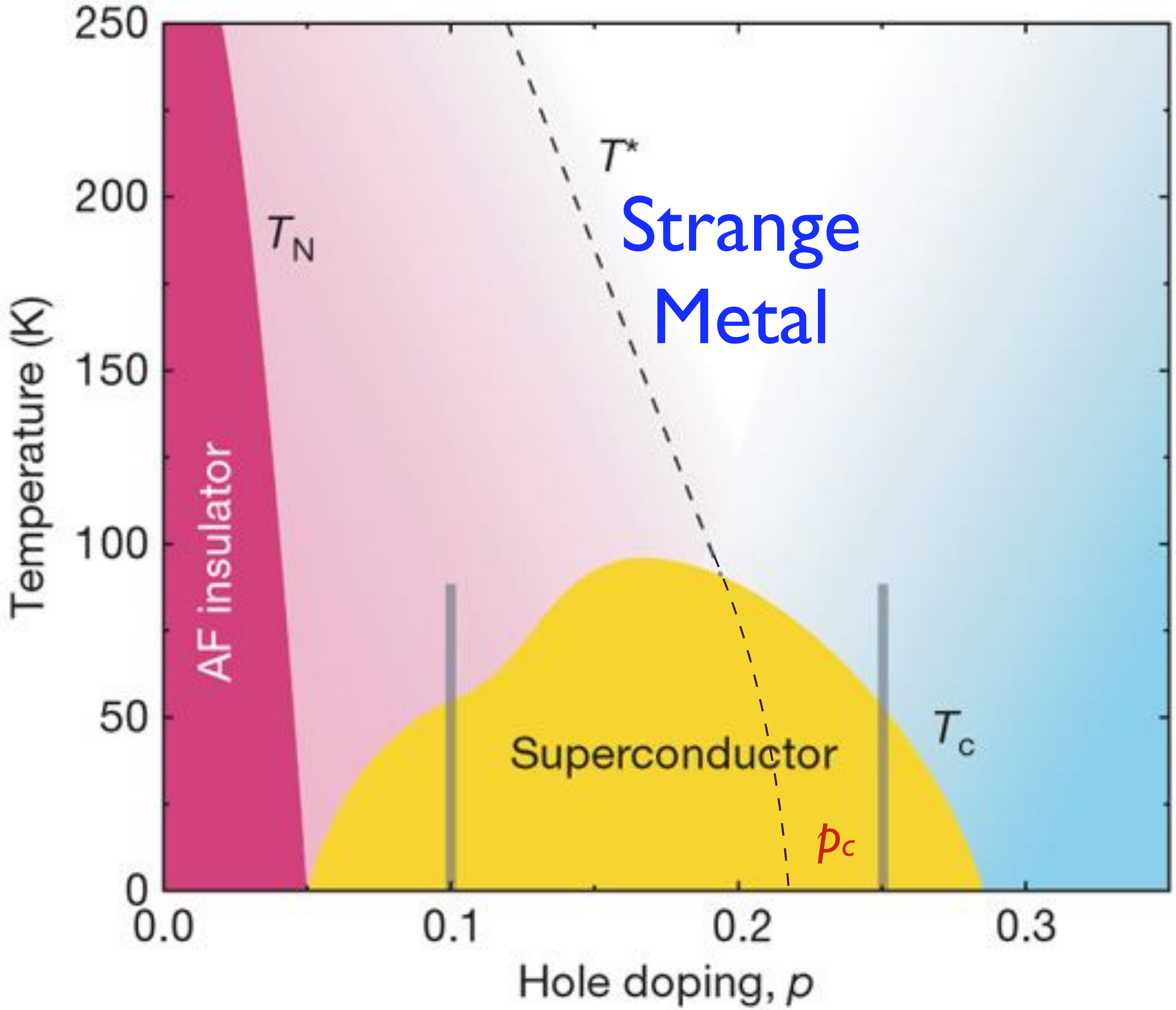
Universal theory of transport in strange metals

Statphys Kolkata XII
S. N. Bose National Center for Basic Sciences, Kolkata
December 20, 2023

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD


250
200
150
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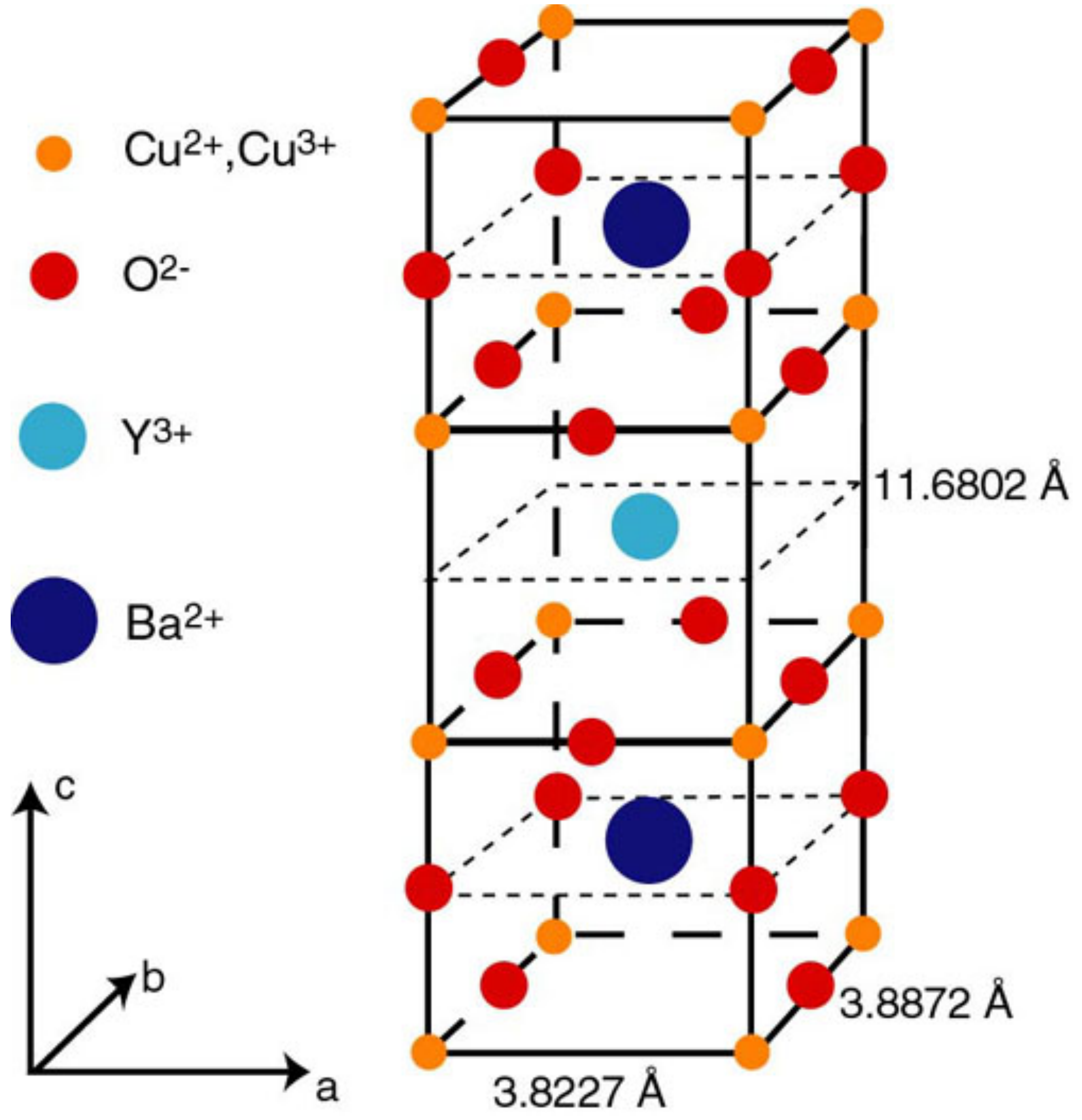
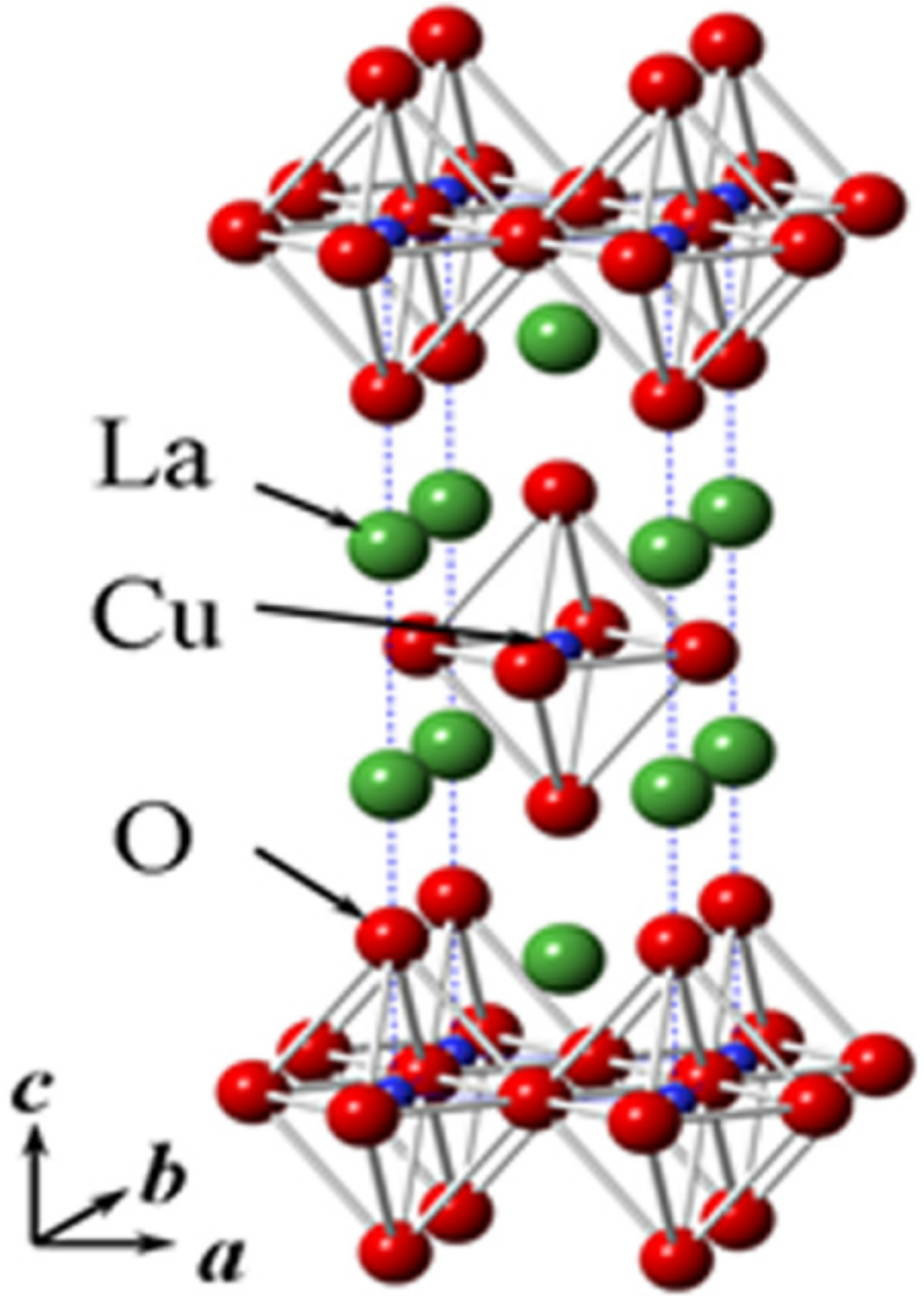
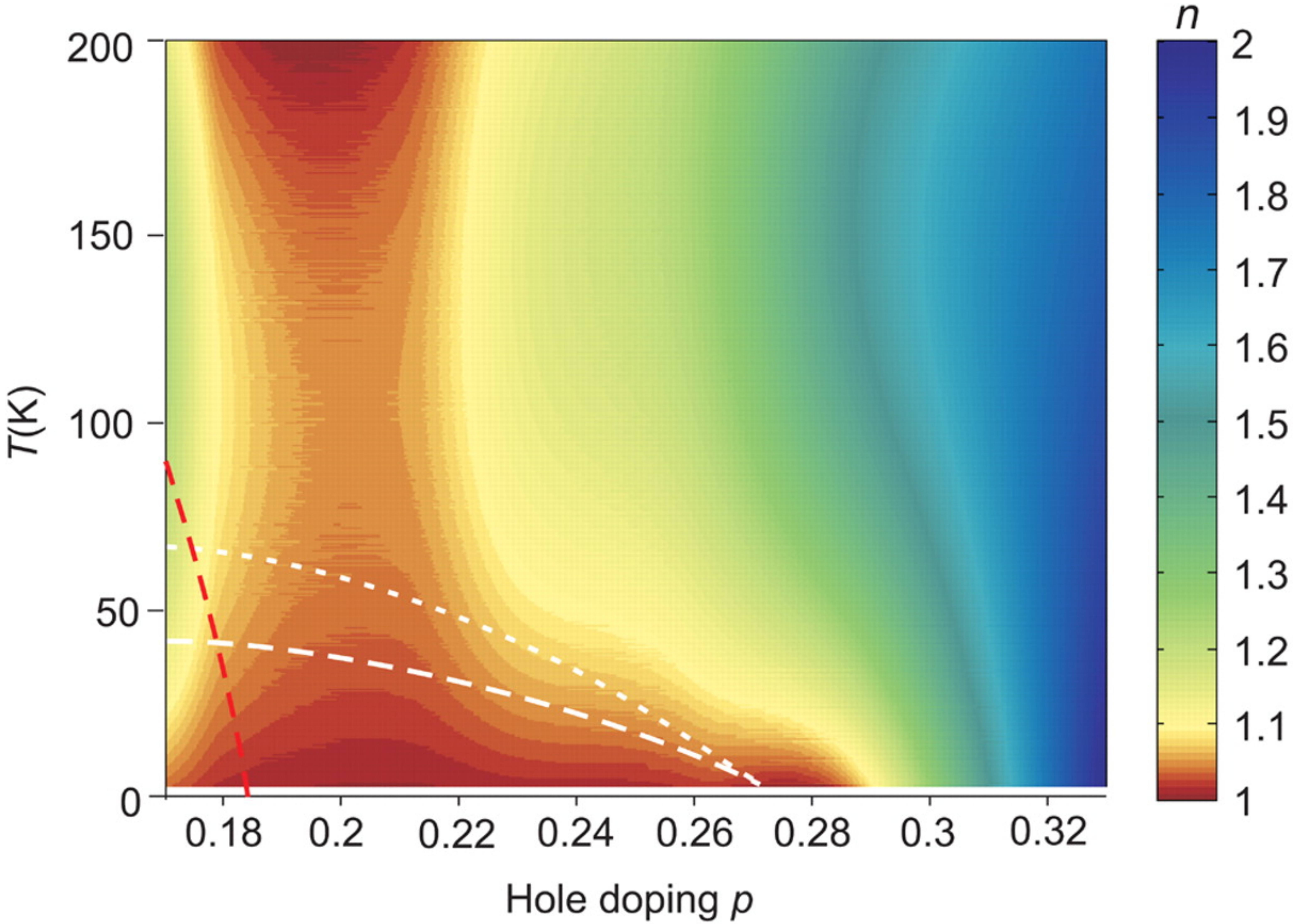
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Superconductor
T_c
p_c

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

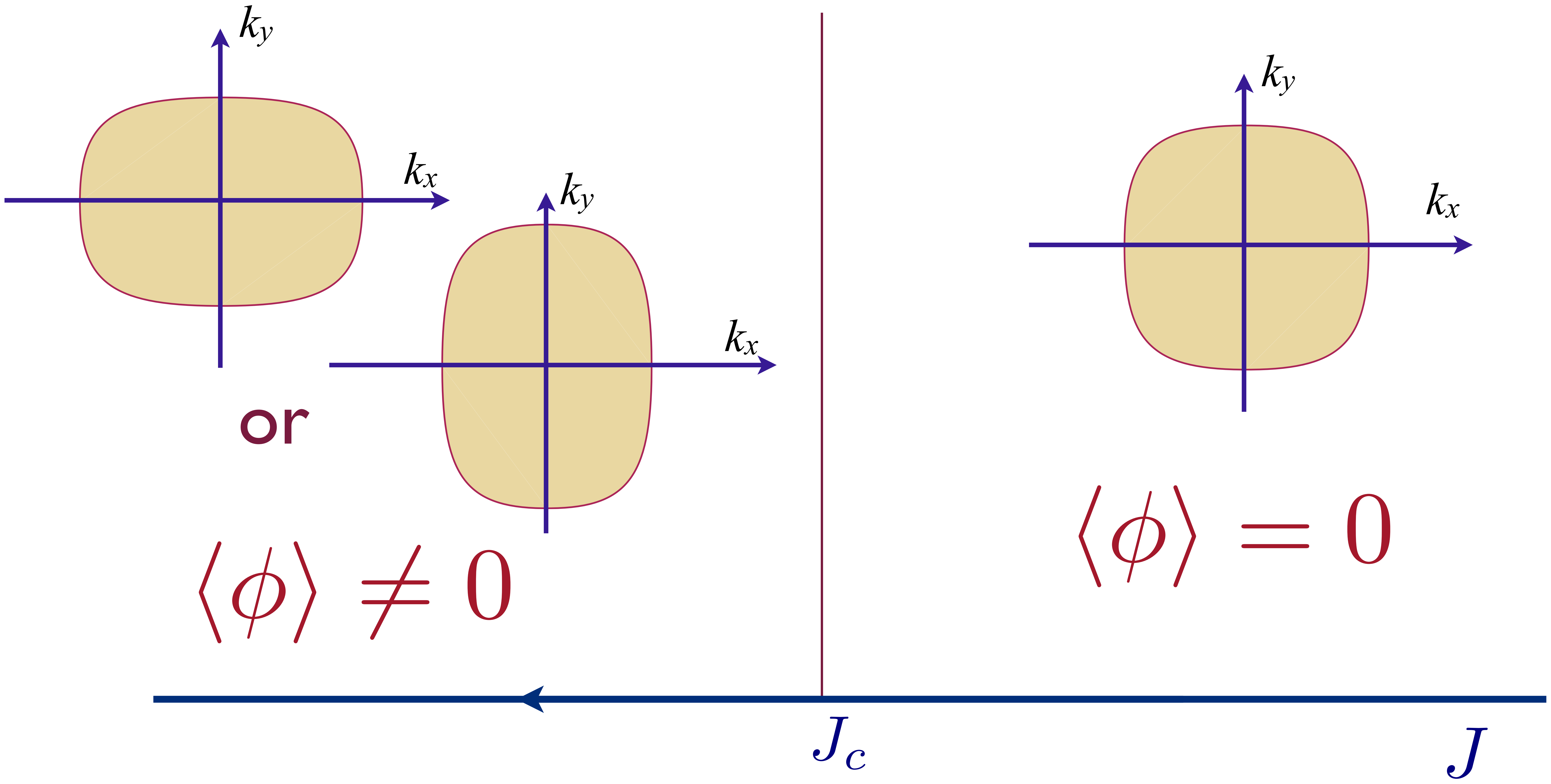
R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 **603** 2009



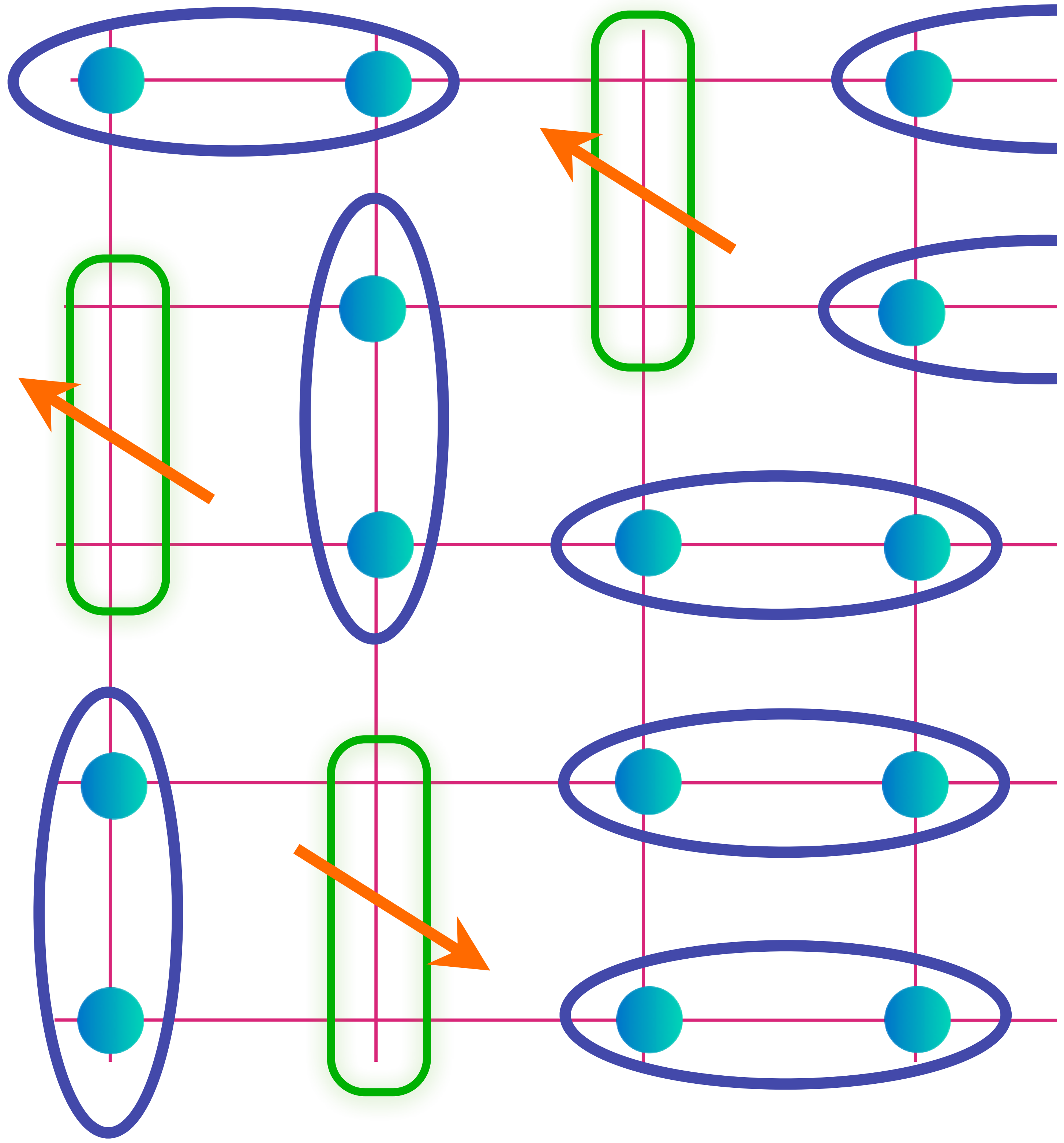


Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling J

Pseudogap metal to Fermi liquid in single band model



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

Φ Higgs boson with the fundamental gauge charge of an emergent SU(2) gauge field.

