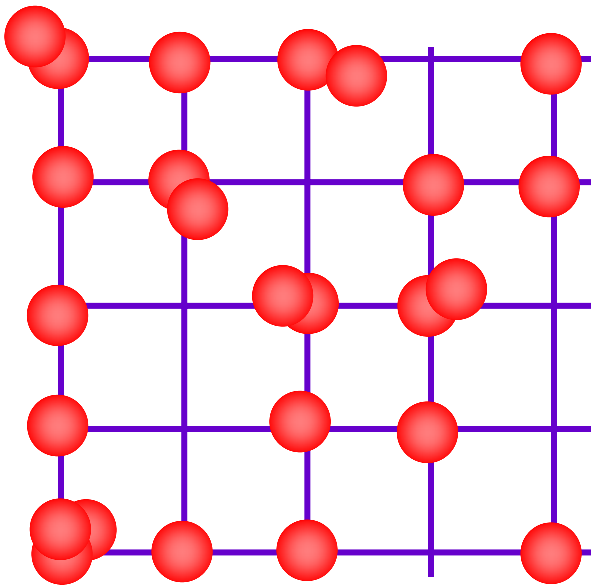

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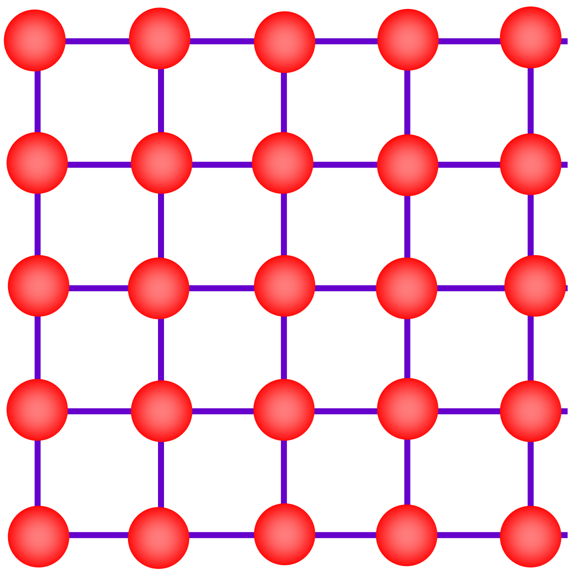
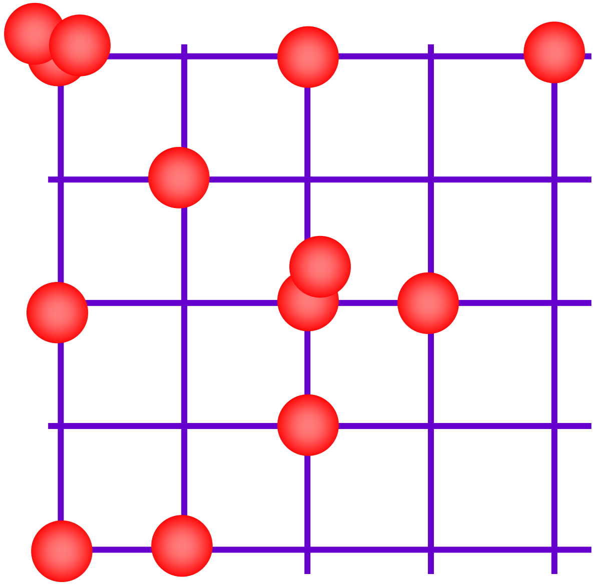


$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

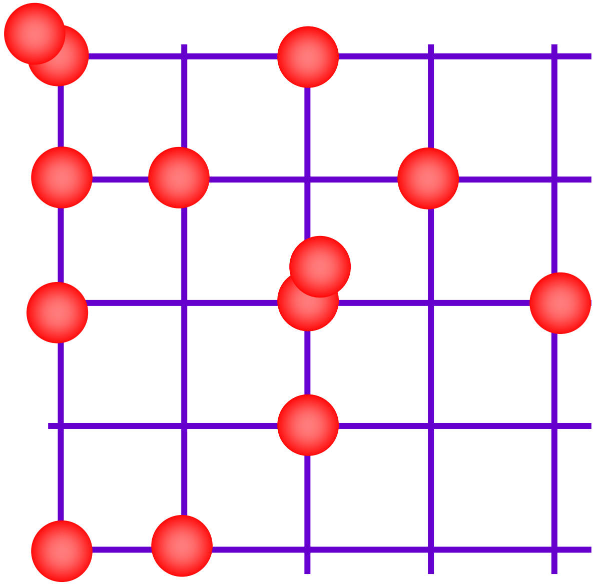
Bosons at filling fraction f = 1/2



$$\langle \Psi \rangle \neq 0$$

Weak interactions: superfluidity

Bosons at filling fraction $f = 1/2$



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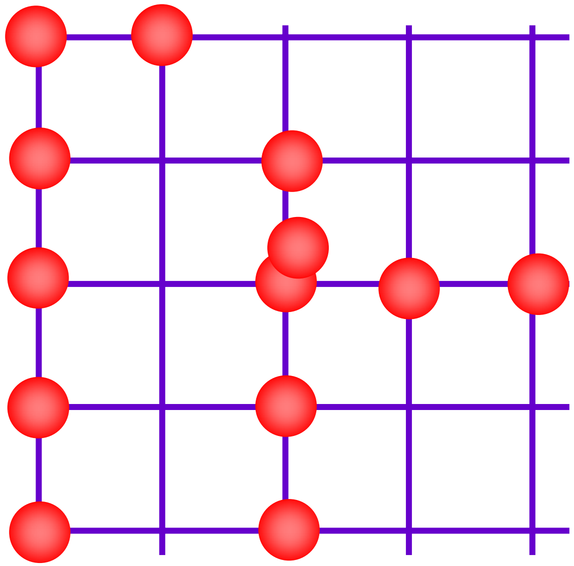

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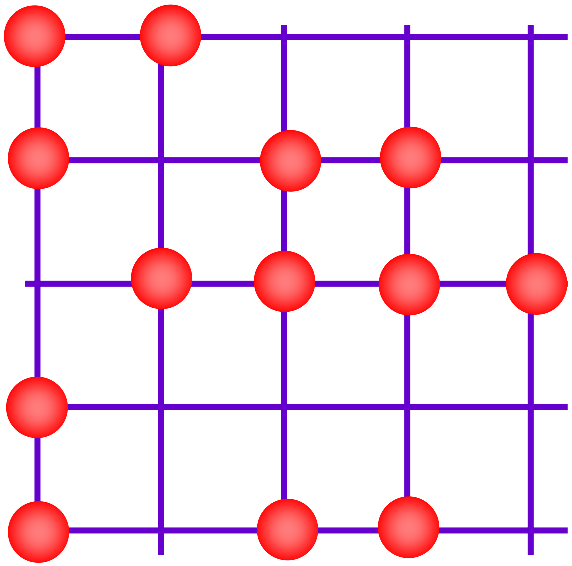

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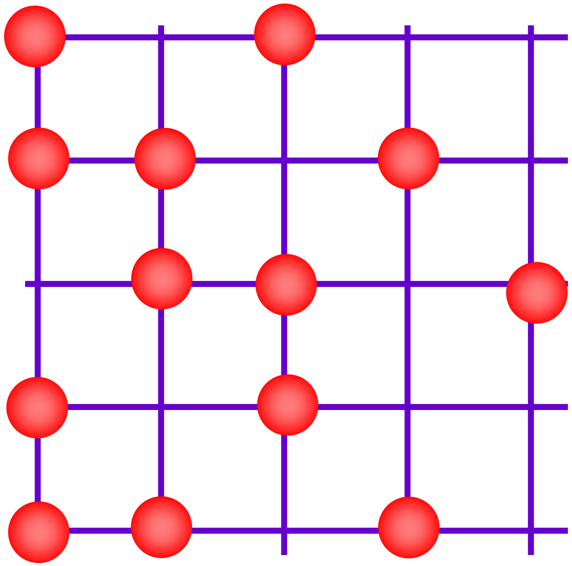

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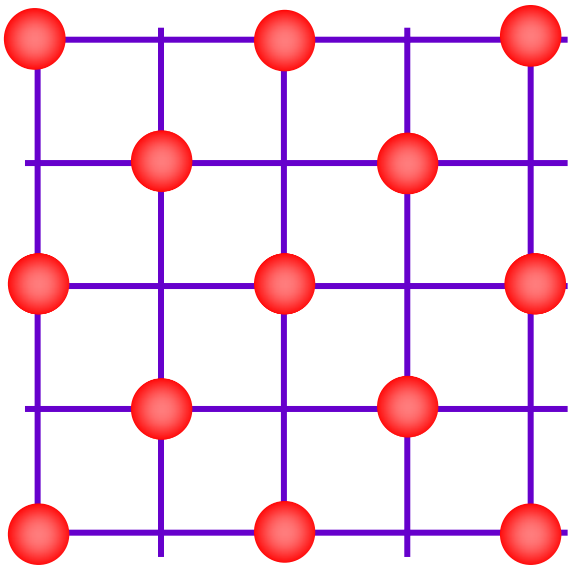

