Quantum phase transitions: from Mott insulators to the cuprate superconductors

Colloquium article in *Reviews of Modern Physics* **75**, 913 (2003)

Talk online:
Google Sachdev

Outline

A. “Dimerized” Mott insulators

Landau-Ginzburg-Wilson (LGW) theory

B. Mott insulators with spin S=1/2 per unit cell

Berry phases, bond order, and the breakdown of the LGW paradigm

C. Cuprate Superconductors

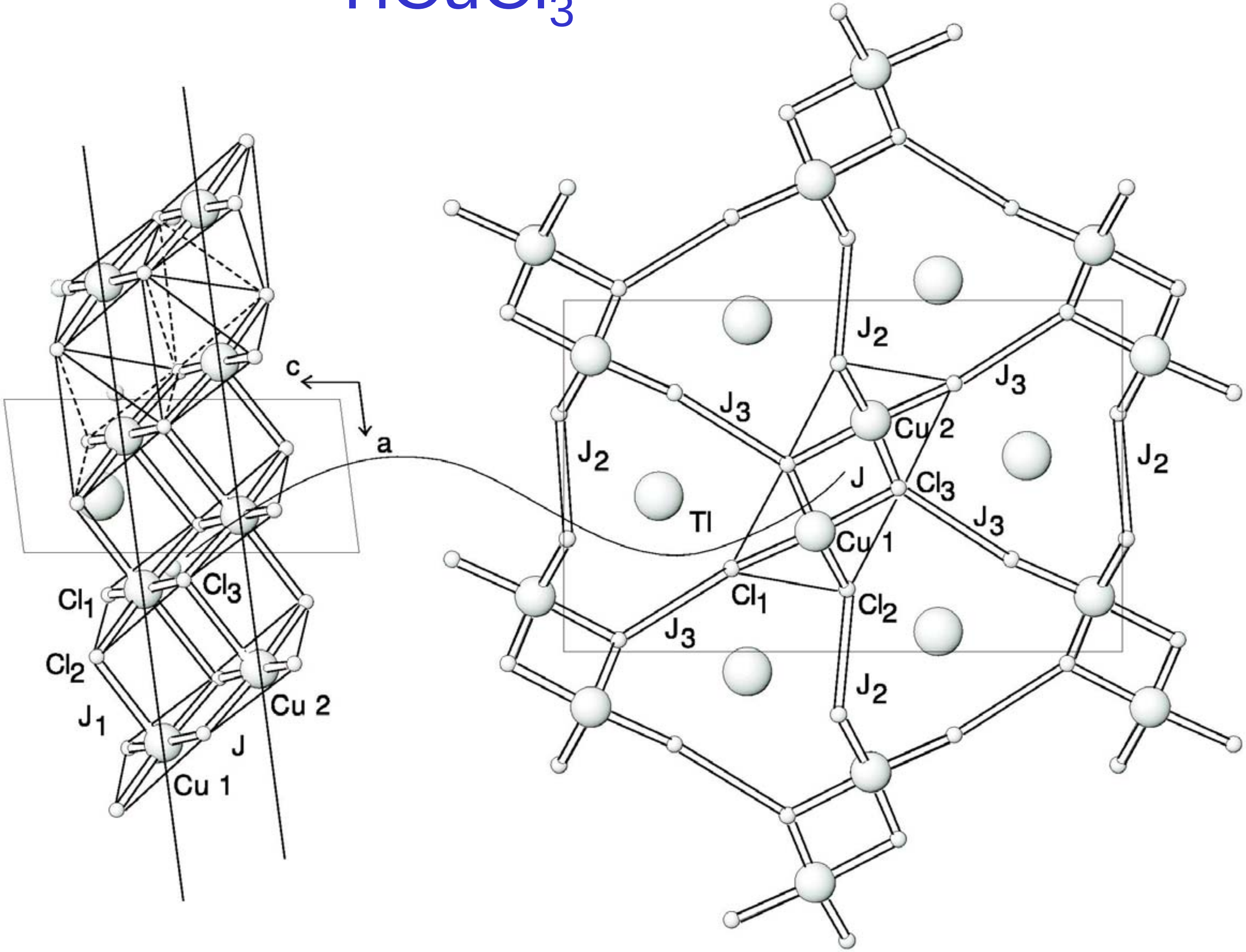
Competing orders and recent experiments

“Dimerized” Mott insulators:

Landau-Ginzburg-Wilson (LGW) theory:

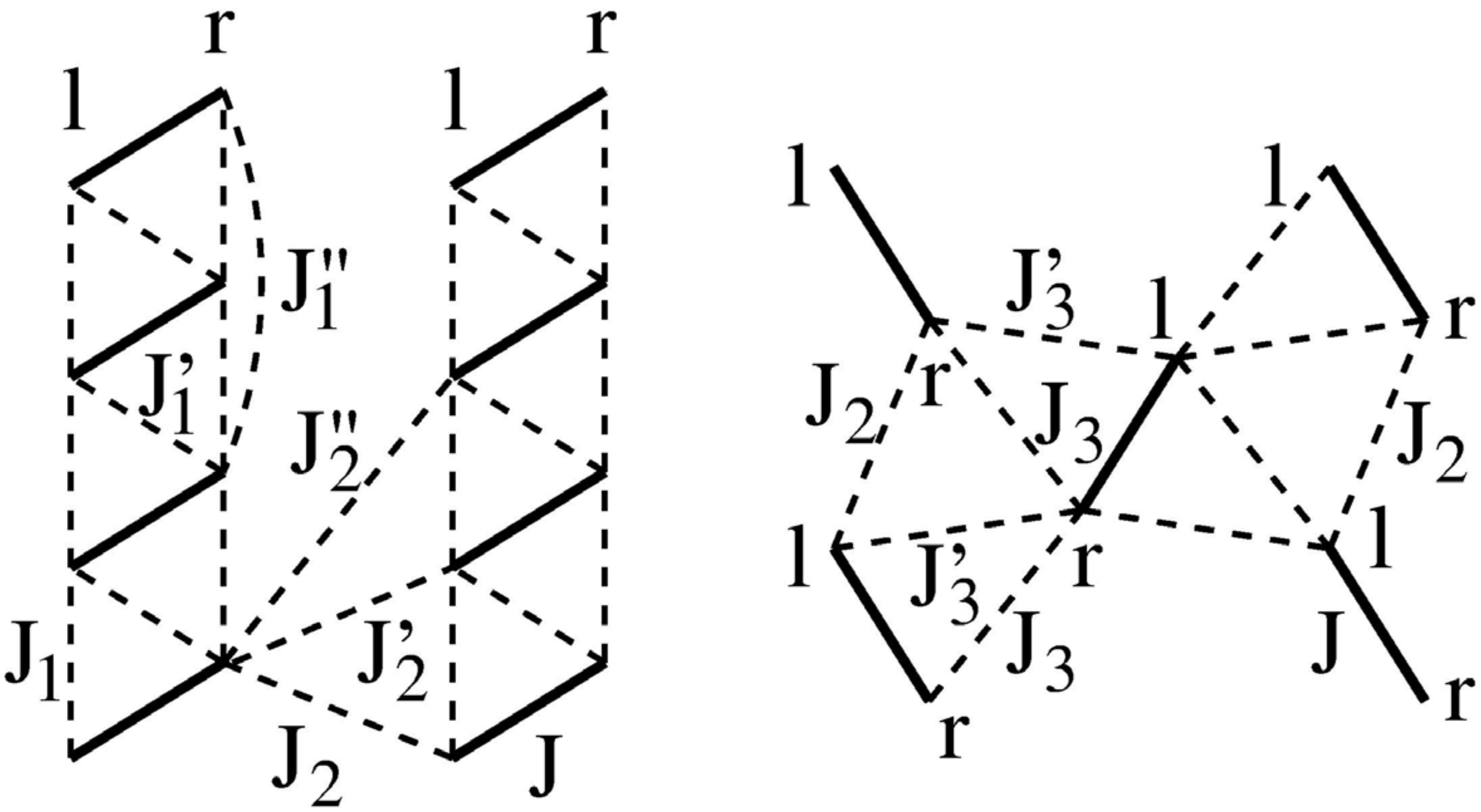
Second-order phase transitions described by fluctuations of an ***order parameter*** *associated with a* ***broken symmetry***

$TlCuCl_3$



M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.

$TlCuCl_3$



(a) (b)

M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.

Coupled Dimer Antiferromagnet

M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
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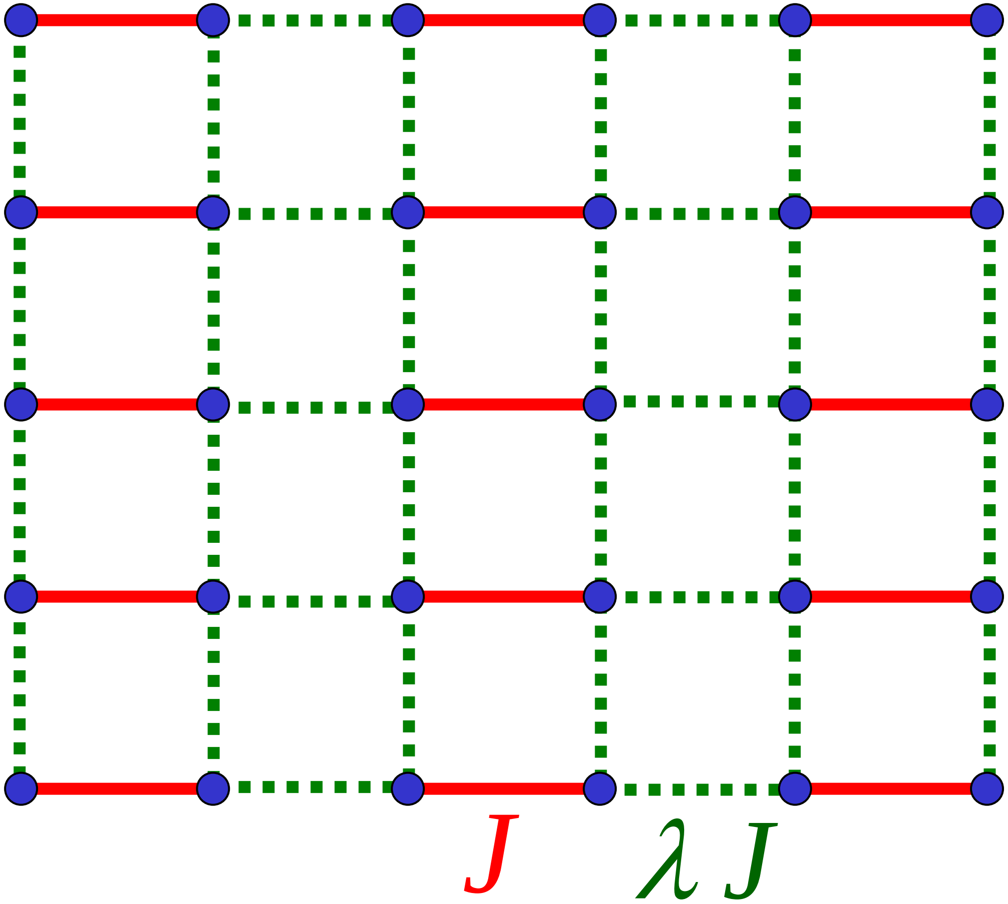
S=1/2 spins on coupled dimers

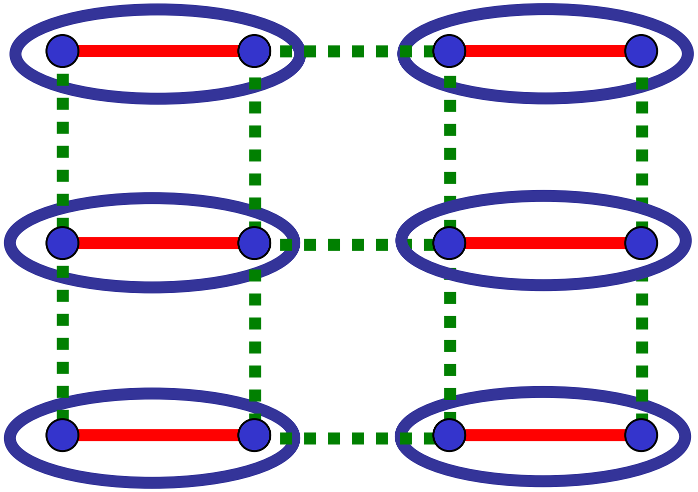


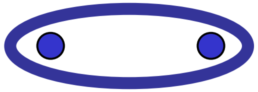
$$H = \sum_{<ij>} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 0

Weakly coupled dimers



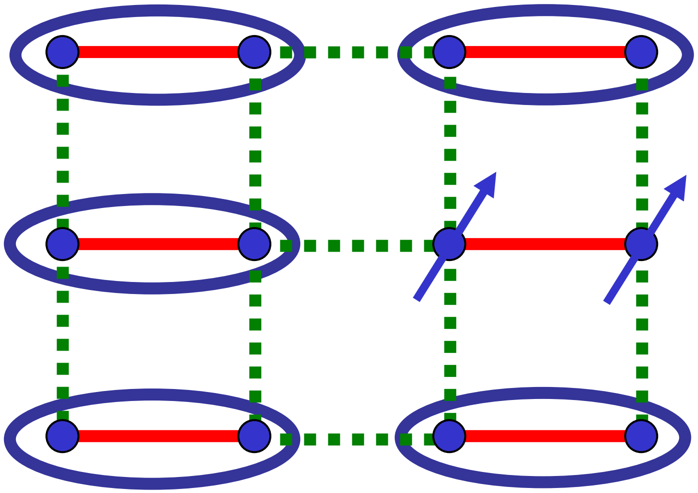


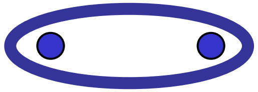
$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Paramagnetic ground state $\langle \vec{S}_i \rangle = 0$

λ close to 0

Weakly coupled dimers

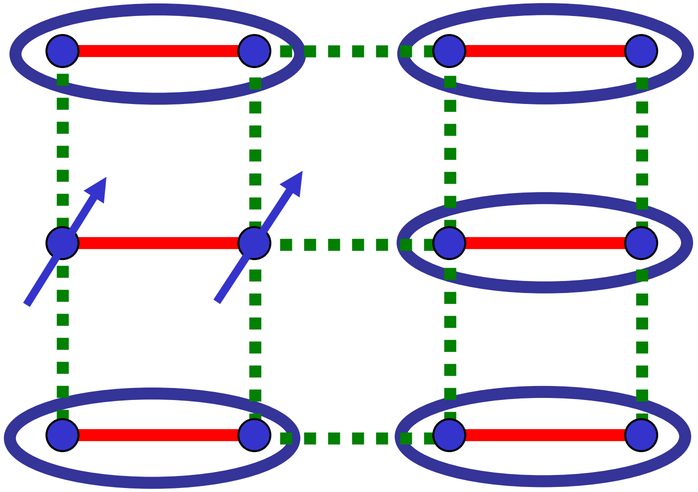


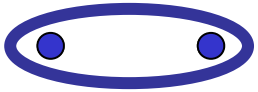
 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers





$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

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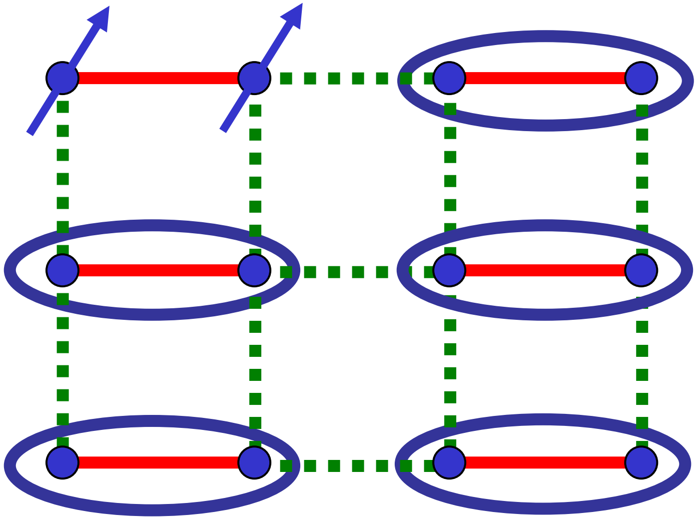
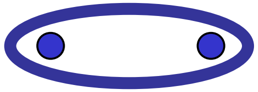
Weakly coupled dimers




$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

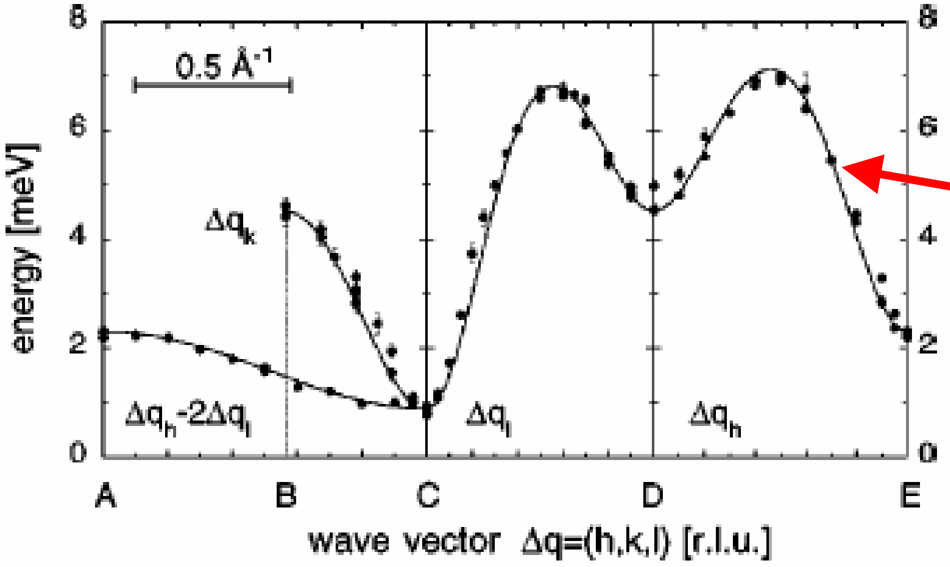
Excitation: *S*=1 ***triplon*** (***exciton***, spin collective mode)

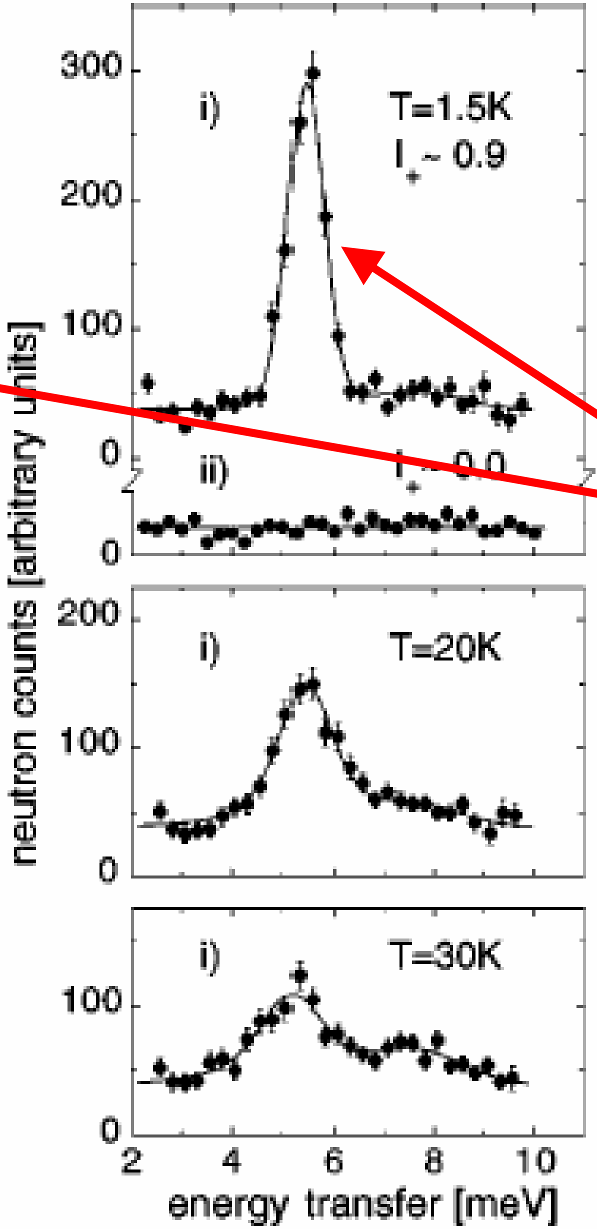
Energy dispersion away from
antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

TlCuCl_3





“triplon” or spin exciton

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

FIG. 1. Measured neutron profiles in the a^*c^* plane of TlCuCl_3 for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

Coupled Dimer Antiferromagnet

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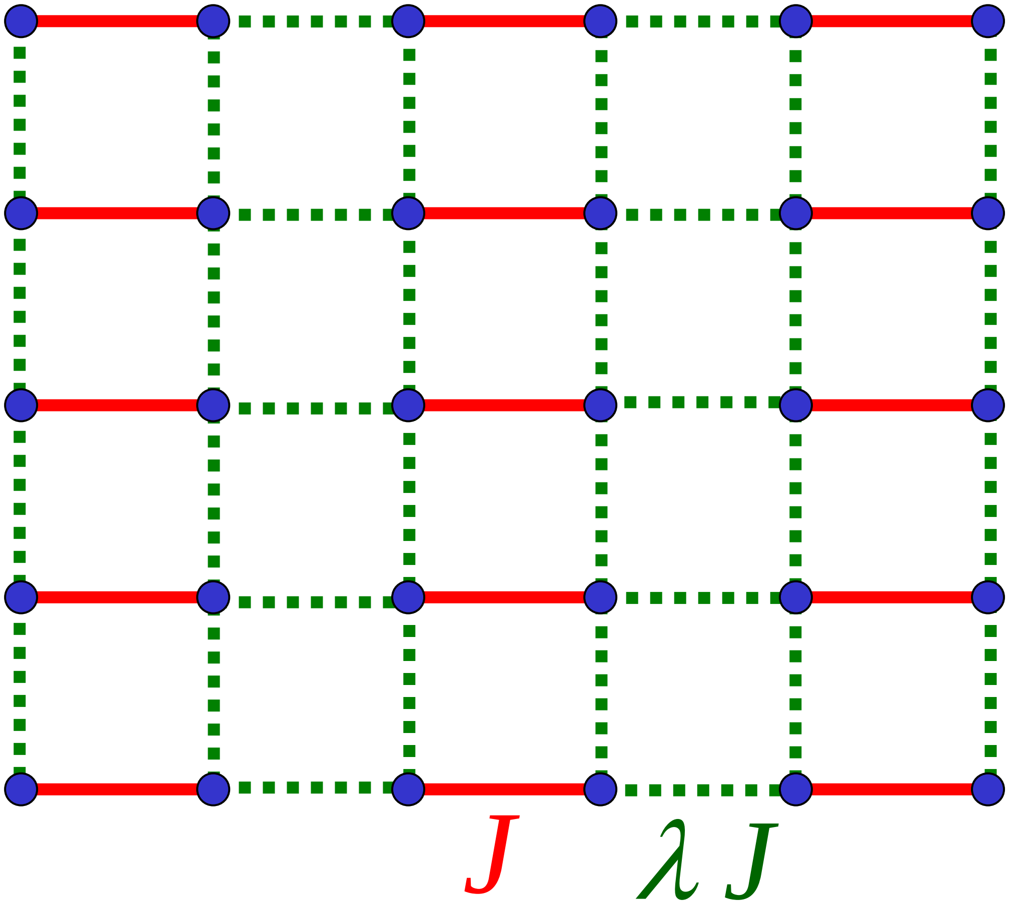
S=1/2 spins on coupled dimers



