

# Gauge theories of ultra quantum metals

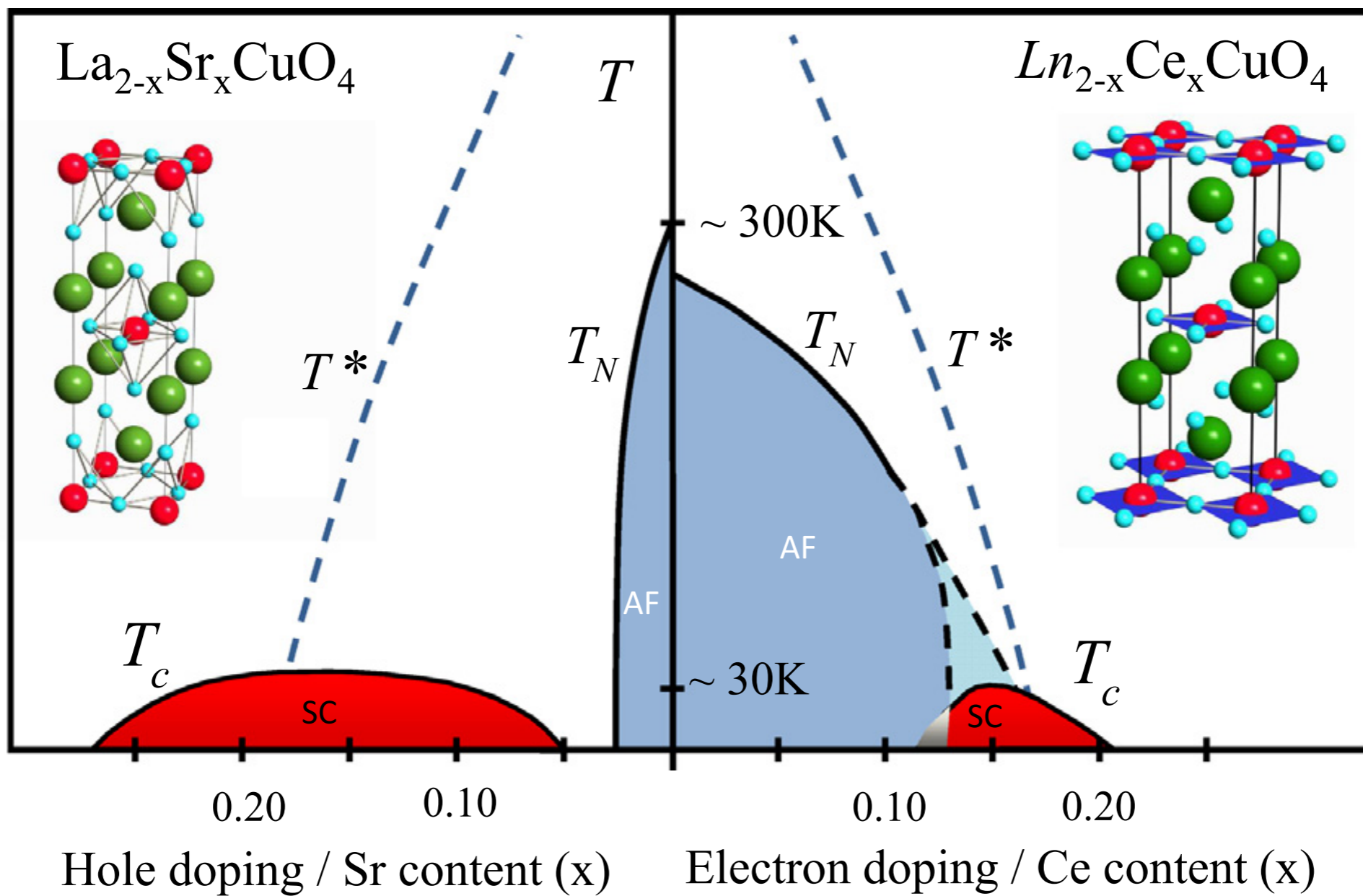
MPS Conference on Ultra Quantum Matter II  
Simons Foundation, New York  
August 23, 2018

Subir Sachdev



Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)





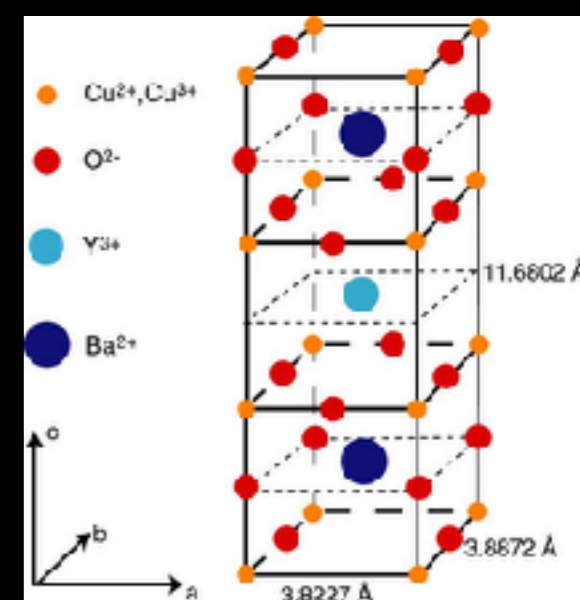
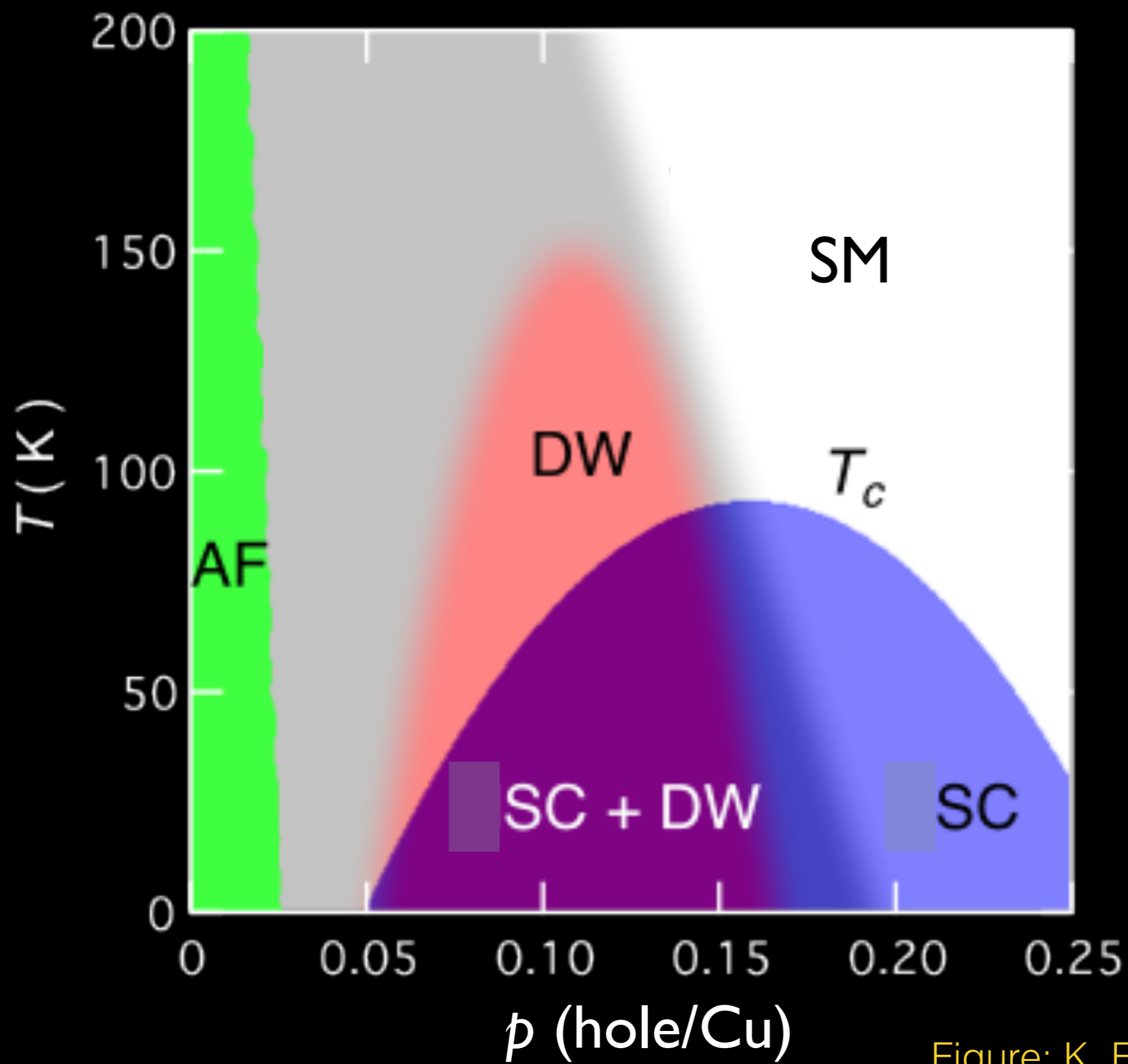
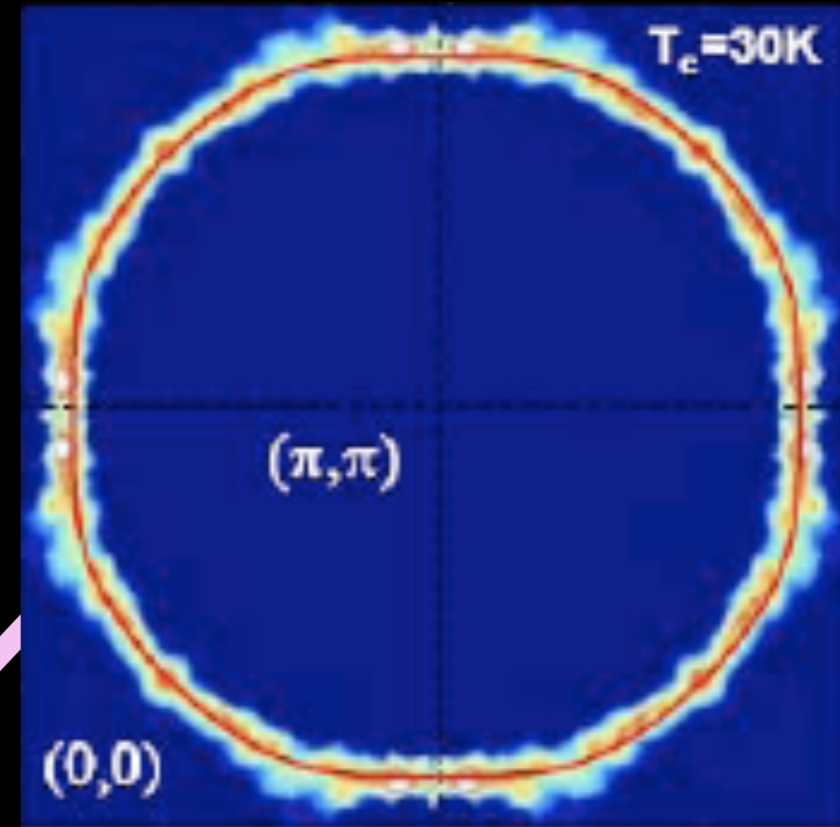
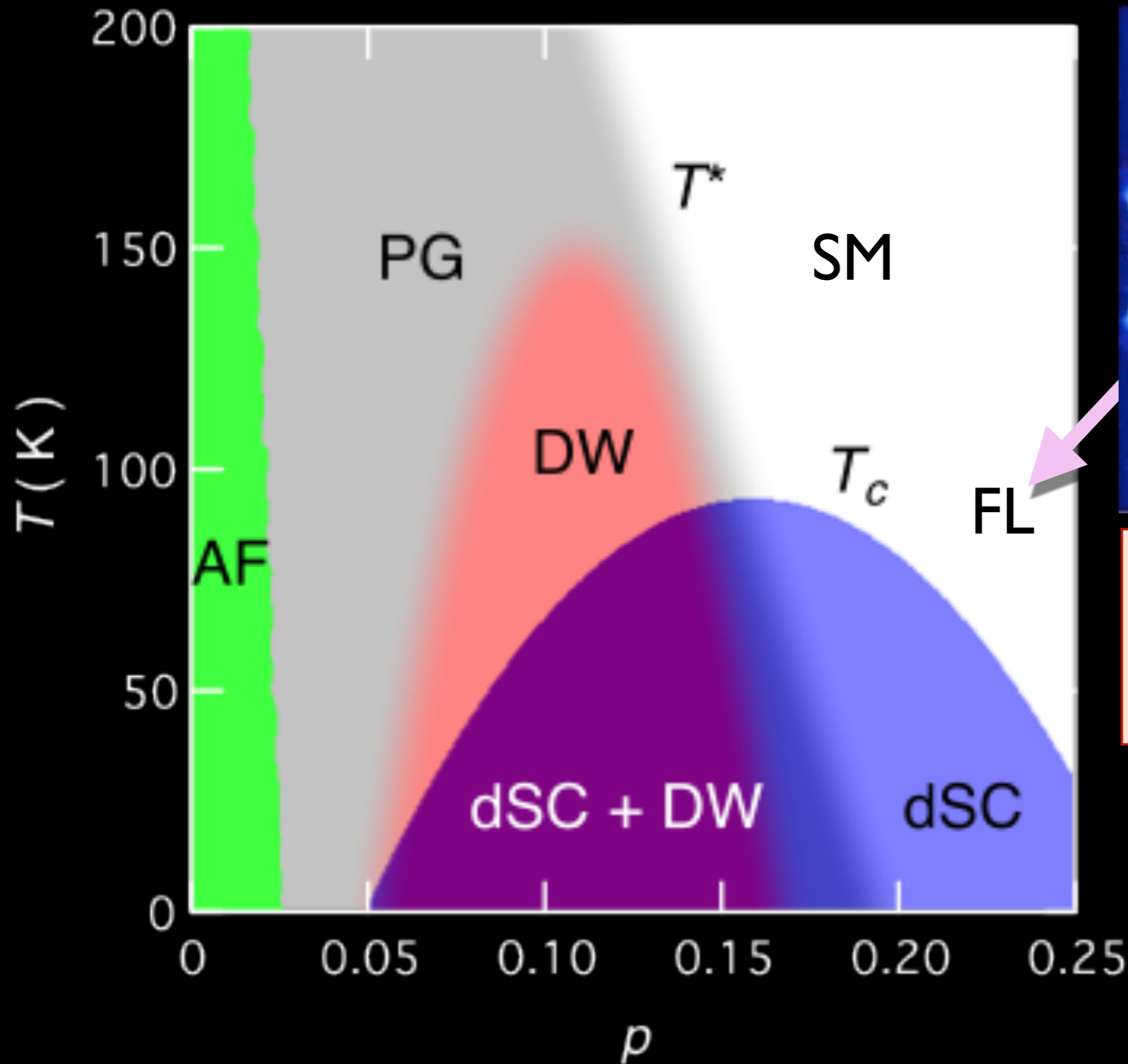


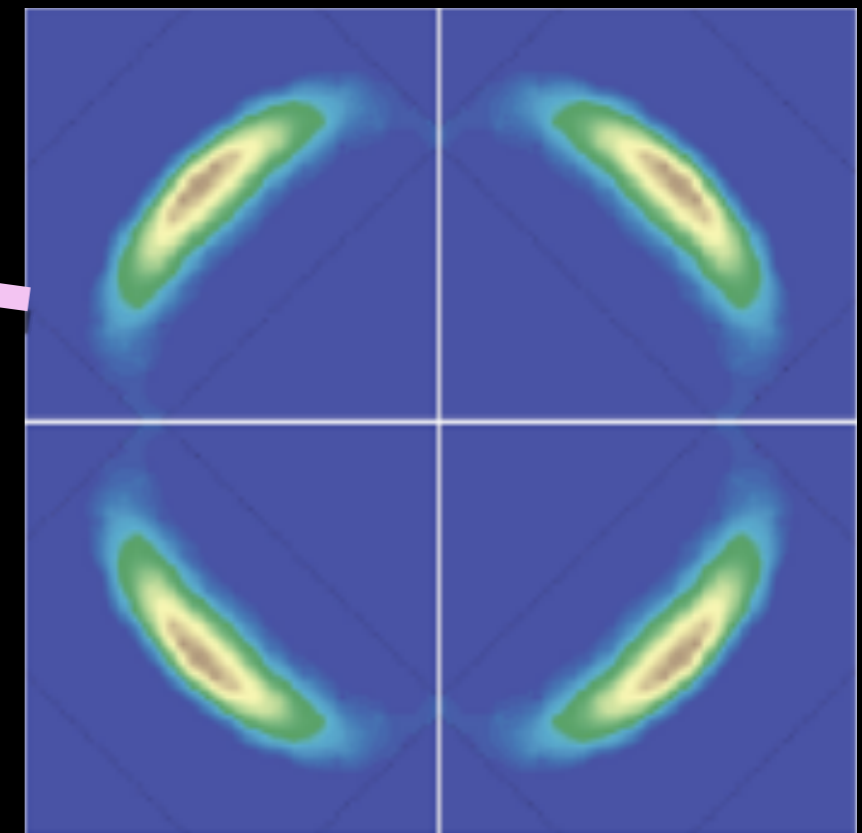
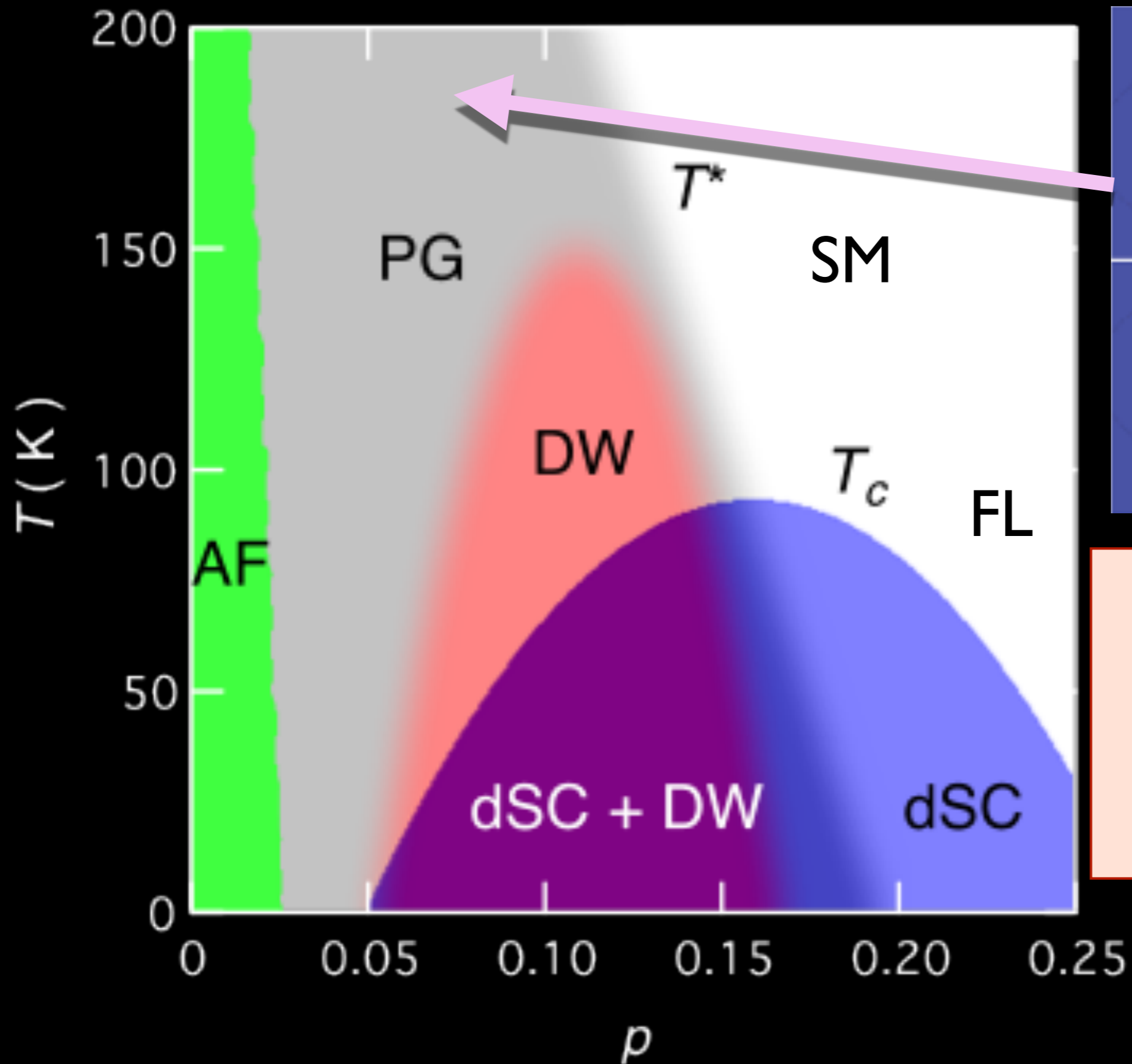
Figure: K. Fujita and J. C. Seamus Davis

M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

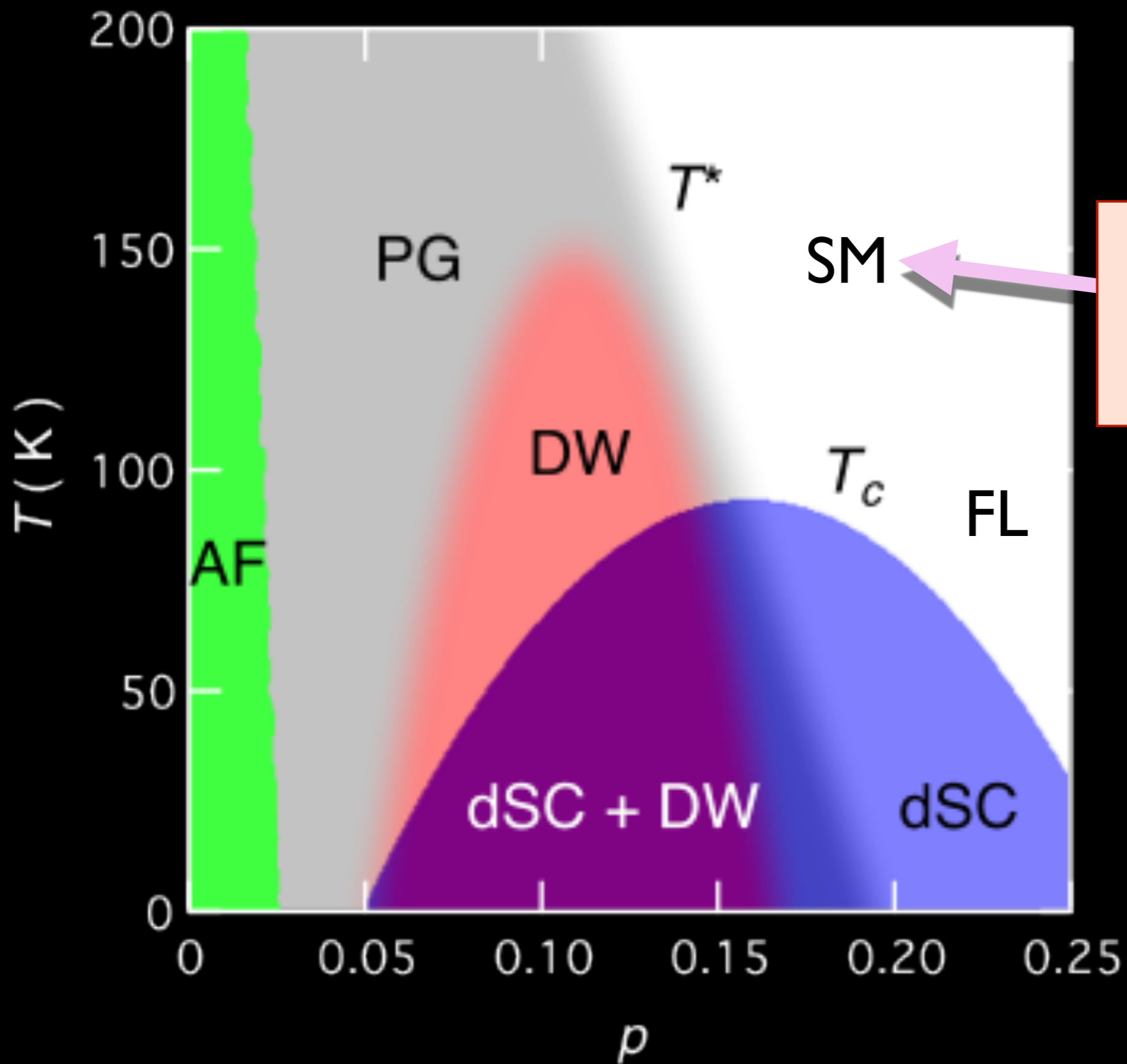


Fermi liquid  
Area enclosed by  
Fermi surface =  $1+p$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)



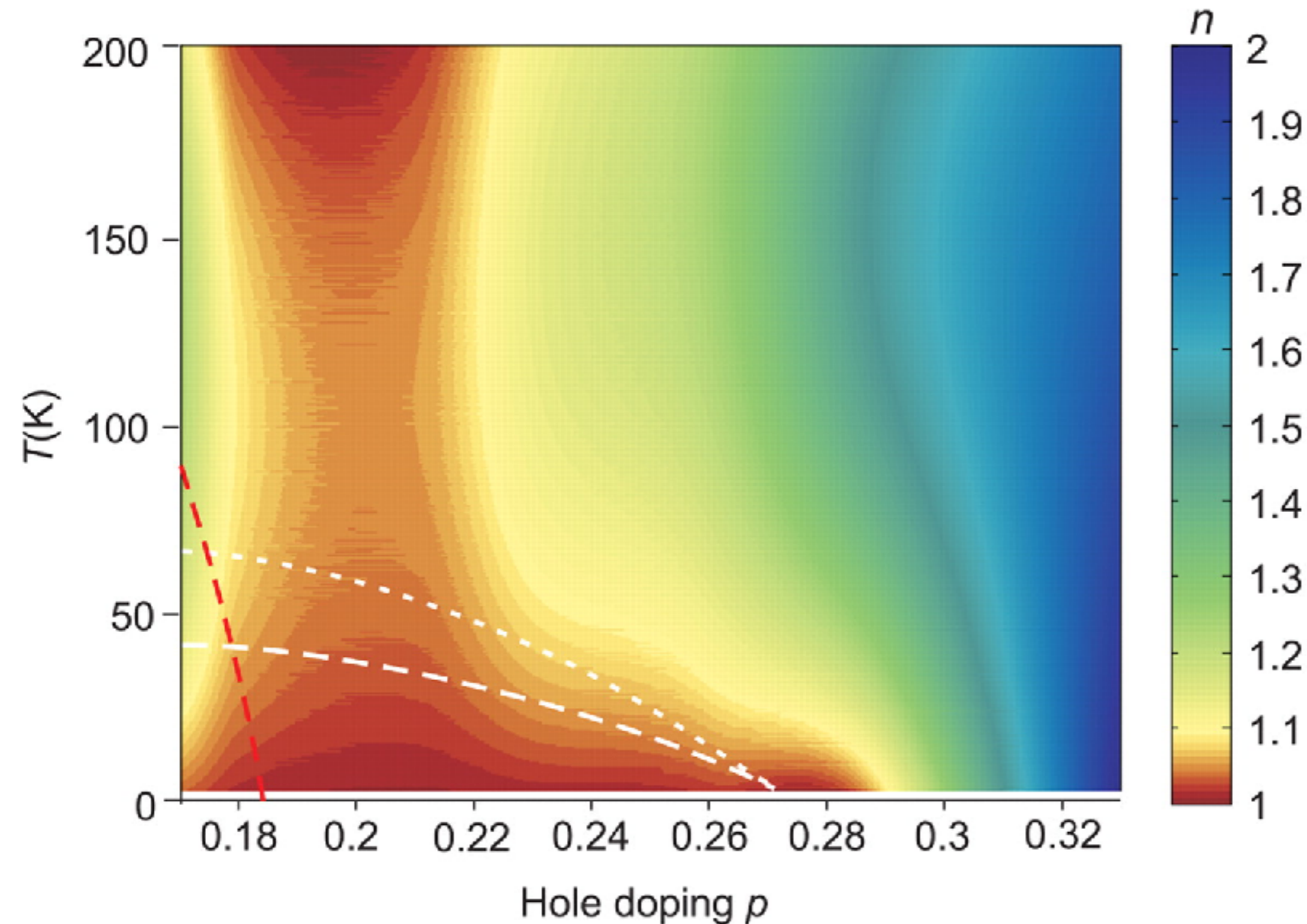
Pseudogap  
metal  
at low  $p$



Strange  
Metal

# Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,<sup>1</sup> Y. Wang,<sup>1</sup> B. Vignolle,<sup>2</sup> O. J. Lipscombe,<sup>1</sup> S. M. Hayden,<sup>1</sup> Y. Tanabe,<sup>3</sup> T. Adachi,<sup>3</sup> Y. Koike,<sup>3</sup> M. Nohara,<sup>4\*</sup> H. Takagi,<sup>4</sup> Cyril Proust,<sup>2</sup> N. E. Hussey<sup>1†</sup>



# Universal $T$ -linear resistivity and Planckian limit in overdoped cuprates

arXiv:1805.02512

A. Legros<sup>1,2</sup>, S. Benhabib<sup>3</sup>, W. Tabis<sup>3,4</sup>, F. Laliberté<sup>1</sup>, M. Dion<sup>1</sup>, M. Lizaire<sup>1</sup>,  
B. Vignolle<sup>3</sup>, D. Vignolles<sup>3</sup>, H. Raffy<sup>5</sup>, Z. Z. Li<sup>5</sup>, P. Auban-Senzier<sup>5</sup>,  
N. Doiron-Leyraud<sup>1</sup>, P. Fournier<sup>1,6</sup>, D. Colson<sup>2</sup>, L. Taillefer<sup>1,6</sup>, and C. Proust<sup>3,6</sup>

From the resistivity, they determined the value of the number  $\alpha$  defined by

$$\rho(T) = \rho_0 + \alpha \frac{h}{2e^2} \left( \frac{T}{T_F} \right)$$

where  $T_F = (\pi\hbar^2/k_B)(n/m^*)$  and  $m^*$  is determined from the specific heat. This expression is obtained from the Drude form  $\rho = m^*/(ne^2\tau)$  and  $\hbar/\tau = \alpha k_B T$ .

# Universal $T$ -linear resistivity and Planckian limit in overdoped cuprates

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Material		$n$ ( $10^{27} \text{ m}^{-3}$ )	$m^*$ ( $m_0$ )	$A_1 / d$ ( $\Omega / \text{K}$ )	$h / (2e^2 T_F)$ ( $\Omega / \text{K}$ )	$\alpha$
Bi2212	$p = 0.23$	6.8	$8.4 \pm 1.6$	$8.0 \pm 0.9$	$7.4 \pm 1.4$	$1.1 \pm 0.3$
Bi2201	$p \sim 0.4$	3.5	$7 \pm 1.5$	$8 \pm 2$	$8 \pm 2$	$1.0 \pm 0.4$
LSCO	$p = 0.26$	7.8	$9.8 \pm 1.7$	$8.2 \pm 1.0$	$8.9 \pm 1.8$	$0.9 \pm 0.3$
Nd-LSCO	$p = 0.24$	7.9	$12 \pm 4$	$7.4 \pm 0.8$	$10.6 \pm 3.7$	$0.7 \pm 0.4$
PCCO	$x = 0.17$	8.8	$2.4 \pm 0.1$	$1.7 \pm 0.3$	$2.1 \pm 0.1$	$0.8 \pm 0.2$
LCCO	$x = 0.15$	9.0	$3.0 \pm 0.3$	$3.0 \pm 0.45$	$2.6 \pm 0.3$	$1.2 \pm 0.3$
TMTSF	$P = 11 \text{ kbar}$	1.4	$1.15 \pm 0.2$	$2.8 \pm 0.3$	$2.8 \pm 0.4$	$1.0 \pm 0.3$

**Slope of  $T$ -linear resistivity vs Planckian limit in seven materials.**

1. Review of spin-density-wave theory
2.  $SU(2)$  gauge theory of fluctuating spin density waves
3. Quantum oscillations and photoemission on the electron-doped cuprates
4. Strange metals and SYK models
5. Review of SYK models: Schwarzian theory
6. Connections to black holes with  $AdS_2$  horizons

1. Review of spin-density-wave theory

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# The Hubbard Model

$$H = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left( n_{i\uparrow} - \frac{1}{2} \right) \left( n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

$t_{ij} \rightarrow$  “hopping”.  $U \rightarrow$  local repulsion,  $\mu \rightarrow$  chemical potential

Spin index  $\alpha = \uparrow, \downarrow$

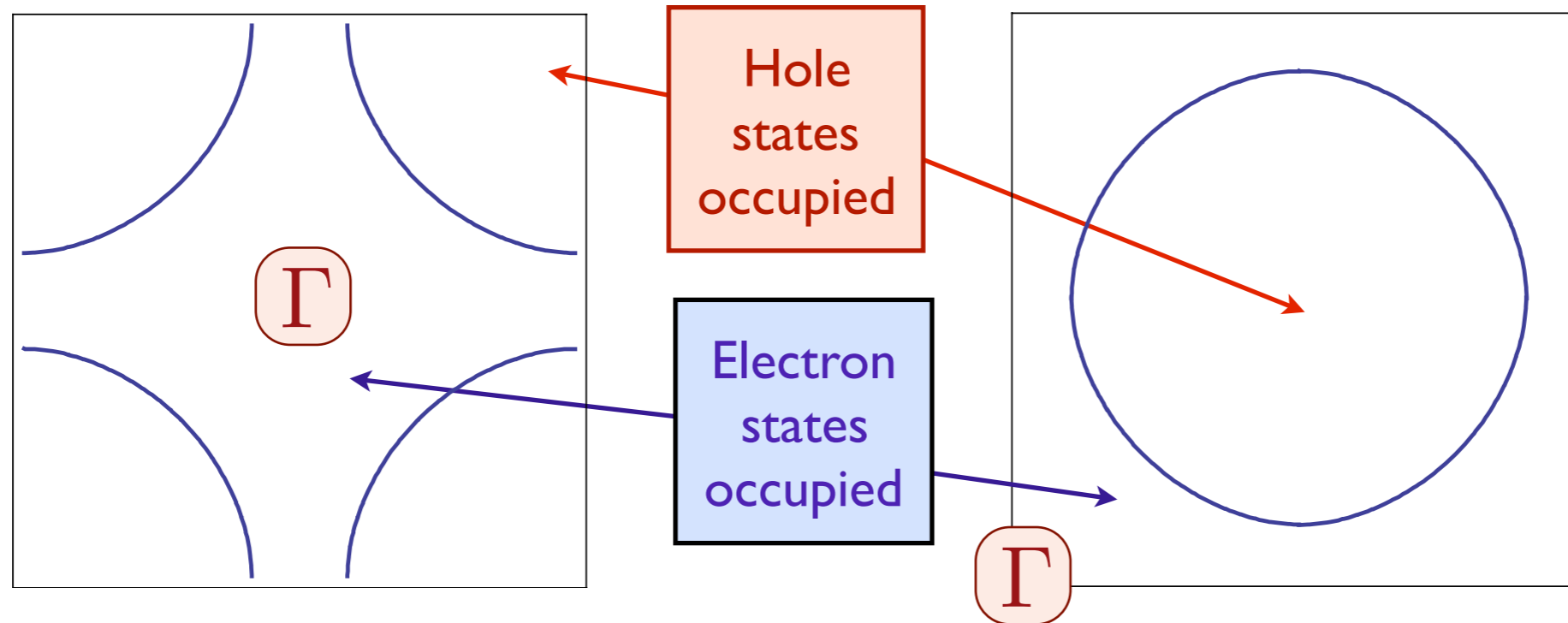
$$n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}$$

$$c_{i\alpha}^\dagger c_{j\beta} + c_{j\beta} c_{i\alpha}^\dagger = \delta_{ij} \delta_{\alpha\beta}$$

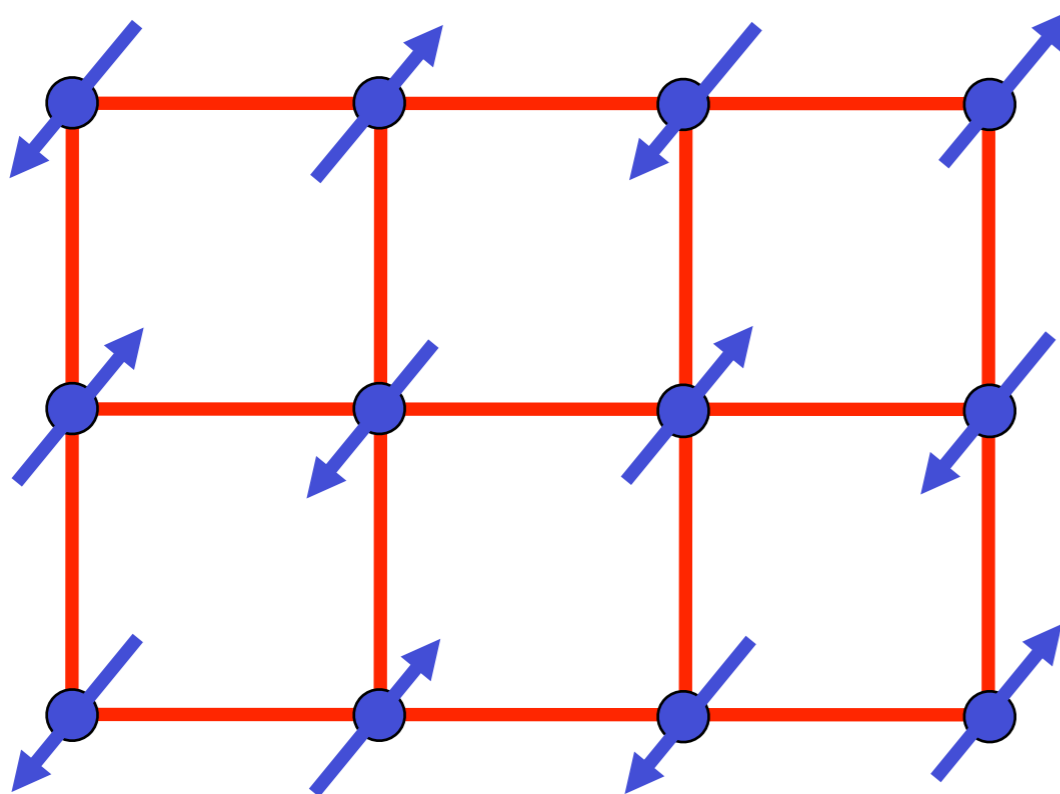
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

**Will study on the square lattice**

# Fermi surface+antiferromagnetism



+



The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\Phi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where  $\mathbf{K} = (\pi, \pi)$  is the ordering wavevector.

