The quantum phases of matter

Solvay Meeting, Brussels,
October 20, 2011

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PHYSICS
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Classify zero temperature ground states
of infinite quantum systems in *d* spatial dimensions
(on a lattice or in the continuum)
with translational invariance.

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1. Gapped systems without zero energy excitations

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1. Gapped systems without zero energy excitations

2. "Relativistic" systems with zero energy excitations at isolated points in momentum space

3. "Compressible" systems with zero energy excitations on d-1 dimensional surfaces in momentum space.

Outline

1. Gapped quantum matter

   Insulators, quantum Hall states

2. Conformal quantum matter

   Graphene, antiferromagnets, ultracold atoms

3. Compressible quantum matter

   Metals, superconductors, strange metals

4. Connections to string theory

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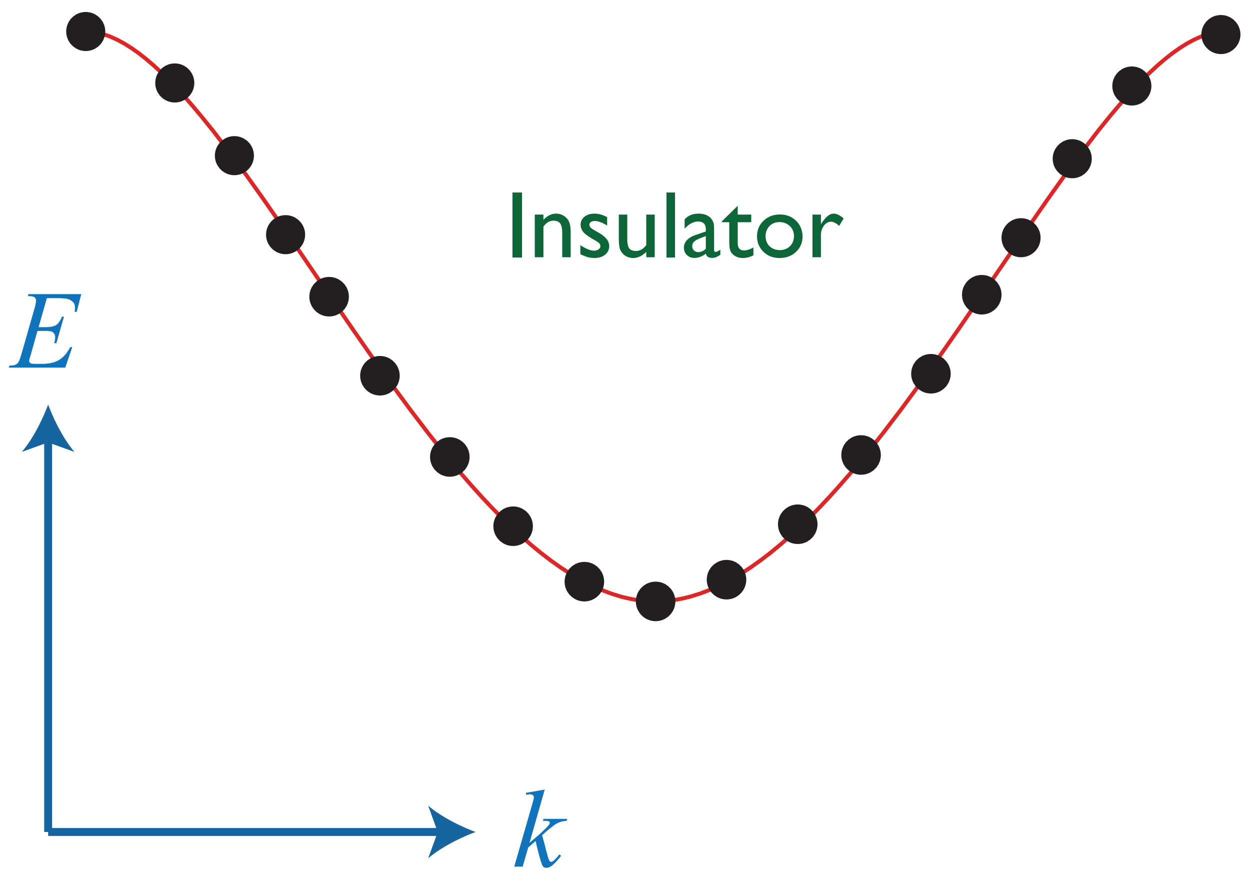
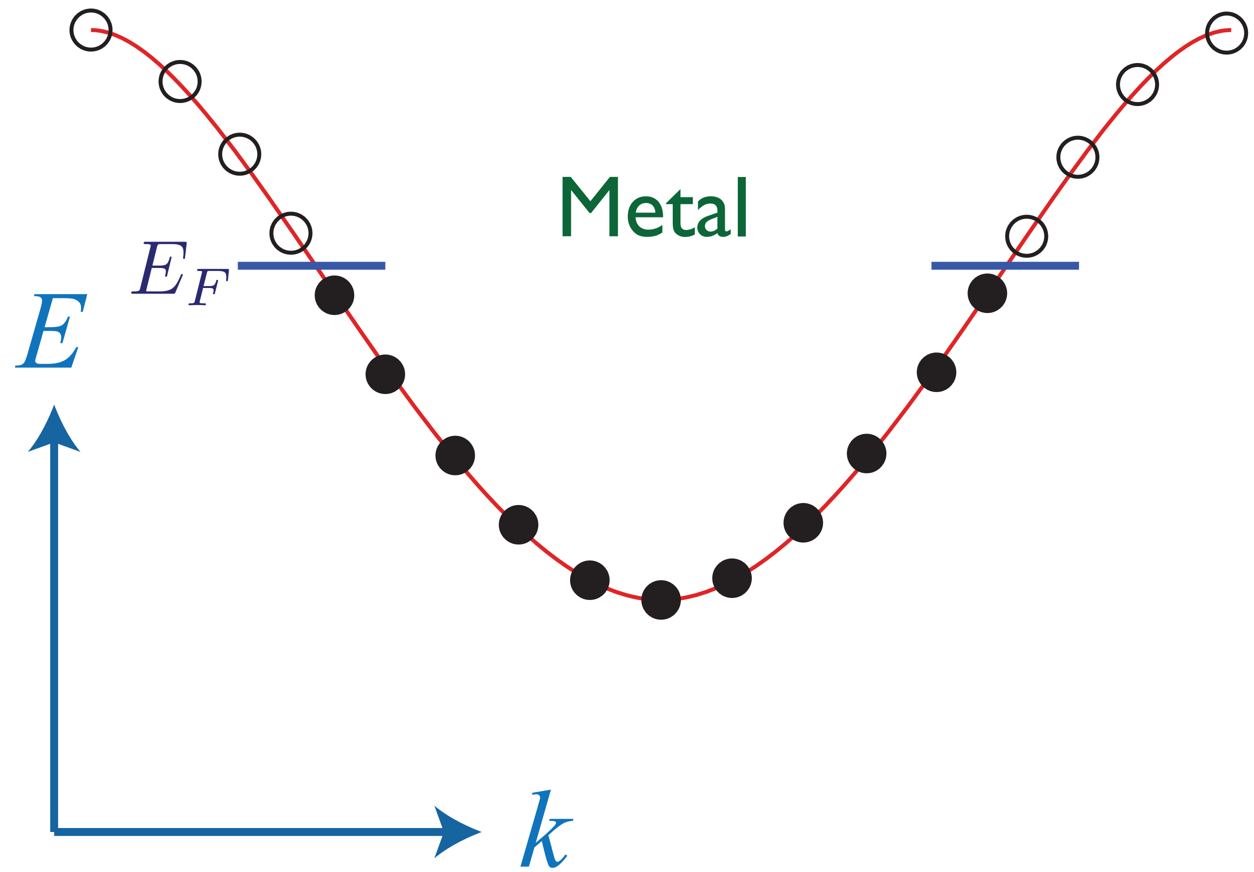
Band insulators



An even number of electrons per unit cell

Metals



An odd number of electrons per unit cell - will be discussed later

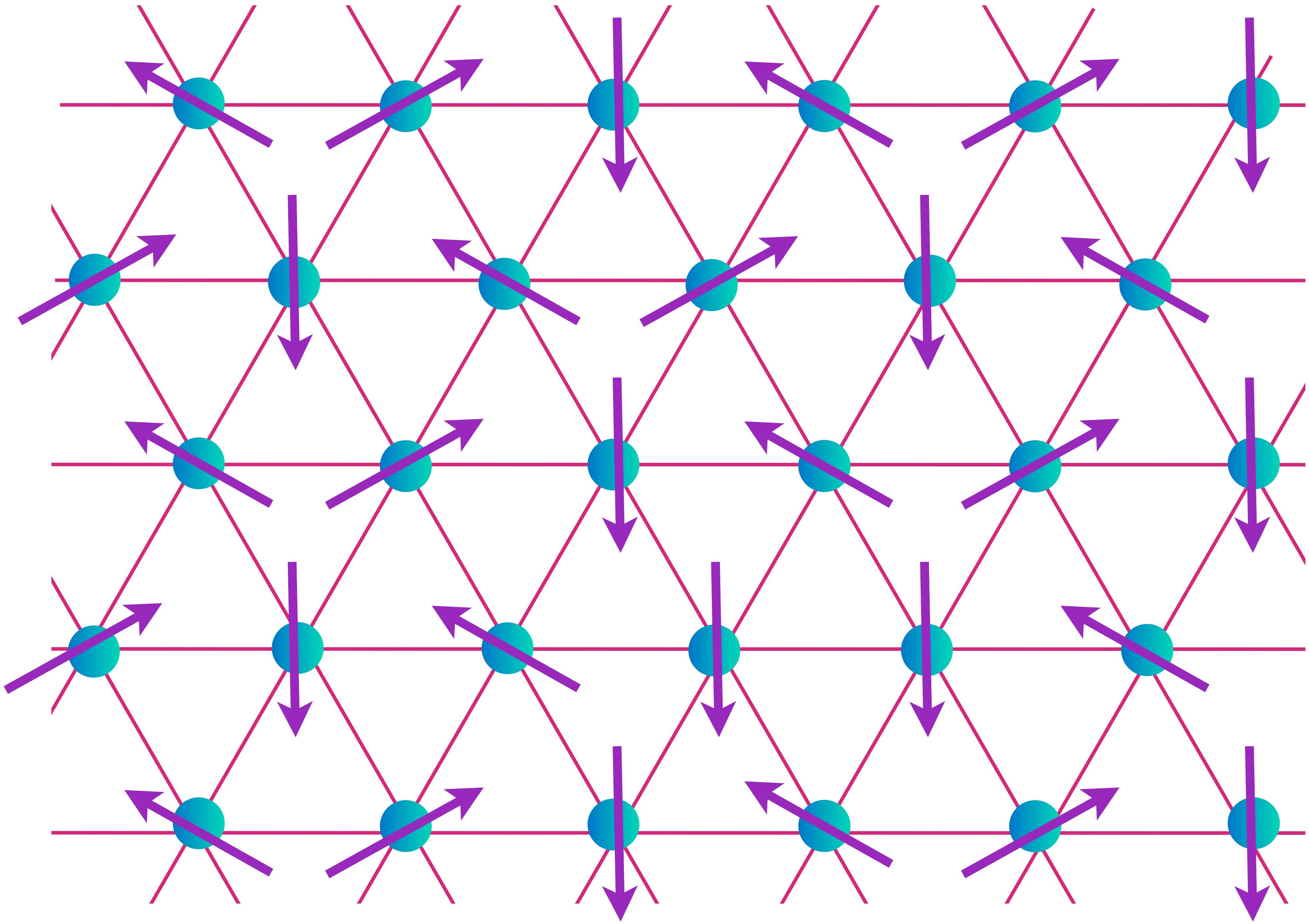
Mott insulator

Emergent excitations

An odd number of electrons per unit cell
but electrons are localized by Coulomb repulsion

Mott insulator: Triangular lattice antiferromagnet

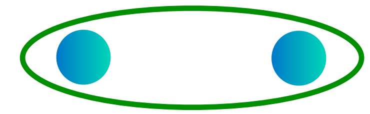
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Mott insulator: Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with S=1/2 per unit cell



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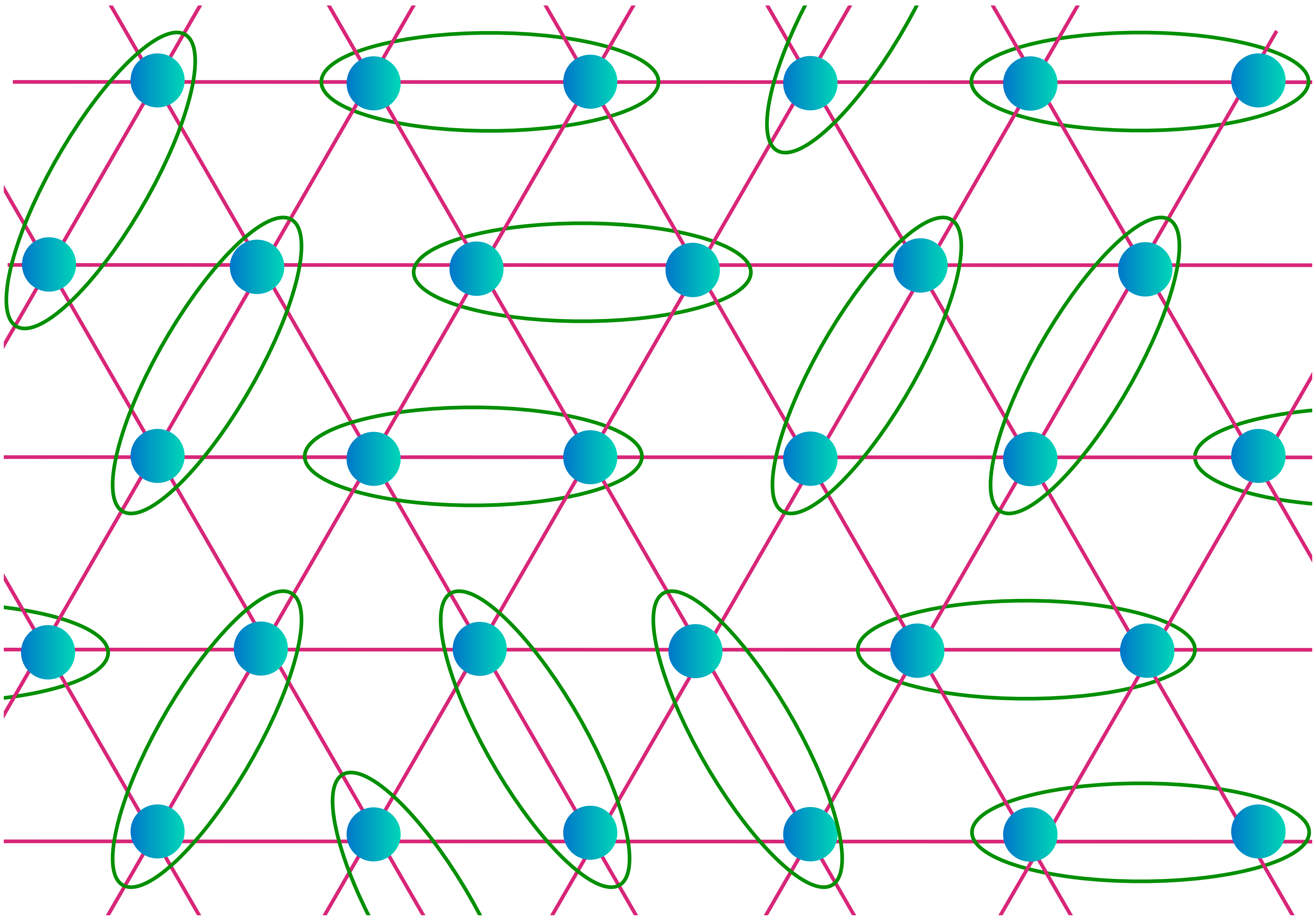

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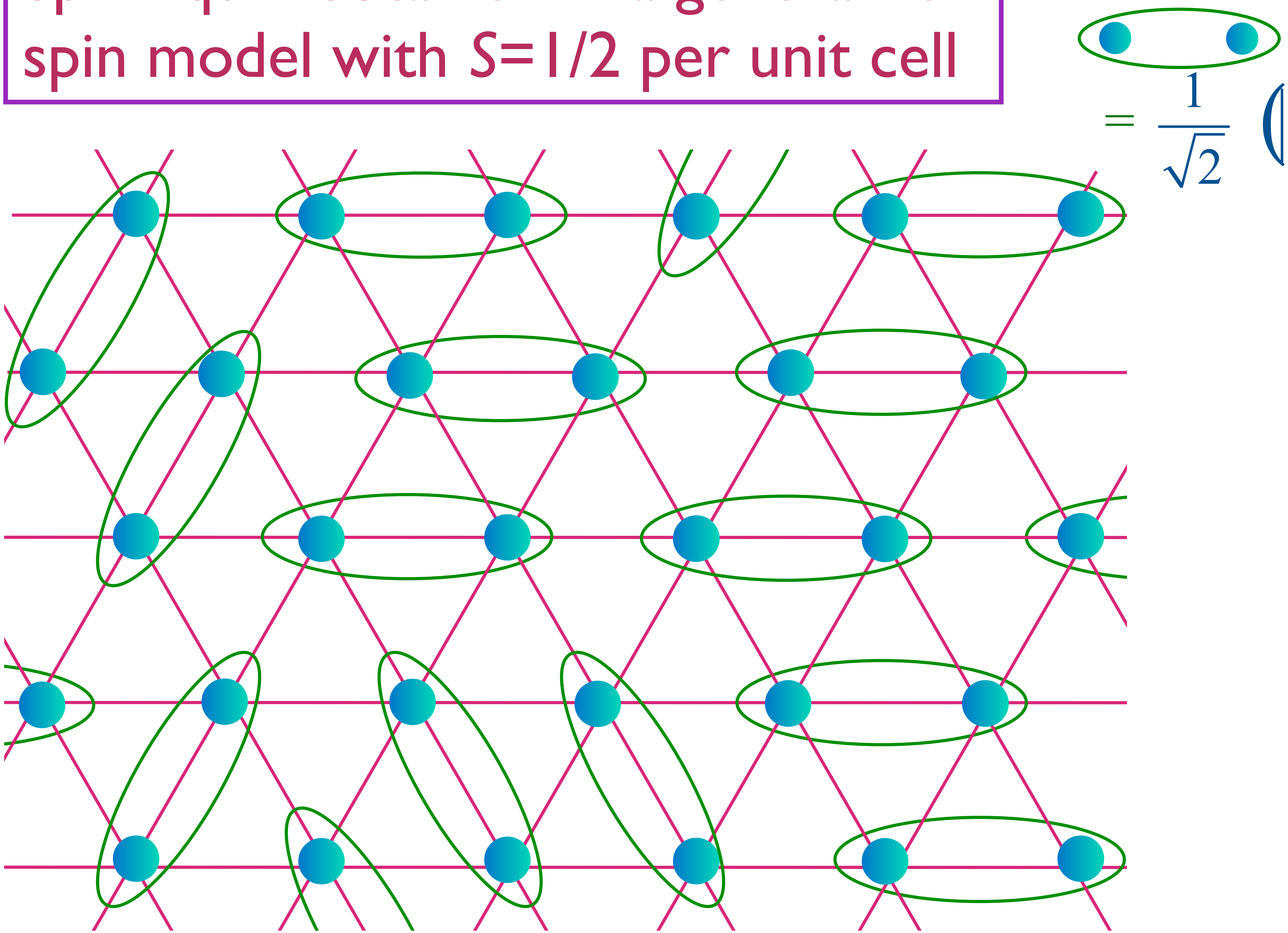

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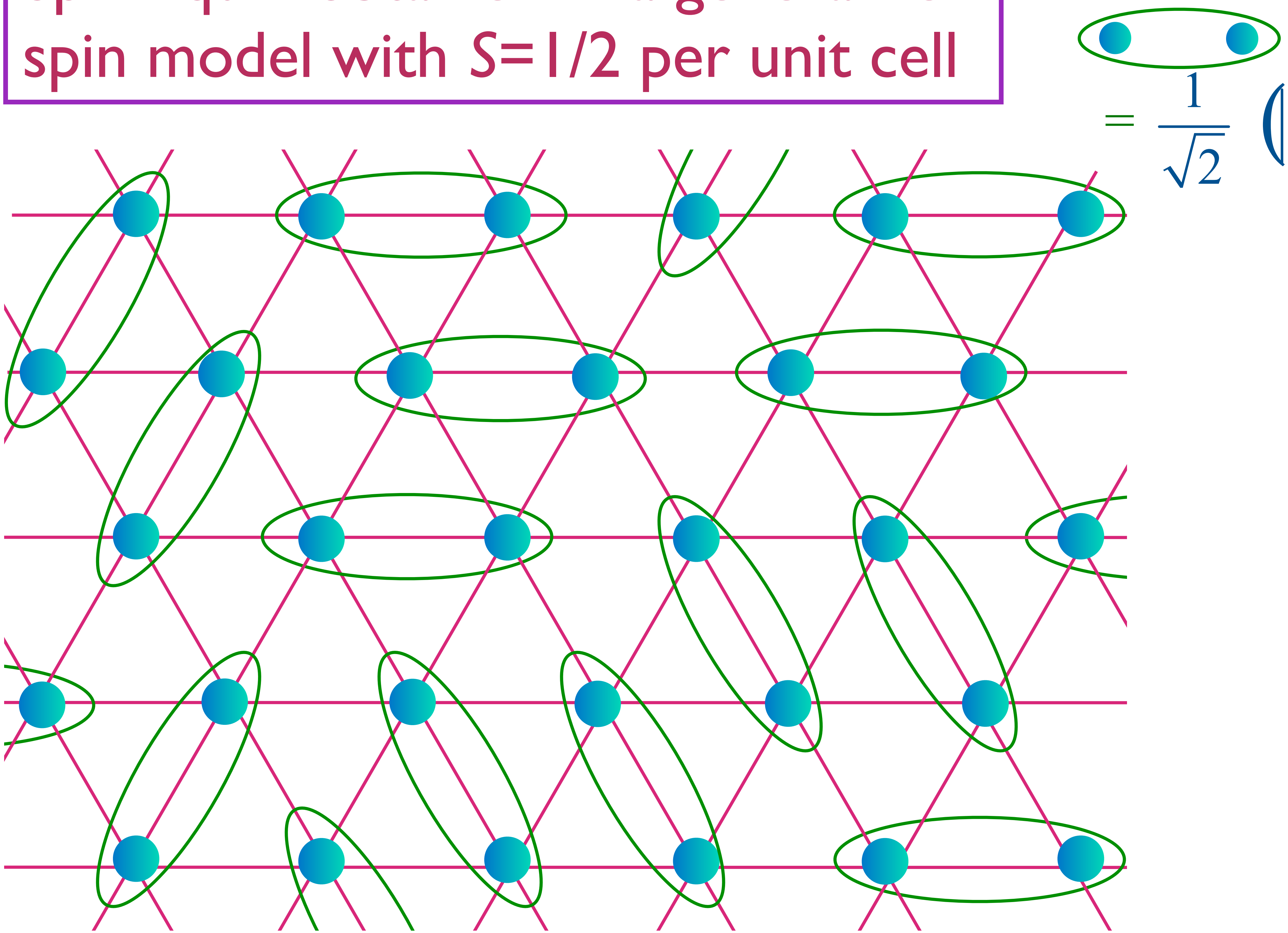

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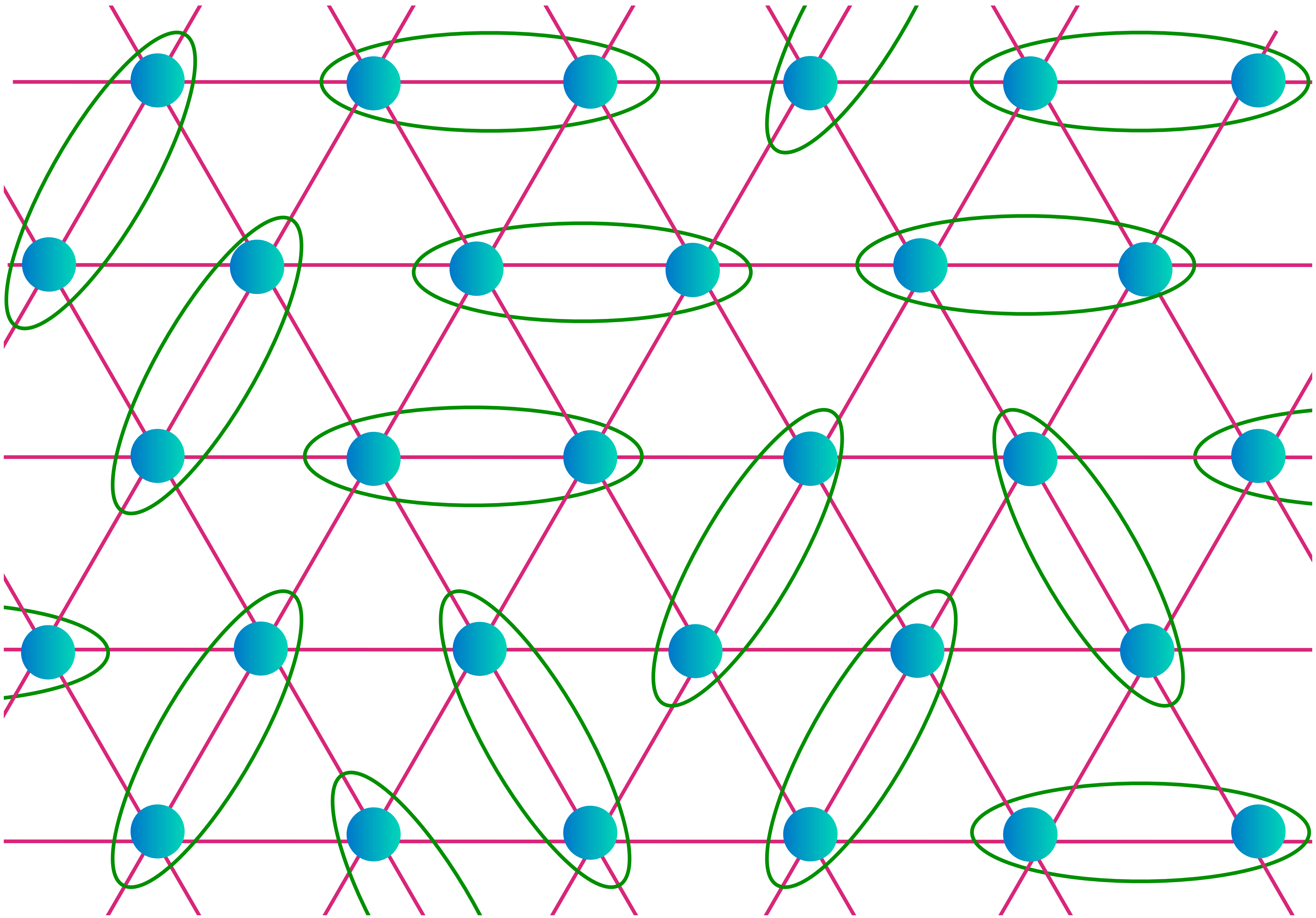

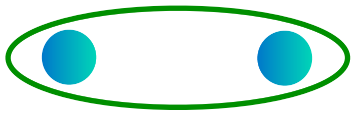

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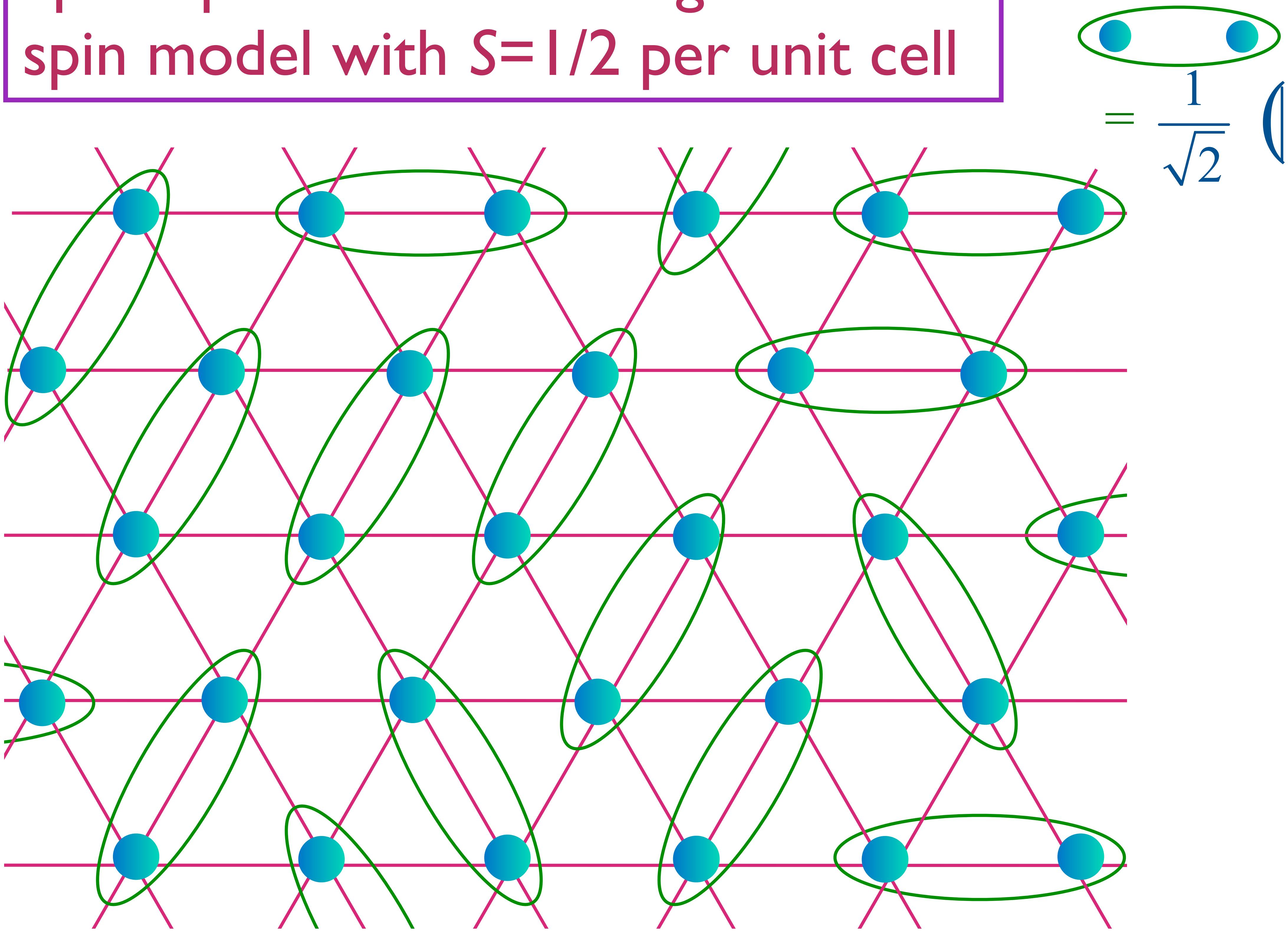

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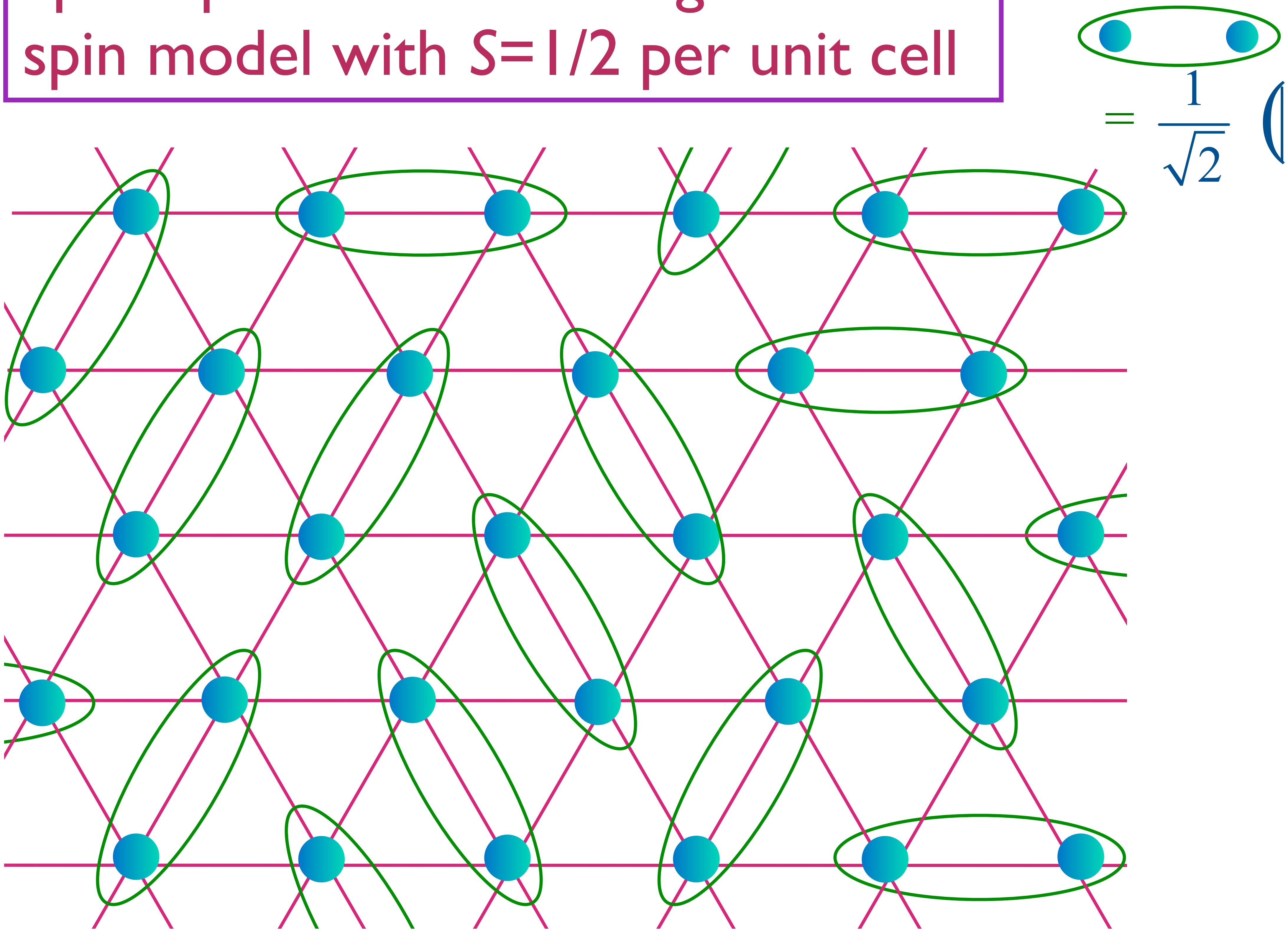

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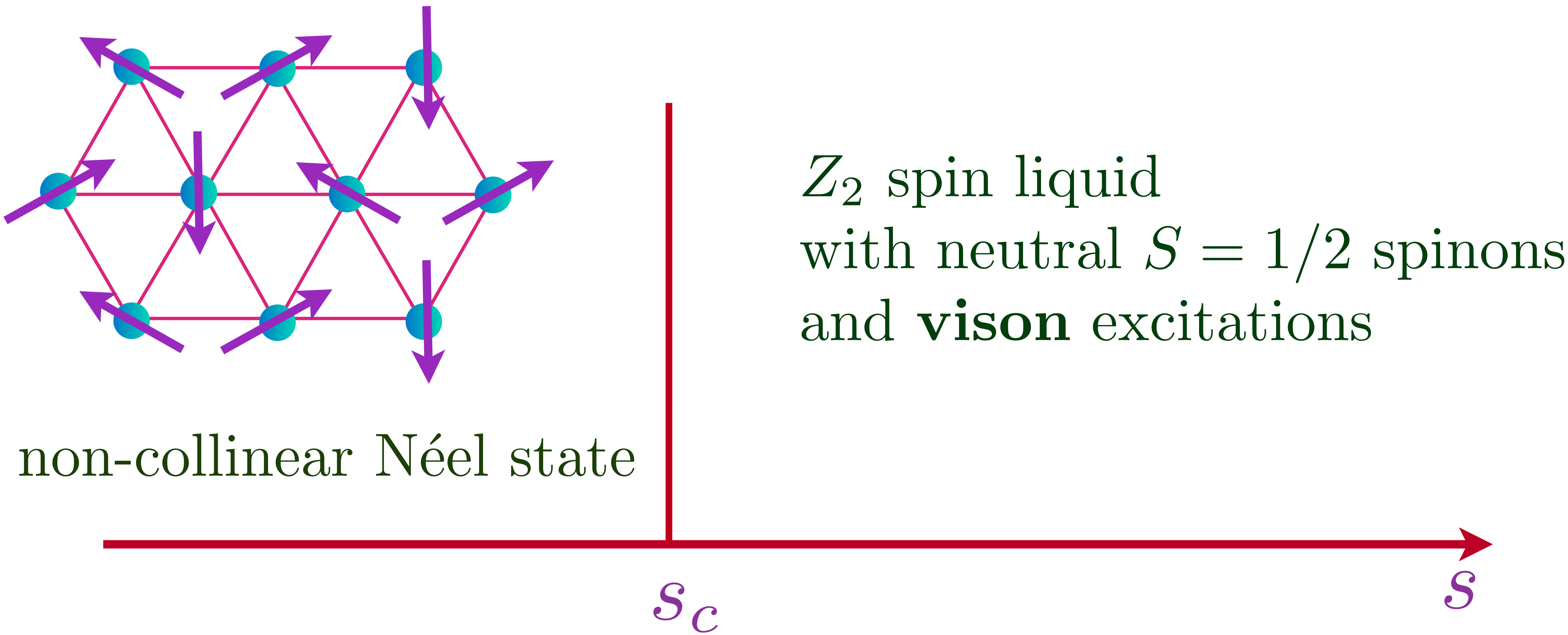
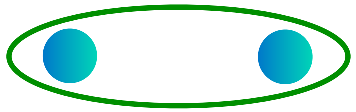


N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Excitations of the Z_2 Spin liquid



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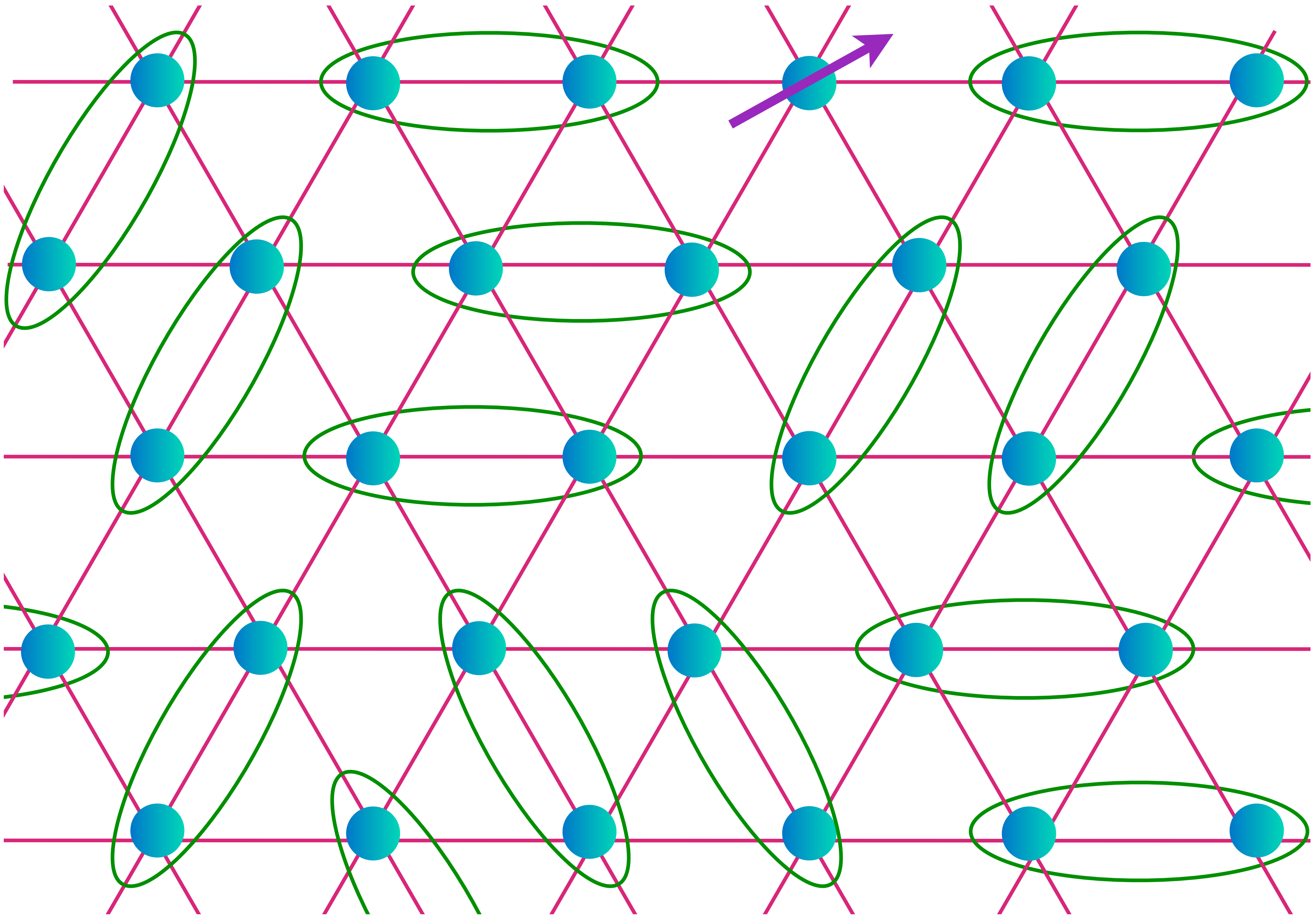
Spinon: S=1/2, charge 0



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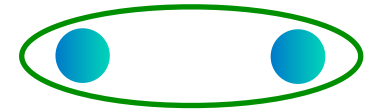

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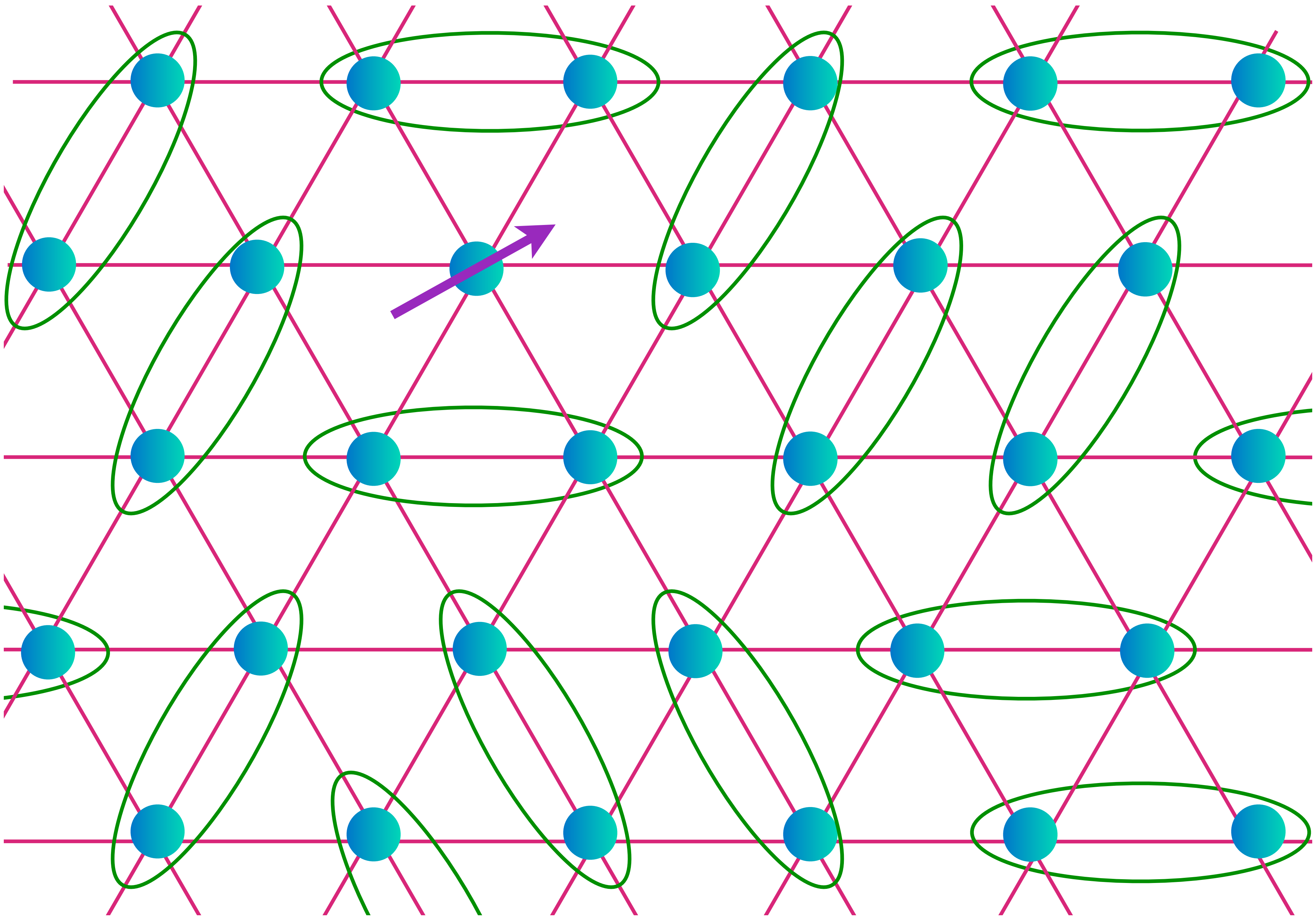