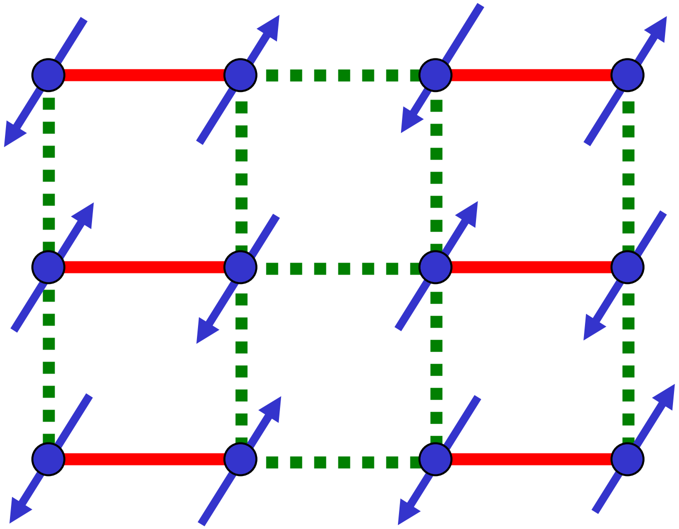
$$H = \sum_{<ij>} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4

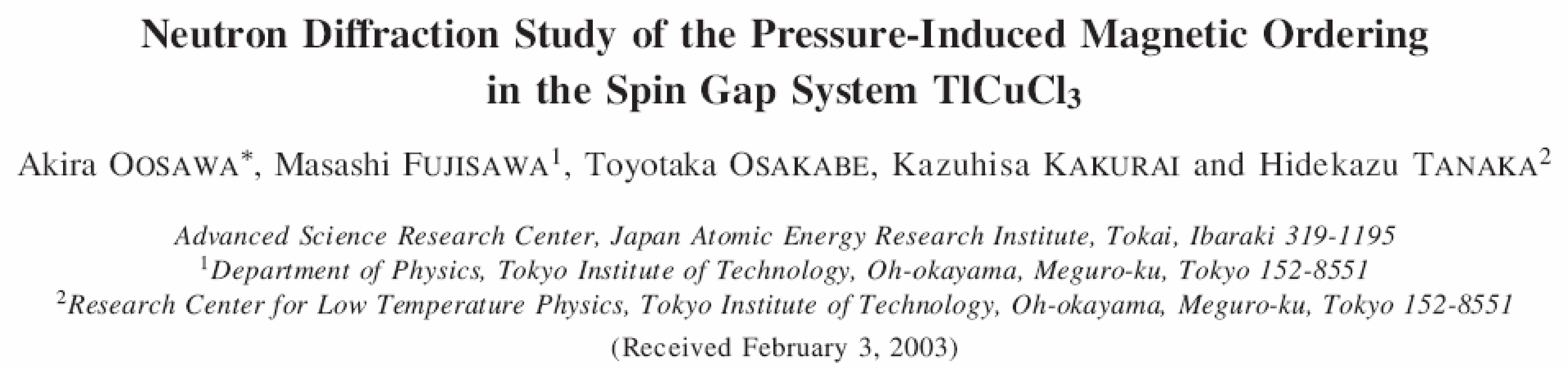


Ground state has long-range magnetic (Neel or spin density wave) order

$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x+i_y} \vec{\varphi} \neq 0$$

Excitations: 2 spin waves (***magnons***) $\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$

TlCuCl$_3$

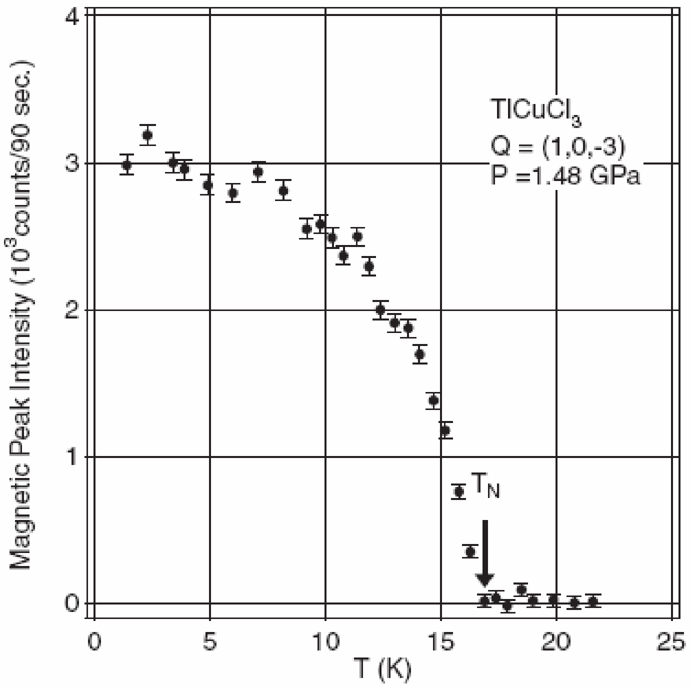


Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System TlCuCl$_3$

Akira OOSAWA*, Masashi FUJISAWA[1], Toyotaka OSAKABE, Kazuhisa KAKURAI and Hidekazu TANAKA[2]

Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195
[1]*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*
[2]*Research Center for Low Temperature Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*

(Received February 3, 2003)



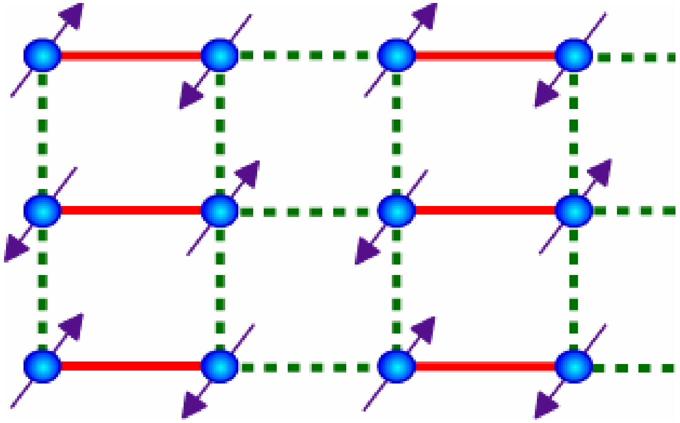
ig. 3. Temperature dependence of the magnetic Bragg peak intensity for $Q = (1, 0, -3)$ reflection measured at $P = 1.48$ GPa in TlCuCl$_3$.

J. Phys. Soc. Jpn **72**, 1026 (2003)

T=0

$\lambda_c = 0.52337(3)$

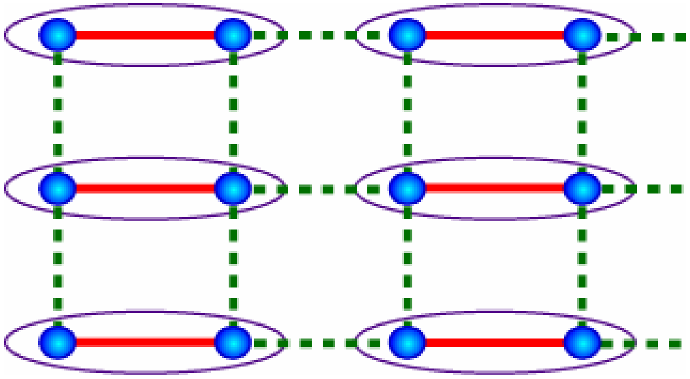
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



Neel state

$$\left\langle \vec{S} \right\rangle = \pm\vec{\varphi}$$

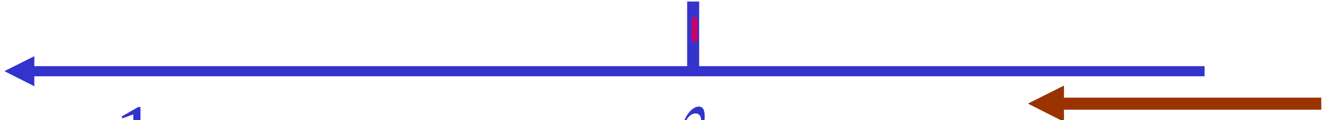
$$\vec{\varphi} \neq 0$$



Quantum paramagnet

$$\left\langle \vec{S} \right\rangle = 0$$

$$\vec{\varphi} = 0$$



λ 1 λ_c

Pressure in $TlCuCl_3$

The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett.* **89**, 077203 (2002))

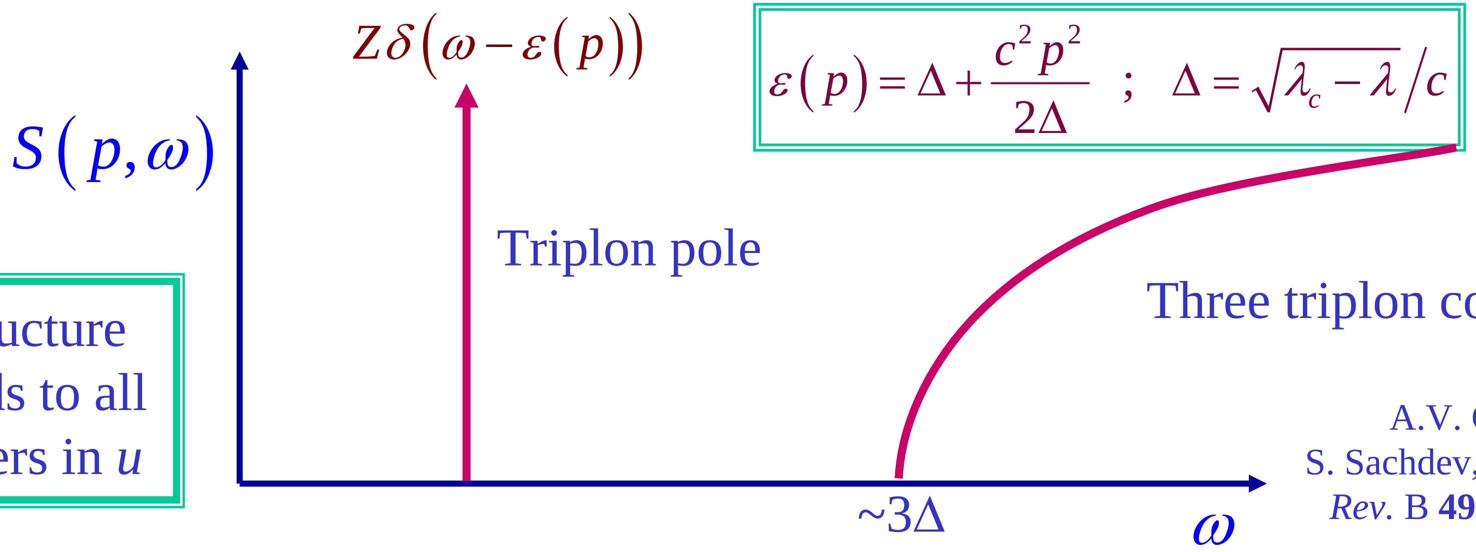
LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x\,d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + c^2 \left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right) \vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

For $\lambda < \lambda_c$ oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ lead to the following structure in the dynamic structure factor $S(p,\omega)$



$$\varepsilon(p) = \Delta + \frac{c^2 p^2}{2\Delta} \quad ; \quad \Delta = \sqrt{\lambda_c - \lambda}/c$$

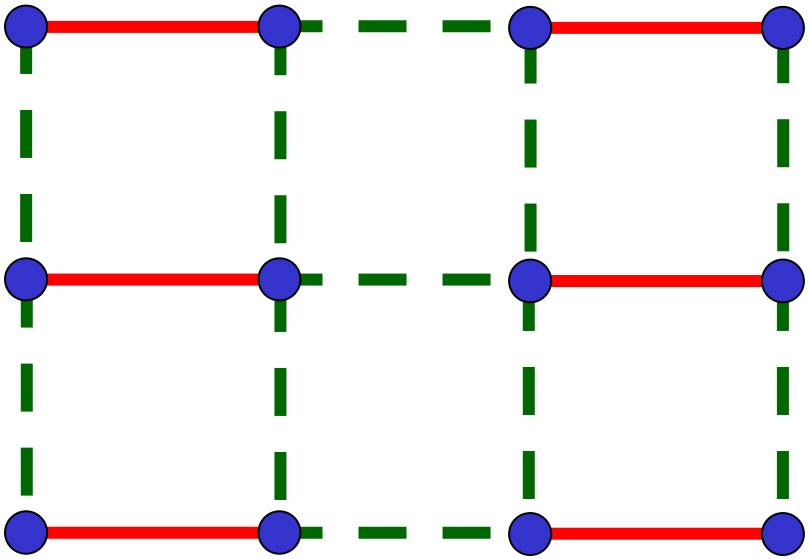
Structure holds to all orders in *u*

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

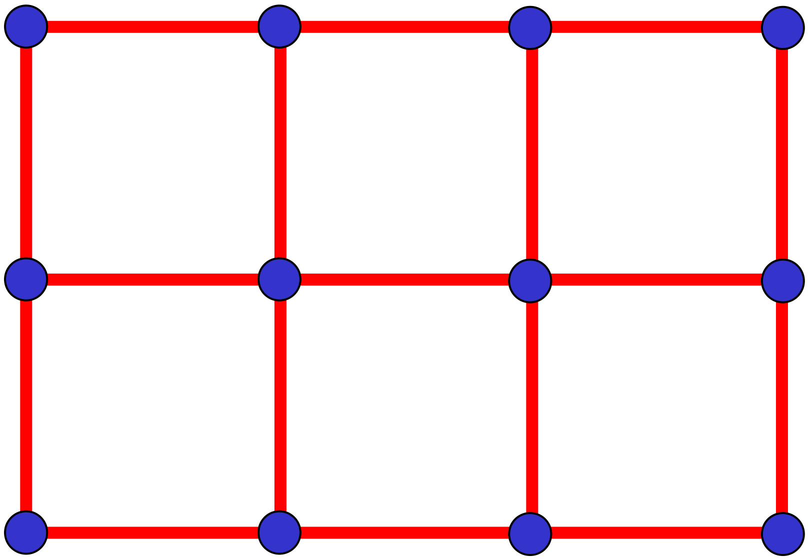
Mott insulators with spin S=1/2 per unit cell:

Berry phases, bond order, and the breakdown of the LGW paradigm

Mott insulator with two S=1/2 spins per unit cell



Mott insulator with one *S*=1/2 spin per unit cell



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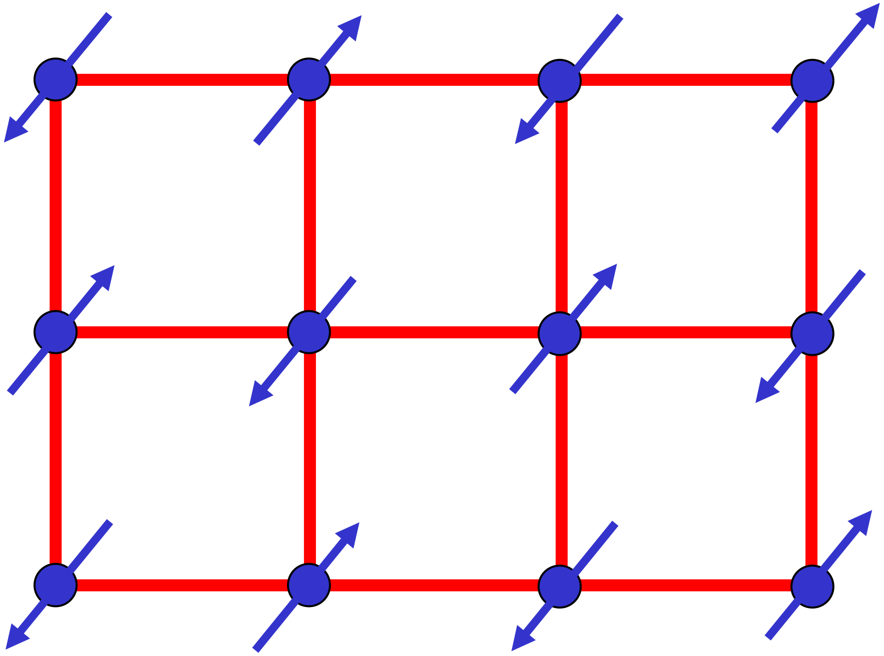
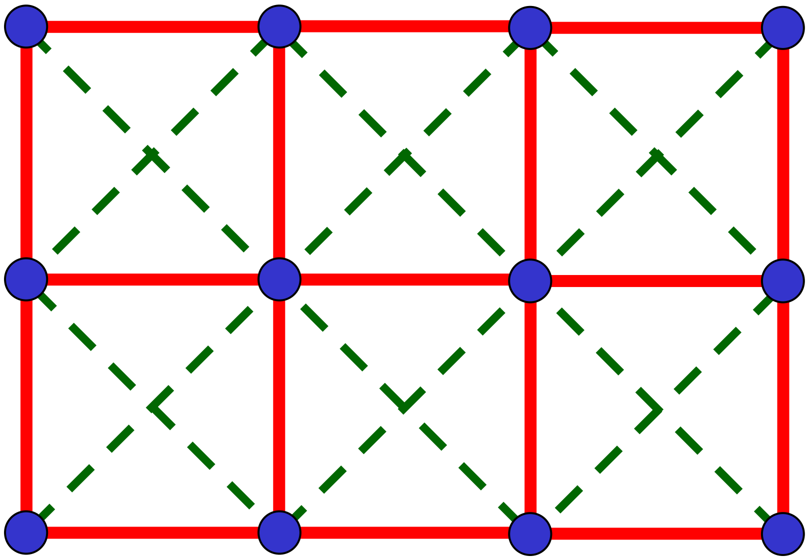


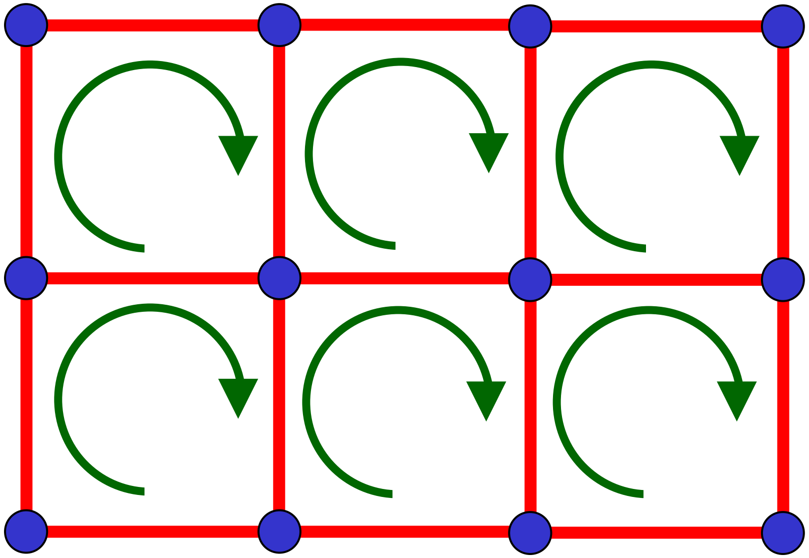
Ground state has Neel order with $\vec{\varphi} \neq 0$

Mott insulator with one *S*=1/2 spin per unit cell



Destroy Neel order by perturbations which
preserve full square lattice symmetry
e.g. second-neighbor or ring exchange

Mott insulator with one S=1/2 spin per unit cell



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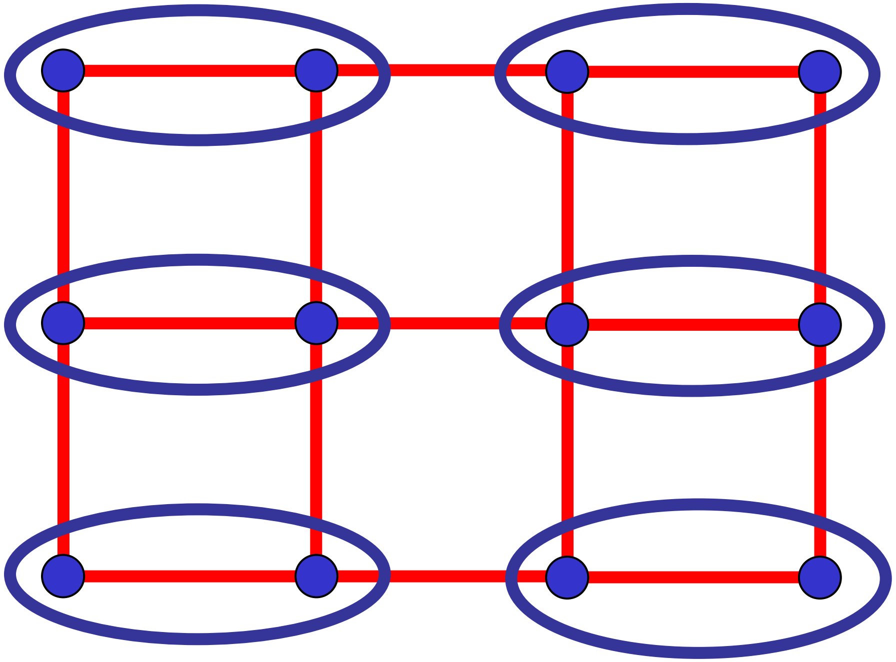
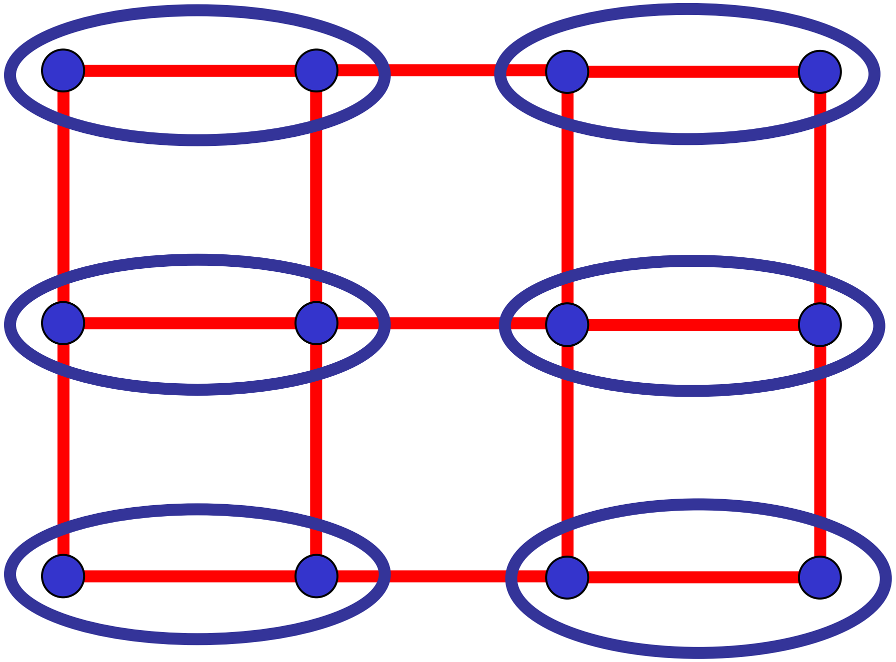


Possible paramagnetic ground state with $\vec{\varphi} = 0$

Mott insulator with one *S*=1/2 spin per unit cell

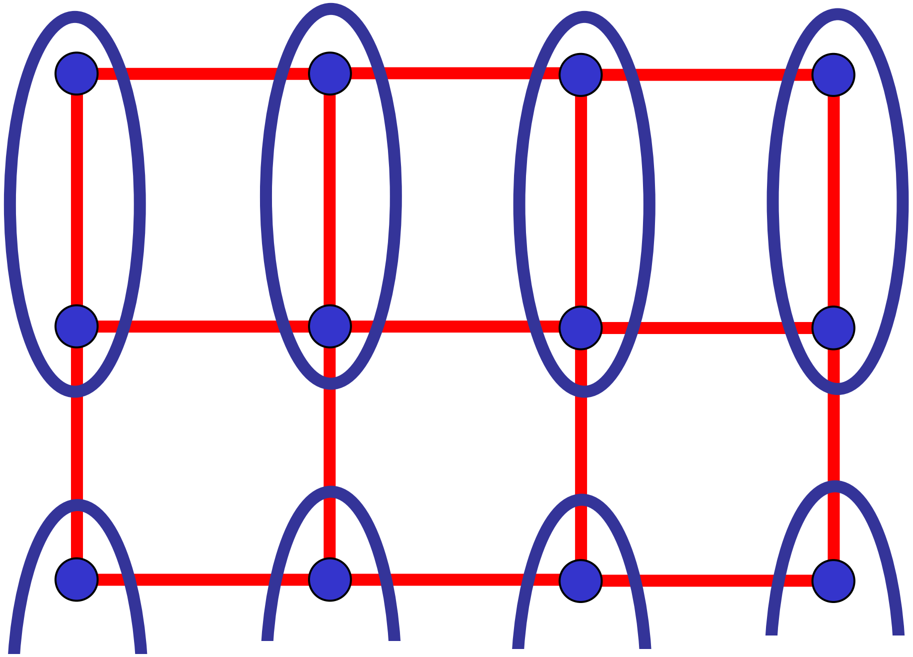


Possible paramagnetic ground state with $\vec{\varphi} = 0$

Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\Psi \neq 0$, where Ψ is the ***bond order parameter***

($\Psi \rightarrow \Psi e^{in\pi/2}$ under the lattice rotation).

