Strange metals and black holes

Rutgers University
September 9, 2015

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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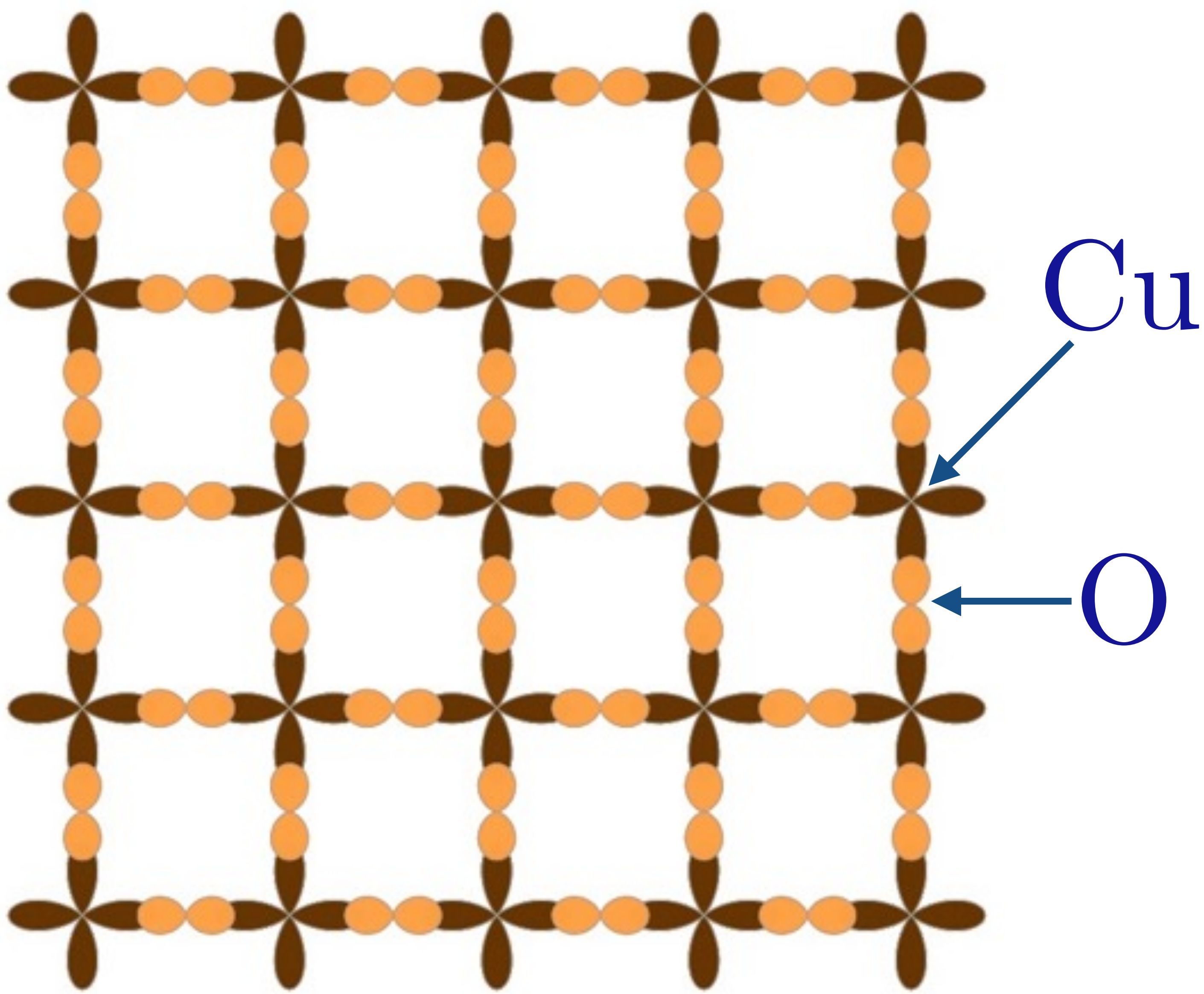
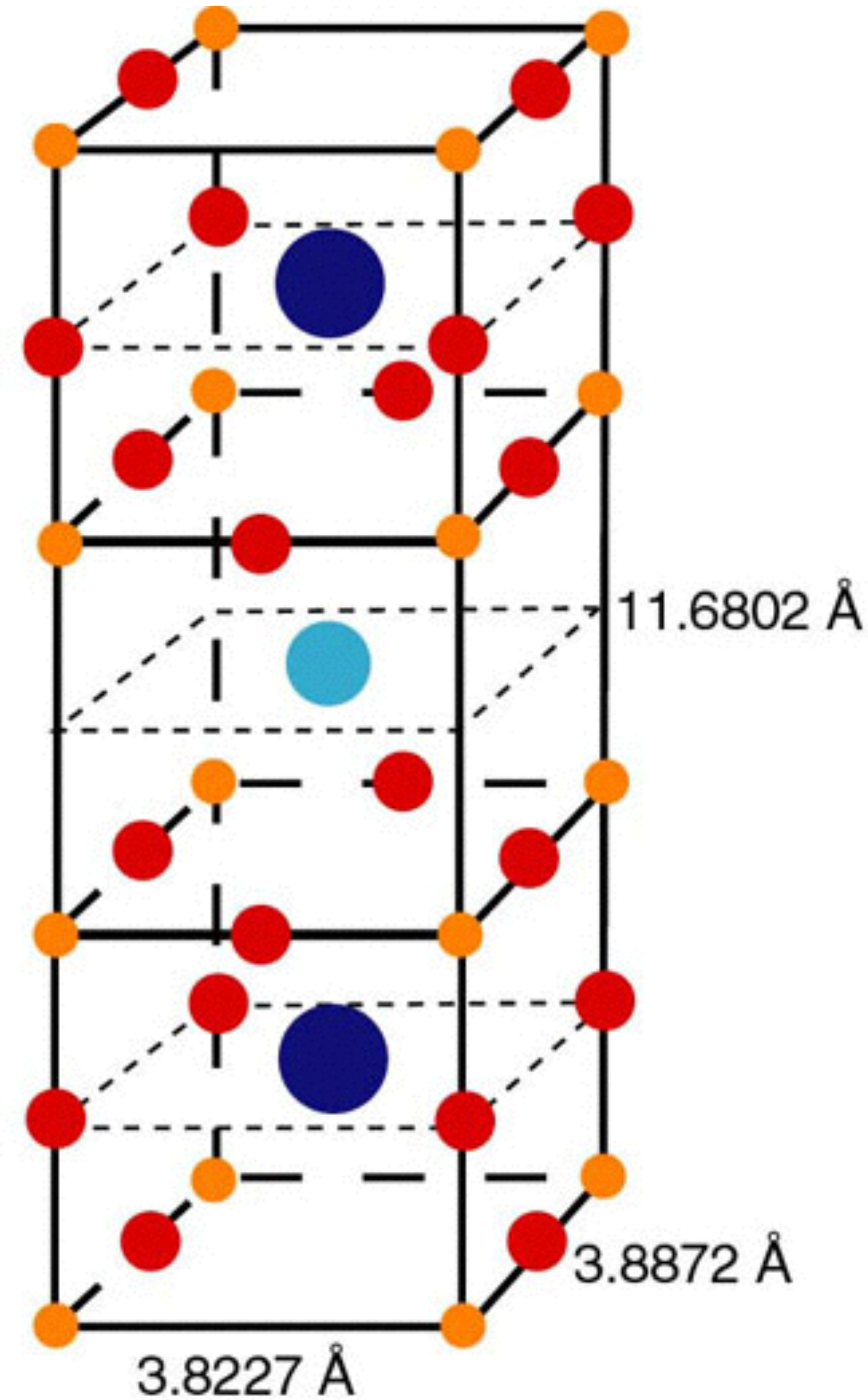
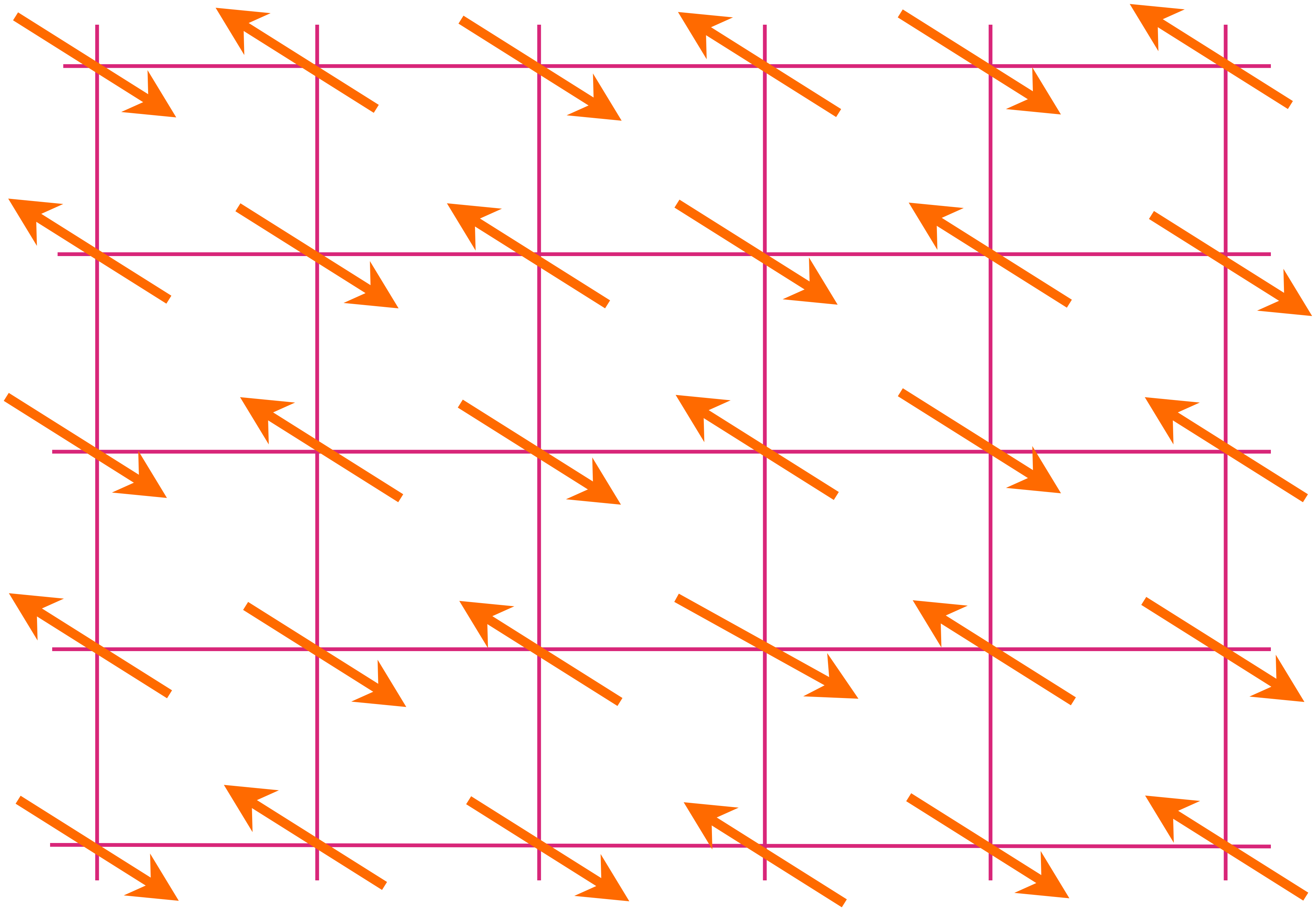
High temperature superconductors



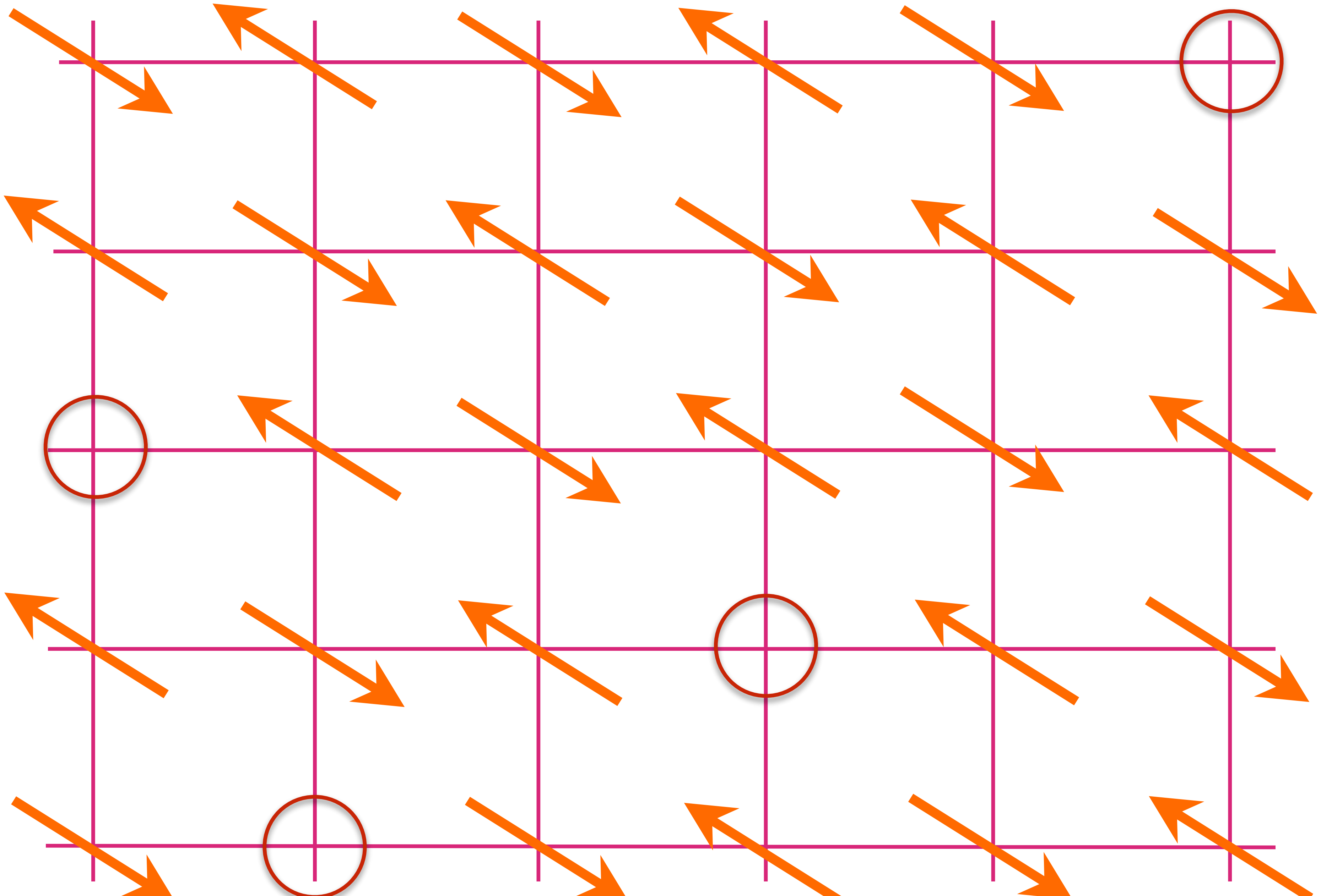
CuO_2 plane



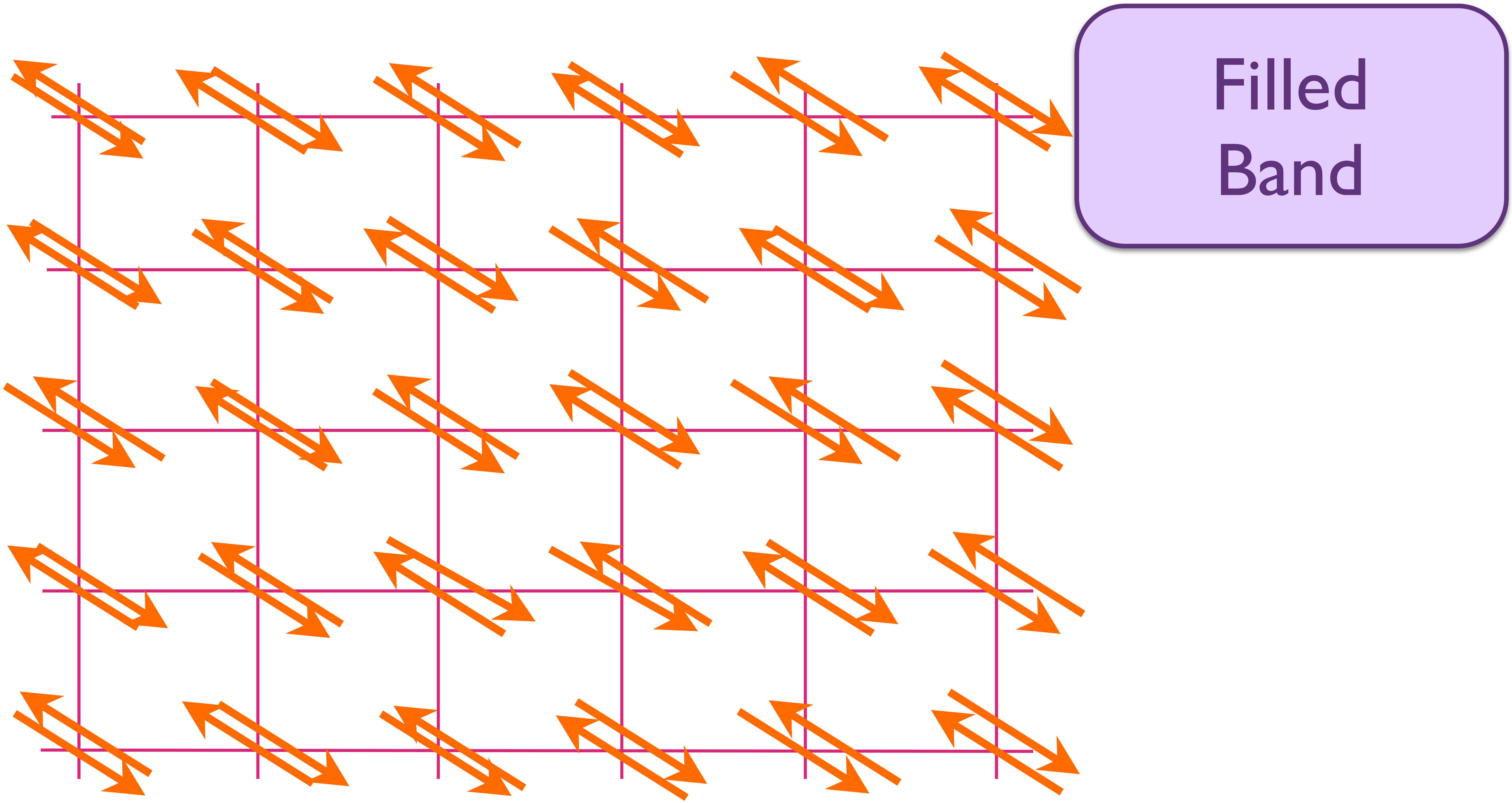
$YBa_2Cu_3O_{6+x}$

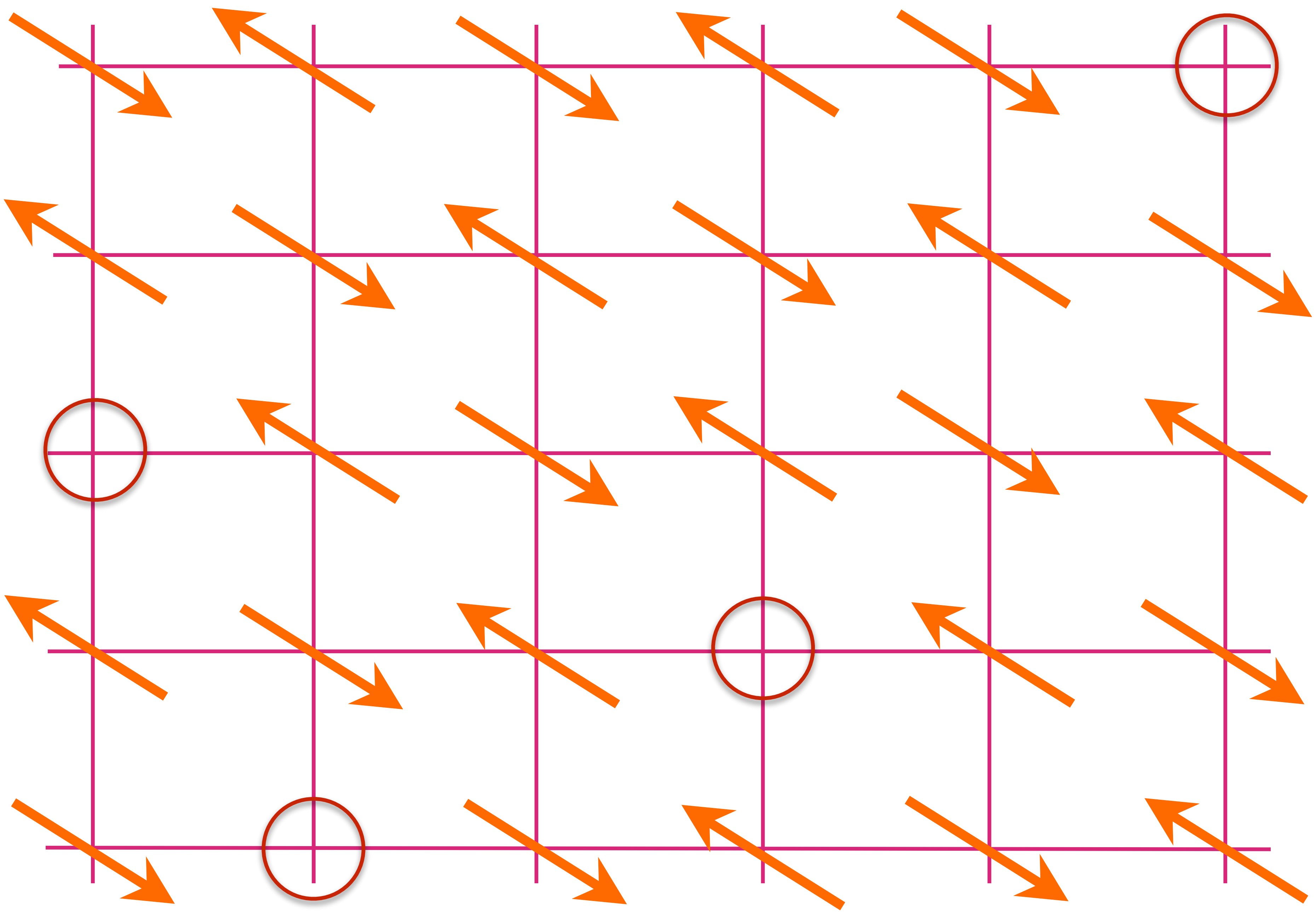


“Undoped” Anti-ferromagnet



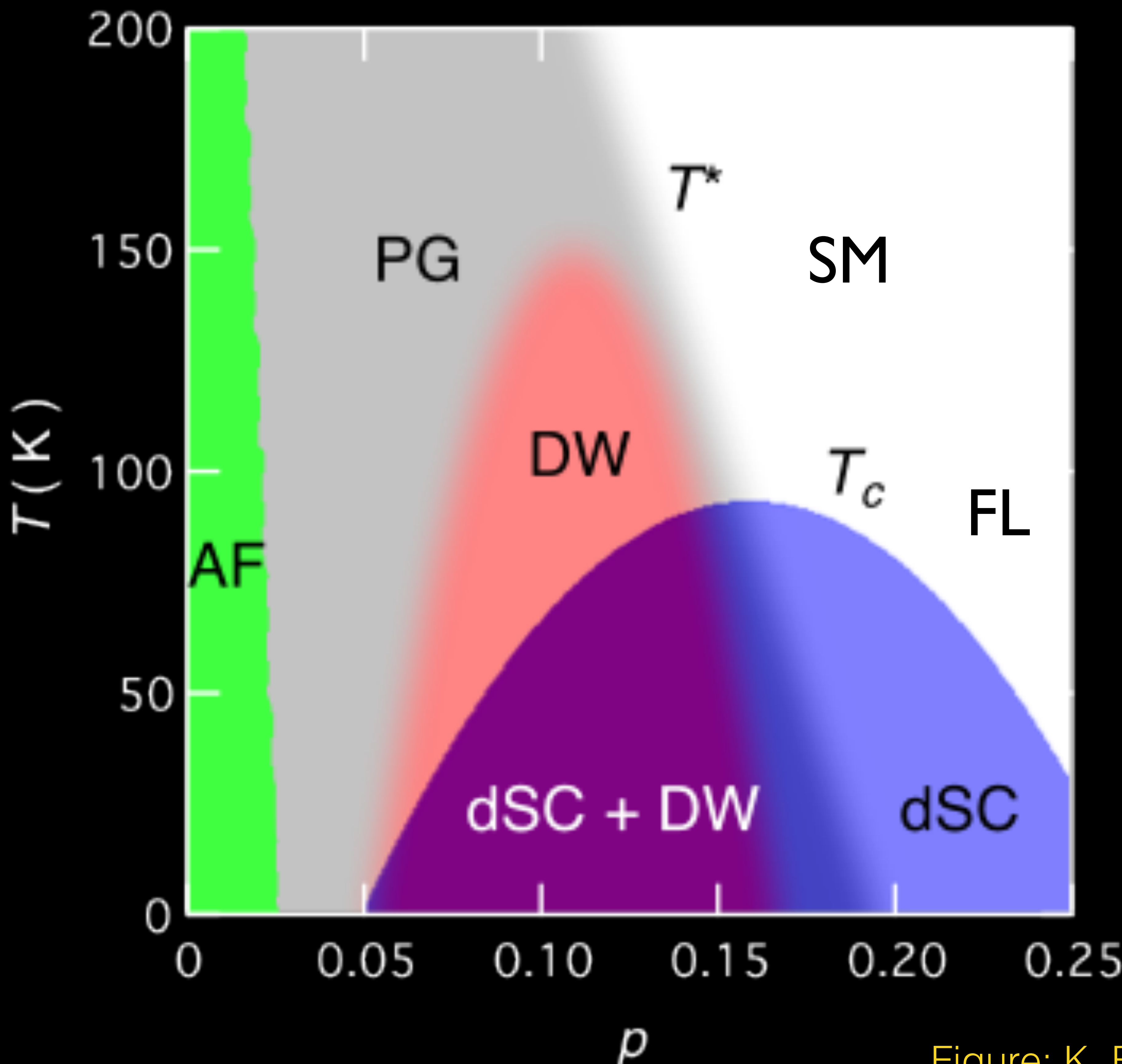
Anti-ferromagnet with p holes per square

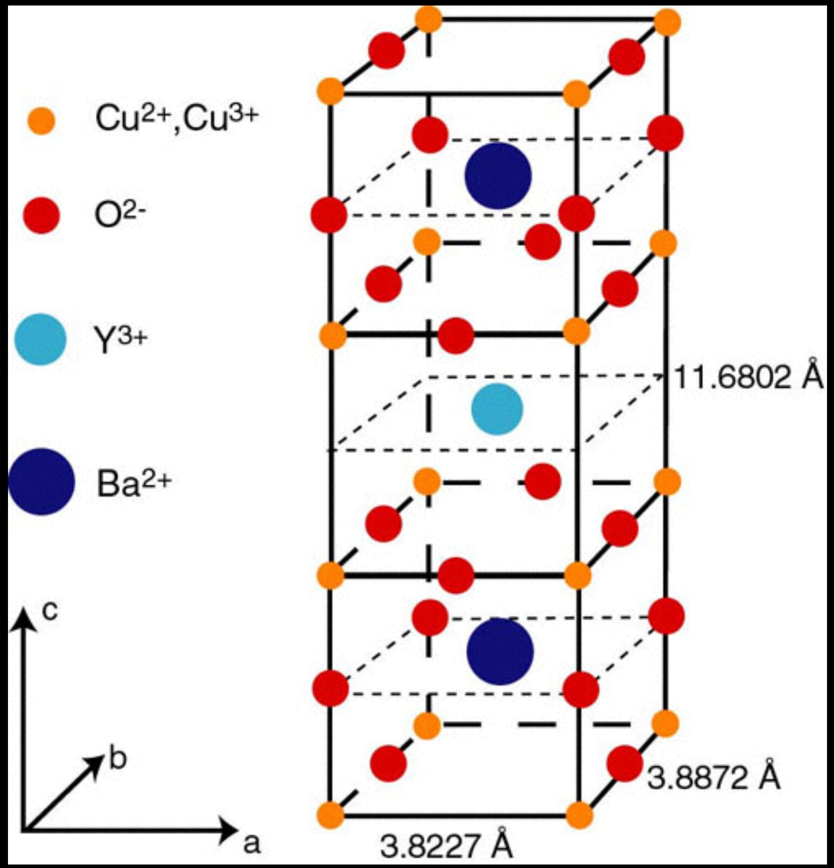

Filled
Band



Anti-ferromagnet with p holes per square

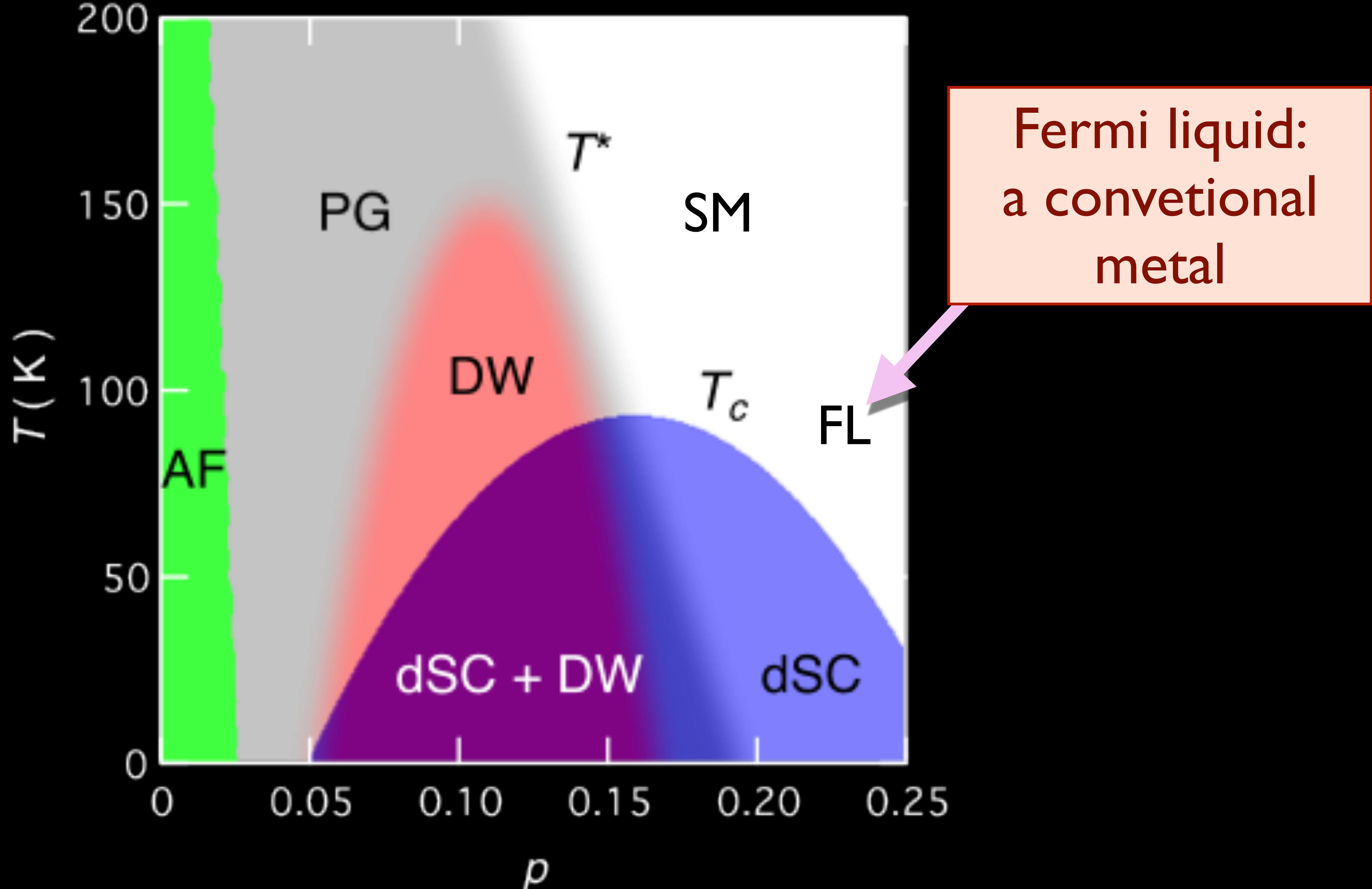
But relative to the band insulator, there are $1 + p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1 + p$