Small Fermi surface of size p + spin liquid.

FL*

$\langle \Phi \rangle \neq 0$

Large Fermi surface of size $1+p$

FL

$\langle \Phi \rangle = 0$

doping p

Ya-Hui Zhang and S.S. *Phys. Rev. Research* **2**, 023172 (2020); *Phys. Rev. B* **102**, 155124 (2020)

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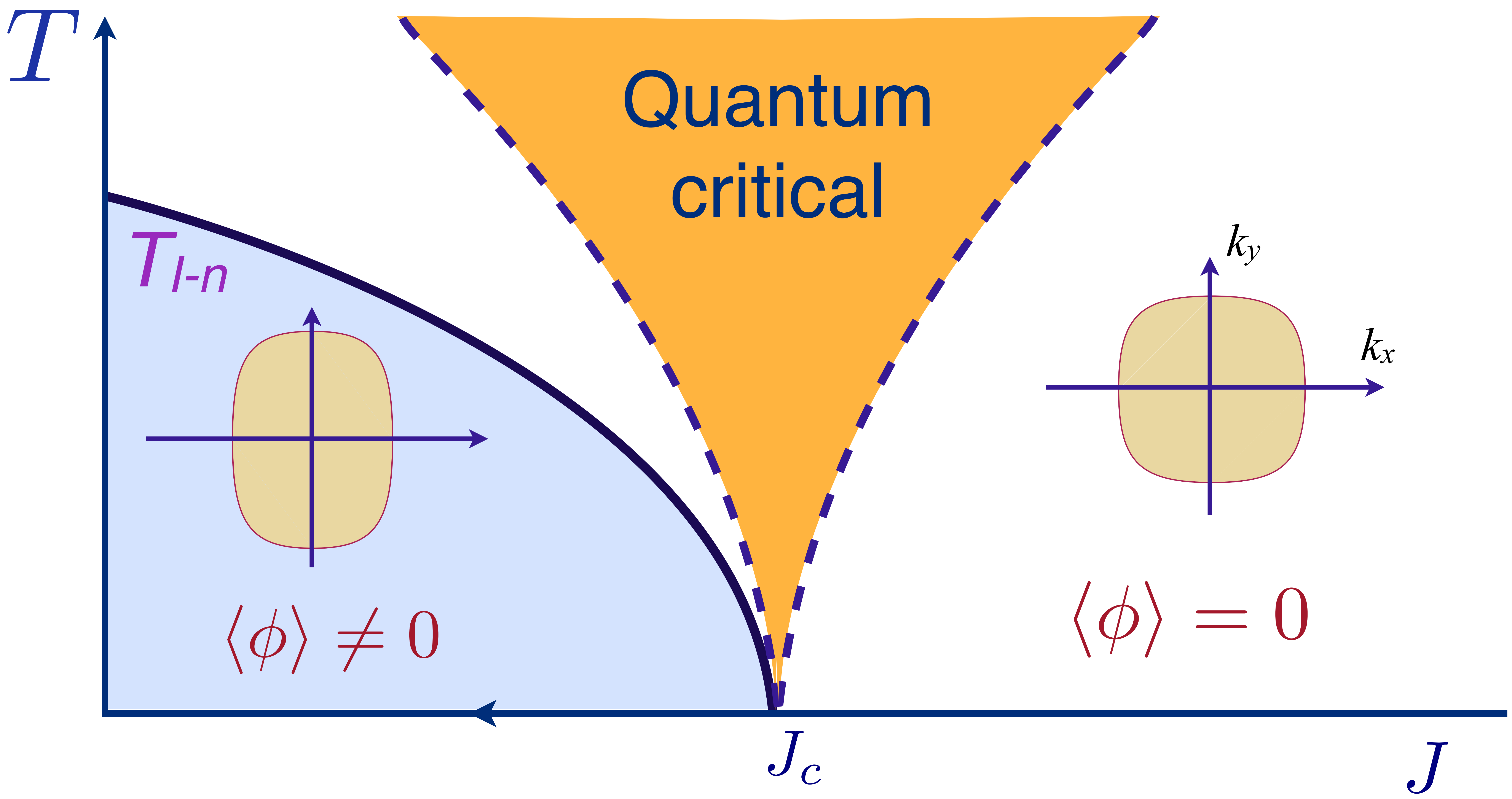

Phase diagram as a function of T and J

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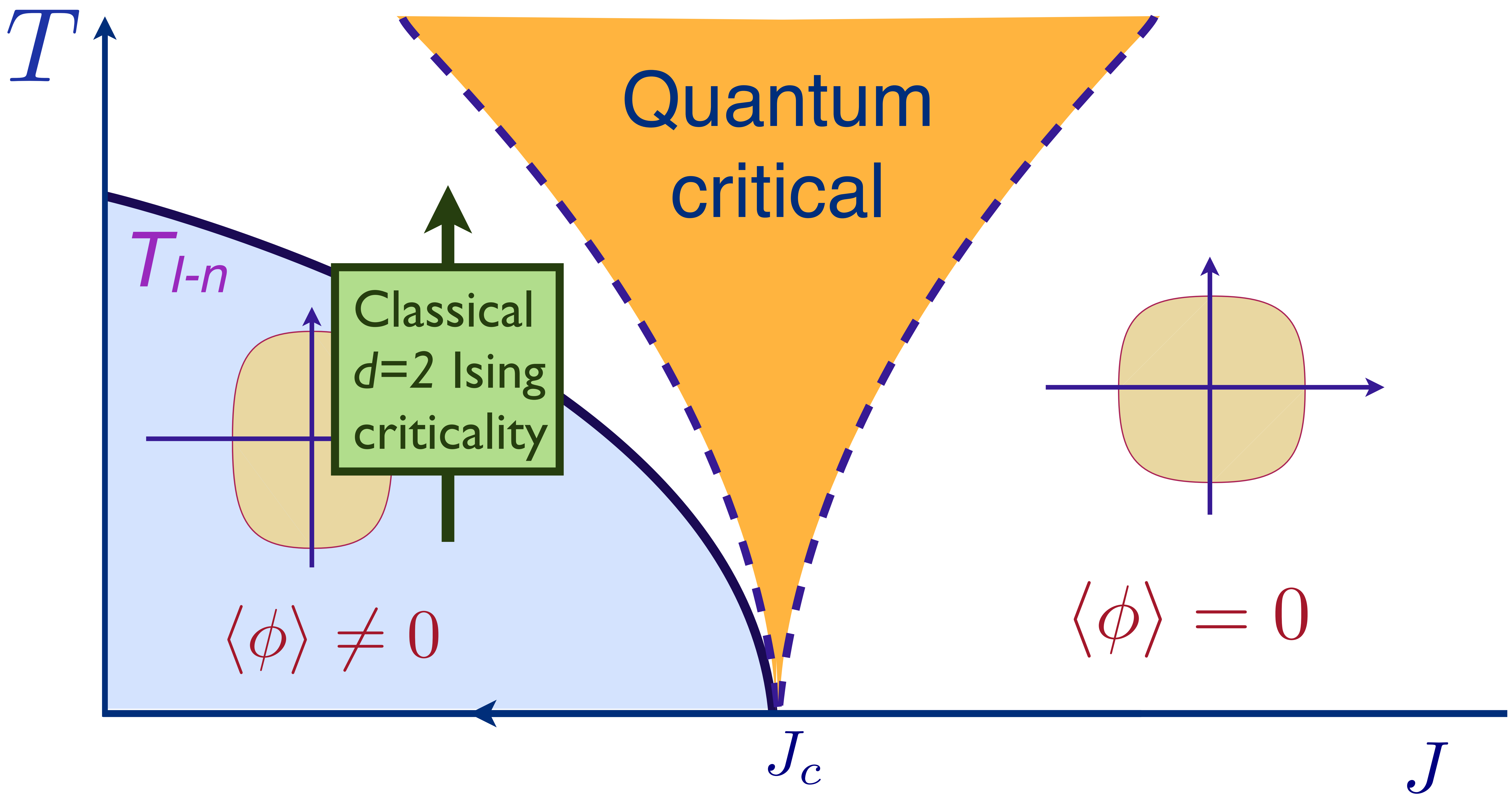

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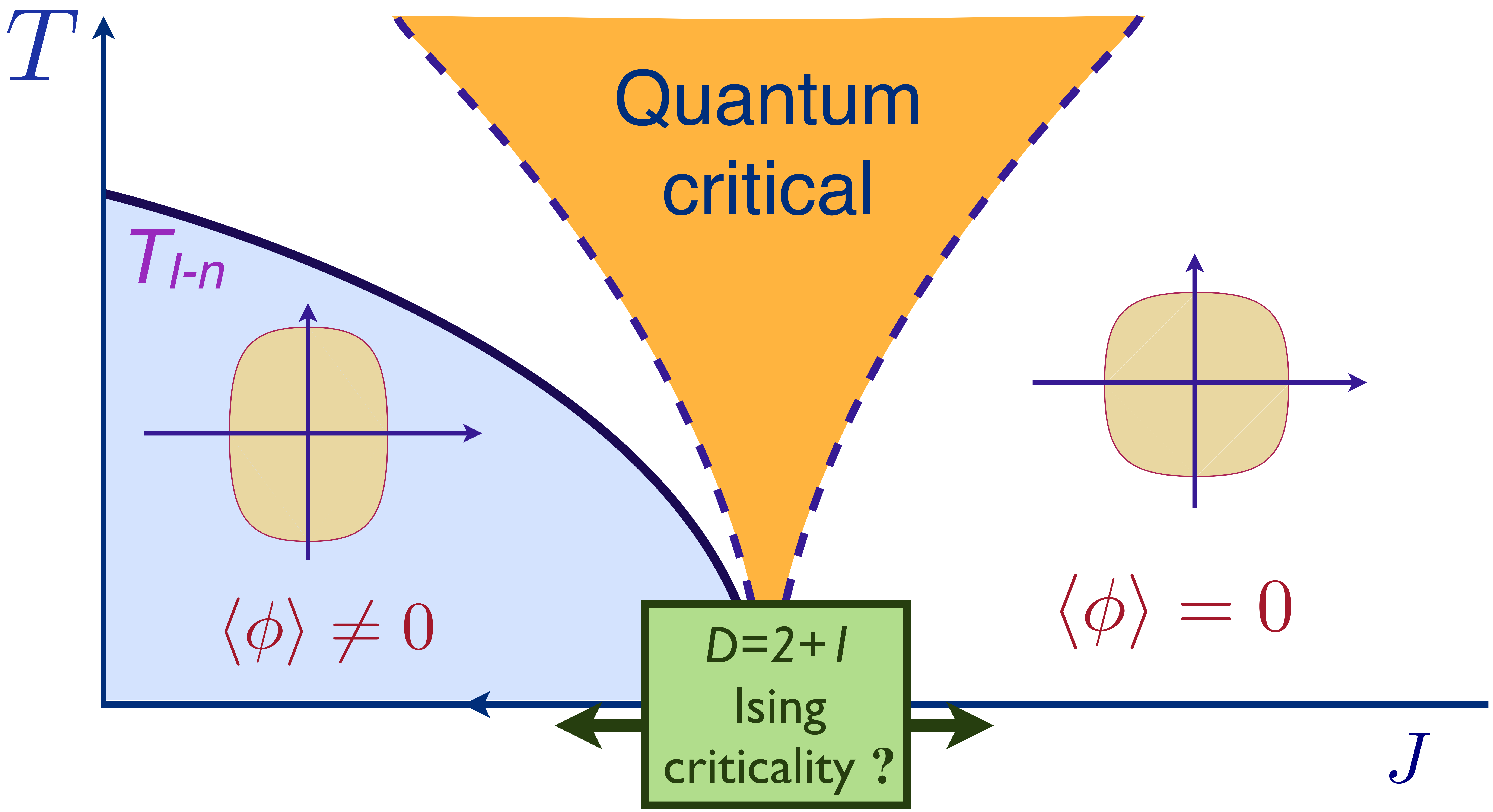

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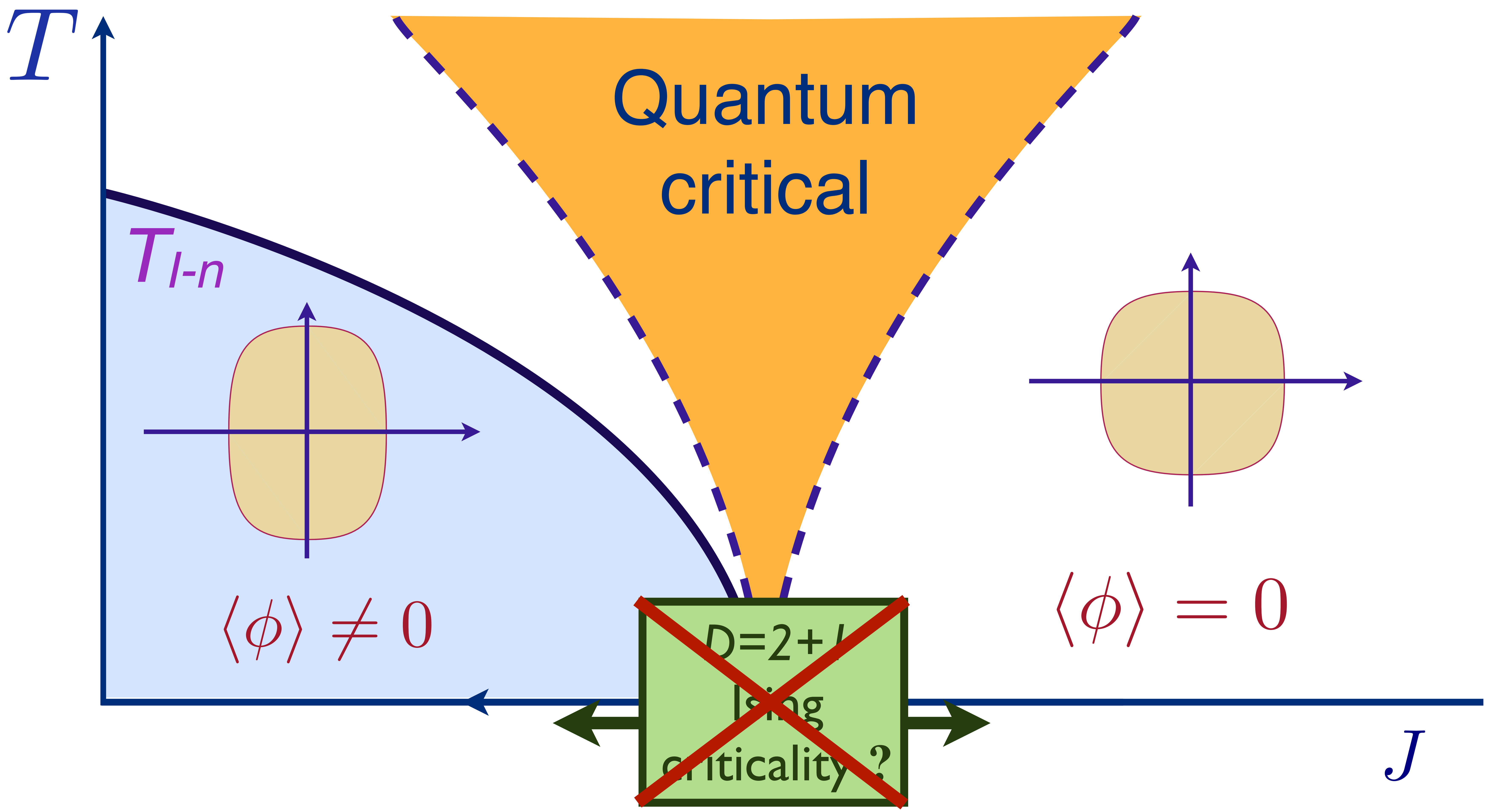

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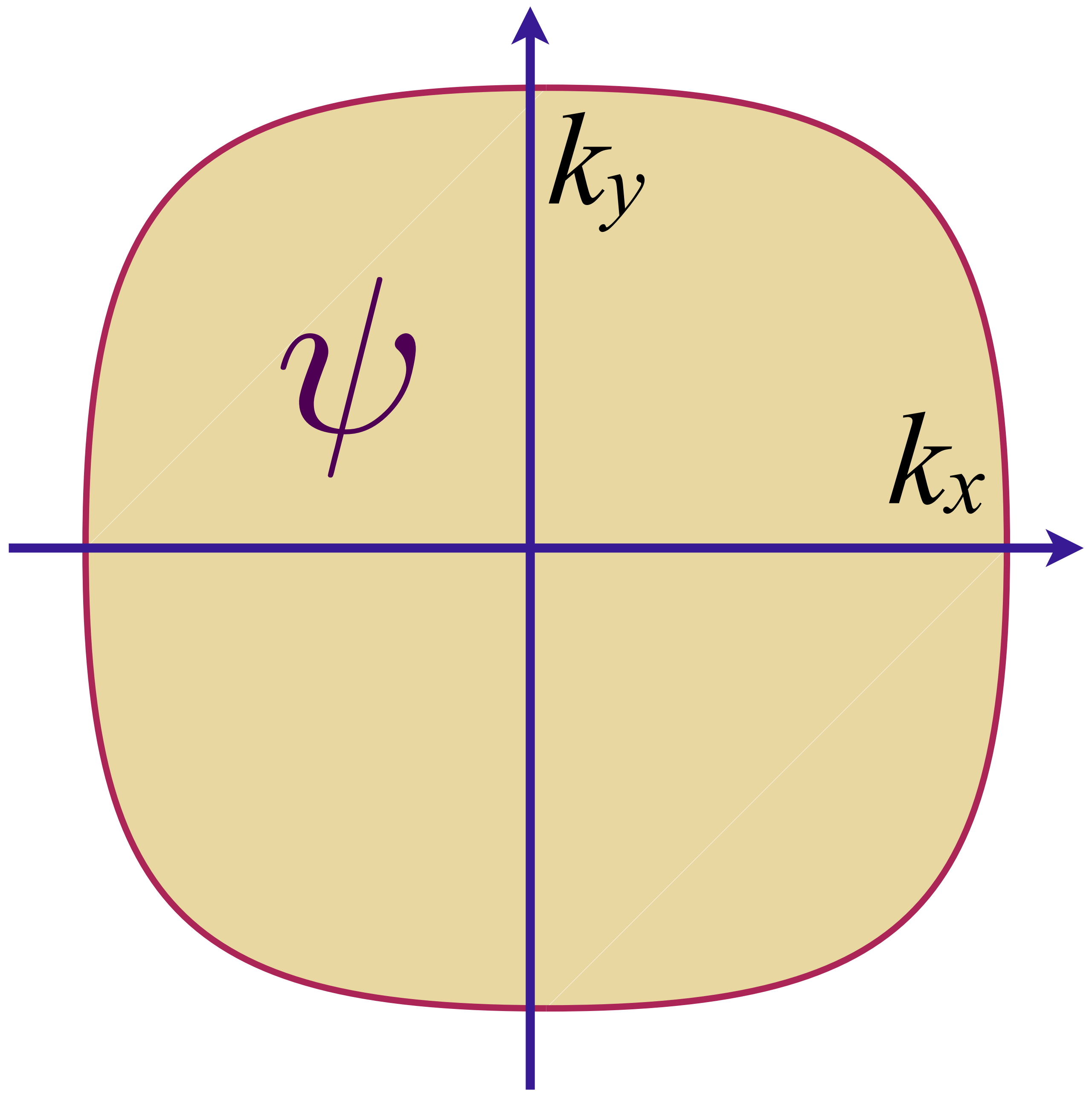
Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



