Quantum Criticality and Black Holes

Talk online: sachdev.physics.harvard.edu

Particle theorists

Sean Hartnoll, KITP

Christopher Herzog, Princeton

Pavel Kovtun, Victoria

Dam Son, Washington

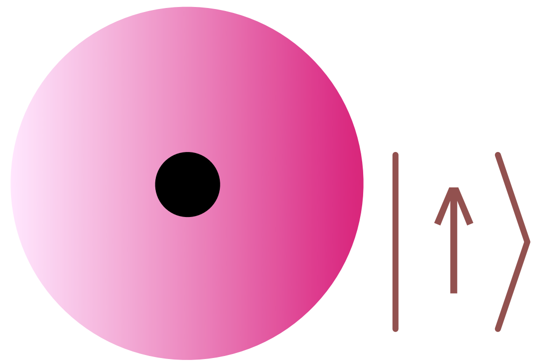
Condensed matter theorists

Markus Mueller, Harvard

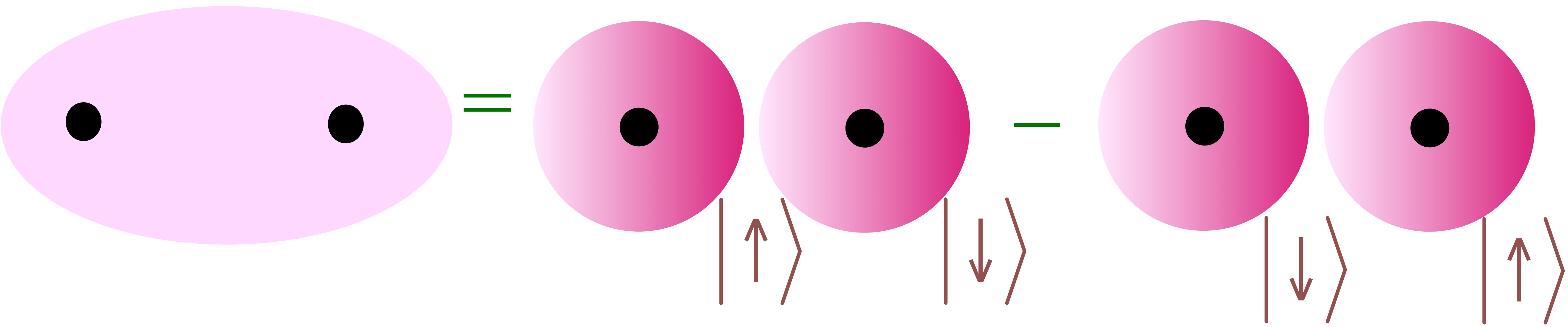
Subir Sachdev, Harvard

Quantum Entanglement

Hydrogen atom:



Hydrogen molecule:



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Superposition of two electron states leads to non-local correlations between spins

Quantum Phase Transition

Change in the nature of entanglement in a macroscopic quantum system.

Familiar phase transitions, such as water boiling to steam, also involve macroscopic changes, but in thermal motion

Quantum Criticality

The complex and non-local entanglement at the critical point between two quantum phases

Outline

1. Entanglement of spins

   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics

   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors

   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene

   Hydrodynamic cyclotron resonance and Nernst effect

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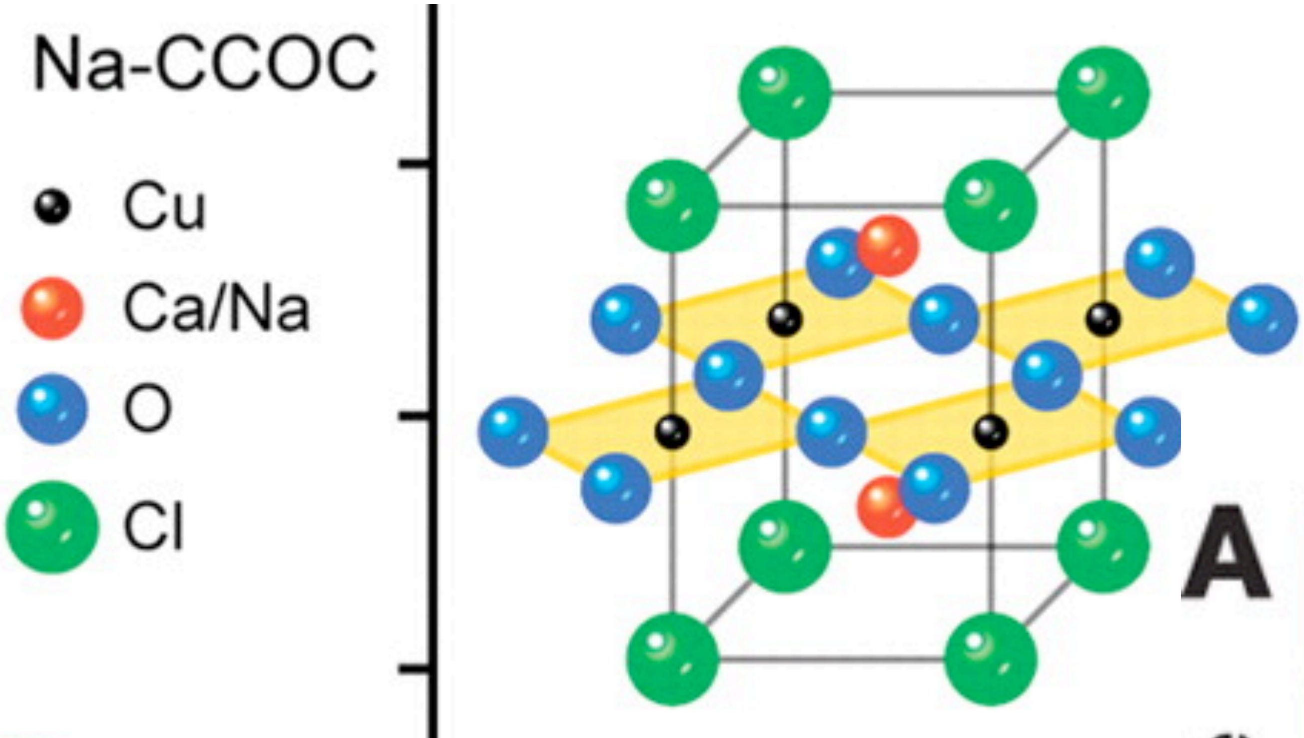
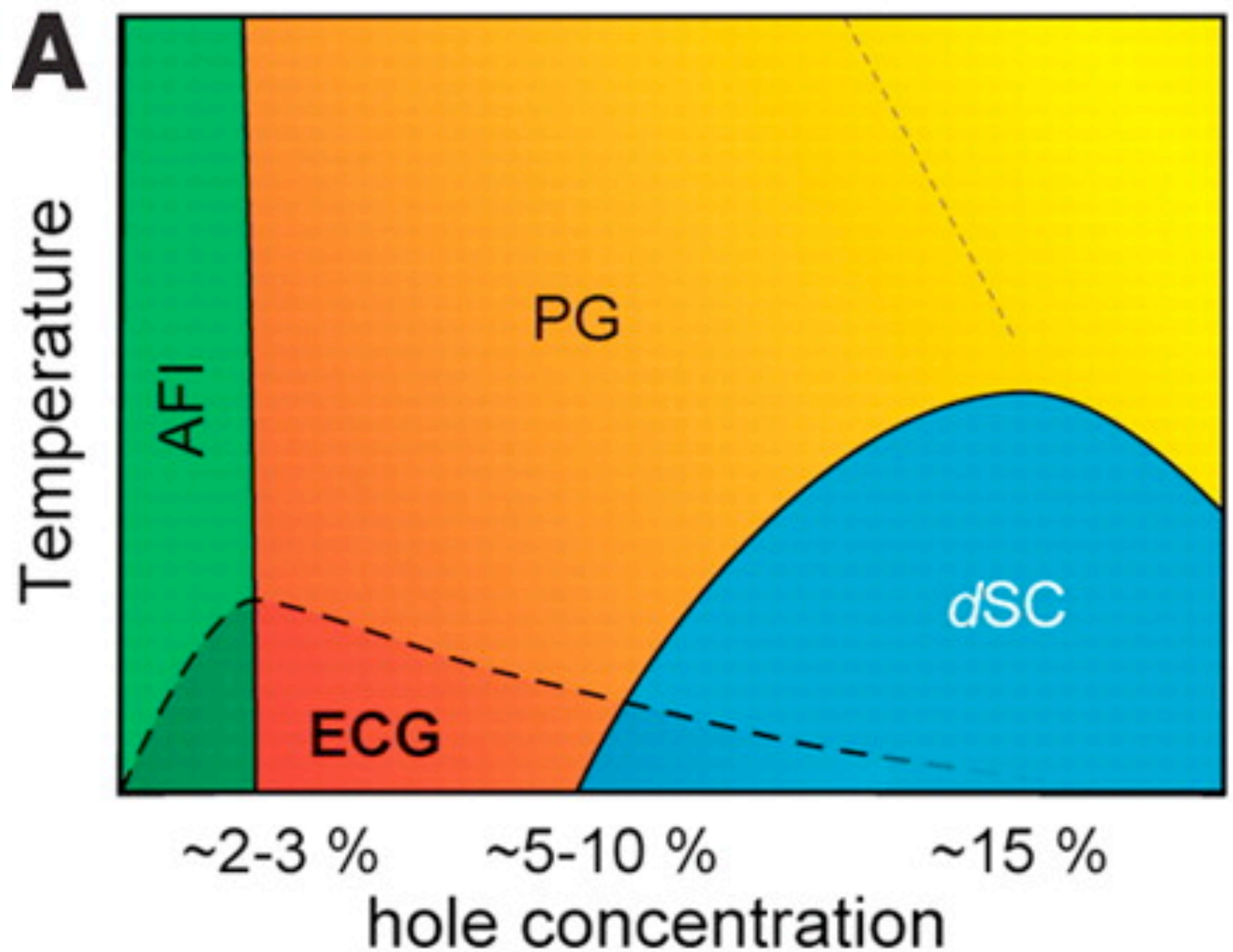
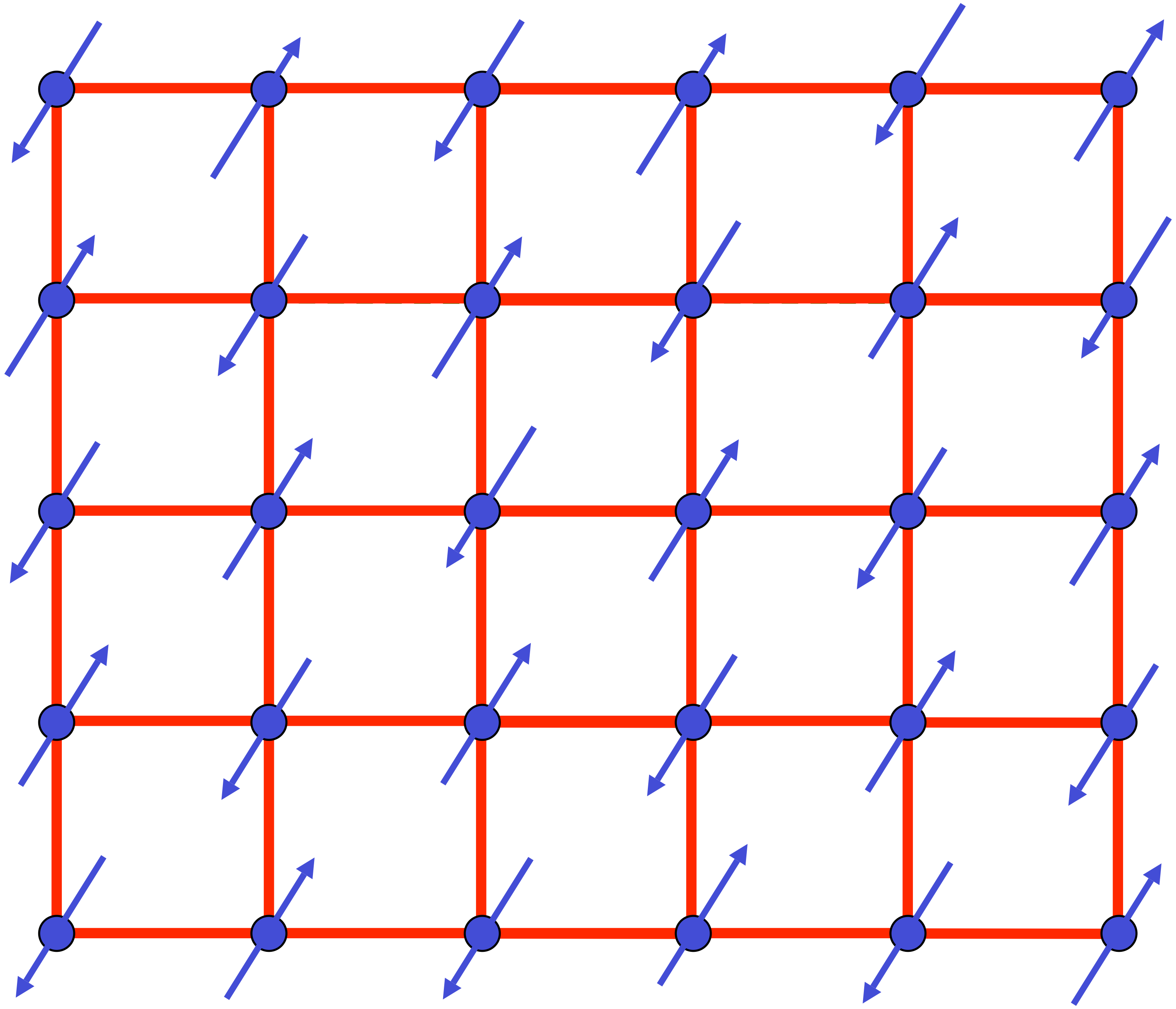
The cuprate superconductors





Antiferromagnetic (Neel) order in the insulator

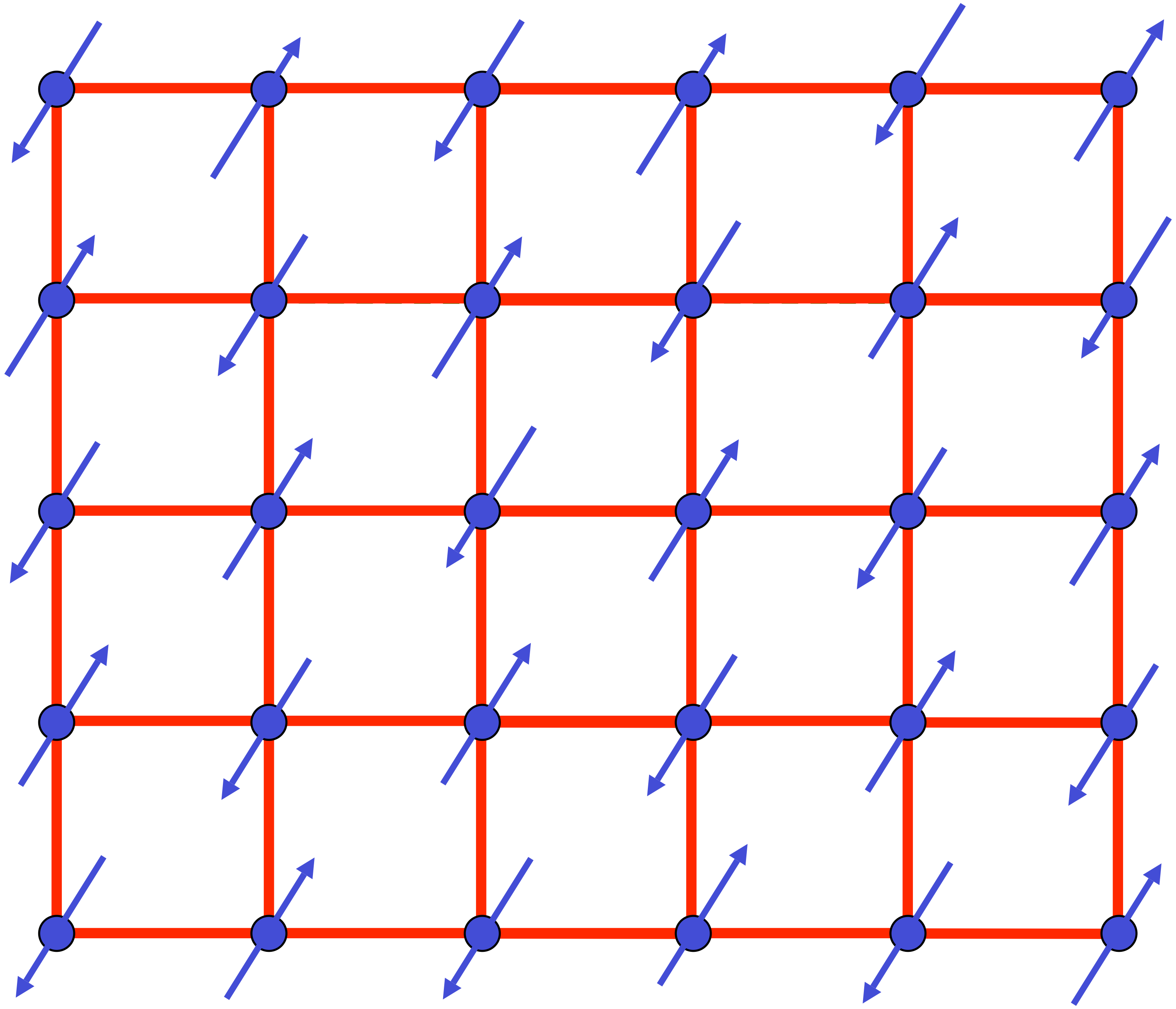


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$

No entanglement of spins

Antiferromagnetic (Neel) order in the insulator



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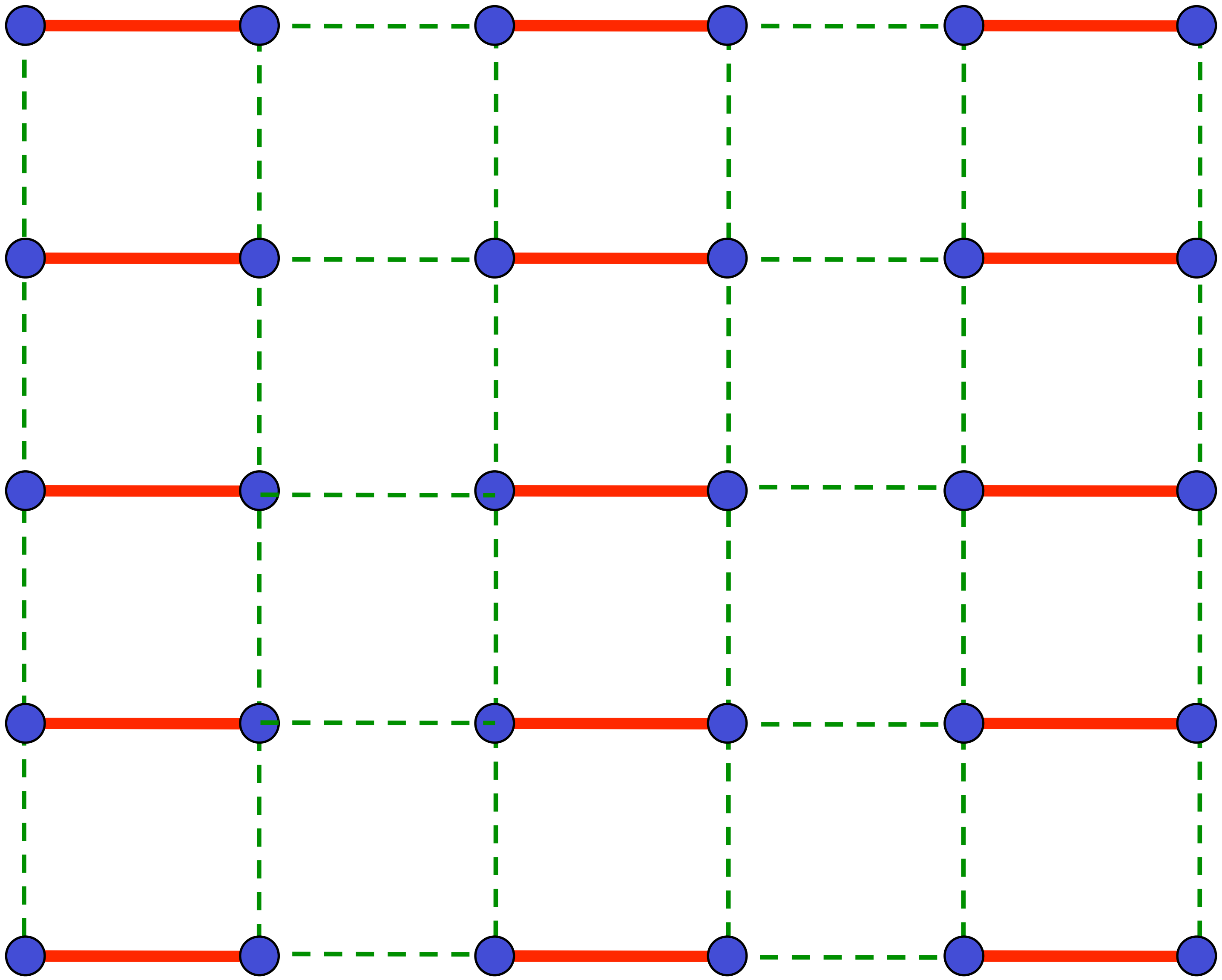
Excitations: 2 spin waves (Goldstone modes)



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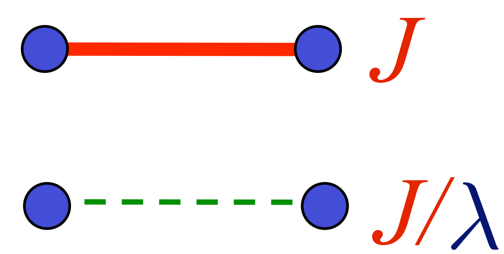
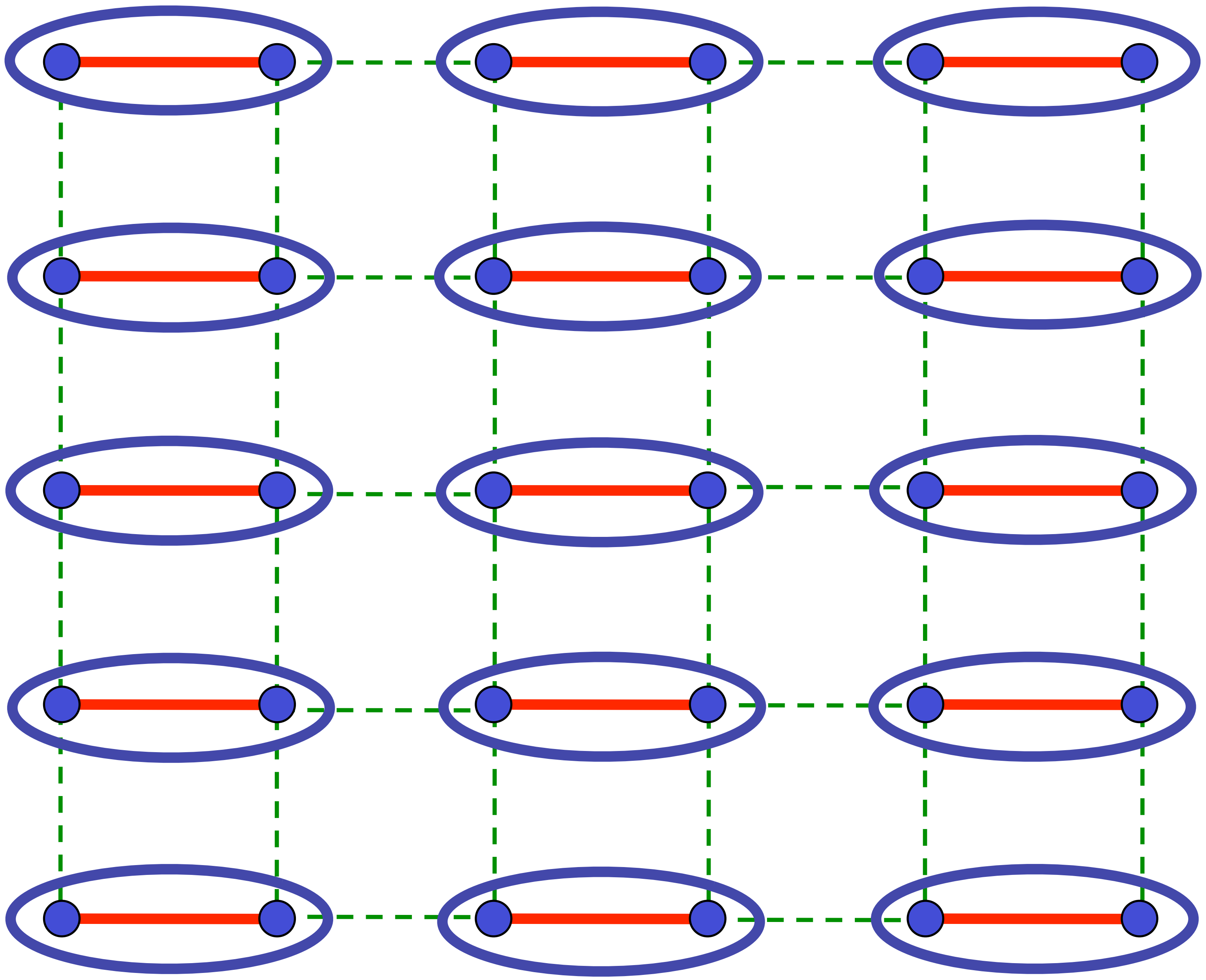


Weaken some bonds to induce spin entanglement in a new quantum phase



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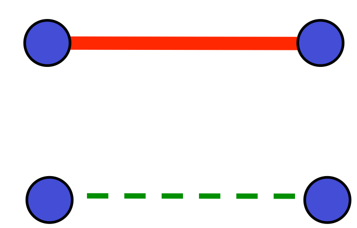
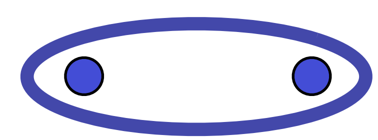


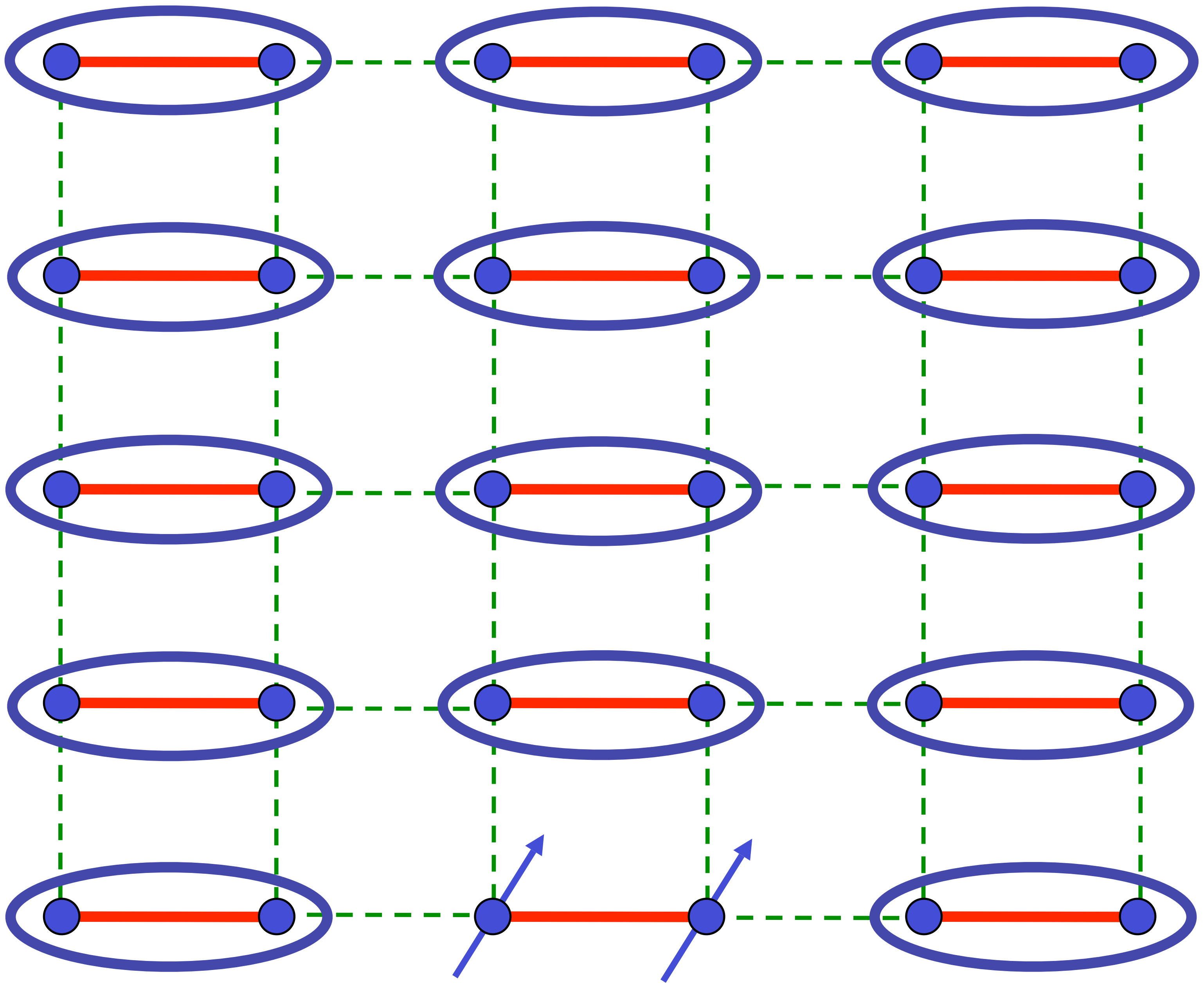
J

J/λ



$$= \frac{1}{\sqrt{2}} \Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \Big)$$

Ground state is a product of pairs of entangled spins.



