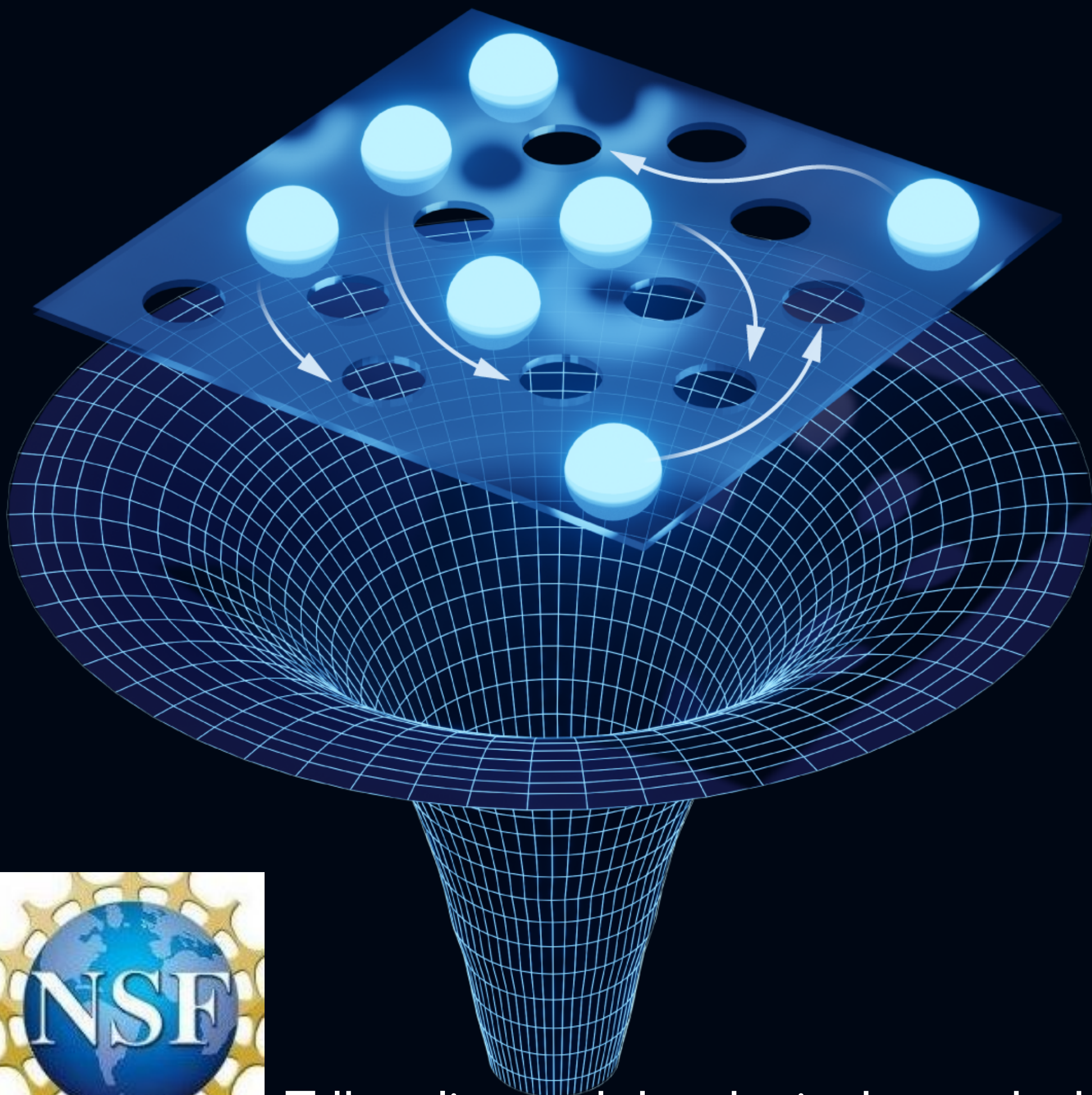


When nature entangles millions of particles: from quantum materials to black holes

2023 Jacques Solvay International Chair in Physics

Inaugural Lecture
June 20, 2023

Subir Sachdev



Talk online: sachdev.physics.harvard.edu



What is entropy
and temperature ?

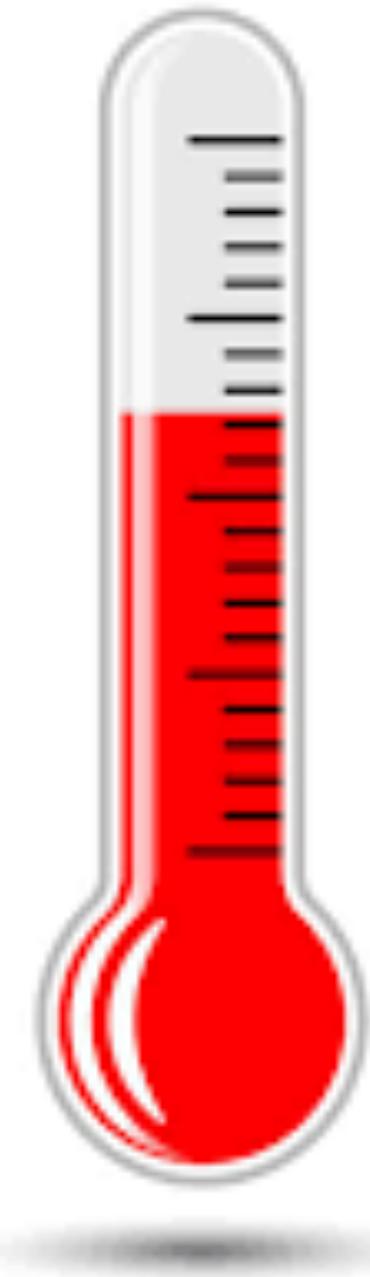
Clausius (1865):
Second Law of Thermodynamics:
Every macroscopic system has an
“entropy” which cannot decrease.

➔ No perpetual motion machines!



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Temperature

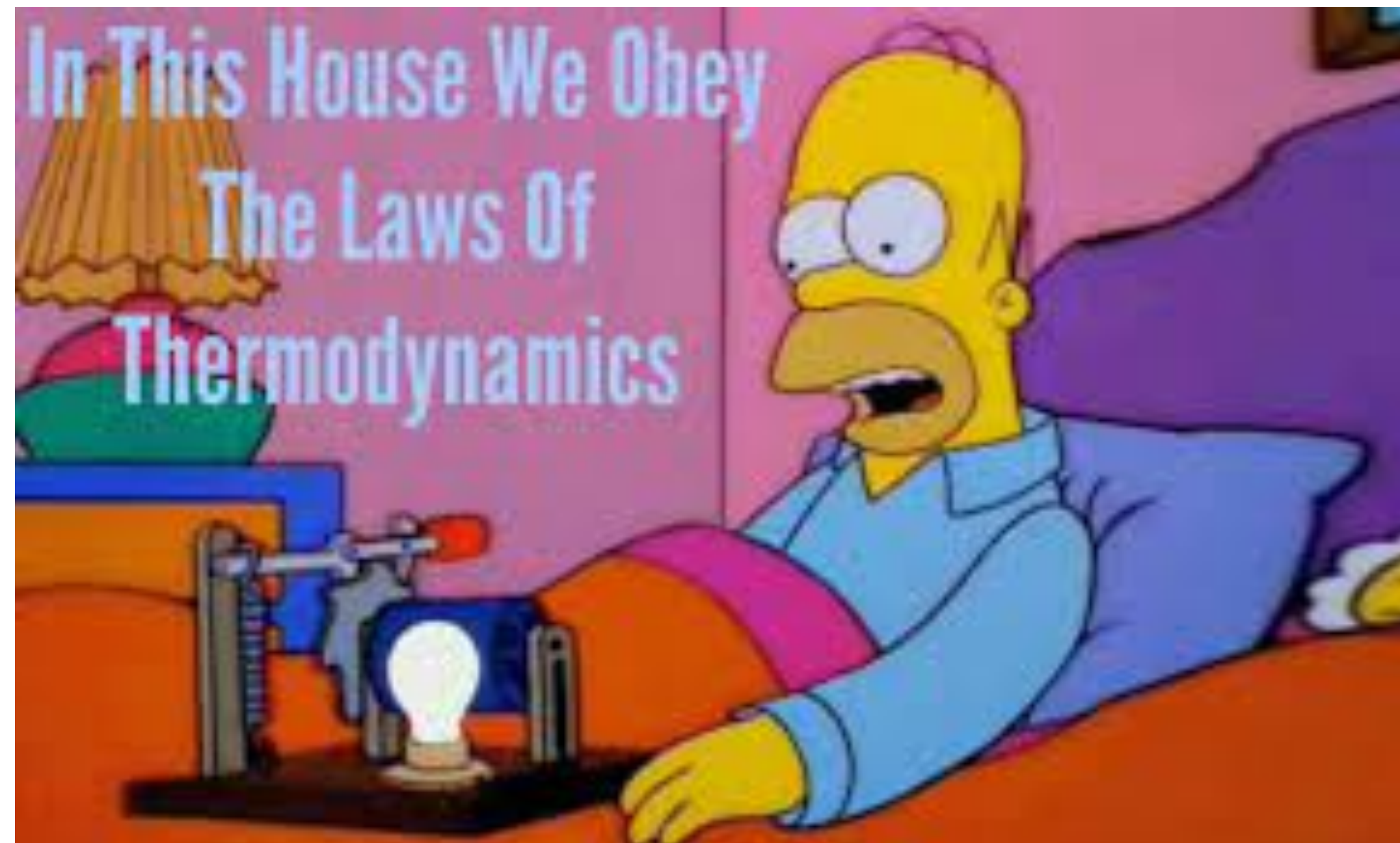
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Every macroscopic system has an “entropy” which cannot decrease.

➔ No perpetual motion machines!

Heat always flows
from a
hot body
to a
cold body



Statistical interpretation of entropy (1870)

$$S = k_B \log W$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

Statistical interpretation of entropy (1870)

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$$\frac{1}{T} = \frac{dS}{dE}$$



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Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E)$

$$S(E) = k_B \log D(E)$$

$$\frac{1}{T} = \frac{dS}{dE}$$



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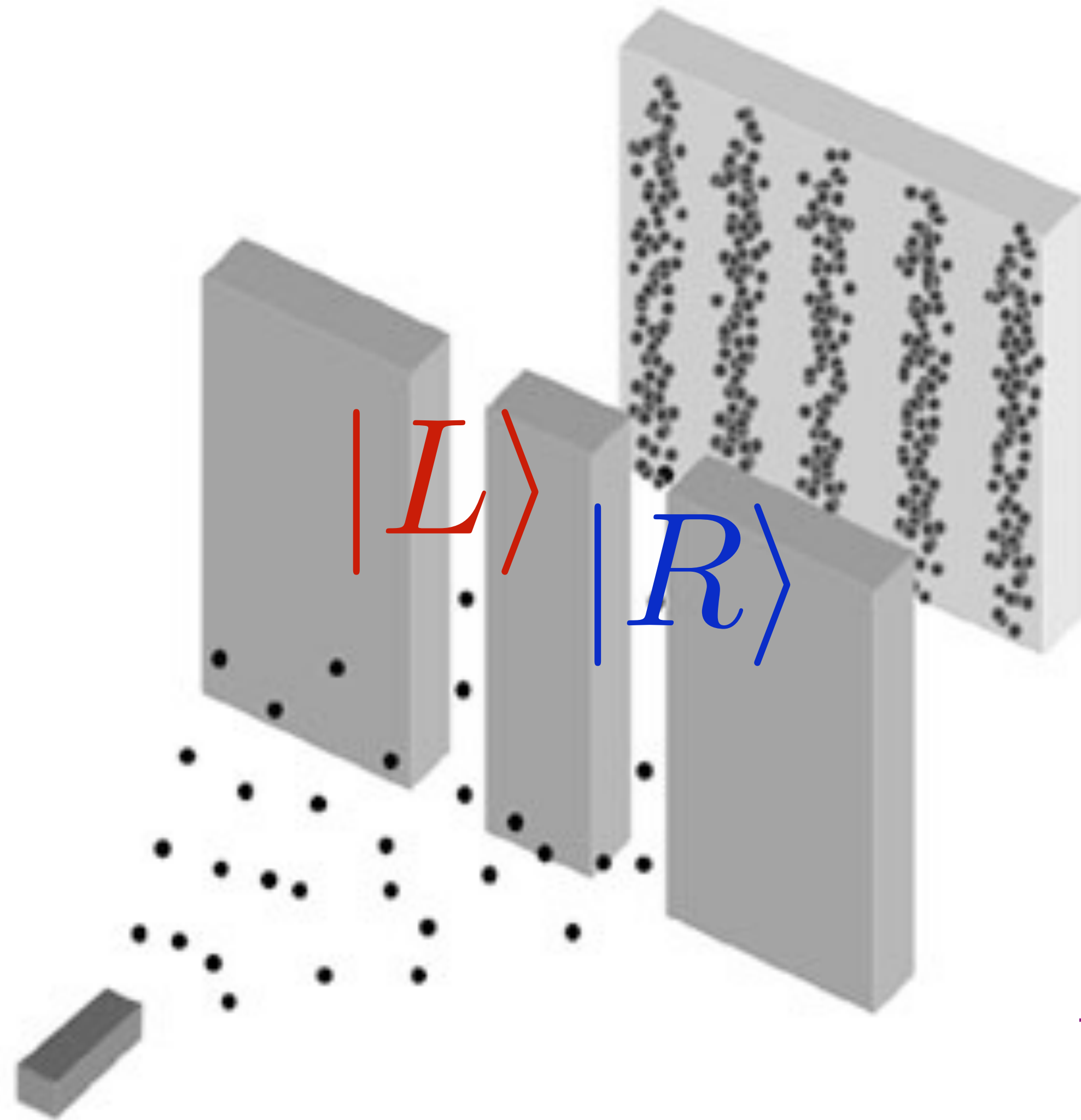
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Vienna, Austria

What is
quantum entanglement?

The most remarkable new idea in the quantum theory is the
principle of superposition:
a physical system can be in a
superposition of two (or more) distinct states.

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

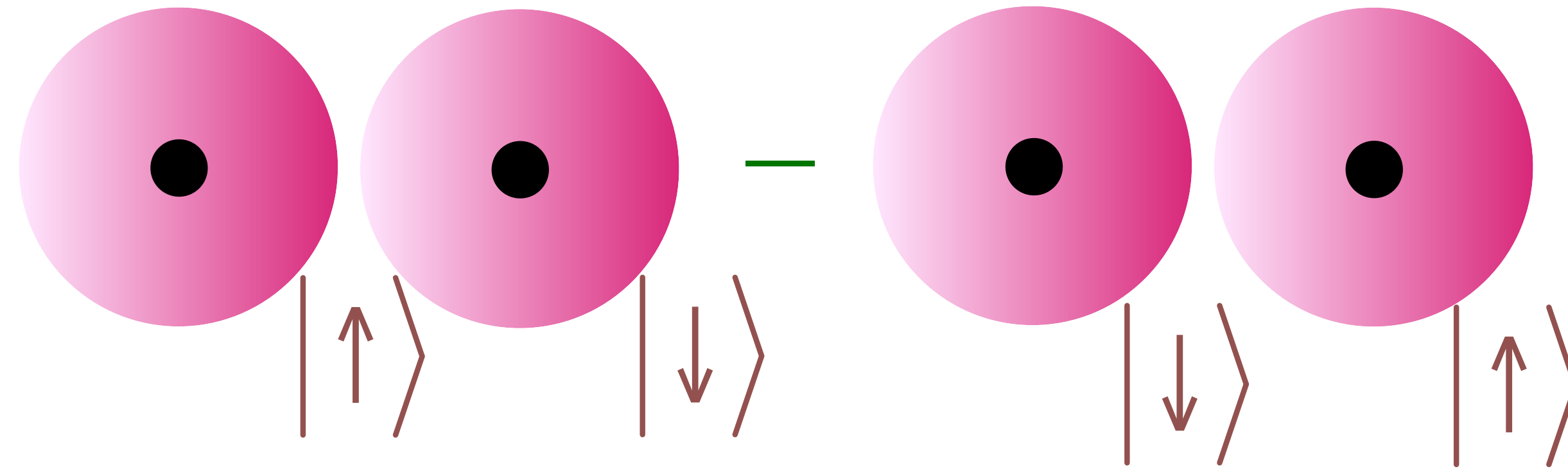
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

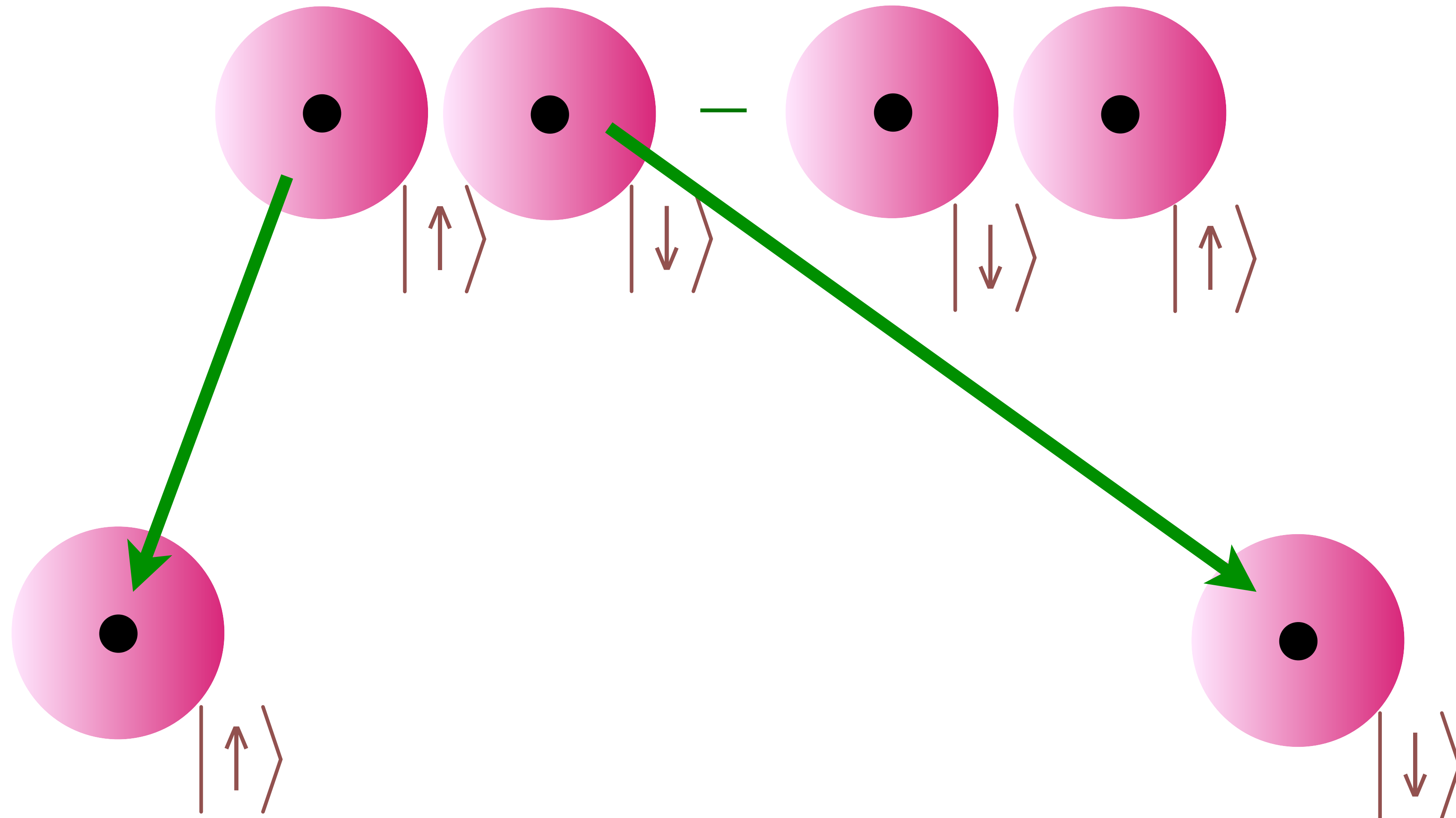
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



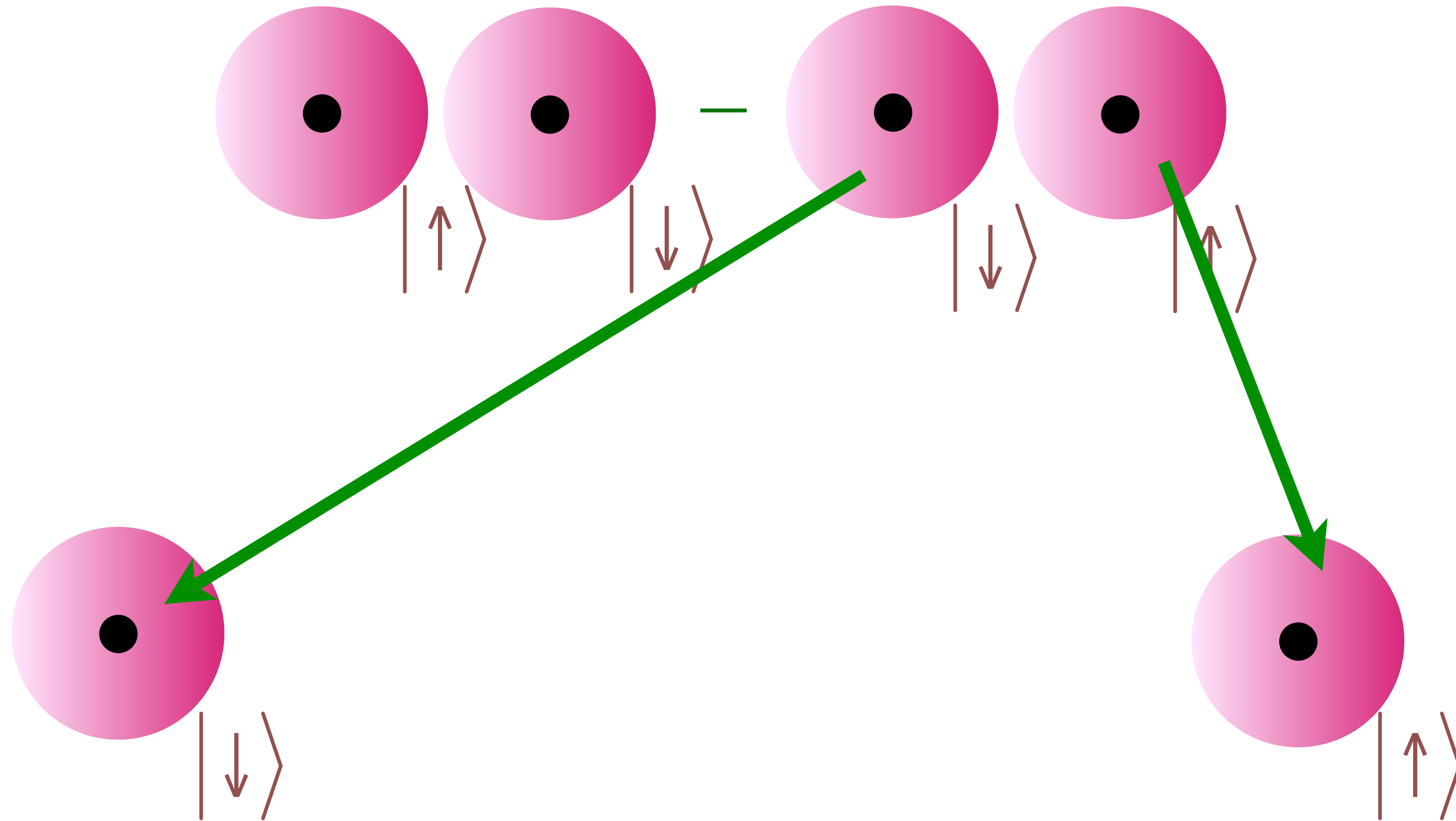
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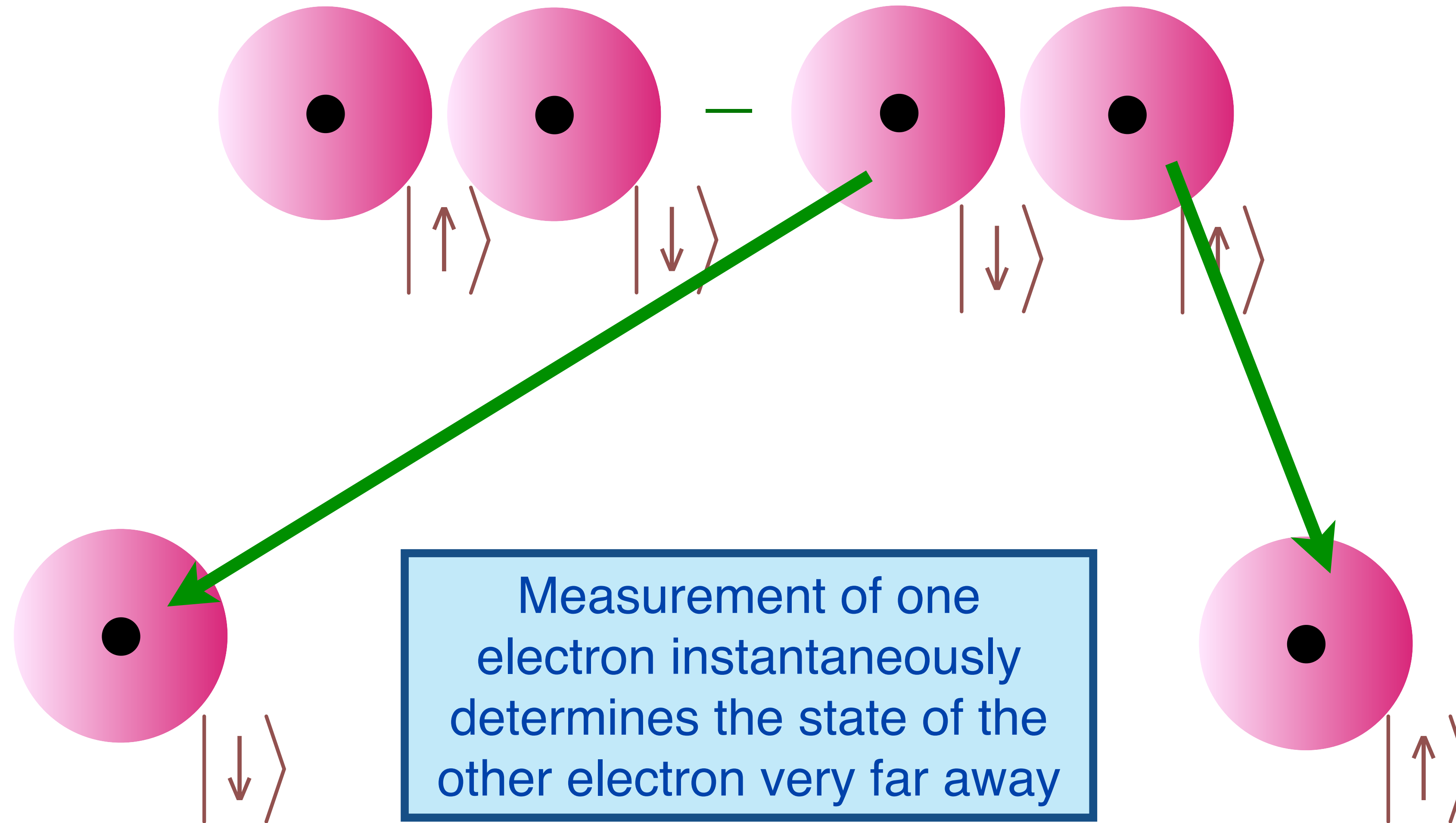
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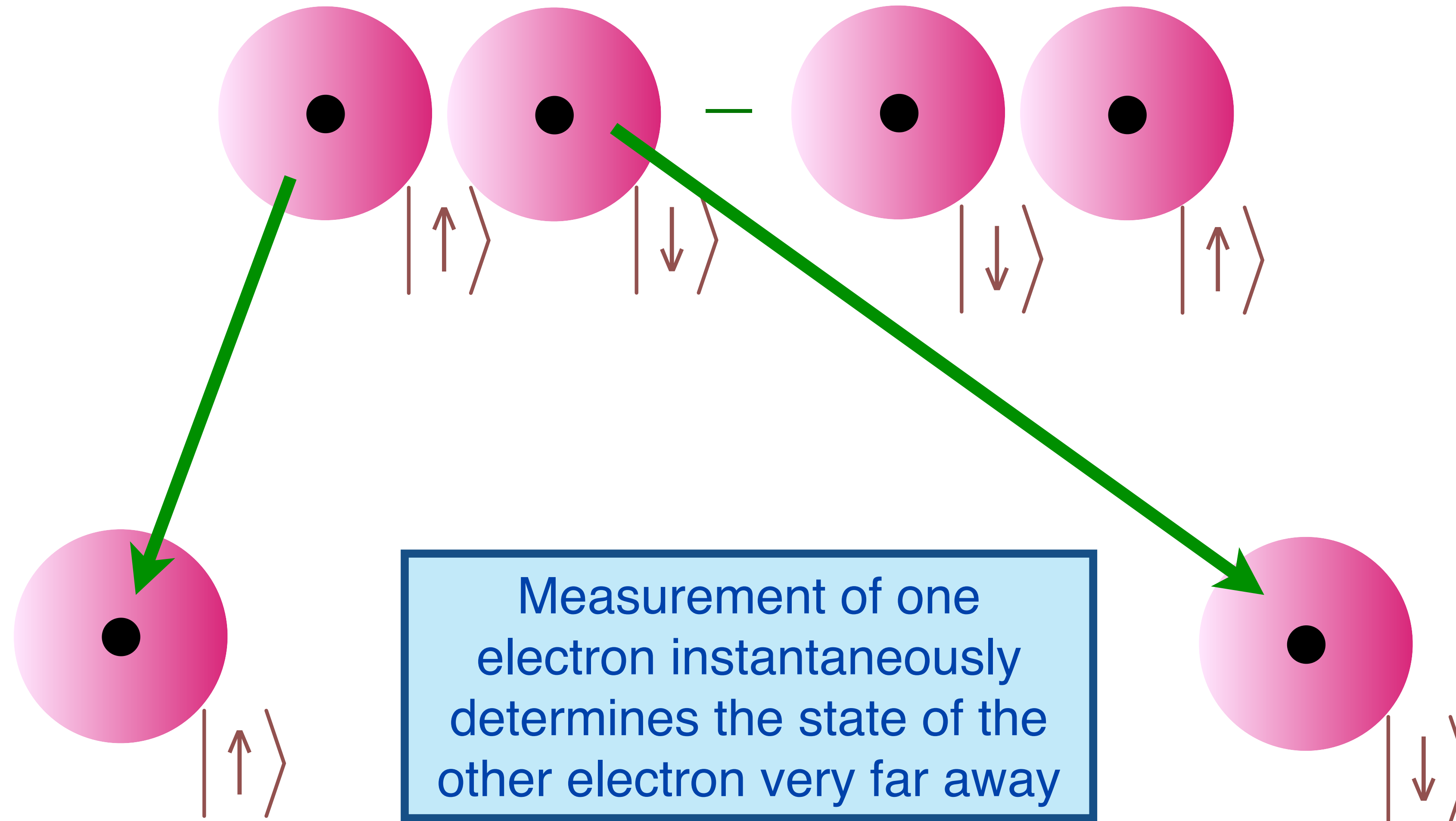
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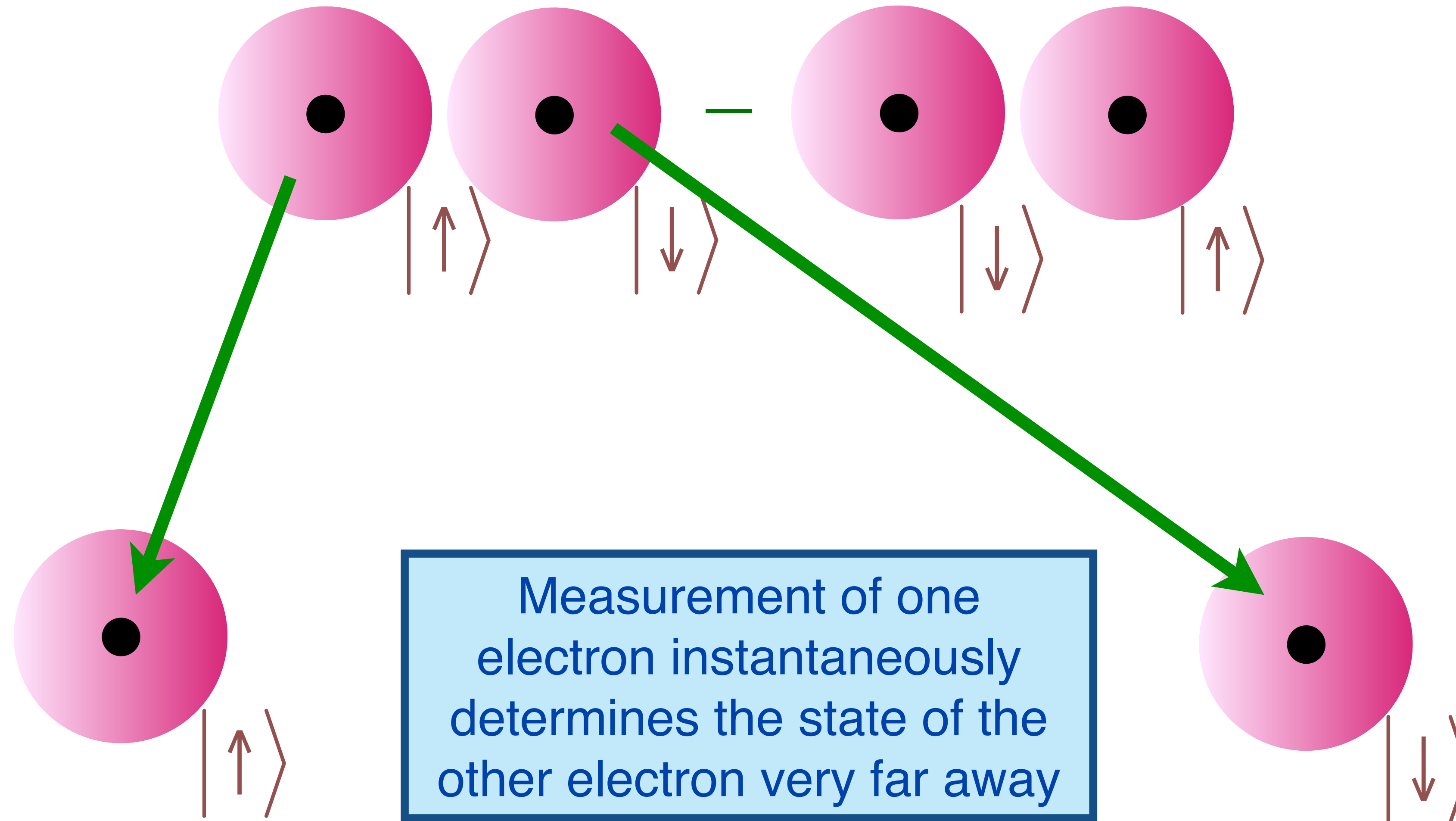
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Quantum Entanglement

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Spooky action at a distance !

natürlicher
deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt daß es wirklich mit der Theorie

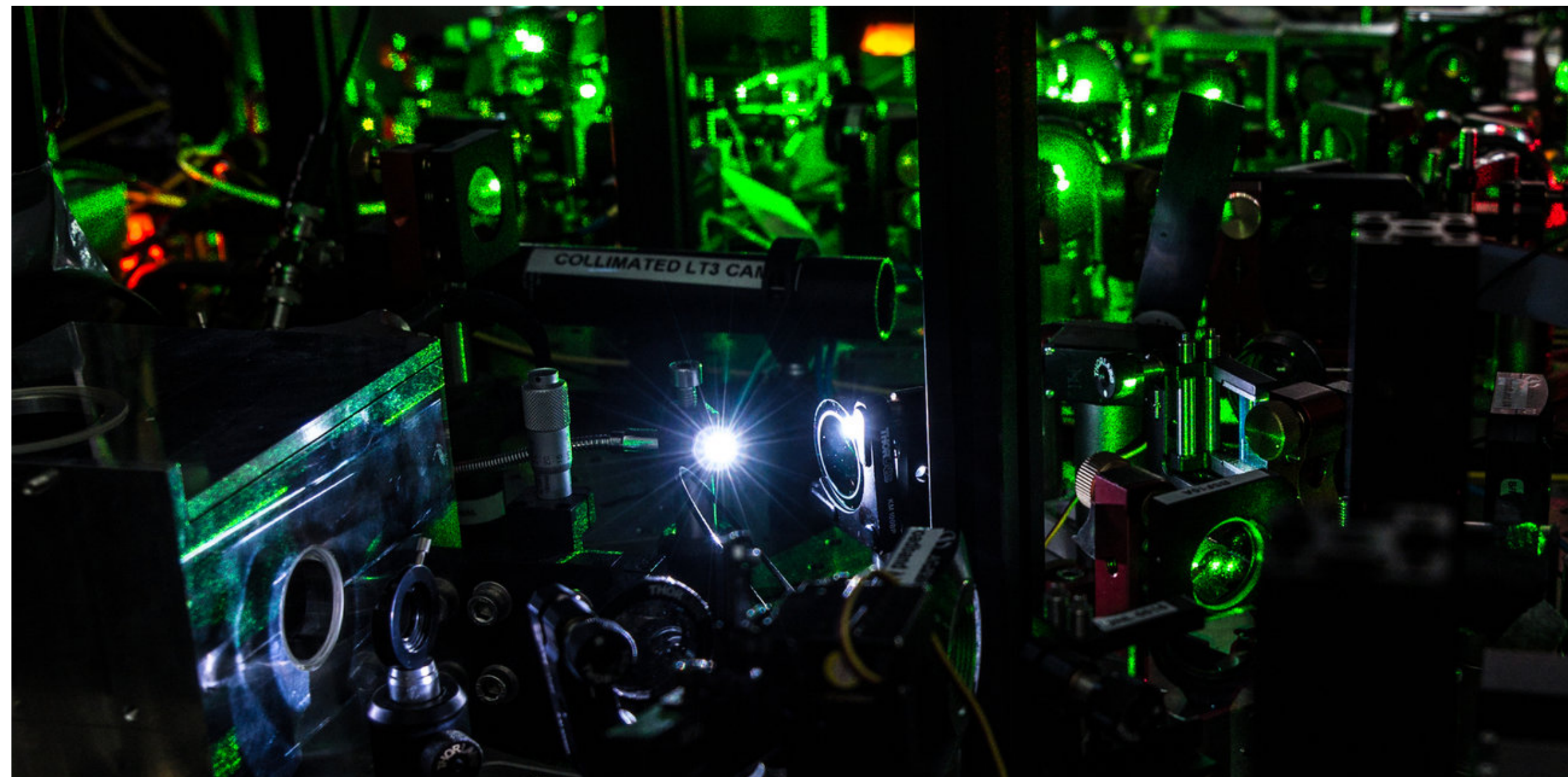
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

The New York Times

Sorry, Einstein. Quantum Study Suggests 'Spooky Action' Is Real.

By **JOHN MARKOFF** OCT. 21, 2015



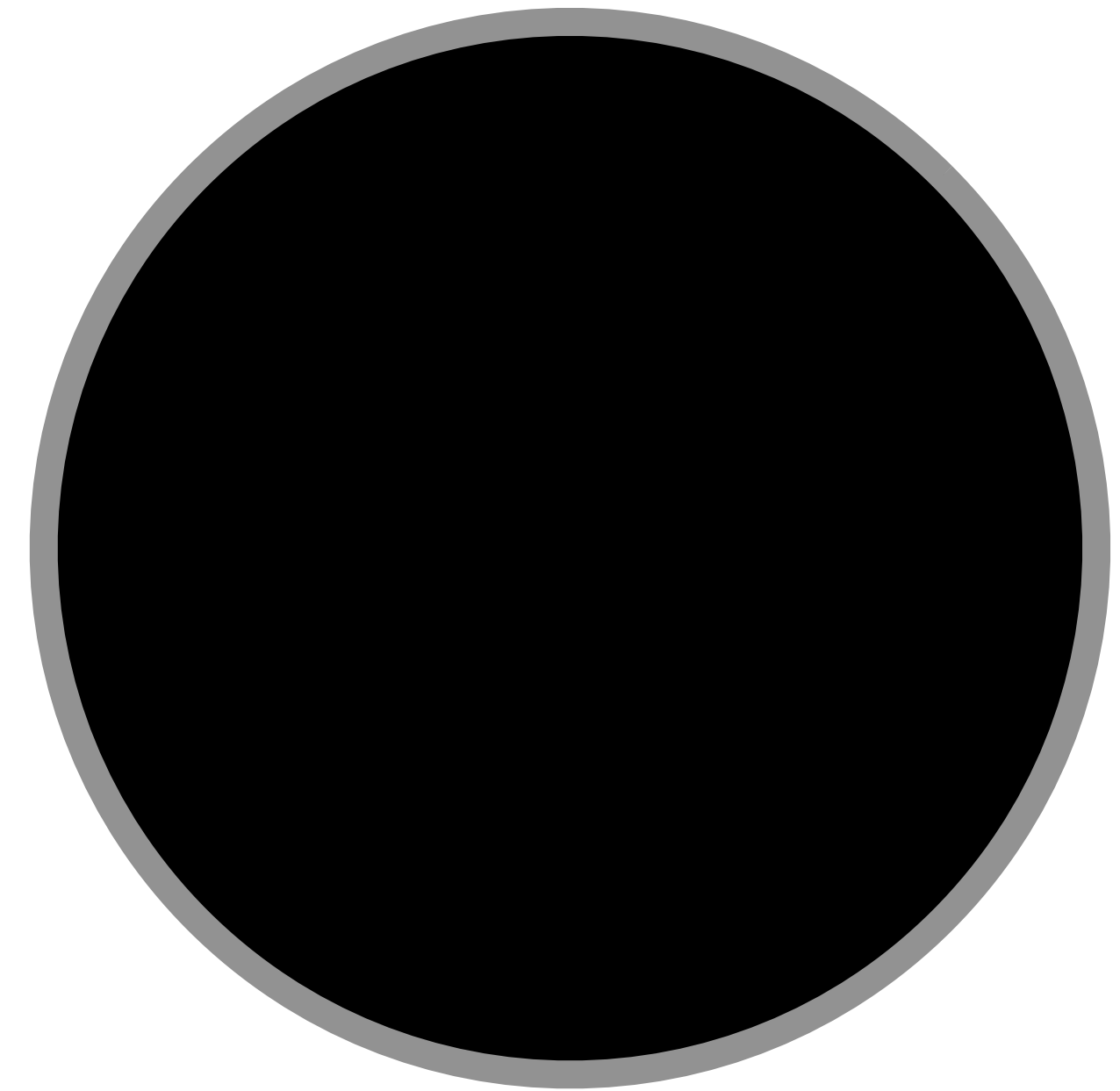
Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

**Black
holes**

Black Holes

Objects so dense that light is gravitationally bound to them.

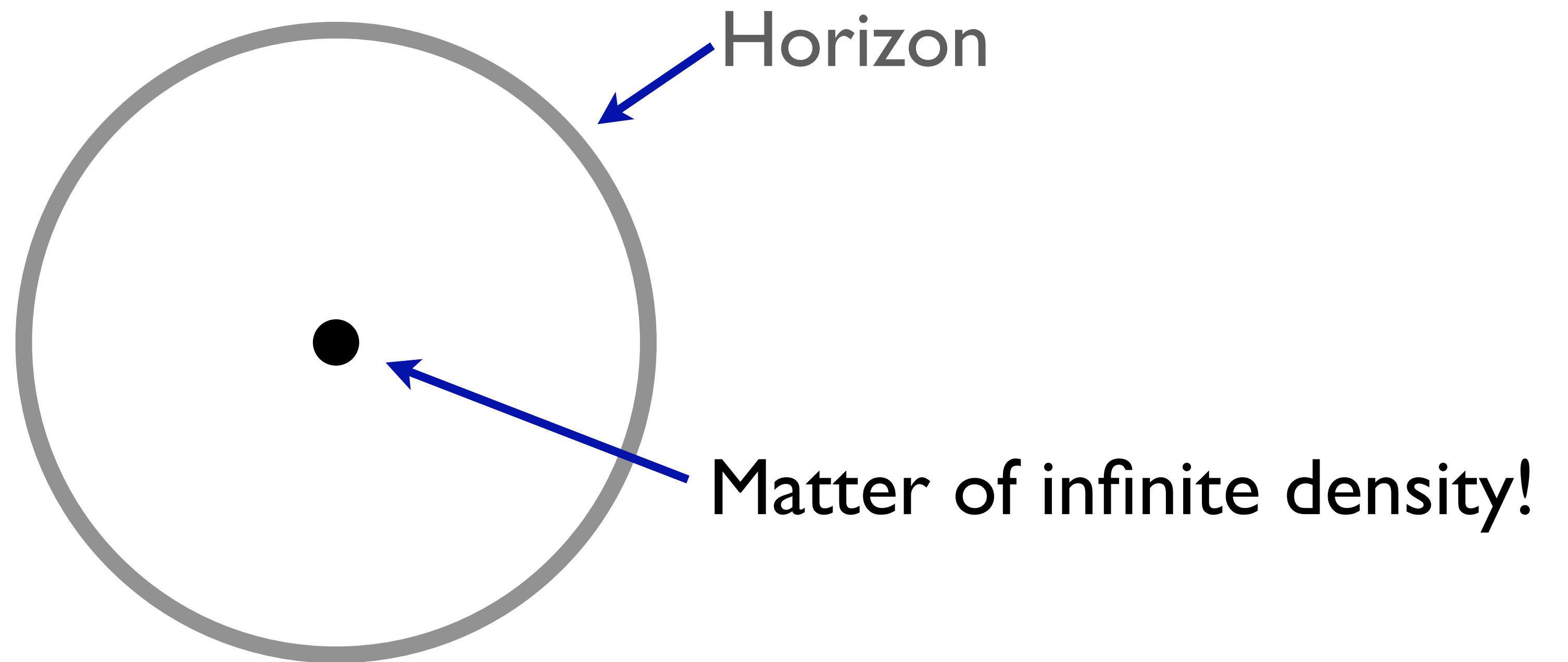
Horizon radius $R = \frac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole
For $M = \text{earth's mass}$, $R \approx 9 \text{ mm!}$

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



What is inside a black hole ???