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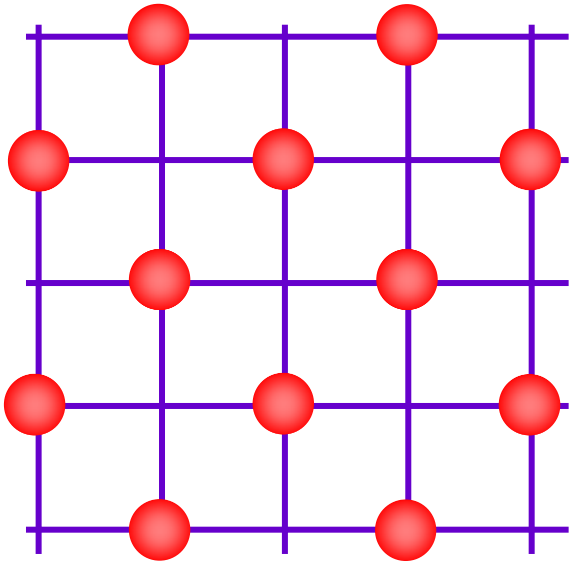

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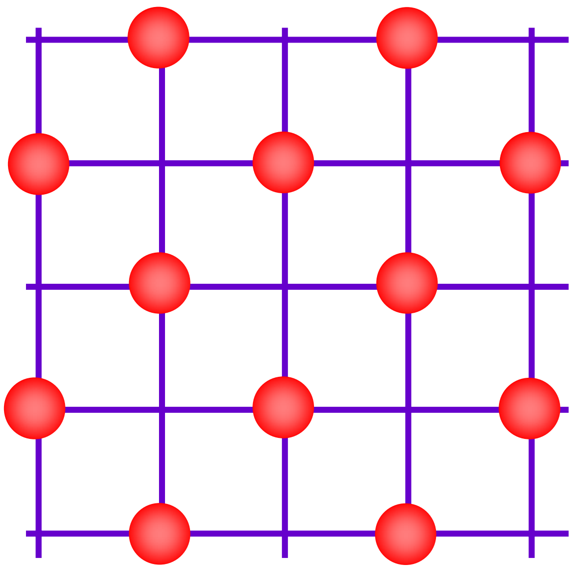
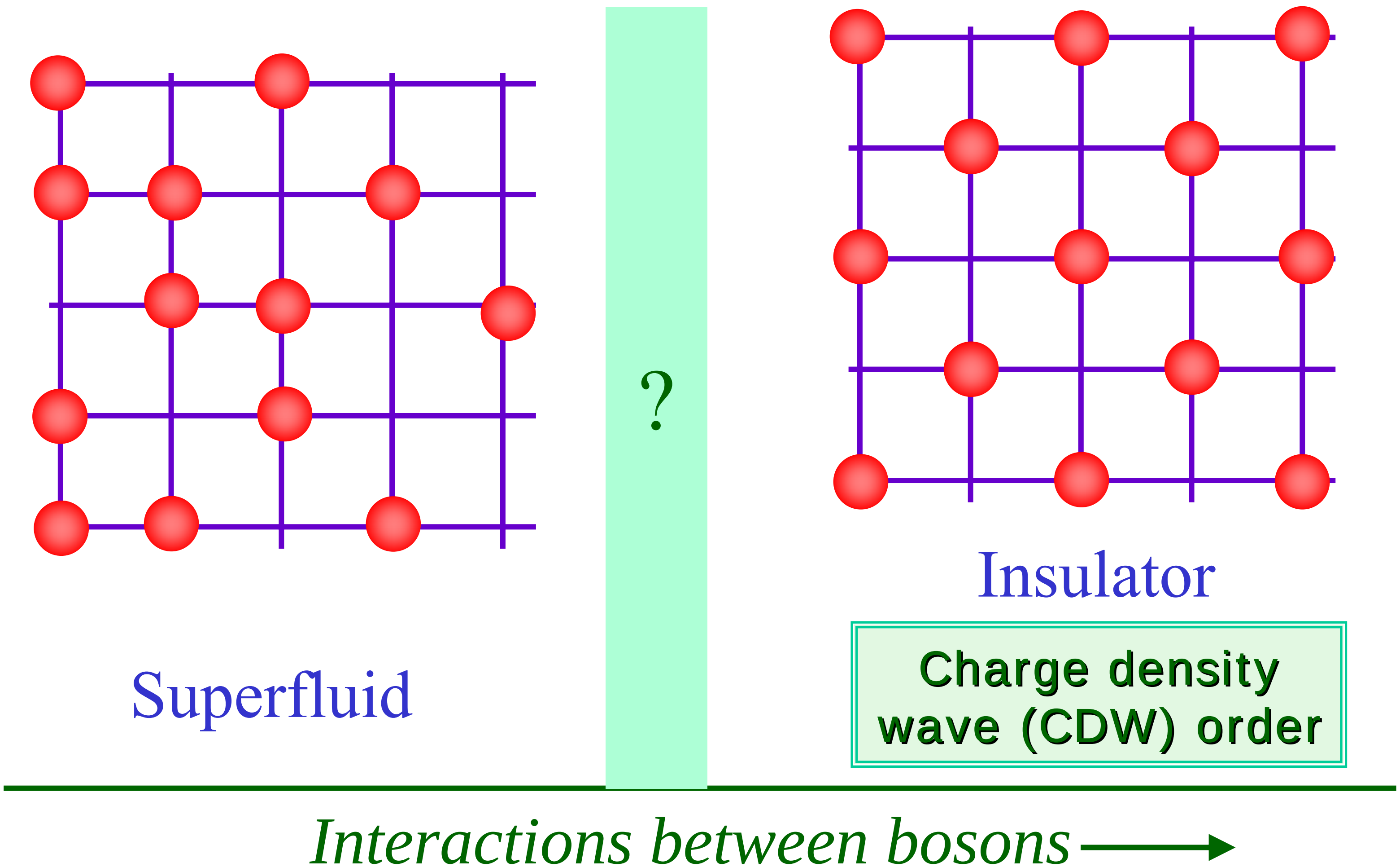


$$\langle \Psi \rangle = 0$$

Strong interactions: insulator

Insulator has "density wave" order

Bosons on the square lattice at filling fraction f=1/2



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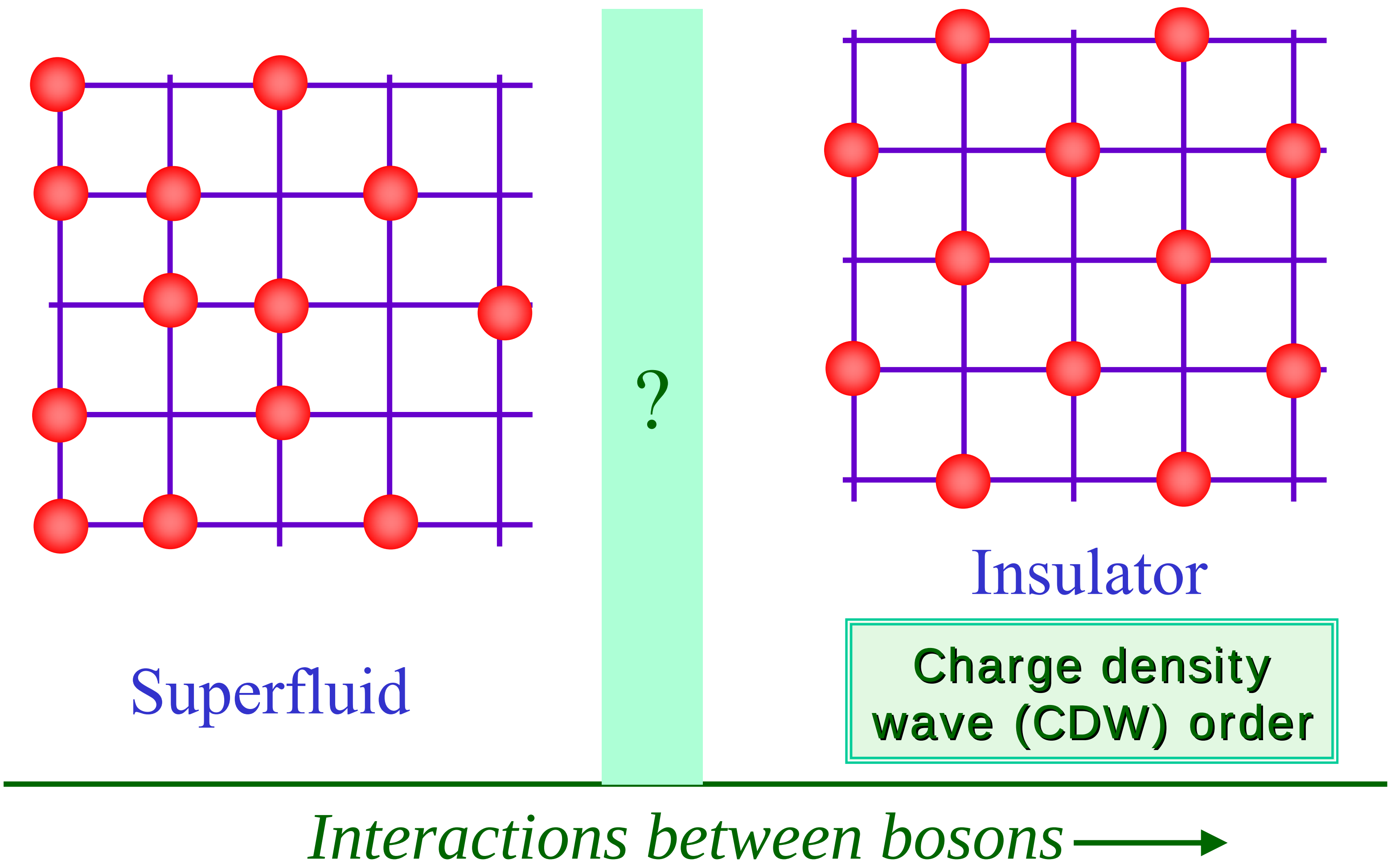

