Quantum critical phenomena

Talk online: sachdev.physics.harvard.edu

Outline

1. Coupled dimer antiferromagnets

   Landau-Ginzburg quantum criticality

2. Spin density waves in metals

   Paramagnon quantum criticality

3. Spin liquids and valence bond solids

   Schwinger-boson mean-field theory and U(1) gauge theory

References

- Exotic phases and quantum phase transitions: model systems and experiments, Rapporteur talk at the 24th Solvay Conference on Physics, "Quantum Theory of Condensed Matter", arXiv:0901.4103

- Quantum magnetism and criticality, *Nature Physics* **4**, 173 (2008), arXiv:0711.3015

- Quantum phases and phase transitions of Mott insulators, arXiv:cond-mat/0401041

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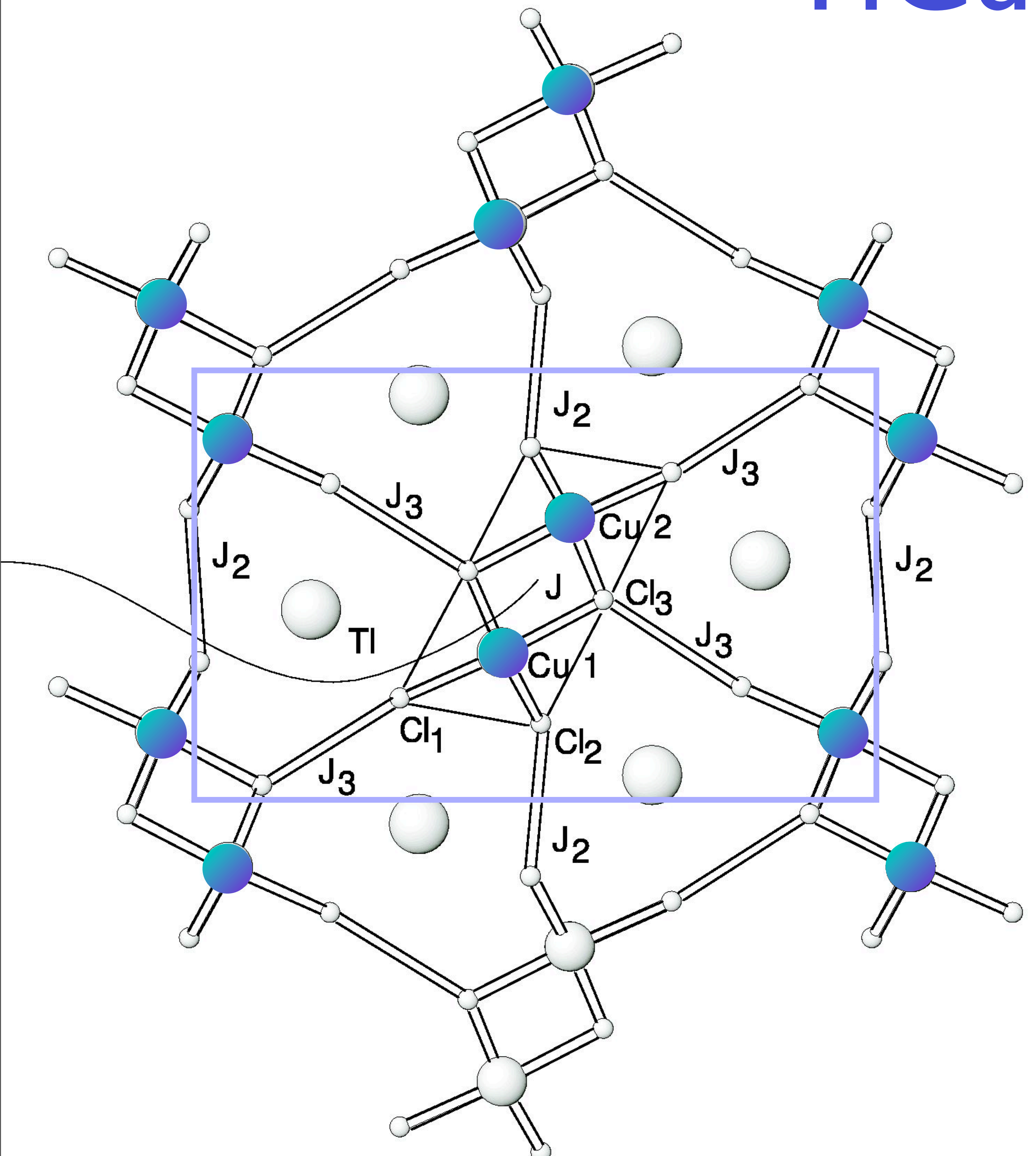
$TlCuCl_3$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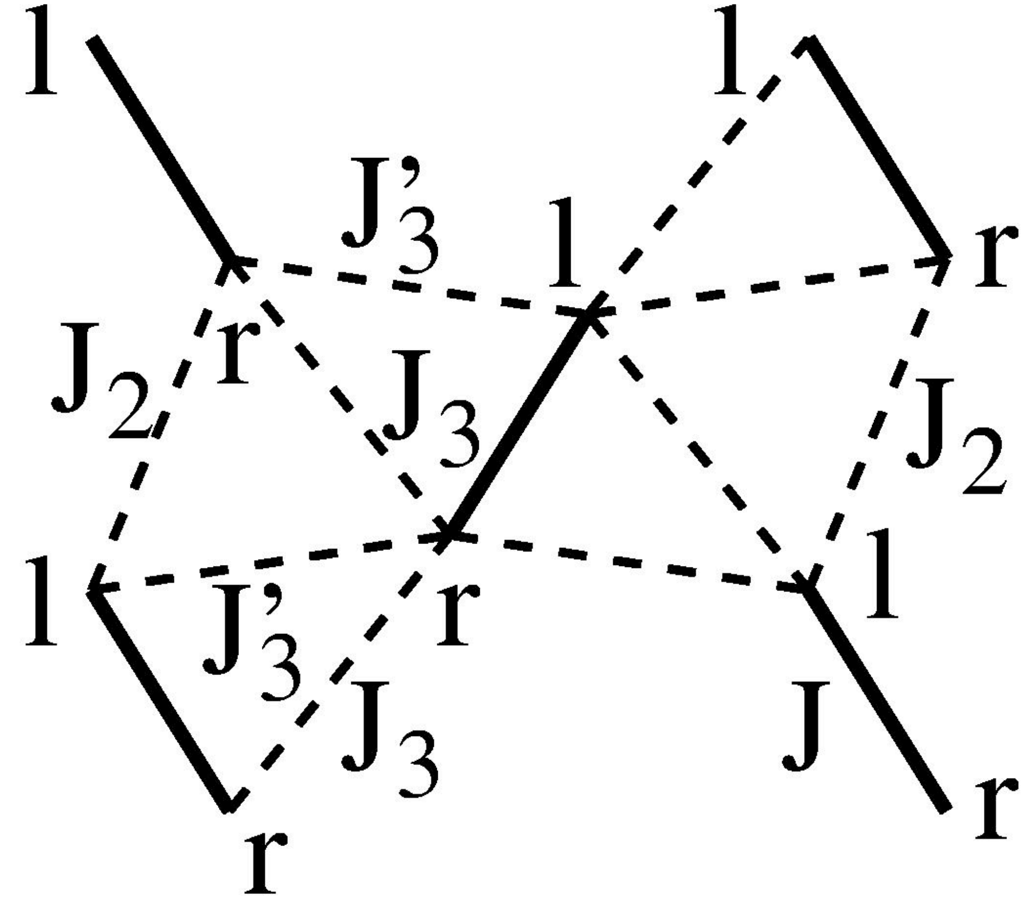
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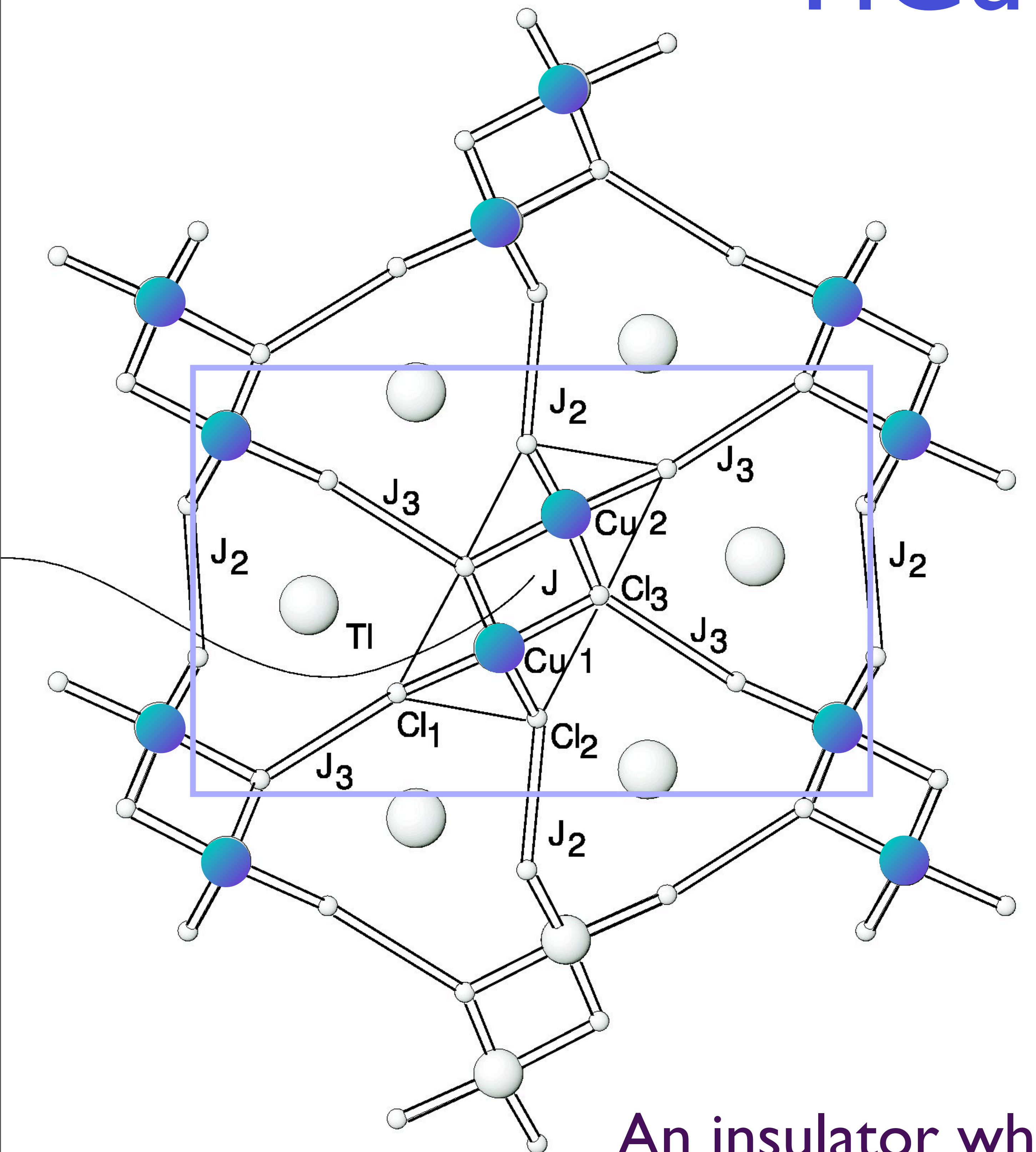
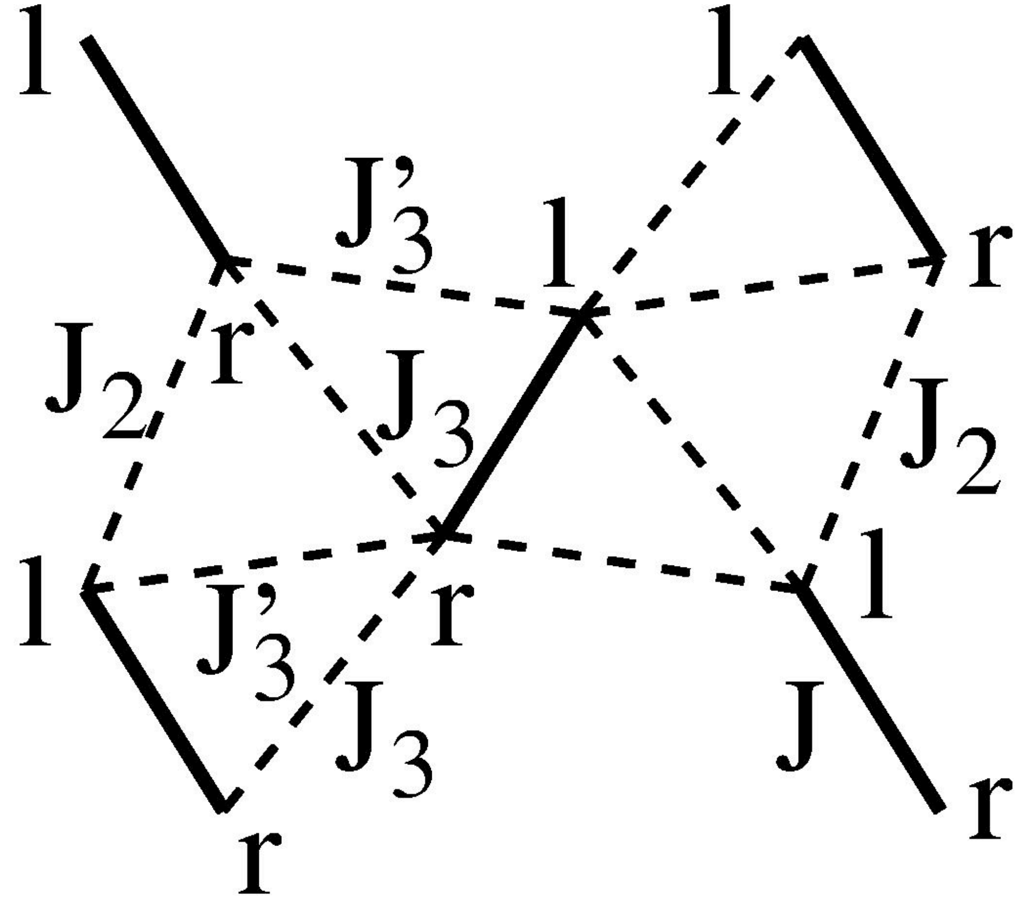
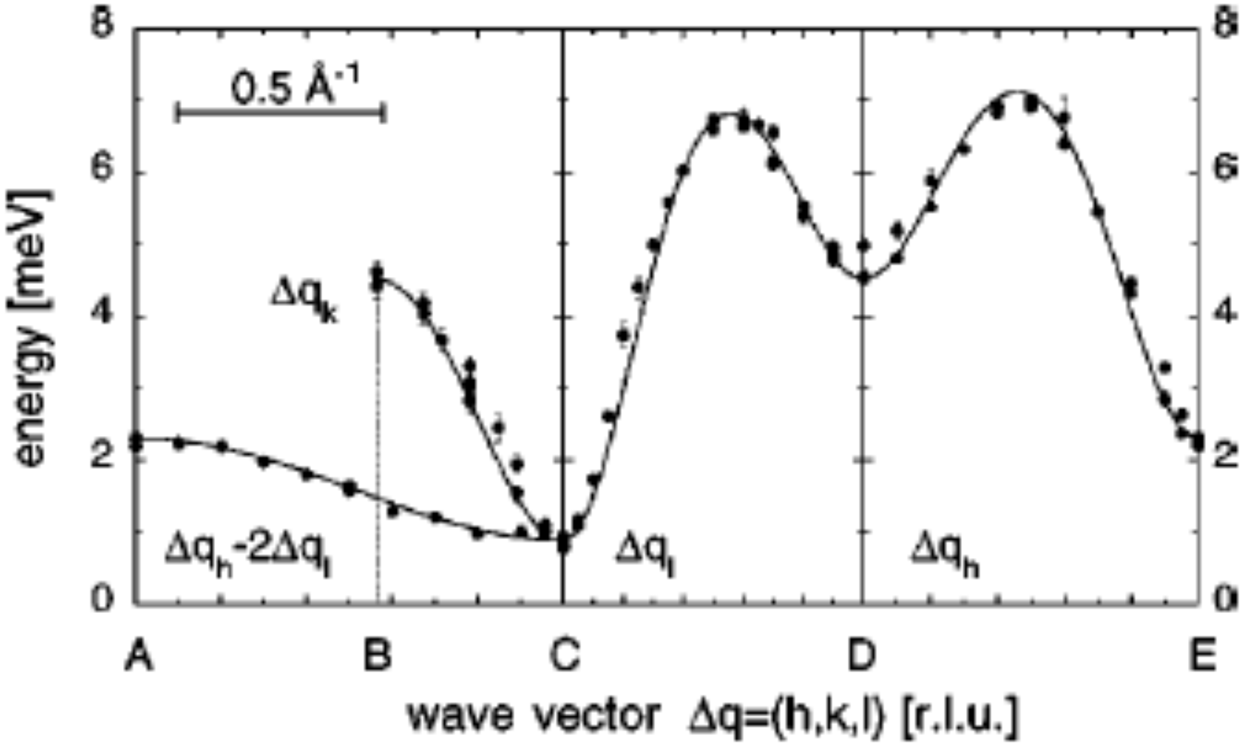
$TlCuCl_3$





