Quantum entanglement and the phases of matter

Twelfth Arnold Sommerfeld Lecture Series
January 31 - February 3, 2012

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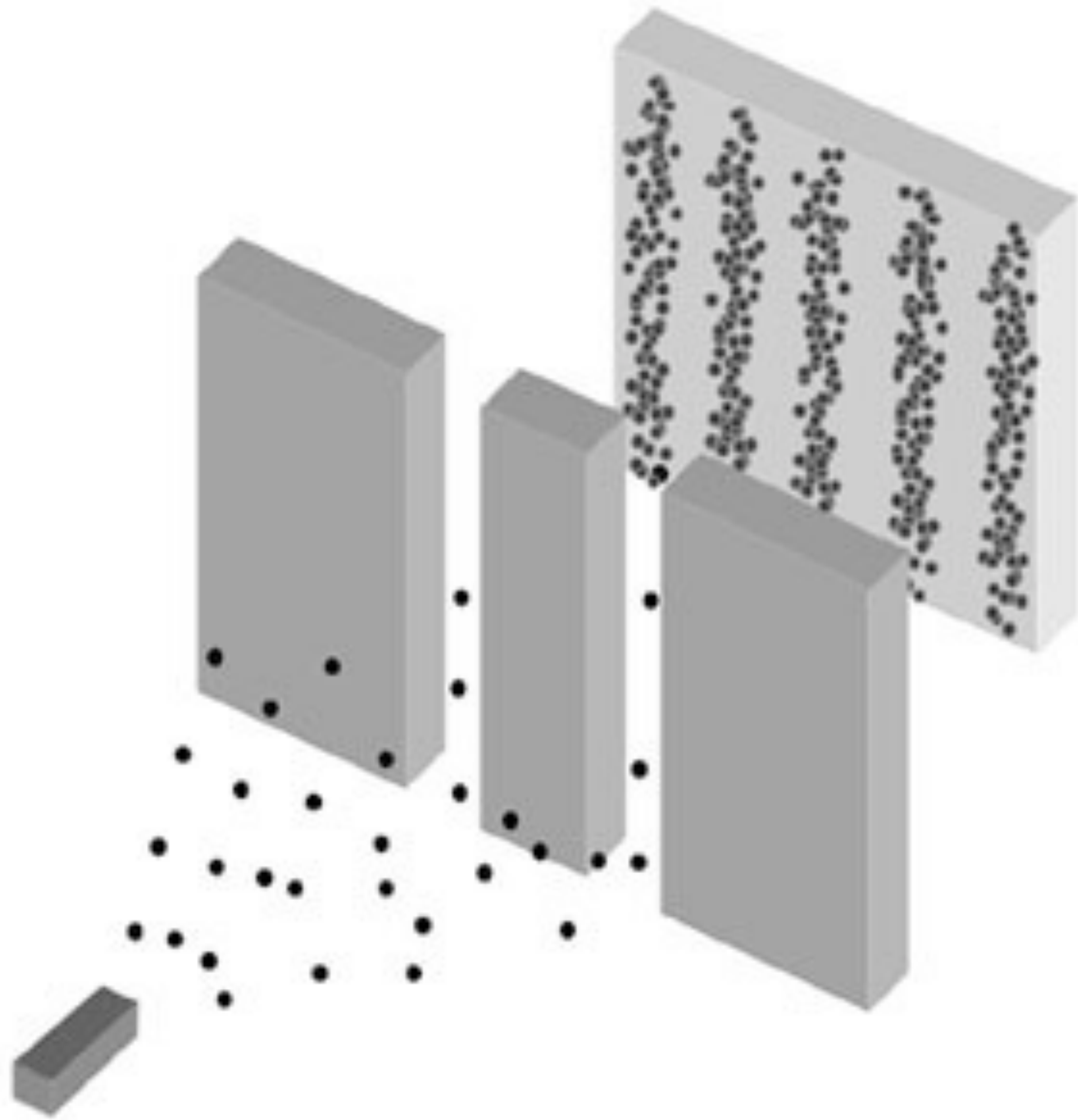
Quantum superposition and entanglement

Quantum Superposition

The double slit experiment



Interference of electrons

Quantum Superposition

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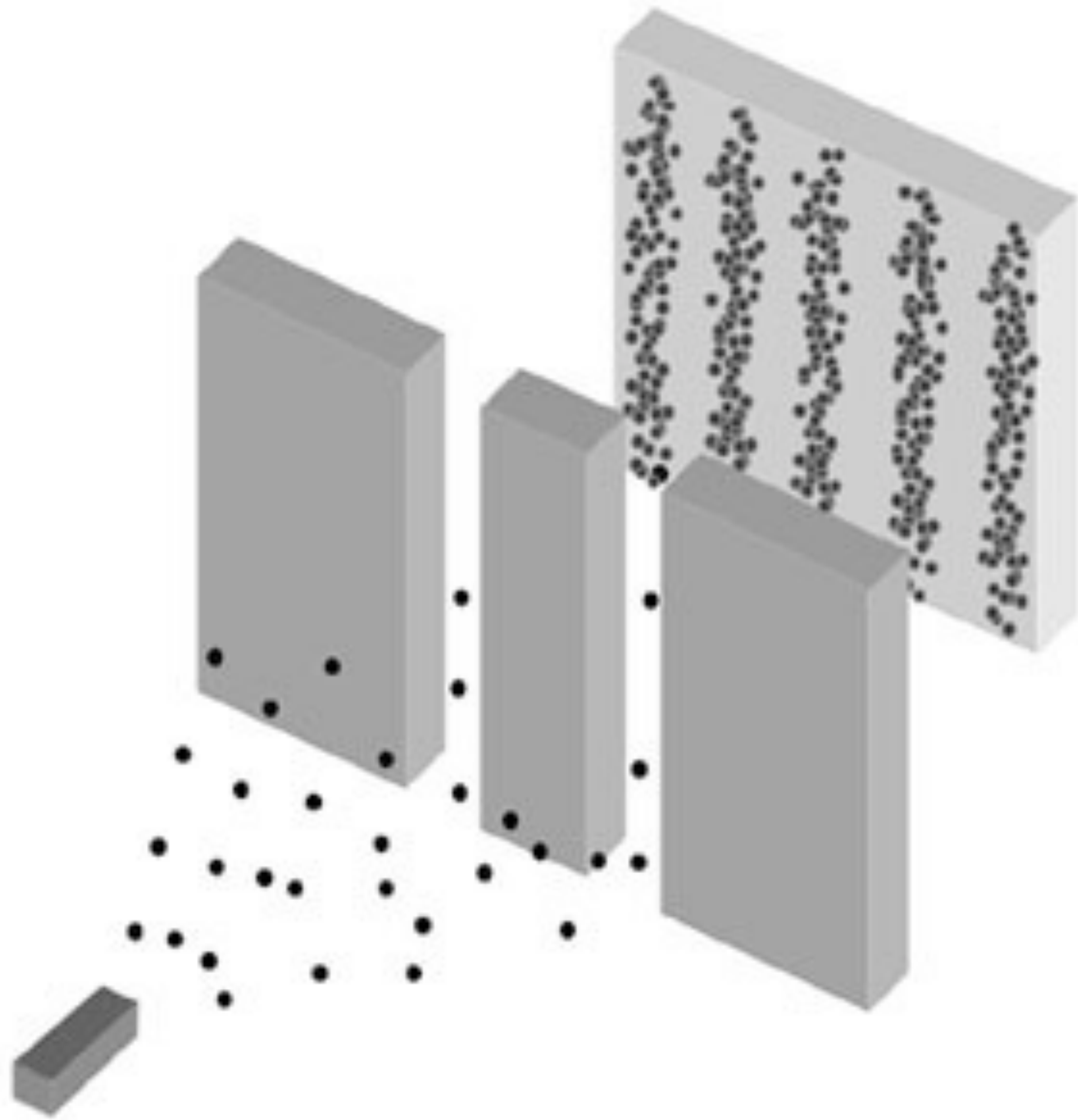

Each electron passes through both slits

Interference of electrons

Quantum Superposition

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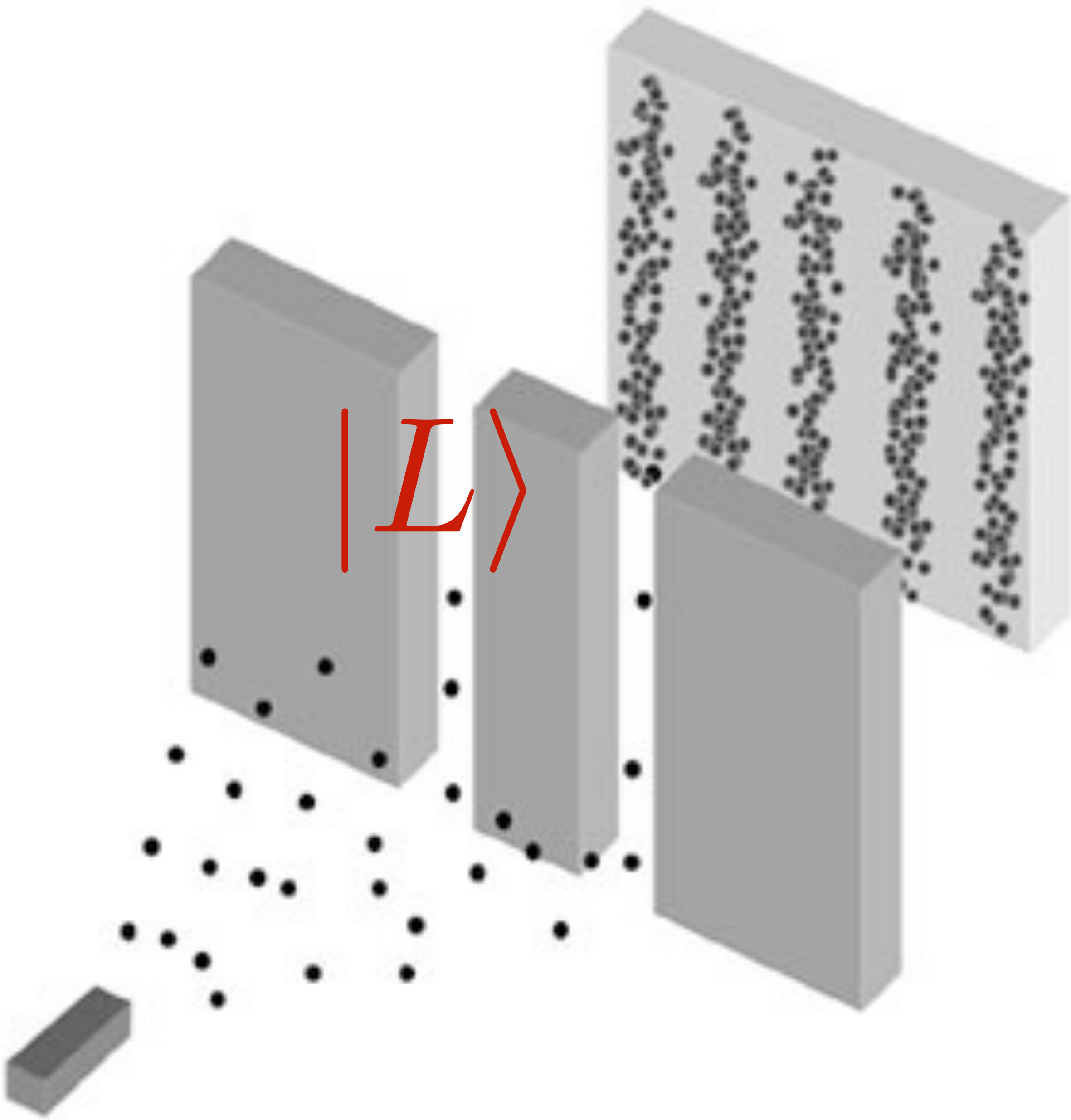
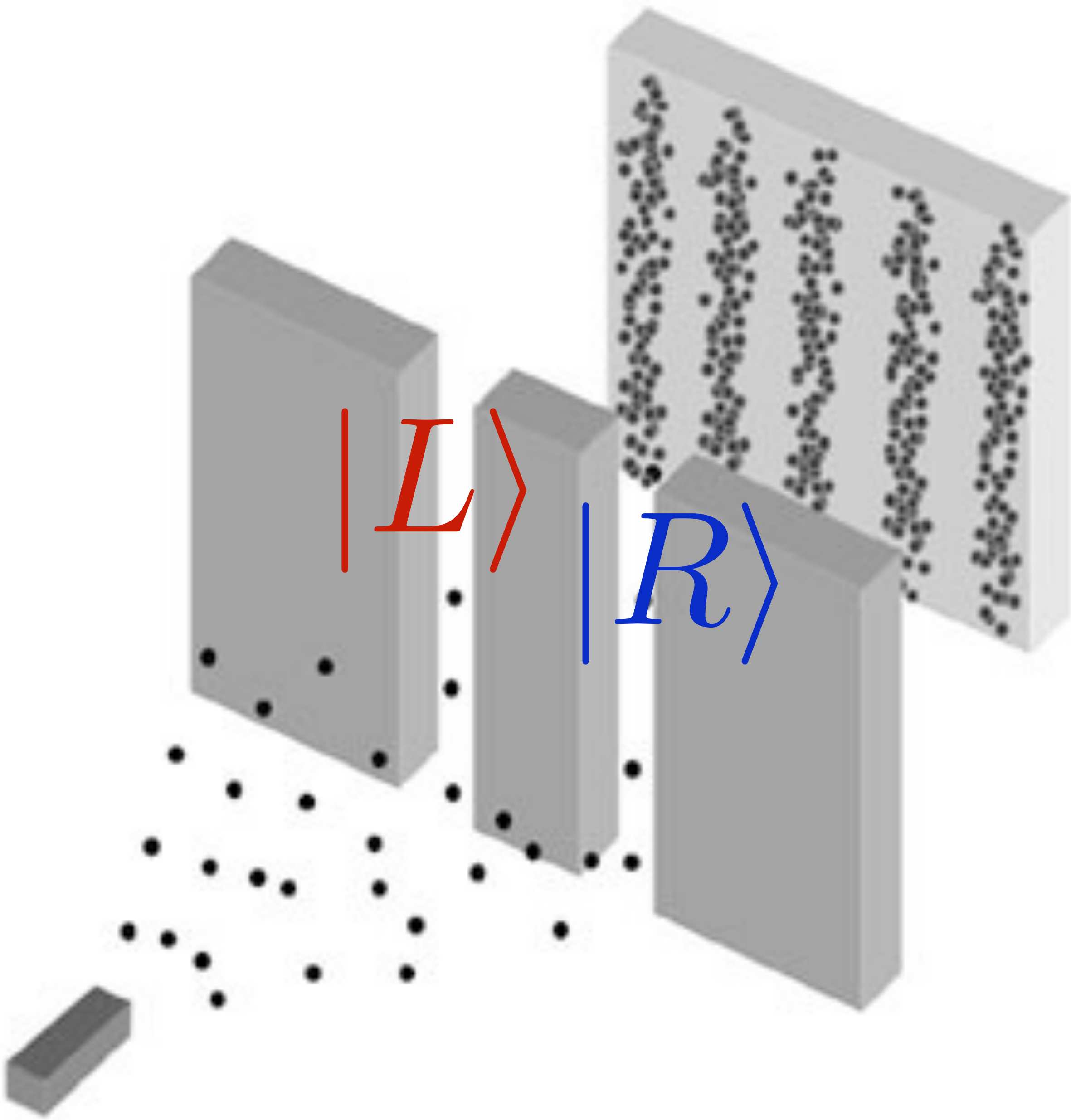


Let $|L\rangle$ represent the state with the electron in the left slit

Quantum Superposition

The double slit experiment

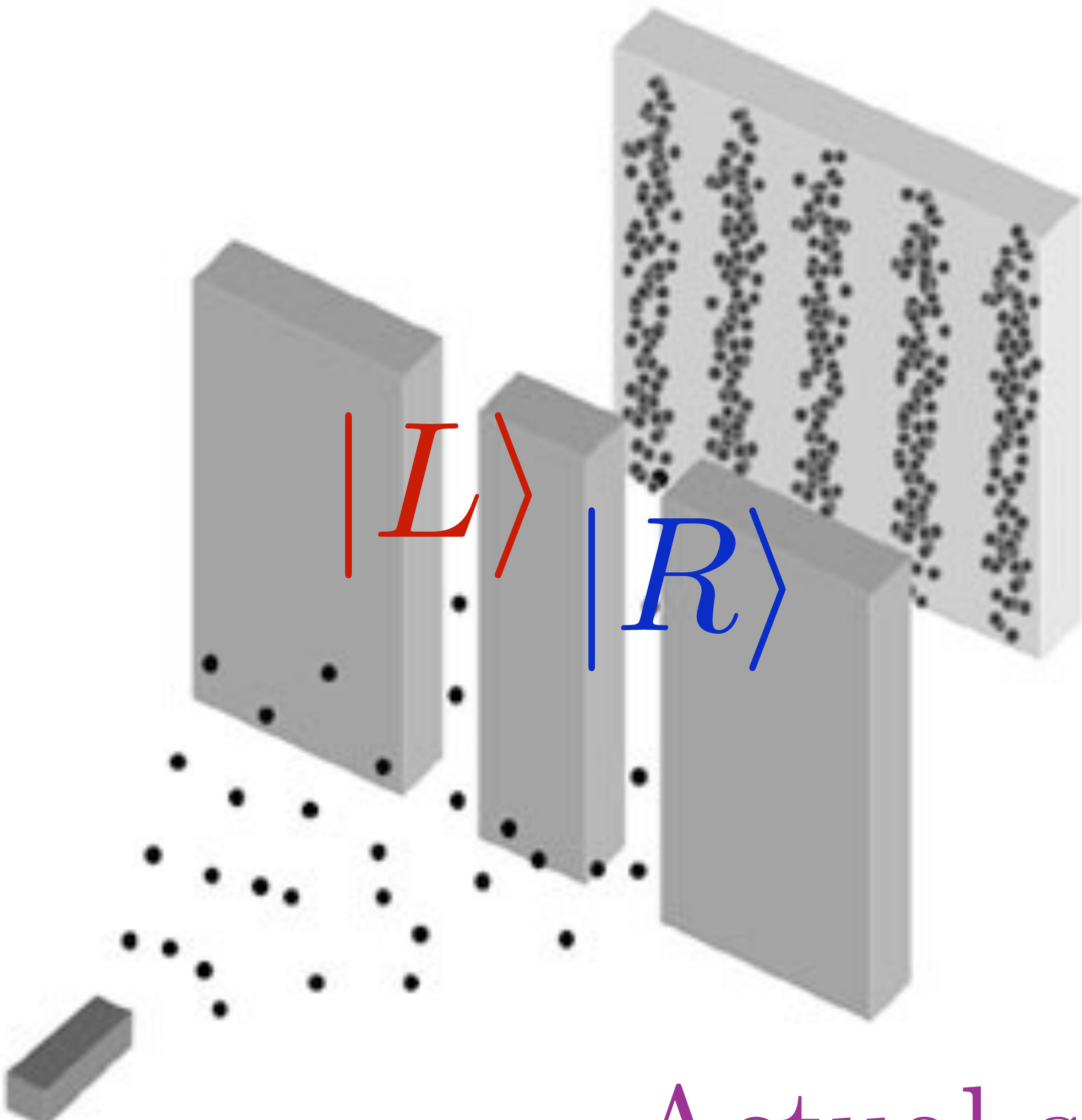


Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Quantum Superposition

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Actual state of the electron is

$$|L\rangle + |R\rangle$$

Quantum Entanglement: quantum superposition with more than one particle

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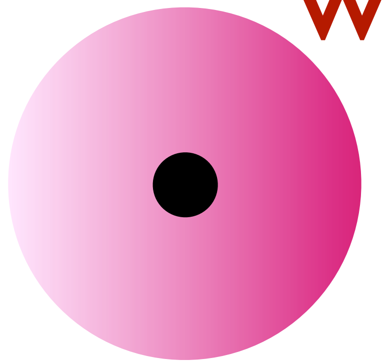
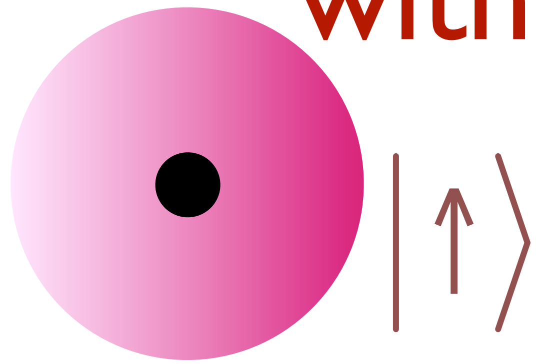
Hydrogen atom:



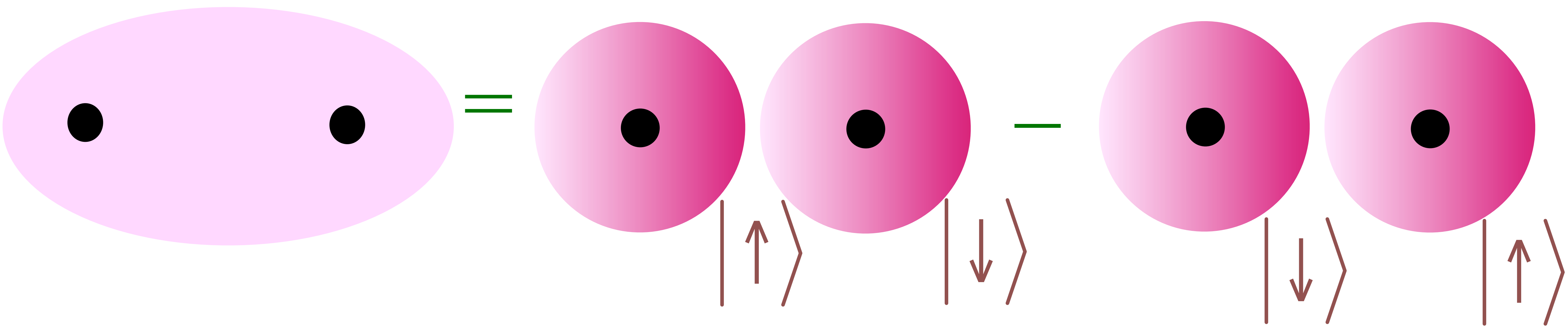
$$|\uparrow\rangle$$

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:



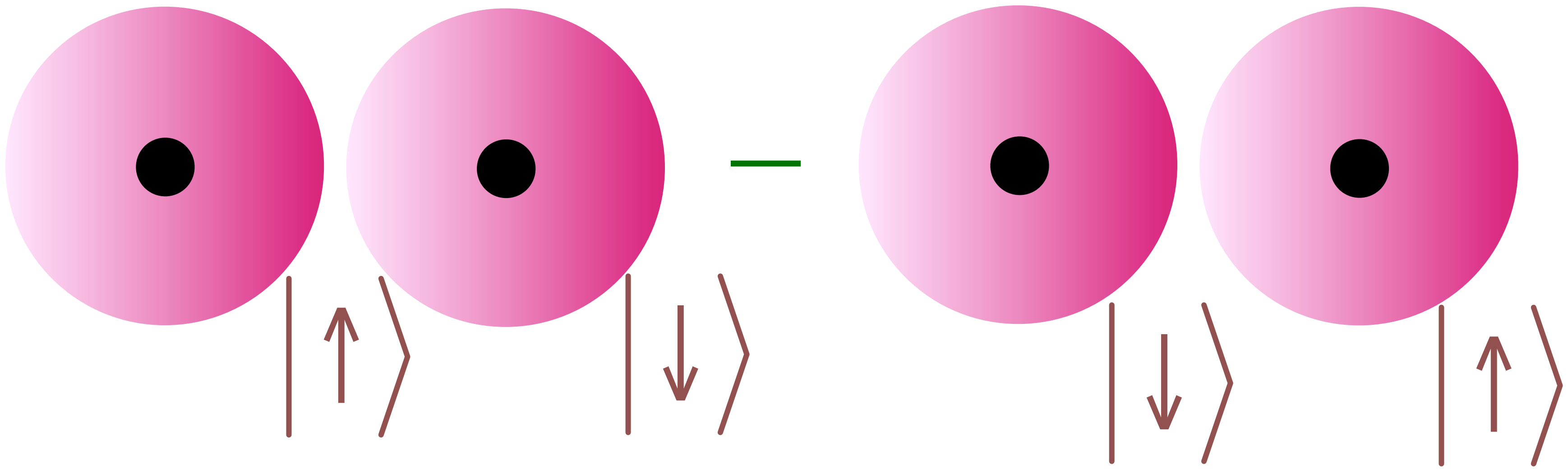
Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

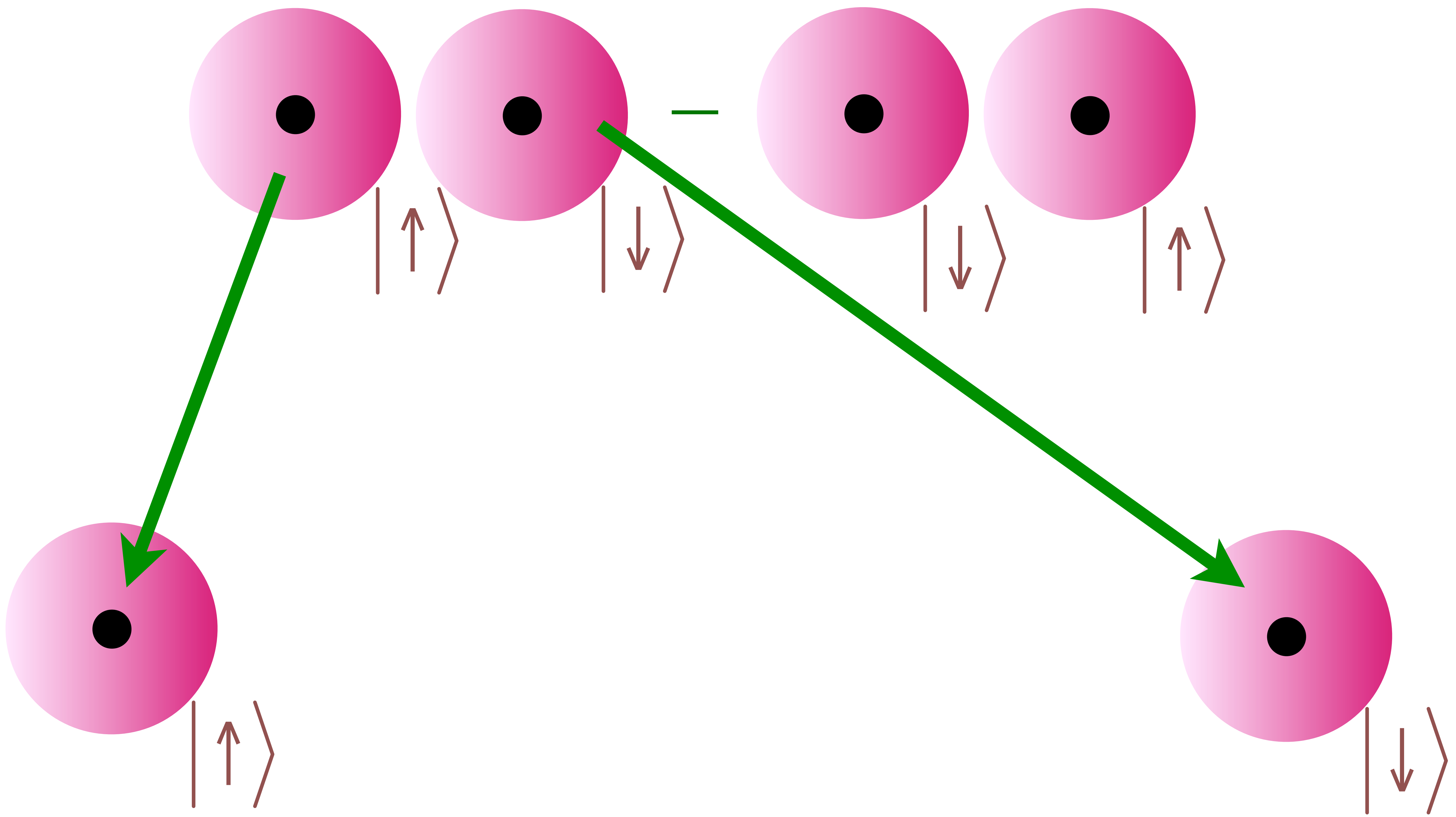
Superposition of two electron states leads to non-local correlations between spins

Quantum Entanglement: quantum superposition with more than one particle



$$|\uparrow\rangle \; |\downarrow\rangle \; - \; |\downarrow\rangle \; |\uparrow\rangle$$

Quantum Entanglement: quantum superposition with more than one particle



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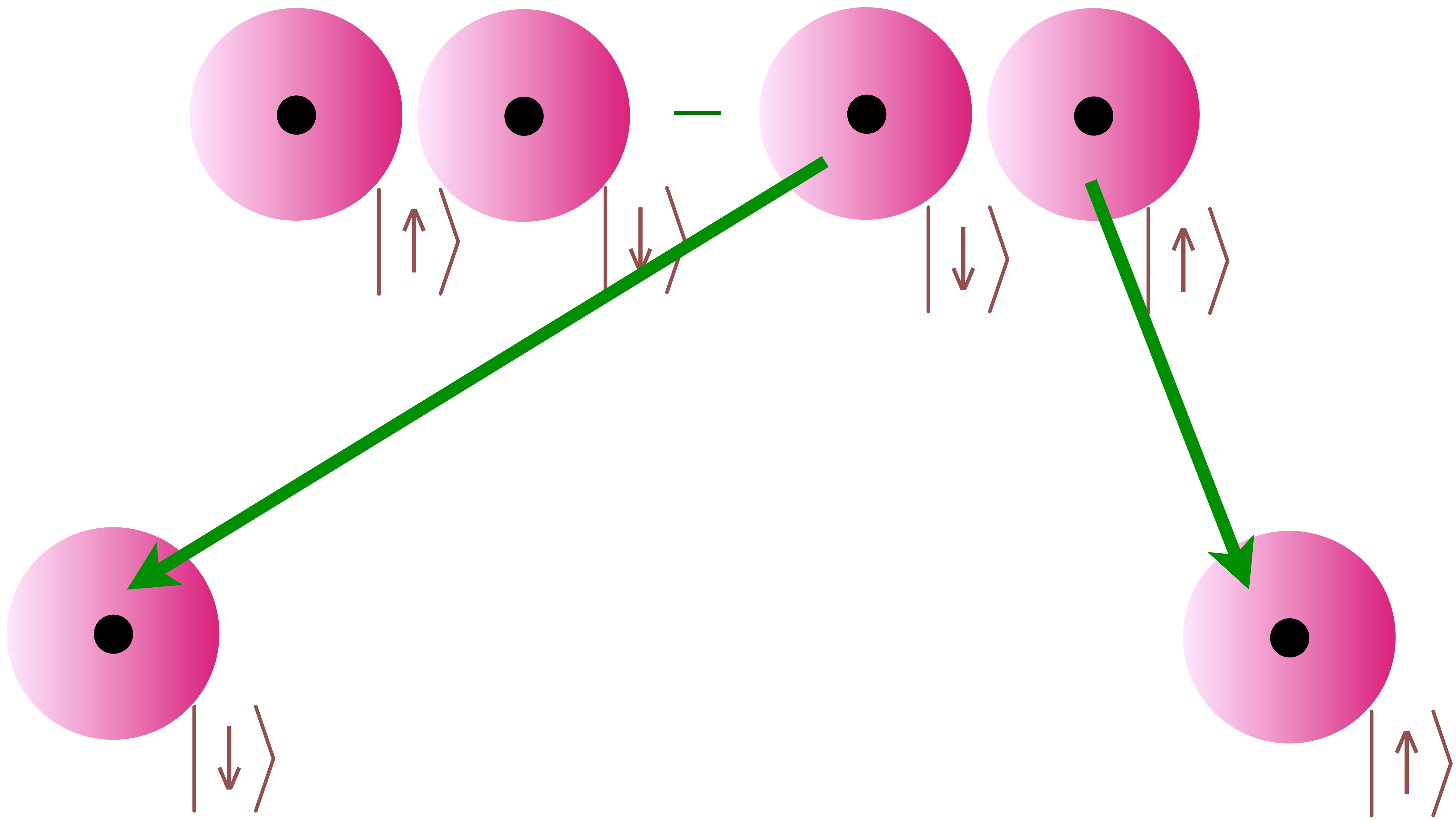

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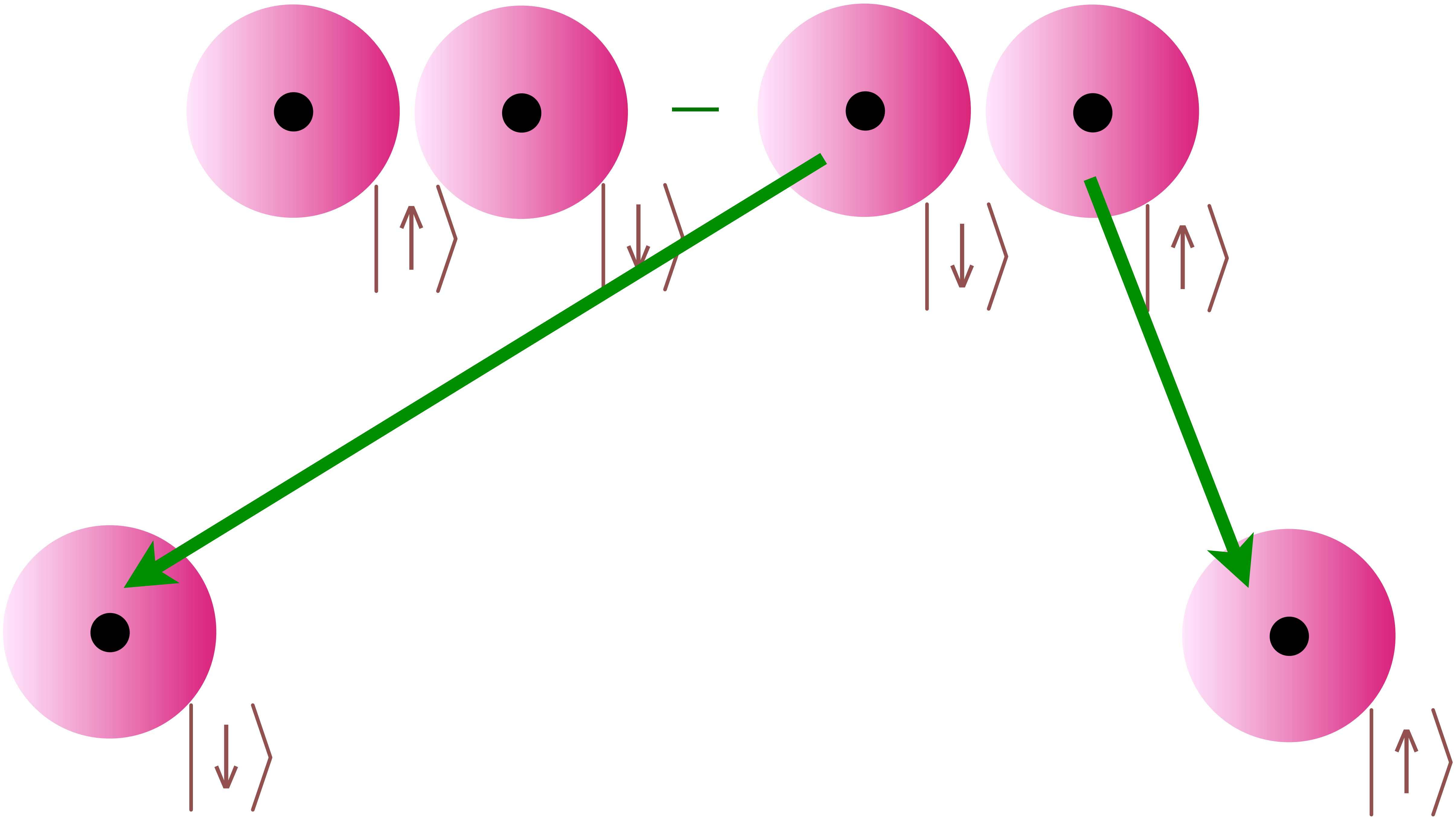


Einstein-Podolsky-Rosen "paradox": Non-local correlations between observations arbitrarily far apart

Quantum superposition and entanglement

Quantum superposition and entanglement

Quantum critical points of electrons in crystals

String theory and black holes

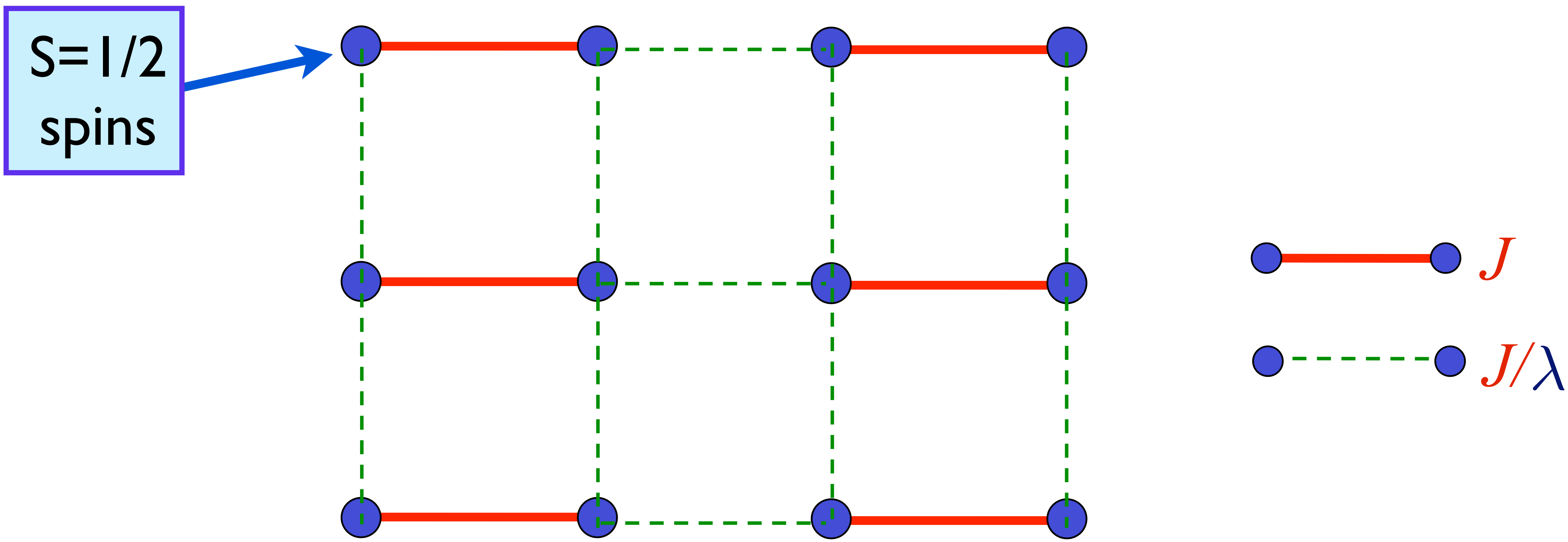
Quantum superposition and entanglement

Quantum critical points of electrons in crystals

String theory and black holes

Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

Spinning electrons localized on a square lattice

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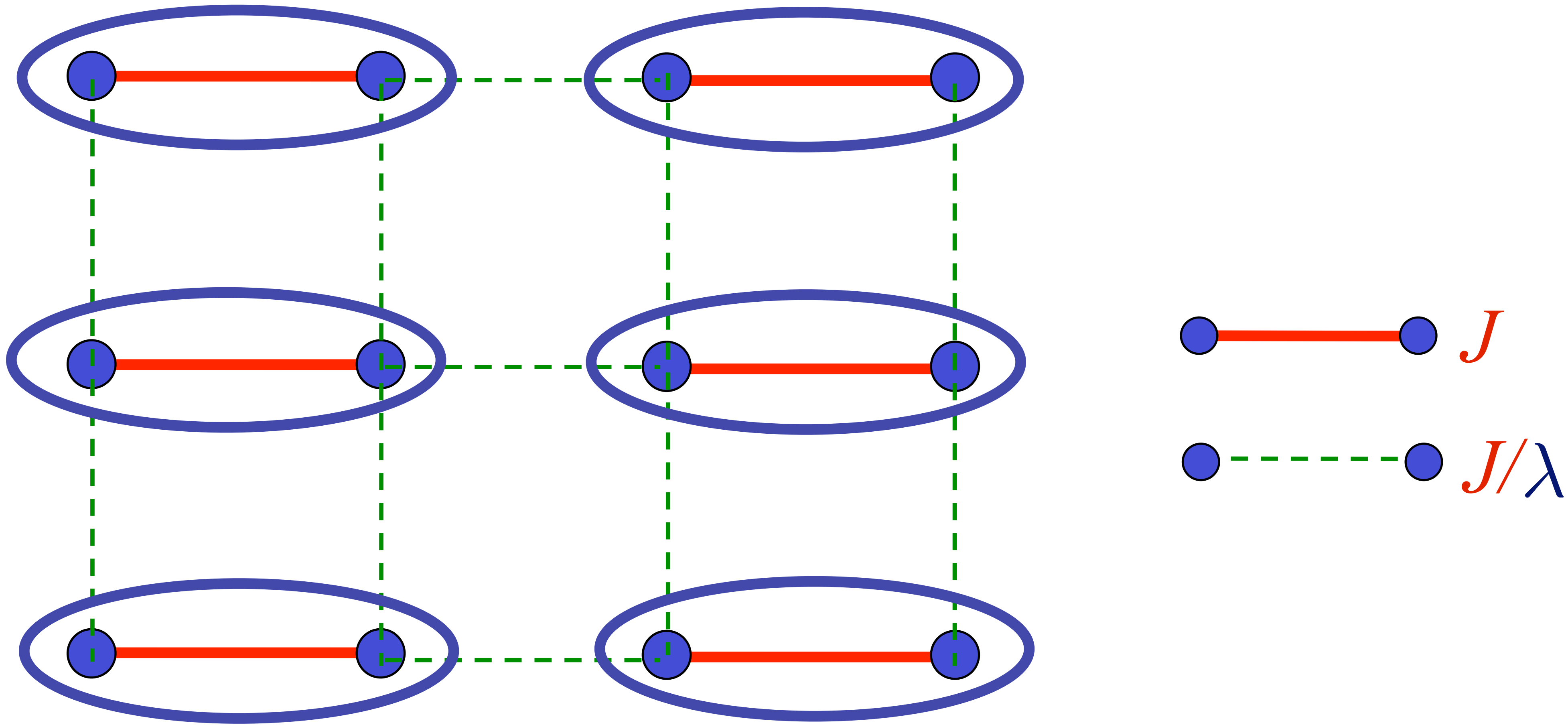