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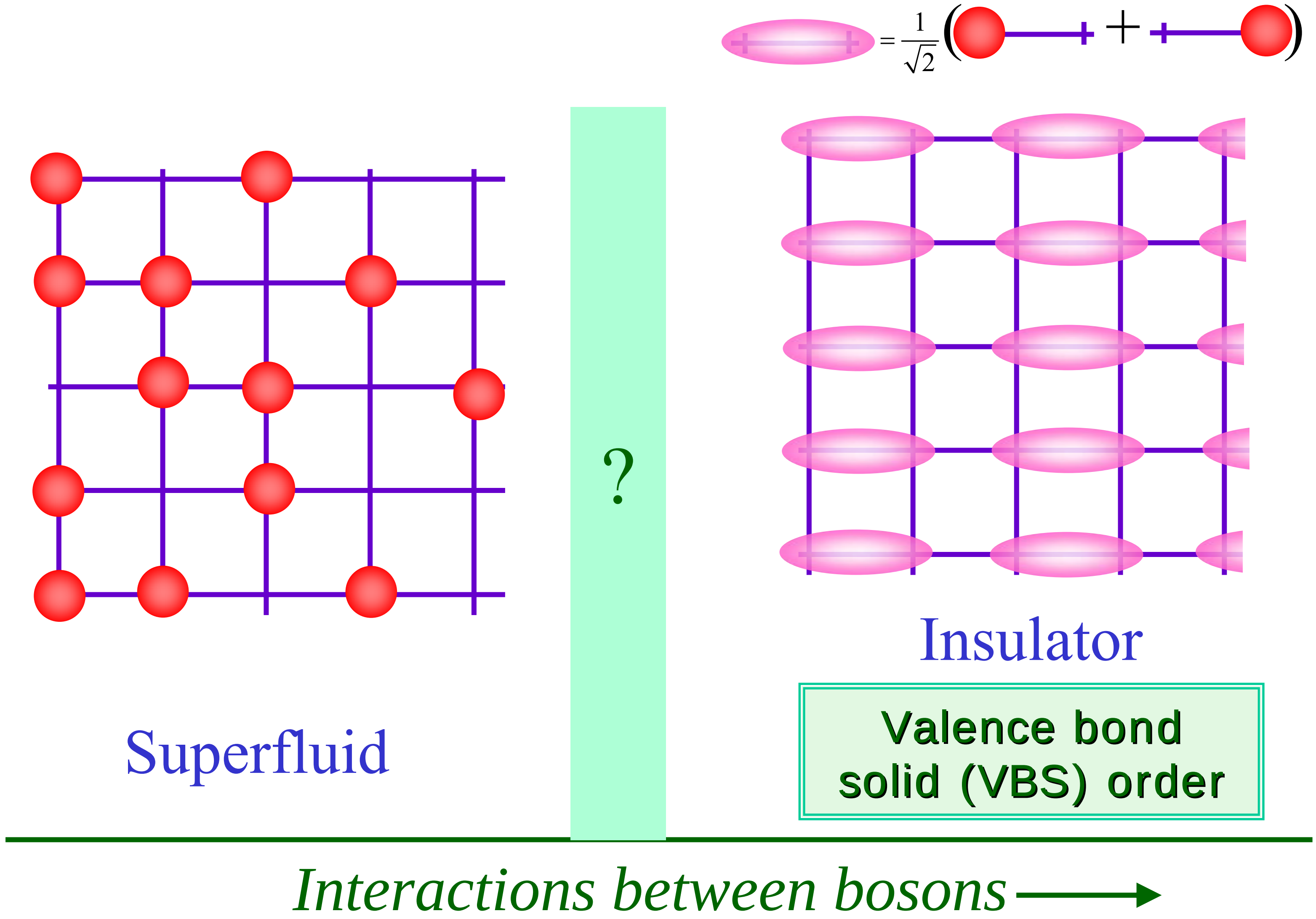

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

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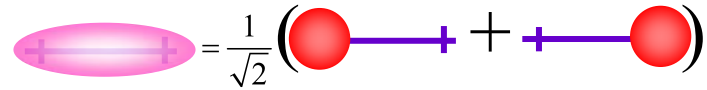

