

# The onset of antiferromagnetism in metals: from the cuprates to the heavy fermion compounds

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**Max Metlitski**



**Matthias Punk**



**Erez Berg**

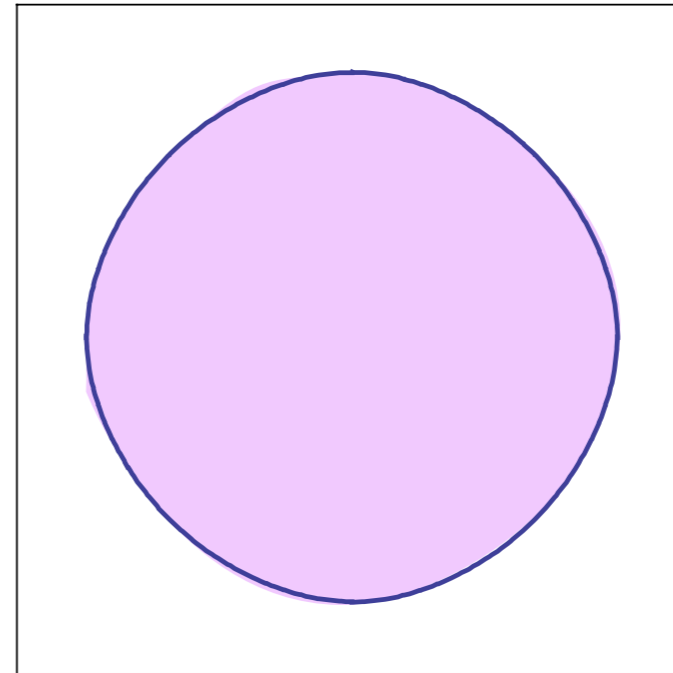


1. Experimental motivations from cuprates and pnictides
2. Conventional theory and its breakdown in two spatial dimensions
3. Fermi surface reconstruction: onset of unconventional superconductivity
4. Fermi surface reconstruction *without* symmetry breaking: metals with “topological” order and the heavy fermion compounds

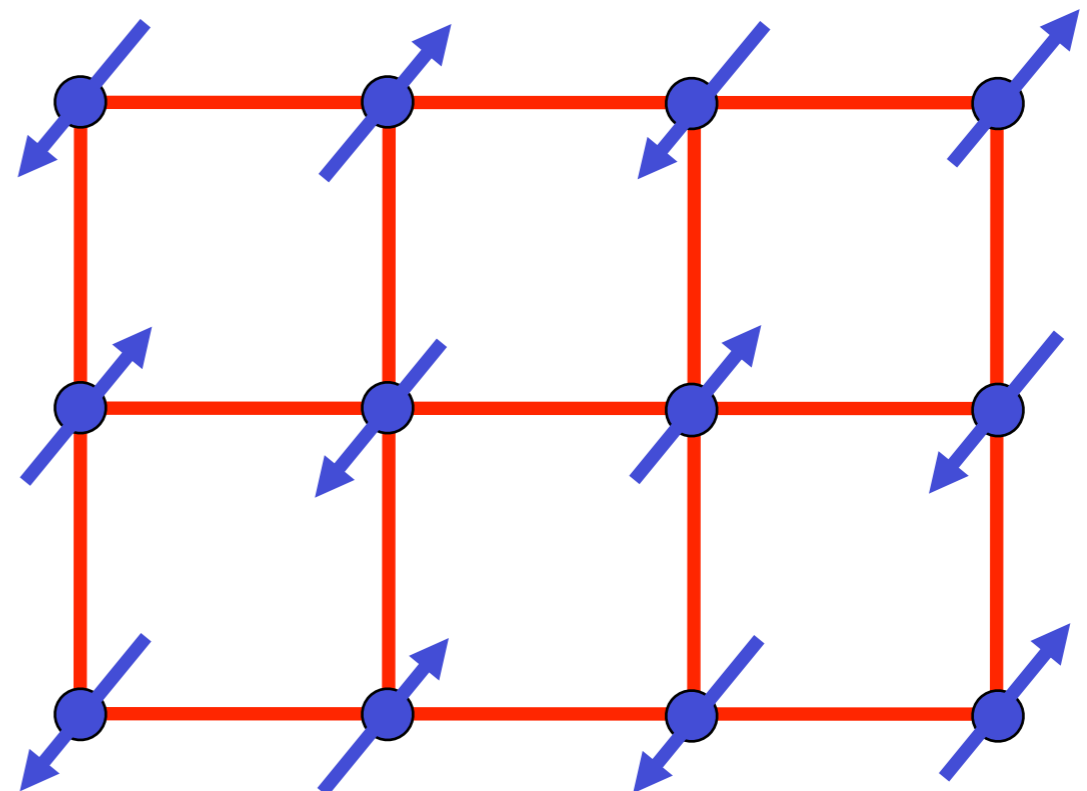
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# Fermi surface+antiferromagnetism

Metal with “large”  
Fermi surface



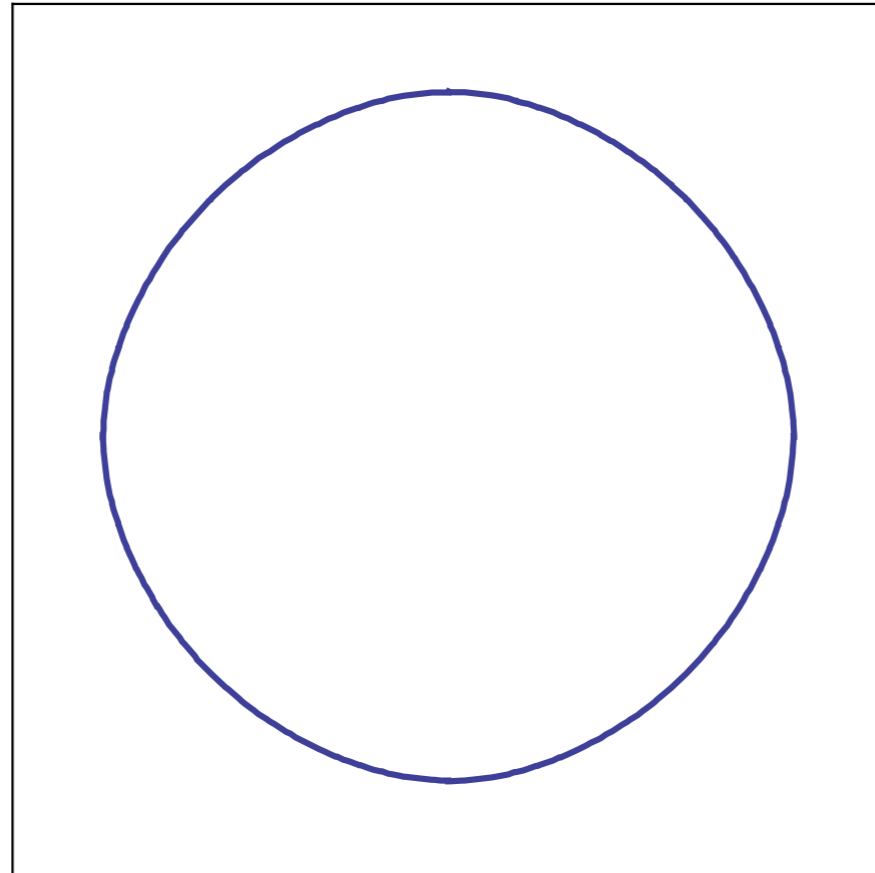
+



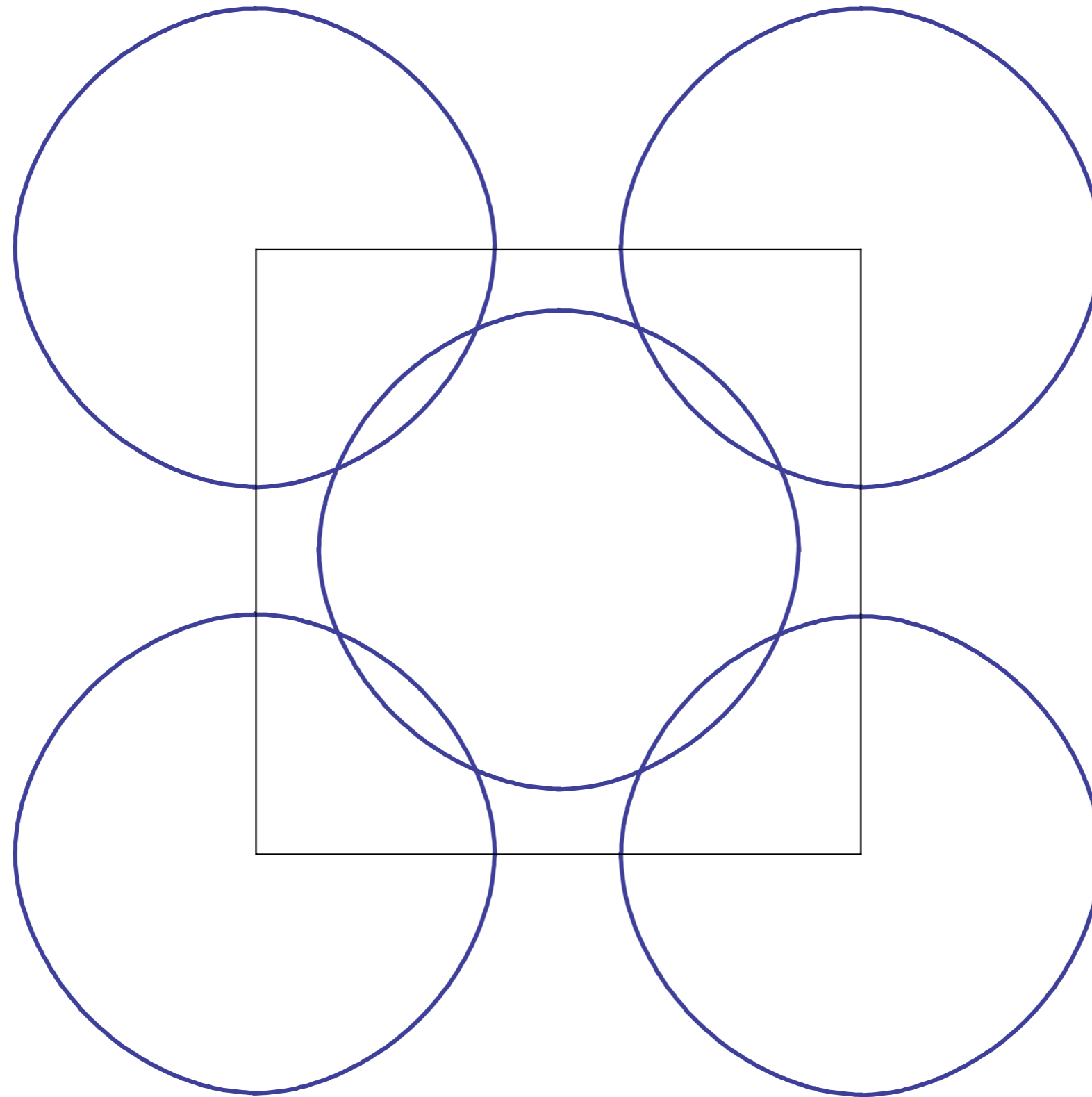
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

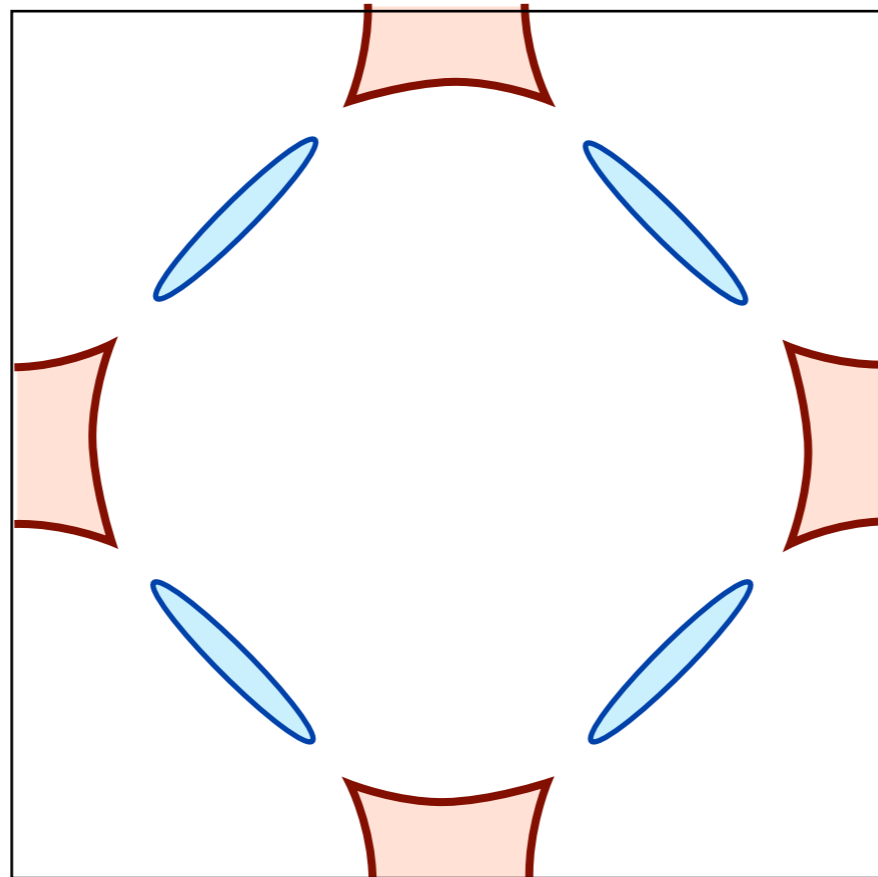
where  $\mathbf{K}$  is the ordering wavevector.



**Metal with “large” Fermi surface**

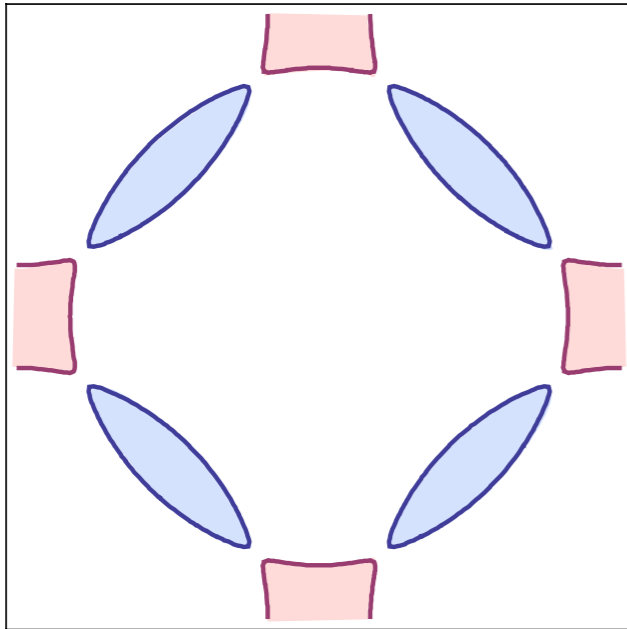


Fermi surfaces translated by  $\mathbf{K} = (\pi, \pi)$ .



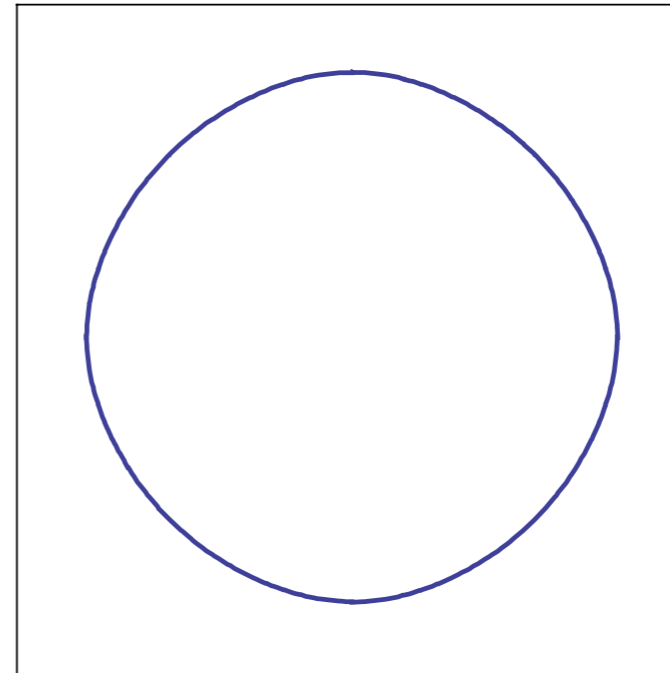
Fermi surface reconstruction  
into electron and hole pockets in  
antiferromagnetic phase with  $\langle \vec{\varphi} \rangle \neq 0$

# Quantum phase transition with Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron  
and hole pockets



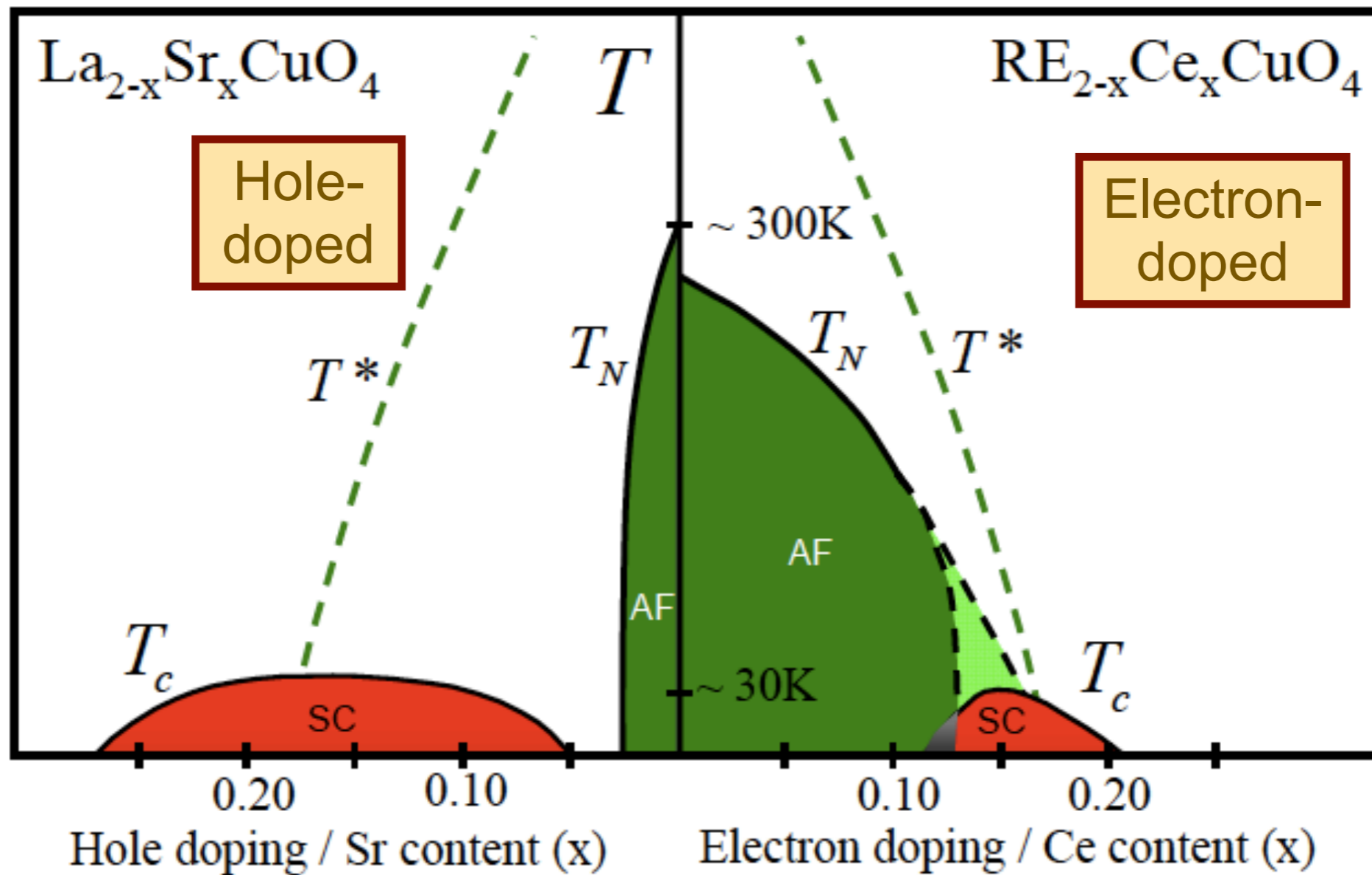
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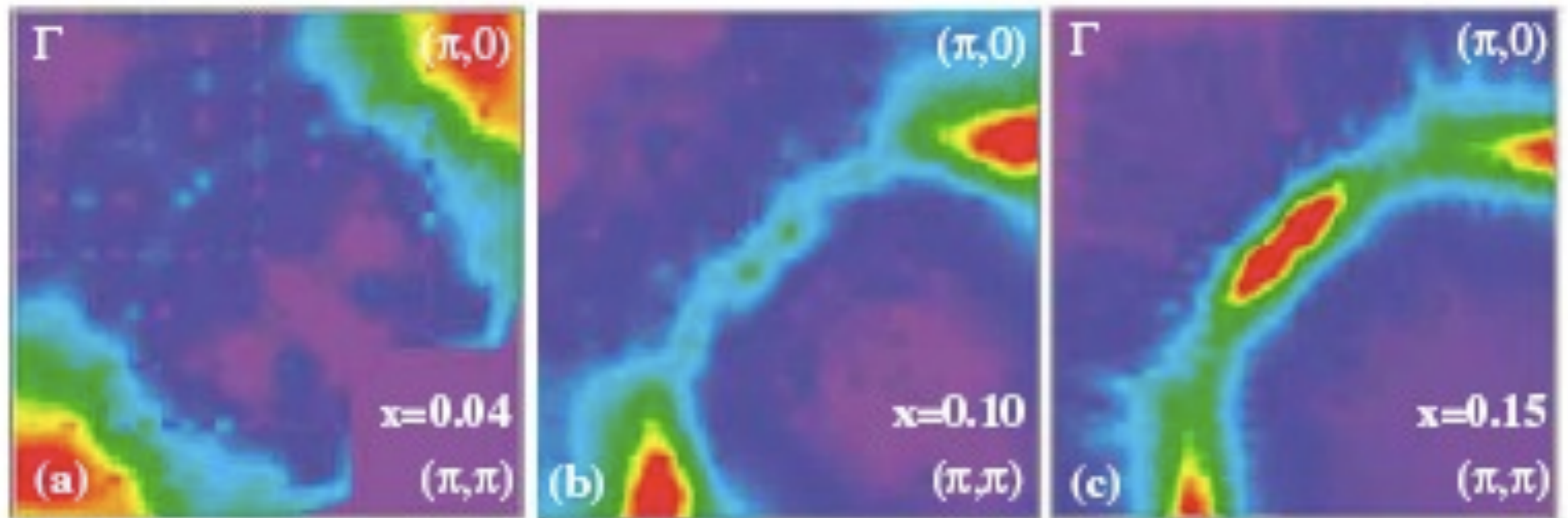
← Increasing interaction