Mott insulator with one S=1/2 spin per unit cell



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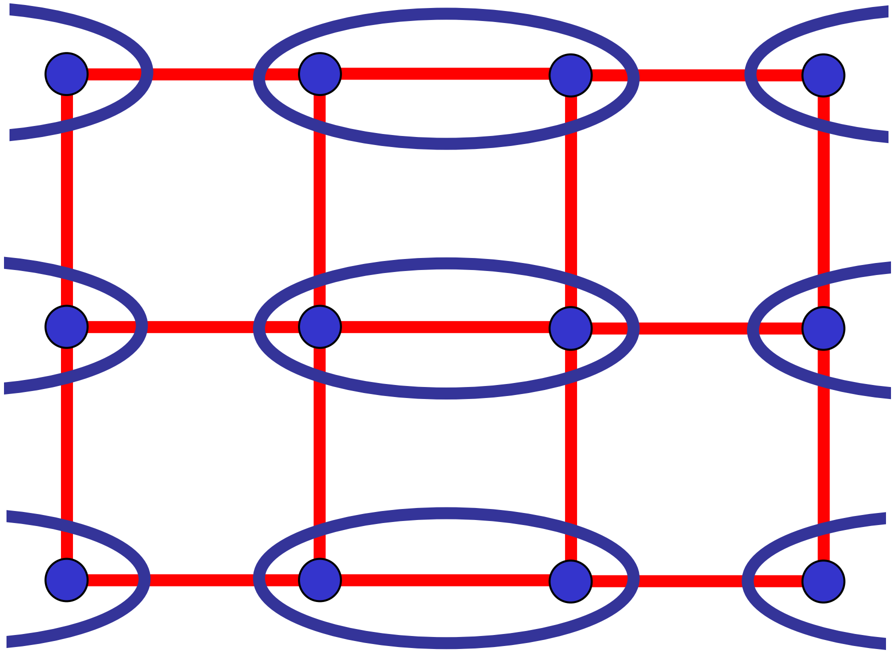

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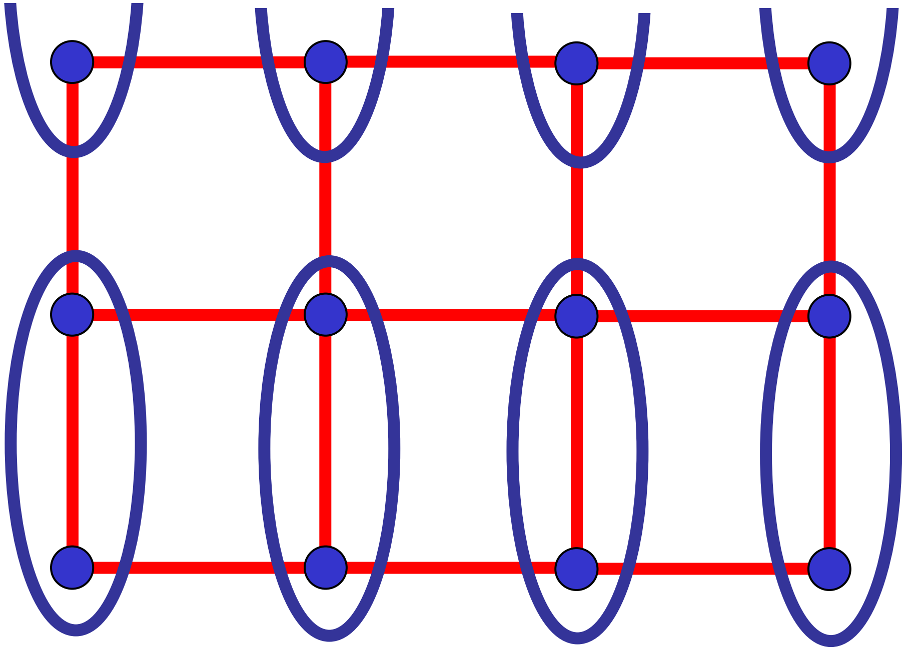

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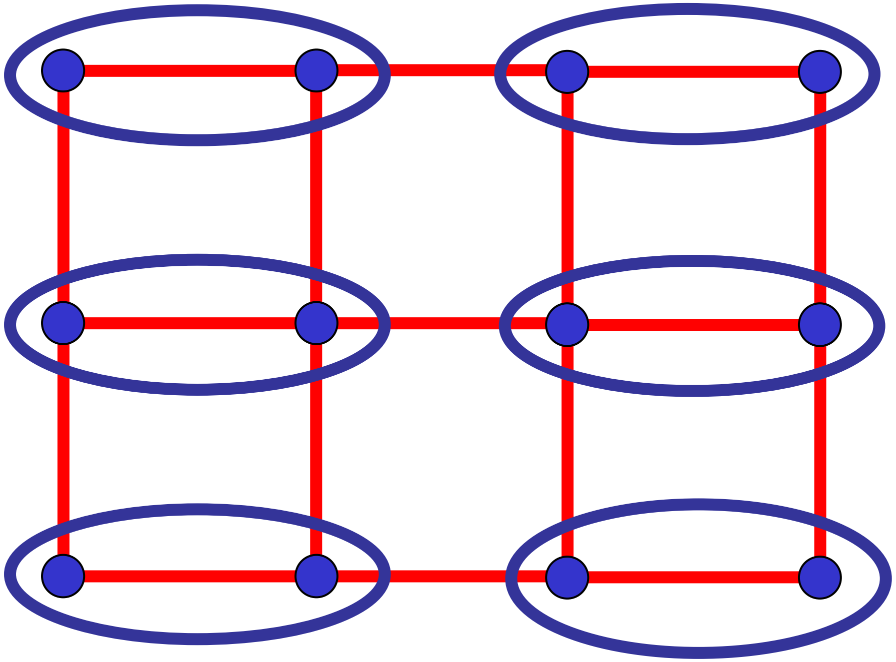

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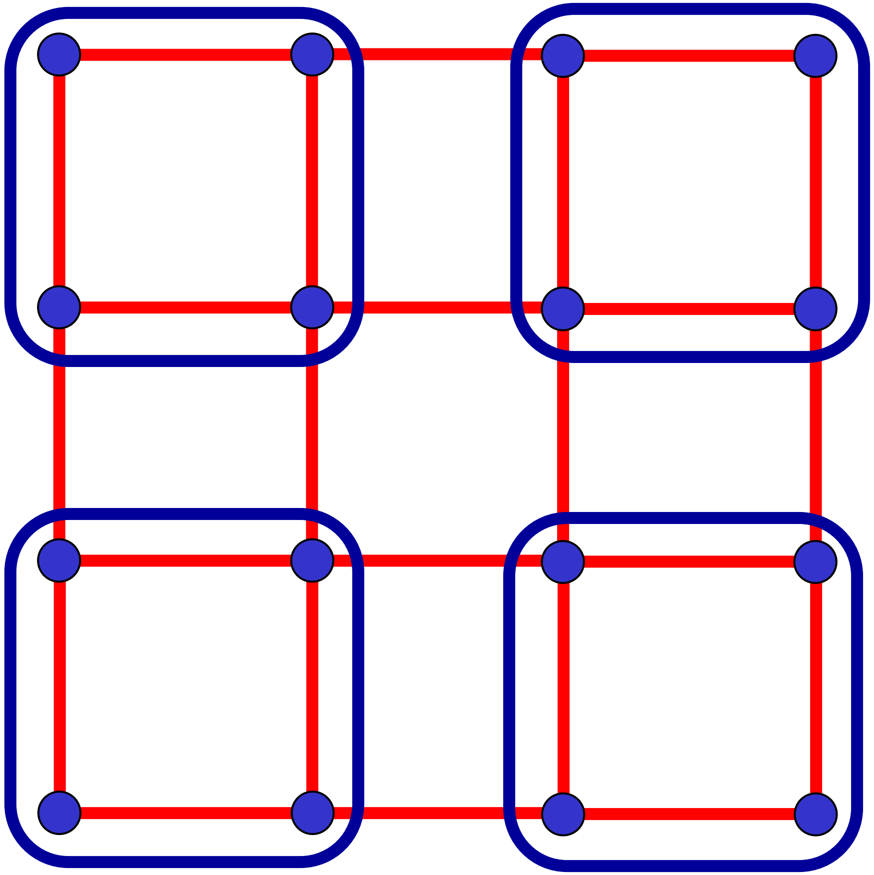
Mott insulator with one S=1/2 spin per unit cell



Possible paramagnetic ground state with $\vec{\varphi} = 0$

Another state that breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\Psi \neq 0$, where Ψ is the ***bond order parameter***

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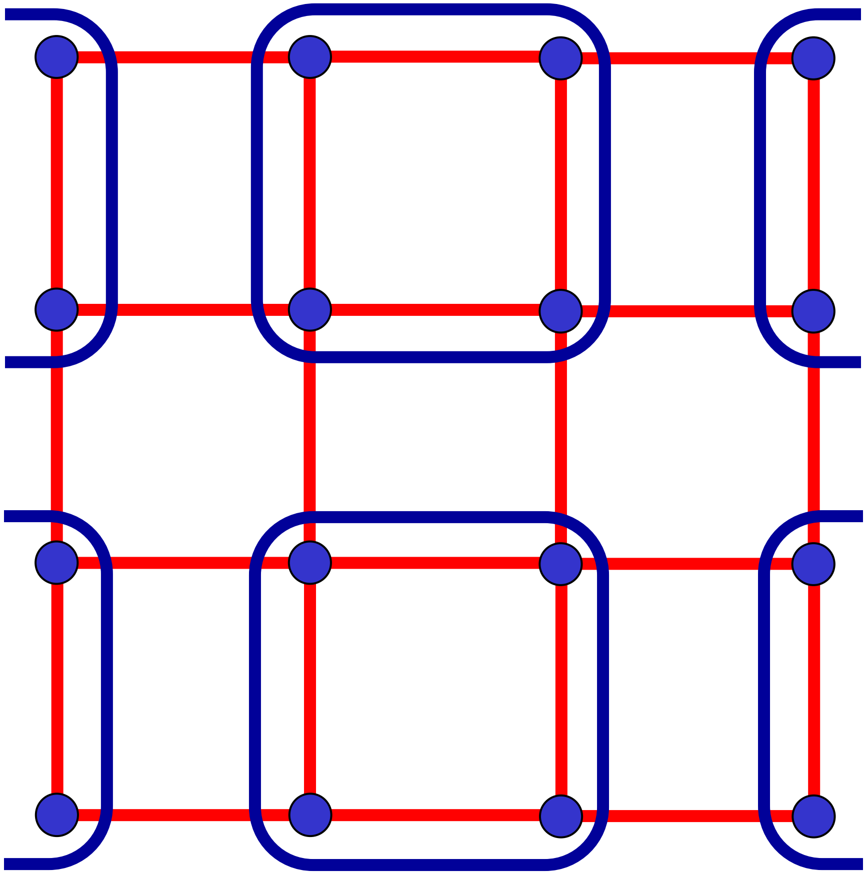
Mott insulator with one S=1/2 spin per unit cell



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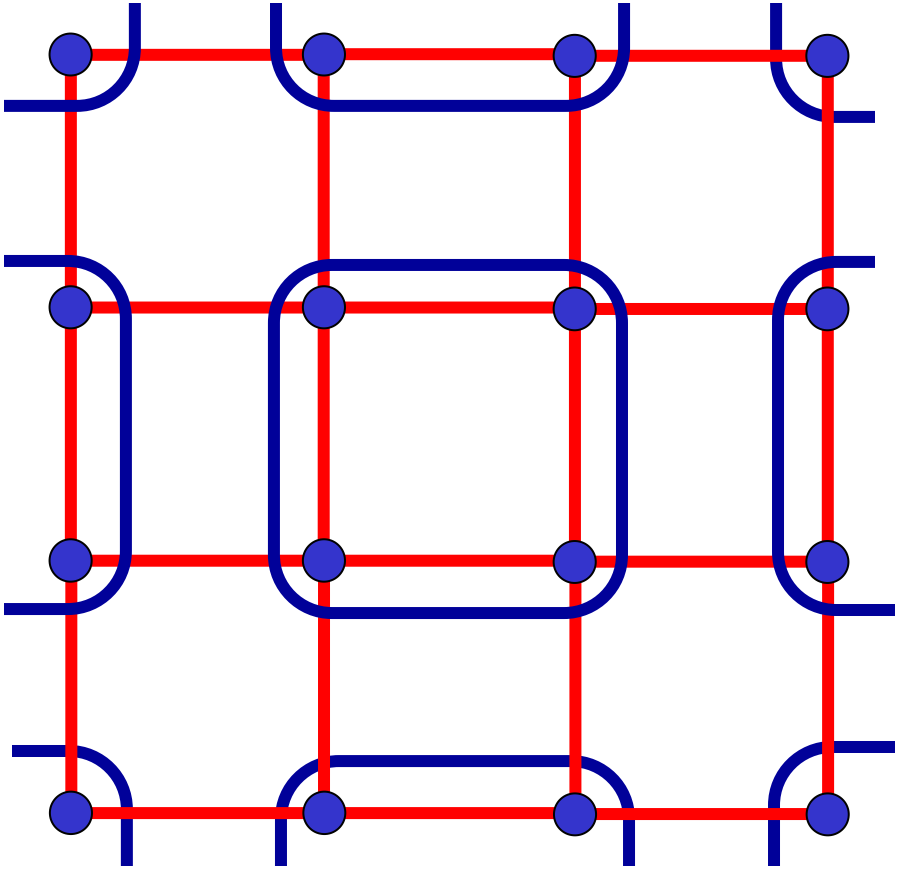

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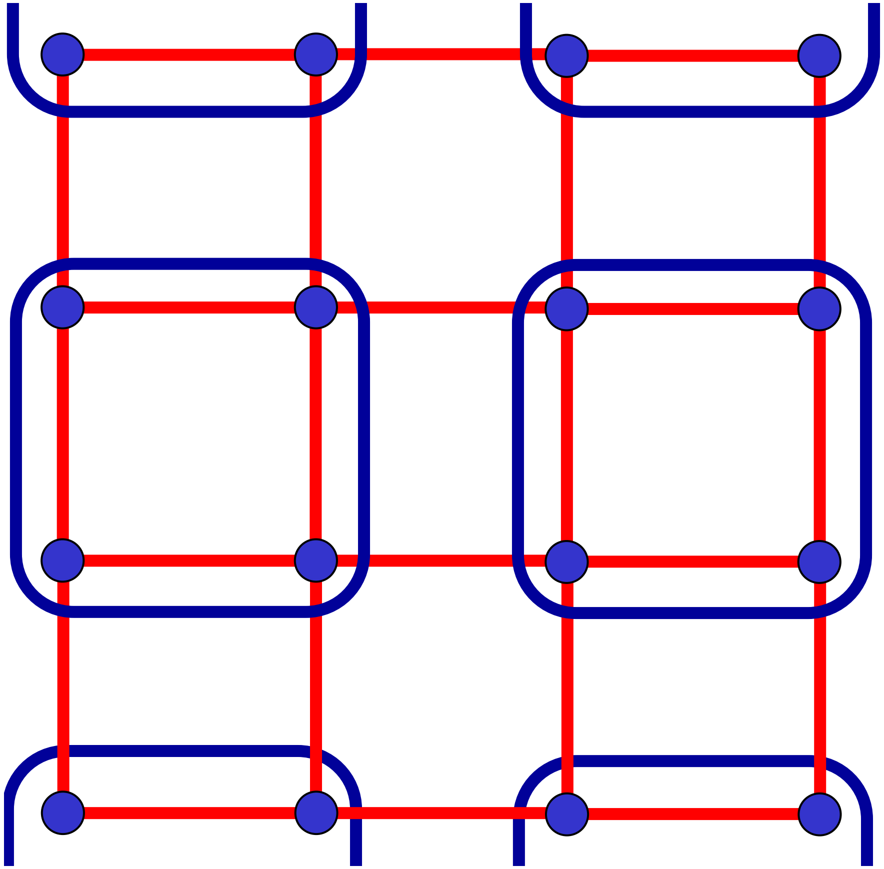

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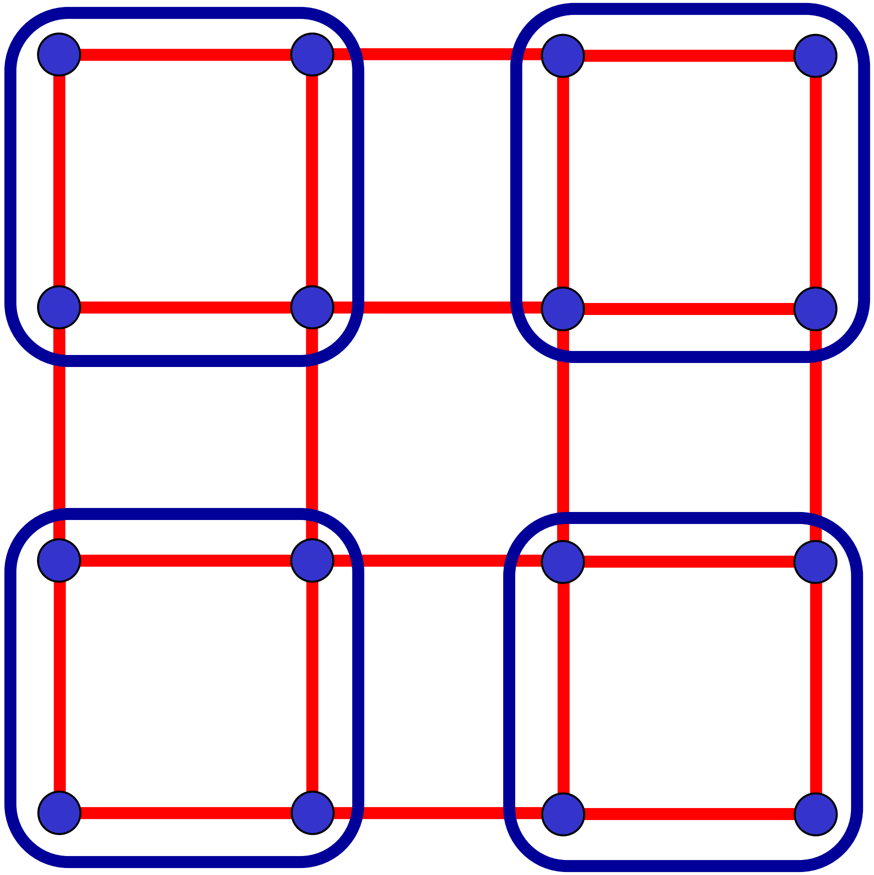

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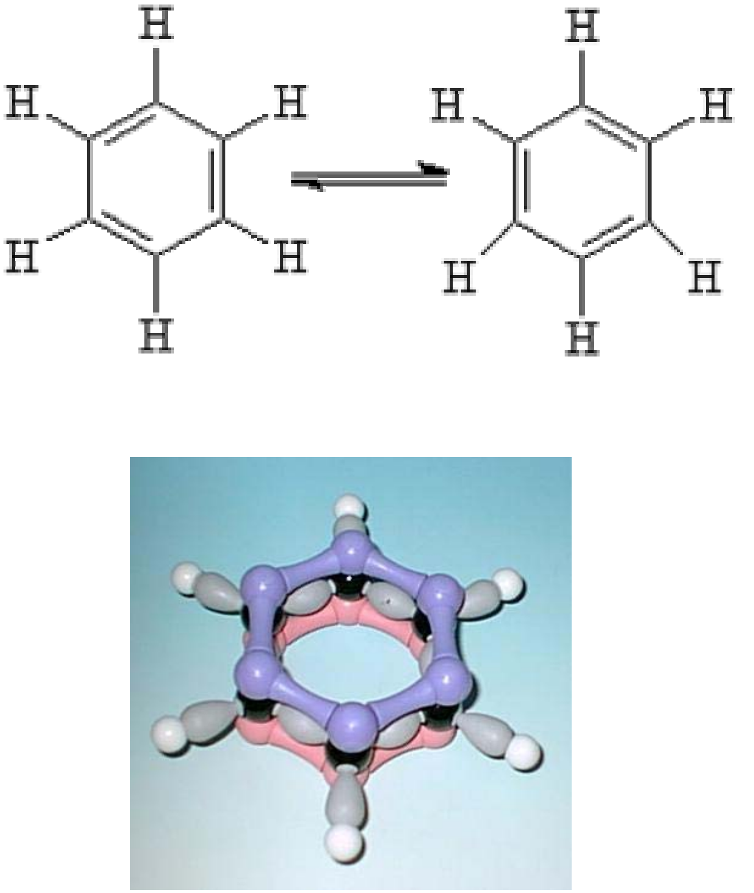
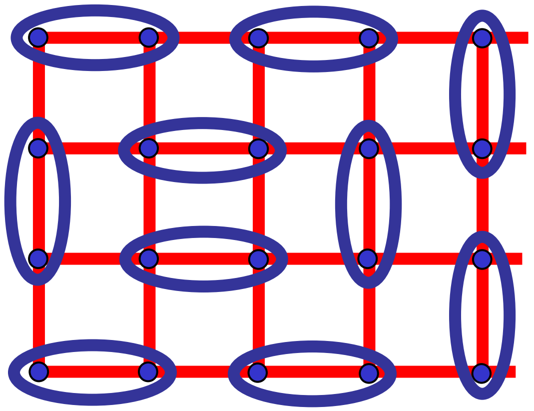
Resonating valence bonds



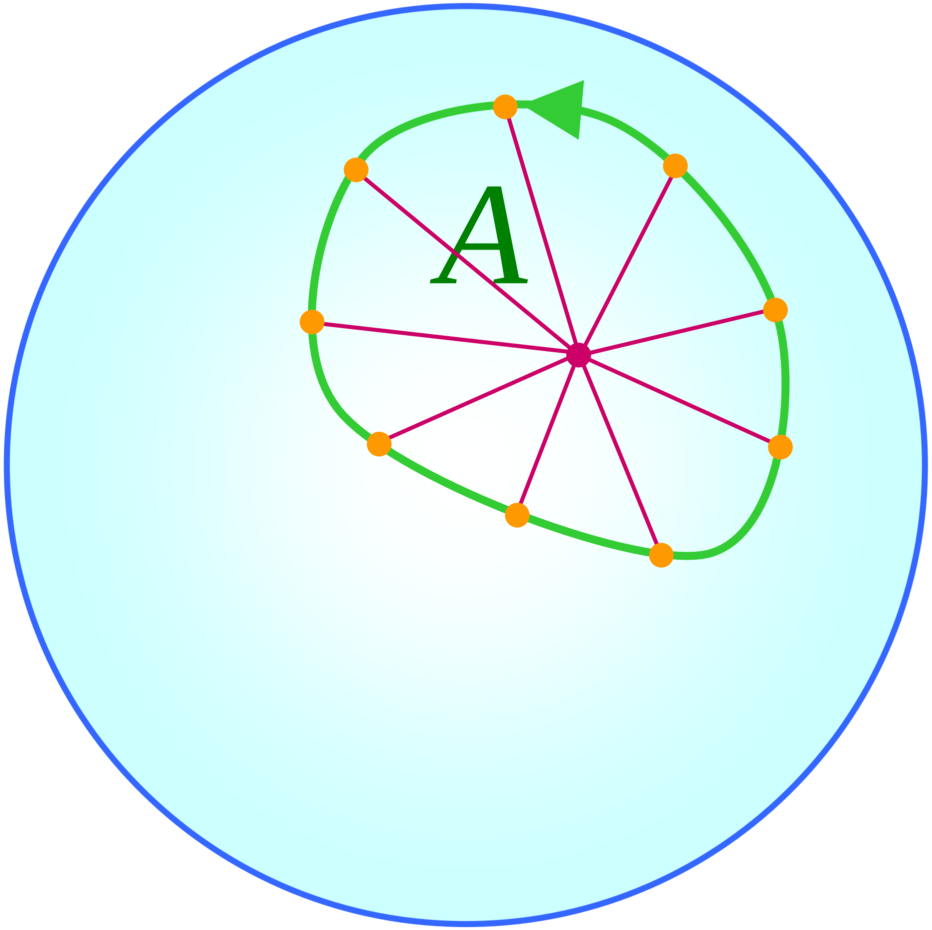
Resonance in benzene leads to a symmetric configuration of valence bonds
(*F. Kekulé, L. Pauling*)



