Z_2 topological order near the Neel state on the square lattice

Simons Conference on

Quantum Entanglement

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Subir Sachdev

Talk online: sachdev.physics.harvard.edu

Shubhayu Chatterjee

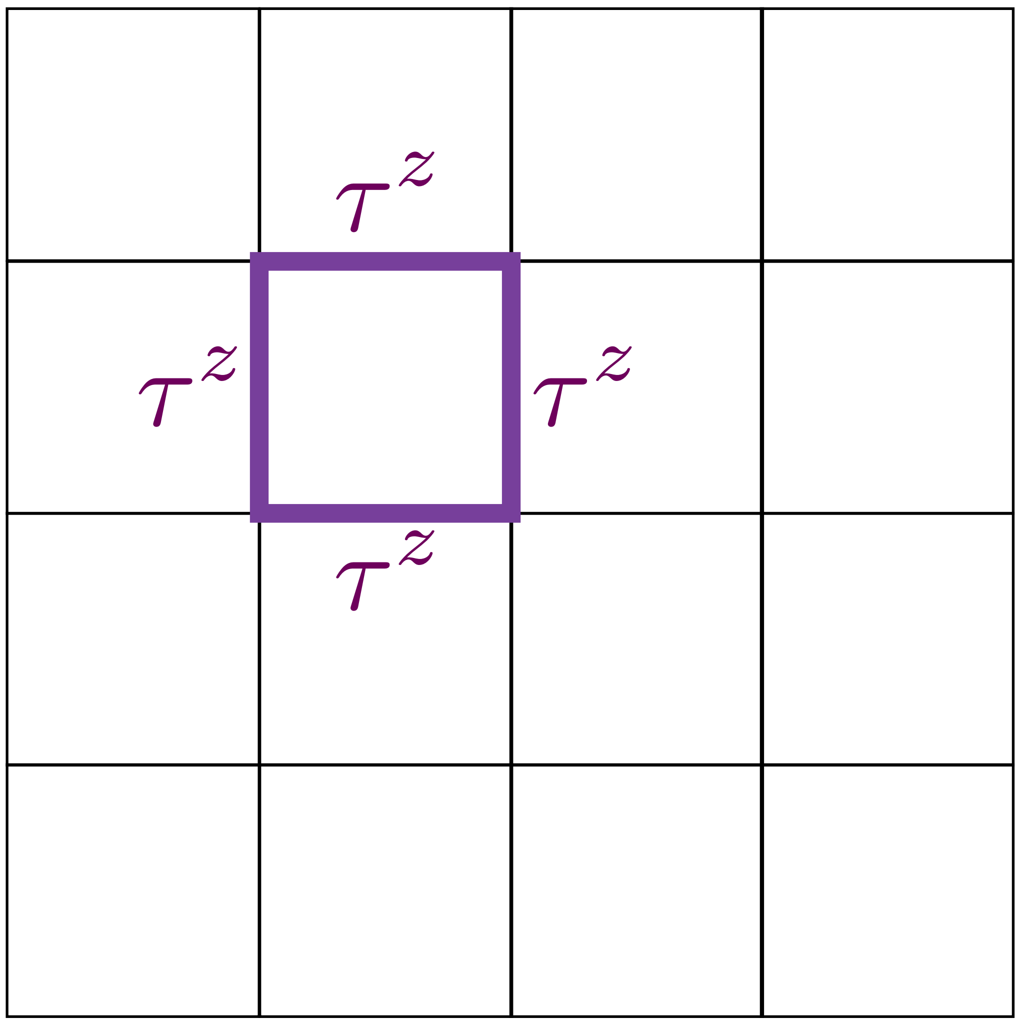
Mathias Scheurer

Alex Thomson

1. Z_2 lattice gauge theory and topological order

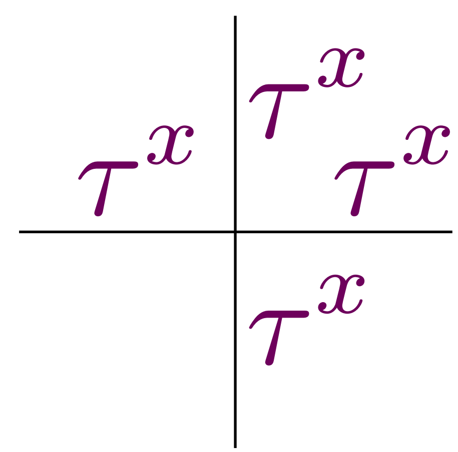
2. CP^1 theory of square lattice antiferromagnets and Z_2 topological order

Z_2 lattice gauge theory (Wegner, 1971)



$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

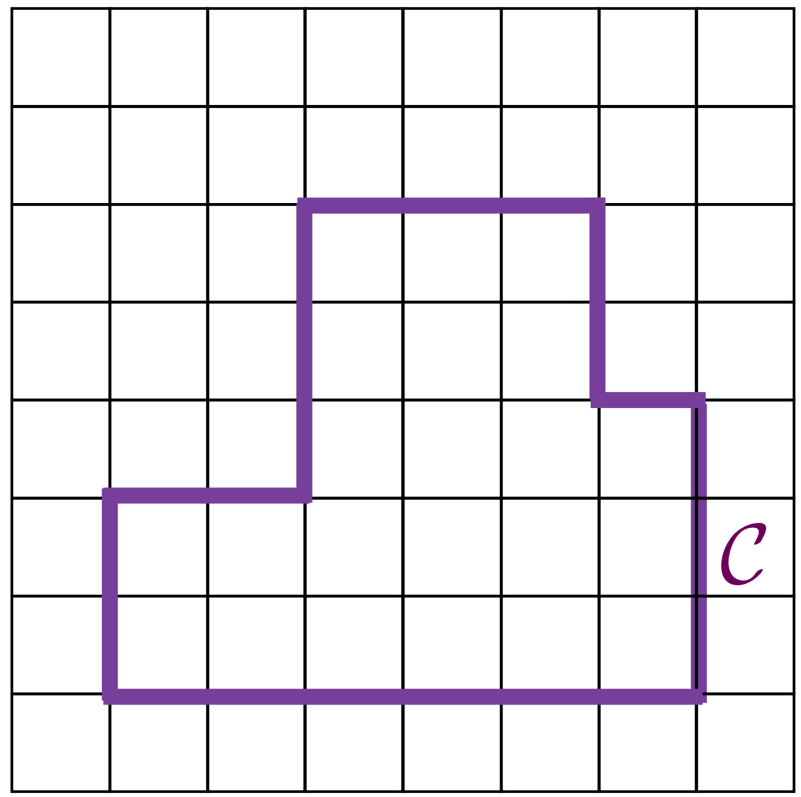
$G_i =$



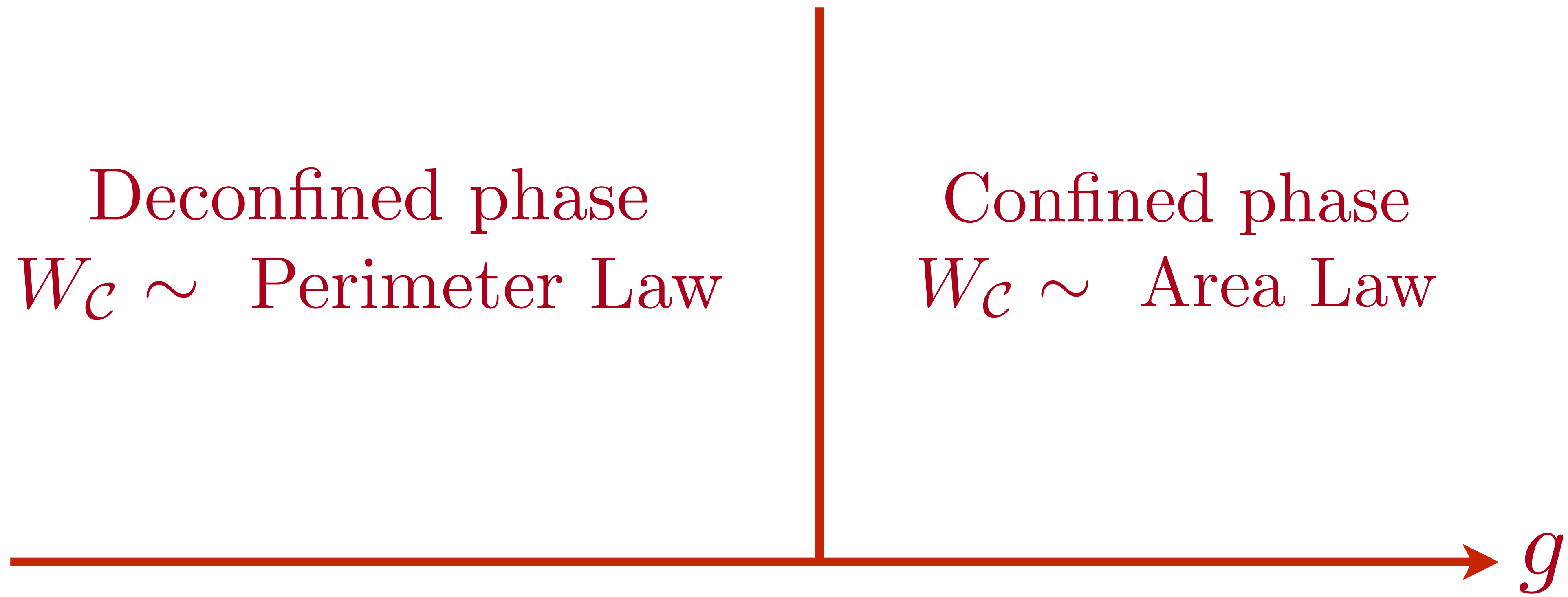
Gauss's Law: $[H, G_i] = 0$, $\boxed{G_i = 1}$

Z_2 lattice gauge theory

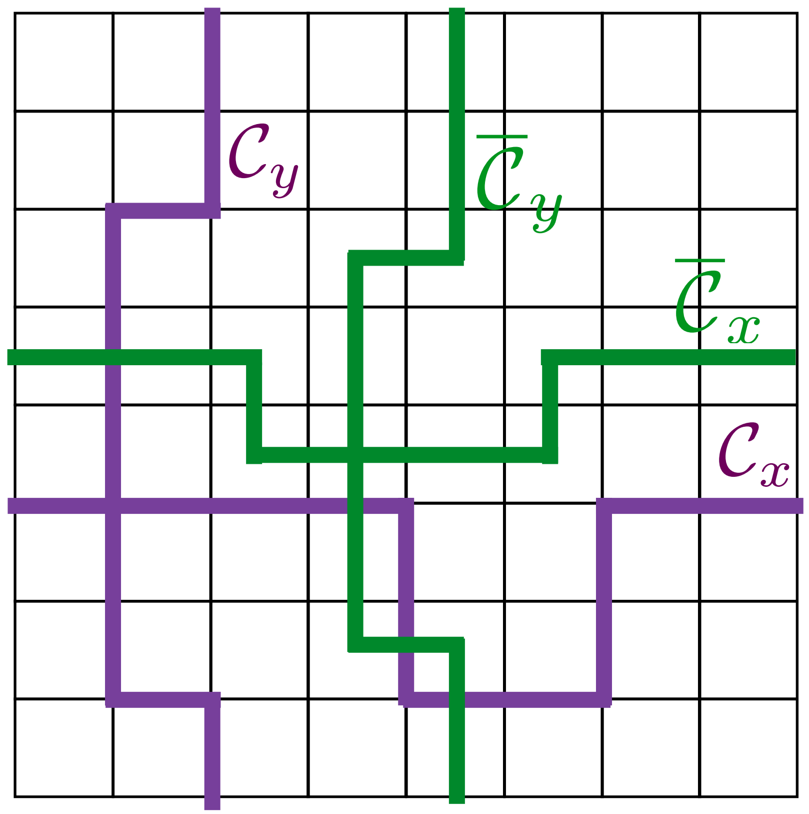
(Wegner, 1971)



$$W_{\mathcal{C}} = \prod_{\mathcal{C}} \tau^z$$



Topological order



Torus

$$V_x = \prod_{\overline{\mathcal{C}}_x} \tau^x \quad , \quad V_y = \prod_{\overline{\mathcal{C}}_y} \tau^x$$

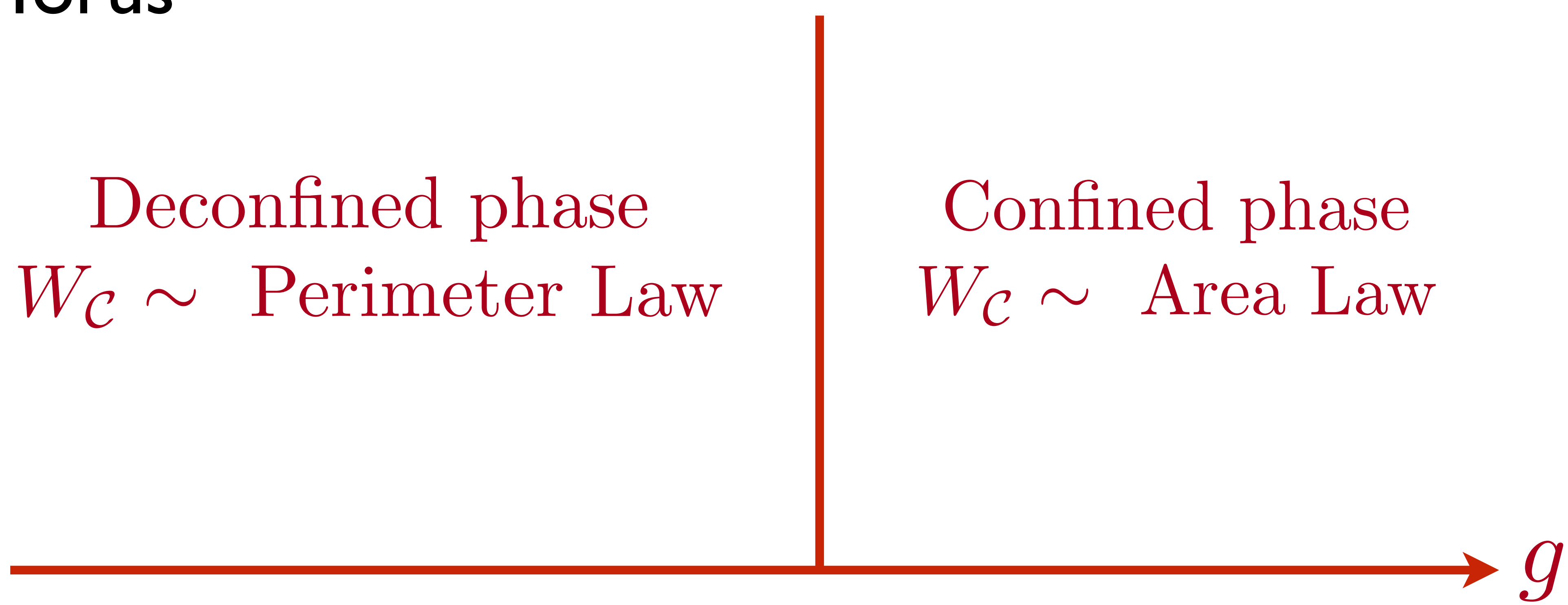
$$W_x = \prod_{\mathcal{C}_x} \tau^z \quad , \quad W_y = \prod_{\mathcal{C}_y} \tau^z$$

$$V_x W_y = -W_y V_x \quad , \quad V_y W_x = -W_x V_y$$

and all other pairs commute.