# Fermi surface+antiferromagnetism

We use the operator equation (valid on each site  $i$ ):

$$U \left( n_{\uparrow} - \frac{1}{2} \right) \left( n_{\downarrow} - \frac{1}{2} \right) = -\frac{2U}{3} \vec{S}^2 + \frac{U}{4} \quad (1)$$

Then we decouple the interaction via

$$\exp \left( \frac{2U}{3} \sum_i \int d\tau \vec{S}_i^2 \right) = \int \mathcal{D}\vec{J}_i(\tau) \exp \left( - \sum_i \int d\tau \left[ \frac{3}{8U} \vec{J}_i^2 - \vec{J}_i \vec{S}_i \right] \right) \quad (2)$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for  $\vec{J}_i$ . At the saddle-point we find that the lowest energy is achieved when the vector has opposite orientations on the A and B sublattices. Anticipating this, we look for a continuum limit in terms of a field  $\vec{\Phi}_i$  where

$$\vec{J}_i = \vec{\Phi}_i e^{i\mathbf{K}\cdot\mathbf{r}_i} \quad (3)$$

# Fermi surface+antiferromagnetism

In this manner, we obtain the “spin-fermion” model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\Phi} \exp(-\mathcal{S})$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left( \frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c_{i\alpha}^\dagger \vec{\Phi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2r \left[ \frac{1}{2} \left( \nabla_r \vec{\Phi} \right)^2 + \frac{1}{2} \left( \partial_\tau \vec{\Phi} \right)^2 + \frac{s}{2} \vec{\Phi}^2 + \frac{u}{4} \vec{\Phi}^4 \right]$$

# Fermi surface+antiferromagnetism

In the Hamiltonian form (ignoring, for now, the time dependence of  $\vec{\Phi}$ ), the coupling between  $\vec{\Phi}$  and the electrons takes the form

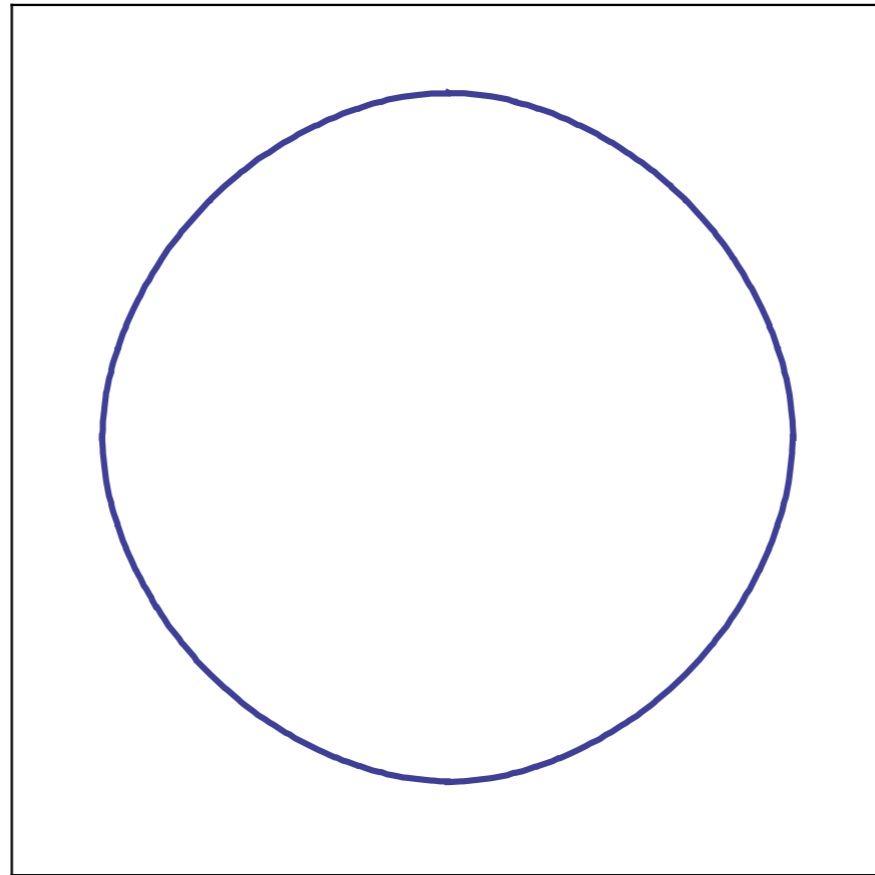
$$H_{\text{sdw}} = \lambda \sum_{\mathbf{k}, \mathbf{q}, \alpha, \beta} \vec{\Phi}_{\mathbf{q}} \cdot c_{\mathbf{k}+\mathbf{q}, \alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}, \beta}$$

where  $\vec{\sigma}$  are the Pauli matrices, the boson momentum  $\mathbf{q}$  is small, while the fermion momentum  $\mathbf{k}$  extends over the entire Brillouin zone. In the antiferromagnetically ordered state, we may take  $\vec{\Phi} \propto (0, 0, 1)$ , and the electron dispersions obtained by diagonalizing  $H_0 + H_{\text{sdw}}$  are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \lambda^2 |\vec{\Phi}|^2}$$

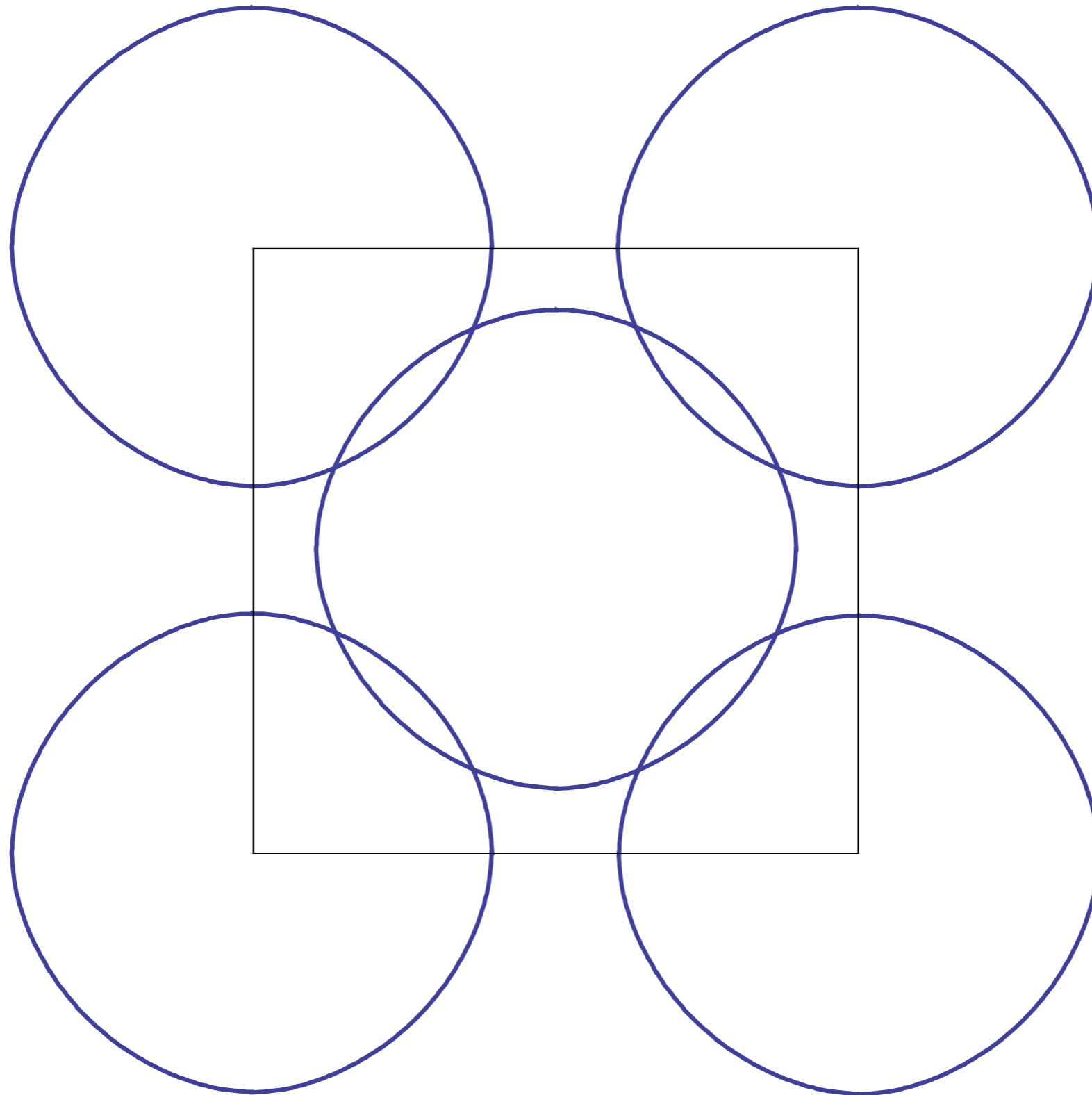
This leads to the Fermi surfaces shown in the following slides as a function of increasing  $|\vec{\Phi}|$ .

# Fermi surface+antiferromagnetism



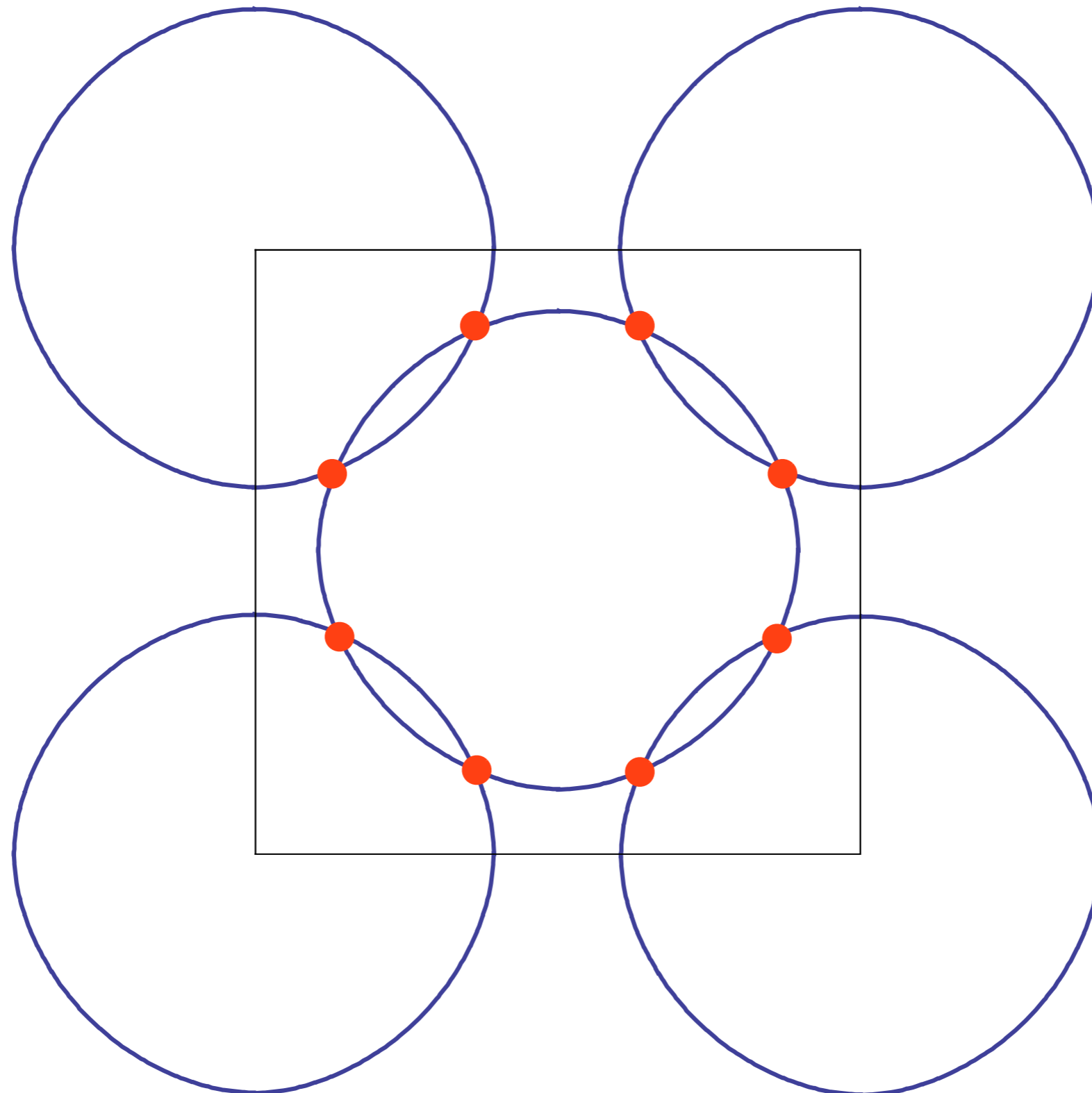
**Metal with “large” Fermi surface**

# Fermi surface+antiferromagnetism



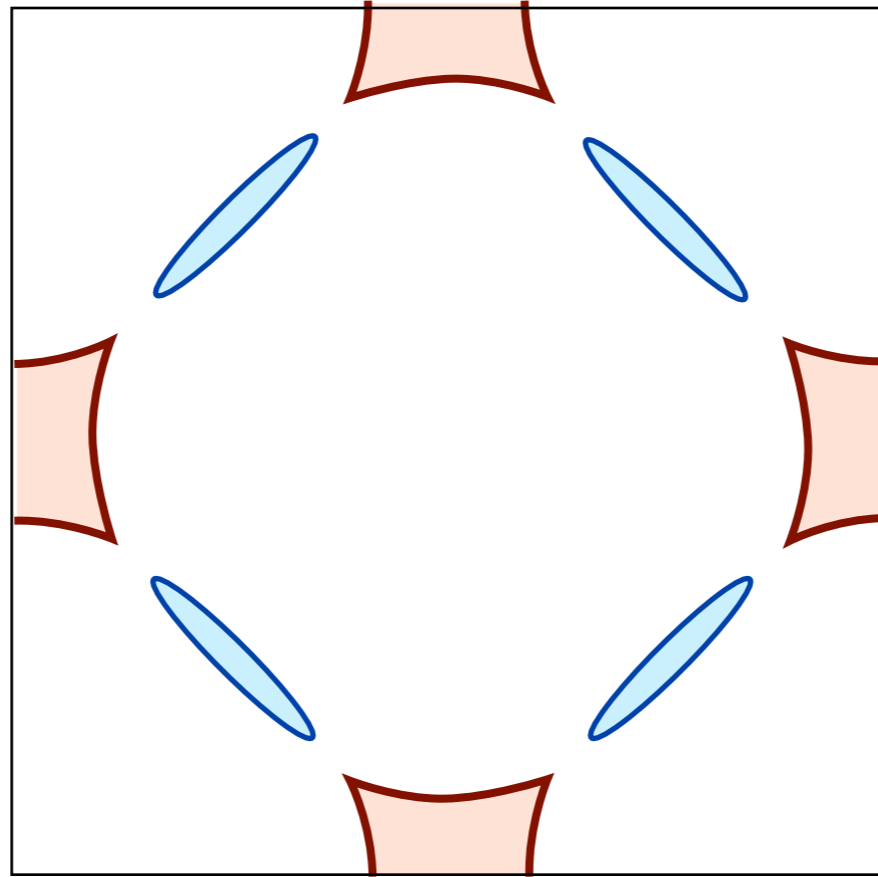
Fermi surfaces translated by  $\mathbf{K} = (\pi, \pi)$ .

# Fermi surface+antiferromagnetism



“Hot” spots

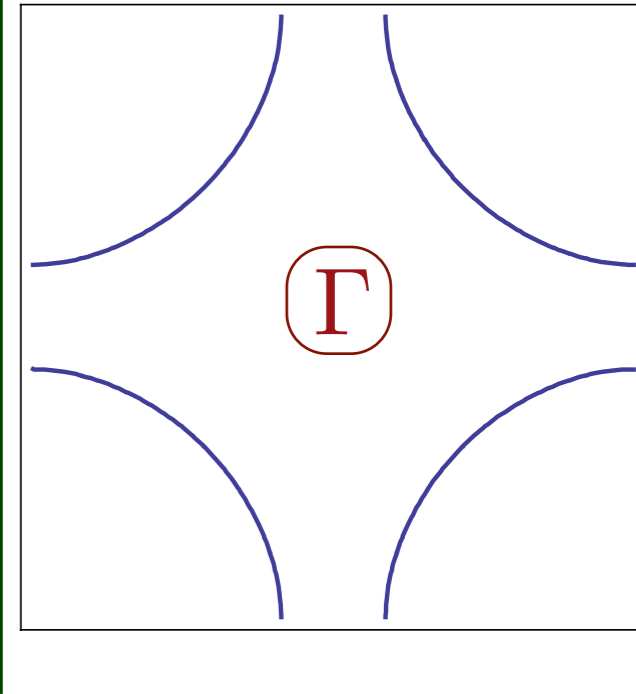
# Fermi surface+antiferromagnetism



Electron and hole pockets in  
antiferromagnetic phase with  $\langle \vec{\Phi} \rangle \neq 0$

# Square lattice Hubbard model with hole doping

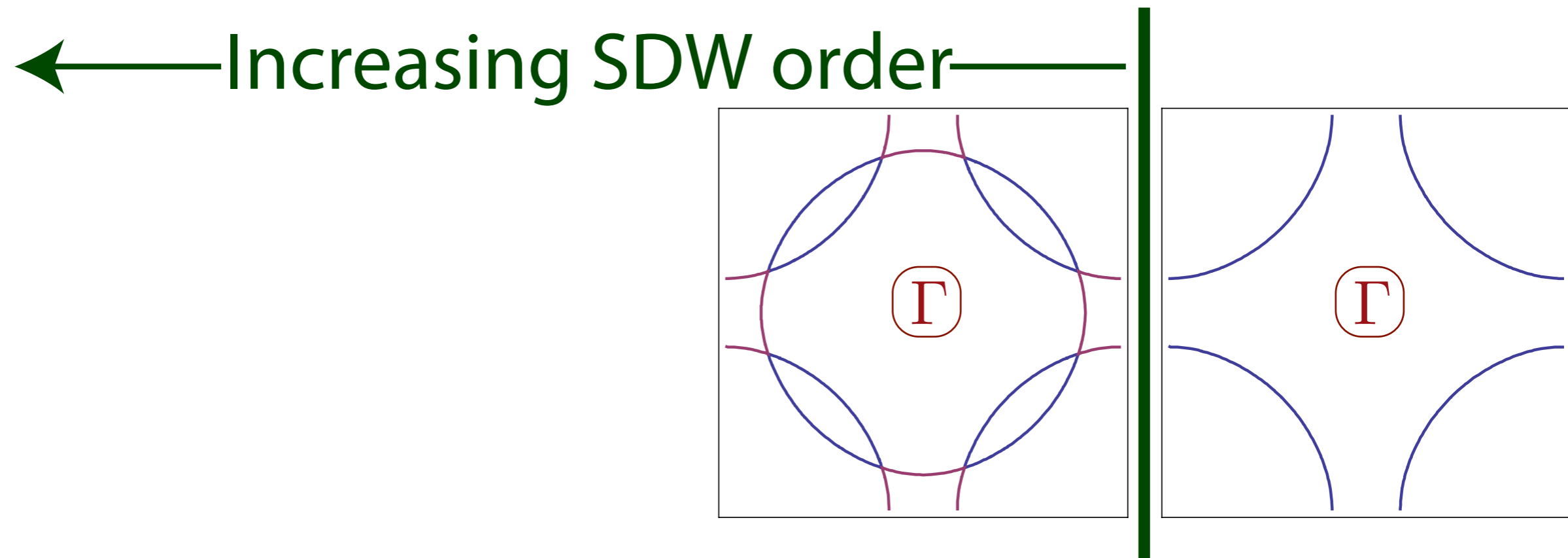
← Increasing SDW order →



S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).

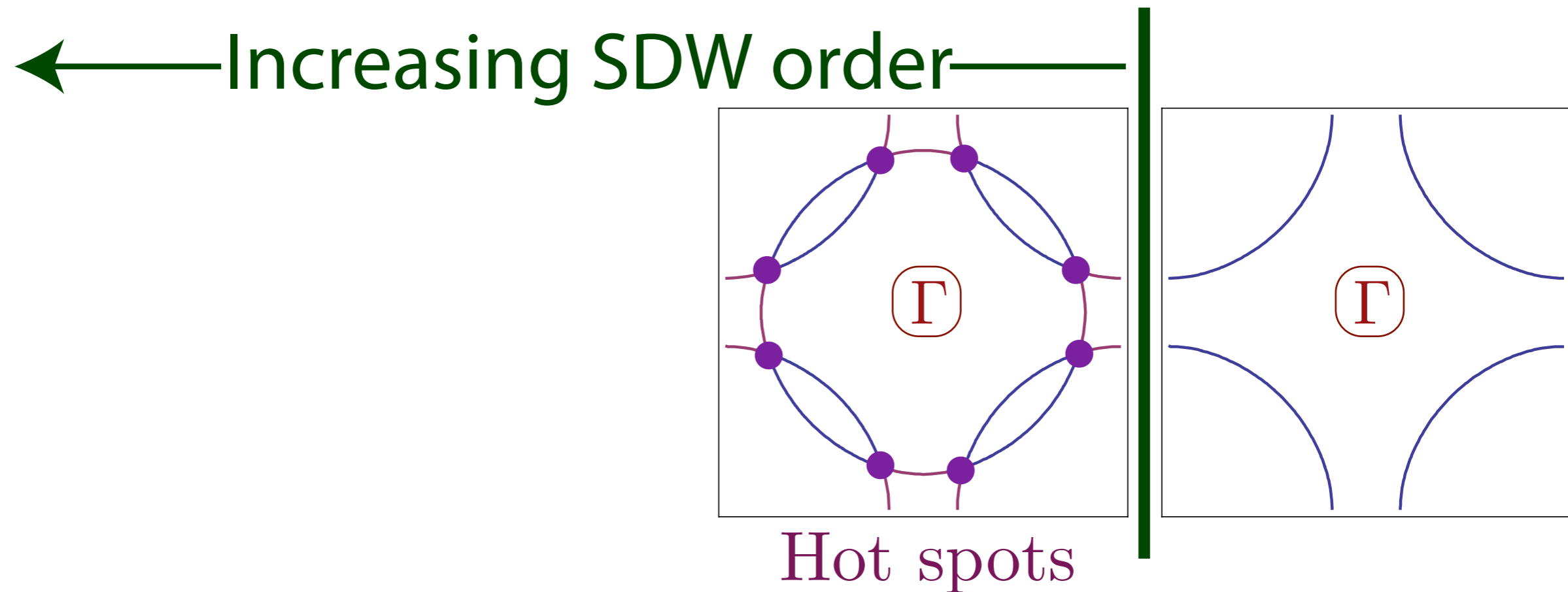
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

# Square lattice Hubbard model with hole doping



S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).  
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

# Square lattice Hubbard model with hole doping

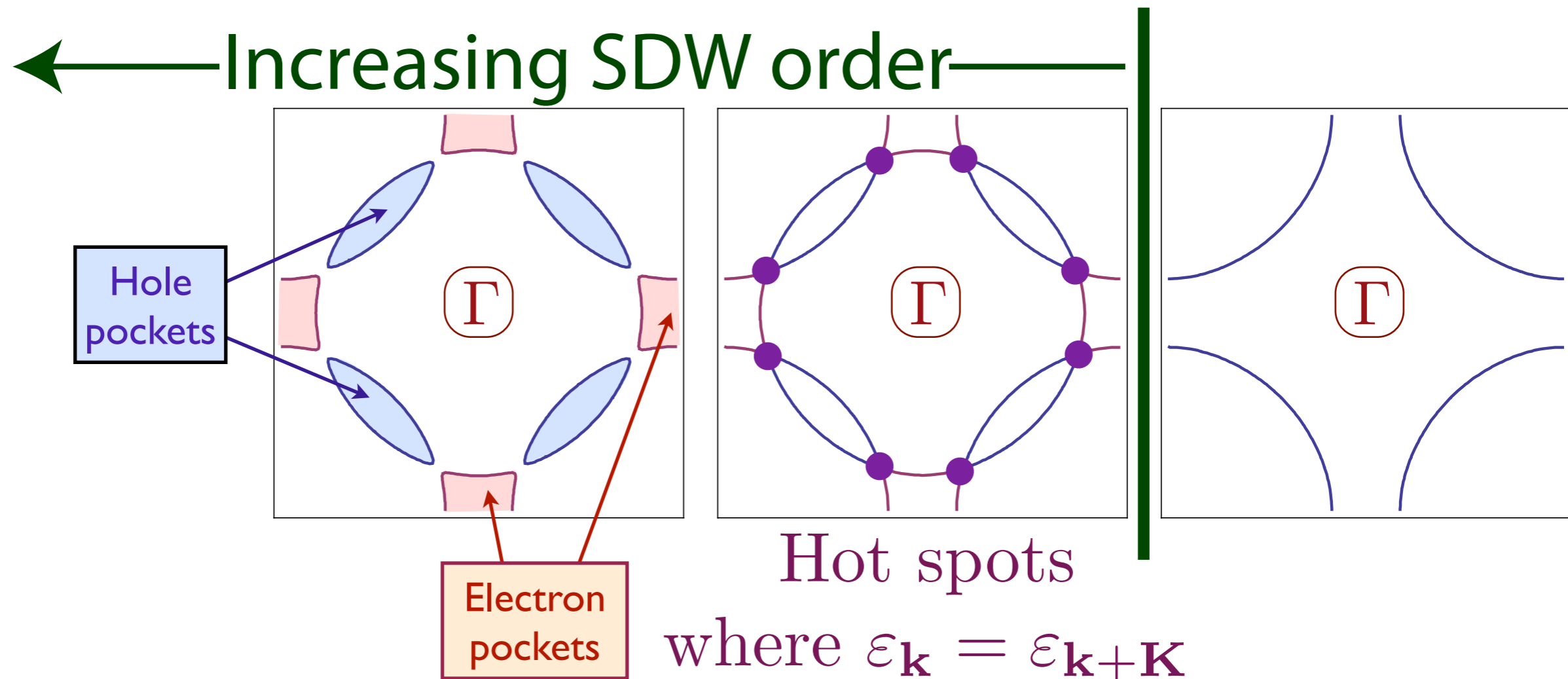


where  $\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}+\mathbf{K}}$

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).

A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

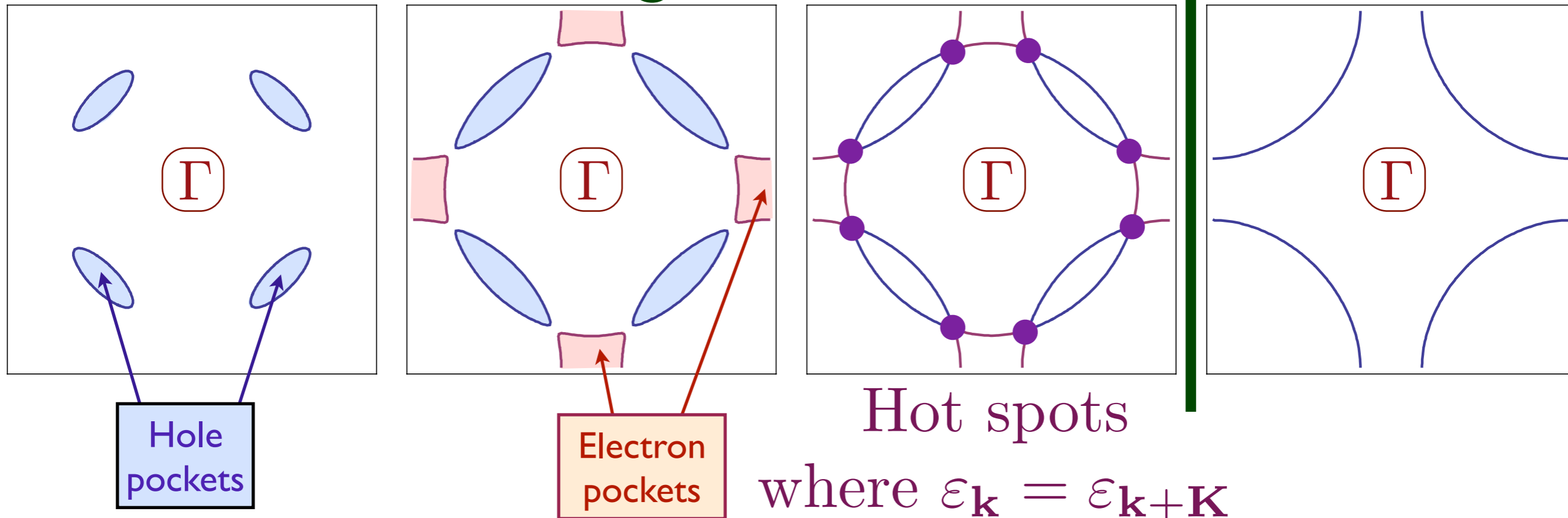
# Square lattice Hubbard model with hole doping



Fermi surface breaks up at hot spots  
into electron and hole “pockets”

# Square lattice Hubbard model with hole doping

← Increasing SDW order →

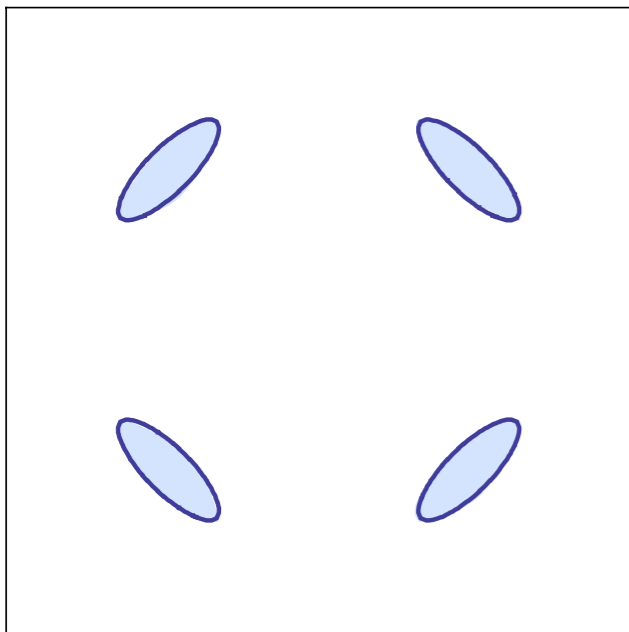


Fermi surface breaks up at hot spots  
into electron and hole “pockets”

# Square lattice Hubbard model with hole doping

$$\langle \vec{\Phi} \rangle \neq 0$$

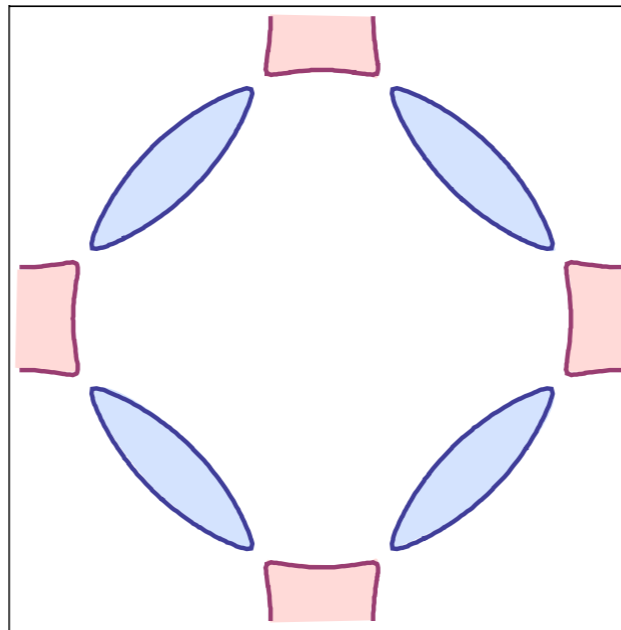
and large



Metal with  
hole pockets

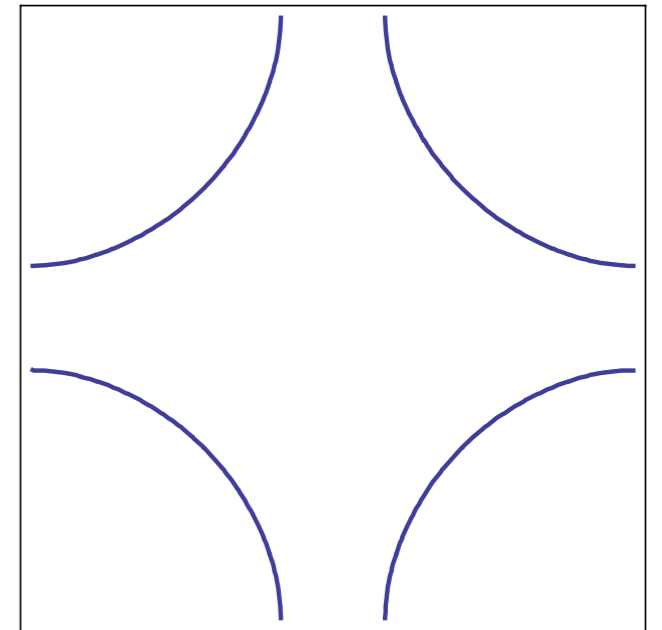
$$\langle \vec{\Phi} \rangle \neq 0$$

and small



Metal with  
electron and  
hole pockets

$$\langle \vec{\Phi} \rangle = 0$$

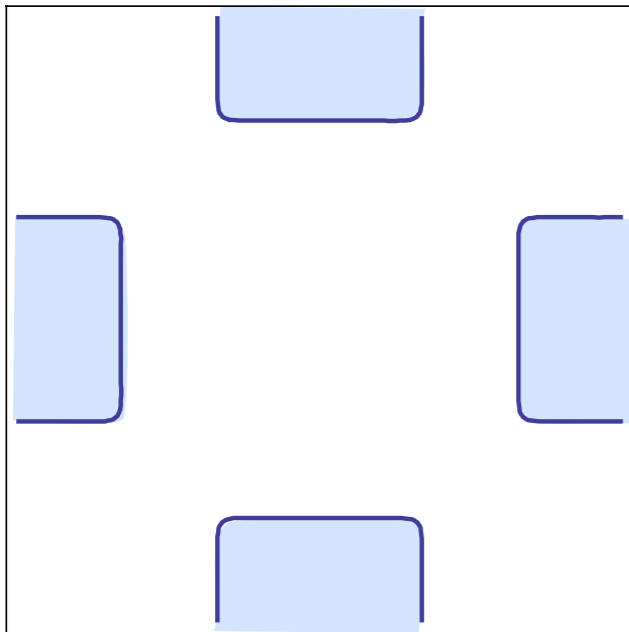


Metal with  
“large” Fermi  
surface

# Square lattice Hubbard model with electron doping

$$\langle \vec{\Phi} \rangle \neq 0$$

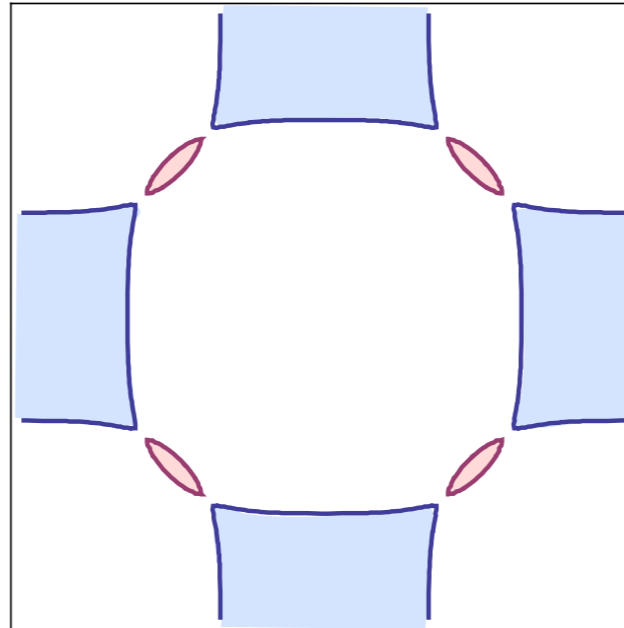
and large



Metal with  
electron pockets

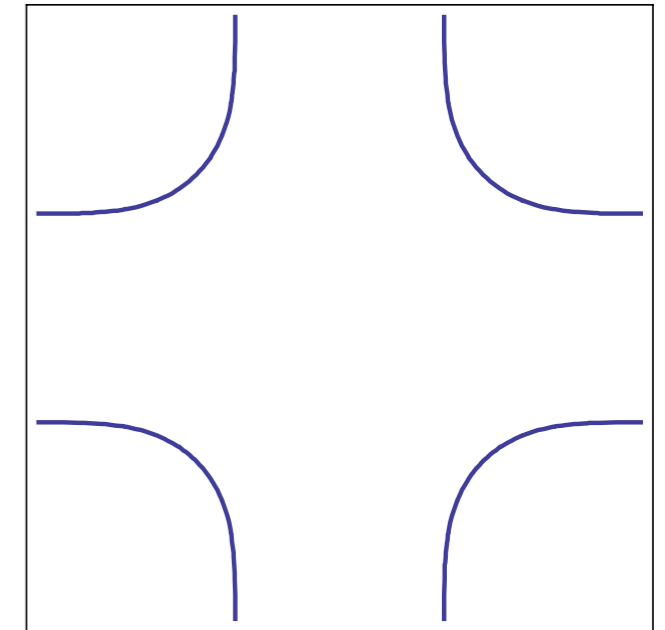
$$\langle \vec{\Phi} \rangle \neq 0$$

and small

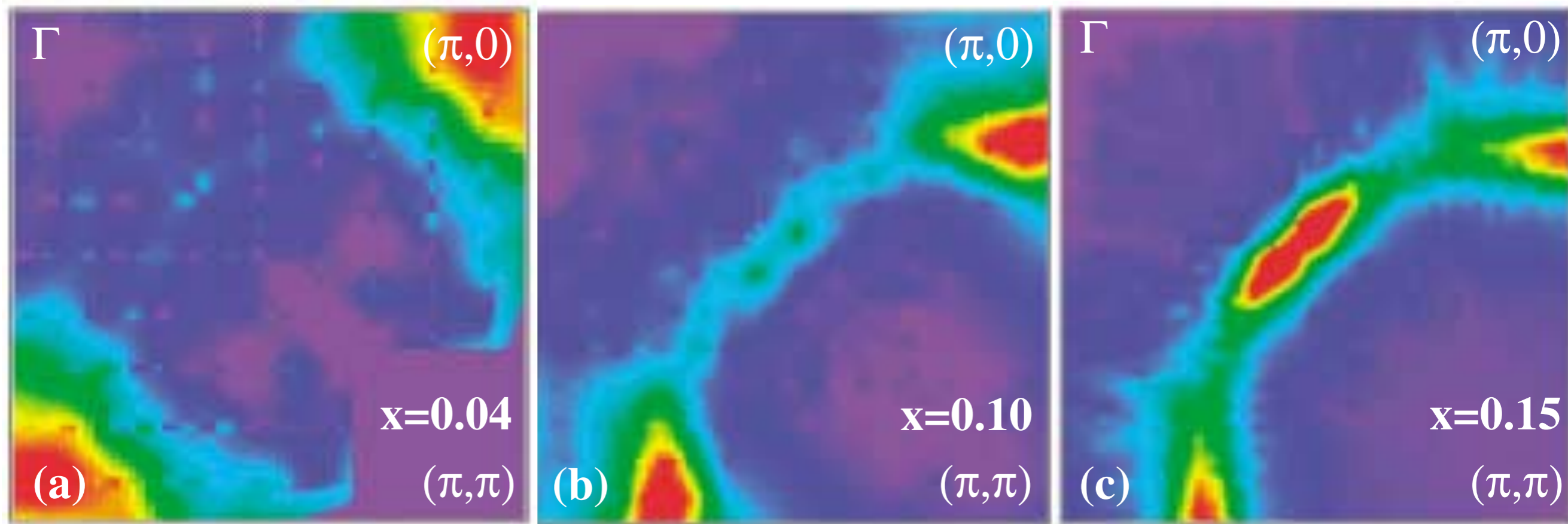


Metal with  
electron and  
hole pockets

$$\langle \vec{\Phi} \rangle = 0$$



Metal with  
“large” Fermi  
surface



## Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura  
Phys. Rev. Lett. **88**, 257001 (2002)

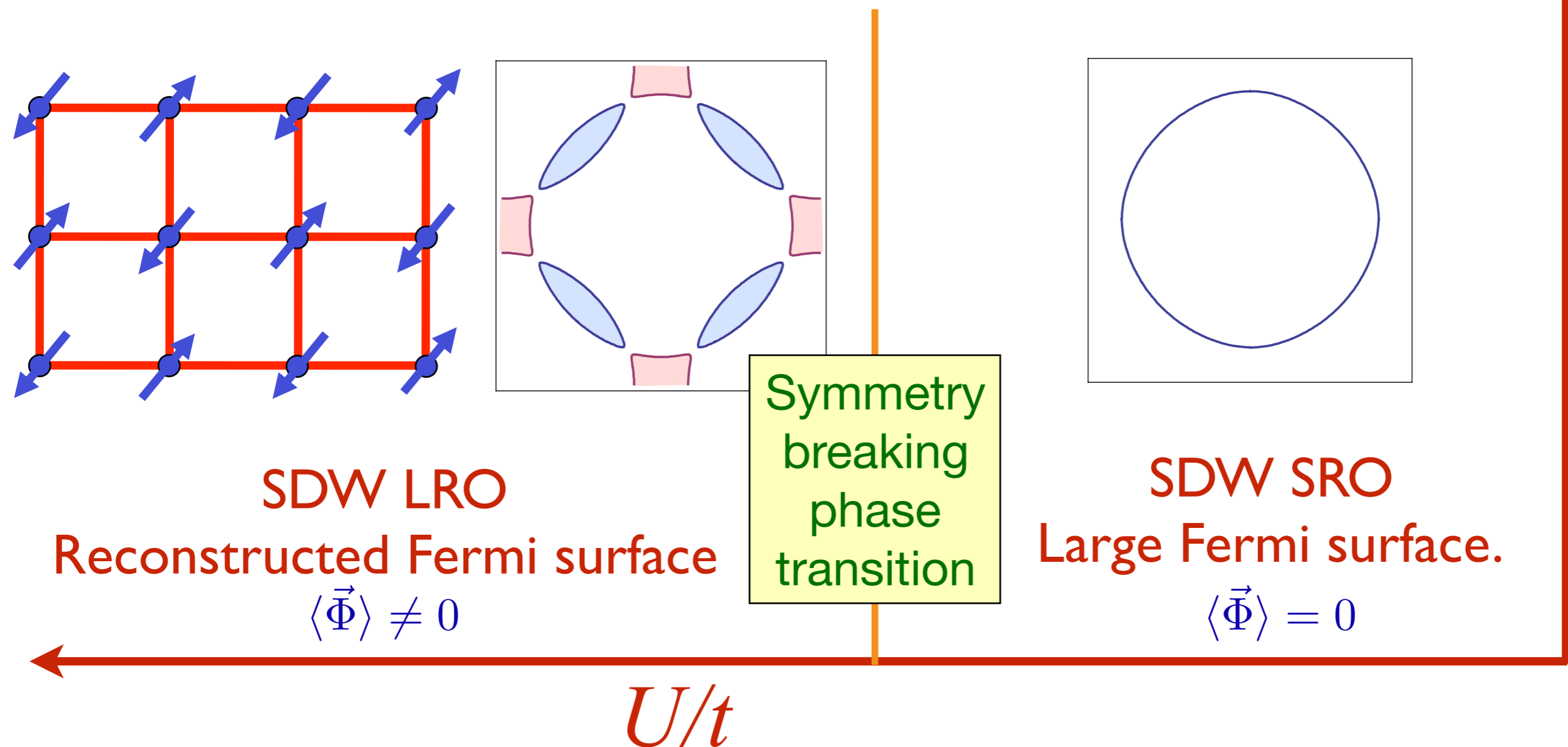
# Antiferromagnetism in the Hubbard Model

$$H = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left( n_{i\uparrow} - \frac{1}{2} \right) \left( n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

$t_{ij} \rightarrow$  "hopping".  $U \rightarrow$  local repulsion,  $\mu \rightarrow$  chemical potential

Mean-field theory with a spin density wave (SDW)

$$\text{order parameter } \vec{\Phi}_i = (-1)^{i_x+i_y} \langle c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta} \rangle / 2$$



# SDW SRO

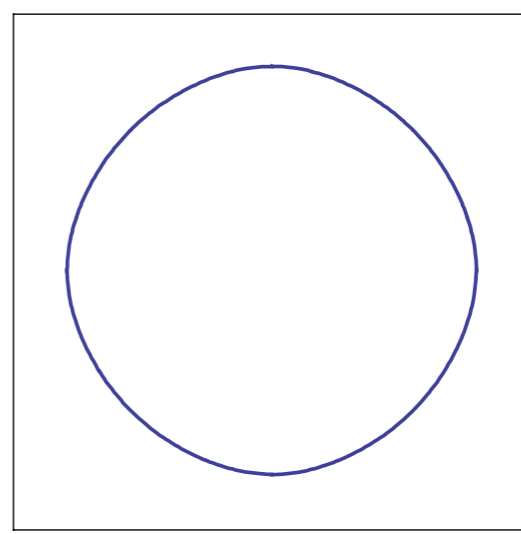
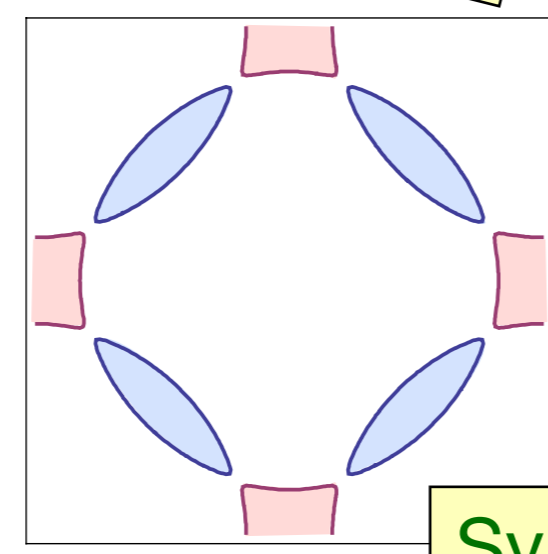
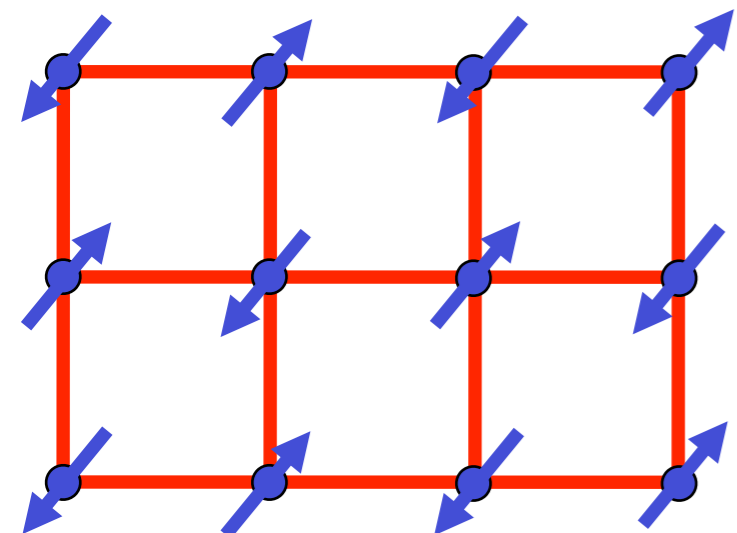
Emergent gauge fields  
and “topological order”.  
Reconstructed Fermi surface.

$$\langle \vec{\Phi} \rangle = 0$$

Symmetry breaking and  
topological phase transition

Topological  
phase transition

$g$



# SDW LRO

Reconstructed Fermi surface

$$\langle \vec{\Phi} \rangle \neq 0$$

Symmetry  
breaking  
phase  
transition

# SDW SRO

Large Fermi surface.

$$\langle \vec{\Phi} \rangle = 0$$

$U/t$



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5. Review of SYK models: Schwarzian theory

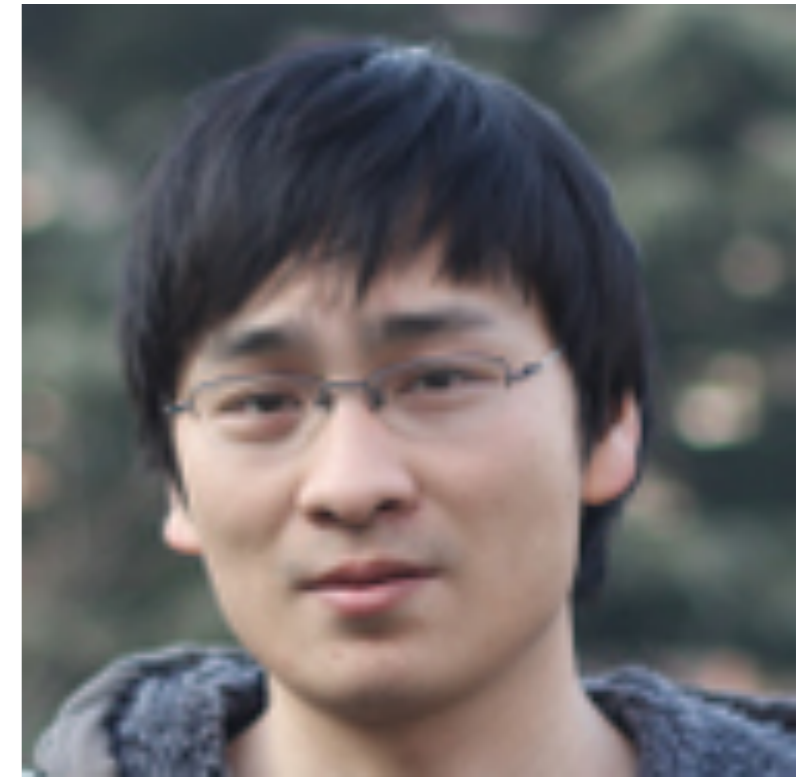
6. Connections to black holes with  $AdS_2$  horizons



**Mathias Scheurer**



**Shubhayu Chatterjee**



**Wei Wu**



**Michel Ferrero**

M. S. Scheurer, S. Chatterjee, Wei Wu,  
M. Ferrero, A. Georges, and S. Sachdev,  
Proceedings of the National Academy of  
Sciences **115**, E3665 (2018)

Wei Wu, M. S. Scheurer, S. Chatterjee,  
S. Sachdev, A. Georges, and M. Ferrero,  
Physical Review X **8**, 021048 (2018)



**Antoine Georges**

We can (exactly) transform the Hubbard model to the “spin-fermion” model: **electrons**  $c_{i\alpha}$  on the square lattice with dispersion

$$\mathcal{H}_c = - \sum_{i,\rho} t_\rho \left( c_{i,\alpha}^\dagger c_{i+\mathbf{v}_\rho,\alpha} + c_{i+\mathbf{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) - \mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\text{int}}$$

are coupled to an **antiferromagnetic SDW order parameter**  $\Phi^\ell(i)$ ,  $\ell = x, y, z$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i \Phi^\ell(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^\ell c_{i,\beta} + V_\Phi$$

where  $\eta_i = \pm 1$  on the two sublattices. (For suitable  $V_\Phi$ , integrating out the  $\Phi^\ell$  yields back the Hubbard model).