An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

$TlCuCl_3$ at ambient pressure



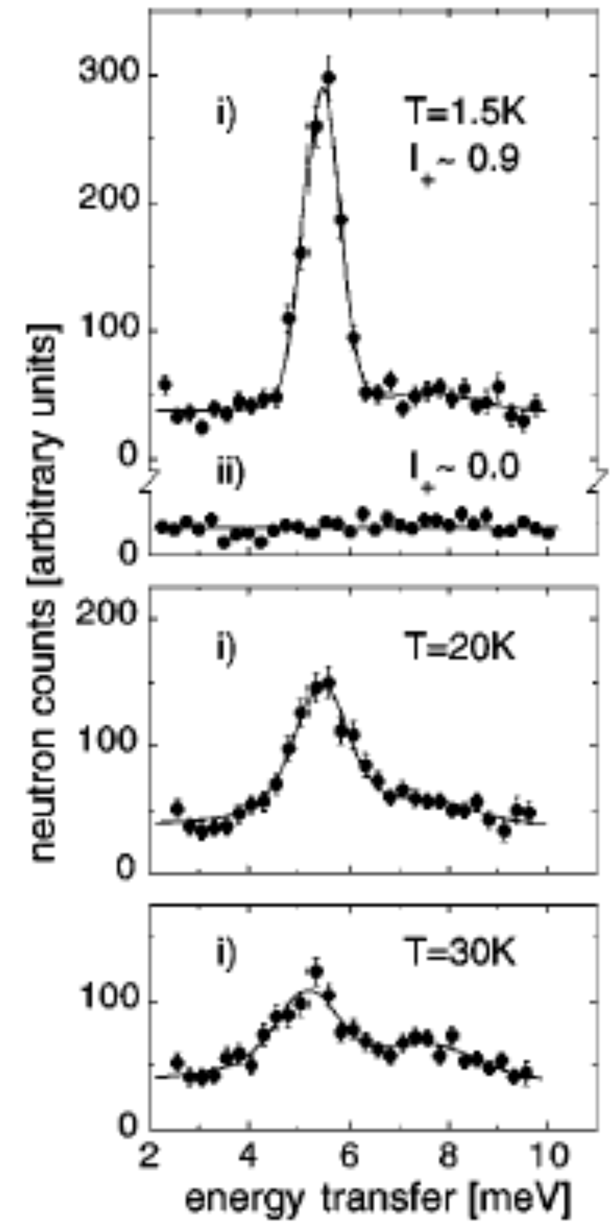
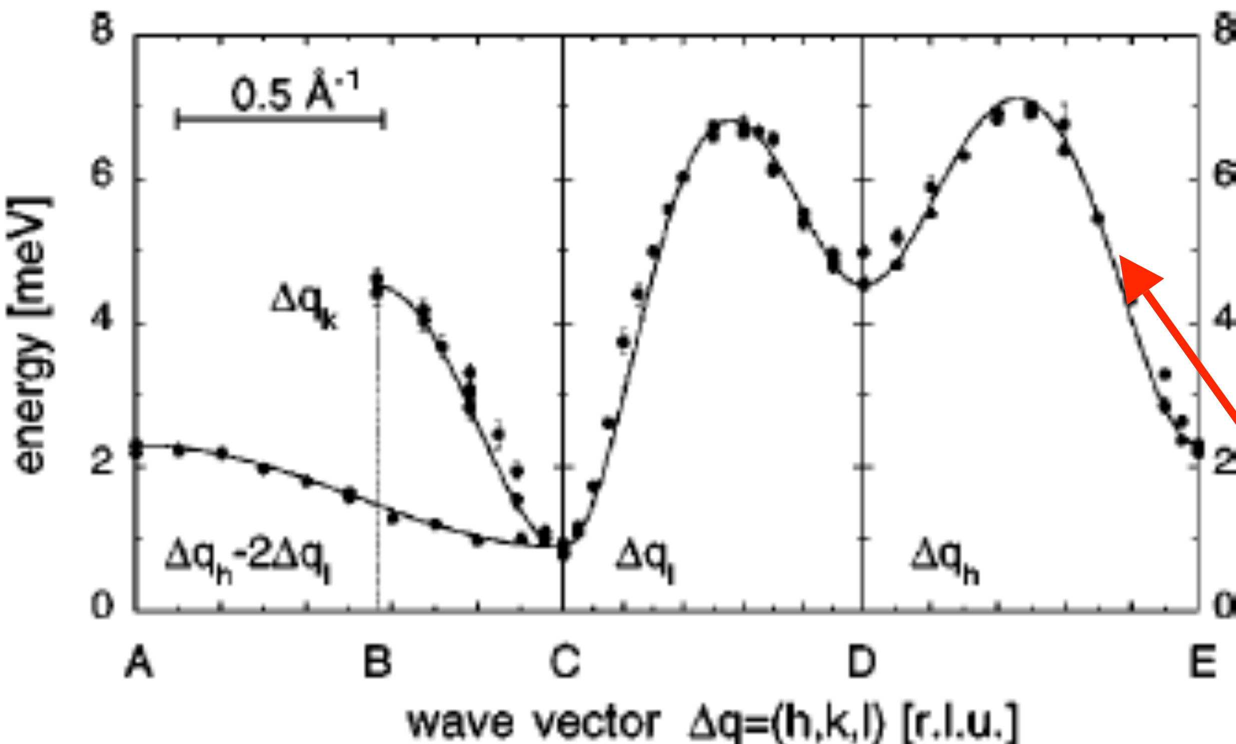
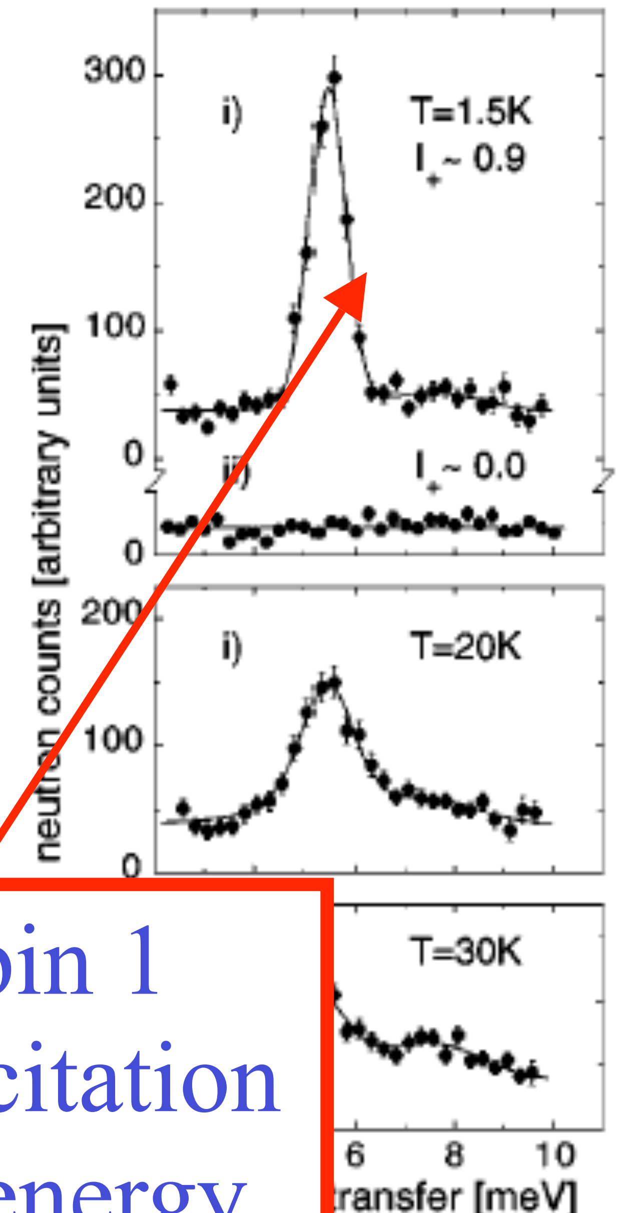


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

$TlCuCl_3$ at ambient pressure





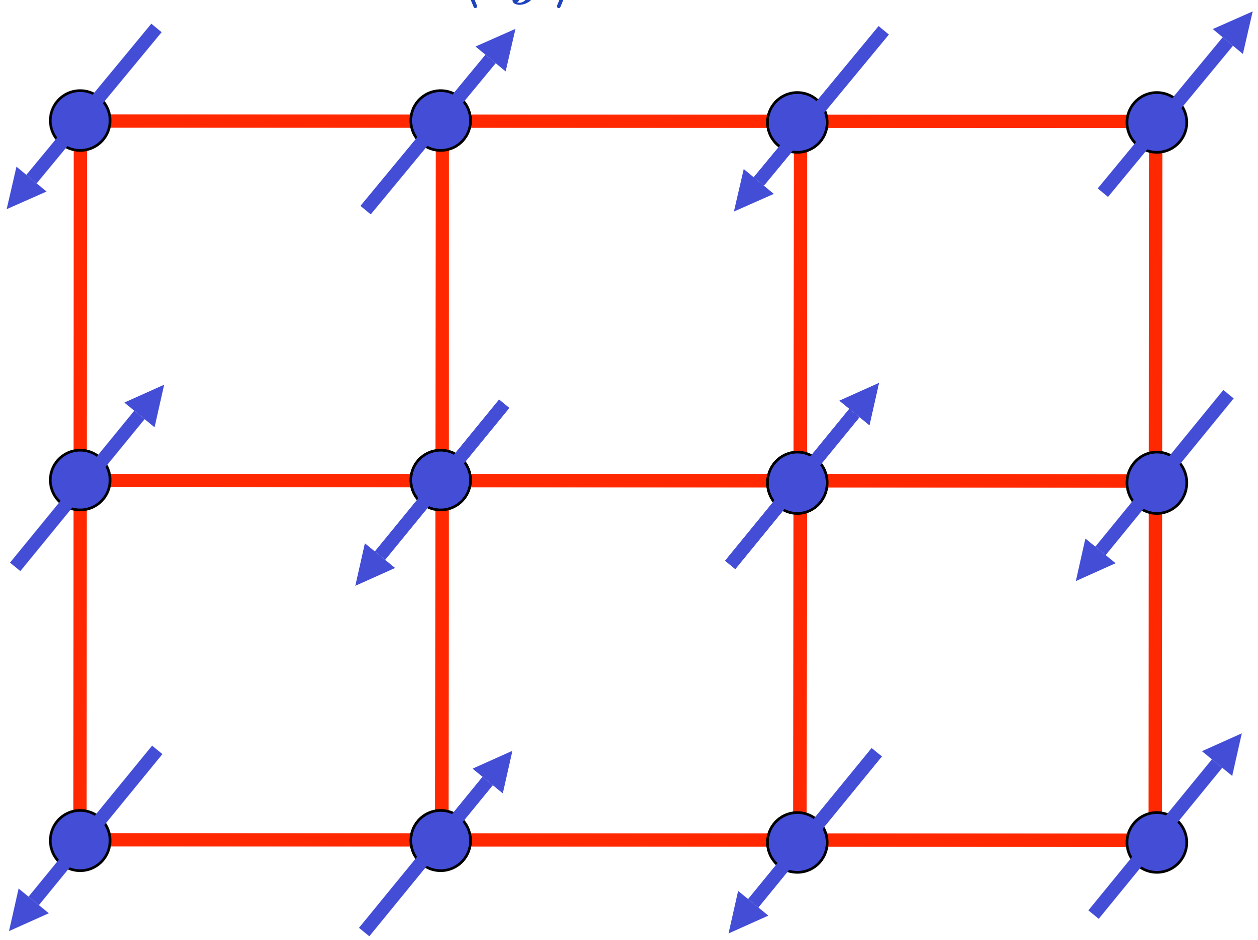
...les in the a^*c^* plane of $TlCuCl_3$
...r.l.u]. The spectrum at $T = 1.5$ K

Sharp spin 1 particle excitation above an energy gap (spin gap)