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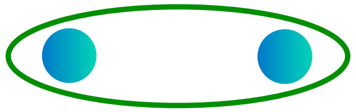

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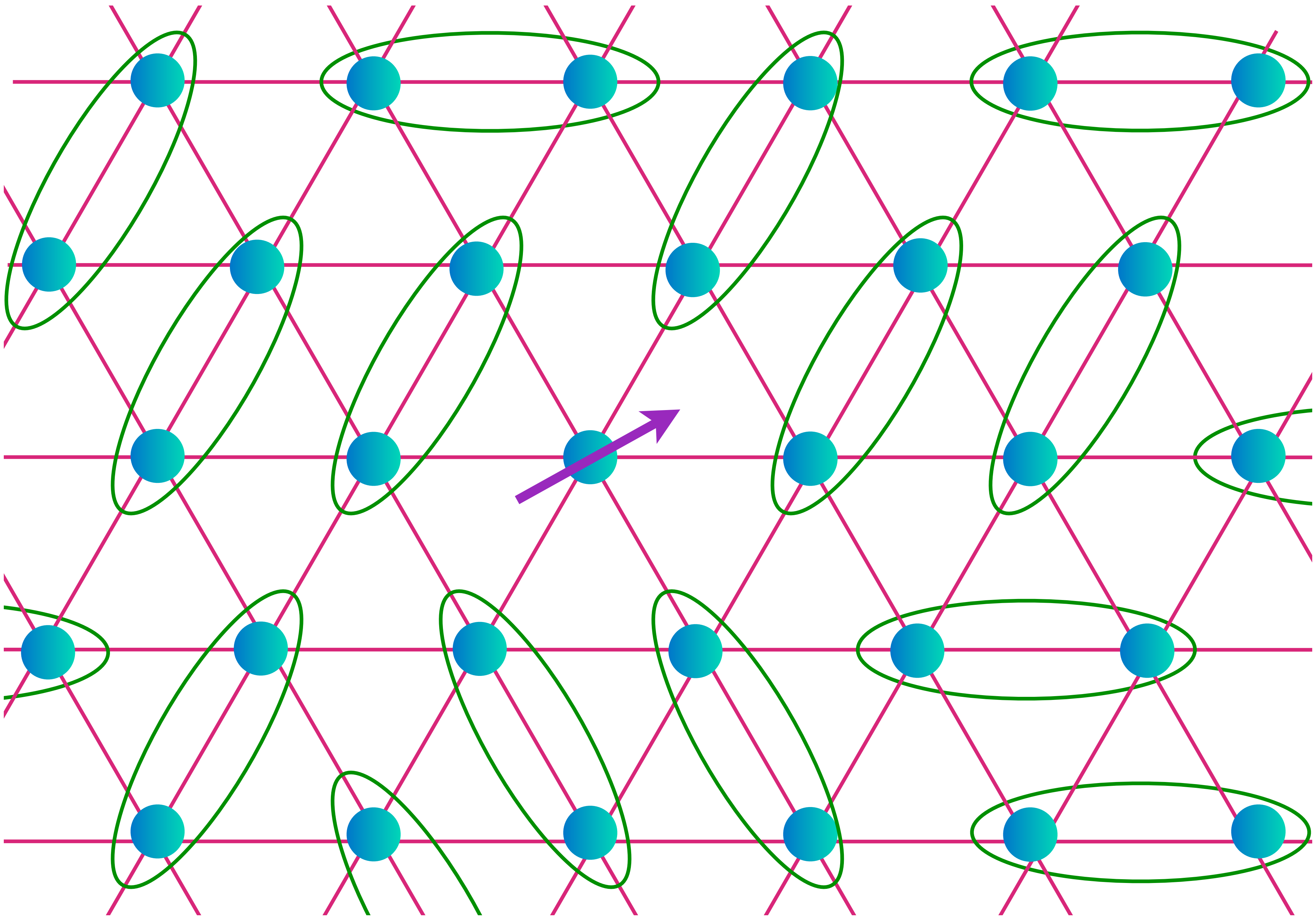

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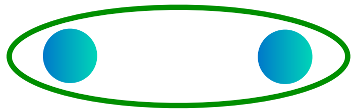

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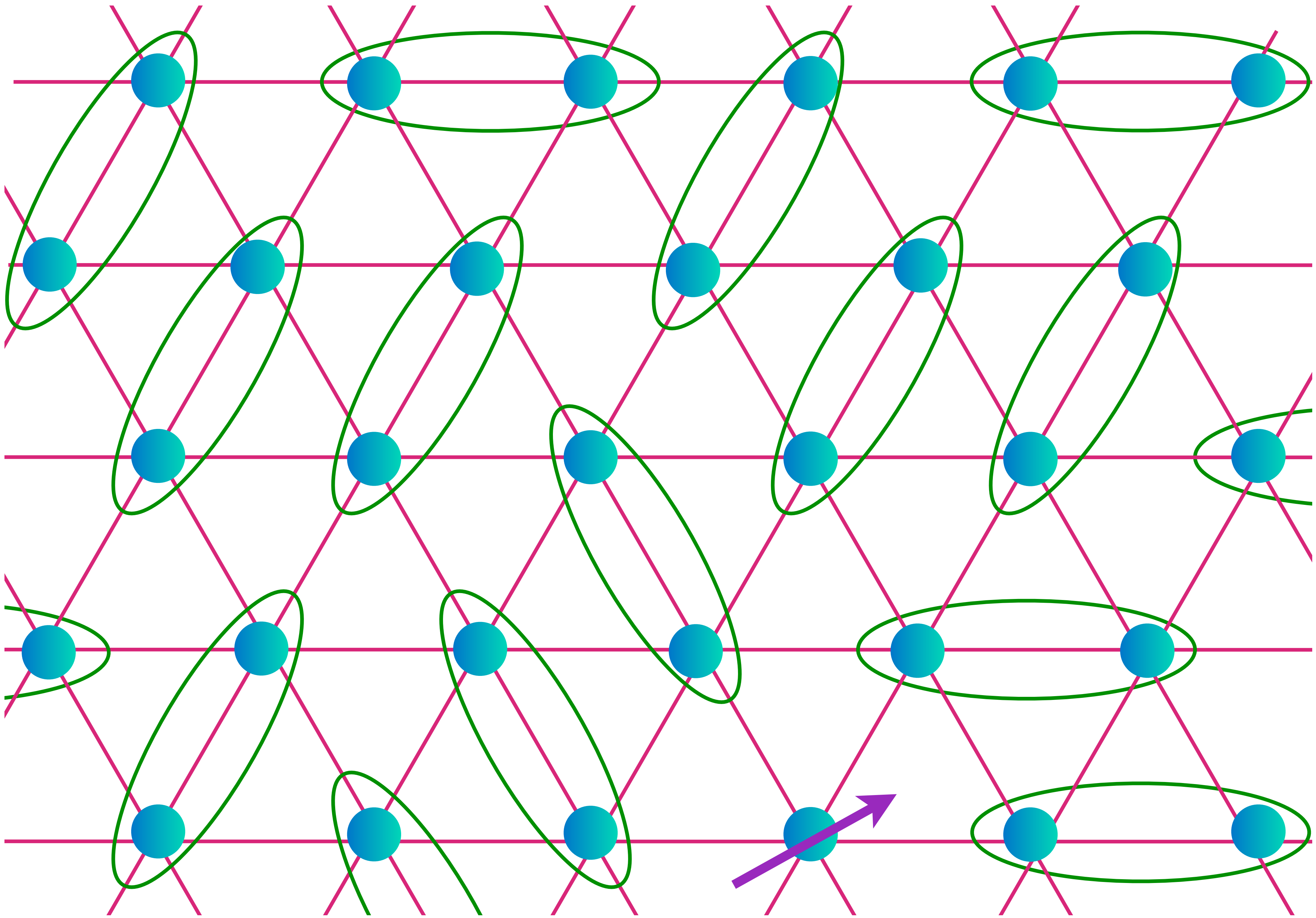

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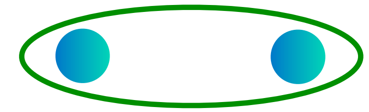
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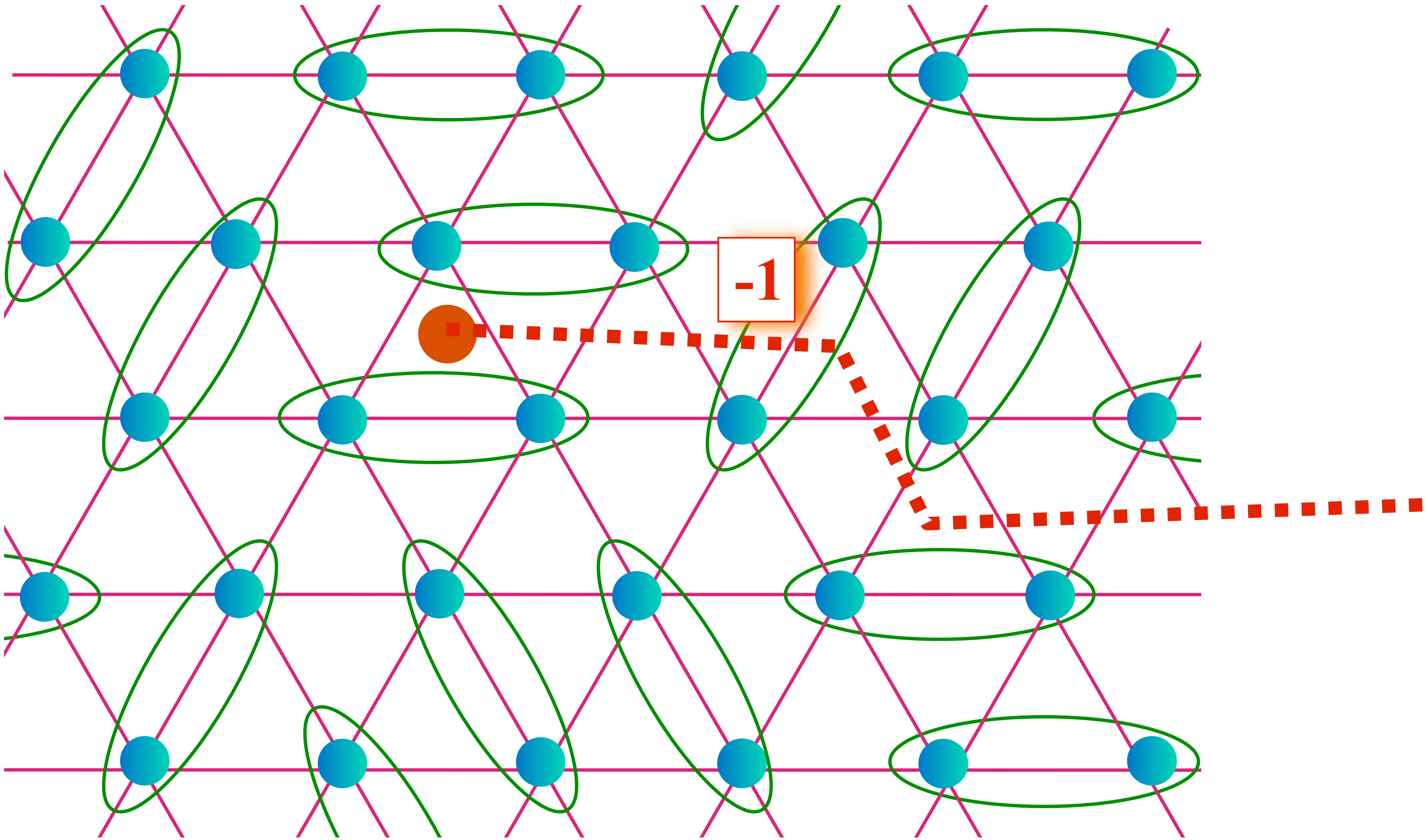
A vison



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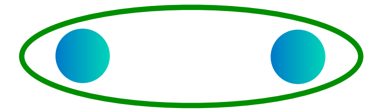

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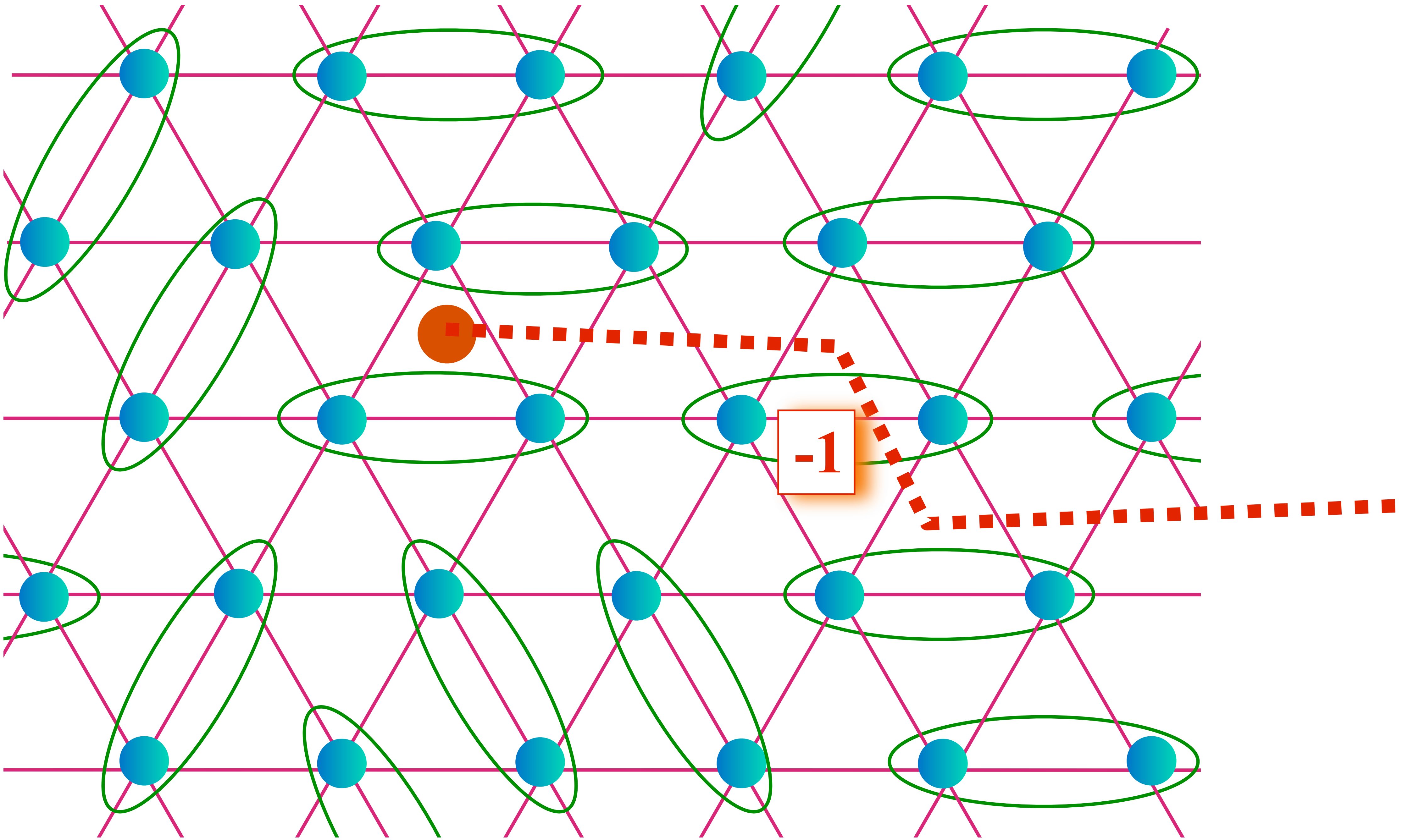

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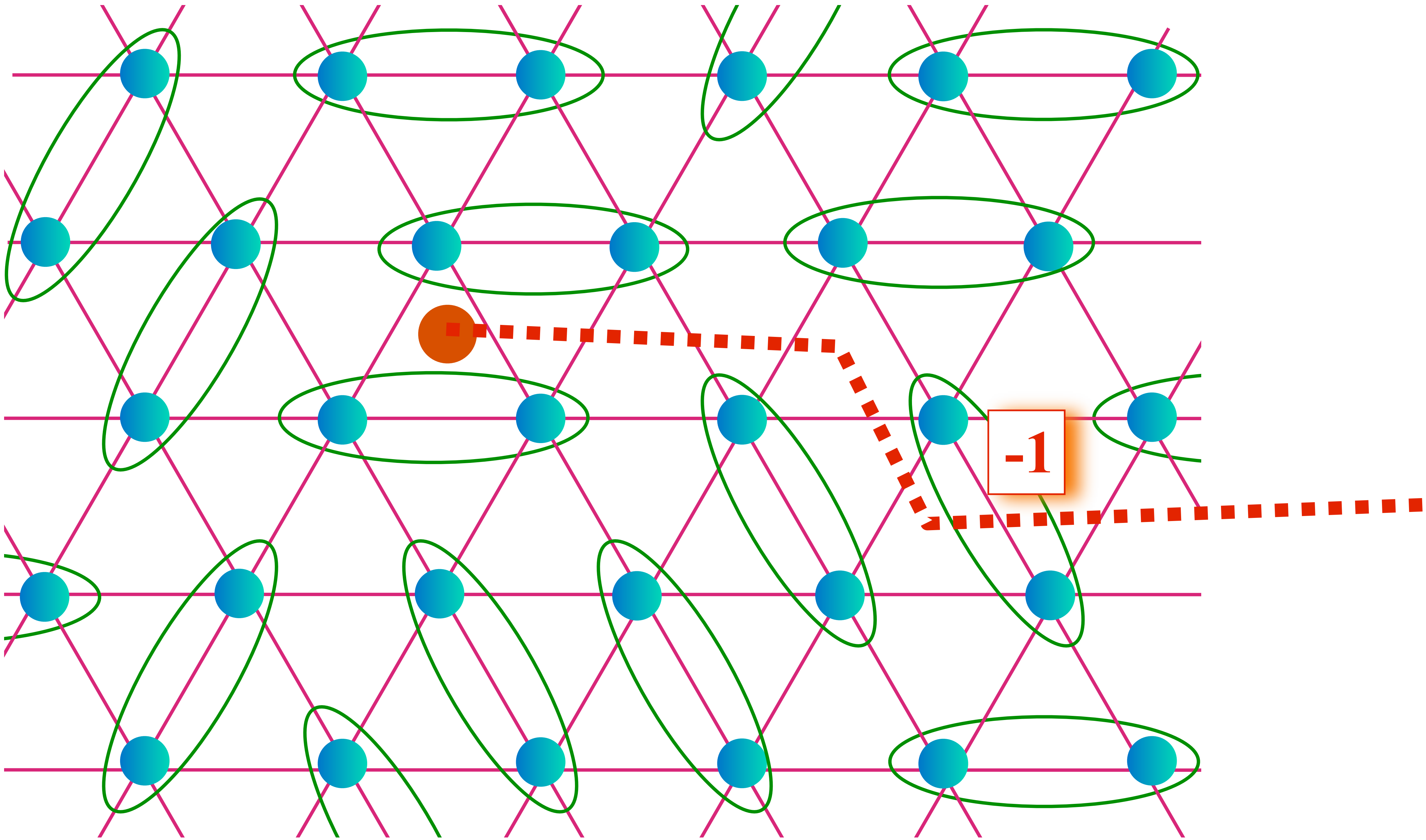

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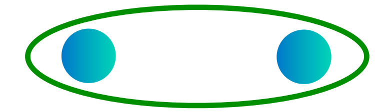

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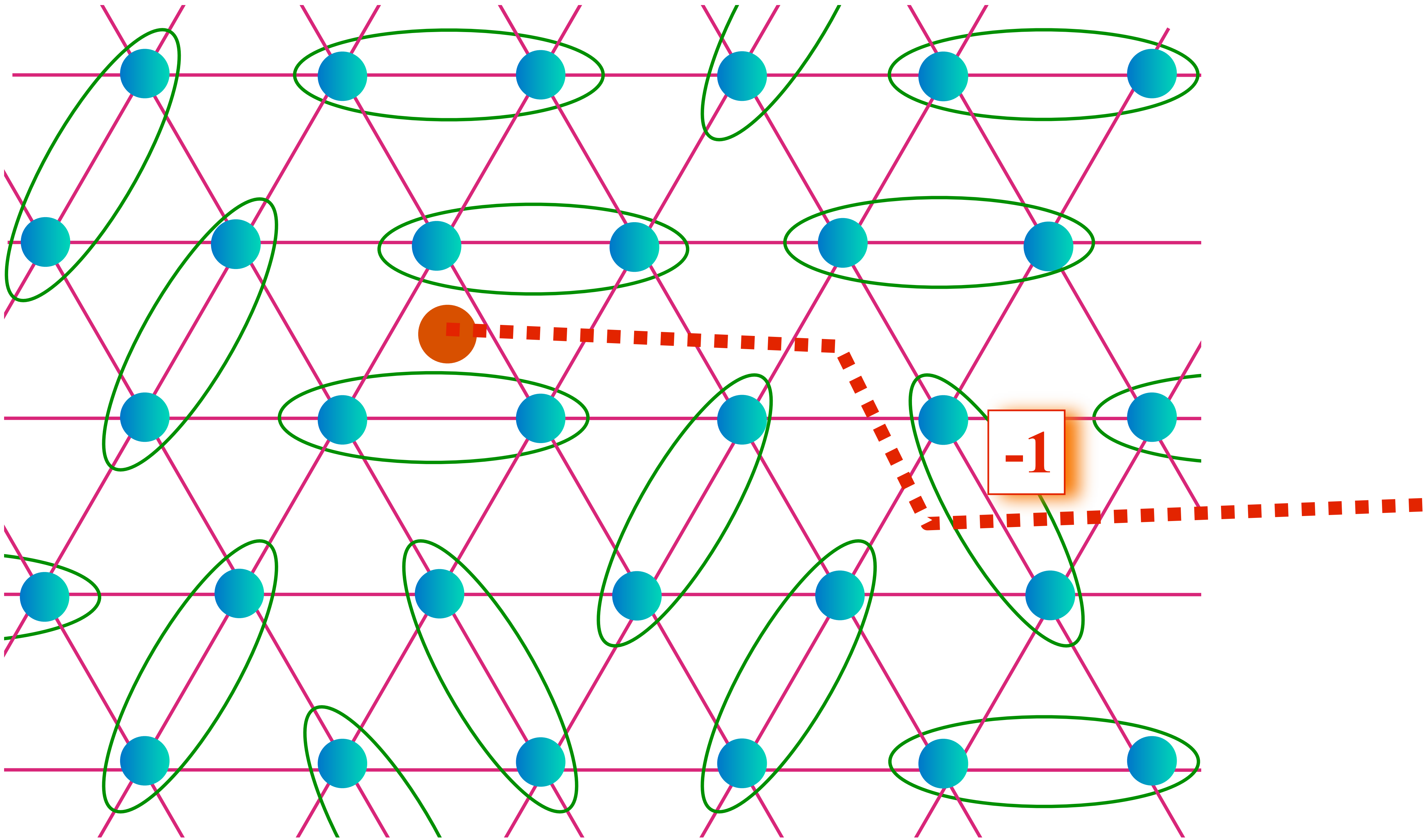

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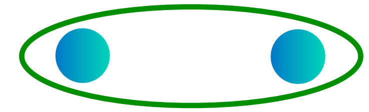

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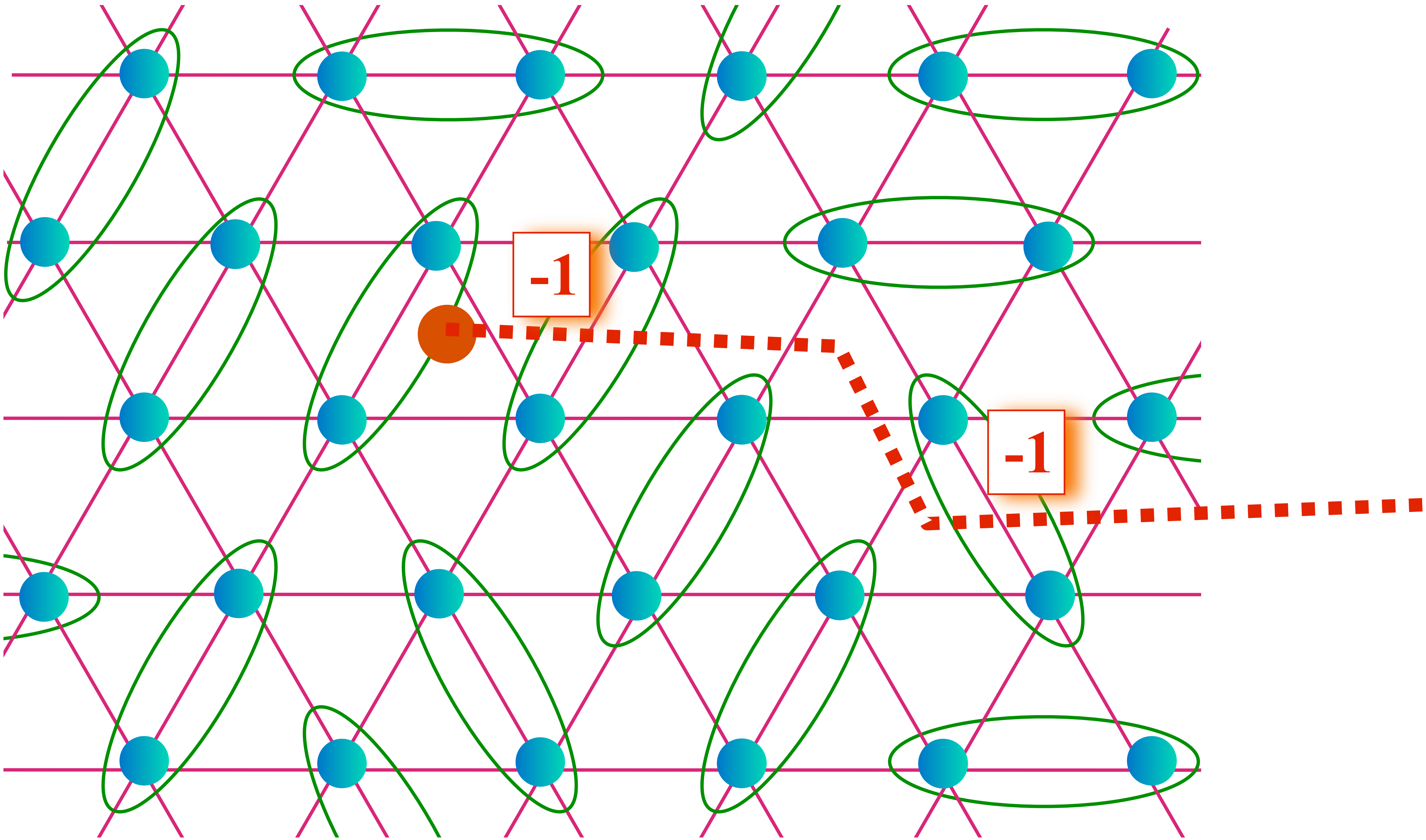

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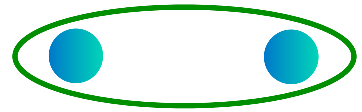

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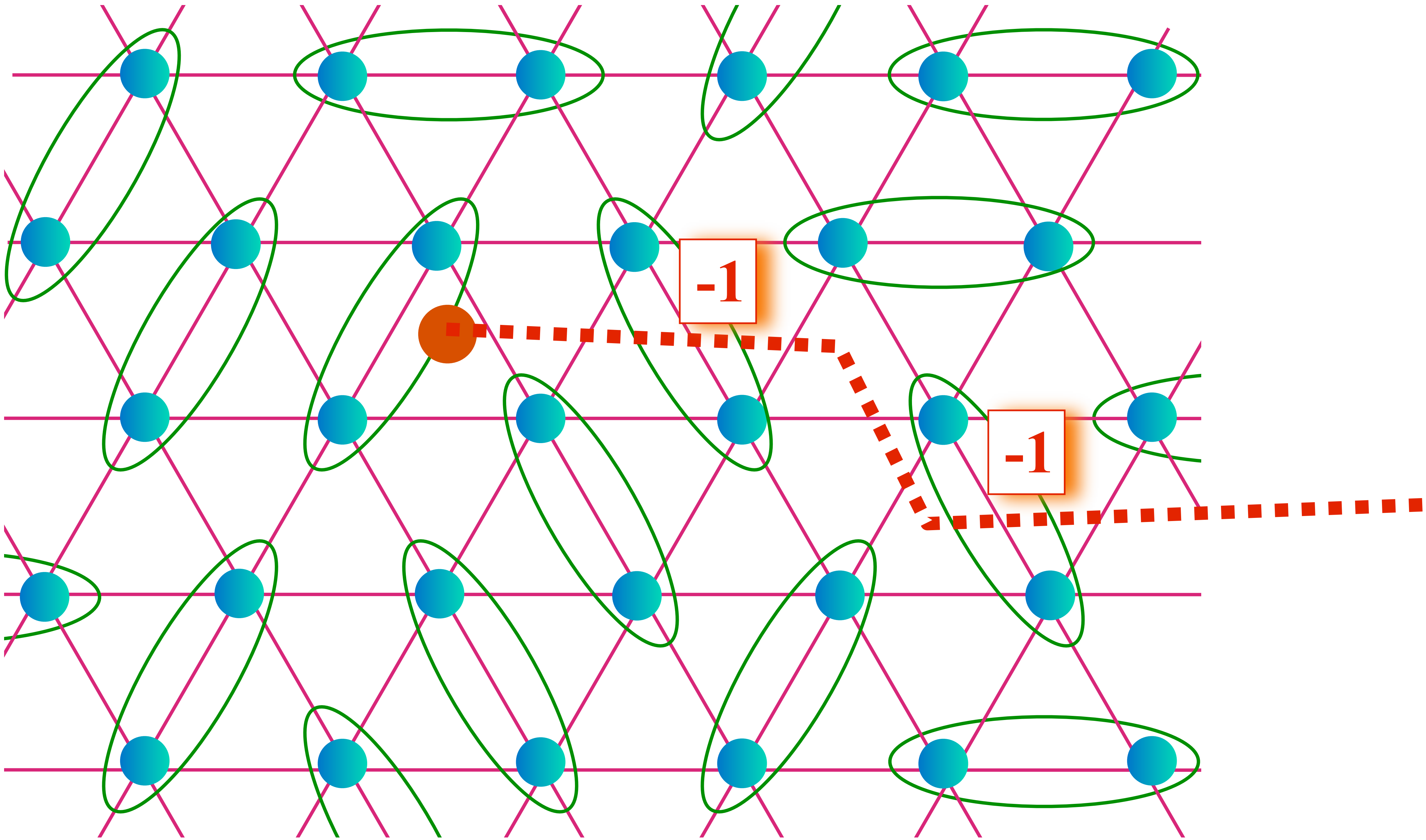

Excitations of the Z_2 Spin liquid

A vison

- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and B. Chakraborty, *Phys. Rev. B* **40,** 7133 (1989)
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001)

Topological order in the Z_2 spin liquid ground state

4-fold degeneracy on the torus

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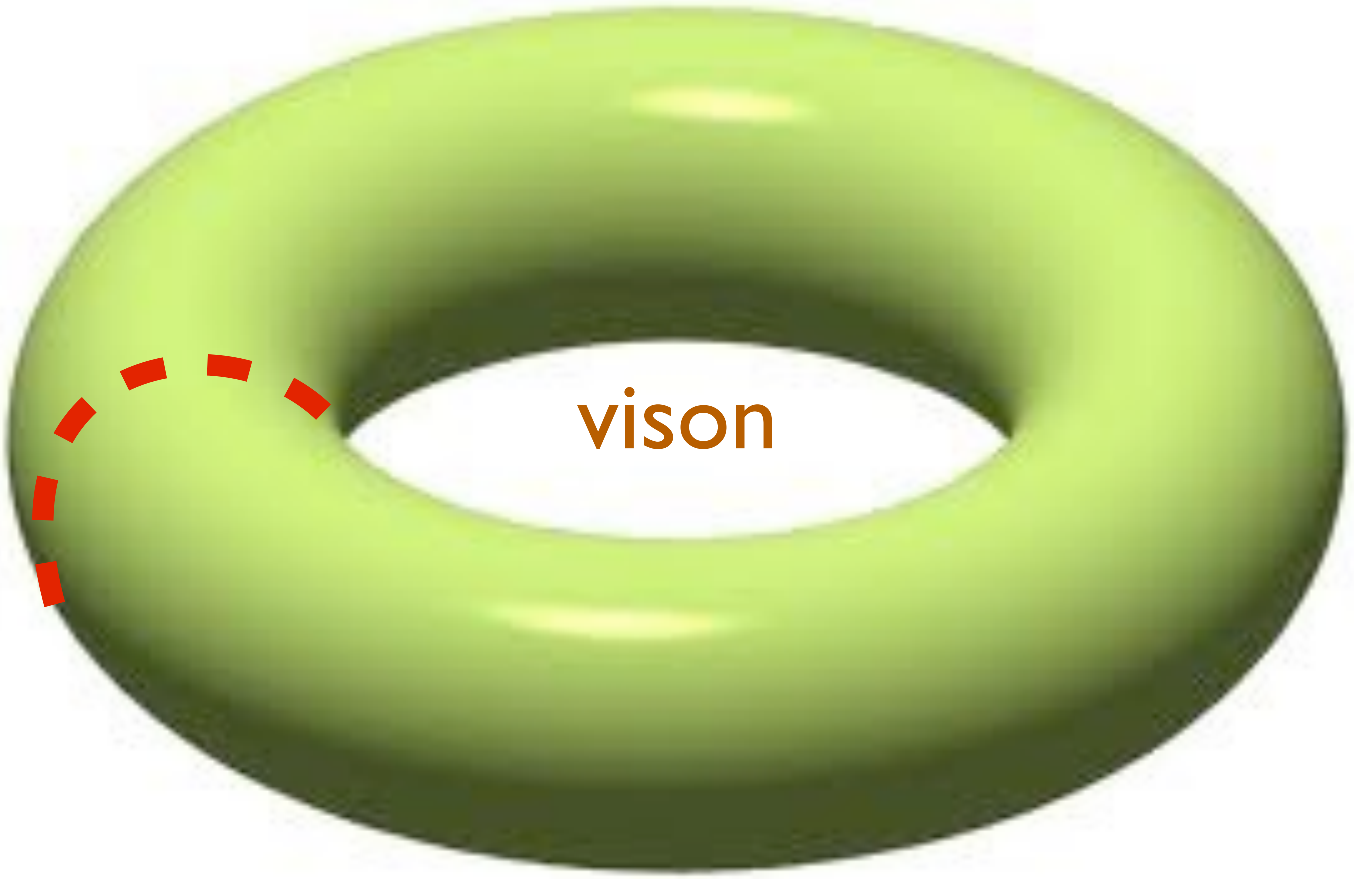

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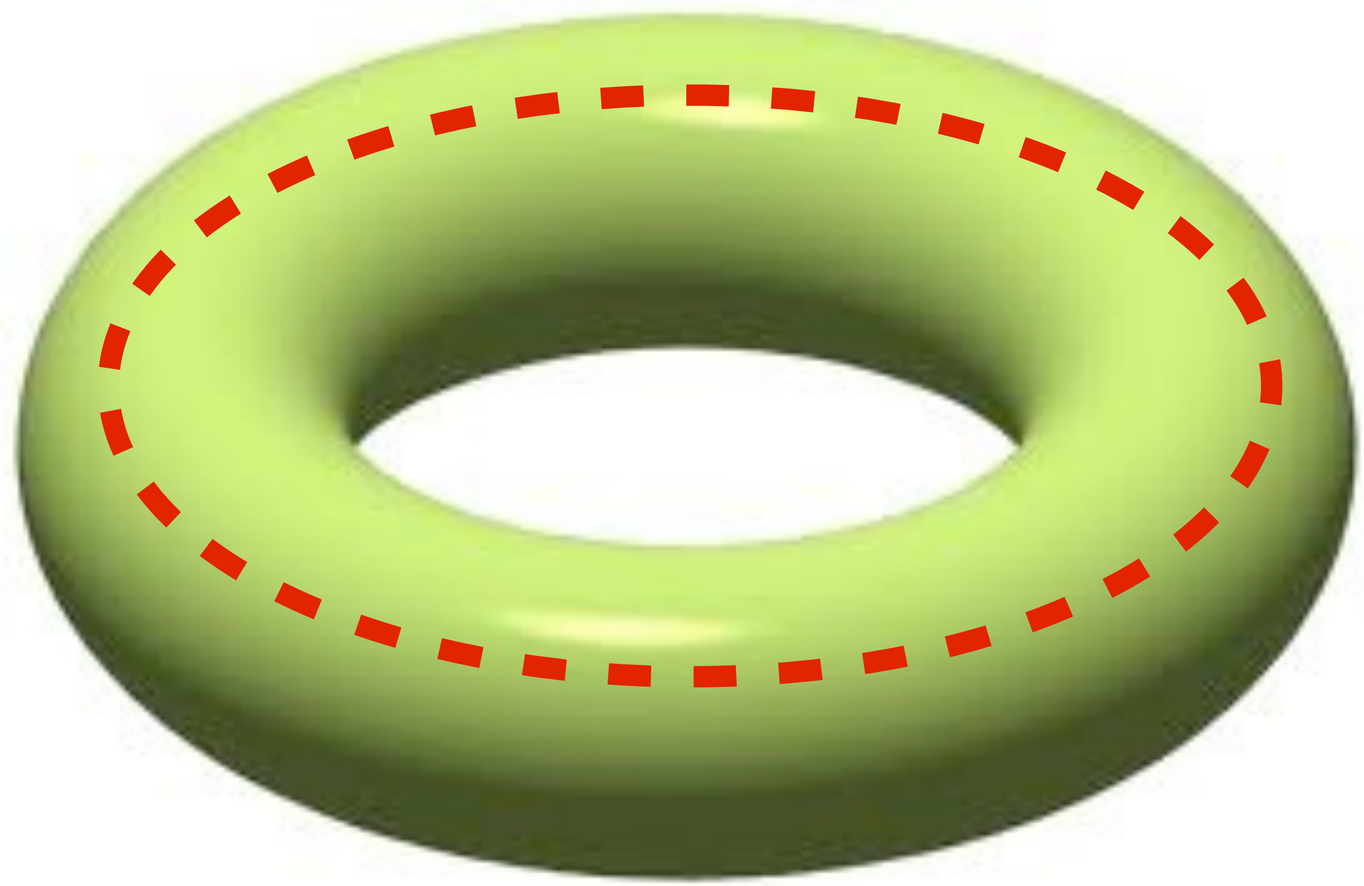

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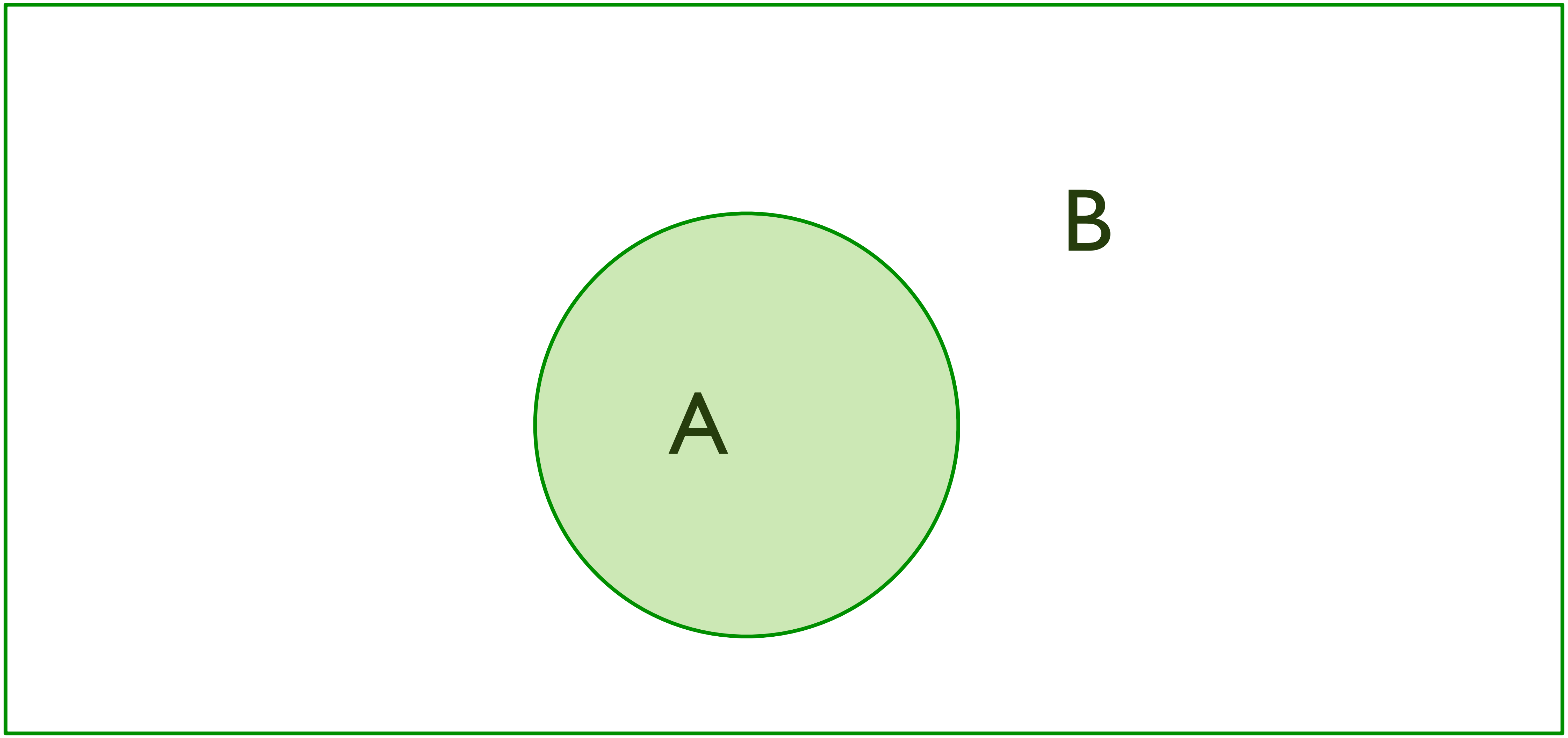
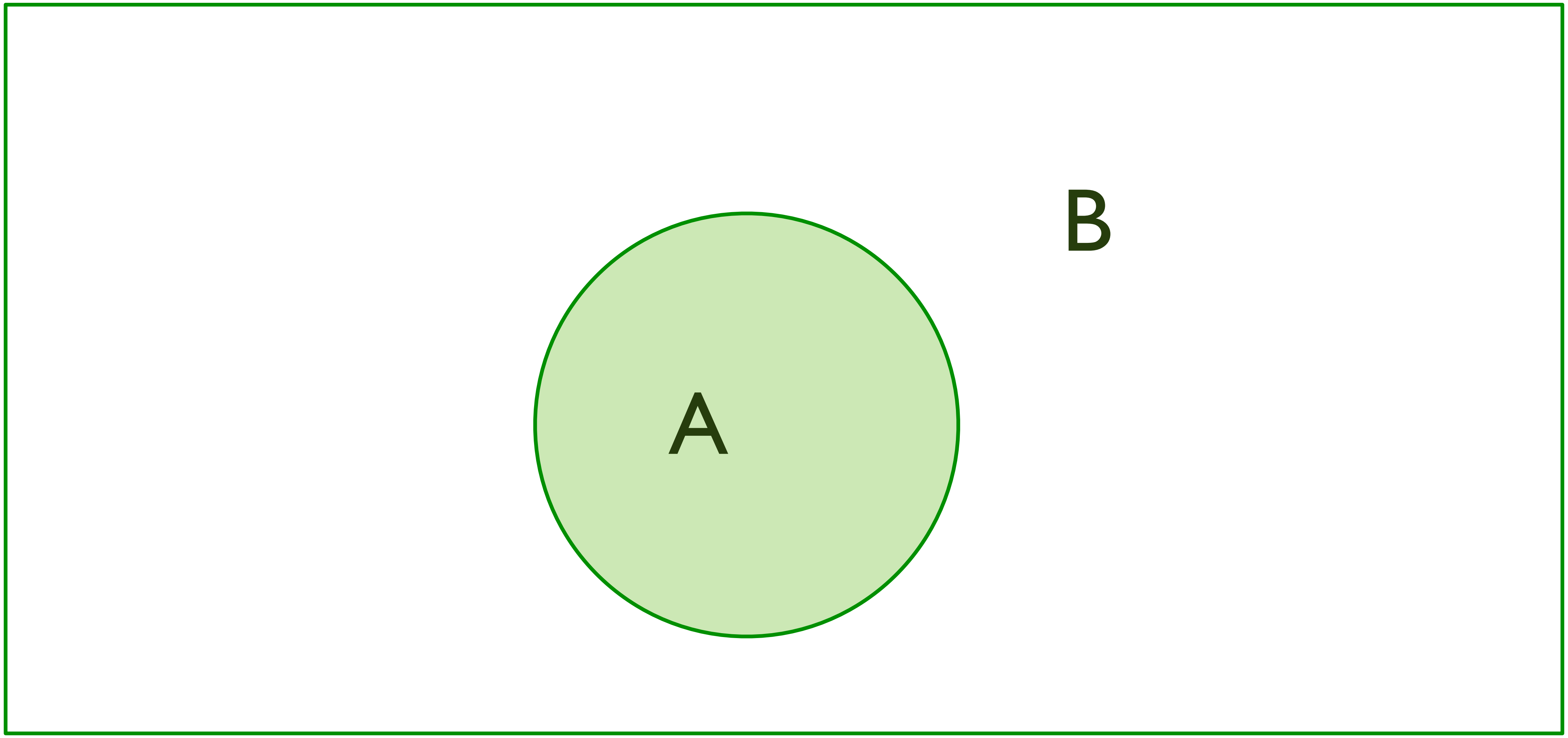


$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Topological order in the Z_2 spin liquid ground state



Entanglement entropy of a Z_2 spin liquid:

$$S_{EE} = aL - \ln(2)$$

where L is the perimeter of the boundary between A and B.
The $\ln(2)$ is a universal characteristic of the Z_2 spin liquid,
and implies *long-range* quantum entanglement.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

Topological order in the Z_2 spin liquid ground state

These properties of the ground state can be described by effective theories:

- deconfined phase of a Z_2 gauge theory

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001)

- topological doubled Chern-Simons gauge theory

J. Maldacena, G. Moore, and N. Seiberg, JHEP 0110:005 (2001).
M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, *Annals of Physics* **310**, 428 (2004).