For (fluctuating) SDW SRO, we transform to a **rotating reference frame** using the SU(2) rotation  $R_i$

$$\begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} = R_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix},$$

in terms of fermionic “chargons”  $\psi_s$  and a **Higgs field**  $H^a(i)$

$$\sigma^\ell \Phi^\ell(i) = R_i \sigma^a H^a(i) R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

For (fluctuating) SDW SRO, we transform to a **rotating reference frame** using the SU(2) rotation  $R_i$

$$\begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix} = R_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix},$$

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$$\sigma^\ell \Phi^\ell(i) = R_i \sigma^a H^a(i) R_i^\dagger$$

**The Higgs field is the SDW order in the rotating reference frame.**

Note that this representation is ambiguous up to a SU(2) gauge transformation,  $V_i$

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \rightarrow V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \sigma^b H^b(i) V_i^\dagger.$$

# Fluctuating SDW

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the **SDW order replaced by the Higgs field**.

$$\mathcal{H}_\psi = - \sum_{i,\rho} t_\rho \left( \psi_{i,s}^\dagger \psi_{i+\mathbf{v}_{\rho,s}} + \psi_{i+\mathbf{v}_{\rho,s}}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\text{int}}$$

$$\mathcal{H}_{\text{int}} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V_H$$

# Particle content of theories

Field	Symbol	Statistics	$SU(2)_{\text{gauge}}$	$SU(2)_{\text{spin}}$	$U(1)_{\text{e.m.charge}}$
Electron	$c$	fermion	<b>1</b>	<b>2</b>	-1
AF order	$\Phi$	boson	<b>1</b>	<b>3</b>	0
Chargon	$\psi$	fermion	<b>2</b>	<b>1</b>	-1
Spinon	$R$ or $z$	boson	<b><math>\bar{2}</math></b>	<b>2</b>	0
Higgs	$H$	boson	<b>3</b>	<b>1</b>	0

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)

D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

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Spin density wave theory

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)

D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

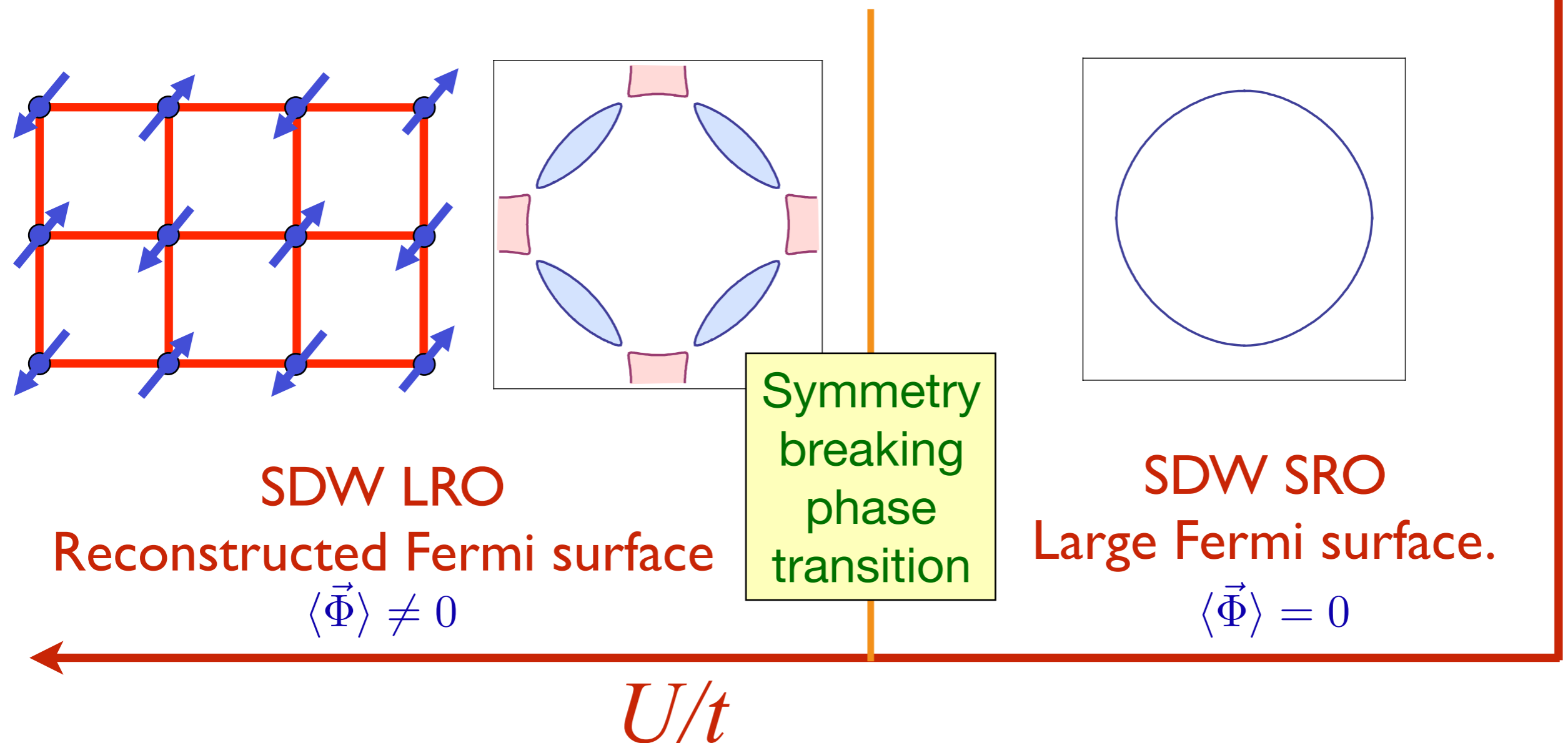
# Antiferromagnetism in the Hubbard Model

$$H = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left( n_{i\uparrow} - \frac{1}{2} \right) \left( n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

$t_{ij} \rightarrow$  "hopping".  $U \rightarrow$  local repulsion,  $\mu \rightarrow$  chemical potential

Mean-field theory with a spin density wave (SDW)

$$\text{order parameter } \vec{\Phi}_i = (-1)^{i_x+i_y} \langle c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta} \rangle / 2$$



# Particle content of theories

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**SU(2) gauge theory**



S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)

D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

SDW SRO

## Higgs phase

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 $Z_2$  vortices or hedgehogs expelled.

$$\langle \Phi^\ell \rangle = 0$$

$$\langle H^a \rangle \neq 0$$

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Symmetry breaking and  
topological phase transition

Topological  
phase transition

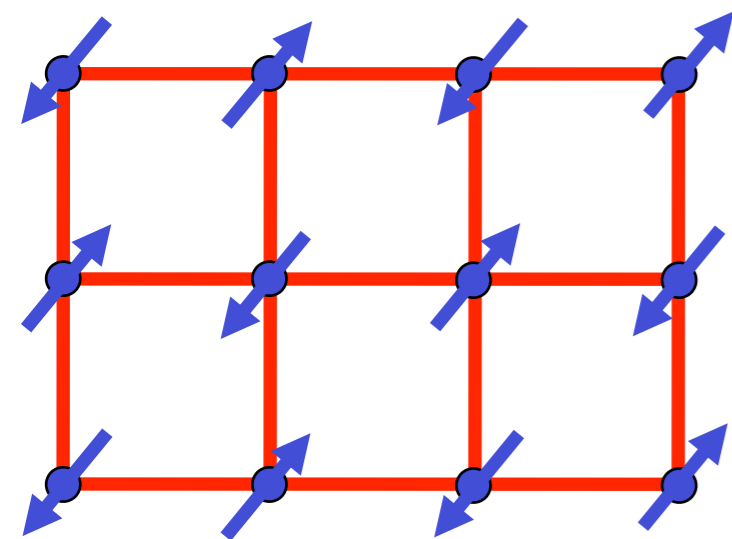
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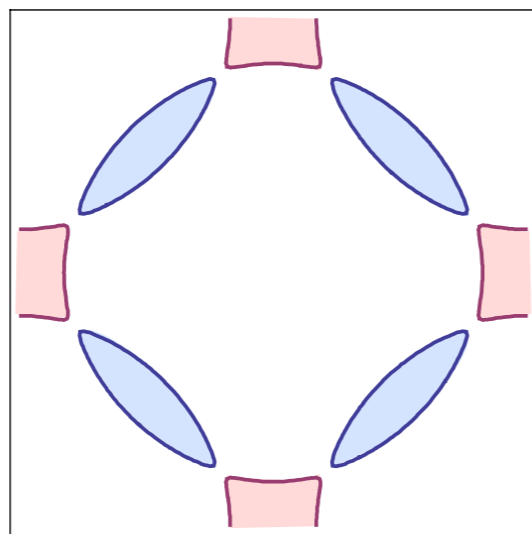
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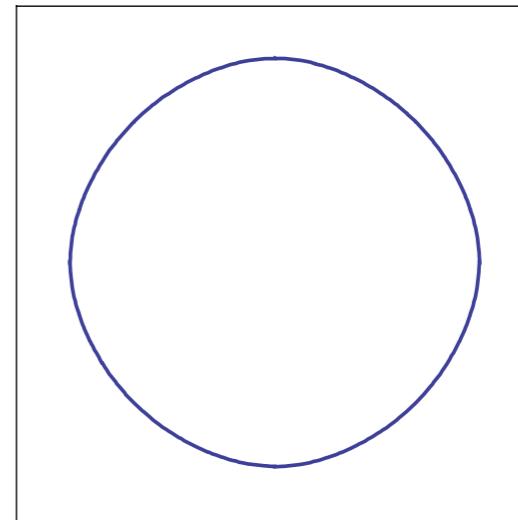
$g$



SDW LRO



Symmetry  
breaking  
phase  
transition



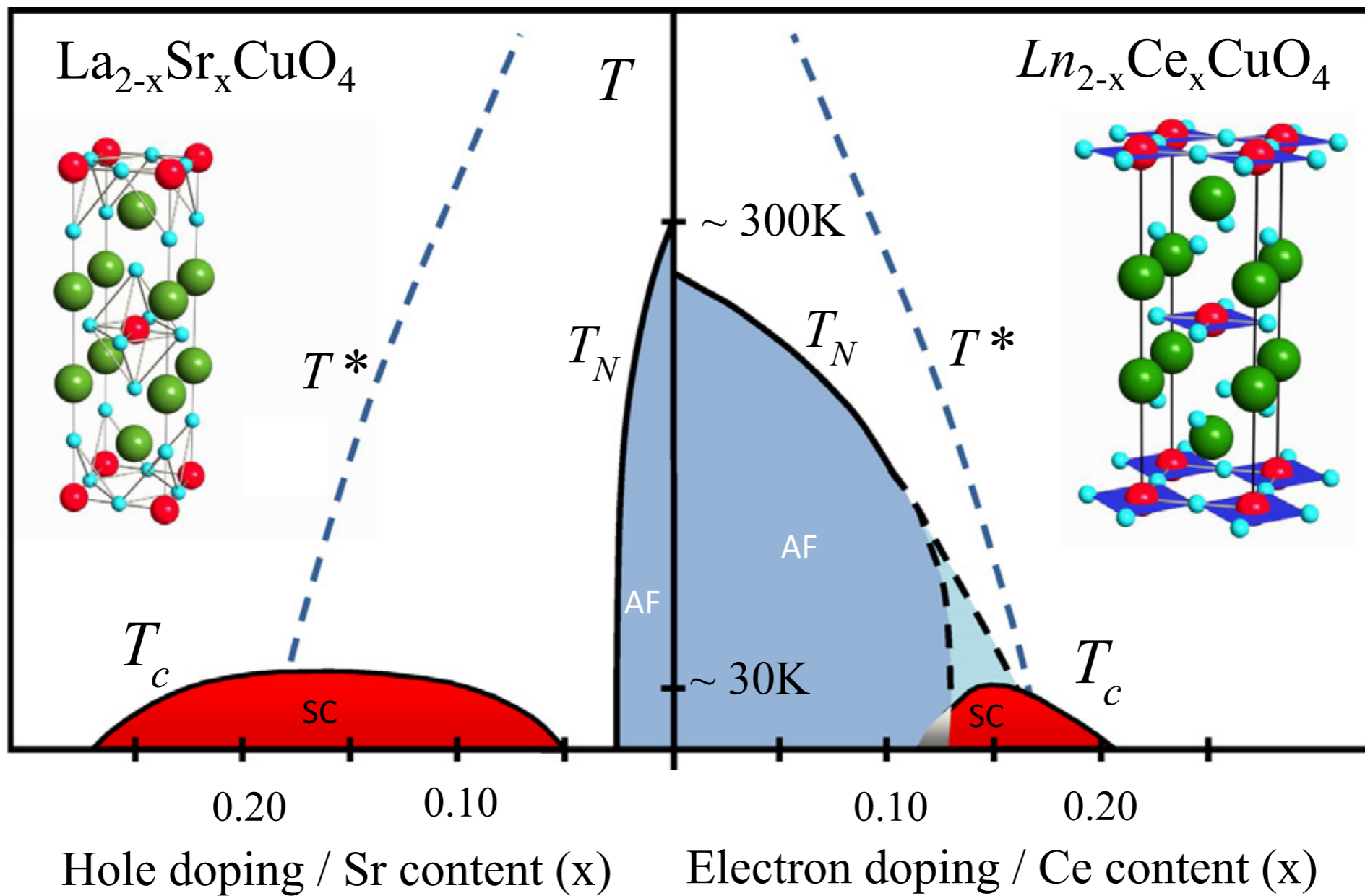
SDW SRO

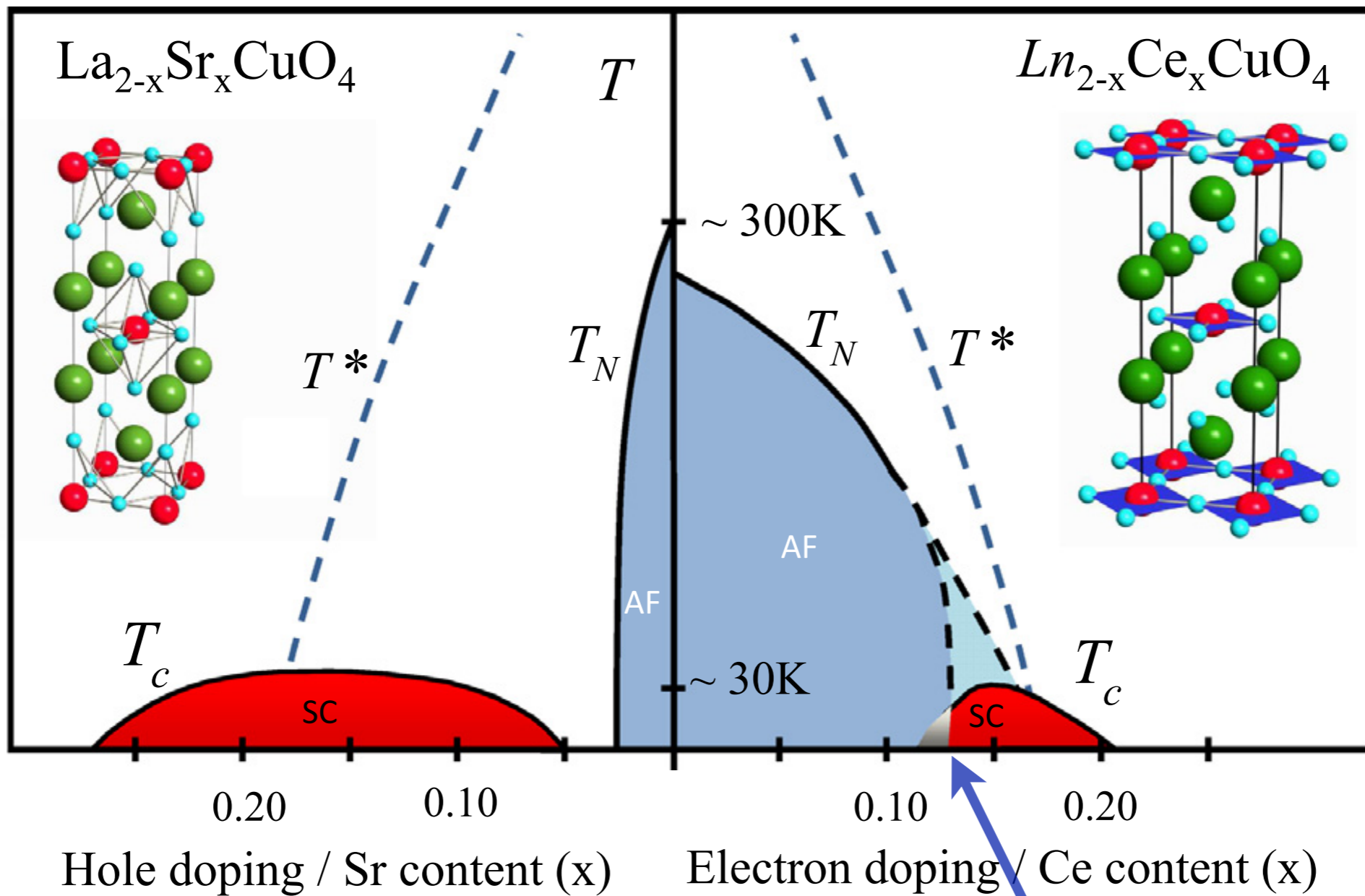
## Confinement

No topological order.

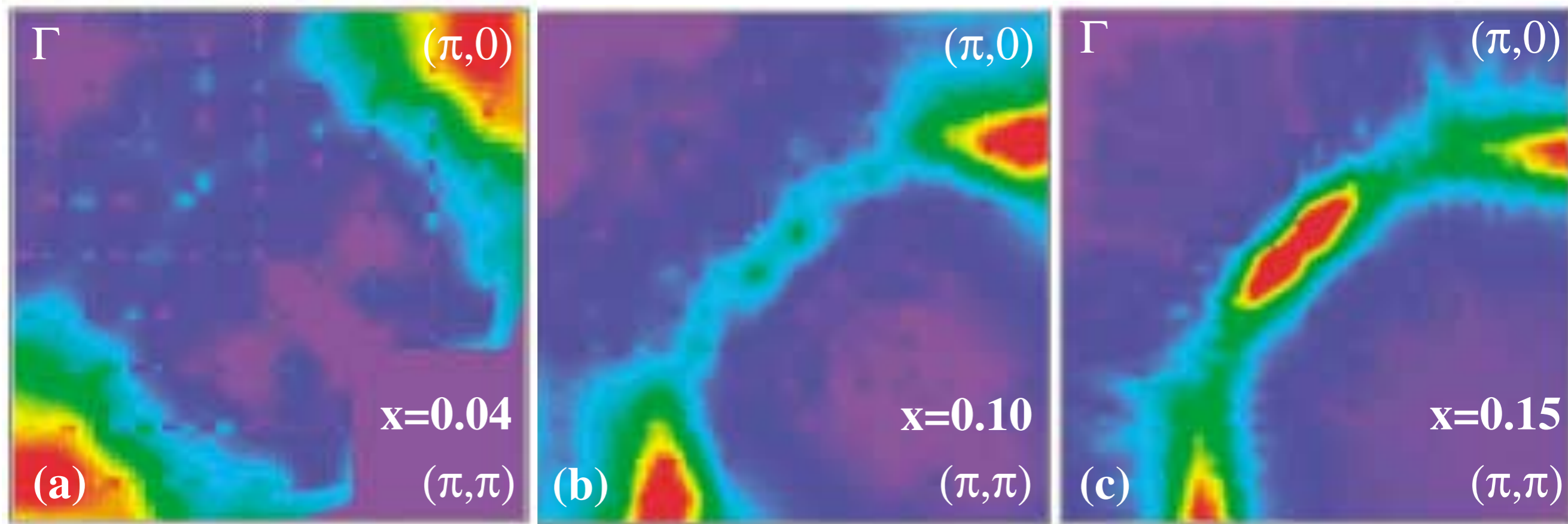
$U/t$

1. Review of spin-density-wave theory
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Antiferromagnetic order disappears beyond  $x=0.14$



## Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura  
Phys. Rev. Lett. **88**, 257001 (2002)

# Correlation between Fermi surface transformations and superconductivity in the electron-doped high- $T_c$ superconductor $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$

T. Helm,<sup>1,\*</sup> M. V. Kartsovnik,<sup>1,†</sup> C. Proust,<sup>2</sup> B. Vignolle,<sup>2</sup> C. Putzke,<sup>3,‡</sup> E. Kampert,<sup>3</sup> I. Sheikin,<sup>4</sup> E.-S. Choi,<sup>5</sup> J. S. Brooks,<sup>5</sup> N. Bittner,<sup>1,§</sup> W. Biberacher,<sup>1</sup> A. Erb,<sup>1,6</sup> J. Wosnitza,<sup>3</sup> and R. Gross<sup>1,6,||</sup>

<sup>1</sup>Walther-Meißner-Institut, Bayerische Akademie der Wissenschaften, D-85748 Garching, Germany

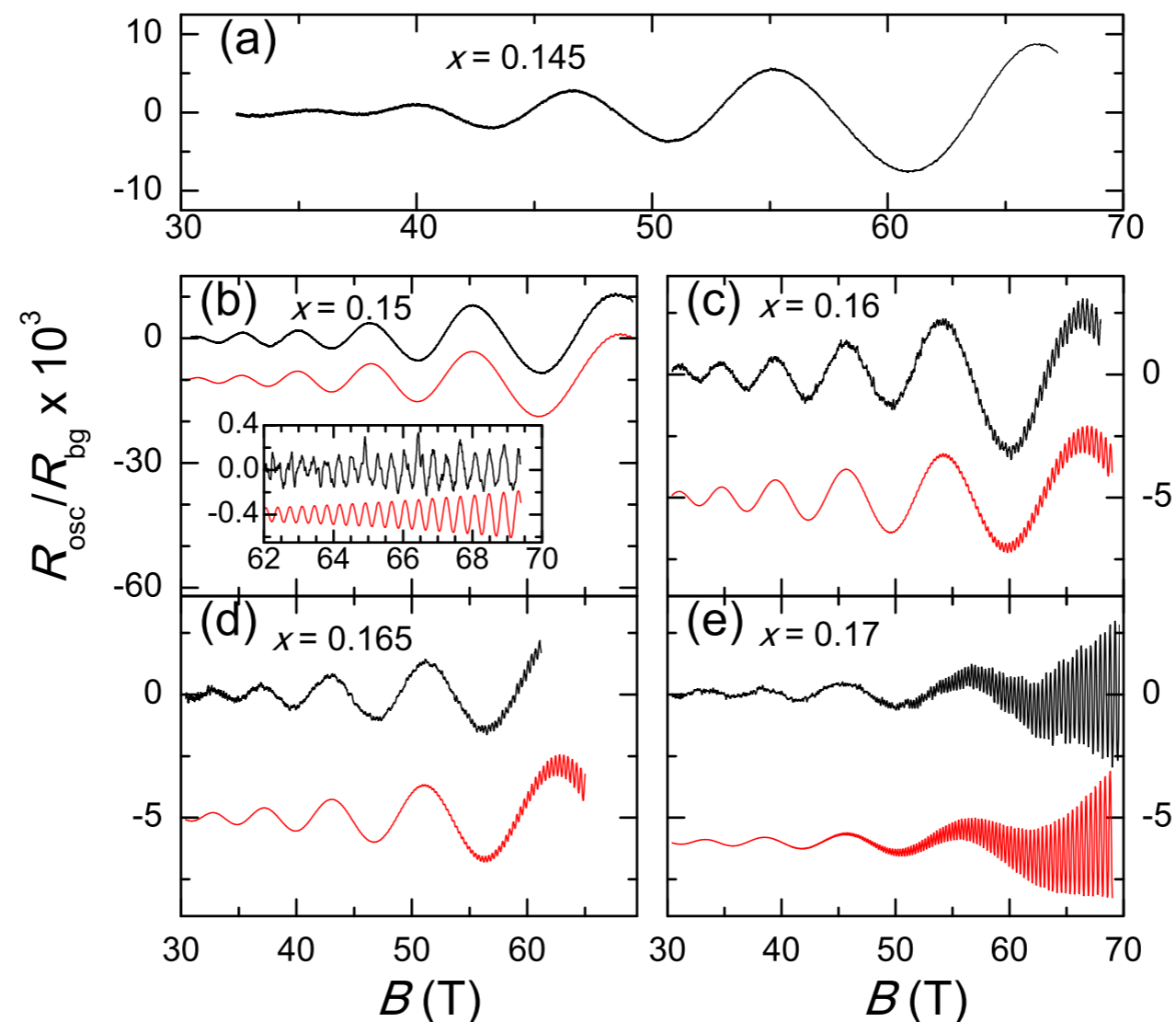
<sup>2</sup>Laboratoire National des Champs Magnétiques Intenses (LNCMI-EMFL), CNRS, INSA, UJF, UPS, F-31400 Toulouse, France

<sup>3</sup>Hochfeld-Magnetlabor Dresden (HLD-EMFL), Helmholtz-Zentrum Dresden-Rossendorf, D-01328 Dresden, Germany

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<sup>5</sup>National High Magnetic Field Laboratory and Department of Physics, Florida State University, Tallahassee, Florida 32310, USA

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- First, the quantitative analysis of the Shubnikov-de Haas effect shows that the weak superlattice potential responsible for the Fermi surface reconstruction in the overdoped regime extrapolates to zero at the doping level  $x_c = 0.175$  corresponding to the onset of superconductivity.

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- Second, the high-field Hall coefficient exhibits a sharp drop right below optimal doping  $x_{\text{opt}} = 0.145$  where the superconducting transition temperature is maximum. This drop is most likely caused by the onset of long-range antiferromagnetic ordering.

# Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen



- Quantum oscillation measurements show the presence of small hole pockets in the electron-doped superconductor  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$  at doping levels into the overdoped regime until  $x = 0.17$  (which corresponds to the end of the superconducting dome).

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- It has been assumed that this discrepancy could be explained by field-induced antiferromagnetic order.

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- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.
- The energy gap between the electron and hole pockets collapses near  $x = 0.17$  like an order parameter.
- “The totality of the data points to a mysterious order between  $x = 0.14$  and  $x = 0.17$ , whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin.”



SDW SRO

## Higgs phase

Emergent  $Z_2$  or  $U(1)$  gauge fields.  
 $Z_2$  vortices or hedgehogs expelled.

$$\langle \Phi^\ell \rangle = 0$$

$$\langle H^a \rangle \neq 0$$

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Symmetry breaking and  
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Topological  
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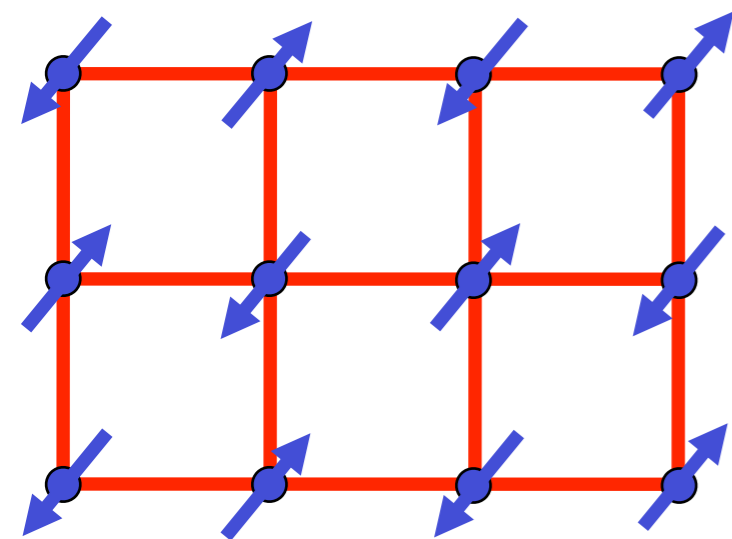
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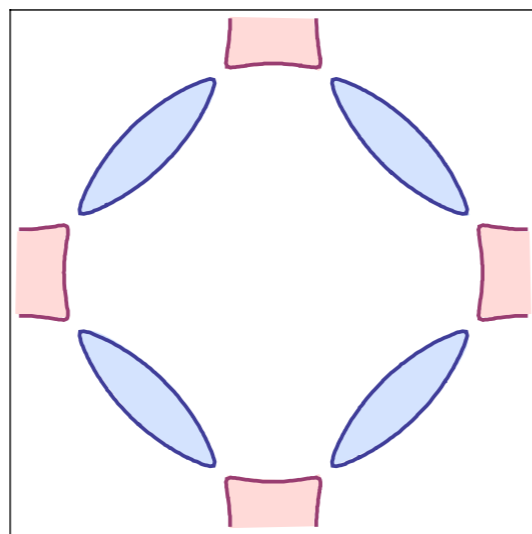
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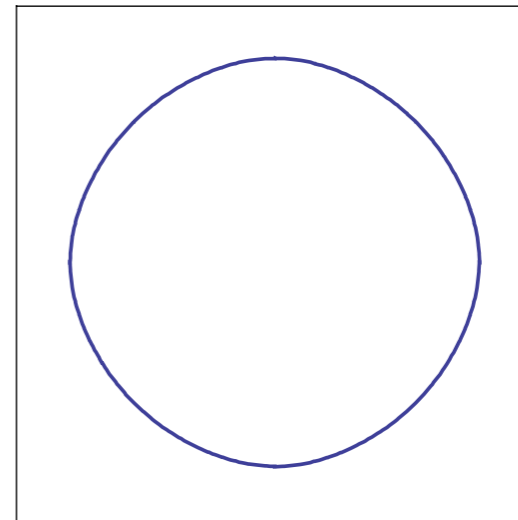
$g$



SDW LRO



Symmetry  
breaking  
phase  
transition



SDW SRO

## Confinement

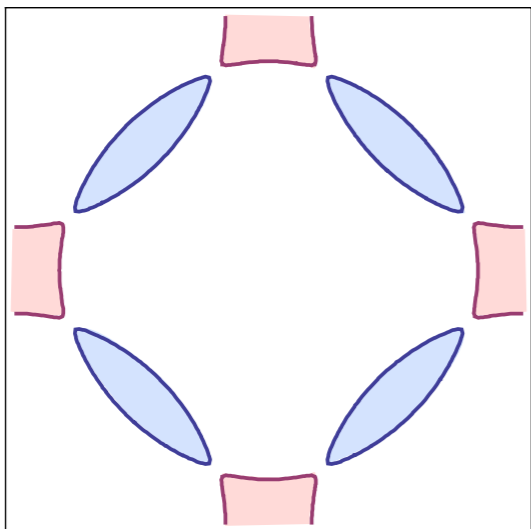
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$U/t$

SDW SRO

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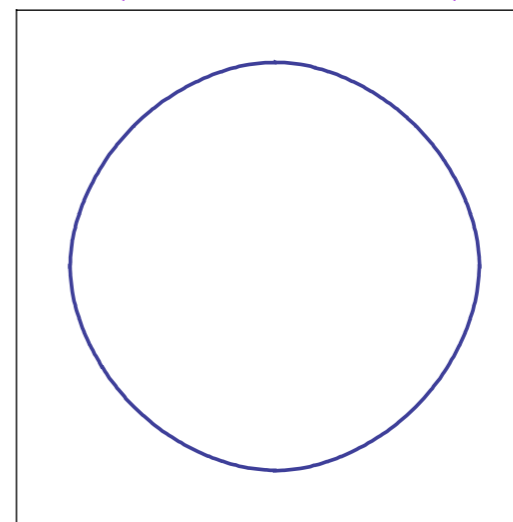
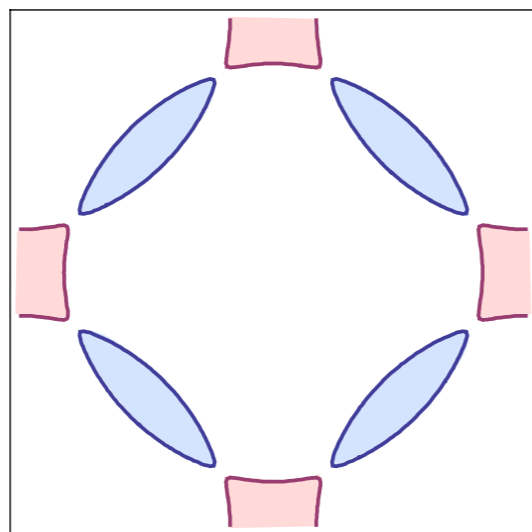
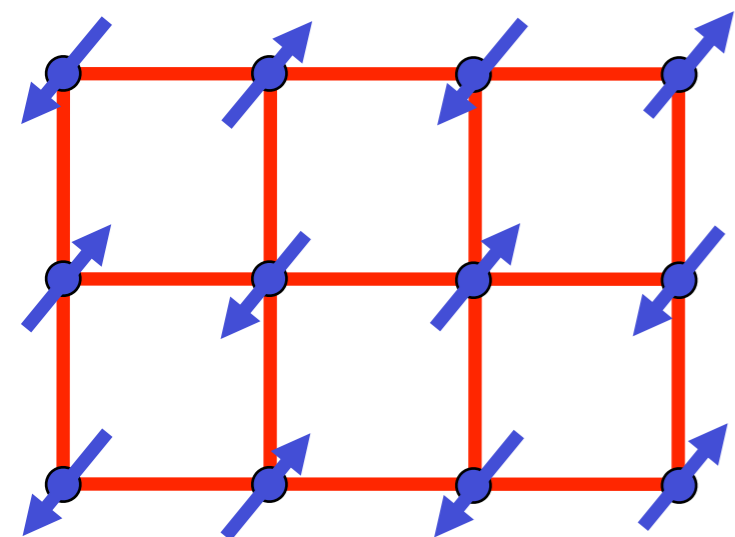
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$g$



