

Statistical mechanics of metals without quasiparticles and charged black holes

Boltzmann lecture 2022

Scuola Internazionale Superiore di Studi Avanzati
Trieste, February 21, 2022

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INSTITUTE FOR
ADVANCED STUDY

PHYSICS



HARVARD



1. Introduction to metals with and without quasiparticles

2. The SYK model

3. T -linear resistivity from spatially random interactions
in a two-dimensional quantum-critical metal

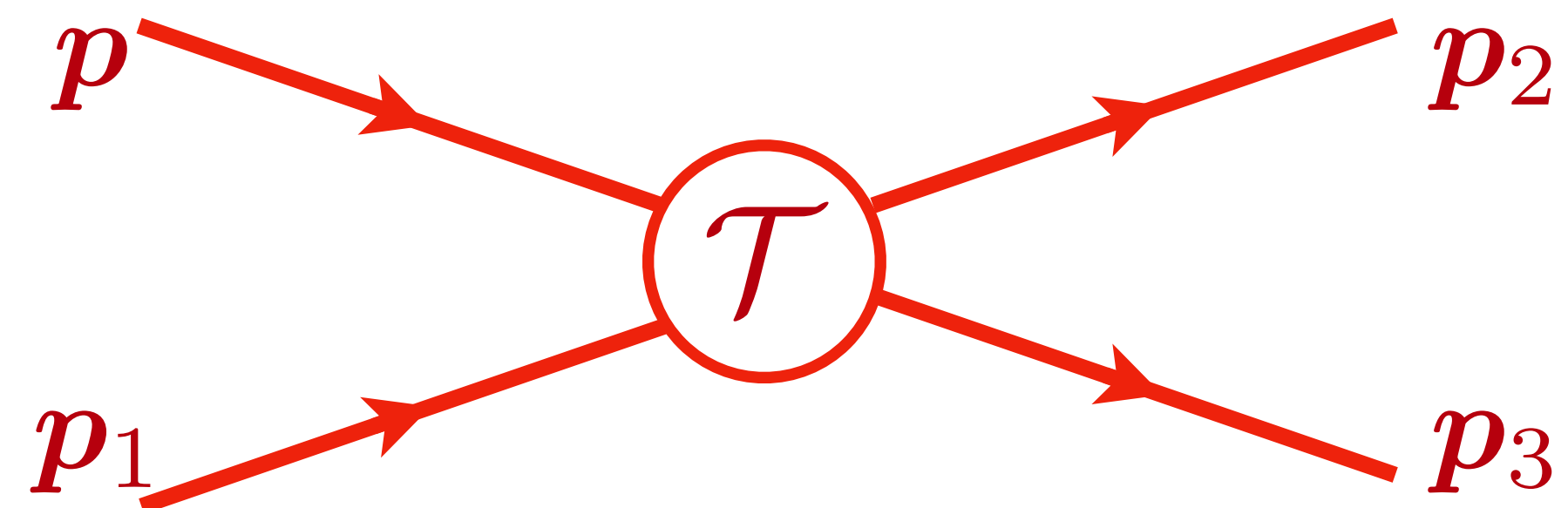
4. Introduction to black holes

5. Progress on the theory of black holes

Boltzmann equation (1872)

Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \nabla_{\mathbf{r}} f_{\mathbf{p}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} f_{\mathbf{p}} =$$
$$- 2\pi \int_{\mathbf{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) \delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)$$
$$\times [f_{\mathbf{p}} f_{\mathbf{p}_1} - f_{\mathbf{p}_2} f_{\mathbf{p}_3}]$$

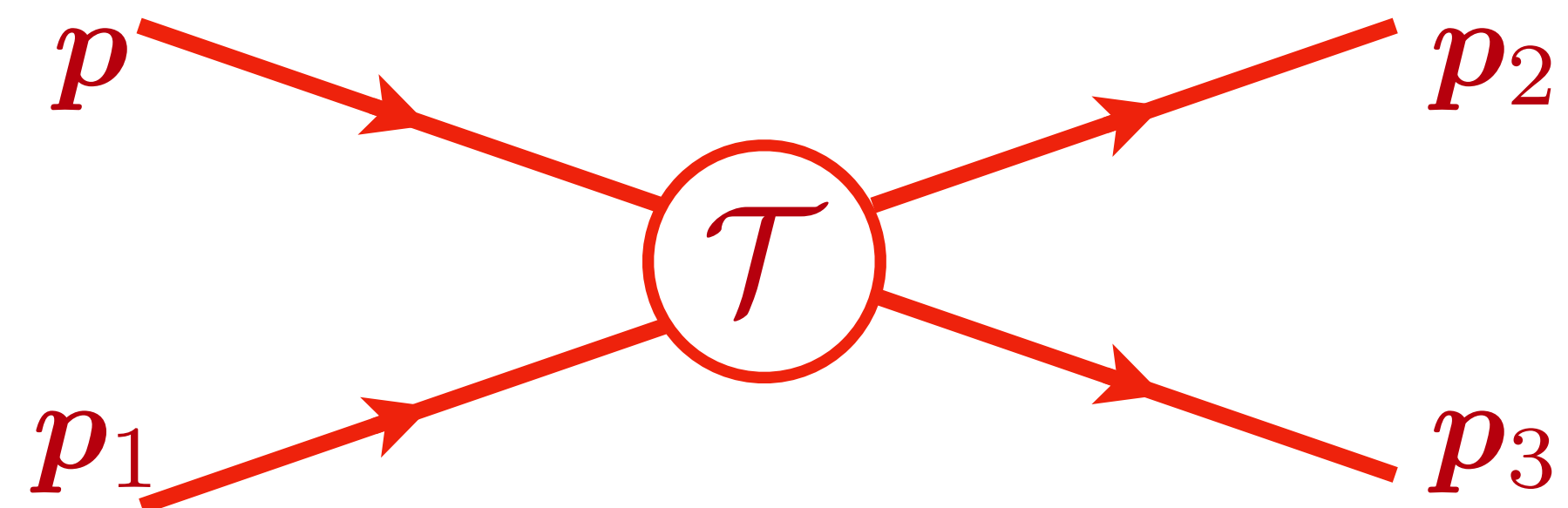


Vienna, Austria

Quantum Boltzmann equation (fermions)

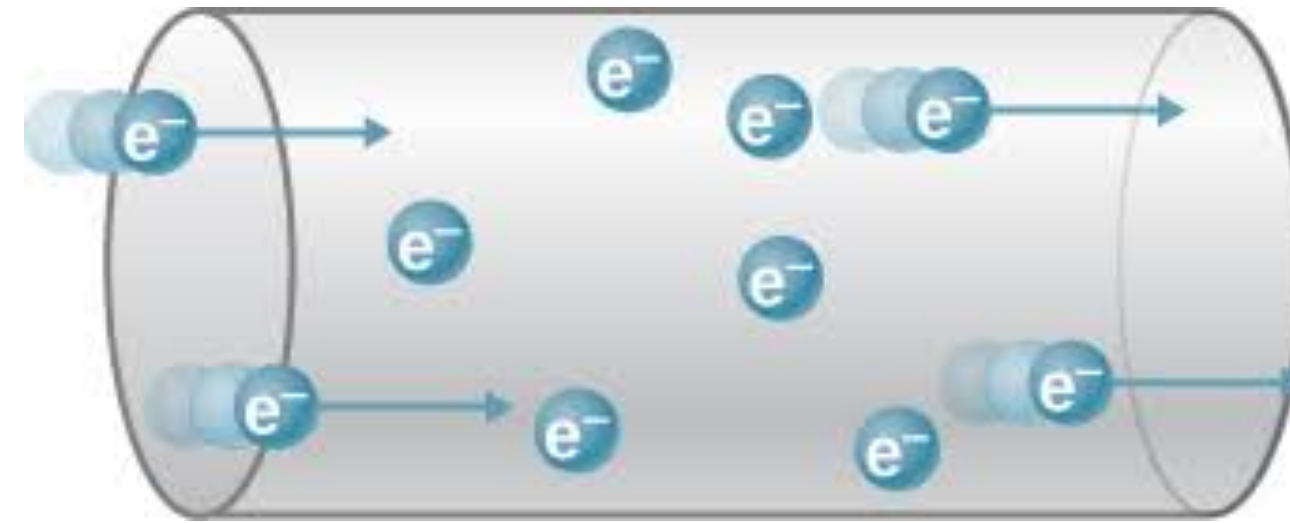
Neglects quantum interference (entanglement) between successive collisions

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \nabla_{\mathbf{r}} f_{\mathbf{p}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} f_{\mathbf{p}} = - 2\pi \int_{\mathbf{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) \delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) \times [f_{\mathbf{p}} f_{\mathbf{p}_1} (1 - f_{\mathbf{p}_2}) (1 - f_{\mathbf{p}_3}) - f_{\mathbf{p}_2} f_{\mathbf{p}_3} (1 - f_{\mathbf{p}}) (1 - f_{\mathbf{p}_1})]$$



Vienna, Austria

Current flow with quasiparticles in Copper

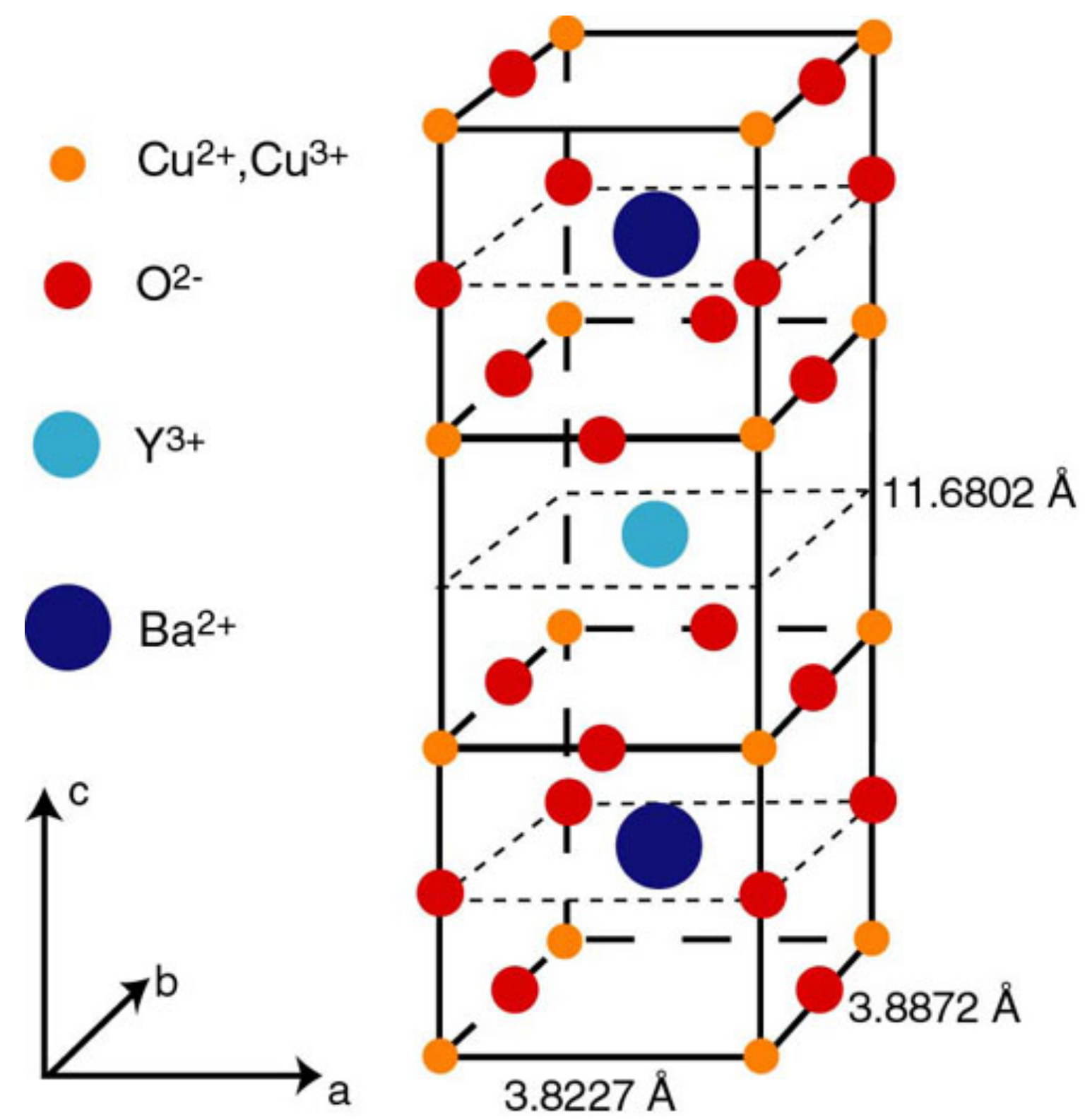
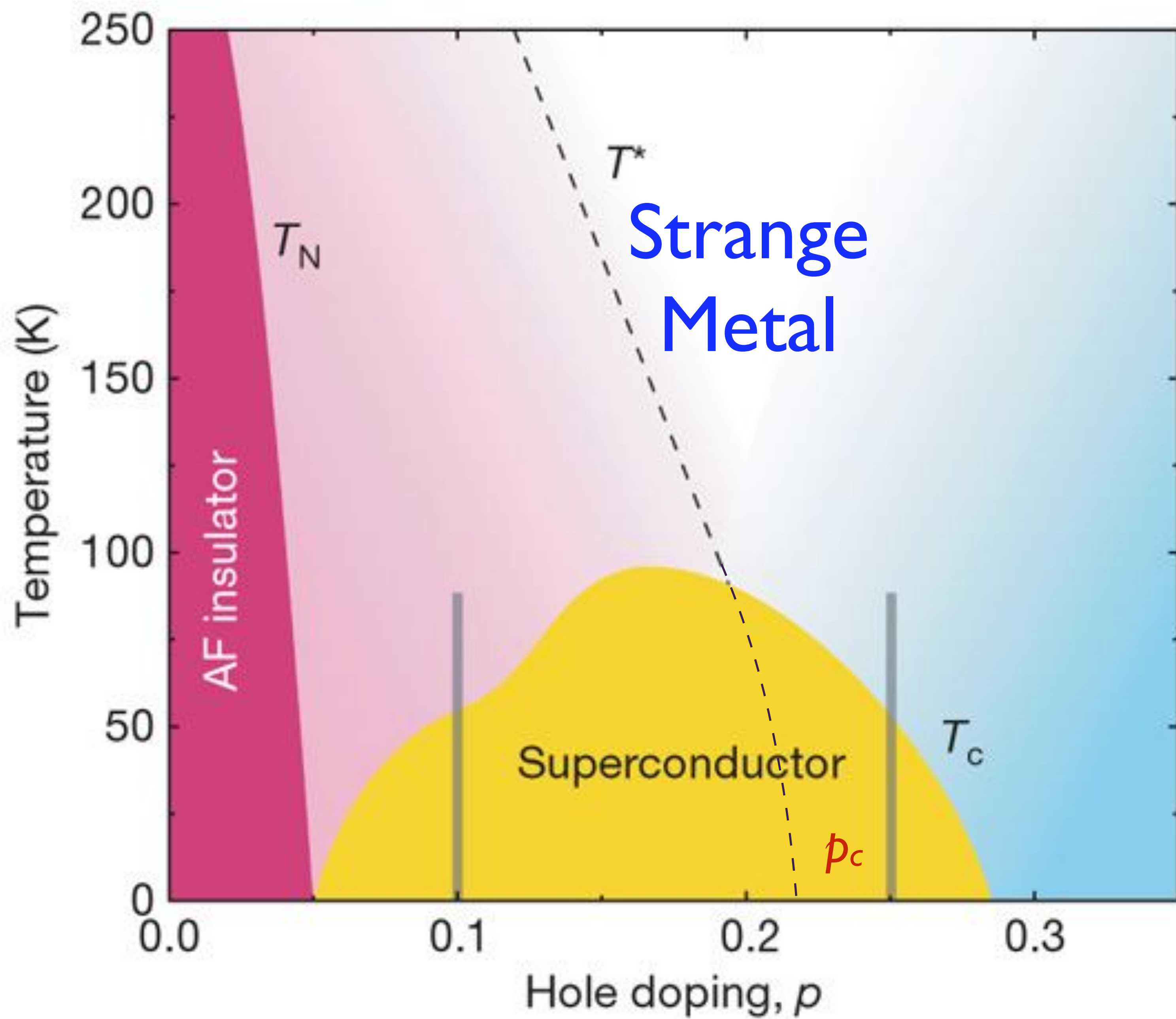


Flow of quasiparticles described by Boltzmann equation \Rightarrow
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\frac{\hbar}{k_B T}$.

The long scattering time implies that quasiparticles are well-defined.

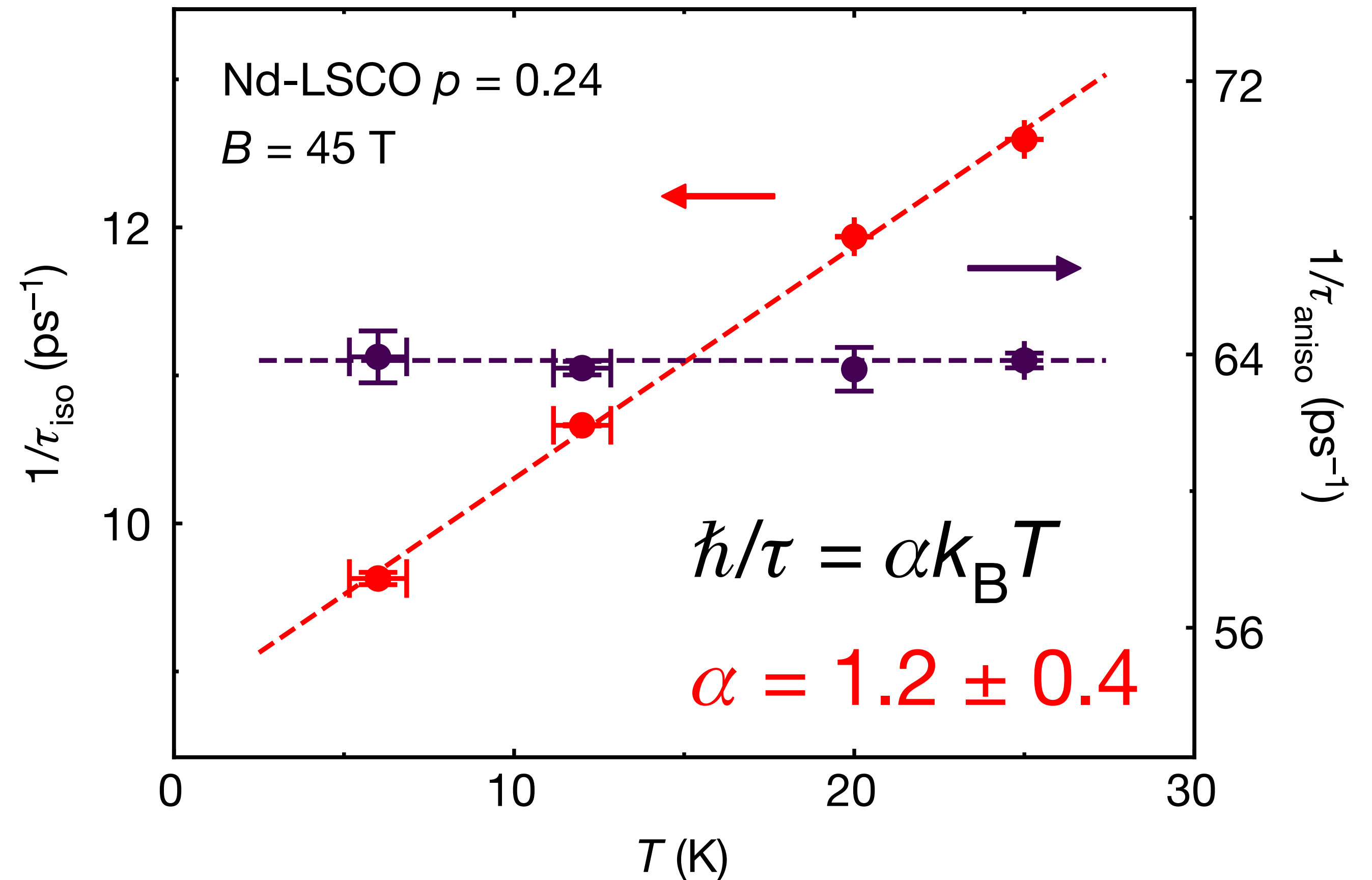
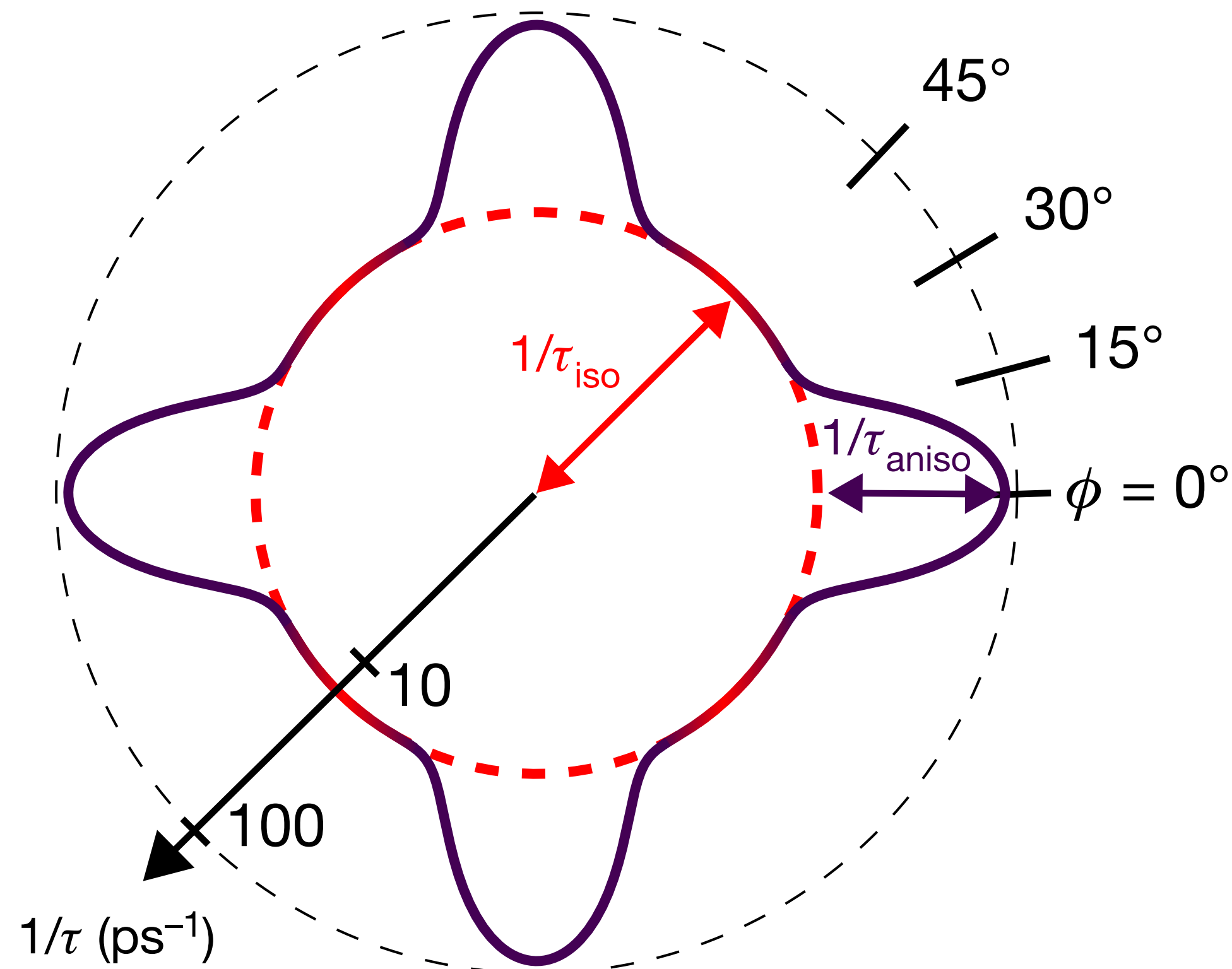
The motion of quasiparticles is 'ballistic' or 'integrable'
up to the long time τ , after which it is chaotic.



Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

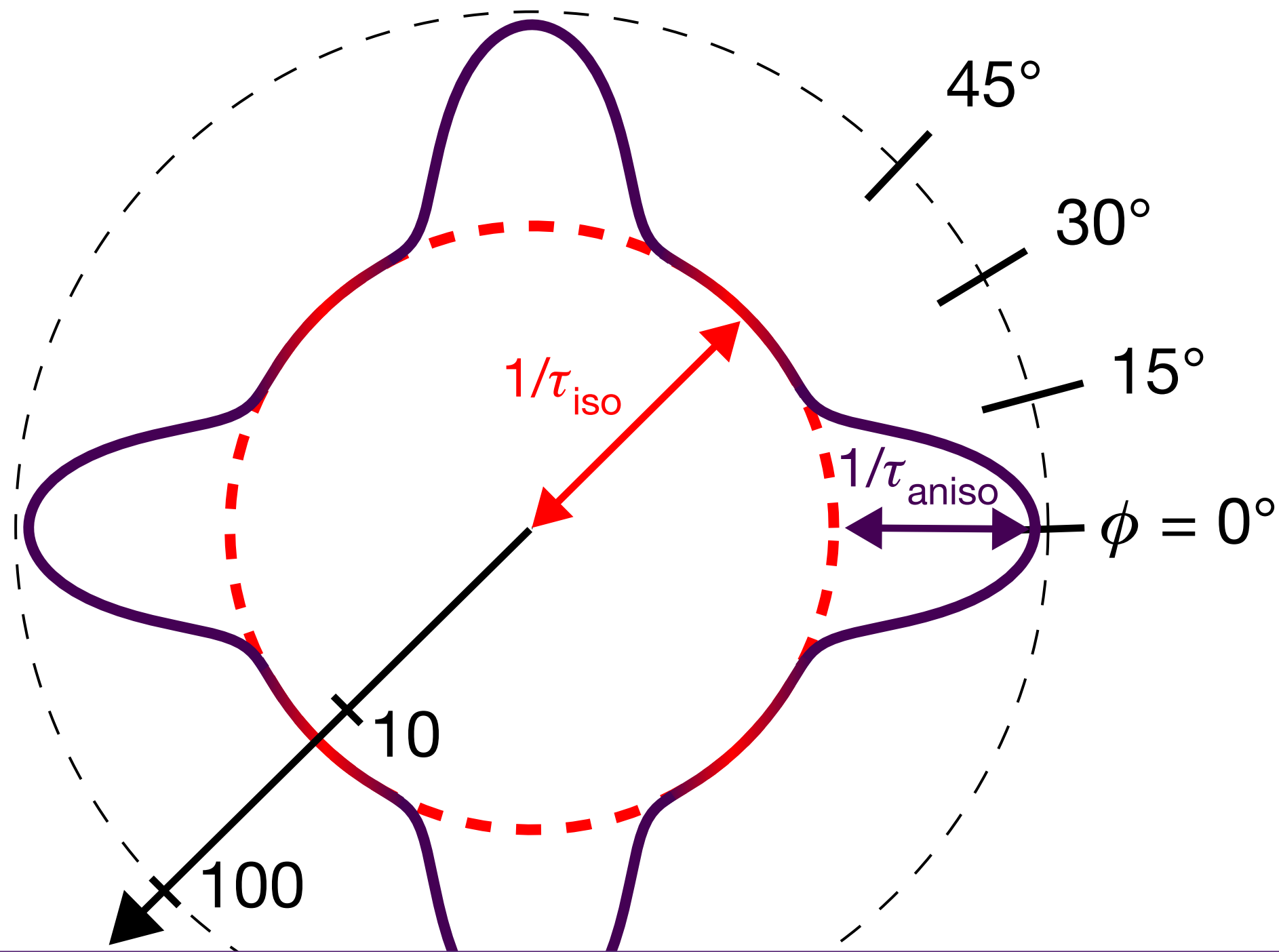
G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw