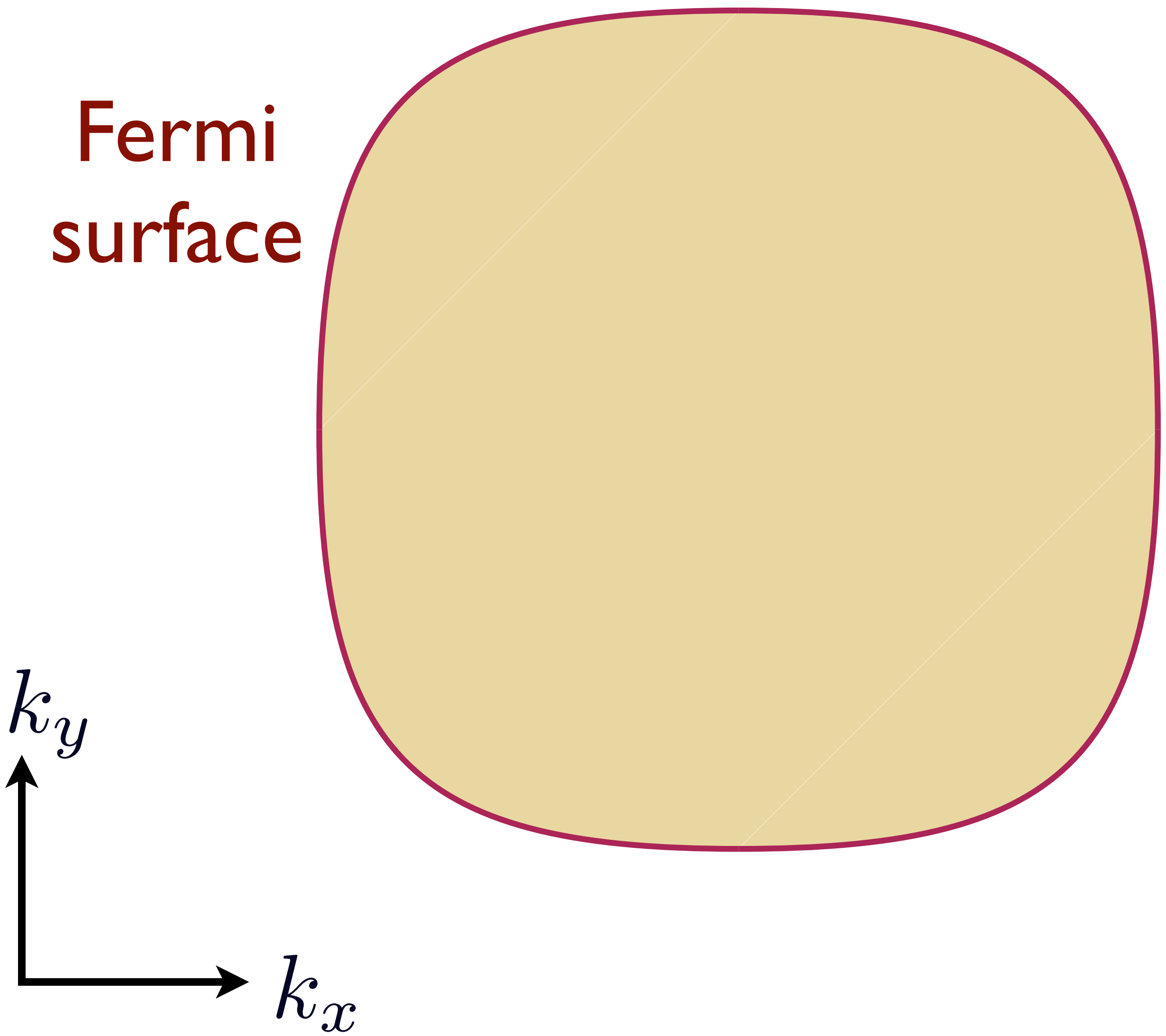


$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

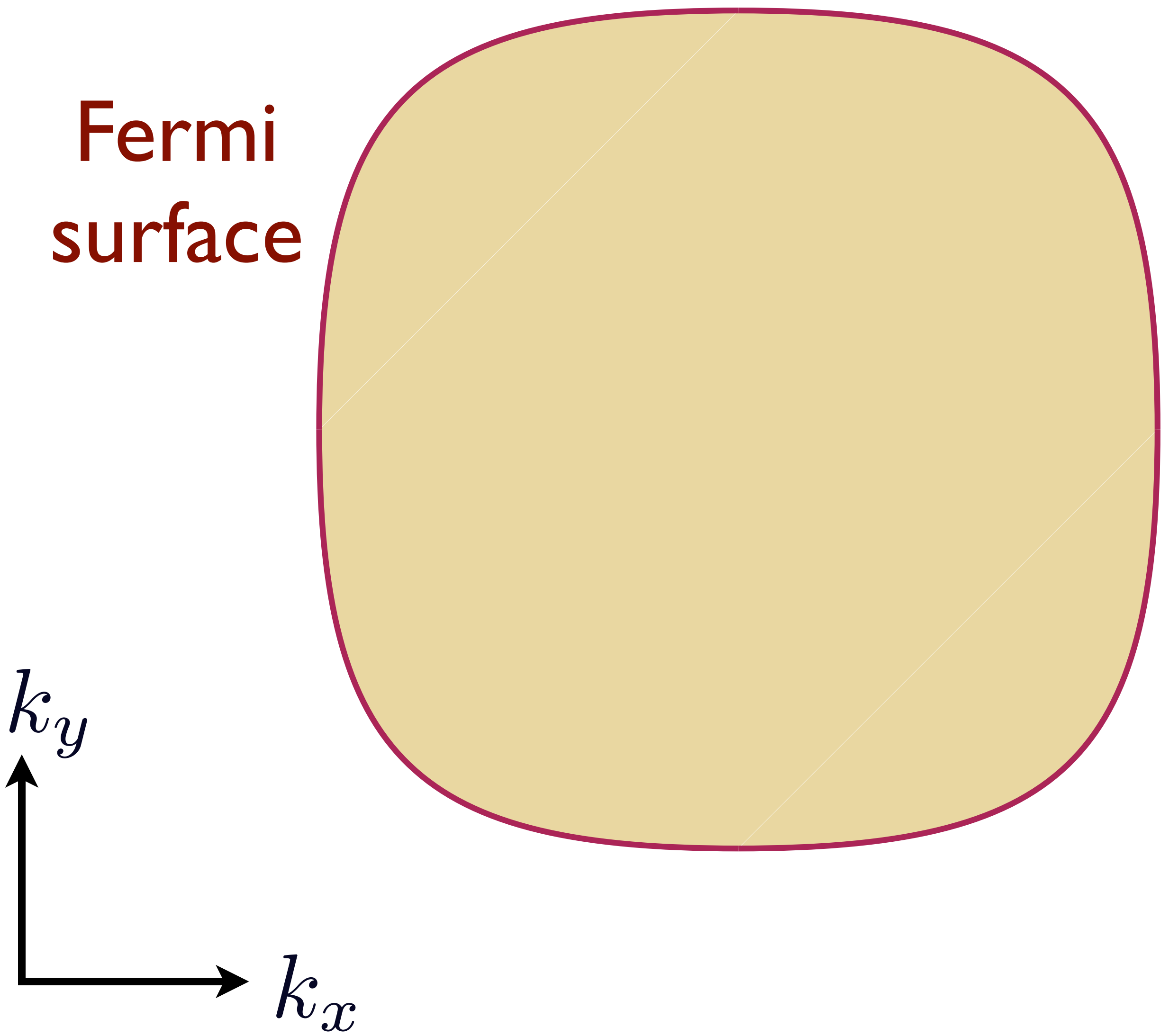

Fermi liquid:
a convetional
metal
200
150
100
50
0
T (K)
0
0.05
0.10
0.15
0.20
0.25
p
AF
PG
T*
SM
DW
Tc
FL
dSC + DW
dSC

Ordinary quantum matter: the Fermi liquid (FL)



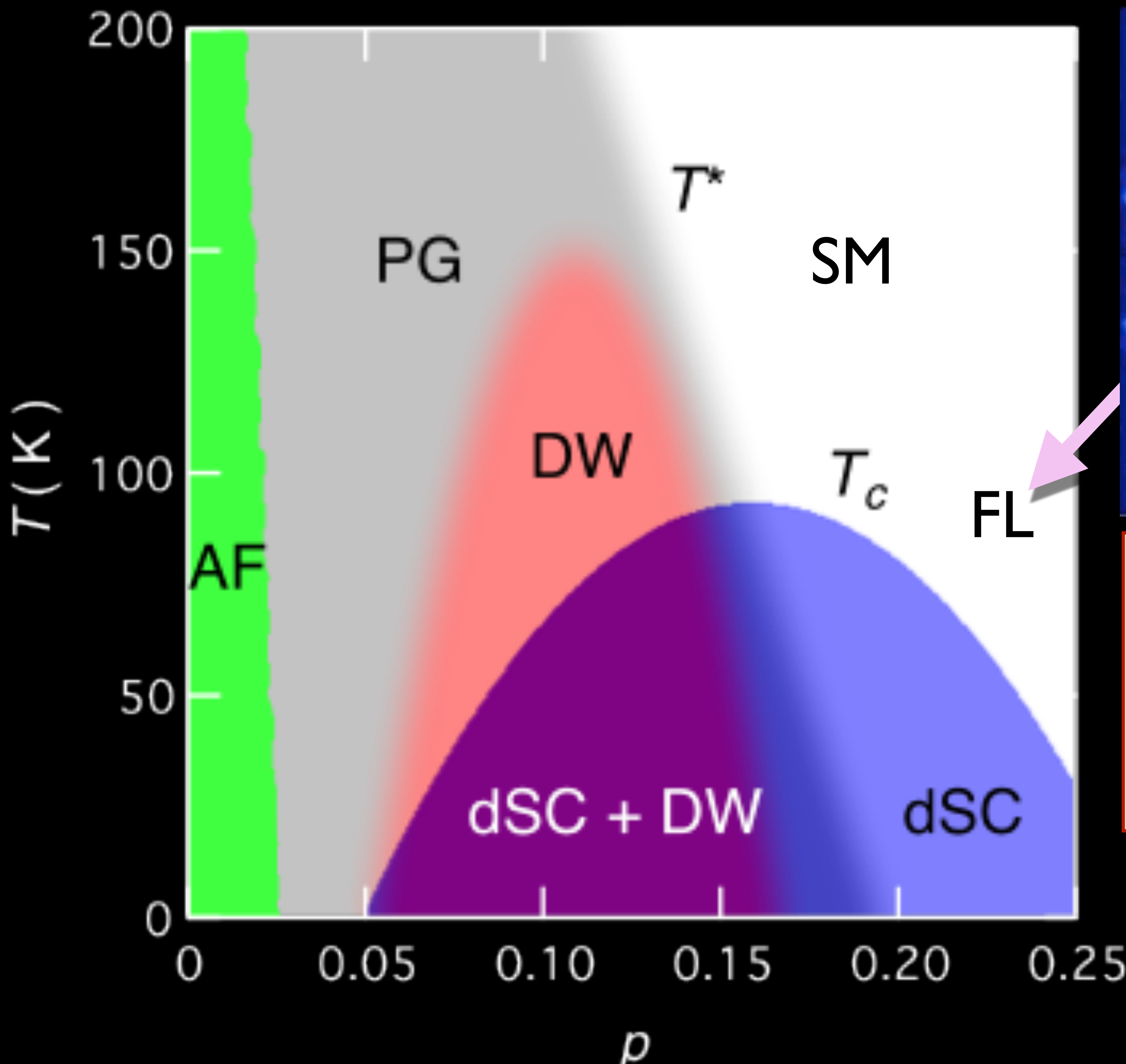
- Fermi surface separates empty and occupied states in momentum space.

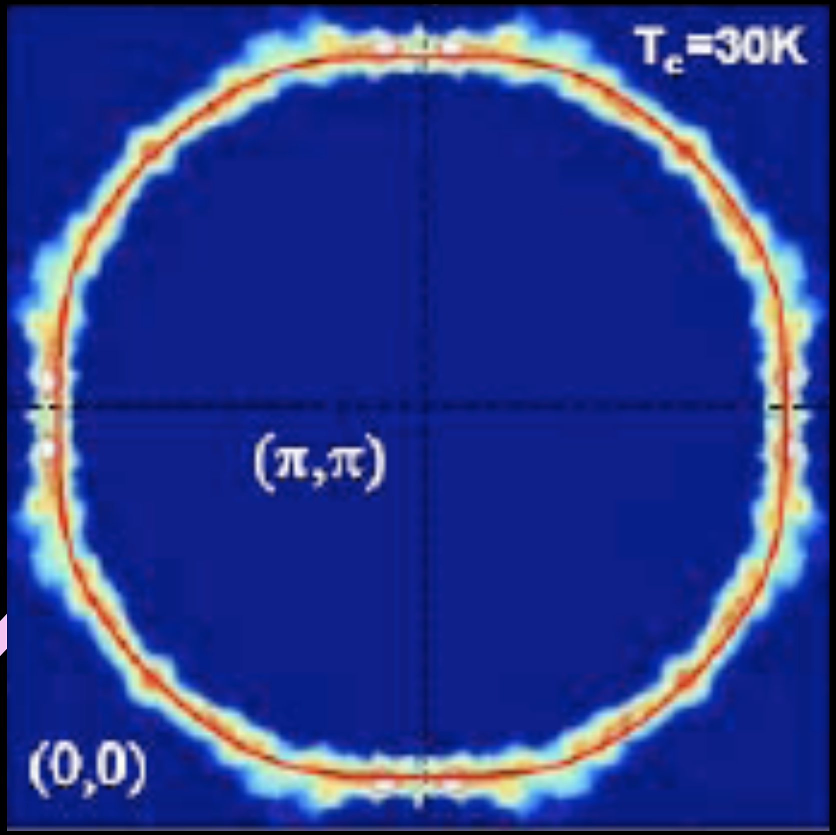
Ordinary quantum matter: the Fermi liquid (FL)



- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2) = $1+p$.

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

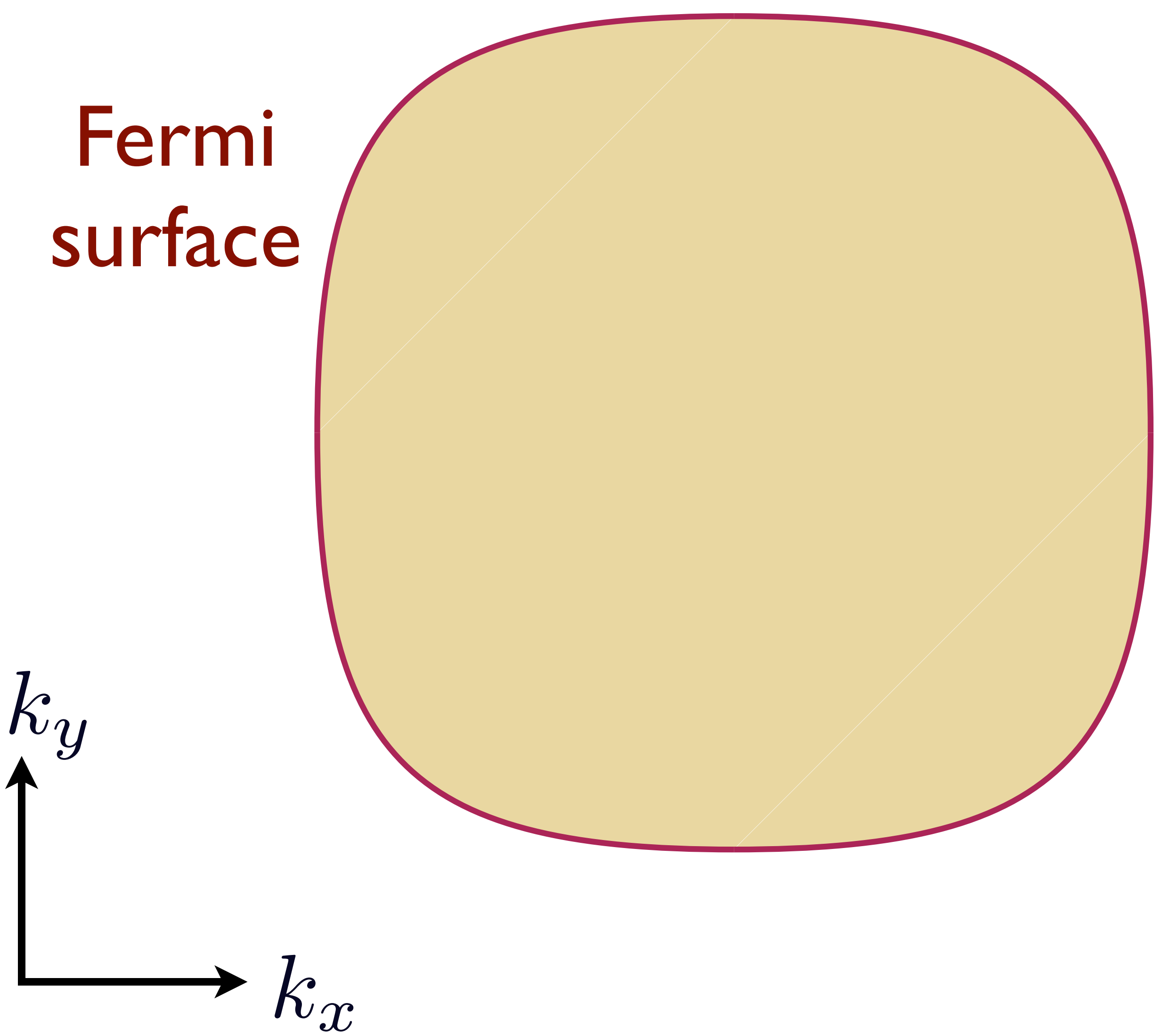




1. Conventional metal

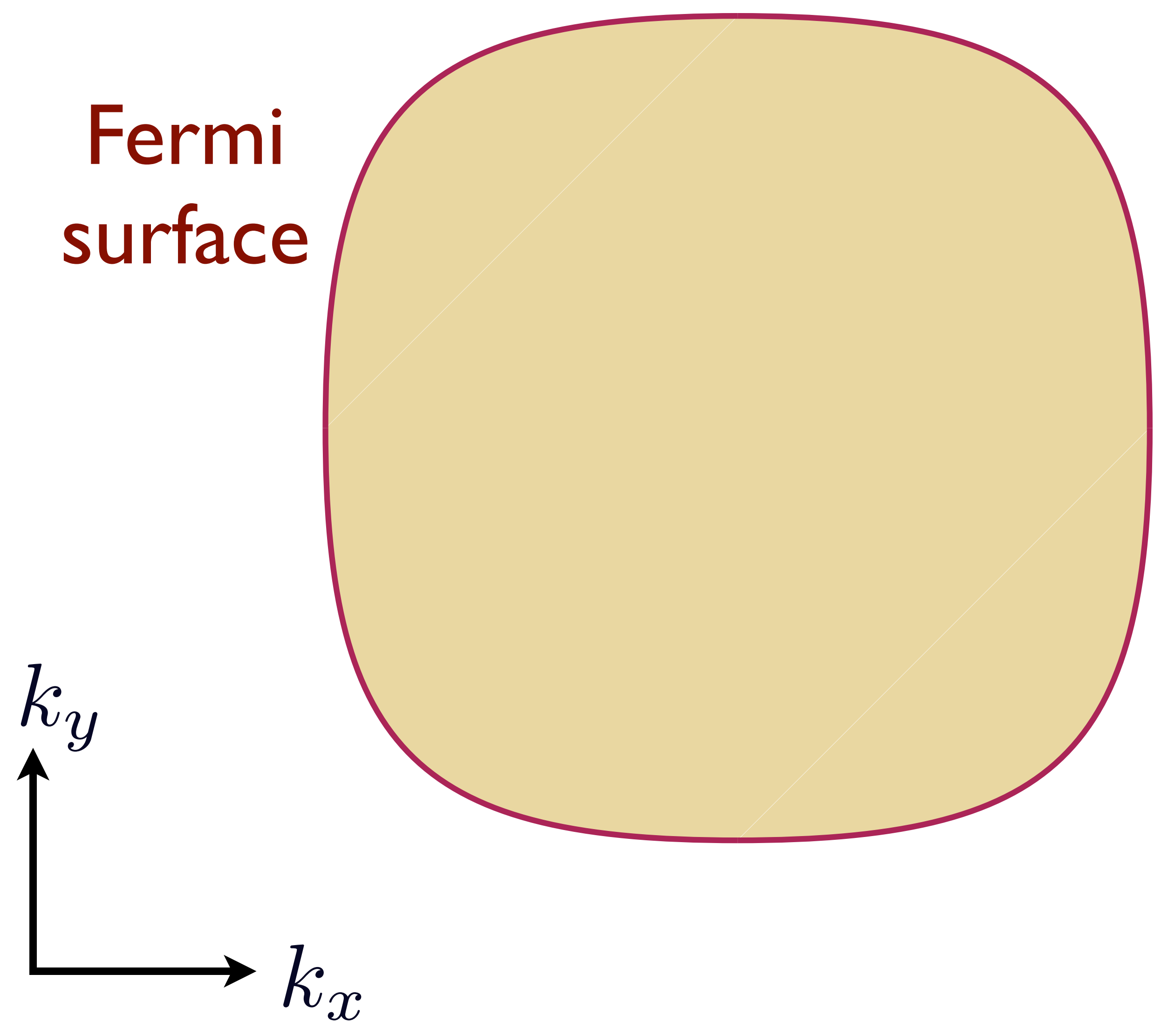
Area enclosed by Fermi surface $=1+p$

Ordinary quantum matter: the Fermi liquid (FL)



- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2) $= 1+p$.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

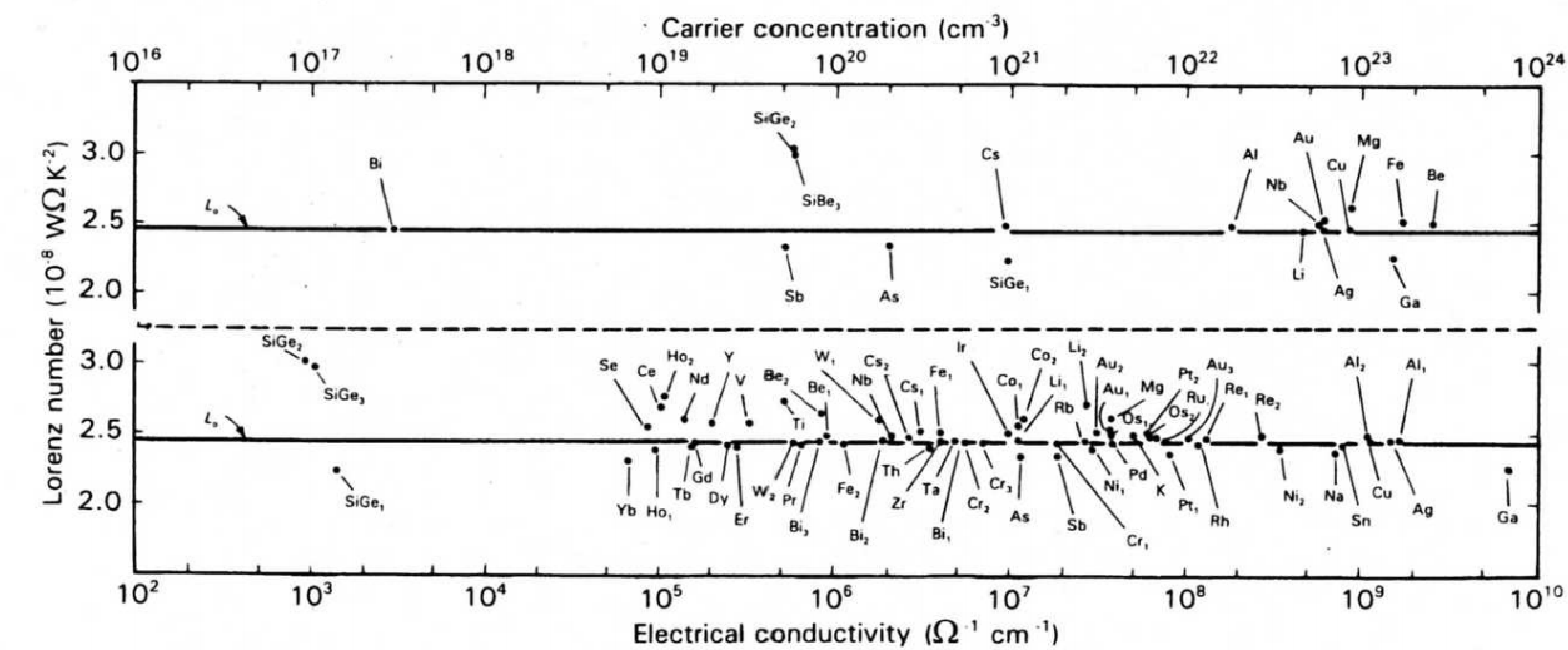
Ordinary quantum matter: the Fermi liquid (FL)