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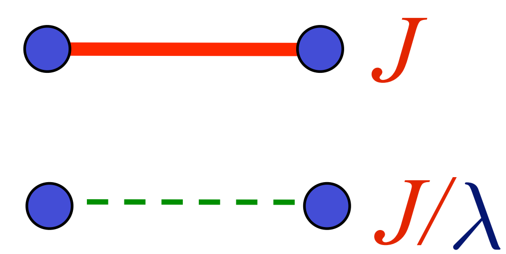
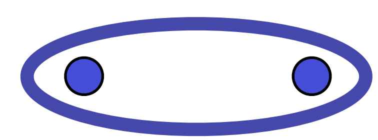
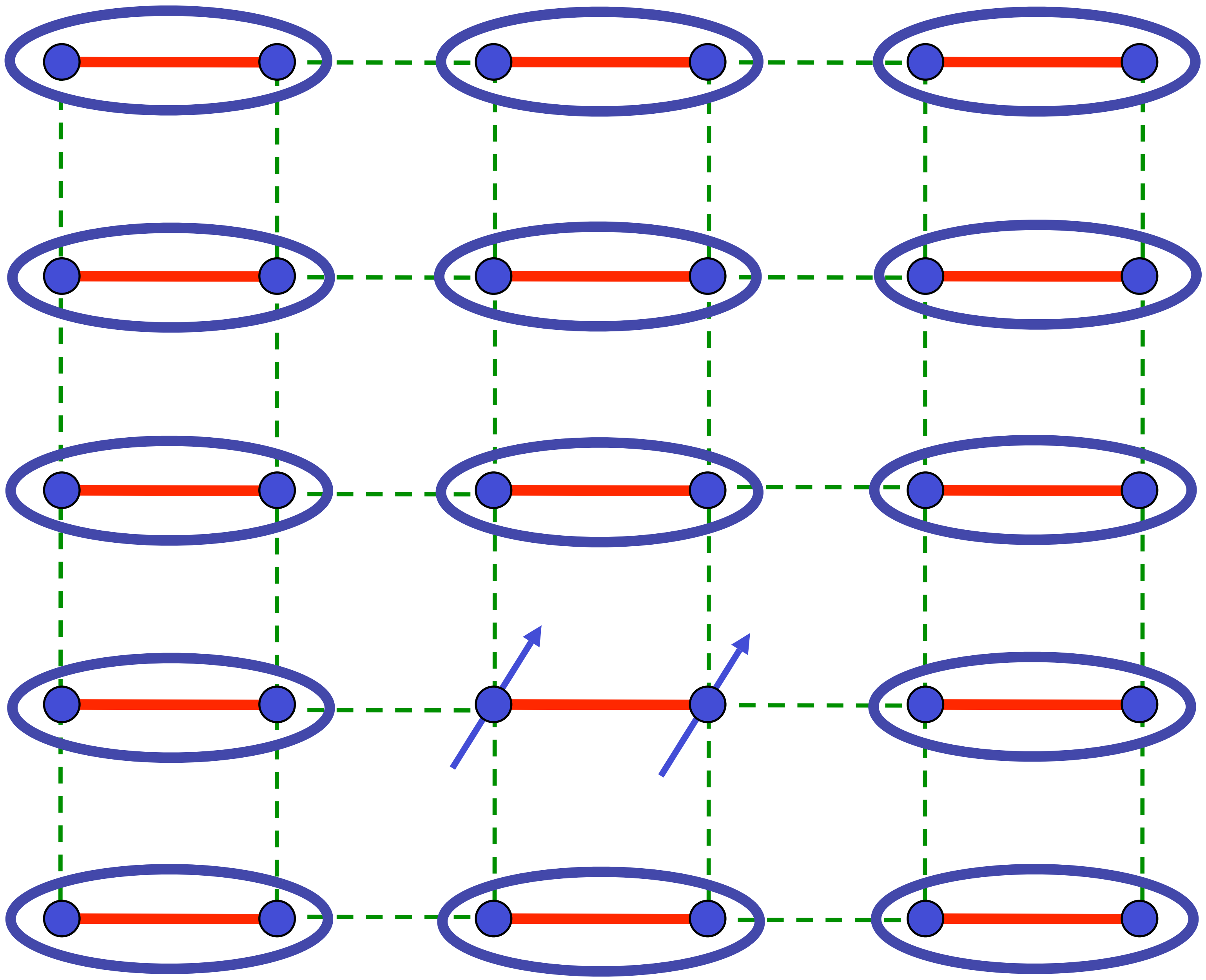




$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Excitations: 3 *S=1* triplons



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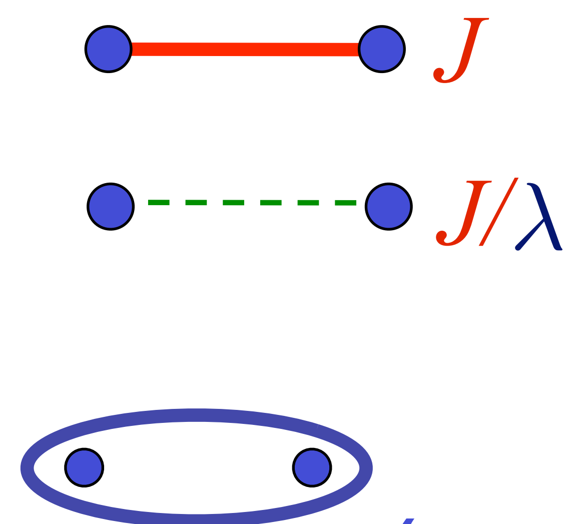
$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



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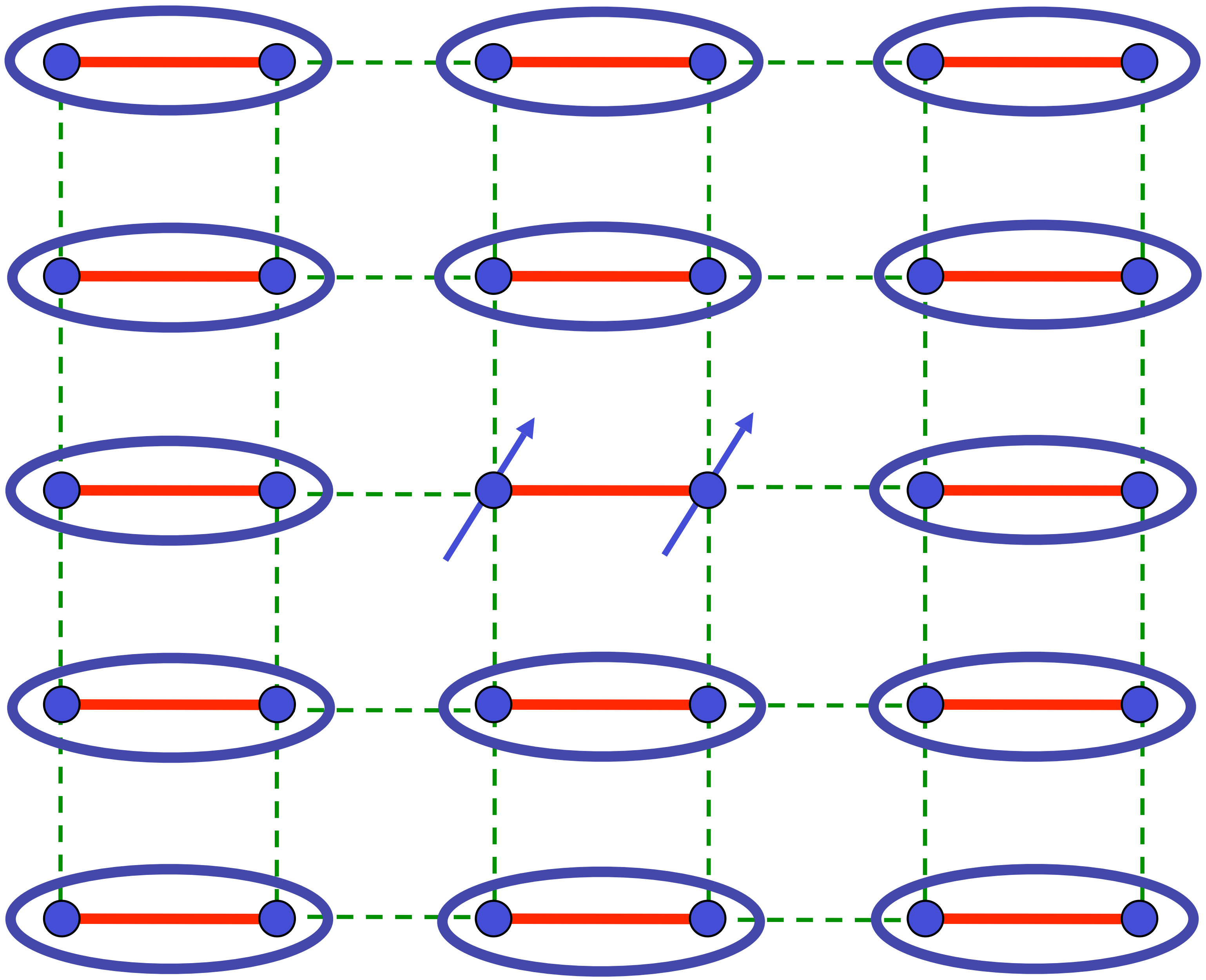

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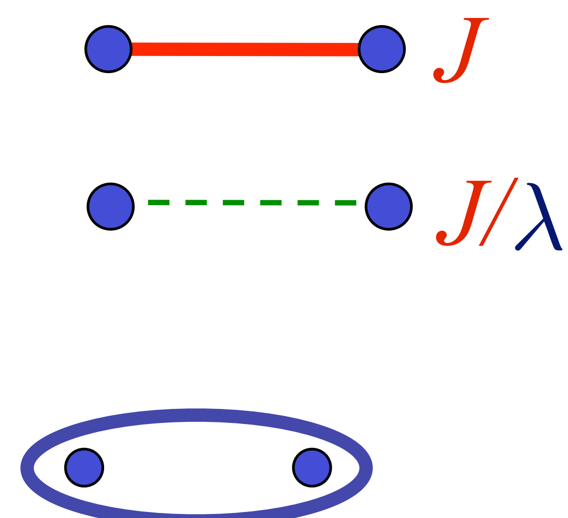

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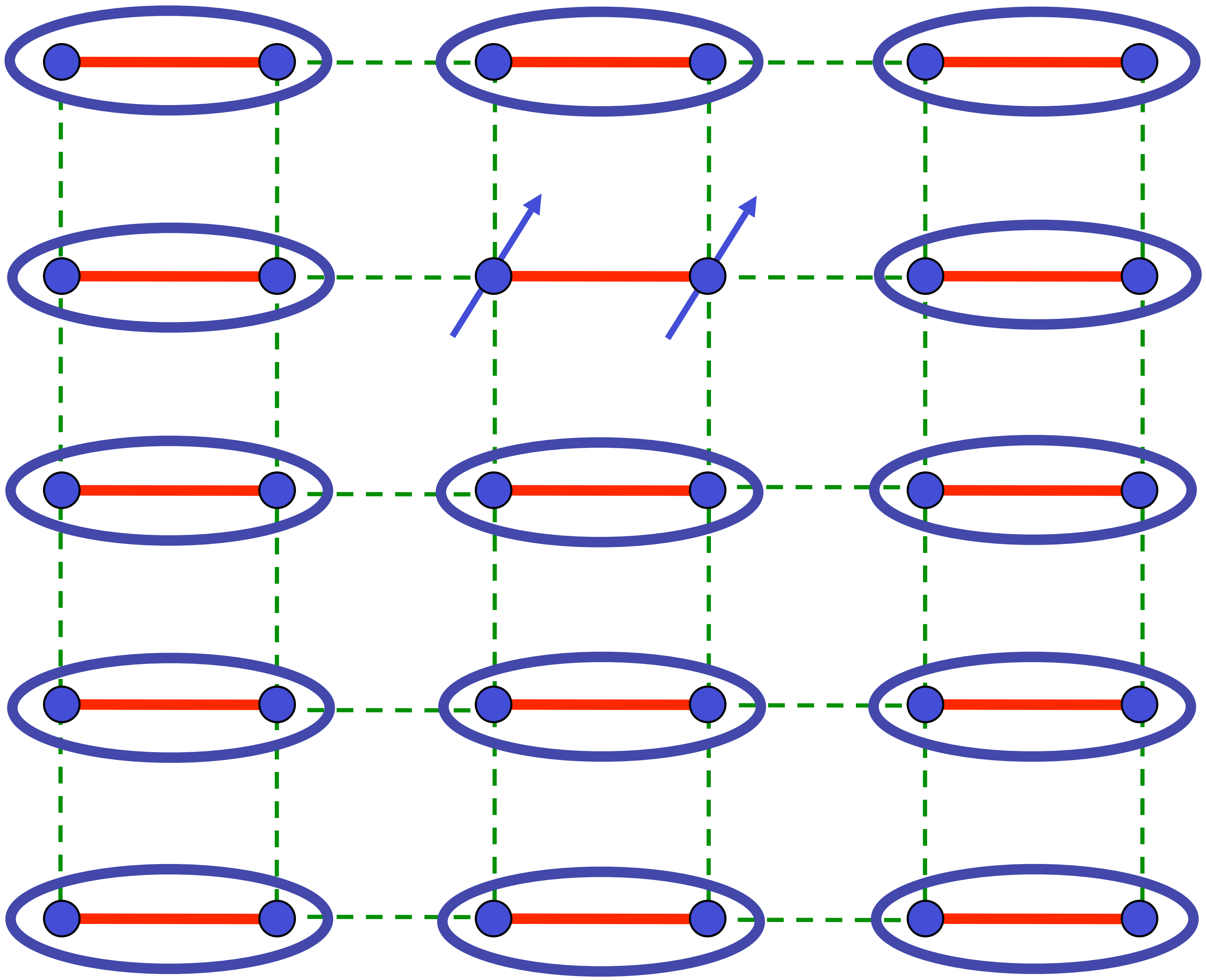

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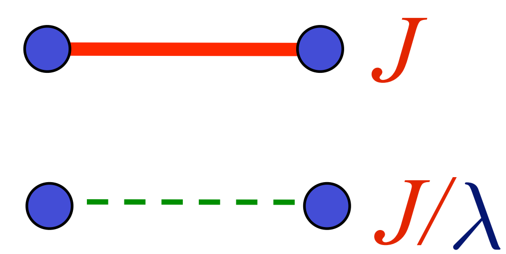

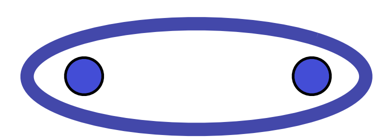

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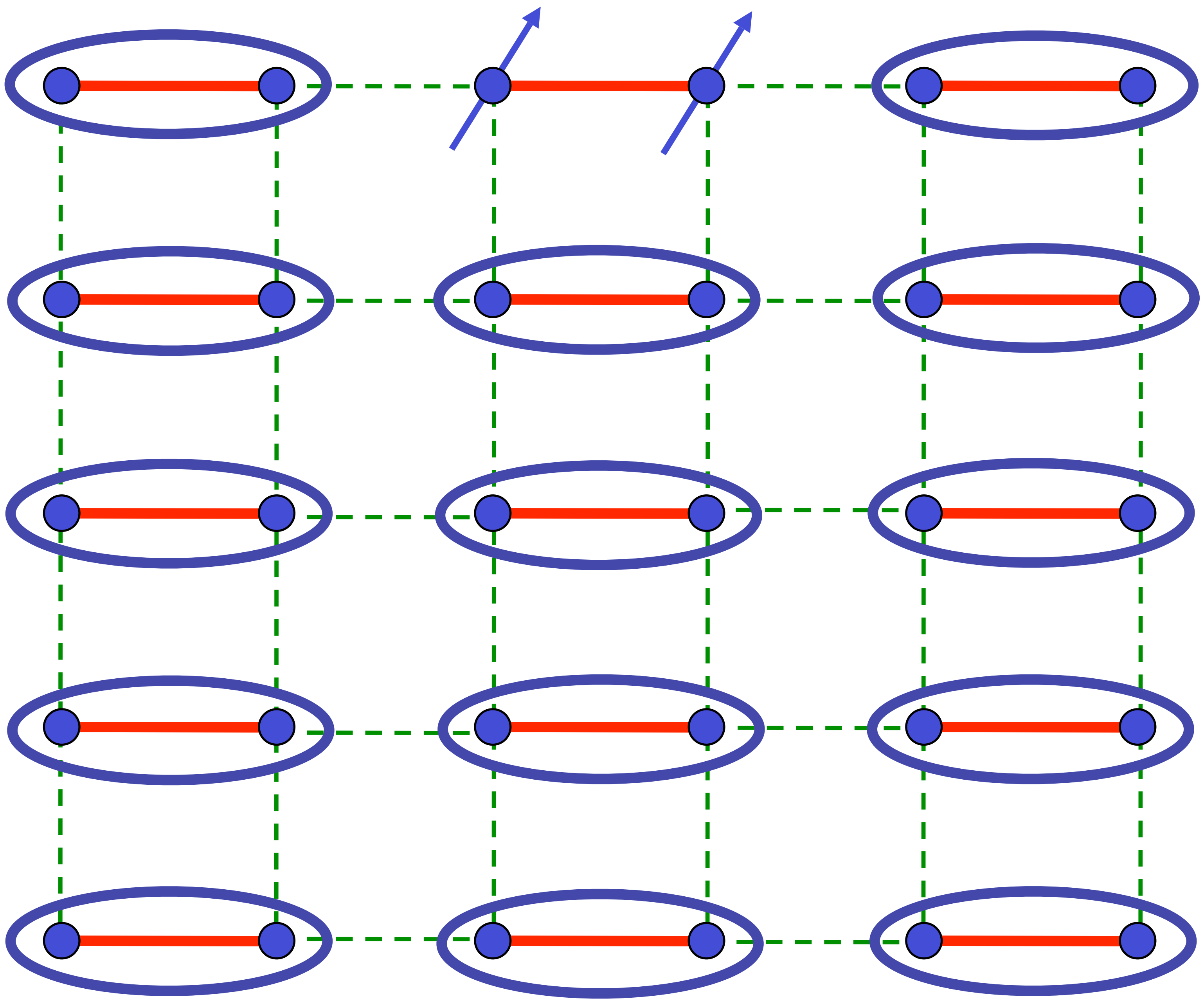
Excitations: 3 *S=1* triplons



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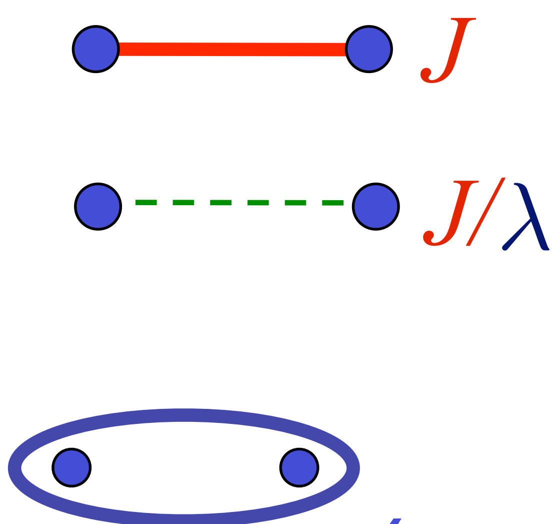

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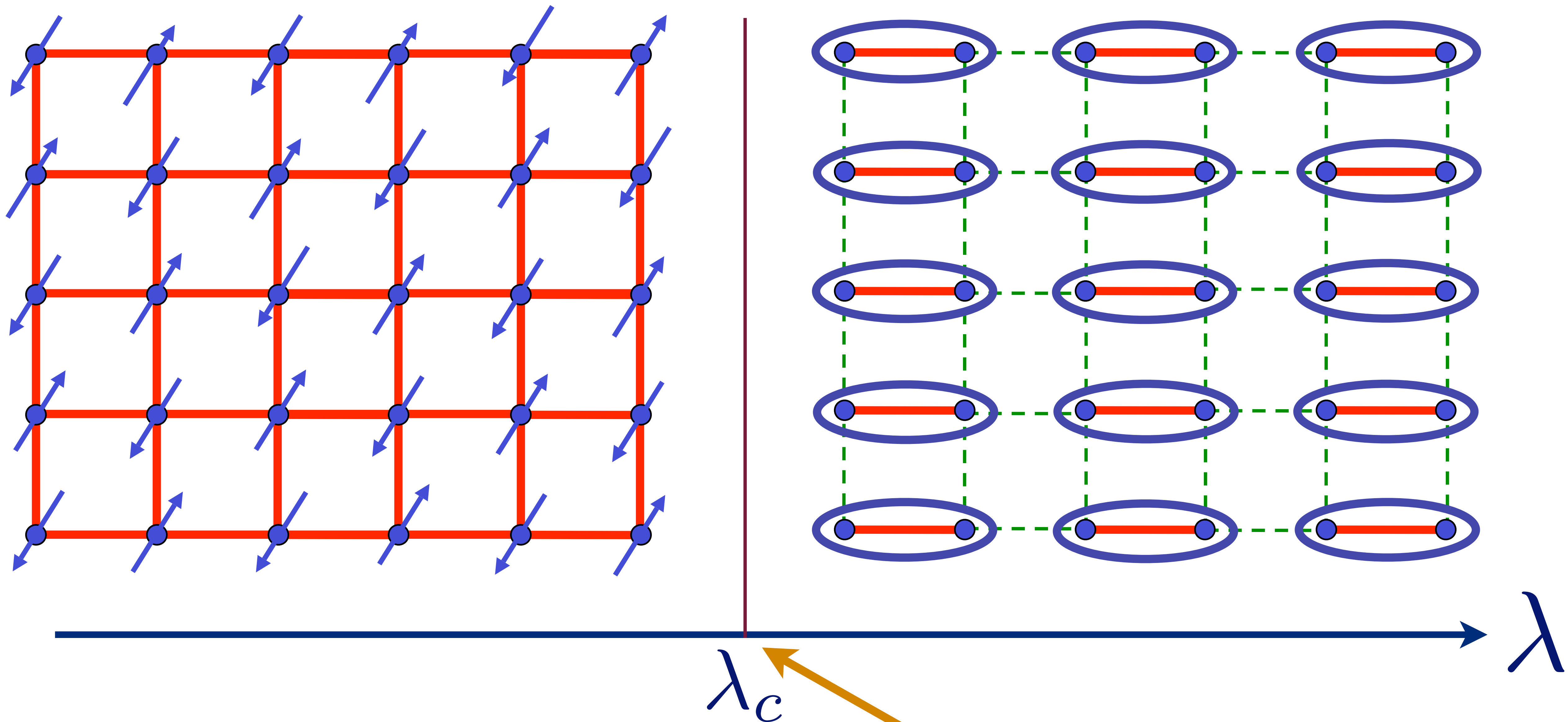
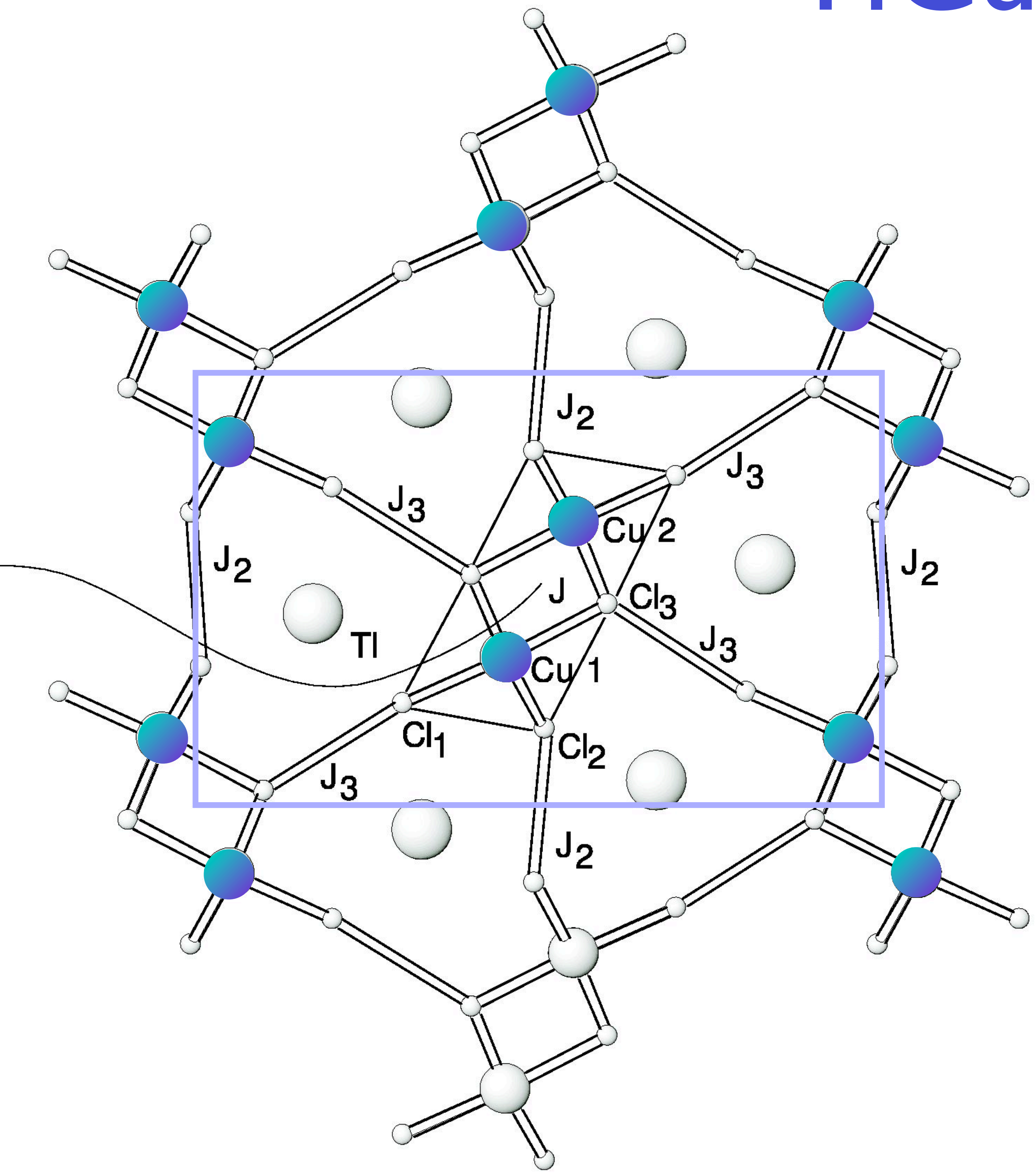
Phase diagram as a function of the ratio of exchange interactions, λ

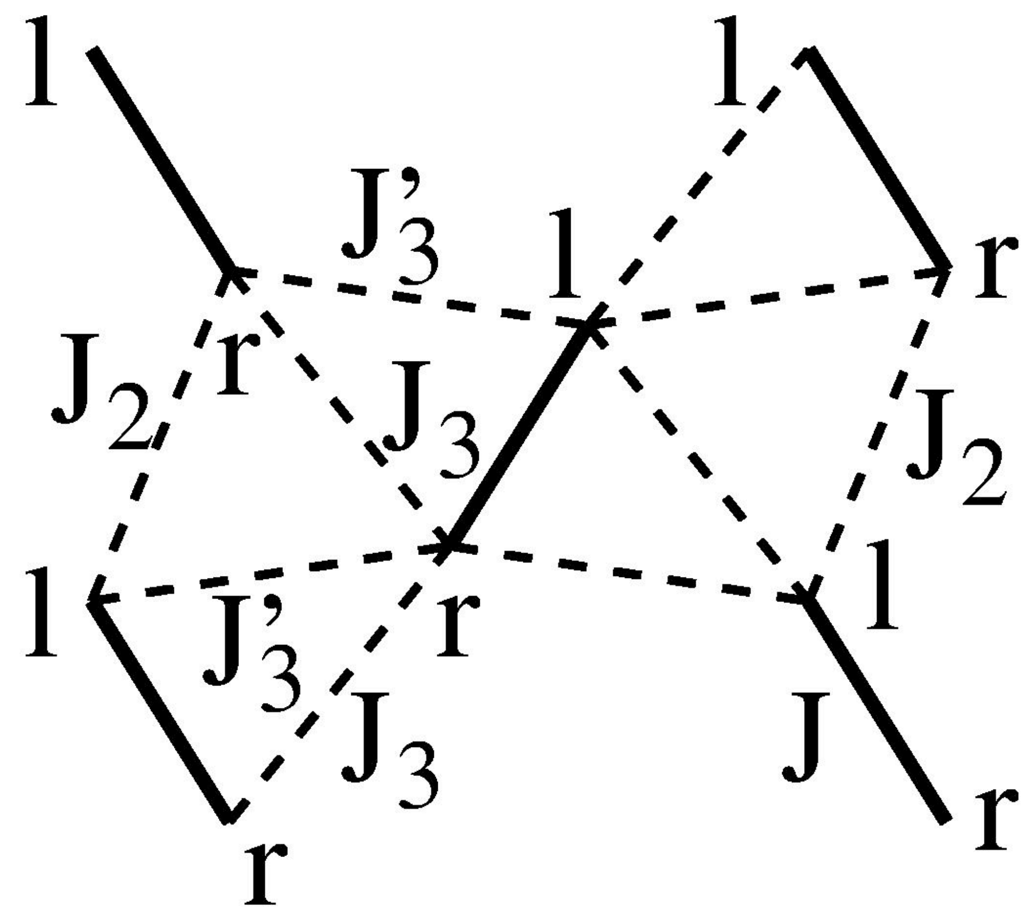


Quantum critical point with non-local entanglement in spin wavefunction

$TlCuCl_3$

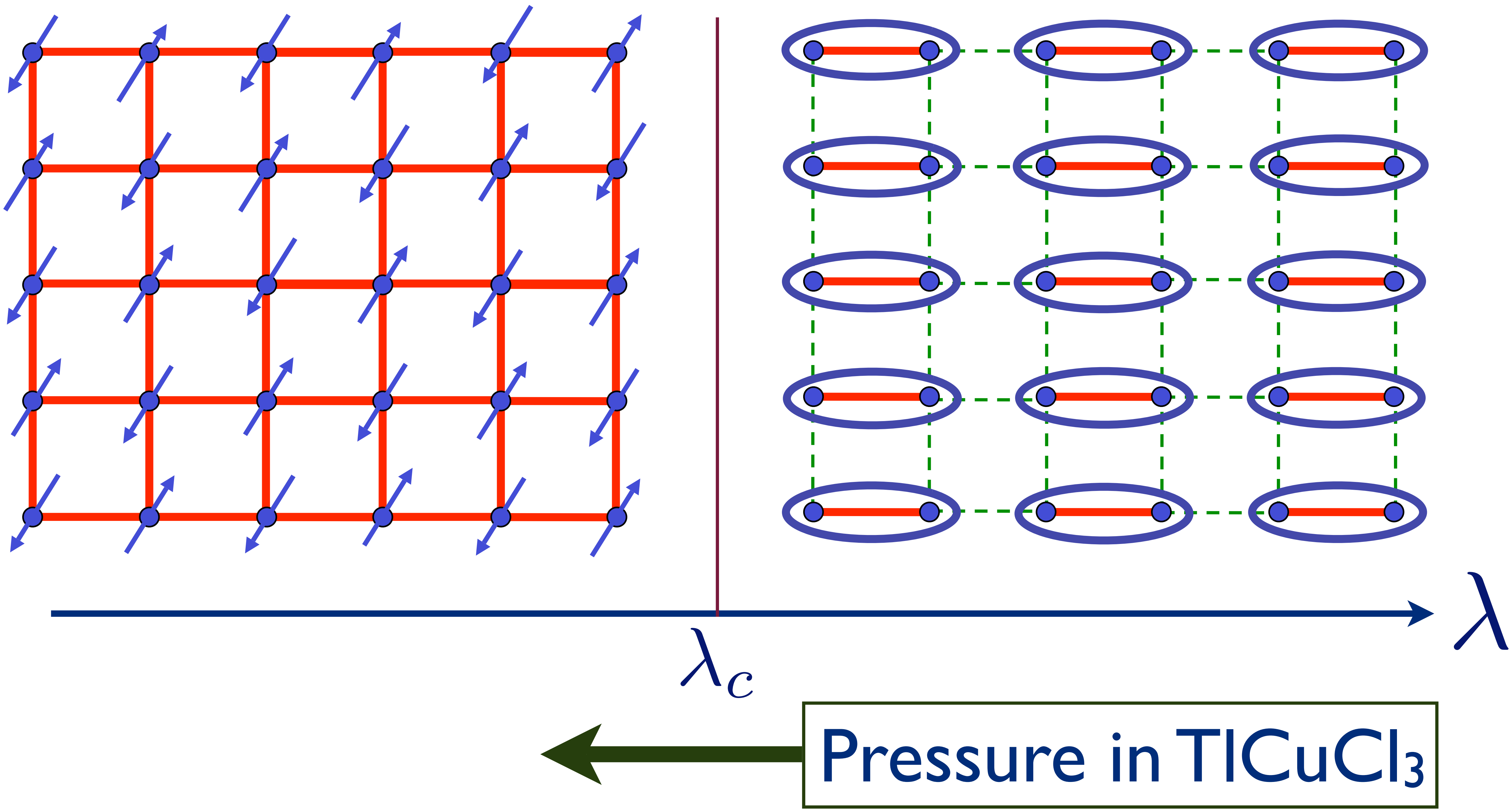


J2
J3
J3
J2
J2
Cu 2
J
Cl3
Tl
J3
Cu 1
Cl1
Cl2
J3
J2



l
l
J'3
l
r
J2
r
J3
J2
l
r
l
J'3
r
J3
J
r
r

Phase diagram as a function of the ratio of exchange interactions, λ



$TlCuCl_3$ at ambient pressure

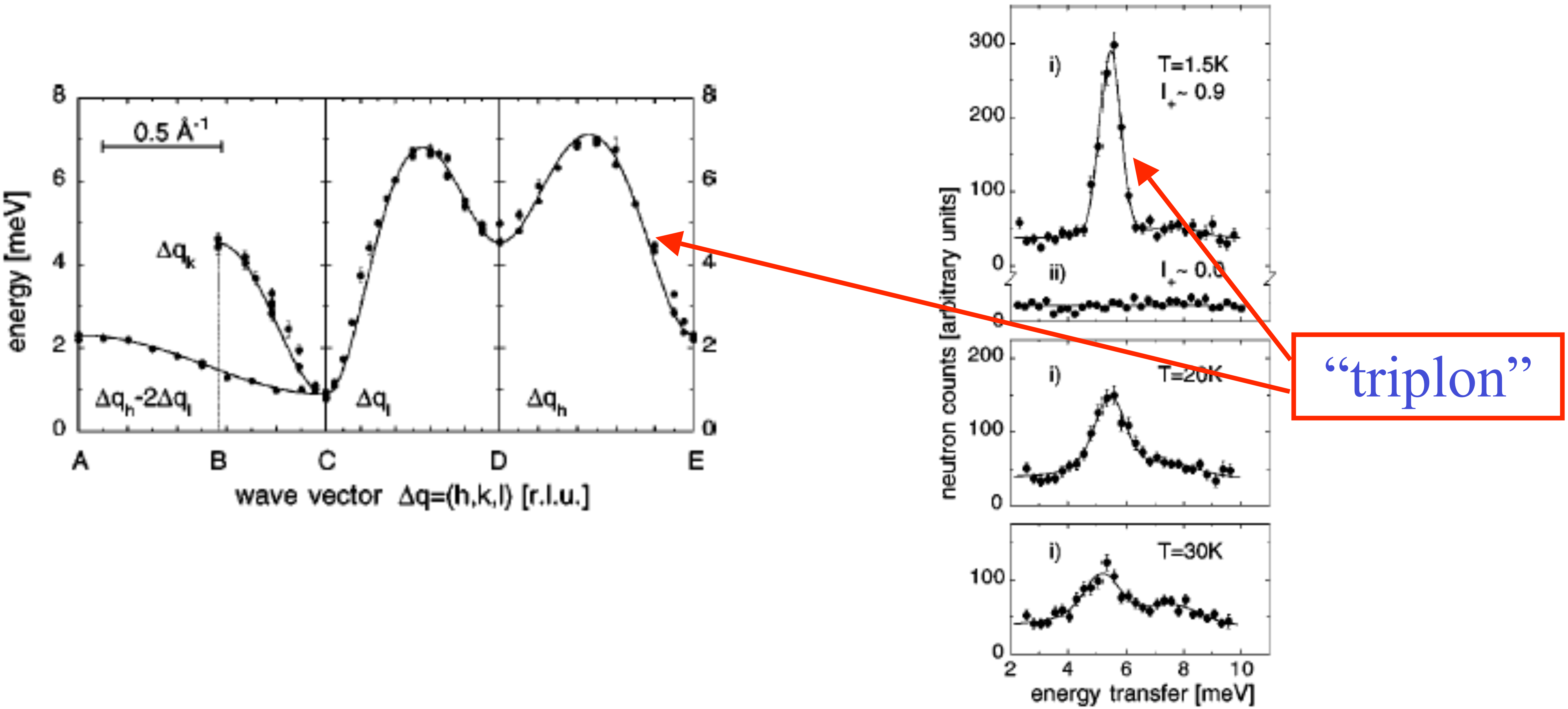
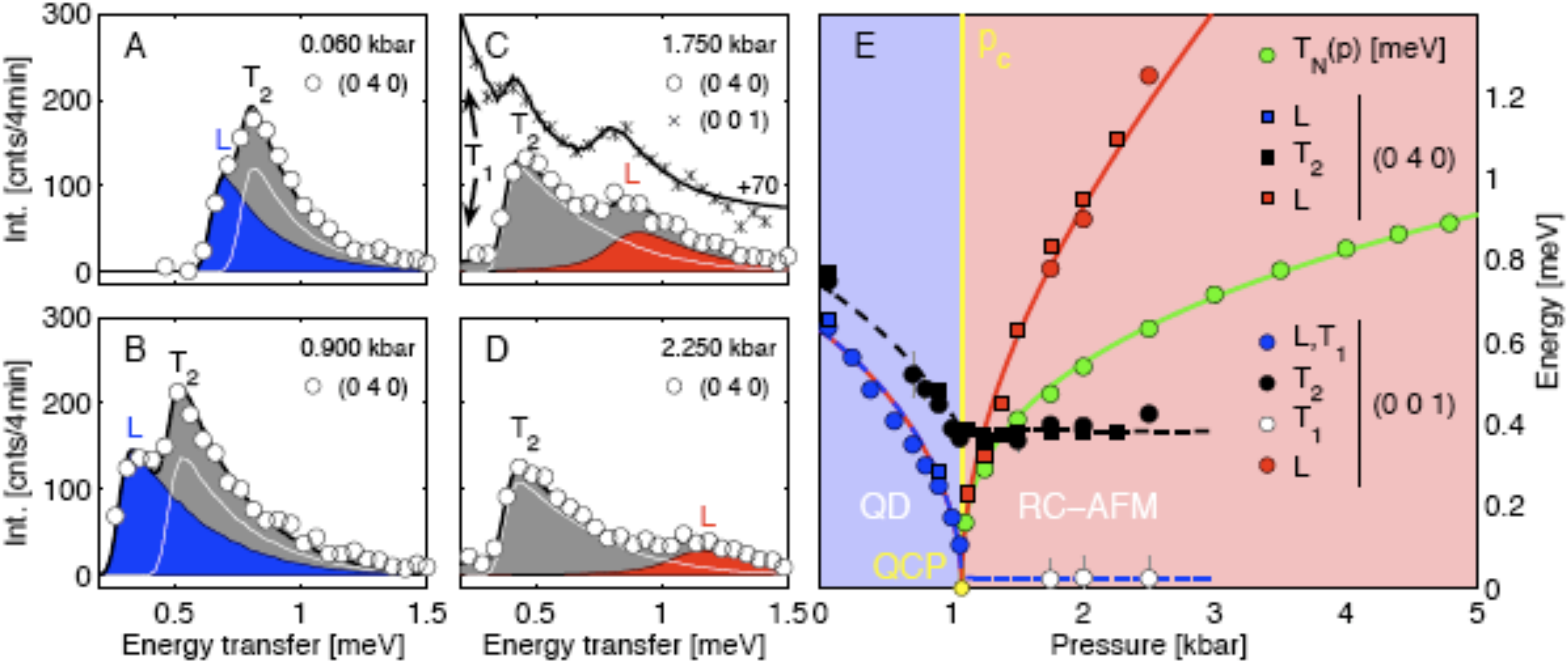