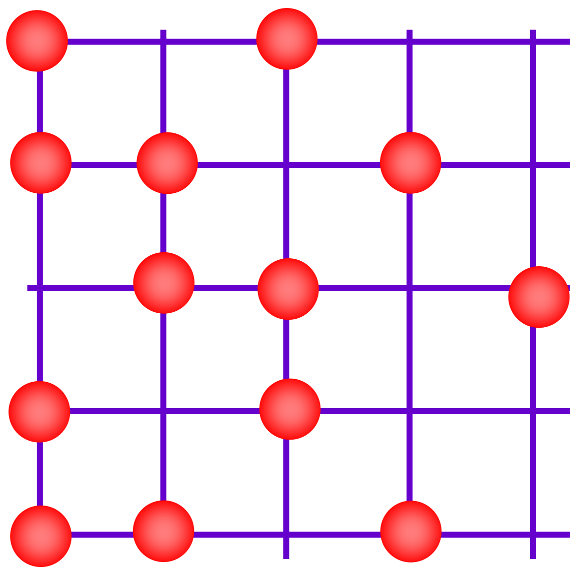

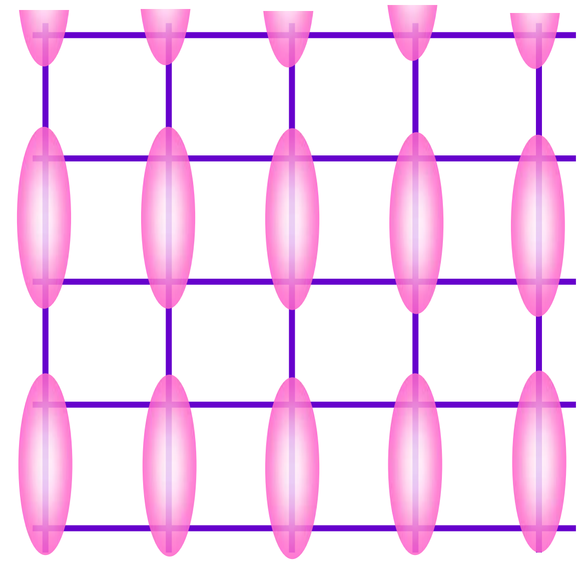

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Superfluid

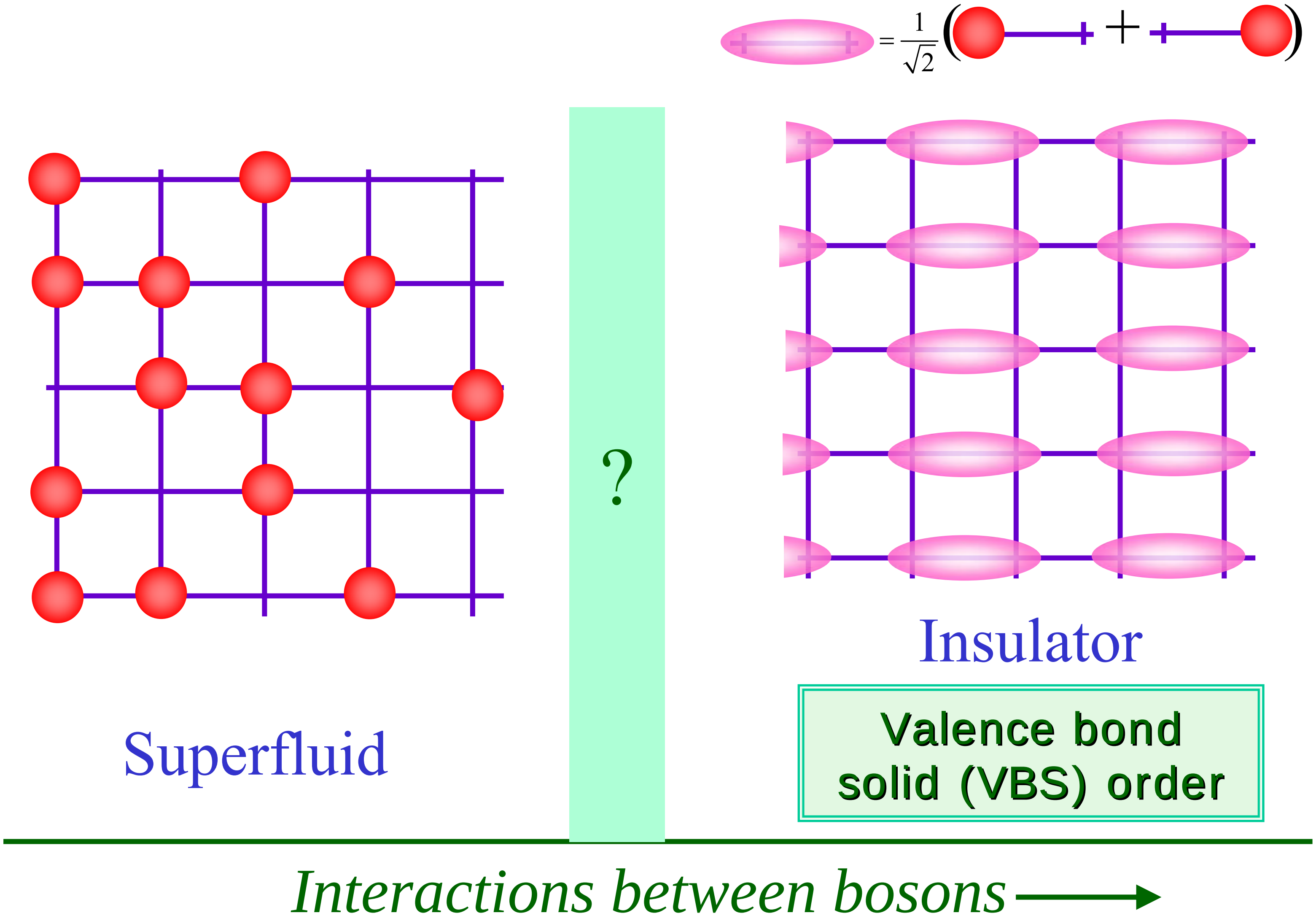
Insulator

Valence bond solid (VBS) order

Interactions between bosons →

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

Bosons on the square lattice at filling fraction f=1/2



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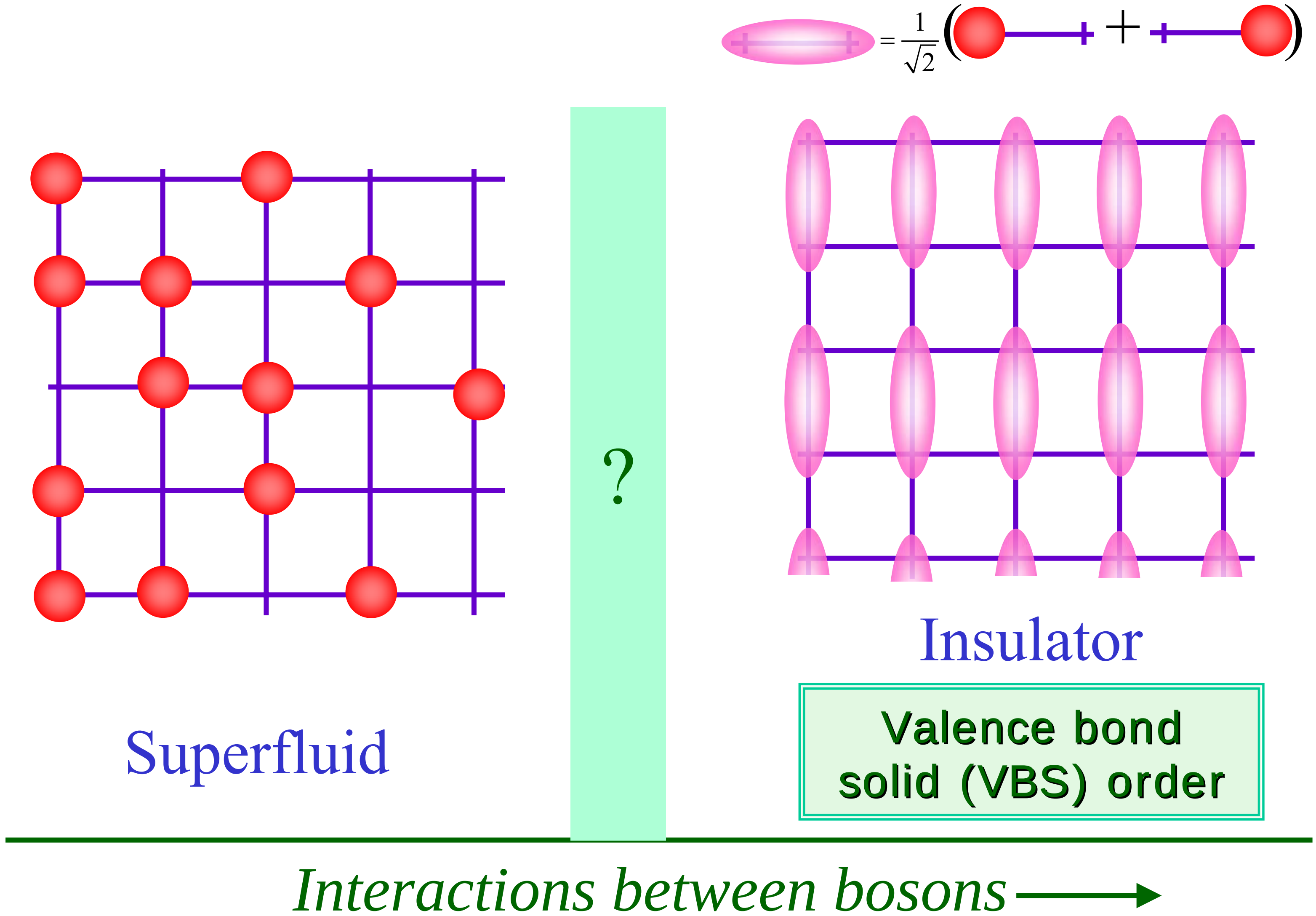

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The superfluid-insulator quantum phase transition

Key difficulty: Multiple order parameters (Bose-Einstein condensate, charge density wave, valence-bond-solid order…) not related by symmetry, but clearly physically connected. Standard methods only predict strong first order transitions (for generic parameters).

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Key theoretical tool: *Duality*

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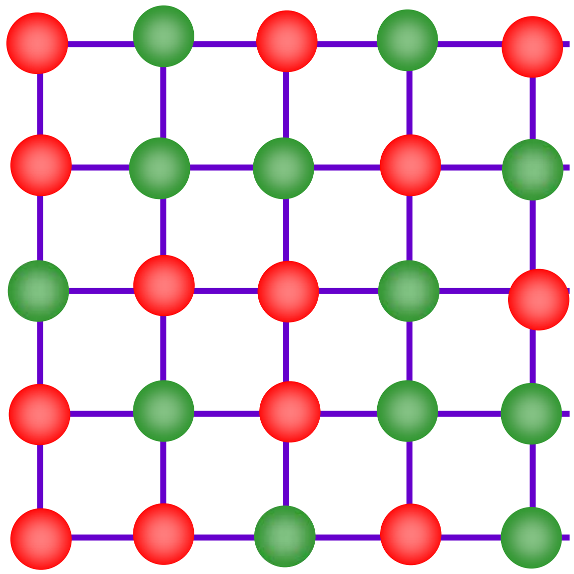
Classical Ising model on the square lattice

$$Z = \sum_{\{\sigma_i = \pm 1\}} \exp\left(K \sum_{\langle ij \rangle} \sigma_i \sigma_j \right)$$



High temperature

$K = 1$

Classical Ising model on the square lattice

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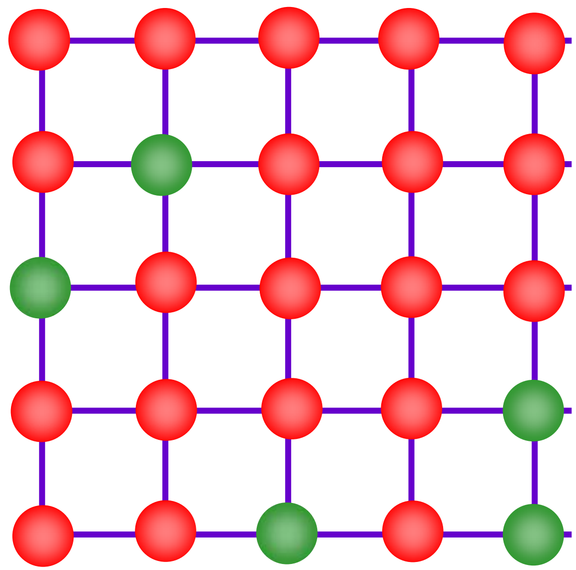
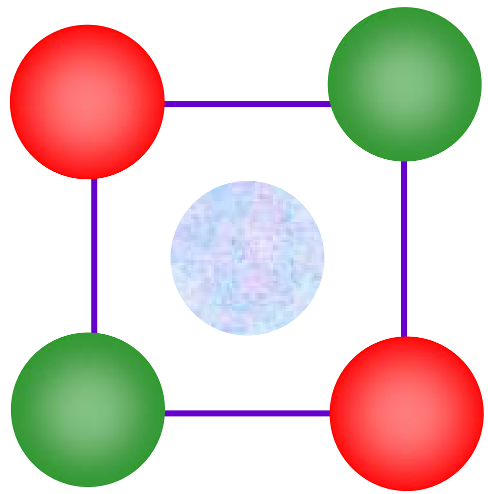