The paramagnet on the square lattice should also allow other valence bond pairings, and this implies a “resonating valence bond liquid” with $\Psi=0$
(*P.W. Anderson, 1987*)

Quantum theory for destruction of Neel order

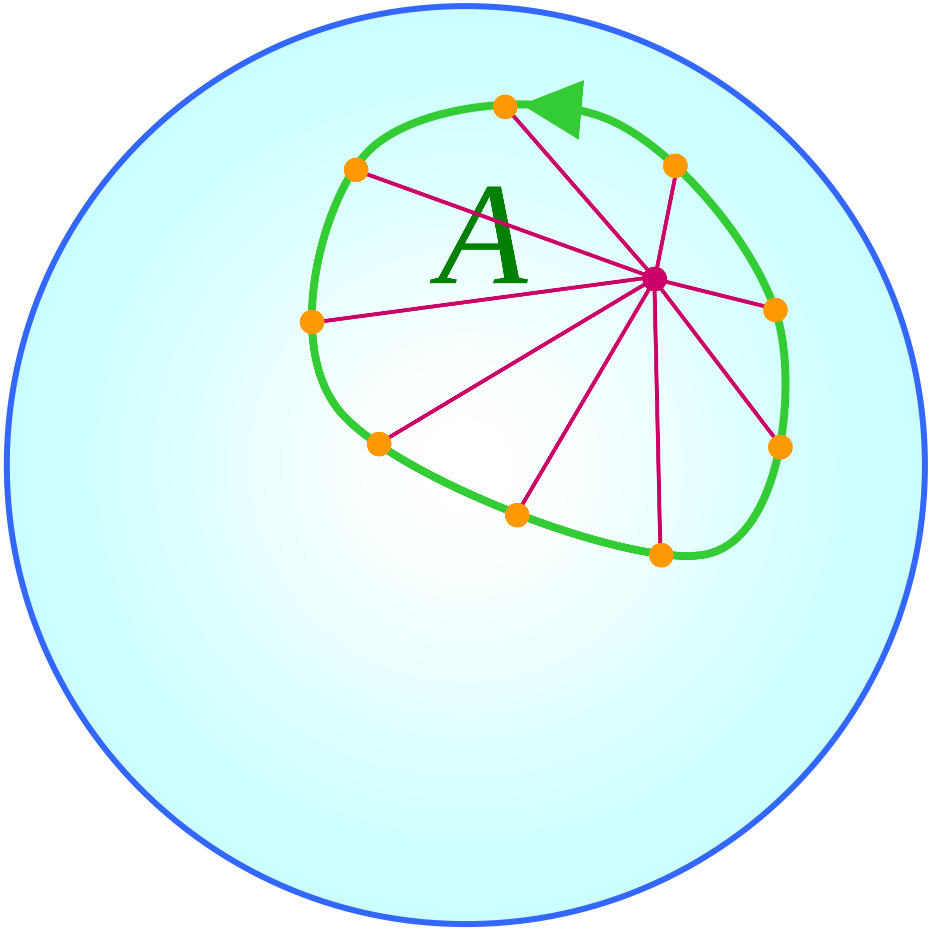
Ingredient missing from LGW theory: Spin Berry Phases



$$e^{iSA}$$

Quantum theory for destruction of Neel order

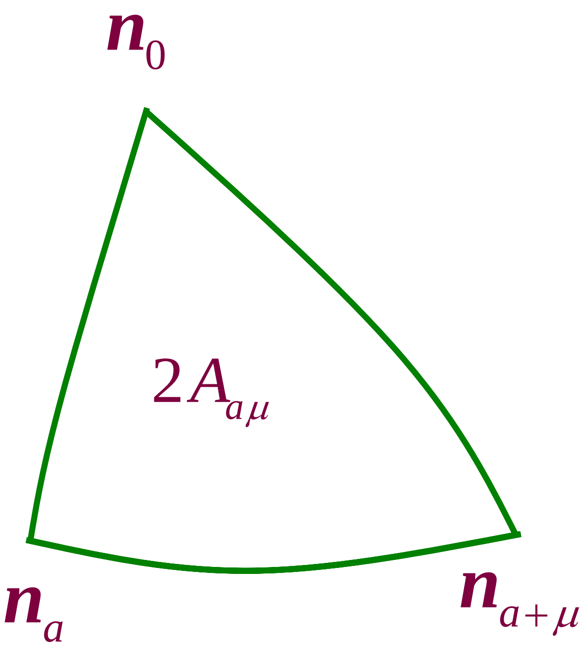
Ingredient missing from LGW theory: Spin Berry Phases



$$e^{iSA}$$

Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*



$\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$\eta_\mathrm{a} \rightarrow \pm 1$ on two square sublattices ;

$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

Quantum theory for destruction of Neel order

Partition function on cubic lattice

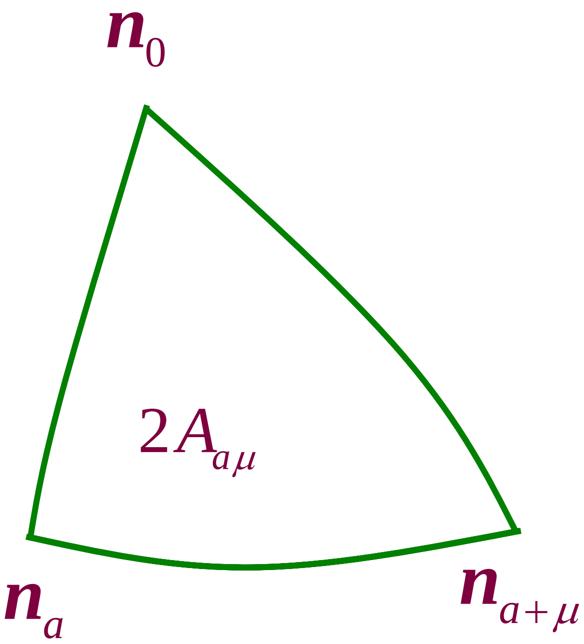
$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - i \sum_a \eta_a A_{a\tau} \right)$$

Modulus of weights in partition function: those of a classical ferromagnet at “temperature” *g*

Small $g \Rightarrow$ ground state has Neel order with $\left\langle \boldsymbol{n}_a \right\rangle = \vec{\varphi} \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\left\langle \boldsymbol{n}_a \right\rangle = 0$

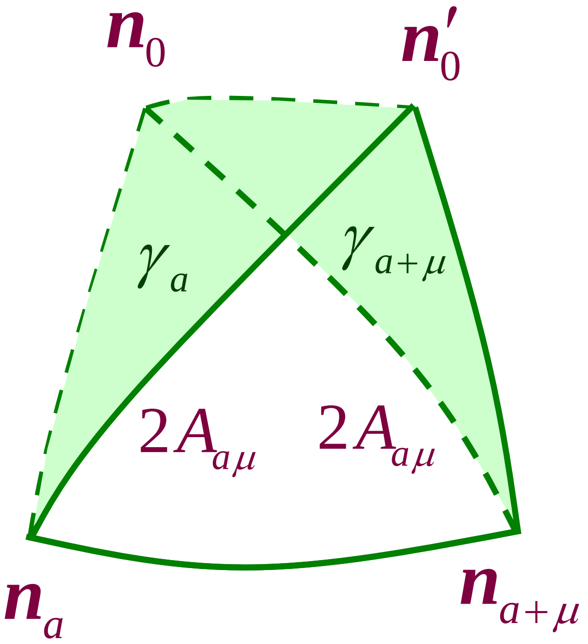
Berry phases lead to large cancellations between different time histories → need an effective action for $A_{a\mu}$ at large g


$\boldsymbol{n}_0$
$2A_{a\mu}$
$\boldsymbol{n}_a$
$\boldsymbol{n}_{a+\mu}$

Change in choice of $\boldsymbol{n}_0$ is like a “gauge transformation”

$$2A_{a\mu} \to 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{n}_a$ and the two choices for $\boldsymbol{n}_0$).



The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \to A_{a\mu} + 2\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$ which provides description of the large g phase

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) - i \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 3 spacetime dimensions with static charges ± 1 on two sublattices.

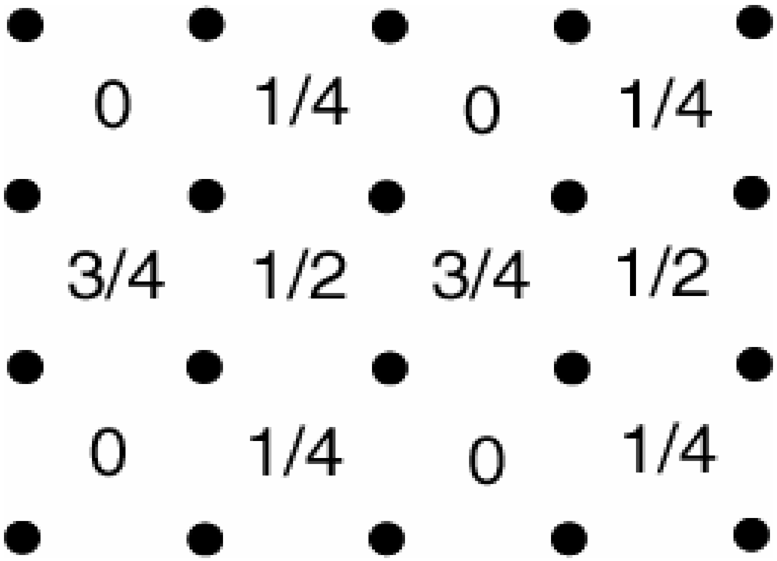
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
S. Sachdev and K. Park, *Annals of Physics* **298**, 58 (2002).

Exact duality transform on a periodic Gaussian ("Villain") action for compact QED + Berry phases leads to a representation in terms of a "height" model

$$Z_{\text{dual}} = \sum_{\{h_{\bar\jmath}\}} \exp\left(-\frac{e^2}{2} \sum_{\bar\jmath} \left(\Delta_\mu h_{\bar\jmath} - \Delta_\mu \mathcal{X}_{\bar\jmath} \right)^2 \right)$$

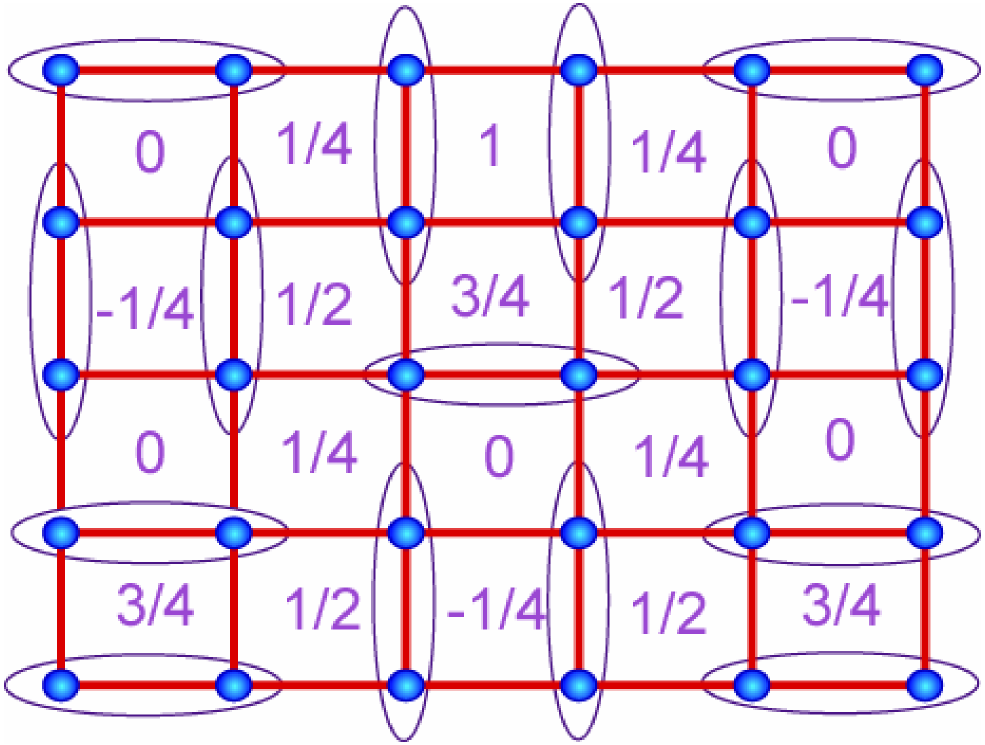
with the $h_{\bar\jmath}$ integer heights.

The Berry phases lead to height 'offsets' $\mathcal{X}_{\bar\jmath} = 0, 1/4, 1/2, 3/4$ on the four dual sublattices.



N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

For large e^2 , low energy height configurations are in exact one-to-one correspondence with nearest-neighbor valence bond pairings of the sites square lattice

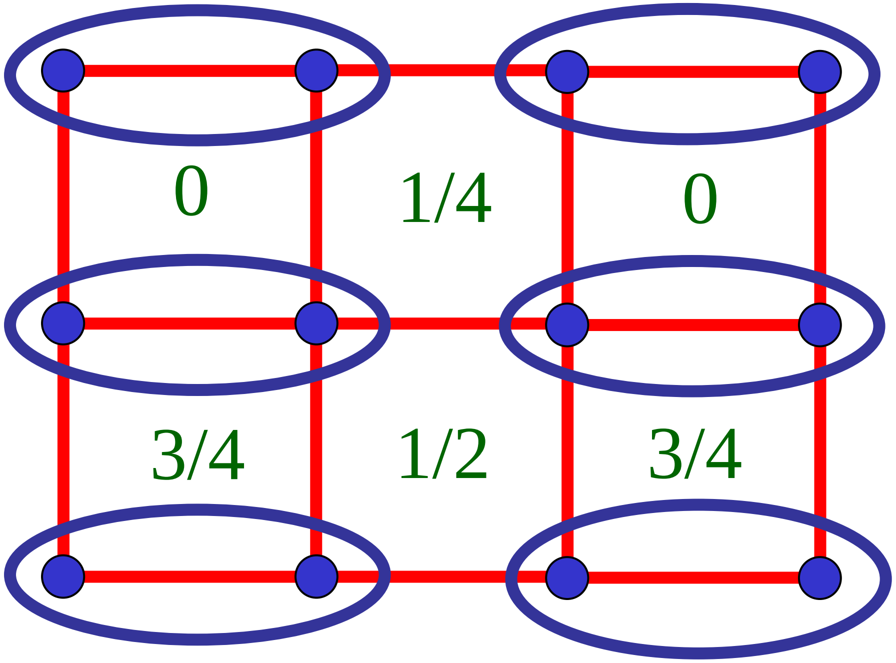


There is no roughening transition for three dimensional interfaces, which are smooth for all couplings

$\Longrightarrow$ There is a definite average height of the interface

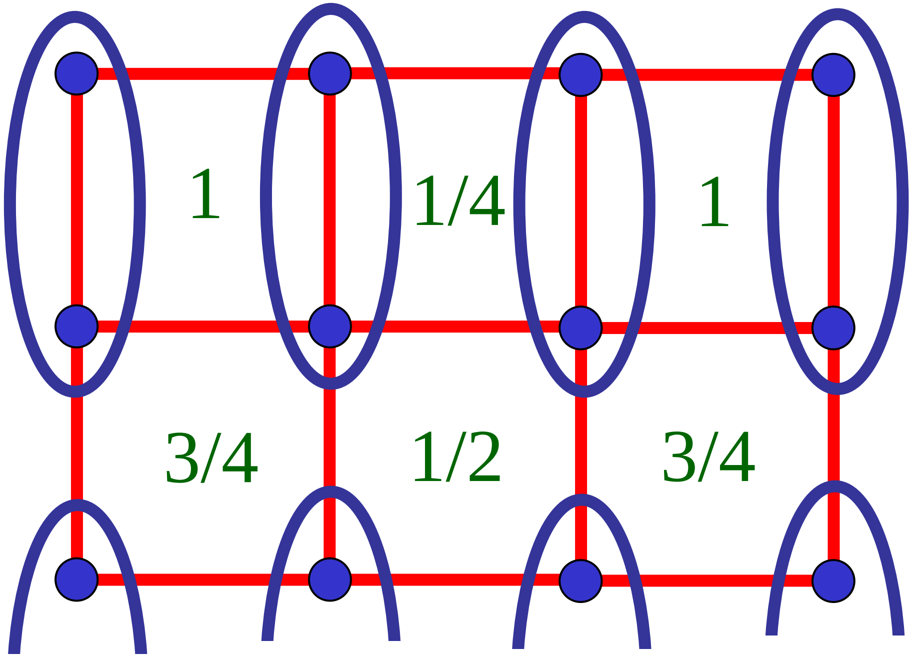
$\Longrightarrow$ **Ground state has bond order.**

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).



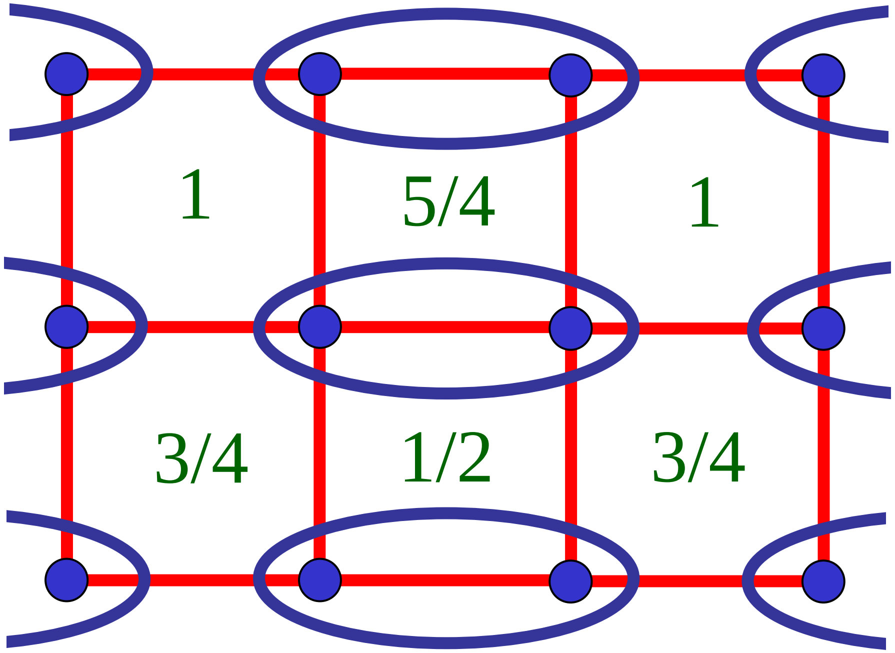
Smooth interface with average height 3/8

W. Zheng and S. Sachdev, *Phys. Rev*. B **40**, 2704 (1989)



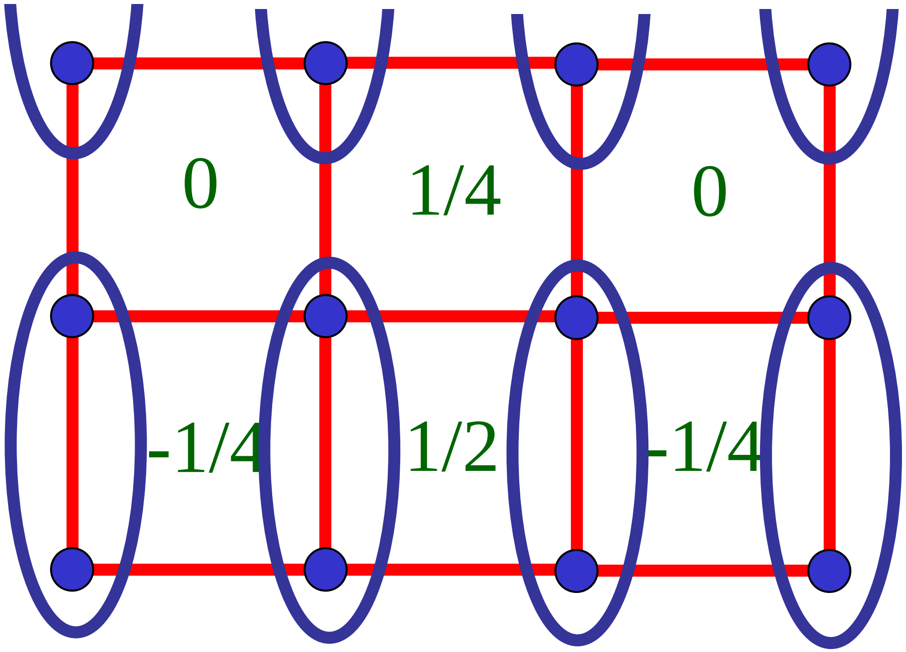
Smooth interface with average height 5/8

W. Zheng and S. Sachdev, *Phys. Rev*. B **40**, 2704 (1989)



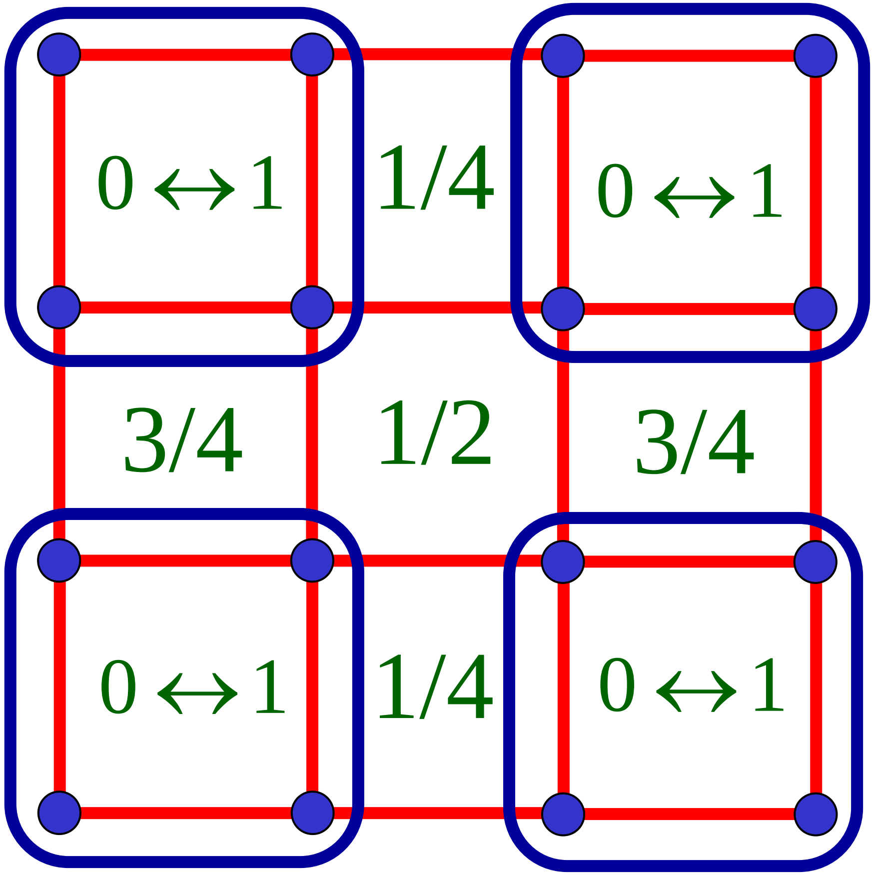
Smooth interface with average height 7/8