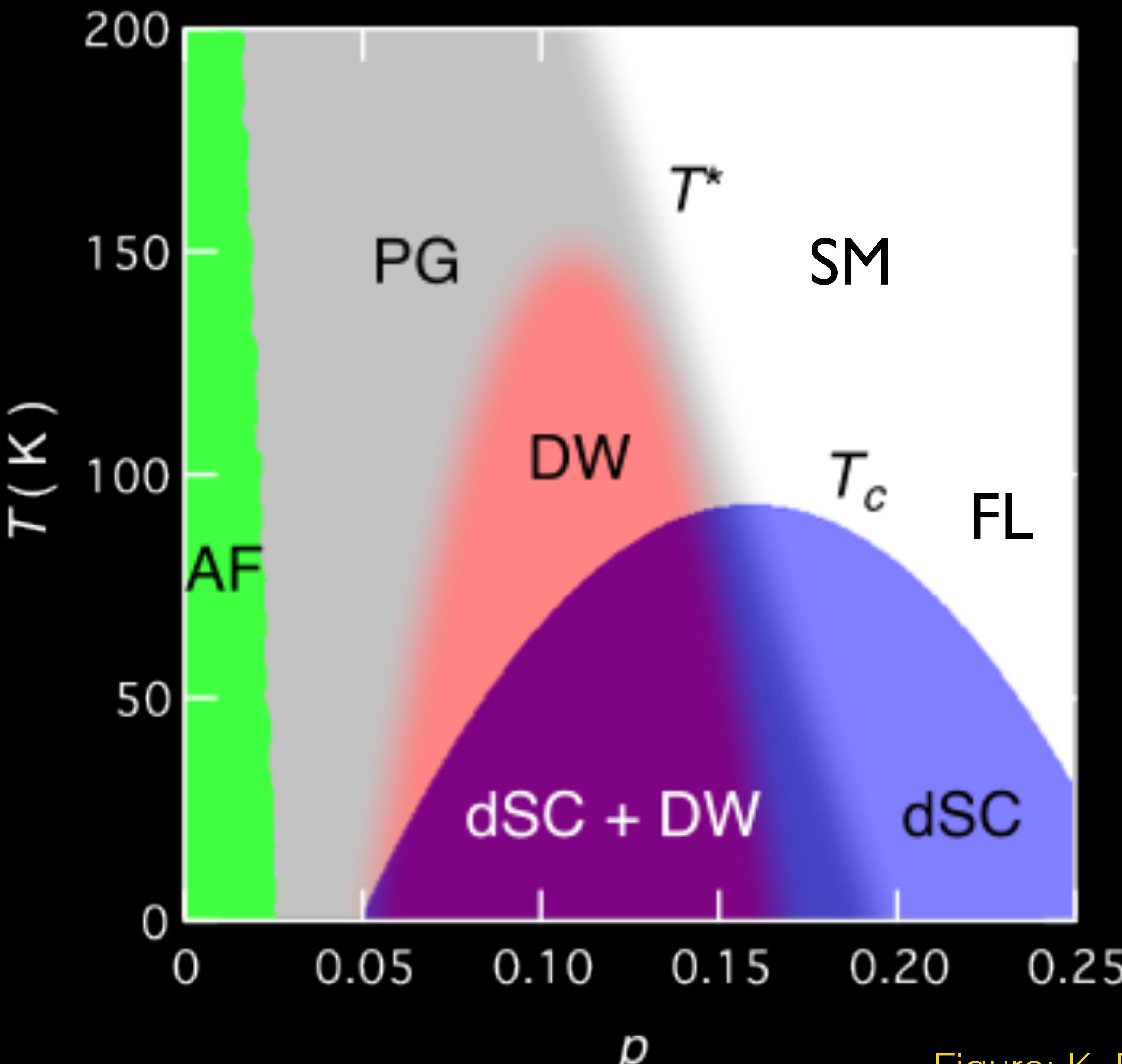


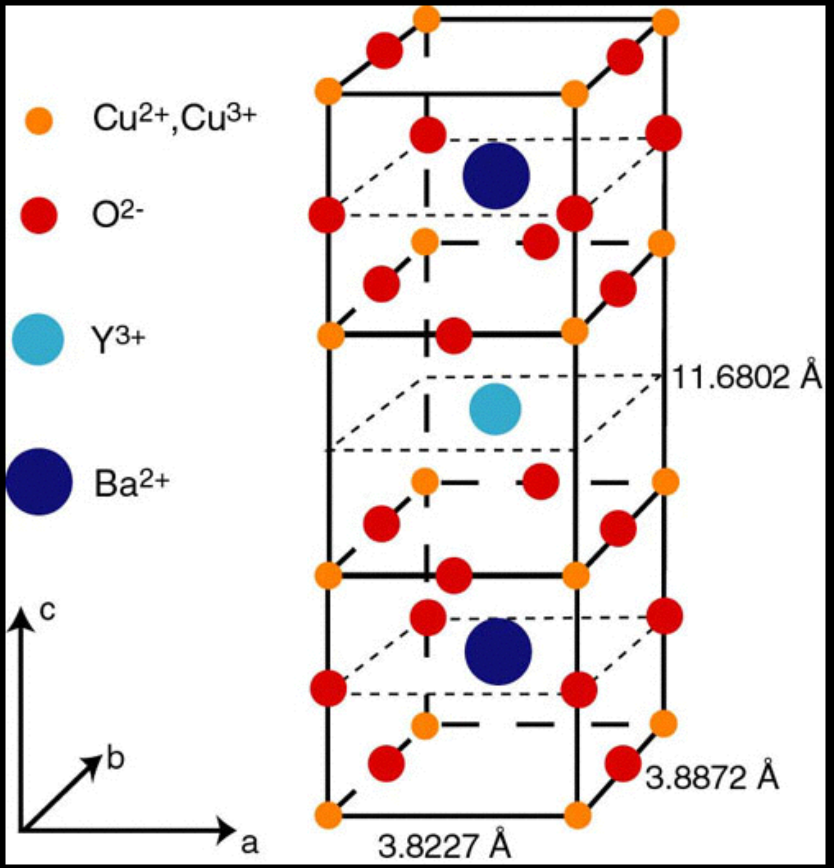
- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2) = $1+p$.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.
- $$\frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}} = \frac{\pi^2 k_B^2}{3e^2}$$

- Wiedemann-Franz law in a Fermi liquid:

$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$

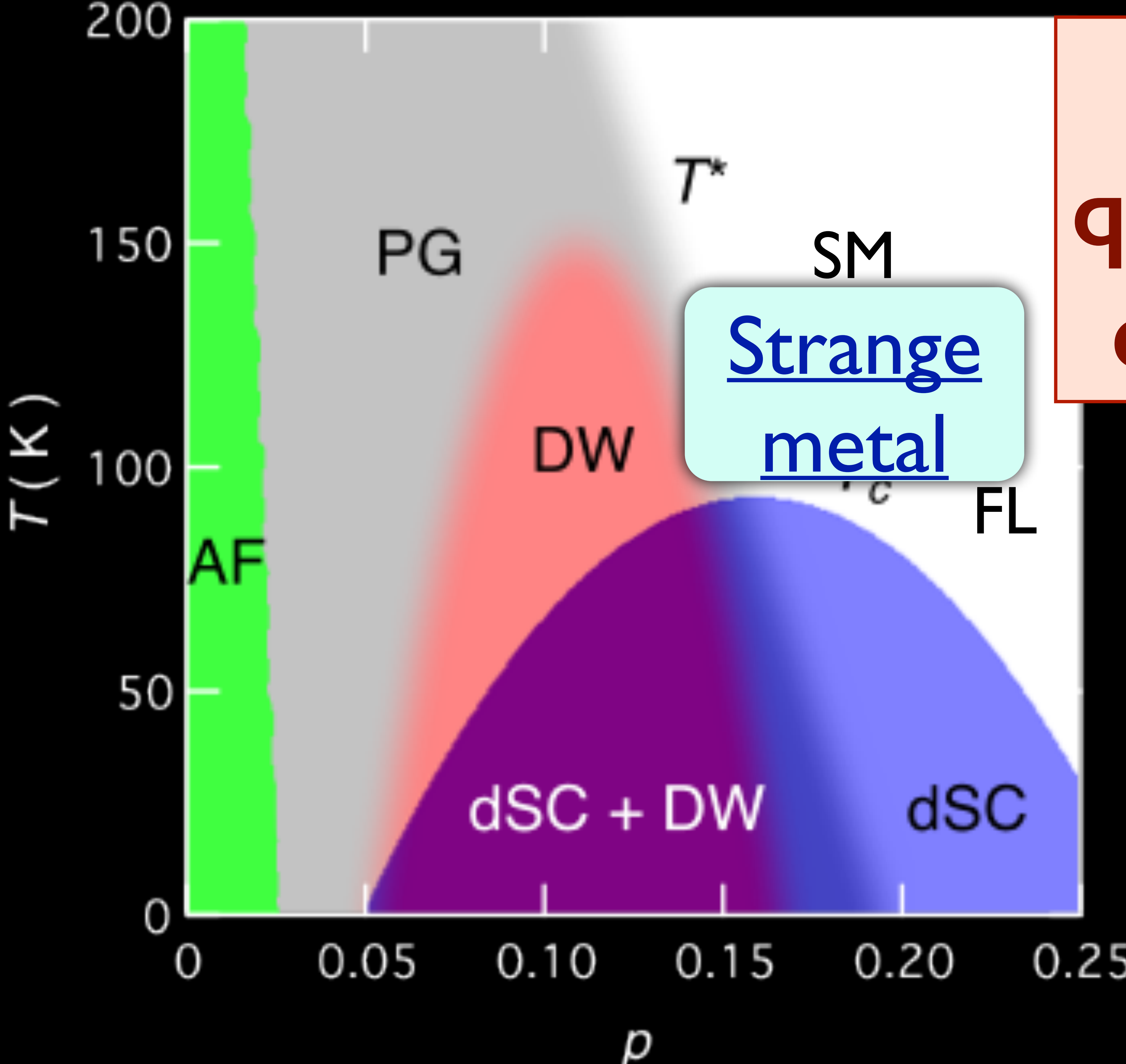






$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis



No quasiparticle excitations

Quantum matter without quasiparticles

1. A solvable model of a strange metal
2. Holography and charged black holes
3. Theory of transport in strange metals
4. The (slightly less) strange metal in graphene

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Infinite-range model of a Fermi liquid

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{t_{ij}^2} = J^2$

Fermions occupying the eigenstates of a
N x *N* random matrix

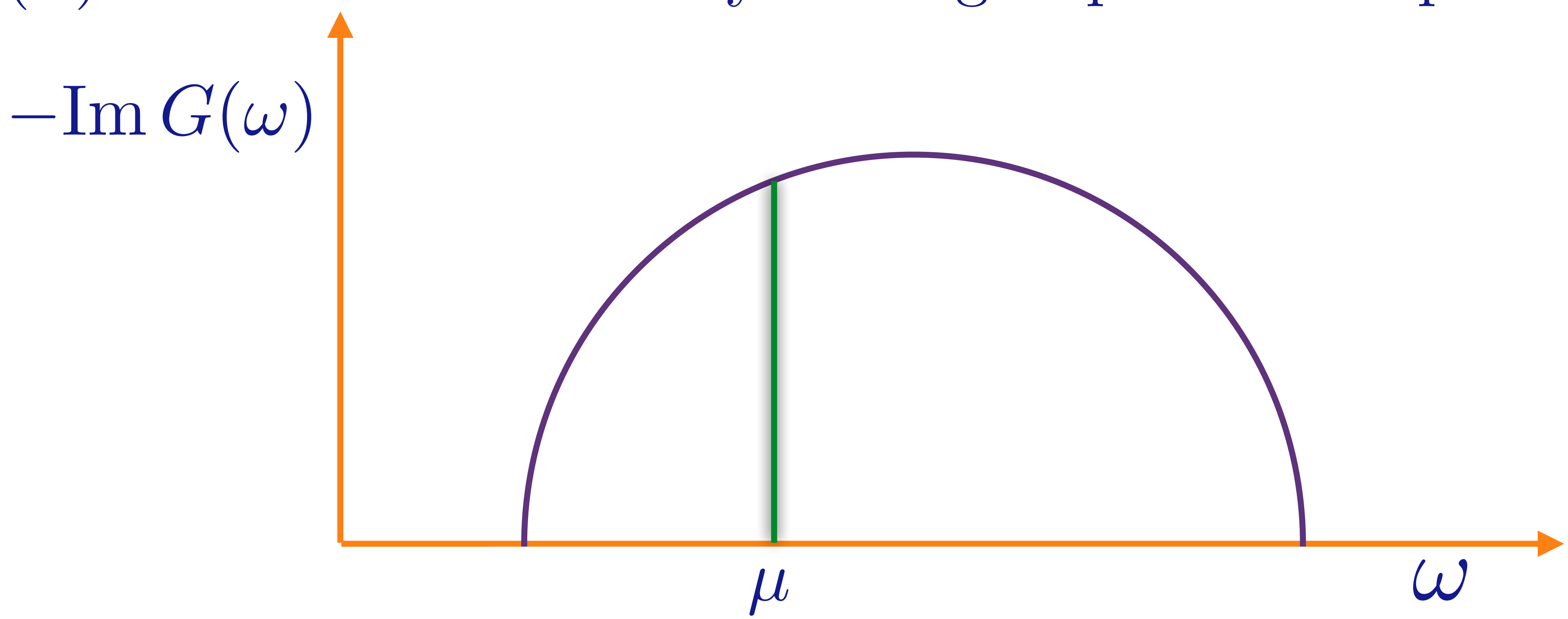
Infinite-range model of a Fermi liquid

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Fermions occupying eigenstates with a "semi-circular" density of states

Infinite-range model of a strange metal

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c^\dagger_{i\alpha} c_{i\beta} c^\dagger_{j\beta} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c^\dagger_{j\beta} + c^\dagger_{j\beta} c_{i\alpha} = \delta_{ij}\delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c^\dagger_{i\alpha} c_{i\alpha} = \mathcal{Q}$$

J_{ij} are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$
$N \to \infty$ at $M = 2$ yields spin-glass ground state.
$N \to \infty$ and then $M \to \infty$ yields critical strange metal

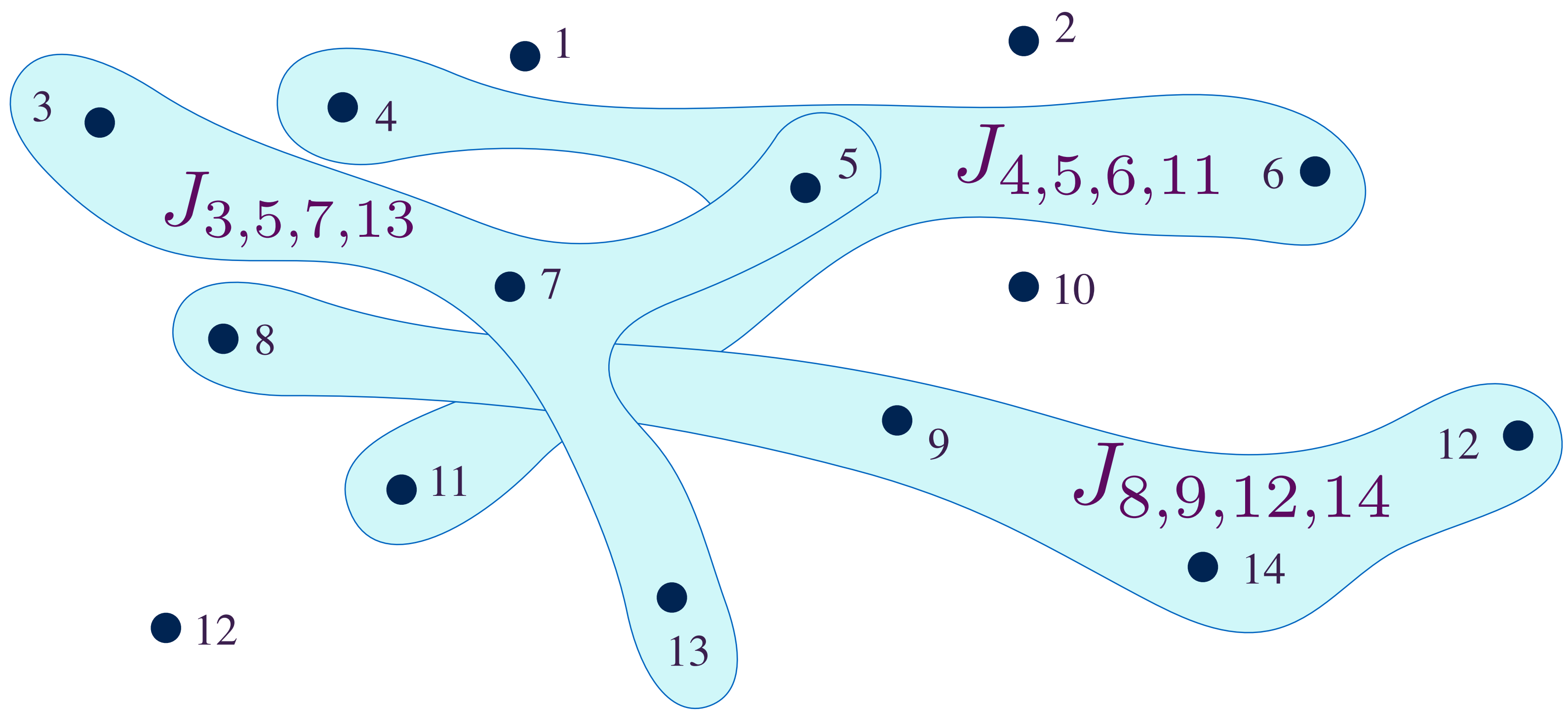
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields <u>same</u> critical strange metal; simpler to study numerically

A. Kitaev, unpublished; S. Sachdev, arXiv:1506.05111

Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. Let us also define $\widetilde{\Sigma}(z) = \Sigma(z) - \mu$.

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Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \; \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

$\mathcal{E}$ encodes the particle-hole asymmetry

The value of $\mathcal{E}$ is determined by the fermion density $\mathcal{Q}$, with particle-like excitations (positive frequencies) for $\mathcal{Q} \to 0$, and hole-like excitations (negative frequencies) for $\mathcal{Q} \to 1$:

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi} \tan^{-1}\left(e^{2\pi\mathcal{E}}\right).$$

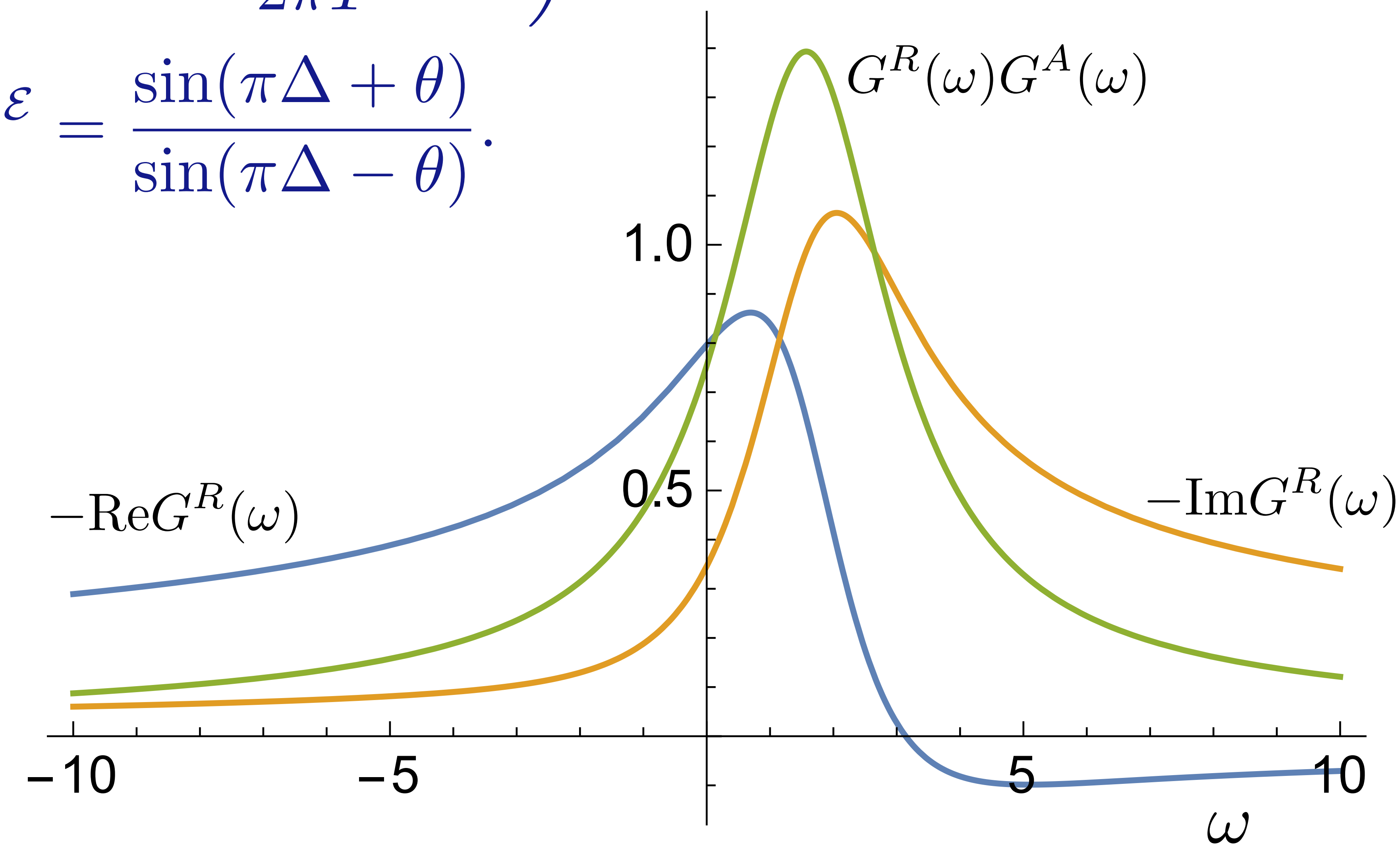
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

At non-zero temperature, T, we obtain the Green's function

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $\quad e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.



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A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$, and can be viewed as each site acquiring the universal boundary entropy of the multichannel Kondo problem.

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta Phys. Rev. B **58**, 3794 (1998)
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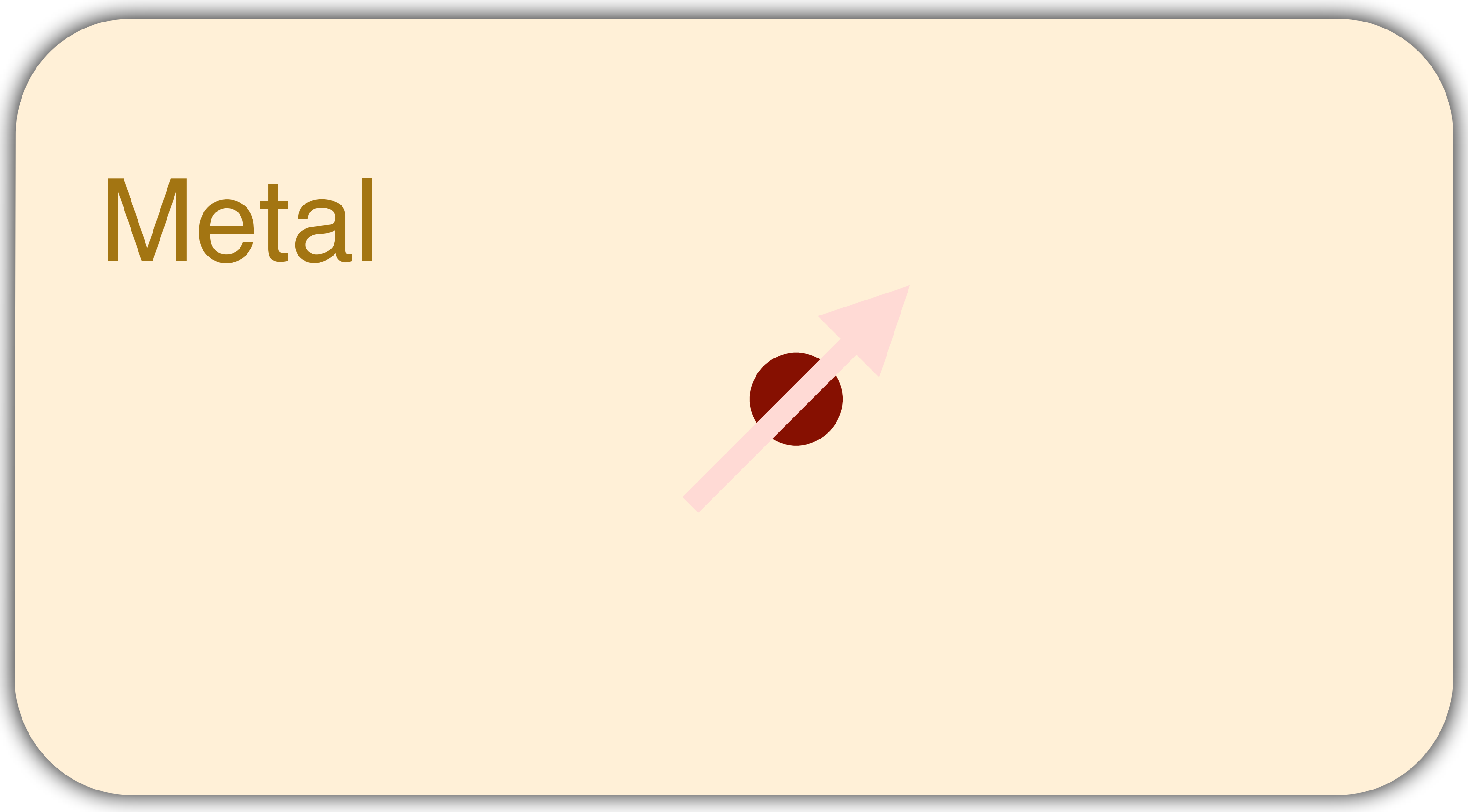
Free spin has entropy $\ln(2S+1)$

Kondo-screened
spin has
no entropy



N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
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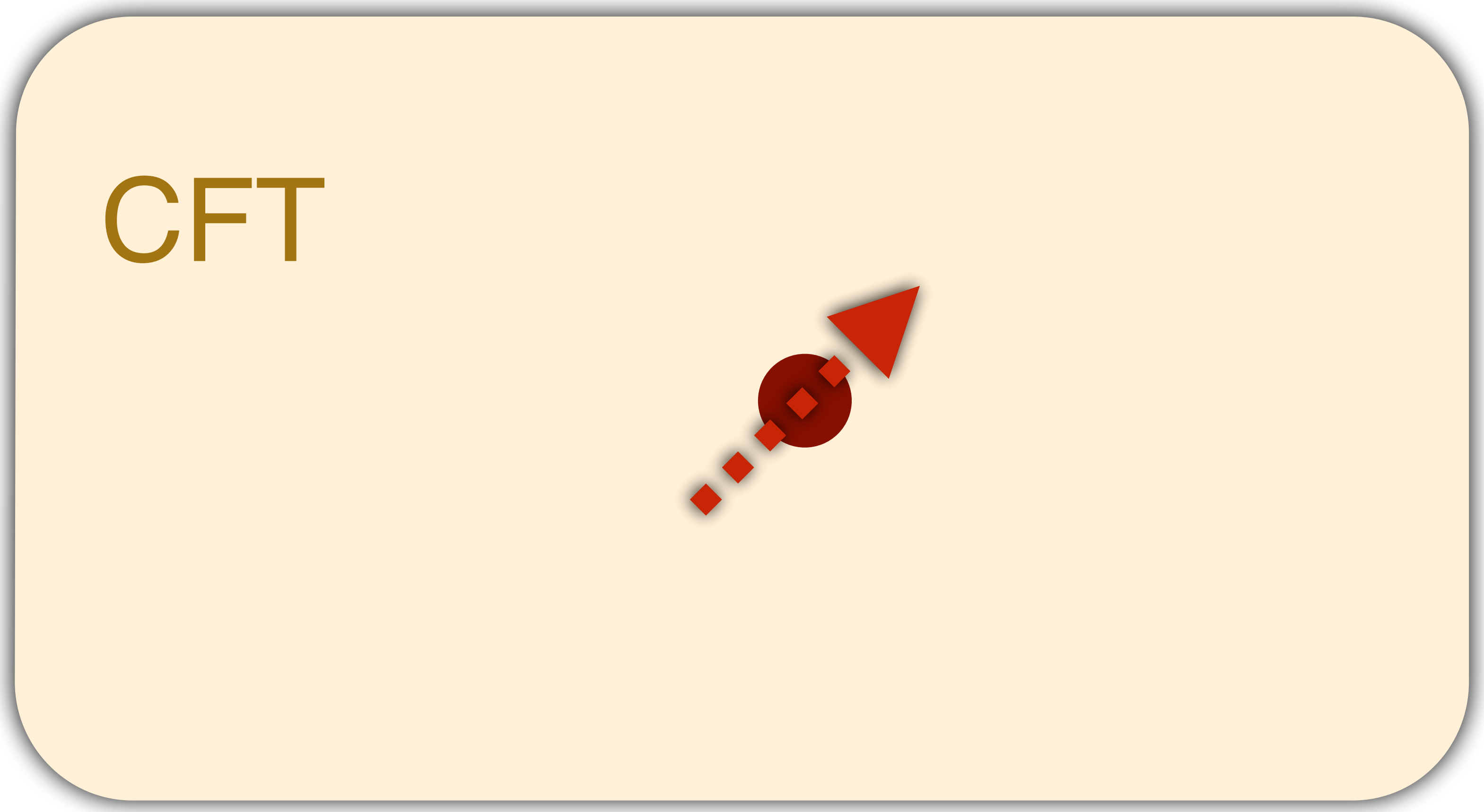
"Critically-screened" spin has "irrational" entropy