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This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

An artist's impression of the supermassive black hole Sagittarius A* at the center of the Milky Way. The image shows a dark, circular event horizon surrounded by a glowing accretion disk. The disk is depicted with a color gradient from dark red and orange near the inner edge to bright yellow and white further out. The background is a dark, starry field.

The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A* contains about 4.3 million solar masses

$$R = 1.3 \times 10^{11} \text{ m}$$

\approx earth's orbit

An artist's impression of Sagittarius A*, the supermassive black hole at the heart of the Milky Way. Image: International Gemini Observatory/NOIRLab/NSF/AURA/J. da Silva/(Spaceengine); M. Zamani (NSF's NOIRLab)

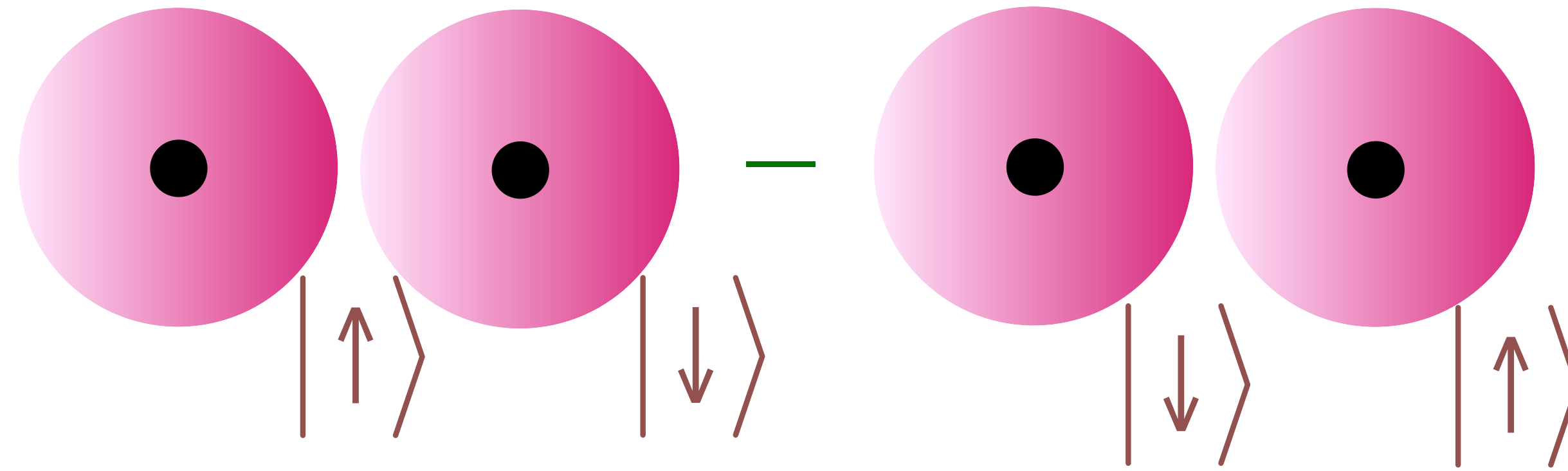
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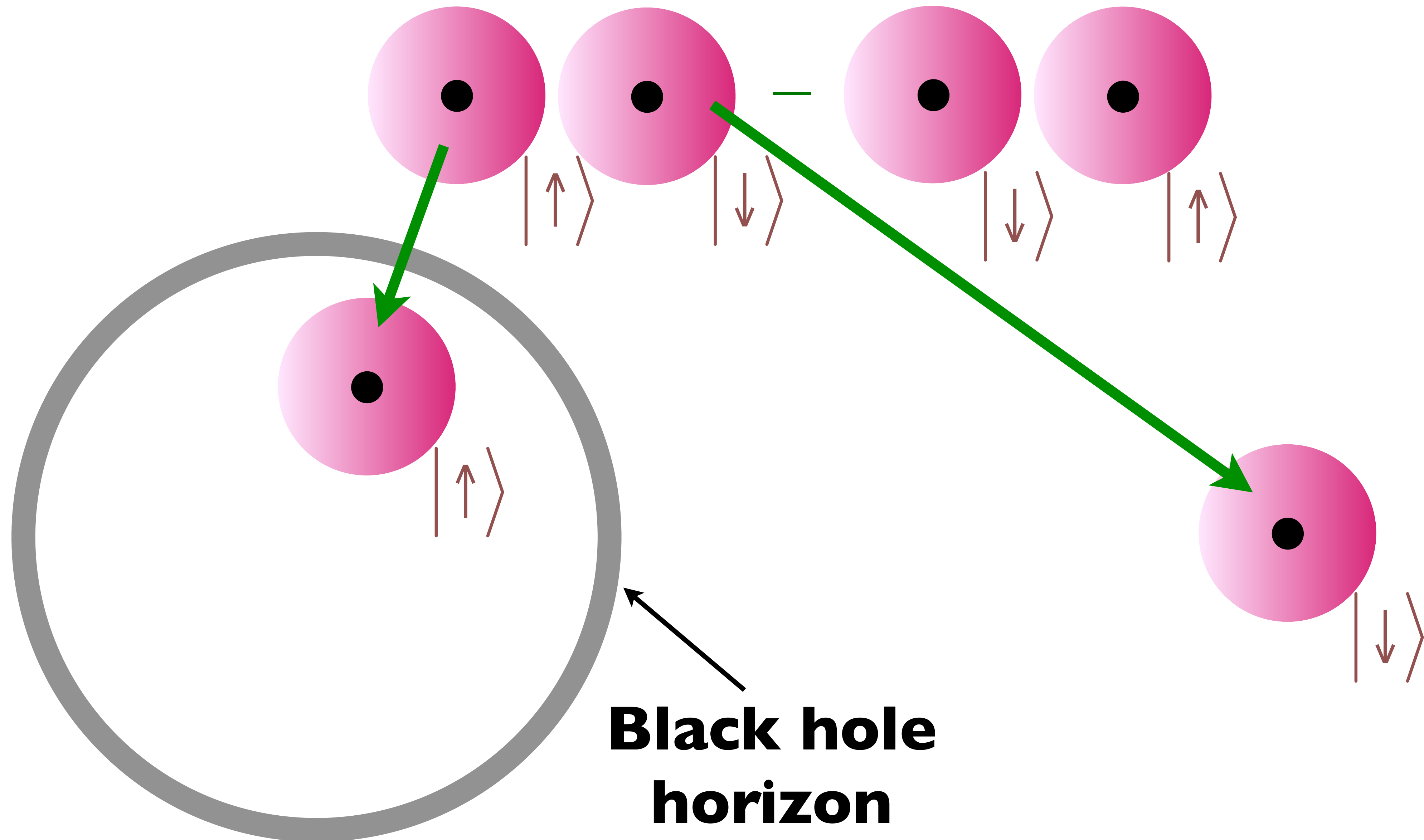
This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

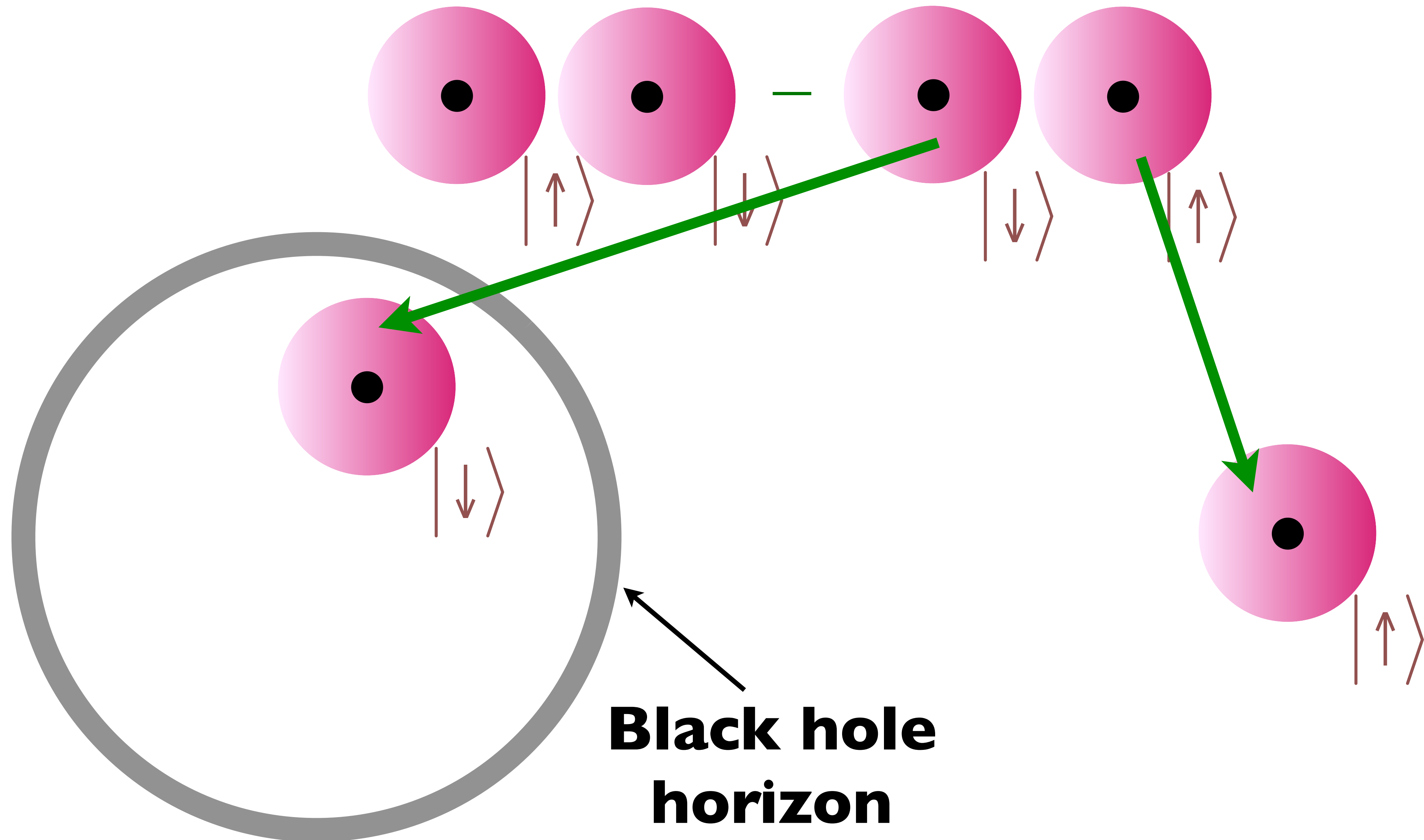
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

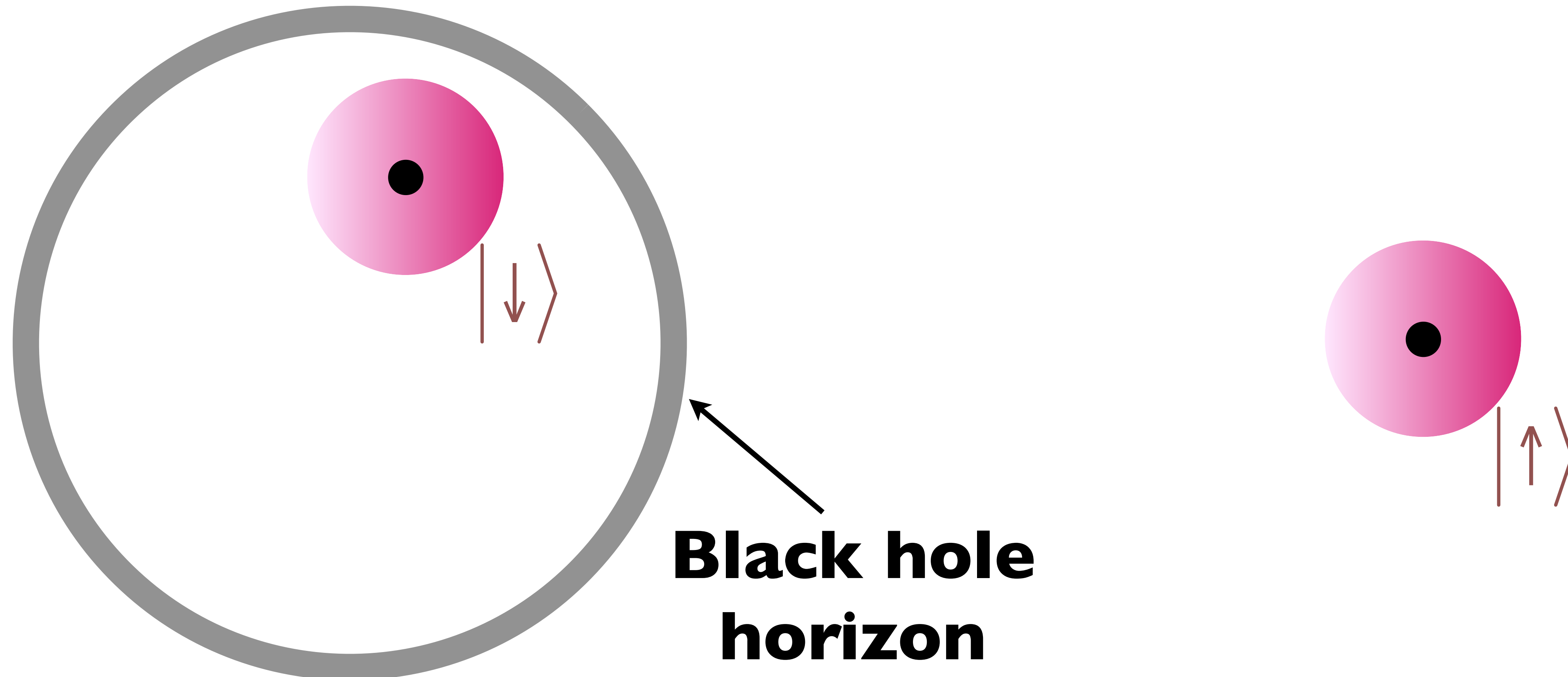


Quantum Entanglement across a black hole horizon



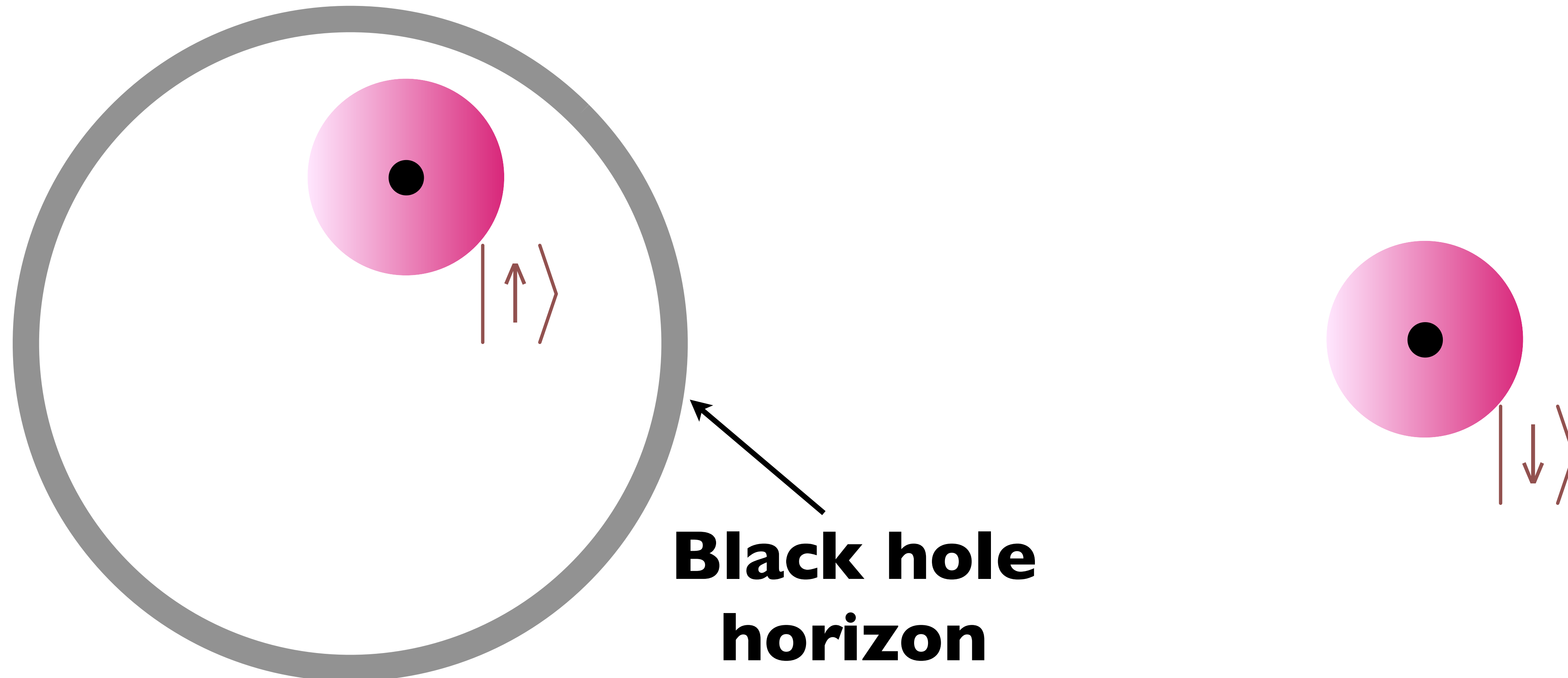
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

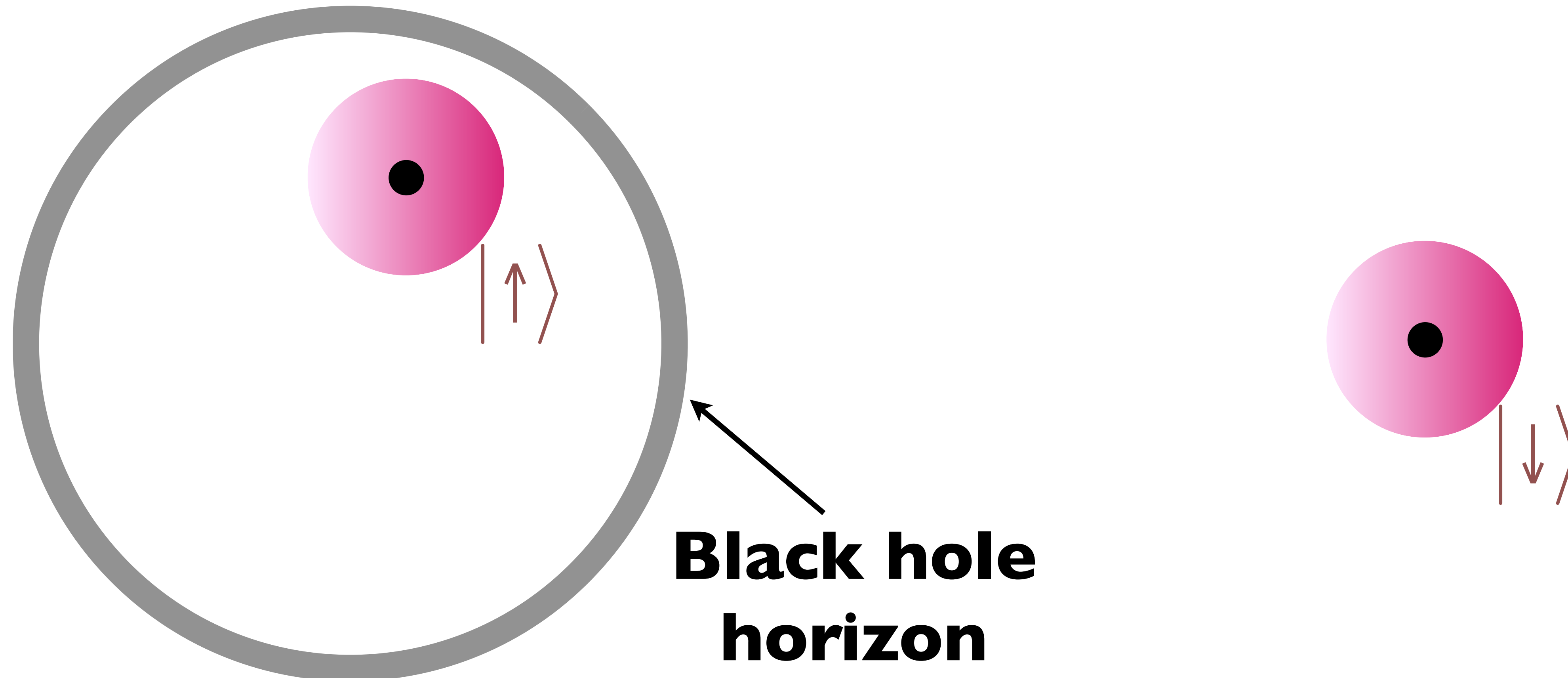
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Quantum Entanglement across a black hole horizon

Hawking (1975): Black holes have a temperature and an entropy!

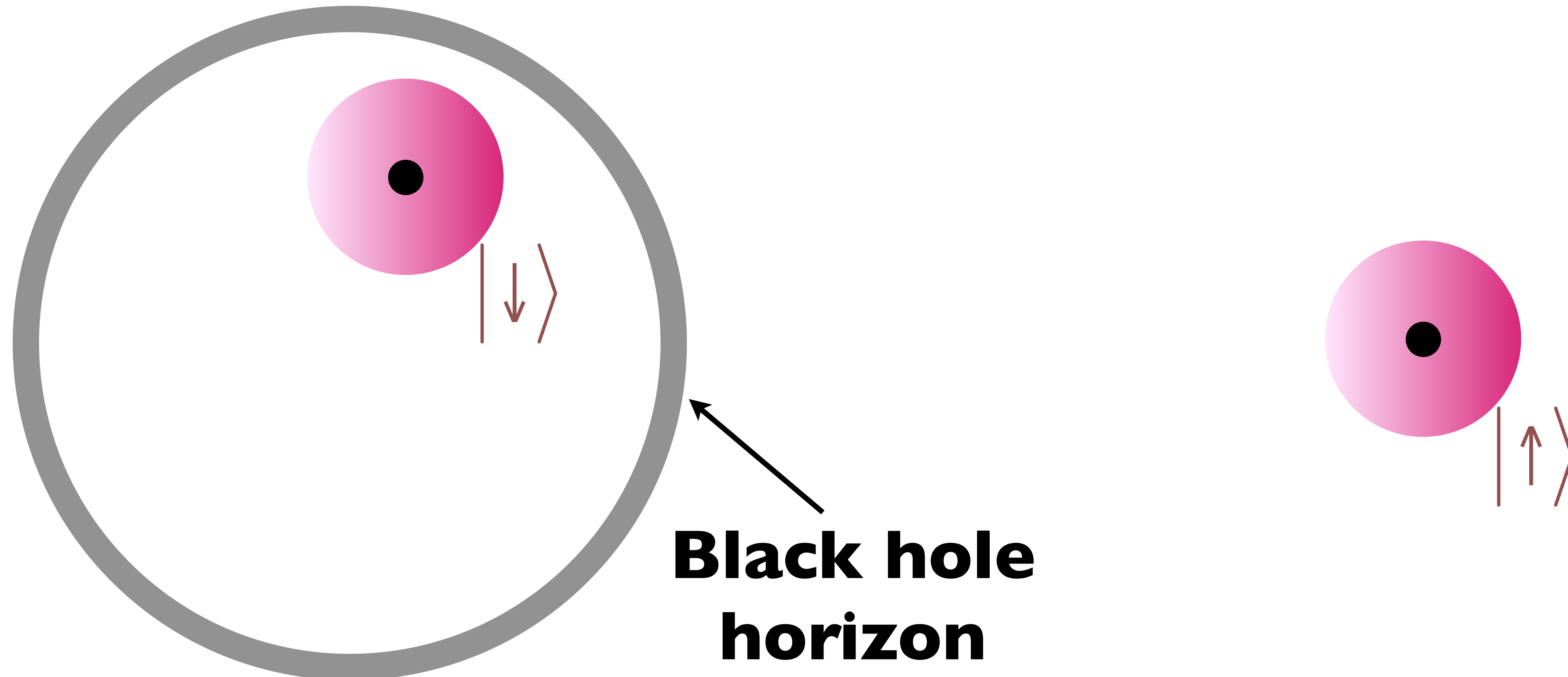
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

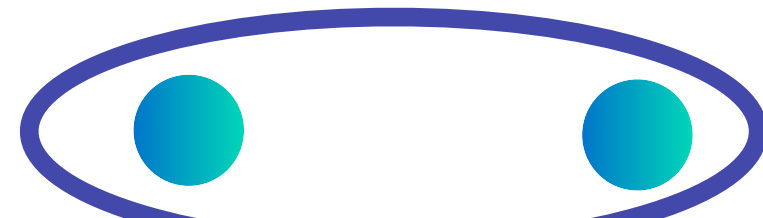
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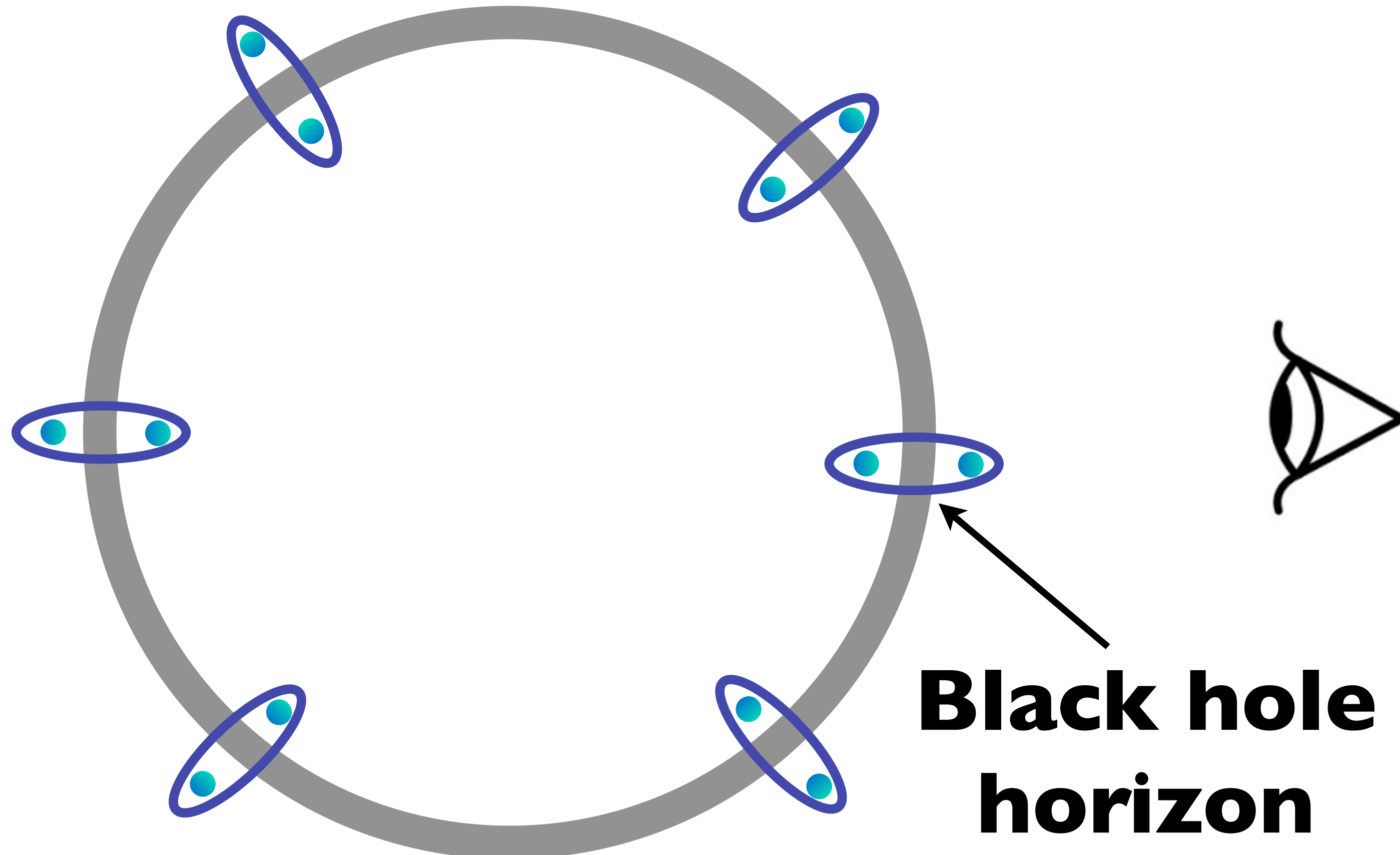
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Quantum Entanglement across a black hole horizon

Quantum entanglement
on the surface


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



By computations *outside*
the black hole,
Hawking obtained

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the
black hole horizon.

All other systems have
entropy proportional to
their volume.

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ matches Hawking's entropy, in accordance with Boltzmann's principles of statistical mechanics?

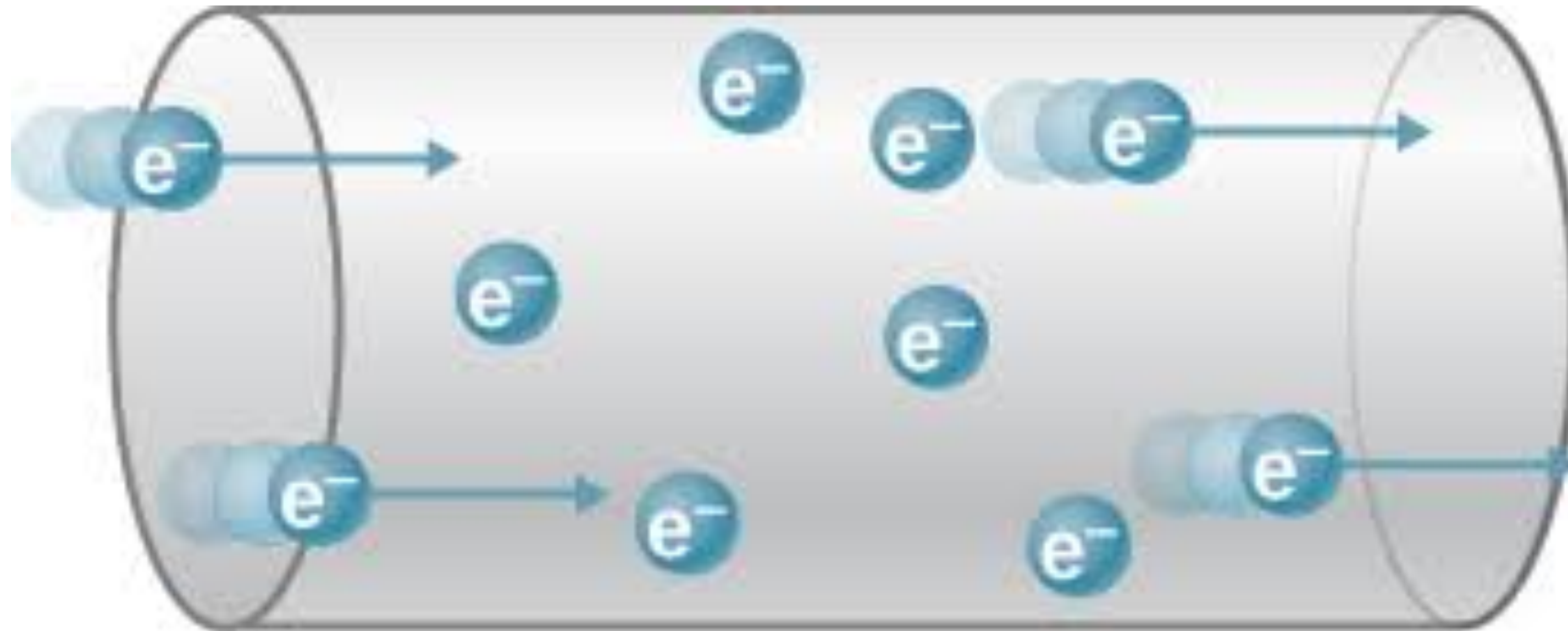
Quantum Black Holes

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- Answer from string theory for 'supersymmetric' charged black holes: $D(E) = e^S \delta(E)$ *i.e.* all the states required by Hawking's entropy have exactly the same energy.

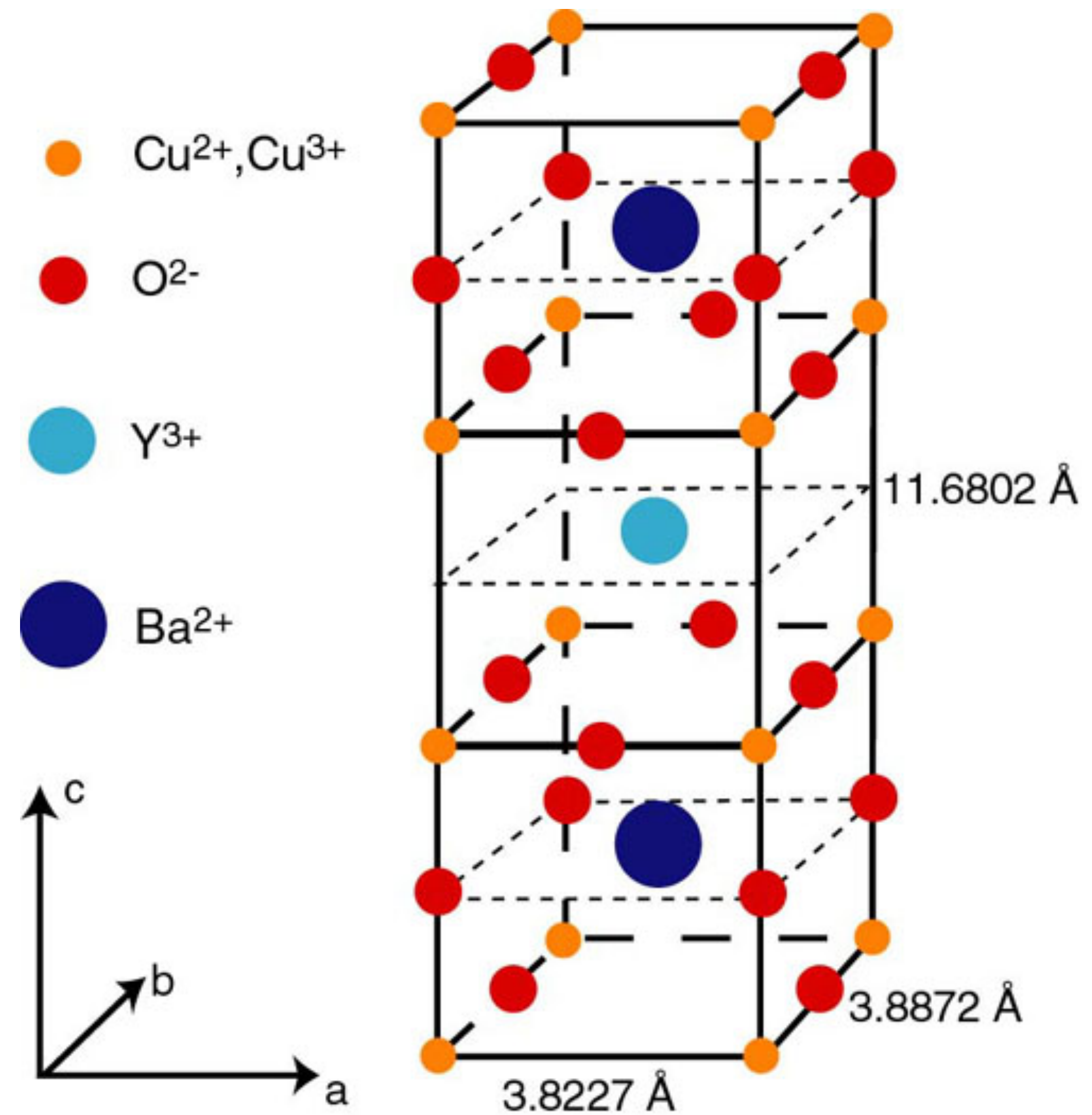
Strominger, Vafa (1996)

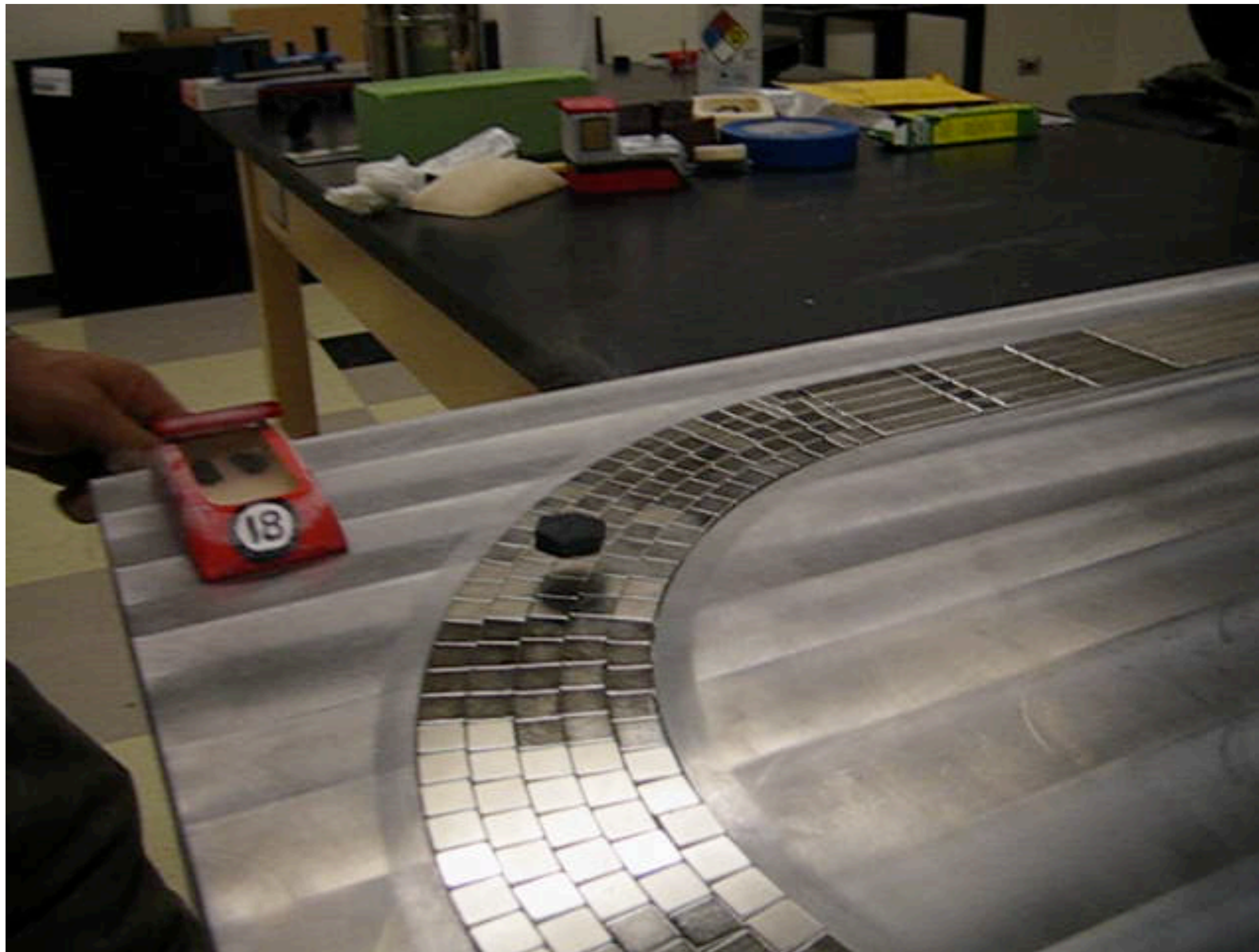
Quantum matter:
ordinary metals,
and
strange metals