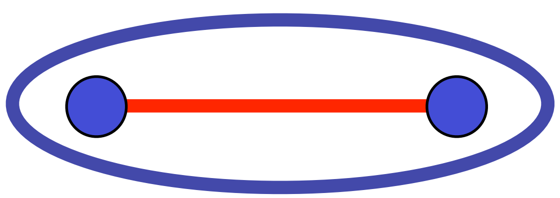

$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

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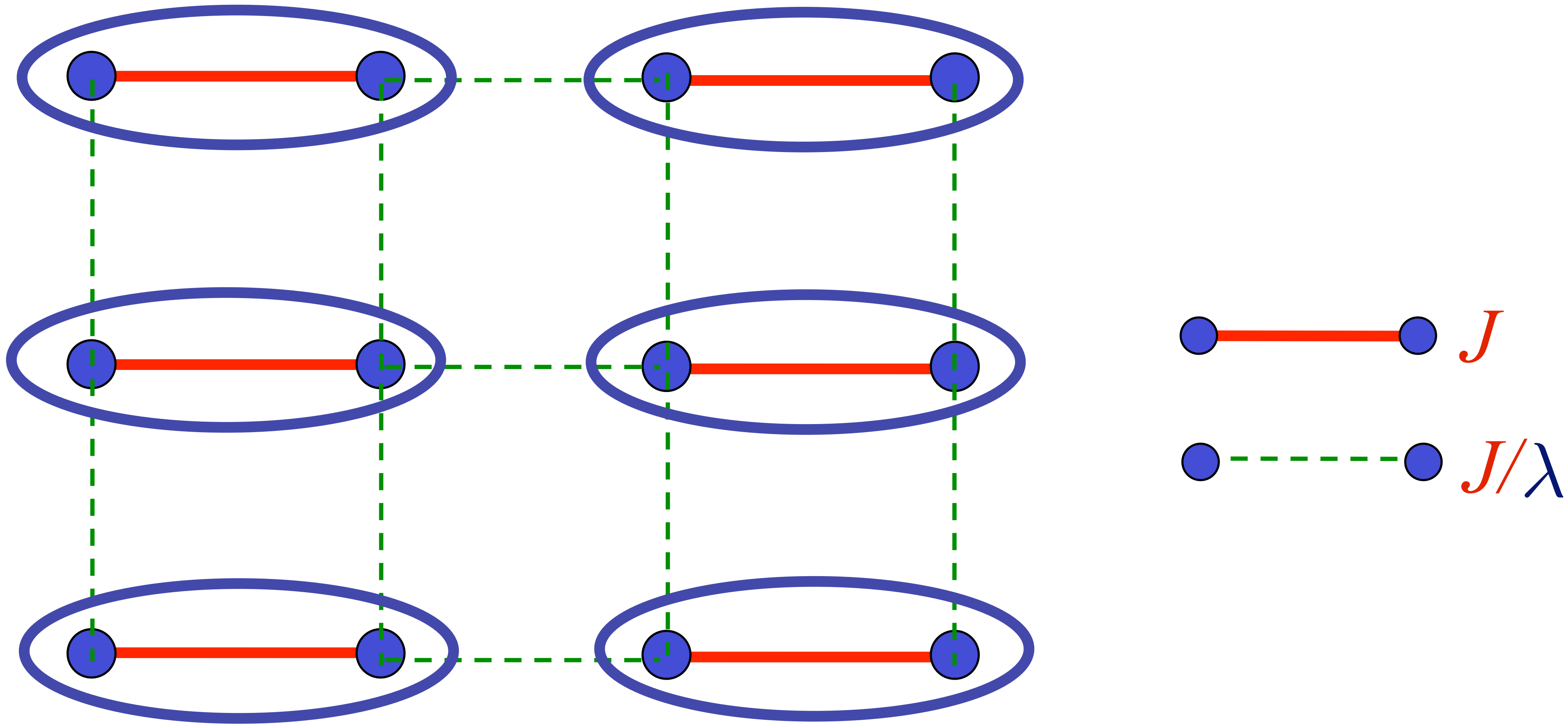
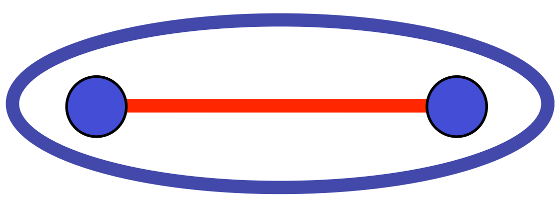


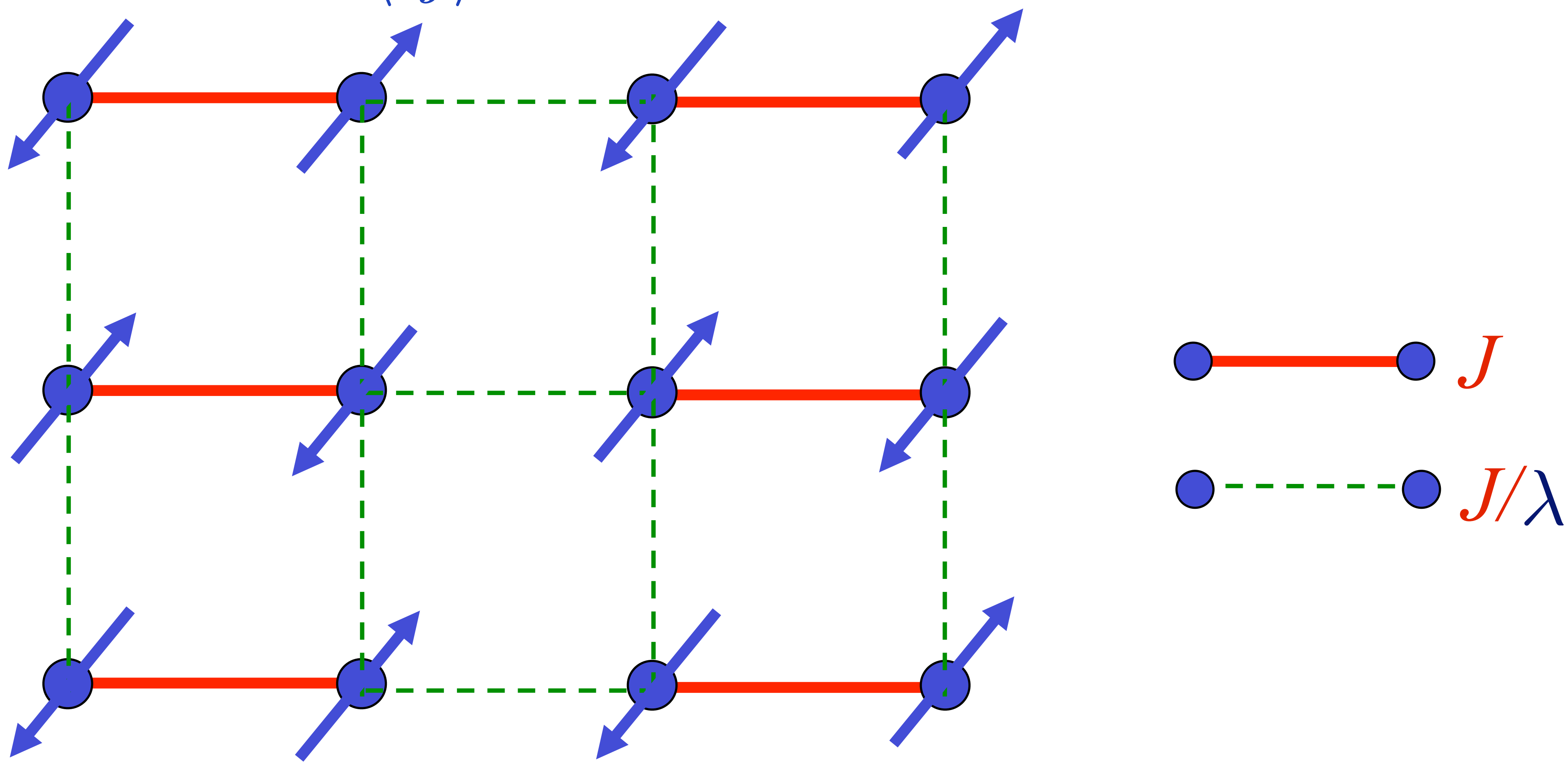


$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

Spinning electrons localized on a square lattice

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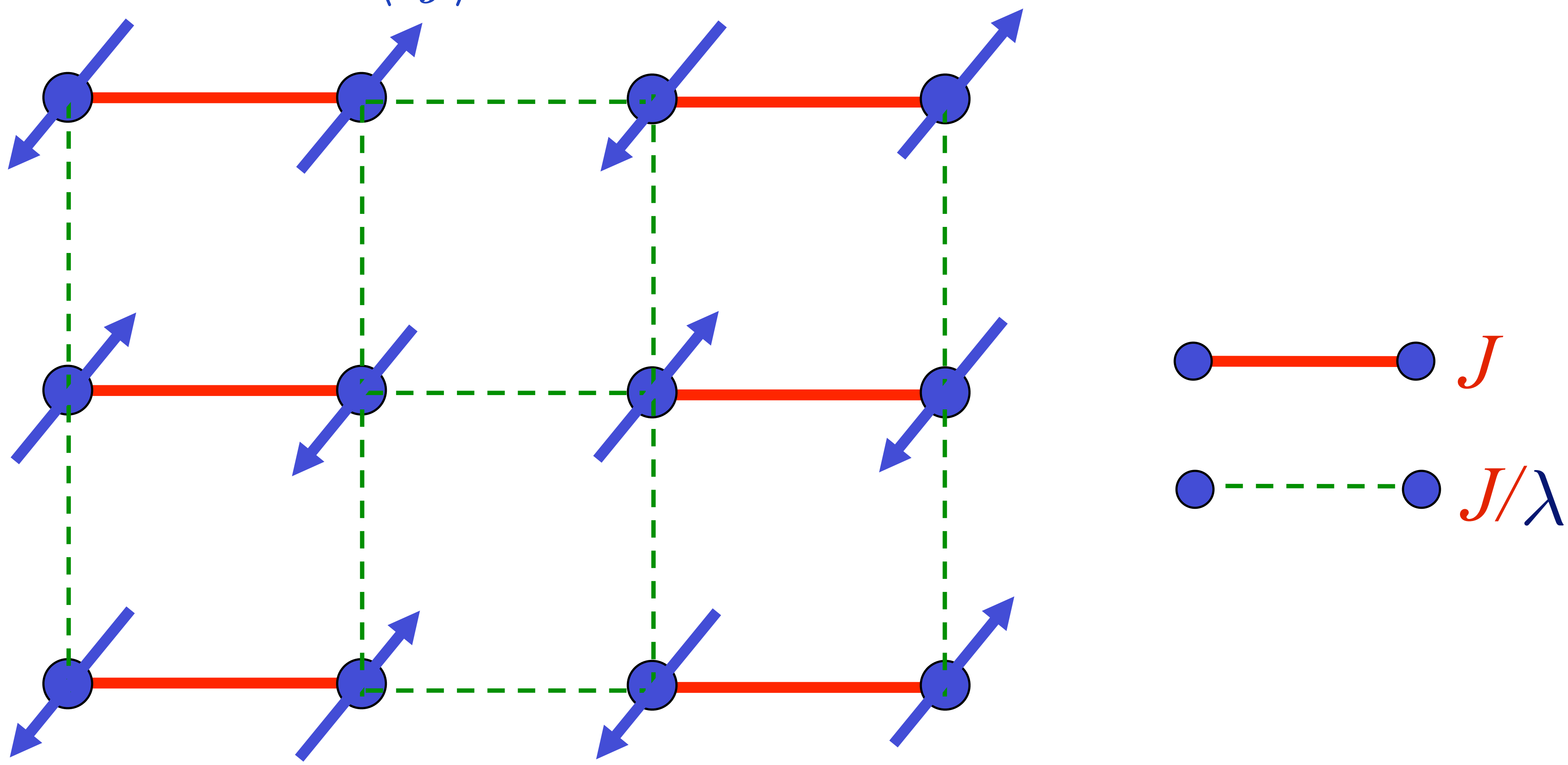

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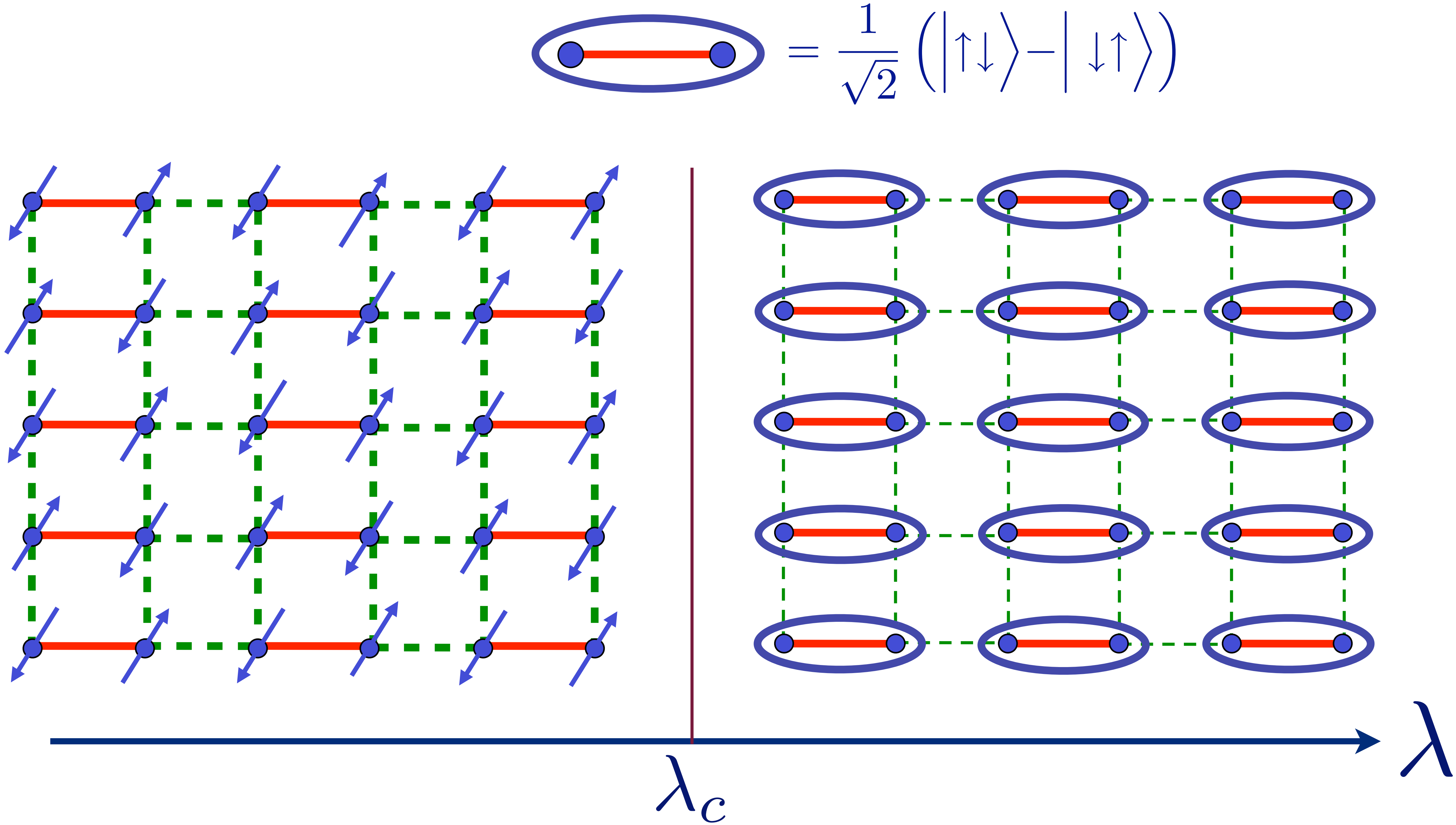
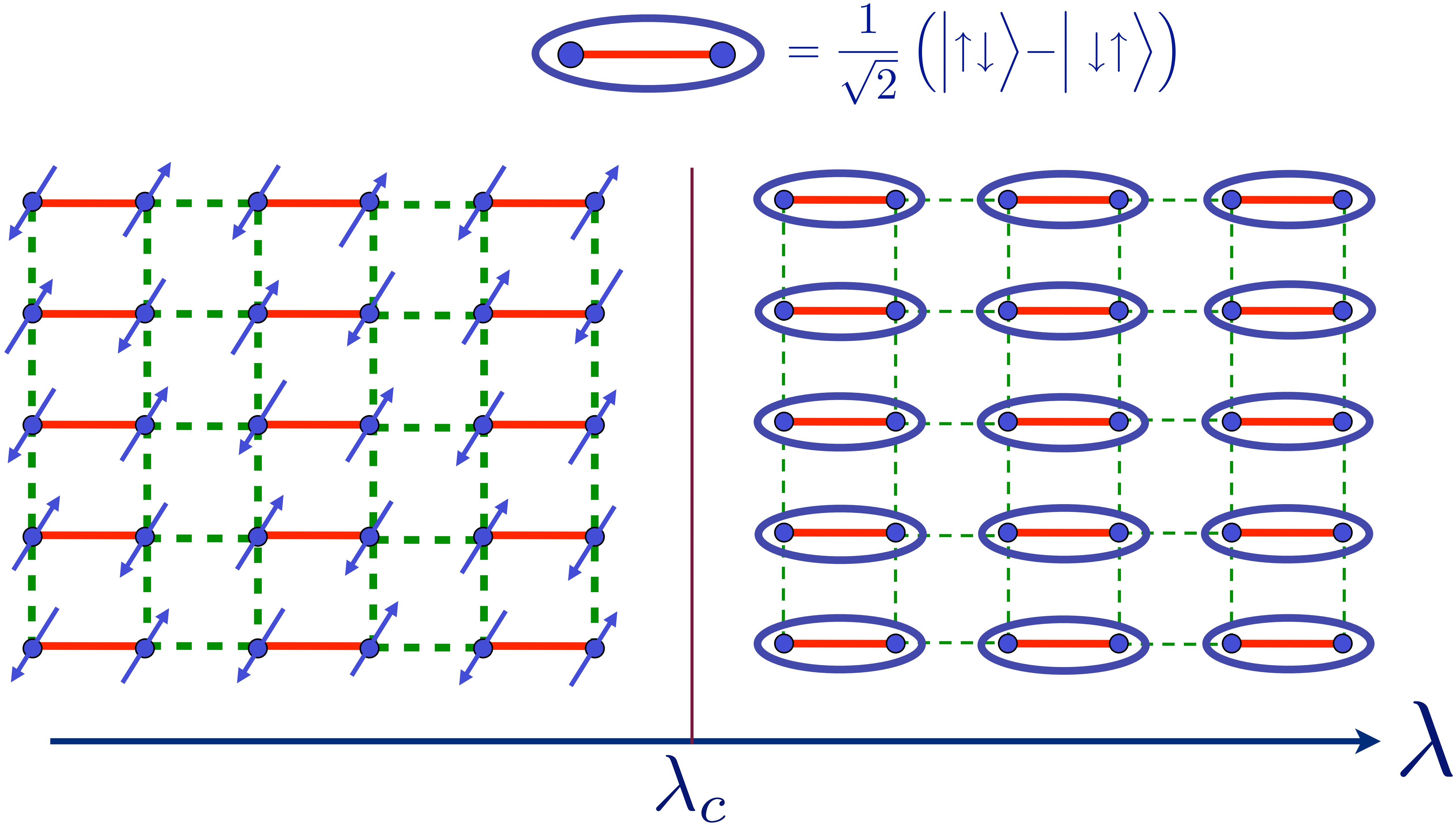
No EPR pairs

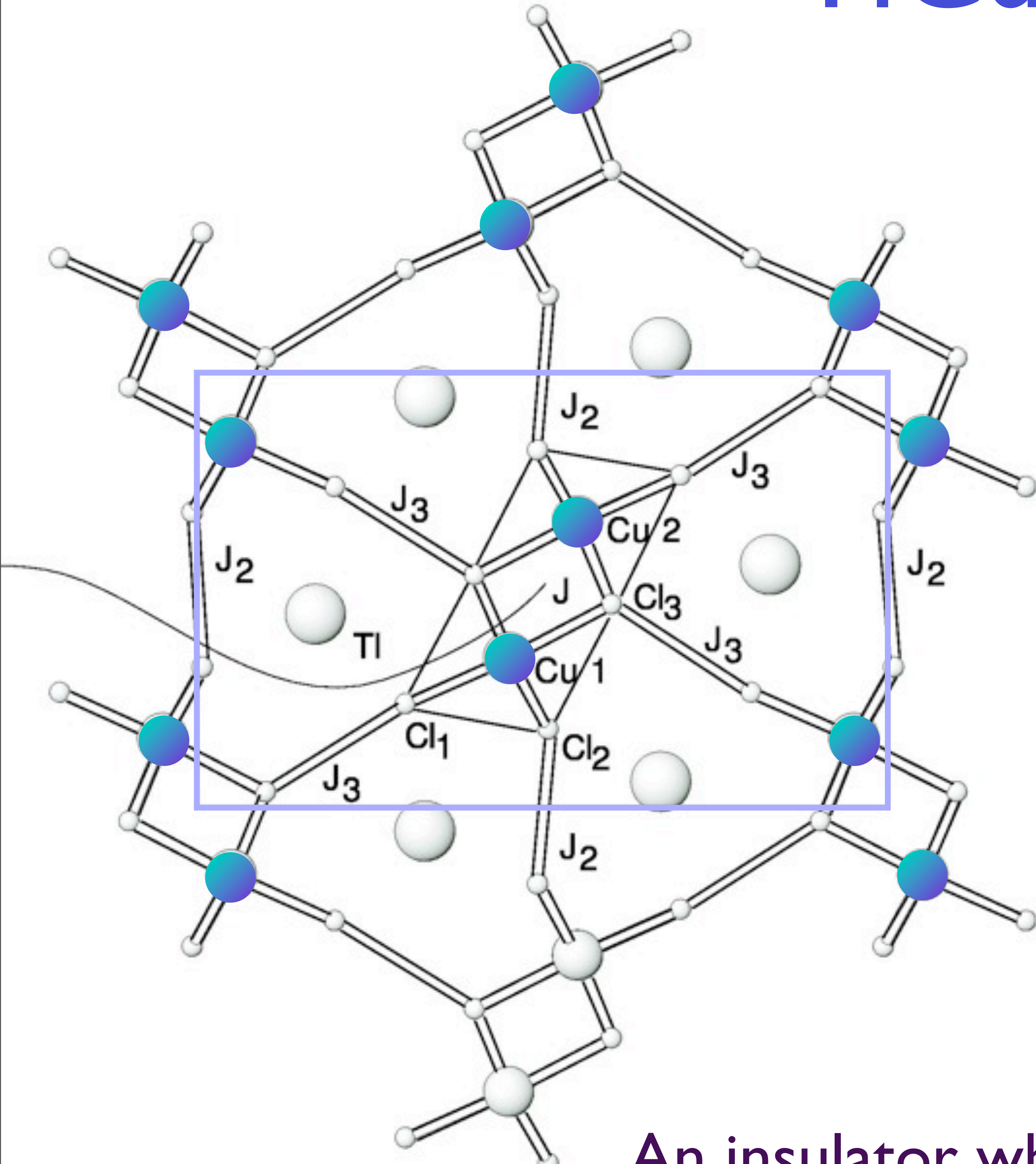

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ

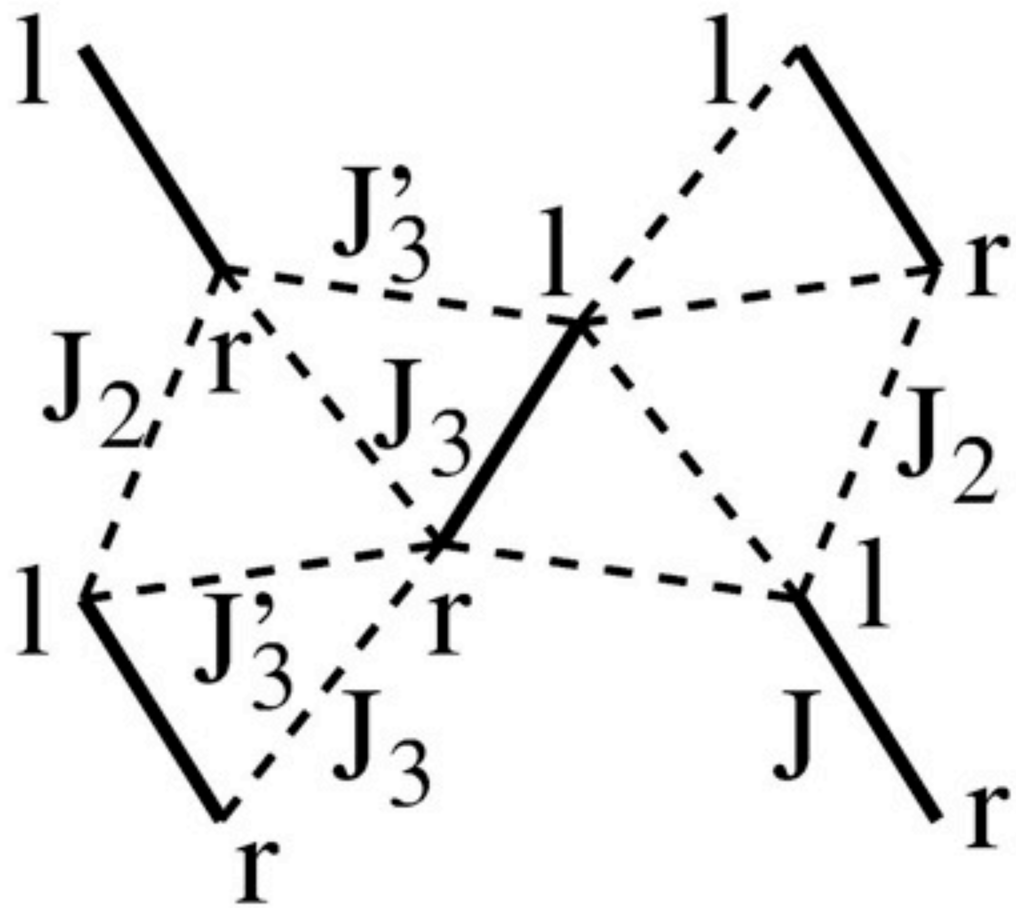


Pressure in $TlCuCl_3$

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

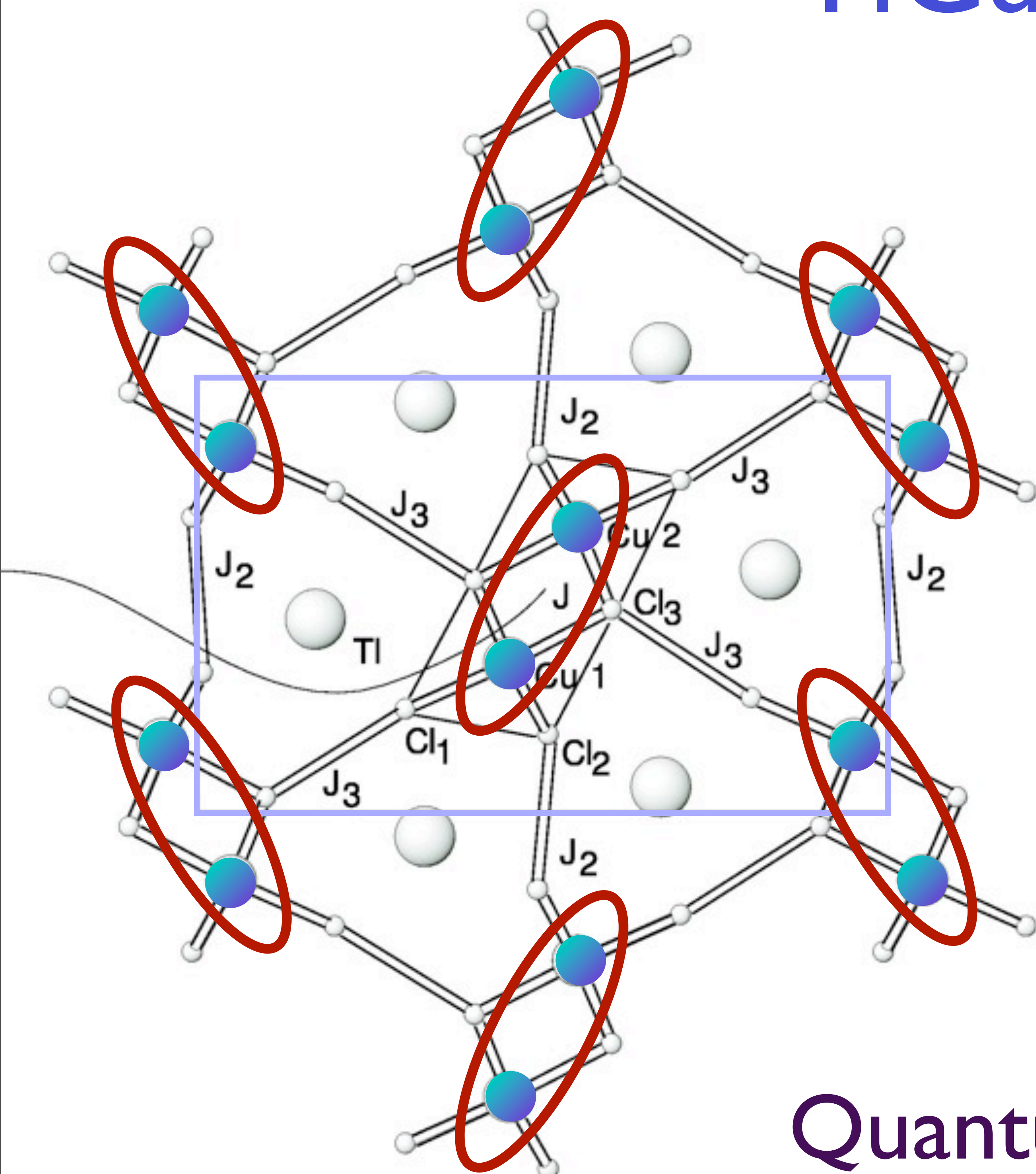
$TlCuCl_3$

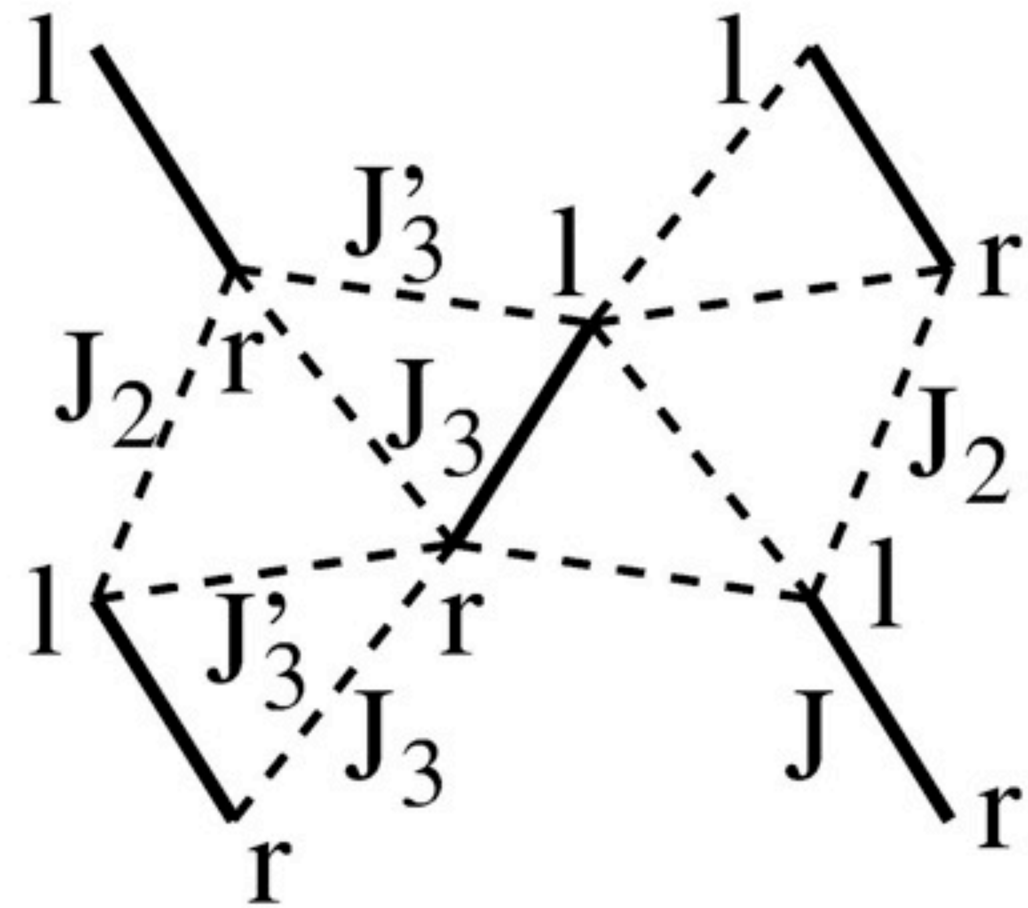




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

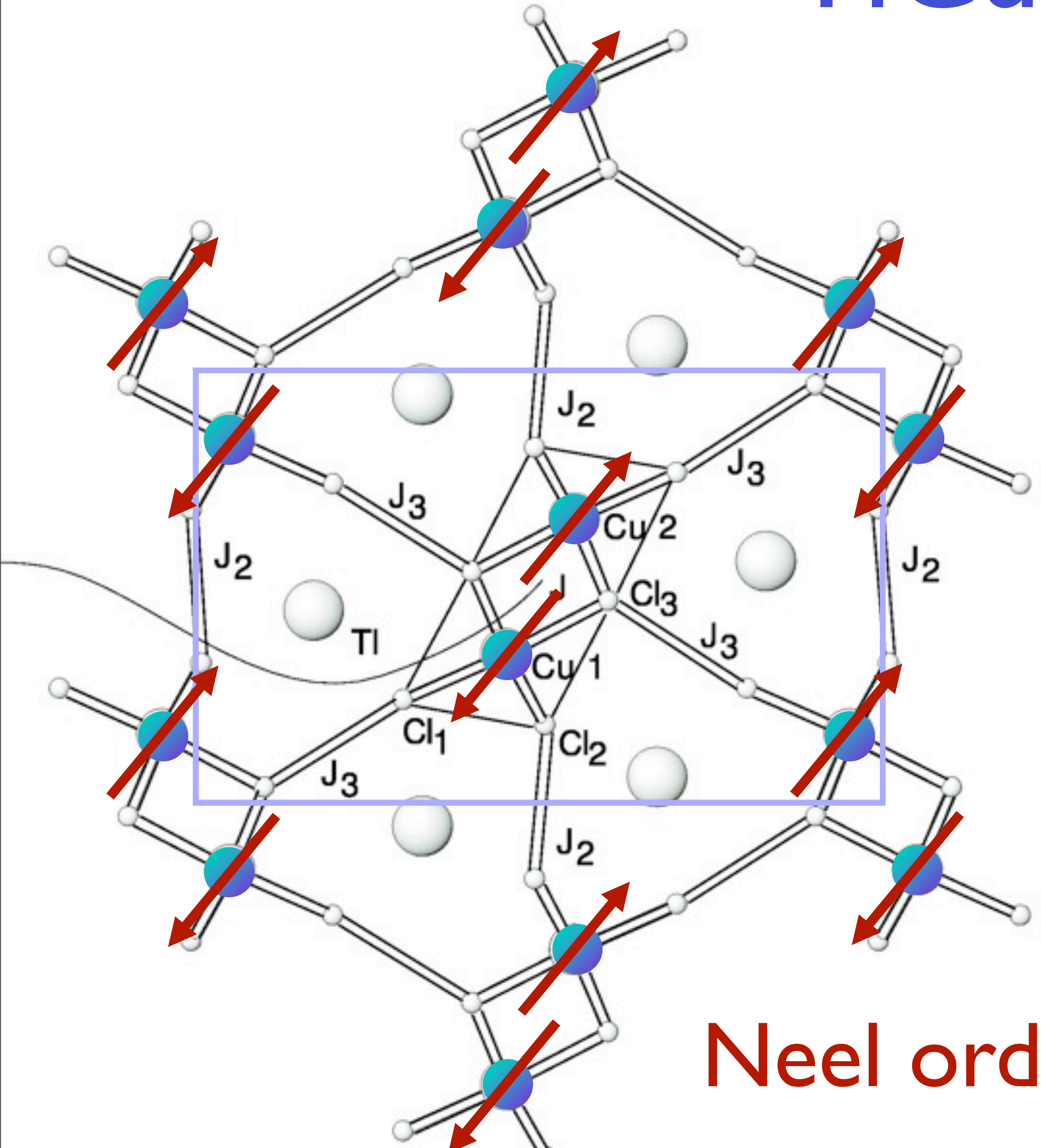
$TlCuCl_3$

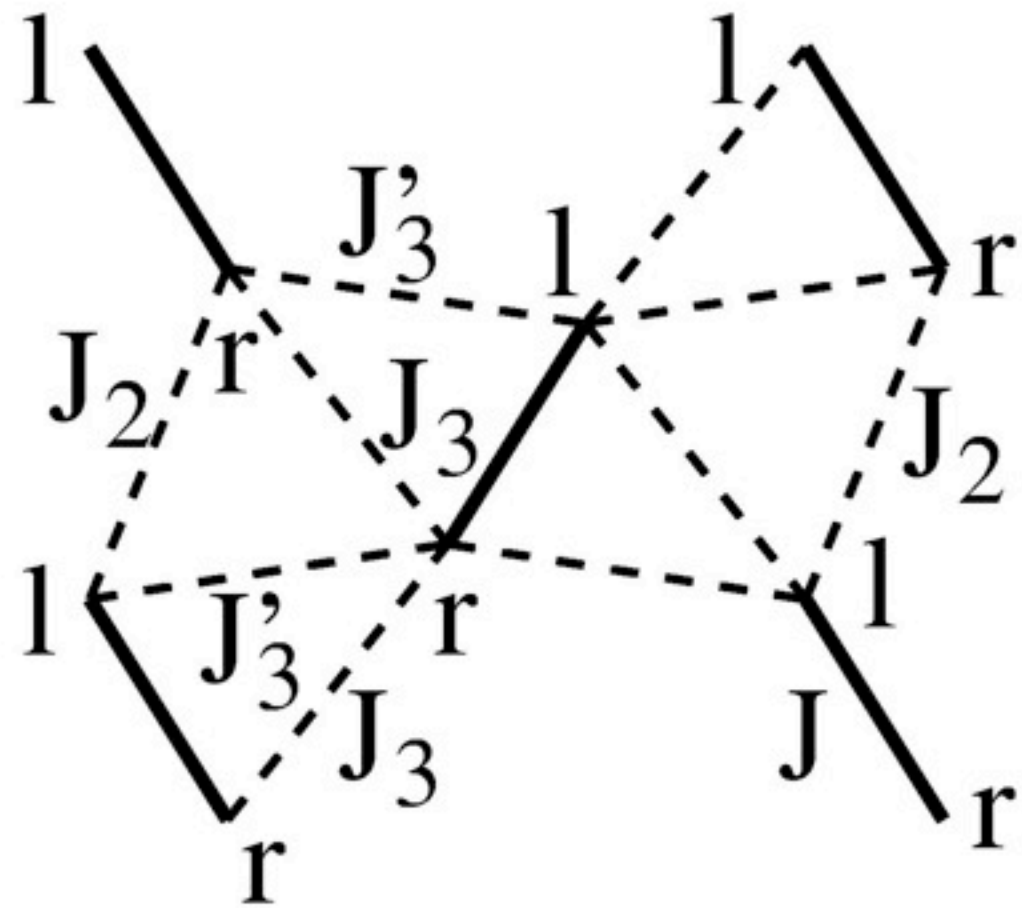




Quantum paramagnet at ambient pressure

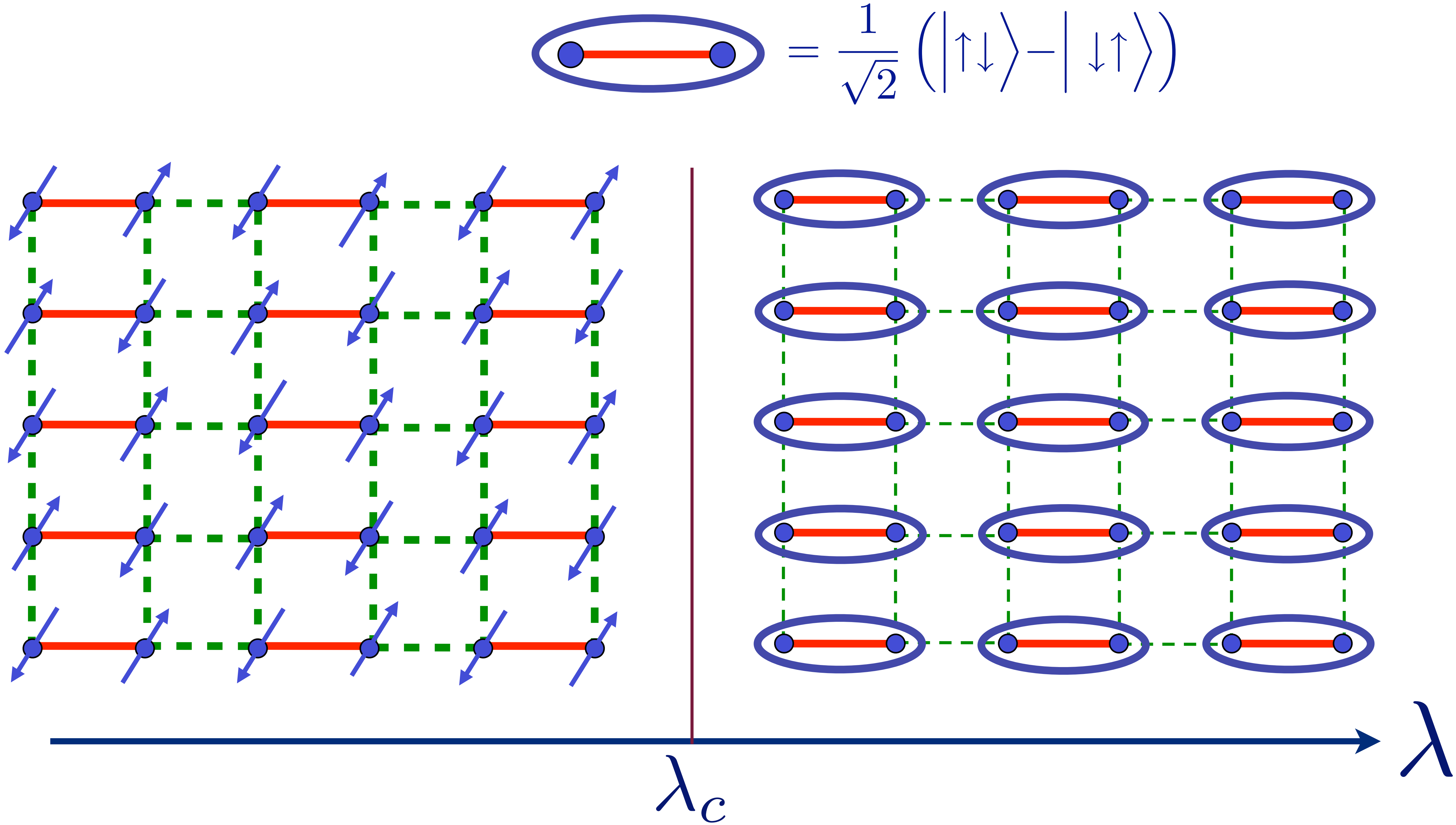
$TlCuCl_3$





Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).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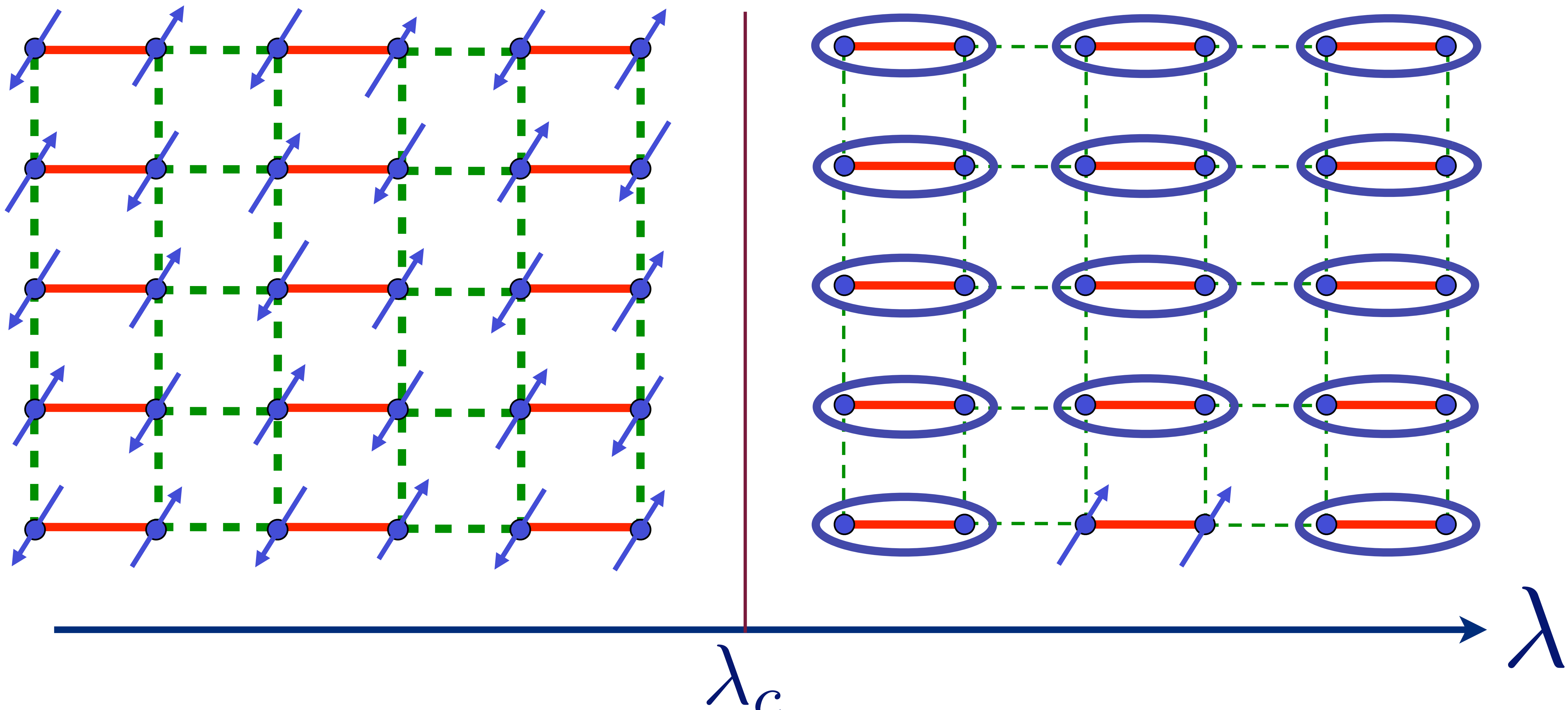
λ_c
λ

