Quantum Entanglement:
from quantum matter
to
string theory

Caneel Bay, February 4-8, 2013

Subir Sachdev

PHYSICS

HARVARD

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement

Gapped quantum matter

Spin liquids, quantum Hall states....

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

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Vishwanath, Wen, Senthil, Oshikawa

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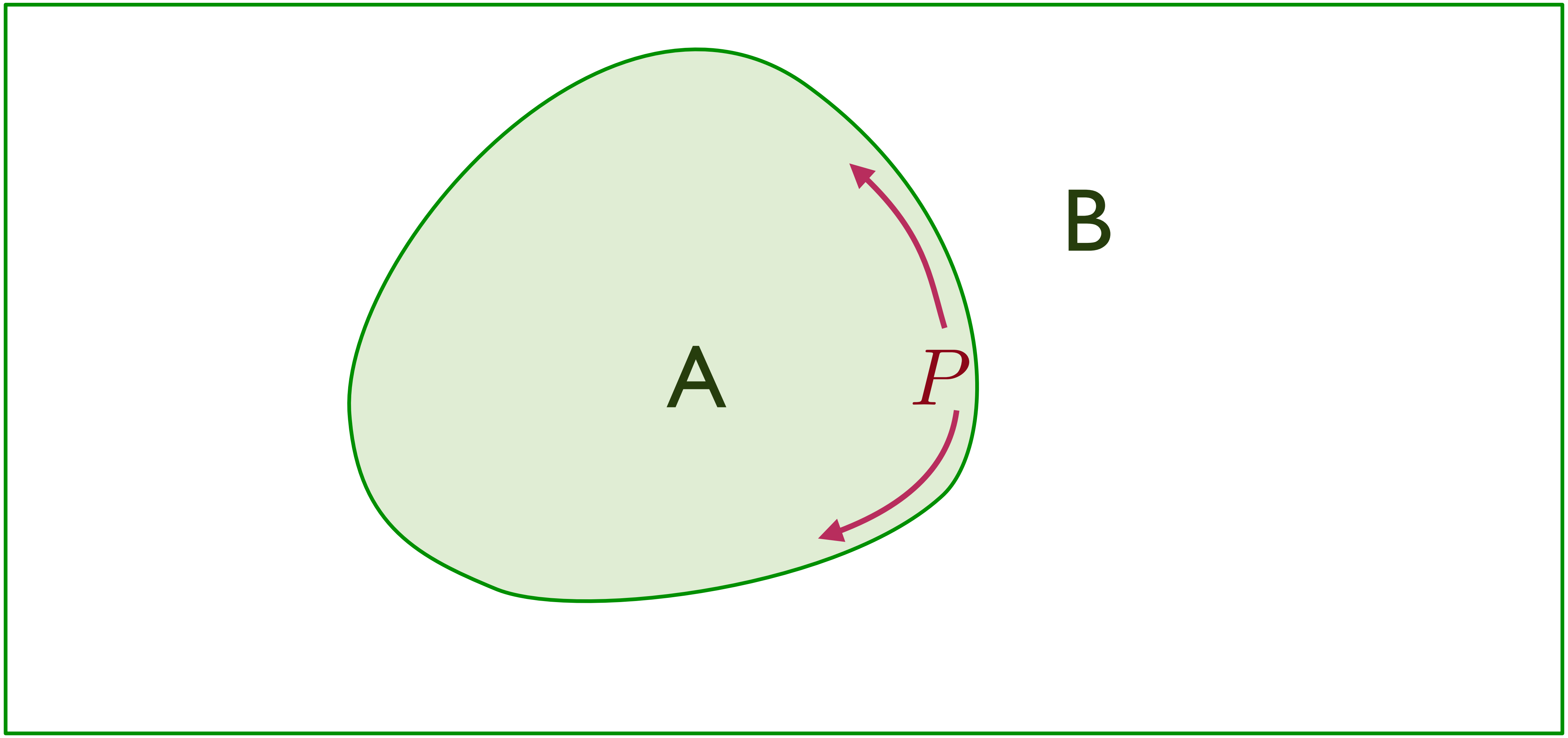
Entanglement entropy



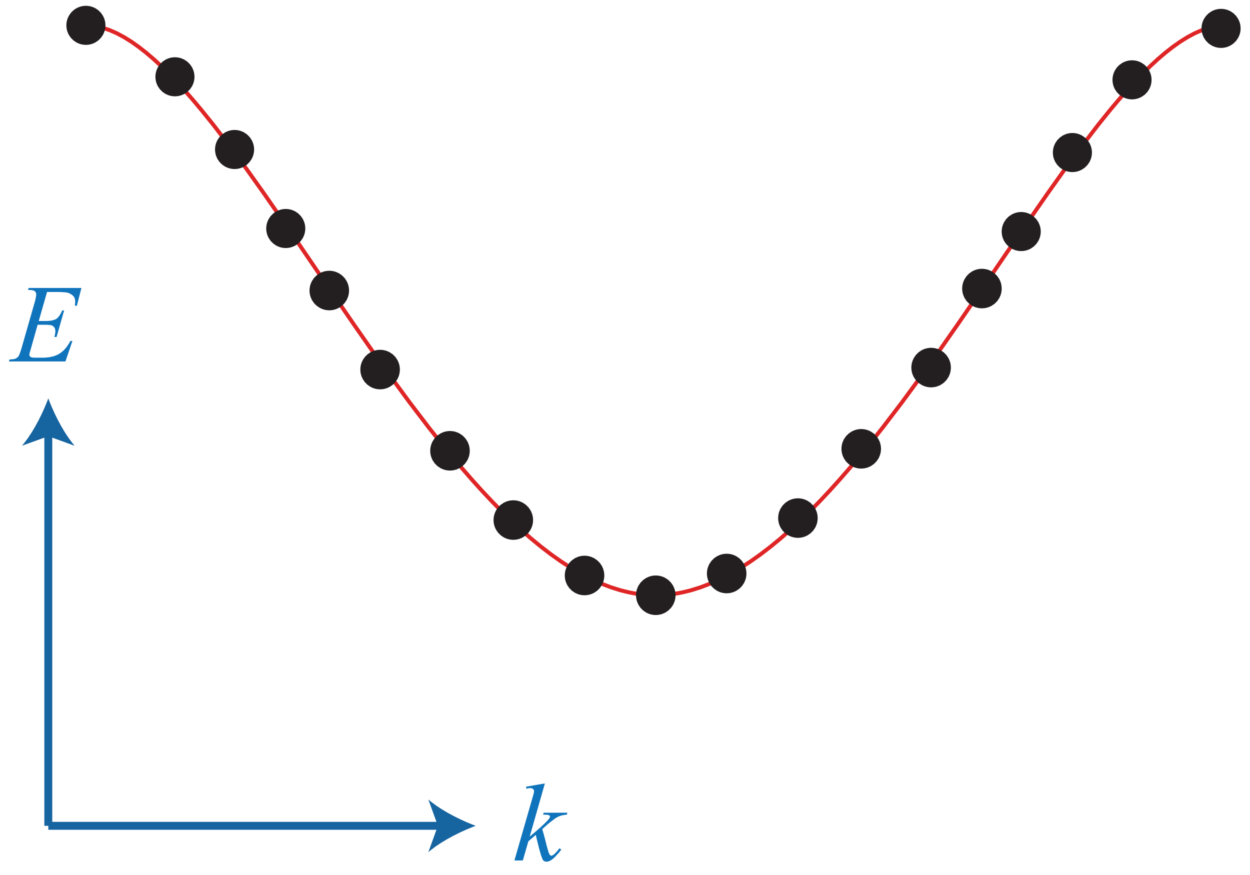
$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

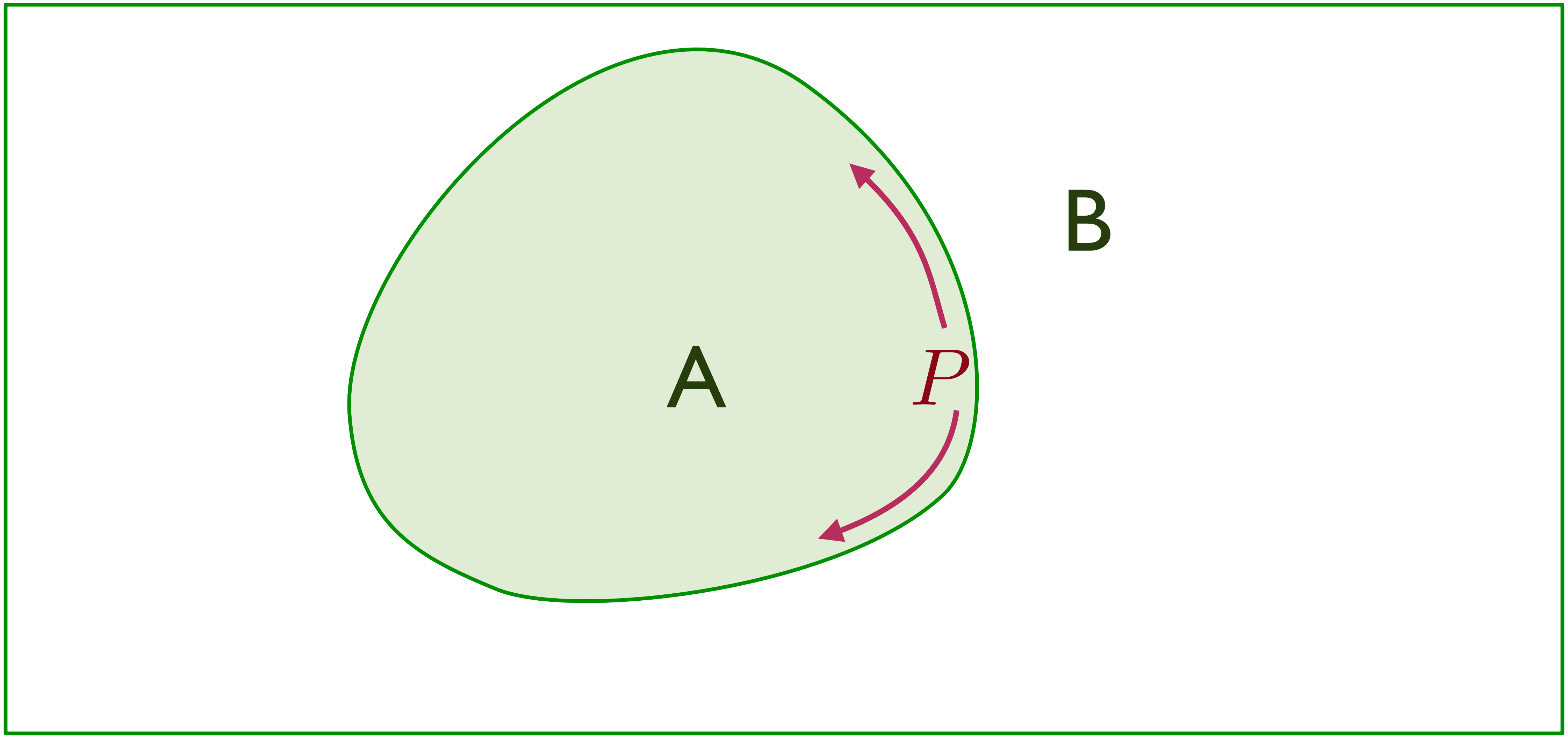
Entanglement entropy of a band insulator

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator

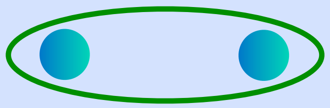


$$\boxed{S_E = aP - b\exp(-cP)}$$

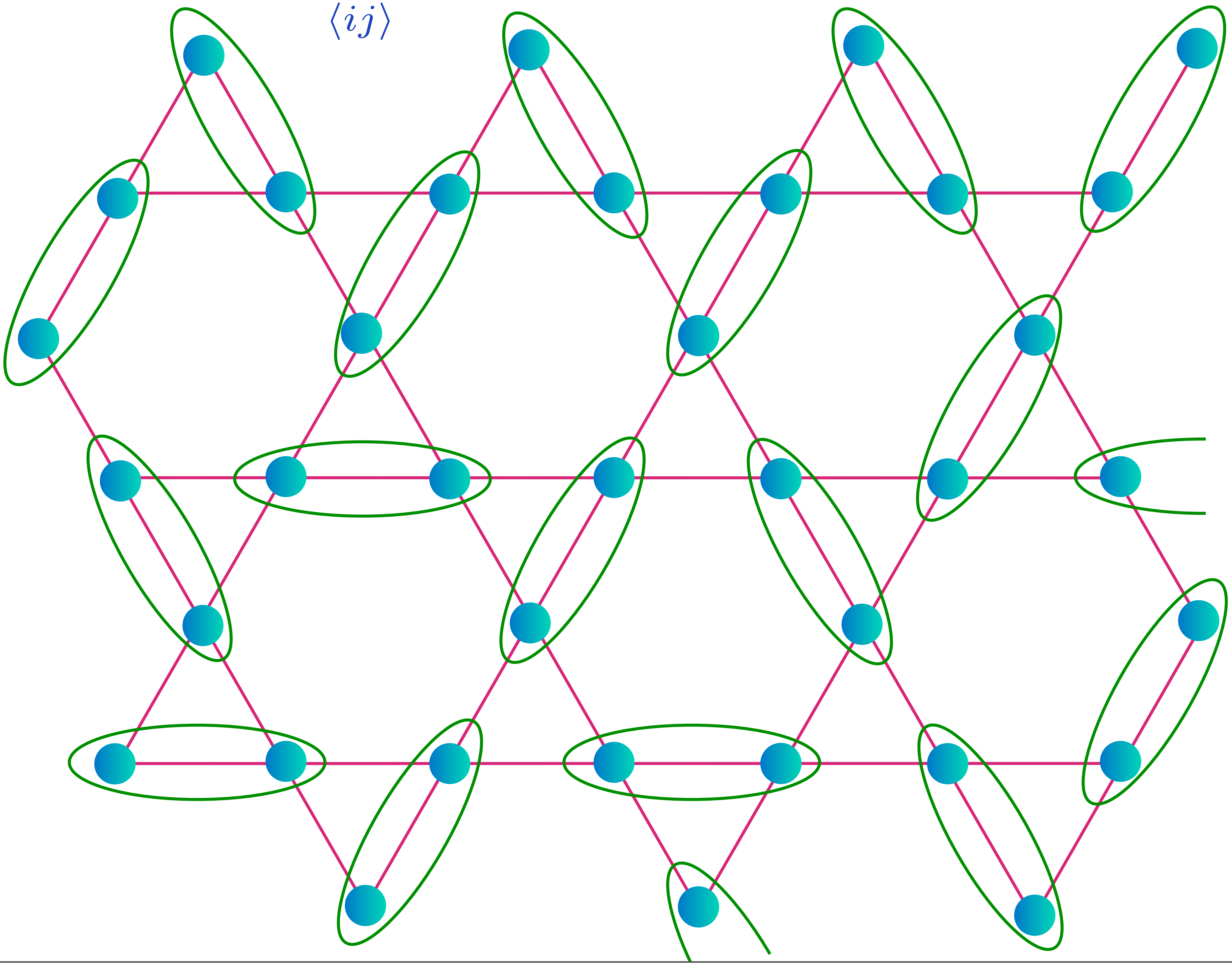
where P is the surface area (perimeter) of the boundary between A and B.

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



$$= \frac{1}{\sqrt{2}} \Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \Big)$$



P. Fazekas and
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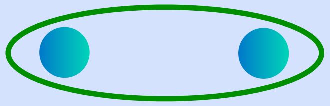

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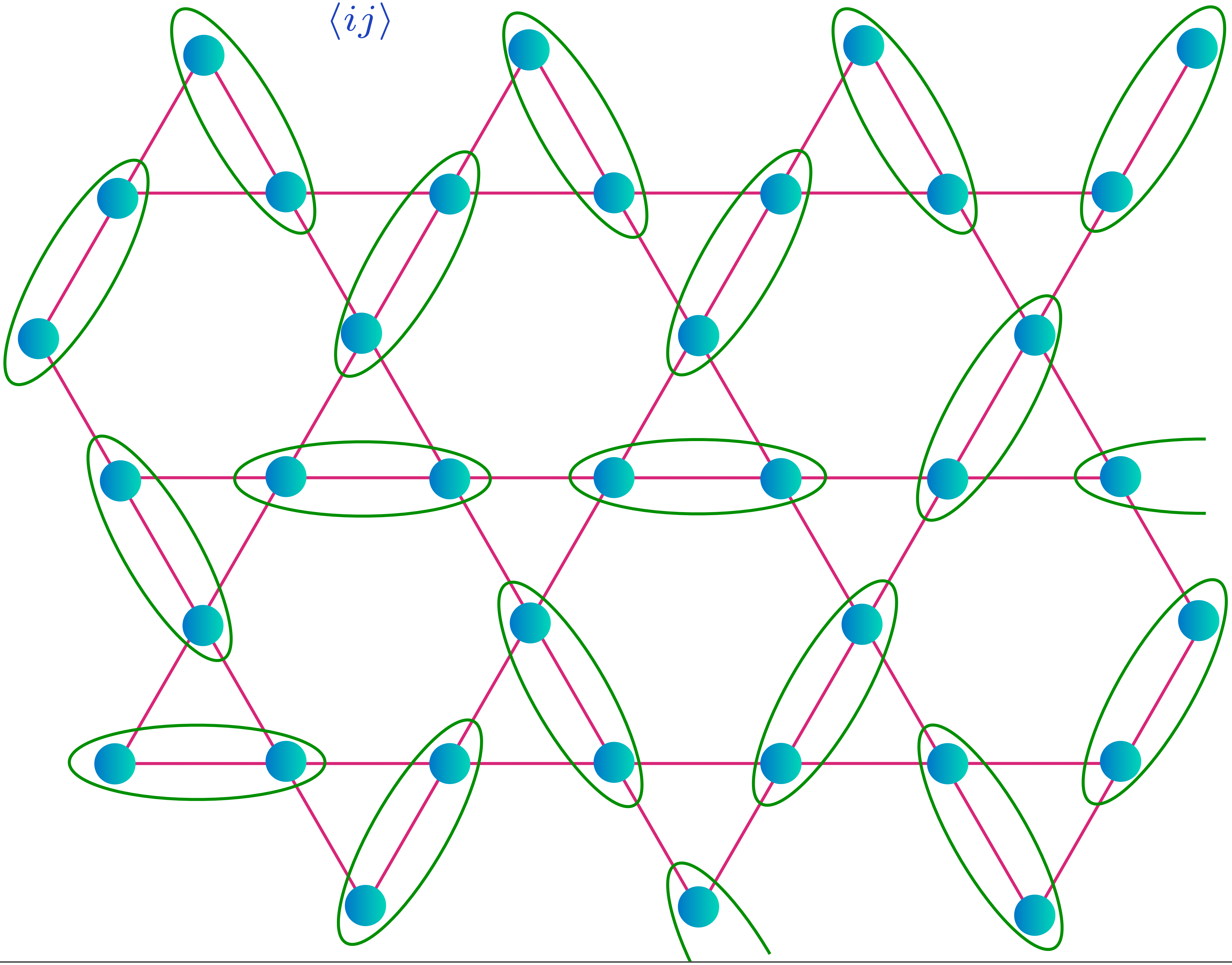

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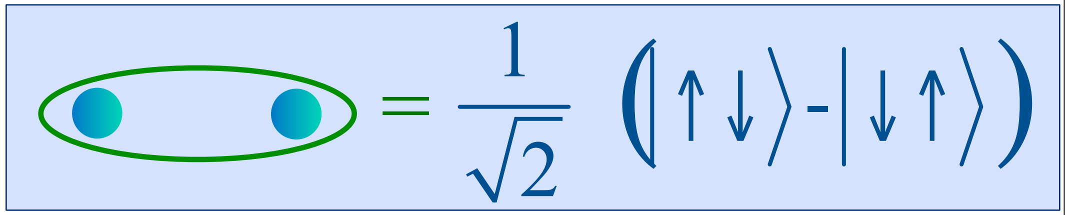

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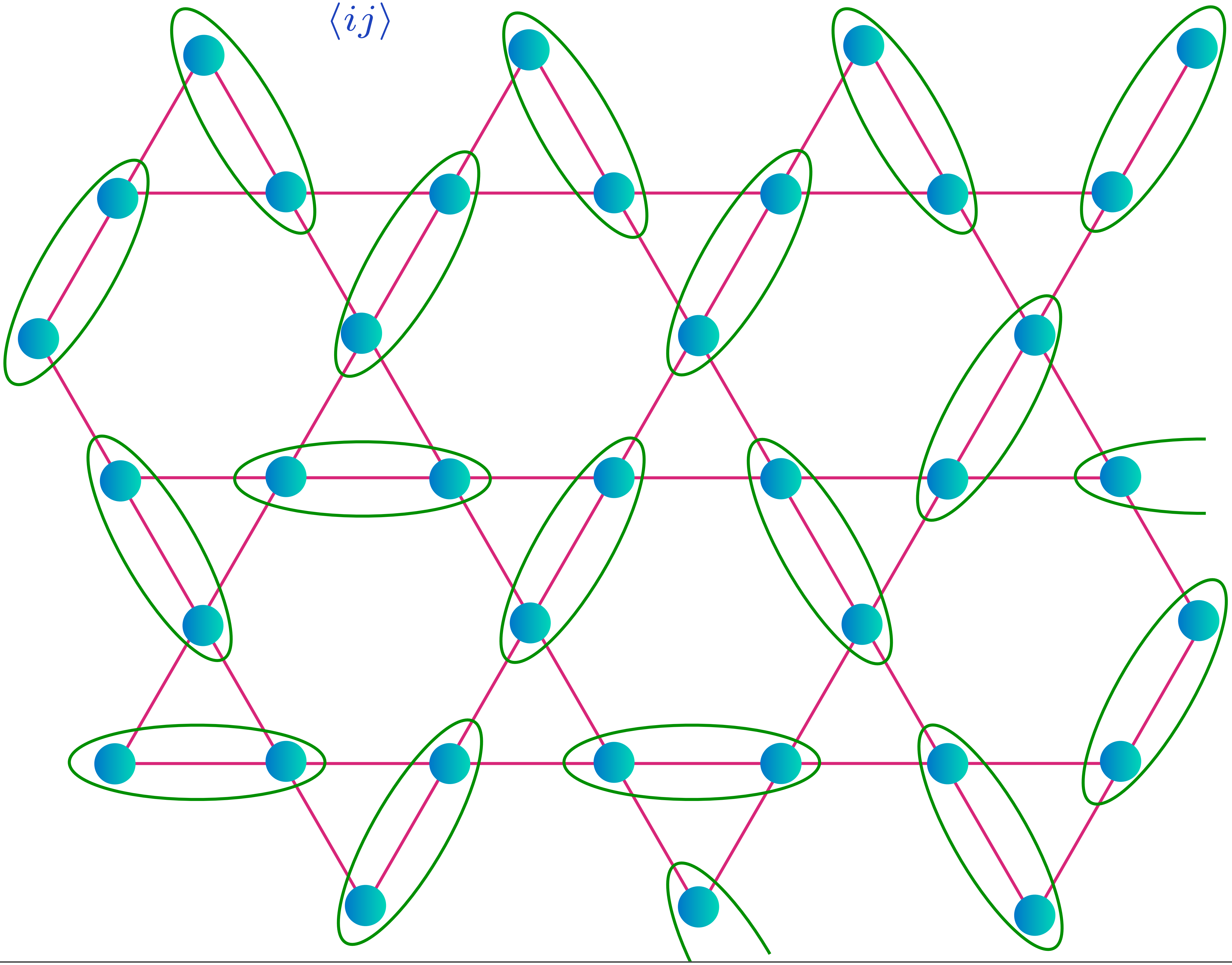

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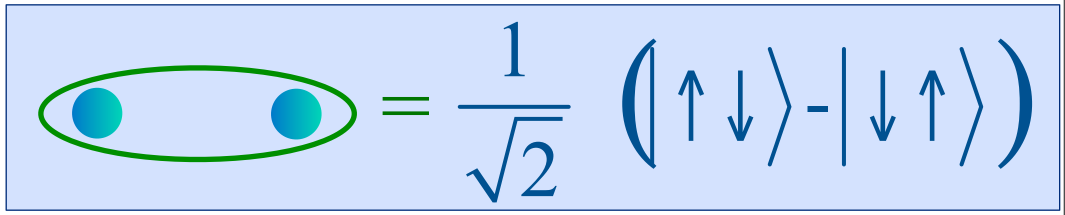

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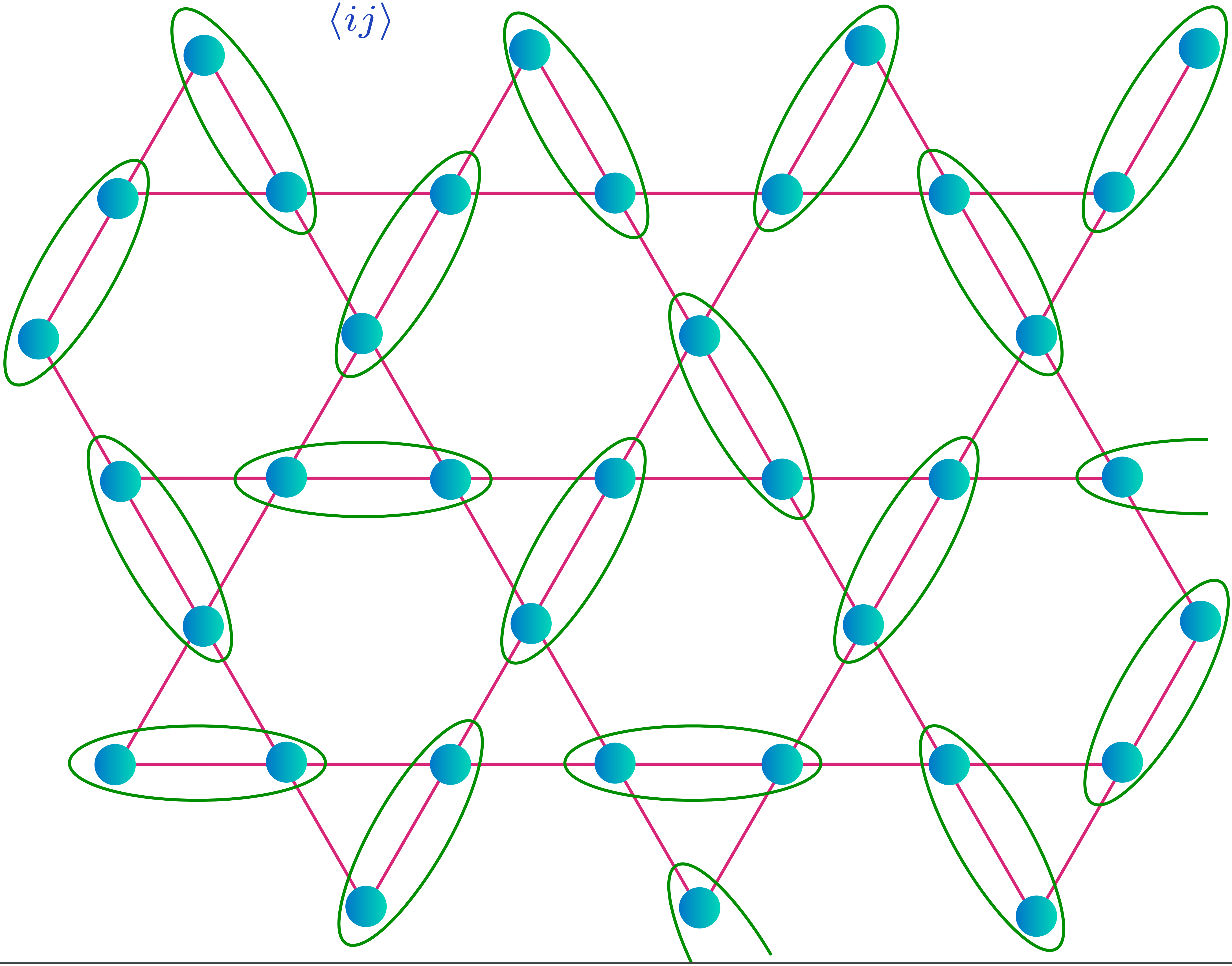

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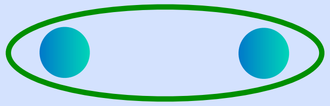

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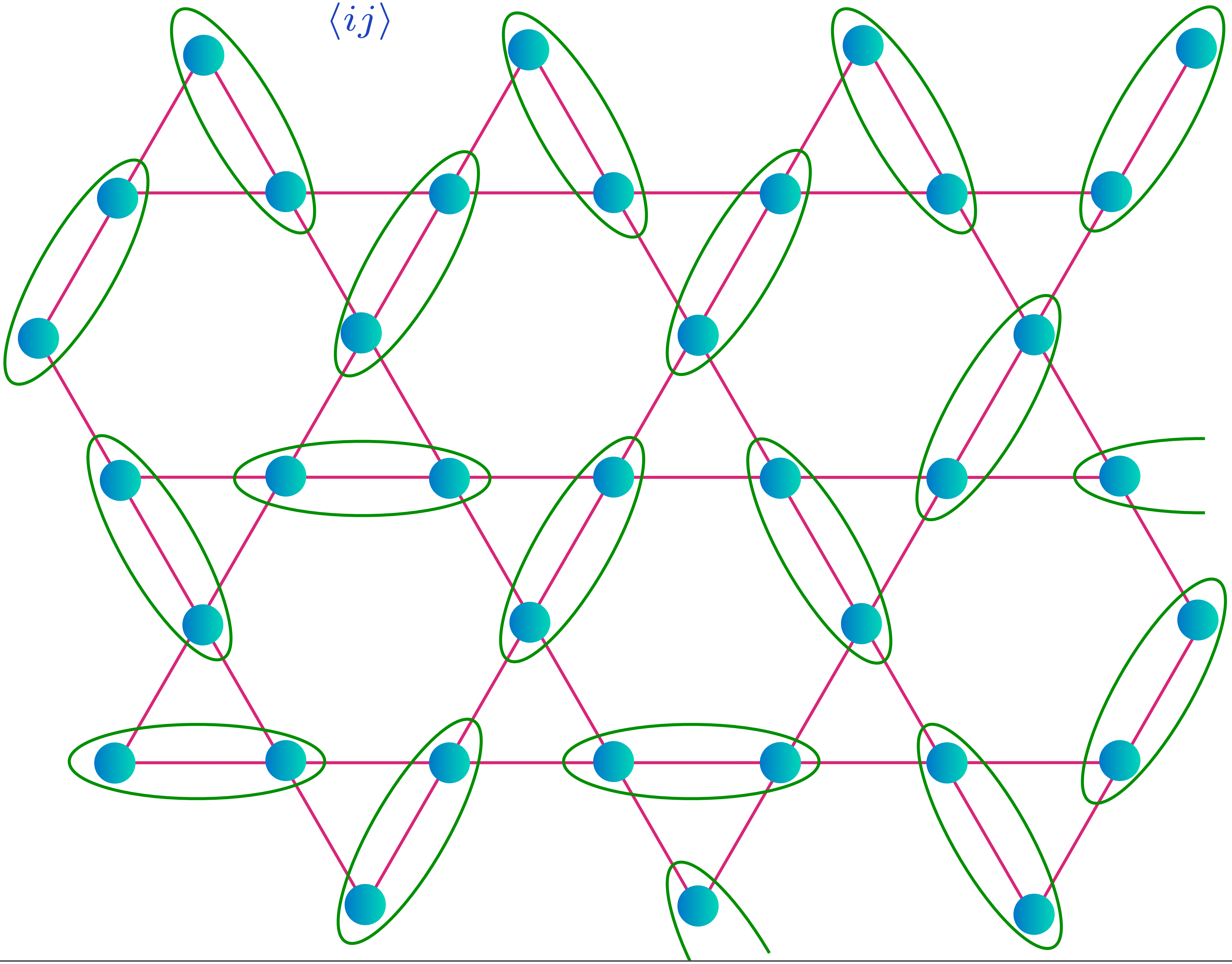

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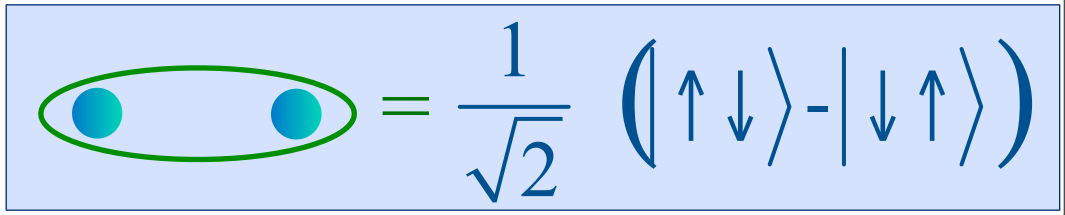

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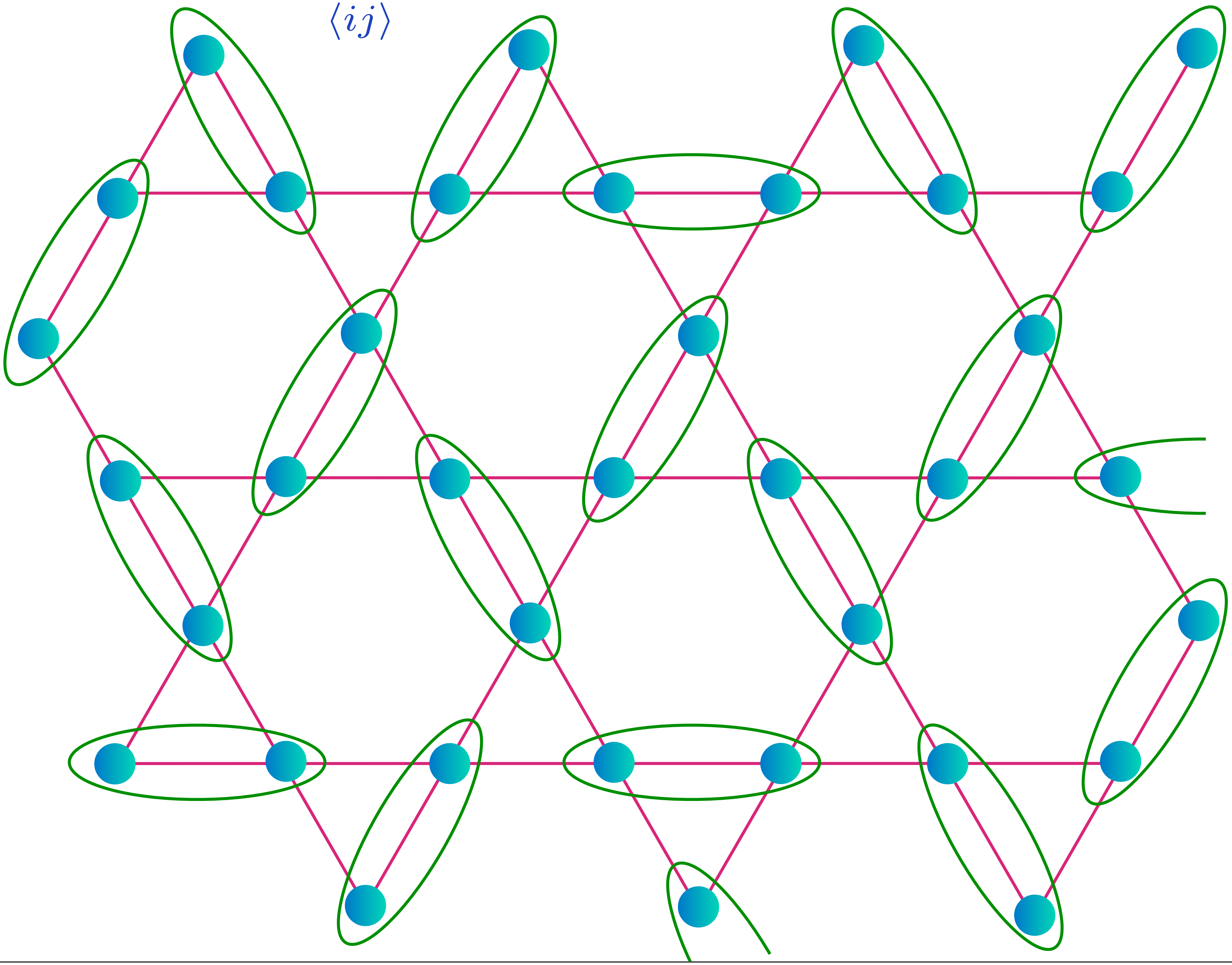

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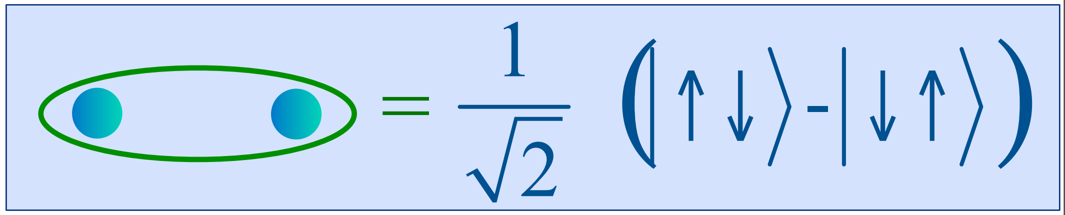

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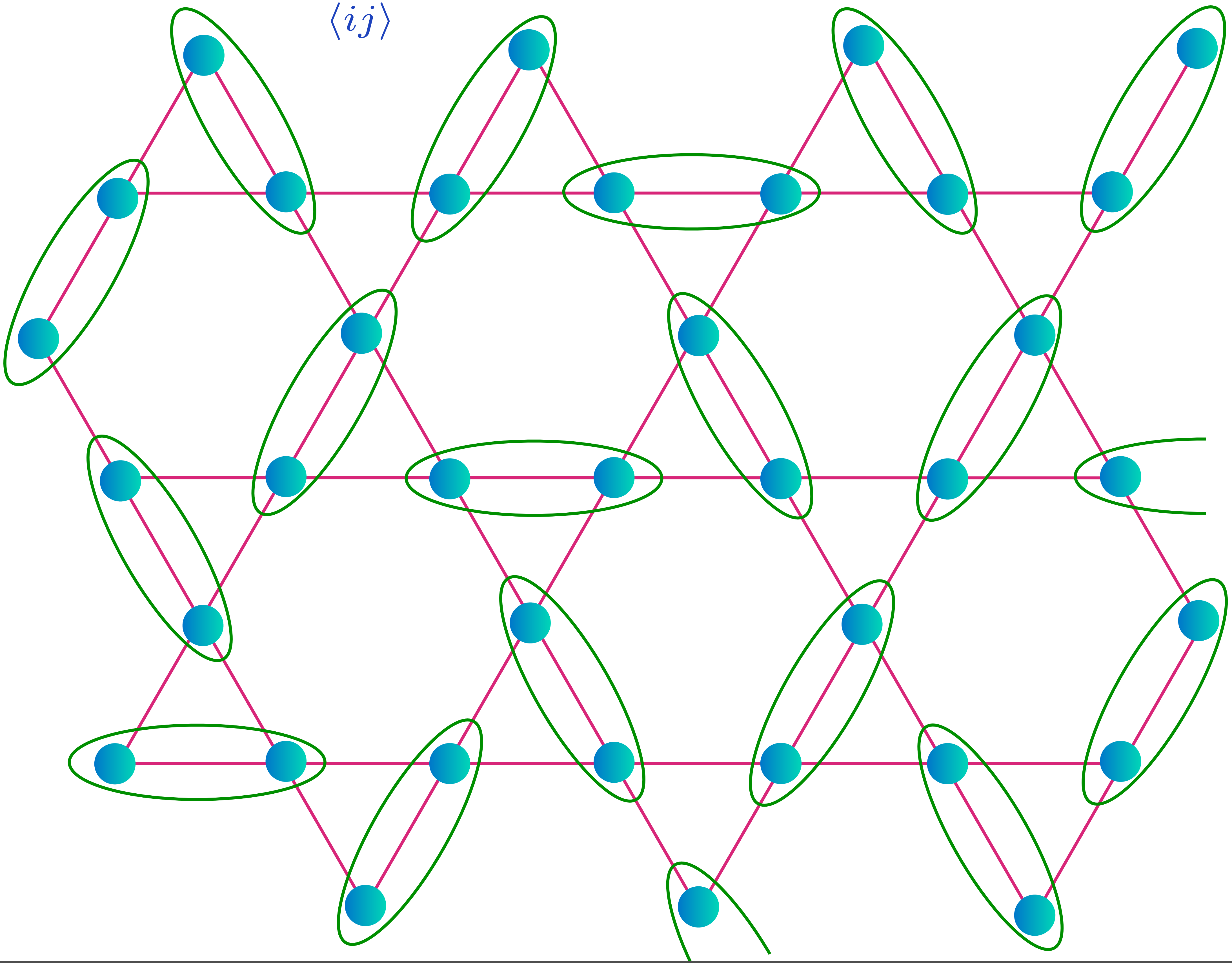

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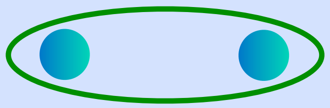

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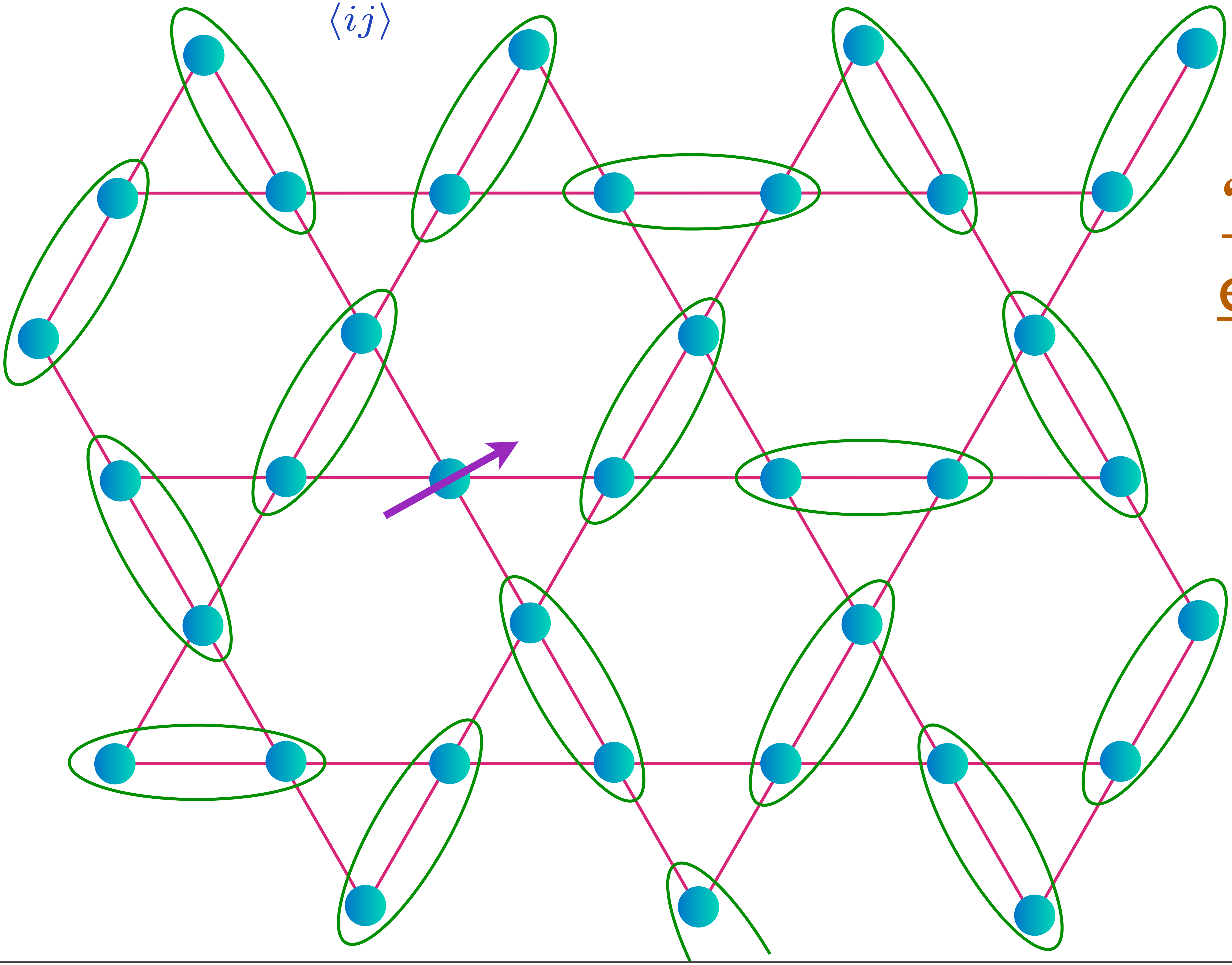

“Electric”
excitation
S=1/2,
charge 0

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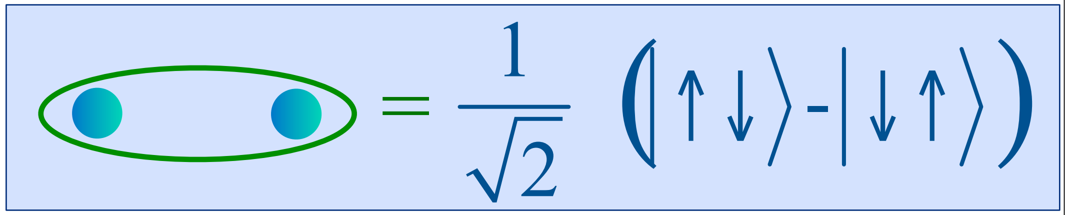

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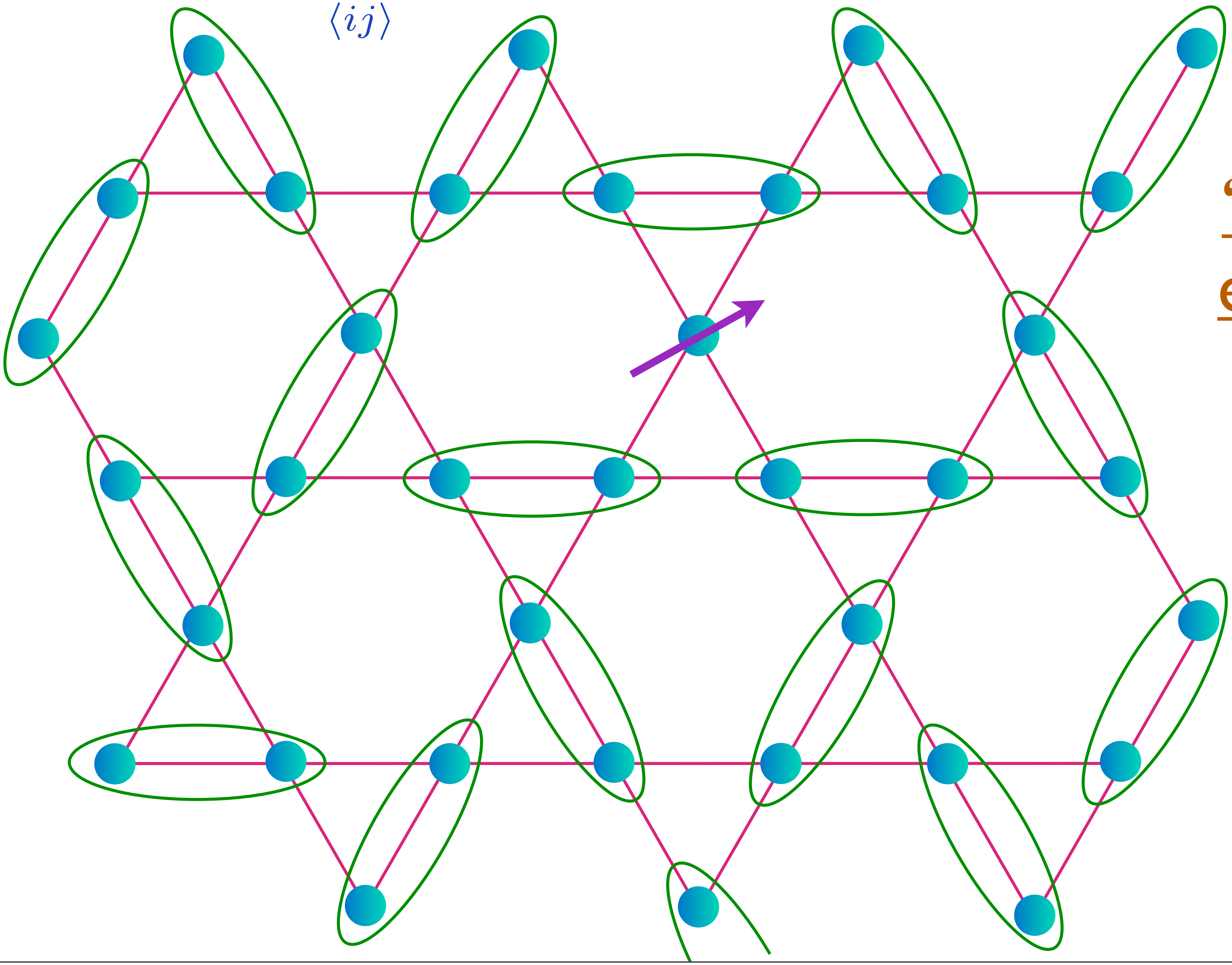