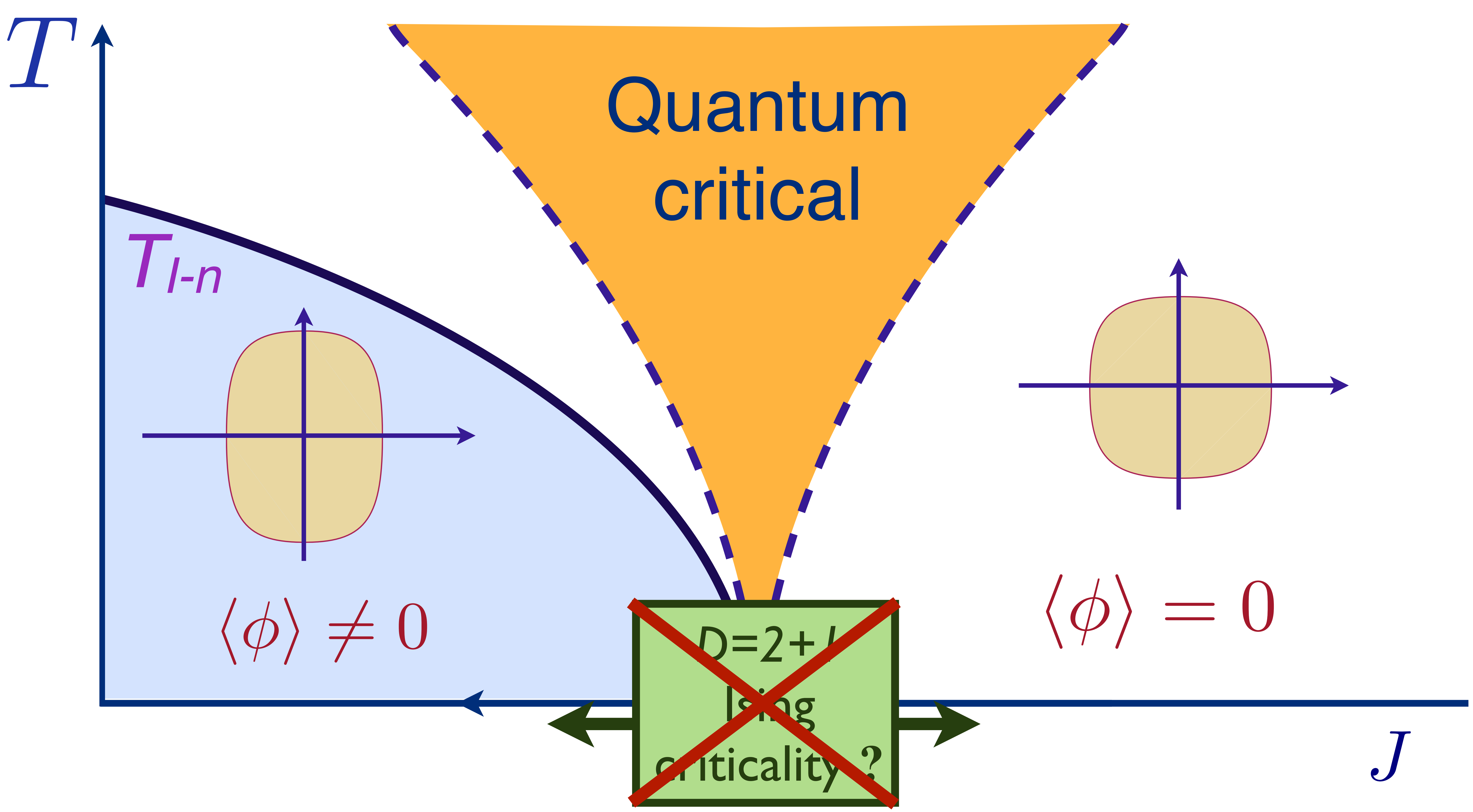
A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 + \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

$$s \sim J_c - J$$

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J

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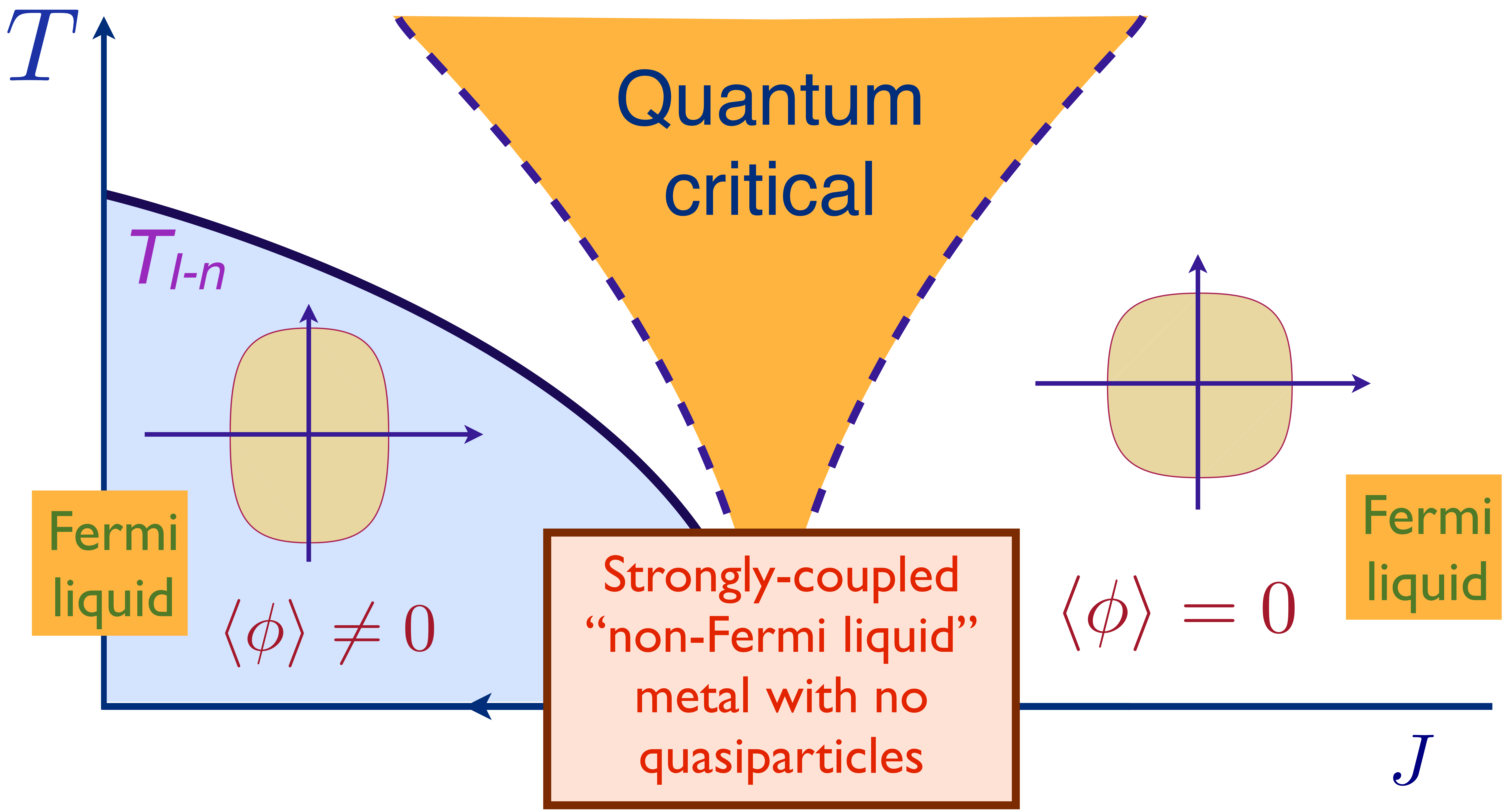

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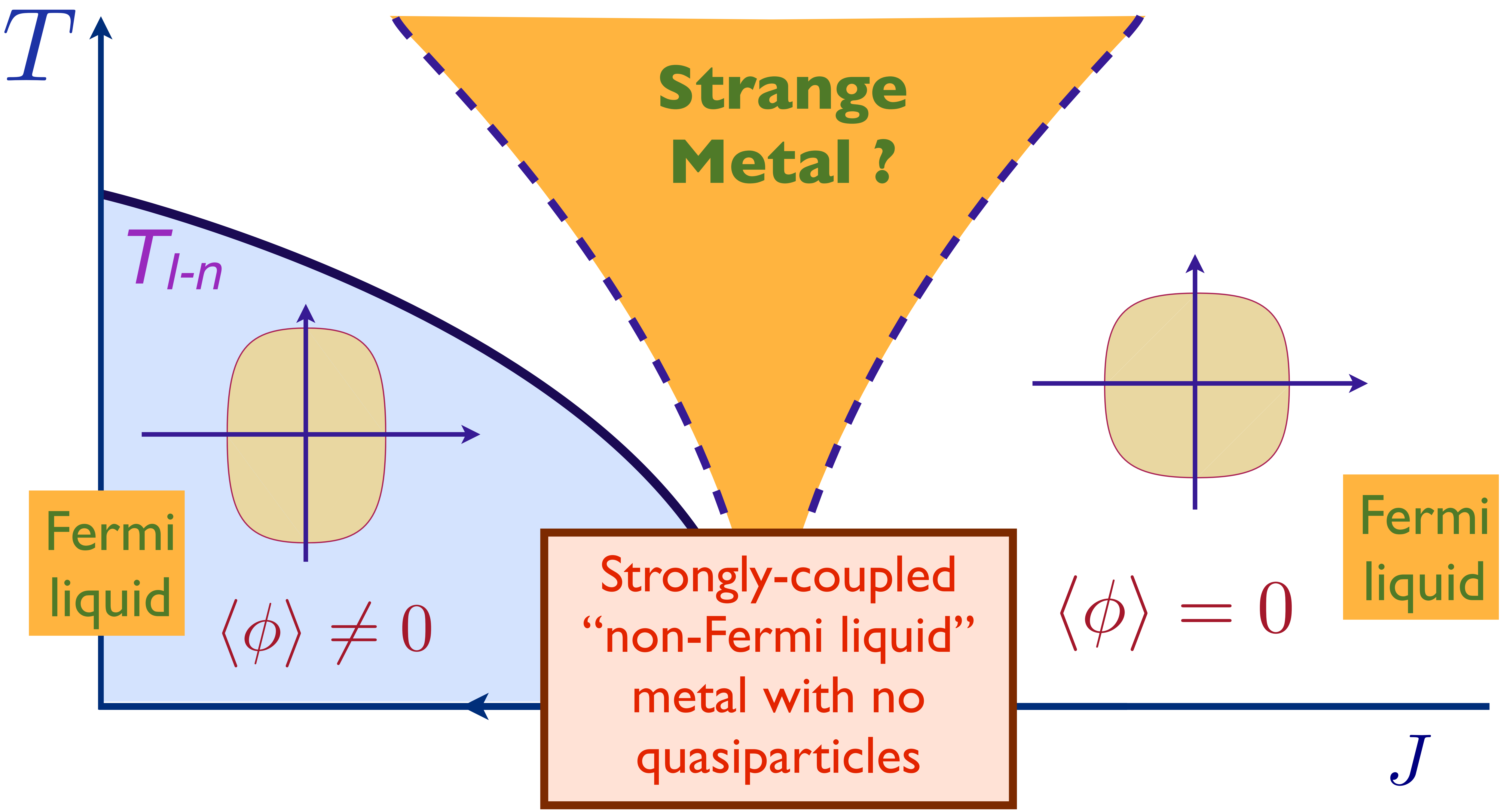

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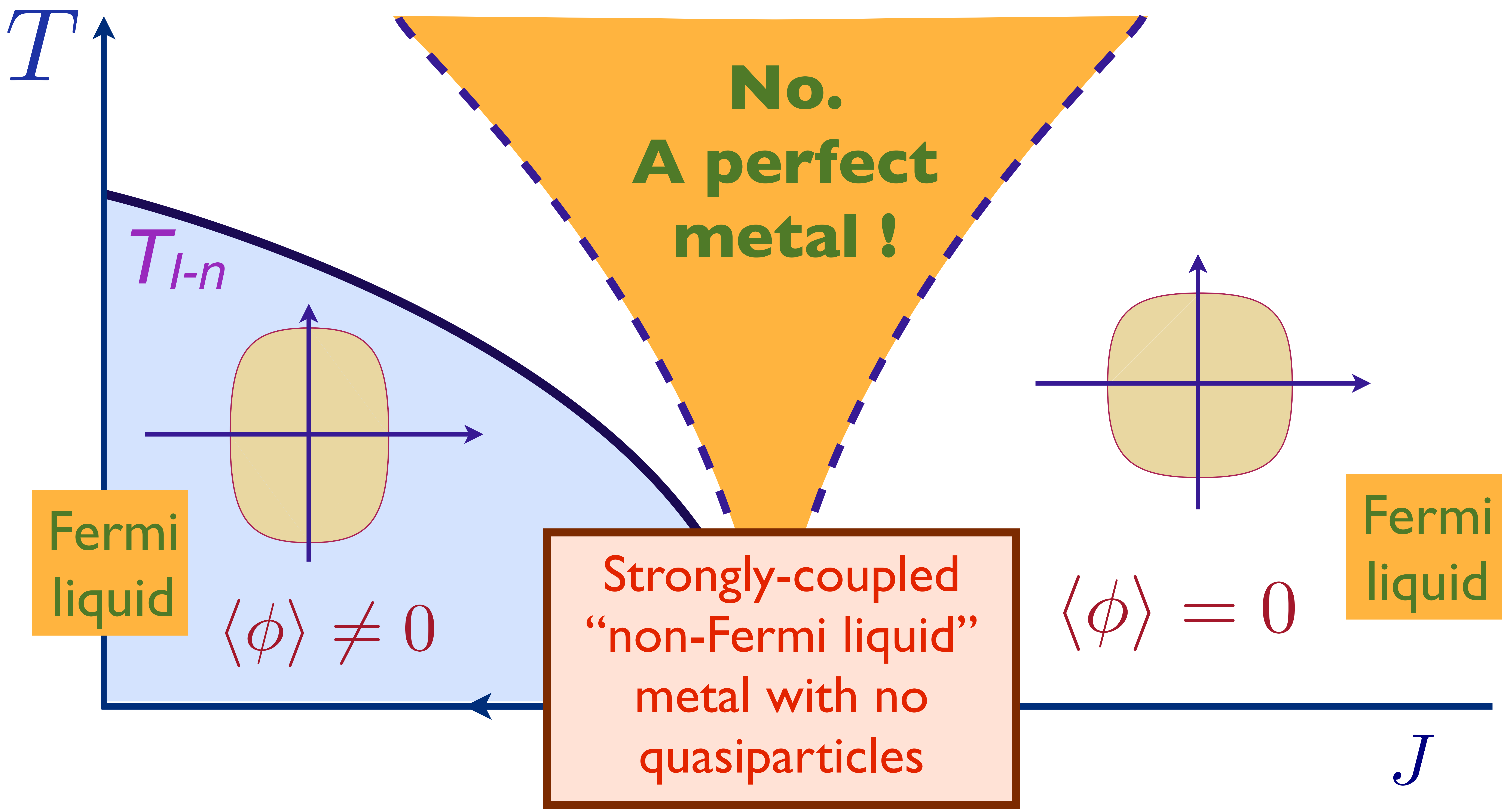


Phase diagram as a function of T and J

Kohn's Theorem

PHYSICAL REVIEW VOLUME 123, NUMBER 4 AUGUST 15, 1961

Cyclotron Resonance and de Haas-van Alphen Oscillations of an Interacting Electron Gas*

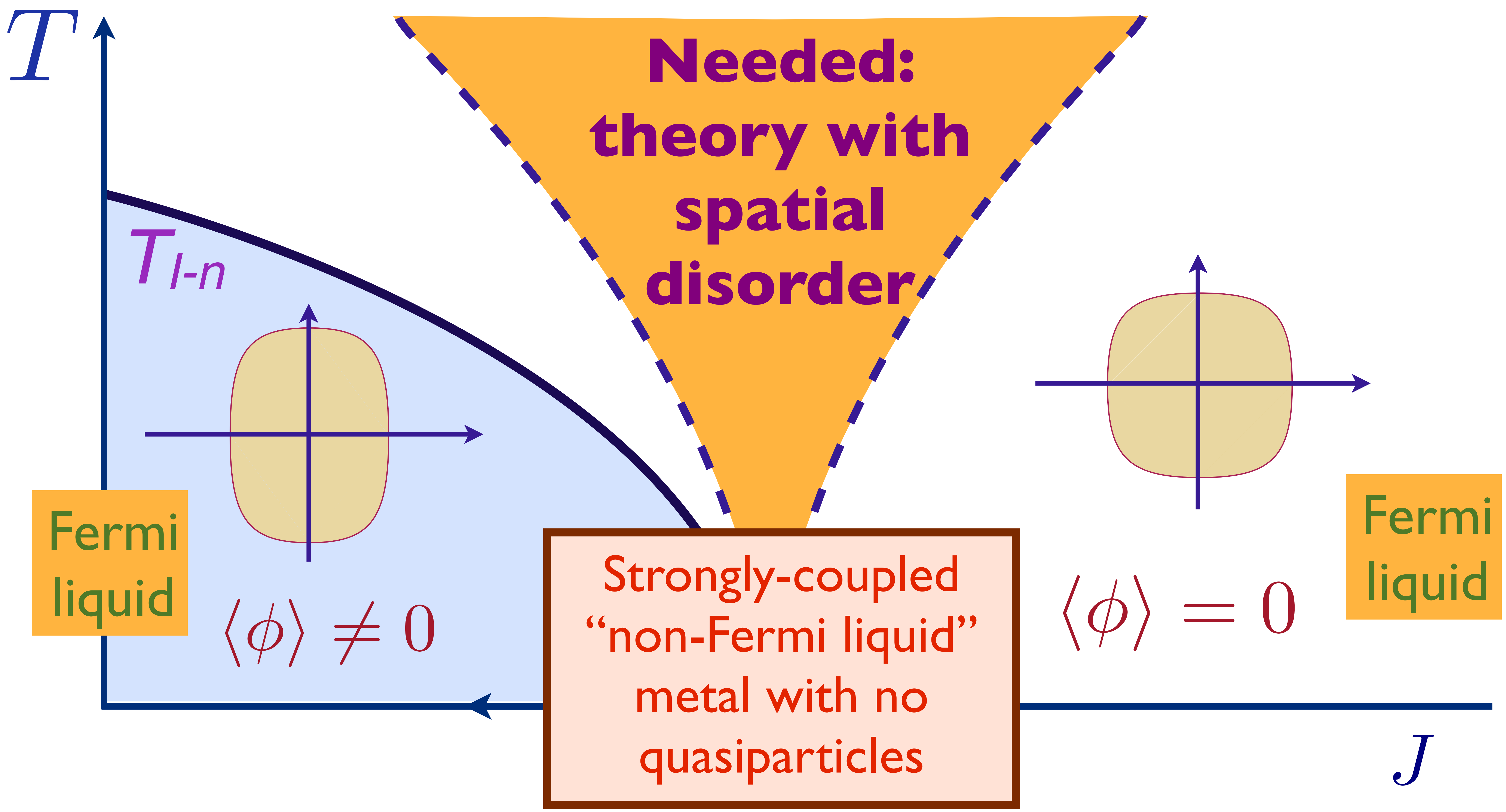
WALTER KOHN
University of California at San Diego, La Jolla, California
(Received April 5, 1961)

An electron gas with short-range interactions is considered in the presence of a uniform magnetic field. It is shown that (1) the cyclotron resonance frequency is independent of the interaction; (2) for a two-dimensional gas, the de Haas–van Alphen period is independent of the interaction. The low-lying excited states are briefly discussed.

In the absence of umklapp and impurities,
the Fermi liquid is a *perfect metal*.

$$\sigma(\omega) = iD/(\omega - \omega_c)$$

Quantum criticality of Ising-nematic ordering in a metal



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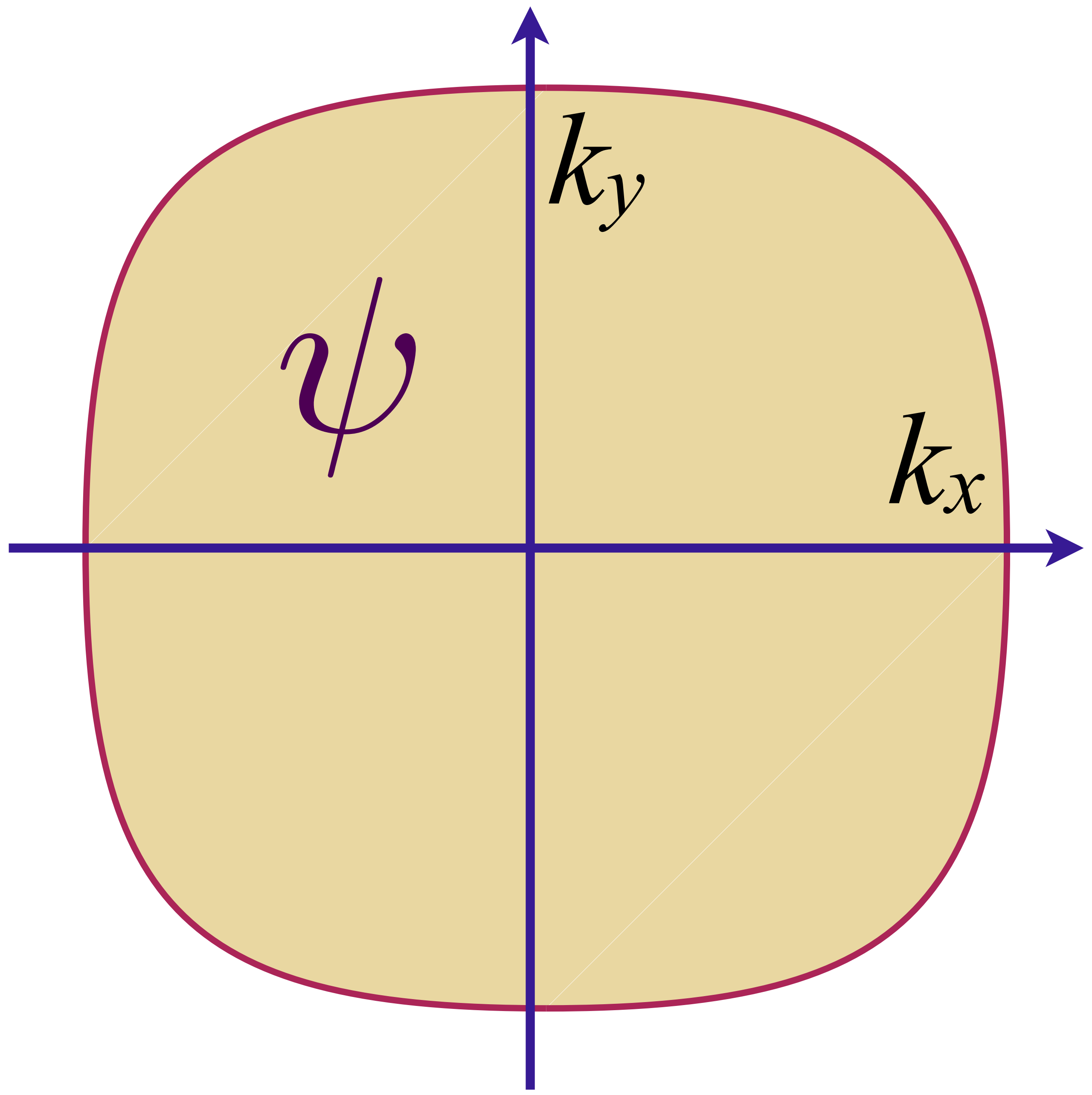