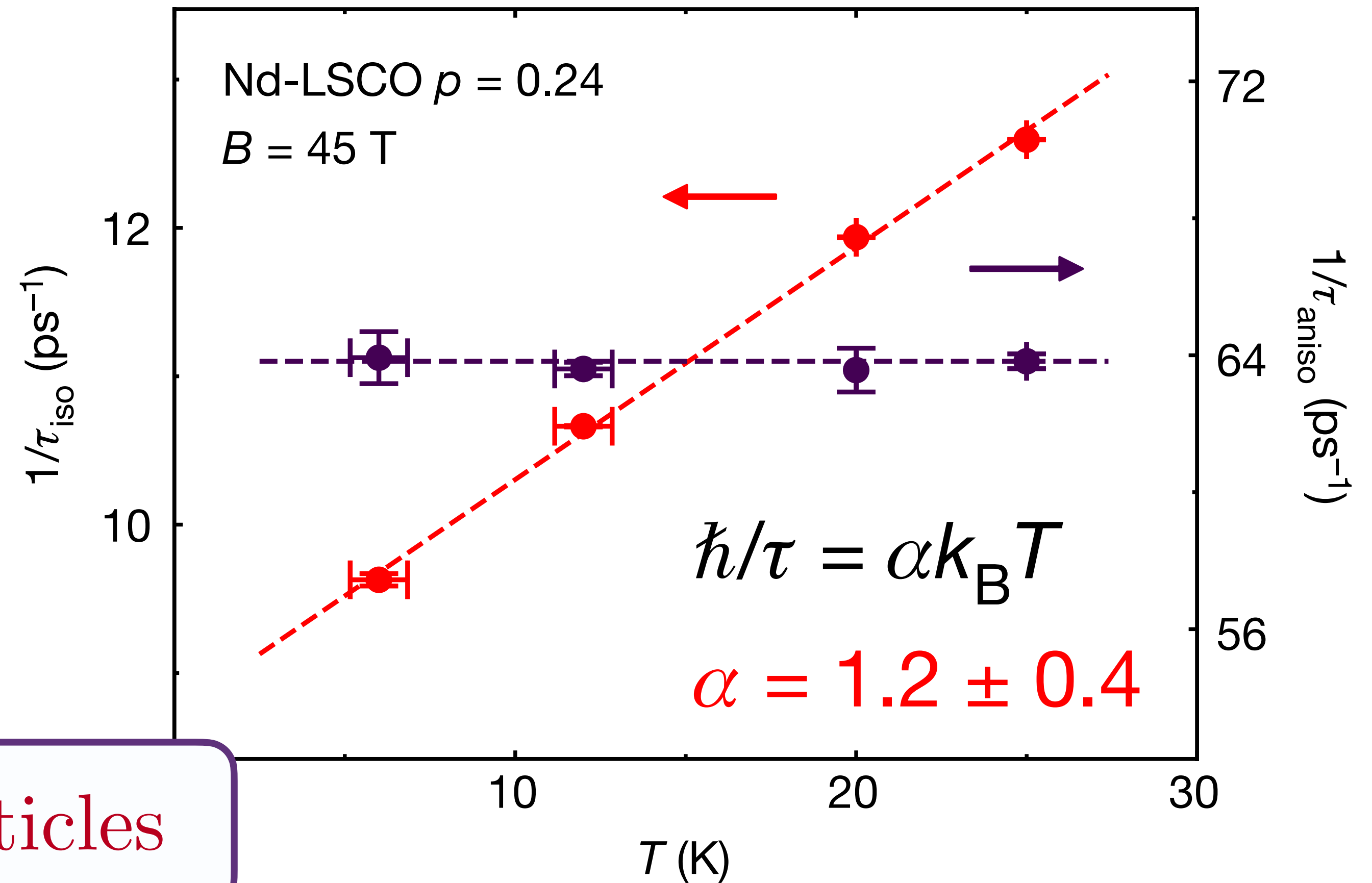
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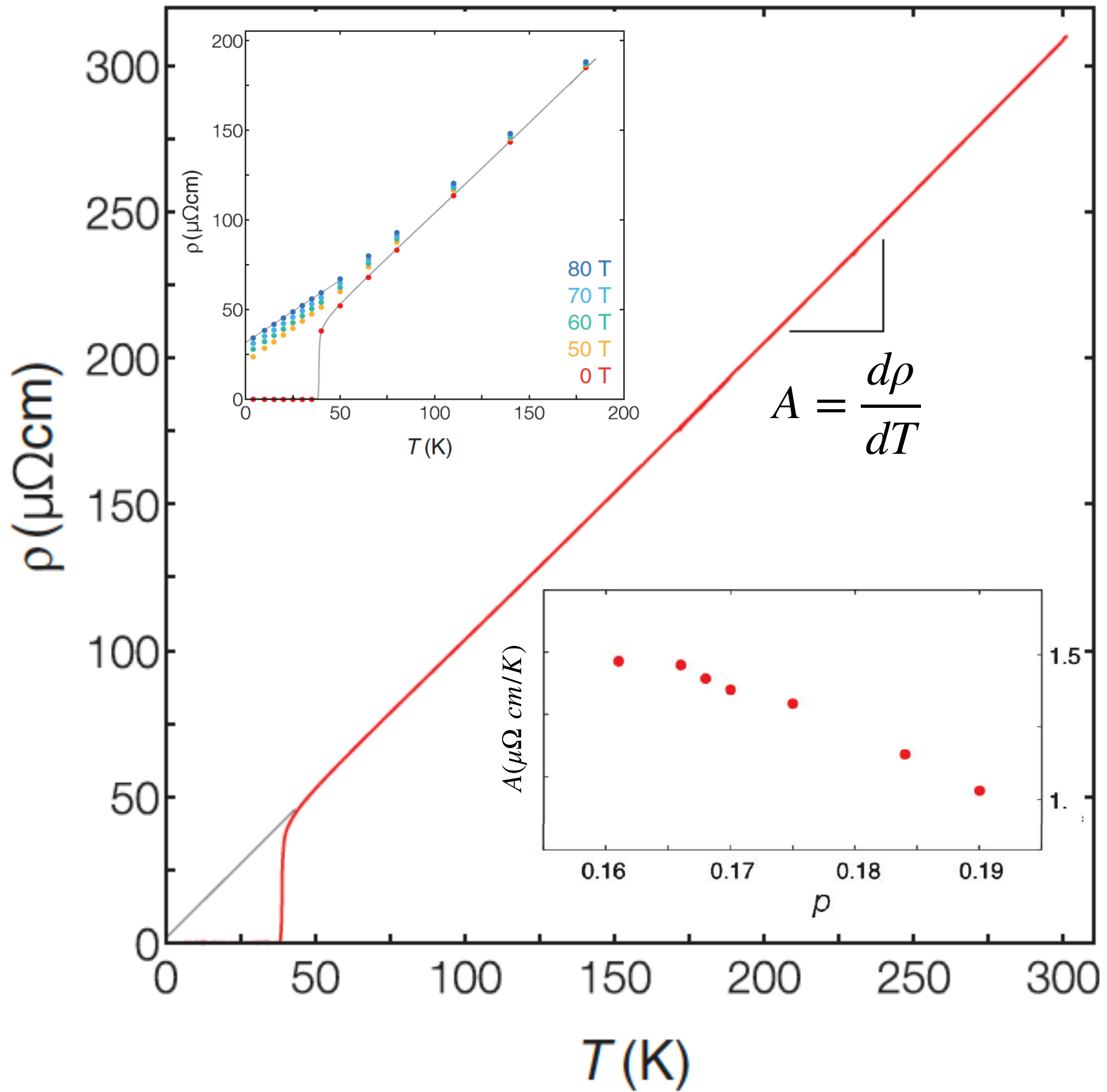
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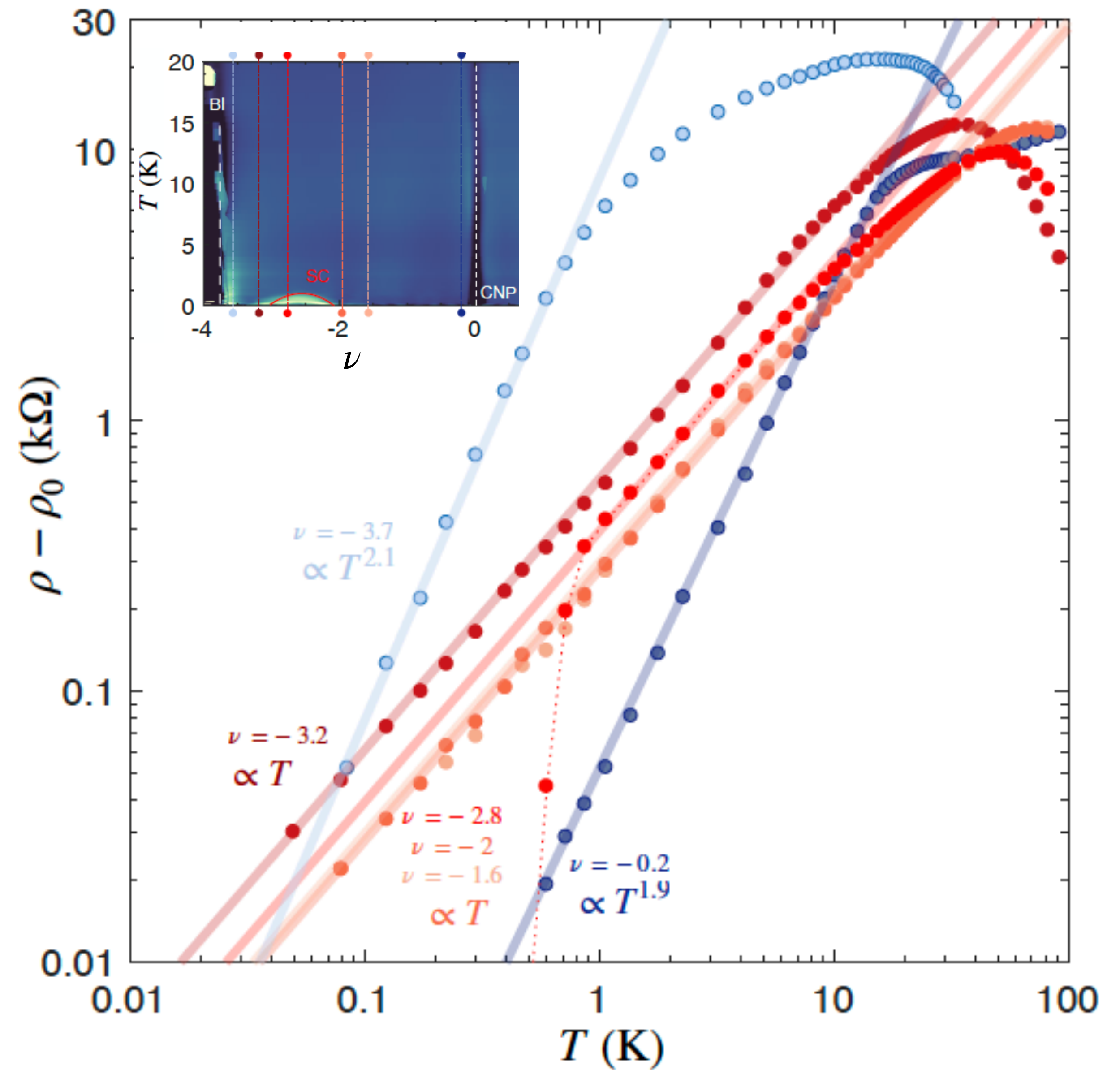


Current flow without quasiparticles





LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Questions

- Theory for a fermion system with variable density without quasiparticles, and relaxation time $\sim \hbar/(k_B T)$.
- Needed: theory for collision time in resistivity $\sim \hbar/(k_B T)$.
- Needed: theory for the appearance of superconductivity (and other broken symmetries) in such a ‘strange metal’.

1. Introduction to metals with and without quasiparticles

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Needed:

A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

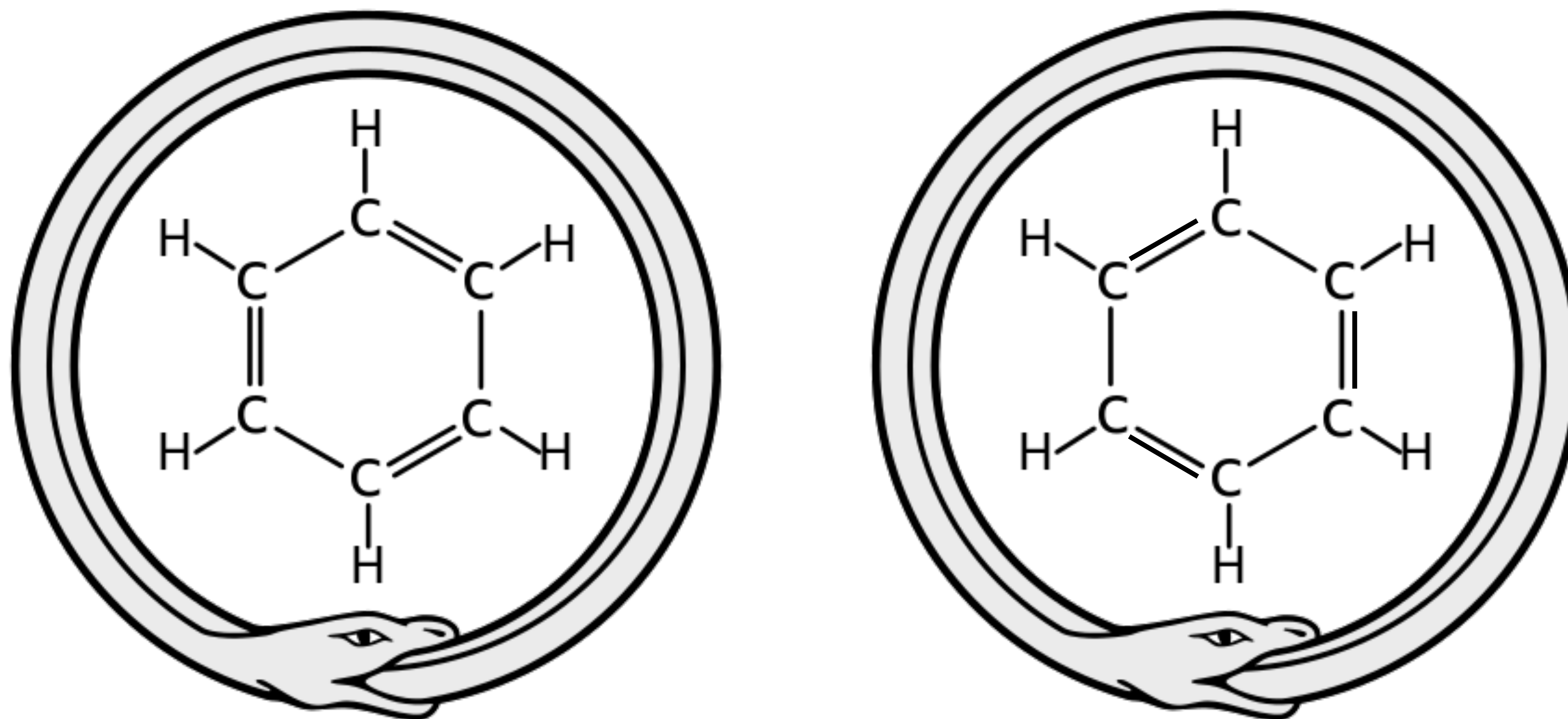
leading to a compressible quantum state with
no quasiparticle excitations

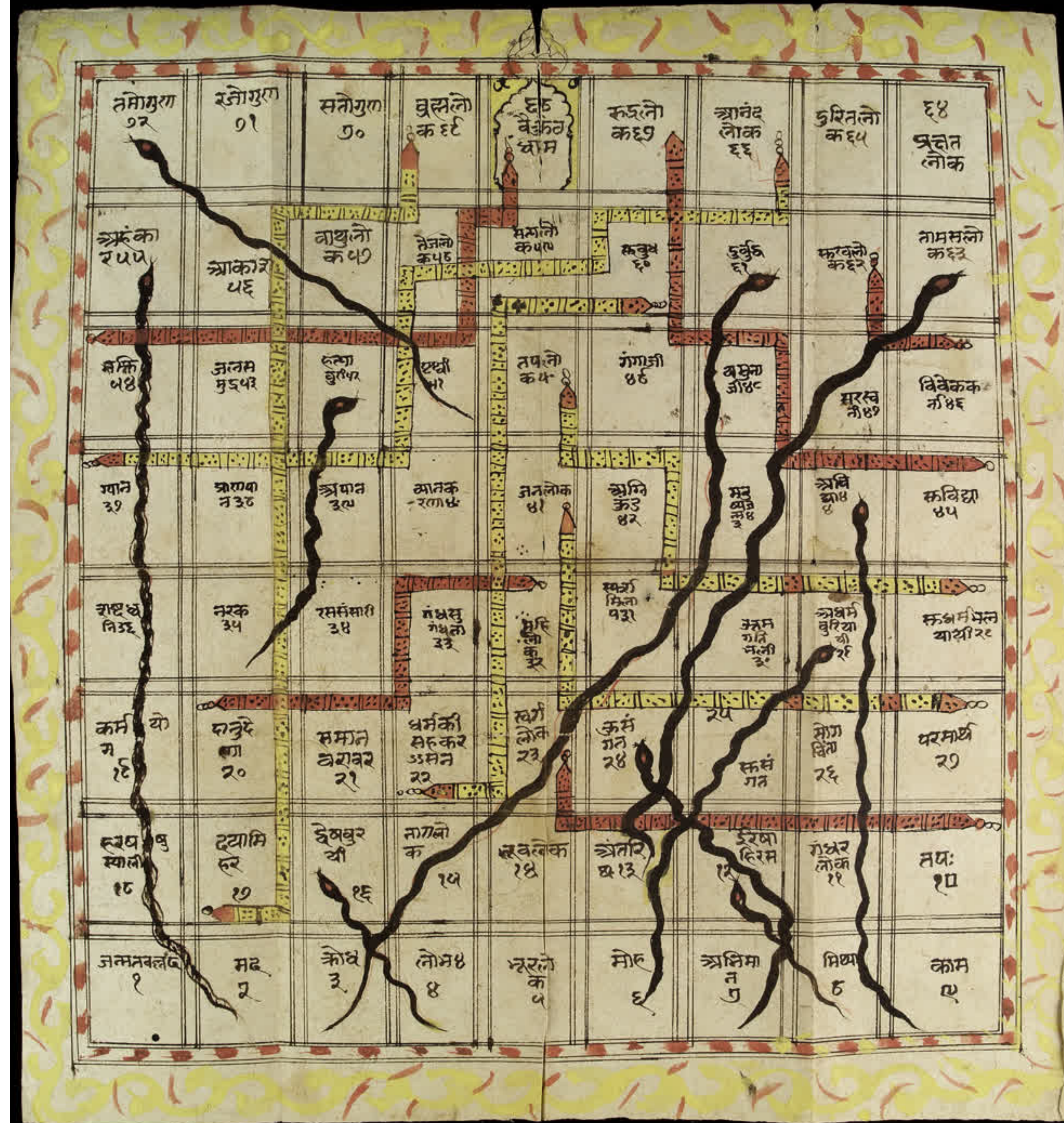


August Kekule, theory of the benzene molecule, 1865

Kekulé's dream

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*





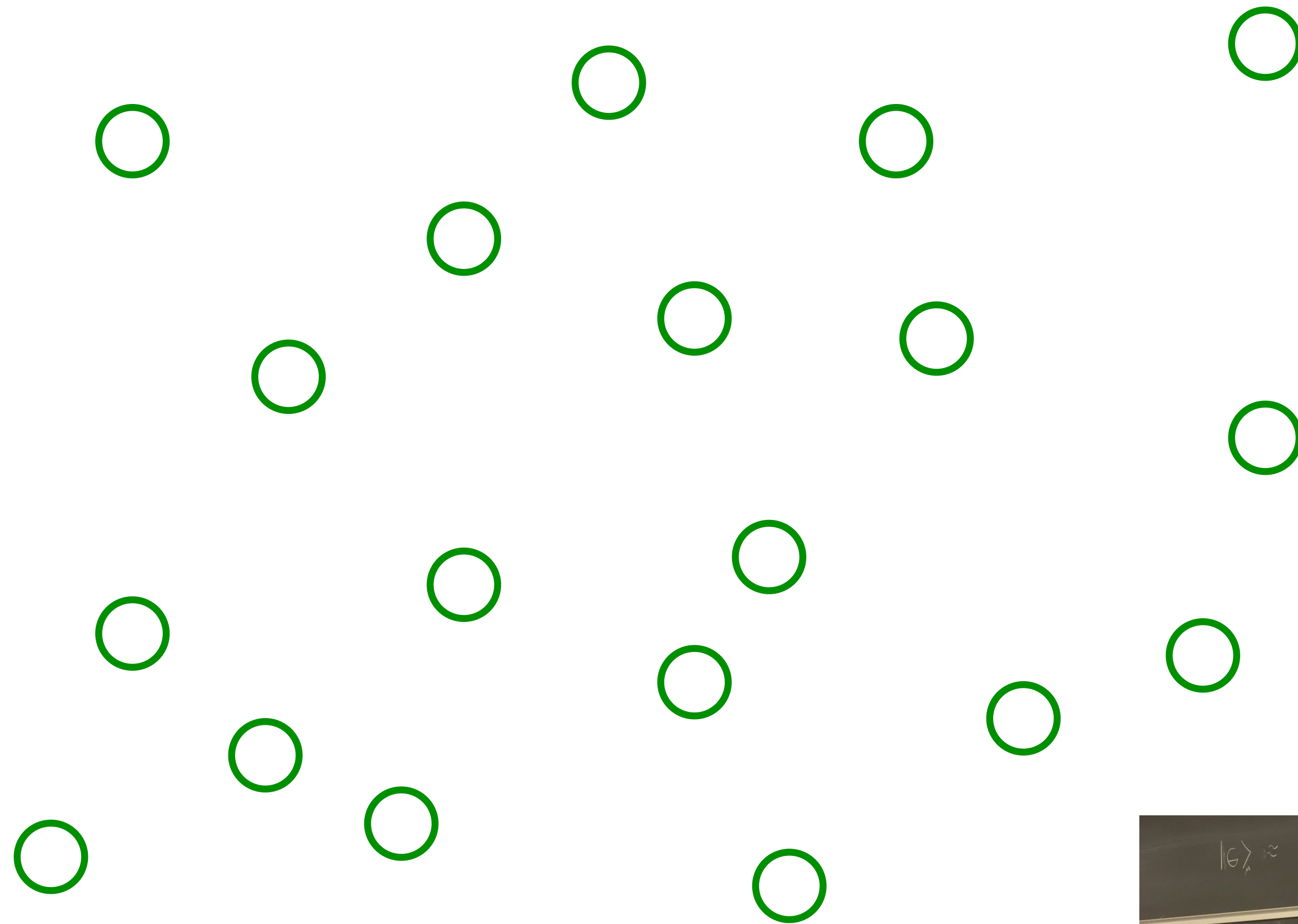
My dream*

Snakes and ladders

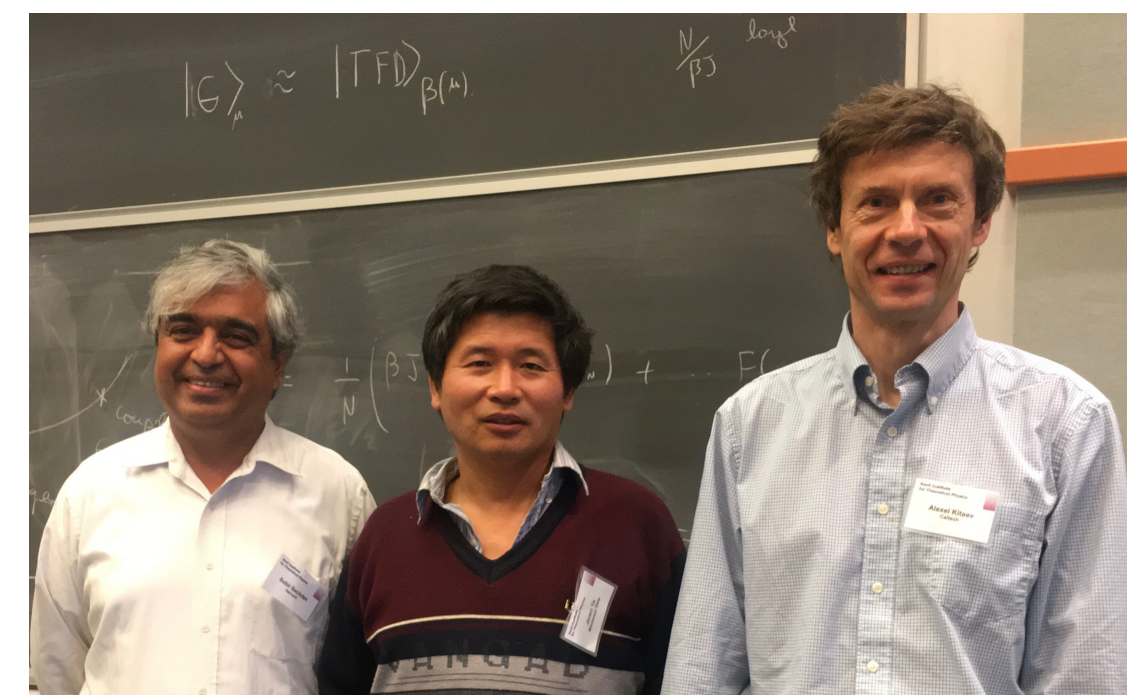
*Not true

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

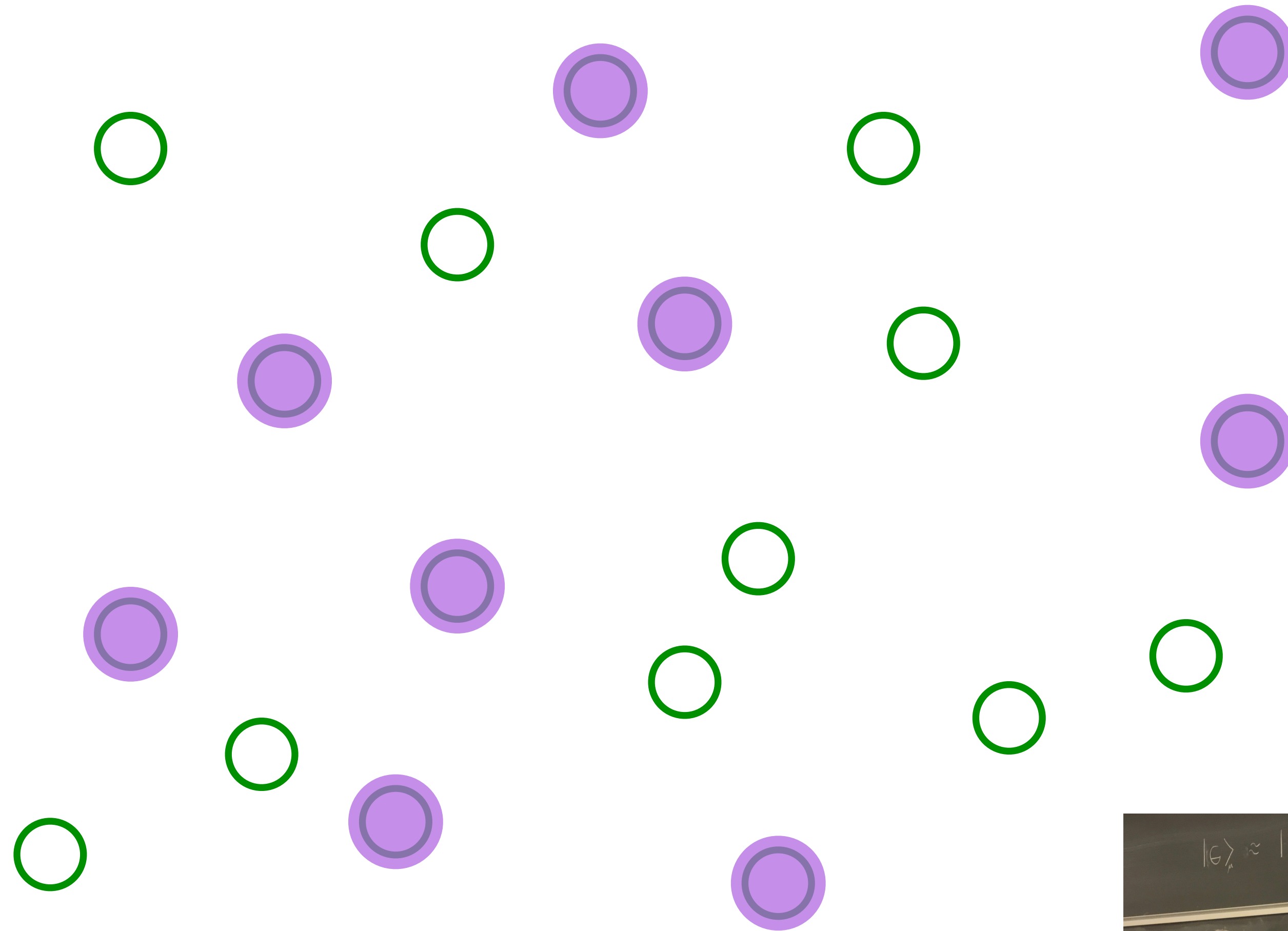


Pick a set of random positions

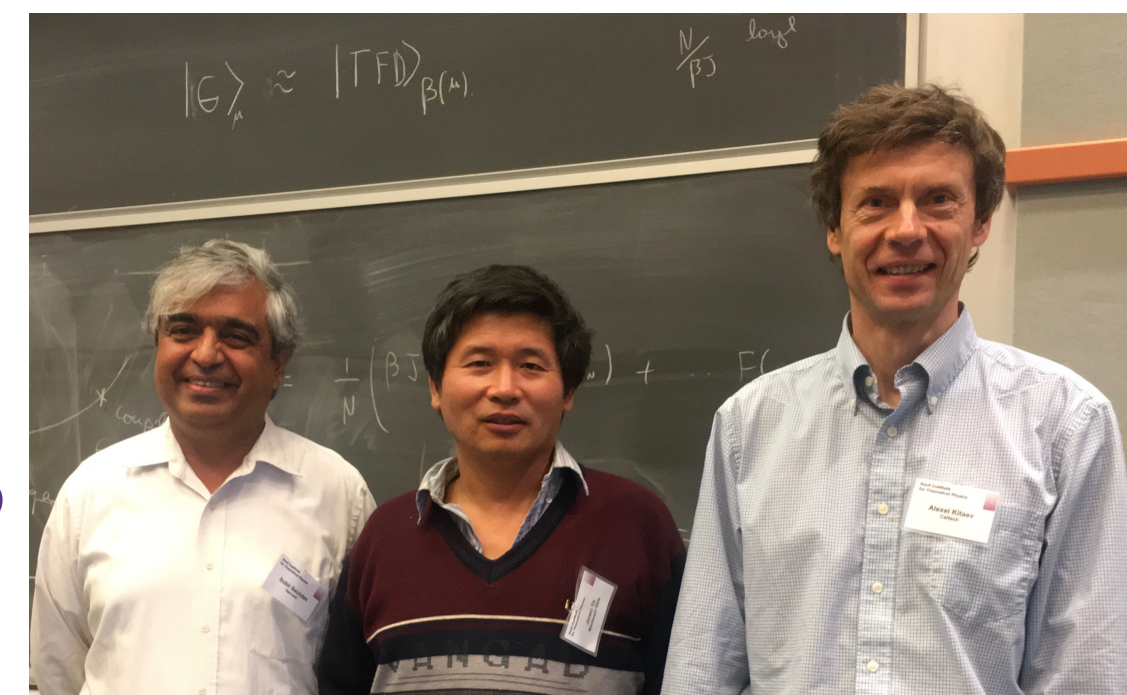


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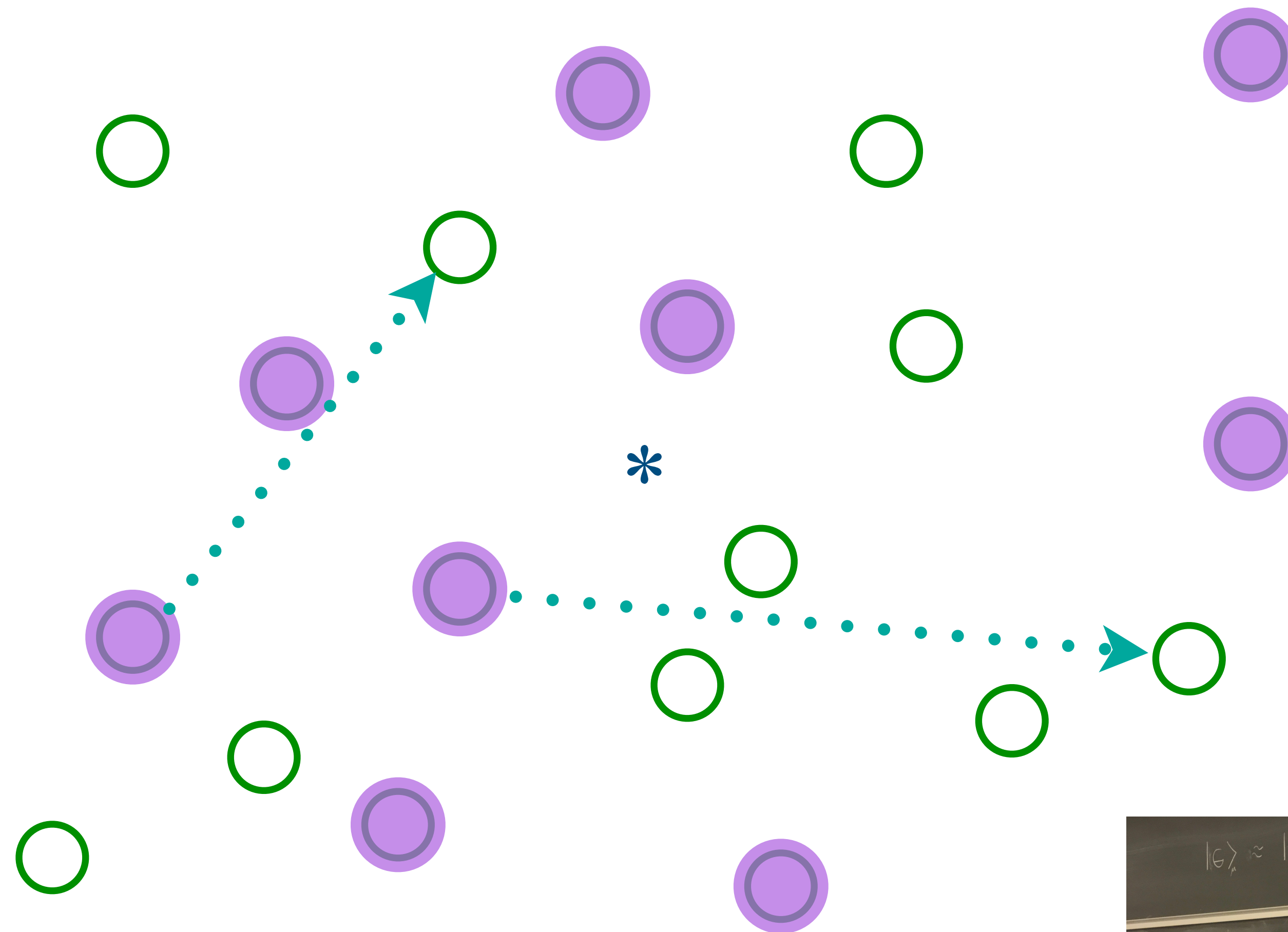


Place electrons randomly on some sites

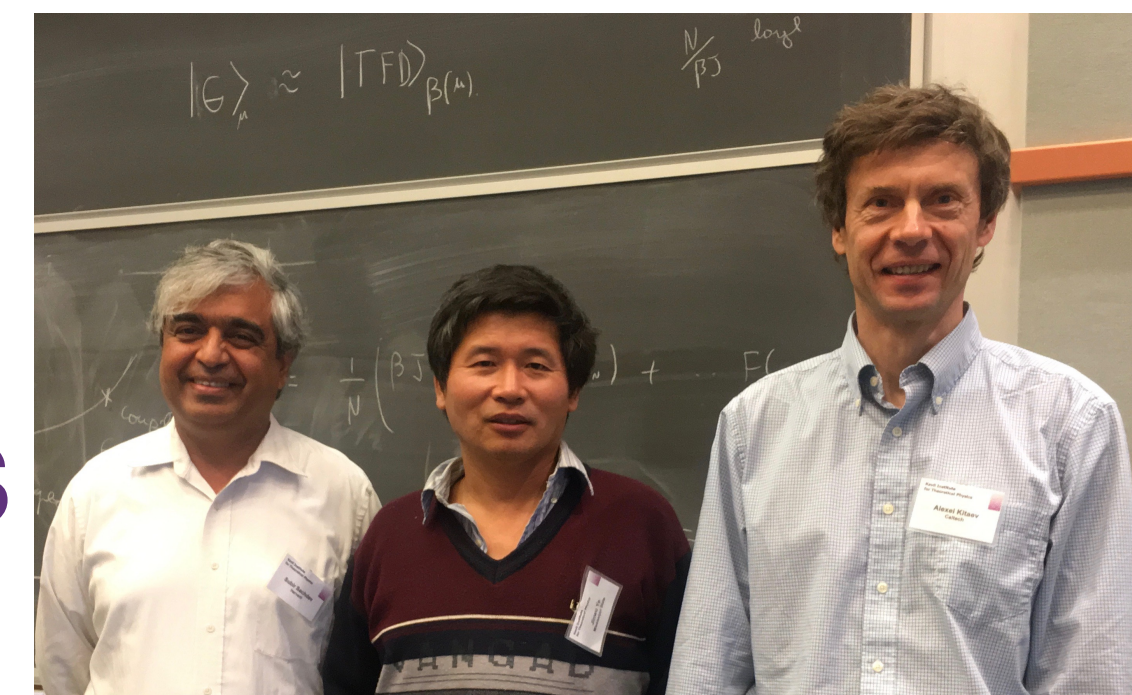


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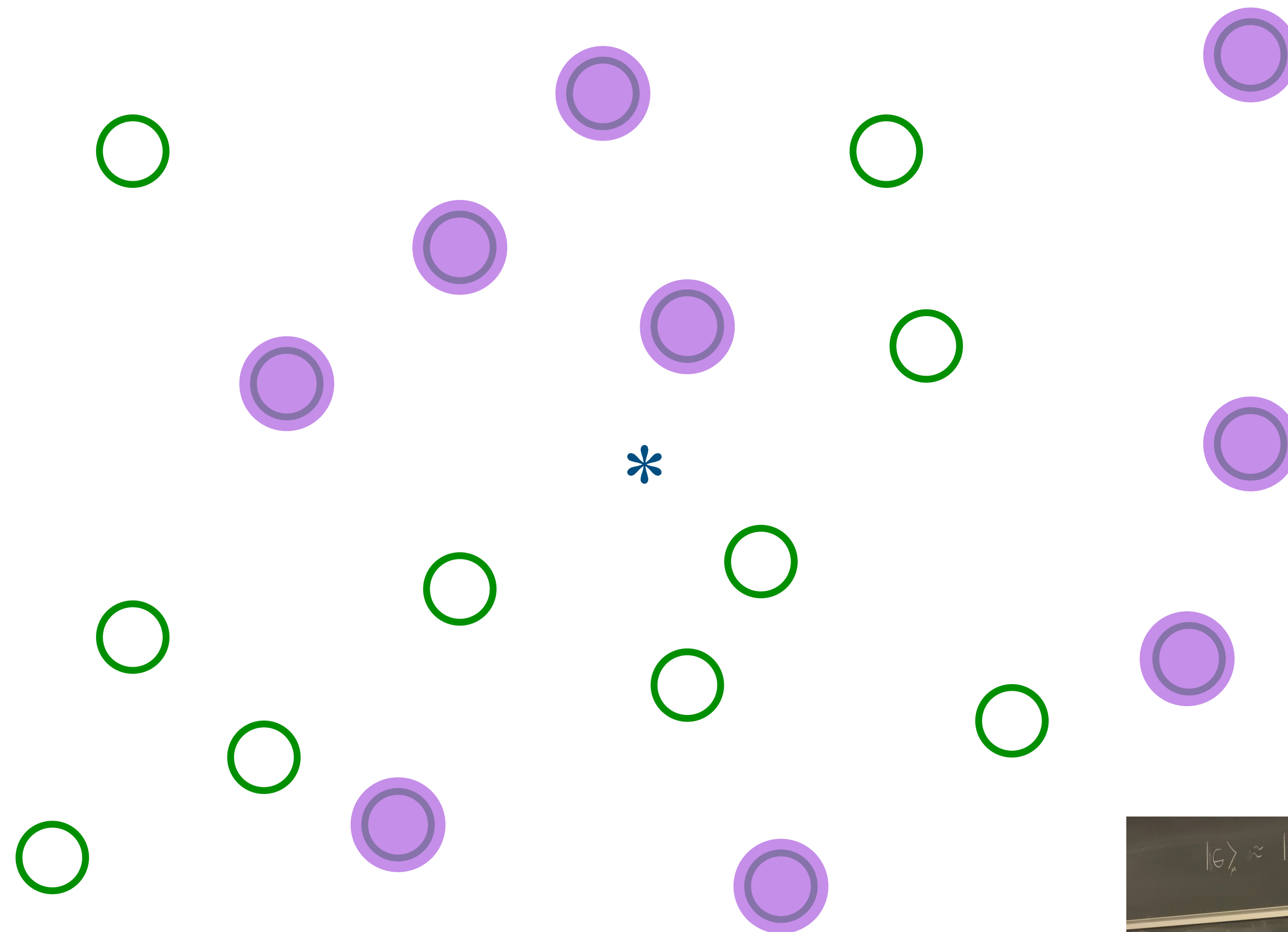