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This entropy obeys

$$\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

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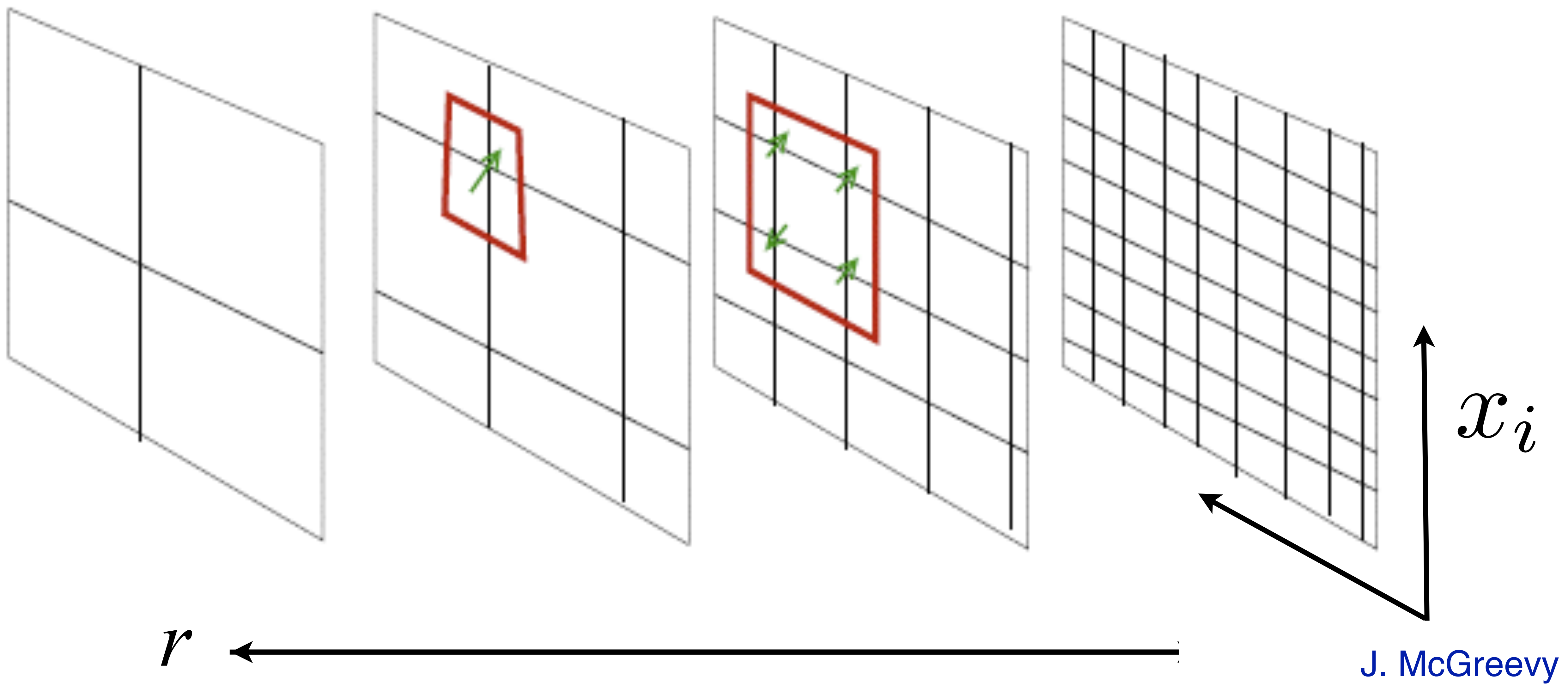
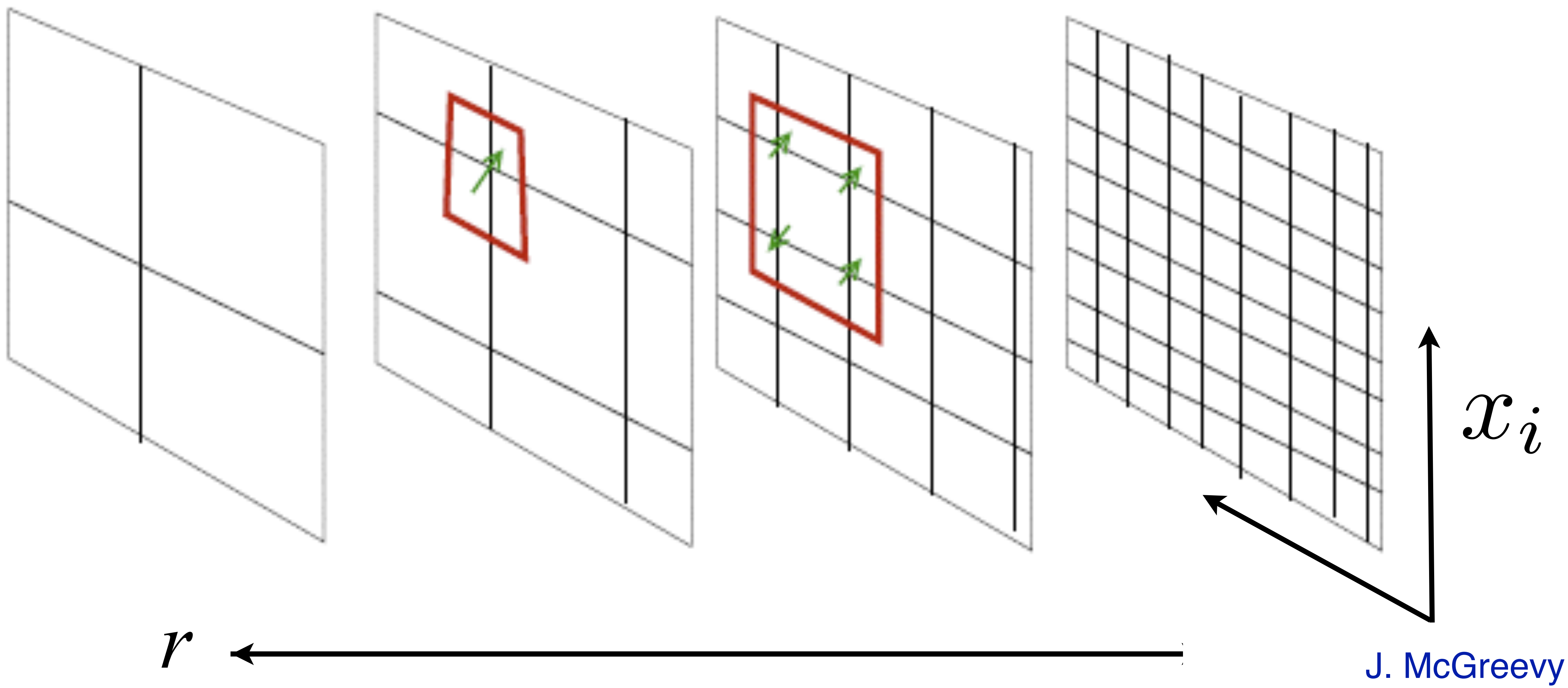
Holography



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

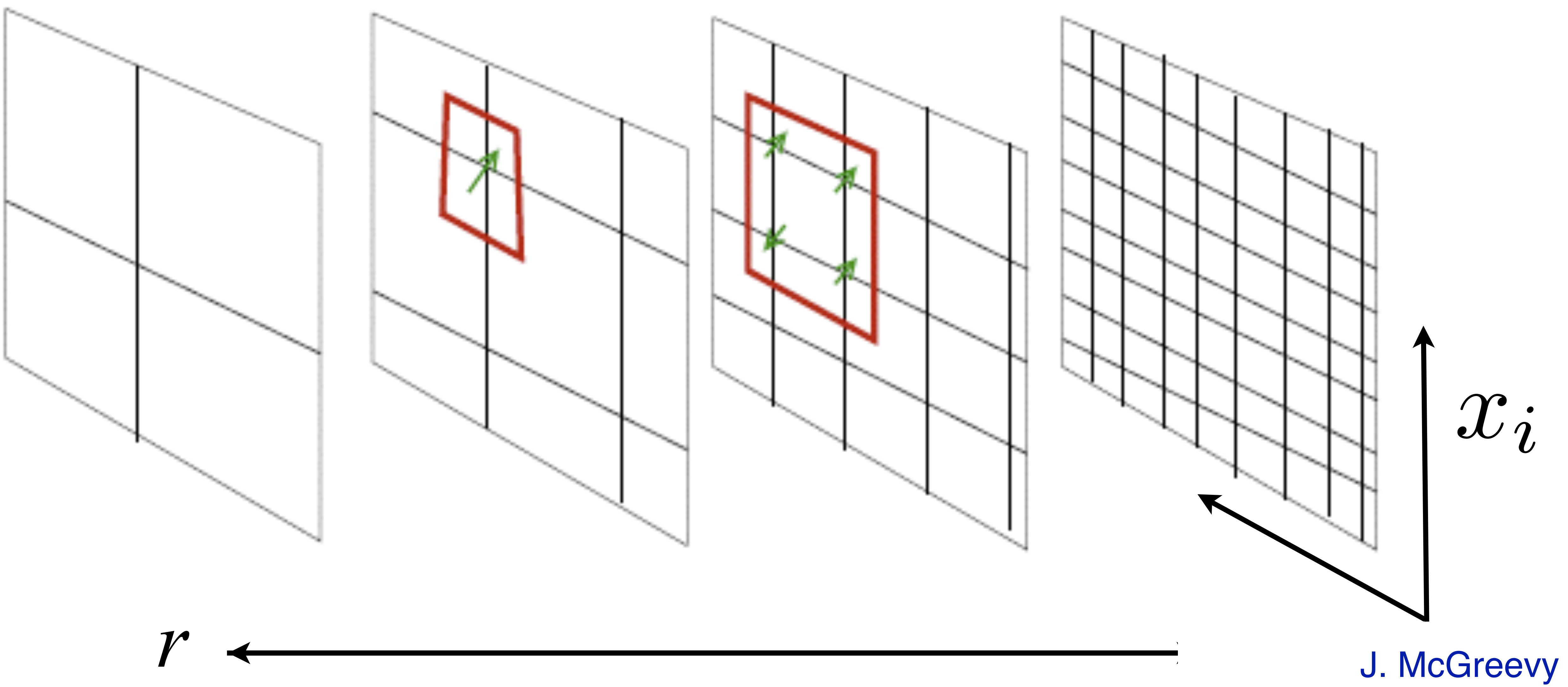
Holography



For a relativistic CFT in d spatial dimensions, the proper length, ds, in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography



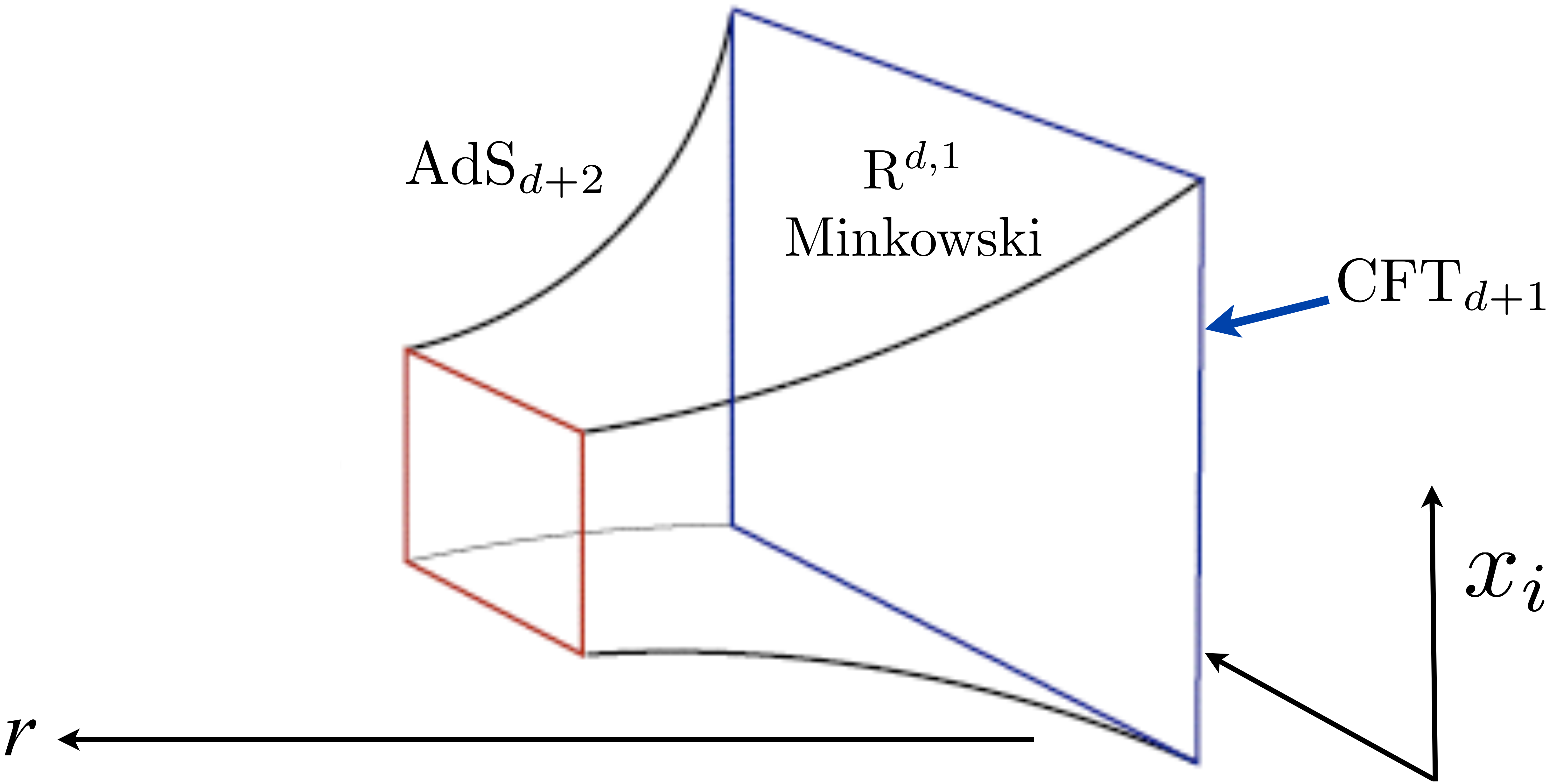
This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS_{d+2}.

AdS/CFT correspondence at zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + d\vec{x}^2\right]$$

AdS/CFT correspondence at non-zero temperature

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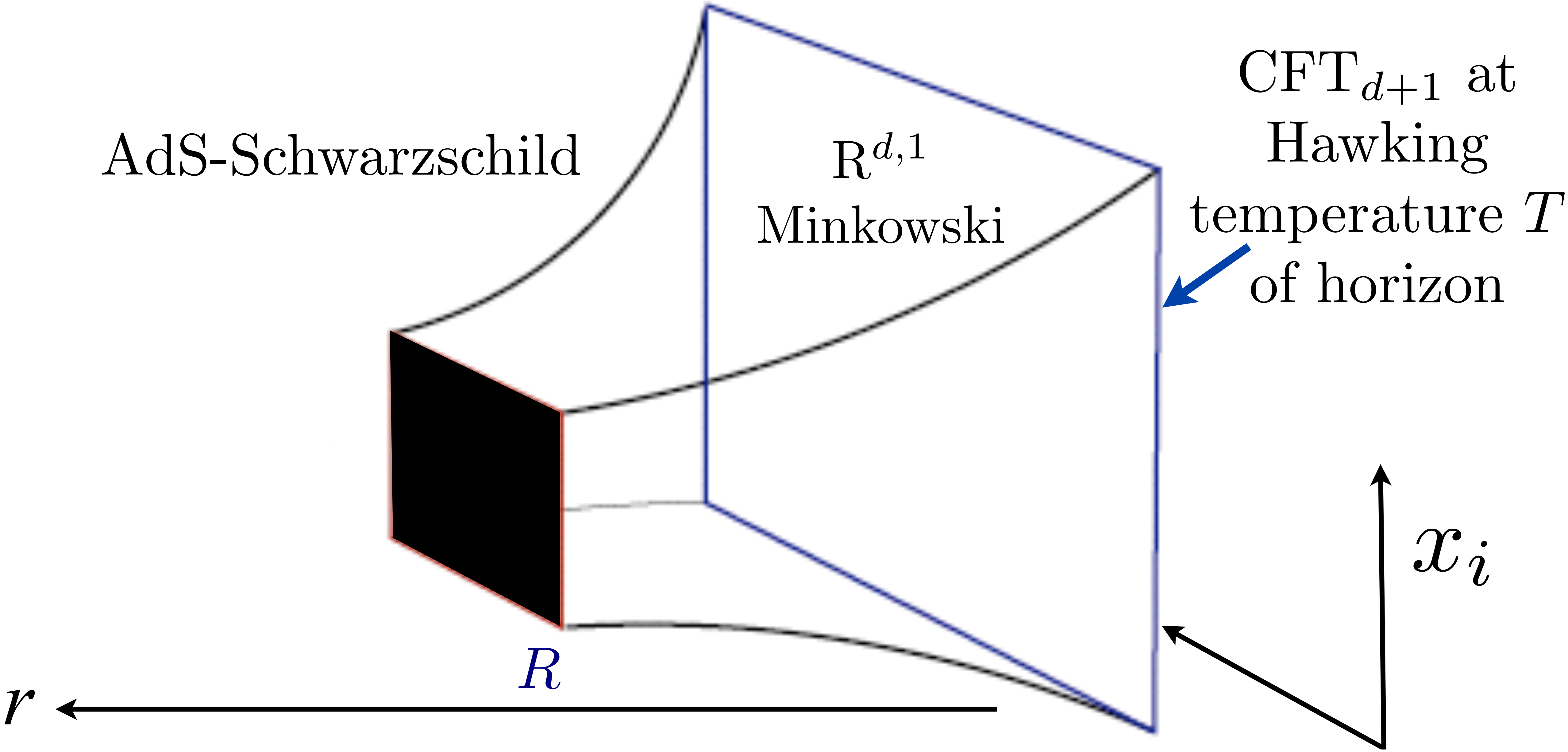

$$ds^2 = \left(\frac{L}{r}\right)^2\left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right]$$

with $f(r) = 1 - (r/R)^{d+1}$ and $T = (d+1)/(4\pi R)$.

AdS/CFT correspondence at non-zero temperature

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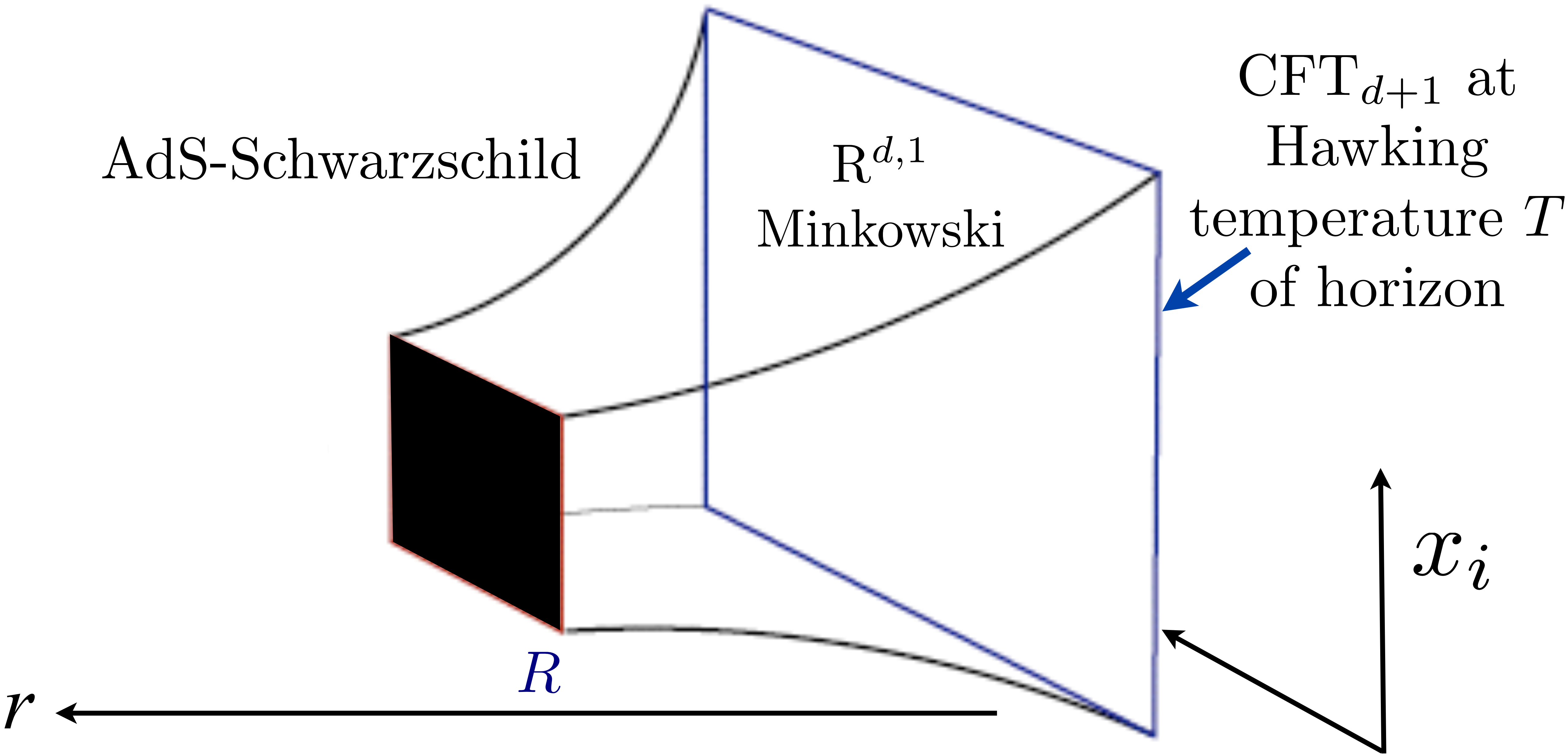
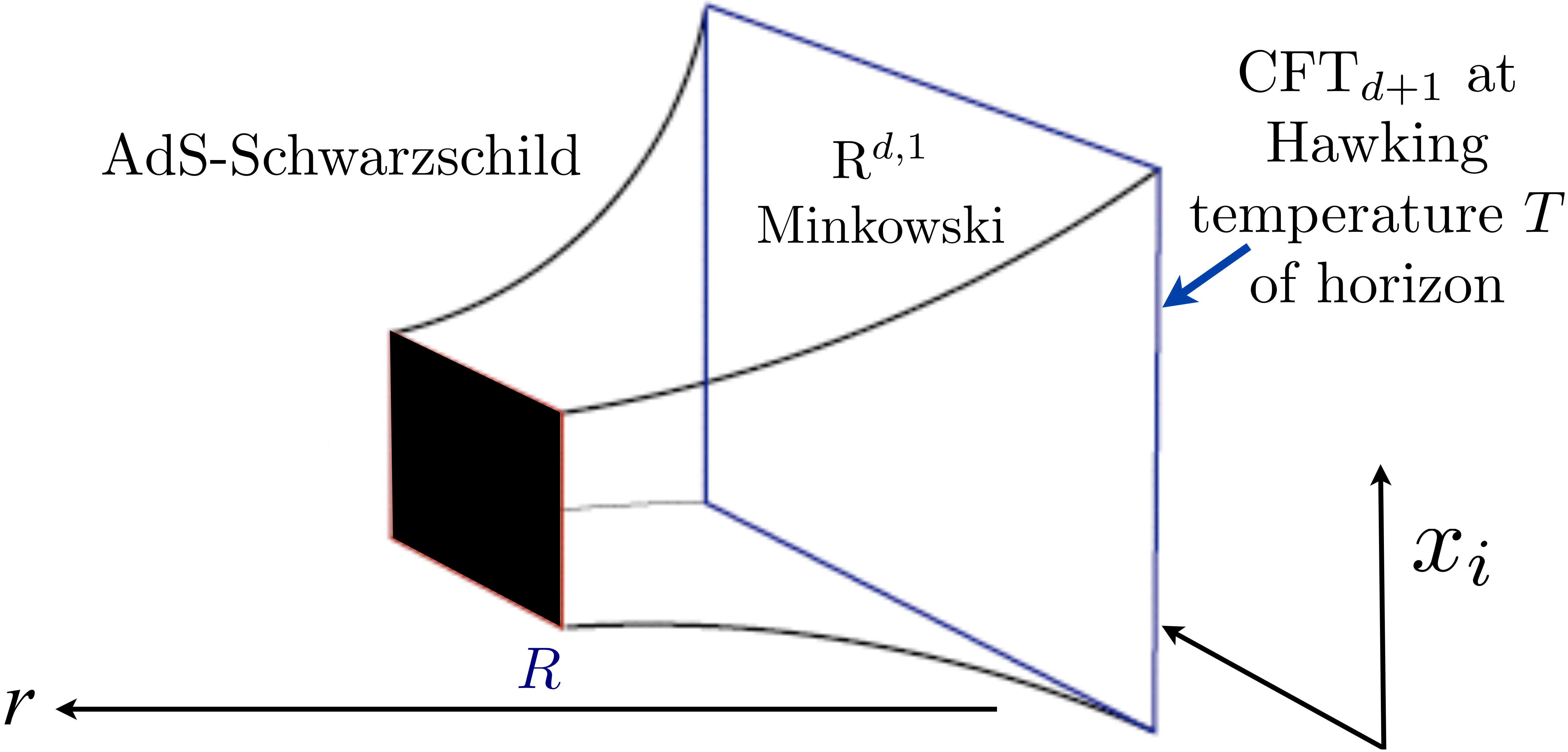


Entropy density of CFT_{d+1}, $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\mathrm{BH}} \sim T^d$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$

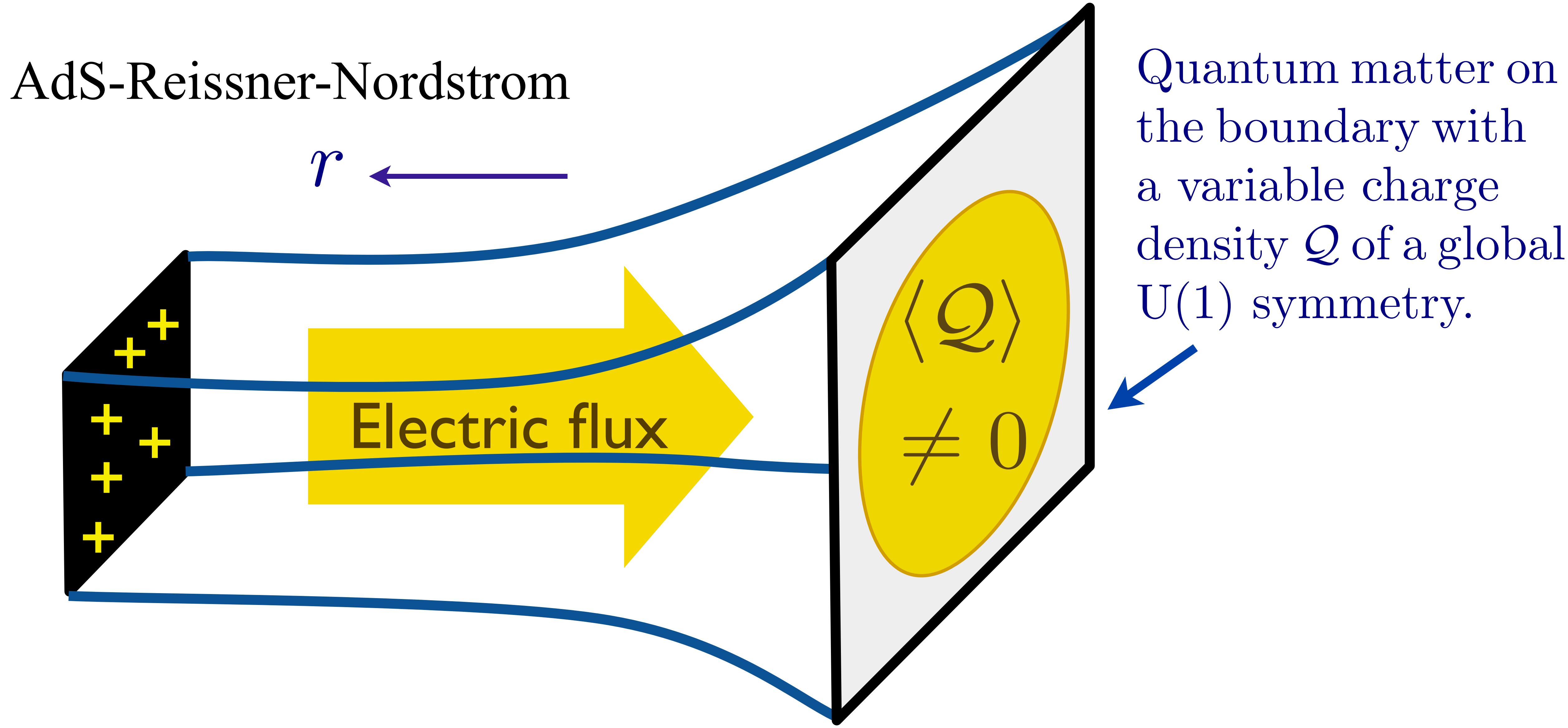


For SU(N) SYM in $d=3$, $\mathcal{S}_{\rm BH} = (\pi^2/2)N^2T^3$. But there is (still) no confirmation of this from a field-theory computation on SYM.