W. Zheng and S. Sachdev, *Phys. Rev*. B **40**, 2704 (1989)



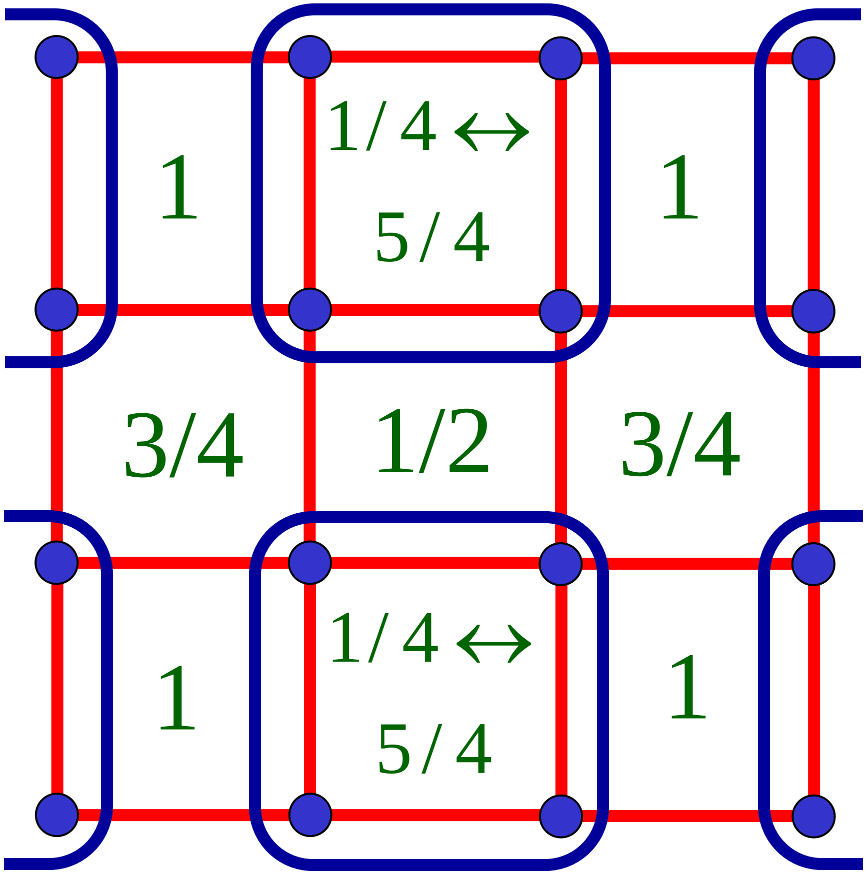
Smooth interface with average height 1/8

W. Zheng and S. Sachdev, *Phys. Rev*. B **40**, 2704 (1989)



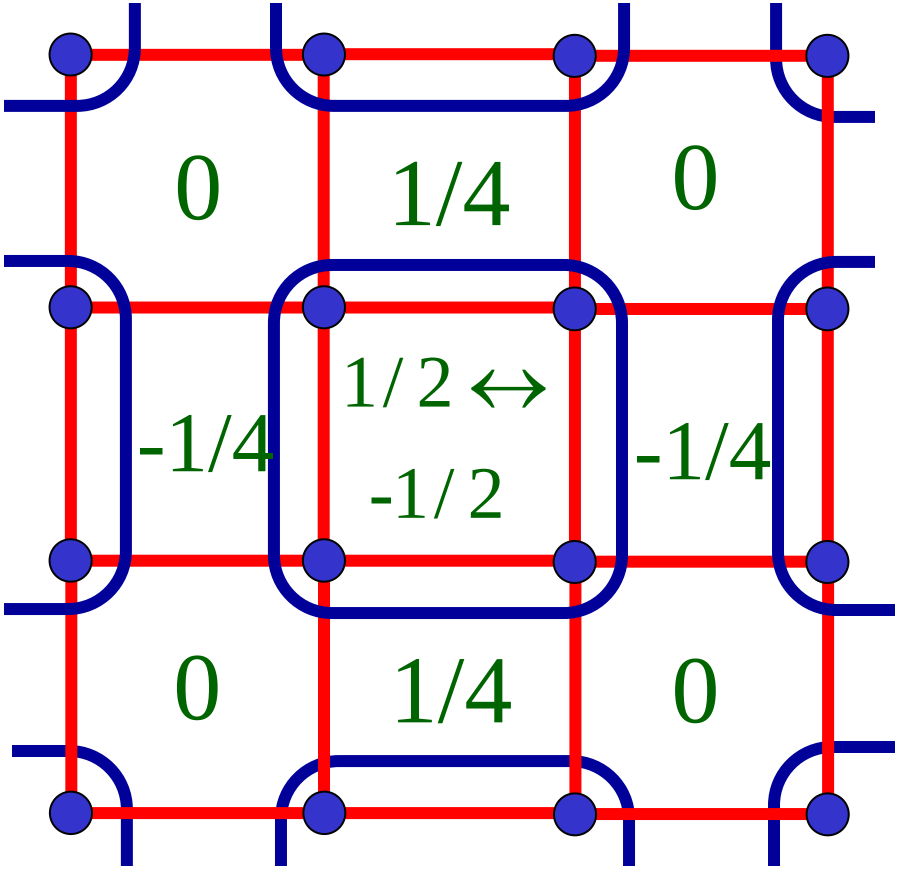
"Disordered-flat" interface with average height 1/2

W. Zheng and S. Sachdev, *Phys. Rev*. B **40**, 2704 (1989)



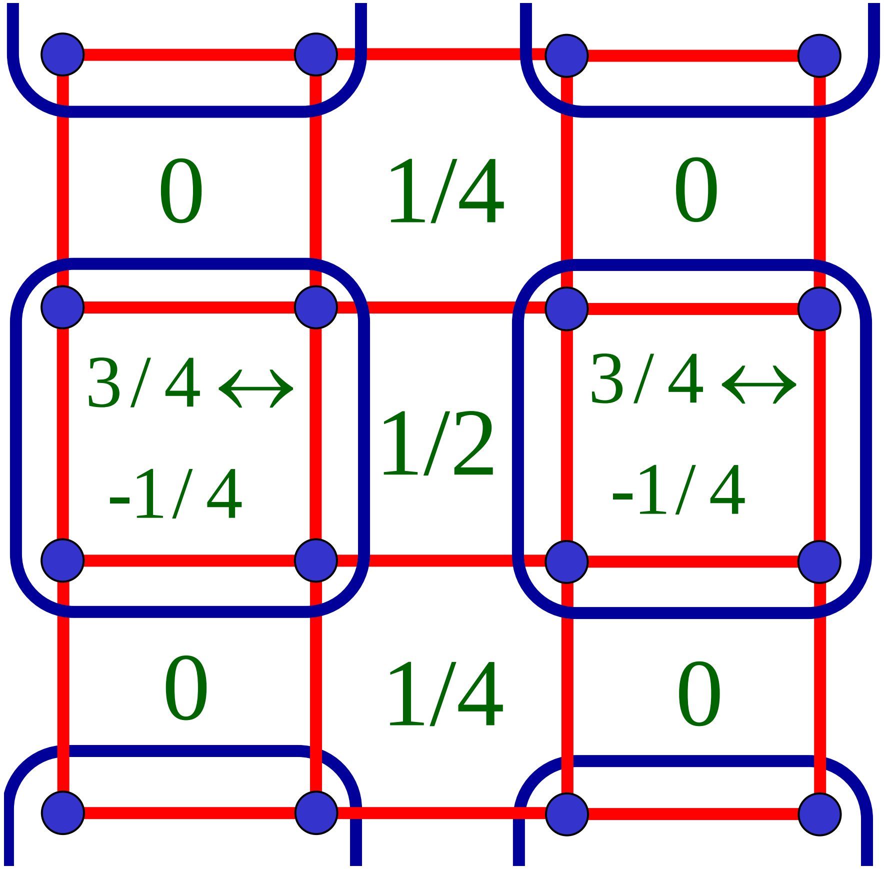
“Disordered-flat” interface with average height 3/4

W. Zheng and S. Sachdev, *Phys. Rev*. B **40**, 2704 (1989)



“Disordered-flat” interface with average height 0

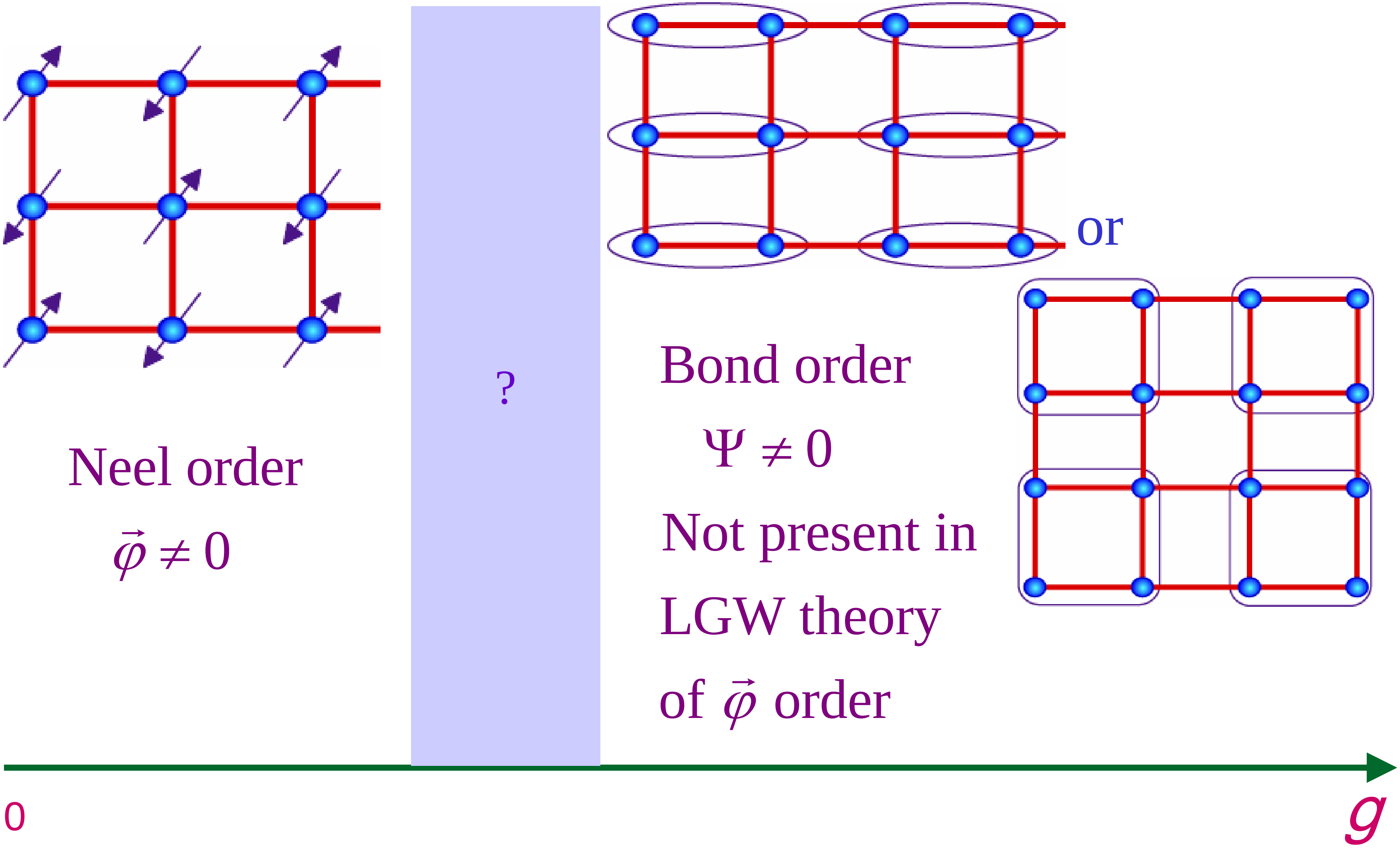
W. Zheng and S. Sachdev, *Phys. Rev*. B **40**, 2704 (1989)



“Disordered-flat” interface with average height 1/4

W. Zheng and S. Sachdev, *Phys. Rev*. B **40**, 2704 (1989)

$$Z = \prod_a \int d\mathbf{n}_a \delta\left(\mathbf{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \mathbf{n}_a \cdot \mathbf{n}_{a+\mu} - i \sum_a \eta_a A_{a\tau} \right)$$



Neel order

$\vec{\varphi} \neq 0$

?

or

Bond order

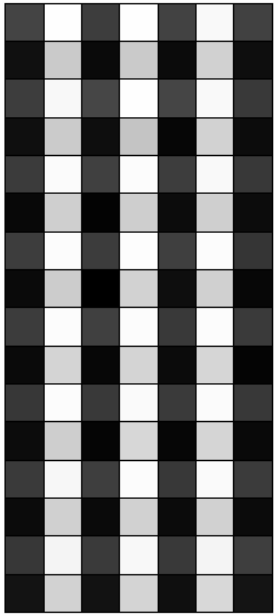
$\Psi \neq 0$

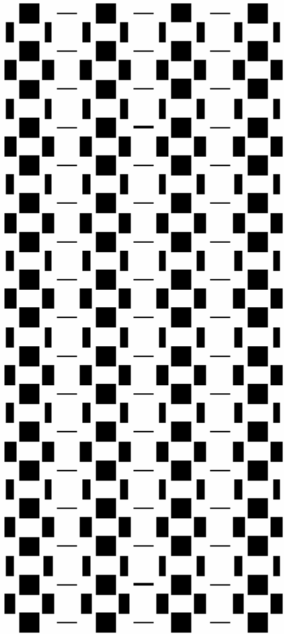
Not present in LGW theory of $\vec{\varphi}$ order

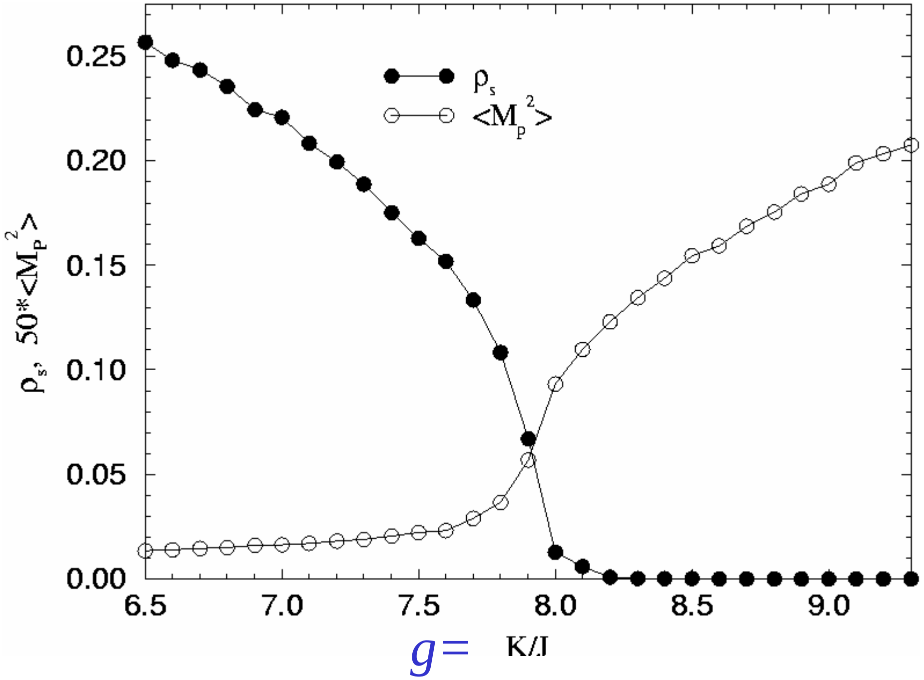
Bond order in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett*. **89**, 247201 (2002)

First *large scale* numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry

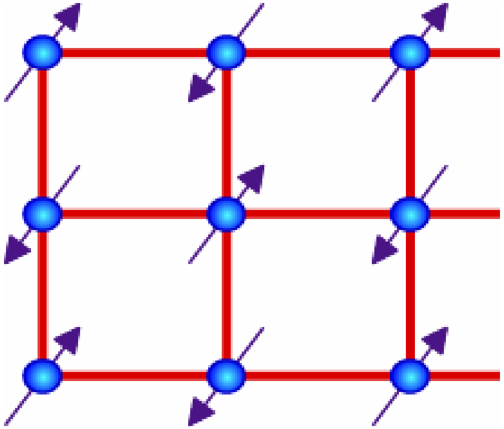






$$H = 2J\sum_{\langle ij\rangle}\left(S_i^x S_j^x + S_i^y S_j^y\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+\right)$$

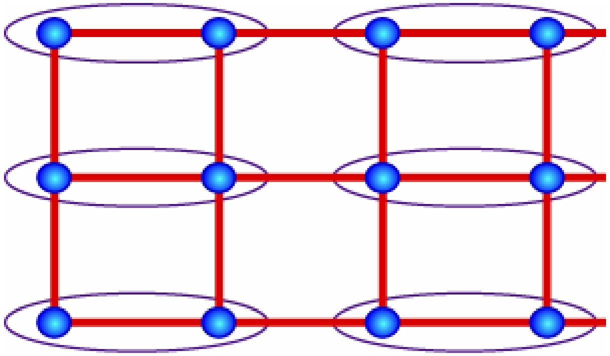
$$Z=\prod_a\int d\boldsymbol{n}_a\,\delta\left(\boldsymbol{n}_a^2-1\right)\exp\left(\frac{1}{g}\sum_{a,\mu}\boldsymbol{n}_a\cdot\boldsymbol{n}_{a+\mu}-i\sum_a\eta_a A_{a\tau}\right)$$



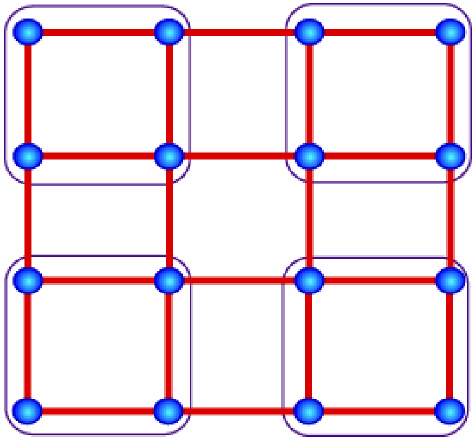
Neel order

$\vec{\varphi}\neq 0$

?



or



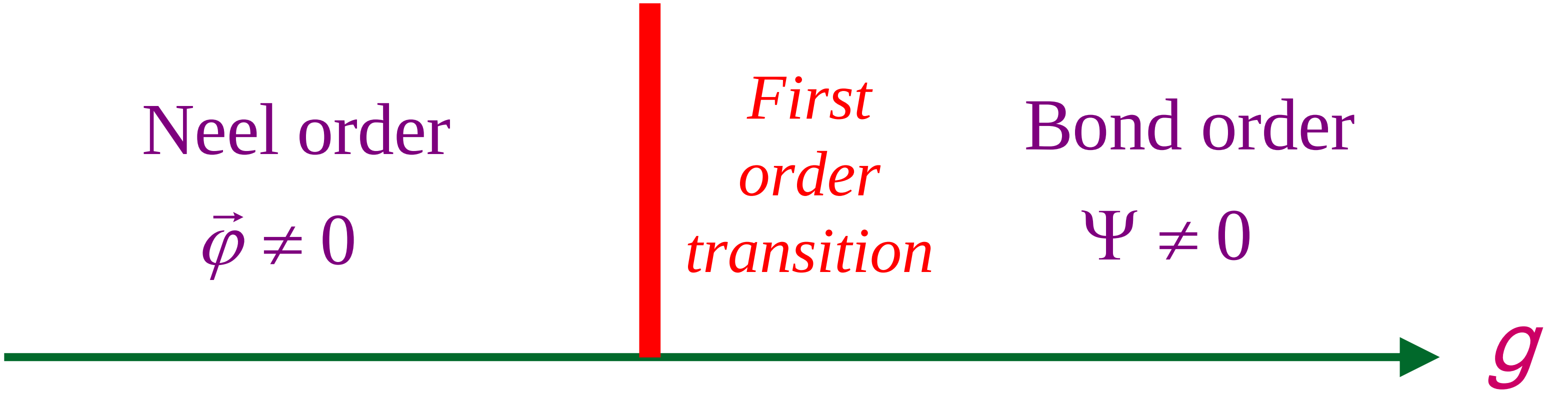
Bond order

$\Psi\neq 0$

Not present in LGW theory of $\vec{\varphi}$ order

0 $\longrightarrow$ g

Naïve approach: add bond order parameter to LGW theory “by hand”



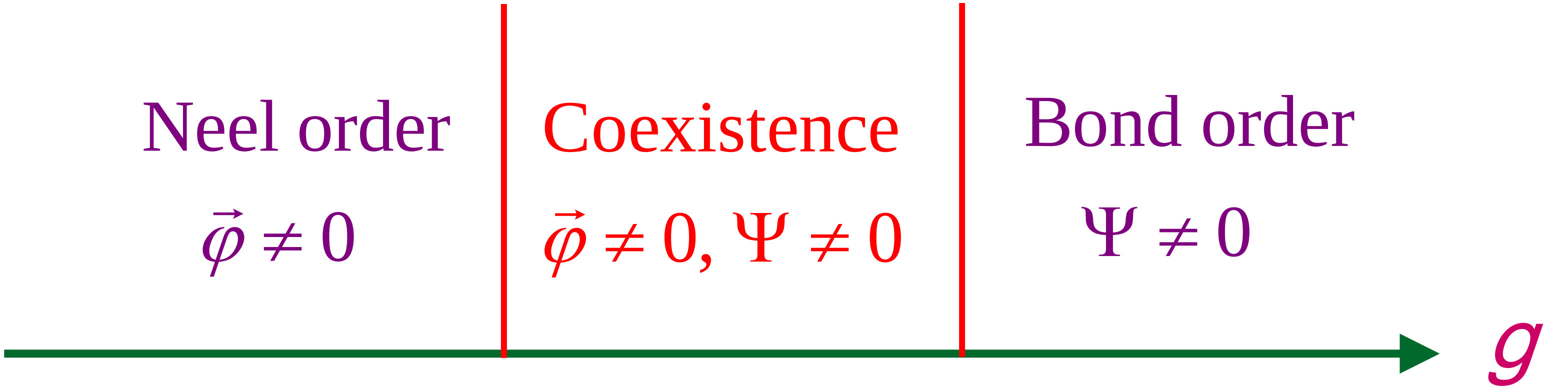
Neel order

$\vec{\varphi} \neq 0$

First order transition

Bond order

$\Psi \neq 0$



Neel order

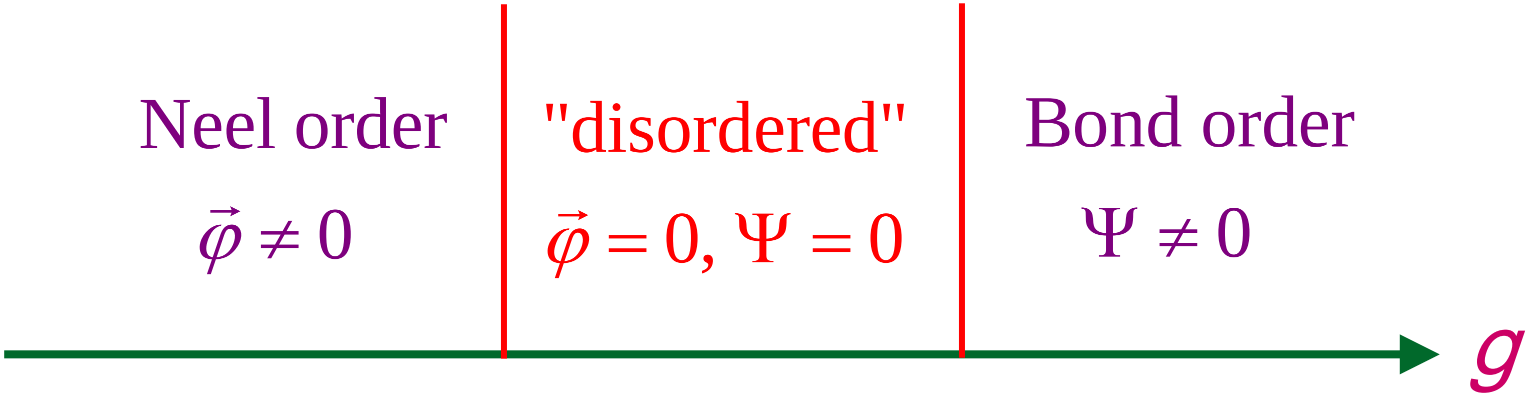
$\vec{\varphi} \neq 0$

Coexistence

$\vec{\varphi} \neq 0, \Psi \neq 0$

Bond order

$\Psi \neq 0$



Neel order

$\vec{\varphi} \neq 0$

"disordered"

$\vec{\varphi} = 0, \Psi = 0$

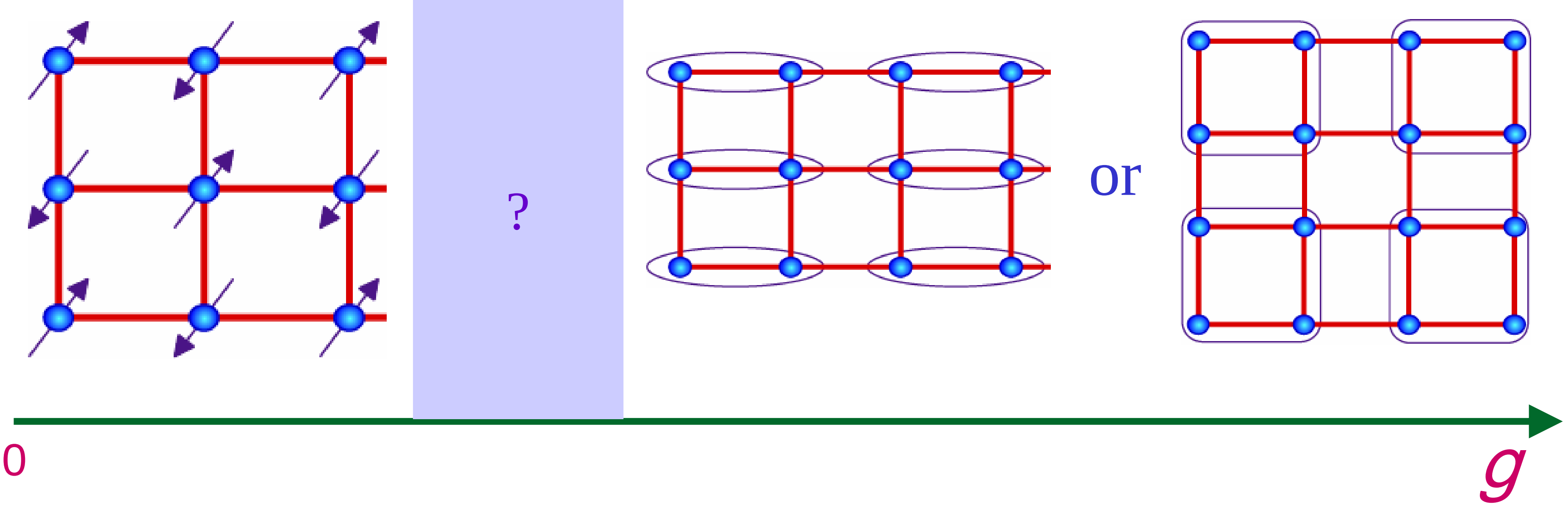
Bond order

$\Psi \neq 0$

Alternative formulation to describe transition:

Express theory in terms of a complex spinor $z_{a\alpha}$, $\alpha = \uparrow, \downarrow$, with

$$
\begin{aligned}
\mathbf{n}_a &= z^*_{a\alpha} \sigma_{\alpha\beta} z_{a\beta} \\
Z &= \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(|z_{a\alpha}|^2 - 1\right) \\
&\qquad \exp\left(\frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + \text{c.c.} + i \sum_a \eta_a A_{a\tau} \right)
\end{aligned}
$$



S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
S. Sachdev and K. Park, *Annals of Physics* **298**, 58 (2002).

