

# Planckian metals

Aavishkar Patel and Subir Sachdev  
PRL **123**, 066601 (2019)



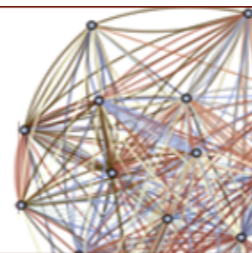
## Applications of Random Matrix Theory to Many Body Physics

Simons Center for Geometry and Physics,  
Stony Brook, September 19, 2019

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)



Simons Collaboration on  
Ultra-Quantum Matter



# 5 Many Body Random Matrix models

## The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} + \epsilon \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta;\gamma\delta}$  are independent random variables

with  $\overline{U_{\alpha\beta;\gamma\delta}} = 0$  and  $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$

# 5 Many Body Random Matrix models

## SYK lattice model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$  is a random function of  $\alpha\beta\gamma\delta$   
 $\epsilon_k$  has a bandwidth  $W$ .

$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = \\ U^2 \left[ \delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right]$$

Independent random  $U_{i,\alpha\beta\gamma\delta}$  on each site  $i$ .

# 5 Many Body Random Matrix models

## Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$  is a random function of  $\alpha\beta\gamma\delta$   
 $\epsilon_k$  has a bandwidth  $W$ .

$$\overline{U(k_1, k_2, k_3, k_4)U^*(k_5, k_6, k_7, k_8)} = \\ U^2 \left[ \delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right] \\ \times \left[ \delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} + \epsilon_{k_6} - \epsilon_{k_7} - \epsilon_{k_8}) \right]$$

Power-law decay of inter-site correlations of  $U_{ijkl,\alpha\beta\gamma\delta}$ .

# 5 Many Body Random Matrix models

## Random $J$ model

$$\frac{1}{\sqrt{N}} \sum_{i < j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha} = 1$$

$J_{ij}$  random

Original SY model with  $SU(2) \Rightarrow SU(M \rightarrow \infty)$

# 5 Many Body Random Matrix models

## Random $t$ - $J$ - $U$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$

$$t_{ij}, J_{ij} \text{ random}, \quad U > 0$$

# Universal $T$ -linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142-147

A. Legros<sup>1,2</sup>, S. Benhabib<sup>3</sup>, W. Tabis<sup>3,4</sup>, F. Laliberté<sup>1</sup>, M. Dion<sup>1</sup>, M. Lizaire<sup>1</sup>, B. Vignolle<sup>3</sup>, D. Vignolles<sup>3</sup>, H. Raffy<sup>5</sup>, Z. Z. Li<sup>5</sup>, P. Auban-Senzier<sup>5</sup>, N. Doiron-Leyraud<sup>1</sup>, P. Fournier<sup>1,6</sup>, D. Colson<sup>2</sup>, L. Taillefer<sup>1,6\*</sup> and C. Proust<sup>3,6\*</sup>

arXiv:1902.01034

## Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,<sup>1,2</sup> Tristin Metz,<sup>2</sup> Christopher Eckberg,<sup>2</sup> Kevin Kirshenbaum,<sup>2</sup> Alex Hughes,<sup>2</sup> Renxiong Wang,<sup>2</sup> Limin Wang,<sup>2</sup> Shanta R. Saha,<sup>2</sup> I-Lin Liu,<sup>2,3,4</sup> Nicholas P. Butch,<sup>2,4</sup> Zhonghao Liu,<sup>5,6</sup> Sergey V. Borisenko,<sup>5</sup> Peter Y. Zavalij,<sup>7</sup> and Johnpierre Paglione<sup>2,8</sup>

## Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,<sup>1,\*</sup> Debanjan Chowdhury,<sup>1,\*</sup> Daniel Rodan-Legrain,<sup>1</sup> Oriol Rubies-Bigordà,<sup>1</sup> Kenji Watanabe,<sup>2</sup> Takashi Taniguchi,<sup>2</sup> T. Senthil,<sup>1,†</sup> and Pablo Jarillo-Herrero<sup>1,†</sup>

arXiv:1901.03710

## Bad metallic transport in a cold atom Fermi-Hubbard system

*Science* **363**, 379–382 (2019)

Peter T. Brown<sup>1</sup>, Debayan Mitra<sup>1</sup>, Elmer Guardado-Sanchez<sup>1</sup>, Reza Nourafkan<sup>2</sup>, Alexis Reymbaut<sup>2</sup>, Charles-David Hébert<sup>2</sup>, Simon Bergeron<sup>2</sup>, A.-M. S. Tremblay<sup>2,3</sup>, Jure Kokalj<sup>4,5</sup>, David A. Huse<sup>1</sup>, Peter Schauf<sup>1\*</sup>, Waseem S. Bakr<sup>1†</sup>

Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity,  $\rho$ , is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

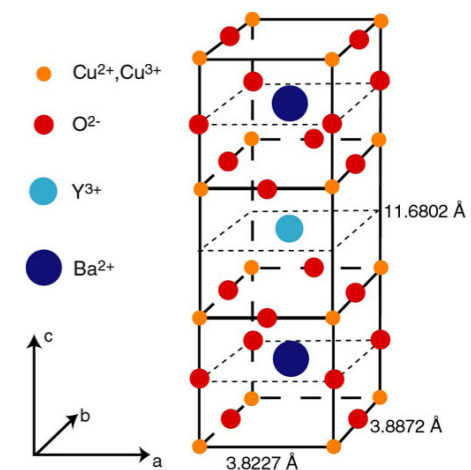
independent of the strength of interactions!



Material		$n$ ( $10^{27} \text{ m}^{-3}$ )	$m^*$ ( $m_0$ )	$A_1 / d$ ( $\Omega / \text{K}$ )	$h / (2e^2 T_F)$ ( $\Omega / \text{K}$ )	$\alpha$
Bi2212	$p = 0.23$	6.8	$8.4 \pm 1.6$	$8.0 \pm 0.9$	$7.4 \pm 1.4$	$1.1 \pm 0.3$
Bi2201	$p \sim 0.4$	3.5	$7 \pm 1.5$	$8 \pm 2$	$8 \pm 2$	$1.0 \pm 0.4$
LSCO	$p = 0.26$	7.8	$9.8 \pm 1.7$	$8.2 \pm 1.0$	$8.9 \pm 1.8$	$0.9 \pm 0.3$
Nd-LSCO	$p = 0.24$	7.9	$12 \pm 4$	$7.4 \pm 0.8$	$10.6 \pm 3.7$	$0.7 \pm 0.4$
PCCO	$x = 0.17$	8.8	$2.4 \pm 0.1$	$1.7 \pm 0.3$	$2.1 \pm 0.1$	$0.8 \pm 0.2$
LCCO	$x = 0.15$	9.0	$3.0 \pm 0.3$	$3.0 \pm 0.45$	$2.6 \pm 0.3$	$1.2 \pm 0.3$
TMTSF	$P = 11 \text{ kbar}$	1.4	$1.15 \pm 0.2$	$2.8 \pm 0.3$	$2.8 \pm 0.4$	$1.0 \pm 0.3$

### Slope of $T$ -linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, *Nature Physics* **15**, 142 (2019)

1. SYK lattice models
2. Planckian metal ansatz
3. Resonant SYK models
4. Random  $t$ - $J$ - $U$  models

1. SYK lattice models

2. Planckian metal ansatz

3. Resonant SYK models

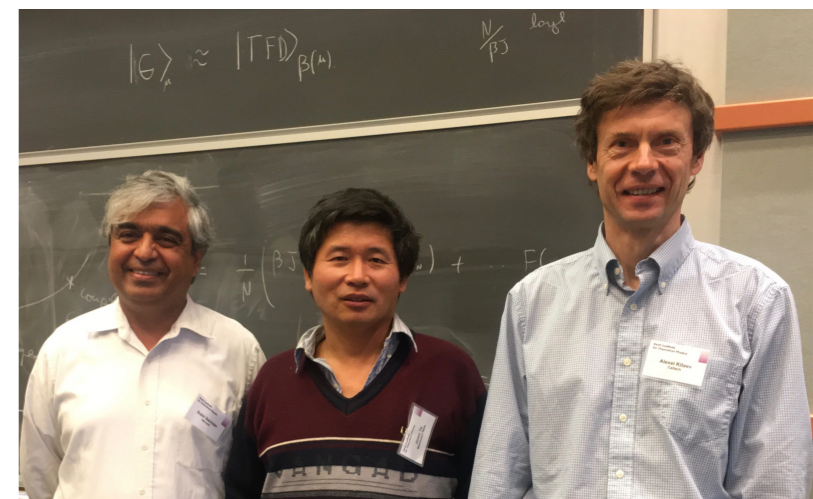
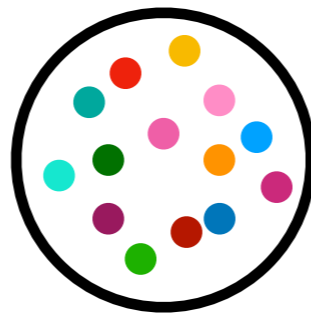
4. Random  $t$ - $J$ - $U$  models

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$U_{\alpha\beta;\gamma\delta}$  are independent random variables

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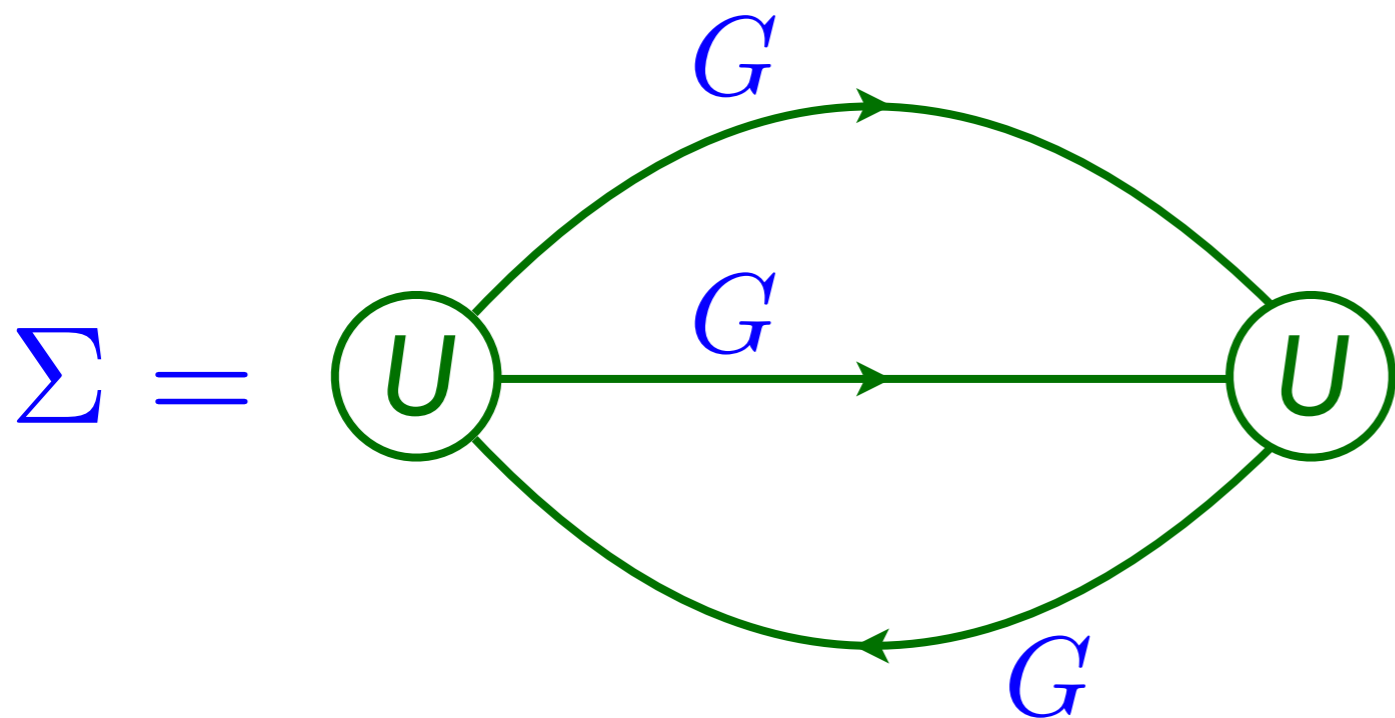
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

# The complex SYK model

Feynman graph expansion in  $U_{\alpha\beta;\gamma\delta}$ , and graph-by-graph average, yields exact equations in the large  $N$  limit:

$$G(i\omega) = \frac{1}{i\omega - \epsilon - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = Q.$$



S. Sachdev and J. Ye,  
PRL **70**, 3339 (1993)



# The complex SYK model

There is a one-parameter family of critical solutions with varying  $Q$ , characterized by a dimensionless parameter  $\mathcal{E}$ .

For long times  $\tau > 0$

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle = e^{\pi\mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$$\langle c_\alpha^\dagger(\tau) c_\alpha(0) \rangle = e^{-\pi\mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$\mathcal{E}$  determines the particle-hole asymmetry, and  $A(\mathcal{E})$  is a known function.

$\mathcal{E}$  is determined by  $\epsilon/U$ .

For small  $\epsilon$ ,  $\mathcal{E} = \mathbb{C} \epsilon/U$  with  $\mathbb{C} = 0.41$

In a Fermi liquid,

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle = \langle c_\alpha^\dagger(\tau) c_\alpha(0) \rangle = \tilde{A}/\tau$$

# The complex SYK model

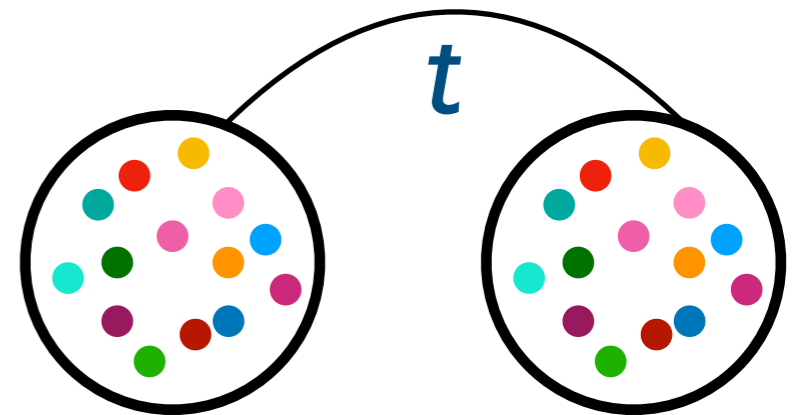
There is a one-parameter family of critical solutions with varying  $Q$ , characterized by a dimensionless parameter  $\mathcal{E}$ .