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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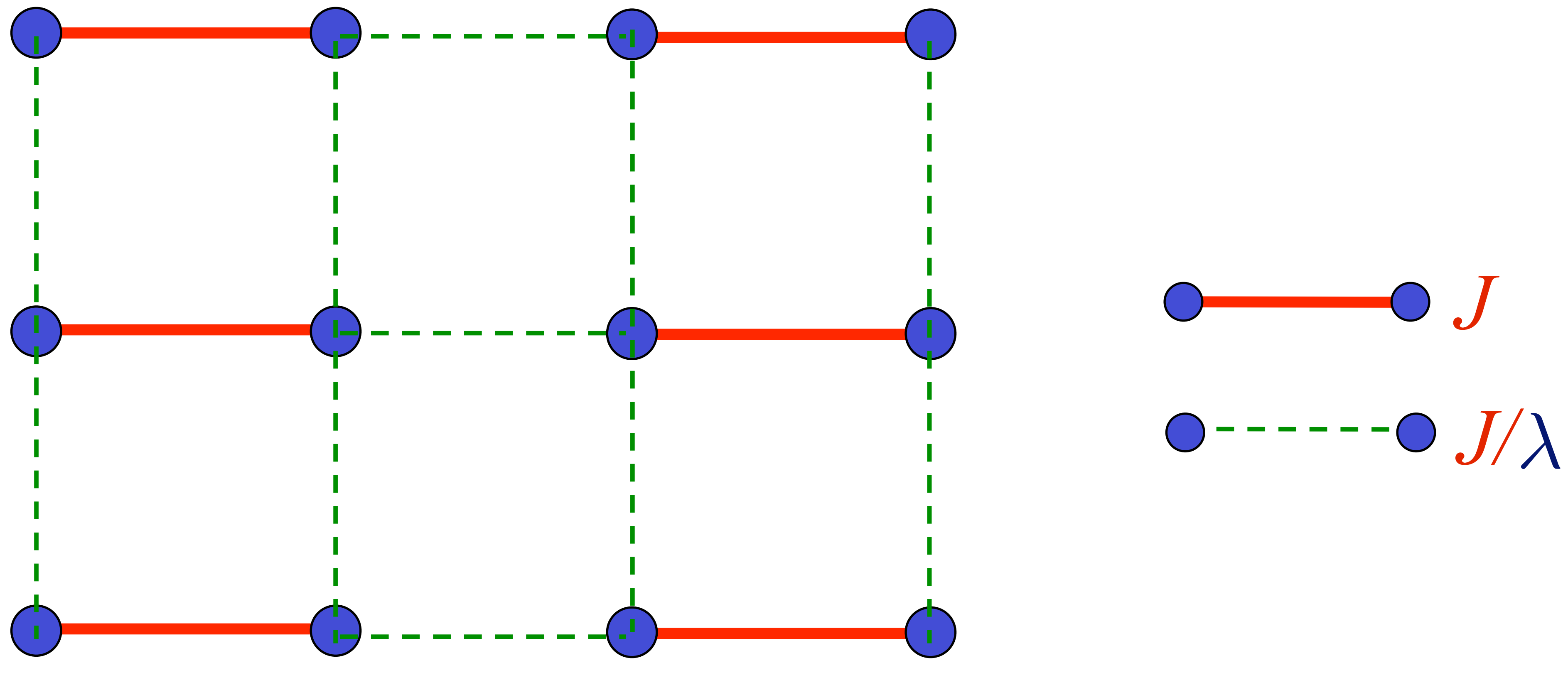

Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

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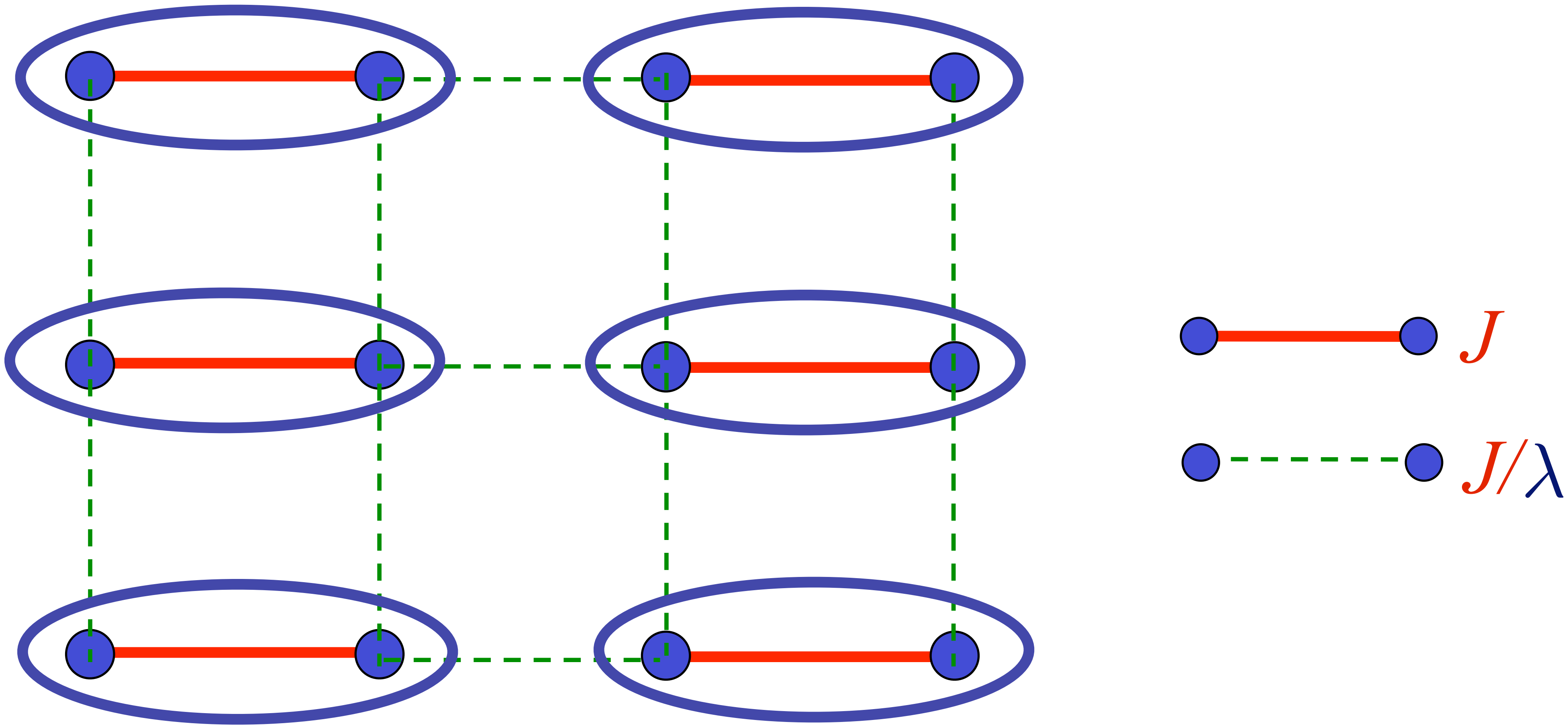
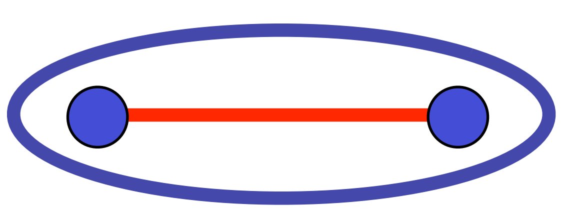
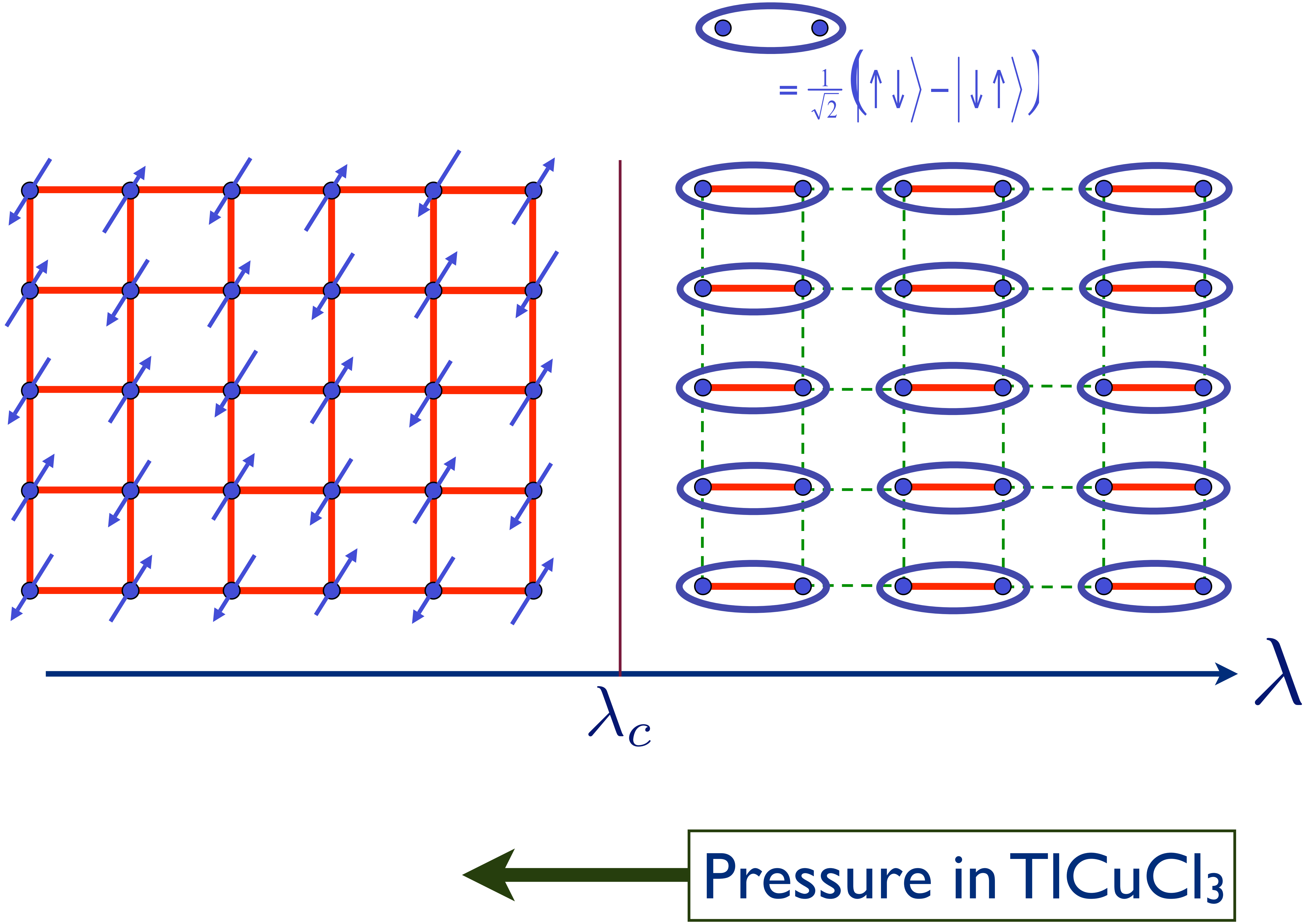