Symmetry  
breaking  
phase  
transition

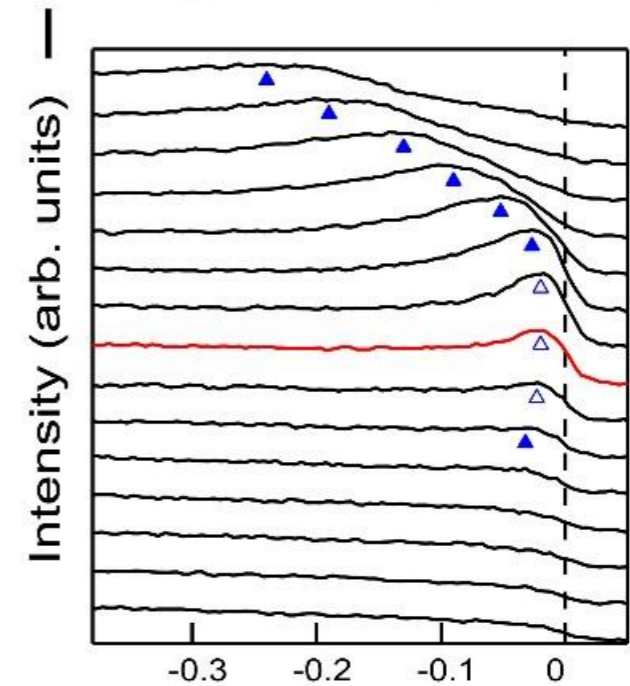
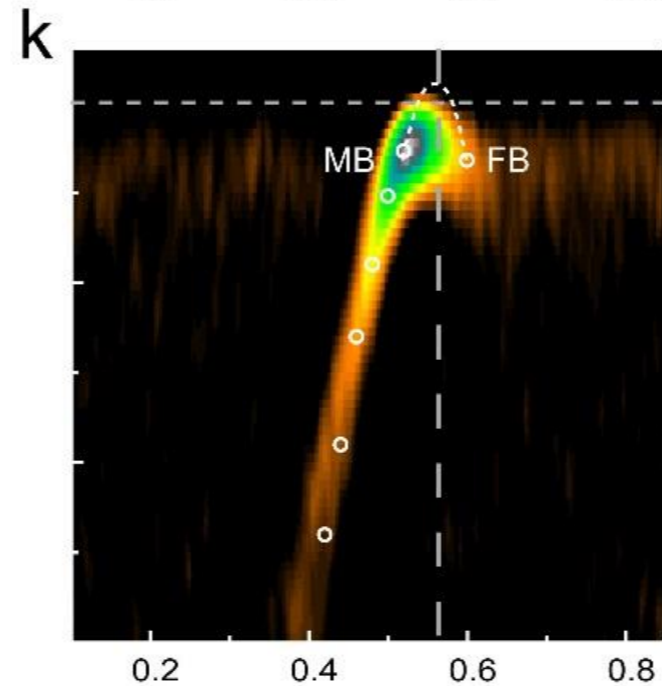
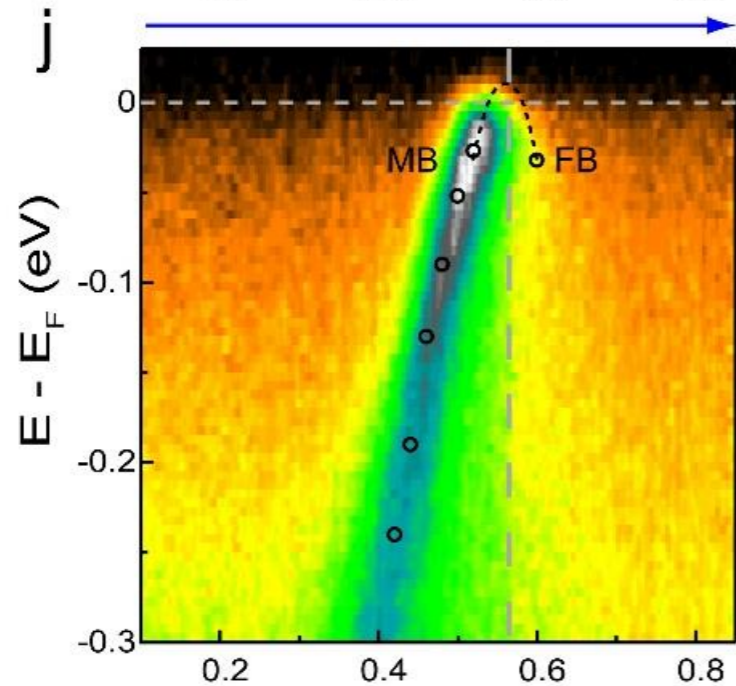
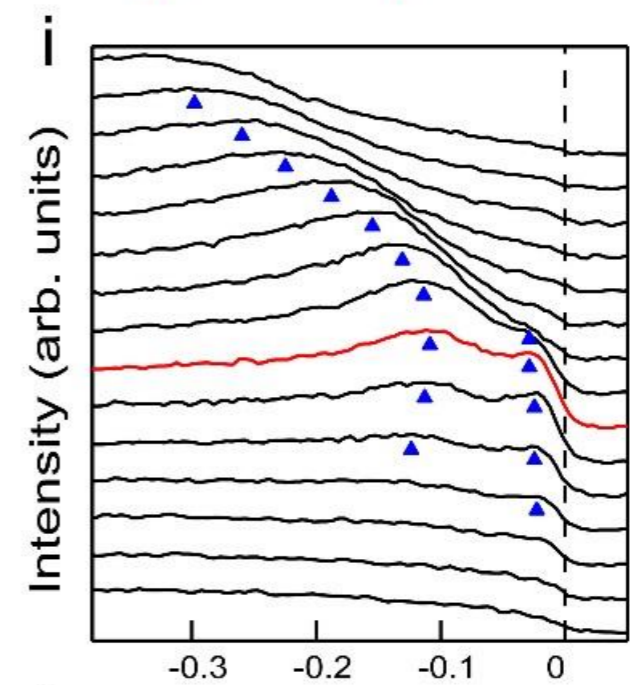
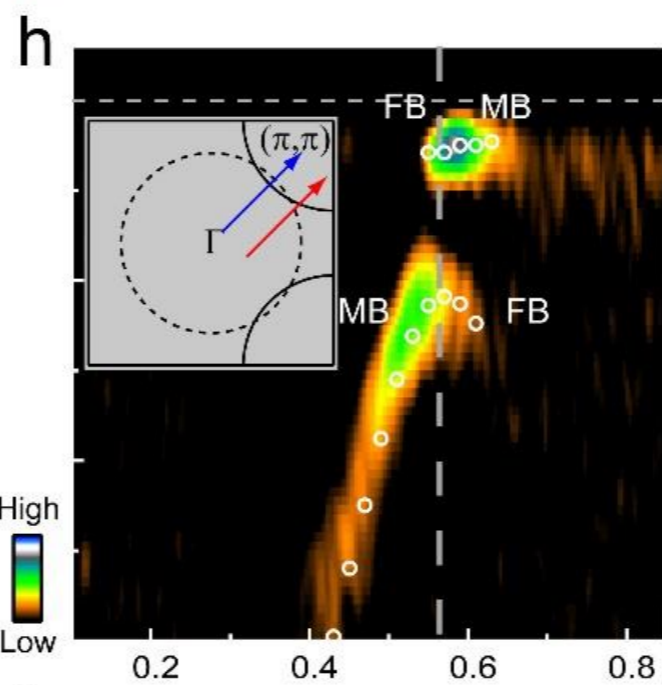
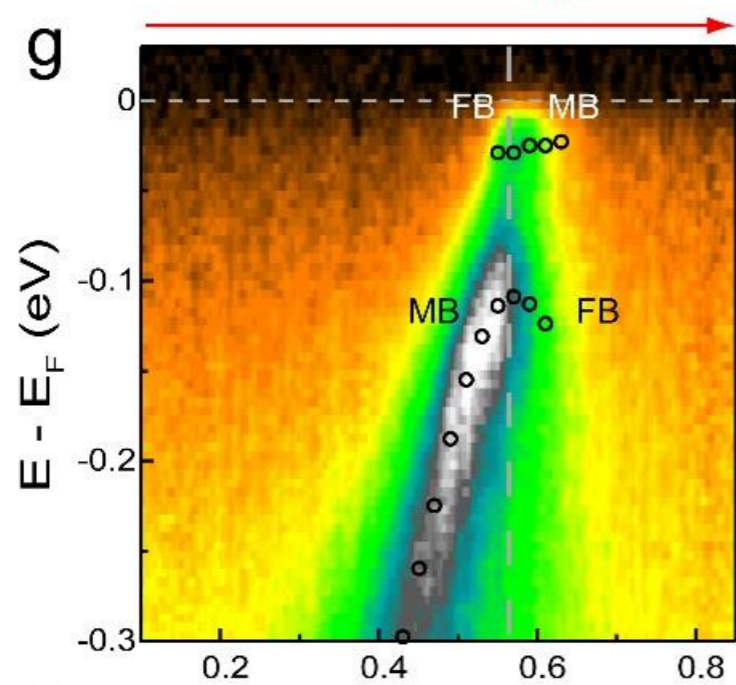
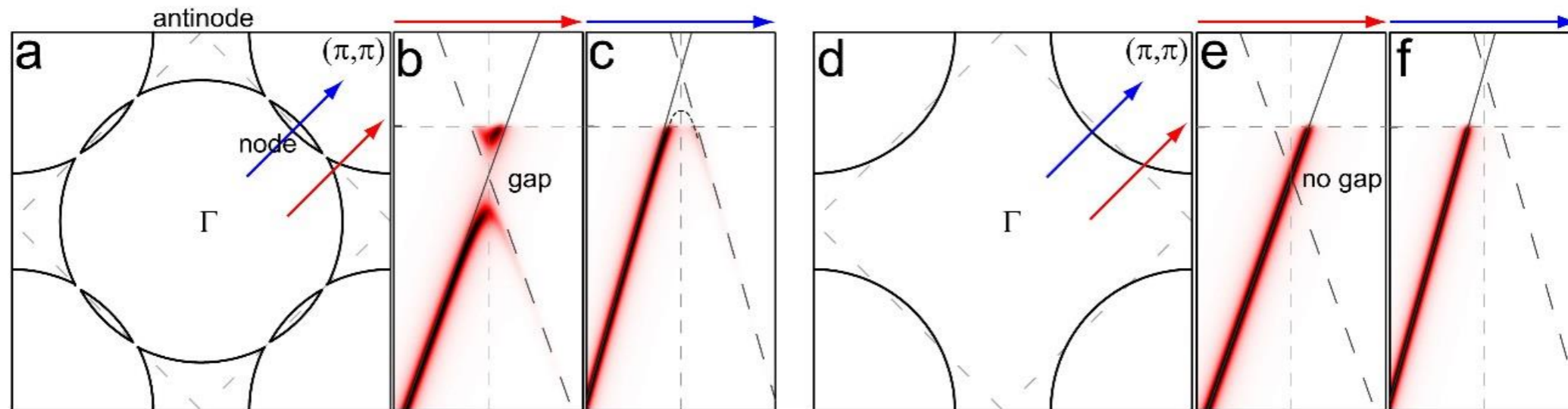
SDW LRO

SDW SRO

# Confinement

No topological order.

$U/t$

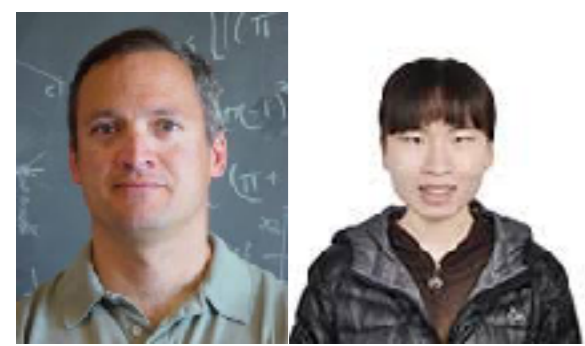


Momentum ( $1/\text{\AA}$ )

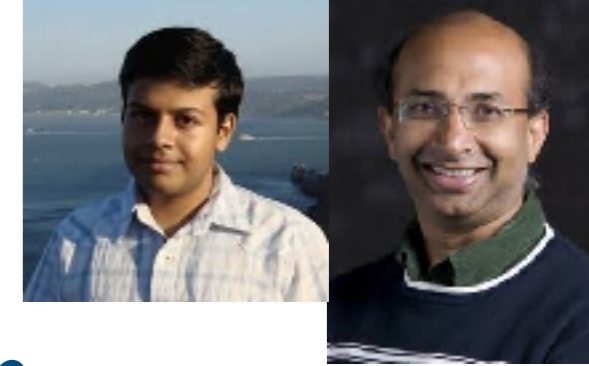
$E - E_F$  (eV)



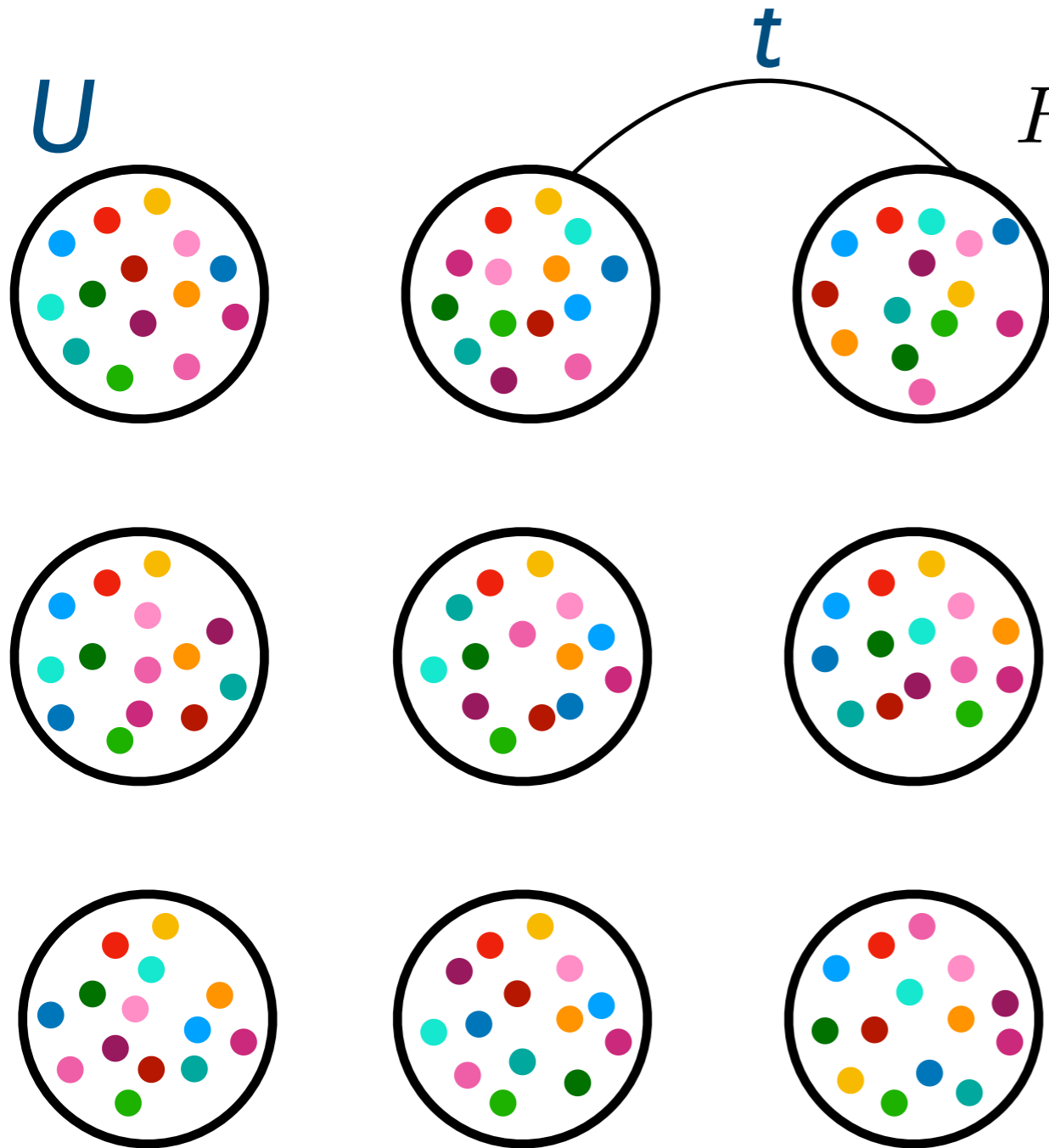
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# Coupled SYK Islands



SYK quantum islands of electrons with random or regular hopping between them.



$$H = \sum_x \sum_{i < j, k < l} U_{ijkl,x} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c_{i,x}^\dagger c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3}$$

$$\overline{|t_{ij,xx'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

Pengfei Zhang, PRB **96**, 205138 (2017)

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 021049 (2018)

Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

# Assessment of SYK lattice models

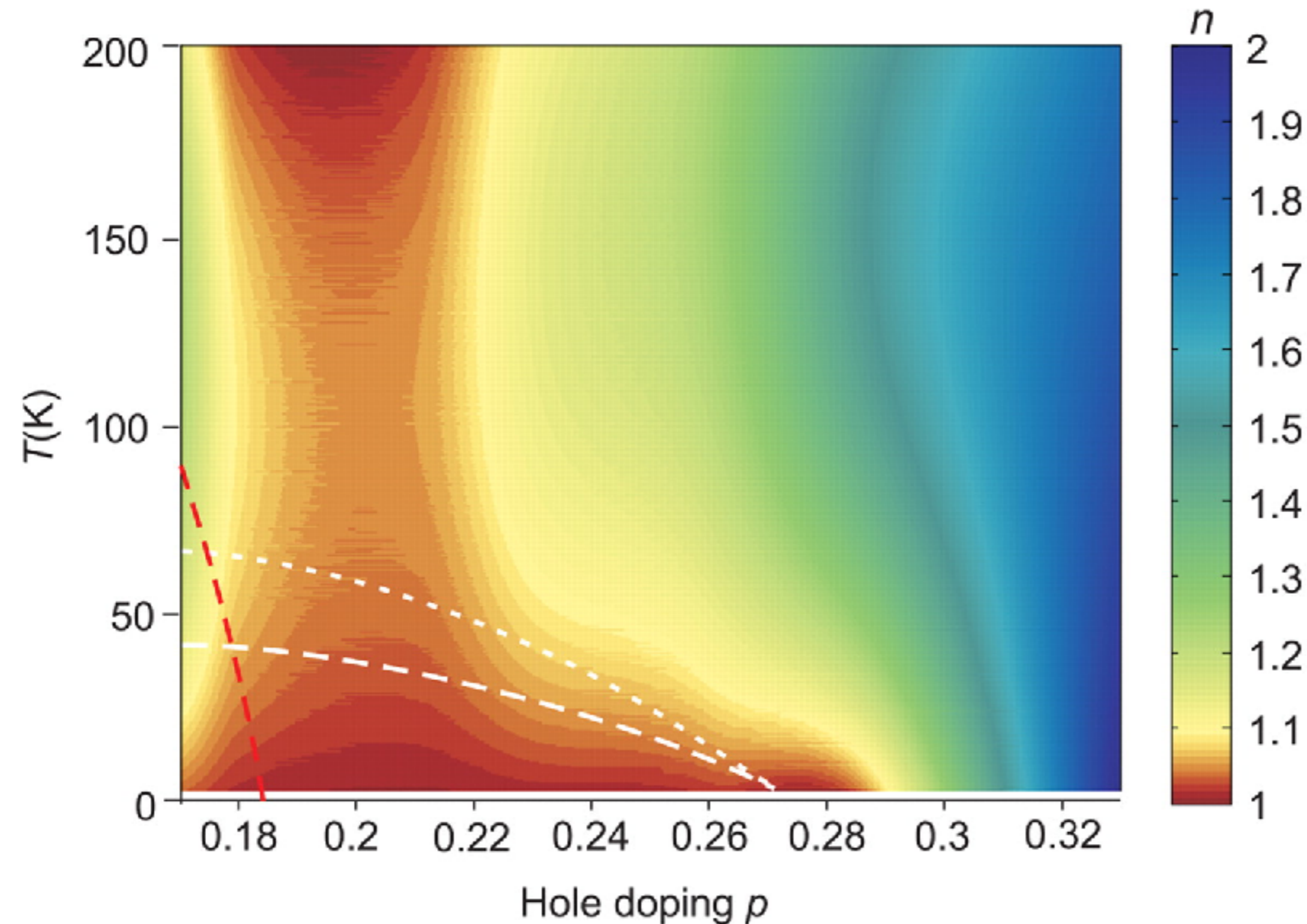
- Reasonable model of incoherent metal behavior for  $t^2/U < T < t$ .
- Strange metal (marginal Fermi-liquid) at smaller  $T$  has linear in  $B$  magnetoresistance, and  $B/T$  scaling with mesoscopic disorder.

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- Strange metal (marginal Fermi-liquid) at smaller  $T$  has linear in  $B$  magnetoresistance, and  $B/T$  scaling with mesoscopic disorder.
- Strange metal (marginal Fermi-liquid) behavior as  $T \rightarrow 0$  only for small ( $p$ ) carrier density, rather than large  $(1 + p)$ .
- No special role for quantum criticality at  $p = p_c$ .
- What is the origin of the “foot” at overdoping ??

# Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

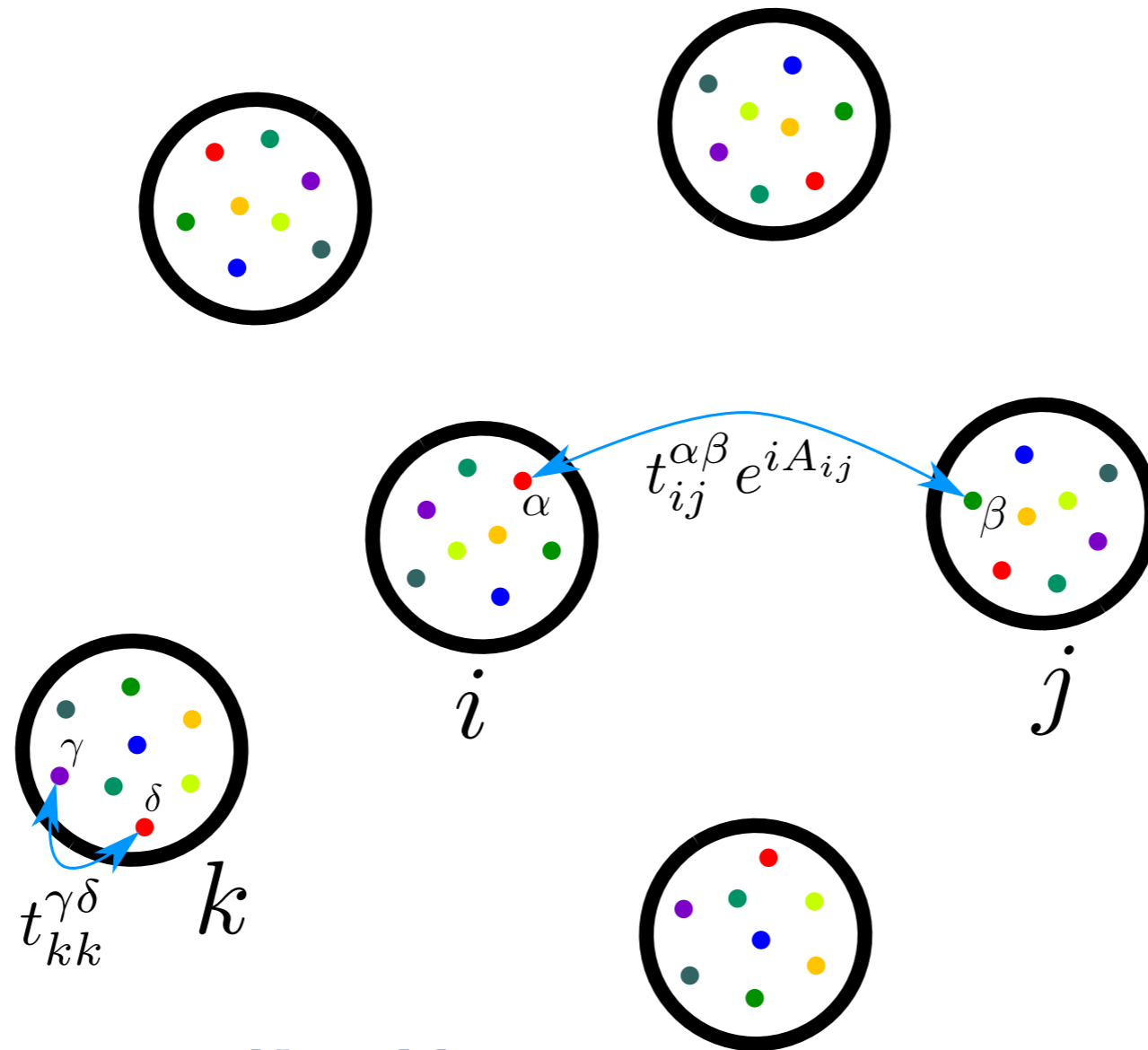
R. A. Cooper,<sup>1</sup> Y. Wang,<sup>1</sup> B. Vignolle,<sup>2</sup> O. J. Lipscombe,<sup>1</sup> S. M. Hayden,<sup>1</sup> Y. Tanabe,<sup>3</sup> T. Adachi,<sup>3</sup> Y. Koike,<sup>3</sup> M. Nohara,<sup>4\*</sup> H. Takagi,<sup>4</sup> Cyril Proust,<sup>2</sup> N. E. Hussey<sup>1†</sup>



# Fermions (chargons) with random hopping coupled to a fluctuating U(1) gauge field



Aavishkar Patel

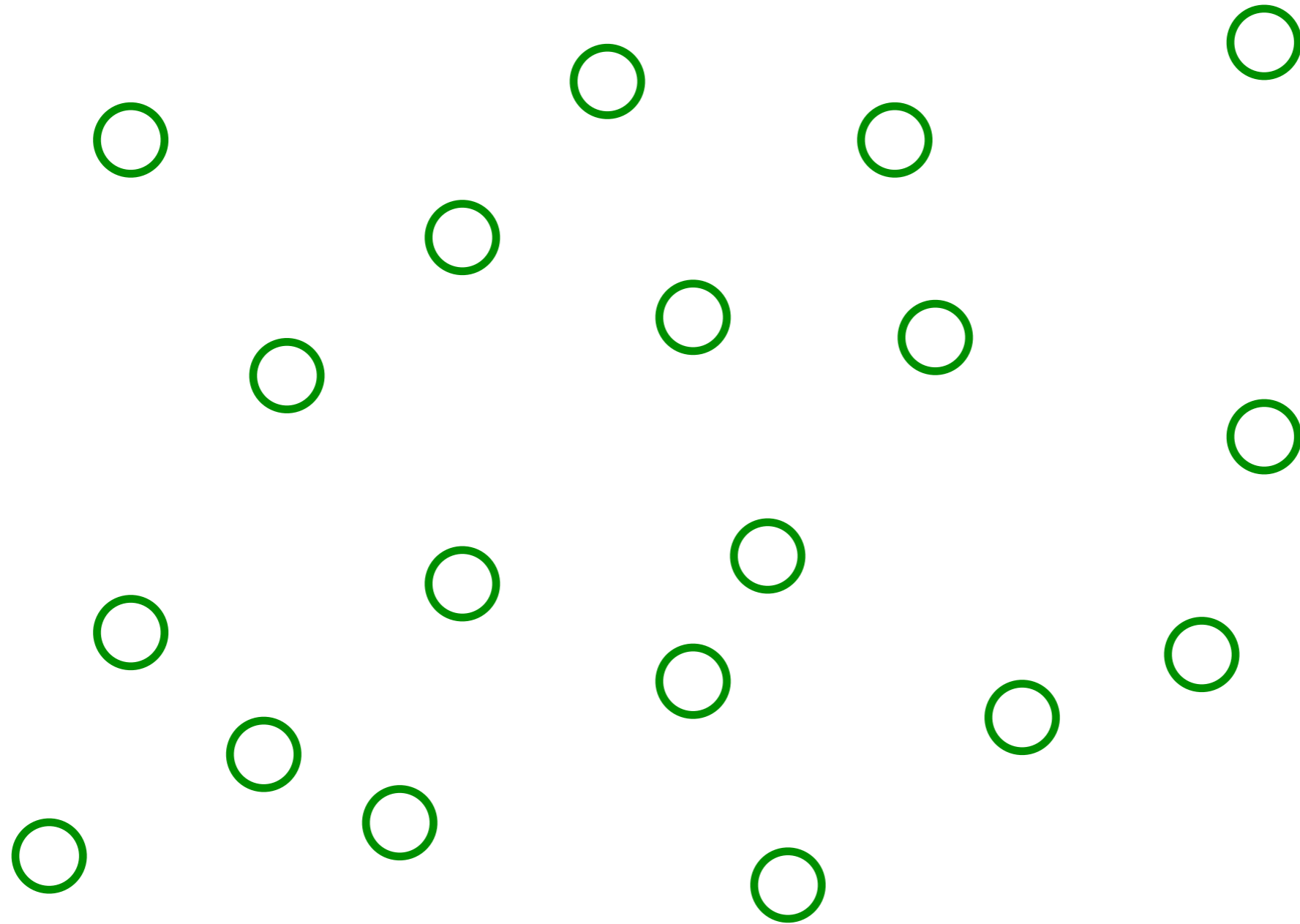


$$H = -\frac{1}{(MN)^{1/2}} \sum_{ij=1}^N \sum_{\alpha\beta=1}^M \left[ t_{ij}^{\alpha\beta} e^{iA_{ij}} \psi_{i\alpha}^\dagger \psi_{j\beta} + (MN)^{1/2} \mu \delta_{ij}^{\alpha\beta} \psi_{i\alpha}^\dagger \psi_{i\alpha} \right],$$

$$\ll t_{ij}^{\alpha\beta} t_{ji}^{\beta\alpha} \gg = \ll |t_{ij}^{\alpha\beta}|^2 \gg = t^2, \quad A_{ji} = -A_{ij}.$$

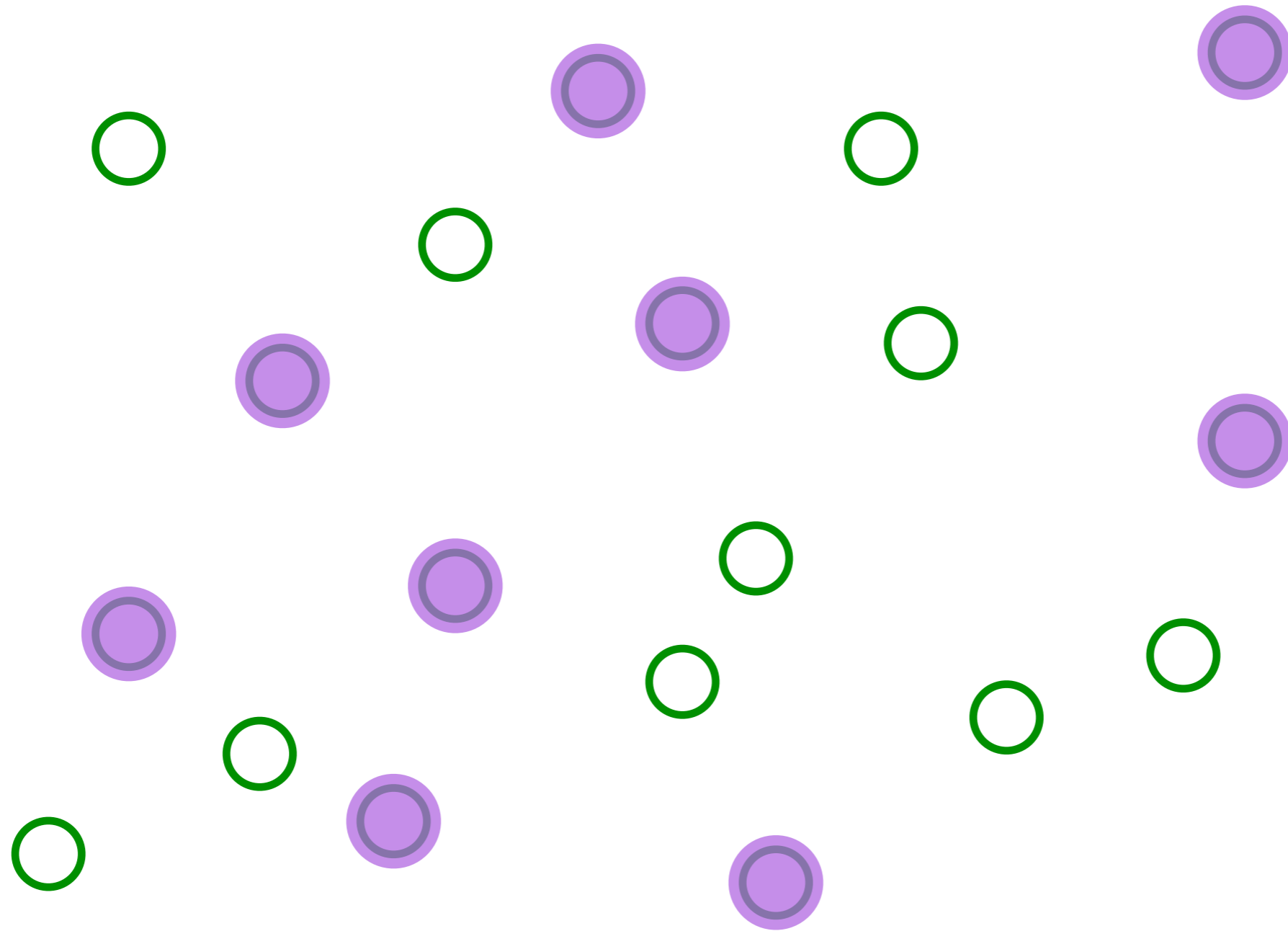
1. Review of spin-density-wave theory
2.  $SU(2)$  gauge theory of fluctuating spin density waves
3. Quantum oscillations and photoemission on the electron-doped cuprates
4. Strange metals and SYK models
5. Review of SYK models: Schwarzian theory
6. Connections to black holes with  $AdS_2$  horizons

# A simple model of a metal with quasiparticles



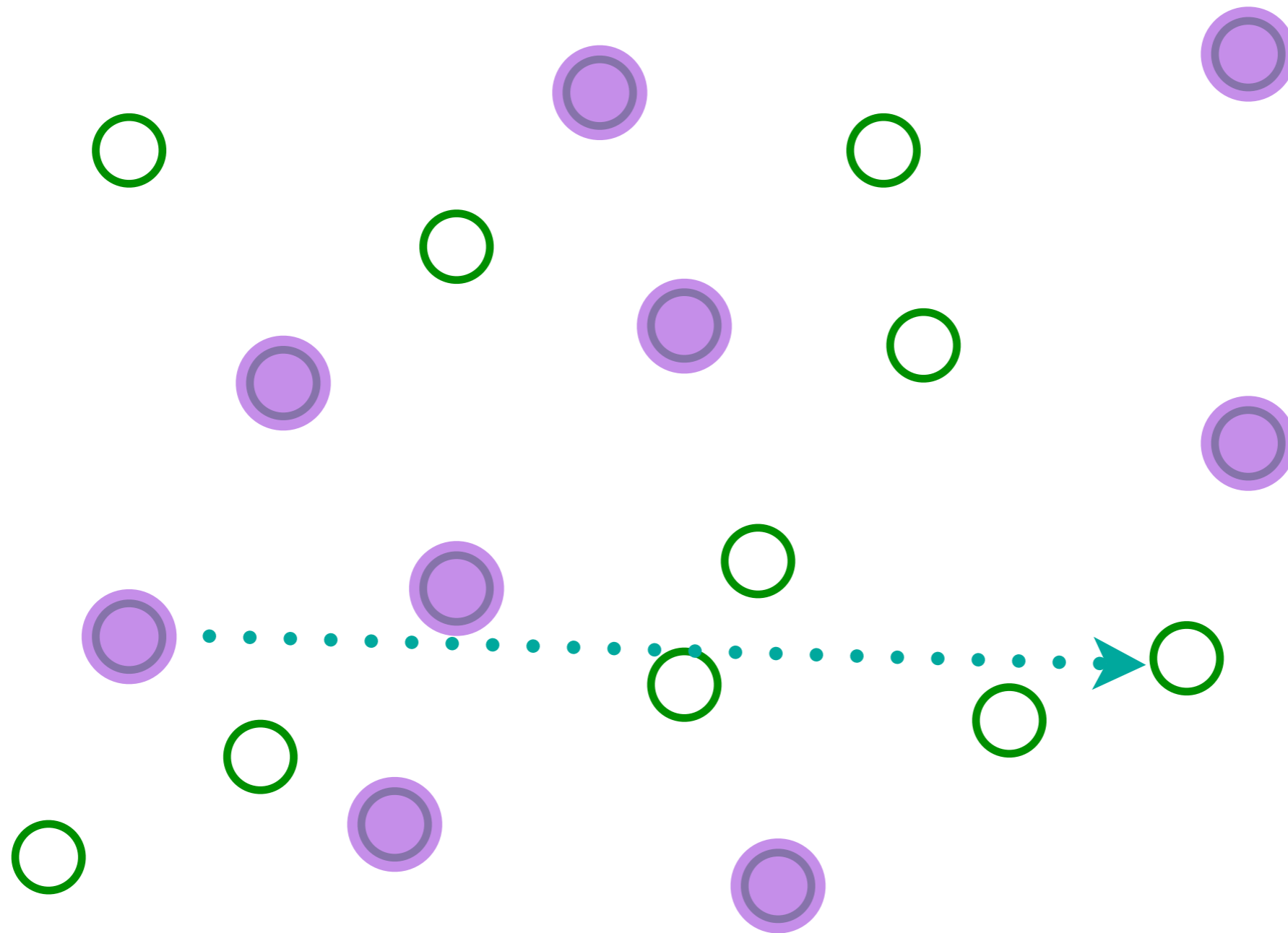
Pick a set of random positions

# A simple model of a metal with quasiparticles



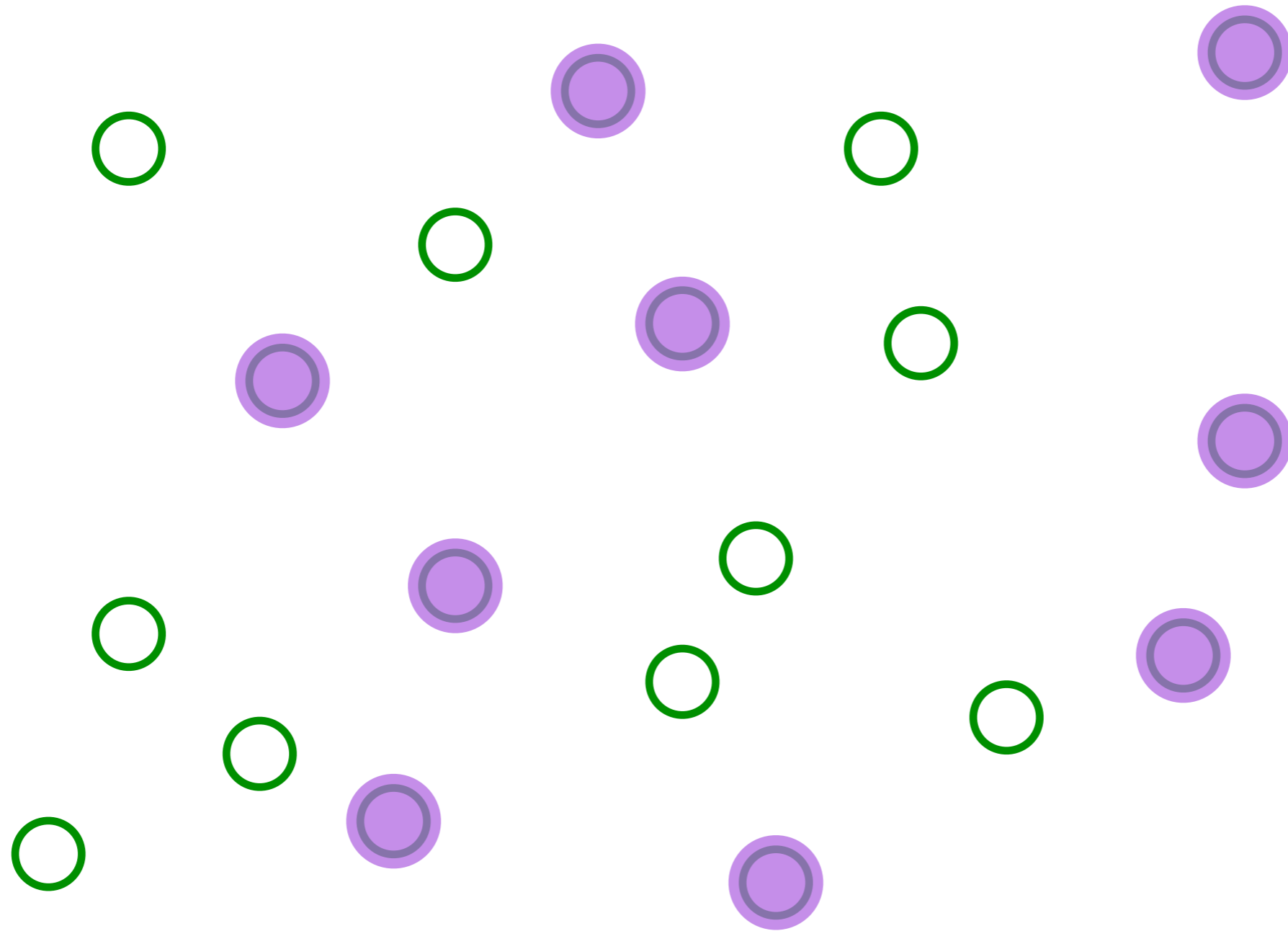
Place electrons randomly on some sites

# A simple model of a metal with quasiparticles



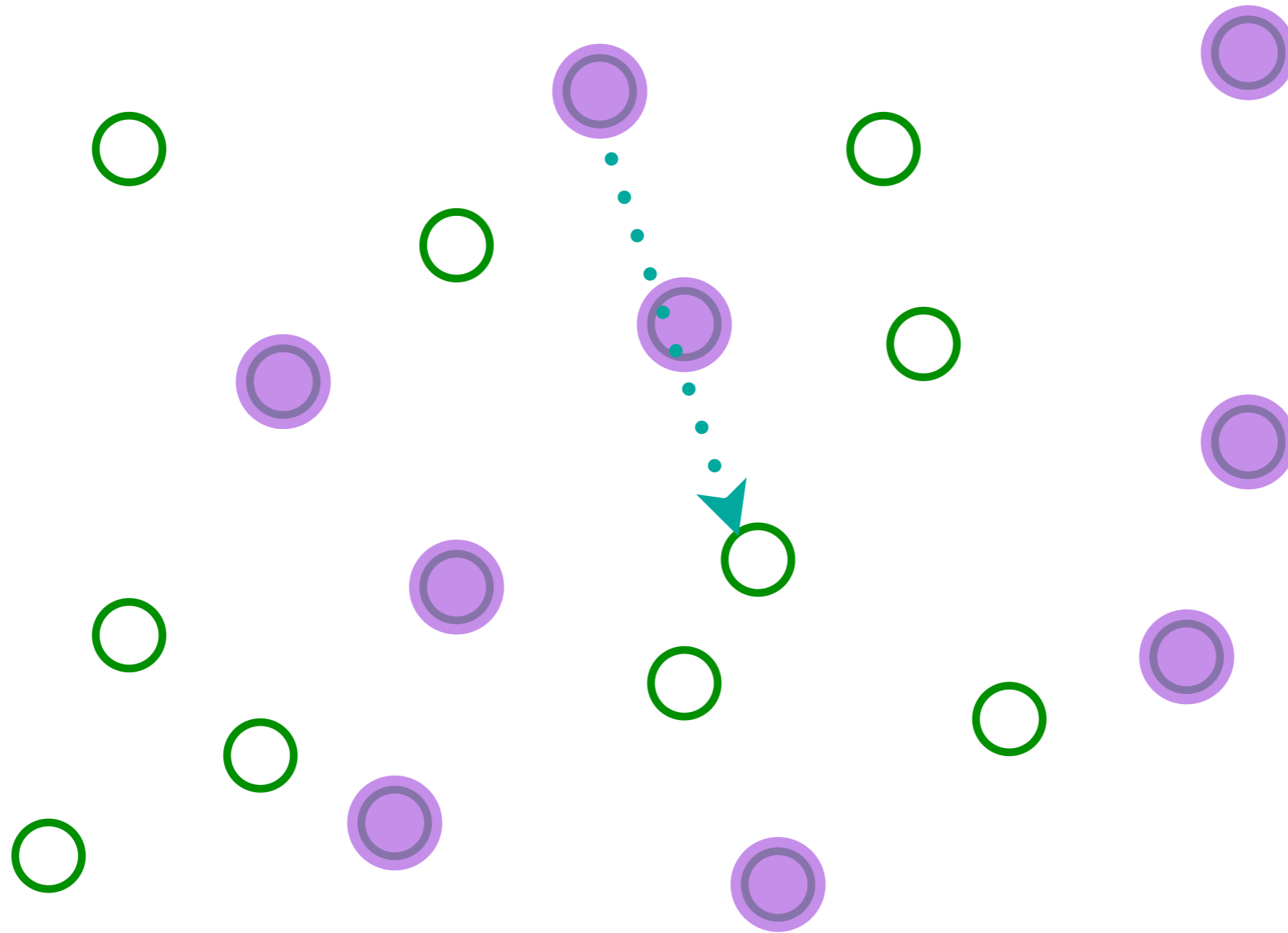
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# A simple model of a metal with quasiparticles