Excitation spectrum in the paramagnetic phase



Spin $S = 1$
"triplon"

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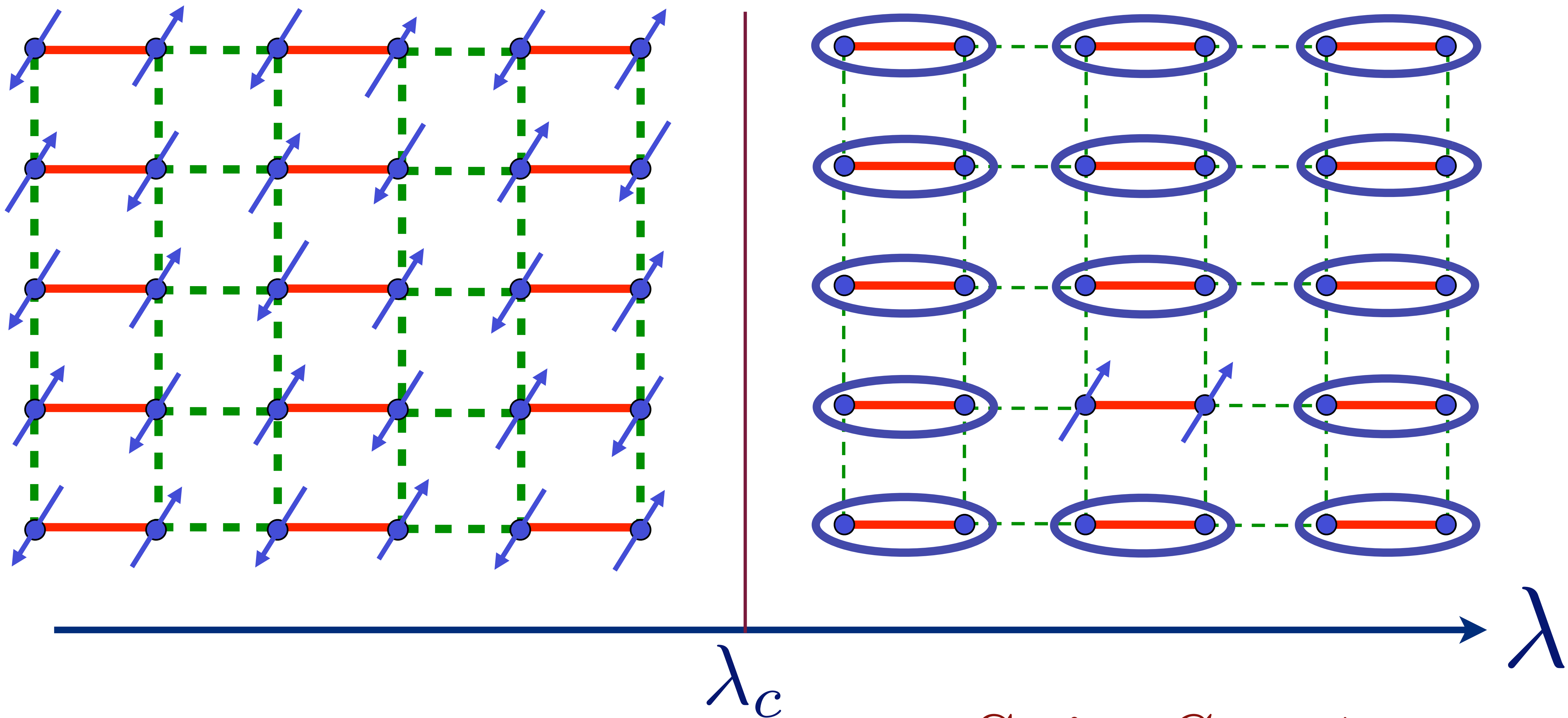

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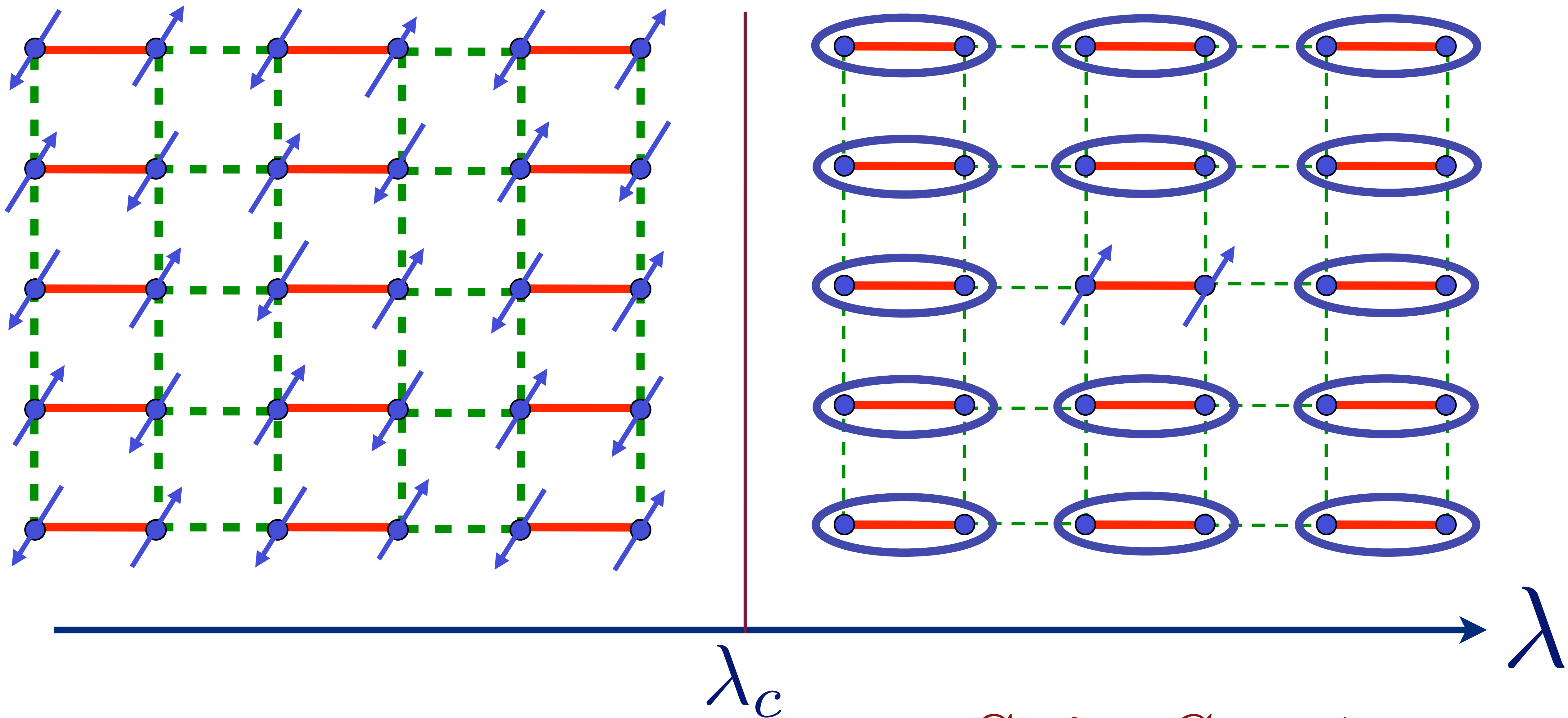

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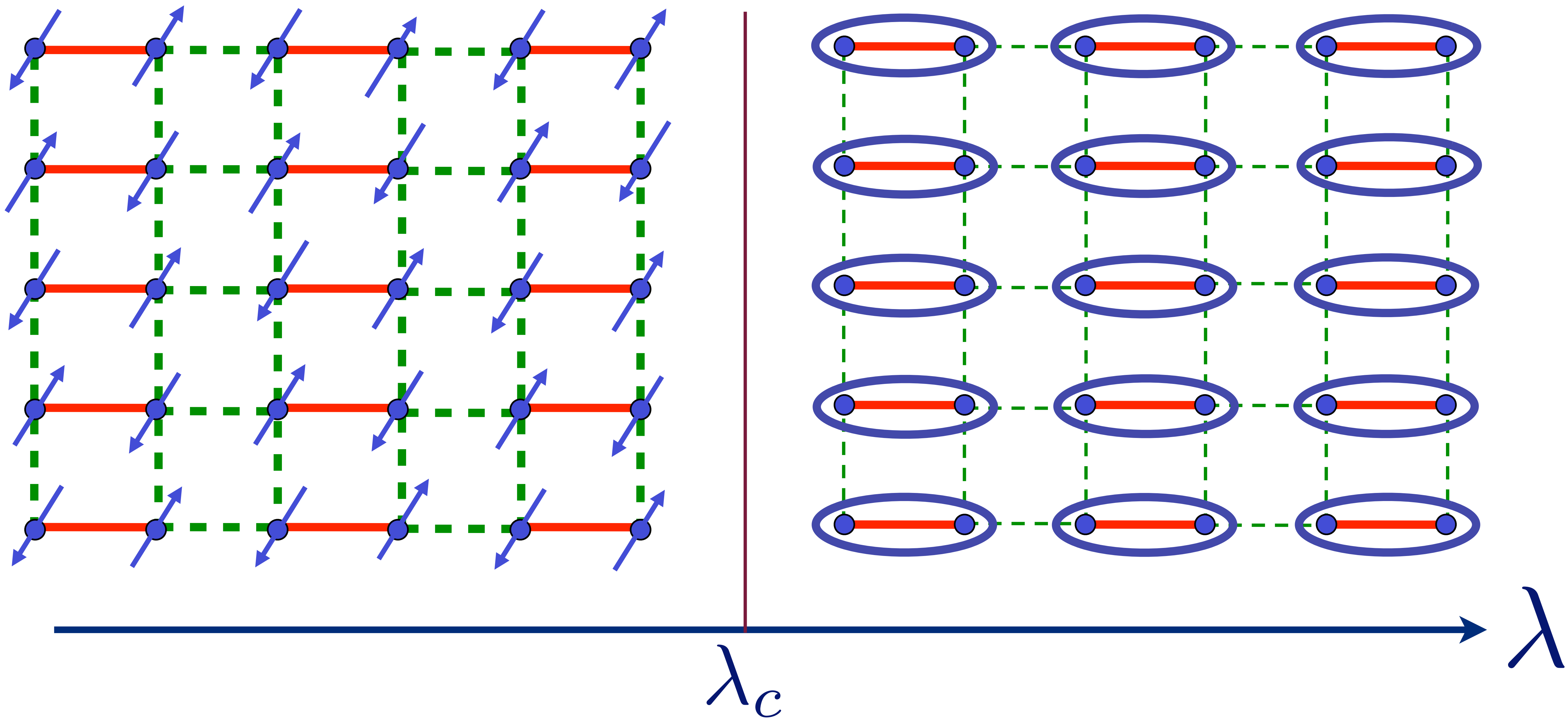
Excitation spectrum in the Néel phase



Spin waves

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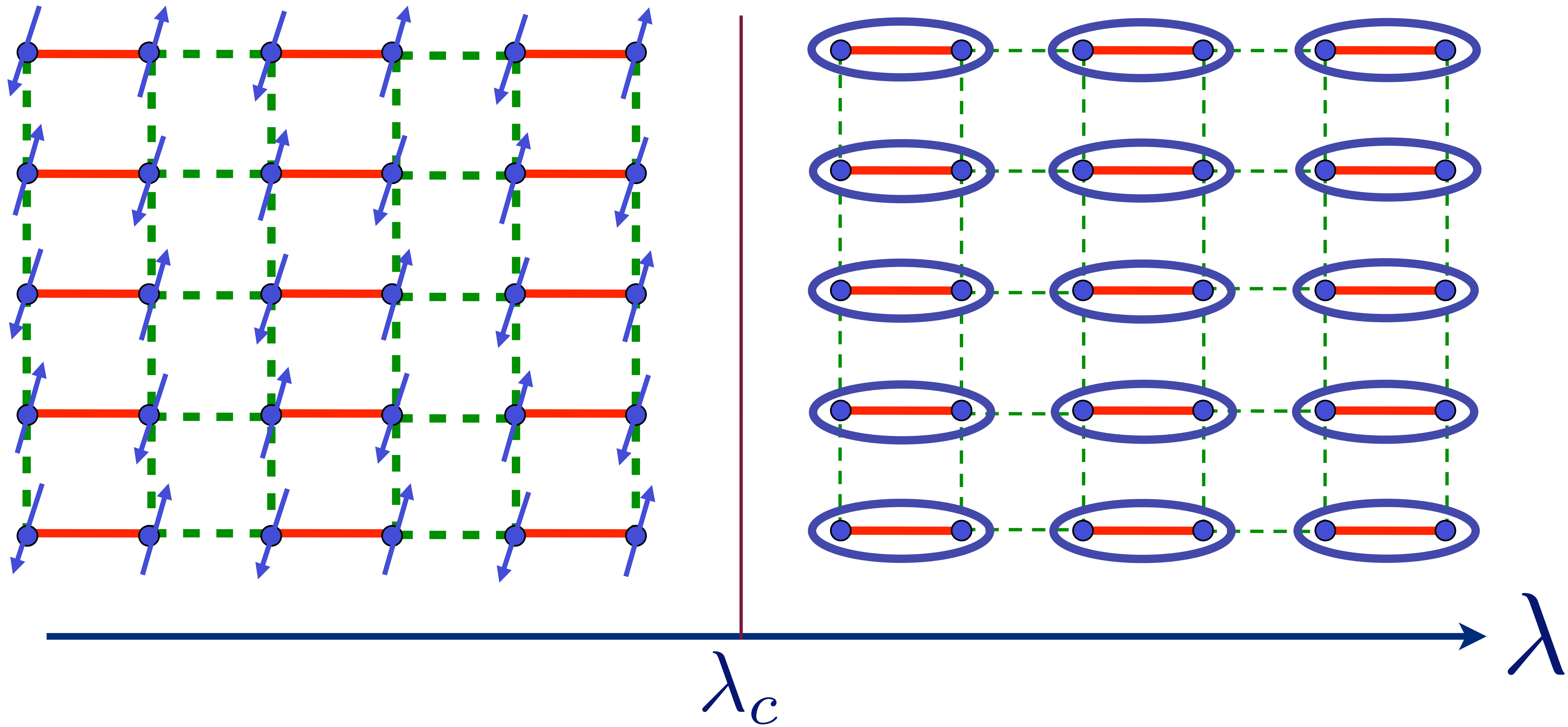

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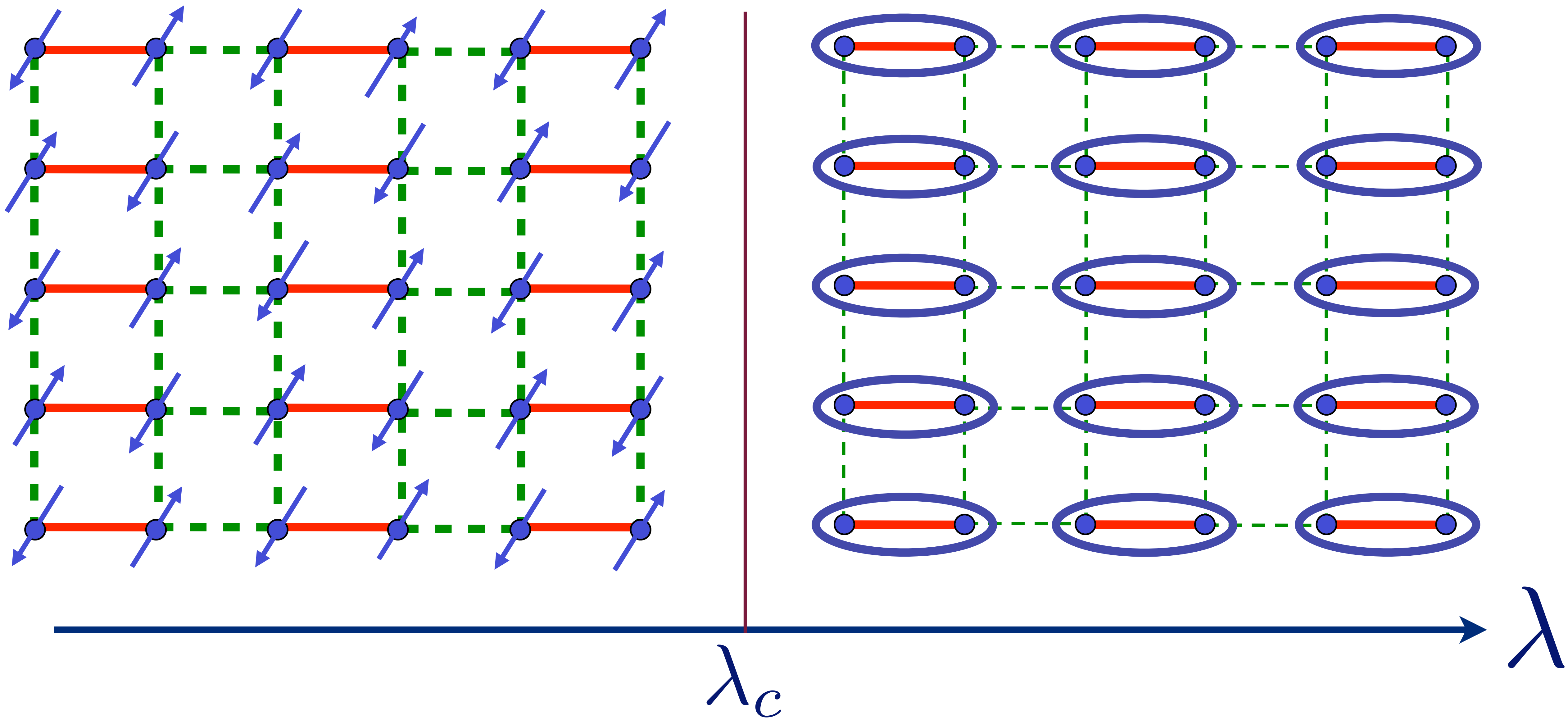
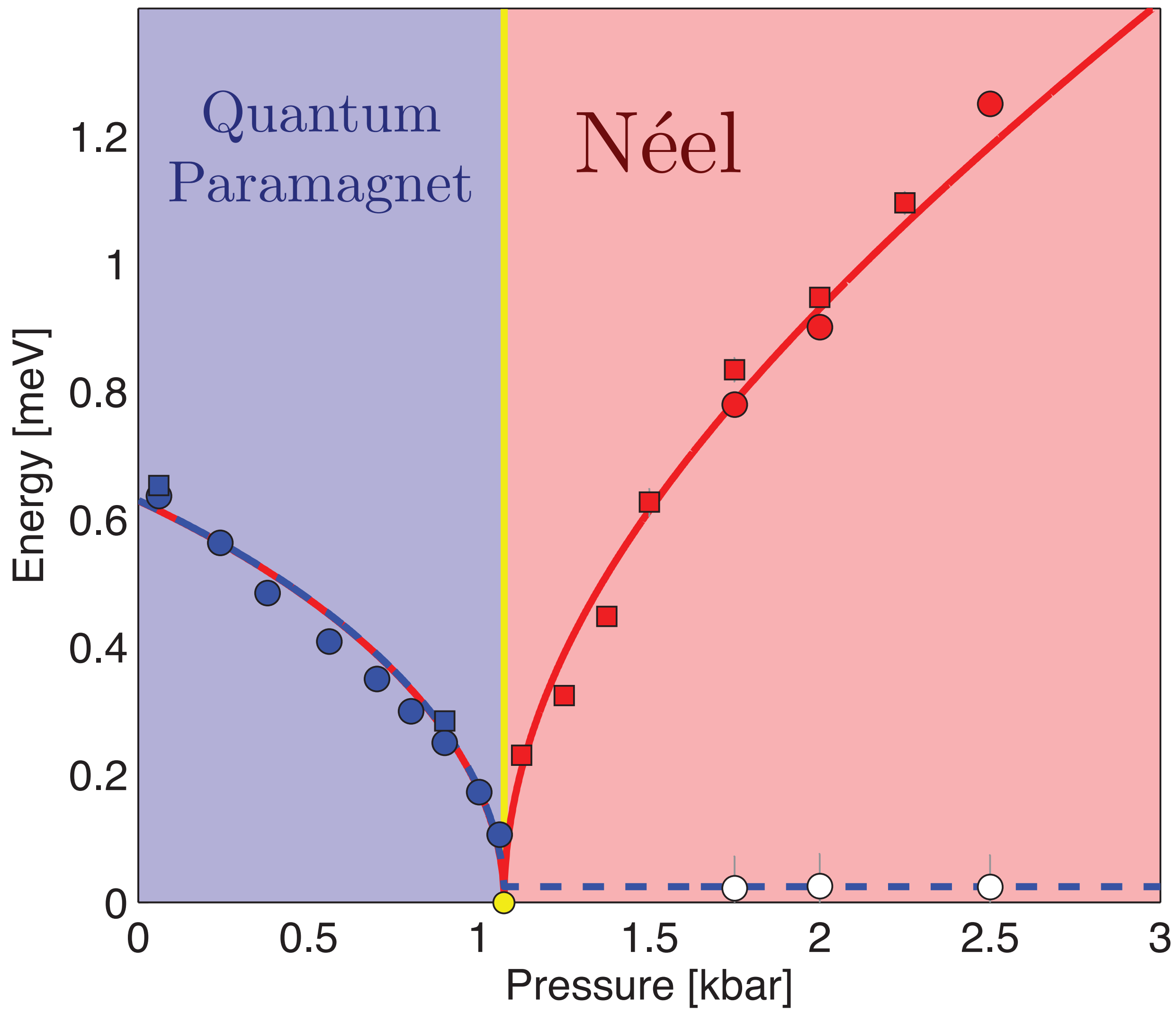


Spin waves

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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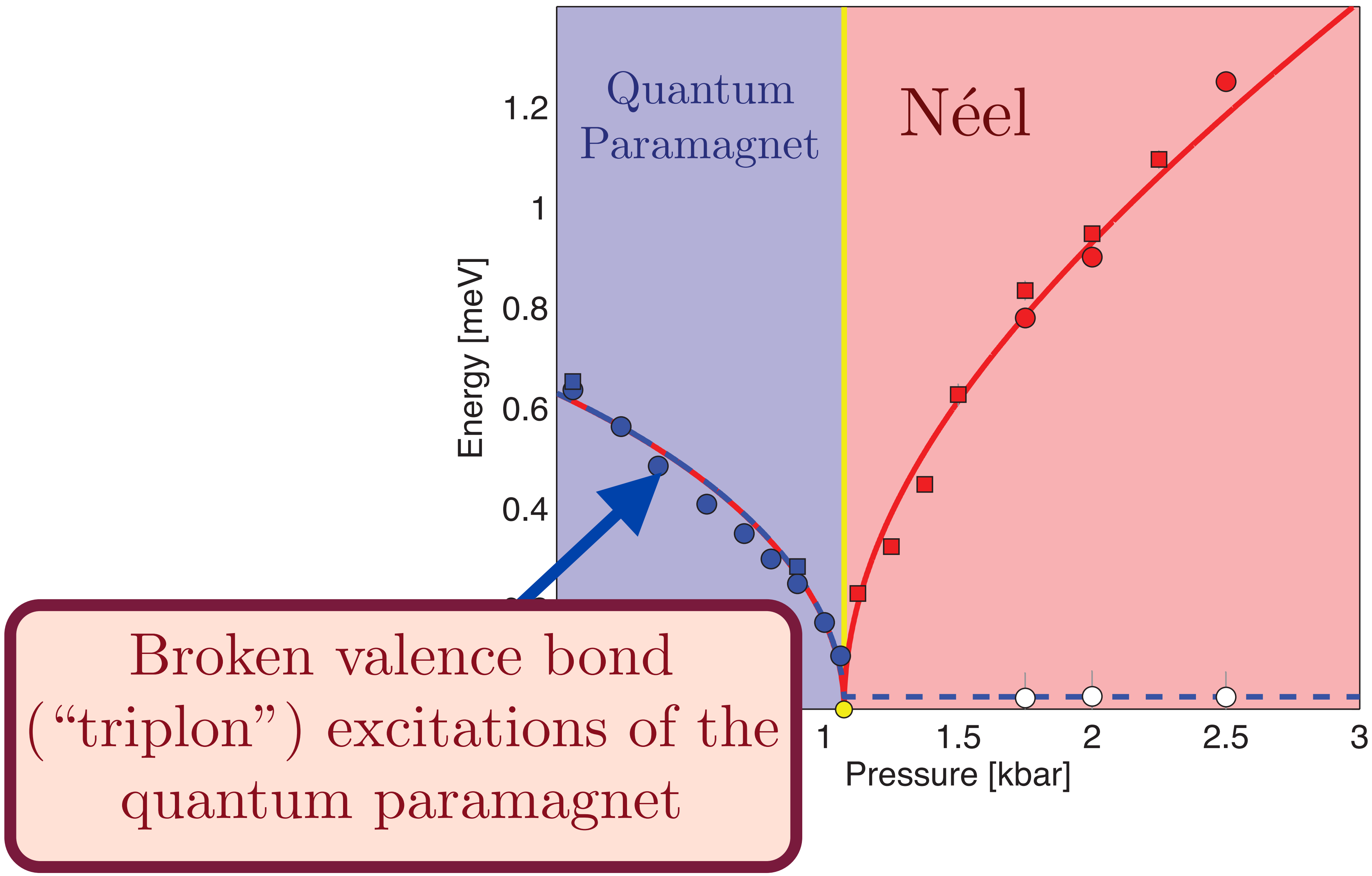

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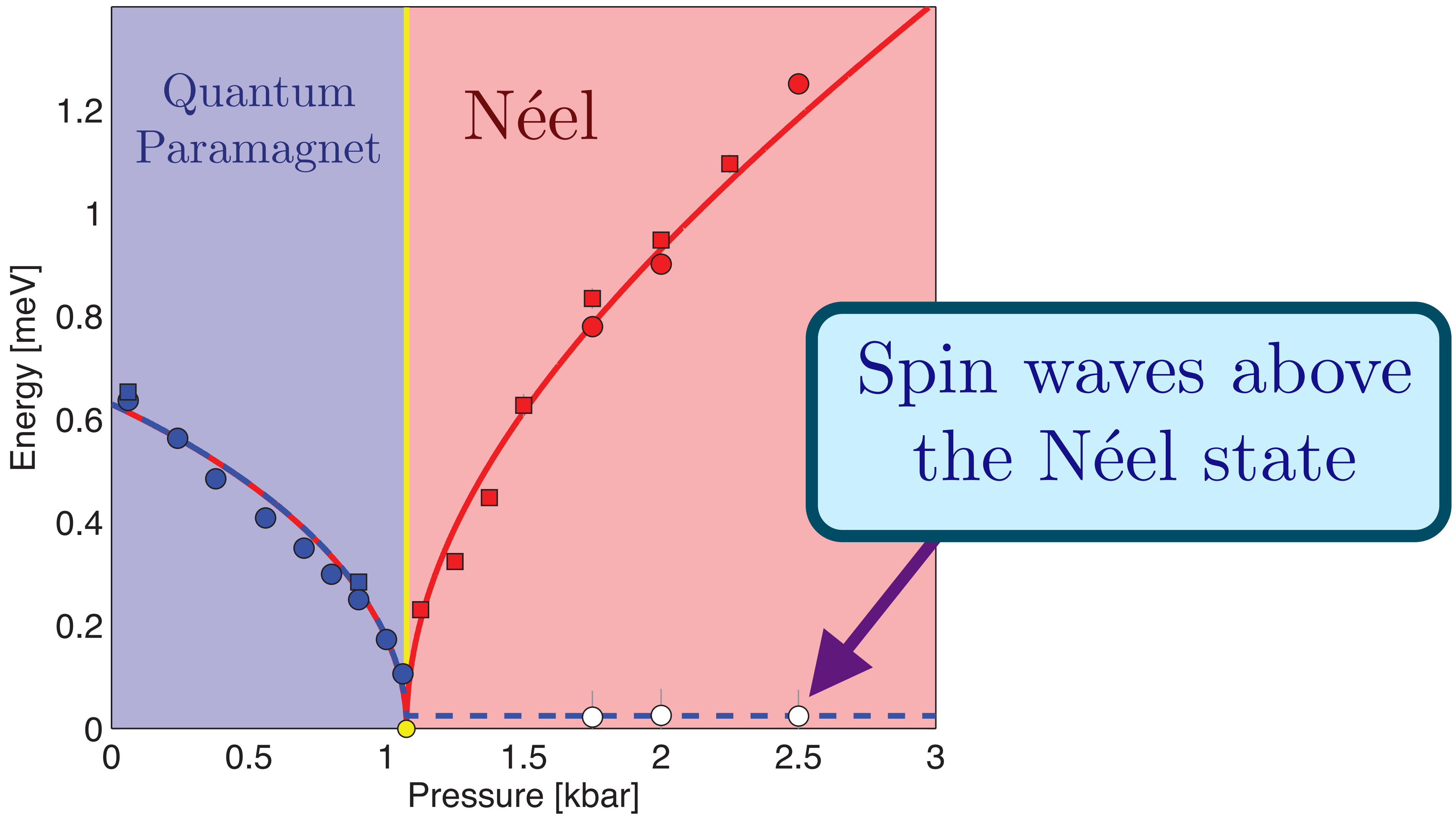

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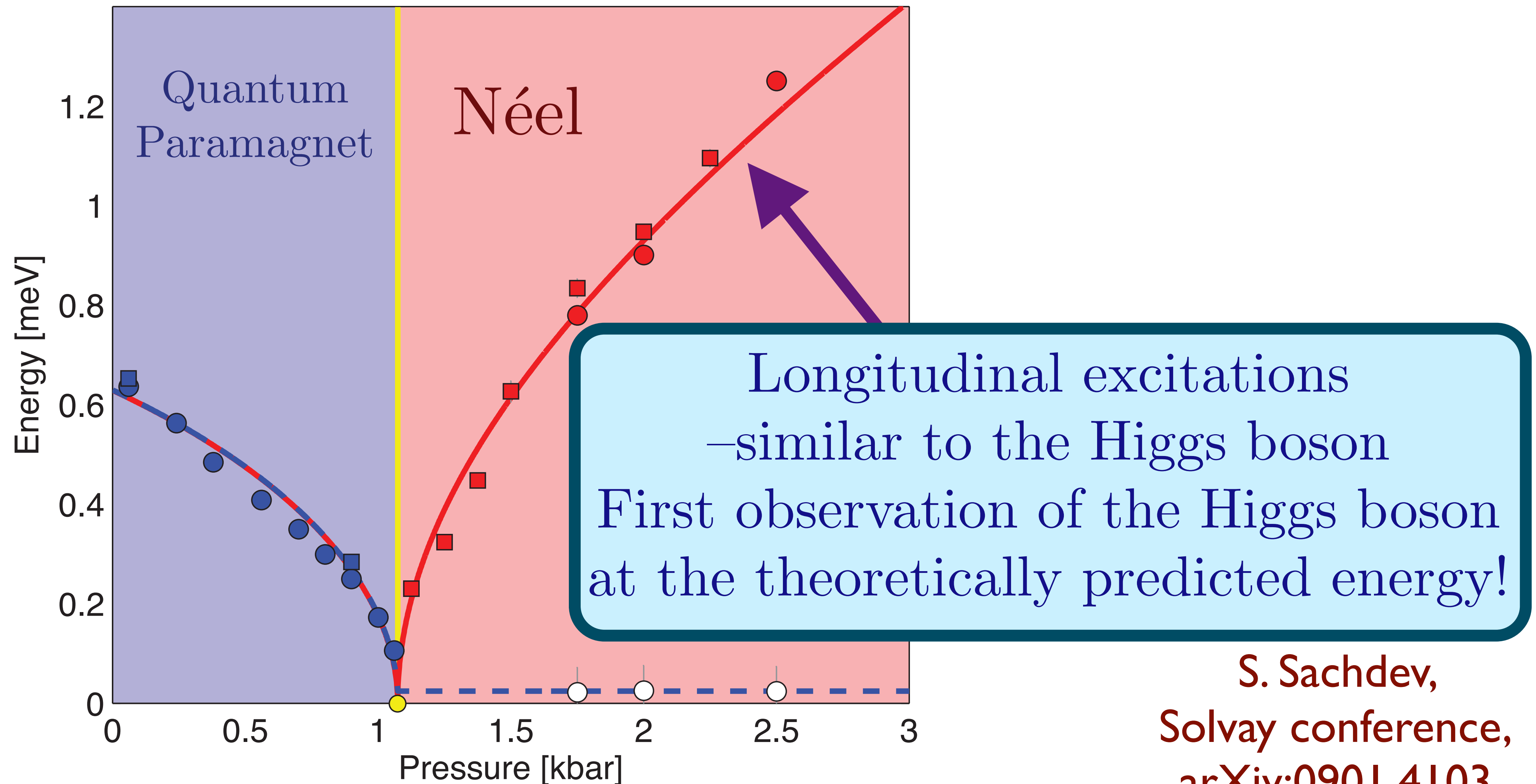

S. Sachdev,
Solvay conference,
arXiv:0901.4103

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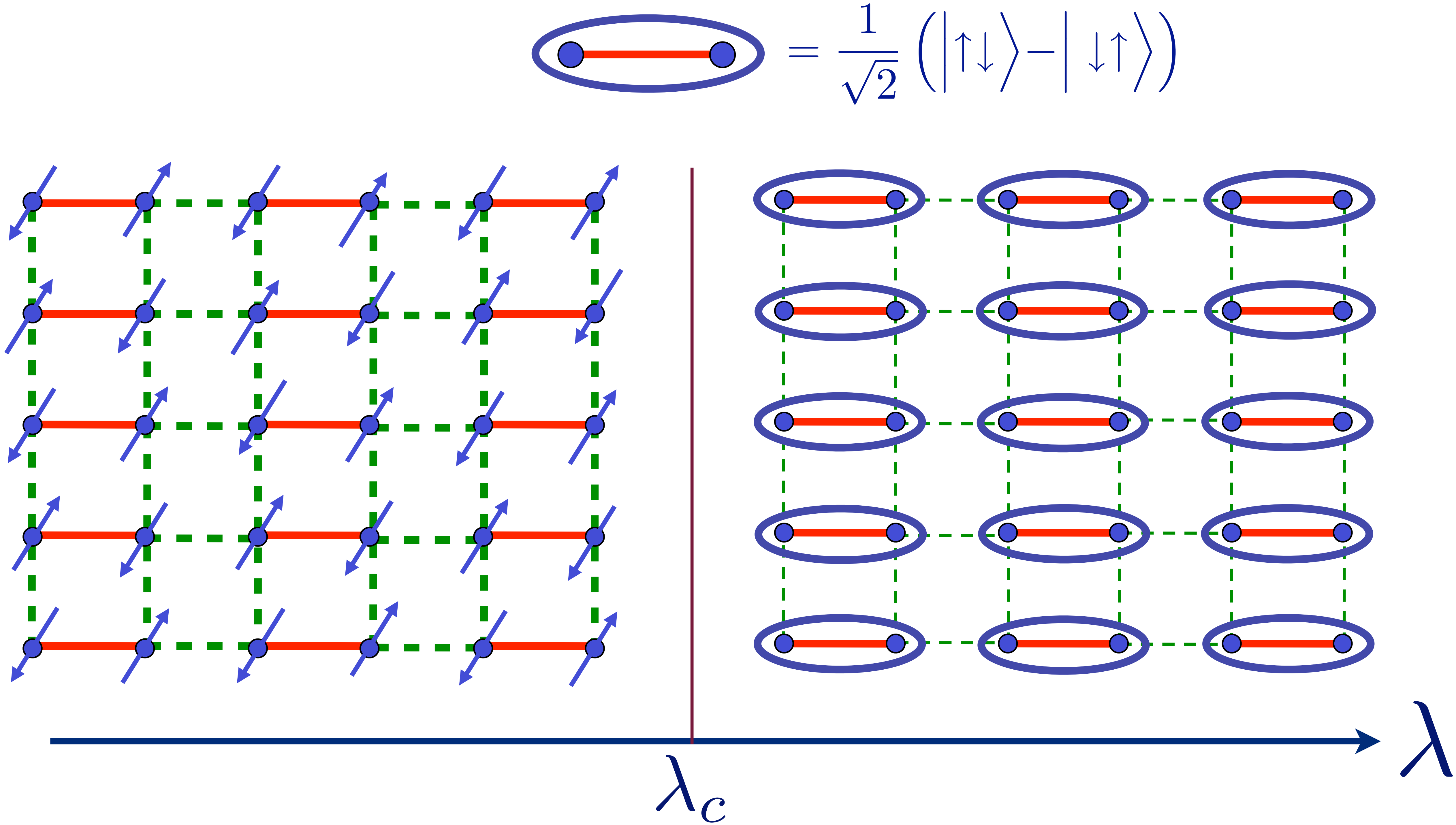
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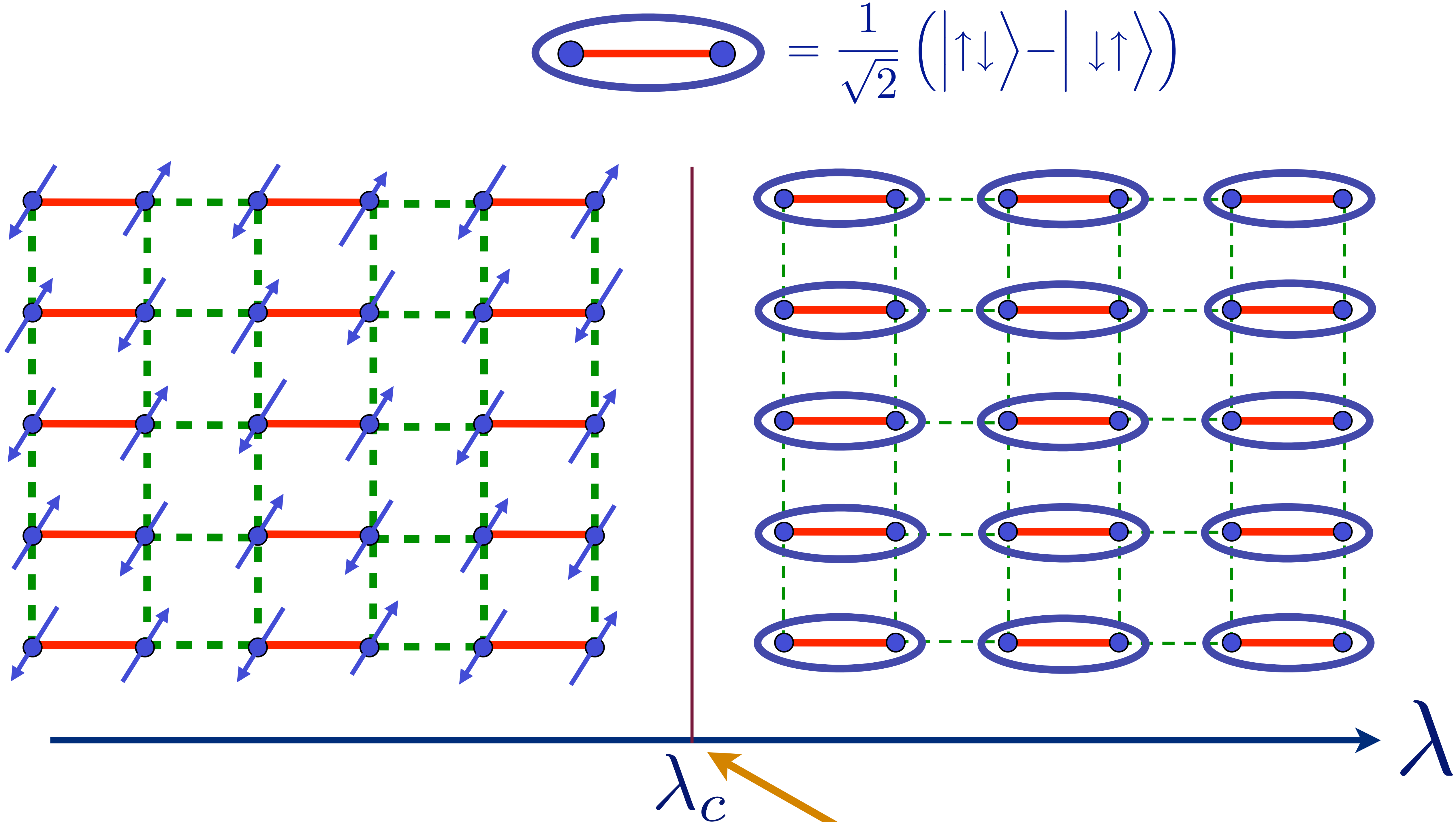
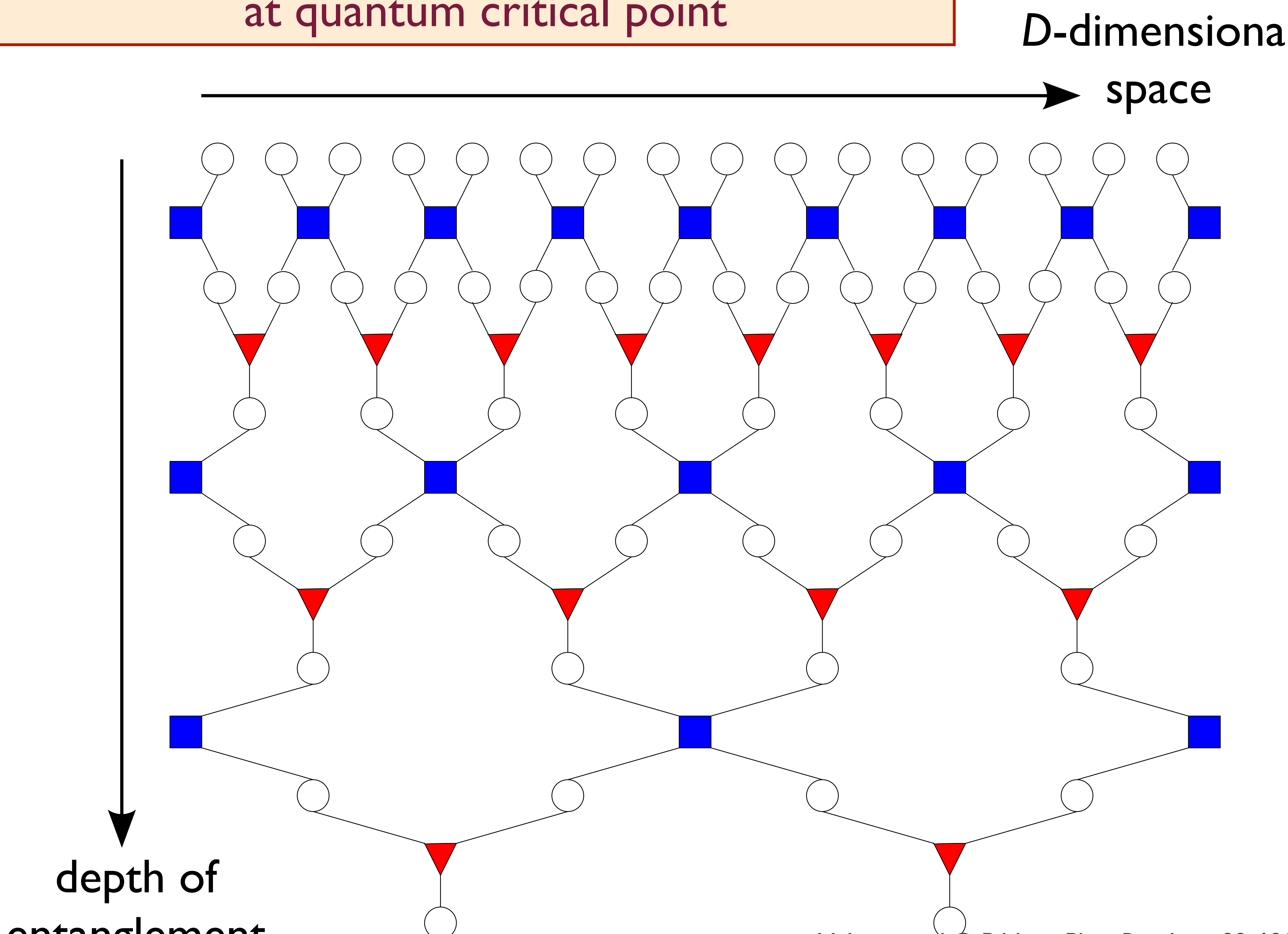


Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)

F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

Characteristics of quantum critical point

- Long-range entanglement

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- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.

