



Theory of the pairbreaking superconductor-metal transition in nanowires

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Adrian Del Maestro, Bernd Rosenow, Nayana Shah, and Subir Sachdev, *Physical Review B* **77**, 180501 (2008).

Adrian Del Maestro, Bernd Rosenow, Markus Mueller, and Subir Sachdev, *Physical Review Letters* **101**, 035701 (2008).

Adrian Del Maestro, Bernd Rosenow, and Subir Sachdev, *Annals of Physics* **324**, 523 (2009).

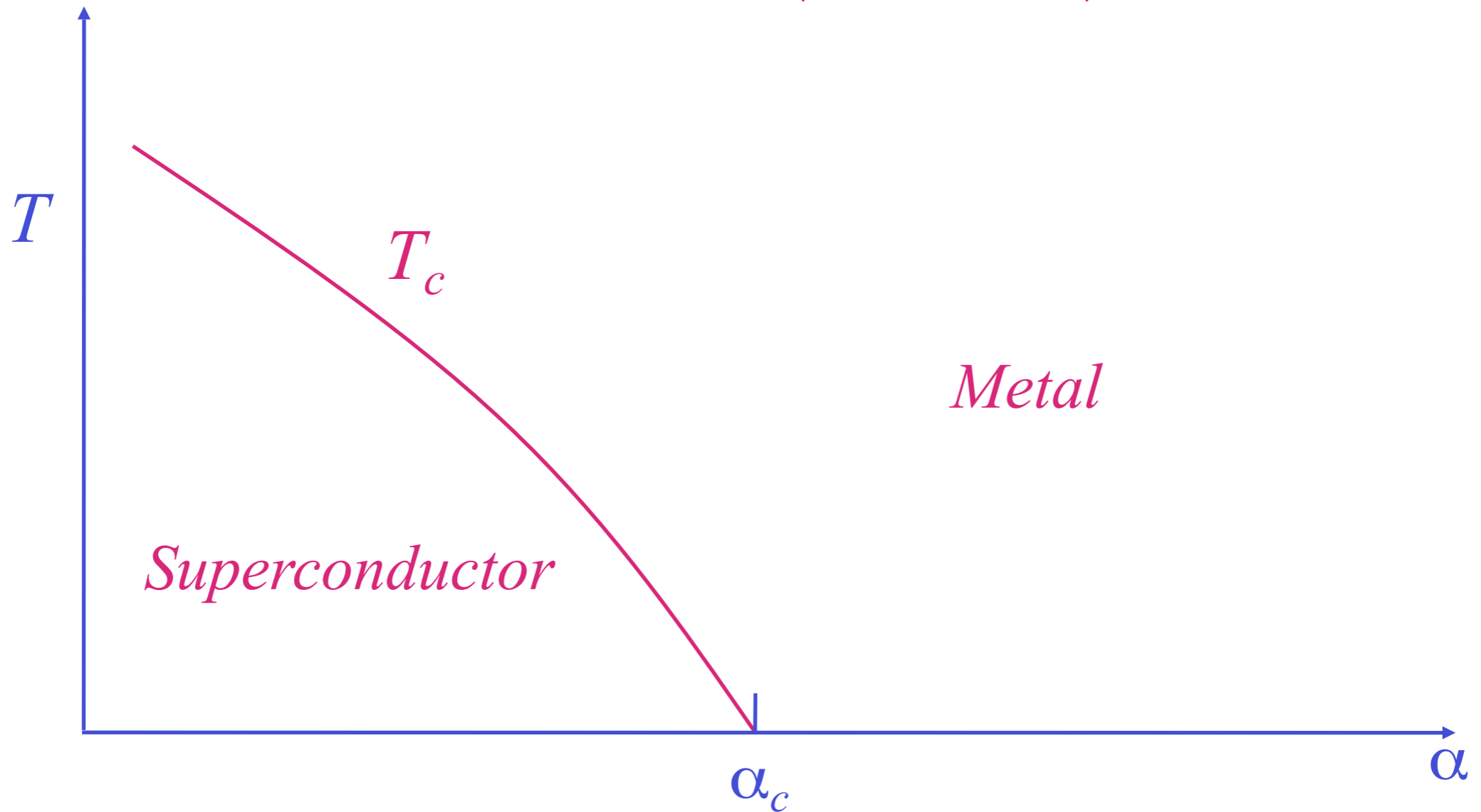




Adrian Del Maestro
University of British Columbia

Standard Abrikosov-Gorkov theory for the suppression of the mean-field BCS critical temperature, T_{c0} , of a superconductor by a pair-breaking frequency α :

$$\ln \left(\frac{T_c}{T_{c0}} \right) = \psi \left(\frac{1}{2} \right) - \psi \left(\frac{1}{2} + \frac{\hbar\alpha}{2\pi k_B T_c} \right)$$



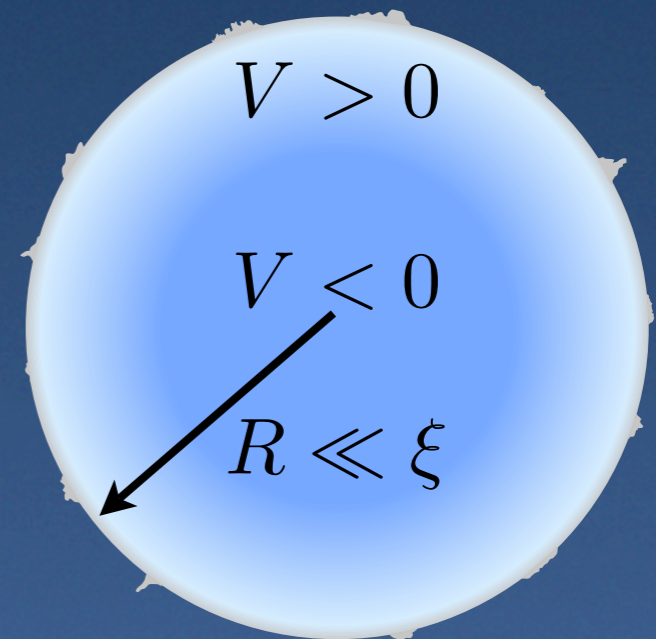
Pair-breaking in nanowires

- Evidence for **magnetic moments** on the wire's surface

A. Rogachev et al., PRL (2006)

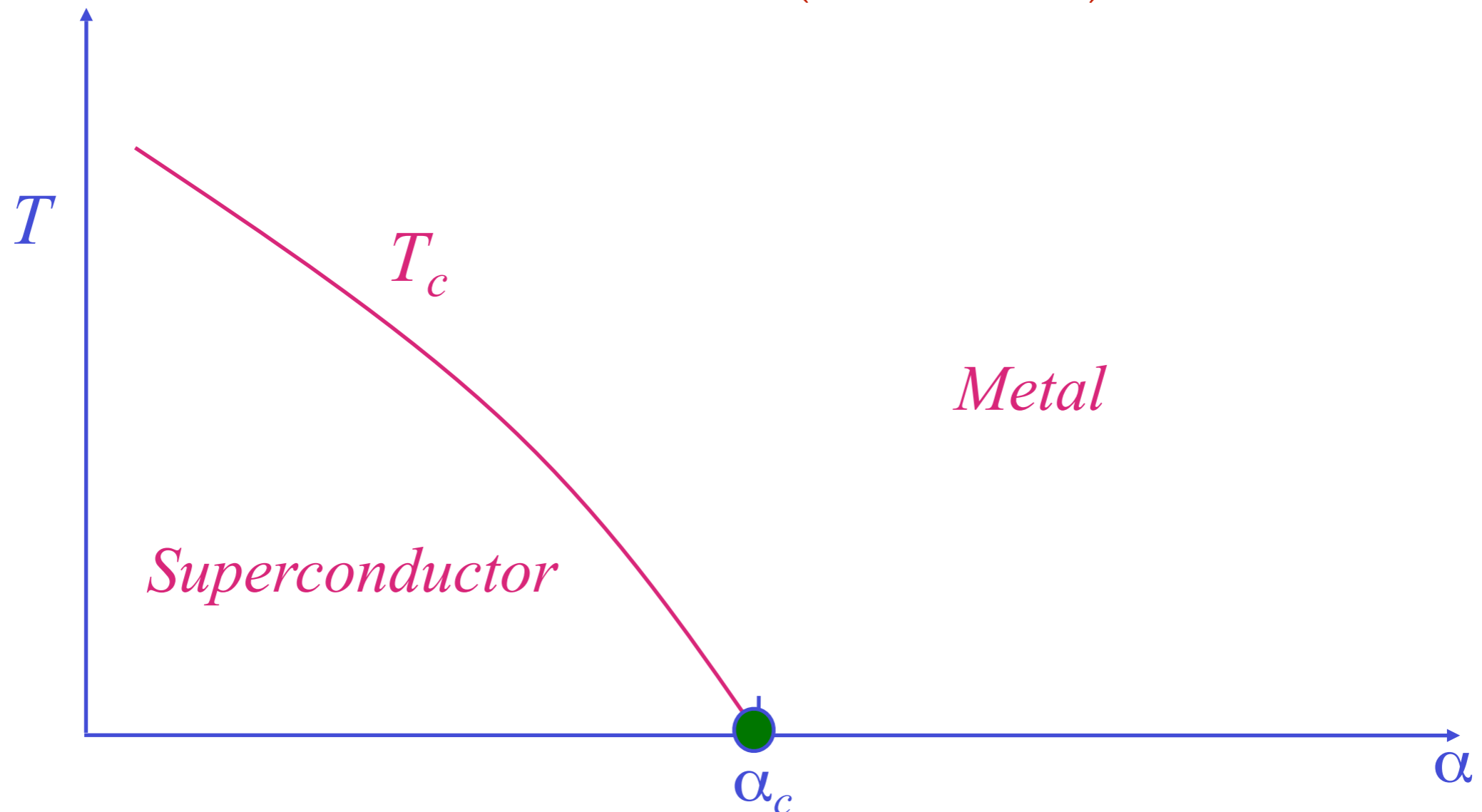
- Inhomogeneity in the pairing interaction

B. Spivak et. al, PRB (2001)







Standard Abrikosov-Gorkov theory for the suppression of the mean-field BCS critical temperature, T_{c0} , of a superconductor by a pair-breaking frequency α :

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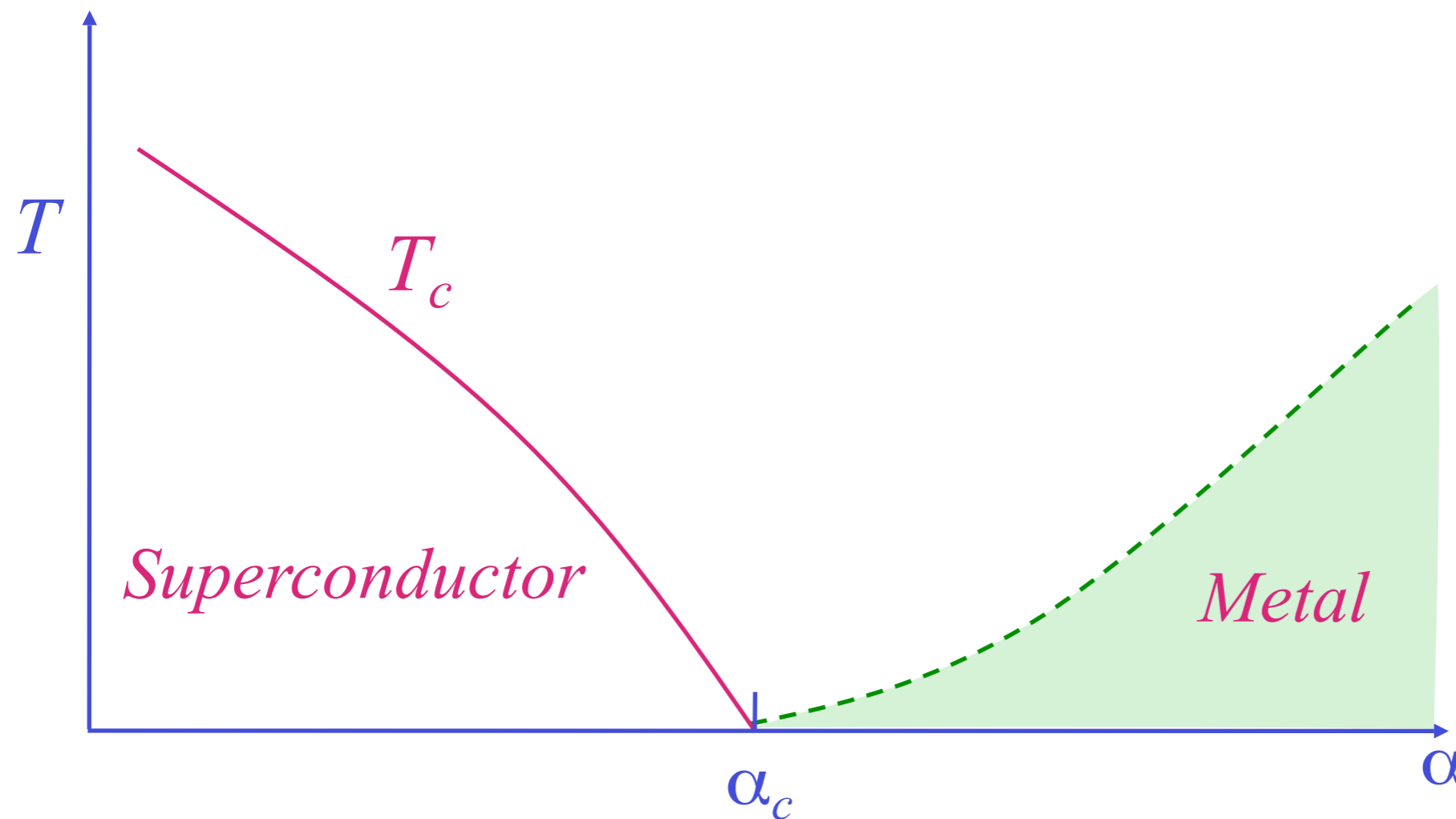


There is a critical $\alpha = \alpha_c$ such that $T_c = 0$ for $\alpha > \alpha_c$. We are interested in the nature of the crossovers near the quantum phase transition at $\alpha = \alpha_c$ especially in spatial dimensions $d = 1, 2$.