Place electrons randomly on some sites

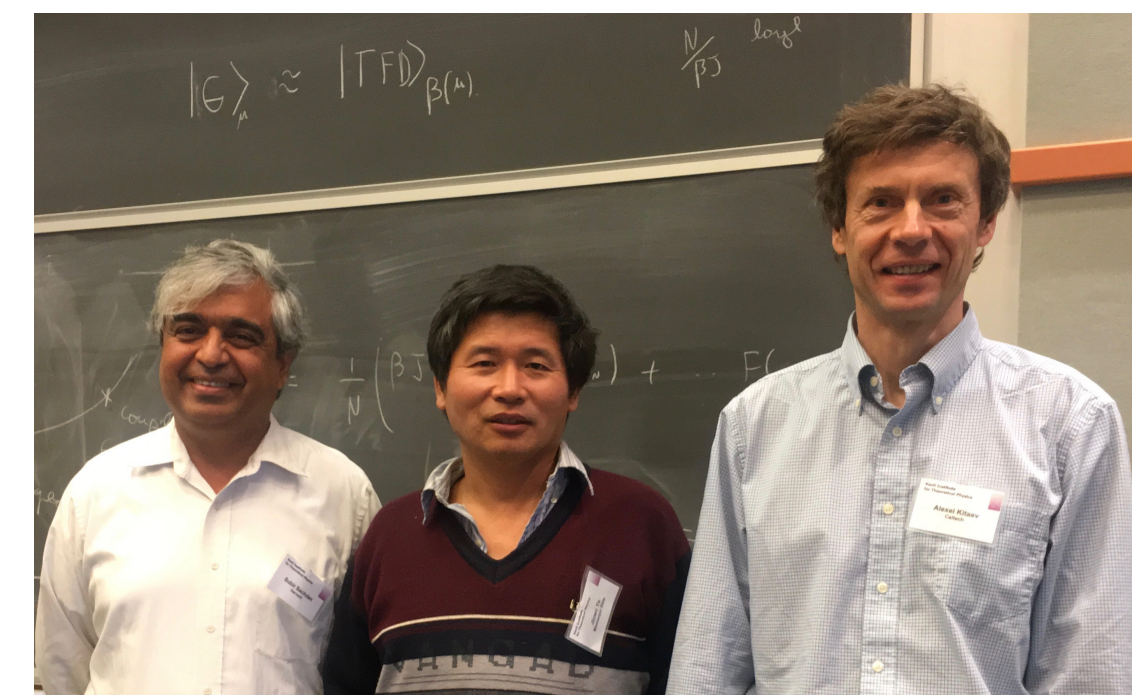


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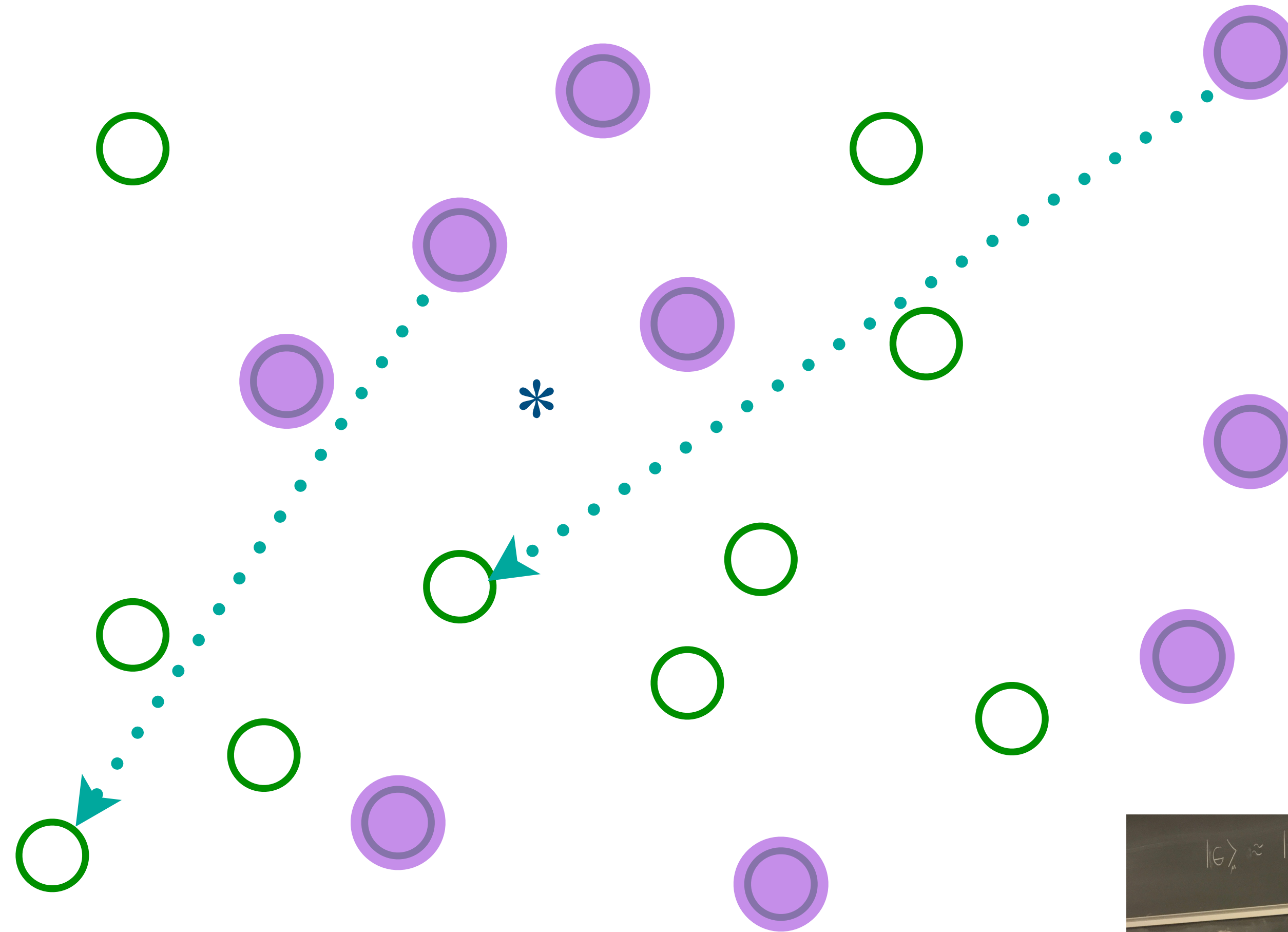


Entangle electrons pairwise randomly

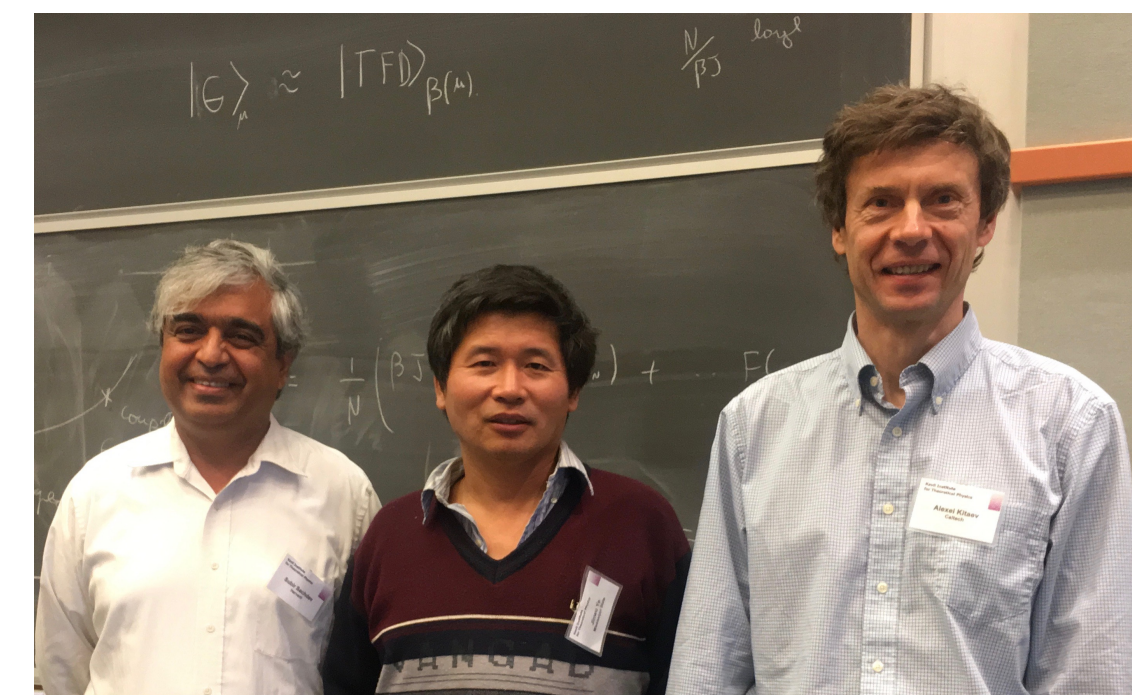


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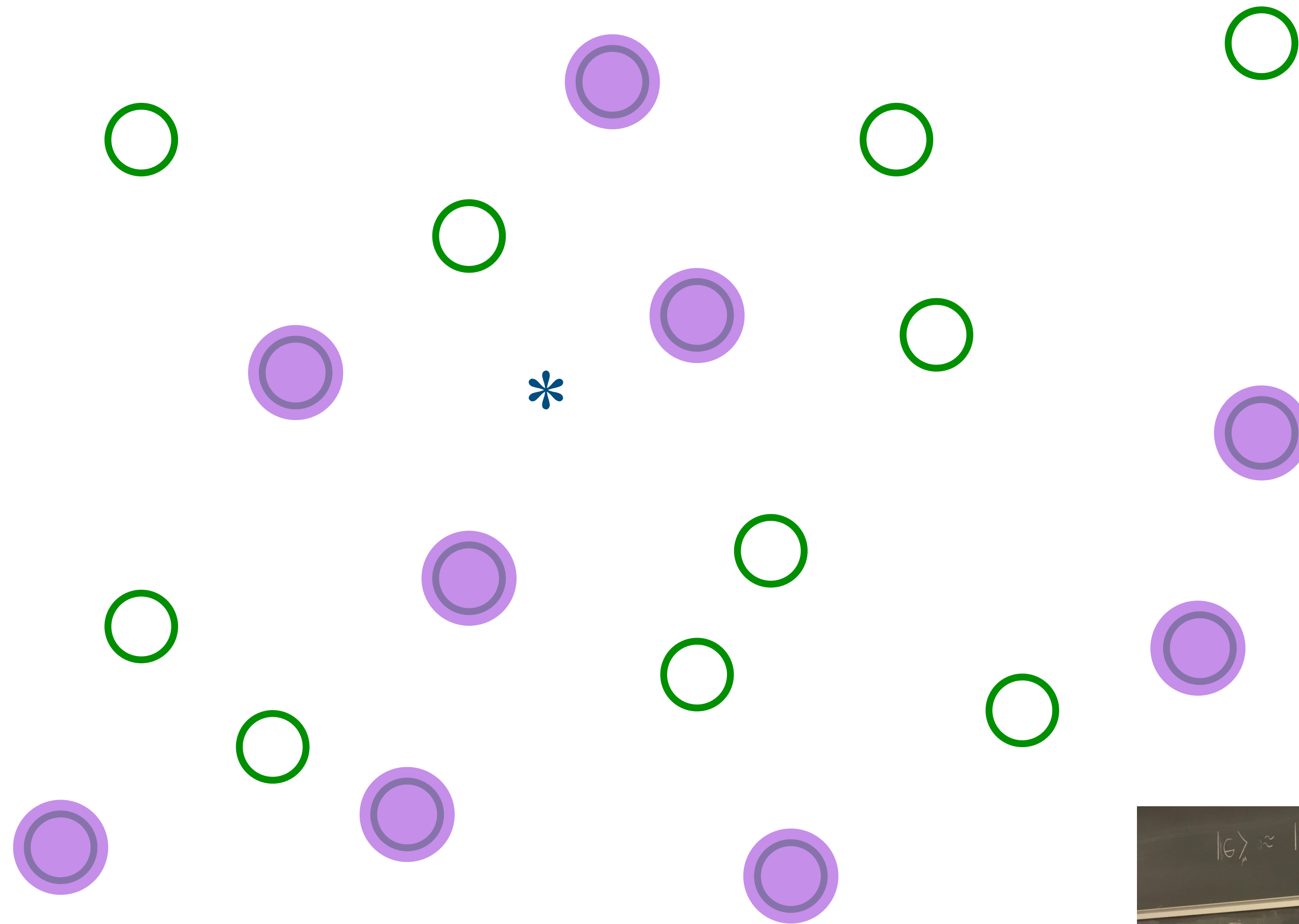


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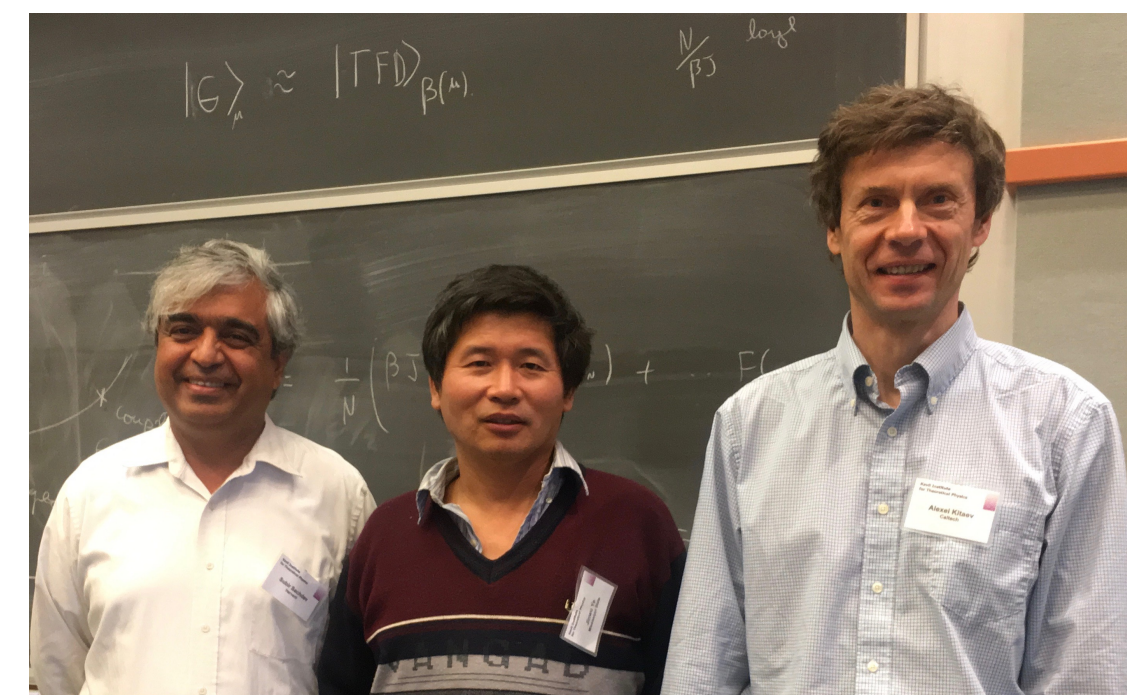


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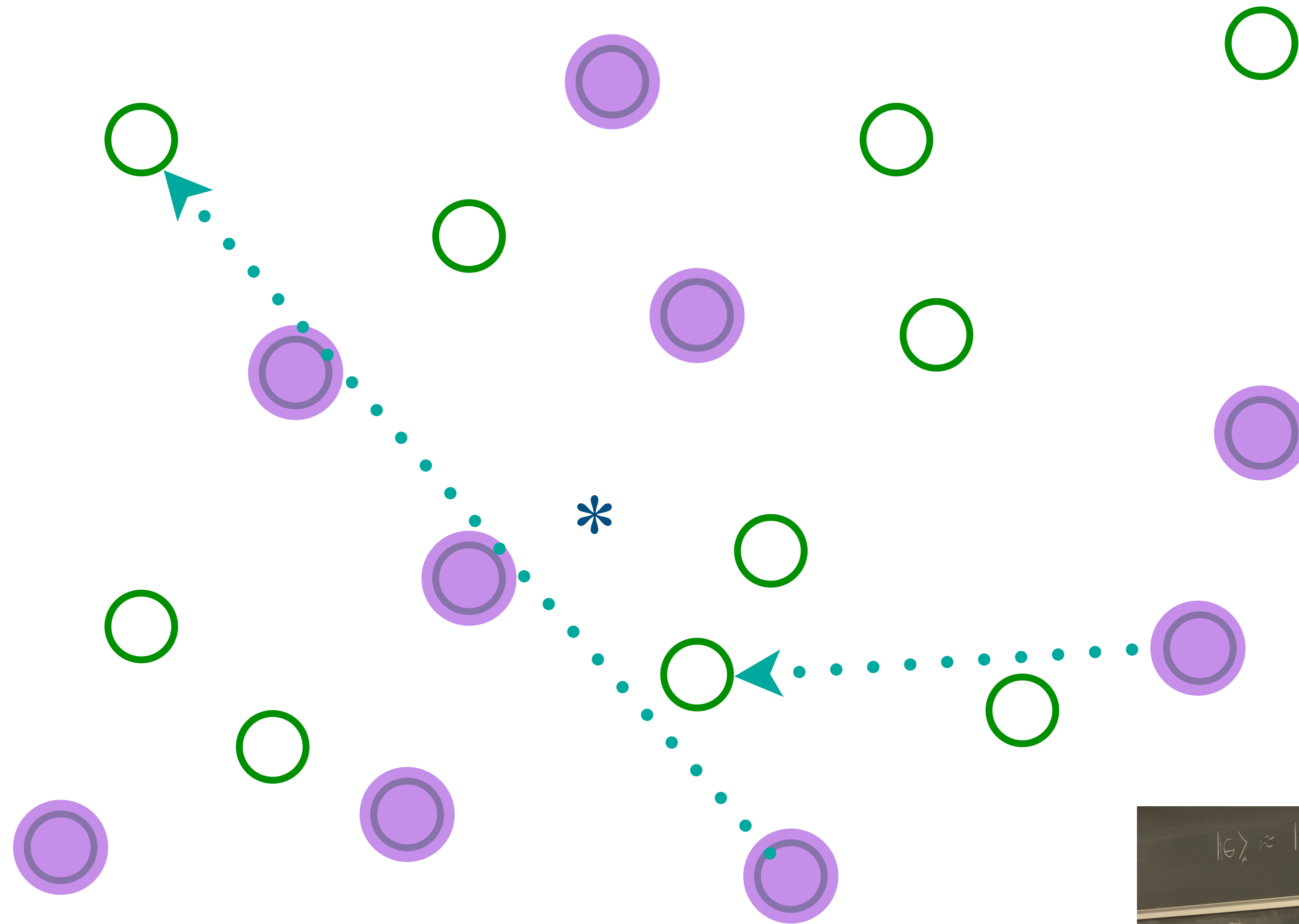


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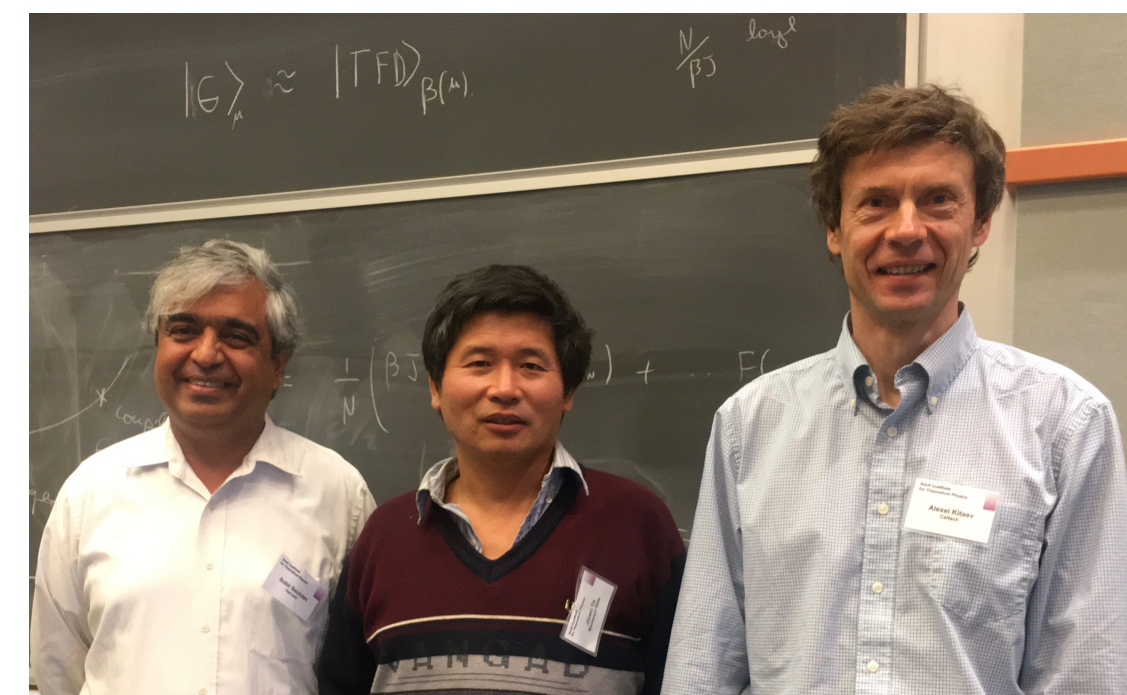


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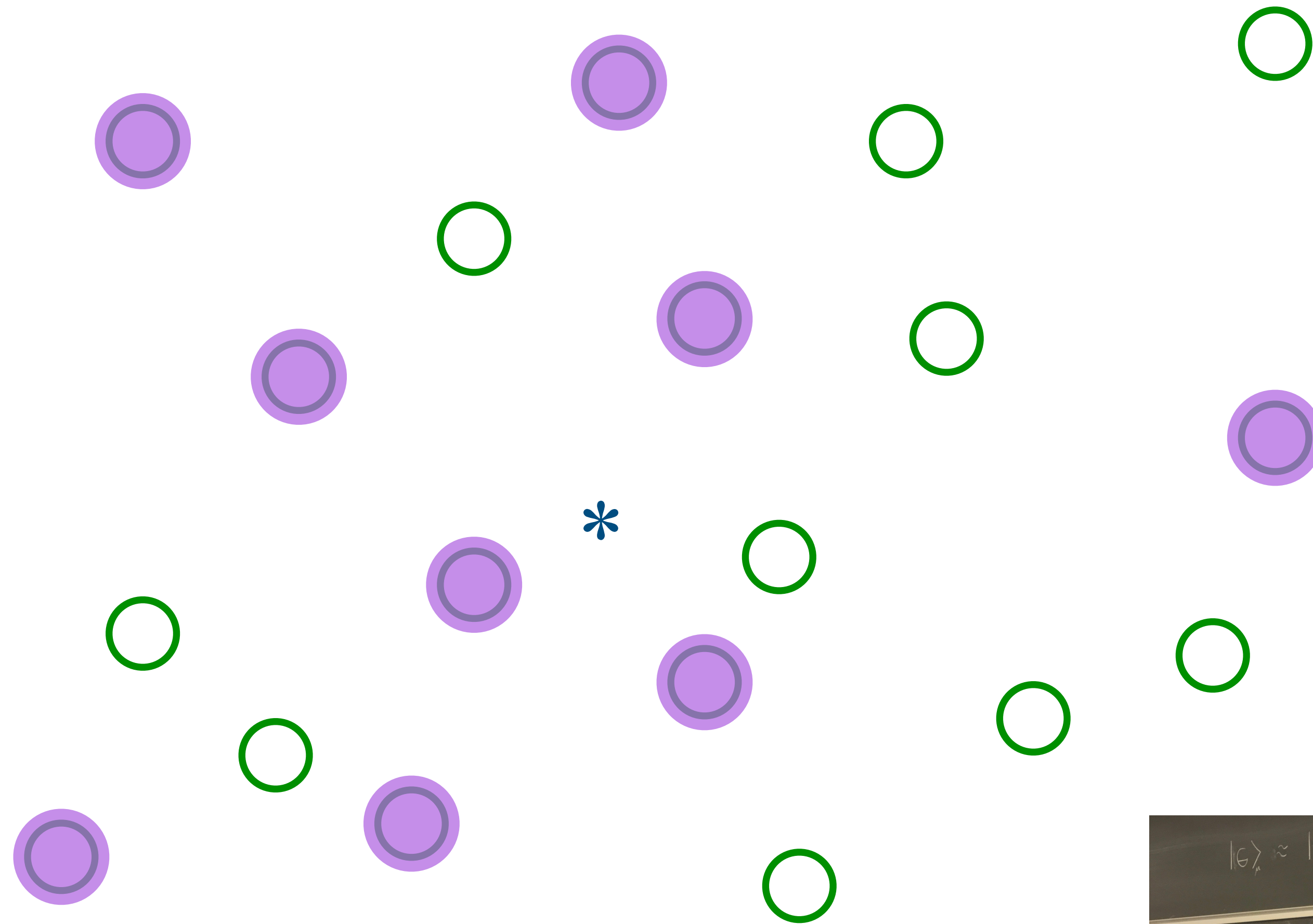


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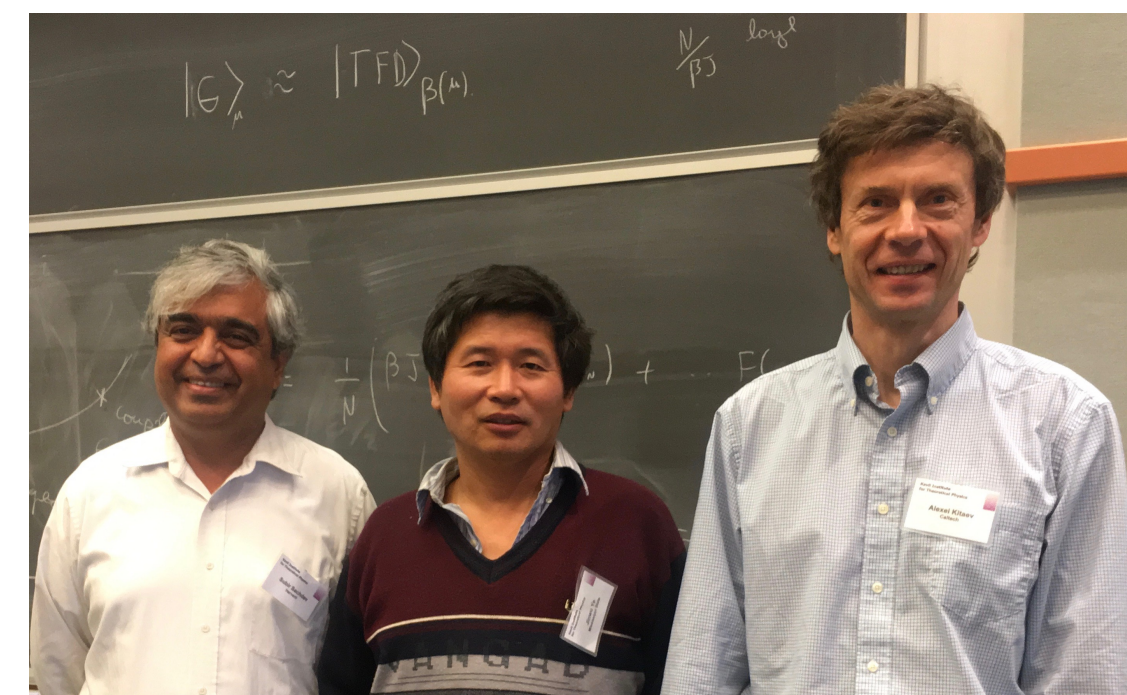


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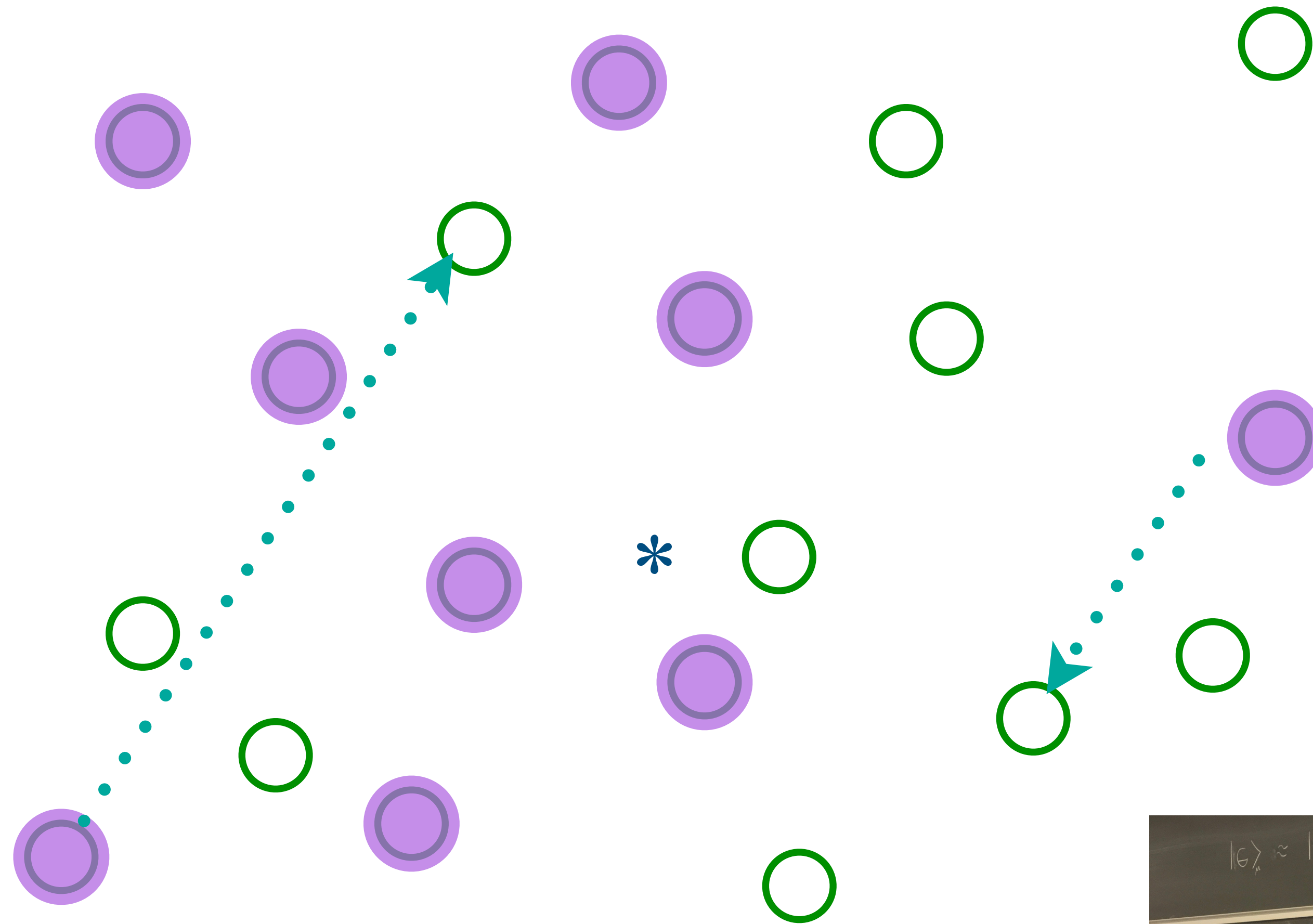