On a torus, there are two additional independent operators, V_x and V_y which commute with the Hamiltonian:

$$[H, V_x] = [H, V_y] = 0$$



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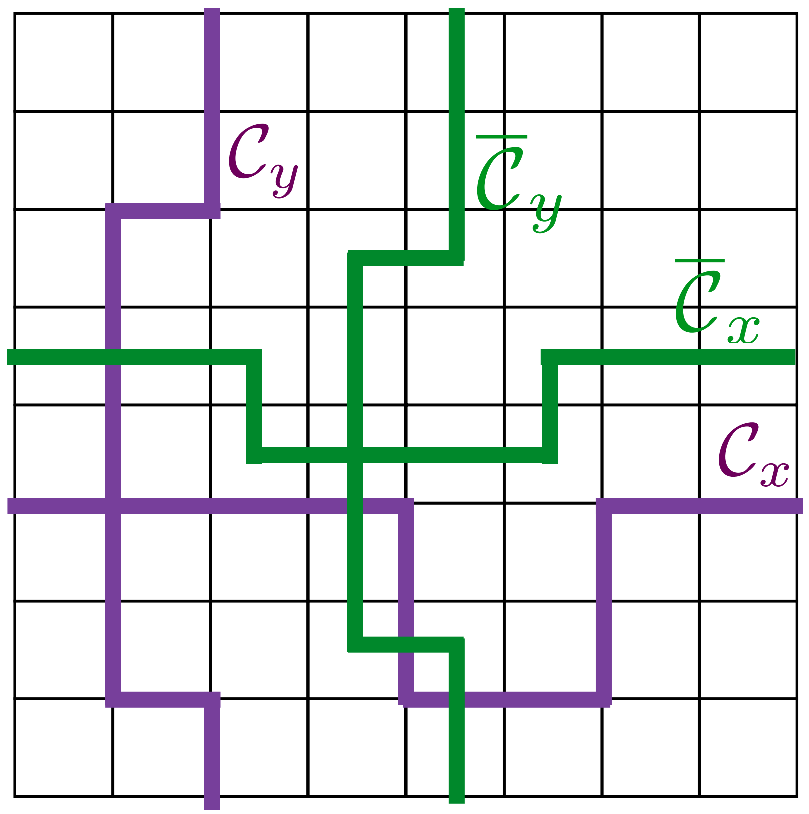

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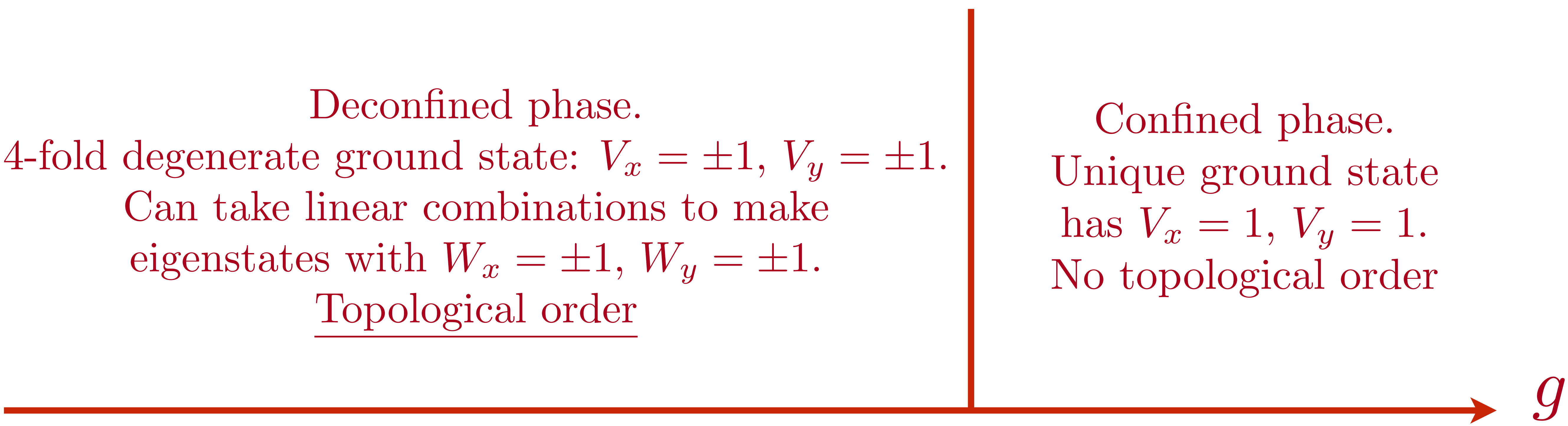

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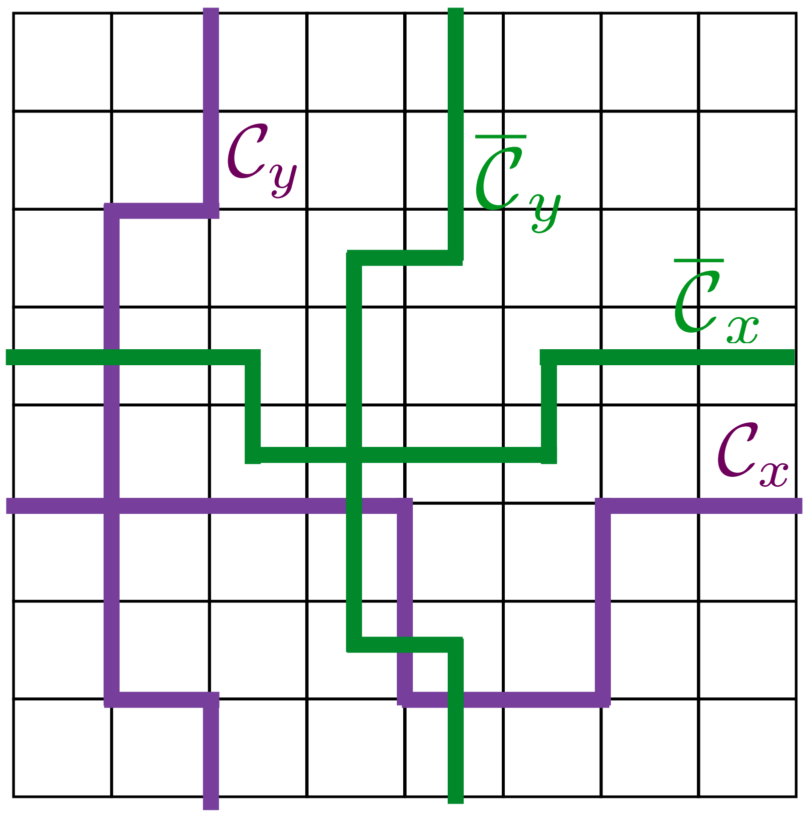

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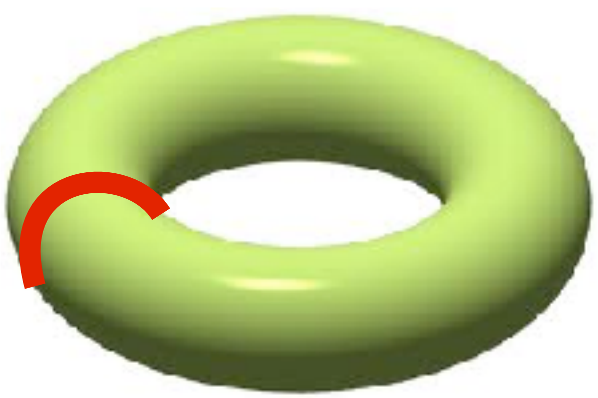

(N. Read and S.S., 1991
Freedman, Nayak, Shtengel, Walker, Wang, 2003)

Topological quantum field theory describes degenerate states with Z_2 flux $W = \pm 1$ through the holes of the torus

Confined phase.
Unique ground state has $V_x = 1$, $V_y = 1$.
No topological order

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This criterion can distinguish the phases when dynamical (or even gapless) matter fields are present

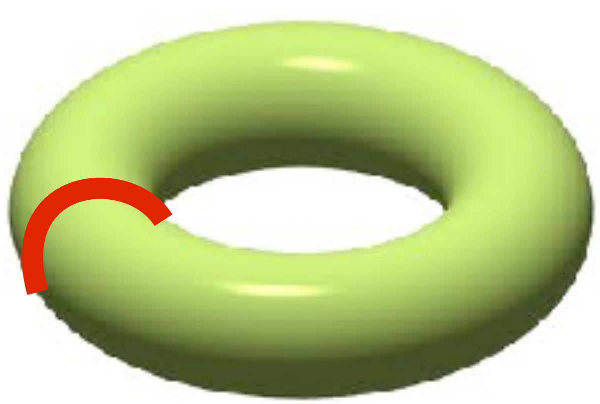
Z_2 lattice gauge theory

$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x \quad , \quad G_i = 1$$

$$\mathcal{L} = |(\partial_\mu - 2ia_\mu)\Phi|^2 + |\Phi|^4$$
$$+ \text{ relevant monopoles.}$$

Ising* criticality

(Fradkin and Shenker, 1979)



Higgs state with $\langle \Phi \rangle \neq 0$: The phase of Φ winds by 2π around the cycle of the torus, trapping U(1) flux π in the hole of the torus. This leads to 4-fold degeneracy

Confined phase. Unique ground state has $V_x = 1$, $V_y = 1$. No topological order

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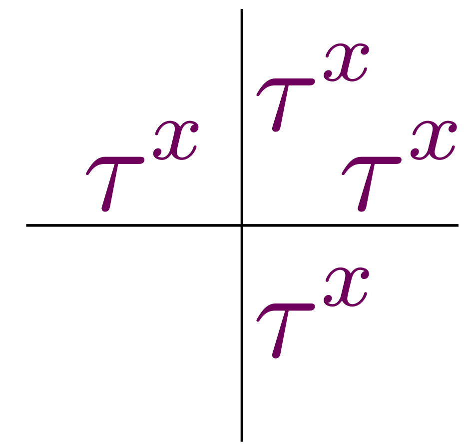
(N. Read and S.S., 1991; X.-G. Wen, 1991
T. Senthil and M.P.A. Fisher, 2000
J. Maldacena, G. Moore, N. Seiberg, 2001)

Symmetry-enriched topological (SET) order and deconfined criticality

$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x \quad , \quad \boxed{G_i = -1}$$