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Quantum superposition and entanglement

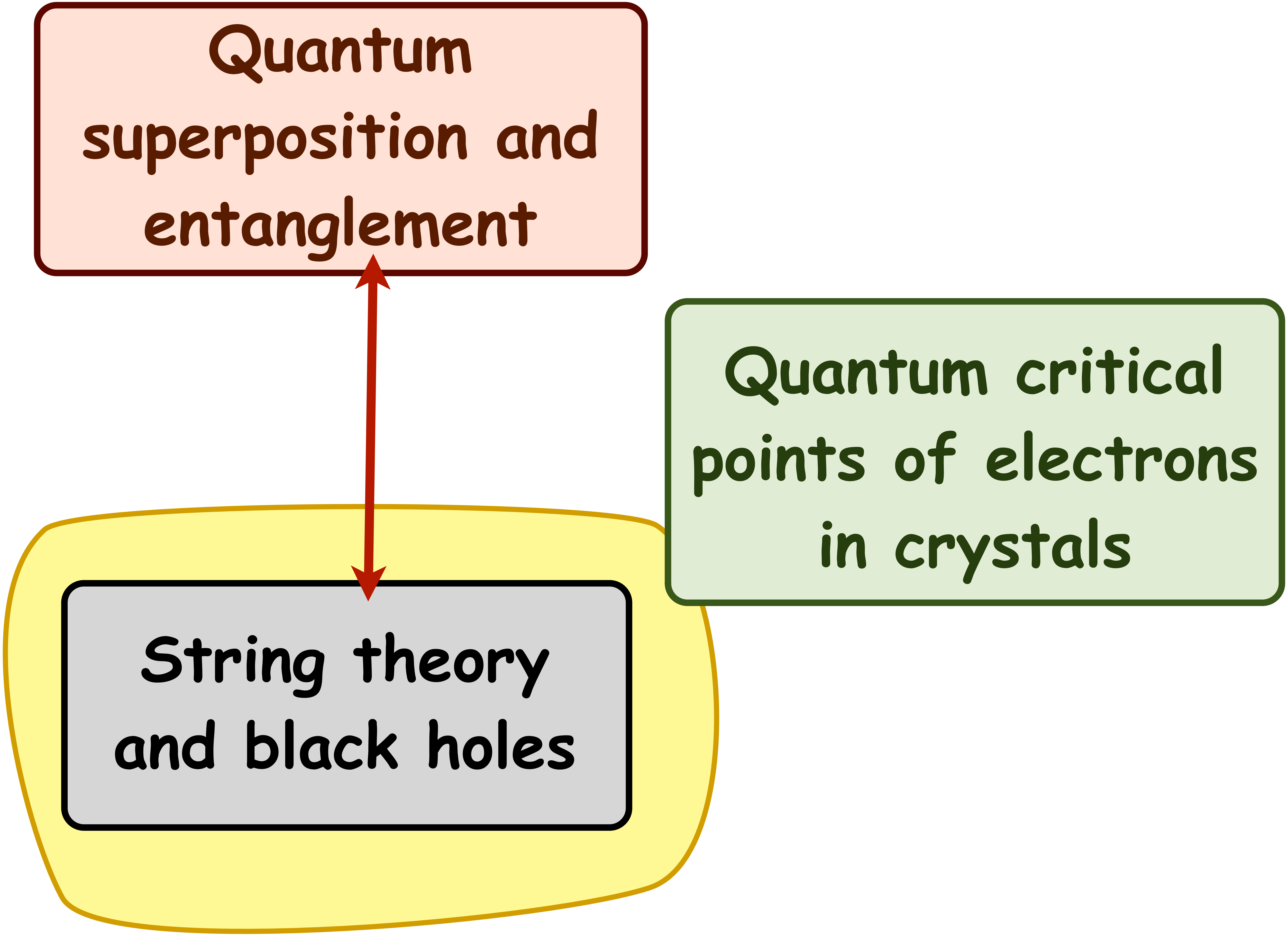
Quantum critical points of electrons in crystals

String theory and black holes

Quantum superposition and entanglement

Quantum critical points of electrons in crystals

String theory and black holes


Quantum superposition and entanglement
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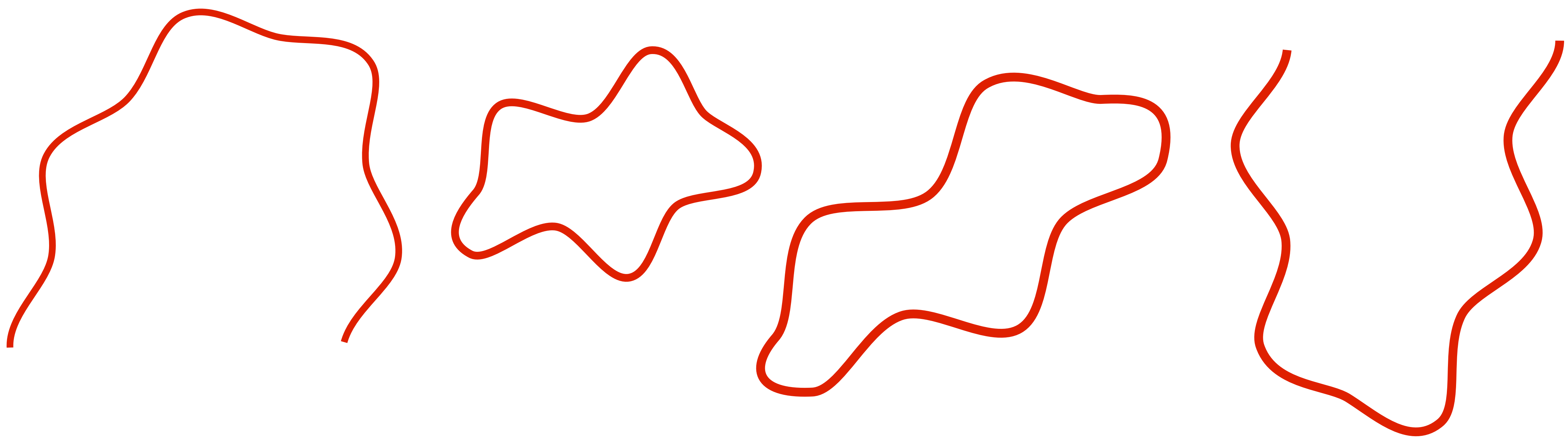
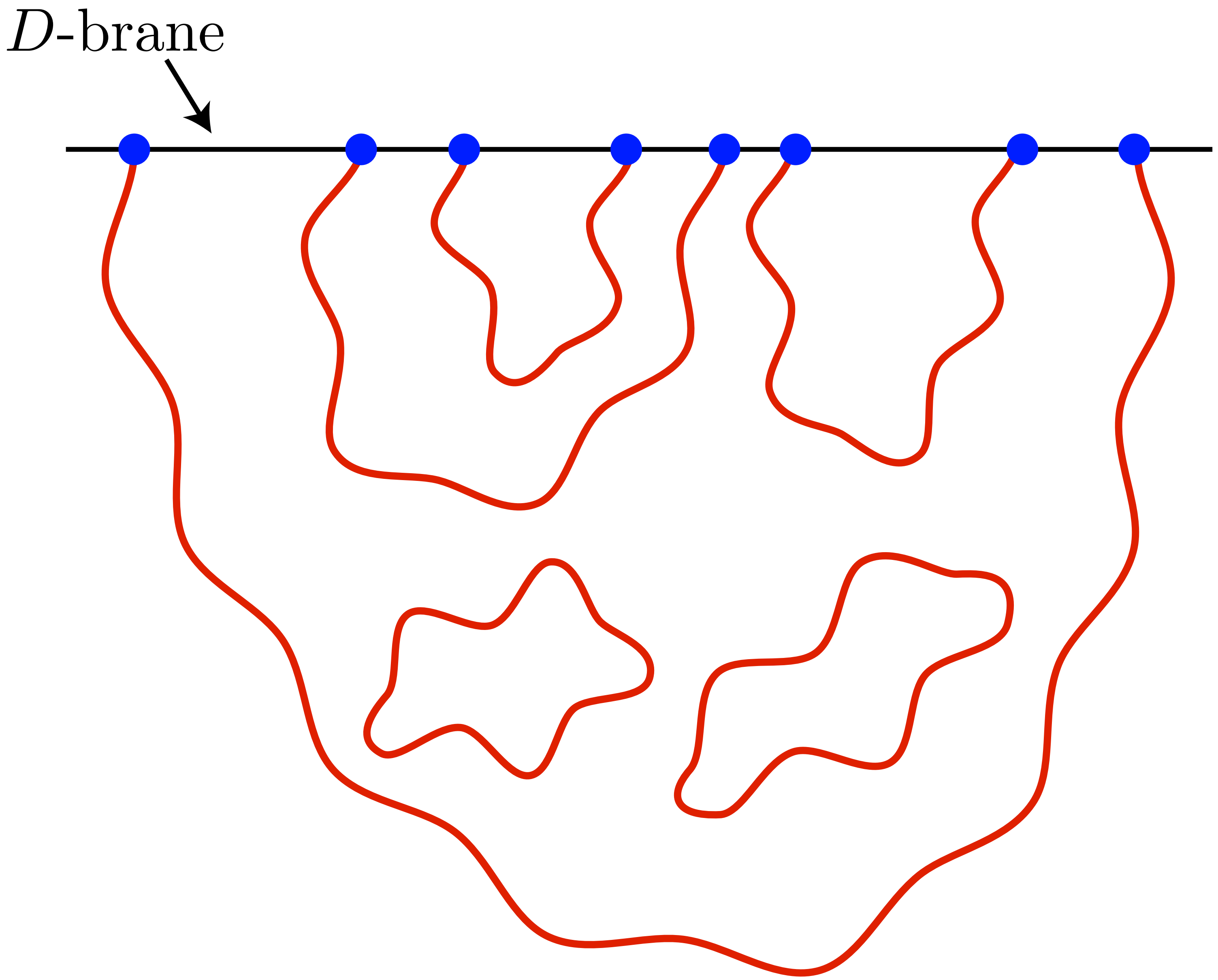
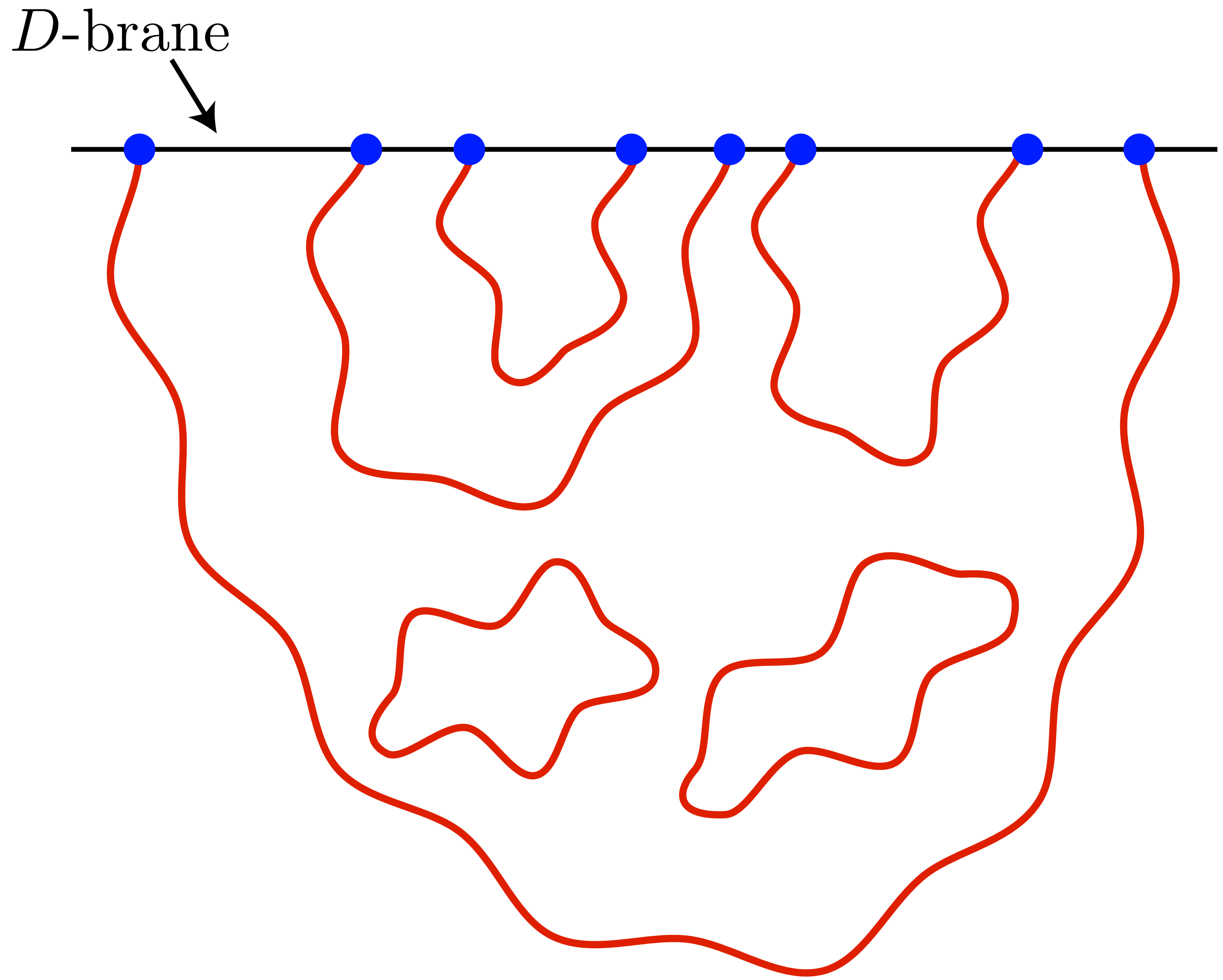
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



- A D-brane is a D-dimensional surface on which strings can end.



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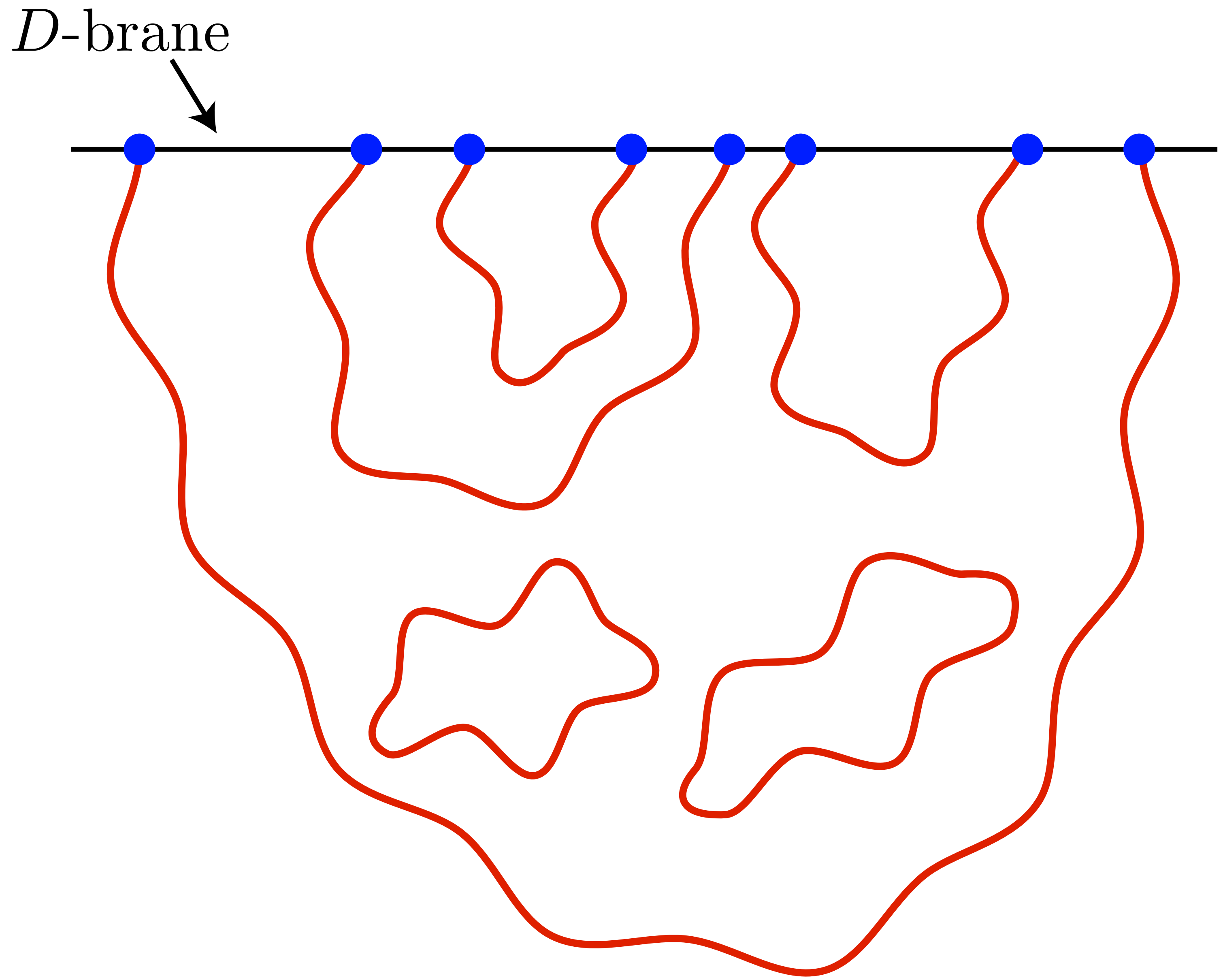
- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us.



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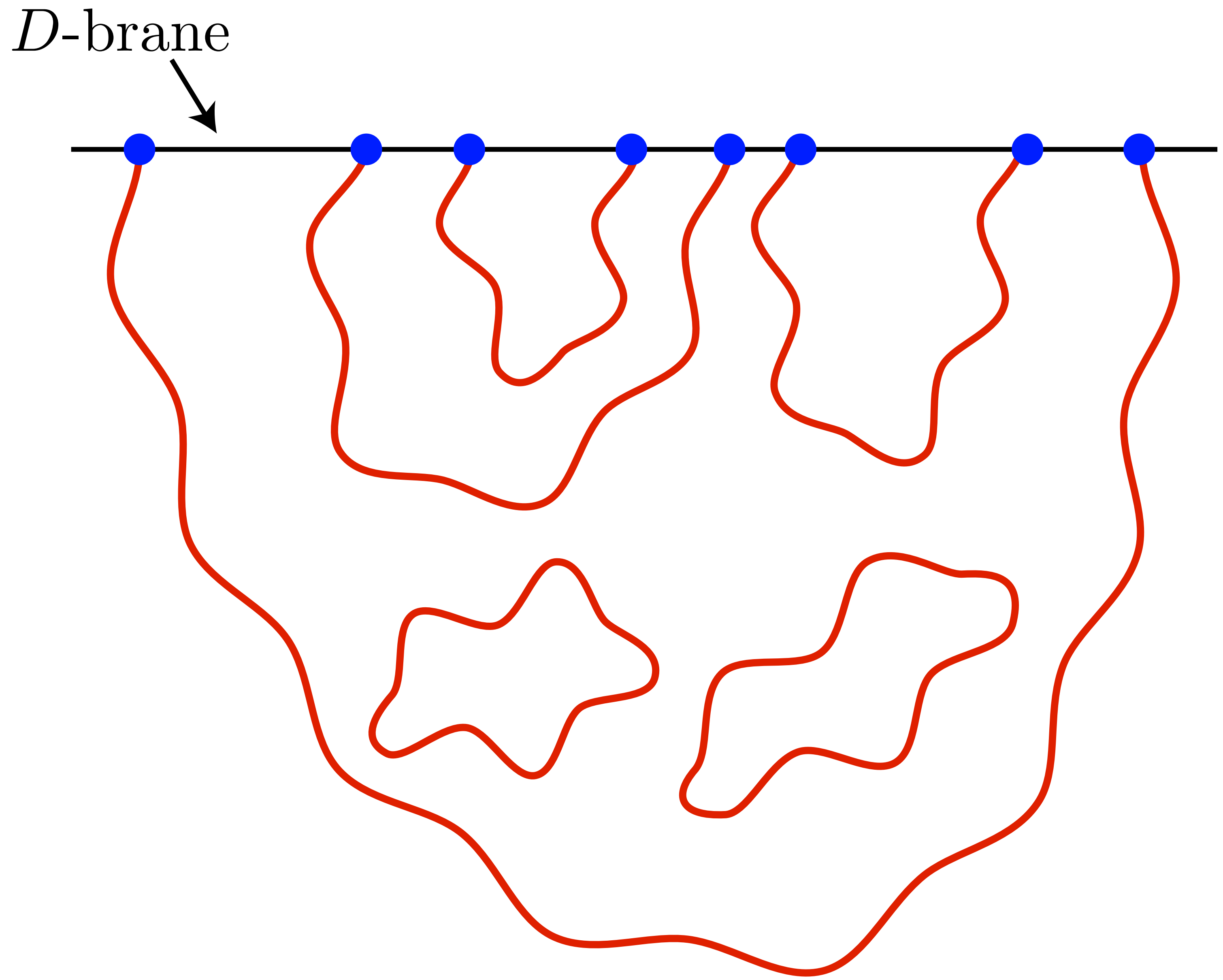

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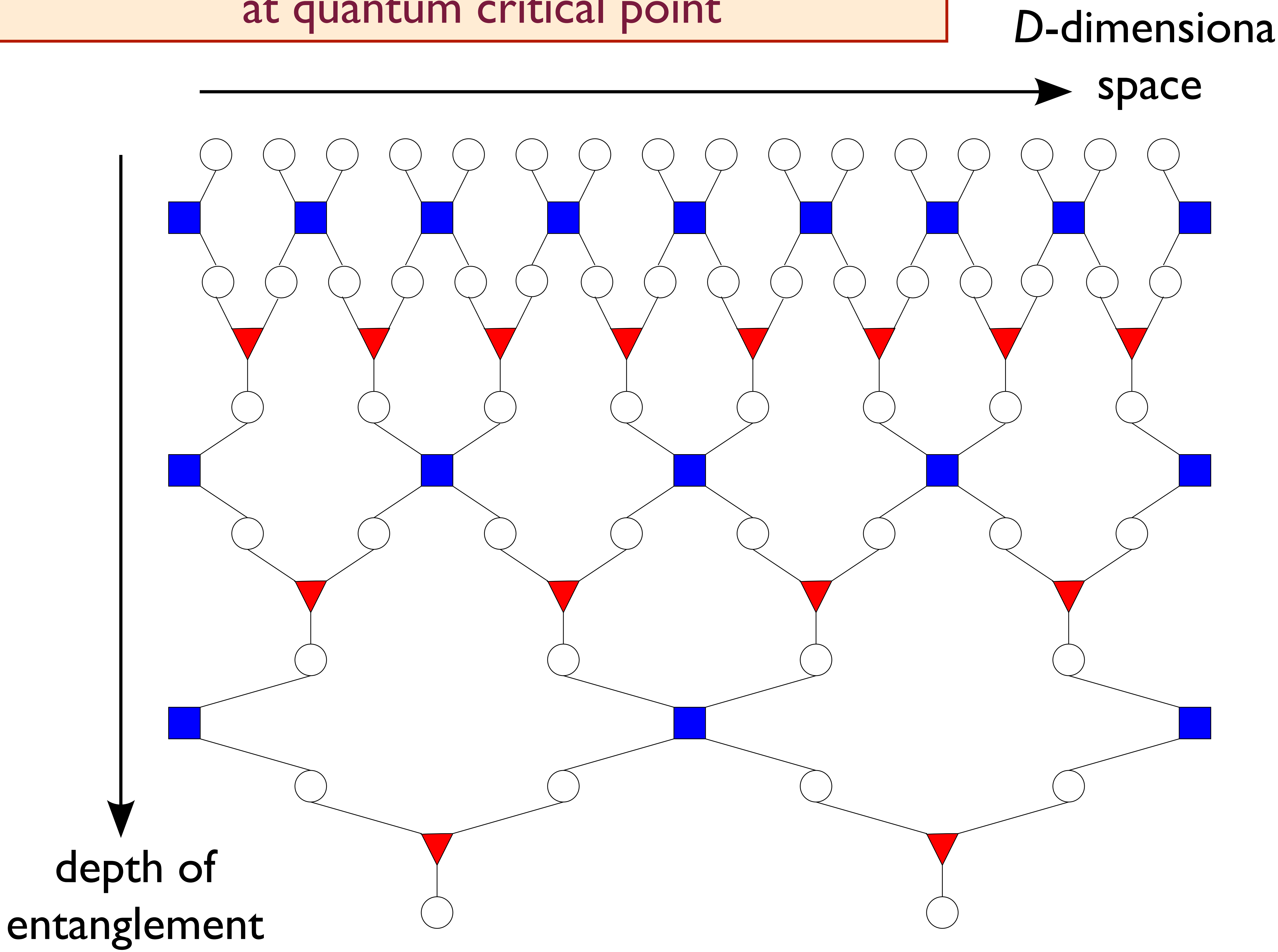
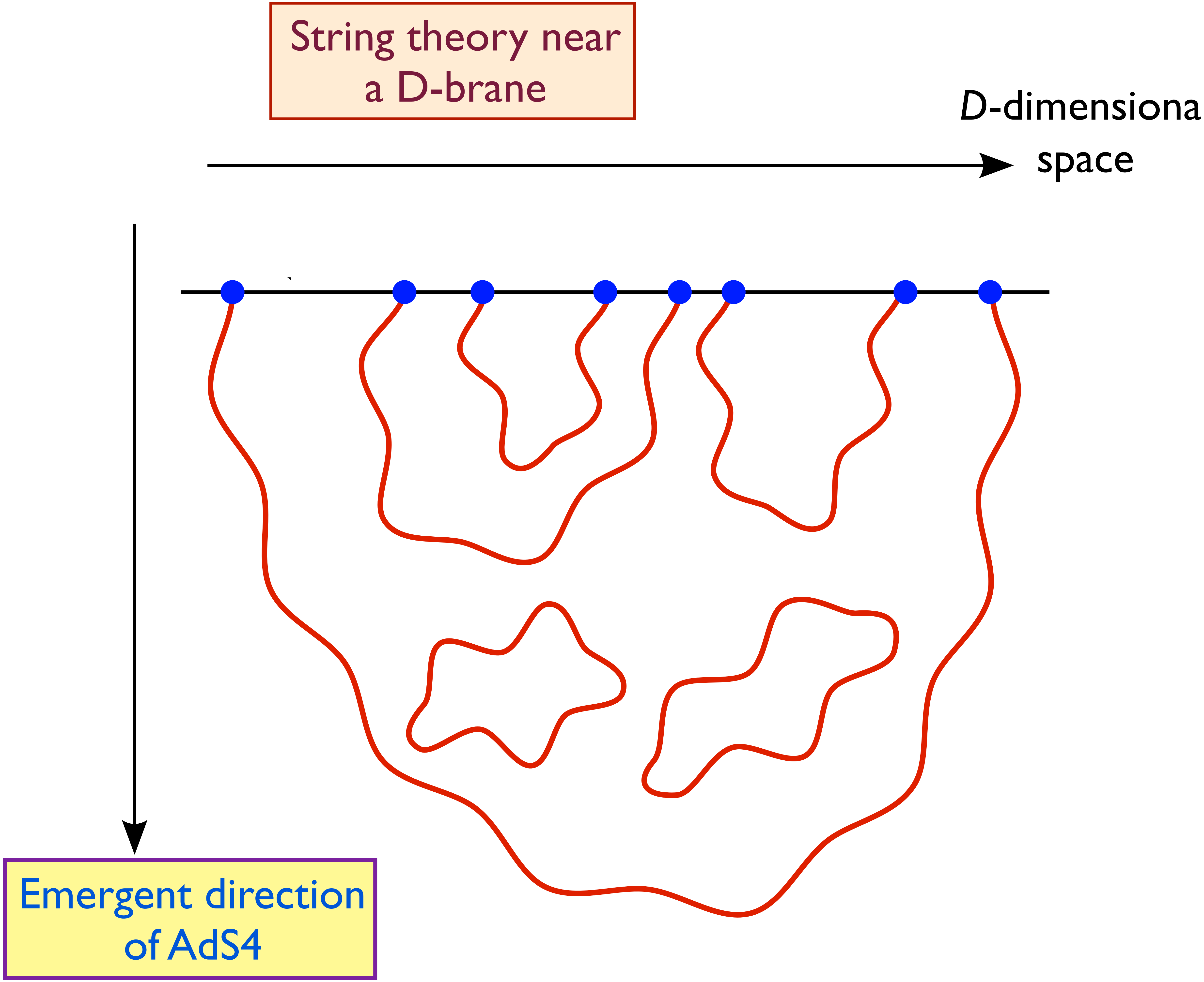
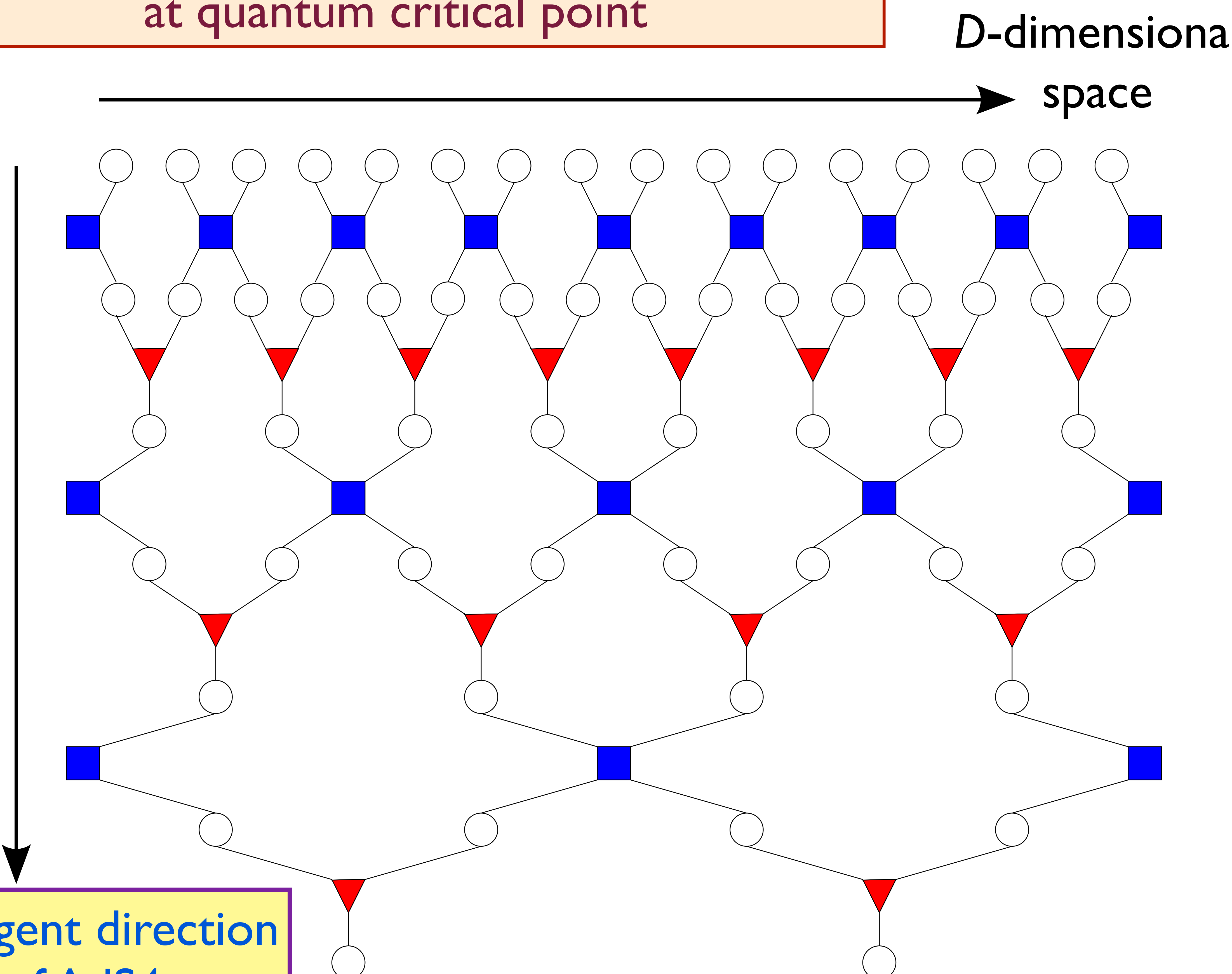


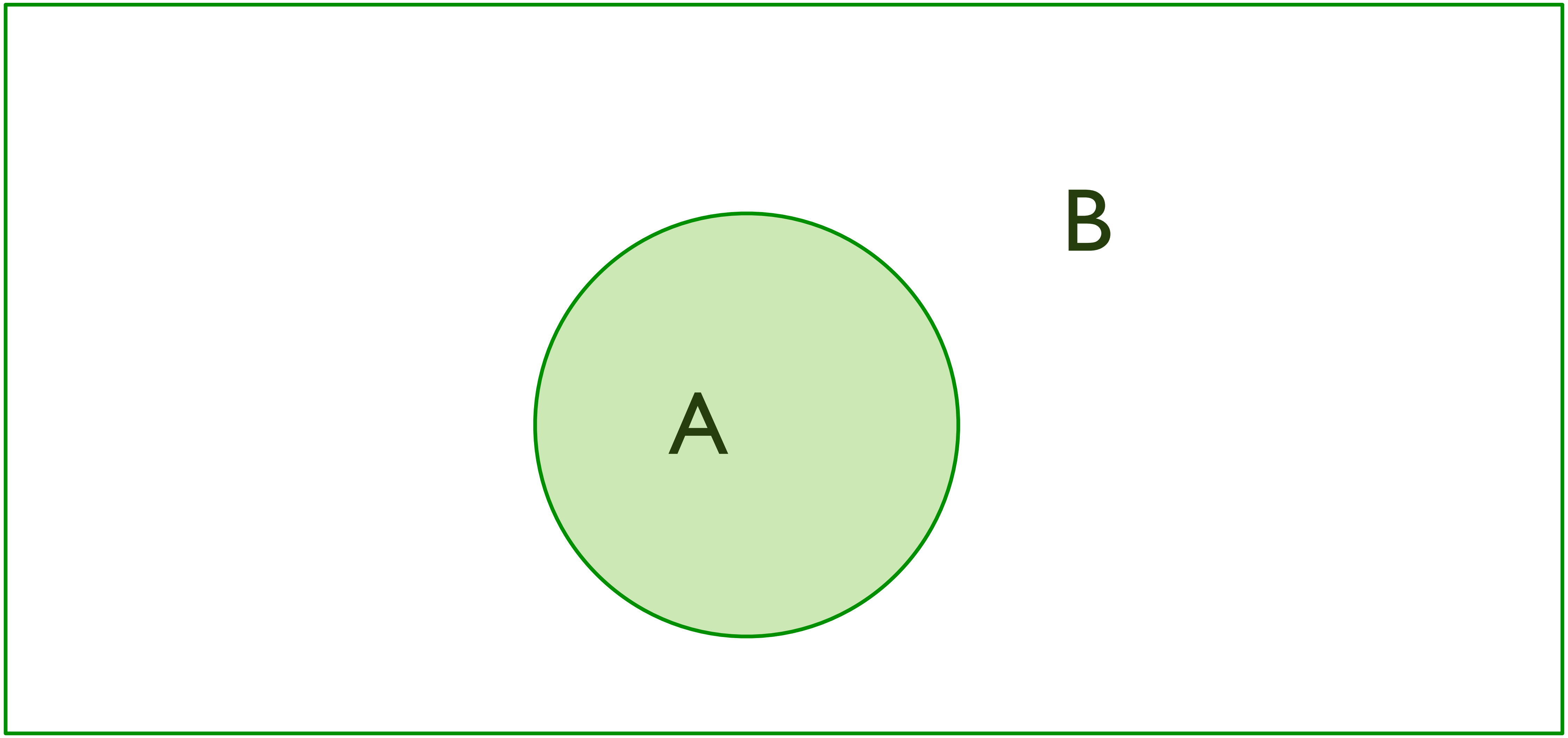

String theory near a D-brane
D-dimensional space
Emergent direction of AdS4

Tensor network representation of entanglement at quantum critical point



Brian Swingle, arXiv:0905.1317

Entanglement entropy

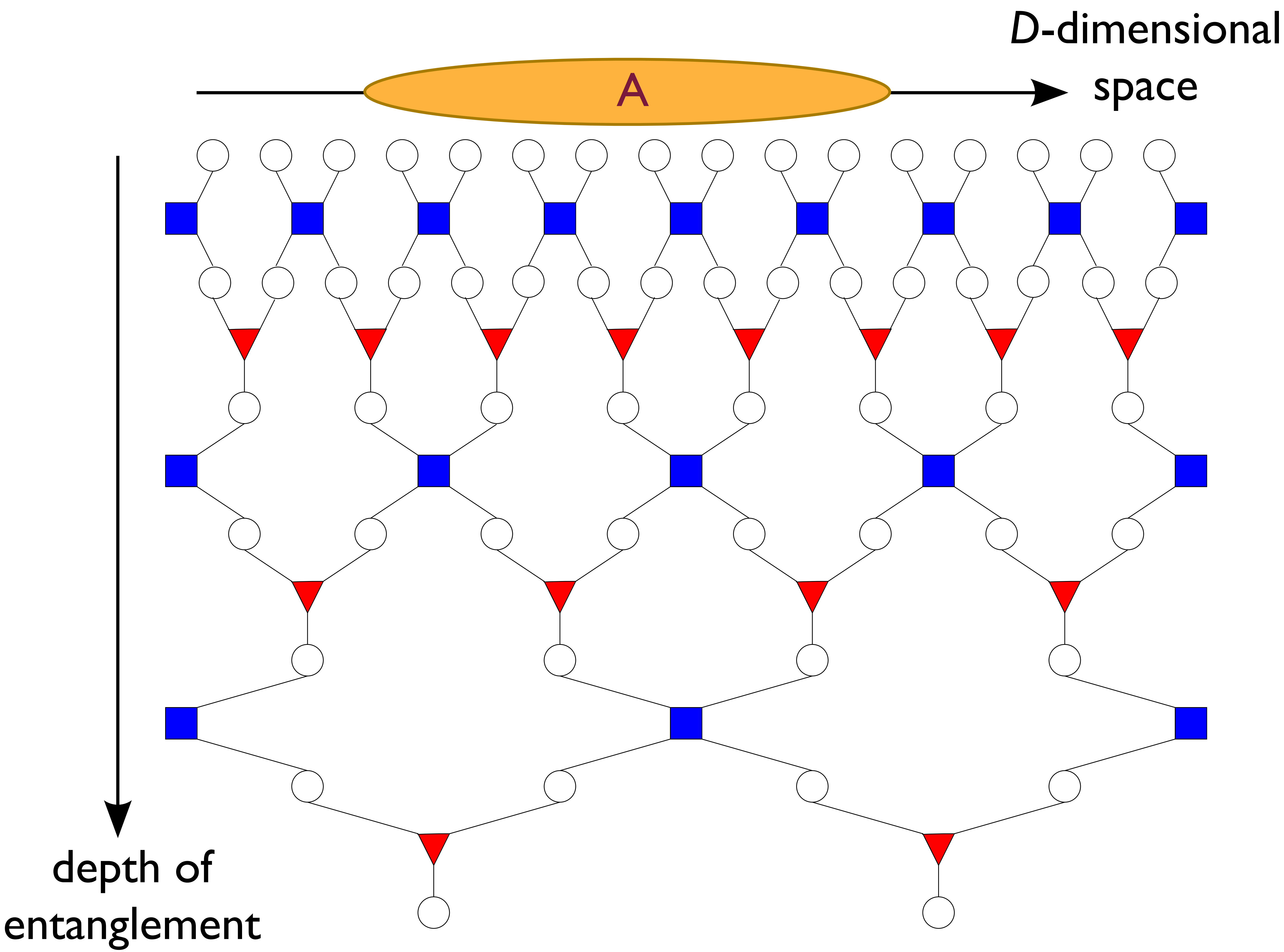


Measure strength of quantum entanglement of region A with region B.