There is an exact “Luttinger” relation (anomaly matching) between  $Q$  and the particle-hole asymmetry parameter  $\mathcal{E}$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$
$$Q = \frac{1}{2} - \frac{\theta}{\pi} - \frac{1}{4} \sin(2\theta)$$

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

Equivalent to an  
 “eternal traversable wormhole”  $U$   
 between two black holes with  
 AdS<sub>2</sub> horizons



J. Maldacena and Xiao-Liang Qi, arXiv:1804.00491

# A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

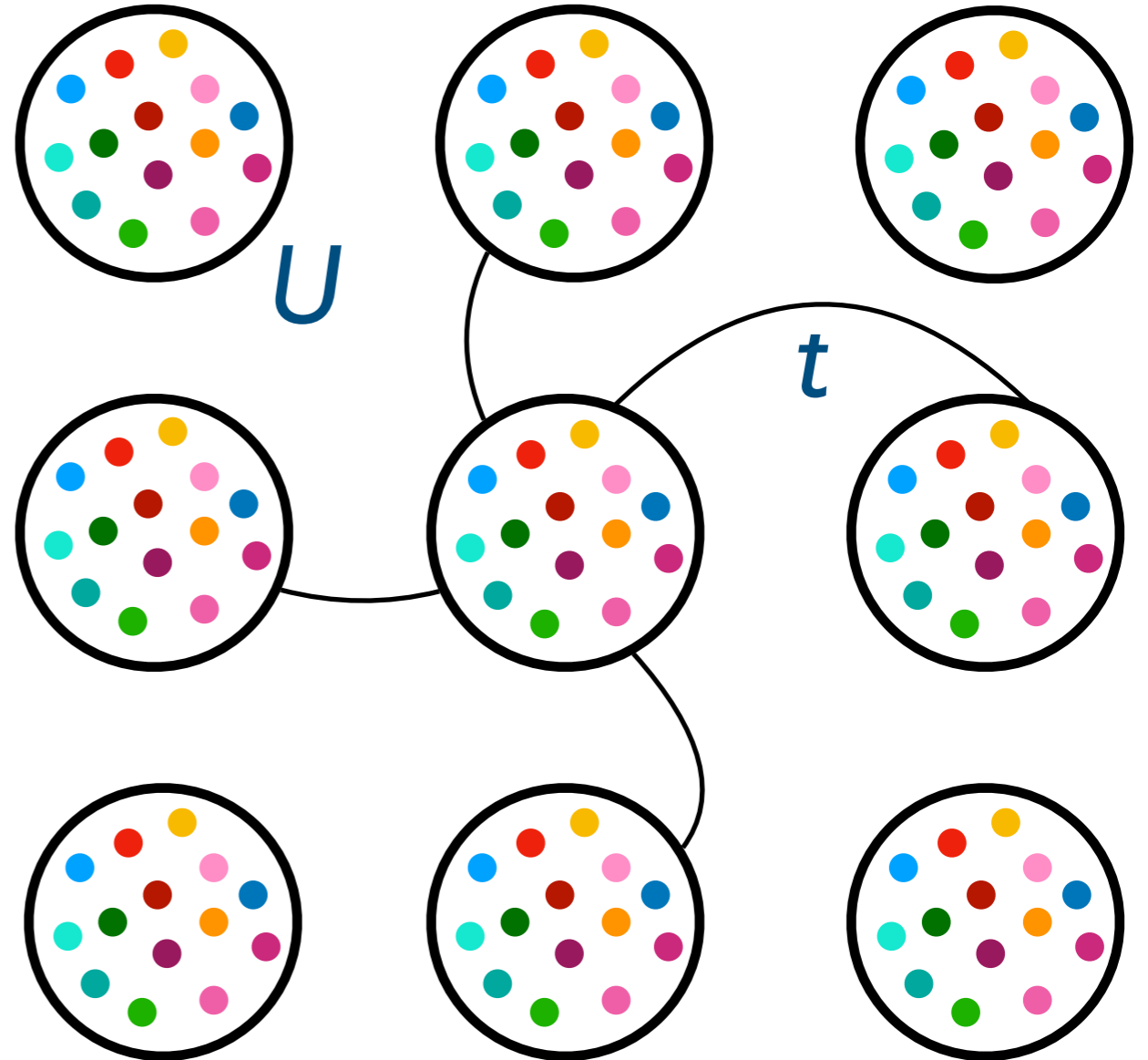
Choose  $U \gg t$  on-site,  
and the same on all sites;  
yields ‘incoherent metal’  
with no Fermi surface  
for  $t^2/U \ll k_B T \ll U$  with

$$G(\mathbf{k}, \omega) = G_{\text{SYK}}(\epsilon, \hbar\omega / (k_B T))$$

independent of  $\mathbf{k}$ .

There is linear-in- $T$  resistivity  
but only with bad metal  
behavior with  $\rho > h/e^2$ , and  
co-efficient dependent upon  $U$ :

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017);  
Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman,  
Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy,  
Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)  
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999);  
Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

# A lattice SYK model

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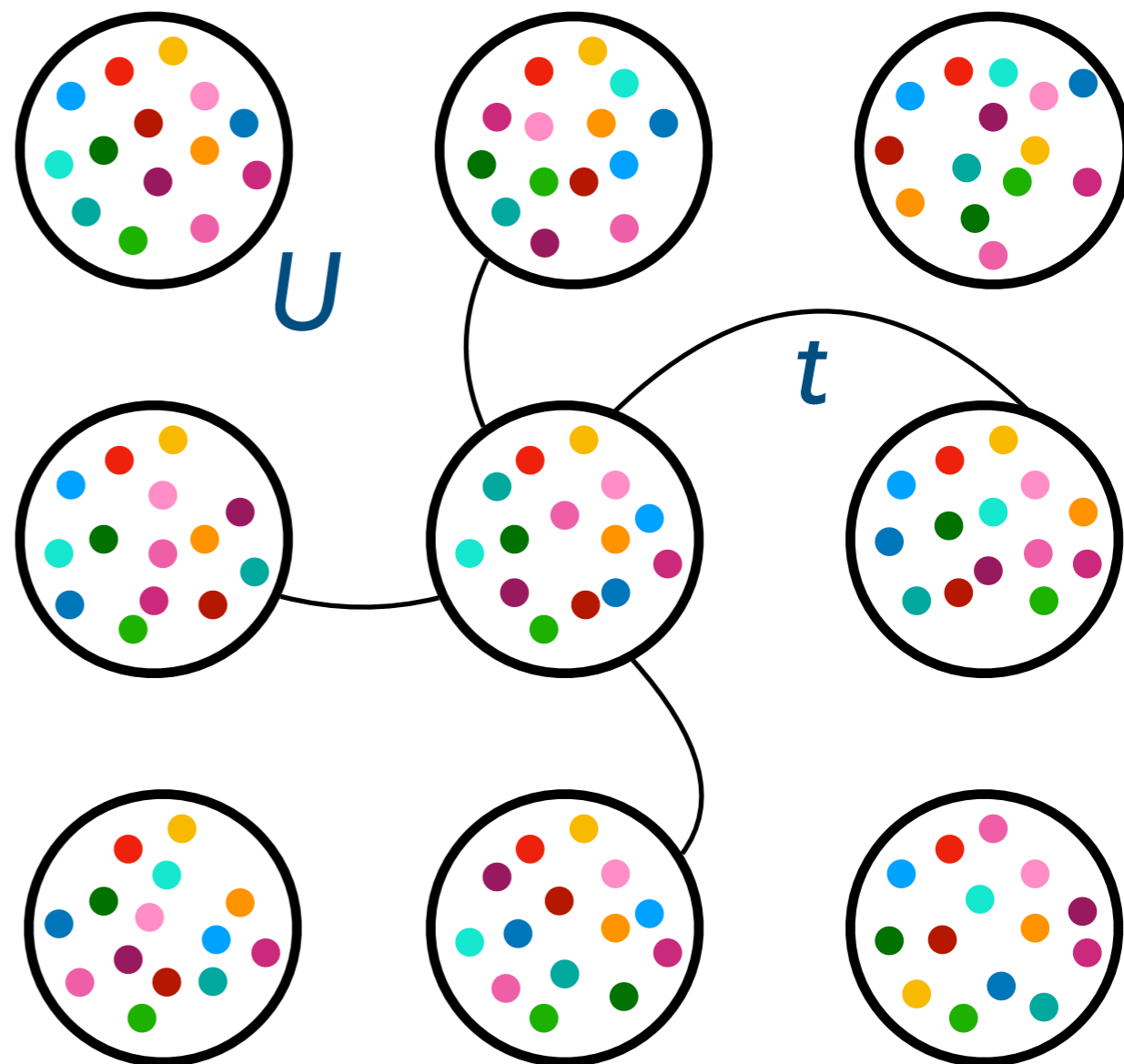
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1. SYK lattice models

2. Planckian metal ansatz

3. Resonant SYK models

4. Random  $t$ - $J$ - $U$  models

# Incoherent metal

For long times  $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U \tau}}$$
$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U \tau}}$$

The parameter  $\mathcal{E}$  is independent of  $k$ ,  
and determined by the total density.  
For small  $\epsilon$ ,  $\mathcal{E} = \mathbb{C} \epsilon / U$  with  $\mathbb{C} = 0.41$ .

# Planckian metal ansatz with remnant Fermi surface

For long times  $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathcal{E}_k} \frac{A(\mathcal{E}_k)}{\sqrt{U\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathcal{E}_k} \frac{A(\mathcal{E}_k)}{\sqrt{U\tau}}$$



We choose a linear function  $\mathcal{E}_k = \mathbb{C} \epsilon_k / U$ :  
the particle-hole asymmetry changes as  
we cross the Fermi surface.



# Planckian metal ansatz with remnant Fermi surface

For long times  $\tau > 0$

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We choose a linear function  $\mathcal{E}_k = \mathbb{C} \epsilon_k / U$ :

No free parameters—everything is determined by the (underlying) quasiparticle dispersion  $\epsilon_k$ , and the interaction strength  $U$ .



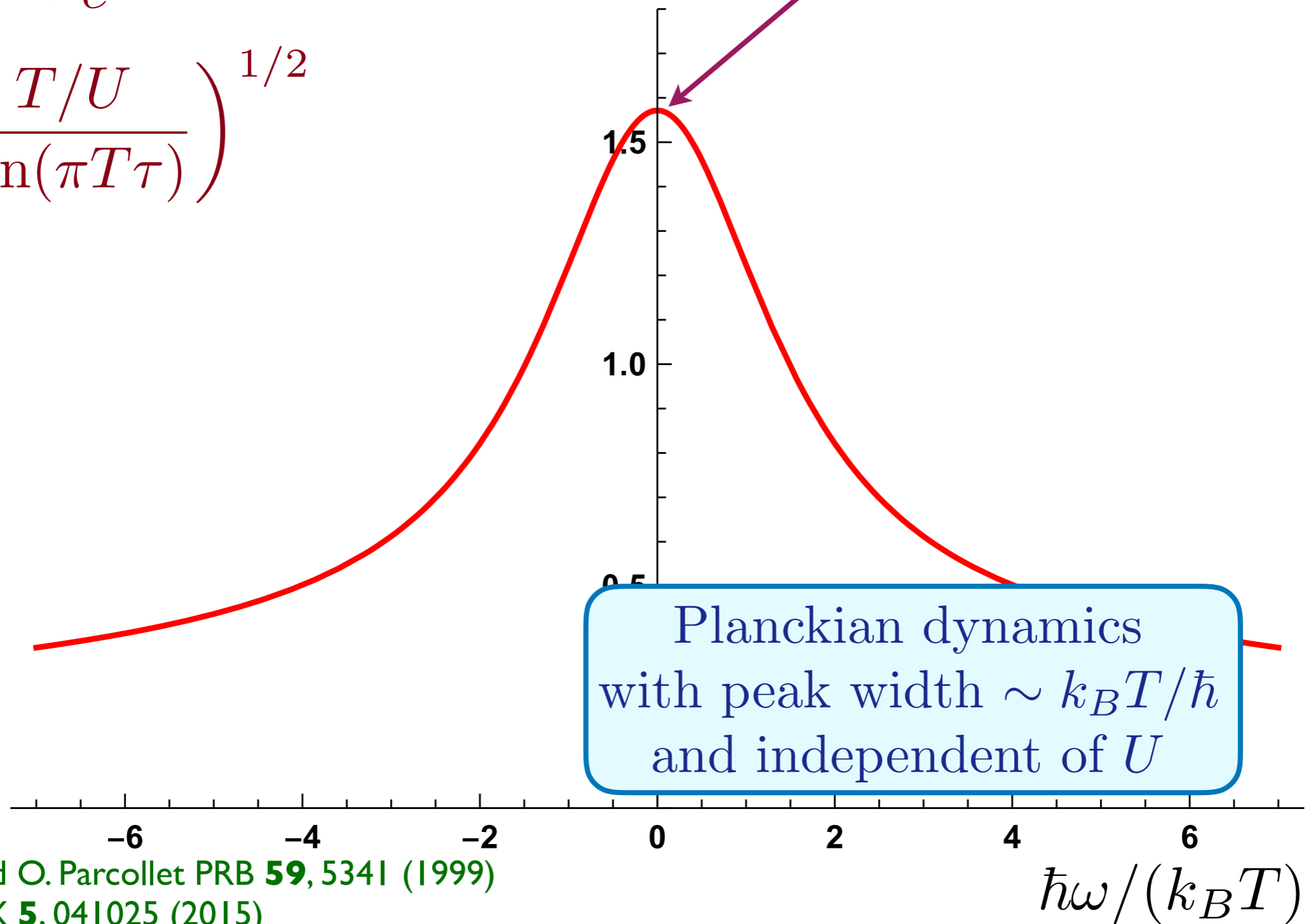
$G^R(\epsilon_k, \omega)$  is the Fourier transform of

$$G(\epsilon_k, \tau) \sim e^{-2\pi\mathcal{E}_k T \tau}$$

$$\times \left( \frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

$$\mathcal{E}_k = \mathbb{C} \frac{\epsilon_k}{U}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E}_k = 0$$



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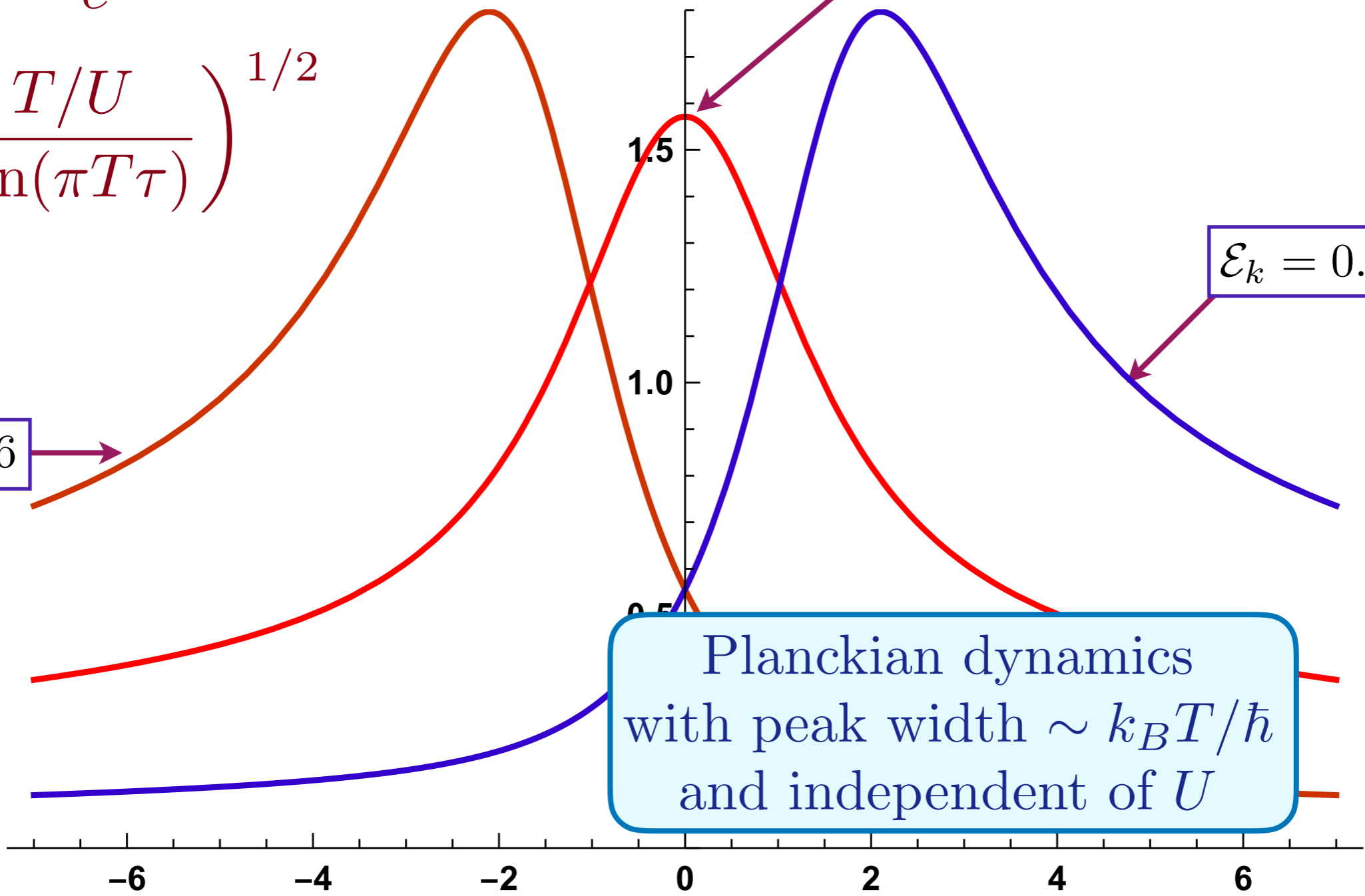
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$$\mathcal{E}_k = \mathbb{C} \frac{\epsilon_k}{U}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E}_k = 0$$

$$\mathcal{E}_k = 0.26$$

$$\mathcal{E}_k = -0.26$$



Planckian dynamics with peak width  $\sim k_B T / \hbar$  and independent of  $U$