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

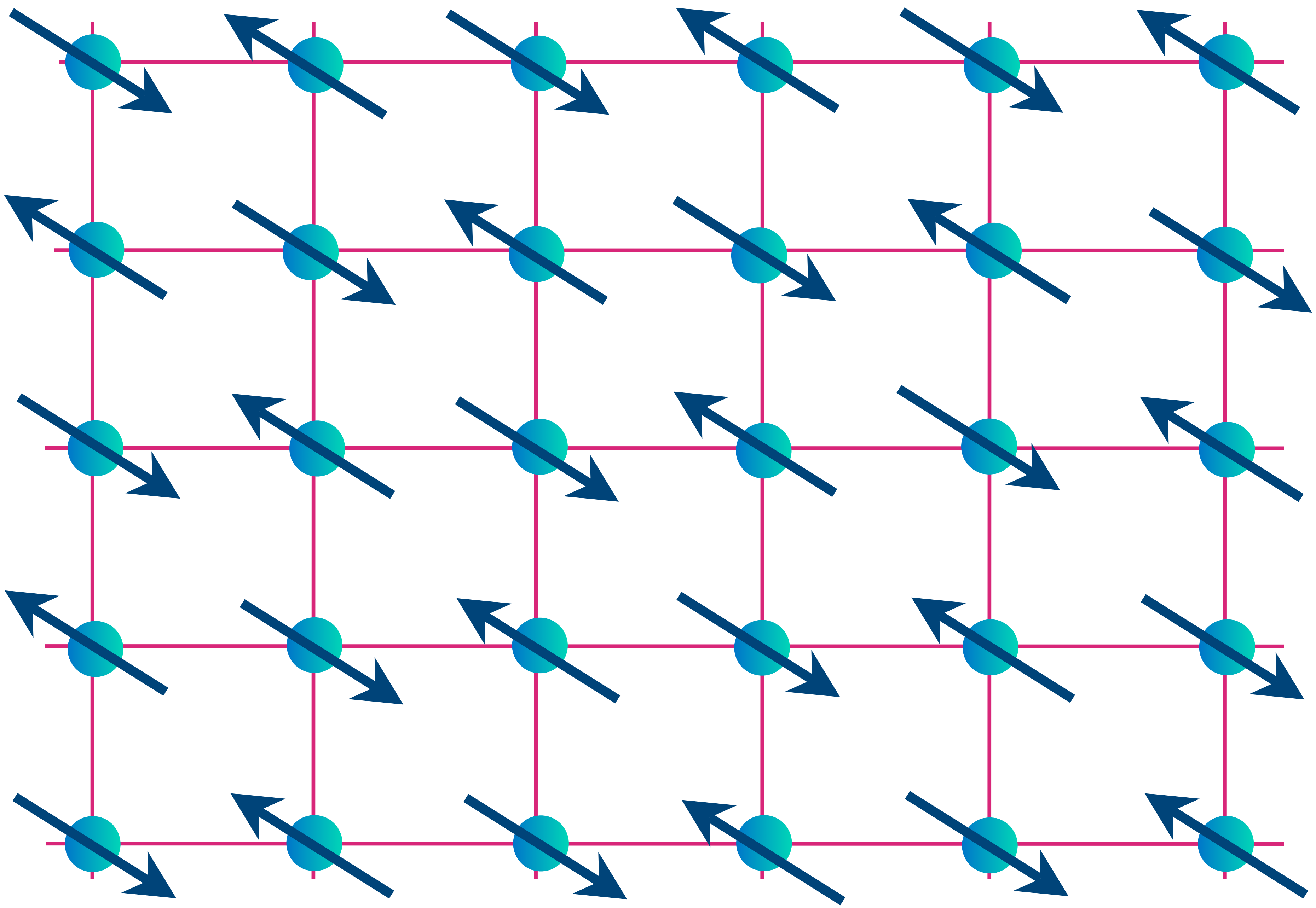
$TlCuCl_3$ with varying pressure



Observation of 3 → 2 low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point

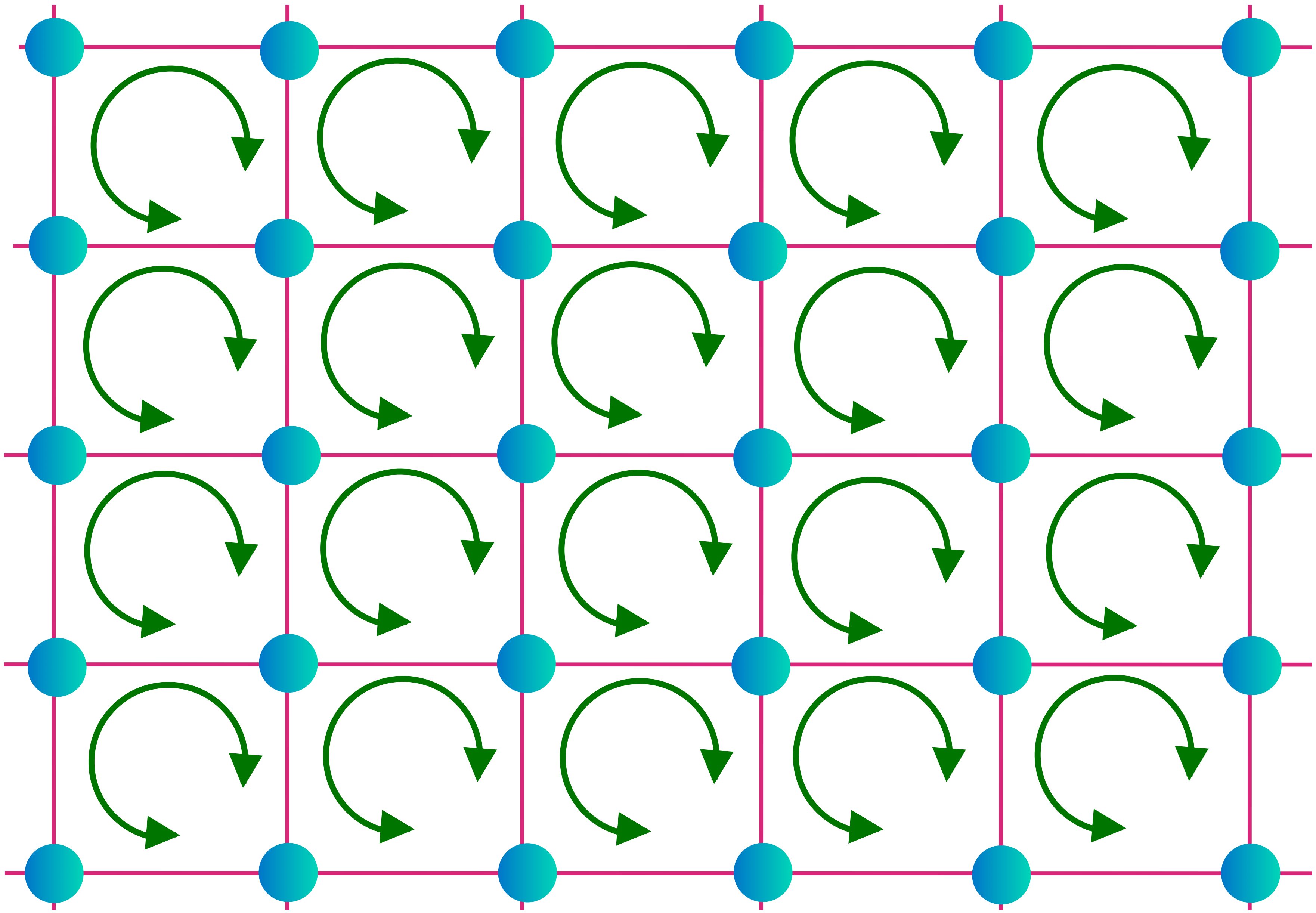
Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, arXiv:0803.3720

Quantum phase transition with full square lattice symmetry



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \quad ; \quad \vec{S}_i \Rightarrow \text{ spin operator with } S = 1/2$$

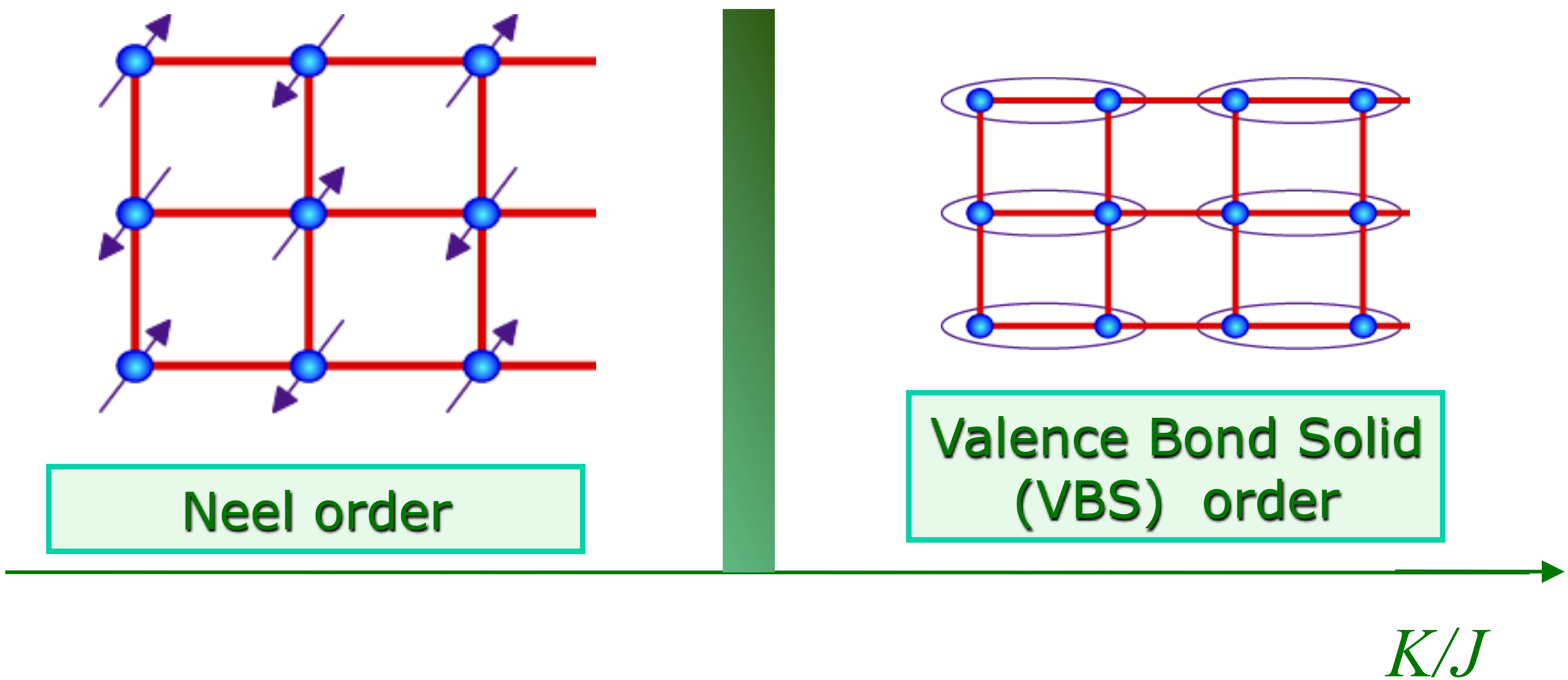
Quantum phase transition with full square lattice symmetry



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{four spin exchange}$$

A. W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007)

Quantum phase transition with full square lattice symmetry



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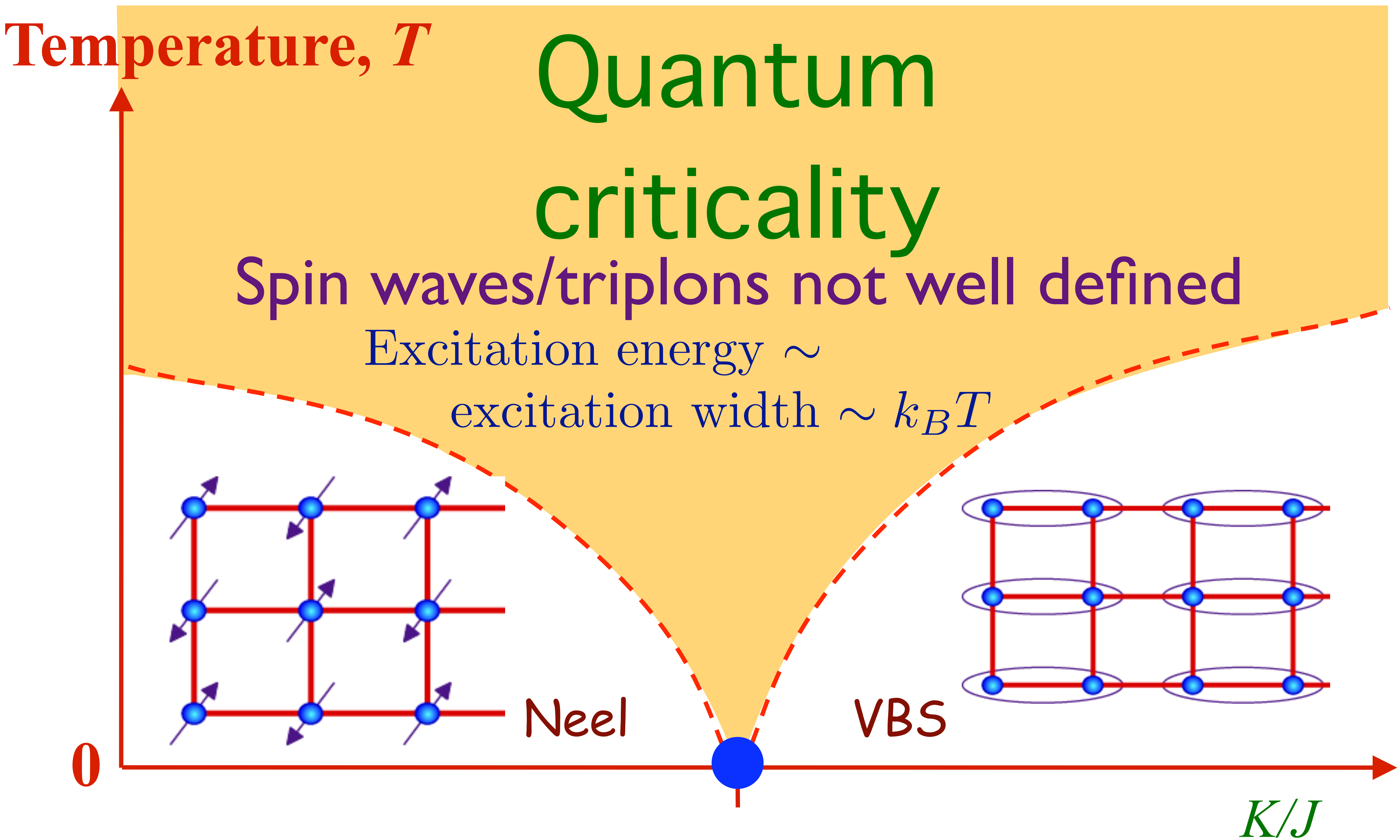
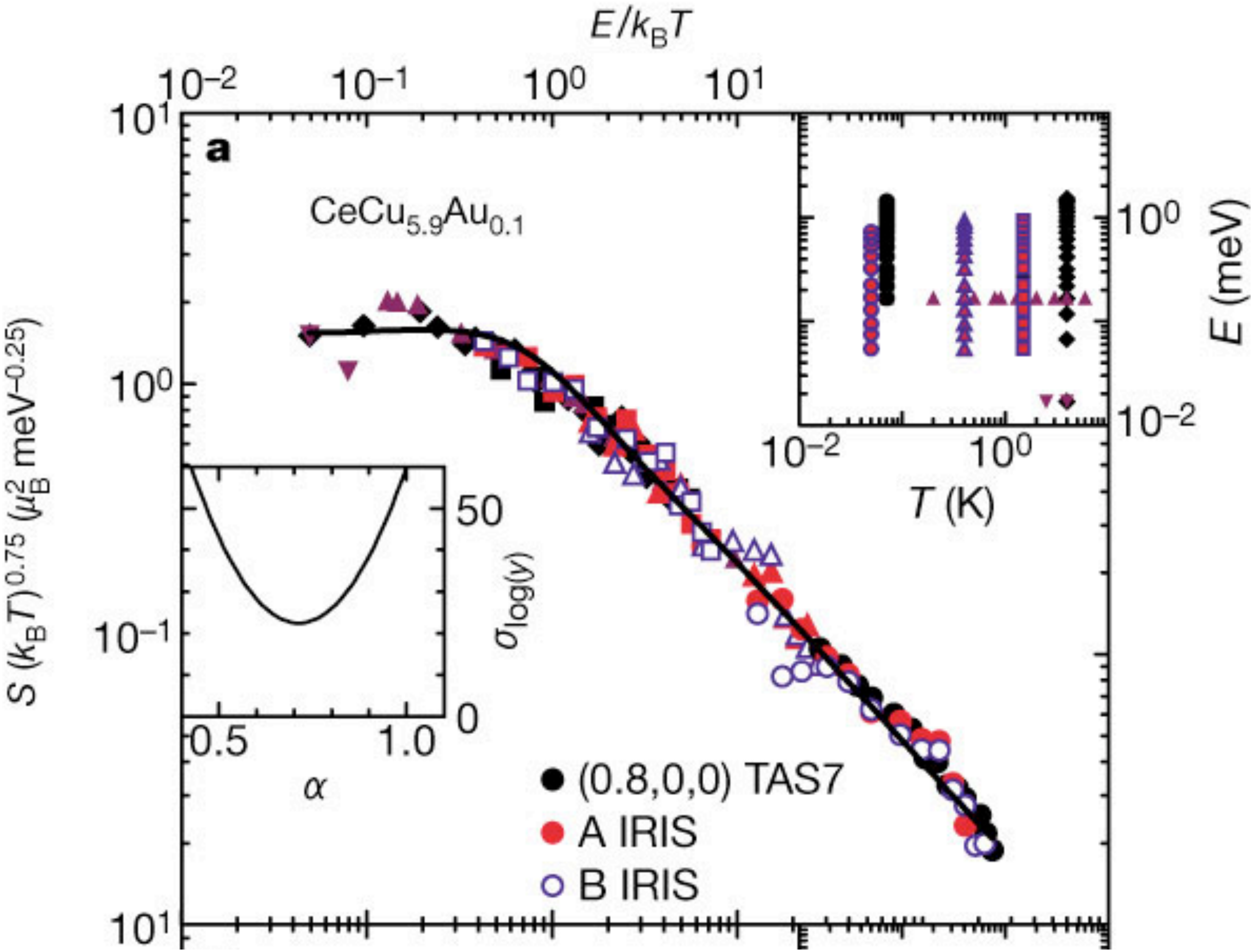
A. W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007)
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

Why should we care about the entanglement at an isolated critical point in the parameter space ?

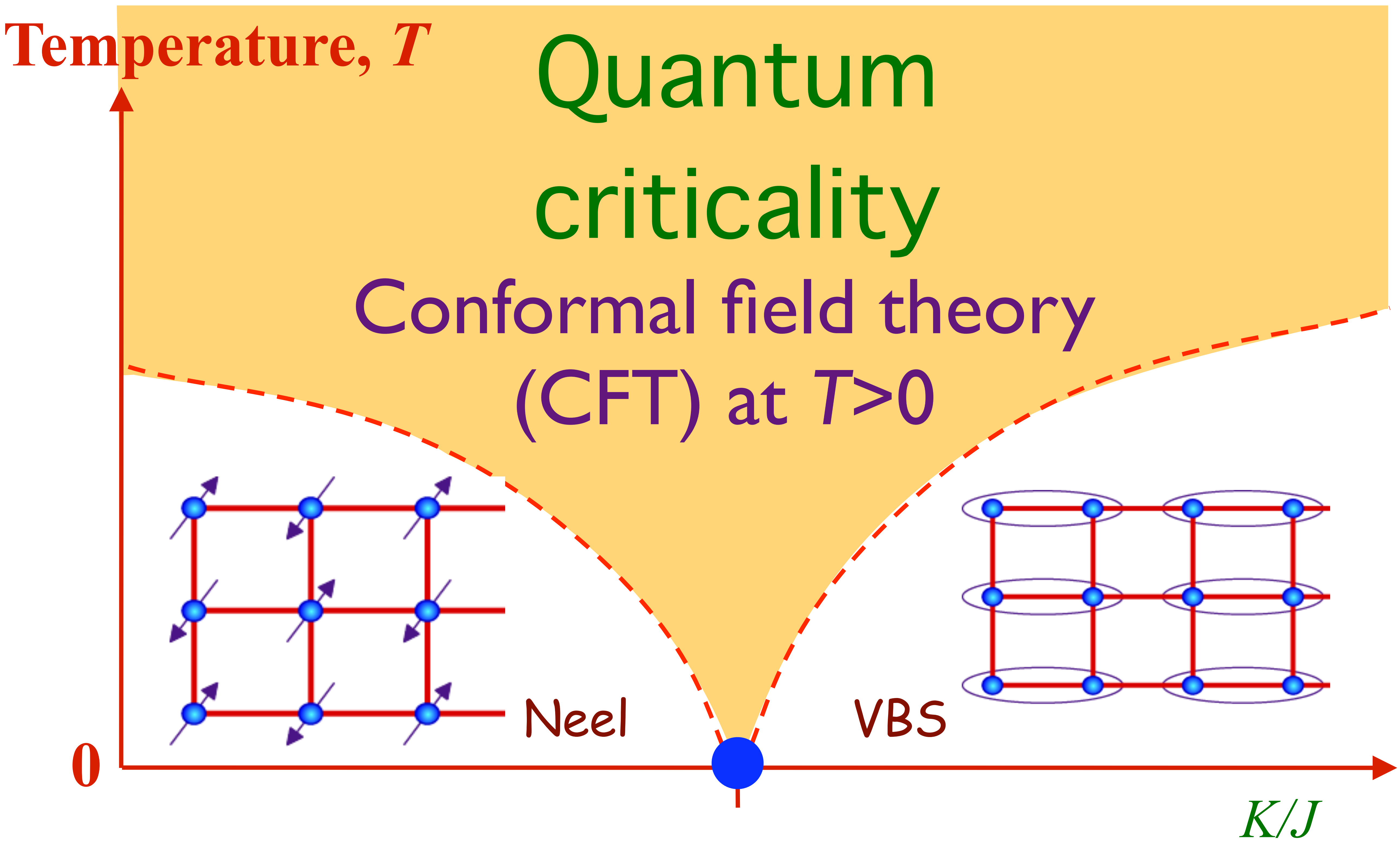


Temperature, T
Quantum criticality
Spin waves/triplons not well defined
Excitation energy $\sim$ excitation width $\sim k_B T$
Neel
VBS
0
K/J



Onset of antiferromagnetism in heavy-fermion metals

A. Schröder, G. Aeppli, R. Coldea, M. Adams, O. Stockert, H.v. Löhneysen, E. Bucher, R. Ramazashvili and P. Coleman, *Nature* **407**, 351 (2000)



Temperature, *T*
Quantum criticality
Conformal field theory (CFT) at *T*>0
Neel
VBS
0
K/J

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Black Holes

Objects so massive that light is gravitationally bound to them.

Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole horizon is causally disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole $S = \dfrac{k_B A}{4\ell_P^2}$

where A is the area of the horizon, and

$\ell_P = \sqrt{\dfrac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$

Black Hole Thermodynamics

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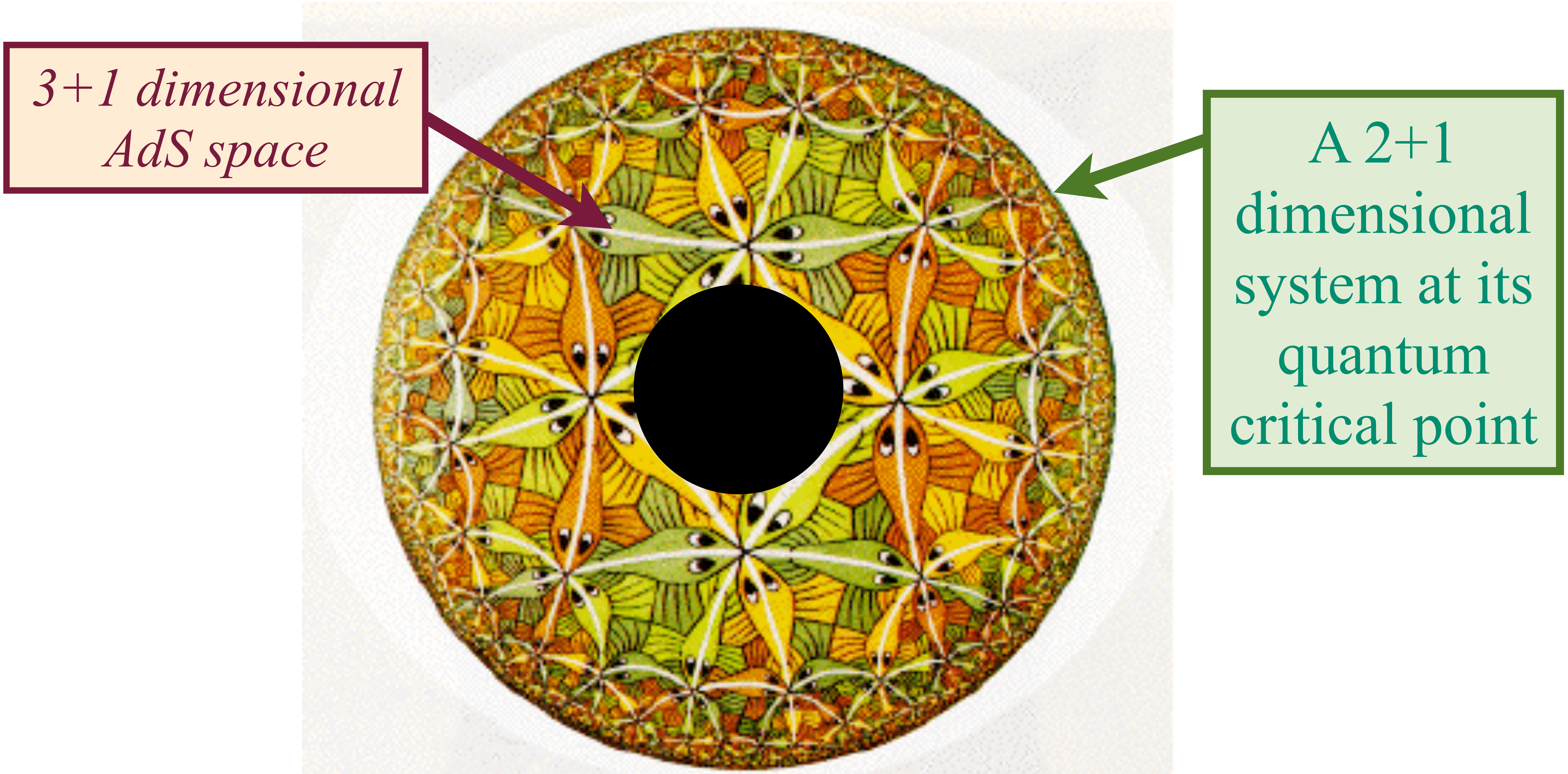
$$\text{Horizon temperature: } 4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$$

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



Maldacena, Gubser, Klebanov, Polyakov, Witten

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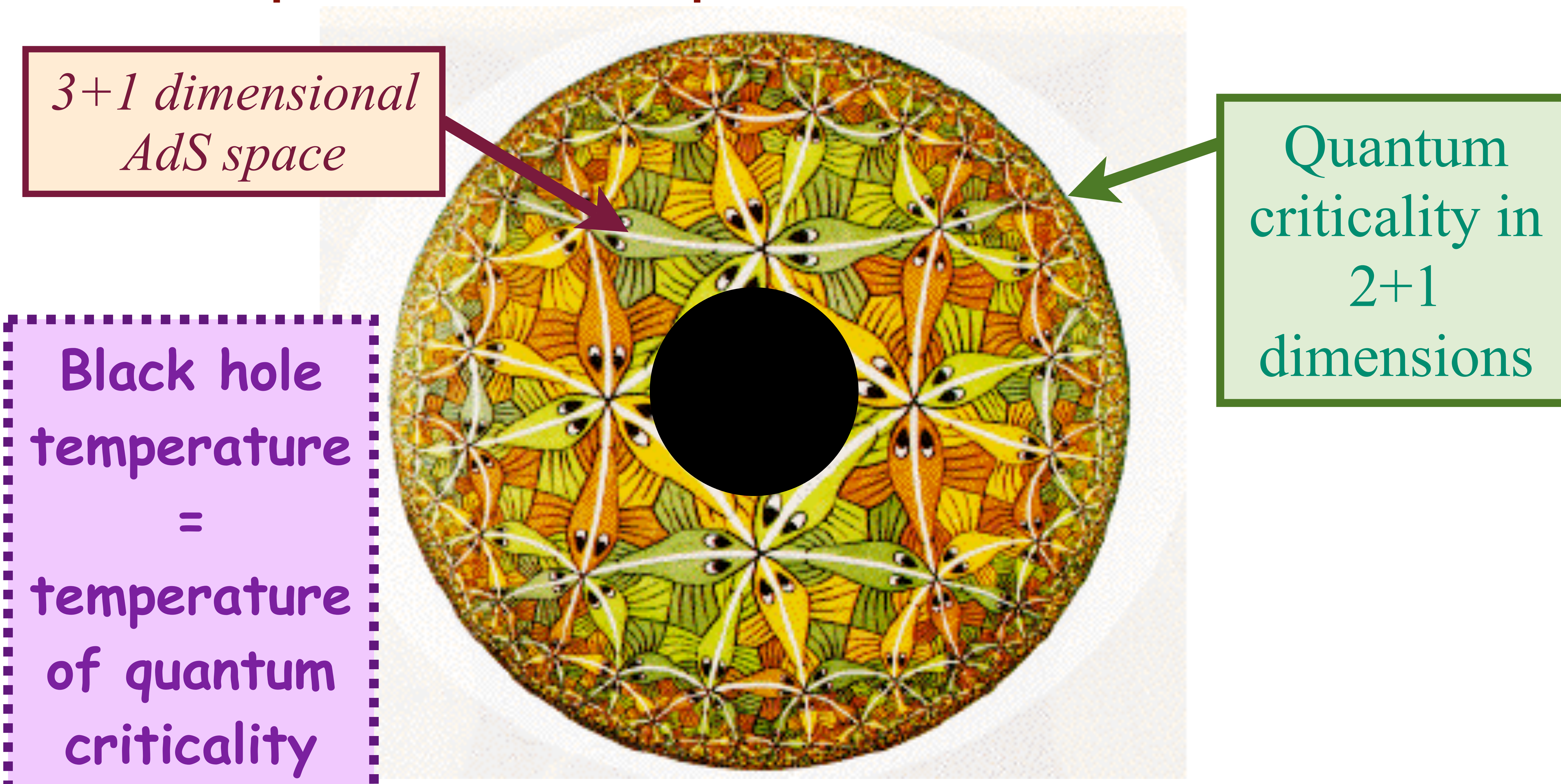