Quantum Hall states

Similar topological properties,
but no time-reversal symmetry:

- ground state degeneracy on a torus
- universal entanglement entropy
- gapless edge states on spaces with boundaries (can also happen for some spin liquids)
- topological Chern-Simons gauge theories

Haldane, Wen

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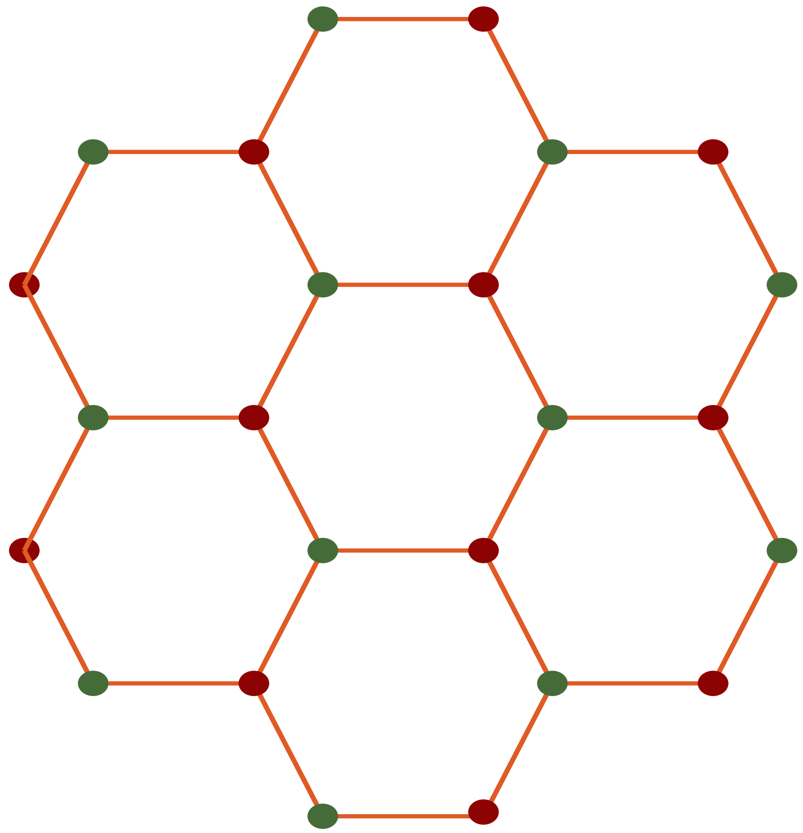
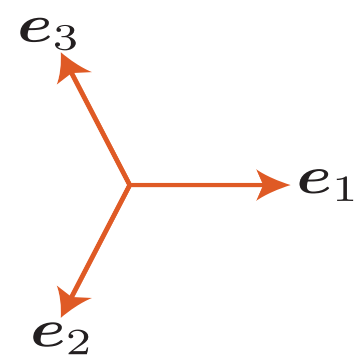
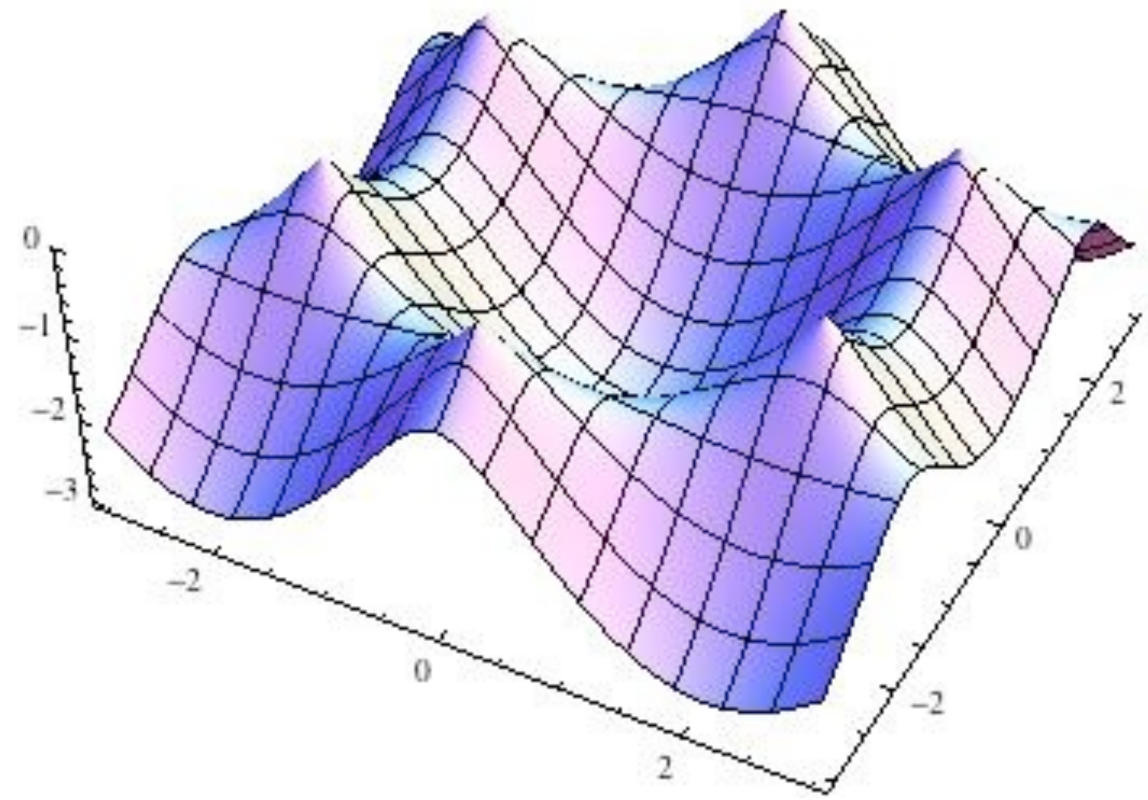
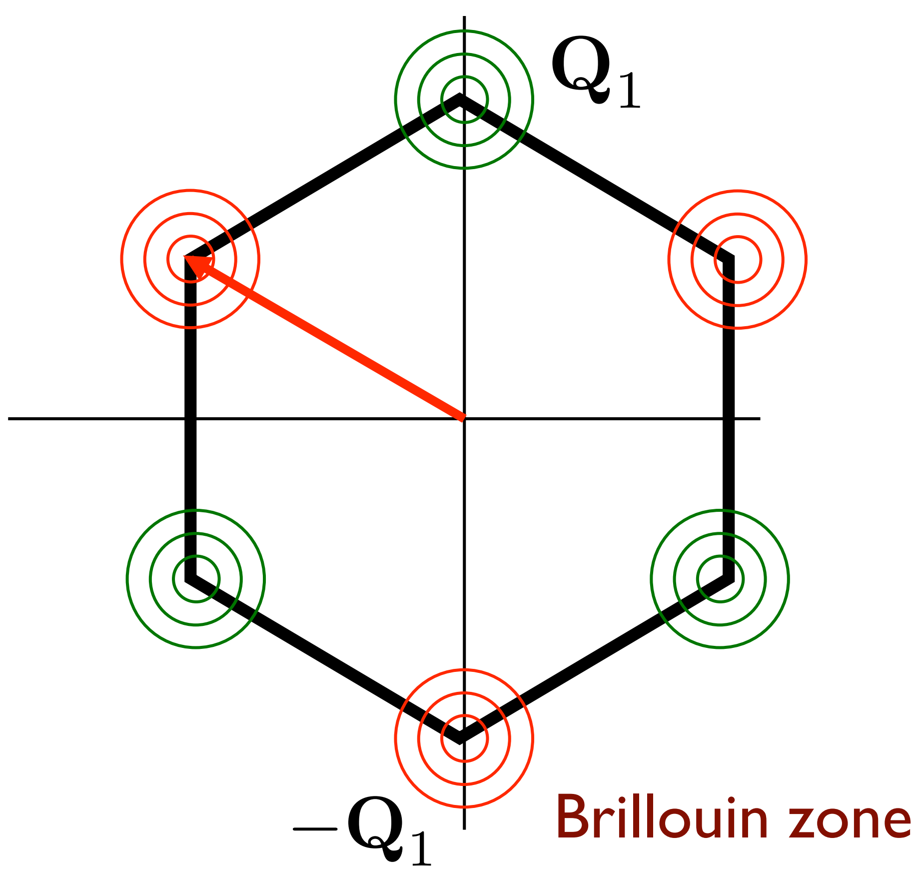
Graphene

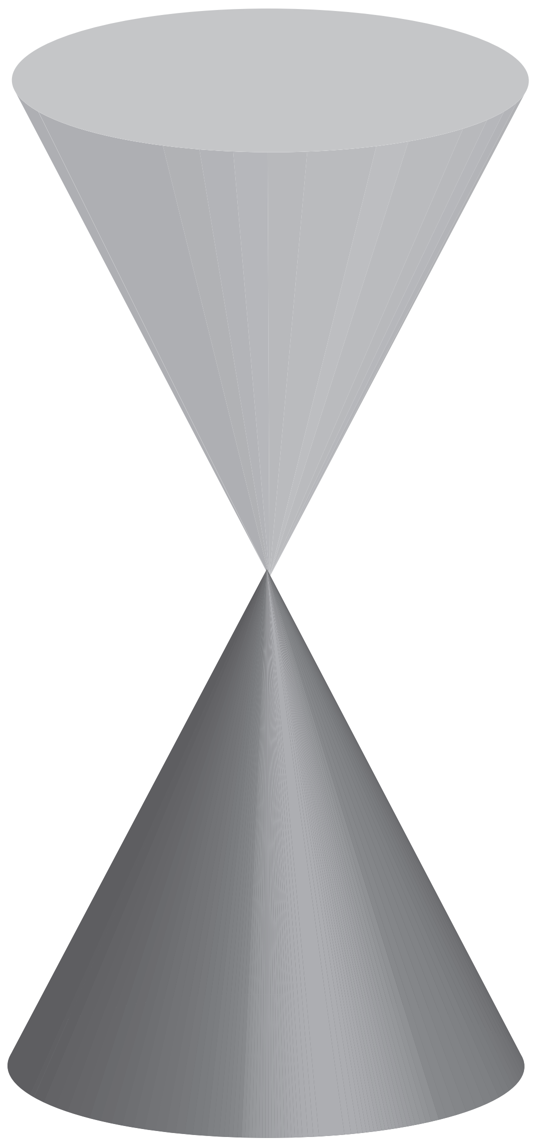






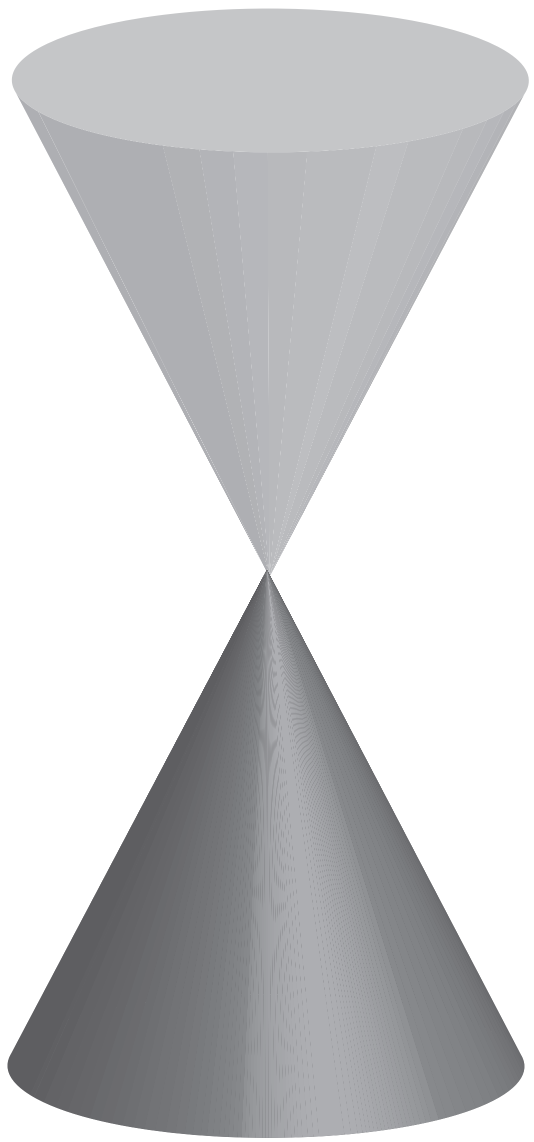


Semi-metal with
massless Dirac fermions



Graphene

Dassarma

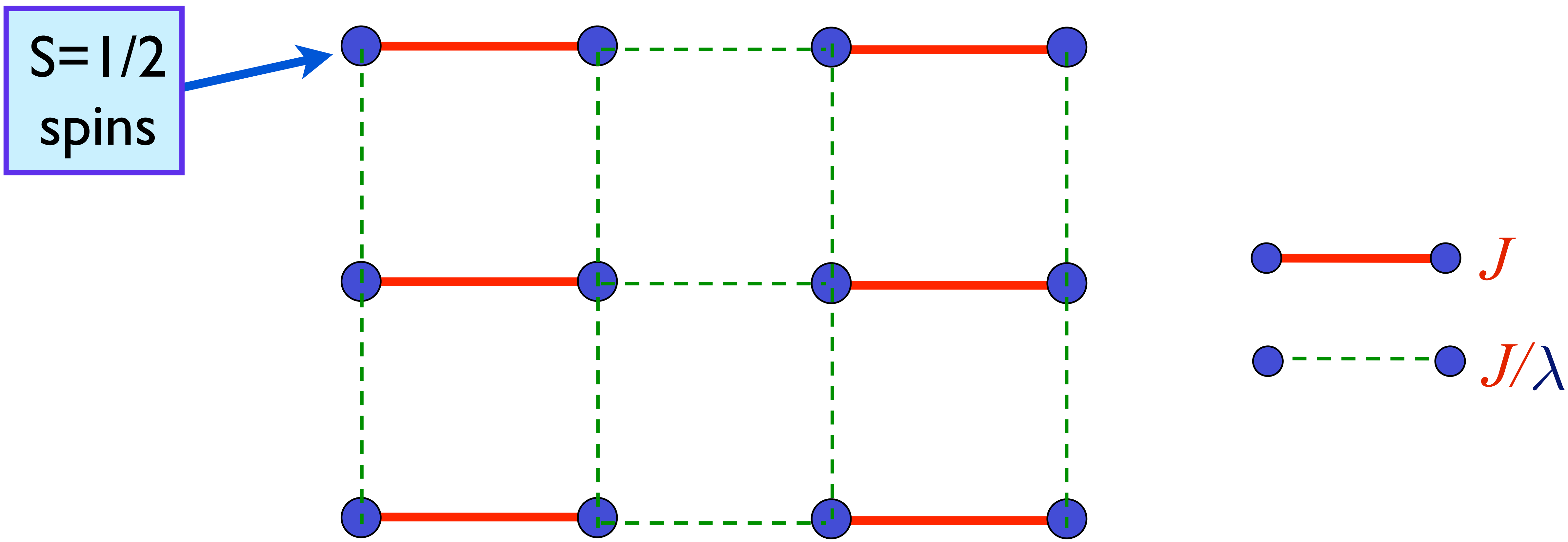


Surfaces of 3d topological insulators

Balents

Square lattice antiferromagnet

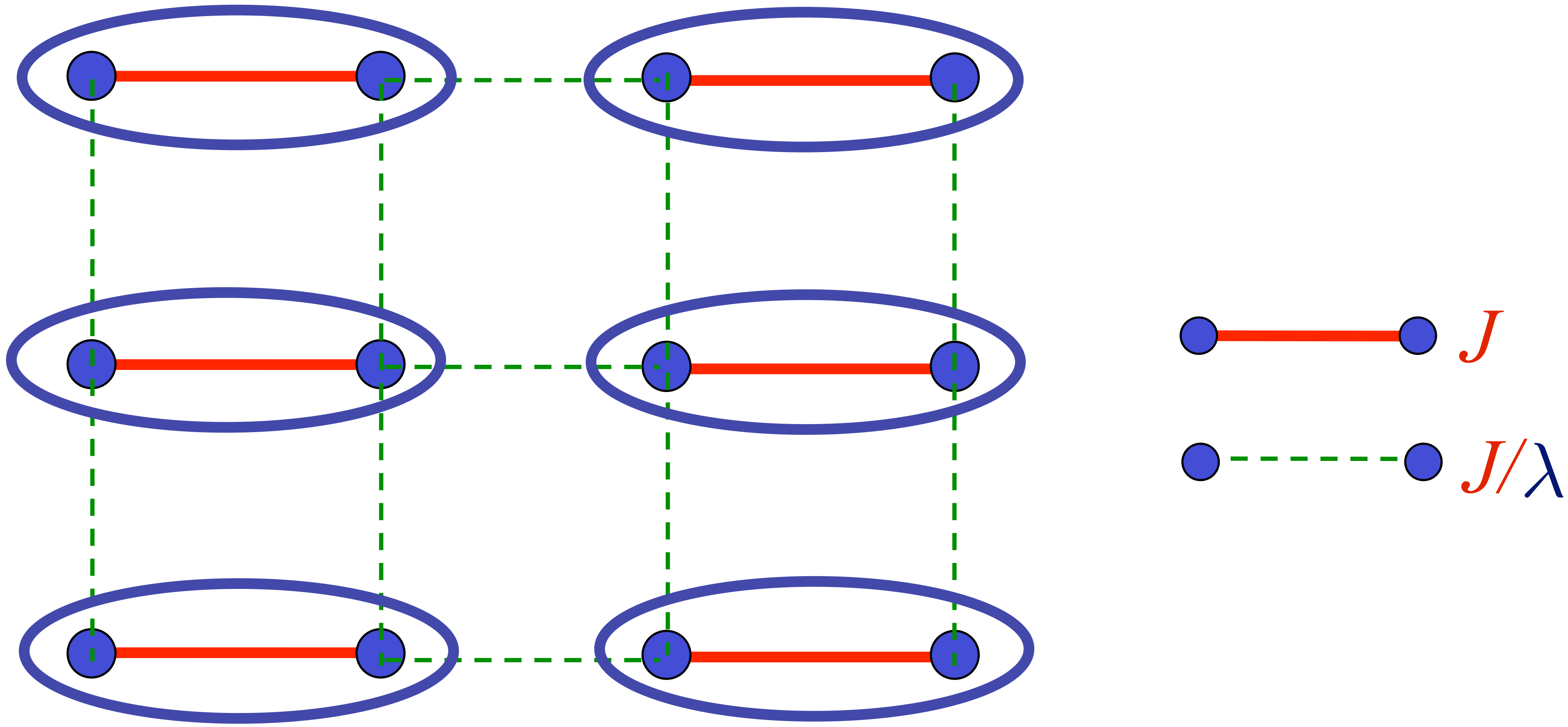
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

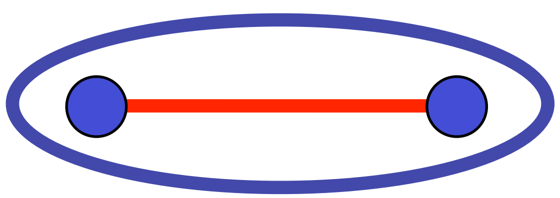


Examine ground state as a function of λ

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

Square lattice antiferromagnet

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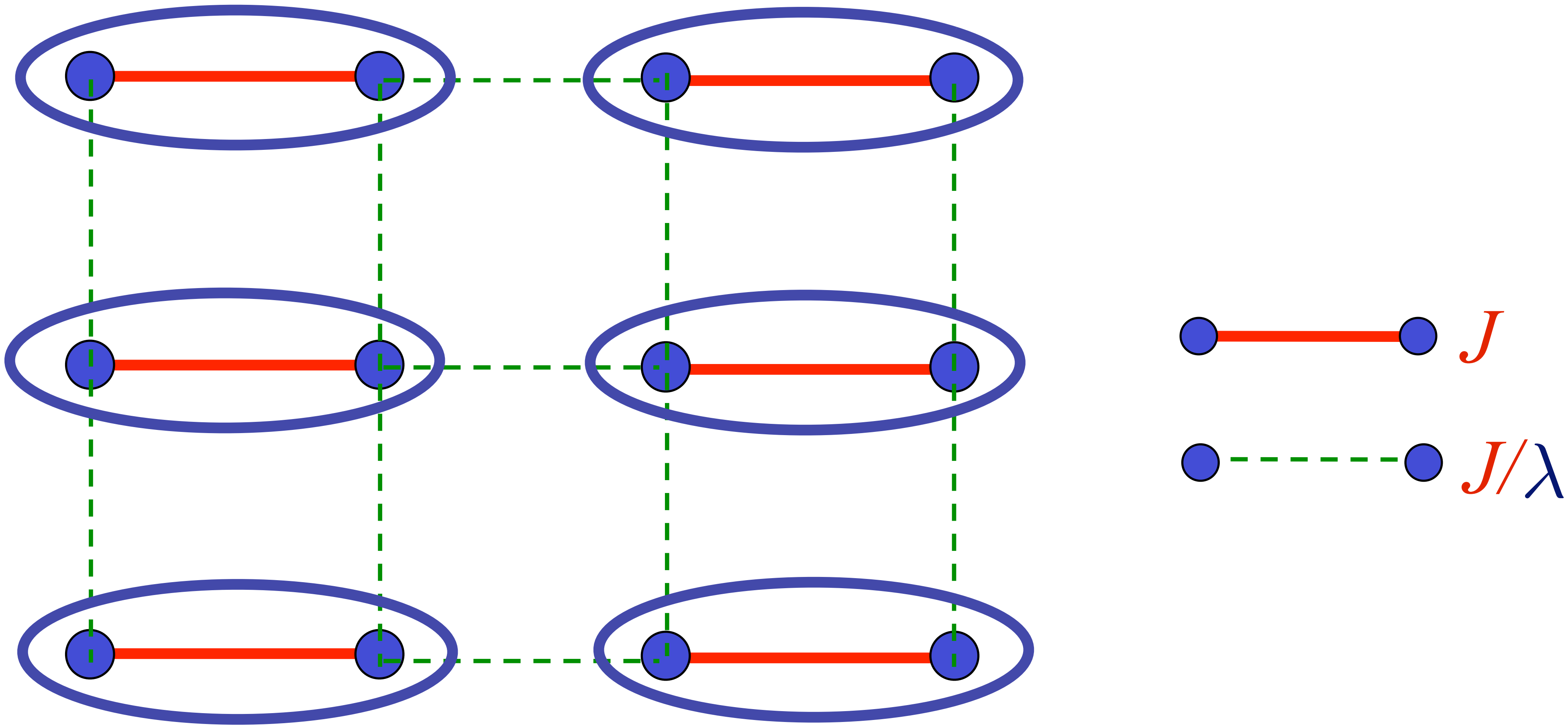

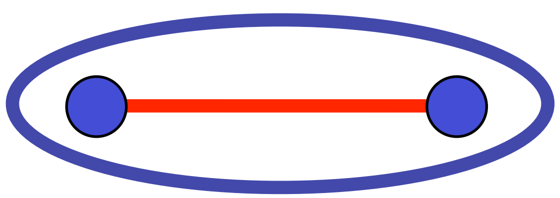 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$

Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Square lattice antiferromagnet

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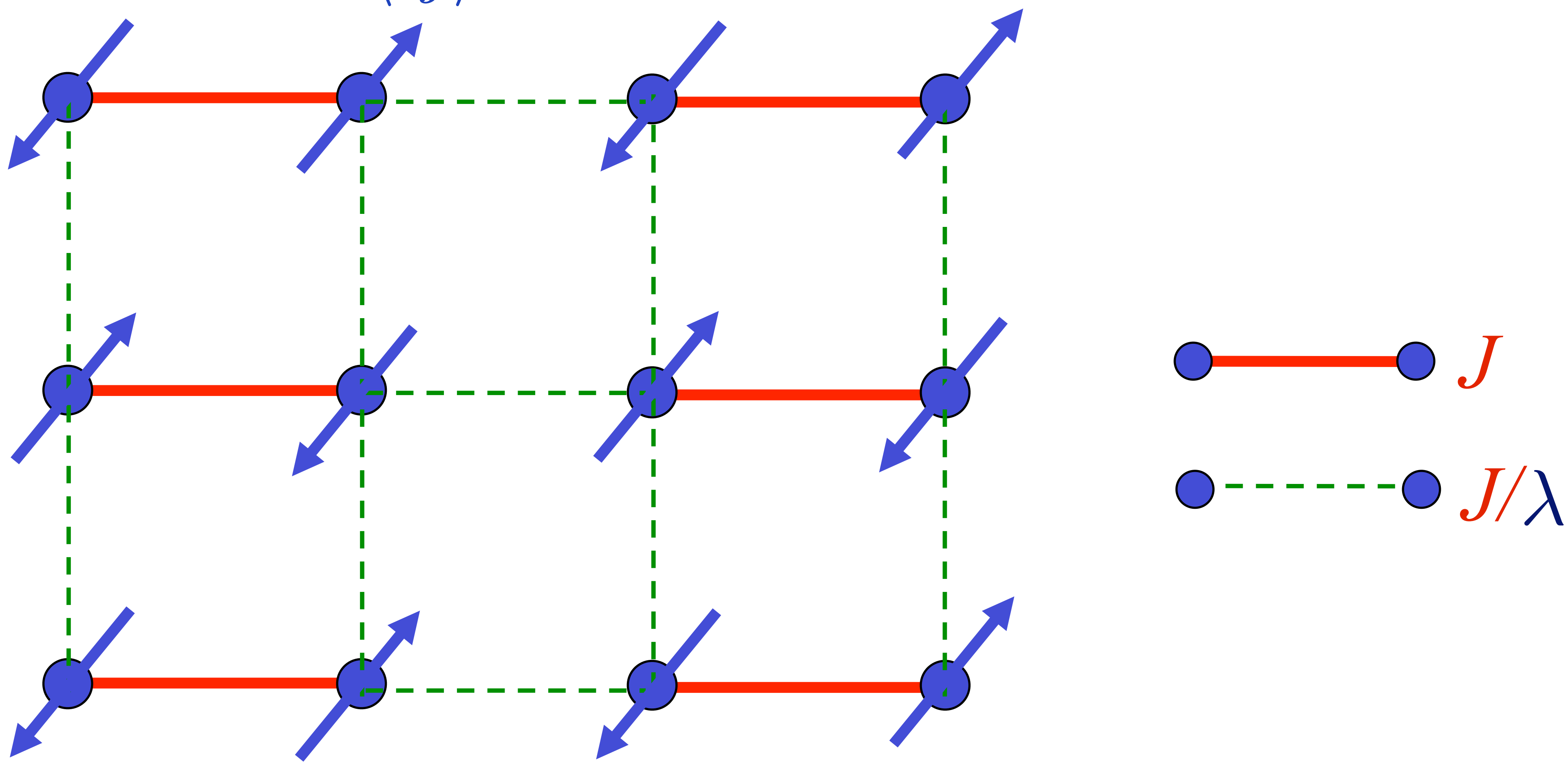
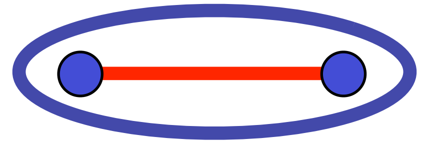


For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern



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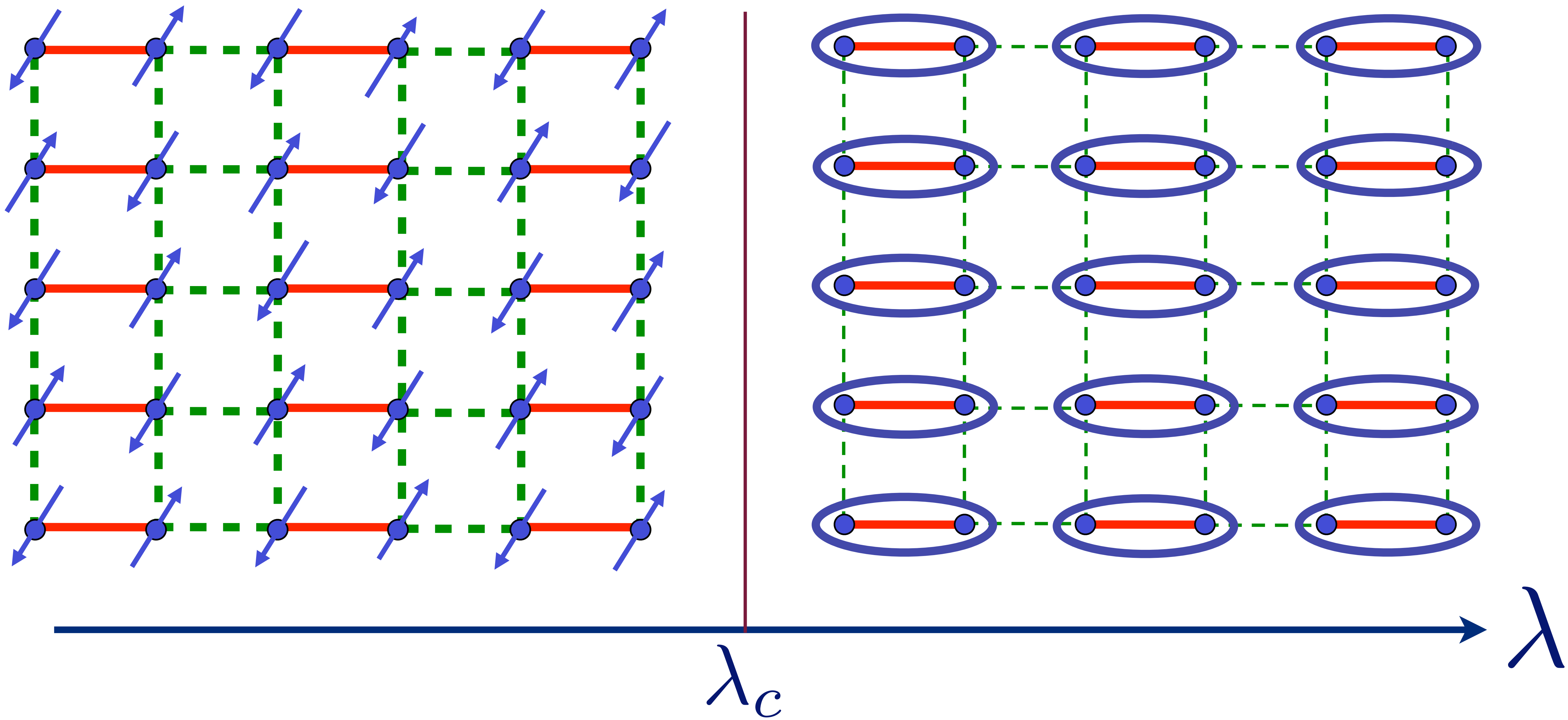
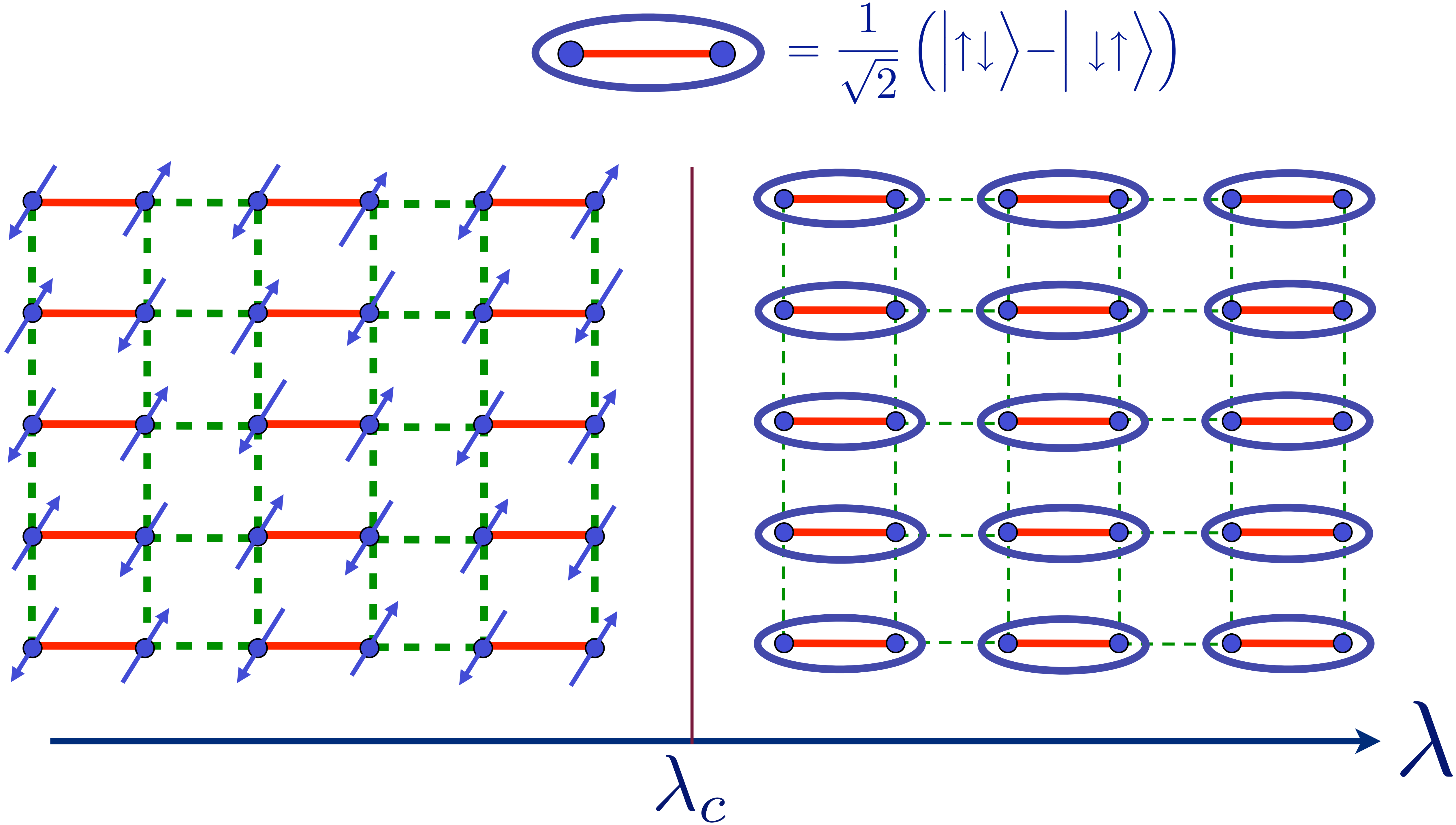
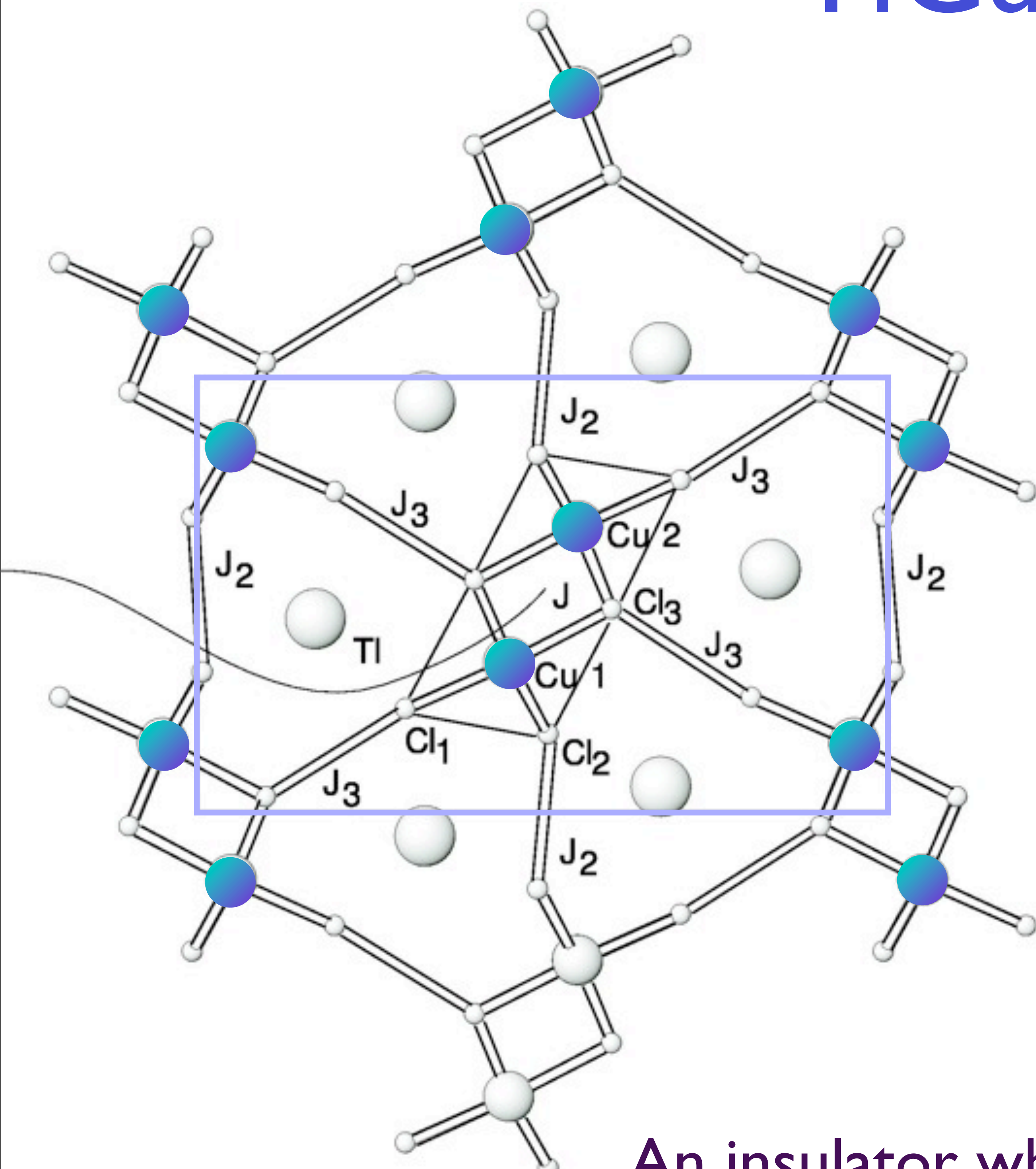
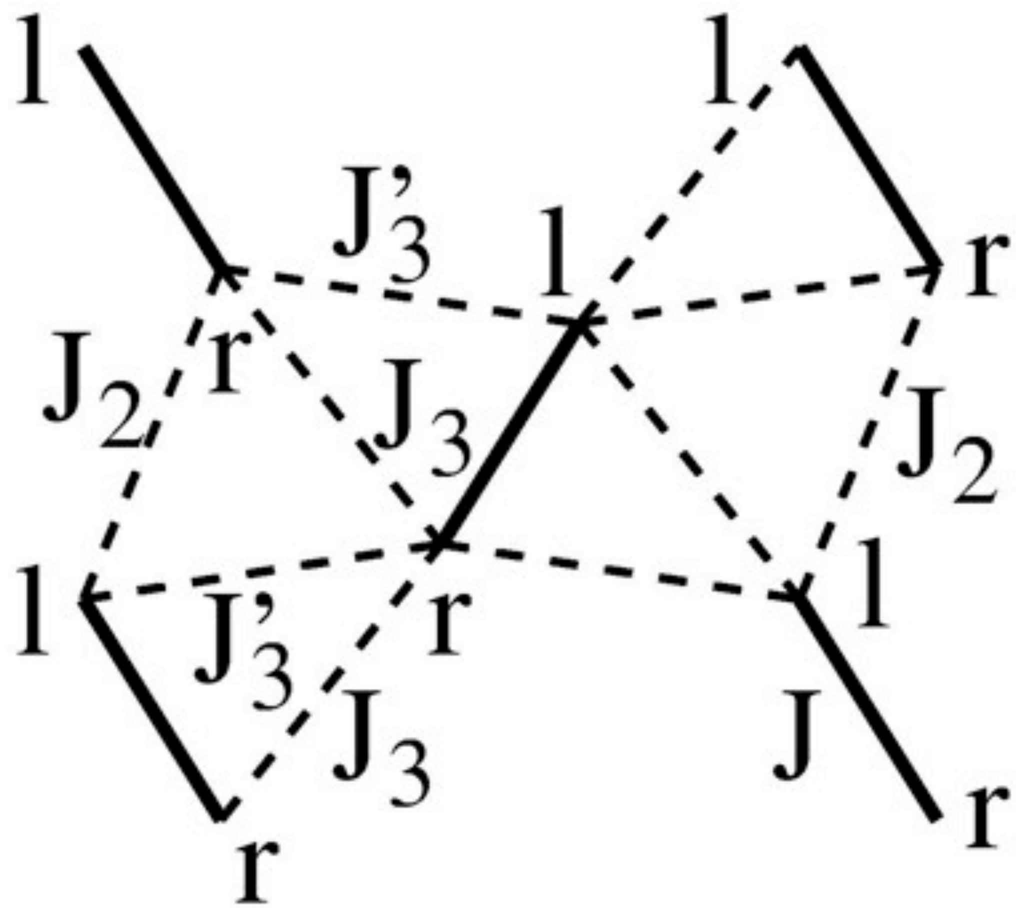