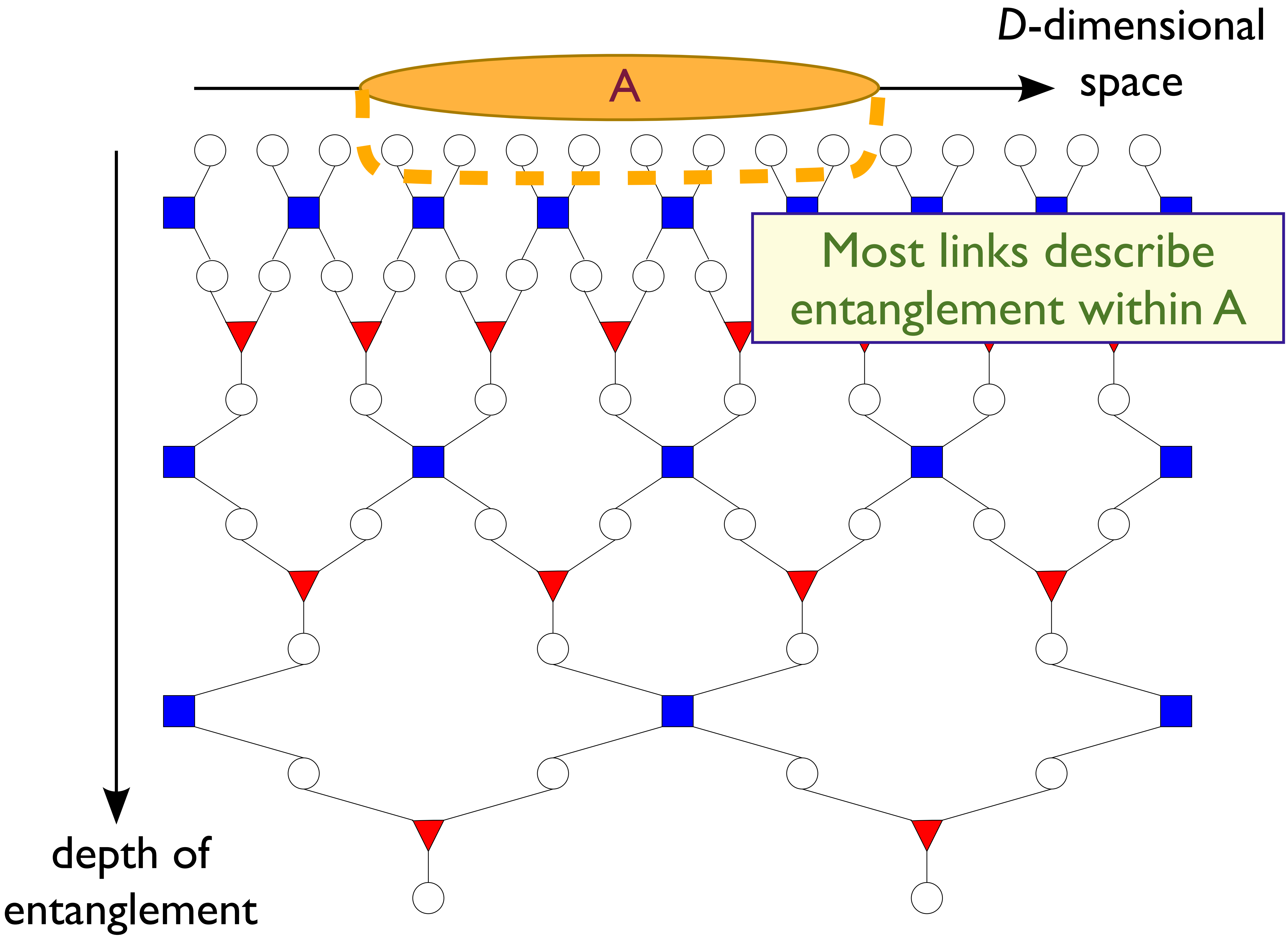
$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

Entanglement entropy $S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

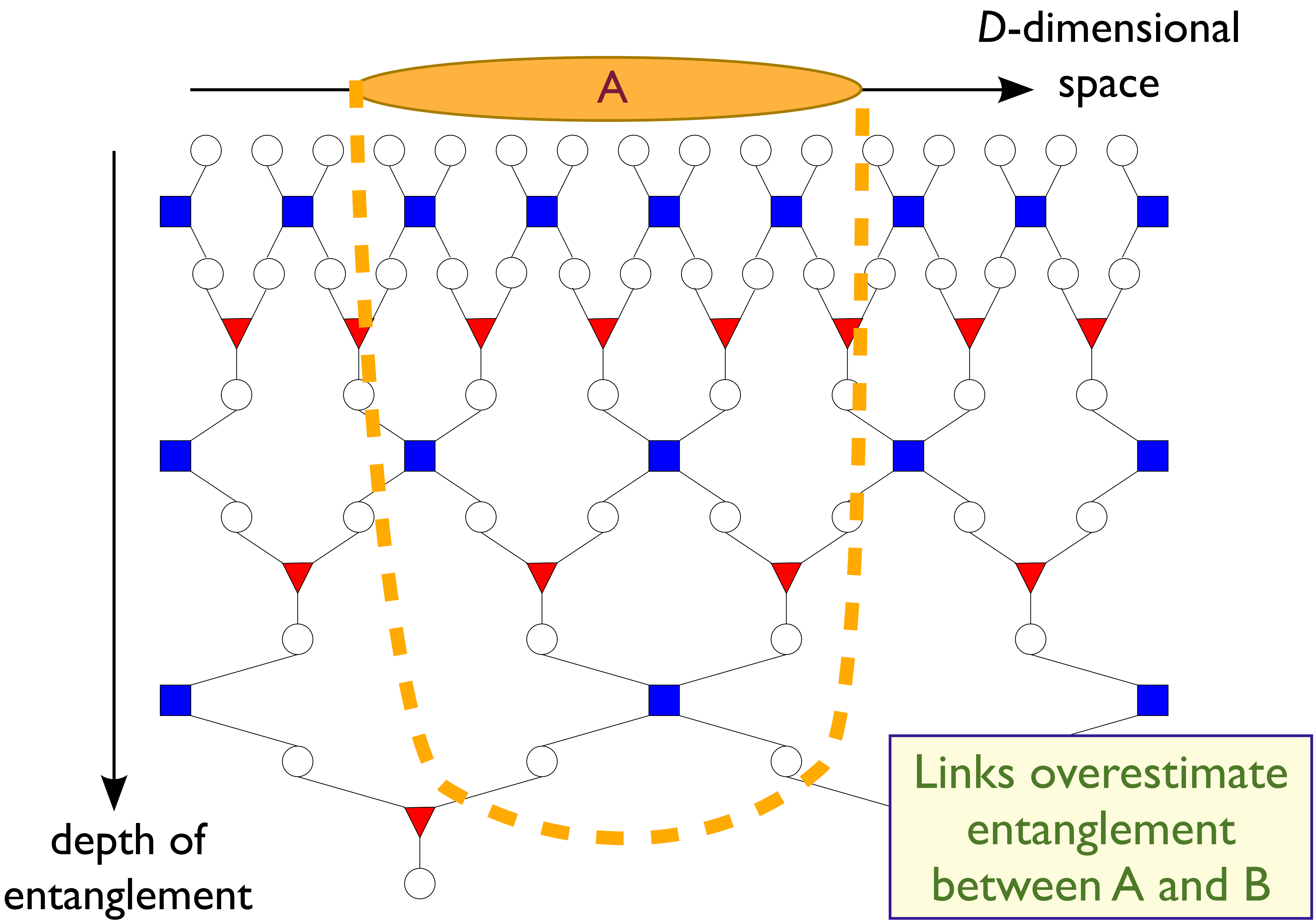
Entanglement entropy



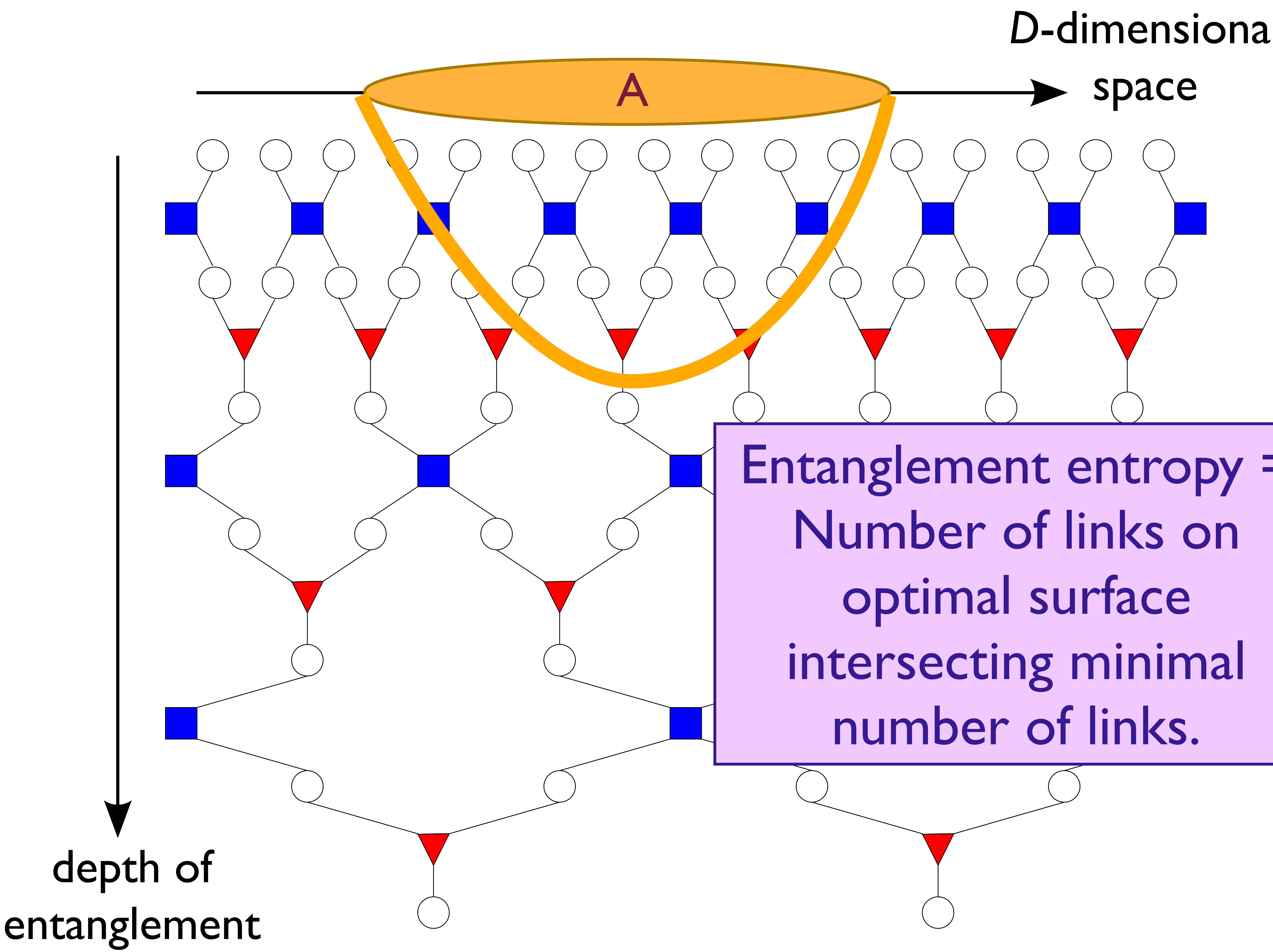
Entanglement entropy



Entanglement entropy



Entanglement entropy



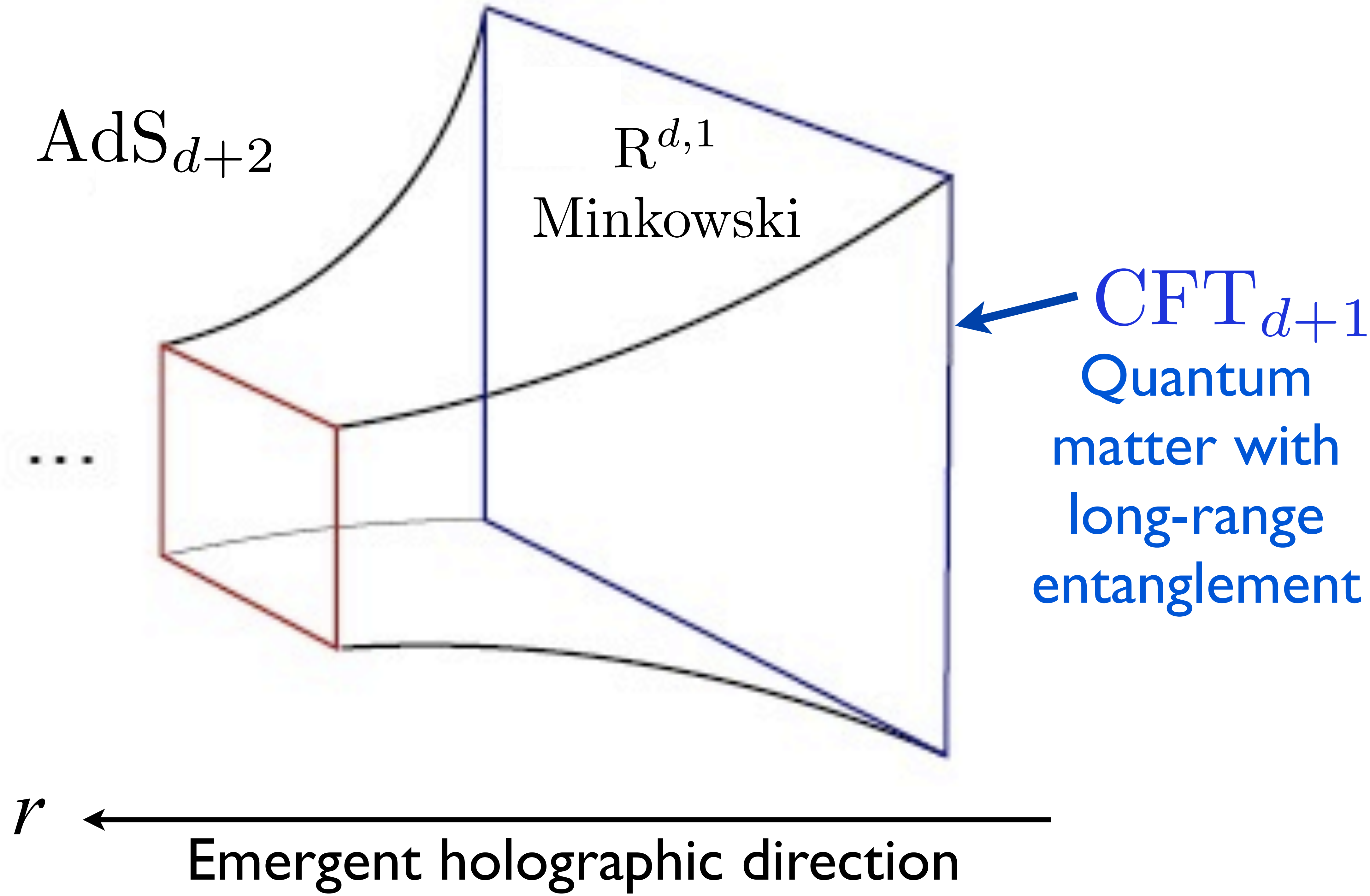
Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Entanglement entropy

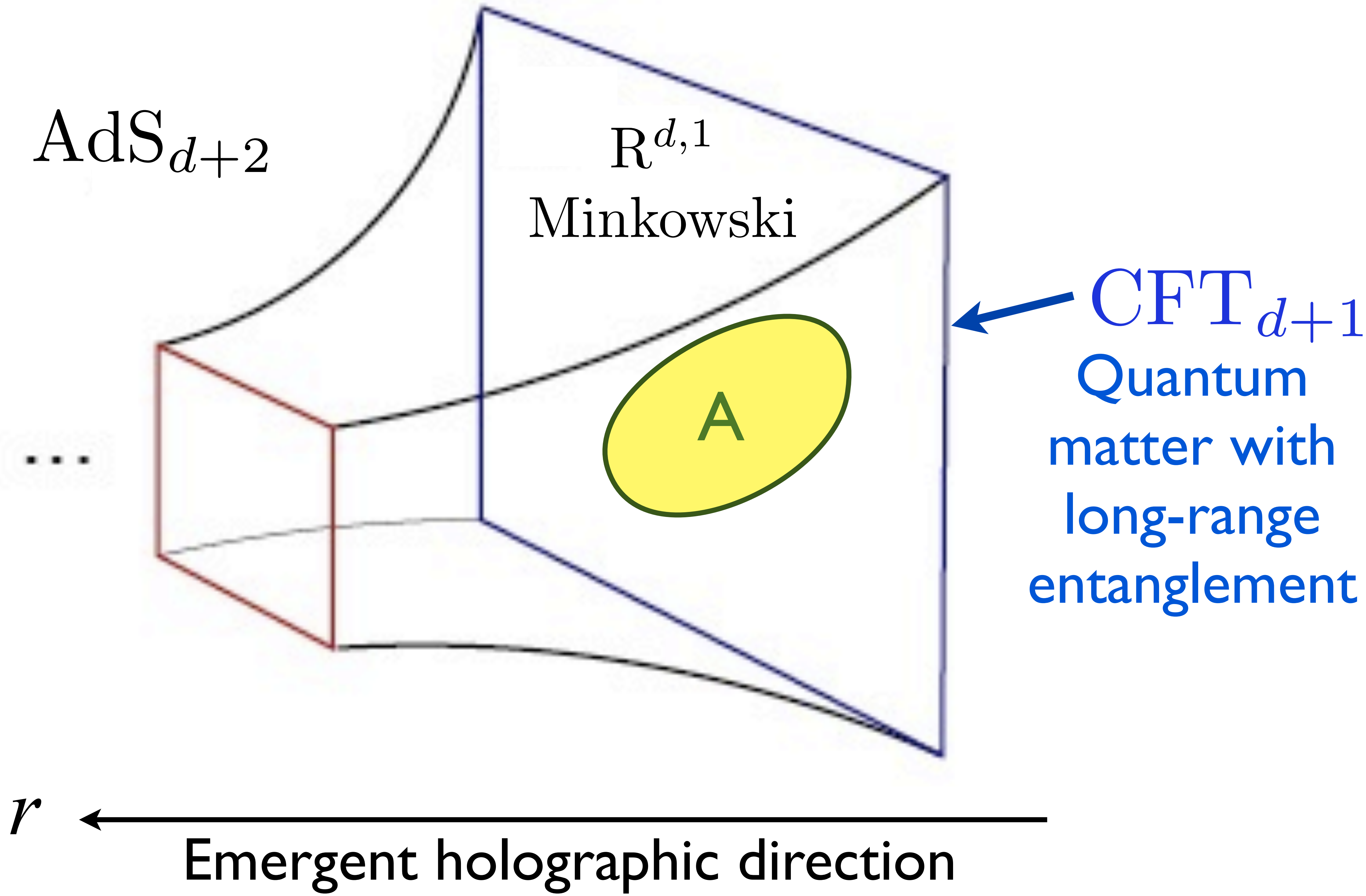
The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

This can be seen both the string and tensor-network pictures

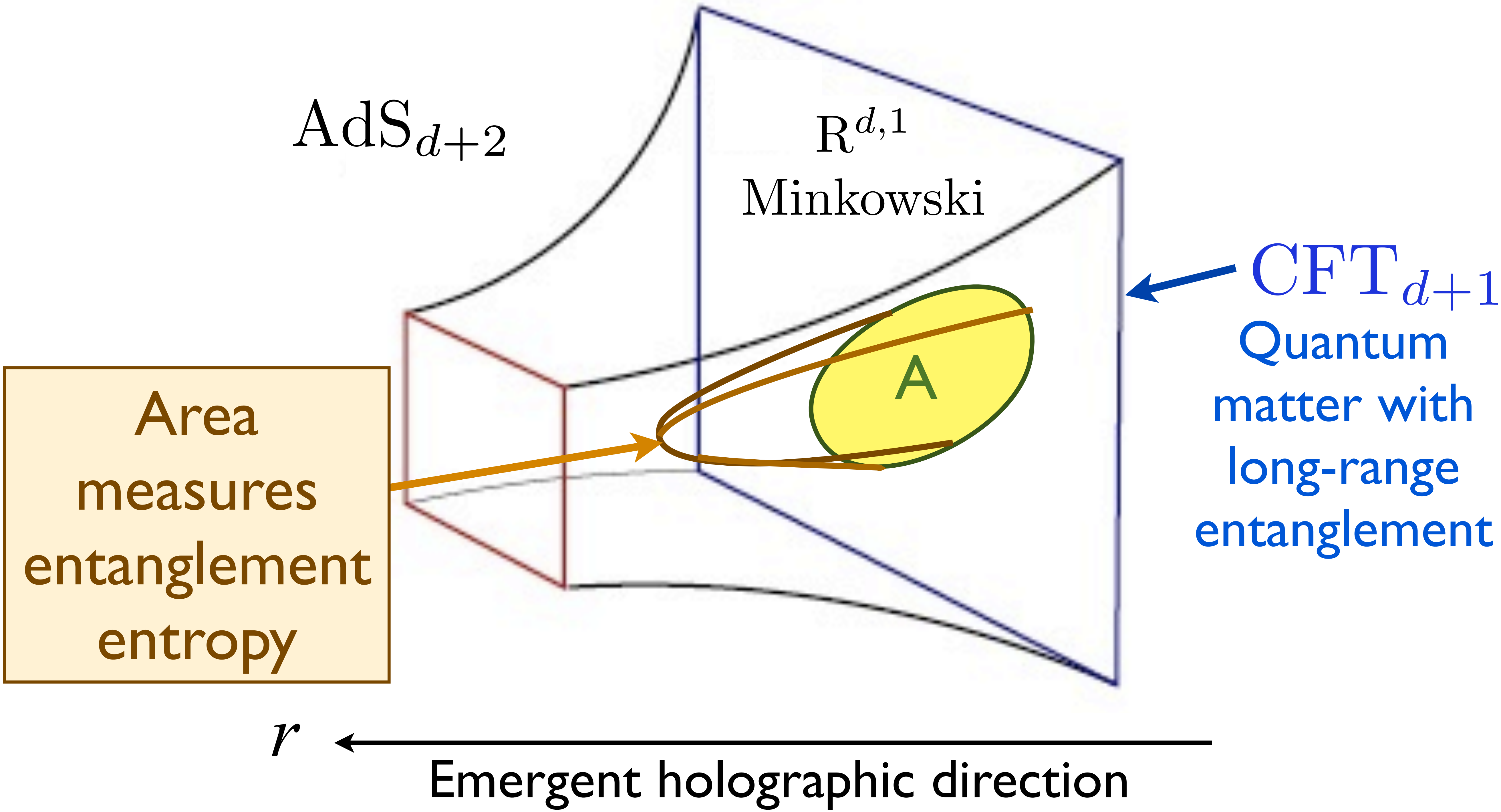
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317



J. McGreevy, arXiv0909.0518



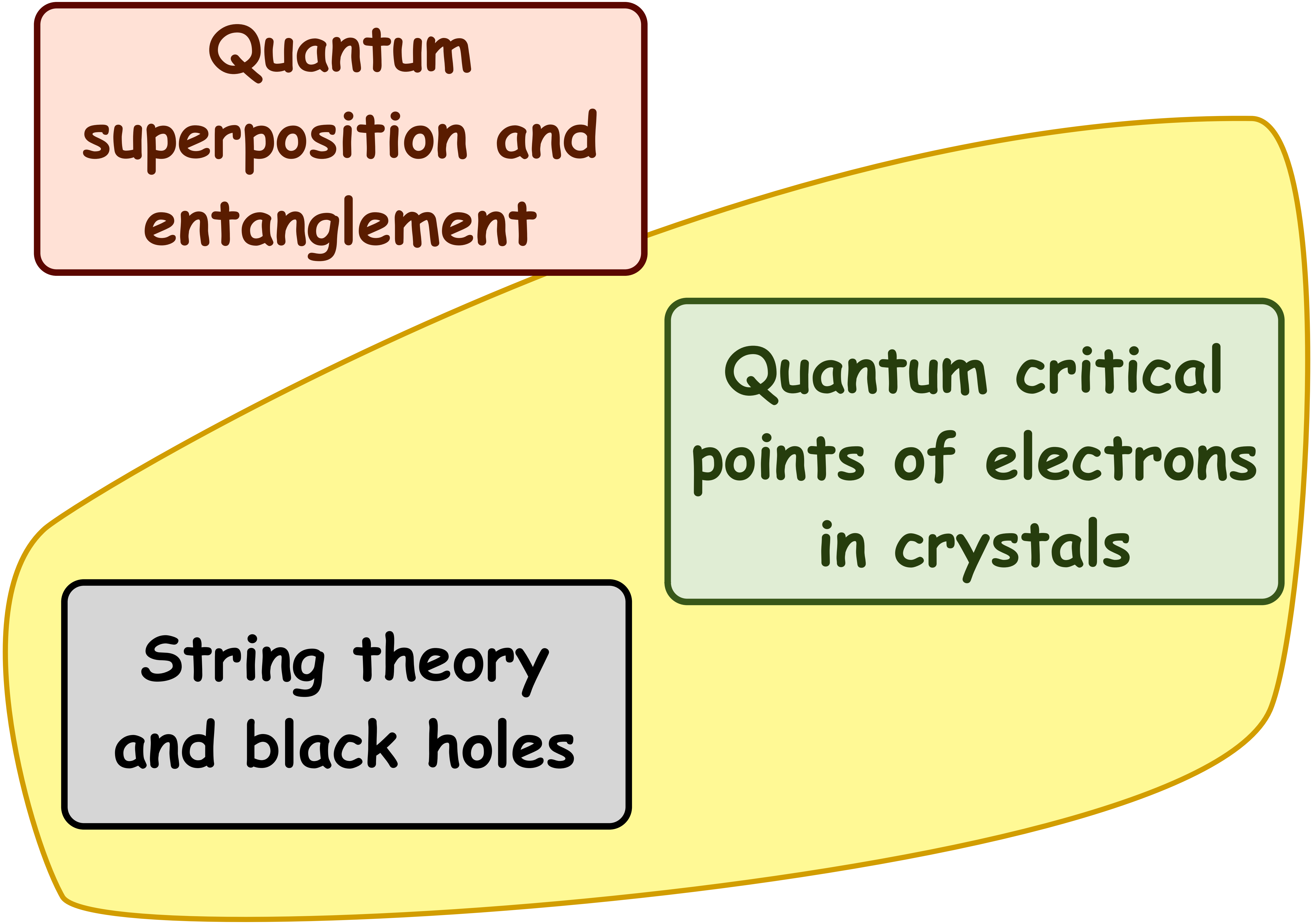
AdS_{d+2}
$R^{d,1}$
Minkowski
A
CFT_{d+1}
Quantum matter with long-range entanglement
r
Emergent holographic direction


AdS_{d+2}
$R^{d,1}$
Minkowski
CFT_{d+1}
Quantum matter with long-range entanglement
A
Area measures entanglement entropy
r
Emergent holographic direction

Quantum superposition and entanglement

Quantum critical points of electrons in crystals

String theory and black holes


Quantum superposition and entanglement
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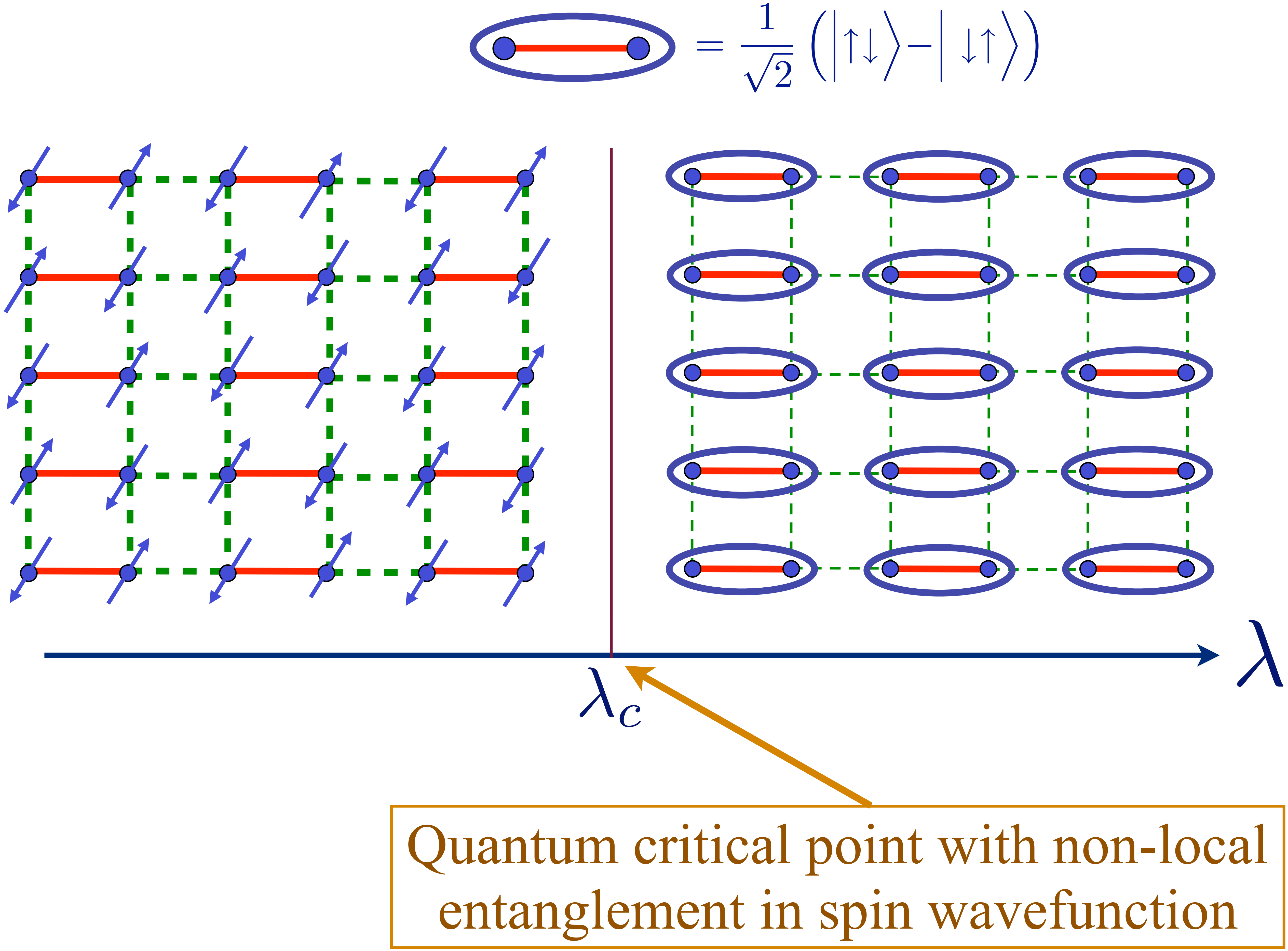
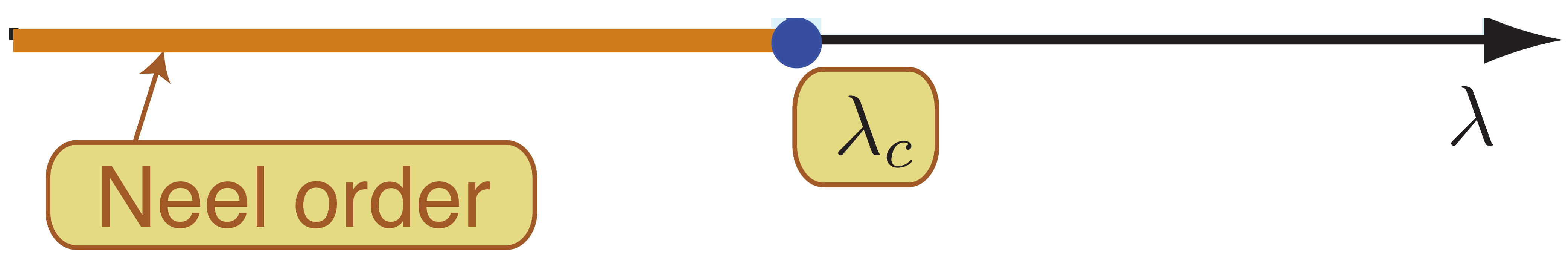
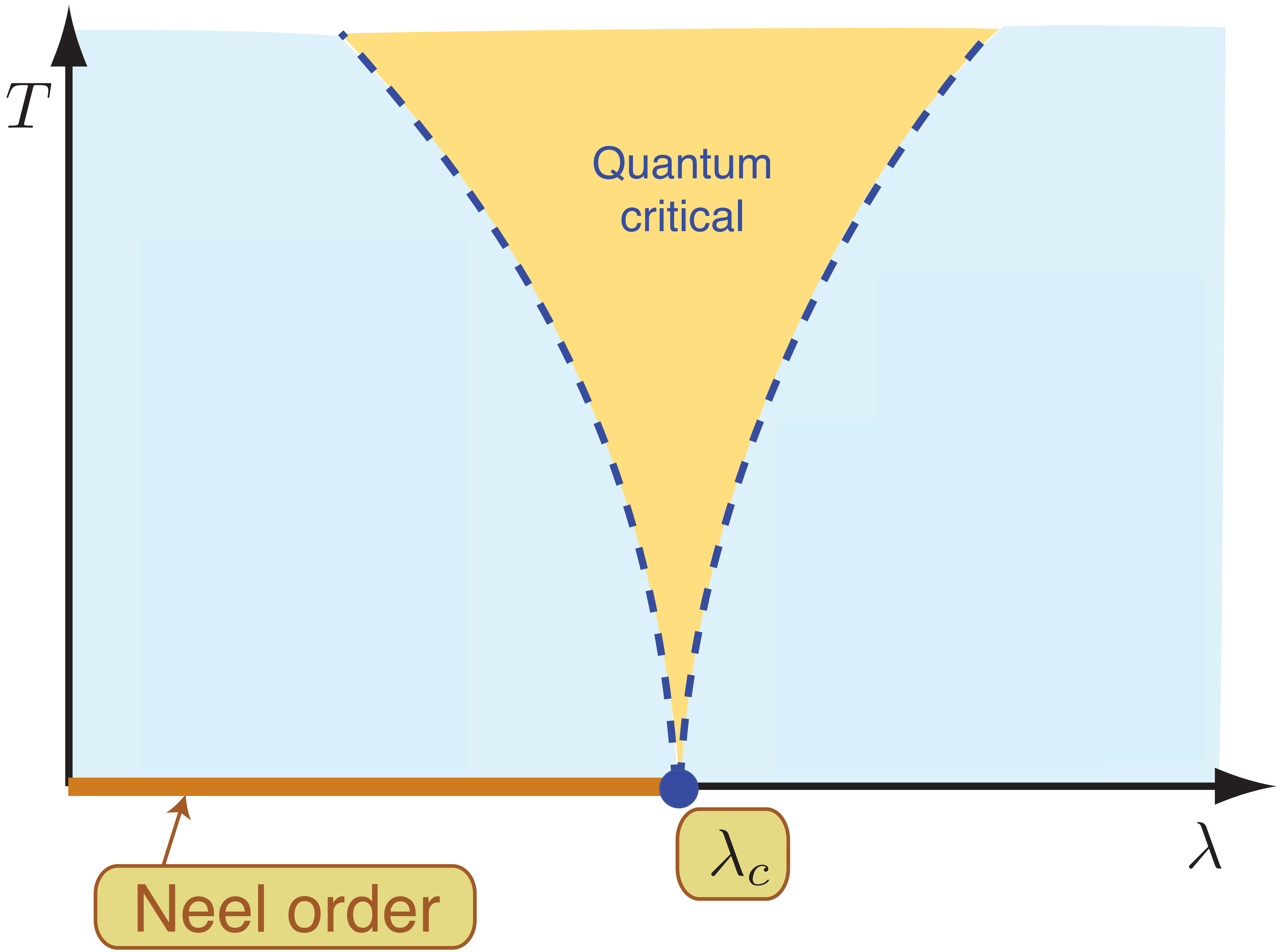
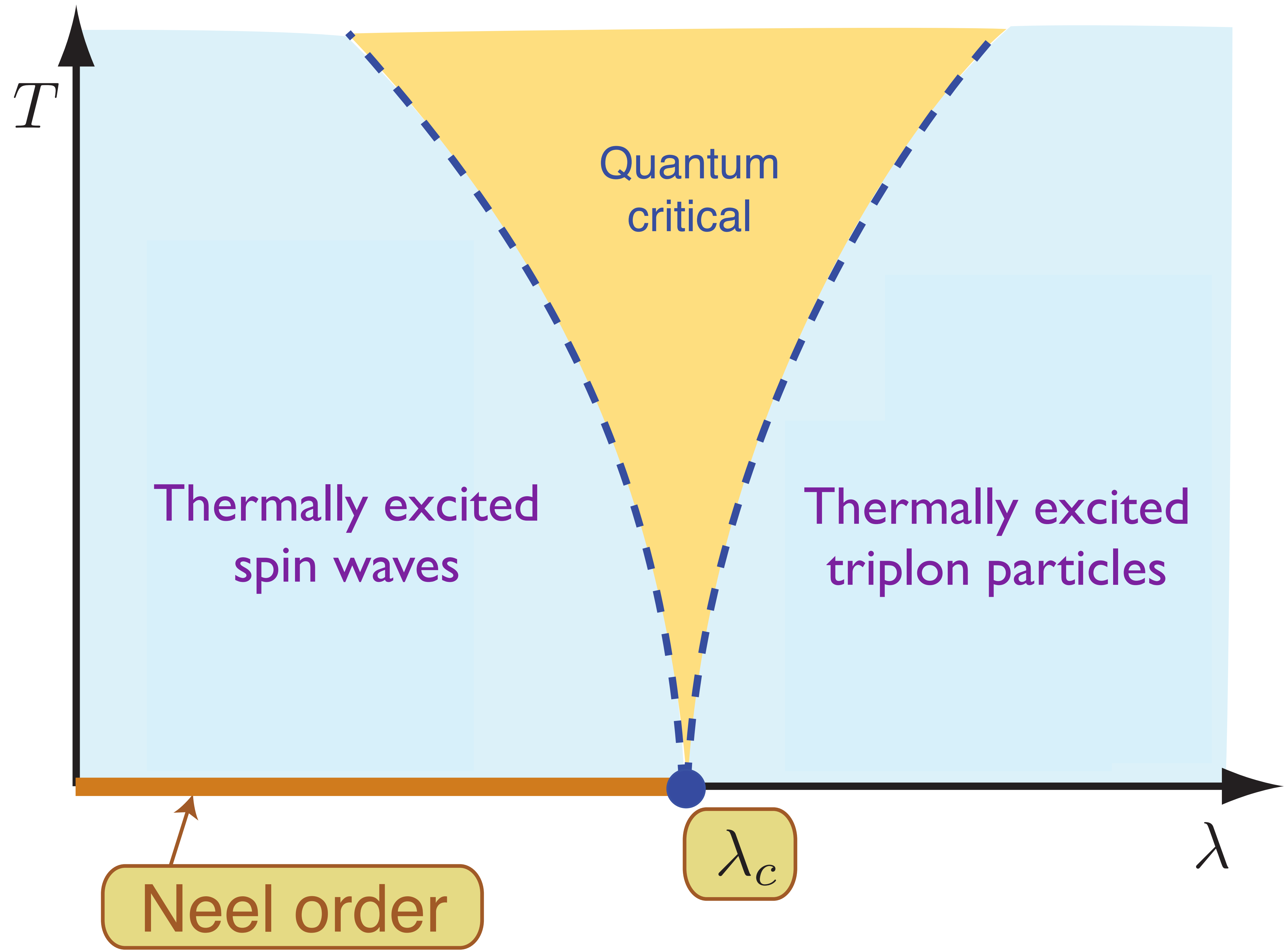
String theory and black holes


$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ
λ_c
Quantum critical point with non-local entanglement in spin wavefunction


Neel order
λ_c
λ


T
Quantum critical
Neel order
λ_c
λ

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



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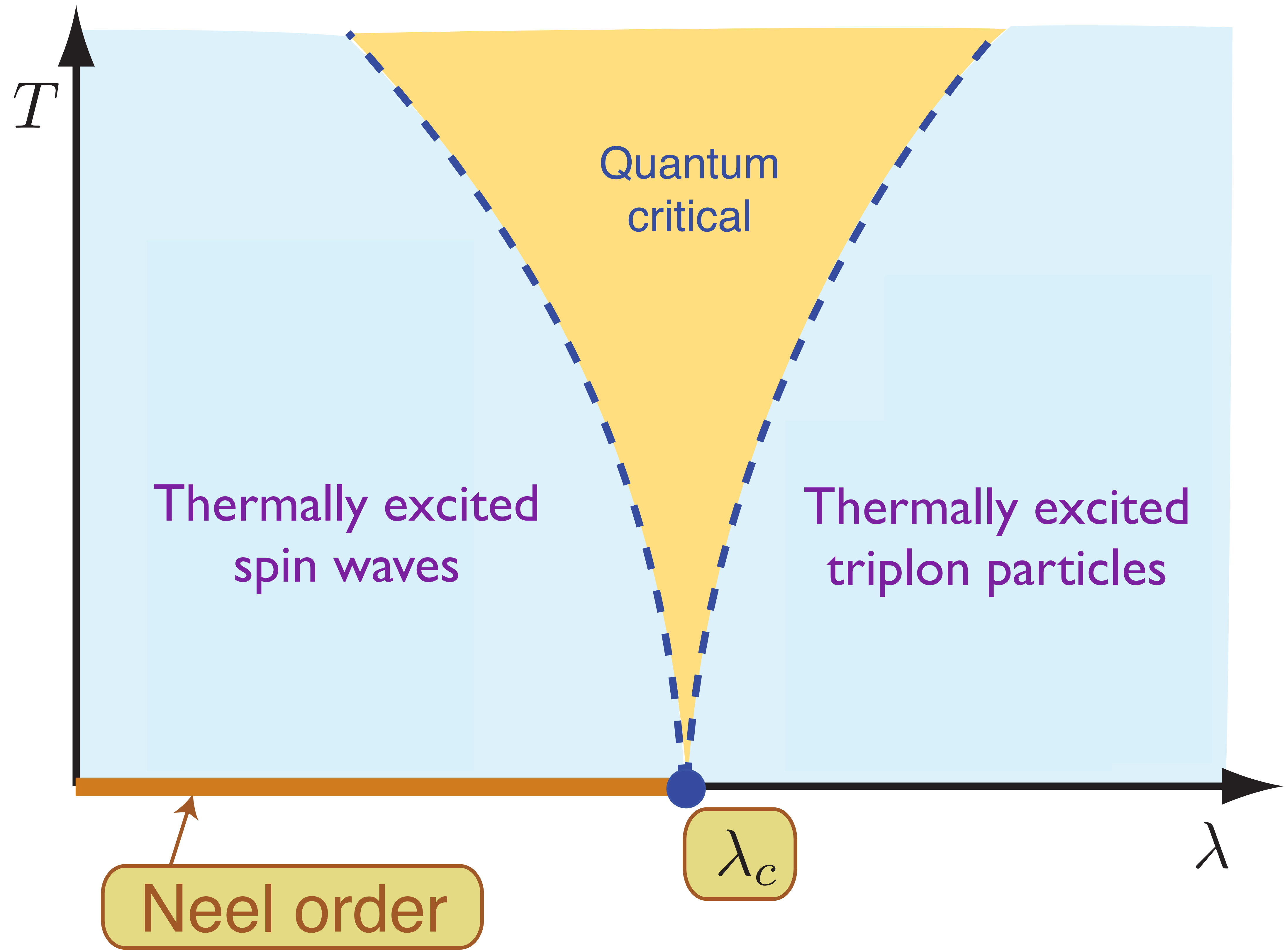