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).  
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

# Hole and electron-doped cuprate superconductors



# Photoemission in $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$

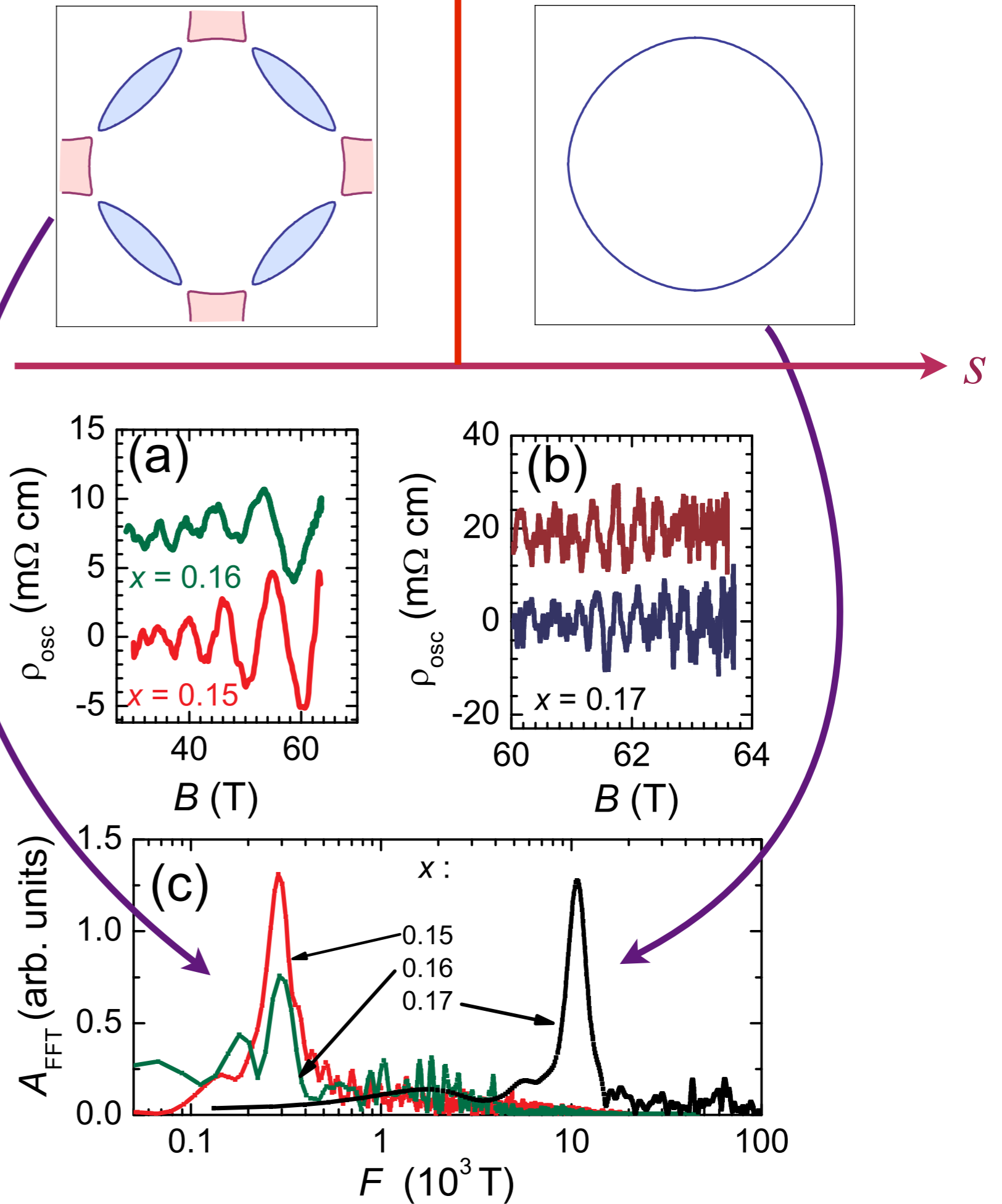


N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

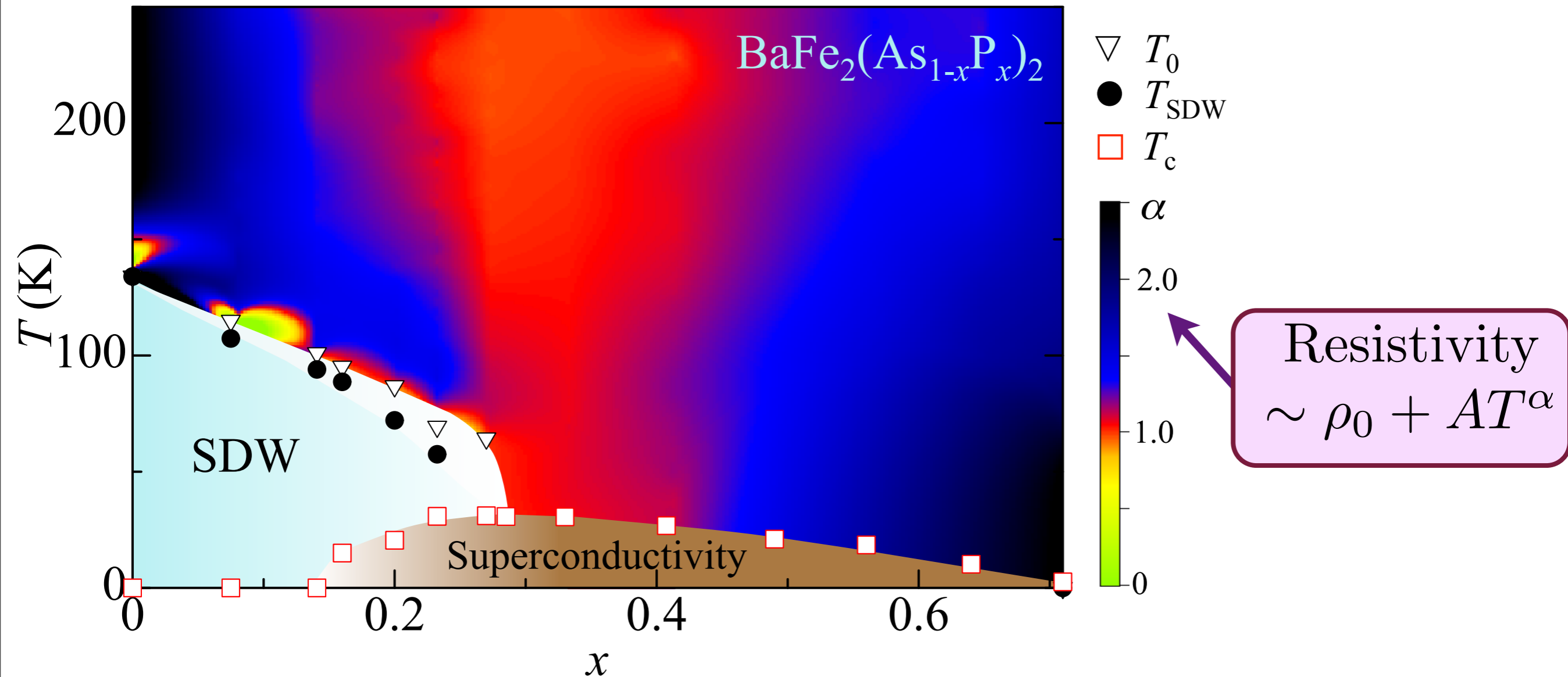
# Quantum oscillations



T. Helm, M.V. Kartsovnik,  
M. Bartkowiak, N. Bittner,  
M. Lambacher, A. Erb, J. Wosnitza,  
and R. Gross,  
*Phys. Rev. Lett.* **103**, 157002 (2009).



# Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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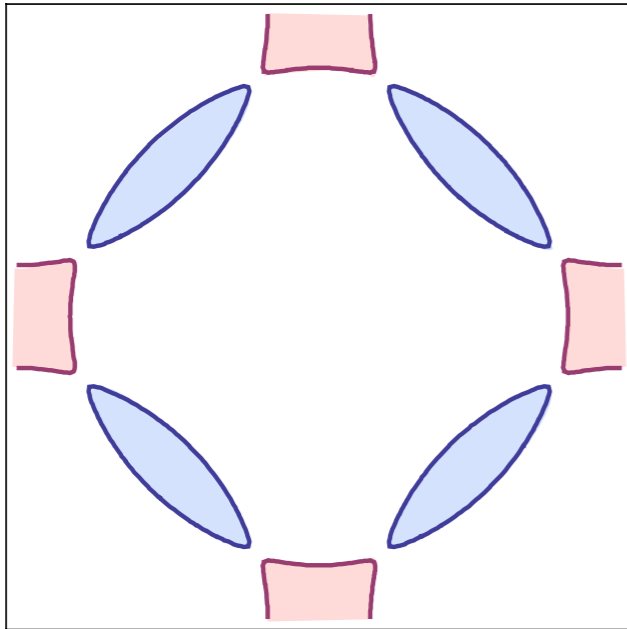
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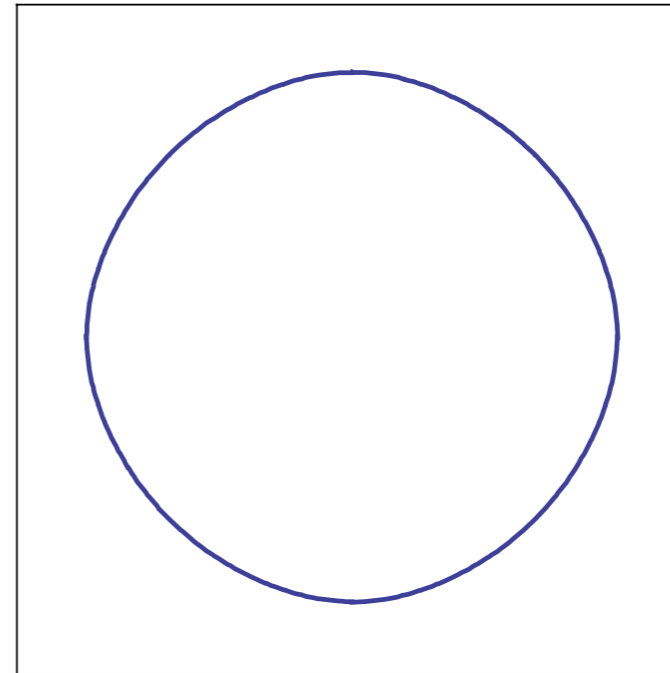
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# Quantum phase transition with Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron  
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# Boson-fermion theory for both phases

$$\mathcal{S} = \int d^2r d\tau [\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi}]$$

$$\mathcal{L}_c = c_a^\dagger \varepsilon (-i \nabla) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla \varphi_\alpha)^2 + \frac{r}{2} \varphi_\alpha^2 + \frac{u}{4} (\varphi_\alpha^2)^2$$

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$$\mathcal{L}_{c\varphi} = \lambda \varphi_\alpha e^{i\mathbf{K}\cdot\mathbf{r}} c_a^\dagger \sigma_{ab}^\alpha c_b.$$

“Yukawa” coupling between fermions and antiferromagnetic order:

$\lambda^2 \sim U$ , the Hubbard repulsion

# Hertz-Moriya-Millis theory

- Integrate out Fermi surface quasiparticles and obtain an effective theory for the order parameter  $\vec{\varphi}$  alone.

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- This is dangerous, and will lead to non-local in the  $\vec{\varphi}$  theory. Hertz focused on only the simplest such non-local term.
- However, there are an infinite number of non-local terms at higher order, and these lead to a breakdown of the Hertz theory in two spatial dimensions.

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

- In  $d = 2$ , we *must* work in local theories which keeps both the order parameter and the Fermi surface quasiparticles “alive”.

Sung-Sik Lee, *Phys. Rev. B* **80**, 165102 (2009)

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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- The theories can be organized in a  $1/N$  expansion, where  $N$  is the number of fermion “flavors”.
- At subleading order, resummation of all “planar” graphics is required (at least): this theory is even more complicated than QCD.

Sung-Sik Lee, *Phys. Rev. B* **80**, 165102 (2009)

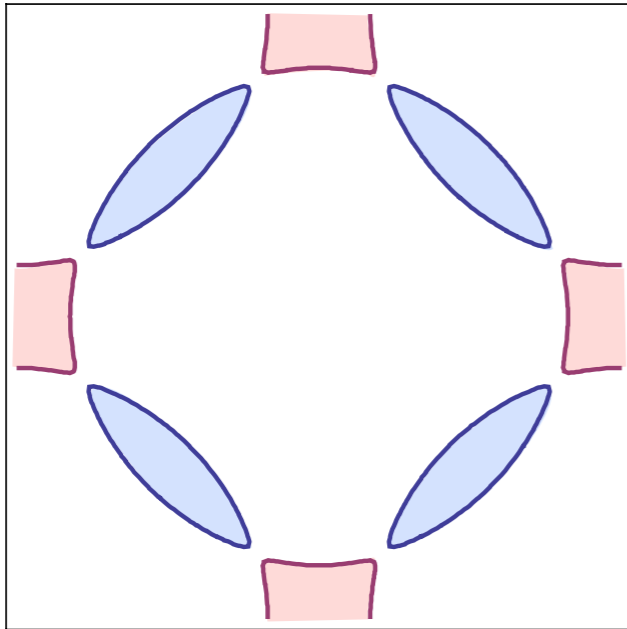
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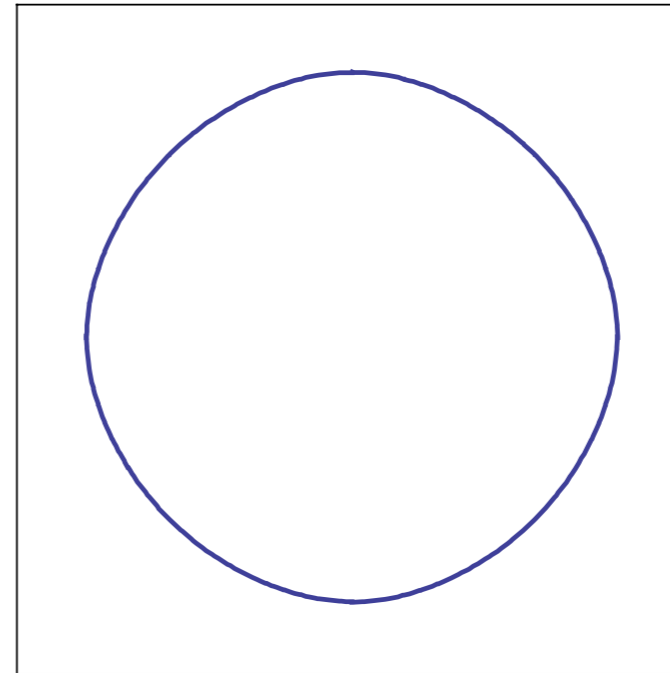
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A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Can antiferromagnetic fluctuations,  
represented by the boson  $\varphi_\alpha$ ,  
provide the pairing glue which leads  
to high temperature  
superconductivity ?

There is an instability in weak-coupling,  
but  $T_c$  is low where the theory is reliable:

### ***d*-wave pairing near a spin-density-wave instability**

D. J. Scalapino, E. Loh, Jr.,\* and J. E. Hirsch†

*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106*

(Received 23 June 1986)

We investigate the three-dimensional Hubbard model and show that paramagnon exchange near a spin-density-wave instability gives rise to a strong singlet *d*-wave pairing interaction. For a cubic band the singlet ( $d_{x^2-y^2}$  and  $d_{3z^2-r^2}$ ) channels are enhanced while the singlet ( $d_{xy}, d_{xz}, d_{yz}$ ) and triplet *p*-wave channels are suppressed. A unique feature of this pairing mechanism is its sensitivity to band structure and band filling.

Physical Review B **34**, 8190 (1986)

At stronger coupling,  
different effects compete:

- Pairing glue becomes stronger.



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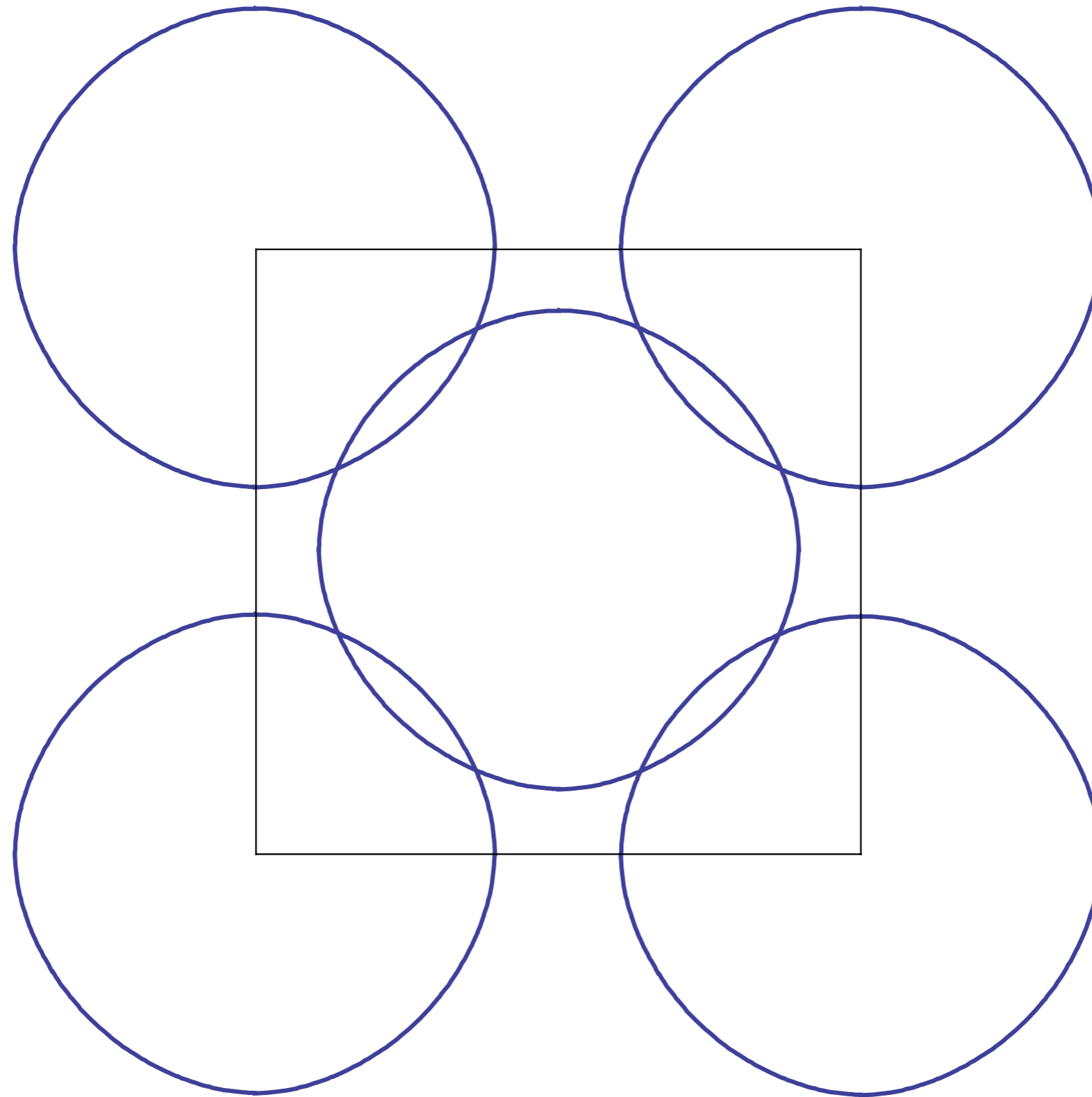
- Pairing glue becomes stronger.
- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.