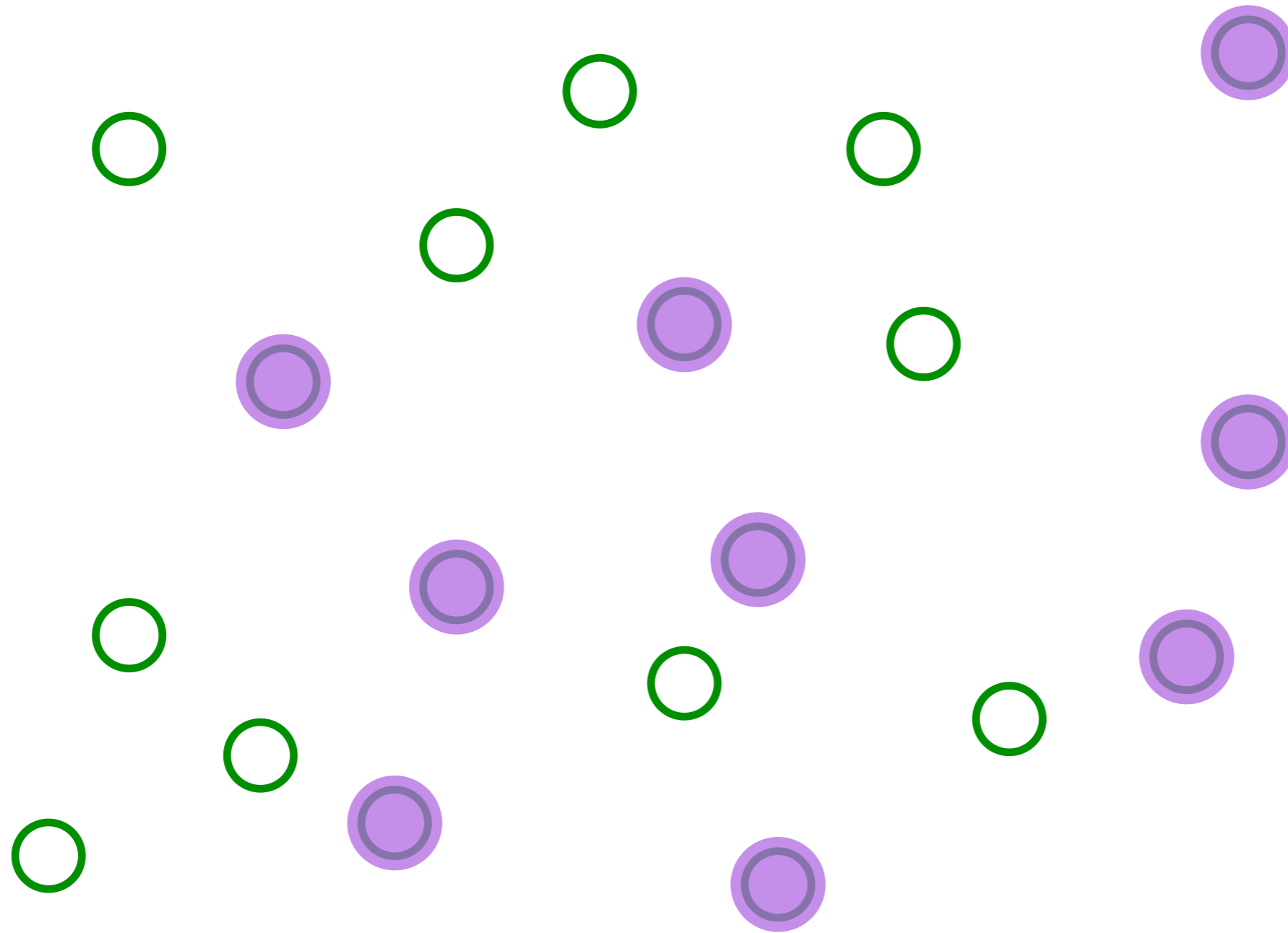
Electrons move one-by-one randomly

# A simple model of a metal with quasiparticles



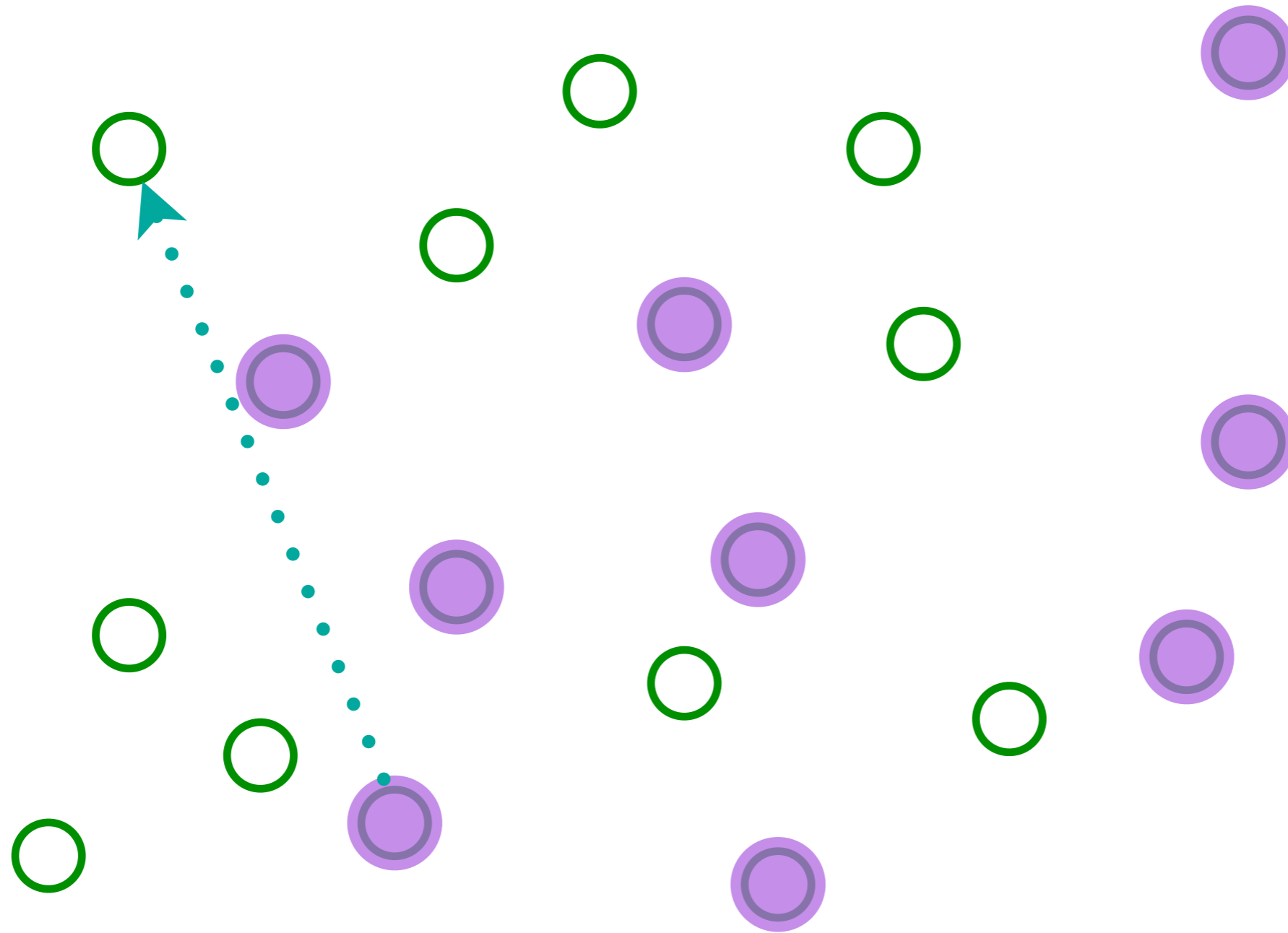
Electrons move one-by-one randomly

# A simple model of a metal with quasiparticles



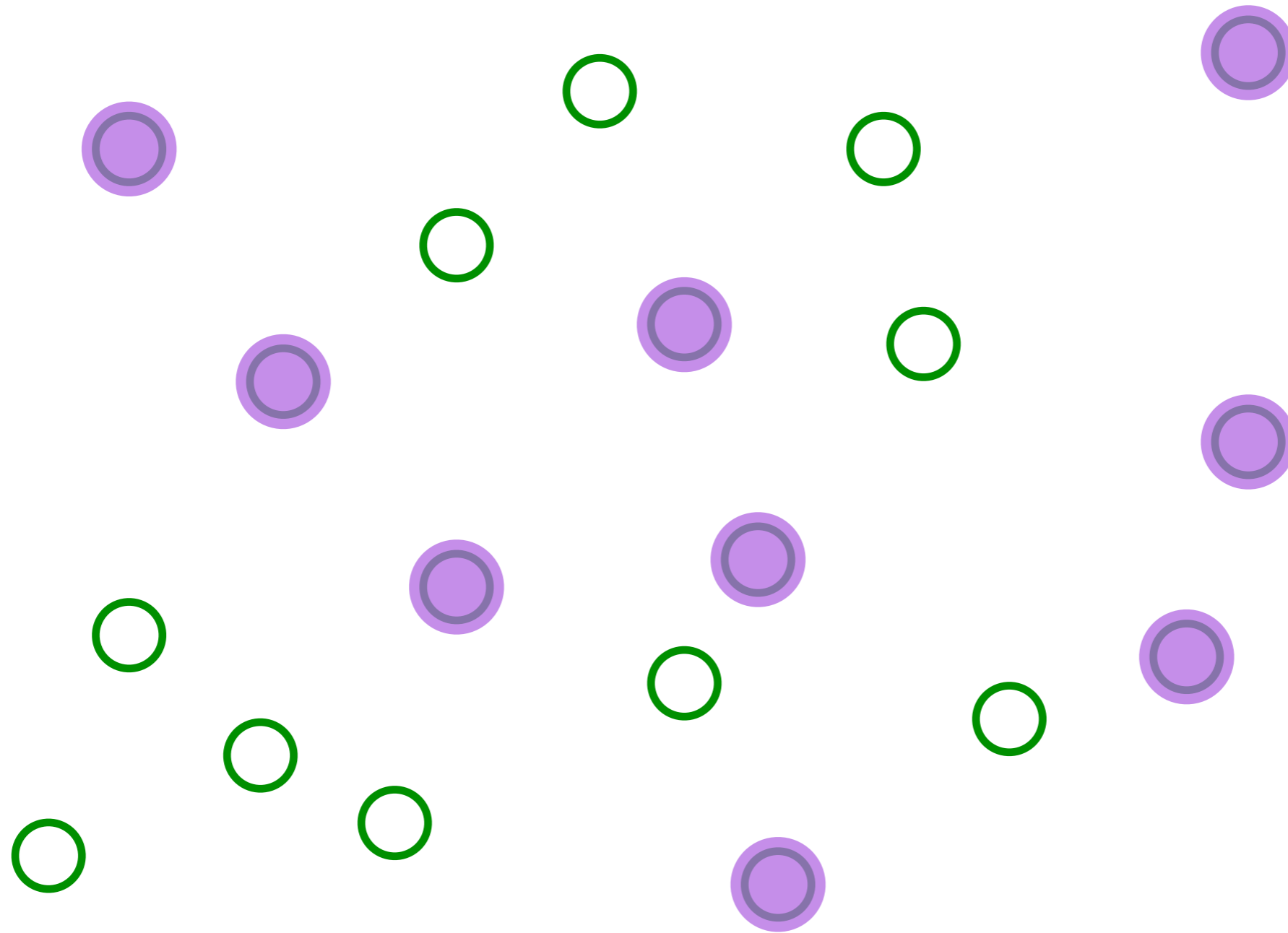
Electrons move one-by-one randomly

# A simple model of a metal with quasiparticles



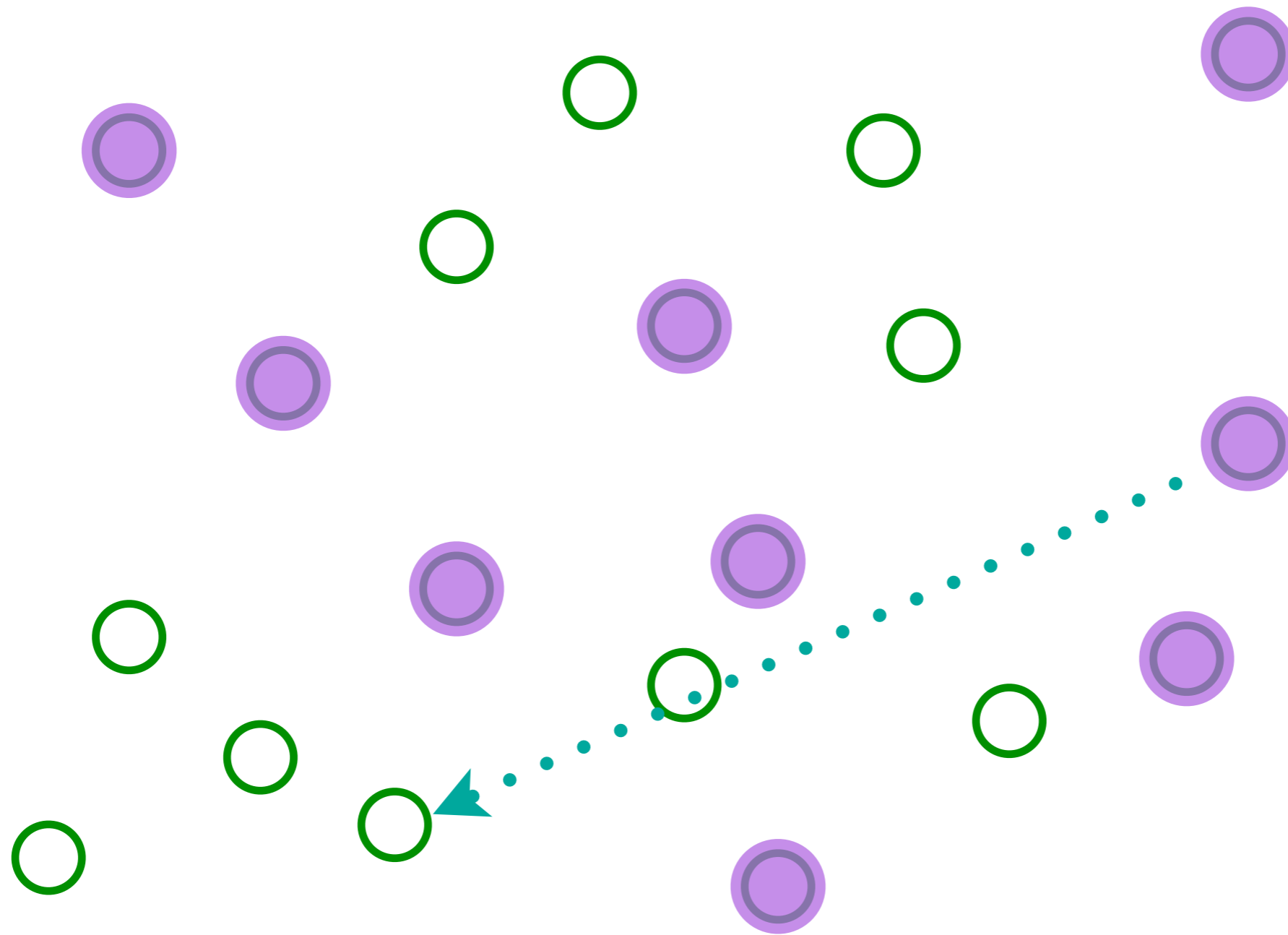
Electrons move one-by-one randomly

# A simple model of a metal with quasiparticles



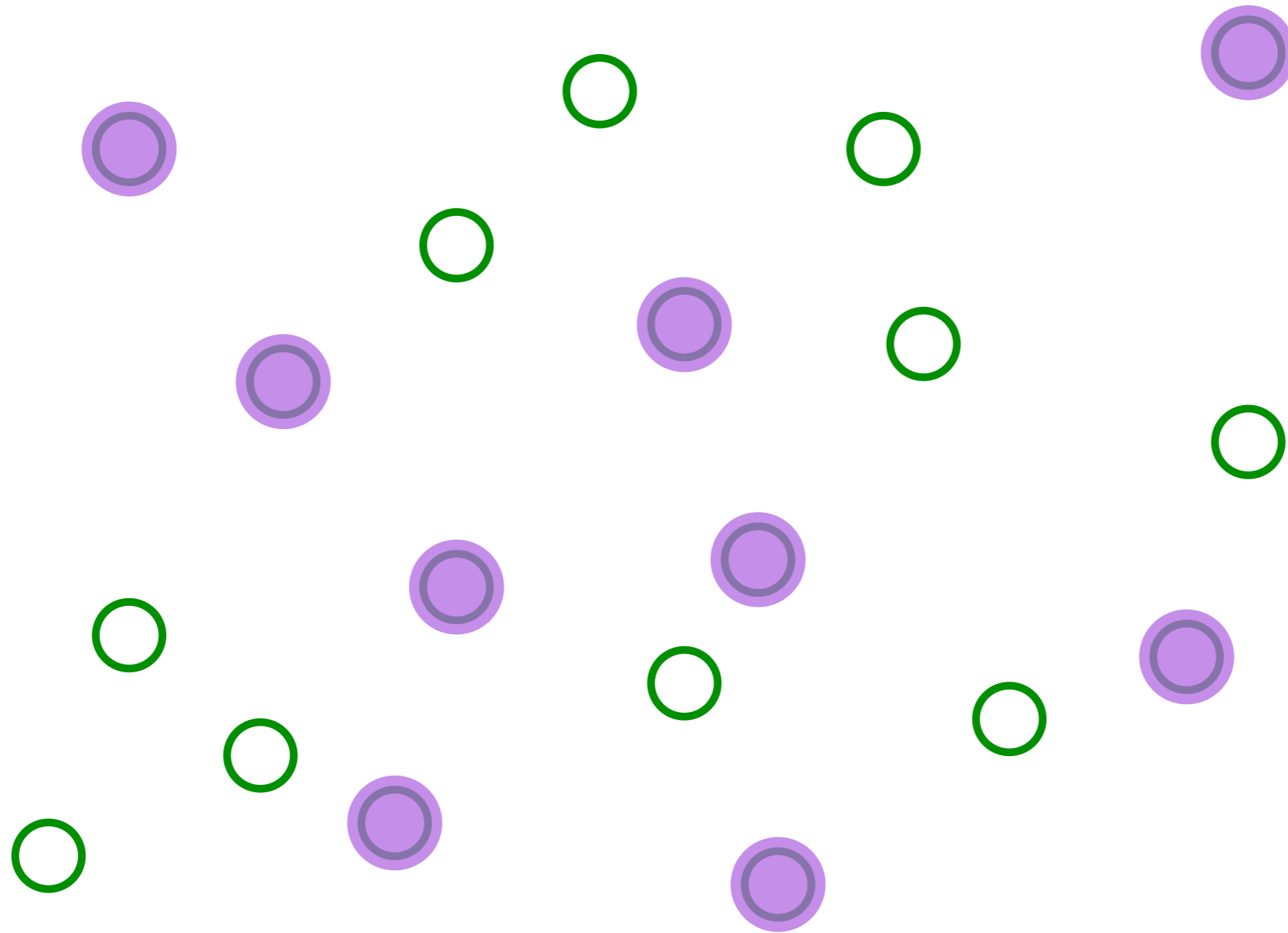
Electrons move one-by-one randomly

# A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

# A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

# A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

$t_{ij}$  are independent random variables with  $\overline{t_{ij}} = 0$  and  $\overline{|t_{ij}|^2} = t^2$

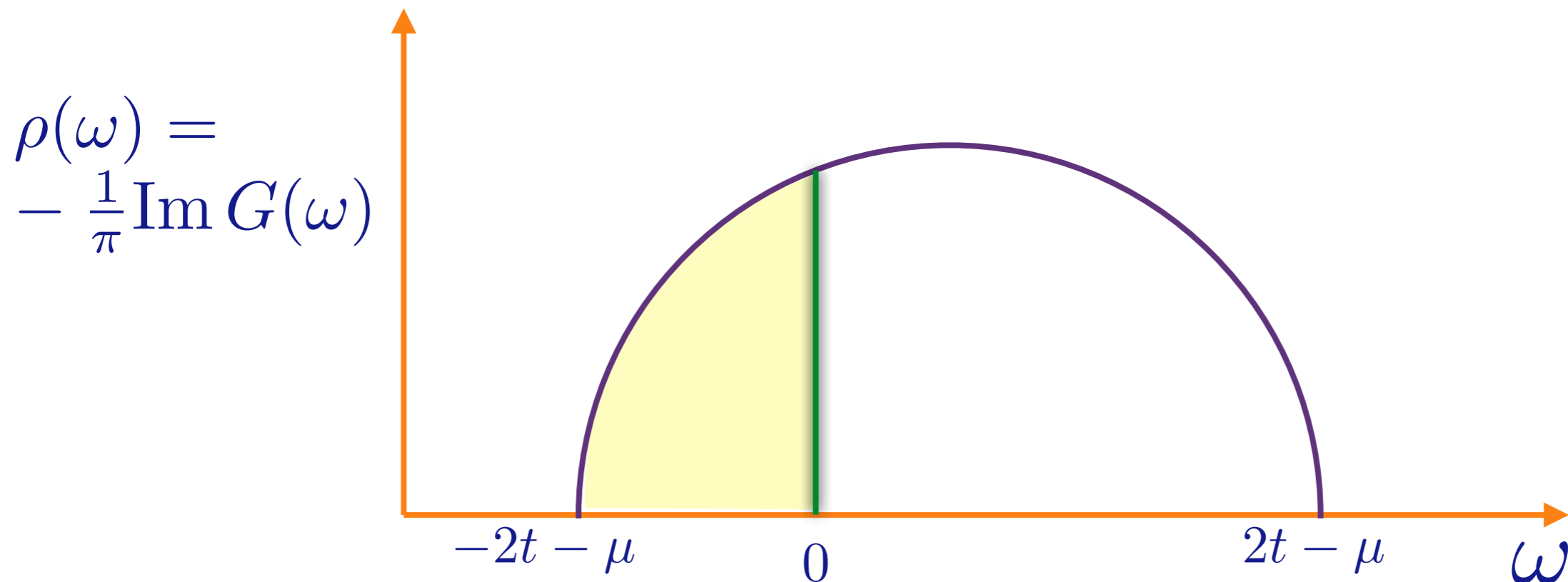
**Fermions occupying the eigenstates of a  
 $N \times N$  random matrix**

# A simple model of a metal with quasiparticles

Feynman graph expansion in  $t_{ij..}$ , and graph-by-graph average, yields exact equations in the large  $N$  limit:

$$G(\tau) \equiv -T_\tau \left\langle c_i(\tau) c_i^\dagger(0) \right\rangle$$
$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = Q.$$

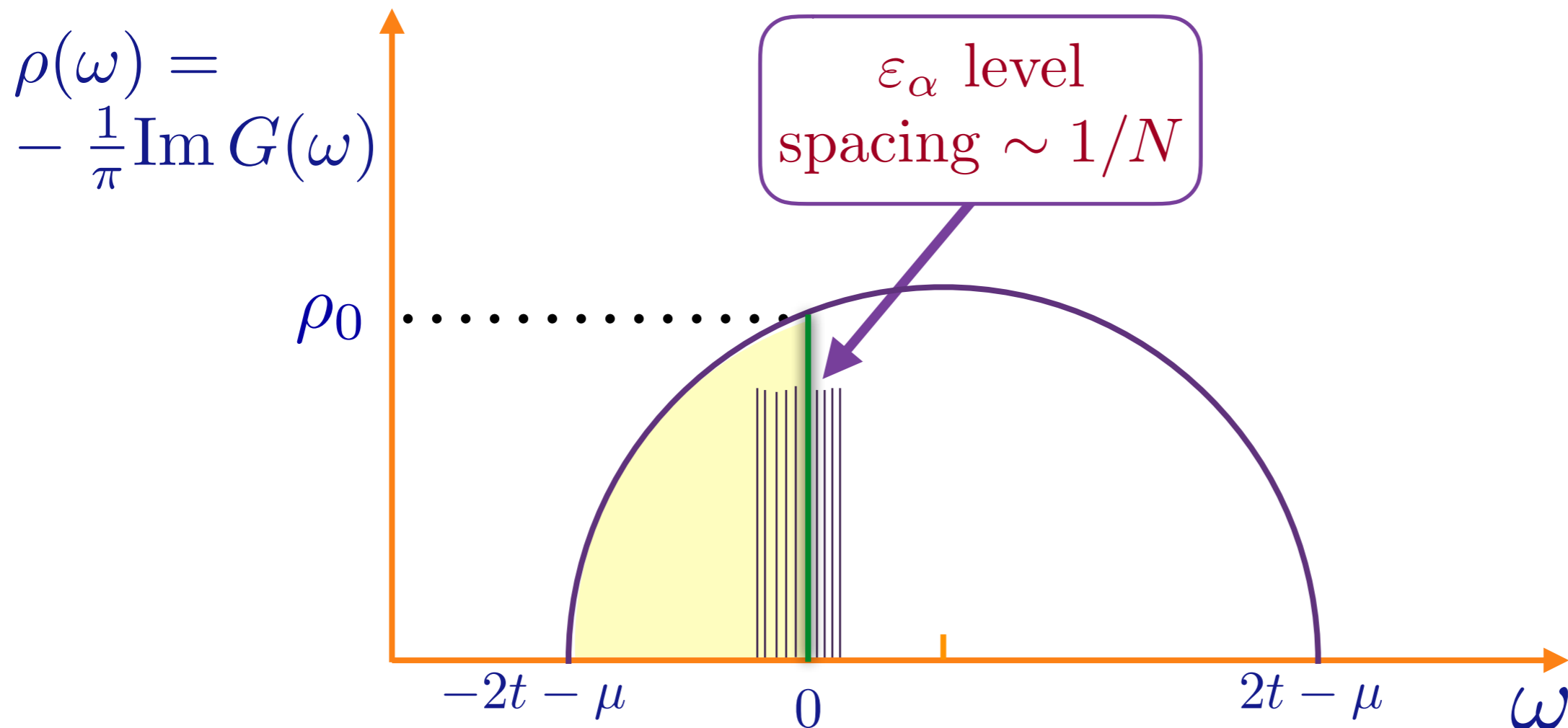
$G(\omega)$  can be determined by solving a quadratic equation.



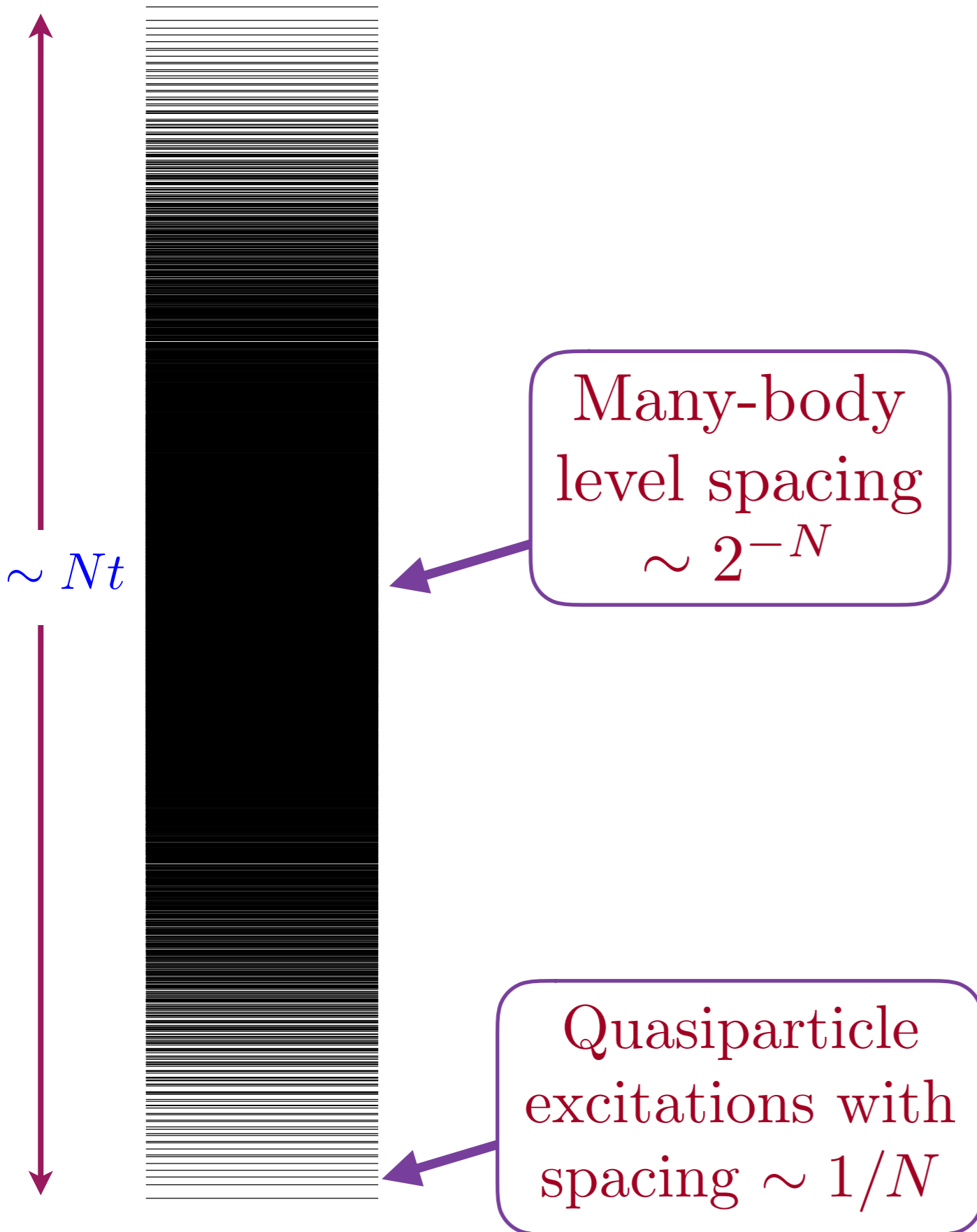
# A simple model of a metal with quasiparticles

Let  $\varepsilon_\alpha$  be the eigenvalues of the matrix  $t_{ij}/\sqrt{N}$ . The fermions will occupy the lowest  $NQ$  eigenvalues, upto the Fermi energy  $E_F$ . The single-particle density of states is

$$\rho(\omega) = (1/N) \sum_\alpha \delta(\omega - \varepsilon_\alpha), \text{ and } \rho_0 \equiv \rho(\omega = 0).$$



# A simple model of a metal with quasiparticles



There are  $2^N$  many body levels with energy

$$E = \sum_{\alpha=1}^N n_{\alpha} \varepsilon_{\alpha},$$

where  $n_{\alpha} = 0, 1$ . Shown are all values of  $E$  for a single cluster of size  $N = 12$ . The  $\varepsilon_{\alpha}$  have a level spacing  $\sim 1/N$ .

# A simple model of a metal with quasiparticles

The grand potential  $\Omega(T)$  at low  $T$  is (from the Sommerfeld expansion)

$$\Omega(T) - E_0 = N \left( -\frac{\pi^2}{6} \rho_0 T^2 + \mathcal{O}(T^4) \right) + \dots$$

where  $\rho_0 \equiv \rho(0)$  is the *single* particle density of states at the Fermi level.

We can also define the *many* body density of states,  $D(E)$ , via

$$Z = e^{-\Omega(T)/T} = \int_{-\infty}^{\infty} dE D(E) e^{-E/T}$$

The inversion from  $\Omega(T)$  to  $D(E)$  has to be performed with care (it does not commute with the  $1/N$  expansion), and we obtain

$$D(E) \sim \exp \left( \pi \sqrt{\frac{2N\rho_0(E - E_0)}{3}} \right), \quad E > E_0, \quad \frac{1}{N} \ll \rho_0(E - E_0) \ll N$$

and  $D(E) = 0$  for  $E < E_0$ . This is related to the asymptotic growth of the partitions of an integer,  $p(n) \sim \exp(\pi\sqrt{2n/3})$ . Near the lower bound, there are large sample-to-sample fluctuations due to variations in the lowest quasiparticle energies.

# A simple model of a metal with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N U_{ij;k\ell} c_i^\dagger c_j^\dagger c_k c_\ell$$

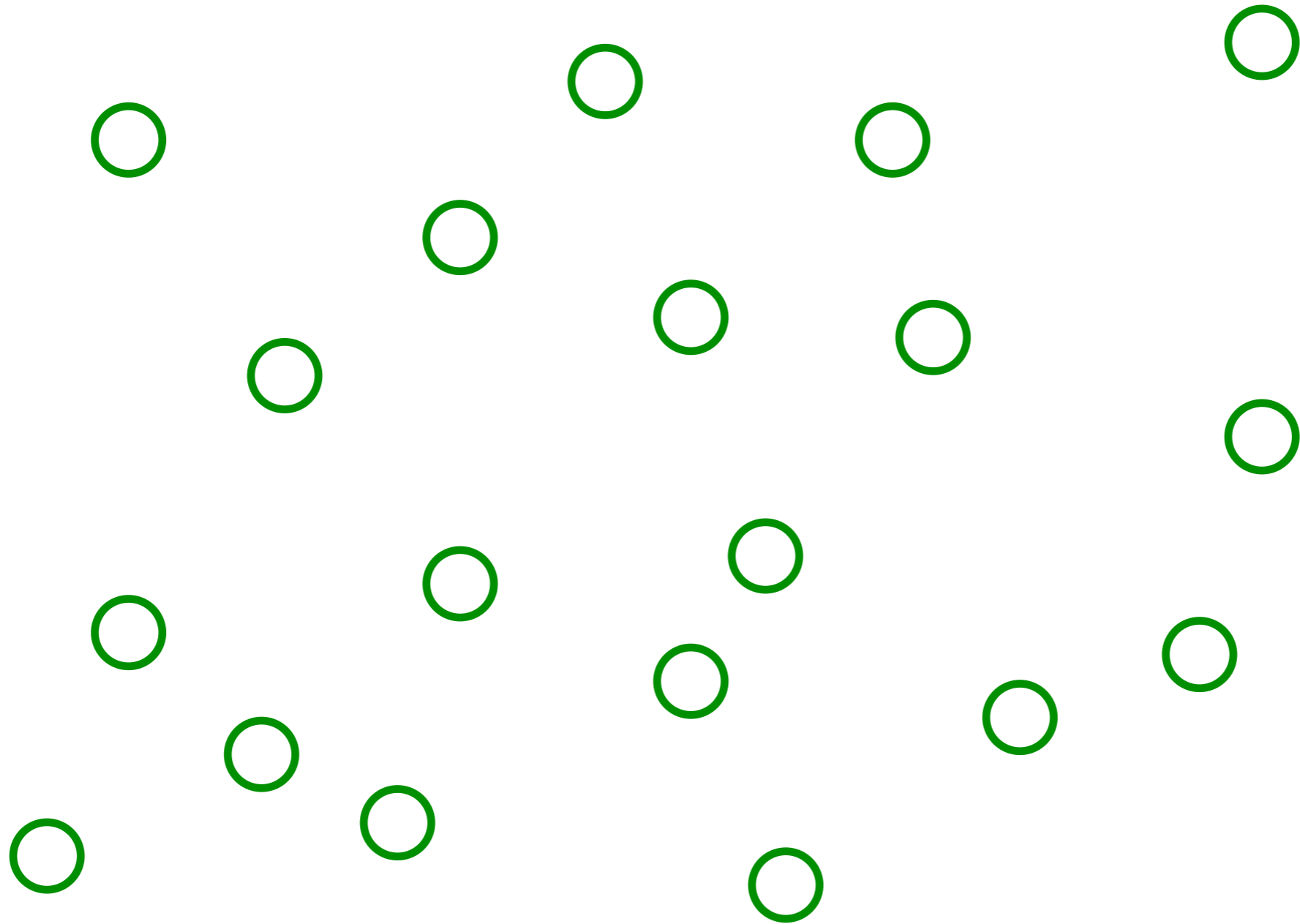
$U_{ij;k\ell}$  are independent random variables with  $\overline{U_{ij;k\ell}} = 0$  and  $|\overline{U_{ij;k\ell}}|^2 = U^2$ . We compute the lifetime of a quasiparticle,  $\tau_\alpha$ , in an exact eigenstate  $\psi_\alpha(i)$  of the free particle Hamiltonian with energy  $\varepsilon_\alpha$ . By Fermi's Golden rule, for  $\varepsilon_\alpha$  at the Fermi energy

$$\begin{aligned} \frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta)) \delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^2}{4} T^2 \end{aligned}$$

where  $\rho_0$  is the density of states at the Fermi energy, and  $f(\varepsilon) = 1/(e^{\varepsilon/T} + 1)$  is the Fermi function.

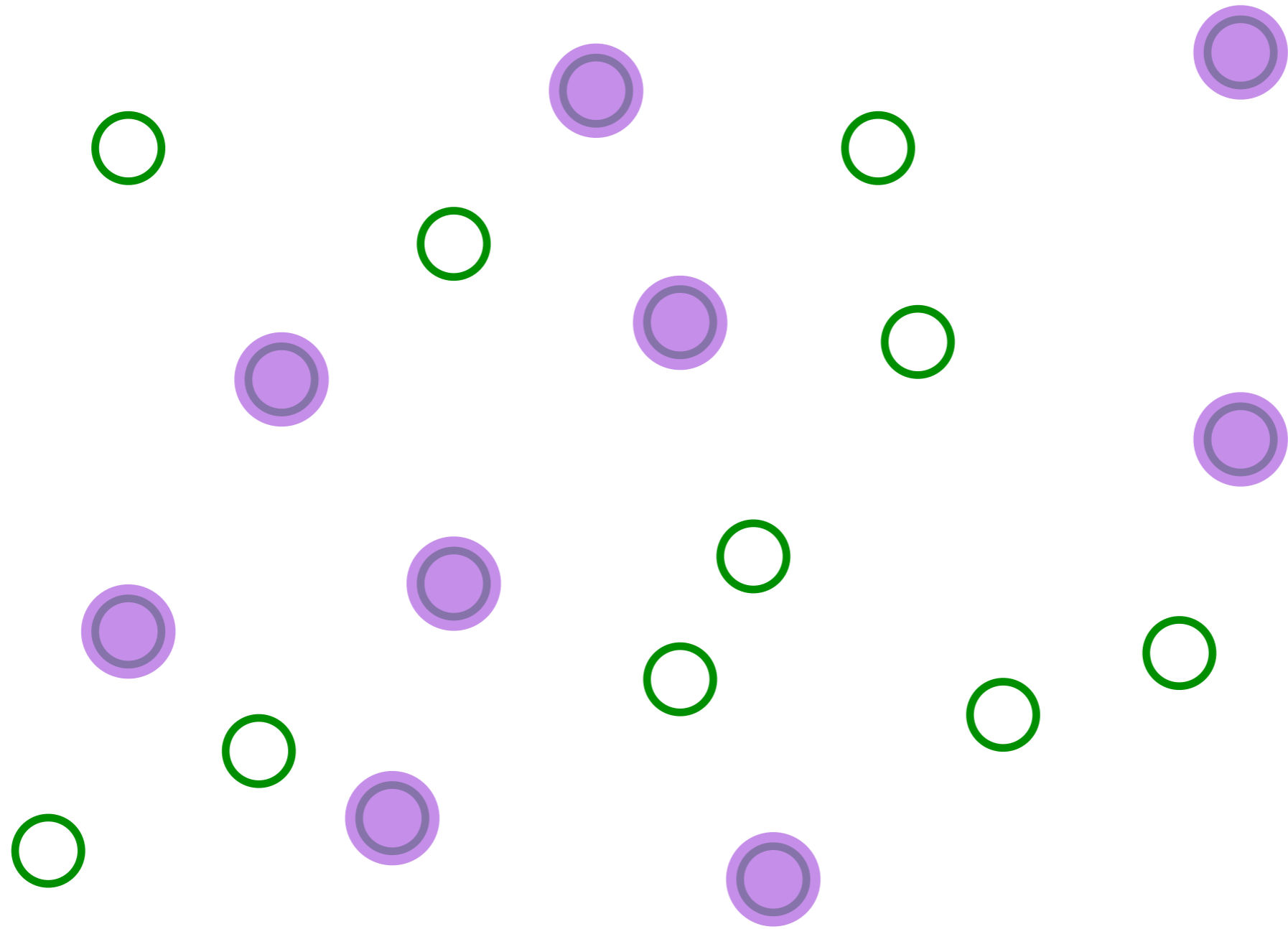
Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as  $\sim T^{-2}$  at the Fermi level.