Copper: ordinary metal quantum matter



Cuprate high temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

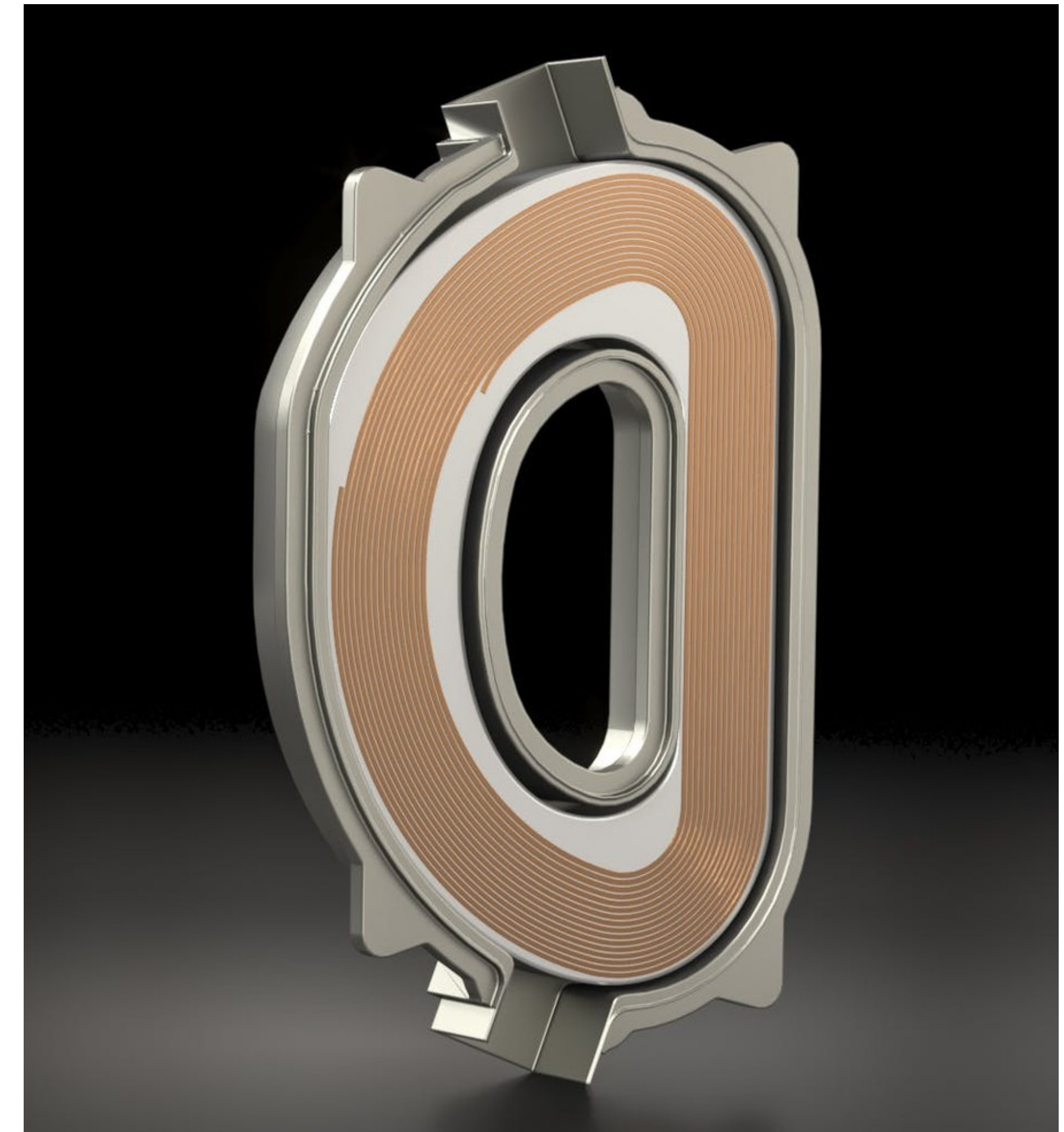
HTS Magnets: Enabling Technology

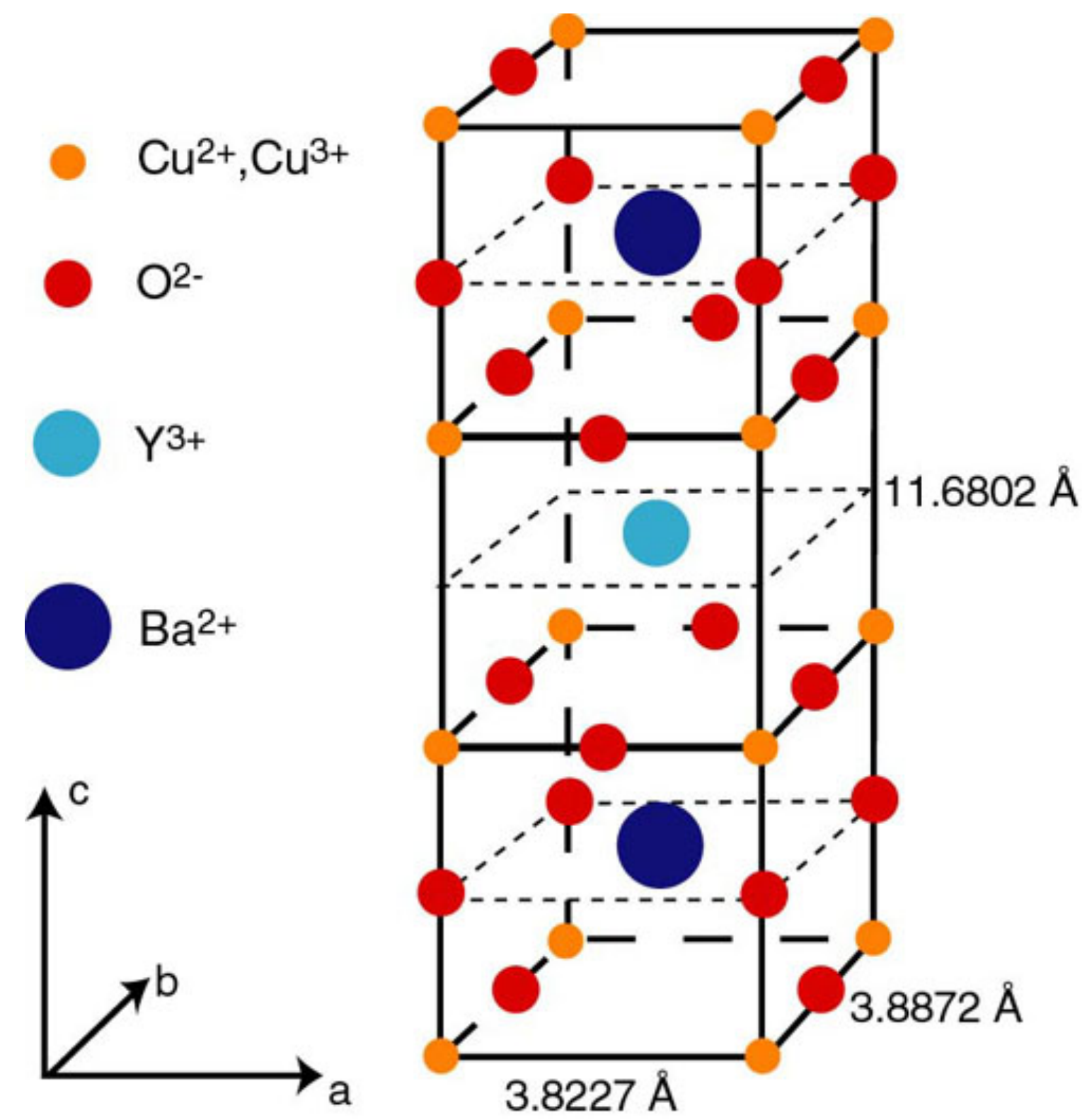
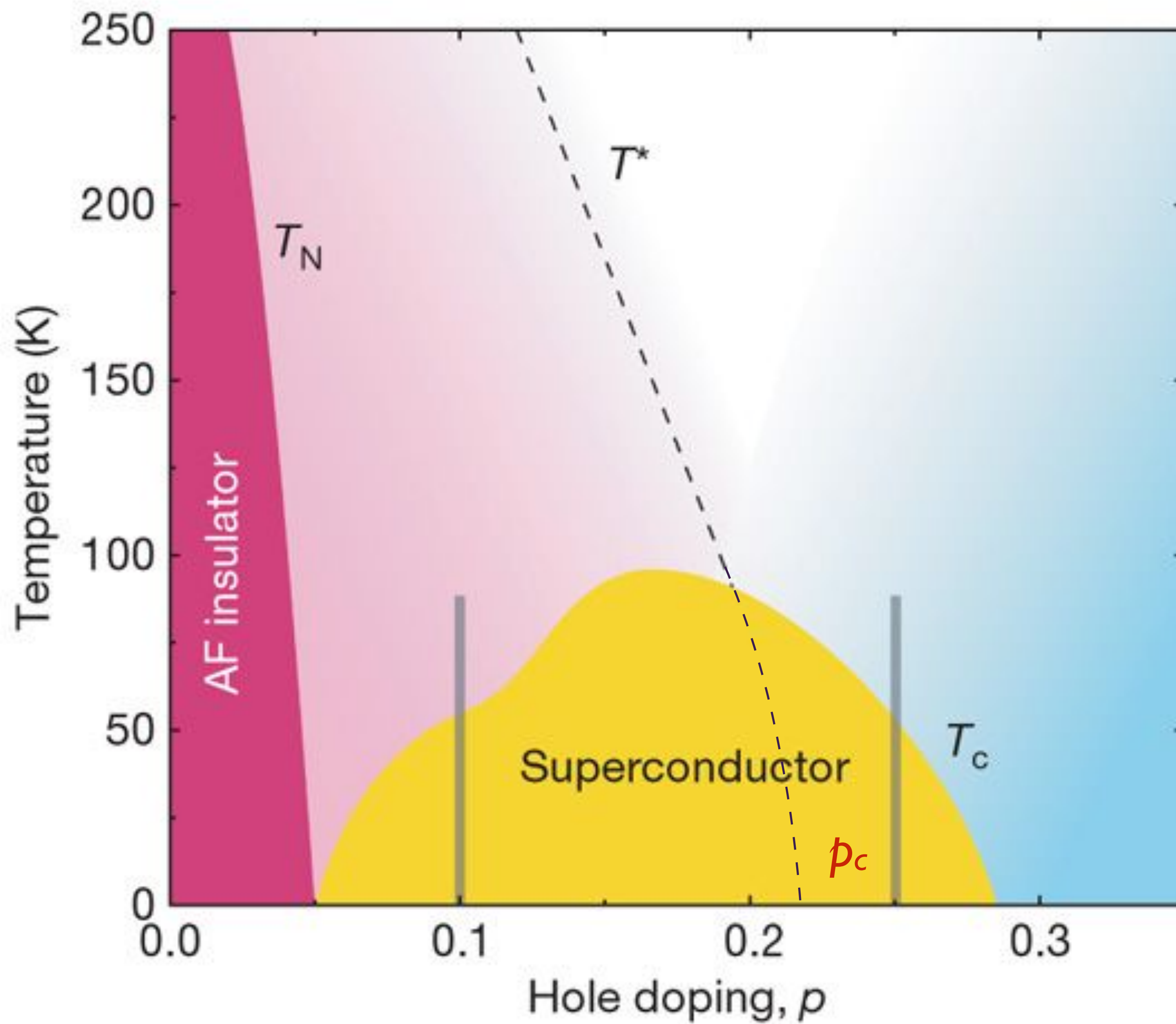
The surest path to limitless,
clean, fusion energy

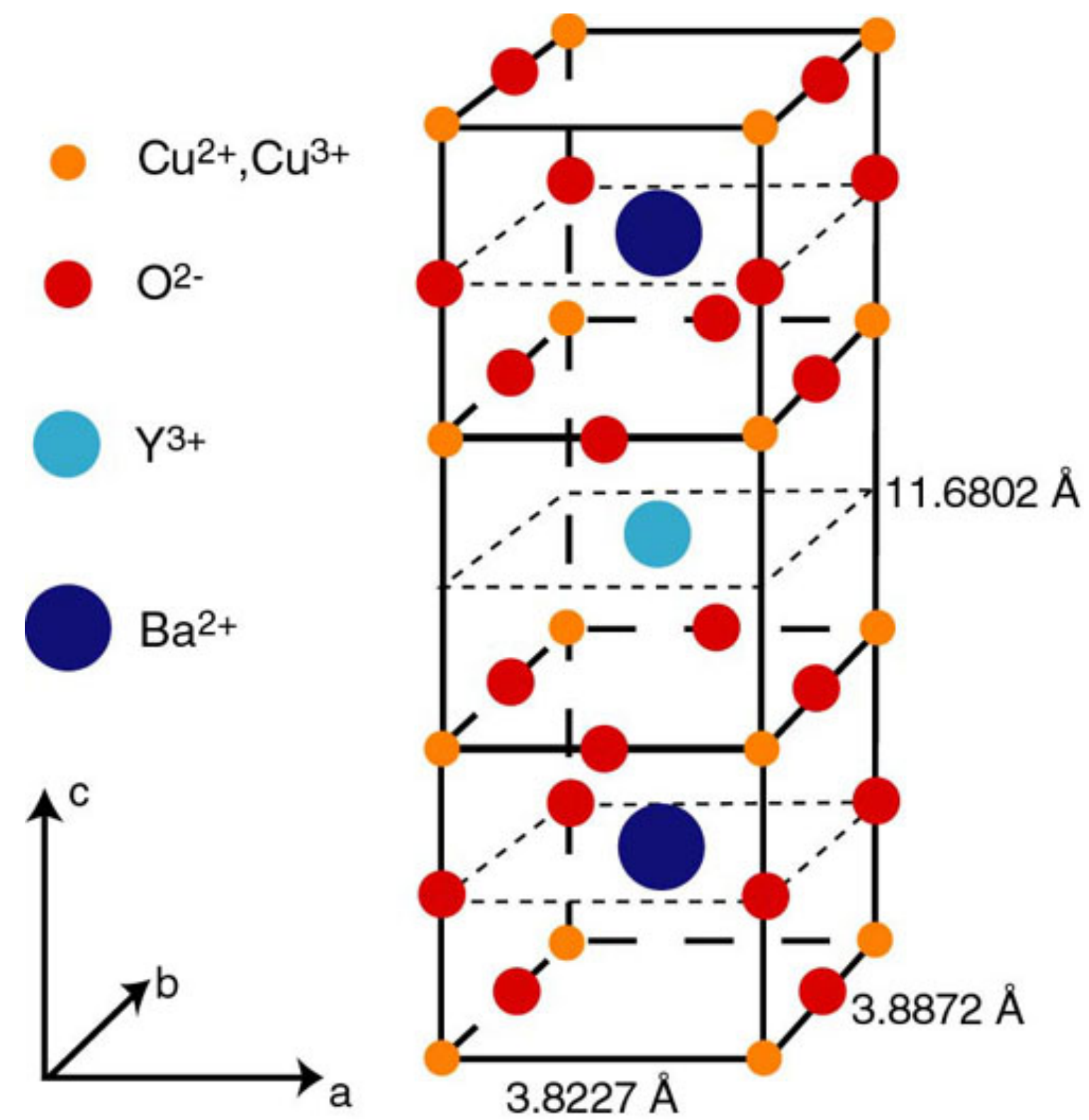
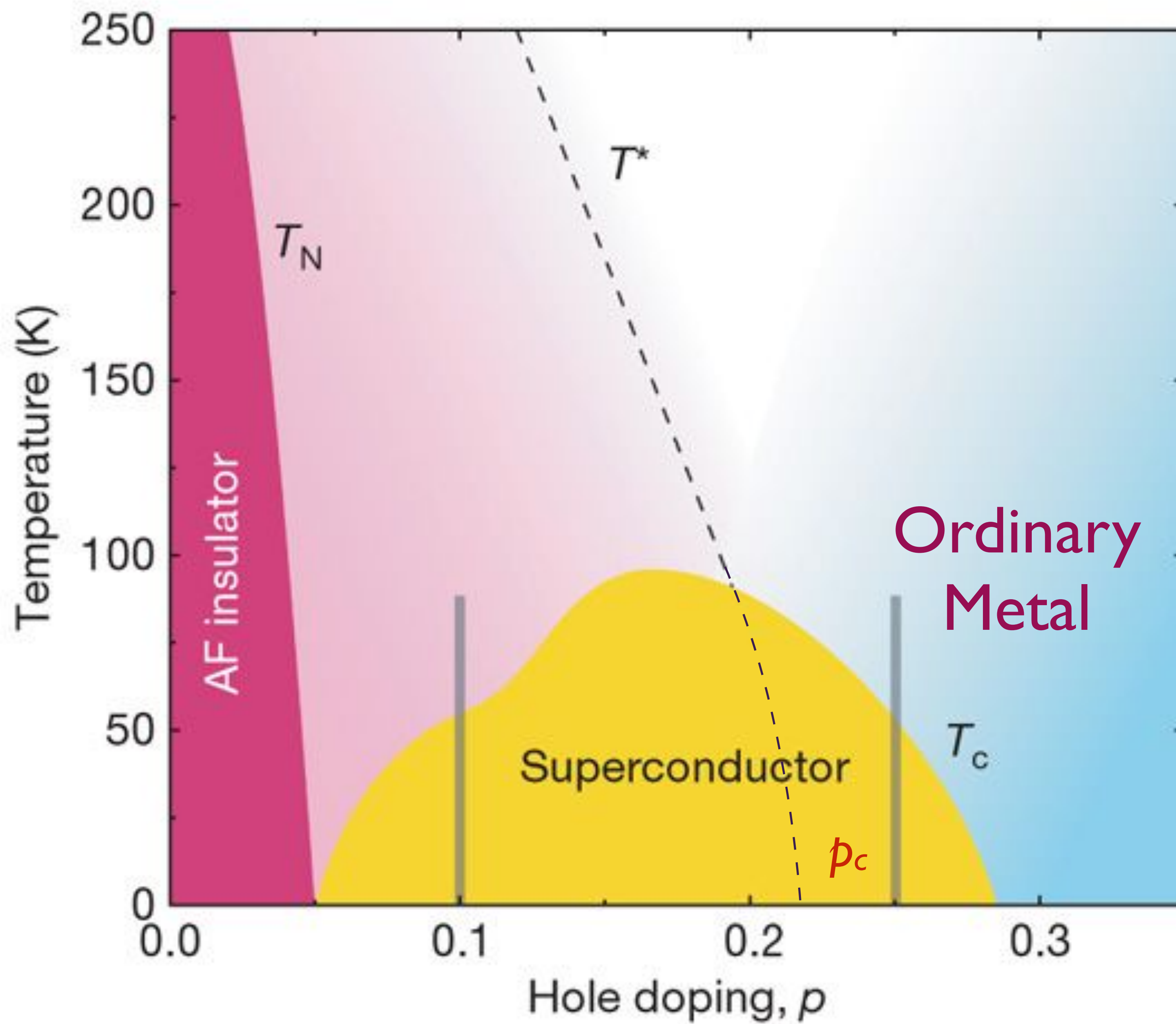
YBCO magnets allow for smaller,
faster, and less expensive
tokamaks for plasma fusion

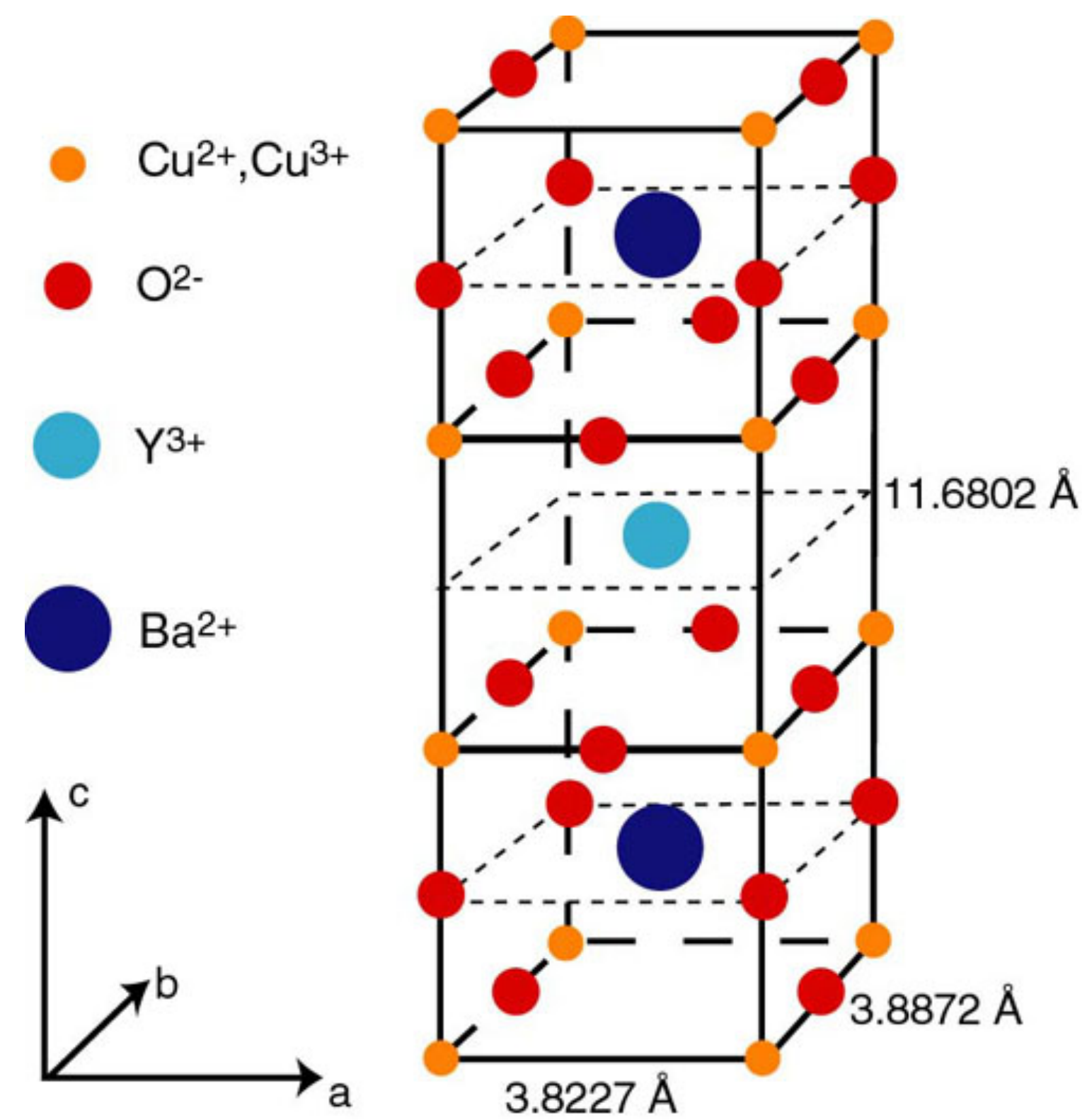
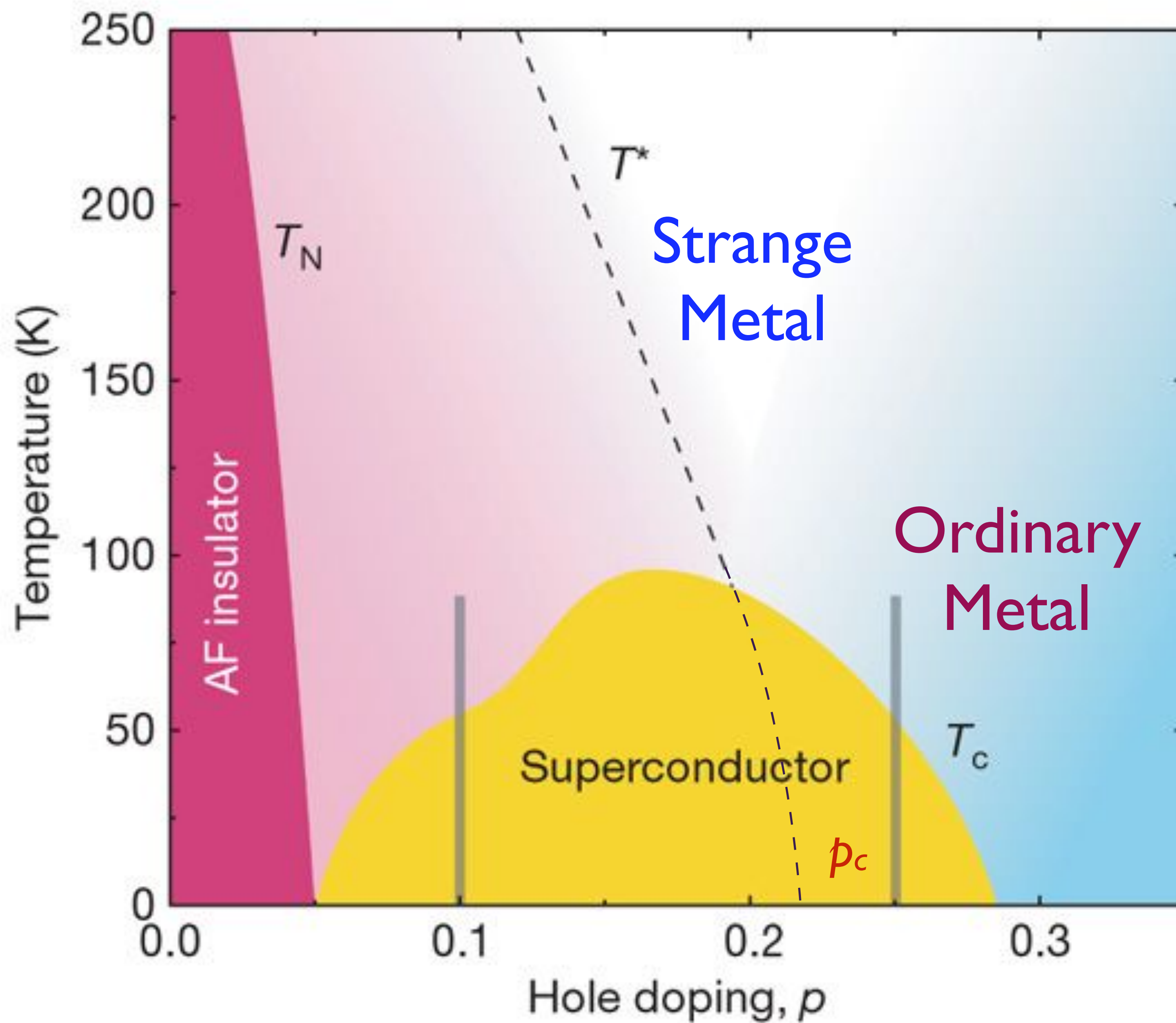


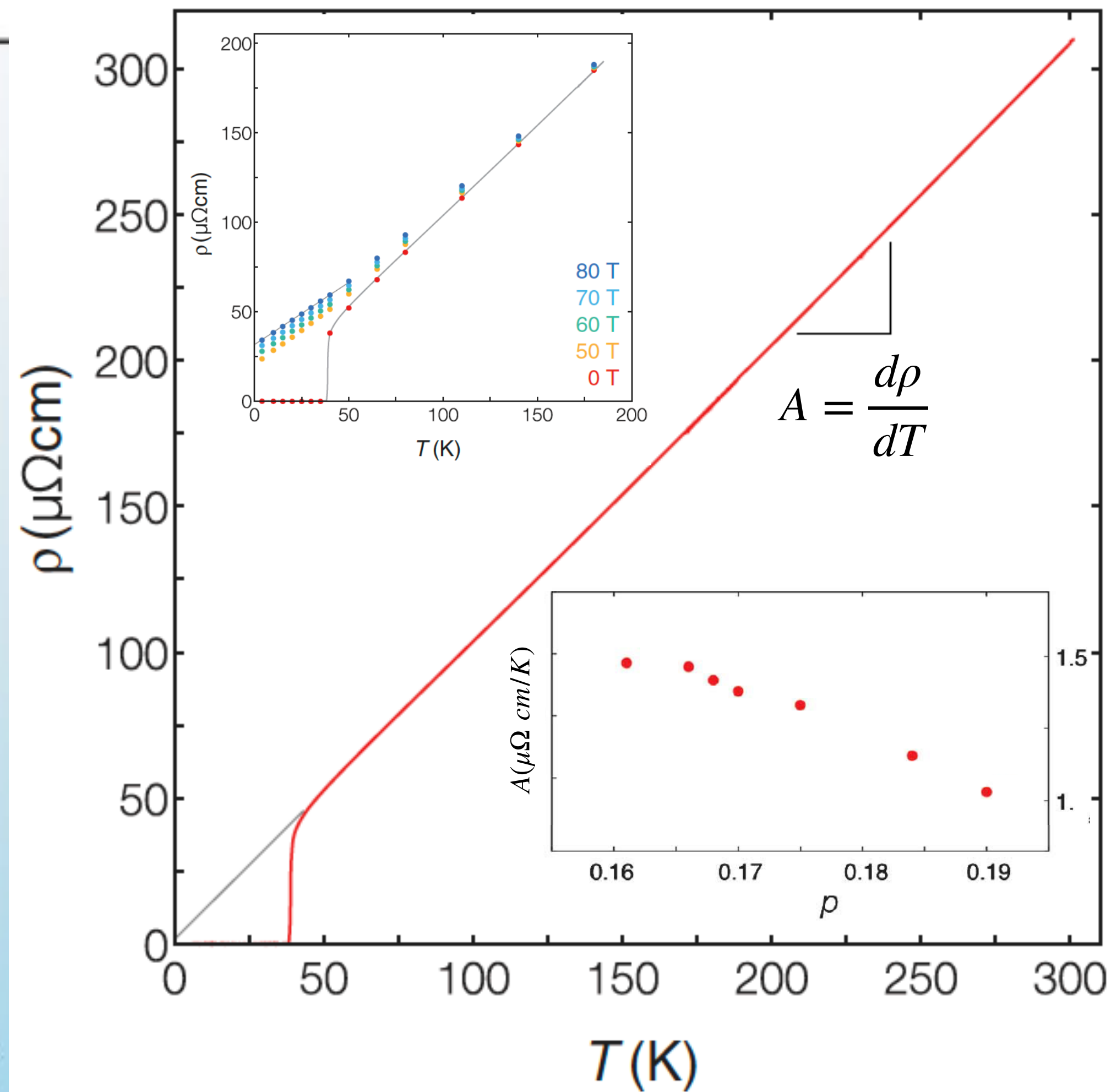
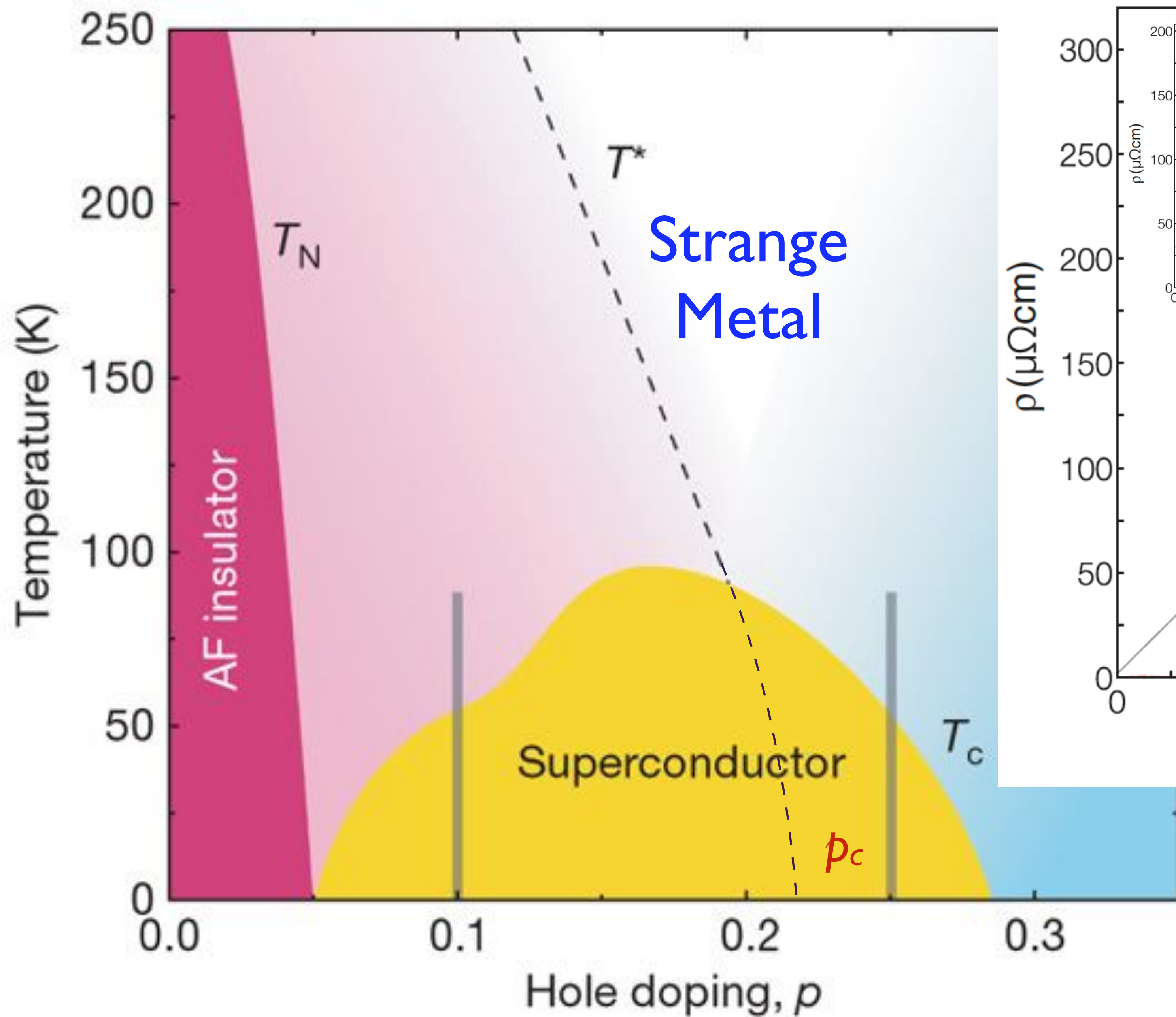
Commonwealth
Fusion Systems



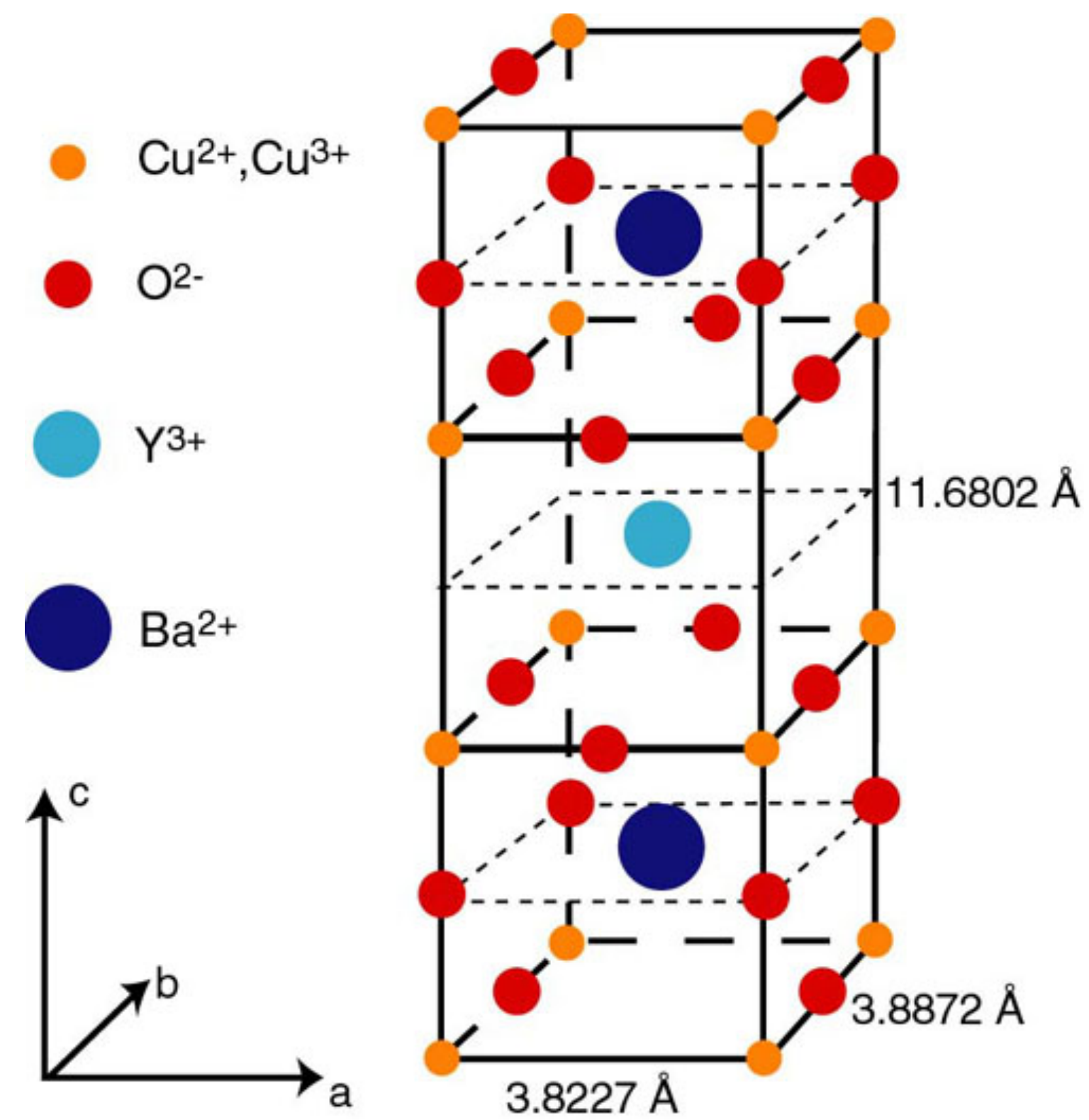
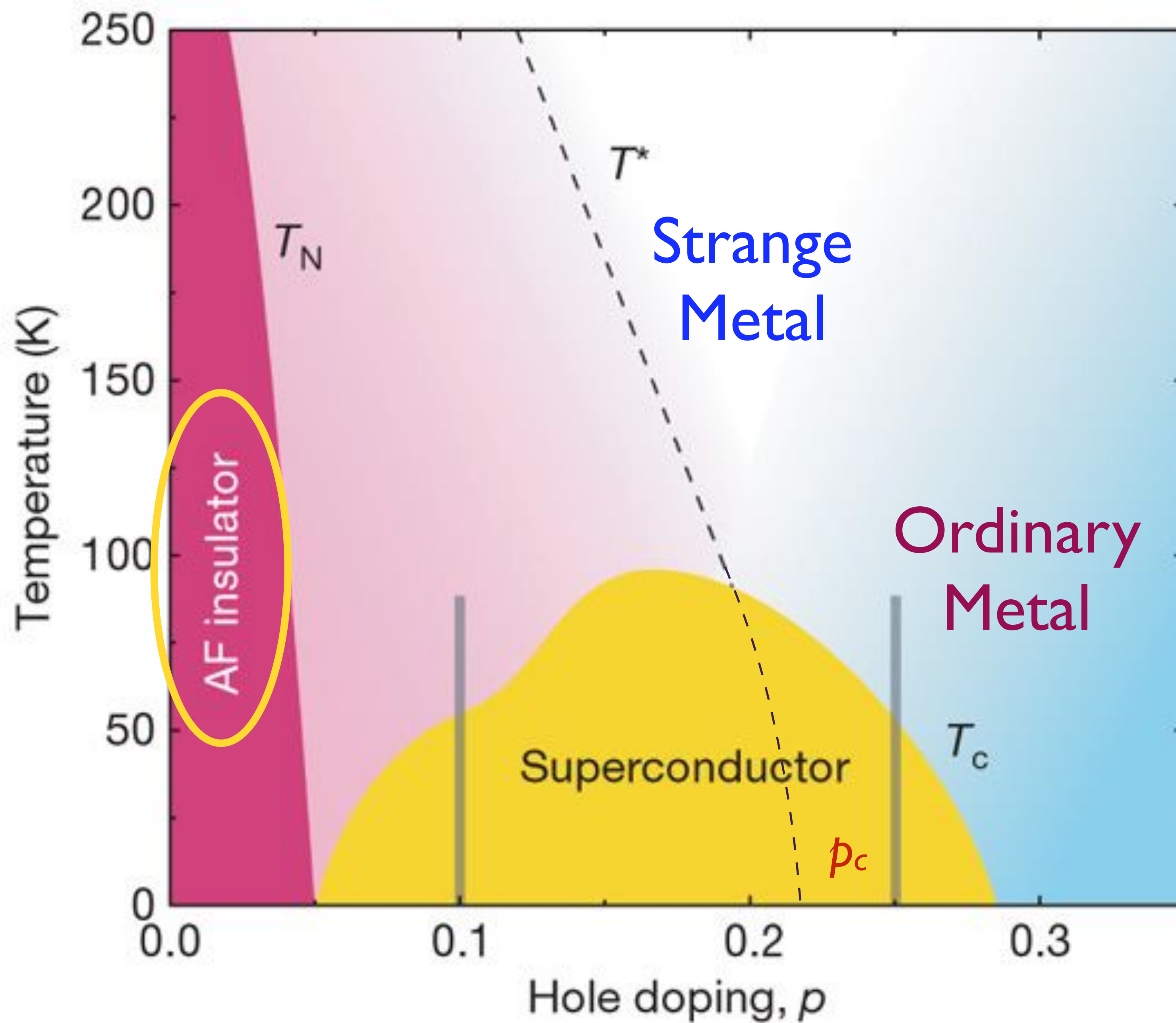




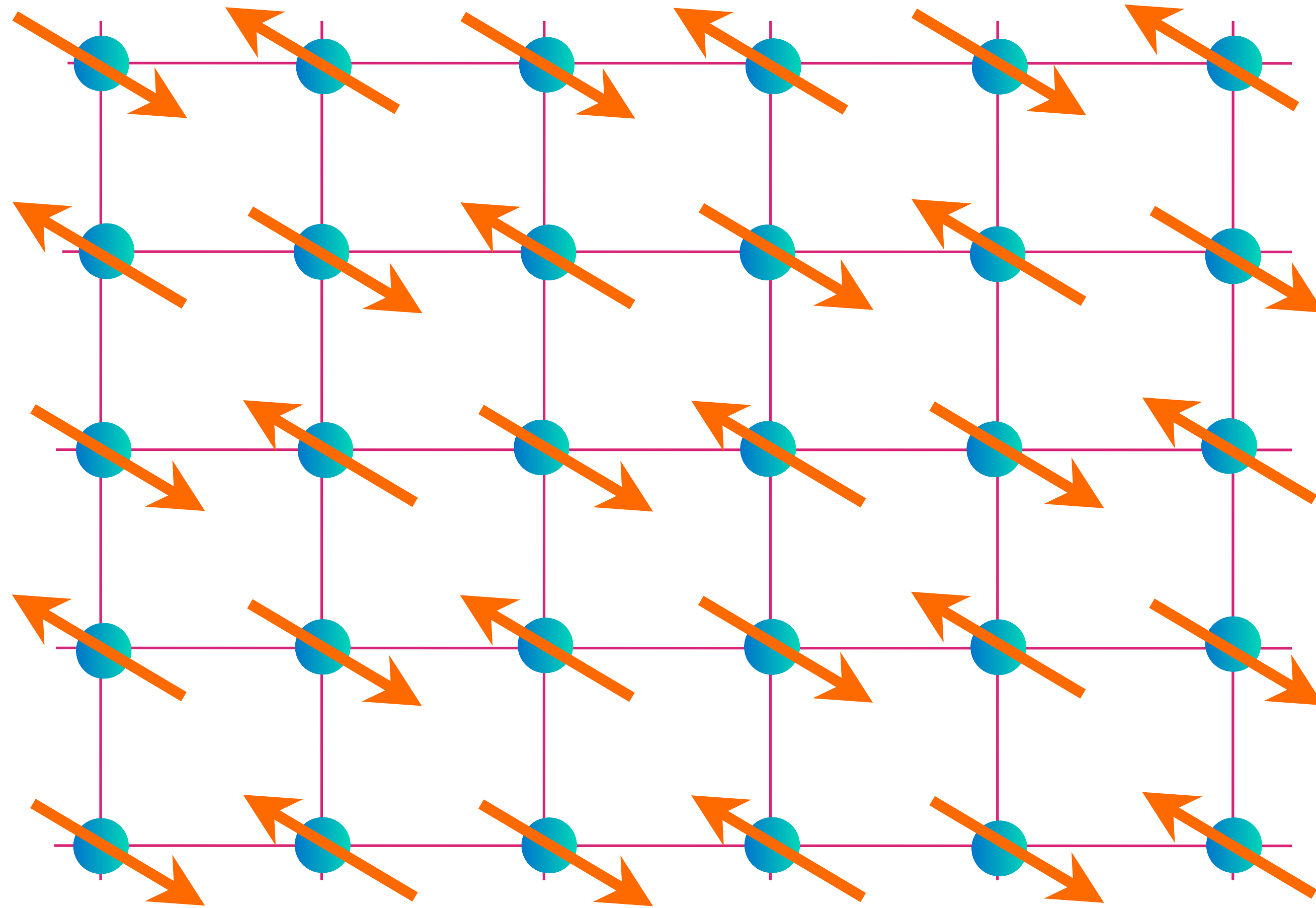




LSCO: Giraldo-Gallo et al. 2018

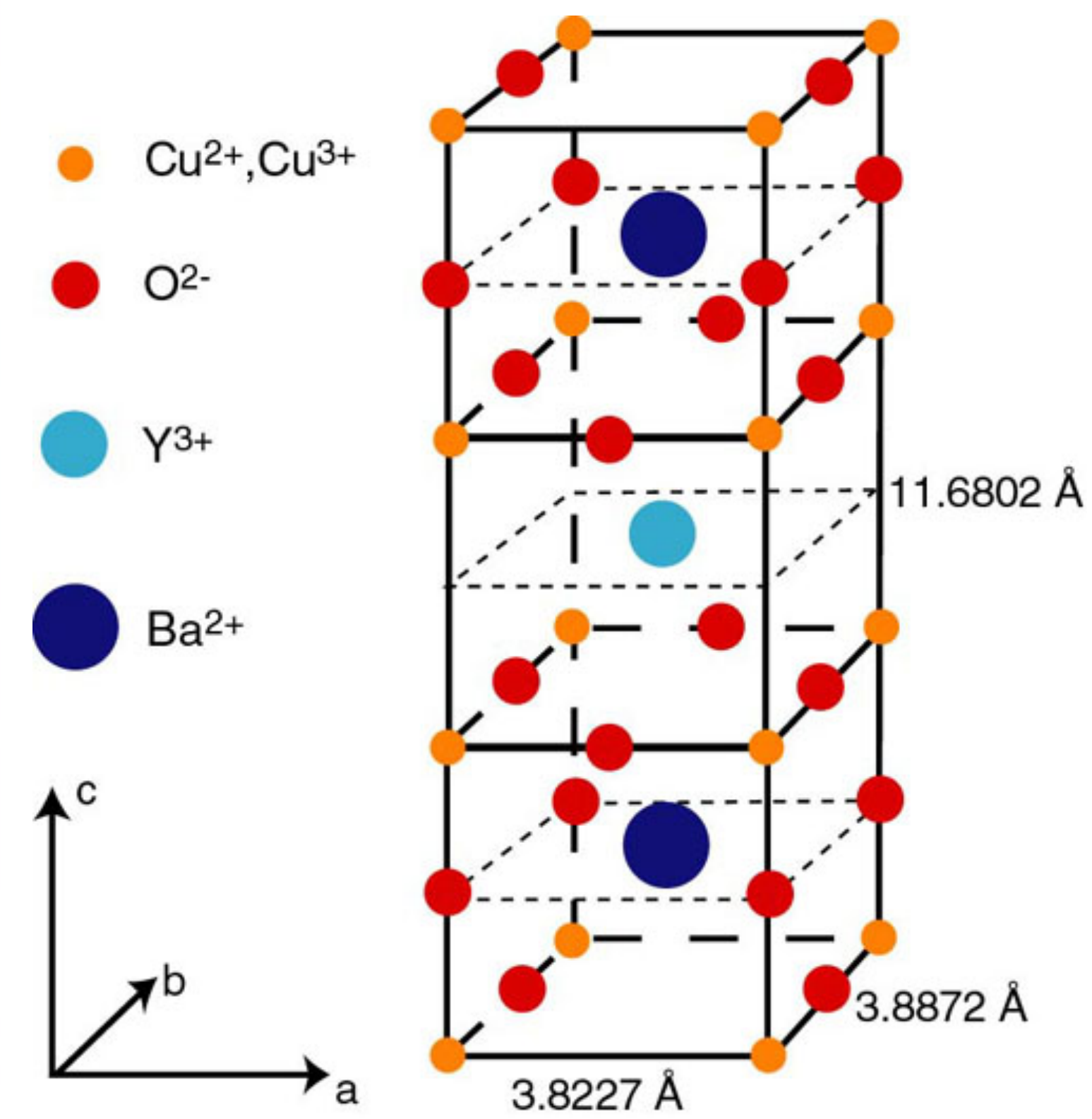
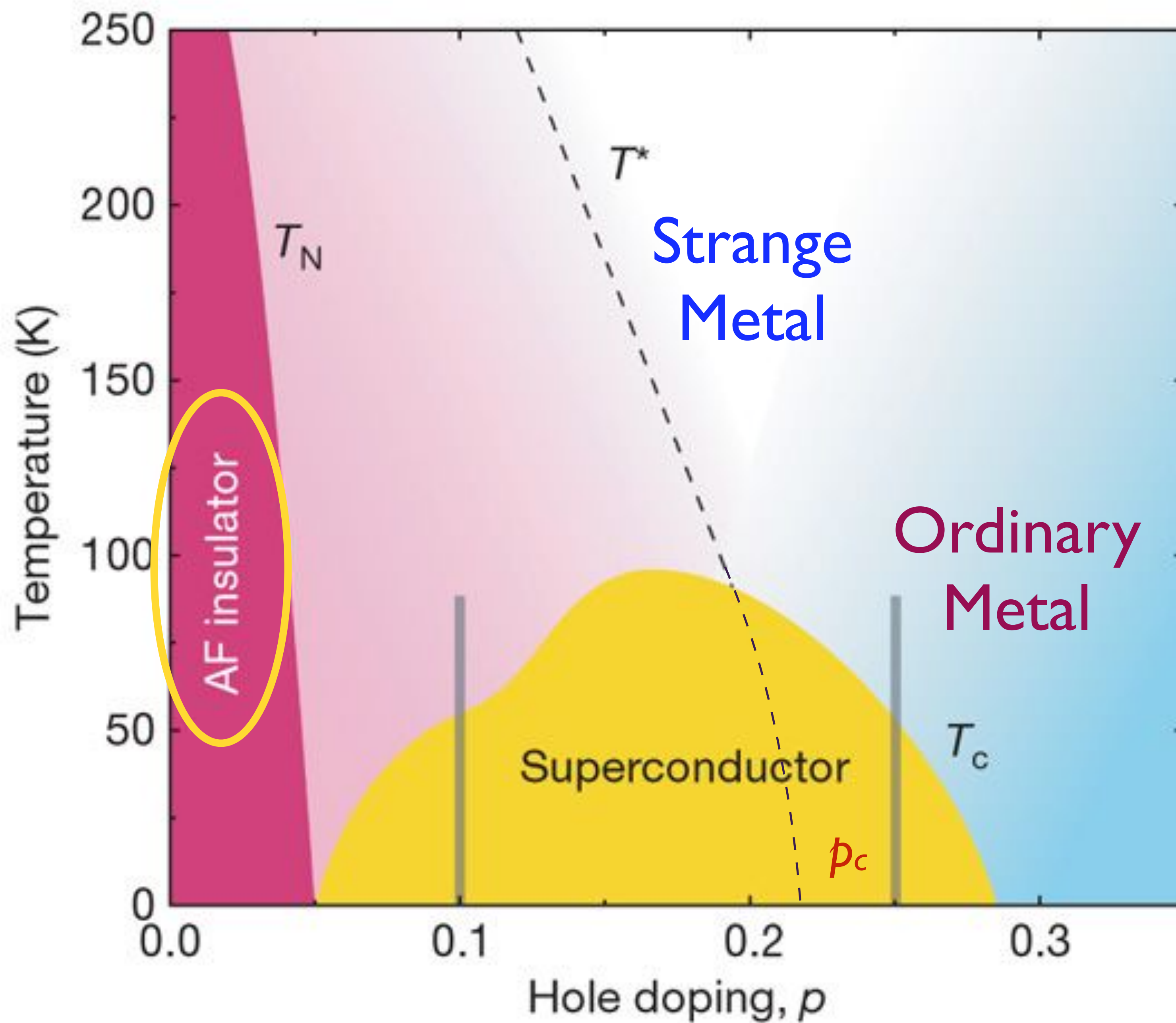