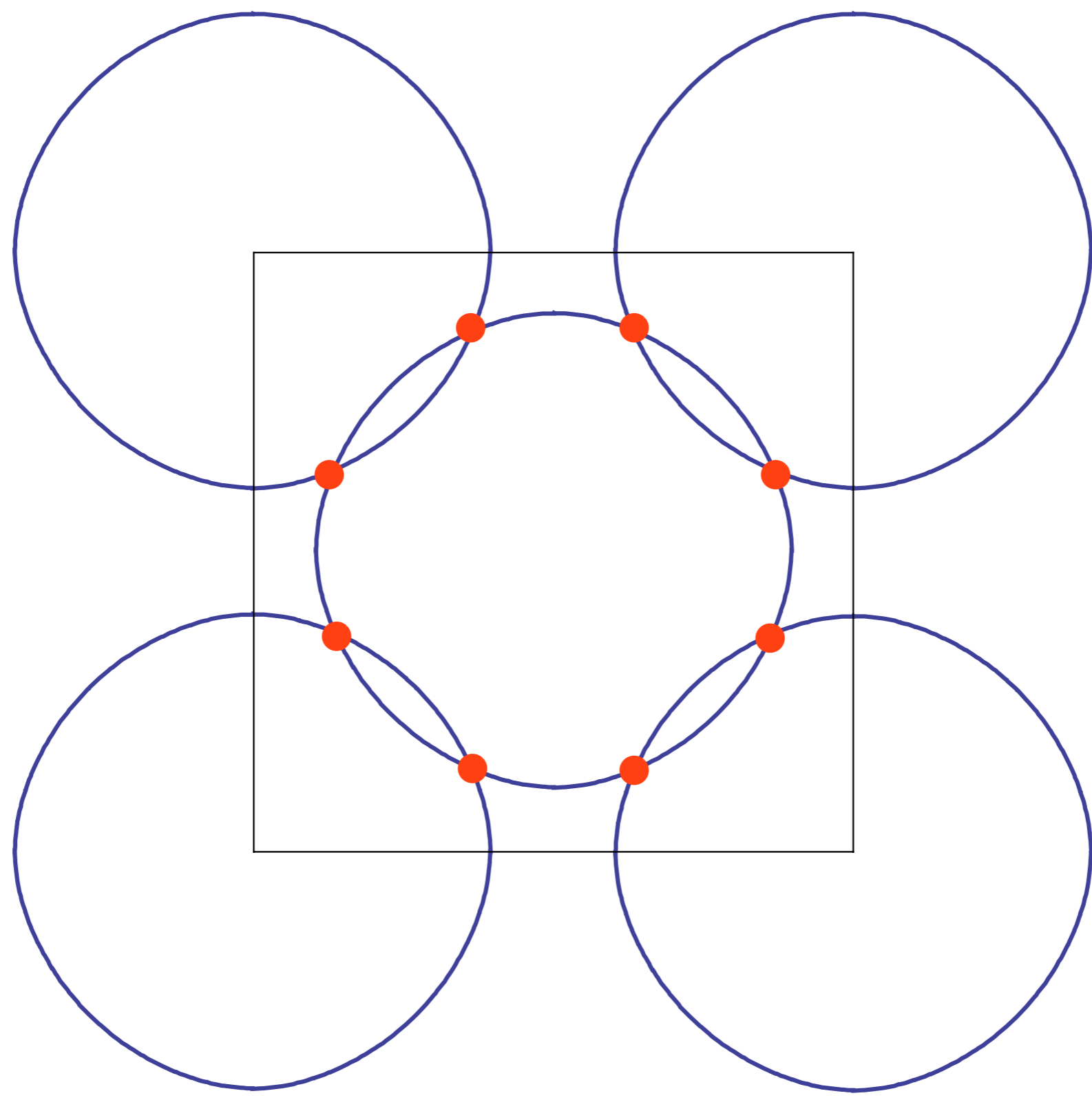
At stronger coupling,  
different effects compete:

- Pairing glue becomes stronger.
- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.
- Other instabilities can appear *e.g.* to charge density waves/stripe order.

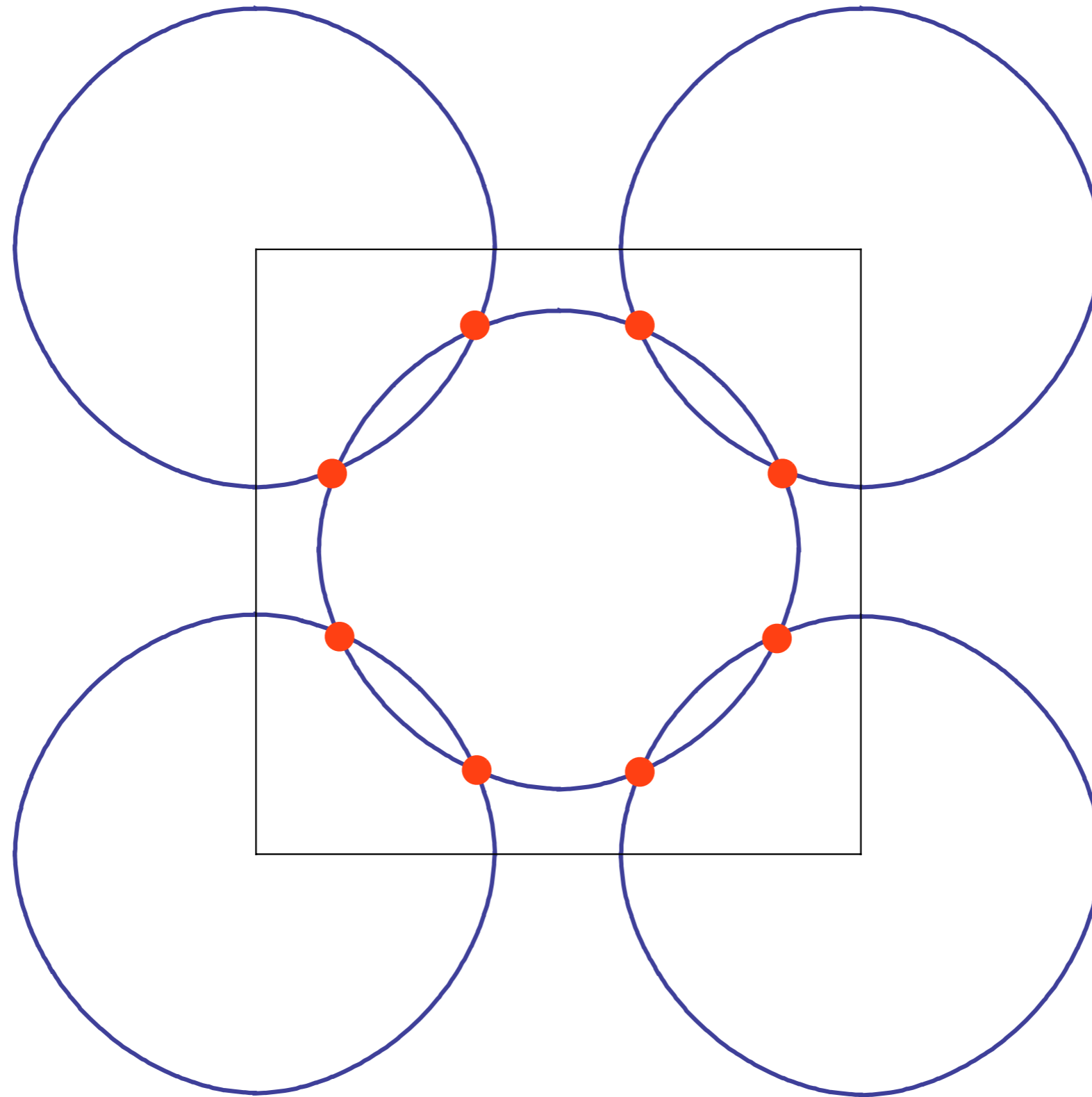




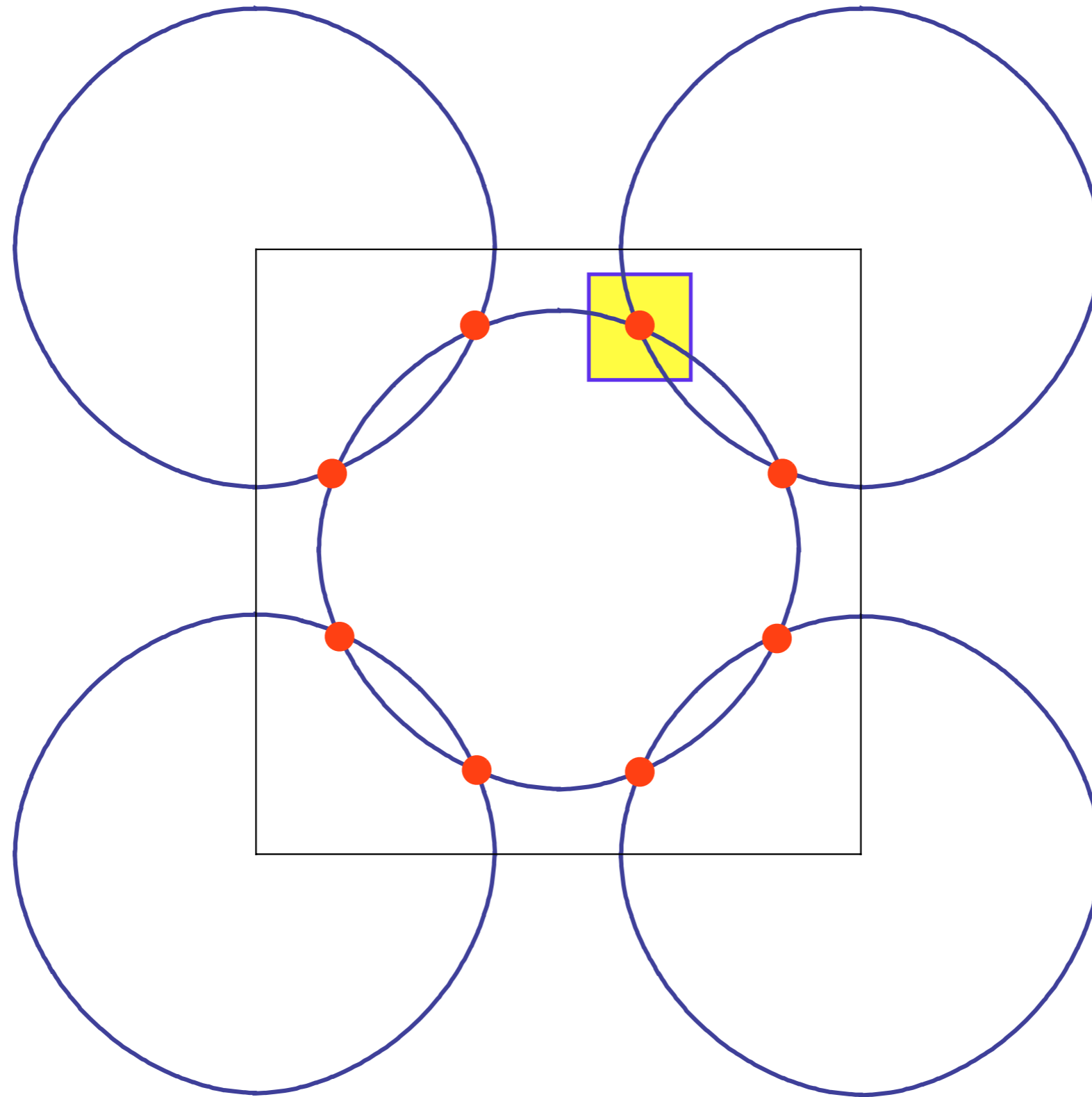
Fermi surfaces translated by  $\mathbf{K} = (\pi, \pi)$ .



**“Hot” spots**

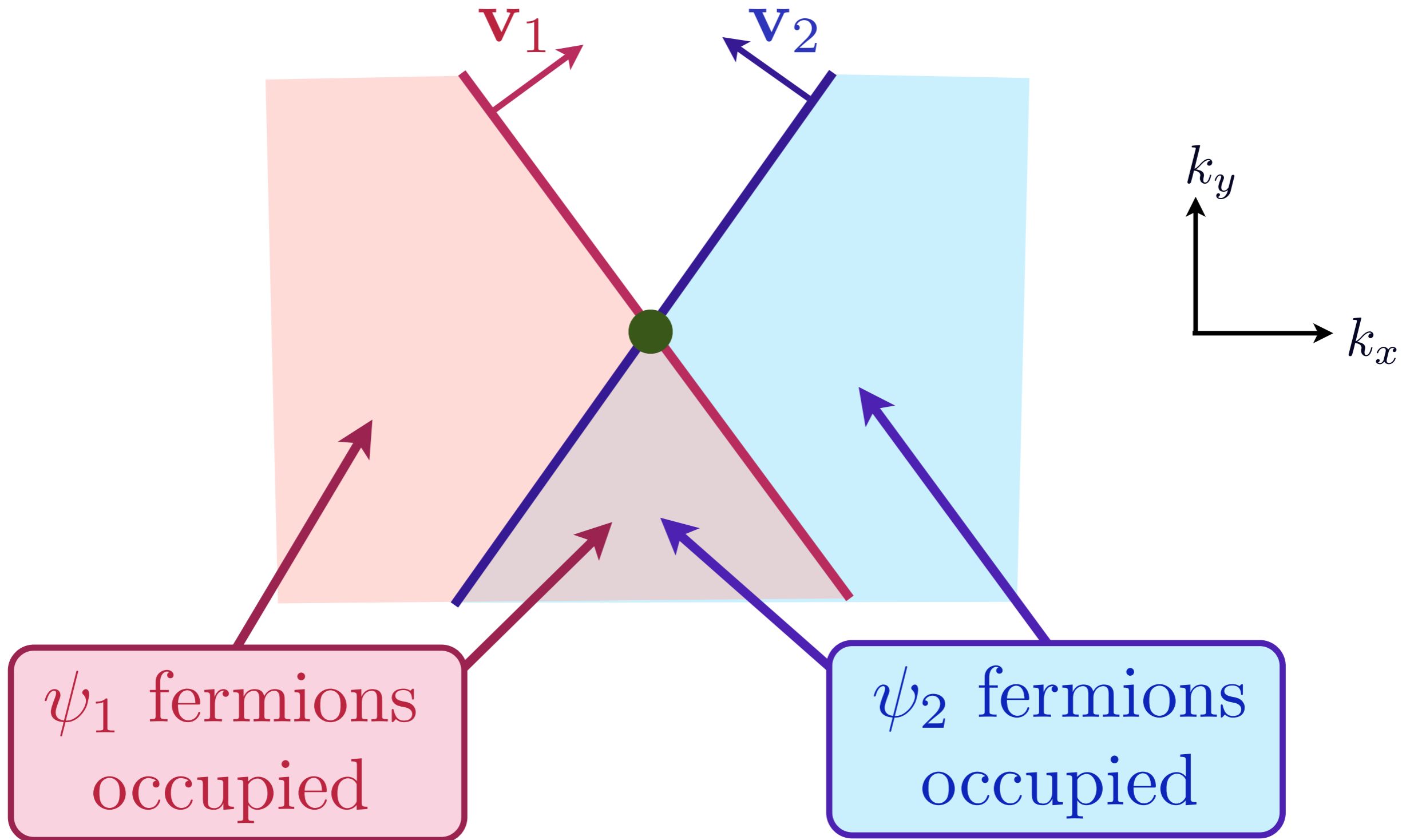


# Low energy theory for critical point near hot spots



# Low energy theory for critical point near hot spots

Theory has fermions  $\psi_{1,2}$  (with Fermi velocities  $\mathbf{v}_{1,2}$ ) and boson order parameter  $\vec{\varphi}$ , interacting with coupling  $\lambda$



# Low energy theory for critical point near hot spots

$$\mathcal{S} = \int d^2r d\tau [\mathcal{L}_\psi + \mathcal{L}_\varphi + \mathcal{L}_{\psi\varphi}]$$

$$\mathcal{L}_\psi = \psi_{1a}^\dagger \left( \frac{\partial}{\partial\tau} - i\mathbf{v}_1 \cdot \nabla \right) \psi_{1a} \\ + \psi_{2a}^\dagger \left( \frac{\partial}{\partial\tau} - i\mathbf{v}_2 \cdot \nabla \right) \psi_{2a}$$

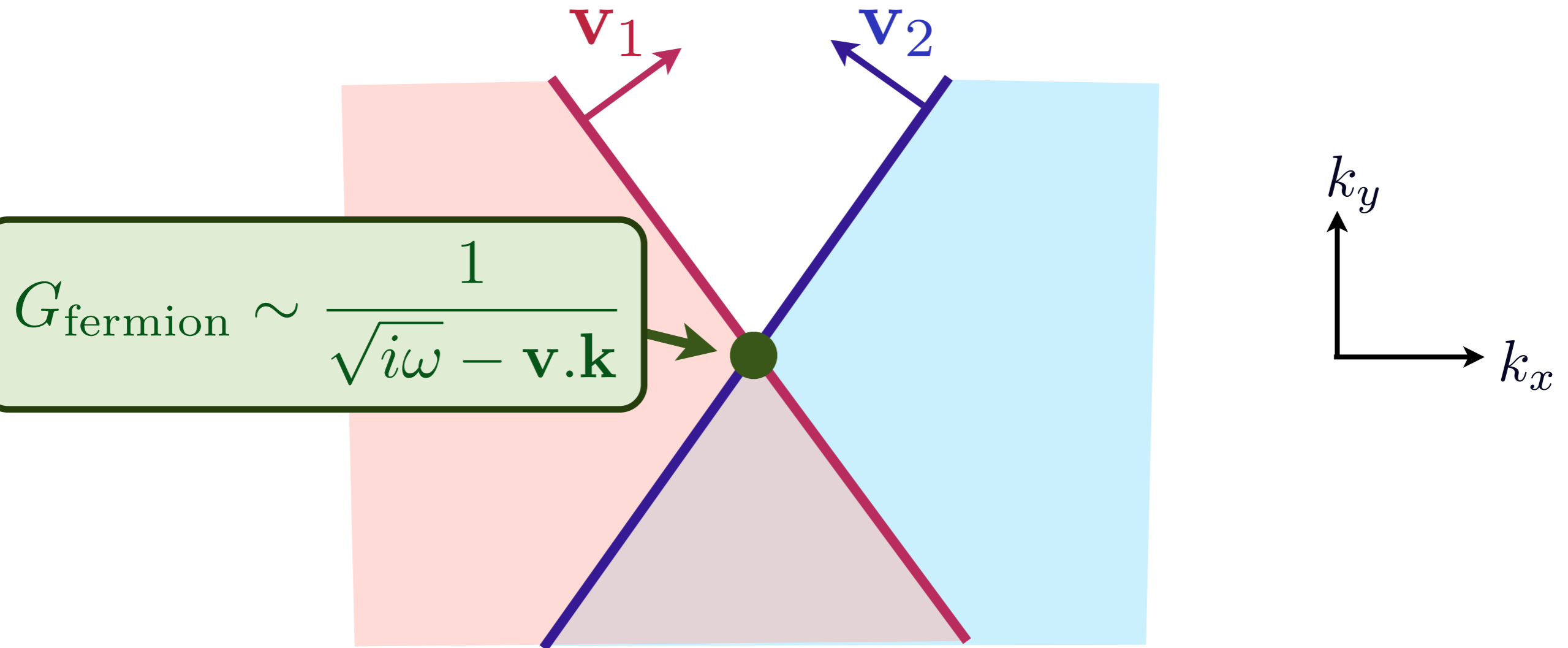
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla \varphi_\alpha)^2 + \frac{r}{2} \varphi_\alpha^2 + \frac{u}{4} (\varphi_\alpha^2)^2$$

$$\mathcal{L}_{\psi\varphi} = \lambda \varphi_\alpha \sigma_{ab}^\alpha \left( \psi_{1a}^\dagger \psi_{2b} + \psi_{2a}^\dagger \psi_{1b} \right).$$