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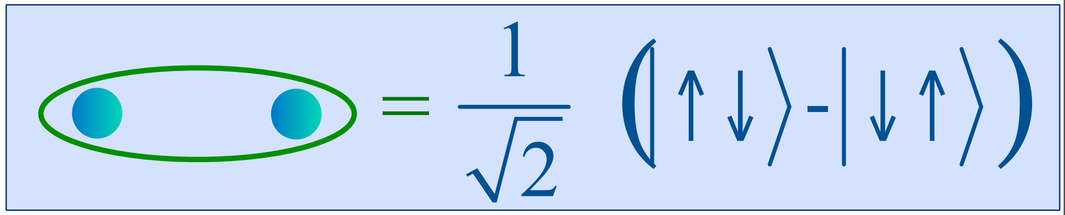

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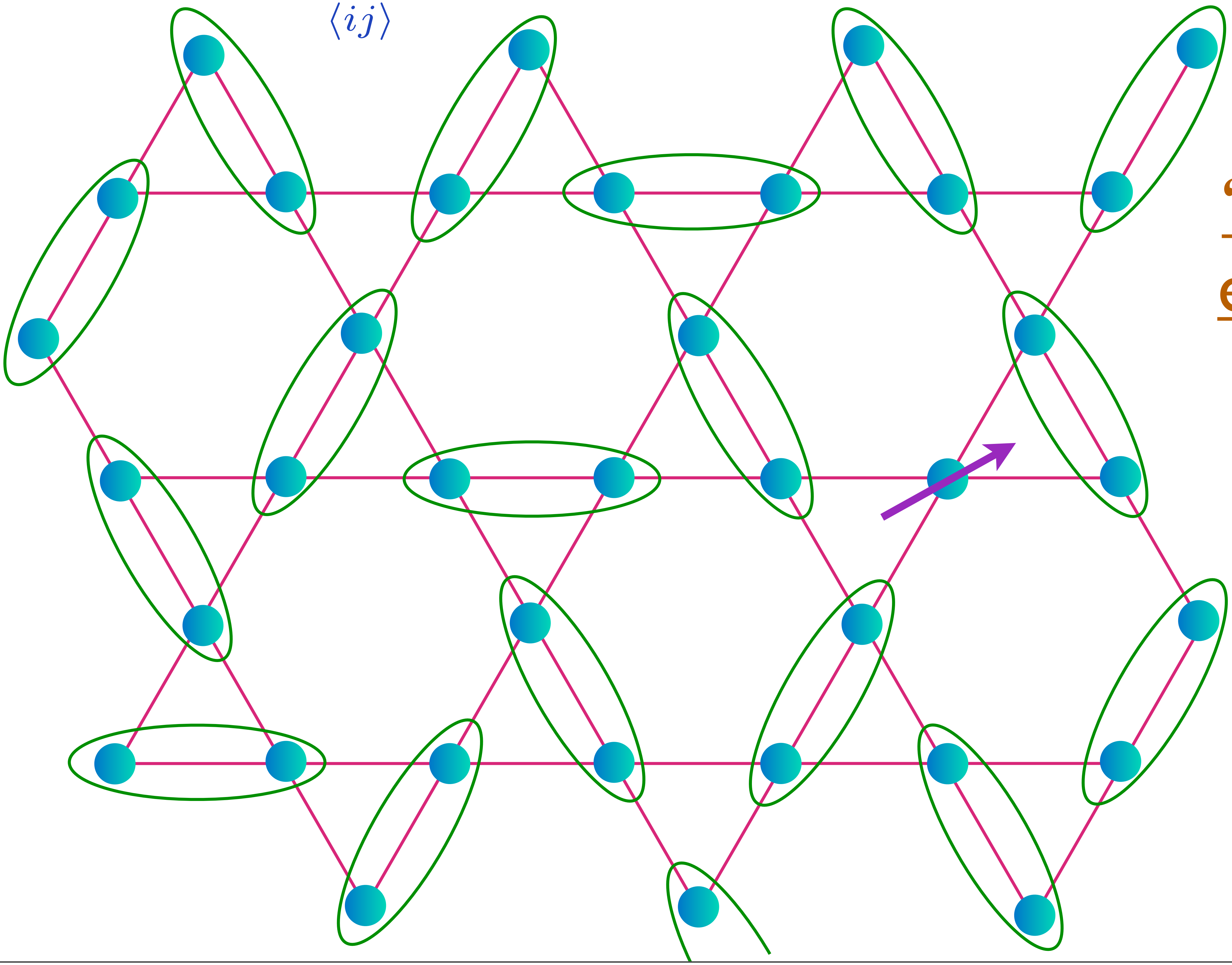

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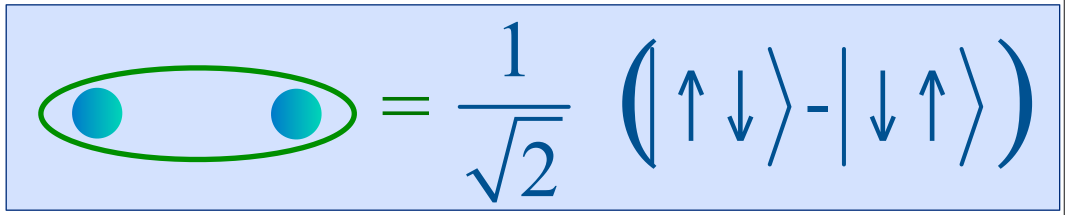

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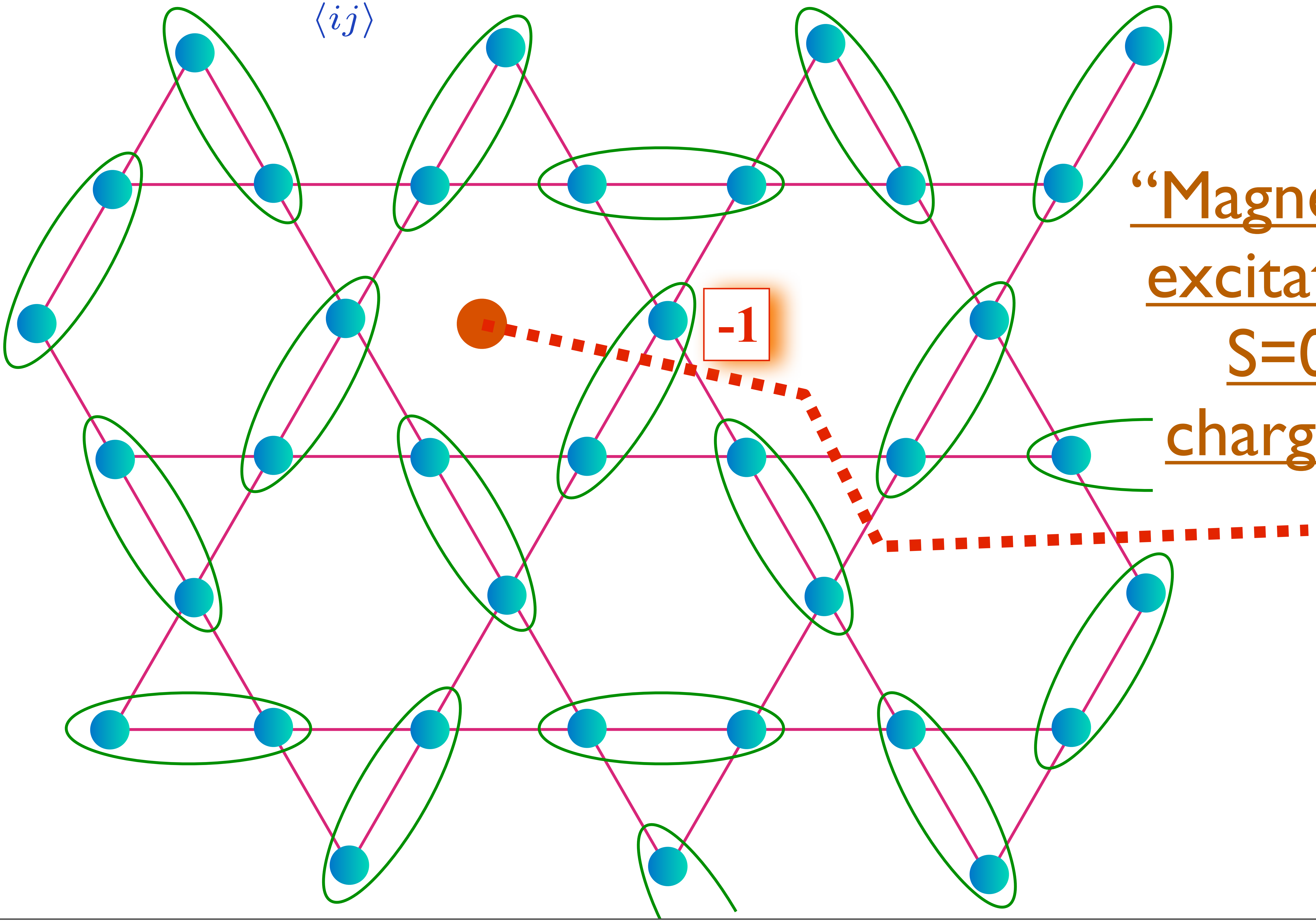

“Magnetic” excitation
S=0,
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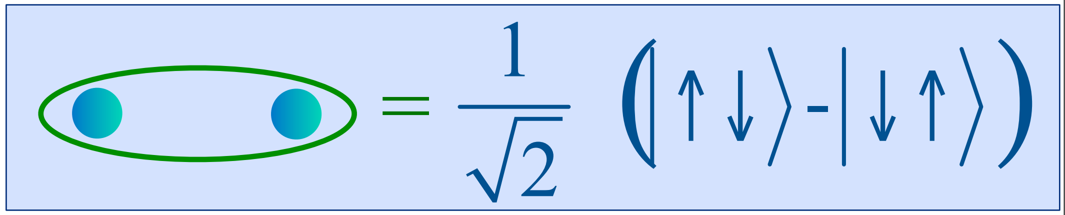

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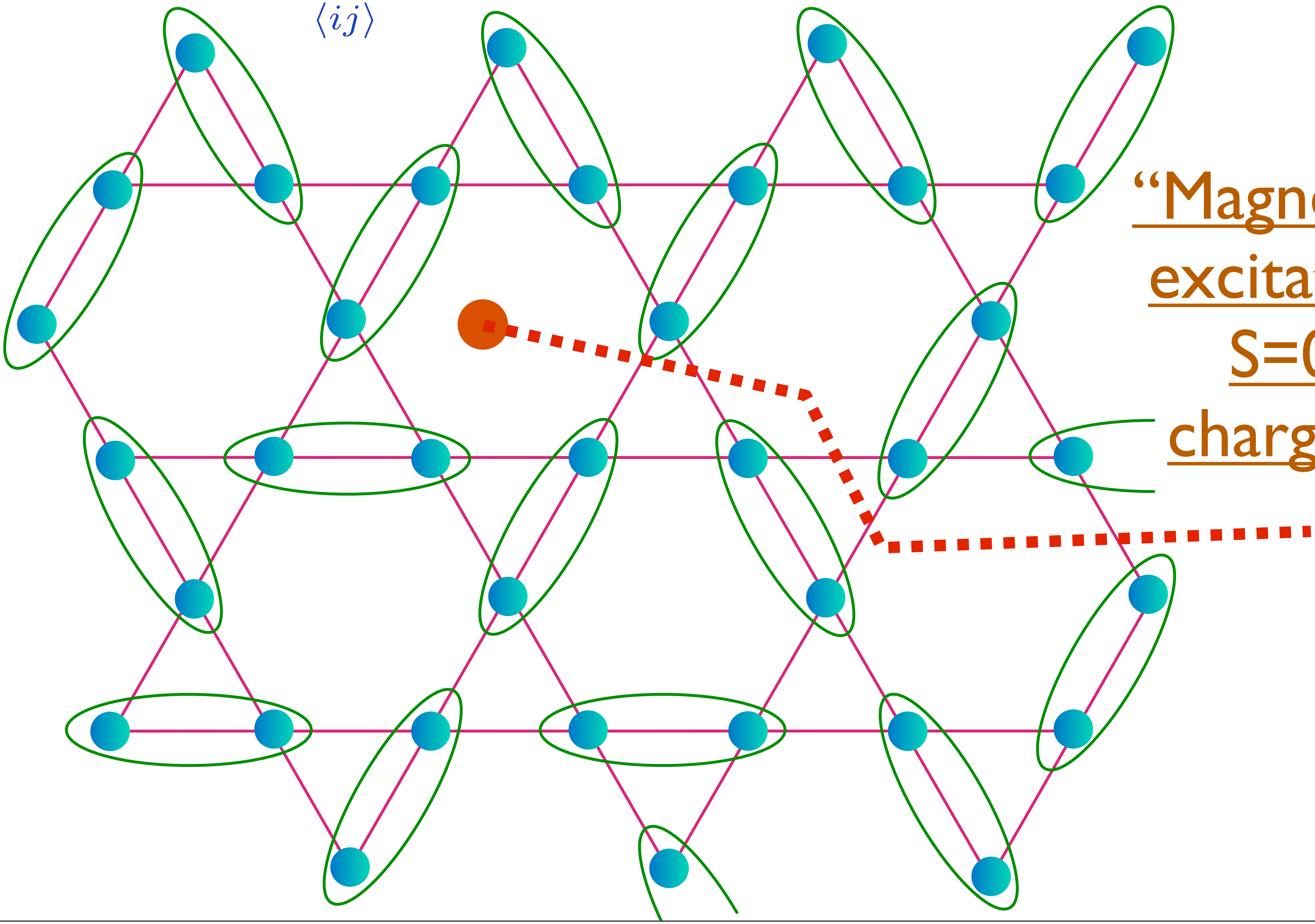

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S=0, charge 0

Monday, February 4, 13

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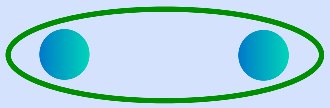

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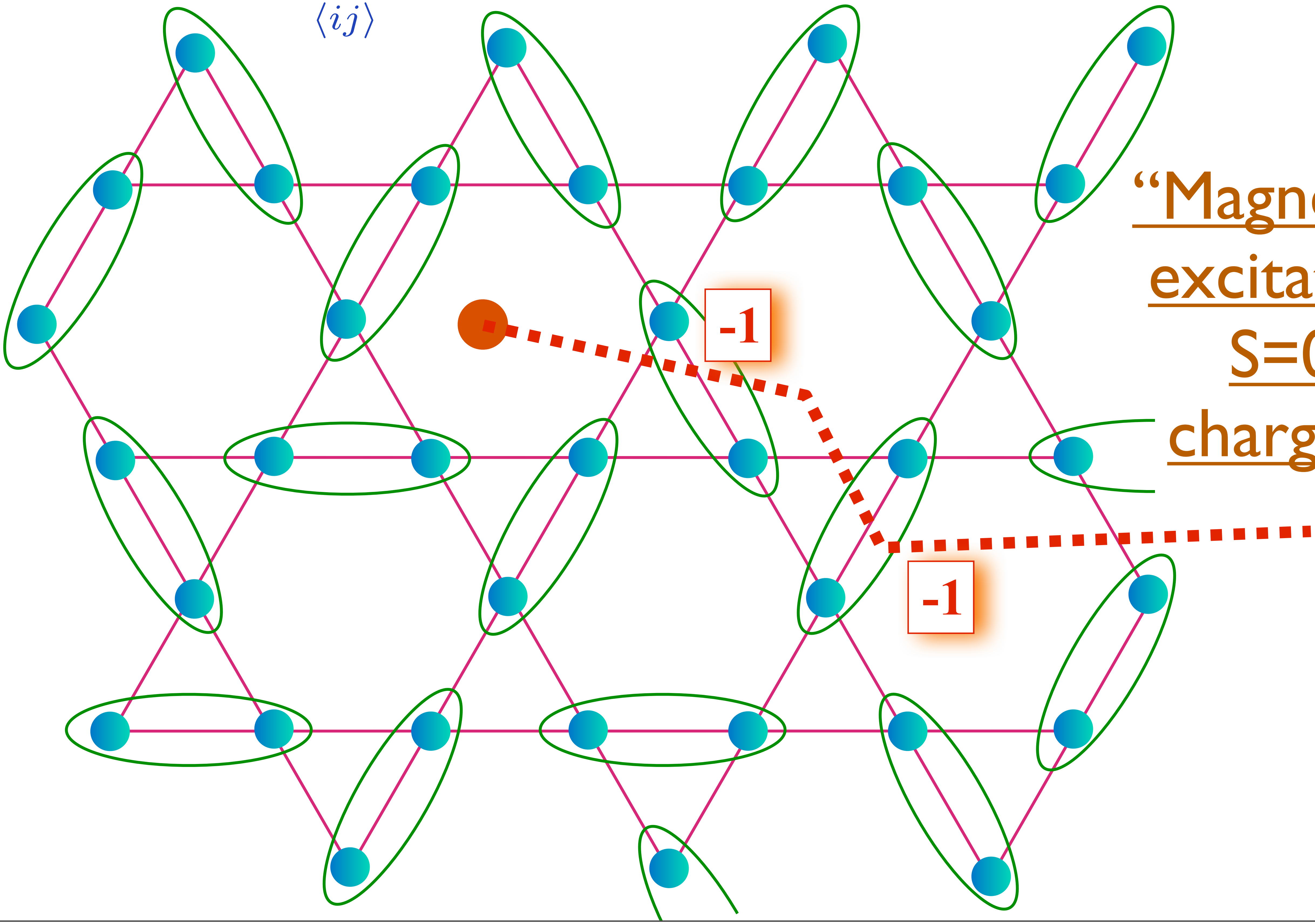
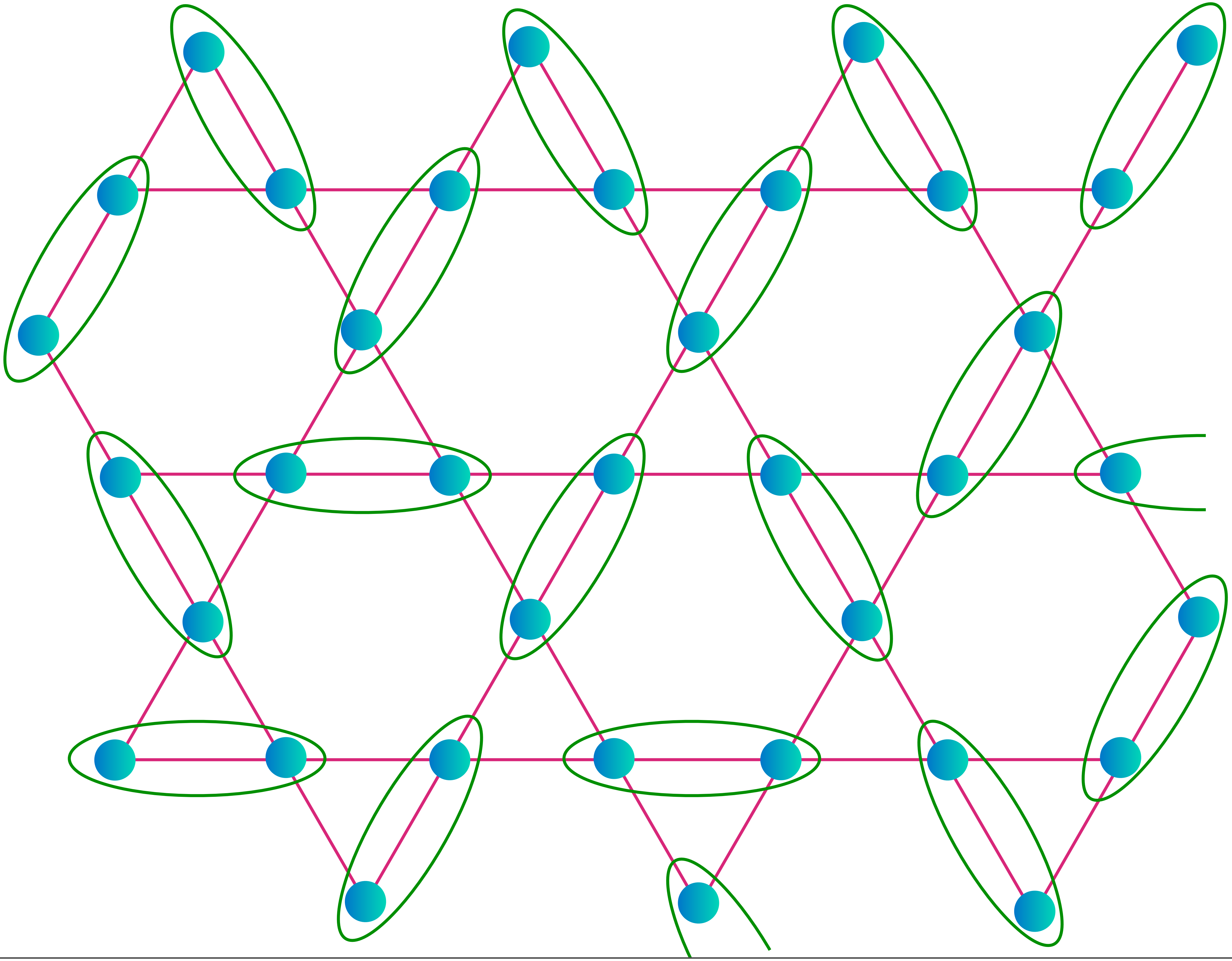


“Magnetic” excitation S=0, charge 0

Mott insulator: Kagome antiferromagnet

Alternative view

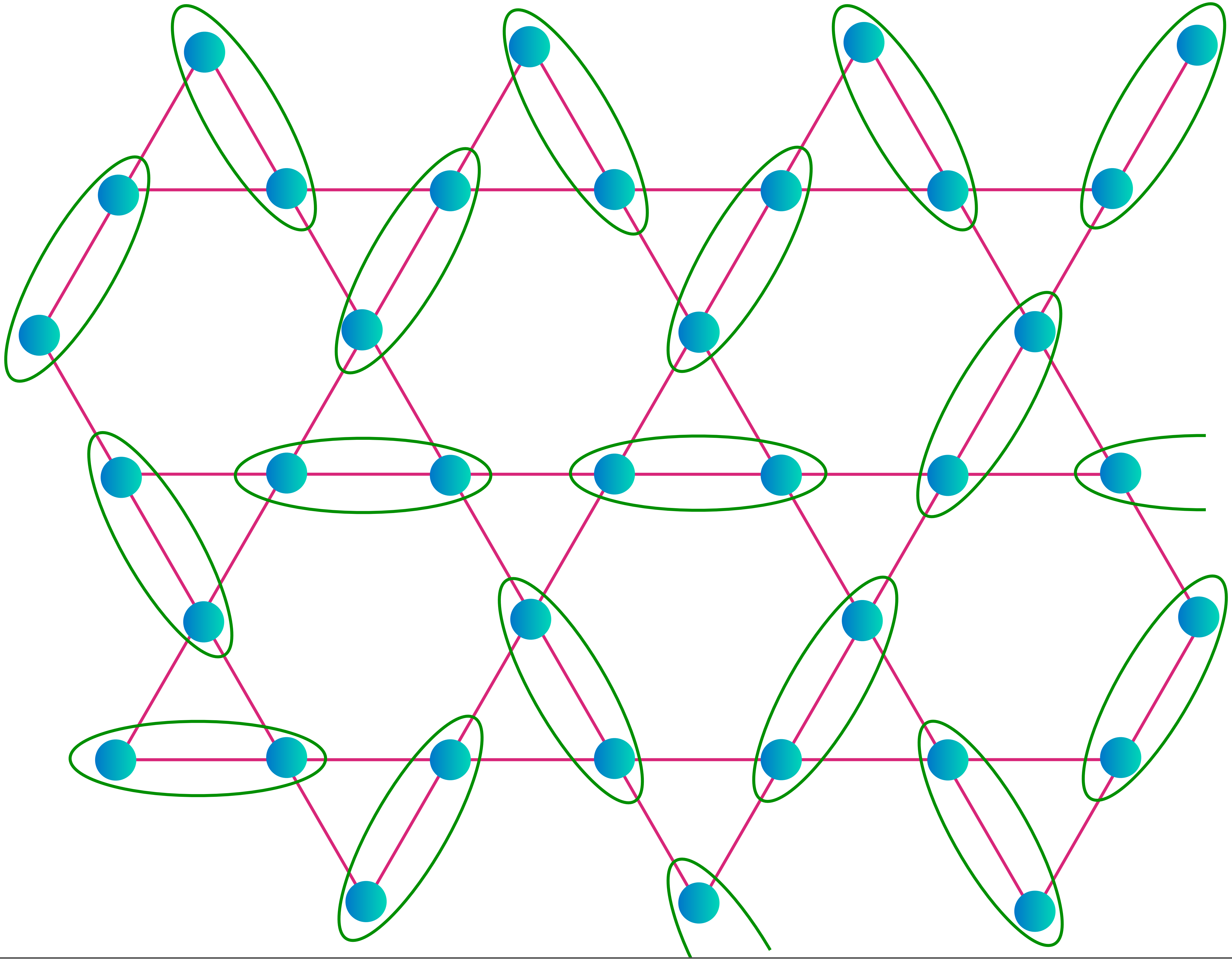
Pick a reference configuration



Mott insulator: Kagome antiferromagnet

Alternative view

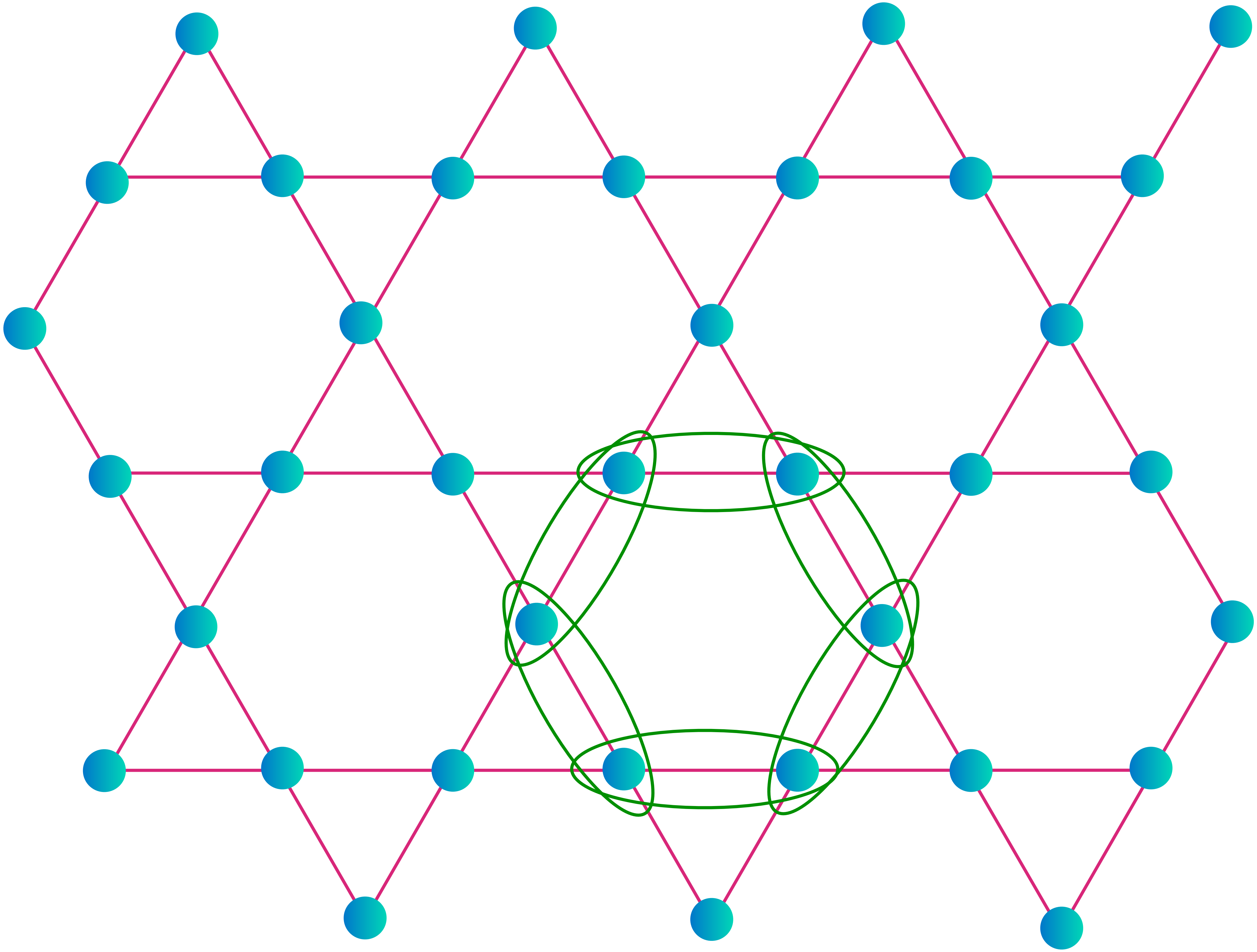
A nearby configuration



Mott insulator: Kagome antiferromagnet

Alternative view

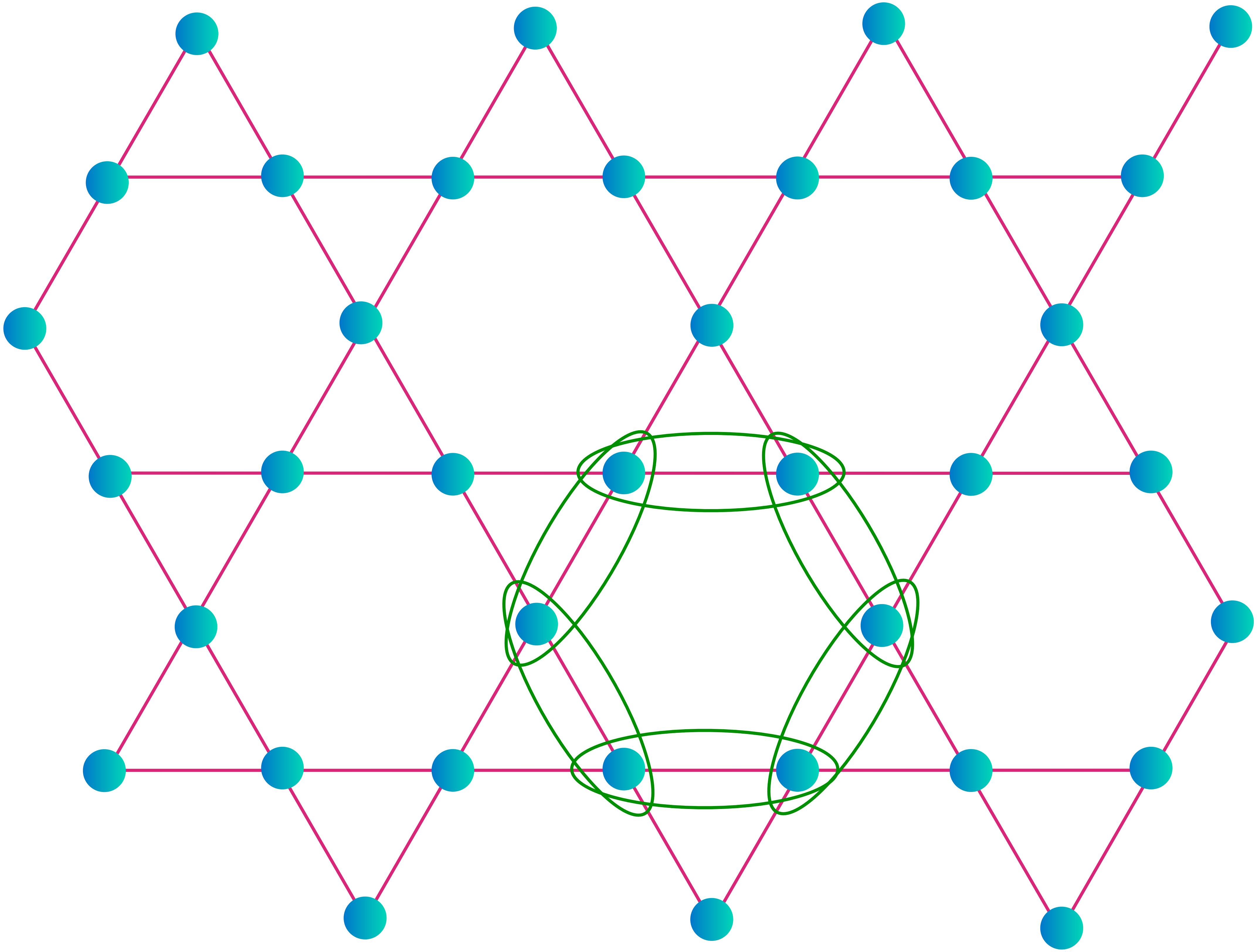
Difference: a closed loop



Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops



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Alternative view

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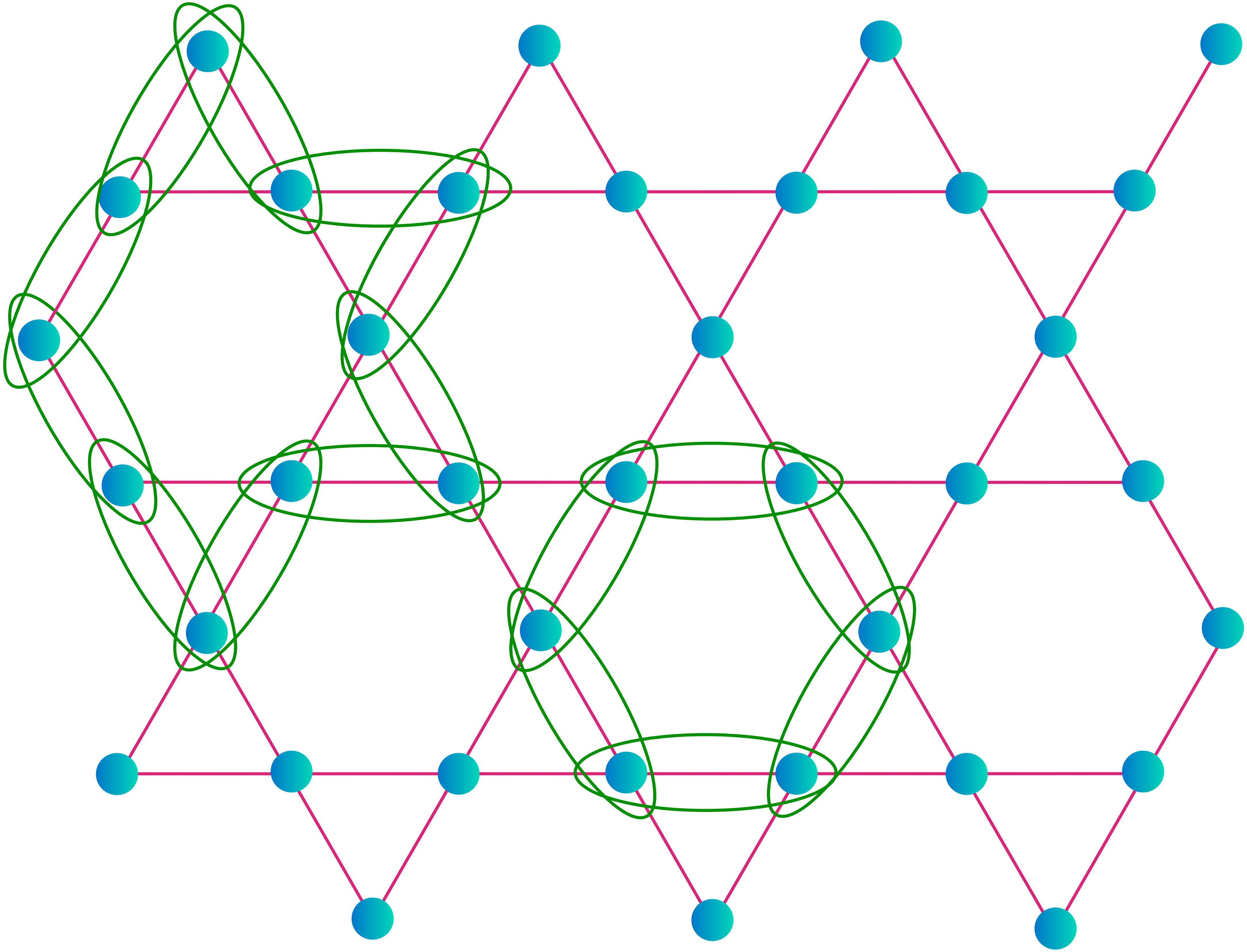
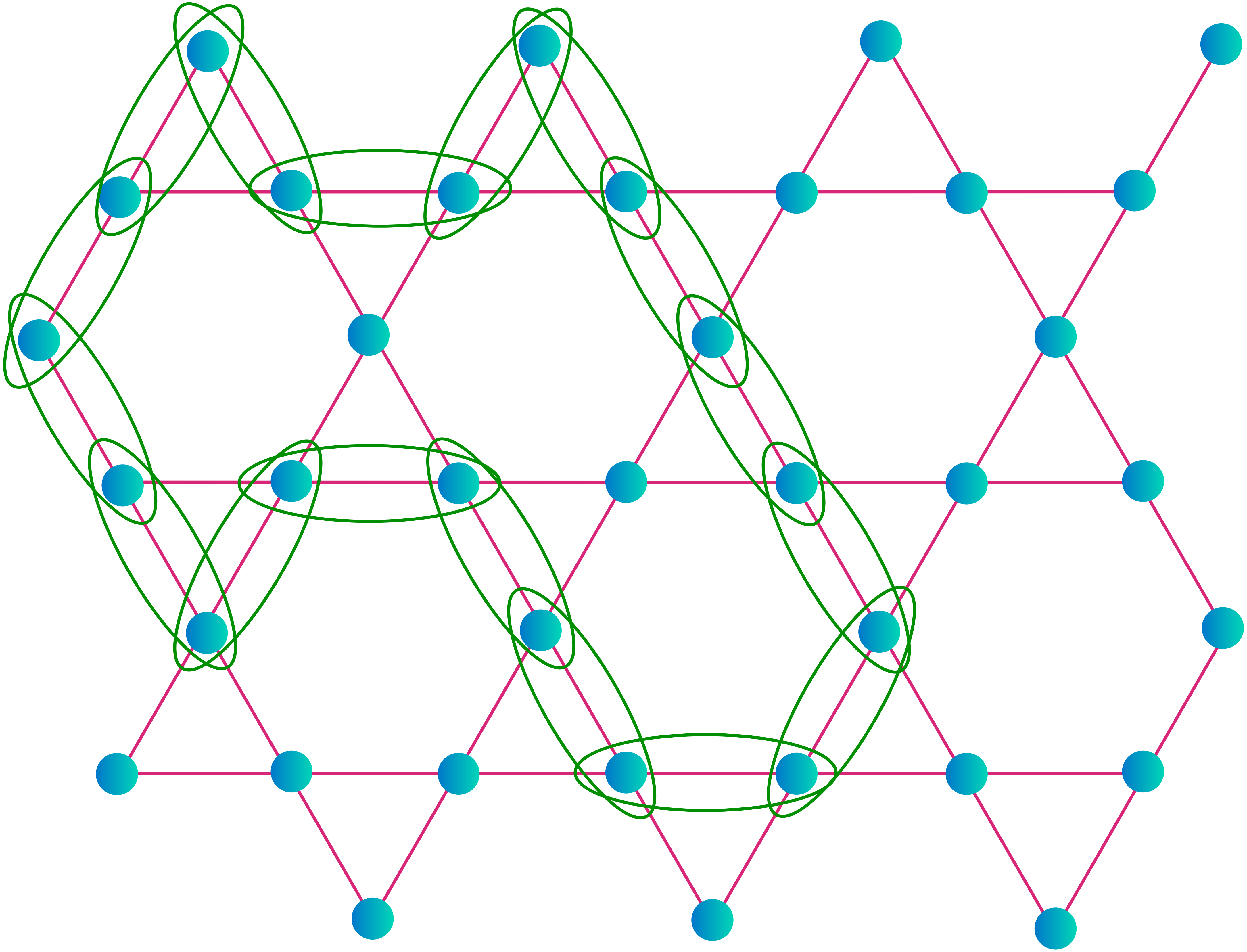


Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops



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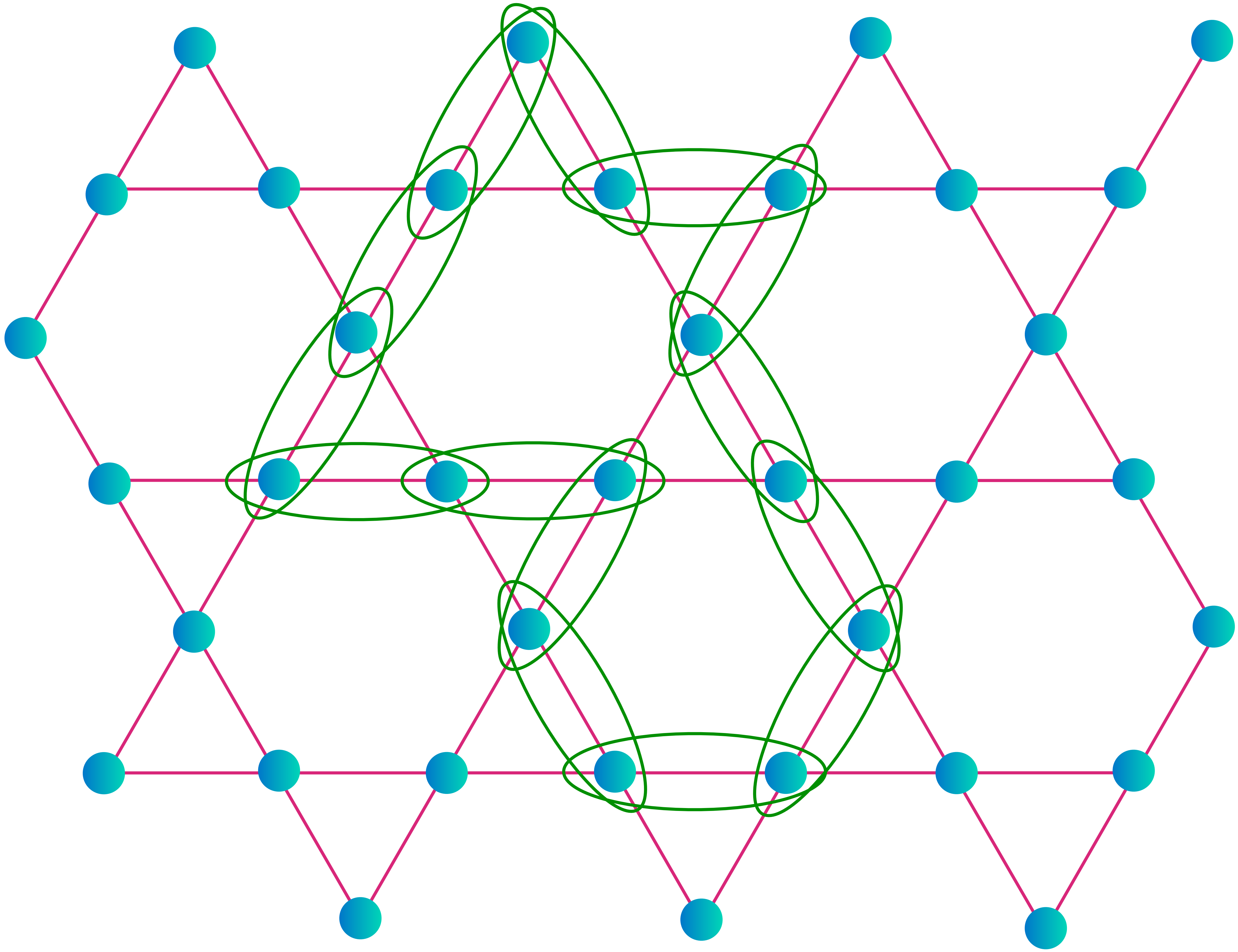
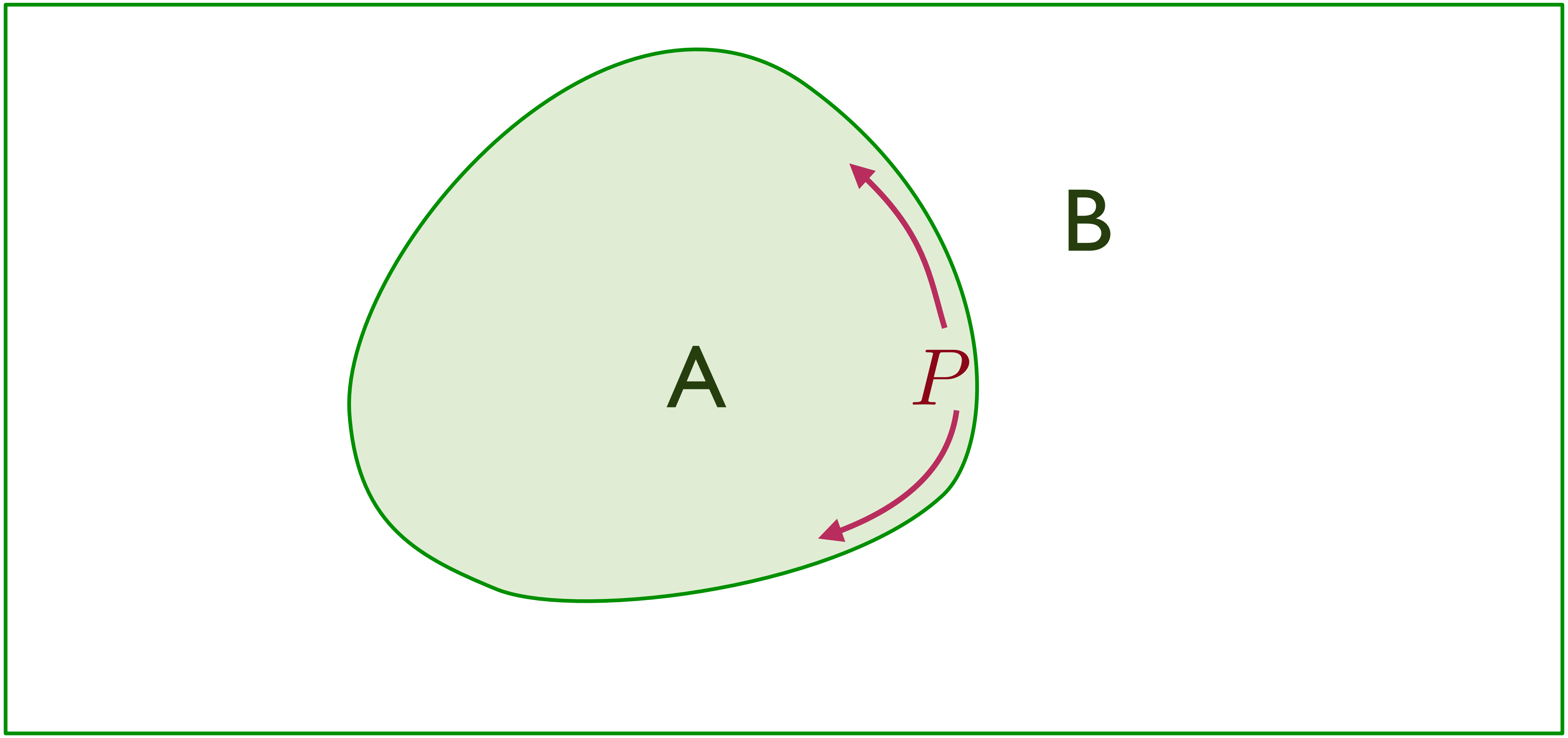