# The Sachdev-Ye-Kitaev (SYK) model



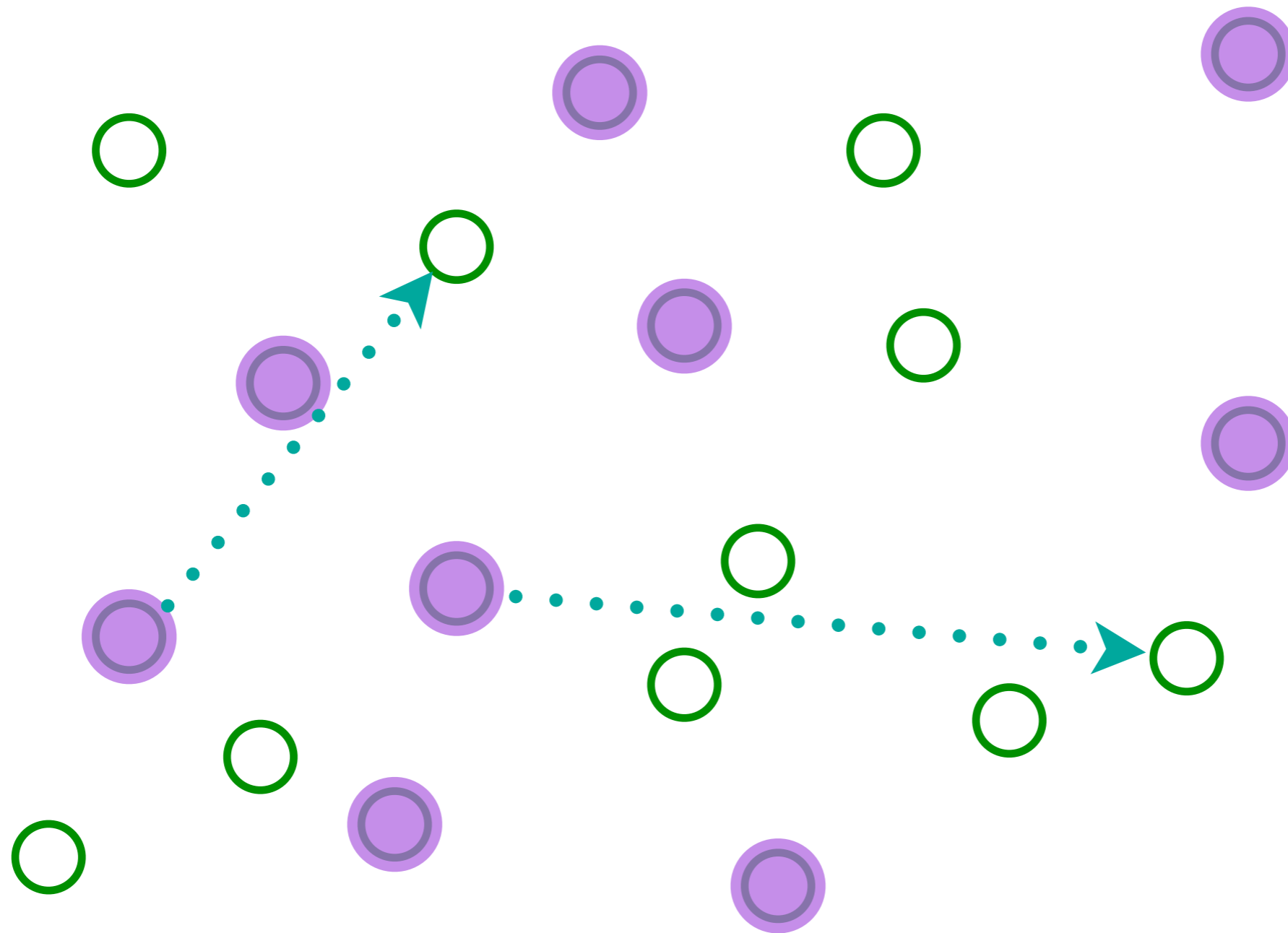
Pick a set of random positions

# The SYK model



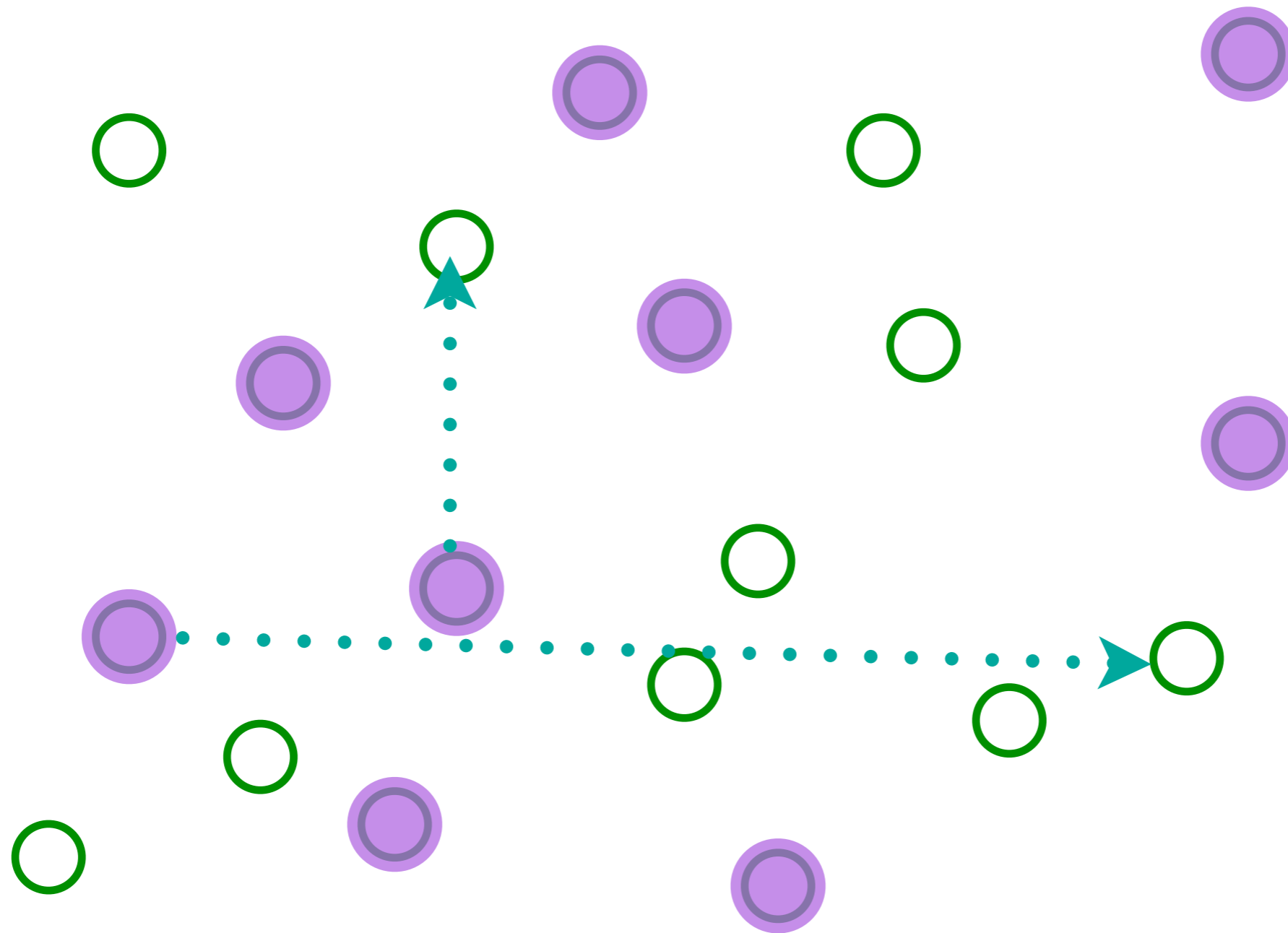
Place electrons randomly on some sites

# The SYK model



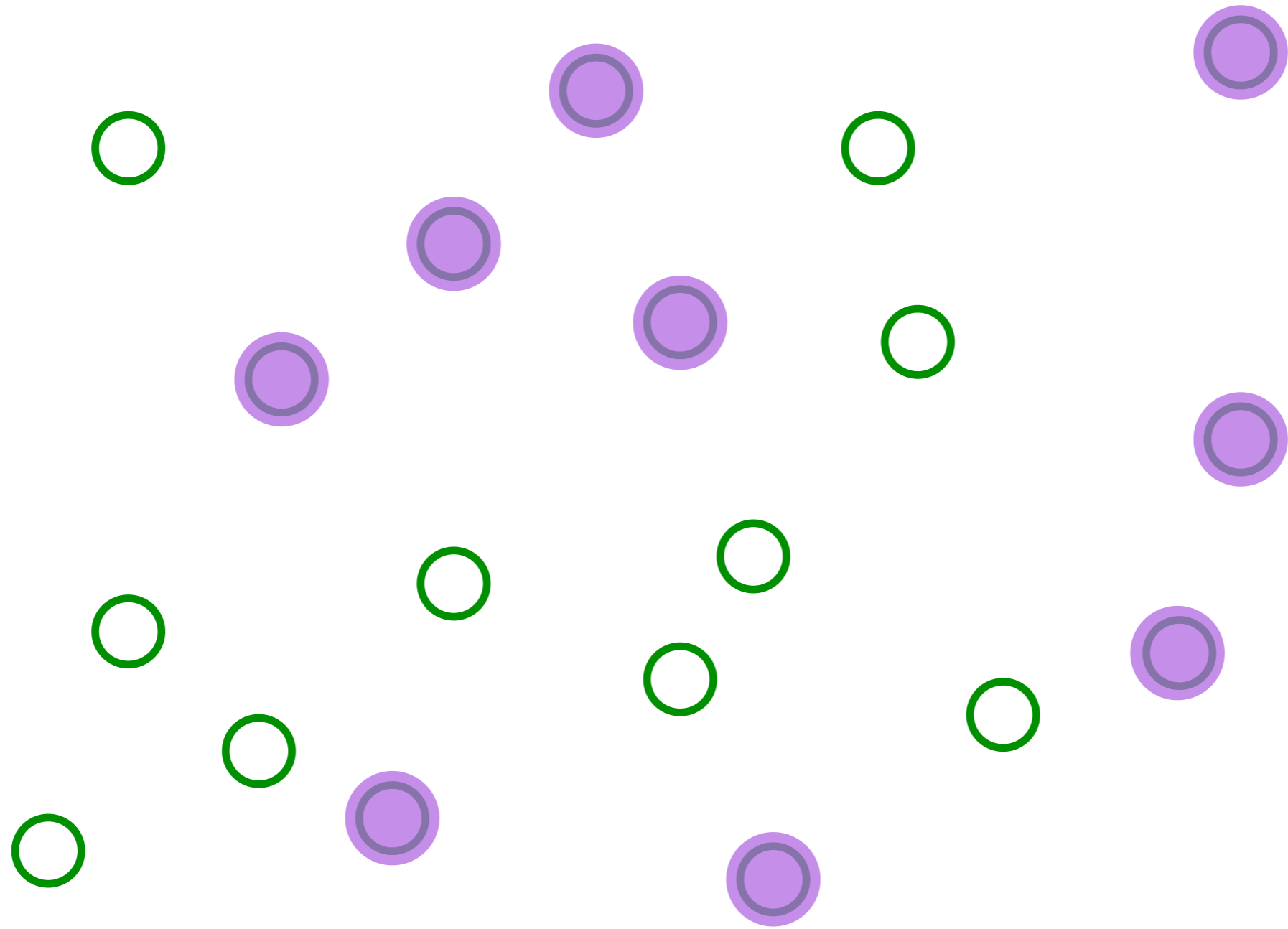
Place electrons randomly on some sites

# The SYK model



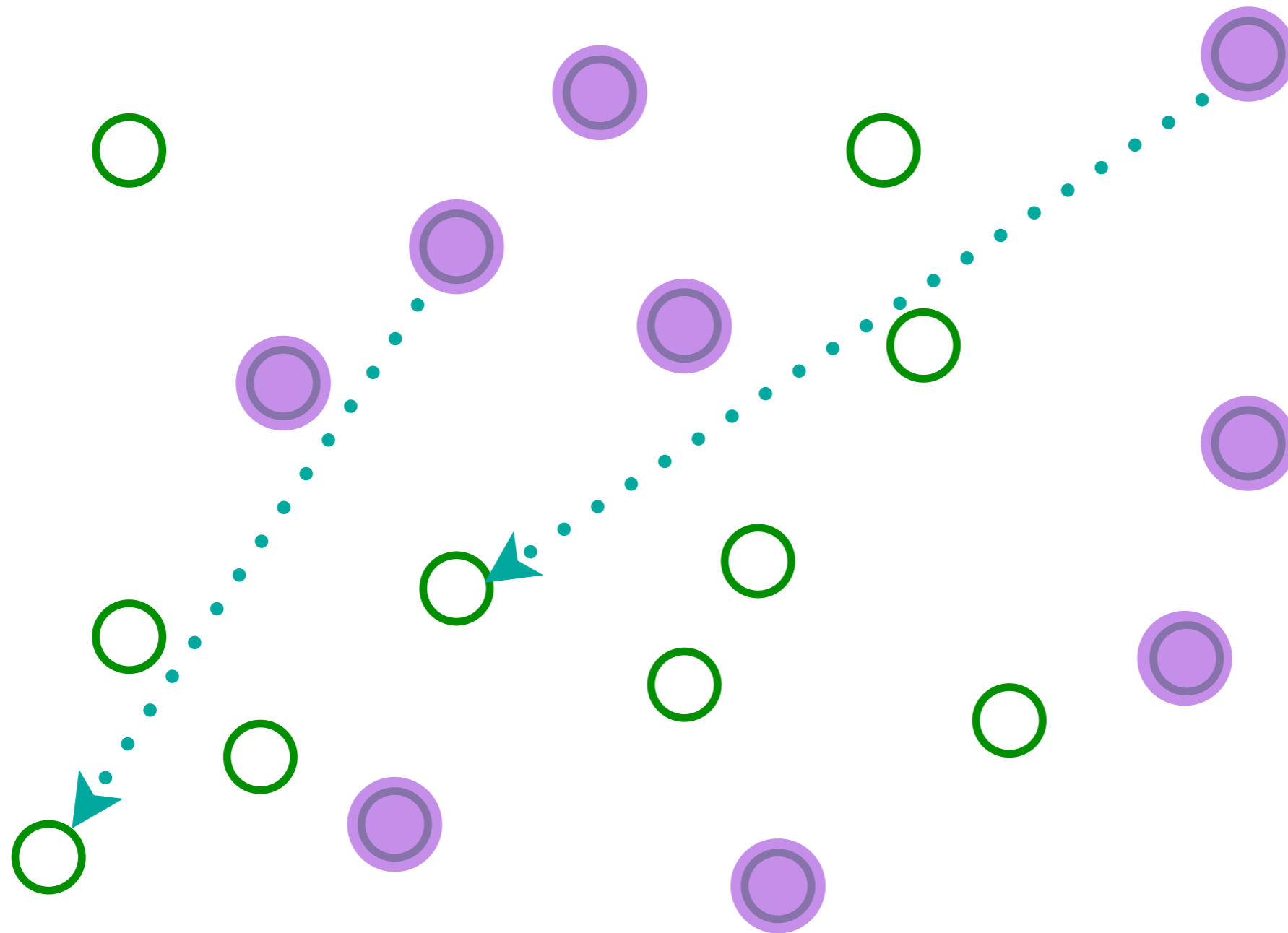
Place electrons randomly on some sites

# The SYK model



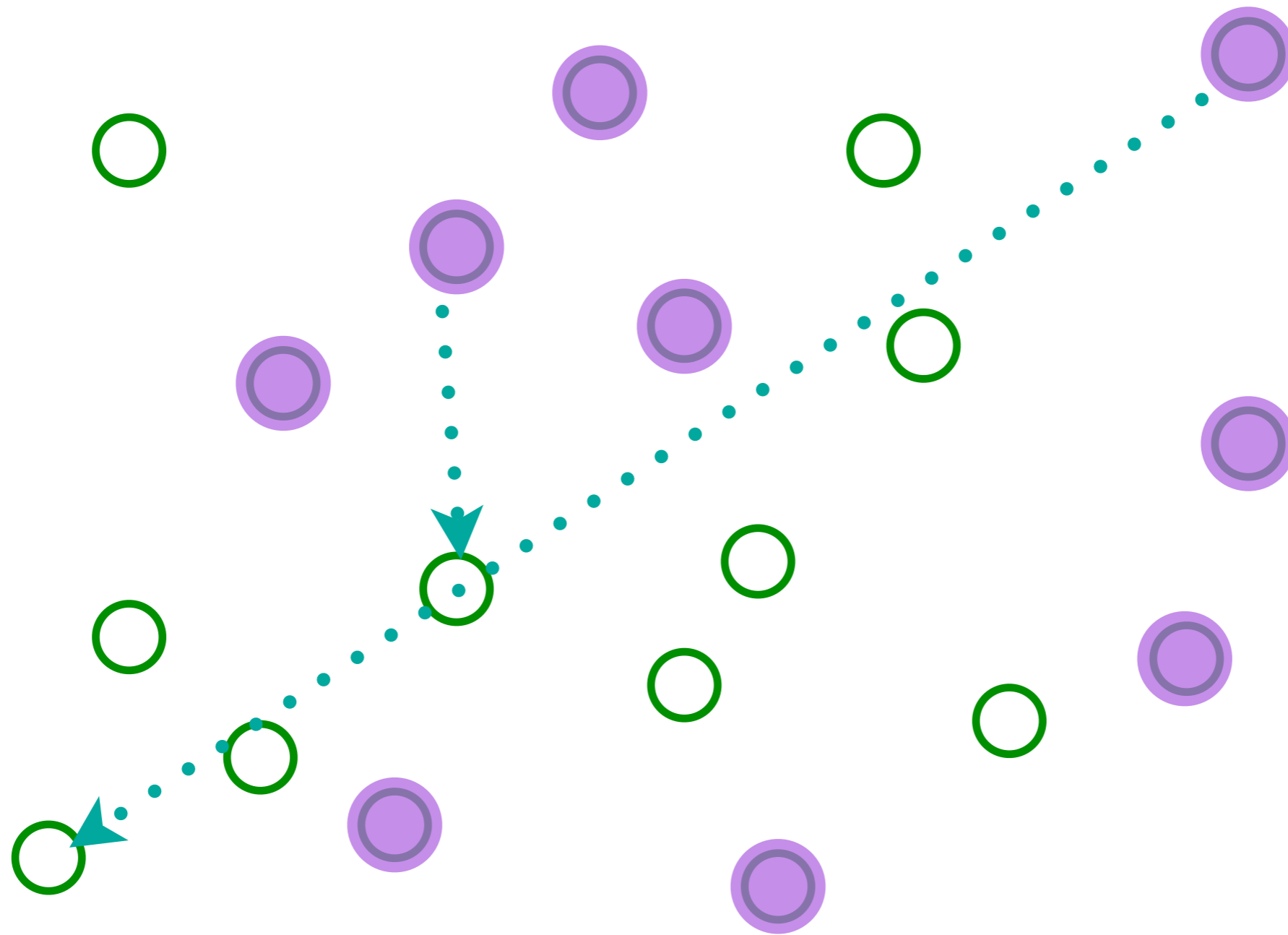
Entangle electrons pairwise randomly

# The SYK model



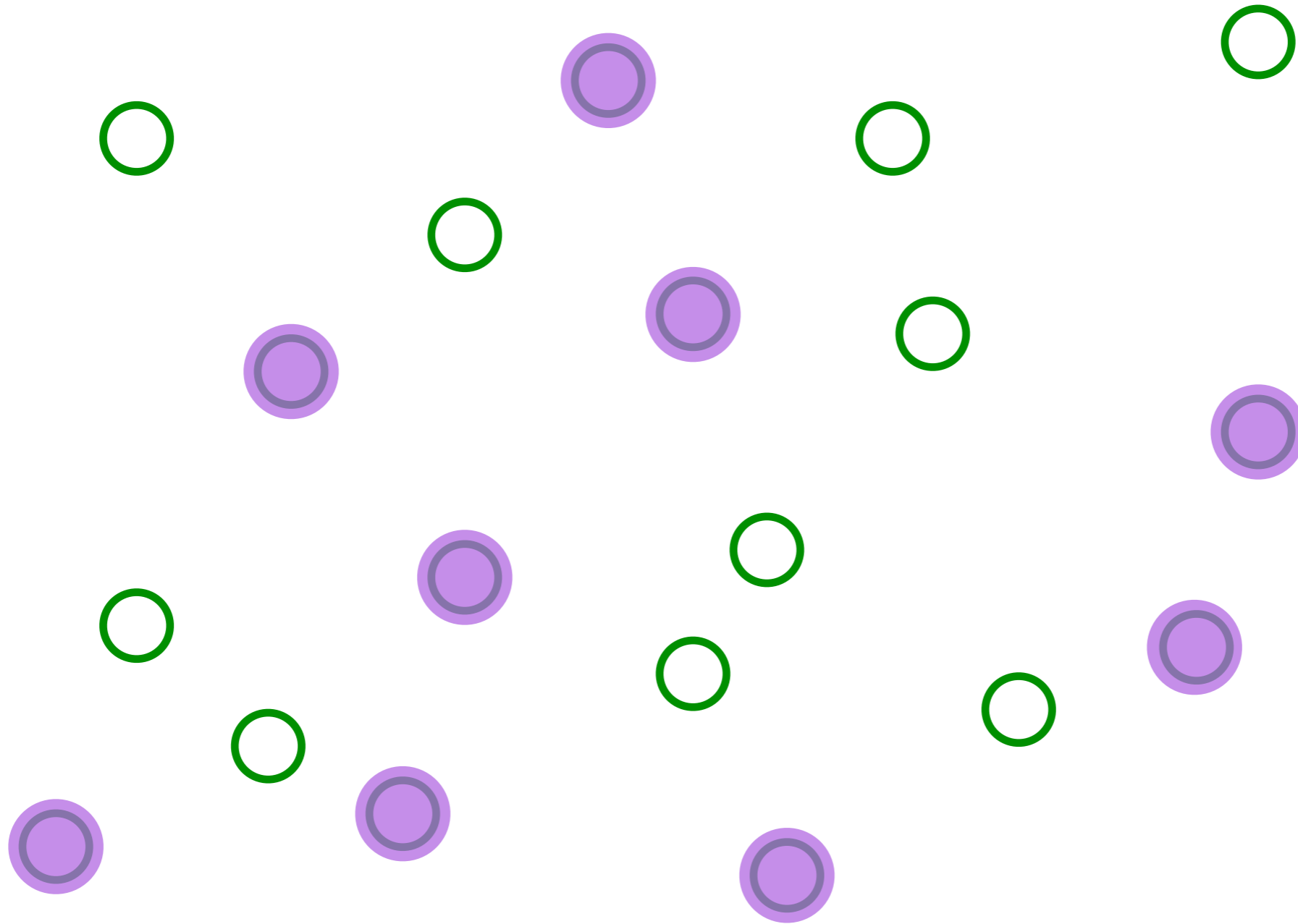
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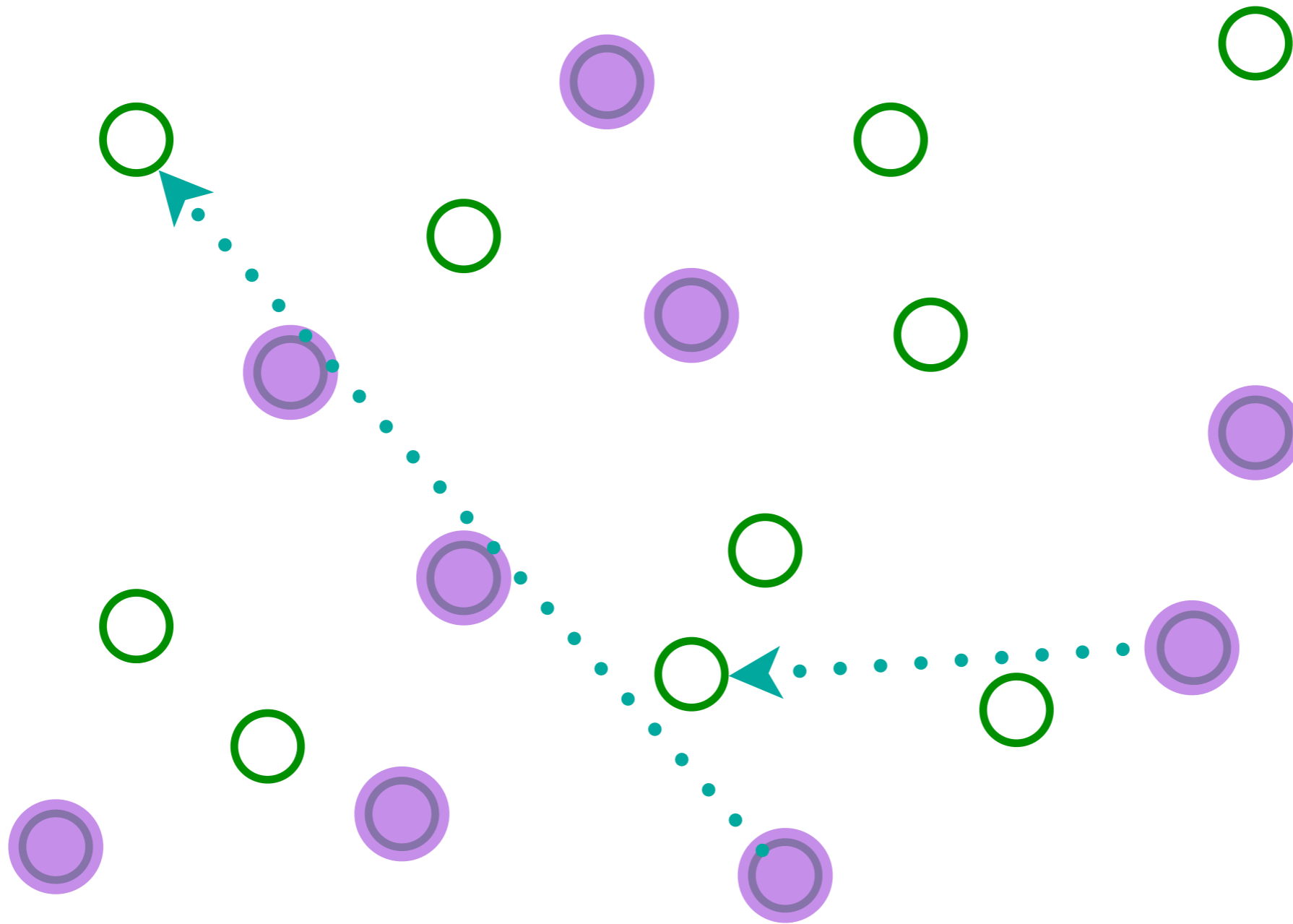
Entangle electrons pairwise randomly

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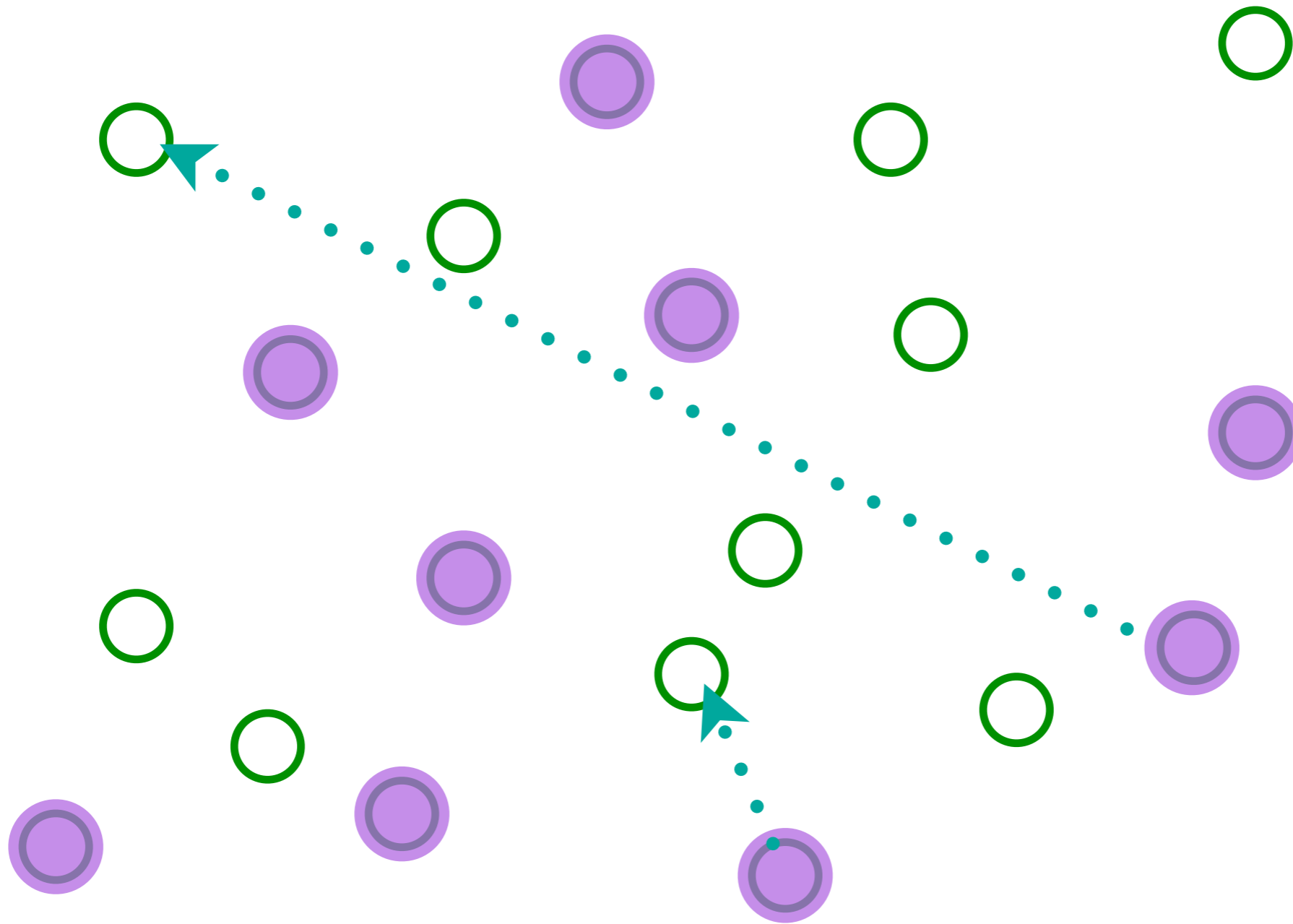
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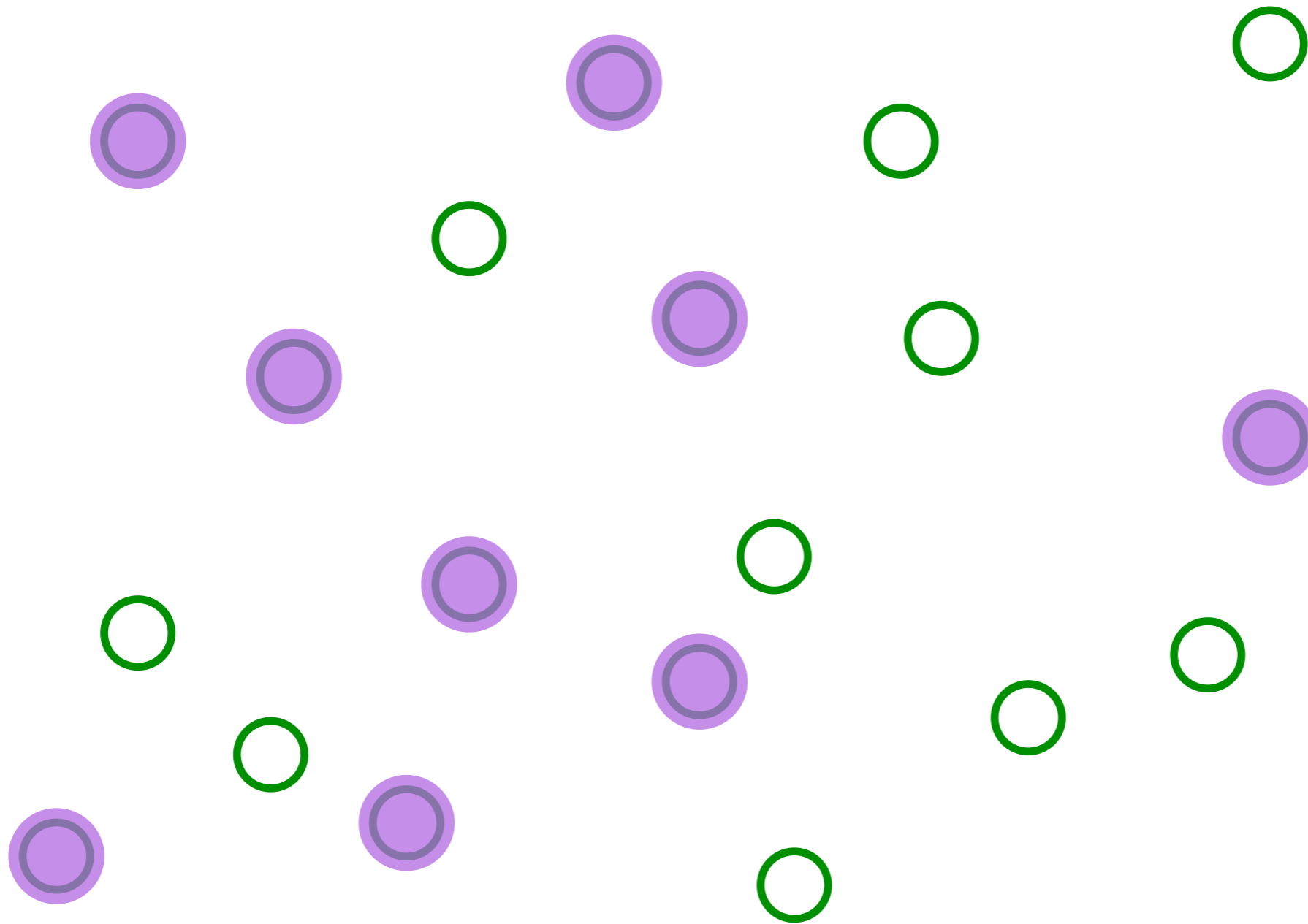
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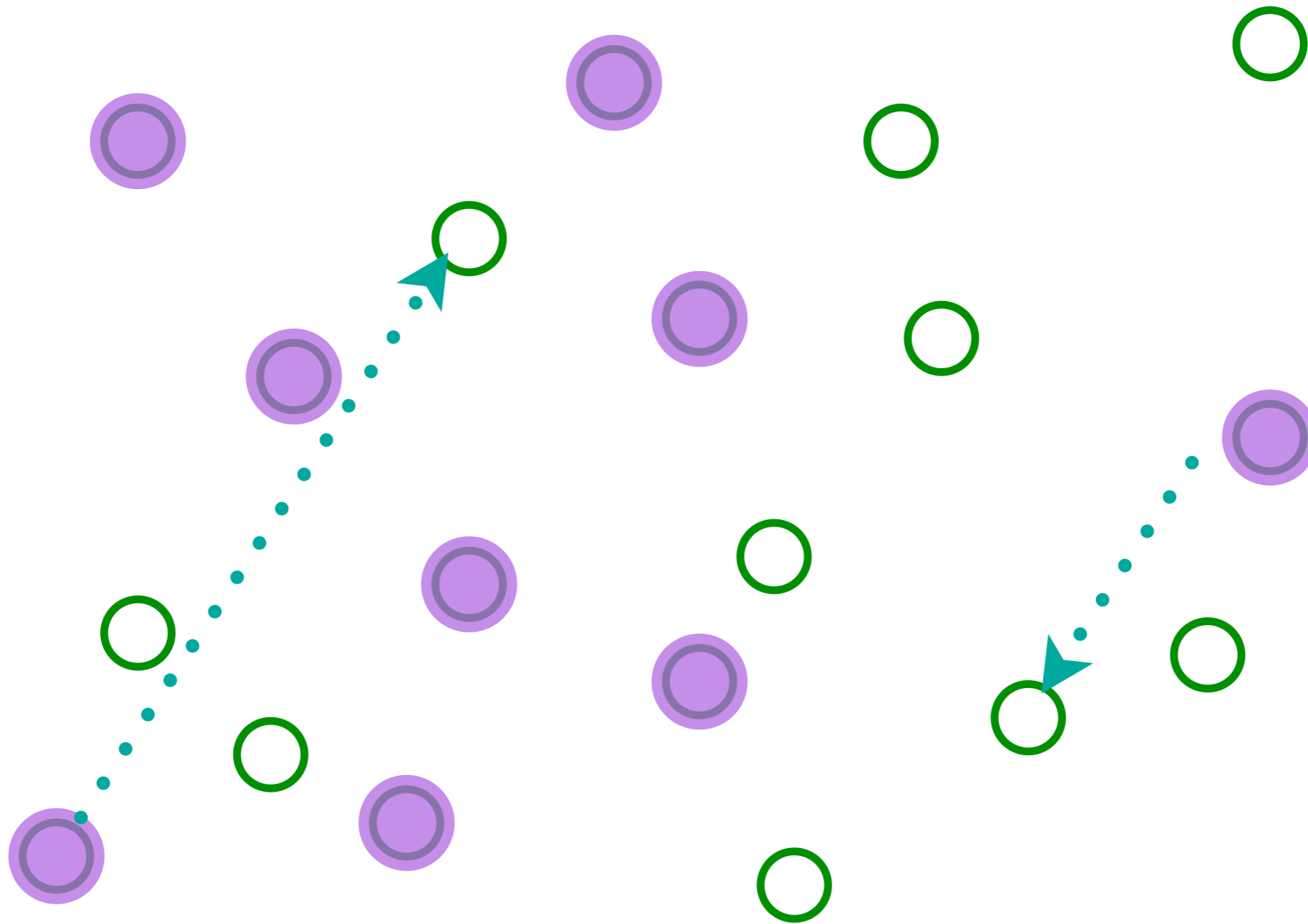
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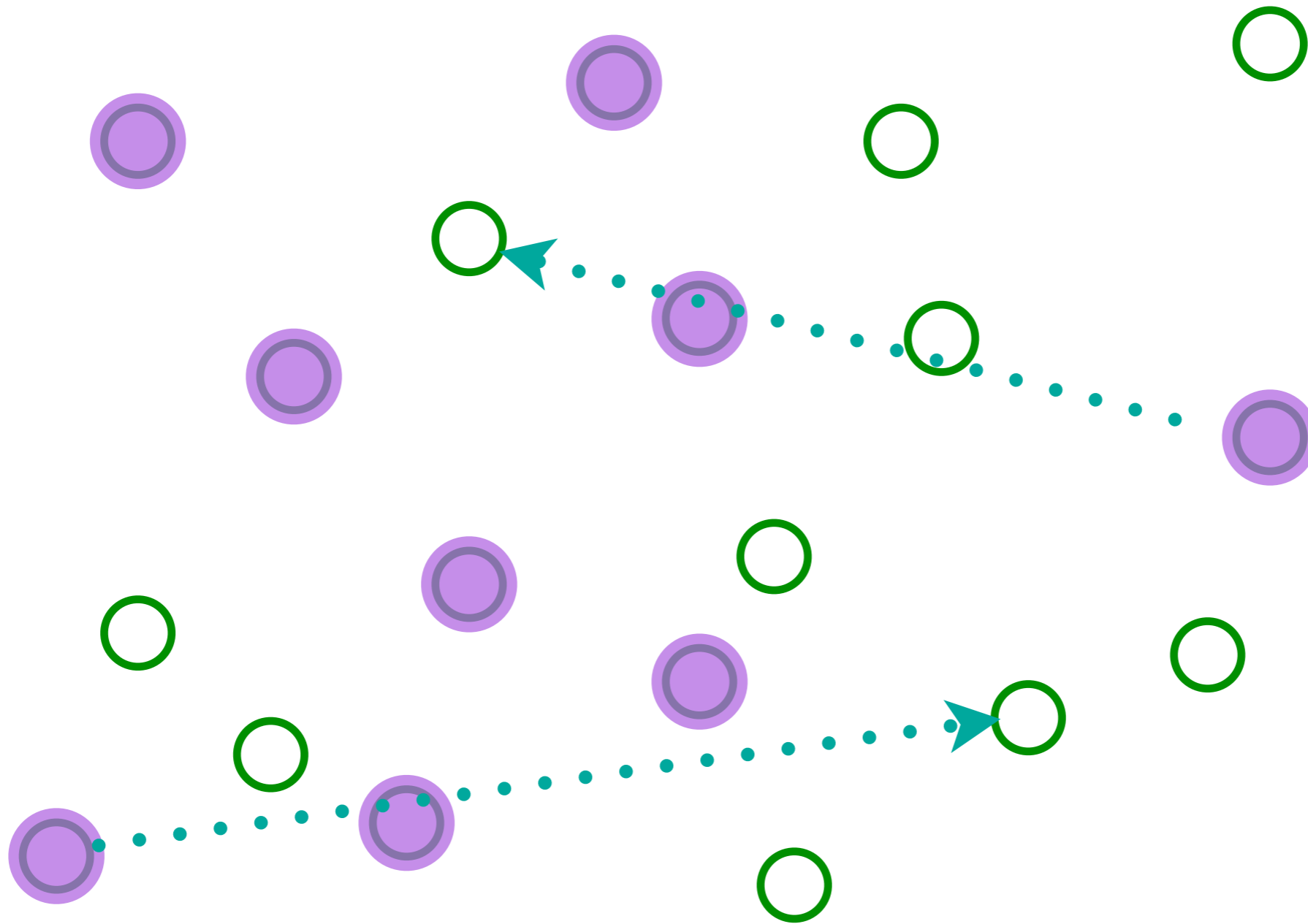
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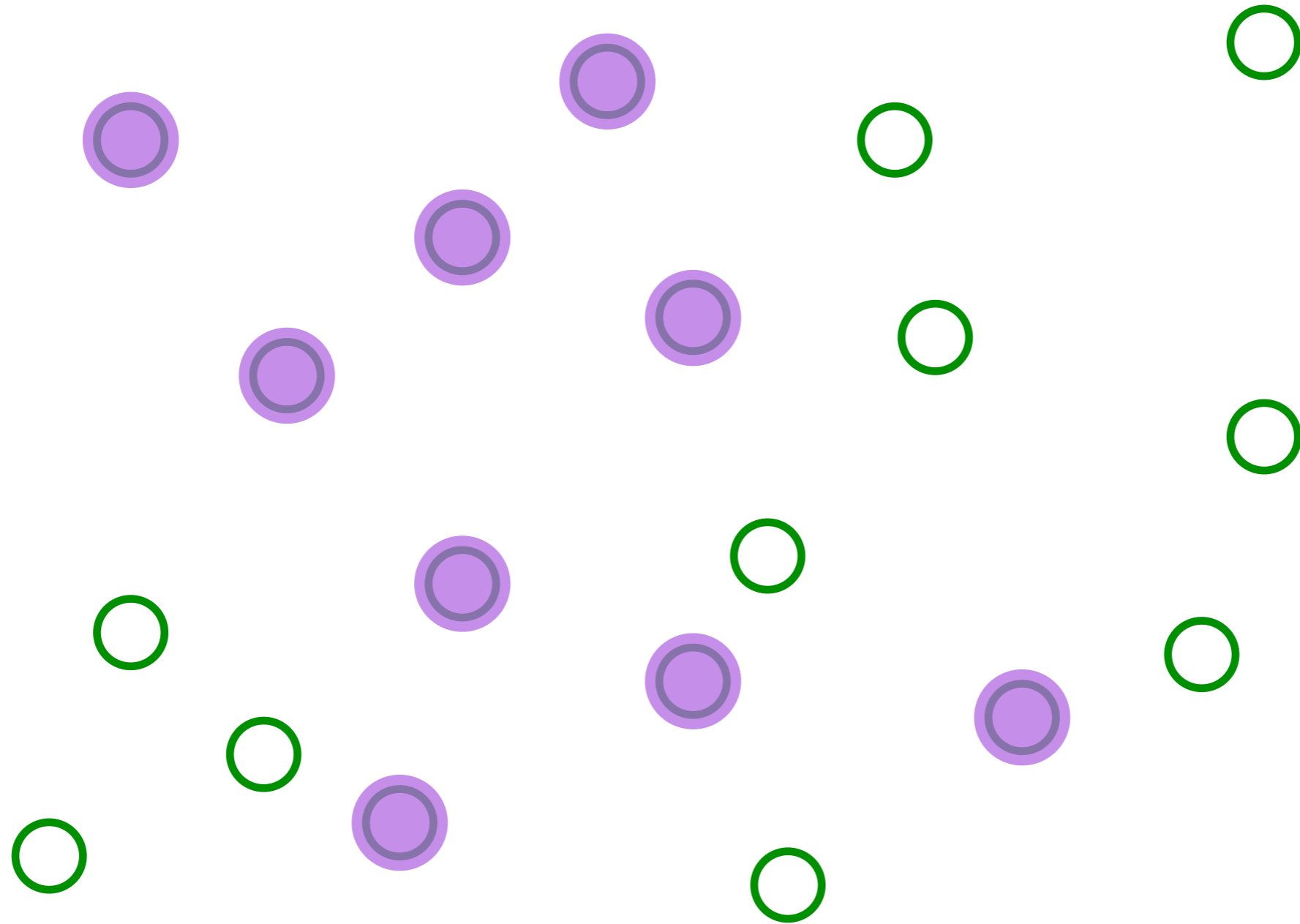
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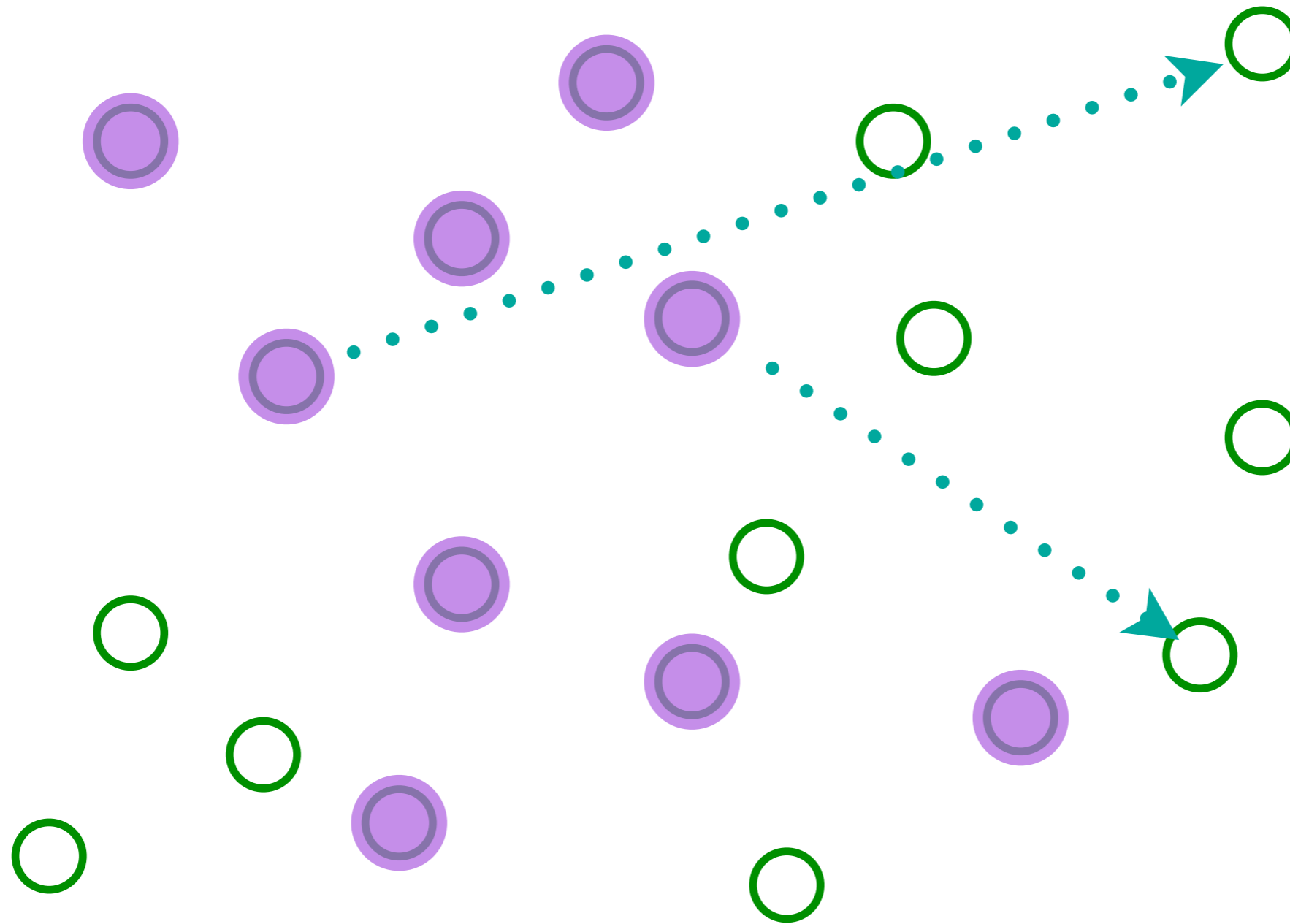
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# The SYK model