A critical boson ϕ
e.g. Ising-nematic order,
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$$+s\,[\phi(\boldsymbol{r})]^2 + \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

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Fermi surface + critical boson with potential and interaction disorder

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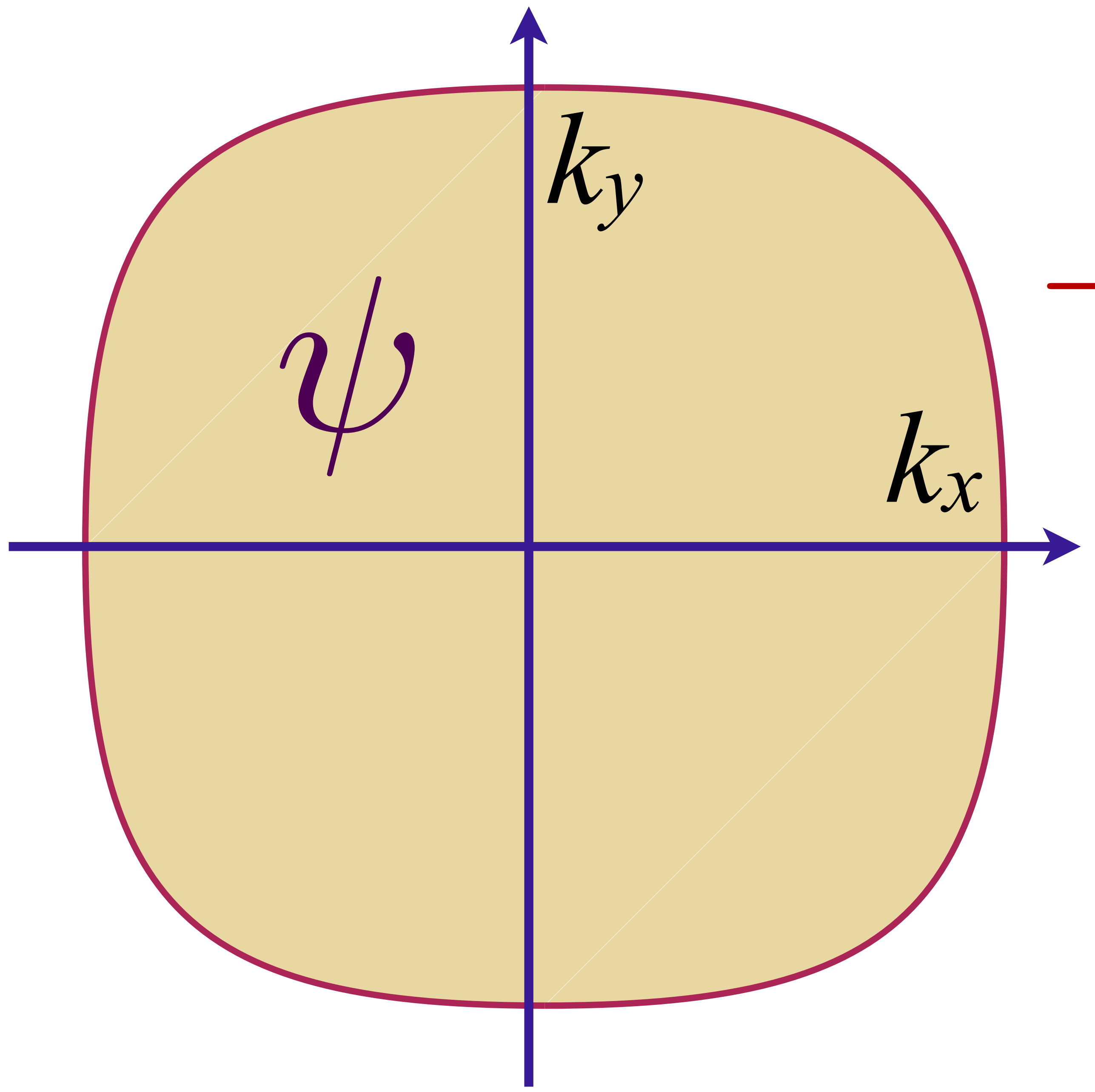

$$+[s + \delta s(\boldsymbol{r})]\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 \;+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})} = 0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')} = \delta s^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Yukawa-SYK model

Universal Yukawa-SYK theory in d=2 spatial dimensions

Random "mass" disorder, andmapping to random transverse-field Ising model (RTFIM)

Yukawa-SYK model

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in d=2 spatial dimensions

Random "mass" disorder,
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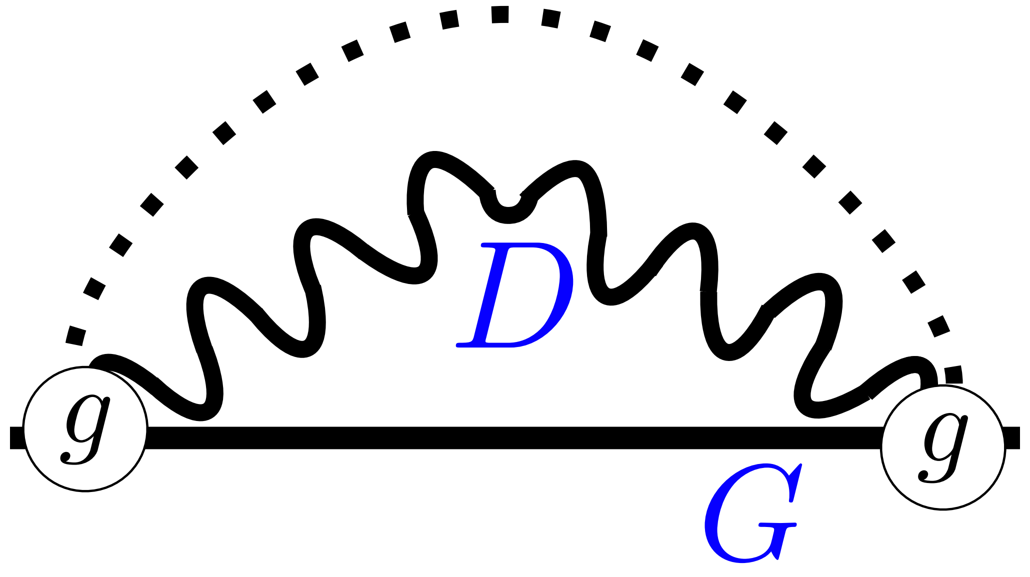
Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

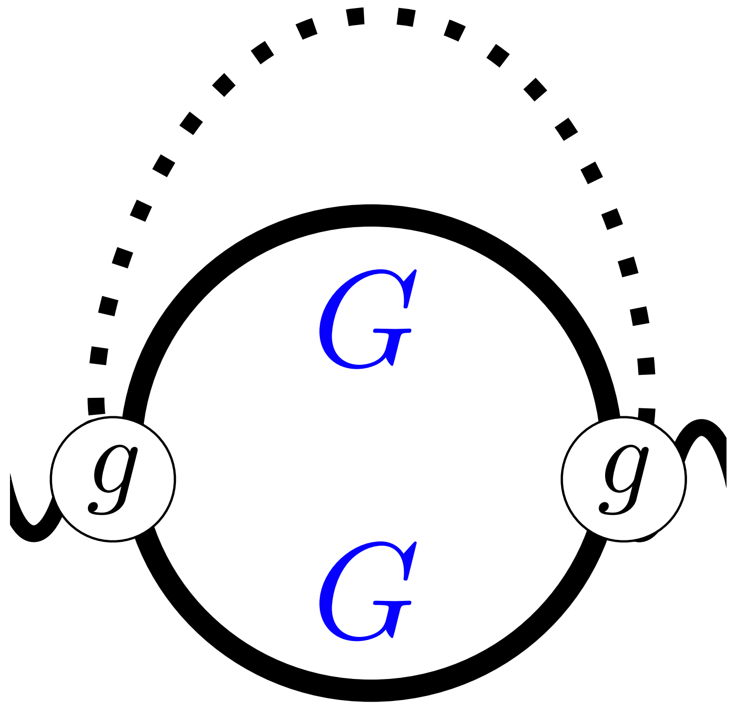
with $g_{ij\ell}$ independent random numbers with $\overline{g_{ij\ell}} = 0$, $\overline{g_{ij\ell}^2} = g^2$.

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

$\Sigma =$



$\Pi =$



W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

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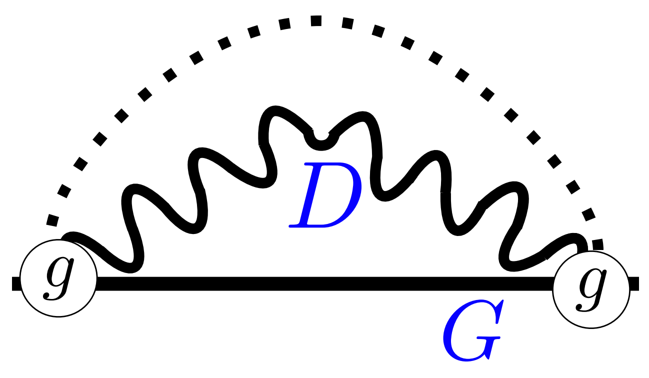
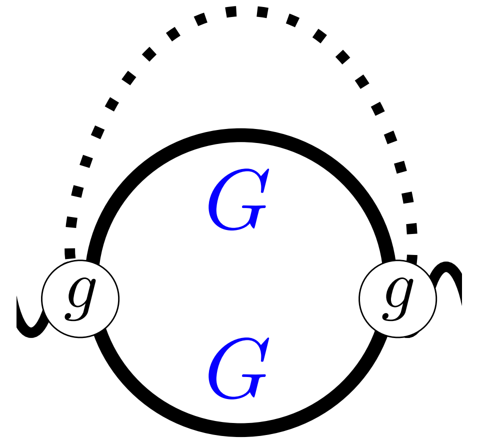
with $g_{ij\ell}$ independent random numbers with $\overline{g_{ij\ell}} = 0$, $\overline{g_{ij\ell}^2} = g^2$. Large N saddle-point equations:

$\Sigma =$



$$\boxed{\begin{aligned} G(i\omega_n) &= \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} \\ \Sigma(\tau) &= g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau) \end{aligned}}$$

$\Pi =$



Make the low frequency ansatz

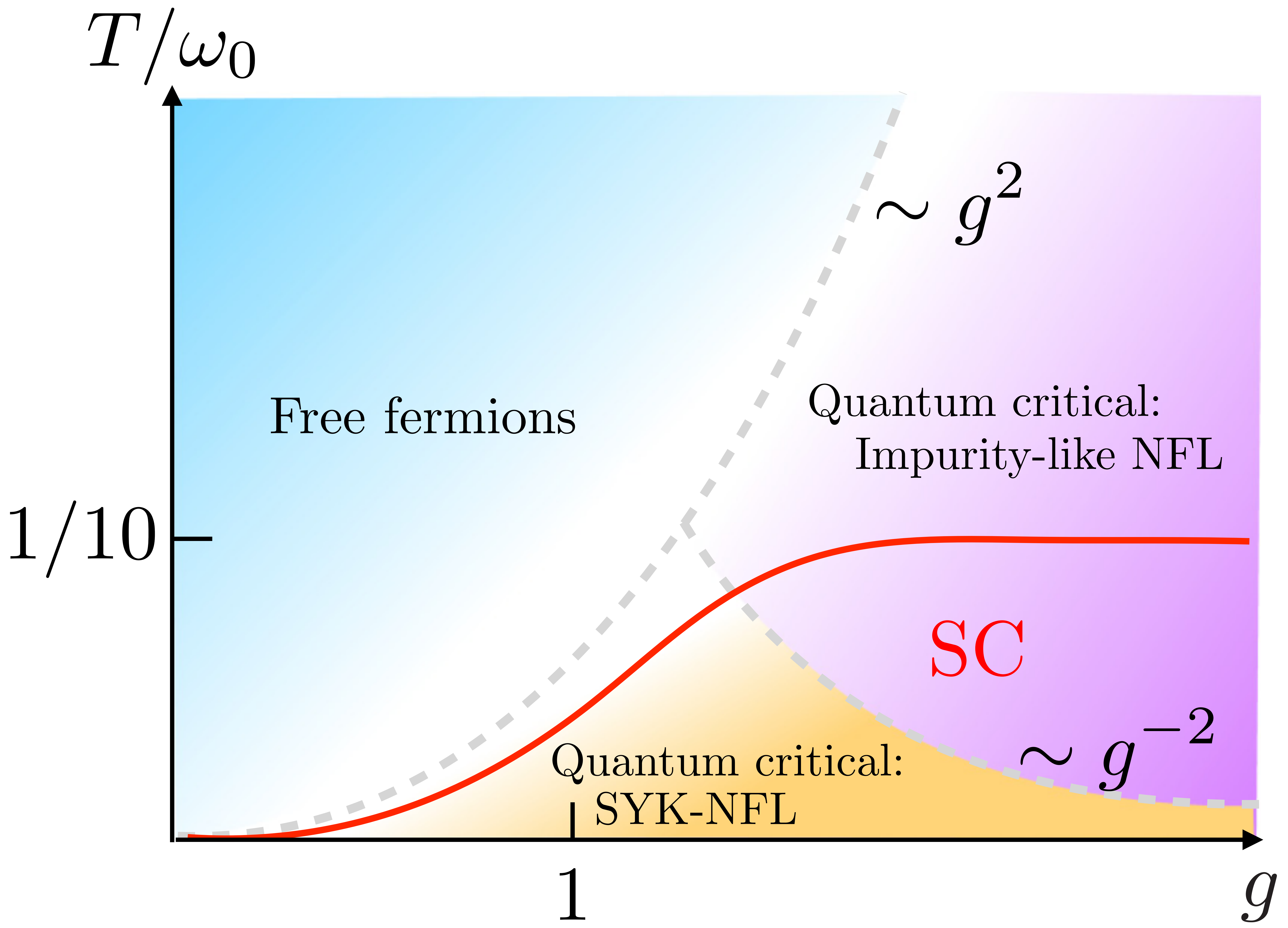
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK models



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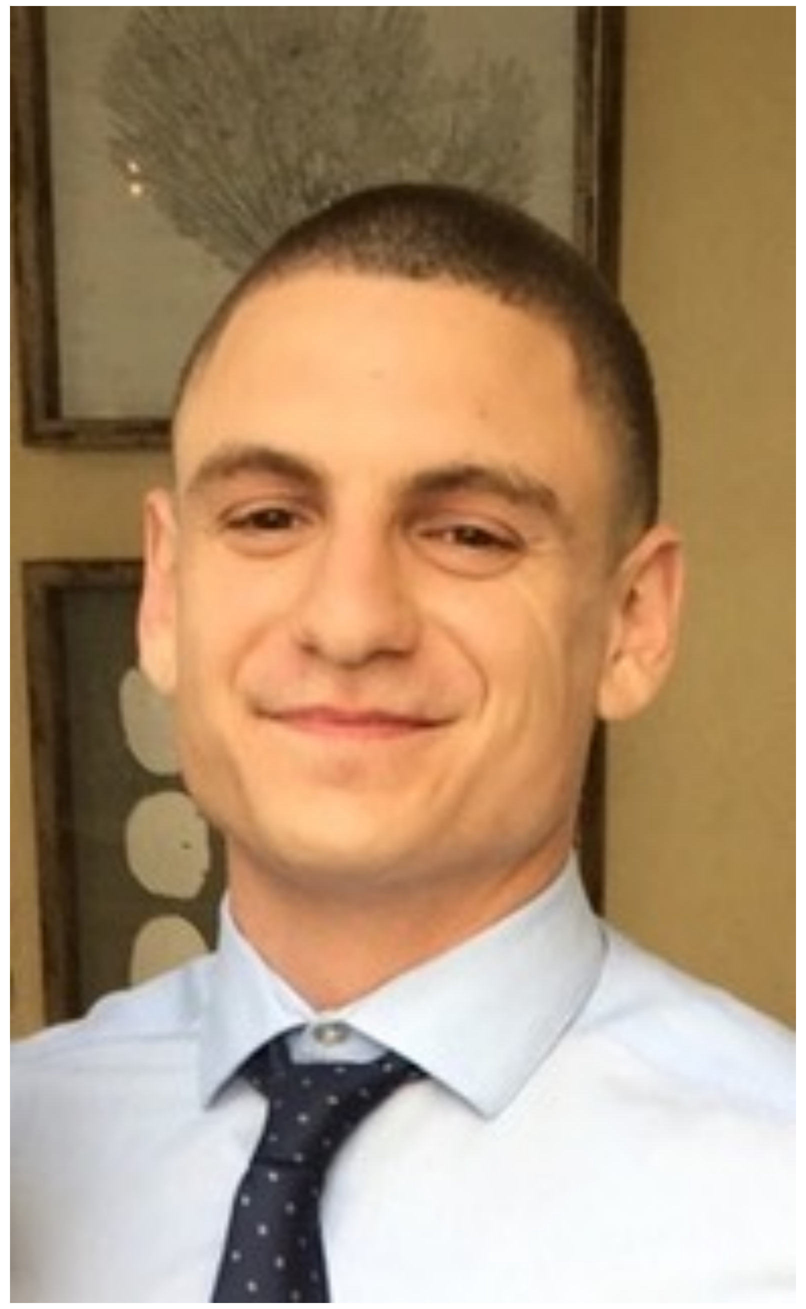
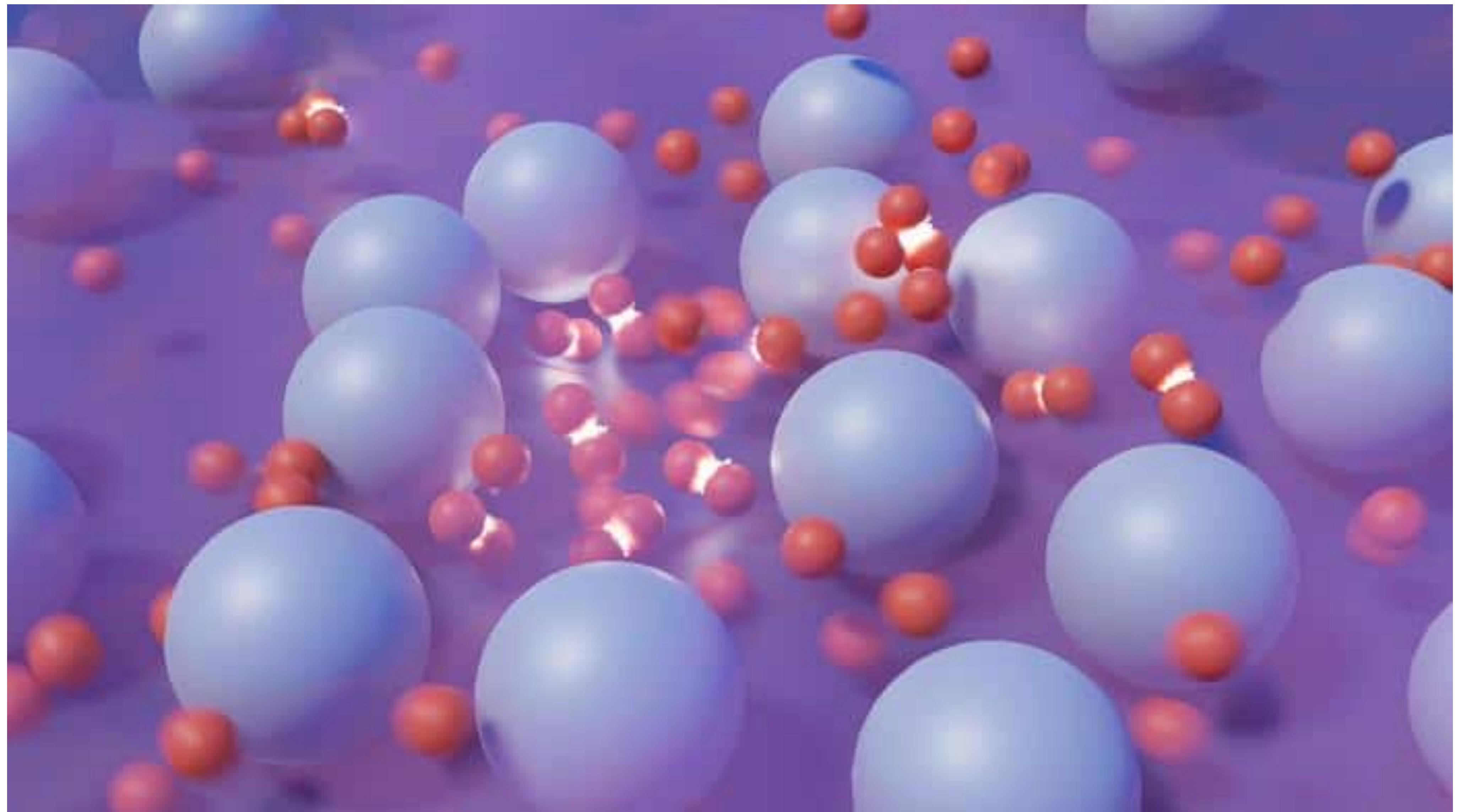


Aavishkar Patel
Flatiron Institute, NYC

Haoyu Guo
Cornell

Ilya Esterlis
Wisconsin



Universal theory of strange metals from spatially random interactions, Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, and S. Sachdev, *Science* **381**, 790 (2023)