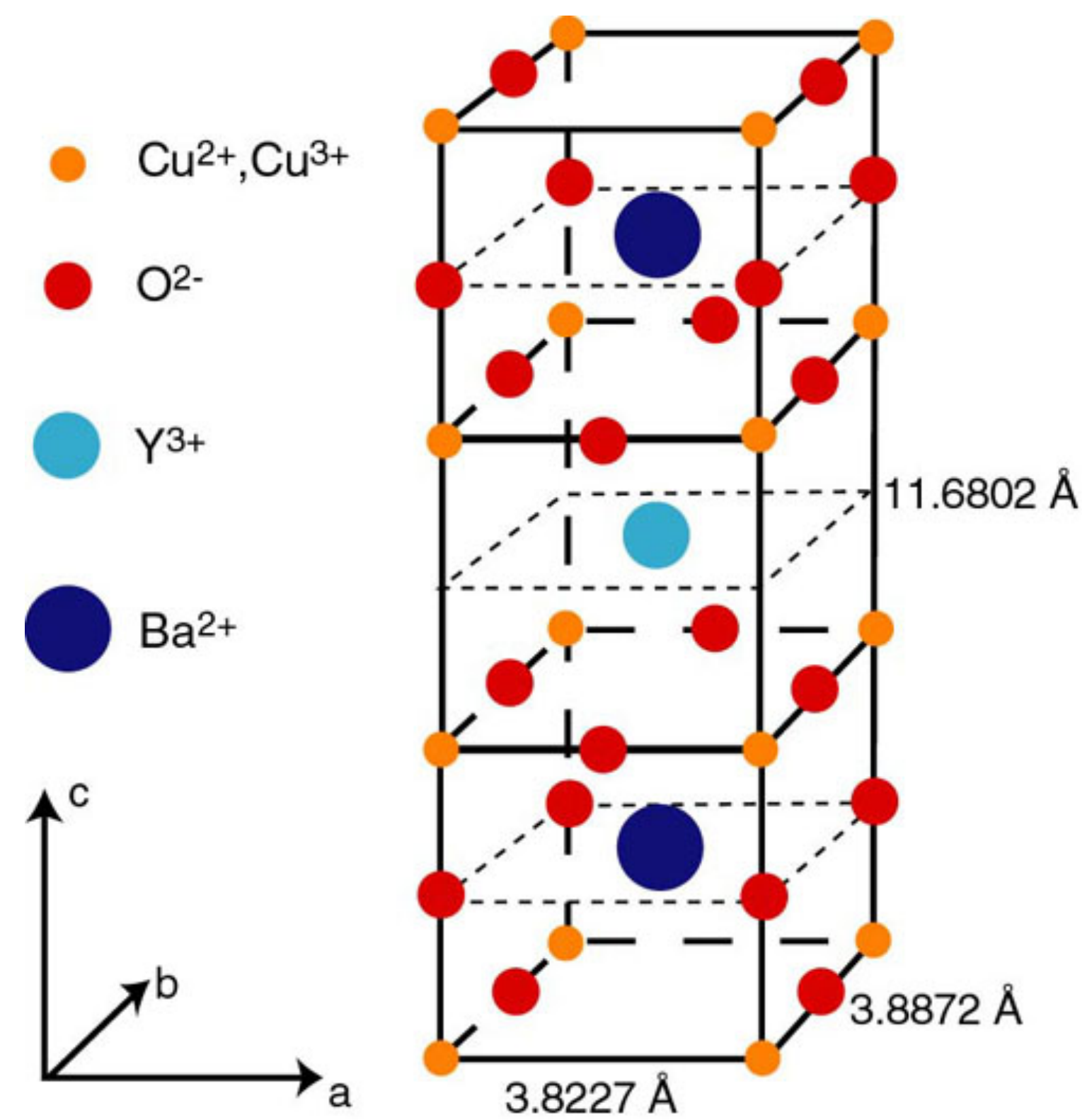
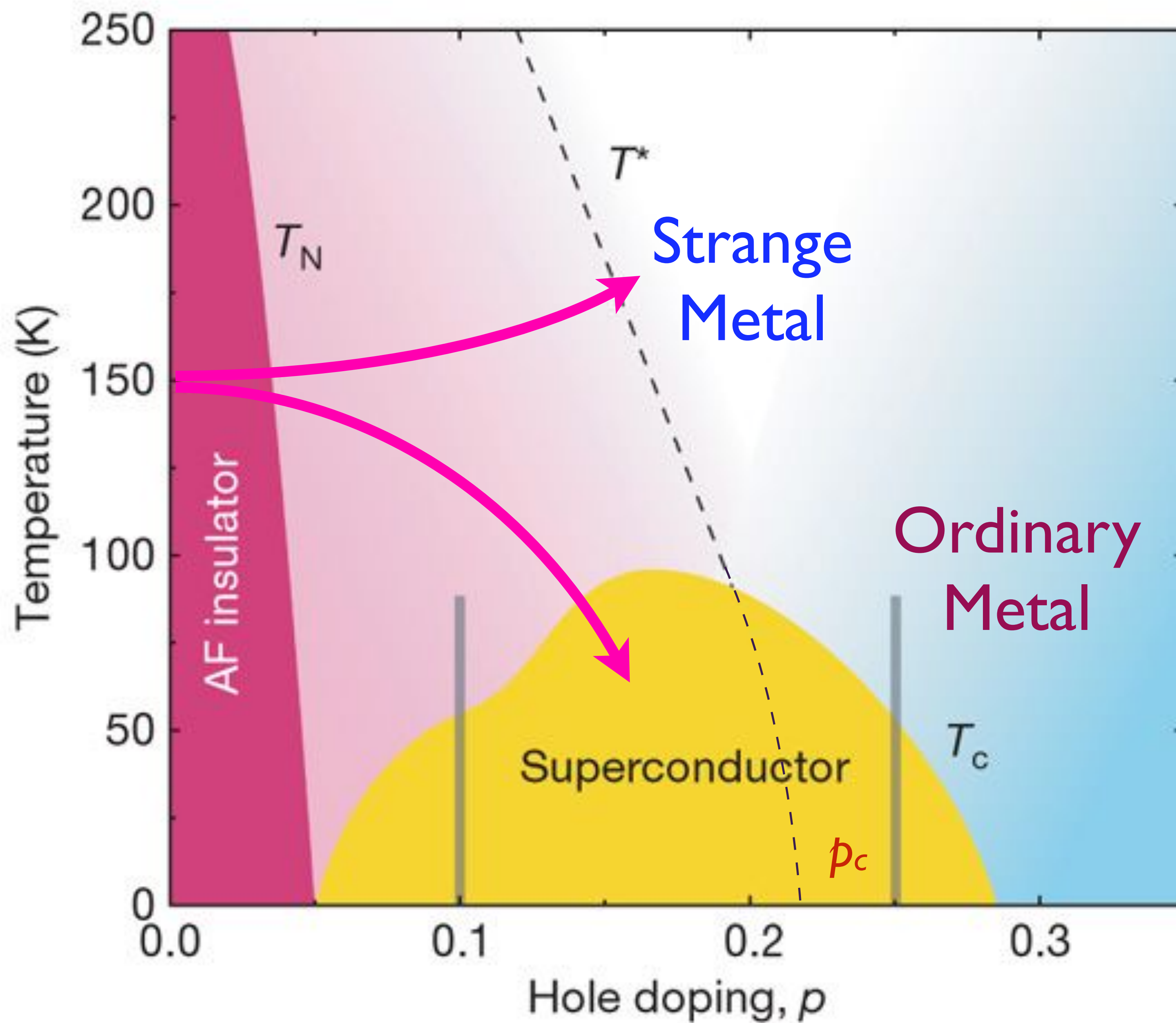
The dance of electrons on Cu atoms in YBCO



Antiferromagnetism

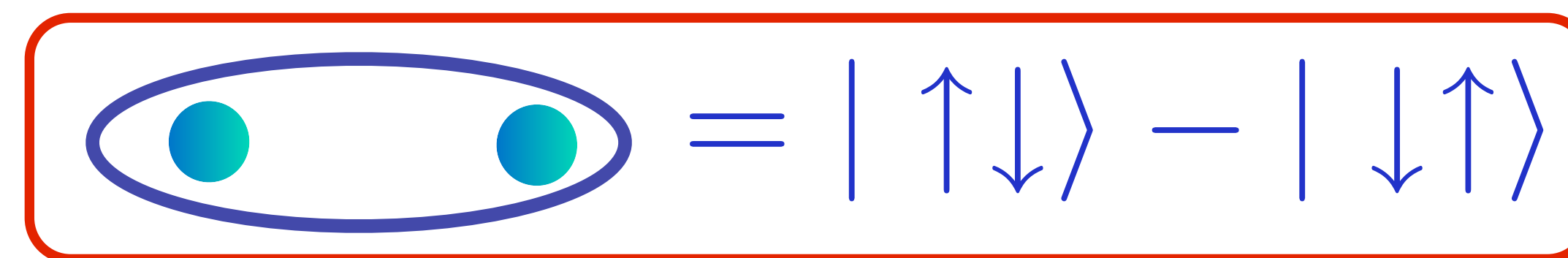
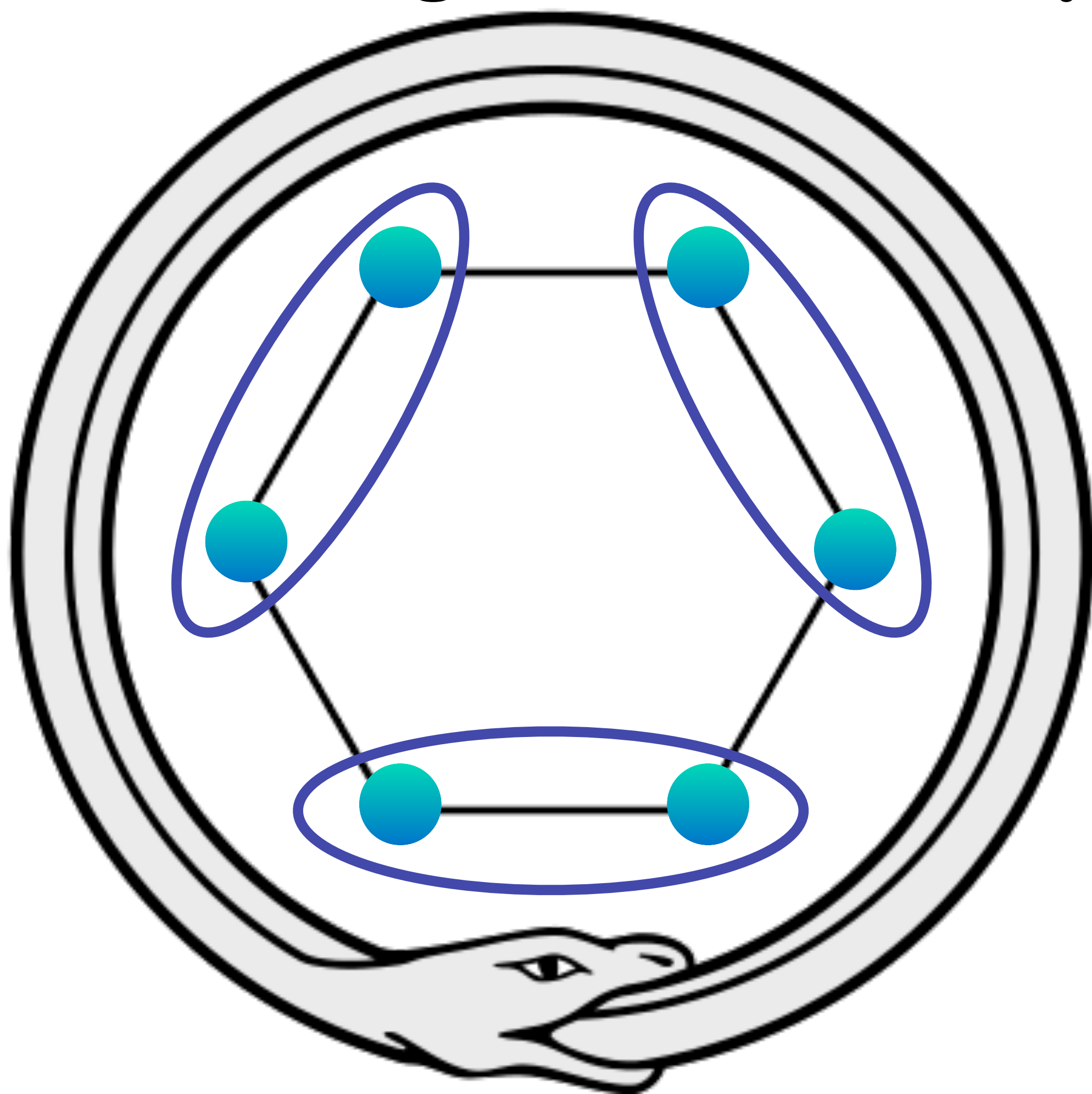
All nearest-neighbor pairs of electrons have opposite spins





Kekulé's spooky dream (1865)

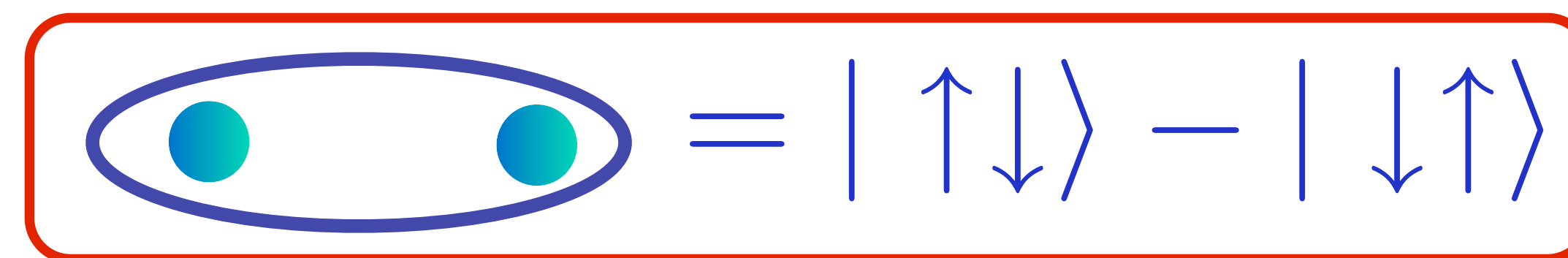
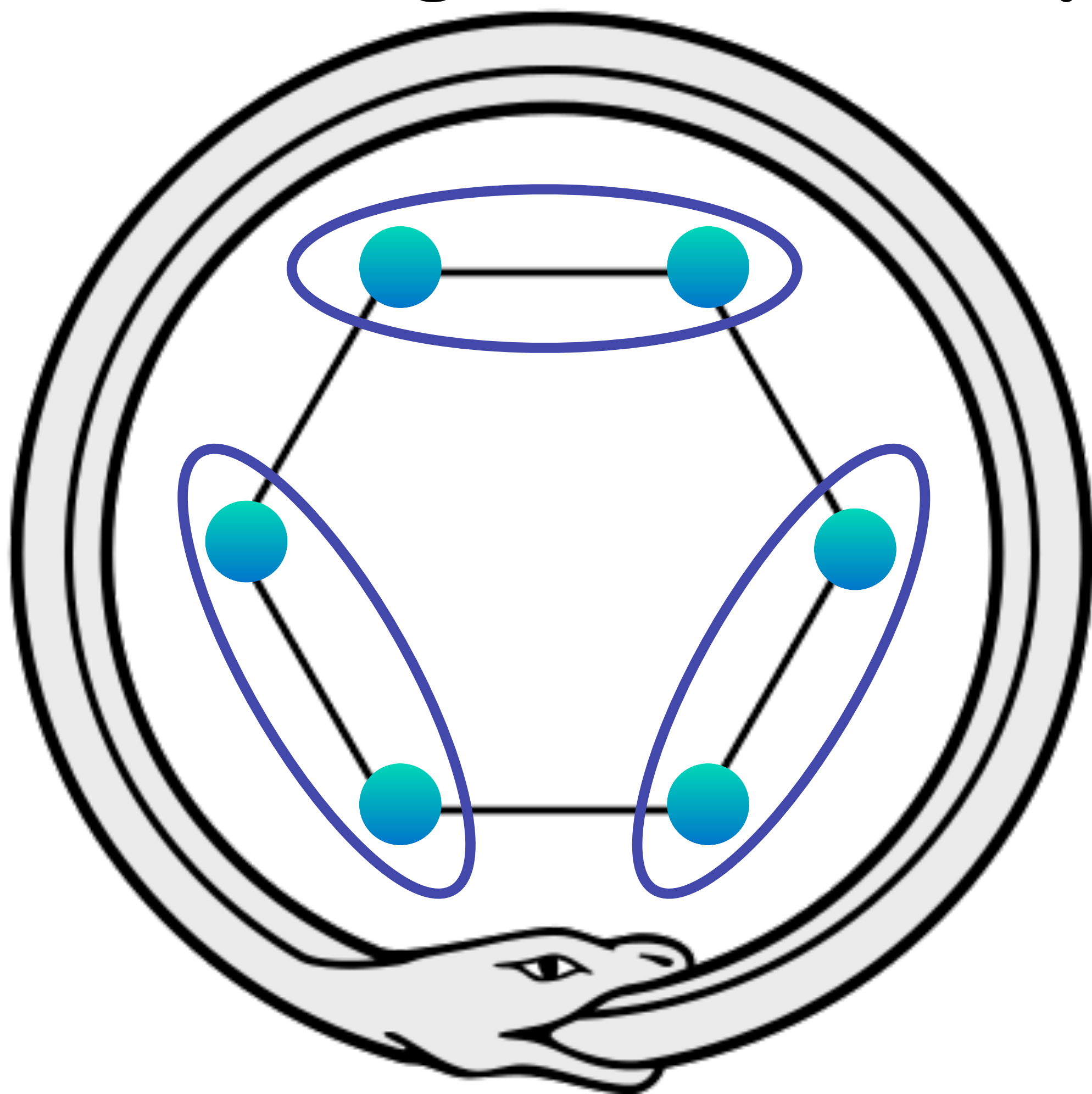
Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



Benzene

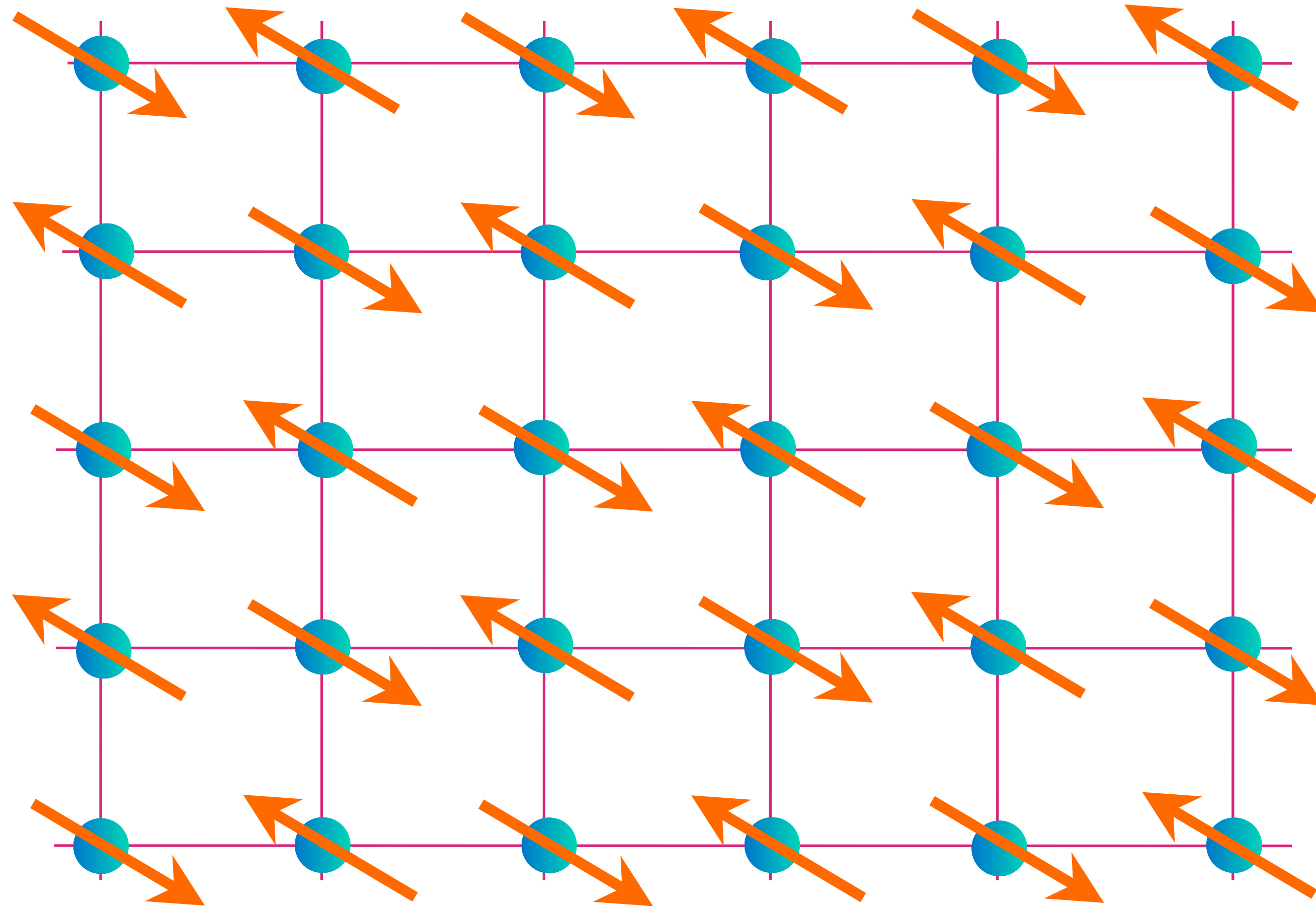
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Benzene

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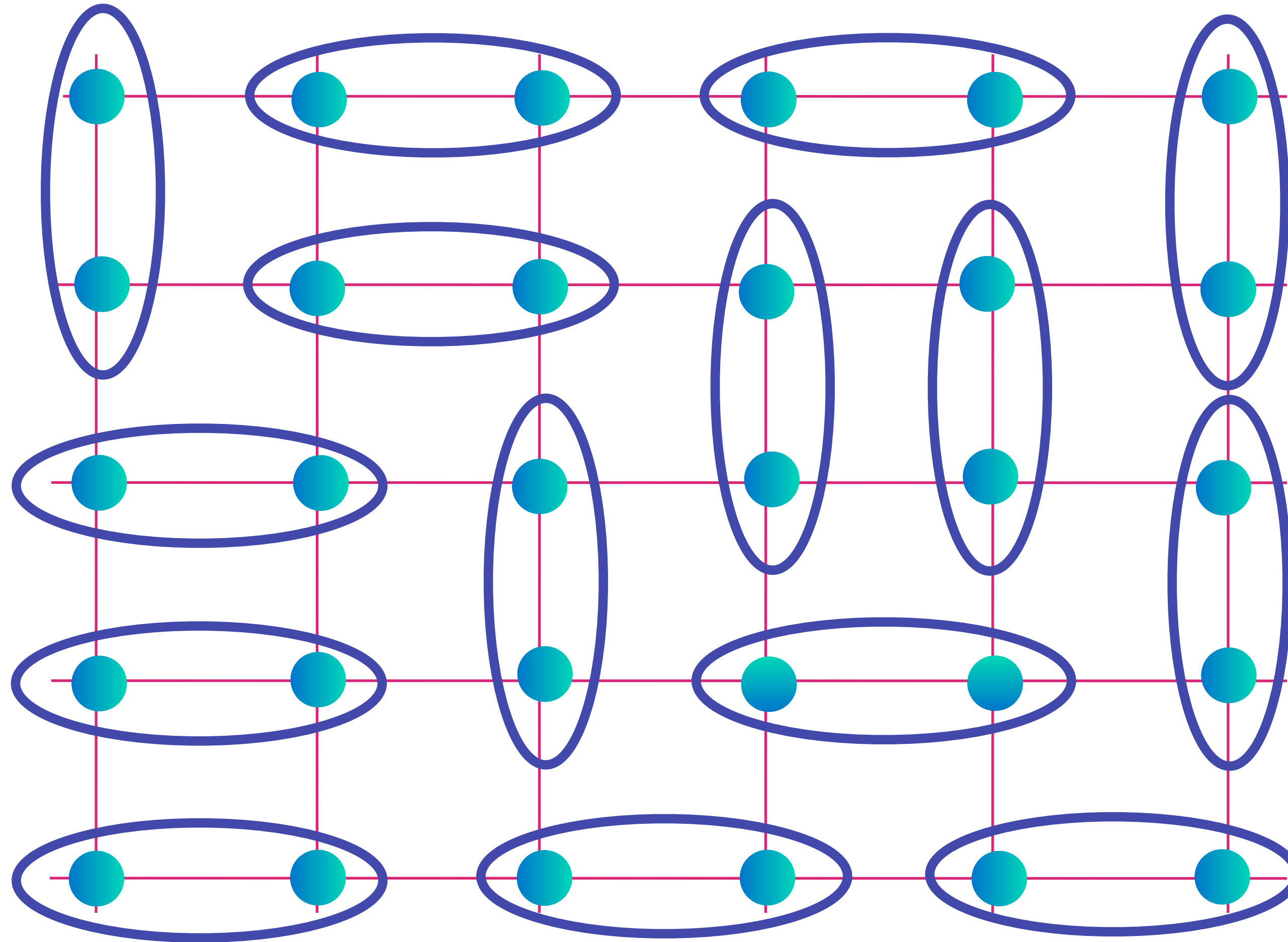
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The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



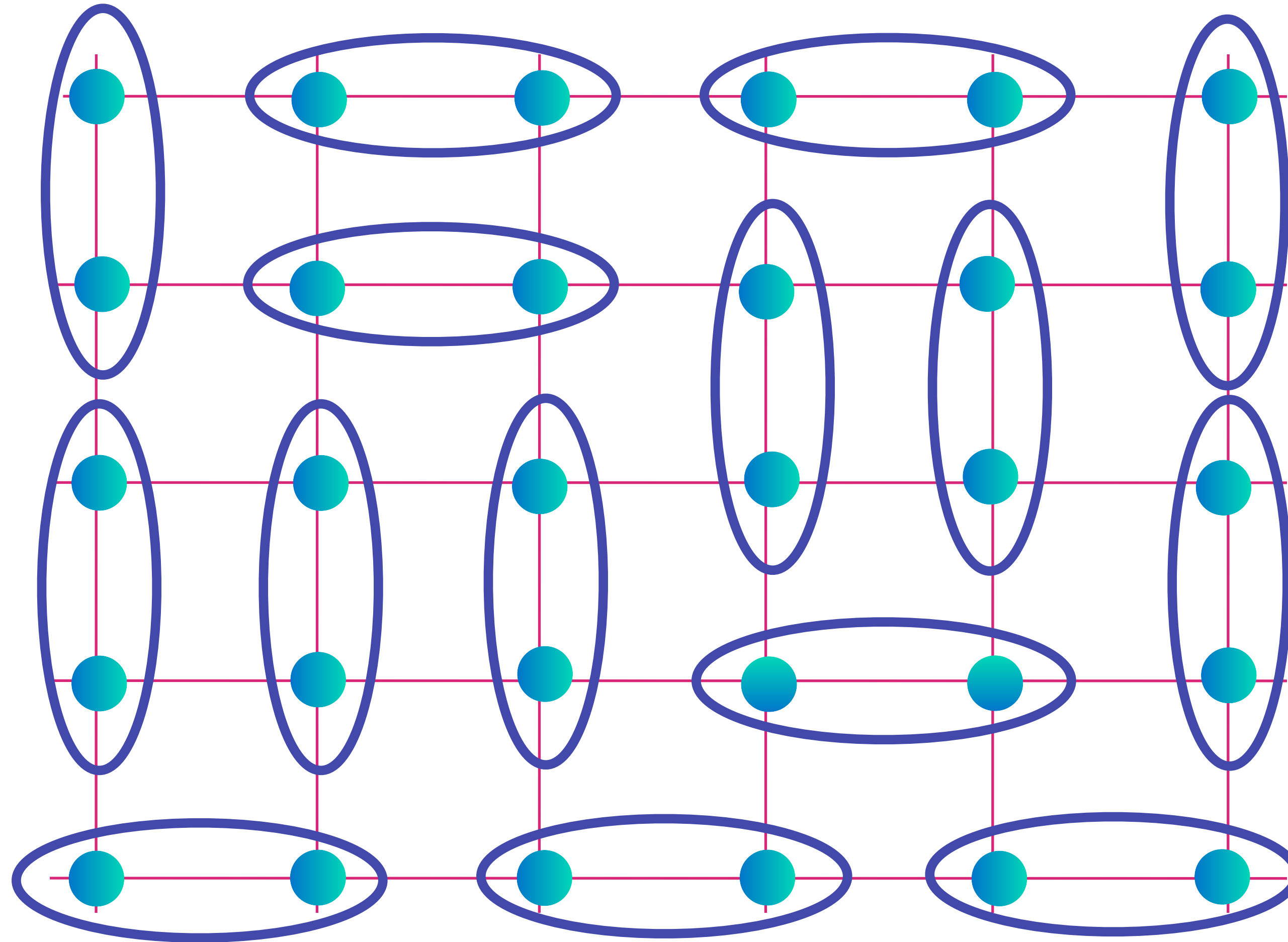
Electrons form entangled pairs, and the pairs entangle across the entire sample

$$\text{[Diagram of two electrons in an oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