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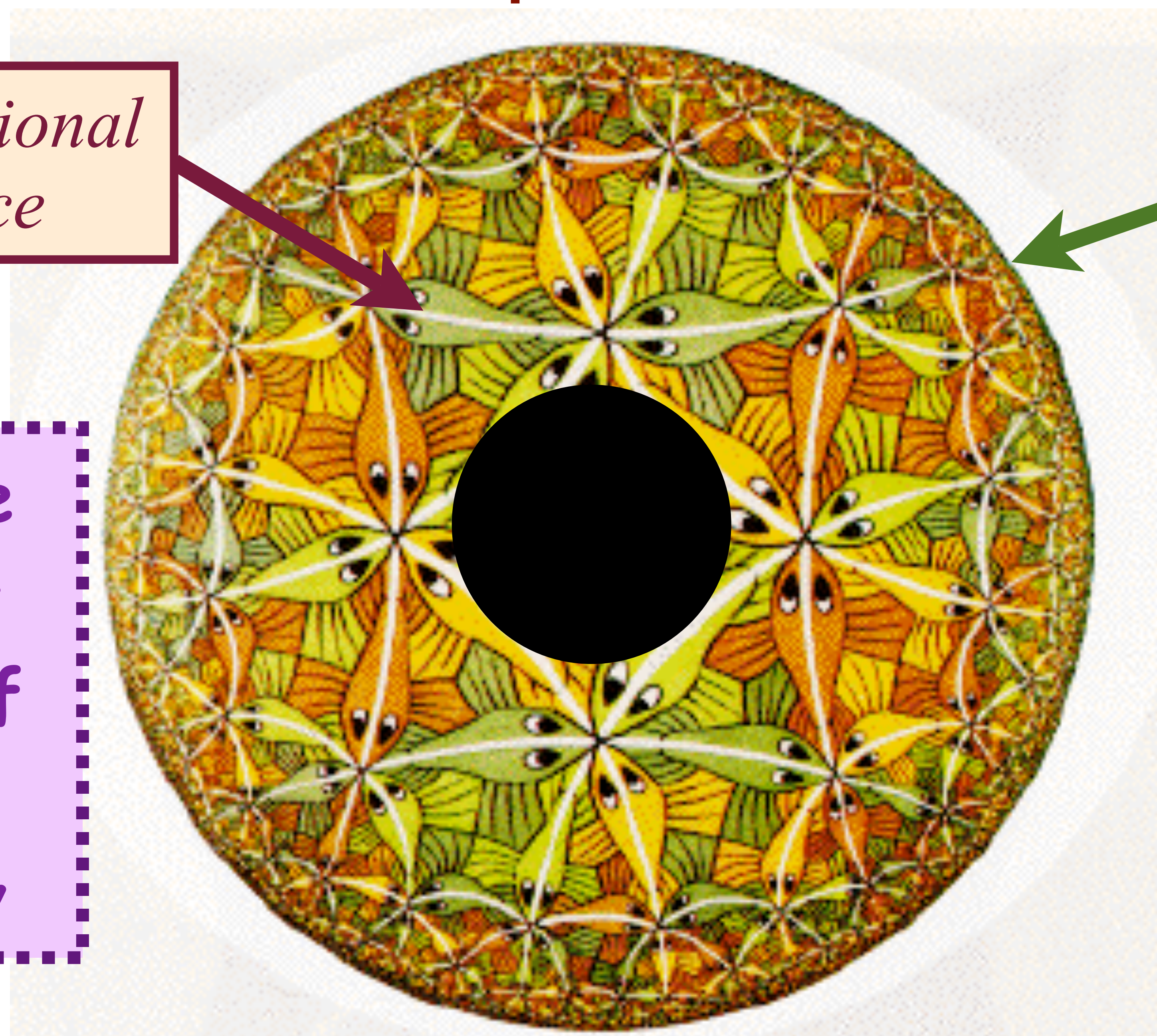
3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions



Black hole entropy = entropy of quantum criticality

Strominger, Vafa

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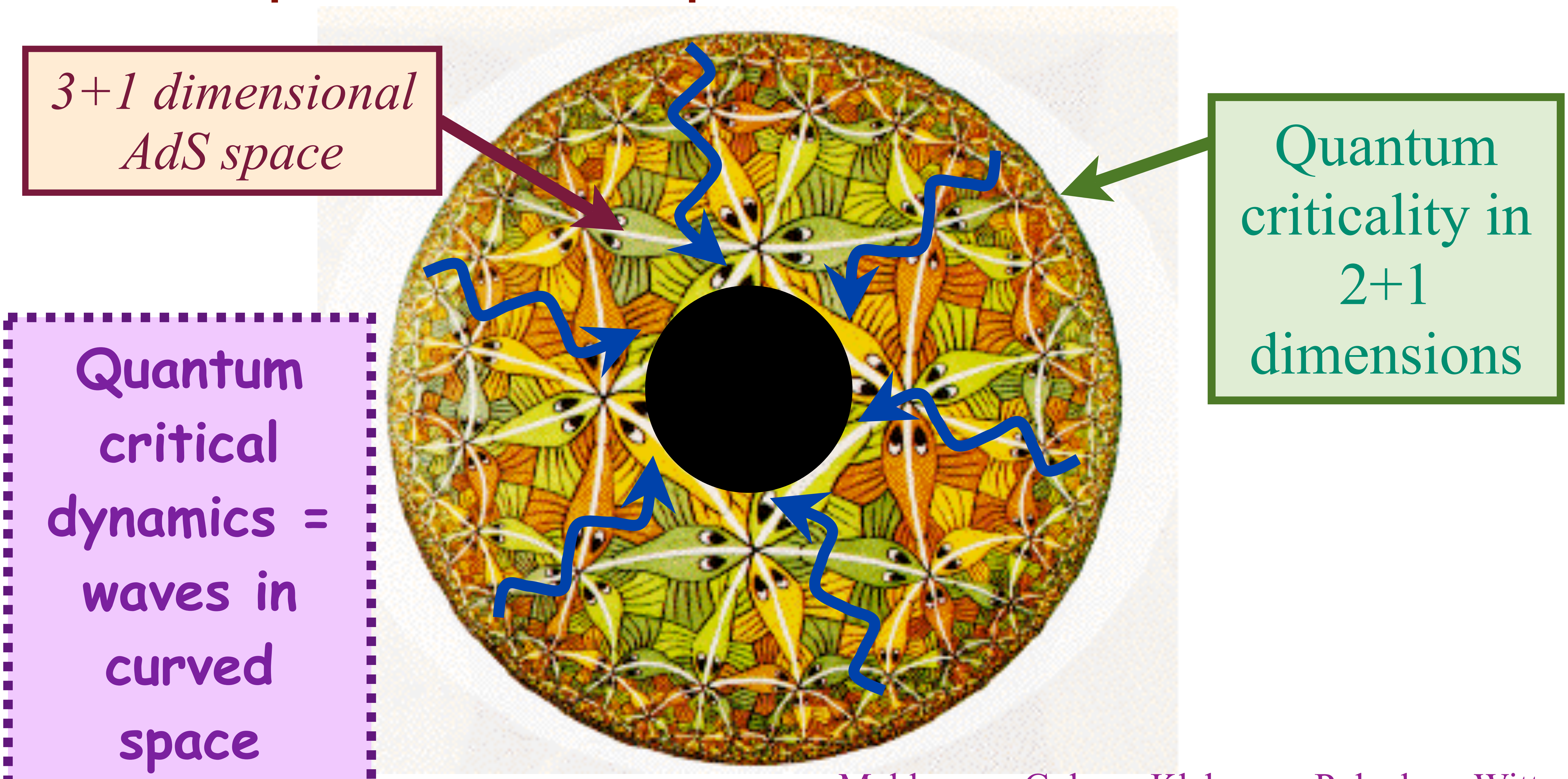

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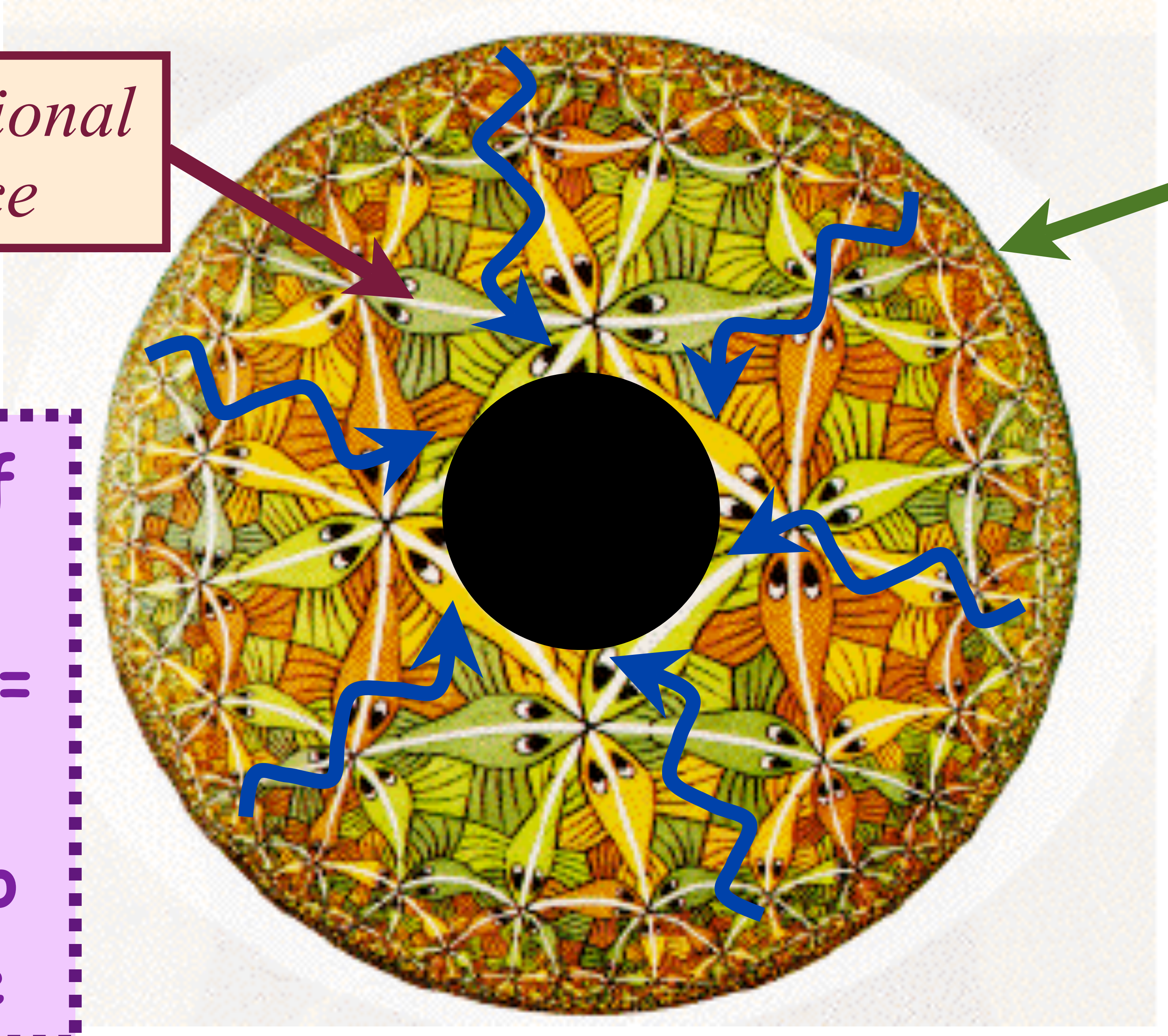

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Friction of quantum criticality = waves falling into black hole

Son

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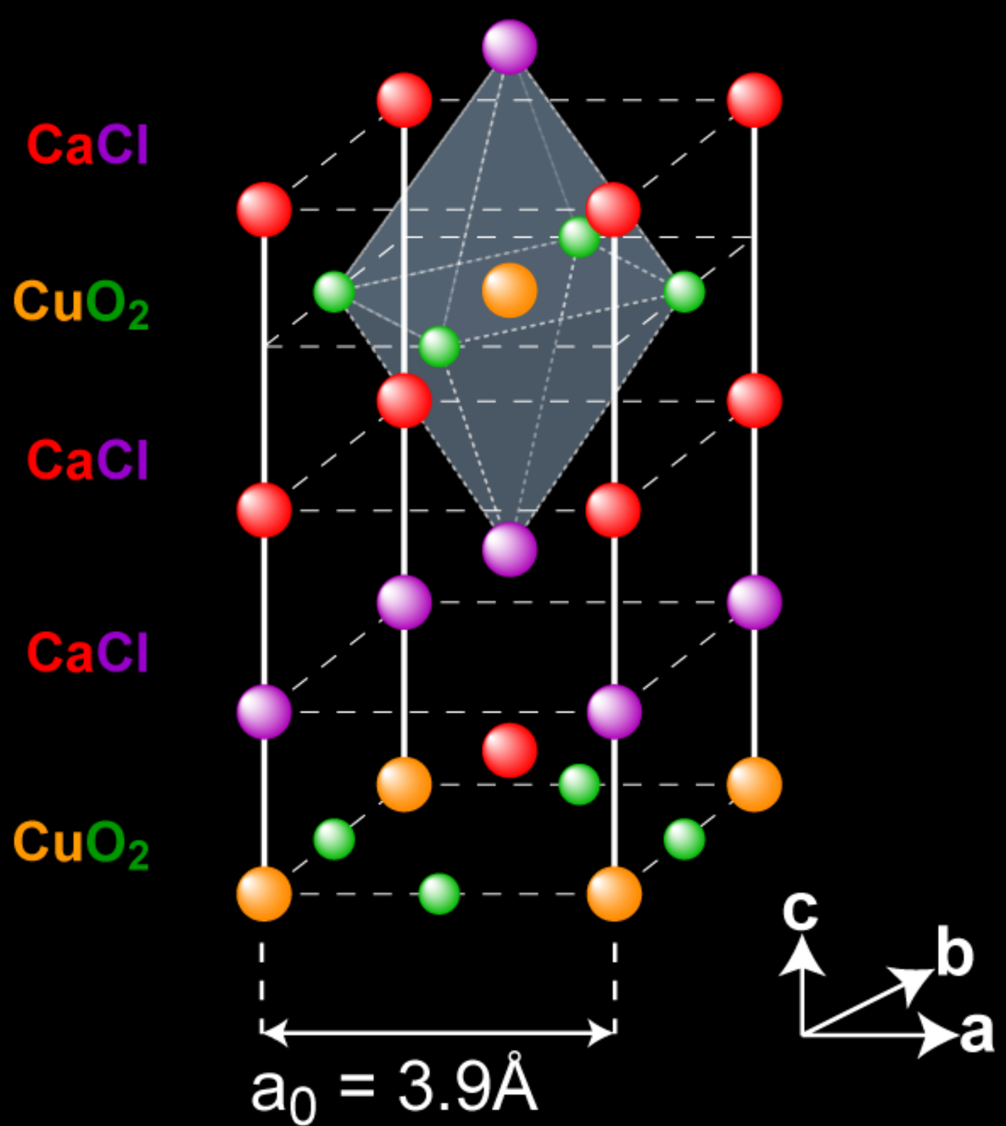
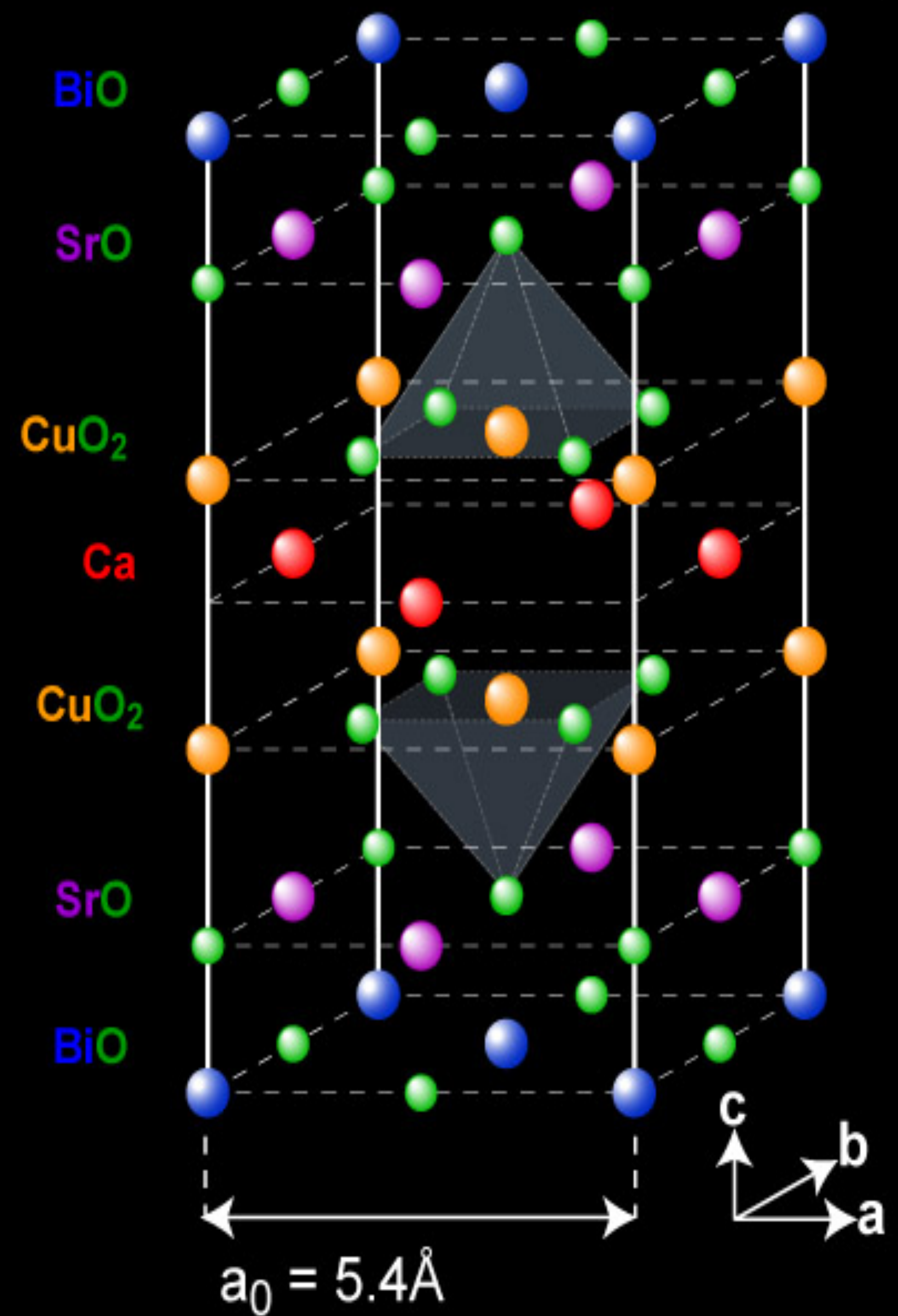
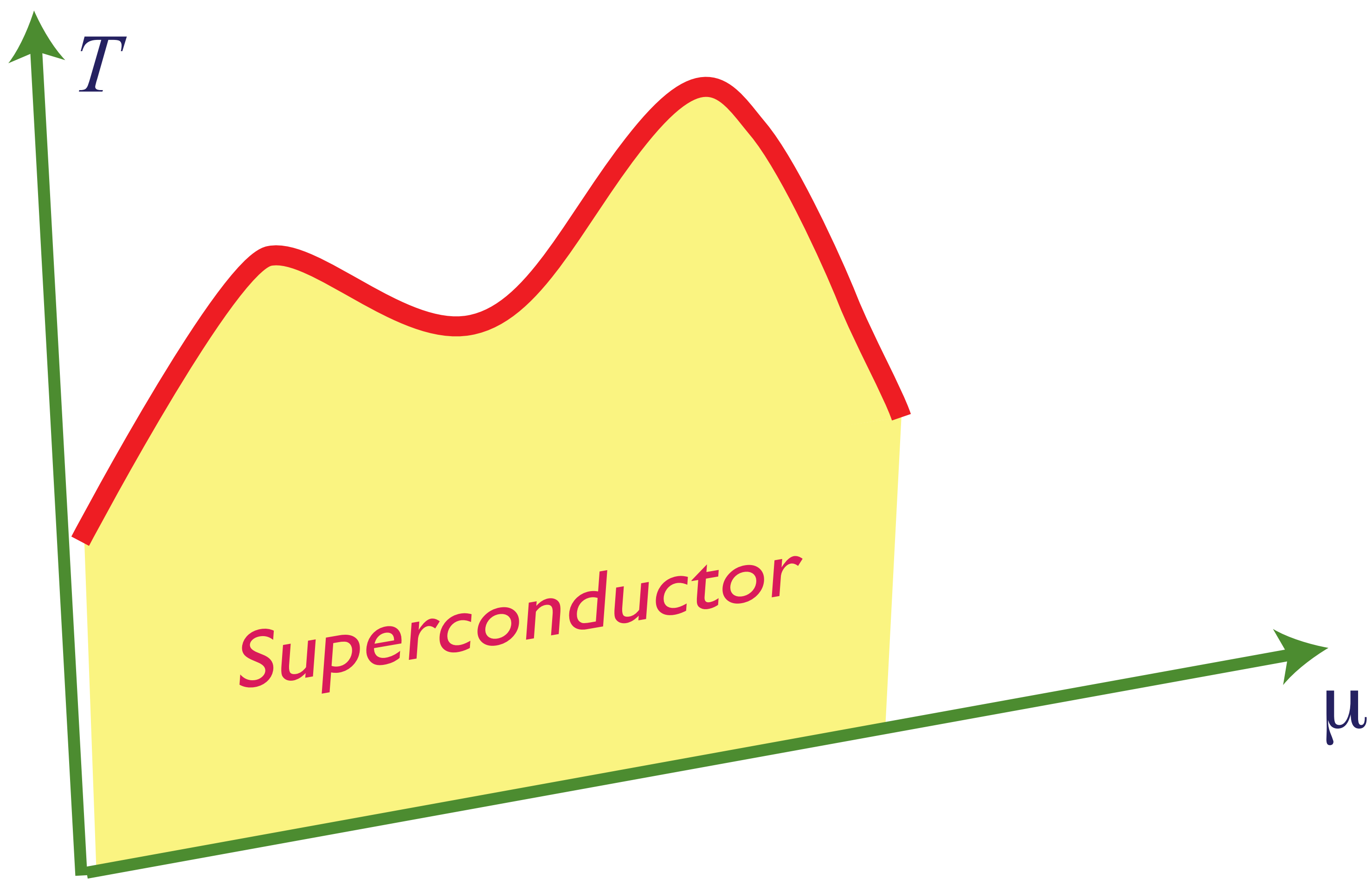
Dope the antiferomagnets with charge carriers of density *x* by applying a chemical potential μ

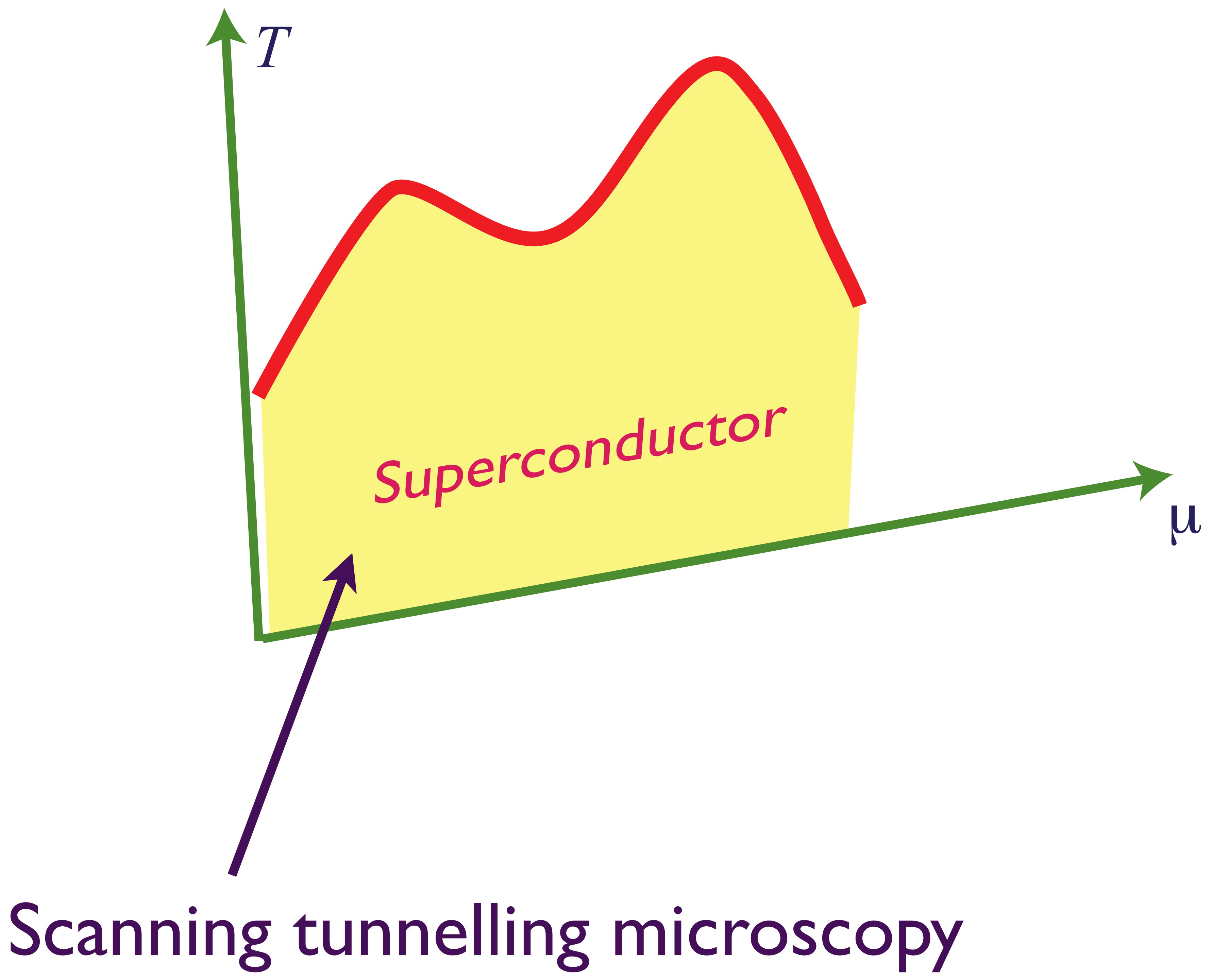
$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