Gubser, Klebanov, Peet 96

Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2} F^2 \right) \right]$



A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

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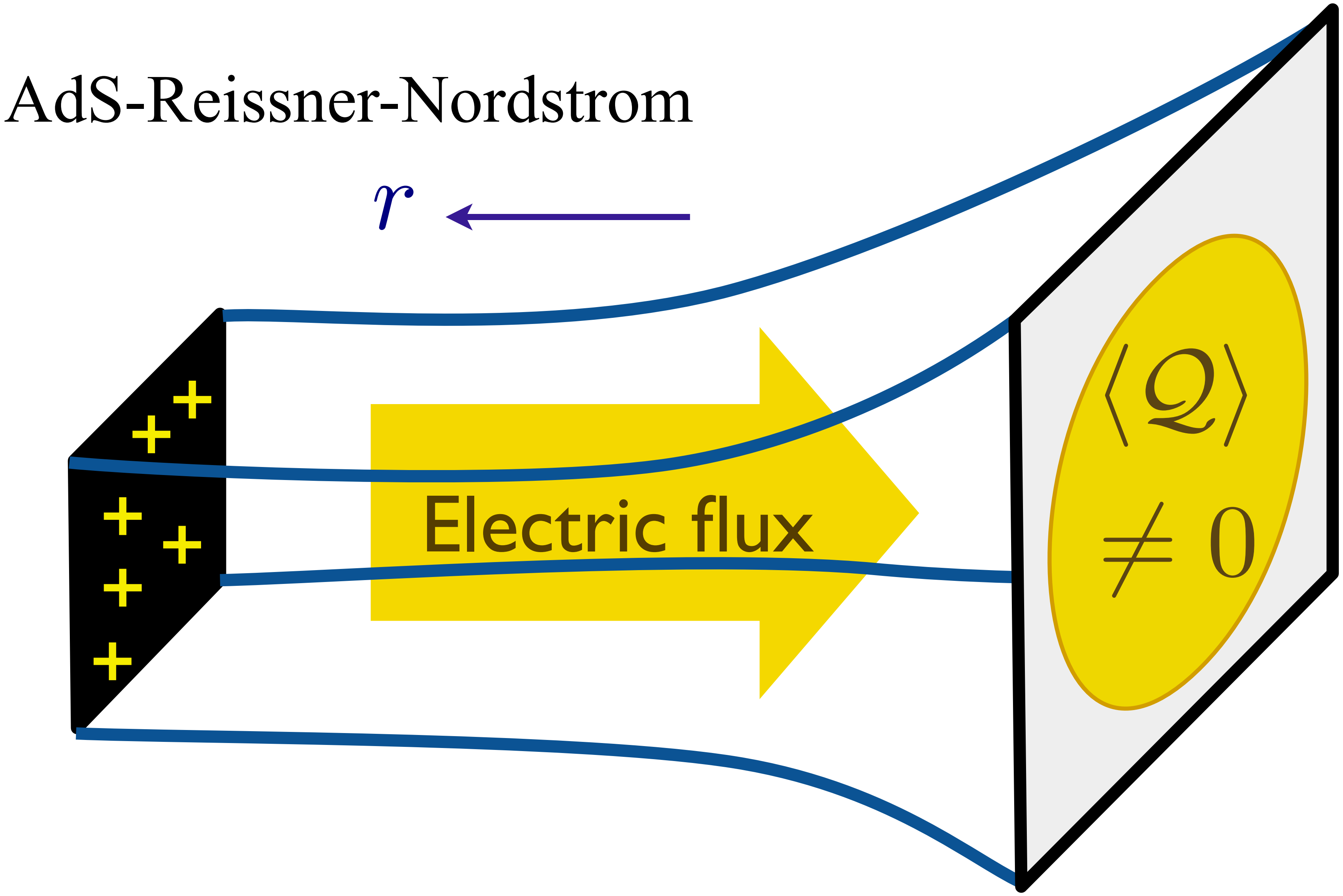
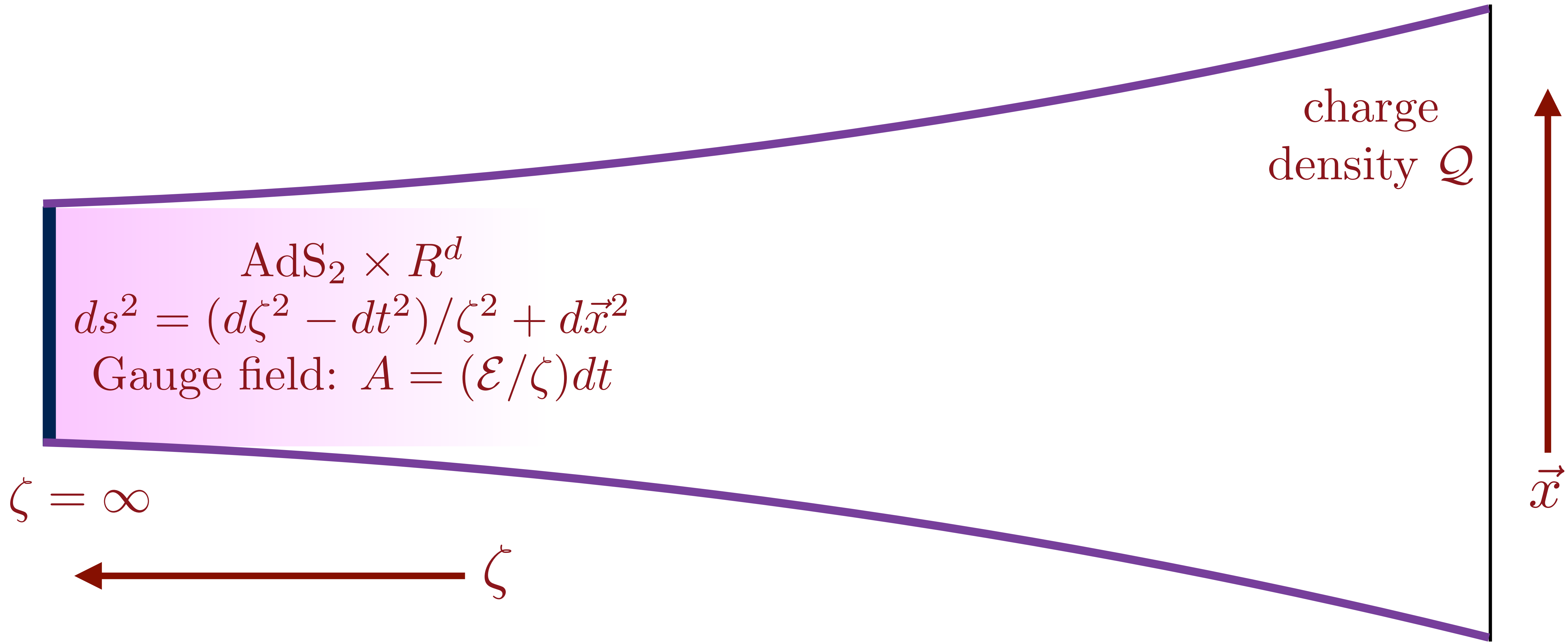


Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

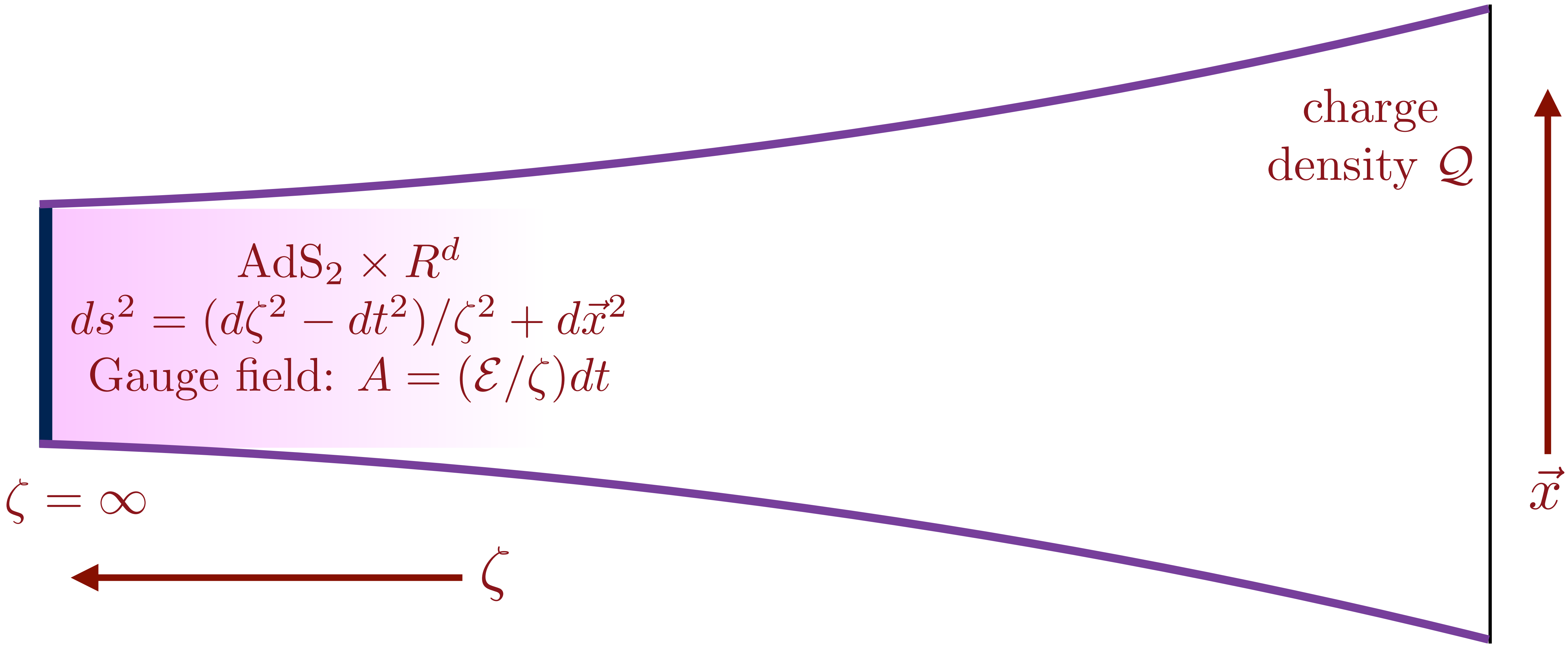
A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

General Relativity of charged black branes



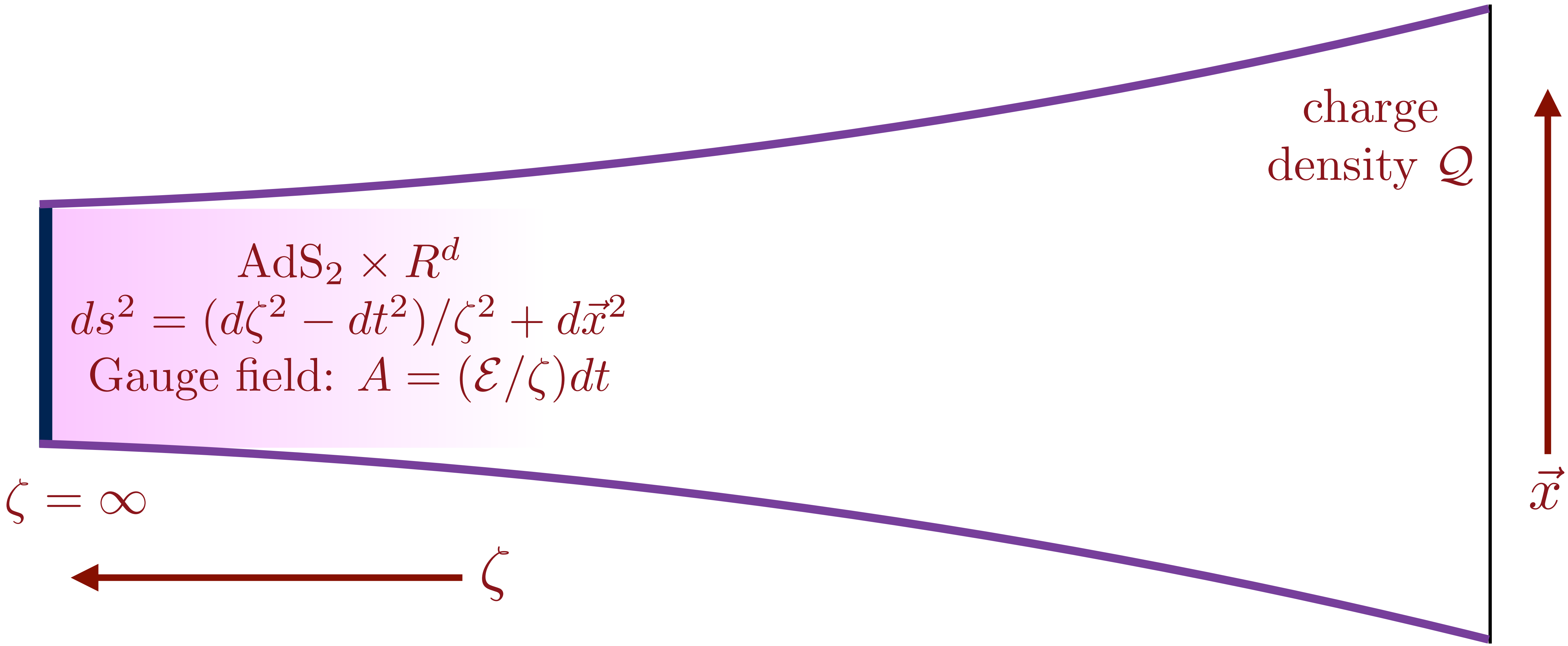
- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.

General Relativity of charged black branes



- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.
- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{\mathrm{BH}}$

General Relativity of charged black branes



- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.
- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{\mathrm{BH}}$
- Both $\mathcal{E}$ and $\mathcal{S}_{\mathrm{BH}}$ are determined by $\mathcal{Q}$, and both vanish as $\mathcal{Q} \to 0$.

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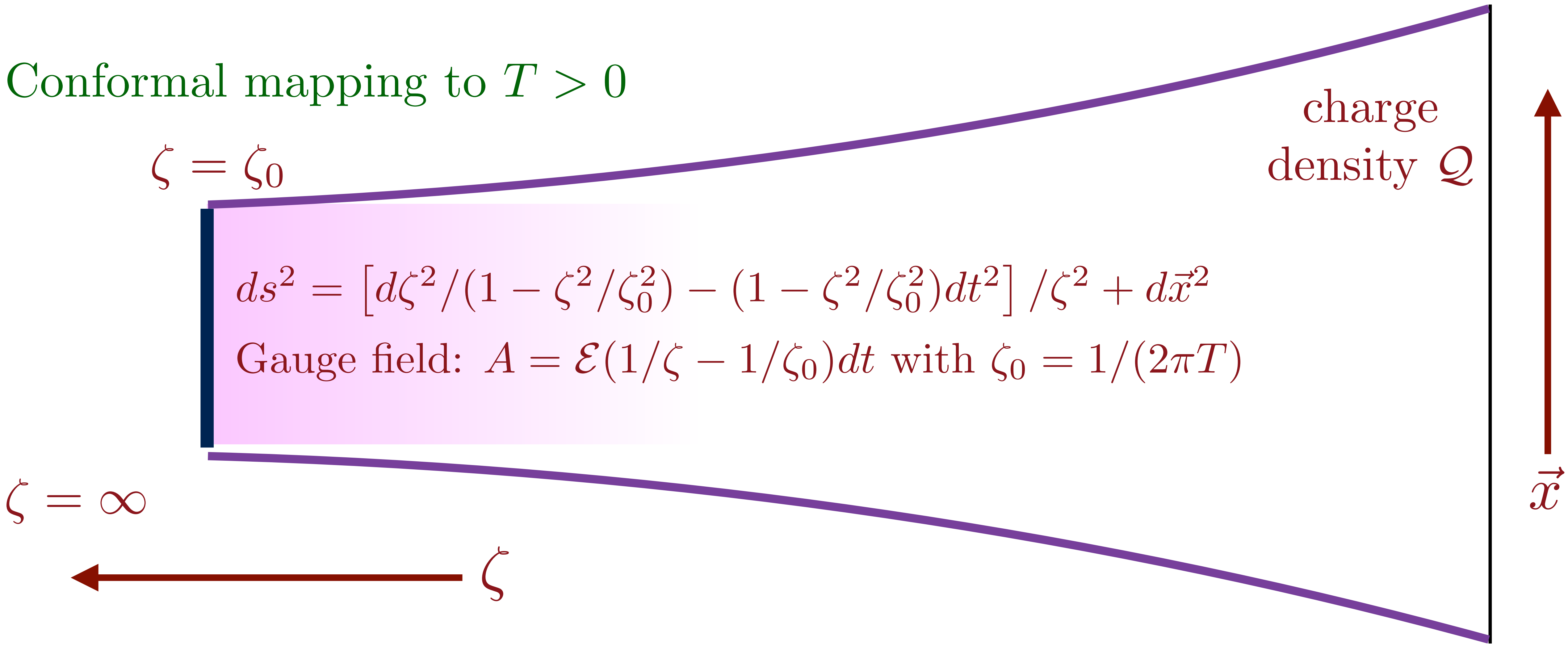

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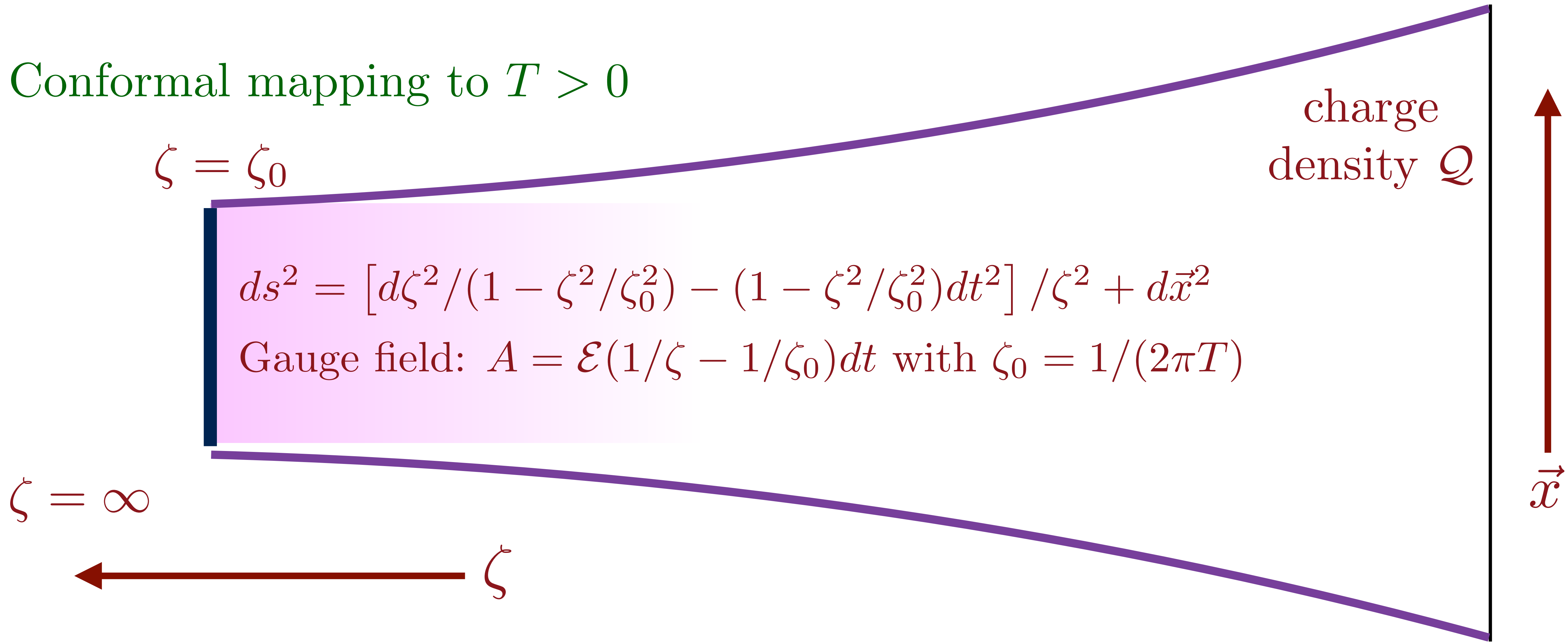

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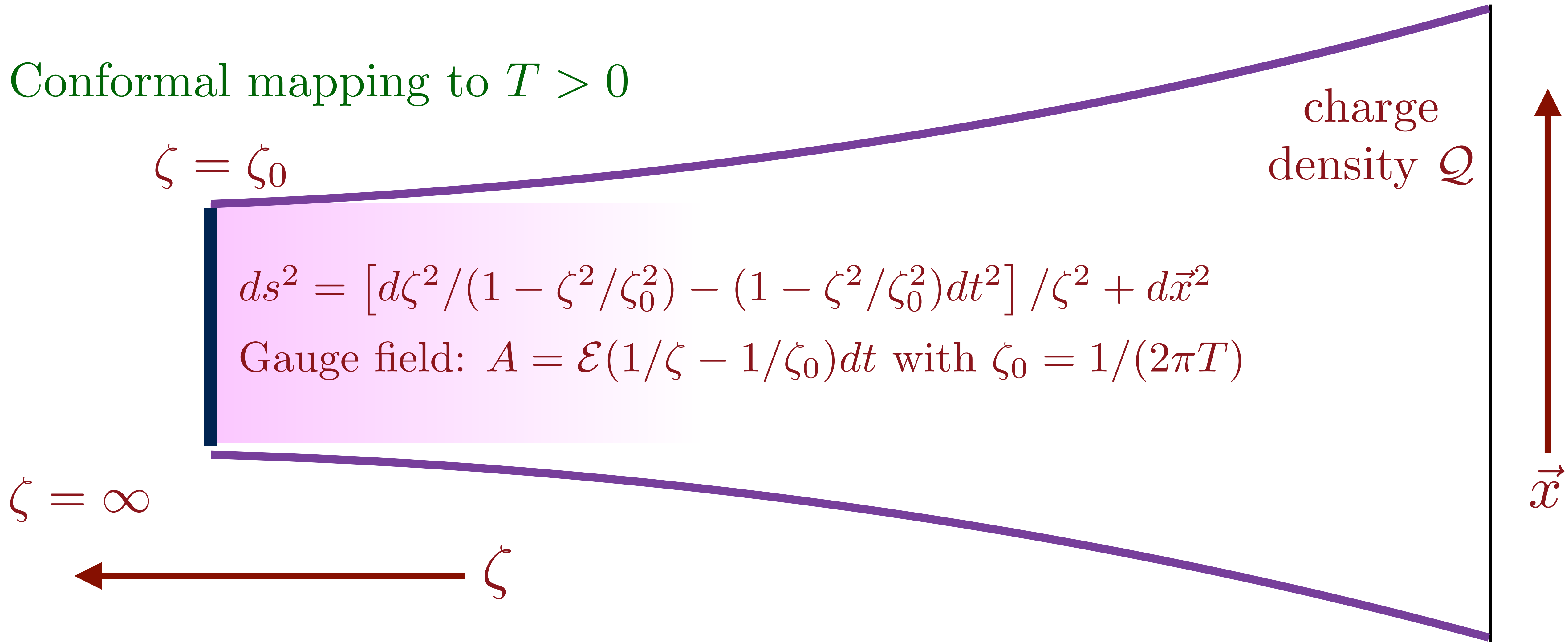

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$$\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

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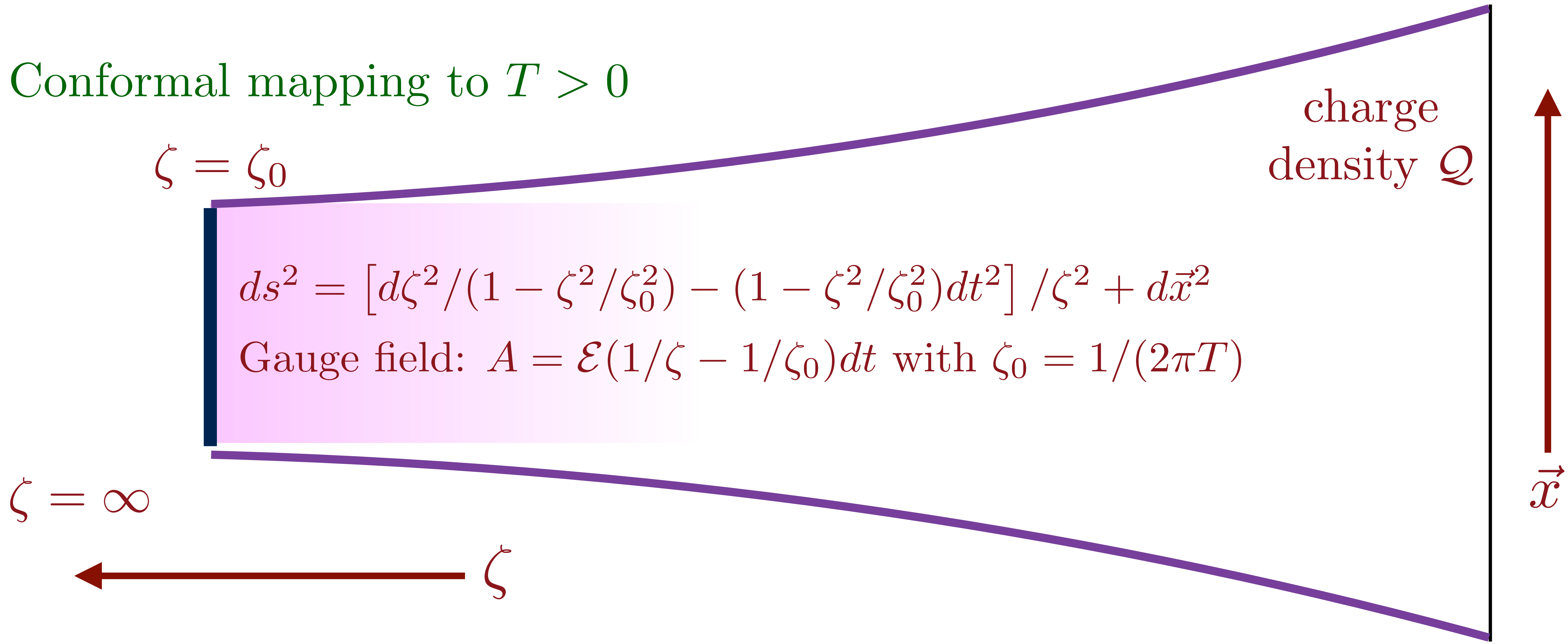

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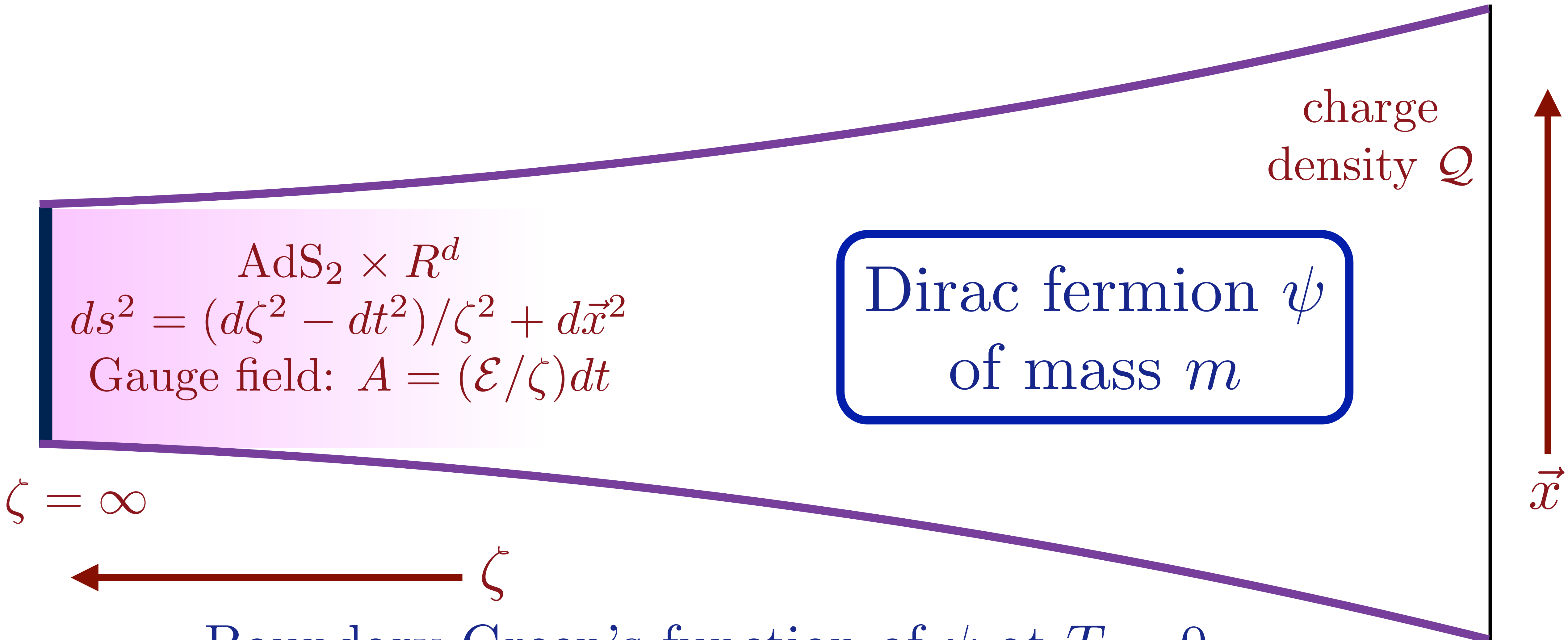
- Also obeyed by the Wald entropy in higher derivative gravity.