“Yukawa” coupling between fermions and antiferromagnetic order:

$\lambda^2 \sim U$ , the Hubbard repulsion

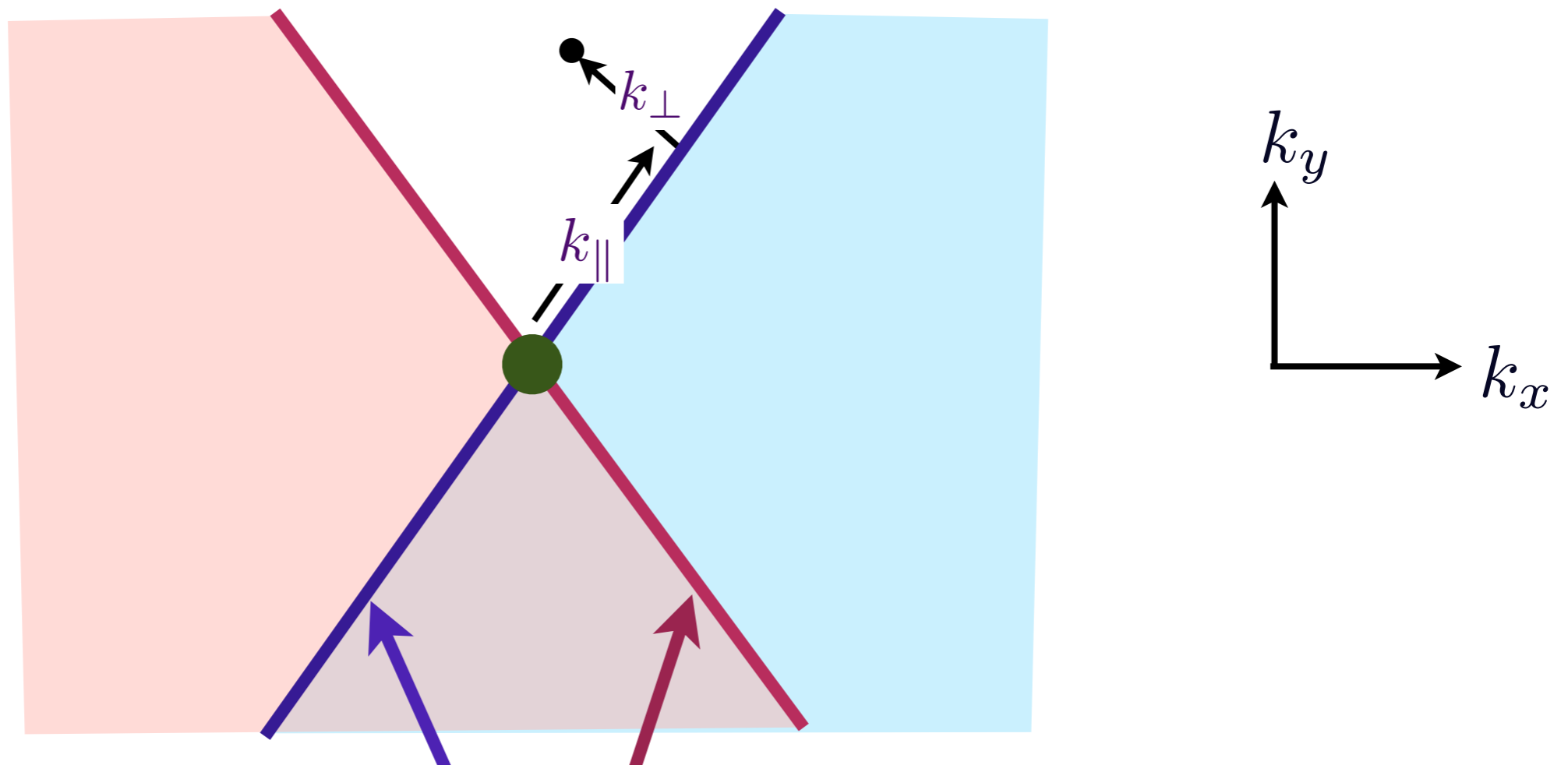
# Critical point theory is strongly coupled in $d = 2$



A. J. Millis, *Phys. Rev. B* **45**, 13047 (1992)

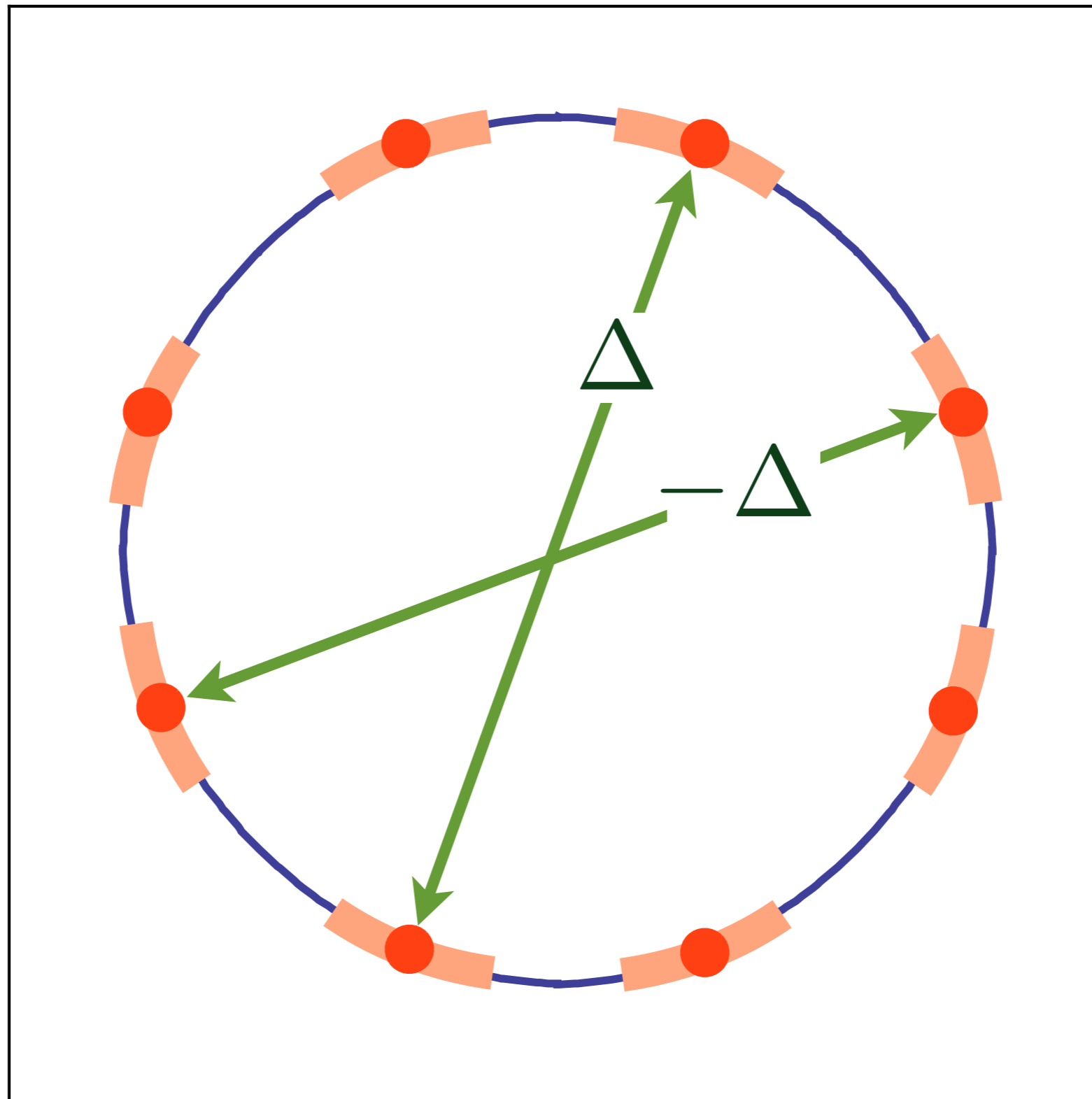
Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004)

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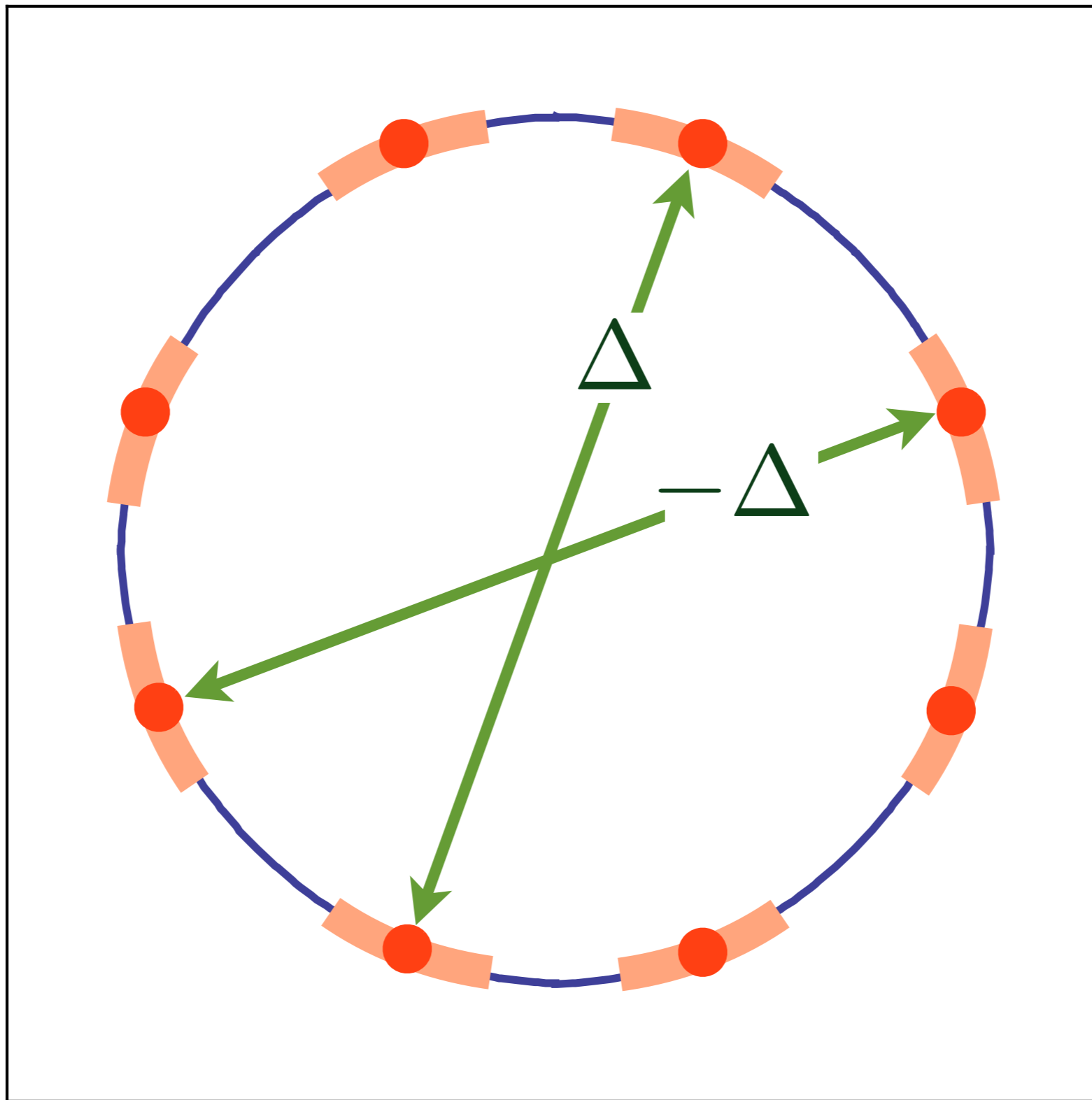
$$G_{\text{fermion}} = \frac{Z(k_{\parallel})}{\omega - v_F(k_{\parallel})k_{\perp}}, \quad Z(k_{\parallel}) \sim v_F(k_{\parallel}) \sim k_{\parallel}$$

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



Unconventional pairing at and near hot spots

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



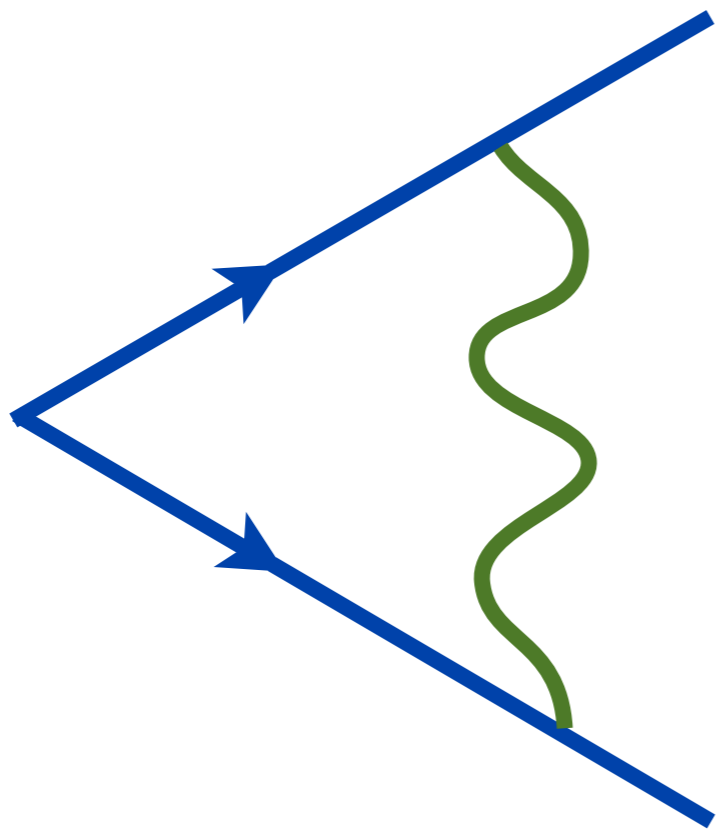
Unconventional pairing at and near hot spots

# BCS theory

$$1 + \lambda_{\text{e-ph}} \log \left( \frac{\omega_D}{\omega} \right)$$



Cooper  
logarithm



# BCS theory

$$1 + \lambda_{\text{e-ph}} \log \left( \frac{\omega_D}{\omega} \right)$$

Electron-phonon  
coupling

Debye  
frequency

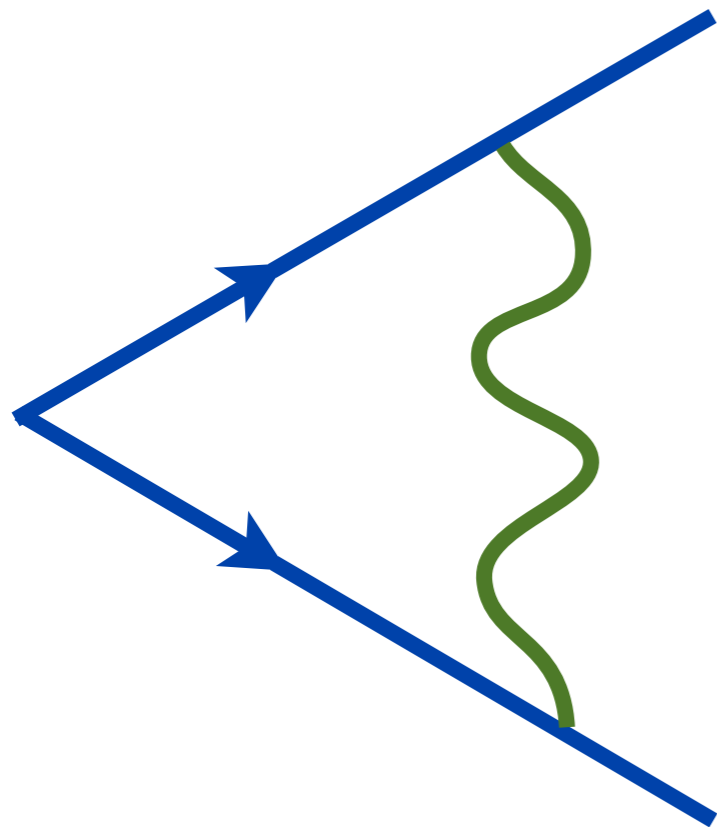
Implies

$$T_c \sim \omega_D \exp(-1/\lambda)$$

# Enhancement of pairing susceptibility by interactions

## Antiferromagnetic fluctuations: weak-coupling

$$1 + \left(\frac{U}{t}\right)^2 \log\left(\frac{E_F}{\omega}\right)$$



Cooper  
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V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

S. Raghu, S. A. Kivelson, and D. J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

# Enhancement of pairing susceptibility by interactions

## Antiferromagnetic fluctuations: weak-coupling

$$1 + \left(\frac{U}{t}\right)^2 \log\left(\frac{E_F}{\omega}\right)$$

Applies in a Fermi liquid  
as repulsive interaction  $U \rightarrow 0$ .