Theory of a second-order quantum phase transition between Neel and bond-ordered phases

At the quantum critical point:

- Dual interface becomes *rough*.
- A_μ gauge field is effectively non-compact (monopoles are *dangerously irrelevant*)

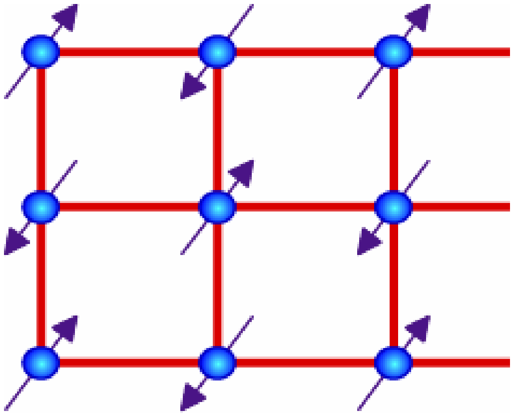
$\Rightarrow$ Total gauge flux is conserved.

- Fractionalized $S=1/2$ z_α are globally propagating degrees of freedom.

Second-order critical point described by emergent fractionalized degrees of freedom (A_μ and z_α); Order parameters (φ and Ψ) are "composites" and of secondary importance

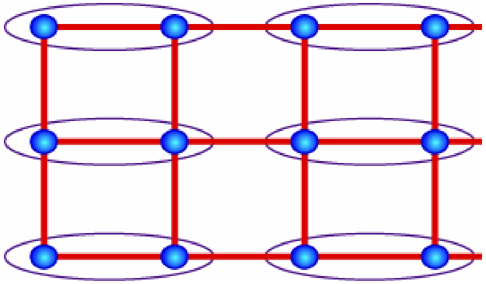
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science, March 5, 2004*

Phase diagram of S=1/2 square lattice antiferromagnet

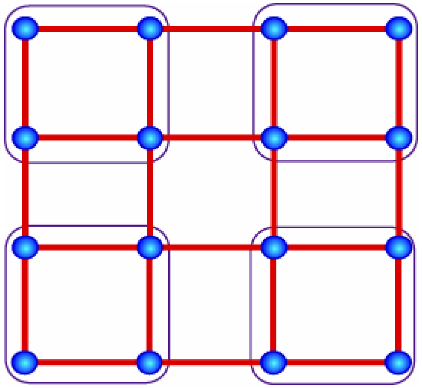


Neel order

$\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \neq 0$



or



Bond order $\Psi \neq 0$,

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

Second-order critical point described by

$$\mathcal{S}_{\mathrm{critical}} = \int d^2 x d\tau \left[\left| (\partial_\mu - i A_\mu) z_\alpha \right|^2 + r \left| z_\alpha \right|^2 + \frac{u}{2} \left(|z_\alpha|^2 \right)^2 + \frac{1}{4e^2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 \right]$$

where $\mathbf{n} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$, z_α are bosonic spinors, and A_μ is *non-compact*

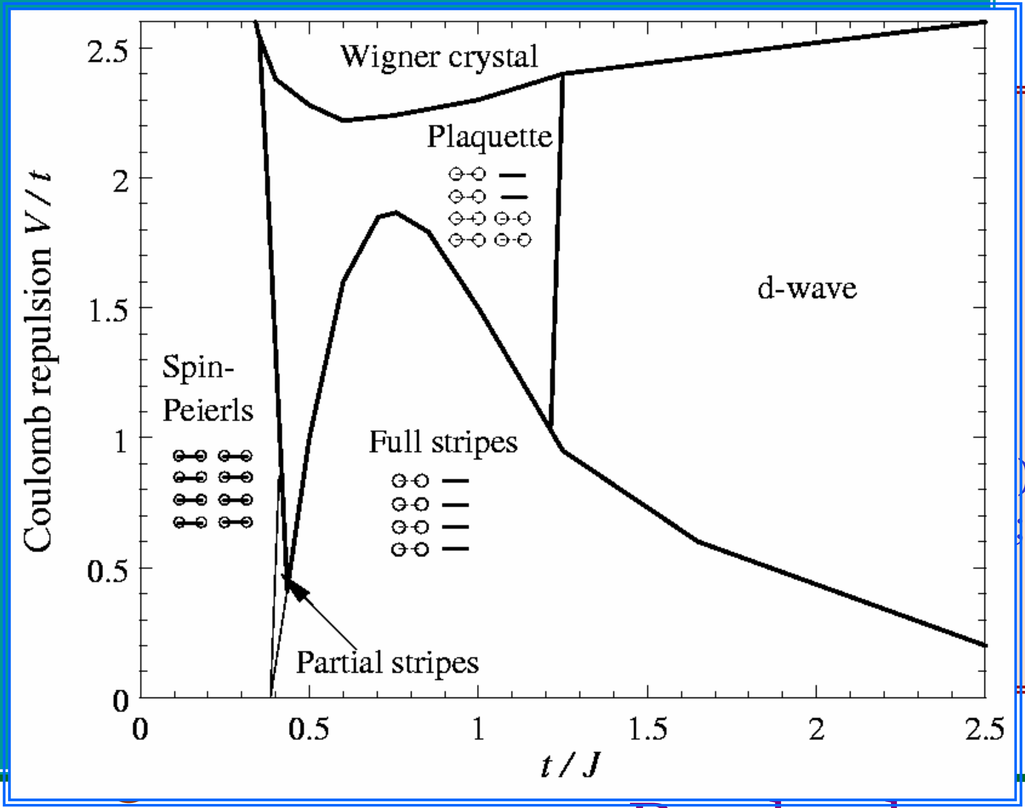
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science, March 5, 2004*

Cuprate superconductors:
Competing orders and recent experiments

Main idea: *one* of the effects of doping mobile carriers is to increase the value of g

δ

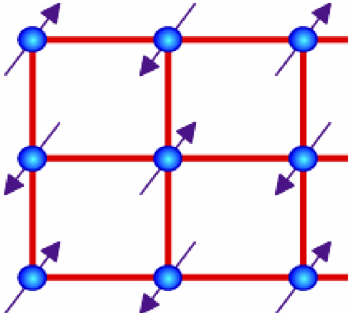
Magnetic,
bond
and super-
conducting
order



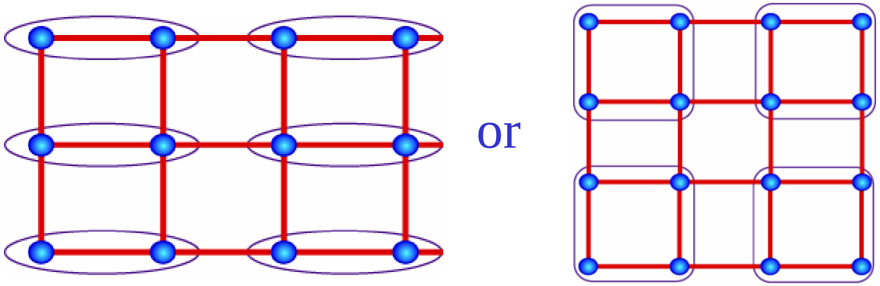
g

La$_2$CuO$_4$

Neel order

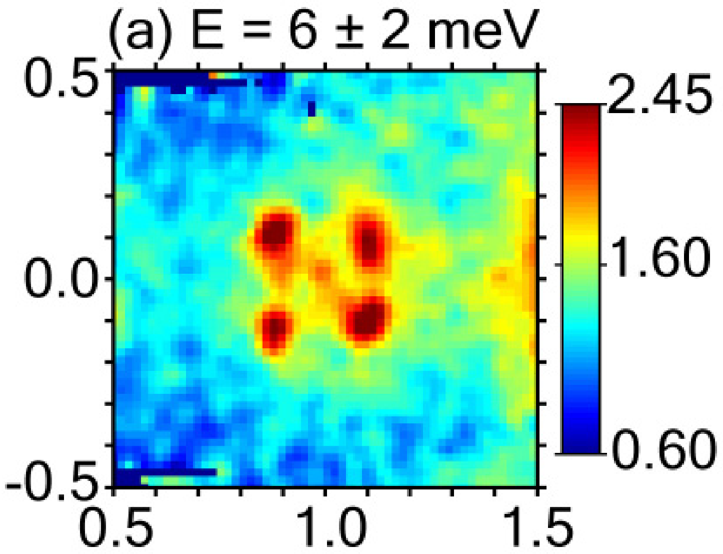


Bond order

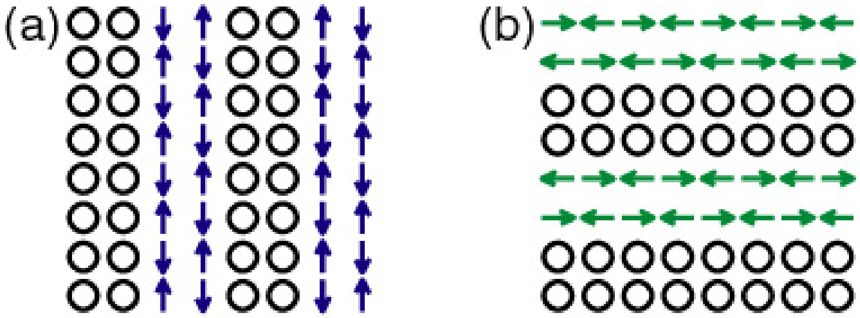


Neutron scattering measurements of $La_{1.875}Ba_{0.125}CuO_4$ (Zurich oxide)

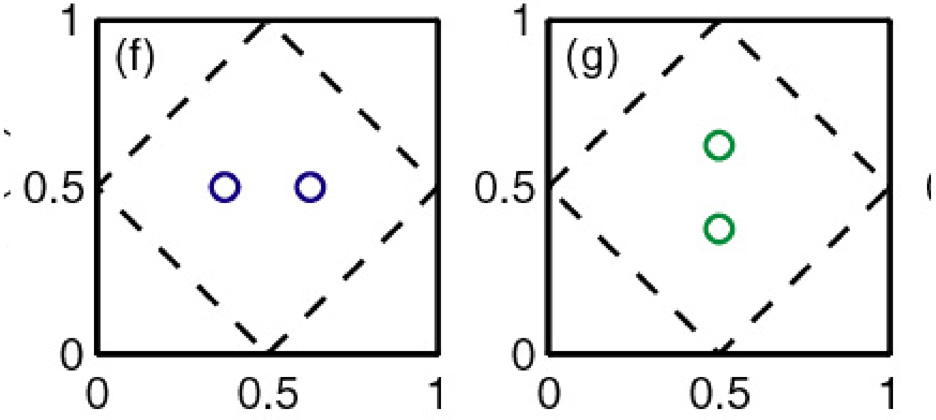
J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



Spin density wave of
8 lattice spacings
along the principal
square lattice axes



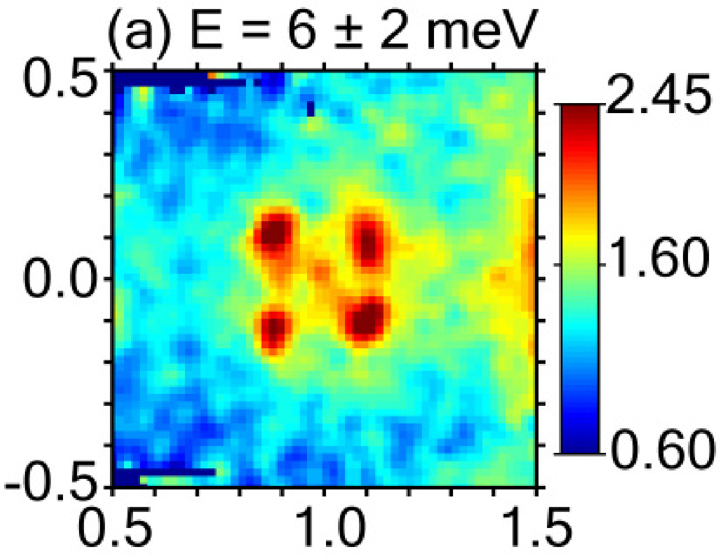
Possible microscopic picture



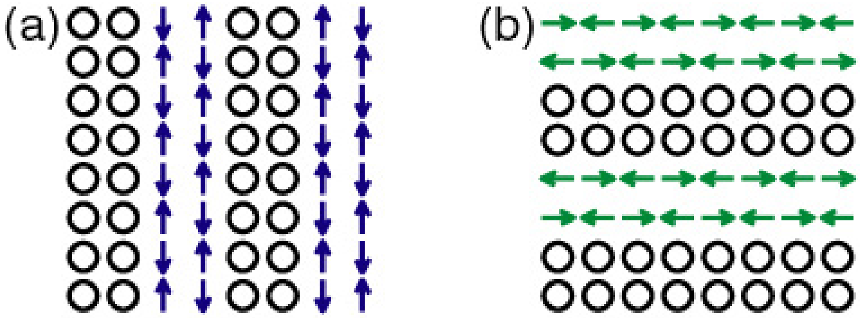
Bragg diffraction off static spin order

Neutron scattering measurements of $La_{1.875}Ba_{0.125}CuO_4$ (Zurich oxide)

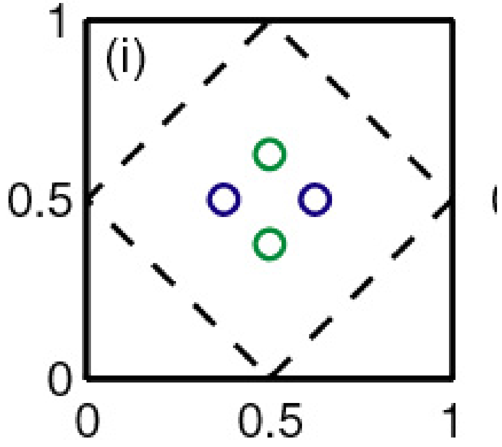
J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



Spin density wave of 8 lattice spacings along the principal square lattice axes



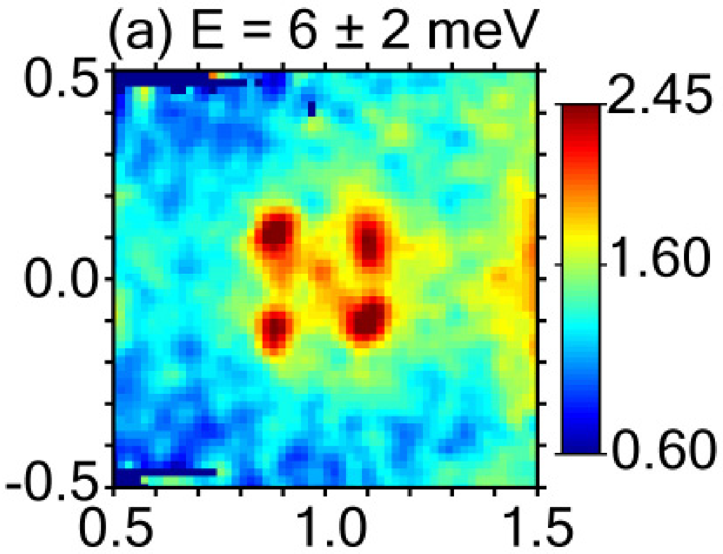
Possible microscopic picture



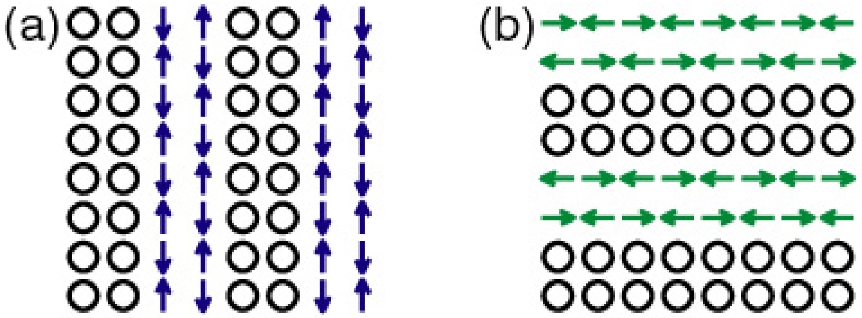
Bragg diffraction off static spin order with multiple domains

Neutron scattering measurements of $La_{1.875}Ba_{0.125}CuO_4$ (Zurich oxide)

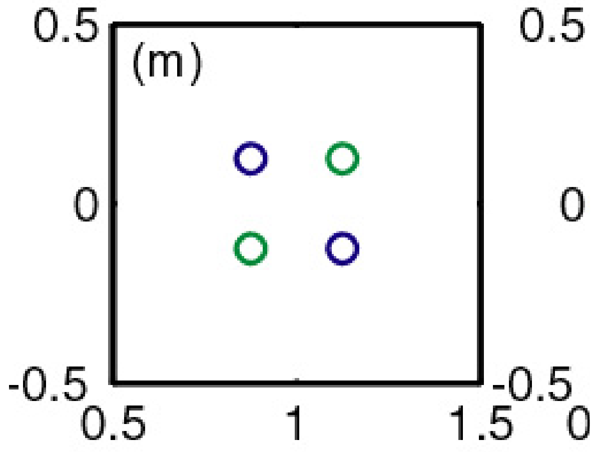
J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



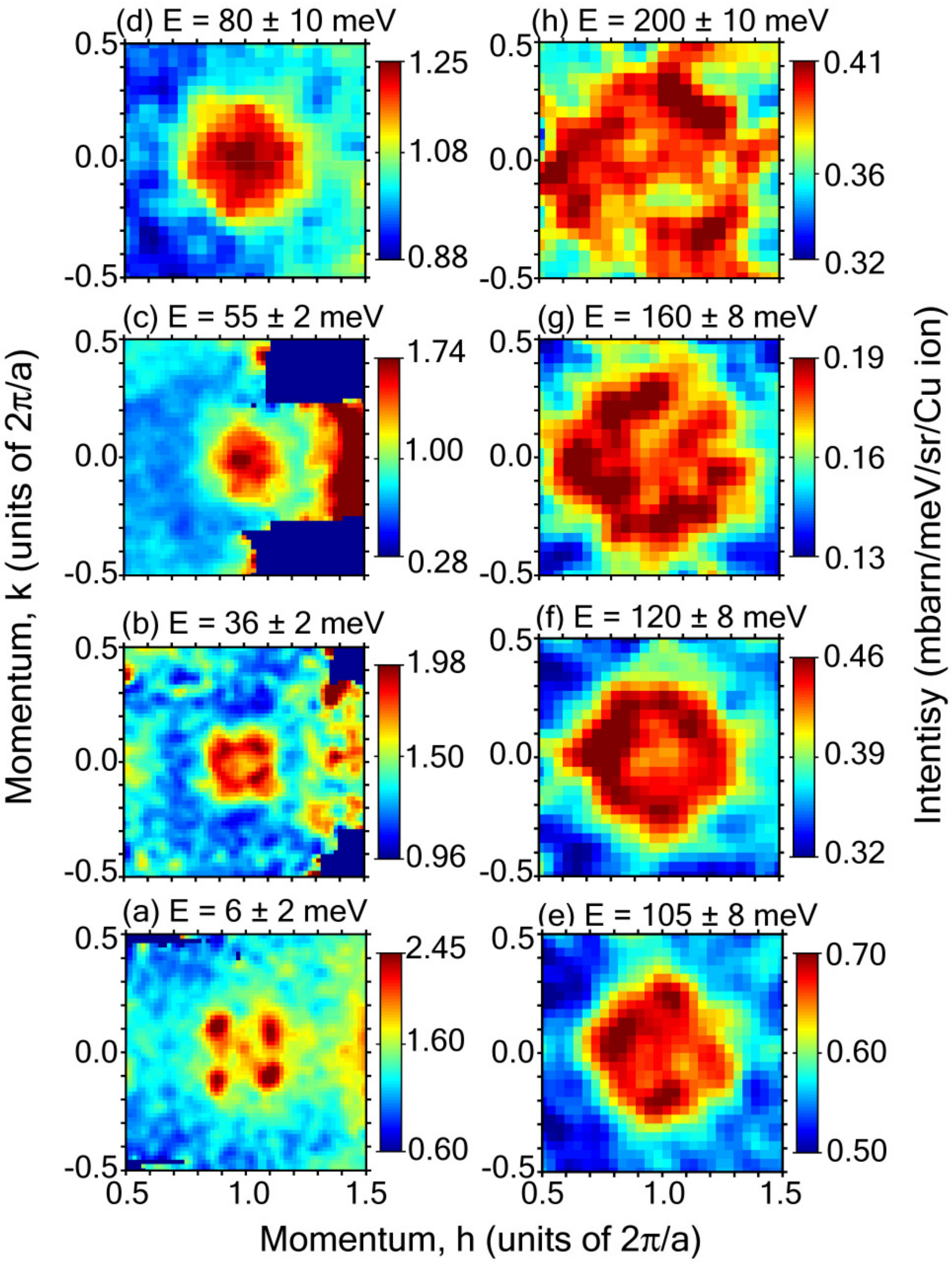
Spin density wave of
8 lattice spacings
along the principal
square lattice axes

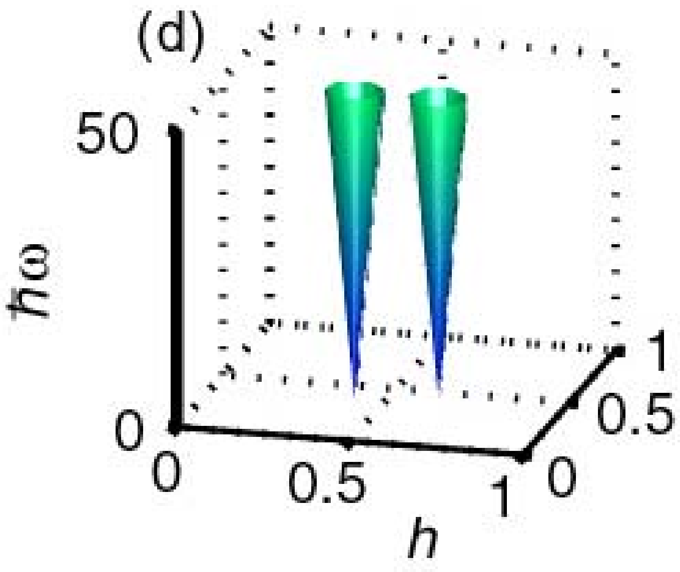


Possible microscopic picture



Bragg diffraction off static spin order
with multiple domains (after rotation by 45°)





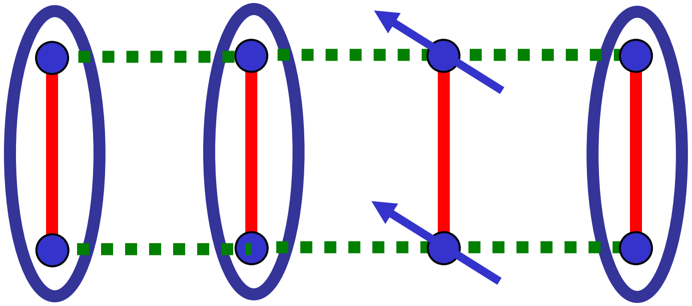
At higher energies, expect “spin-wave cones”.