Quantum fields on charged black branes



Boundary Green's function of ψ at $T = 0$

$$\mathrm{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\,|\omega|^{-(1-2\Delta)} & , \ \omega < 0. \end{cases}$$

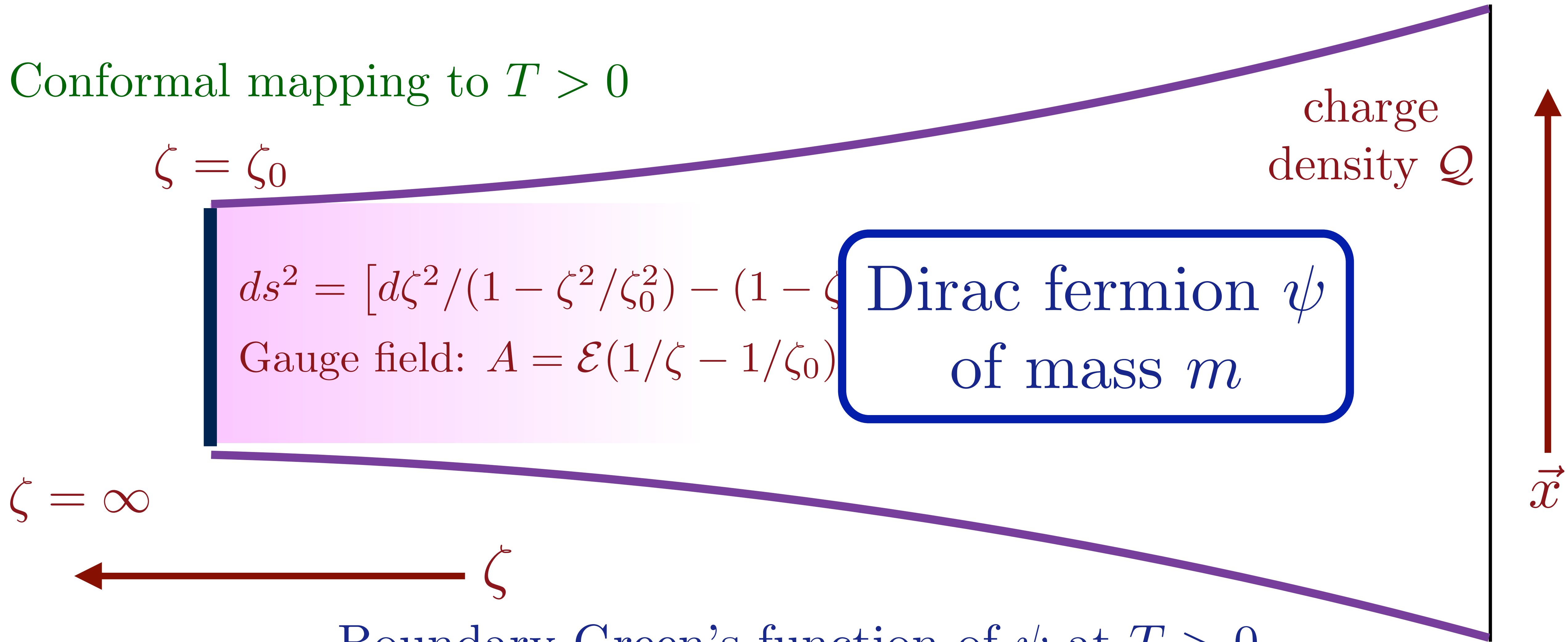
where the fermion scaling dimension Δ is a function of m

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

Quantum fields on charged black branes

Conformal mapping to $T > 0$



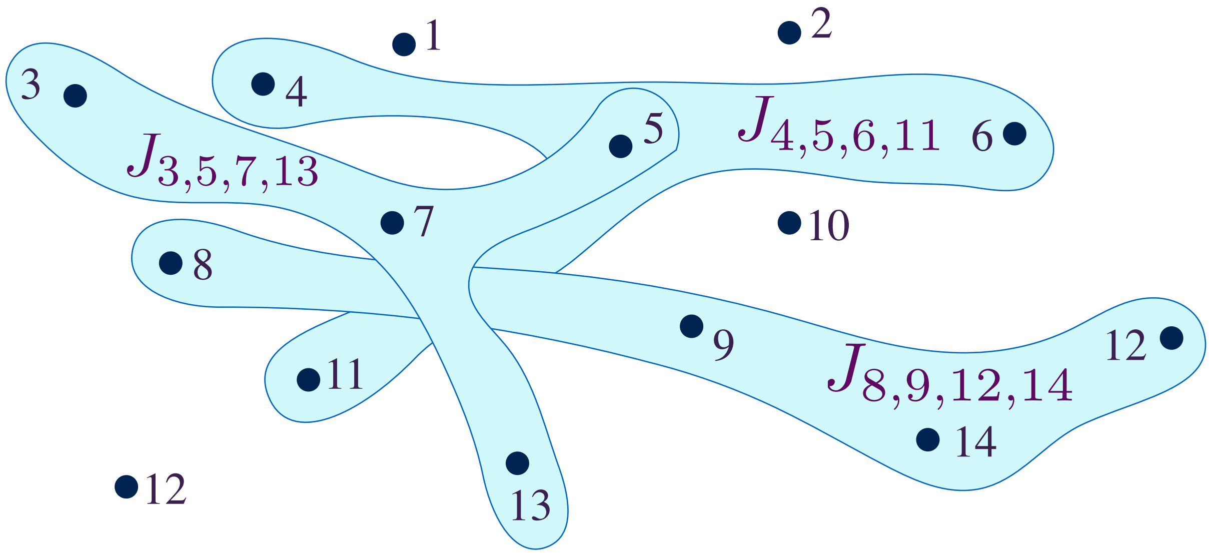
Boundary Green's function of ψ at $T > 0$

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell} \, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

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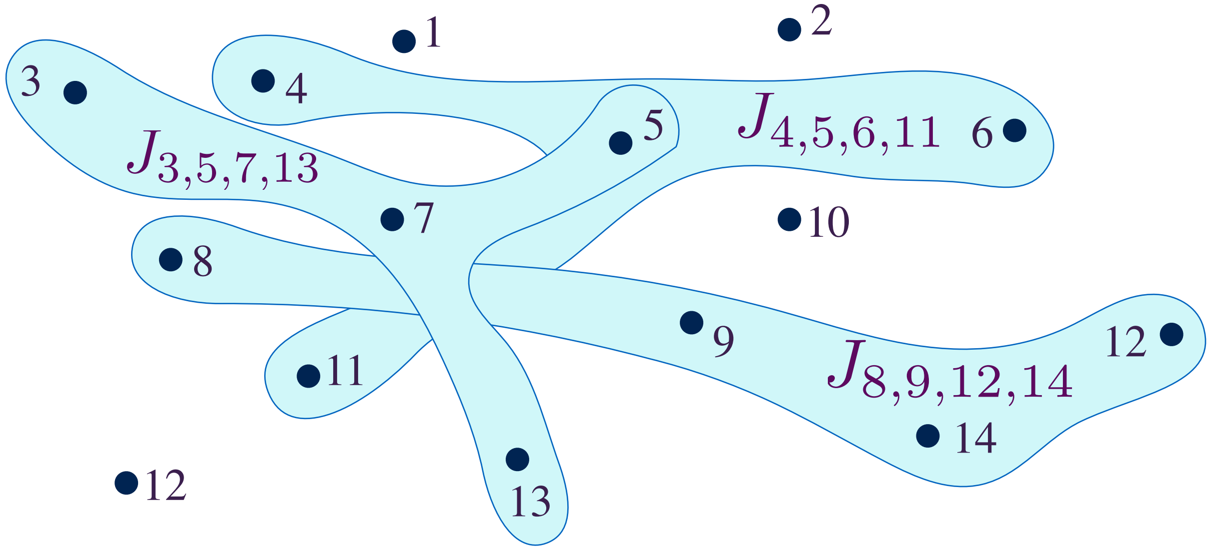

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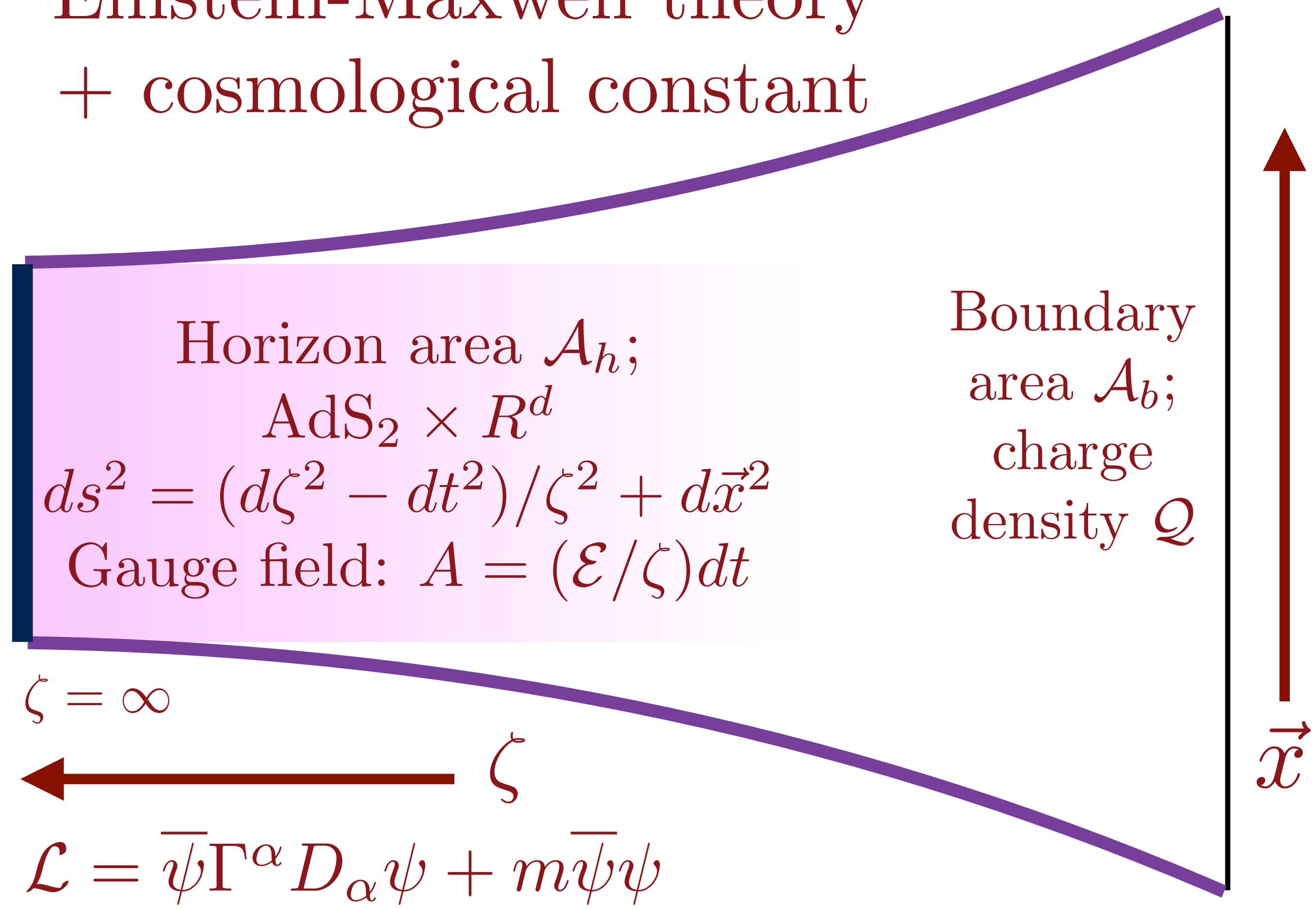
Einstein-Maxwell theory
+ cosmological constant



$$\mathcal{L} = \overline{\psi} \Gamma^\alpha D_\alpha \psi + m \overline{\psi} \psi$$

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‘Equation of state’ relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical general relativity) yields

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A. Sen, arXiv:hep-th/0506177; S. Sachdev, arXiv:1506.0511

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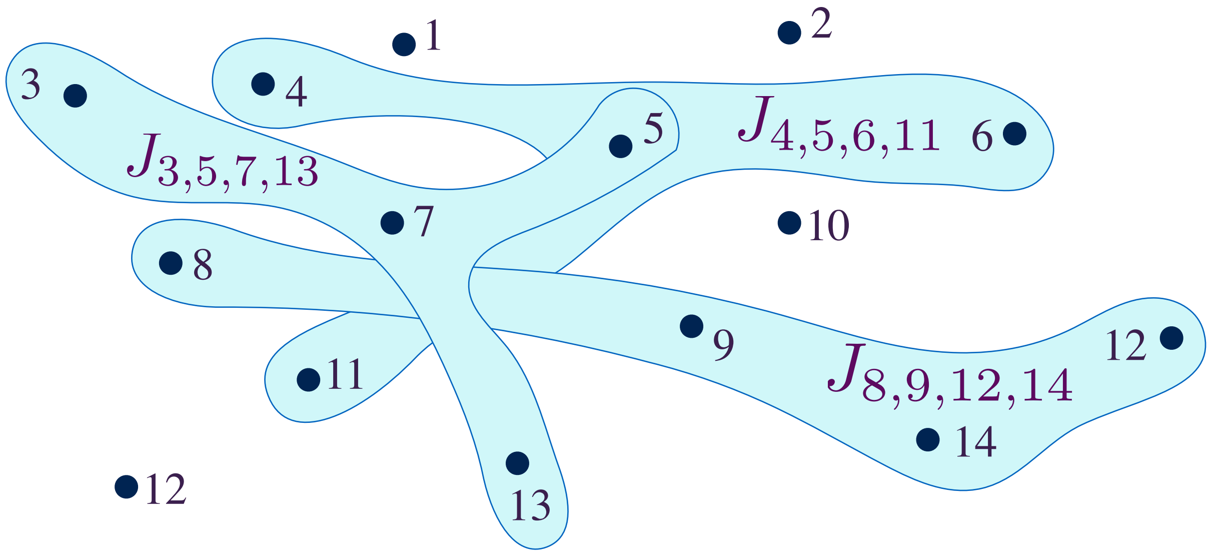

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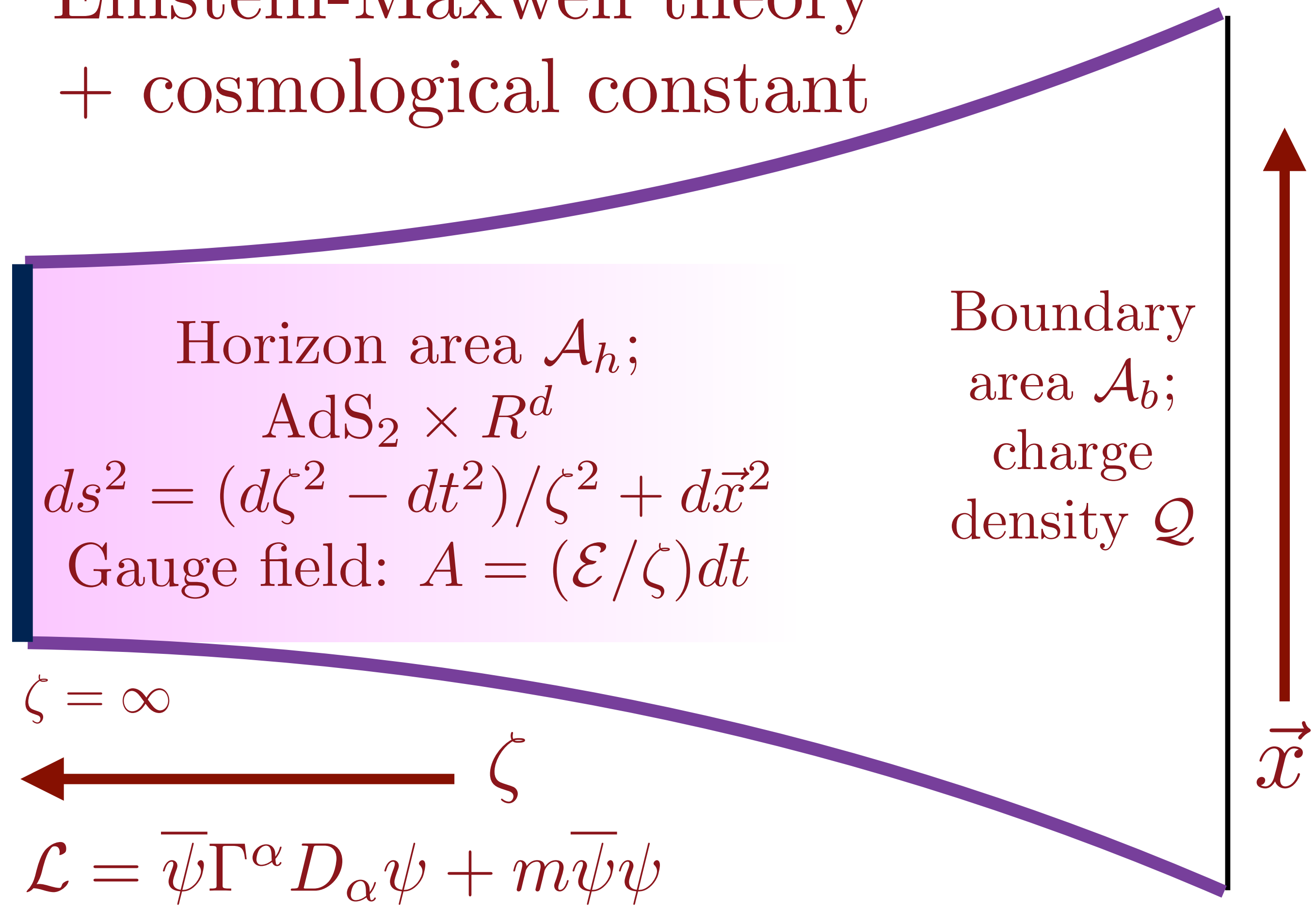

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Evidence for AdS_2 gravity dual of H

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S. Sachdev, arXiv:1506.05111

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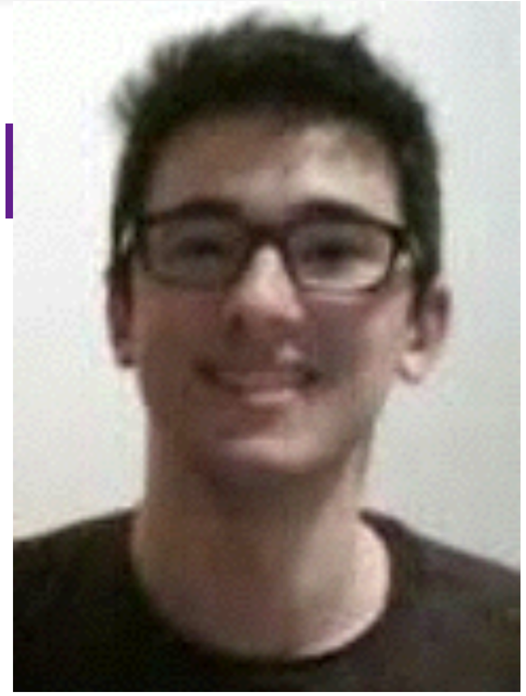
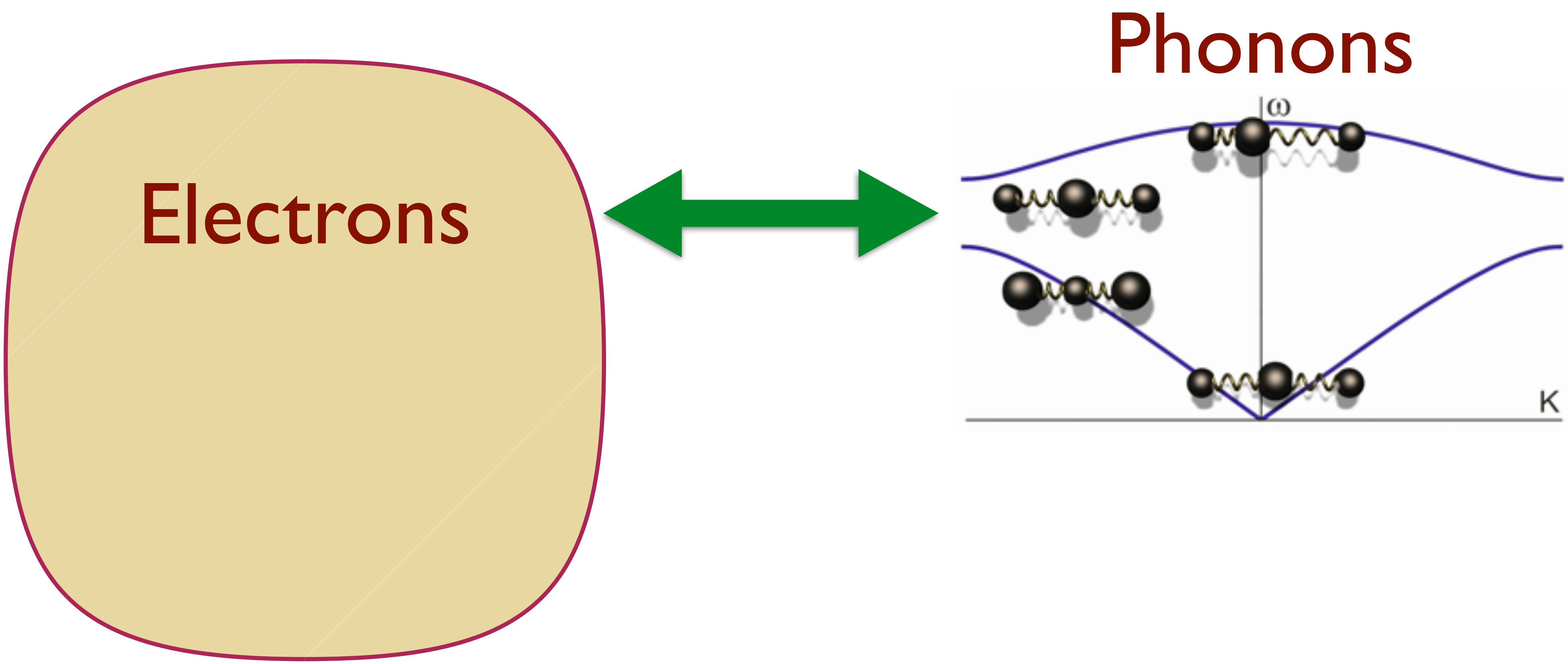


Andrew Lucas

Quasiparticle transport in metals:

- Compute the scattering rate of charged quasiparticles off phonons: this leads to Bloch's law (1930) : a resistivity $\rho(T) \sim T^5$.



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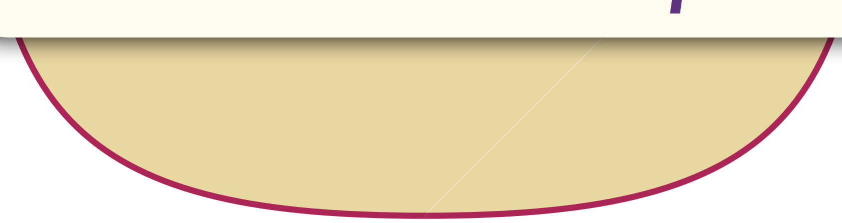
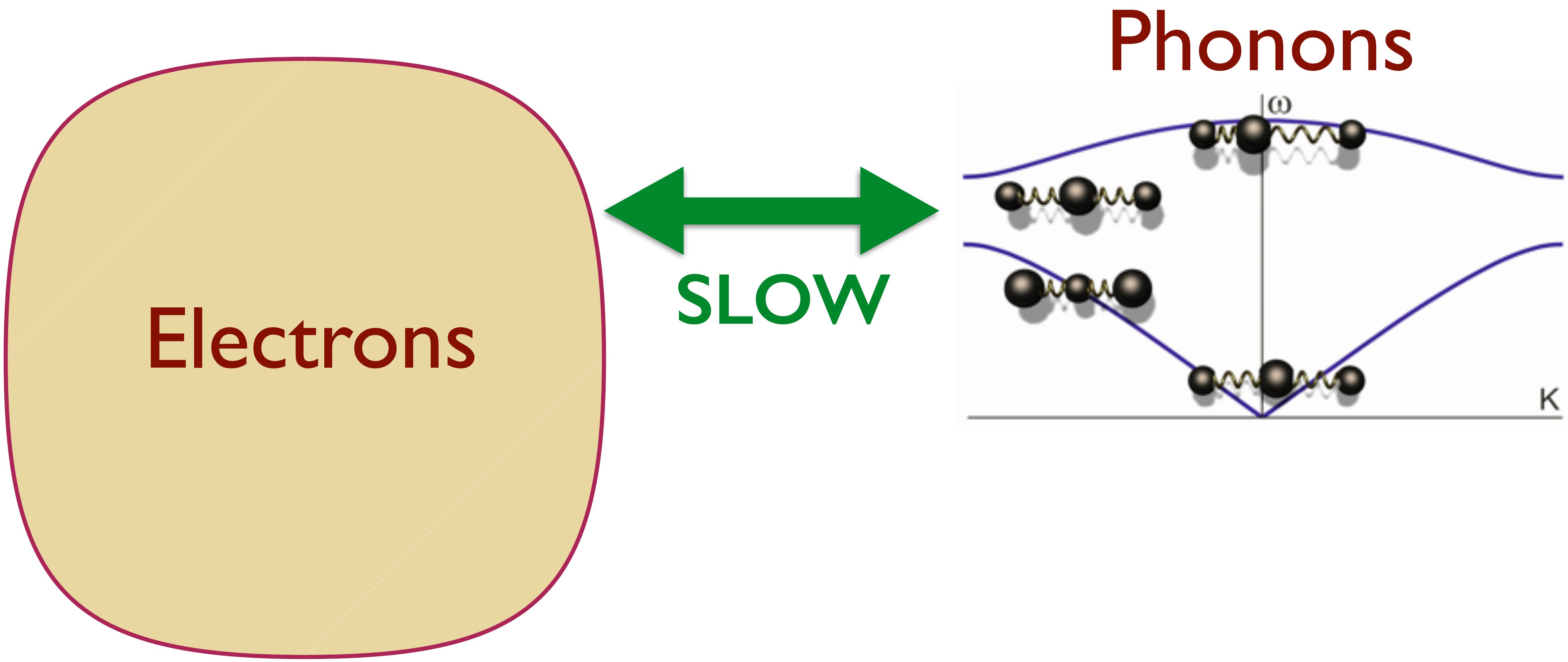
Peierls (1932) argued this was incorrect, and the resistivity was even smaller. Although Peierls was right, it is Bloch's law that is observed in experiments on metals.

It turns out that Peierls' point is far more important for strange metals



Rates of Momentum Flow



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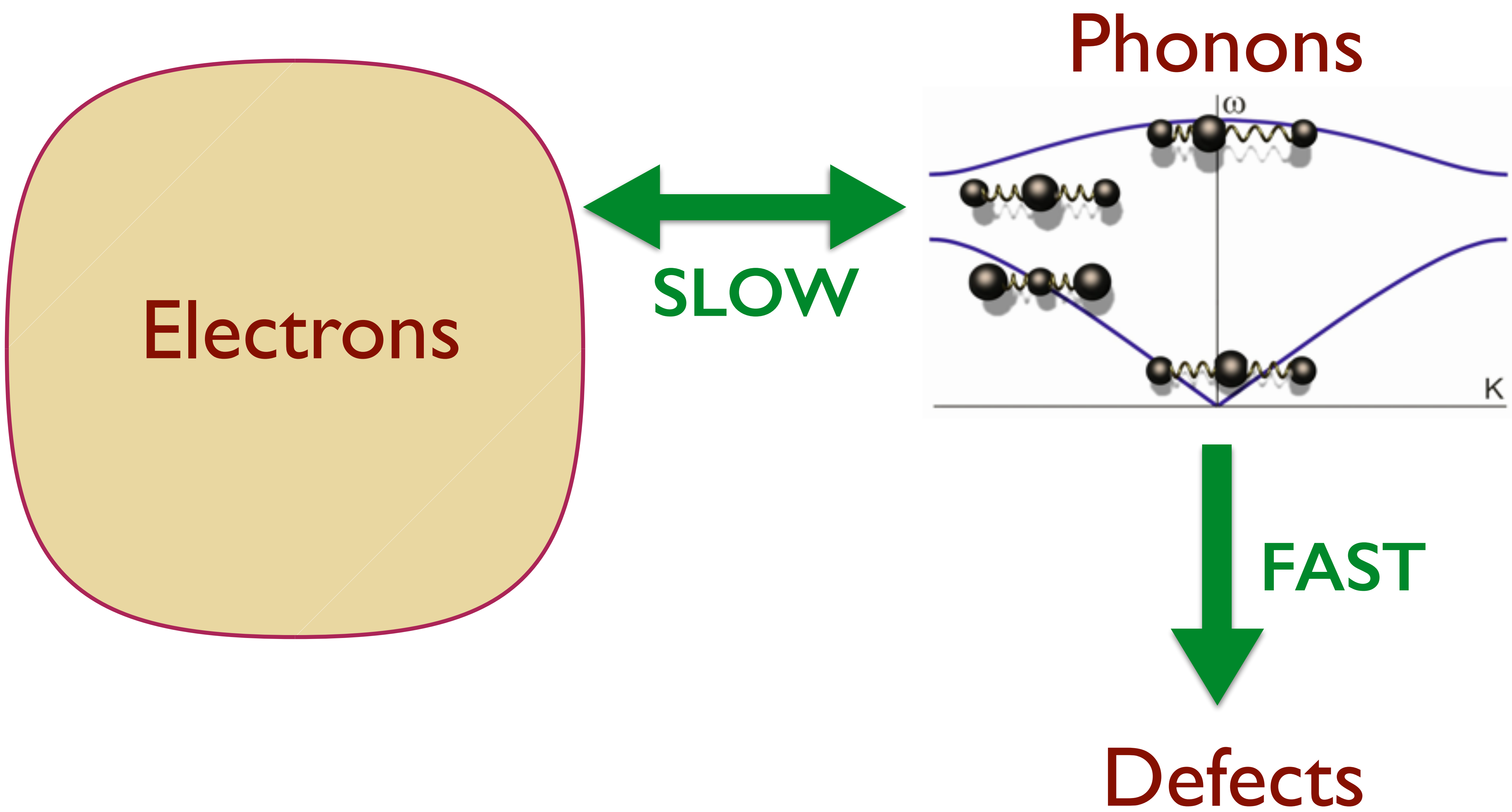

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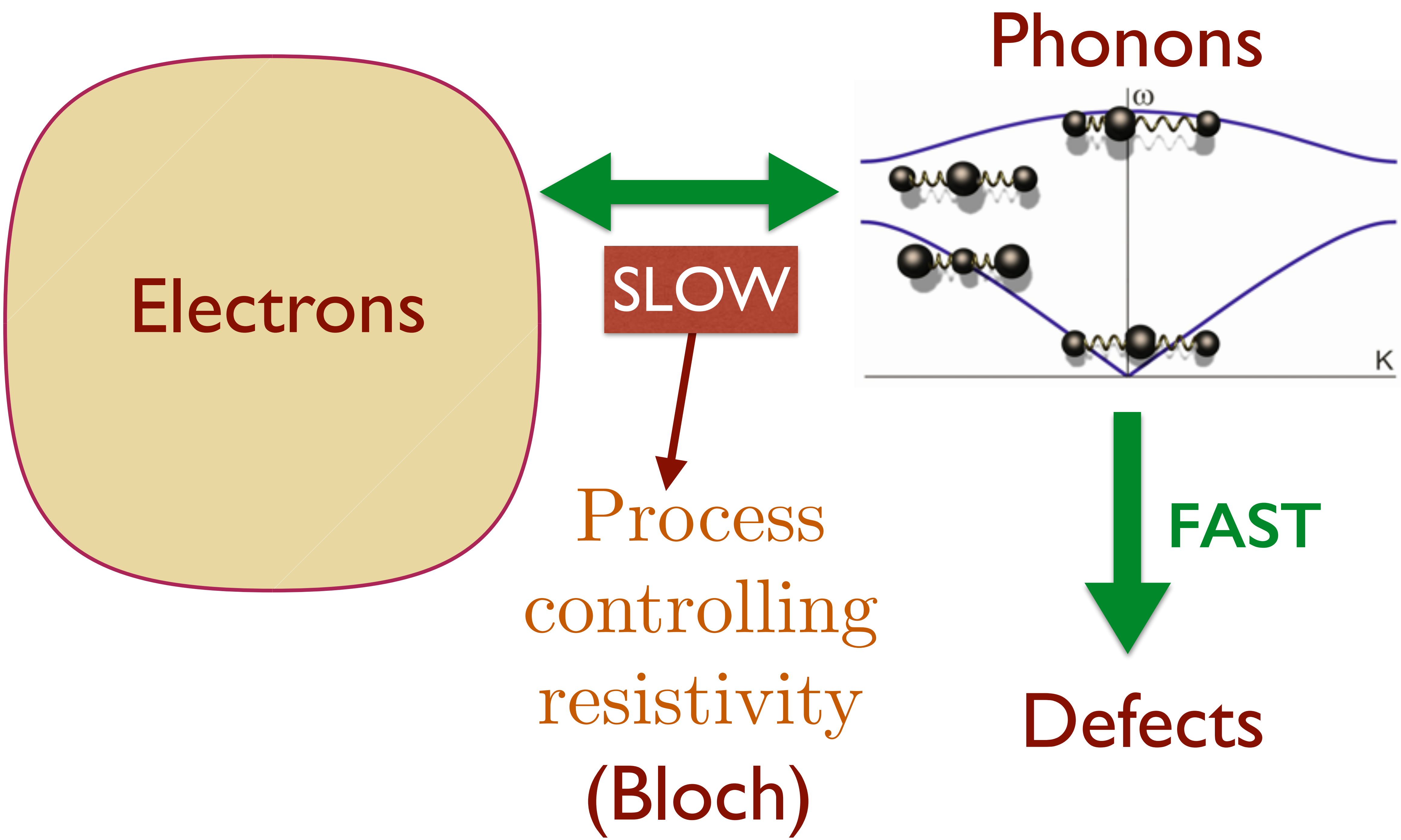