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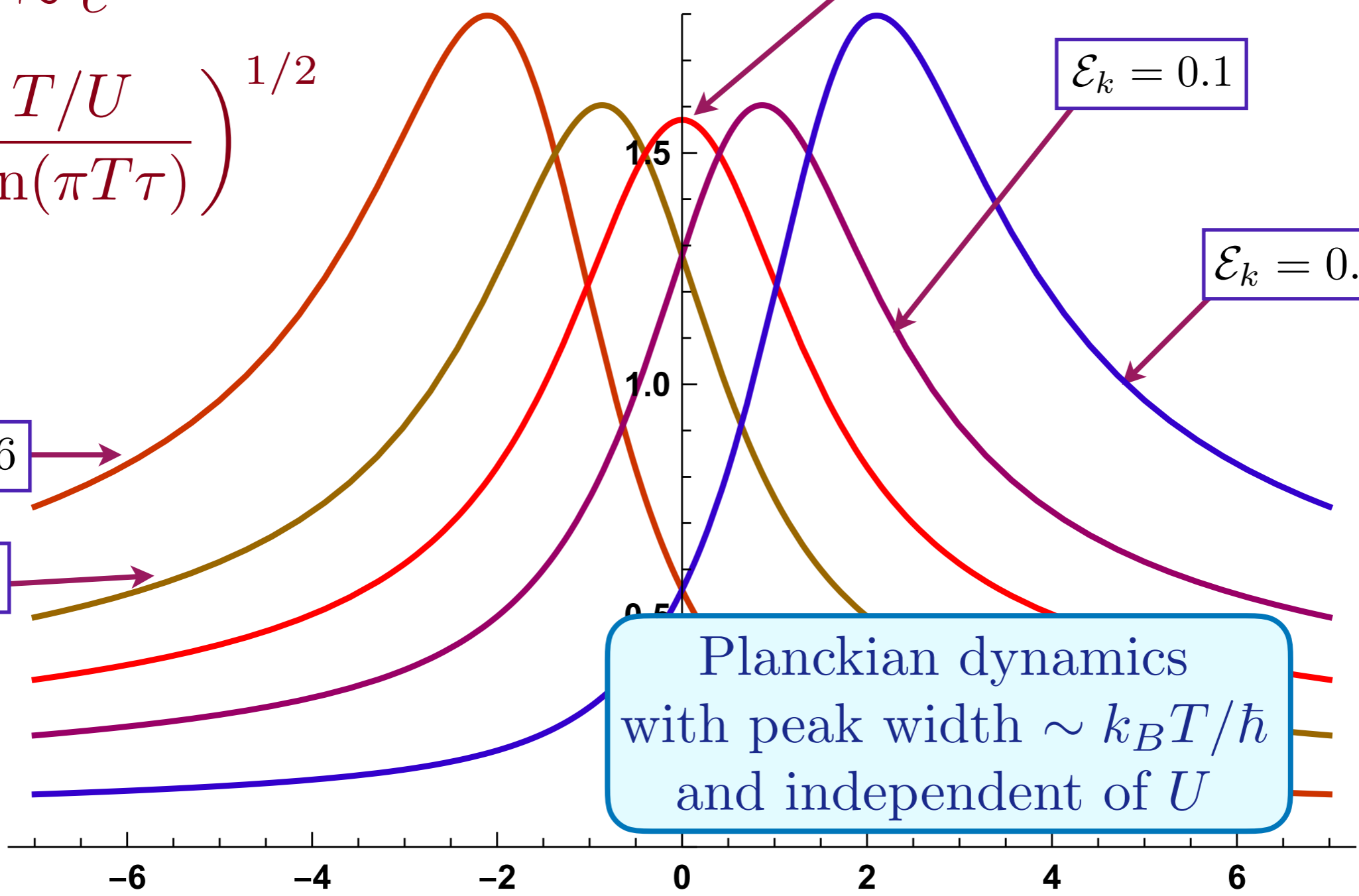
$$\mathcal{E}_k = 0.1$$

$$\mathcal{E}_k = 0.26$$

$$\mathcal{E}_k = -0.26$$

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Planckian dynamics  
with peak width  $\sim k_B T / \hbar$   
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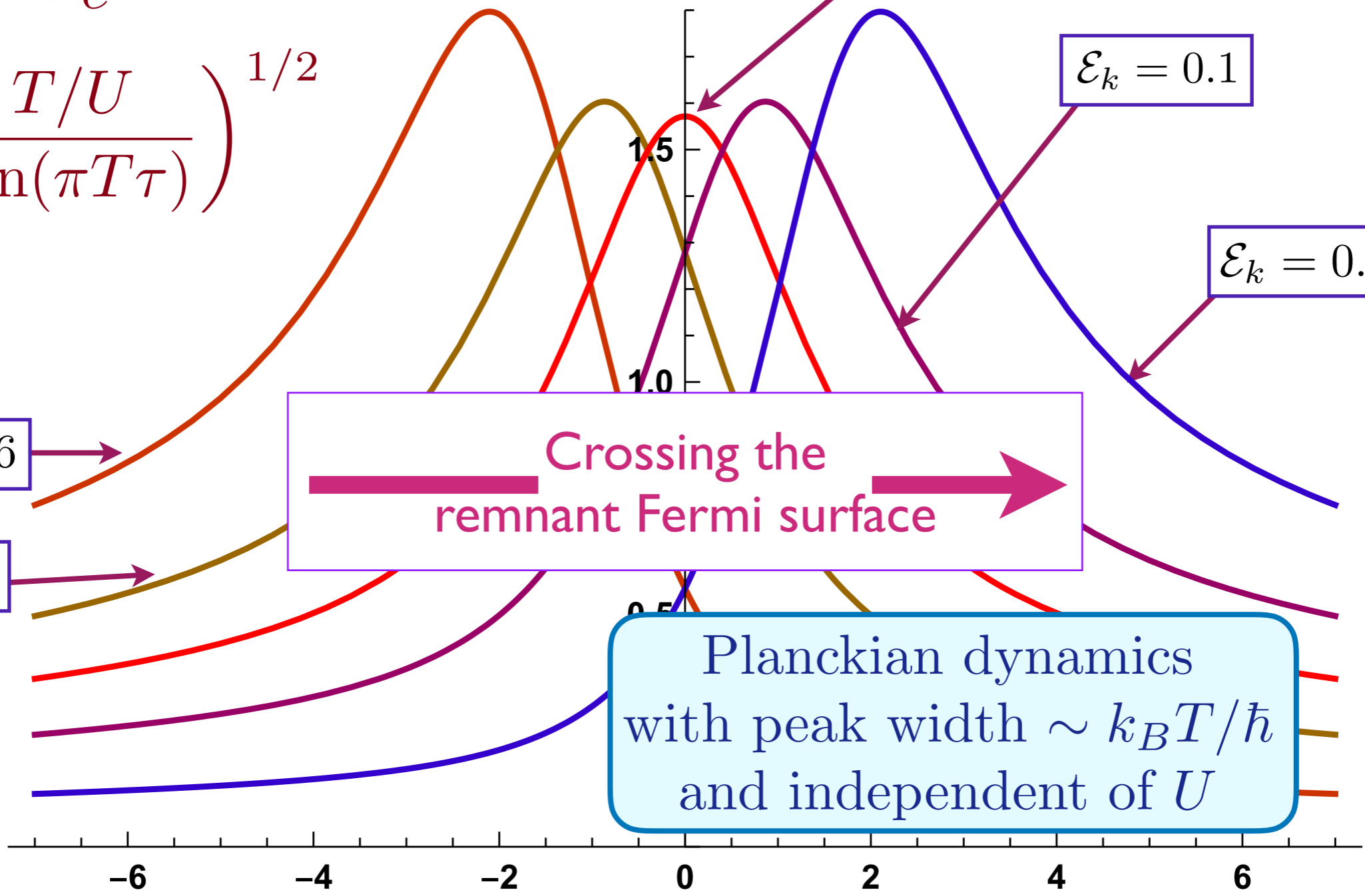
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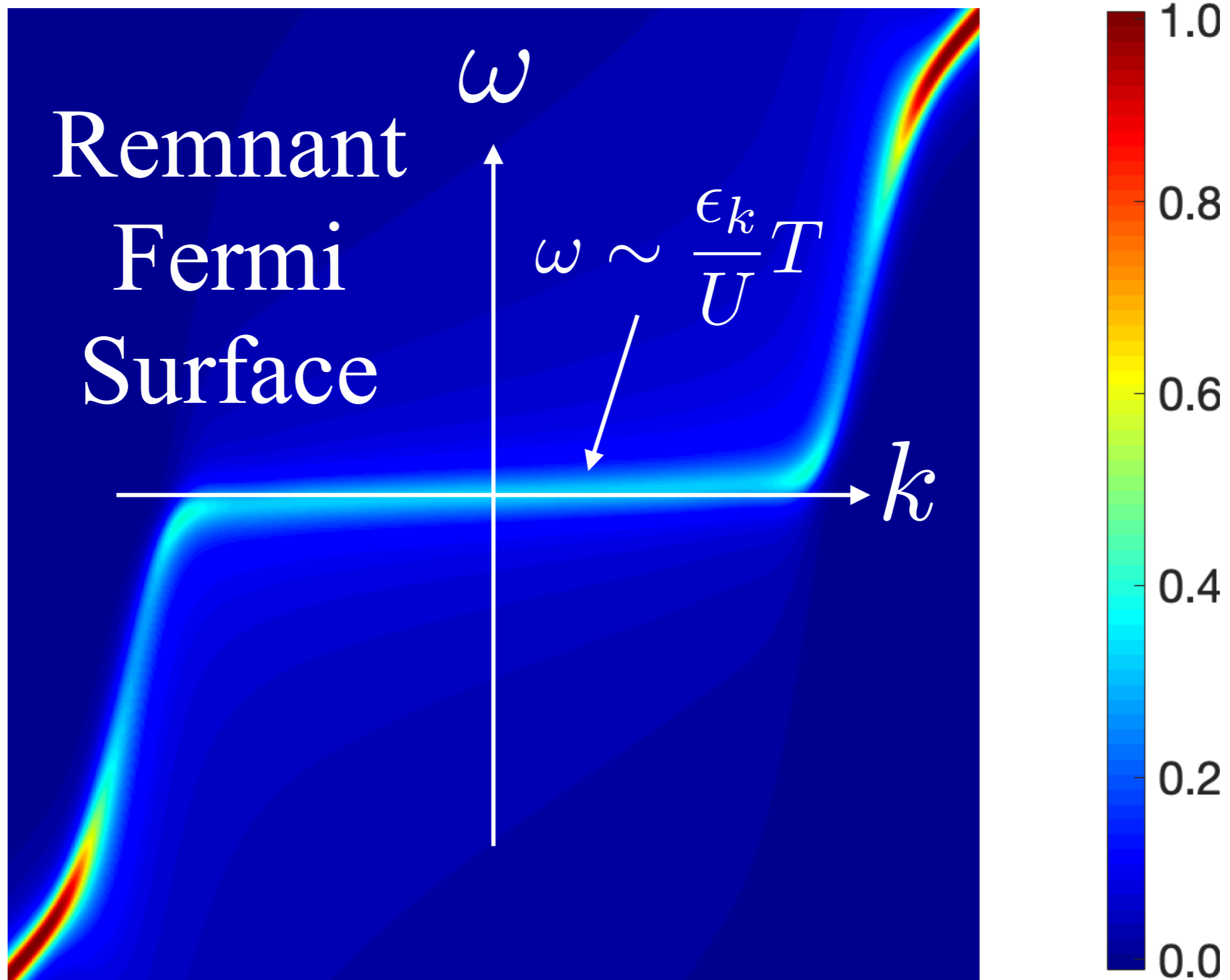
Crossing the remnant Fermi surface

Planckian dynamics with peak width  $\sim k_B T / \hbar$  and independent of  $U$



A. Georges and O. Parcollet PRB **59**, 5341 (1999)  
 S. Sachdev, PRX **5**, 041025 (2015)

$$\hbar\omega / (k_B T)$$



## Resistivity of a Planckian metal as $T \rightarrow 0$

From the Kubo formula,

$$\sigma = \frac{e^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[ \text{Im} G_{\text{SYK}}^R \left( \epsilon, \frac{\omega}{T} \right) \right]^2 \text{sech}^2 \left( \frac{\omega}{2T} \right)$$

where the Fermi surface is defined by  $\epsilon_k = 0$ ,  $\mathbf{v}_F = \nabla_{\mathbf{k}} \epsilon_k$  on the Fermi surface, and

$$m^* = \frac{d V_{FS}}{\oint_{FS} |\mathbf{v}_F|},$$

with  $d$  the spatial dimensionality, and  $V_{FS}$  is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual  $m^*$ .

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Evaluating the integrals, we find

$$\rho = \frac{m^*}{n e^2} 2.71 \mathbb{C} \frac{k_B T}{\hbar}, \quad \text{using } \mathcal{E} = \mathbb{C} \epsilon / U,$$

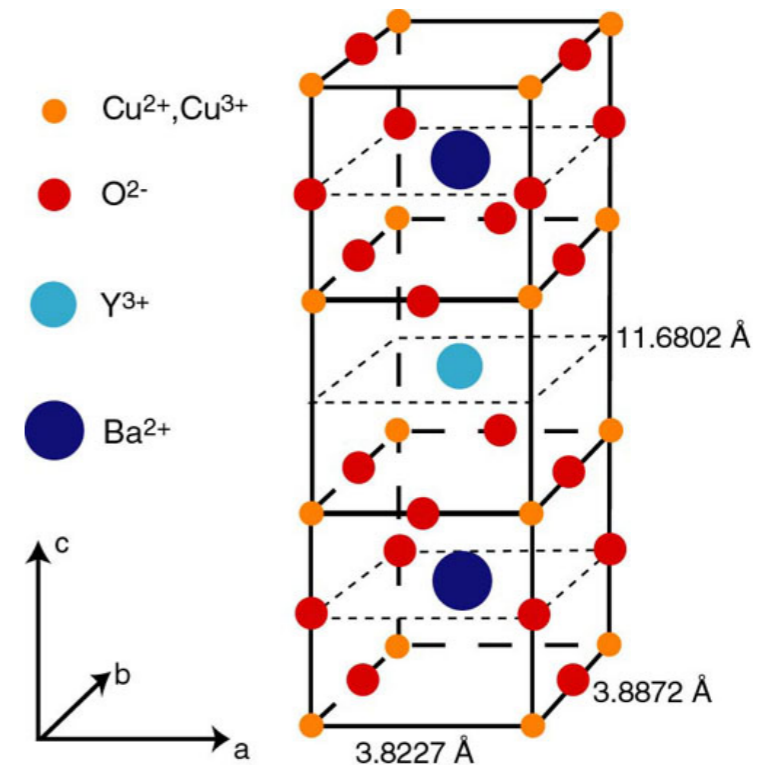
where  $n = V_{FS} / (2\pi)^d$  is the density.