Fermi surface + critical boson with potential and interaction disorder

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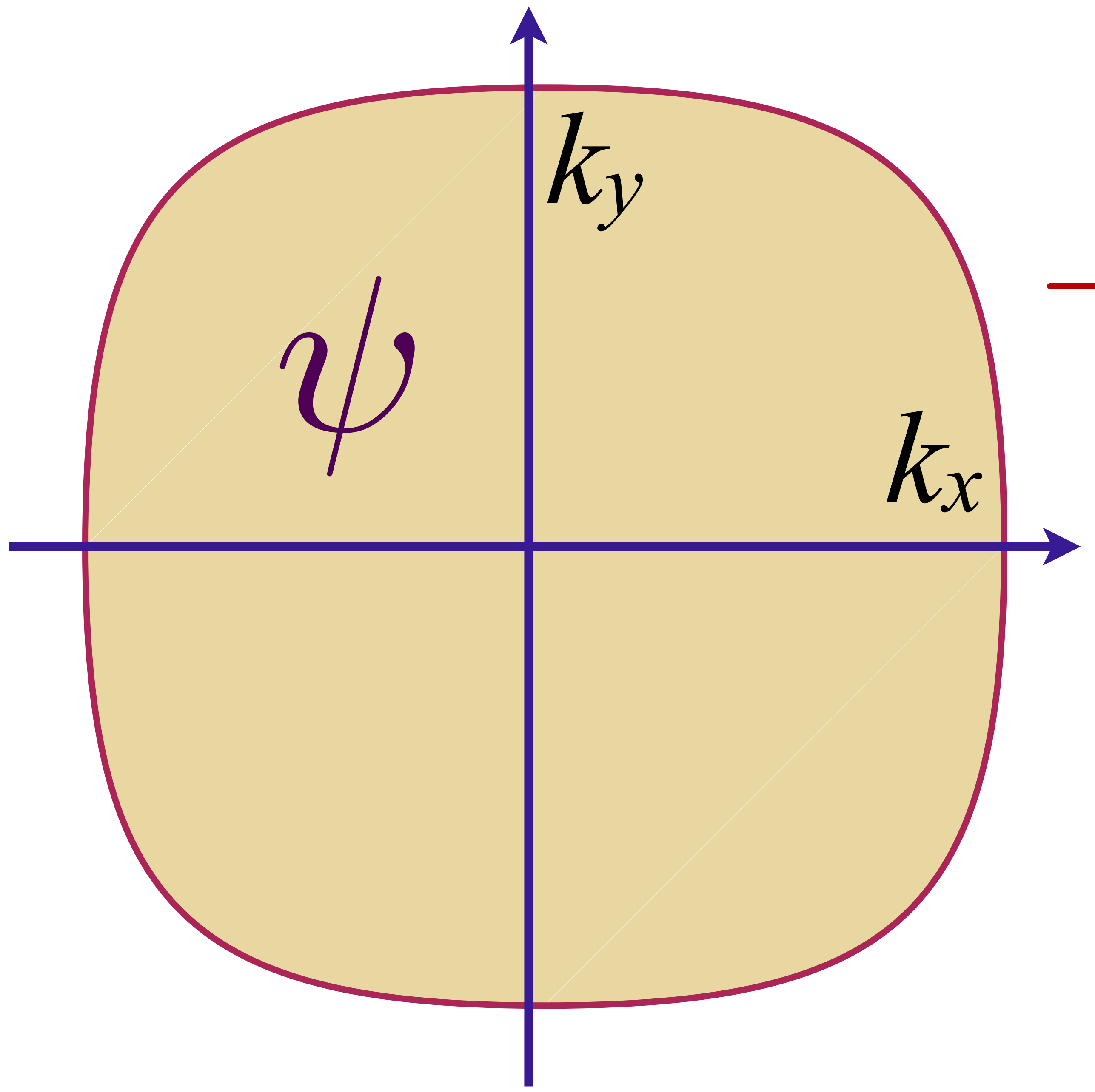

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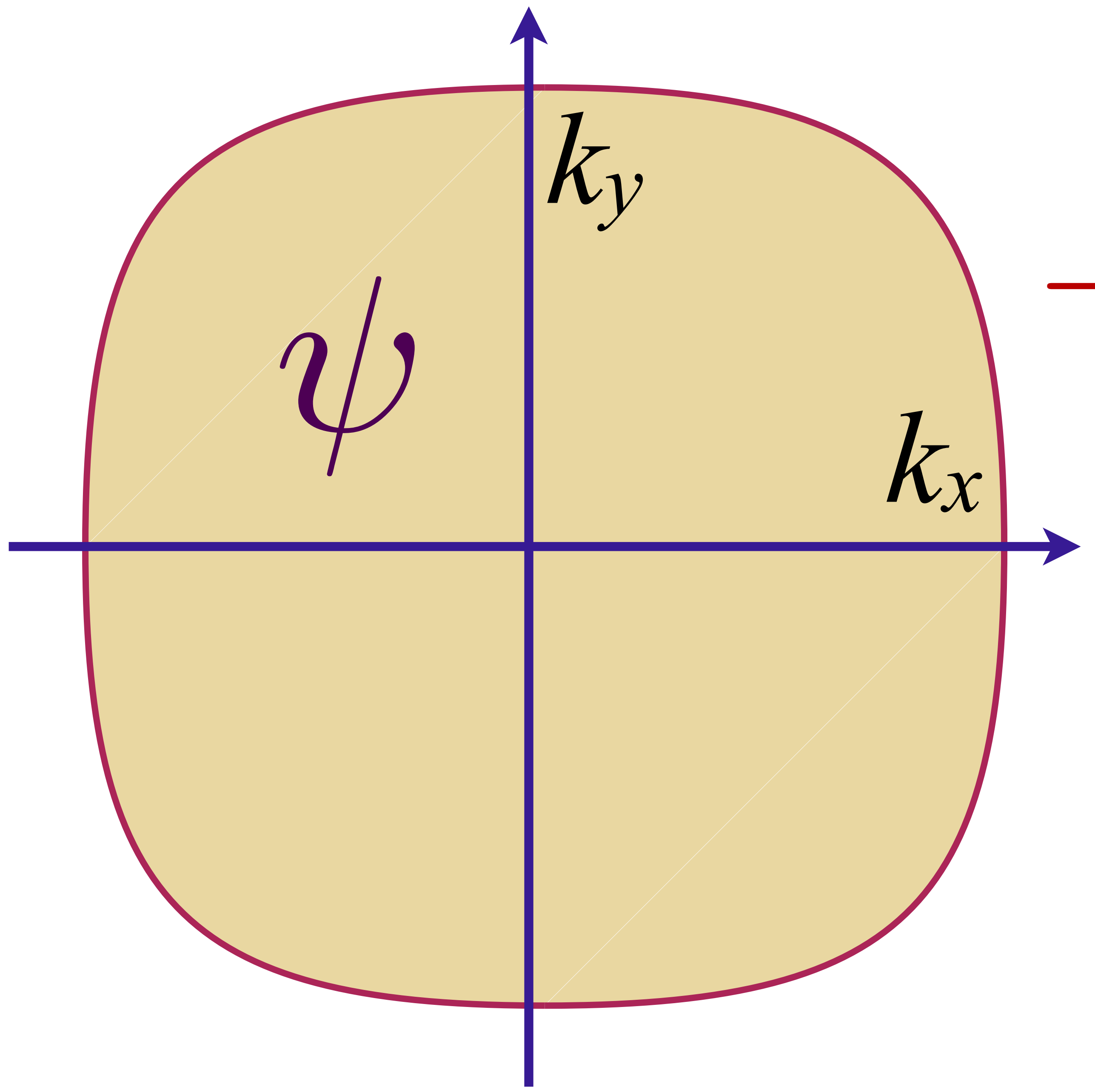

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Rescale $\phi(\boldsymbol{r})$ to absorb random mass $\delta s(\boldsymbol{r})$ into
the random Yukawa coupling $g'(\boldsymbol{r})$
and analyze with self-averaging as in Yukawa-SYK model.

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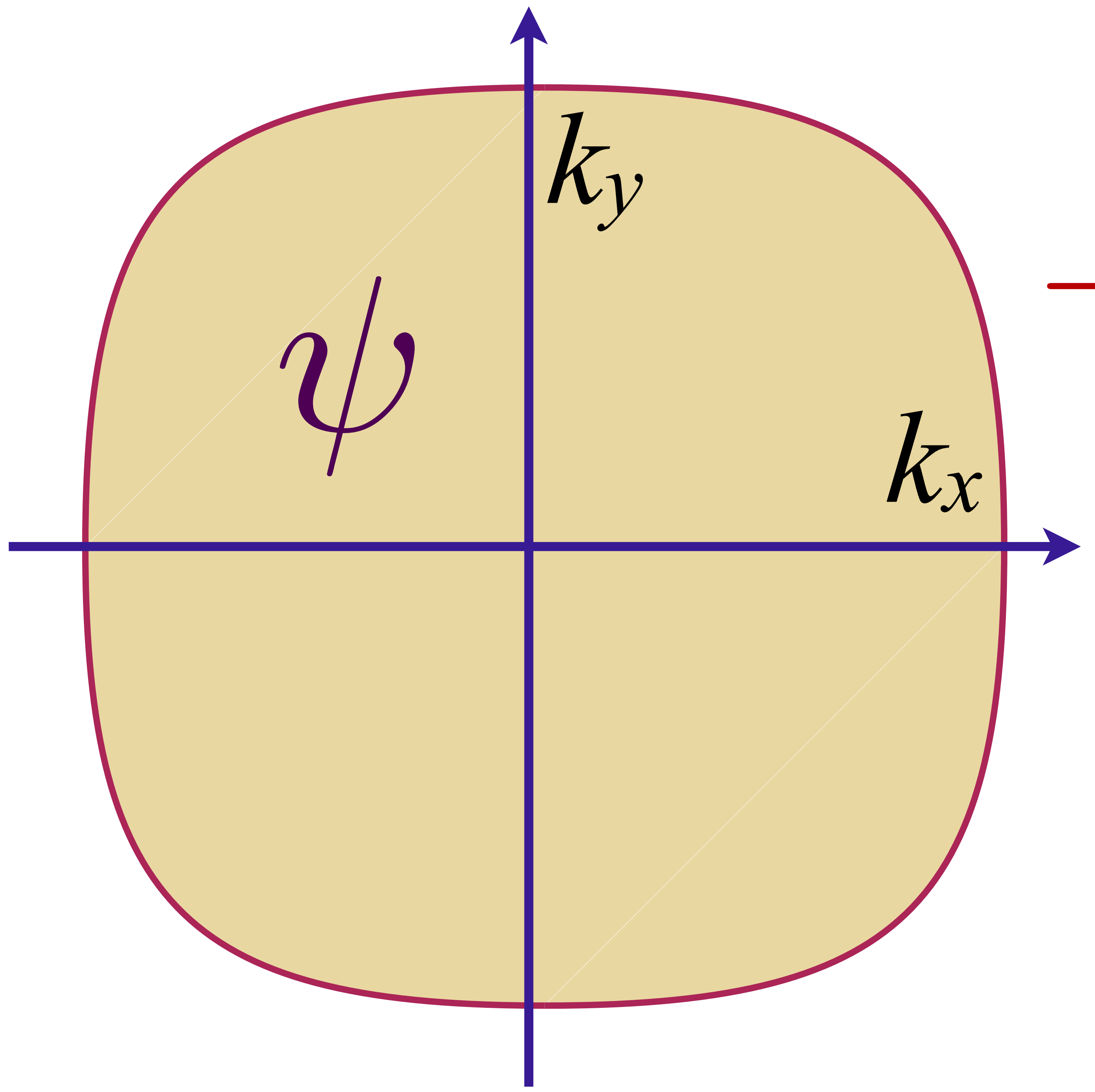

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Rescale $\phi(\boldsymbol{r})$ to absorb random mass $\delta s(\boldsymbol{r})$ into
the random Yukawa coupling $g'(\boldsymbol{r})$
and analyze with self-averaging as in Yukawa-SYK model.

Should be applicable as long as eigenmodes of $\phi(\boldsymbol{r})$ are extended.

Fermi surface + critical boson with potential and interaction disorder

SYK-type self-consistent equations

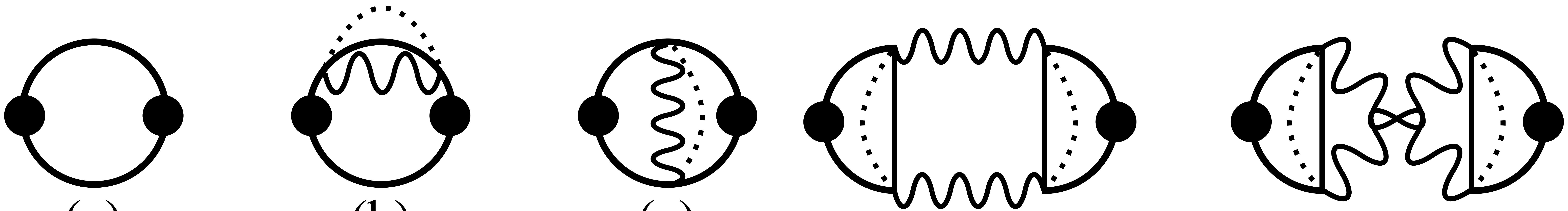
$$\Sigma(\tau,\mathbf{r}) = g^2 D(\tau,\mathbf{r})G(\tau,\mathbf{r}) + v^2 G(\tau,\mathbf{r})\delta^2(\mathbf{r}) + {g'}^2 G(\tau,\mathbf{r})D(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$

$$\Pi(\tau,\mathbf{r}) = -g^2 G(-\tau,-\mathbf{r})G(\tau,\mathbf{r}) - {g'}^2 G(-\tau,\mathbf{r})G(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$

$$G(i\omega,\mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega,\mathbf{k})},$$

$$D(i\Omega,\mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega,\mathbf{q})}.$$

Conductivity:



(a) σ_v (b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$ (c) $\sigma_{V,g}$ (d) (e)

+ all ladders and bubbles…..

Fermi surface + critical boson with potential and interaction disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}}$$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)}\right)\mathrm{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

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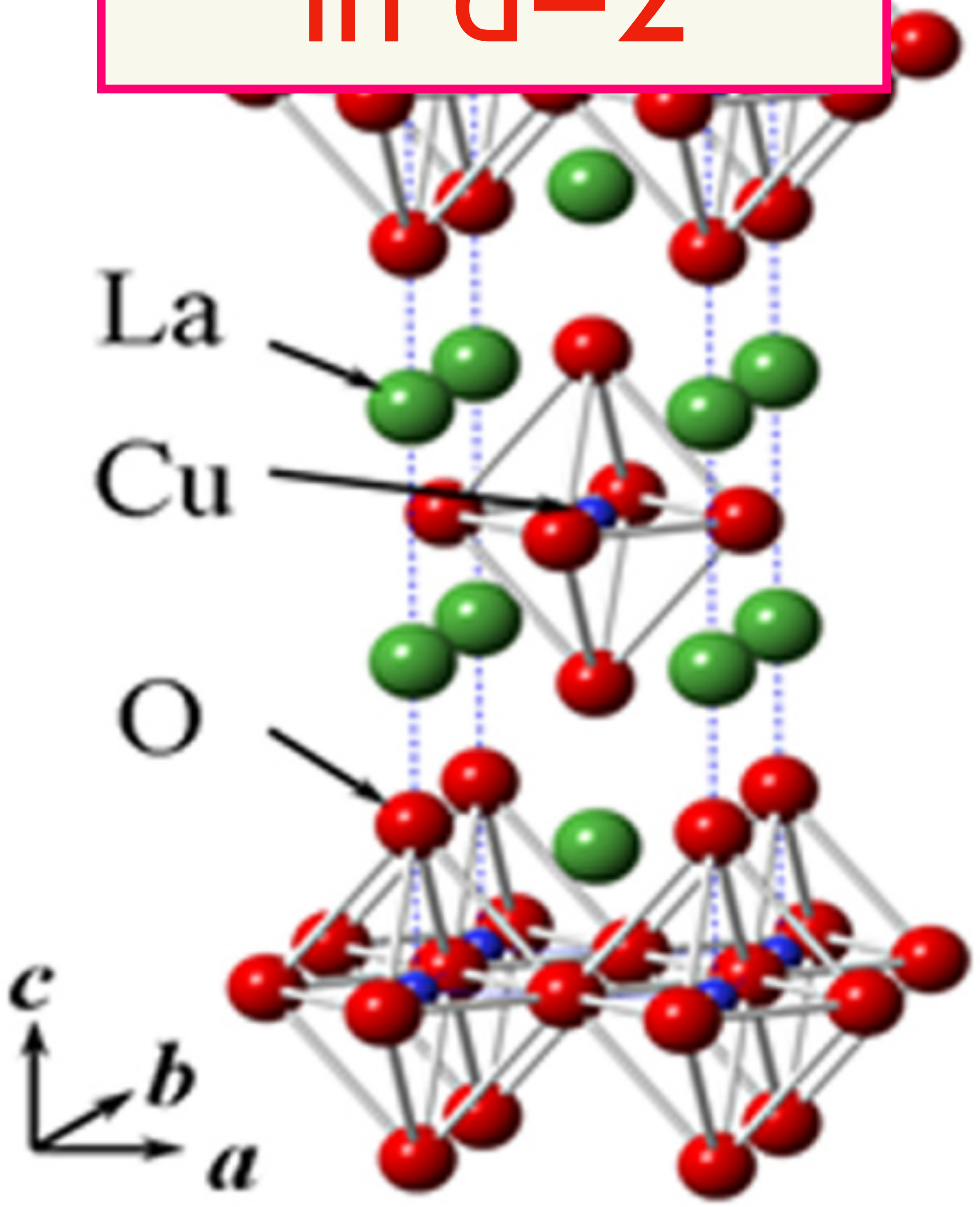
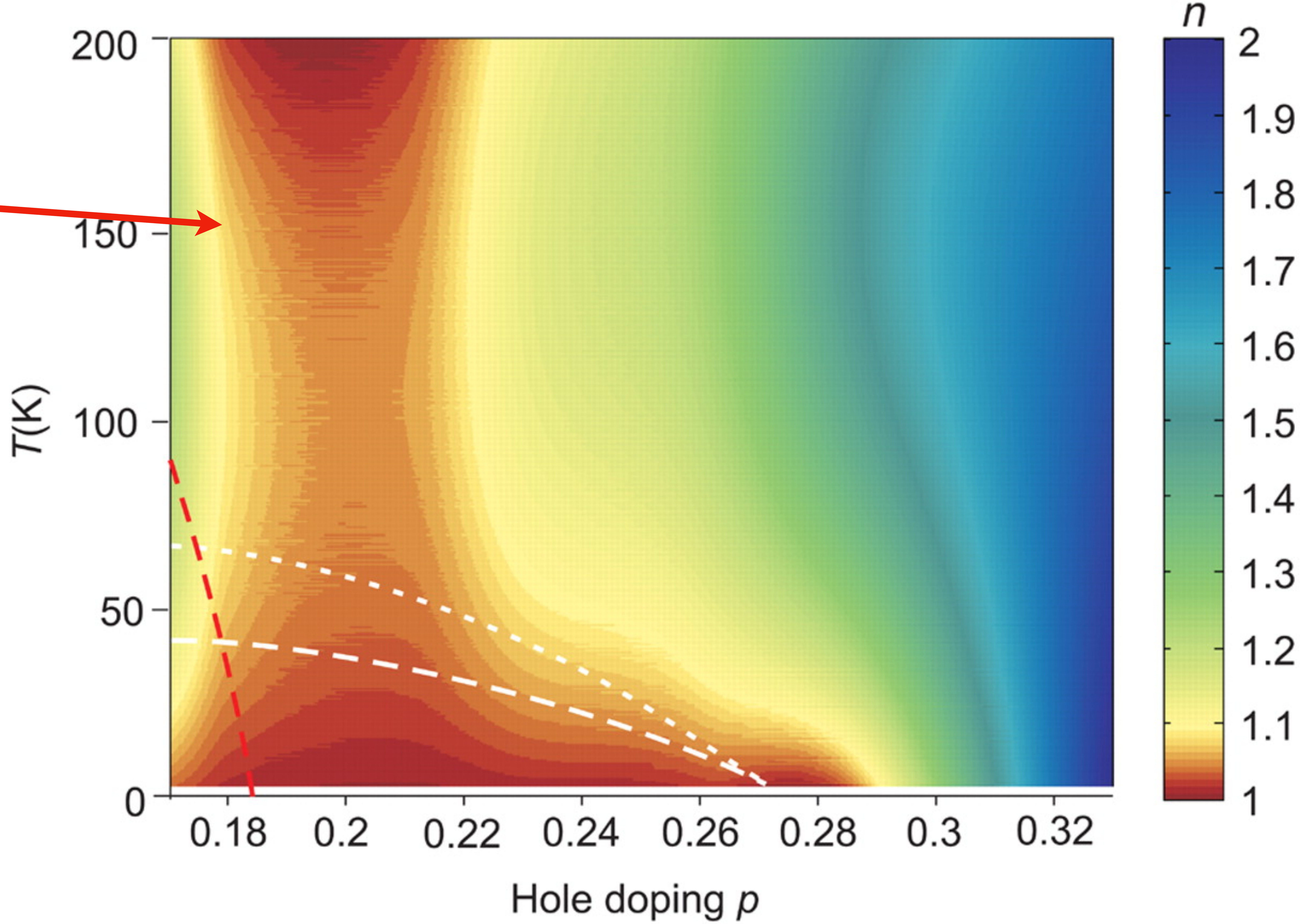
SCIENCE VOL 323 **603** 2009