Entanglement in the Z_2 spin liquid



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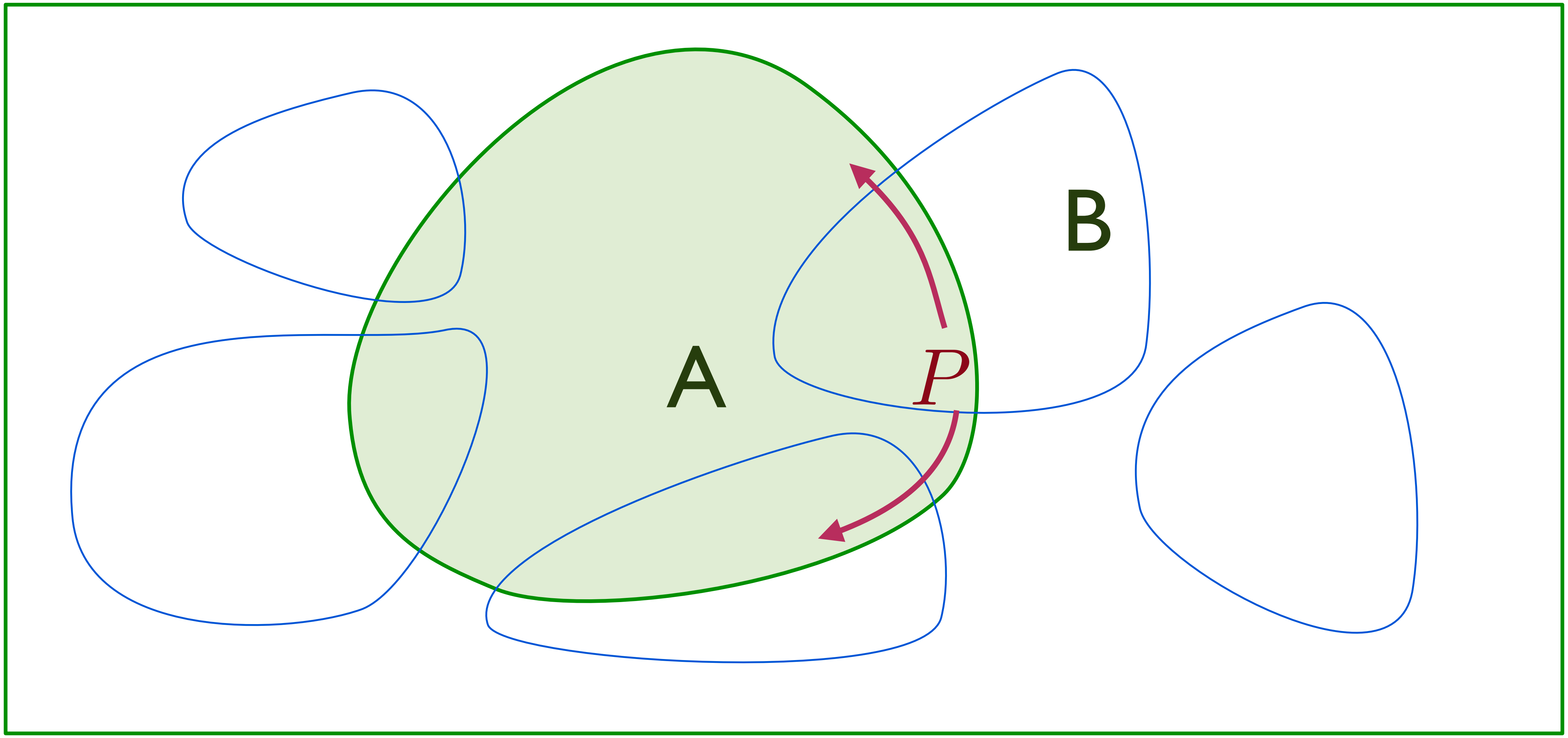

Sum over closed loops: only an even number of links cross the boundary between A and B

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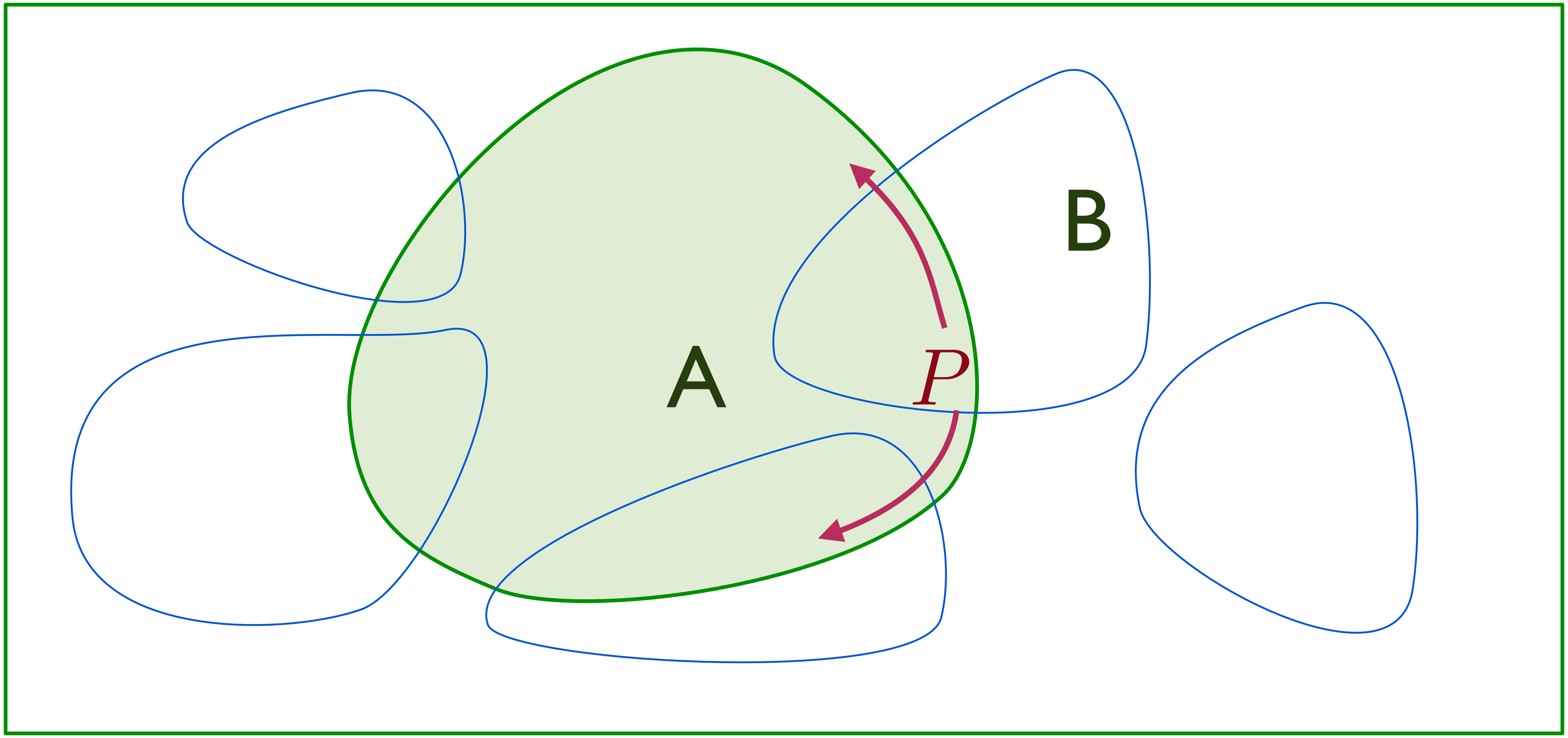

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A. Hamma, R. Ionicioiu, and P. Zanardi, Phys. Rev. A **71**, 022315 (2005)
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006); A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006)
Y. Zhang, T. Grover, and A. Vishwanath, Phys. Rev. B **84**, 075128 (2011)

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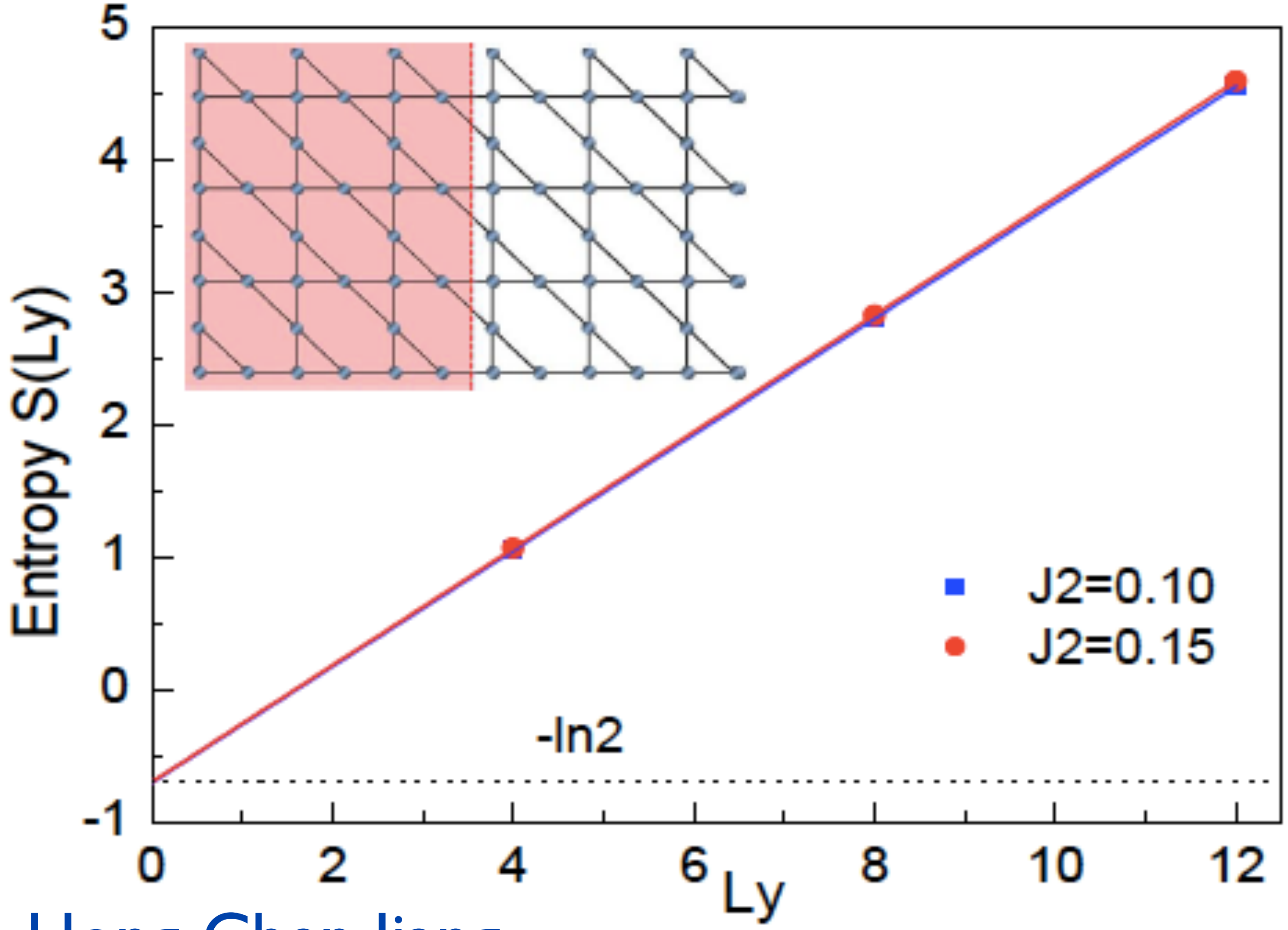
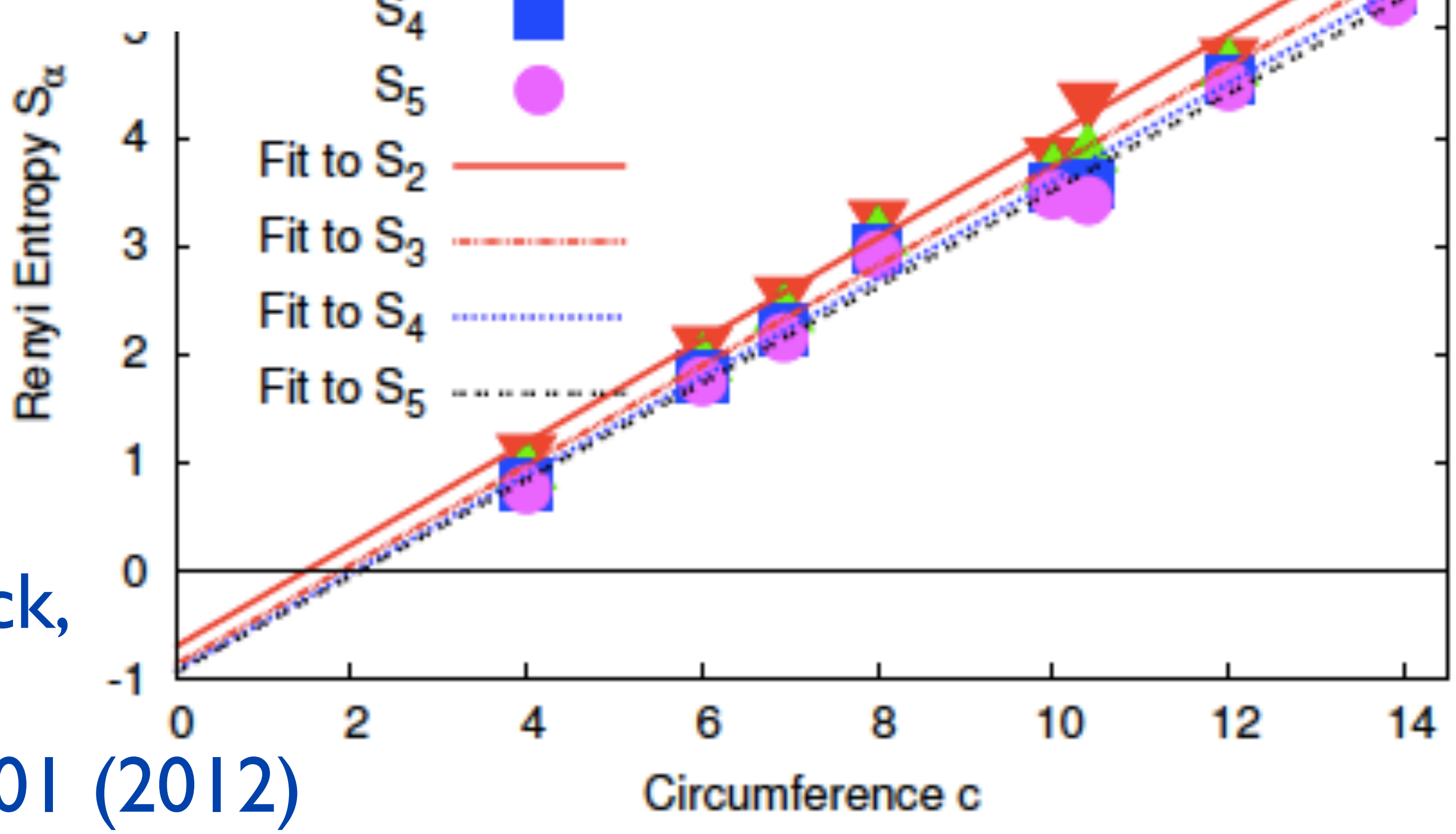


Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, *Nature Physics* **8**, 902 (2012)



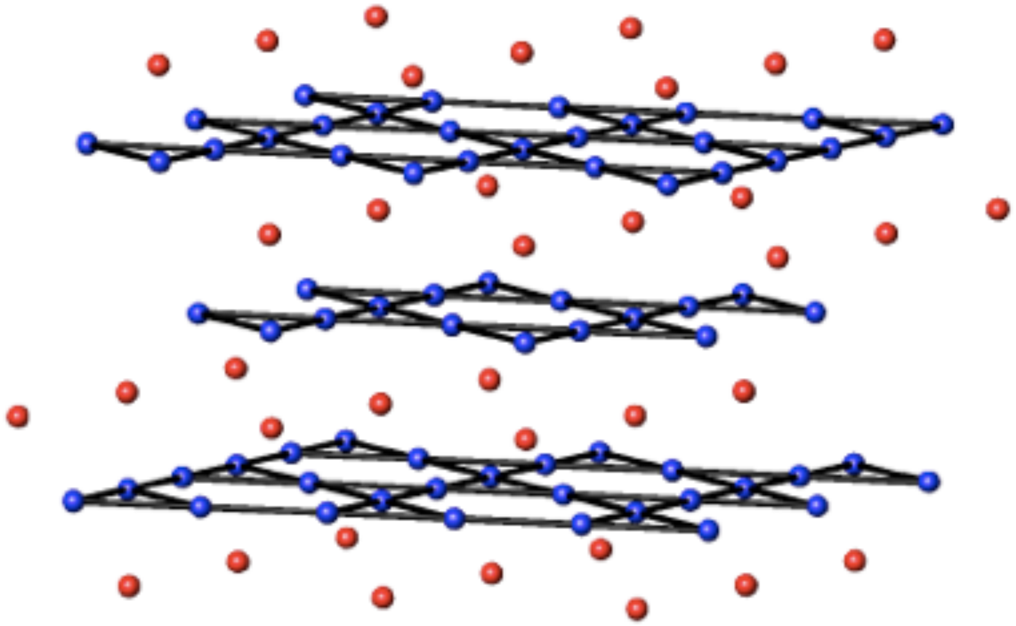
S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, *Physical Review Letters* **109**, 067201 (2012)

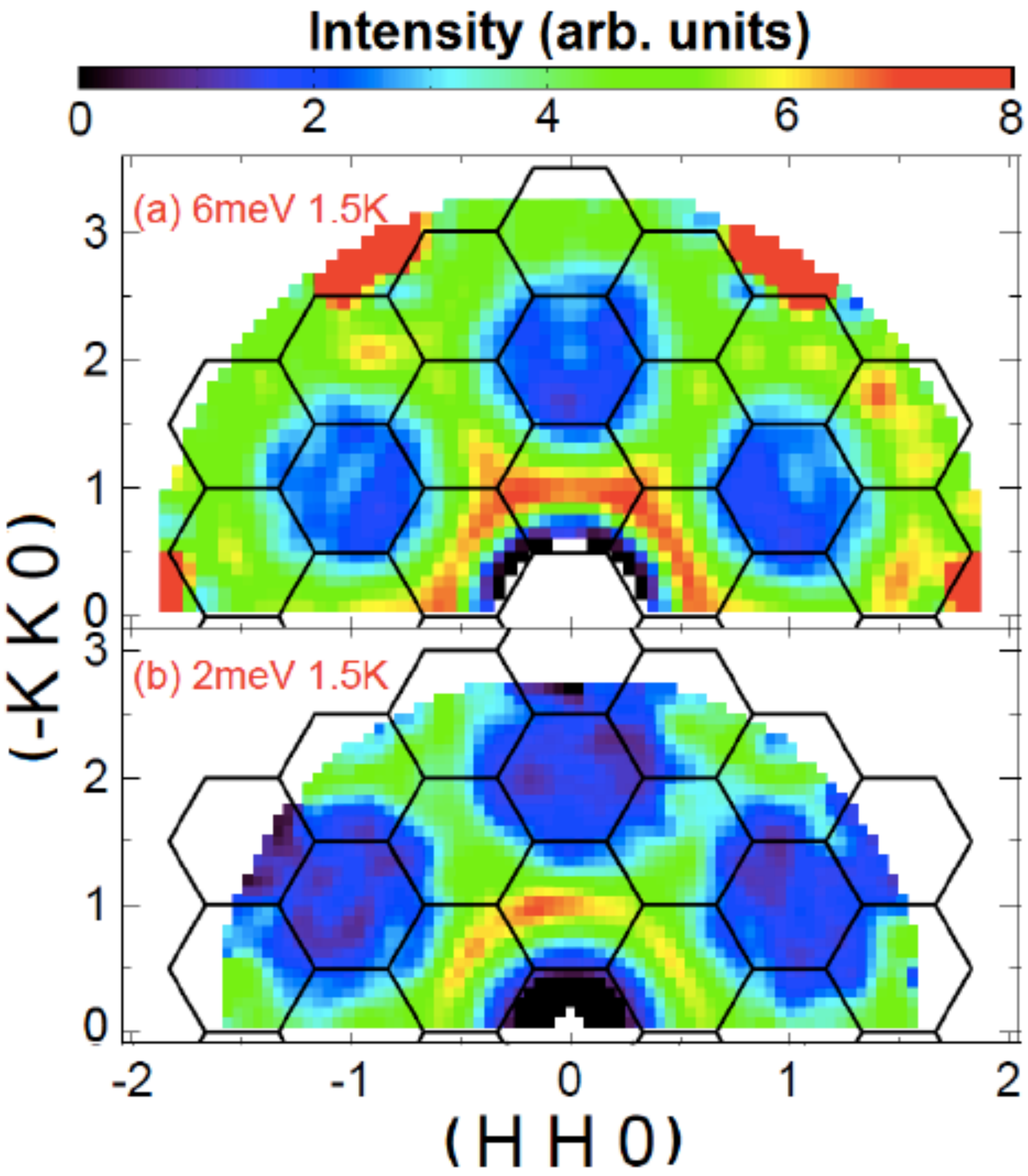
Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han[1], Joel S. Helton[2], Shaoyan Chu[3], Daniel G. Nocera[4], Jose A. Rodriguez-Rivera[2,5], Collin Broholm[2,6] & Young S. Lee[1]

Nature **492**, 406 (2012)

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





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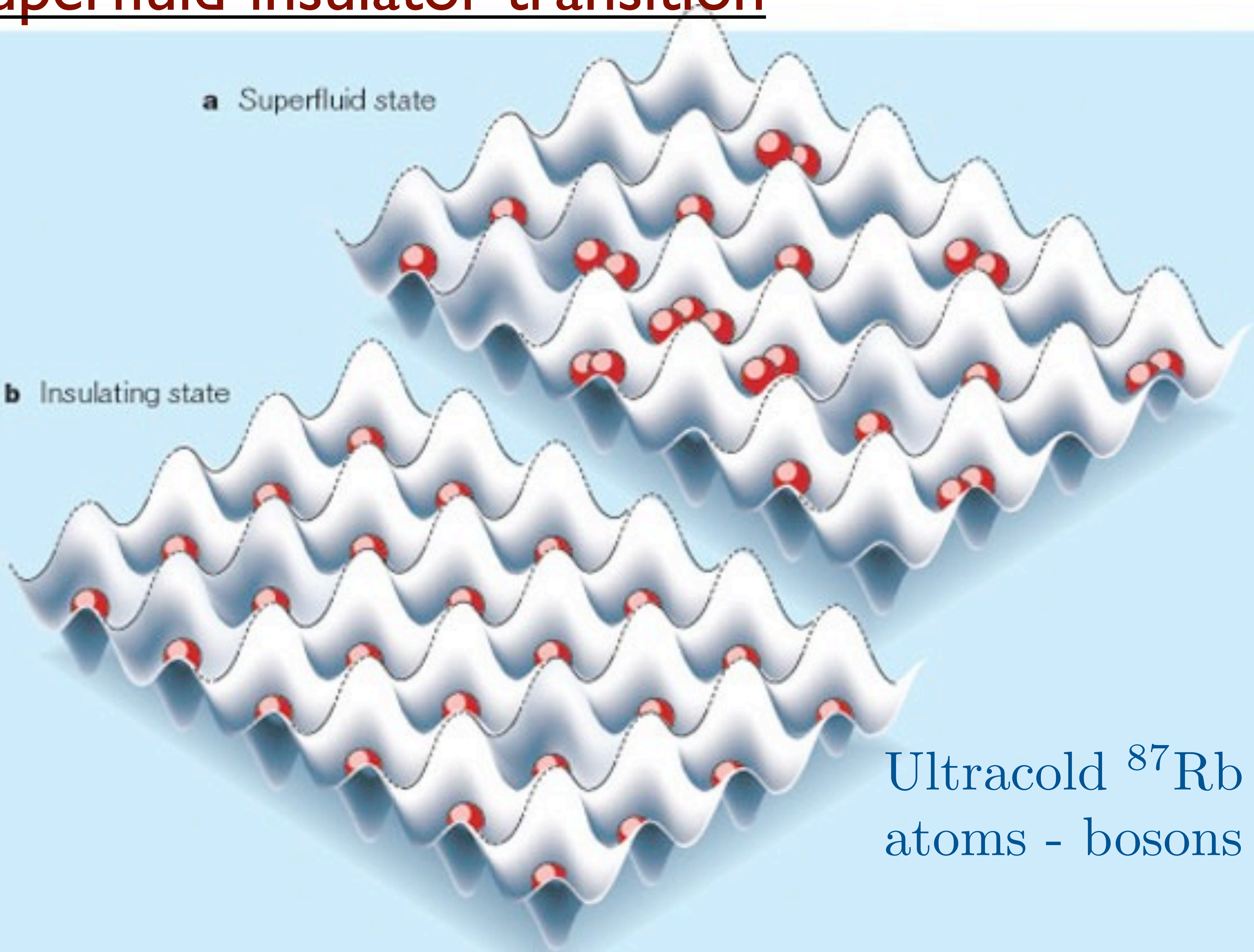
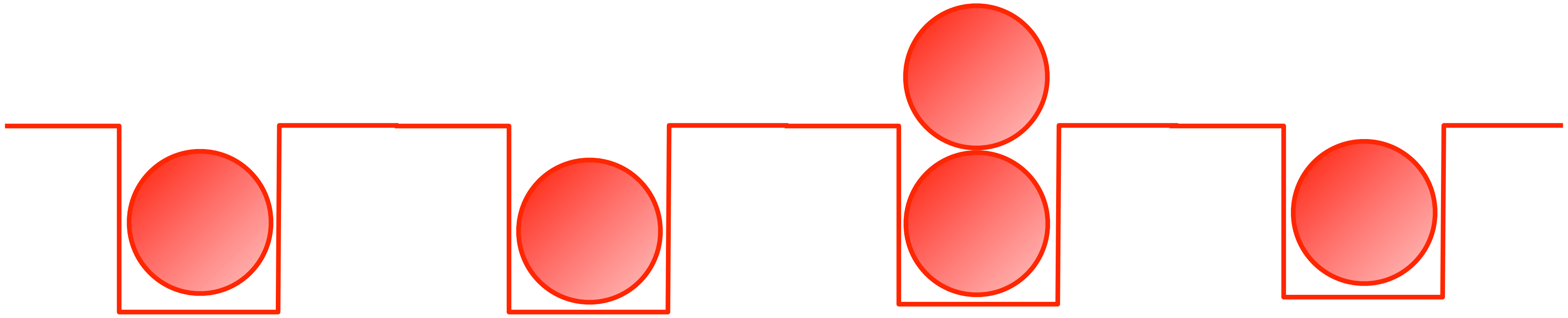
Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Excitations of the insulator:



Particles $\sim \Psi^\dagger$

Holes $\sim \Psi$

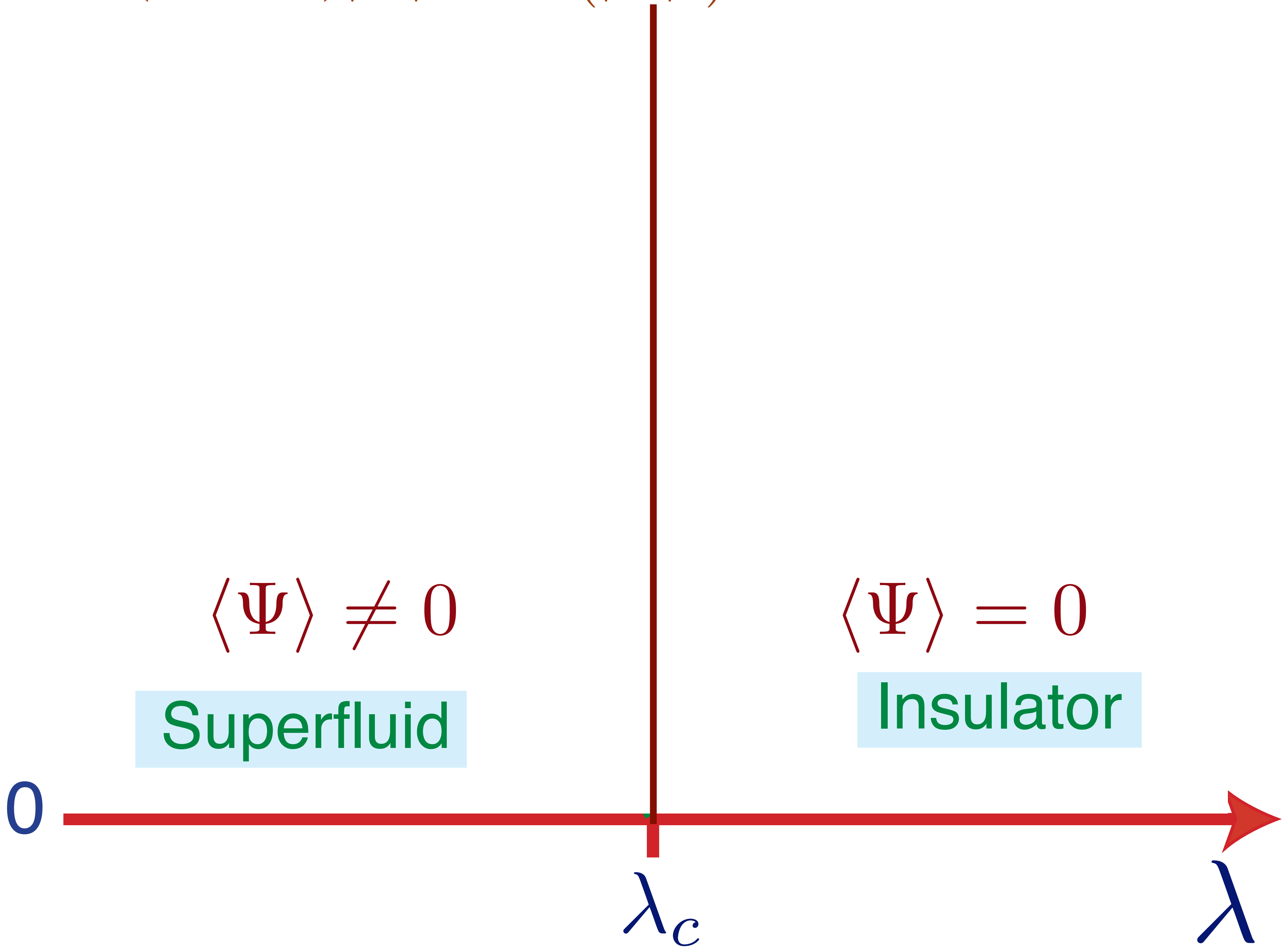
Density of particles = density of holes $\Rightarrow$
"Relativistic" field theory for Ψ:

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



$\langle \Psi \rangle \neq 0$

Superfluid

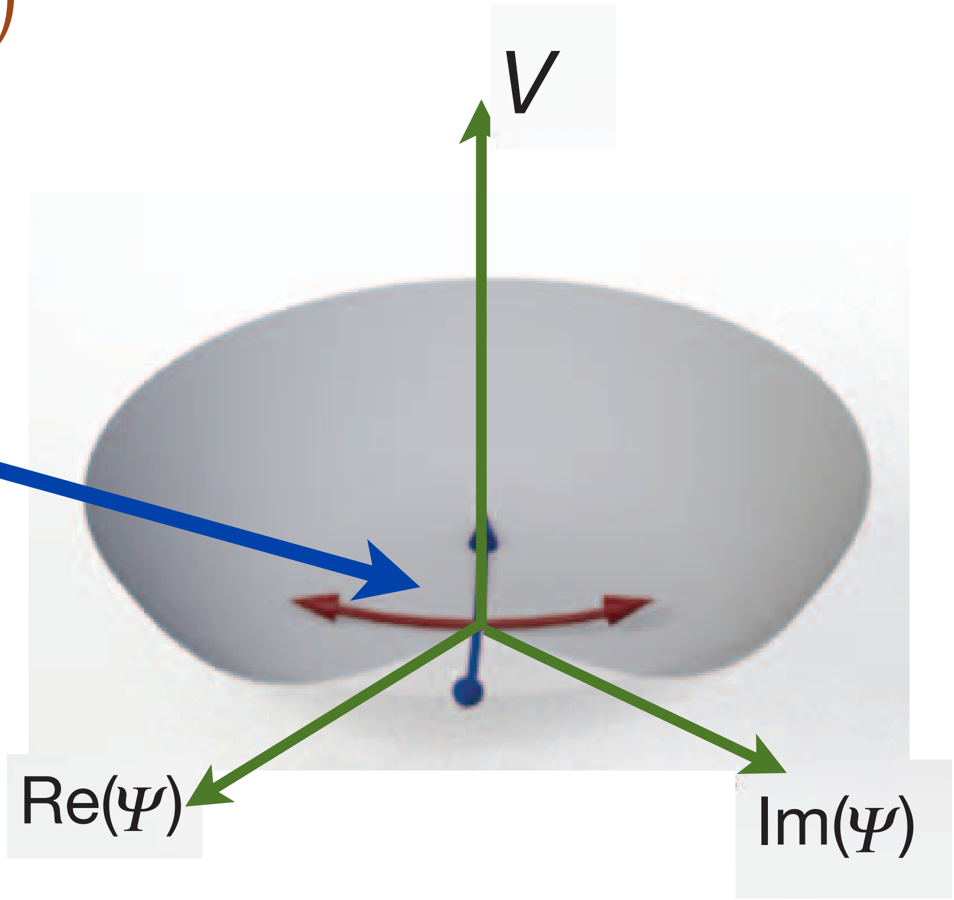
$\langle \Psi \rangle = 0$

Insulator

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$

Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

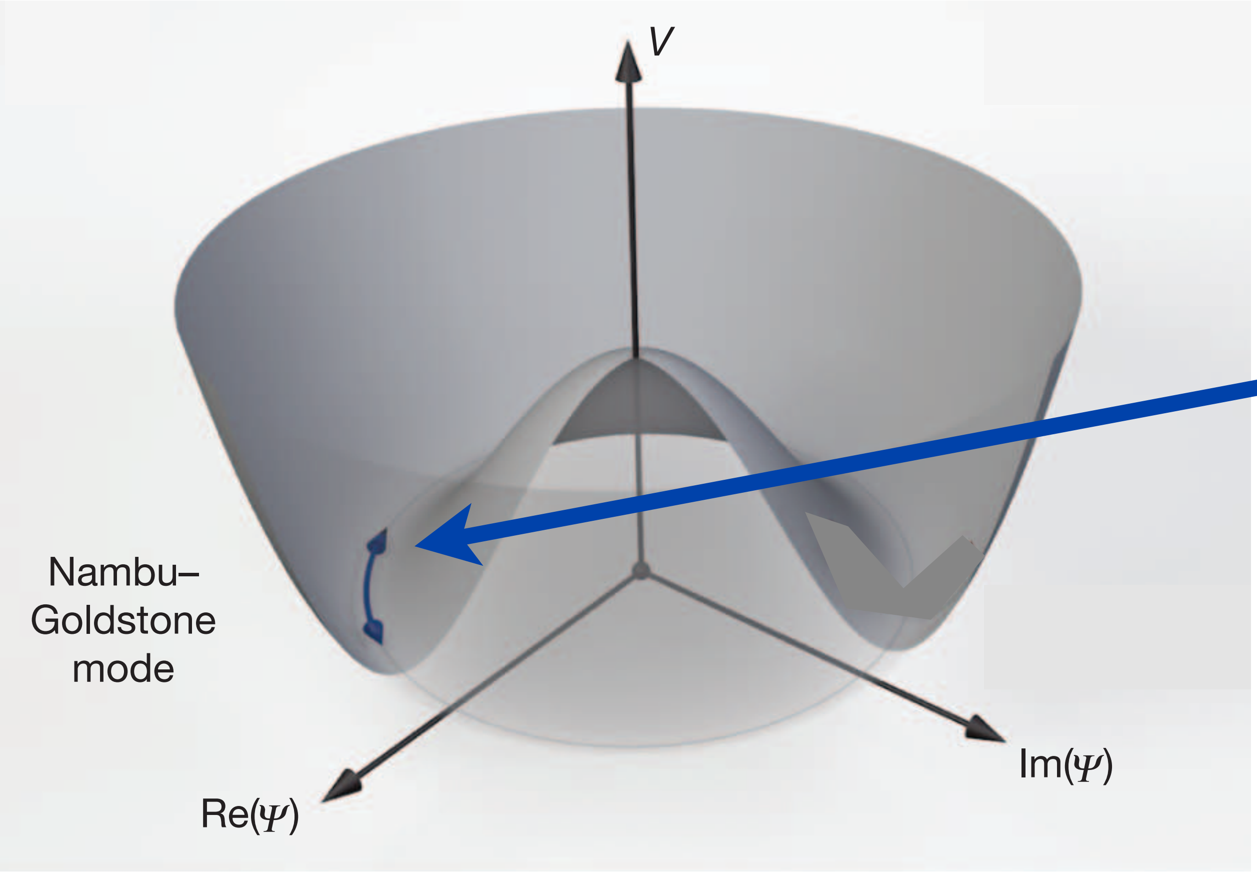
0

λ_c

λ

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



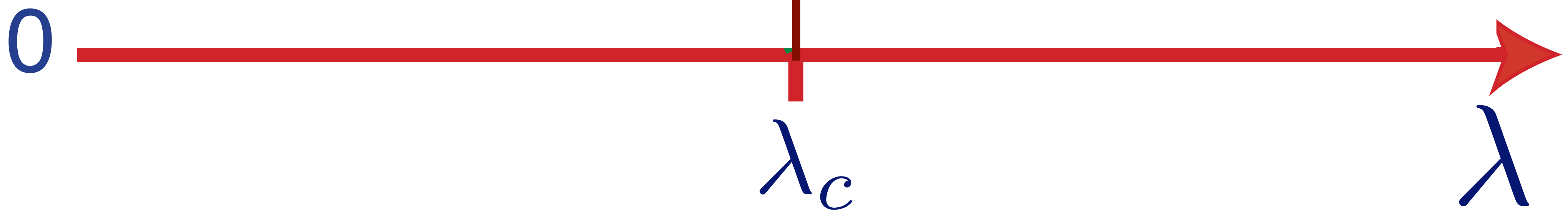
Nambu-Goldstone mode is the oscillation in the phase Ψ at a constant non-zero $|\Psi|$.

$\langle \Psi \rangle \neq 0$

Superfluid

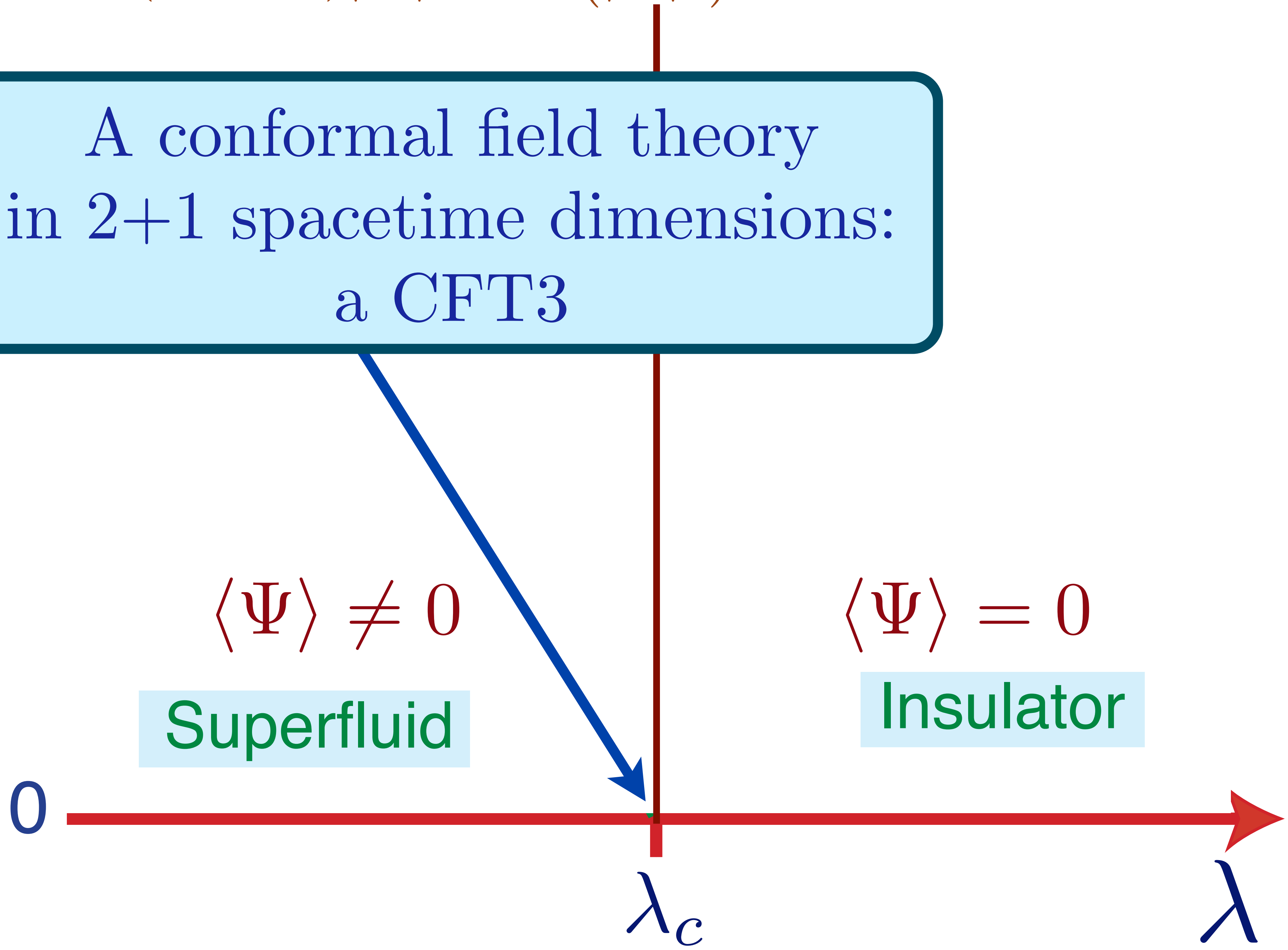
$\langle \Psi \rangle = 0$

Insulator



$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



A conformal field theory
in 2+1 spacetime dimensions:
a CFT3

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

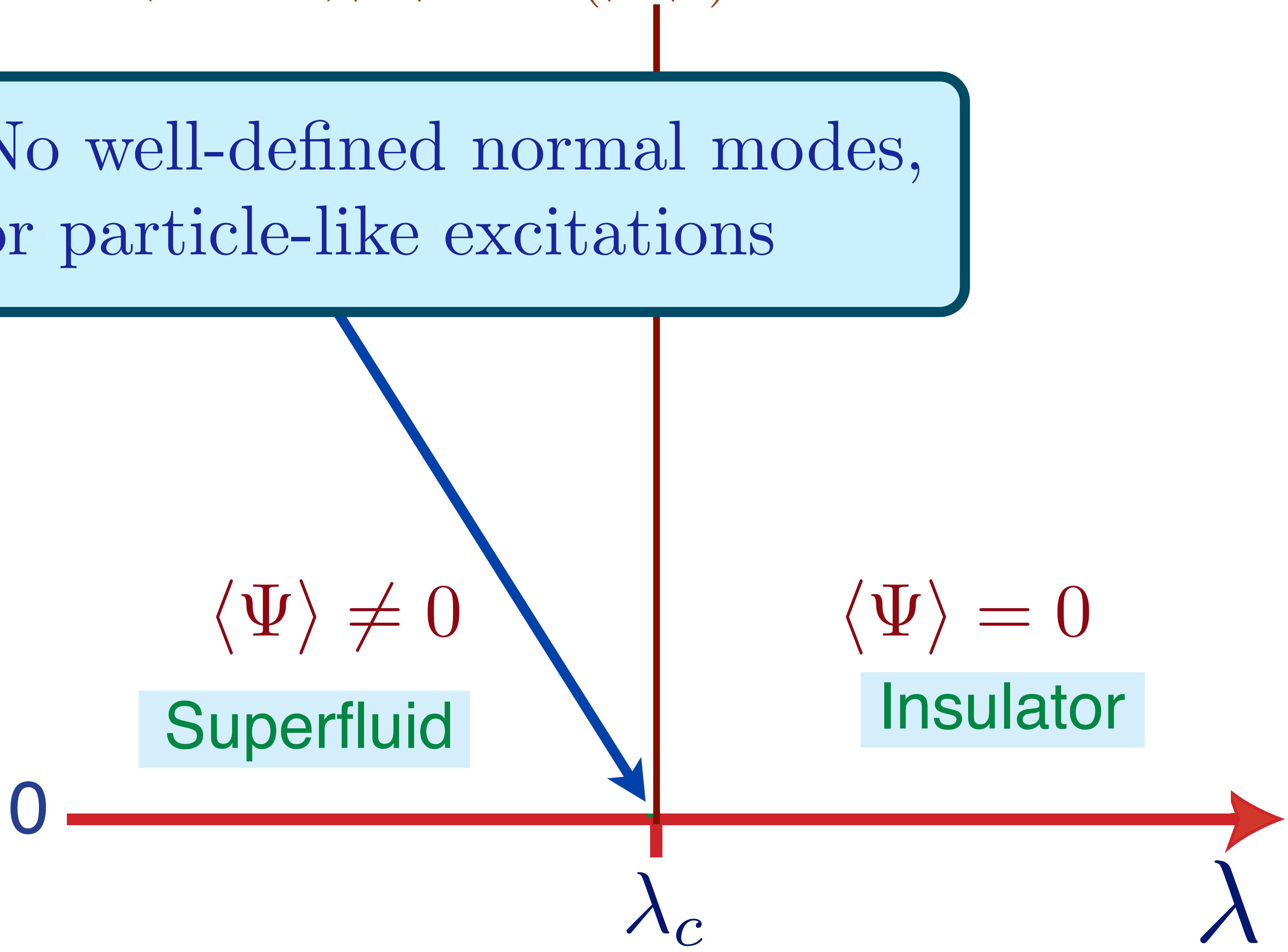
0

λ_c

λ

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



No well-defined normal modes, or particle-like excitations

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

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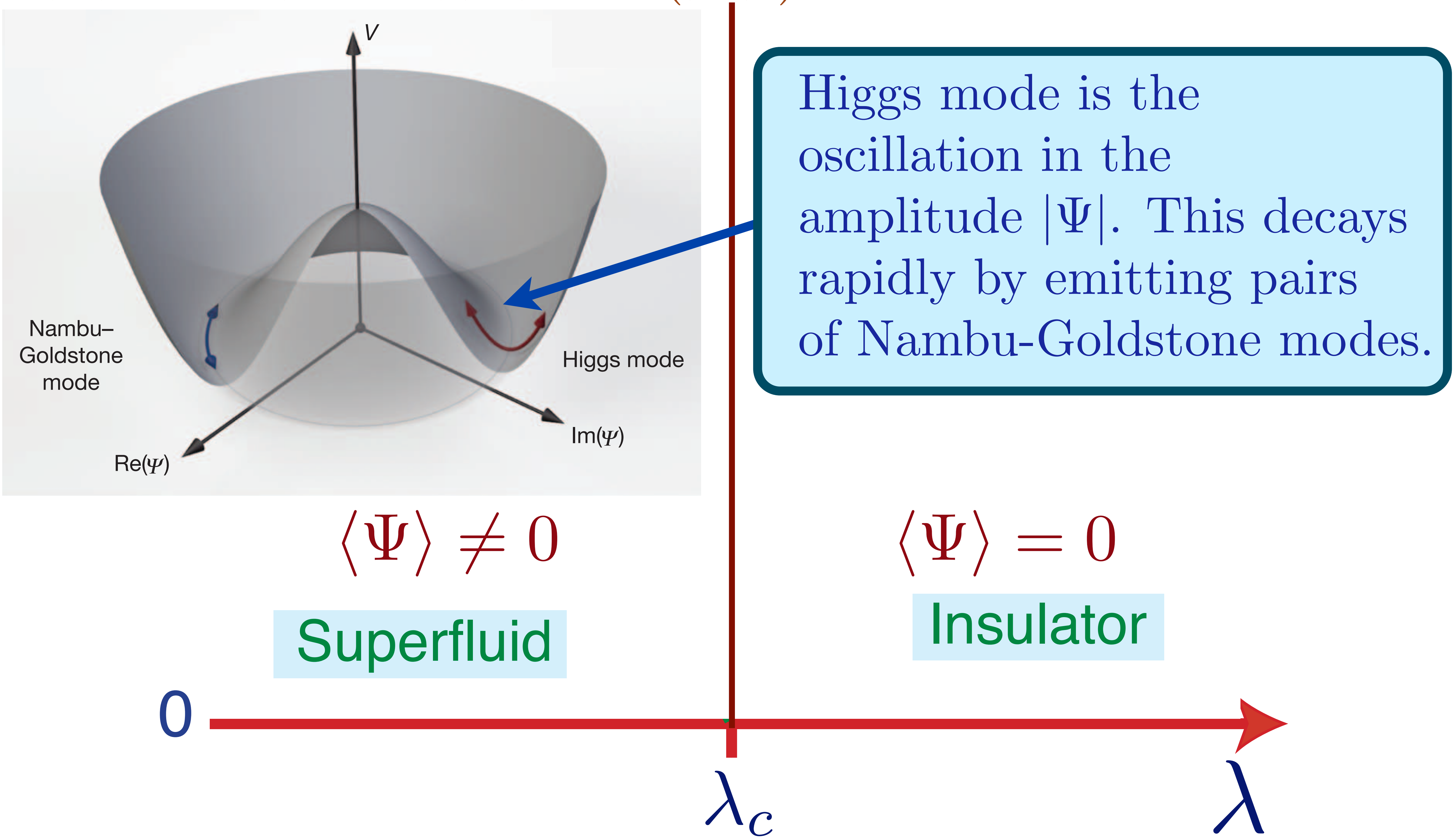


Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes.