Fermi  
energy

Implies

$$T_c \sim E_F \exp\left(-\left(t/U\right)^2\right)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)

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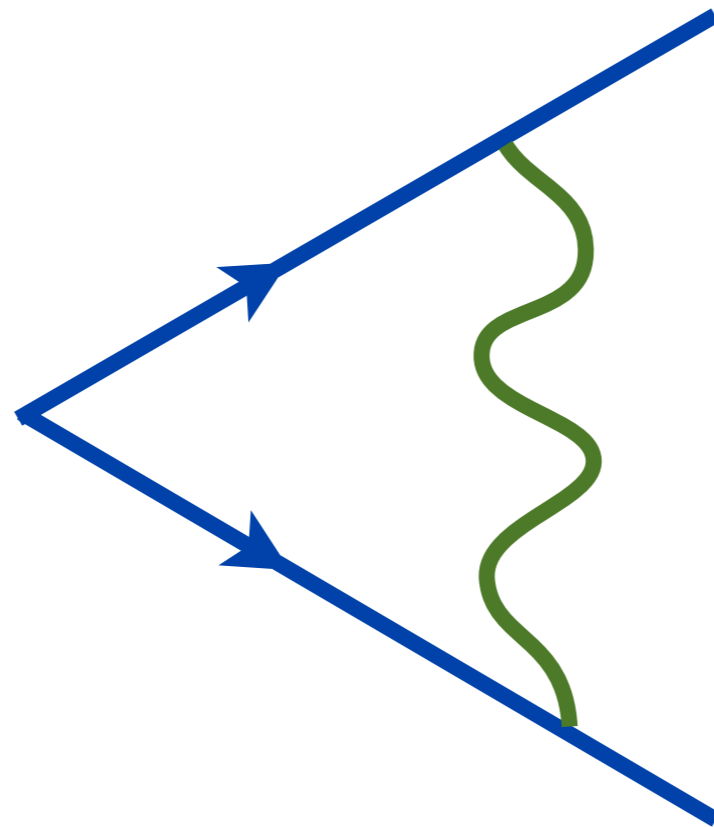
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## Antiferromagnetic critical point

$$1 + \frac{\sin \theta}{2\pi} \log^2 \left( \frac{E_F}{\omega} \right)$$



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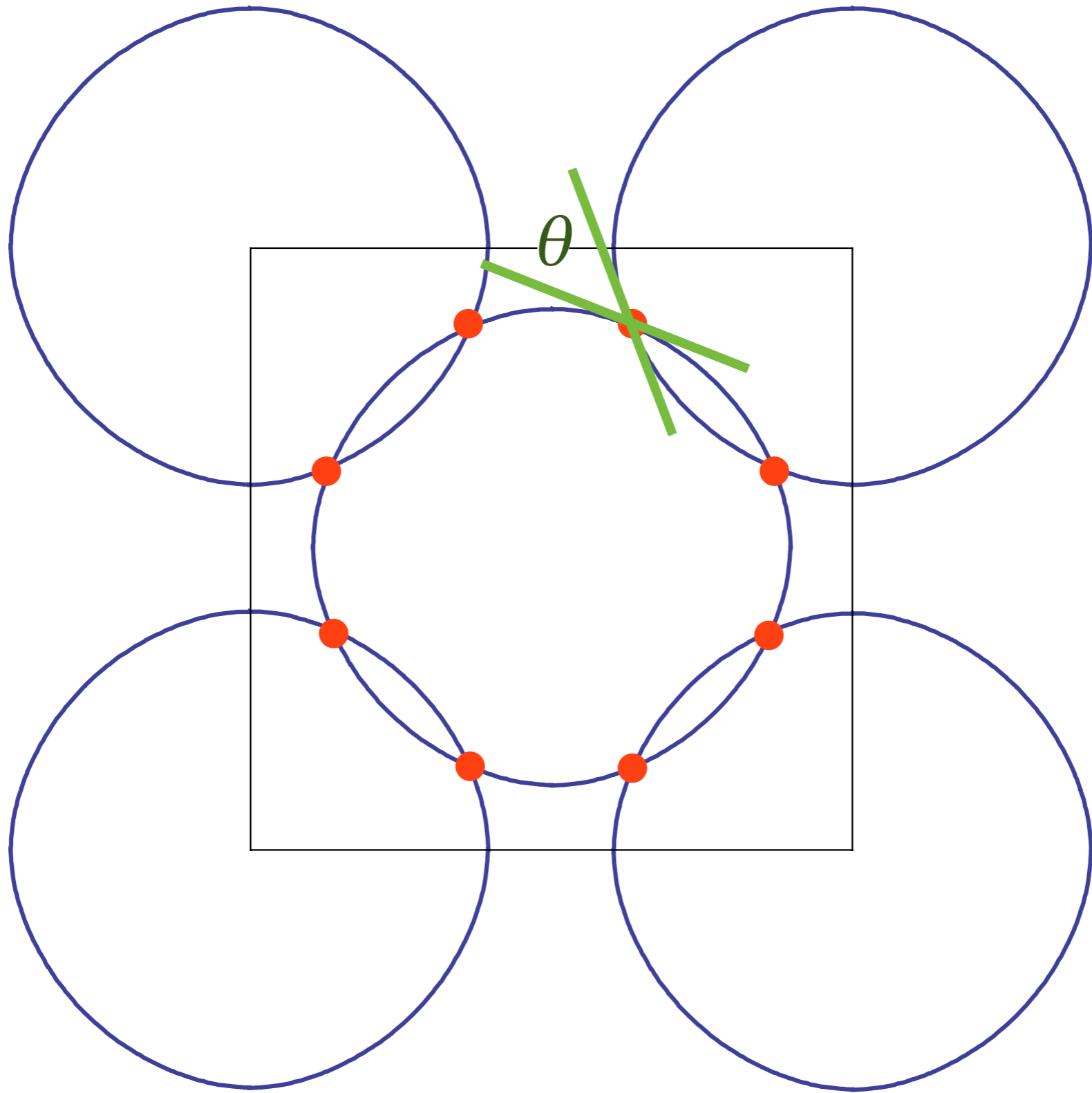
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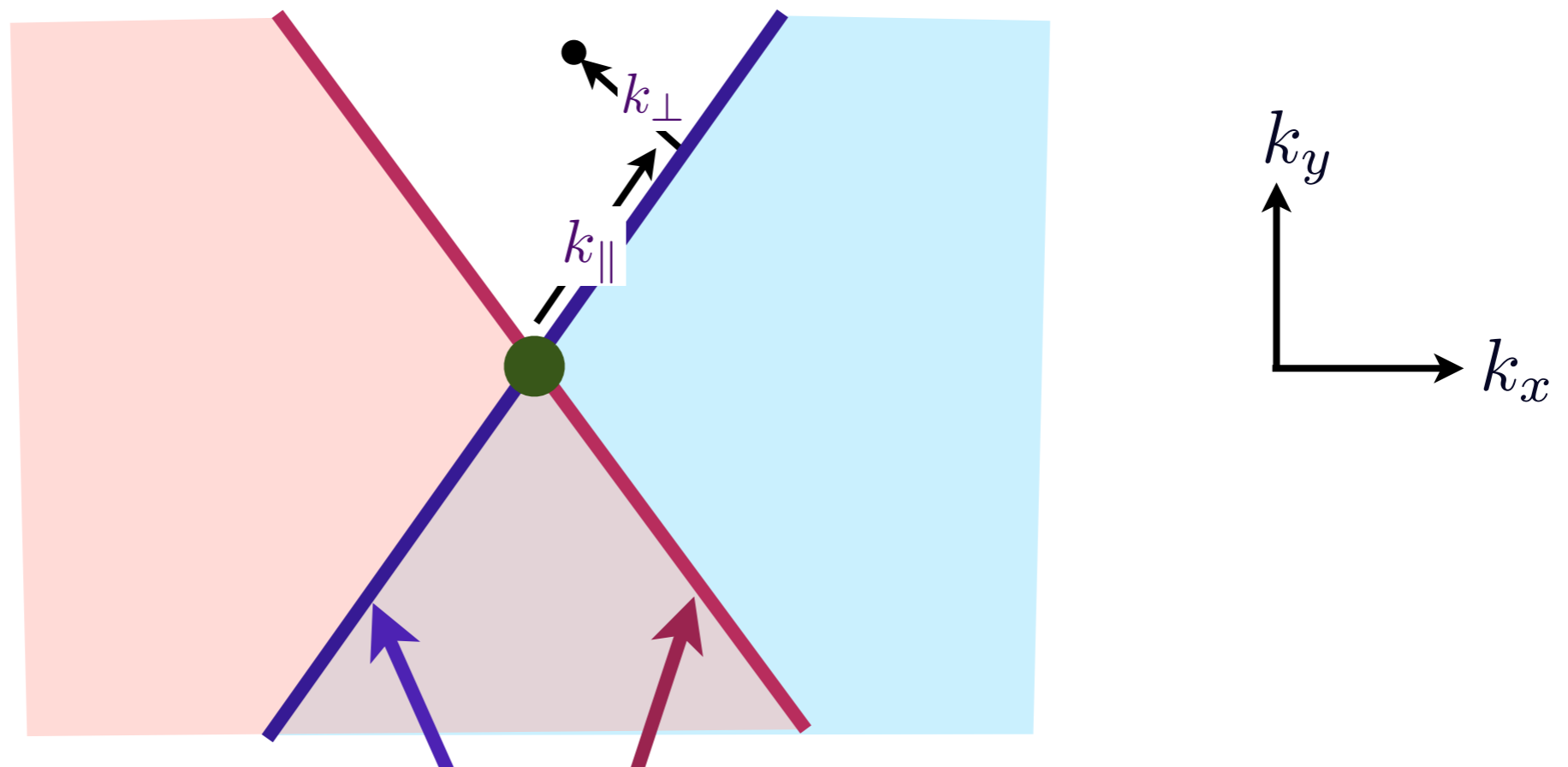
Fermi  
energy

$\theta$  is the angle between Fermi lines.  
Independent of interaction strength  
 $U$  in 2 dimensions.

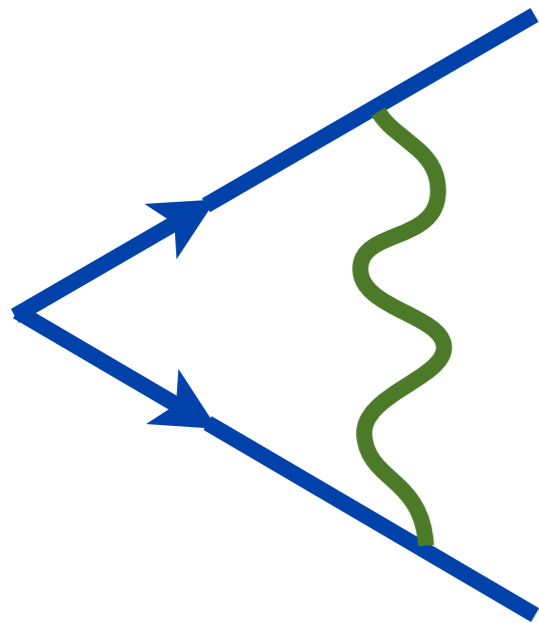
(see also Ar. Abanov, A. V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))  
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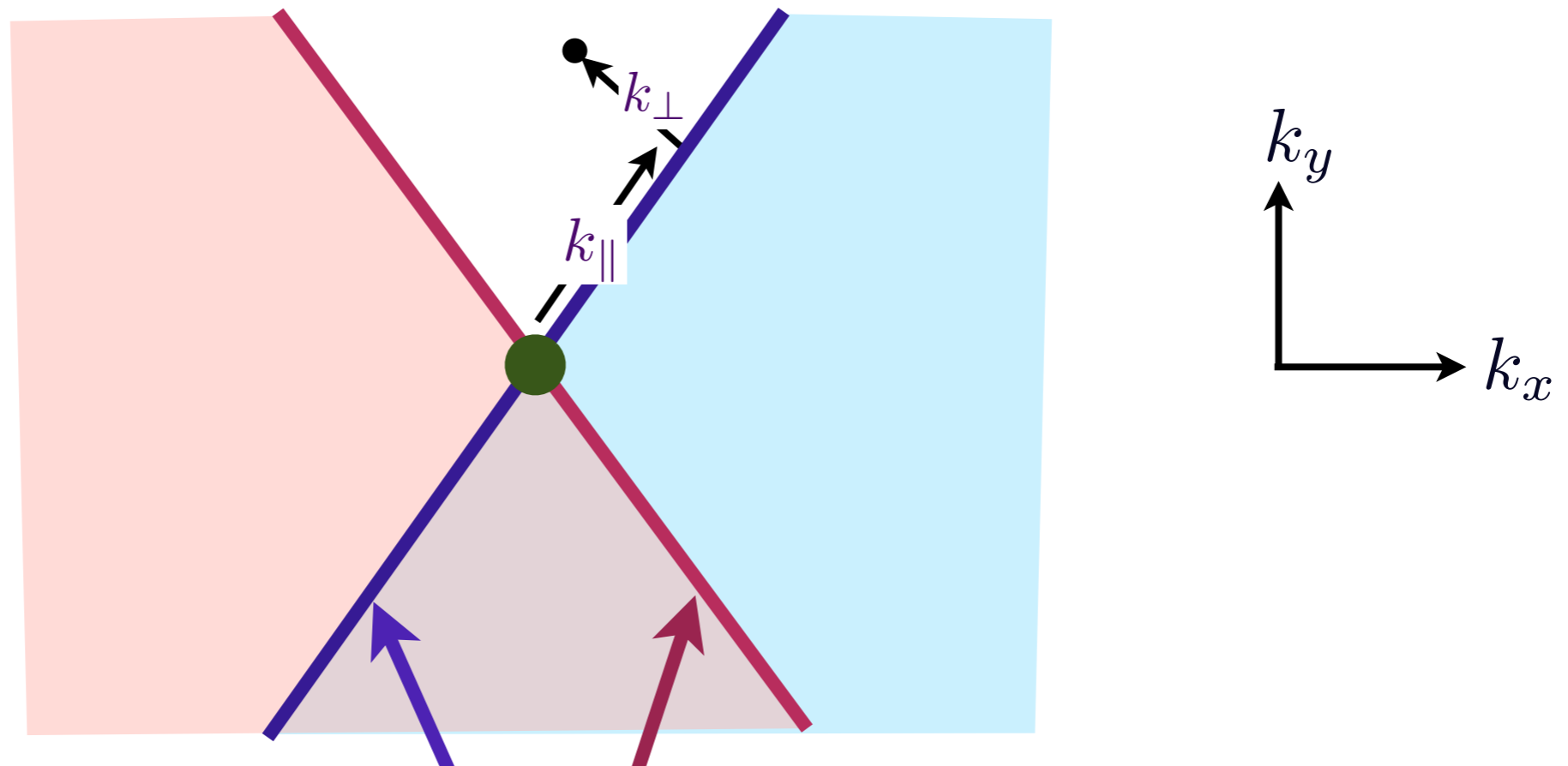


$$G_{\text{fermion}} = \frac{Z(k_{\parallel})}{i\omega - v_F(k_{\parallel})k_{\perp}}, \quad Z(k_{\parallel}) \sim v_F(k_{\parallel}) \sim k_{\parallel}$$

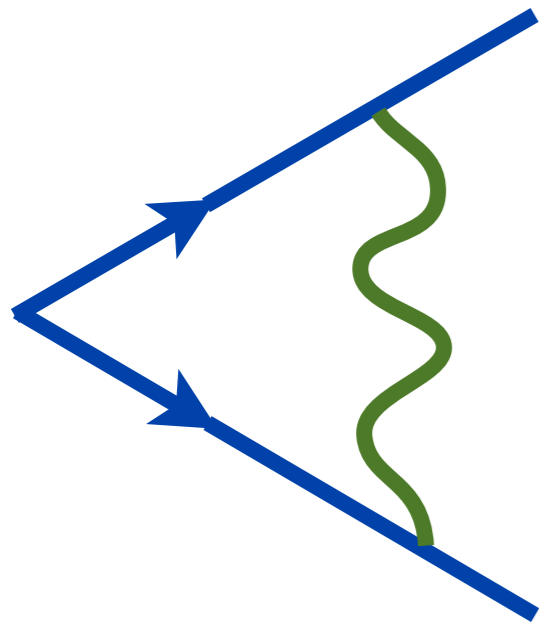


$$\int dk_{\parallel} \frac{1}{k_{\parallel}^2} \left( \frac{Z^2(k_{\parallel})}{v_F(k_{\parallel})} \right) \log \frac{k_{\parallel}^2}{\omega}$$

M.A. Metlitski  
and S. Sachdev,  
*Phys. Rev. B* **85**,  
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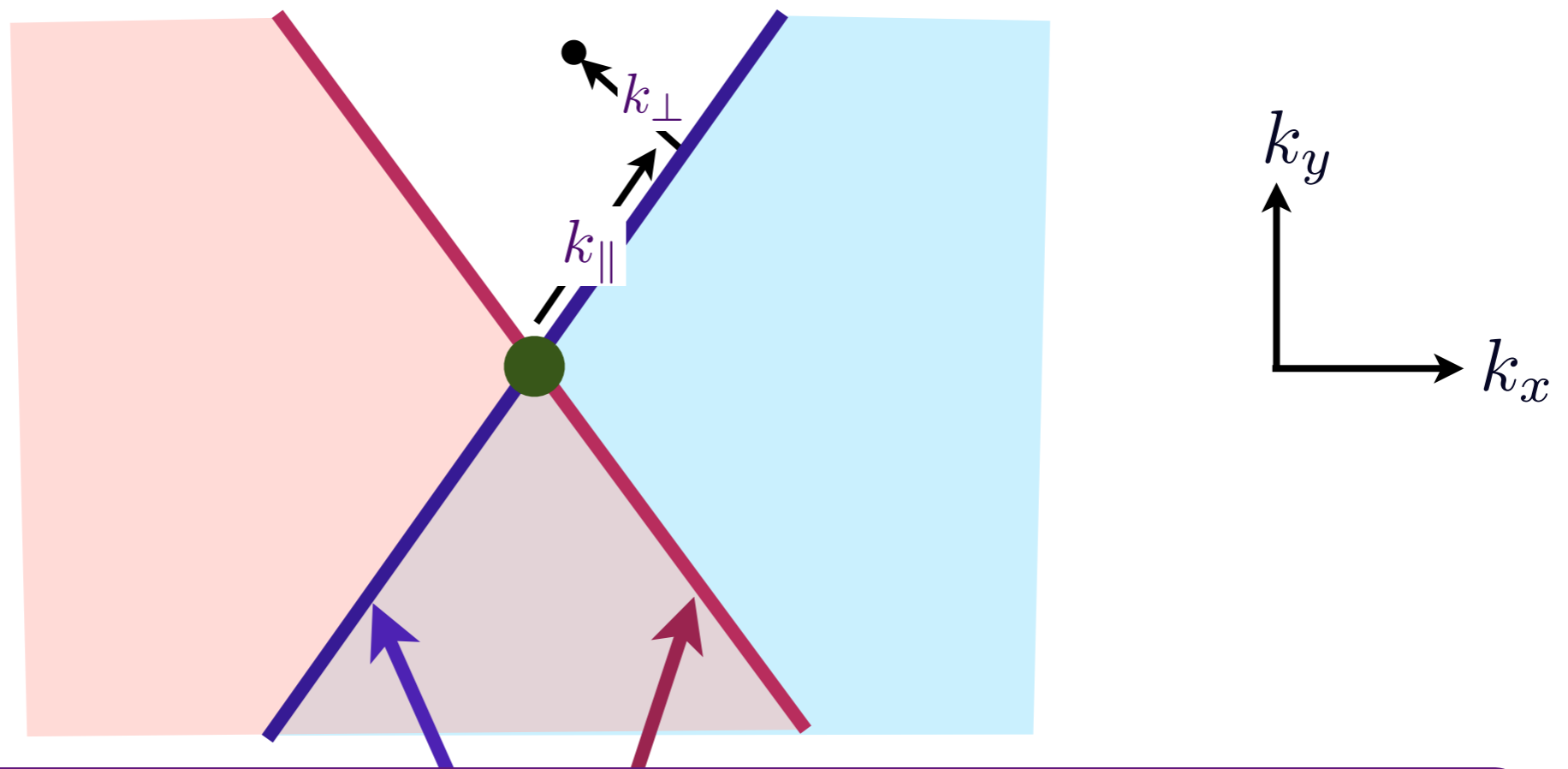
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Cooper  
logarithm

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Spin fluctuation propagator

Cooper logarithm

# Enhancement of pairing susceptibility by interactions

## Antiferromagnetic critical point

$$1 + \frac{\sin \theta}{2\pi} \log^2 \left( \frac{E_F}{\omega} \right)$$



- $\log^2$  singularity arises from Fermi lines; singularity *at* hot spots is weaker.
- Interference between BCS and quantum-critical logs.
- Momentum dependence of self-energy is crucial.
- Not suppressed by  $1/N$  factor in  $1/N$  expansion.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Is there a  $\log^2$  towards any  
other instability ?

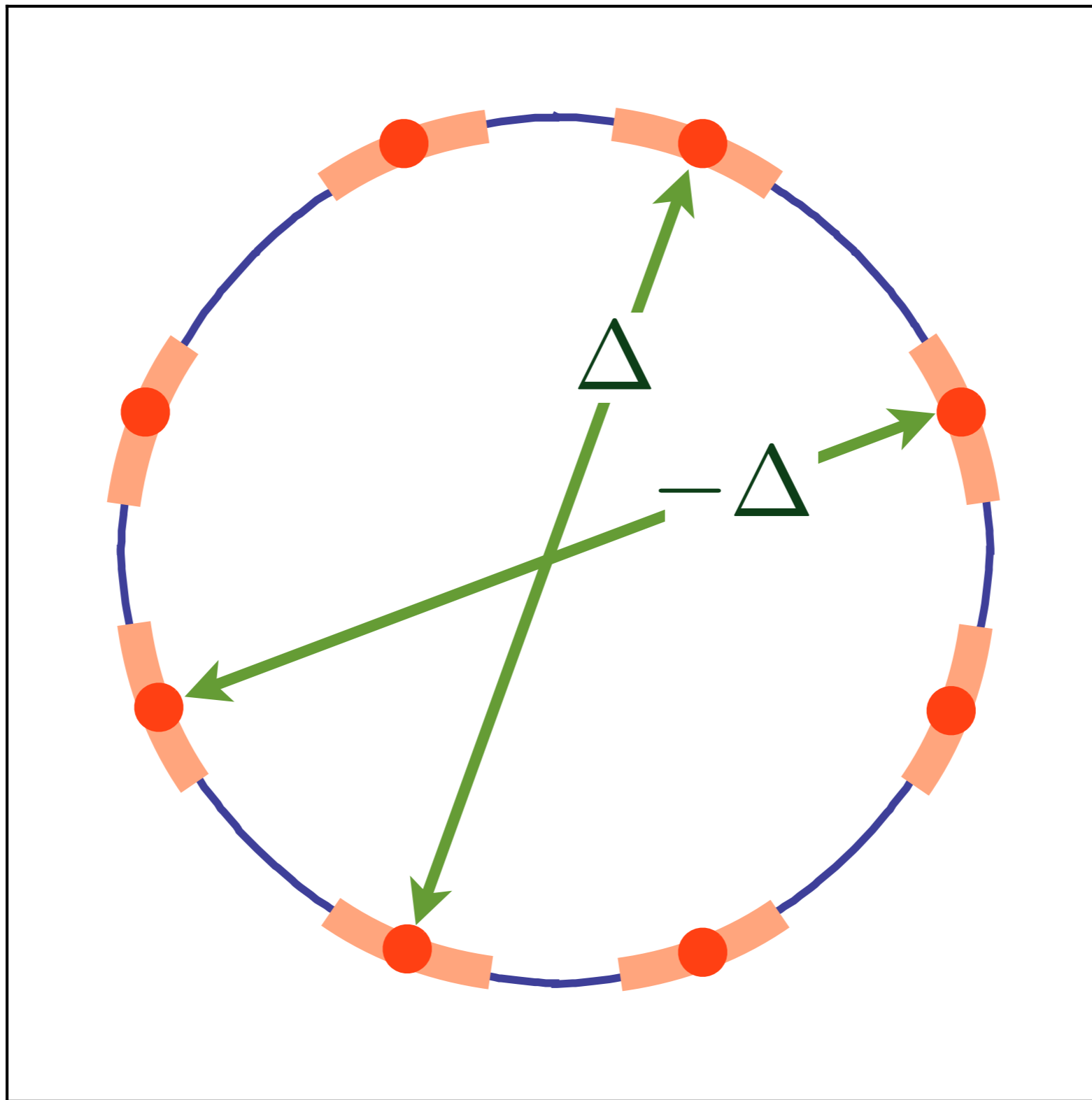
Is there a  $\log^2$  towards any other instability ?