Outline

-  Field theory for pair-breaking superconductor-metal transition
-  Finite temperature transport (weak disorder)
-  Dirty wires and the strong disorder renormalization group
-  Evidence for an infinite randomness fixed point

Computation of fluctuation conductivity in metal at low temperatures

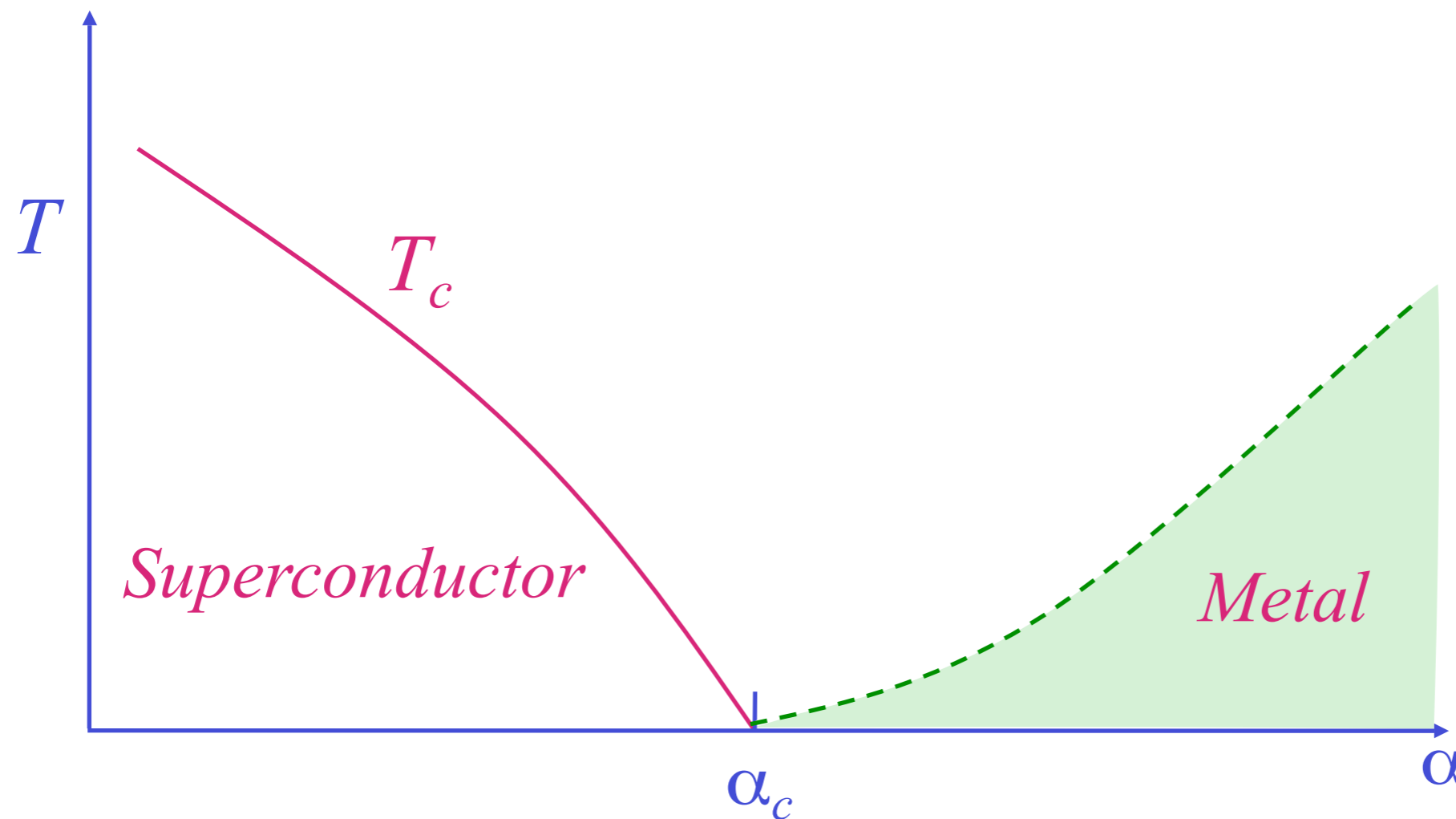


At $T = 0$, the Maki-Thomson and density of states corrections to the conductivity, $\delta\sigma$, *increase* with increasing α (negative magnetoresistance):

$$\delta\sigma \sim (\alpha - \alpha_c)$$

(A. V. Lopatin, N. Shah, and V. M. Vinokur, Phys. Rev. Lett. **94**, 037003 (2005)).

Computation of fluctuation conductivity in metal at low temperatures



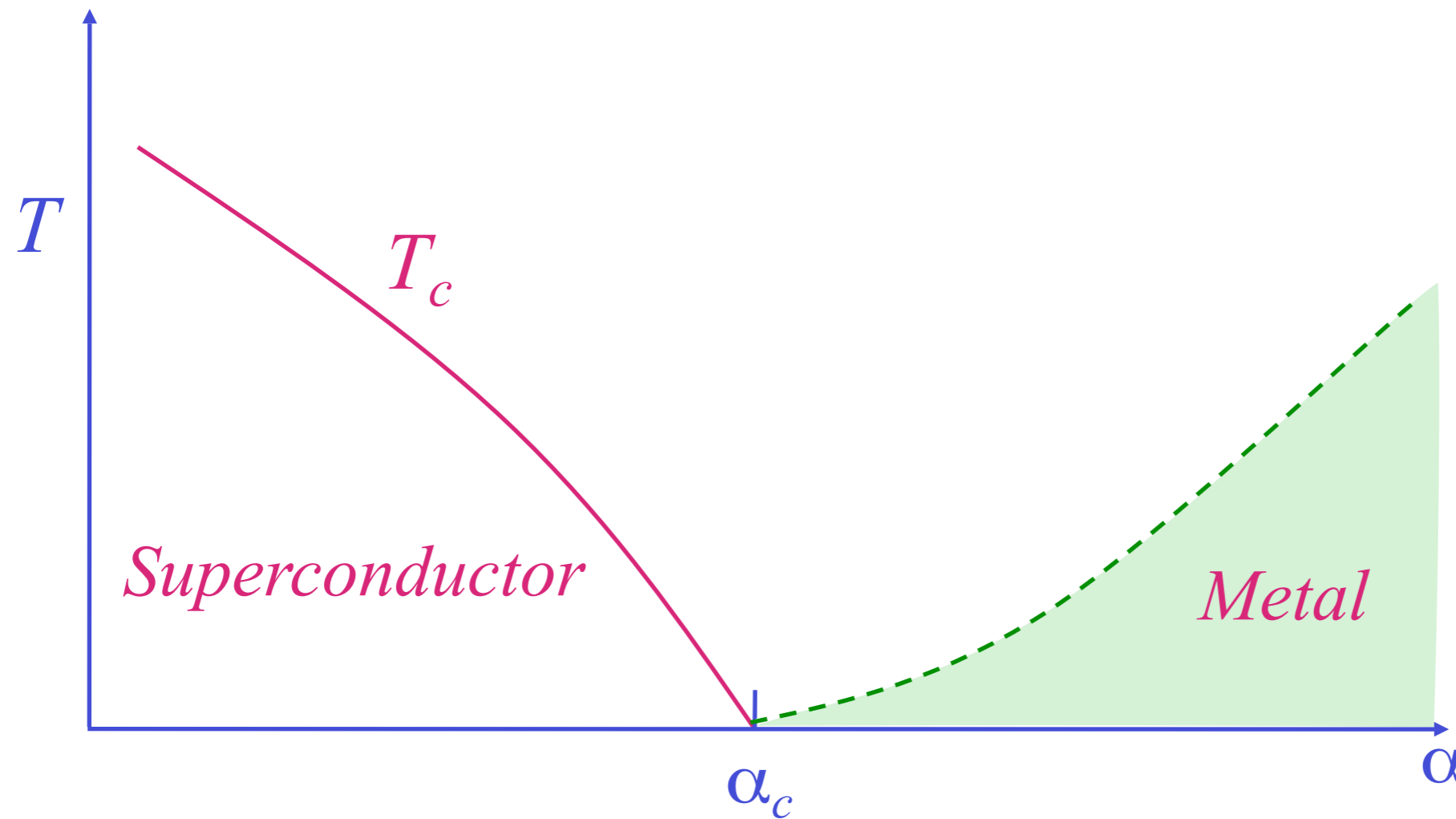
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We will argue that these are corrections to scaling to the theory of the quantum critical point. These corrections are *dangerously irrelevant*, because they dominate at low T .

Computation of fluctuation conductivity in metal at low temperatures

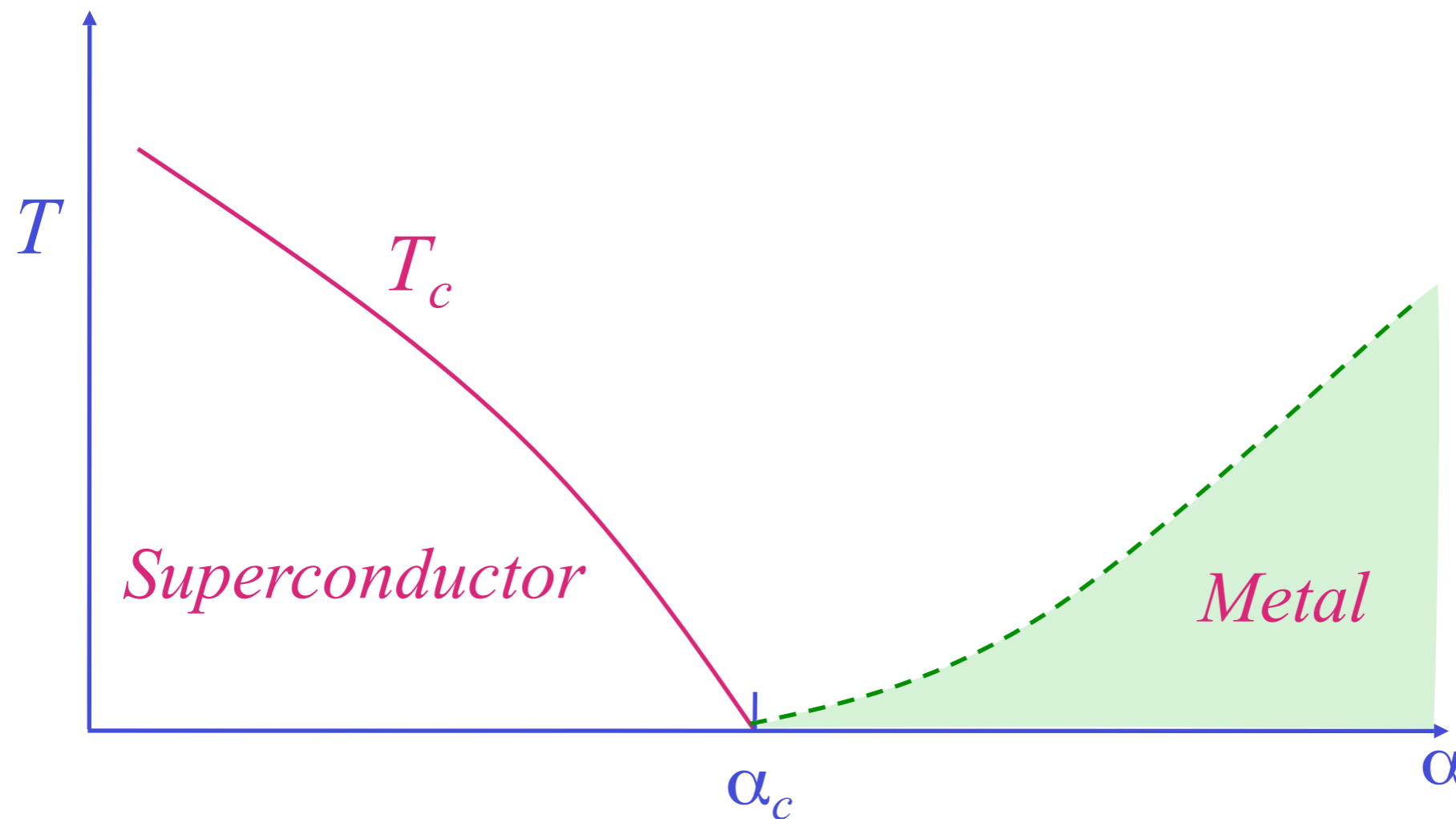


At $T > 0$, Aslamazov-Larkin corrections lead to

$$\delta\sigma \sim \frac{4e^2}{h} \frac{D^{2-d} (k_B T / \hbar)^2}{(\alpha - \alpha_c)^{(6-d)/2}}$$

(A. V. Lopatin, N. Shah, and V. M. Vinokur, Phys. Rev. Lett. **94**, 037003 (2005))

Computation of fluctuation conductivity in metal at low temperatures



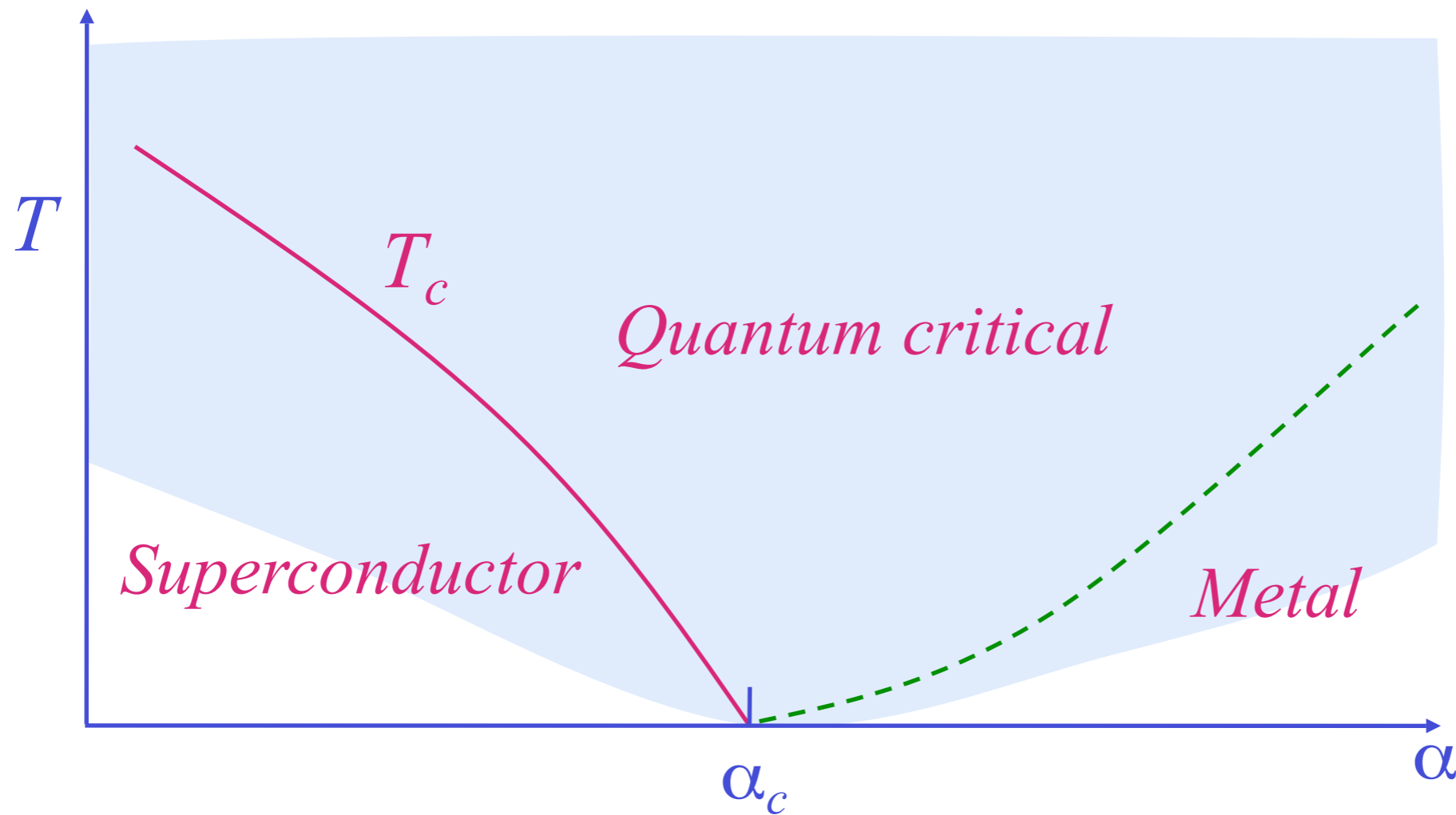
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We will argue these are contained in the quantum critical theory. Note, however, the leading critical fluctuations vanish at $T = 0$ for $\alpha > \alpha_c$. This leads to a non-monotonic T dependence in critical theory.