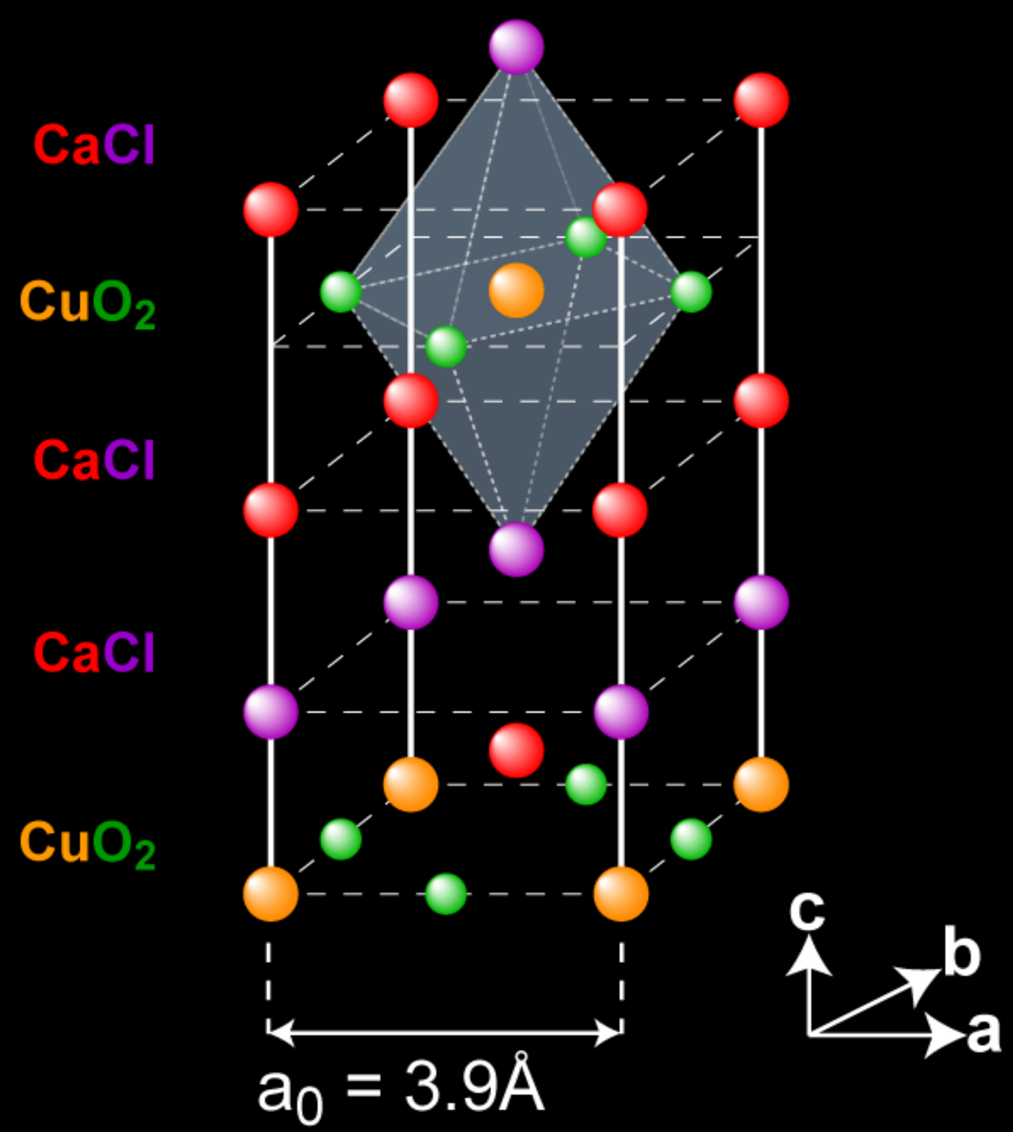



T
Superconductor
μ


T
Superconductor
μ
Scanning tunnelling microscopy

STM studies of the underdoped superconductor

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



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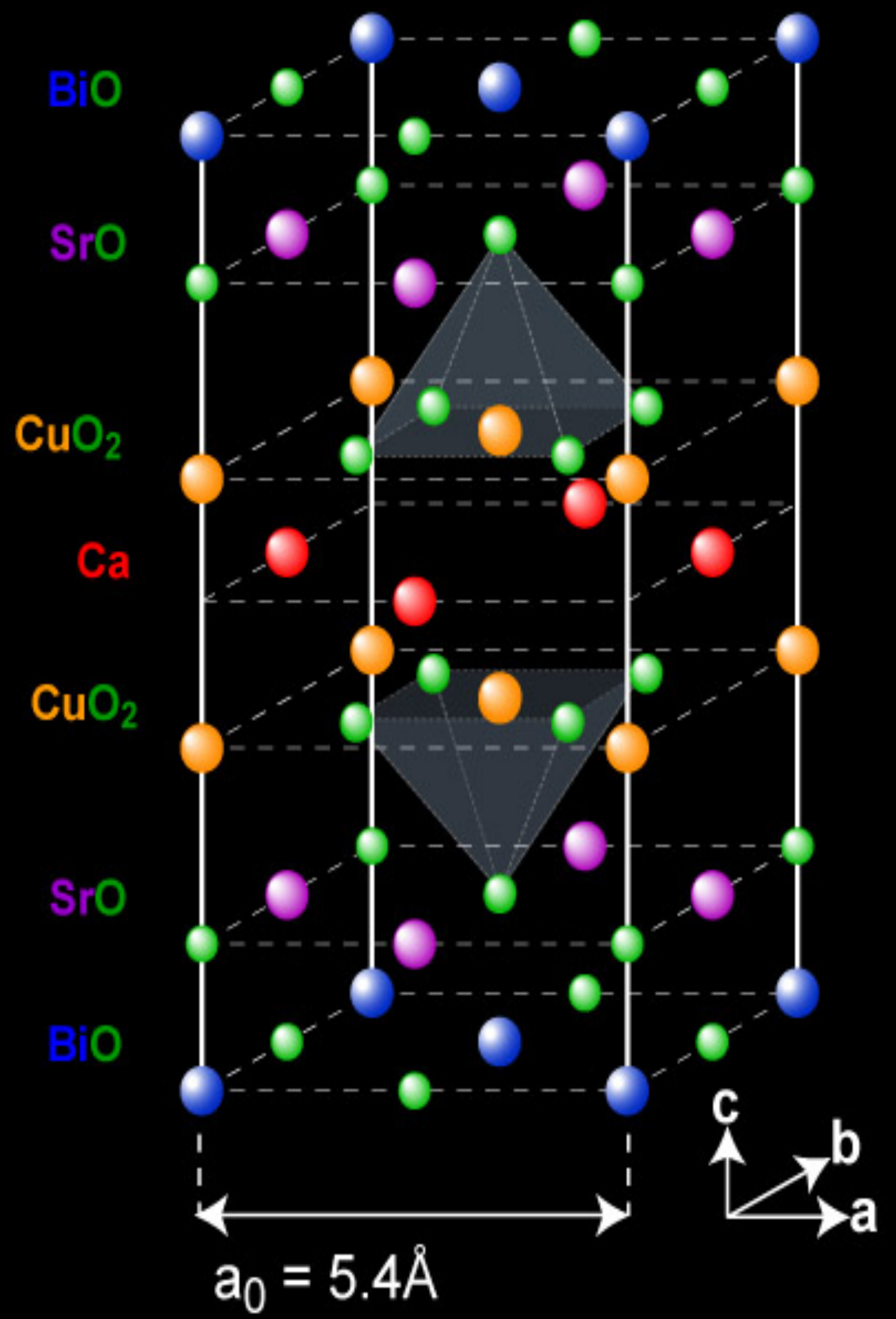
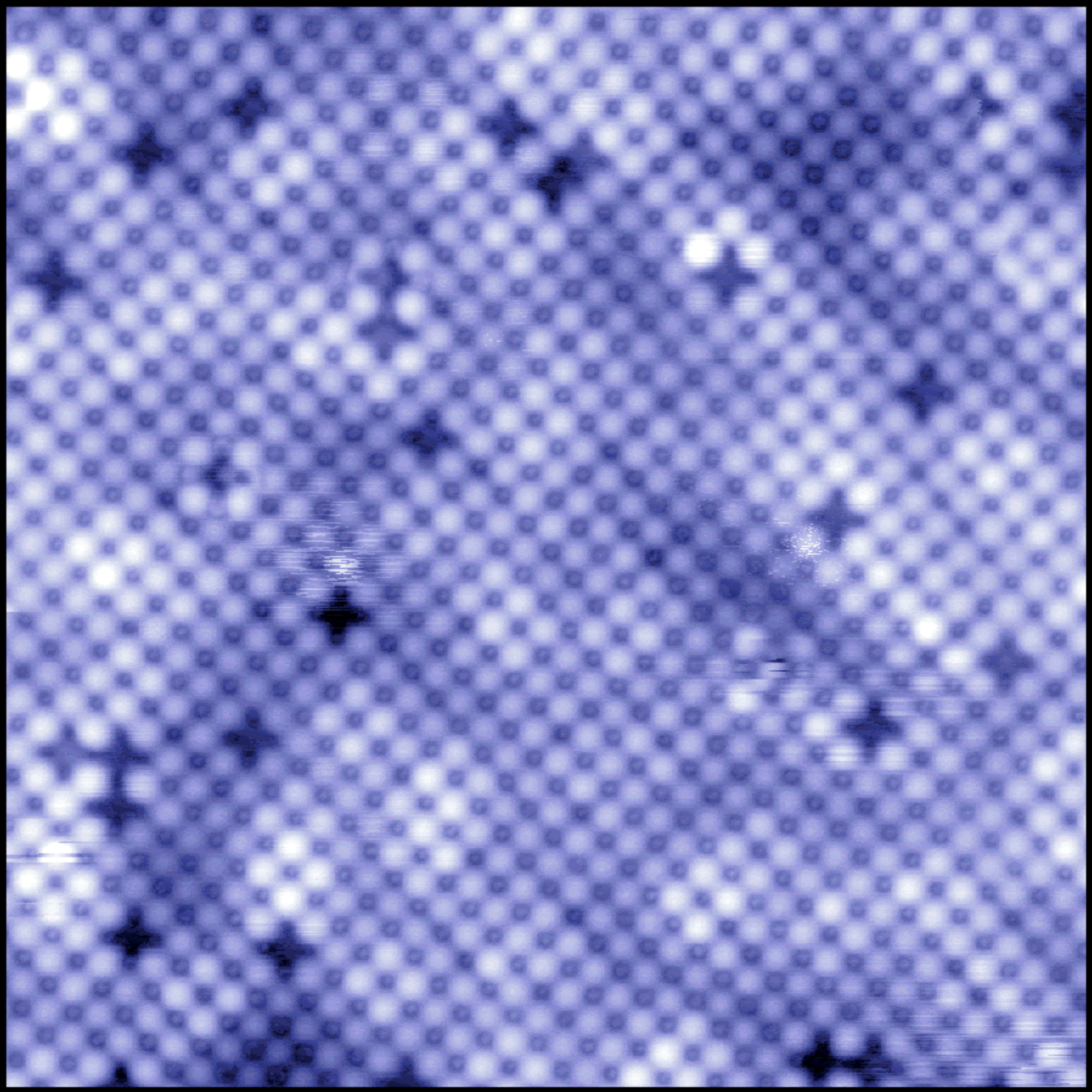
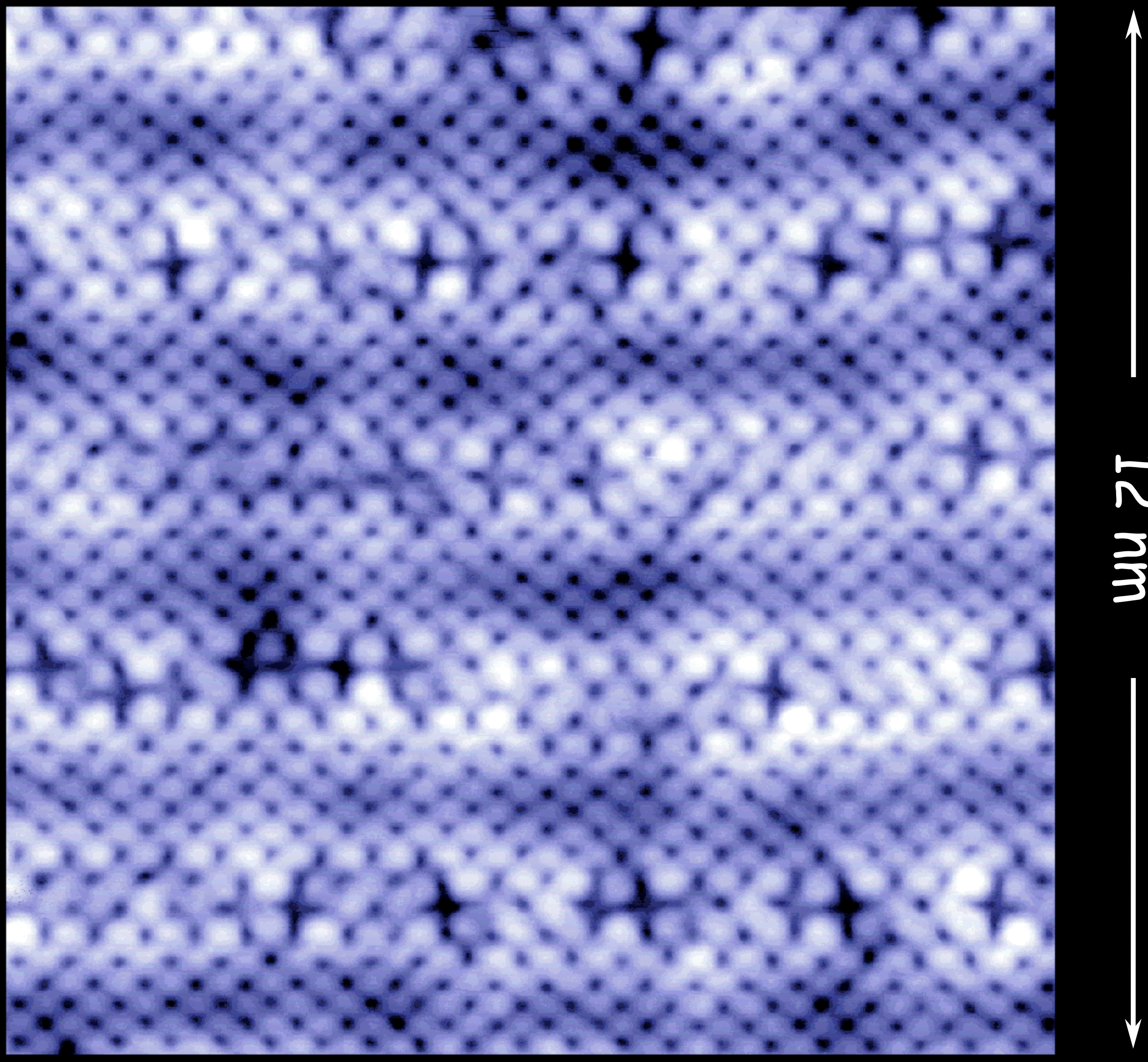


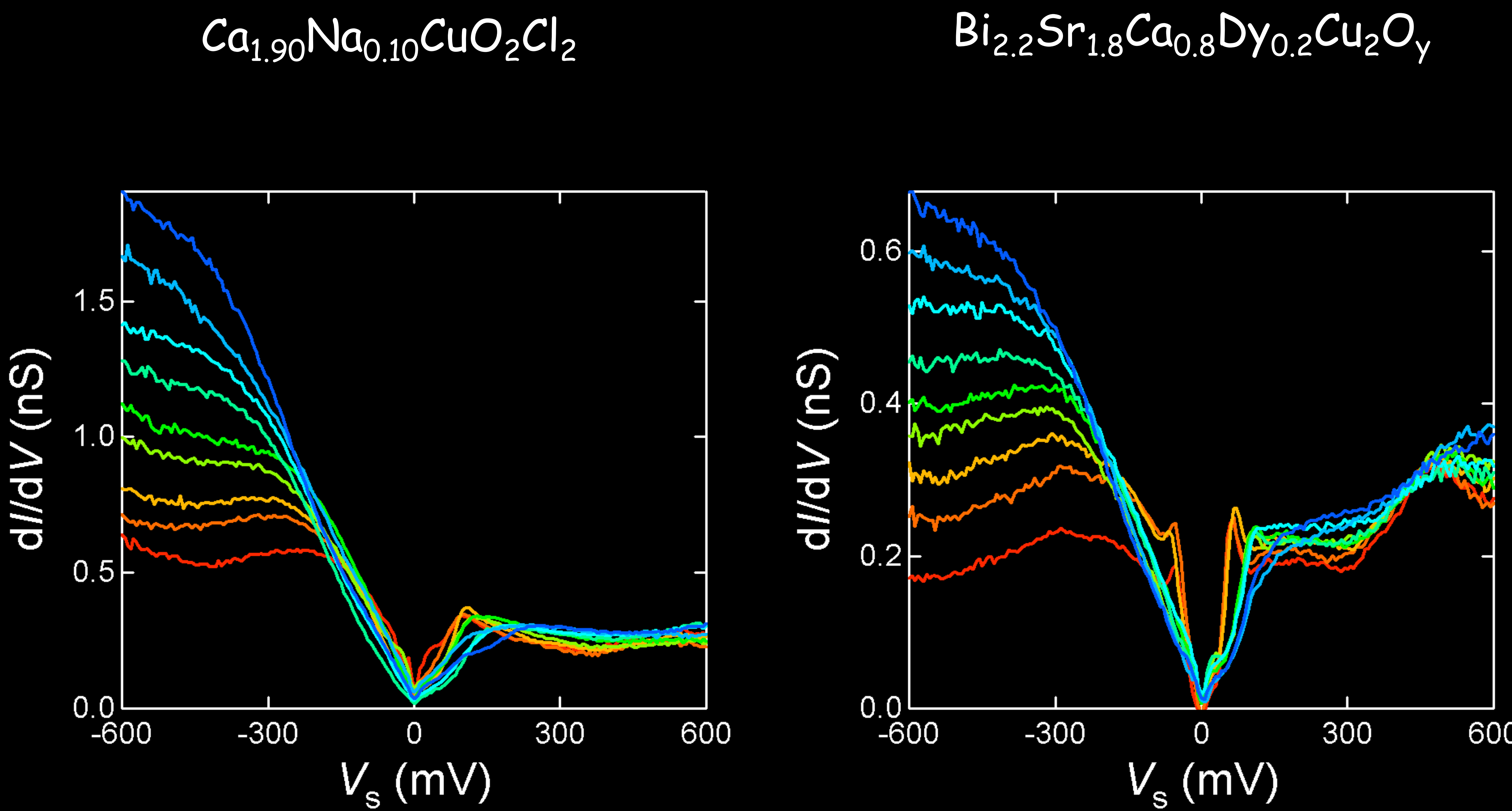
Topograph





Y. Kohsaka et al. Science 315, 1380 (2007)

dI/dV Spectra

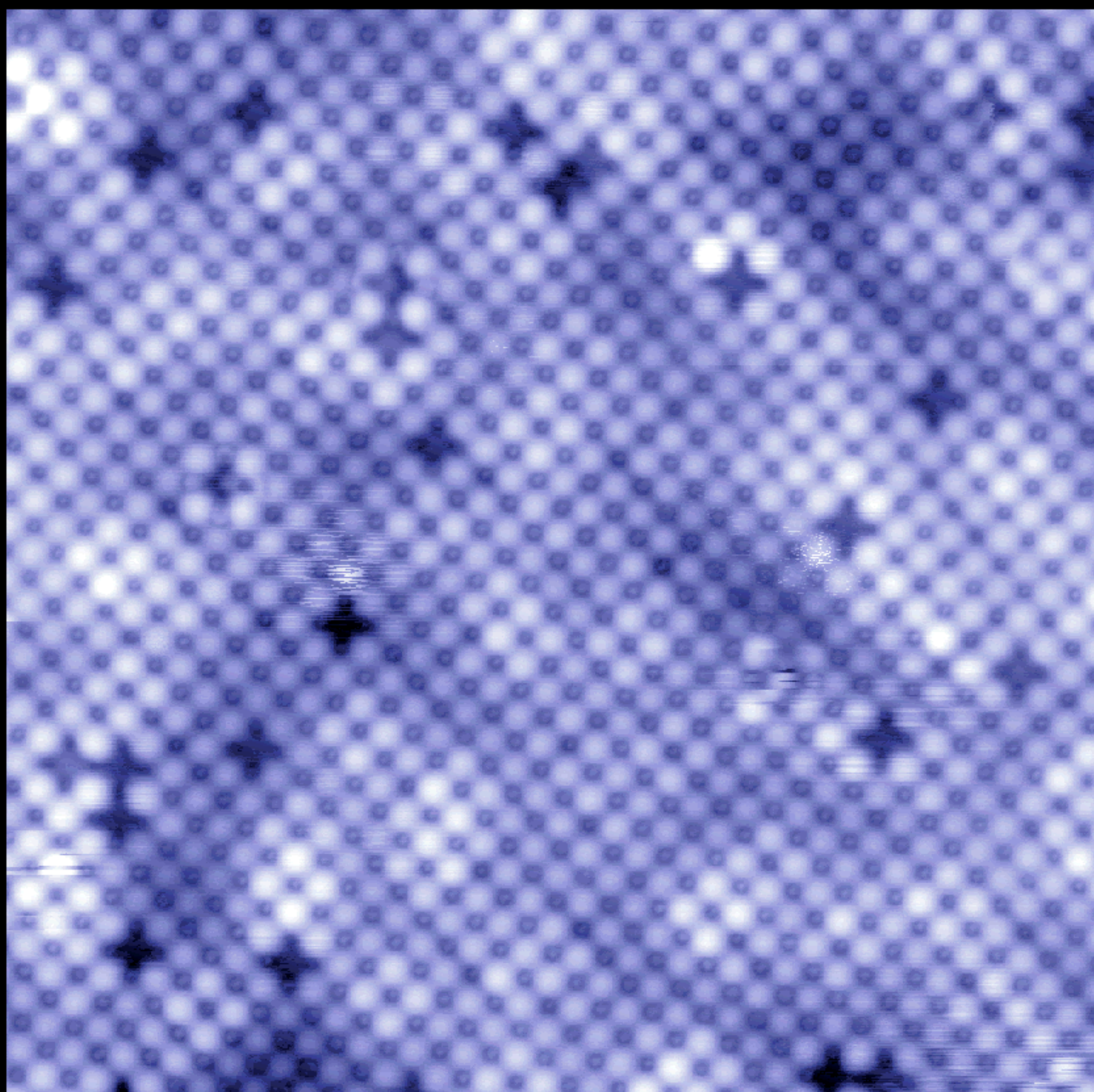


Intense Tunneling-Asymmetry (TA)
variation are highly similar

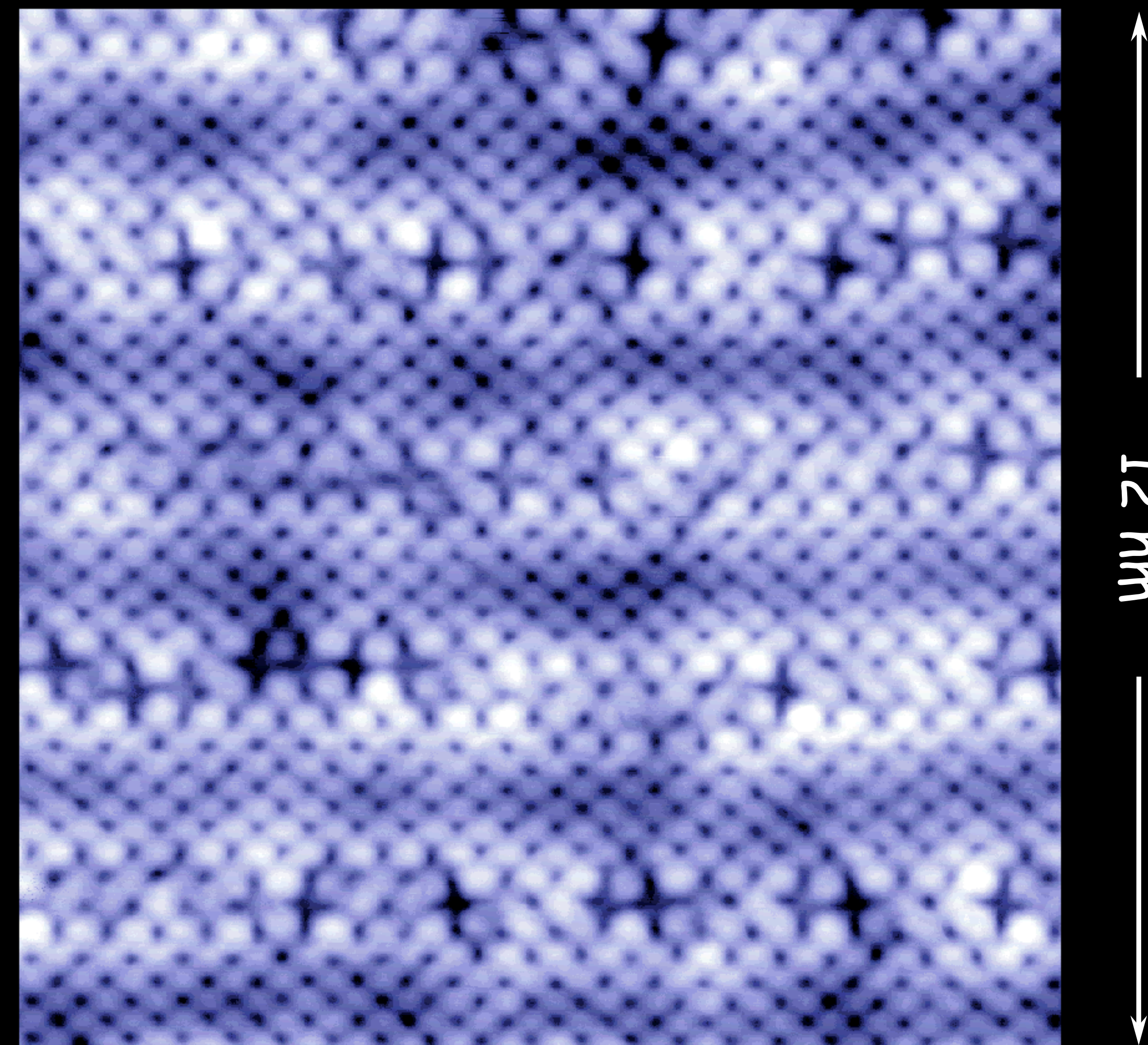
Y. Kohsaka et al. *Science* **315**, 1380 (2007)

Topograph

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



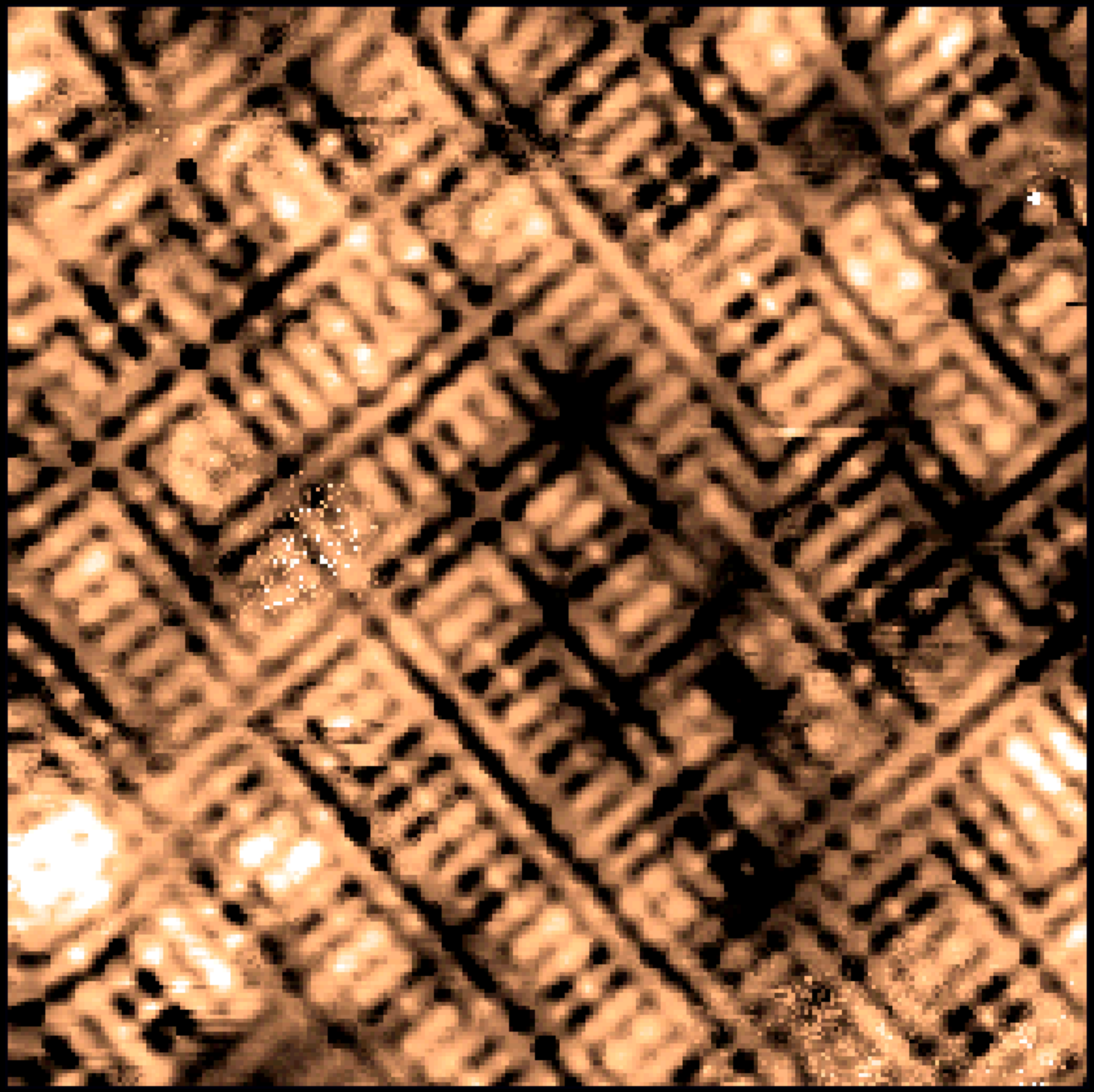
$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Y. Kohsaka et al. Science 315, 1380 (2007)

Tunneling Asymmetry (TA)-map at E=150meV

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



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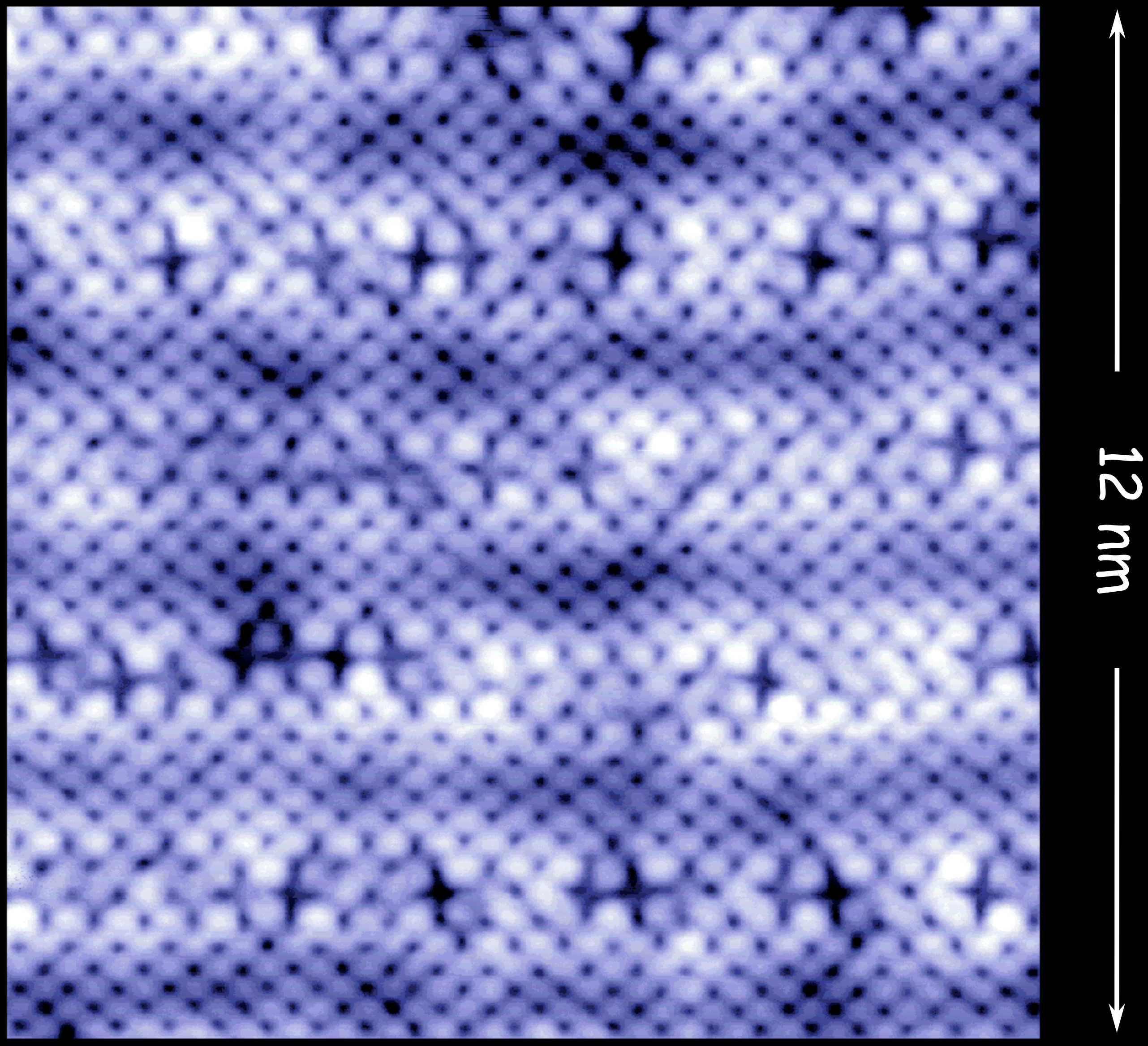

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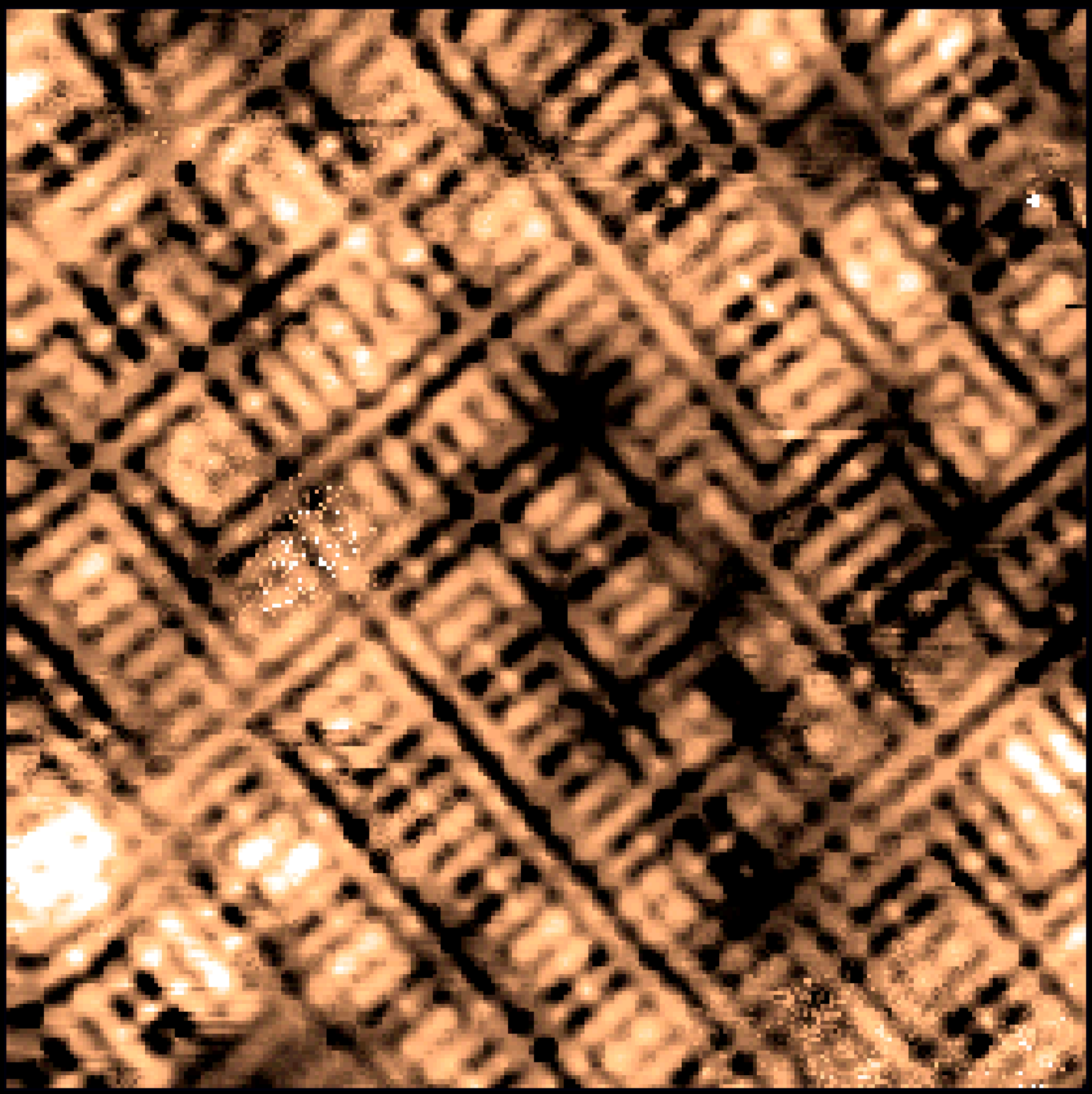