A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

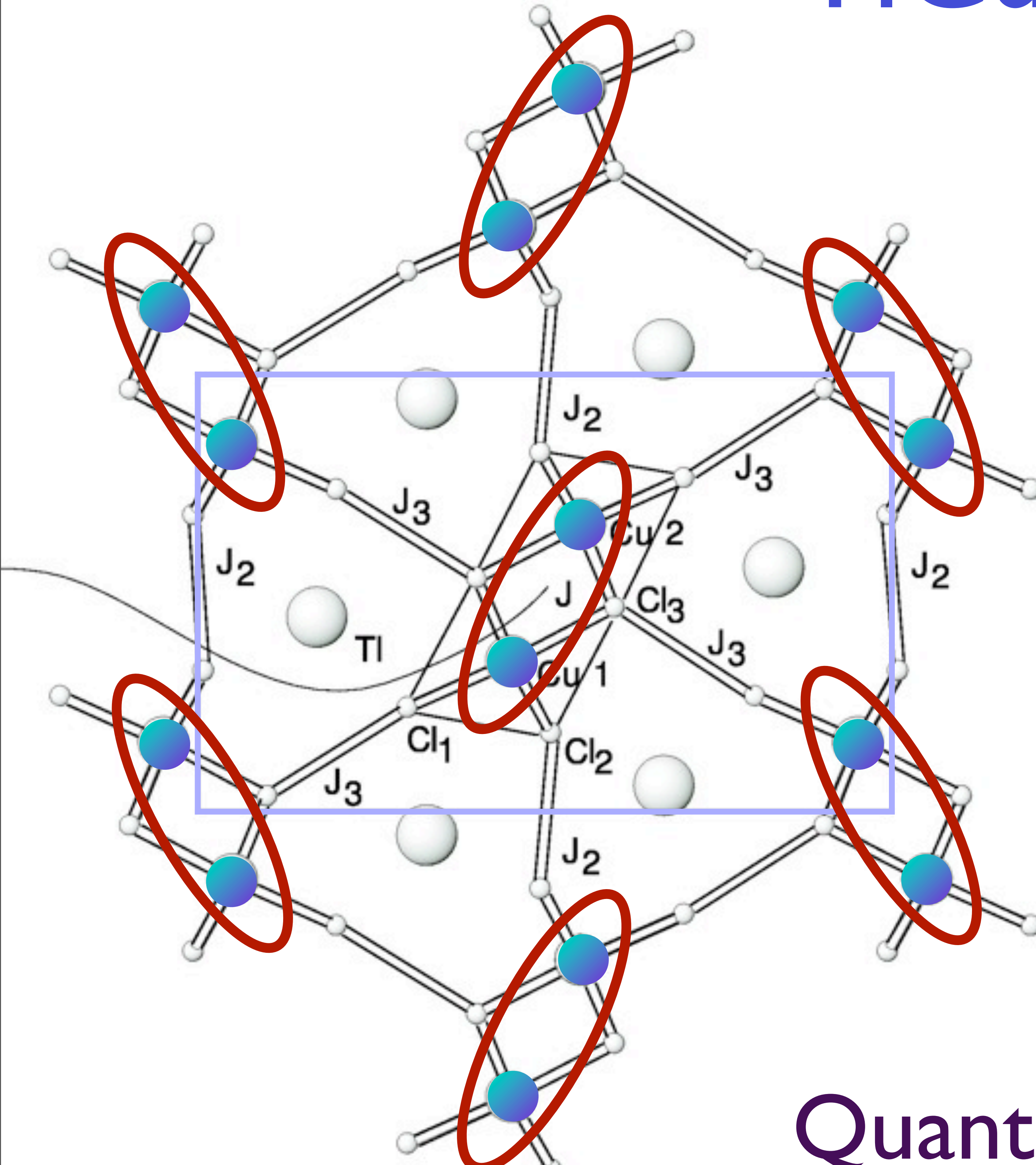
$TlCuCl_3$

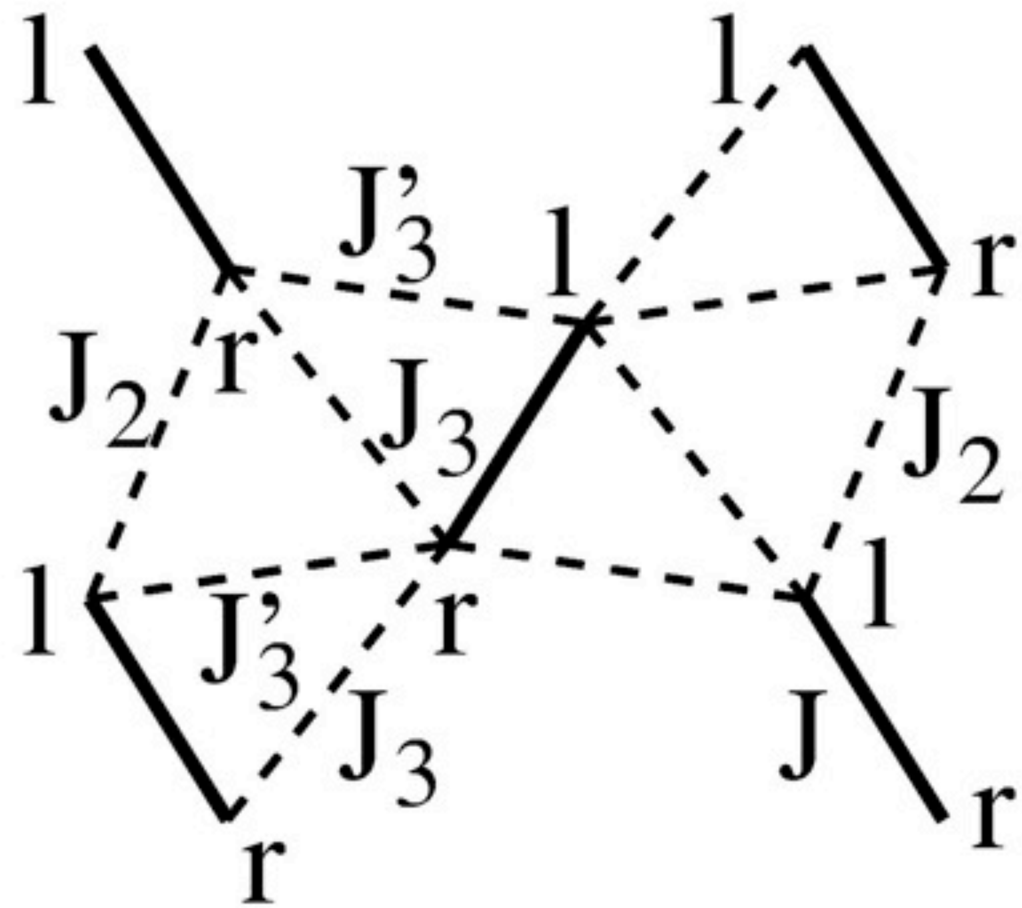




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

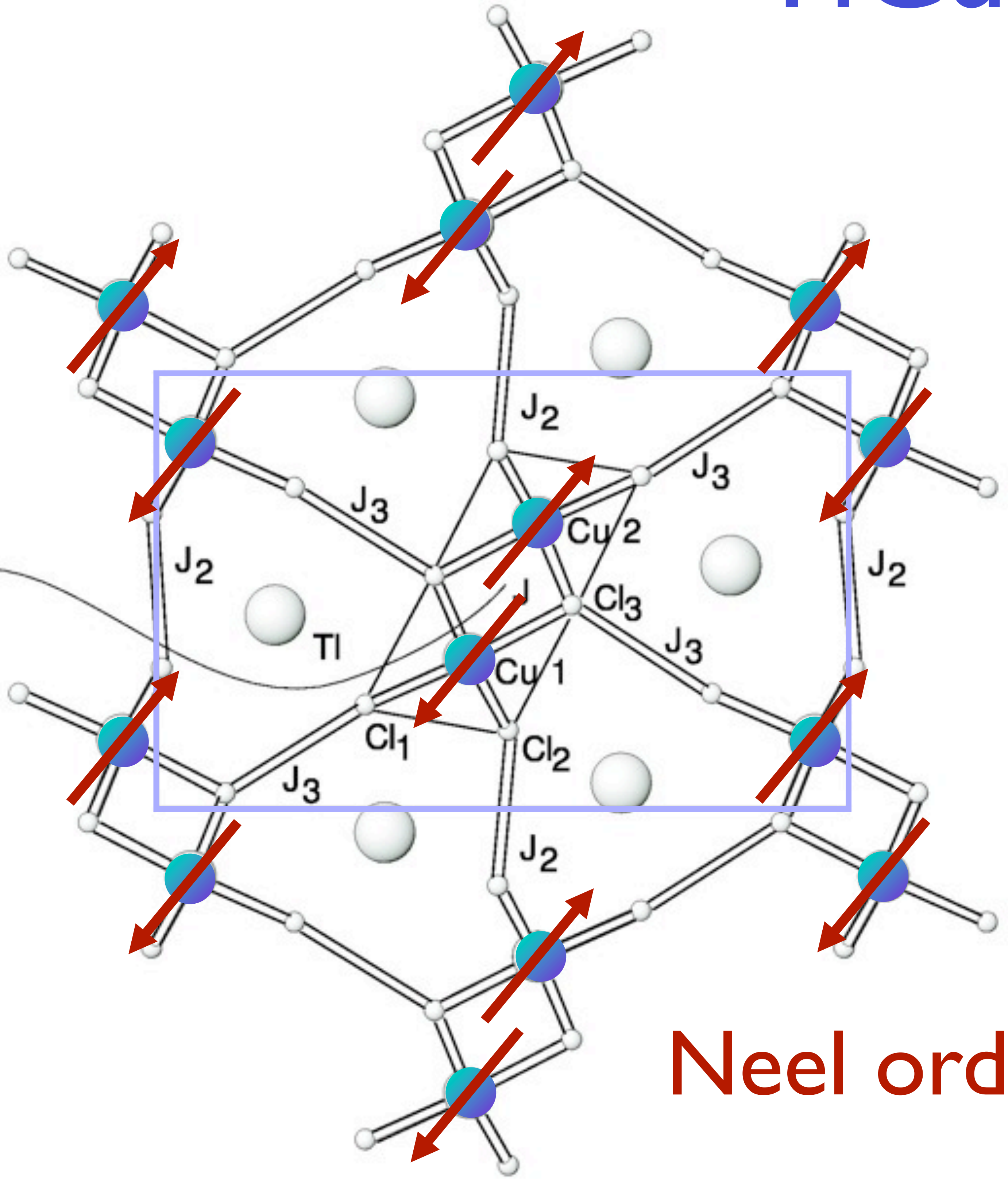
$TlCuCl_3$

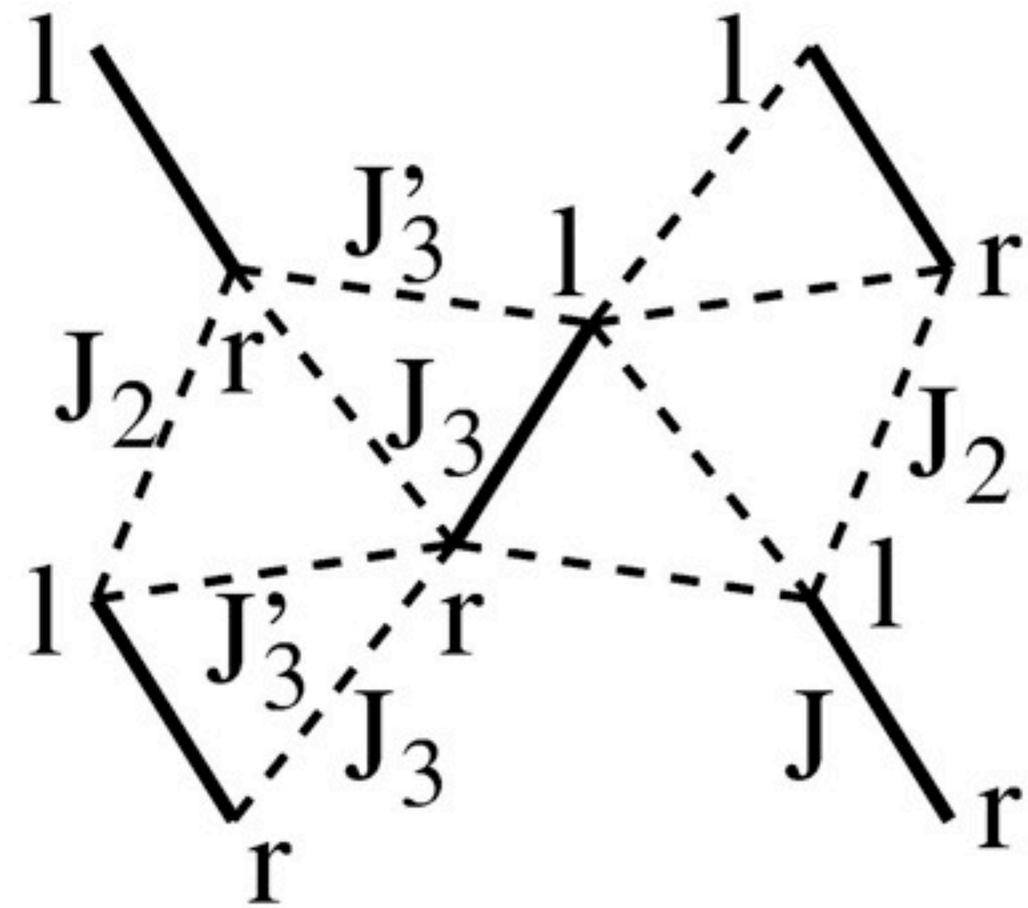




Quantum paramagnet at ambient pressure

$TlCuCl_3$

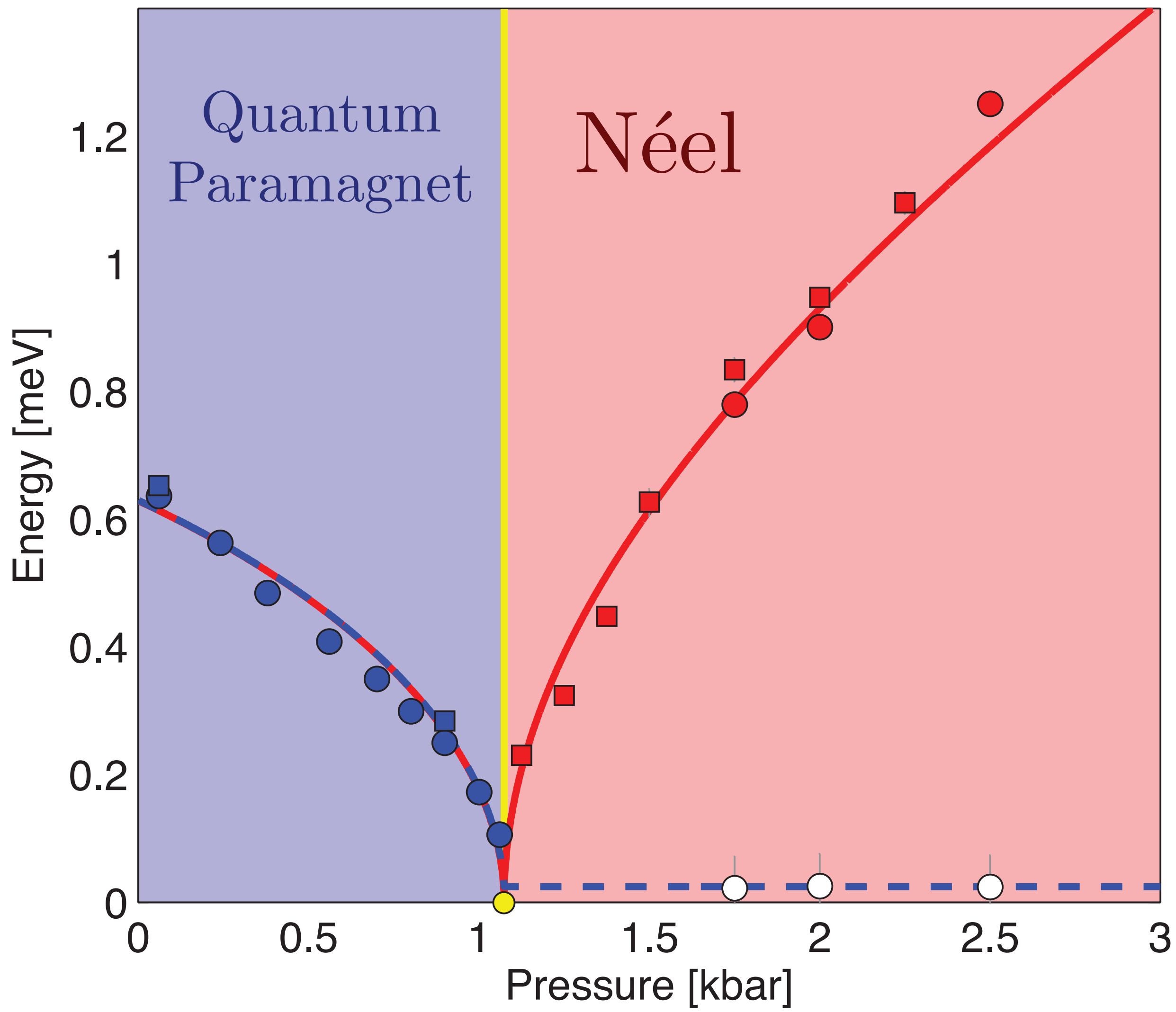




Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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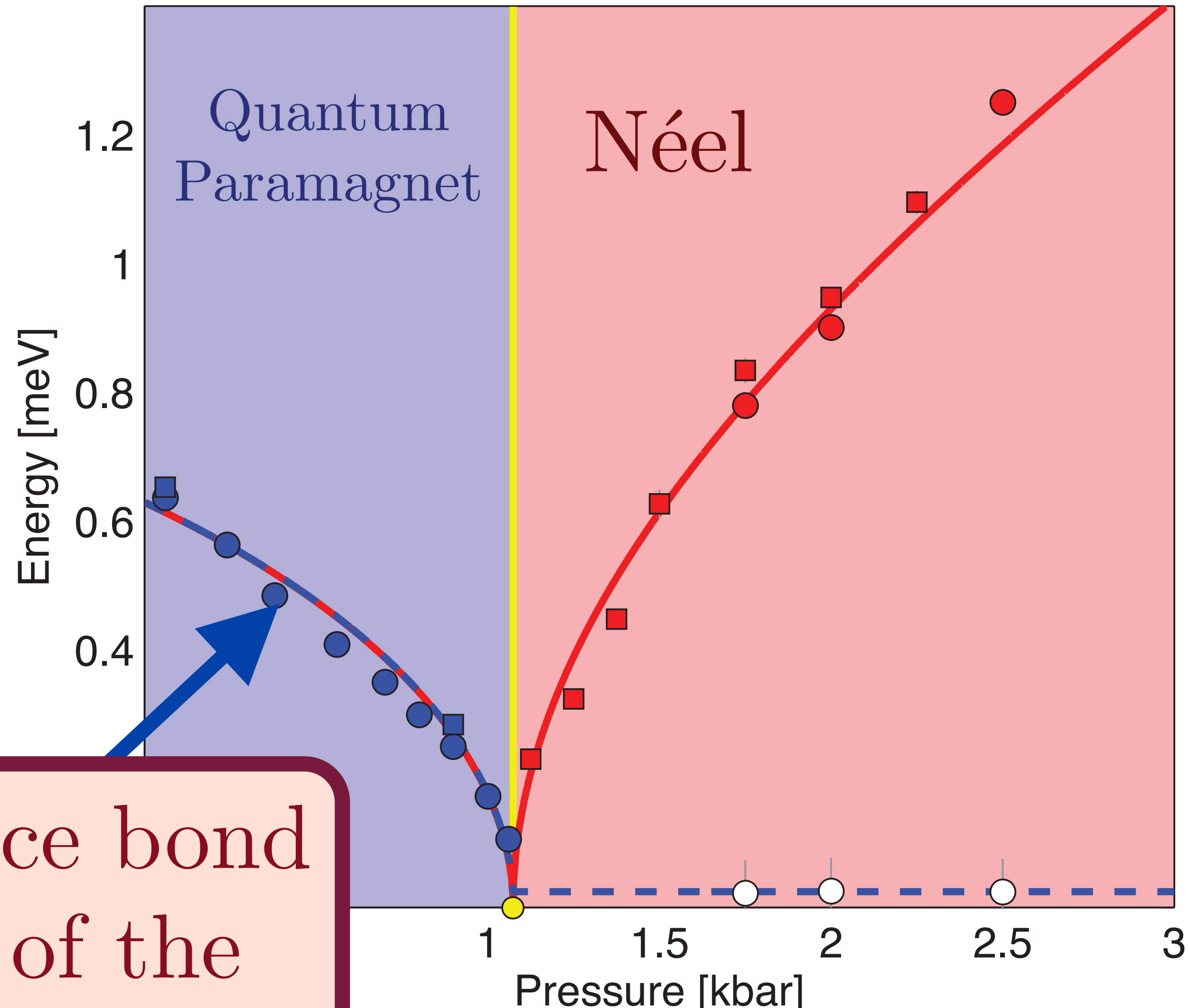
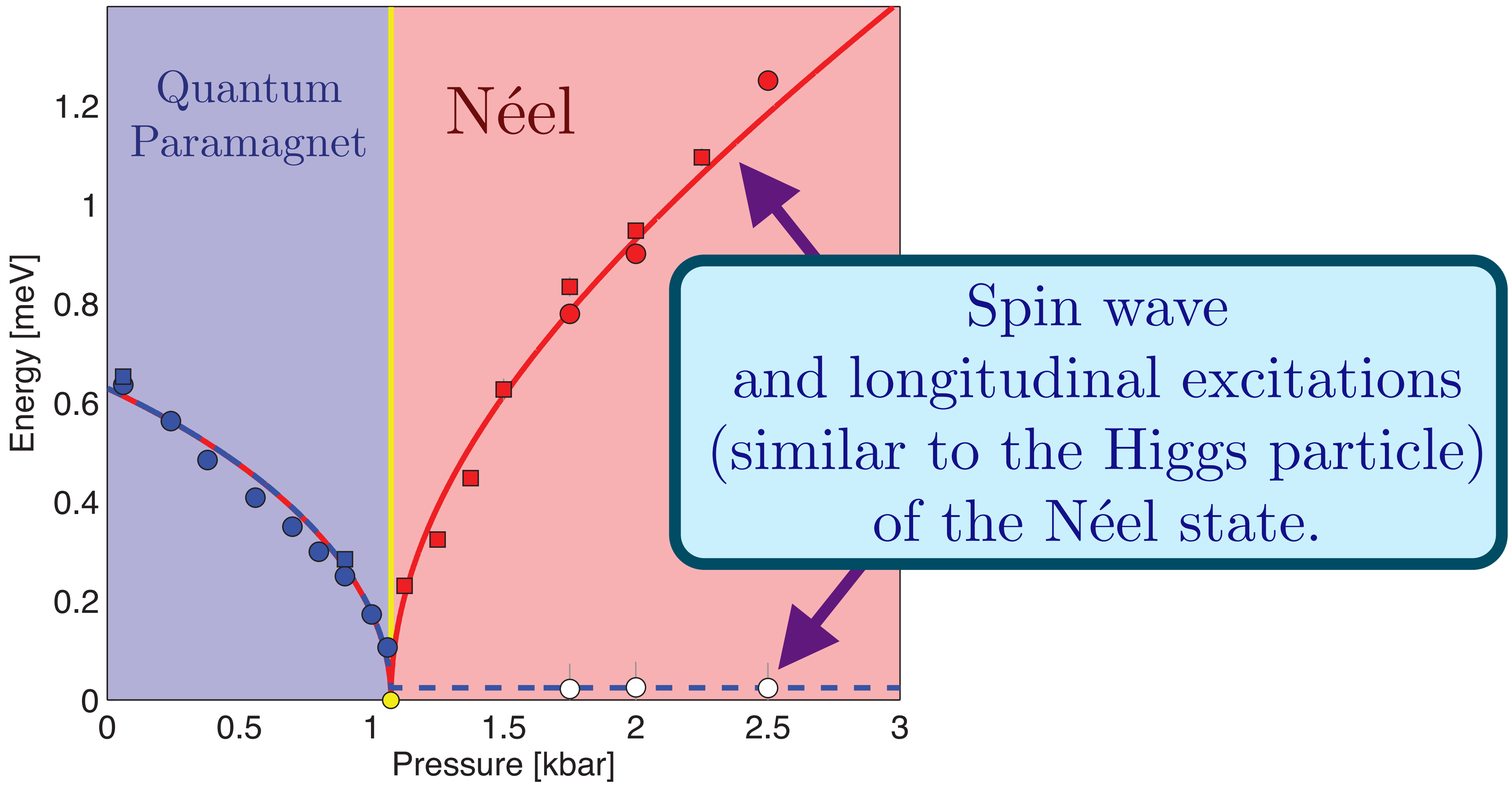


Broken valence bond excitations of the quantum paramagnet

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Excitations of $TlCuCl_3$ with varying pressure



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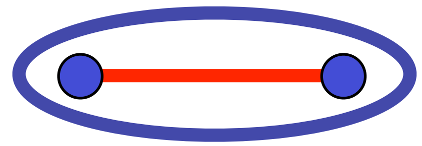
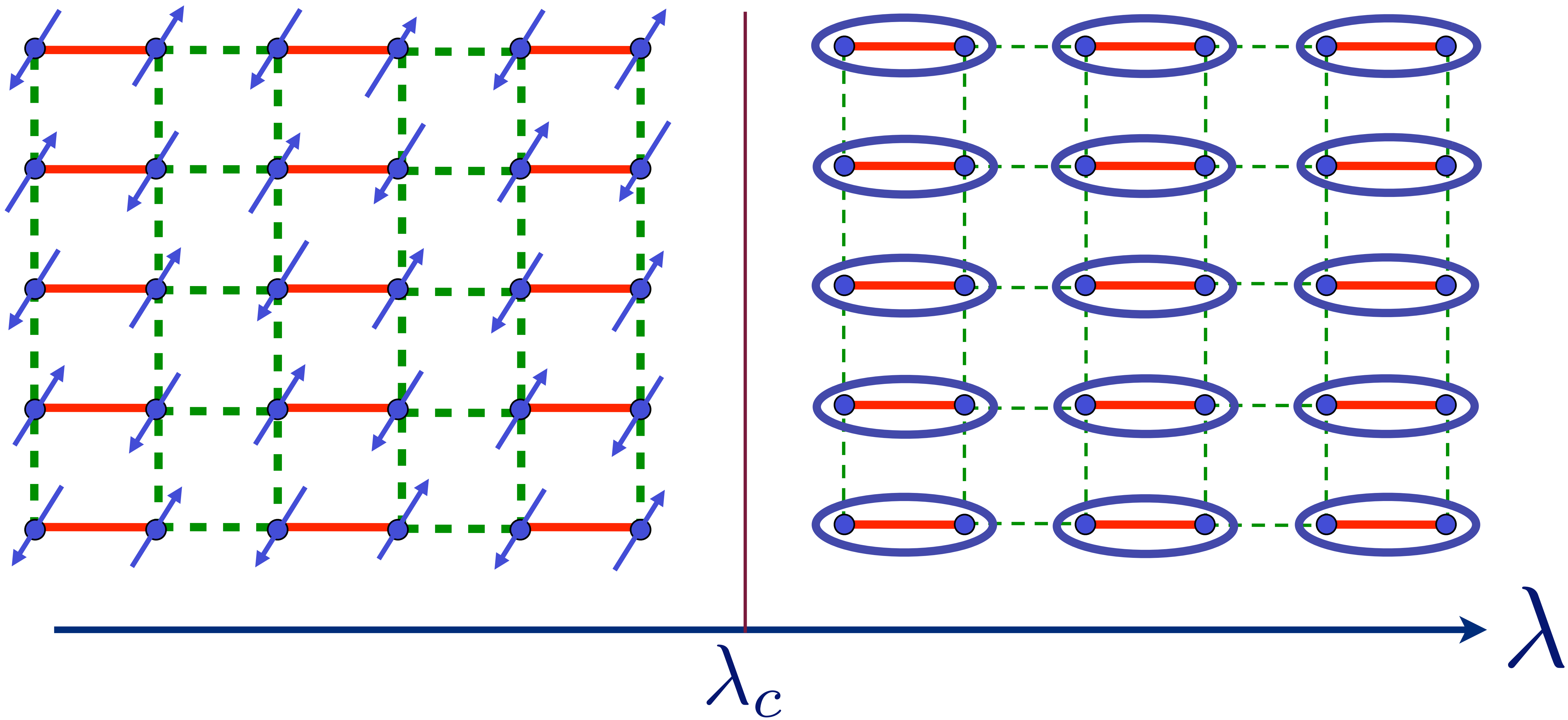
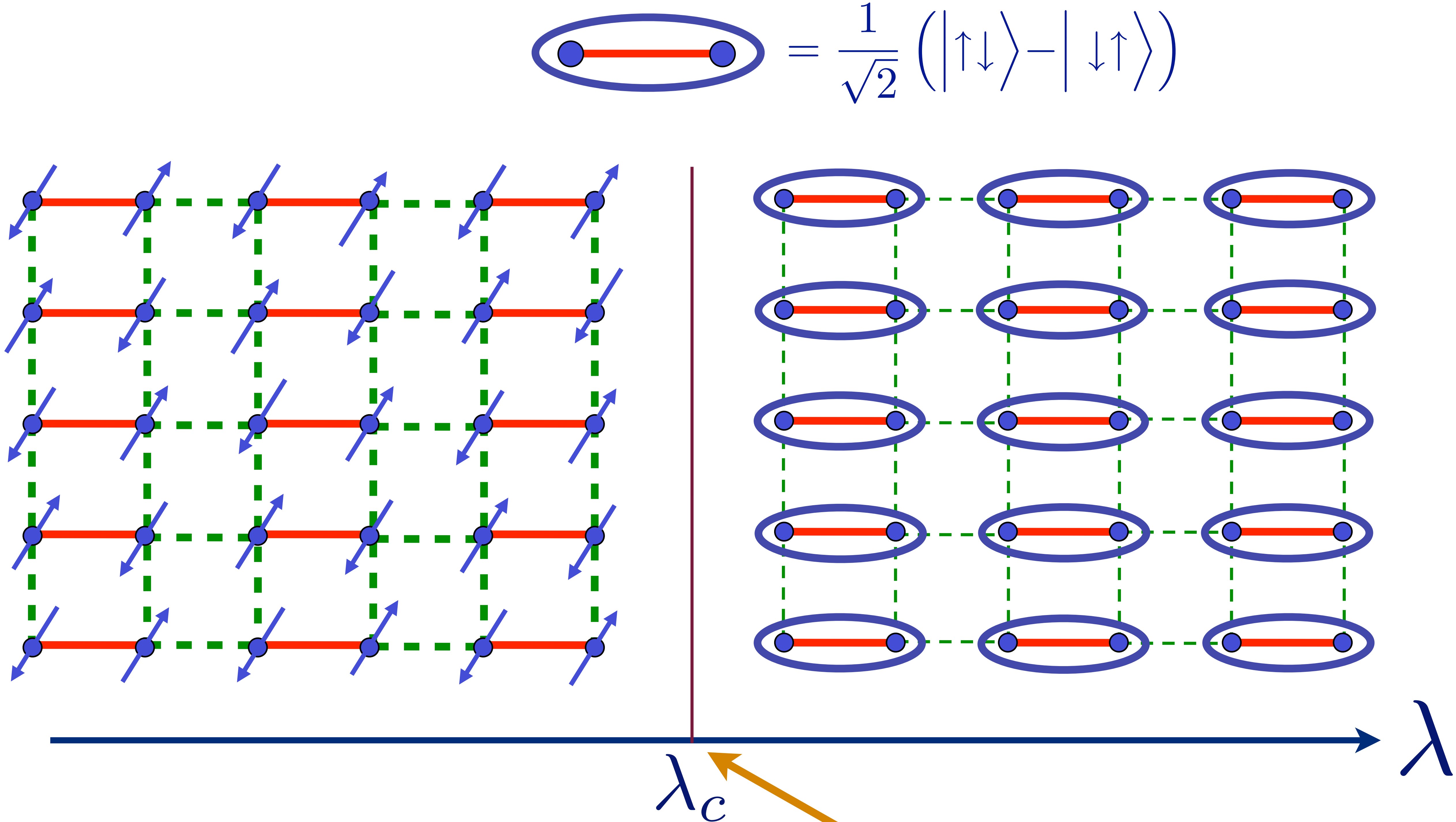


$$= \frac{1}{\sqrt{2}} \Big(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \Big)$$

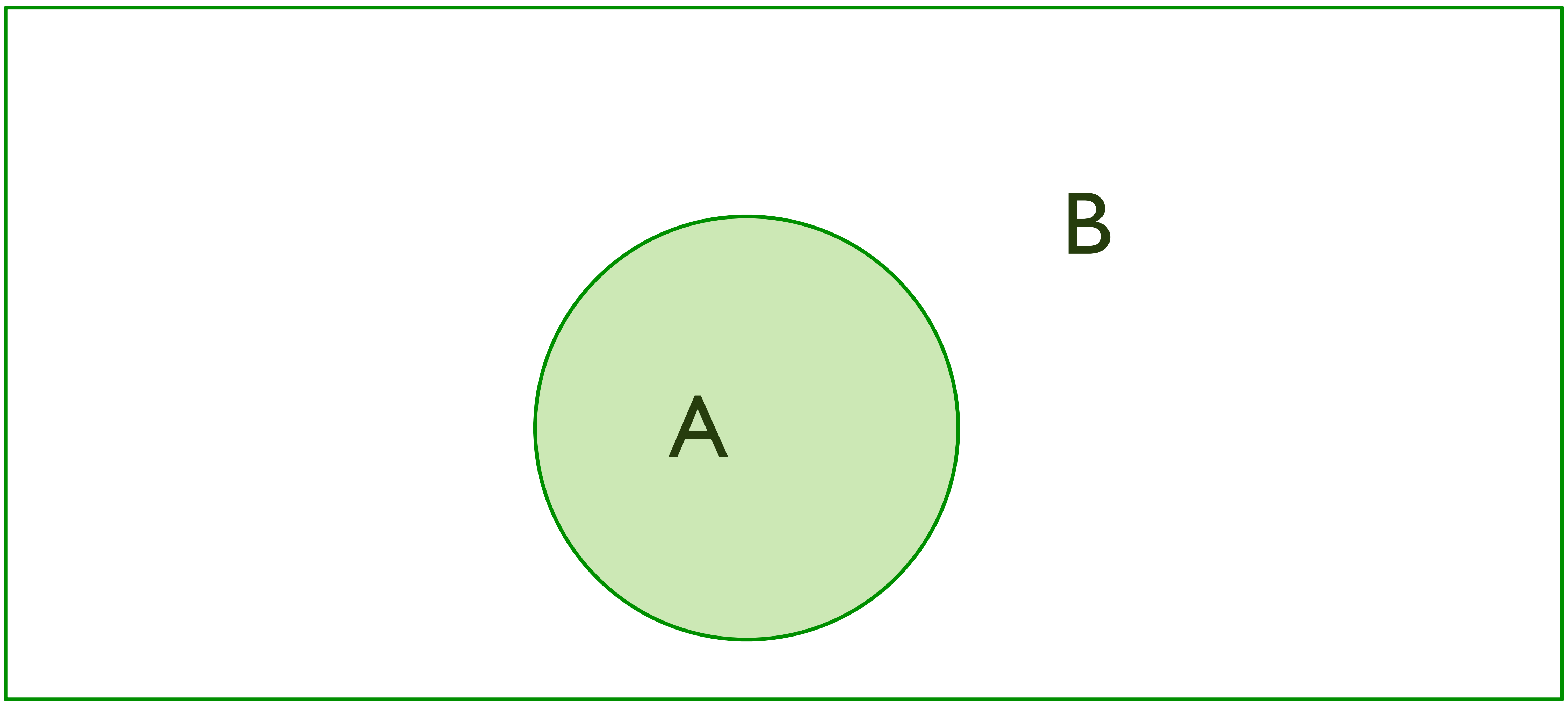




Quantum critical point with long-range entanglement in spin wavefunction

Characteristics of quantum critical point

- Long-range entanglement: entanglement entropy obeys $S_{EE} = aL - \gamma$, where γ is a universal number associated with the quantum critical point.



M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

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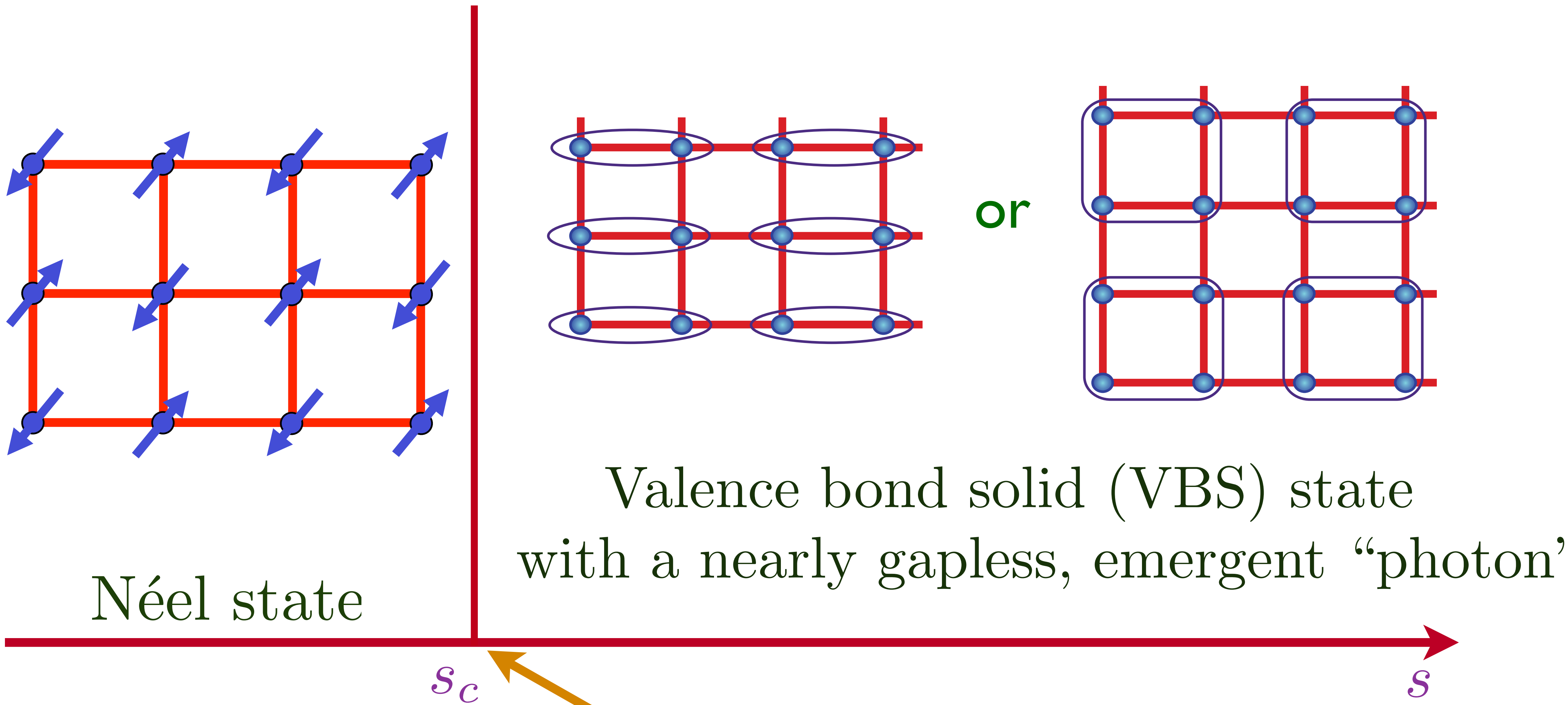
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").