# Resistivity of a Planckian metal as $T \rightarrow 0$

$$\rho = \frac{m^*}{ne^2} 2.71\mathbb{C} \frac{k_B T}{\hbar}$$

Note that all explicit dependence on  $U$  has cancelled out!

Choosing  $\mathbb{C} = 0.41$  as in the SYK model, we have the prefactor  $2.71\mathbb{C} = 1.11$ .



Aavishkar Patel

1. SYK lattice models

2. Planckian metal ansatz

3. Resonant SYK models

4. Random  $t$ - $J$ - $U$  models

# Generalized SYK models

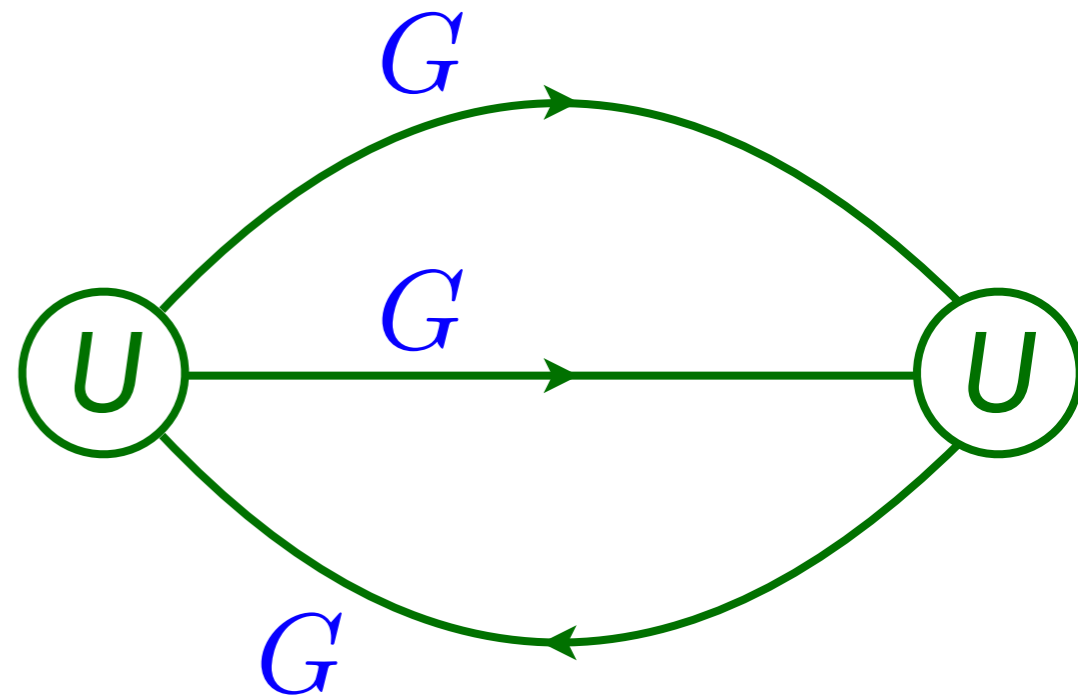
$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$  is a random function of  $\alpha\beta\gamma\delta$   
 $\epsilon_k$  has a bandwidth  $W$ .

The large  $N$  limit is still given by the sum of “melon” diagrams.

$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)}$$

$$\Sigma =$$



# Generalized SYK models

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For many generic models in this class with  $U \gg W$ ,  
 $\hbar\omega/(k_B T)$  scaling of SYK holds for  $W^2/U \ll k_B T \ll U$ ,  
and Fermi liquid theory is recovered for  $k_B T \ll W^2/U$ .

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017);  
Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Verman,  
Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy,  
Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

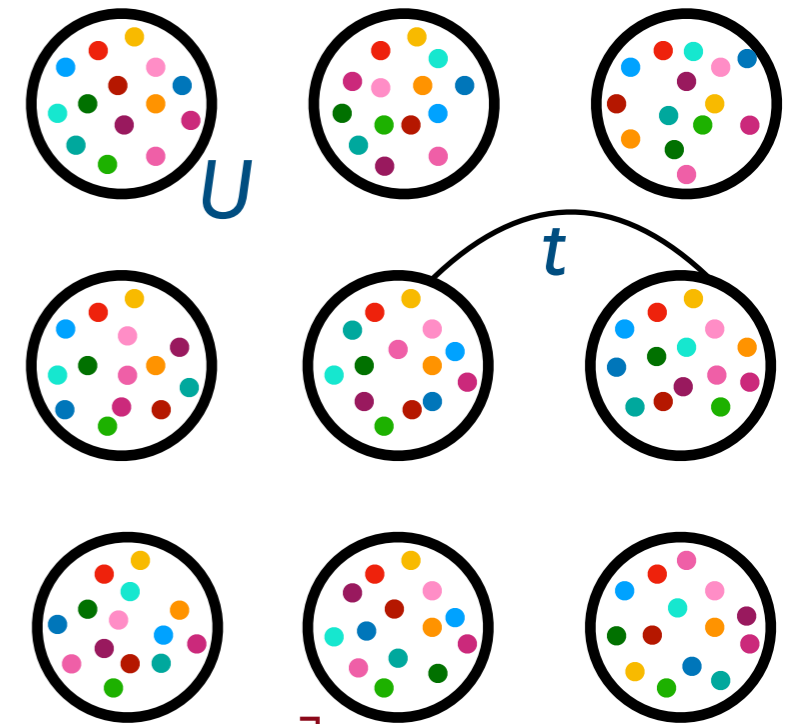
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999); Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

# A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$  is a random function of  $\alpha\beta\gamma\delta$   
 $\epsilon_k$  has a bandwidth  $W$ .

Rewriting of lattice model of incoherent and bad metal in momentum space



$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = U^2 \left[ \delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right]$$

# Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$  is a random function of  $\alpha\beta\gamma\delta$   
 $\epsilon_k$  has a bandwidth  $W$ .

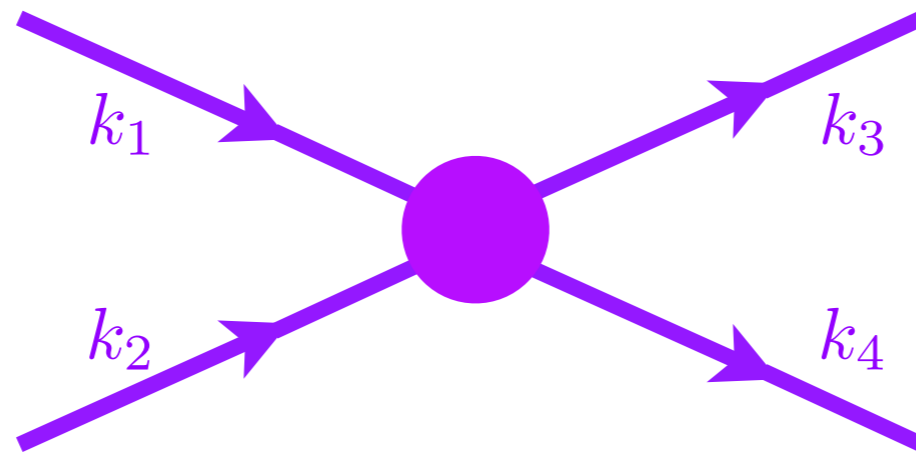
We examine a model with weaker  $W \lesssim U$ , but impose a **resonance condition**.

This leads to a solution which obeys the Planckian ansatz as  $T \rightarrow 0$ .

$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = \\ U^2 \left[ \delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right] \\ \times \left[ \delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} + \epsilon_{k_6} - \epsilon_{k_7} - \epsilon_{k_8}) \right]$$

This implies off-site interactions with correlations which decay with a power-law in space.

# Resonant SYK model



Interactions with  $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$  are non-resonant: we “integrate these out” in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion  $\epsilon_k$ , which we have already accounted for.

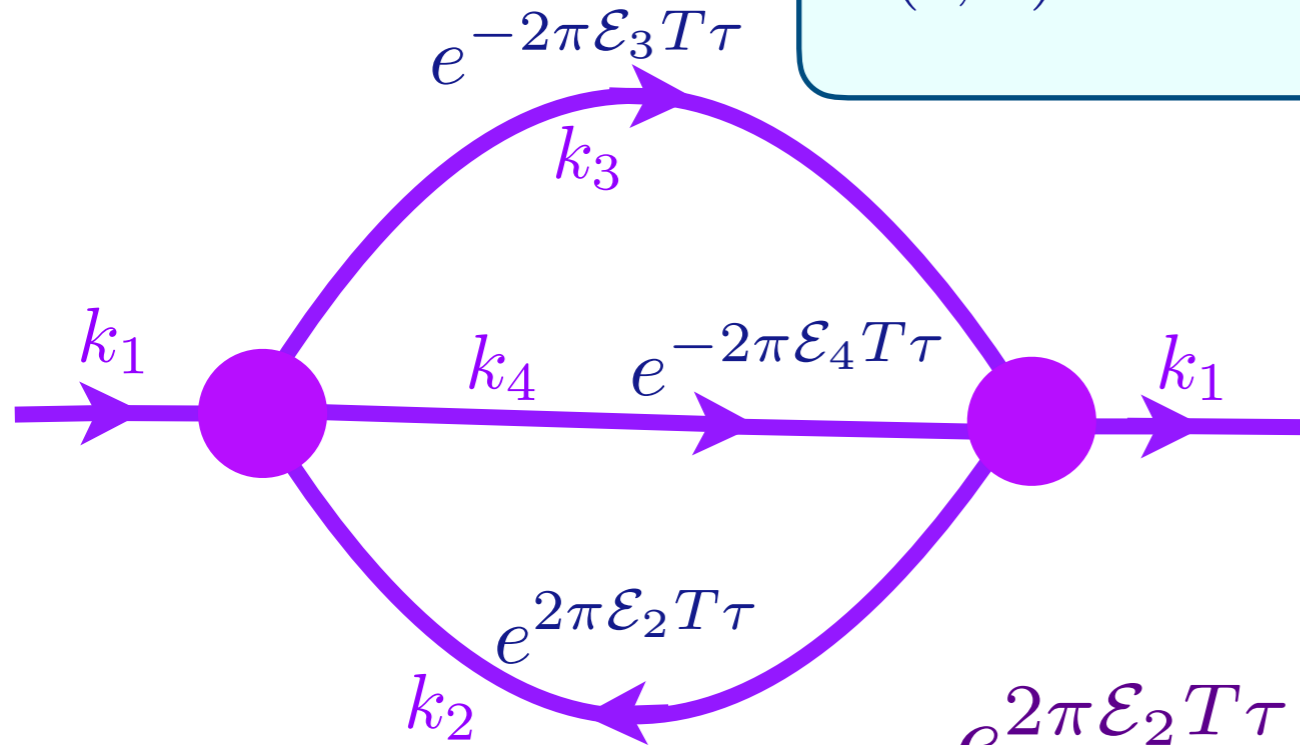
Keep only the interactions resonant in the bare quasiparticle energy with  $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$  and account for them with a self-consistent SYK-like analysis.

This is precisely the effective Hamiltonian method, when low energy states are separated from high energy states by a gap; we are assuming it can also apply in a gapless system.

# Resonant SYK model

Conformal Green's function at  $T > 0$  must have the form

$$G(\epsilon, \tau) \sim e^{-2\pi\mathcal{E}T\tau} \left( \frac{T/U}{\sin(\pi T\tau)} \right)^{1/2}, \quad 0 < \tau < 1/T.$$



$$e^{2\pi\mathcal{E}_2 T\tau} e^{-2\pi\mathcal{E}_3 T\tau} e^{-2\pi\mathcal{E}_4 T\tau} = e^{-2\pi\mathcal{E}_1 T\tau}$$

if

$$\mathcal{E}_a = \mathbb{C}\epsilon_a/U$$

and

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$$

SYK behavior in a Planckian metal as  $T \rightarrow 0$  with a remnant Fermi surface:  
 $G(k, \omega) = G_{\text{SYK}}(\epsilon_k, \hbar\omega/(k_B T))$ ,  
 with  $\mathcal{E}_k = \mathbb{C}\epsilon_k/U$

# Planckian metals with a remnant Fermi surface

- Resonant SYK models are compressible and dispersive quantum systems with  $\hbar\omega/(k_B T)$  scaling as  $T \rightarrow 0$ .
- The resonance condition is supported by a RG argument: non-resonant interactions mainly renormalize the underlying quasi-particle dispersion  $\epsilon_k$ , while resonant interactions have to be treated self-consistently.
- The resonance is a single ‘fine-tuning’ condition designed to obtain  $\hbar\omega/(k_B T)$  scaling as  $T \rightarrow 0$ . However, then many other nice features follow: we obtain a Planckian metal with remnant large Fermi surface at  $\epsilon_k = 0$ , and an effective mass  $m^*$  defined by the dispersion of  $\epsilon_k$ , with a resistivity  $\rho \sim (m^*/(ne^2))k_B T/\hbar$  independent of the strength of interactions.



Aavishkar Patel (Harvard → Miller Fellow at Berkeley)