$\langle \Psi \rangle \neq 0$

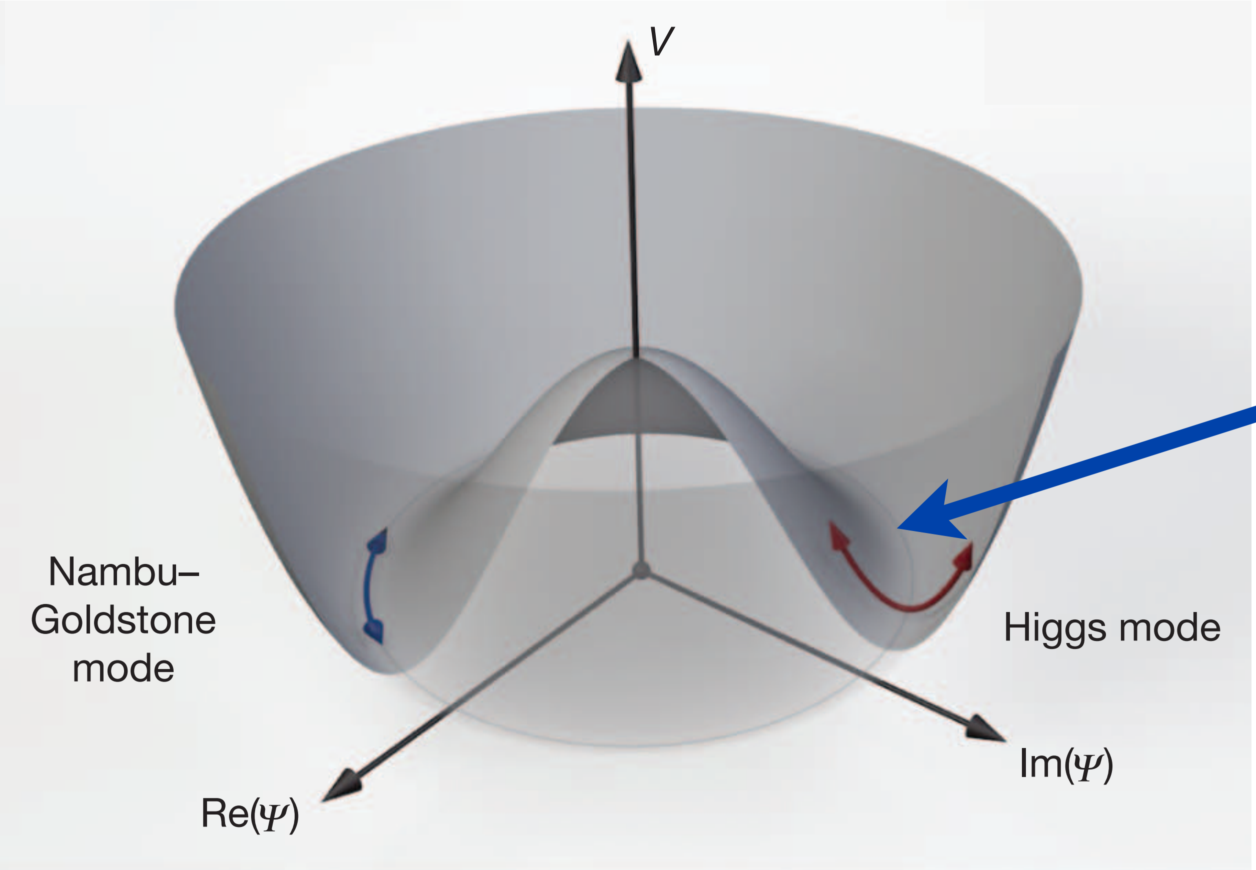
Superfluid

$\langle \Psi \rangle = 0$

Insulator

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



Despite rapid decay,
there is a well-defined
Higgs "quasi-normal mode".
This is associated with a pole
in the lower-half
of the complex frequency plane.

$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0

λ_c

λ

A. V. Chubukov, S. Sachdev, and J. Ye, PRB **49**, 11919 (1994). S. Sachdev, PRB **59**, 14054 (1999).
W. Zwerger, PRL **92**, 027203 (2004).
D. Podolsky, A. Auerbach, and D. P. Arovas, PRB **84**, 174522 (2011).
D. Podolsky and S. Sachdev, PRB **86**, 054508 (2012). L. Pollet and N. Prokof'ev, PRL **109**, 010401 (2012).

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

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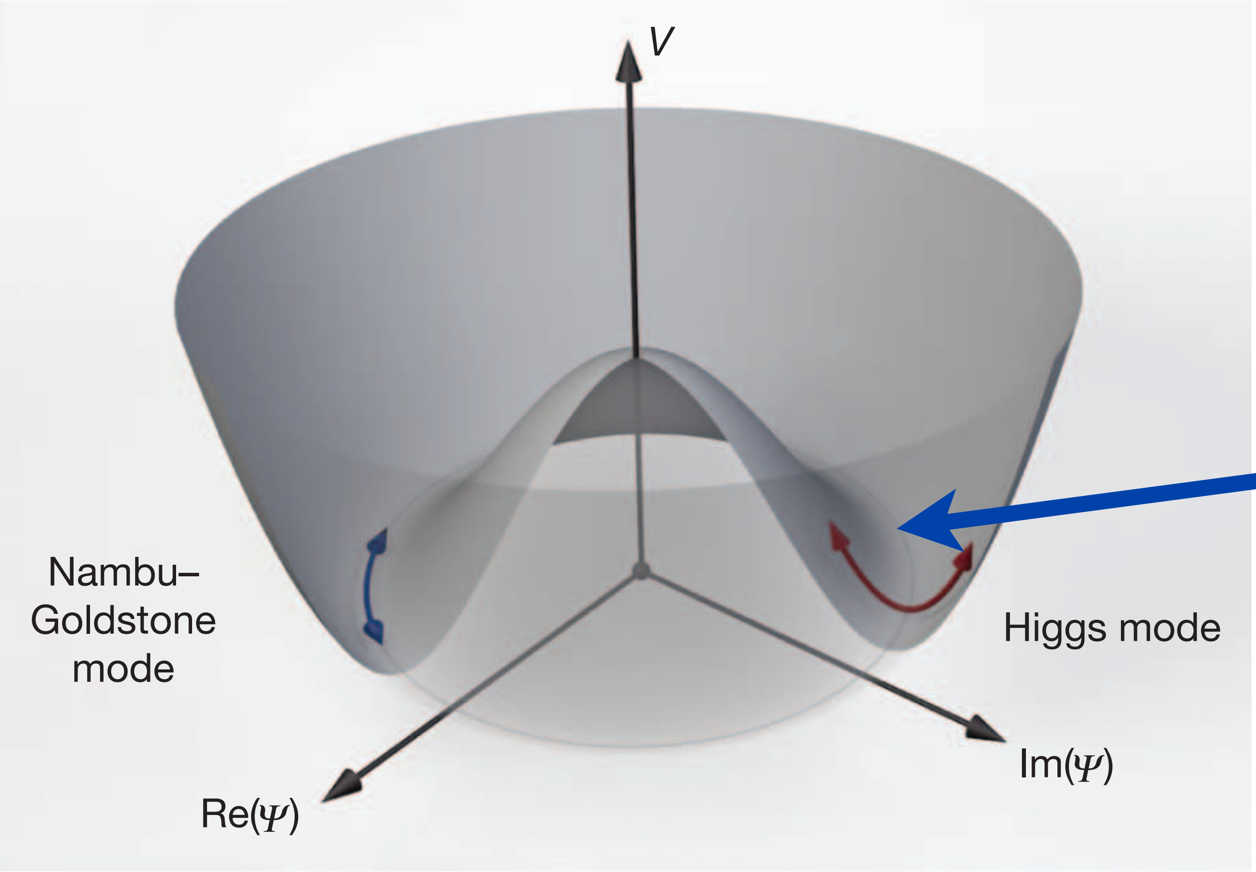

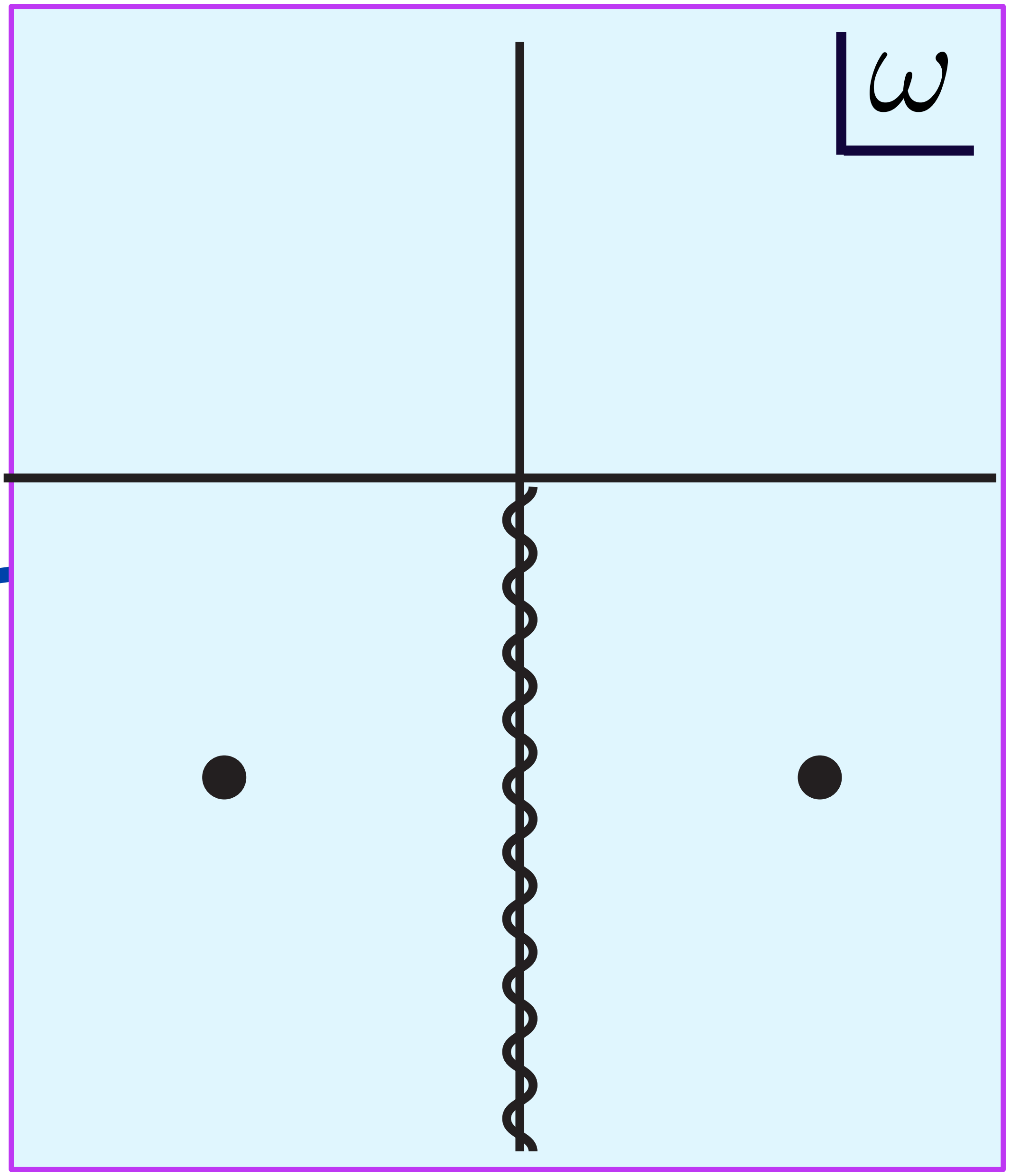

D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

The Higgs quasi-normal mode is at the frequency

$$\frac{\omega_{\rm pole}}{\Delta} = -i\,\frac{4}{\pi} + \frac{1}{N}\left(\frac{16\left(4+\sqrt{2}\log\left(3-2\sqrt{2}\right)\right)}{\pi^2} + 2.46531203396\,i\right) + \mathcal{O}\left(\frac{1}{N^2}\right)$$

where Δ is the particle gap at the complementary point in the “paramagnetic” state with the same value of $|\lambda - \lambda_c|$, and $N = 2$ is the number of vector components of Ψ. The universal answer is a consequence of the strong interactions in the CFT3

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right]$$

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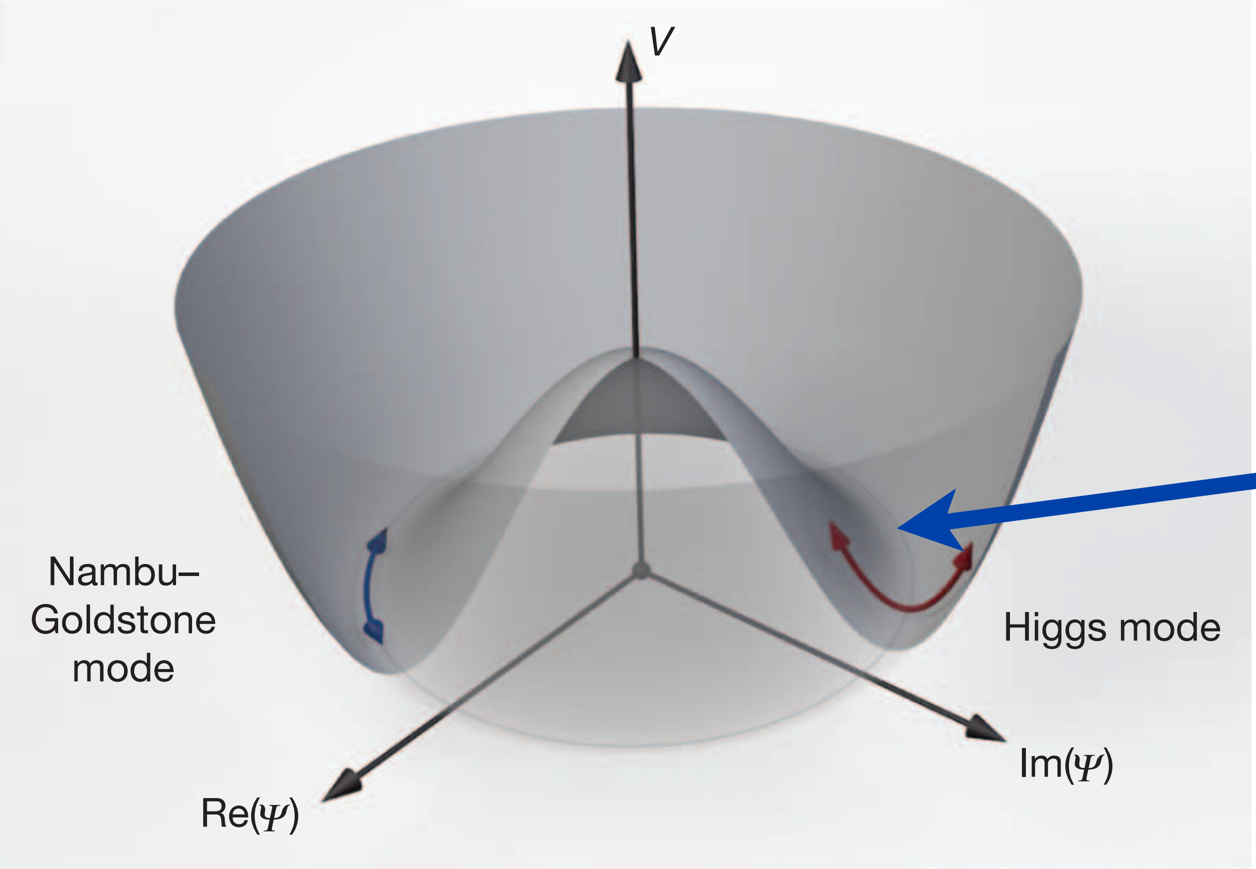

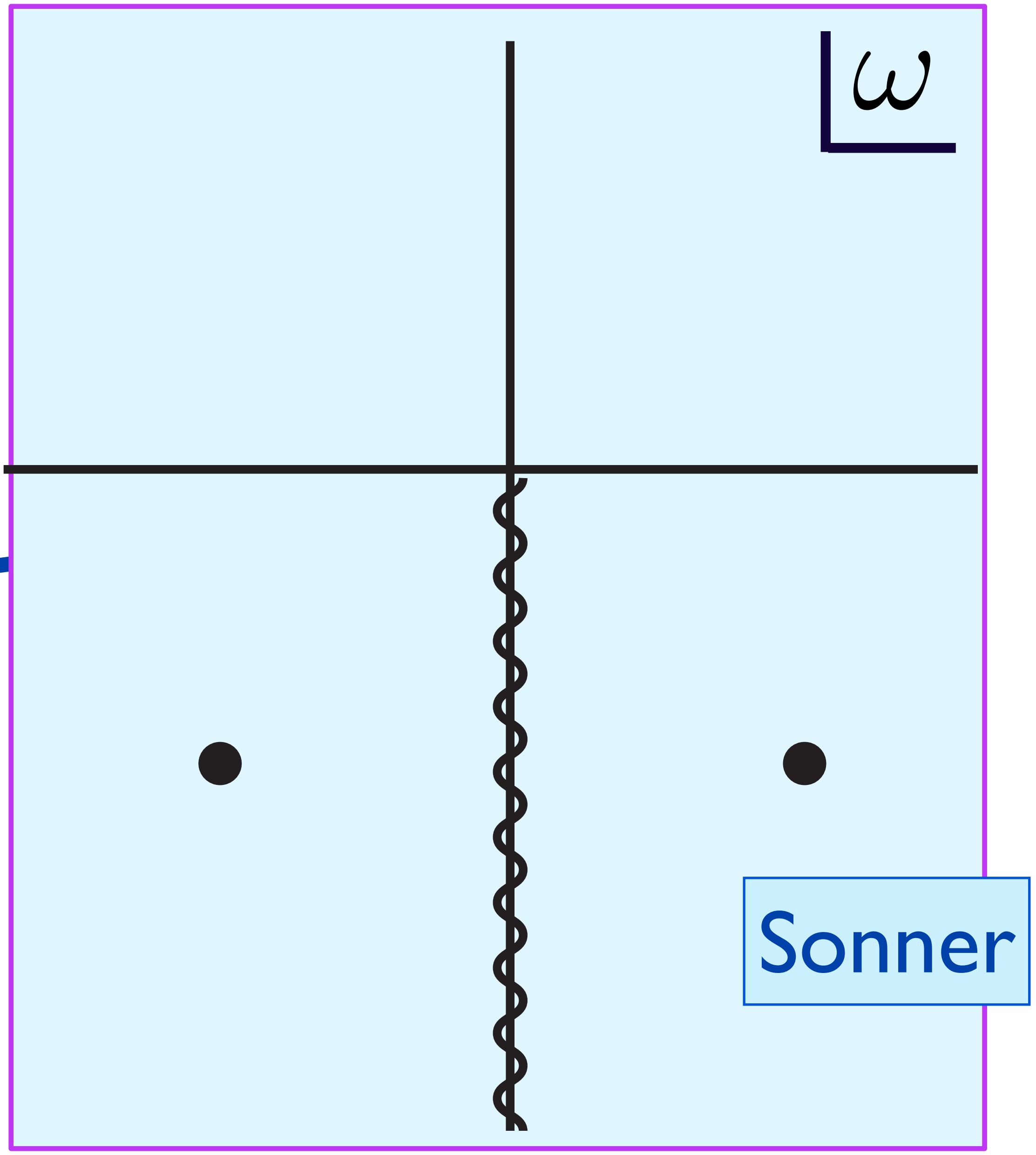

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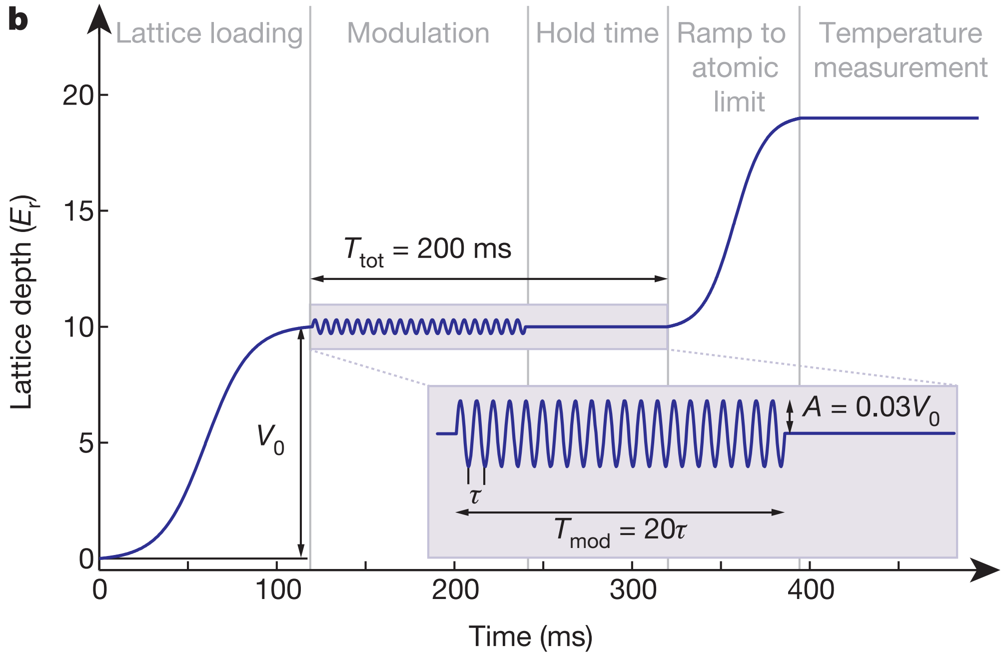
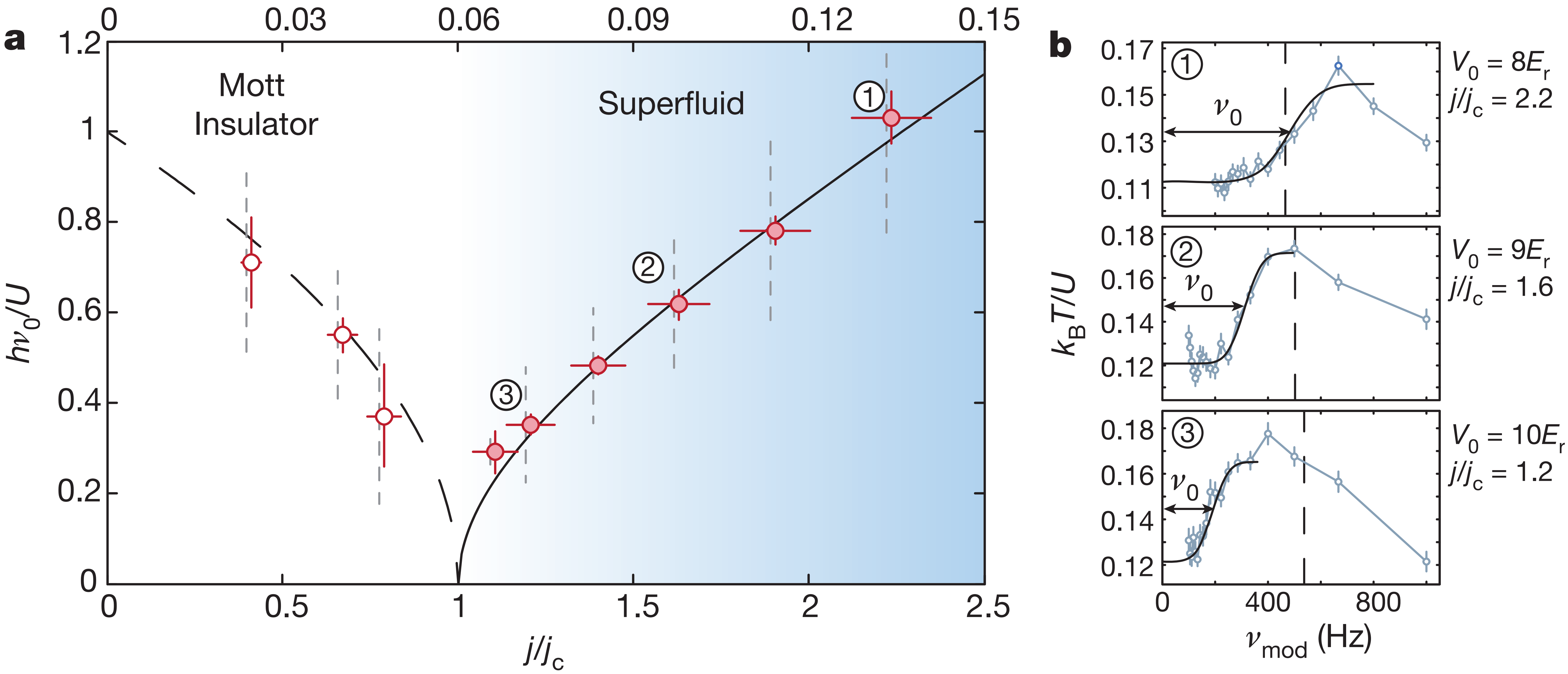
Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole



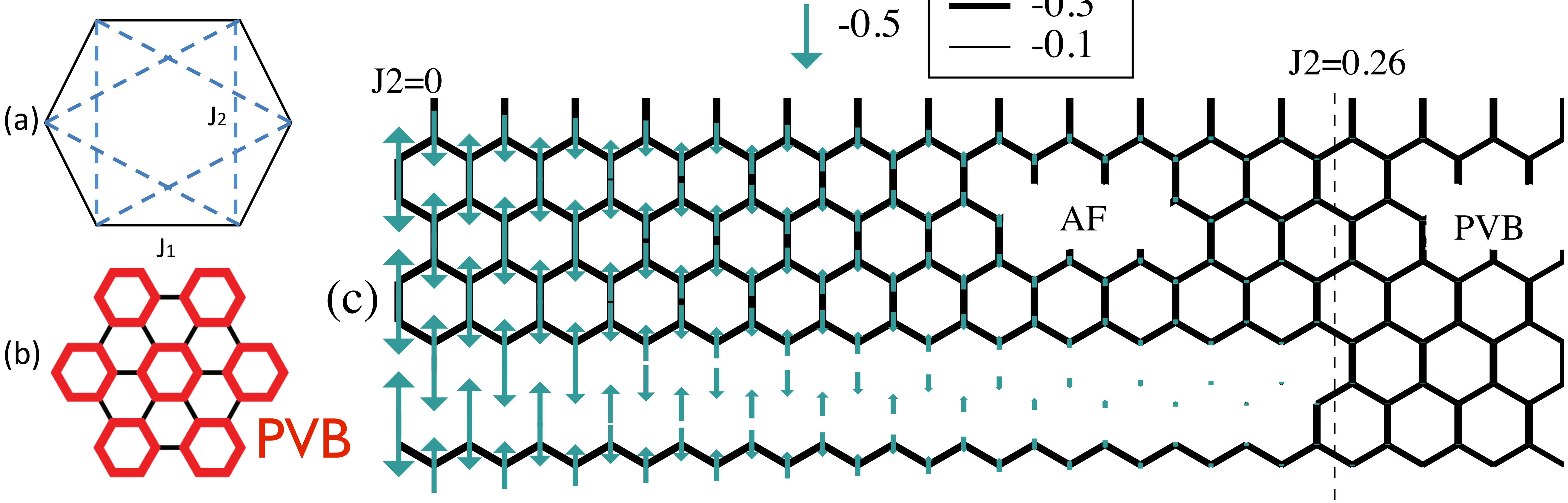


Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Honeycomb lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

R. Ganesh, J. van den Bring, and S. Nishimoto, arXiv:1301.0853
Z. Zhu, D.A. Huse, and S.R. White, arXiv:1212.6322



N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)
O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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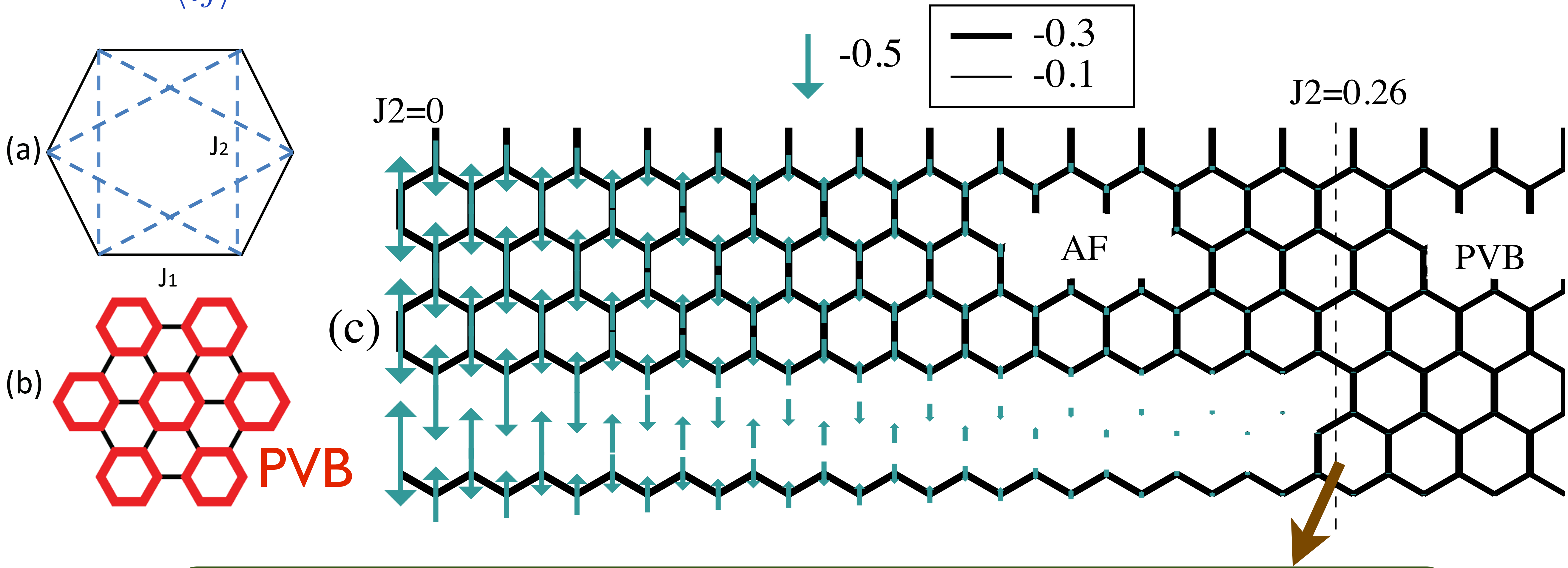

Critical theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

Néel order $\sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$; PVB order $\sim$ monopoles in A_μ

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)
O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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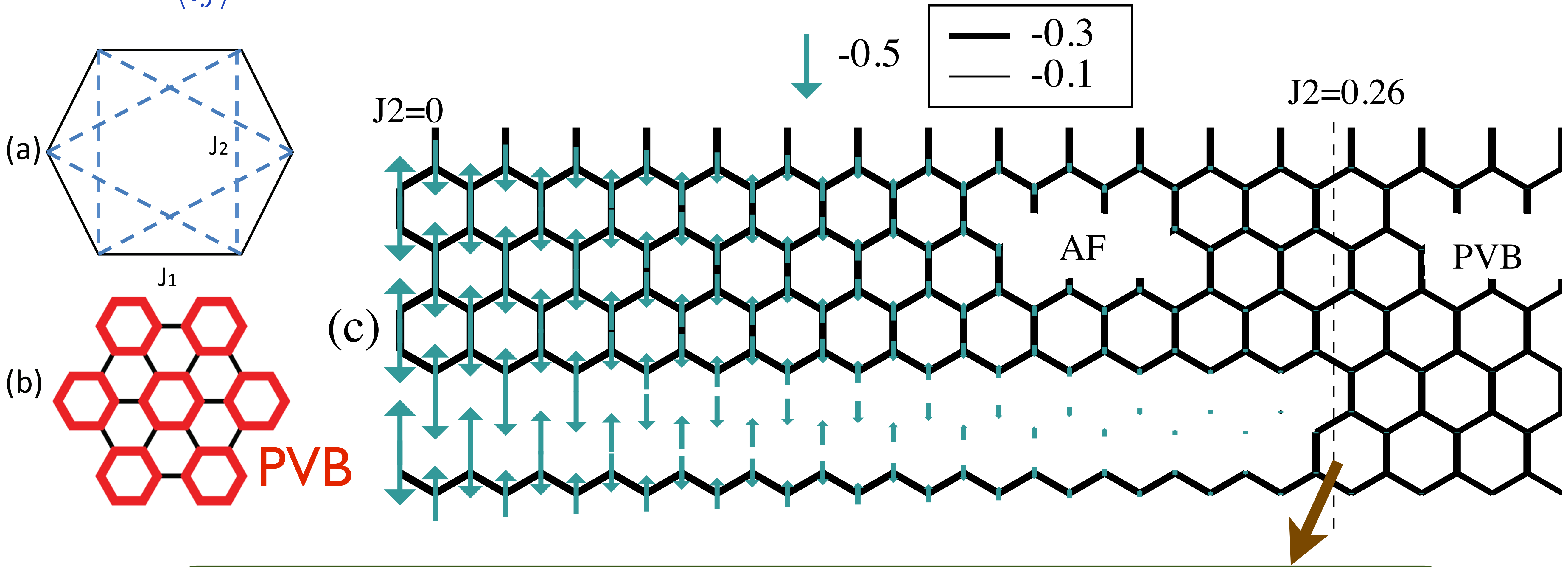
Z. Zhu, D.A. Huse, and S.R. White, arXiv:1212.6322



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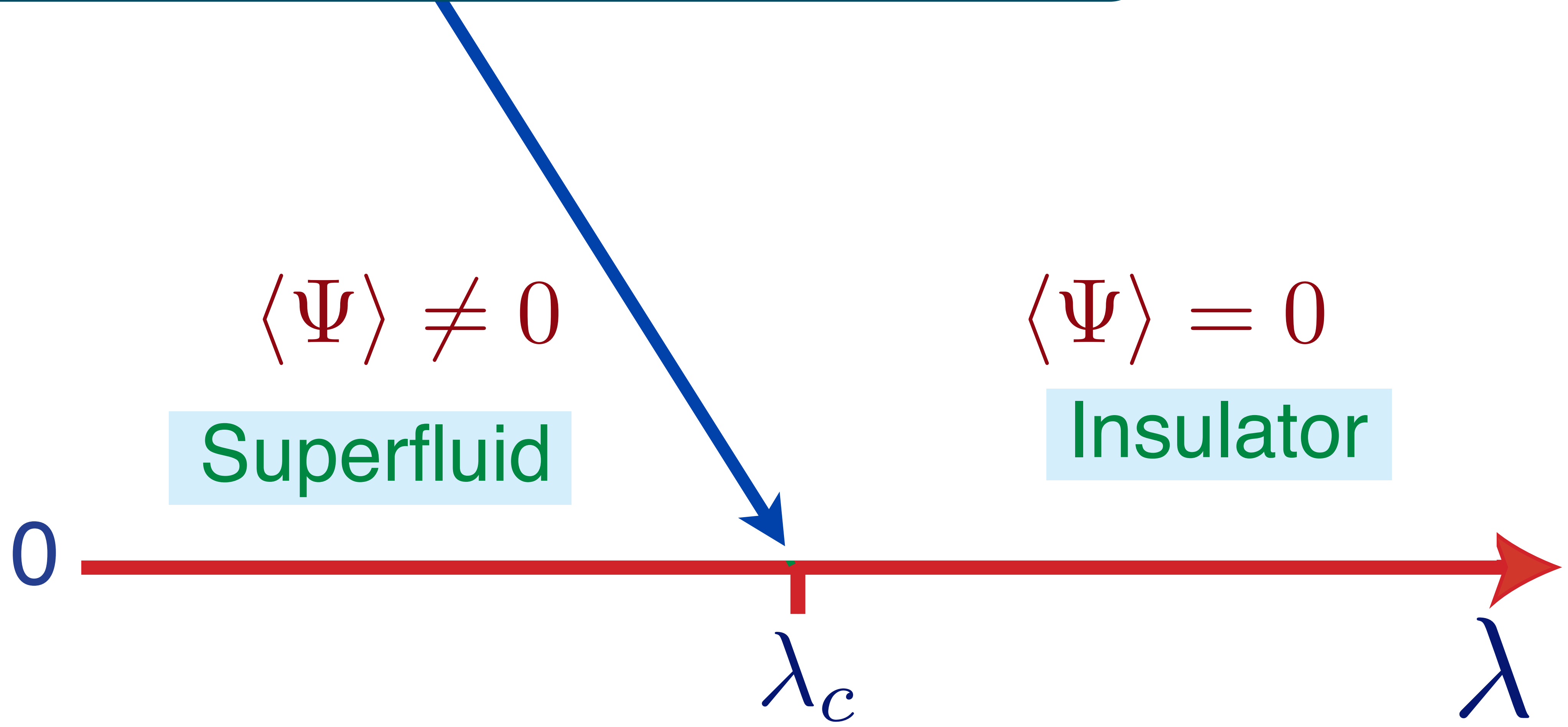
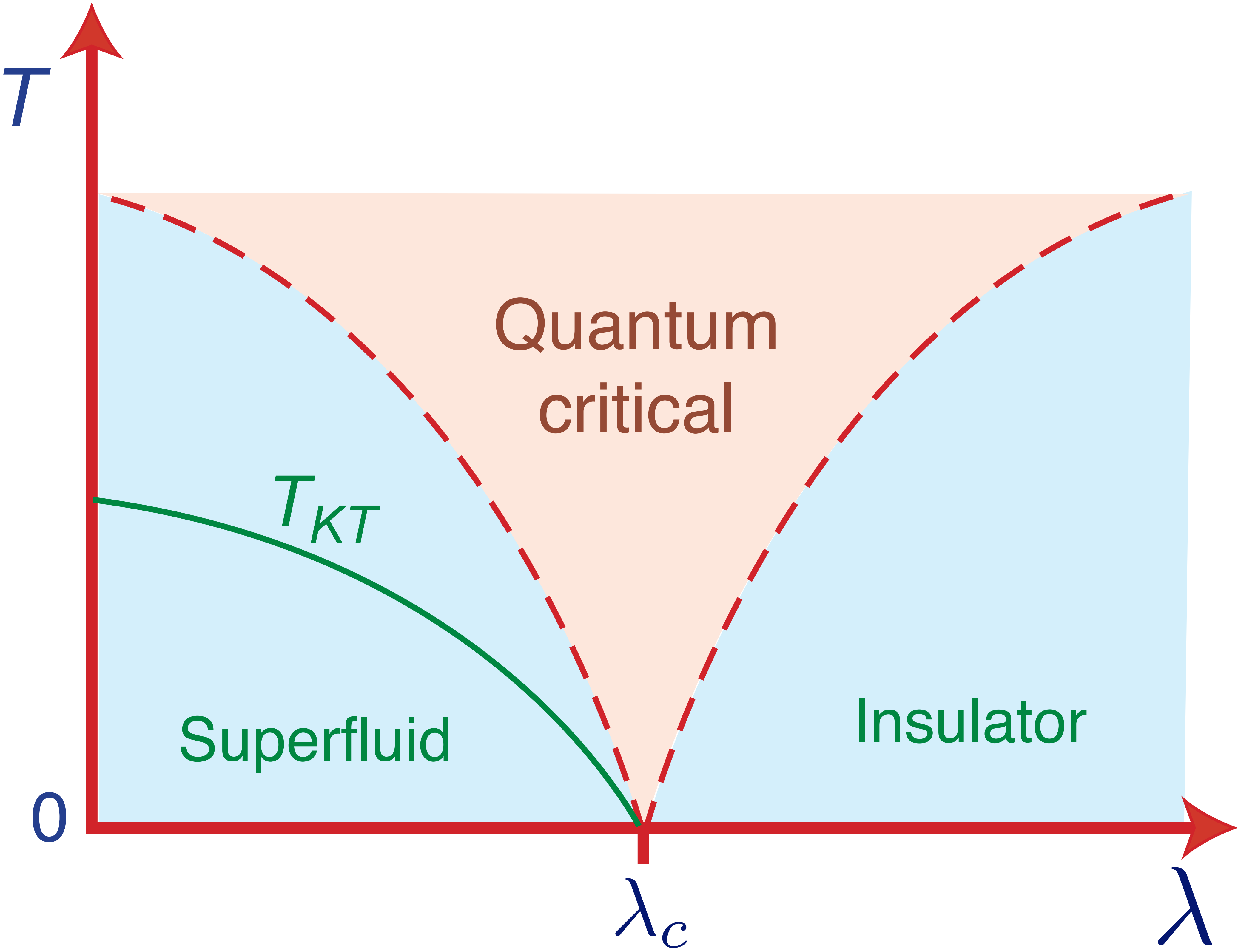
Critical point is a conformal gauge theory for SU(*N*>*4*) antiferromagnets. Evidence for long correlation lengths and emergent gauge fluctuations for SU(2)

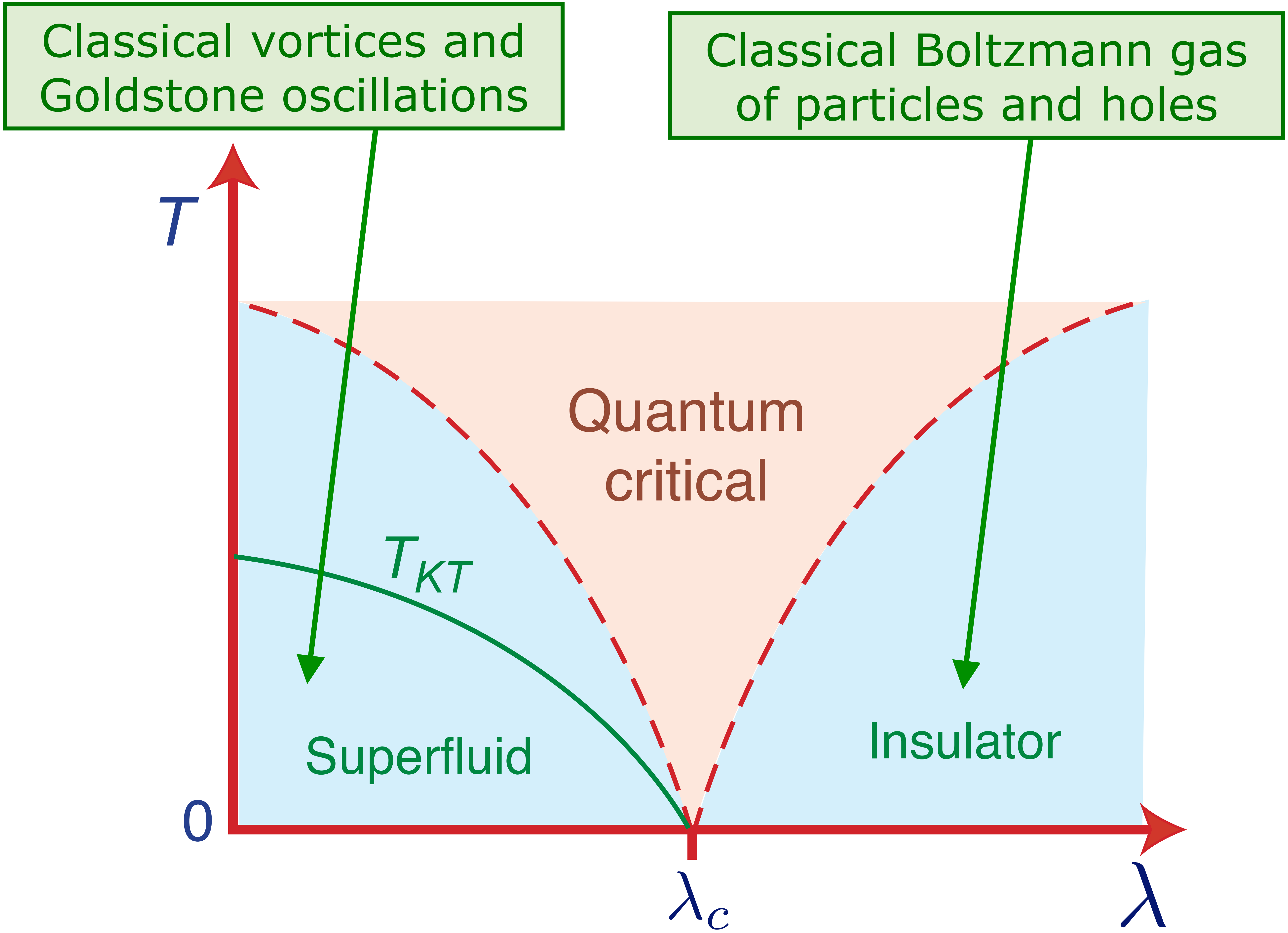
1694 (1989)
104 (2004).
490 (2004).

$$
\begin{aligned}
\mathcal{S} &= \int d^2r d\tau \left[|\partial_\tau \Psi|^2 + c^2 |\nabla_r \Psi|^2 + V(\Psi) \right] \\
V(\Psi) &= (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2
\end{aligned}
$$

A conformal field theory
in 2+1 spacetime dimensions:
a CFT3

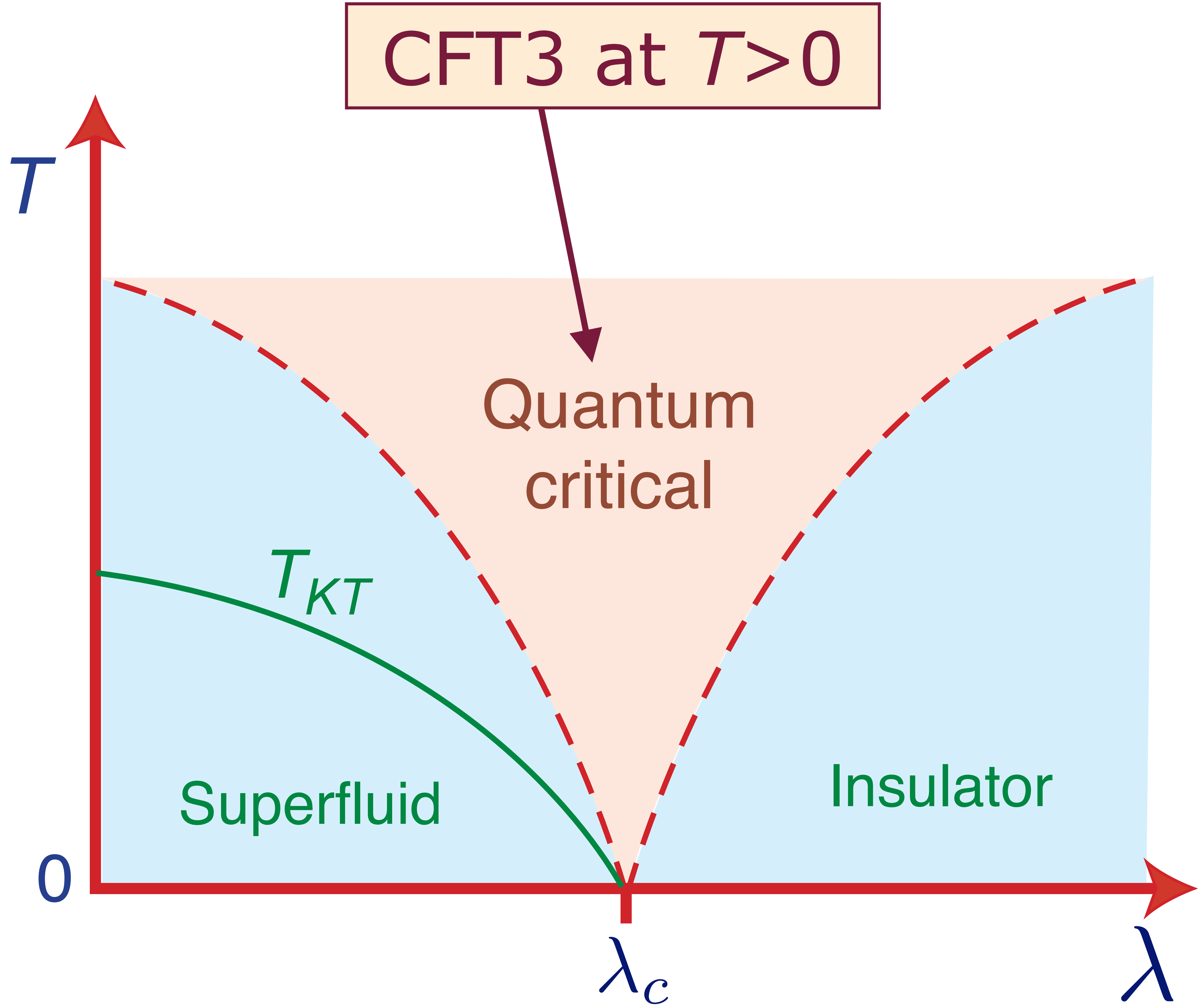



T
Quantum critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ



Classical vortices and Goldstone oscillations

Classical Boltzmann gas of particles and holes


CFT3 at $T>0$
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ

Quantum critical dynamics

Quantum *"nearly perfect fluid"*
with shortest possible *local* equilibration time, $\tau_{\rm eq}$

$$\boxed{\tau_{\rm eq} = \mathcal{C}\frac{\hbar}{k_B T}}$$

where $\mathcal{C}$ is a *universal* constant.