- The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a **CFT3**

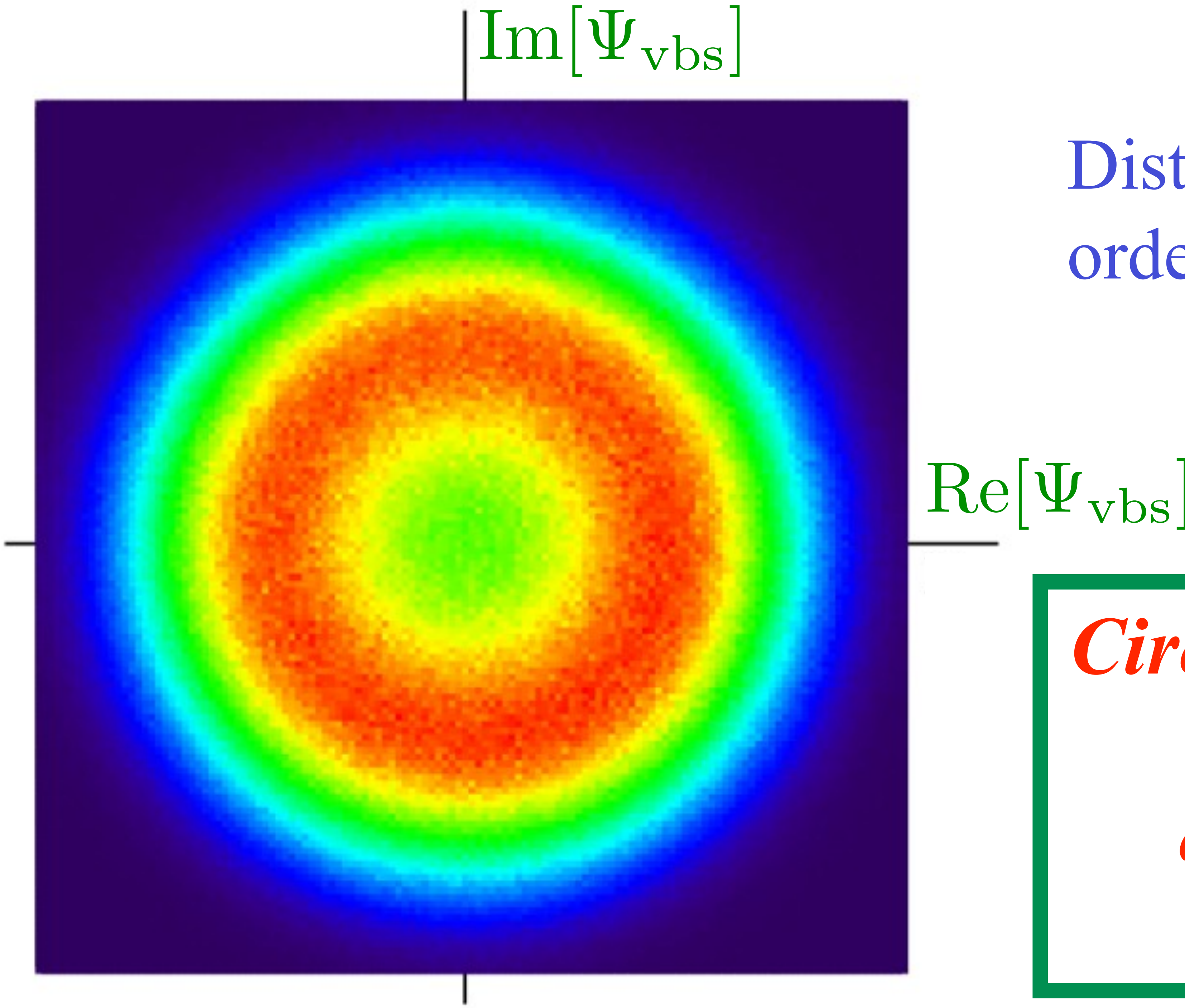
Quantum critical point in a frustrated square lattice antiferromagnet



Long-range entanglement described by a quantum field theory with an emergent U(1) “photon”

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right)\left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$

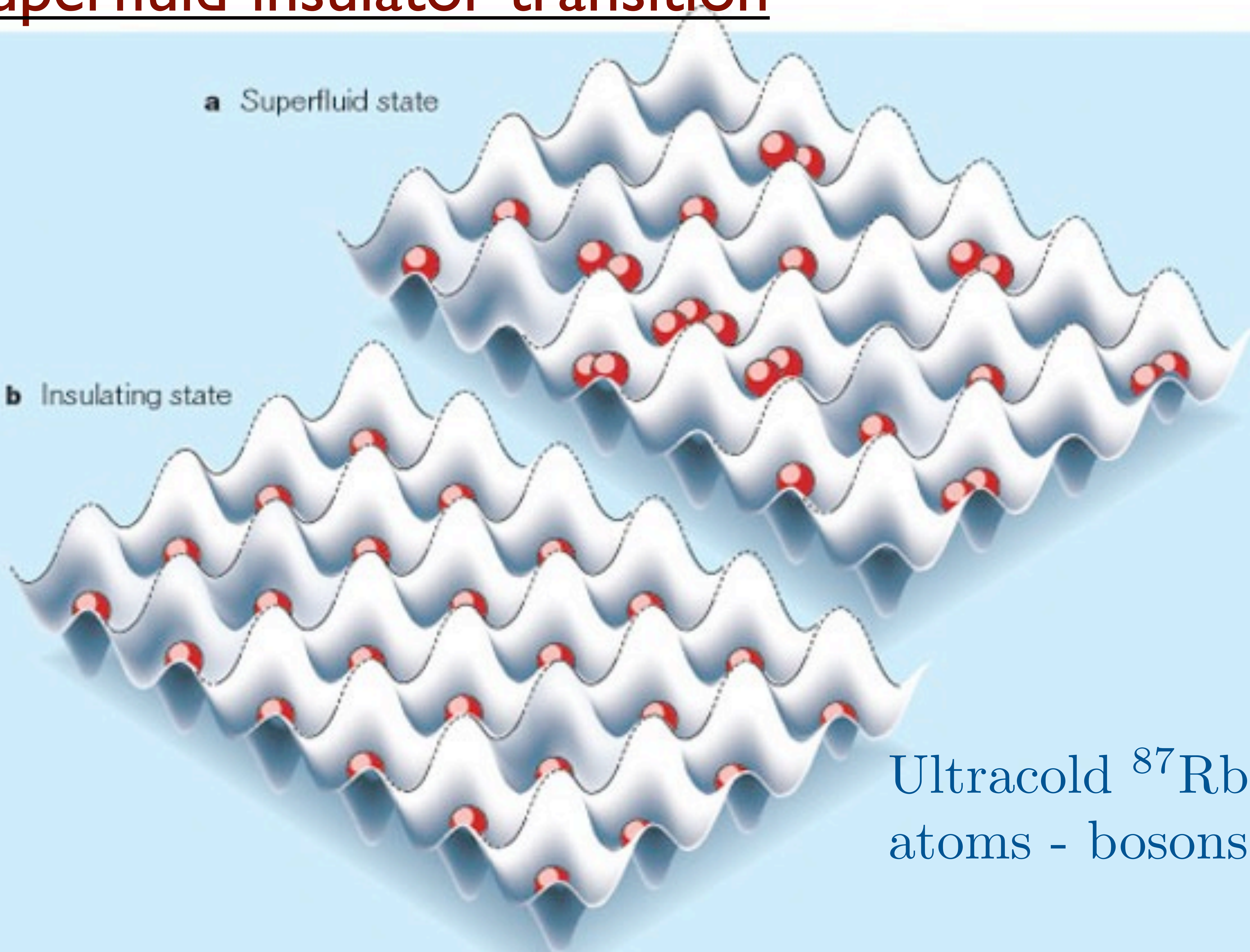


Distribution of VBS order Ψ_{vbs} at large Q

Circular symmetry is evidence for emergent U(1) photon

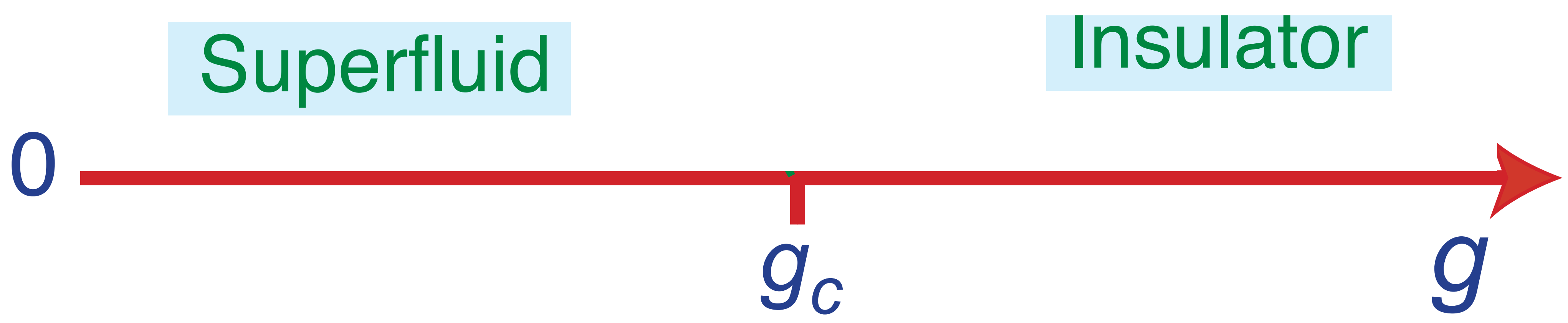
A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

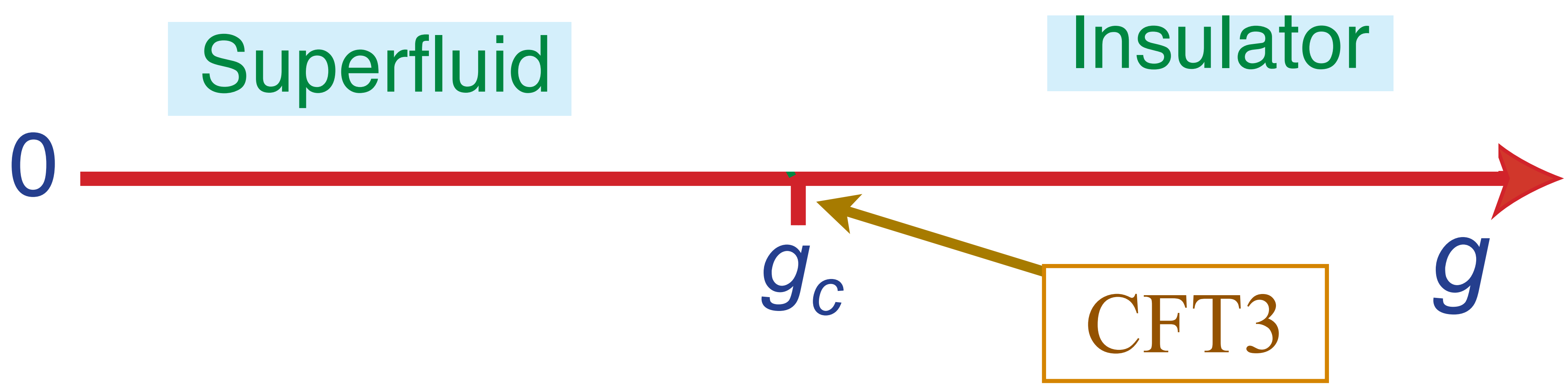
Superfluid-insulator transition



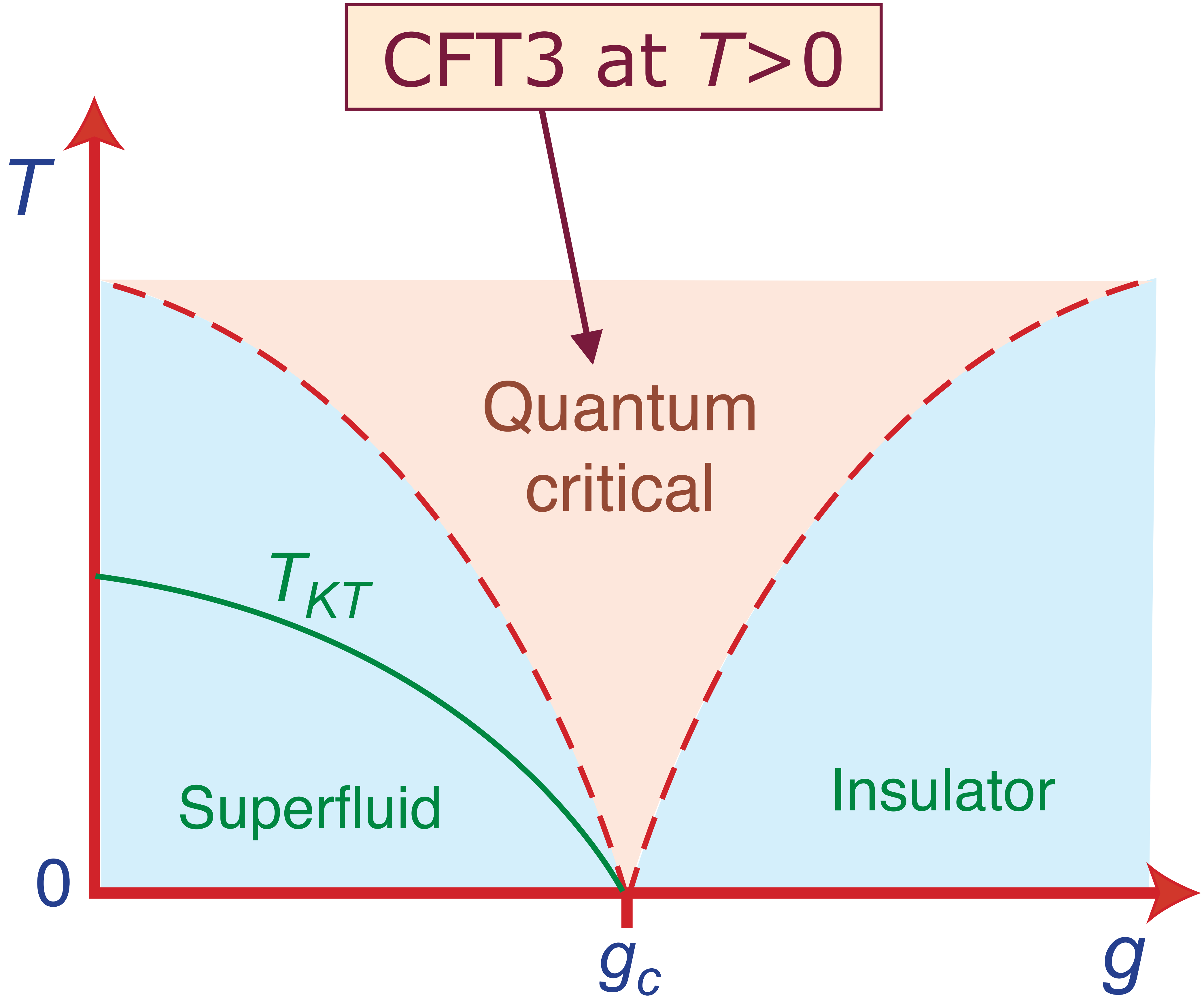
Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).


Superfluid
Insulator
0
g_c
g



M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, $\tau_{\rm eq}$

$$\boxed{\tau_{\rm eq} = \mathcal{C} \frac{\hbar}{k_B T}}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\boxed{\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]}$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Outline

1. Gapped quantum matter
   Insulators, quantum Hall states

2. Conformal quantum matter
   Graphene, antiferromagnets, ultracold atoms

3. Compressible quantum matter
   Metals, superconductors, strange metals

4. Connections to string theory

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

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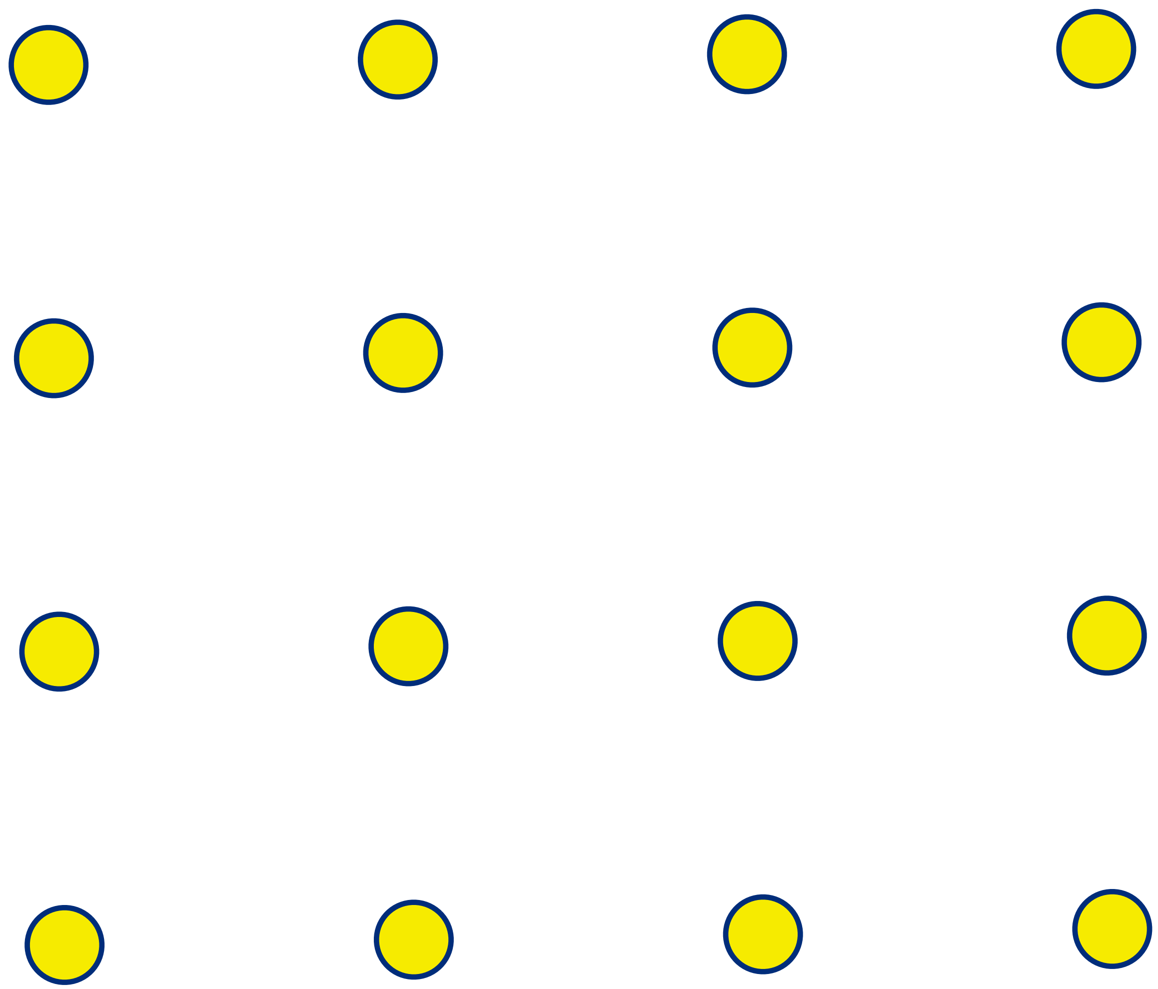
- Conformal systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

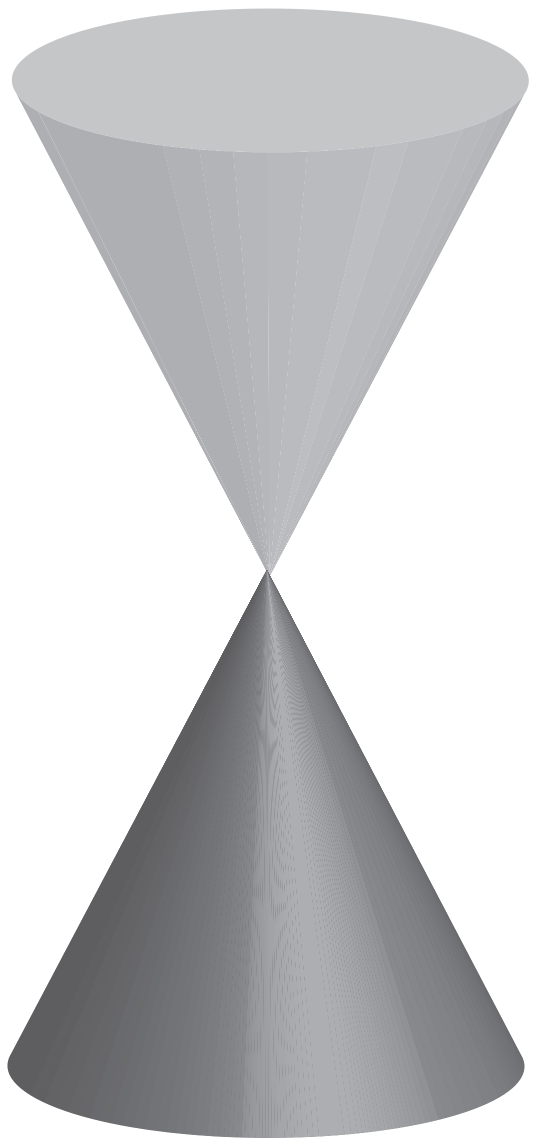
One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

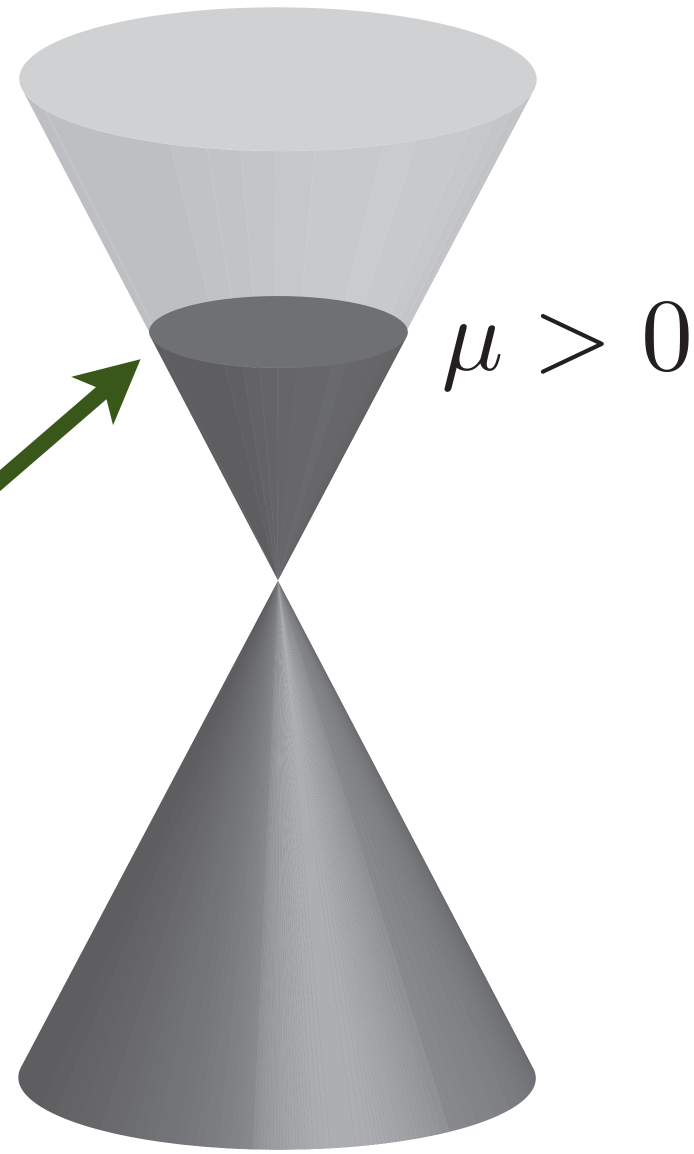
Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs



Graphene

The only other familiar compressible phase is a **<u>Fermi Liquid</u>** with a **<u>Fermi surface</u>**



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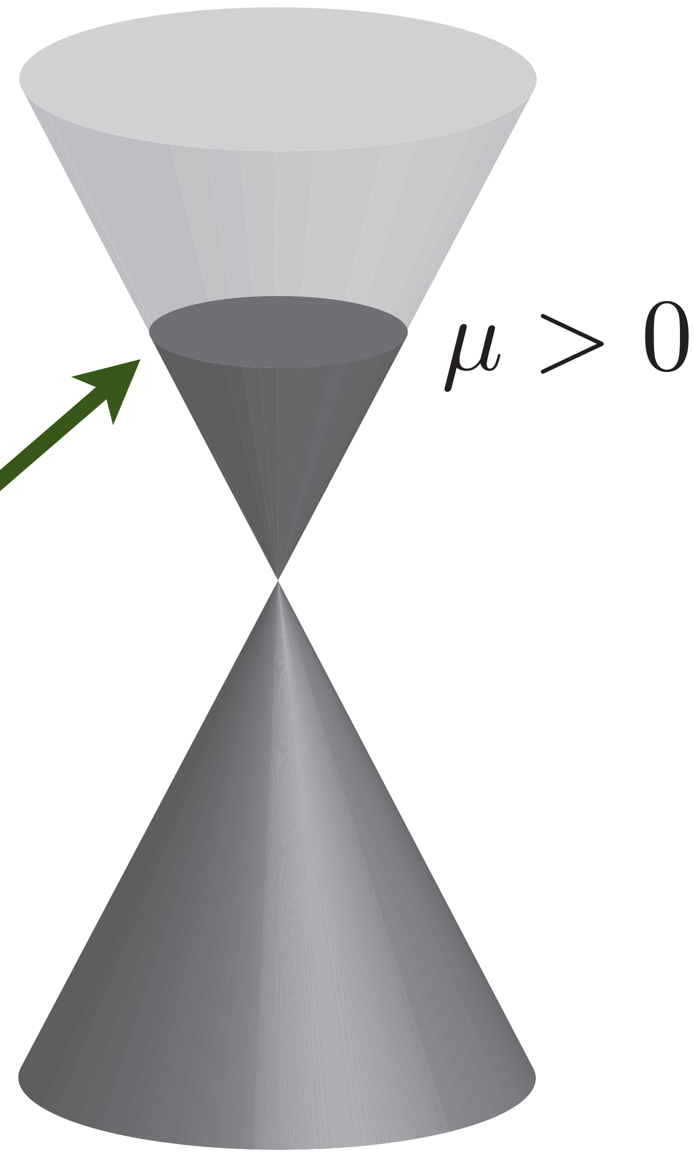
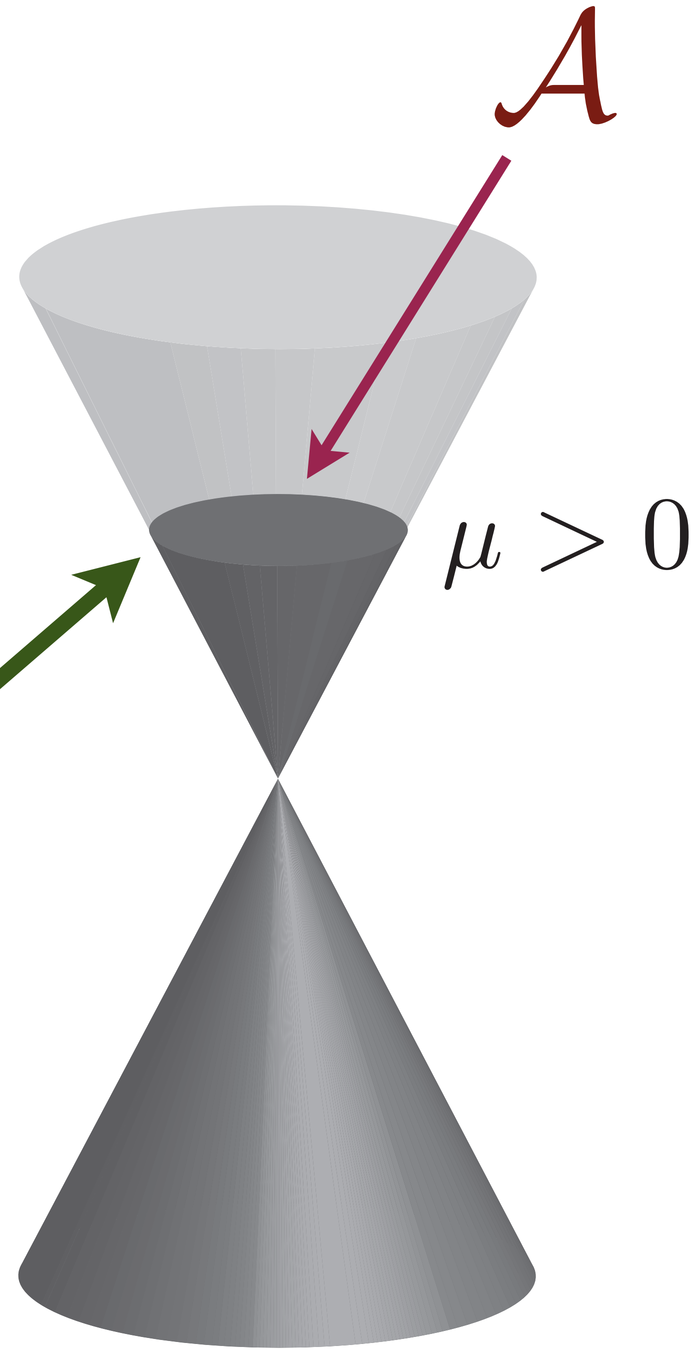


- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

The only other familiar compressible phase is a **<u>Fermi Liquid</u>** with a **<u>Fermi surface</u>**



- **Luttinger relation:** The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$.

Classify states of compressible quantum matter in continuum theories which preserve translational invariance.

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Are there any other compressible phases, apart from superfluids and Fermi liquids ?

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Yes

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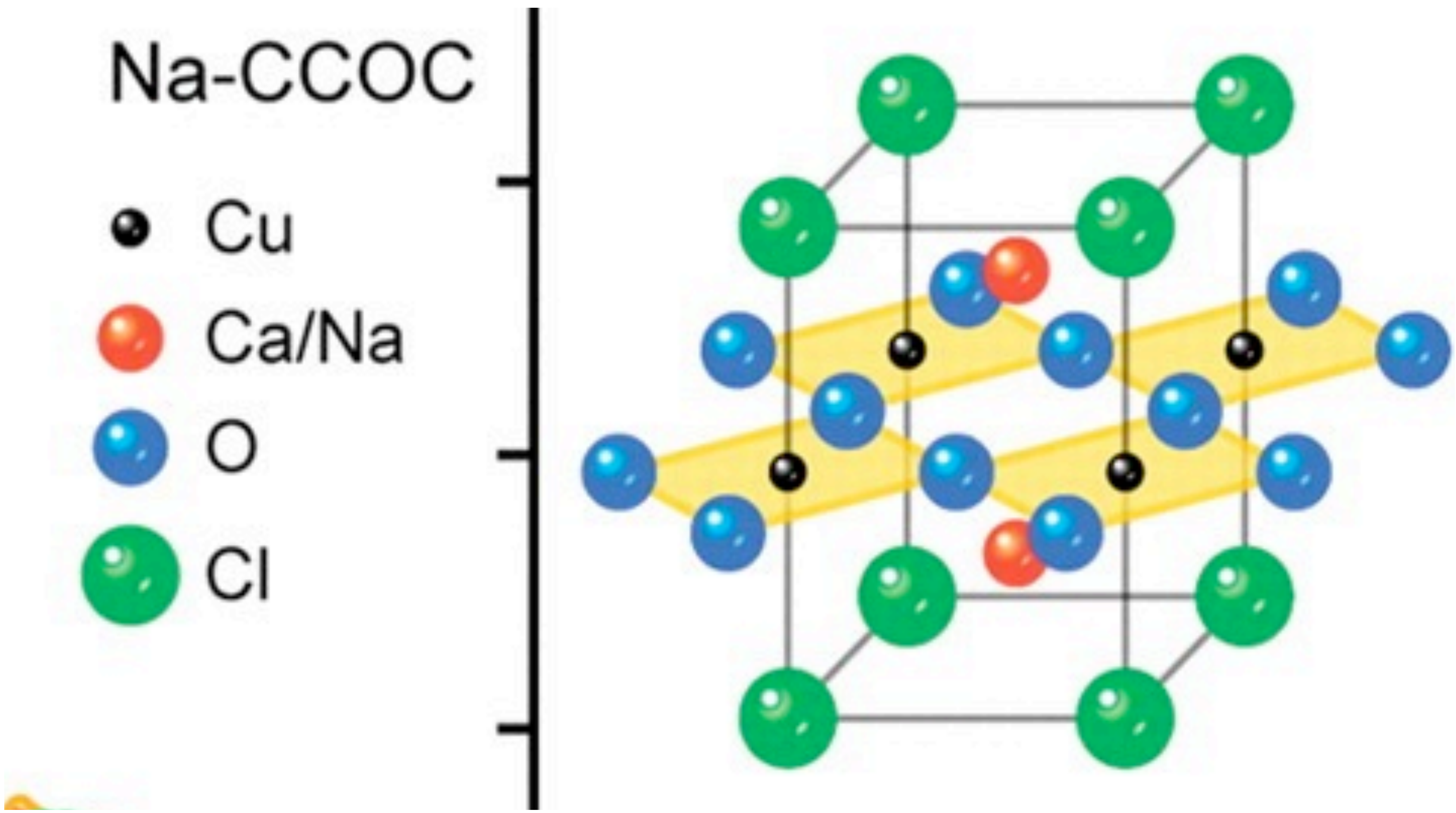
Are there any other compressible phases, apart from superfluids and Fermi liquids ?

Yes

Can they explain experiments on the new quantum materials ?

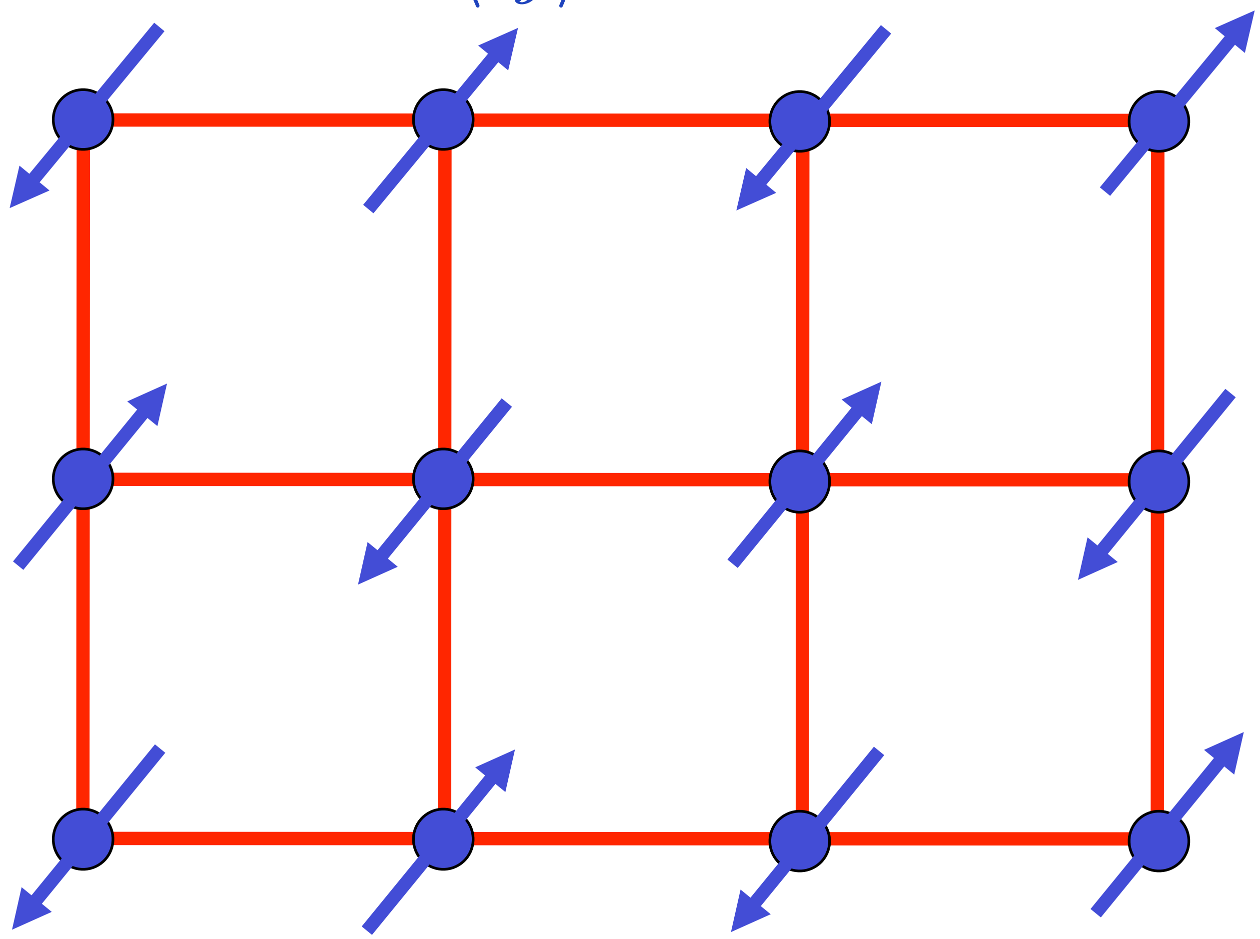
The cuprate superconductors



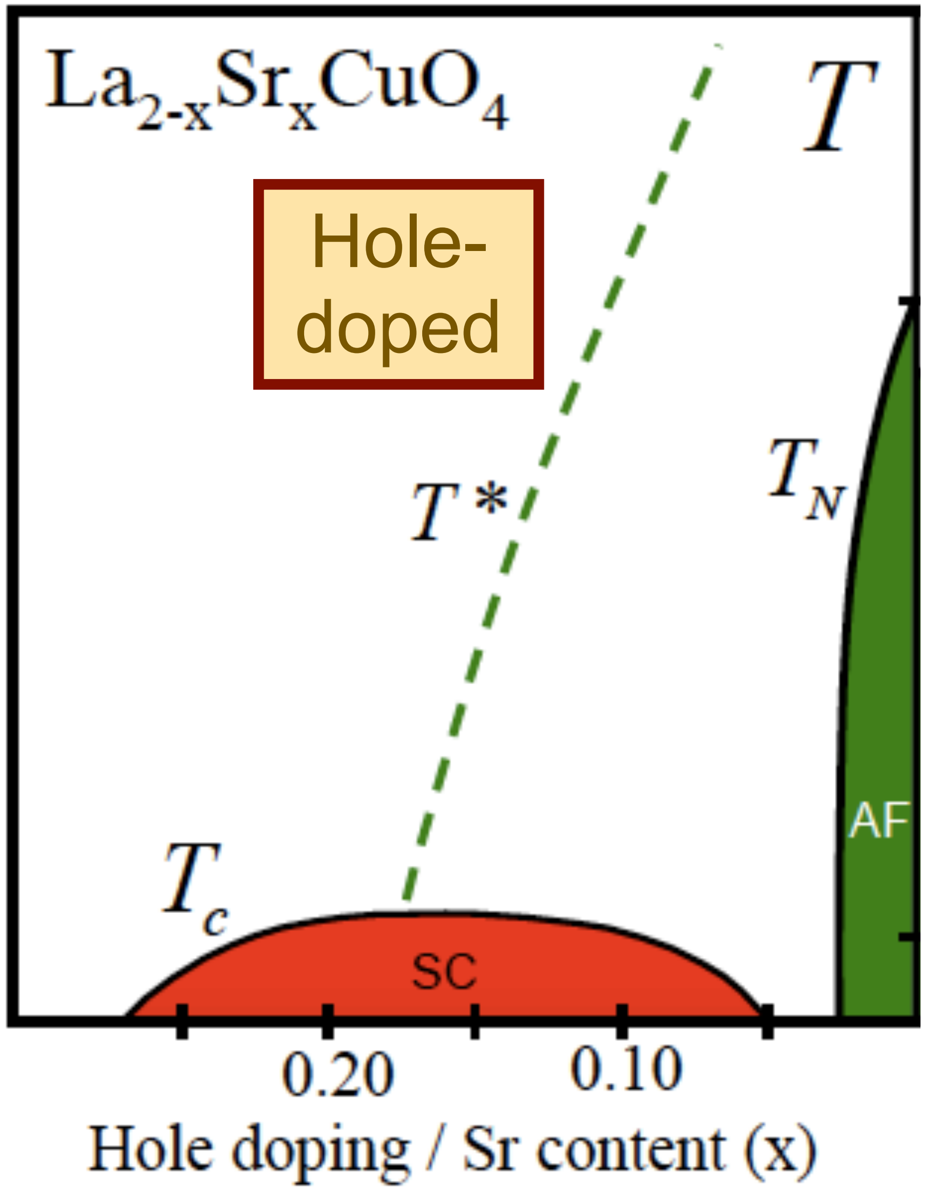
Davis

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

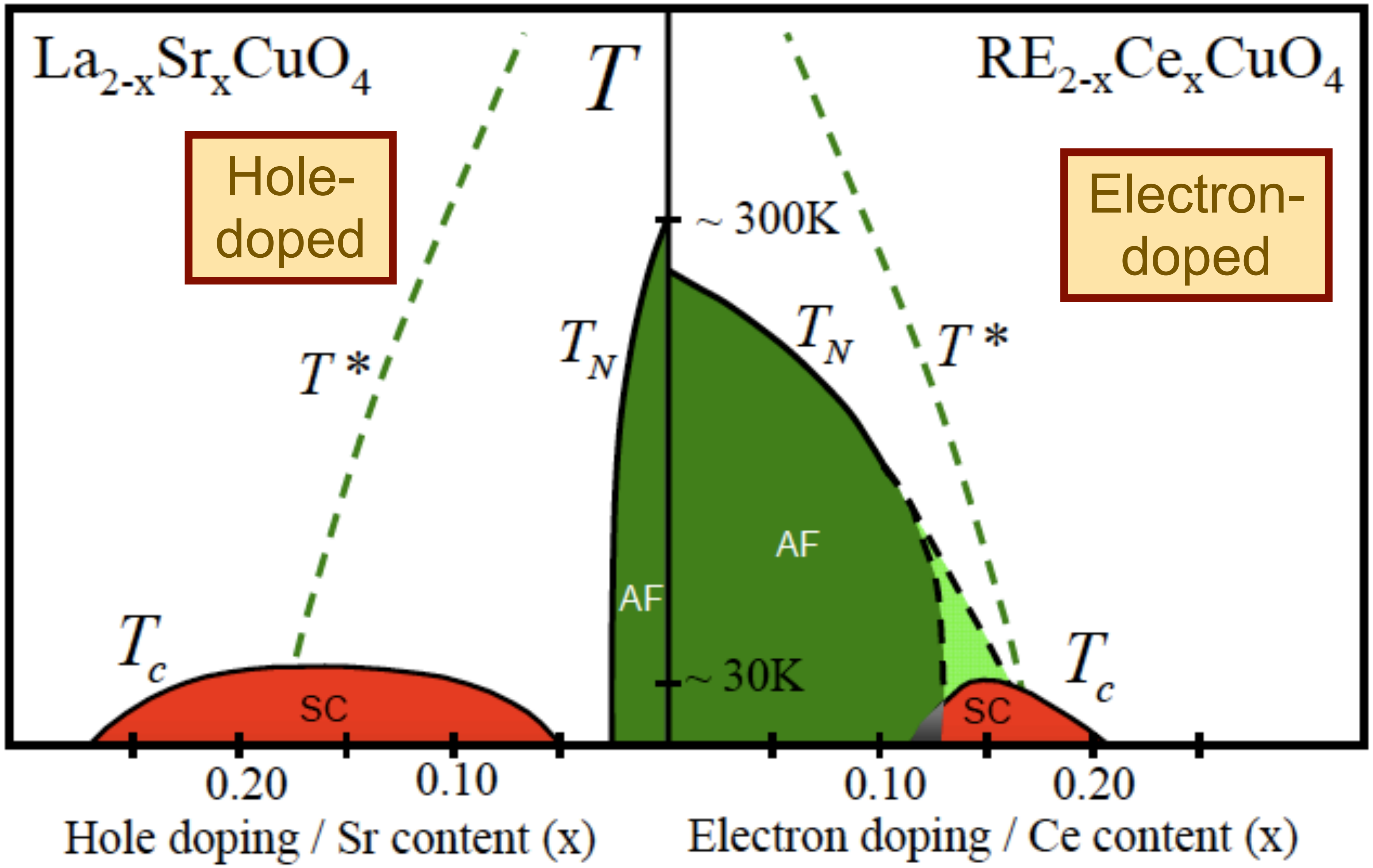


Ground state has long-range Néel order



La2-xSrxCuO4
T
Hole-doped
T*
TN
AF
Tc
SC
0.20
0.10
Hole doping / Sr content (x)

Electron-doped cuprate superconductors



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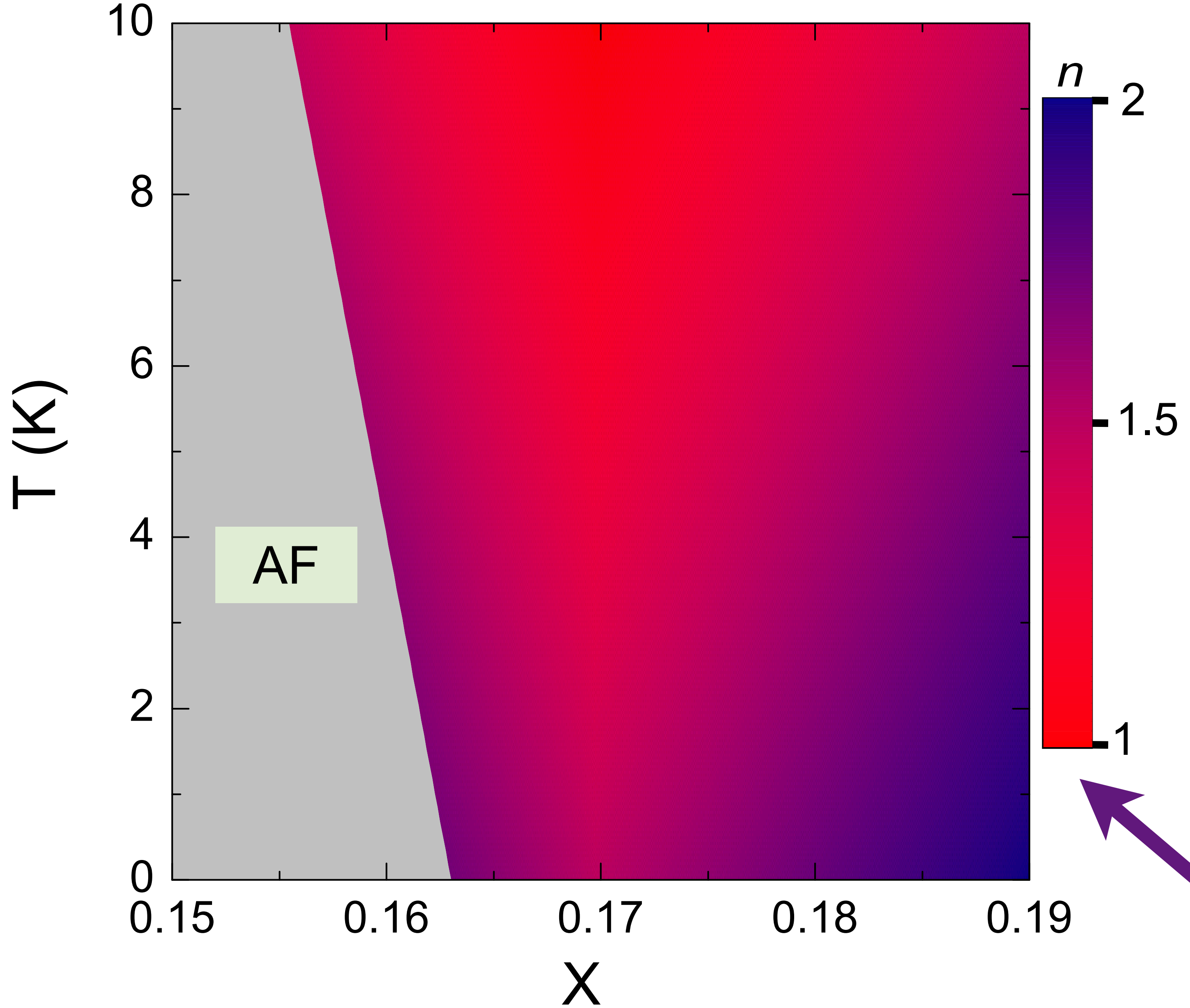
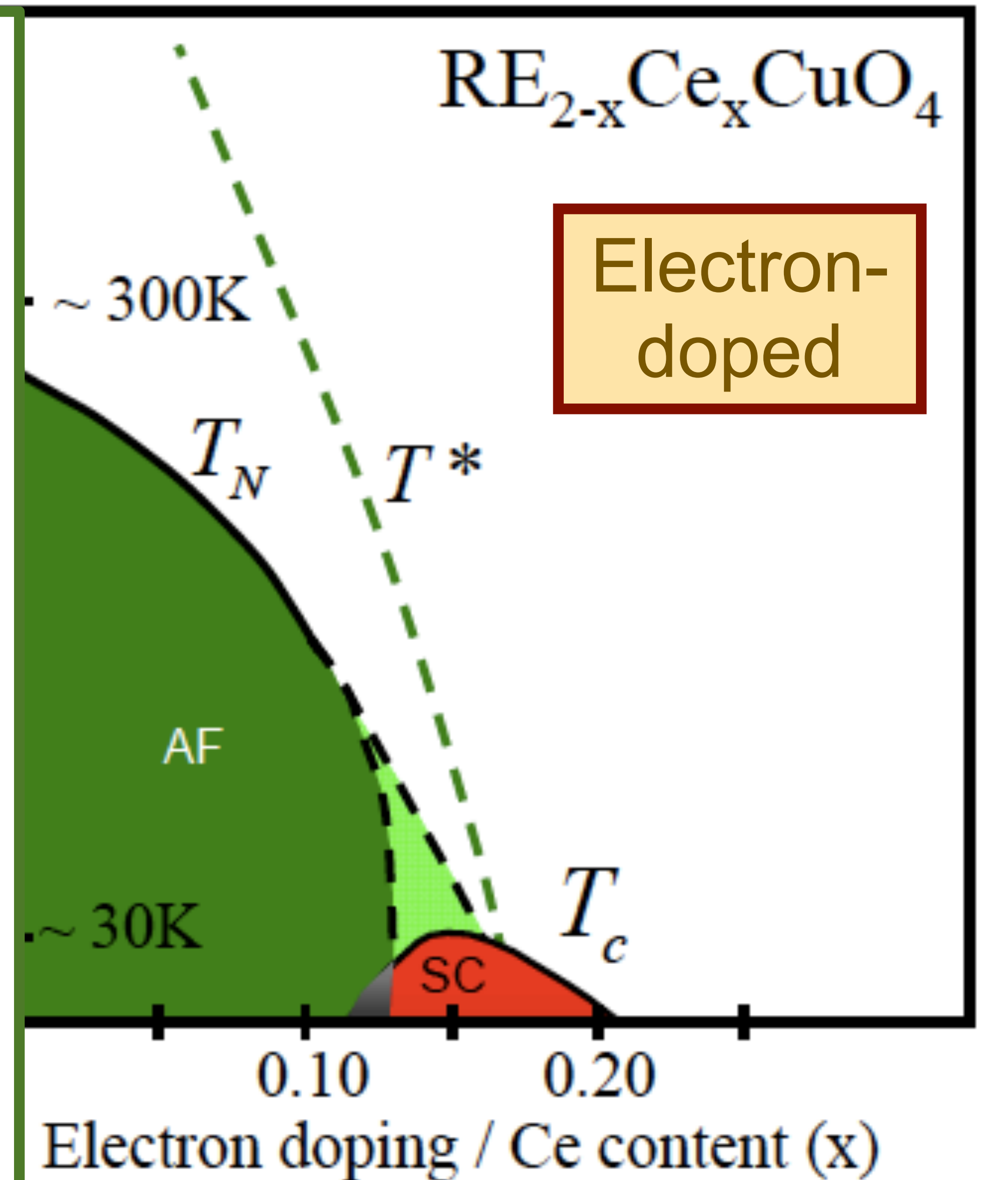
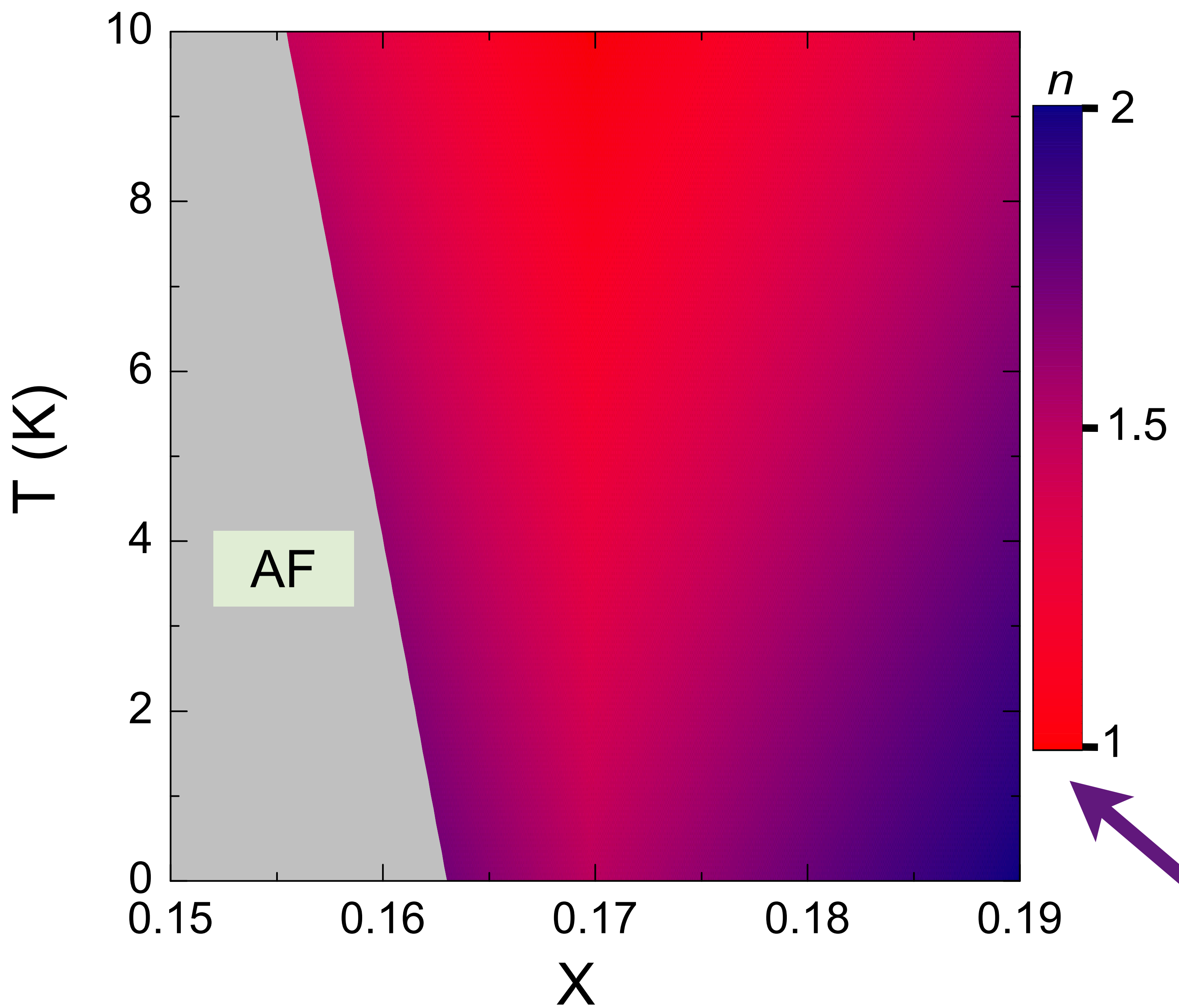


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).