$G_i =$



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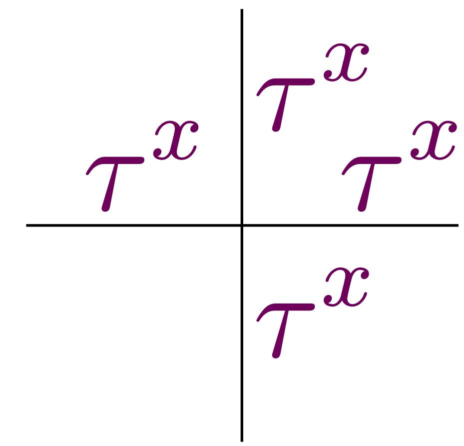
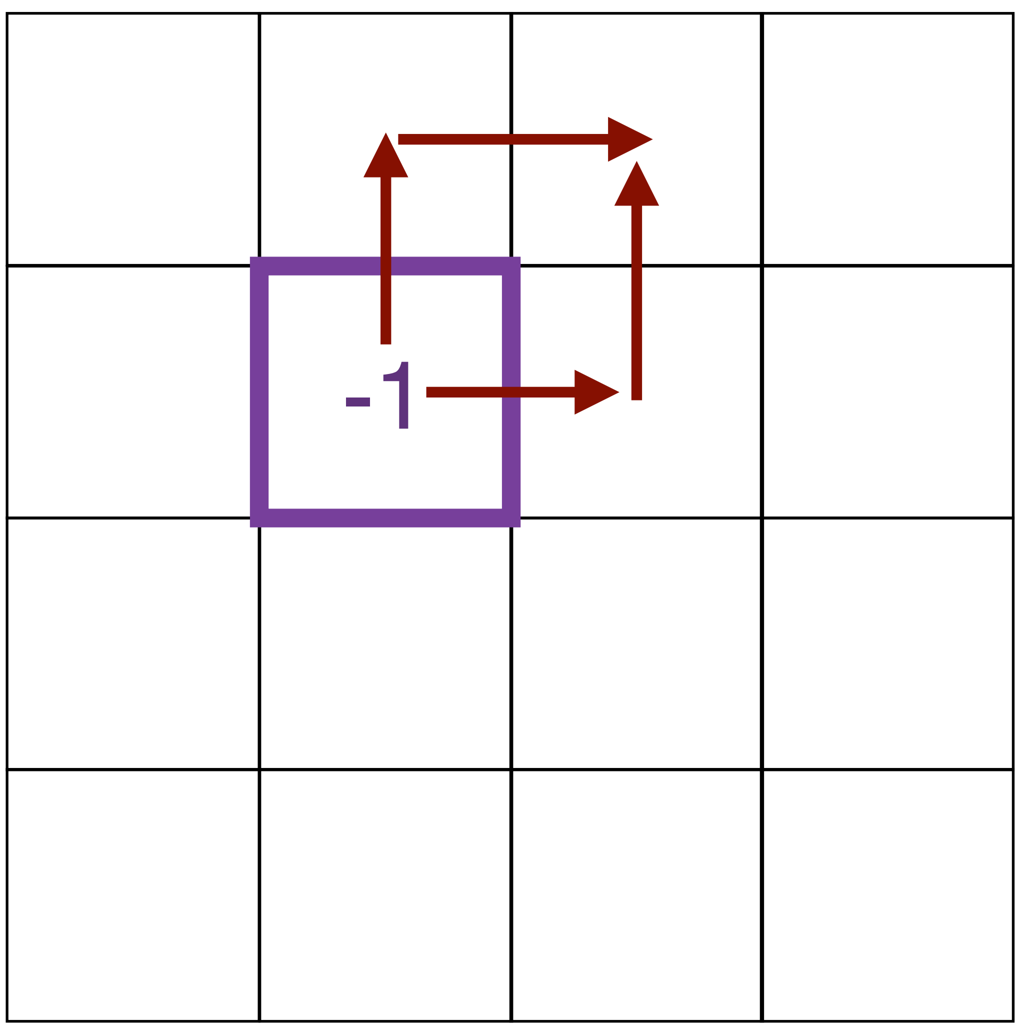
$G_i =$ 



Symmetry fractionalization:
Single spacing translations anti-commute

$$T_x T_y = -T_y T_x$$

when acting on
'fractionalized' states with Z_2 flux -1.

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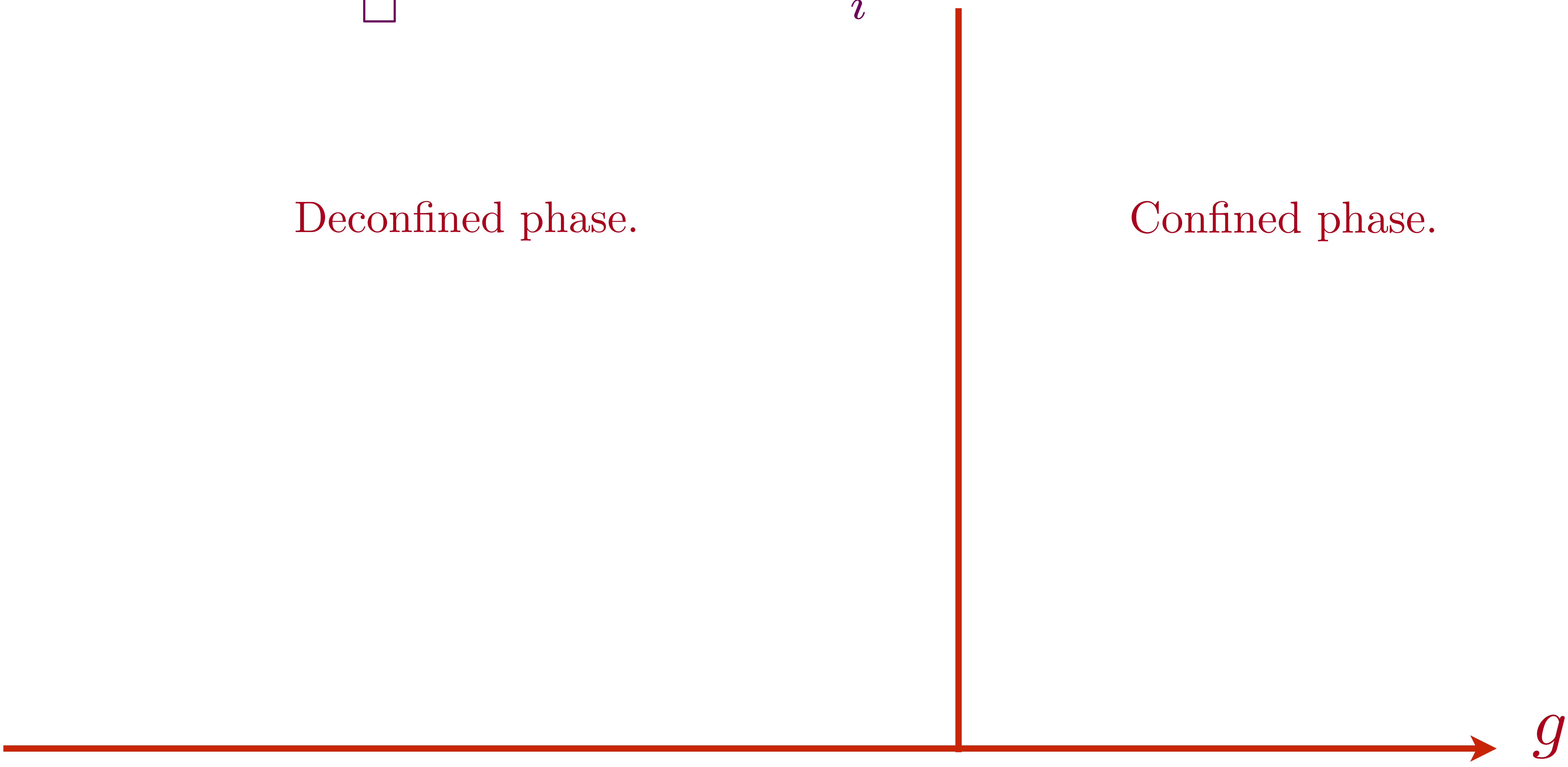

(R. Jalabert and S.S., 1991; T. Senthil and M.P.A. Fisher, 2000; T. Senthil, A. Vishwanath, L. Balents, S. S. and M.P.A. Fisher, 2004)

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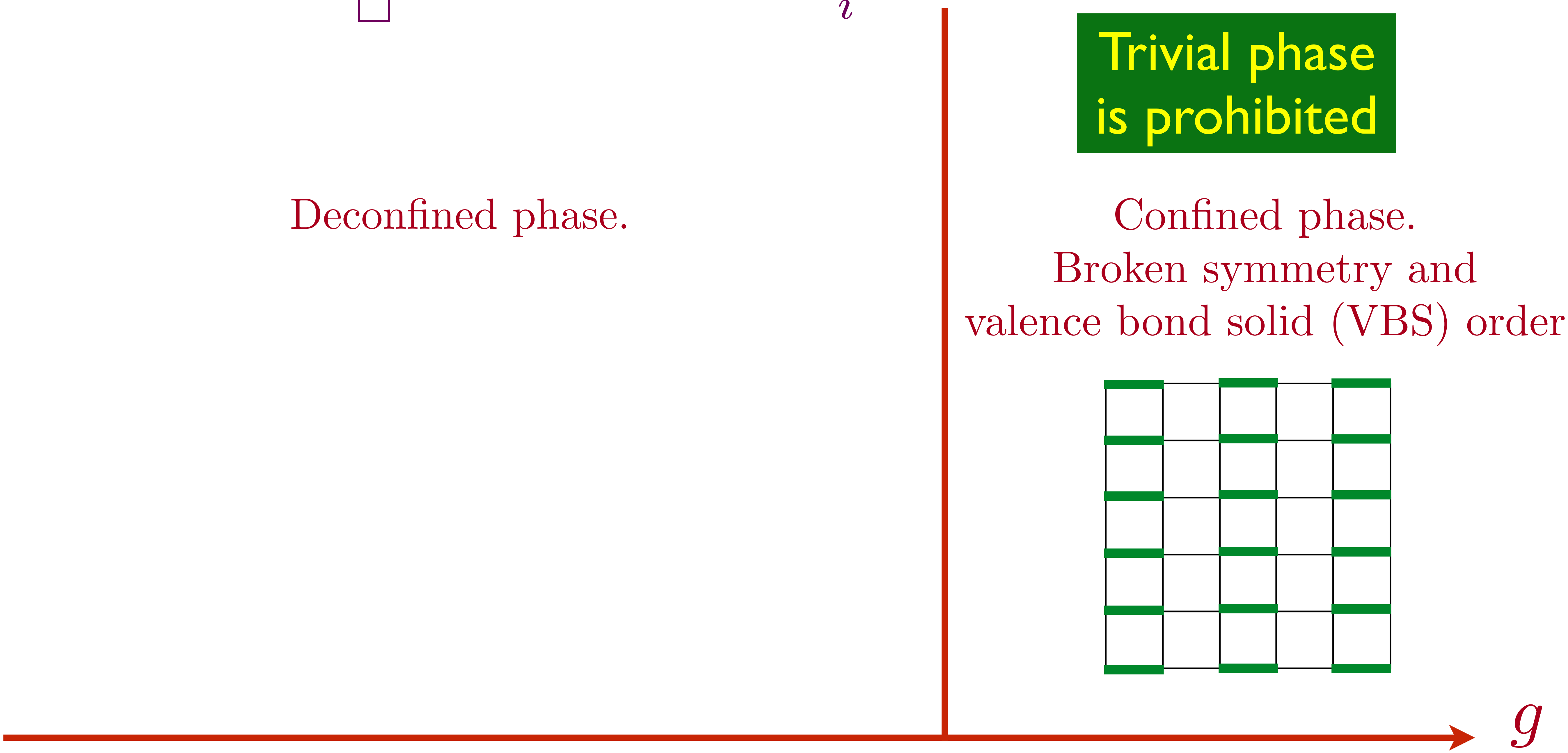

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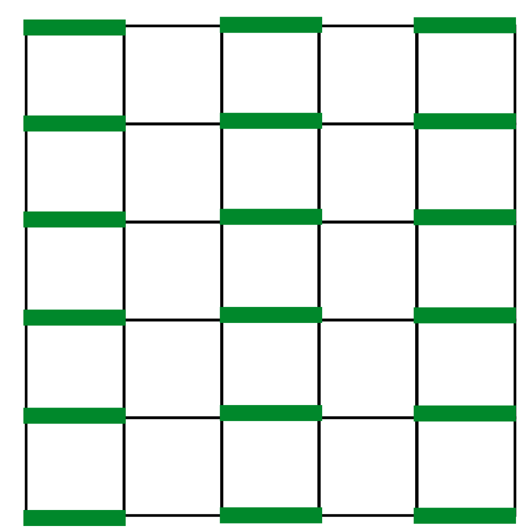
Deconfined phase.
Topological order
Particles with $\mathbb{Z}_2$ flux have a degenerate spectrum which realizes
$T_x T_y = -T_y T_x$

Confined phase.
Broken symmetry and valence bond solid (VBS) order



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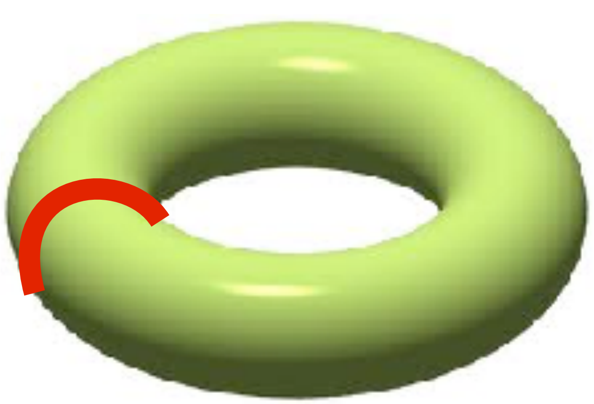
Trivial phase is prohibited

Deconfined quantum criticality:
$$\mathcal{L} = |(\partial_\mu - 2ia_\mu)\Phi|^2 + |\Phi|^4$$
$+$ irrelevant quadrupled monopoles



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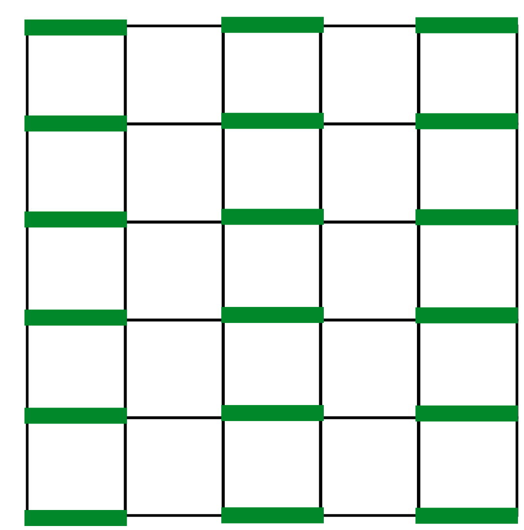