Only one other:  
to a  $2k_F$  bond-nematic order,  
which is smaller by a factor of 3.

$$1 + \frac{\sin \theta}{6\pi} \log^2 \left( \frac{E_F}{\omega} \right)$$

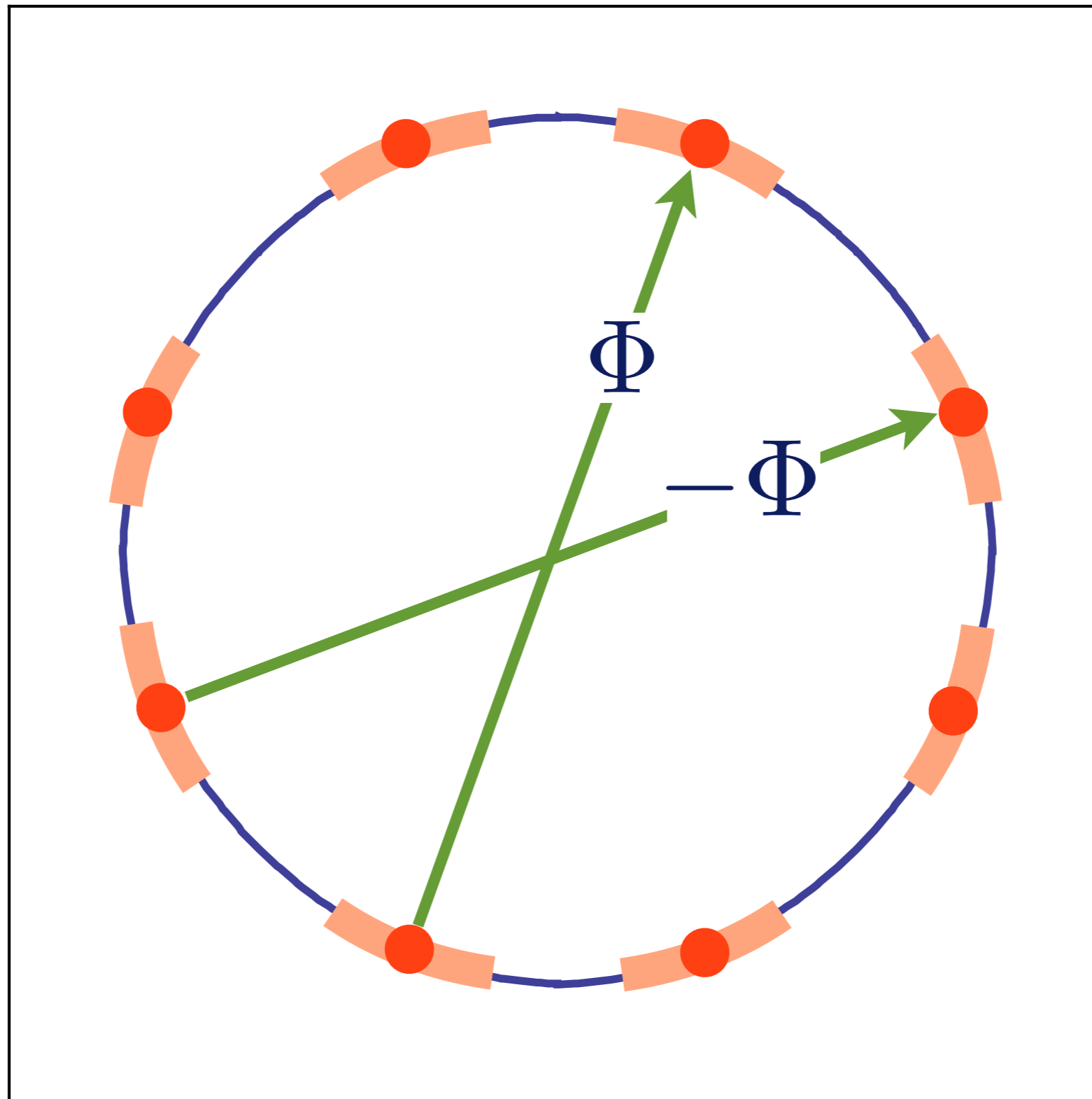


$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at and near hot spots

$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$



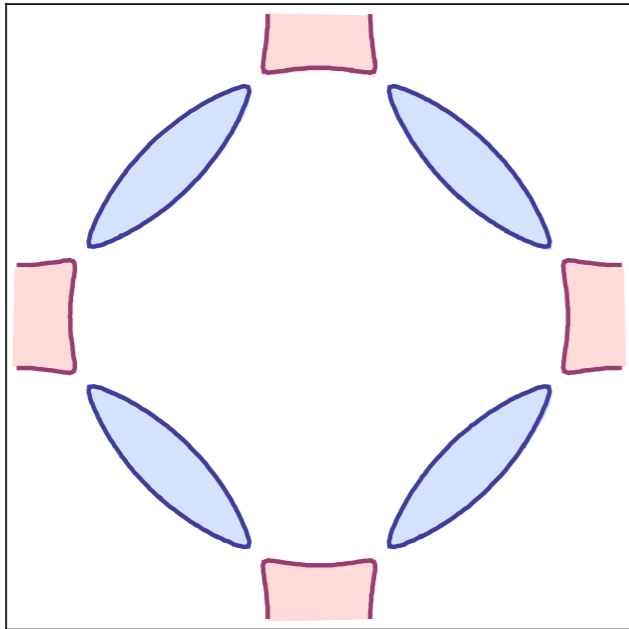
$\mathbf{Q}$  is ' $2k_F$ '  
wavevector

Unconventional particle-hole pairing at and near hot spots

1. Experimental motivations from cuprates and pnictides
2. Conventional theory and its breakdown in two spatial dimensions
3. Fermi surface reconstruction: onset of unconventional superconductivity

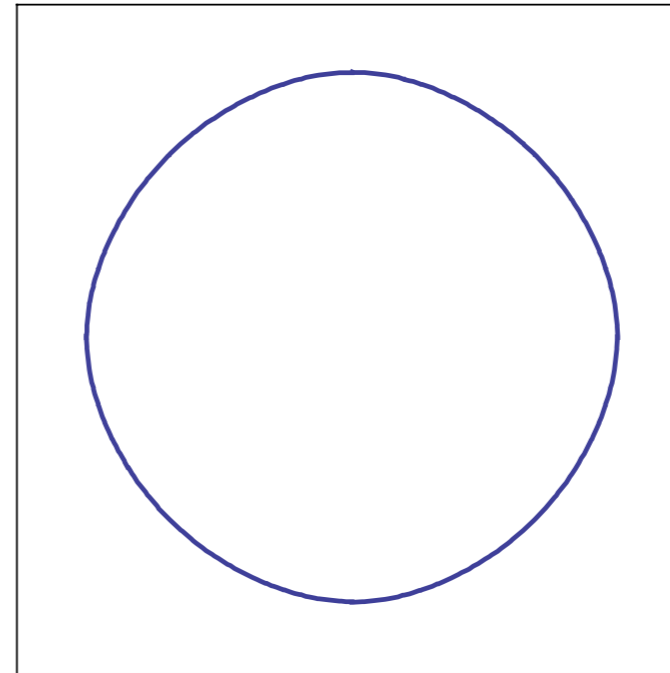
4. Fermi surface reconstruction *without* symmetry breaking: metals with “topological” order and the heavy fermion compounds

# Quantum phase transition with Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron  
and hole pockets

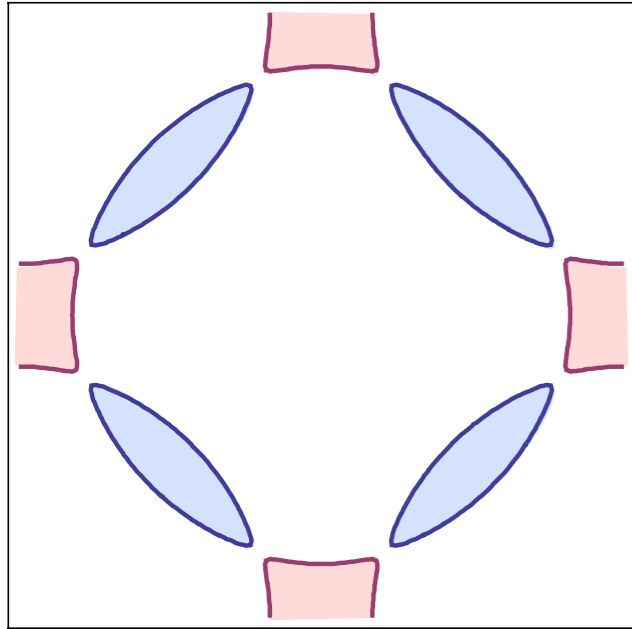


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”  
Fermi surface

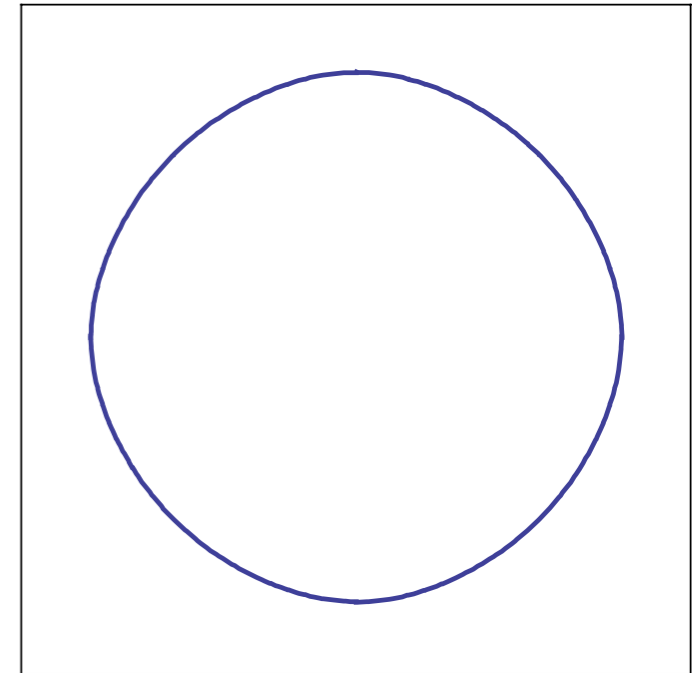


# Separating onset of SDW order and Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron  
and hole pockets

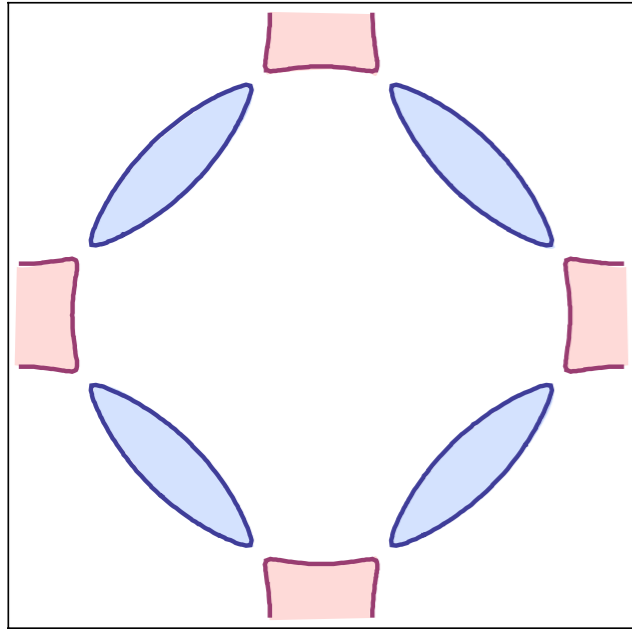


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”  
Fermi surface



# Separating onset of SDW order and Fermi surface reconstruction

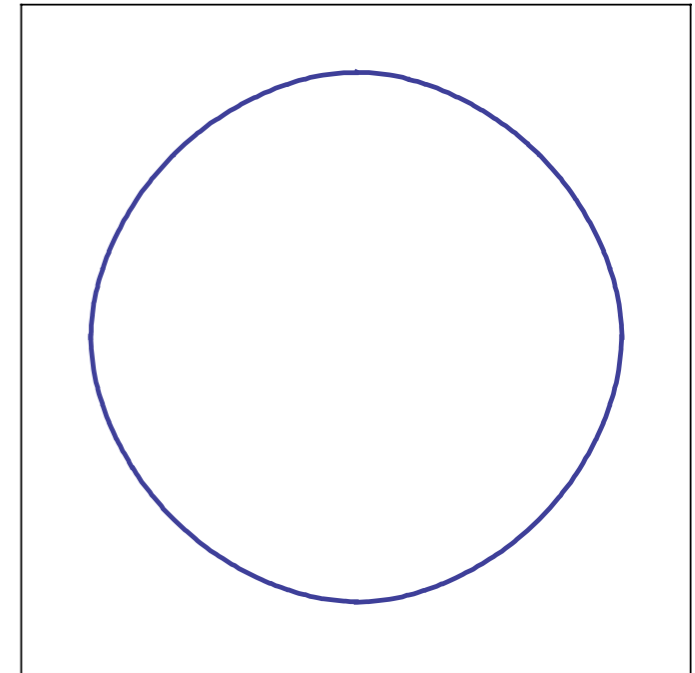


$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron  
and hole pockets

Electron and/or hole  
Fermi pockets form in  
“local” SDW order, but  
quantum fluctuations  
destroy long-range  
SDW order

$$\langle \vec{\varphi} \rangle = 0$$

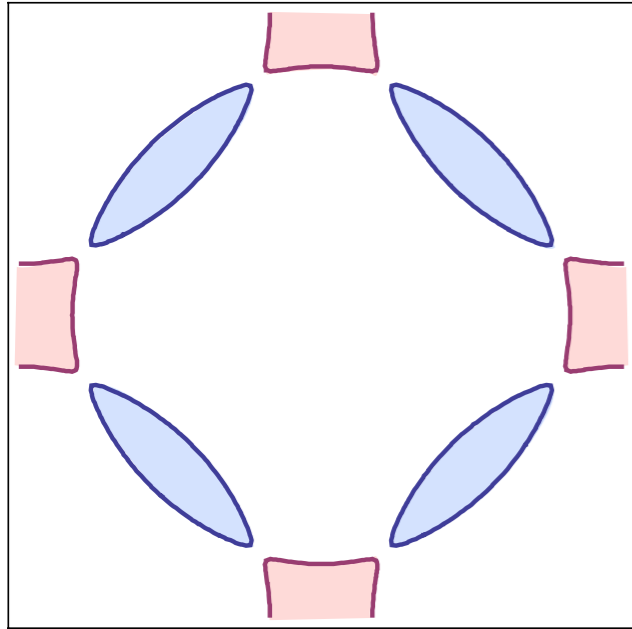


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”  
Fermi surface

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

# Separating onset of SDW order and Fermi surface reconstruction



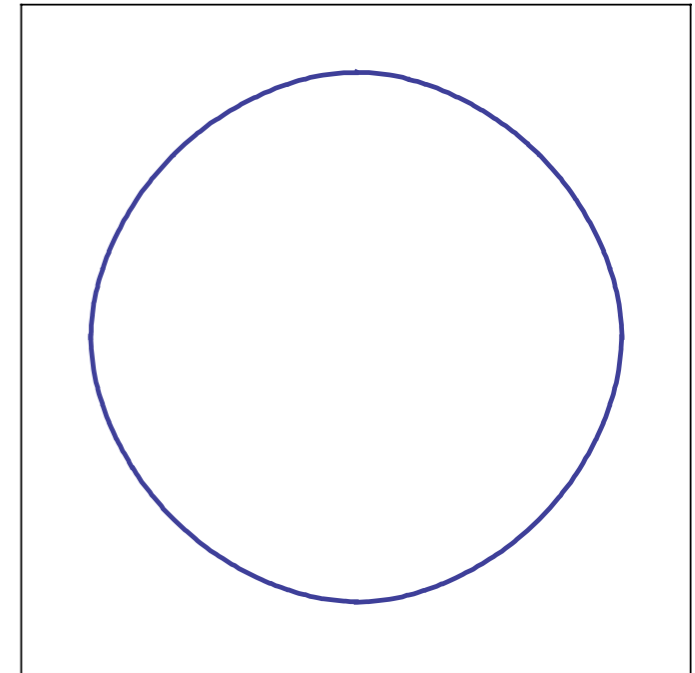
$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron  
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Electron and/or hole  
Fermi pockets form in  
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quantum fluctuations  
destroy long-range  
SDW order

$$\langle \vec{\varphi} \rangle = 0$$

Fractionalized Fermi  
liquid (FL\*) phase  
with no symmetry  
breaking and “small”  
Fermi surface