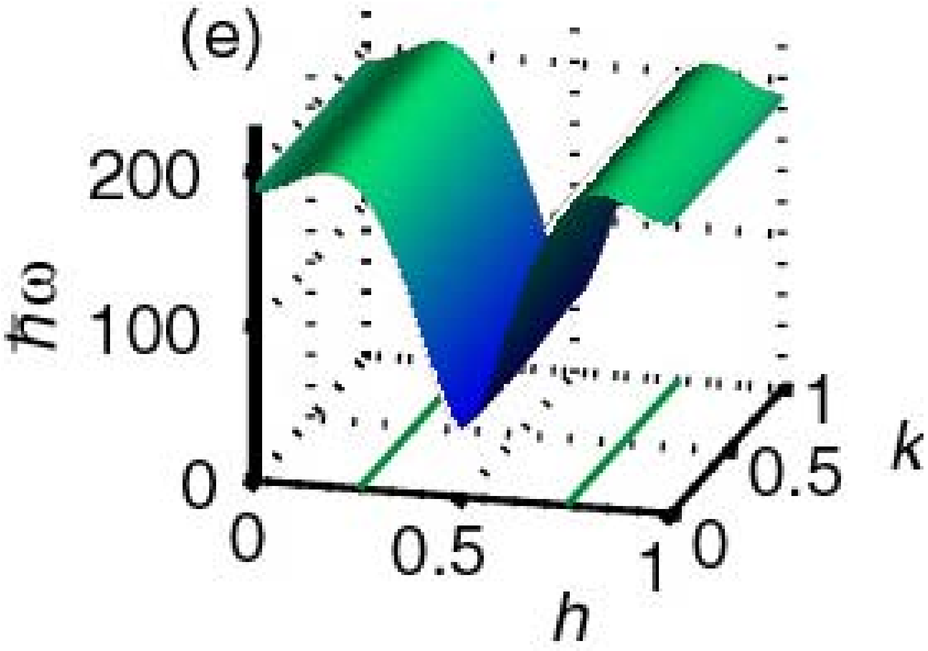
Only seen at relatively low energies.

Proposal of J. M. Tranquada *et al.,* cond-mat/0401621

High energy spectrum is the triplon excitation of two-leg spin ladders

⇒ *presence of bond order*

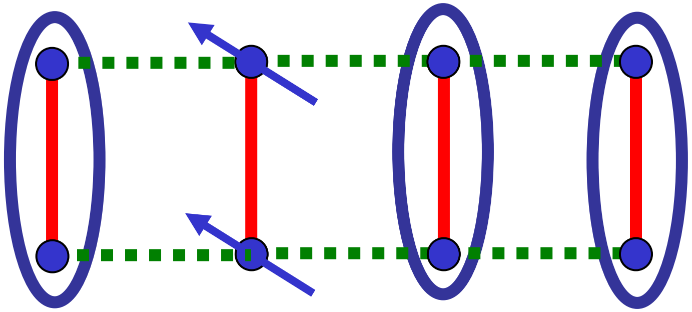


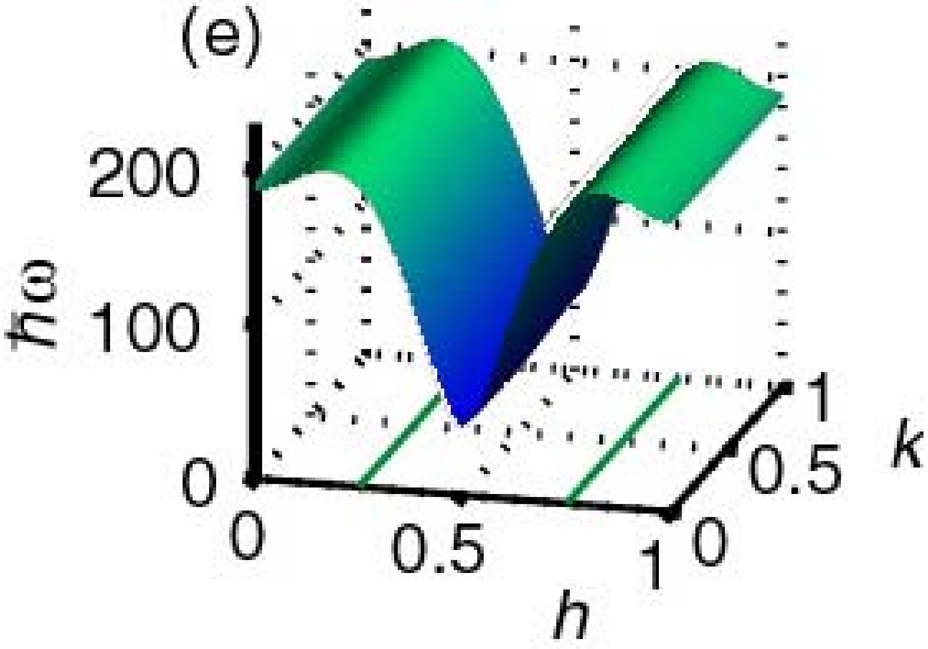


Proposal of J. M. Tranquada *et al.*, cond-mat/0401621

High energy spectrum is the triplon excitation of two-leg spin ladders

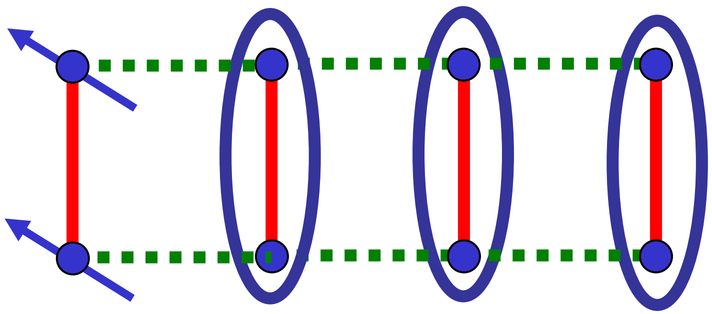
$\Rightarrow$ *presence of bond order*

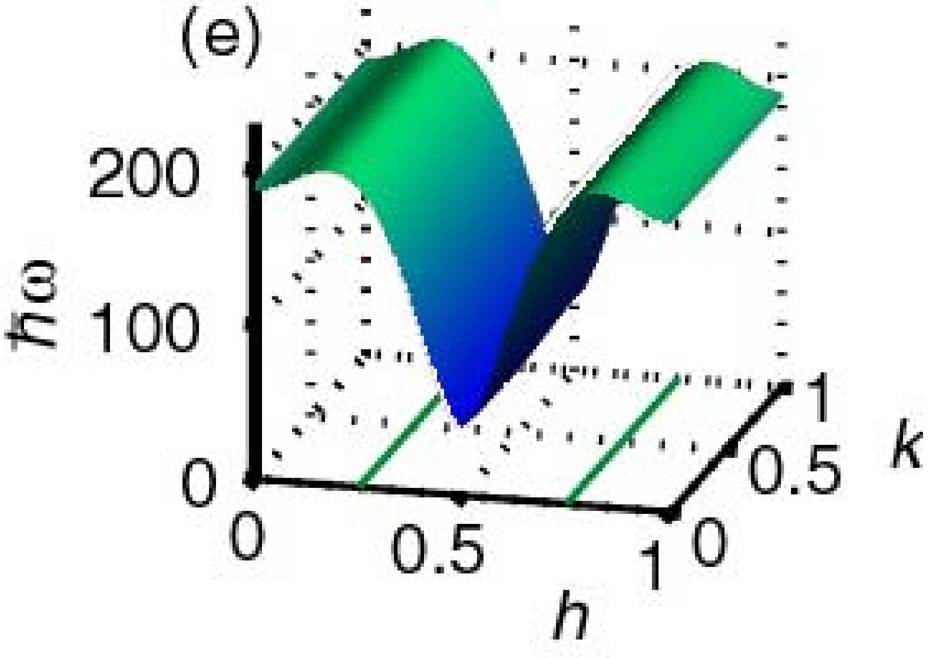


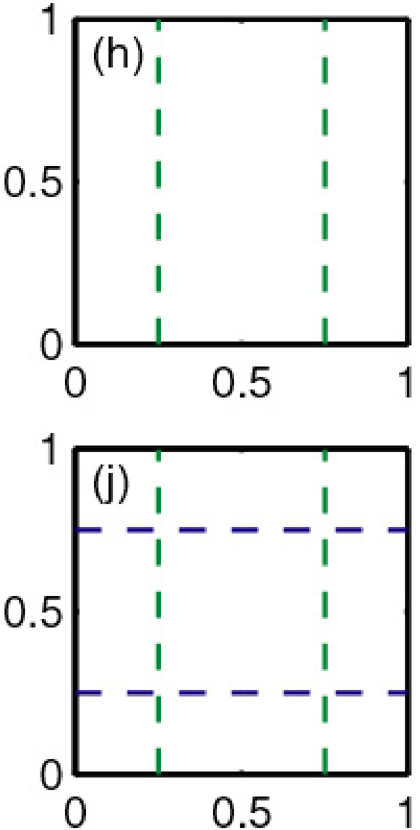


Proposal of J. M. Tranquada *et al.*, cond-mat/0401621

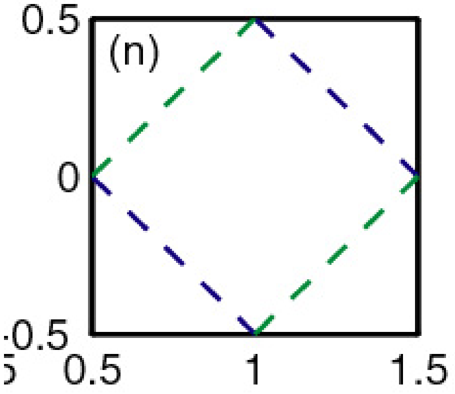
High energy spectrum is the triplon excitation of two-leg spin ladders
⇒ *presence of bond order*



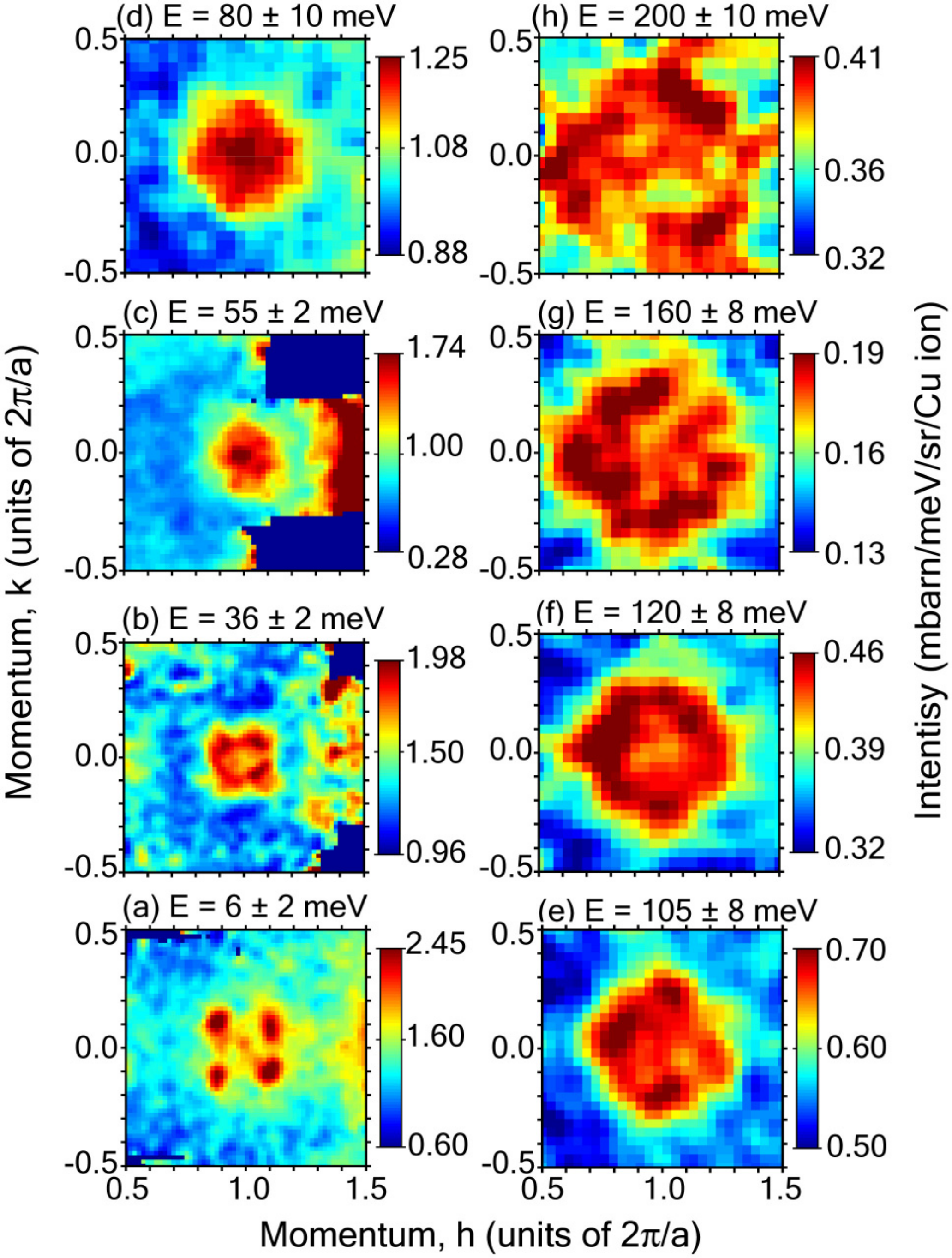


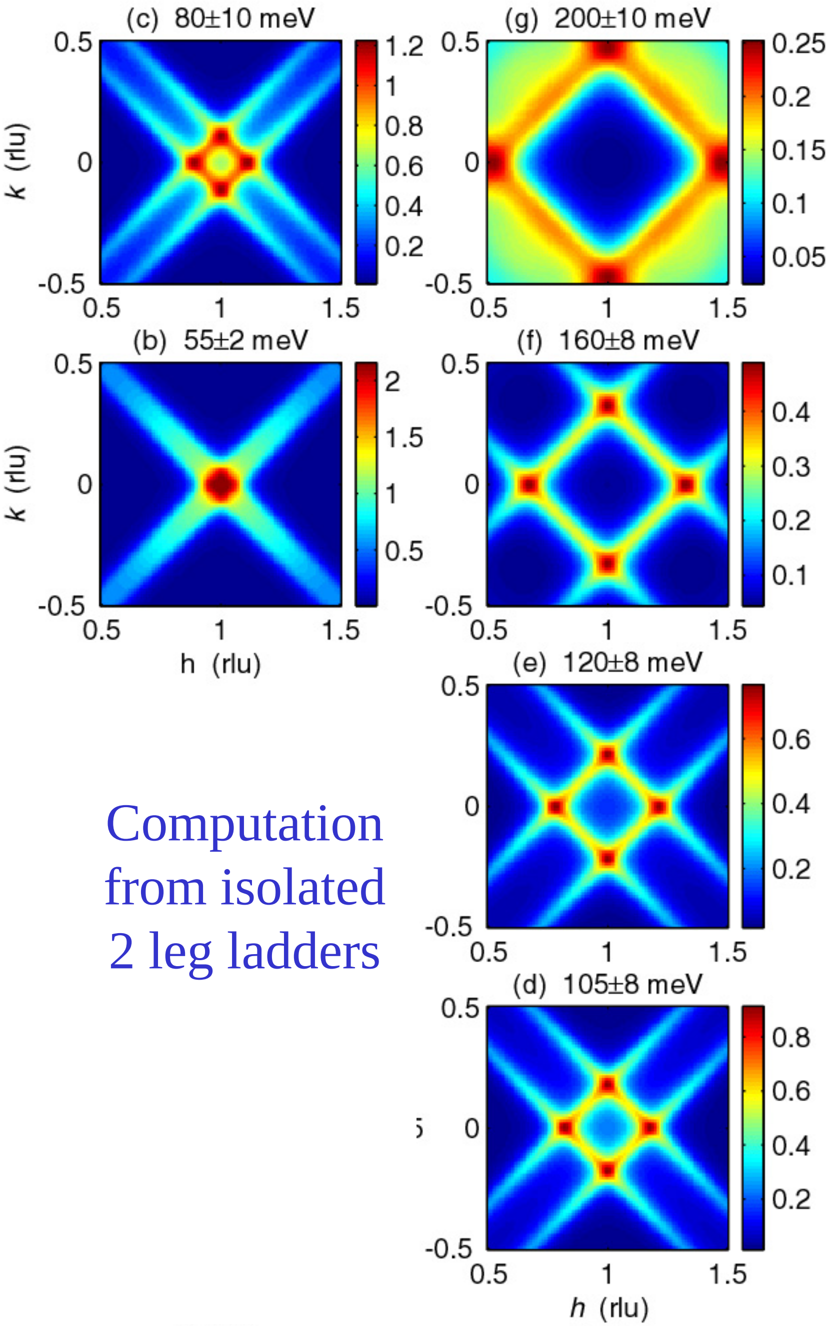


Location of maximum energy excitations



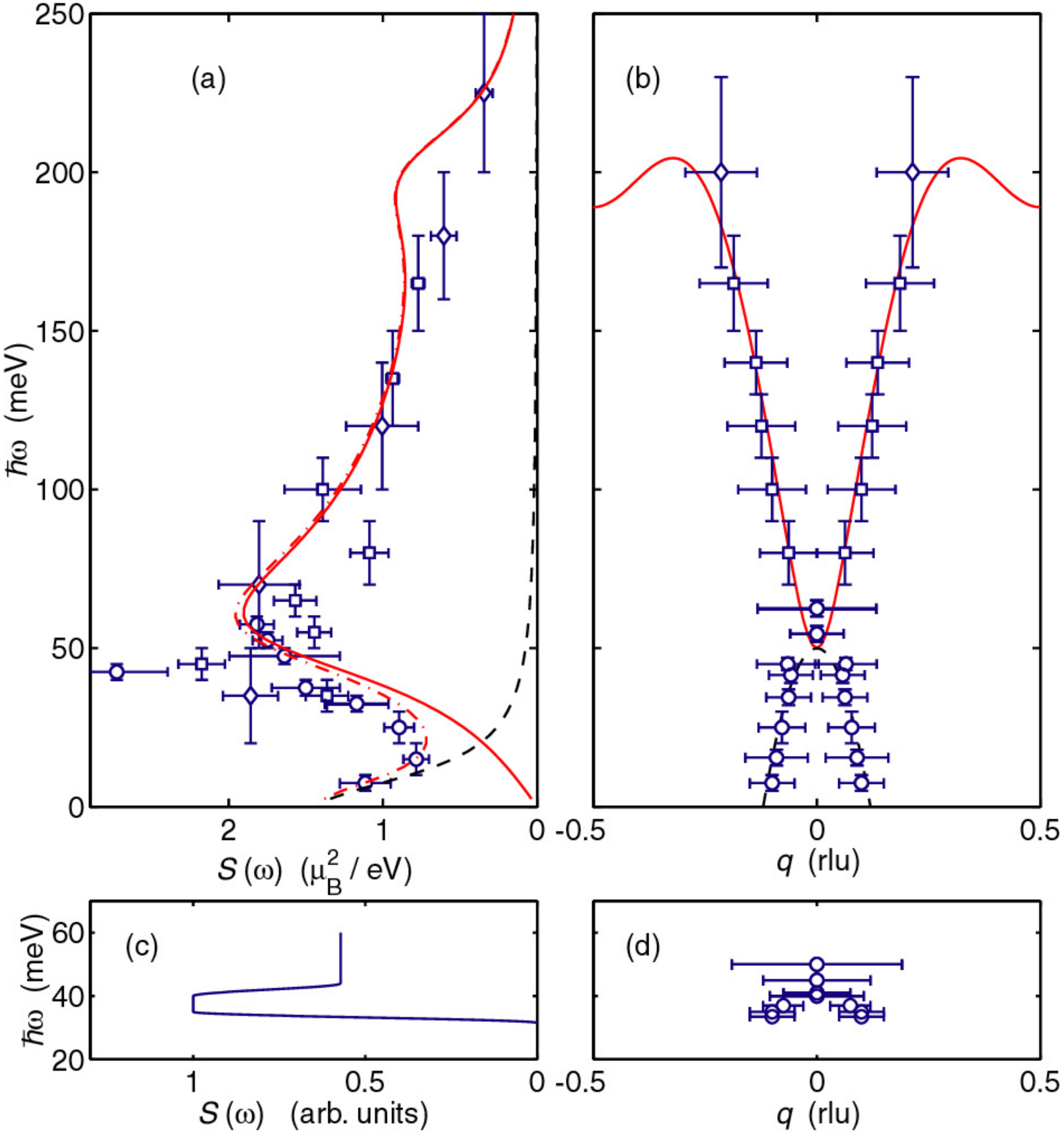
Superposition and rotation by 45 degrees