Theory for quantum-critical region, and beyond

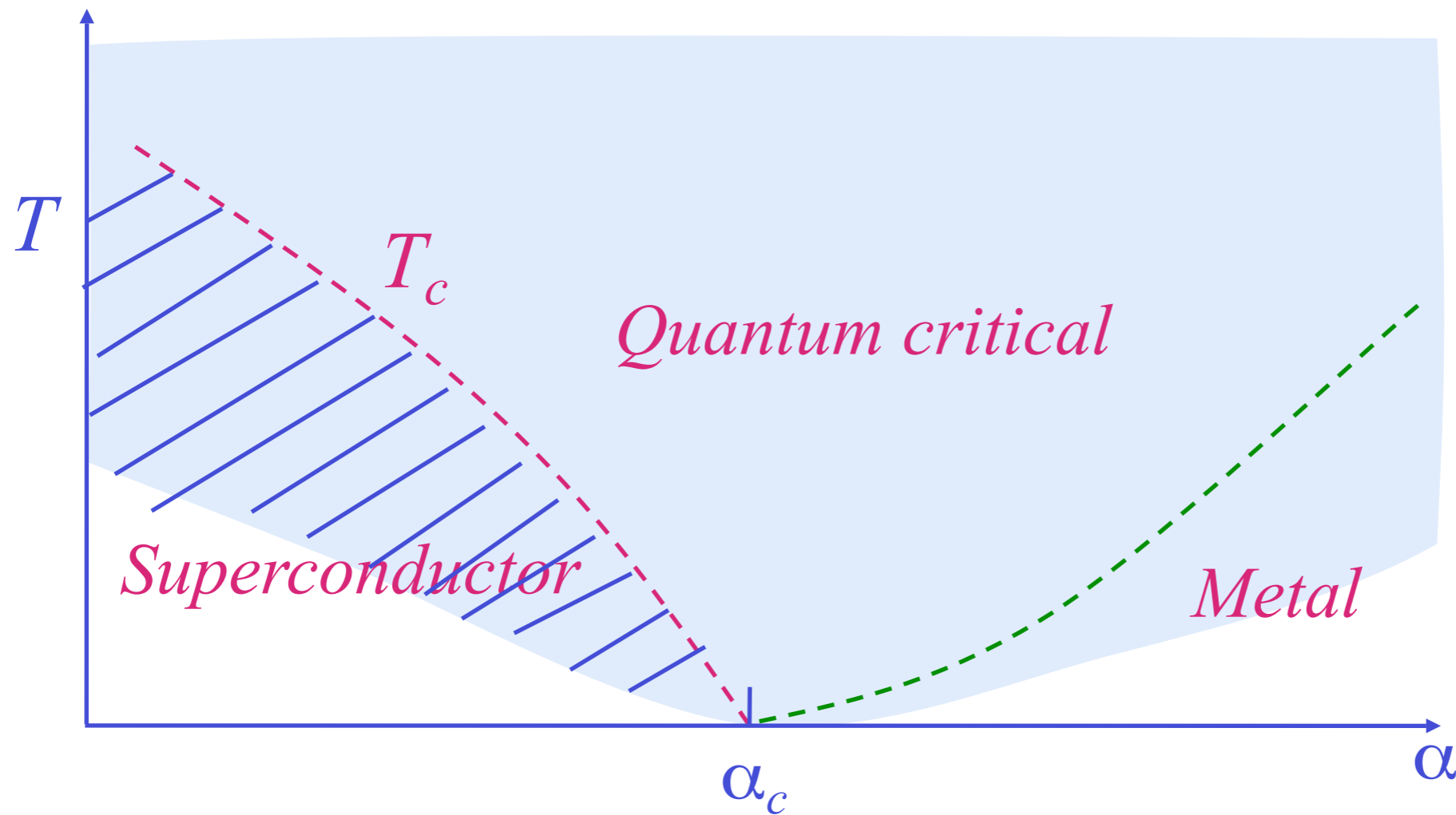


Cooperon fluctuations have propagator $\sim 1/(Dq^2 + |\omega| + \alpha)$. Self-interactions between such fluctuations are described by

$$\mathcal{S}_{\text{bulk}} = \int d^d x \left[\int \frac{d\omega}{2\pi} \left(D |\nabla_x \psi(x, \omega)|^2 + (|\omega| + \alpha) |\psi(x, \omega)|^2 \right) + \frac{u}{2} \int d\tau |\psi(x, \tau)|^4 \right],$$

R. Ramazashvili and P. Coleman, Phys. Rev. Lett. **79**, 3752 (1997); I. F. Herbut, Phys. Rev. Lett. **85**, 1532 (2000); D. Dalidovich and P. Phillips, Phys. Rev. Lett. **84**, 737 (2000); B. Spivak, A. Zyuzin, and M. Hruska, Phys. Rev. B **64**, 132502 (2001)

Theory for quantum-critical region, and beyond



In one dimension, theory reduces to the Langer-Ambegaokar-McCumber-Halperin theory (Model A dynamics), near mean-field T_c

$$\frac{\partial \psi}{\partial t} = - \left[-D \partial_x^2 \psi + \alpha \psi + u |\psi|^2 \psi \right] \\ + \text{thermal Langevin noise}$$

Role of charge conservation in quantum critical theory

Dynamics of quantum theory (and model A) does not conserve total charge.

Analogous to the Fermi-liquid/spin-density-wave transition (Hertz theory), where dynamics of critical theory does not conserve total spin.

Role of charge conservation in quantum critical theory

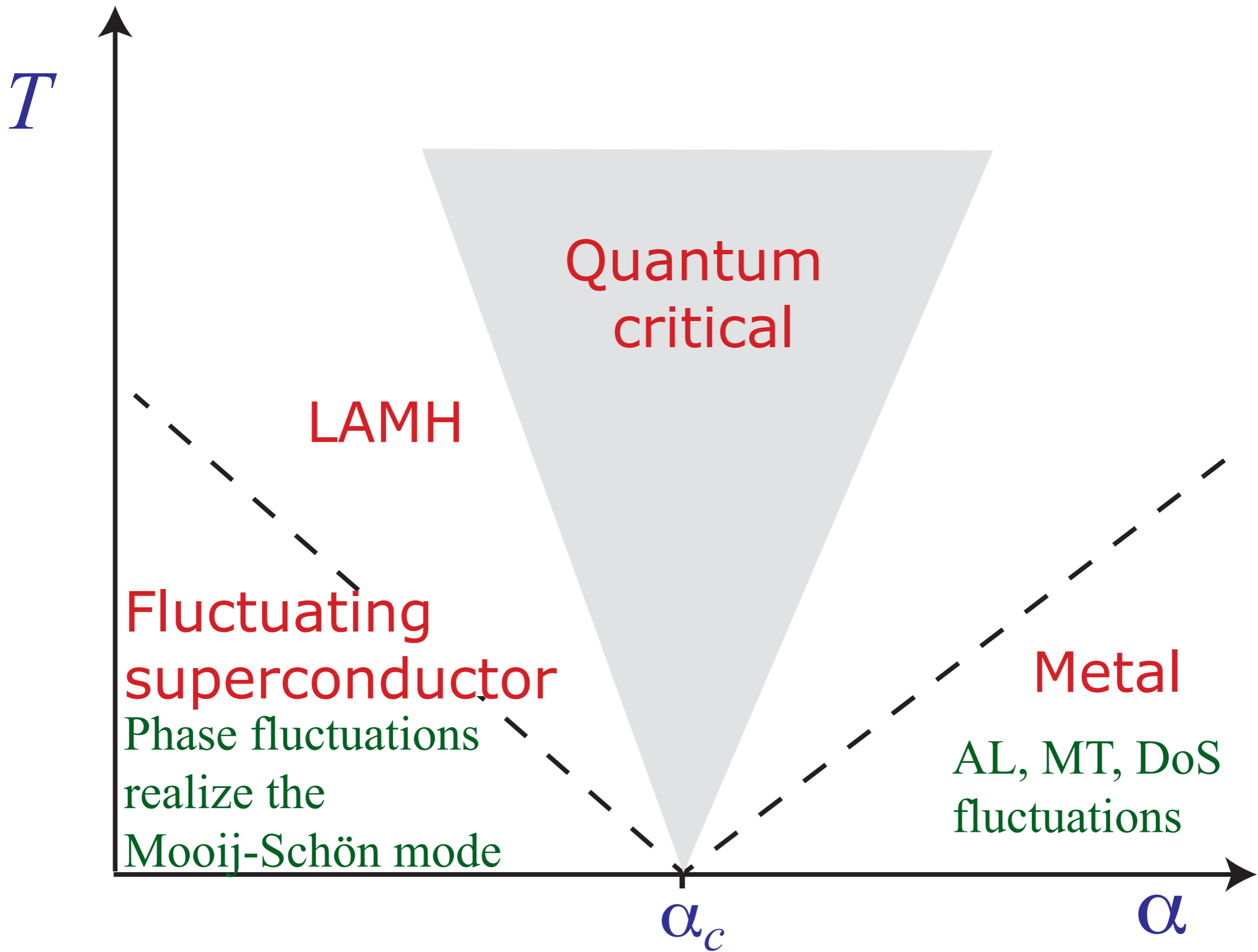
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Analogous to the Fermi-liquid/spin-density-wave transition (Hertz theory), where dynamics of critical theory does not conserve total spin.





Conservation laws place strong constraints for $\omega/q \rightarrow \infty$, but can be ignored in the critical regime, where $\omega/q \rightarrow 0$.

L. B. Ioffe and A. J. Millis, Phys. Rev. B **51**, 16151 (1995)

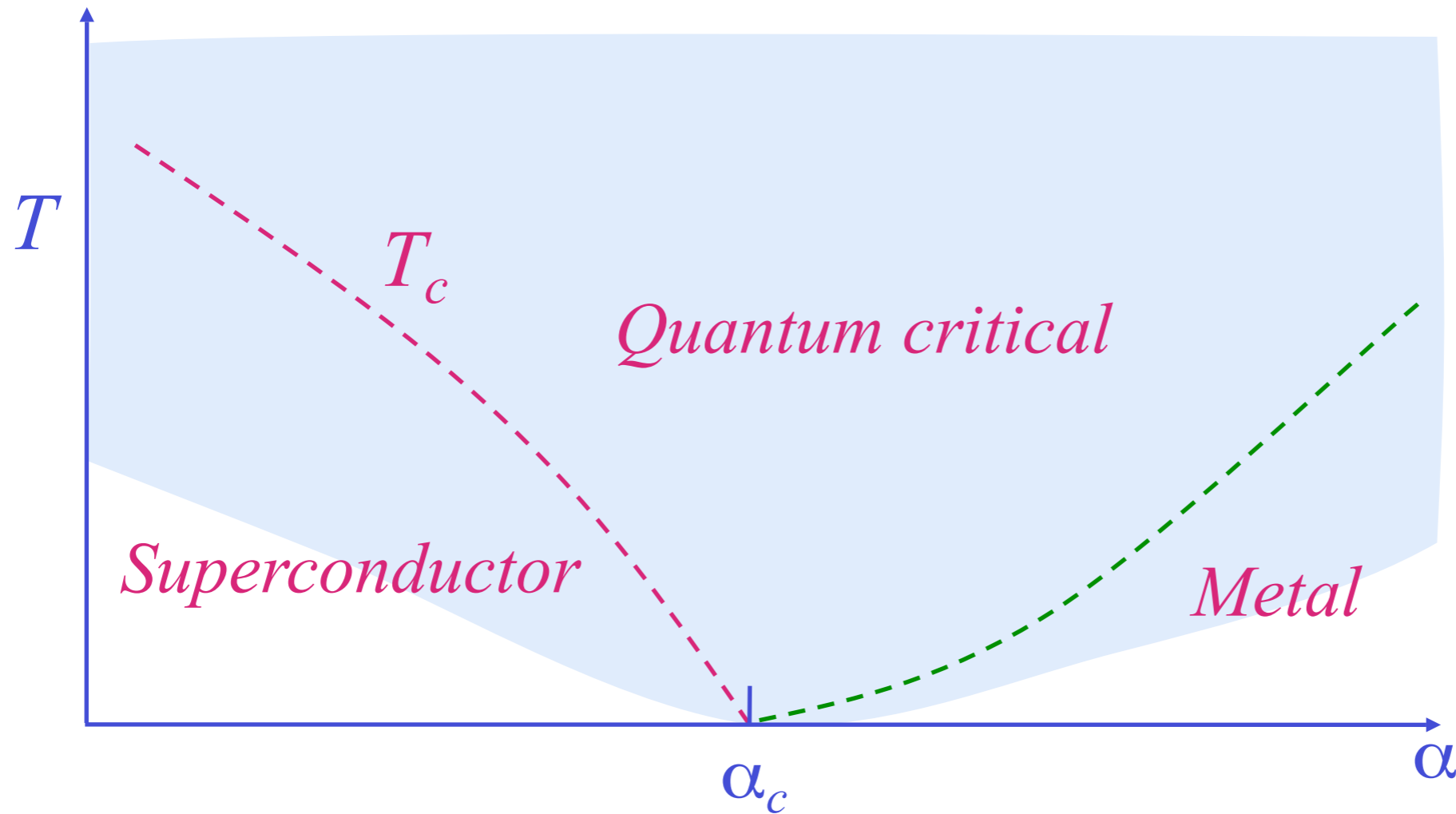
Cooper pairs (SDW) fluctuations decay into fermionic excitations at a finite rate, before any appreciable phase precession due to changes in chemical potential (magnetic field).