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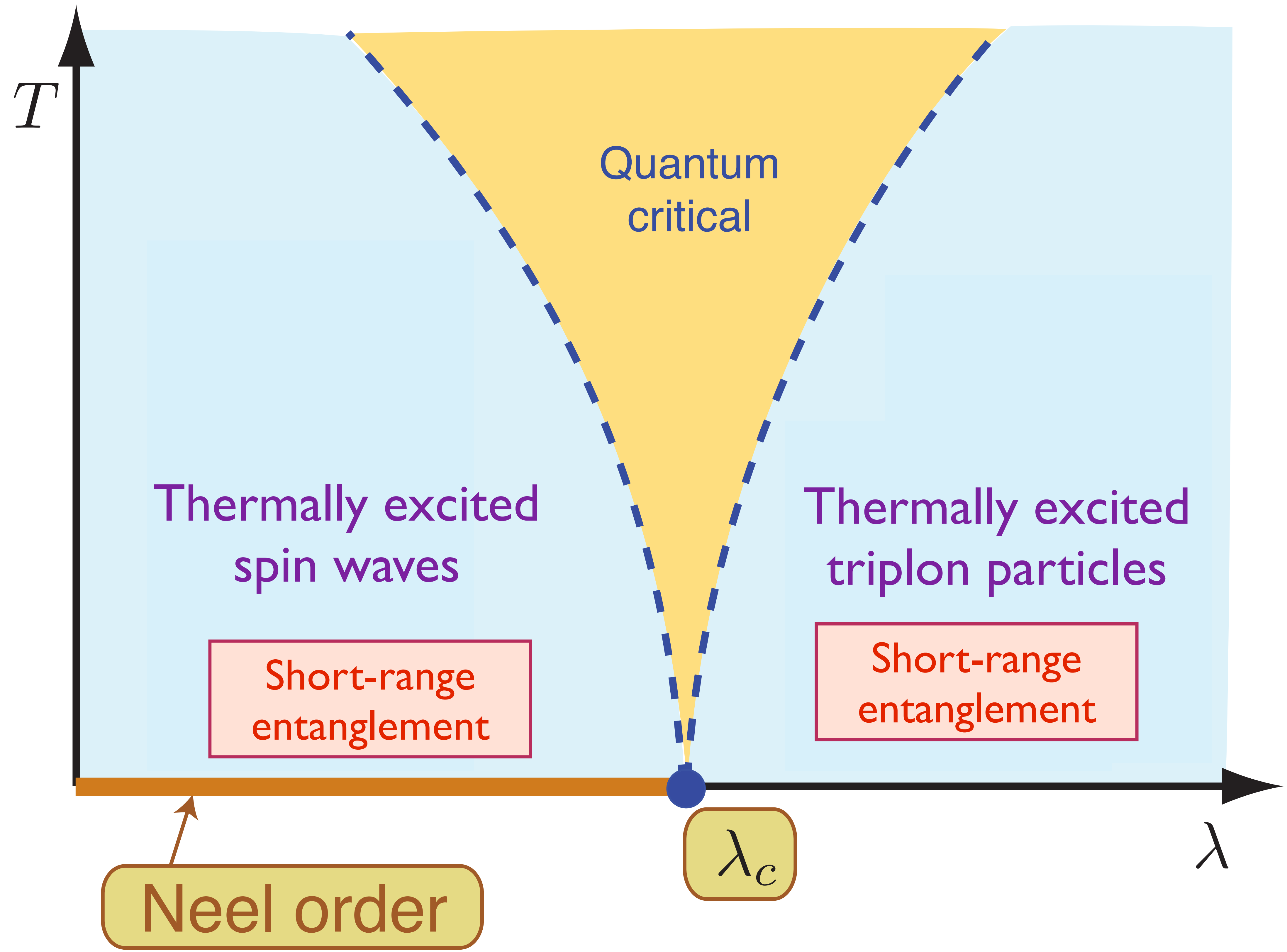

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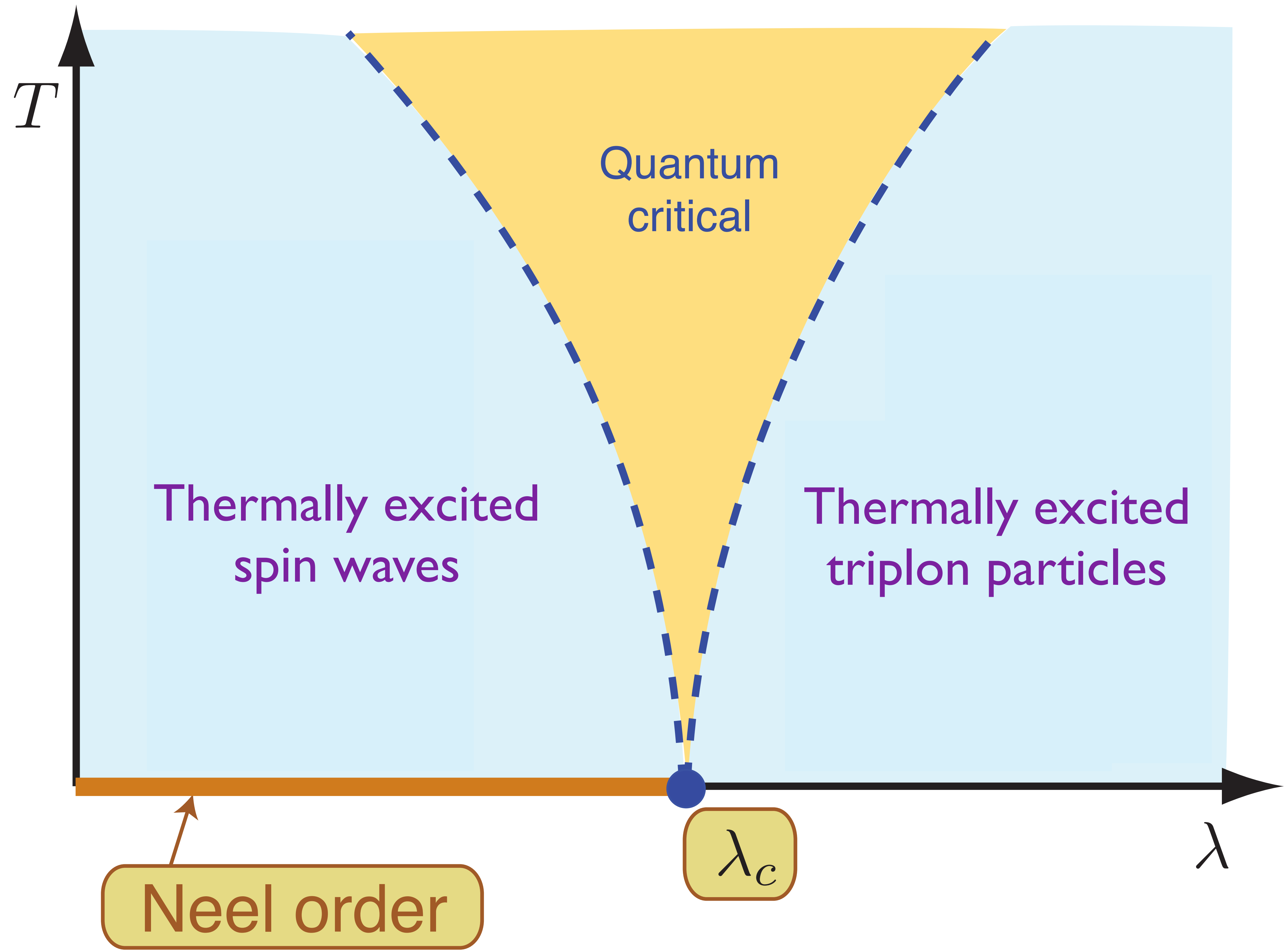

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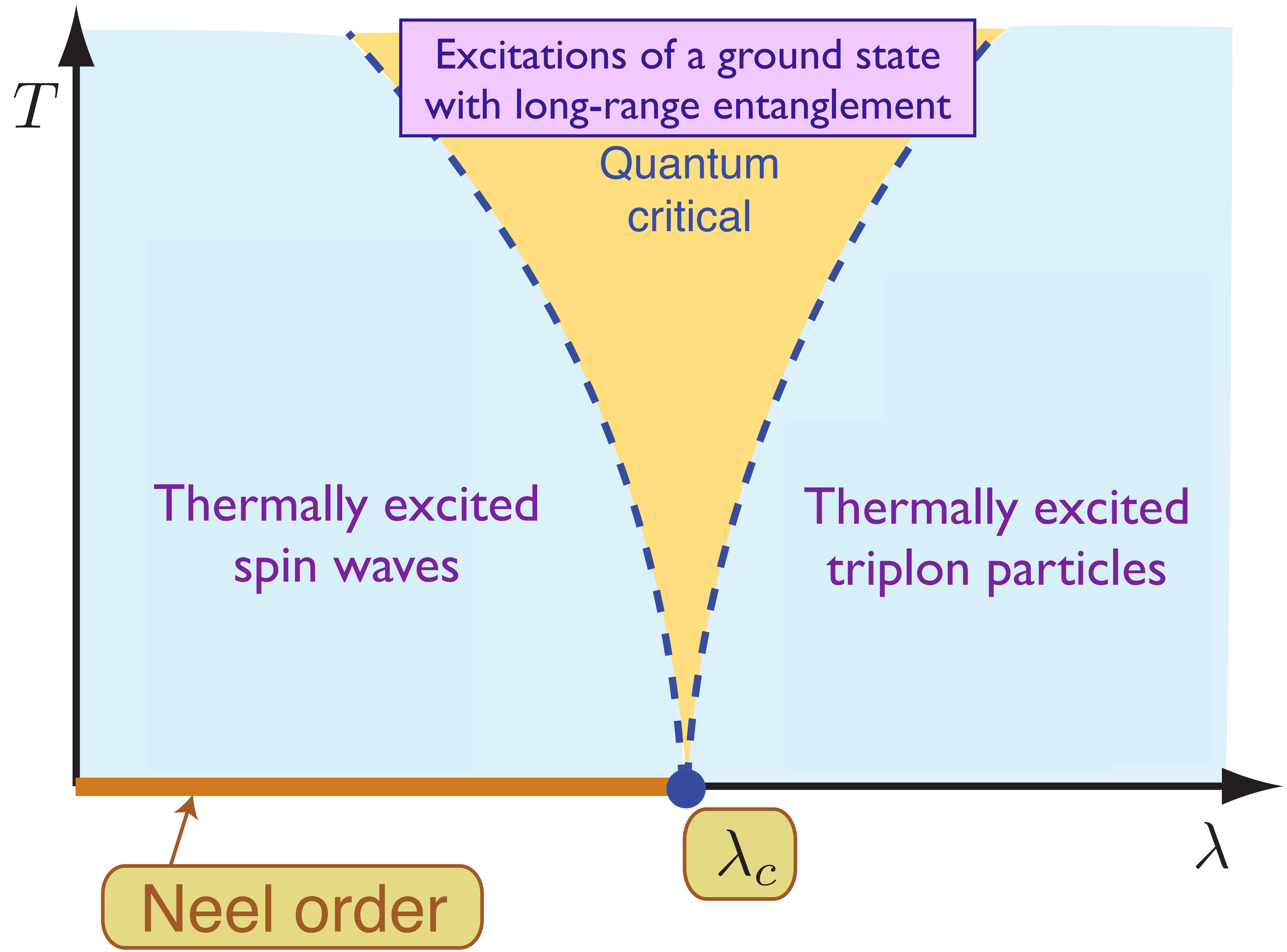

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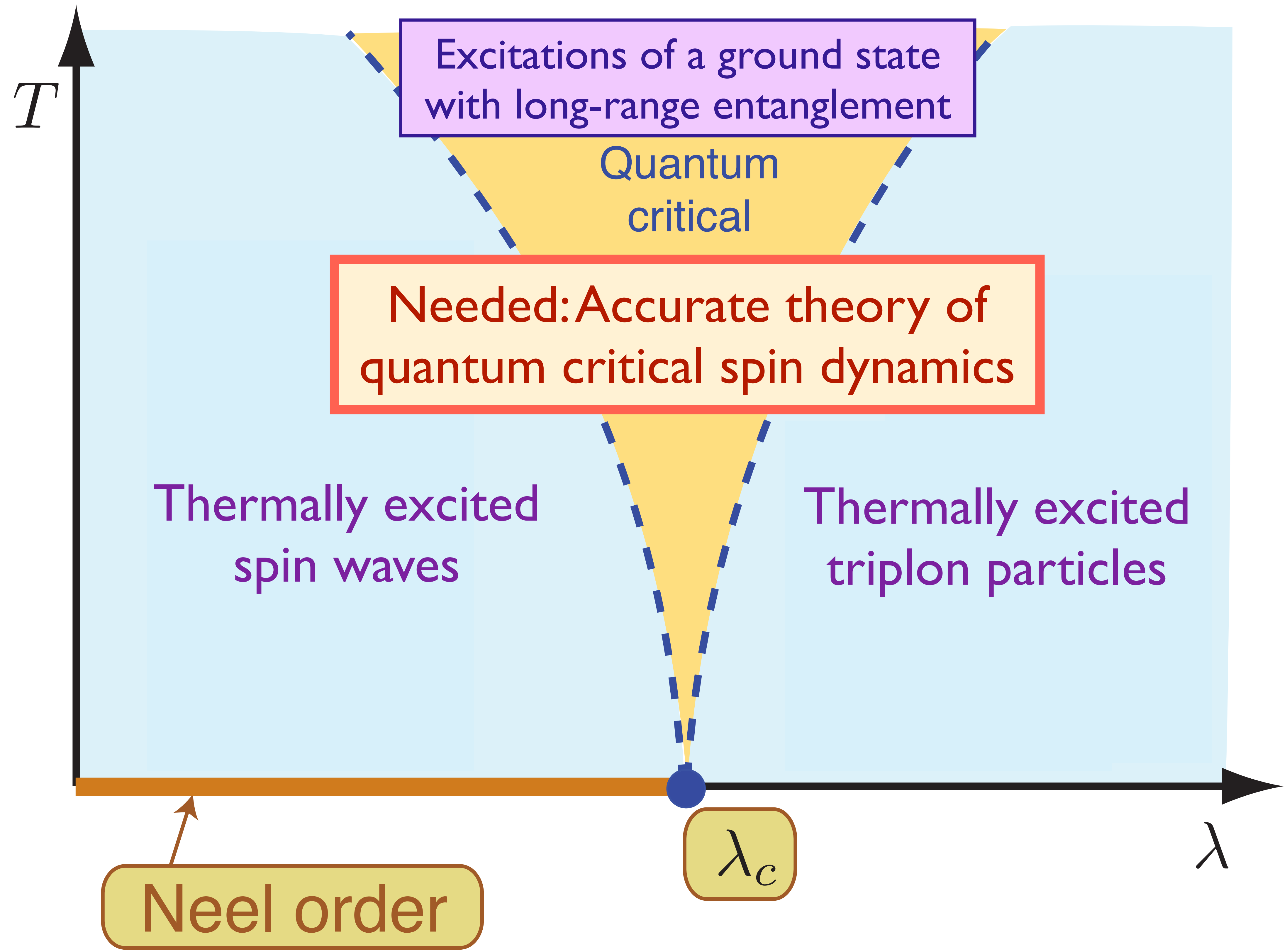
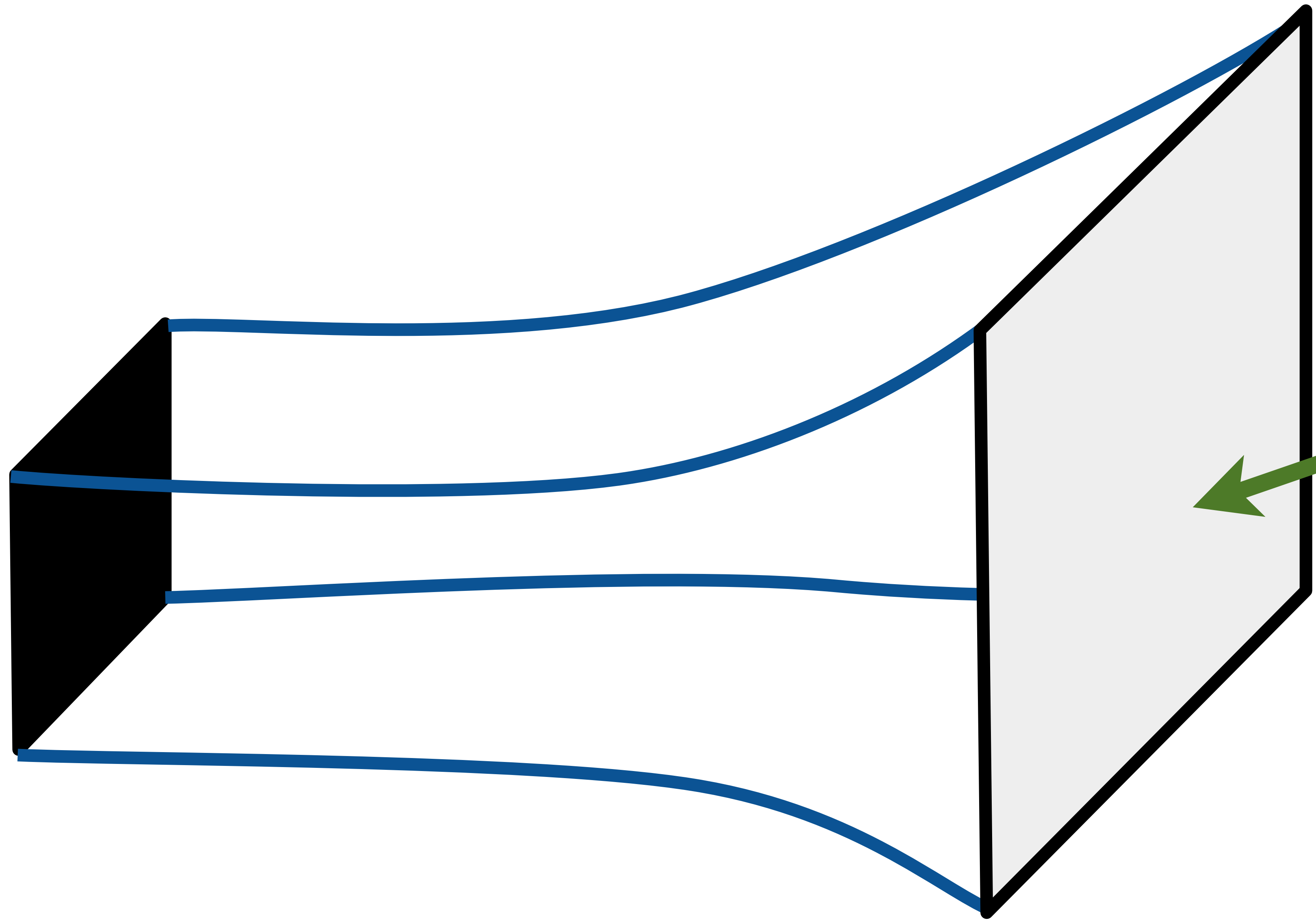
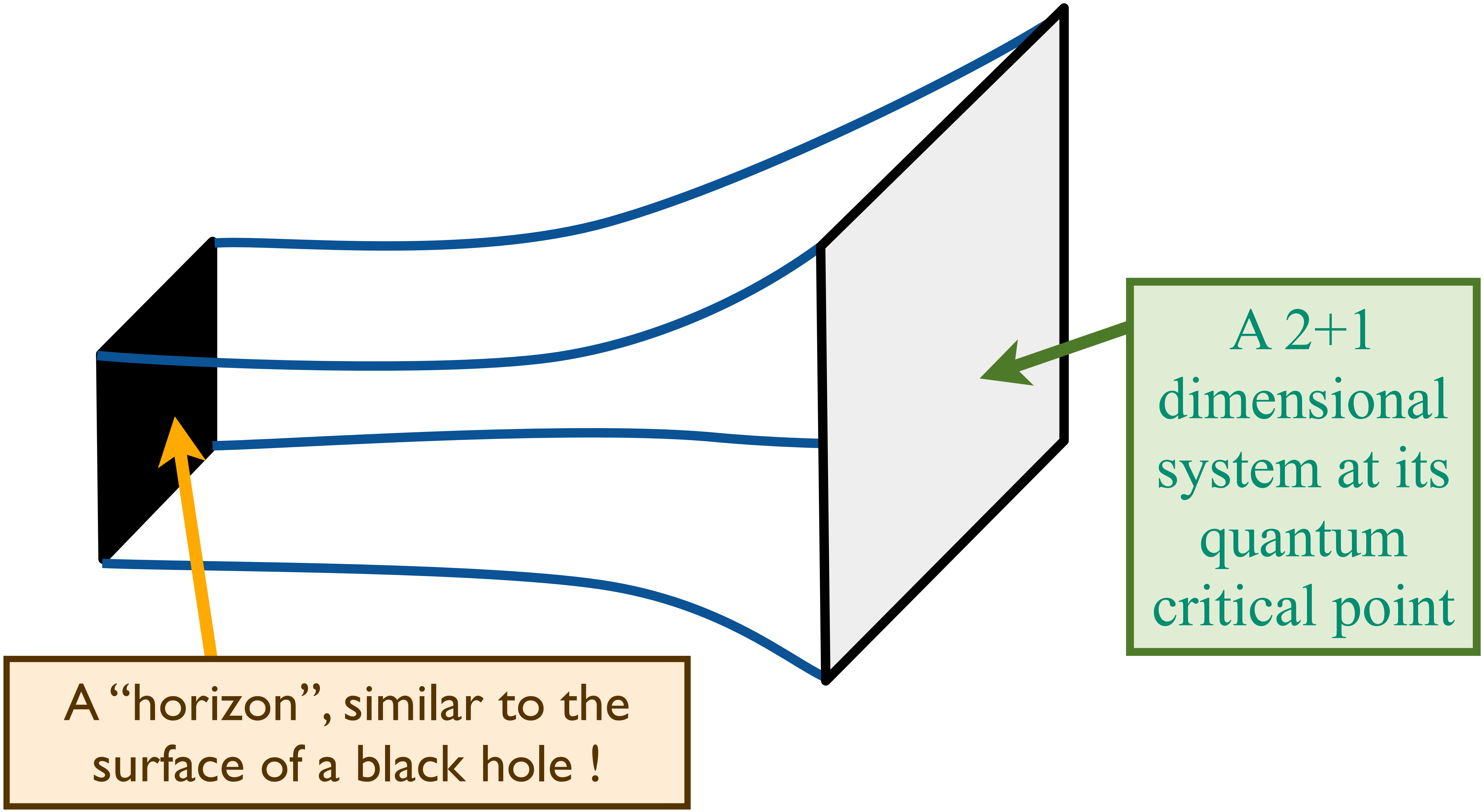


String theory at non-zero temperatures



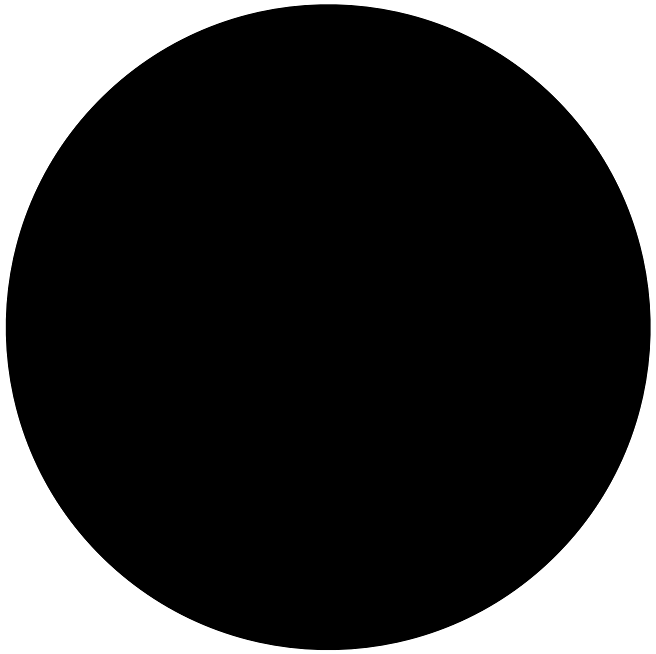
A 2+1 dimensional system at its quantum critical point

String theory at non-zero temperatures



Black Holes

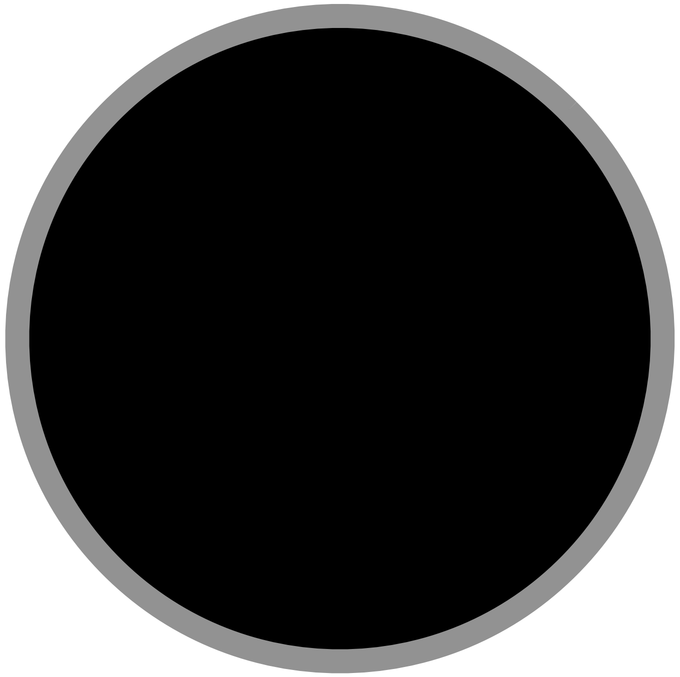
Objects so massive that light is gravitationally bound to them.



Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.

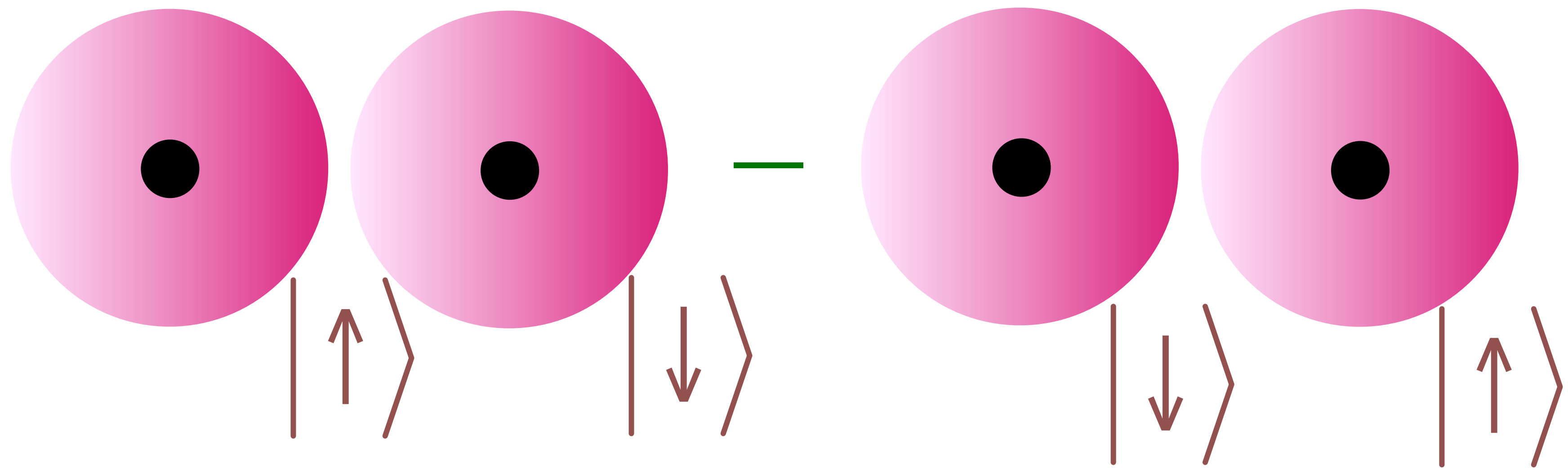


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

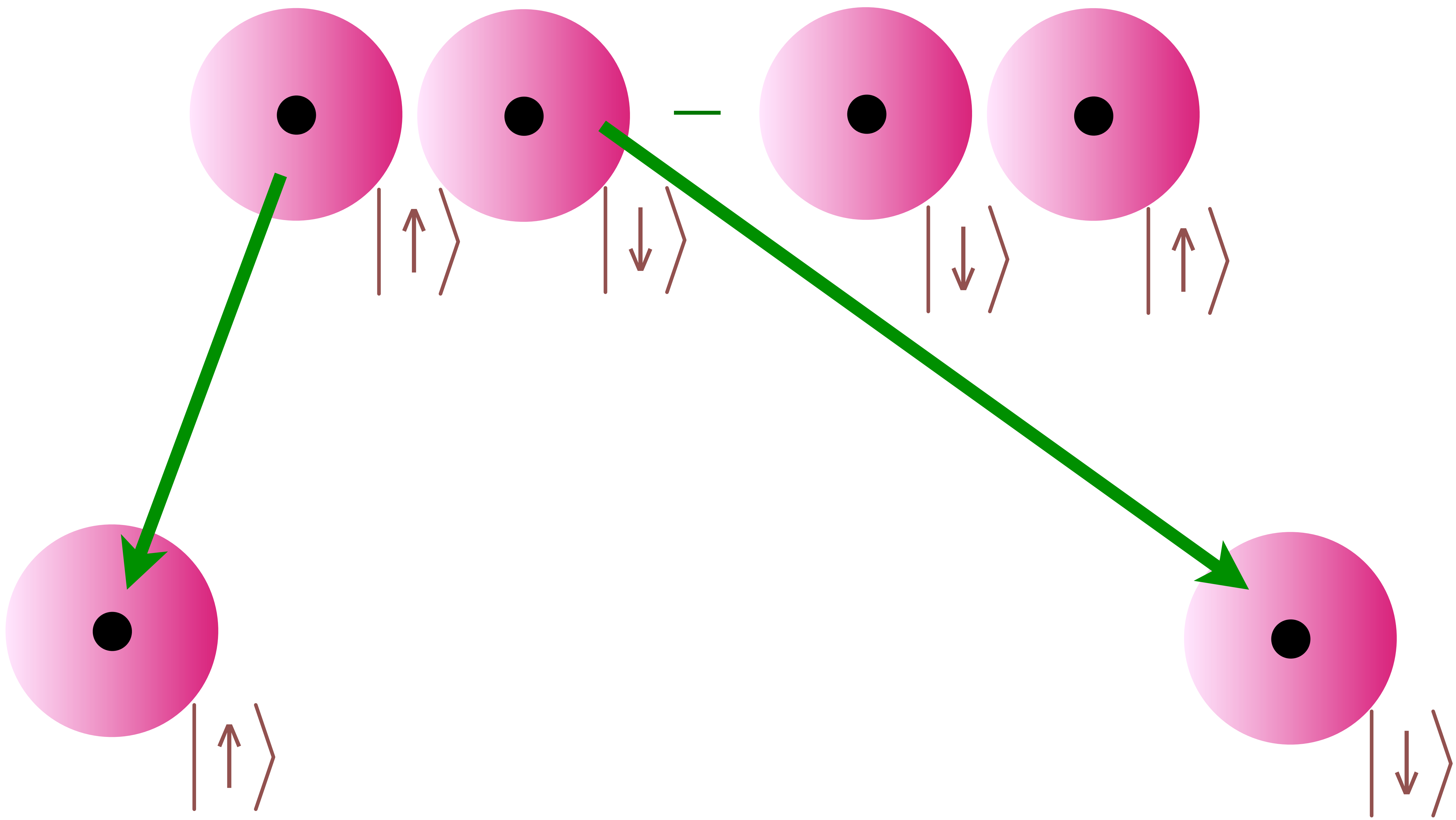
Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

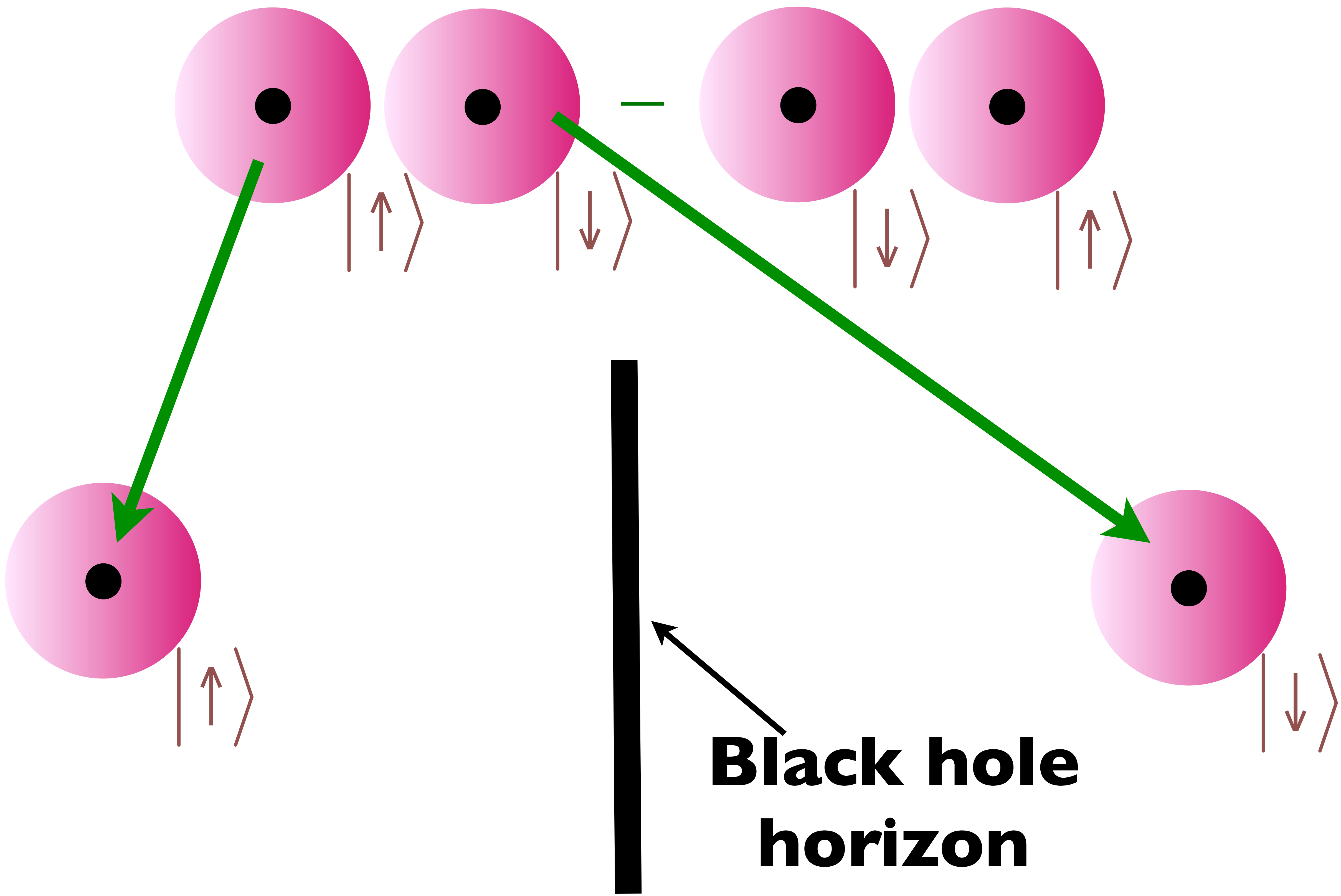
Quantum Entanglement across a black hole horizon



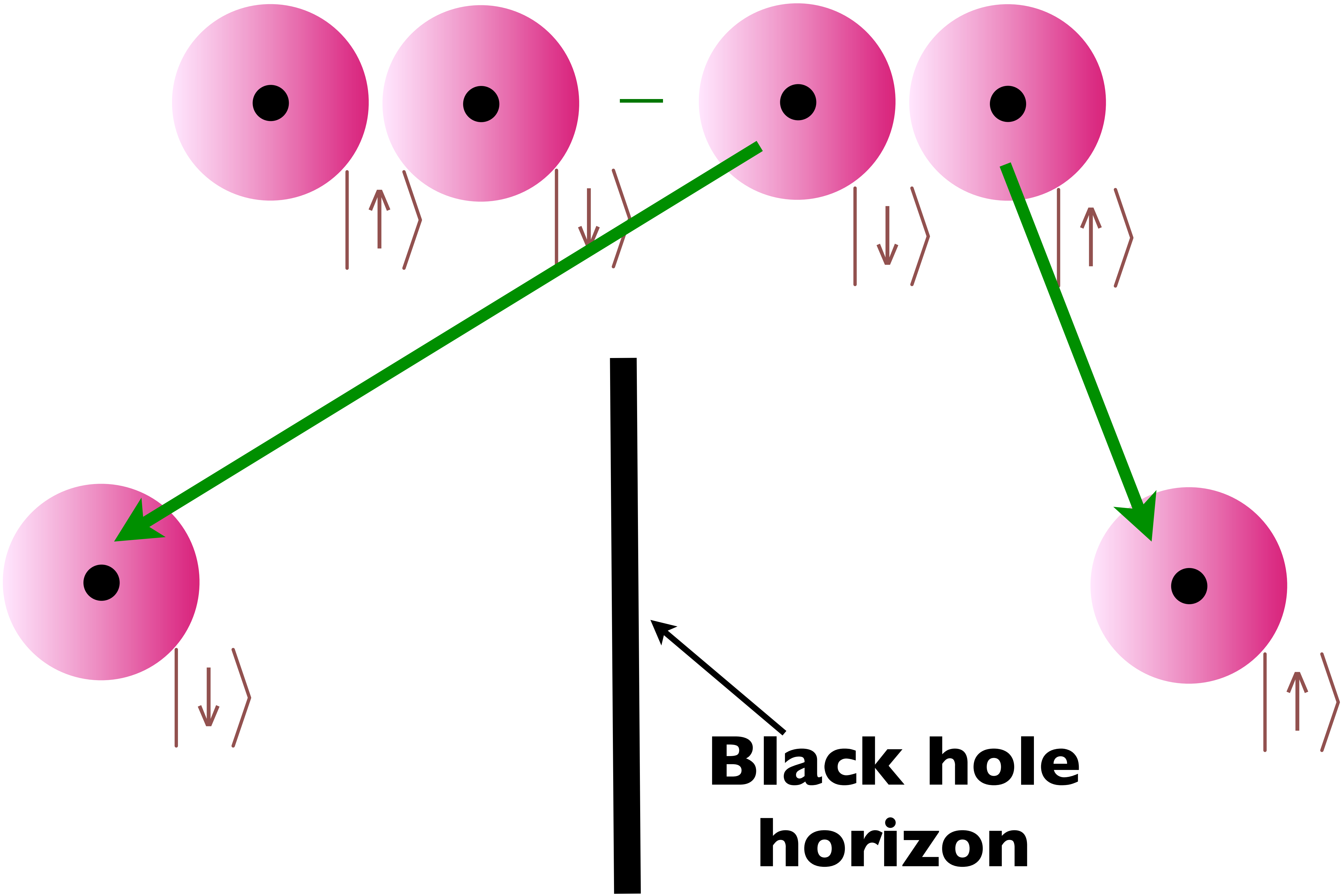
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