Resistivity
$\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors



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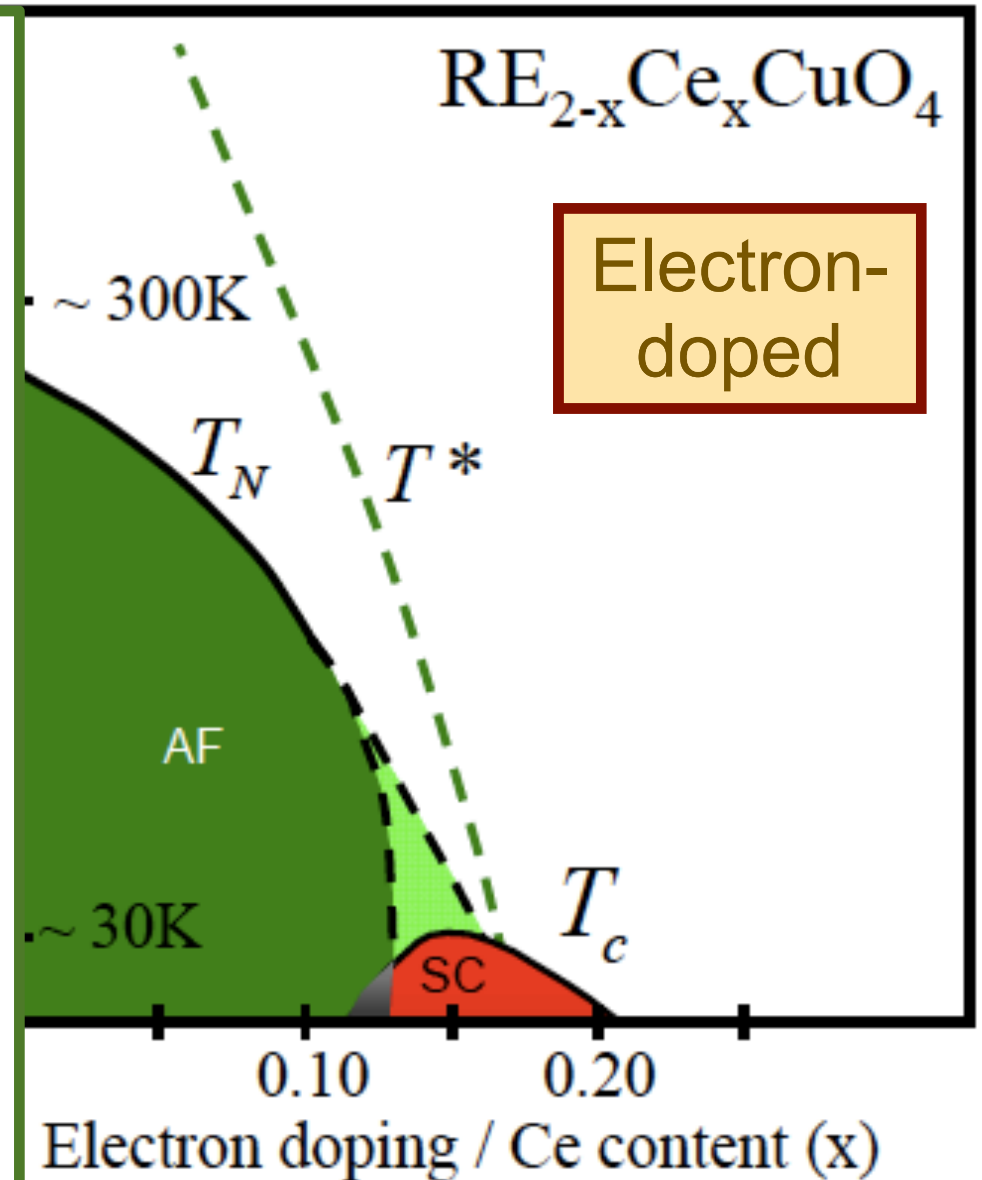

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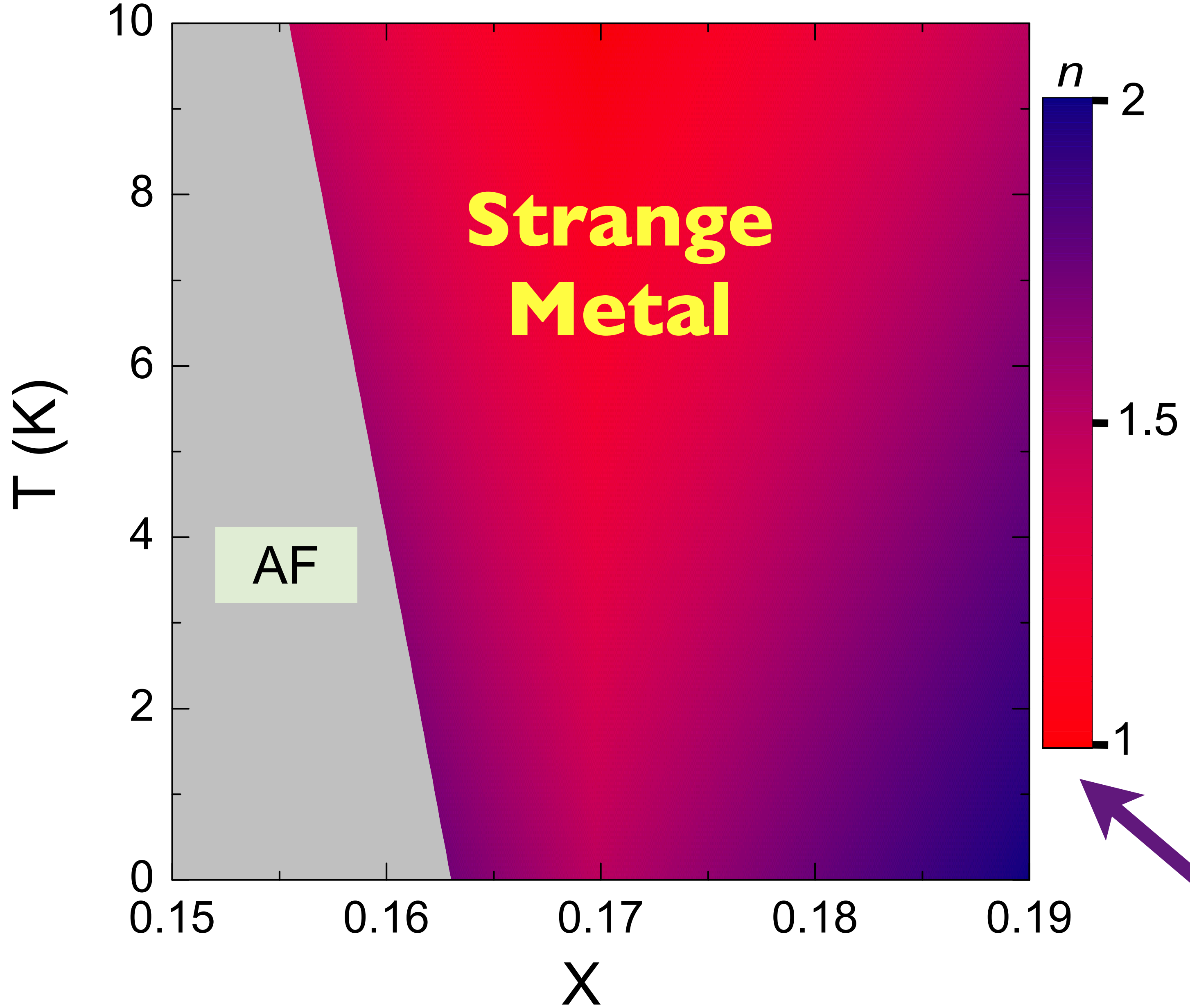

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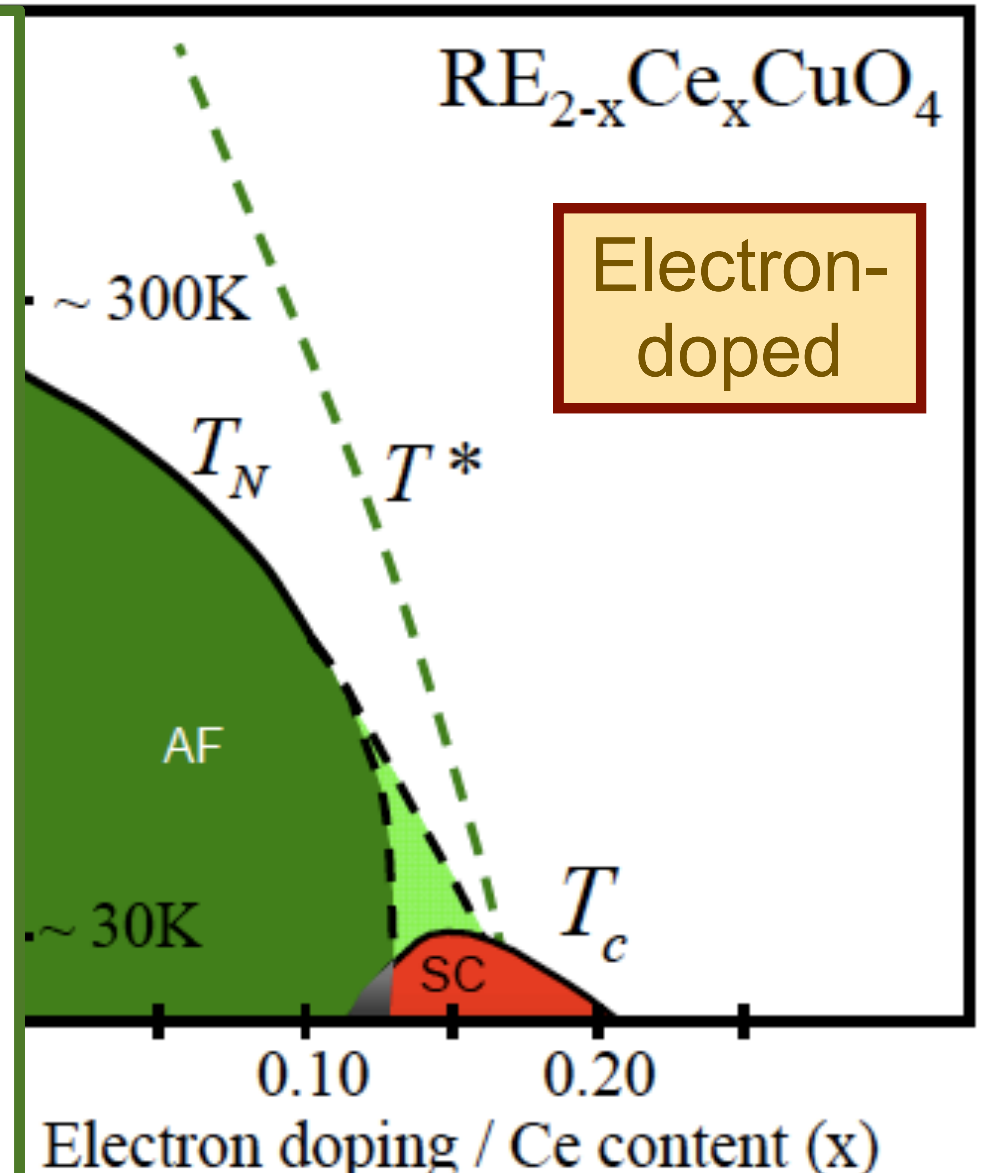

Resistivity
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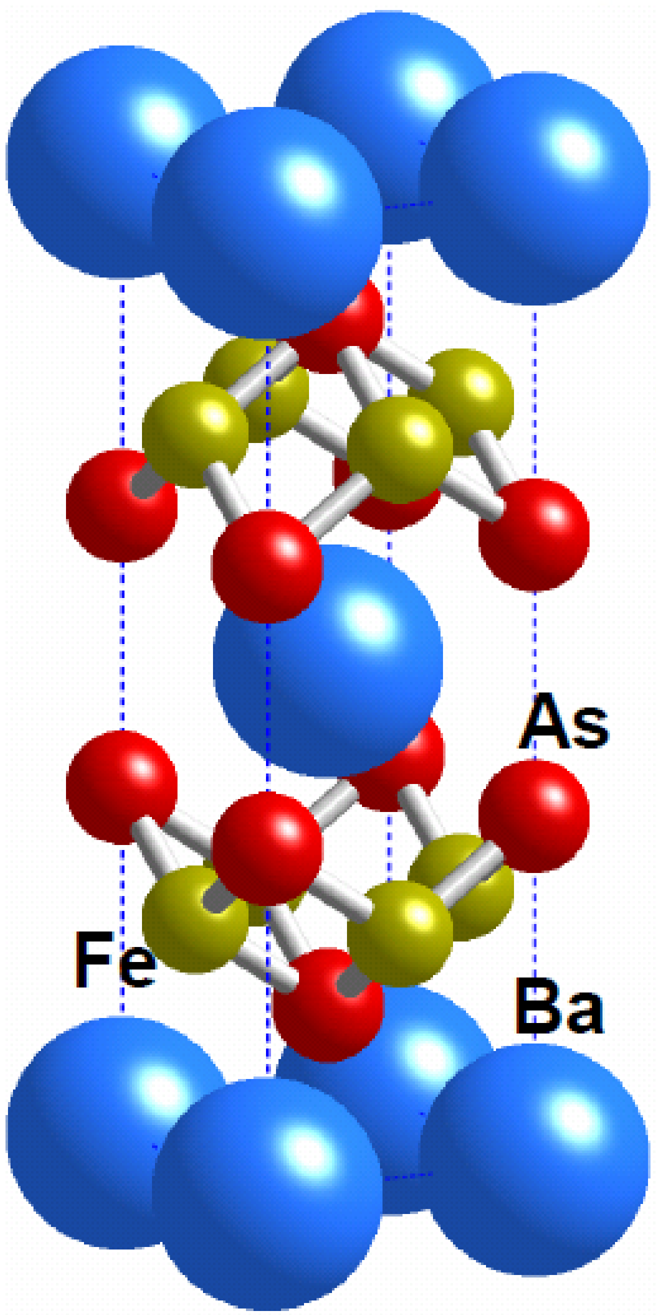
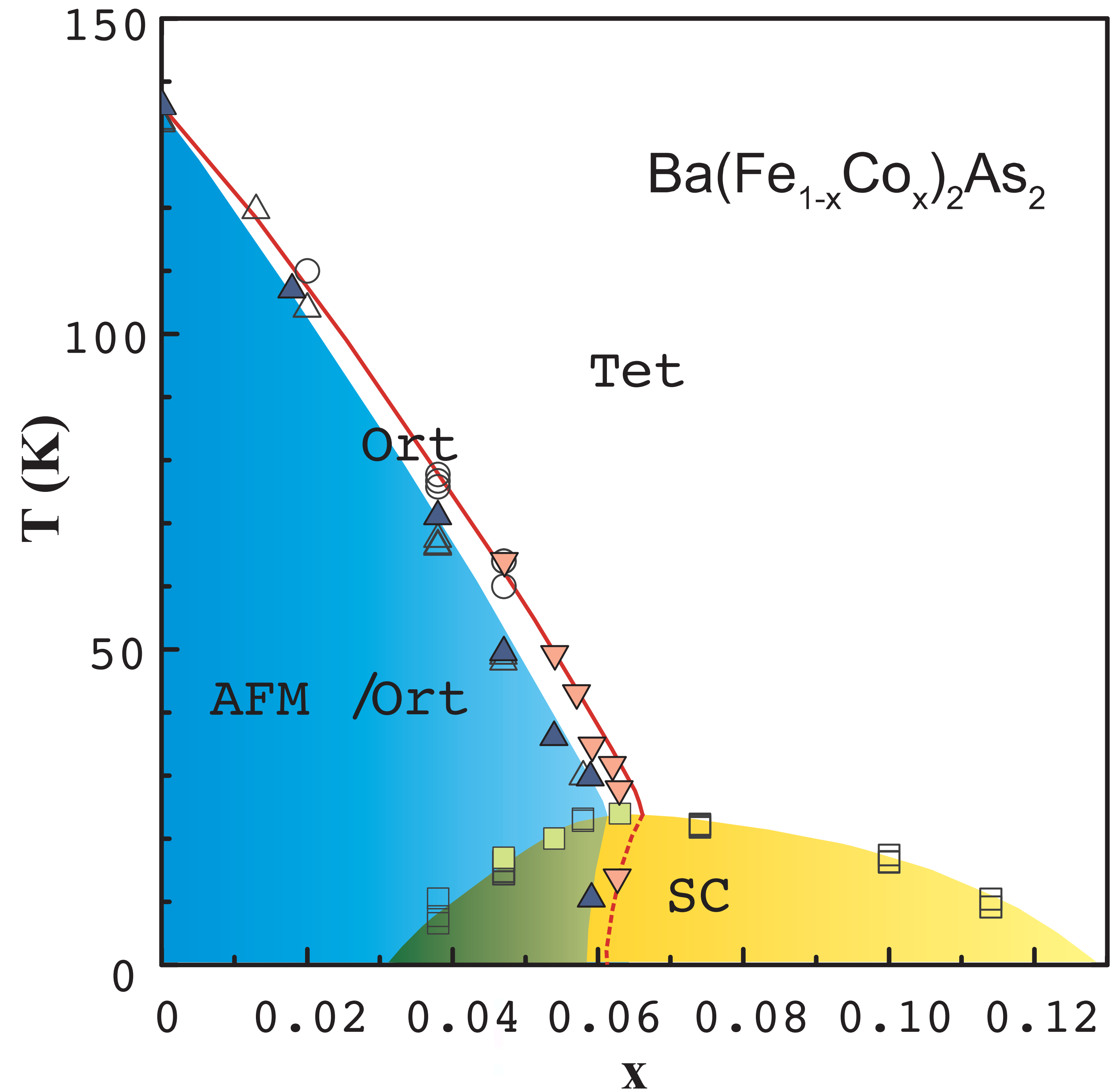
Iron pnictides:

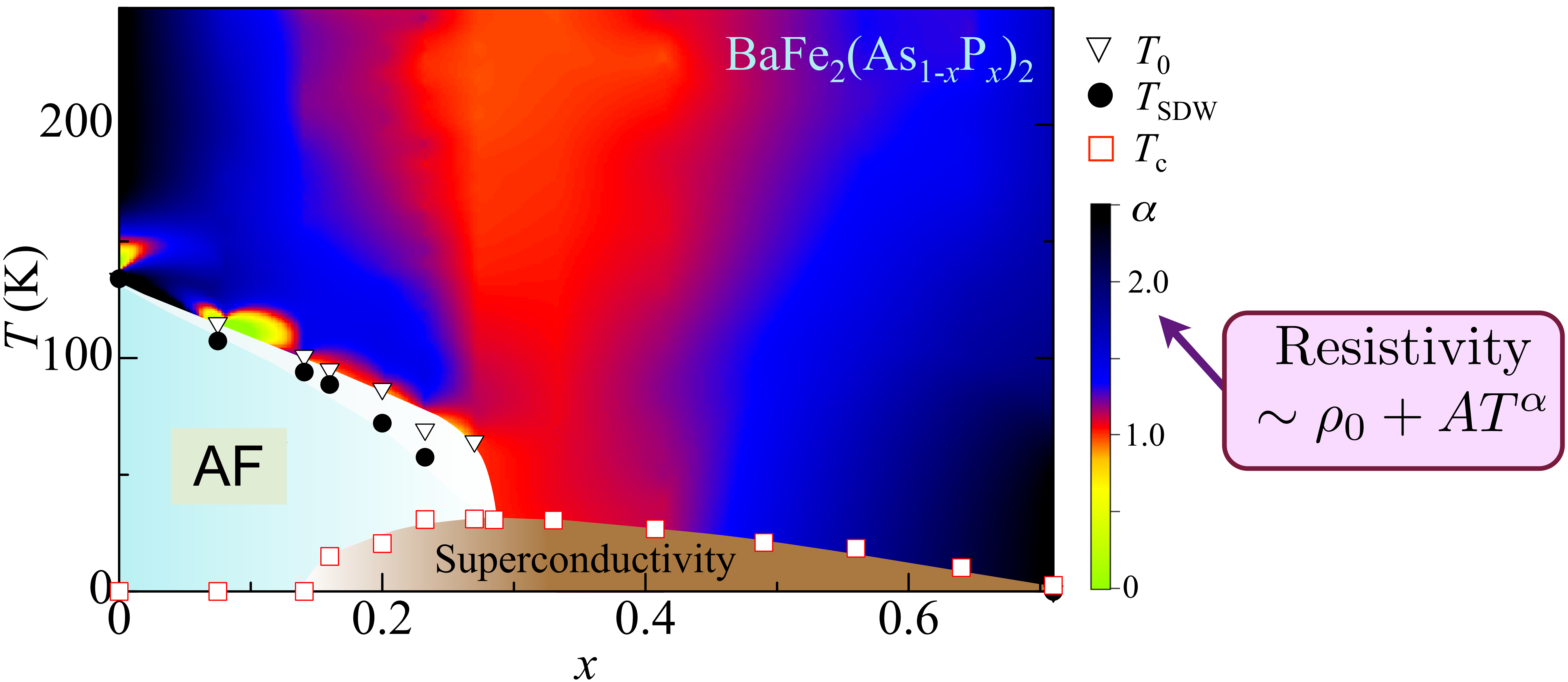
a new class of high temperature superconductors





S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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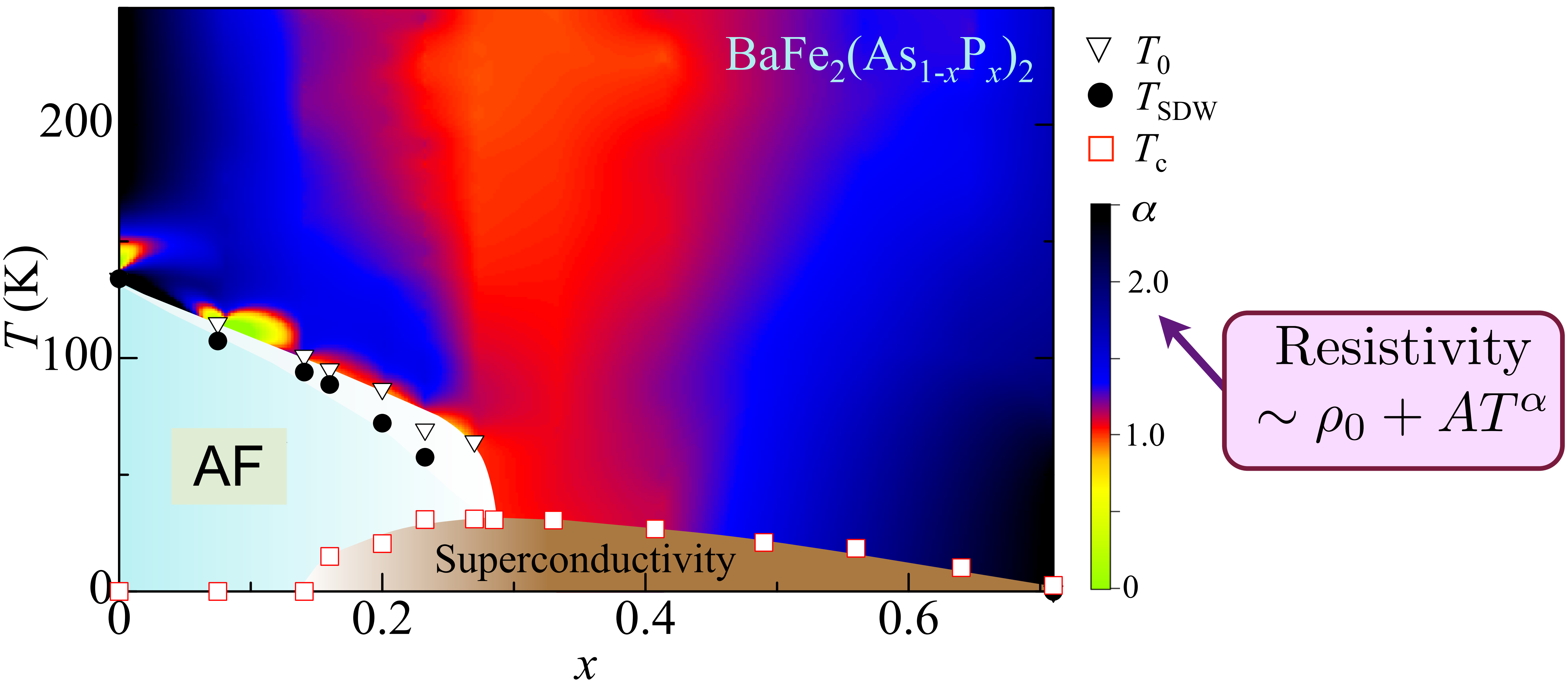

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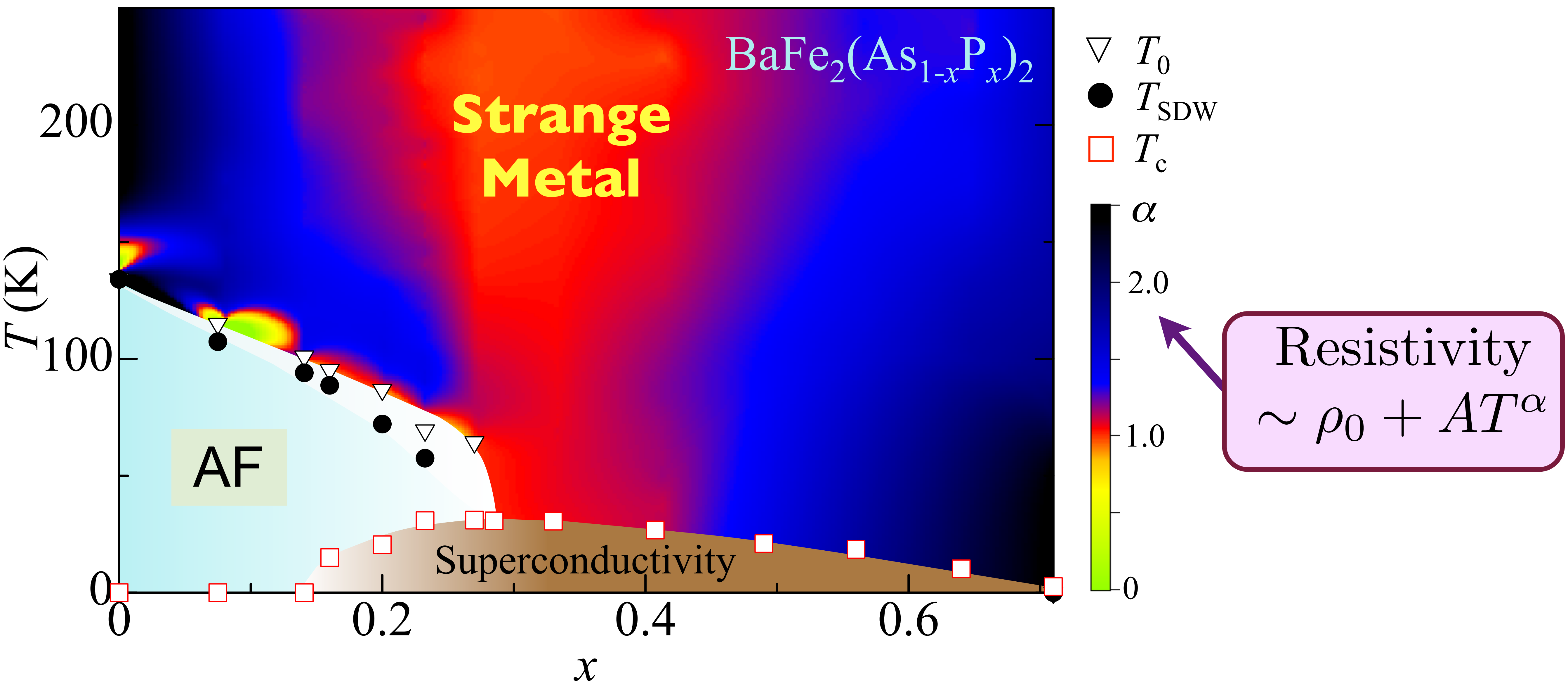

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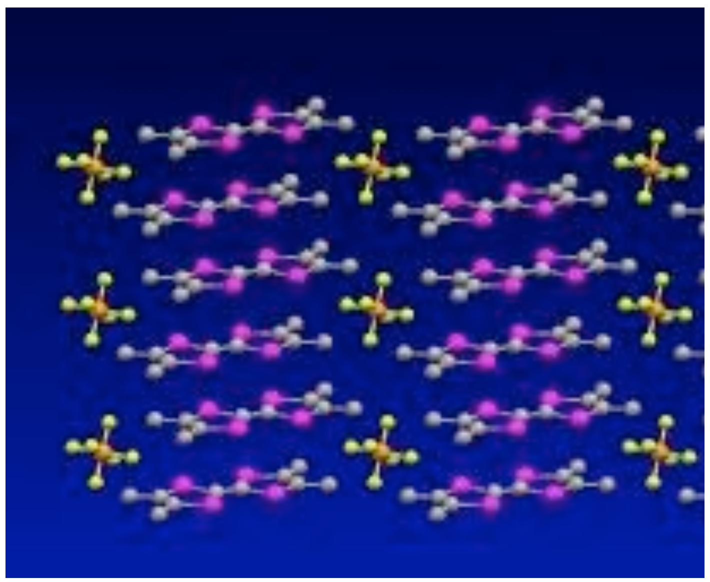
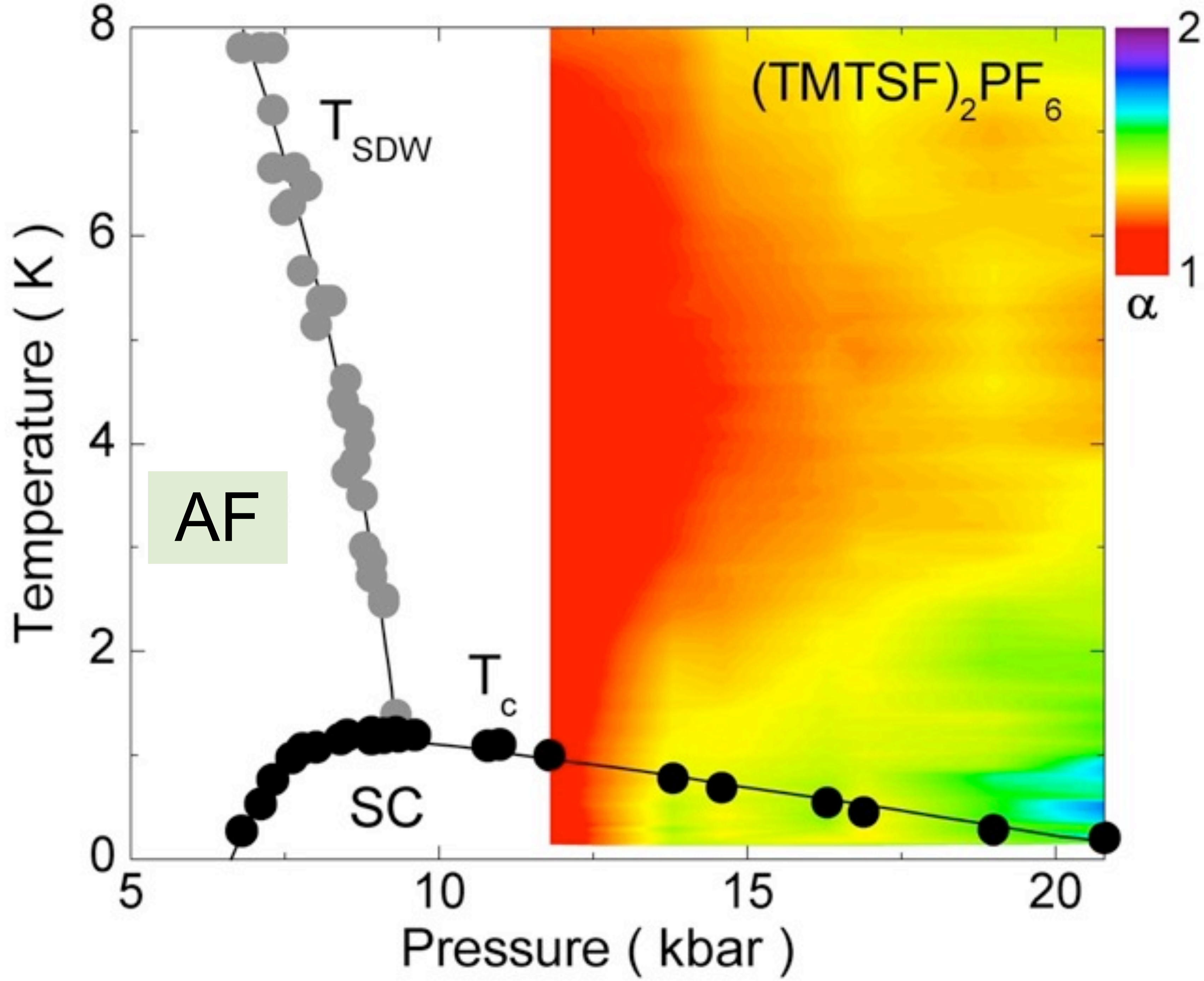
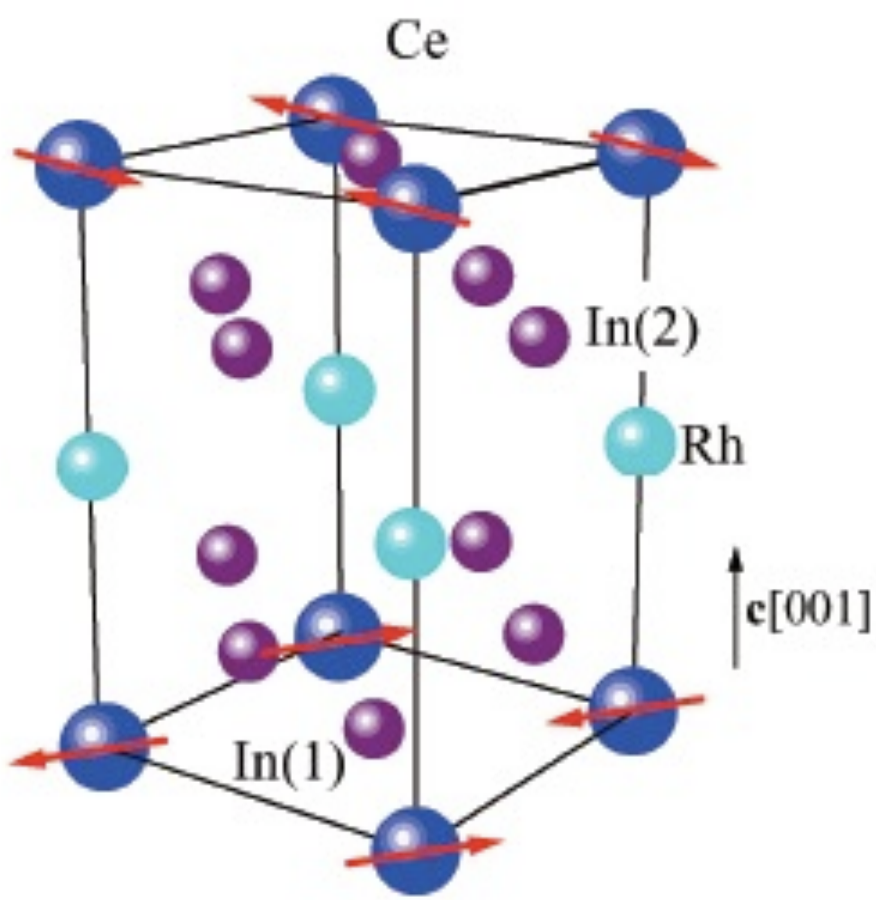
Temperature-pressure phase diagram of an organic superconductor

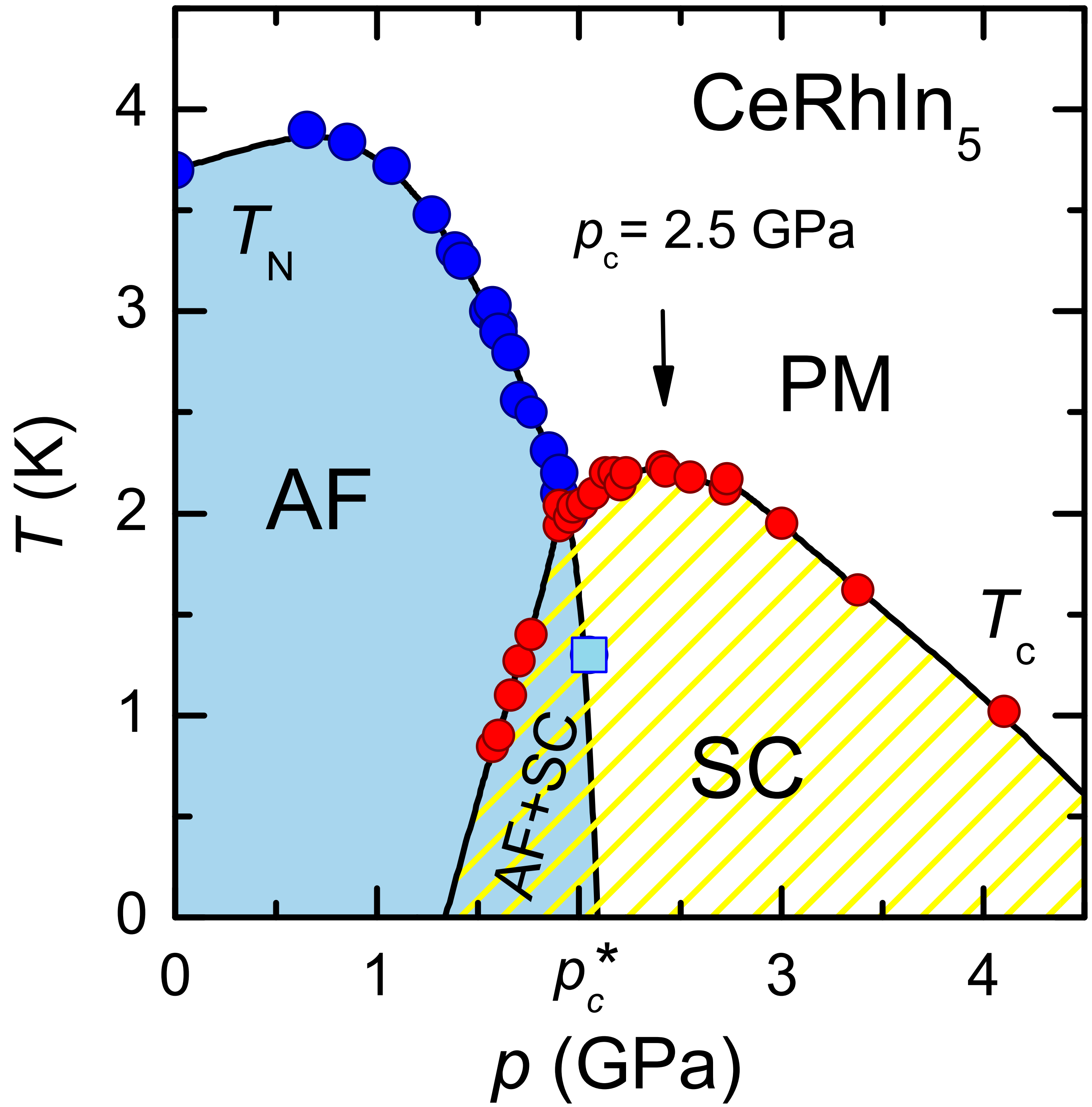




N. Doiron-Leyraud, P. Auban-Senzier, S. Rene de Cotret, A. Sedeki, C. Bourbonnais, D. Jerome, K. Bechgaard, and Louis Taillefer, Physical Review B 80, 214531 (2009)