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Theory for quantum-critical region, and beyond in $d=1$

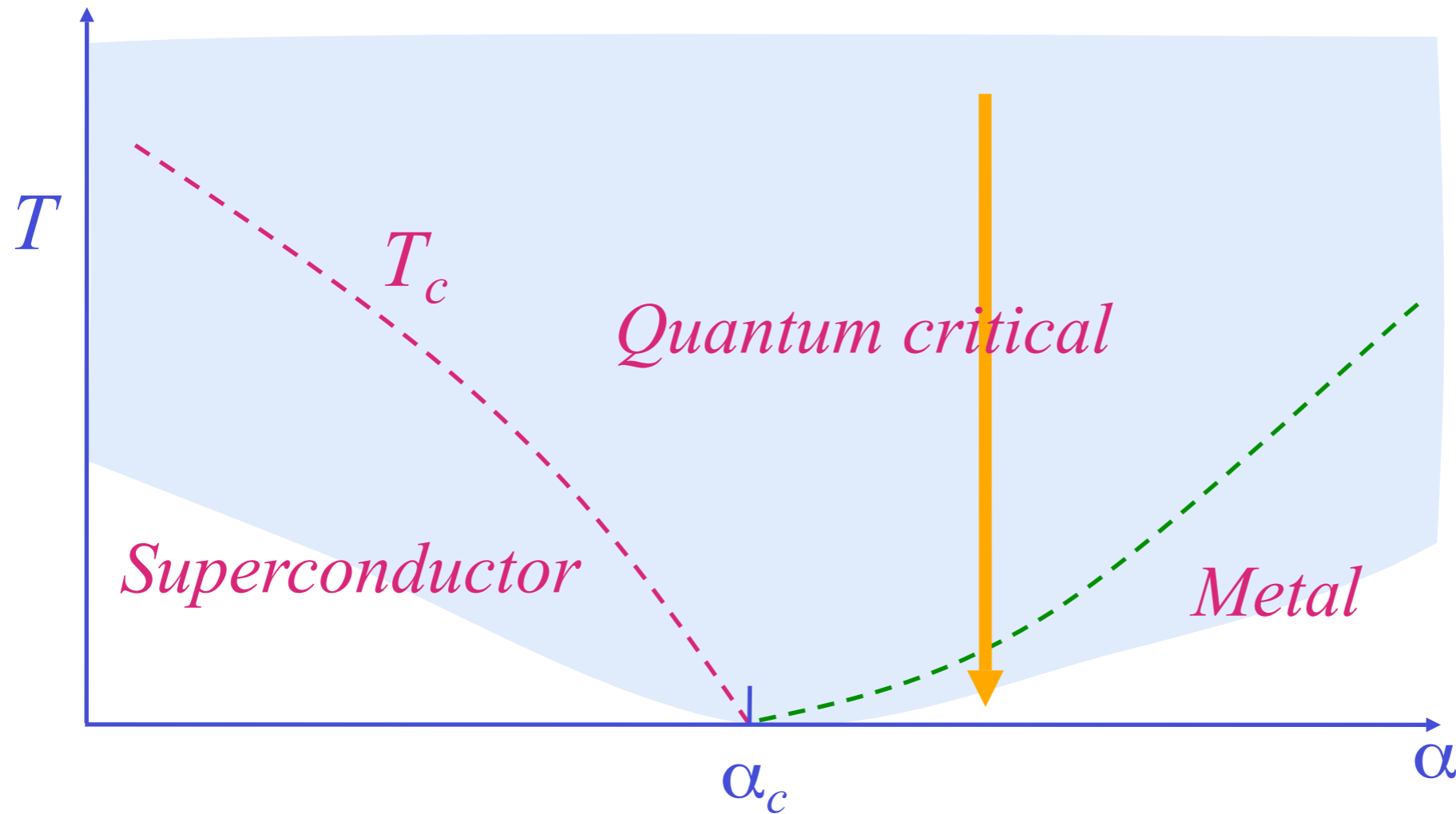


In $d = 1$, conductivity of critical theory obeys universal scaling form:

$$\delta\sigma = \frac{4e^2}{h} \left(\frac{\hbar D}{k_B T} \right)^{1/z} \Phi_\sigma \left(\frac{\alpha - \alpha_c}{T^{1/(z\nu)}} \right)$$

where Φ_σ is a scaling function.

Theory for quantum-critical region, and beyond in $d=1$



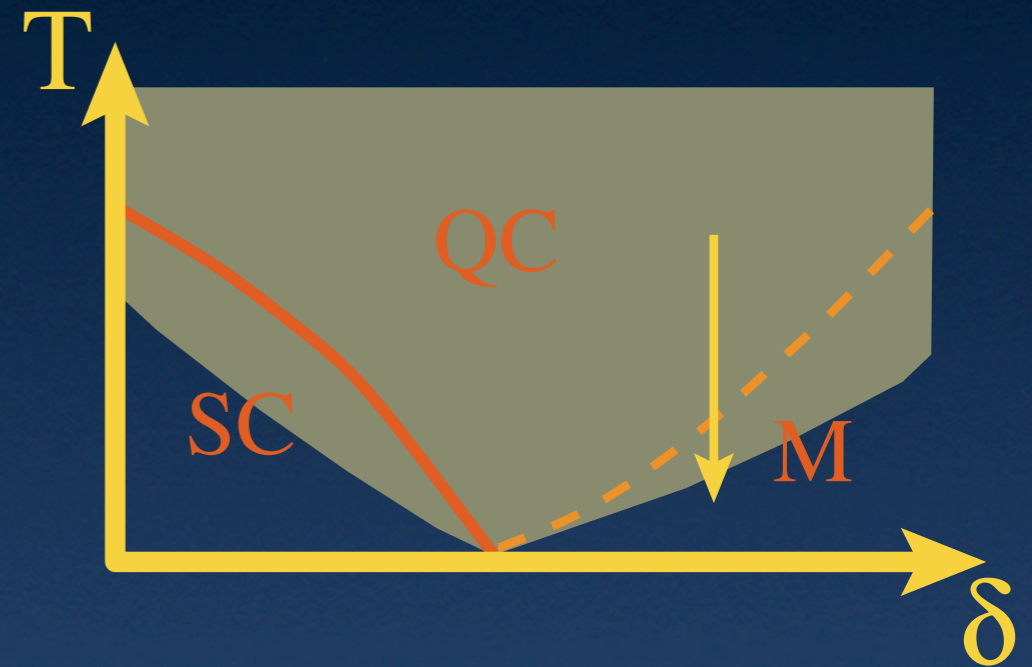
Quantum critical T dependence in $d = 1$:

$$\delta\sigma \sim \begin{cases} \frac{1}{T^{1/z}} & \text{for } T > (\alpha - \alpha_c)^{z\nu} \\ \frac{T^2}{(\alpha - \alpha_c)^{(2z+1)\nu}} & \text{for } T < (\alpha - \alpha_c)^{z\nu} \end{cases}$$

Non-monotonic dependence on T .

Accuracy of large-N expansion

- In the metallic phase, **exact agreement** is found between the large-N and microscopic calculations for the **electrical and thermal conductivity**



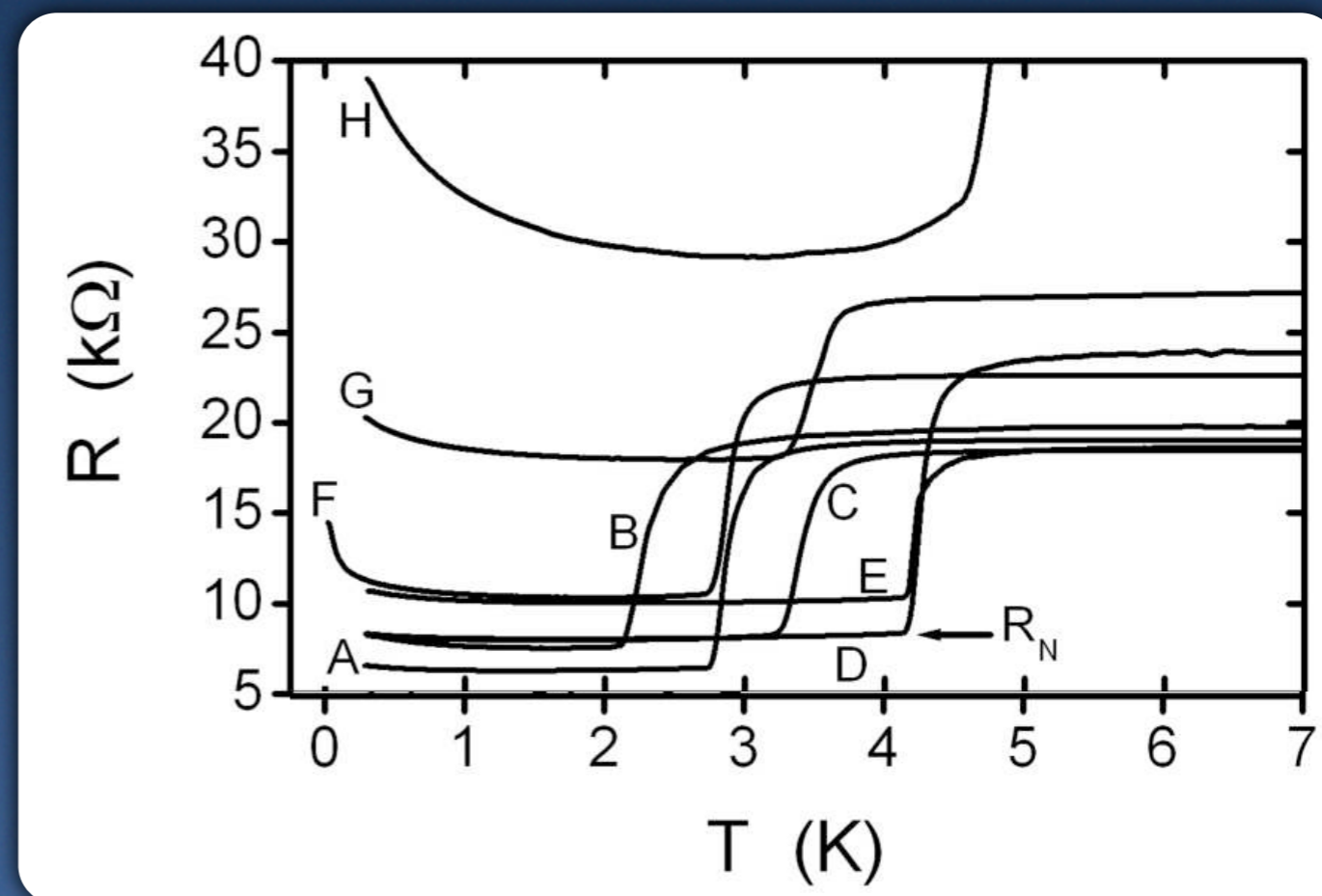
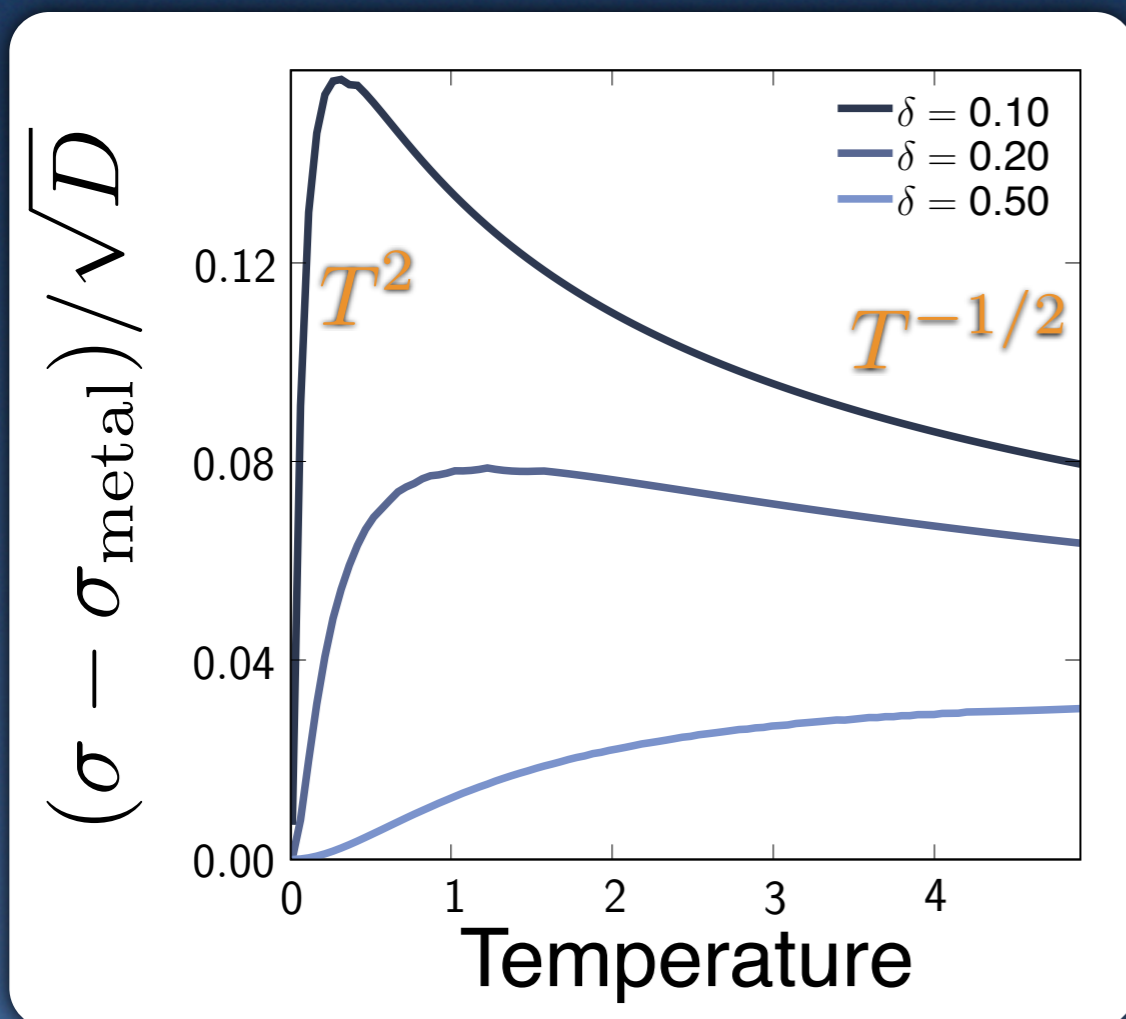
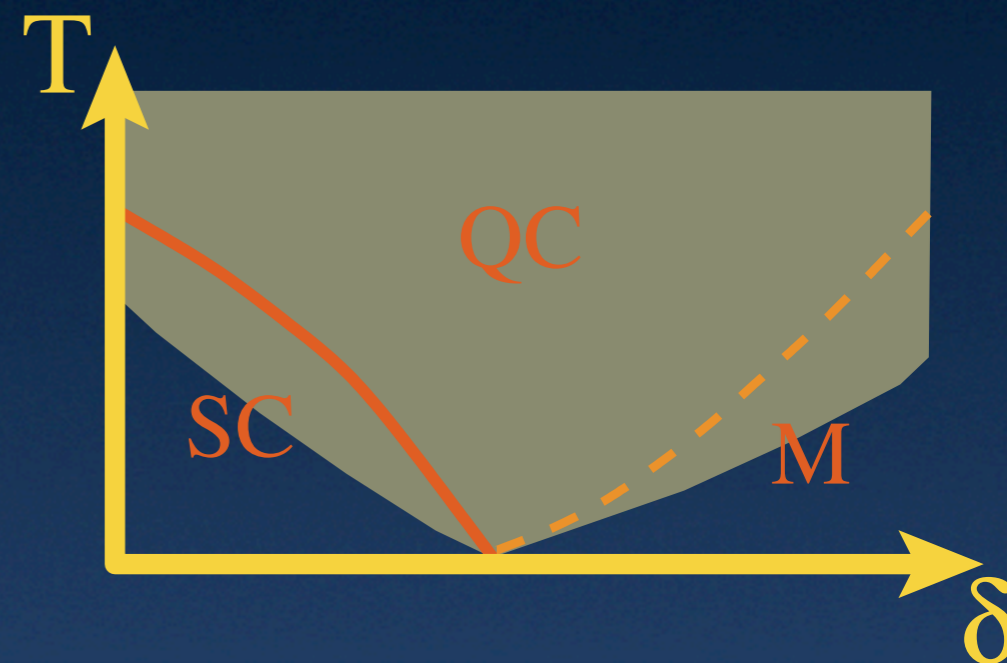
Lopatin, Shah and Vinokur (2005,2007)