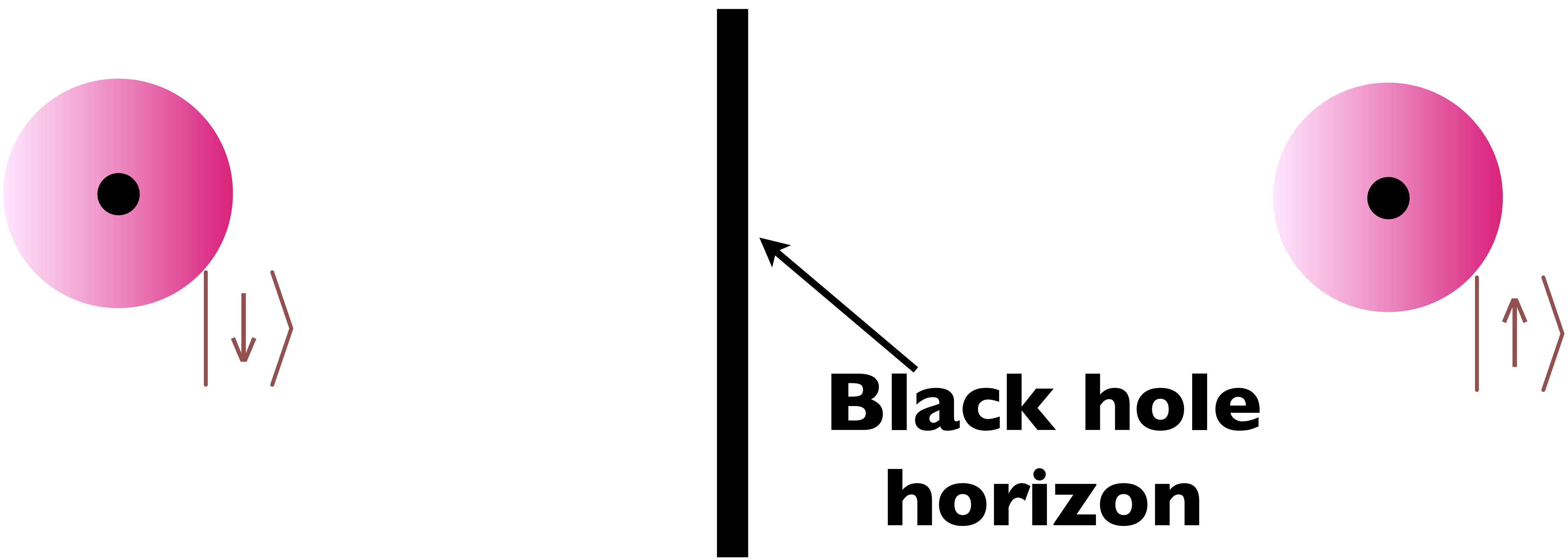


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

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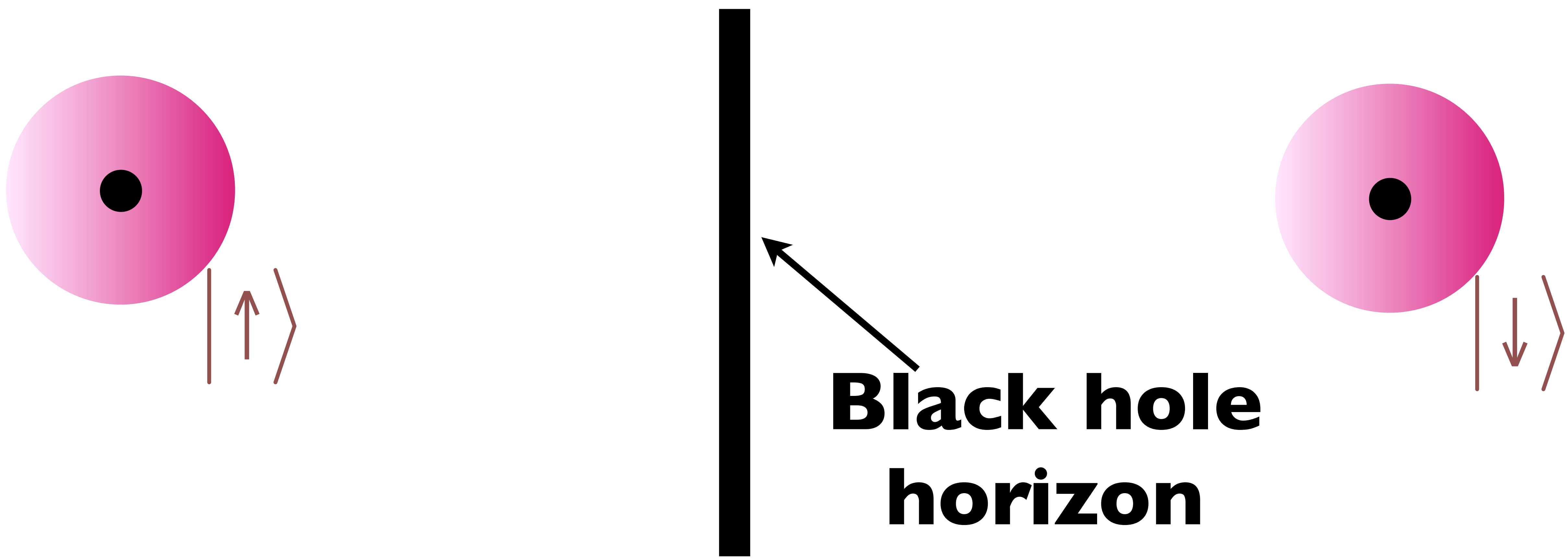
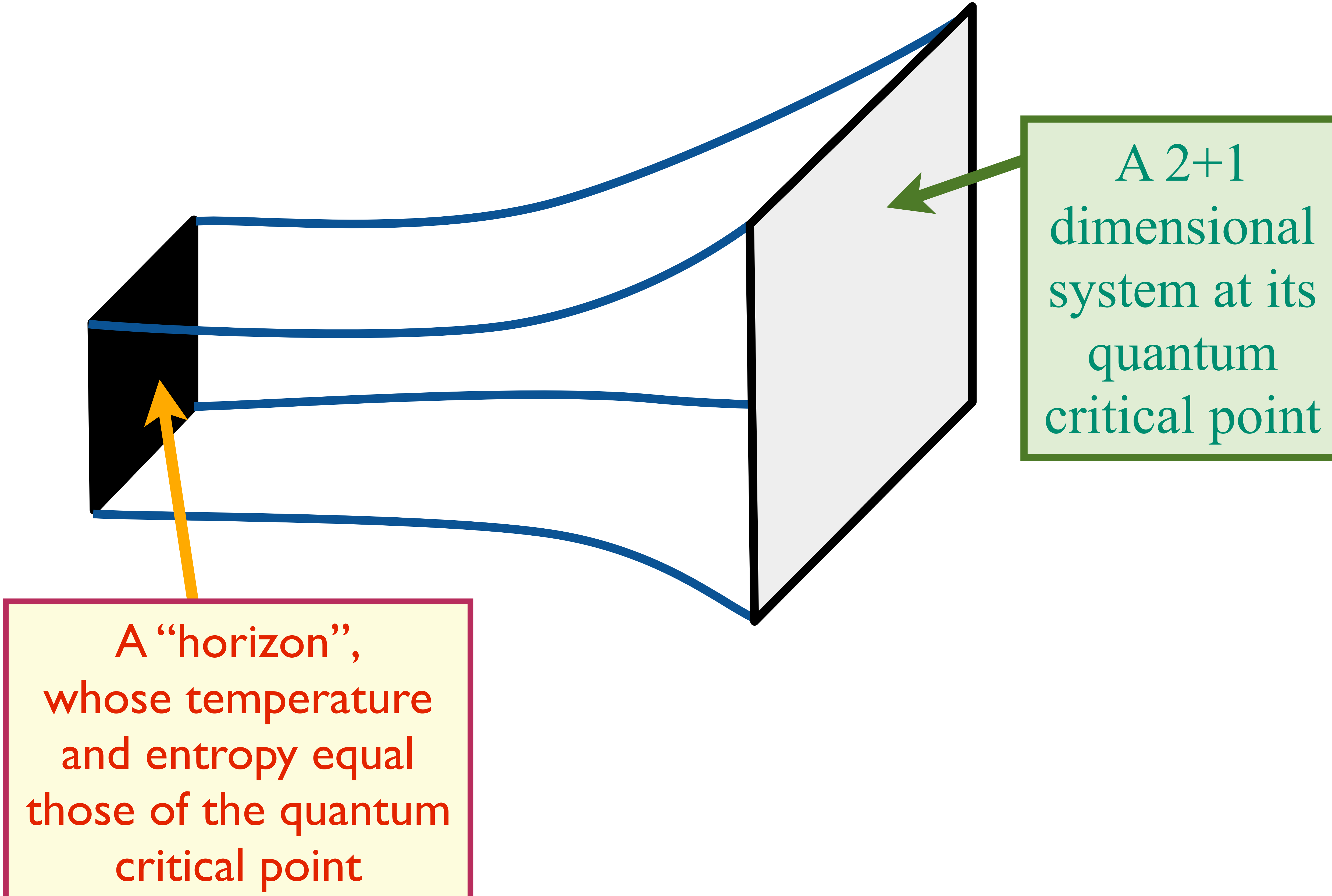


Quantum Entanglement across a black hole horizon

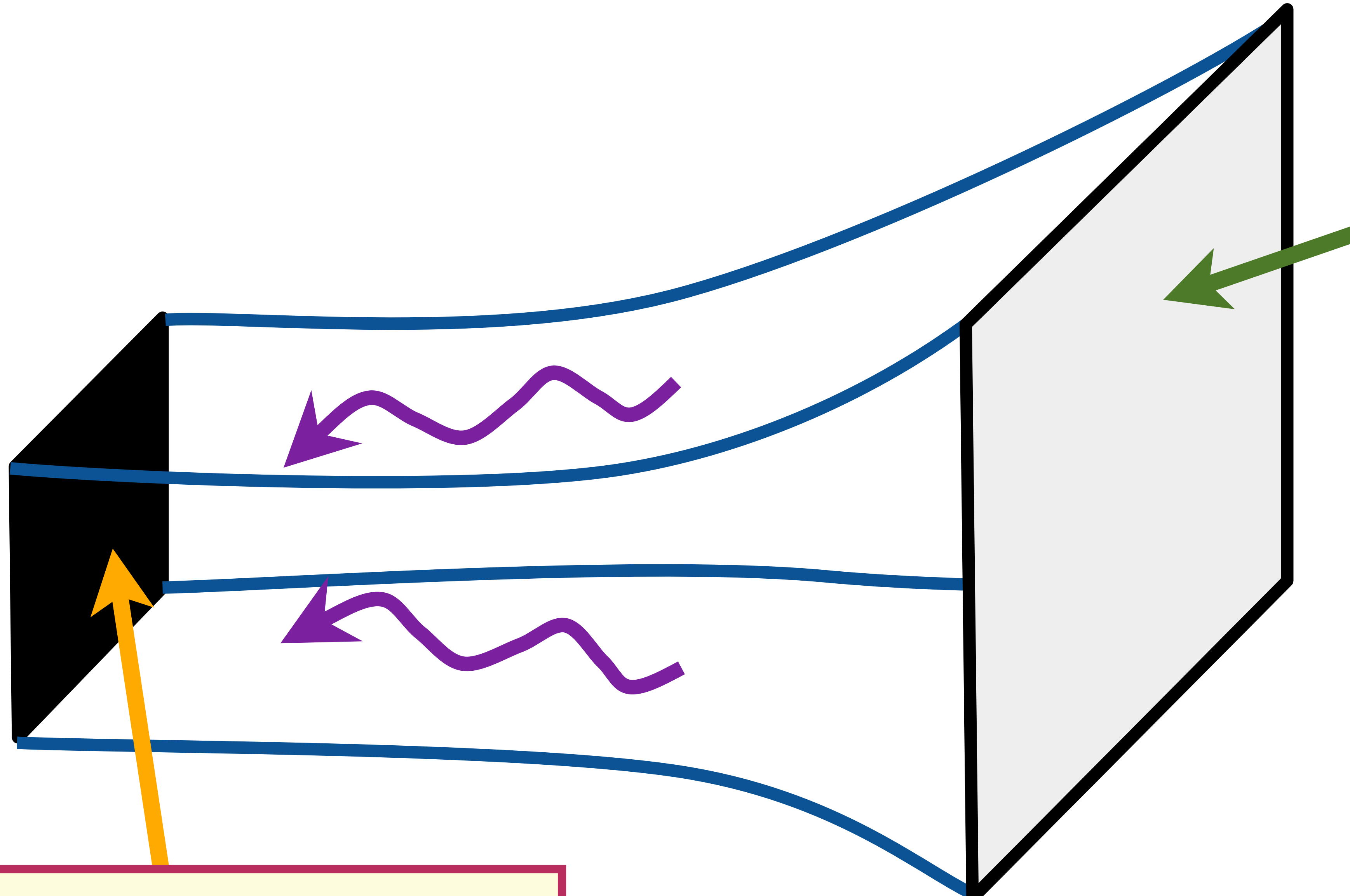
There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

String theory at non-zero temperatures



String theory at non-zero temperatures

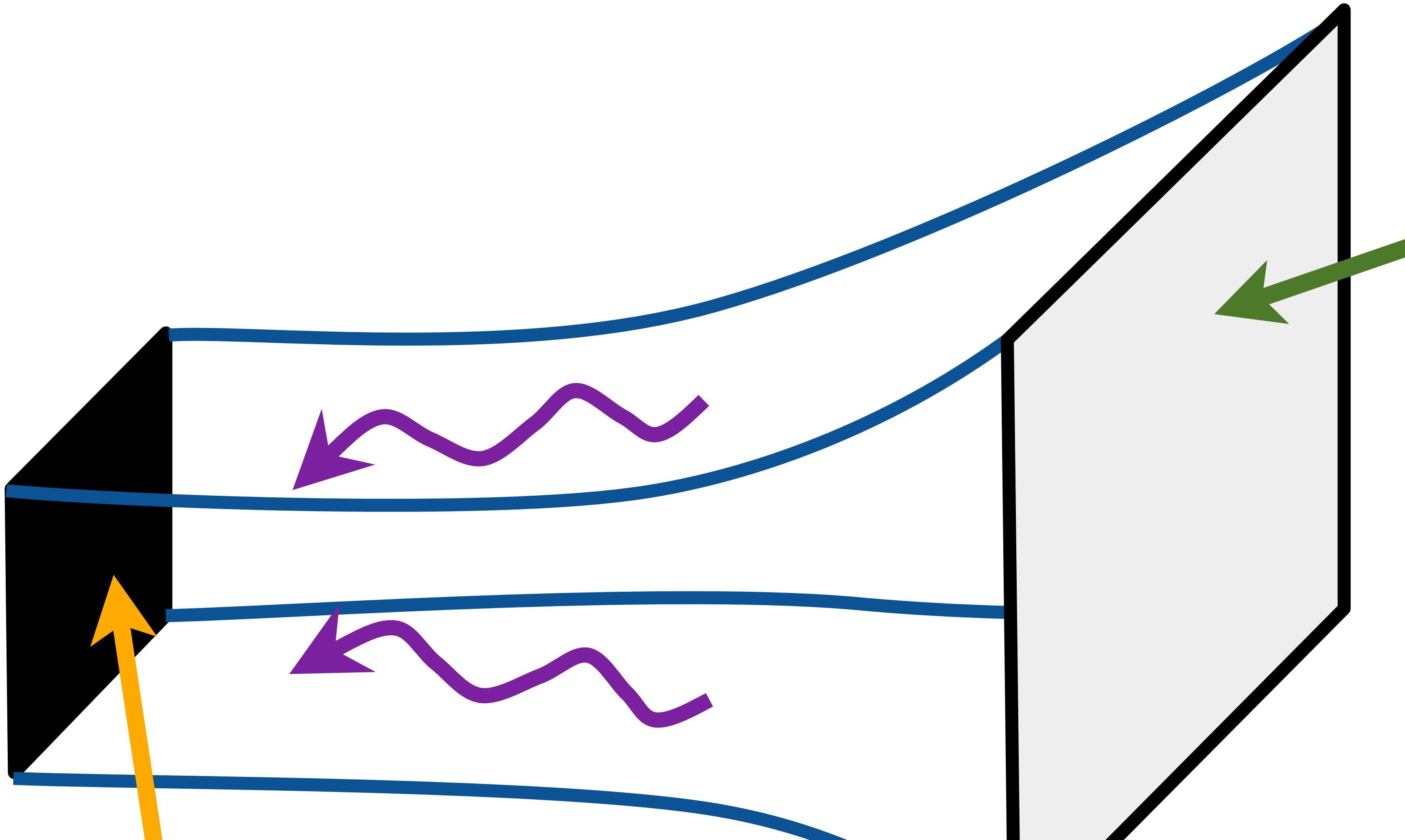


A 2+1 dimensional system at its quantum critical point

A “horizon”, whose temperature and entropy equal those of the quantum critical point

Friction of quantum criticality = waves falling into black brane

String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

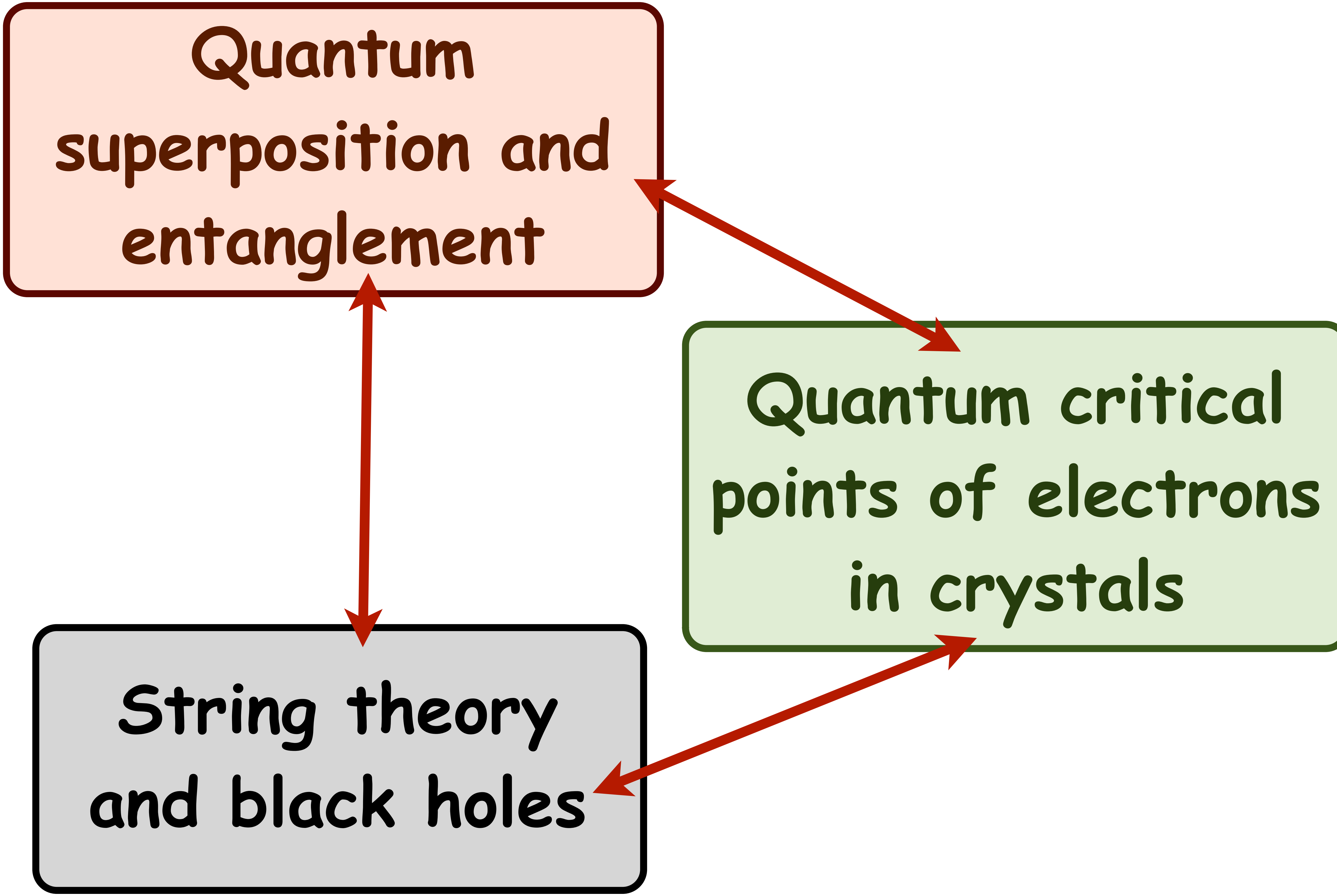
A "horizon", whose temperature and entropy equal those of the quantum critical point

An (extended) Einstein-Maxwell provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply)

Quantum superposition and entanglement

Quantum critical points of electrons in crystals

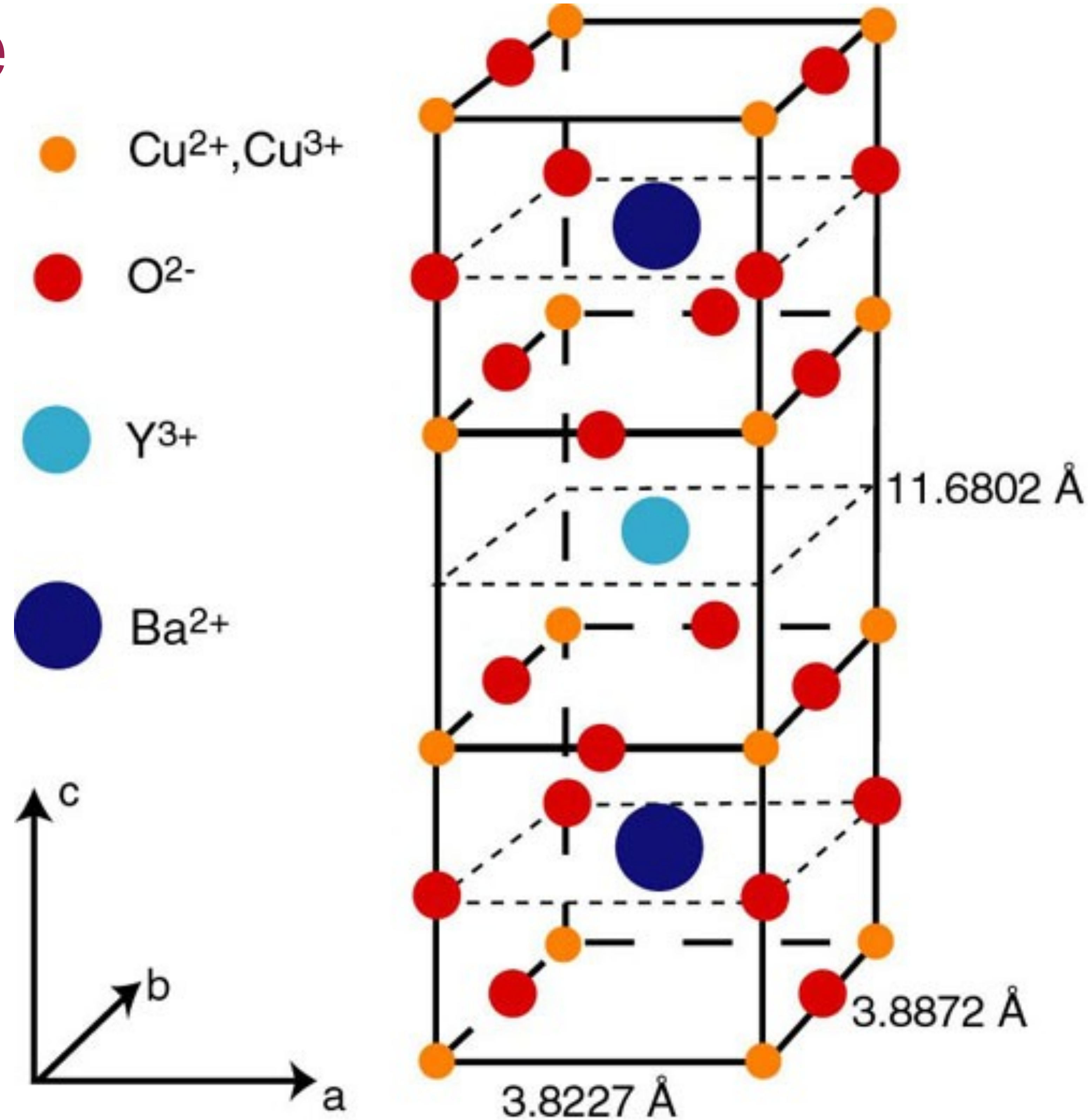
String theory and black holes


Quantum superposition and entanglement
Quantum critical points of electrons in crystals
String theory and black holes

Metals, “strange metals”, and high temperature superconductors

Insights from gravitational “duals”

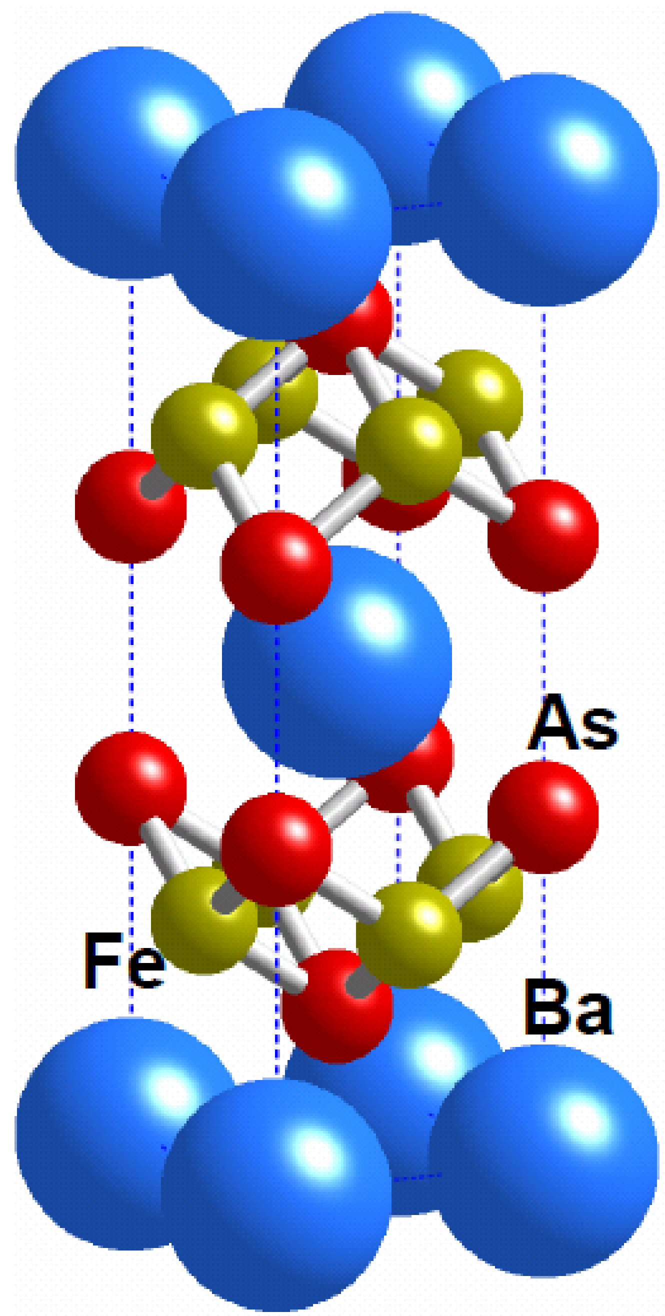
High temperature superconductors



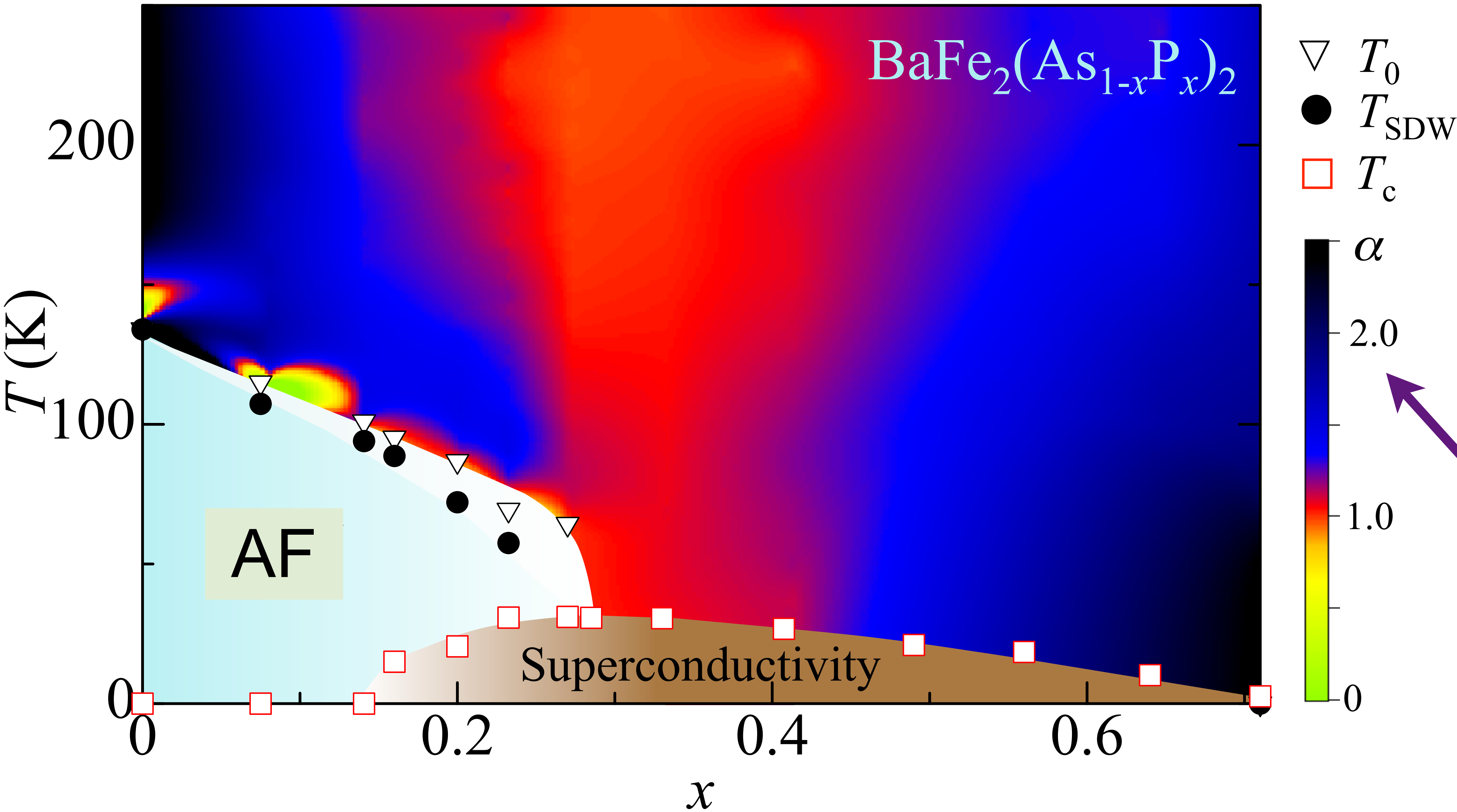
$$YBa_2Cu_3O_{6+x}$$

Iron pnictides:

a new class of high temperature superconductors



Temperature-doping phase diagram of the iron pnictides:



Resistivity
$\sim \rho_0 + AT^\alpha$

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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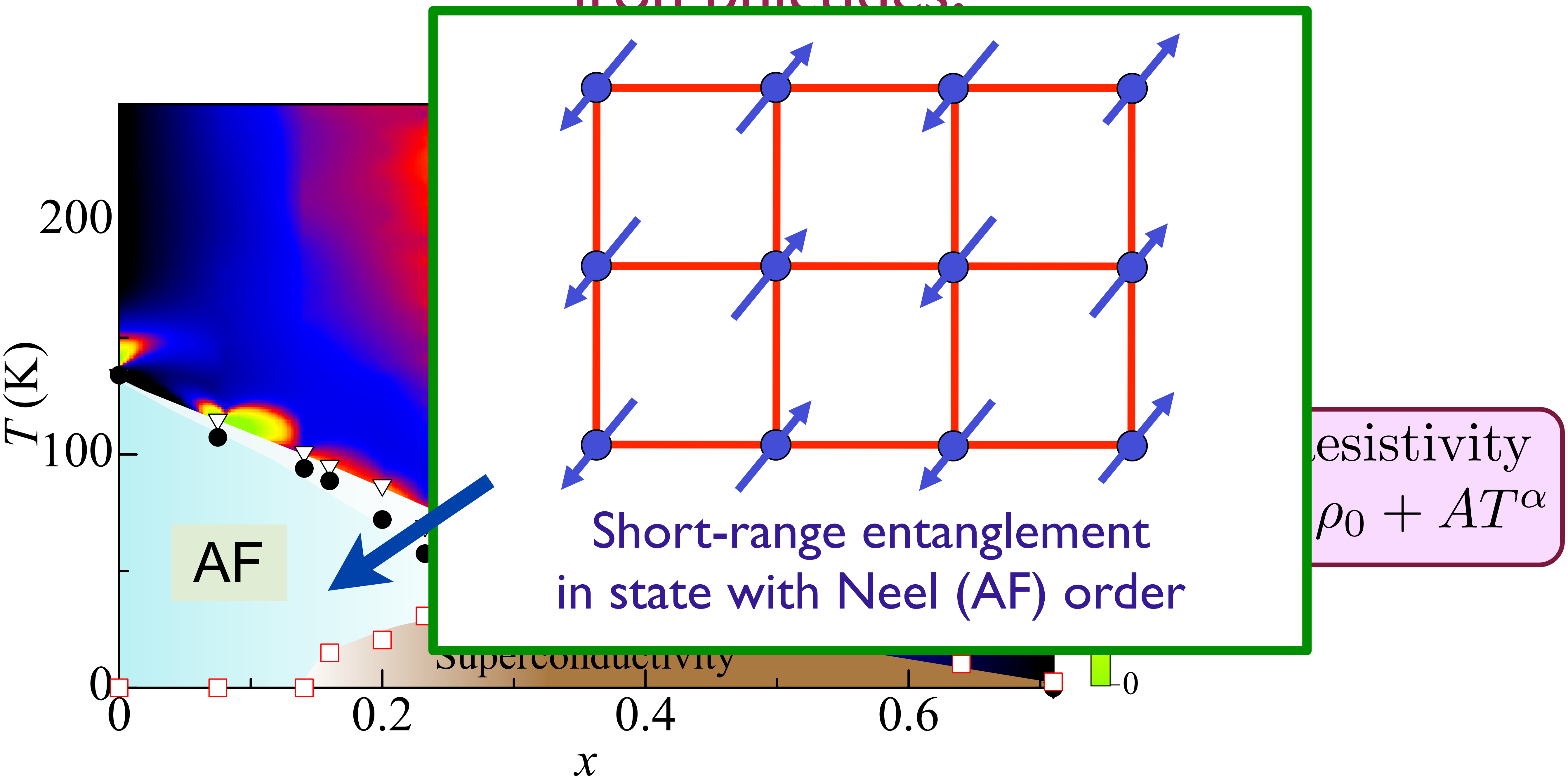

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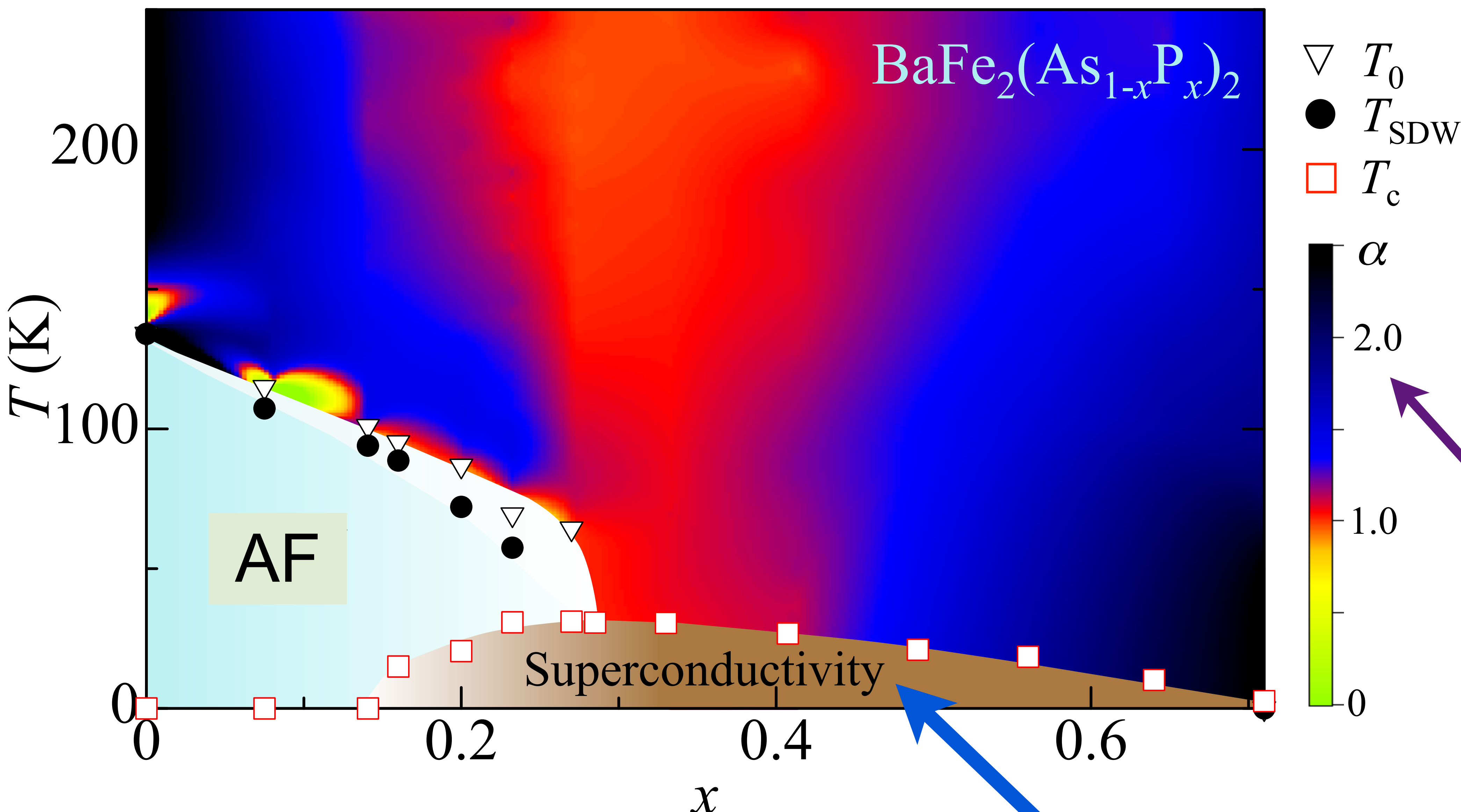
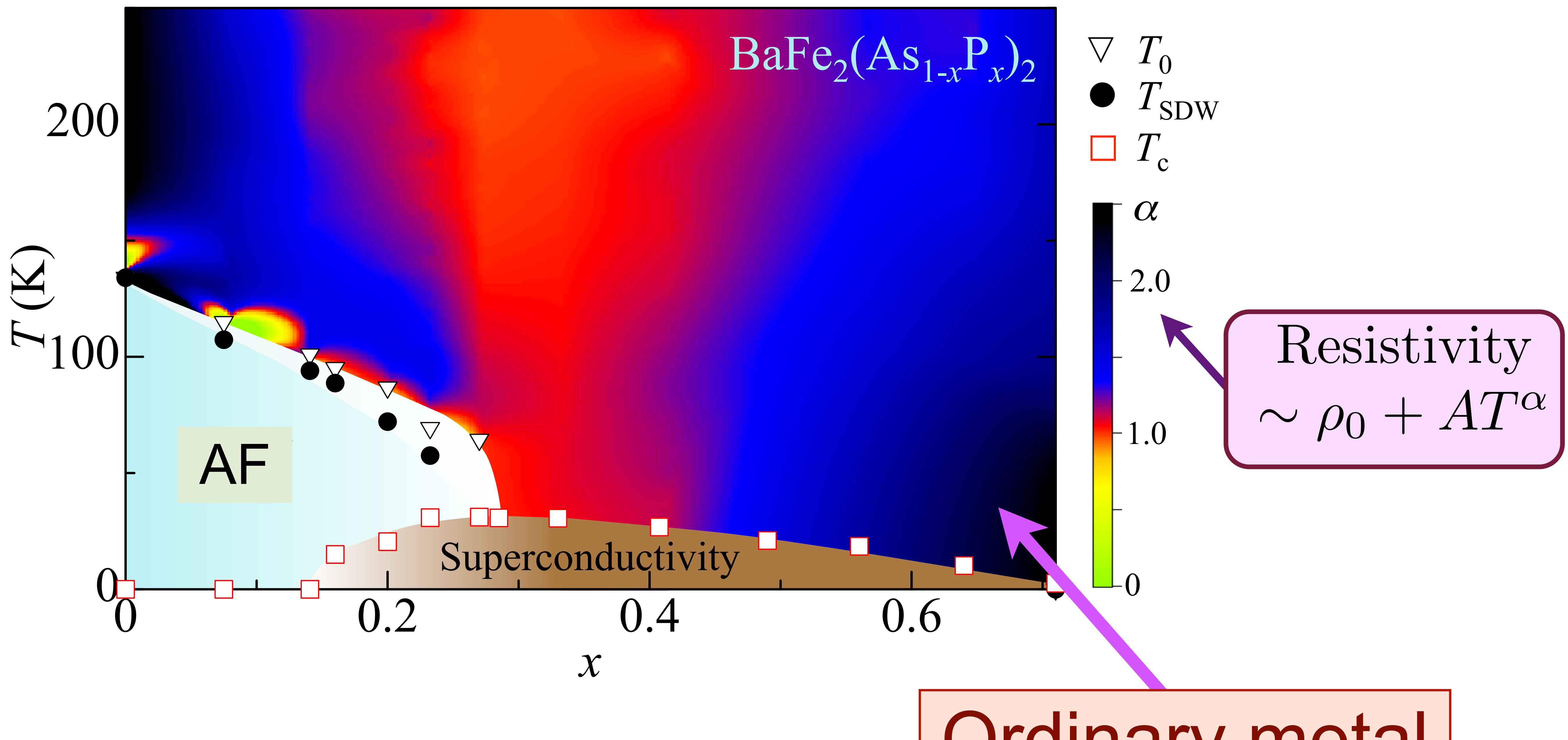


Resistivity
$\sim \rho_0 + AT^\alpha$

S. Kasahara, T. Shib
H. Ik

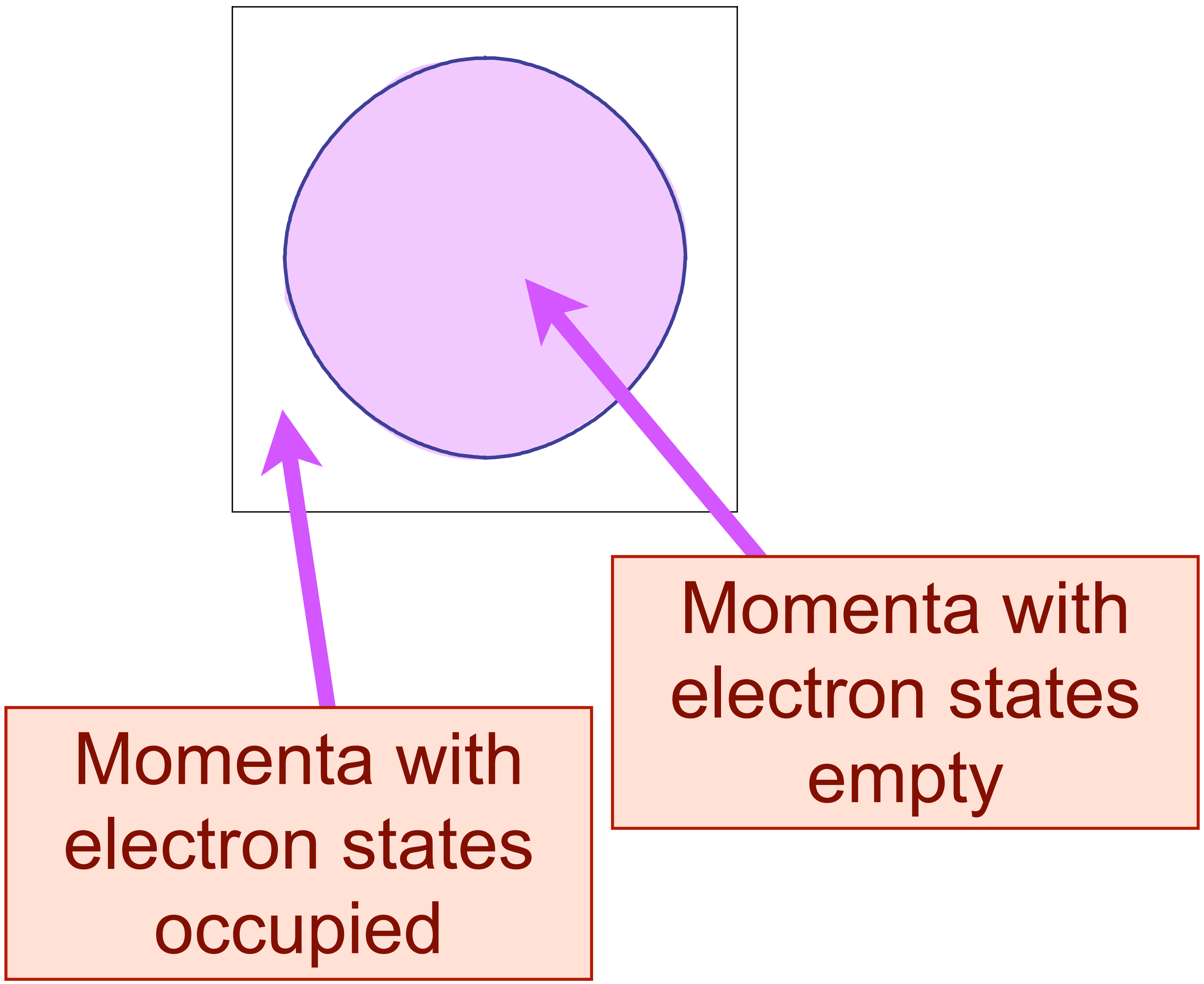
Superconductor
Bose condensate of pairs of electrons
Short-range entanglement

Temperature-doping phase diagram of the iron pnictides:

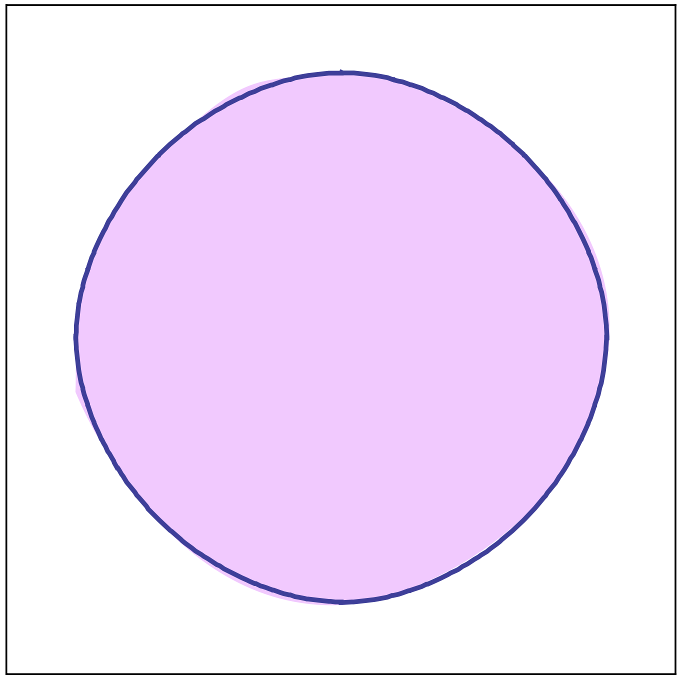


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S
H. Ikeda, H. Takeya, K. Hirata, T. Teras
Physical Review B **81**, 184519 (2010)

Sommerfeld theory of ordinary metals



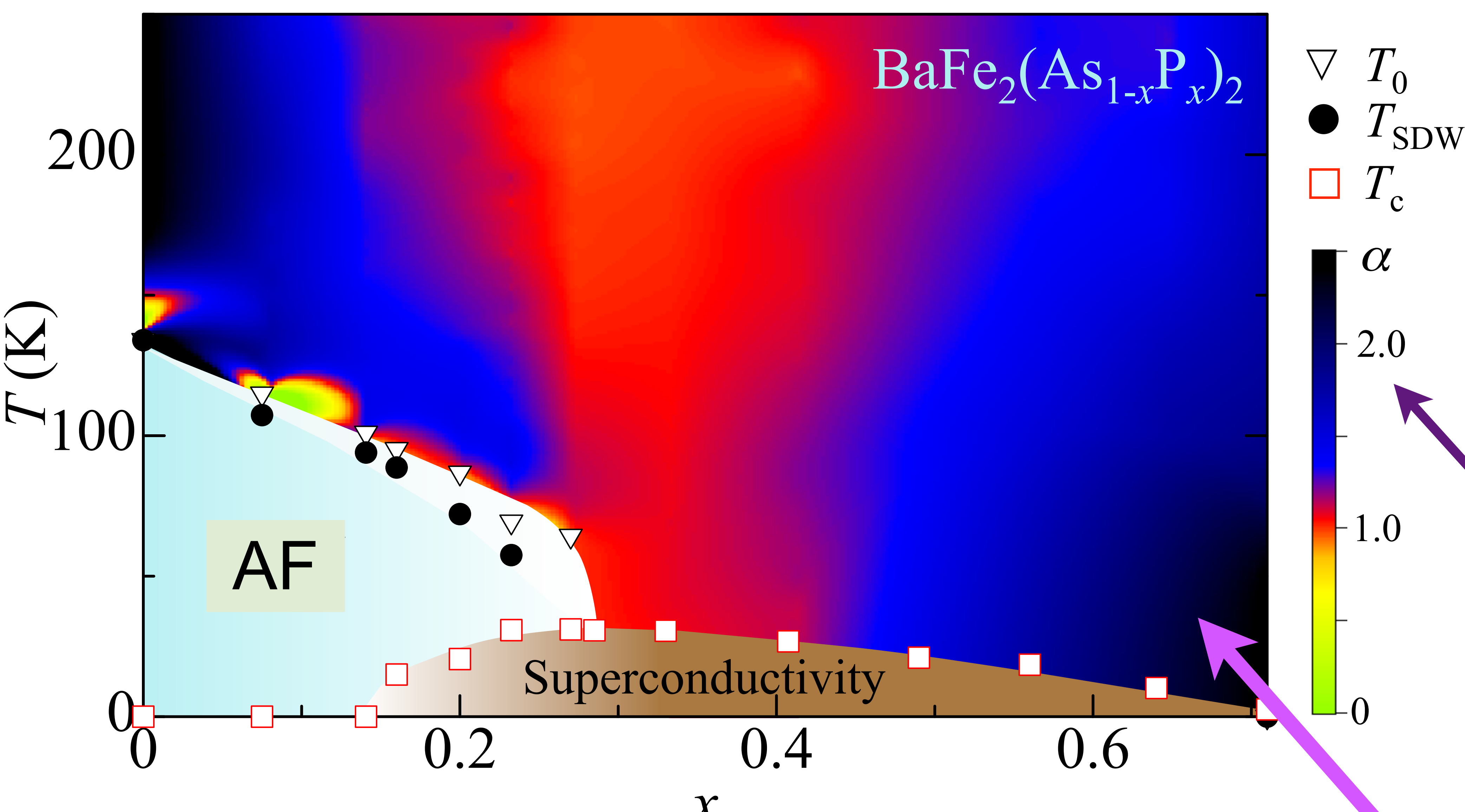
Sommerfeld theory of ordinary metals



Key feature of the Sommerfeld theory: the Fermi surface

The Fermi surface separates regions of occupied and empty electron states, and is responsible for most of the familiar properties of ordinary metals, such as resistivity $\sim T^2$.