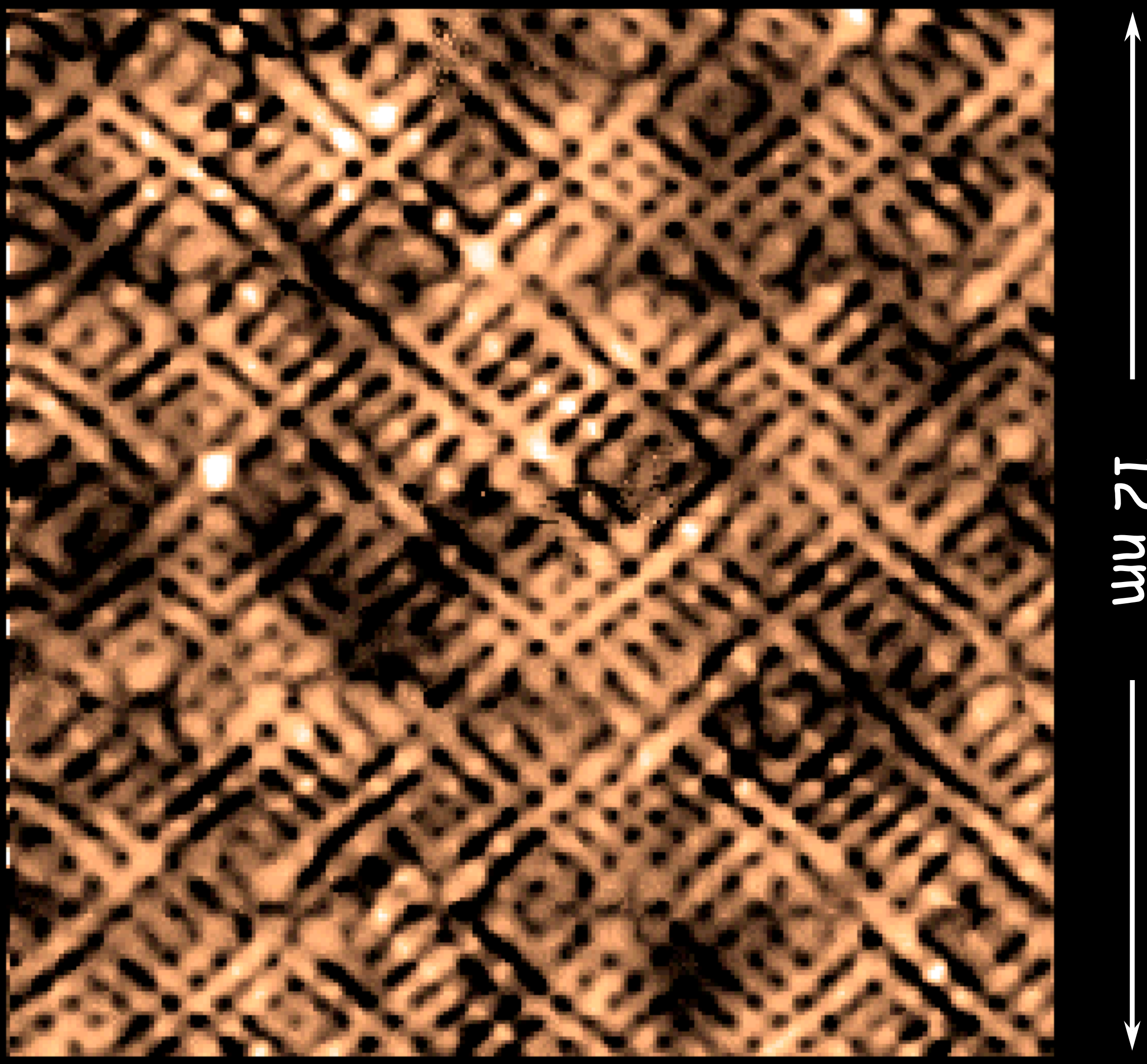

Y. Kohsaka et al. Science 315, 1380 (2007)

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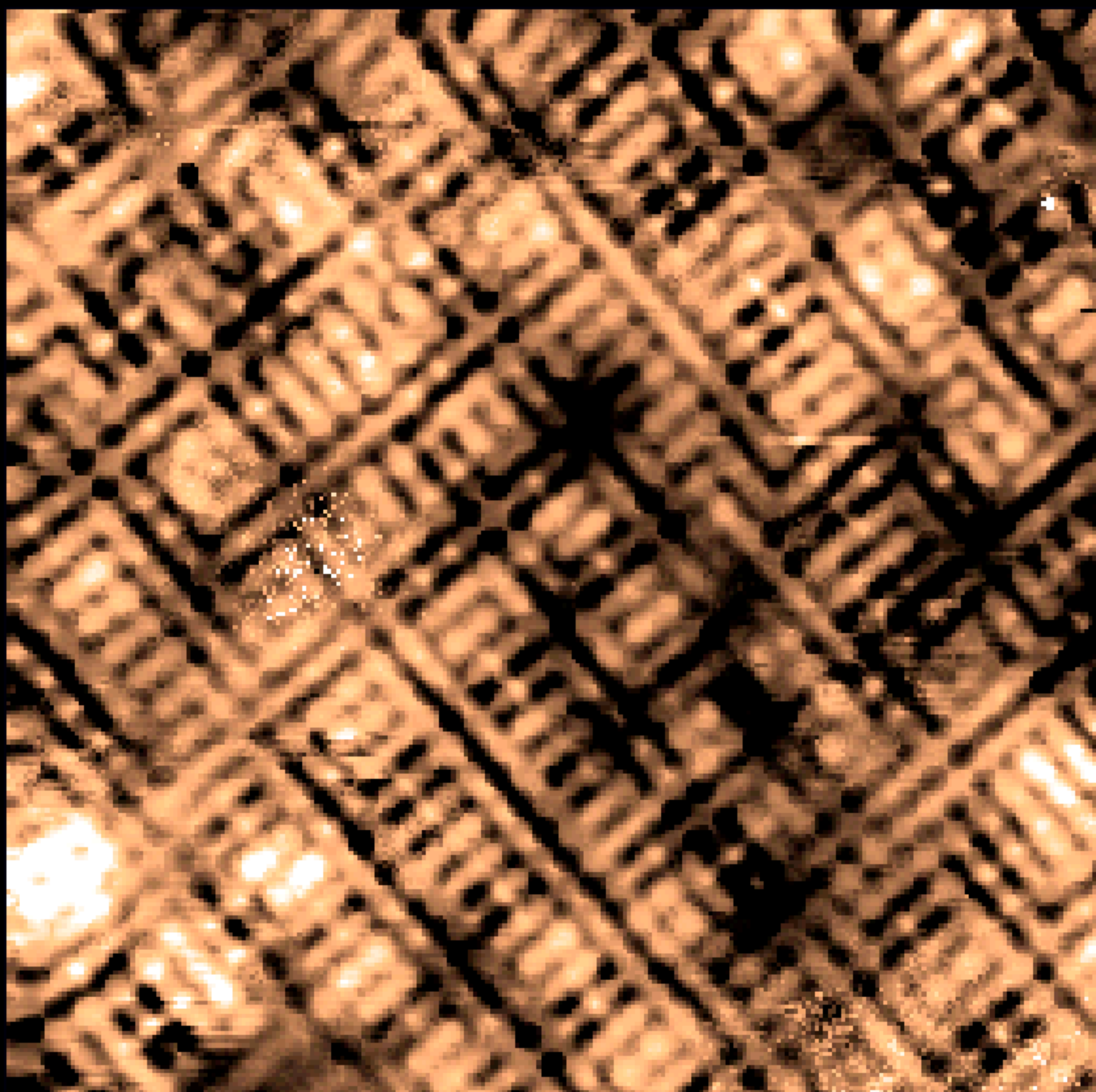

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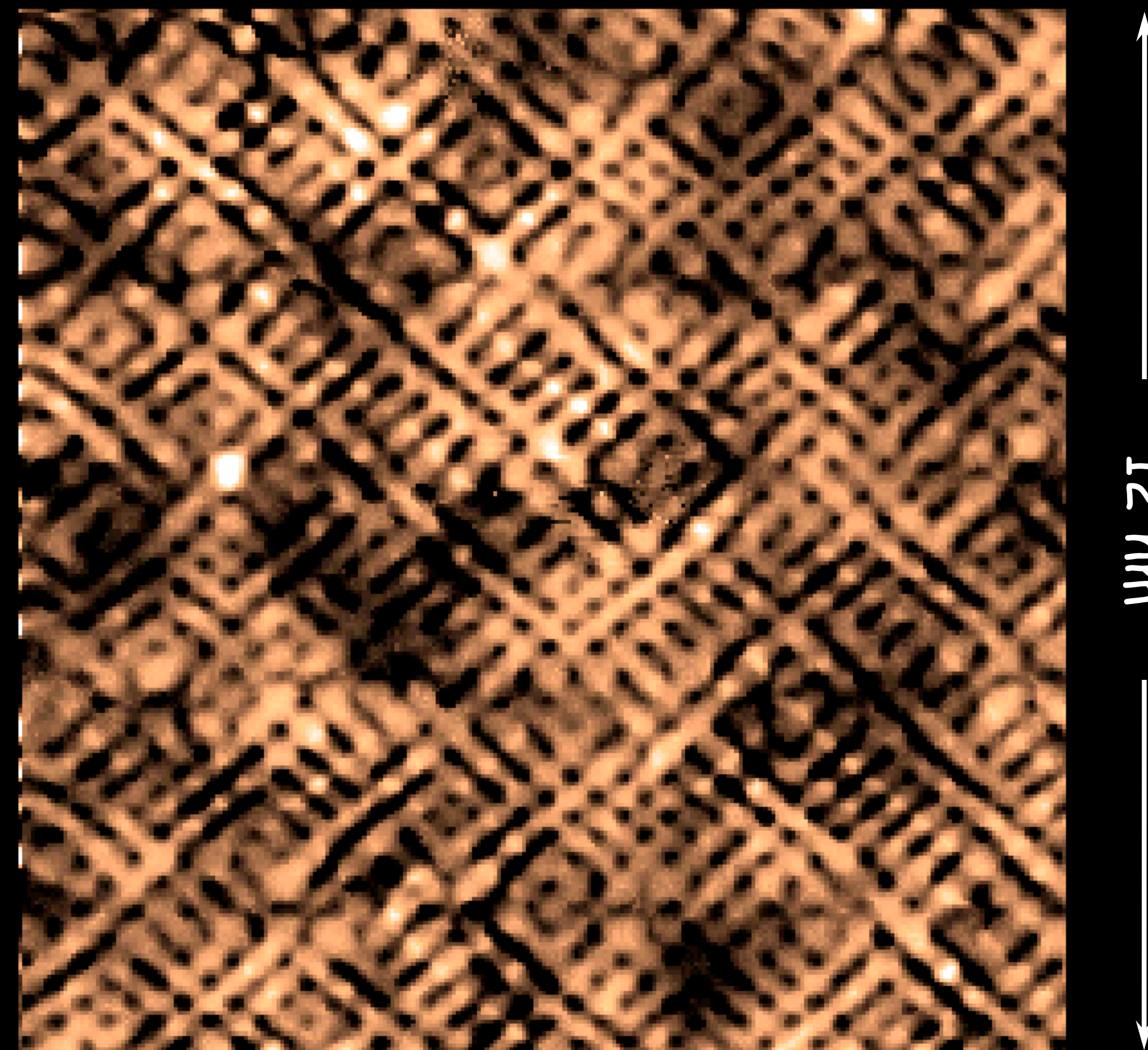
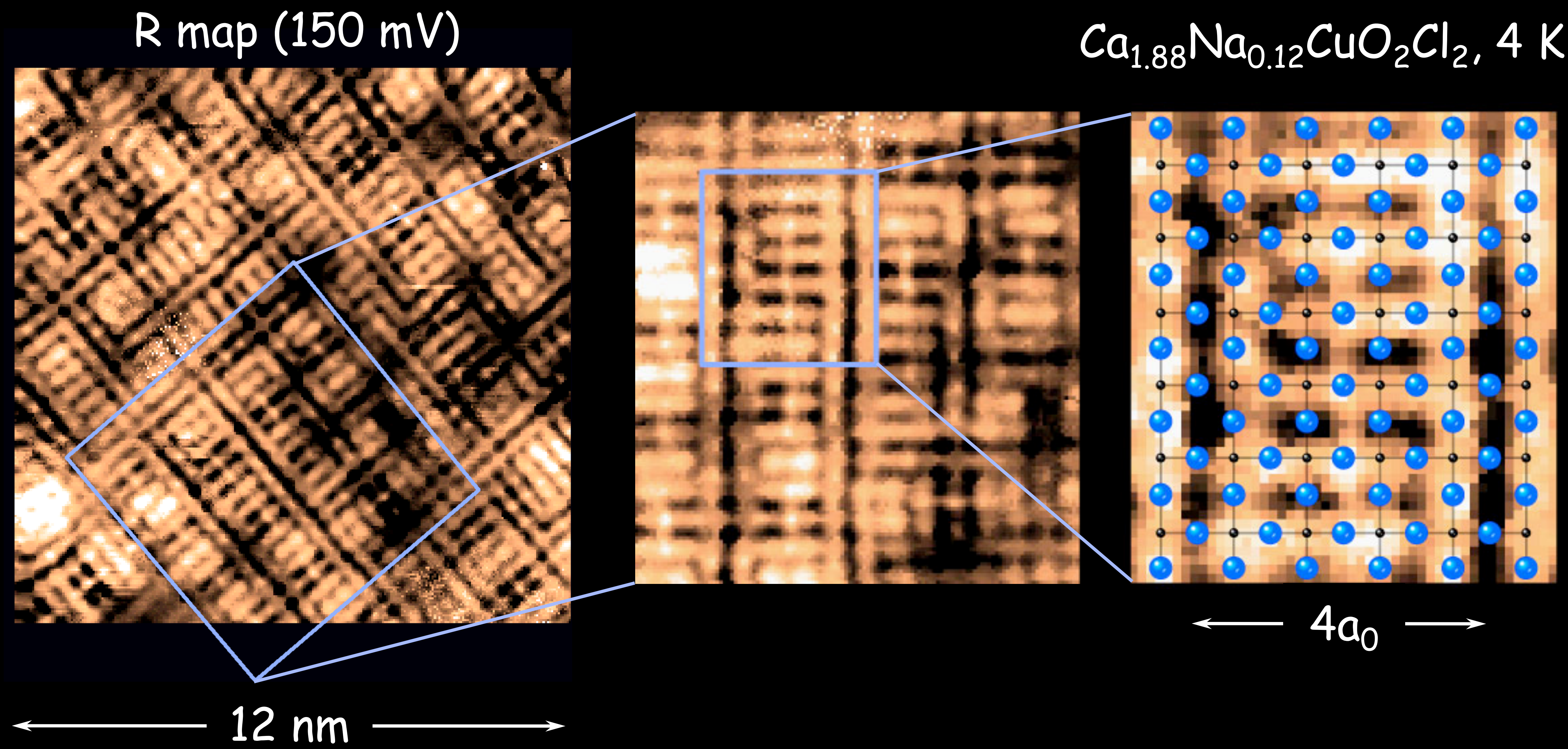


Indistinguishable bond-centered TA contrast

with disperse $4a_0$-wide nanodomains

Y. Kohsaka et al. Science 315, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



Y. Kohsaka et al. Science 315, 1380 (2007)

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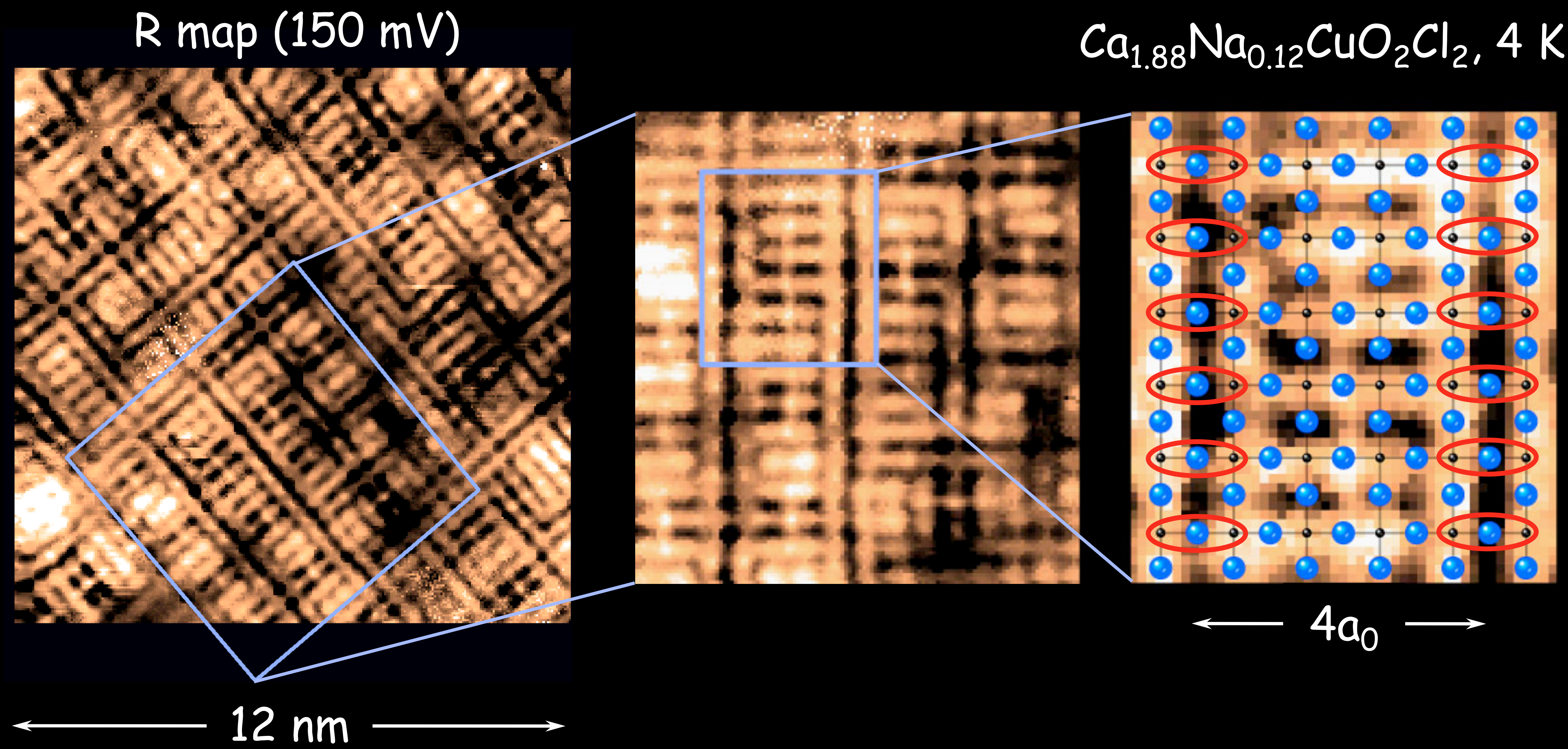
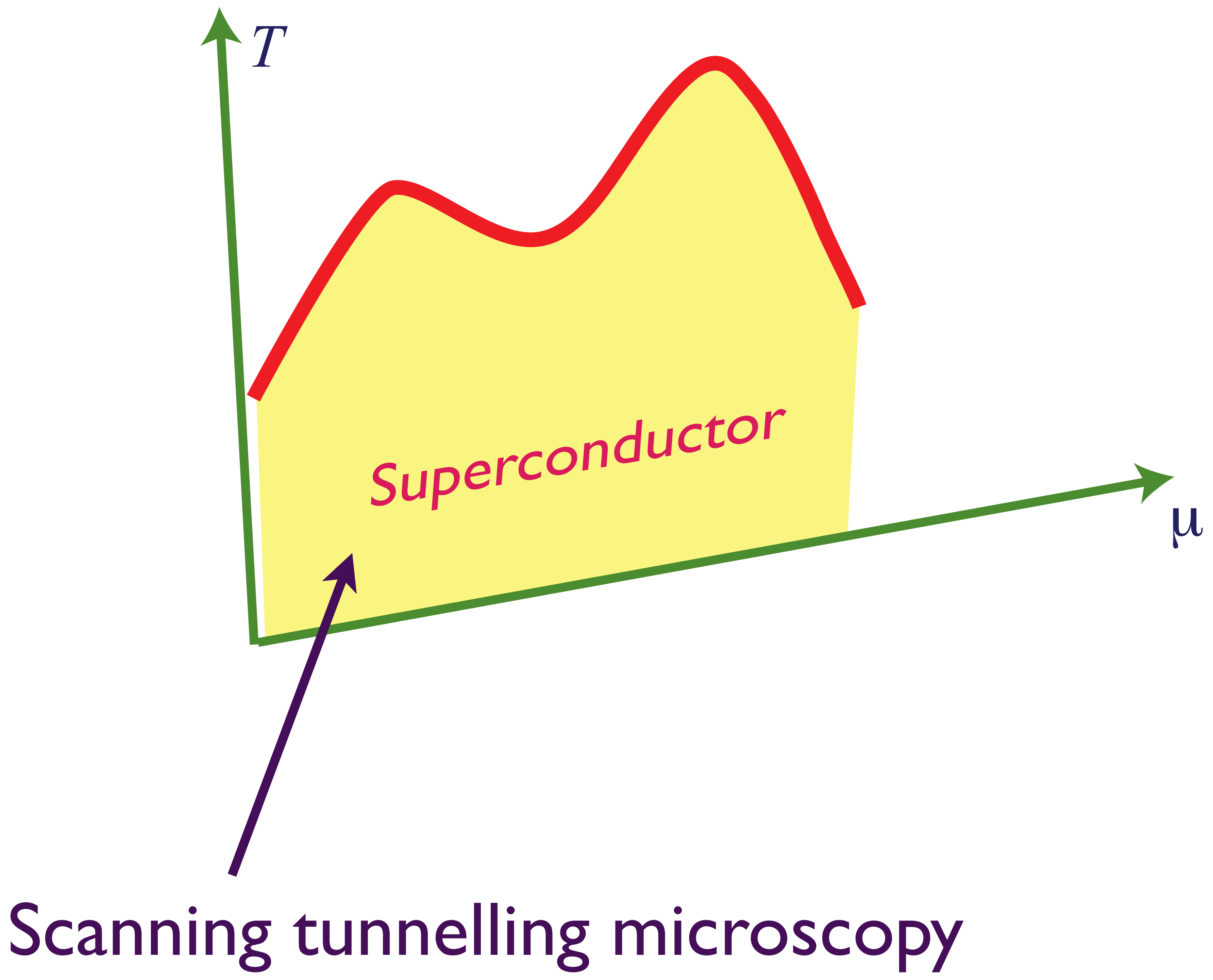
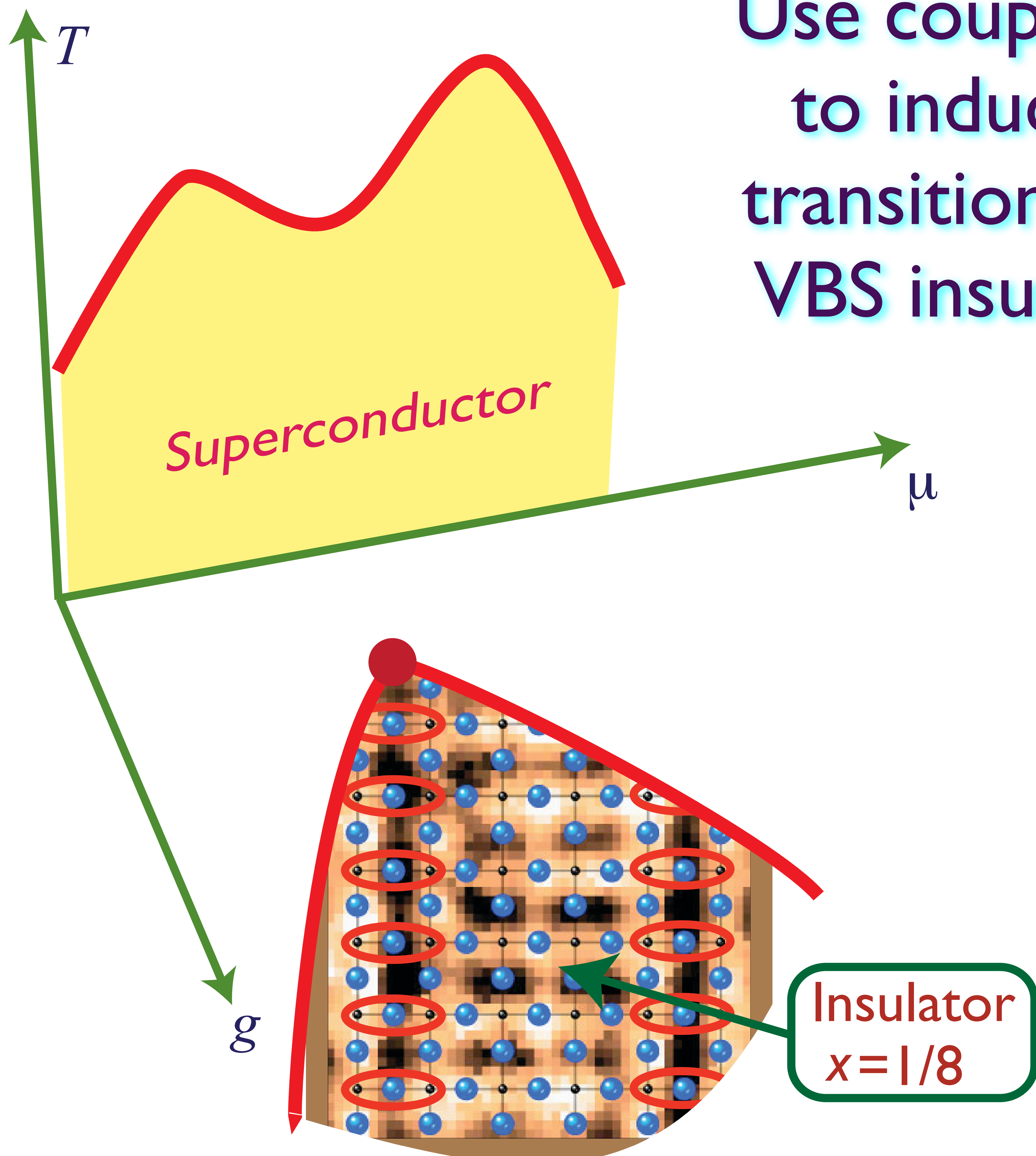


Evidence for a predicted valence bond supersolid

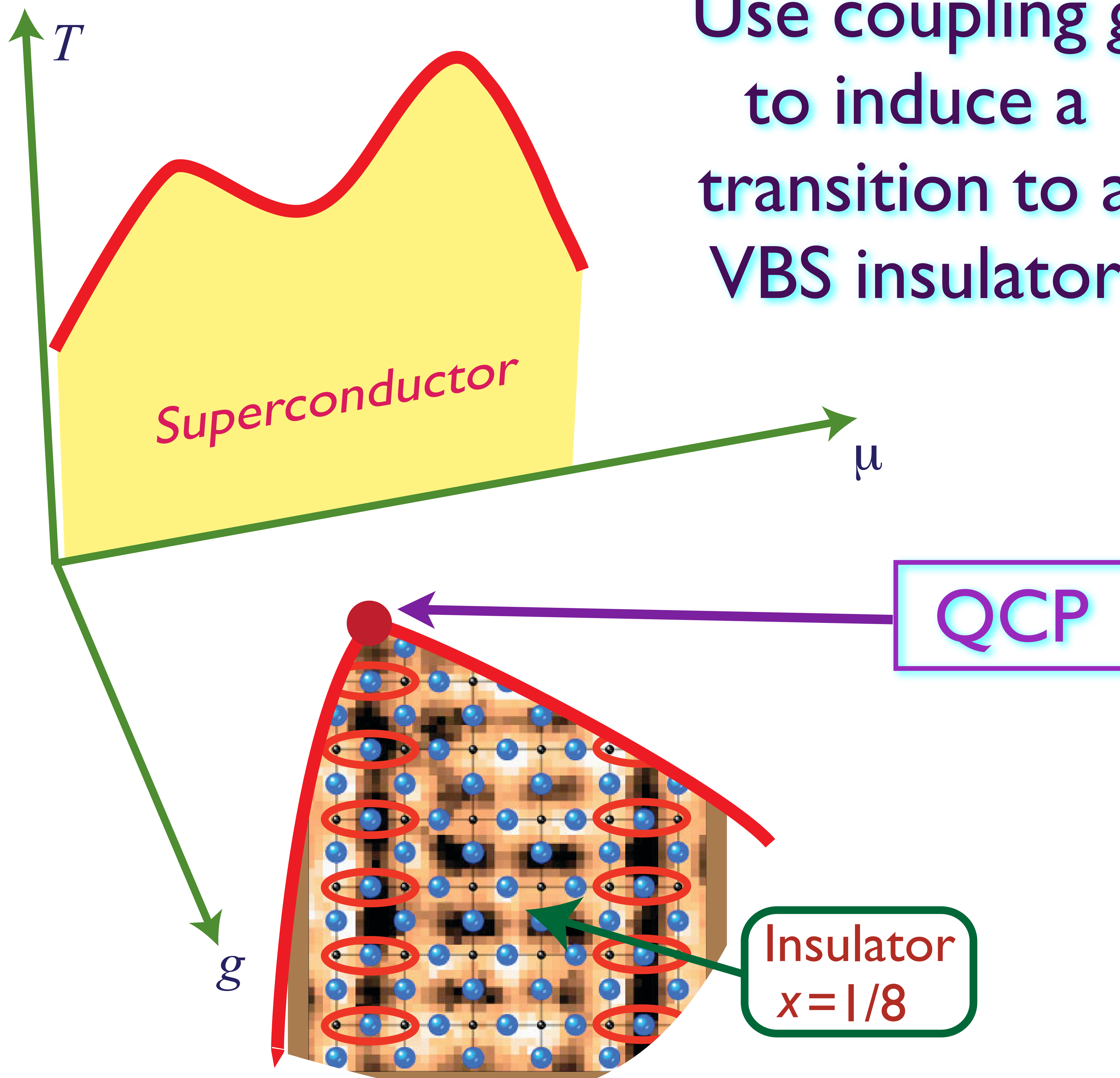
S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999).