Computation
from isolated
2 leg ladders

J. M. Tranquada *et al.*, cond-mat/0401621

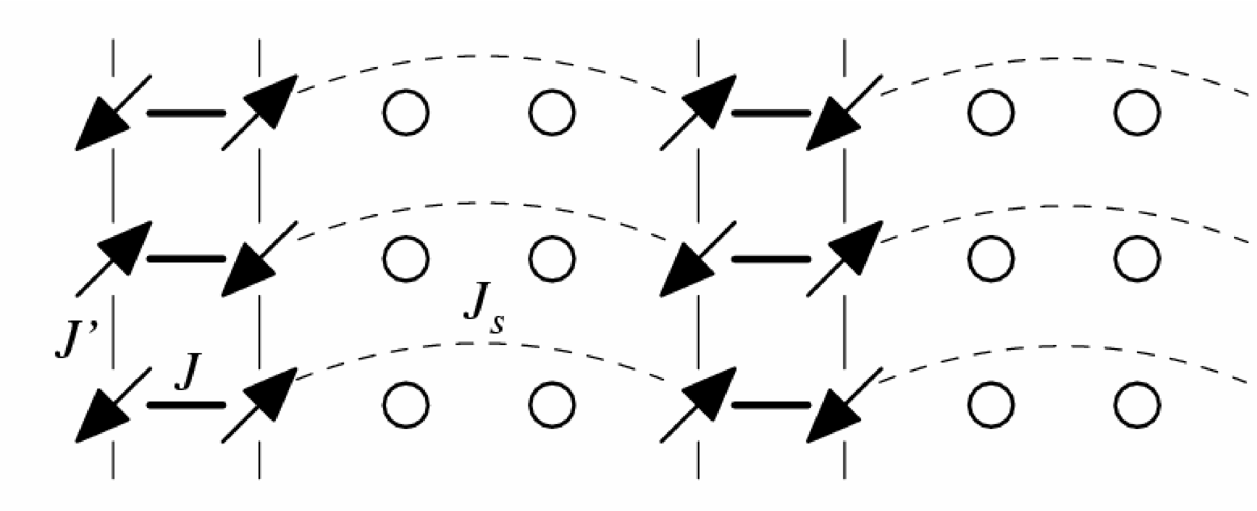


$La_{1.875}Ba_{0.125}CuO_4$

$YBa_2Cu_3O_{6.85}$

J. M. Tranquada *et al.*, cond-mat/0401621

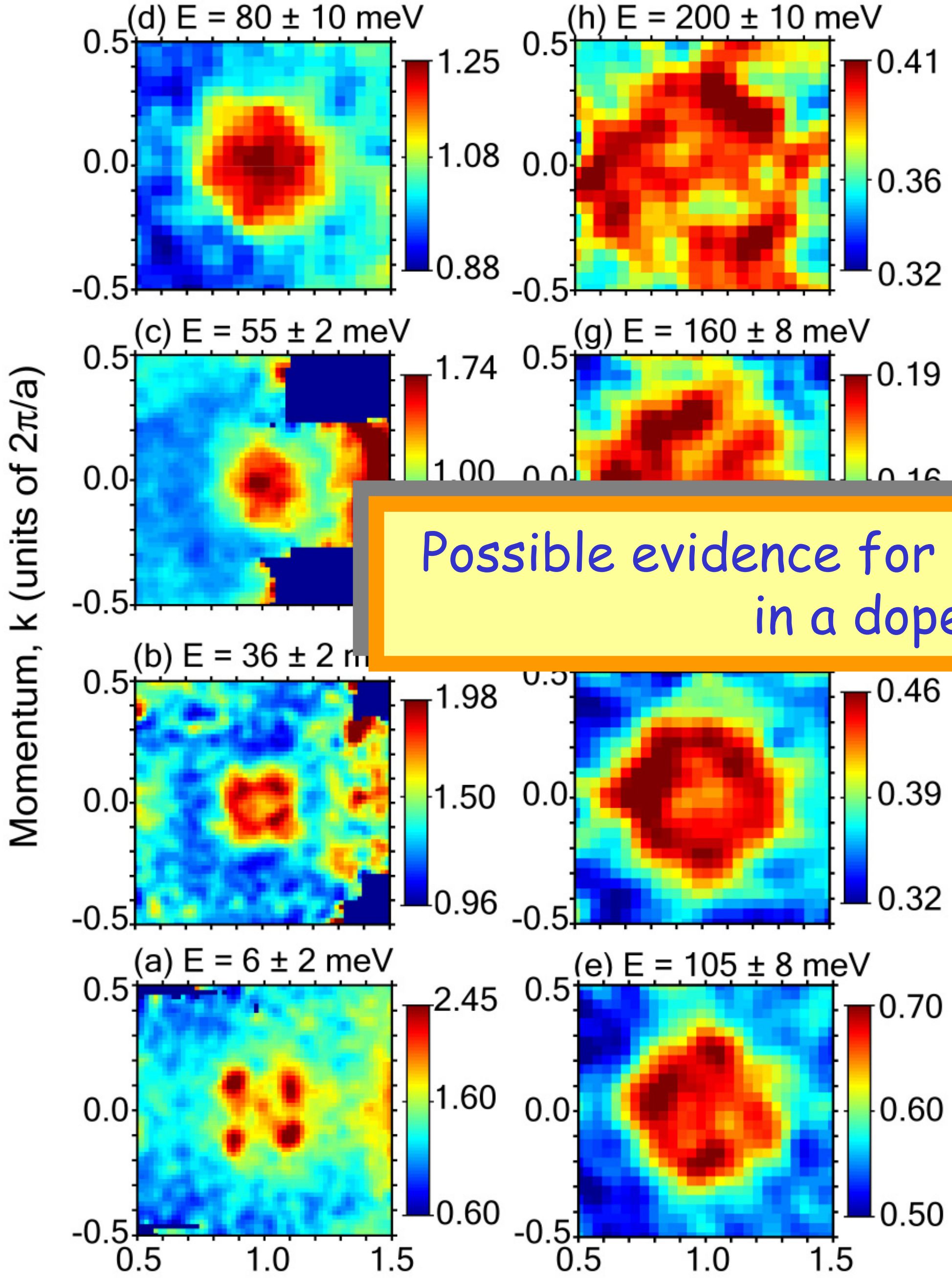
Understanding spectrum at all energies requires coupling between ladders, just past the quantum critical point to the onset of long-range magnetic order



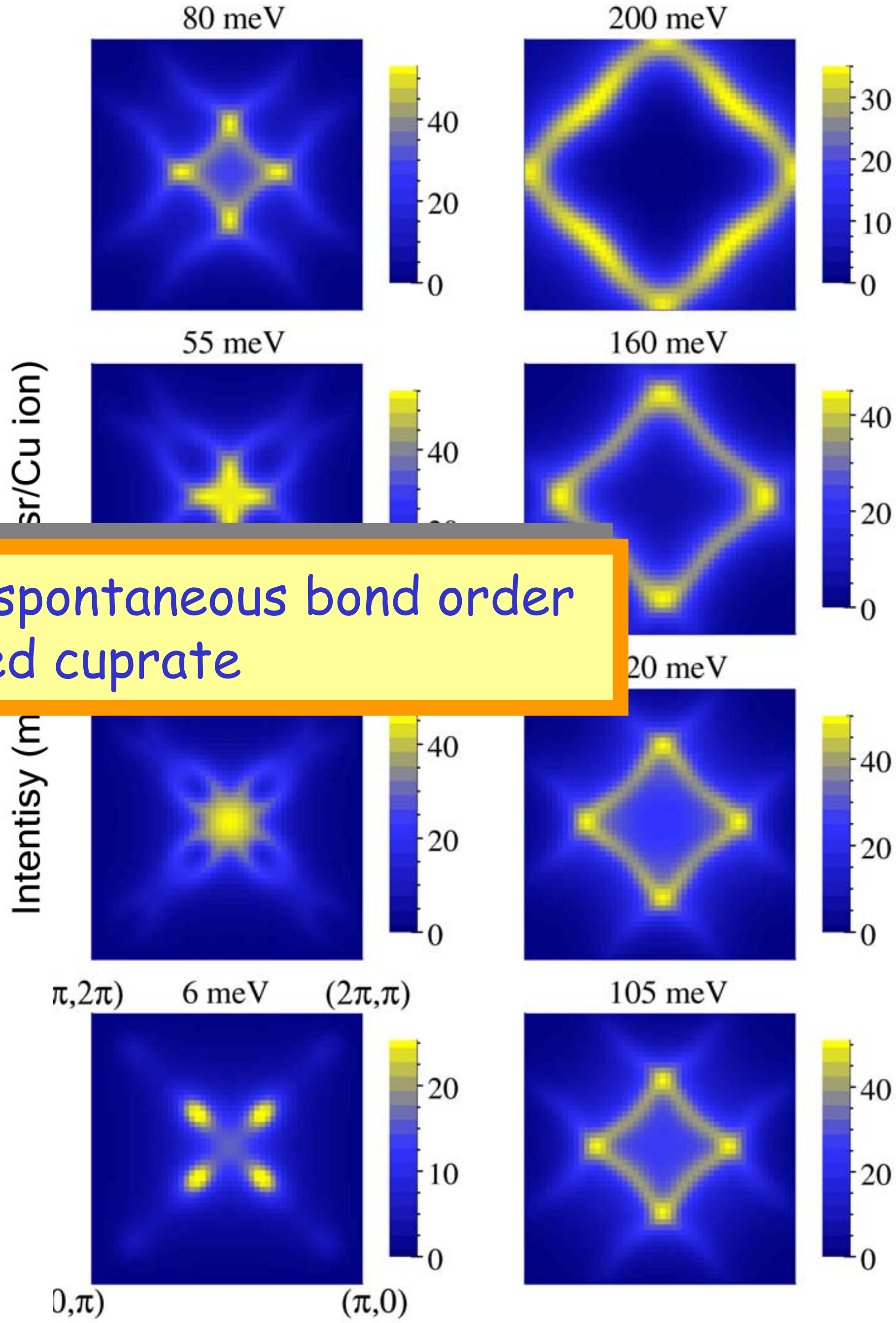
Use bond-operator method (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) to compute crossover from spin-waves at low energies to triplons at high energies

M. Vojta and T. Ulbricht, cond-mat/0402377

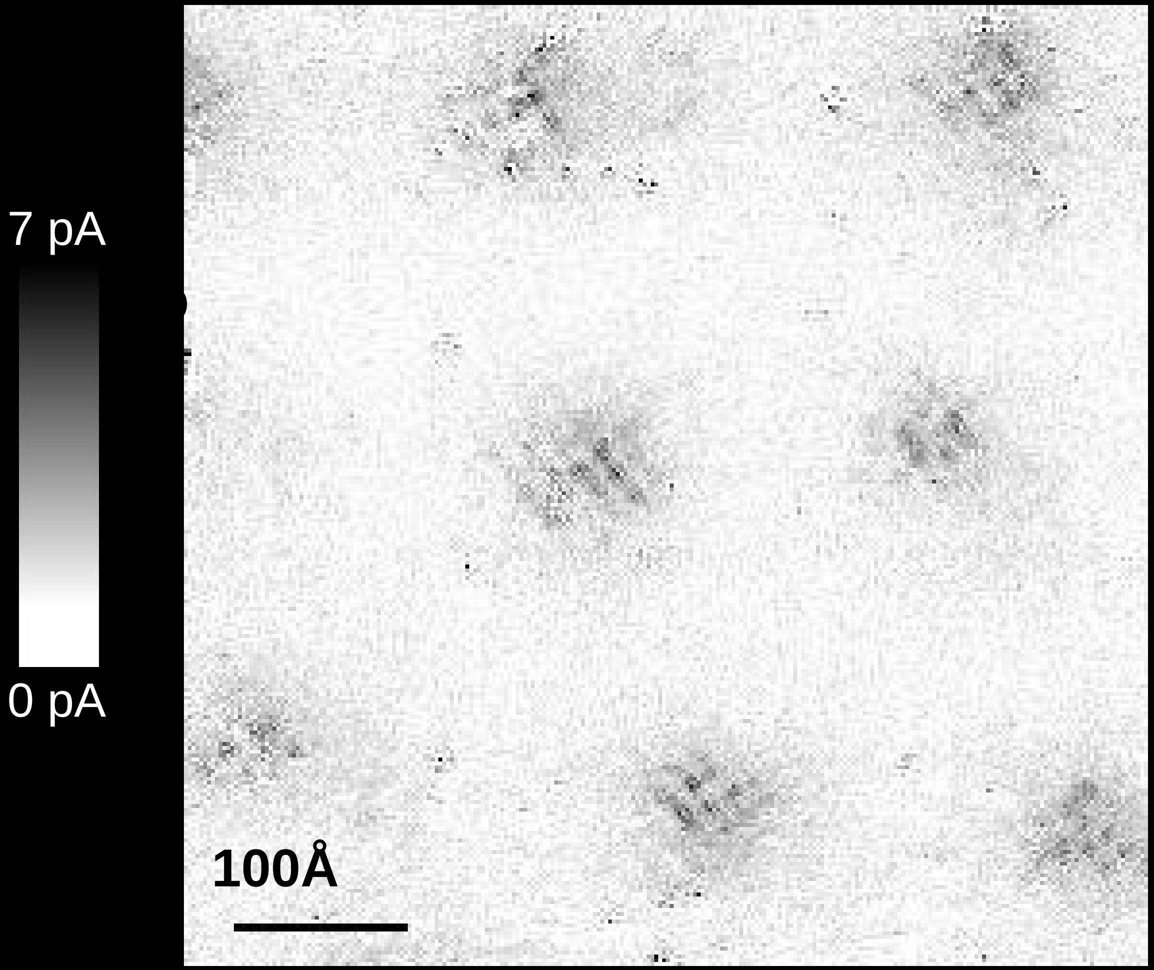
Possible evidence for spontaneous bond order in a doped cuprate



J. M. Tranquada *et al.*, cond-mat/0401621



M. Vojta and T. Ulbricht, cond-mat/0402377

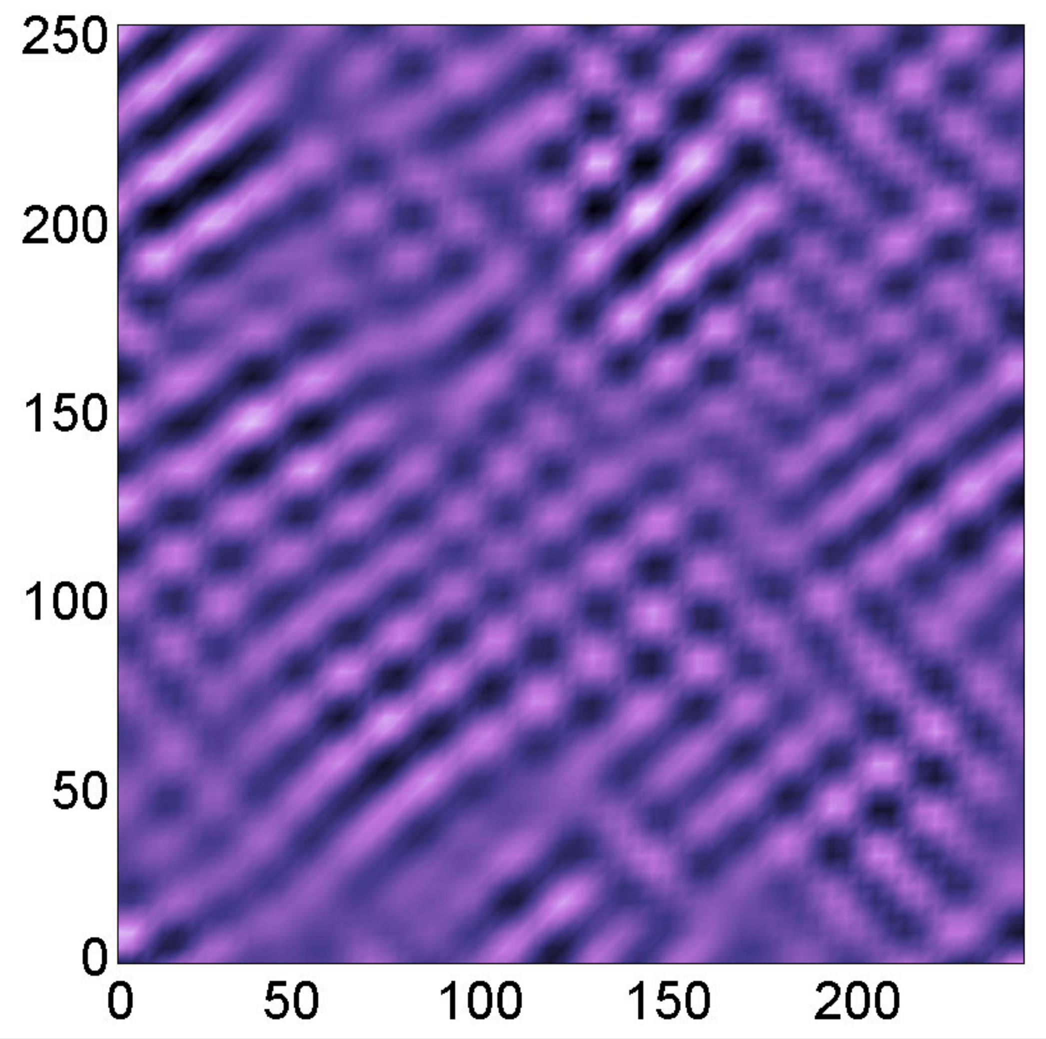


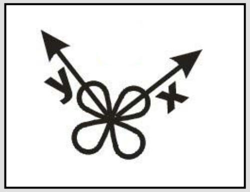
Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Our interpretation: STM evidence for fluctuating spin density/bond order pinned by vortices/impurities

A. Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, *Int. J. Mod. Phys.* B **16**, 3156 (2002)





STM image of LDOS modulations (after filtering in Fourier space) in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field

C. Howald, H. Eisaki, N. Kaneko, M. Greven,and A. Kapitulnik,
Phys. Rev. B **67**, 014533 (2003).

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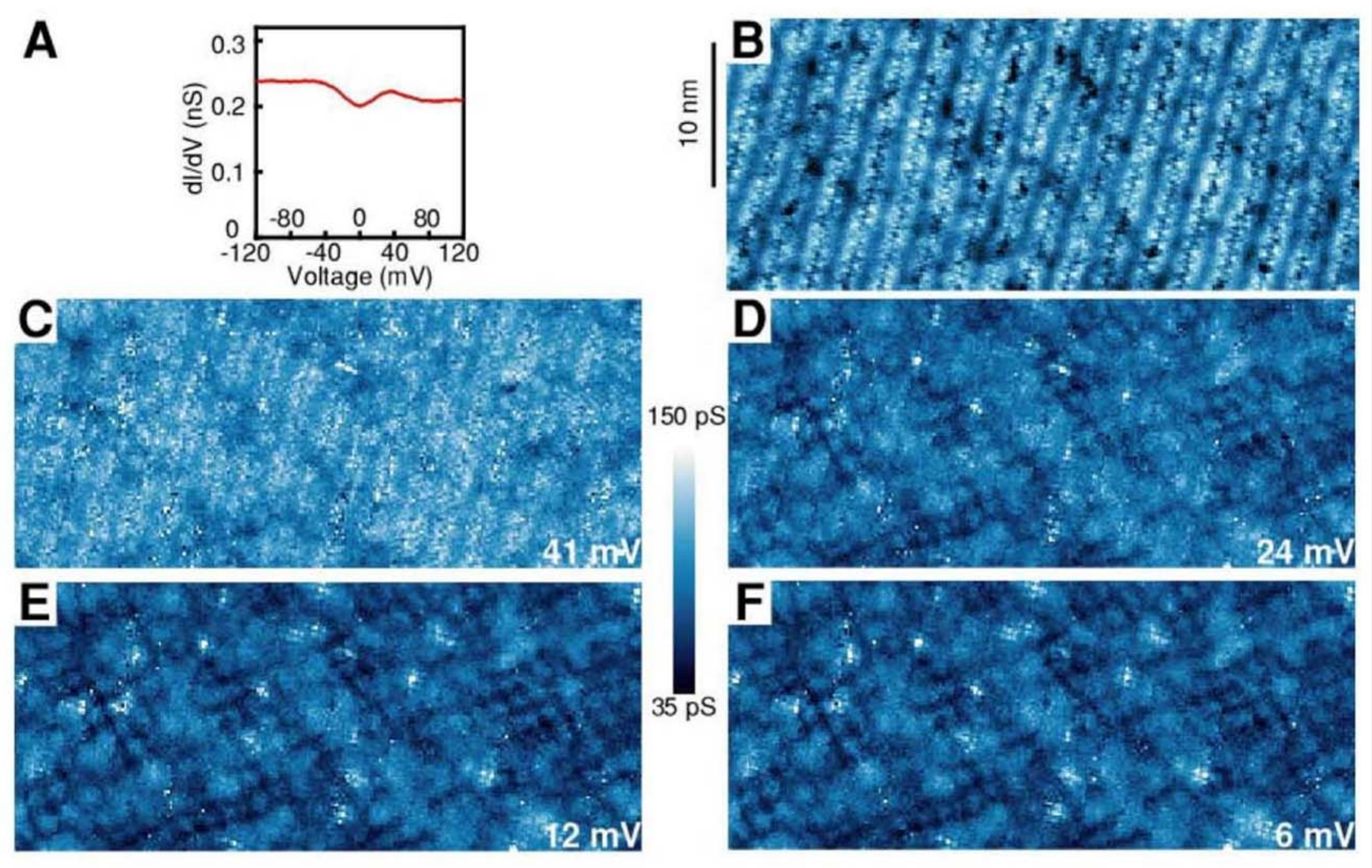
A. Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, *Int. J. Mod. Phys.* B **16**, 3156 (2002)

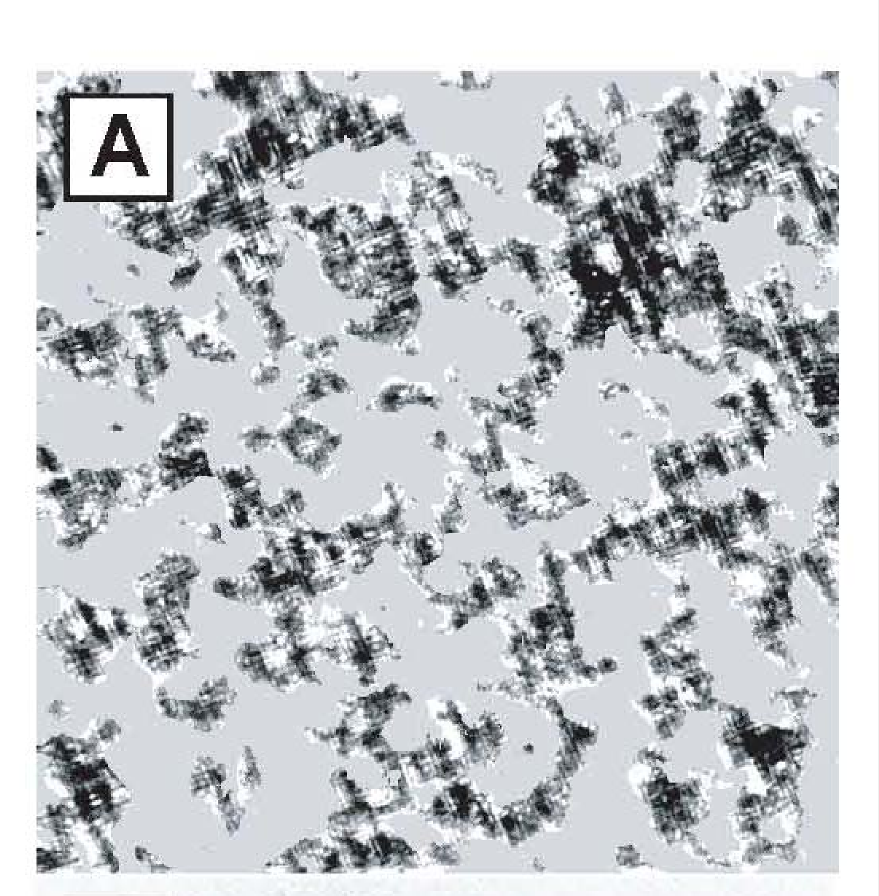


LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ at 100 K.

M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, 12 Feb 2004.

Our interpretation: STM evidence for fluctuating spin density/bond order pinned by vortices/impurities

A. Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, *Int. J. Mod. Phys*. B **16**, 3156 (2002)



Energy integrated LDOS (between 65 and 150 meV) of strongly underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ at low temperatures, showing only regions without superconducting "coherence peaks"

K. McElroy, D.-H. Lee, J. E. Hoffman, K. M. Lang, J. Lee, E. W. Hudson, H. Eisaki, S. Uchida, and J.C. Davis, cond-mat/0402xxx.

Our interpretation: STM evidence for fluctuating spin density/bond order pinned by vortices/impurities

A. Polkovnikov, S. Sachdev, M. Vojta, and E. Demler, *Int. J. Mod. Phys.* B **16**, 3156 (2002)

Conclusions

I. Theory of quantum phase transitions between magnetically ordered and paramagnetic states of Mott insulators:

  A. *Dimerized Mott insulators*: Landau-Ginzburg-Wilson theory of fluctuating magnetic order parameter.

  B. *S=1/2 square lattice*: Berry phases induce bond order, and LGW theory breaks down. Critical theory is expressed in terms of emergent fractionalized modes, and the
  order parameters are secondary.

II. Preliminary evidence for spin density/bond orders in superconducting cuprates