$$\sigma = \frac{\pi e^2}{3 h} \frac{\sqrt{DT}^2}{R^{5/2}}$$

$$T < \delta^2$$

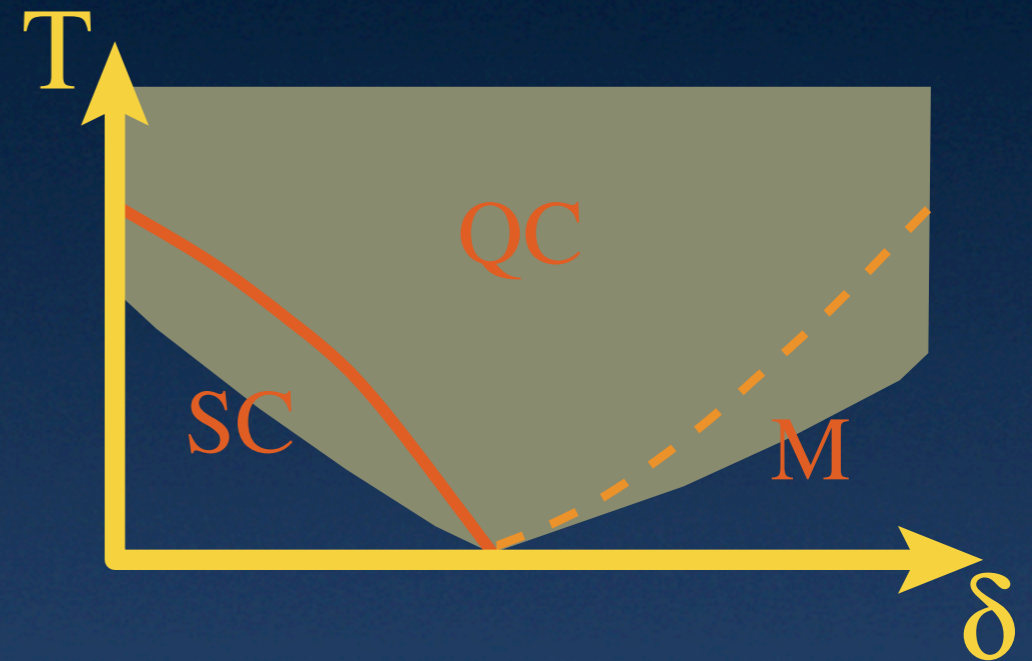
Non-monotonic temp. dependance

- A **crossover** is observed as the temperature is reduced for $\delta > 0$



The Wiedemann-Franz law

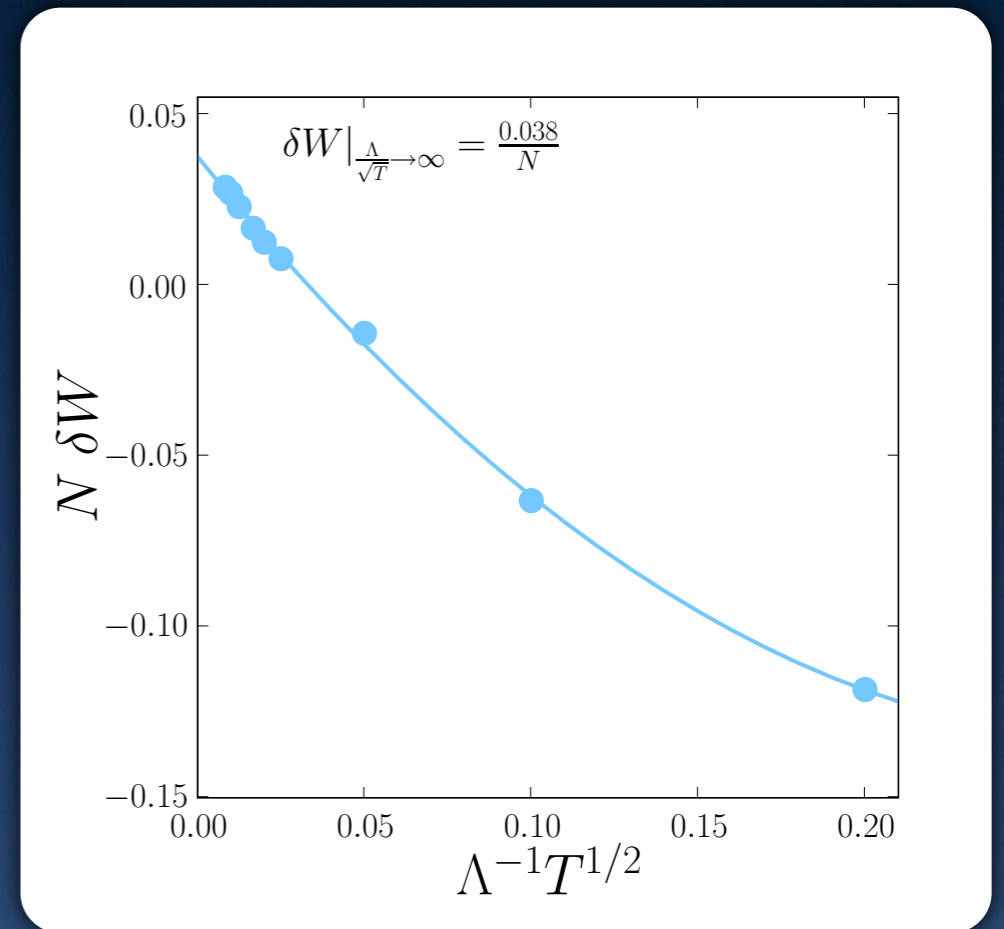
- The large-N results can be used to evaluate both the **electrical** and **thermal** conductivity



	QC	M
σ	$T^{-1/2}$	T^2
κ/T	$T^{-1/2}$	T^2





Corrections to the WF law

- All couplings between bosons and fermions scale to universal values and we are left with a **universal** correction to the WF law



$$\delta W = \left(\frac{k_B}{e} \right)^2 \left(0.2820 + \frac{0.0376}{N} \right)$$

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Spatially dependent random couplings

$$\mathcal{S} = \int dx \int d\tau \left[D(x) |\partial_x \Psi(x, \tau)|^2 + \alpha(x) |\Psi(x, \tau)|^2 + \frac{u(x)}{2} |\Psi(x, \tau)|^4 \right] + \int \frac{d\omega}{2\pi} \gamma(x) |\omega| |\Psi(x, \omega)|^2$$

$T = 0$

Hoyos, Kotabage and Vojta, PRL 2007

- Real space RG predicts the flow to a strong randomness fixed point for $z = 2$

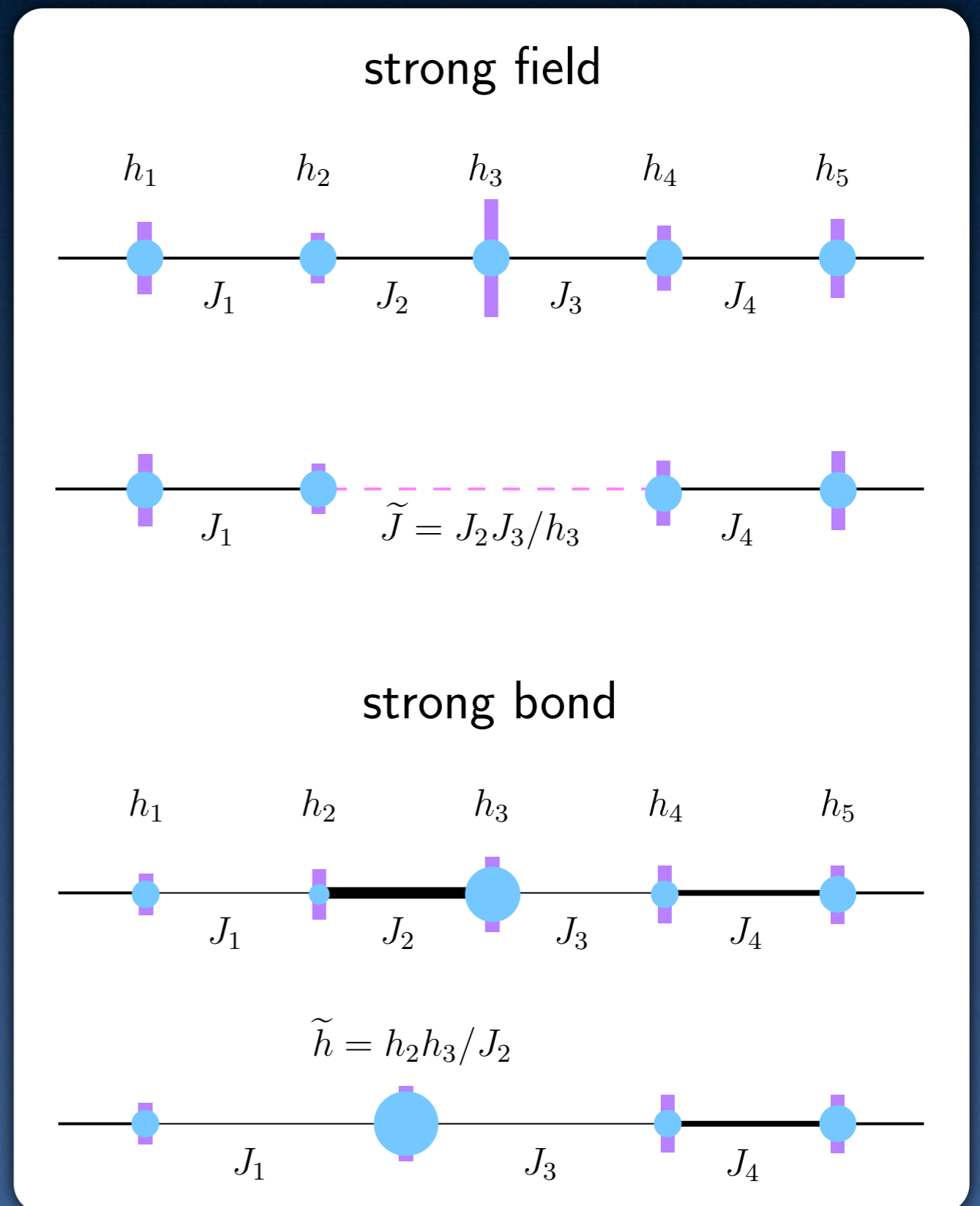
Fisher's
RTFIM

Strong disorder renormalization group

- First developed by Ma, Dasgupta and Hu, applied to the **RTFIM** by D.S. Fisher

$$\mathcal{H} = - \sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x$$

- Clusters** are **created** or **decimated** as the energy is reduced



Manifestations of strong disorder

- Under renormalization, probability distributions for observables become extremely **broad**
- Dynamics are highly anisotropic in space and time: **activated scaling**

$$\ln \xi_{\tau} \sim \xi^{\psi}$$

- Averages become dominated by **rare events** (spurious disorder configurations)