T
Superconductor
μ
Scanning tunnelling microscopy

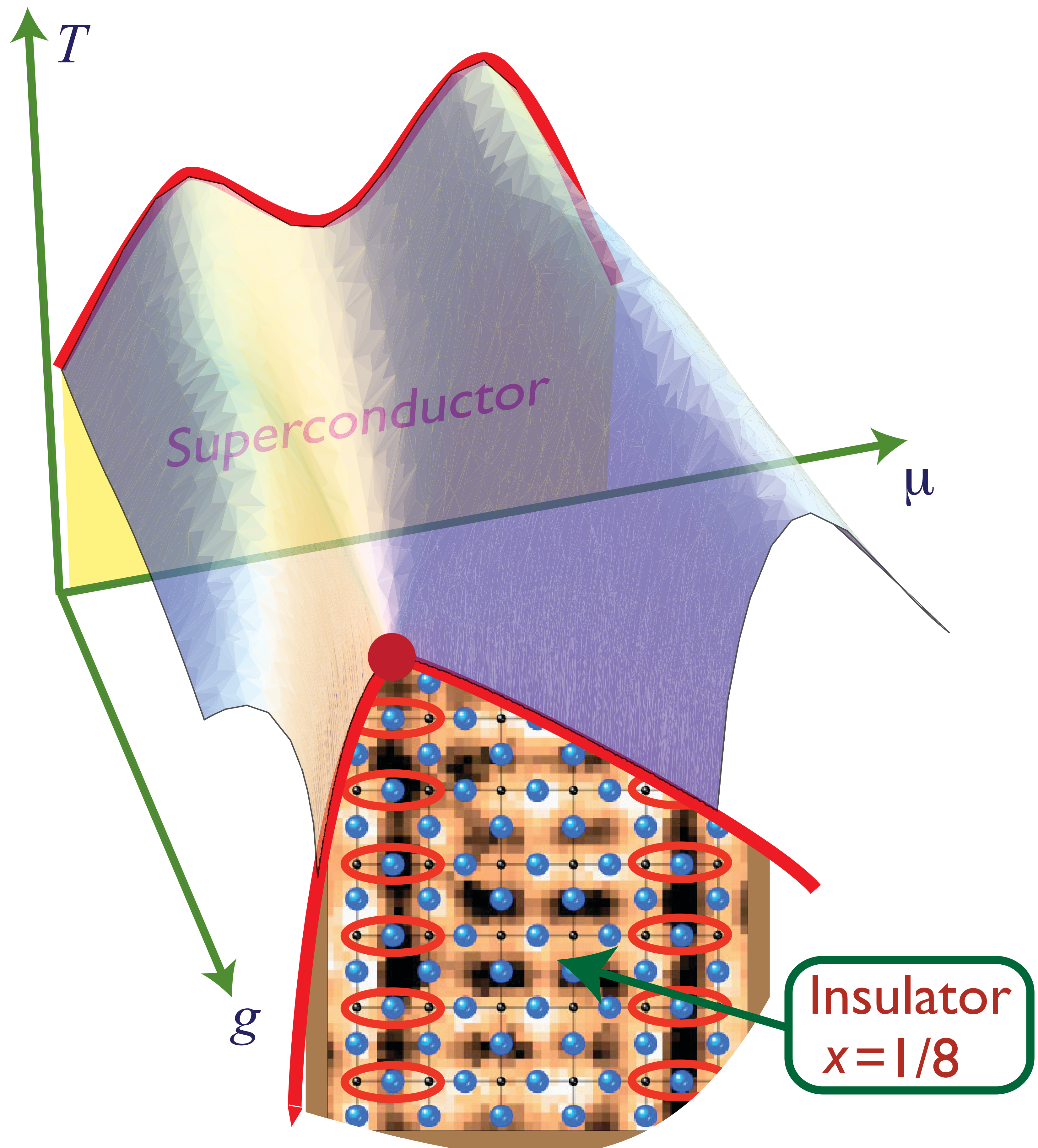
Use coupling g to induce a transition to a VBS insulator

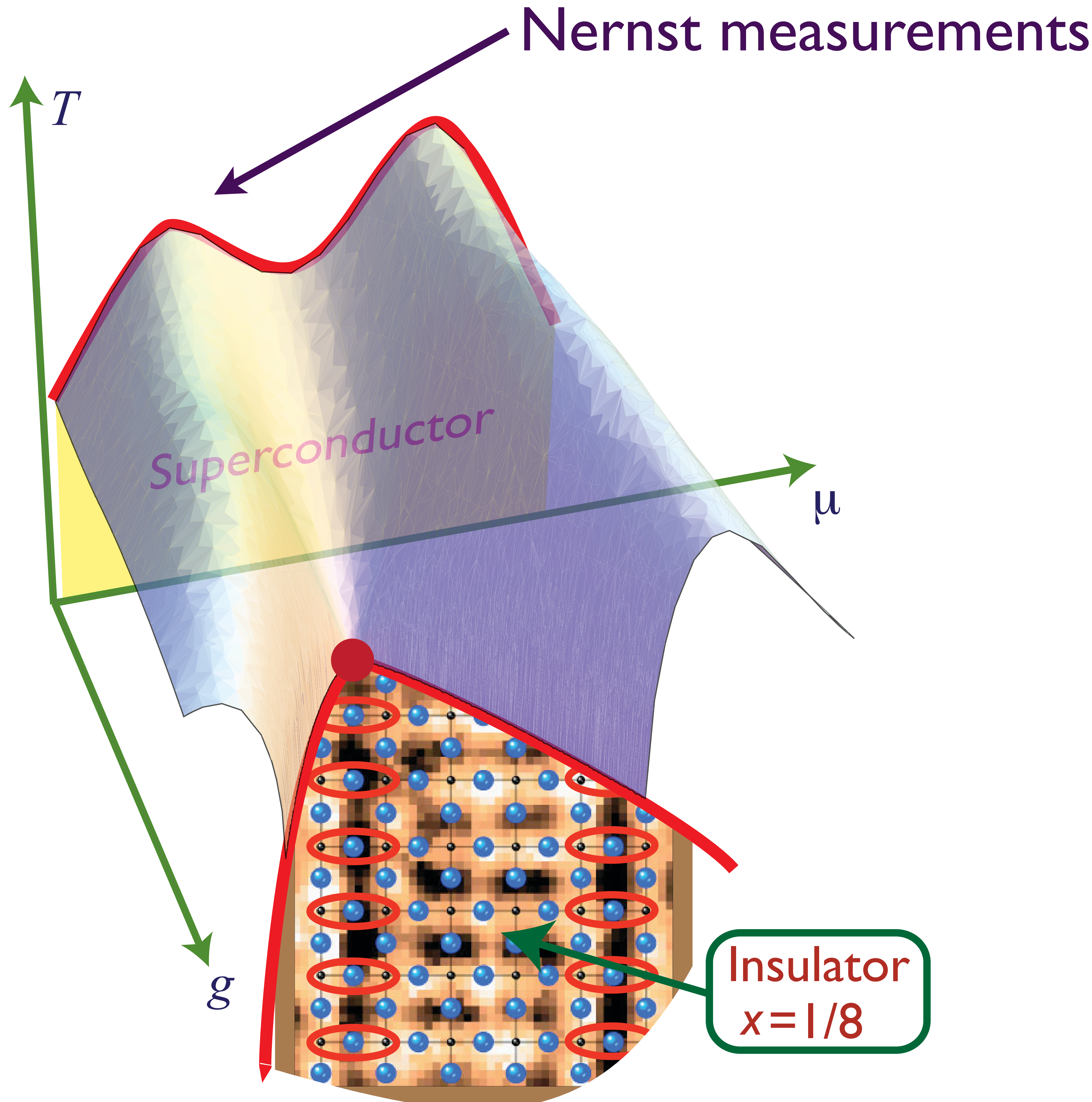


Use coupling g to induce a transition to a VBS insulator



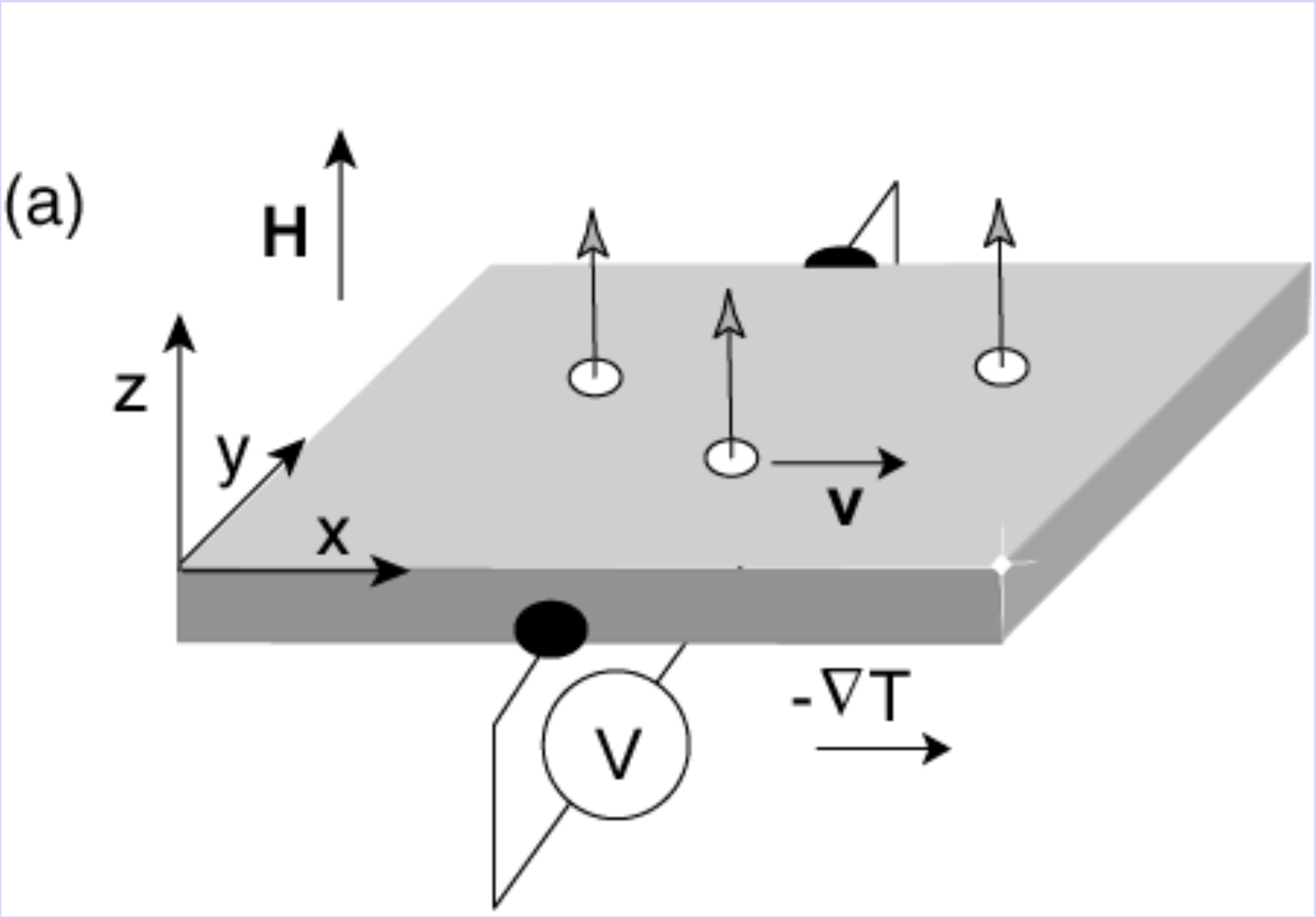
Proposed generalized phase diagram

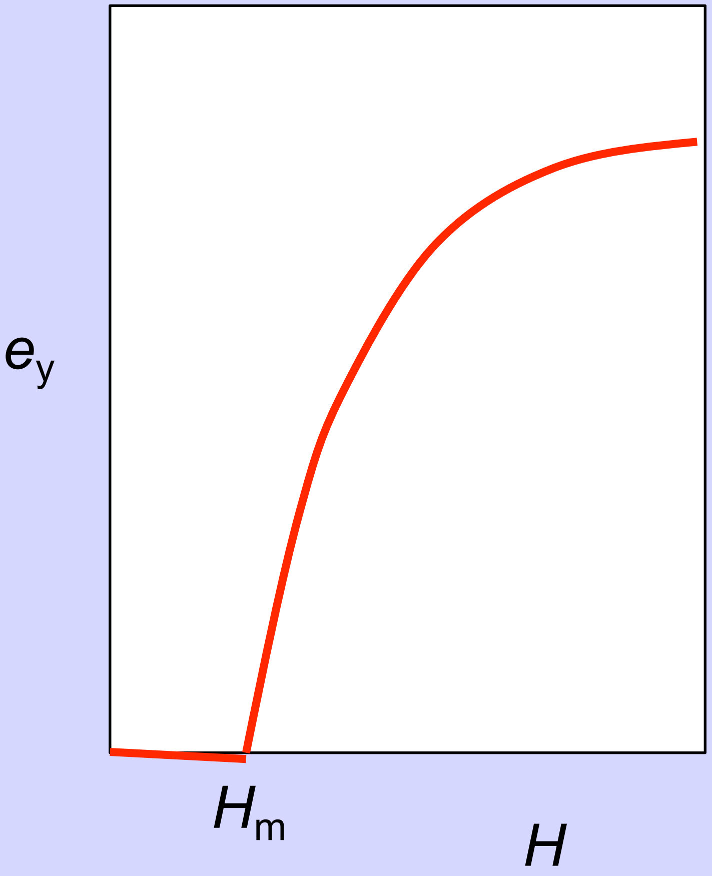


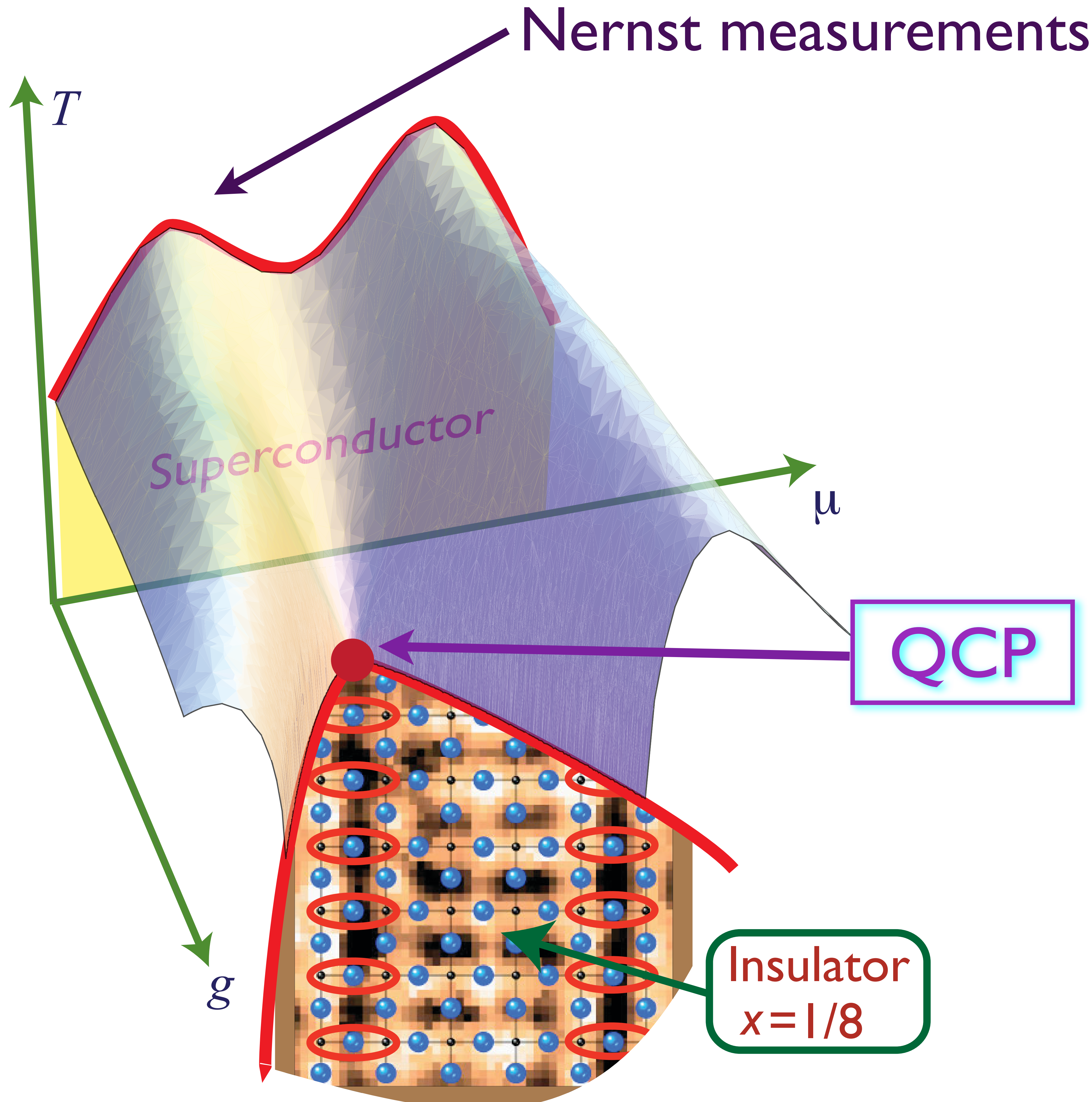


Nernst measurements
T
Superconductor
μ
g
Insulator
x=1/8

Nernst experiment

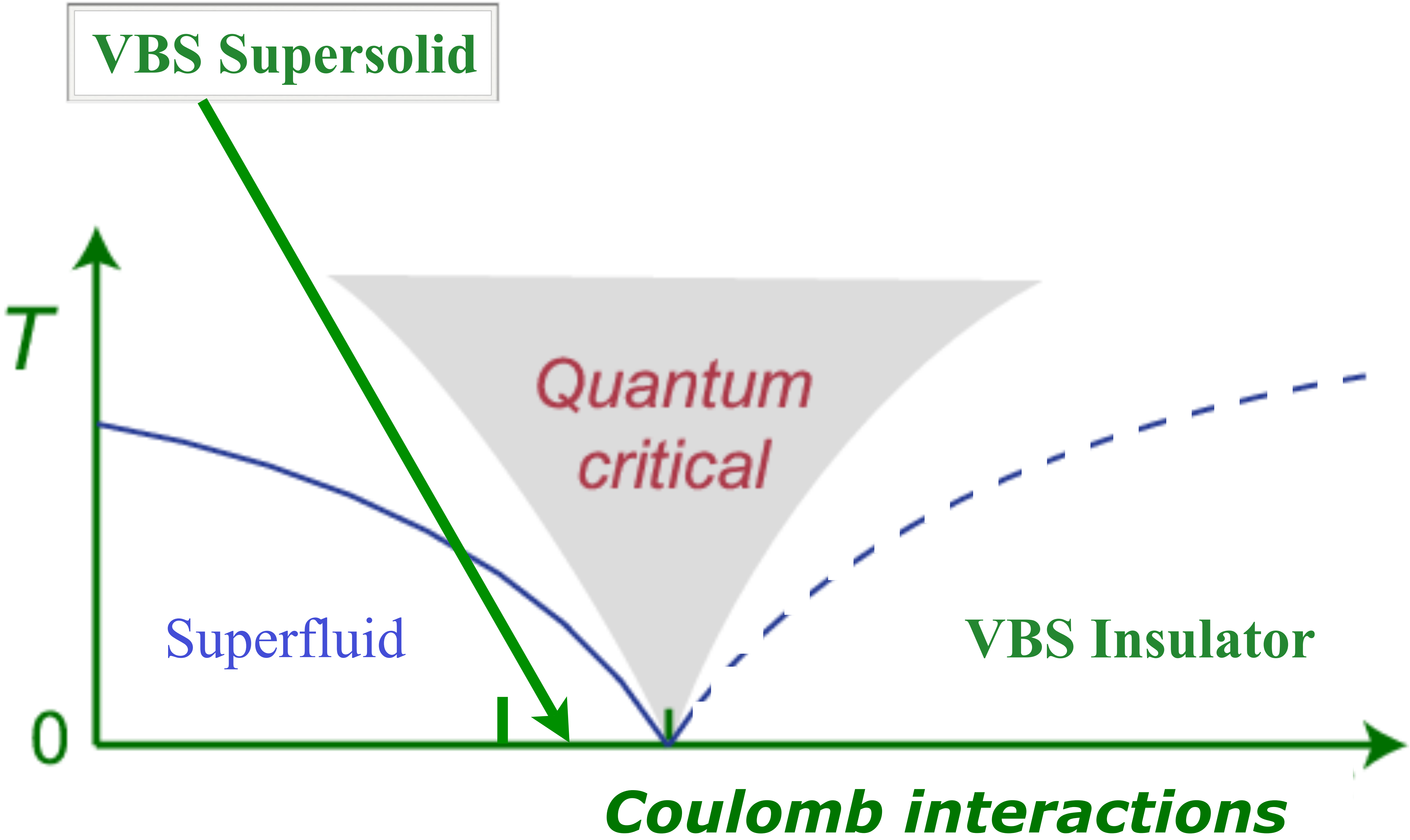




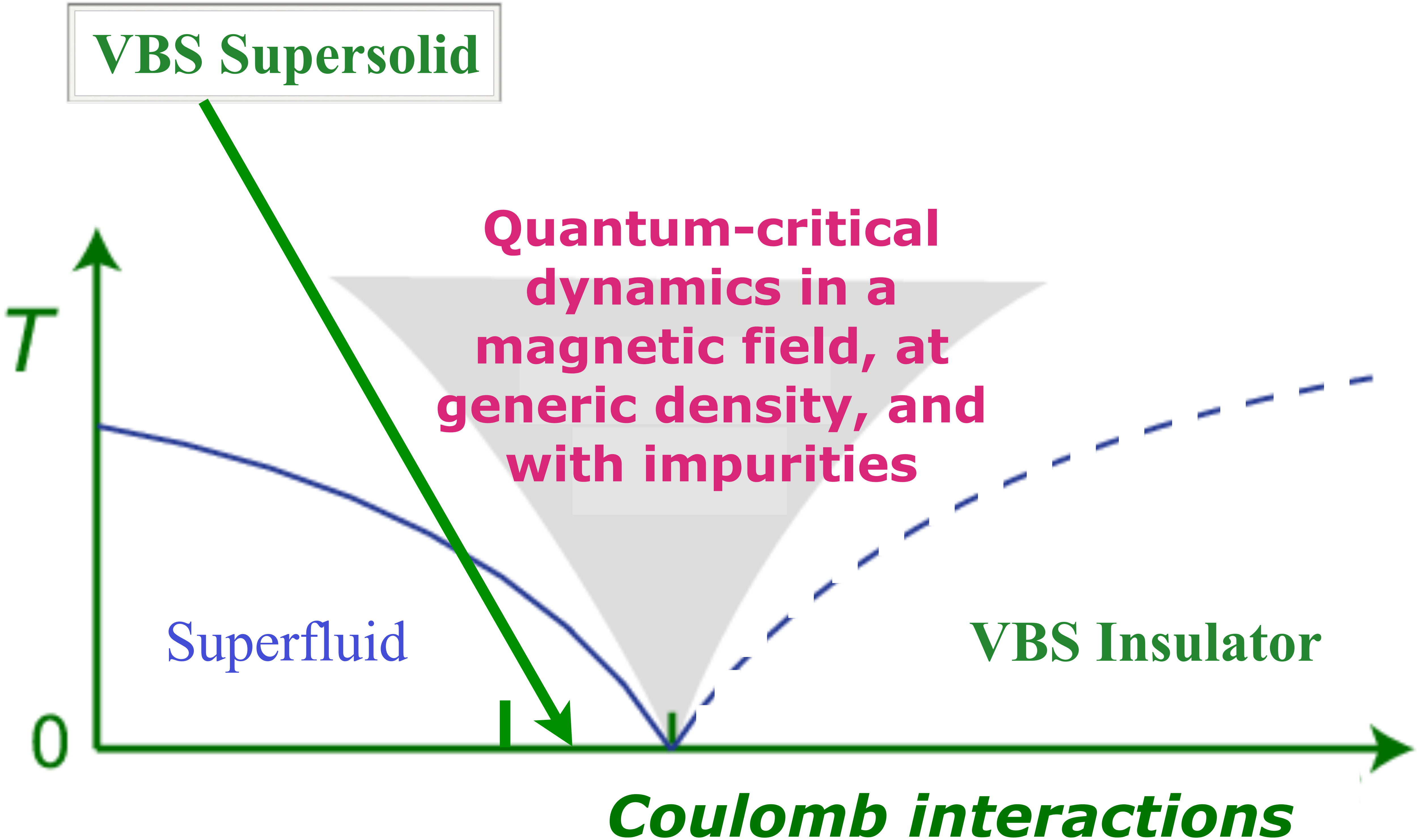


Nernst measurements
T
Superconductor
μ
QCP
g
Insulator
x=1/8

Non-zero temperature phase diagram



Non-zero temperature phase diagram



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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, Phys. Rev. B (2007)

In the hydrodynamic regime, $\hbar\omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.
The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^μ, the current,
- ρ, the local number density,
- ε, the local energy density,
- P, the local pressure,
- u^μ, the local velocity, and
- σ_Q, a universal conductivity, which is the **single transport co-efficient**.

The dependence of ε, P, σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \end{aligned}$$

Conservation laws/equations of motion

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\ T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\ J^\mu &= \rho u^\mu \end{aligned}$$

Constitutive relations which follow from Lorentz transformation to moving frame

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]
\end{aligned}
$$

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} \left(\delta^\mu_\nu + u^\mu u_\nu\right) T^{\nu\gamma} u_\gamma \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda\right) + \mu \frac{\partial_\mu T}{T} \right]
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Momentum relaxation from impurities

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Transverse thermoelectric co-efficient

$$\left(\frac{h}{2ek_B}\right)\alpha_{xy} = \Phi_s \overline{B}\,(k_B T)^2 \left(\frac{2\pi\tau_{\rm imp}}{\hbar}\right)^2 \frac{\overline{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon+P}(k_B T)^3\,\hbar/2\pi\tau_{\rm imp}}{\Phi_{\varepsilon+P}^2(k_B T)^6 + \overline{B}^2\overline{\rho}^2(2\pi\tau_{\rm imp}/\hbar)^2},$$

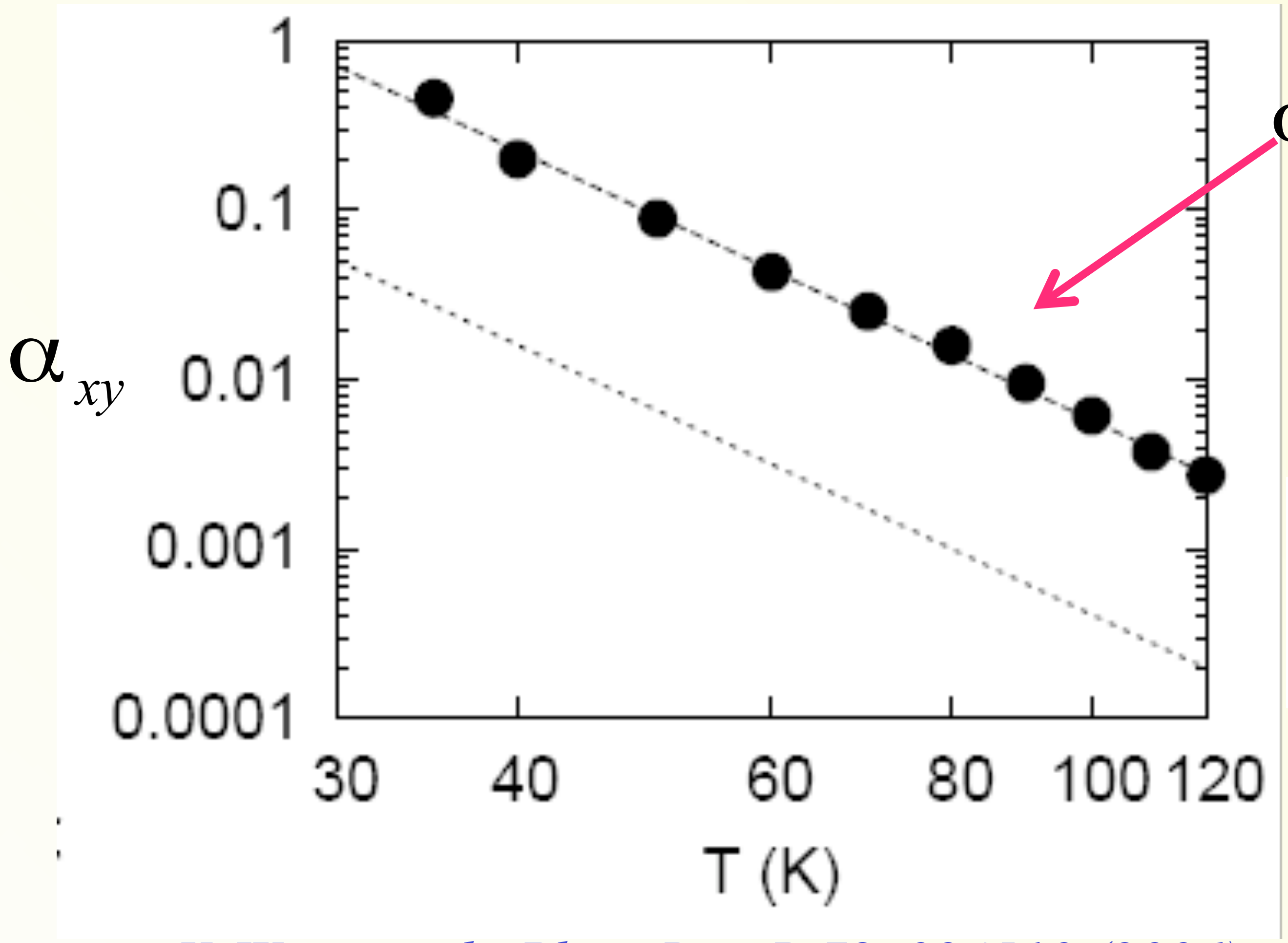
where

$$B = \overline{B}\phi_0/(\hbar v)^2 \quad ; \quad \rho = \overline{\rho}/(\hbar v)^2.$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xx} e_N$



Y. Wang et al., Phys. Rev. B ***73****, 024510 (2006).*

$$\alpha_{xy} \propto \frac{1}{T^4}$$

$$\alpha_{xy} \propto \frac{BT^2\left(\#\rho^2\tau_{imp} + \#T^3\right)}{T^6 + \#B^2\rho^2\tau_{imp}^2}$$

(T small)

$$\frac{\alpha_{xy}}{B}(B \to 0) \approx \left(\frac{2ek_B}{h\phi_0}\right)\frac{\Phi_s}{\Phi_{\varepsilon+P}^2}\left(\frac{2\pi\tau_{\rm imp}}{\hbar}\right)^2 \frac{\rho^2(\hbar v)^6}{(k_B T)^4}$$

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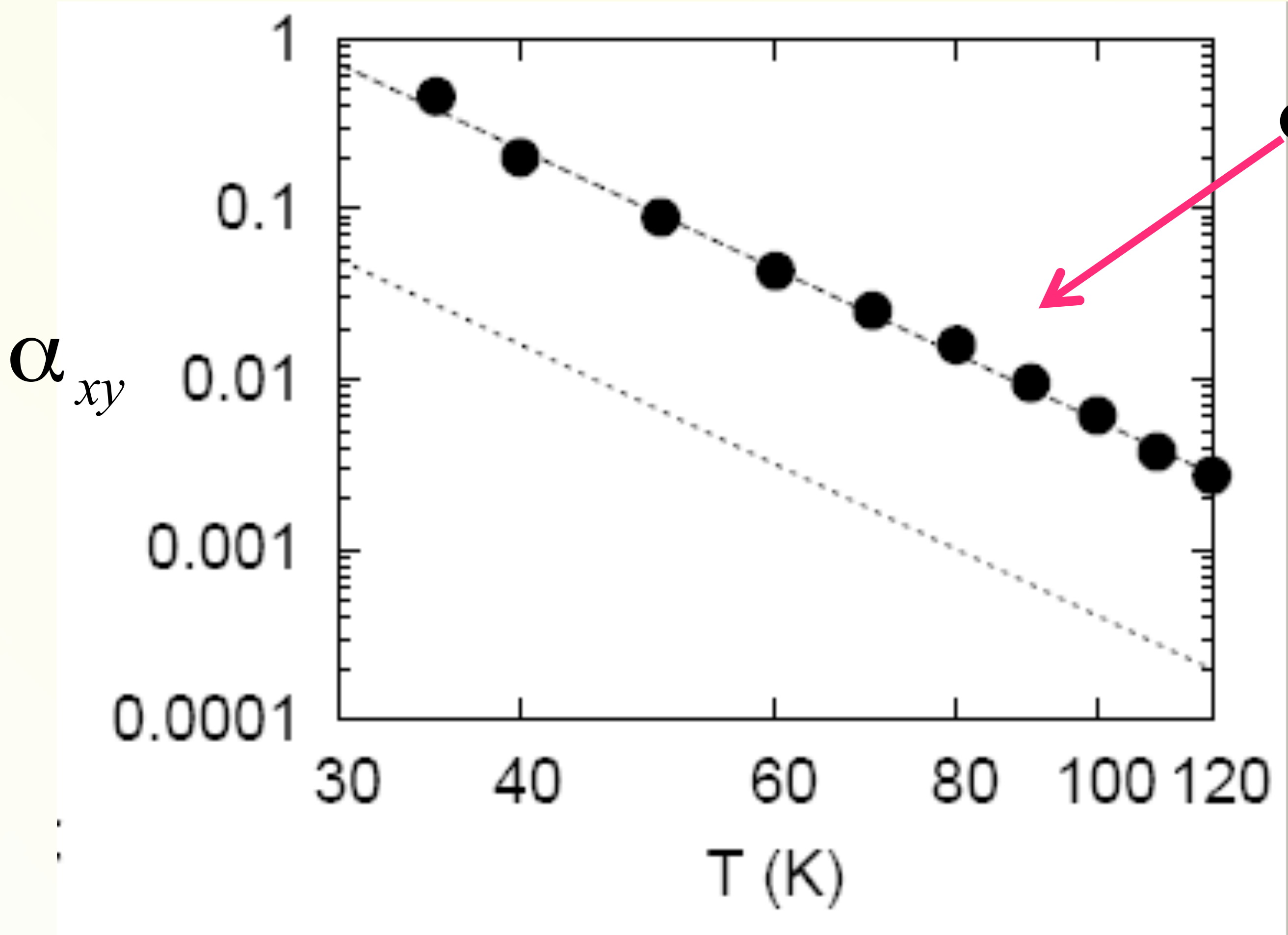

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$\hbar v \approx 47 \text{meV Å}$

$v \approx 2.5 \times 10^{-5} c$

$\tau_{imp} \approx 10^{-12}\text{s}$

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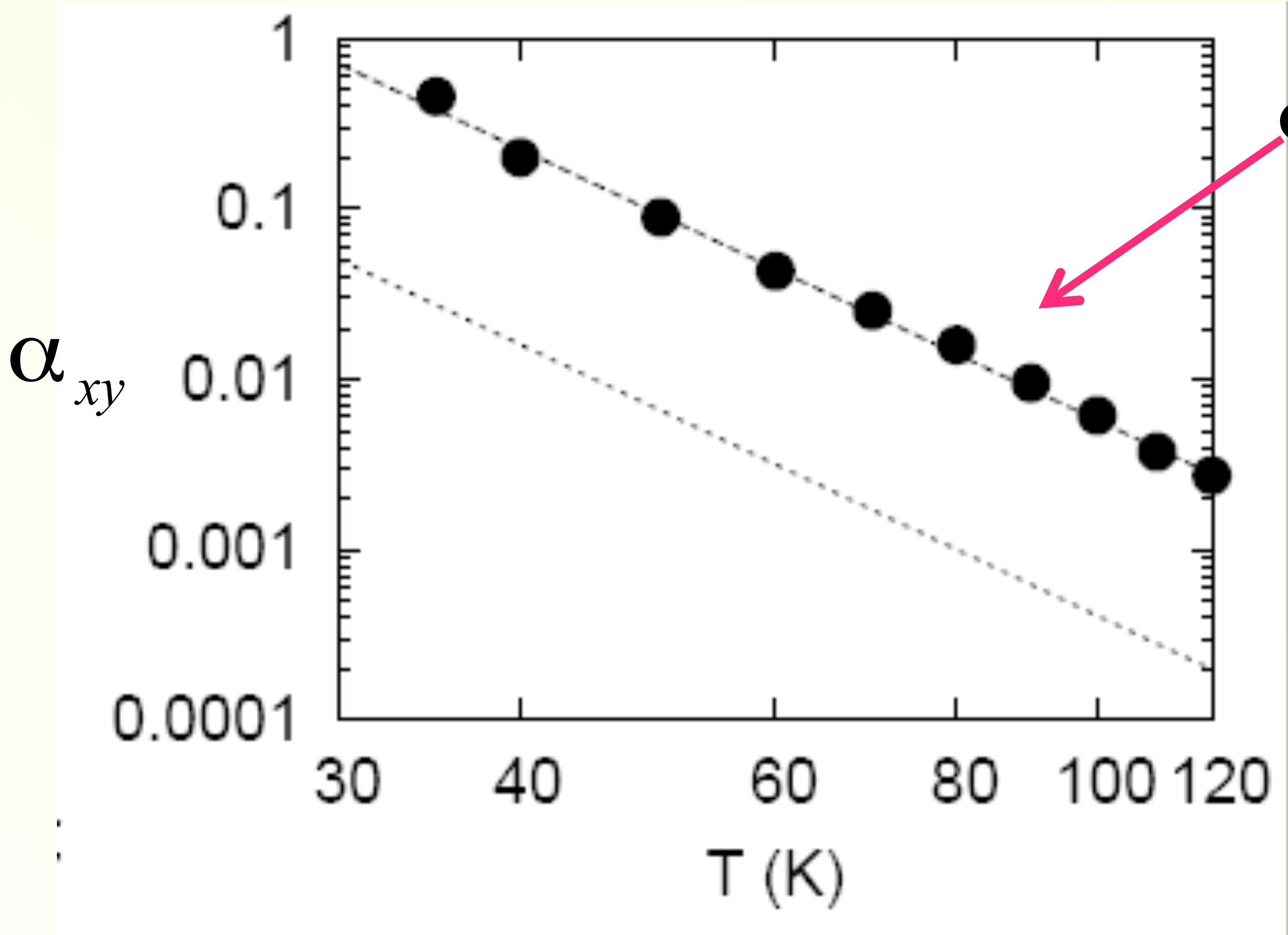