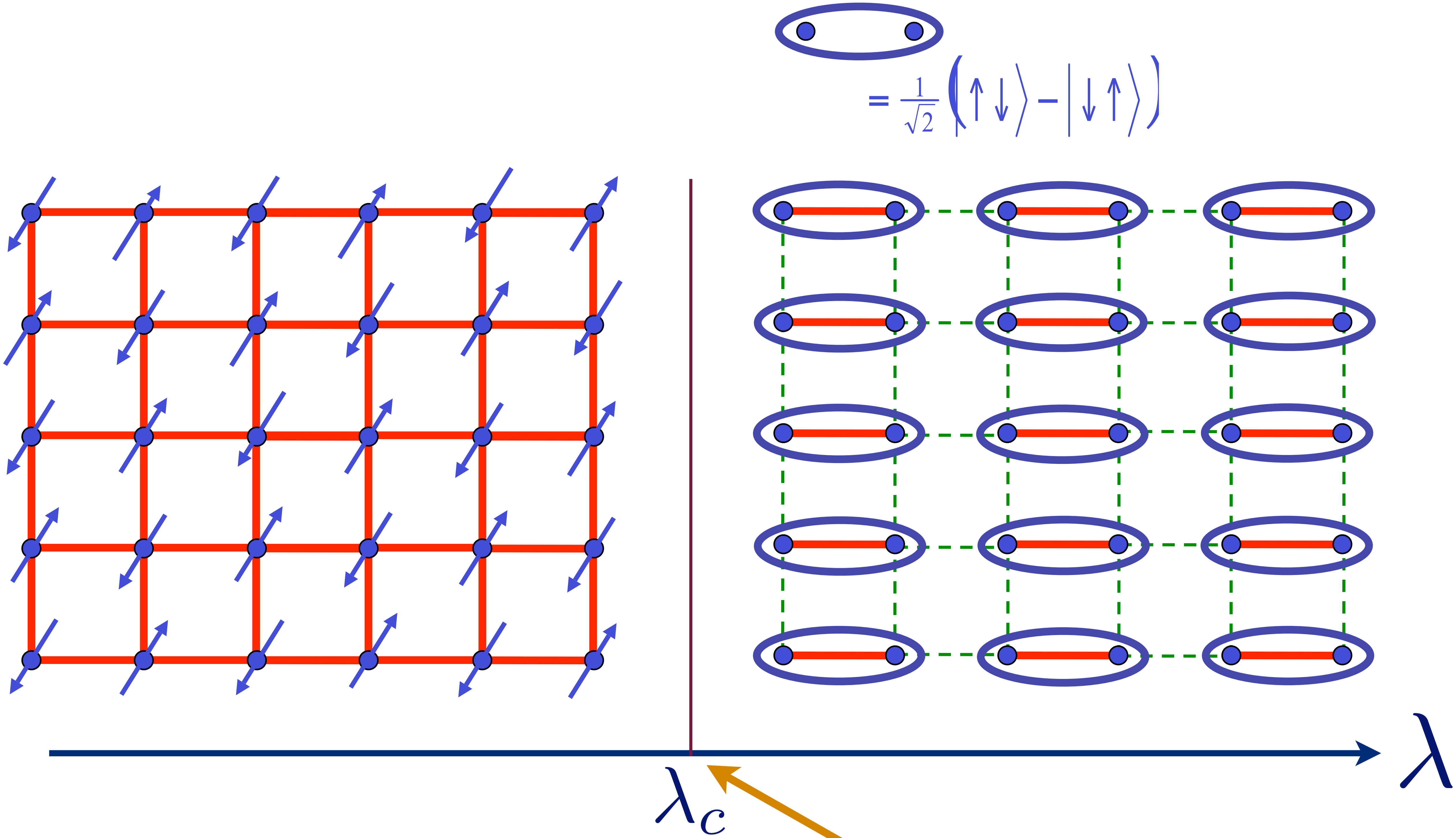
Ground state is a "quantum paramagnet" with spins locked in valence bond singlets



$$= \frac{1}{\sqrt{2}} \Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle \Big)$$