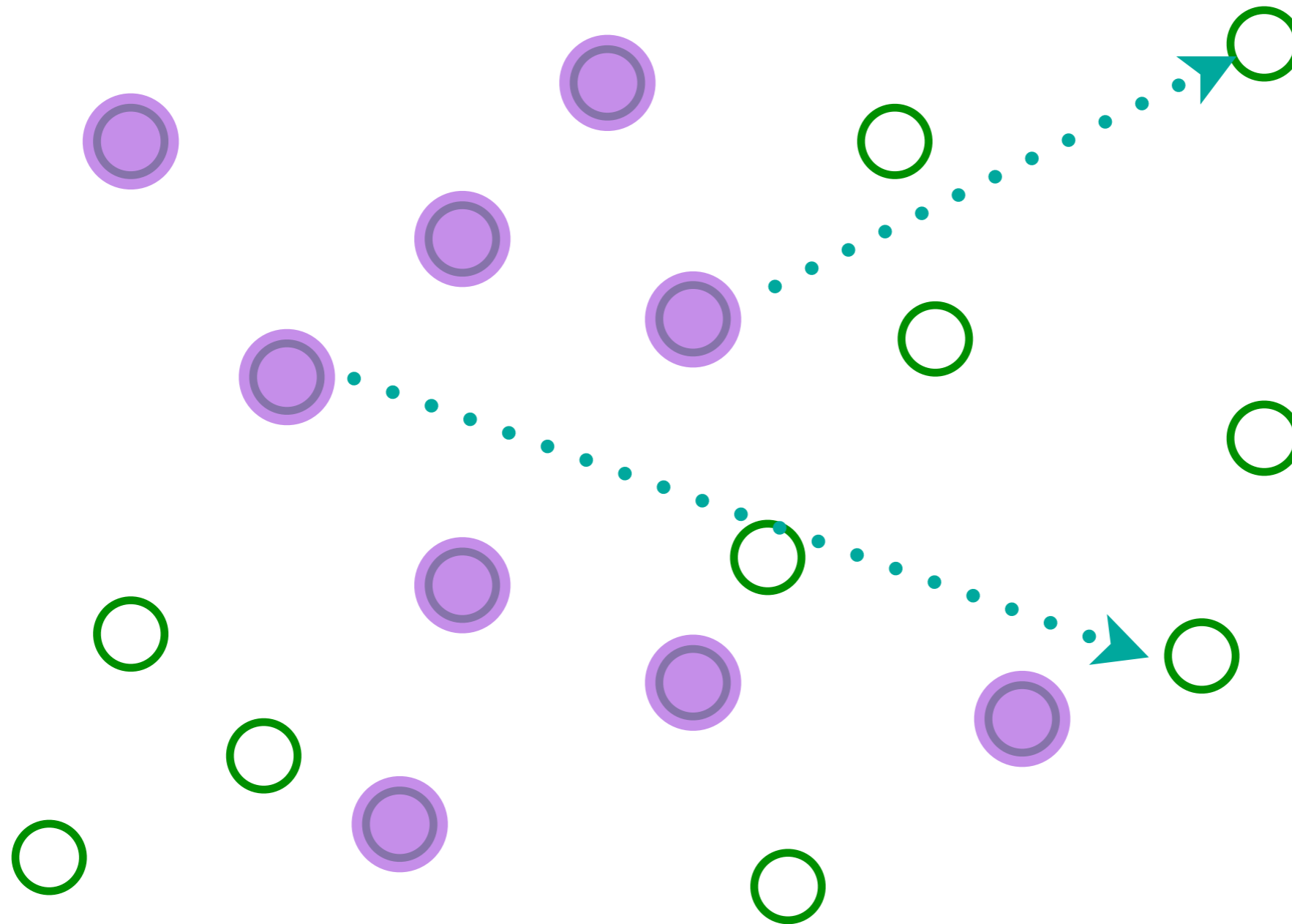
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# The SYK model



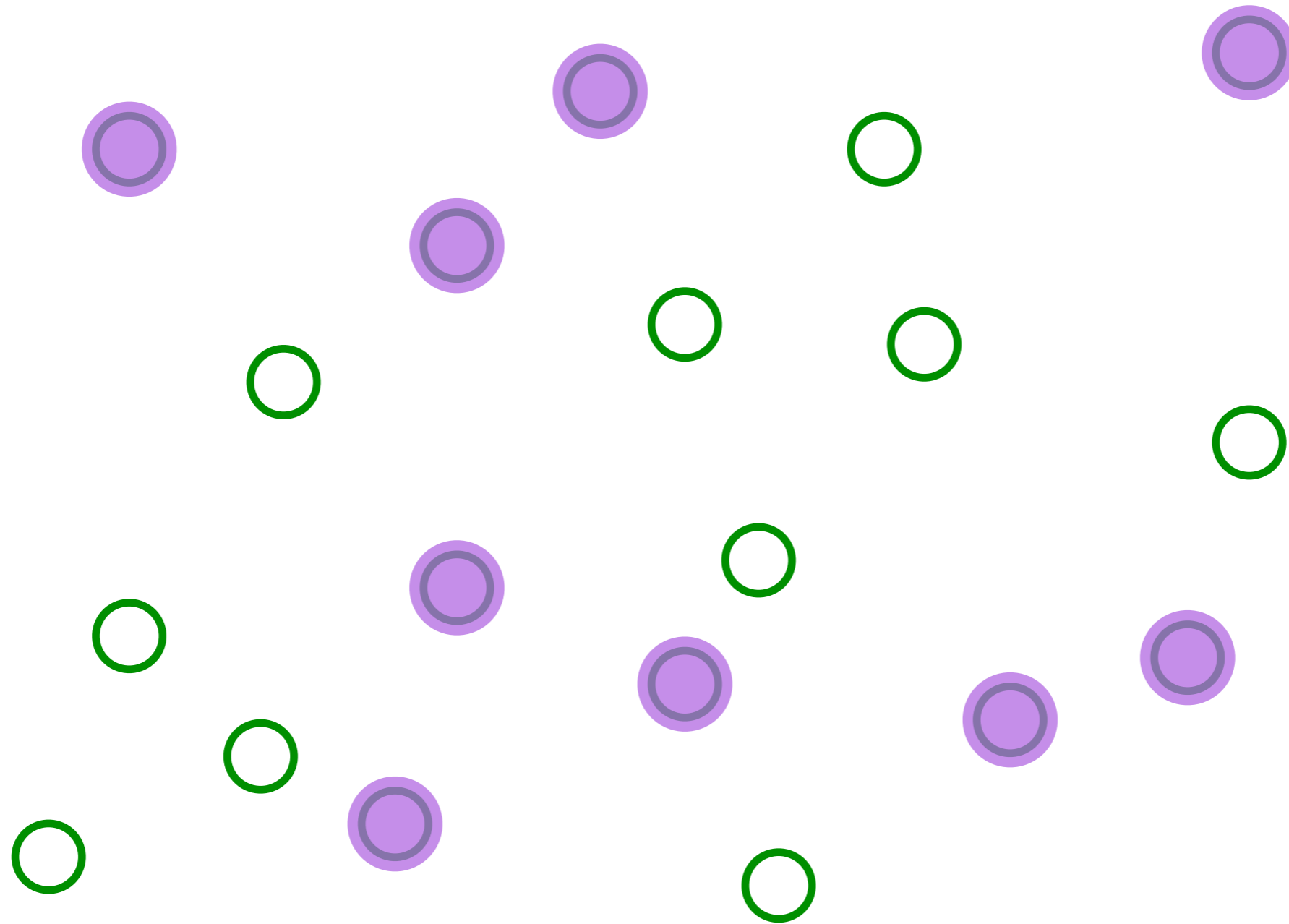
Entangle electrons pairwise randomly

# The SYK model



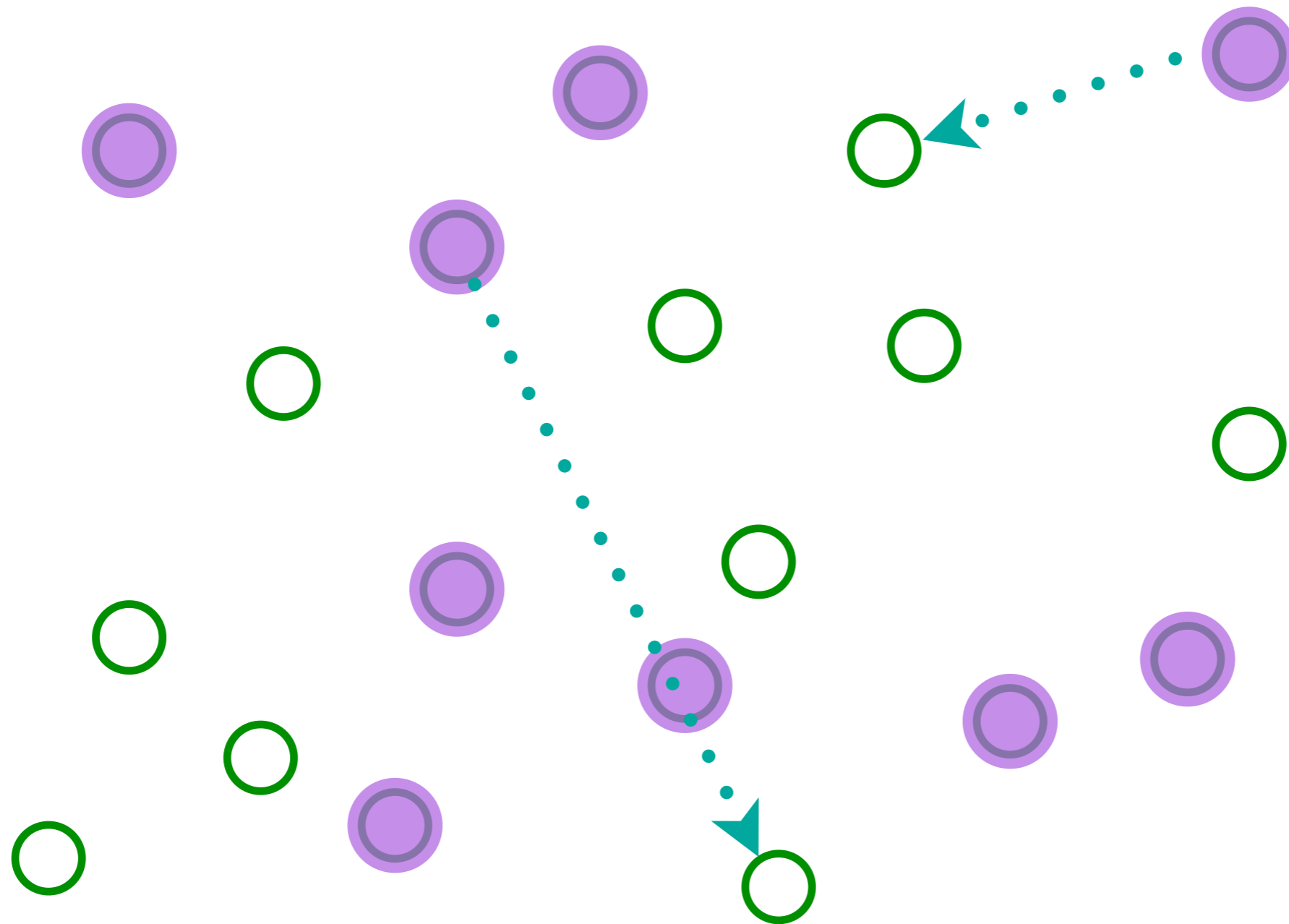
Entangle electrons pairwise randomly

# The SYK model



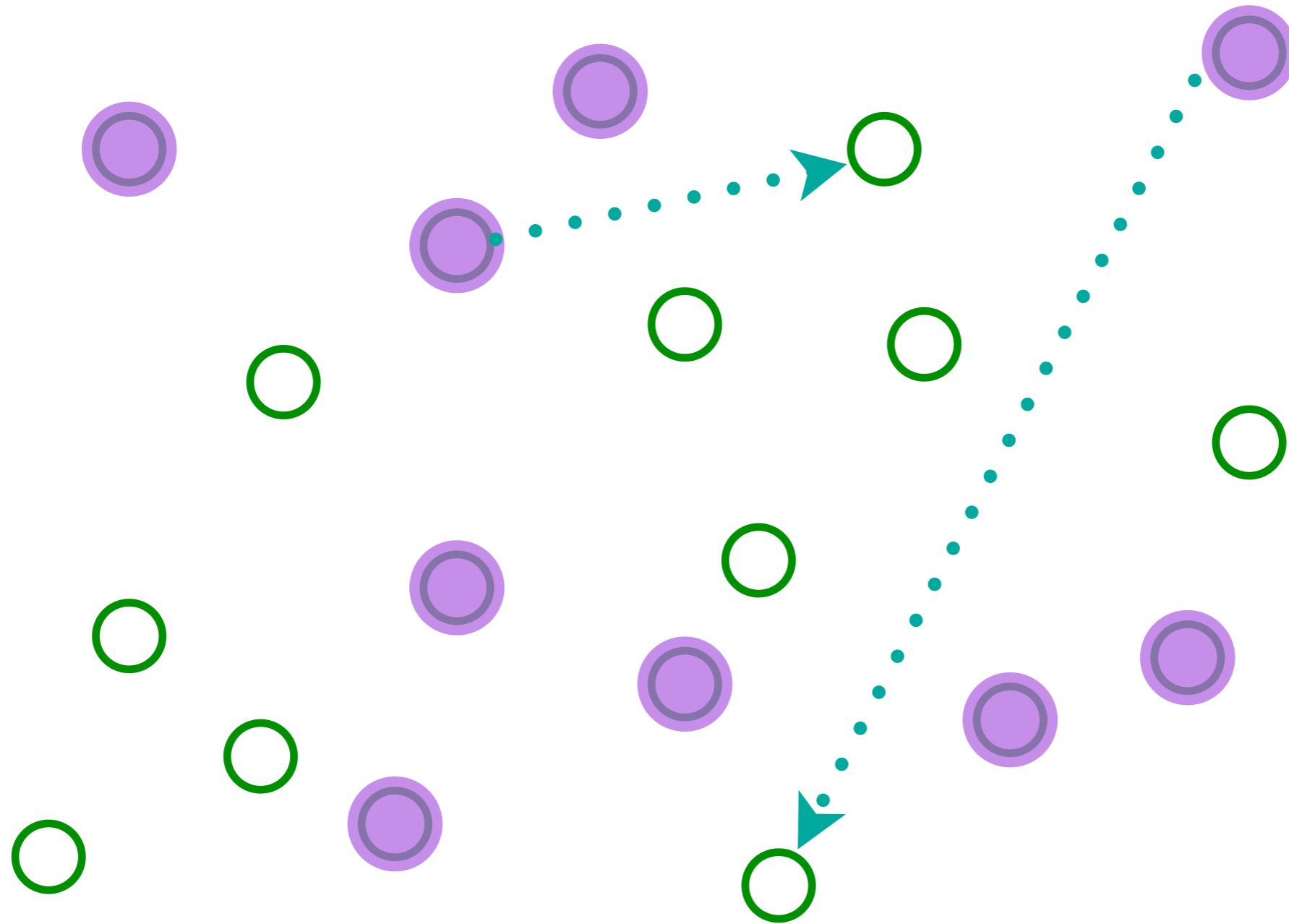
Entangle electrons pairwise randomly

# The SYK model



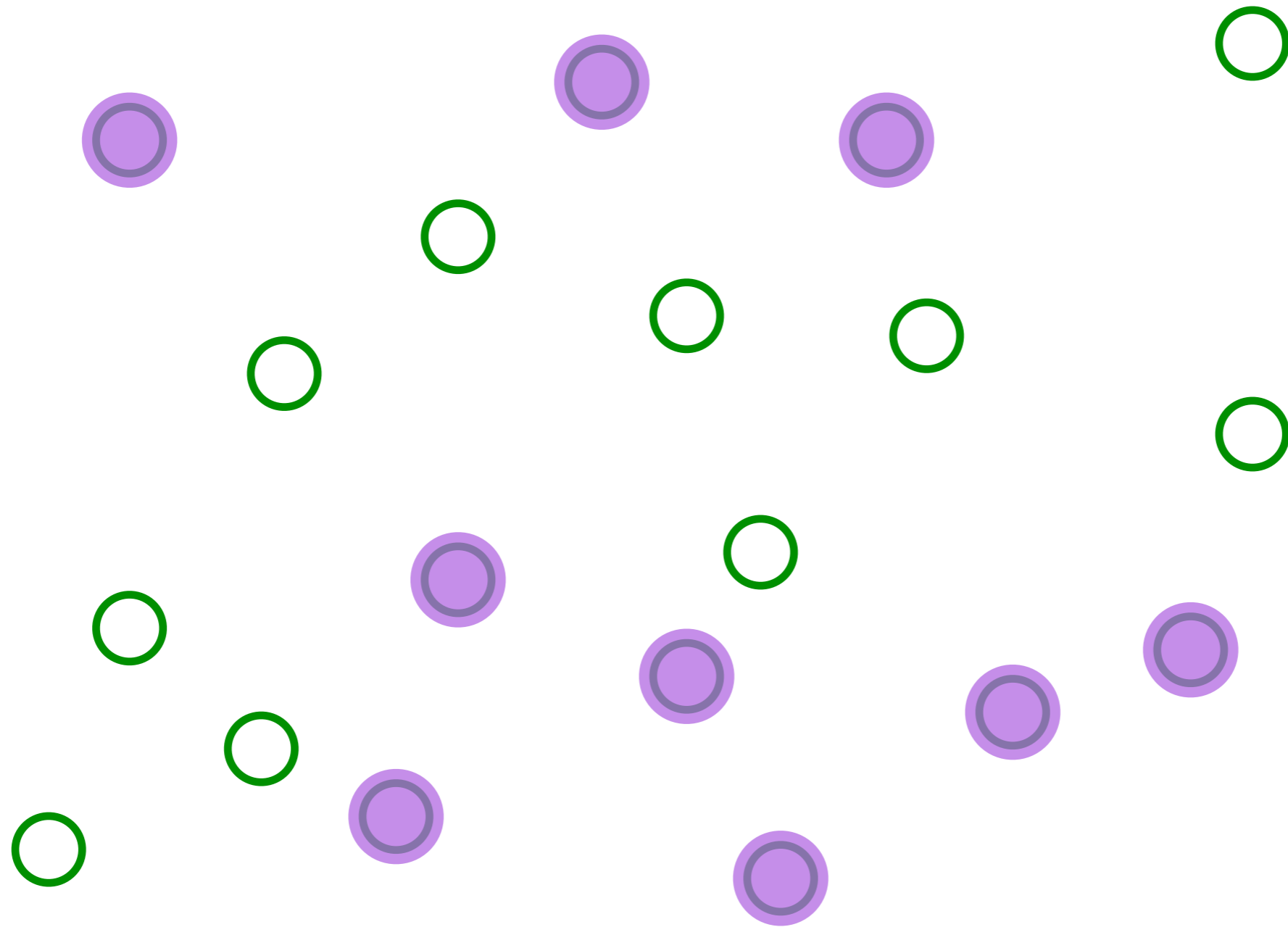
Entangle electrons pairwise randomly

# The SYK model



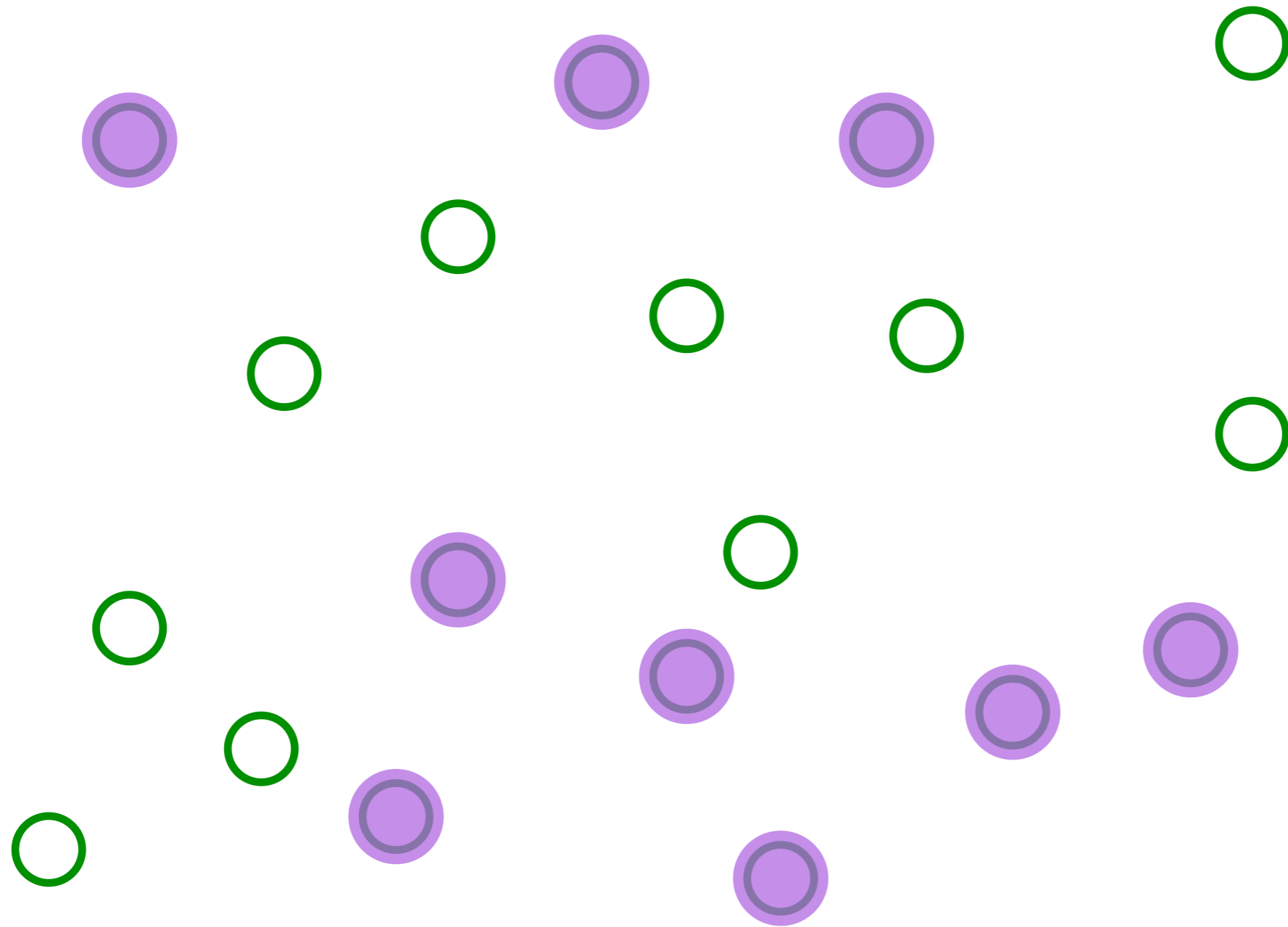
Entangle electrons pairwise randomly

# The SYK model



Entangle electrons pairwise randomly

# The SYK model



This describes both a strange metal and a black hole!

# The SYK model

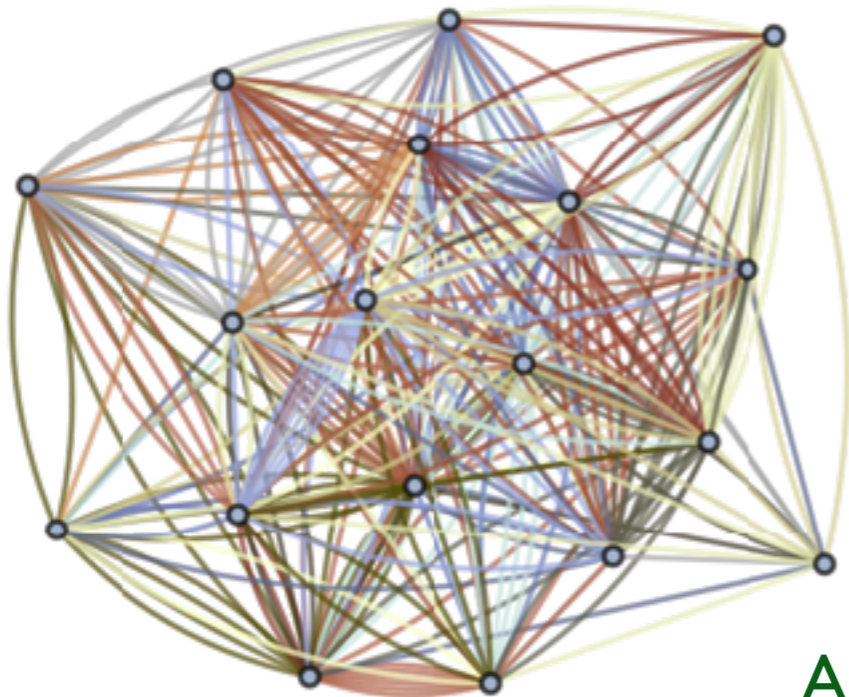
(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large  $N$  limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N U_{ij;k\ell} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$  are independent random variables with  $\overline{U_{ij;k\ell}} = 0$  and  $\overline{|U_{ij;k\ell}|^2} = U^2$   
 $N \rightarrow \infty$  yields critical strange metal.



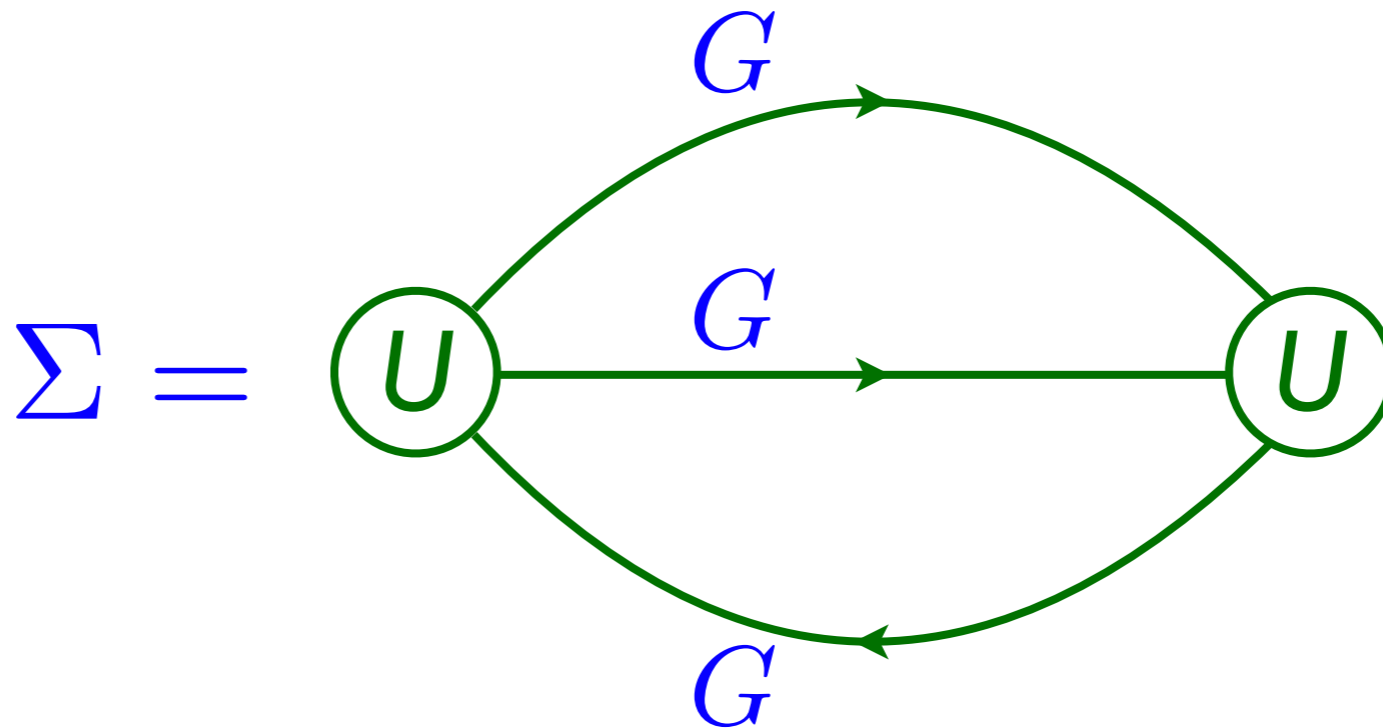
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

# The SYK model

Feynman graph expansion in  $U_{ijkl}$ , and graph-by-graph average, yields exact equations in the large  $N$  limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$



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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{e^{i(\pi/4+\theta)}}{A} \sqrt{z} + \dots \quad , \quad G(z) = \frac{A e^{-i(\pi/4+\theta)}}{\sqrt{z}}$$

where  $A = (\pi/U^2 \cos(2\theta))^{1/4}$ . The value of  $\theta$  is universally related to  $\mathcal{Q}$  by a Luttinger-Ward functional analysis similar to that used to establish the Luttinger theorem of Fermi liquid theory:

$$\mathcal{Q} = \frac{1}{2} - \frac{\theta}{\pi} - \frac{\sin(2\theta)}{4}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = Q.$$

At  $T > 0$ , we obtain a solution with a conformal structure

$$G(\tau) = -A \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left( \frac{T}{\sin(\pi T\tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \quad ,$$

where the ‘particle-hole asymmetry’ is determined by  $\mathcal{E}$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)} .$$

# The SYK model



There are  $2^N$  many body levels with energy  $E$ . Shown are all values of  $E$  for a single cluster of size  $N = 12$ . The  $T \rightarrow 0$  state has an entropy  $S_{GPS} = N s_0$ , where  $s_0 < \ln 2$  is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi\mathcal{E}.$$

At  $Q = 1/2$ ,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\dots$$

where  $G$  is Catalan's constant.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

$\sim NU$

Many-body level spacing  $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing  $\sim e^{-N s_0}$

# The SYK model

$$\Omega(T) - E_0 = N \left[ -s_0 T - \frac{1}{2}(\gamma + 4\pi^2 \mathcal{E}^2 K) T^2 + \mathcal{O}(T^3) \right] + 2T \ln \left( \frac{U}{T} \right) \dots$$

is the grand potential, where  $K = d\mathcal{Q}/d\mu \sim 1/U$  is the compressibility/ $N$ ,  $\gamma \sim 1/U$  will appear later in the co-efficient of the Schwarzian, and the  $N^0$  term arises from fluctuations about the large  $N$  theory described by the Schwarzian.

The inversion from  $\Omega(T)$  to the *many*-body density of states,  $D(E)$ ,

$$Z = e^{-\Omega(T)/T} = \int_{-\infty}^{\infty} dE D(E) e^{-E/T}$$

requires terms in  $\Omega(T)$  which are exponentially small in  $N$  (not shown above) from the Schwarzian action, yielding terms which are not small in  $D(E)$ . We obtain

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, arXiv:1611.04650;  
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849 ;  
A.M. Garcia-Garcia and J.J.M. Verbaarschot, arXiv:1701.06593; D. Bagrets, A. Altland, and A. Kamenev, arXiv:1702.08902;  
D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S.J. Suh, arXiv:1711.08467; Yingfei Gu and S. Sachdev, unpublished.

# The SYK model

$$D(E) = \sum_{p=-\infty}^{\infty} e^{2\pi p \mathcal{E}} d\left(E - \frac{p^2}{2NK}\right)$$

where  $N\mathcal{Q} + p$  is the integer fermion number, and  $d(E)$  is the density of states at *fixed* fermion number. We have  $d(E) = 0$  for  $E < E_0$ , and

$$d(E) \sim \exp(Ns_0) \sinh\left(\sqrt{2N\gamma(E - E_0)}\right), \quad E > E_0, \quad e^{-cN} \ll \gamma(E - E_0) \ll N$$

There are exponentially more low energy states than for the quasiparticle case, and  $D(E)$  self-averages down to energies exponentially small in  $N$ .

We can understand the dependence on the integer charge  $p$  by the relationship  $ds_0/d\mathcal{Q} = 2\pi\mathcal{E}$ , and hence  $Ns_0(\mathcal{Q} + p/N) \approx Ns_0 + 2\pi p\mathcal{E}$ .

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$$\Omega(T) - E_0 = N \left( -\frac{\pi^2}{6} \rho_0 T^2 + \mathcal{O}(T^4) \right) + \dots$$

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The inversion from  $\Omega(T)$  to  $D(E)$  has to be performed with care (it does not commute with the  $1/N$  expansion), and we obtain

$$D(E) \sim \exp \left( \pi \sqrt{\frac{2N\rho_0(E - E_0)}{3}} \right), \quad E > E_0, \quad \frac{1}{N} \ll \rho_0(E - E_0) \ll N$$

and  $D(E) = 0$  for  $E < E_0$ . This is related to the asymptotic growth of the partitions of an integer,  $p(n) \sim \exp(\pi\sqrt{2n/3})$ . Near the lower bound, there are large sample-to-sample fluctuations due to variations in the lowest quasiparticle energies.

# The SYK model

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We can understand the dependence on the integer charge  $p$  by the relationship  $ds_0/d\mathcal{Q} = 2\pi\mathcal{E}$ , and hence  $Ns_0(\mathcal{Q} + p/N) \approx Ns_0 + 2\pi p\mathcal{E}$ .

# The SYK model

$$D(E) = \sum_{p=-\infty}^{\infty} e^{2\pi p \mathcal{E}} d\left(E - \frac{p^2}{2NK}\right)$$

where  $N\mathcal{Q} + p$  is the integer fermion number, and  $d(E)$  is the density of states at *fixed* fermion number. We have  $d(E) = 0$  for  $E < E_0$ , and

$$d(E) \sim \boxed{\exp(Ns_0)} \sinh\left(\sqrt{2N\gamma(E - E_0)}\right), \quad E > E_0, \quad e^{-cN} \ll \gamma(E - E_0) \ll N$$

There are exponentially more low energy states than for the quasiparticle case, and  $D(E)$  self-averages down to energies exponentially small in  $N$ .

We can understand the dependence on the integer charge  $p$  by the relationship  $ds_0/d\mathcal{Q} = 2\pi\mathcal{E}$ , and hence  $Ns_0(\mathcal{Q} + p/N) \approx Ns_0 + 2\pi p\mathcal{E}$ .

The  $\exp(Ns_0)$  prefactor suggests that the low-lying eigenstates are separated into analogs of ‘superselection sectors’ which cannot be connected by ‘simple’ operators, in the Schwarzian approximation.

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, arXiv:1611.04650;  
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849 ;  
A.M. Garcia-Garcia and J.J.M. Verbaarschot, arXiv:1701.06593; D. Bagrets, A. Altland, and A. Kamenev, arXiv:1702.08902;  
D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S.J. Suh, arXiv:1711.08467; Yingfei Gu and S. Sachdev, unpublished.

# The SYK model

*No quasiparticles*

- Rapid local thermal equilibration (of fermion correlators) in a ‘Planckian’ time

$$\tau_{\text{eq}} \sim \frac{\hbar}{k_B T} \quad , \quad \text{as } T \rightarrow 0.$$

A. Georges and O. Parcollet  
PRB **59**, 5341 (1999)

A. Eberlein, V. Kasper, S. Sachdev, and  
J. Steinberg, PRB **96**, 205123 (2017)

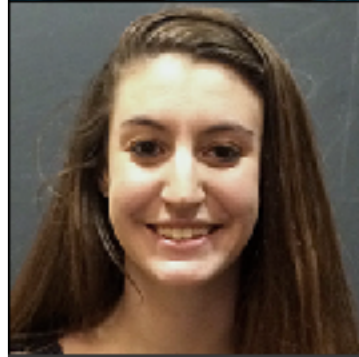
Established by solution of Schwinger-Keldysh equations for a quench.

- Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey

$$\tau_{\text{eq}} > C \frac{\hbar}{k_B T} \quad , \quad \text{as } T \rightarrow 0.$$

S. Sachdev, *Quantum Phase Transitions*,  
Cambridge (1999)

Absence of quasiparticles  $\Leftrightarrow$  Fastest possible thermalization



# The SYK model

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

# The SYK model

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$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies  $\ll U$ , the  $i\omega + \mu$  can be dropped, and without it equations are invariant under the reparametrization and gauge transformations.

The singular part of the self-energy and the Green's function obey

$$\int_0^\beta d\tau_2 \Sigma_{\text{sing}}(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma_{\text{sing}}(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

# The SYK model

$$\int_0^\beta d\tau_2 \Sigma(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$
$$\Sigma(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

These equations are invariant under

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{\Sigma}(\sigma_1, \sigma_2)$$

where  $f(\sigma)$  and  $g(\sigma)$  are arbitrary functions.

By using  $f(\sigma) = \tan(\pi T \sigma) / (\pi T)$  we can

now obtain the  $T > 0$  solution from the  $T = 0$  solution.

## Basics of conformal field theory

In a space with metric tensor  $g_{\mu\nu}$  and proper distance

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu$$

after the co-ordinate transformation  $x_\mu \rightarrow x'_\mu$ , the new metric tensor is

$$g'_{\mu\nu} = g_{\rho\lambda} \frac{\partial x_\rho}{\partial x'_\mu} \frac{\partial x_\lambda}{\partial x'_\nu}.$$

A conformal transformation is one which preserves all angles and so

$$g'_{\mu\nu}(x') = \Lambda(x) g_{\mu\nu}(x).$$

In a conformal field theory, two-point correlators of scalar fields transform as

$$\langle \phi(x_1) \phi(x_2) \rangle = \left| \det \left[ \frac{\partial x'_1}{\partial x_1} \right] \right|^{\Delta/d} \left| \det \left[ \frac{\partial x'_2}{\partial x_2} \right] \right|^{\Delta/d} \langle \phi(x'_1) \phi(x'_2) \rangle$$

# The SYK model

Let us write the large  $N$  saddle point solutions of  $S$  as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

The saddle point will be invariant under a reparamaterization  $f(\tau)$  when choosing  $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$  leads to a transformed  $\tilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$  (and similarly for  $\Sigma$ ). It turns out this is true only for the  $SL(2, \mathbb{R})$  transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to  $SL(2, \mathbb{R})$  by the saddle point.