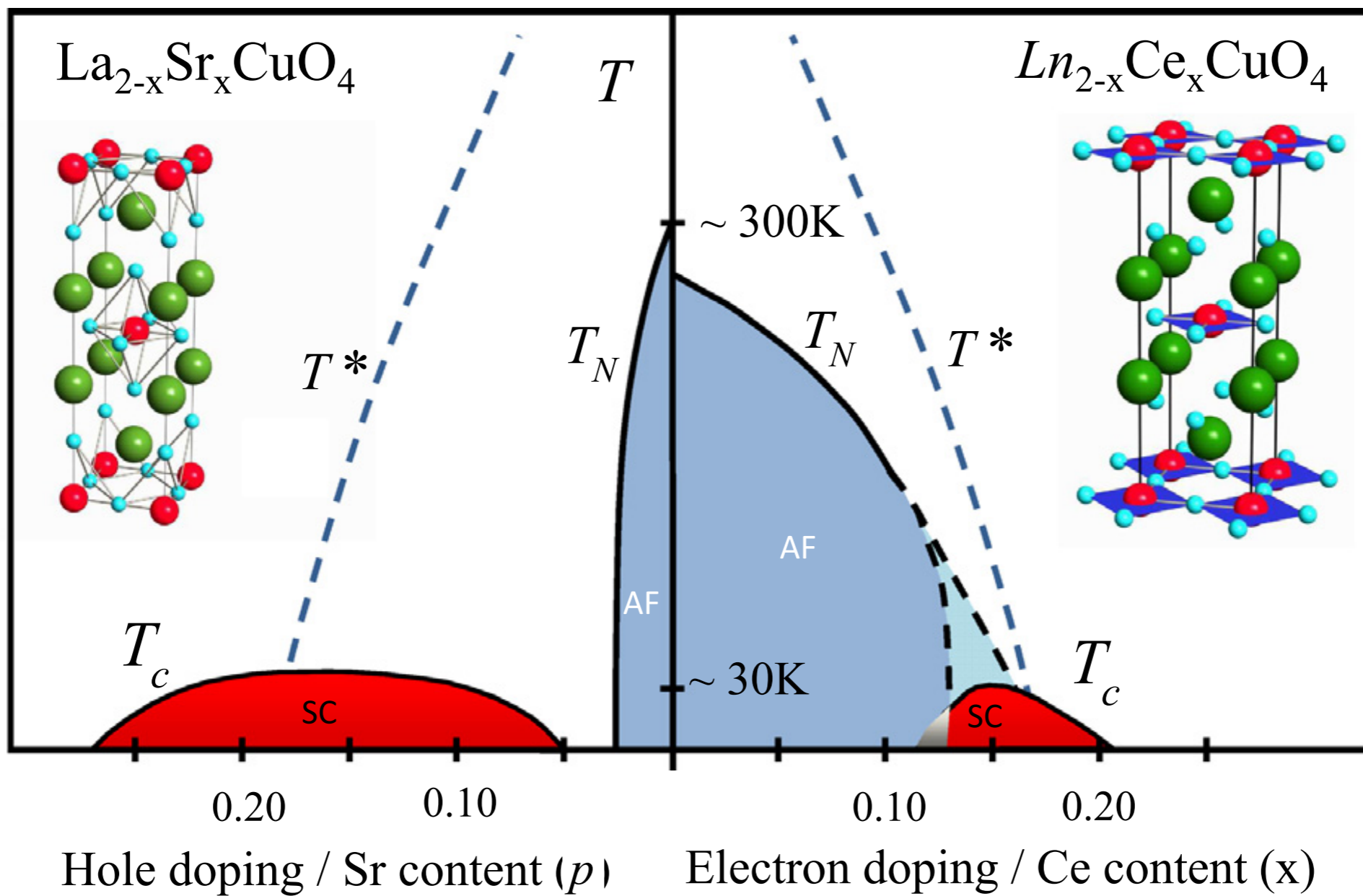
1. SYK lattice models

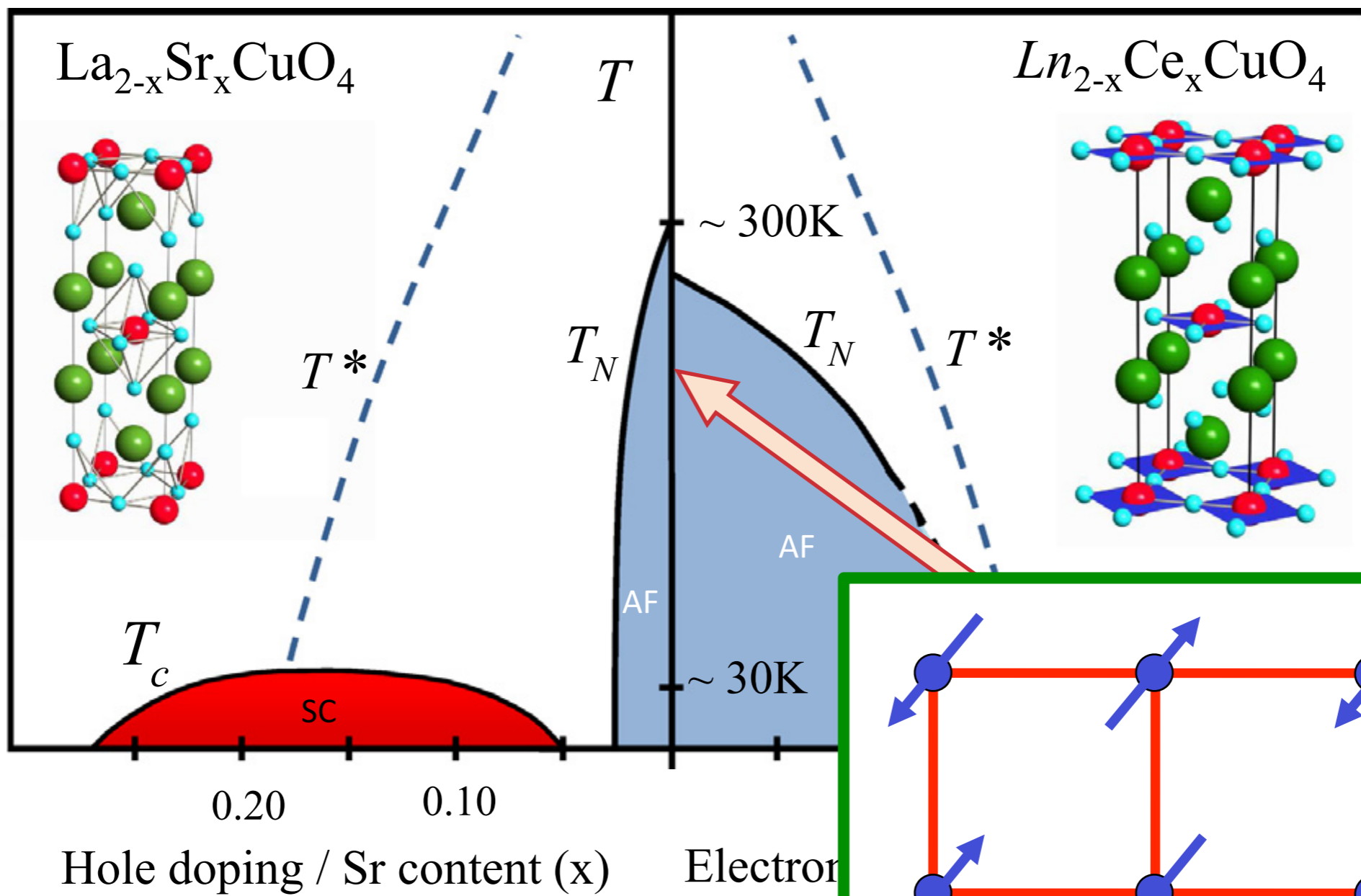
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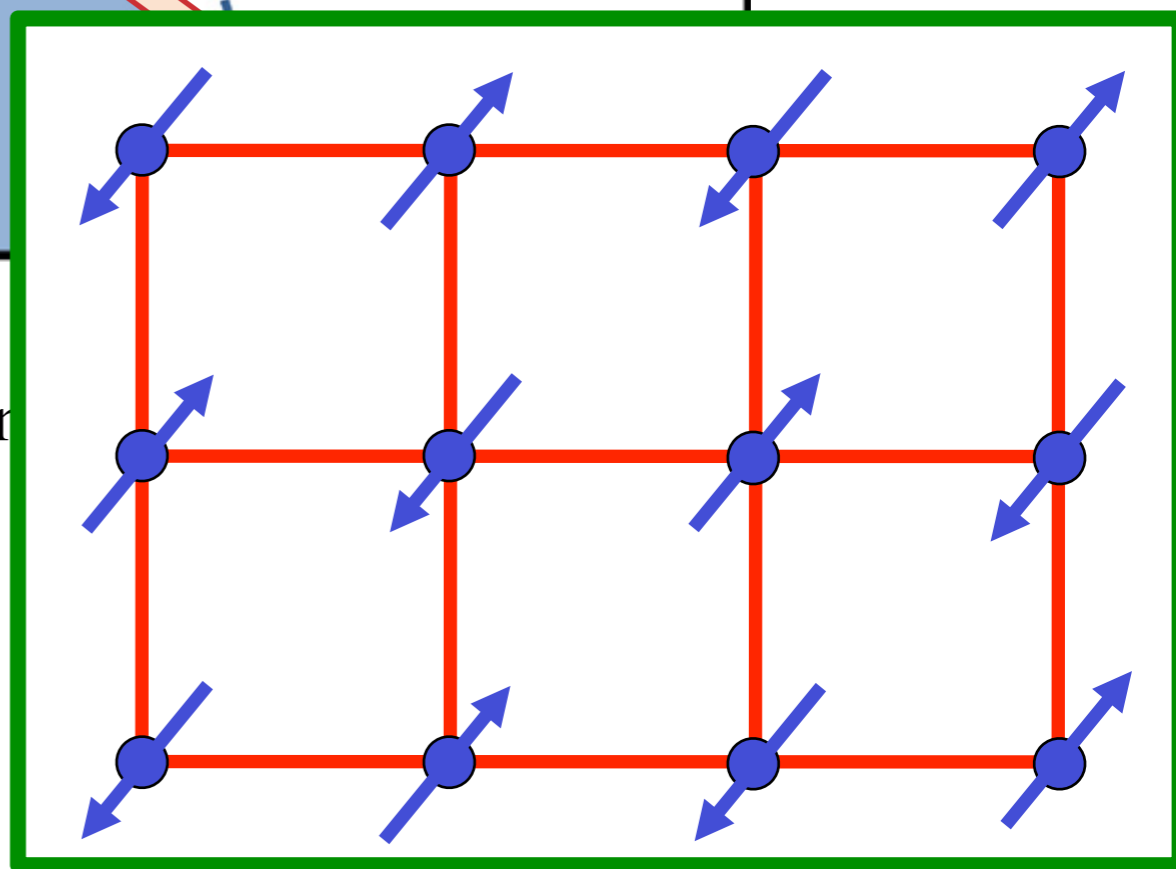
4. Random  $t$ - $J$ - $U$  models

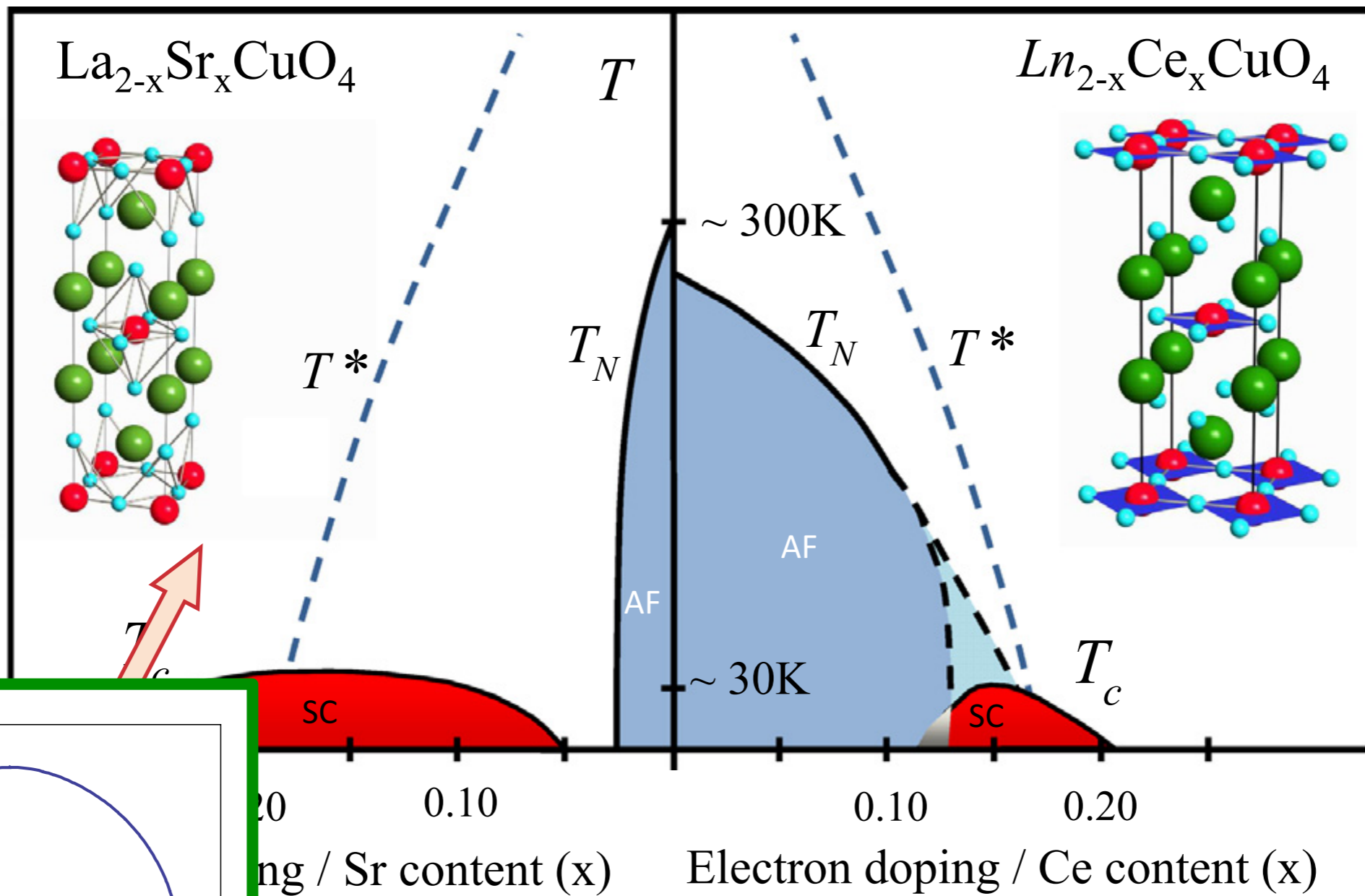
Ongoing work with Antoine Georges, Yingfei Gu,  
Darshan Joshi, Chenyuan Li, Olivier Parcollet,  
Gregory Tarnopolsky