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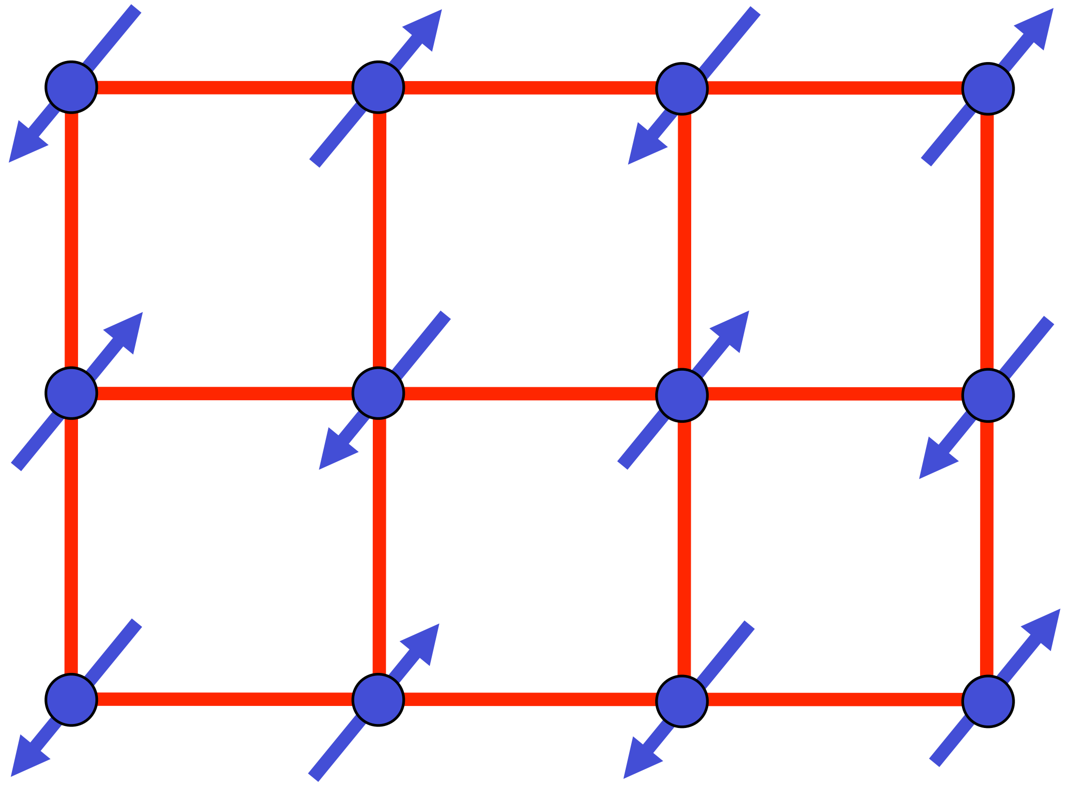
2. CP^1 theory of square lattice antiferromagnets and Z_2 topological order



Insulating Antiferromagnet

Néel order parameter $\mathbf{n}(x_i,\tau) = \eta_i \mathbf{S}_i(\tau)$, where $\eta_i = \pm 1$ on two sublattices.
O(3) non-linear sigma model:

$$S = \frac{1}{2g}\int d^2x d\tau \, (\partial_\mu \mathbf{n})^2 \quad , \quad \mathbf{n}^2 = 1.$$

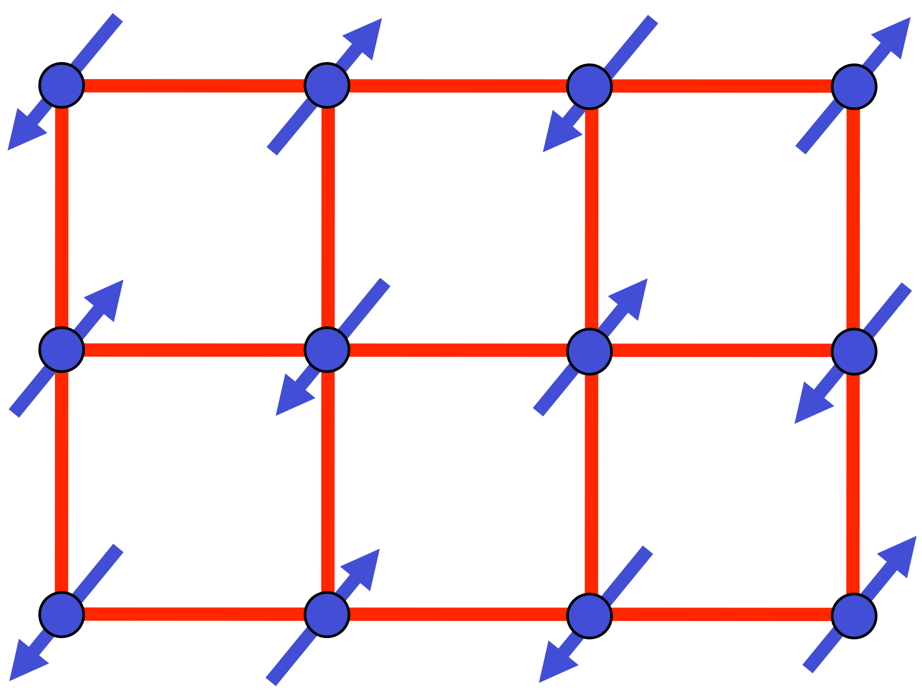
$\mathbb{CP}^1$ model: use $\mathbf{n} = z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta$ with $\alpha, \beta = \uparrow, \downarrow$, and then

$$S = \frac{1}{g}\int d^2x d\tau \, |(\partial_\mu - i a_\mu) z_\alpha|^2 \quad , \quad |z_\alpha|^2 = 1\,,$$

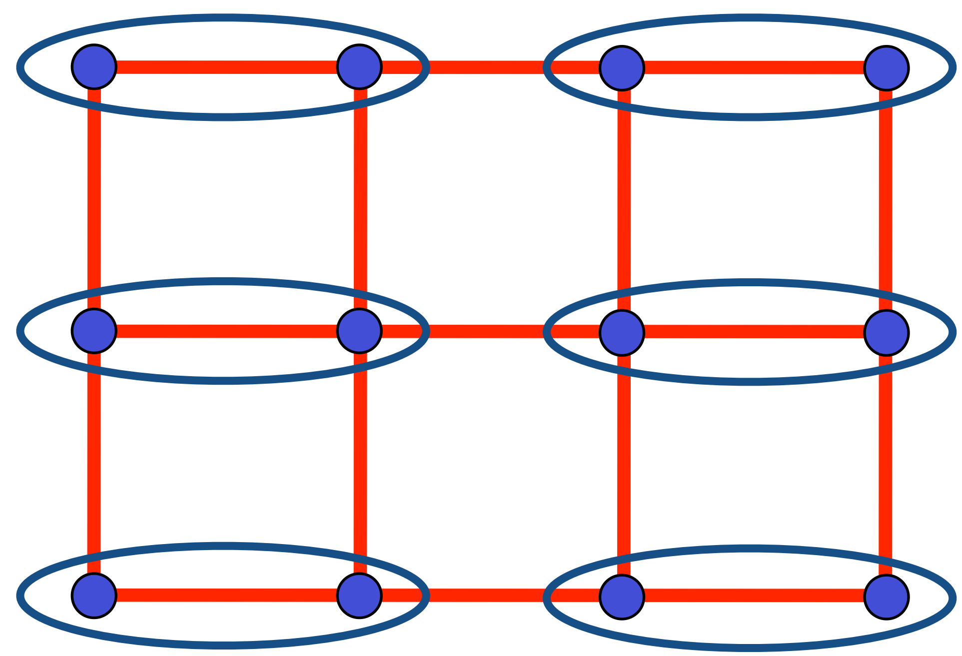
where a_μ is an emergent U(1) gauge field.

Theory for $S = 1/2$ antiferromagnet also has spin Berry phase terms

$$S = \frac{1}{g} \int d^2x d\tau \left| (\partial_\mu - i a_\mu) z_\alpha \right|^2 + i \sum_i \int d\tau \, \eta_i a_{i\tau}$$



Higgs phase with $\langle z_\alpha \rangle \neq 0$
Néel order wih Nambu-Goldstone
(spin-wave) gapless excitations.



Confined phase with $\langle z_\alpha \rangle = 0$
VBS order

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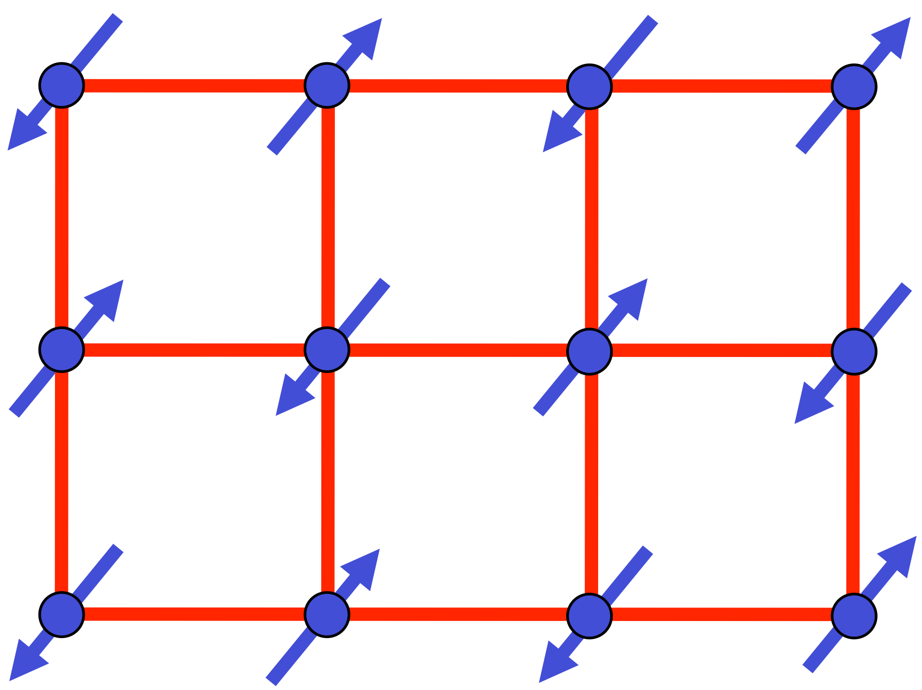

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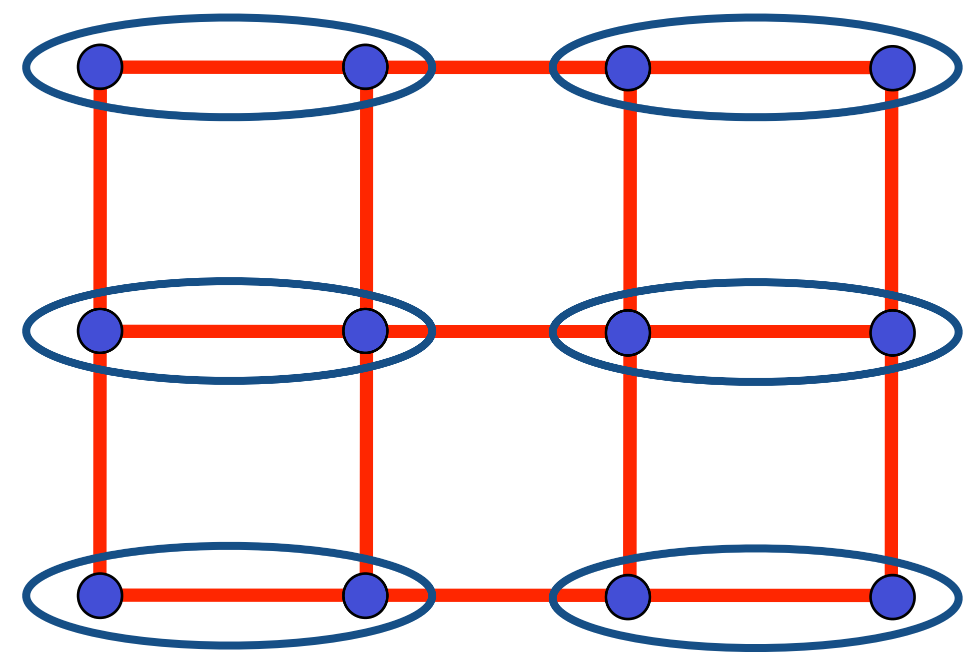

Confined phase with $\langle z_\alpha \rangle = 0$
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(T. Senthil, A. Vishwanath, L. Balents, S. S. and M.P.A. Fisher, 2004)

To obtain a Z_2 deconfined phase, we need to condense a Higgs field with U(1) charge 2. The simplest route is to condense spin-singlet pairs of long-wavelength spinons, z_α. There are two candidates for such Higgs fields, corresponding to the operators

$$\varepsilon_{\alpha\beta} z_\alpha \partial_\tau z_\beta \quad , \quad \varepsilon_{\alpha\beta} z_\alpha \vec{\nabla} z_\beta$$

So we introduce corresponding Higgs fields, P and $\vec{Q}$, and the following effective action with additional tuning parameters s_1 and s_2