# Infinite-range (SYK) model without quasiparticles

After introducing replicas  $a = 1 \dots n$ , and integrating out the disorder, the partition function can be written as

$$Z = \int \mathcal{D}c_{ia}(\tau) \exp \left[ - \sum_{ia} \int_0^\beta d\tau c_{ia}^\dagger \left( \frac{\partial}{\partial \tau} - \mu \right) c_{ia} - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c_{ia}^\dagger(\tau) c_{ib}(\tau') \right|^4 \right].$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D}\Sigma(\tau_1, \tau_2) \exp \left[ -N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left( G(\tau_2, \tau_1) + \frac{1}{N} \sum_i c_i(\tau_2) c_i^\dagger(\tau_1) \right) \right].$$

# Infinite-range (SYK) model without quasiparticles

Then the partition function can be written as a path integral with an action  $S$  analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$
$$S = \ln \det [\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)]$$
$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) [G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)]$$

At frequencies  $\ll U$ , the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where  $f(\sigma)$  and  $g(\sigma)$  are arbitrary functions.

A. Georges and O. Parcollet  
PRB **59**, 5341 (1999)

A. Kitaev, 2015

S. Sachdev, PRX **5**, 041025 (2015)

# The SYK model

## Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1) \mathcal{D}\Sigma(\tau_1, \tau_2) e^{-NS[G, \Sigma]}$$

for a known action  $S[G, \Sigma]$ . We find the saddle point,  $G_s, \Sigma_s$ , and only focus on the “Nambu-Goldstone” modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for  $\Sigma$ ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-NS_{\text{eff}}[f, \phi]}.$$

J. Maldacena and D. Stanford, arXiv:1604.07818;  
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849;  
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;  
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

# The Schwarzian theory of the SYK model

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f, \phi] = \frac{K}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi\mathcal{E}T)\partial_\tau f)^2 - \frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T f(\tau)), \tau \},$$

where  $f(\tau)$  is a monotonic map from  $[0, 1/T]$  to  $[0, 1/T]$ , the couplings  $K$ ,  $\gamma$ , and  $\mathcal{E}$  can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left( \frac{g''}{g'} \right)^2.$$

Specifically, an argument constraining the effective at  $T = 0$  is

$$S_{\text{eff}} \left[ f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0 \right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;  
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849;  
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746;  
Yingfei Gu and S. Sachdev, unpublished

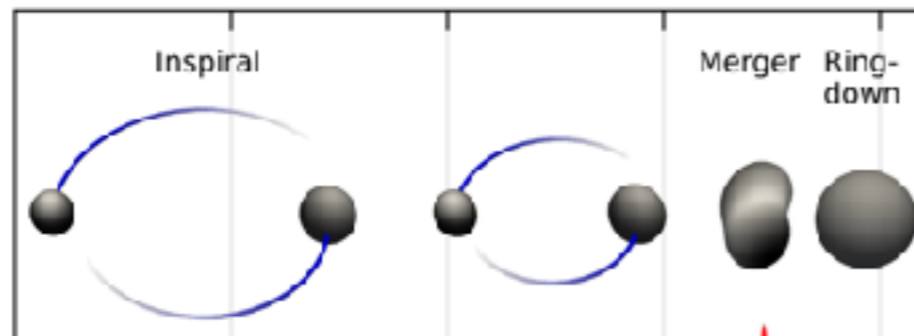
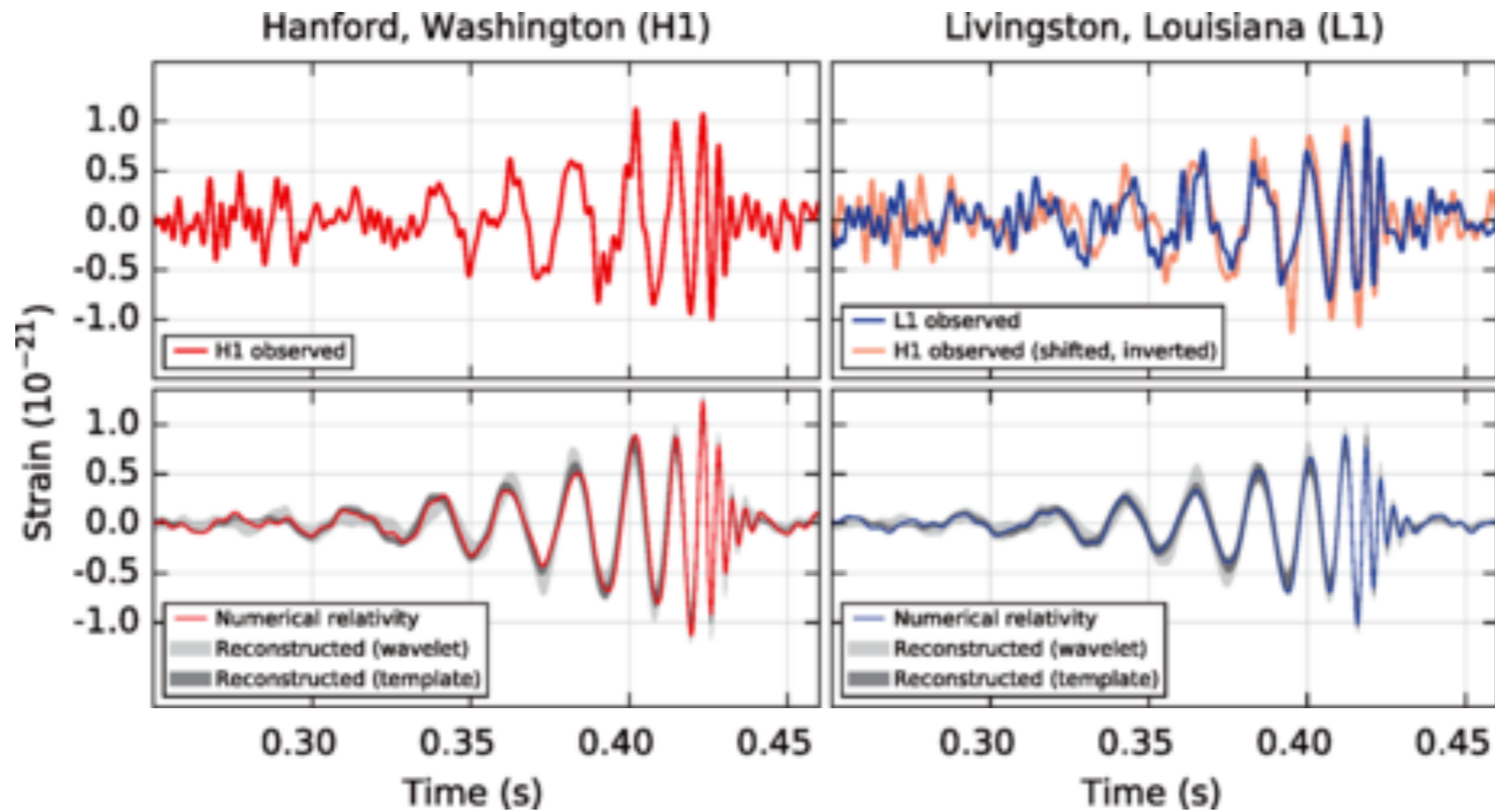
1. Review of spin-density-wave theory
2.  $SU(2)$  gauge theory of fluctuating spin density waves
3. Quantum oscillations and photoemission on the electron-doped cuprates
4. Strange metals and SYK models
5. Review of SYK models: Schwarzian theory
6. Connections to black holes with  $AdS_2$  horizons

# Black holes

- Black holes have an entropy and a temperature,  $T_H = \hbar c^3 / (8\pi G M k_B)$ .
- The entropy is proportional to their surface area.

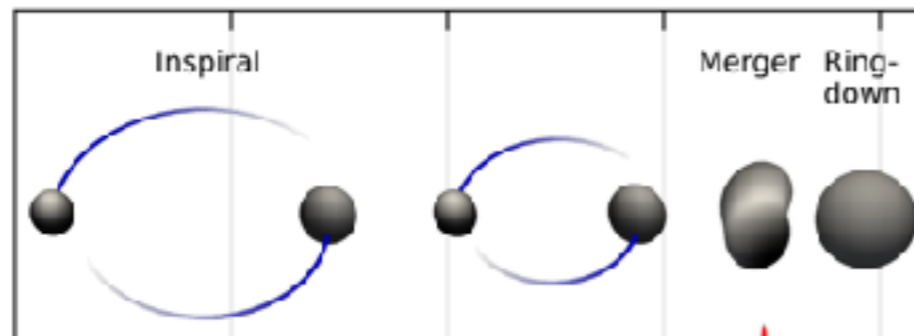
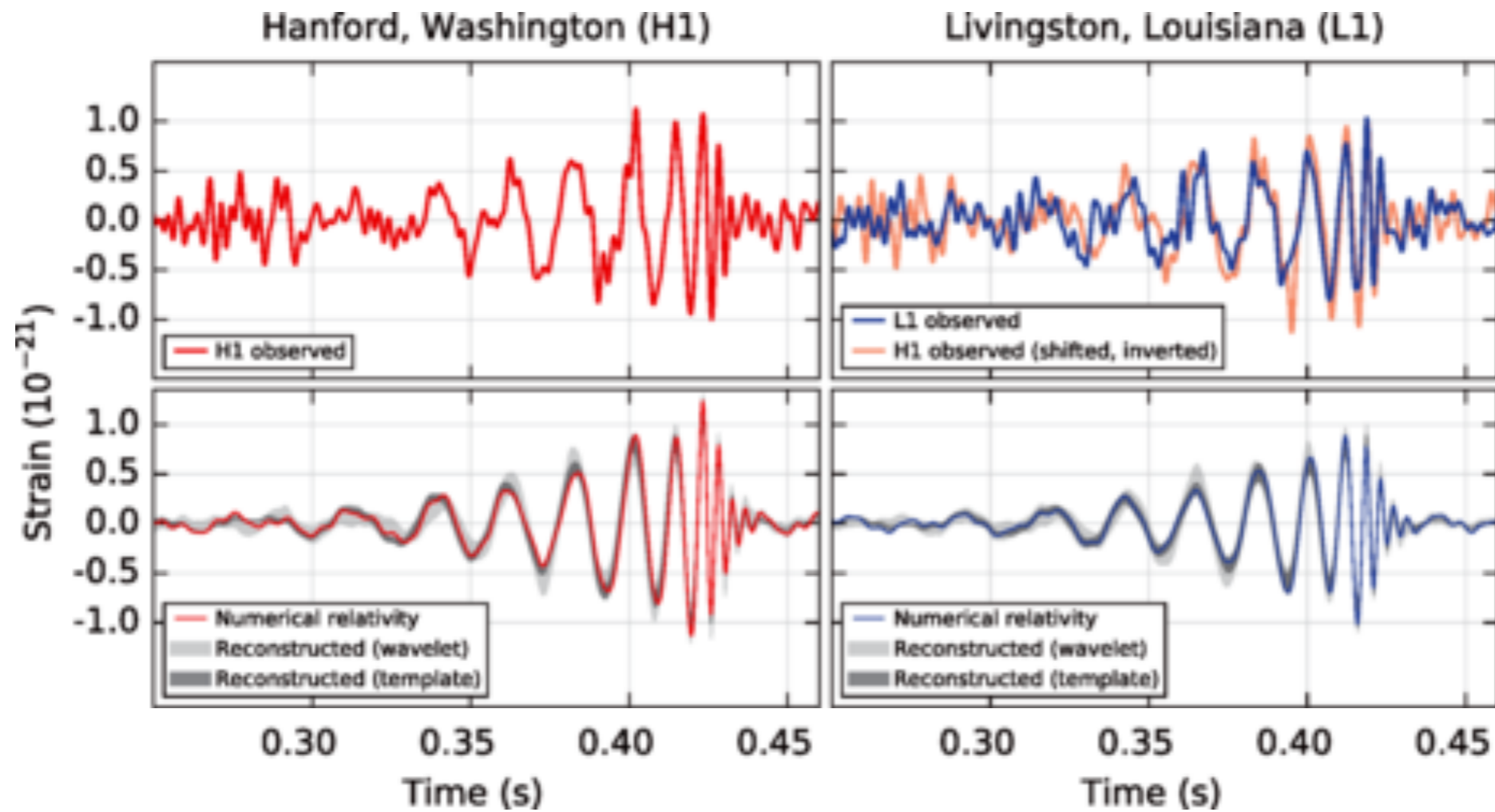
J. D. Bekenstein, PRD **7**, 2333 (1973)  
S.W. Hawking, Nature **248**, 30 (1974)





**LIGO**  
**September 14, 2015**

- The ring-down is predicted by General Relativity to happen in a time  $\frac{8\pi GM}{c^3} \sim 8$  milliseconds.



**LIGO**  
**September 14, 2015**

- The ring-down is predicted by General Relativity to happen in a time  $\frac{8\pi GM}{c^3} \sim 8$  milliseconds. Curiously this happens to equal  $\frac{\hbar}{k_B T_H}$ ; so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

# Black holes

- Black holes have an entropy and a temperature,  $T_H = \hbar c^3 / (8\pi G M k_B)$ .
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time  $\sim \hbar / (k_B T_H)$ .

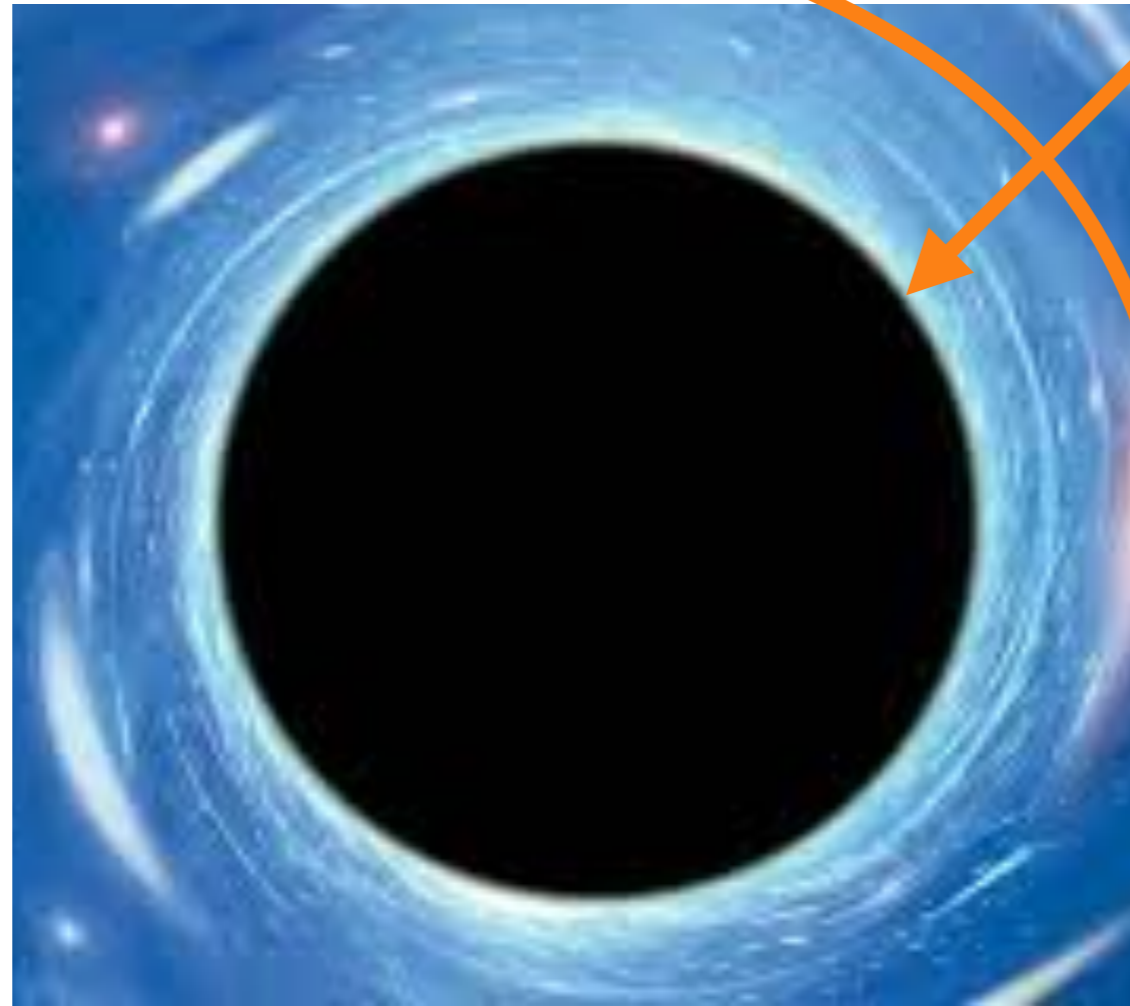


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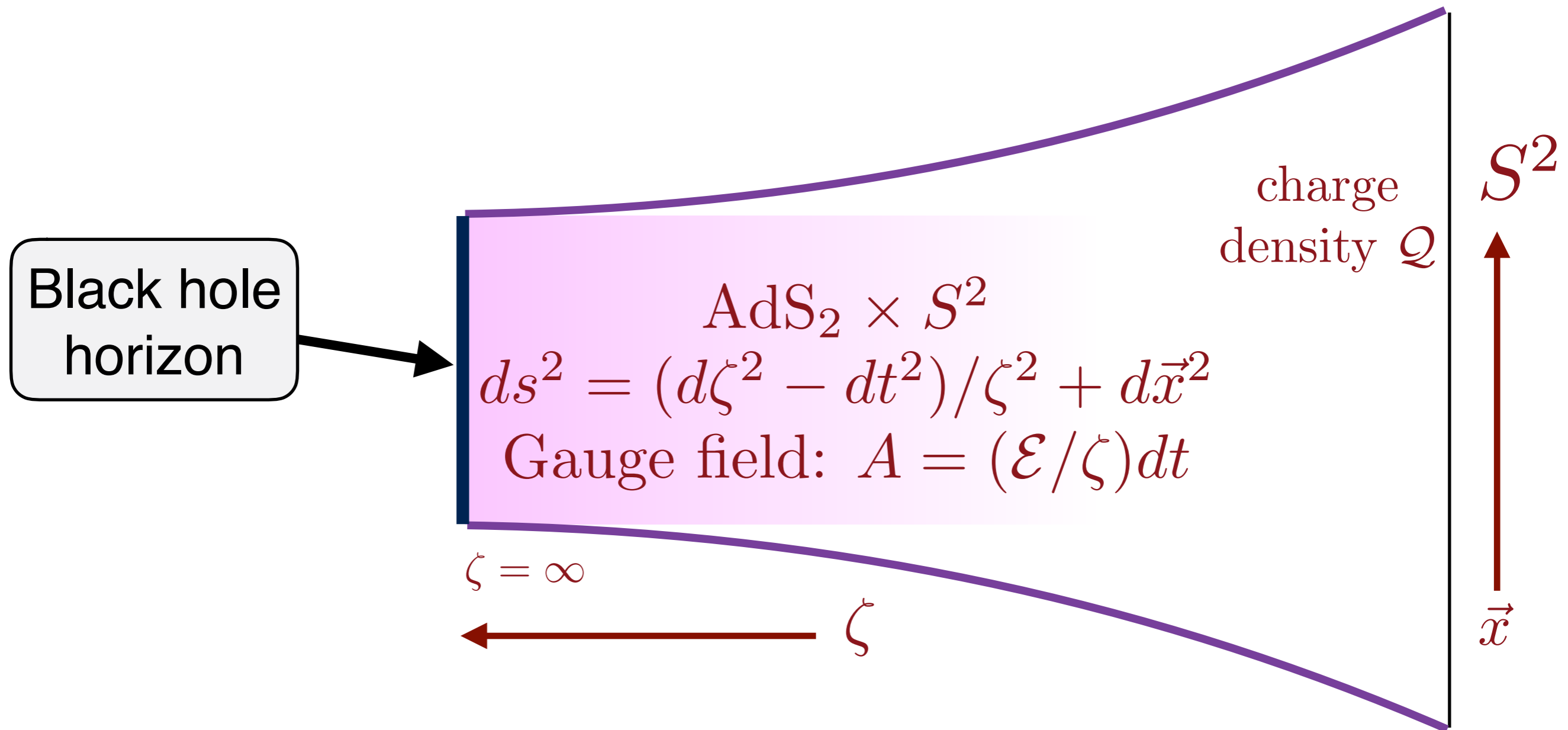
## Holography:

Quantum black holes “look like” quantum many-particle systems without quasiparticle excitations, residing “on” the surface of the black hole



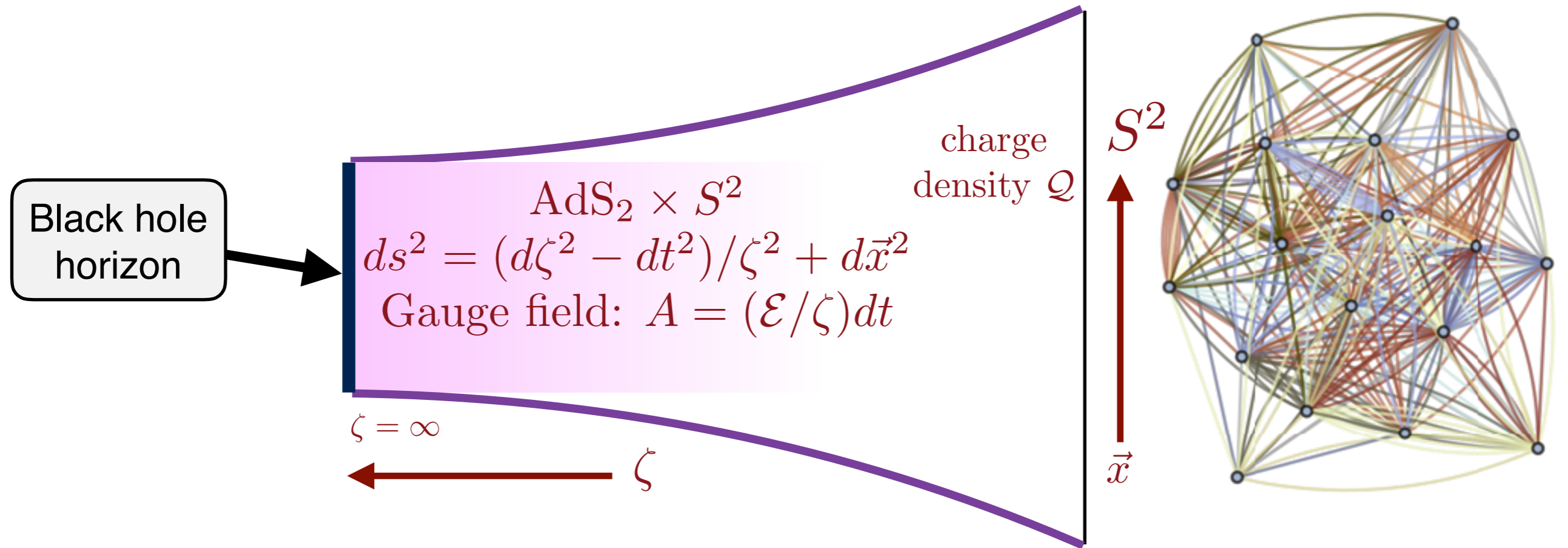
Consider a charged black hole with the smallest possible mass: the extremal limit. Zoom in to the near-horizon region at low energies. In this limit, the quantum theory lives in one space ( $\zeta$ ) and one time dimension

# SYK models and black holes



The near-horizon region of an extremal charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model

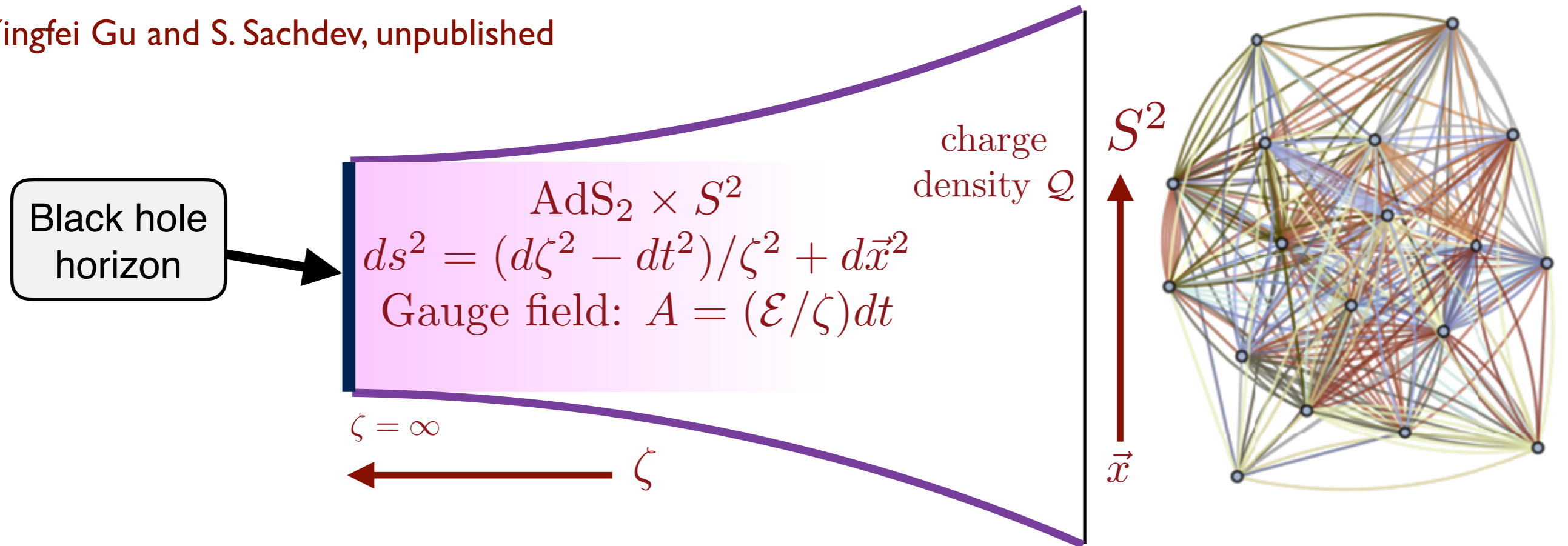
# SYK models and black holes



Bekenstein-Hawking entropy of  $AdS_2$  horizon  
at  $T = 0 \Leftrightarrow N s_0$  entropy of SYK model

# SYK models and black holes

Yingfei Gu and S. Sachdev, unpublished



The correspondence between the complex SYK model and extremal black holes holds also for the low  $T$  thermodynamics and low energy density of states. Both obey

$$\Omega(T) - E_0 = N \left[ -s_0 T - \frac{1}{2}(\gamma + 4\pi^2 \mathcal{E}^2 K) T^2 + \mathcal{O}(T^3) \right] + 2T \ln \left( \frac{U}{T} \right) \dots$$

for the grand potential, and for the density of states at a fixed charge  $\mathcal{Q}$

$$d(E) \sim \exp(Ns_0) \sinh \left( \sqrt{2N\gamma(E - E_0)} \right) \quad , \quad E > E_0 \quad , \quad e^{-cN} \ll \gamma(E - E_0) \ll N$$

with the relation  $\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}$  also obtained from Einstein's equations

# SYK models and black holes

- Reparameterization invariance is a defining property of Einstein's theory of gravity
- In imaginary time,  $AdS_2$  is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under  $SL(2, \mathbb{R})$

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$  is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1.$$



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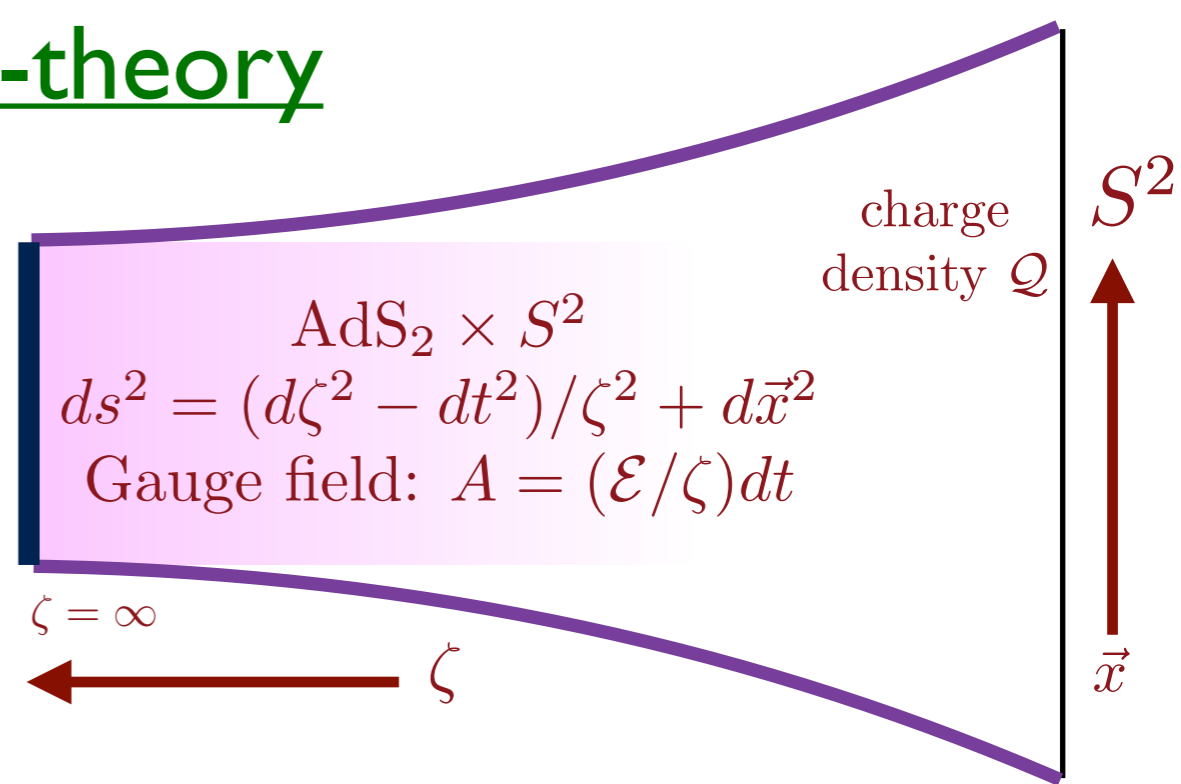
**Their identical symmetries lead to the same low energy quantum theory for the SYK model and extremal charged black holes !**





# Einstein-Maxwell-theory

$$S_{4D} = \int d^4x \sqrt{-\hat{g}} \left( \hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

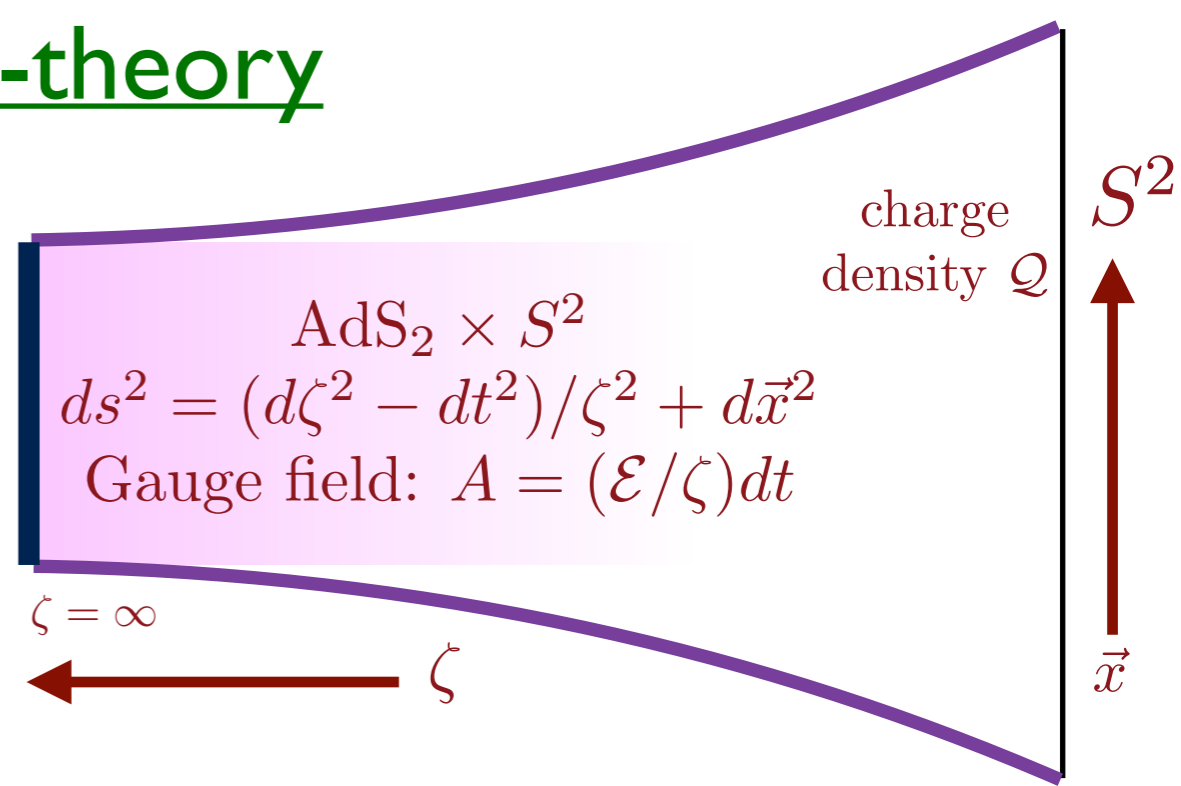


- Has Reissner-Nördstrom-AdS charged black hole solution, with charge density  $\mathcal{Q}$ , a near-horizon  $\text{AdS}_2 \times S^2$  geometry, and surface electric field  $\mathcal{E}$ . (This analysis also applies in asymptotically Minkowski spacetime ( $L \rightarrow \infty$ ) provided the black hole mass is extremal.)



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- From Einstein's equations, the Bekenstein-Hawking black hole entropy  $S_{4D}$  is found to obey the same relation as the entropy of the SYK model

$$\frac{\partial S_{4D}}{\partial \mathcal{Q}} = 2\pi \mathcal{E},$$

A Sen, JHEP **0509**, 038 (2005)

where  $\mathcal{E}$  is identified from the spectral asymmetry of probe particle Green's functions in both cases. This establishes that the SYK entropy  $Ns_0$  maps onto (Area of horizon)/(4G)

S. Sachdev, PRX **5**, 041025 (2015)

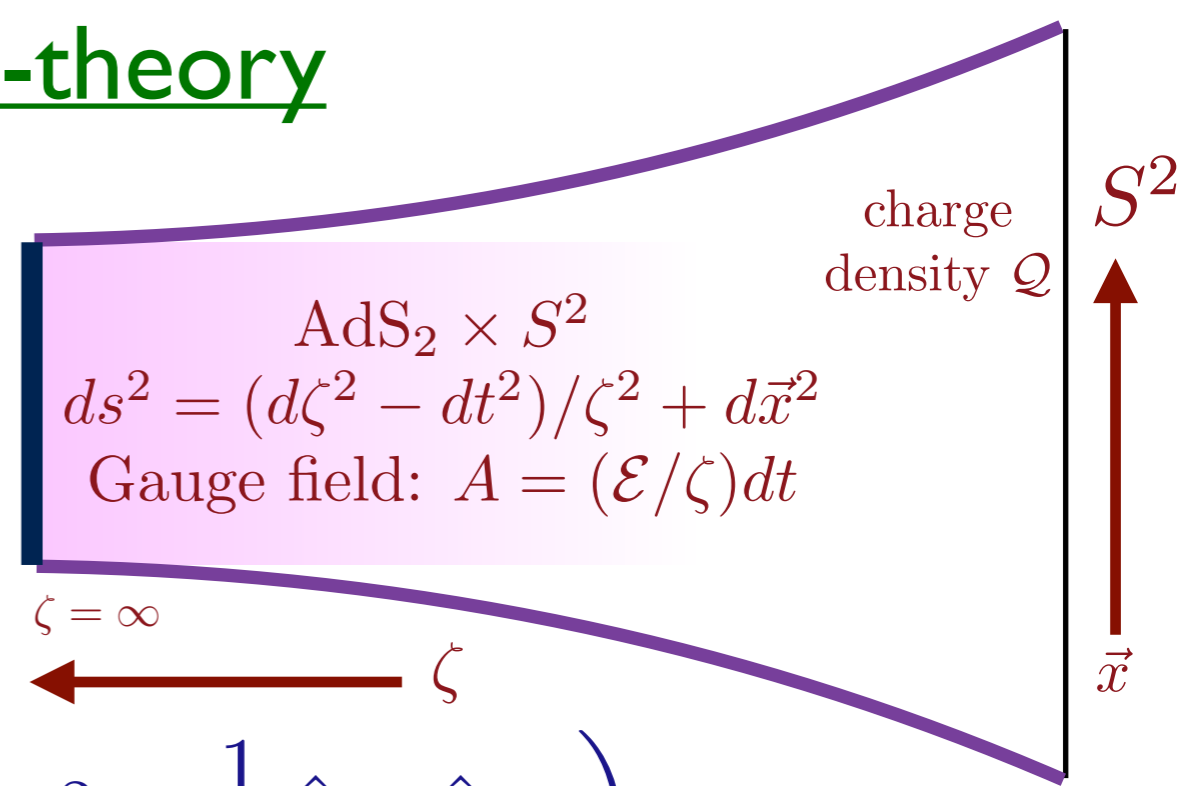


# Einstein-Maxwell-theory

P. Nayak, A. Shukla, R.M. Soni, S.P. Trivedi, and V. Vishal,  
arXiv:1802.09547;

A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia,  
arXiv:1802.07746

U. Moitra, S.P. Trivedi, V. Vishal, arXiv:1808.08239



$$S_{4D} = \int d^4x \sqrt{-\hat{g}} \left( \hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

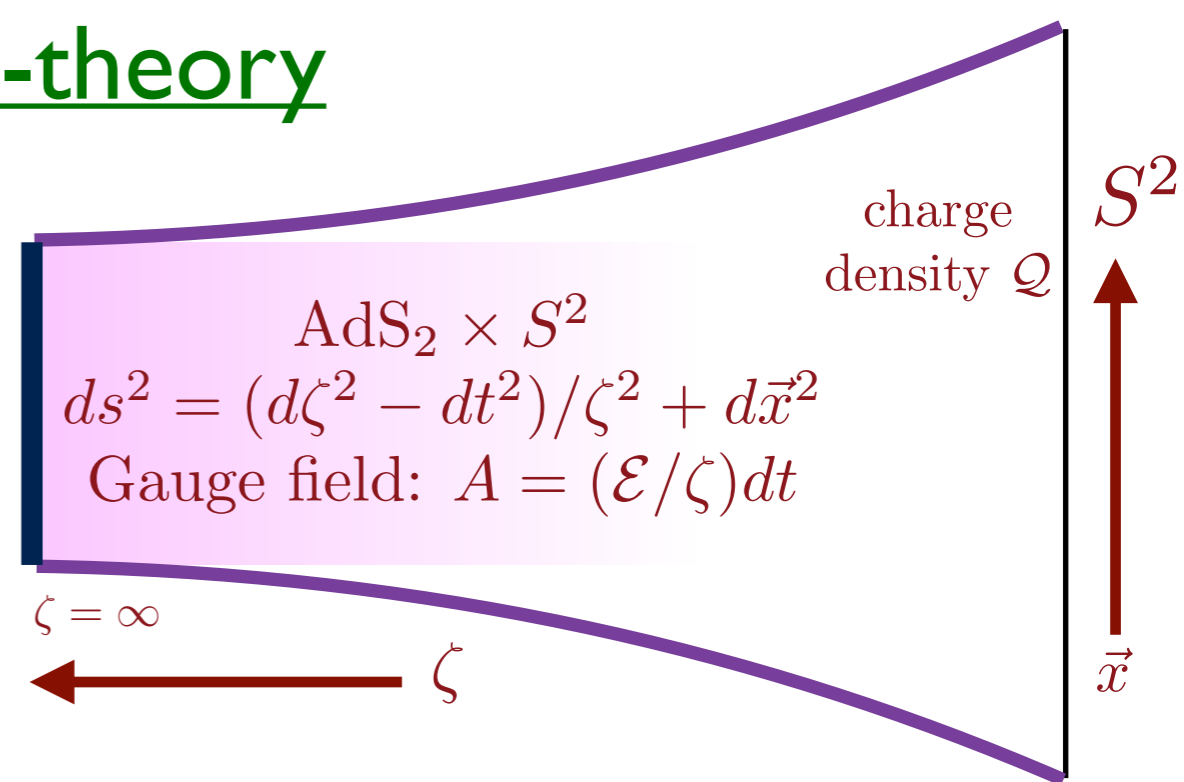
In the small black hole size limit,  $T \ll 1/R$ , where  $R$  is the radius of the black hole, the theory dimensionally reduces to an Einstein-Maxwell-dilaton theory in two dimensions (the Jackiw-Teitelbaum model), along with Maxwell term

$$S_{2D} = N s_0 + \int d^2x \sqrt{-g} \left( \Phi(\mathcal{R} - \Lambda) - \frac{Z(\Phi)}{4} F_{ab} F^{ab} \right).$$

The dilaton  $\Phi$  represents the radial oscillations of the small black hole.



# Einstein-Maxwell-theory



$$S_{2D} = N s_0 + \int d^2x \sqrt{-g} \left( \Phi(\mathcal{R} - \Lambda) - \frac{Z(\Phi)}{4} F_{ab} F^{ab} \right).$$

There are no bulk quantum fluctuations of the metric in two-dimensional gravity, and there a further dimensional reduction to a 0 + 1 dimensional theory representing fluctuations of the AdS<sub>2</sub> boundary: this 0 + 1 dimensional turns out to be *precisely the Schwarzian theory obtained for the SYK model.*

$$S_{\text{eff}}[f, \phi] = \frac{K}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi\mathcal{E}T)\partial_\tau f)^2 - \frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T f(\tau)), \tau \},$$

J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;  
 K. Jensen, arXiv:1605.06098;  
 J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

## *Quantum matter without quasiparticles*

- Planckian dynamics is realized in the ‘solvable’ SYK models
- Black holes thermalize in a time  $\sim \hbar/(k_B T_H)$ , where  $T_H$  is the Hawking temperature.
- A Schwarzian theory of a time reparameterization mode, with  $SL(2, \mathbb{R})$  symmetry, describes the quantum dynamics of
  - the SYK models
  - black holes with near-extremal  $AdS_2$  horizons