Yukawa-SYK
in d=2





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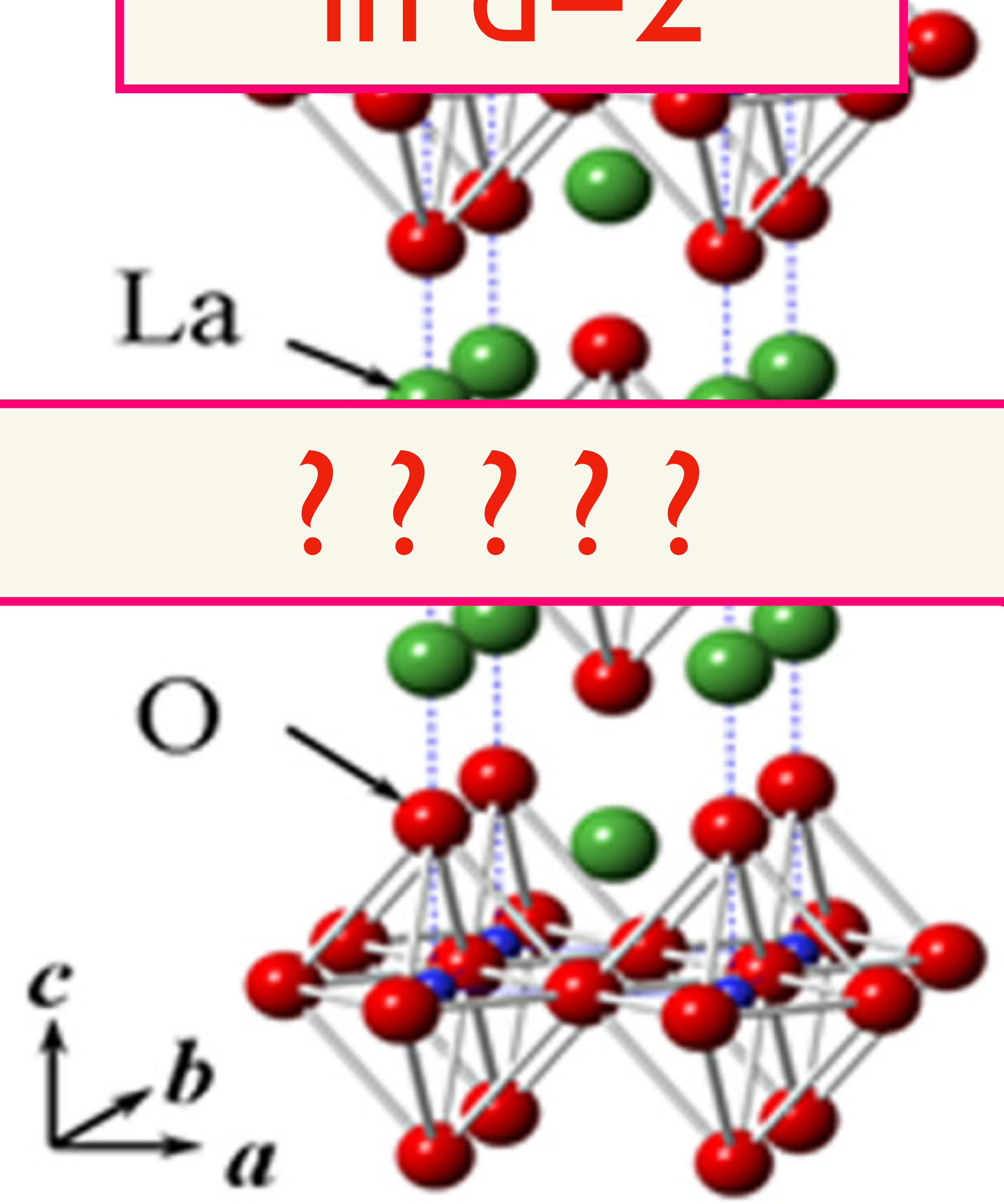
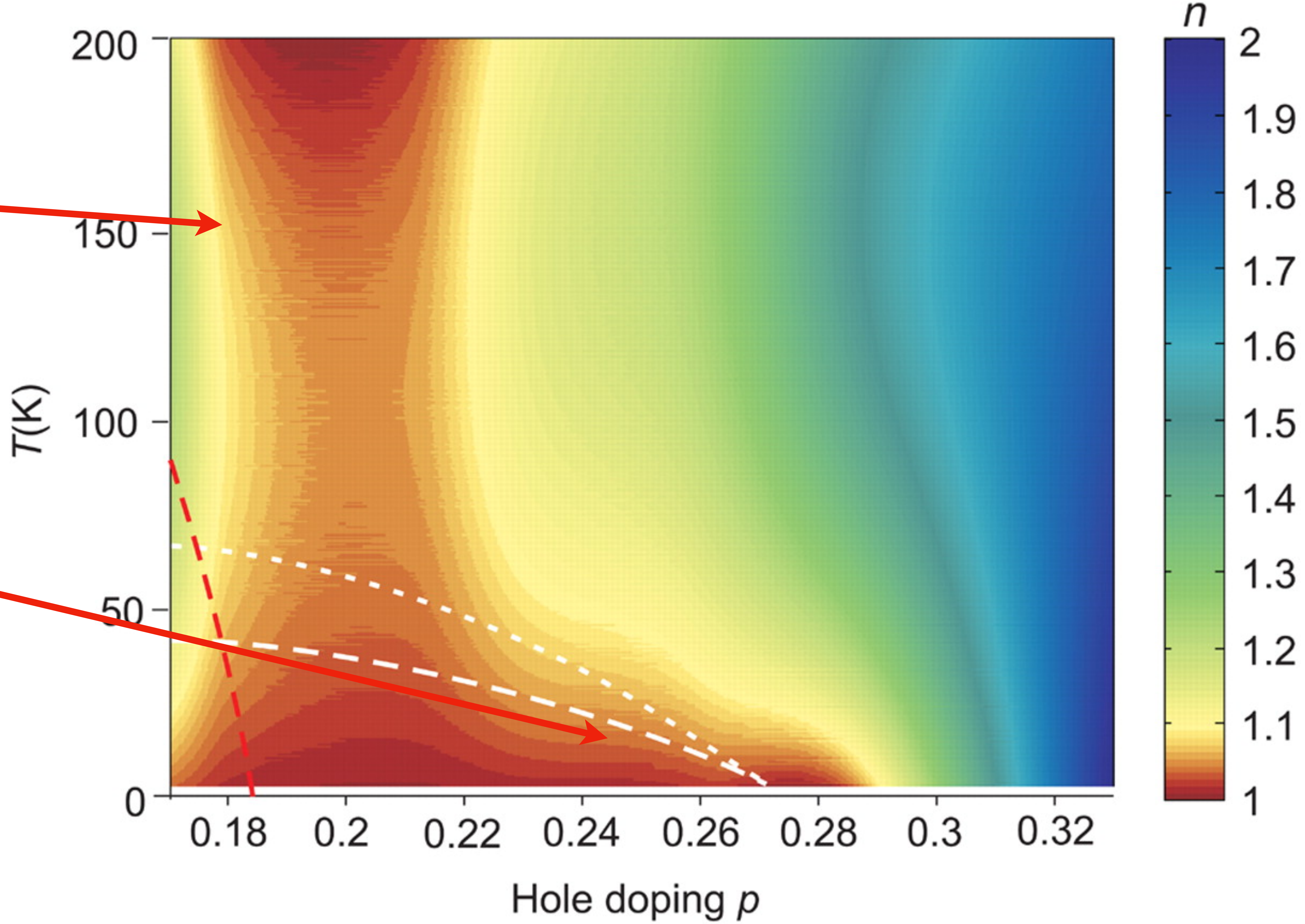
Yukawa-SYK in d=2

? ? ? ? ?





Yukawa-SYK model

Universal Yukawa-SYK theory in d=2 spatial dimensions

Random “mass” disorder,
andmapping to
random transverse-field Ising model (RTFIM)

Strong disorder and transport in strange metals

Aavishkar A. Patel, Peter Lunts, S. Sachdev, *arXiv:2312.06751*

Aavishkar Patel
Flatiron Institute, NYC

Peter Lunts
Harvard

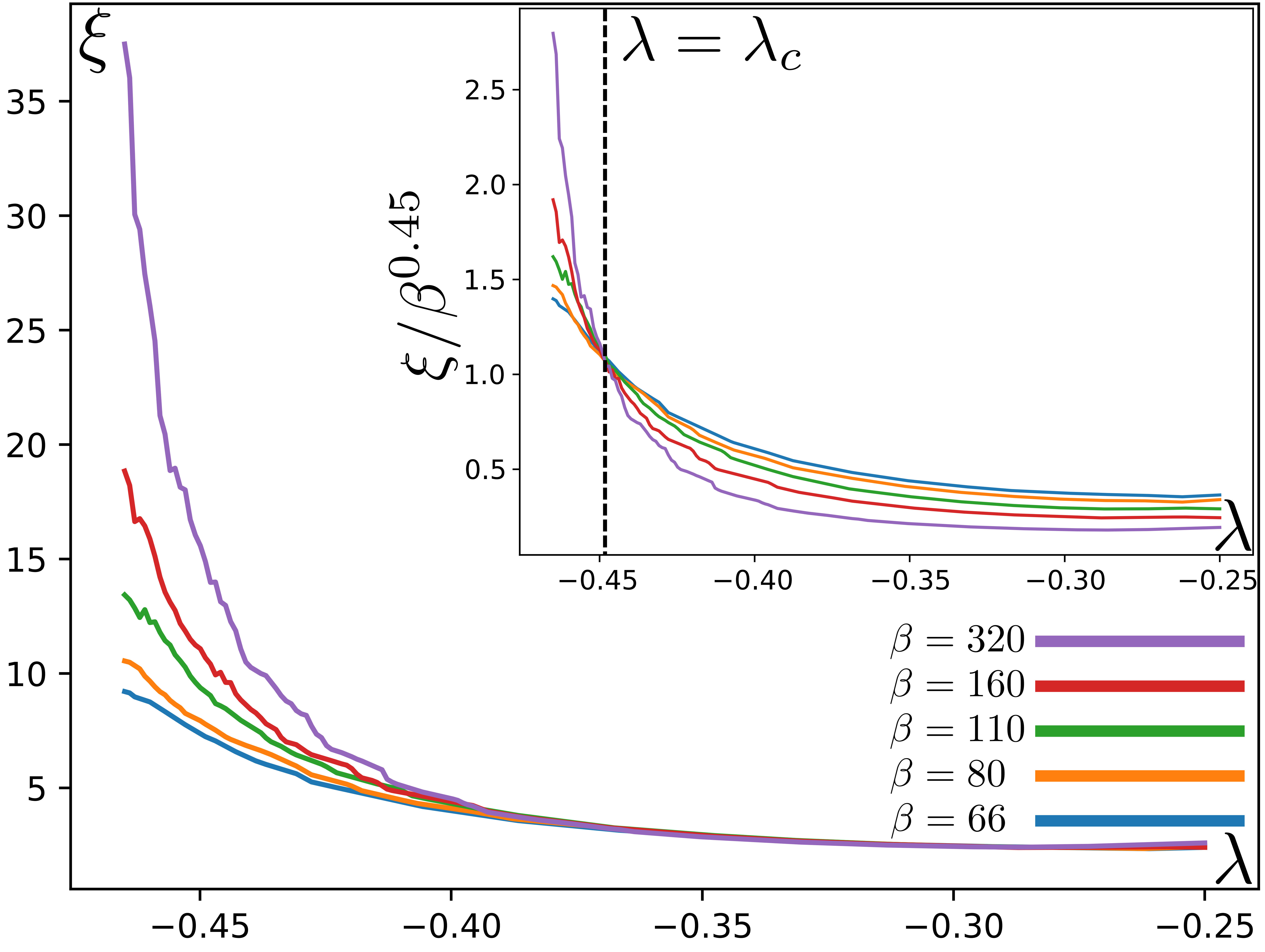
Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$
\begin{aligned}
\mathcal{S}_\phi &= \int d\tau \Big[\frac{J}{2}\sum_{\langle ij\rangle}\left(\phi_{ia}-\phi_{ja}\right)^2 + \sum_j\Big(\frac{\lambda+\lambda'_j}{2}\phi_{ja}^2 + \frac{u}{4M}\left(\phi_{ja}^2\right)^2\Big)\Big] \\
\mathcal{S}_{\phi d} &= \frac{T}{2}\sum_{\Omega}\sum_j\left(\gamma|\Omega| + \Omega^2/c^2\right)|\phi_{ja}(i\Omega)|^2 \, ,
\end{aligned}
$$

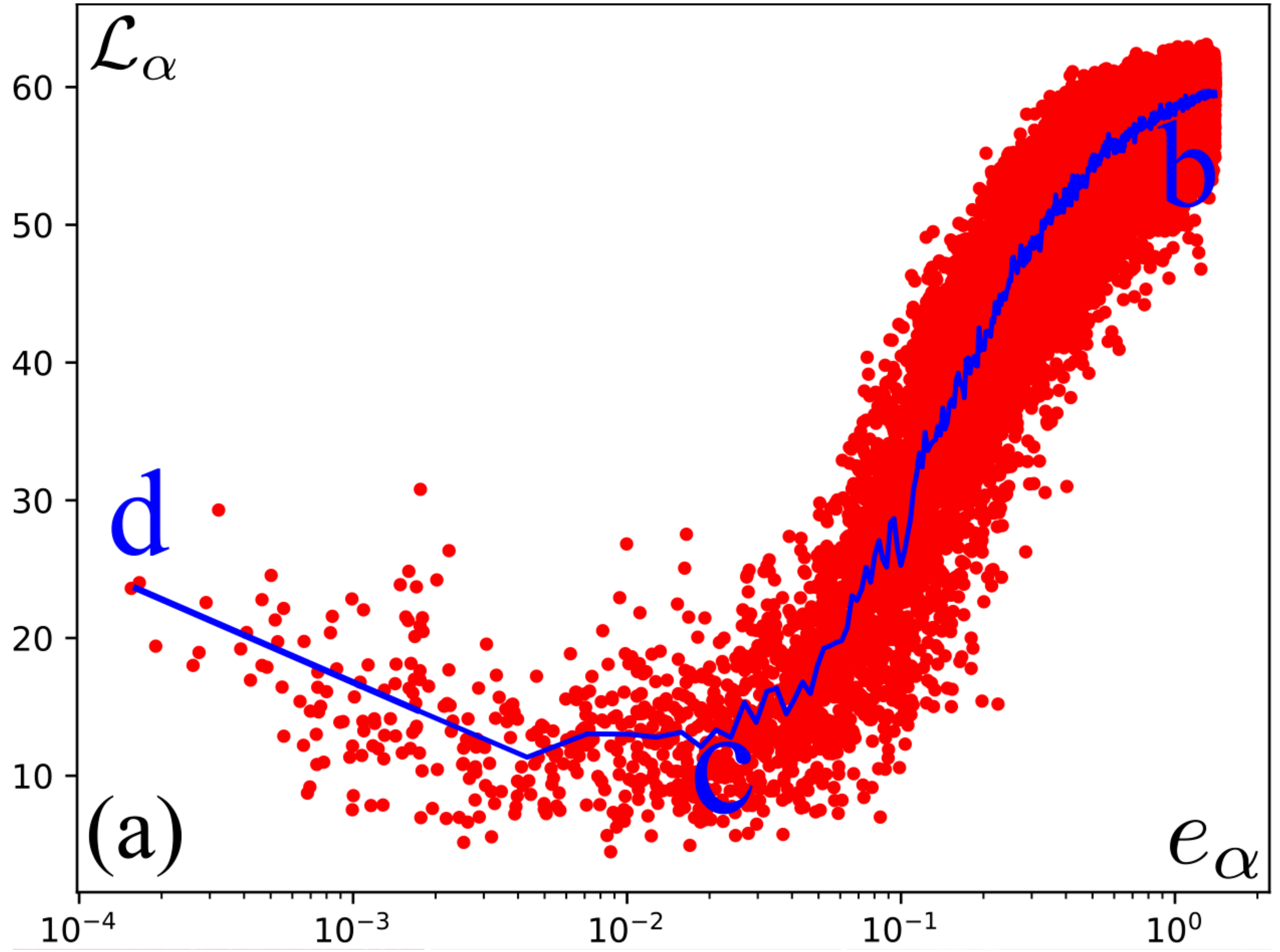
where $a = 1 \ldots N$ is a flavor index for an order parameter with $\mathrm{O}(N)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$
\begin{aligned}
\overline{\mathcal{S}}_\phi &= \int d\tau \Big[\frac{J}{2}\sum_{\langle ij\rangle}\left(\phi_{ia}-\phi_{ja}\right)^2 + \sum_j \frac{\overline{\lambda}'_j}{2}\phi_{ja}^2\Big] \\
\overline{\lambda}'_j &= \lambda + \lambda'_j + \frac{u}{M}\sum_a \langle \phi_{ja}^2 \rangle_{\overline{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}}
\end{aligned}
$$

ϕ correlation length ξ



ϕ eigenmodes localization length $\mathcal{L}_\alpha$



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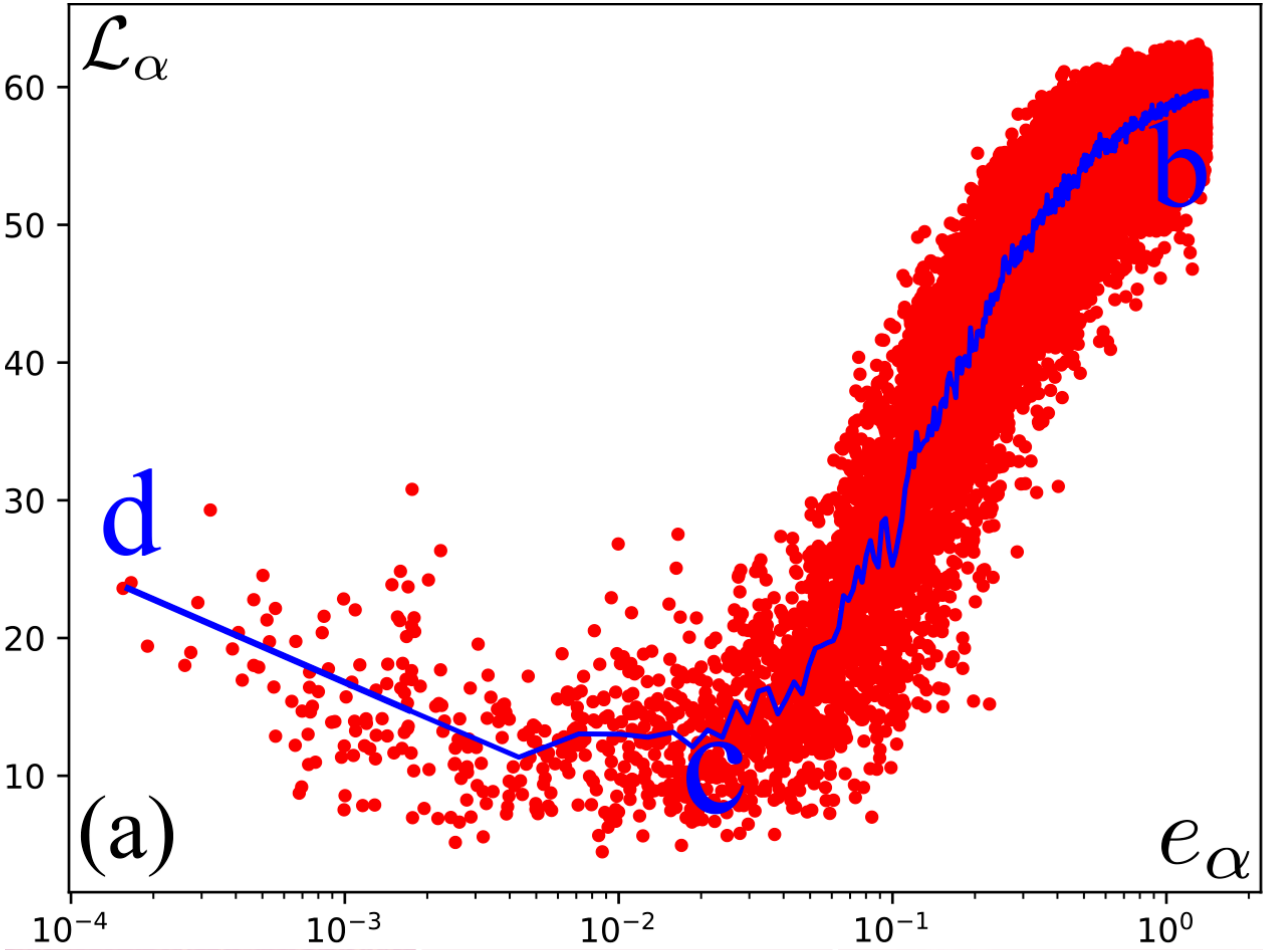