Response functions are characterized by poles in LHP
with $\omega \sim k_B T/\hbar$.
These poles (quasi-normal modes) appear naturally in
the holographic theory.
(Analogs of Higgs quasi-normal mode.)

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Conductivity

$$\boxed{\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]}$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Momentum transport

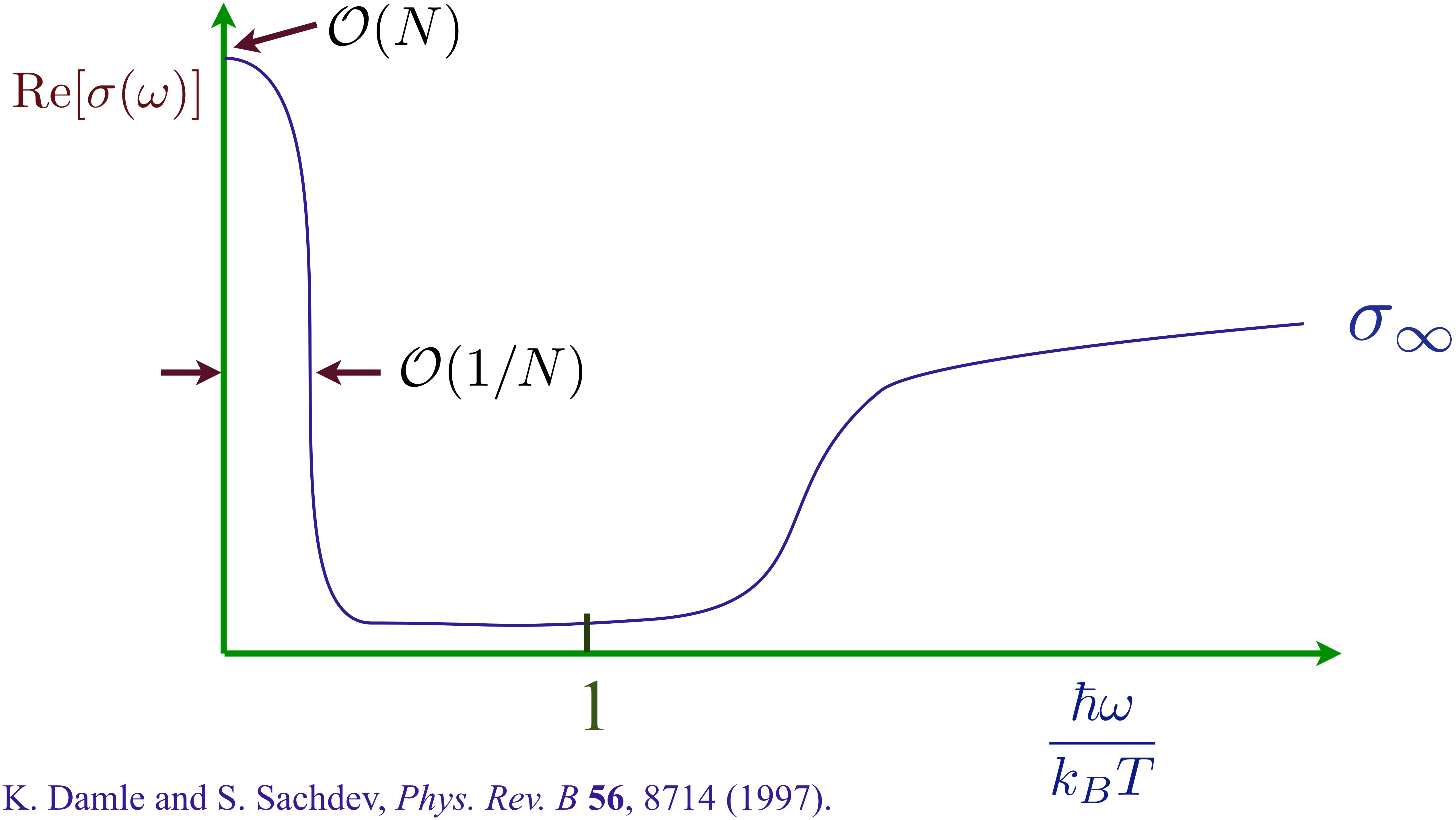
$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1) \;]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Vector large-N expansion for frequency-dependent conductivity of CFT3

$$\sigma(\omega) = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \quad ; \quad \Sigma \rightarrow \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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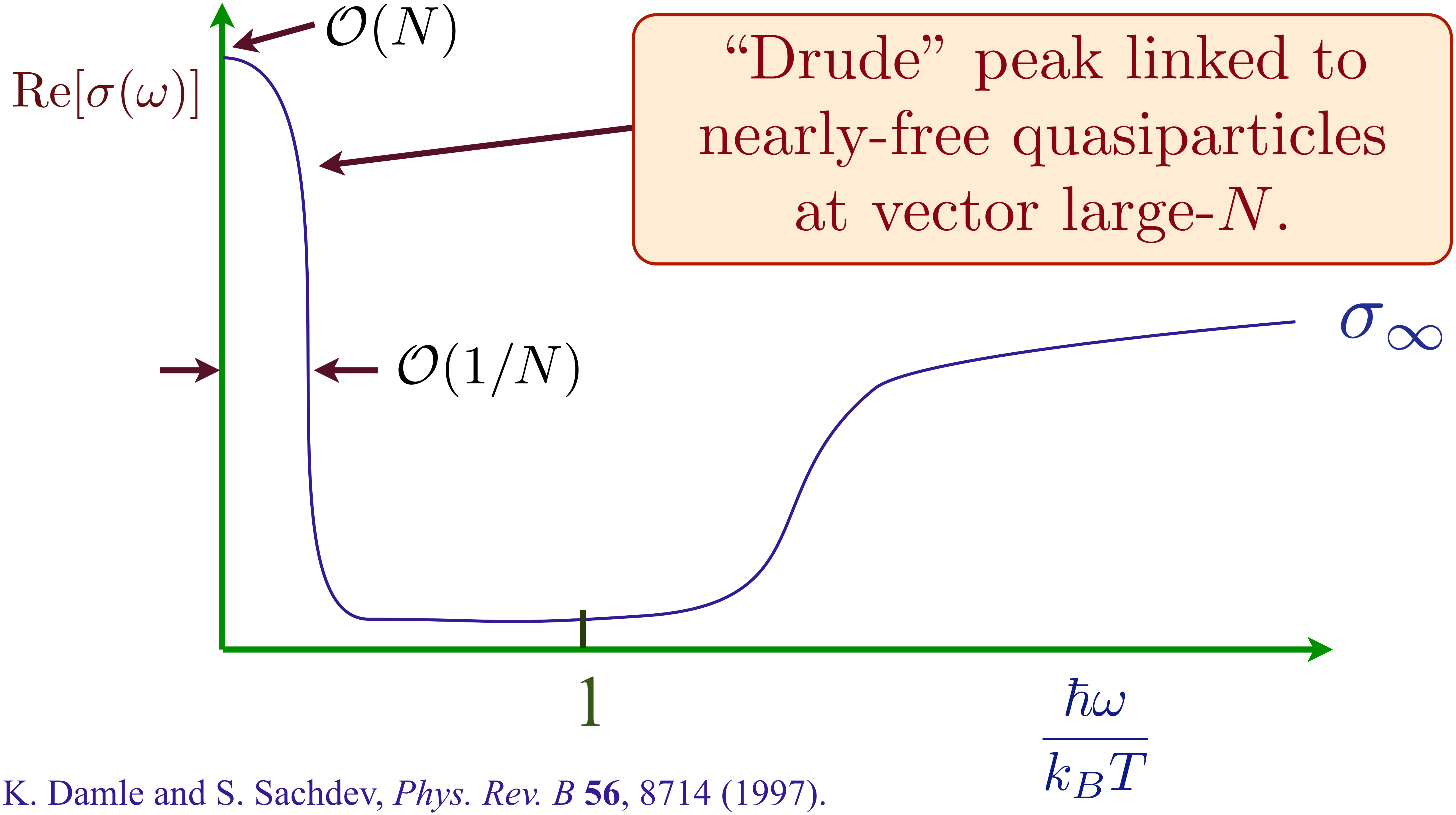
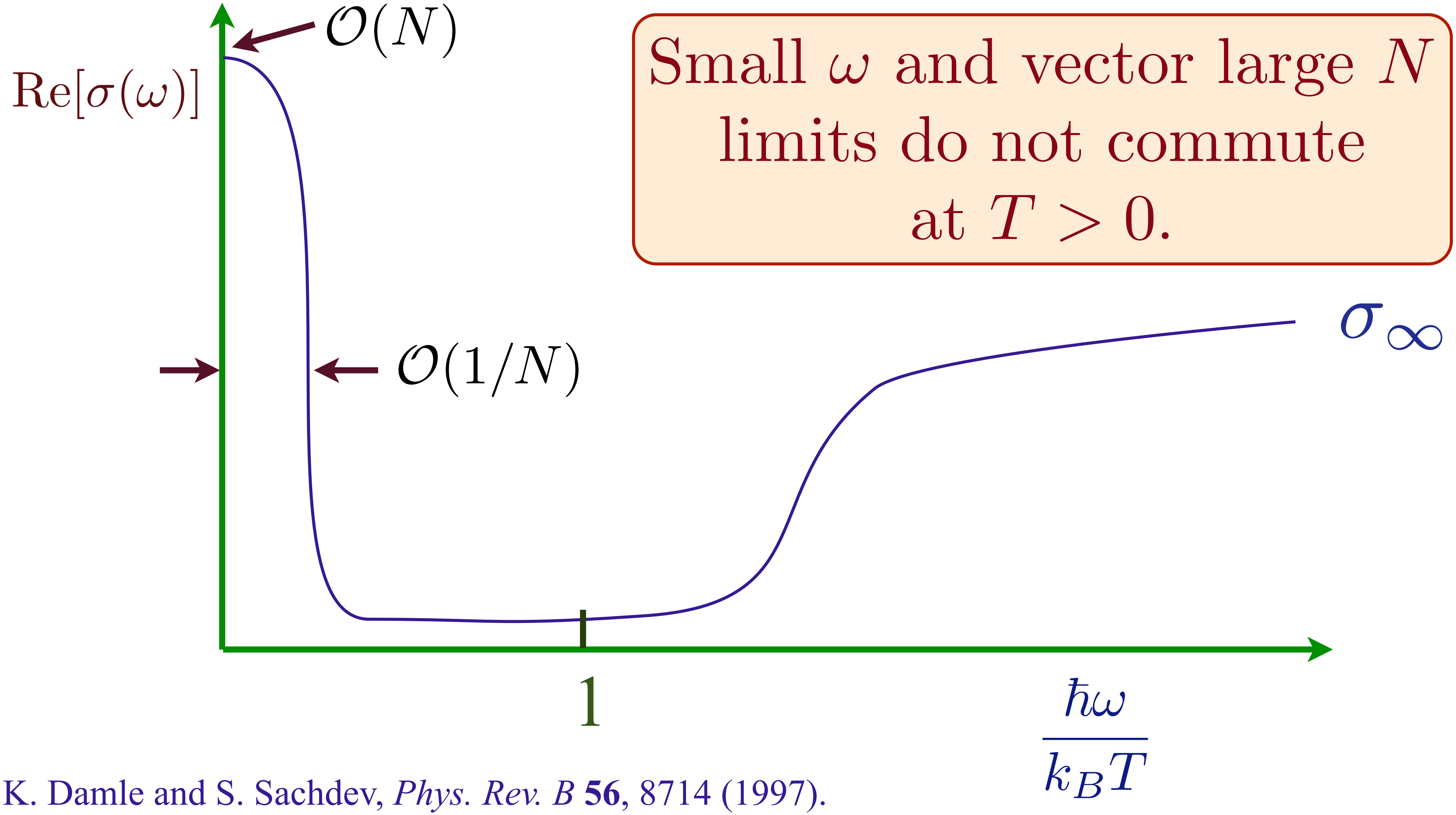


"Drude" peak linked to nearly-free quasiparticles at vector large-N.

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Vector large-N expansion for frequency-dependent conductivity of CFT3

$$\sigma(\omega) = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \quad ; \quad \Sigma \to \text{a universal function}$$



Small ω and vector large N limits do not commute at $T > 0$.

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

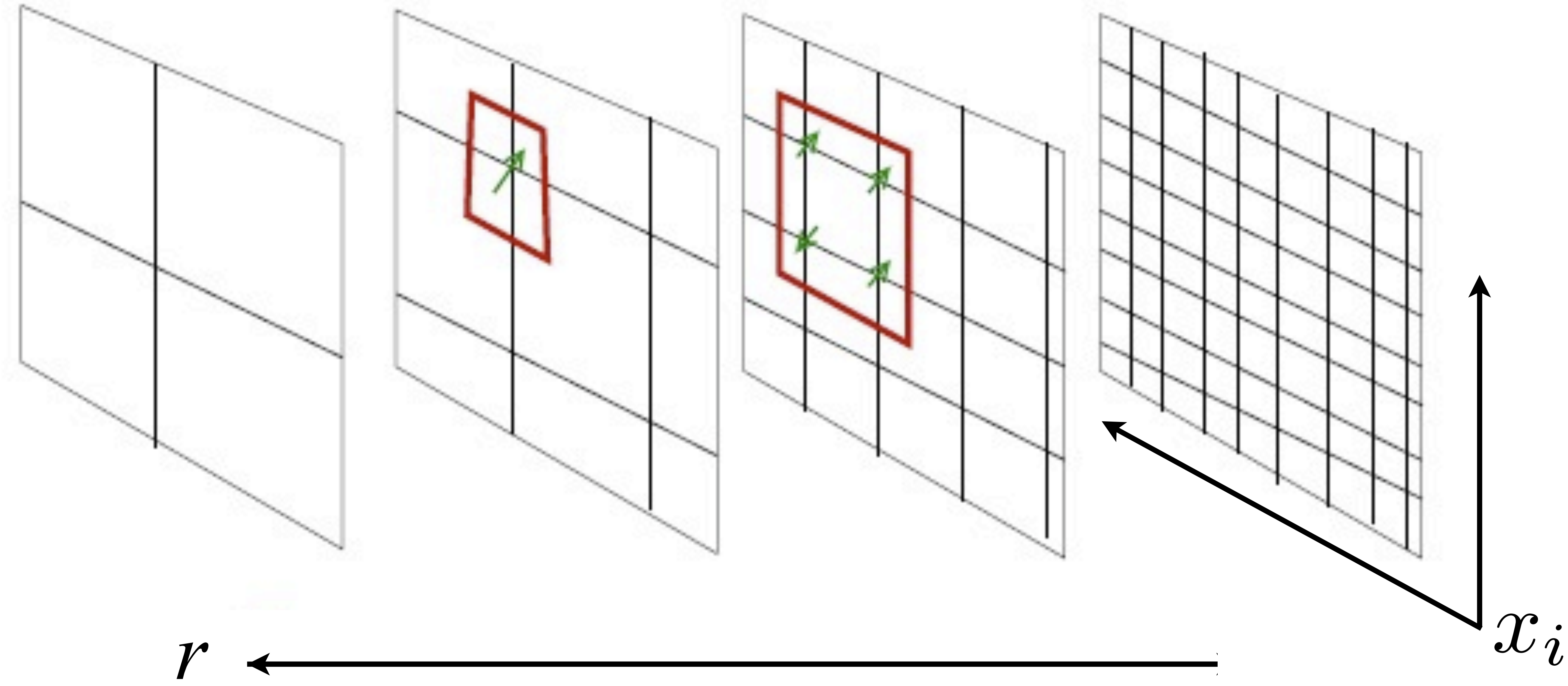
Holography

Field theories in $d+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

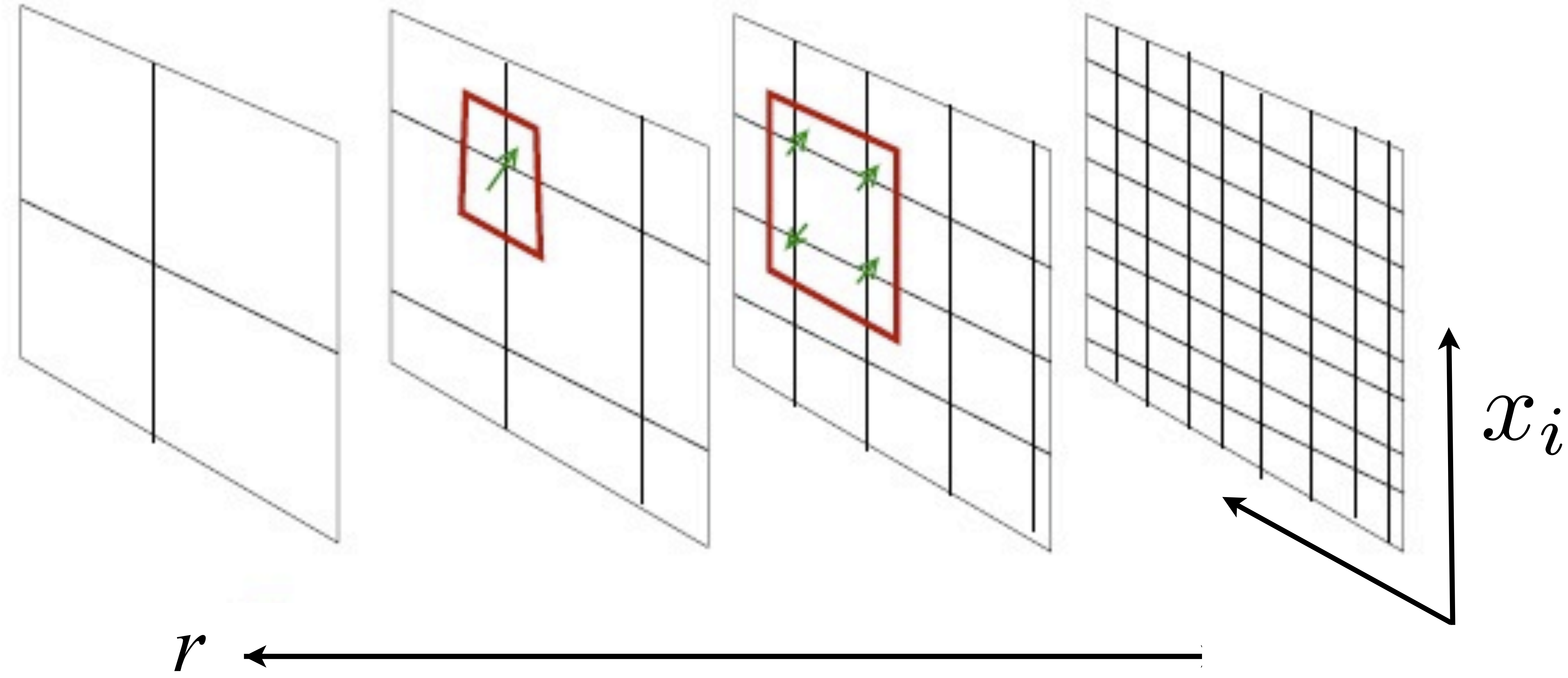
Holography



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

J. McGreevy, arXiv0909.0518

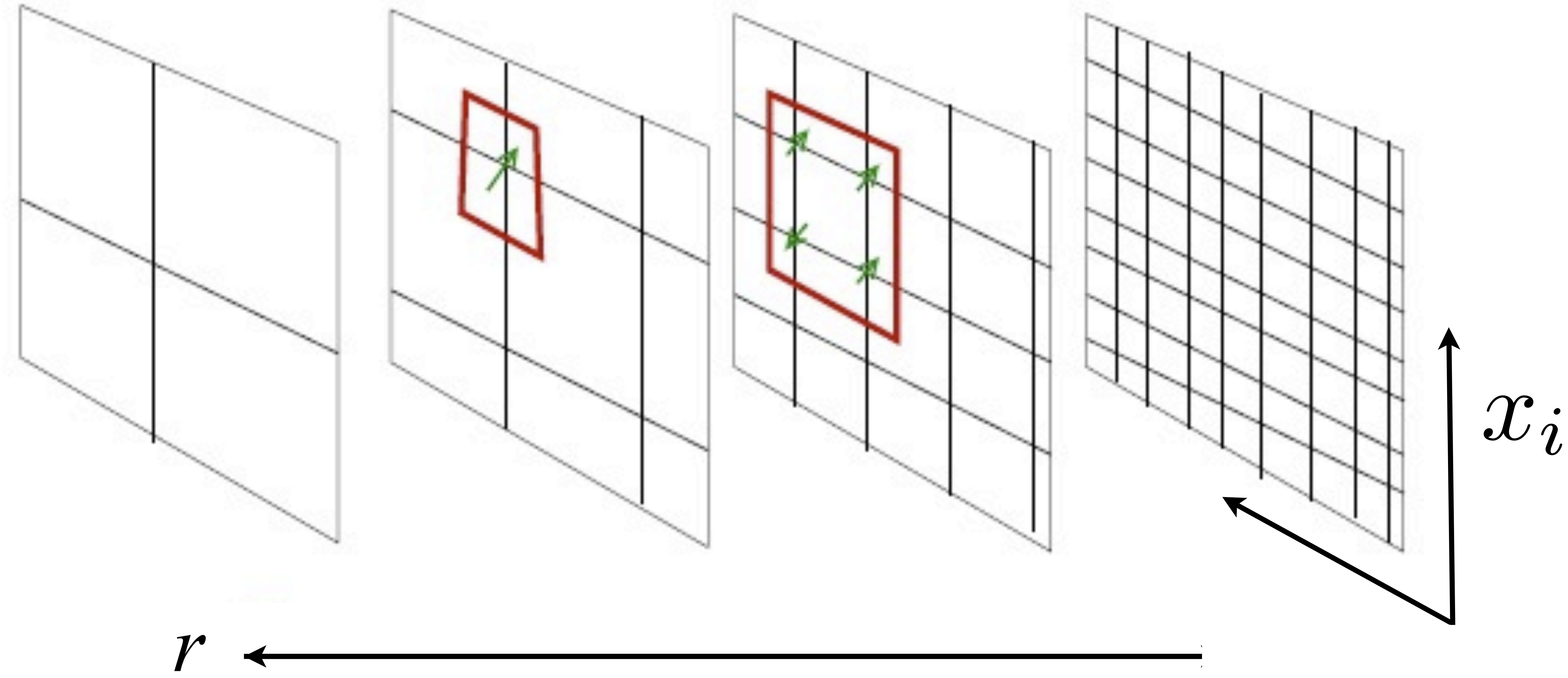
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography

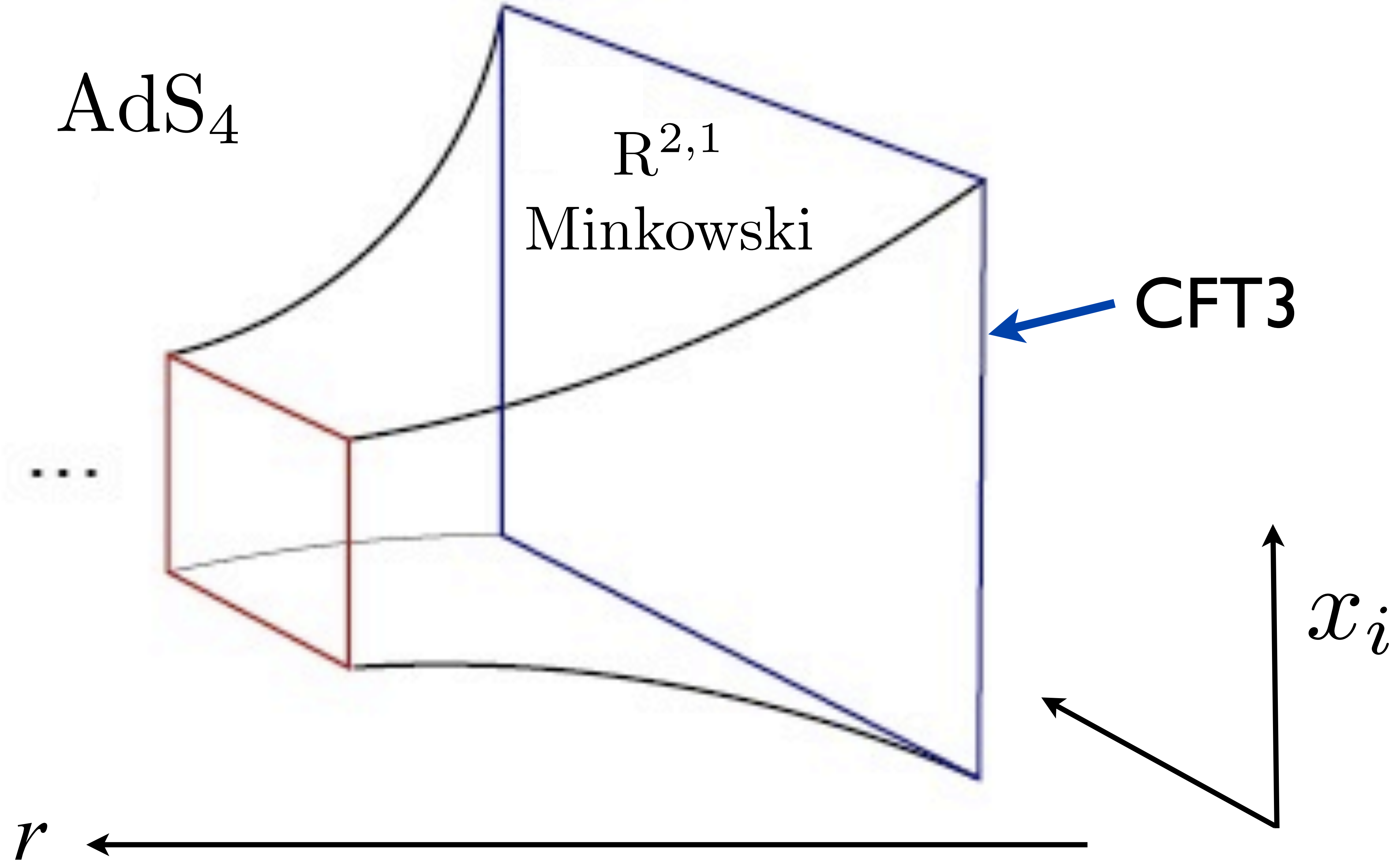


This gives the unique metric

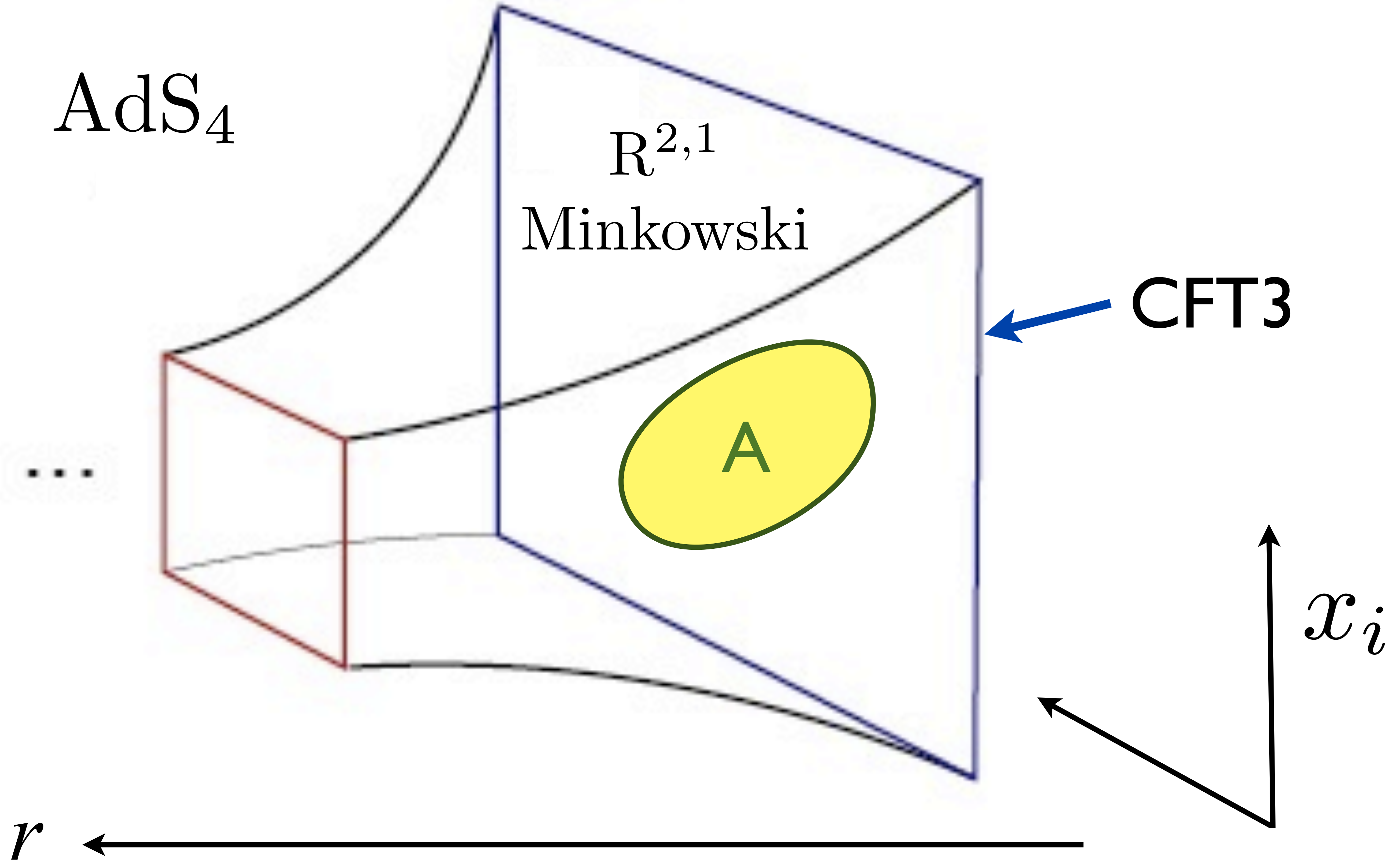
$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS$_{d+2}$.

AdS/CFT correspondence

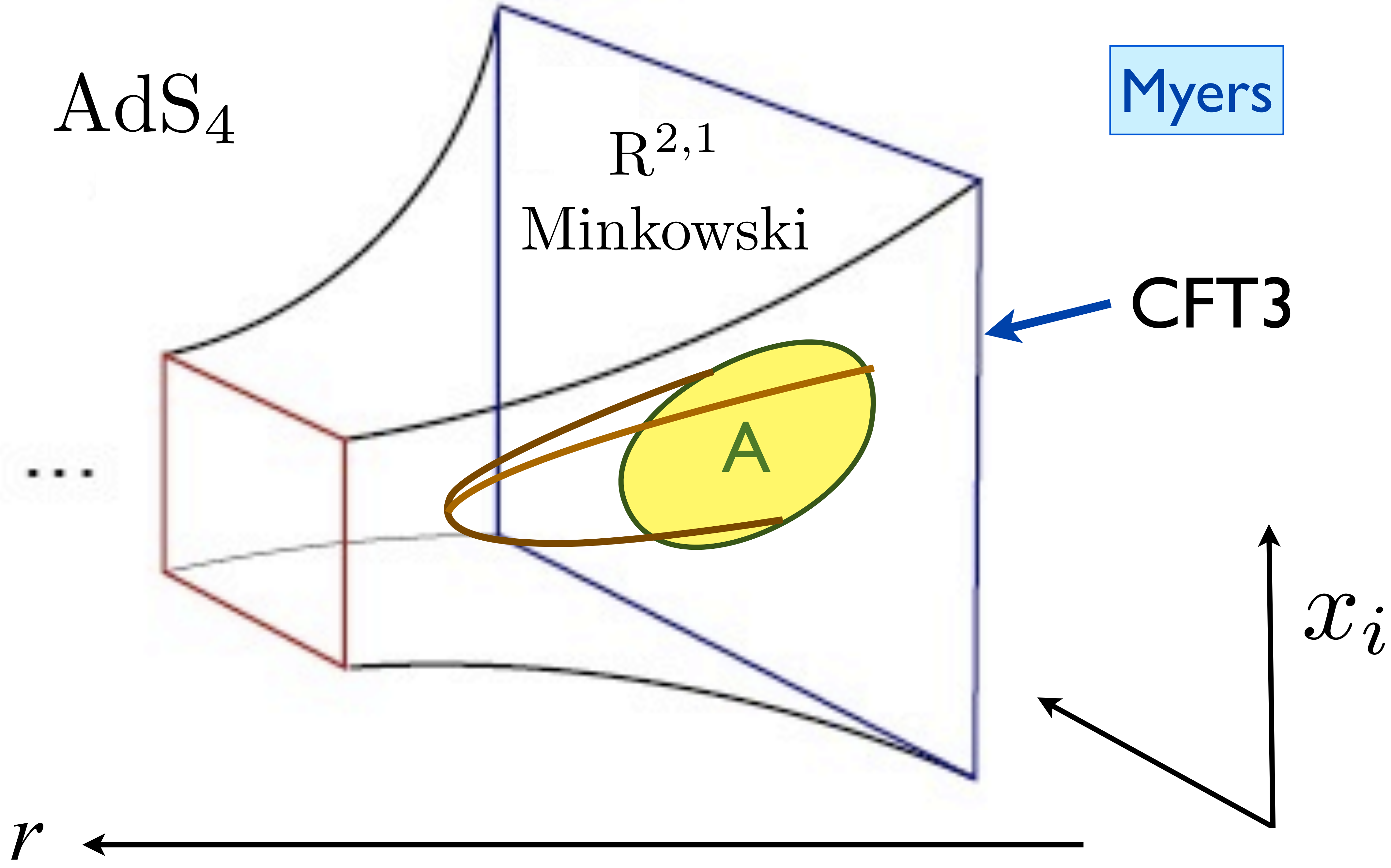


AdS/CFT correspondence



AdS/CFT correspondence

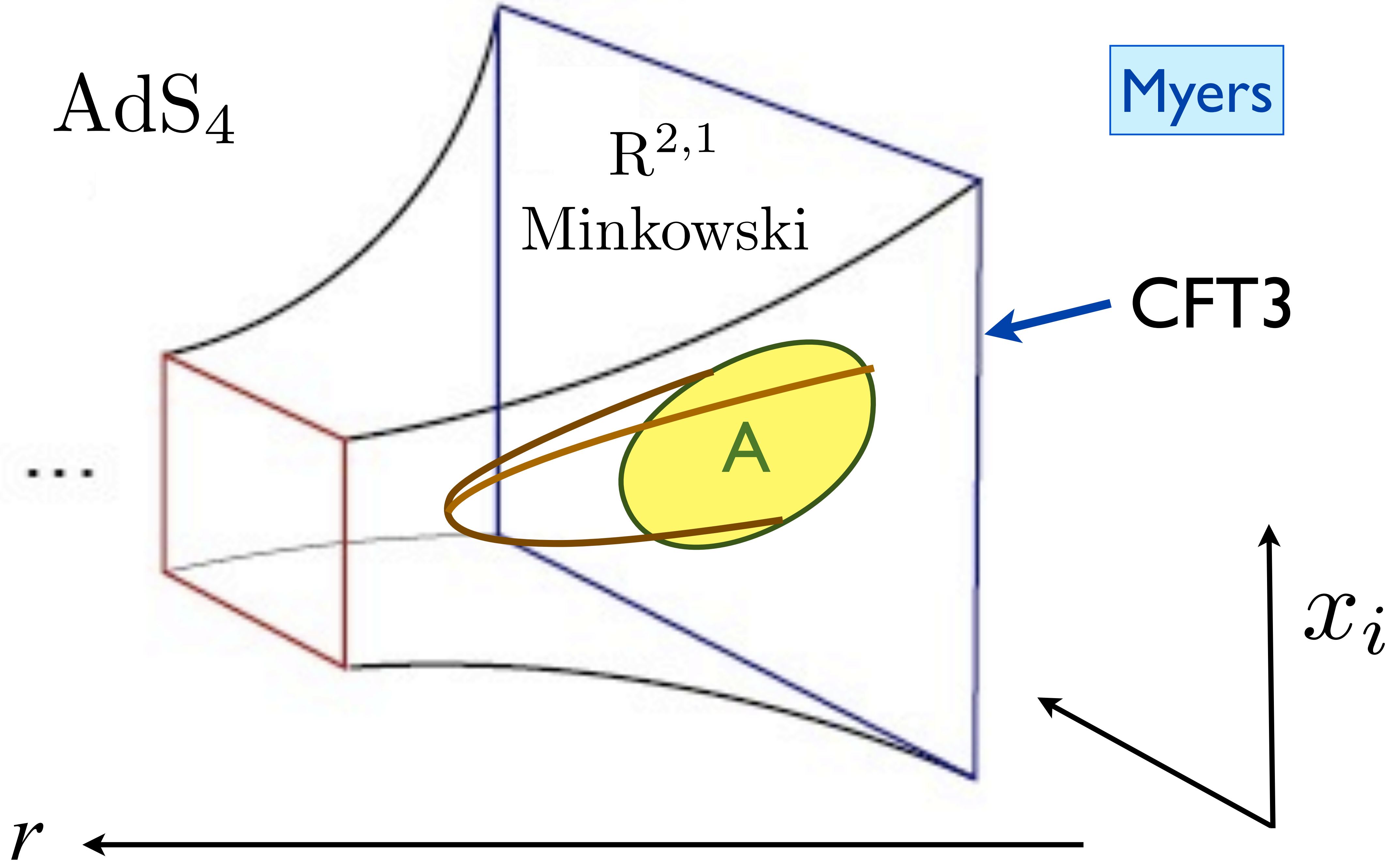
Myers



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence

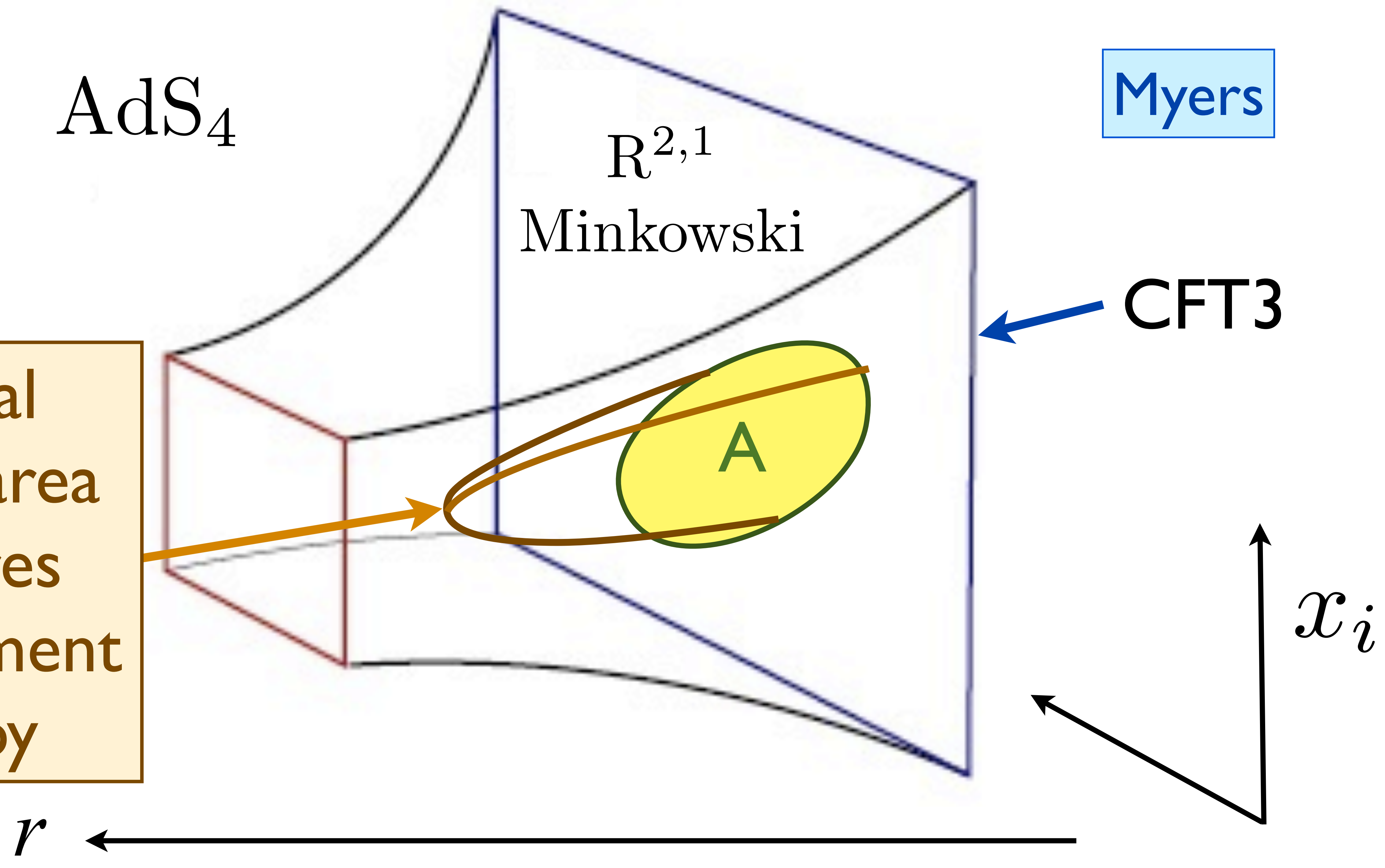


The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence

Myers

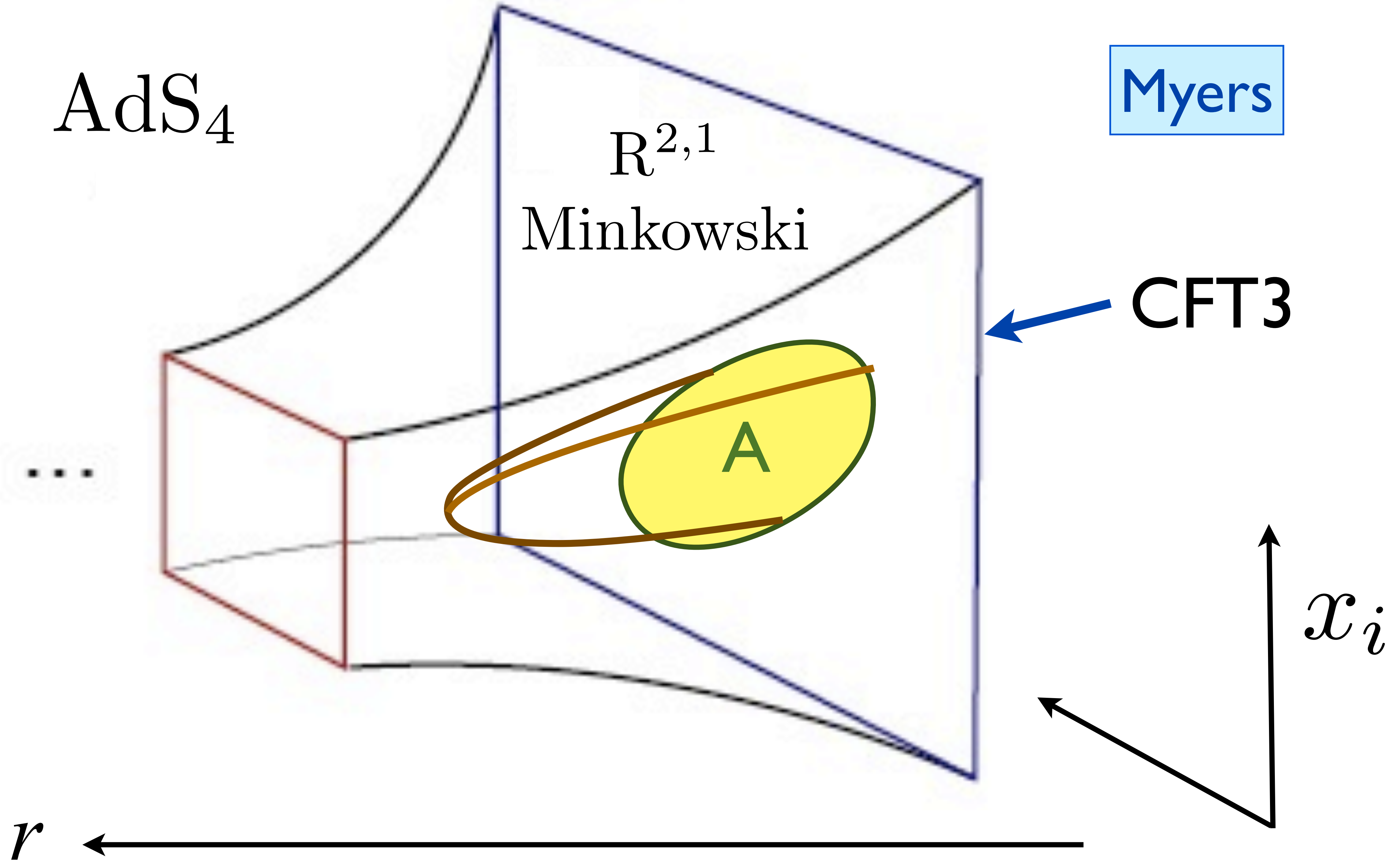


Minimal surface area measures entanglement entropy

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence

Myers



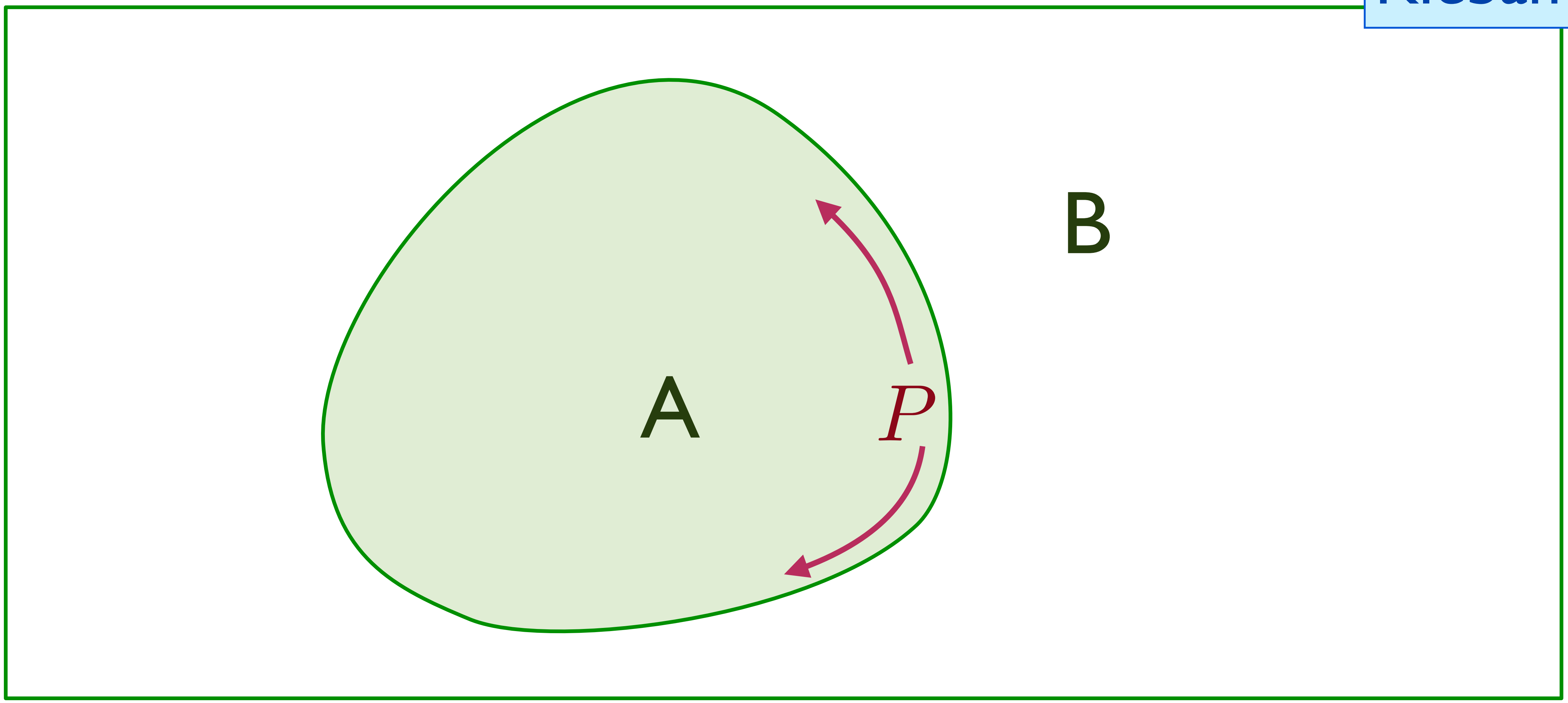
- Computation of minimal surface area yields
$$S_E = aP - \gamma,$$
where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Entanglement entropy from field theory of CFT3

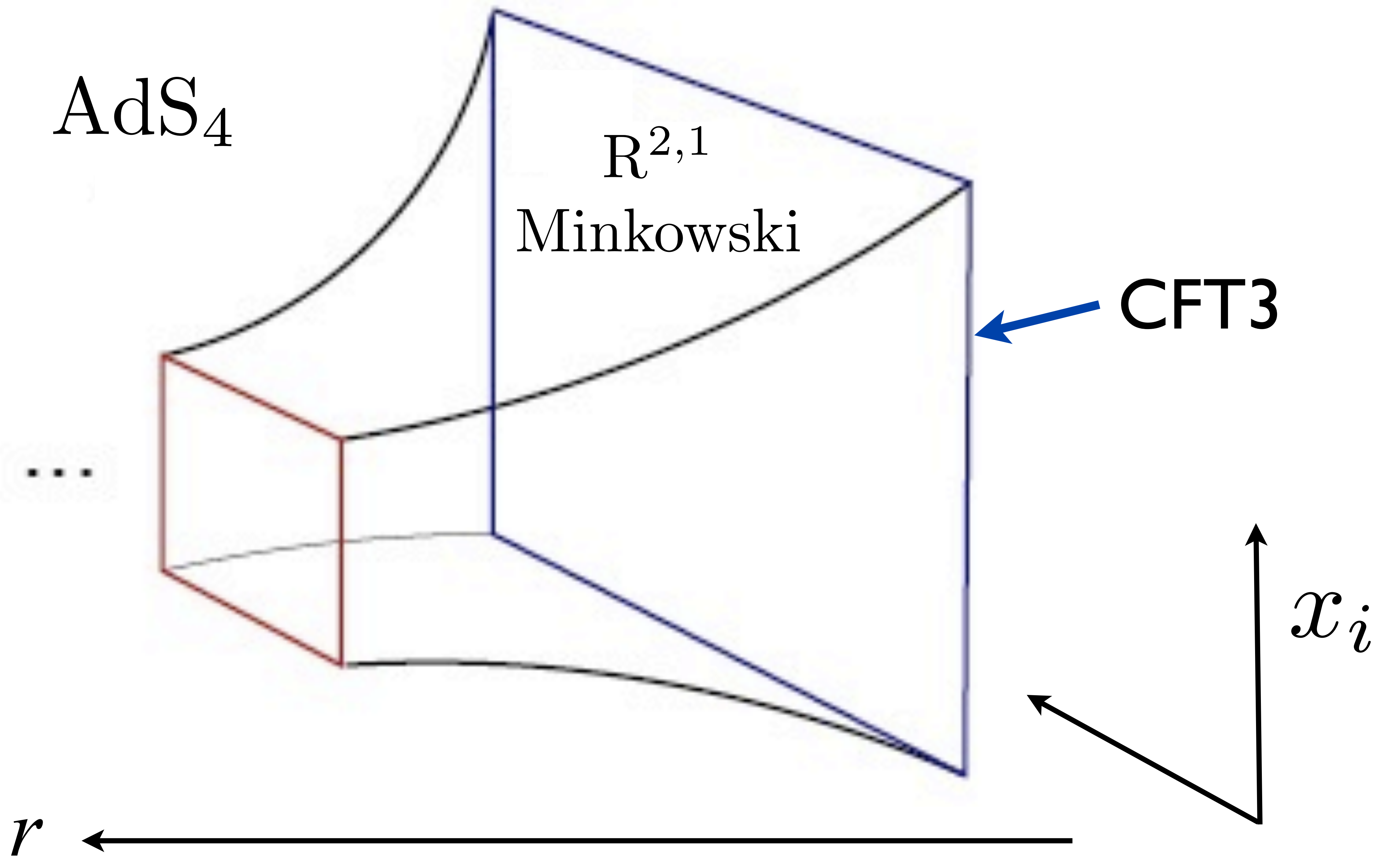
- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.