Exact predictions from the RTFIM

D. Fisher, PRL (1992);
PRB (1995)

$$\mathcal{H} = - \sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^x$$

➤ For a finite size system

$$\ln(1/\Omega) \sim L^\psi$$

$$\mu \sim \ln(1/\Omega)^\phi$$

➤ In the disordered phase

$$\xi \sim |\delta|^{-\nu}$$

critical exponents

$$\psi = 1/2$$

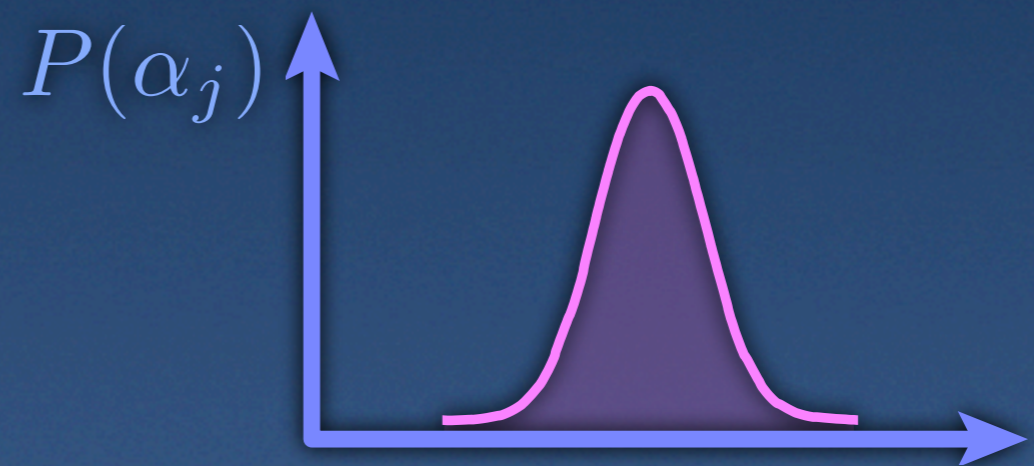
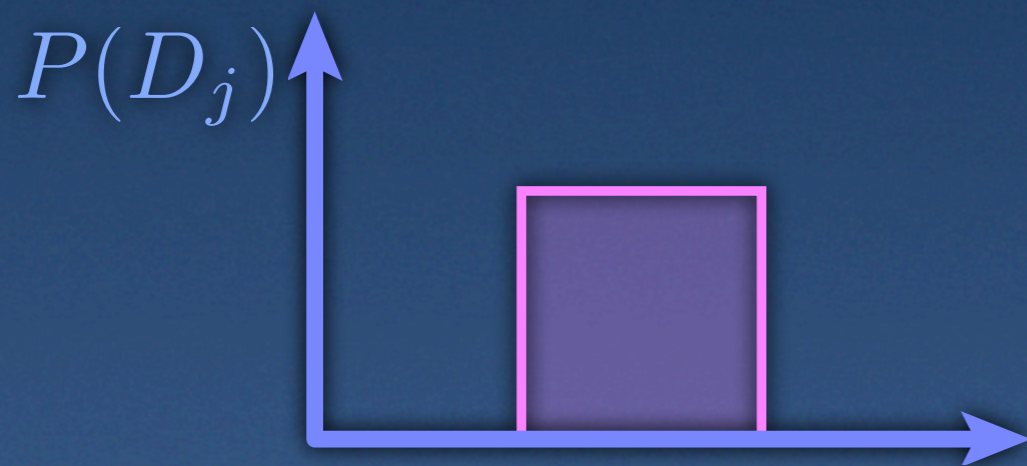
$$\nu = 2$$

$$\phi = (1 + \sqrt{5})/2$$

$$\bar{C}(x) \sim \frac{\exp \left[-(x/\xi) - (27\pi^2/4)^{1/3} (x/\xi)^{1/3} \right]}{(x/\xi)^{5/6}}$$

Discretize to a chain of L sites

$$\mathcal{S} = \int d\tau \left\{ \sum_{j=1}^{L-1} D_j |\Psi_j(\tau) - \Psi_{j+1}(\tau)|^2 + \sum_{j=1}^L \left[\alpha_j |\Psi_j(\tau)|^2 + \frac{u_j}{2} |\Psi_j(\tau)|^4 + c_l |\Psi_1(\tau)|^2 + c_r |\Psi_L(\tau)|^2 \right] \right\} + \int \frac{d\omega}{2\pi} \sum_{j=1}^L \gamma_j |\omega| |\Psi_j(\omega)|^2$$



- Measure all quantities with respect to γ^2 and set $u_j = 1$

Take the large-N limit

- Enforce a large-N constraint by solving the self-consistent saddle point equations numerically

$$\mathcal{S}_N = \int \frac{d\omega}{2\pi} \sum_{i,j=1}^L \Psi_i^*(\omega) [|\omega|\delta_{ij} + M_{ij}] \Psi_j(\omega)$$

$$r_j = \alpha_j + \langle |\Psi_j(\tau)|^2 \rangle \mathcal{S}_N$$

Numerical investigation of observables

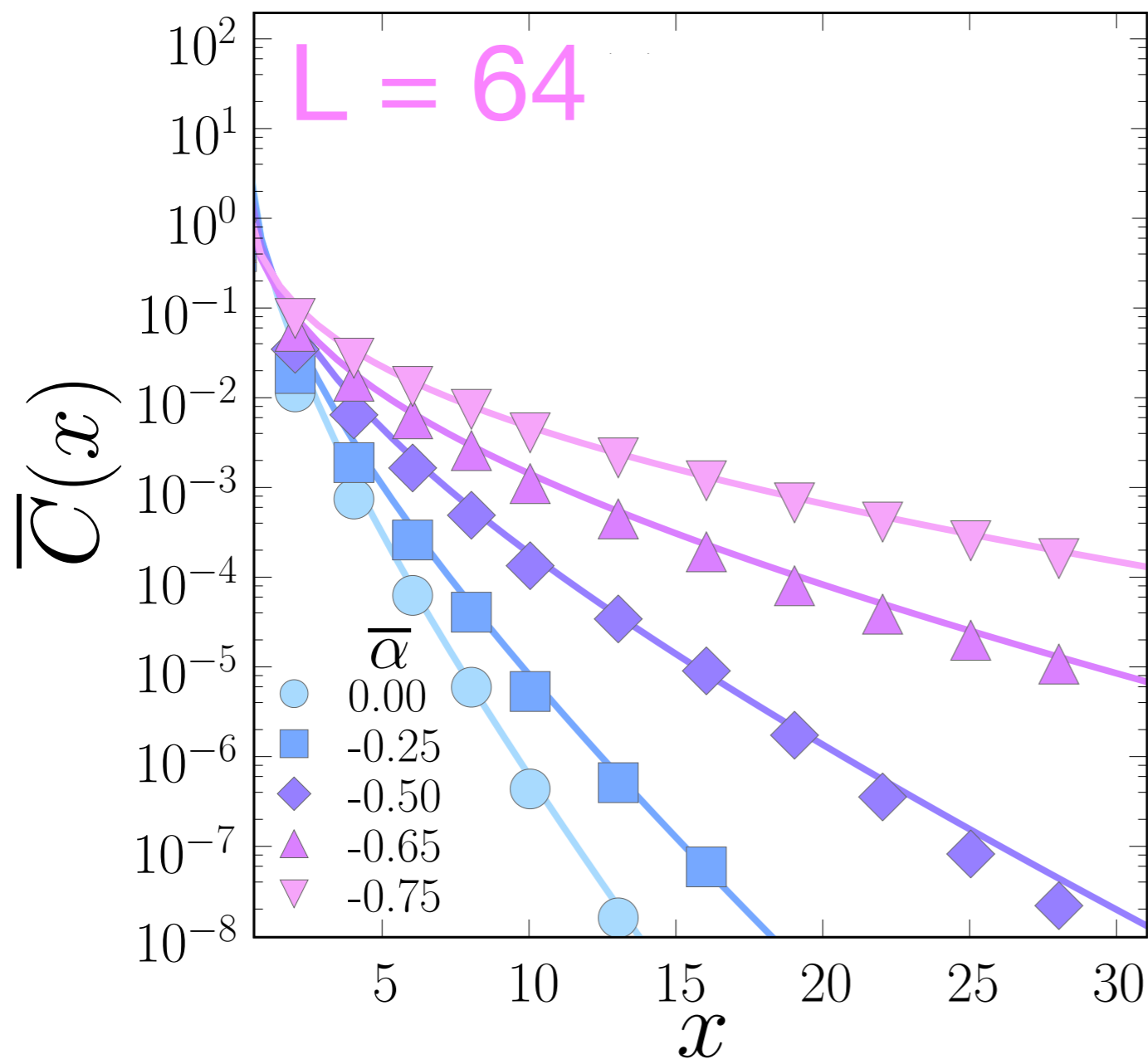
- Numerical solution by solve-join-patch method
- Measure observables for $L = 16, 32, 64, 128$ averaged over 3000 realizations of disorder
- Tune the transition by shifting the mean of the α_j distribution, $\delta \sim \bar{\alpha} - \bar{\alpha}_c$
- Equal time correlators are easily determined from the self-consistency condition

$$\bar{C}(x) = \overline{\langle \Psi_x^*(\tau) \Psi_0(\tau) \rangle}_{\mathcal{S}_N}$$

Equal time correlation functions

Fisher's asymptotic scaling form

$$\overline{C}(x) \sim \frac{\exp \left[-(x/\xi) - (27\pi^2/4)^{1/3} (x/\xi)^{1/3} \right]}{(x/\xi)^{5/6}}$$