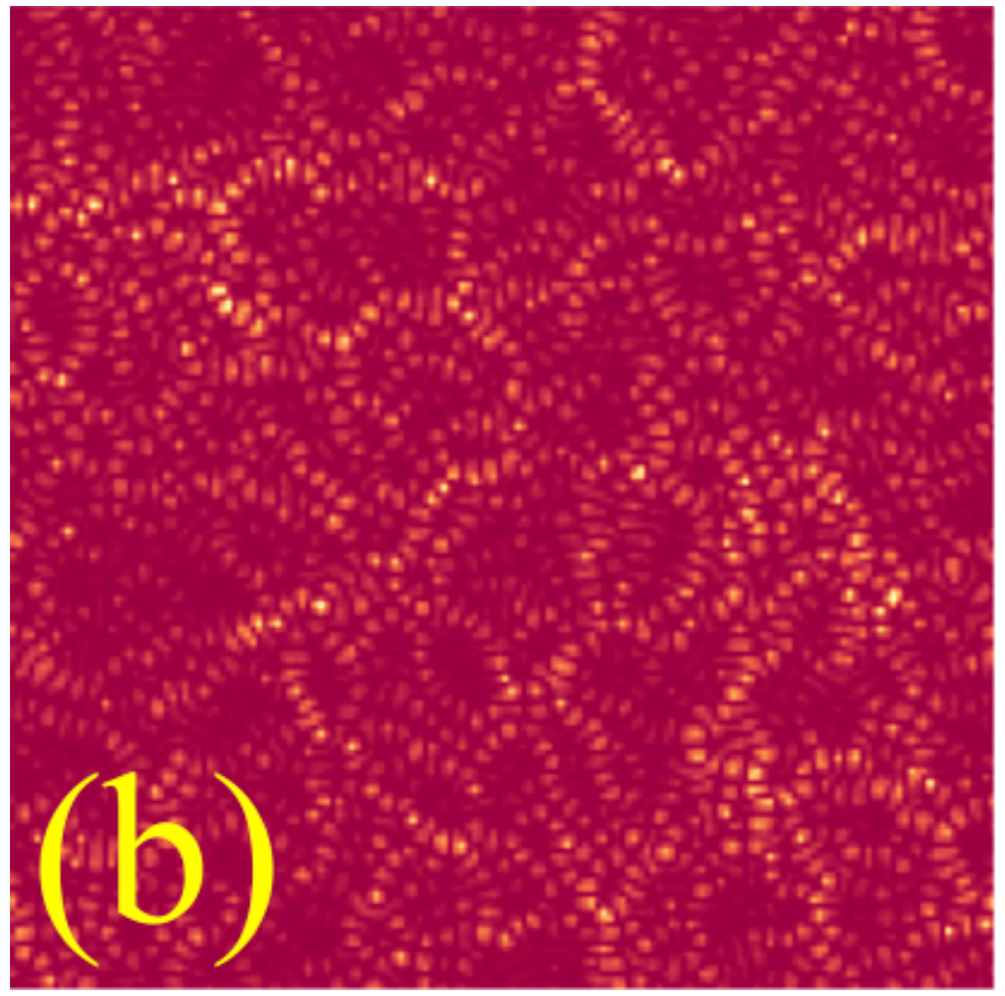

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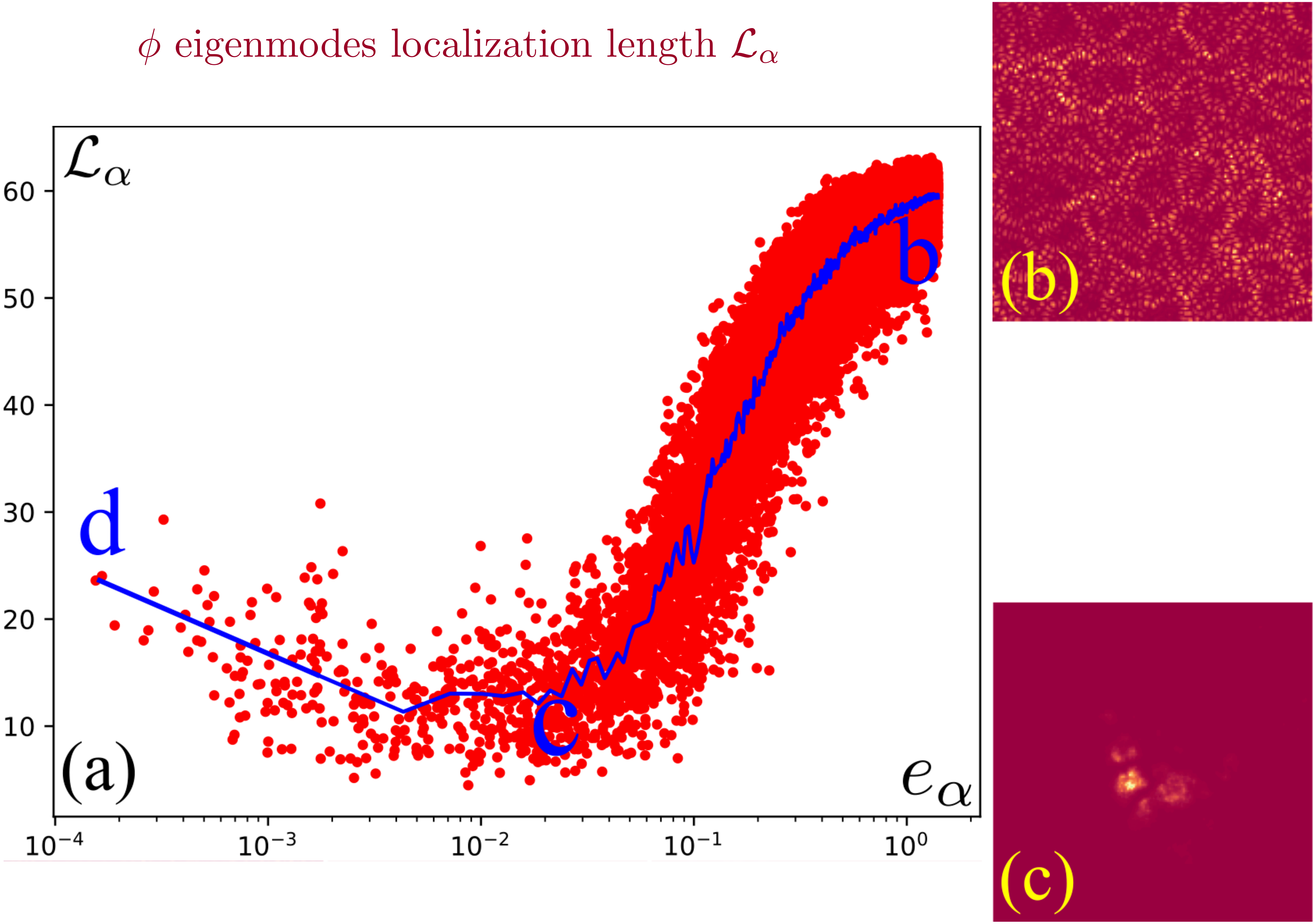

ϕ eigenmodes localization length $\mathcal{L}_\alpha$

T. Vojta and J. Schmalian, PRB **72**, 045438 (2005)

PRL **99,** 230601 (2007) PHYSICAL REVIEW LETTERS

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

(Received 19 May 2007; published 4 December 2007)

We study the effects of dissipation on a disordered quantum phase transition with $O(N)$ order-parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a nonperturbative infinite-randomness critical point

$$\mathcal{S}_b = \int d\tau \left(-\sum_{\langle ij \rangle} J_{ij}\phi_{ia}\phi_{ja} + \sum_j \left[\frac{s_j}{2}\phi_{ja}^2 + \frac{u}{4}\left(\phi_{ja}^2\right)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

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Strong disorder RG identical to that for the RTFIM (D.S. Fisher)

$$\widetilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\widetilde{s}_2 = 2\frac{s_2 s_3}{J_{23}}$$

$$H_{\mathrm{RTFIM}} = -\sum_{\langle ij\rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

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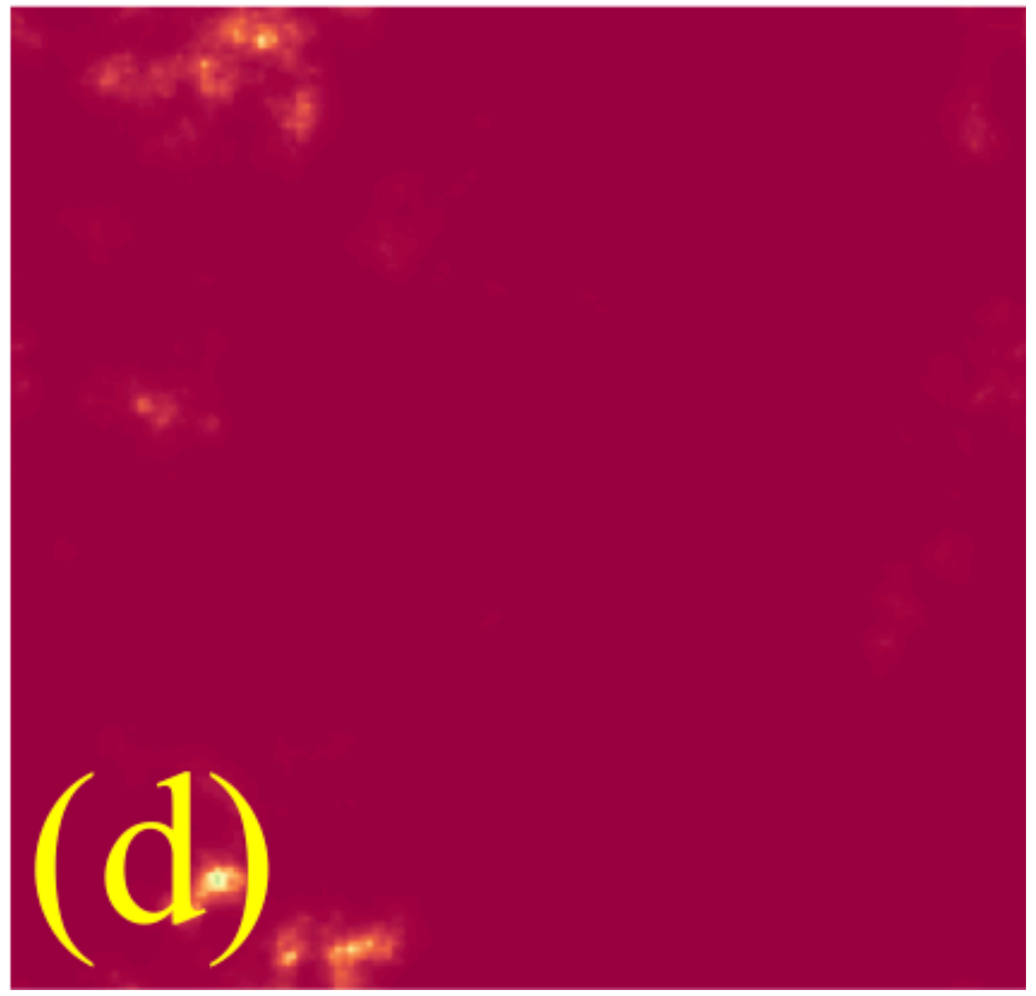
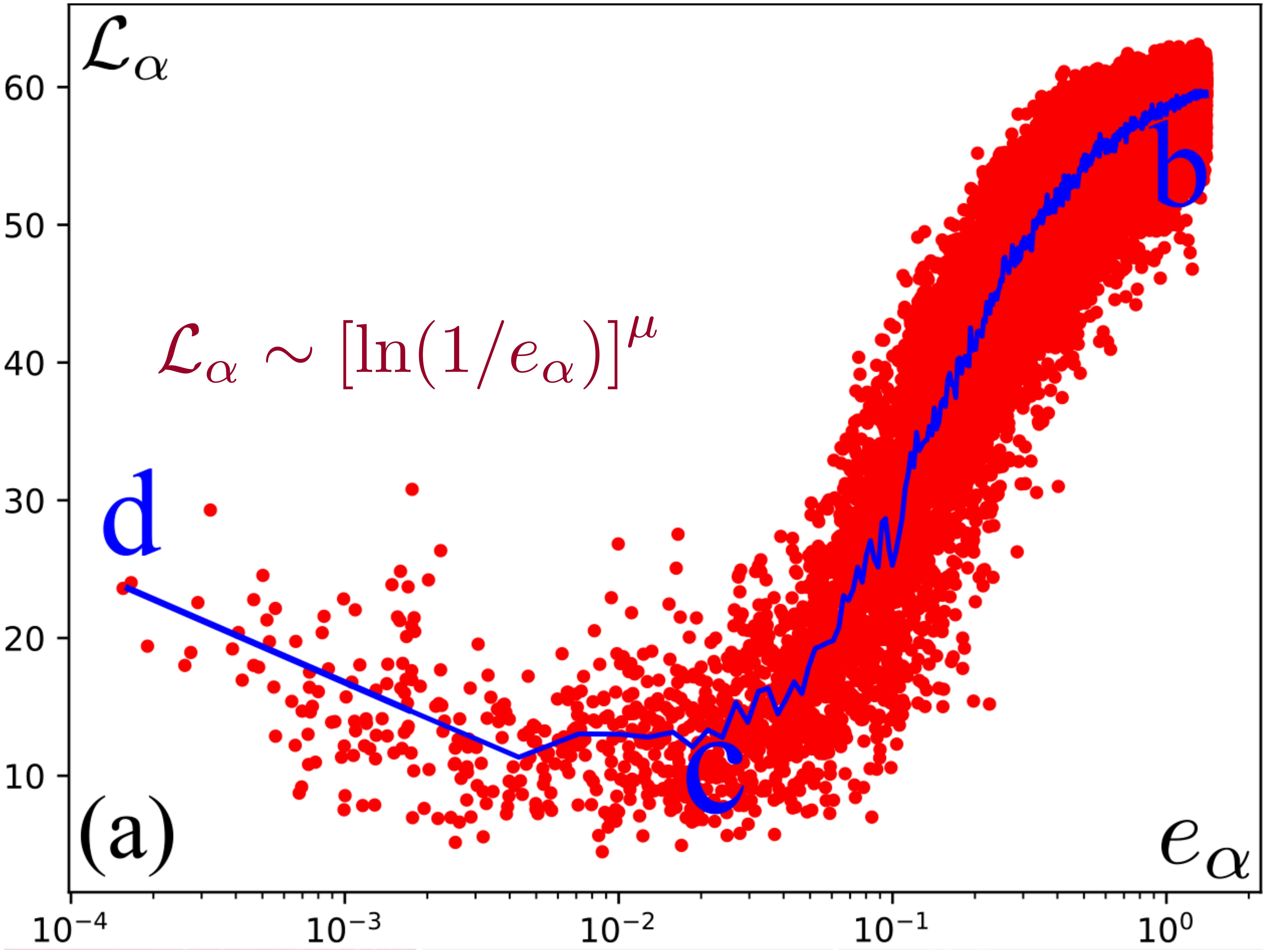
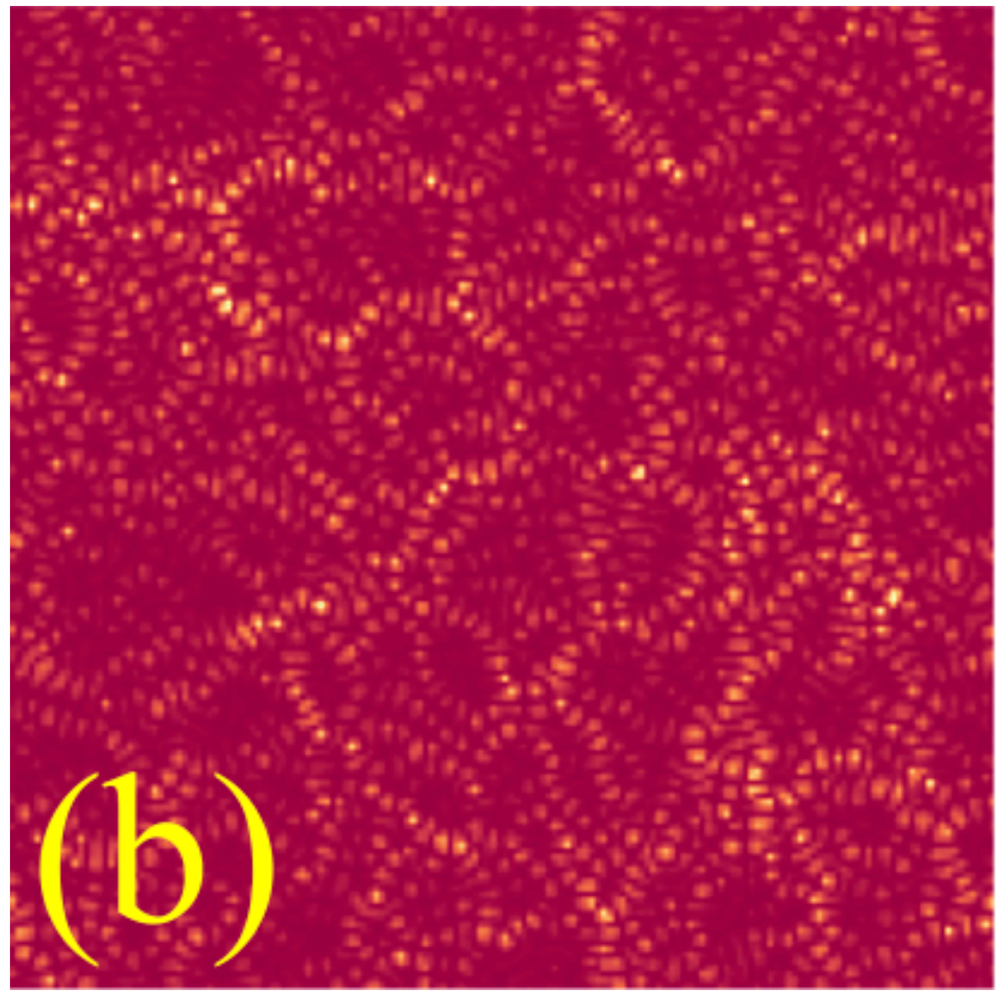
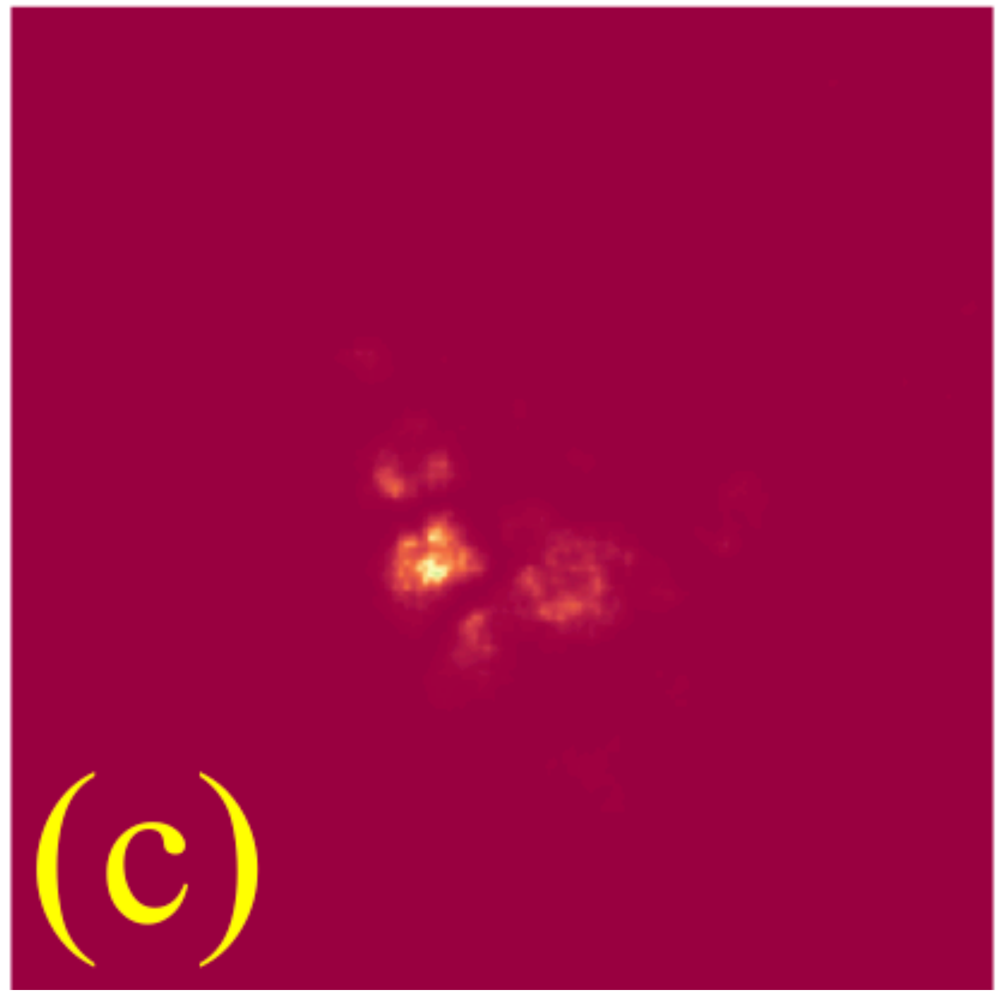
- Each rare region is described by a one-dimensional classical O(N) model with a long-range $1/\tau^2$ interaction.
- For $N \geq 2$, the classical model has an exponentially long correlation time at weak coupling (low 'temperature') - Kosterlitz 1976.
- This is similar to the classical Ising chain with short-range interactions.