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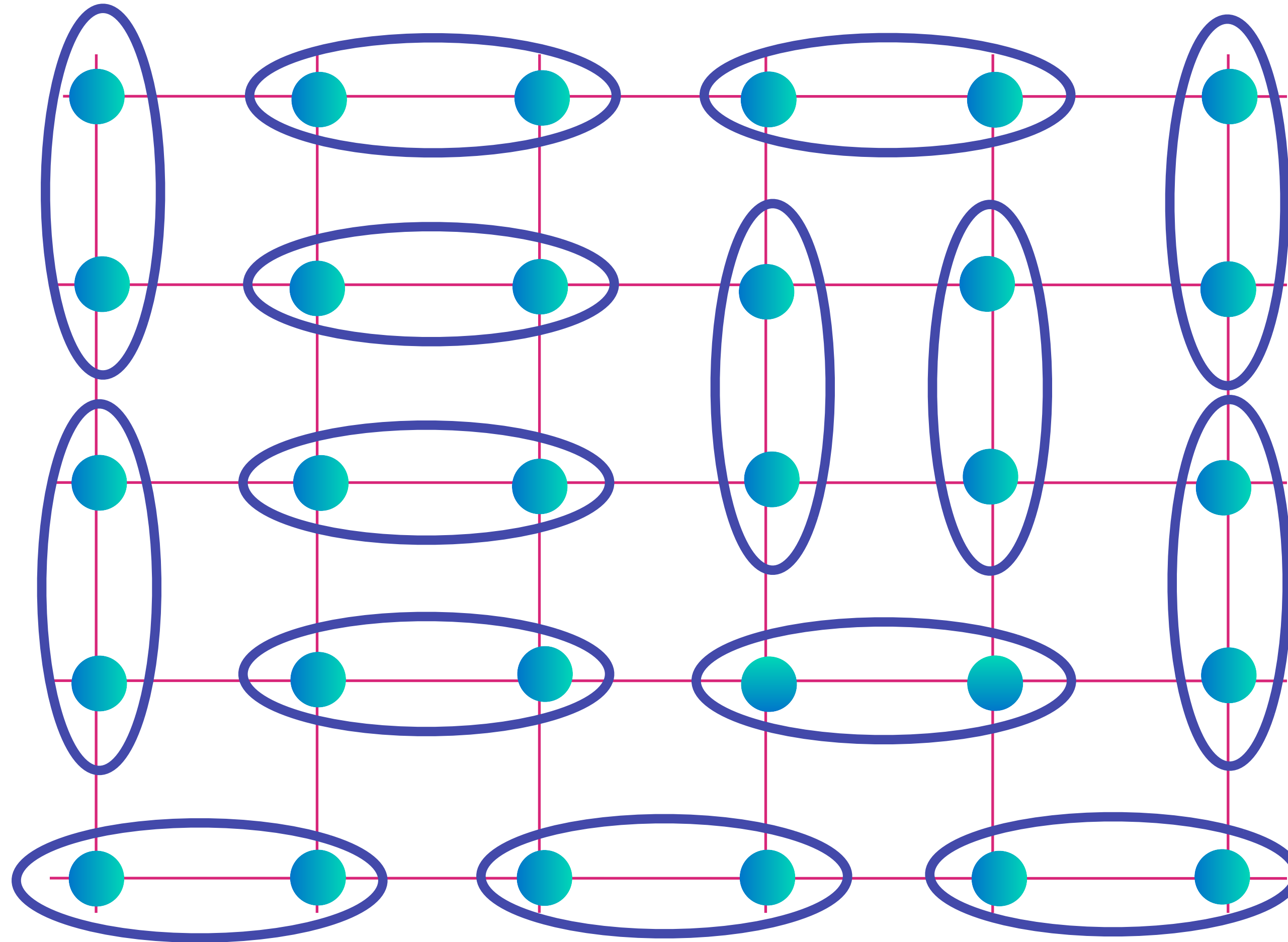
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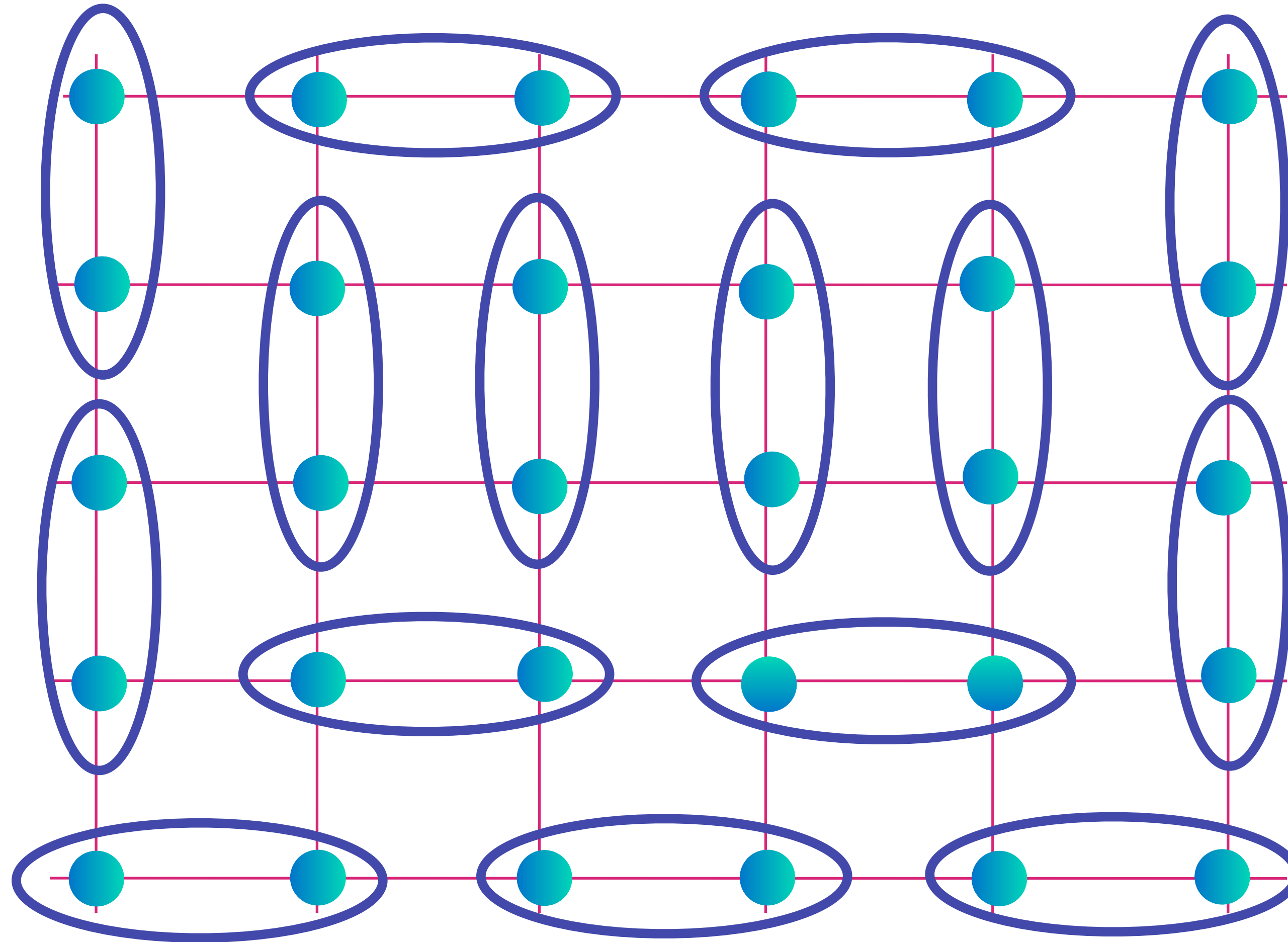
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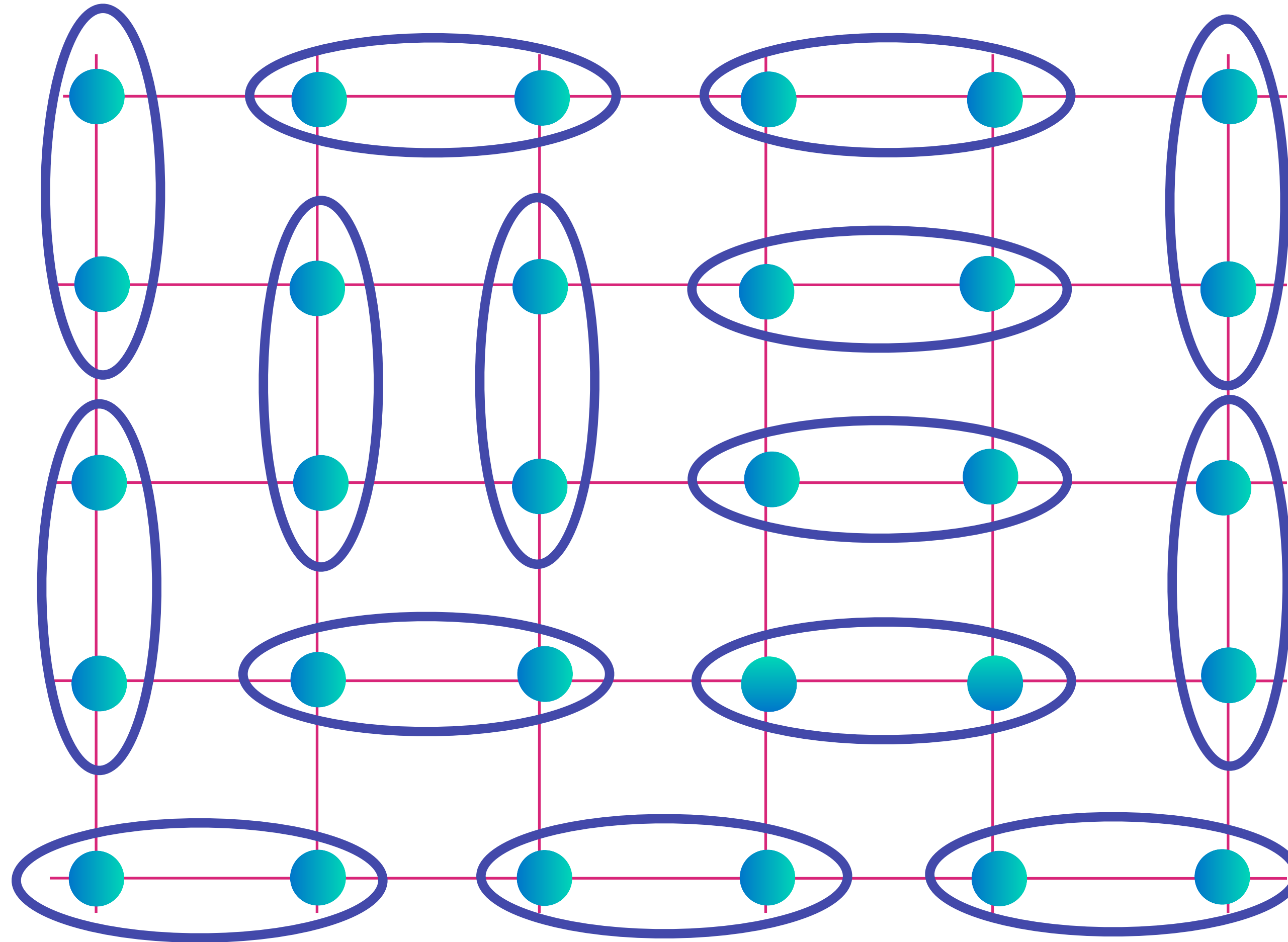
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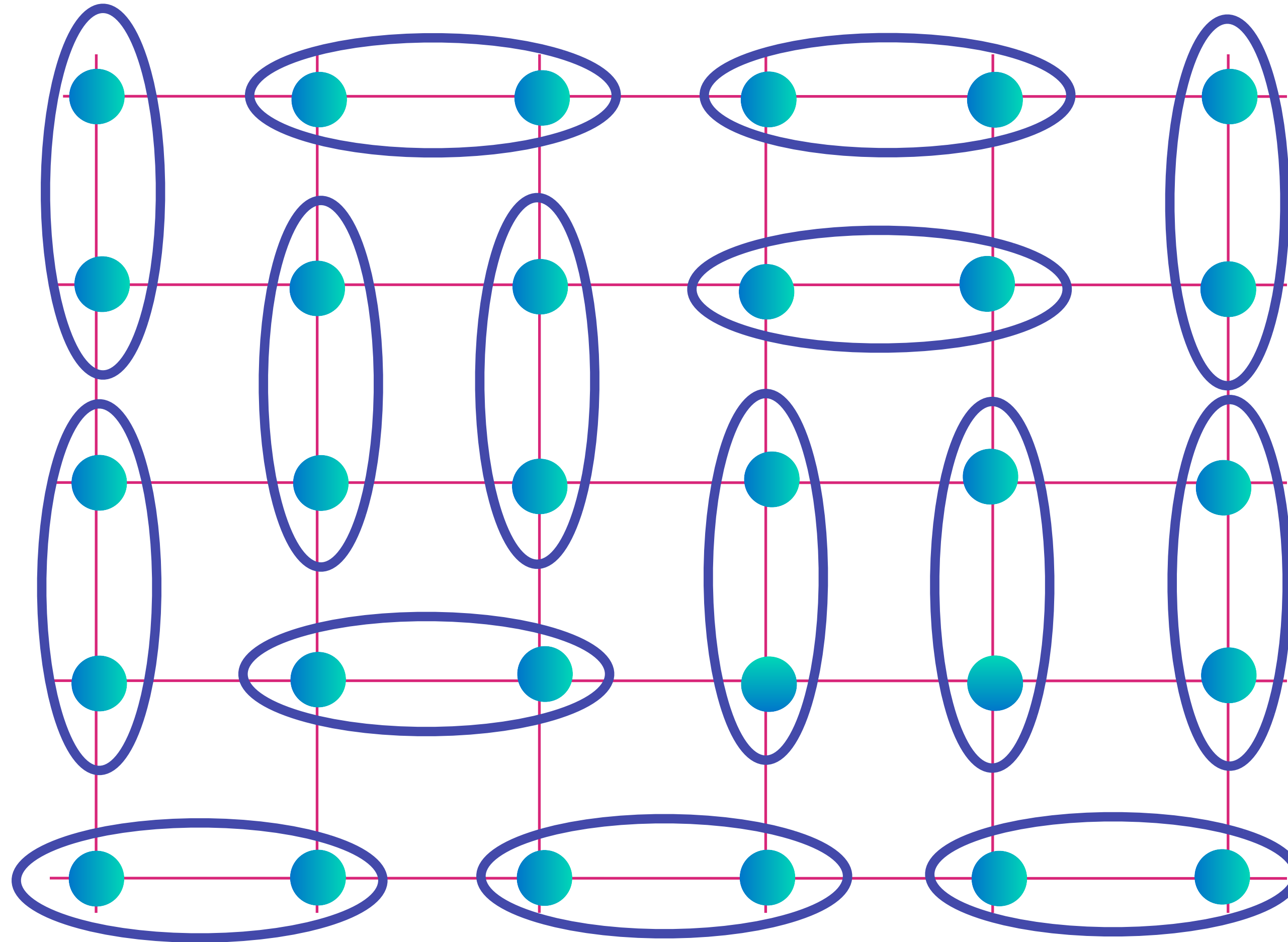
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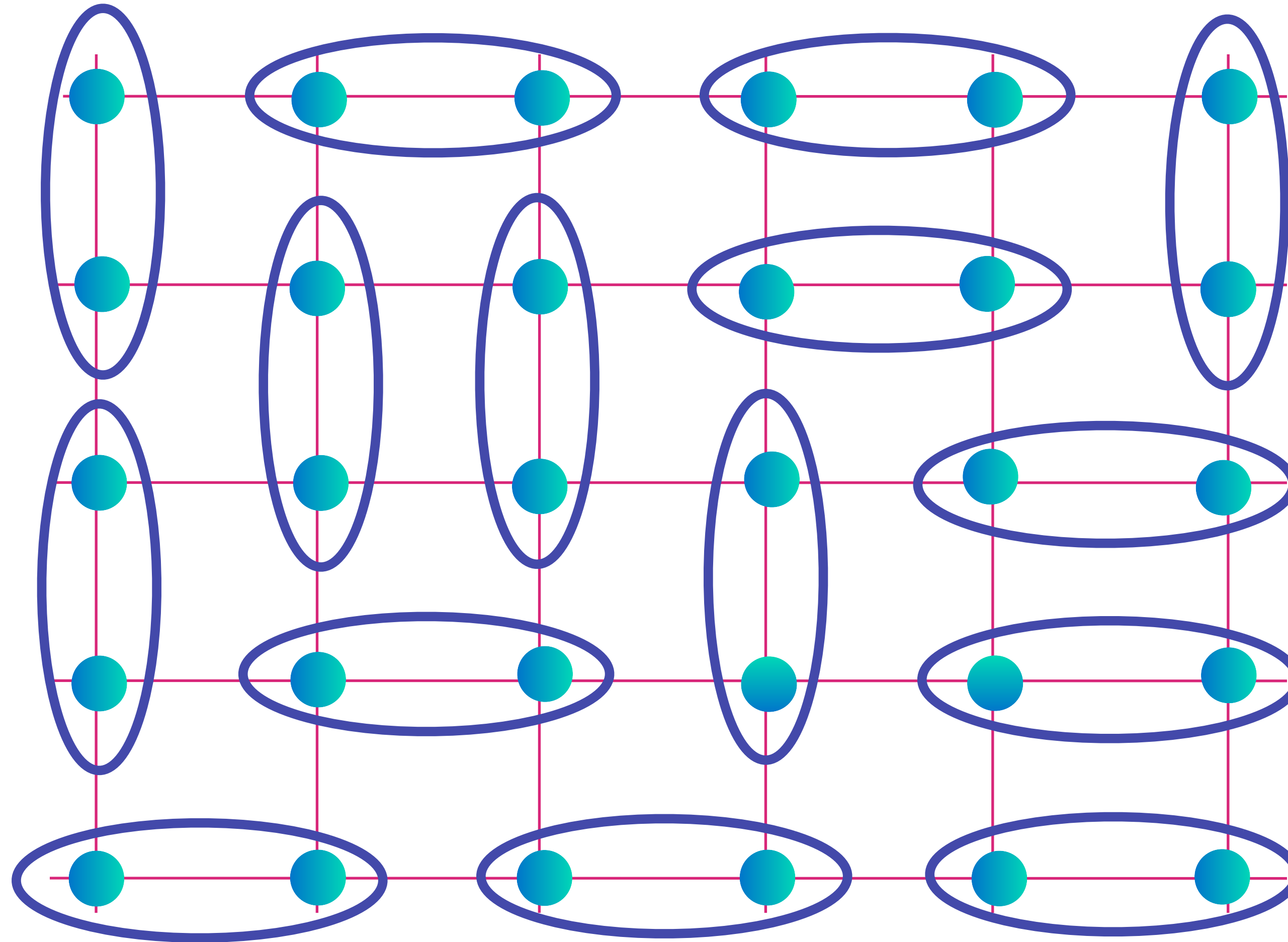
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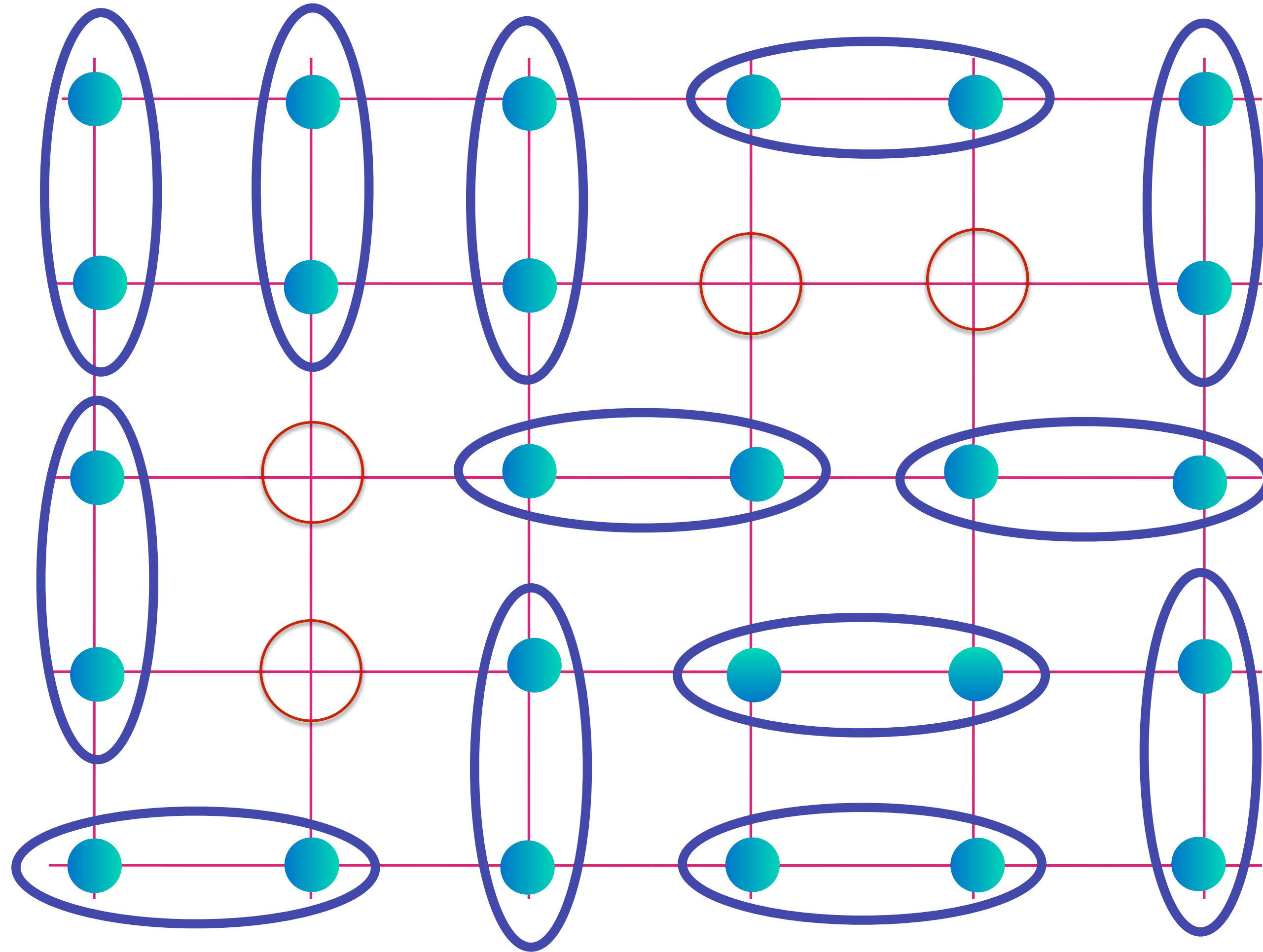
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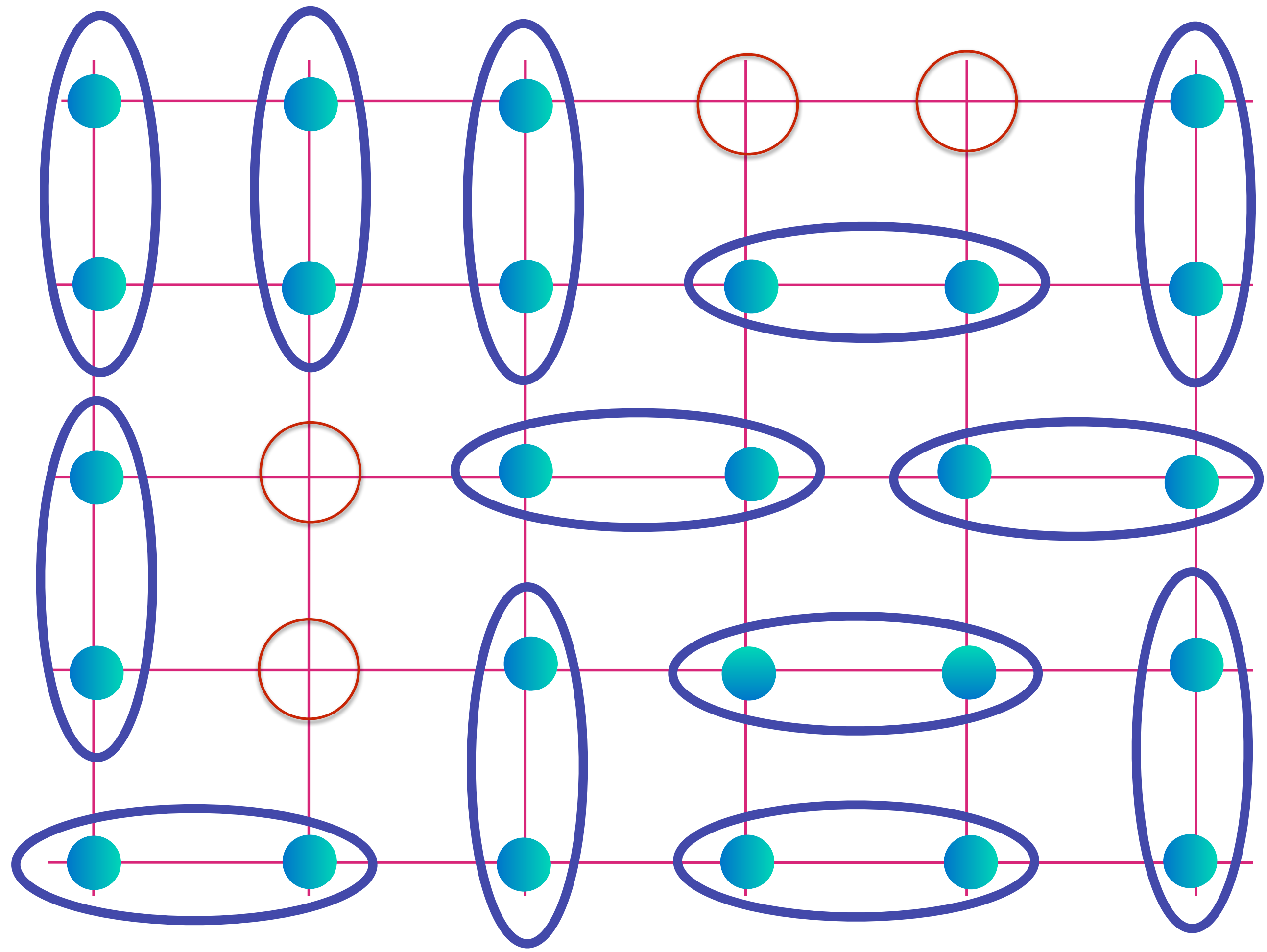


Superconductivity

Mobile
entangled
electron pairs

$$\text{[Diagram of a pair of electrons in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

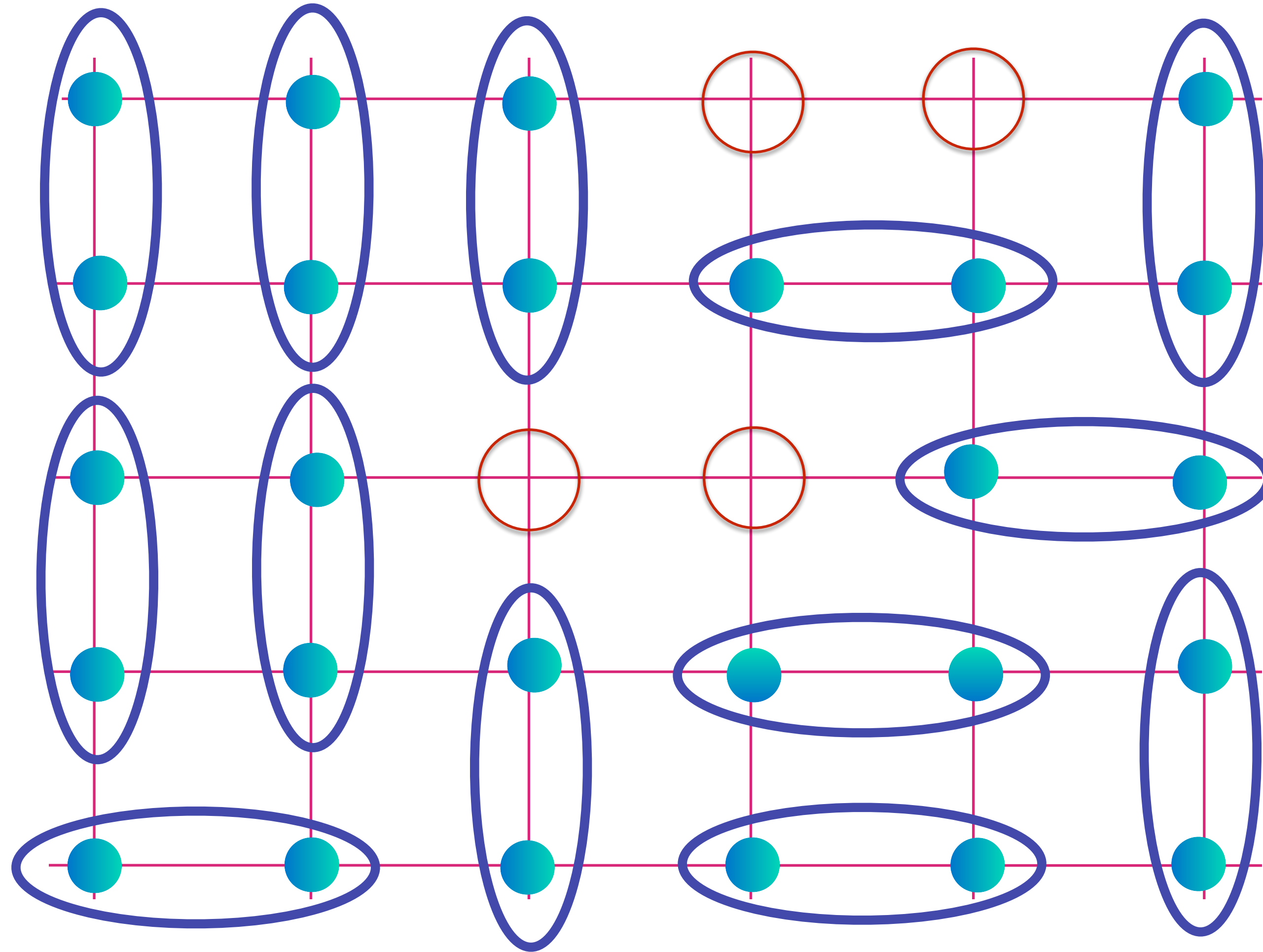


Superconductivity

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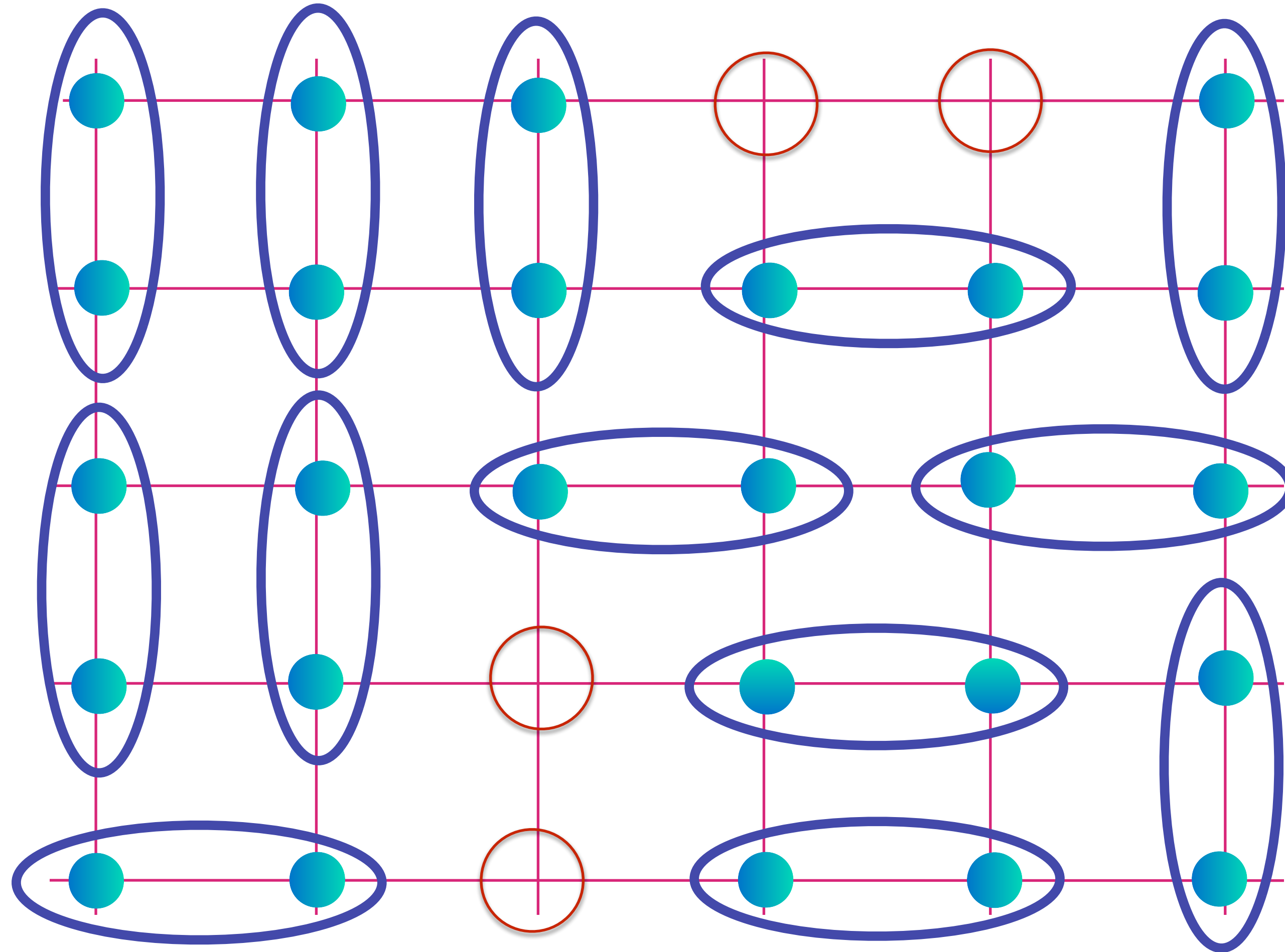


Superconductivity

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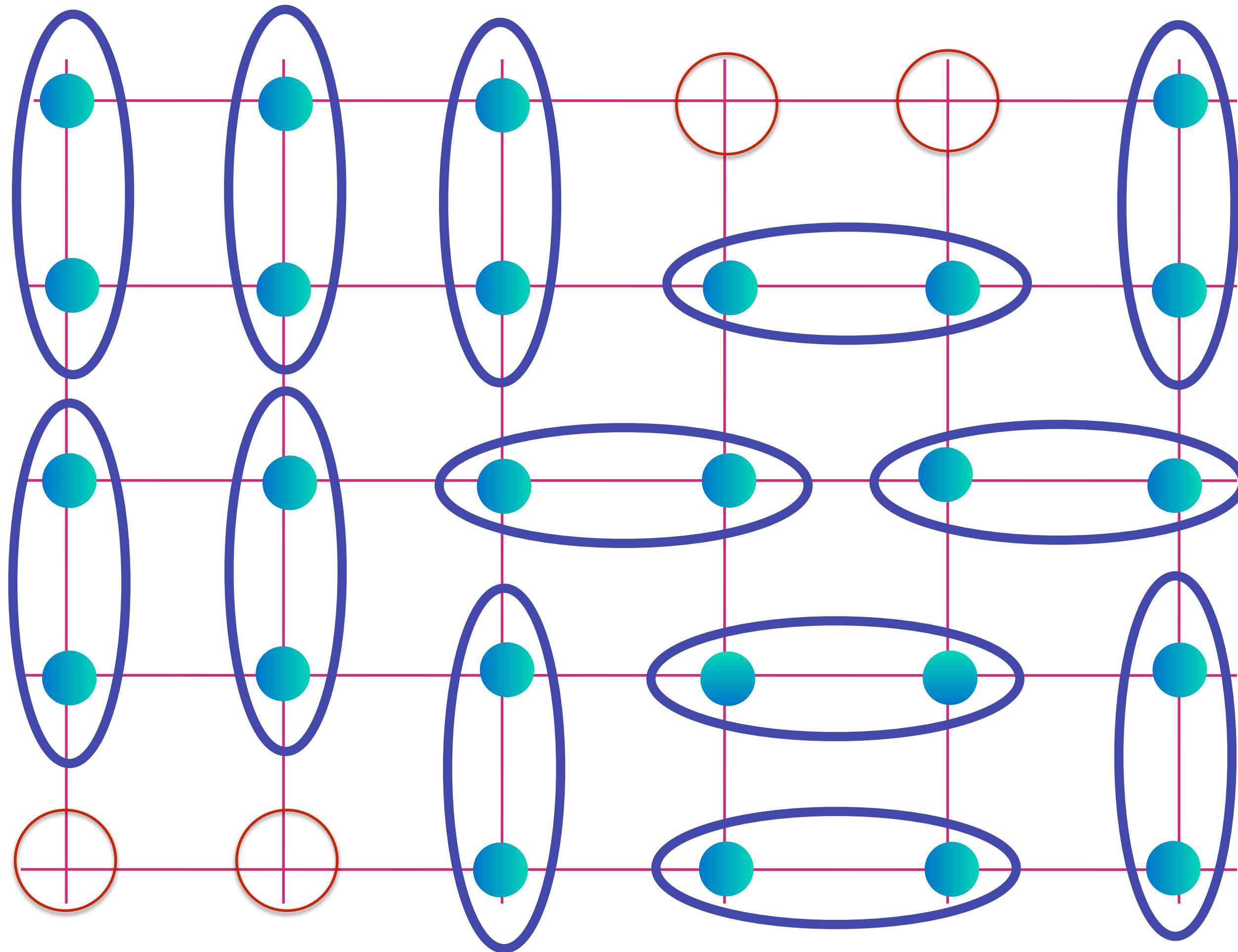


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The dance of electrons on Cu atoms in YBCO



Superconductivity

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$$\text{[Diagram of a pair of electrons in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Needed:

A solvable model of quantum entanglement
of 3, 4, 5, \dots ∞ particles

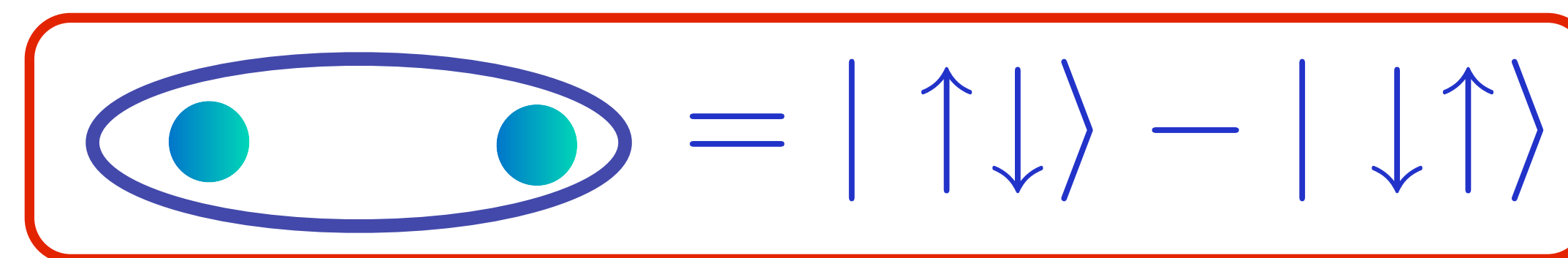
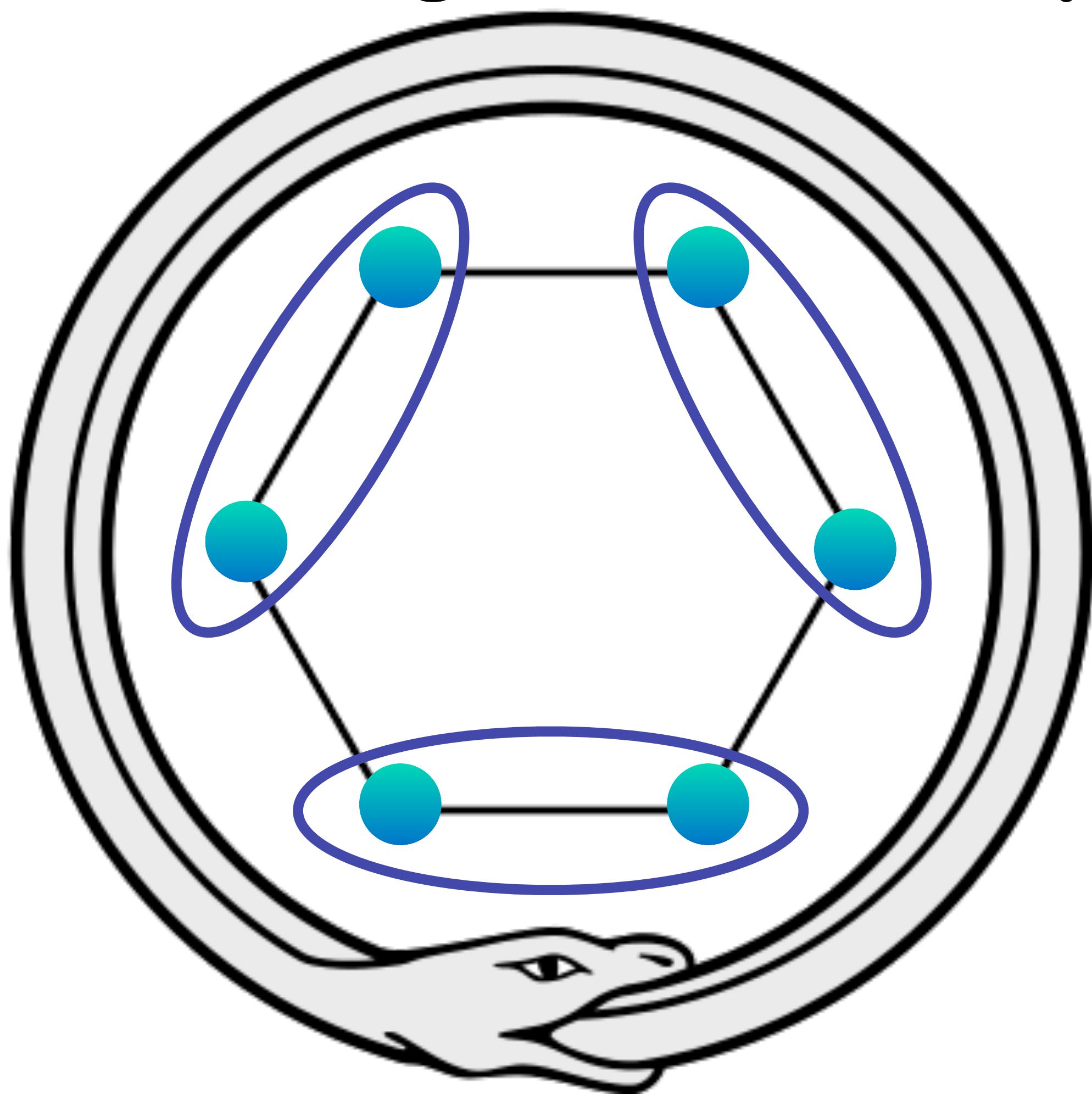
Needed:

A solvable model of quantum entanglement
of 3, 4, 5, ... ∞ particles

**The Sachdev-Ye-Kitaev model
of many-particle entanglement**

Kekulé's spooky dream (1865)

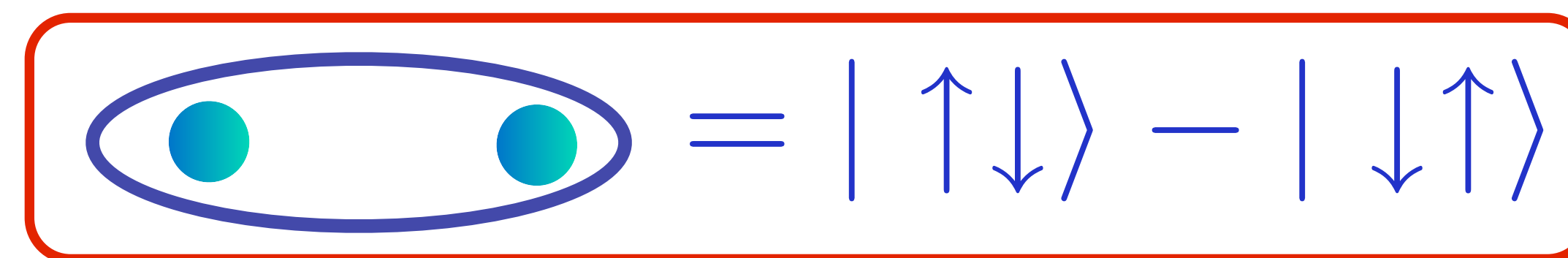
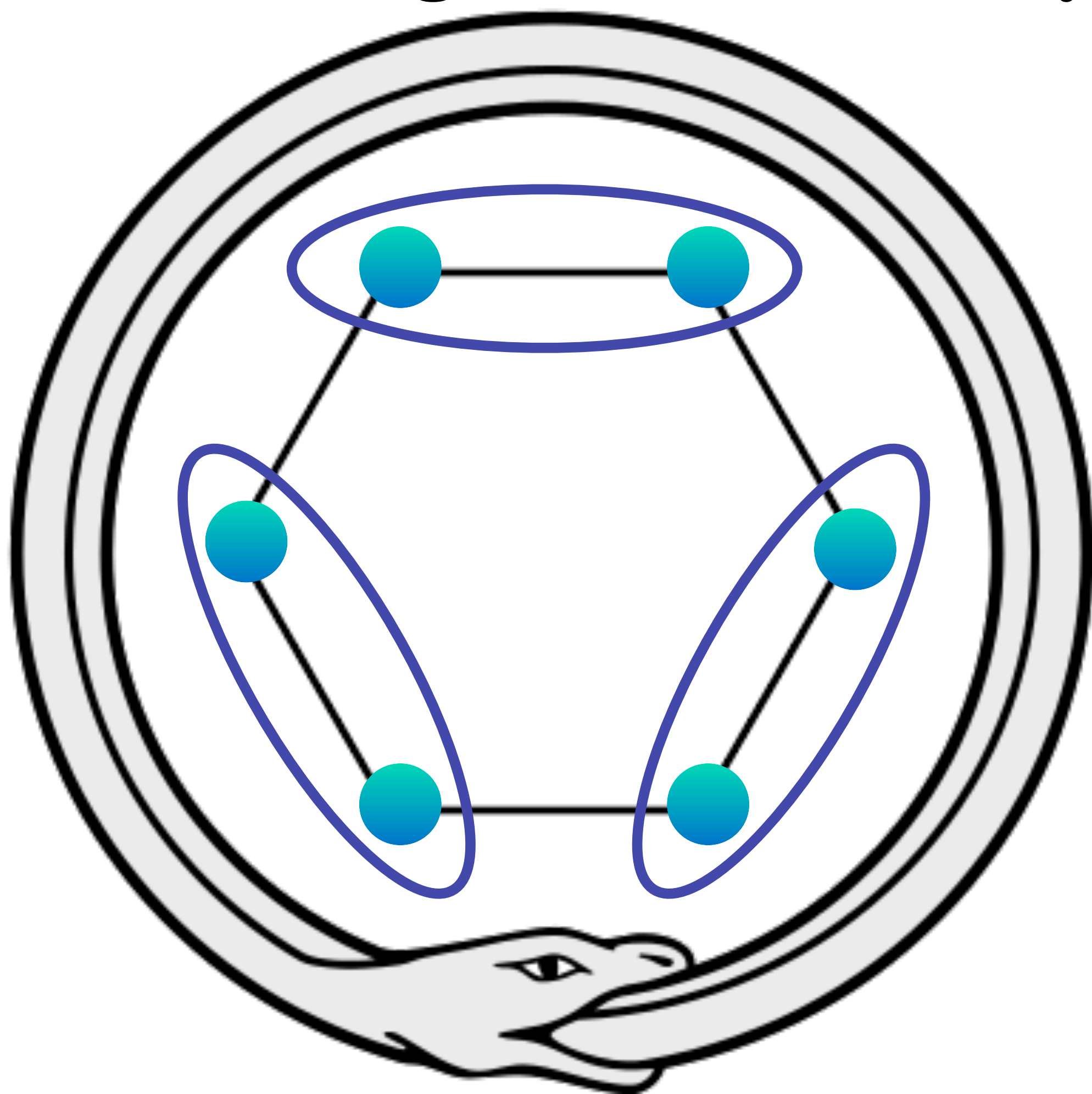
Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



Benzene

Kekulé's spooky dream (1865)

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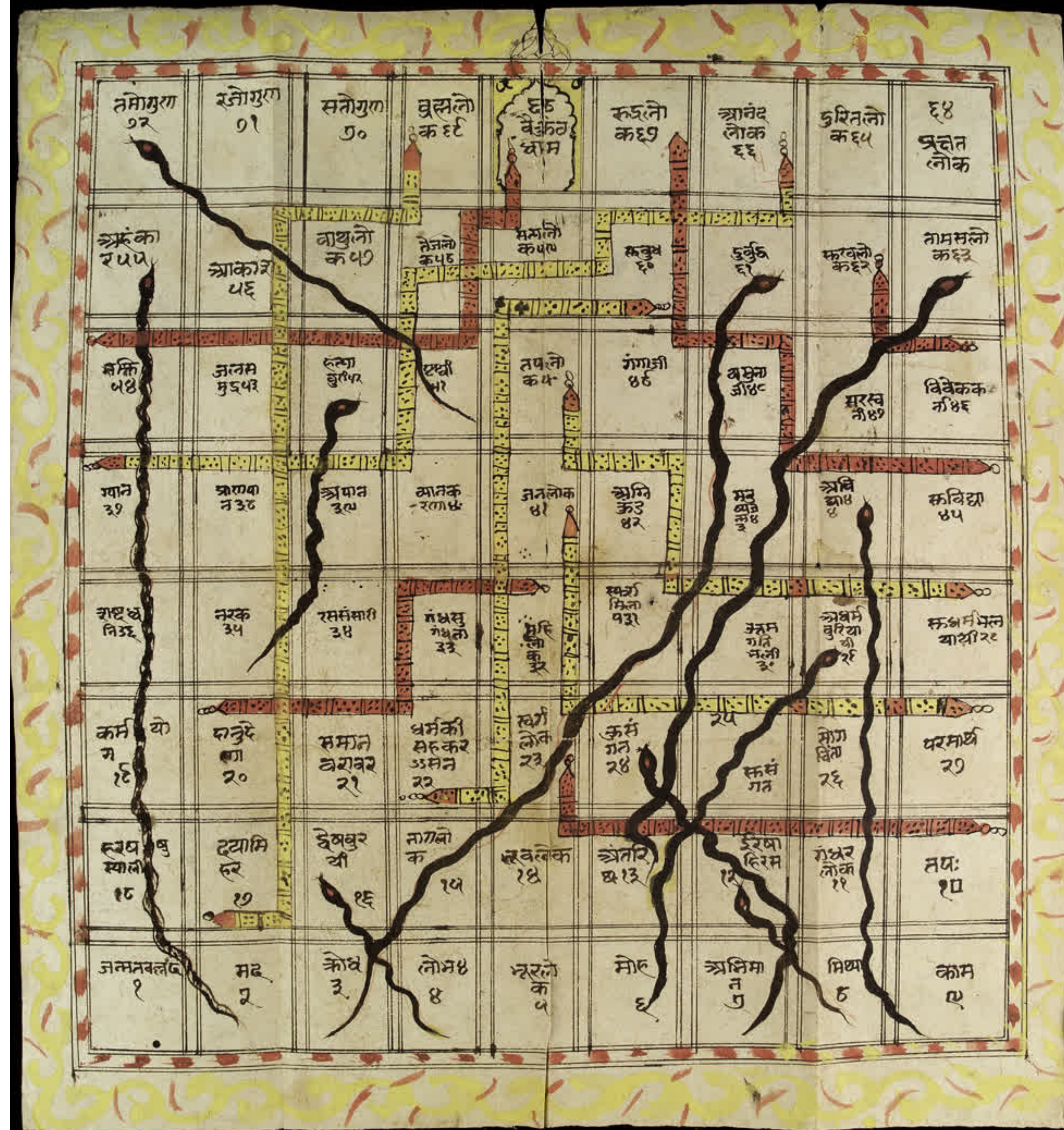


Benzene

My
spooky
dream*

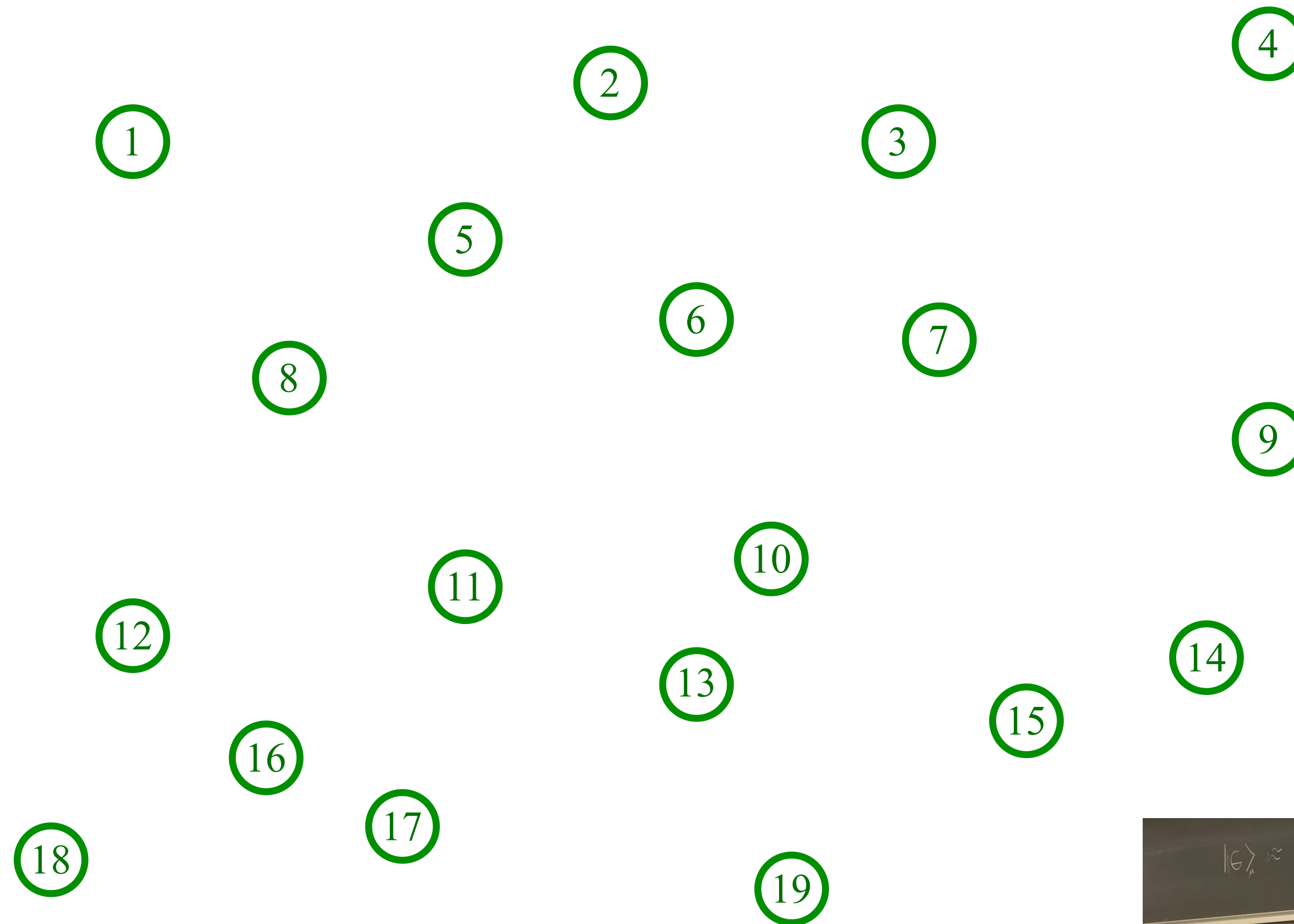
Ancient
Indian
game of
Snakes
and
Ladders