Low temperature

K ? 1

Classical Ising model on the square lattice

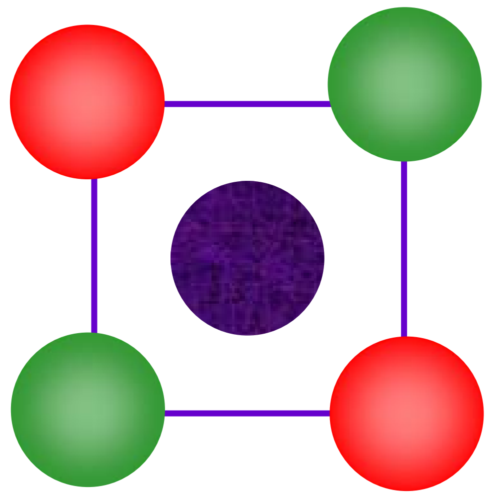
Duality



Kramers-Wannier (1941): introduce a dual "disorder" Ising spin μ_k. This resides on the centers of plaquettes, and is the "Fourier conjugate" variable to the 4 σ_i spins on the vertices of the plaquette.

Classical Ising model on the square lattice

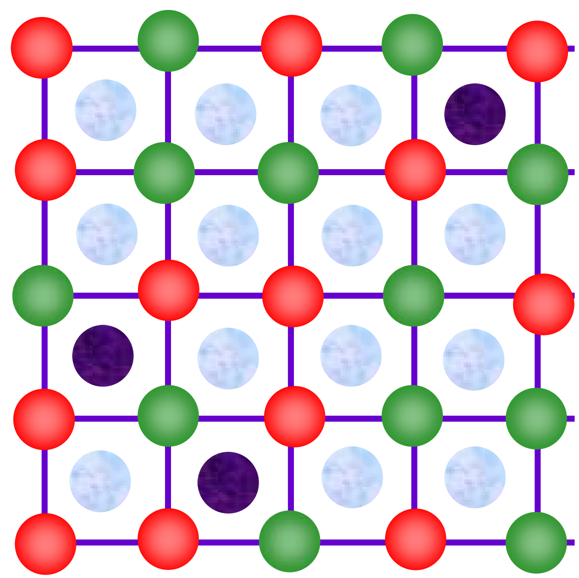
Duality



Kramers-Wannier (1941): introduce a dual "disorder" Ising spin μ_k. This resides on the centers of plaquettes, and is the "Fourier conjugate" variable to the 4 σ_i spins on the vertices of the plaquette.

Classical Ising model on the square lattice

$$Z = \sum_{\{\sigma_i = \pm 1\}} \exp\left(K \sum_{\langle ij \rangle} \sigma_i \sigma_j \right) = \sum_{\{\mu_k = \pm 1\}} \exp\left(K_d \sum_{\langle kl \rangle} \mu_k \mu_l \right)$$



High temperature

$K = 1$

K_d ? 1

Partition function of "disorder" spins μ_k at coupling K_d equals that of σ_i at coupling K with $\sinh(2K)\sinh(2K_d) = 1$

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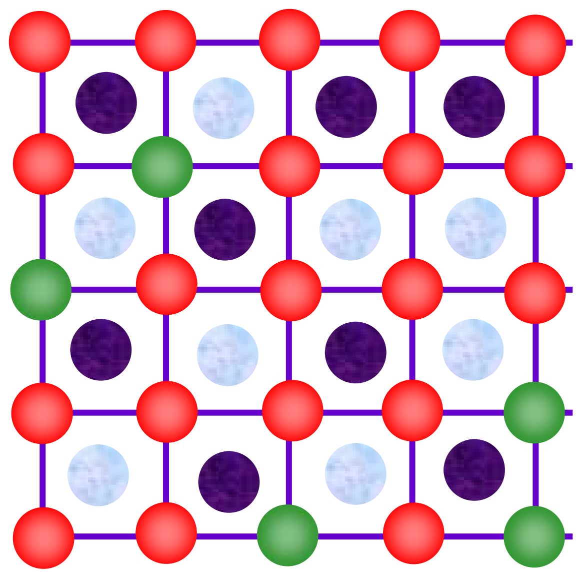

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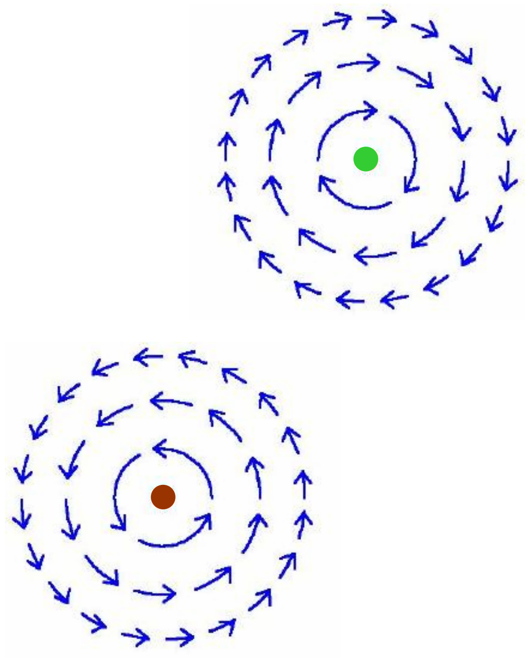
V. The quantum mechanics of vortices near a superfluid-insulator transition

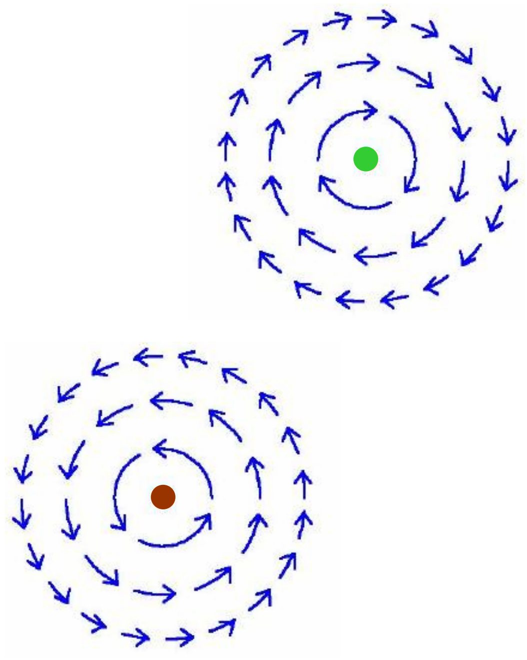
Dual theory of the superfluid-insulator transition as the proliferation of vortex-anti-vortex-pairs

Excitations of the superfluid: **Vortices and anti-vortices**



As a superfluid approaches an insulating state, the decrease in the strength of the condensate will lower the energy cost of creating vortex-anti-vortex pairs.

Excitations of the superfluid: **Vortices and anti-vortices**



Dual picture of the transition to the insulator:

Proliferation of vortex-anti-vortex pairs.

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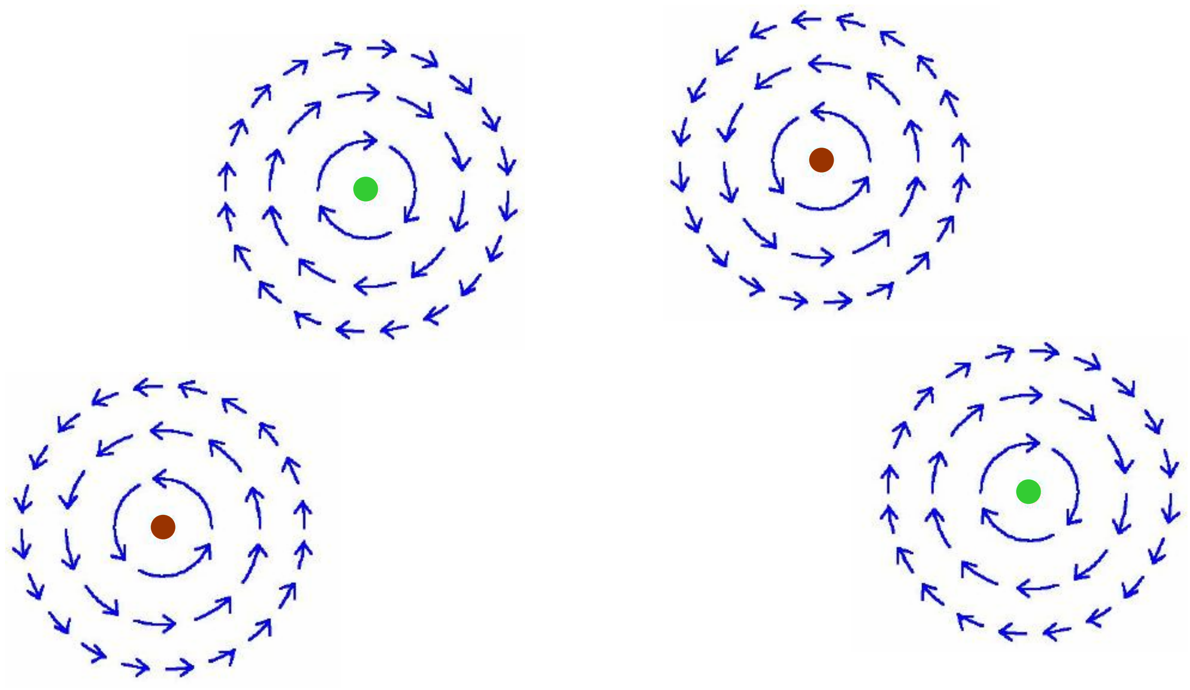