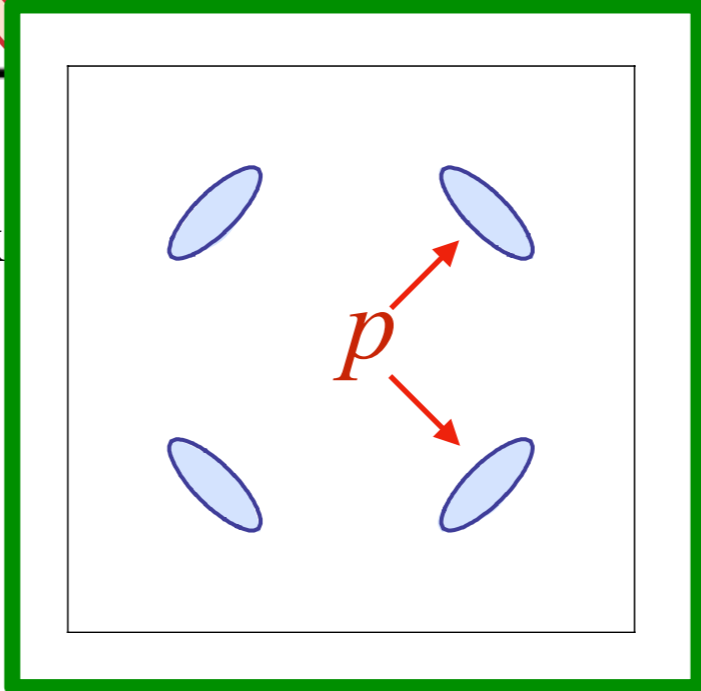
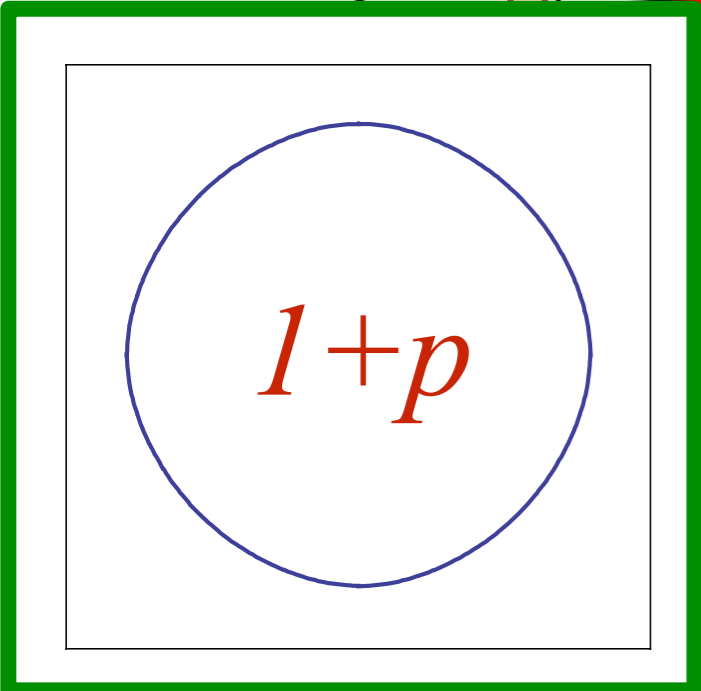
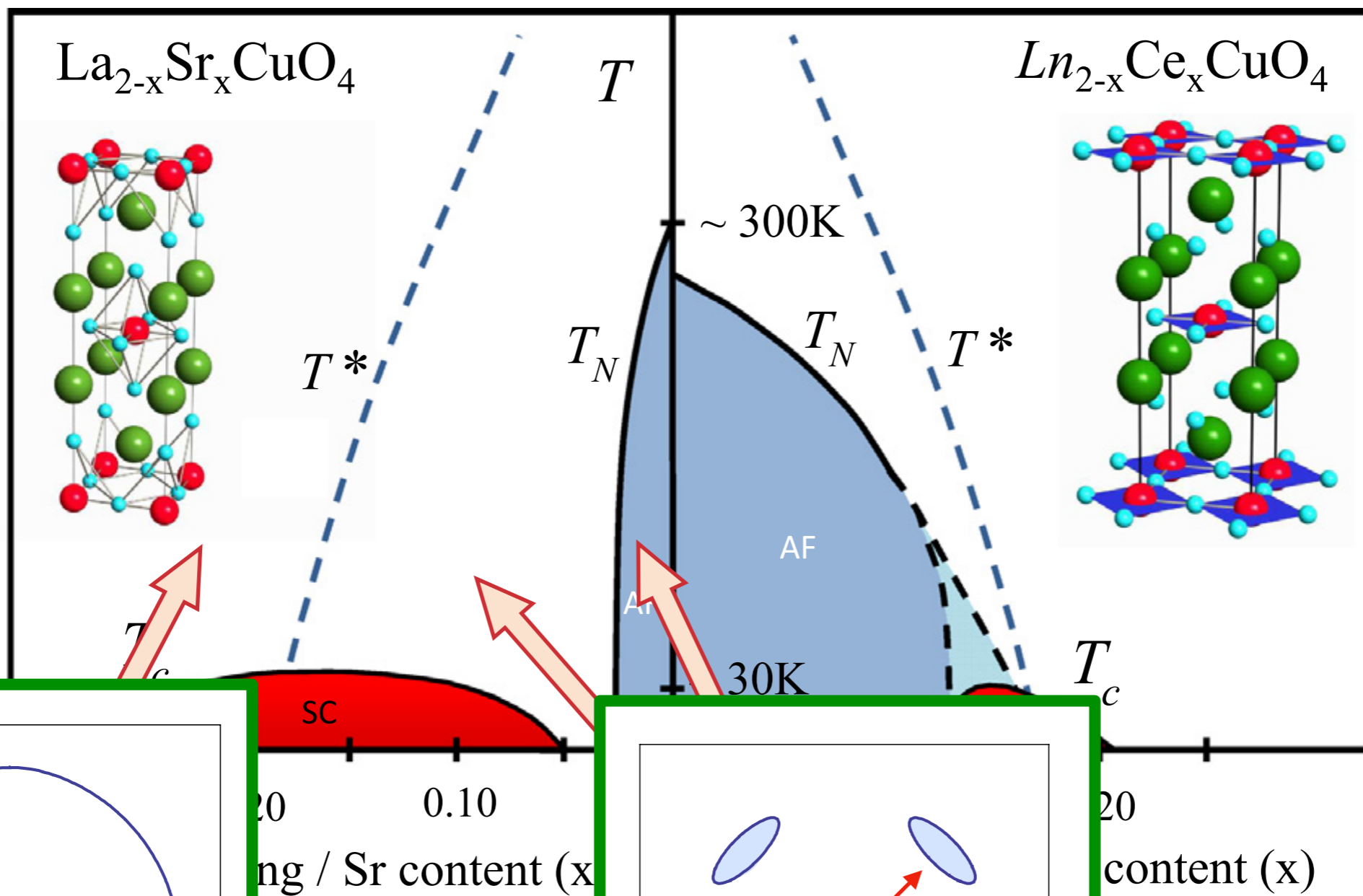
Insulating Antiferromagnet

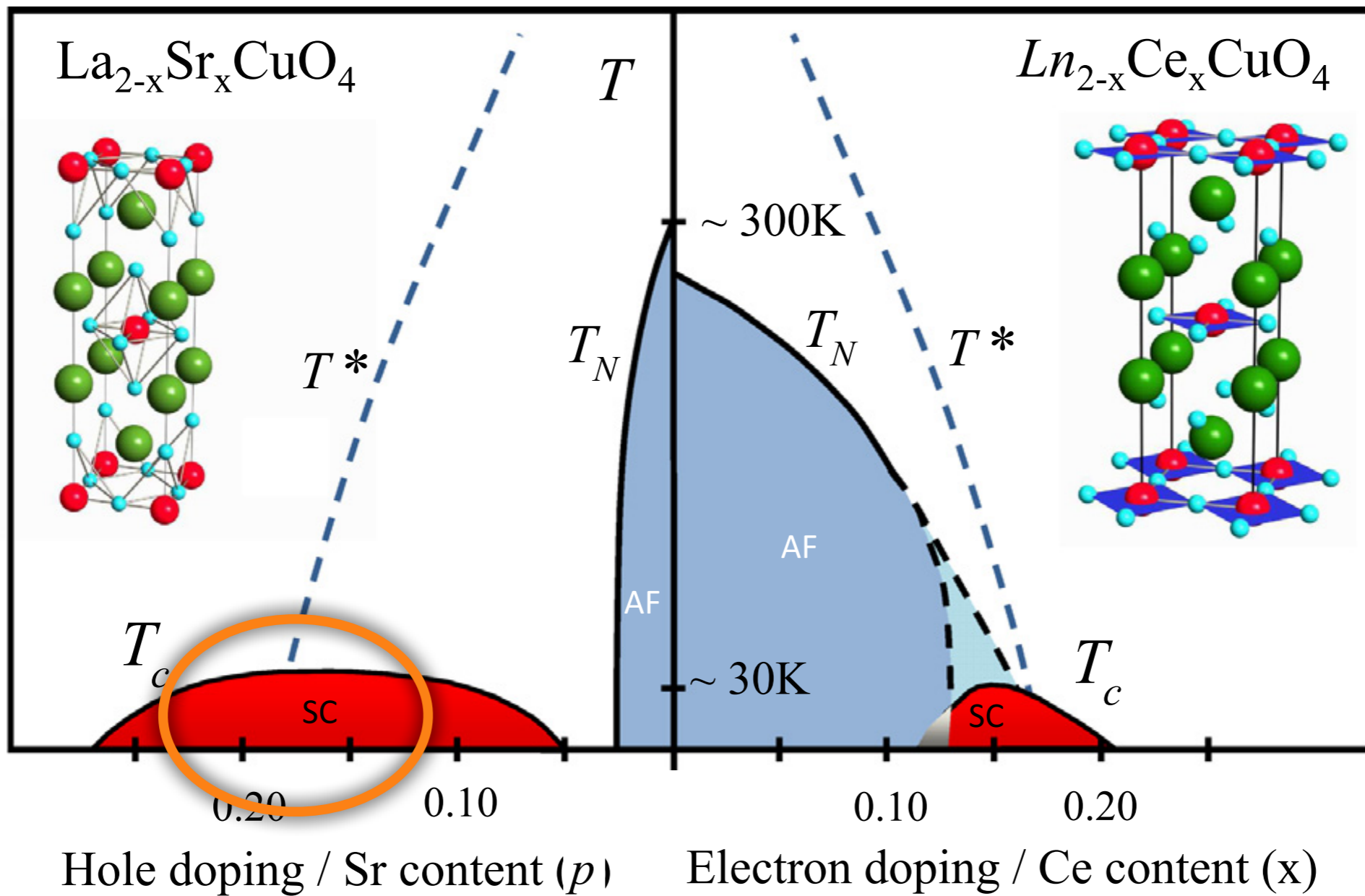




$1+p$

Conventional Fermi liquid

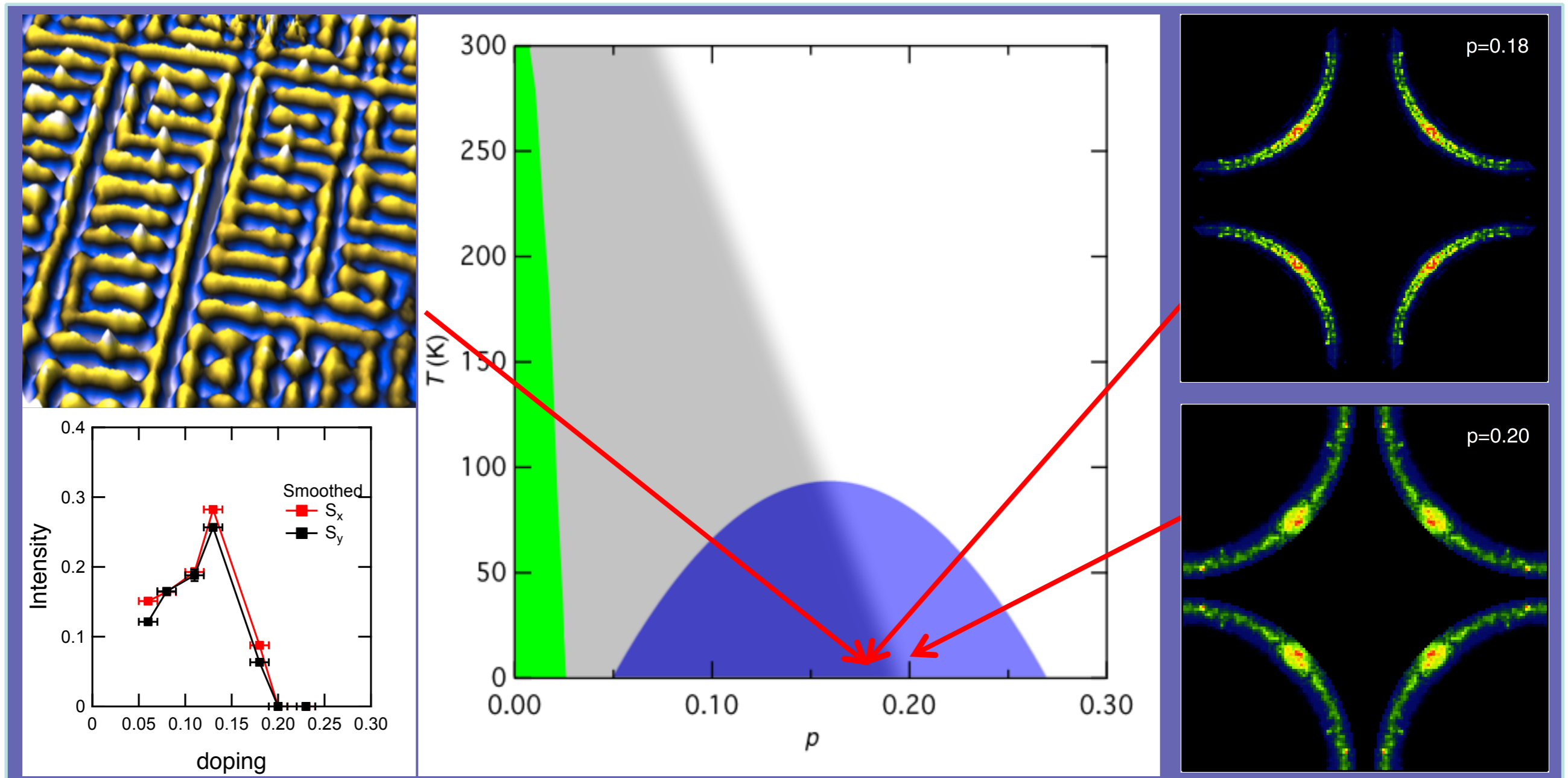




# Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, *Science* **344**, 608 (2014)

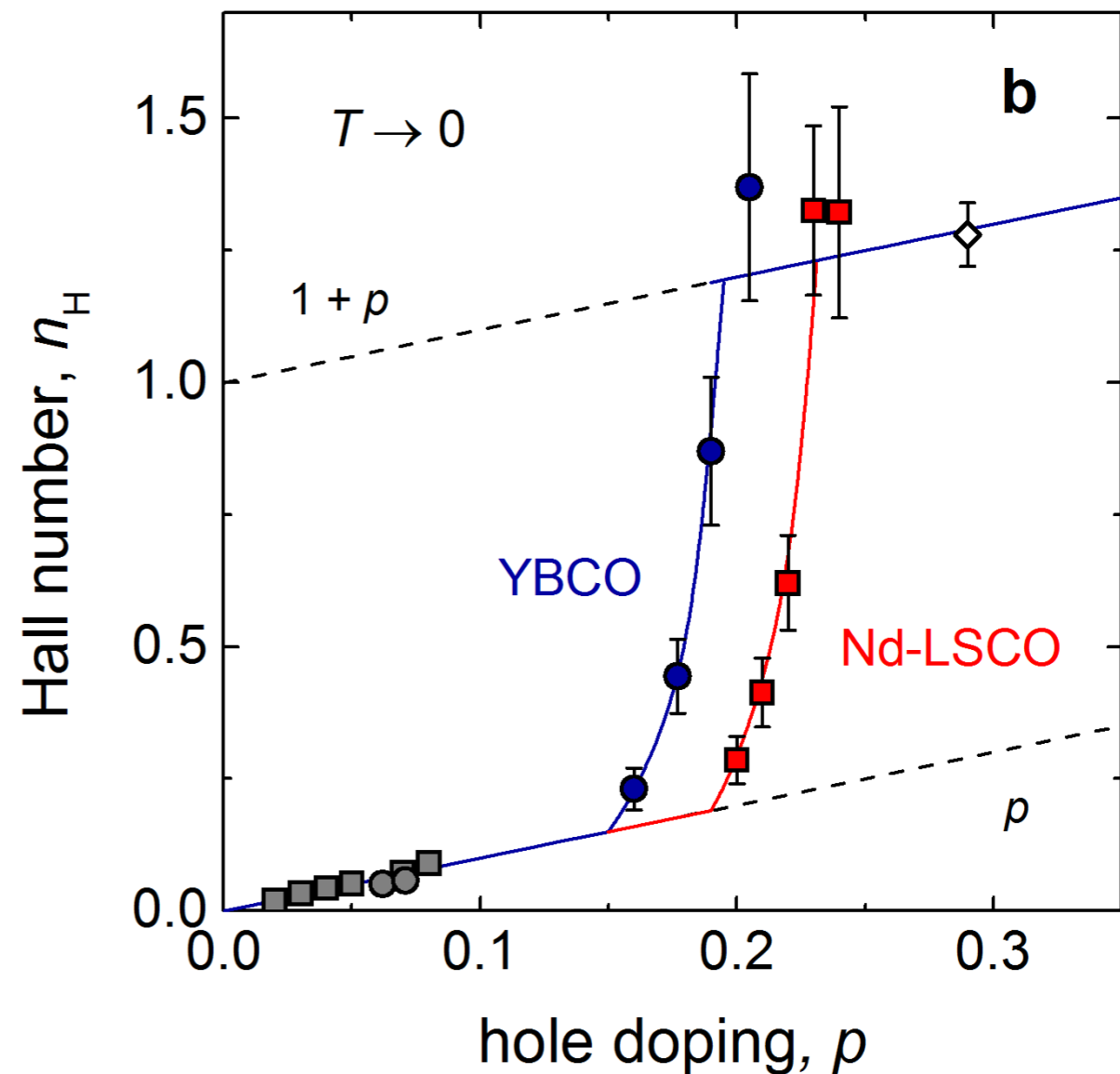
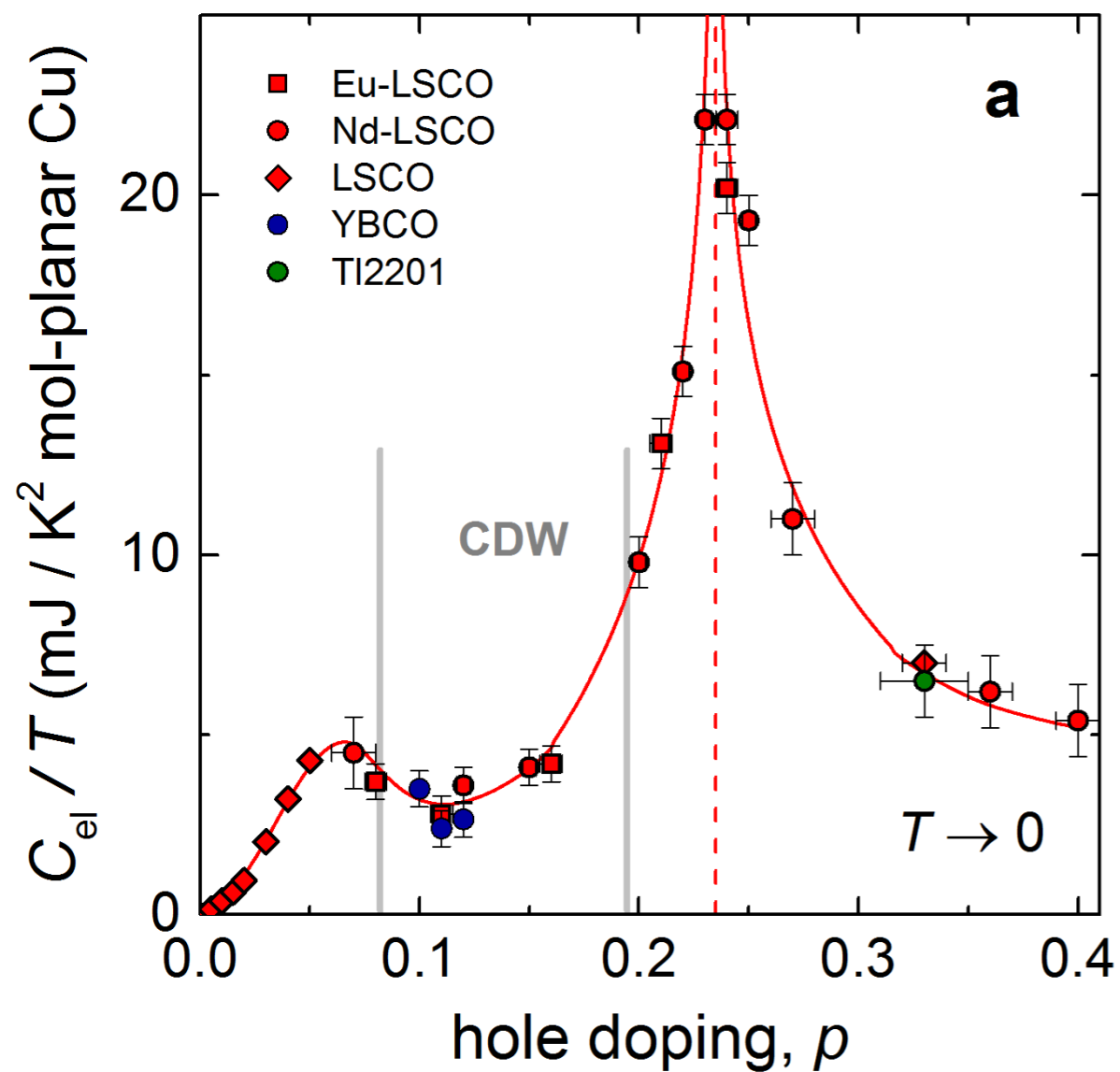
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, *Science* **344**, 612 (2014)