*Not true

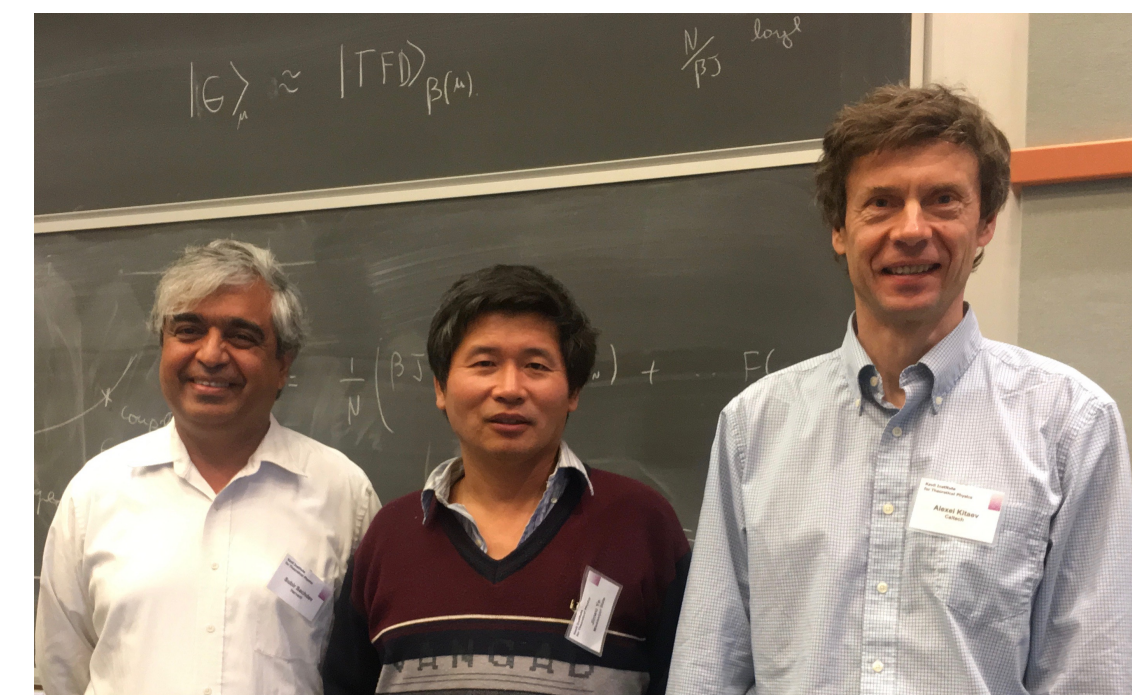


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

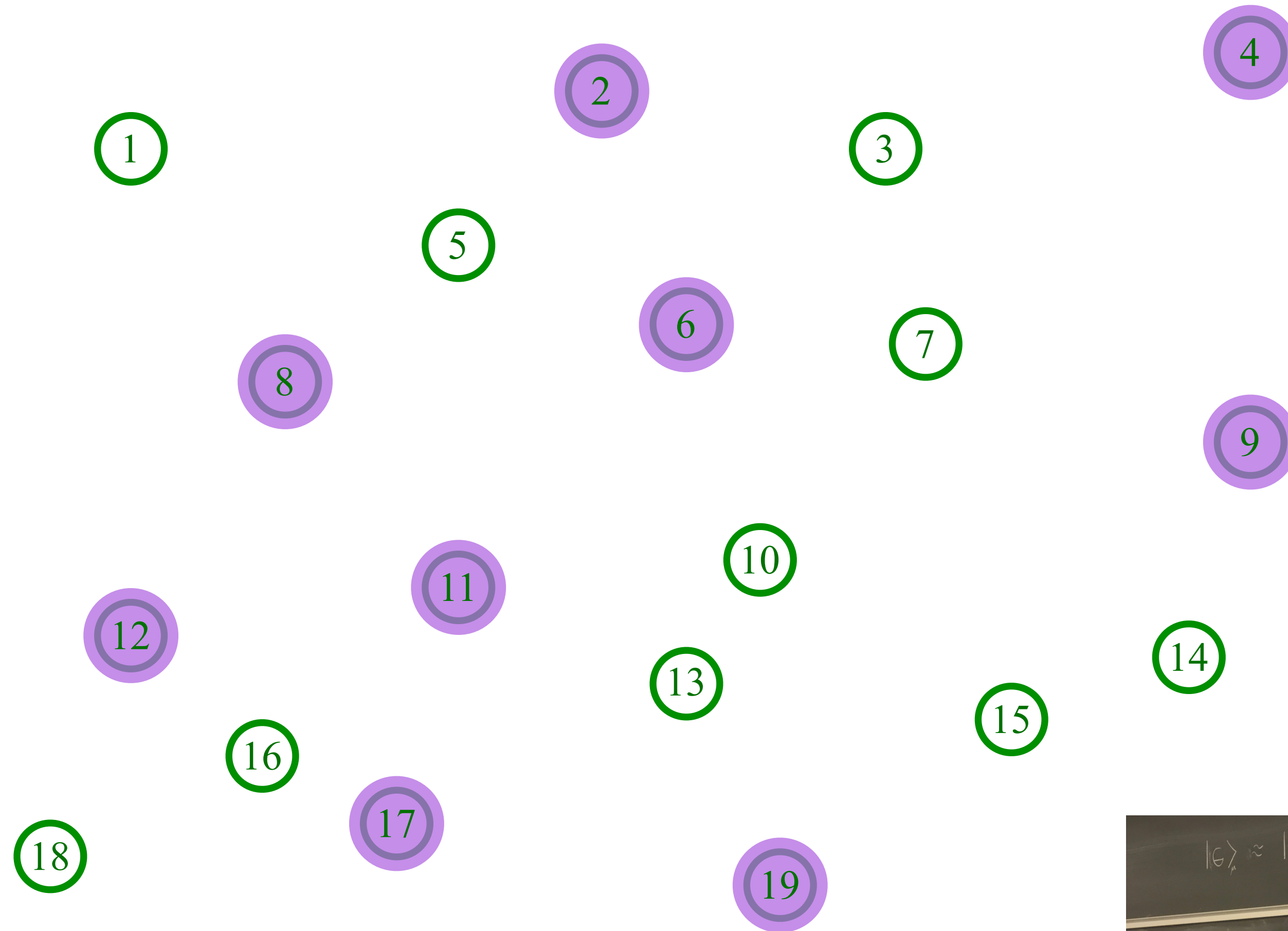


Pick a set of random positions

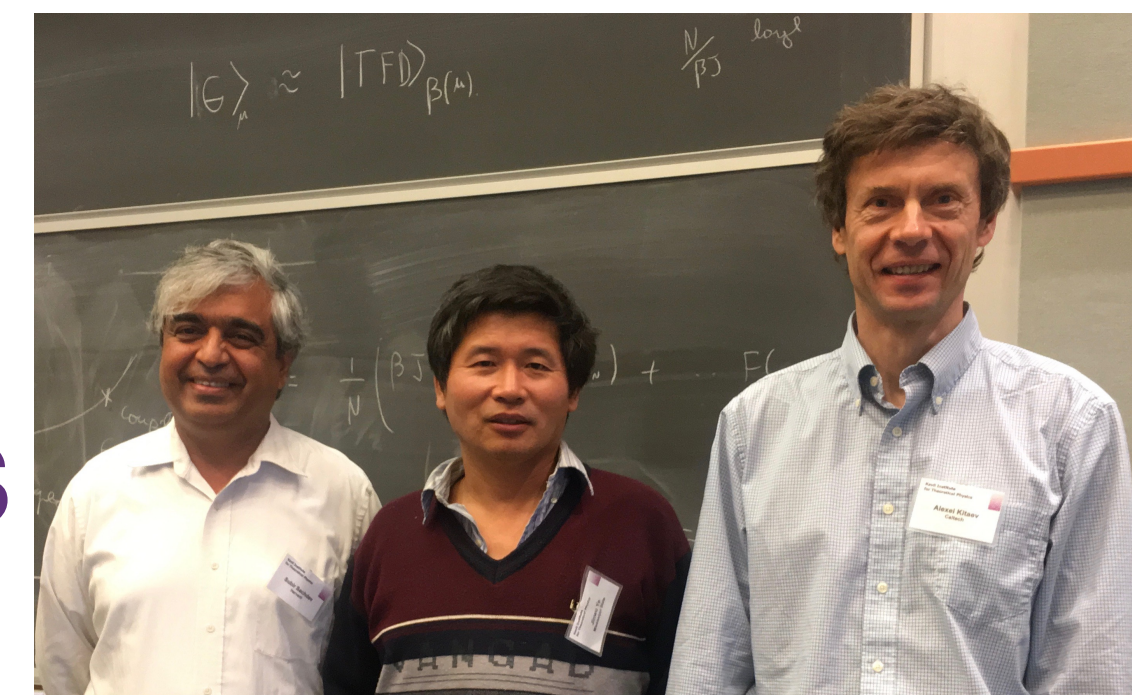


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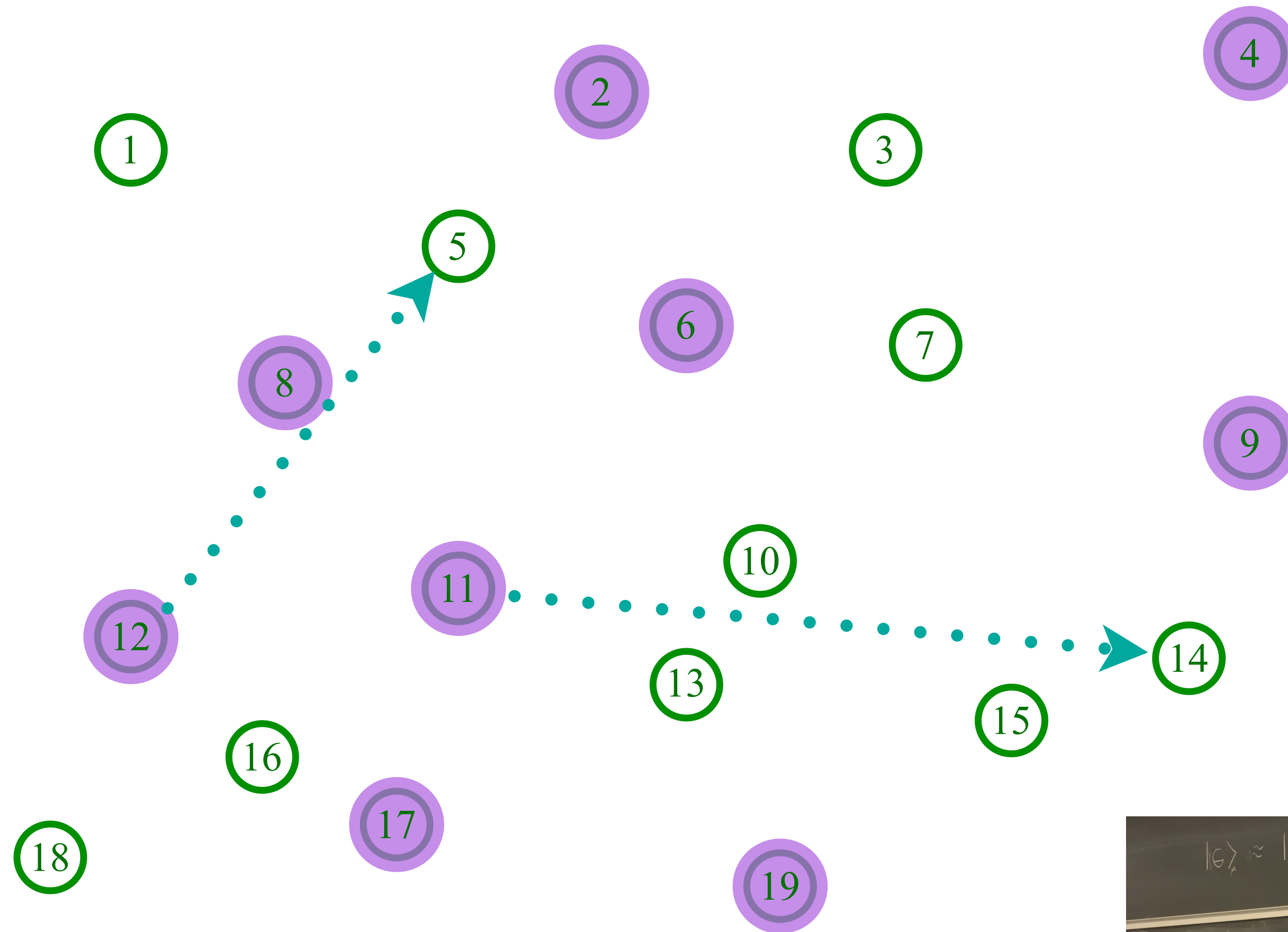
Place electrons randomly on some sites



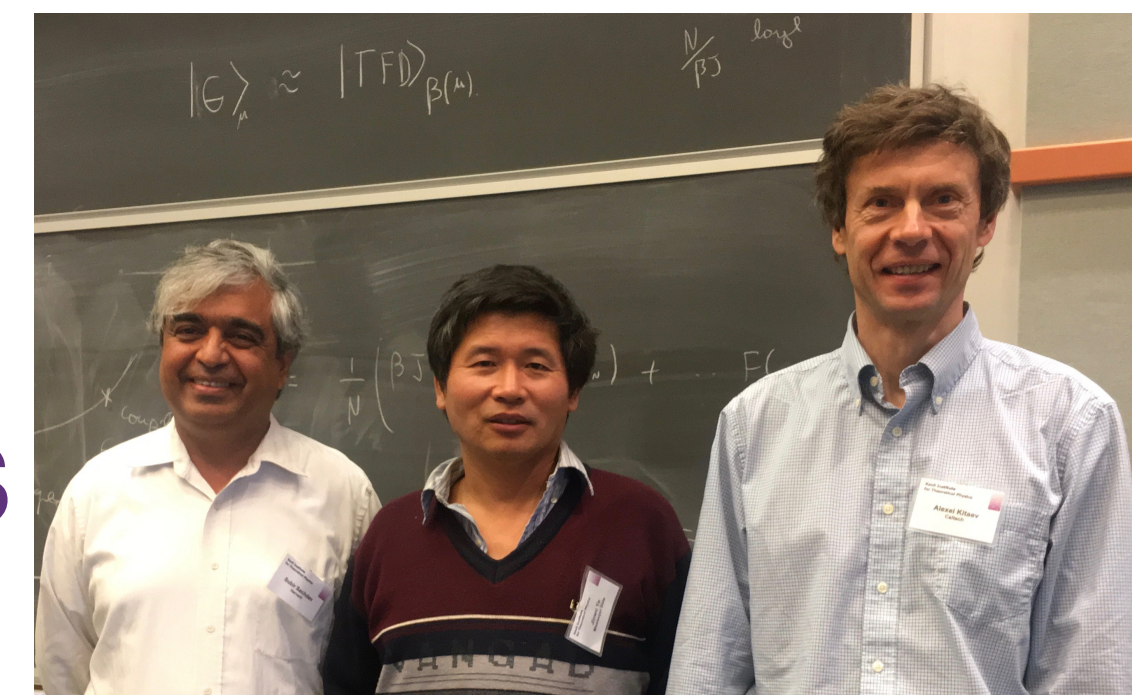
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



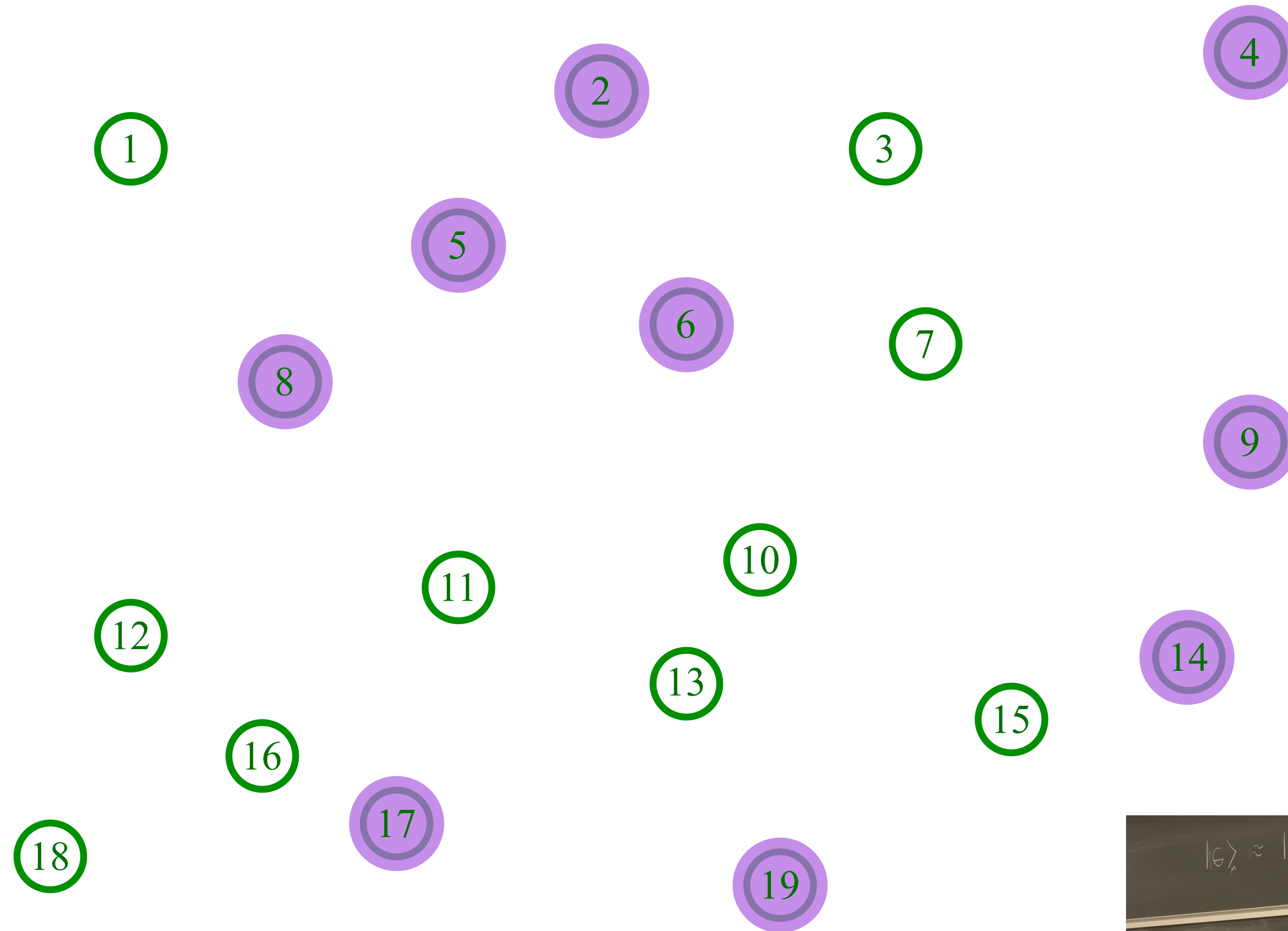
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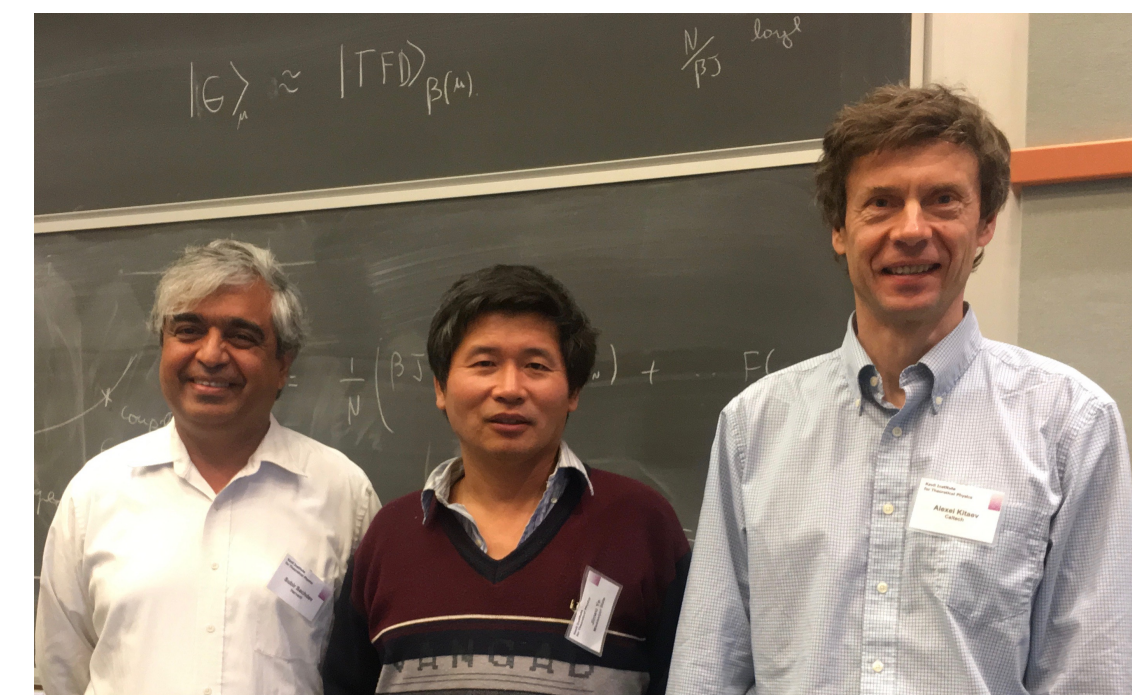
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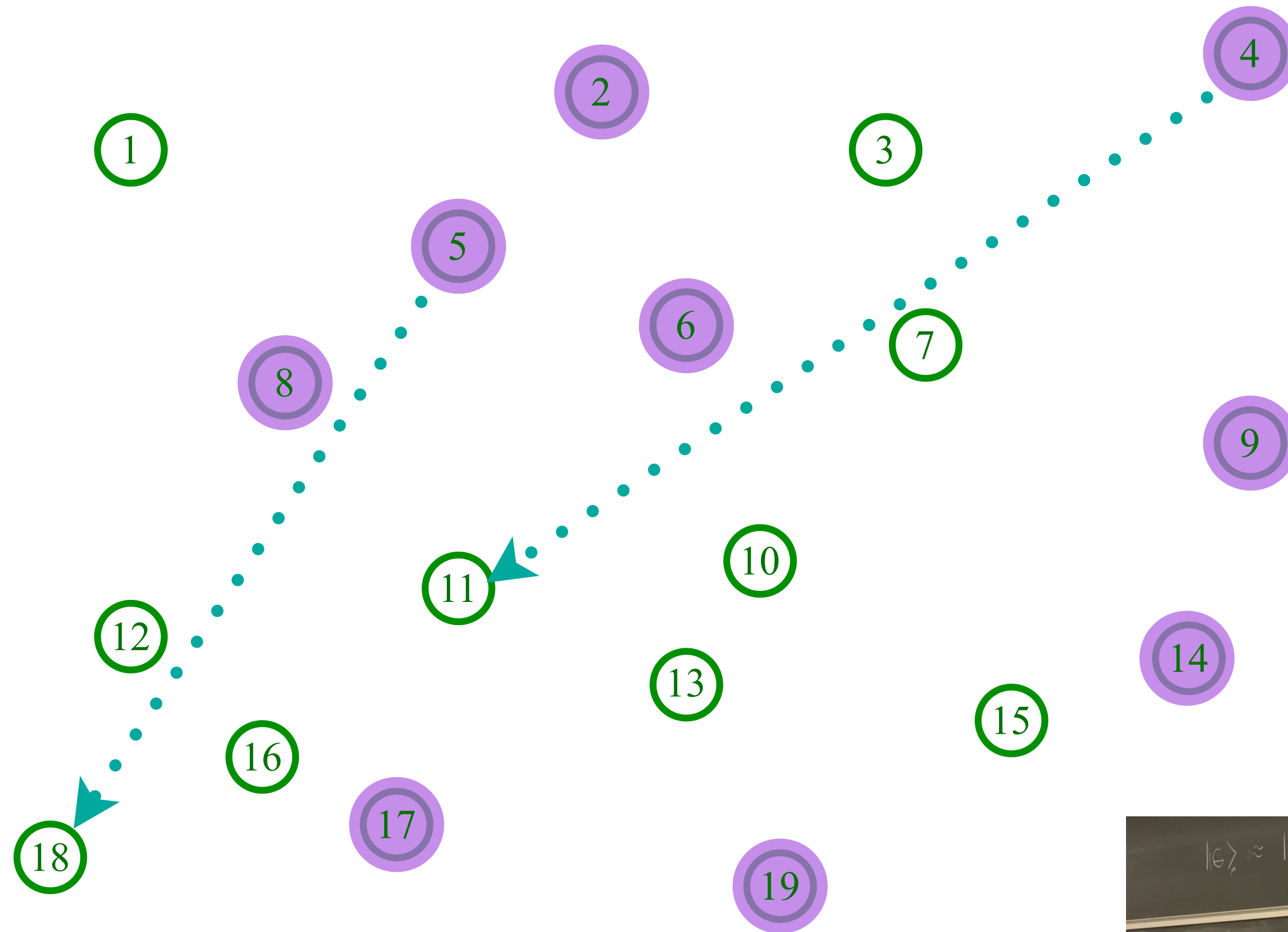
Entangle electrons pairwise randomly



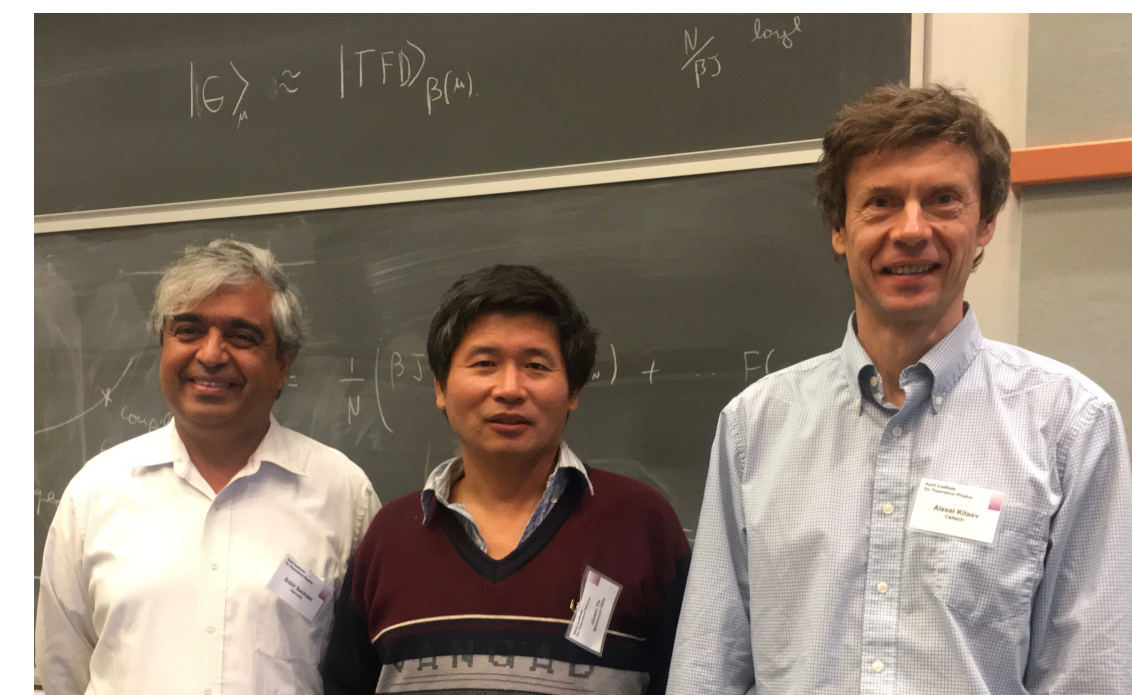
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$$U_{4,5;11,18}$$



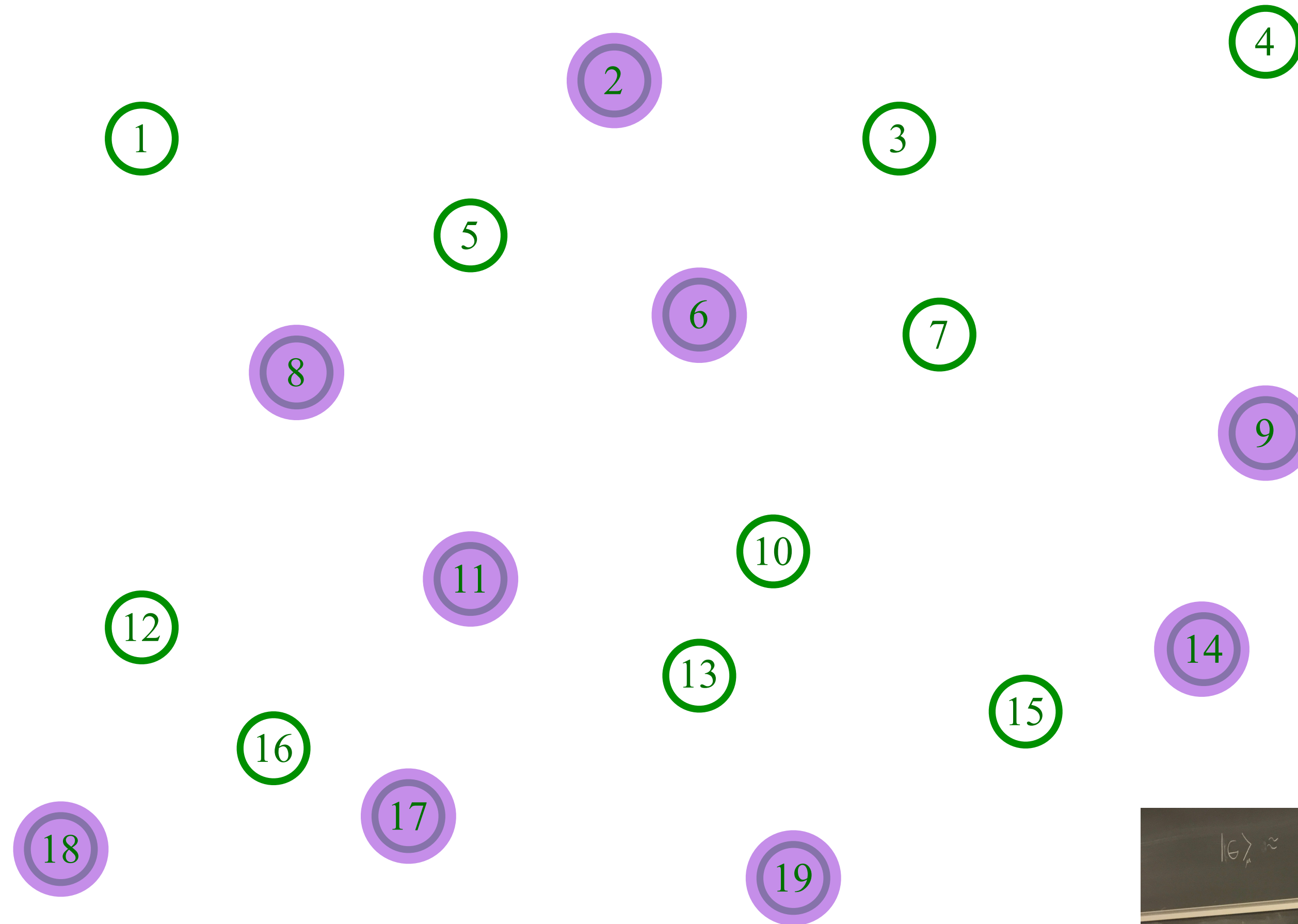
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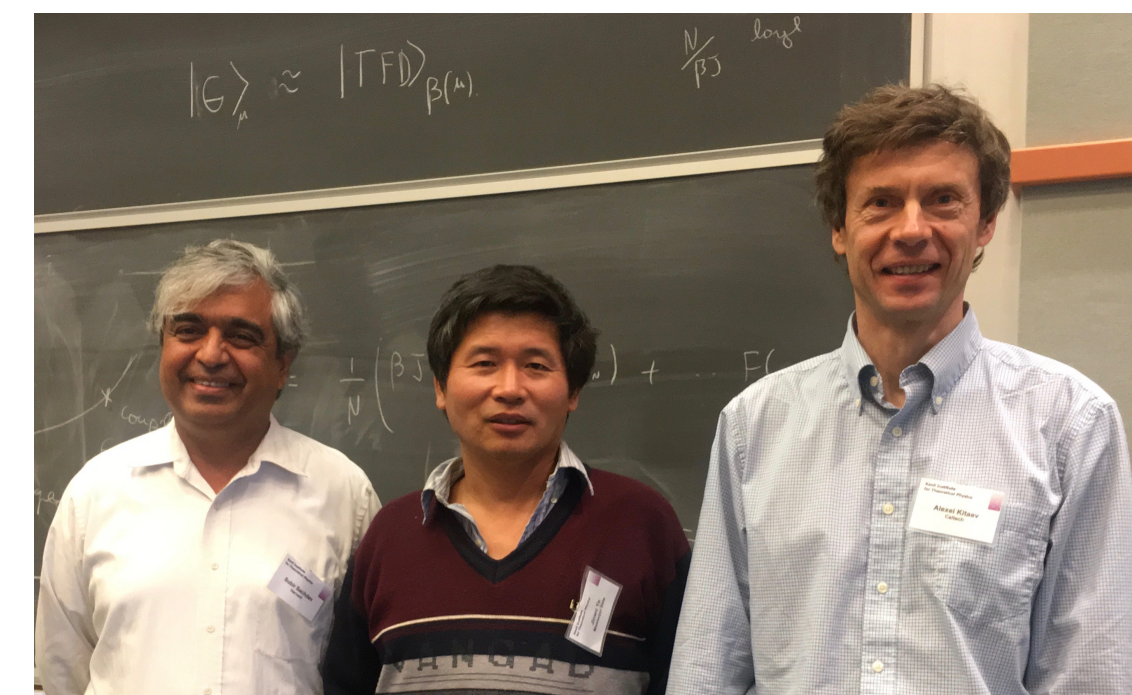
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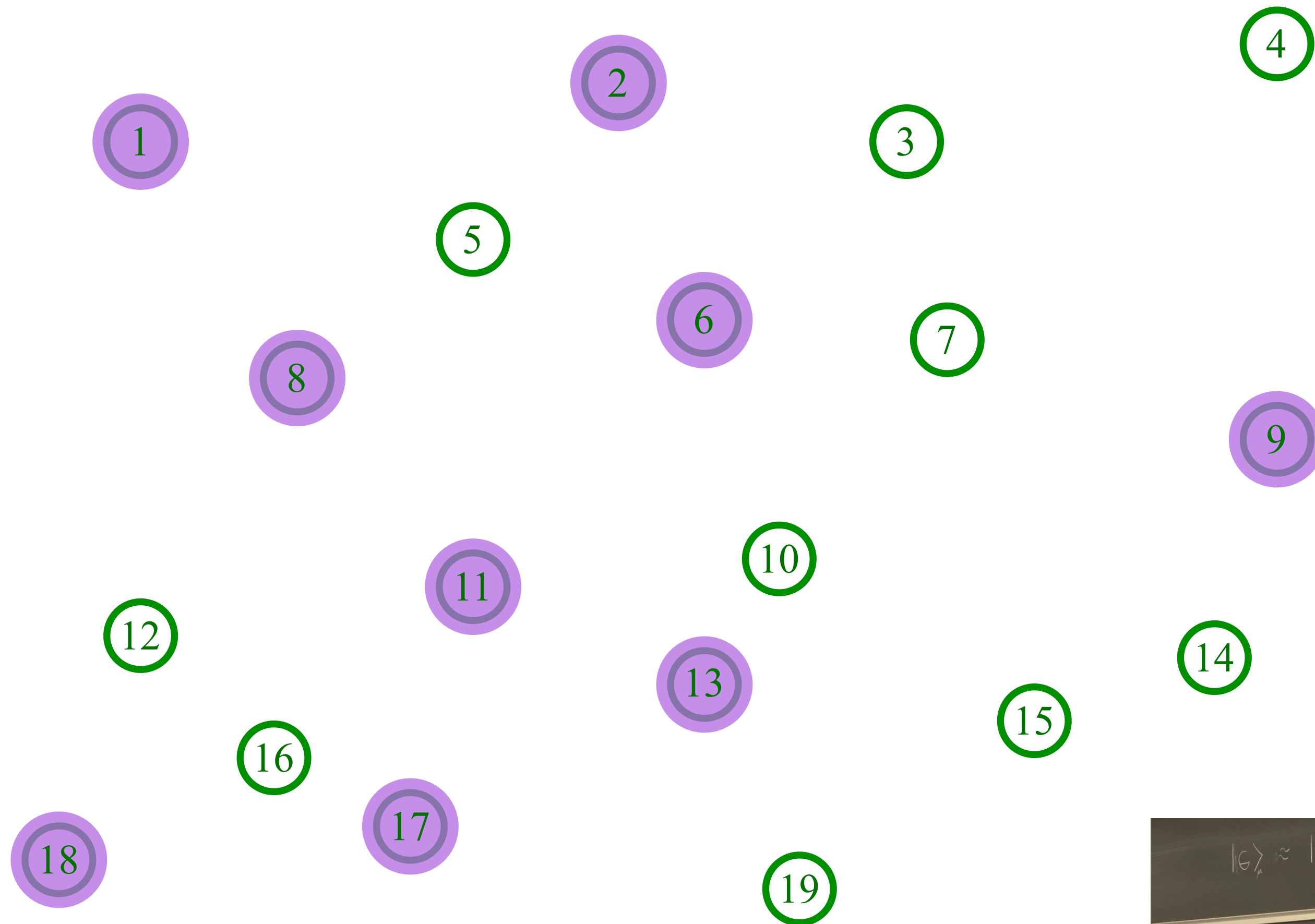
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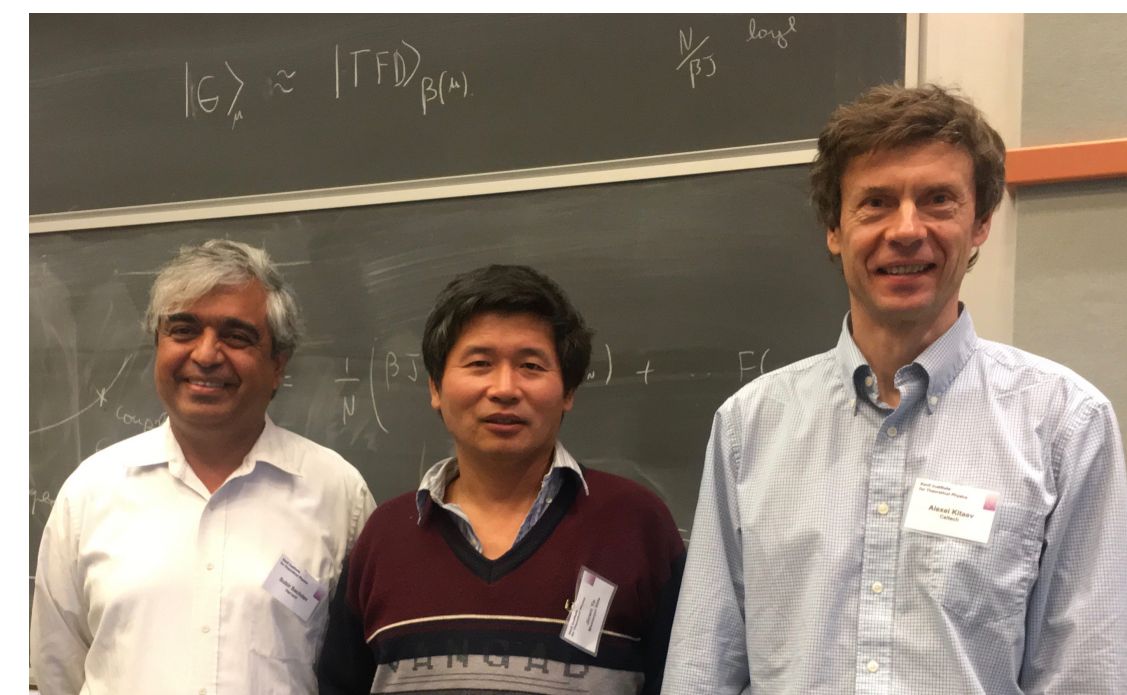
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



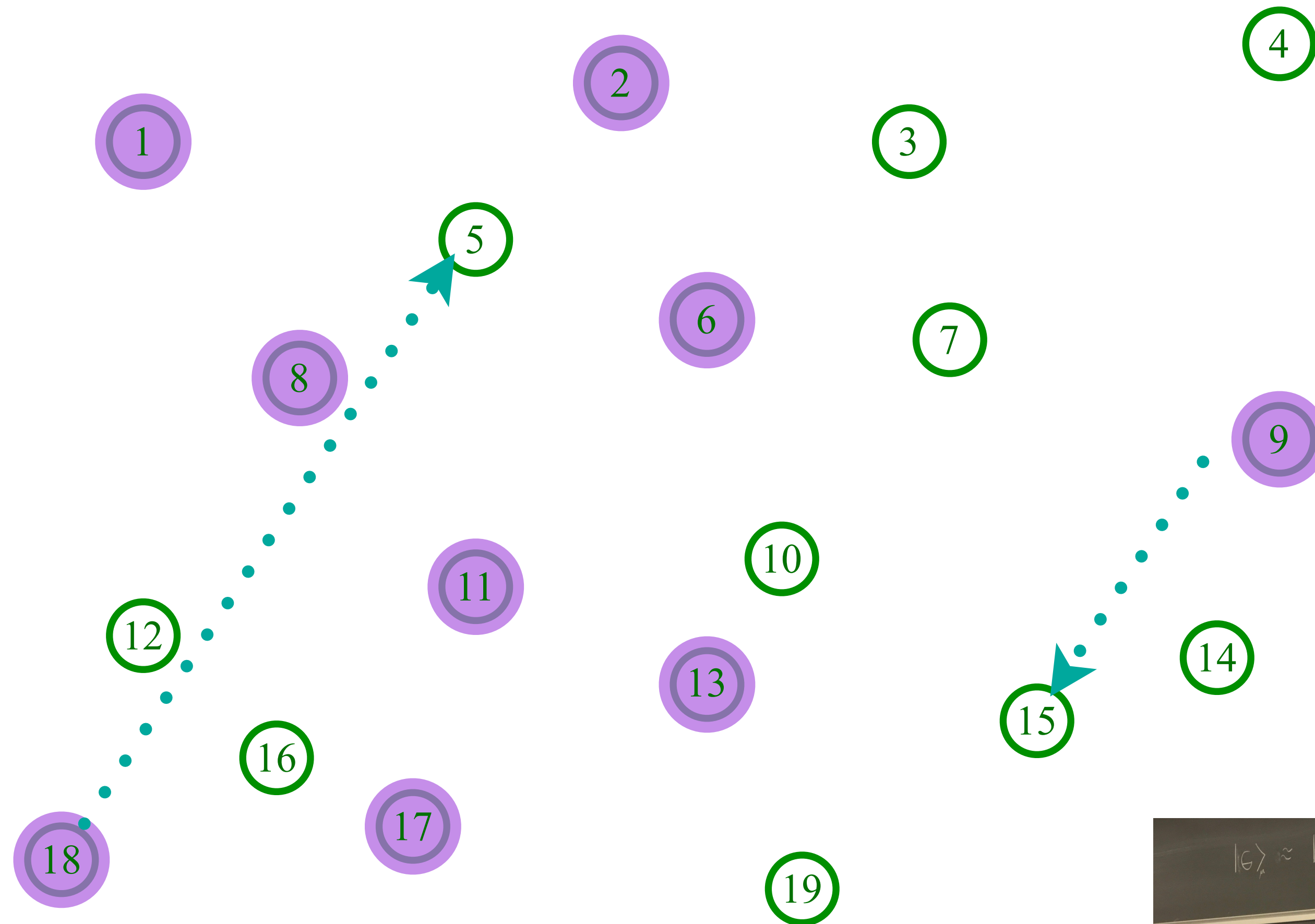
Entangle electrons pairwise randomly



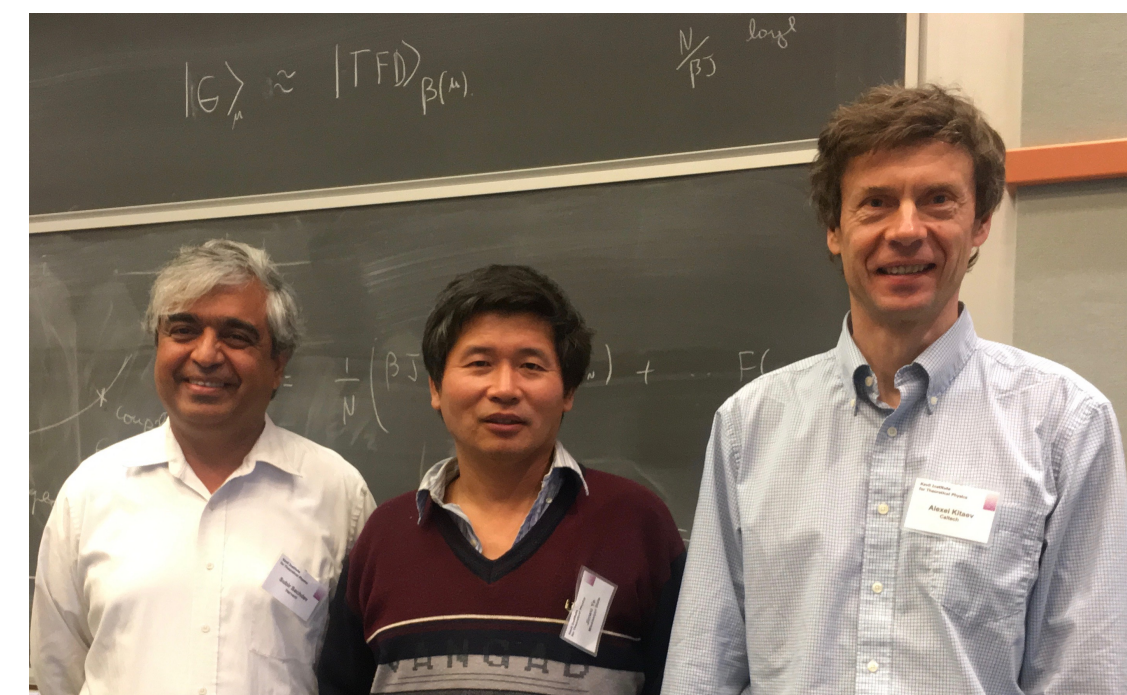
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