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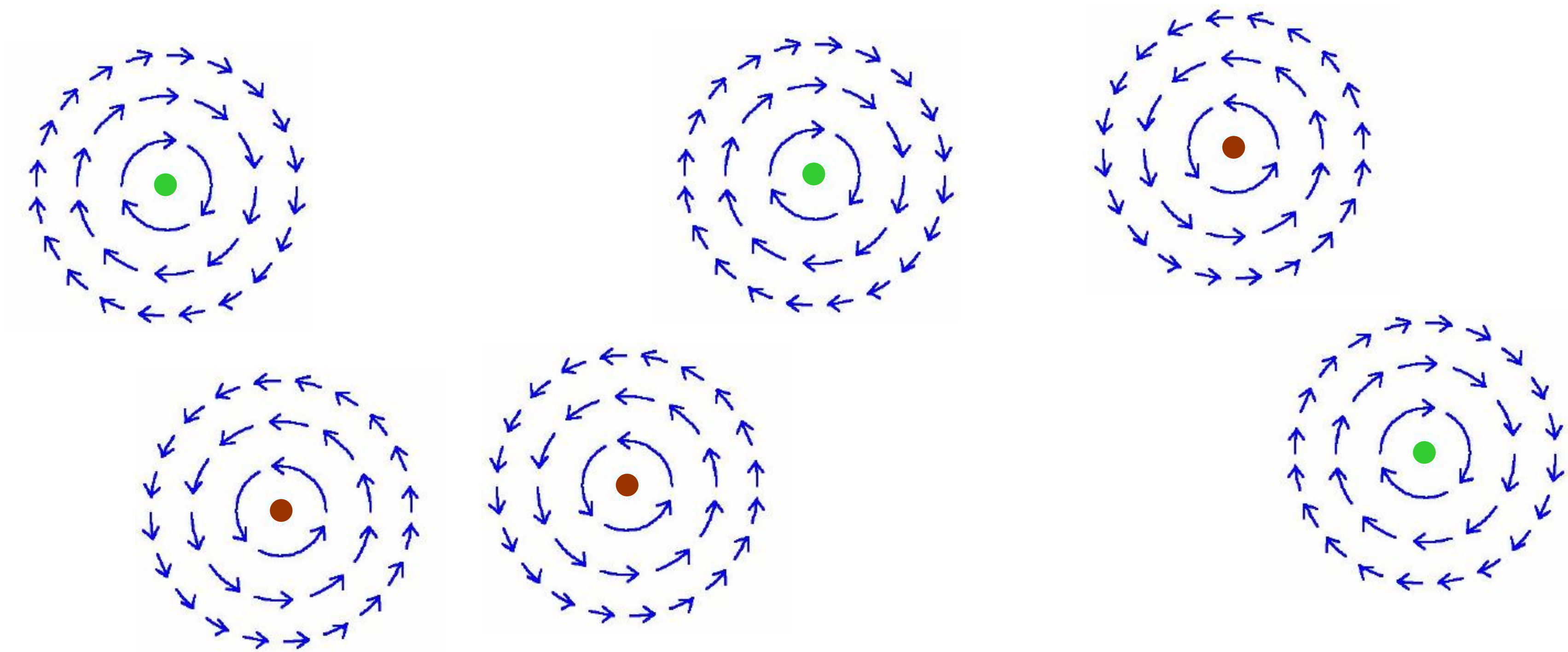

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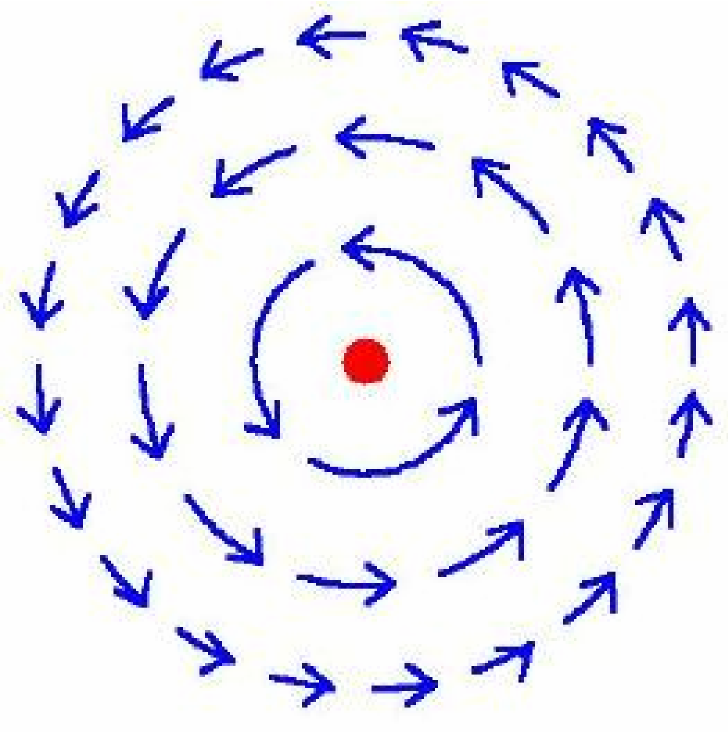
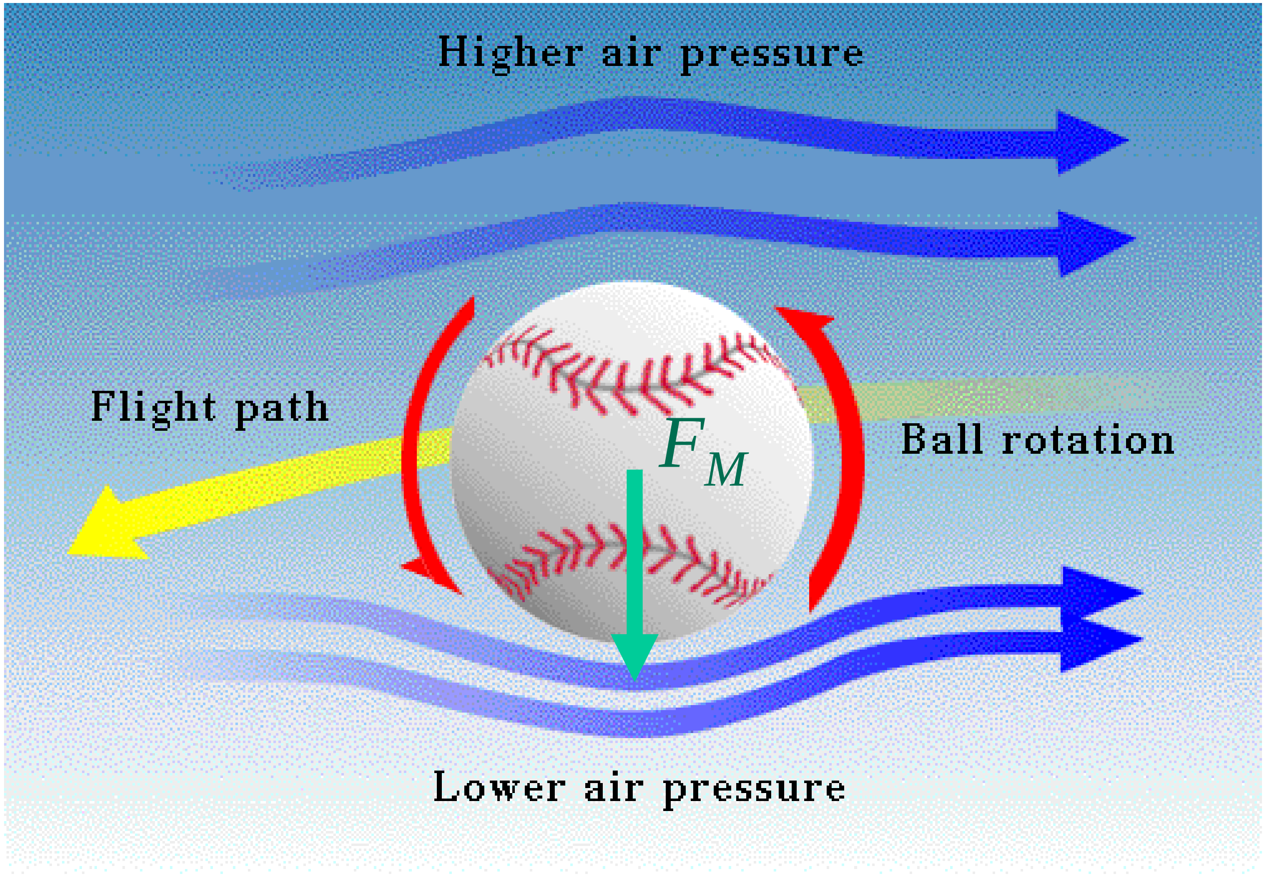
Excitations of the superfluid: **Vortices and anti-vortices**



<u>Central question</u>:
In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these “particles” ?