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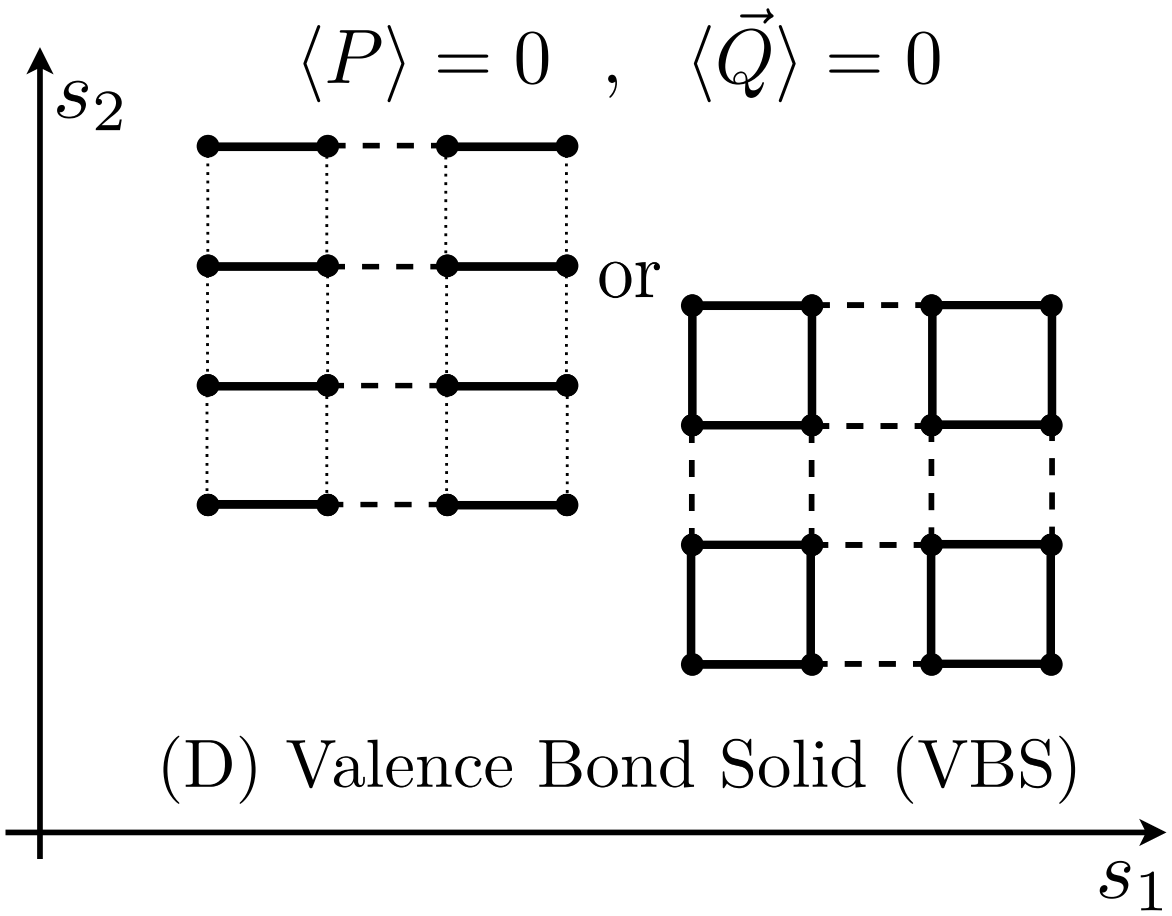
$$\begin{aligned} S = & \frac{1}{g} \int d^2x d\tau \, |(\partial_\mu - i a_\mu) z_\alpha|^2 + i \sum_i \int d\tau \, \eta_i a_{i\tau} \\ & \int d^2x d\tau \Big[|(\partial_\mu - 2i a_\mu) P|^2 + |(\partial_\mu - 2i a_\mu) \vec{Q}|^2 \\ & + i\lambda_1 P^* \varepsilon_{\alpha\beta} z_\alpha \partial_\tau z_\beta + \lambda_2 \vec{Q}^* \cdot \varepsilon_{\alpha\beta} z_\alpha \vec{\nabla} z_\beta + \text{H.c.} \\ & + s_1 |P|^2 + s_2 |\vec{Q}|^2 + u_1 |P|^4 + u_2 |\vec{Q}|^4 + \ldots \Big] \end{aligned}$$

Phase diagram at large g with $\langle z_\alpha \rangle = 0$



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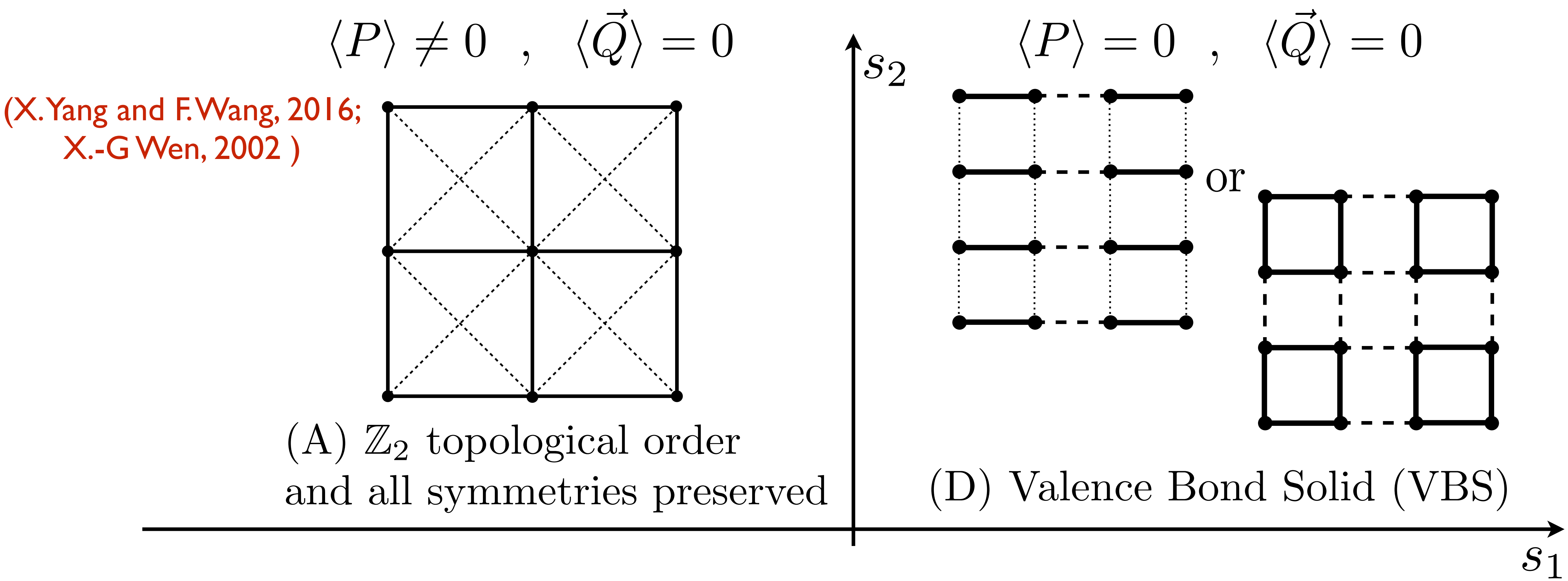

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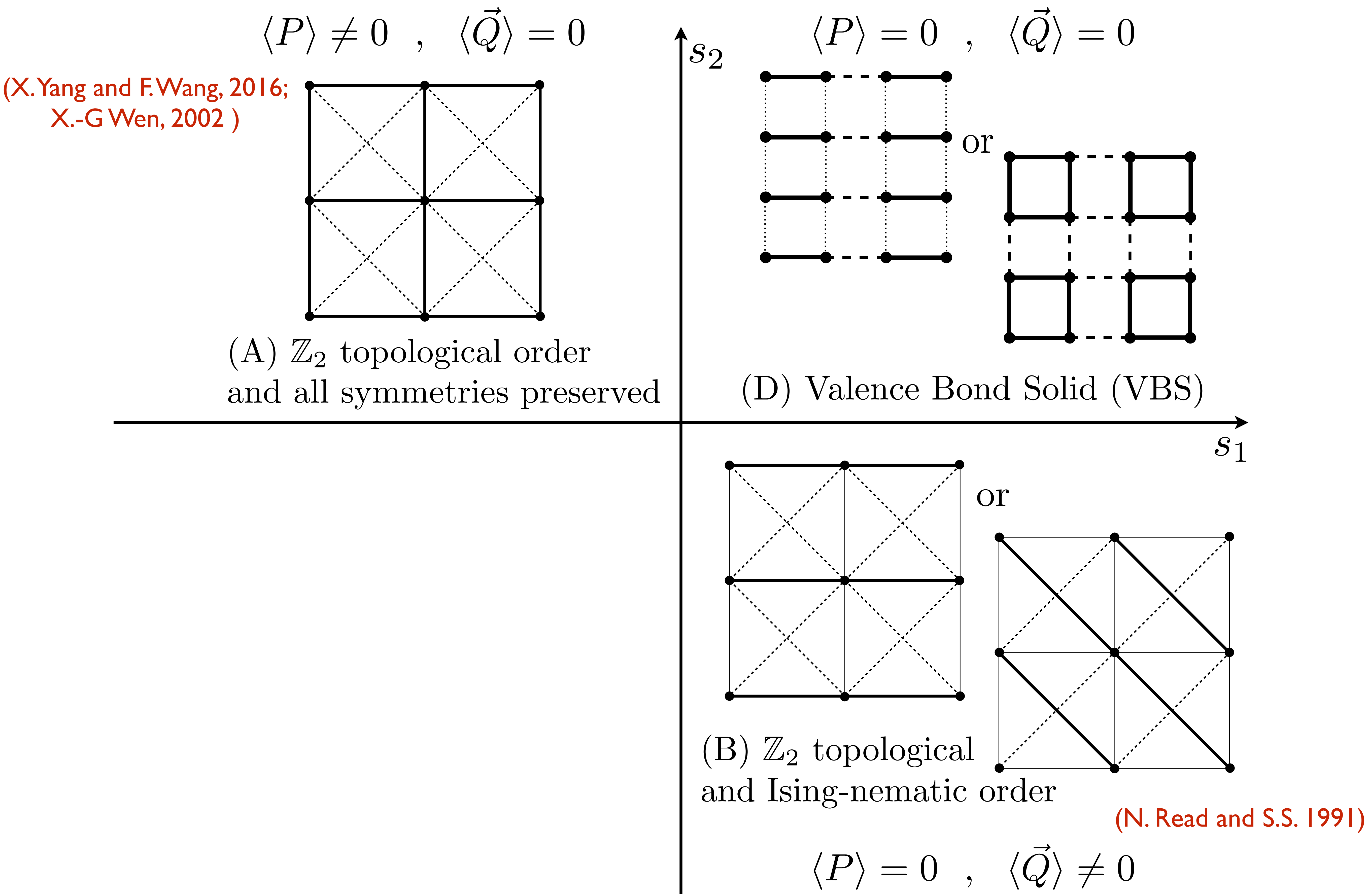
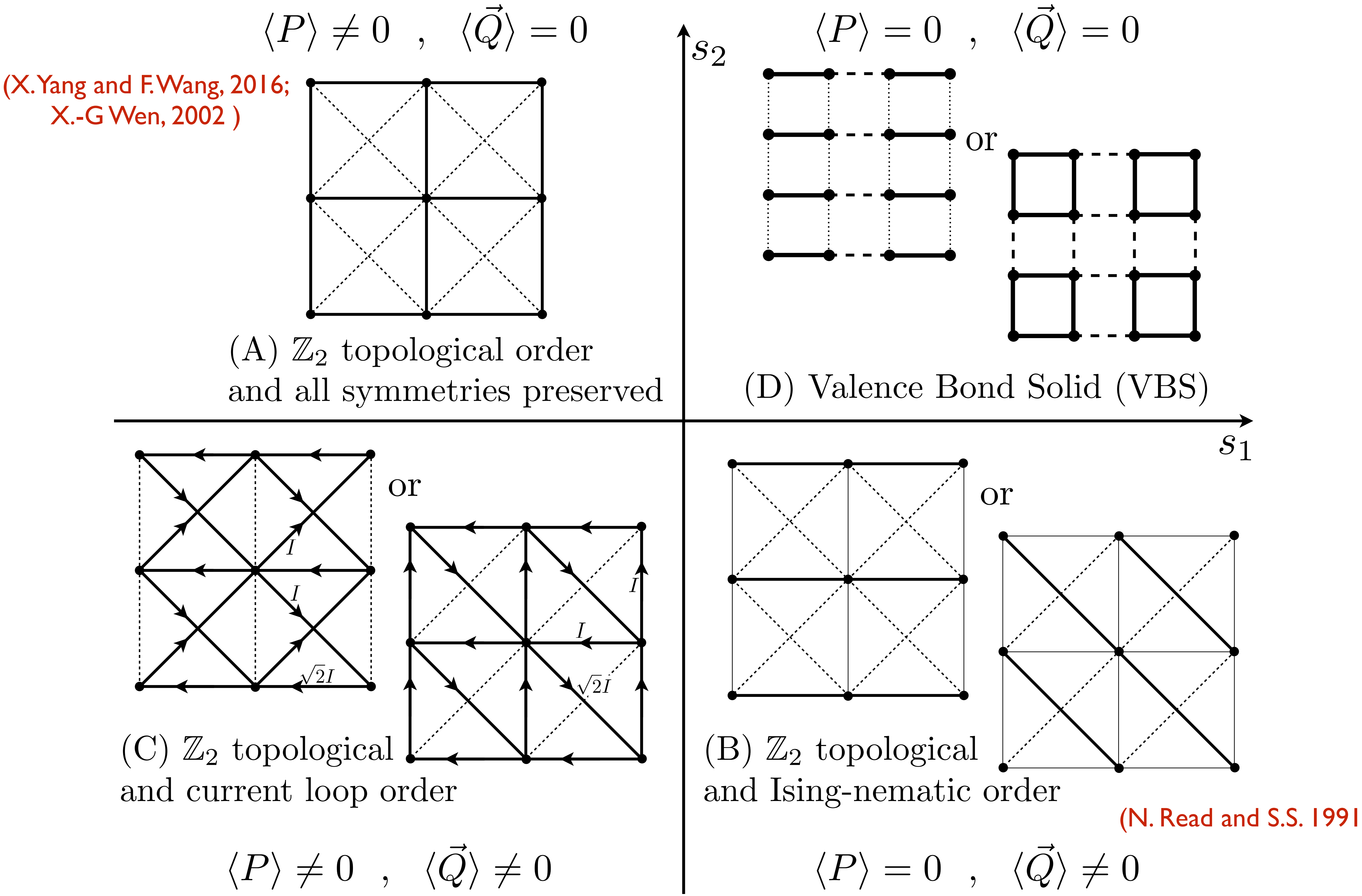