Klebanov



M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\boxed{\mathcal{S}_E = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]}$$

AdS/CFT correspondence

For every primary operator $O(\boldsymbol{x})$ in the CFT, there is a corresponding field $\phi(\boldsymbol{x}, r)$ in the bulk (gravitational) theory. For a scalar operator $O(\boldsymbol{x})$ of dimension Δ, the correlators of the boundary and bulk theories are related by

$$\langle O(\boldsymbol{x}_1) \ldots O(\boldsymbol{x}_n) \rangle_{\mathrm{CFT}} = \\ Z^n \lim_{r \to 0} r_1^{-\Delta} \ldots r_n^{-\Delta} \langle \phi(\boldsymbol{x}_1, r_1) \ldots \phi(\boldsymbol{x}_n, r_n) \rangle_{\mathrm{bulk}}$$

where the “wave function renormalization” factor $Z = (2\Delta - D)$.

AdS/CFT correspondence

For a U(1) conserved current J_μ of the CFT, the corresponding bulk operator is a U(1) *gauge* field A_μ. With a Maxwell action for the gauge field

$$\boxed{\mathcal{S}_M = \frac{1}{4g_M^2}\int d^{D+1}x\,\sqrt{g}F_{ab}F^{ab}}$$

we have the bulk-boundary correspondence

$$\begin{aligned}&\langle J_\mu(\boldsymbol{x}_1)\ldots J_\nu(\boldsymbol{x}_n)\rangle_{\mathrm{CFT}} = \\ &\quad (Zg_M^{-2})^n \lim_{r\to 0} r_1^{2-D}\ldots r_n^{2-D}\,\langle A_\mu(\boldsymbol{x}_1,r_1)\ldots A_\nu(\boldsymbol{x}_n,r_n)\rangle_{\mathrm{bulk}}\end{aligned}$$

with $Z = D-2$.

AdS/CFT correspondence

A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the change in metric of the bulk theory. We write $\delta g_{\mu\nu} = (L^2/r^2)\chi_{\mu\nu}$, and then the bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(\boldsymbol{x}_1)\ldots T_{\rho\sigma}(\boldsymbol{x}_n)\rangle_{\mathrm{CFT}} =$$

$$\left(\frac{ZL^2}{\kappa^2}\right)^n \lim_{r\to 0} r_1^{-D}\ldots r_n^{-D}\,\langle\chi_{\mu\nu}(\boldsymbol{x}_1,r_1)\ldots\chi_{\rho\sigma}(\boldsymbol{x}_n,r_n)\rangle_{\mathrm{bulk}}\,,$$

with $Z = D$.

AdS/CFT correspondence

So the minimal bulk theory for a CFT with a conserved U(1) current is the *Einstein-Maxwell* theory with a cosmological constant

$$\boxed{\begin{aligned}\mathcal{S} = & \frac{1}{4g_M^2}\int d^4x\,\sqrt{g}F_{ab}F^{ab} \\ & + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right].\end{aligned}}$$

This action is characterized by two dimensionless parameters: g_M and L^2/κ^2, which are related to the $T=0$ conductivity $\sigma(\omega)=\sigma_\infty$ and the central charge of the CFT.

AdS/CFT correspondence

This minimal action also fixes multi-point correlators of the CFT: however these do not have the most general form allowed for a CFT. To fix these, we have to allow for higher-gradient terms in the bulk action. For the conductivity, it turns out that only a single 4 gradient term contributes

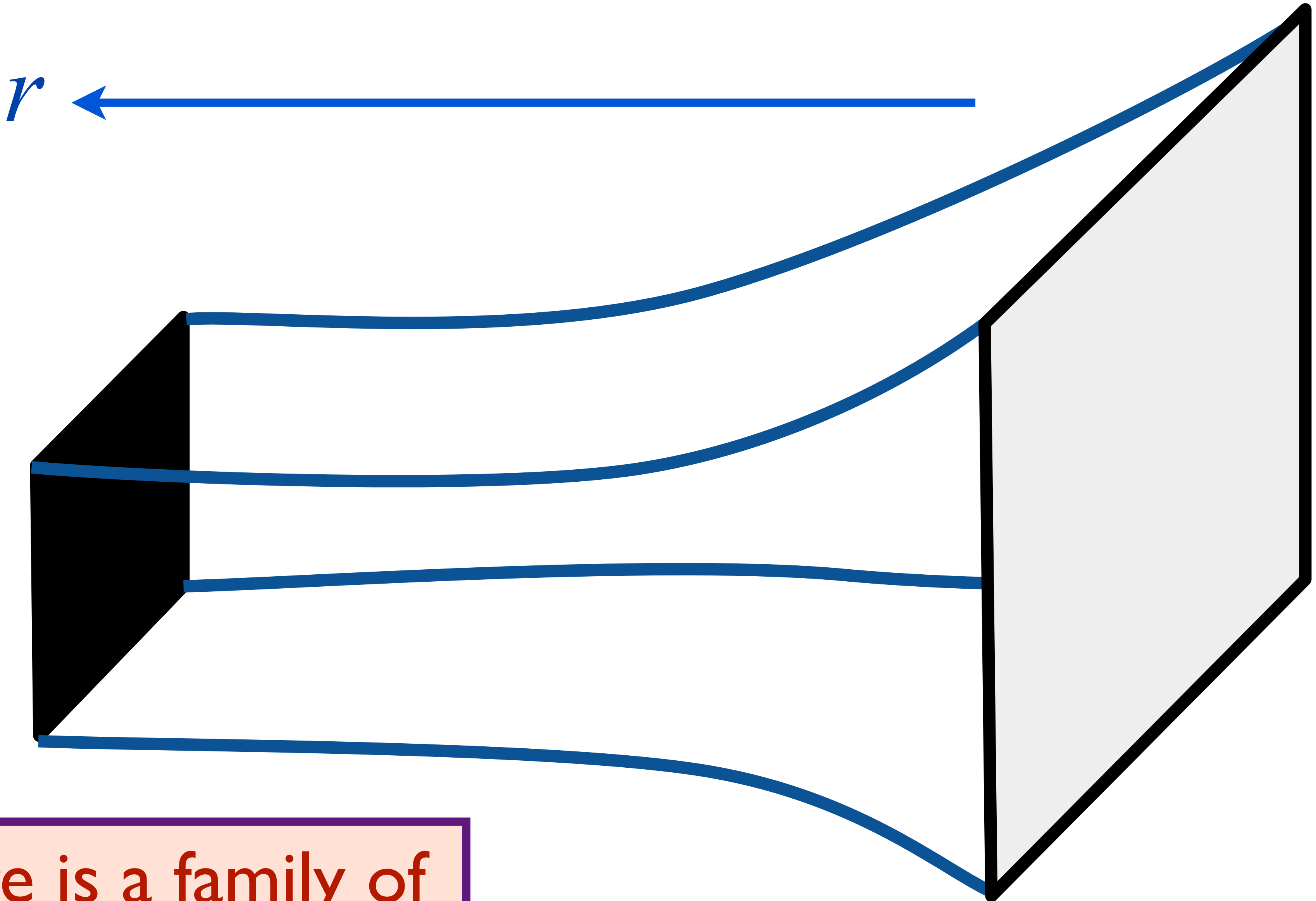
$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[- \frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right],$$

where C_{abcd} is the Weyl tensor. The parameter γ can be related to 3-point correlators of J_μ and $T_{\mu\nu}$. Both boundary and bulk methods show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, arXiv:1210.5247

AdS/CFT correspondence at non-zero temperatures

AdS4-Schwarzschild black-brane

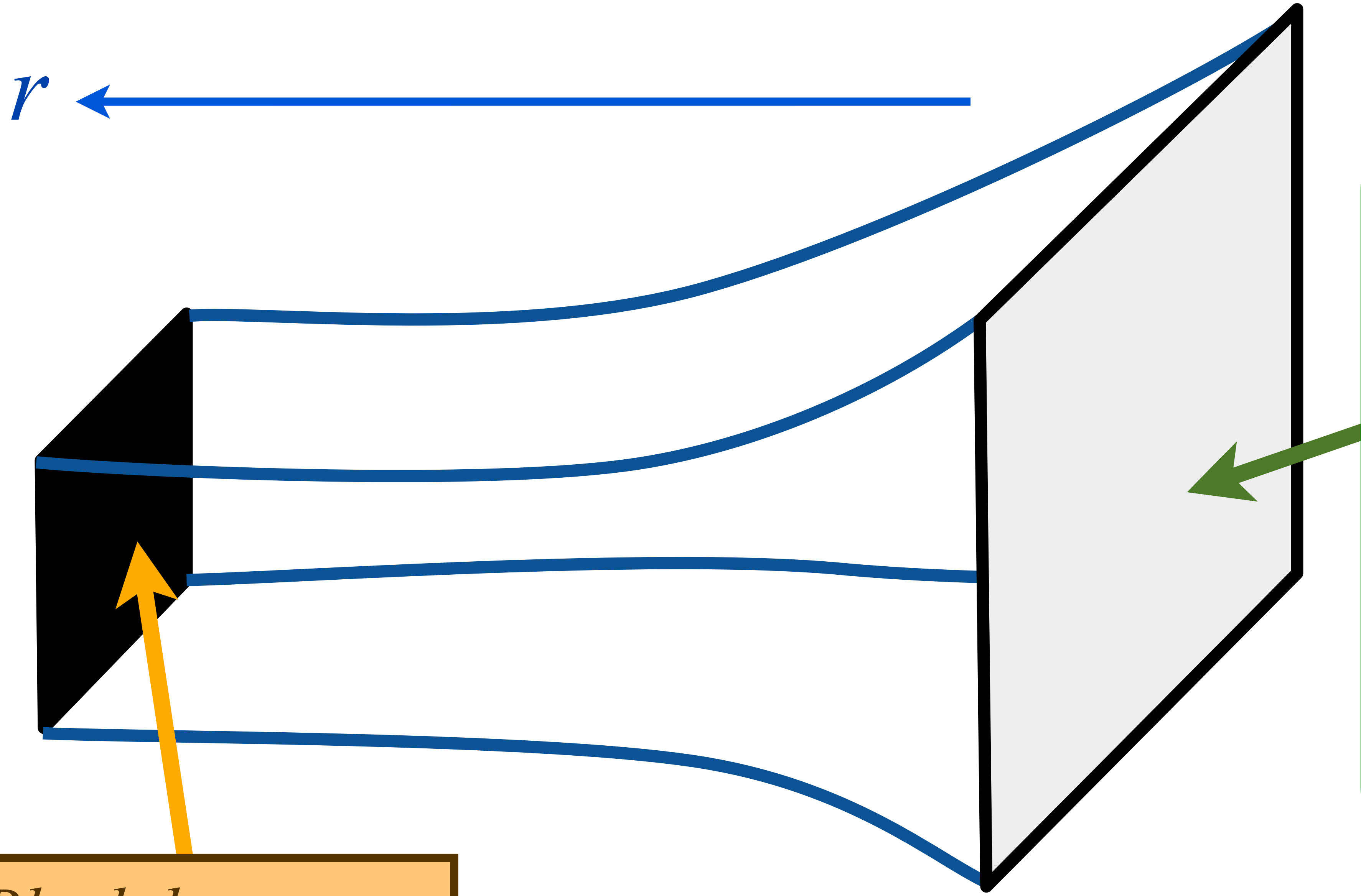


There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)\right]$$

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point:

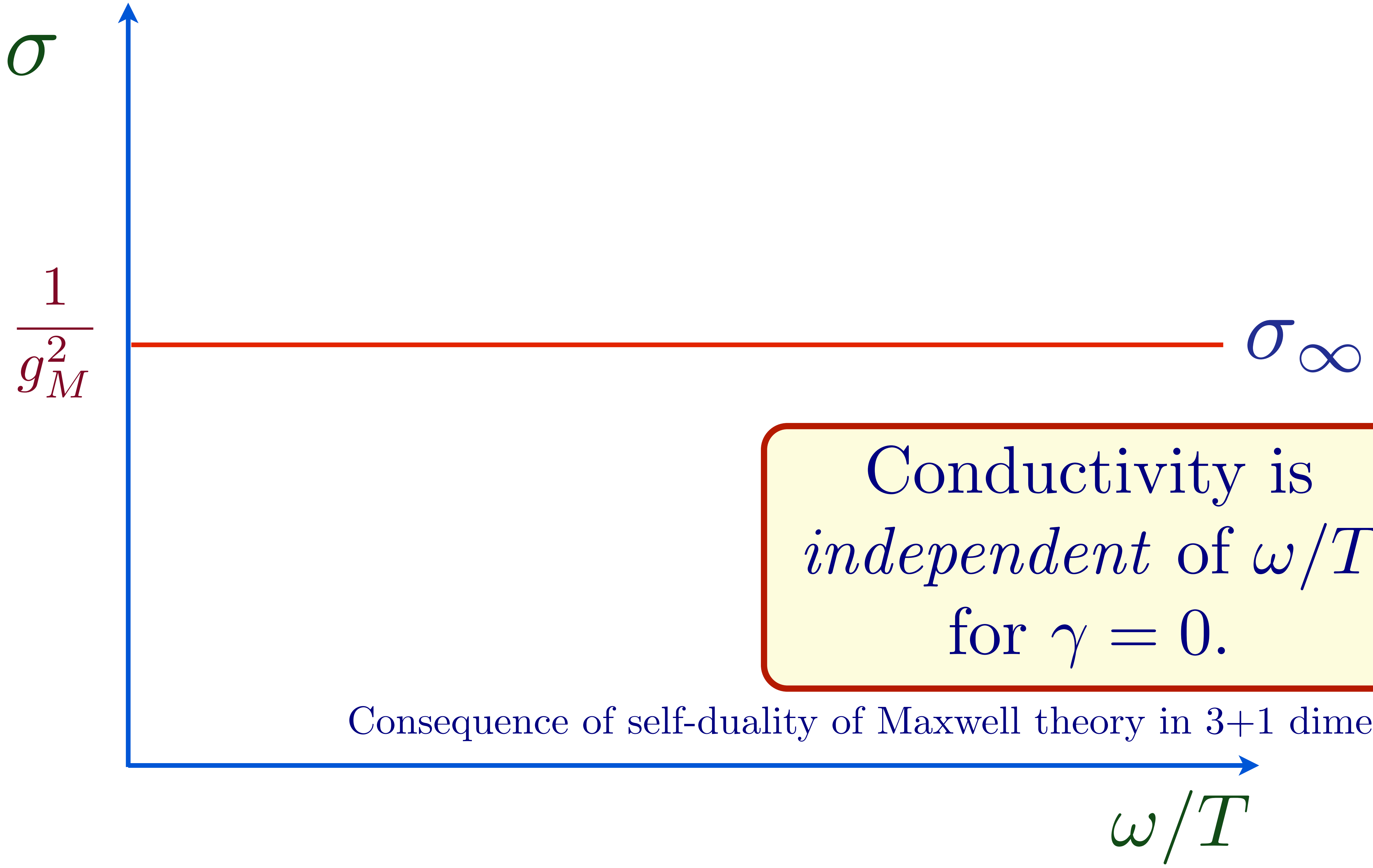
$$k_B T = \frac{3\hbar}{4\pi R}.$$

Black-brane at temperature of 2+1 dimensional quantum critical system

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

AdS$_4$ theory of quantum criticality

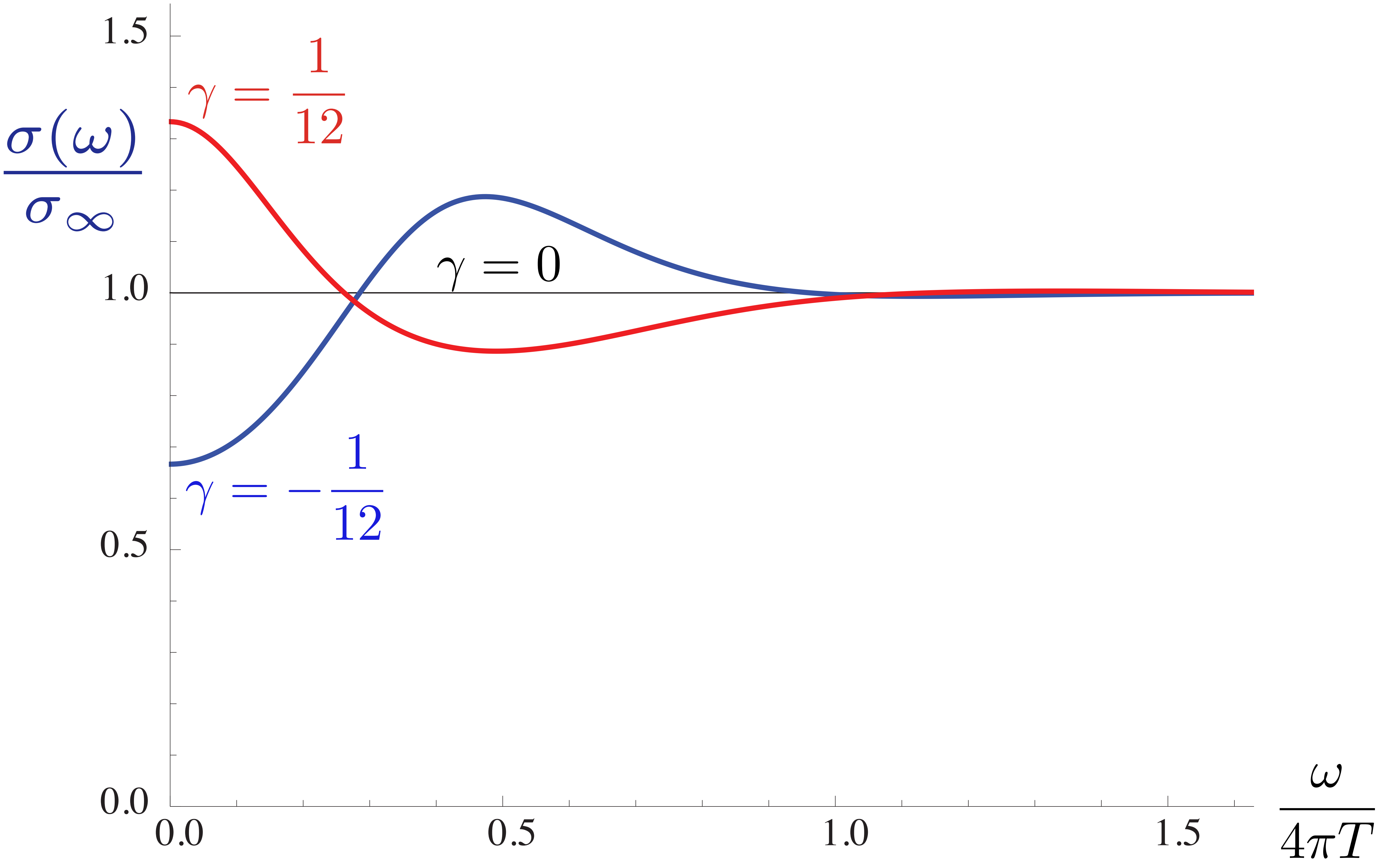


Conductivity is *independent* of ω/T for $\gamma = 0$.

Consequence of self-duality of Maxwell theory in 3+1 dimensions

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

AdS$_4$ theory of quantum criticality



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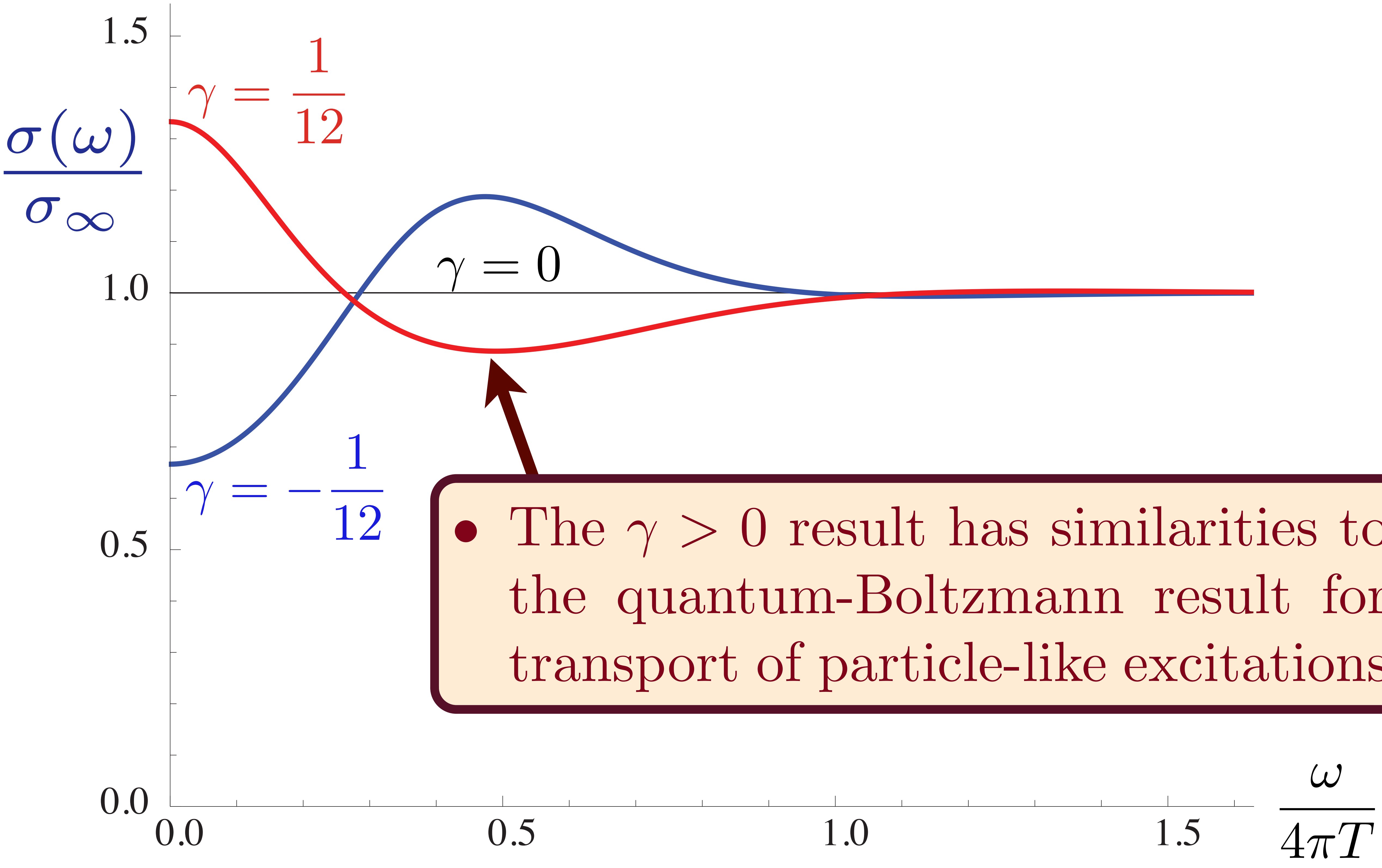
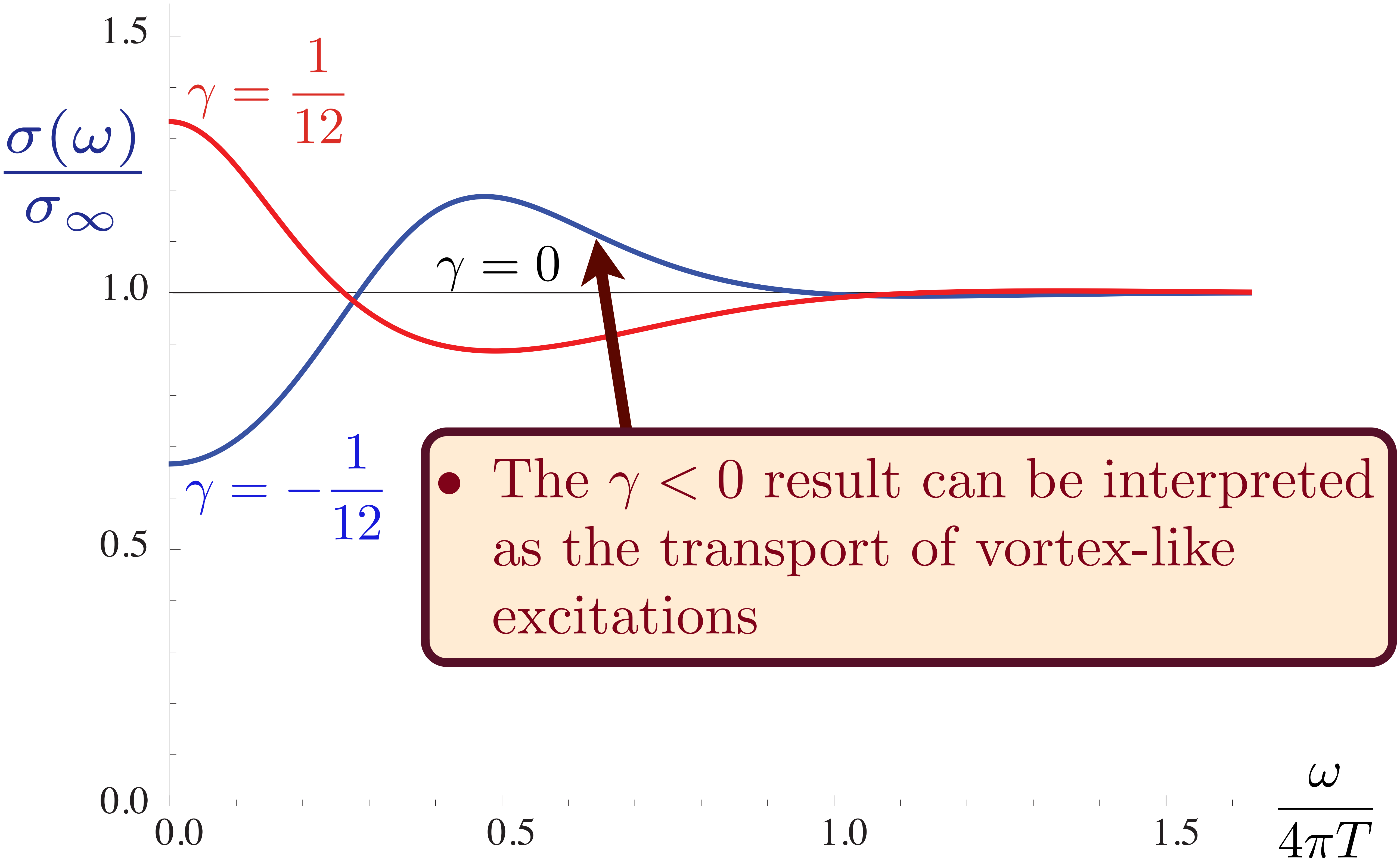


- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

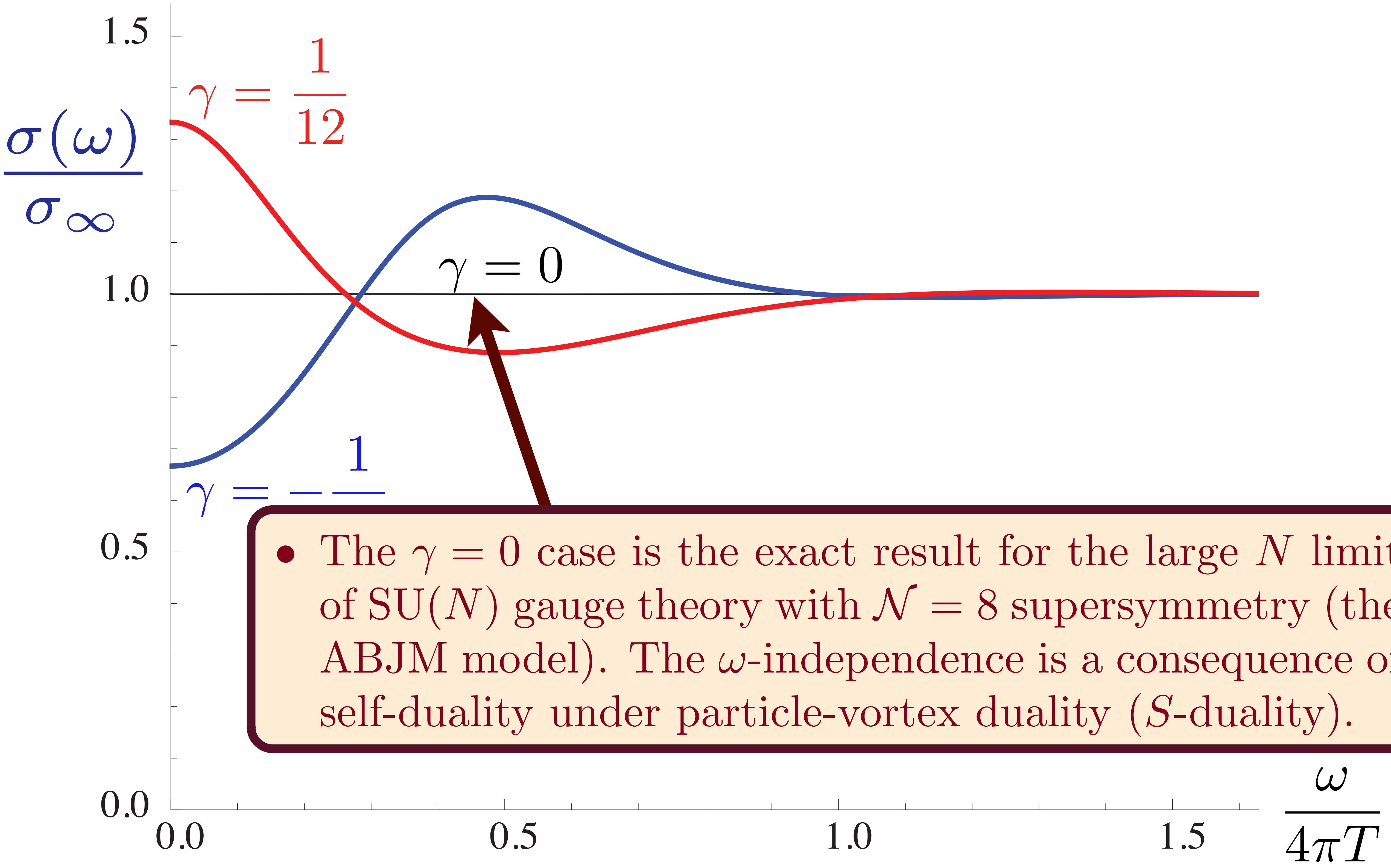
AdS$_4$ theory of quantum criticality



- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

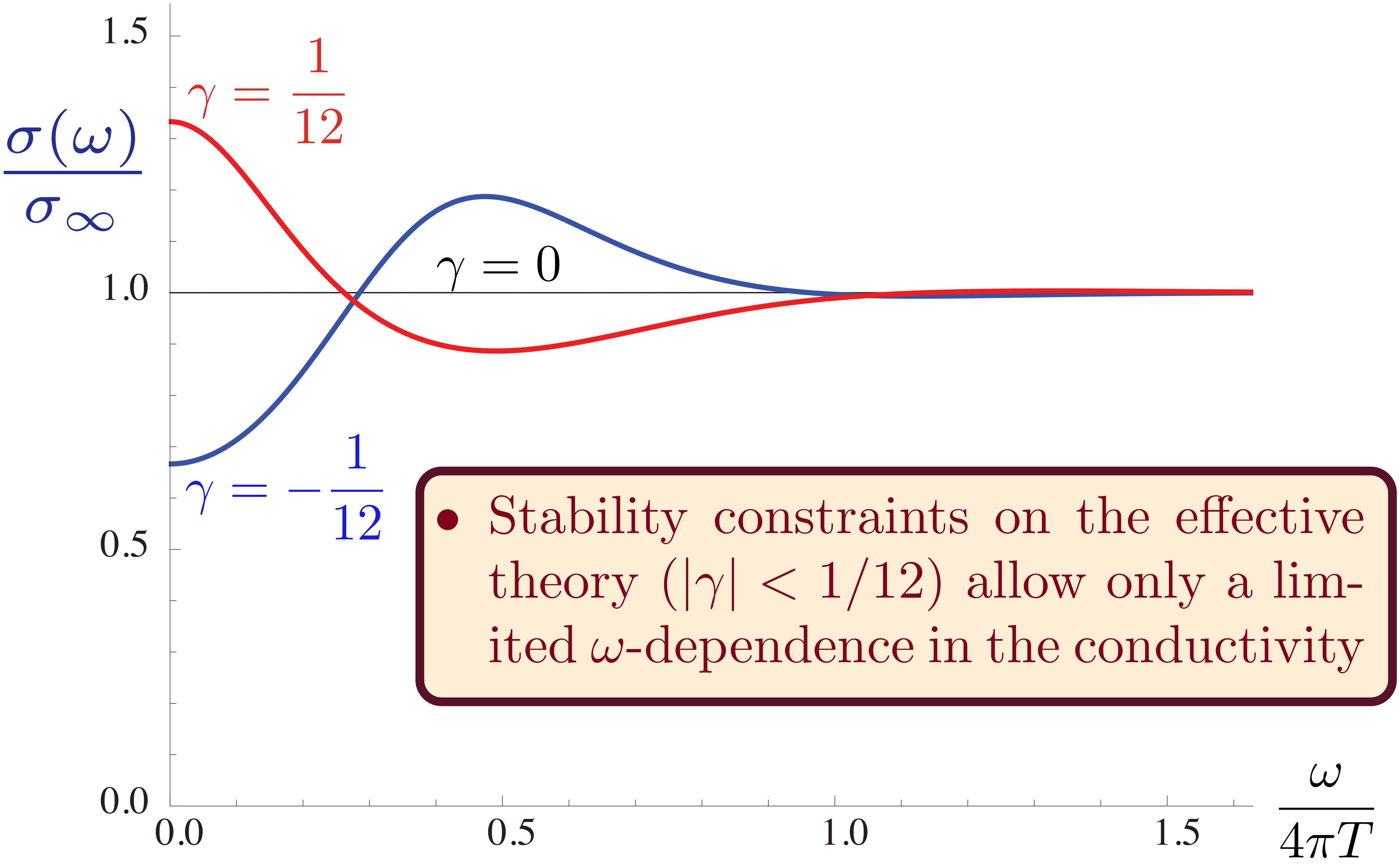
AdS$_4$ theory of quantum criticality



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS₄ theory of quantum criticality



- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of quantum criticality

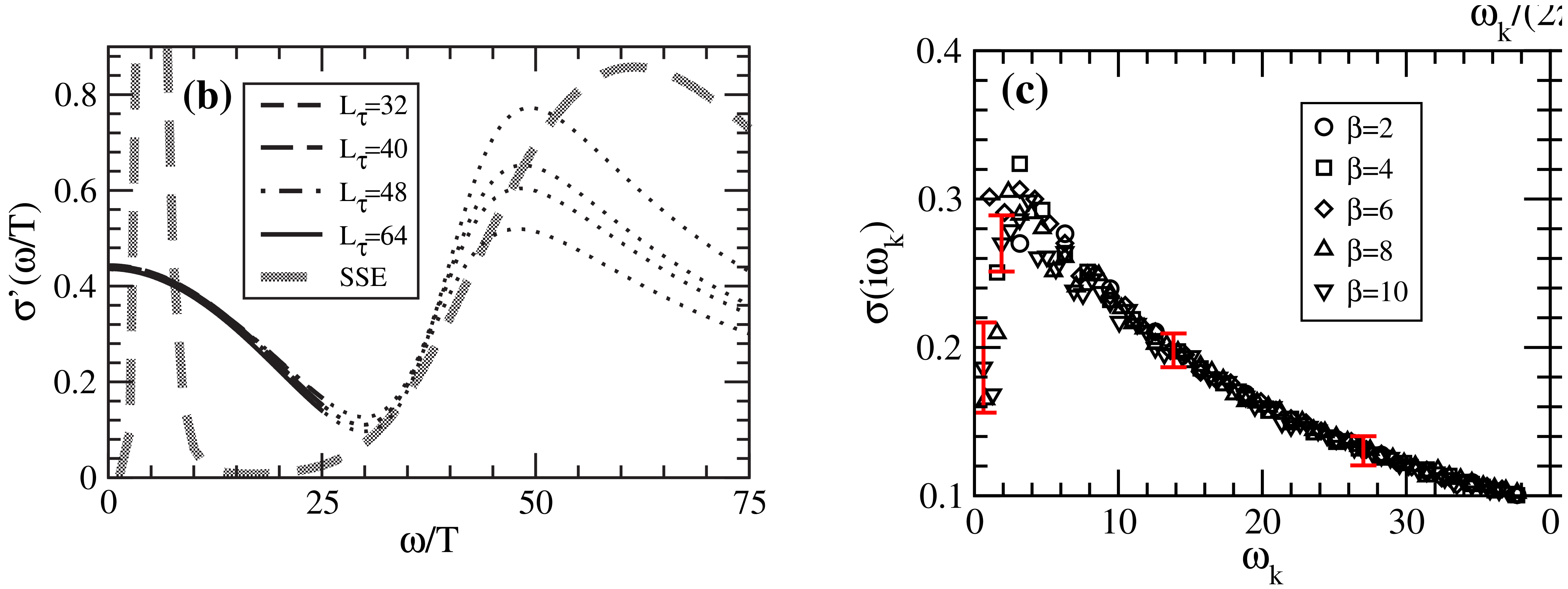
PRL **95,** 180603 (2005) PHYSICAL REVIEW LETTERS week ending 28 OCTOBER 2005

Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

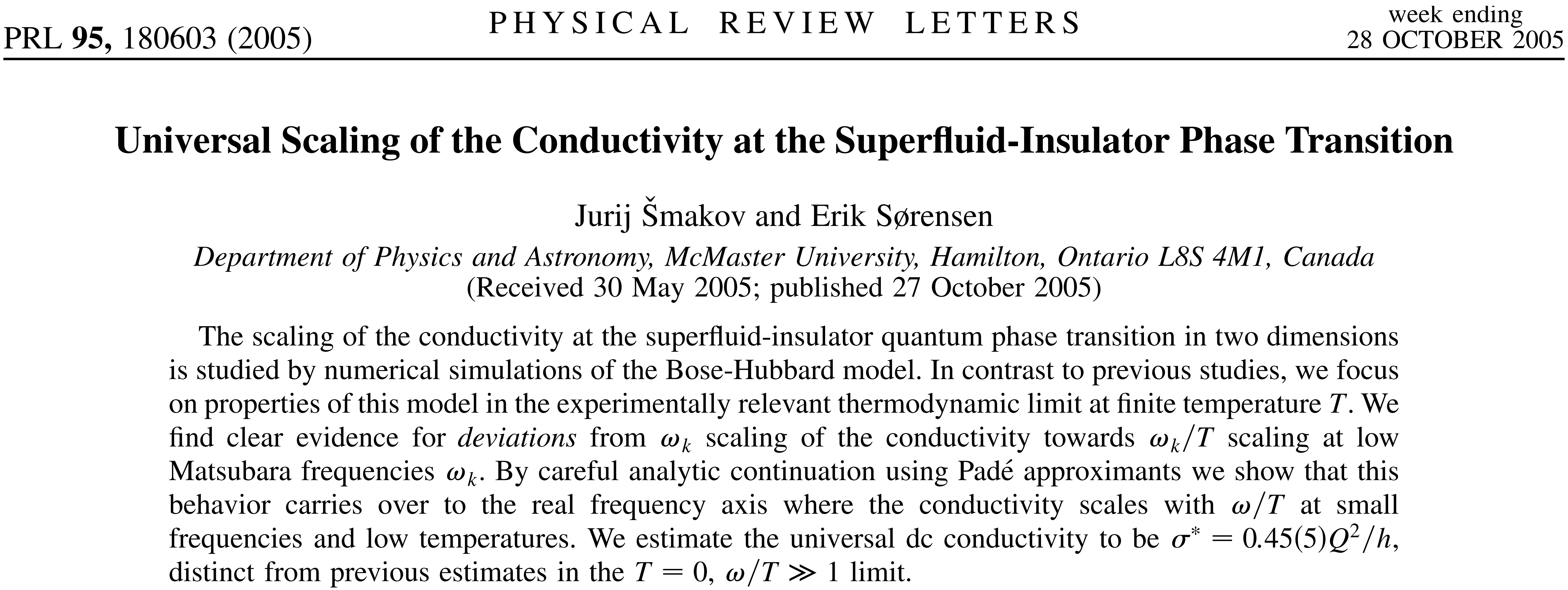
Jurij Šmakov and Erik Sørensen
Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada

(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature T. We find clear evidence for *deviations* from ω_k scaling of the conductivity towards ω_k/T scaling at low Matsubara frequencies ω_k. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with ω/T at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.



AdS$_4$ theory of quantum criticality



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QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

W. Witzack-Krempa and S. Sachdev, to appear

AdS$_4$ theory of quantum criticality

The holographic solutions for the conductivity satisfy two sum rules, valid for all CFT3s.

$$\boxed{\begin{aligned}\int_0^\infty d\omega \mathrm{Re}\left[\sigma(\omega)-\sigma(\infty)\right] &= 0\\ \int_0^\infty d\omega \mathrm{Re}\left[\frac{1}{\sigma(\omega)}-\frac{1}{\sigma(\infty)}\right] &= 0\end{aligned}}$$

The second rule follows from the existence of a EM-dual CFT3.

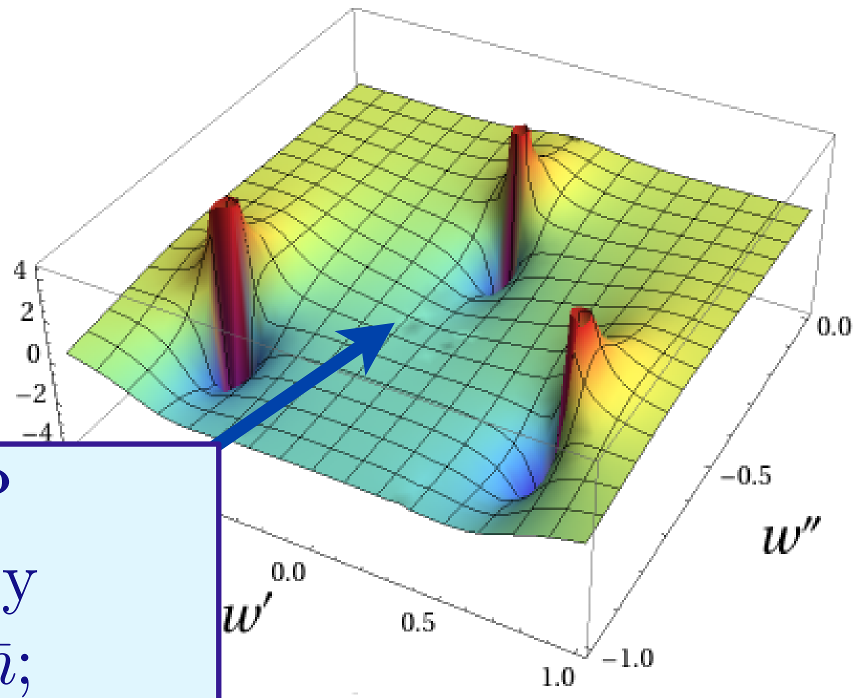
Boltzmann theory chooses a "particle" basis: this satisfies only *one* sum rule but not the other.

Holographic theory satisfies both sum rules.