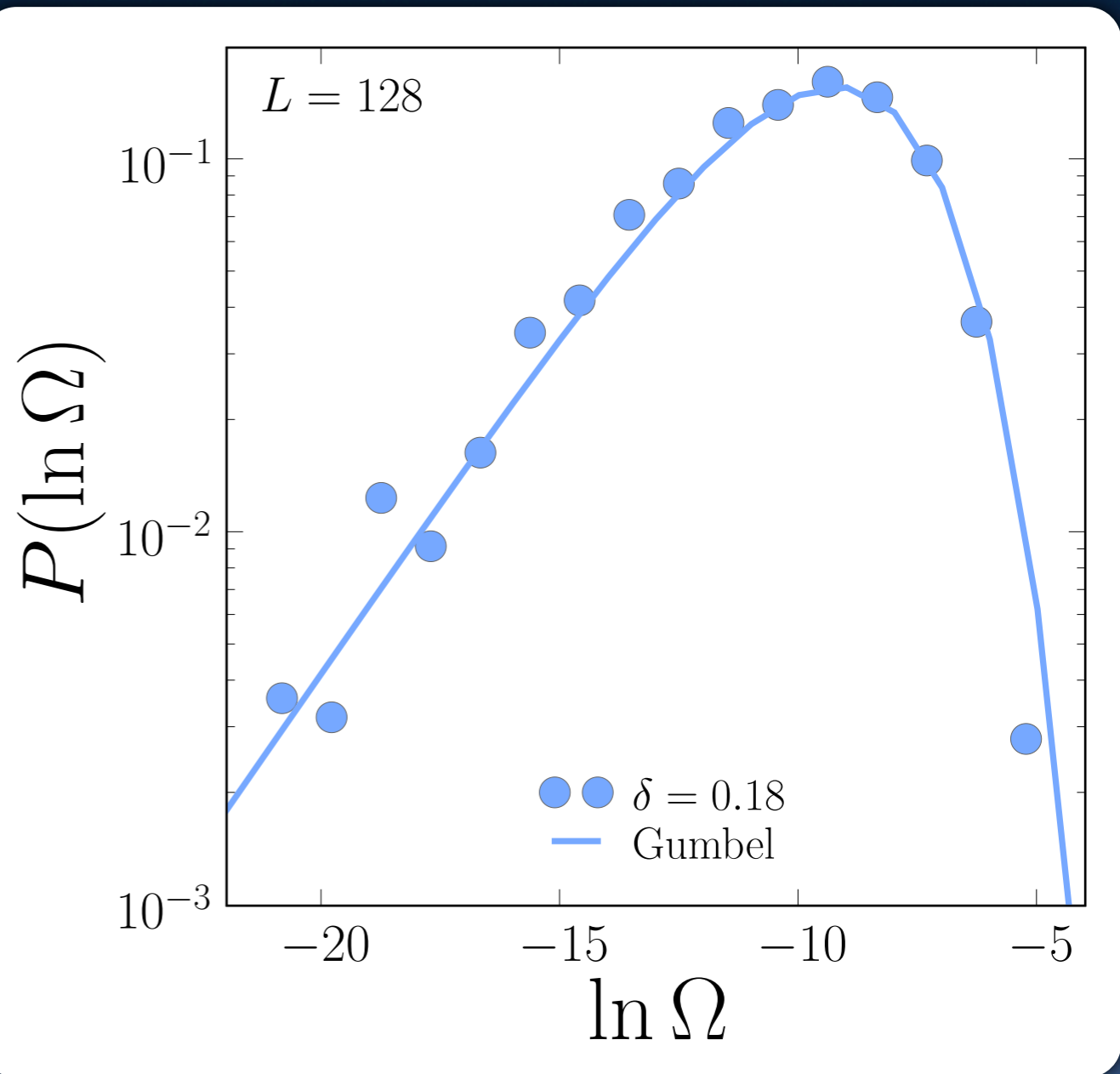
$$\xi \sim \delta^{-\nu}$$

$$\alpha_c \simeq -0.93$$
$$\nu = 1.9(2)$$

Energy gap statistics

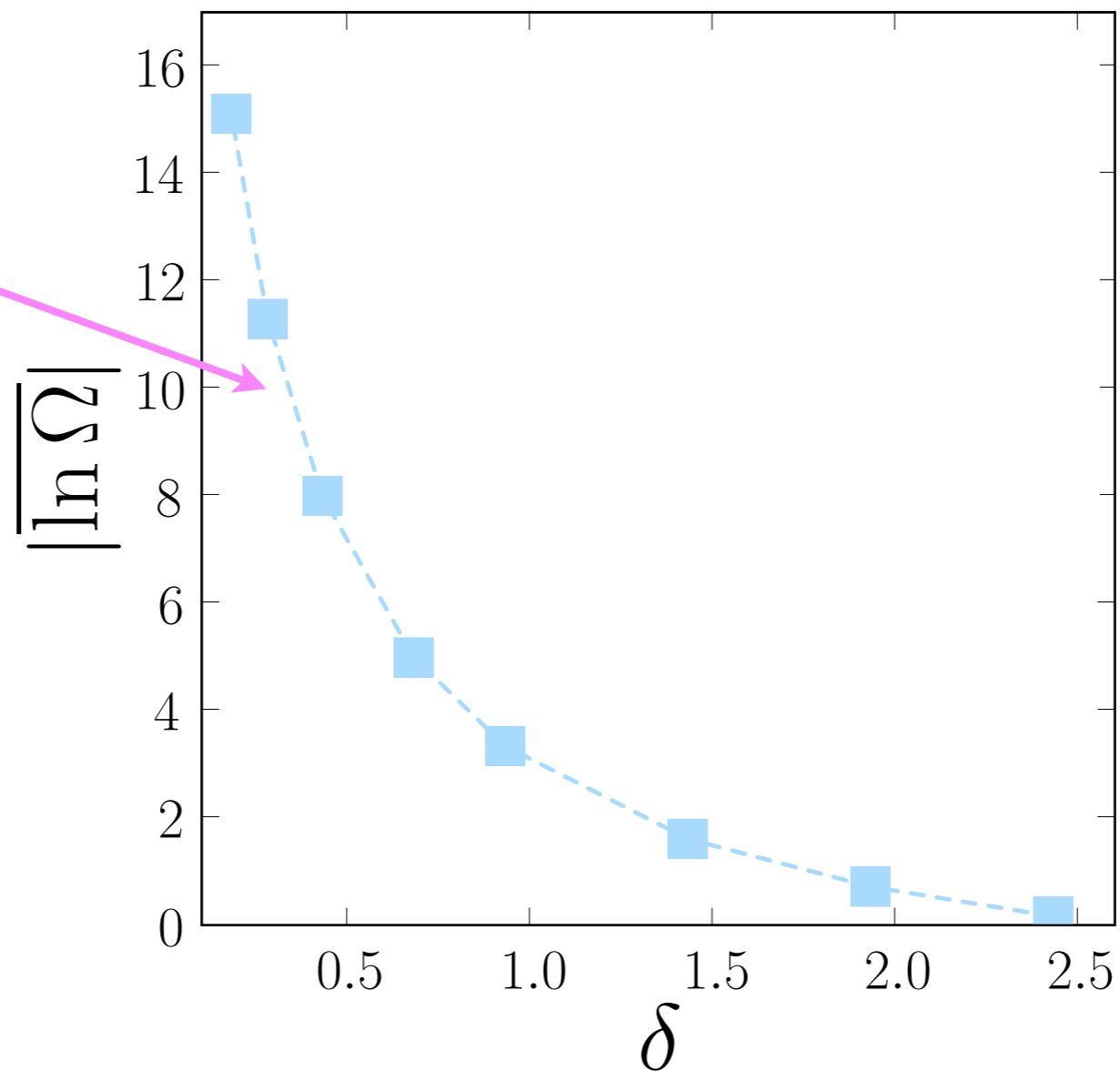
- The minimum excitation energy is due to a **rare event**, an extremal value

$$P(x) = \frac{1}{\beta} e^{-\left(\frac{x-\mu}{\beta}\right)} e^{-e^{-\left(\frac{x-\mu}{\beta}\right)}}$$



Activated scaling

Divergence cannot be fit with any power law!



$$|\ln \Omega| \sim L^\psi \sim \delta^{-\nu\psi}$$

$$\psi = 0.53(6)$$

Dynamic susceptibilities

- We have **direct access** to real dynamical quantities

$$\text{Im } \bar{\chi}(\omega) = \frac{1}{L} \sum_x \text{Im} \overline{\langle \Psi_x^*(i\omega) \Psi_0(i\omega) \rangle}_{\mathcal{S}_N} \Big|_{i\omega \rightarrow \omega + i\epsilon}$$

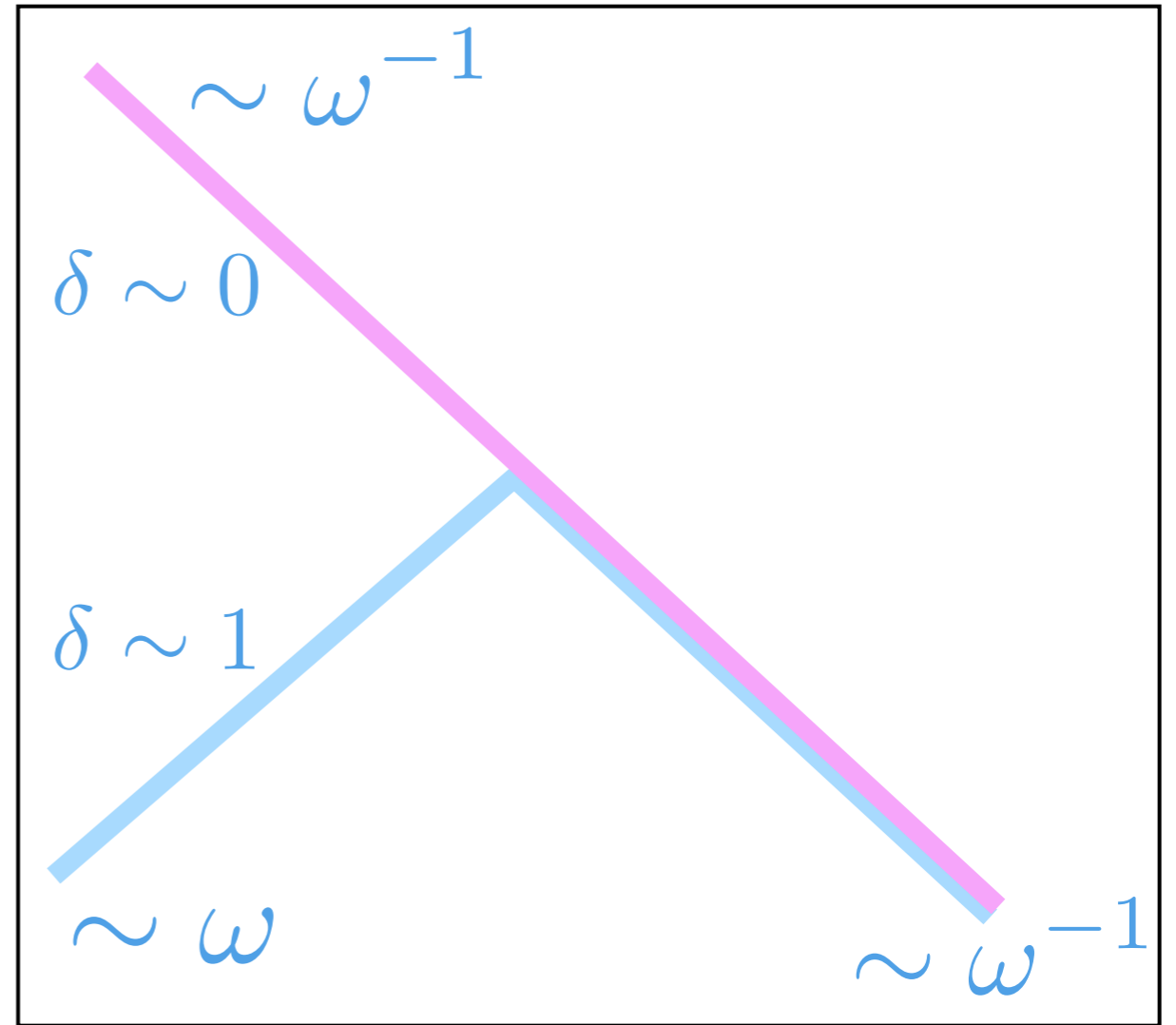
$$\text{Im } \bar{\chi}_{\text{loc}}(\omega) = \text{Im} \overline{\langle \Psi_0^*(i\omega) \Psi_0(i\omega) \rangle}_{\mathcal{S}_N} \Big|_{i\omega \rightarrow \omega + i\epsilon}$$

real frequency

Local order parameter susceptibility

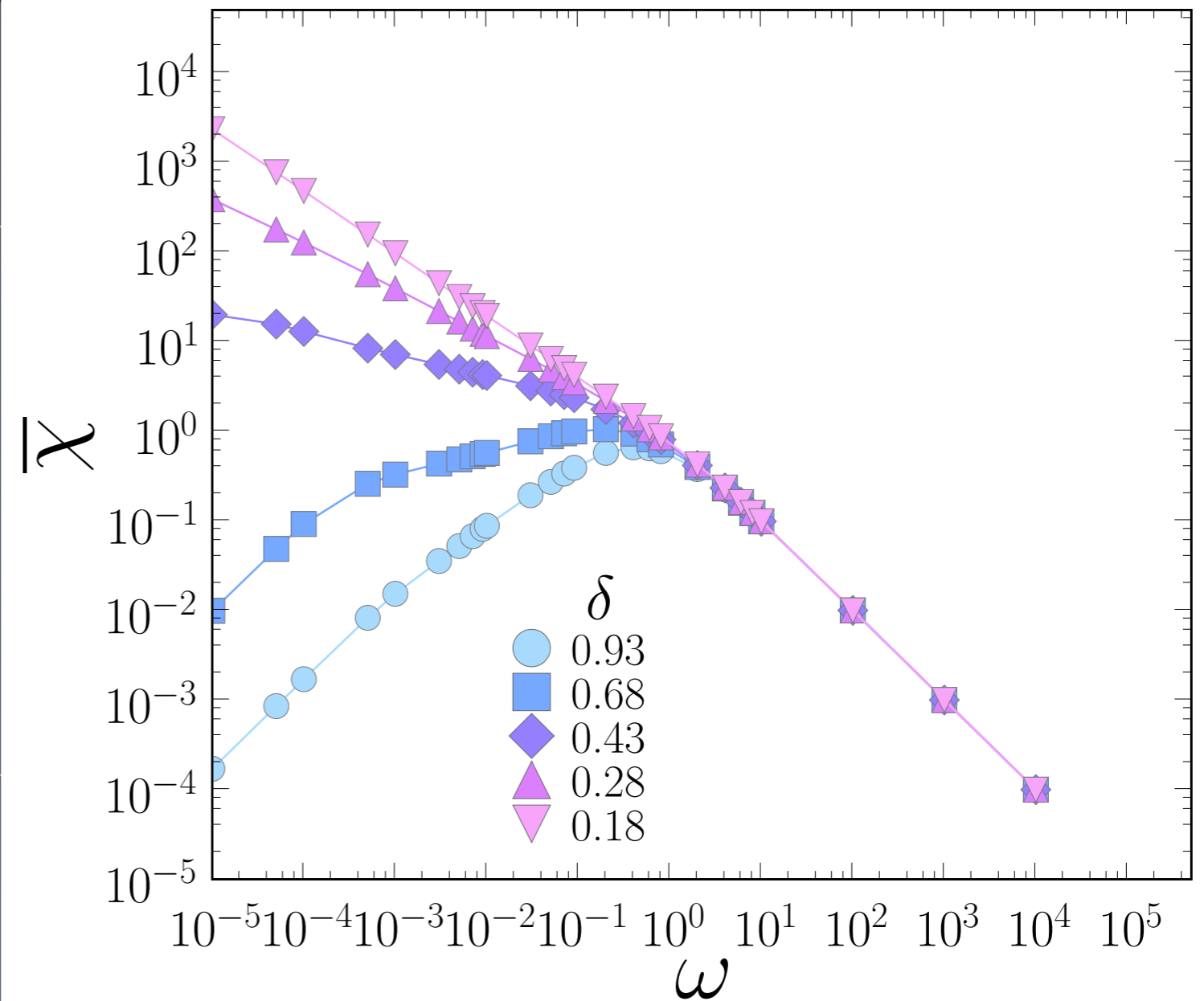
$$\text{Im } \chi_{\text{loc}} = \frac{1}{L} \sum_{i=1}^L \frac{\omega}{\omega^2 + \lambda_i^2}$$

$\bar{\chi}_{\text{loc}}$



Average order parameter susceptibility

$$\text{Im } \chi = \frac{1}{L^2} \sum_{i,j,k=1}^L \frac{\omega V_{ik} V_{jk}}{\omega^2 + \lambda_j^2}$$



Susceptibility scaling forms

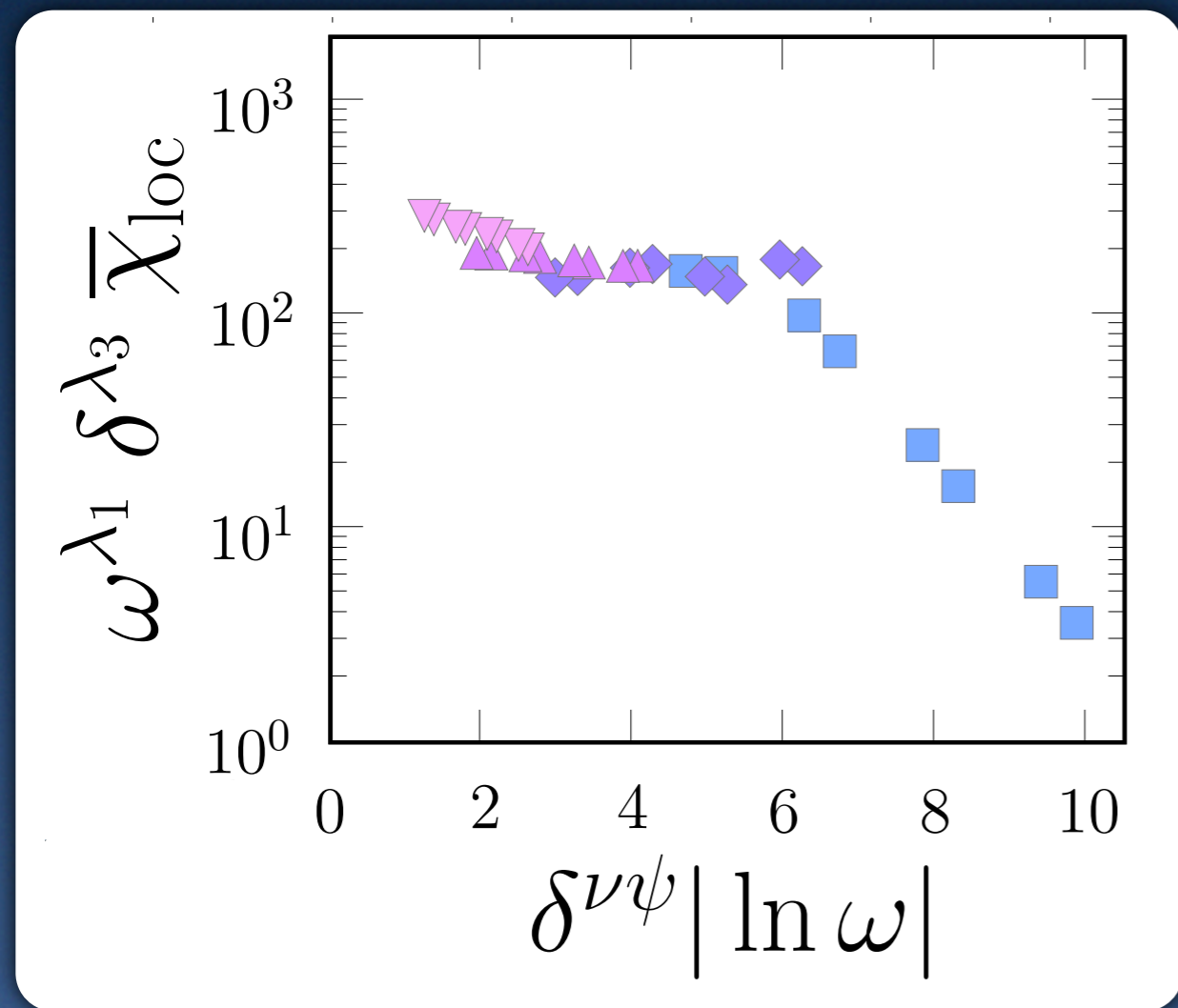
- The scaling forms for the disorder averaged order parameter susceptibilities follow from an analysis of a single cluster

$$\text{Im } \overline{\chi}_{\text{loc}} \sim \frac{\delta^{1/\psi - \phi\nu\delta}}{\omega^{1 - \delta/\psi}} \Phi_{\text{loc}} (\delta^{\nu\psi} |\ln \omega|)$$