Phase diagram at large g with $\langle z_\alpha \rangle = 0$

Three phases with Z_2 topological order



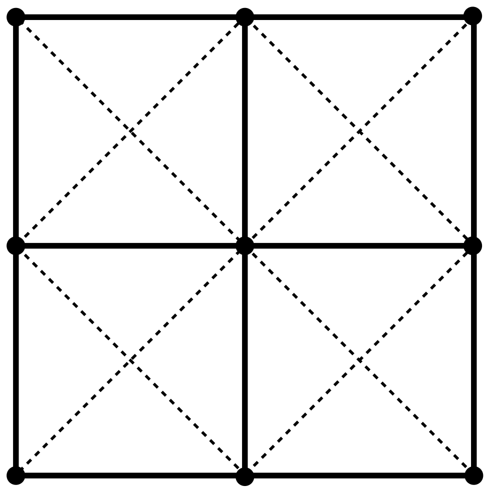
Phase diagram at large g with $\langle z_\alpha \rangle = 0$

Three phases with Z_2 topological order

s_2

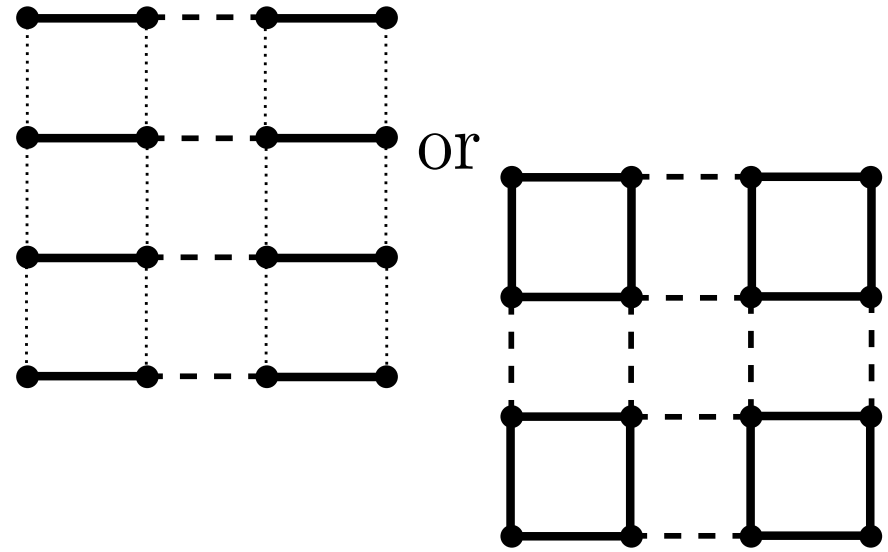
$\langle P \rangle \neq 0 \quad , \quad \langle \vec{Q} \rangle = 0$

(X. Yang and F. Wang, 2016; X.-G Wen, 2002)



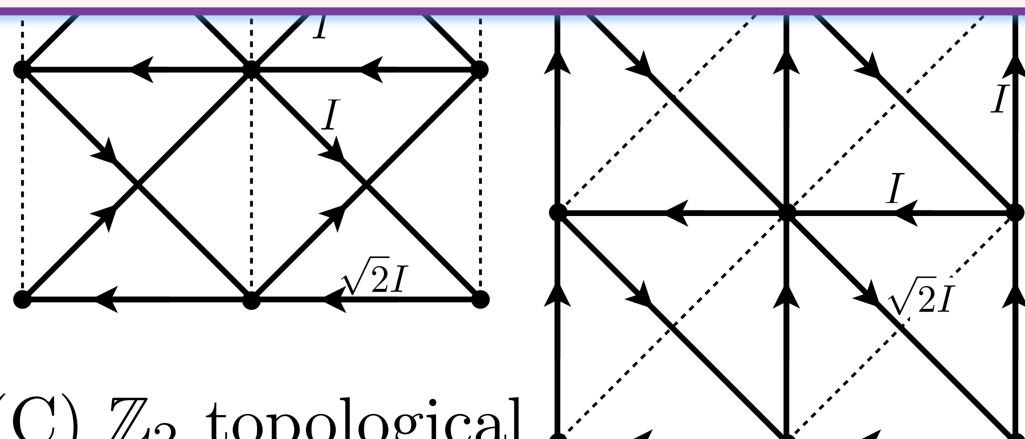
(A) $\mathbb{Z}_2$ topological order and all symmetries preserved

$\langle P \rangle = 0 \quad , \quad \langle \vec{Q} \rangle = 0$



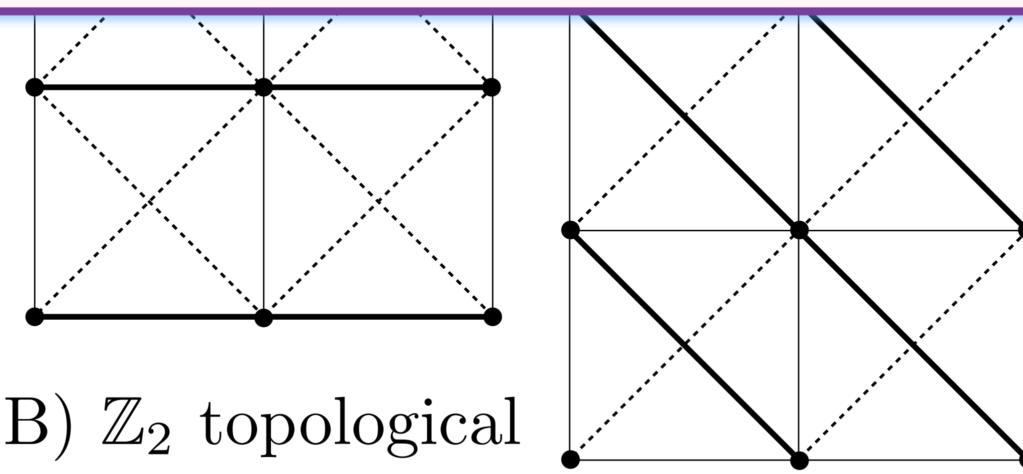
(D) Valence Bond Solid (VBS)

The broken symmetries co-existing with Z_2 topological order are precisely those observed in the pseudogap phase of the cuprates



(C) $\mathbb{Z}_2$ topological and current loop order

$\langle P \rangle \neq 0 \quad , \quad \langle \vec{Q} \rangle \neq 0$



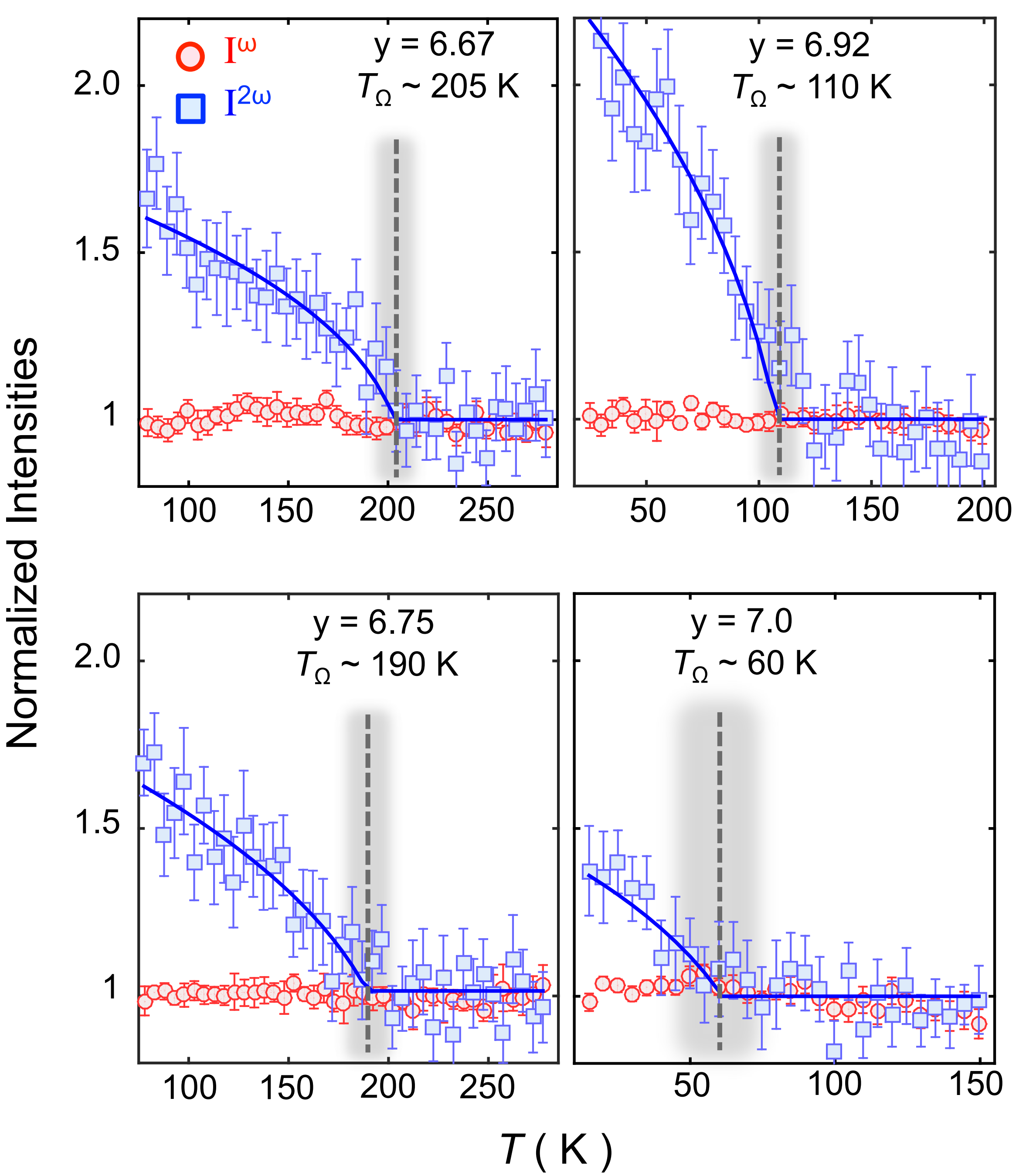
(B) $\mathbb{Z}_2$ topological and Ising-nematic order

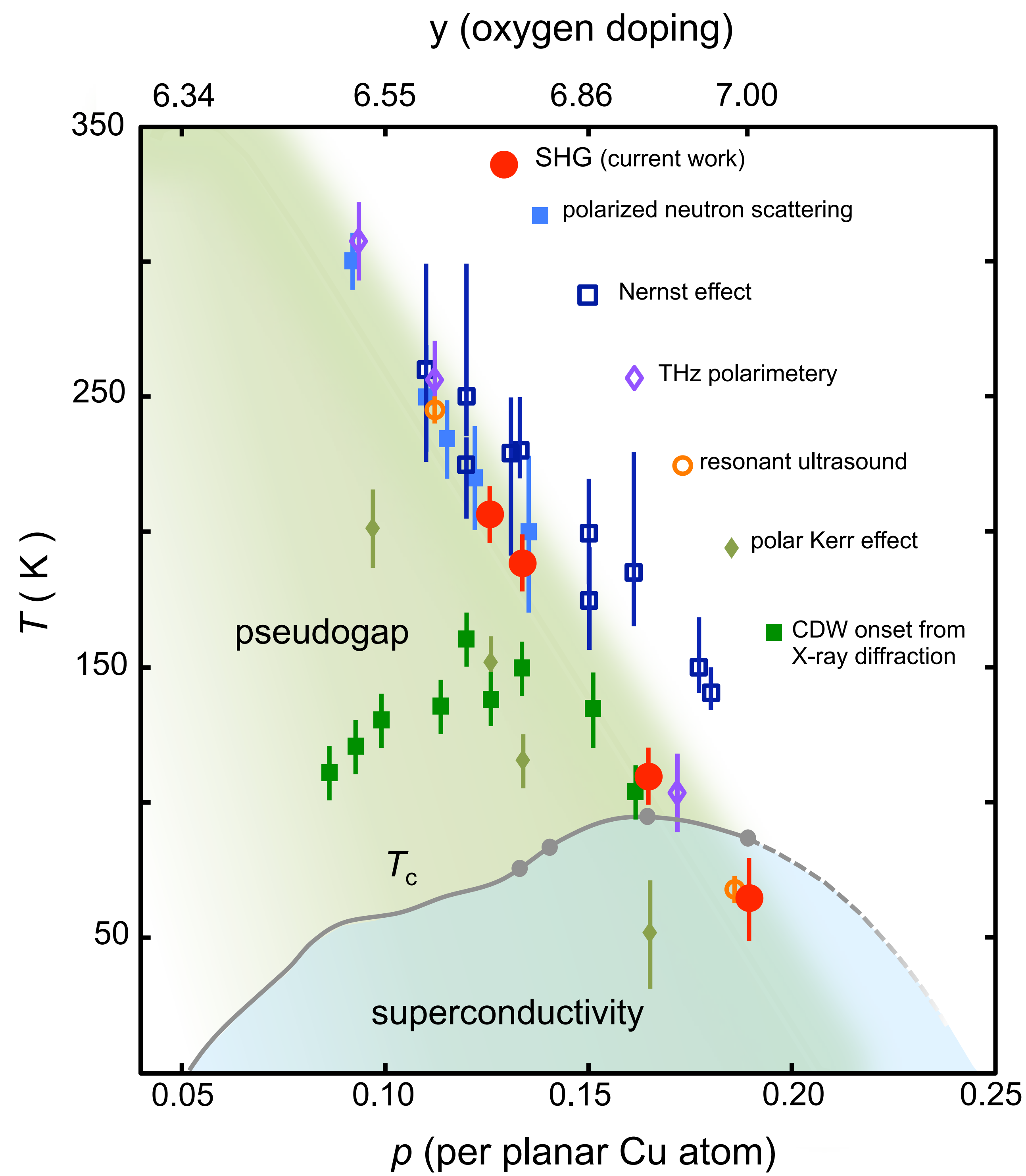
(N. Read and S.S. 1991)