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \cdot (\text{velocity of ball}) \cdot (\text{circulation})$$

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right) \\ &= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\end{aligned}$$

where ρ = number density of bosons

$\mathbf{v}_s$ = local velocity of superfluid

$\mathbf{r}_v$ = position of vortex

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where $\mathbf{E} = \rho\mathbf{v}_s \times \hat{\mathbf{z}}$ and $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

Dual picture:

The vortex is a quantum particle with dual "electric" charge *n*, moving in a dual "magnetic" field of strength = h×(number density of Bose particles)

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989)

Let the Hamiltonian of a single vortex be $\mathcal{H}_v$.

In general, this is a very complicated object, but we can obtain all needed information by symmetry considerations.

The Hamiltonian $\mathcal{H}_v$ should commute with T_x, the operator which translates the square lattice by one site in the x direction (and similarly for T_y):

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

However, T_x and T_y do not commute with each other.

Under translation along a distance $\mathbf{s}$, a vortex picks up a a Aharanov-Bohm phase factor $\exp\left(i\int_0^{\mathbf{s}} d\mathbf{r}\cdot\mathbf{A}\right)$.

Consequently

$$\boxed{T_x T_y = \exp\left(i\phi\right) T_y T_x}$$

where ϕ is the dual "flux" through a unit cell, This "flux" has the value

$$\boxed{\phi = 2\pi f}$$

where f is the filling fraction of bosons (Cooper pairs). We will consider the case of rational filling fraction $f = p/q$, where p, q are relatively prime integers.

Bosons on the square lattice at filling fraction f=p/q

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

$$T_x T_y = \exp\left(2\pi i p/q\right) T_y T_x$$

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$$T_x T_y = \exp\left(2\pi i p/q\right) T_y T_x$$

Theorem:

The ground state of $\mathcal{H}_v$ is at least q-fold degenerate. We can choose a basis, $|m\rangle$ $(m = 0 \ldots (q-1))$, for the ground states such that

$$\begin{aligned} T_x|m\rangle &= |m+1\rangle \\ T_y|m\rangle &= e^{2\pi i m p/q}|m\rangle \end{aligned}$$

Properties of a quantum-fluctuating vortex weakly pinned by an impurity.

- Any impurity breaks translational invariance, and so chooses a prefered orientation in vortex "flavor space". This chooses some linear combination among the ground states: $|G\rangle = \sum_{m=0}^{q-1} c_m |m\rangle$

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- The expectation value of any observable $\mathcal{O}$, $\langle G|\mathcal{O}|G\rangle$ can be related to the matrix of overlaps $\langle m|n\rangle$ which, in turn, are linearly related to quantities ρ_{mn} which transform under T_x, T_y like the Fourier components of a density $\rho_{\mathbf{Q}}$ at the wavevectors $\mathbf{Q} = 2\pi f(m,n)$:

$$T_x : \rho_{\mathbf{Q}} \rightarrow e^{i\mathbf{Q}\cdot\hat{x}} \rho_{\mathbf{Q}} \qquad T_y : \rho_{\mathbf{Q}} \rightarrow e^{i\mathbf{Q}\cdot\hat{y}} \rho_{\mathbf{Q}}$$

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- It can be shown that there is no linear combination $|G\rangle$ for which all the ρ_{mn} are zero.

Properties of a quantum-fluctuating vortex weakly pinned by an impurity.

- Any pinned vortex exhibits modulations in "density"-like observables at the wavevectors Q over the region in which the vortex executes its quantum zero-point motion.

Implications.

- Aharanov-Bohm or Berry phases lead to surprising kinematic duality relations between seemingly distinct orders. These phase factors allow for continuous quantum phase transitions in situations where such transitions are forbidden by Landau-Ginzburg-Wilson theory.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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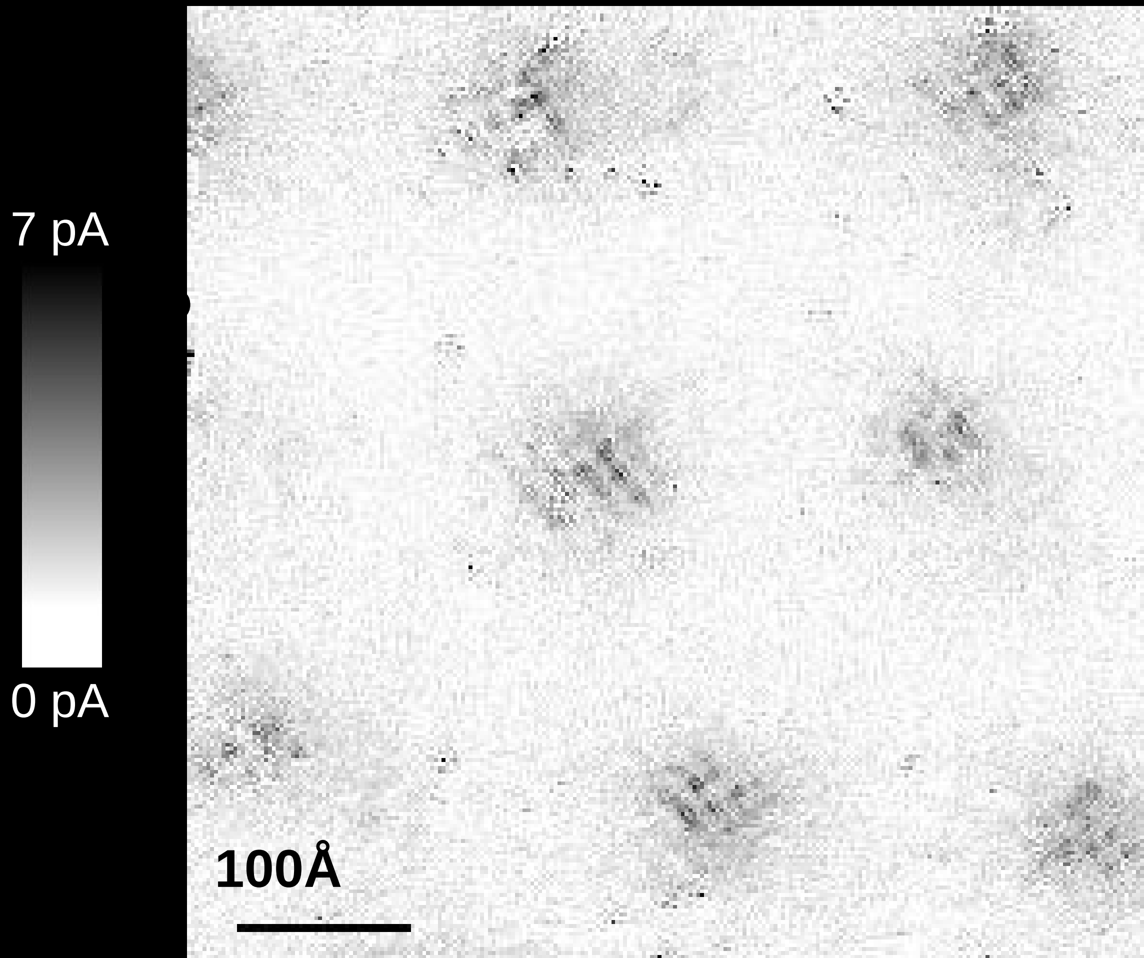
- Vortex zero point motion leads to a natural explanation of STM experiments.