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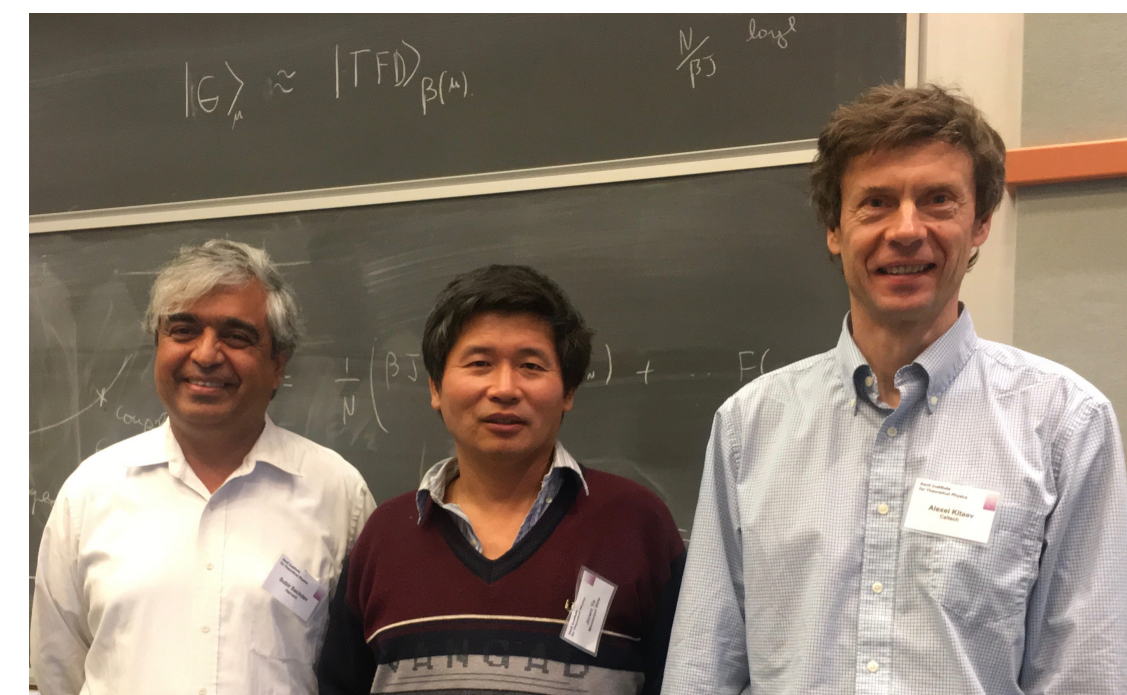


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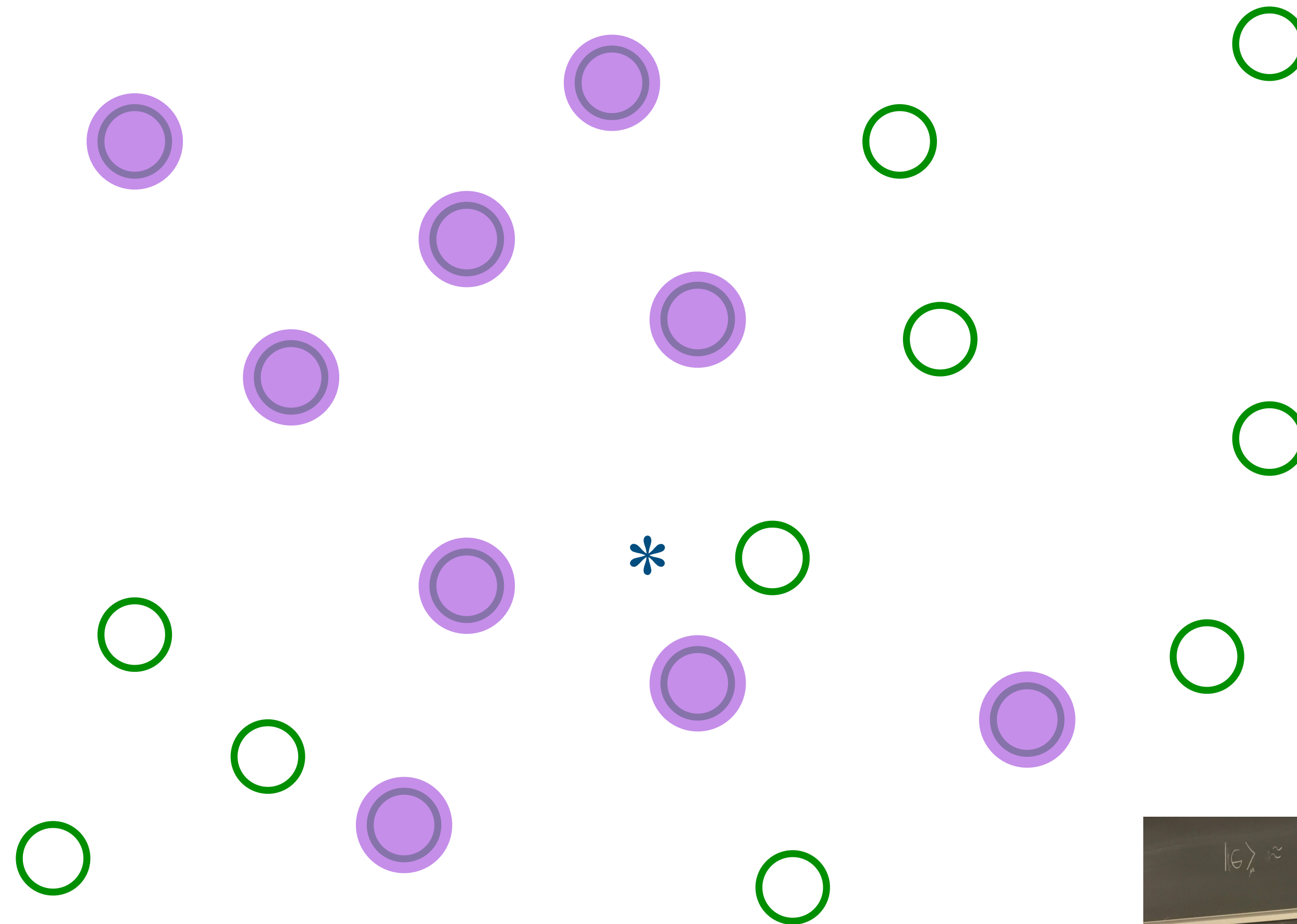


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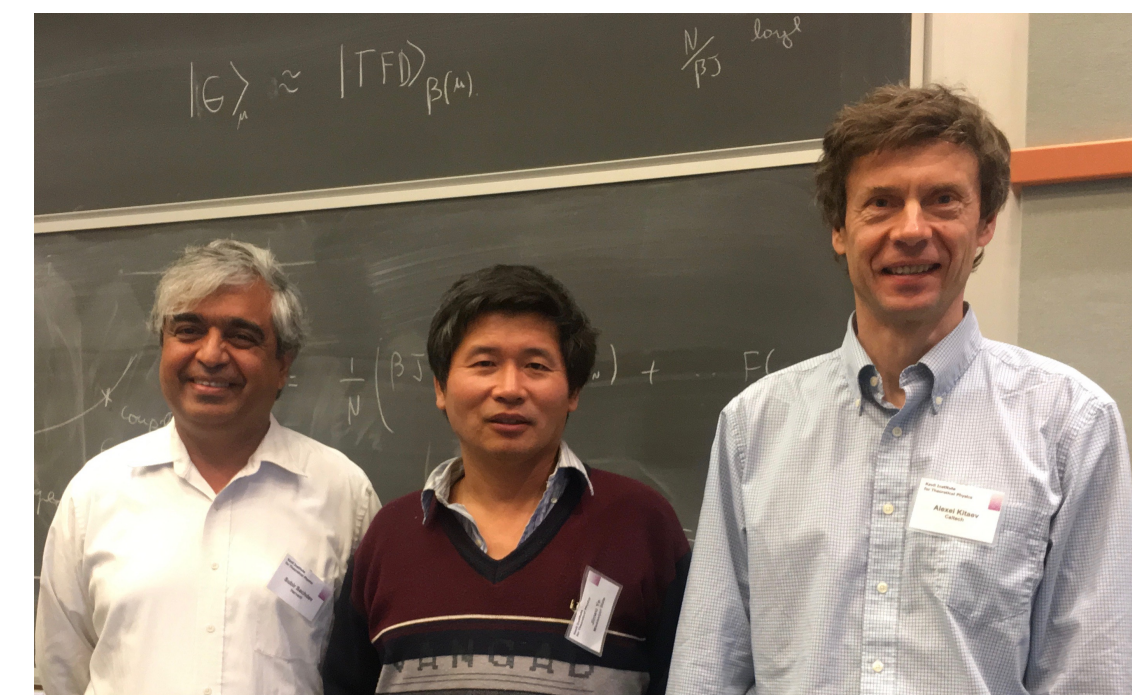


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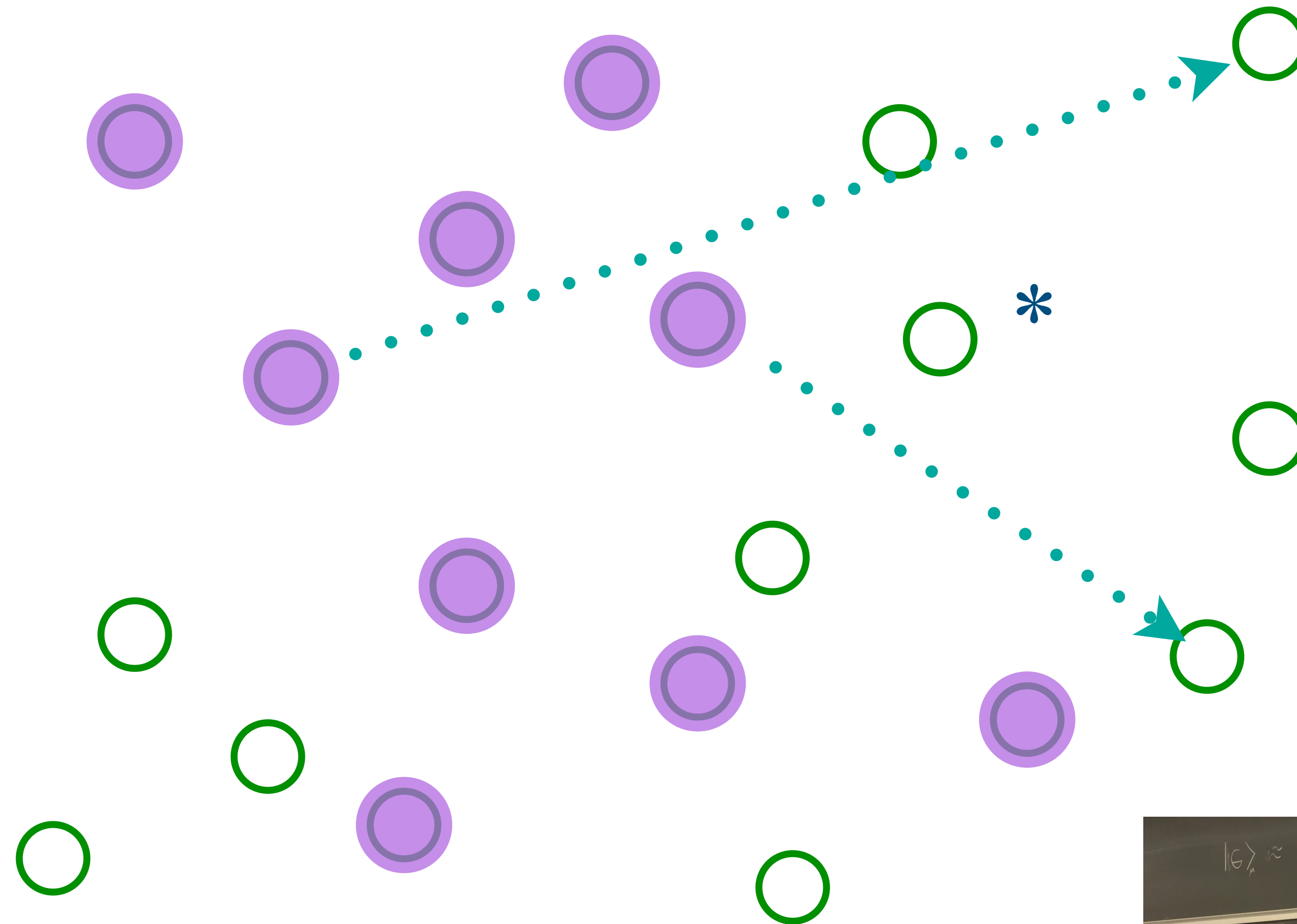


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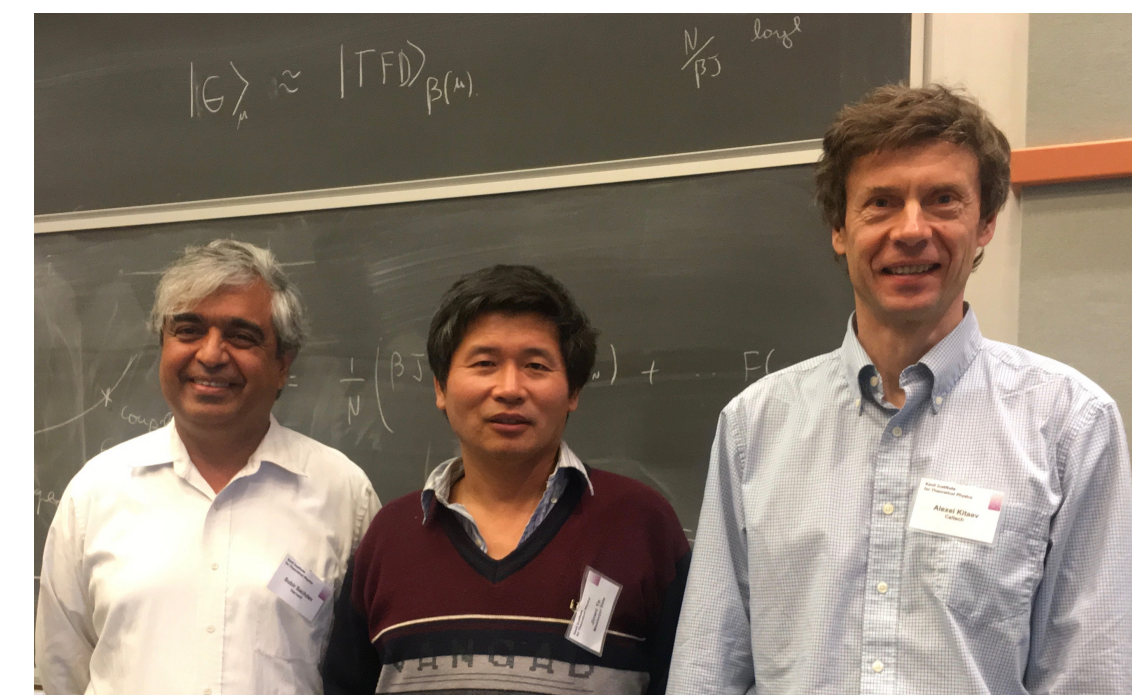


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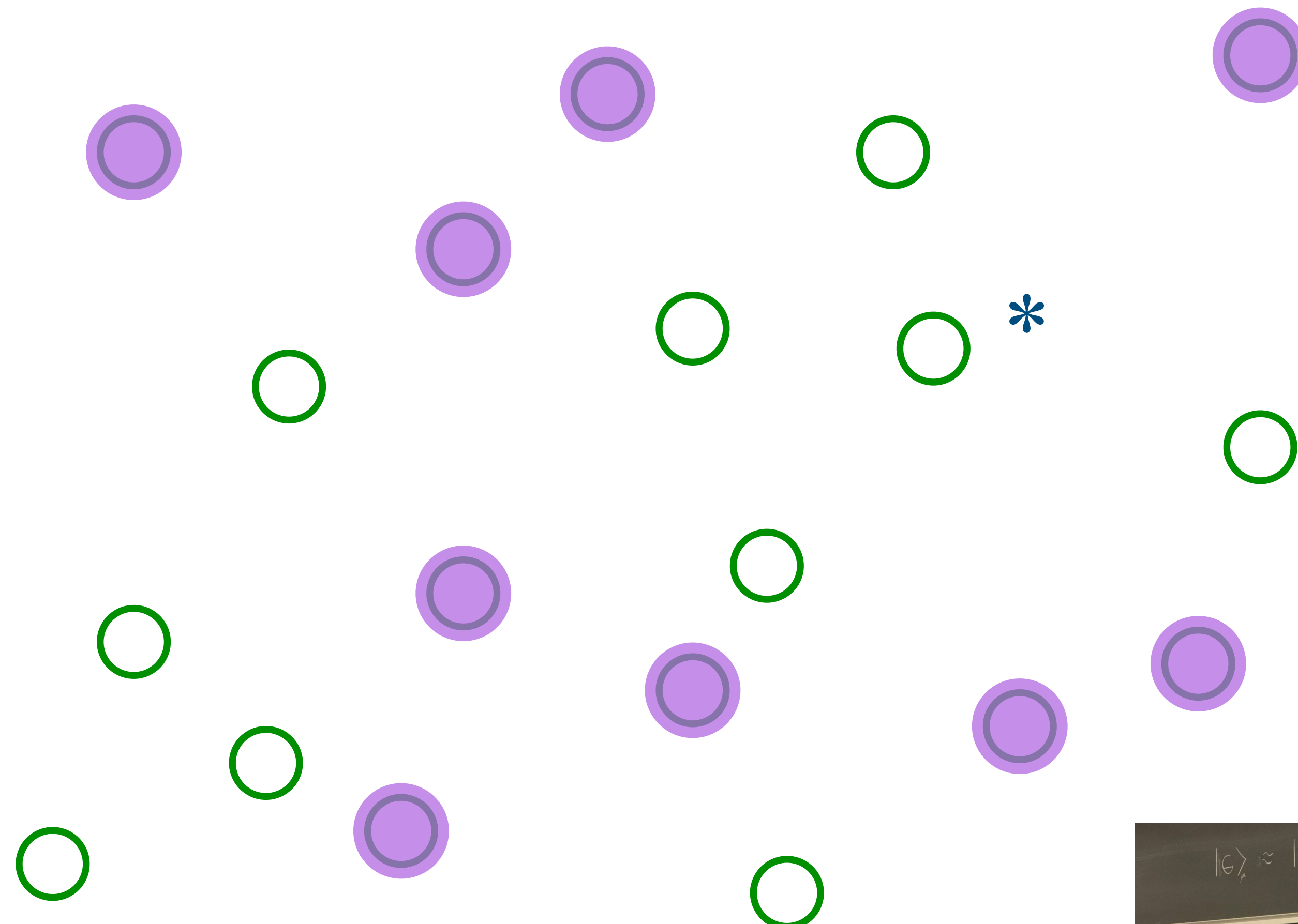


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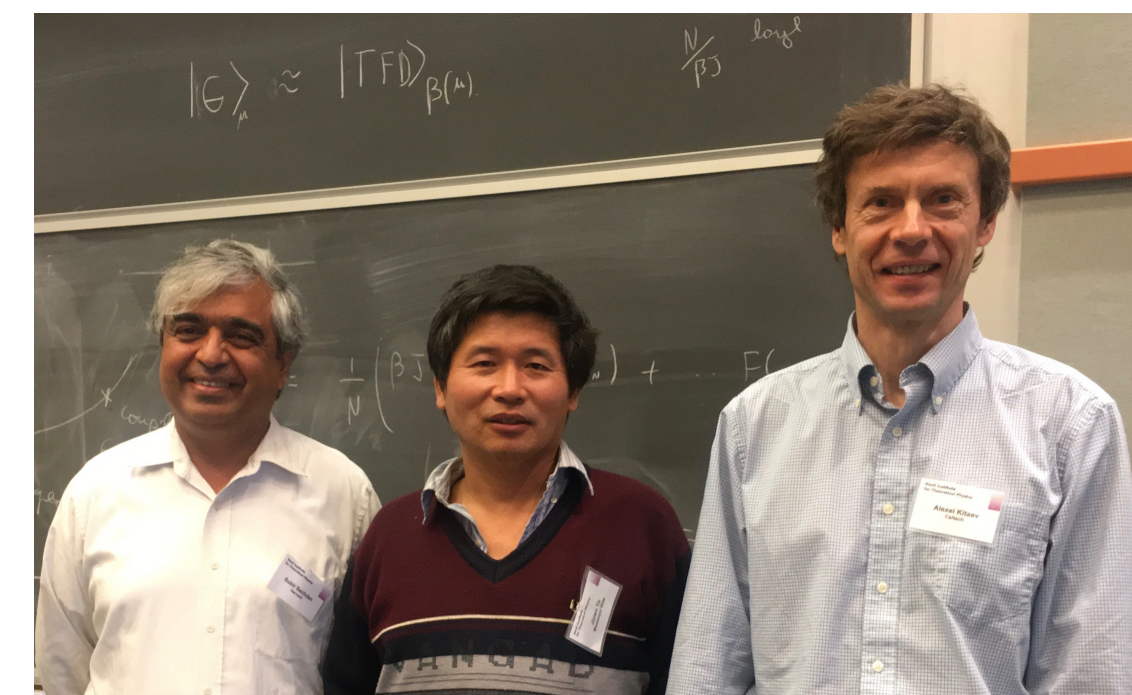


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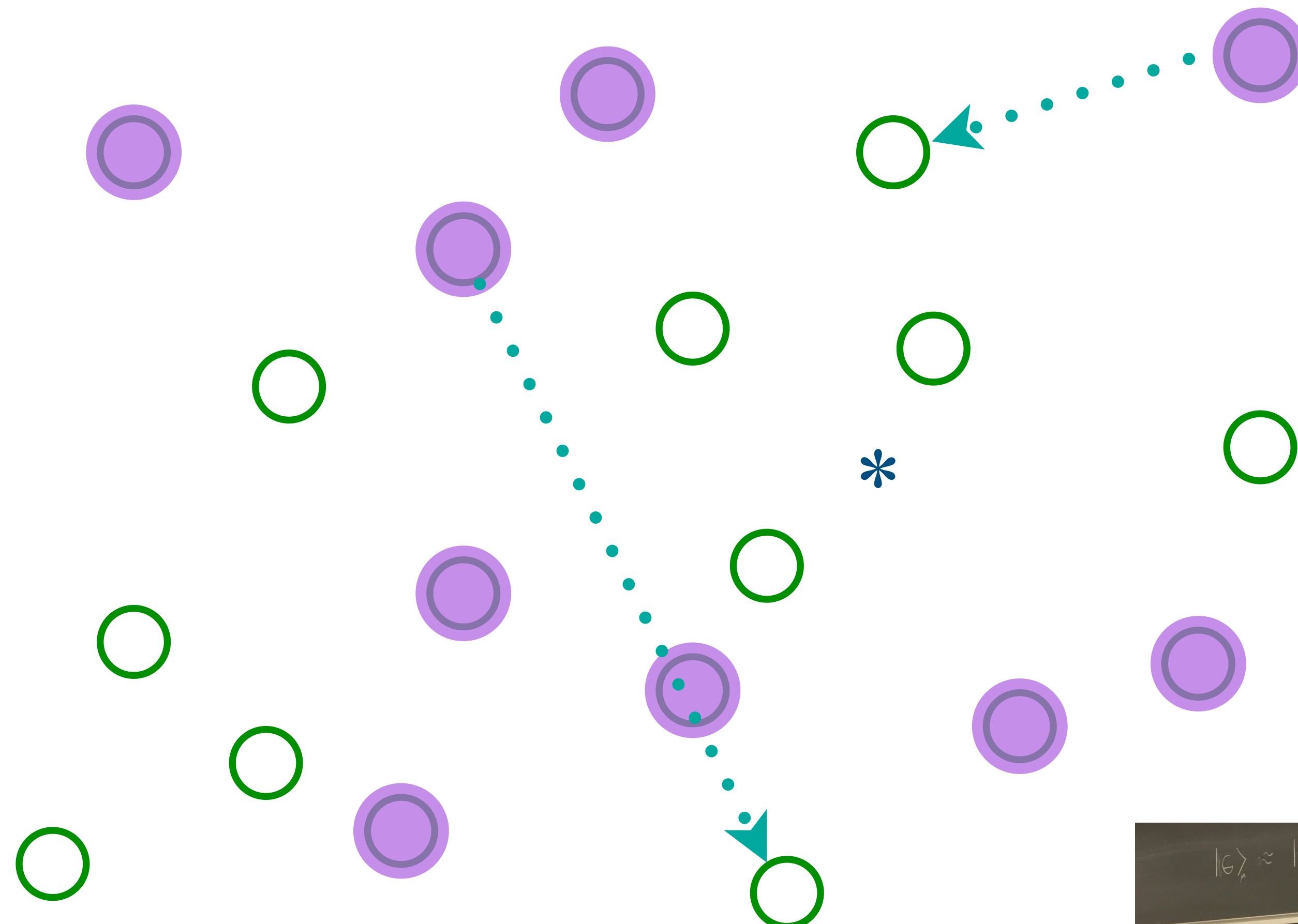


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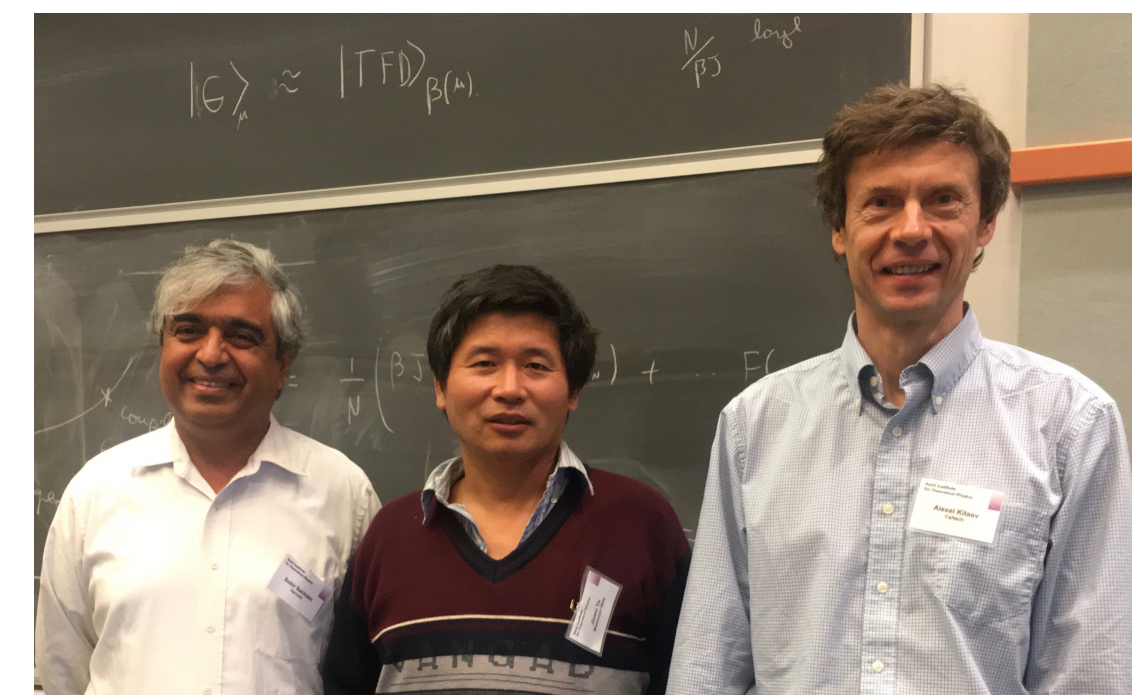


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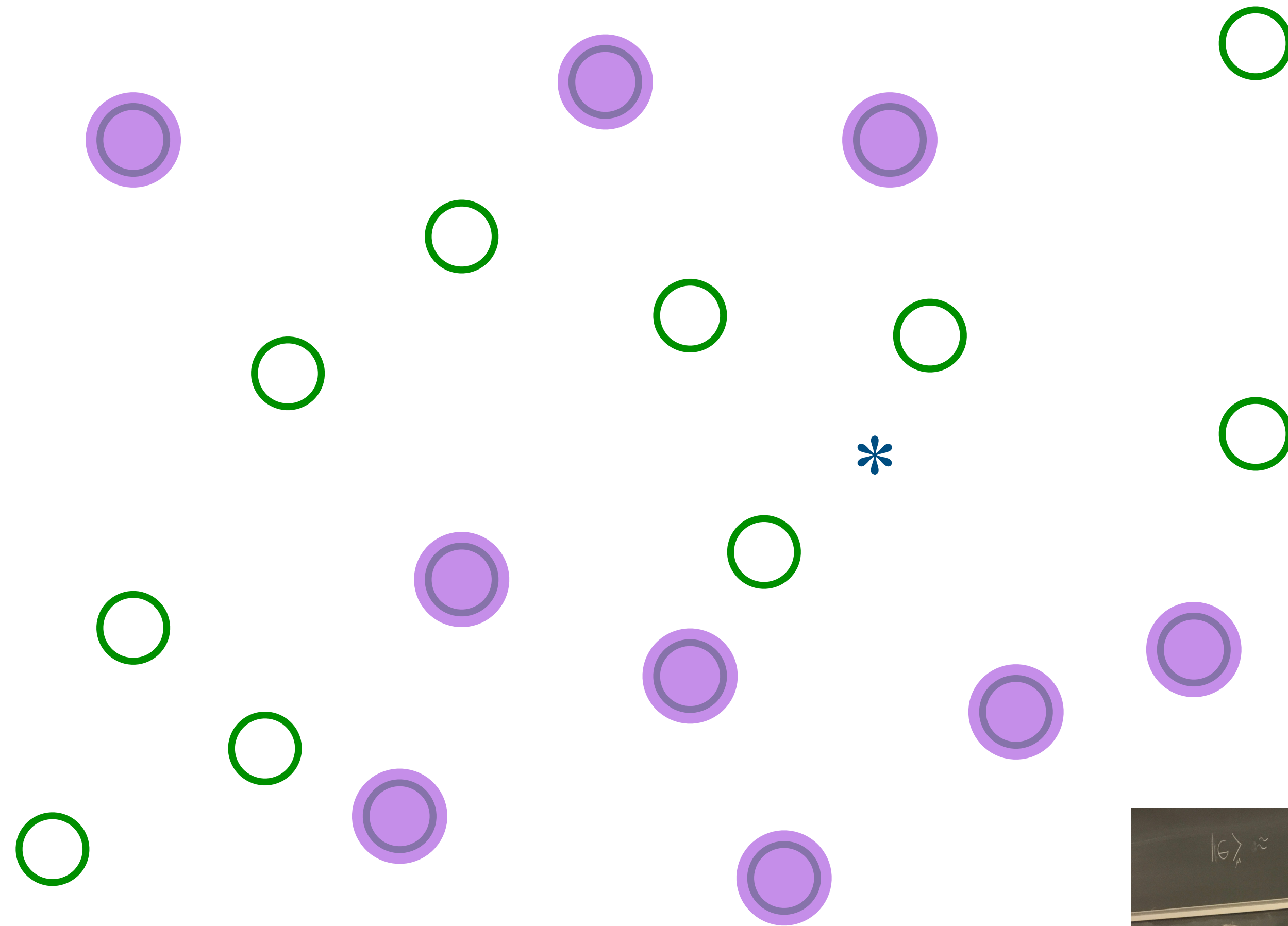


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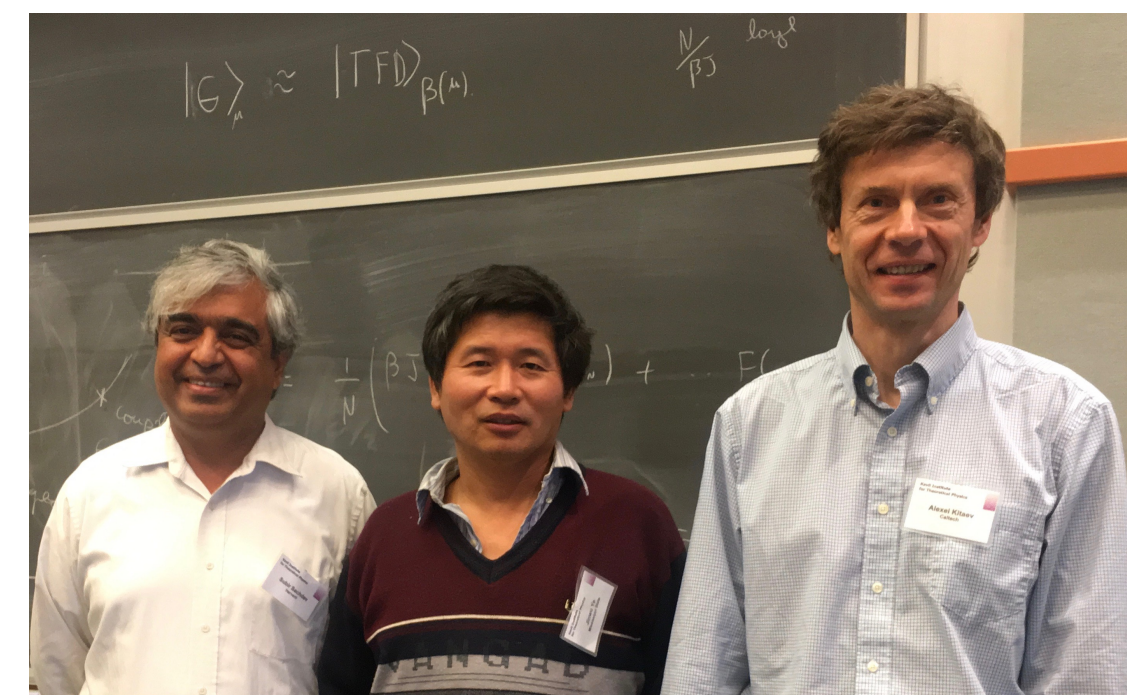