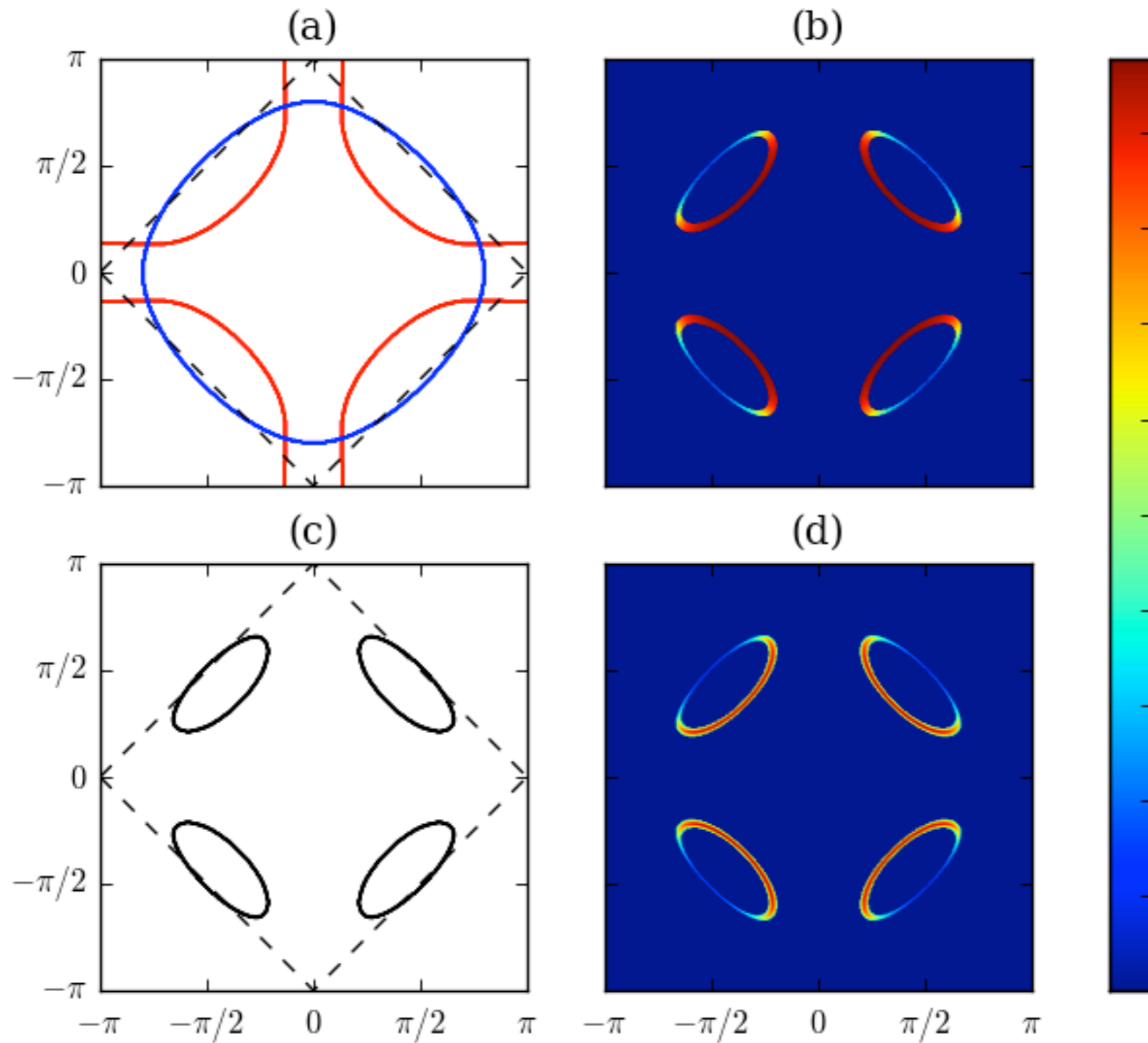


$$\langle \vec{\varphi} \rangle = 0$$

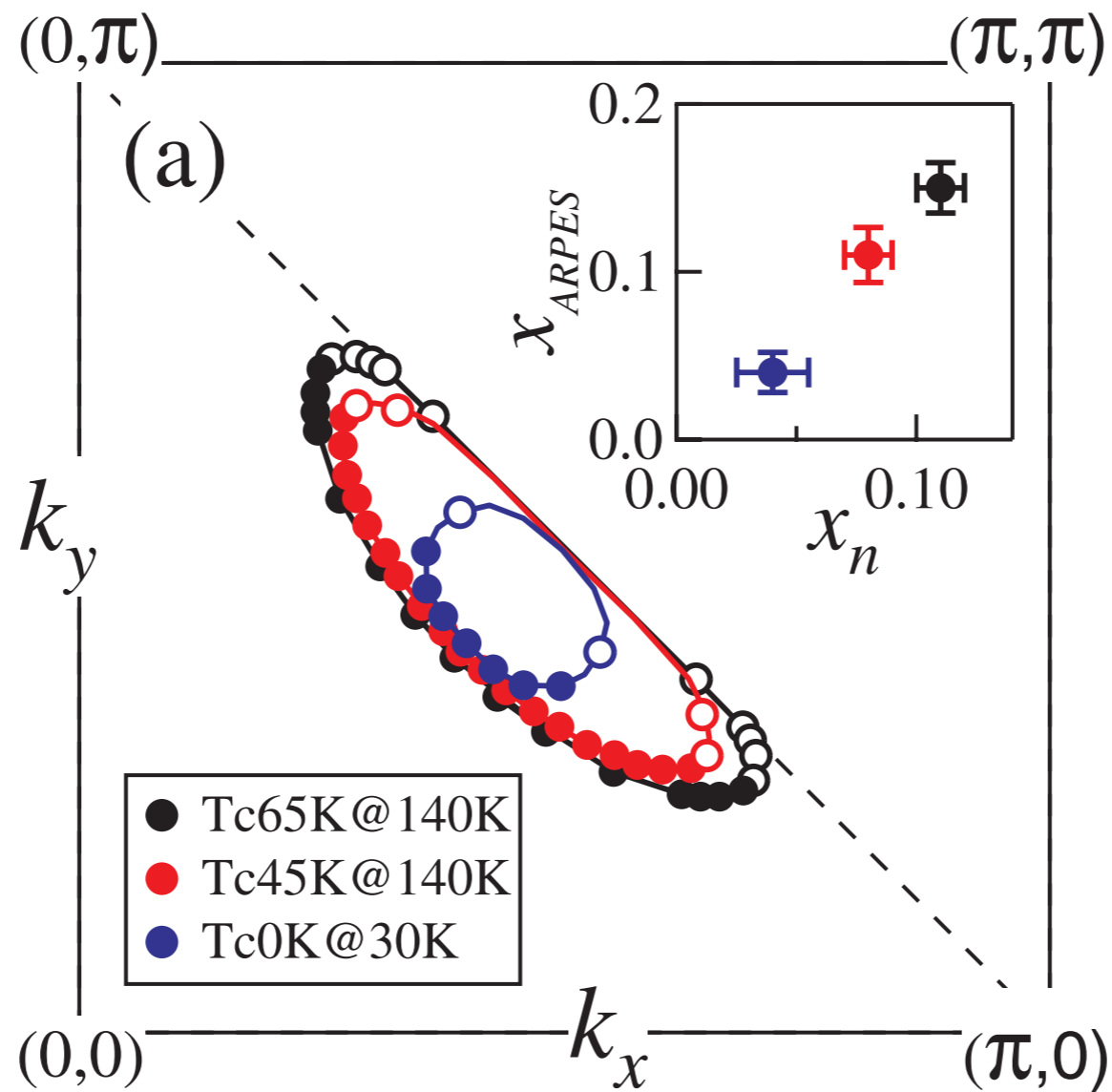
Metal with “large”  
Fermi surface

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

FL\* phase has Fermi pockets without long-range antiferromagnetism, along with emergent gauge excitations



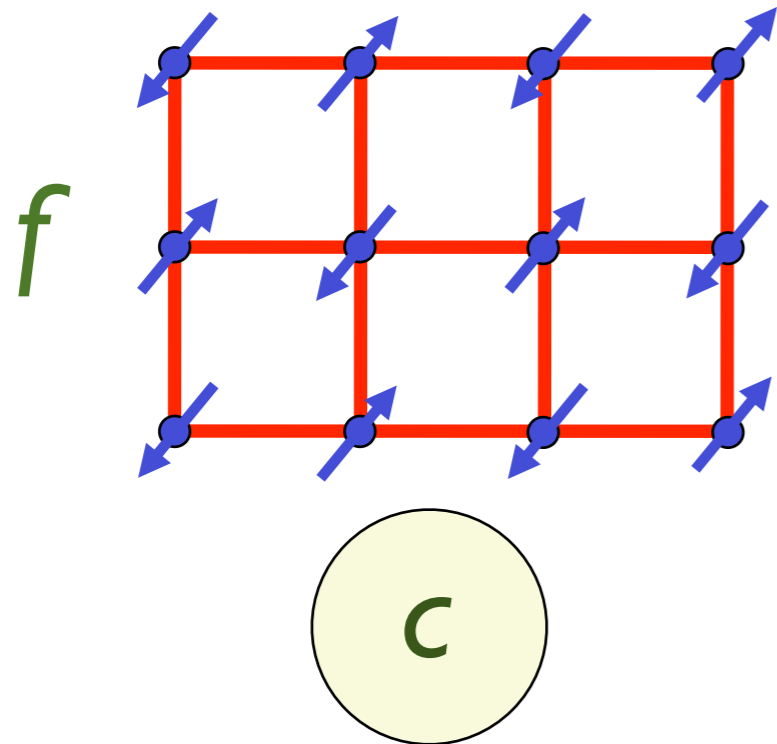
Y. Qi and S. Sachdev, *Physical Review B* **81**, 115129 (2010)



## Reconstructed Fermi Surface of Underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ Cuprate Superconductors

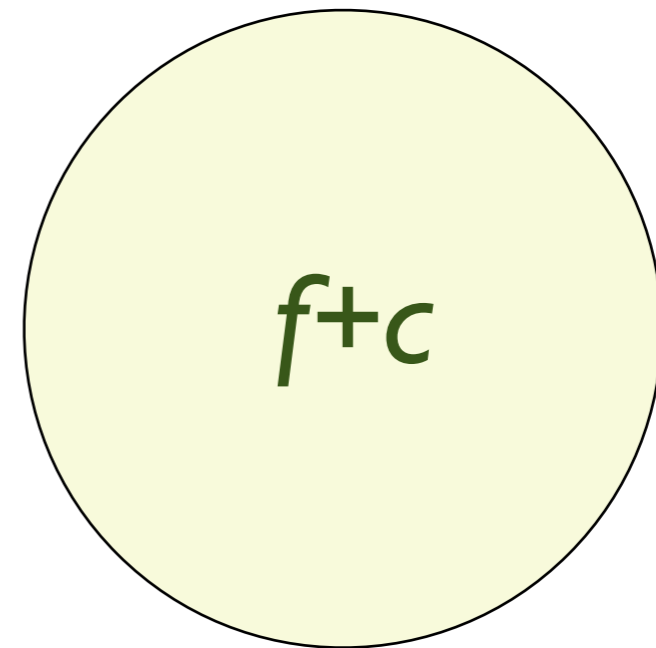
H.-B. Yang,<sup>1</sup> J. D. Rameau,<sup>1</sup> Z.-H. Pan,<sup>1</sup> G. D. Gu,<sup>1</sup> P. D. Johnson,<sup>1</sup> H. Claus,<sup>2</sup> D. G. Hinks,<sup>2</sup> and T. E. Kidd<sup>3</sup>

# Magnetic order and the heavy Fermi liquid in the Kondo lattice



$$\langle \vec{\varphi} \rangle \neq 0$$

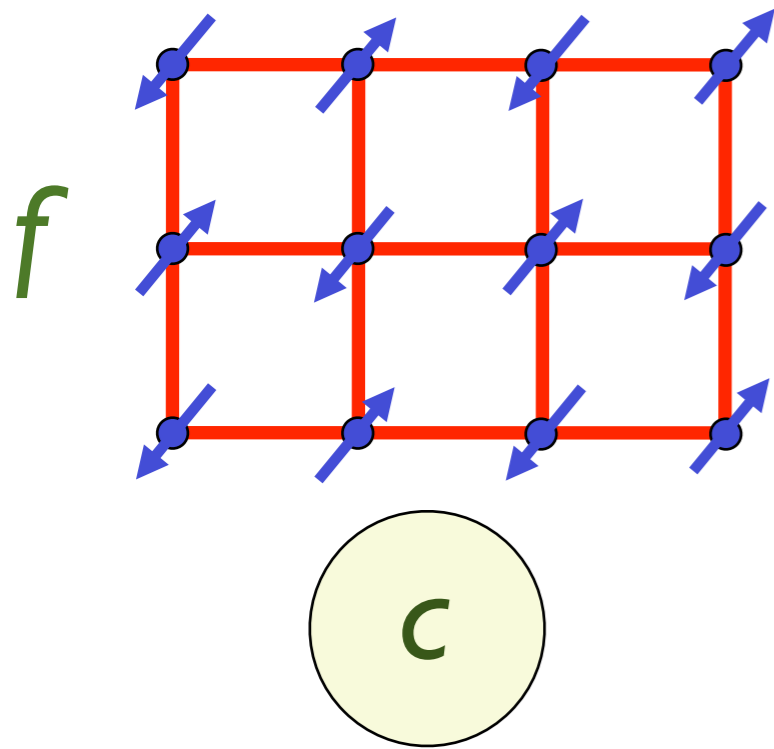
Magnetic Metal:  
f-electron moments  
and  
c-conduction electron  
Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

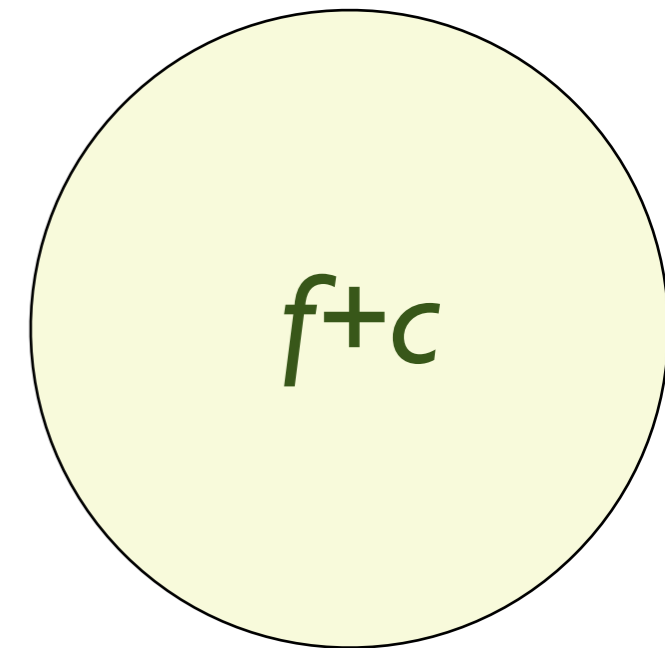
Heavy Fermi liquid  
with “large” Fermi  
surface of  
hybridized f and  
c-conduction  
electrons

# Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:  
f-electron moments  
and  
c-conduction electron  
Fermi surface

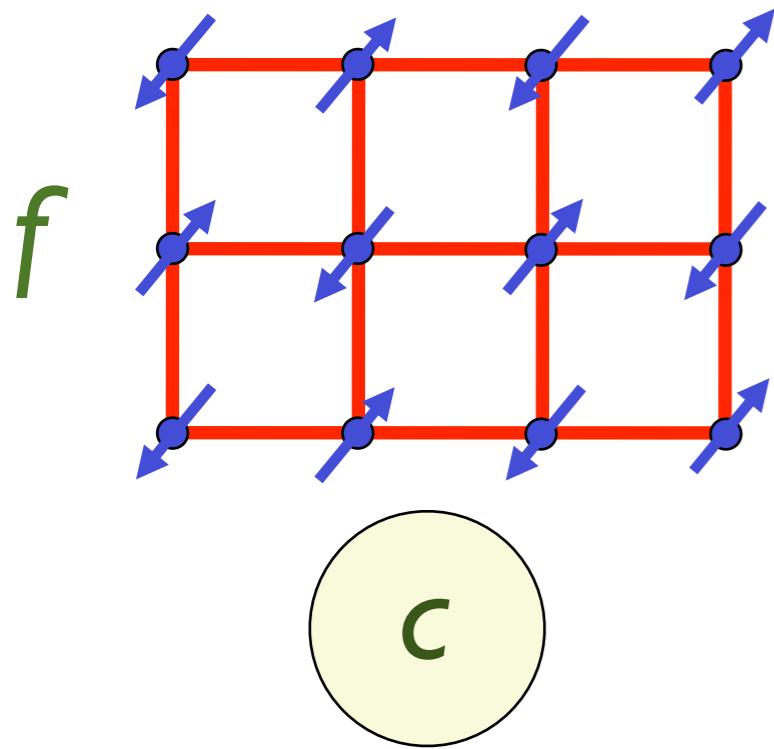


$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid  
with “large” Fermi  
surface of  
hybridized f and  
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electrons

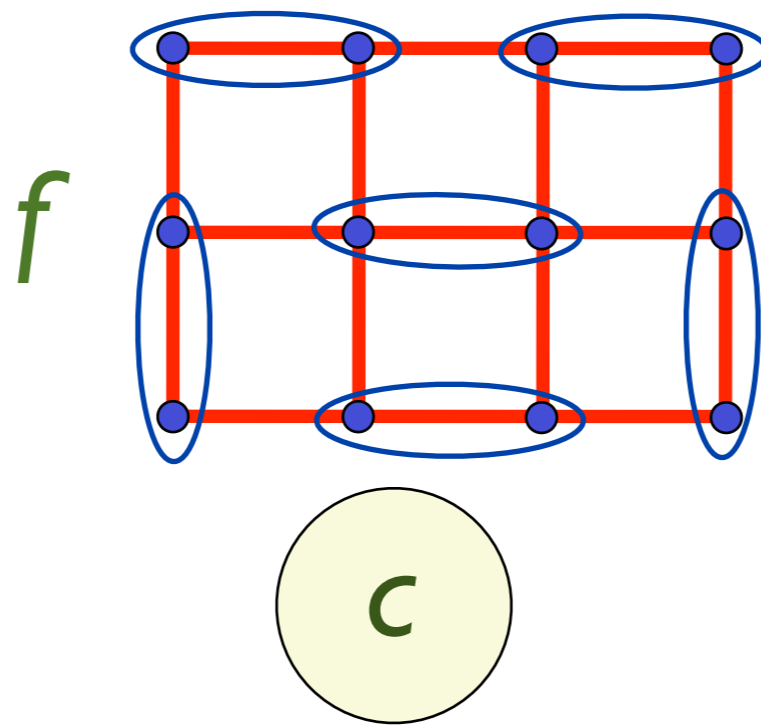
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

# Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



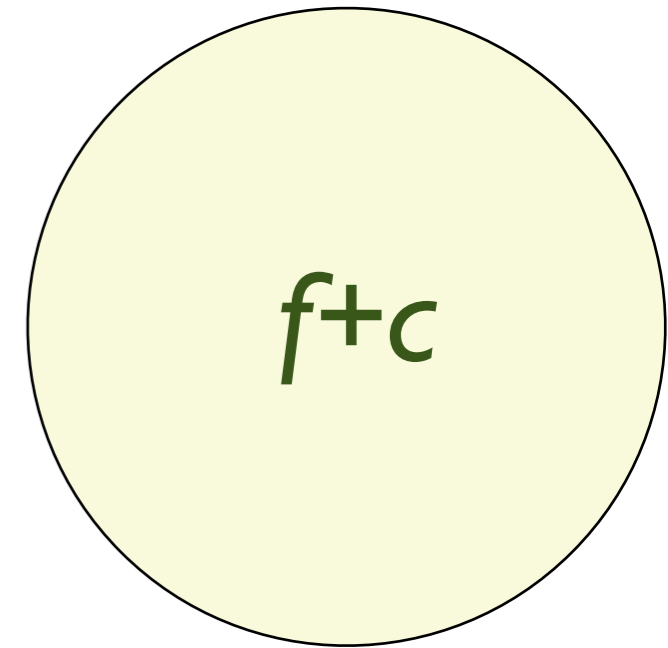
$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:  
f-electron moments  
and  
c-conduction electron  
Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

Conduction electron  
Fermi surface  
and  
spin-liquid of  
f-electrons

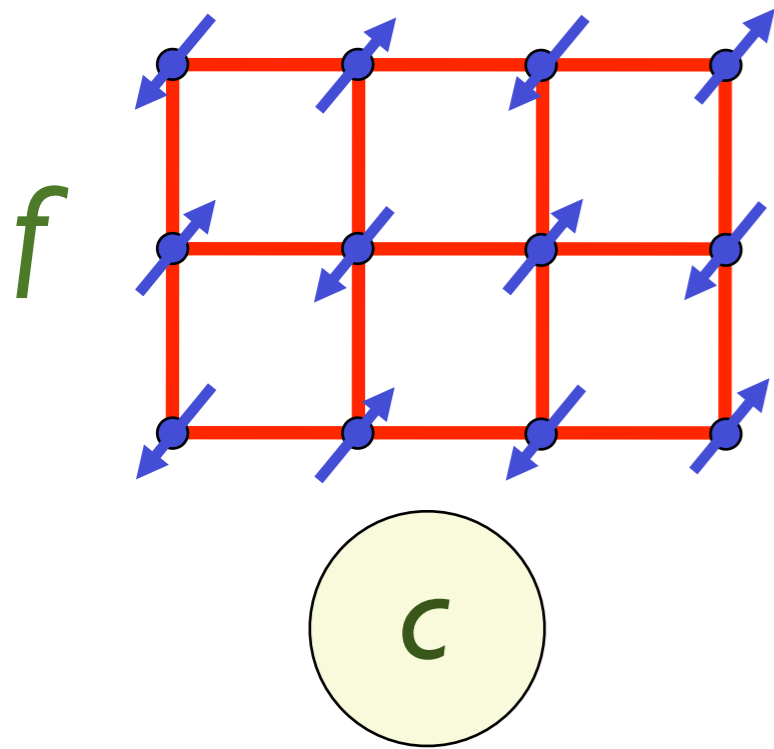


$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid  
with “large” Fermi  
surface of  
hybridized f and  
c-conduction  
electrons