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

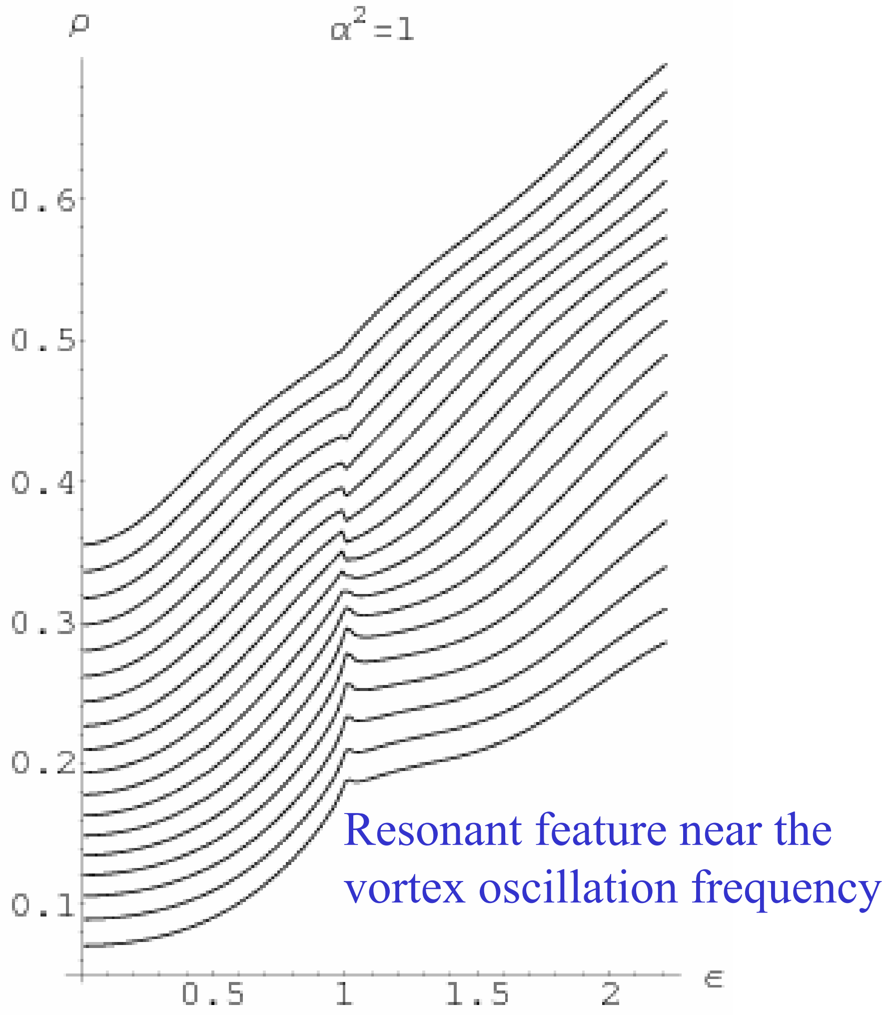
J. Hoffman et al., *Science* 295, 466 (2002).
G. Levy et al., Phys. Rev. Lett. 95, 257005 (2005).

Prediction of periodic LDOS modulations near vortices:
K. Park and S. Sachdev, Phys. Rev. B 64, 184510 (2001).

Experimental implications:

- Size of modulation halo allows estimate of the inertial mass of a vortex
- Direct detection of vortex zero-point motion may be possible in inelastic neutron or light-scattering experiments
- The quantum zero-point motion of the vortices influences the spectrum of the electronic quasiparticles. There is no current theory of the electronic density of states near a vortex in a cuprate superconductor, and the vortex zero-point motion is a promising candidate for resolving this long-standing theoretical puzzle.

Influence of the quantum oscillating vortex on the LDOS



P. Nikolic, S. Sachdev, and L. Bartosch, cond-mat/0606001

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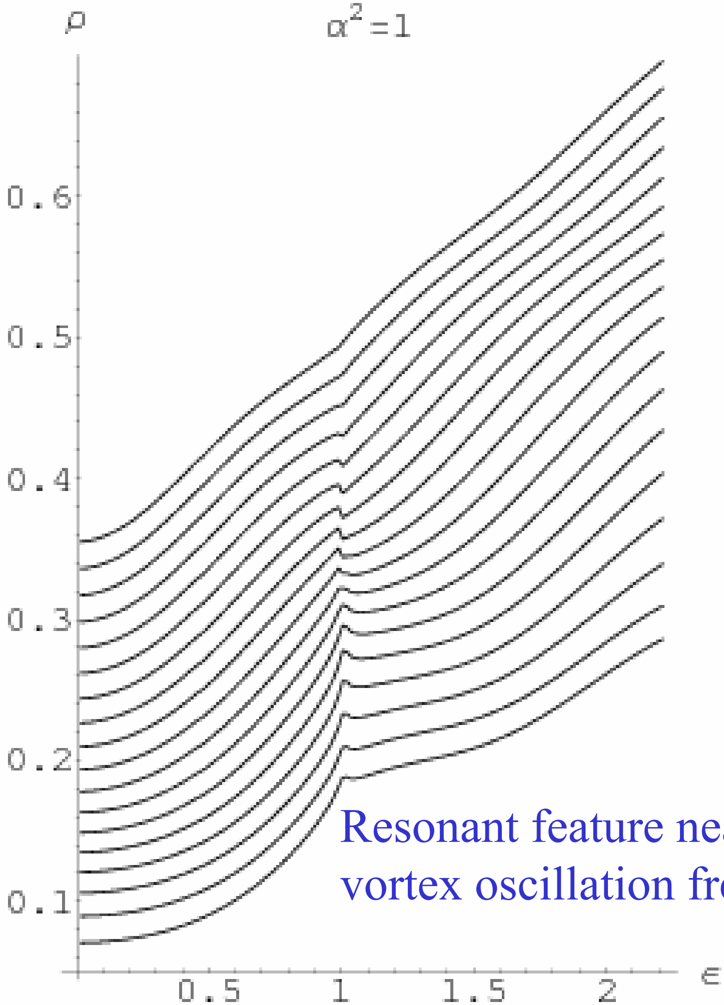
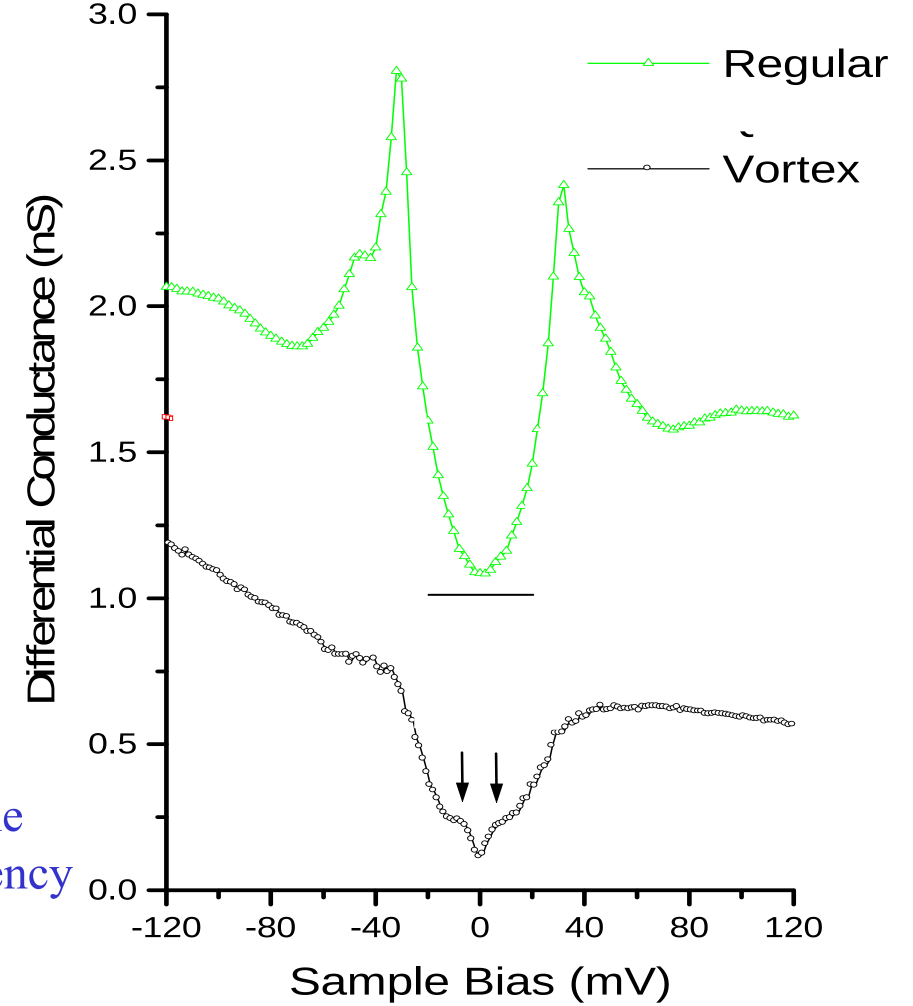


Resonant feature near the vortex oscillation frequency



I. Maggio-Aprile *et al. Phys. Rev. Lett.* **75**, 2754 (1995)
S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

P. Nikolic, S. Sachdev, and L. Bartosch, cond-mat/0606001

Conclusions

- Duality is a powerful tool in investigating the competition between quantum ground states with different types of order.
- "Aharanov-Bohm" or "Berry" phases lead to surprising kinematic duality relations between seemingly distinct orders. These phase factors allow for continuous quantum phase transitions in situations where such transitions are forbidden by Landau-Ginzburg-Wilson theory.
- Evidence that vortices in the cuprate superconductors carry a "flavor" index which encodes the spatial modulations of a proximate insulator. Quantum zero point motion of the vortex provides a natural explanation for LDOS modulations observed in STM experiments.