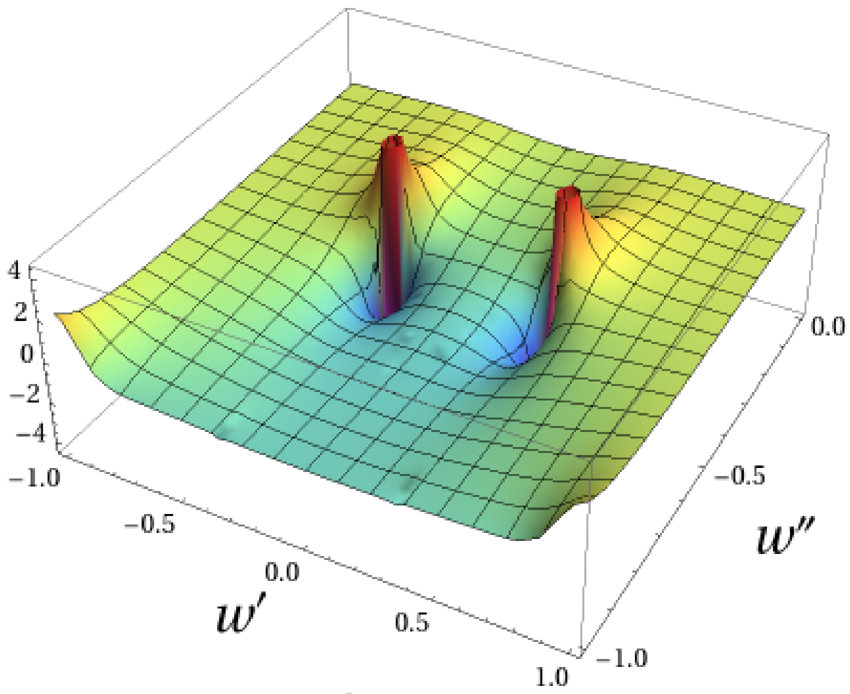
W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of quantum criticality

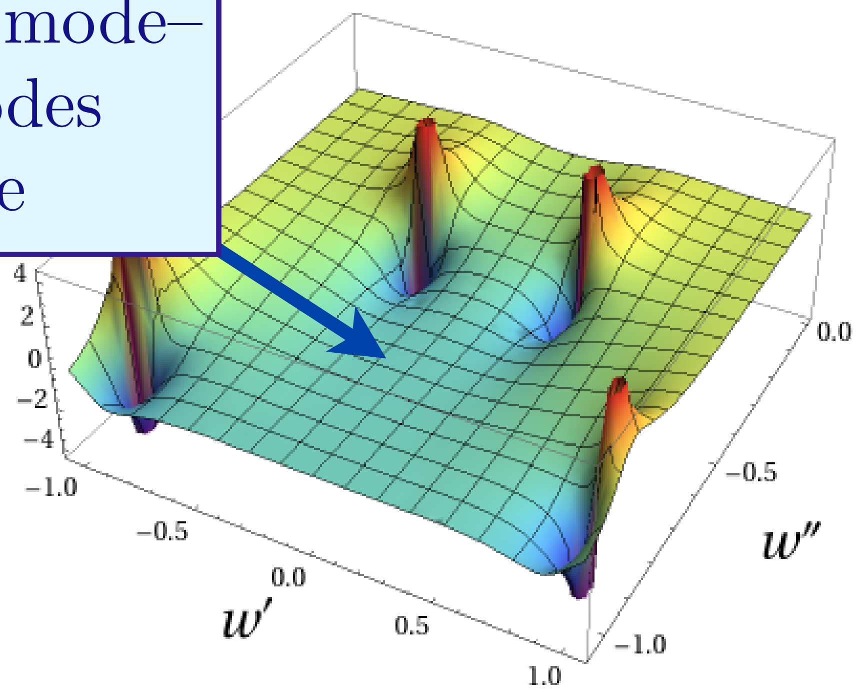
Poles in LHP
of conductivity
at $\omega \sim k_B T/\hbar$;
analog of
Higgs quasinormal mode–
quasinormal modes
of black brane



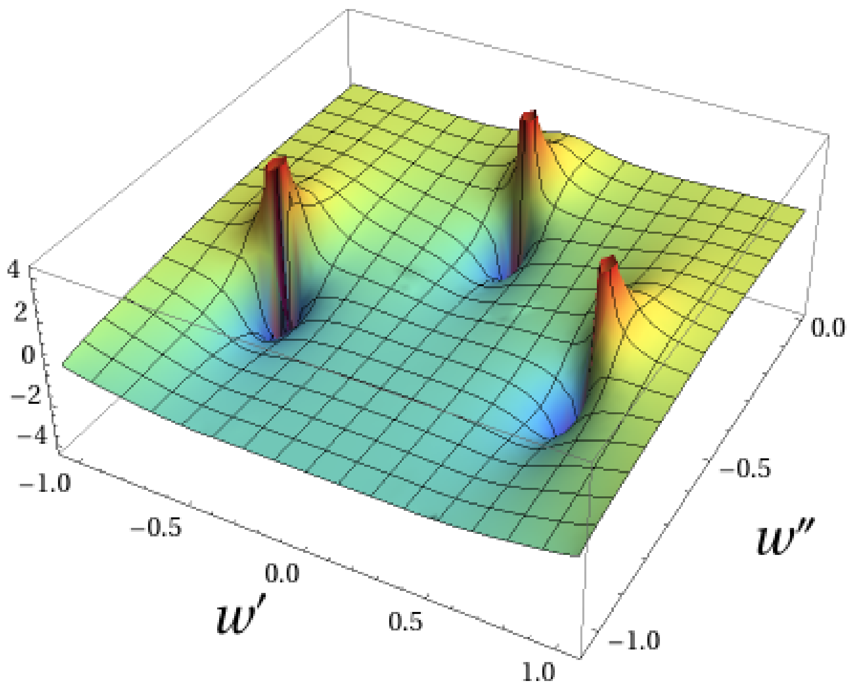
(a)$\Re\{\sigma(w;\gamma = 1/12)\}$



(b)$\Re\{\hat{\sigma}(w;\gamma = 1/12)\}$



(c)$\Re\{\sigma(w;\gamma = -1/12)\}$



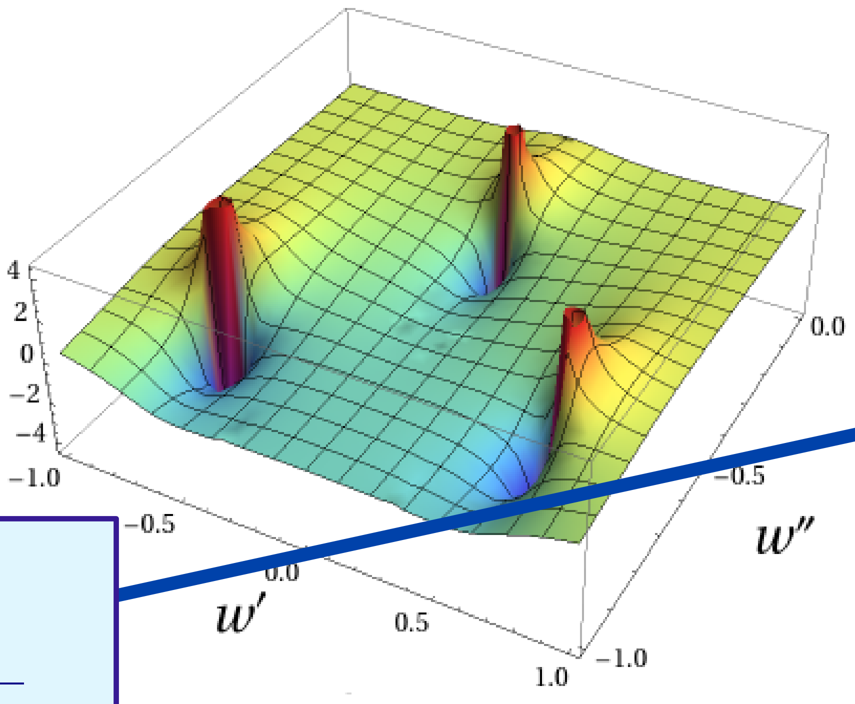
(d)$\Re\{\hat{\sigma}(w;\gamma = -1/12)\}$

Sonner

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

AdS$_4$ theory of quantum criticality

Poles in LHP
of resistivity —
quasinormal modes
of S-dual theory



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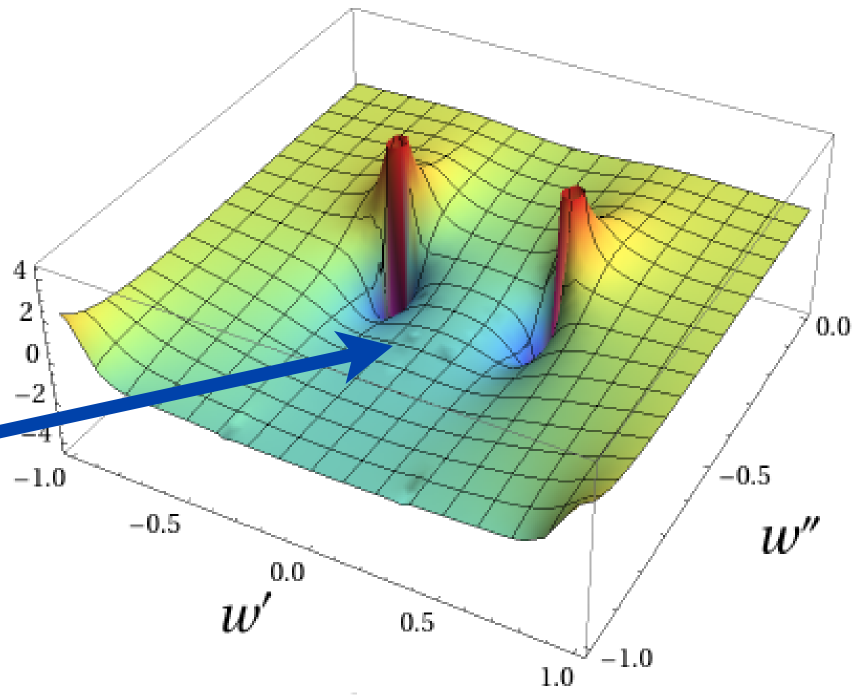
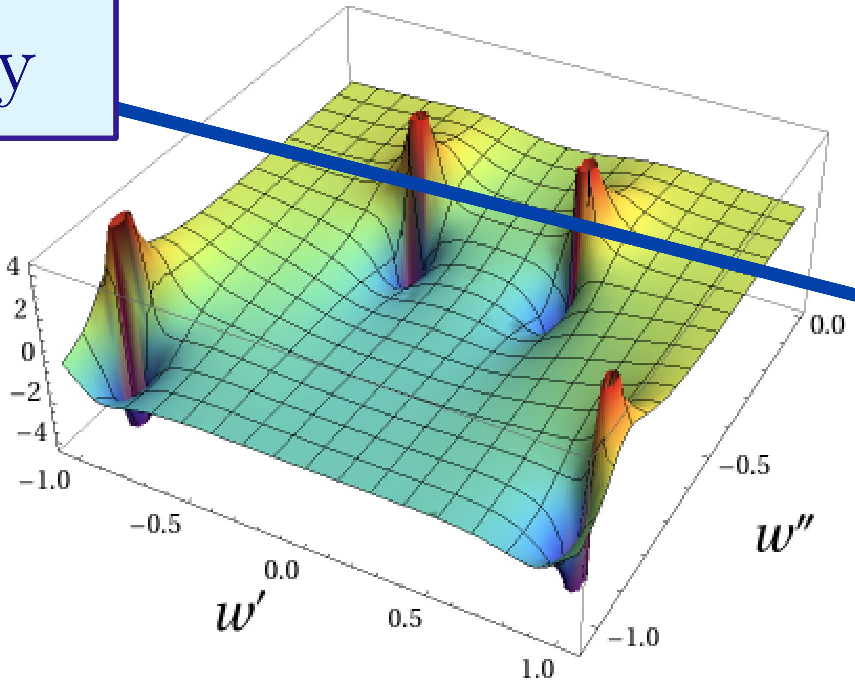


(b)$\Re\{\hat{\sigma}(w;\gamma=1/12)\}$



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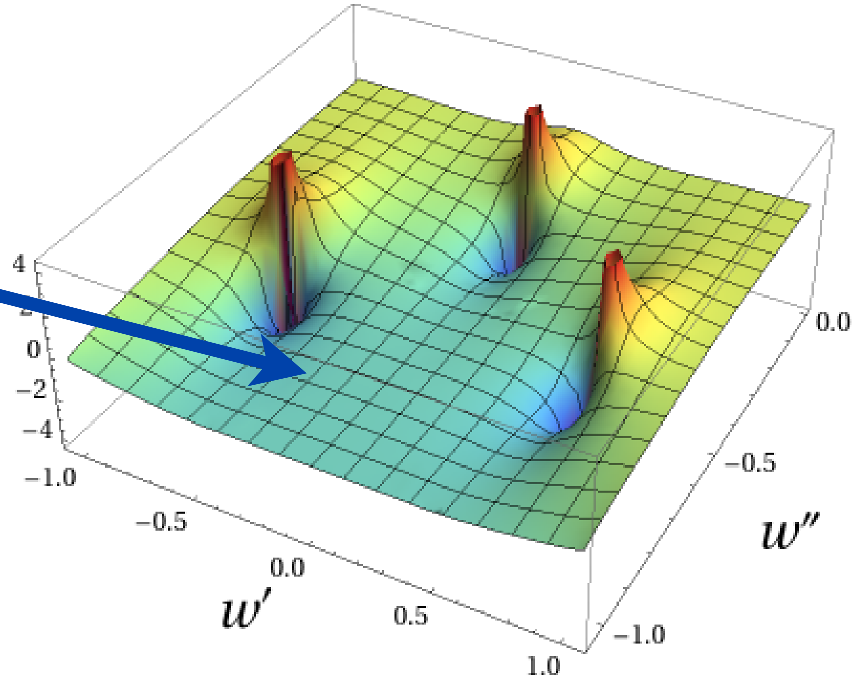

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Sonner

W. Witzack-Krempa and S. Sachdev, *Physical Review D* **86**, 235115 (2012)

Gapped quantum matter

Spin liquids, quantum Hall states....

Vishwanath, Wen, Senthil, Oshikawa

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Myers, Klebanov, Polchinski, Strominger, Swingle, Lee

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

Liu, Hartnoll, McGreevy, Silverstein, Huijse, Zaanen, Horowitz, Sonner, Trivedi, Kachru, Ooguri

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

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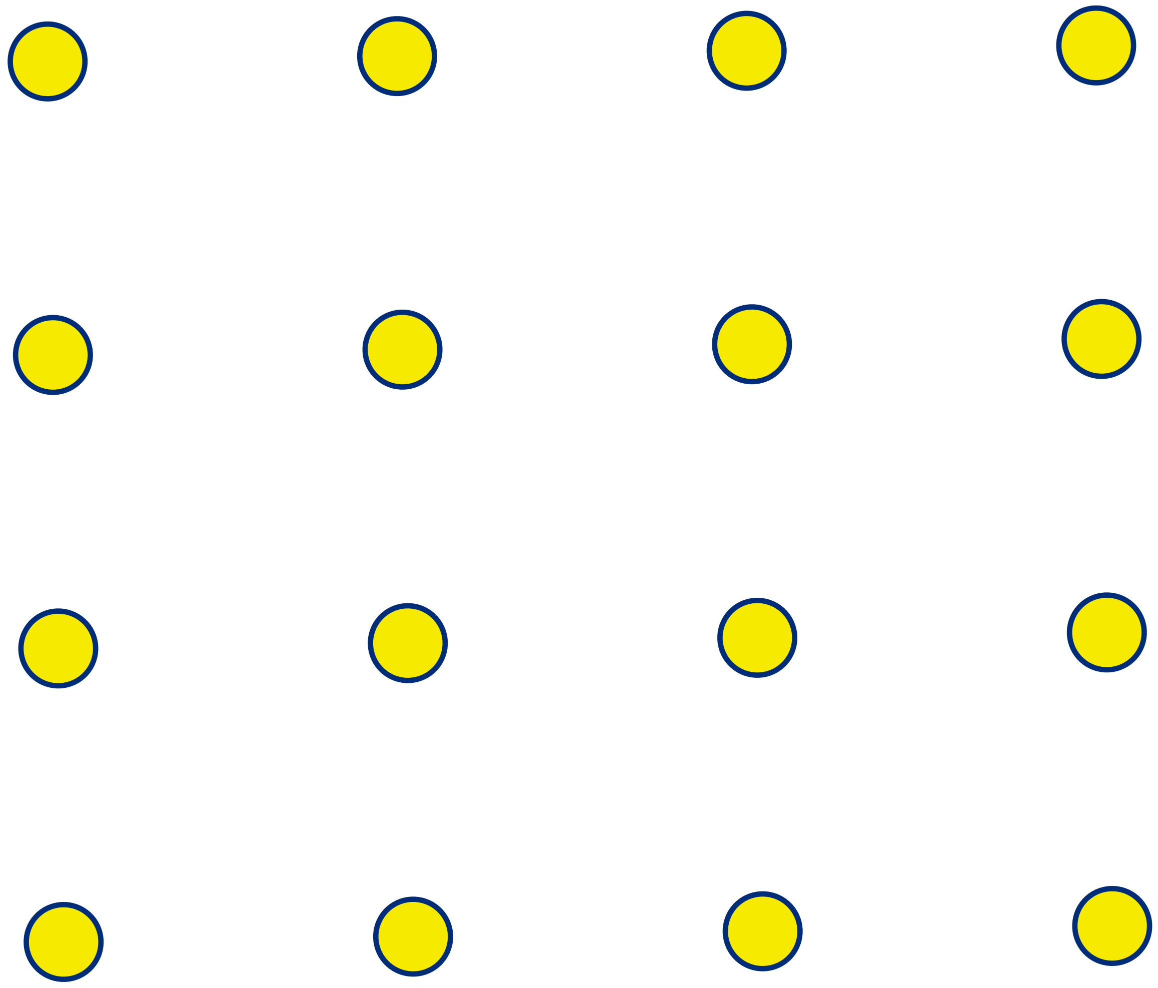
- We will obtain compressible states in holography by studying conformal field theories at a non-zero μ. These are obtained by imposing the boundary condition $A_t(x, t, r \to 0) = \mu$.

Compressible quantum matter

One compressible state is the **solid** (or “Wigner crystal” or “stripe”).
This state breaks translational symmetry.
Has integer number of particles per unit cell



Kachru
Ooguri
Sachdev
Sonner

Compressible quantum matter

Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

Hartnoll
Horowitz

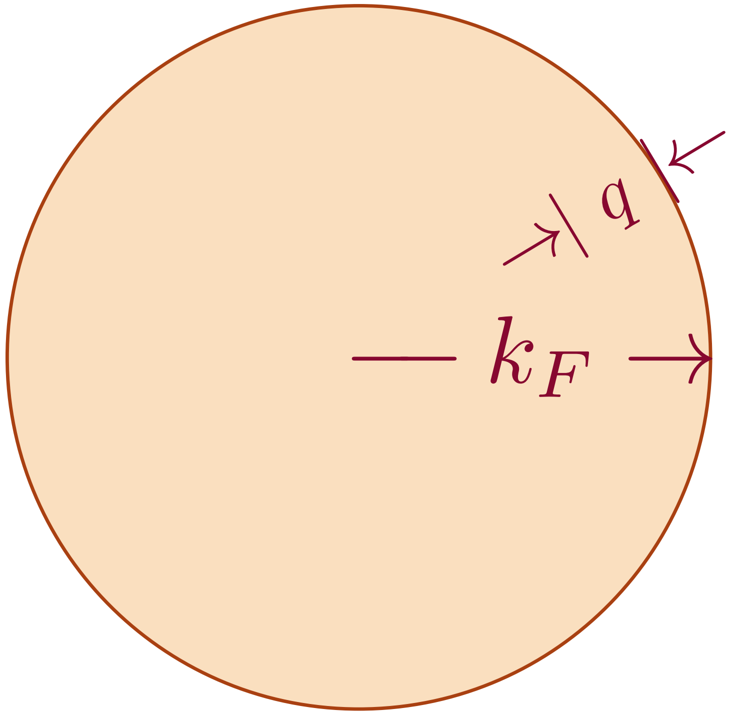
Compressible quantum matter

The only compressible phase of traditional condensed matter physics which does not break the translational or $\mathrm{U}(1)$ symmetries is the Landau Fermi liquid

The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

$$+ \, 4 \text{ Fermi terms}$$

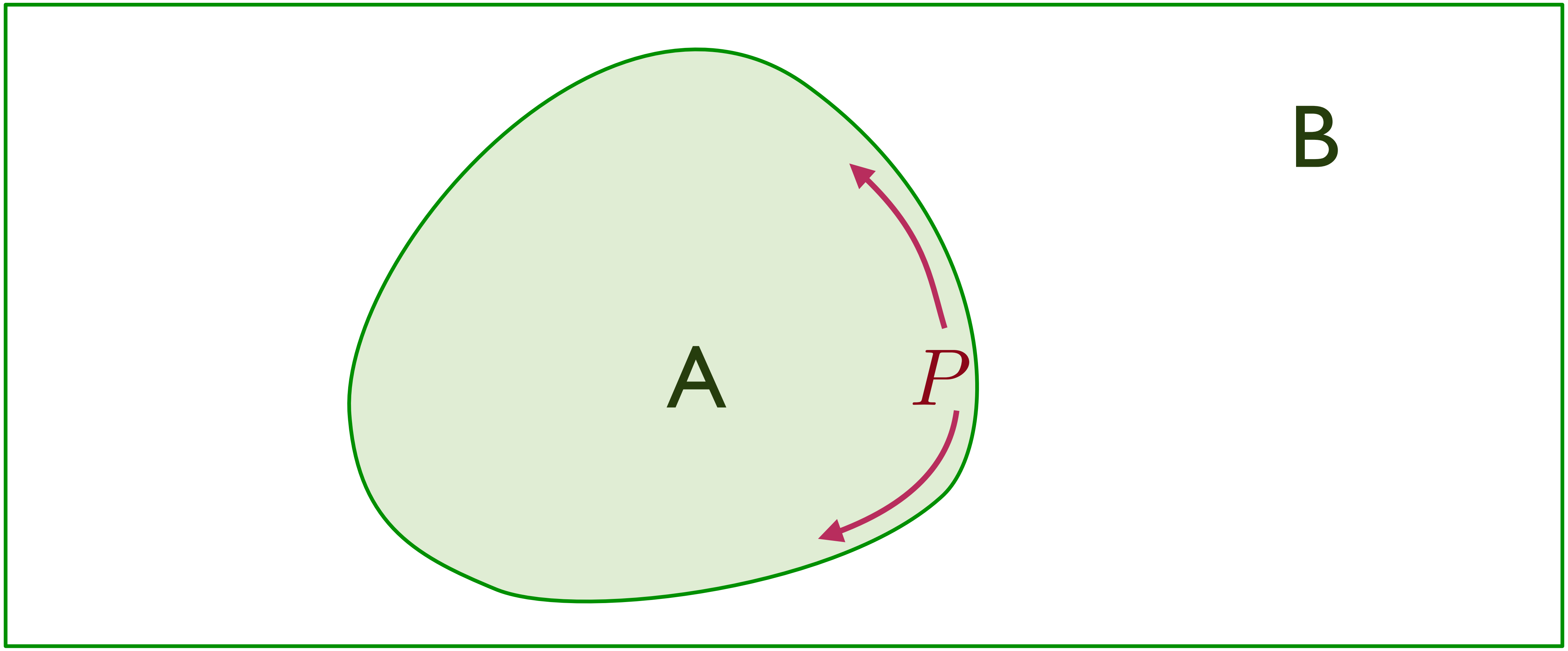


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.

- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T$. It is useful to write this is as $S \sim T^{(d-\theta)/z}$, with violation of hyperscaling exponent $\theta = d - 1$.

Entanglement entropy of the Fermi liquid



Logarithmic violation of "area law": $S_E = \dfrac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F, where P is the perimeter of region A with an arbitrary smooth shape. The prefactor 1/12 is *universal*: it is independent of the shape of the entangling region, and of the strength of the interactions.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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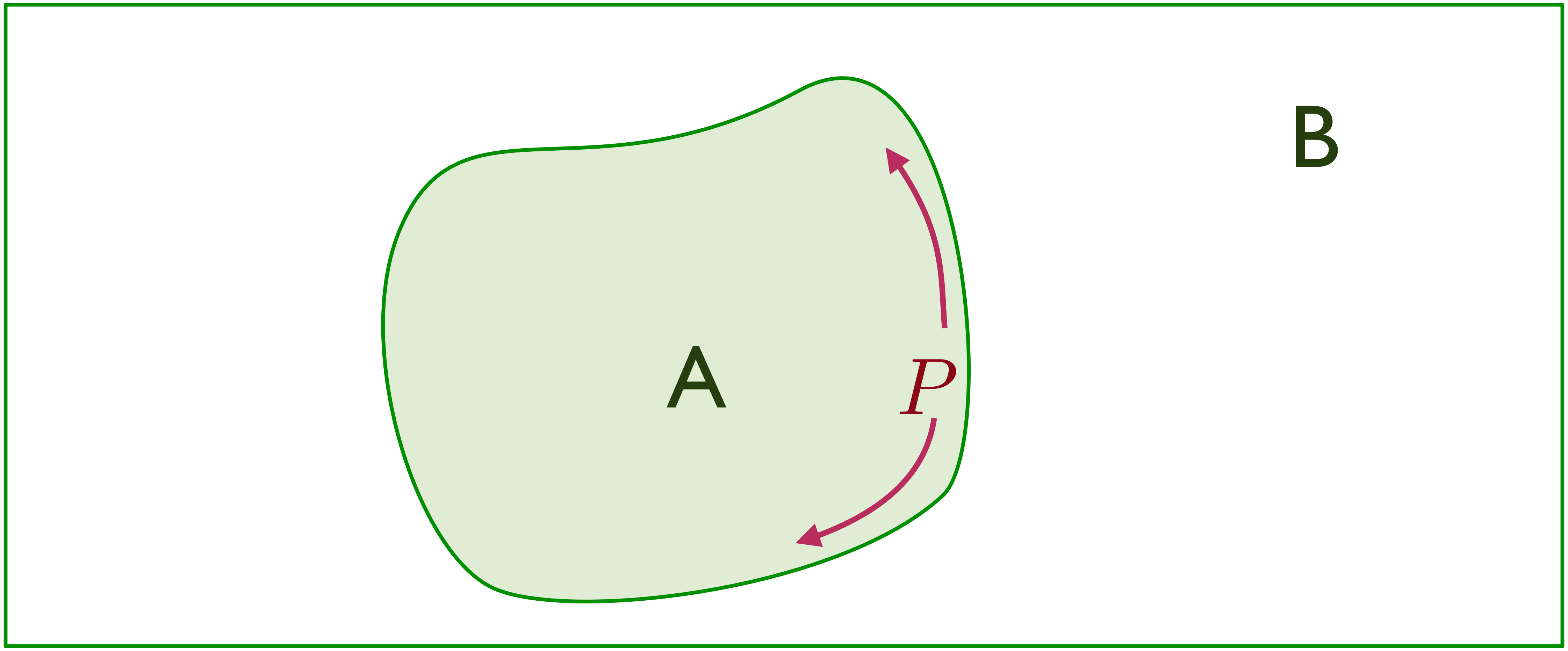

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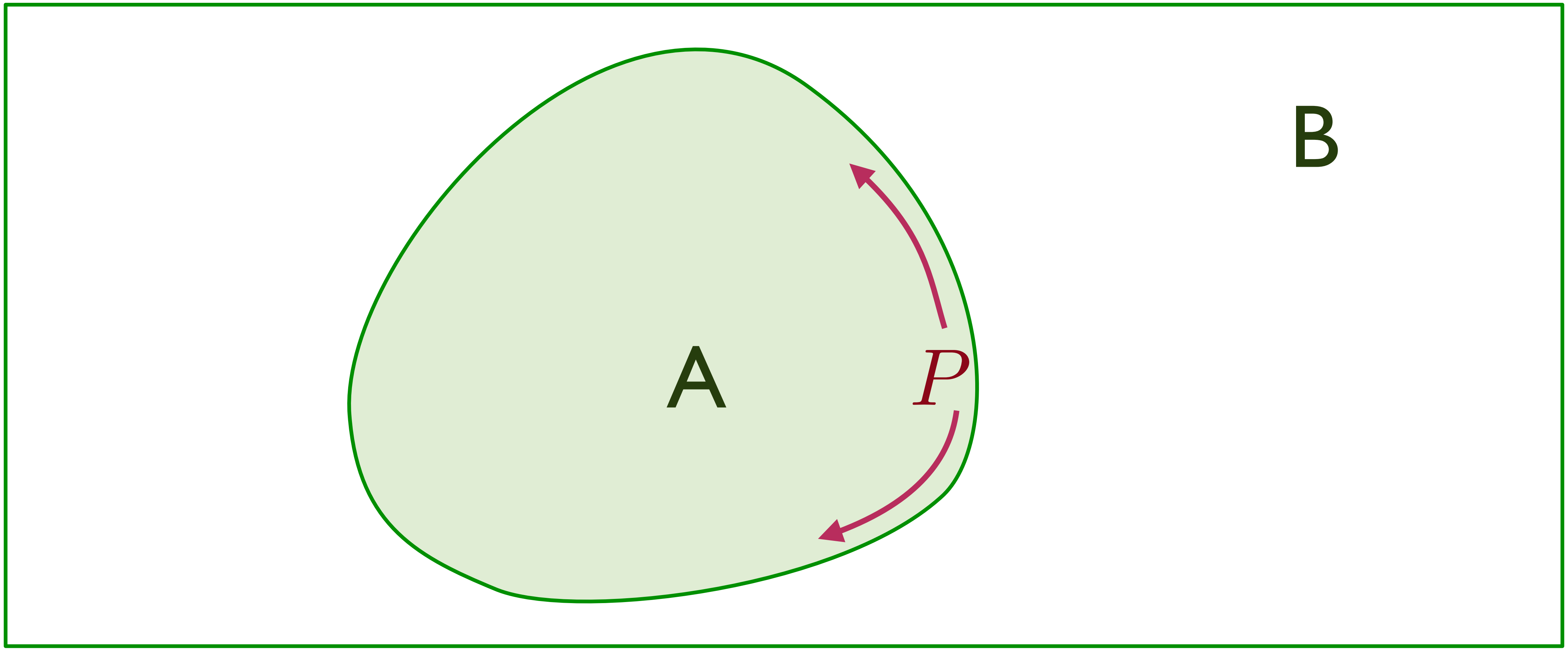

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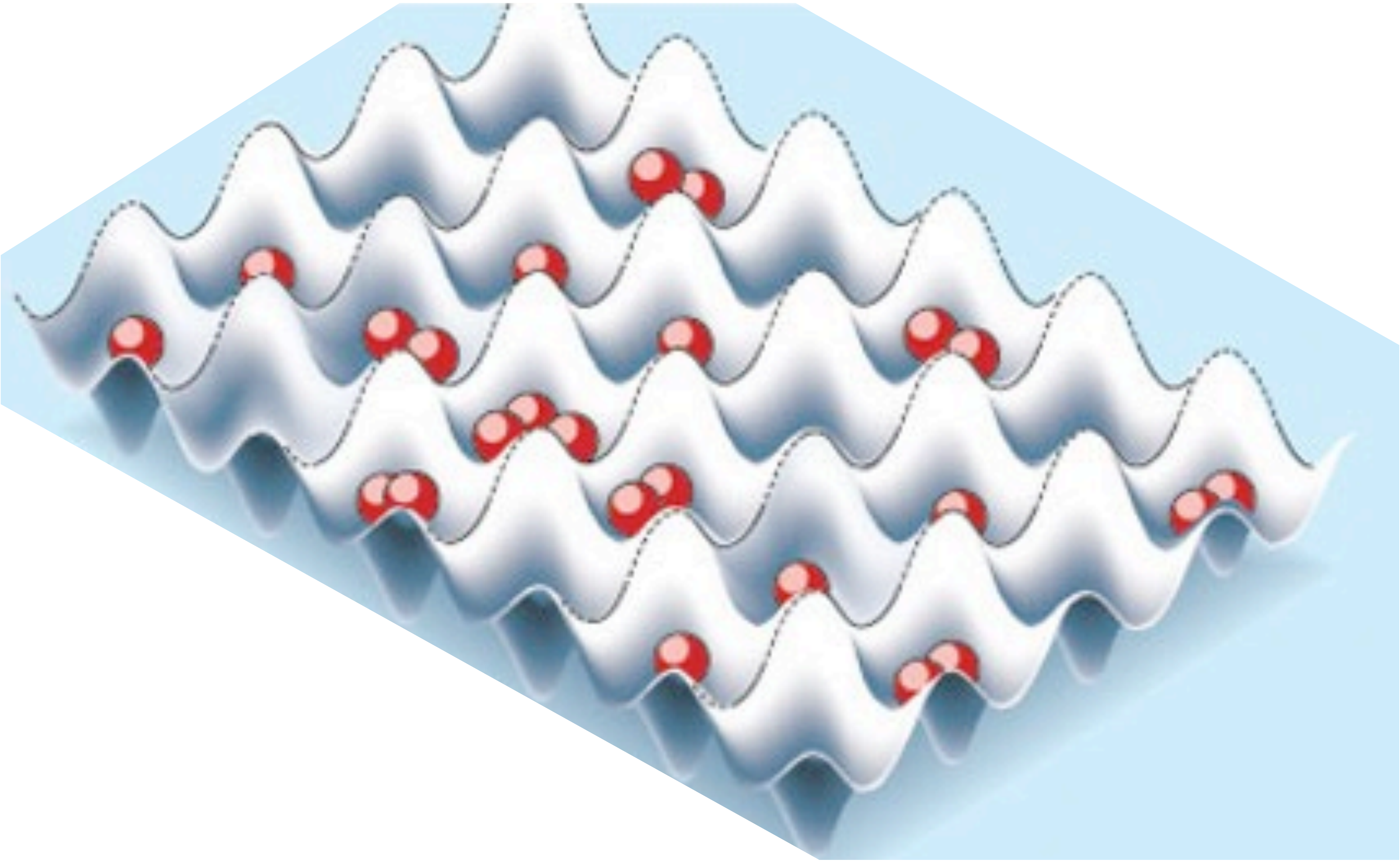
Bosons with correlated hopping

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) + w \sum_{ijk\ell \in \Box} b_i^\dagger b_k^\dagger b_j b_\ell$$



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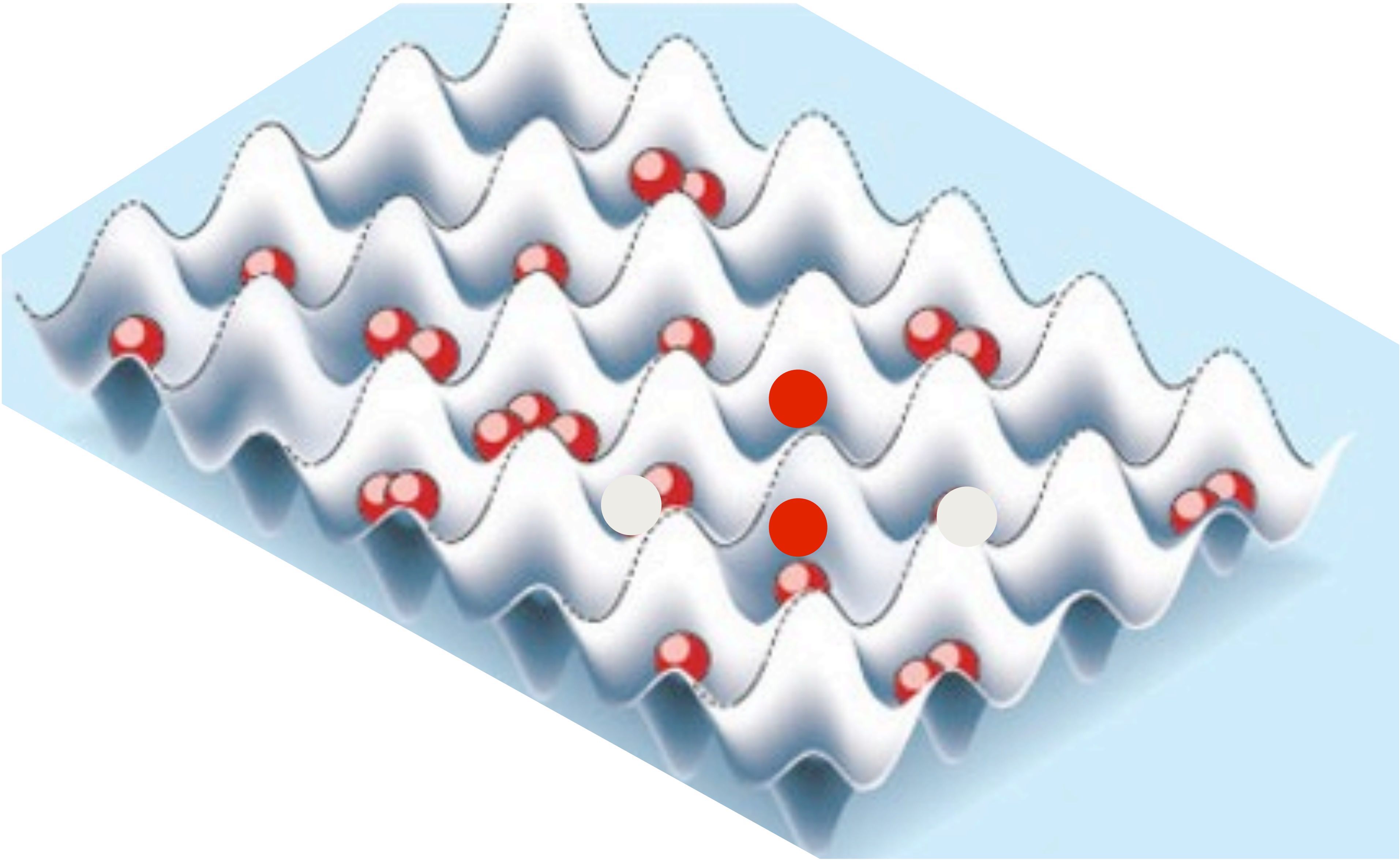

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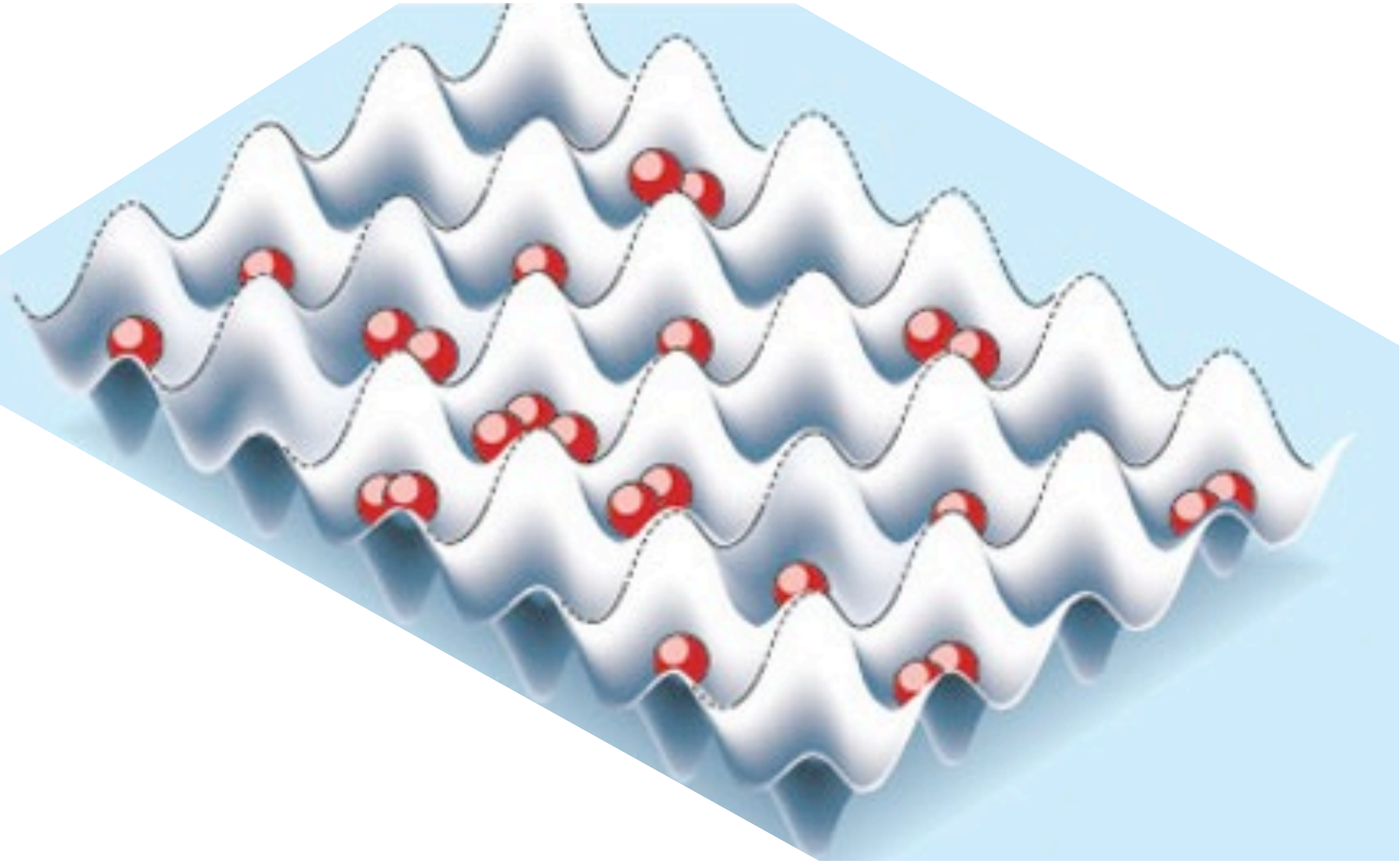

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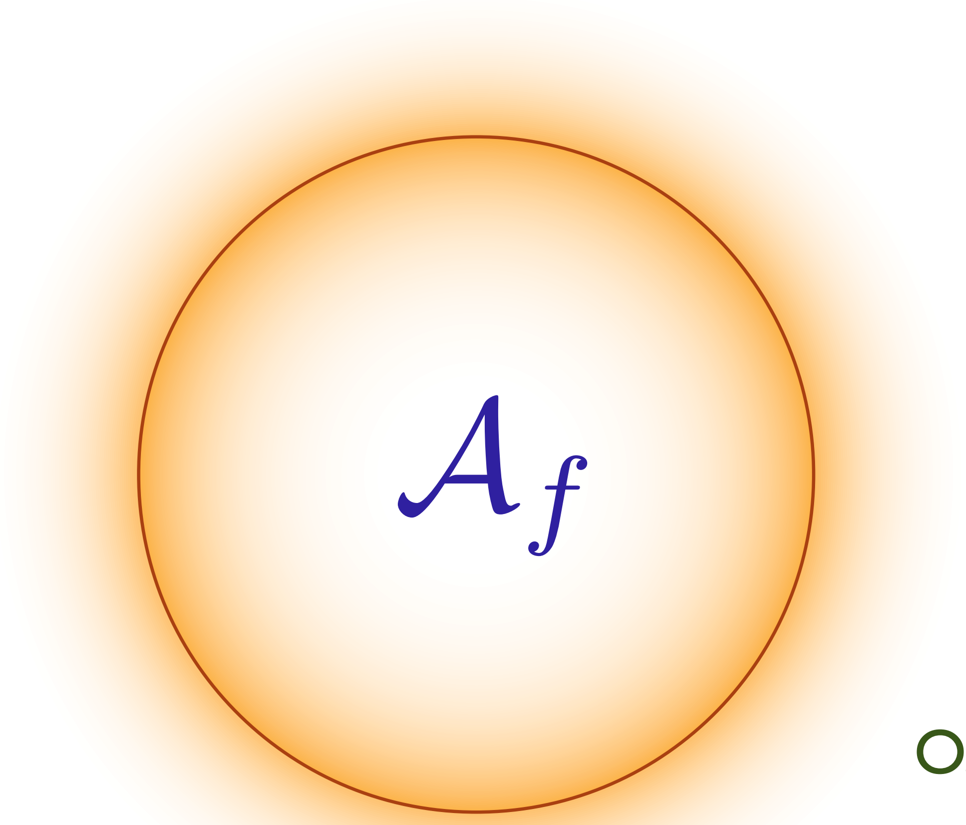
- **NFL Bose metal:** We have the fractionalization $b \to f_1 f_2$, where the f_1, f_2 both form a Fermi surface. Both fermions are gauge-charged, and so the Fermi surfaces are partially "hidden".



$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher, *Phys. Rev.* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Phys. Rev.* D **84**, 026001 (2011)
S. Sachdev, arXiv:1209.1637

Non-Fermi liquid Bose Metal

For suitable interactions, we can have the boson, b, *fractionalize* into two fermions $f_{1,2}$:

$$b \rightarrow f_1 f_2$$

This implies the effective theory for $f_{1,2}$ is invariant under the U(1) gauge transformation

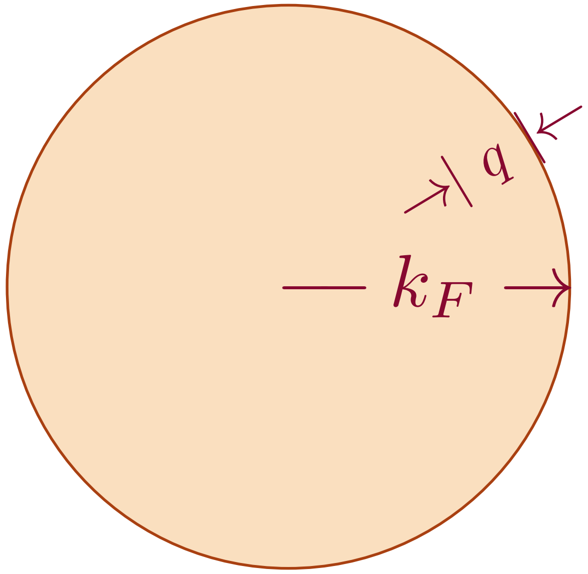
$$f_1 \rightarrow f_1 e^{i\theta(\boldsymbol{x},\tau)} \quad , \quad f_2 \rightarrow f_2 e^{-i\theta(\boldsymbol{x},\tau)}$$

Consequently, the effective theory of the Bose metal has an emergent gauge field A_μ and has the structure

$$\mathcal{L} = f_1^\dagger \left(\partial_\tau - iA_\tau - \frac{(\boldsymbol{\nabla} - i\boldsymbol{A})^2}{2m} - \mu \right) f_1 + f_2^\dagger \left(\partial_\tau + iA_\tau - \frac{(\boldsymbol{\nabla} + i\boldsymbol{A})^2}{2m} - \mu \right) f_2$$

The gauge-dependent $f_{1,2}$ Green's functions have Fermi surfaces obeying $\mathcal{A}_f = \langle \mathcal{Q} \rangle$. However, these Fermi surfaces are not directly observable because it is gauge-dependent. Nevertheless, gauge-independent operators, such as b or $b^\dagger b$, will exhibit *Friedel oscillations* associated with fermions scattering across these hidden Fermi surfaces.