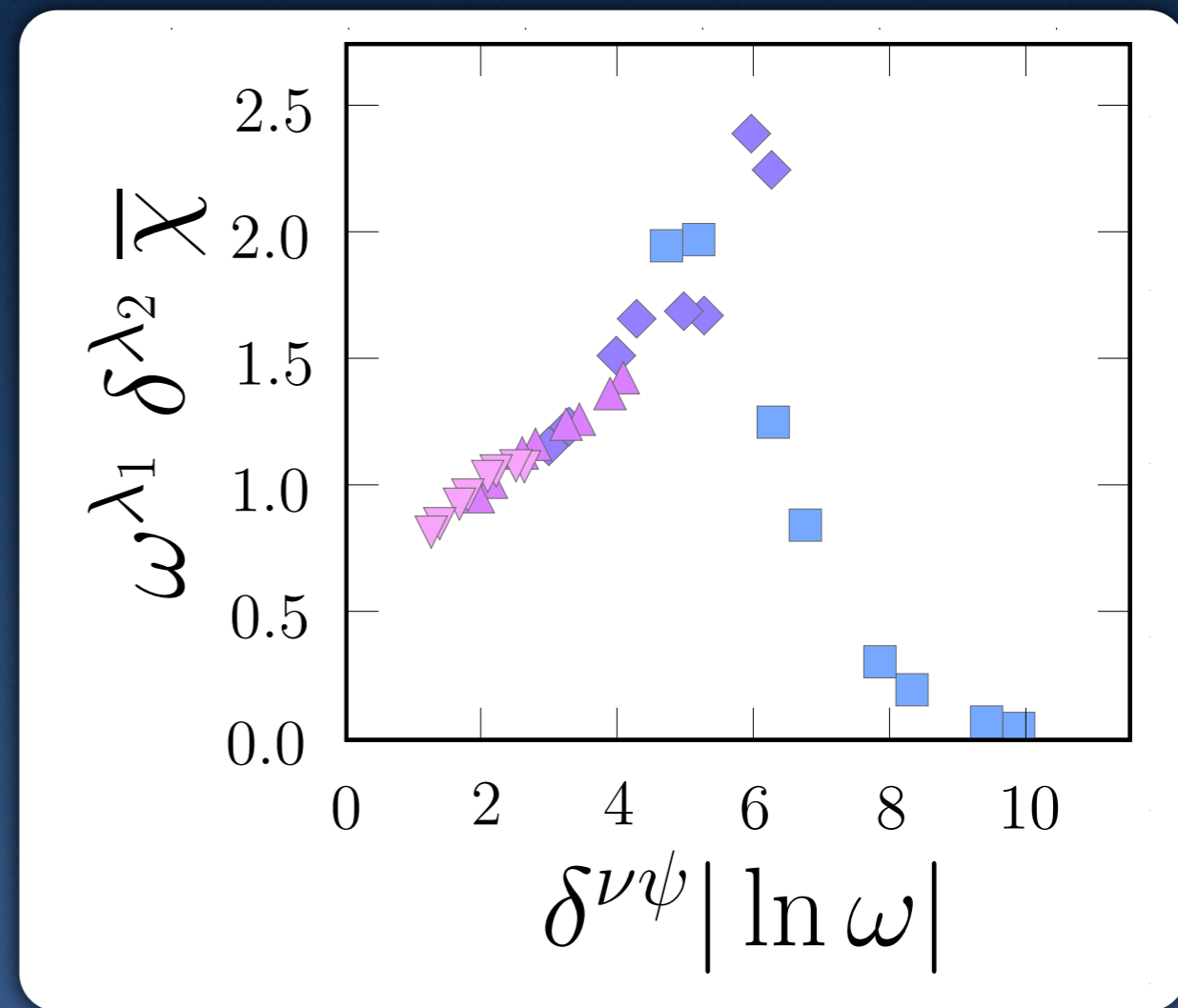
$$\text{Im } \overline{\chi} \sim \frac{\delta^{1/\psi - \phi\nu\psi(1 + \delta/\psi)}}{\omega^{1 - \delta/\psi}} \Phi (\delta^{\nu\psi} |\ln \omega|)$$

Susceptibility scaling collapse

- Observe **data collapse** consistent with expectations from scaling



local
susceptibility



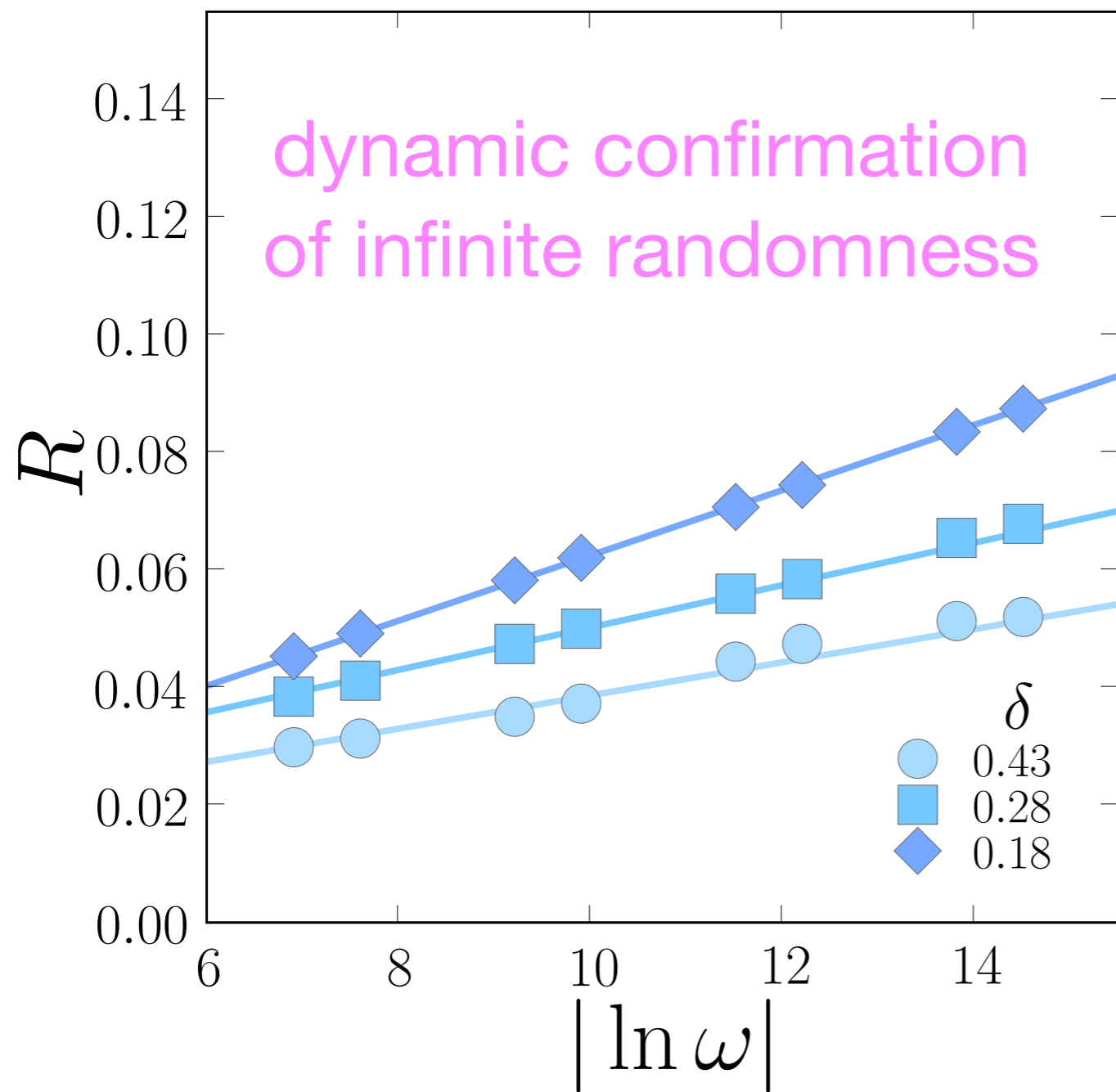
average
susceptibility

Susceptibility ratio

- Too much **parameter freedom** in data collapse to confidently extract the tunneling exponent ϕ
- The ratio of the average to local susceptibility is related to the ***average cluster moment***

$$R(\omega) = \frac{\text{Im}\bar{\chi}(\omega)}{\text{Im}\overline{\chi_{\text{loc}}}(\omega)}$$
$$\sim \delta^{\nu\psi(1-\phi)} |\ln \omega|$$

Activated dynamical scaling



$$R(\omega) = \frac{\text{Im} \bar{\chi}(\omega)}{\text{Im} \bar{\chi}_{\text{loc}}(\omega)}$$
$$\sim \delta^{\nu\psi(1-\phi)} |\ln \omega|$$

$$\phi = 1.6(2)$$

Putting it all together

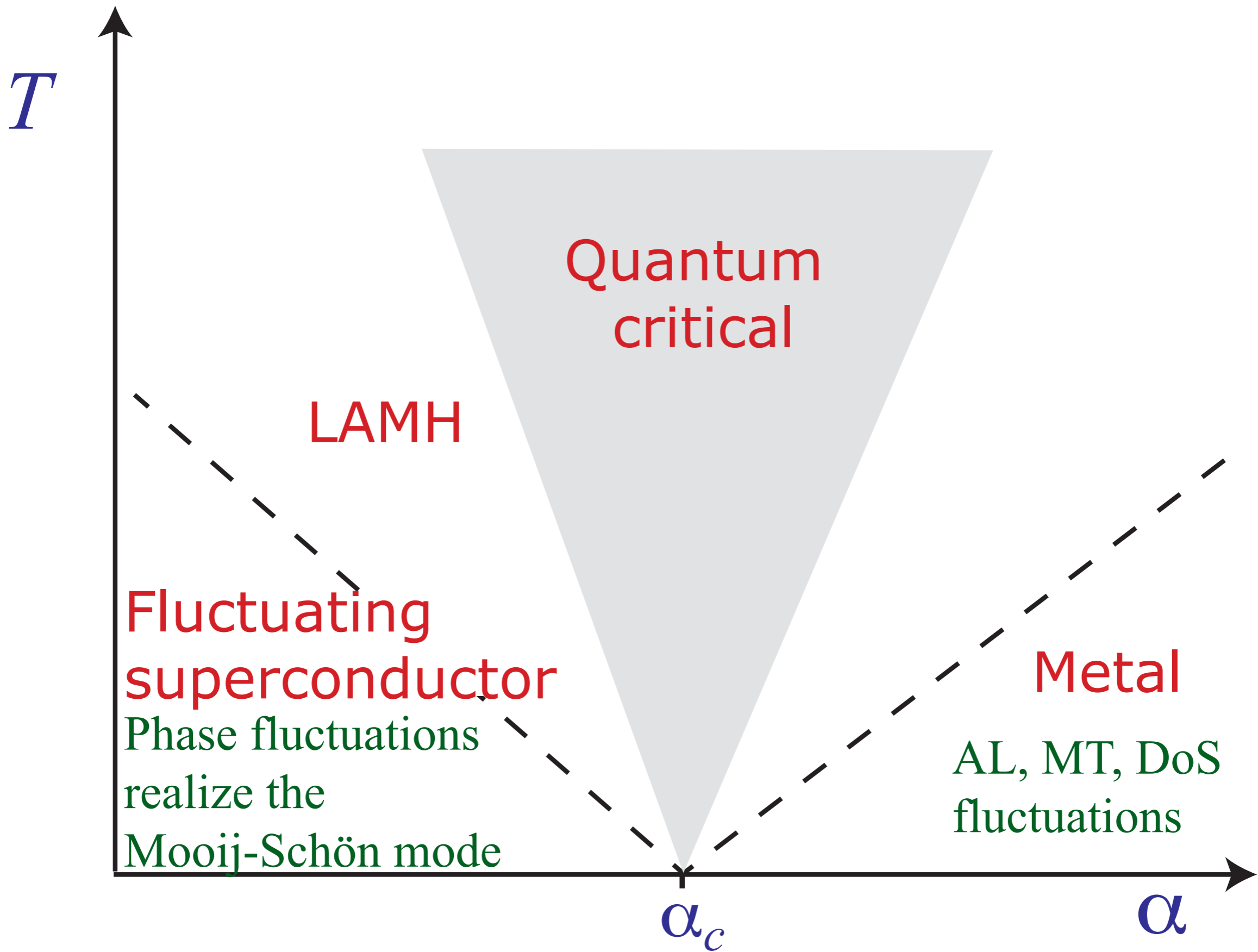
	ν	ψ	ϕ
RTFIM	2	1/2	$(1+\sqrt{5})/2$
SMT	1.9(2)	0.53(6)	1.6(2)

Numerical confirmation of the strong disorder RG calculations of Hoyos, Kotabage and Vojta

Conclusions



Field theory for quantum critical transport near the superconductor-metal transition. Crosses over to phase fluctuation theory (in the superconductor) and conventional pairing fluctuation theory (in the metal) at low temperatures.





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-  New Wiedemann-Franz ratio in the quantum critical region.

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- New Wiedemann-Franz ratio in the quantum critical region.
- First **dynamical confirmation** of activated scaling from numerical simulations in disordered systems.
- SMT** has an infinite randomness fixed point in the **RTFIM** universality class
- Scratching the surface of **transport** calculations near a strong disorder fixed point