$\langle P \rangle = 0 \quad , \quad \langle \vec{Q} \rangle \neq 0$

Broken inversion symmetry below T^* in $YBa_2Cu_3O_y$

No anomalies at T_c, T_{CDW} or T_{Kerr}





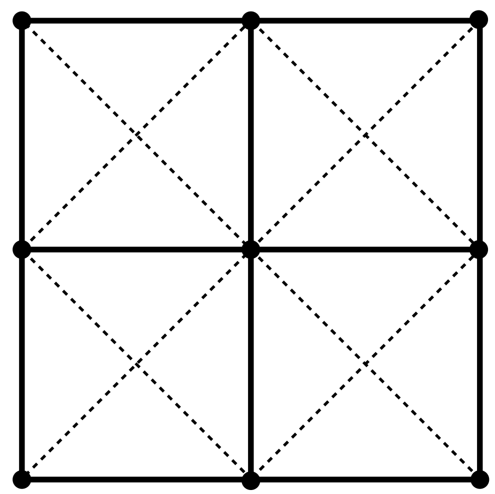
L. Zhao, C. A. Belvin, R. Liang, D. A. Bonn, W. N. Hardy, N. P. Armitage, and D. Hsieh, Nature Physics **13**, 250 (2017)

Phase diagram at large g with $\langle z_\alpha \rangle = 0$

Three phases with Z_2 topological order

$\langle P \rangle \neq 0 \quad , \quad \langle \vec{Q} \rangle = 0$

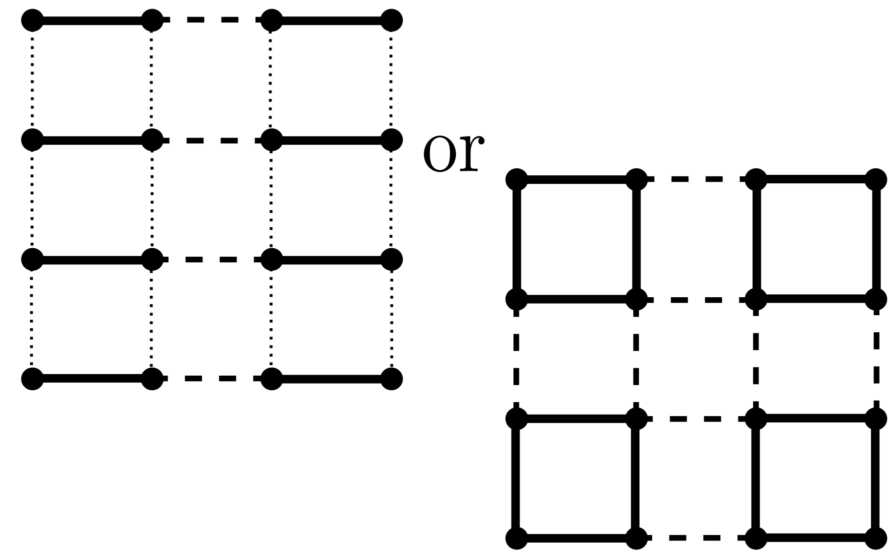
(X. Yang and F. Wang, 2016; X.-G Wen, 2002)



(A) $\mathbb{Z}_2$ topological order and all symmetries preserved

s_2

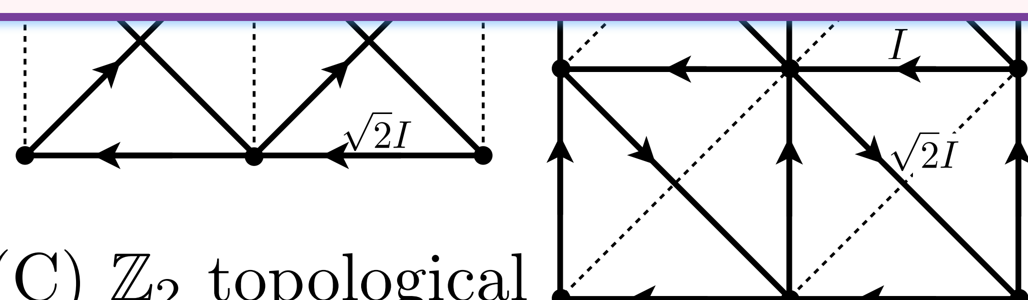
$\langle P \rangle = 0 \quad , \quad \langle \vec{Q} \rangle = 0$



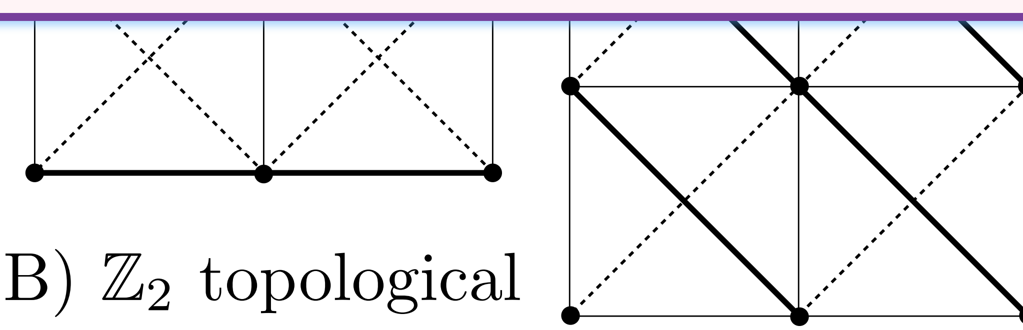
(D) Valence Bond Solid (VBS)

Z_2 topological order can also be present in gapless systems with Fermi surfaces. In the presence of topological order, the volume enclosed by the Fermi surfaces can differ from the Luttinger value.

T. Senthil, M. Vojta, and S. Sachdev, 2004



(C) $\mathbb{Z}_2$ topological and current loop order



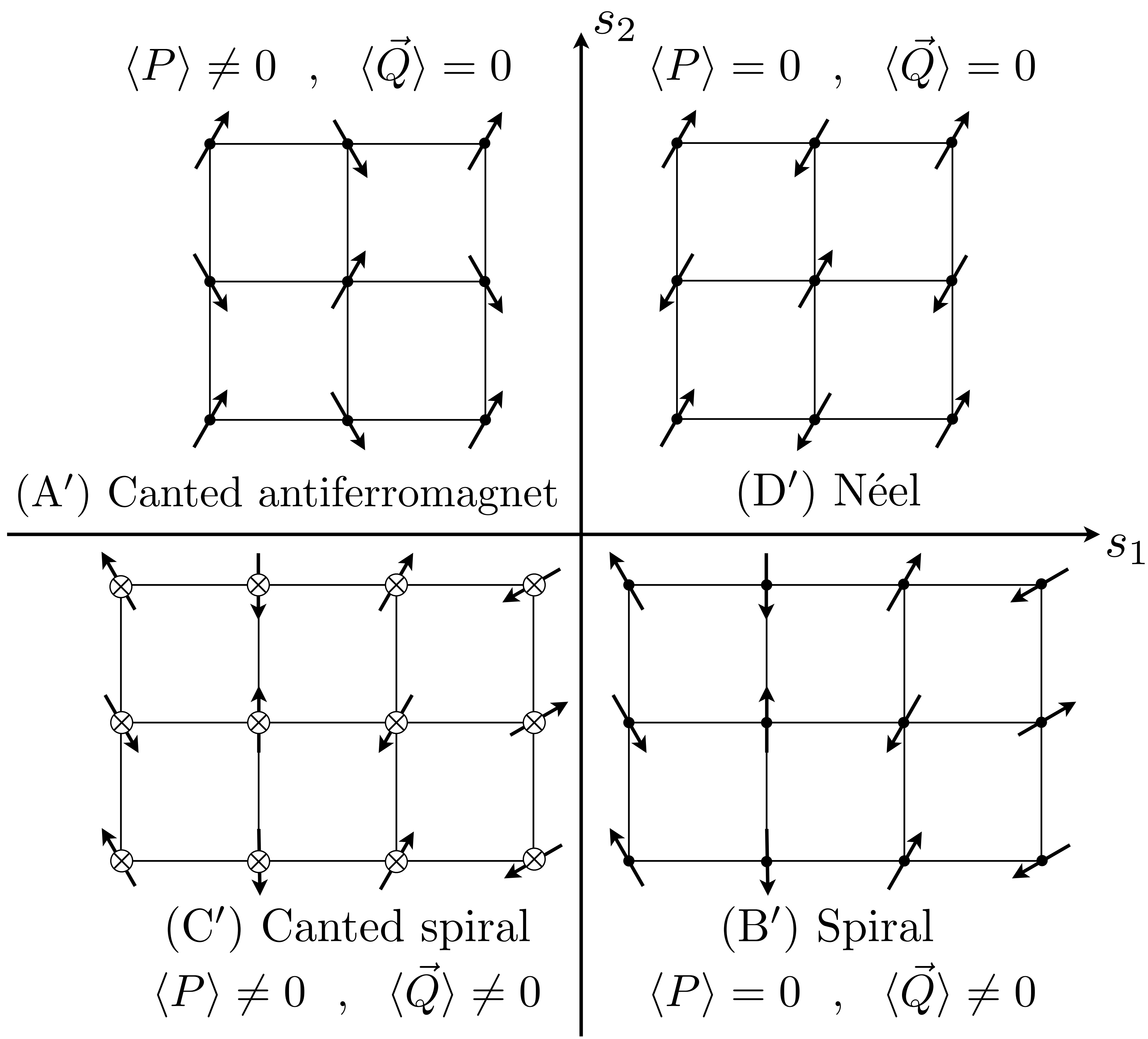
(B) $\mathbb{Z}_2$ topological and Ising-nematic order

(N. Read and S.S. 1991)

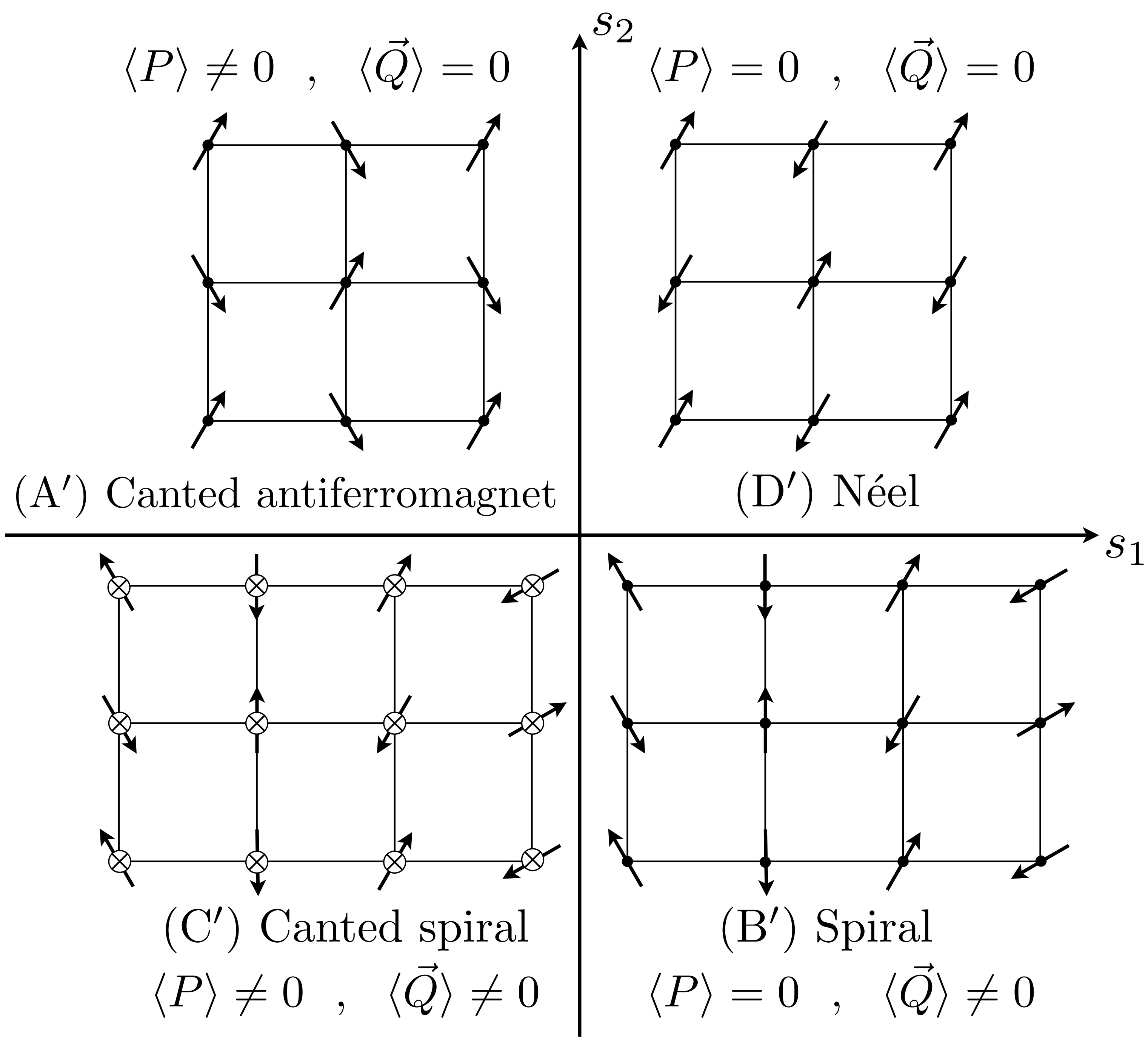
$\langle P \rangle \neq 0 \quad , \quad \langle \vec{Q} \rangle \neq 0$