FL
Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

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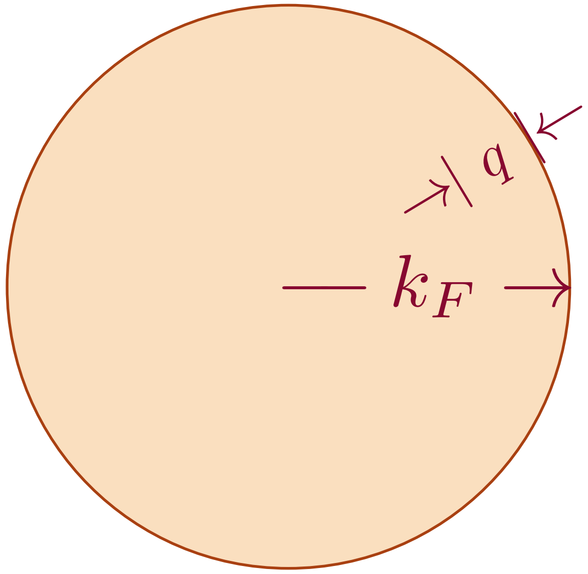

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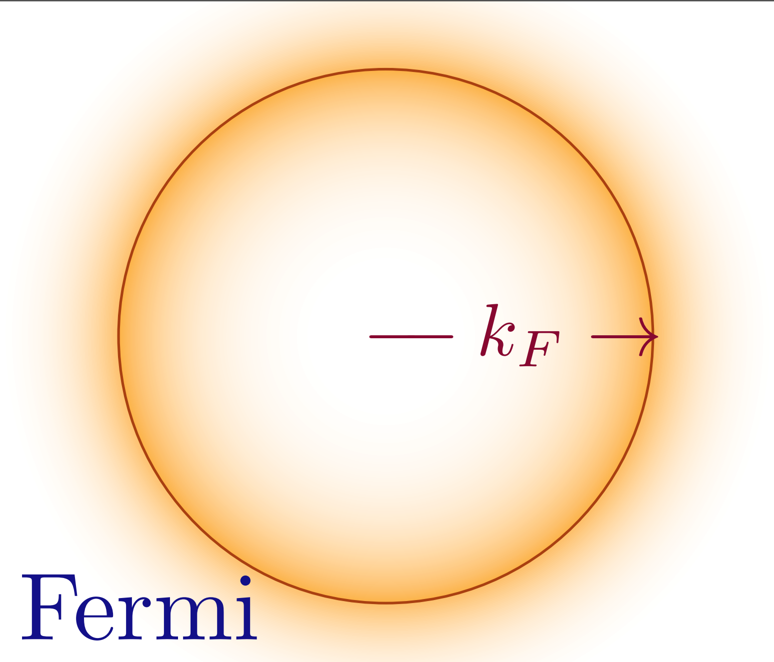
NFL
Bose
metal



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FL
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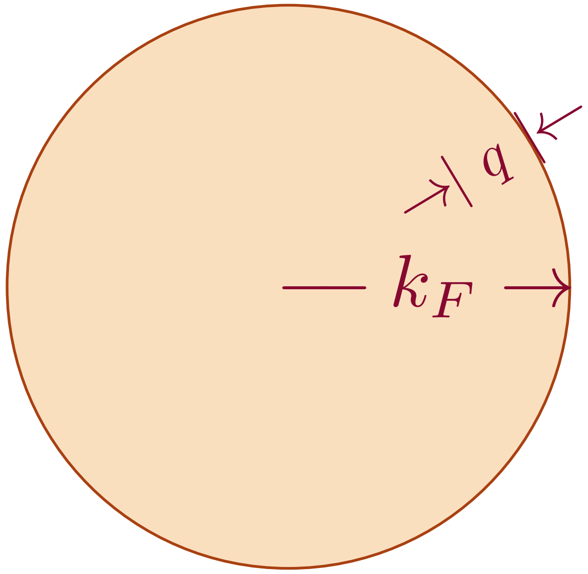

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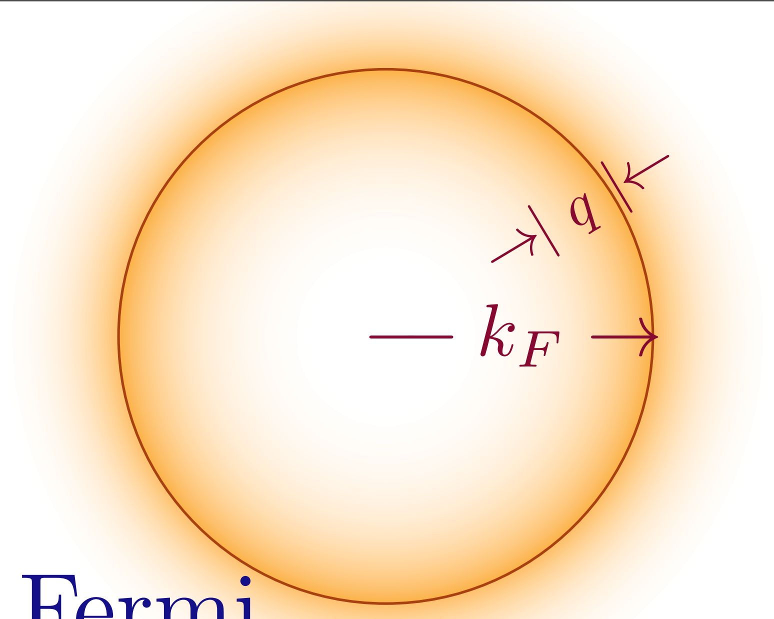

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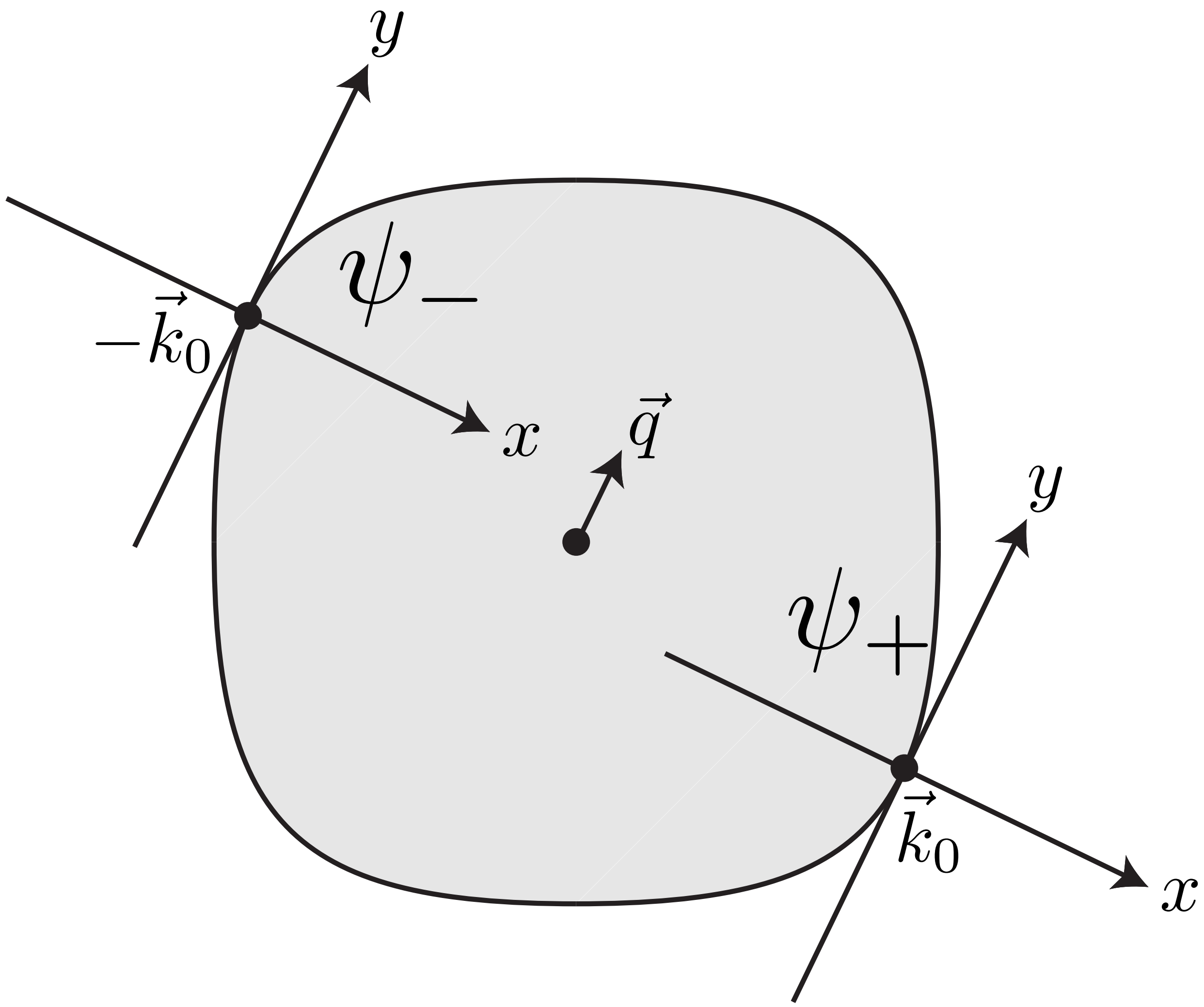
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

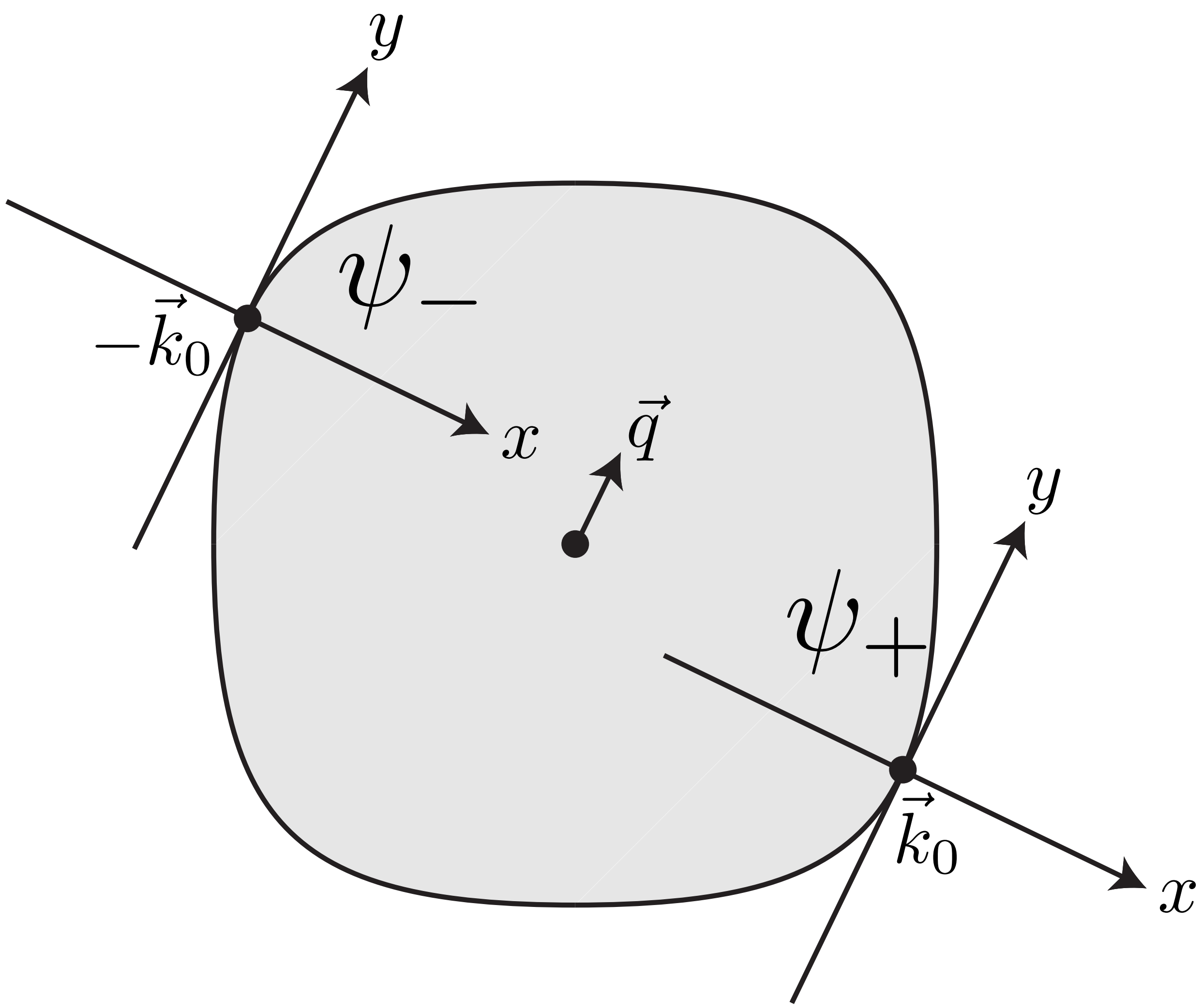
P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Field theory of non-Fermi liquid



- $\vec{A}$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Field theory of non-Fermi liquid



$$\mathcal{L}[\psi_\pm, a] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

Field theory of non-Fermi liquid

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- \\ - a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

Simple scaling argument for $z = 3/2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Field theory of non-Fermi liquid

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Simple scaling argument for $z = 3/2$.

Under the rescaling $x \rightarrow x/s$, $y \rightarrow y/s^{1/2}$, and $\tau \rightarrow \tau/s^z$, we find invariance provided

$$\begin{aligned} a & \rightarrow a\, s \\ \psi & \rightarrow \psi\, s^{(2z+1)/4} \\ g & \rightarrow g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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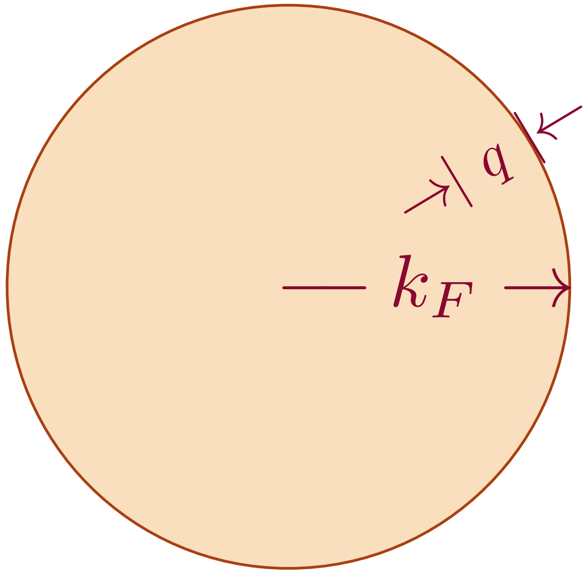

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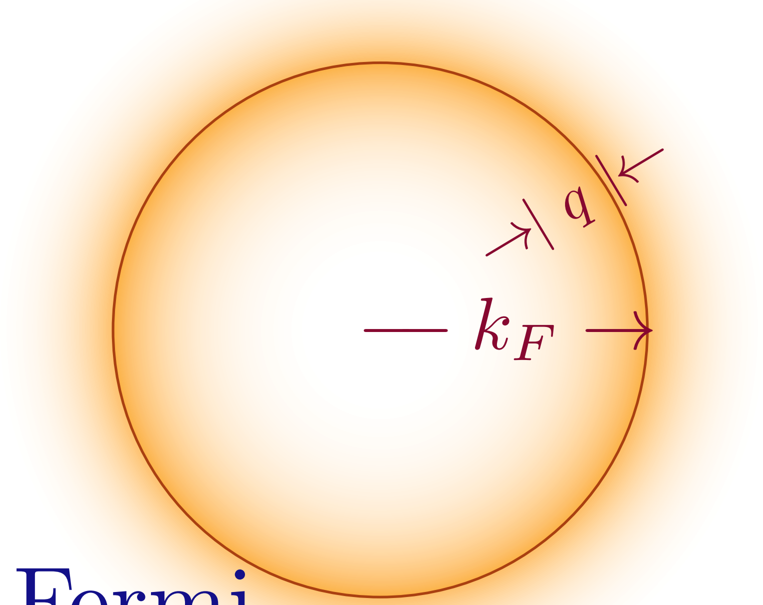
NFL
Bose metal



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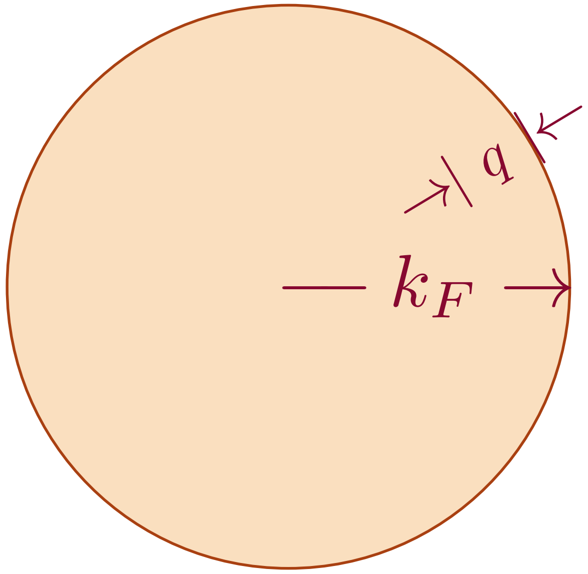
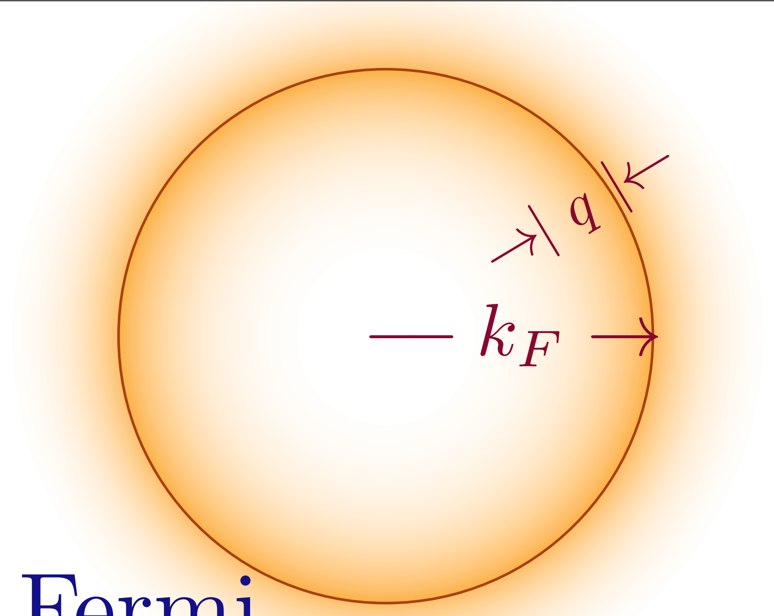
FL Fermi liquid 	**NFL Bose metal** 
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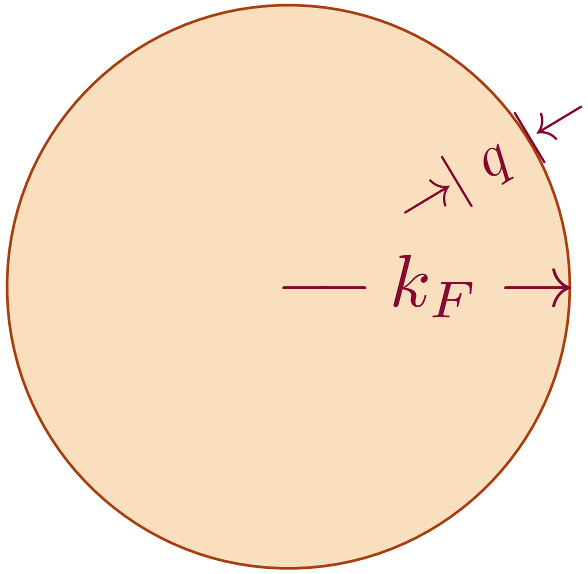
FL
Fermi
liquid



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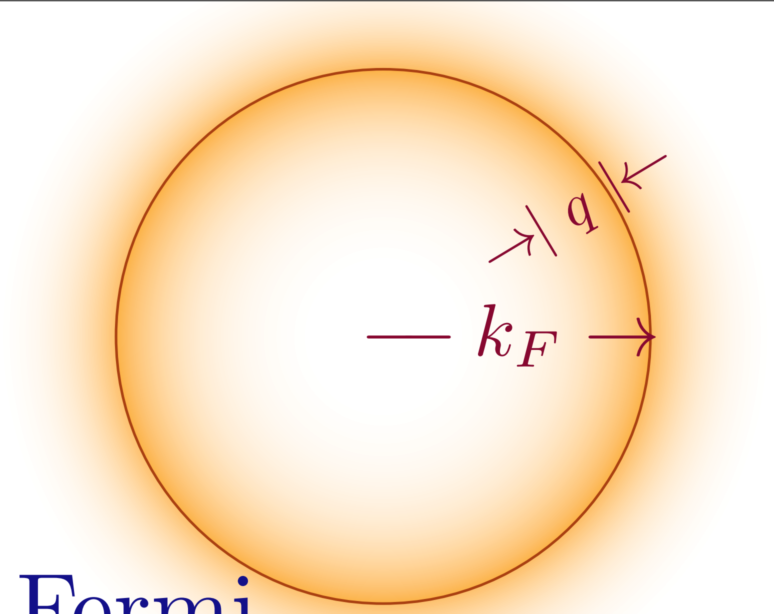

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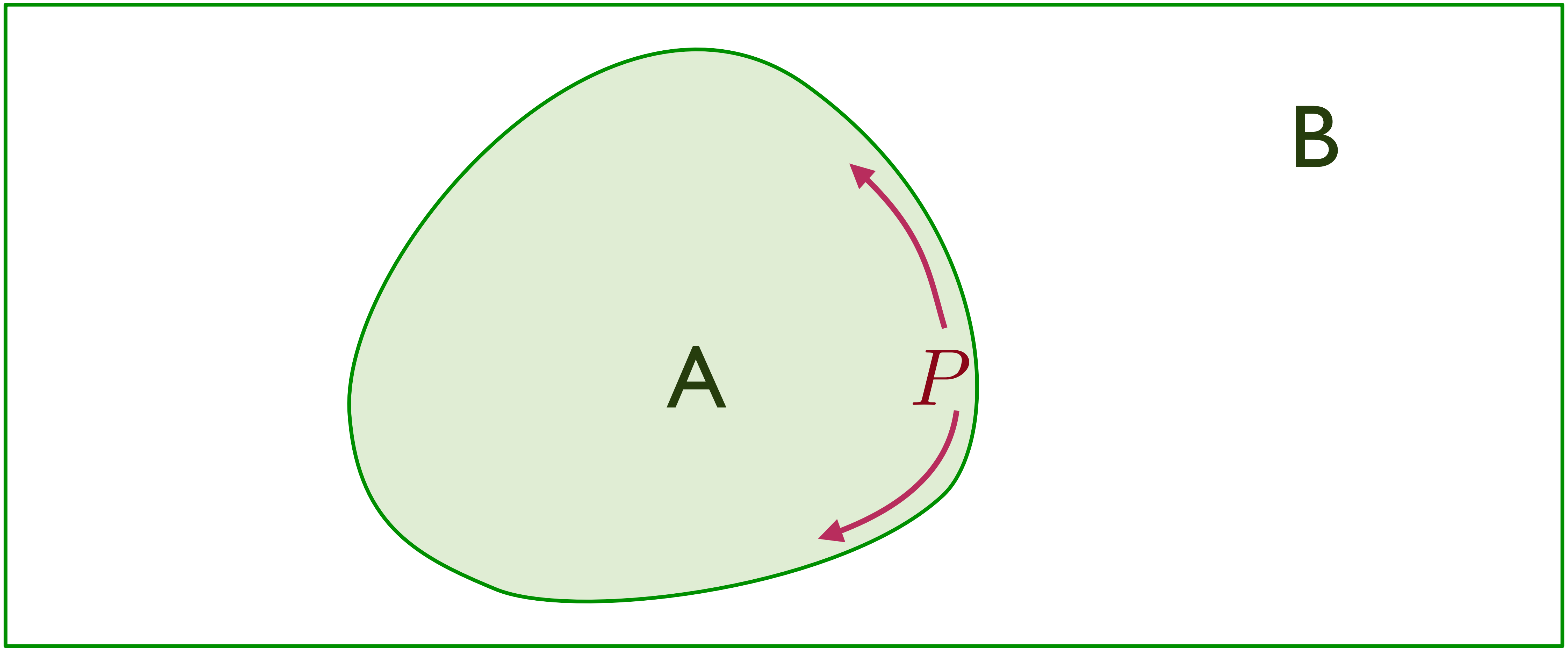
Entanglement entropy of the non-Fermi liquid



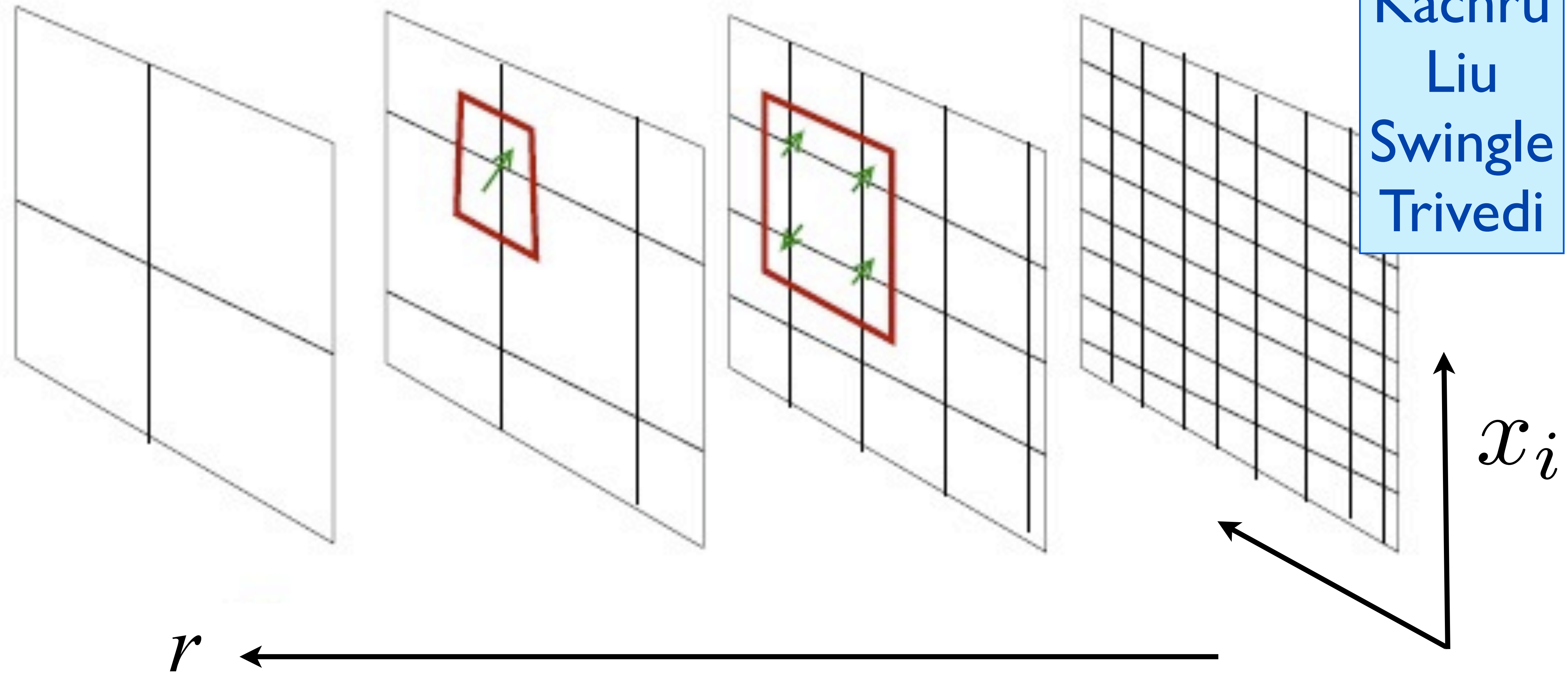
Logarithmic violation of “area law”: $S_E = \mathcal{C}_E\, k_F P \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.
The prefactor $\mathcal{C}_E$ is expected to be universal but $\neq 1/12$:
independent of the shape of the entangling region, and dependent
only on IR features of the theory.

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Holography

Huijse
Kachru
Liu
Swingle
Trivedi



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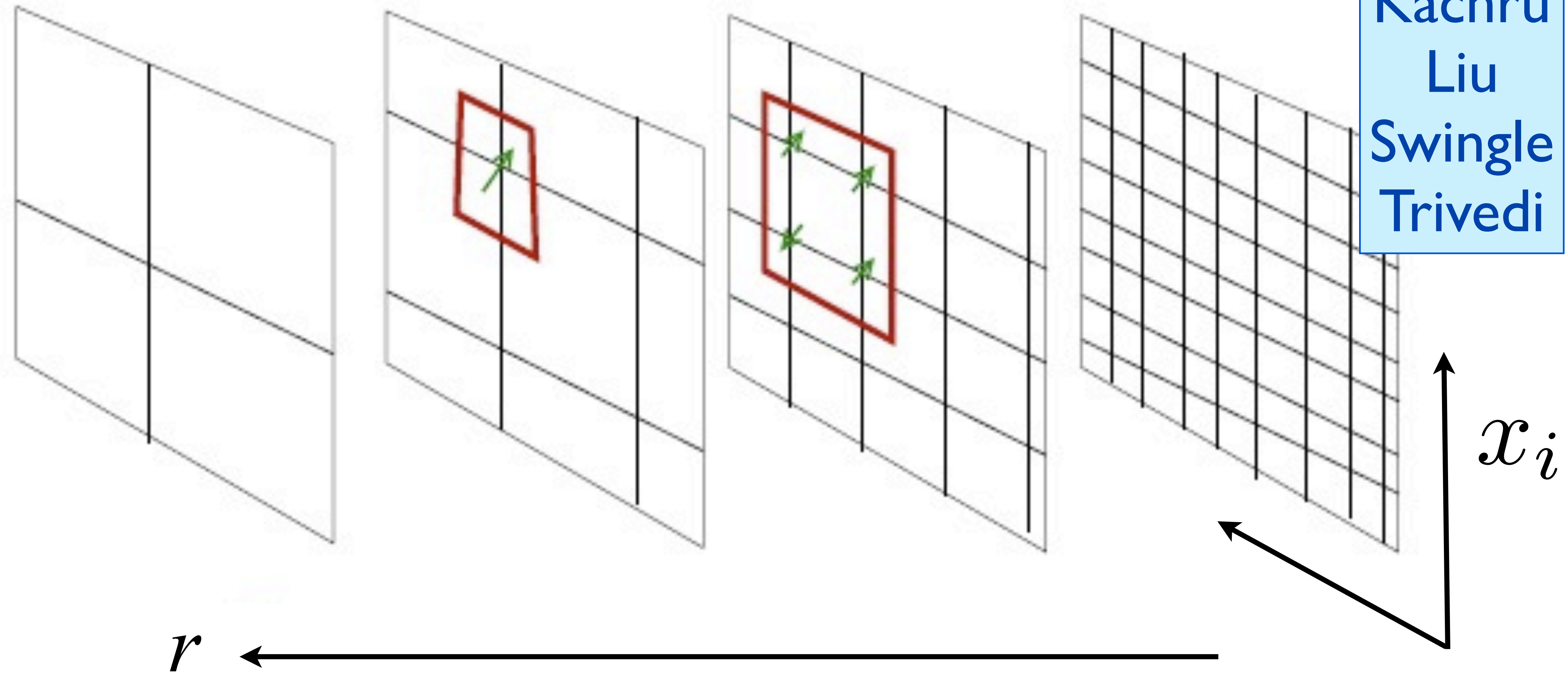

Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Holography

Huijse
Kachru
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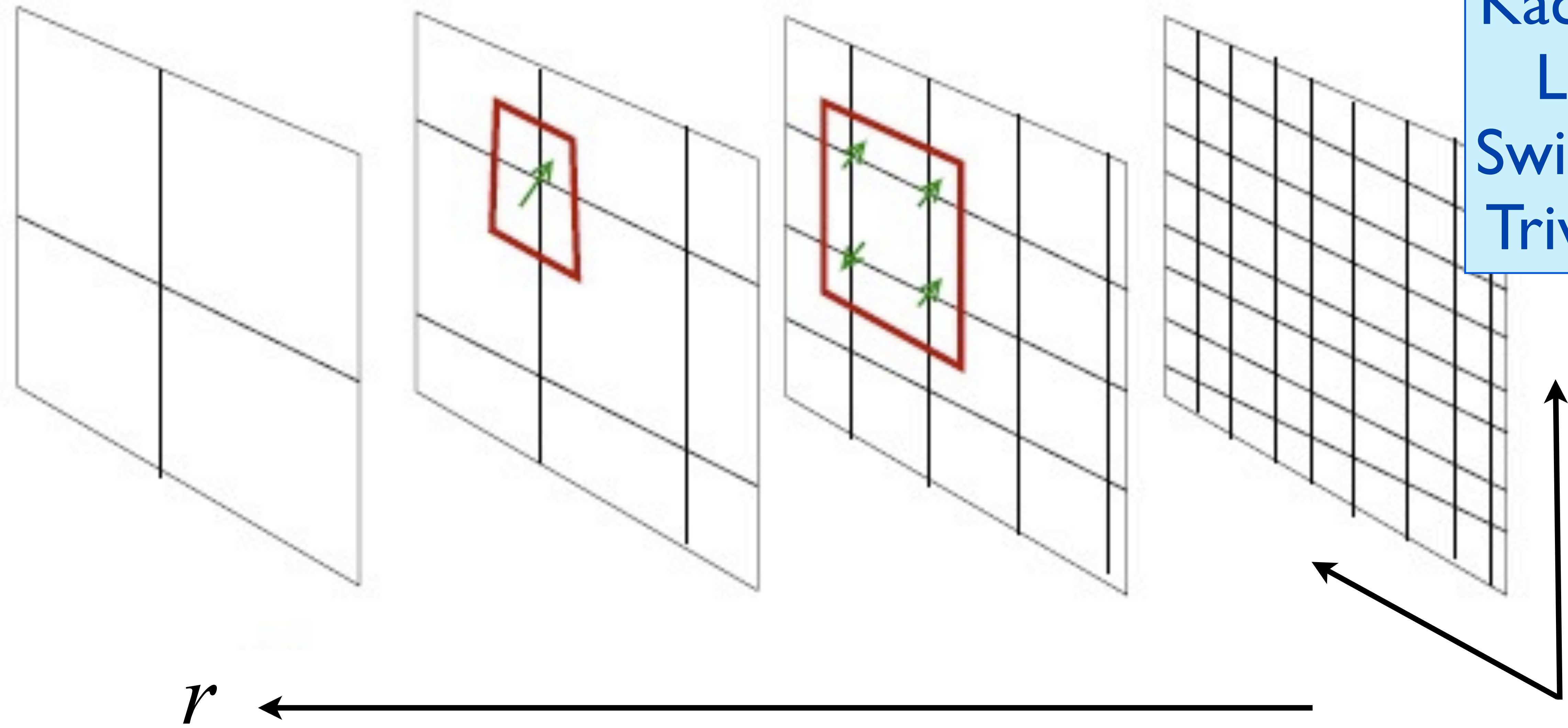

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The value $\theta = d - 1$ reproduces *all* the essential characteristics of the entropy and entanglement entropy of a non-FL.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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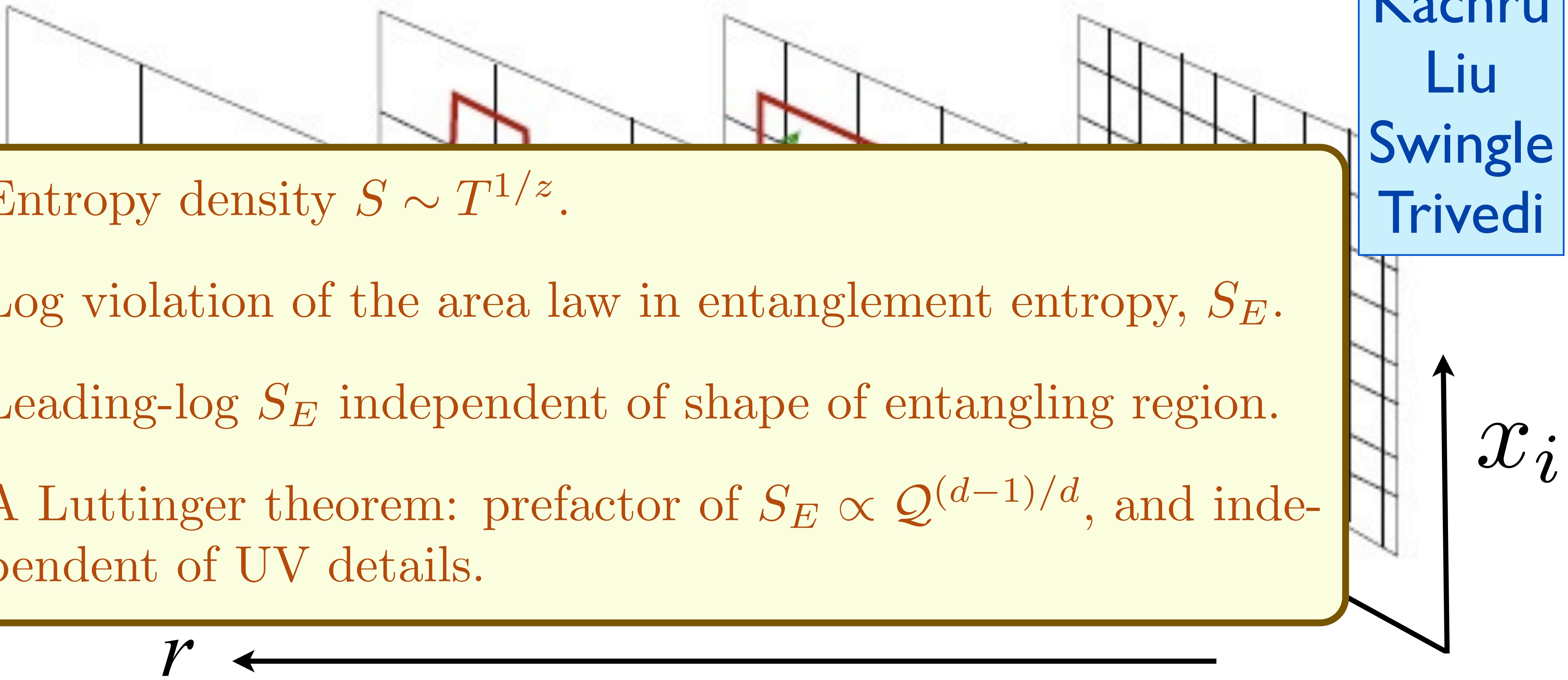

- Entropy density $S \sim T^{1/z}$.
- Log violation of the area law in entanglement entropy, S_E.
- Leading-log S_E independent of shape of entangling region.
- A Luttinger theorem: prefactor of $S_E \propto \mathcal{Q}^{(d-1)/d}$, and independent of UV details.

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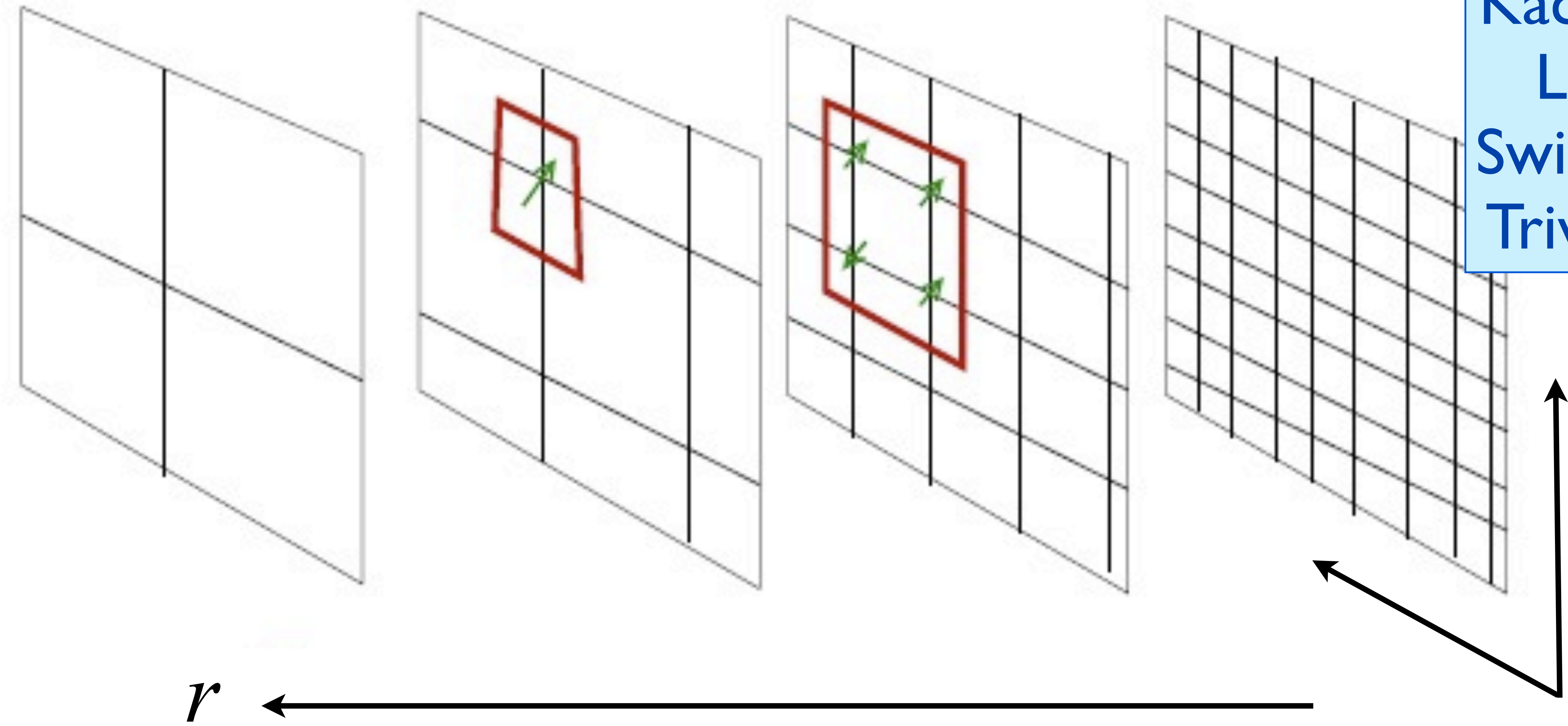

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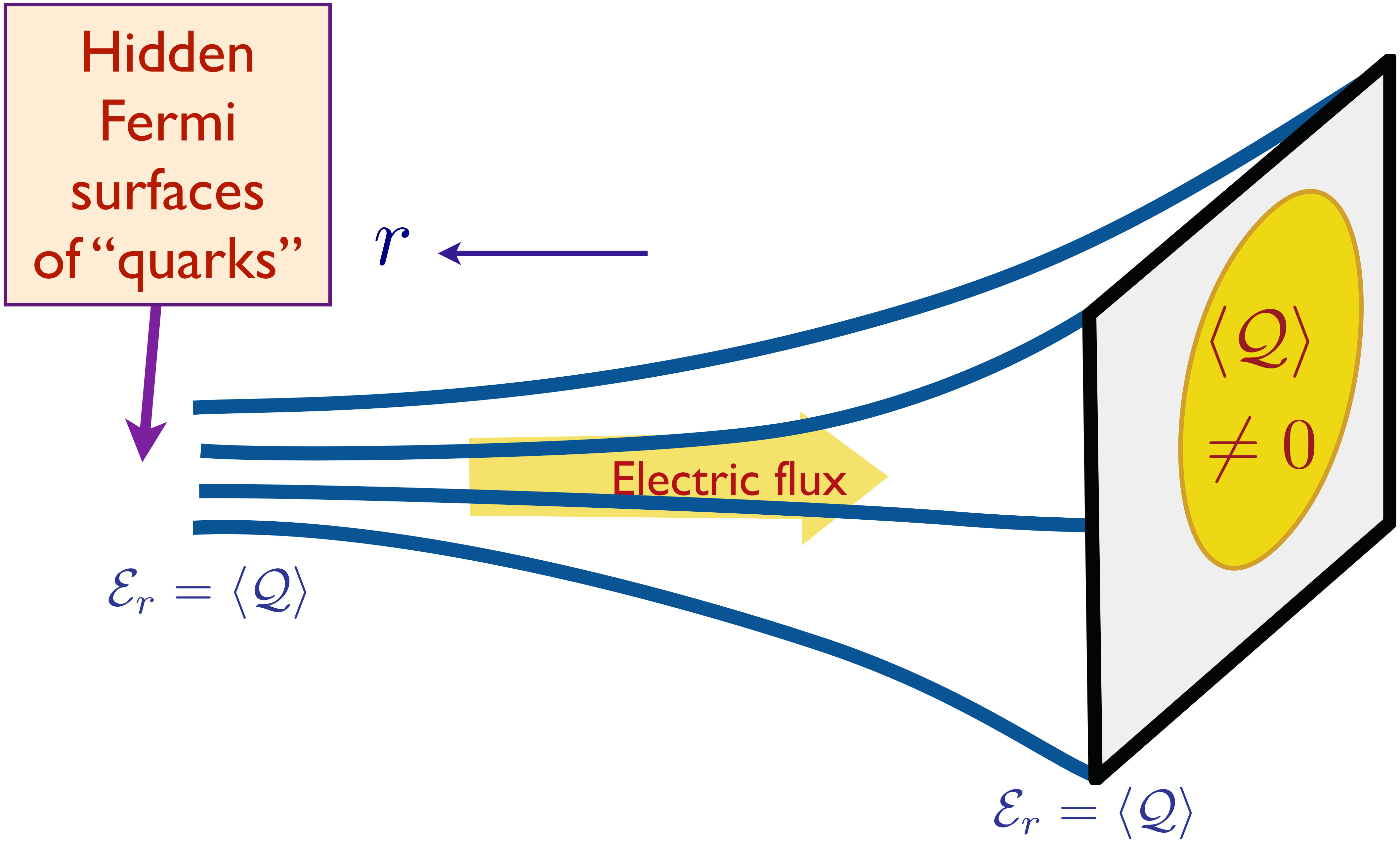
The null-energy condition of gravity yields $z \geq 1 + \theta/d$. In $d = 2$, this leads to $z \geq 3/2$. Field theory on non-FL yields $z = 3/2$ to 3 loops!

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

P. A. Lee, Phys. Rev. Lett. **63**, 680 (1989)
B. Blok and H. Monien, Phys. Rev. B **47**, 3454 (1993)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Holography of a non-Fermi liquid



Hidden Fermi surfaces of "quarks"

This is a "bosonization" of the *hidden* Fermi surface

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

See also: J. Polchinski and E. Silverstein, arXiv:1203.1015

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Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension *d=1*

Monopoles in the 2+1 dimensional bulk U(1) gauge field acquire a Berry phase determined by the boundary U(1) charge density $\mathcal{Q}$, and a dilute gas theory of monopoles leads to Friedel oscillations with

$$\langle \rho(x)\rho(0)\rangle \sim \frac{\cos(2k_F x)}{|x|^{2\Delta_F}}$$

T. Faulkner and N. Iqbal, arXiv:1207.4208

See also: J. Polchinski and E. Silverstein, arXiv:1203.1015

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T. Faulkner and N. Iqbal, arXiv:1207.4208

Exact solution of adjoint Dirac fermions at non-zero density coupled to a SU(N_c) gauge field: low energy theory has an emergent $\mathcal{N} = (2,2)$ supersymmetry, the global U(1) symmetry becomes the R-symmetry, and there are Friedel oscillations with

$$\Delta_F = 1/3 \quad \text{for all } N_c \geq 2$$

R. Gopakumar, A. Hashimoto, I.R. Klebanov, S. Sachdev, and K. Schoutens, arXiv:1206.4719

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension *d=2*

- For every CFT in 2+1 dimensions with a globally conserved U(1), we can define a monopole operator which transforms as a scalar under conformal transformations.
  e.g. for the XY model, we insert a monopole at x_m by including a *fixed* background gauge flux α_μ so that

$$\mathcal{L} = |(\partial_\mu - i\alpha_\mu)\psi|^2 + s|\psi|^2 + u|\psi|^4$$

where the flux $\beta_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu \alpha_\lambda$ obeys

$$\partial_\mu \beta_\mu = 2\pi\delta(x - x_m) \quad , \quad \epsilon_{\mu\nu\lambda}\partial_\nu(\Omega\beta_\nu) = 0$$

where the CFT lives on the conformally flat space with is $ds^2 = \Omega^{-2} dx_\mu^2$.

S. Sachdev, arXiv:1209.1637

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Spatial dimension *d=2*

- In the holographic theory, we have a bulk scalar field Φ_m (conjugate to the monopole operator of the CFT) which carries the charge of the S-dual of the 4-dimensional bulk U(1) gauge field:

$$\mathcal{S}_m = \int d^4x \sqrt{-g} \left[|(\nabla - 2\pi i \widetilde{A})\Phi_m|^2 + \ldots \right]$$

where $\widetilde{F} = d\widetilde{A} = *F = *dA$.

S. Sachdev, arXiv:1209.1637

Holography of a non-Fermi liquid

Can we see the Fermi surface directly via "Friedel oscillations" in density (or related) correlations ?

Kachru

Spatial dimension $d=2$

- When a chemical potential is applied to the boundary CFT, Φ_m experiences a magnetic flux. Consequently condensation of Φ_m leads to a vortex-lattice-like state, which corresponds to the formation of a *crystal* in the CFT. *The crystal has unit $\mathcal{Q}$ charge per unit cell.*
- We expect that a vortex-liquid-like state of the Φ_m will yield the Friedel oscillations of the Fermi surface, with the correct Fermi wavevector.

S. Sachdev, arXiv:1209.1637

Gapped quantum matter

Spin liquids, quantum Hall states....

Vishwanath, Wen, Senthil, Oshikawa

Conformal quantum matter

Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene

Myers, Klebanov, Polchinski, Strominger, Swingle, Lee

Compressible quantum matter

Strange metals in high temperature superconductors, Bose metals

Liu, Hartnoll, McGreevy, Silverstein, Huijse, Zaanen,
Horowitz, Sonner, Trivedi, Kachru, Ooguri