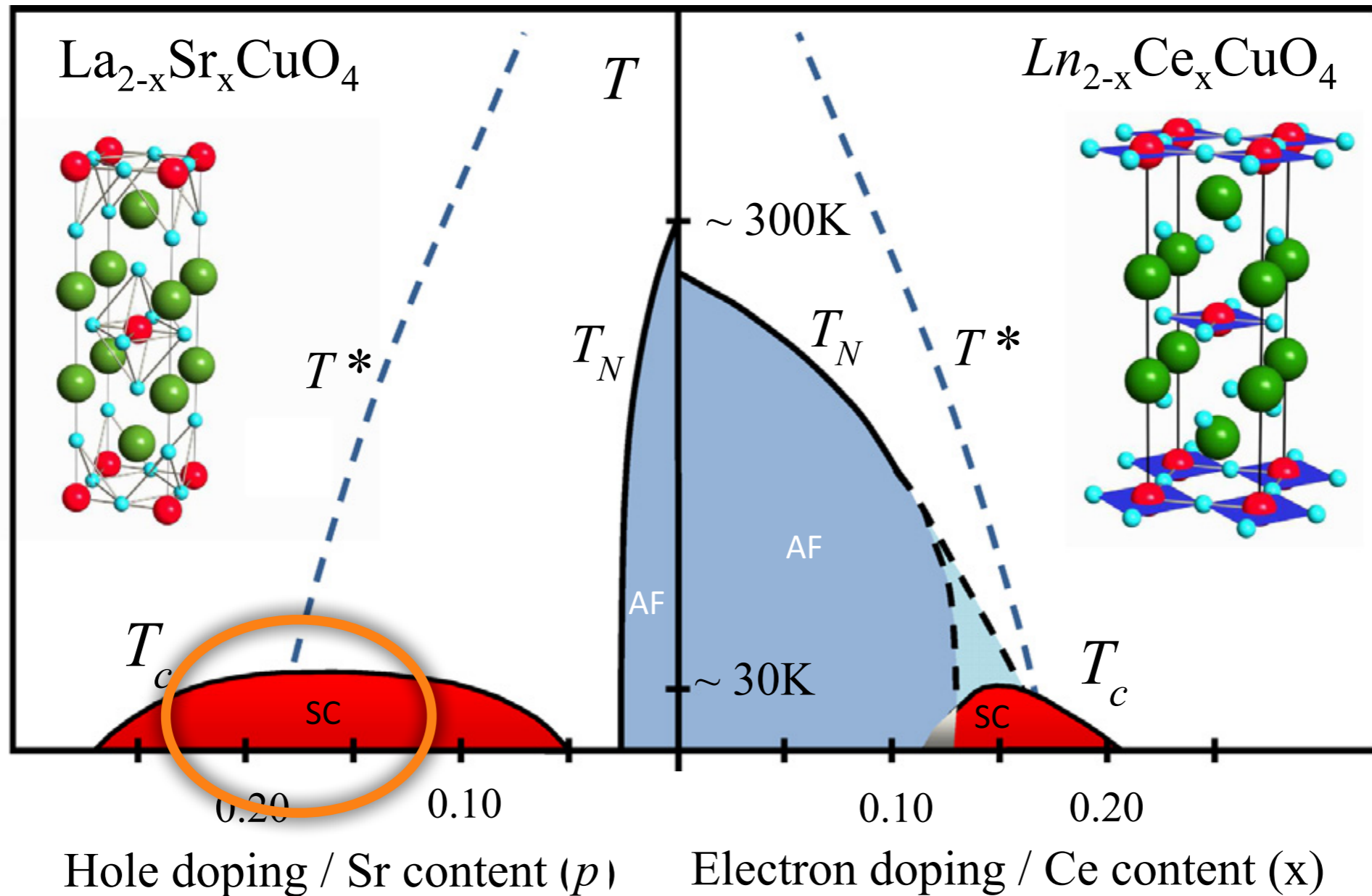


# Hole doped cuprates

## The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507





Is there a quantum critical point near optimal doping, not associated with the onset of antiferromagnetism ?

# Random $t$ - $J$ - $U$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$

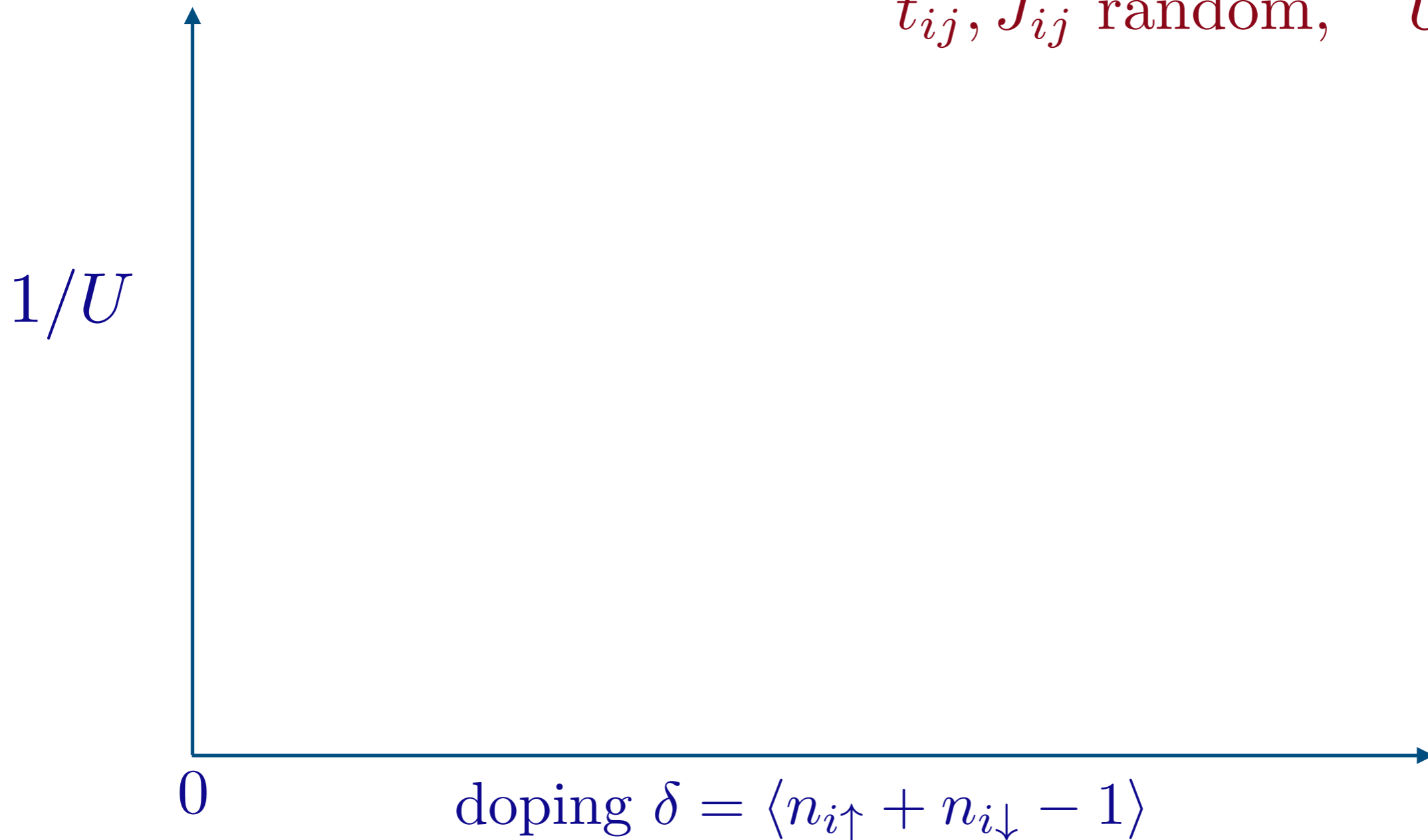
$$t_{ij}, J_{ij} \text{ random}, \quad U > 0$$

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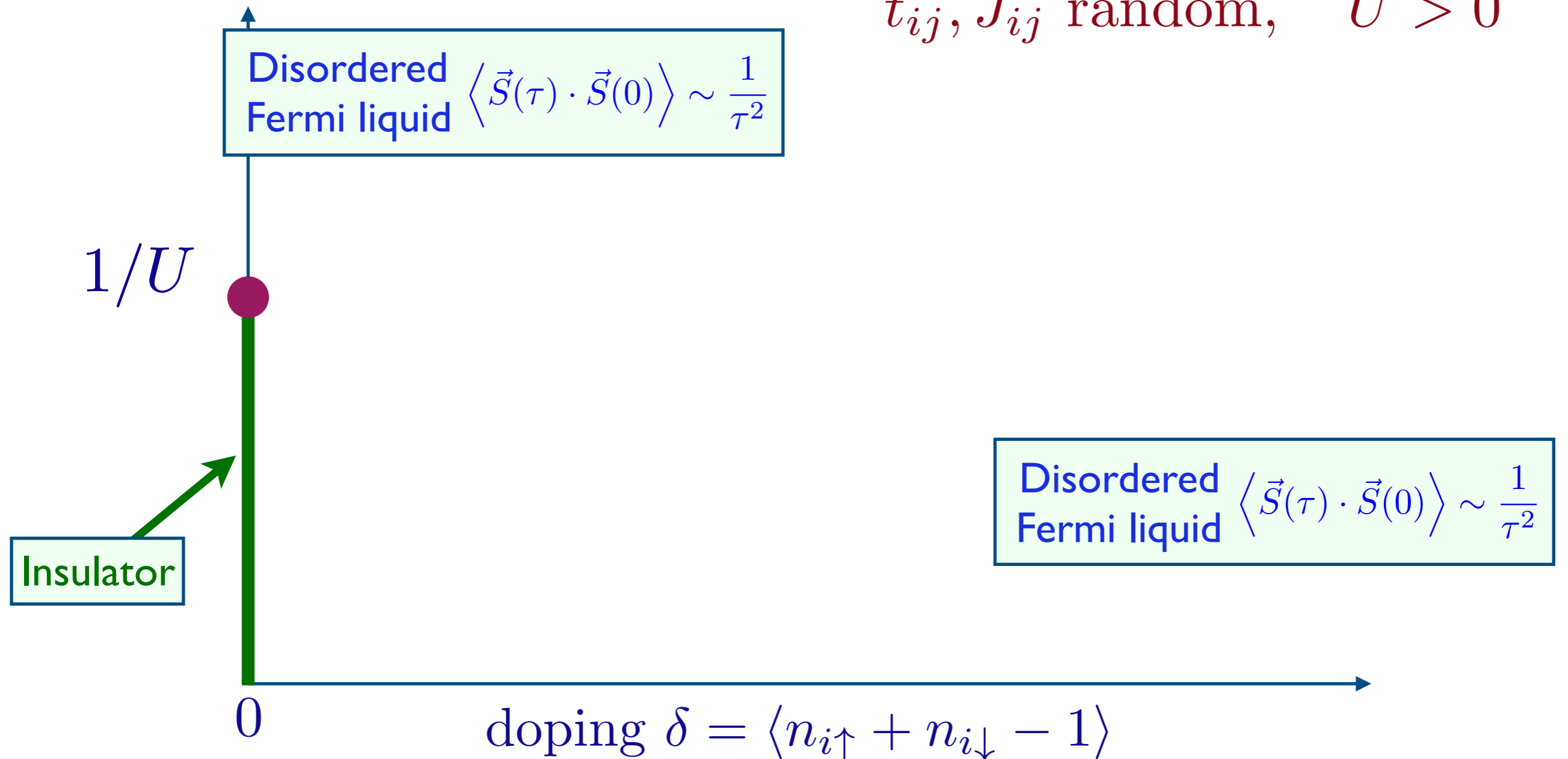