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The Sachdev-Ye-Kitaev (SYK) model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

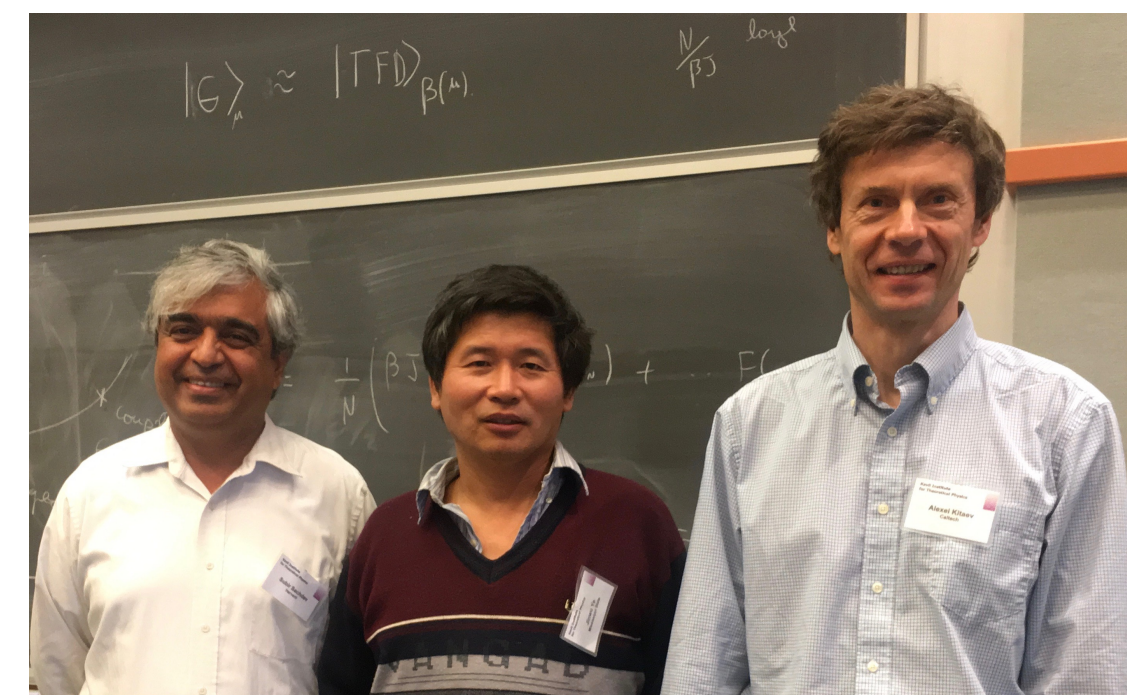
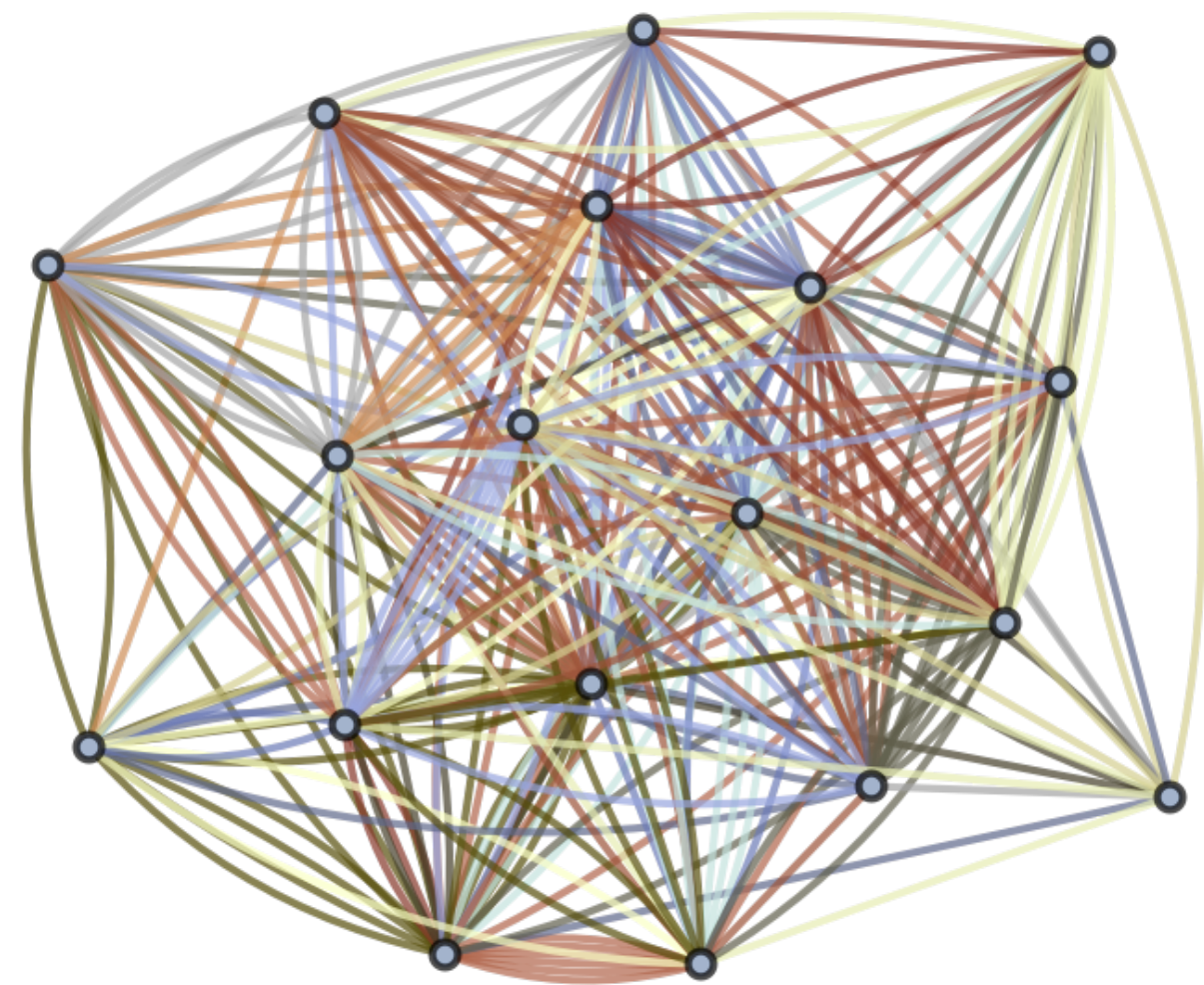
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$Q = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta; \gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta; \gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta; \gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



A simple model of a metal with quasiparticles

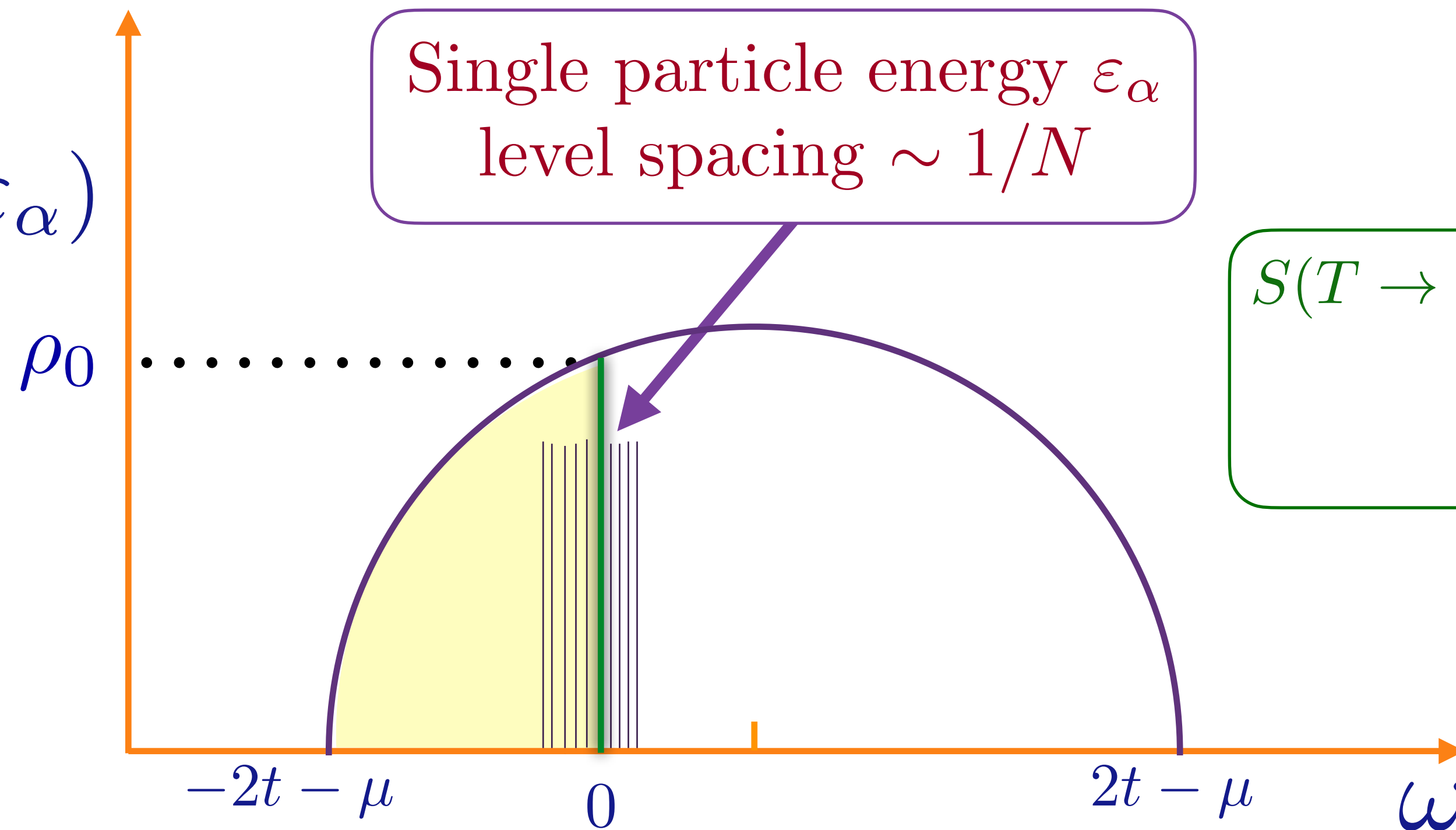
$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

$$\rho(\omega) = \frac{1}{N} \sum_{\alpha} \delta(\omega - \varepsilon_{\alpha})$$

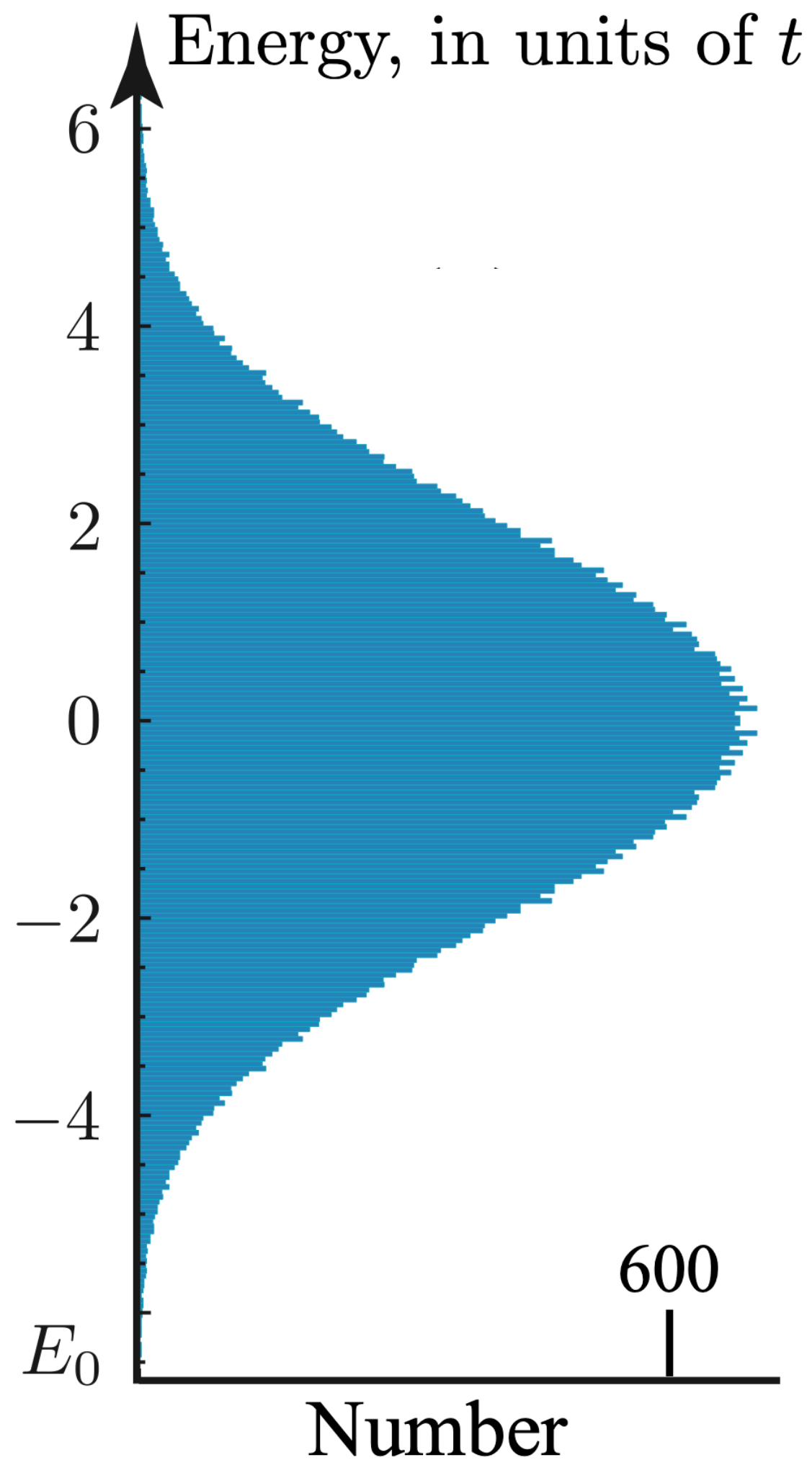


$$S(T \rightarrow 0) = N\gamma T$$

$$\gamma = \frac{\pi^2}{3} \rho_0$$

Random matrix model

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



For random matrix model:

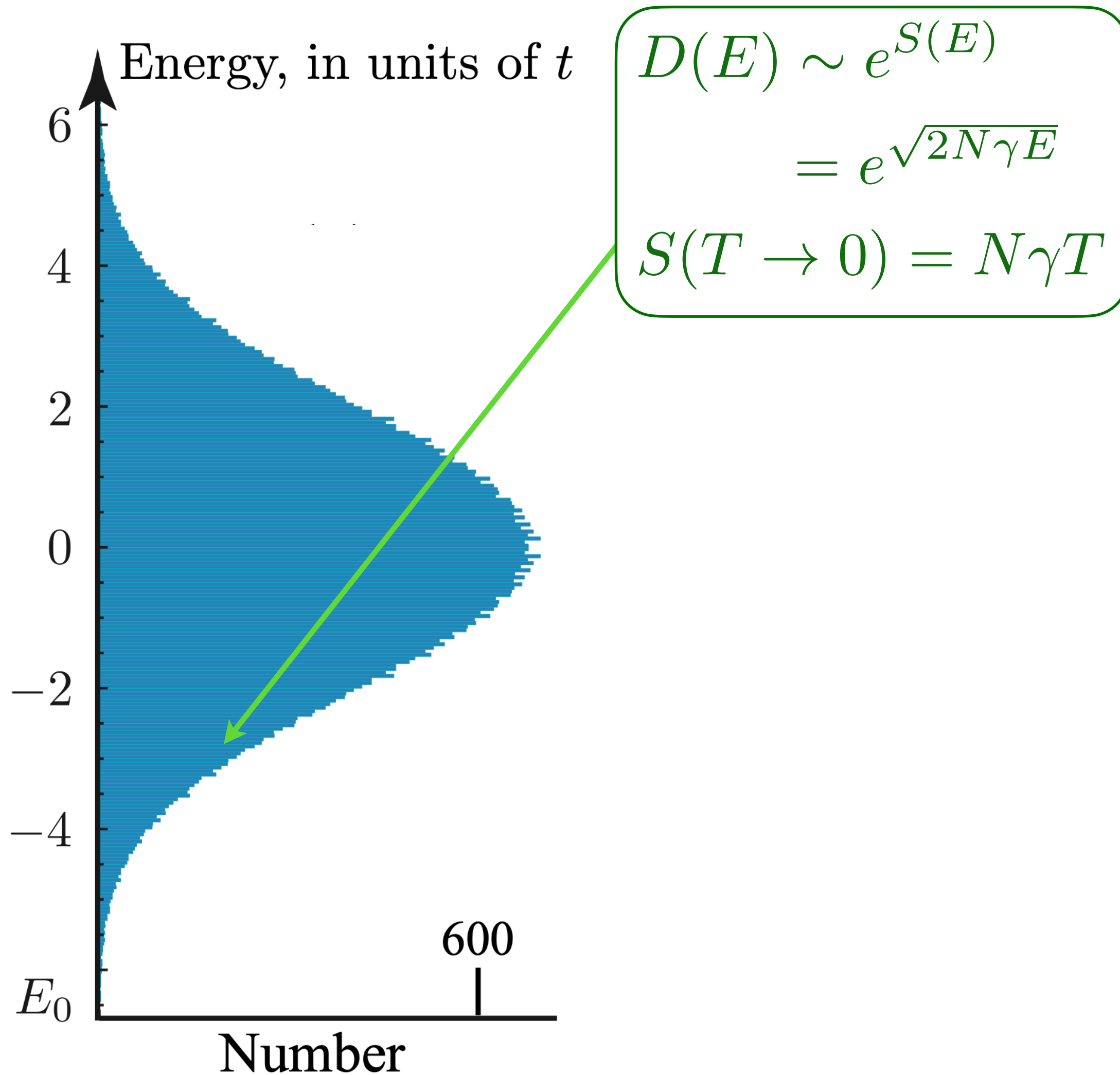
$$E_0 + E_i = \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha}$$

$n_{\alpha} = 0, 1,$
occupation number

Many-body density of states

Random matrix model

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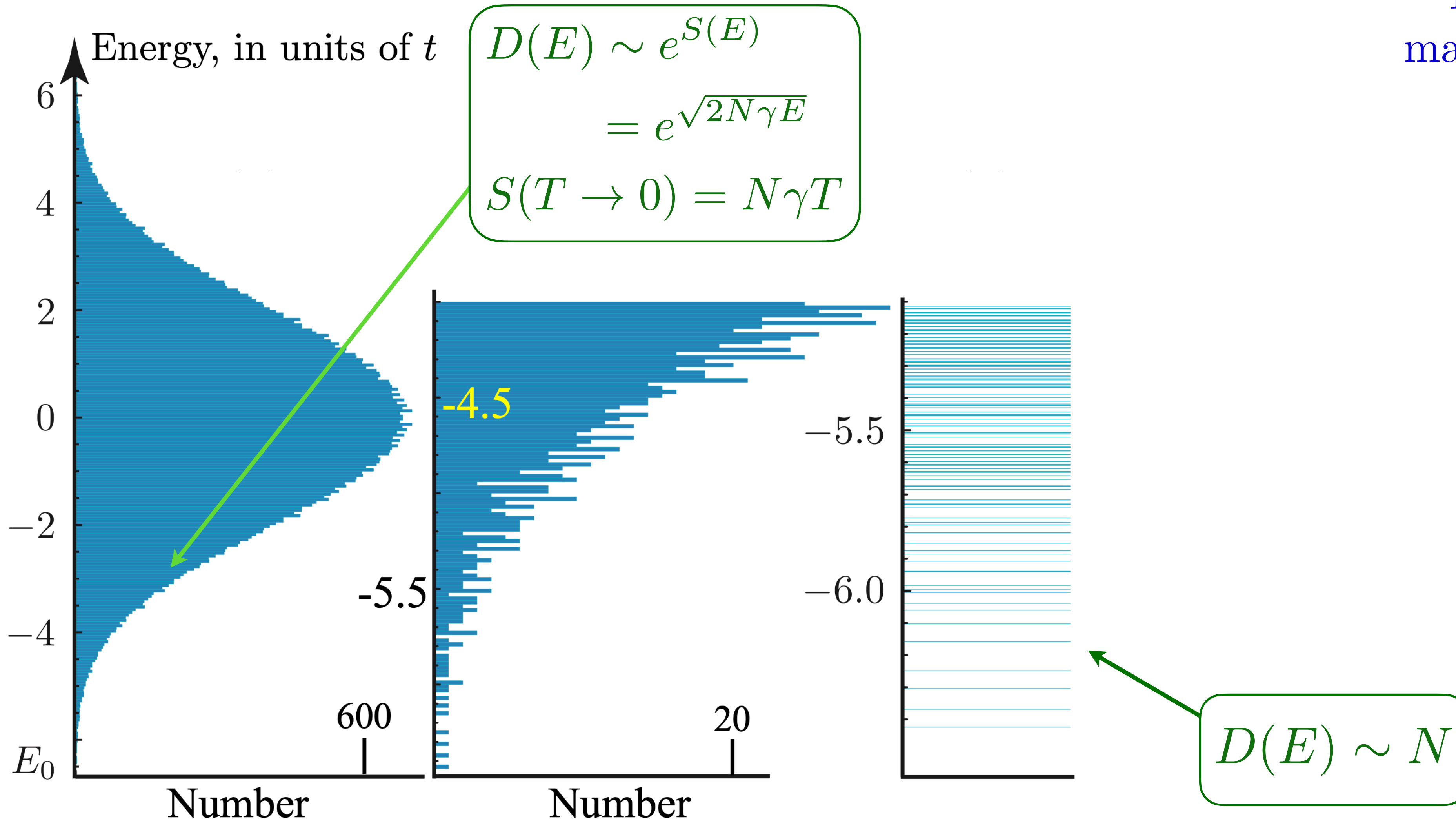
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Many-body density of states

The Sachdev-Ye-Kitaev (SYK) model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

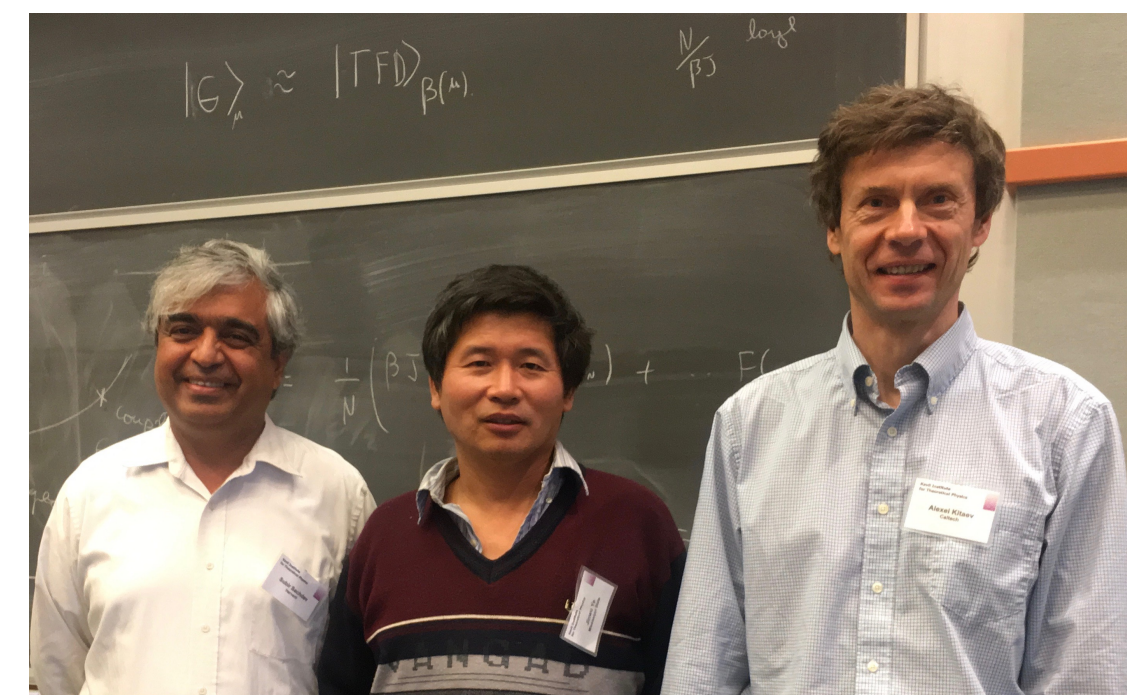
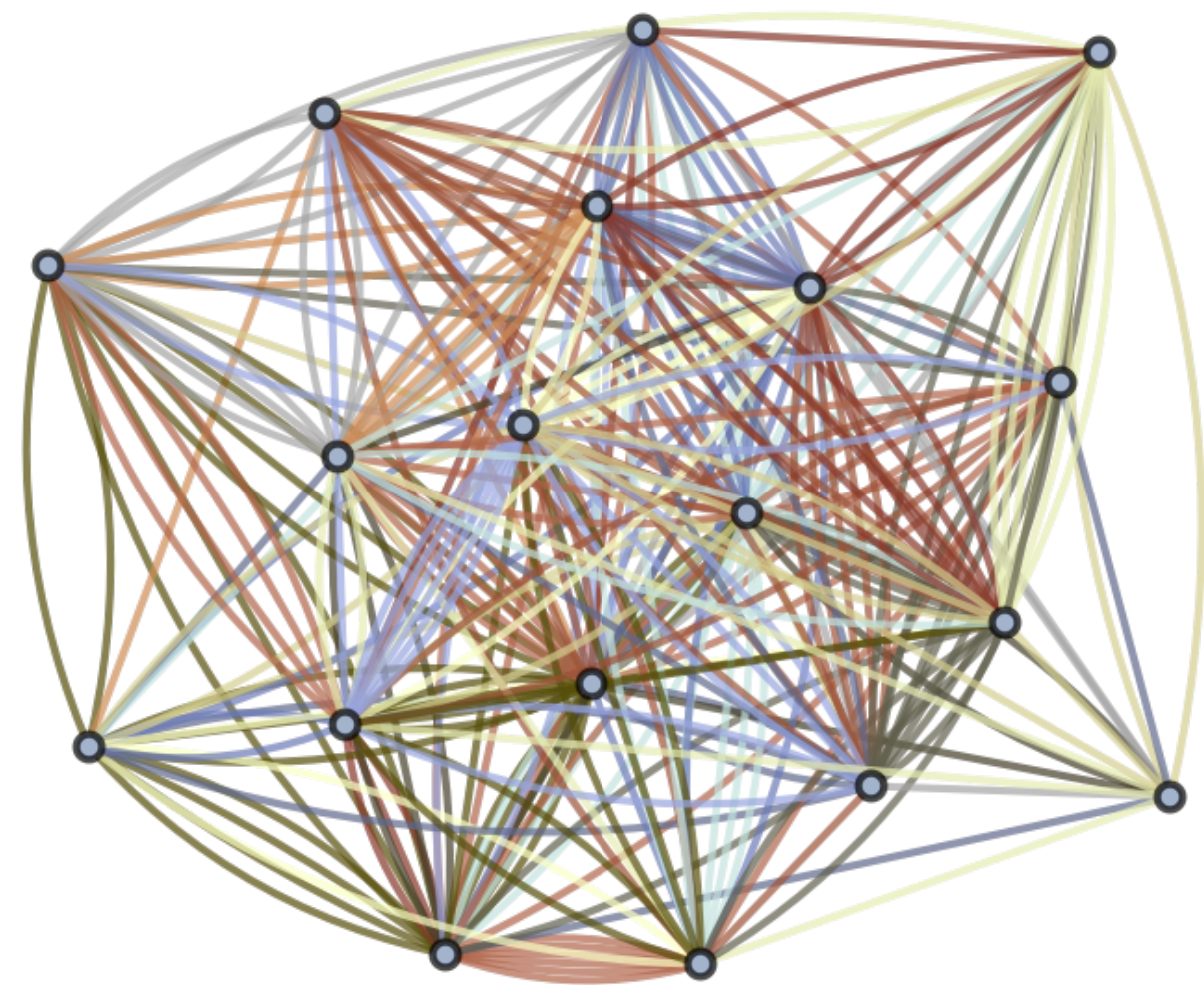
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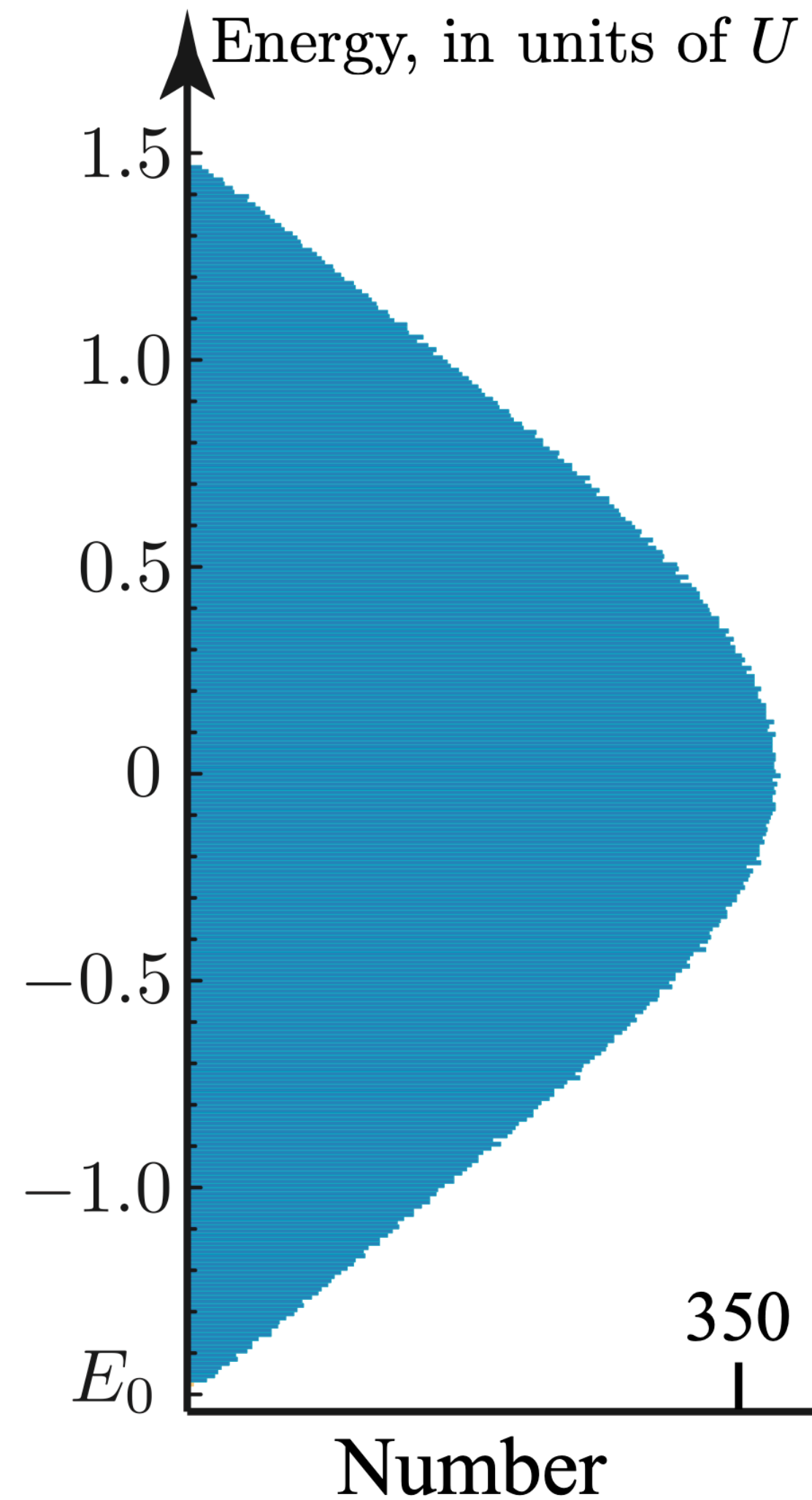
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Complex SYK model