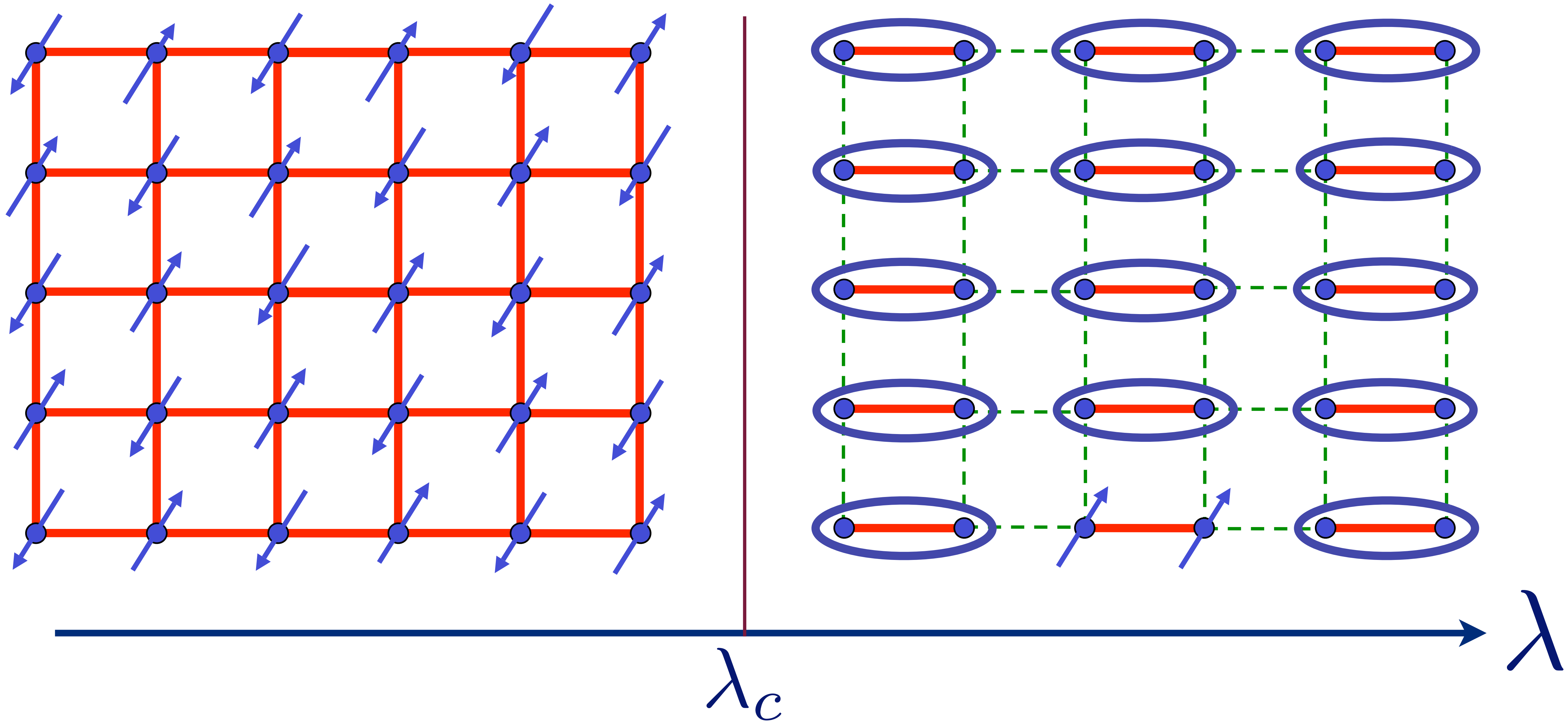


$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ
Pressure in $TlCuCl_3$



Quantum critical point with non-local entanglement in spin wavefunction

Excitation spectrum in the paramagnetic phase



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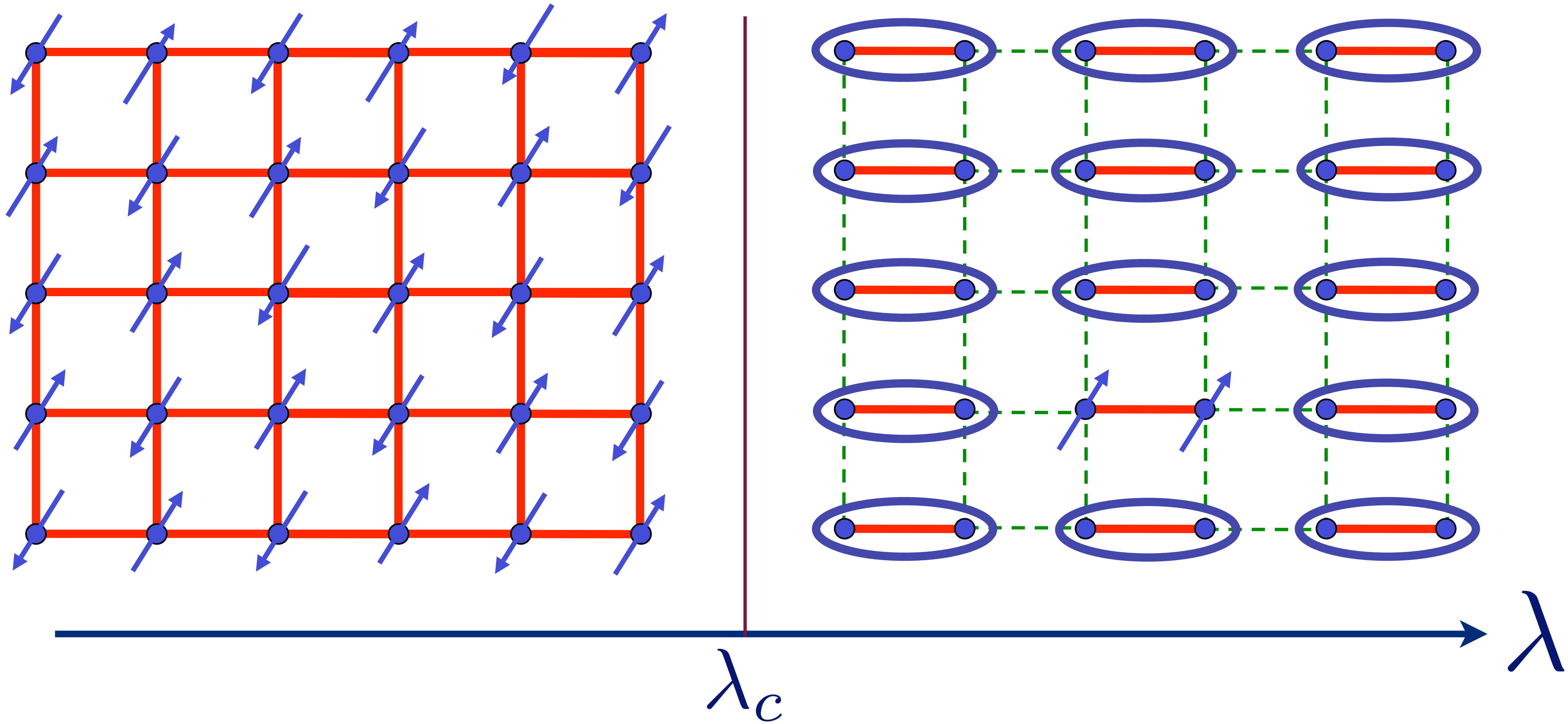

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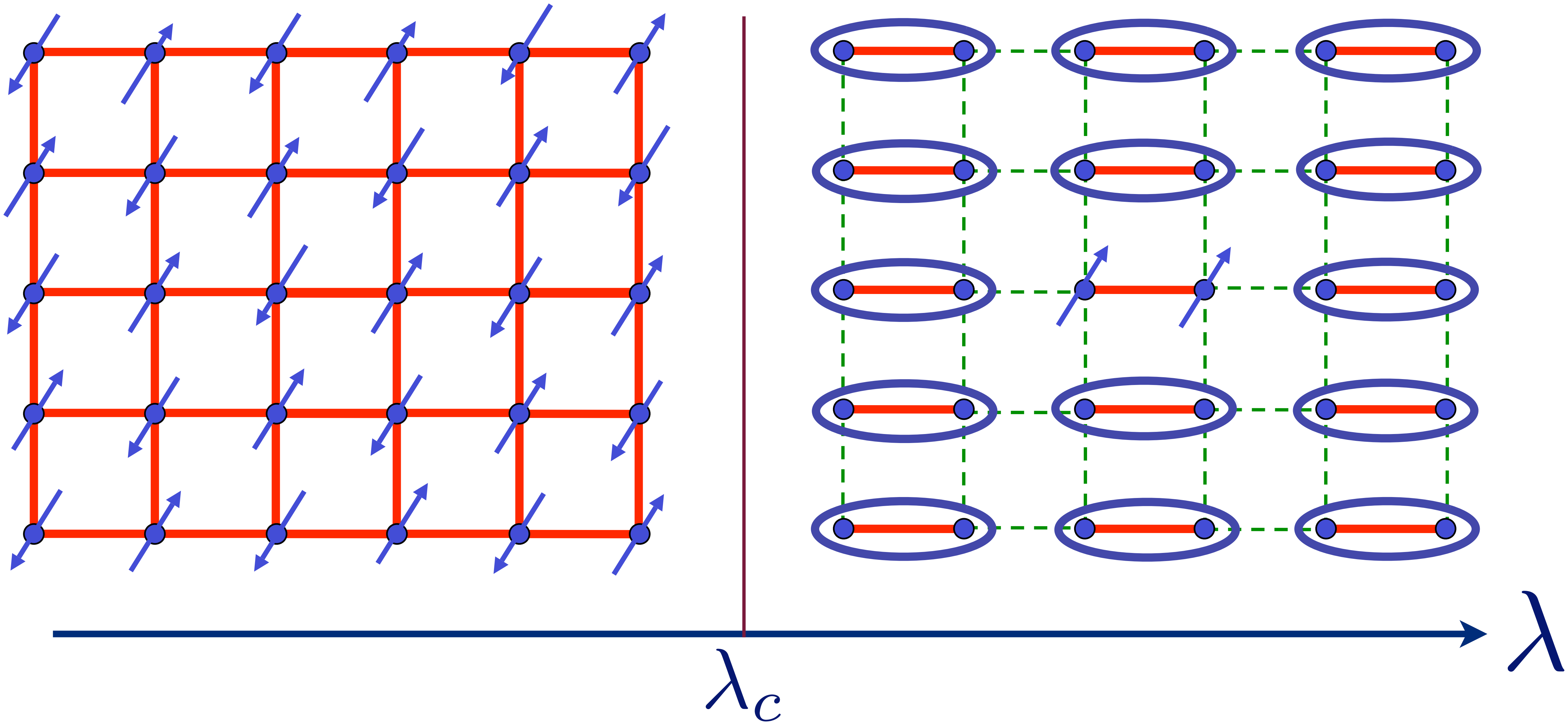

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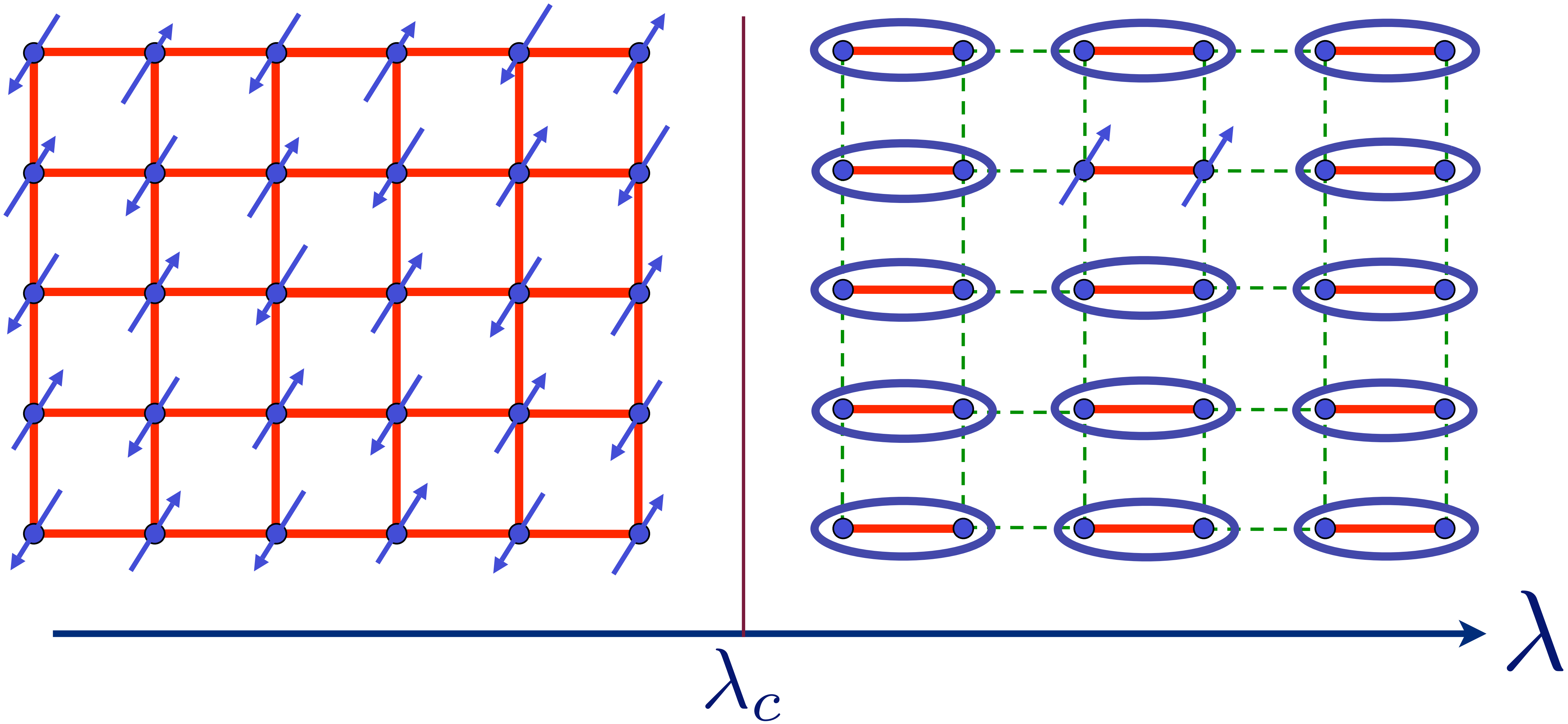