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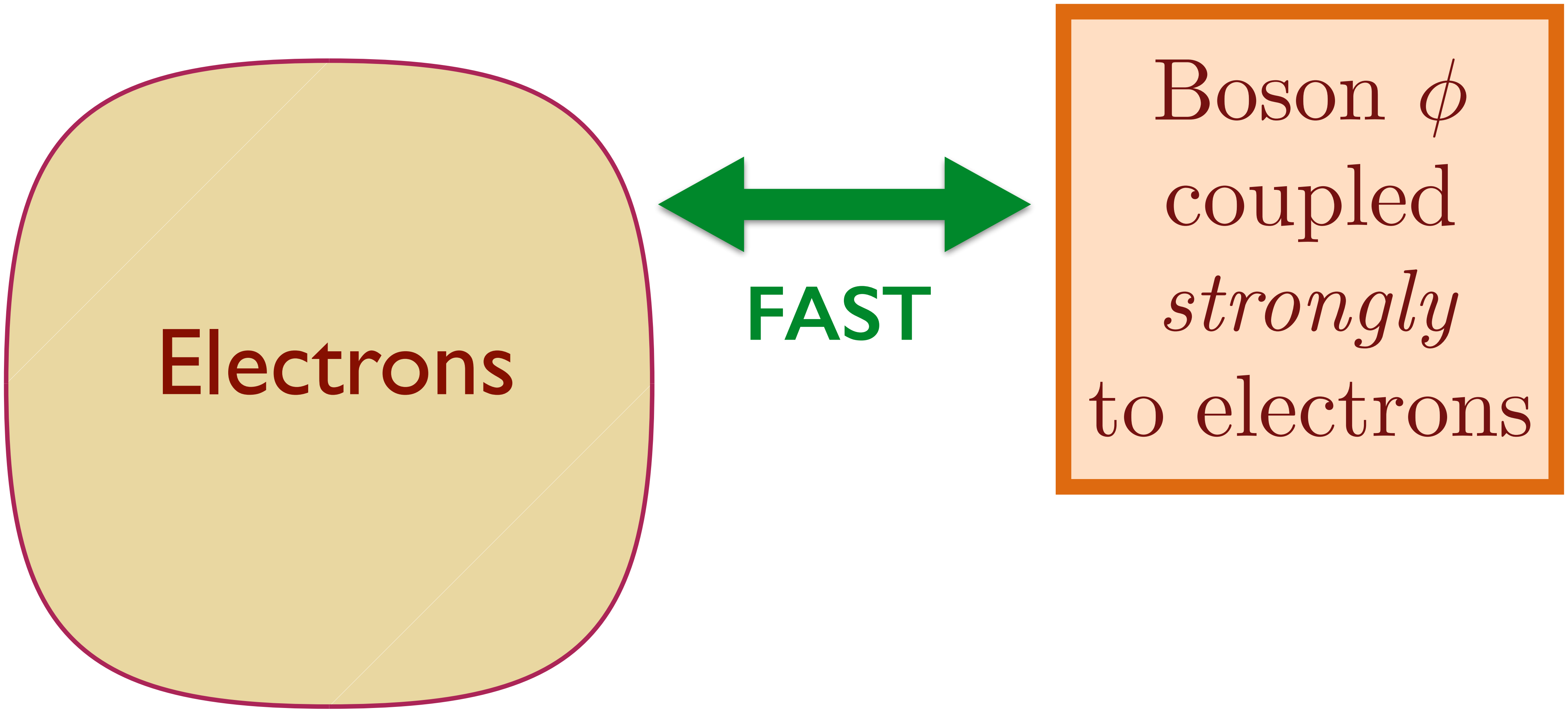

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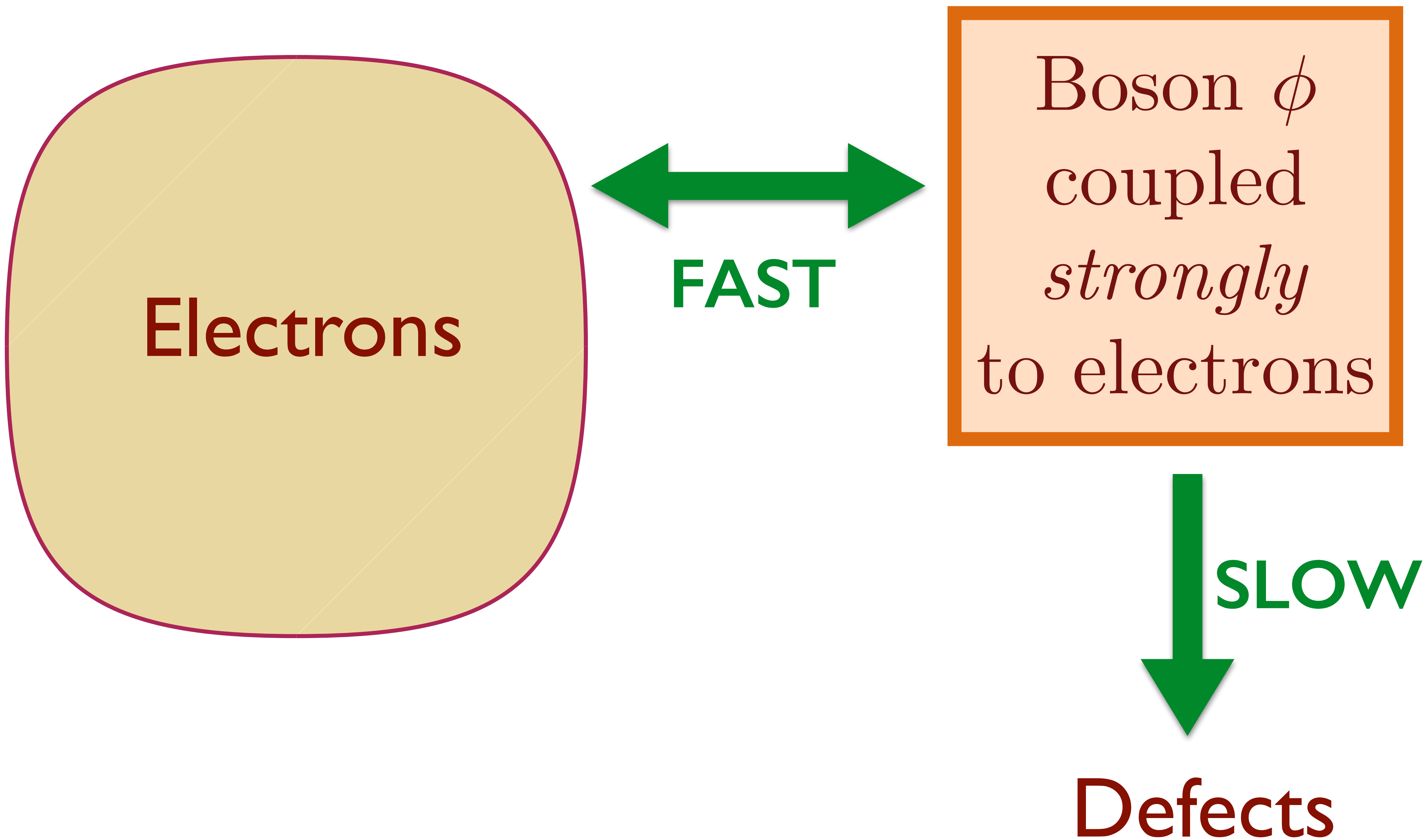

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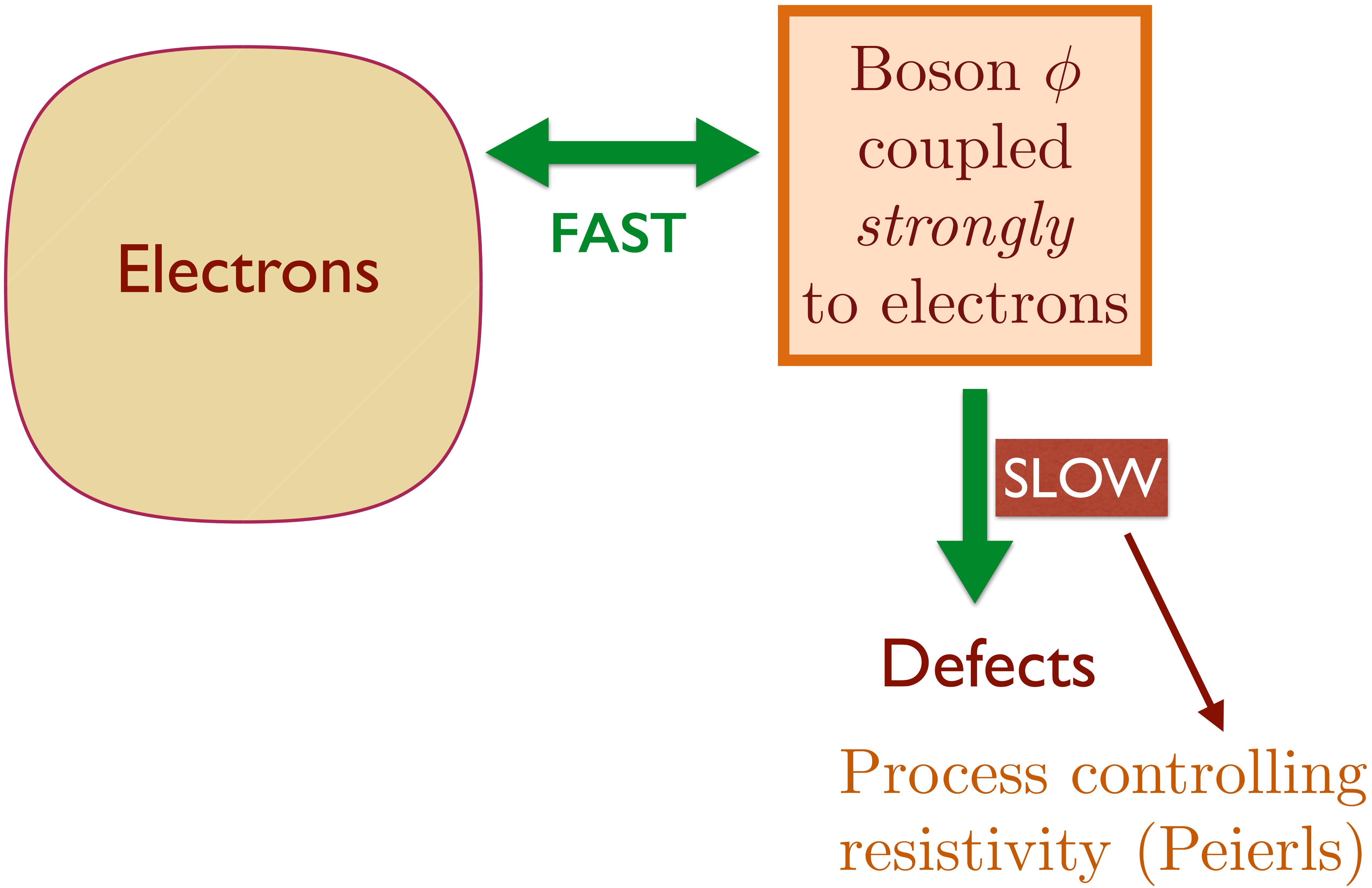

Transport in Strange Metals

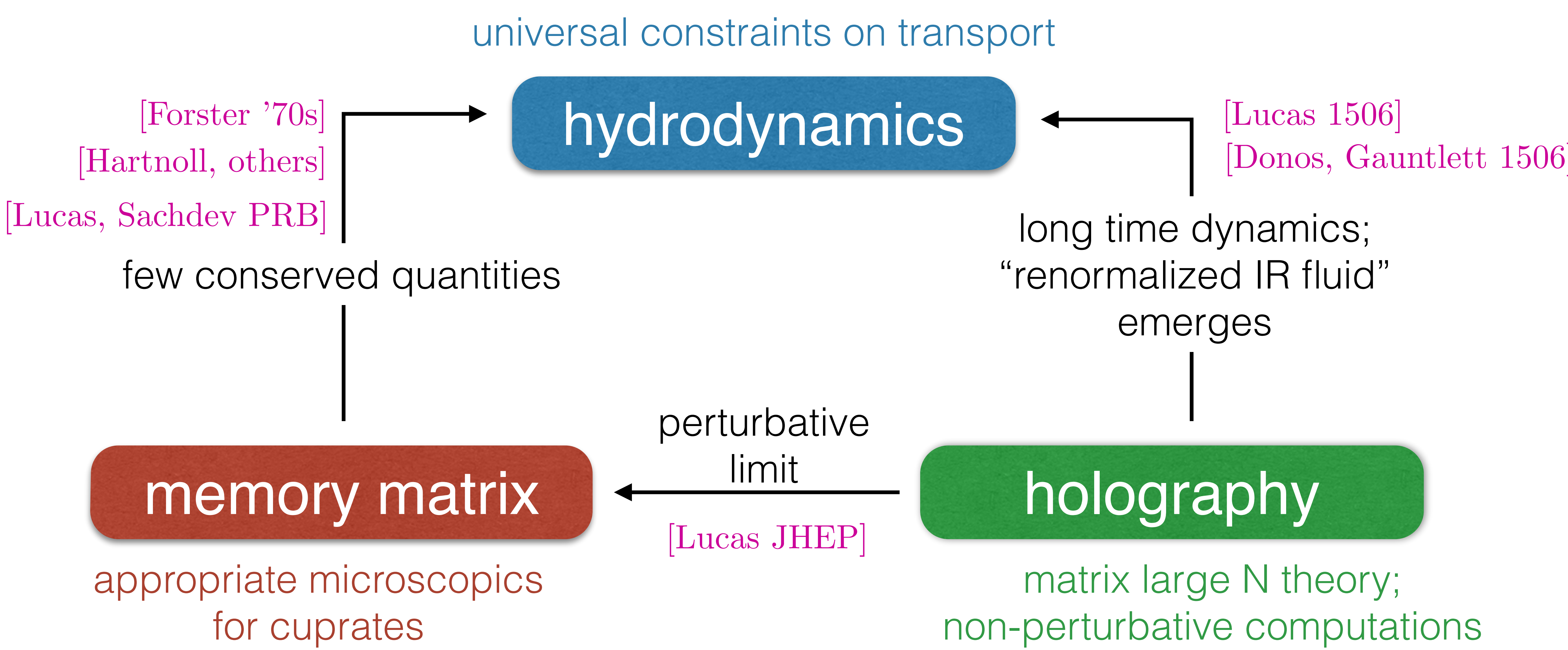

figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Transport in Strange Metals

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by

$$L = \frac{\pi^2 k_B^2}{3e^2}$$

Transport in Strange Metals

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by

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In contrast, for a strange metal with a "relativistic" Hamiltonian, L diverges as the charge density $\mathcal{Q} \to 0$ and the impurity momentum relaxation time $\tau_{\rm imp} \to \infty$:

$$L = \frac{\mathcal{H}\tau_{\rm imp}}{T^2\sigma_Q}\frac{1}{\left(1 + \mathcal{Q}^2\tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2},$$

where $\mathcal{H}$ is the enthalpy density, and σ_Q is an intrinsic "quantum critical" conductivity.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

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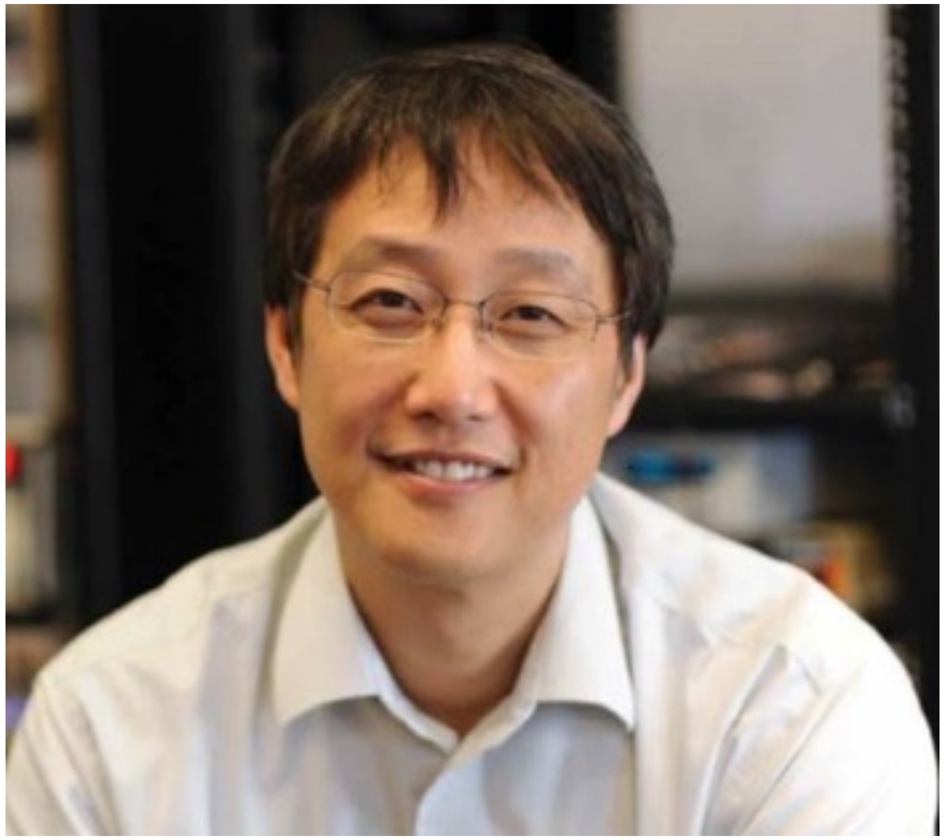
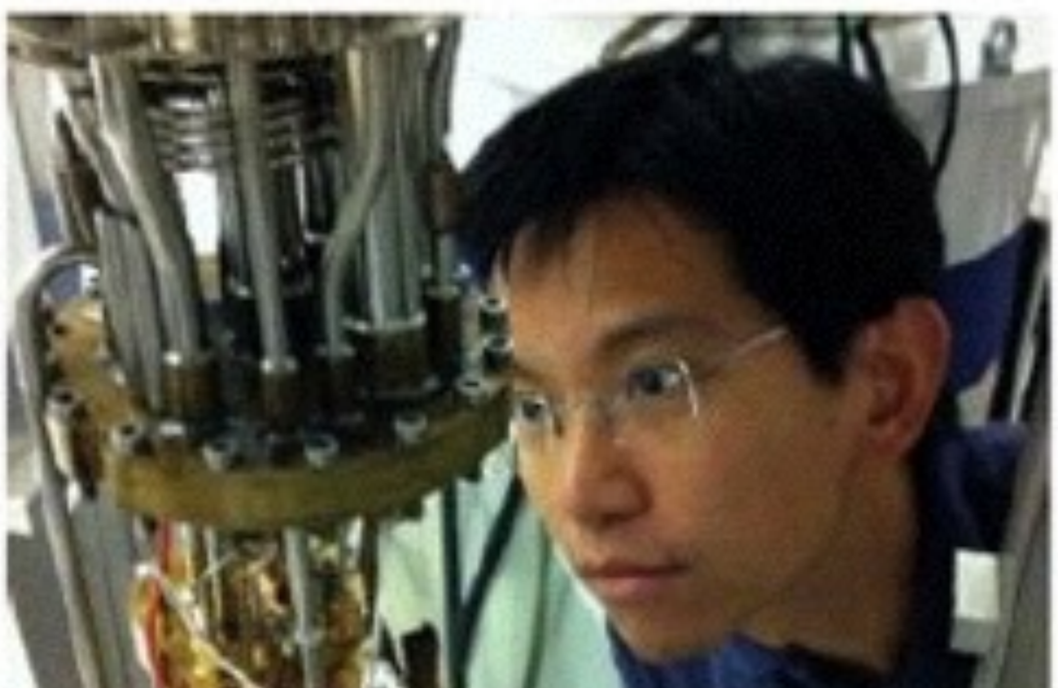
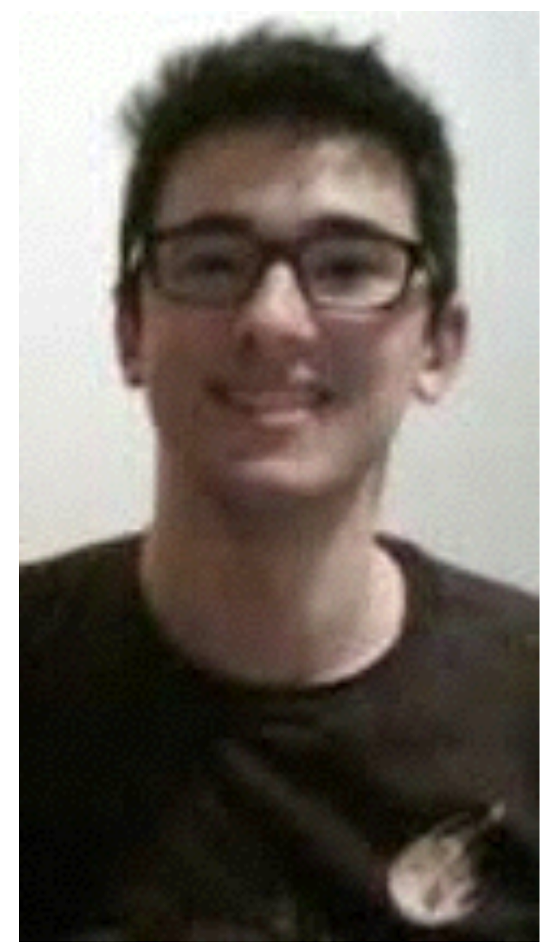