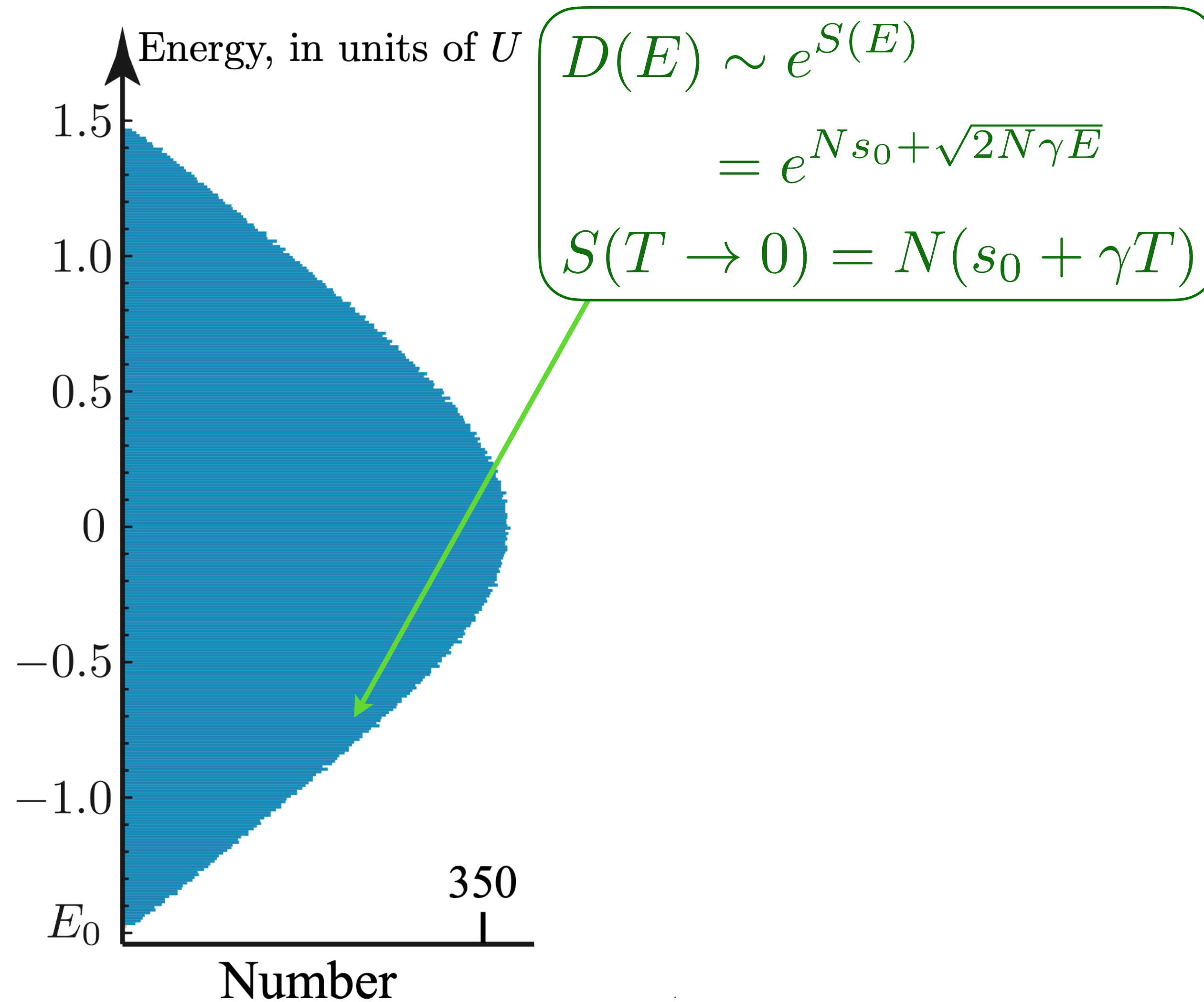
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Many-body density of states

Complex SYK model

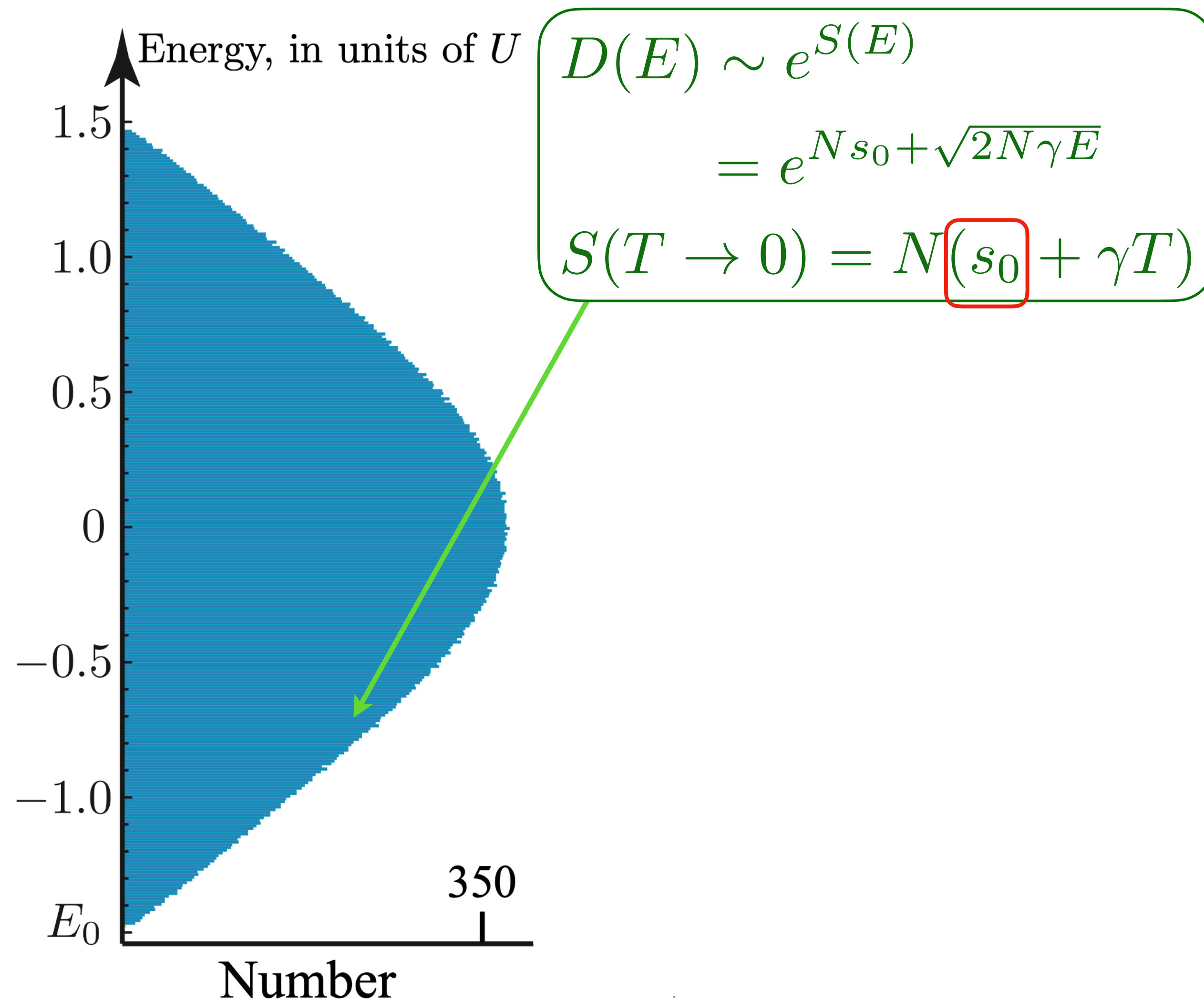
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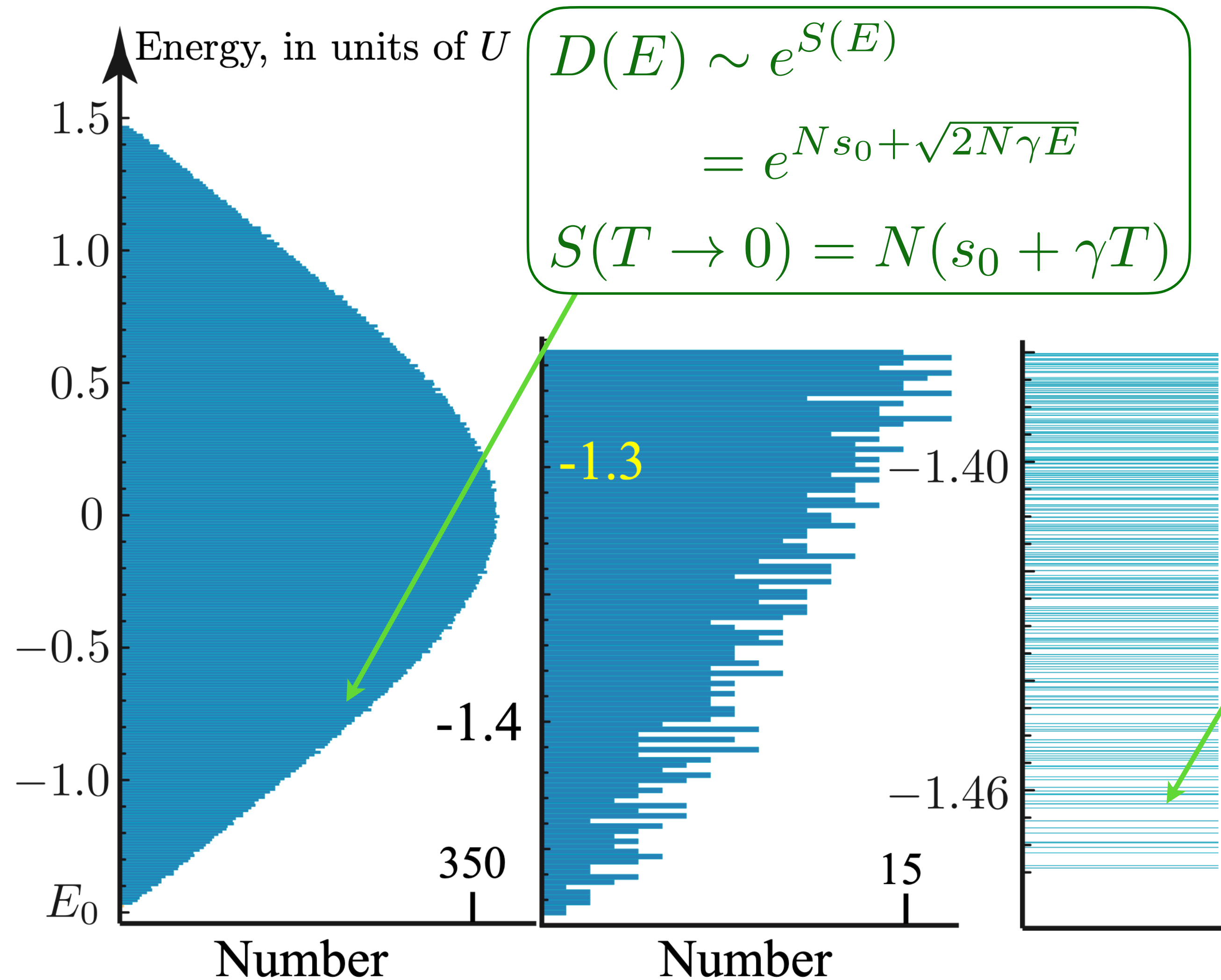
$$s_0 = 0.464848 \dots$$

A. Georges, O. Parcollet, and
S. Sachdev,
PRB **63**, 134406 (2001)

Many-body density of states

Complex SYK model

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2 e^{N s_0} \sqrt{2N\gamma E}$$

No quasiparticle decomposition:
wavefunctions change chaotically
from one state to the next.

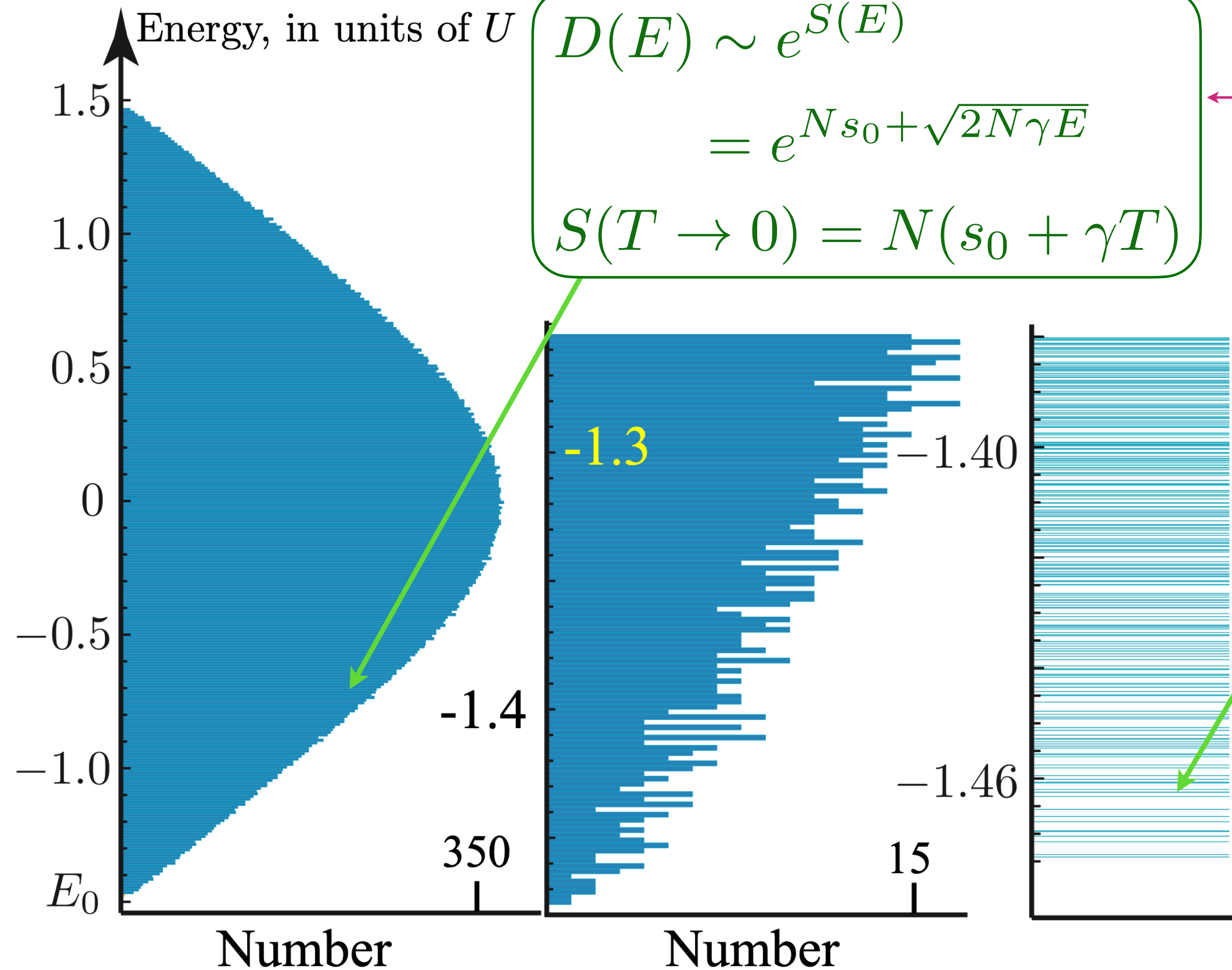
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$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2 e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$S(T) = N(s_0 + \gamma T) - \frac{3}{2} \ln \left(\frac{U}{T} \right)$$

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No quasiparticle decomposition:
wavefunctions change chaotically
from one state to the next.

$$s_0 = 0.464848 \dots$$

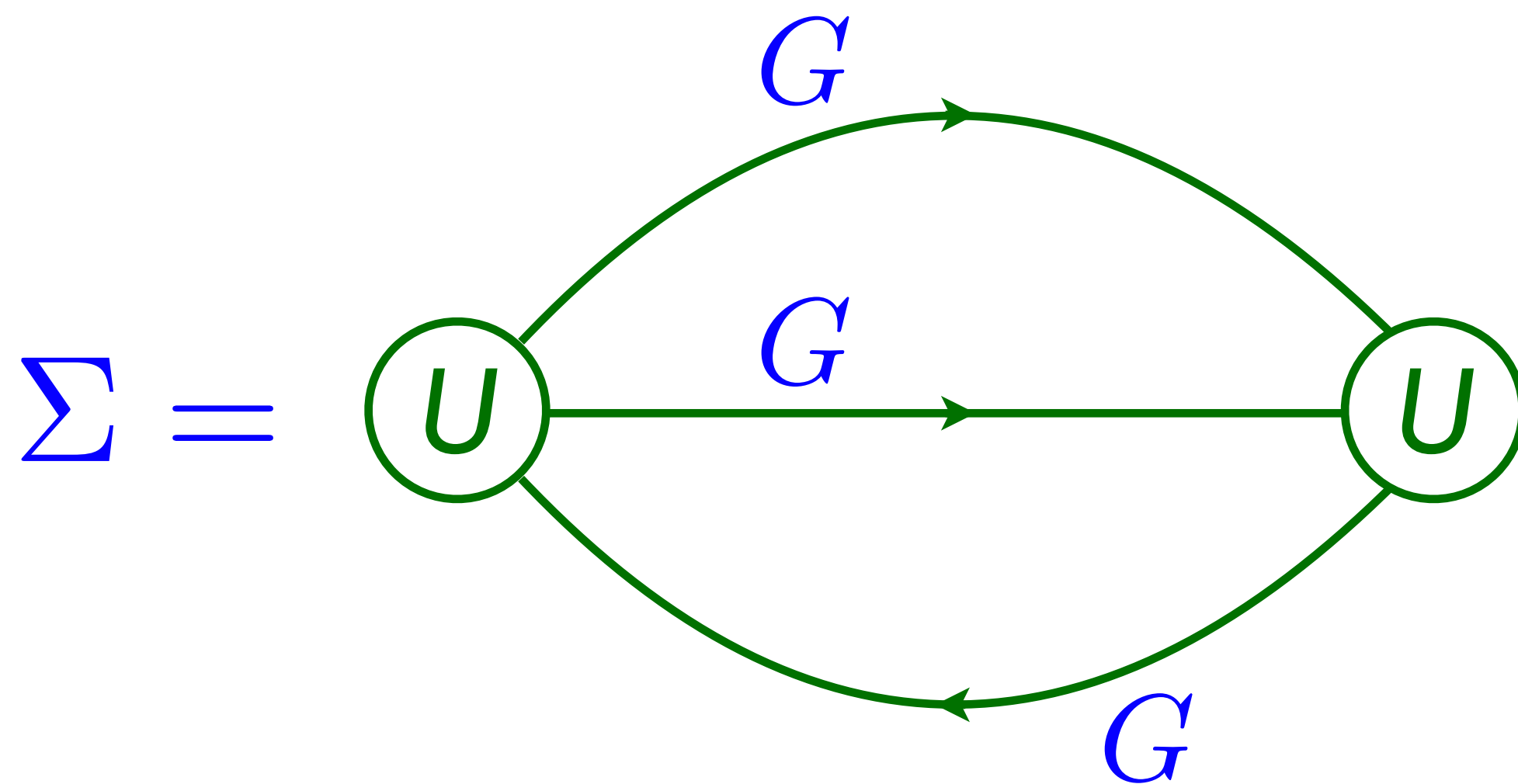
A. Georges, O. Parcollet, and
S. Sachdev,
PRB **63**, 134406 (2001)

Many-body density of states

Complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$



Conformal solution at $\mu = 0$, $G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}}$.

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

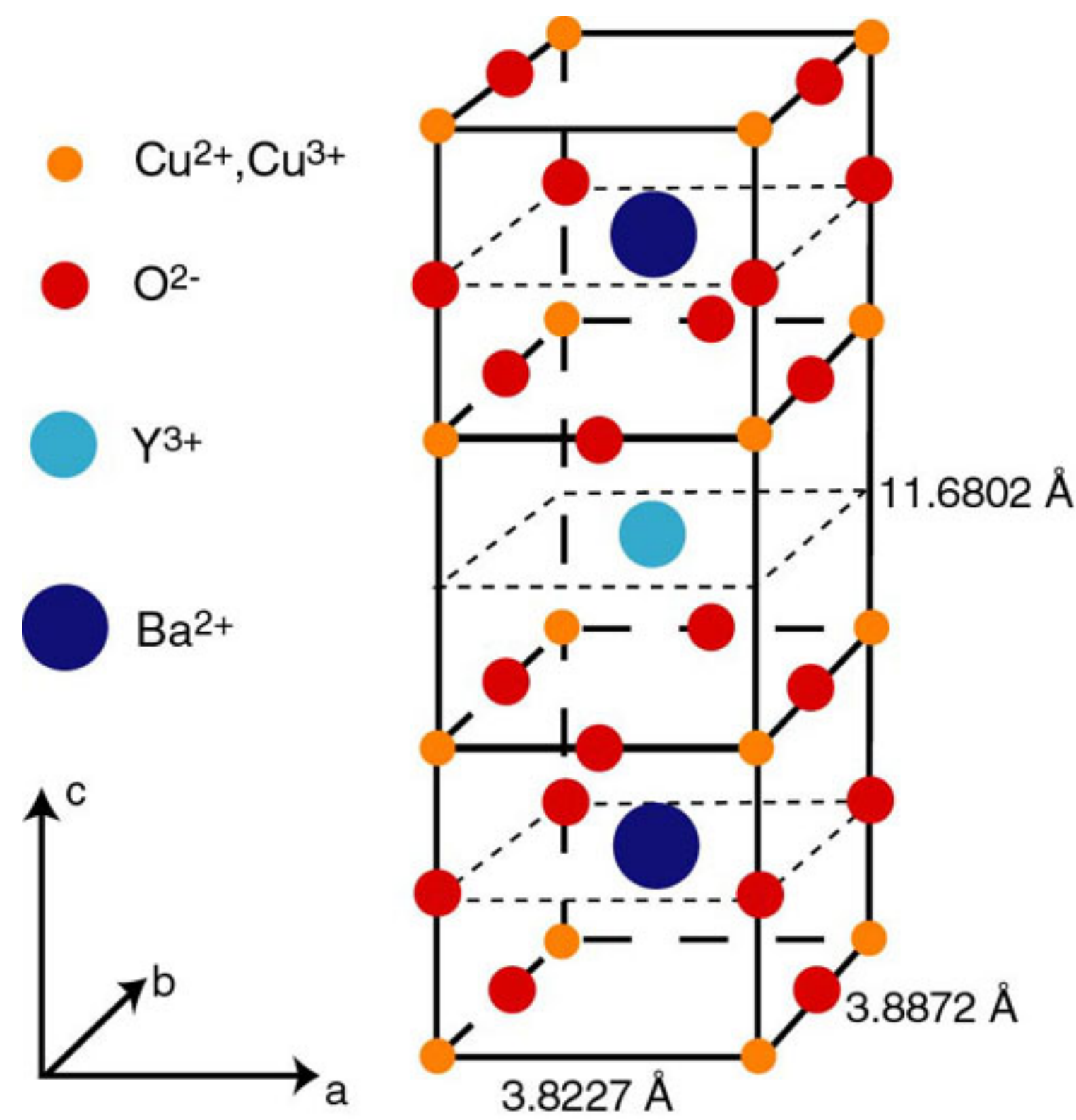
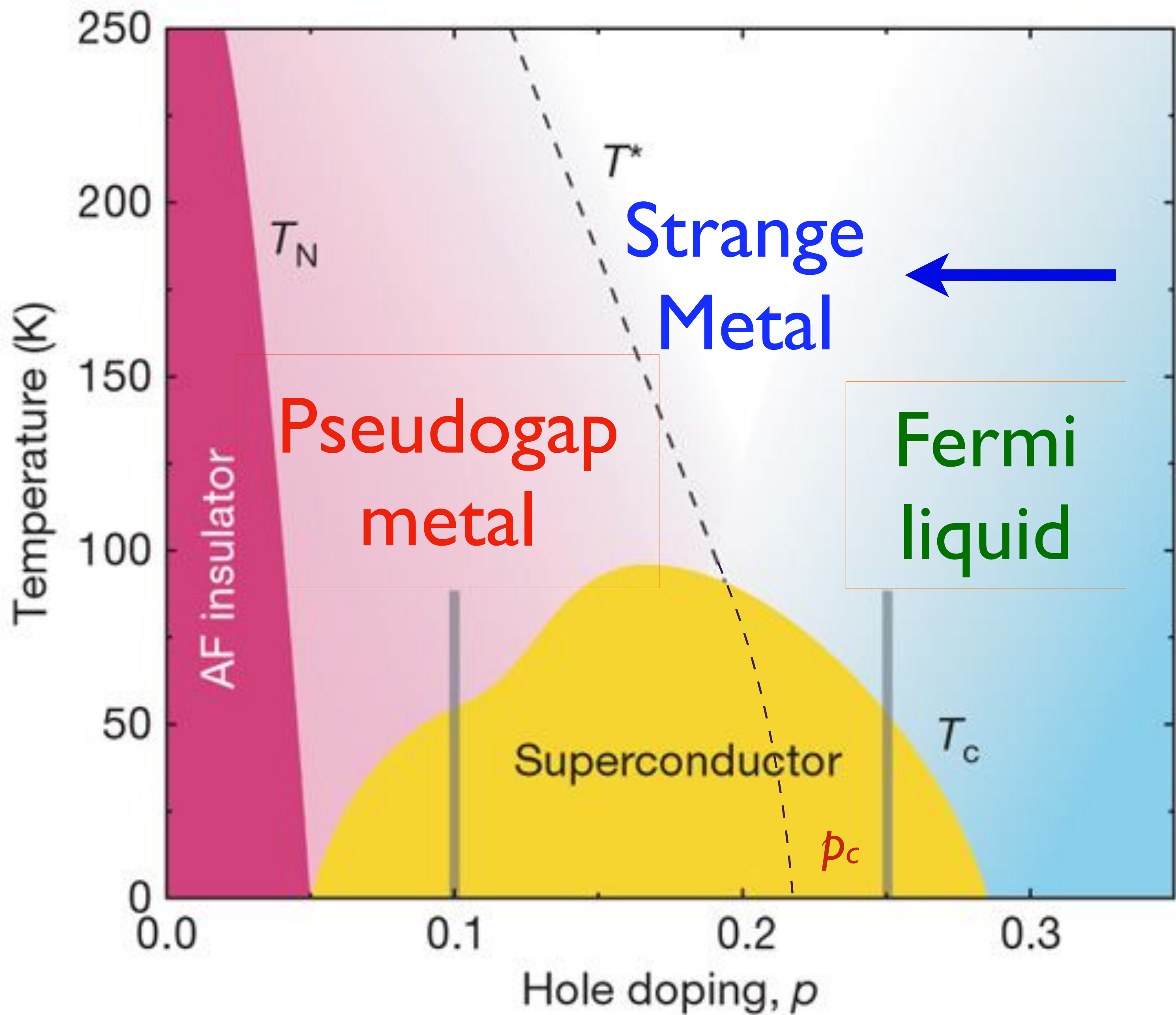


The Sachdev-Ye-Kitaev (SYK) model

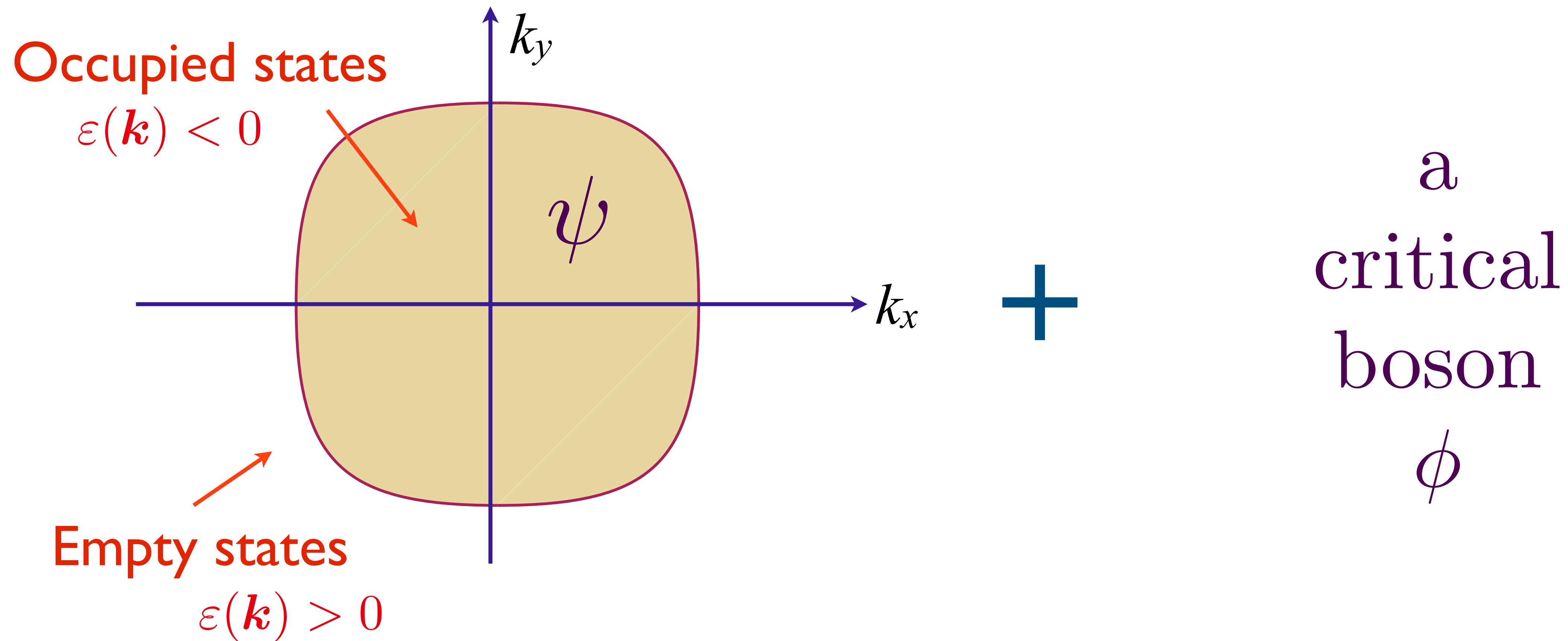
Universal Planckian time dynamics

- Green's function has Planckian time scaling $G(\omega, T) \sim \omega^{-1/2} F(\hbar\omega/k_B T)$.
- Leading (dangerously) irrelevant operator is a time reparameterization soft mode $\tau \rightarrow f(\tau)$.
- The effective action for $f(\tau)$ is the quantum theory of a charged black hole in Einstein-Maxwell theory in 3+1 spacetime dimensions at low T !
This leads to the $\sim \sinh(\sqrt{2N\gamma E})$ density of states.

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Fermi surface coupled to a critical boson



Fermi surface coupled to a critical boson

“Yukawa” coupling: $g \int d^2r d\tau \psi^\dagger(r, \tau) \psi(r, \tau) \phi(r, \tau)$

Yields a state without quasiparticle excitations, but the theory is not systematic at large N

Sung-Sik Lee (2009)

Fermi surface coupled to a critical boson

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$$\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Main idea:

Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

Ilya Esterlis, J. Schmalian, PRB **100**, 115132 (2019)

Yuxuan Wang and A.V. Chubukov, PRR **2**, 033084 (2020)

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

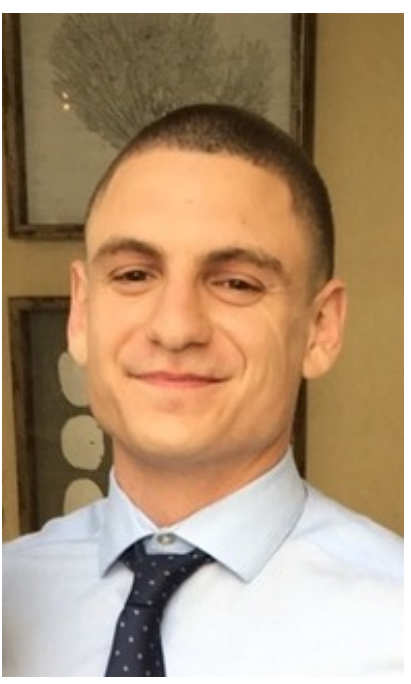
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$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

- Yields a systematic large N theory of a “non-Fermi liquid”: a compressible state without quasiparticle excitations. The Fermi surface is sharp in momentum space, but the spectral functions are diffuse in frequency space.
- There is Planckian dynamics at the Fermi surface:

$$G(k = k_F, \omega, T) \sim \omega^{-2/3} F(\hbar\omega/k_B T)$$

- There is many-body quantum chaos in the out-of-time-order correlator (OTOC) with maximal Lyapunov exponent $\lambda_L = 2\pi k_B T / \hbar$.