Temperature-pressure phase diagram of an heavy-fermion superconductor

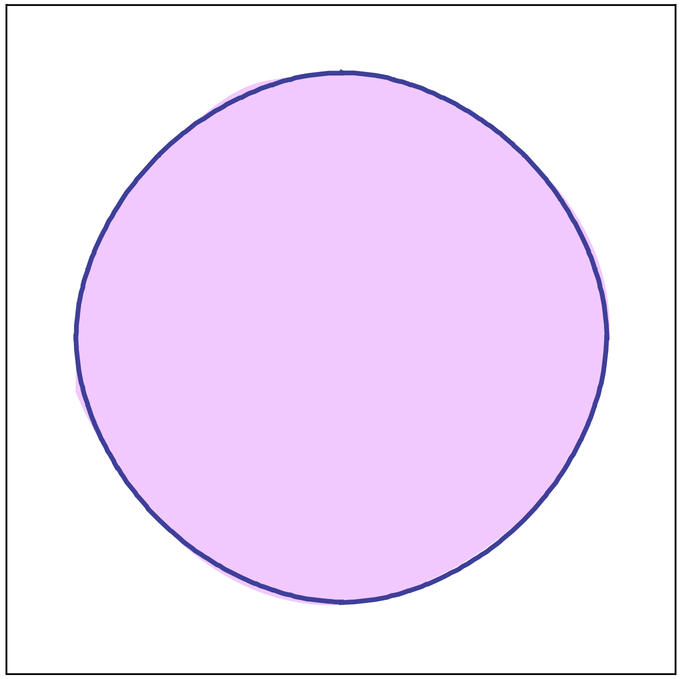




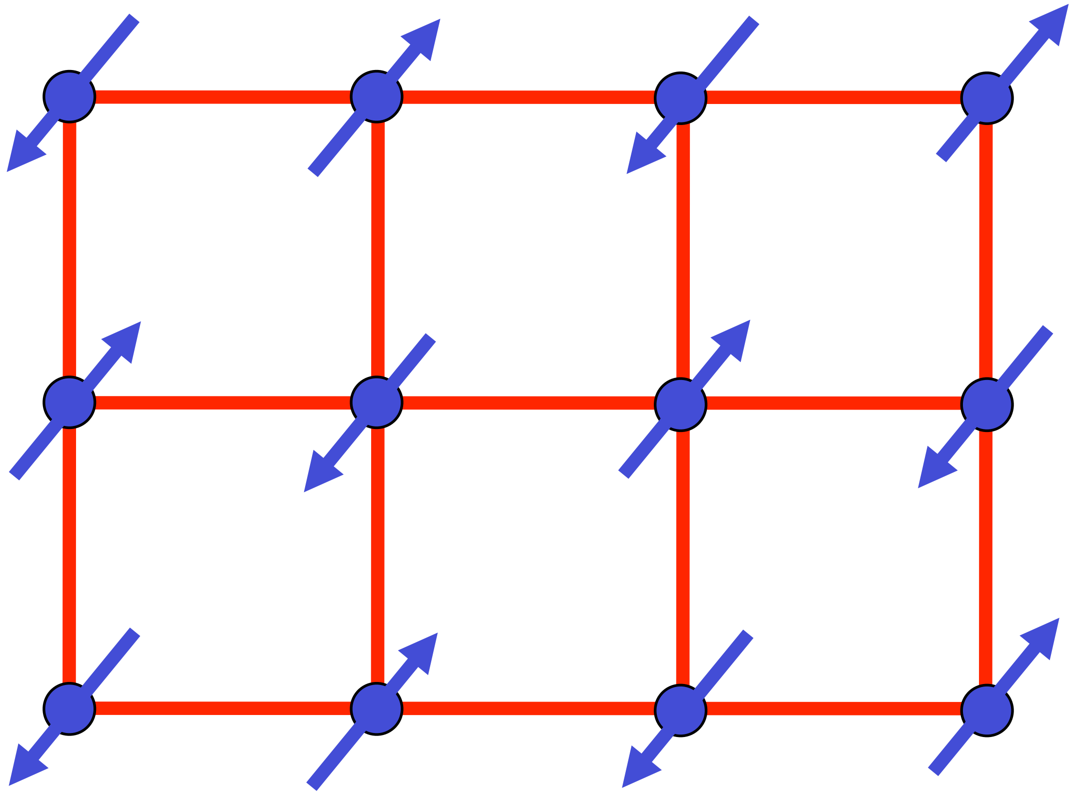
G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.
Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

Fermi surface+antiferromagnetism

Metal with "large" Fermi surface



+

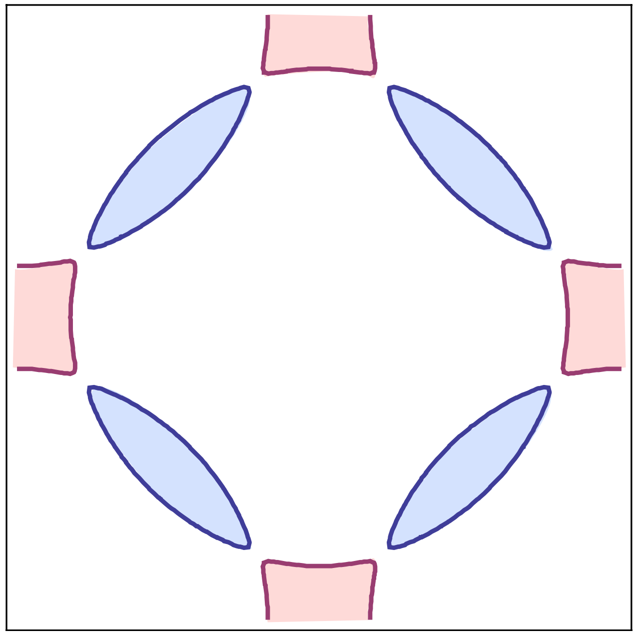


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

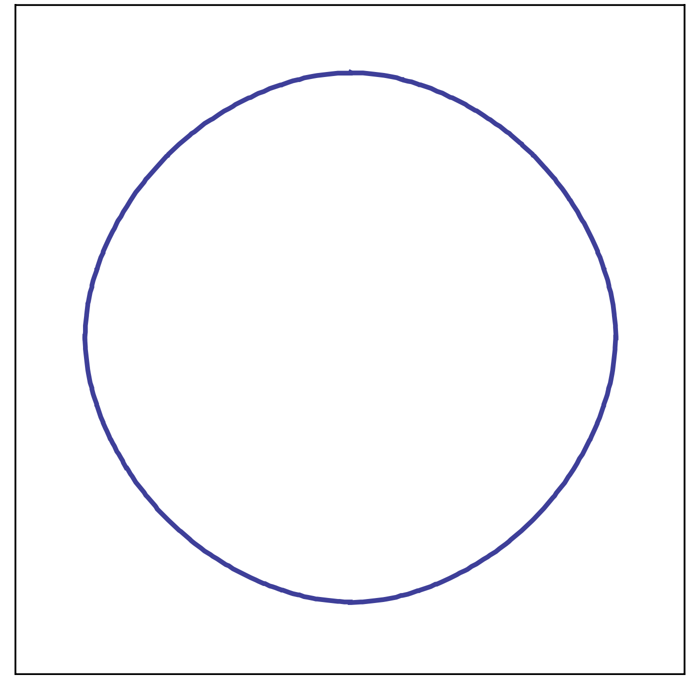
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism



AF $\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

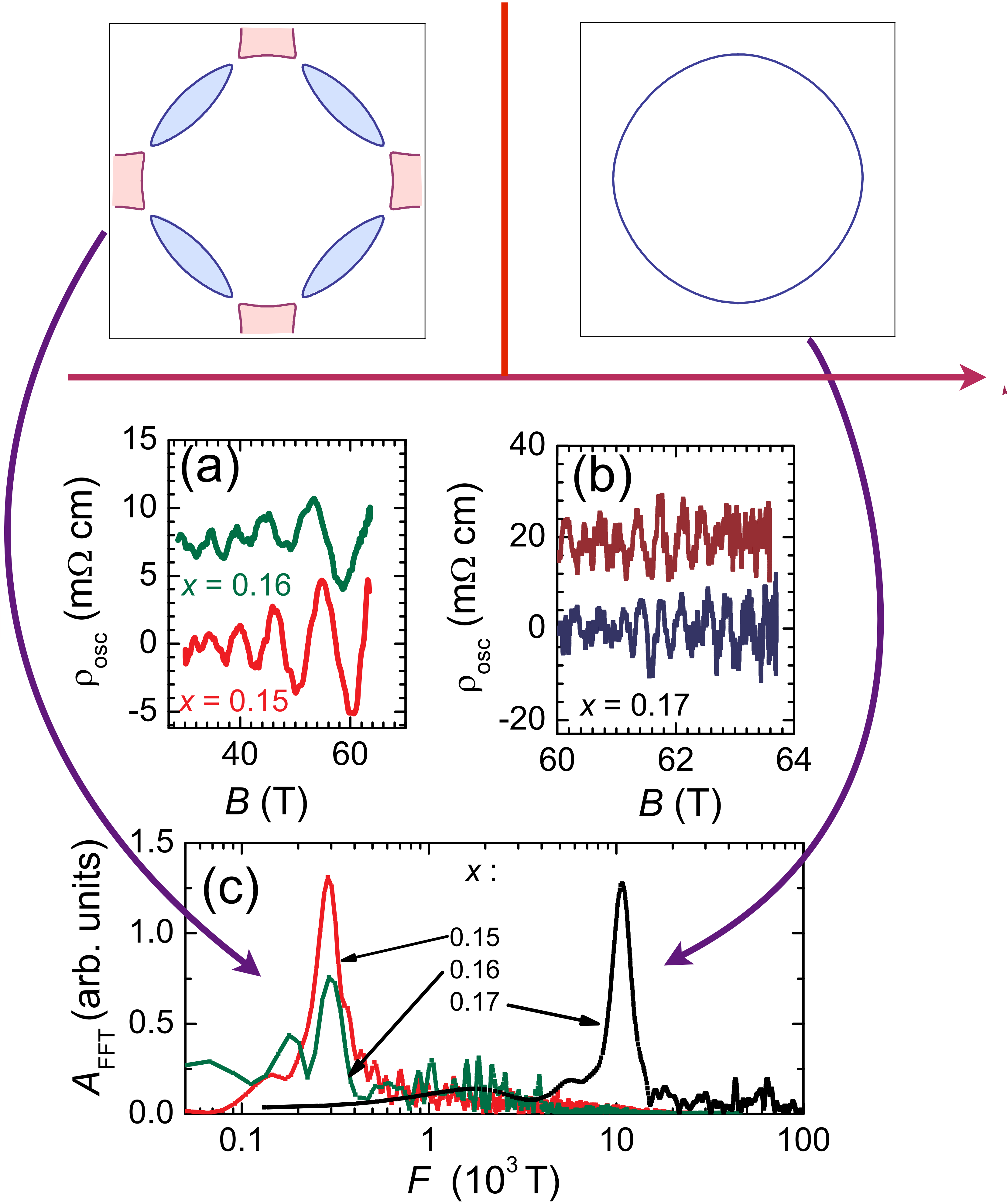
Increasing interaction

Fermi surface reconstruction and onset of antiferromagnetism

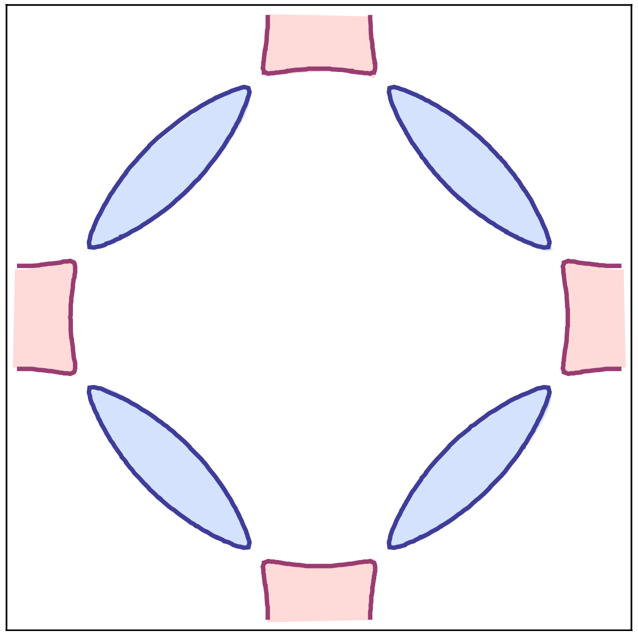
Quantum oscillations

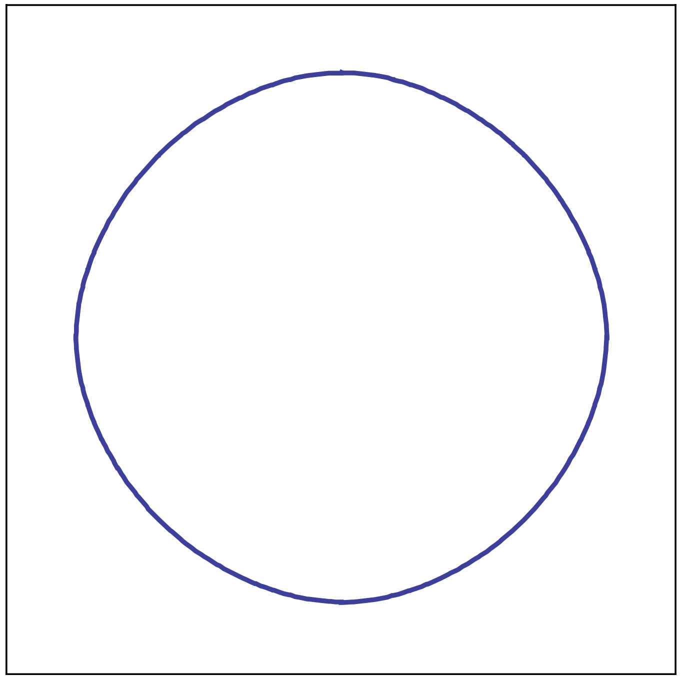
$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).



Fermi surface+antiferromagnetism





AF $\langle \vec{\varphi} \rangle \neq 0$

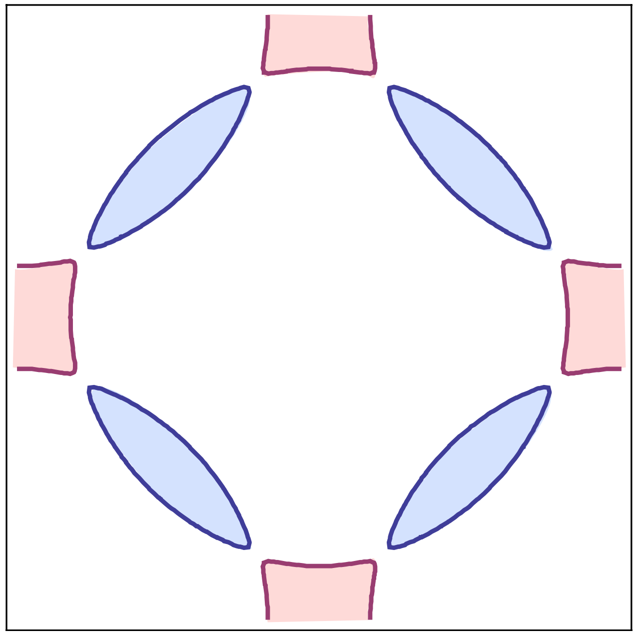
$\langle \vec{\varphi} \rangle = 0$

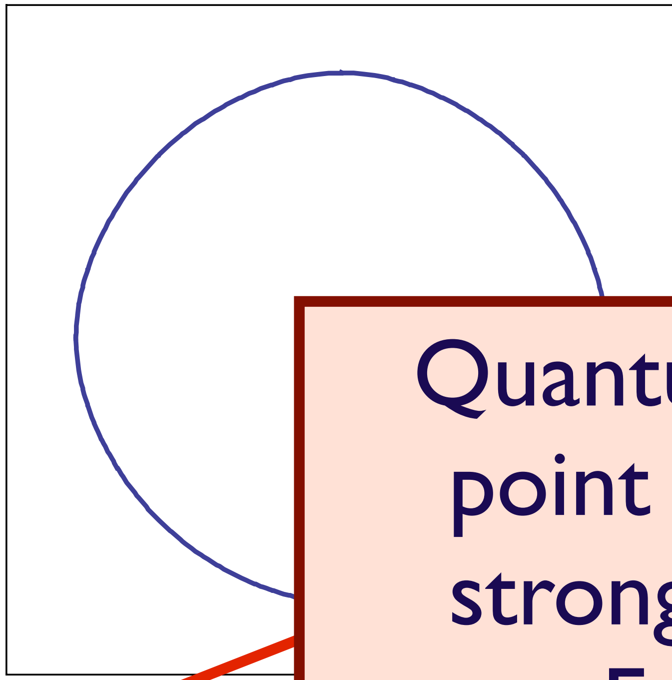
Metal with electron and hole pockets

Metal with "large" Fermi surface

s

Fermi surface+antiferromagnetism





Quantum critical point realizes a strong-coupled non-Fermi liquid compressible phase

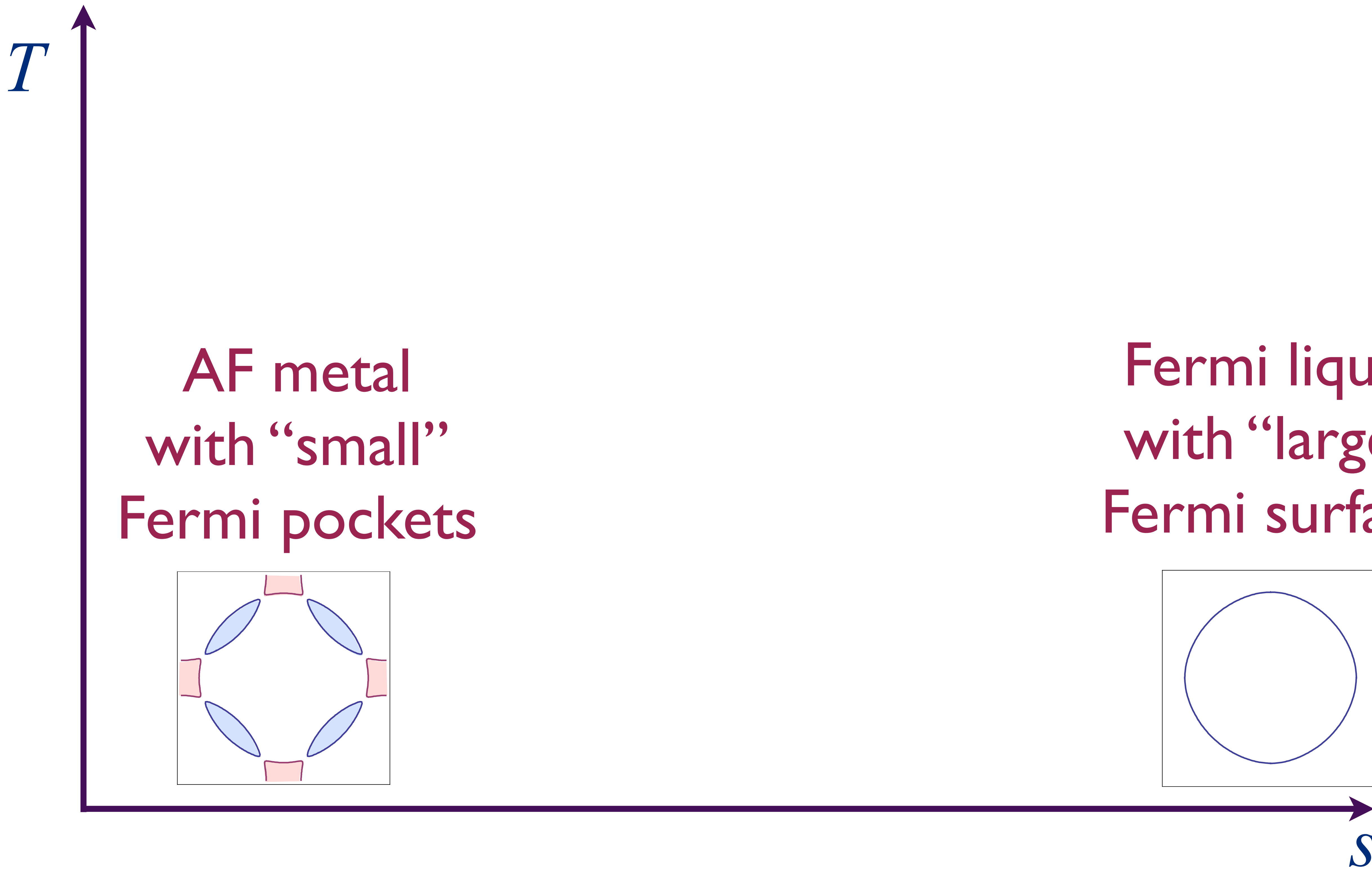
AF $\langle\vec{\varphi}\rangle \neq 0$

$\langle\vec{\varphi}\rangle = 0$

Metal with electron and hole pockets

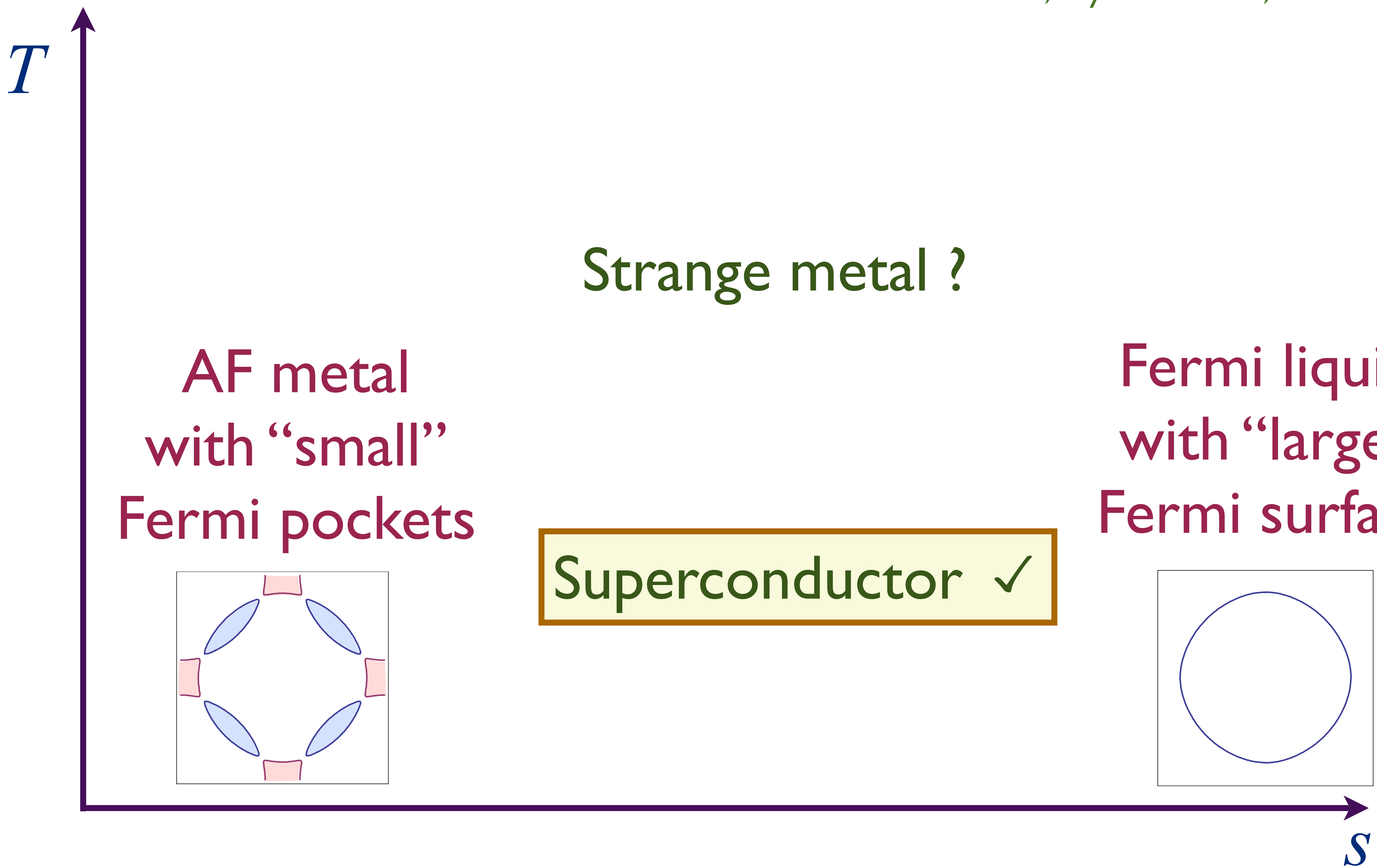
Metal with "large" Fermi surface

s



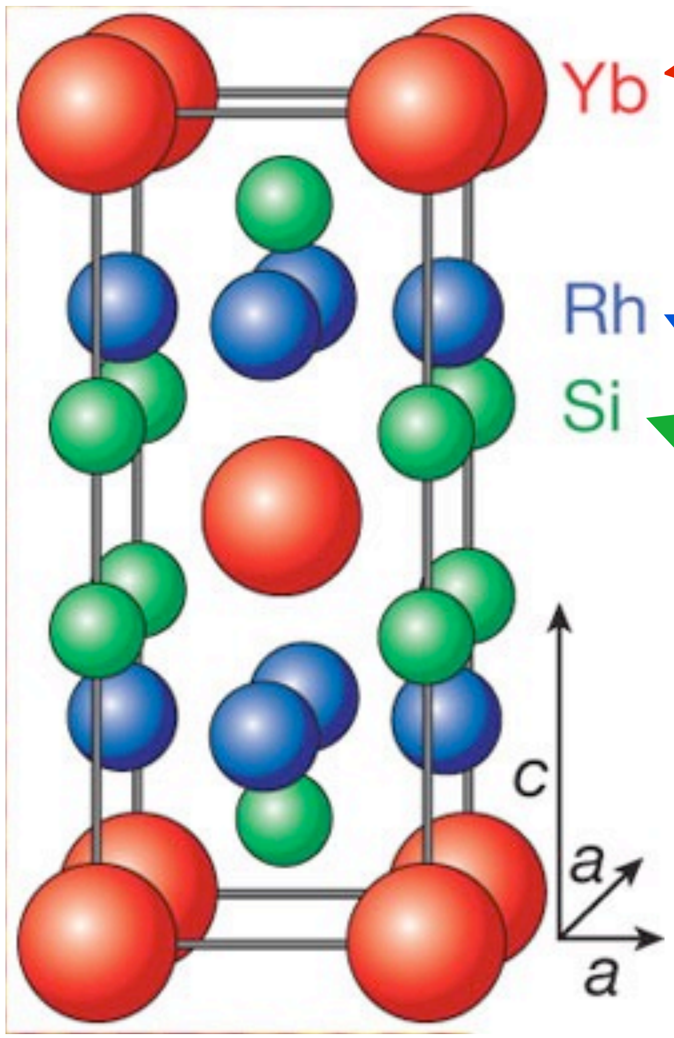
T
AF metal
with "small"
Fermi pockets
Fermi liquid
with "large"
Fermi surface
s

D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
Ar. Abanov, A.V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



Davis

The "heavy fermion" compounds

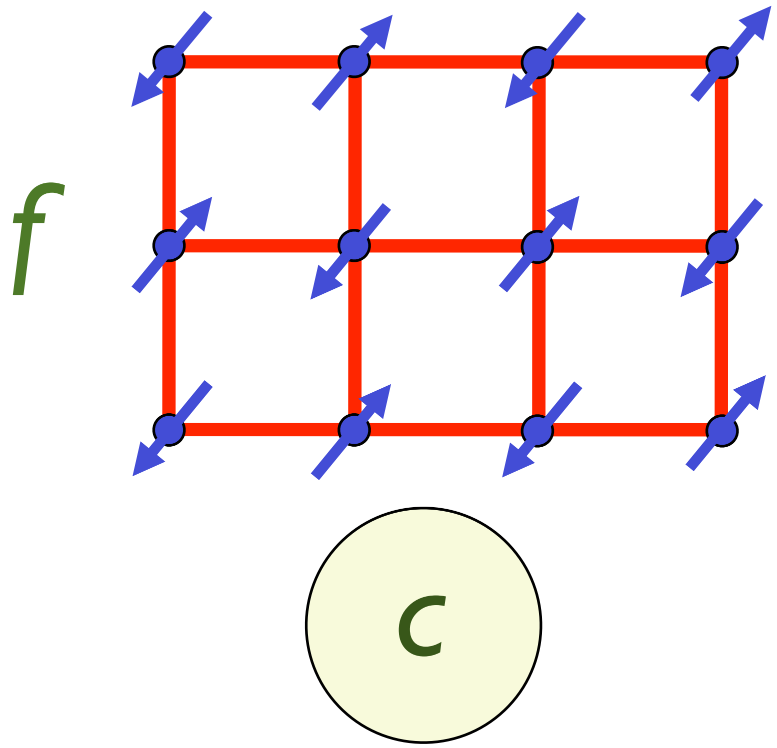


Localized magnetic moments on *f* orbitals

Delocalized conduction electrons

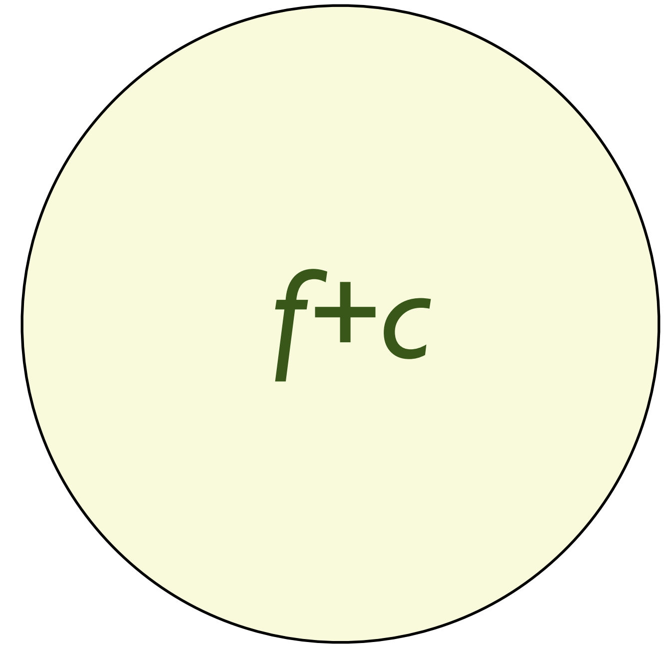
Described by "Kondo lattice" models

Magnetic order and the heavy Fermi liquid in the Kondo lattice



$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:
f-electron moments
and
c-conduction electron
Fermi surface

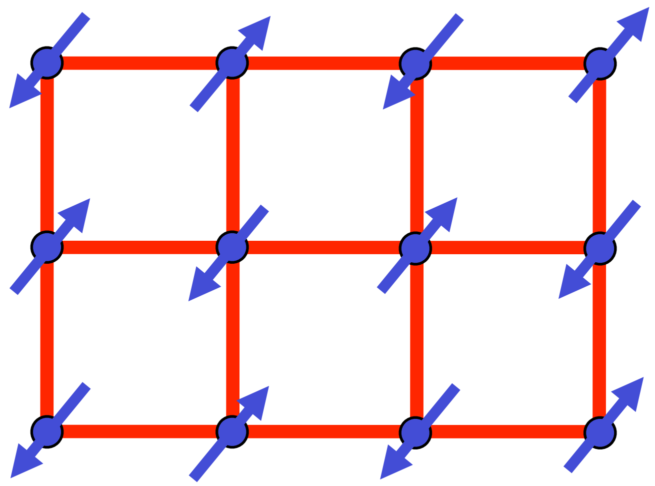


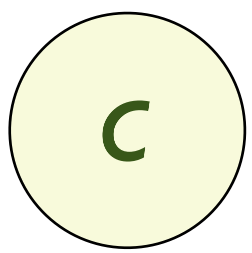
$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid
with “large” Fermi
surface of
hydridized *f* and
c-conduction
electrons

Separating onset of AF order and the heavy Fermi liquid in the Kondo lattice

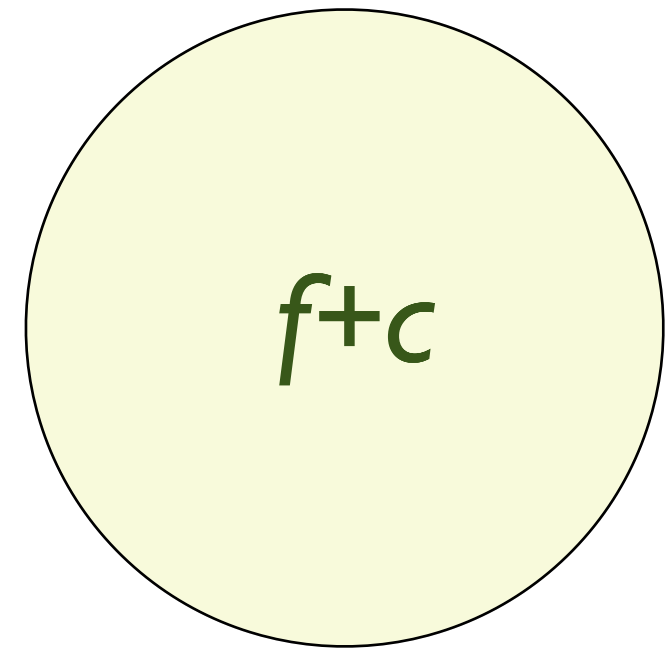
f





$\langle \vec{\varphi} \rangle \neq 0$

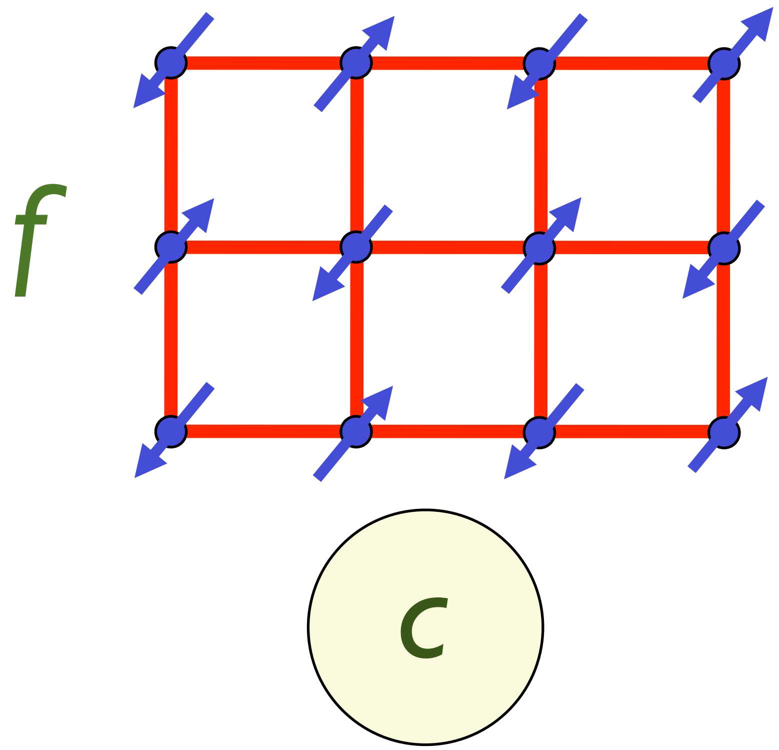
AF Metal:
f-electron moments
and
c-conduction electron
Fermi surface



$\langle \vec{\varphi} \rangle = 0$

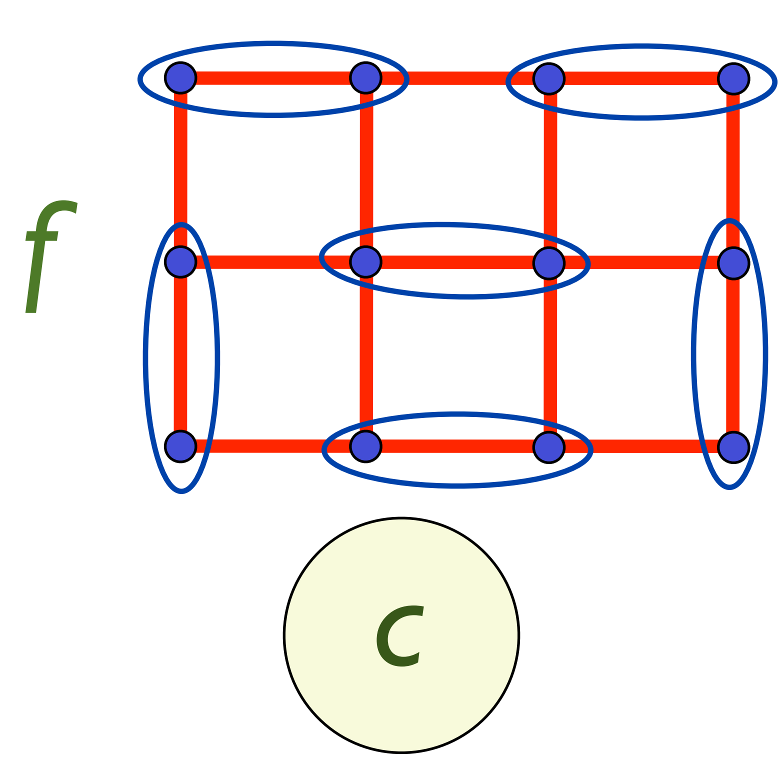
Heavy Fermi liquid
with “large” Fermi
surface of
hydridized *f* and
c-conduction
electrons

Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



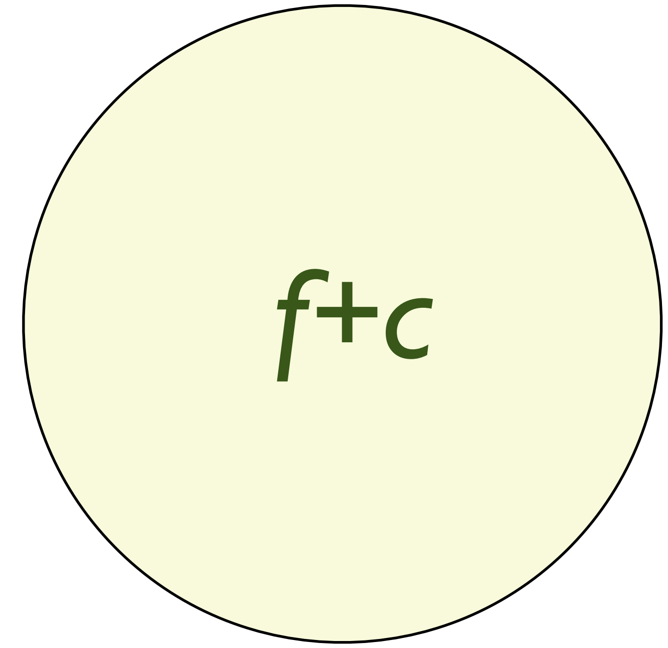
$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal:
f-electron moments
and
c-conduction electron
Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

Conduction electron
Fermi surface
and
spin-liquid of
f-electrons



$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid
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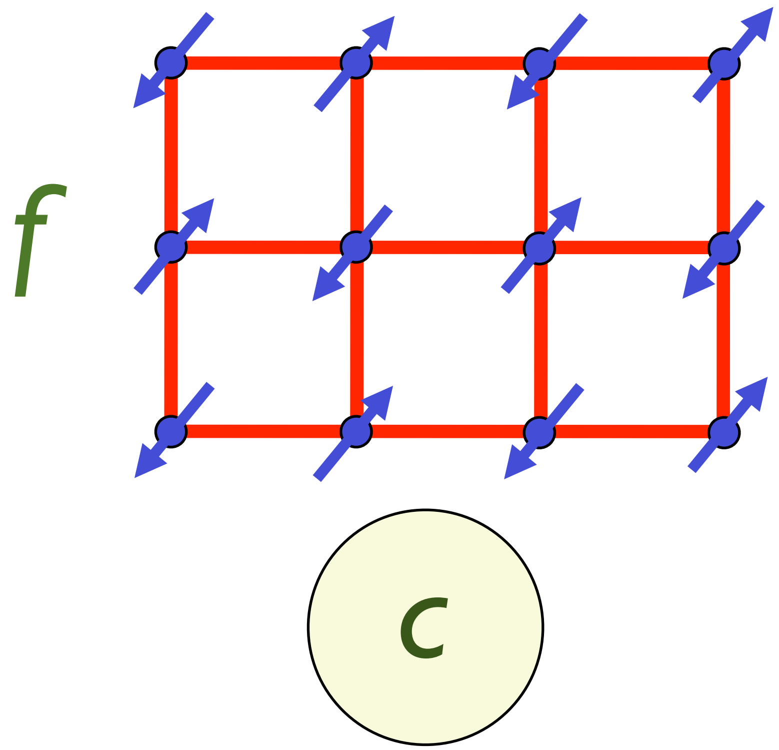
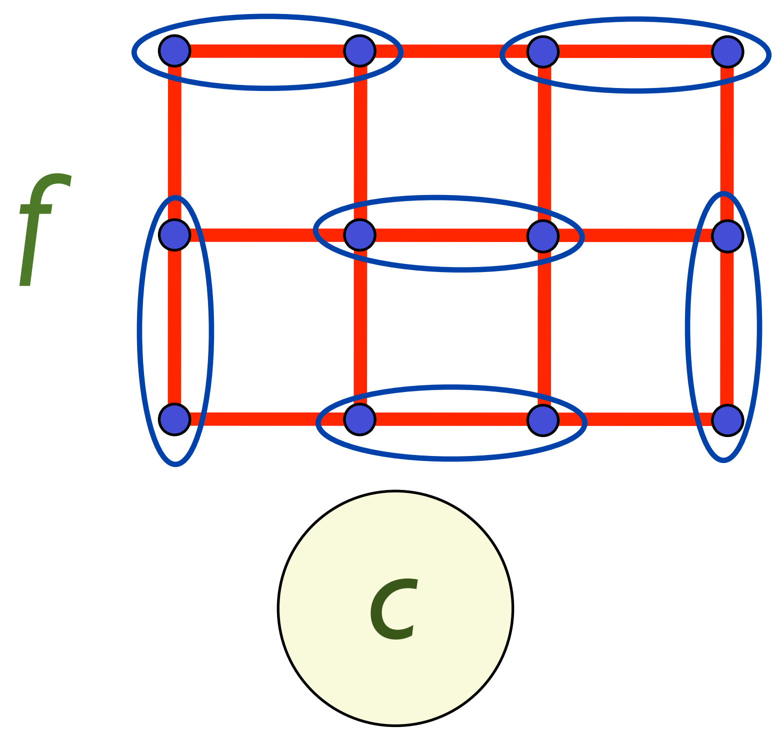