Philip Kim

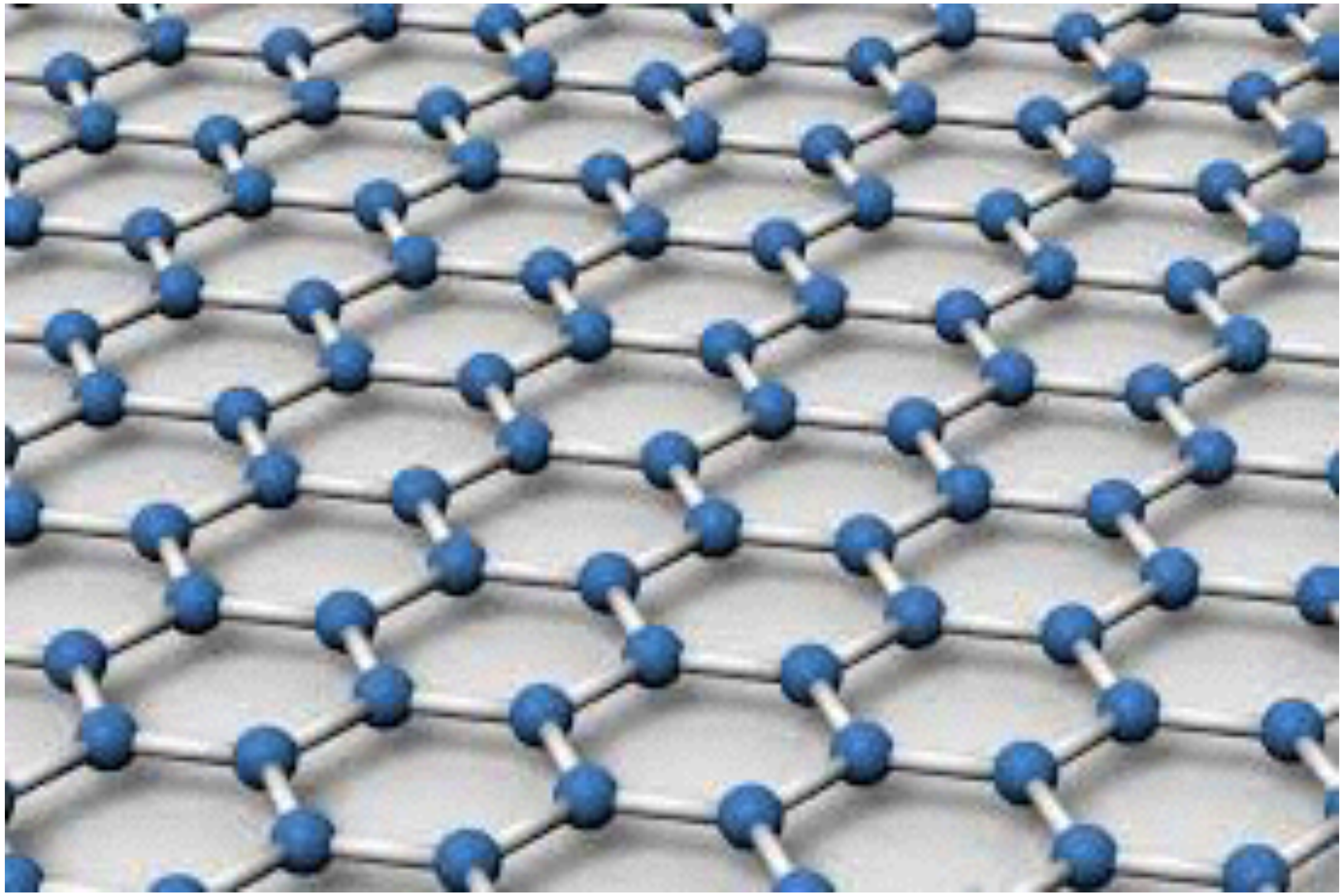
Jesse Crossno

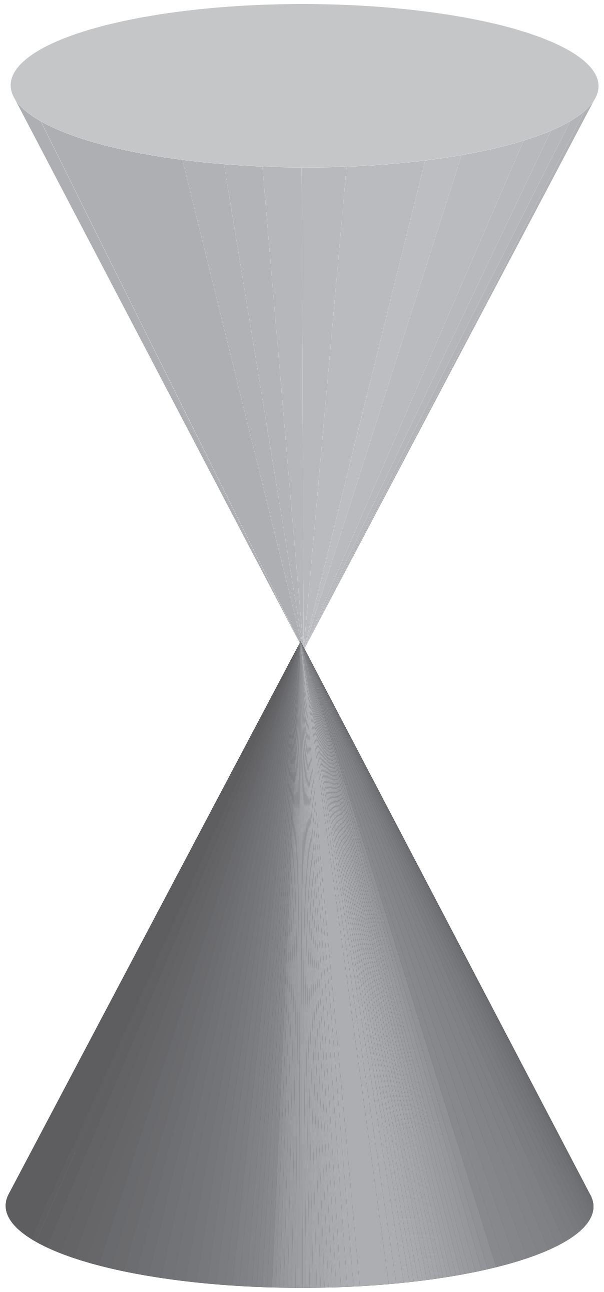


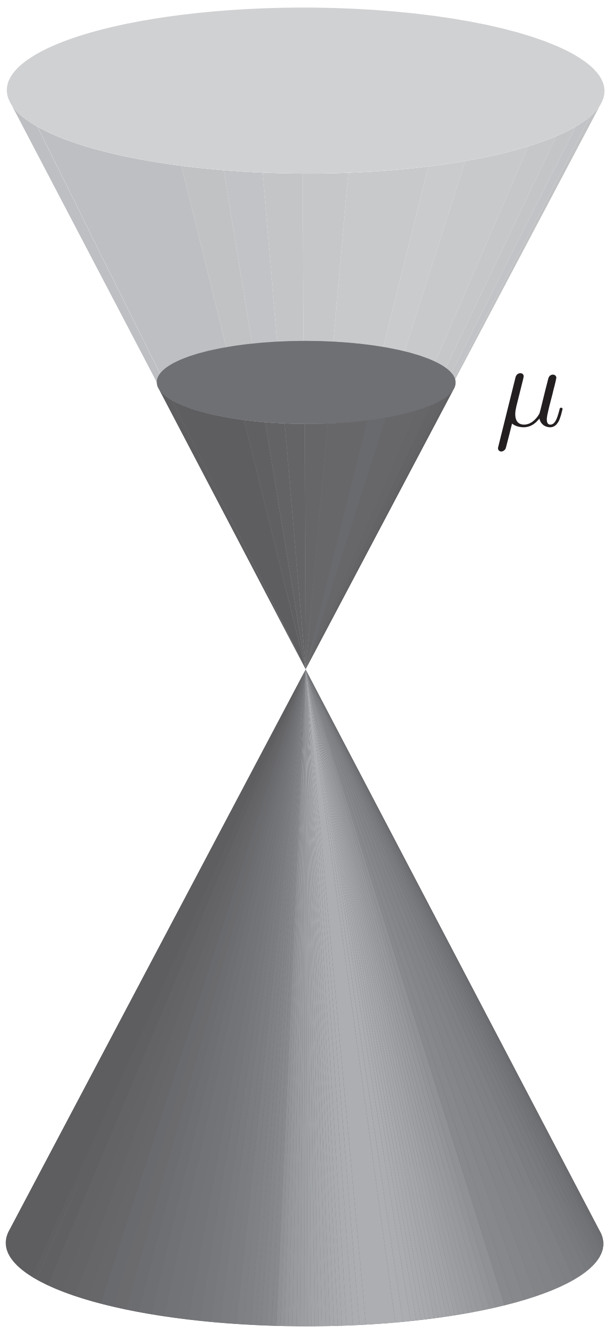
Kin Chung Fong



Andrew Lucas



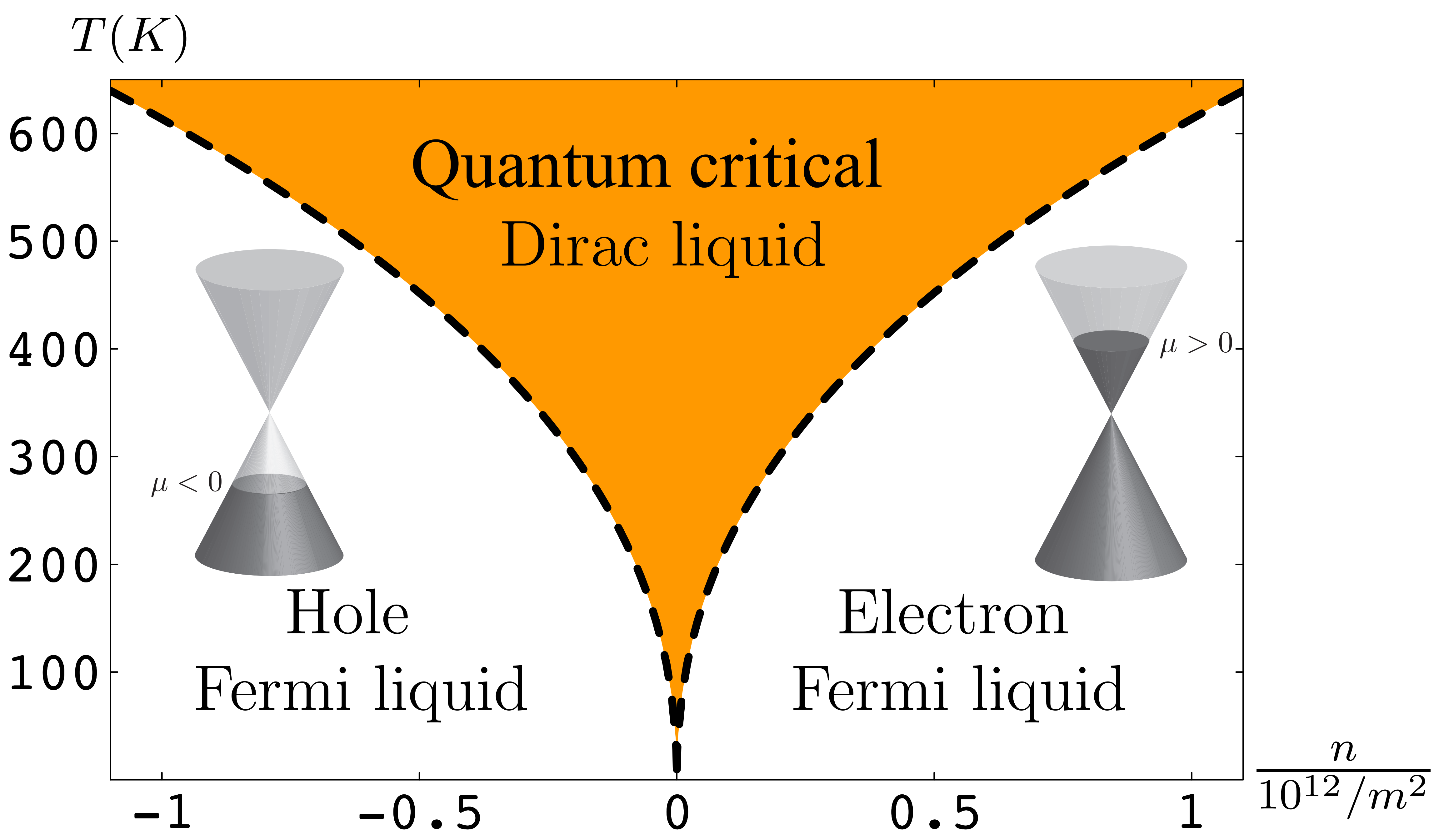




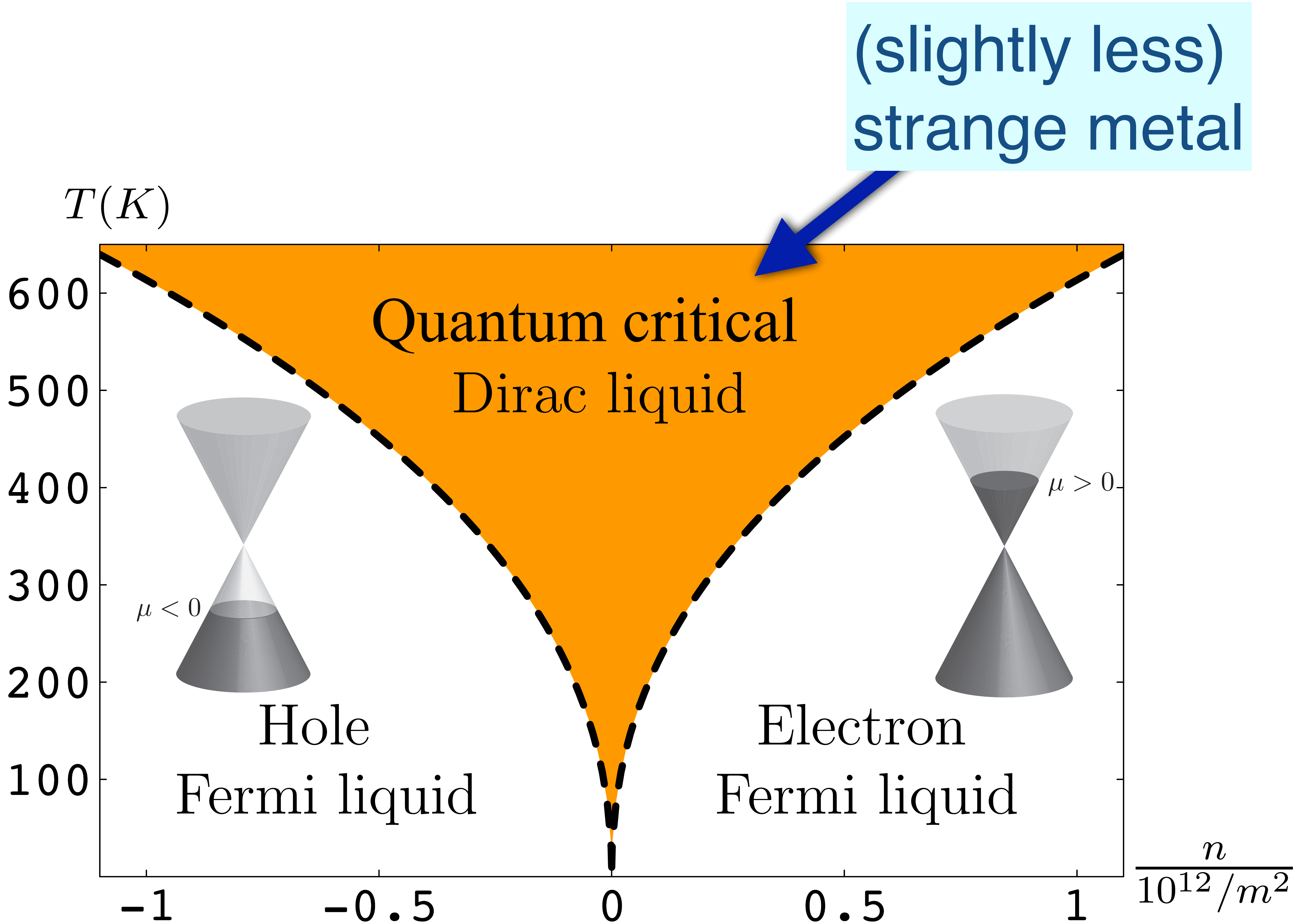
Electron
Fermi surface

Hole
Fermi surface

Electron
Fermi surface



$T(K)$
600
500
400
300
200
100
Quantum critical
Dirac liquid
$\mu < 0$
$\mu > 0$
Hole
Fermi liquid
Electron
Fermi liquid
-1
-0.5
0
0.5
1
$\frac{n}{10^{12}/m^2}$


(slightly less)
strange metal
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μ < 0
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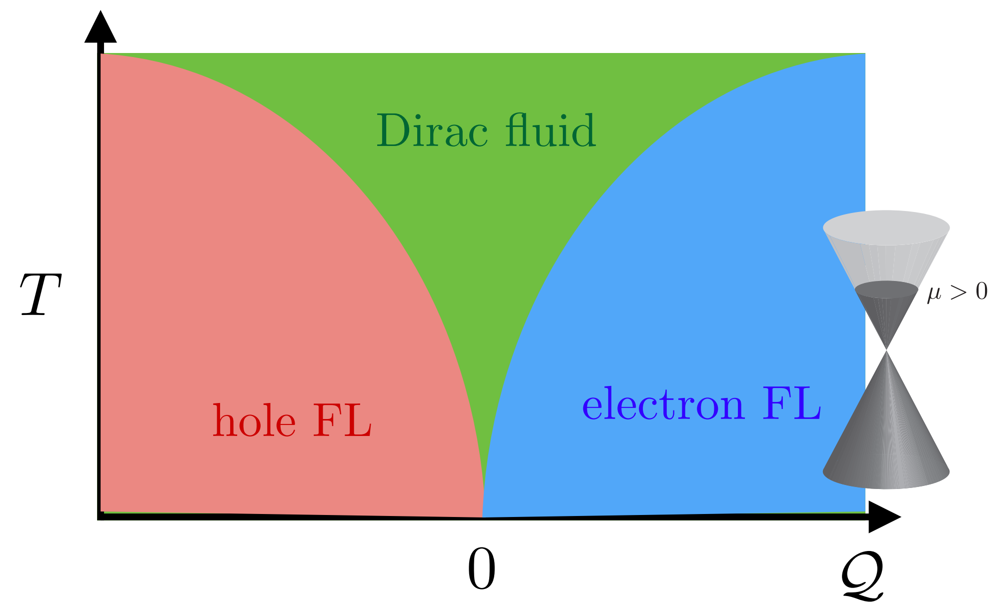
Hole
Fermi liquid
Electron
Fermi liquid
-1
-0.5
0
0.5
1
n / 10^12/m^2

$$\epsilon_{a\sigma} = \hbar v_{\mathrm{F}} k$$
$$+$$
$$V_{\mathrm{int}} = \frac{\alpha_{\mathrm{eff}}}{r}$$



- marginally irrelevant $1/r$ Coulomb interactions:

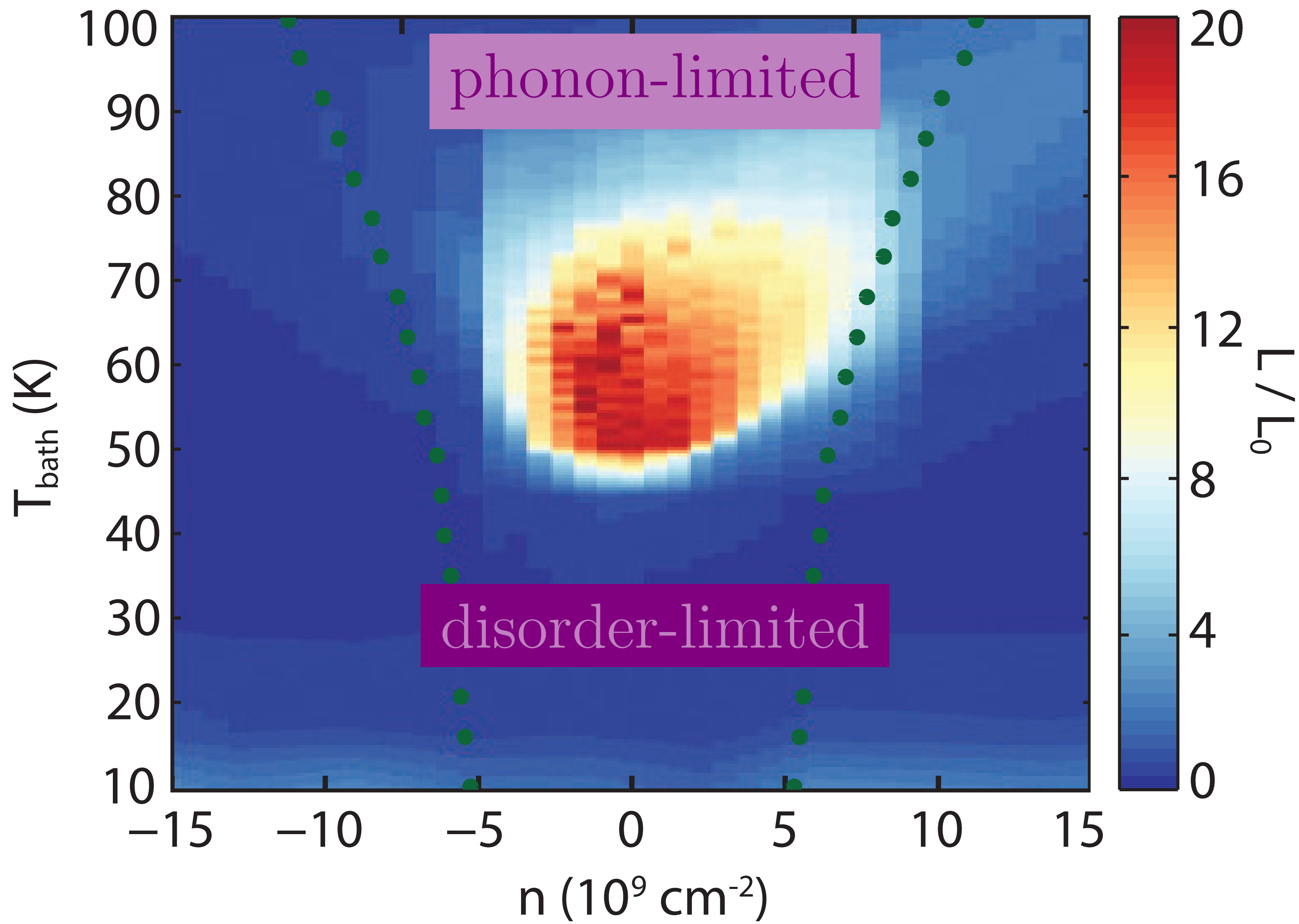
$$\alpha_{\mathrm{eff}} = \frac{\alpha_0}{1 + (\alpha_0/4)\log((10^5\ \mathrm{K})/T)}, \quad \alpha_0 \approx \frac{1}{137}\frac{c}{v_{\mathrm{F}}\epsilon_{\mathrm{r}}} \sim 0.5.$$

- thermo/hydro nearly that of relativistic theory
- $\alpha_{\mathrm{eff}} \sim 0.3$ at $T = 100$ K

[Müller, Fritz, Sachdev, *Physical Review* **B78** 115406 (2008)]

Dirac Fluid in Graphene

Wiedemann-Franz Law Violations in Experiment



[Crossno *et al, submitted*]

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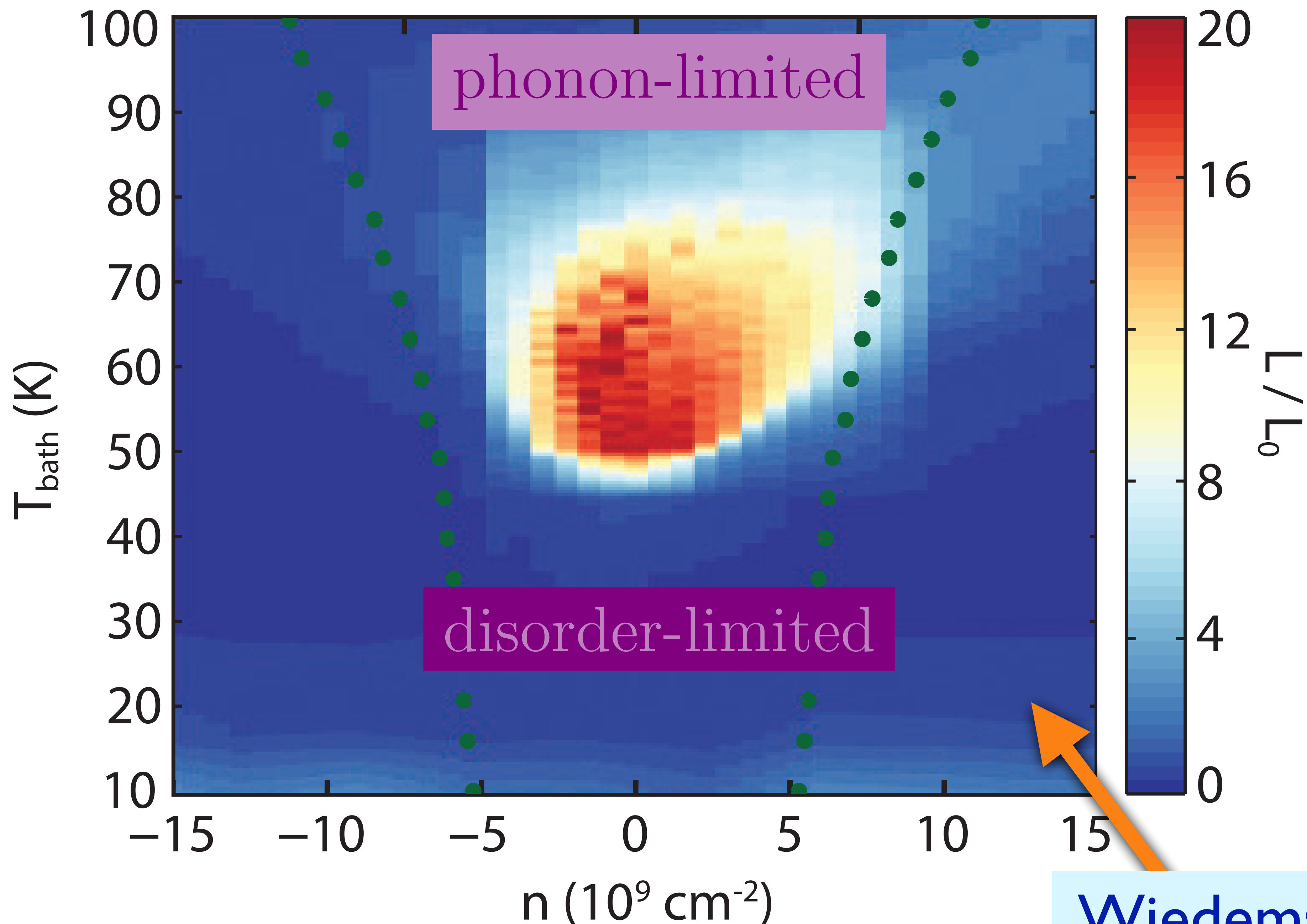

Wiedemann-Franz obeyed

[Crossno *et al, submitted*]

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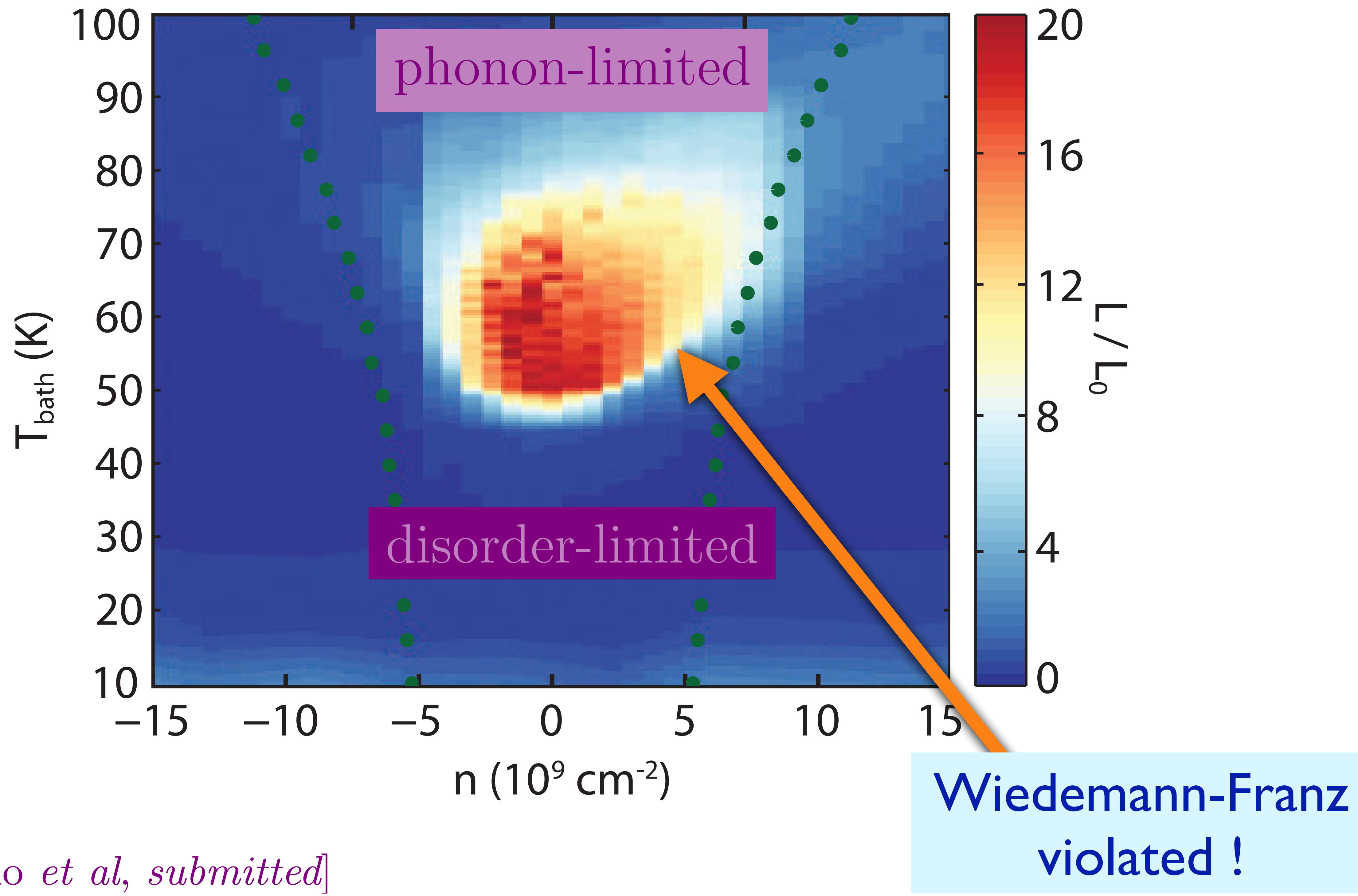
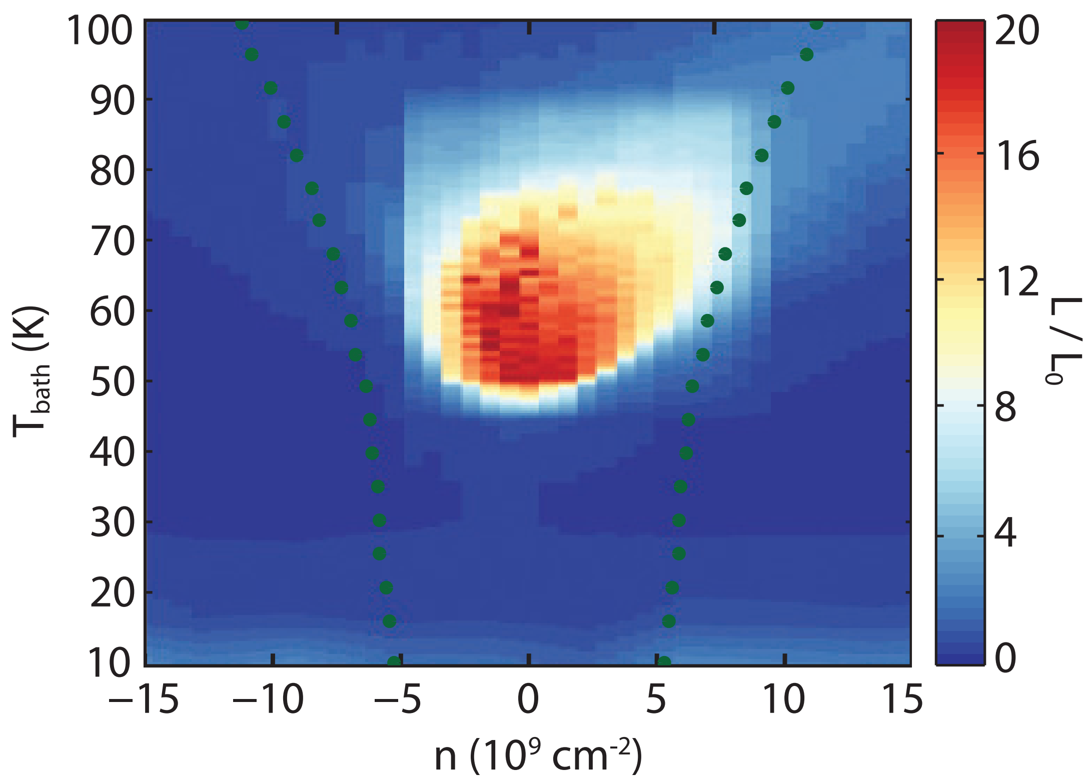


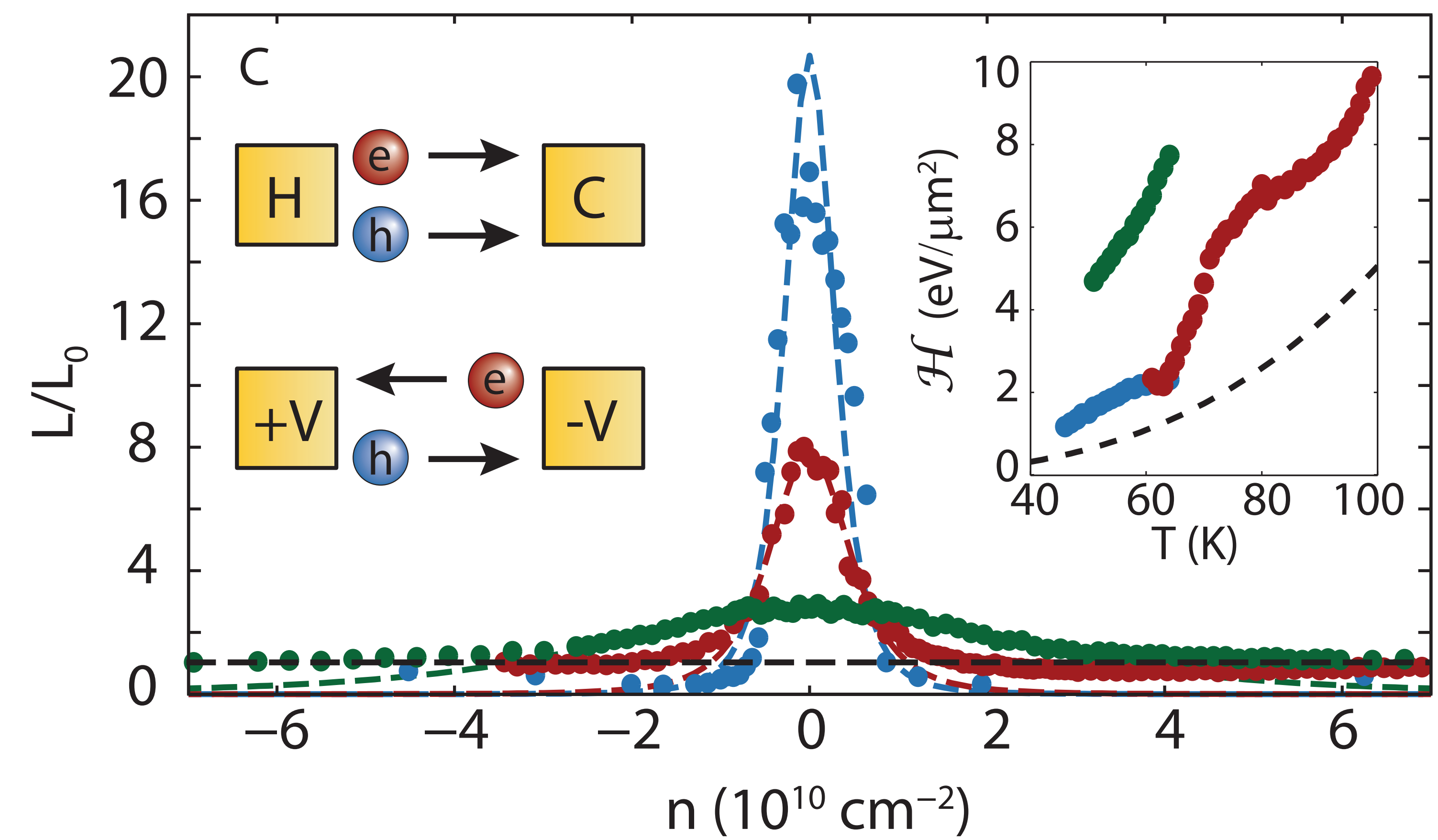
[Crossno *et al, submitted*]

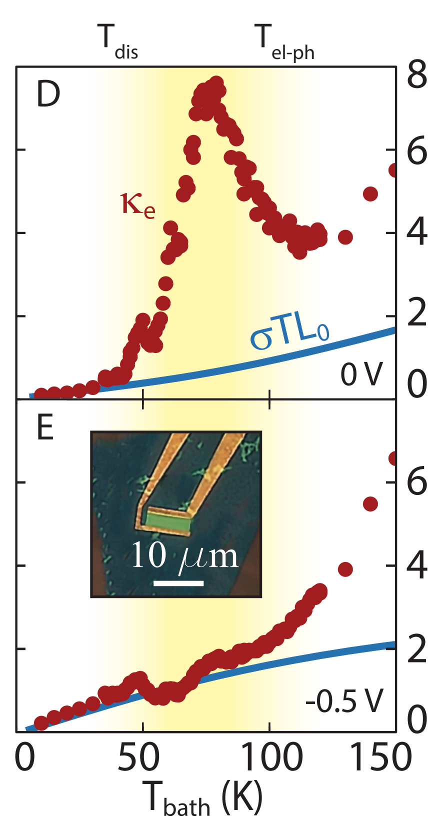
(submitted)

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]







Lorentz ratio $L = \kappa/(T\sigma)$

$$= \frac{\mathcal{H}\tau_{\rm imp}}{T^2\sigma_Q}\frac{1}{\left(1 + \mathcal{Q}^2\tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

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