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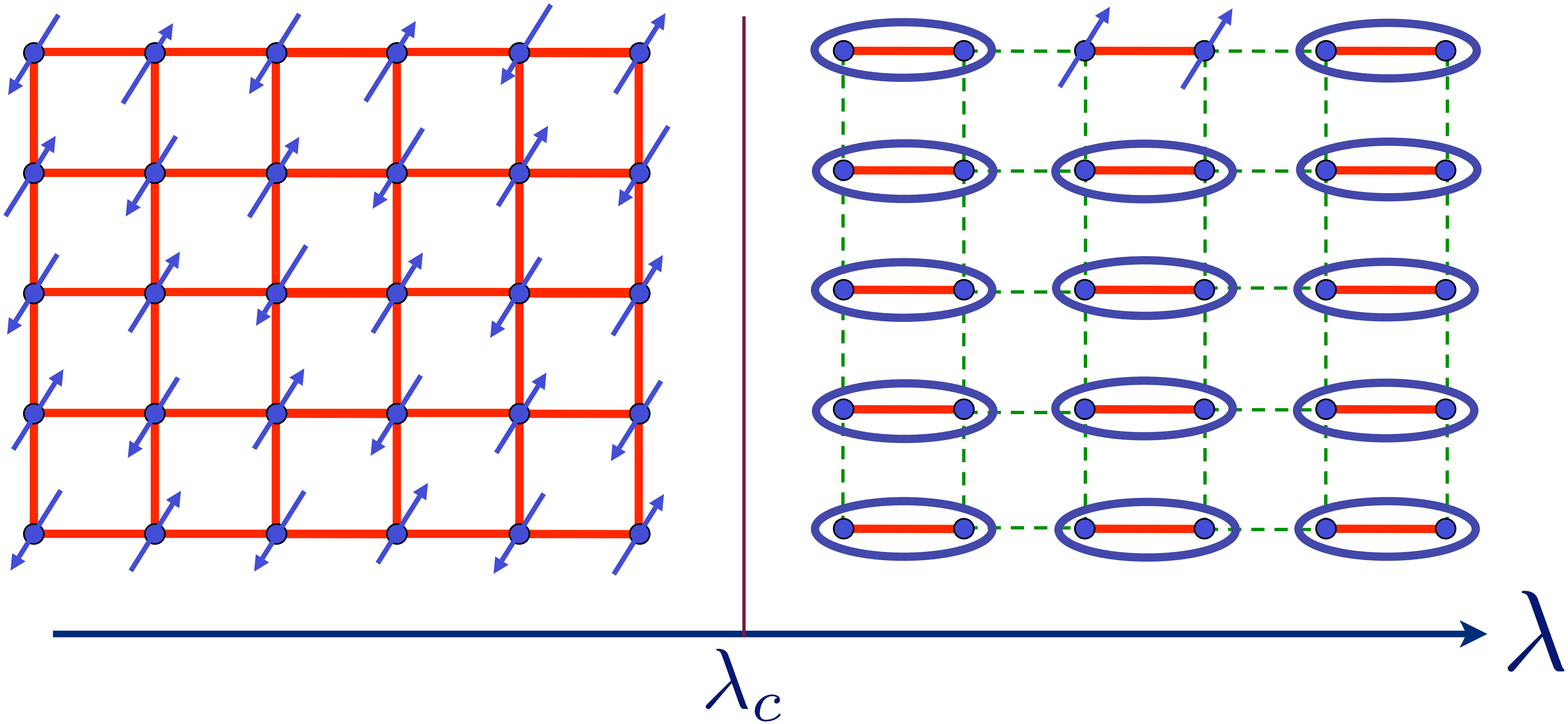
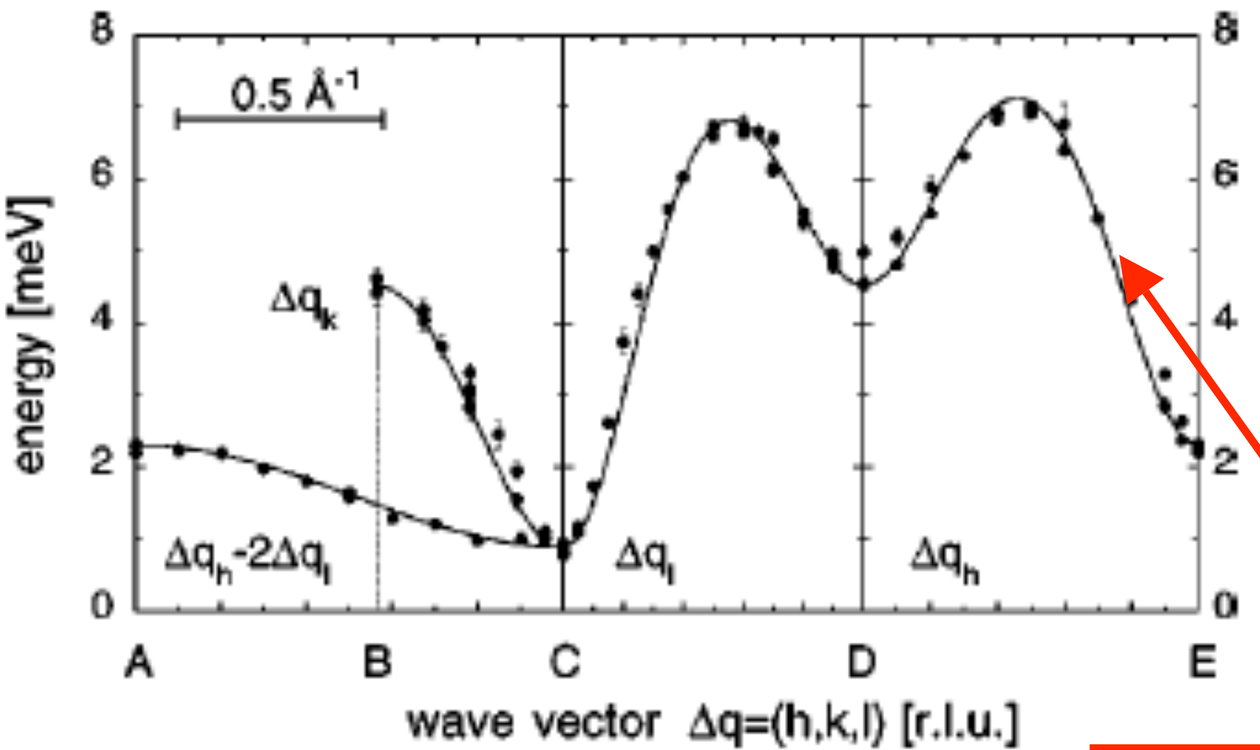
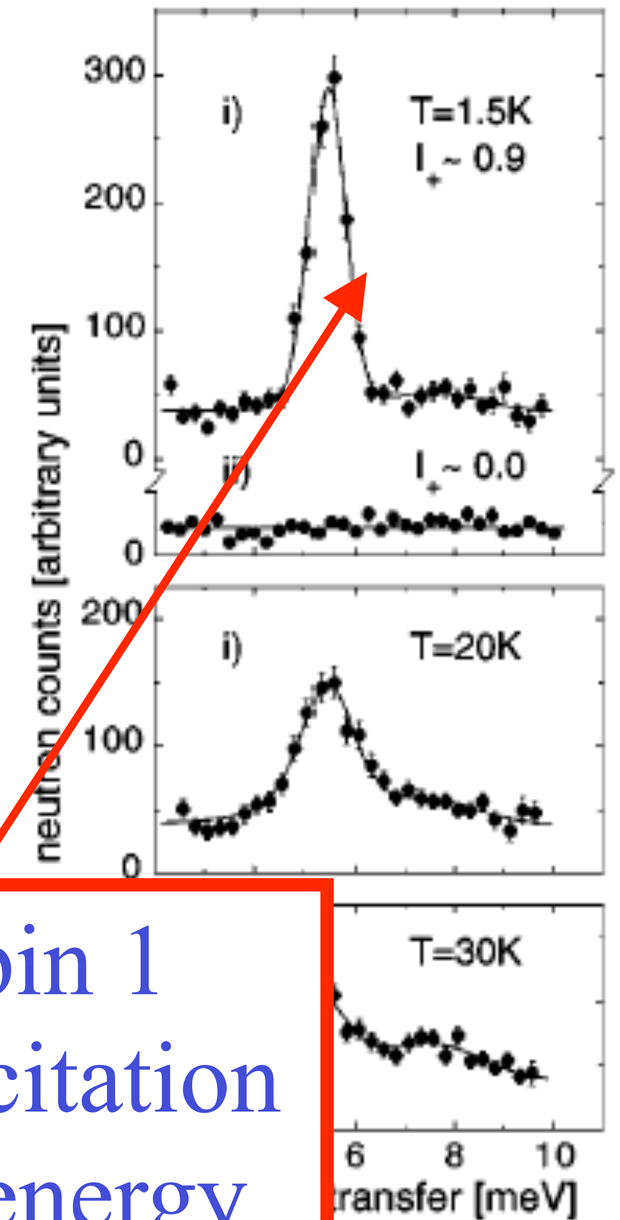