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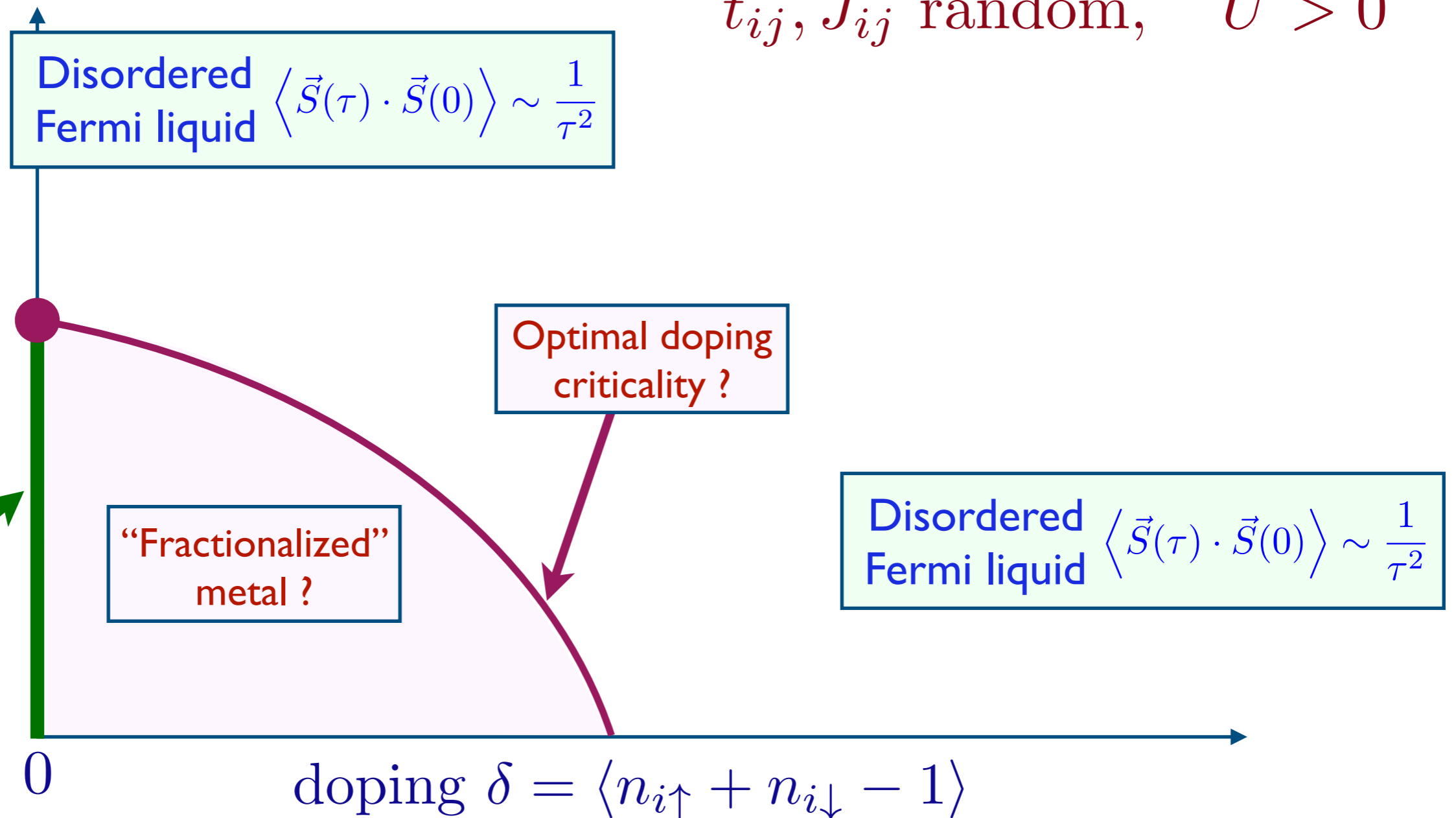


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$t_{ij}, J_{ij}$  random,  $U > 0$



# Large $N$ limit

$$\mathcal{Z} = \int \mathcal{D}c_\alpha(\tau) e^{-\mathcal{S}}$$

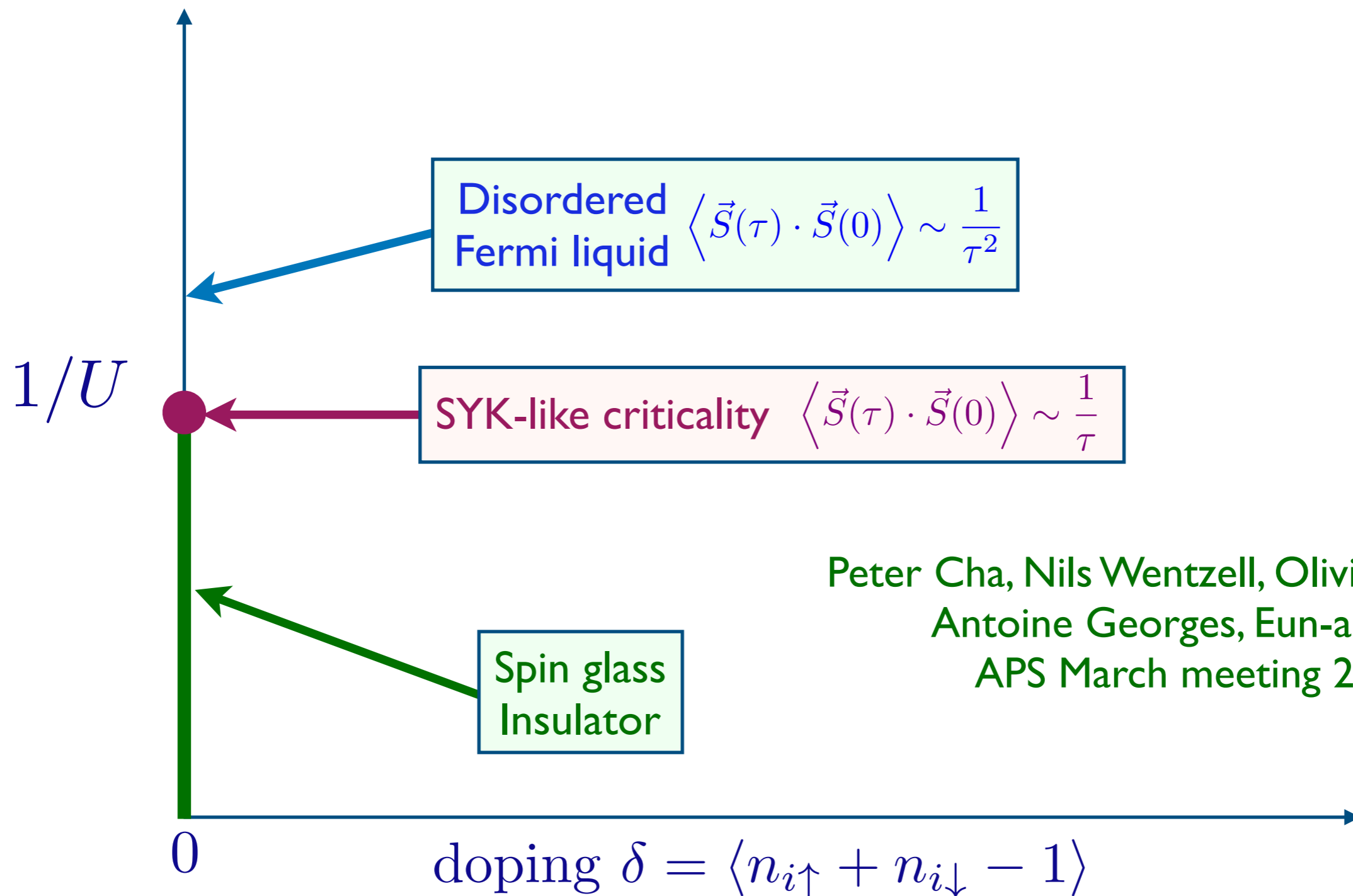
$$\begin{aligned} \mathcal{S} = & \int d\tau \left[ c_\alpha^\dagger(\tau) \left( \frac{\partial}{\partial \tau} - \mu \right) c_\alpha(\tau) + U n_{i\uparrow}(\tau) n_{i\downarrow}(\tau) \right] \\ & - t^2 \int d\tau d\tau' R^*(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') \\ & - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau'). \end{aligned}$$

The functions  $R(\tau)$  and  $Q(\tau)$  have to be determined self-consistently:

$$\begin{aligned} R(\tau - \tau') &= -\frac{1}{2} \langle c_\alpha^\dagger(\tau) c_\alpha(\tau') \rangle_{\mathcal{Z}} \\ Q(\tau - \tau') &= \frac{1}{3} \langle \vec{S}(\tau) \cdot \vec{S}(\tau') \rangle_{\mathcal{Z}} \end{aligned}$$

# Large $N$ limit

Numerical solution (DMFT methods) at half-filling



Peter Cha, Nils Wentzell, Olivier Parcollet,  
Antoine Georges, Eun-ah Kim,  
APS March meeting 2019

# Large $N$ limit

and then a large  $M$  limit (interested in  $M=2$ )

Consider the hole-doped case, in the limit of  $U \rightarrow \infty$ . Each site has 2 states which we map to the space of a boson  $b$  (the holon) and a fermion  $f_\alpha$  (the spinon):

$$\begin{aligned} |0\rangle &\Rightarrow b^\dagger |v\rangle & , & & c_\alpha^\dagger |0\rangle &\Rightarrow f_\alpha^\dagger |v\rangle \\ c_\alpha &= f_\alpha b^\dagger & , & & f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

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To obtain a large  $M$  limit, let  $\alpha = 1 \dots M$ , endow the boson with an ‘orbital’ index  $a = 1 \dots M'$  and send  $M \rightarrow \infty$  at fixed  $k = M'/M$ . Then

$$c_{a\alpha} = f_\alpha b_a^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b_a^\dagger b_a = \frac{M}{2}$$

# Large $N$ limit

and then a large  $M$  limit (interested in  $M=2$ )

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green's functions:

$$\begin{aligned}G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\ \Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\ G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\ \Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k t^2 G_f(\tau) G_b(\tau) G_b(-\tau)\end{aligned}$$

Here  $\mu_f$  and  $\mu_b$  are chemical potentials chosen to satisfy

$$\langle f^\dagger f \rangle = \frac{1}{2} - k\delta \quad , \quad \langle b^\dagger b \rangle = \delta .$$

# Large $N$ limit

and then a large  $M$  limit (interested in  $M=2$ )

A condensed boson state with  $\langle b \rangle \neq 0$ , yields a disordered Fermi liquid.

However, critical states without boson condensation are also possible. These obey:

$$G_f(z) = C_f \frac{e^{-i(\pi\Delta_f + \theta_f)}}{z^{1-2\Delta_f}}, \quad G_b(z) = C_b \frac{e^{-i(\pi\Delta_b + \theta_b)}}{z^{1-2\Delta_b}}, \quad \text{Im}(z) > 0$$

$$\Delta_f + \Delta_b = \frac{1}{2}$$

$$\frac{\theta_f}{\pi} + \left( \frac{1}{2} - \Delta_f \right) \frac{\sin(2\theta_f)}{\sin(2\pi\Delta_f)} = k\delta$$

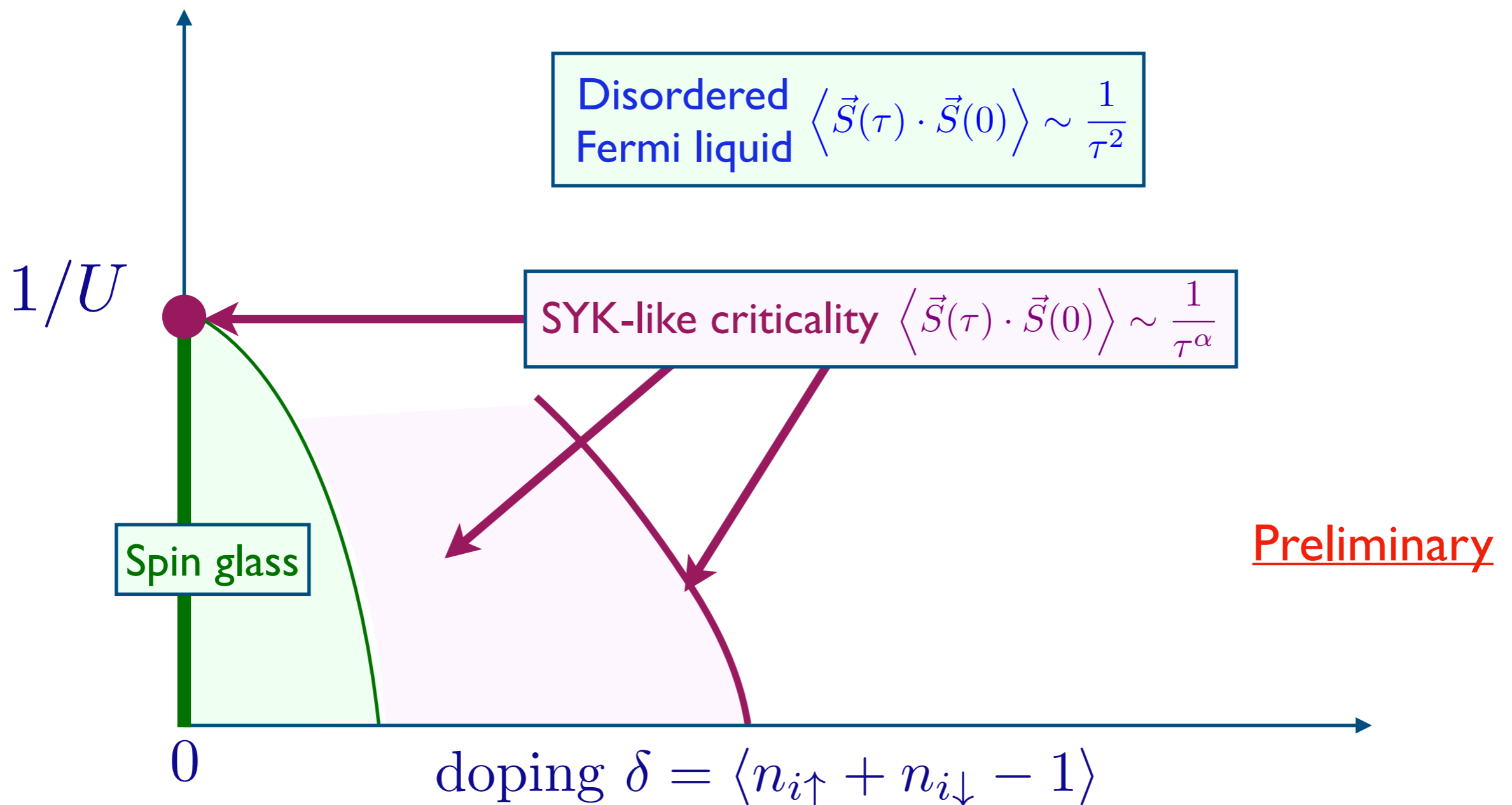
$$\frac{\theta_b}{\pi} + \left( \frac{1}{2} - \Delta_b \right) \frac{\sin(2\theta_b)}{\sin(2\pi\Delta_b)} = \frac{1}{2} + \delta.$$

The last two are analogs of ‘Luttinger’ relations, which follow from an anomaly matching argument (Yingei Gu, A. Kitaev, S. Sachdev, G. Tarnopolsky to appear).

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$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}, \quad t_{ij}, J_{ij} \text{ random}, \quad U > 0$$

Solve as  $N \rightarrow \infty$ : large  $M$  ( $SU(2) \rightarrow SU(M)$ ).



# 3 Many Body Random Matrix models

## The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} + \epsilon \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta;\gamma\delta}$  are independent random variables

with  $\overline{U_{\alpha\beta;\gamma\delta}} = 0$  and  $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$

# 3 Many Body Random Matrix models

## Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$  is a random function of  $\alpha\beta\gamma\delta$   
 $\epsilon_k$  has a bandwidth  $W$ .

$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = \\ U^2 \left[ \delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right] \\ \times \left[ \delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} + \epsilon_{k_6} - \epsilon_{k_7} - \epsilon_{k_8}) \right]$$

# 3 Many Body Random Matrix models

## Random $t$ - $J$ - $U$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$

$$t_{ij}, J_{ij} \text{ random,} \quad U > 0$$