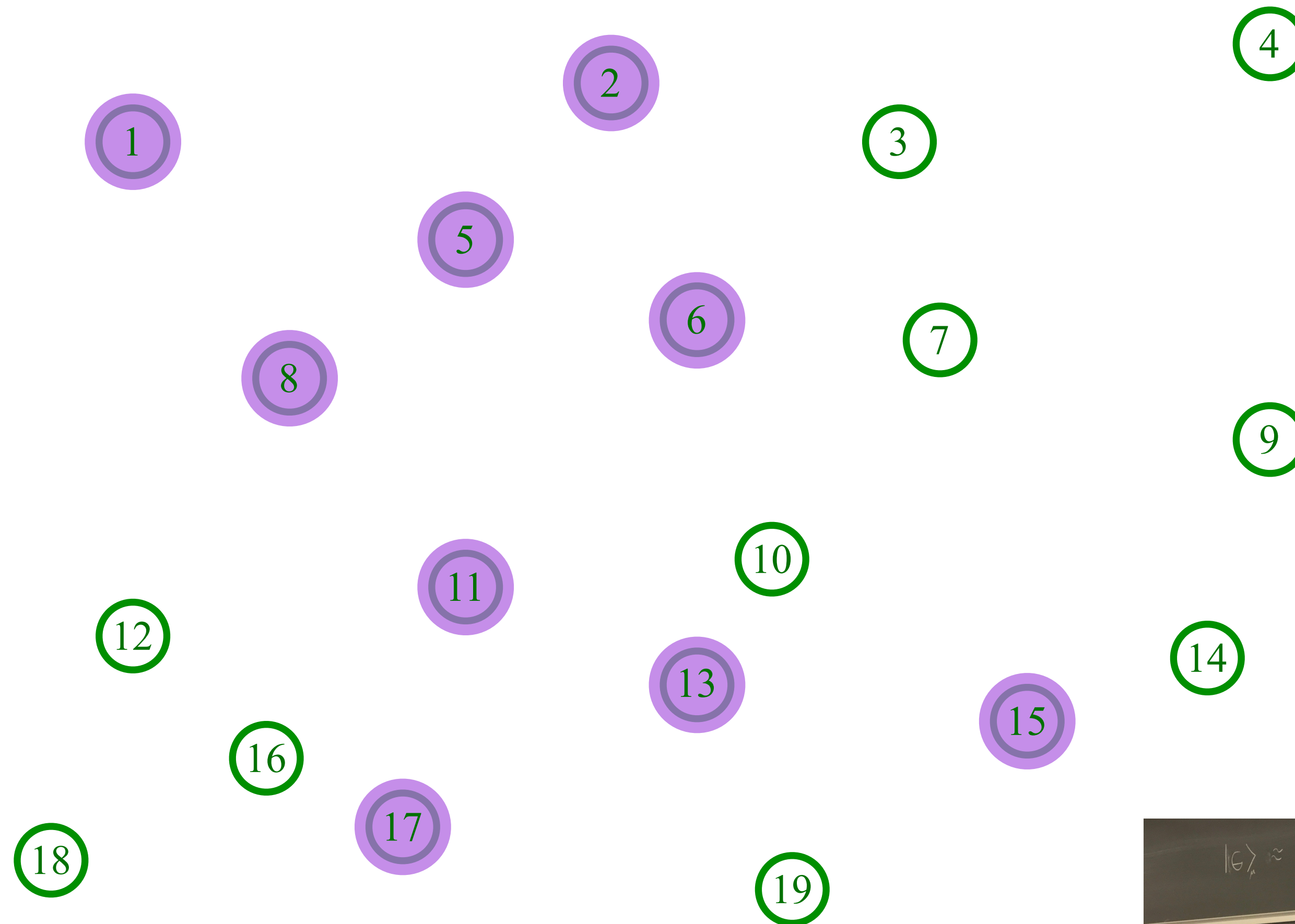
Entangle electrons pairwise randomly



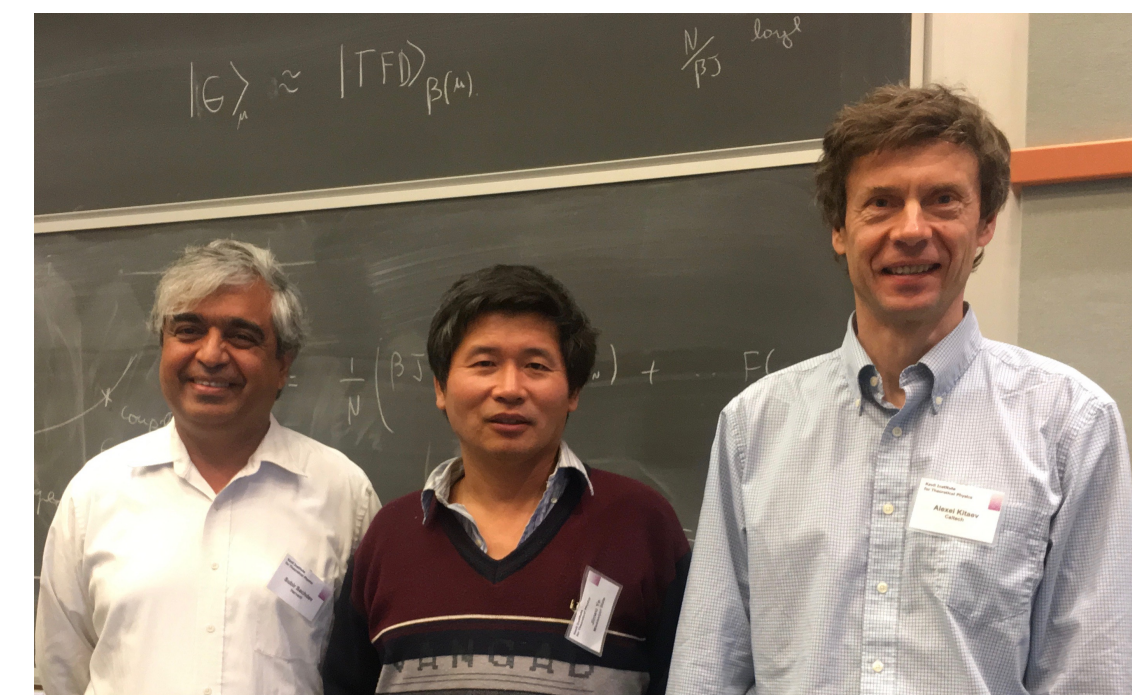
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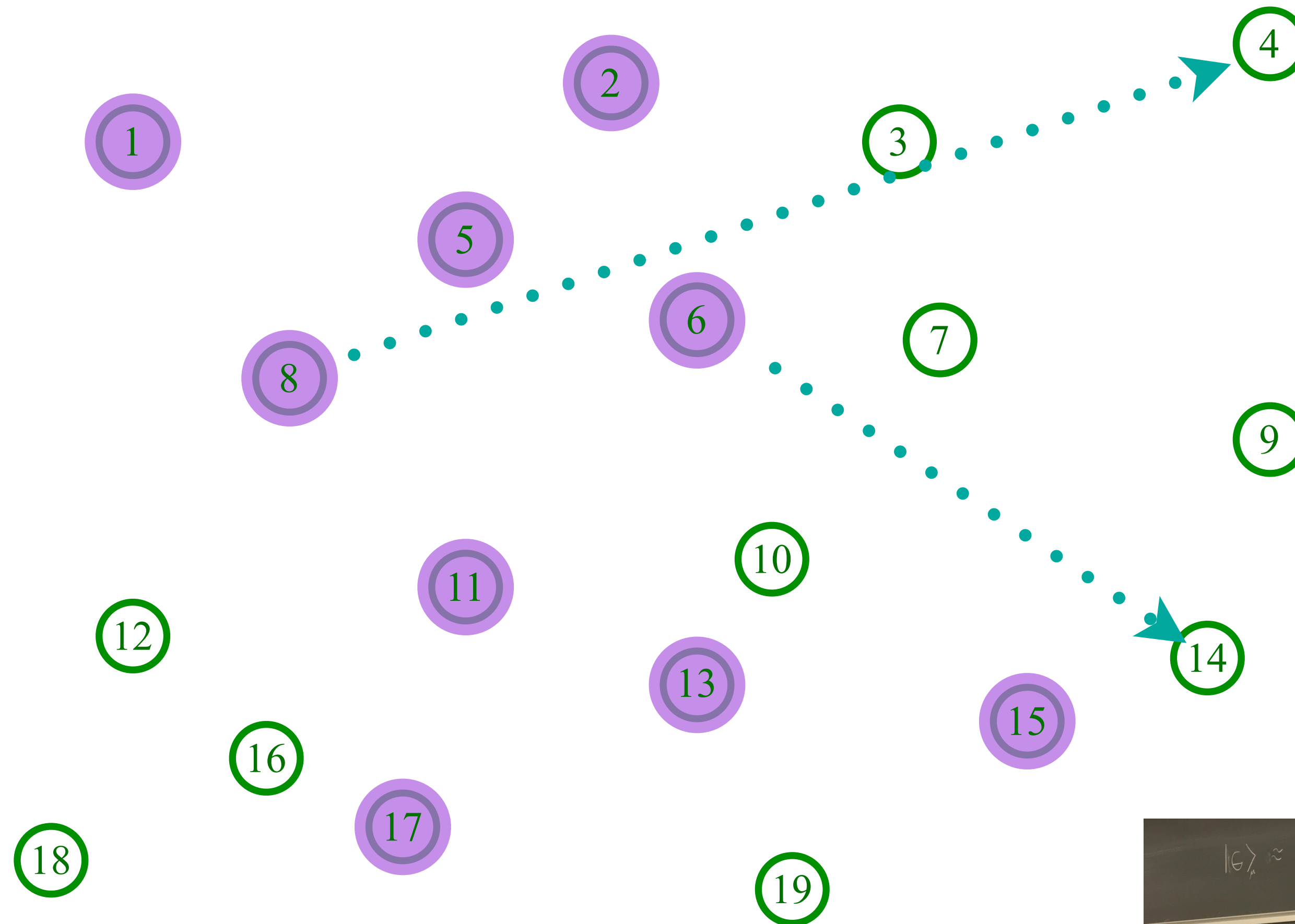
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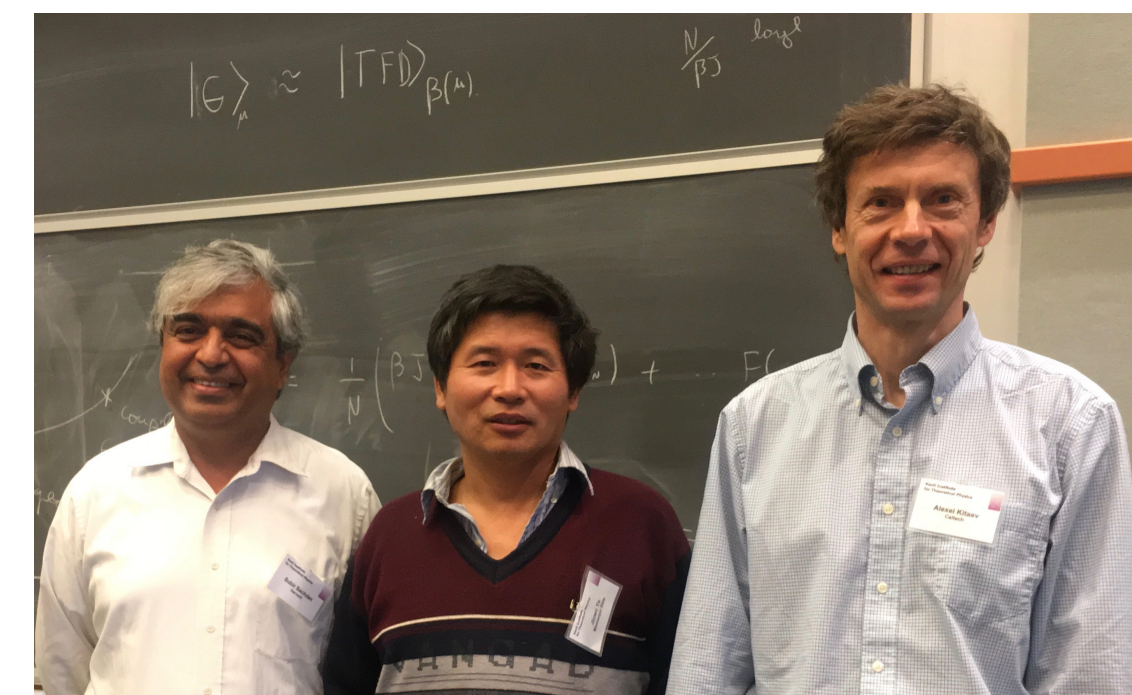
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



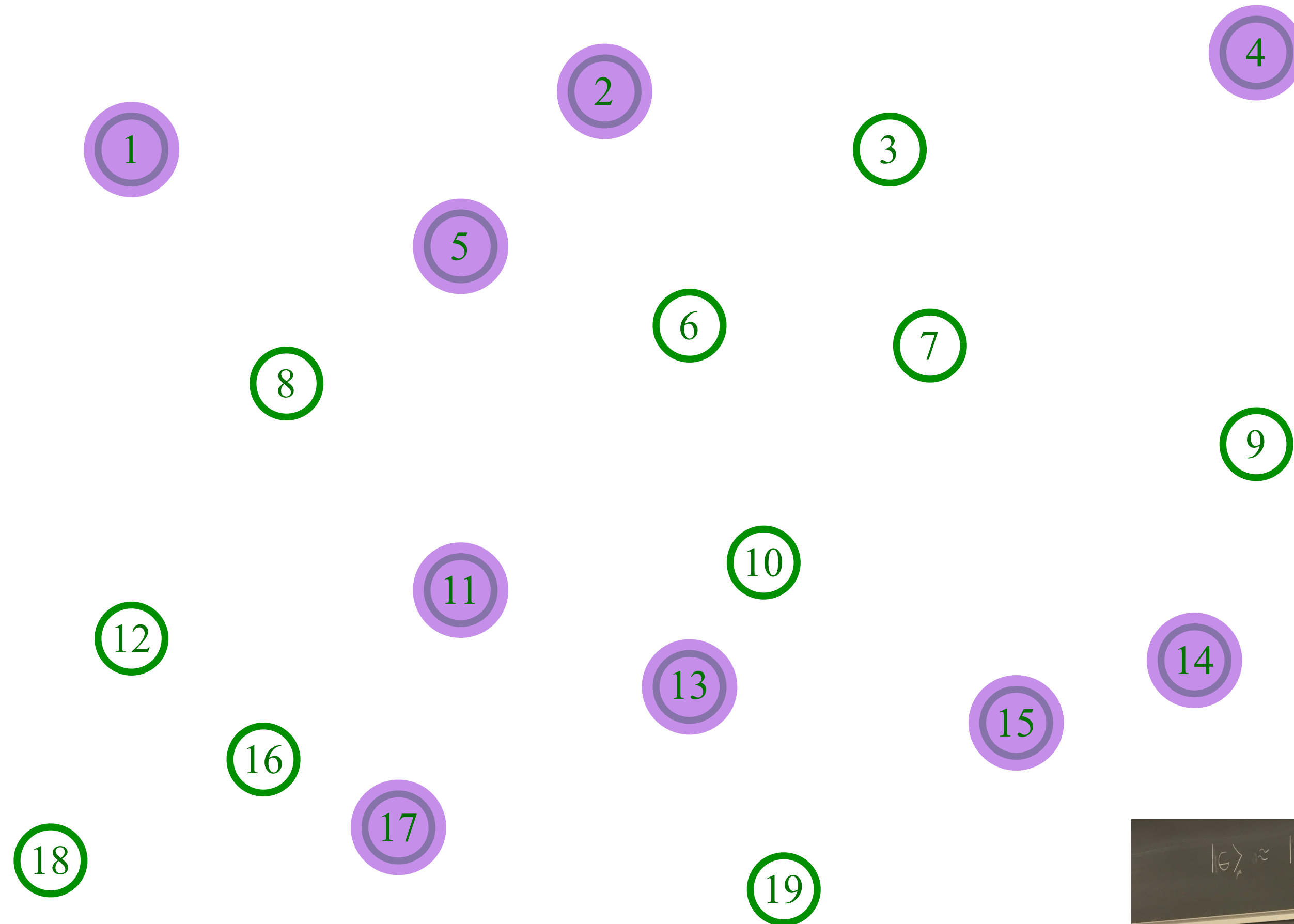
Entangle electrons pairwise randomly



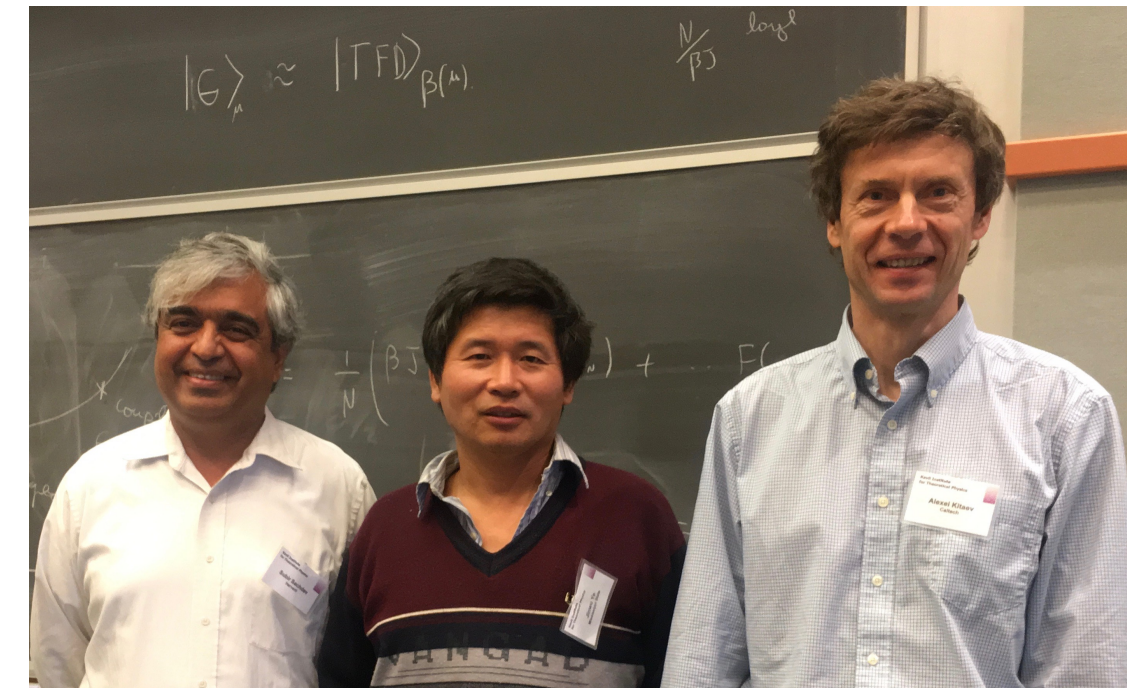
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Sachdev, Ye (1993); Kitaev (2015)

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Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

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A solvable model of multi-particle
quantum entanglement.

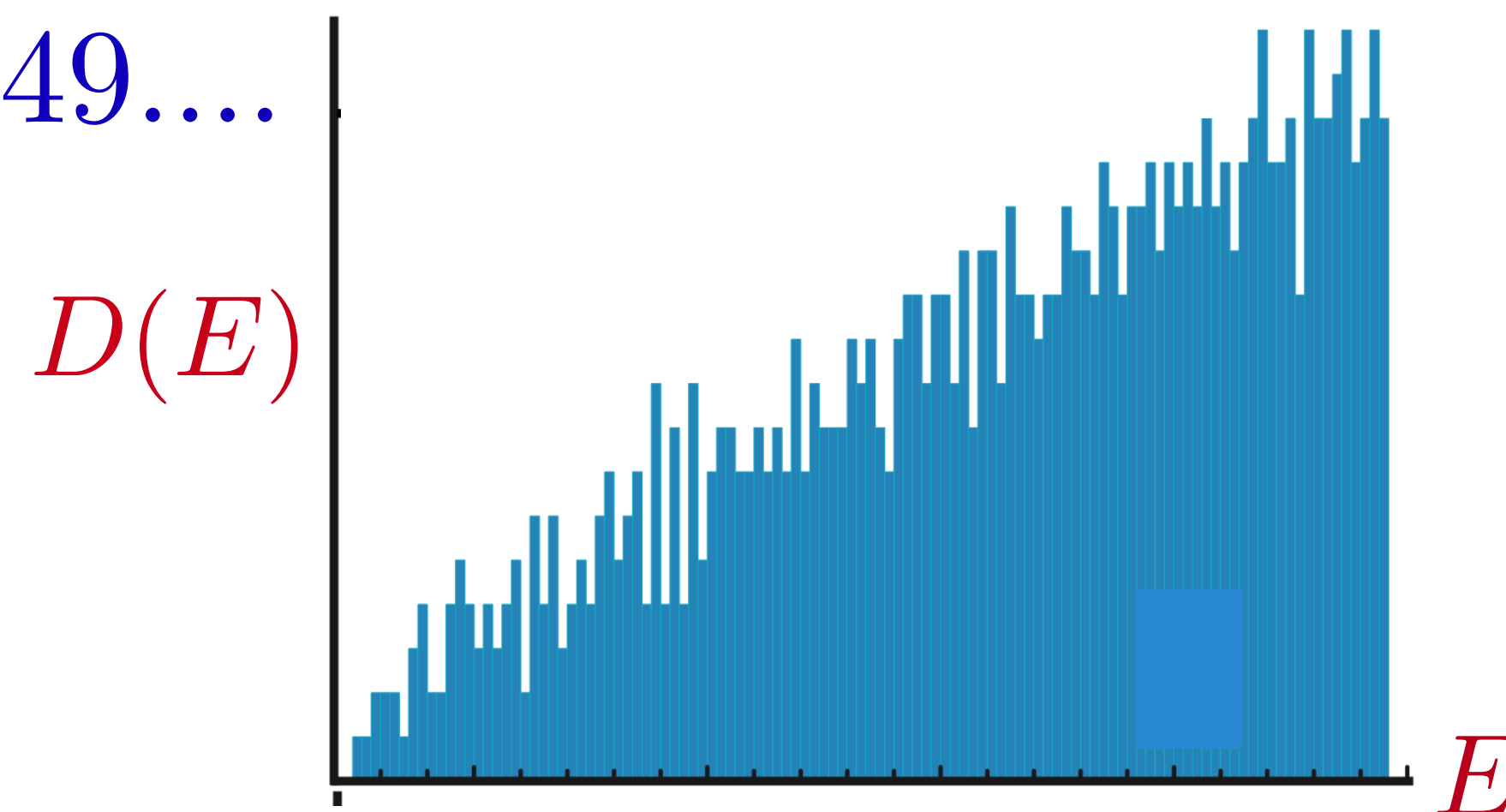
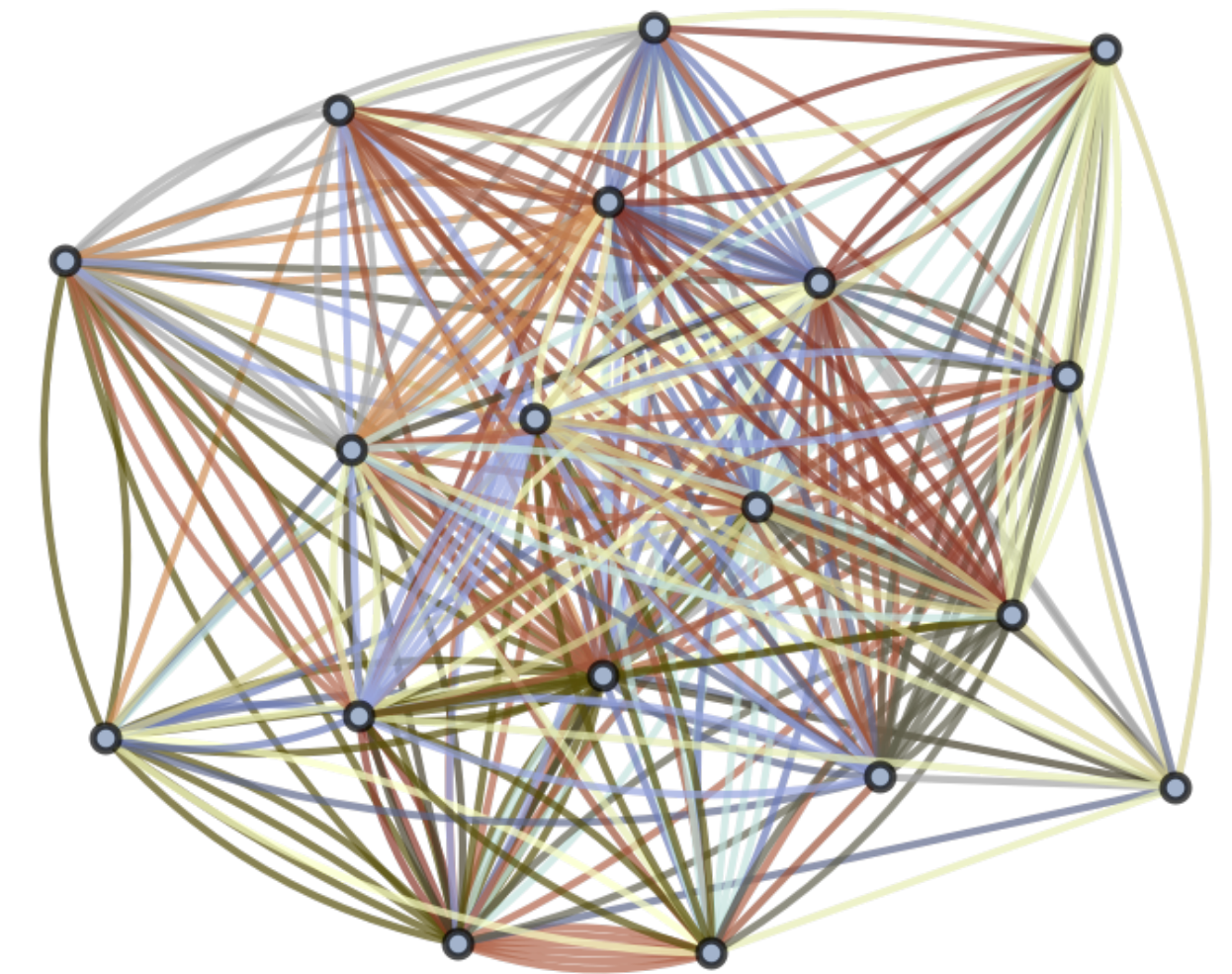
Yields a metal in which current is carried
not by individual electrons,
but by an entangled “quantum soup”

The Sachdev-Ye-Kitaev (SYK) model

- Density of quantum states of the SYK model with N sites

$$D(E) \sim \frac{1}{N} \exp(N s_0) \sinh\left([2N\gamma E]^{1/2}\right)$$

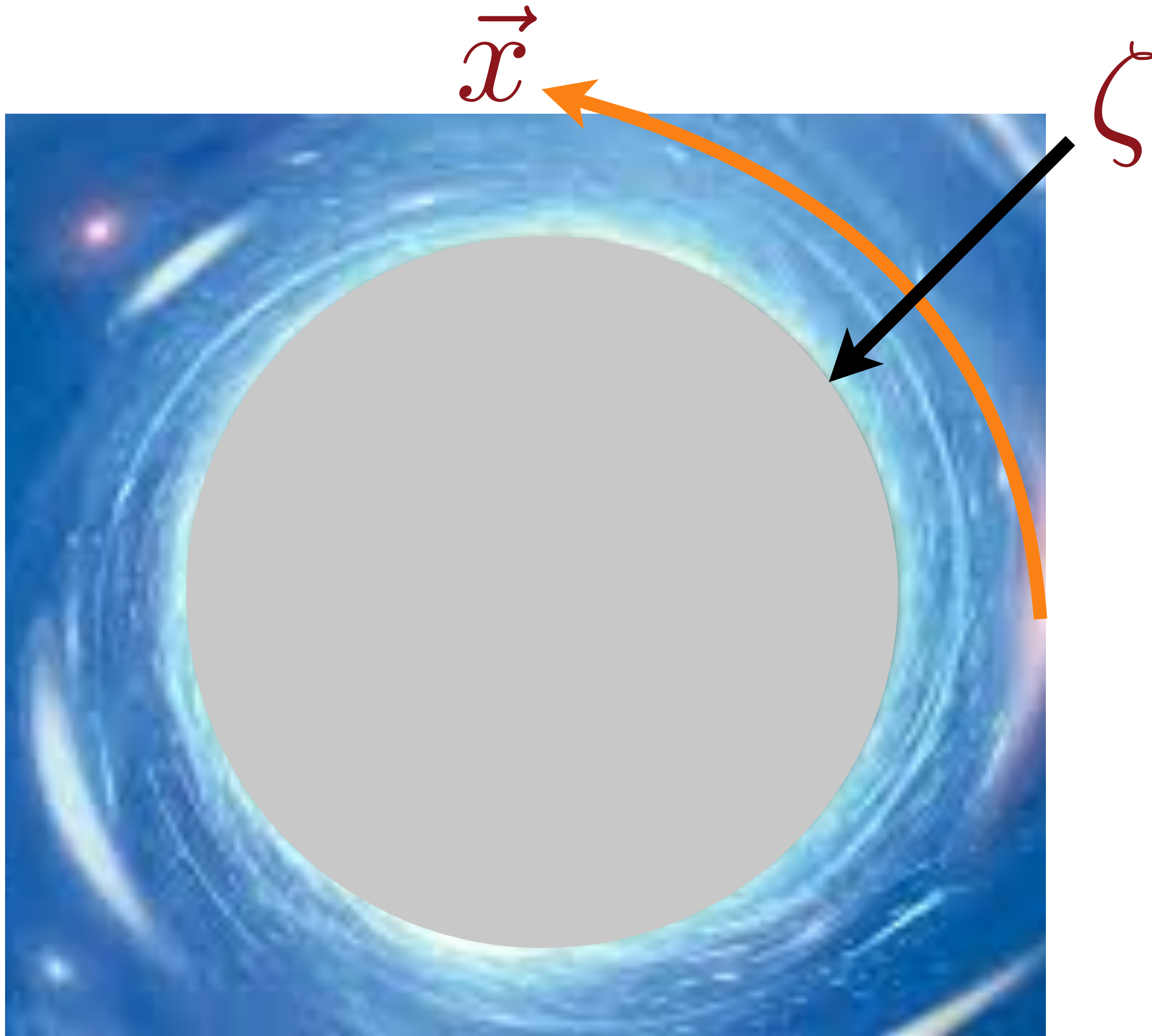
where $s_0 = 0.46484769917080510749\dots$



From the SYK model
to a quantum theory of
charged black holes

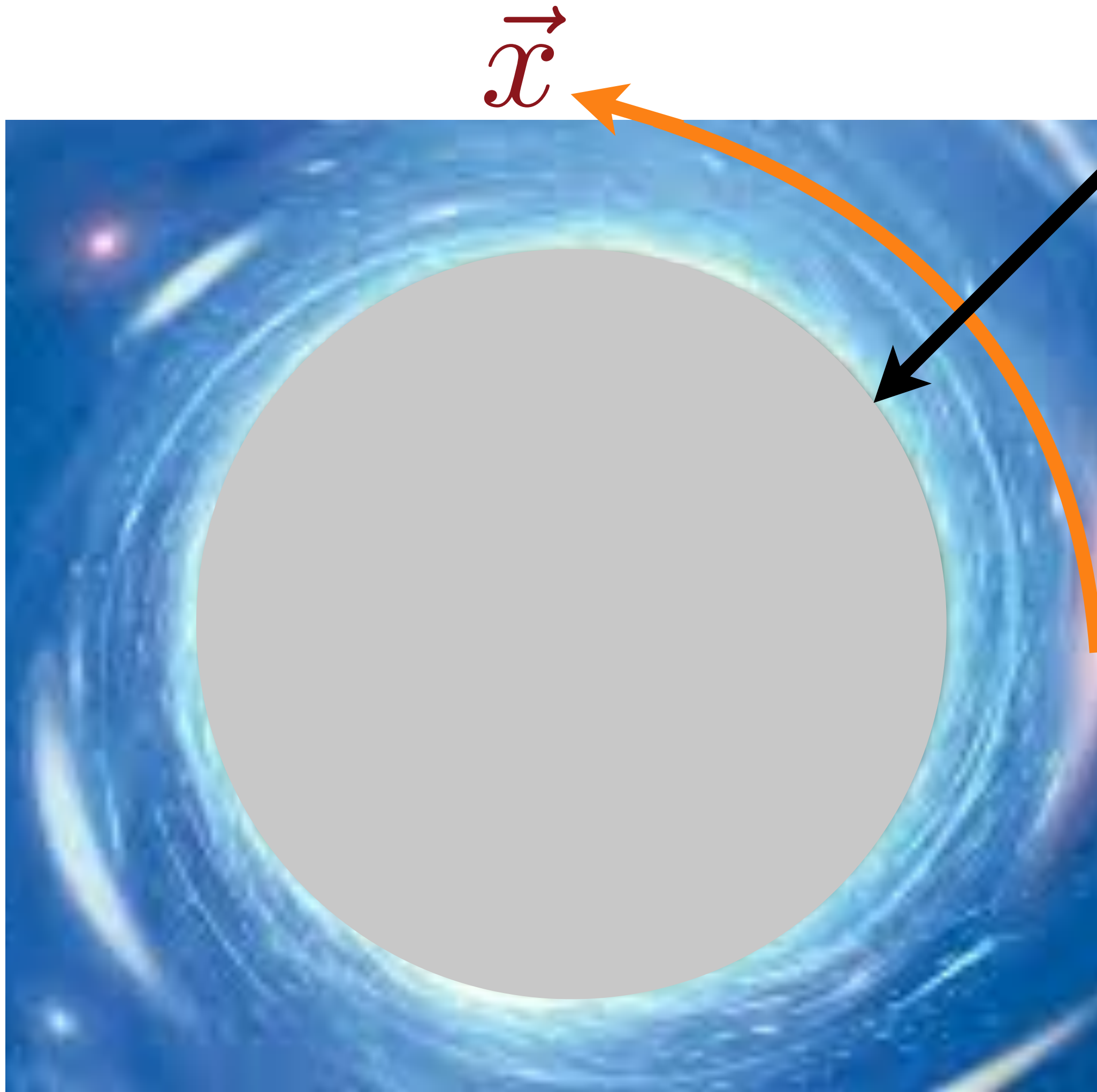


Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge





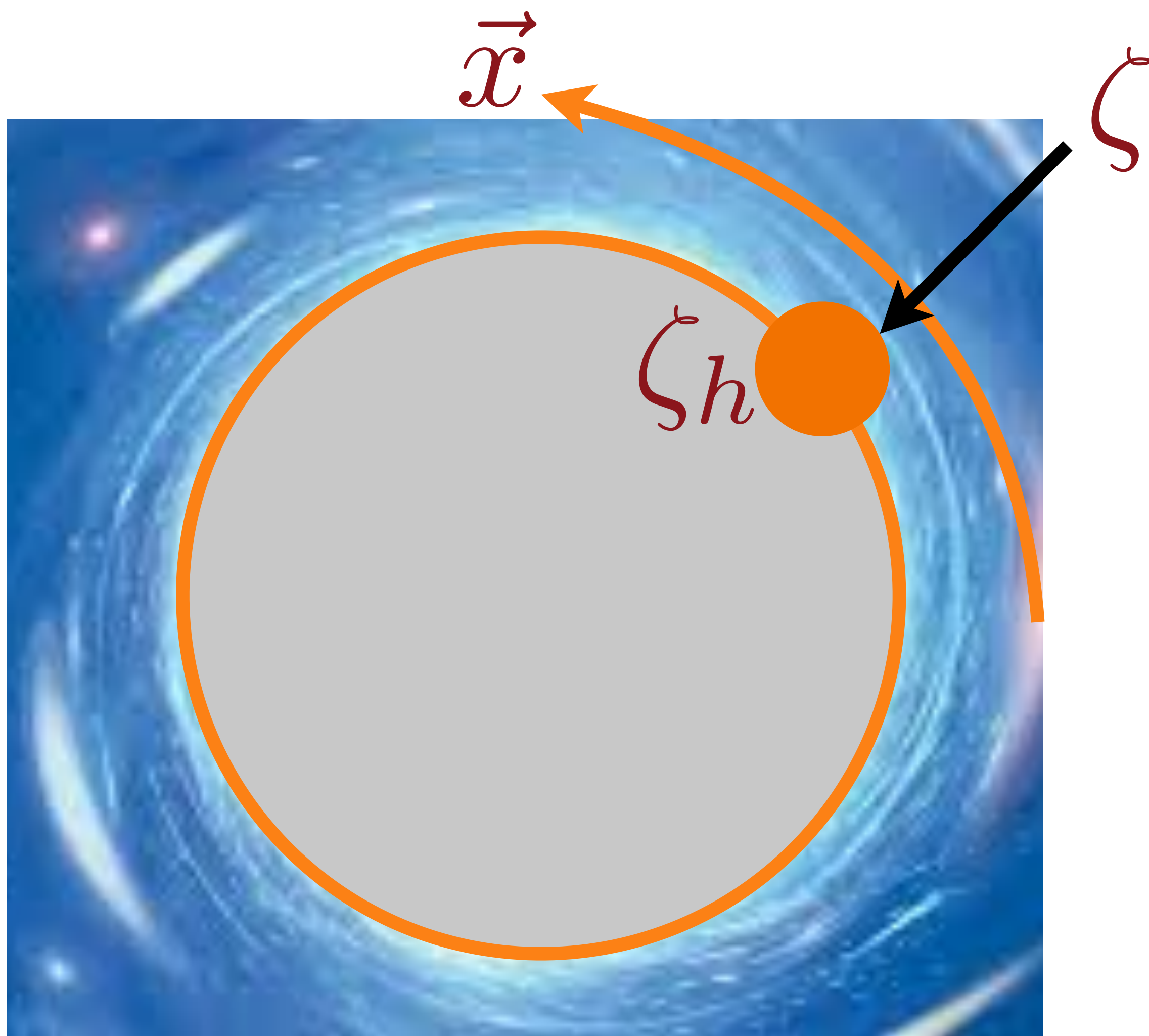
Maxwell's electromagnetism
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Zooming into the
near-horizon region
of a charged black hole
at low temperature,
yields a theory
in one space (ζ) and
one time dimension



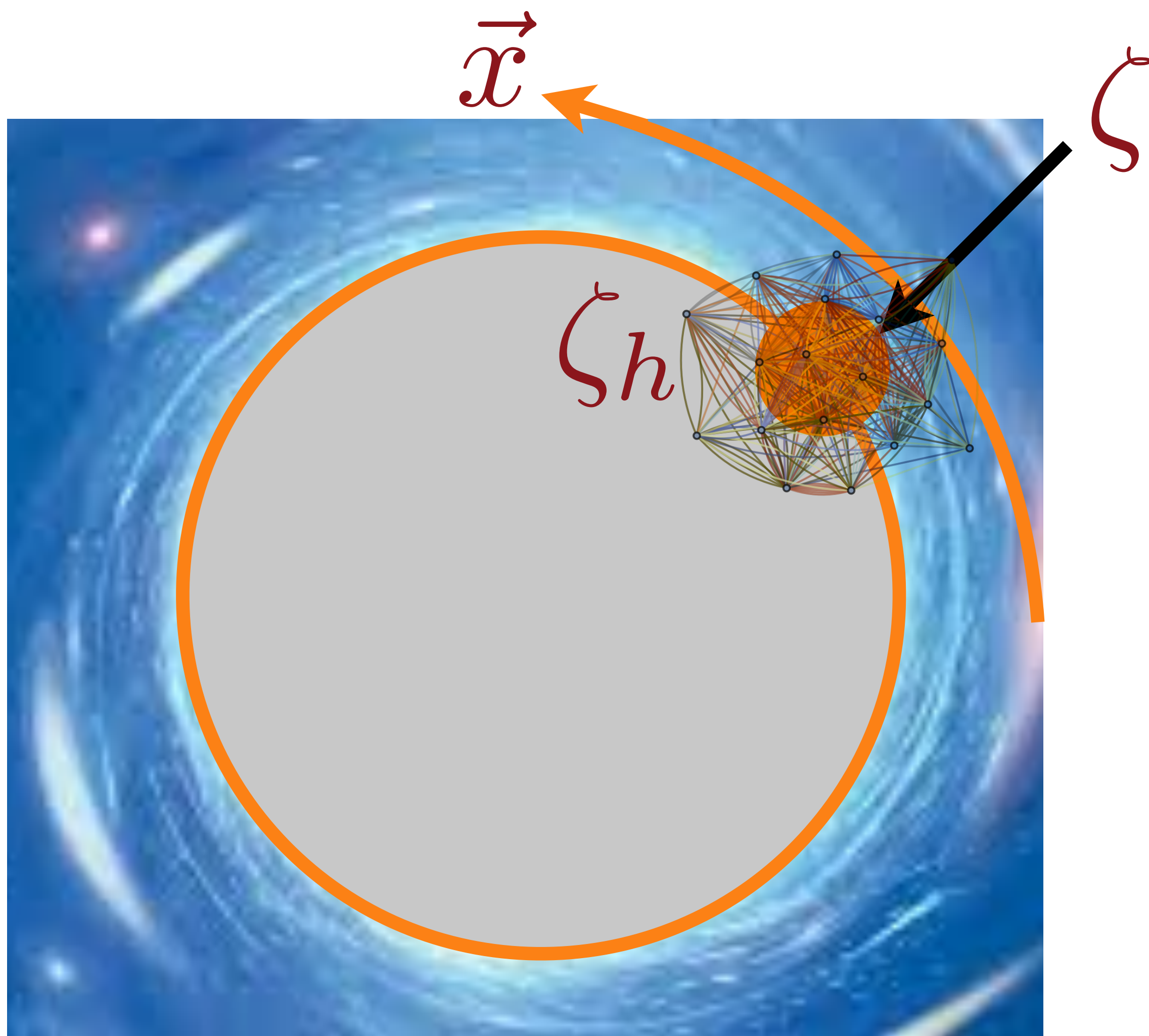
Maxwell's electromagnetism
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So we need only consider
complex entanglement at
one spatial “point”
on the horizon ($\zeta = \zeta_h$),
just as is described
by the SYK model



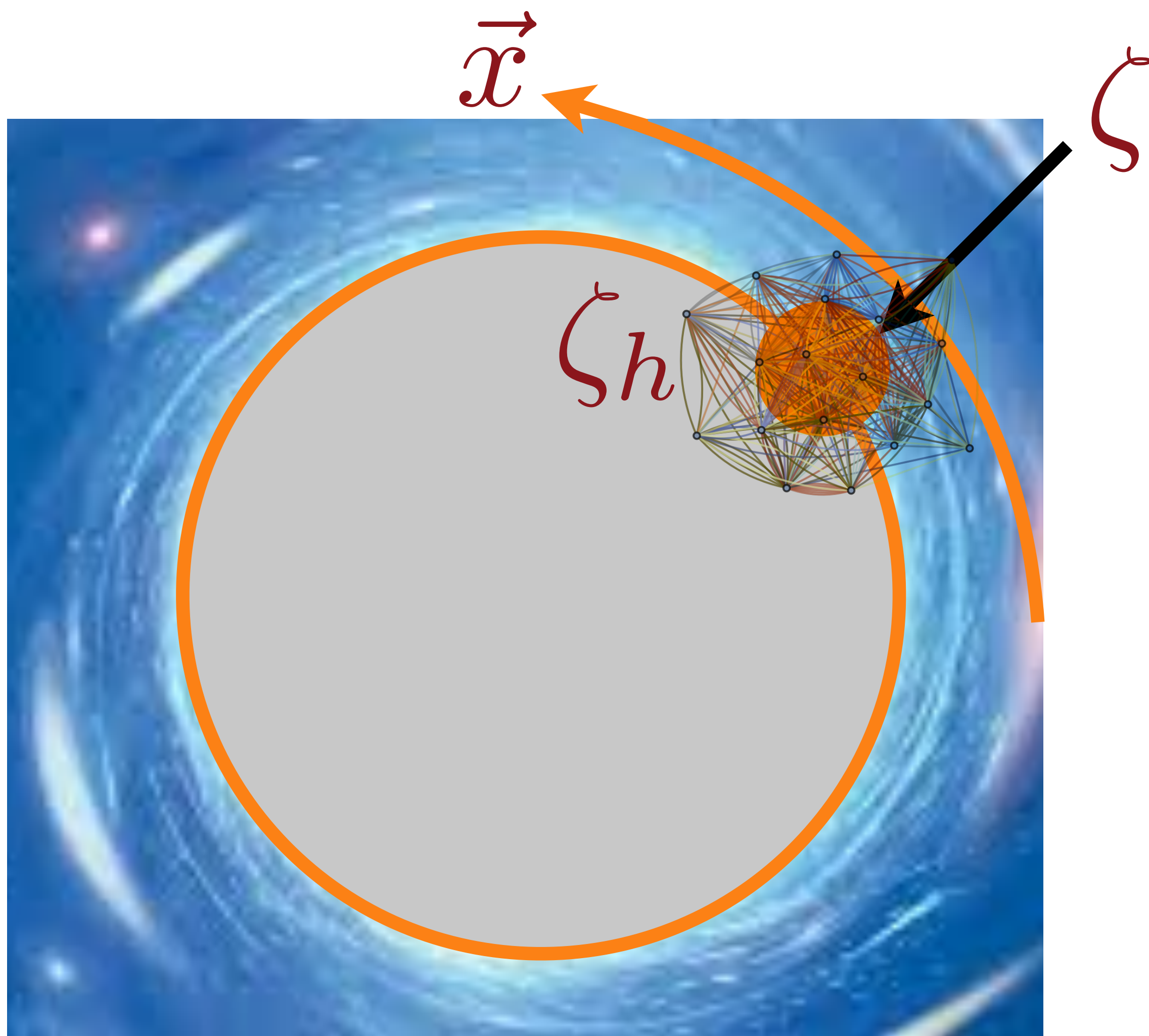
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The quantum versions of
Maxwell's and Einstein's
equations in this
two-dimensional spacetime are
also the equations describing
electron entanglement in the
SYK model!