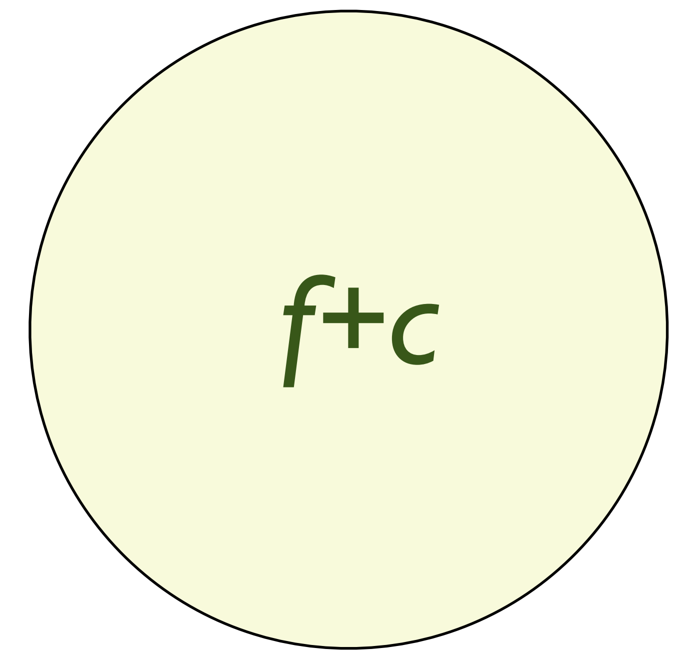
$$\langle \vec{\varphi} \rangle \neq 0$$

Magnetic Metal: *f*-electron moments and *c*-conduction electron Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface

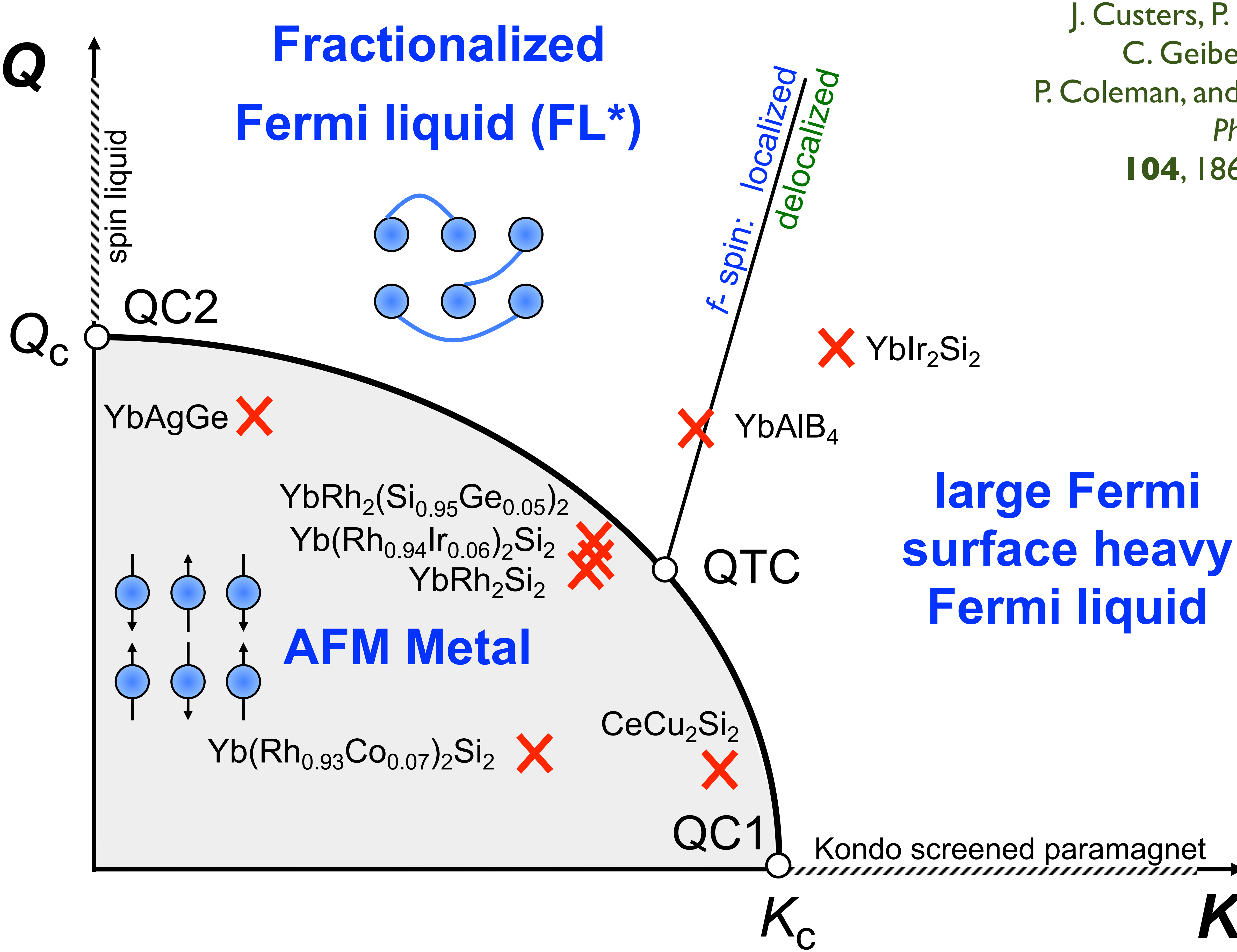


$$\langle \vec{\varphi} \rangle = 0$$

Heavy Fermi liquid with "large" Fermi surface of hydridized *f* and *c*-conduction electrons

Experimental phase diagram

J. Custers, P. Gegenwart,
C. Geibel, F. Steglich,
P. Coleman, and S. Paschen,
Phys. Rev. Lett.
104, 186402 (2010)



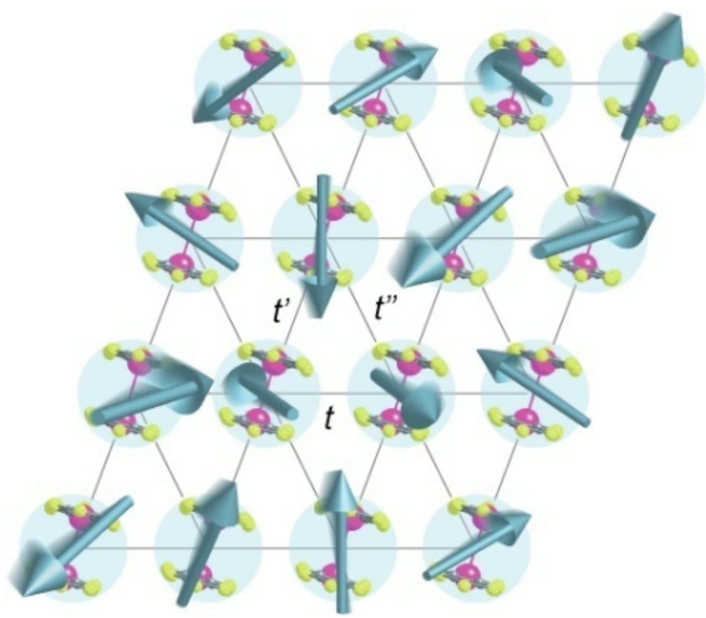
Exotic phases in heavy fermion compounds

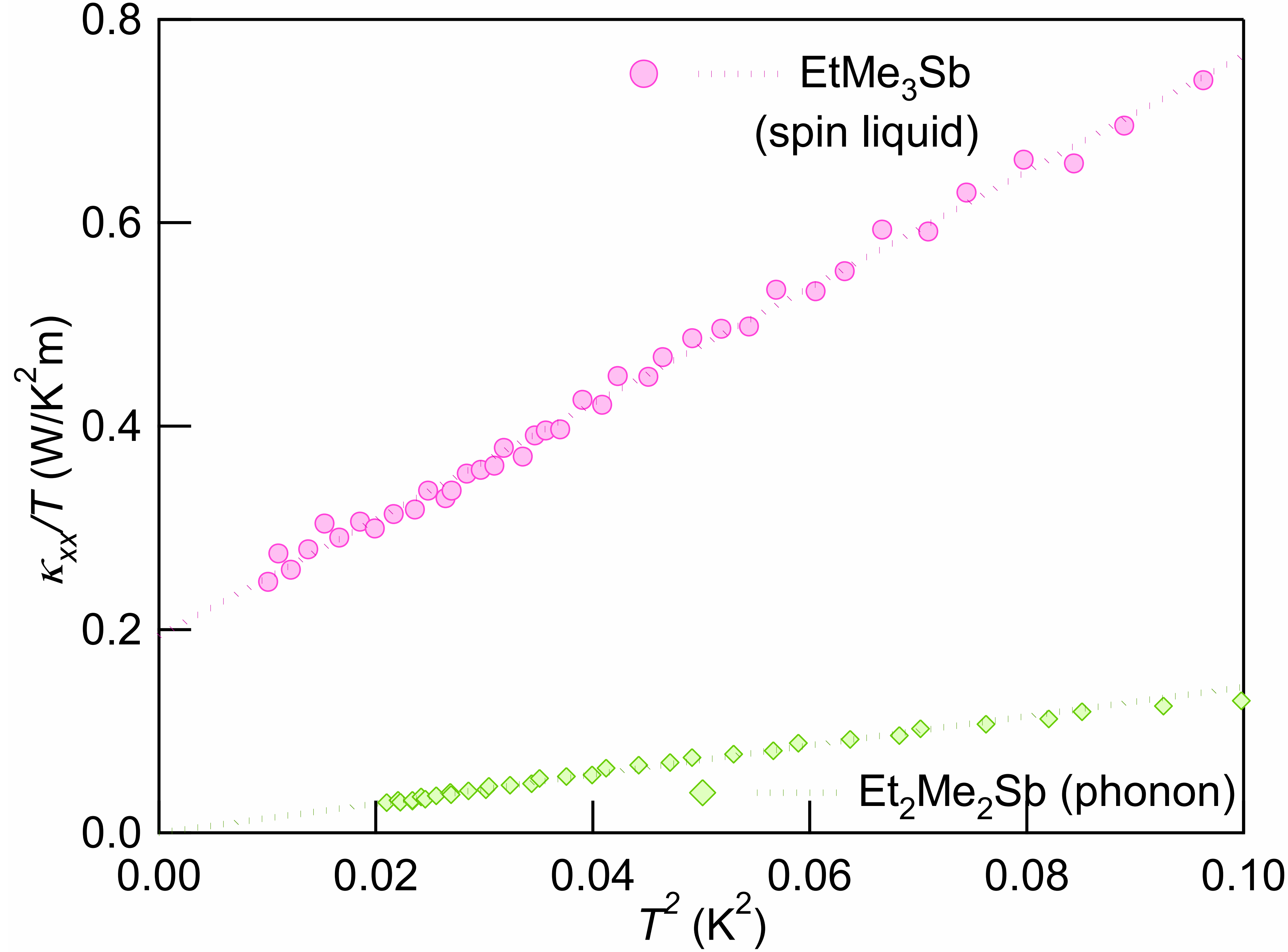
The volume of the Fermi surface is a direct experimental probe of "topological order" and long-range entanglement in a metallic state

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Spin liquid in $EtMe_3Sb[Pd(dmit)_2]_2$

Minoru Yamashita, Norihito Nakata, Yoshinori Senshu, Masaki Nagata, Hiroshi M. Yamamoto, Reizo Kato, Takasada Shibauchi, Yuji Matsuda, Science 328, 1246 (2010).





An exotic compressible "metal" of spin excitations

Fisher

Outline

1. Gapped quantum matter
   Insulators, quantum Hall states

2. Conformal quantum matter
   Graphene, antiferromagnets, ultracold atoms

3. Compressible quantum matter
   Metals, superconductors, strange metals

4. Connections to string theory

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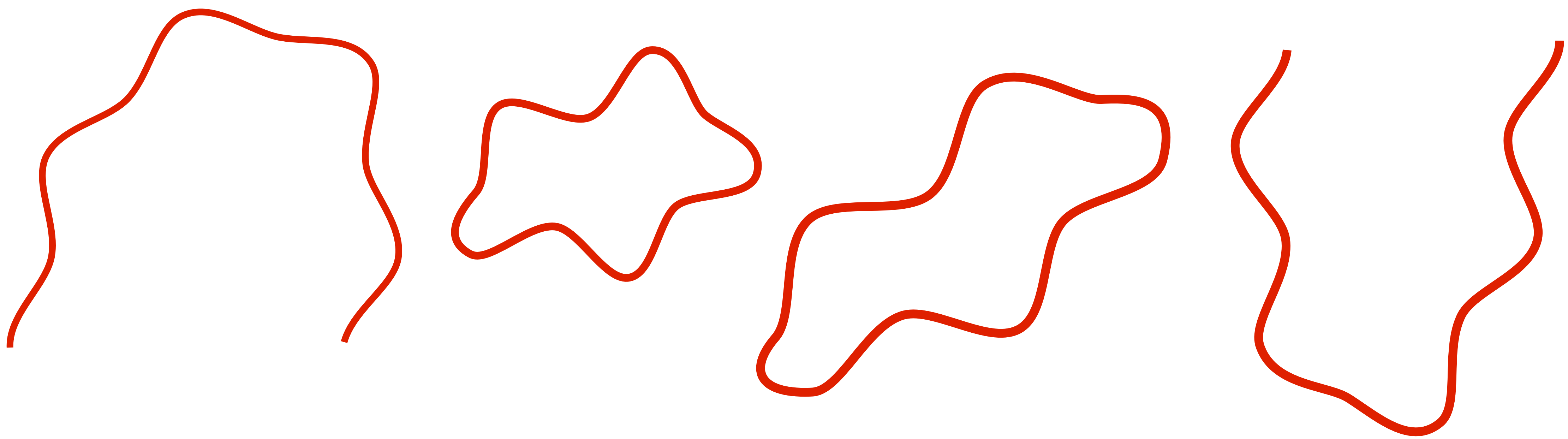
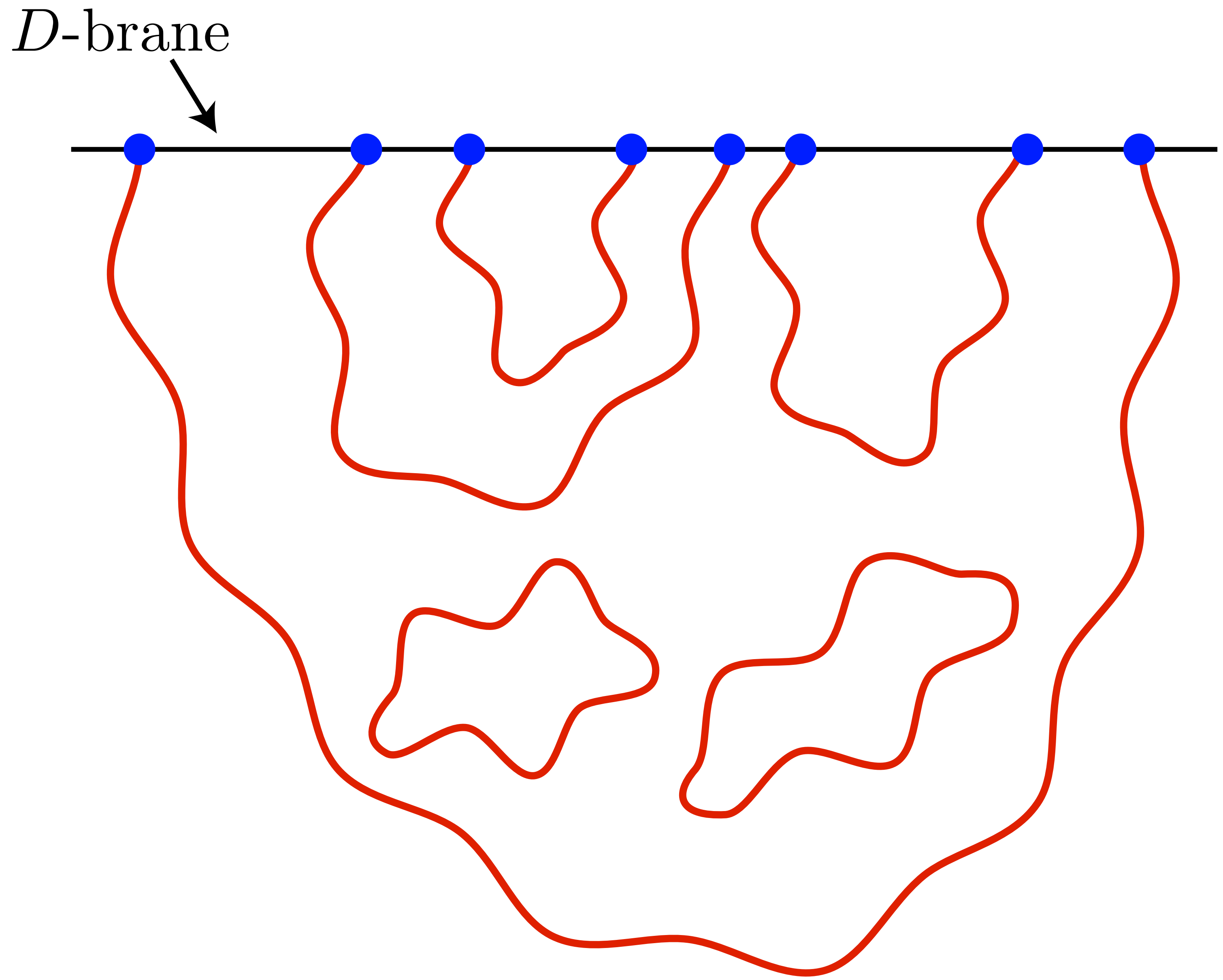
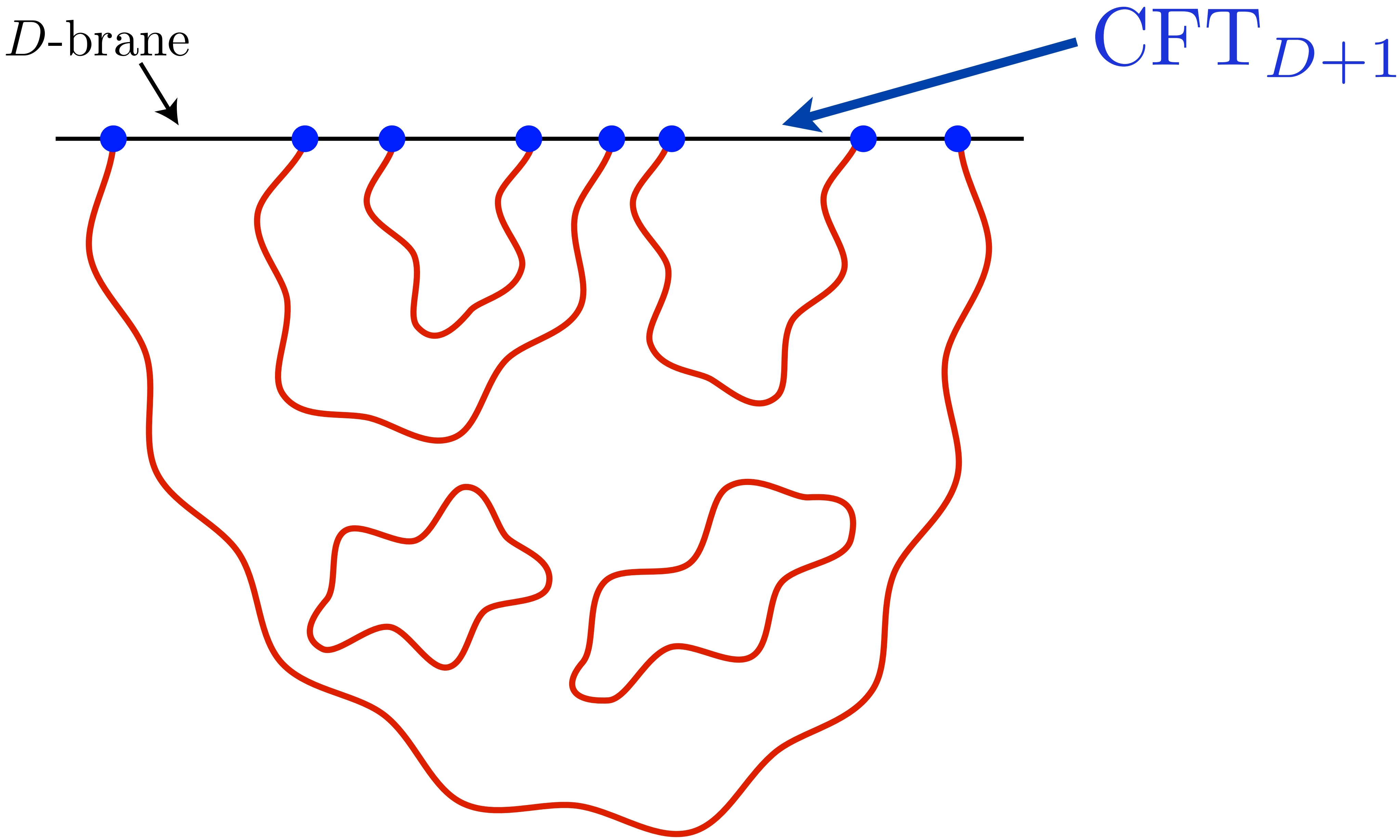
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...

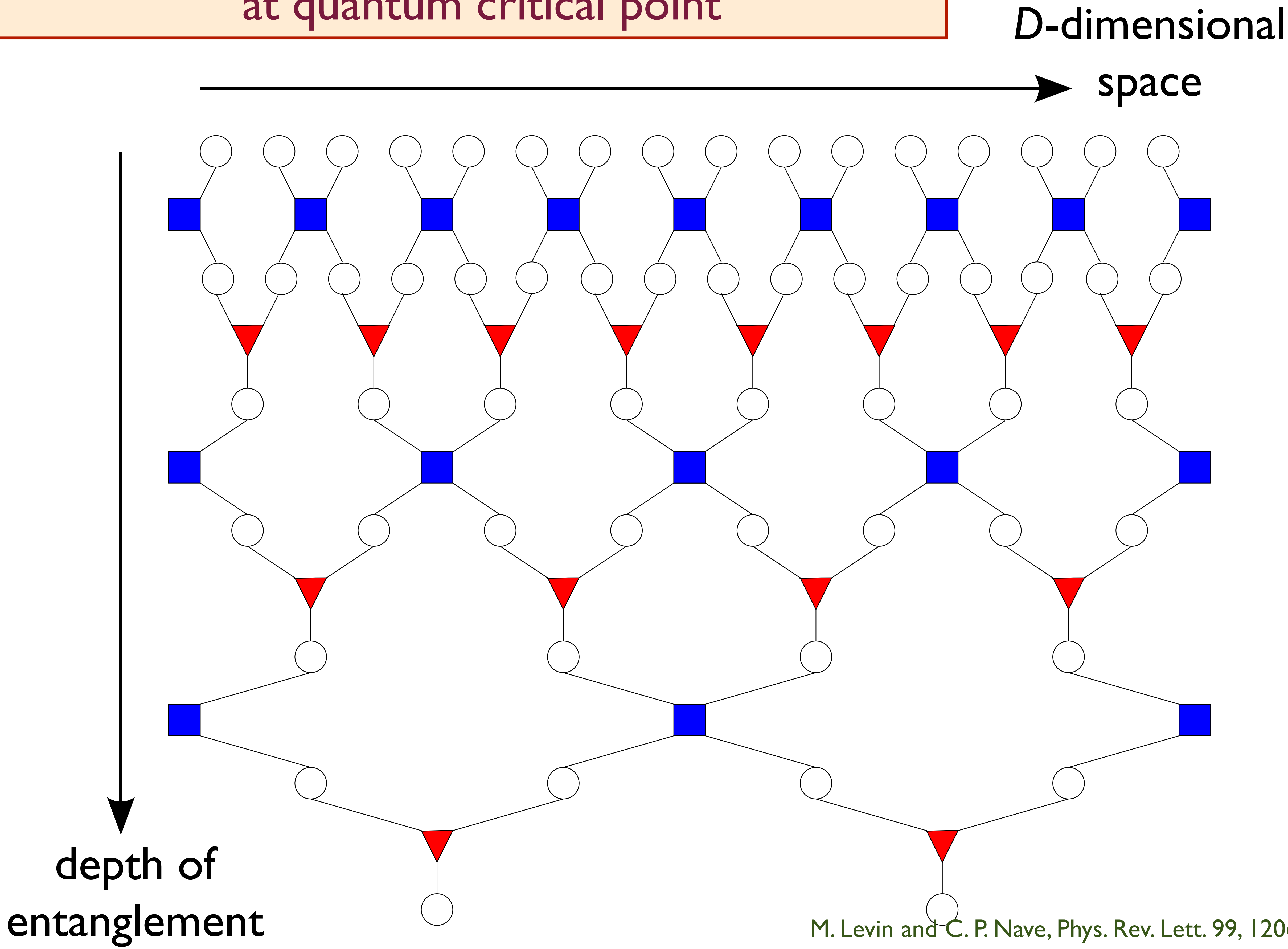


- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.

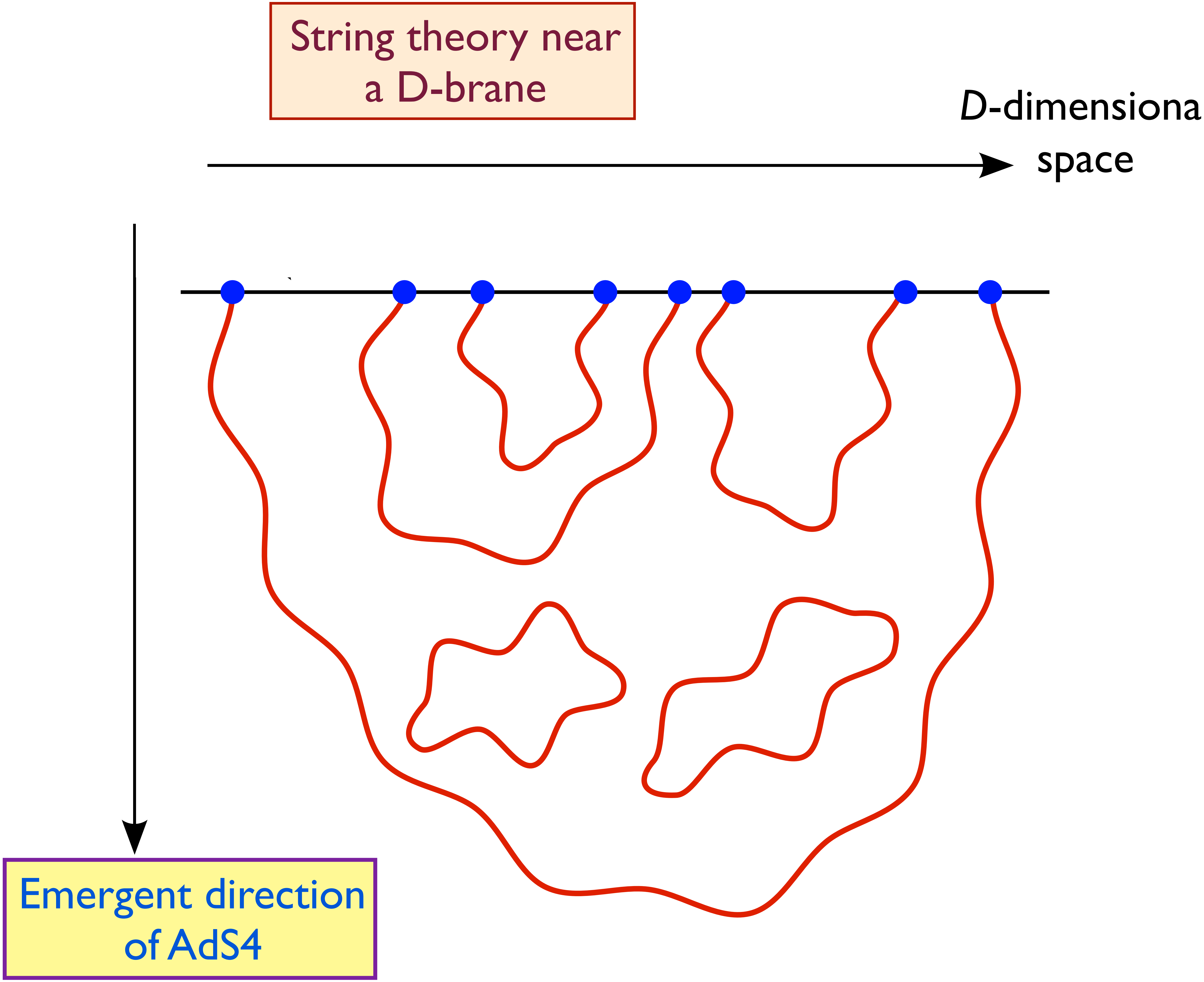


- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.
- In $D = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

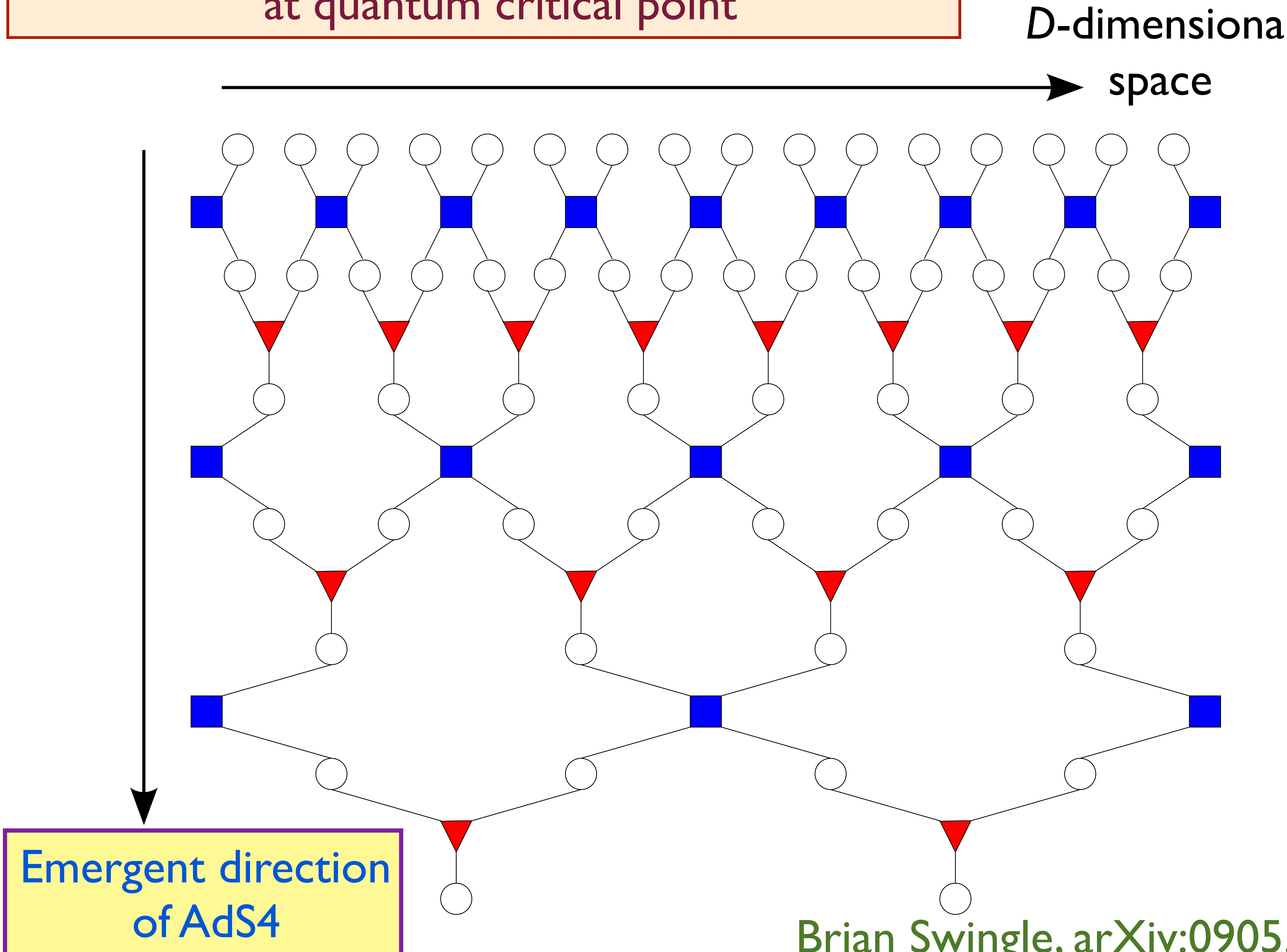
Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

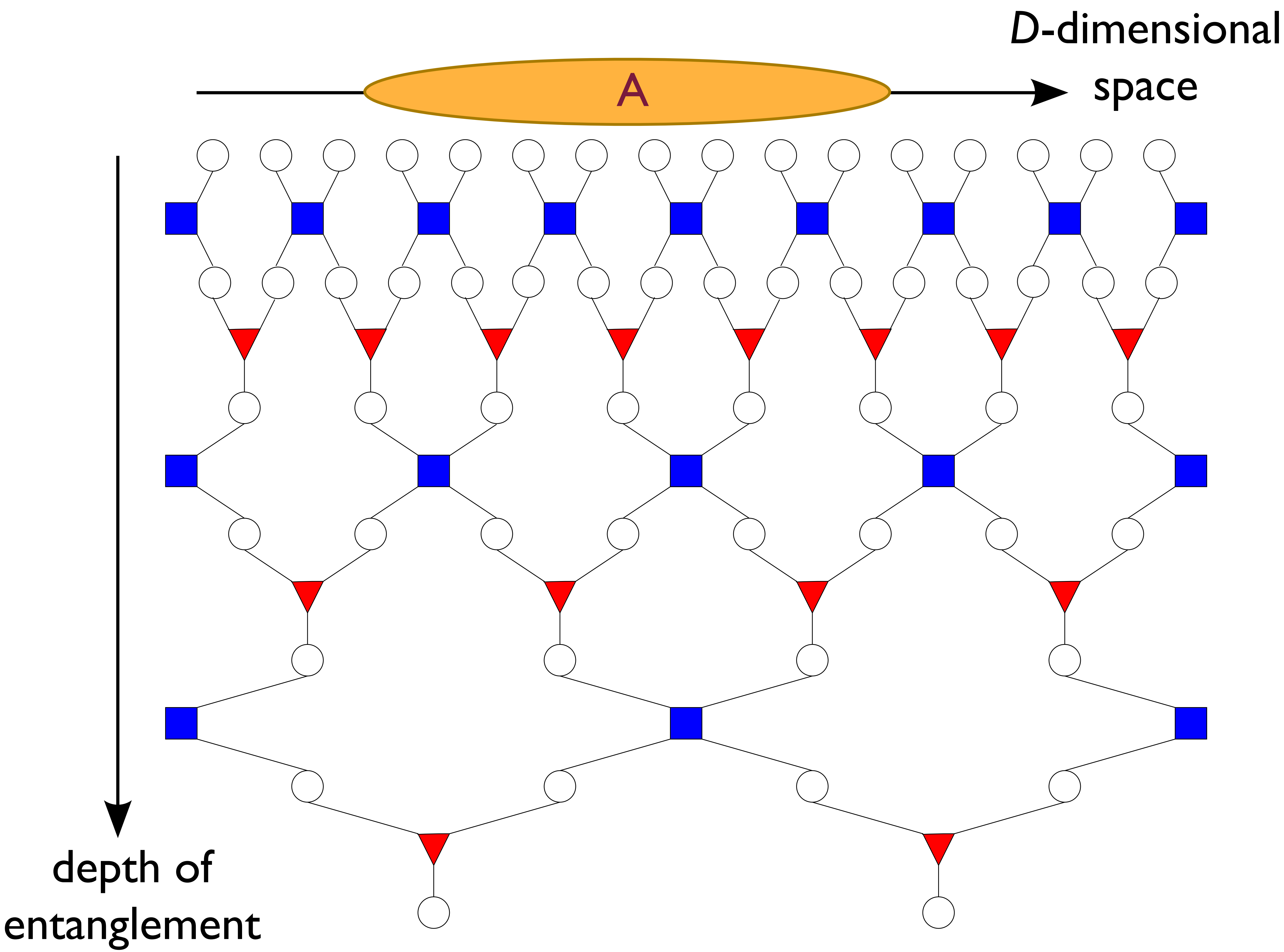

String theory near a D-brane
D-dimensional space
Emergent direction of AdS4

Tensor network representation of entanglement at quantum critical point

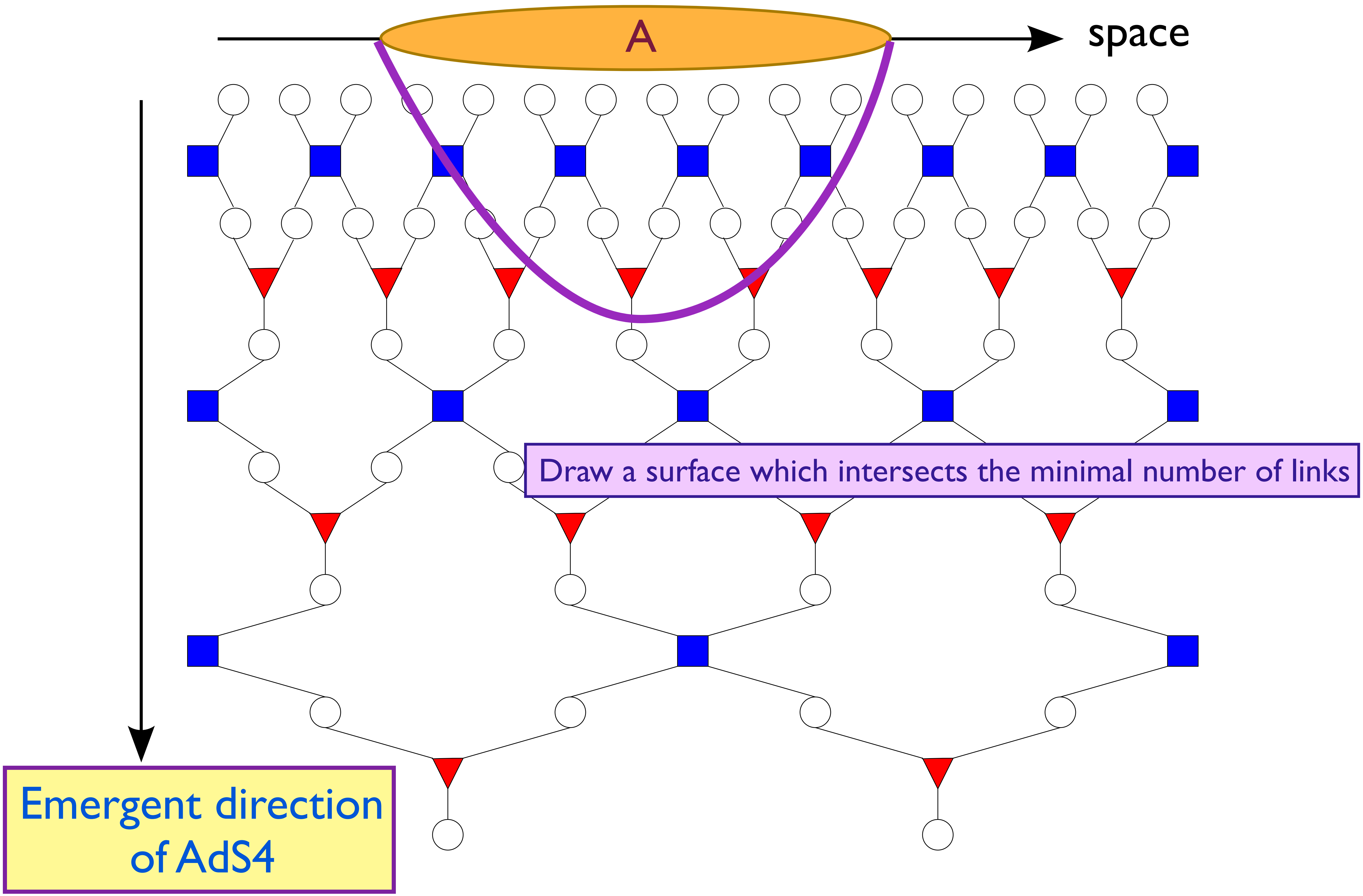


Brian Swingle, arXiv:0905.1317

Entanglement entropy



Entanglement entropy



Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

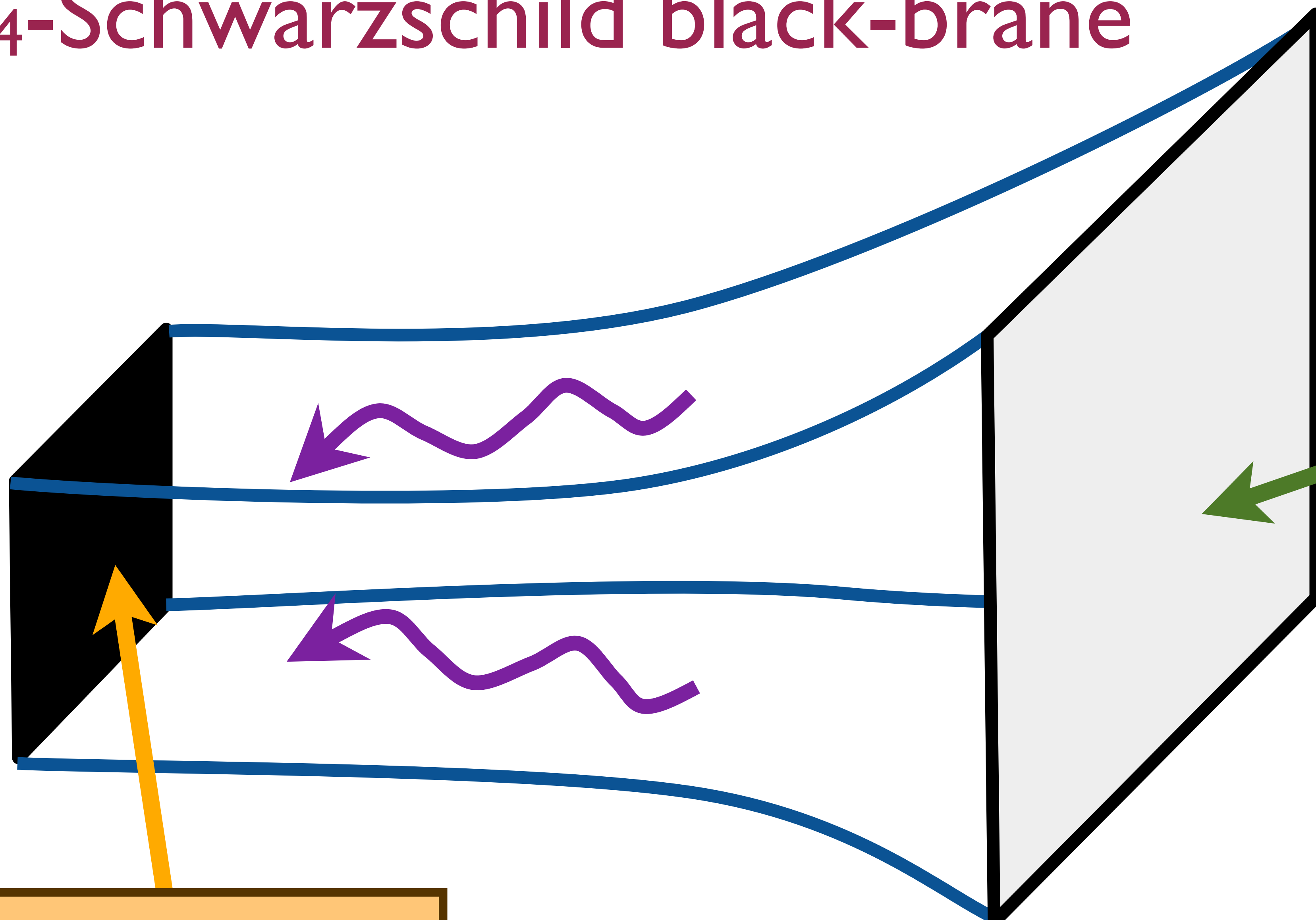
This can be seen both the string and tensor-network pictures

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317

Such string theory approaches have provided a useful description of conformal quantum matter, especially at non-zero temperatures.

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point

Black-brane at temperature of 2+1 dimensional quantum critical system

Provides successful description of many properties of conformal quantum matter

Such string theory approaches have provided a useful description of conformal quantum matter, especially at non-zero temperatures.

Challenge to string theory:

Classify states of compressible quantum matter in continuum theories which preserve translational invariance.

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Can we obtain holographic theories of superfluids and Fermi liquids?

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Yes

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Can we obtain holographic theories of superfluids and Fermi liquids? Yes

Are there any other compressible phases?

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Conclusions

1. Gapped quantum matter

   Spin liquids, quantum Hall states

2. Conformal quantum matter

   Graphene, antiferromagnets, ultracold atoms

3. Compressible quantum matter

   High temperature superconductors, strange metals, heavy fermion metals

4. Connections to string theory