$\langle P \rangle = 0 \quad , \quad \langle \vec{Q} \rangle \neq 0$

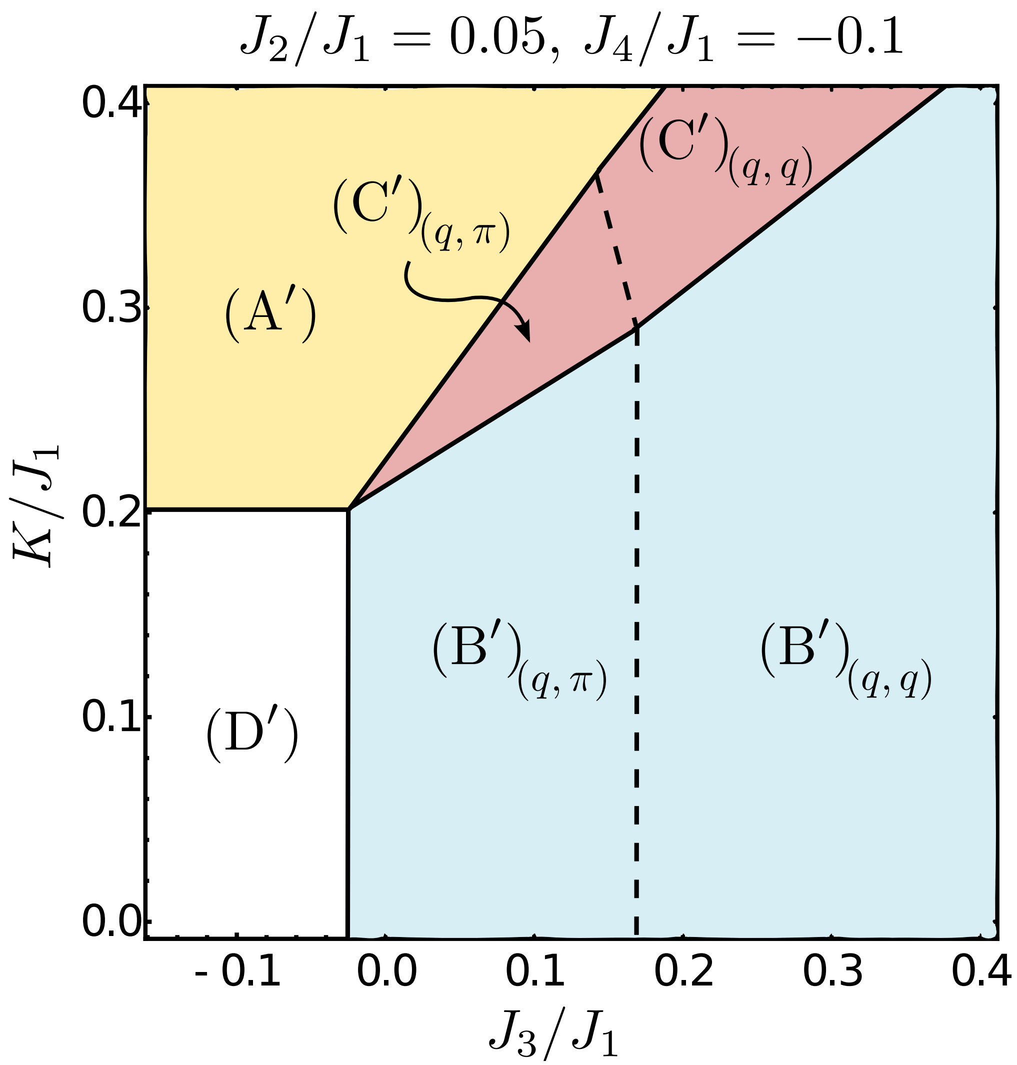
Phase diagram at small g with $\langle z_\alpha \rangle \neq 0$



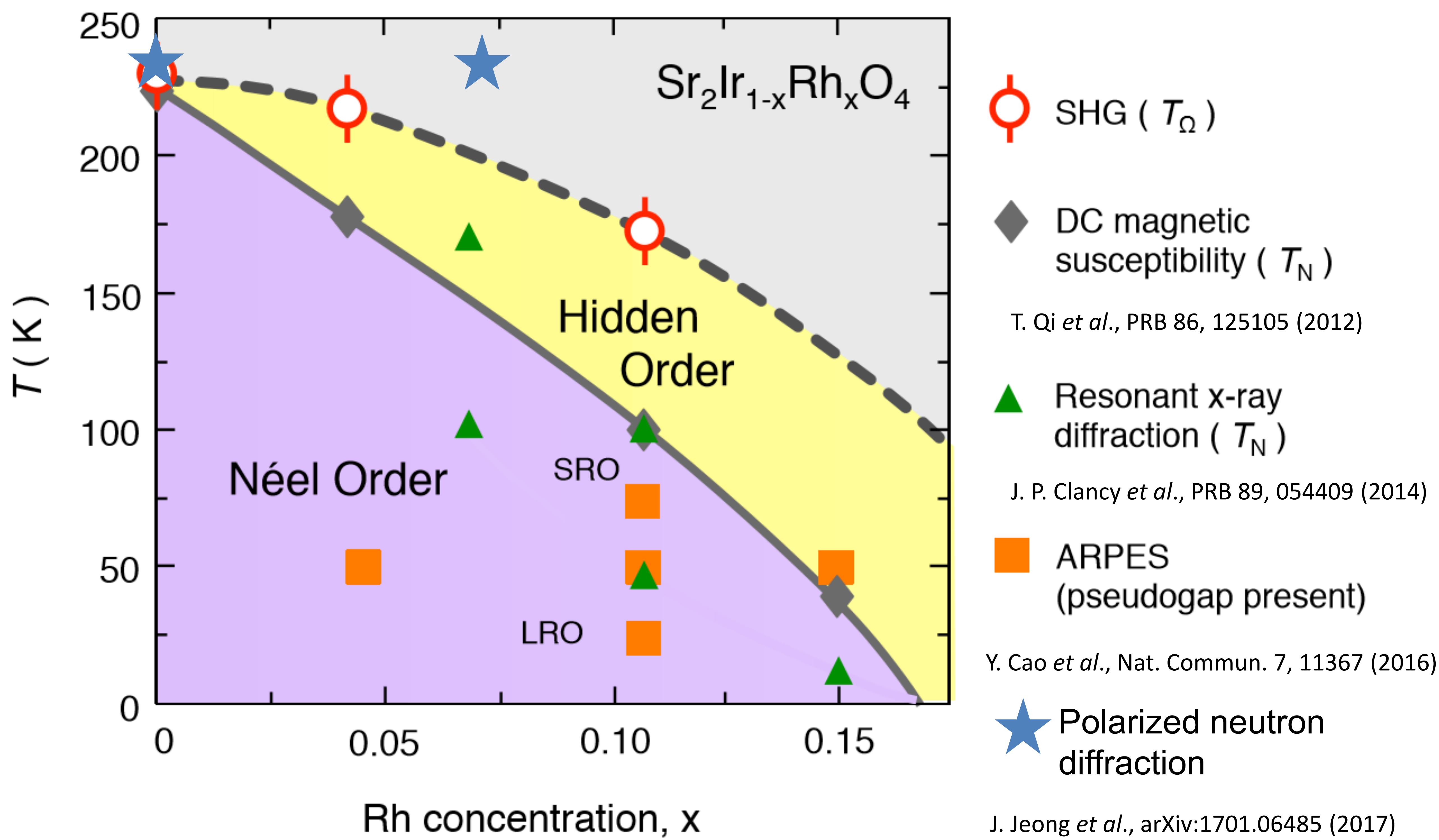
Phase diagram at small g with $\langle z_\alpha \rangle \neq 0$



Classical phase diagram of antiferromagnet with near-neighbor exchange J_p and ring-exchange K



Phase diagram of $Sr_2Ir_{1-x}Rh_xO_4$



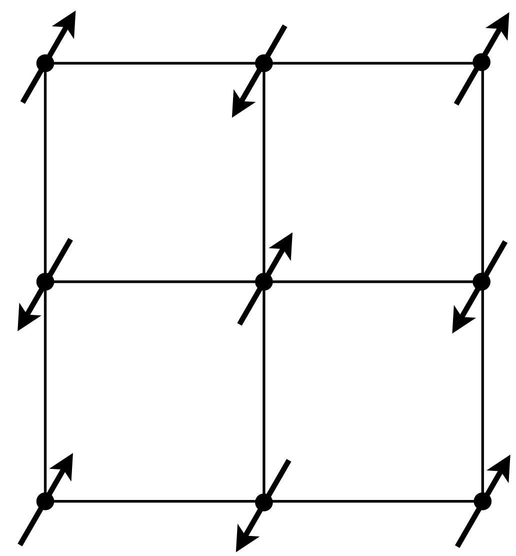
L. Zhao, D. H. Torchinsky, H. Chu, V. Ivanov, R. Lifshitz, R. Flint, T. Qi, G. Cao, and D. Hsieh, Nature Physics **12**, 32 (2016)

An attractive possibility at intermediate g with $\langle z_\alpha \rangle = 0$

s_2

s_1

$\langle P \rangle \neq 0 \quad , \quad \langle \vec{Q} \rangle \neq 0$



$$\left\langle z_\alpha^* \sigma^a_{\alpha\beta} z_\beta \right\rangle \neq 0$$

$$\langle z_\alpha \rangle = 0$$

<u>The AF* state:</u>

co-existing

$\mathbb{Z}_2$ topological

and Néel order.

Gapped $S = 1/2$ spinons

and gapless spin waves.

T.Senthil and M.P.A. Fisher, 2000

Fractional excitations in the square-lattice quantum antiferromagnet

B. Dalla Piazza, M. Mourigal, N. B. Christensen, G. J. Nilsen, P. Tregenna-Piggott, T. G. Perring, M. Enderle, D. F. McMorrow, D. A. Ivanov, and H. M. Rønnow,
Nature Physics **11**, 62 (2015)

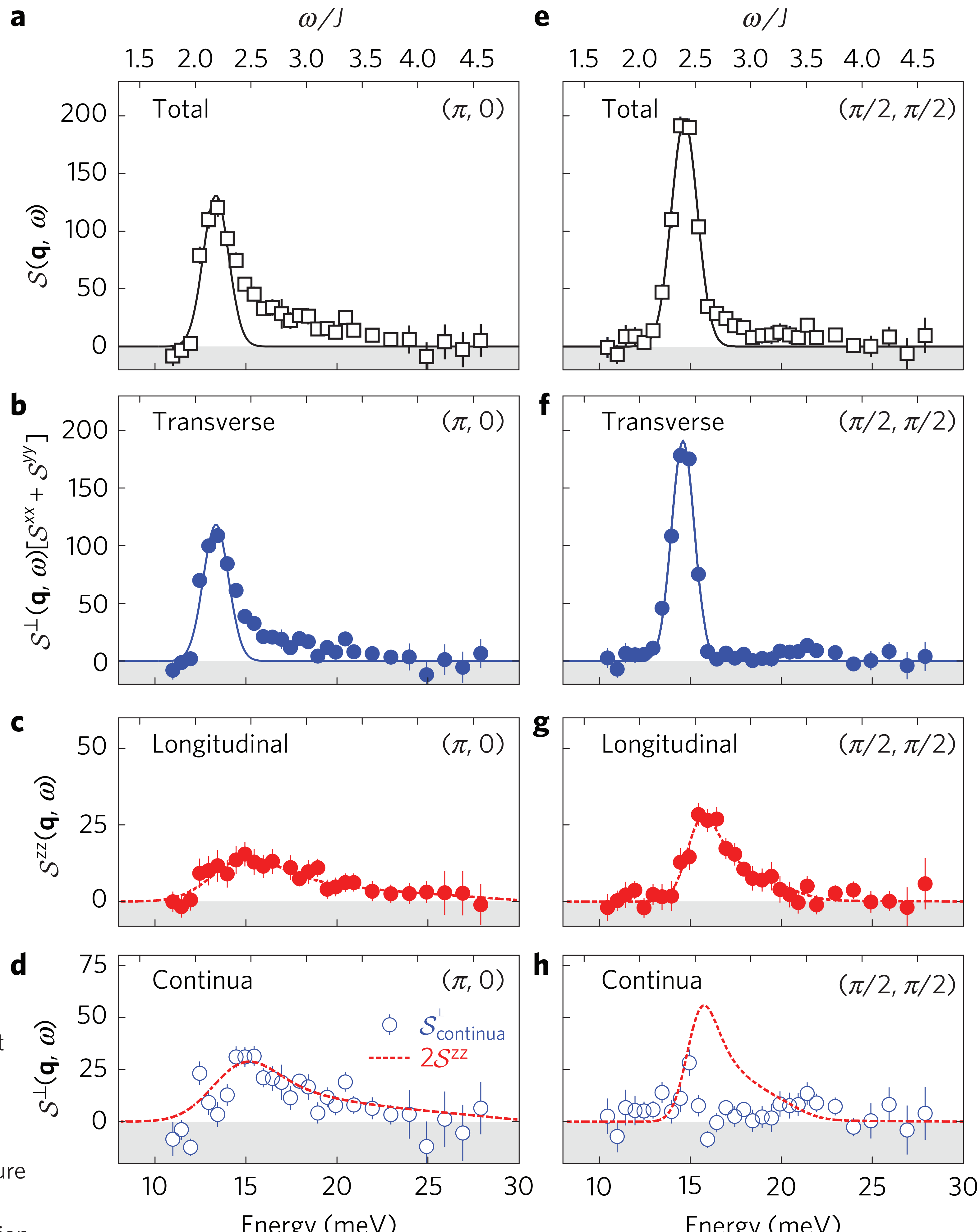


Figure 2 | Summary of the polarized neutron scattering data. **a–c**,**e–g**, Energy dependence of the total, transverse and longitudinal contributions to the dynamic structure factor, respectively, at constant wavevectors $\mathbf{q} = (\pi, 0)$ (**a–c**) and $\mathbf{q} = (\pi/2, \pi/2)$ (**e–g**) measured by polarized neutron scattering on CFTD. The solid lines indicate resolution-limited Gaussian fits, while the dashed lines are empirical lineshapes used as guides-to-the-eye. **d**,**h**, Transverse dynamic structure factor with subtracted resolution-limited Gaussian fits at $(\pi, 0)$ and $(\pi/2, \pi/2)$, respectively. Error bars correspond to one standard deviation.