ϕ eigenmodes localization length $\mathcal{L}_\alpha$





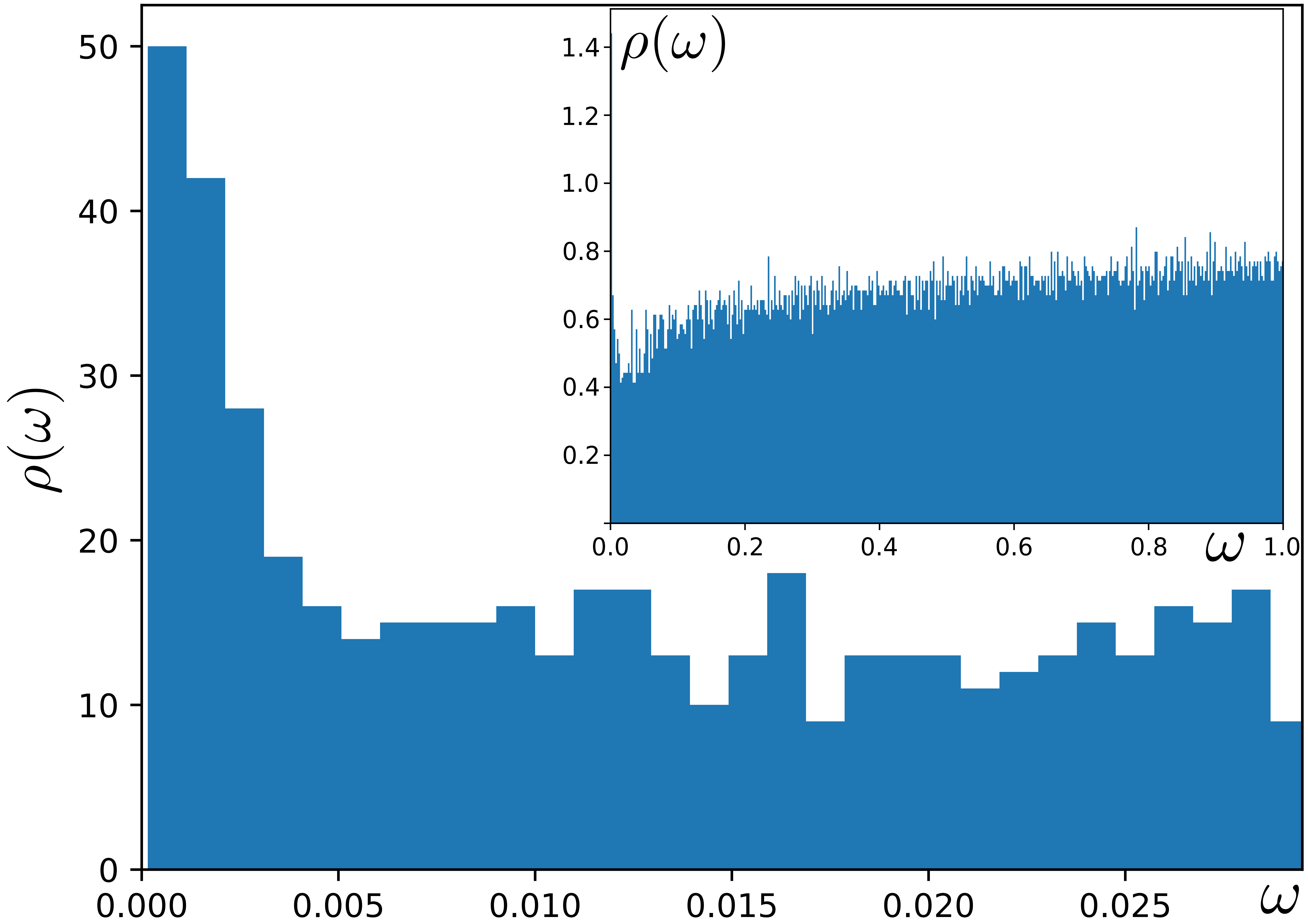




Olexei Motrunich, Siun-Chuon Mau, David A. Huse, Daniel S. Fisher, PRB **61**, 1160 (2000)

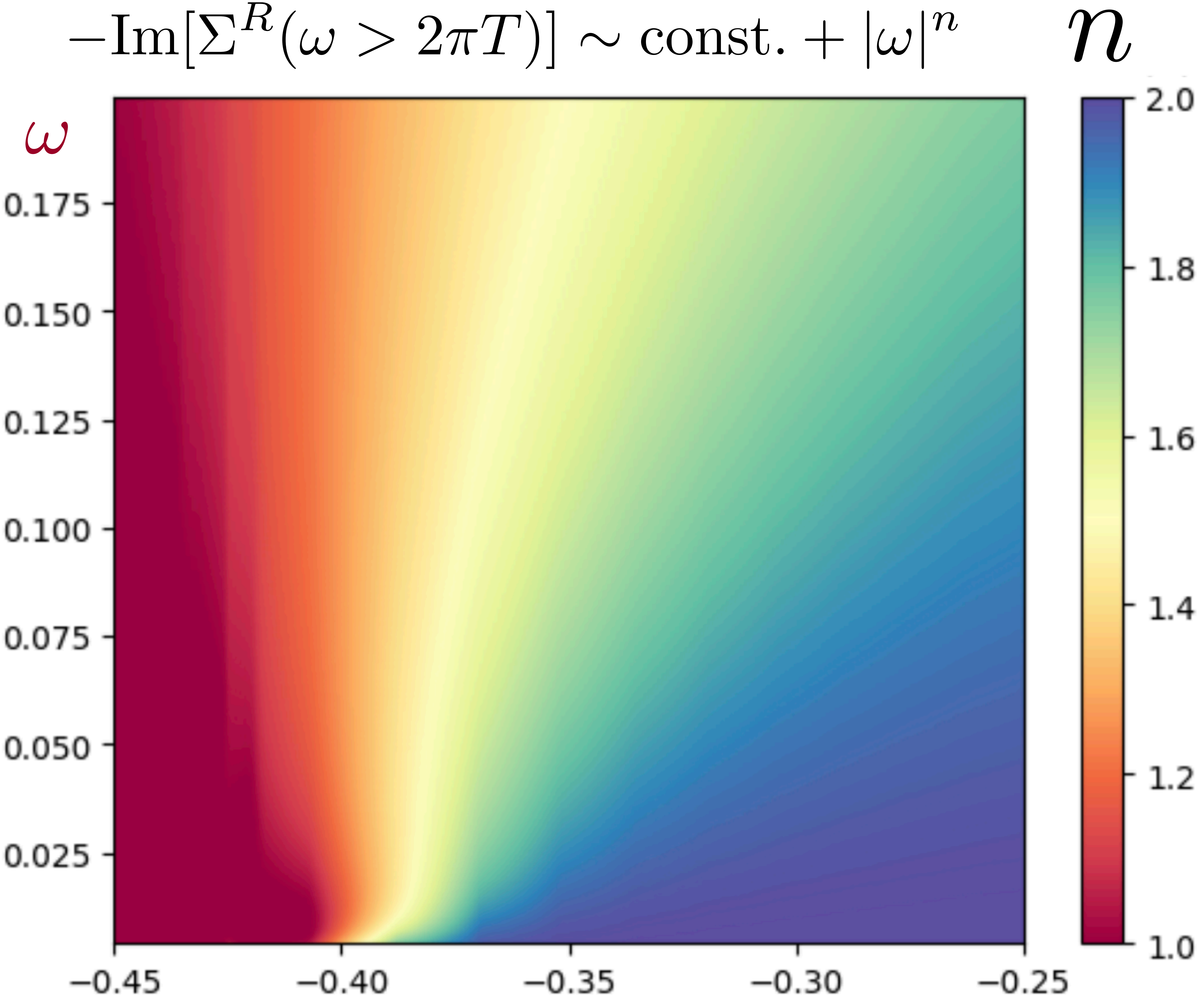
Density of states of ϕ eigenmodes $\rho(\omega)$

Yukawa-SYK theory $\rho(\omega) \sim$ constant.

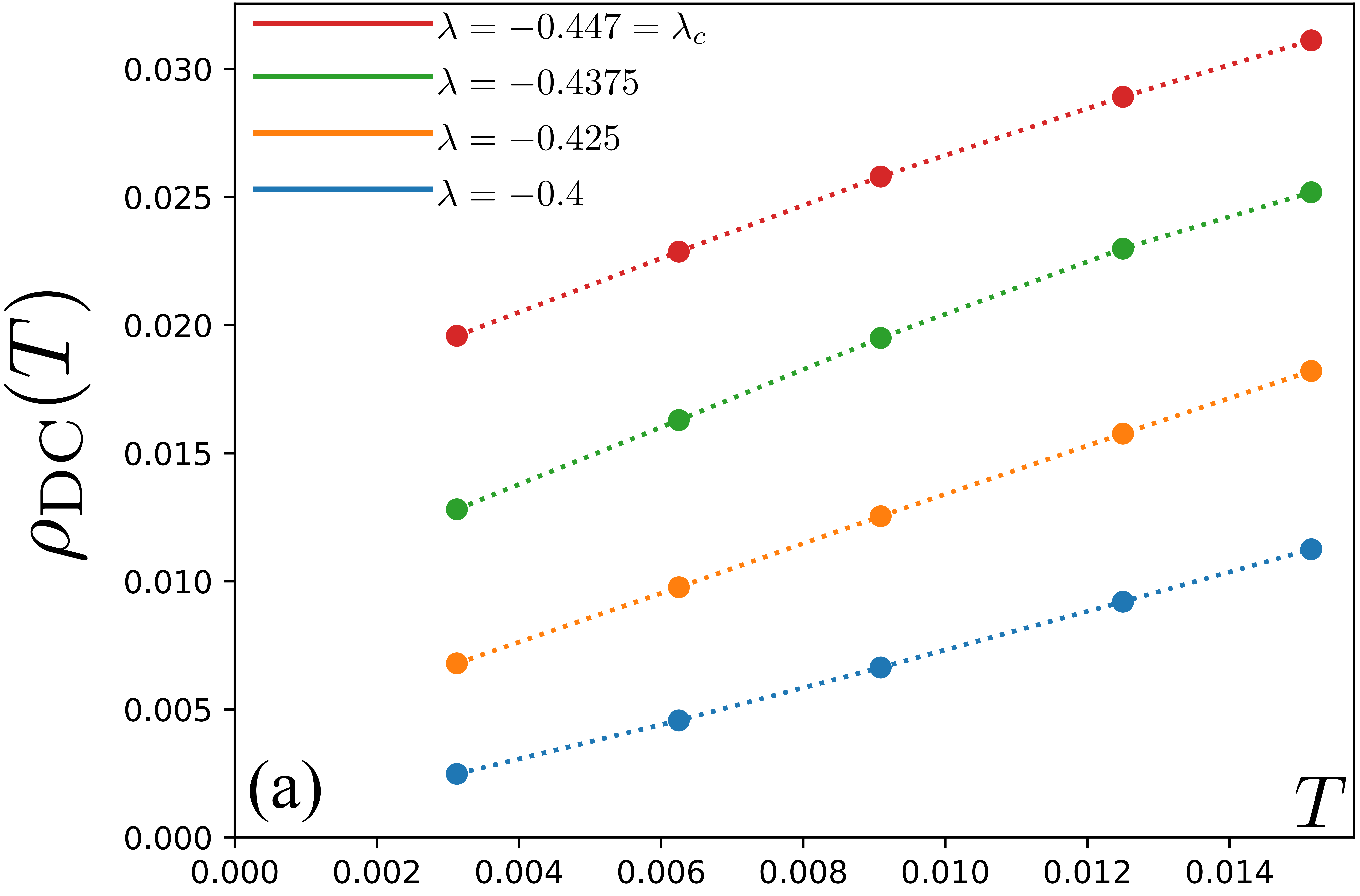


$$-\mathrm{Im}[\Sigma^R(\omega > 2\pi T)] \sim \text{const.} + |\omega|^n$$

Fermion
self energy

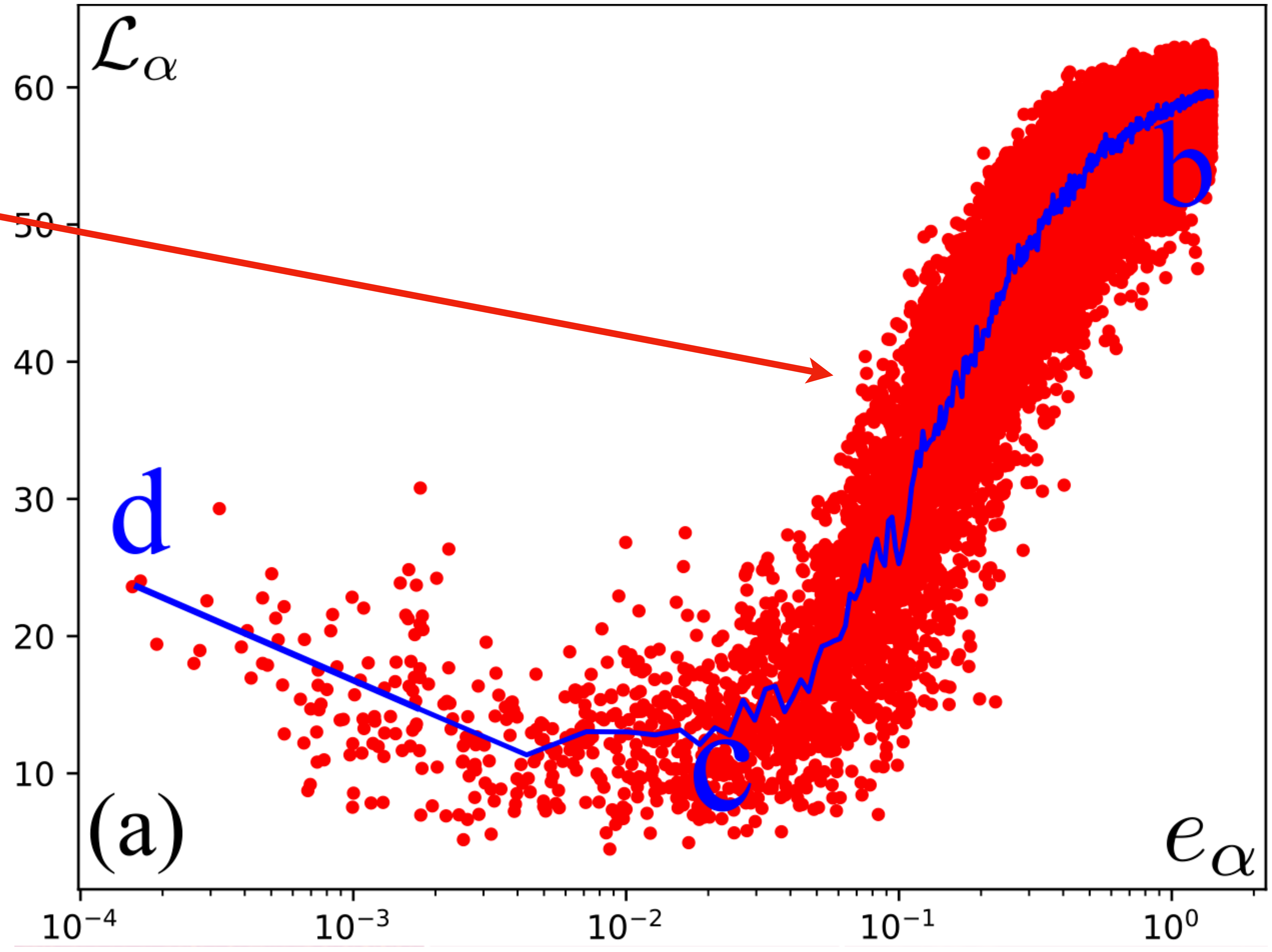


Resistivity



ϕ eigenmodes localization length $\mathcal{L}_\alpha$

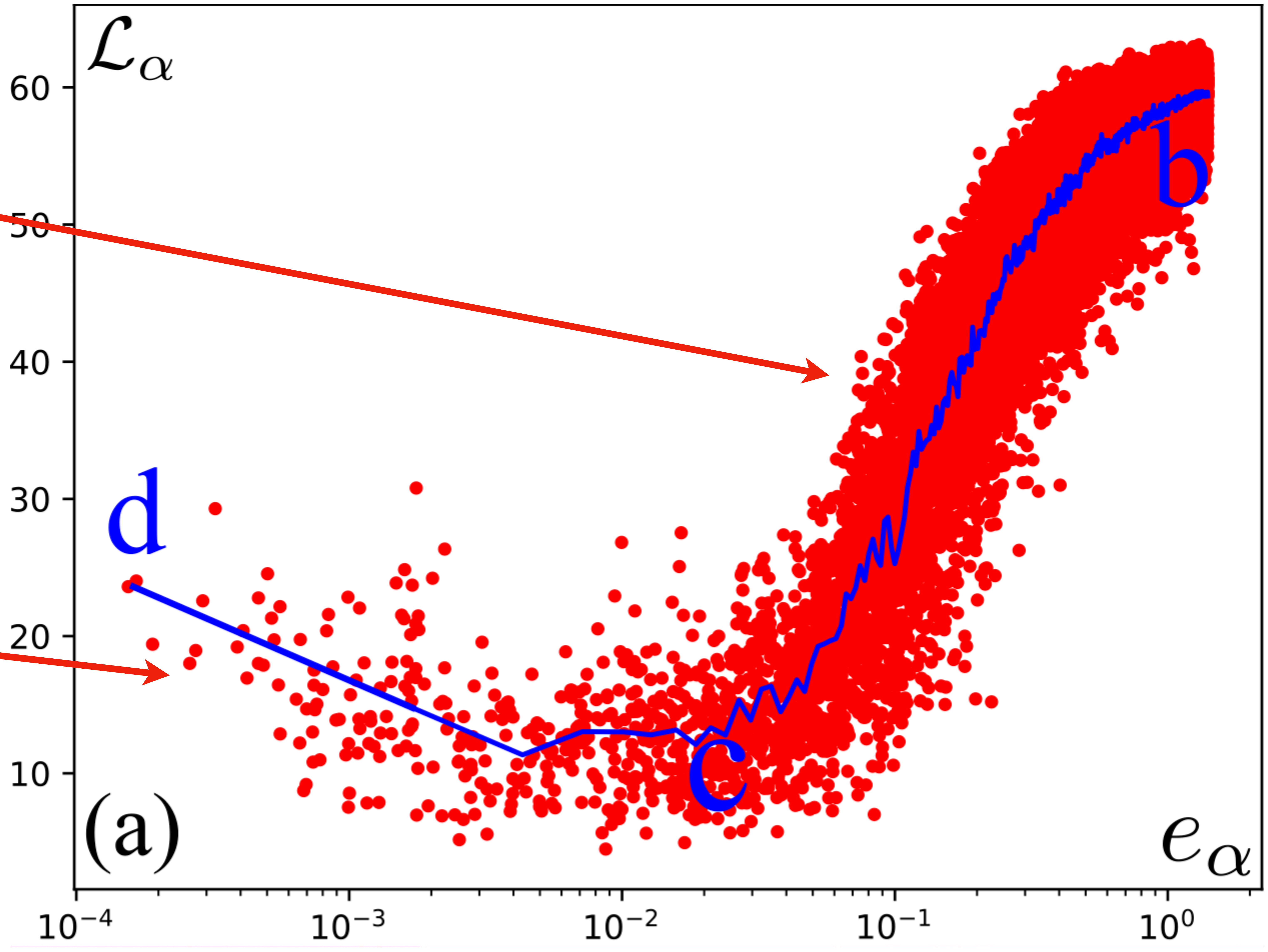
Extended bosons: physics of Yukawa-SYK



ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Extended bosons: physics of Yukawa-SYK

Localized bosons: physics of RTFIM



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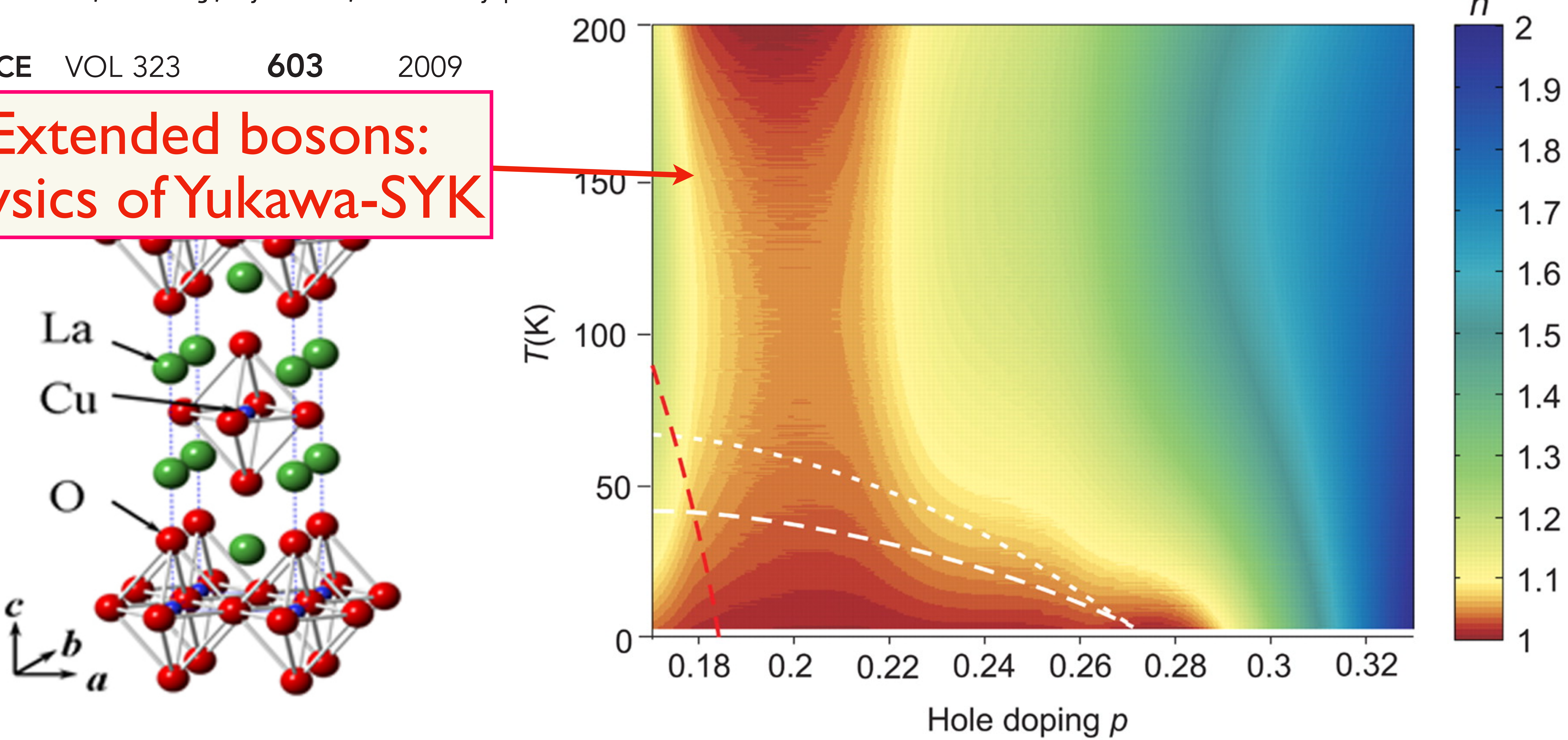

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Extended bosons: physics of Yukawa-SYK

Localized bosons: physics of RTFIM