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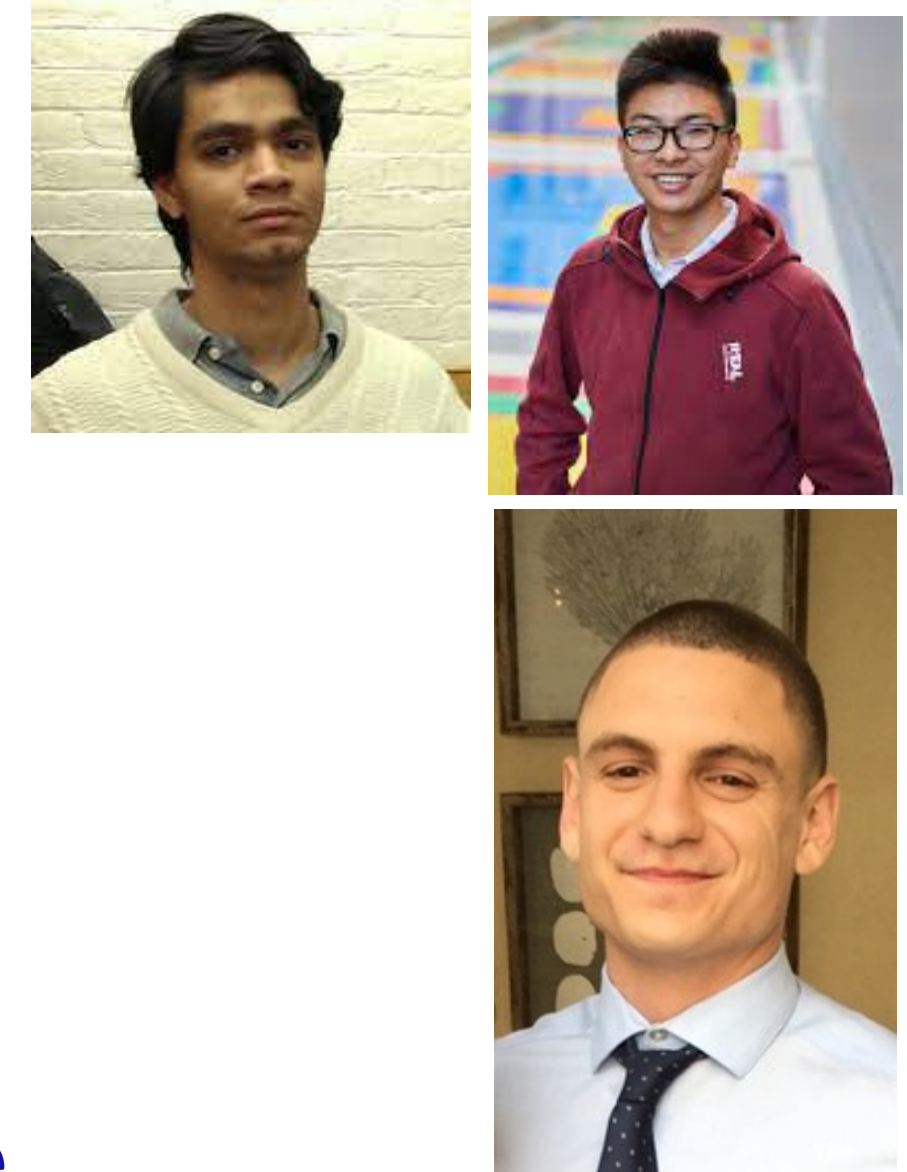
Conservation of momentum implies the d.c. conductivity is infinite

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. S., PRB **76**, 144502 (2007)

$$\text{Re } \sigma(\omega) = D\delta(\omega) + \text{Re } \sigma_{\text{reg}}(\omega)$$

$$\text{Re } \sigma_{\text{reg}}(\omega, T = 0) \sim \frac{1}{\omega^{2/3}}$$

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994)



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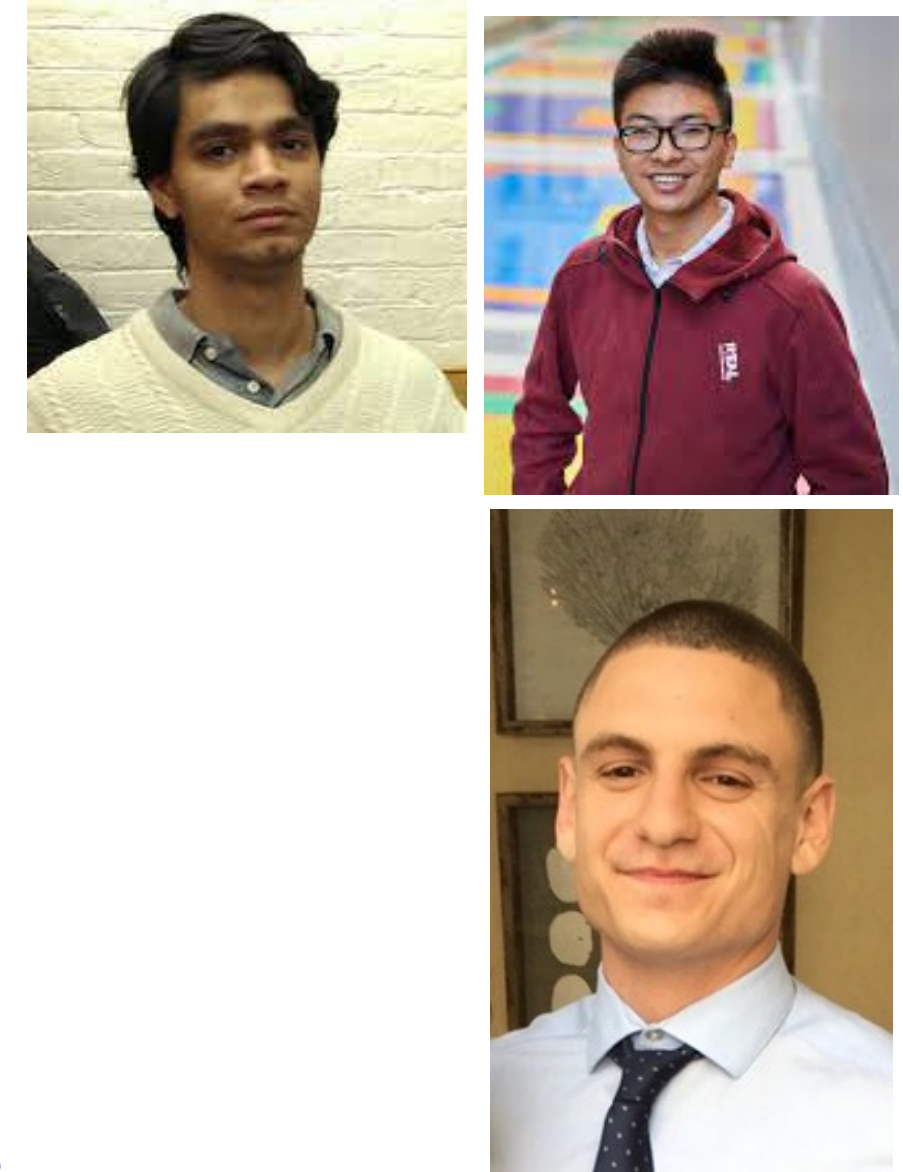
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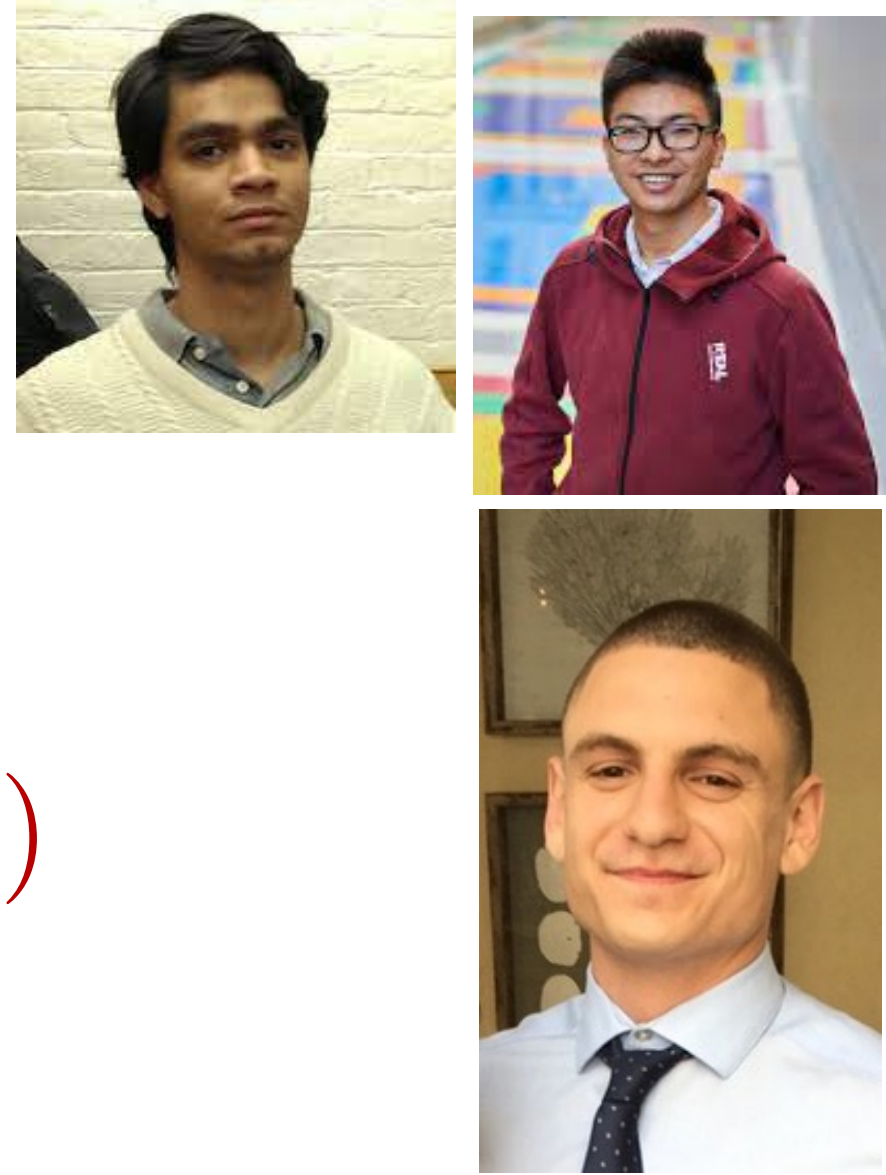
Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994)

Have to include the effects of spatial disorder or umklapp

Aavishkar Patel, Haoyu Guo, Haoyu Guo, Ilya Esterlis, S.S. to appear



Fermi surface coupled to a critical boson



“Yukawa” coupling: $\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$

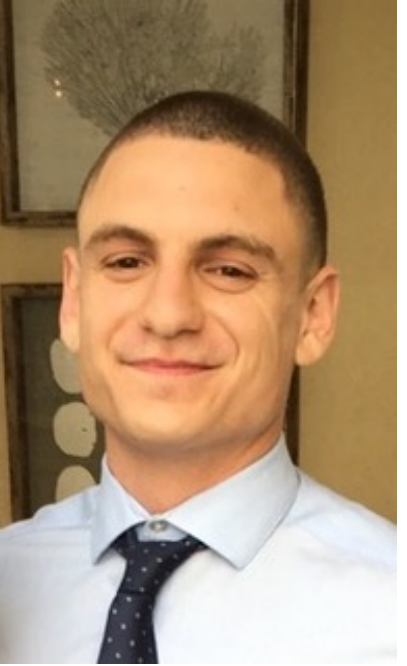
Random potential: $+ \frac{1}{\sqrt{N}} \int d^2r d\tau v_{ij}(r) \psi_i^\dagger(r, \tau) \psi_j(r, \tau)$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g_{ijl}^* g_{abc}} = g^2 \delta_{ia} \delta_{jb} \delta_{lc} \quad , \quad \overline{v_{ij}(r)} = 0 \quad , \quad \overline{v_{ij}^*(r) v_{lm}(r')} = v^2 \delta(r - r') \delta_{il} \delta_{jm}$$

With g and v non-zero, we obtain a non-zero residual resistivity

$$\rho(T) = \rho(0) + AT^2 + \dots$$

Fermi surface coupled to a critical boson



“Yukawa” coupling: $\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$

Random potential: $+\frac{1}{\sqrt{N}} \int d^2r d\tau v_{ij}(r) \psi_i^\dagger(r, \tau) \psi_j(r, \tau)$

Random interactions: $+\frac{1}{N} \int d^2r d\tau g'_{ijl}(r) \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)$

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$$\overline{g'_{ijl}(r)} = 0 \quad , \quad \overline{g'_{ijl}^*(r) g'_{abc}(r')} = g'^2 \delta(r - r') \delta_{ia} \delta_{jb} \delta_{lc}$$

With g , v , and g' non-zero, we obtain a linear- T correction!

$$\rho(T) = \rho(0) + aT + \dots$$

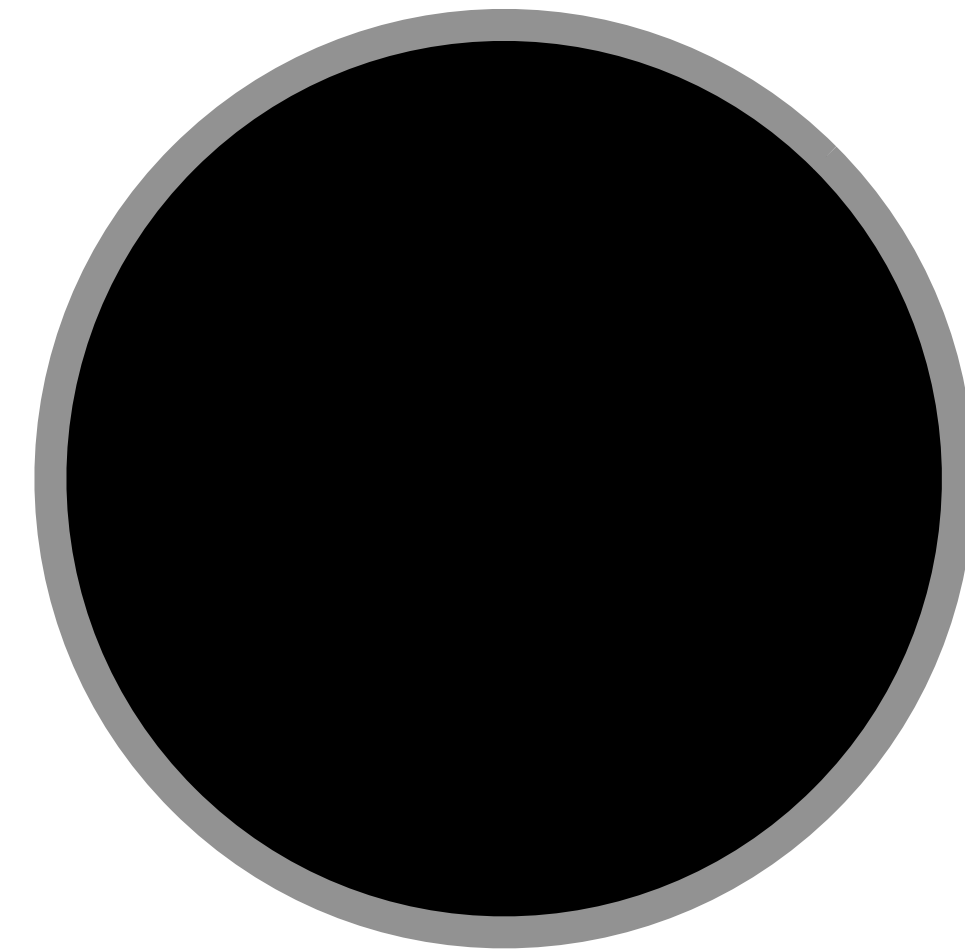
$\Rightarrow \rho(0)$ is determined by v ; a is determined by g' .

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Black Holes

Objects so dense that light is gravitationally bound to them.

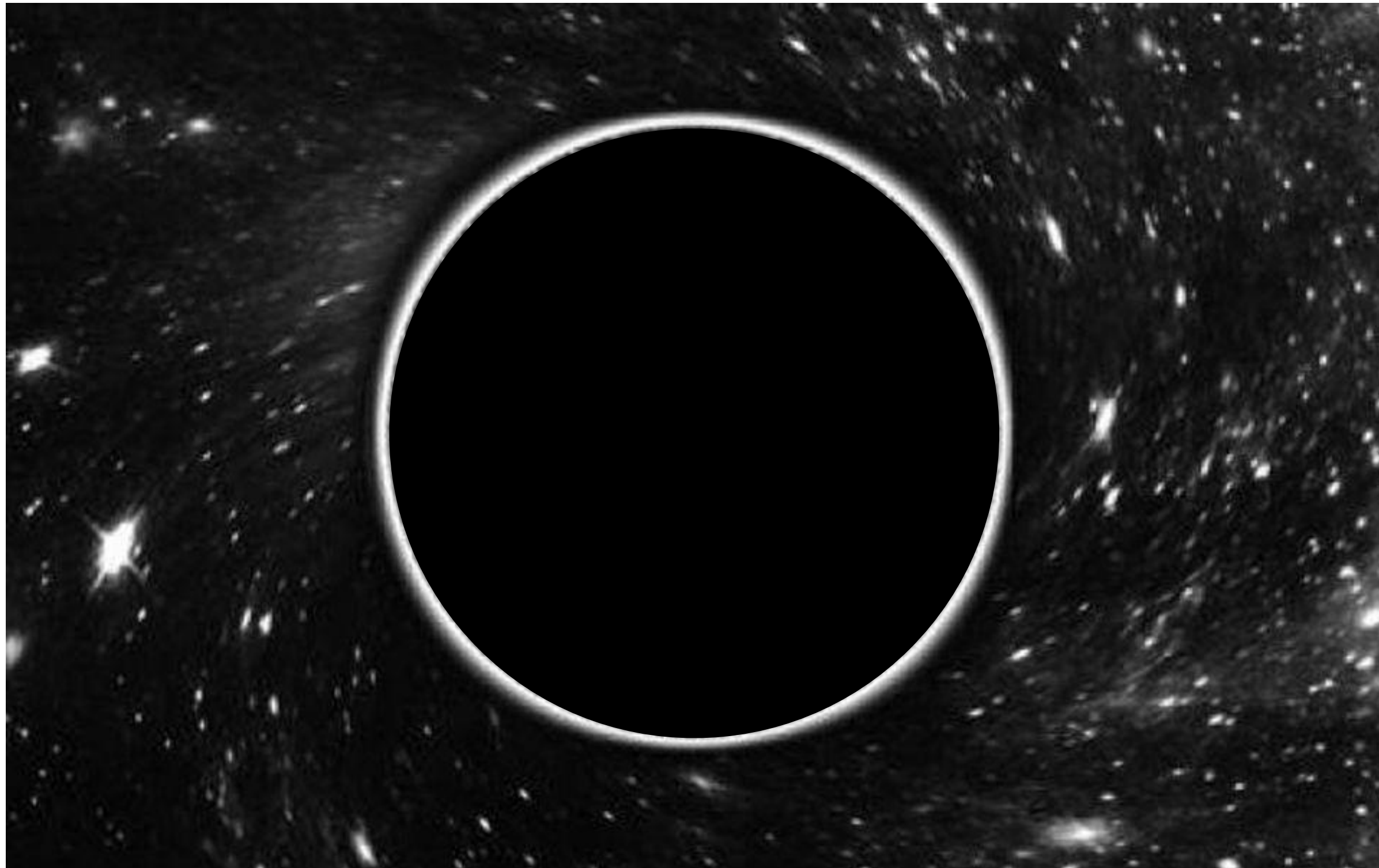
Horizon radius $R = \frac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole
For $M = \text{earth's mass}$, $R \approx 9 \text{ mm!}$

Quantum Black holes

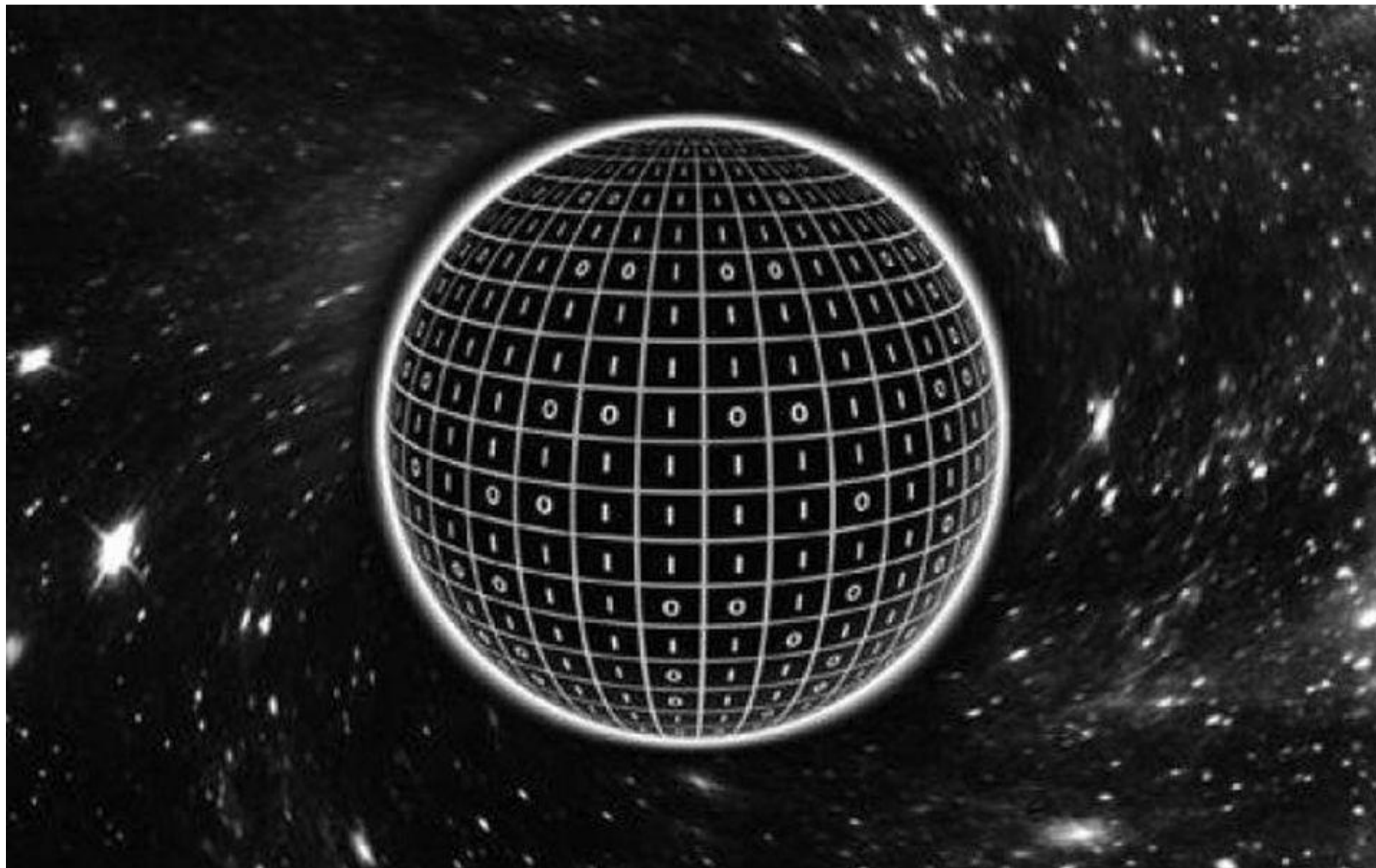
- Black holes have an entropy and a temperature,
 $T_H = \hbar c^3 / (8\pi G M k_B)$.
- The entropy is proportional to their surface area.
 $S = A k_B c^3 / (4G\hbar)$.



J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)

Quantum Black holes

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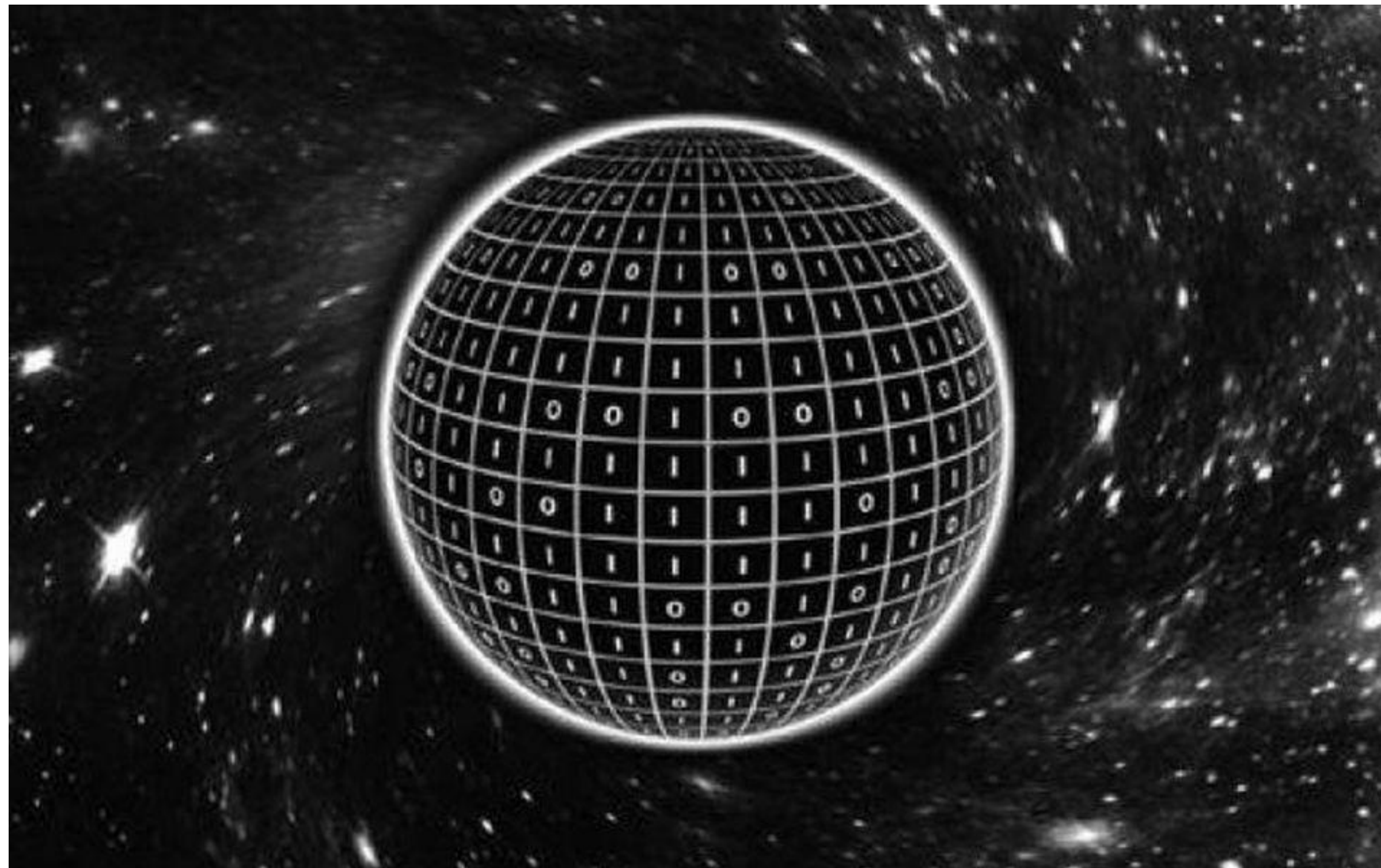
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Remarkable features:

- Entropy is finite.
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- They relax to thermal equilibrium in a time $\sim 8\pi G M / c^3$



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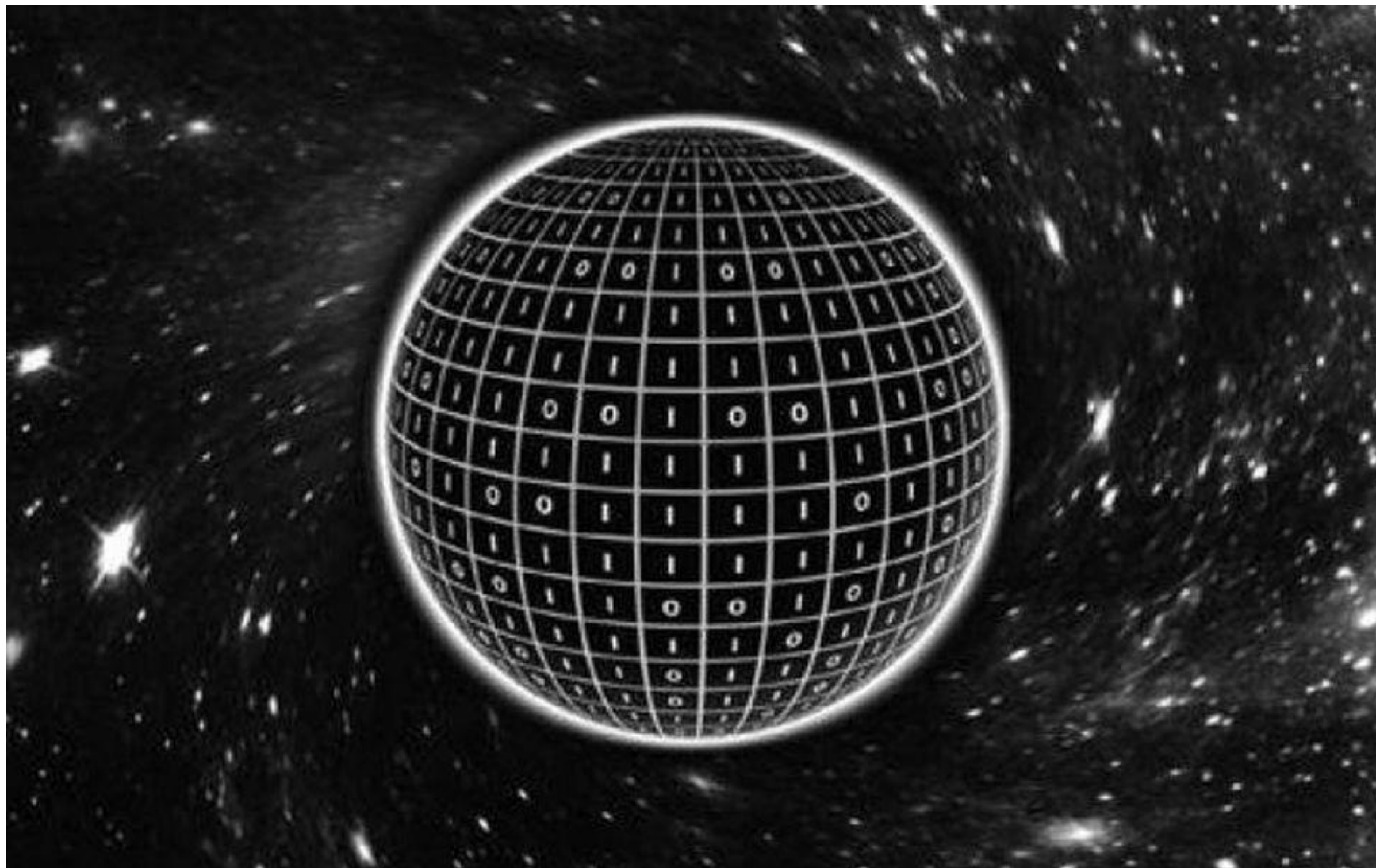
C.V. Vishveshwara, Nature **227**, 936 (1970)

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- The entropy is proportional to their surface area. $S = A k_B c^3 / (4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M / c^3 = \hbar / (k_B T_H)$ which is Planckian!