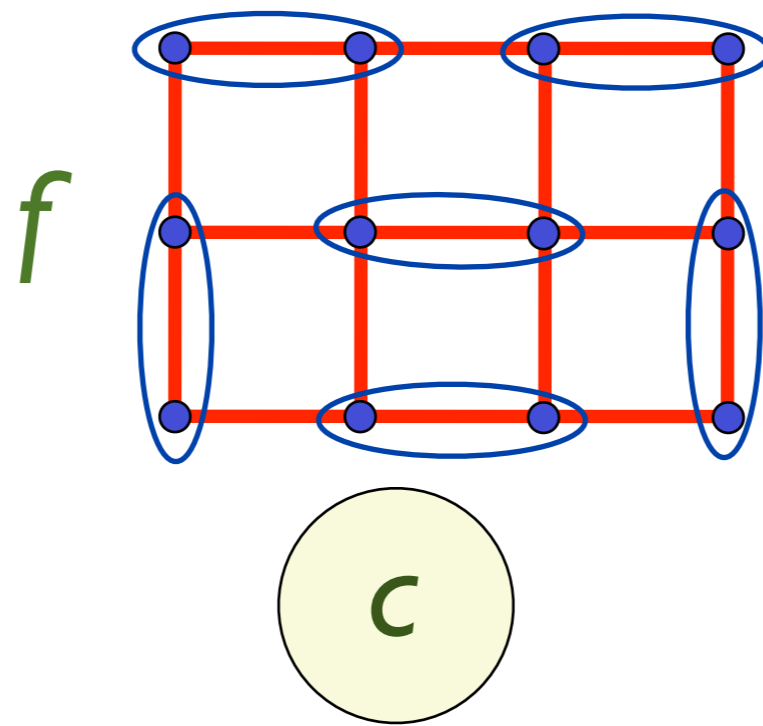
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

# Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



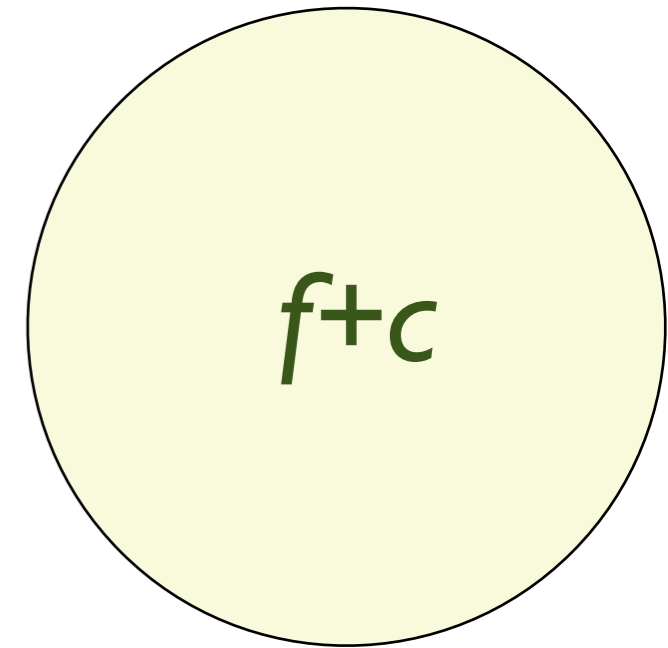
$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:  
f-electron moments  
and  
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Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

Fractionalized Fermi  
liquid (FL\*) phase  
with no symmetry  
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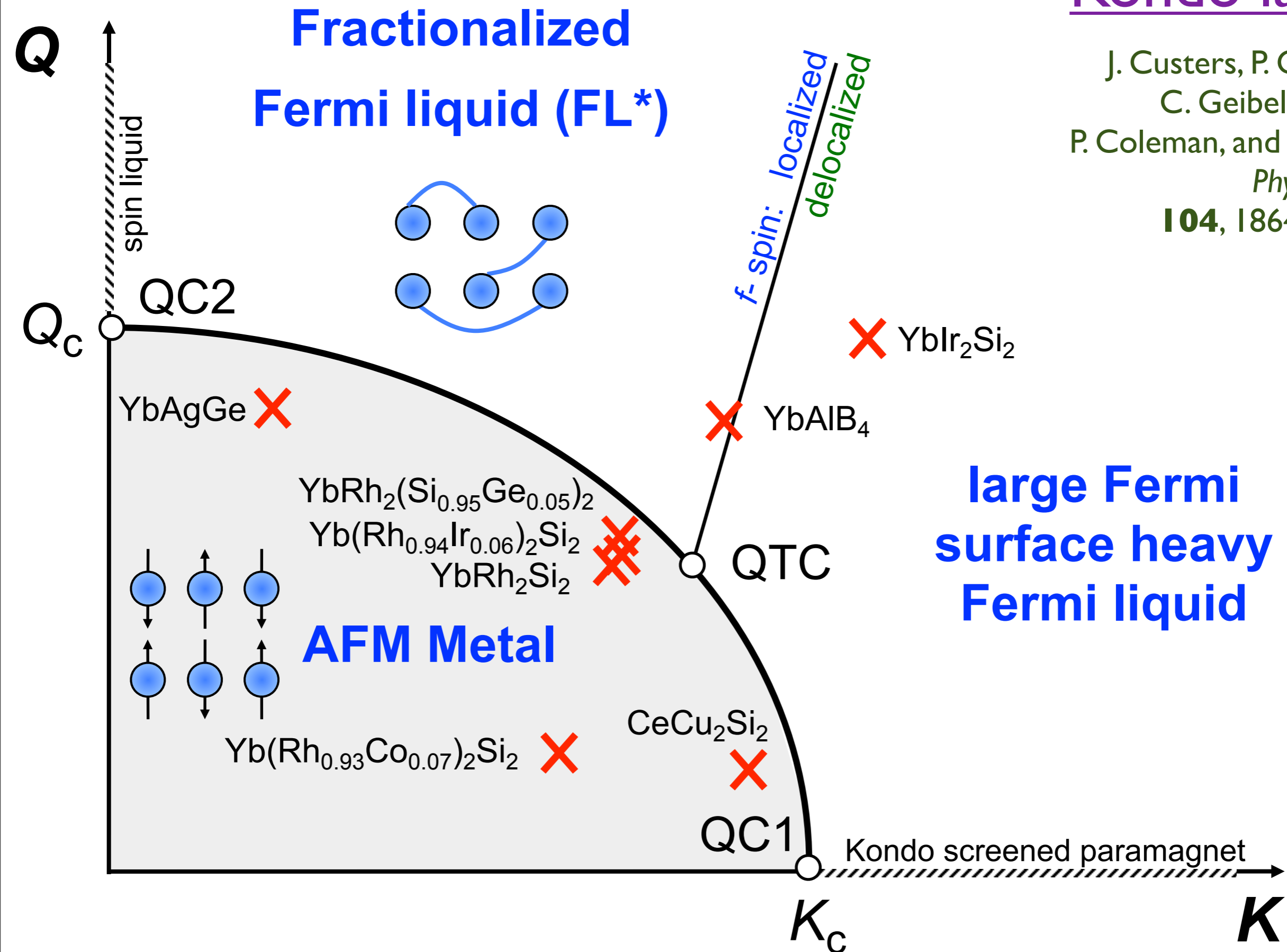
$$\langle \vec{\varphi} \rangle = 0$$

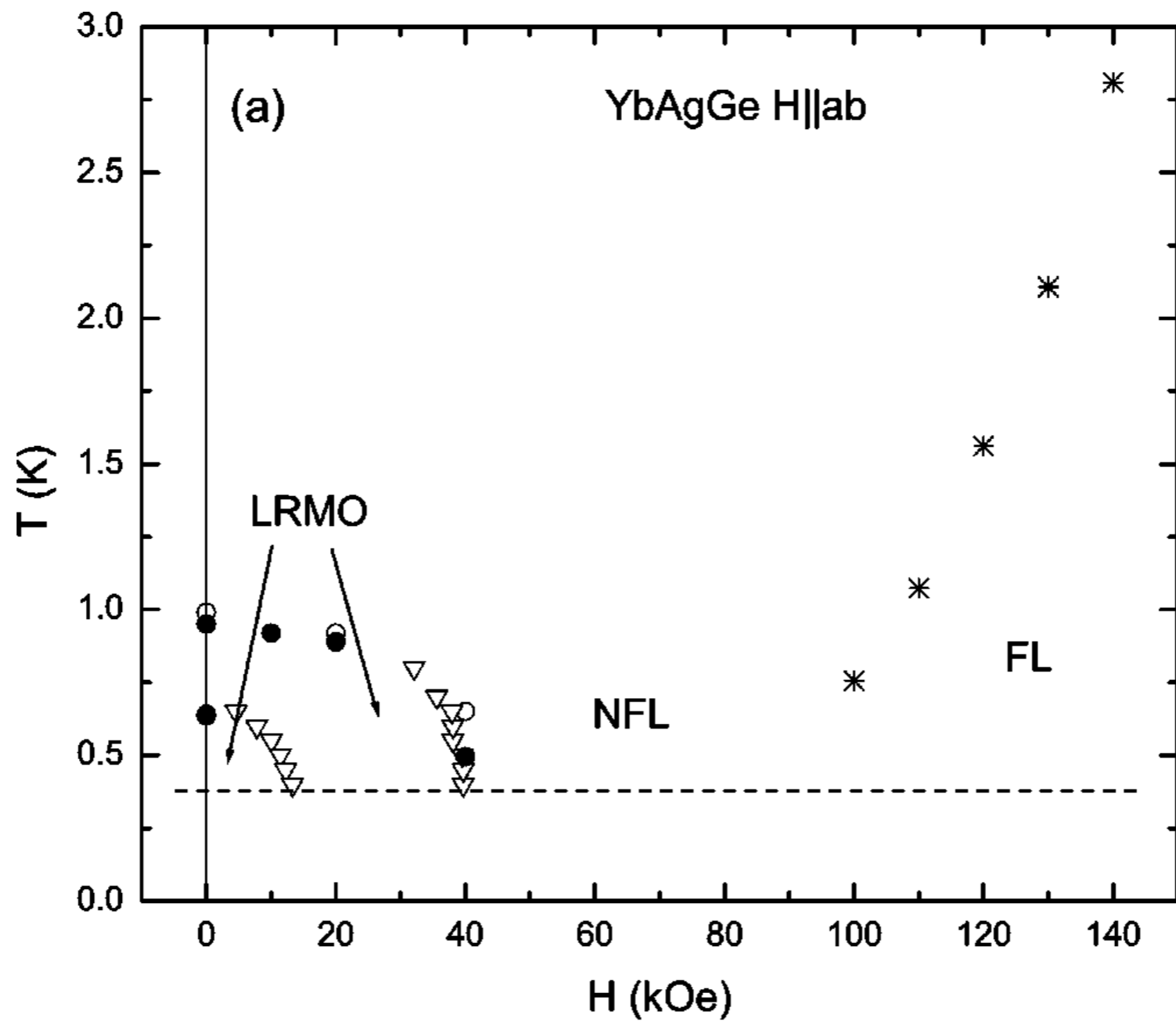
Heavy Fermi liquid  
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surface of  
hybridized f and  
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electrons

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

# Experimental perspective on same phase diagrams of Kondo lattice

J. Custers, P. Gegenwart,  
C. Geibel, F. Steglich,  
P. Coleman, and S. Paschen,  
*Phys. Rev. Lett.*  
**104**, 186402 (2010)

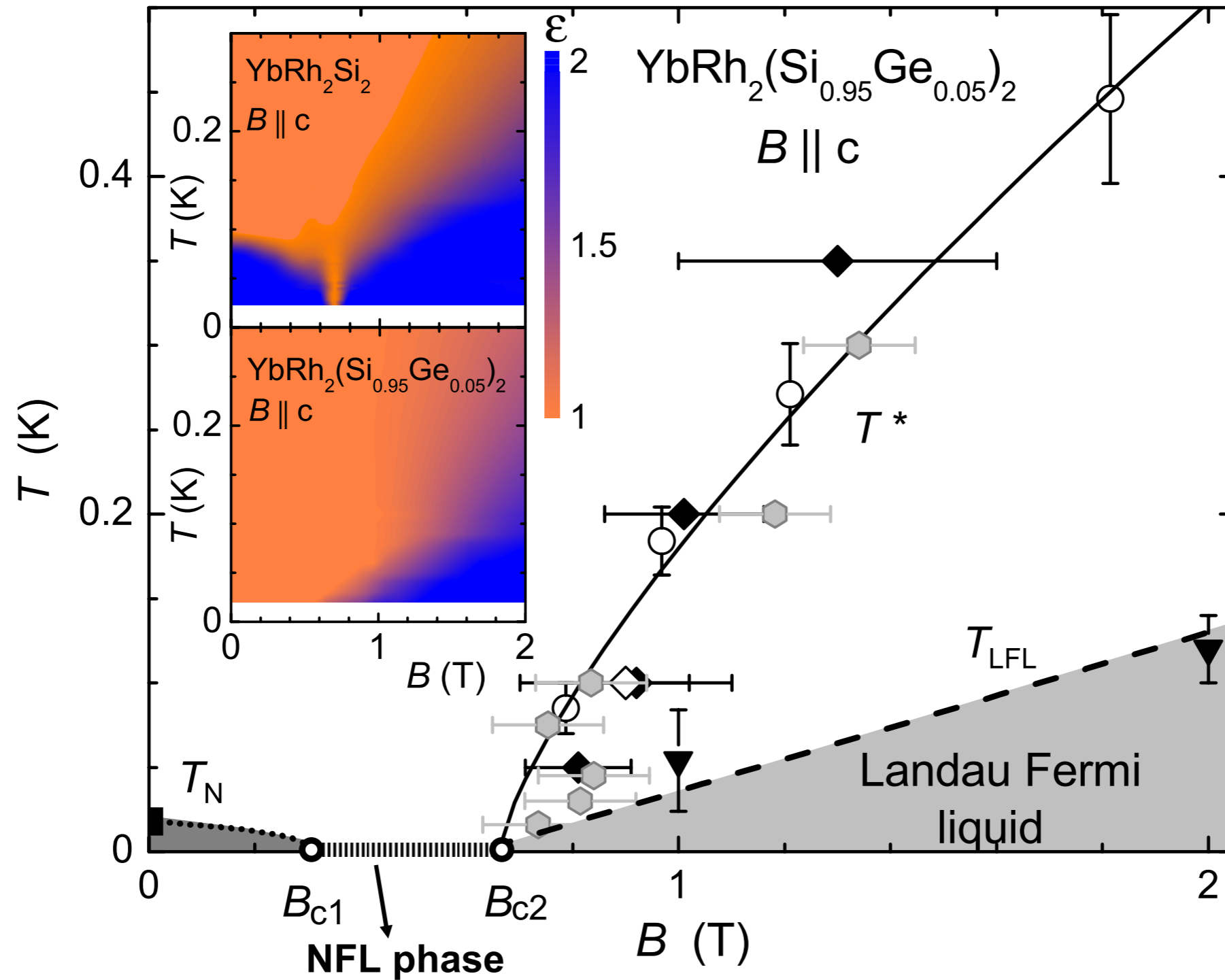




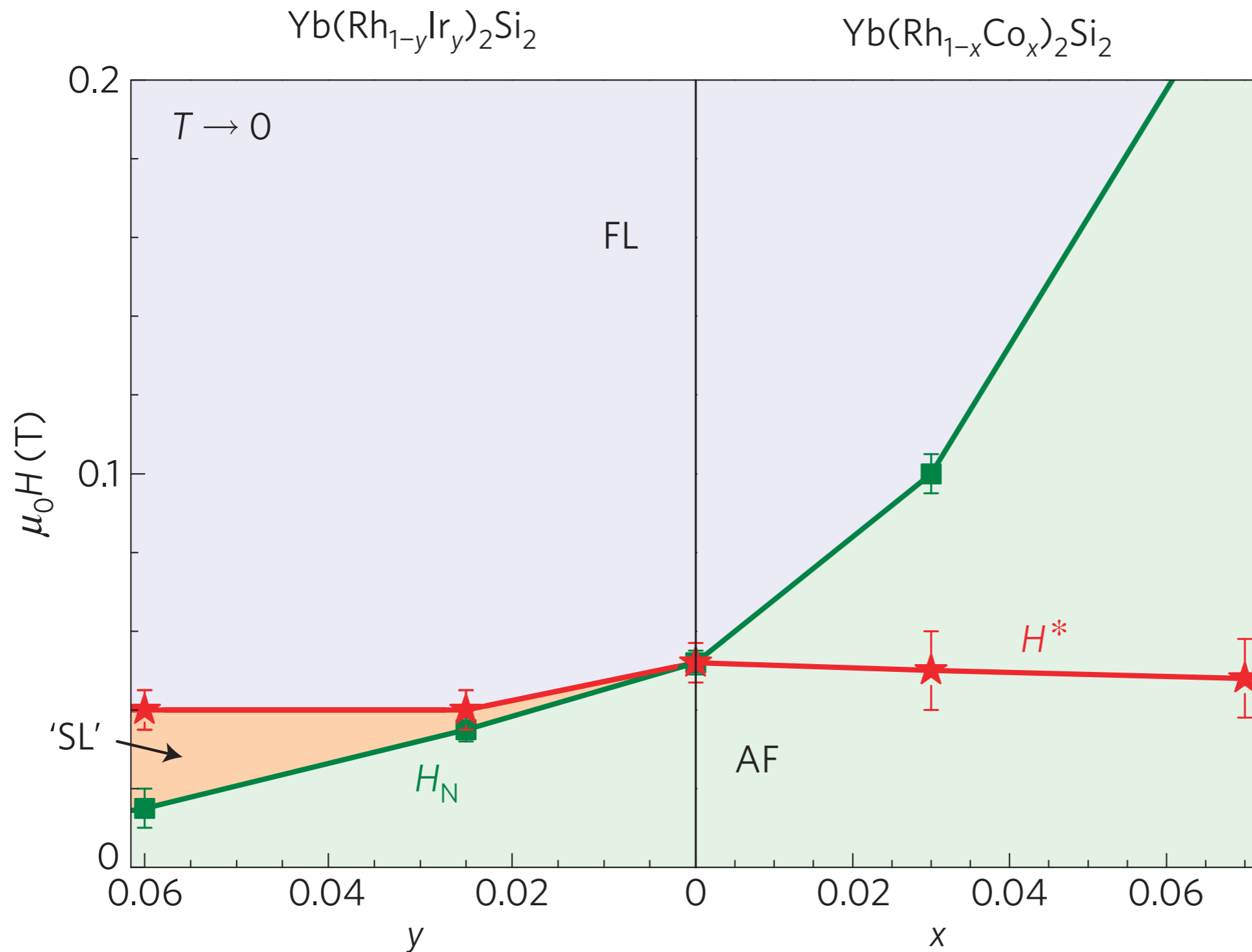
PHYSICAL REVIEW B **69**, 014415 (2004)

## Magnetic field induced non-Fermi-liquid behavior in YbAgGe single crystals

S. L. Bud'ko,<sup>1</sup> E. Morosan,<sup>1,2</sup> and P. C. Canfield<sup>1,2</sup>



J. Custers, P. Gegenwart, C. Geibel, F. Steglich, P. Coleman, and S. Paschen,  
*Phys. Rev. Lett.* **104**, 186402 (2010)



# Detaching the antiferromagnetic quantum critical point from the Fermi-surface reconstruction in YbRh<sub>2</sub>Si<sub>2</sub>

Nature Physics 5, 465 (2009)

S. Friedemann<sup>1\*</sup>, T. Westerkamp<sup>1</sup>, M. Brando<sup>1</sup>, N. Oeschler<sup>1</sup>, S. Wirth<sup>1</sup>, P. Gegenwart<sup>1,2</sup>, C. Krellner<sup>1</sup>, C. Geibel<sup>1</sup> and F. Steglich<sup>1\*</sup>

# Characteristics of FL\* phase

- Fermi surface volume does not count all electrons.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

## Characteristics of FL\* phase

- Fermi surface volume does not count all electrons.
- Such a phase *must* have neutral  $S = 1/2$  excitations (“spinons”), and collective spinless gauge excitations (“topological” order).

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

## Characteristics of FL\* phase

- Fermi surface volume does not count all electrons.
- Such a phase *must* have neutral  $S = 1/2$  excitations (“spinons”), and collective spinless gauge excitations (“topological” order).
- These topological excitations are needed to account for the deficit in the Fermi surface volume, in M. Oshikawa’s proof of the Luttinger theorem.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

## Conclusions

All quantum phase transitions of metals  
in two spatial dimensions  
involving symmetry breaking  
are strongly-coupled  
and  
and very different from the  
“Stoner” mean field theory

# Conclusions

There is an instability of  
*universal* strength to  
unconventional superconductivity  
near the onset of antiferromagnetism  
in a two-dimensional metal

# Conclusions

There can be an intermediate  
*non-Fermi liquid phase*  
between the two Fermi liquids:  
the antiferromagnetic metal  
with “small” Fermi surfaces  
and  
the metal with “large” Fermi surfaces

## Conclusions

This *non-Fermi liquid phase* has neutral  $S=1/2$  excitations, and “topological” gauge excitations, which account for deficits in the Luttinger count of the volume enclosed by the Fermi surfaces