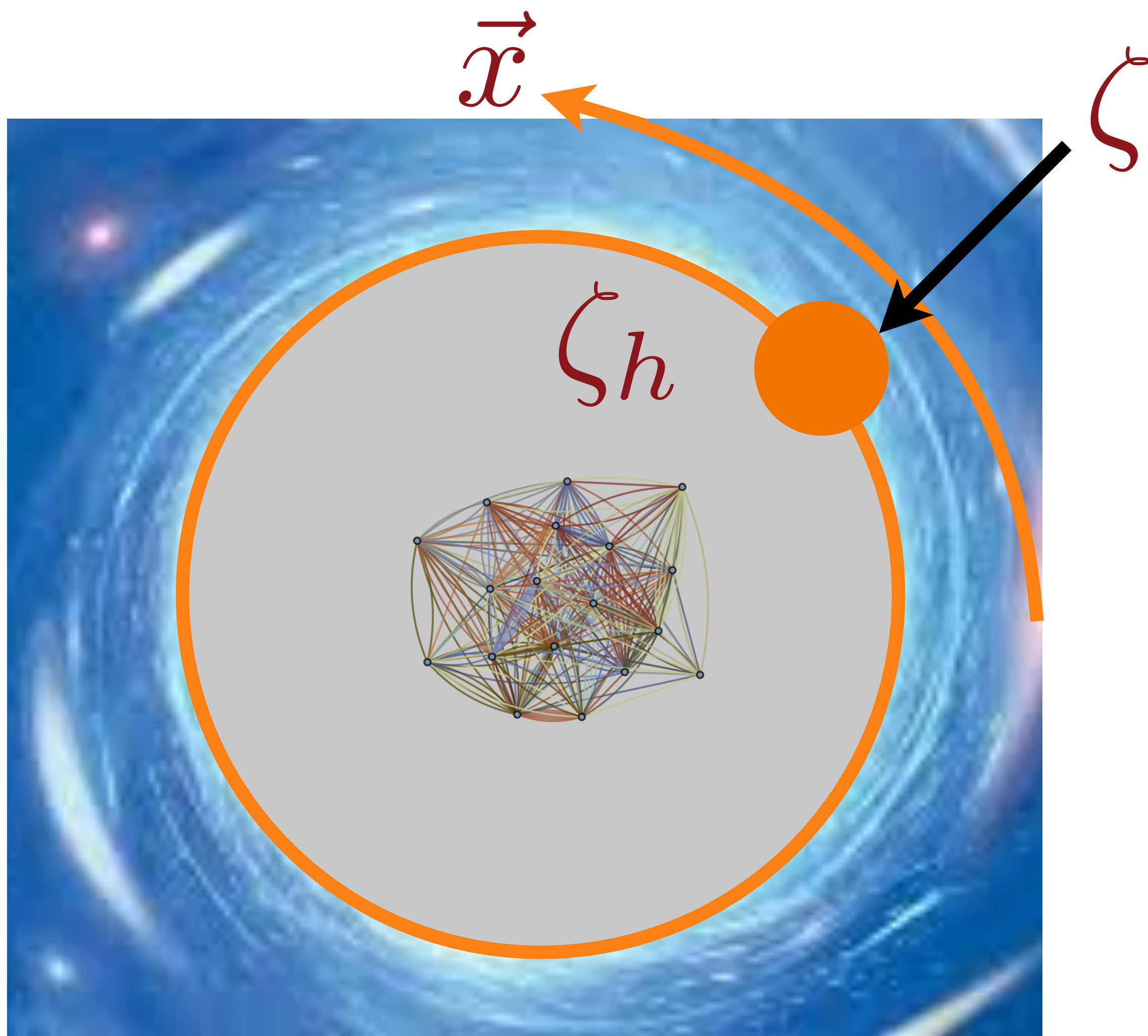
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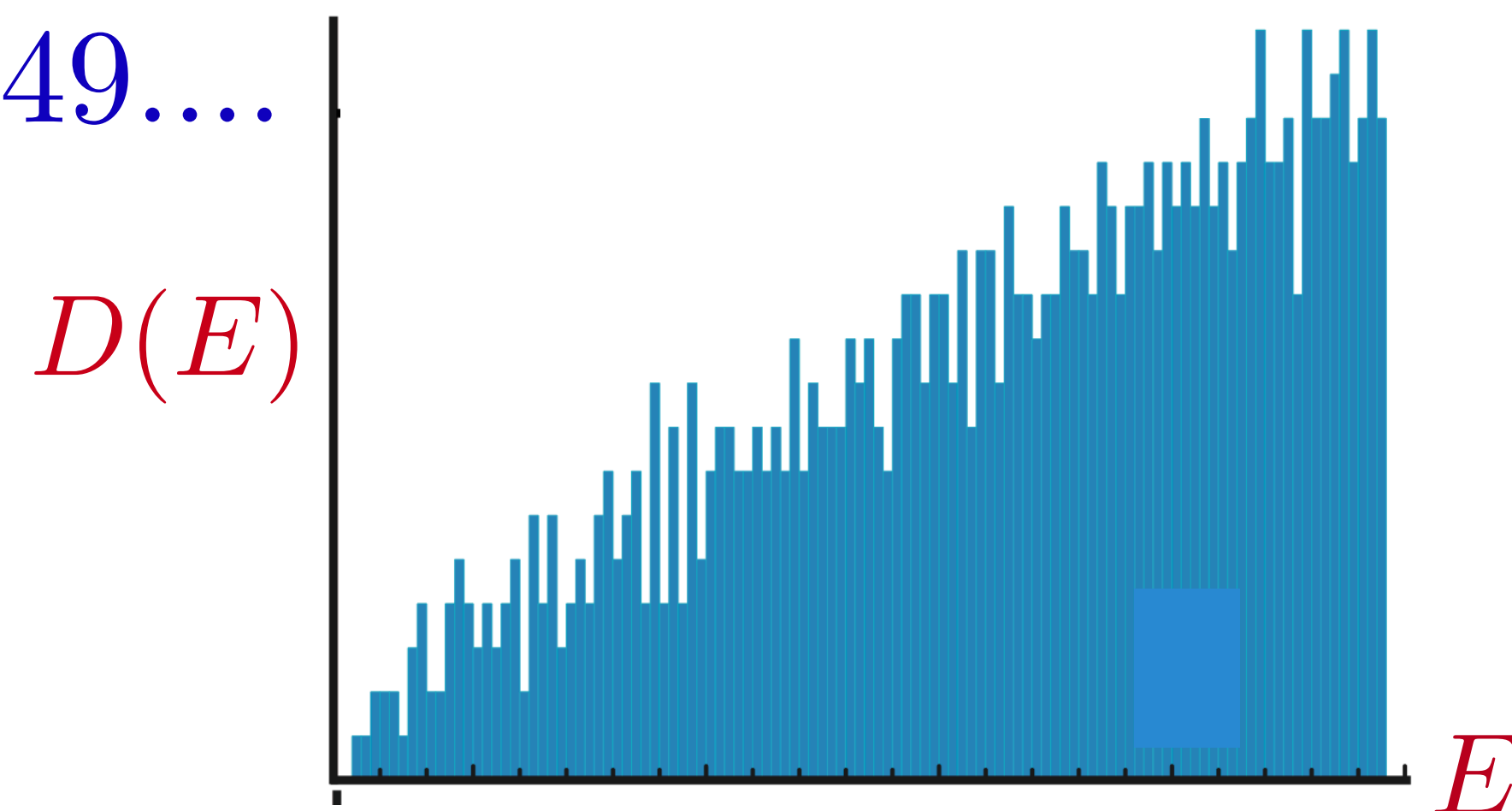
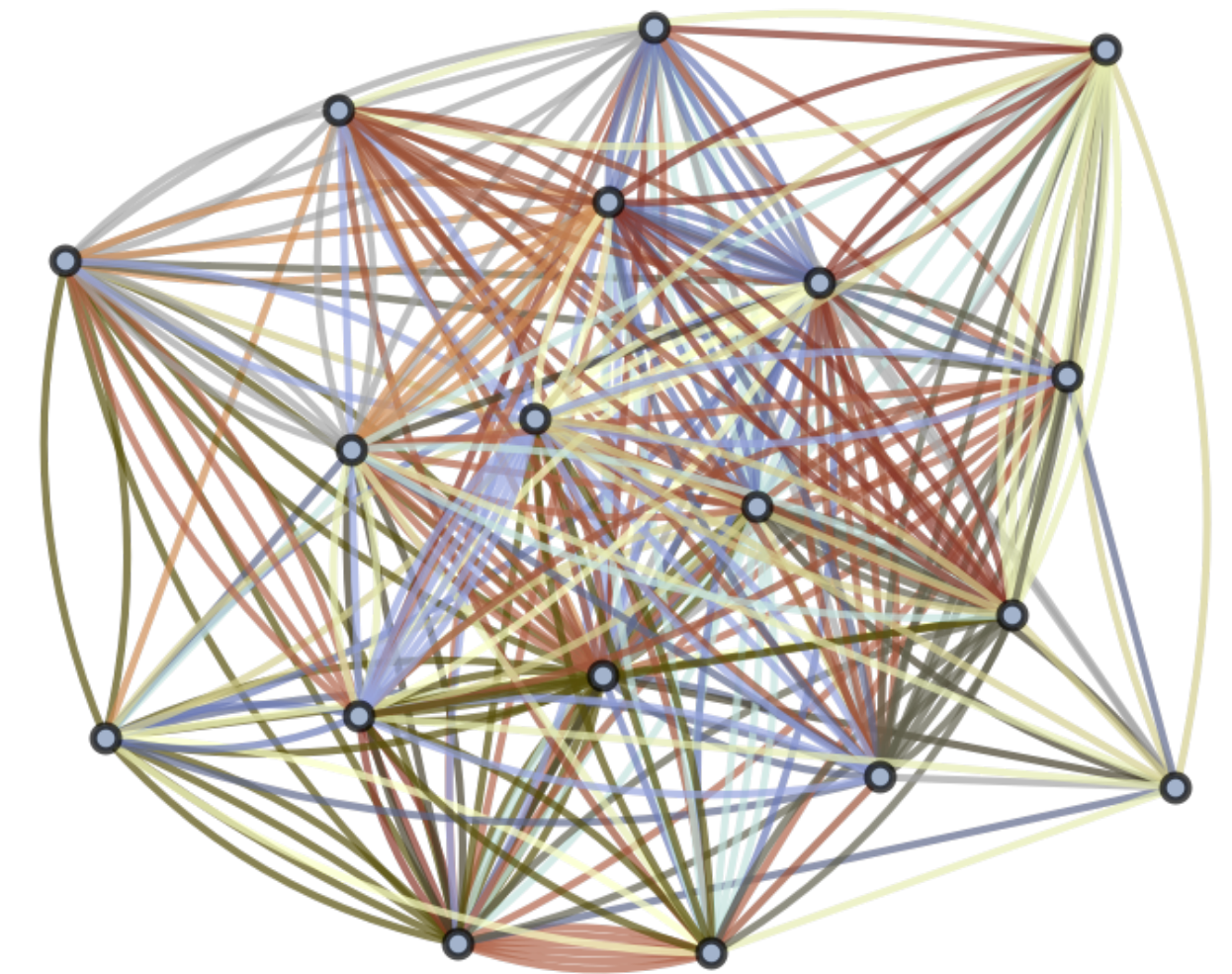
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D(E) of charged black holes from the SYK model

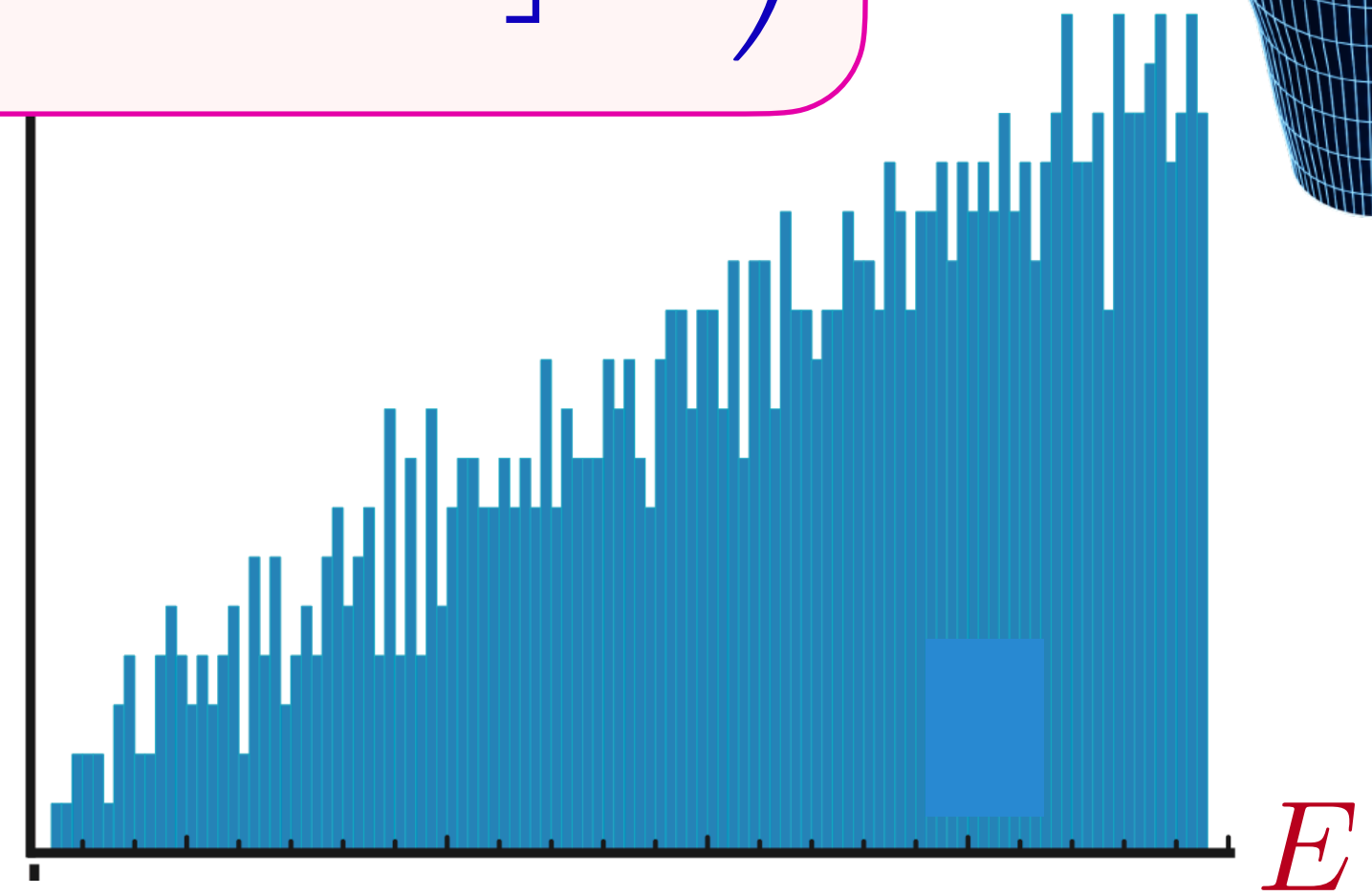
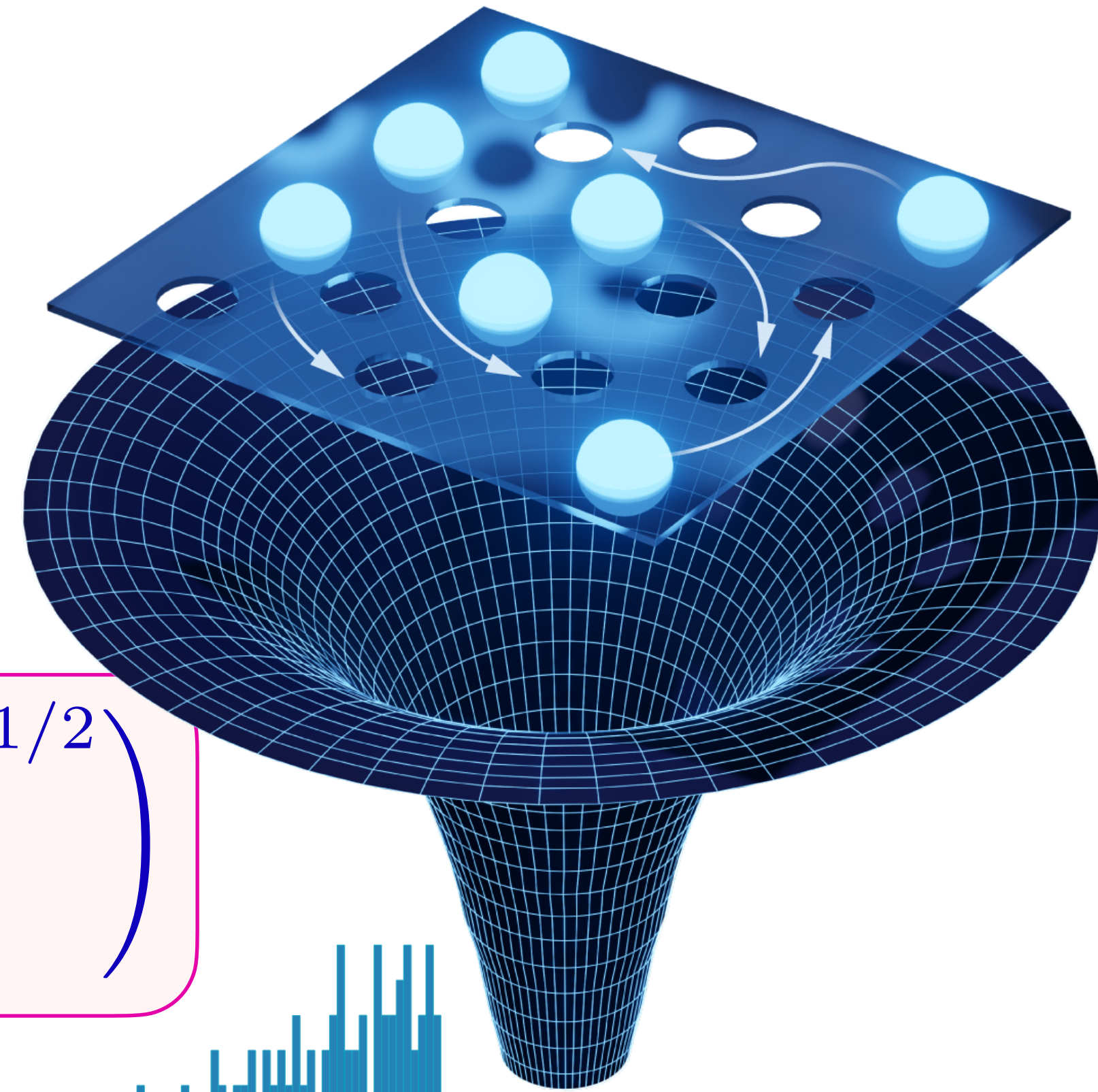
- For generic charged black holes in 3+1 dimensions with horizon area A at $T = 0$ and fixed charge Q ($A = 2GQ^2/c^4$), the density of quantum states at small energy E is

$$D(E) \sim \left(\frac{Ac^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{Ac^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

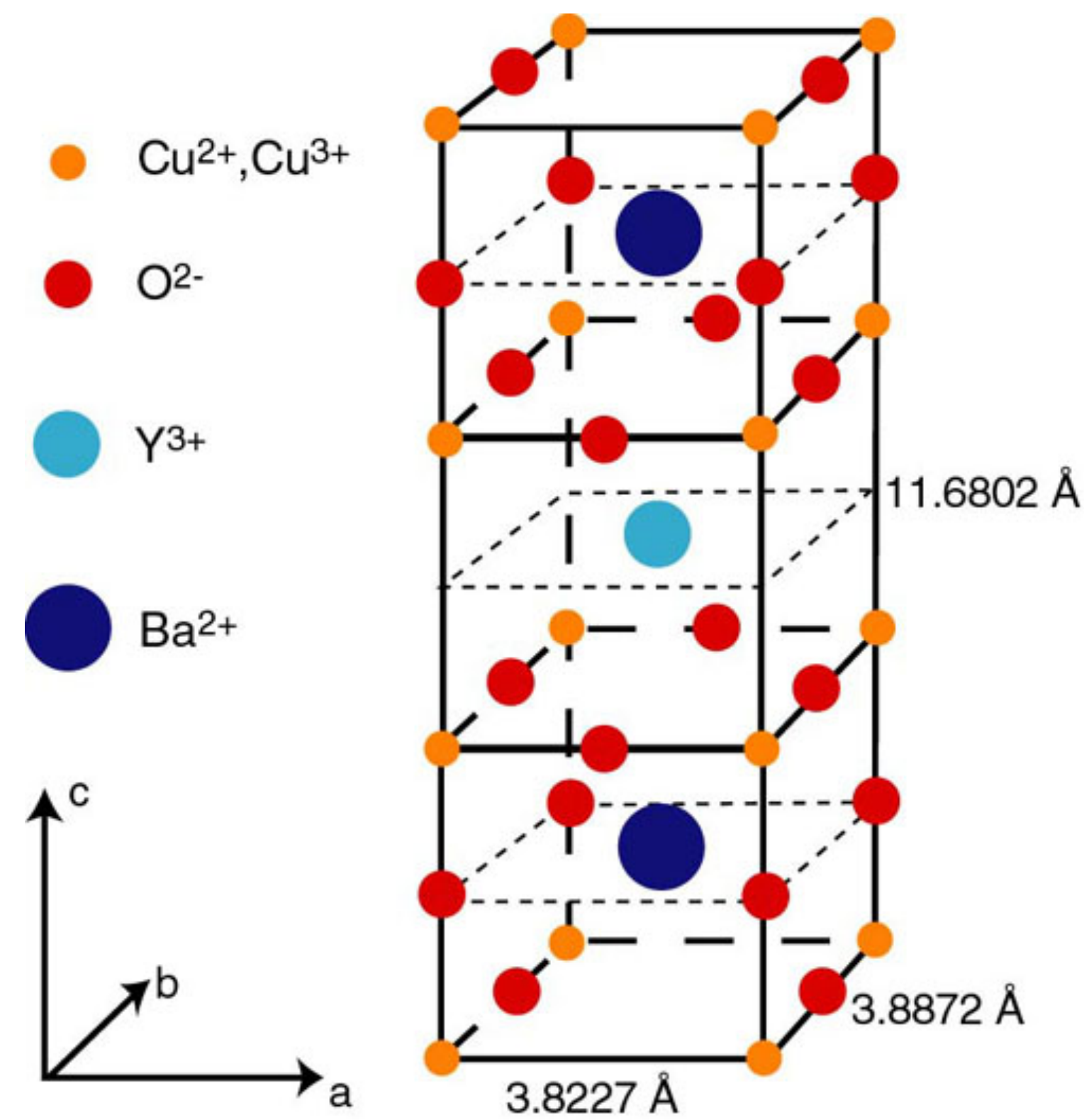
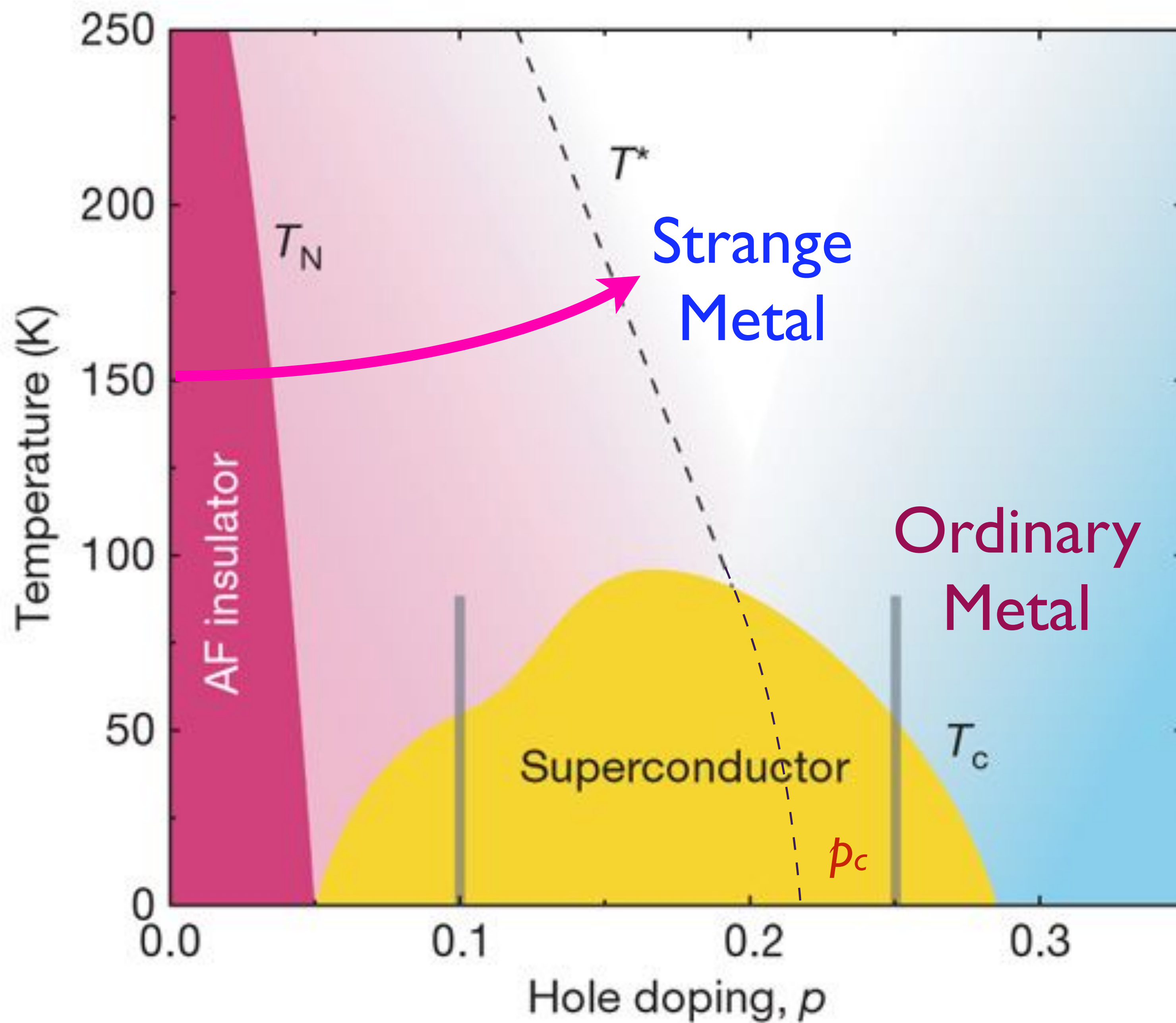
Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

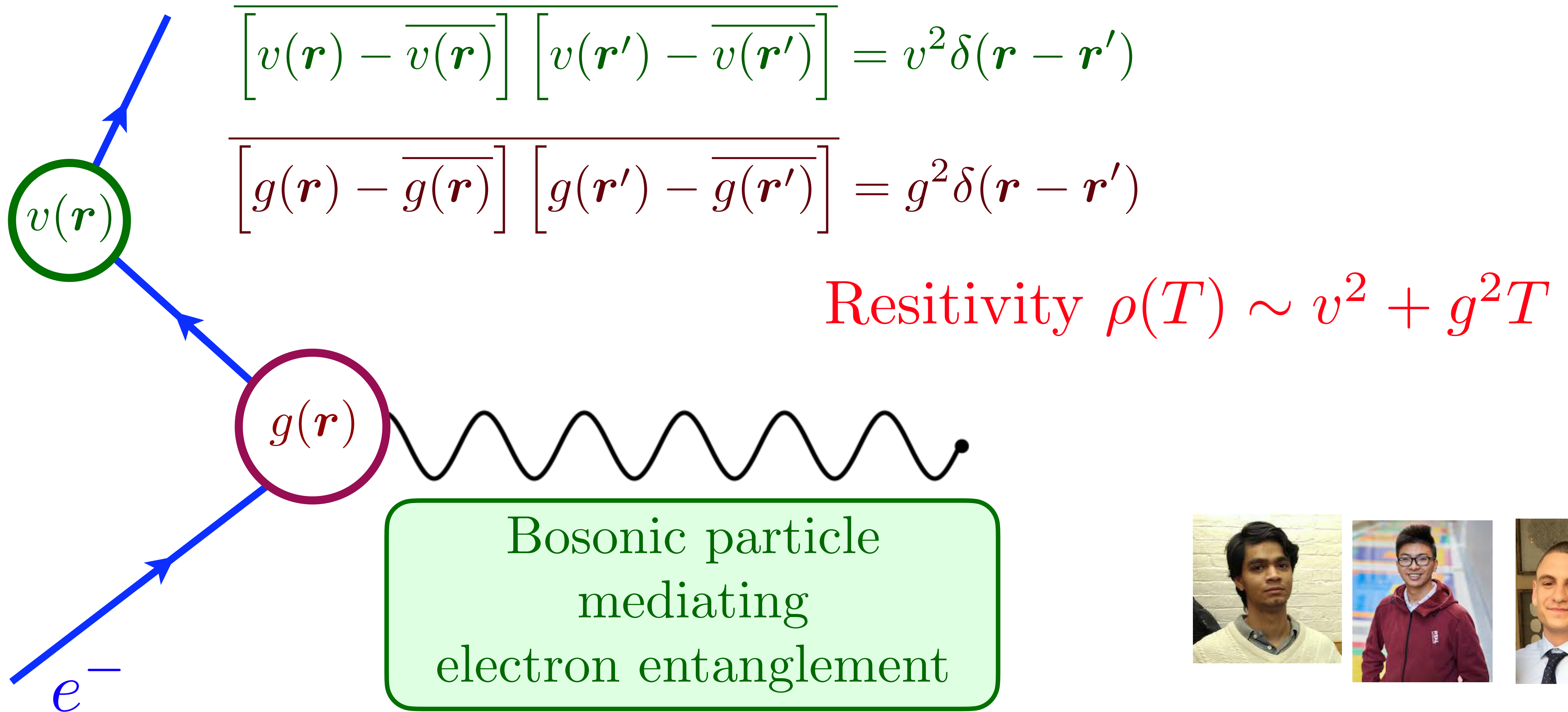
Developments from the SYK model



From the SYK model
to a theory of
strange metals



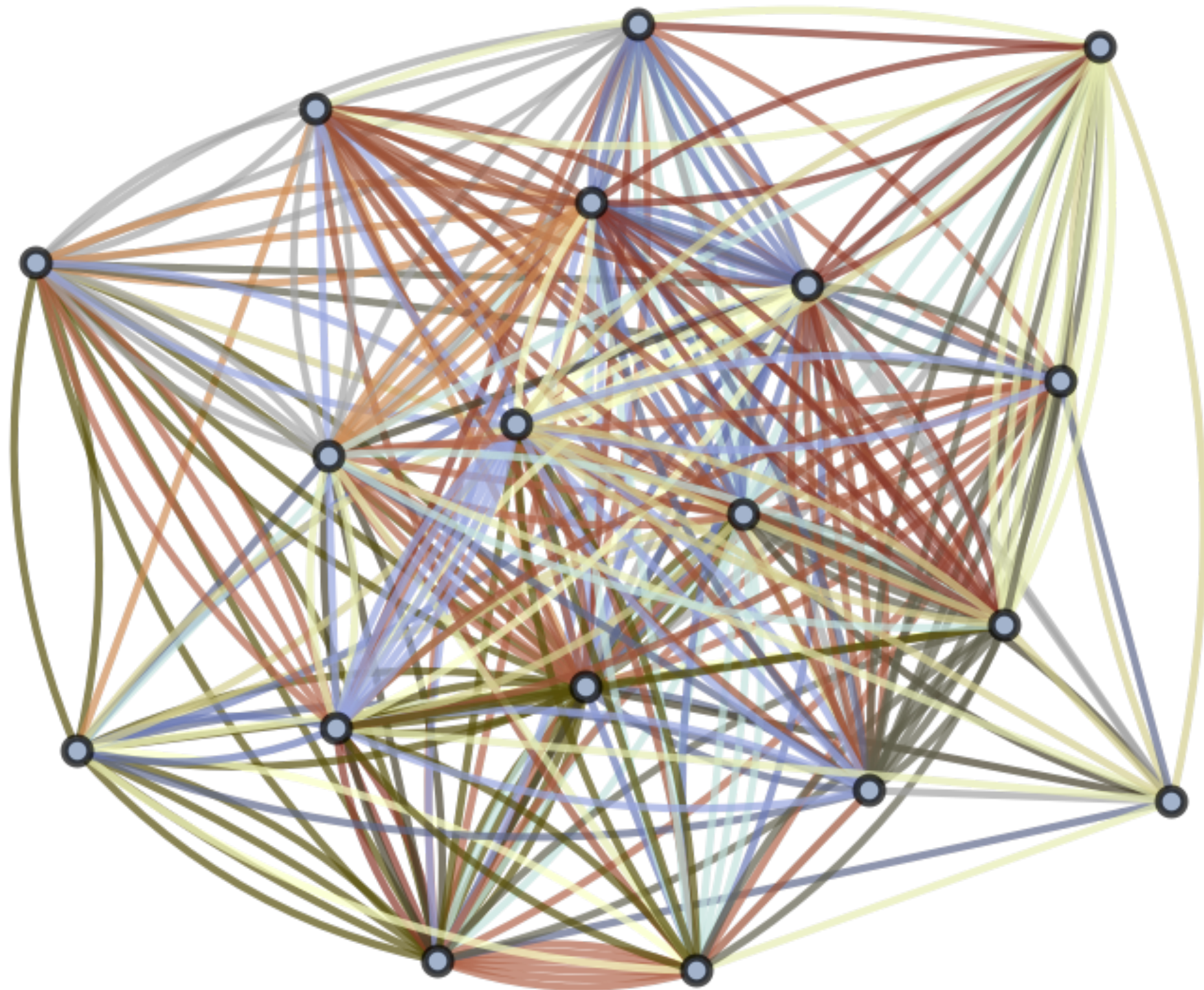
The dance of electrons on Cu atoms in YBCO



Recap

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

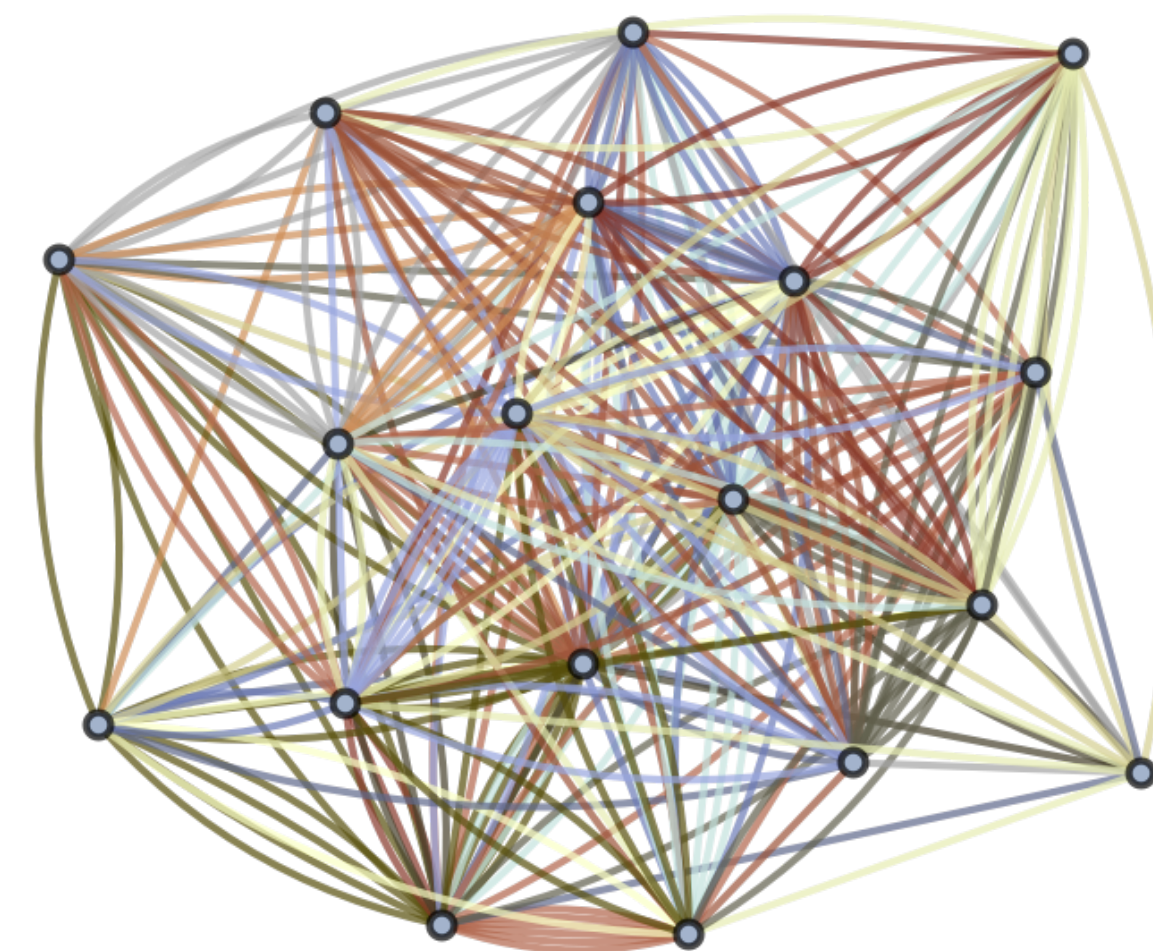
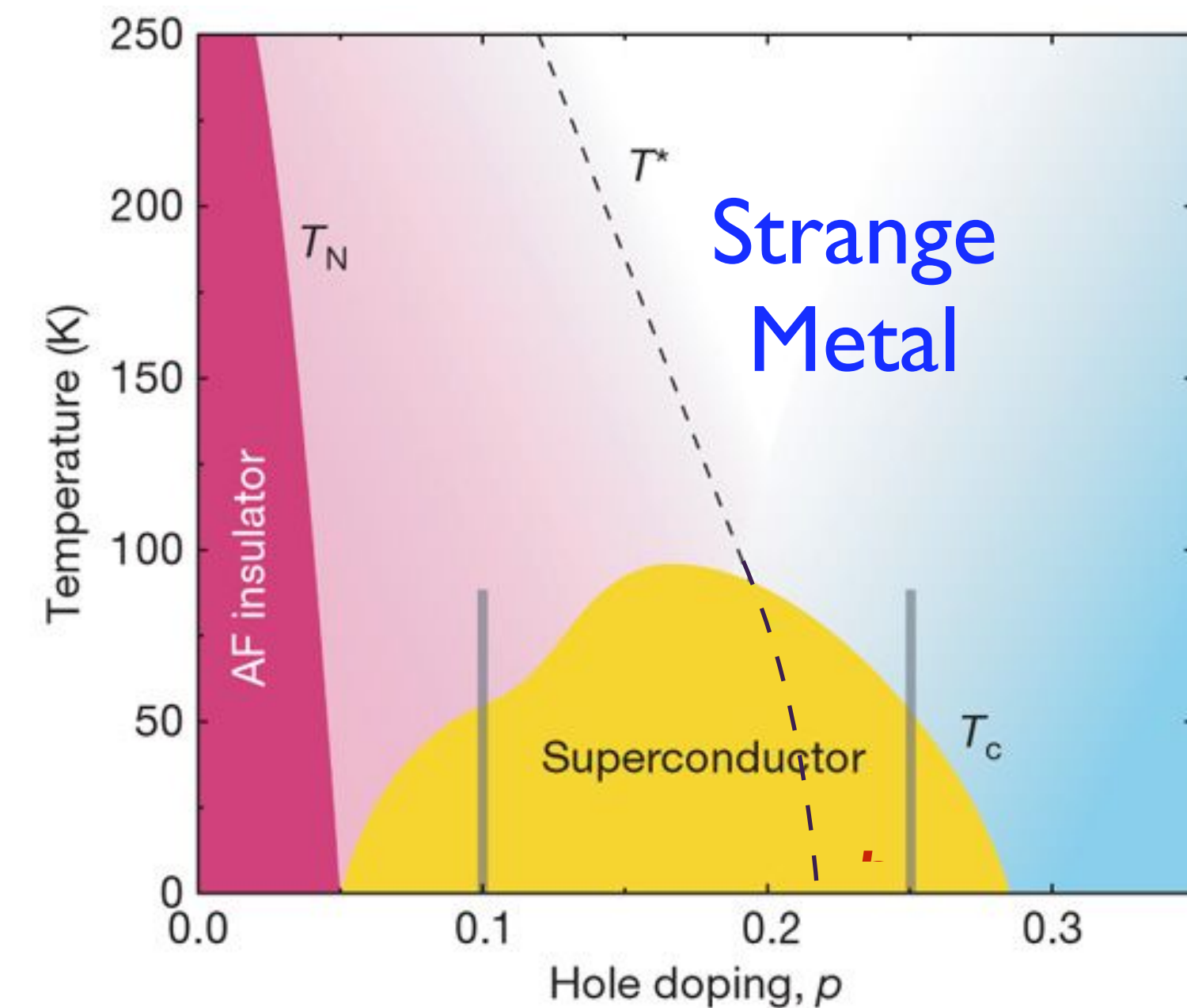


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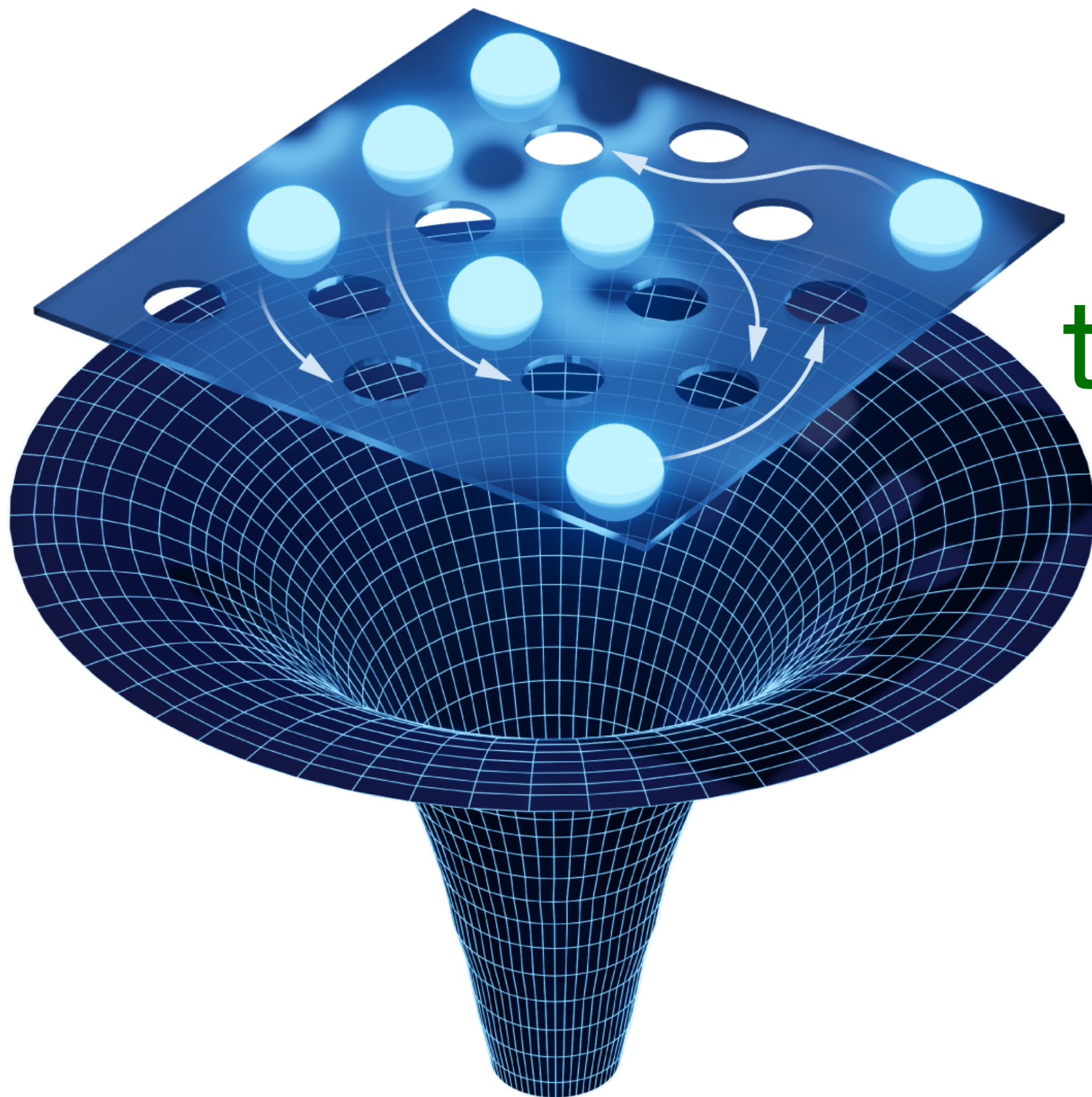
In one set of variables, it helps describe the *strange* electrical properties of YBCO

Sachdev, Ye (1993)



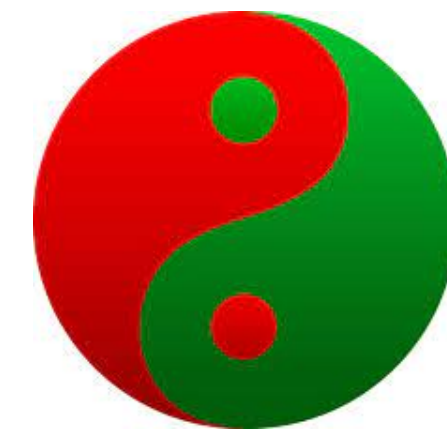
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In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



In a ***dual*** set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)