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S.W. Hawking, Nature **248**, 30 (1974)

C.V. Vishveshwara, Nature **227**, 936 (1970)

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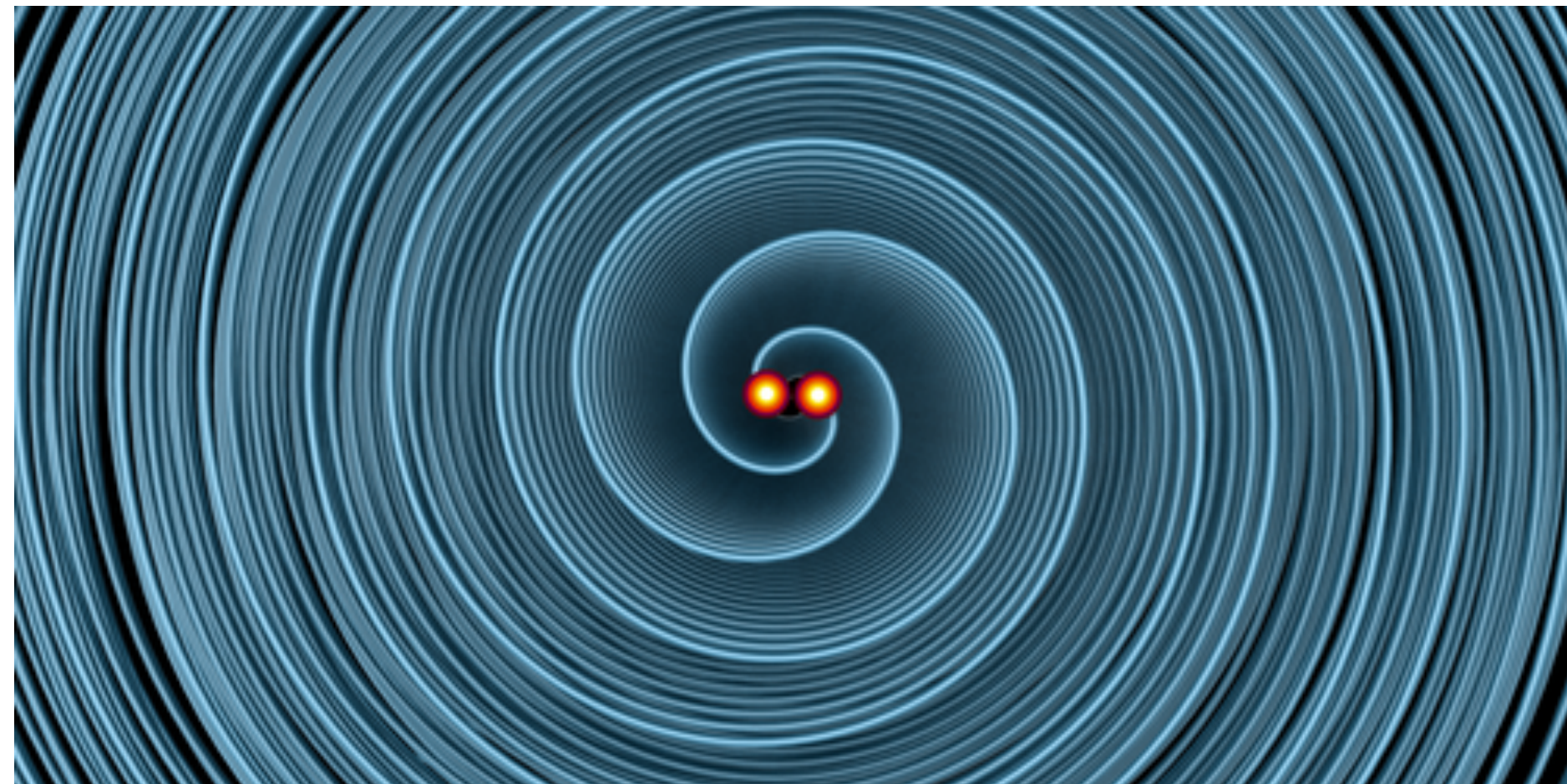
Black Holes Obey Information-Emission Limits

Limits

April 22, 2021 • *Physics 14, s47* –Christopher Crockett

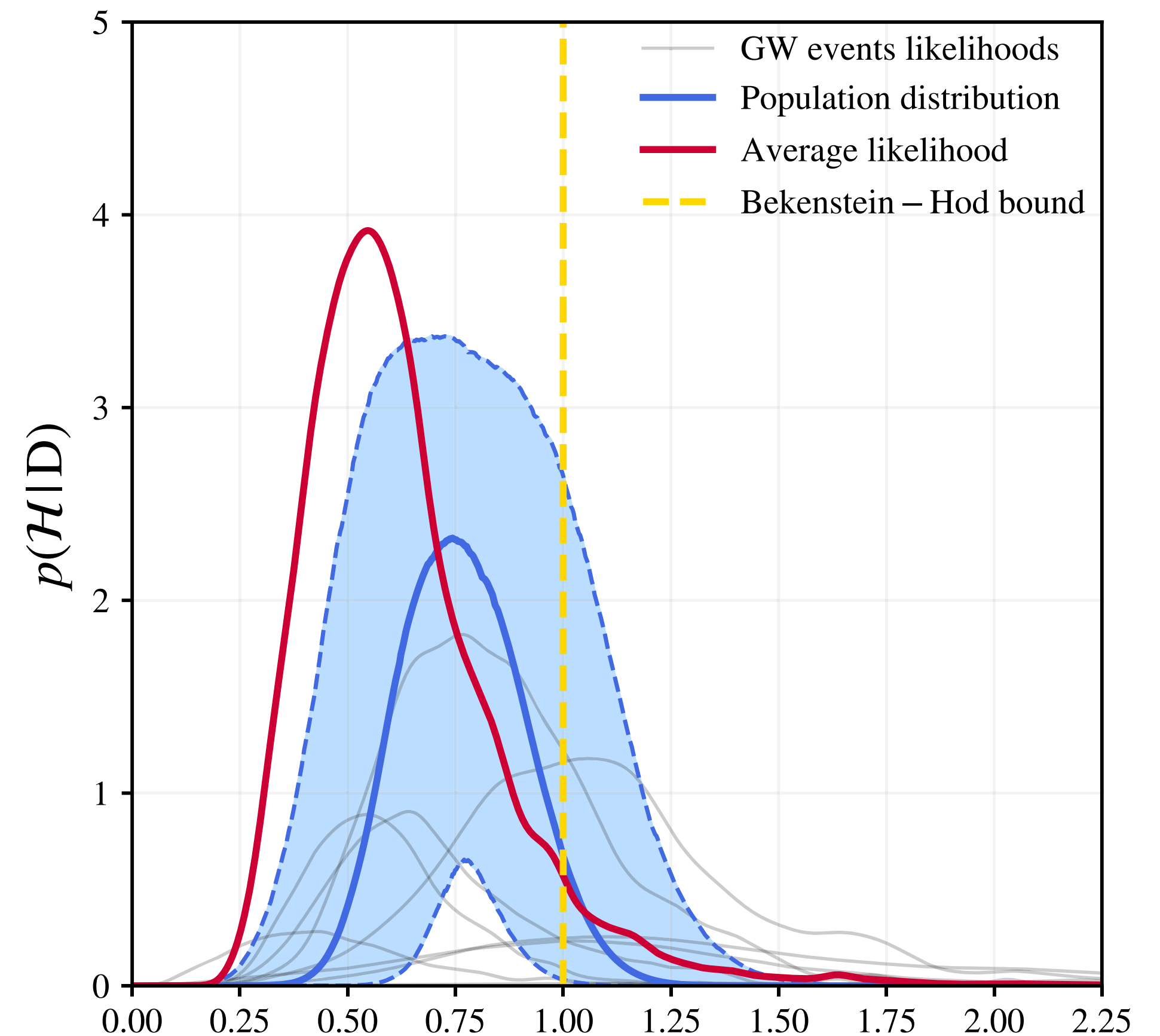
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, *Phys. Rev. Lett.* **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$



$$\mathcal{H} = \frac{1}{\pi} \frac{\hbar/\tau}{k_B T}$$

Thermodynamics of quantum black holes with charge Q :



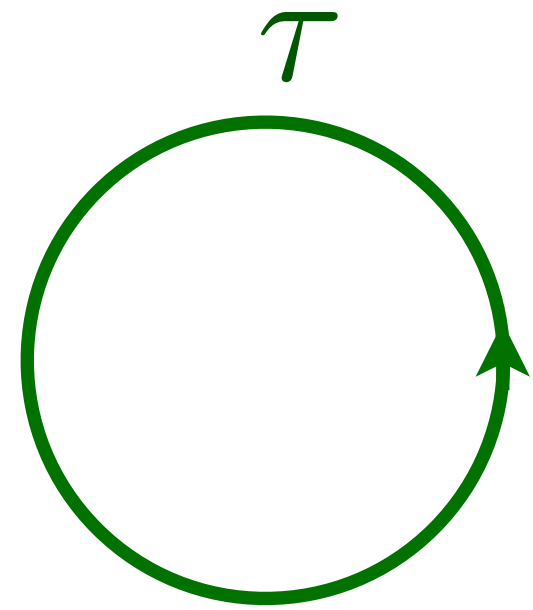
$$\int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$

Metric of
spacetime

Electromagnetic
gauge field

In general, this integral is not well defined, because of an uncontrollably large number of spacetime configurations.

Thermodynamics of quantum black holes with charge Q :



Imaginary
time circle
of length
 $\hbar/(k_B T)$

$$\int \mathcal{D}g_{\mu\nu} \mathcal{D}A_\mu \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_\mu] \right)$$

$$= \exp(S_{BH}) \times \left(\dots????\dots \right)$$

Gibbons, Hawking (1977)

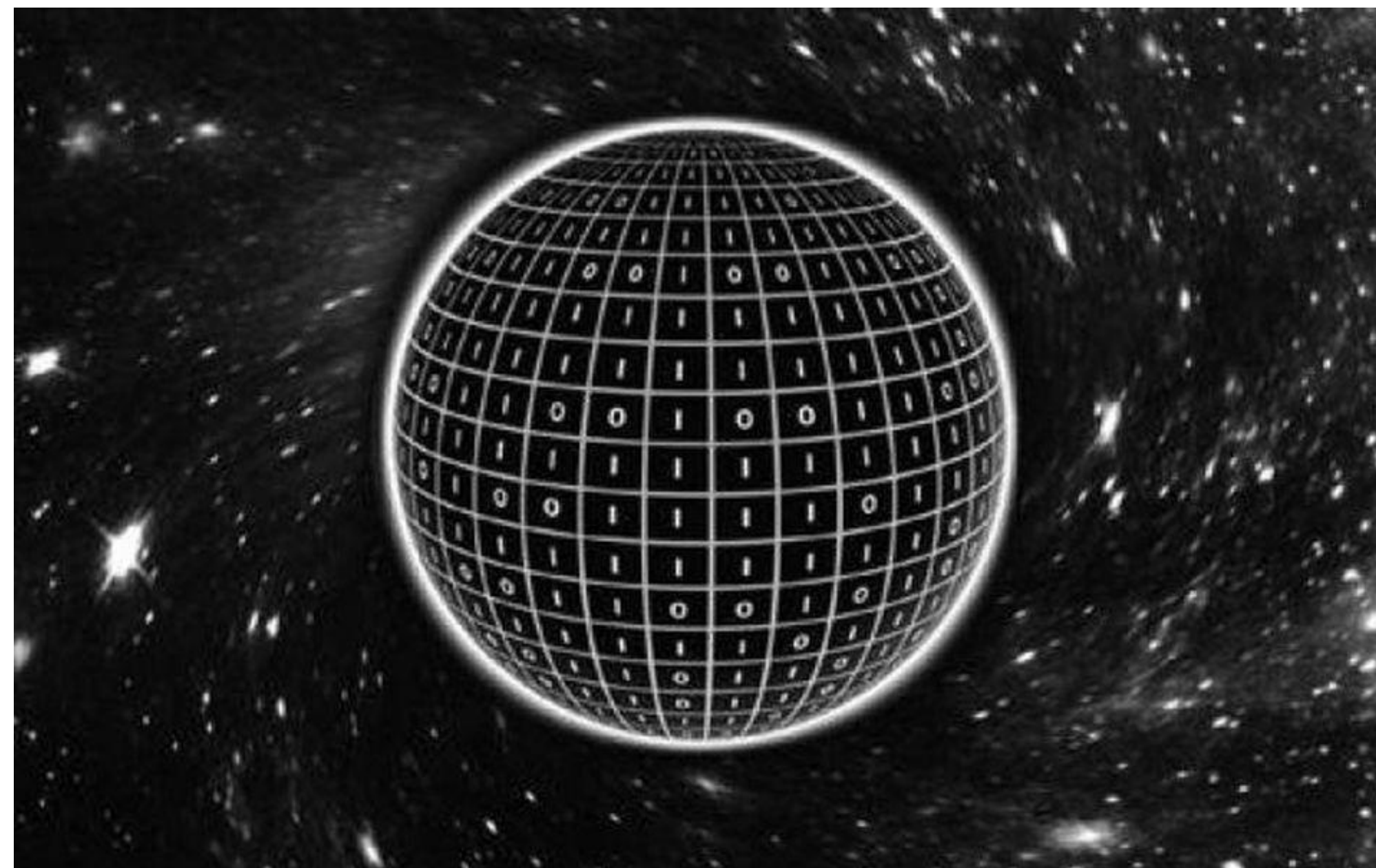
Chamblin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c} \right)$$

A_0 is the area of the charged black hole horizon at $T = 0$.
 Q is the black hole charge. A_0 is a function of Q .

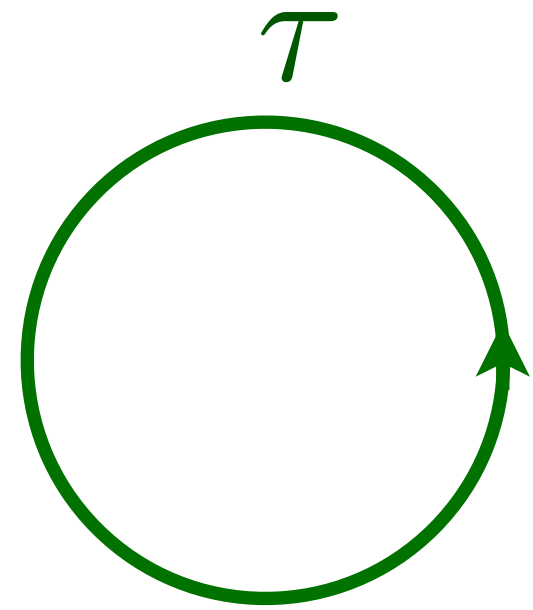
Questions

- Is Einstein-Maxwell theory meaningful beyond the saddle point, and can we compute quantum fluctuation corrections to S_{BH} ?
- Can the resulting entropy be understood as that of a unitary quantum system with a discrete spectrum ?
- Can we compute the evolution of the entropy as the black hole evaporates? Is it that of an evaporating unitary quantum system?



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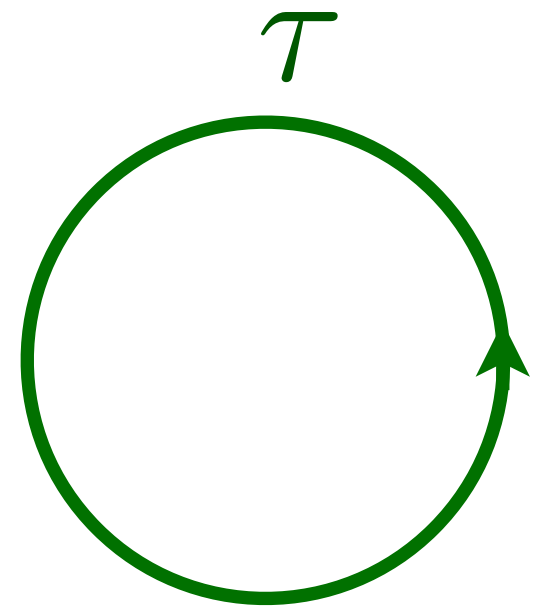
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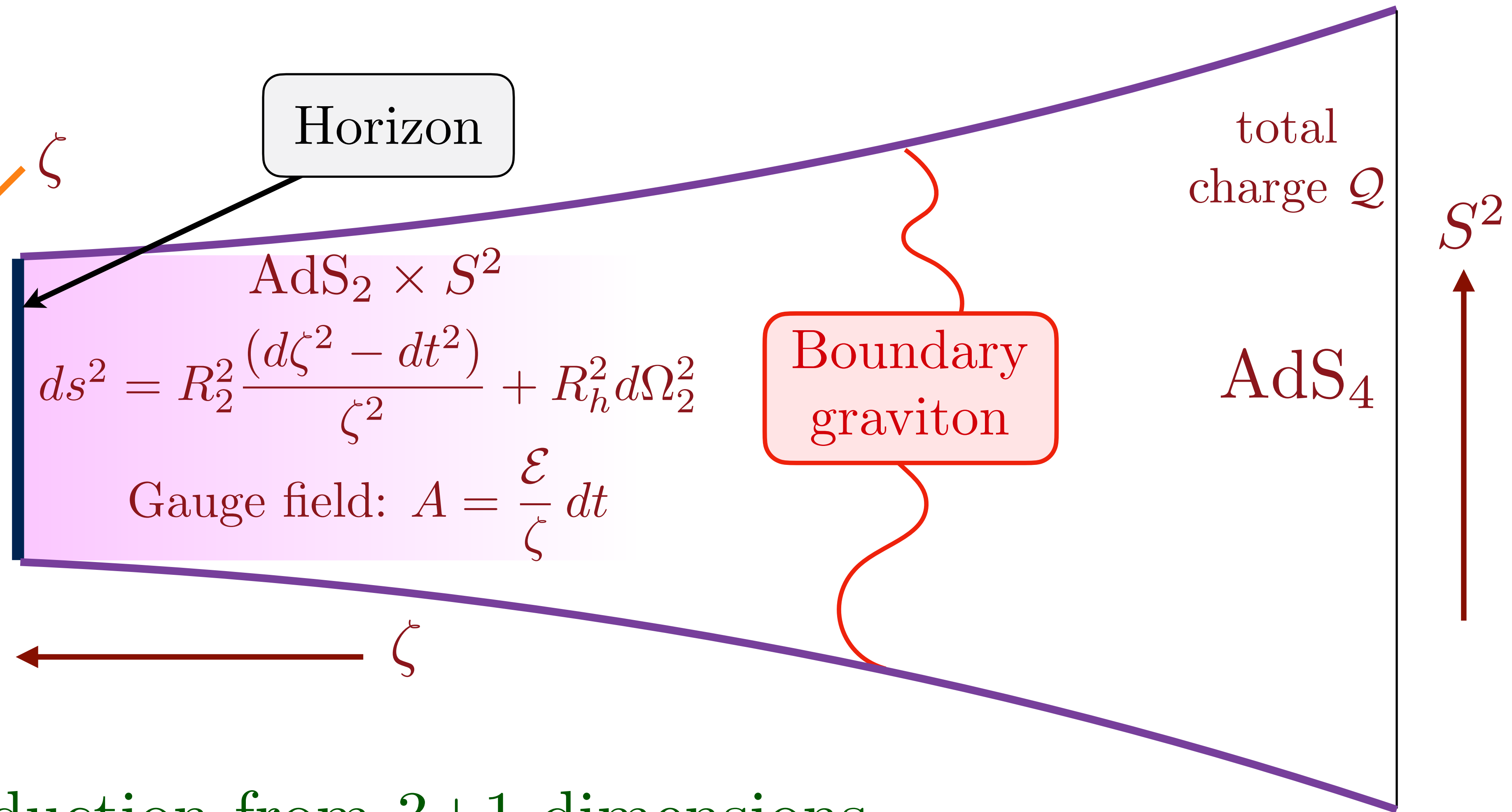
Q is the black hole charge. A_0 is a function of Q .

Note the similarity to the large N entropy of the SYK model

$S = N(s_0 + \gamma T) - (3/2) \ln(U/T) + \dots$ (along with other similarities, (S.S. PRL 2010)).

Will now argue that the $-(3/2) \ln(1/T)$ is also present for charged black holes.

Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Thermodynamics of quantum black holes with charge Q :



$$\int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right) \\ \approx \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{JT gravity of AdS}_2+\text{boundary graviton}}^{(1+1)}[g_{\mu\nu}, A_{\mu}] \right)$$

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$$= \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)] \right)$$

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Thermodynamics of quantum black holes with charge Q :



$$\begin{aligned}
 & \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right) \\
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 \end{aligned}$$

$$S(T \rightarrow 0, Q) = S_{BH} - \frac{3}{4} \ln \left(\frac{\hbar c^5}{GT^2} \right)$$

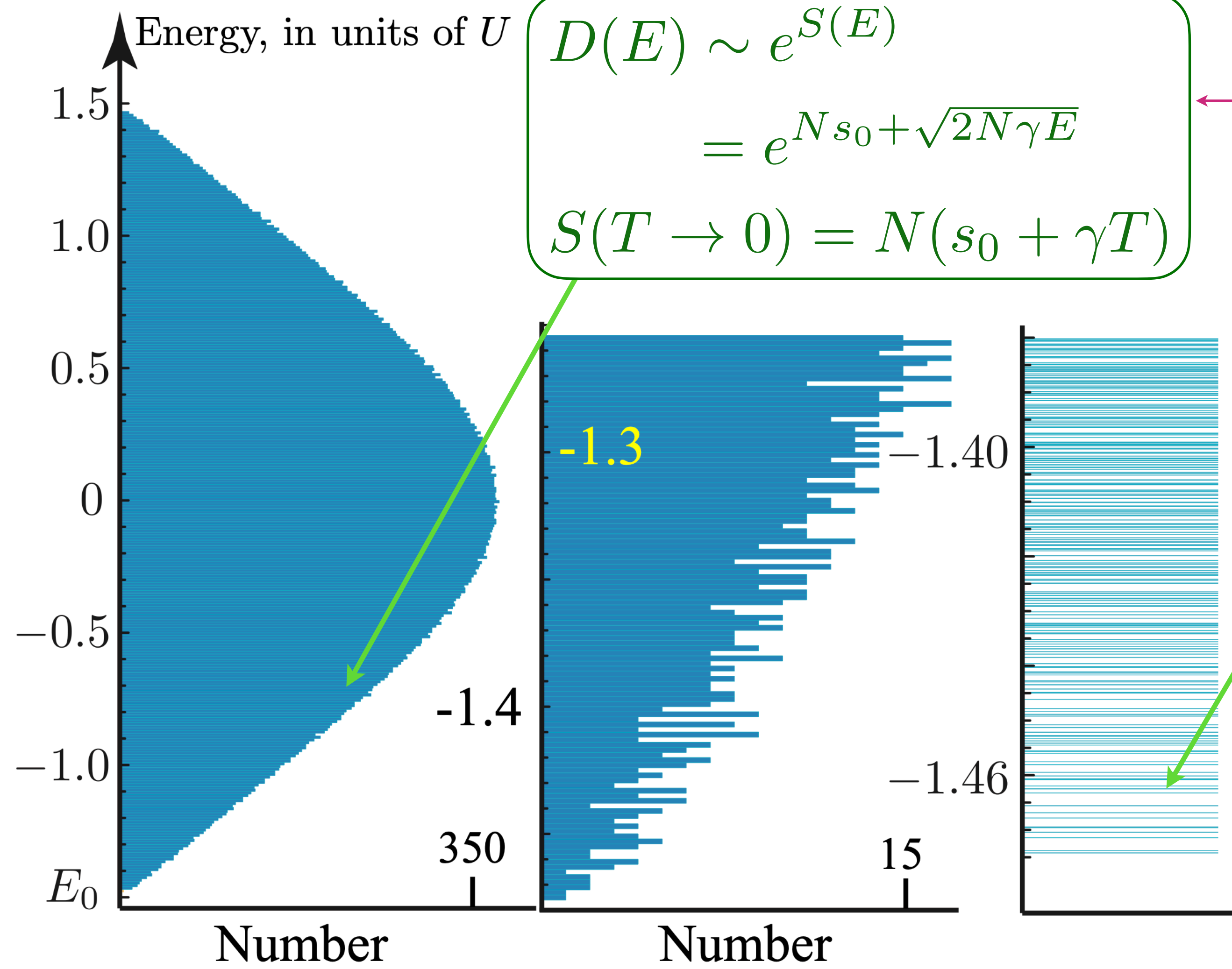
$$S_{BH} = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c} \right)$$

A_0 is the area of the charged black hole horizon at $T = 0$, Q is the black hole charge. The $\ln T$ term is the contribution of the boundary graviton.

(There is also a $-(241/45) \ln(A_0/G)$ correction at $T = 0$
A. Sen 2011)

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2 e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$S(T) = N(s_0 + \gamma T) - \frac{3}{2} \ln \left(\frac{U}{T} \right)$$

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wavefunctions change chaotically
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$$s_0 = 0.464848 \dots$$

A. Georges, O. Parcollet, and S. Sachdev,
PRB **63**, 134406 (2001)

Complex SYK model

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

Same entropy and (coarse-grained) density of states in a model of interacting (fermionic) qubits with a discrete spectrum!

Energy, in units of U

$$D(E) \sim e^{S(E)}$$

$$= e^{Ns_0 + \sqrt{2N\gamma E}}$$

$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2 e^{Ns_0} \sinh(\sqrt{2N\gamma E})$$

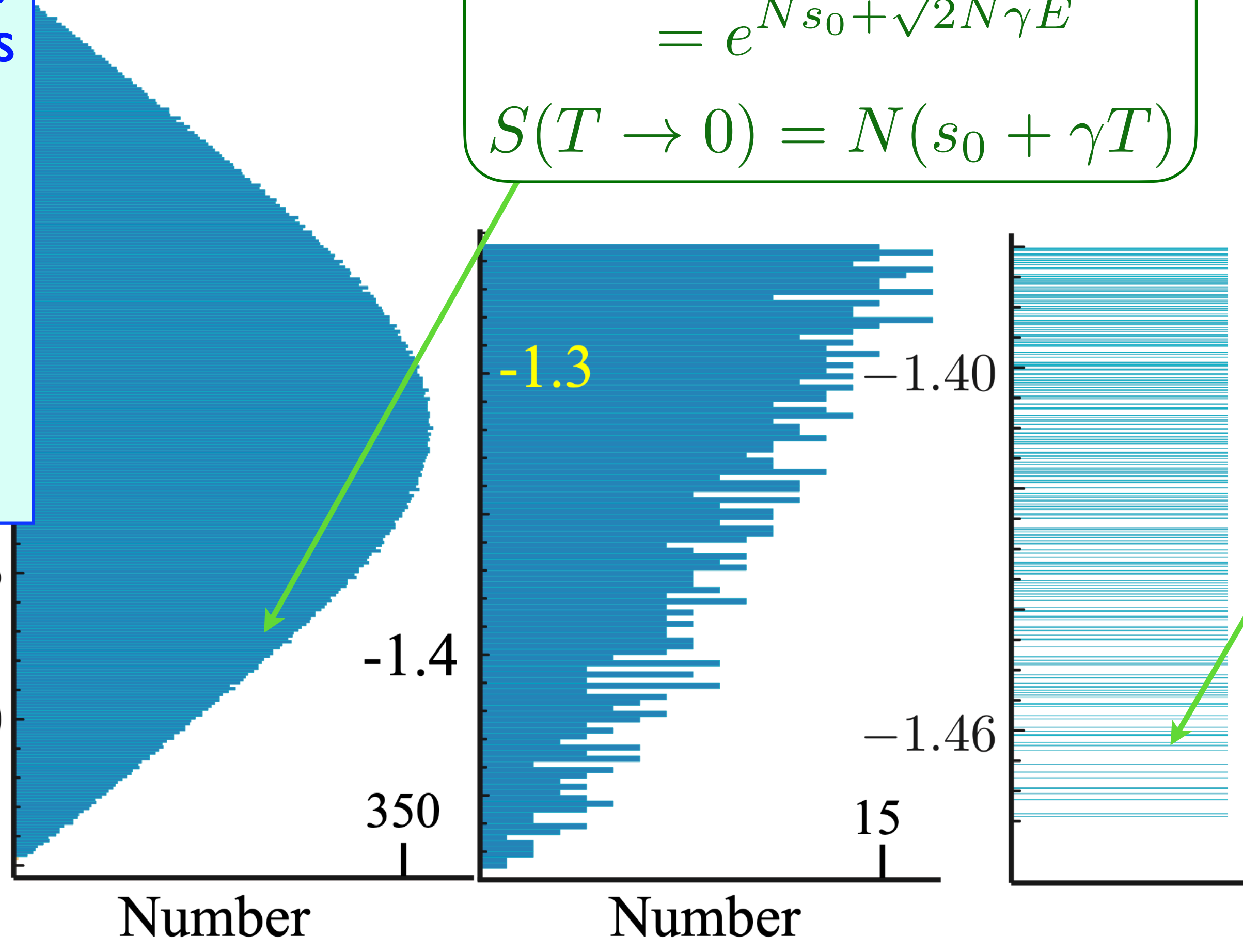
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No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848 \dots$$

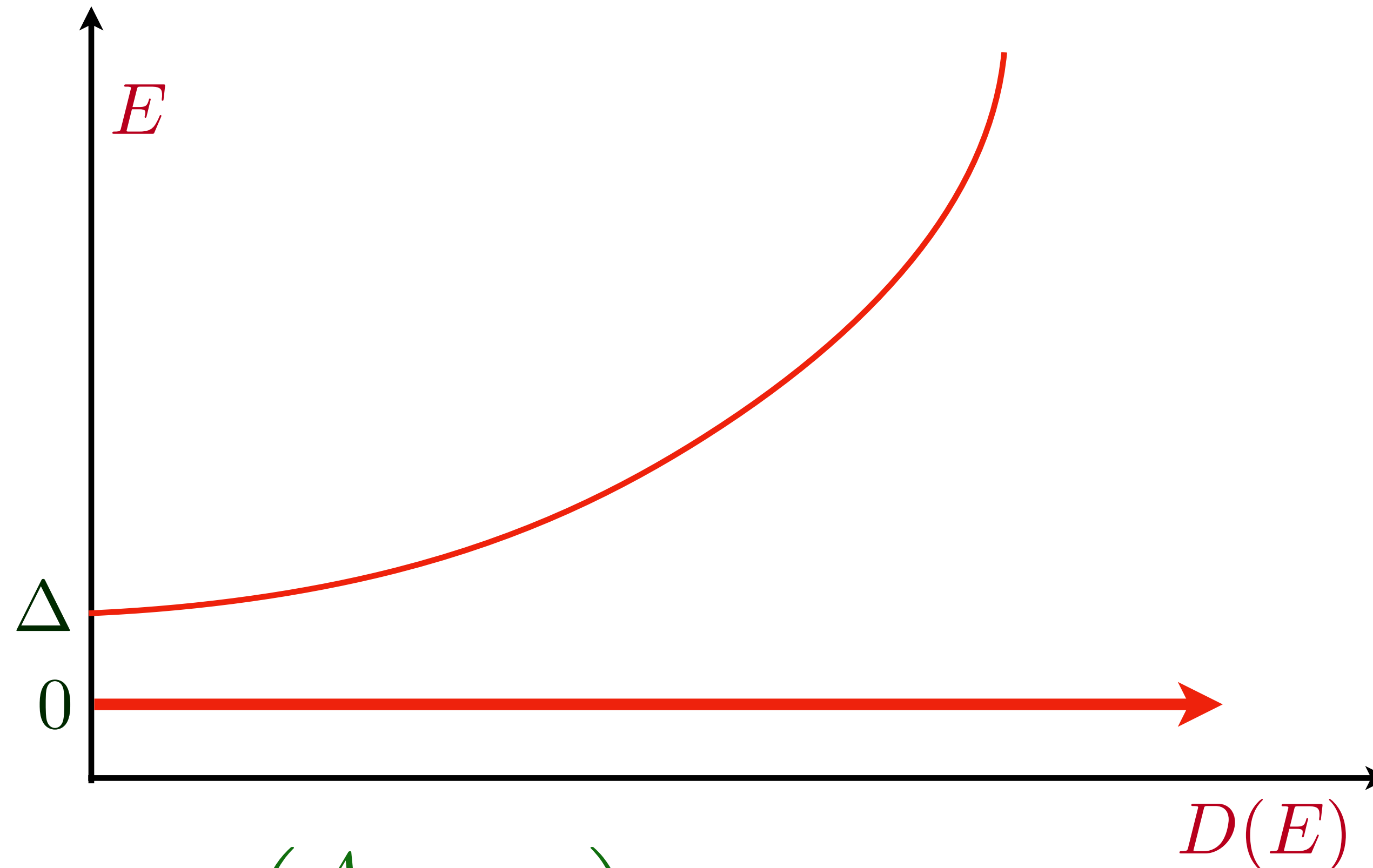
A. Georges, O. Parcollet, and S. Sachdev,
PRB **63**, 134406 (2001)



Complex SYK model

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$D(E) \sim \exp\left(\frac{A_0}{4G} + \dots\right) \delta(E) + f_{\text{reg}}(E - \Delta), \quad \Delta \sim R_h^{-1}$$

Supersymmetric black holes and SYK models

Summary

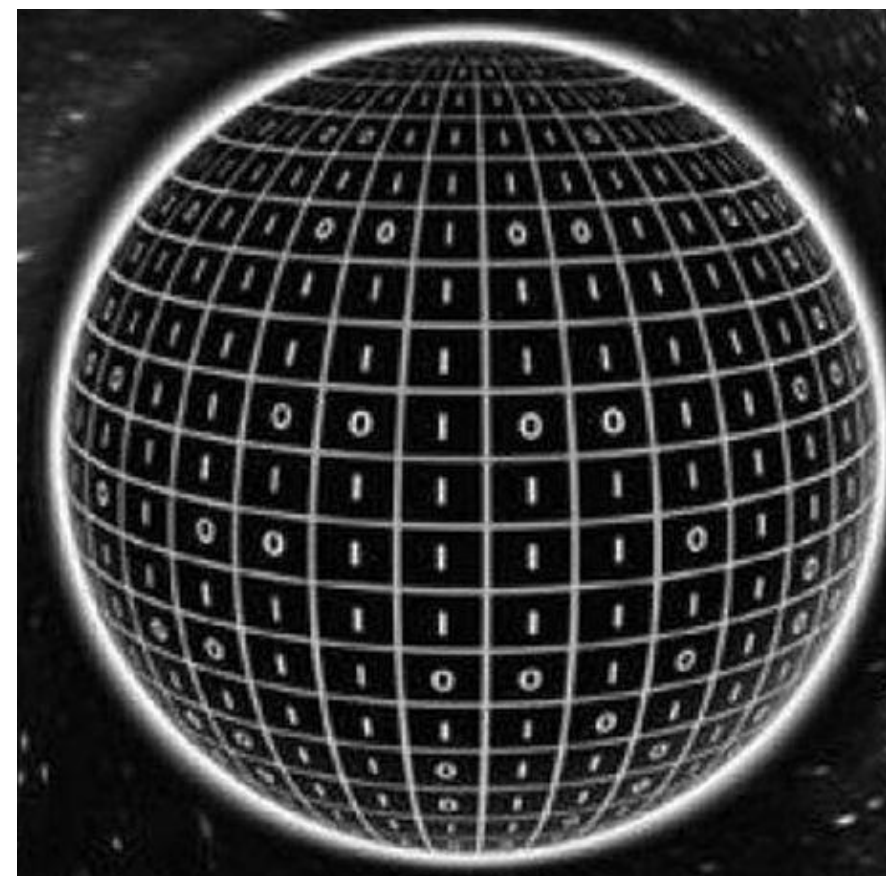
- SYK: a solvable model without quasiparticle excitations, with Planckian time dynamics, and maximal chaos with Lyapunov exponent $2\pi k_B T / \hbar$.

Summary

- SYK: a solvable model without quasiparticle excitations, with Planckian time dynamics, and maximal chaos with Lyapunov exponent $2\pi k_B T / \hbar$.
- Low energy theory of time reparameterizations is the theory of the boundary graviton in 2D quantum gravity on AdS_2 .

Summary

- Boundary graviton leads to universal $-3/2 \ln(1/T)$ correction to Bekenstein-Hawking entropy of low T charged black holes in Einstein gravity, and to the SYK model. So the semiclassical entropy of Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.



Summary

- Two-dimensional Fermi surface coupled to a critical boson has no quasiparticle excitations, and exhibits Planckian time dynamics and maximal chaos with Lyapunov exponent $2\pi k_B T / \hbar$.

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- Two-dimensional Fermi surface coupled to a critical boson has no quasiparticle excitations, and exhibits Planckian time dynamics and maximal chaos with Lyapunov exponent $2\pi k_B T / \hbar$.
- Linear- T resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.