Temperature-doping phase diagram of the iron pnictides:



Resistivity
$\sim \rho_0 + AT^\alpha$

Ordinary metal
(Fermi liquid)

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S[...]
H. Ikeda, H. Takeya, K. Hirata, T. Teras[...]
Physical Review B **81**, 184519 (2010)

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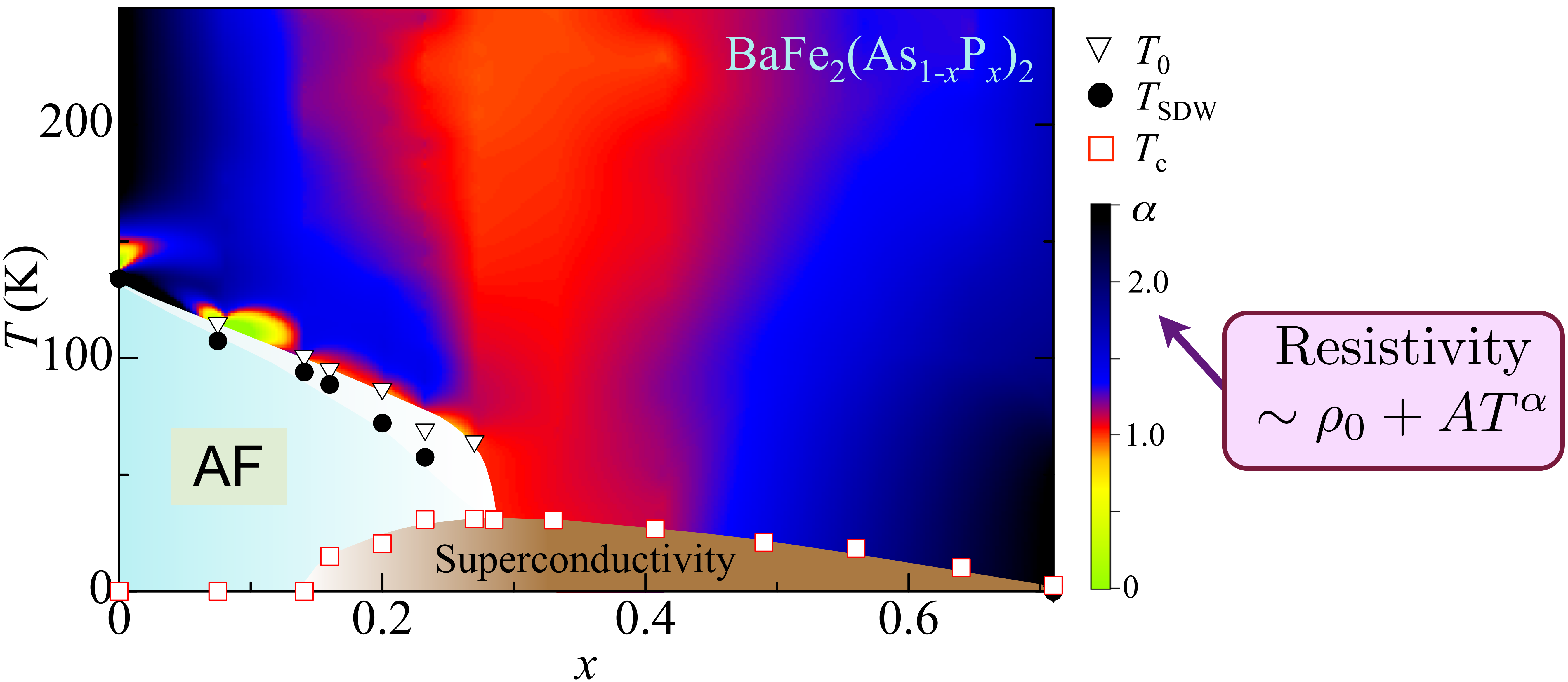

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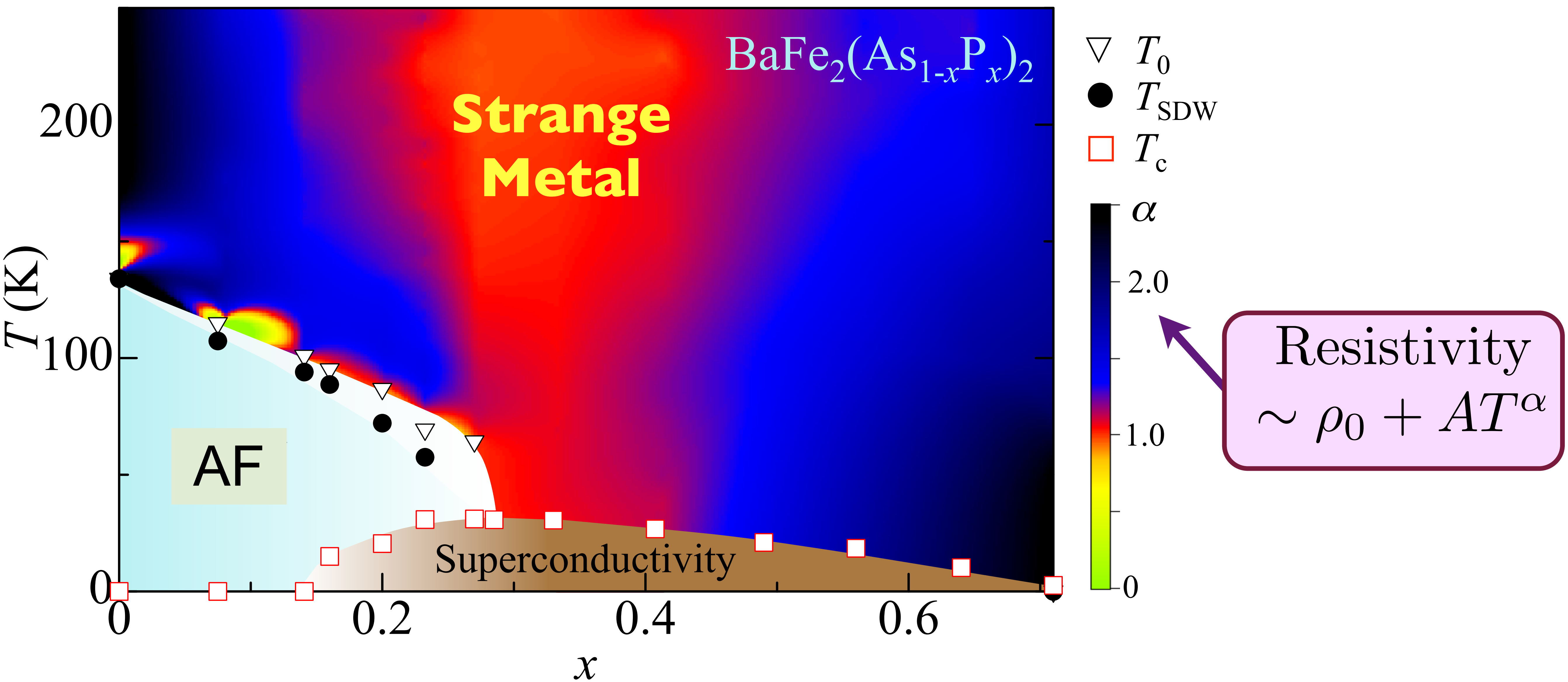

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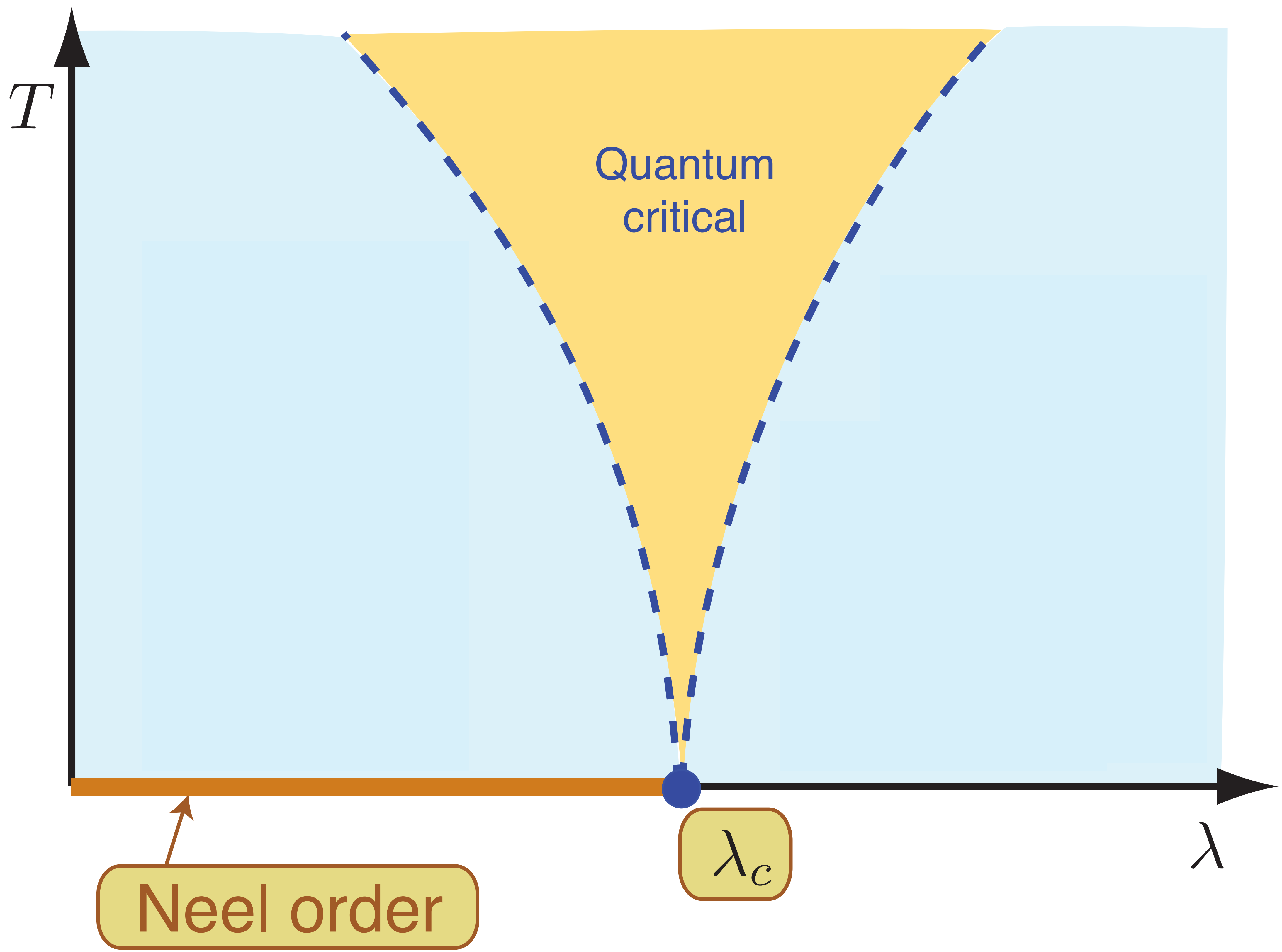
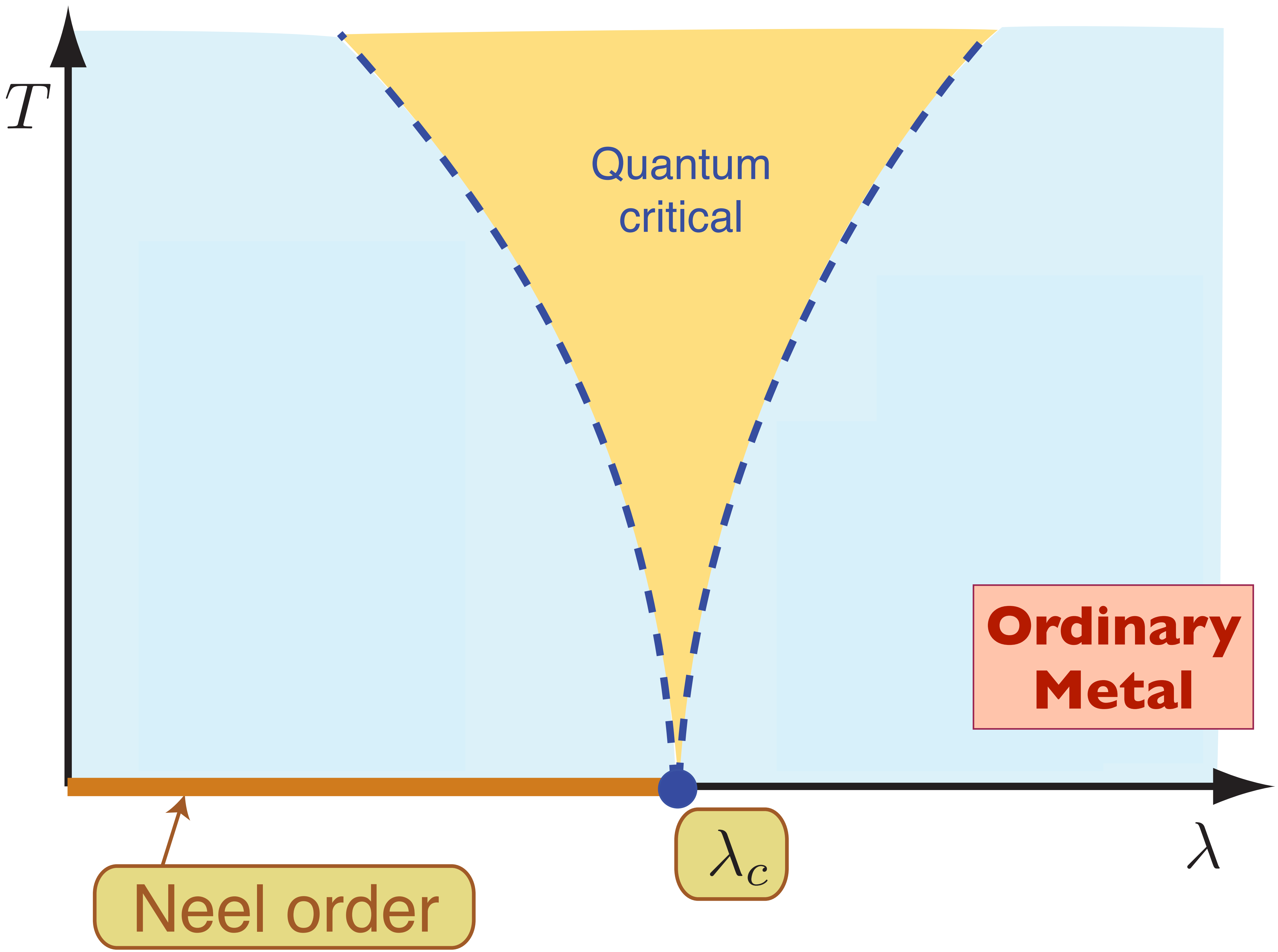
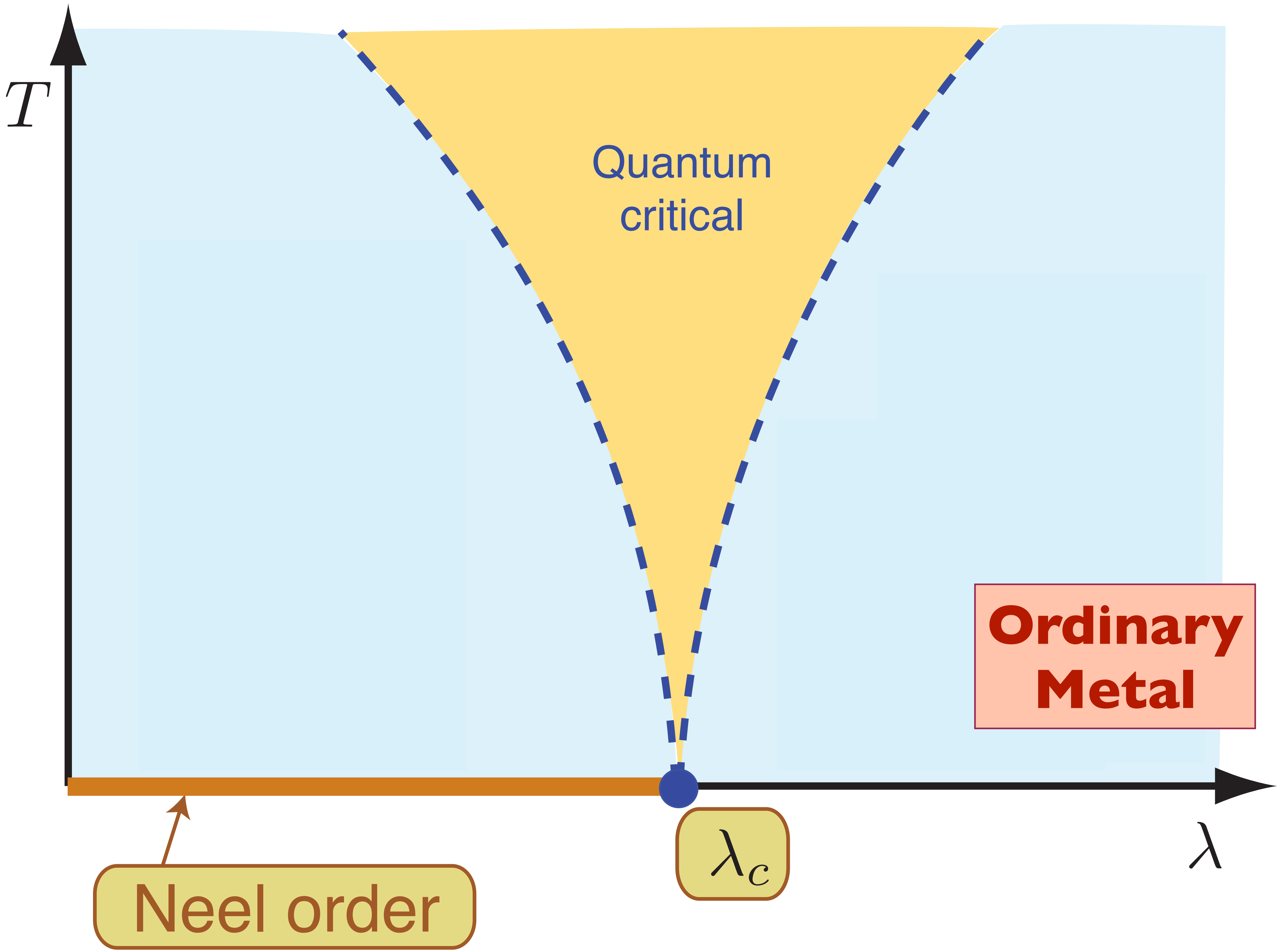
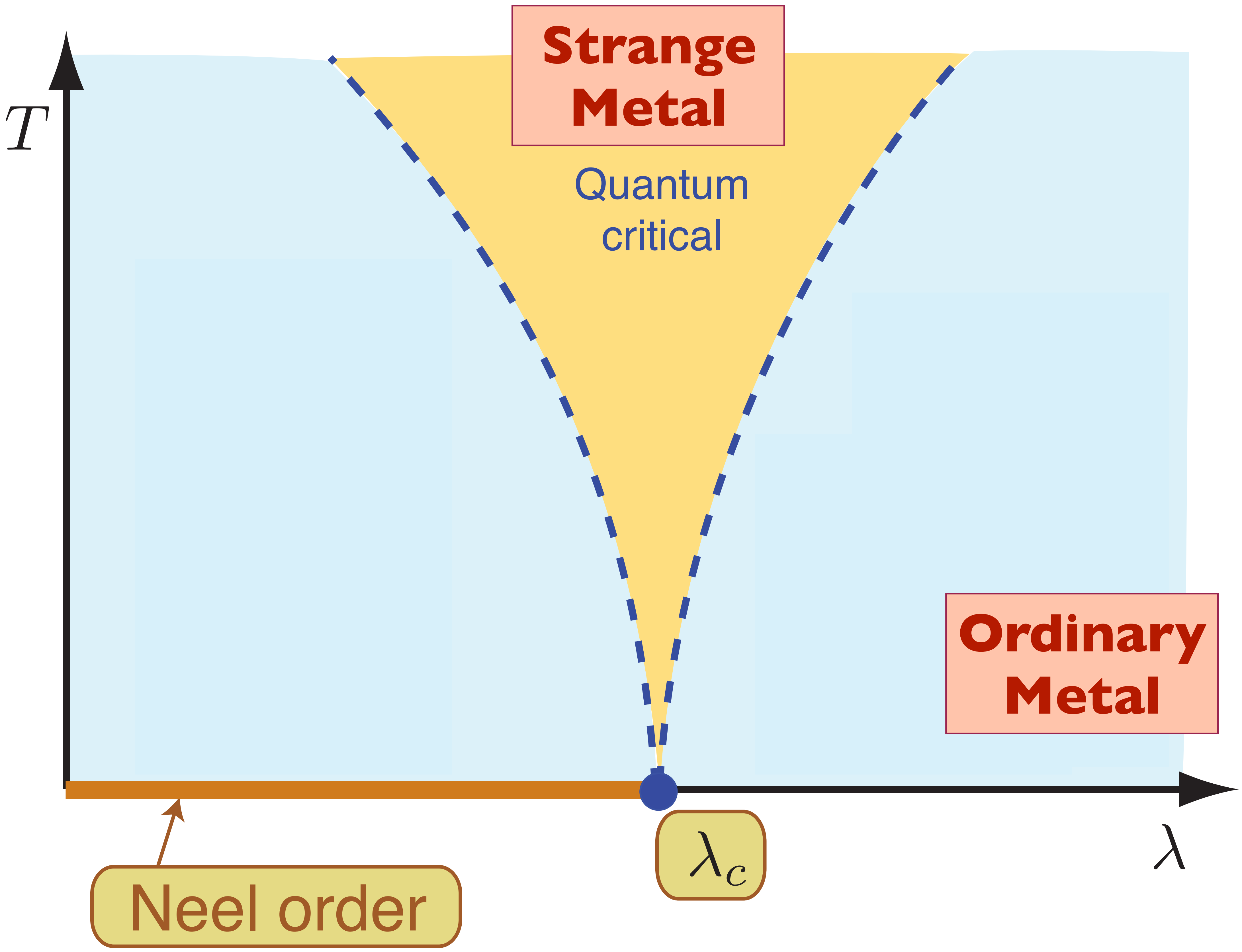



T
Quantum
critical
Ordinary
Metal
Neel order
λ_c
λ

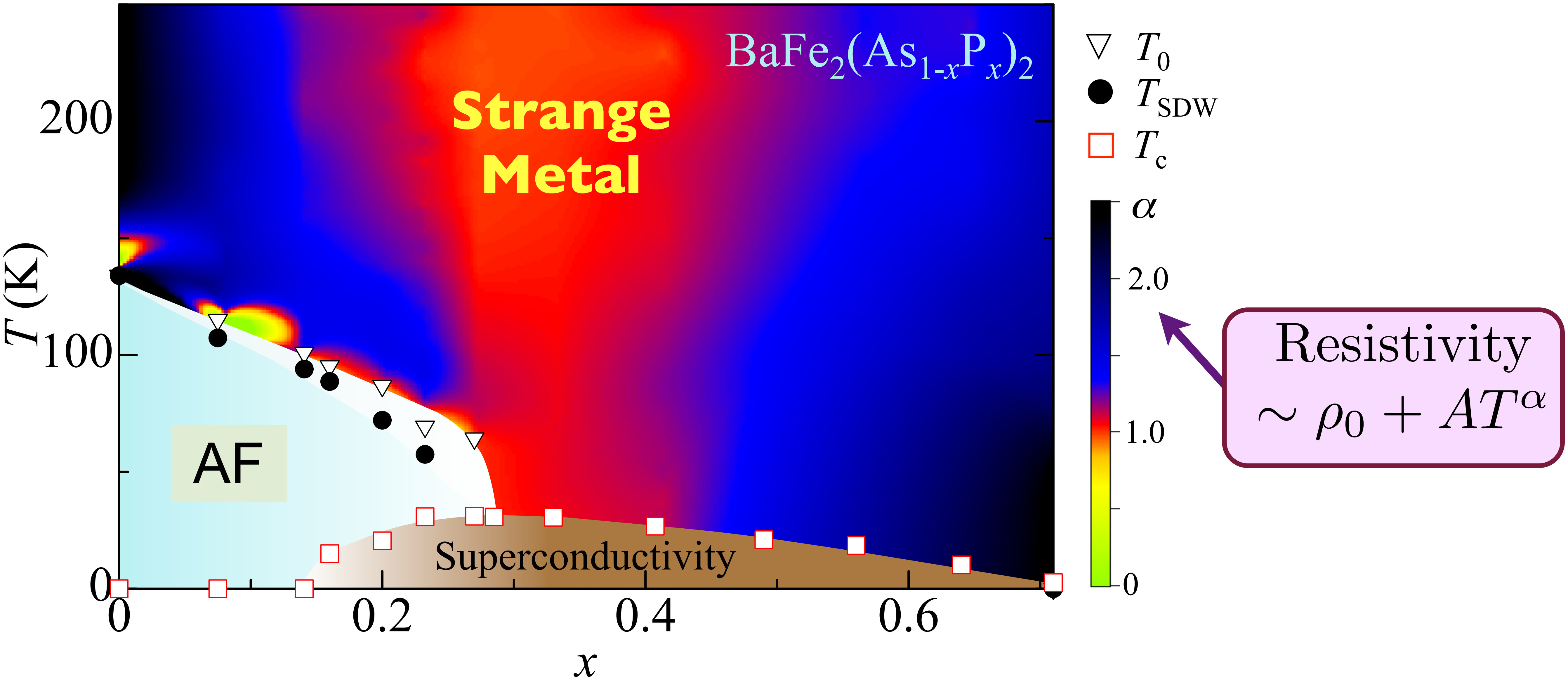


T
Quantum
critical
Ordinary
Metal
λ
Neel order
λ_c



T
Strange Metal
Quantum critical
Ordinary Metal
Neel order
λ_c
λ

Temperature-doping phase diagram of the iron pnictides:



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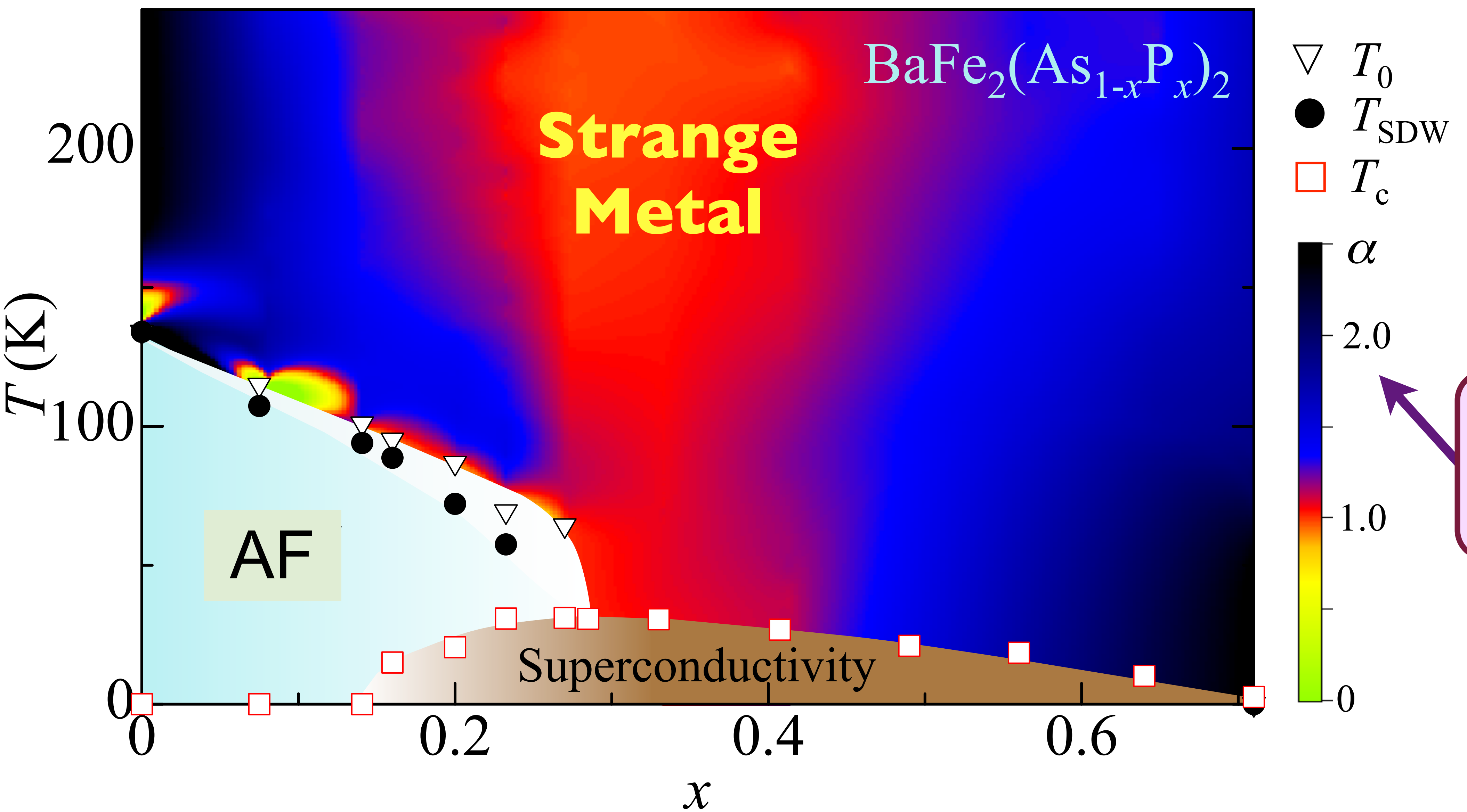

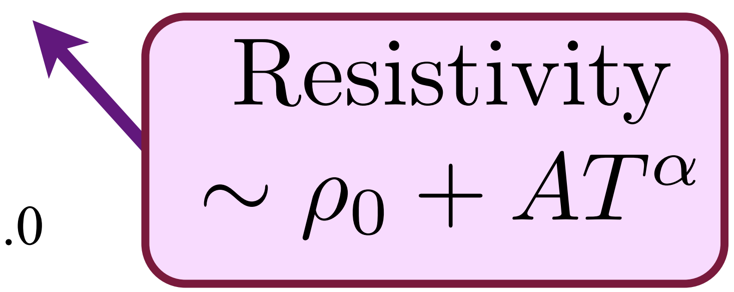

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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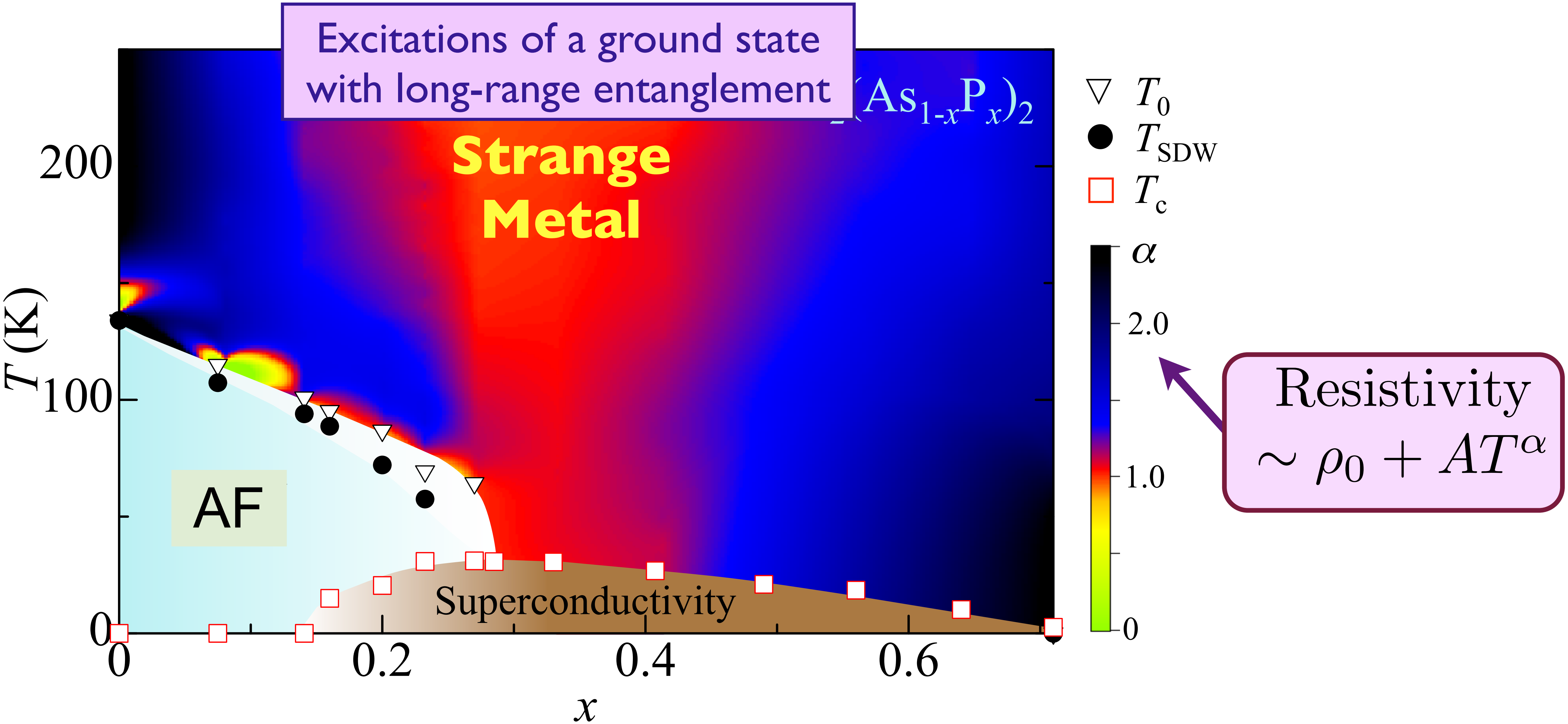
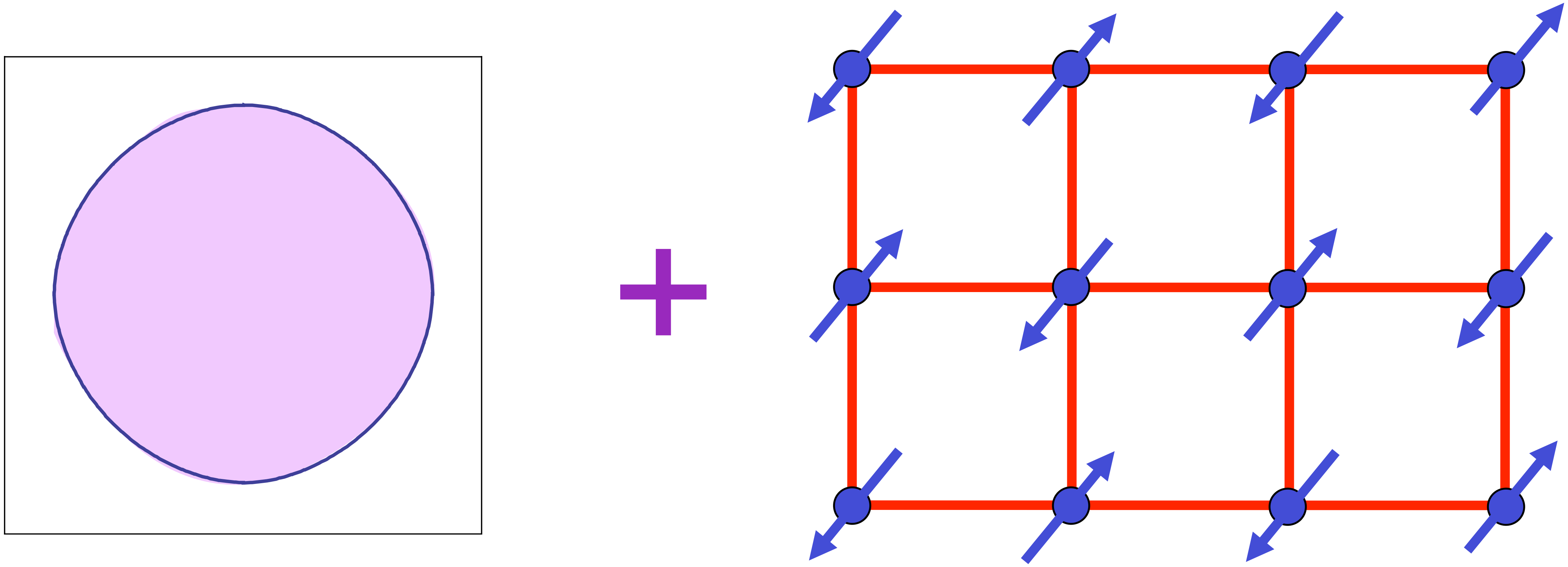


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

<u>Key (difficult) problem:</u>

Describe quantum critical points and phases of systems with Fermi surfaces leading to metals with novel types of long-range entanglement



Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Can we obtain gravitational theories of superconductors and ordinary Sommerfeld metals ?

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Can we obtain gravitational theories of superconductors and ordinary Sommerfeld metals ?

Yes

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do the "holographic" gravitational theories yield metals distinct from ordinary Sommerfeld metals ?

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do the "holographic" gravitational theories yield metals distinct from ordinary Sommerfeld metals ?

Yes, lots of them, with many "strange" properties !

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic "strange metals" have the correct type of long-range entanglement linked to Fermi surfaces ?

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic "strange metals" have the correct type of long-range entanglement linked to Fermi surfaces ?

Yes, a very select subset has the proper logarithmic violation of the area law of entanglement entropy !!
These are now being studied intensively.........

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023; L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.

Conclusions

Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

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String theory and gravity in
emergent dimensions
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Much recent progress offers hope of a
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