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$\hbar v \approx 47 \text{meV Å}$

$v \approx 2.5 \times 10^{-5} c$

$\tau_{imp} \approx 10^{-12}\,\text{s}$

→ Prediction for ω_c:

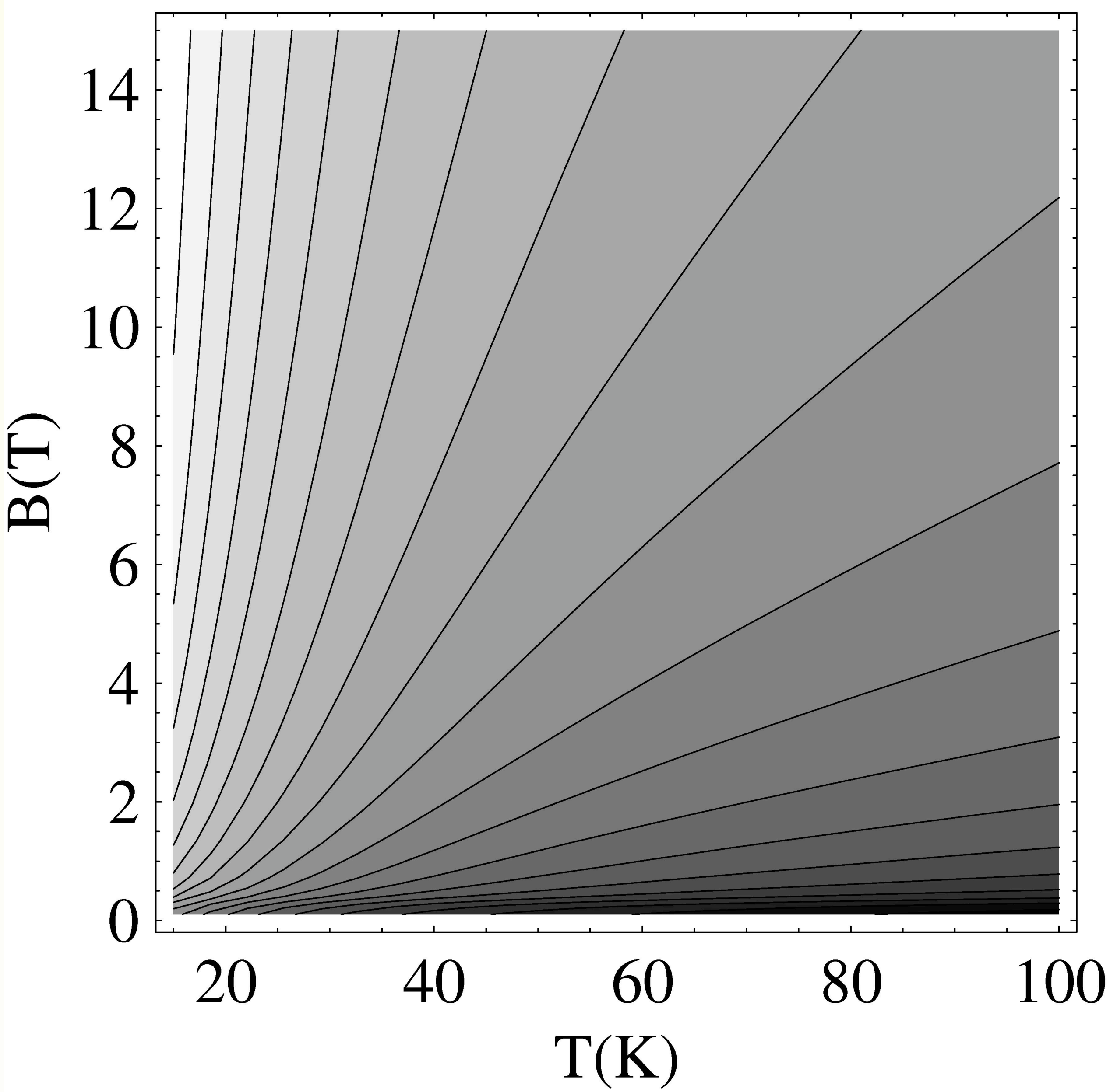
$$\omega_c = 6.2\ \text{GHz}\frac{B}{1\ \text{T}}\left(\frac{35\ \text{K}}{T}\right)^3$$

- T-dependent cyclotron frequency!
- 0.035 times smaller than the cyclotron frequency of free electrons (at T=35 K)
- Only observable in ultra-pure samples where $\tau_{imp}^{-1} \leq \omega_c$

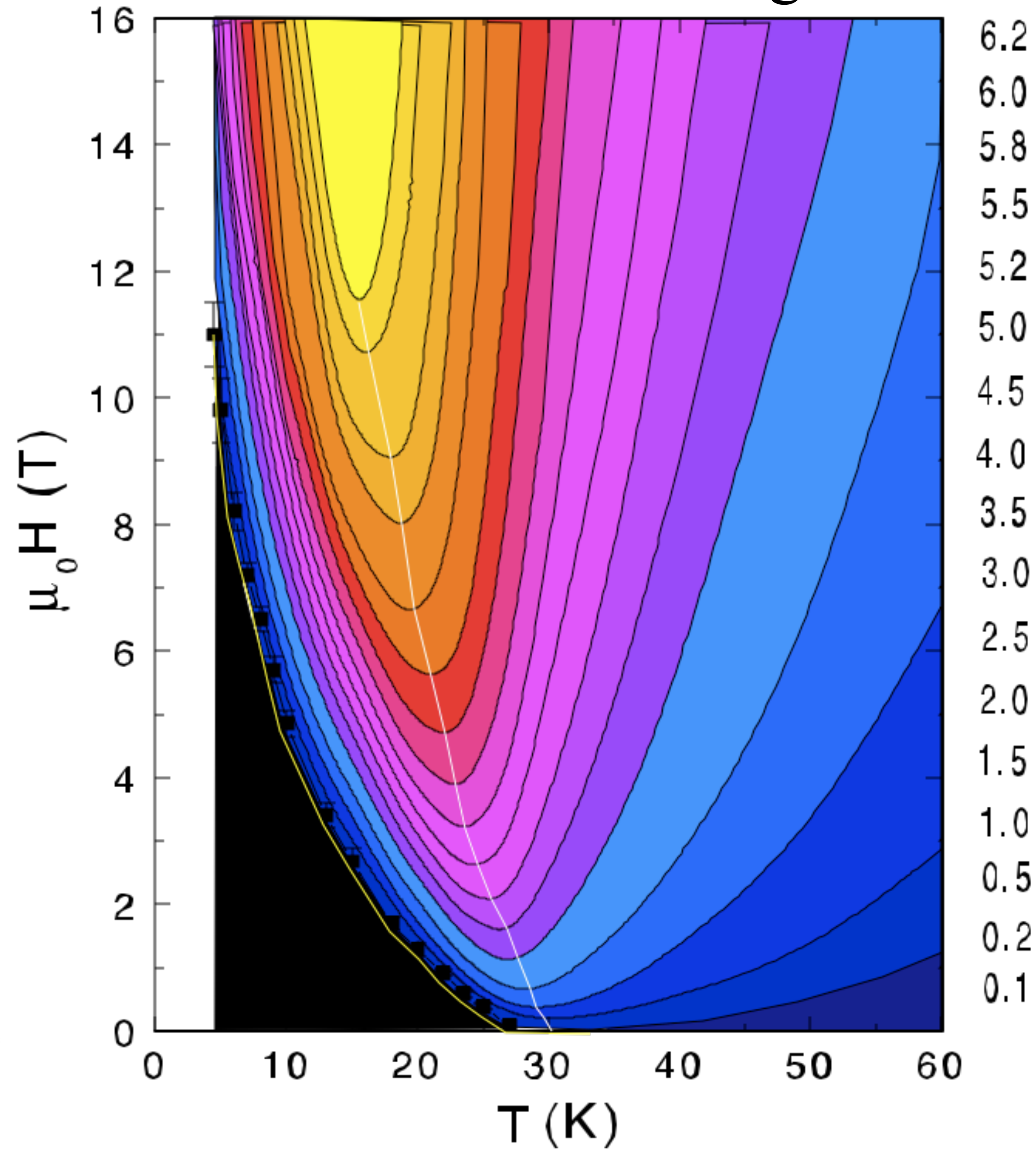
LSCO Experiments

B, T-dependence

Theory for $\alpha_{xy} \approx \sigma_{xx} N$



LSCO-0.12 Nernst signal N



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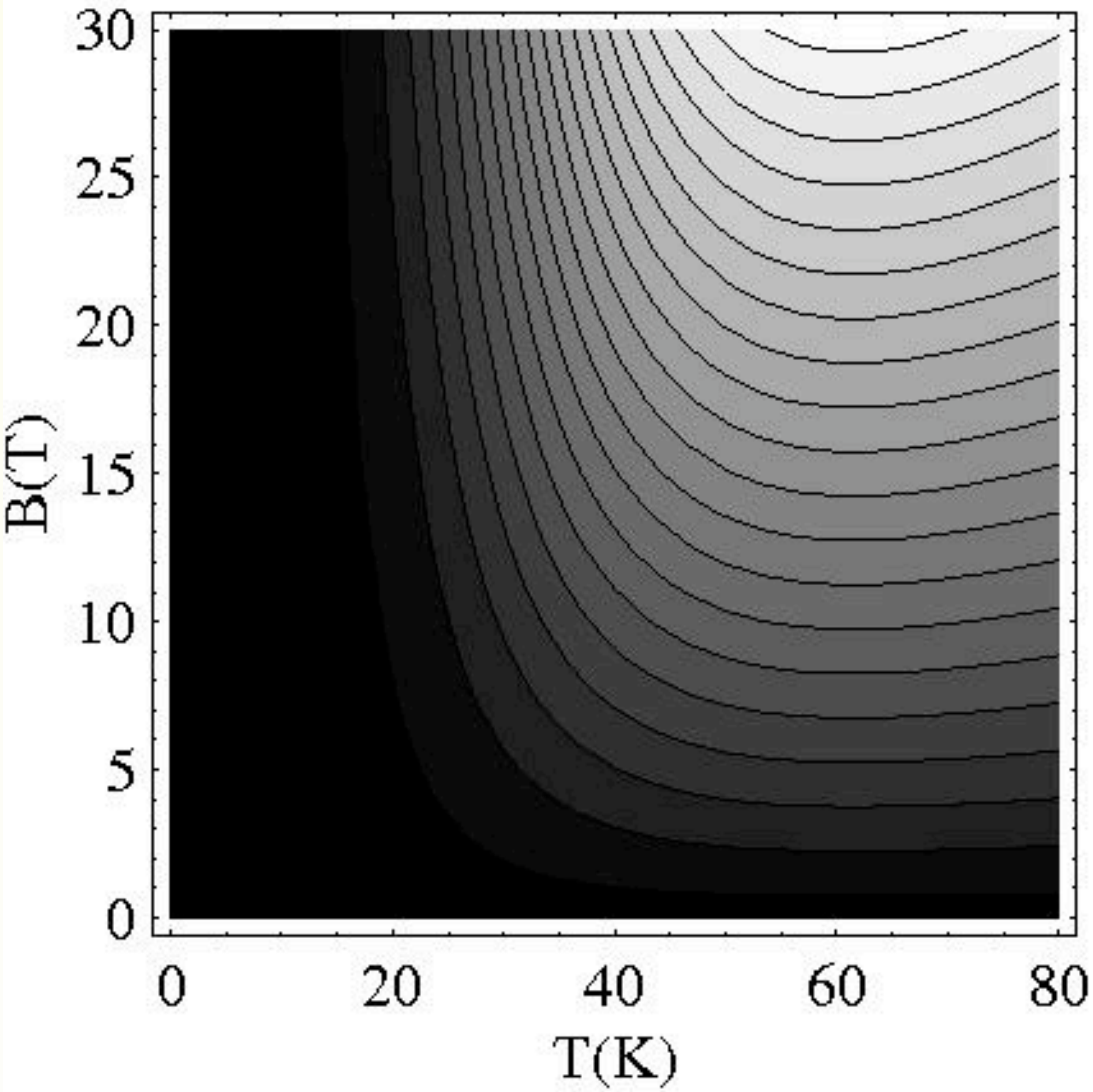
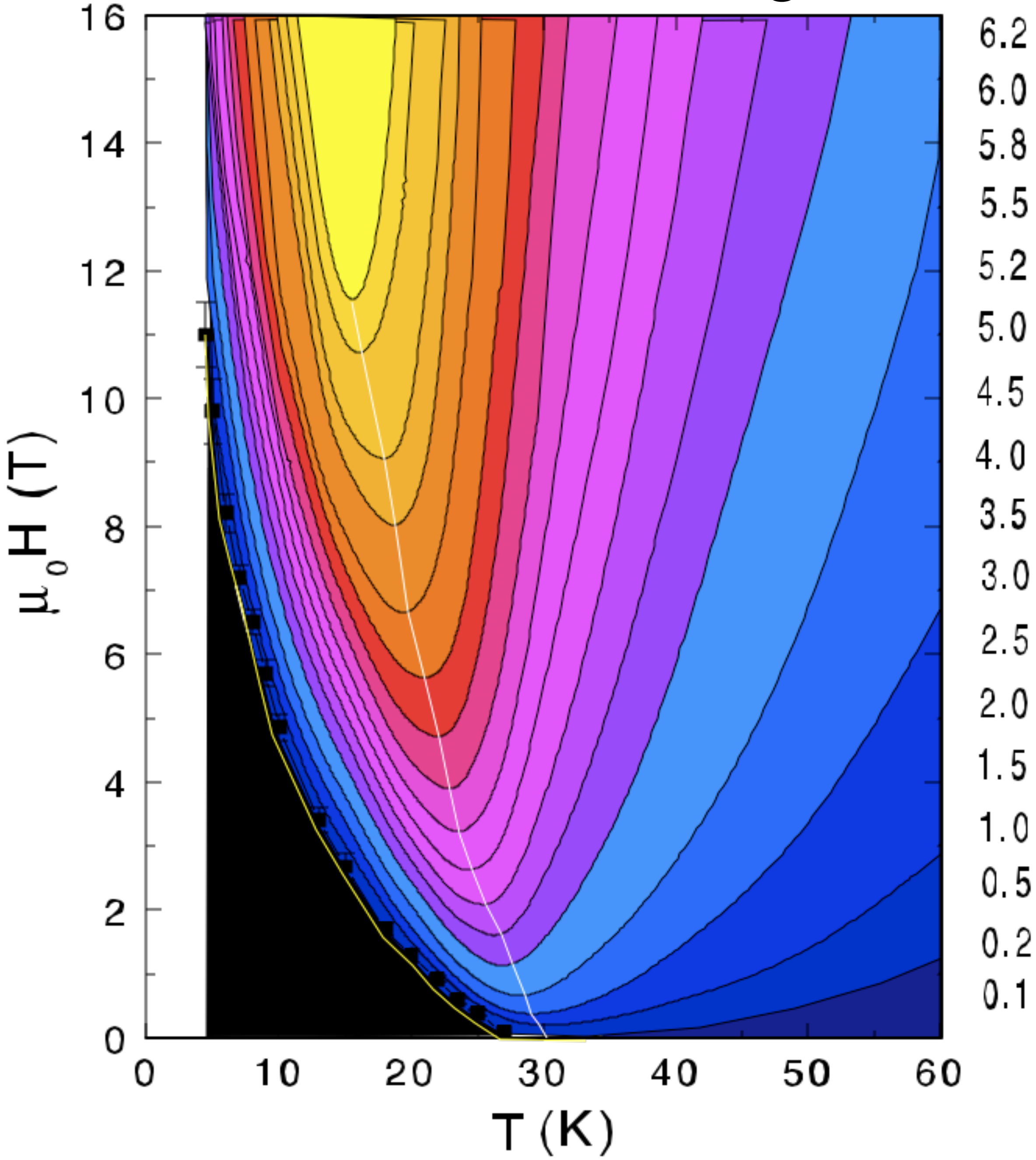
LSCO Experiments

Theory for N





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Outline

1. Entanglement of spins

   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics

   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors

   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene

   Hydrodynamic cyclotron resonance and Nernst effect

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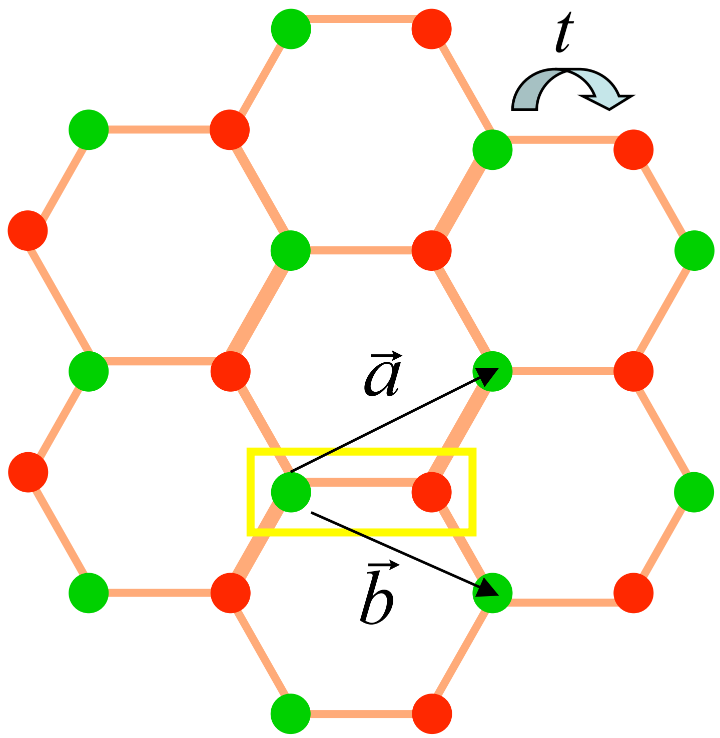
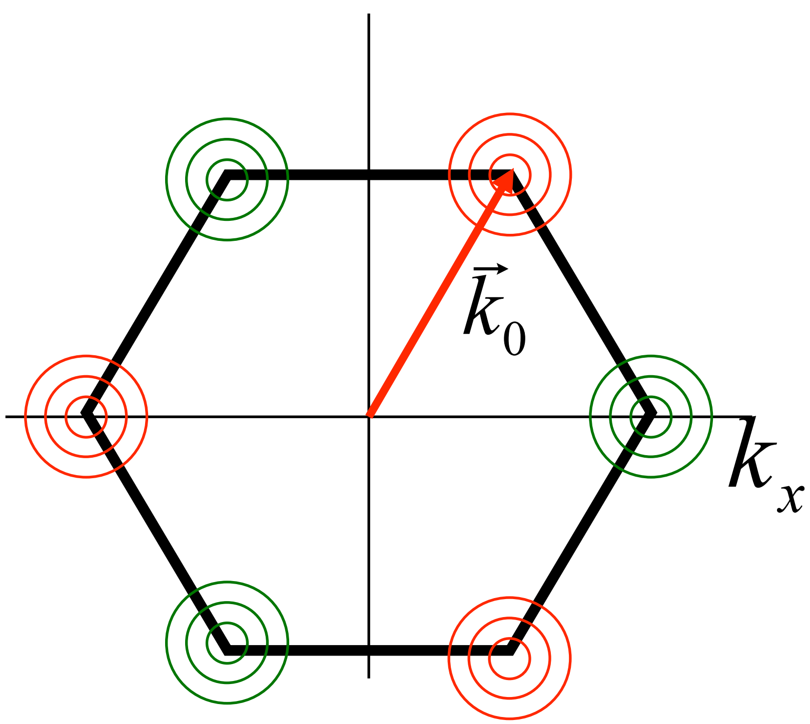
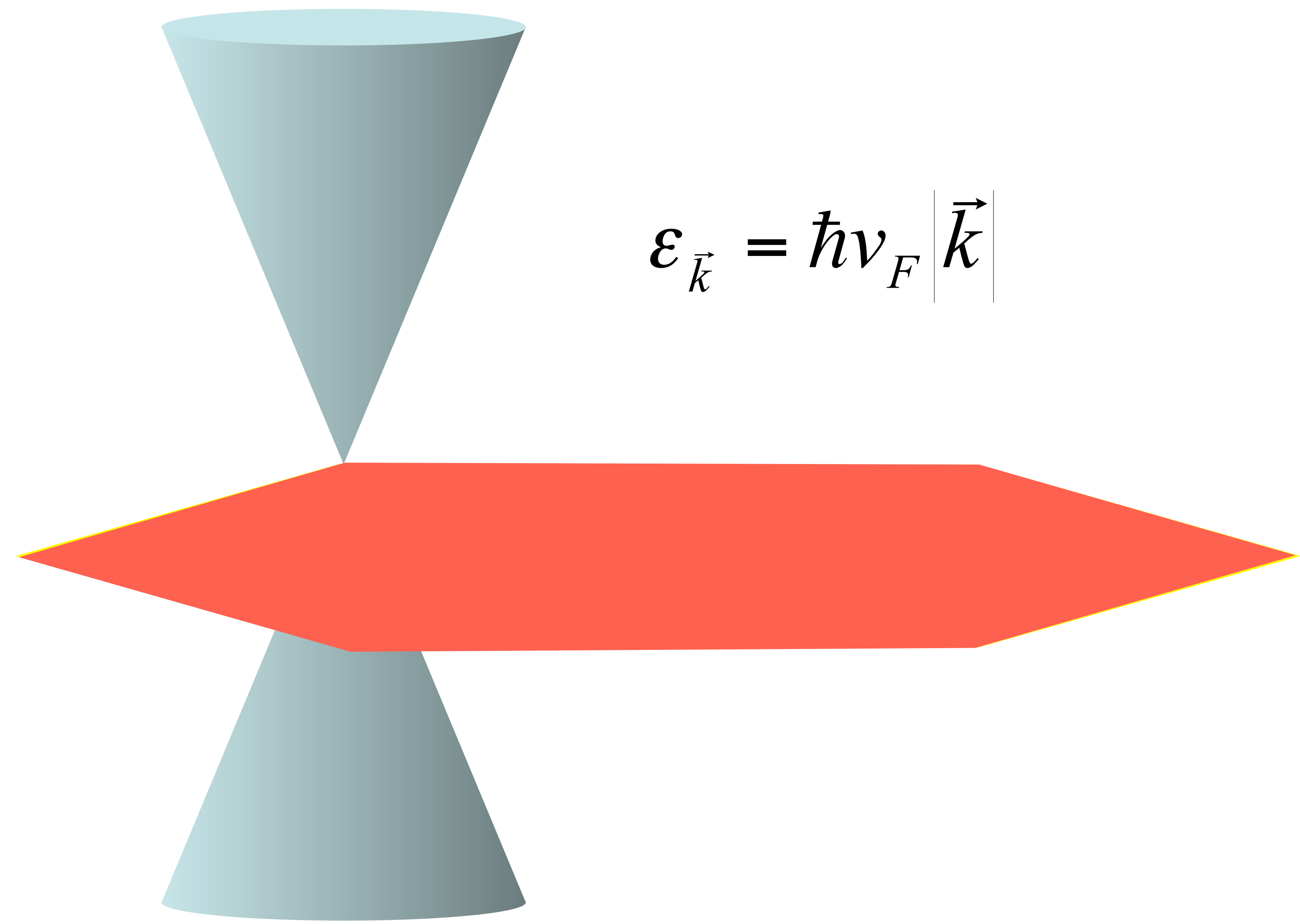
Graphene







$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

Graphene

Low energy theory has 4 two-component Dirac fermions, ψ_α, $\alpha = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\mathcal{S} = \int d^2 r d\tau \psi_\alpha^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\alpha + \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\alpha^\dagger \psi_\alpha(r) \frac{1}{|r - r'|} \psi_\beta^\dagger \psi_\beta(r')$$

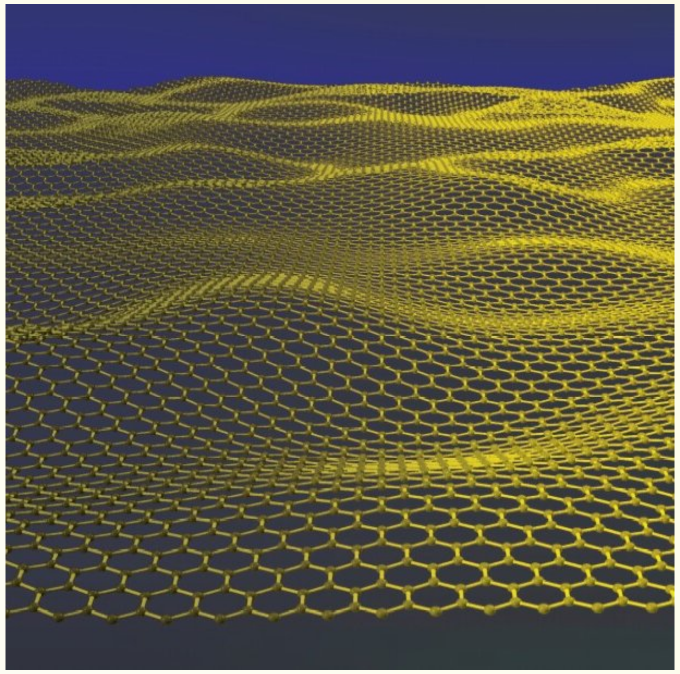
Dimensionless "fine-structure" constant $\lambda = e^2/(4\hbar v_F)$.
RG flow of α:

$$\frac{d\lambda}{d\ell} = -\lambda^2 + \ldots$$

Behavior is similar to a CFT3 with $\lambda \sim 1/\ln(\text{scale})$

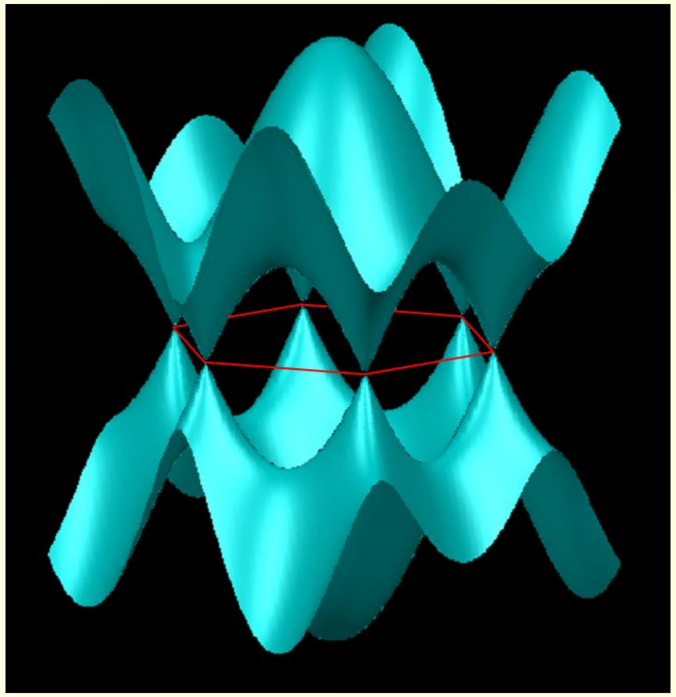
Cyclotron resonance in graphene

M. Mueller, and S. Sachdev, arXiv:0801.2970.



$$\omega = \pm\omega_c^{rel} - i\gamma - i/\tau$$

$$v = 1.1\times 10^6\, m/s \approx c/300$$



Conditions to observe resonance

- Negligible Landau quantization $\quad E_{LL} = \hbar v\sqrt{\dfrac{2eB}{\hbar c}} << k_B T$
- Hydrodynamic, collison-dominated regime $\quad \hbar\omega_c^{rel} << k_B T$
- Negligible broadening $\quad \gamma, \tau^{-1} < \omega_c^{rel}$
- Relativistic, quantum critical regime $\quad \rho \le \rho_{th} = \dfrac{(k_B T)^2}{(\hbar v)^2}$

$T \approx 300\,\mathrm{K}$

$B \approx 0.1\,\mathrm{T}$

$\rho \approx 10^{11}\,\mathrm{cm}^{-2}$

$\omega_c \approx 10^{13}\,\mathrm{s}^{-1}$

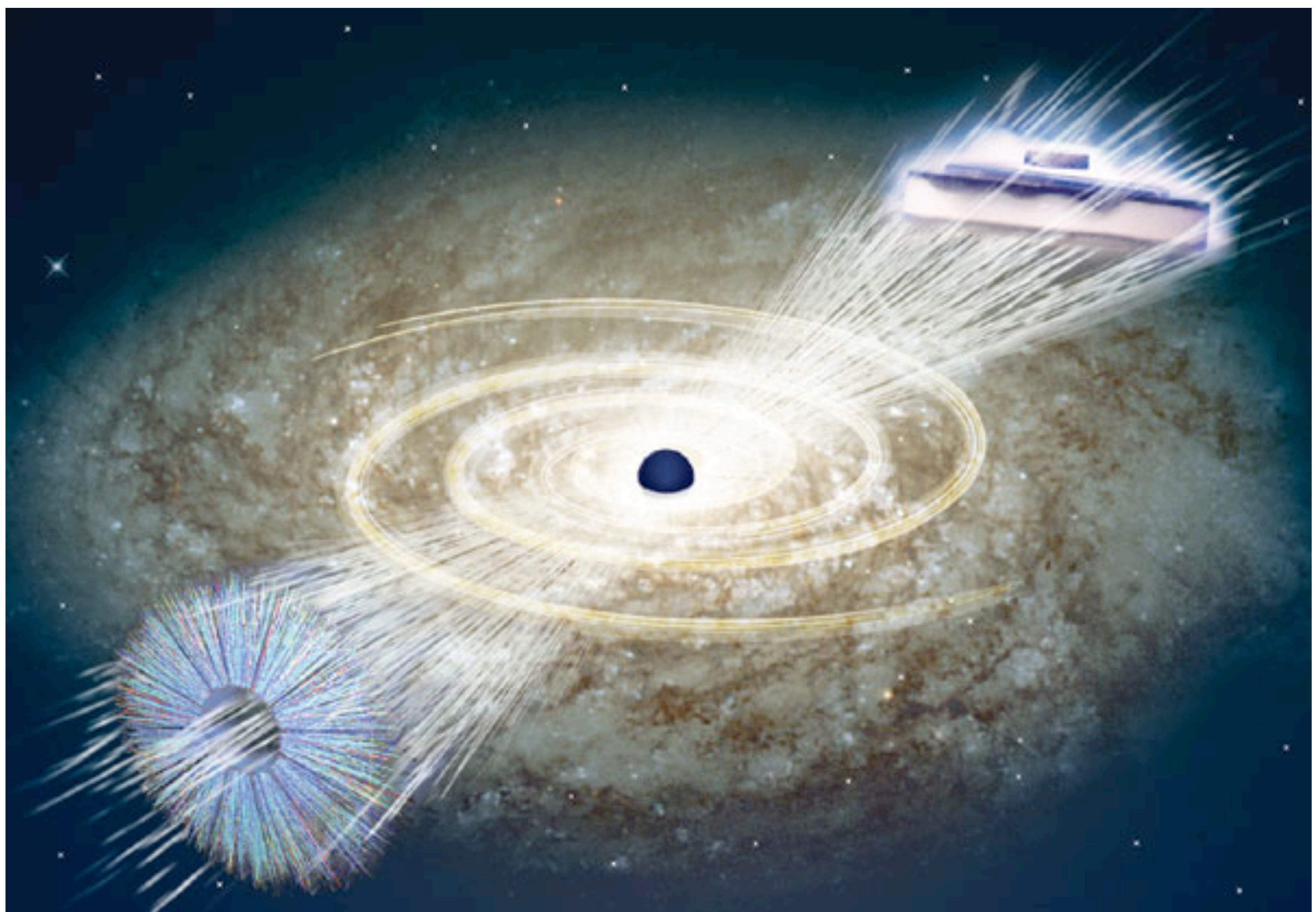
THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007



Conclusions

- Quantum phase transitions in antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in curpates.
- Quantum-critical transport in graphene.