$TlCuCl_3$ at ambient pressure





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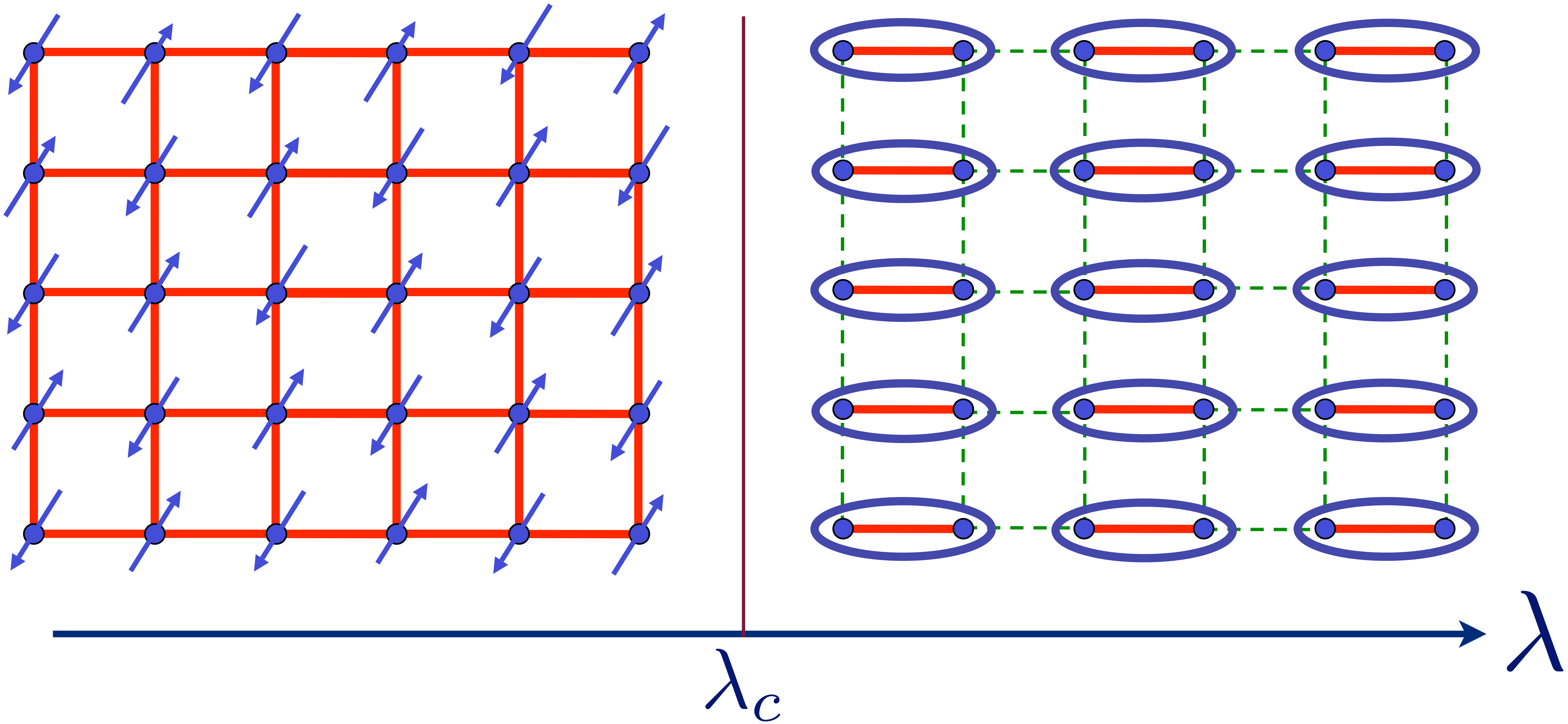
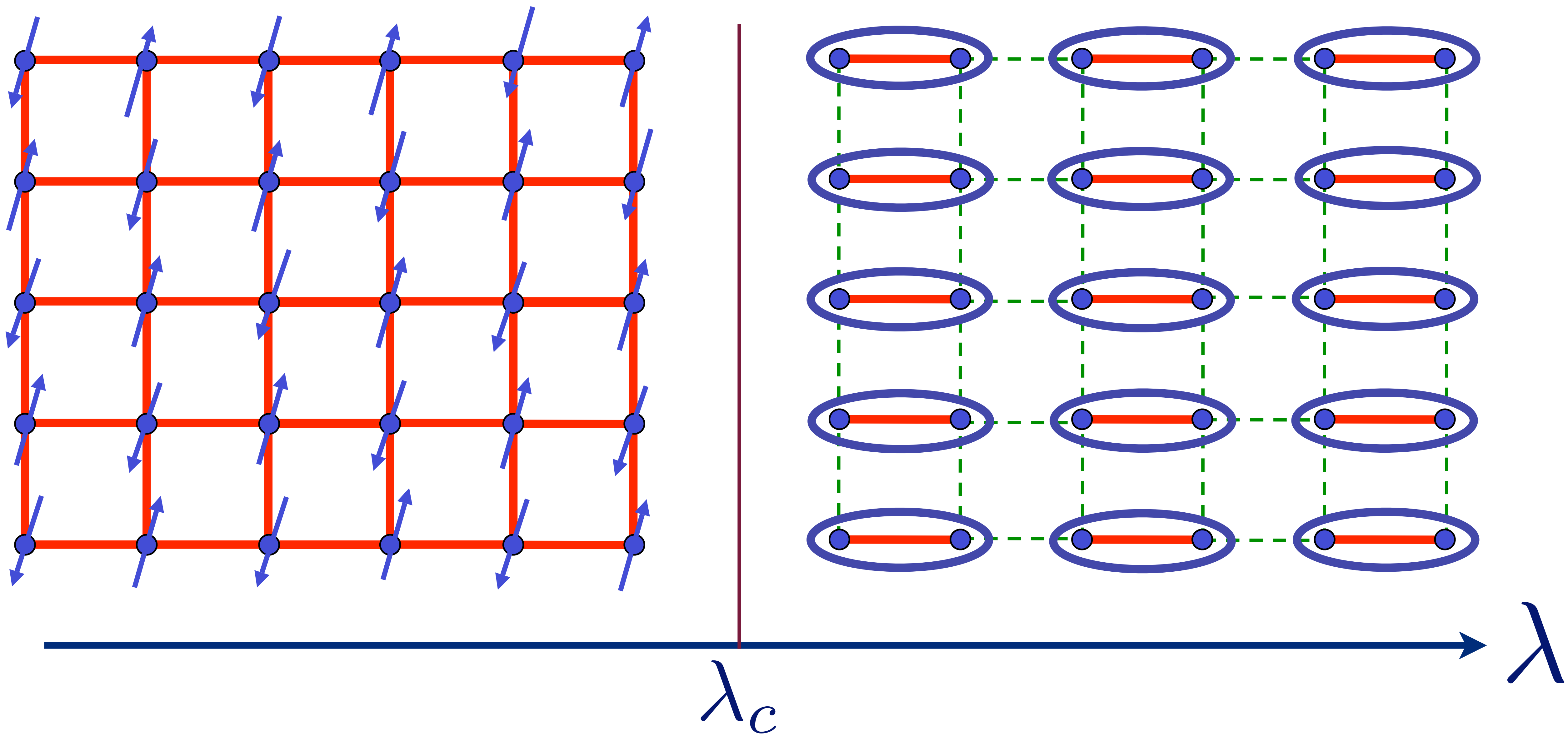
Excitation spectrum in the Néel phase

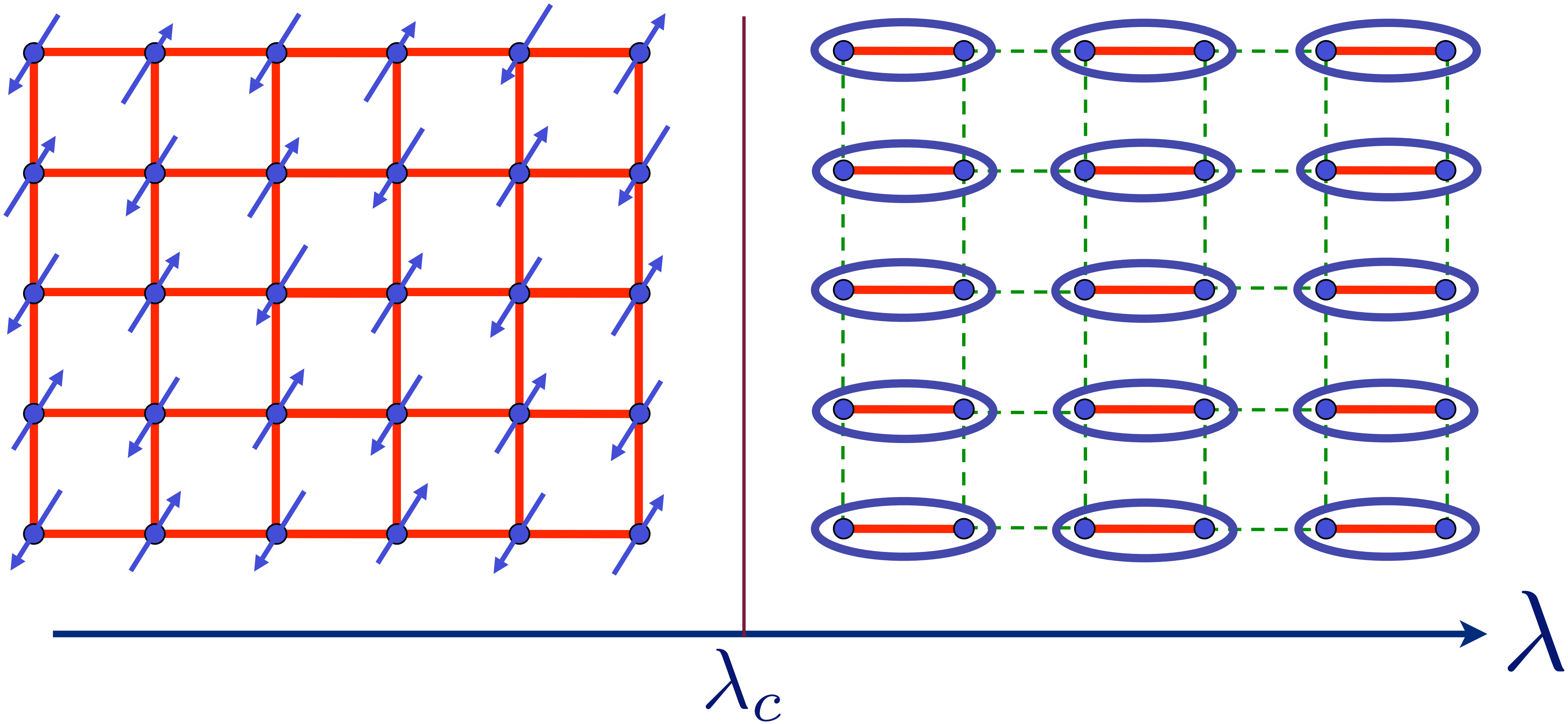


Excitation spectrum in the Néel phase

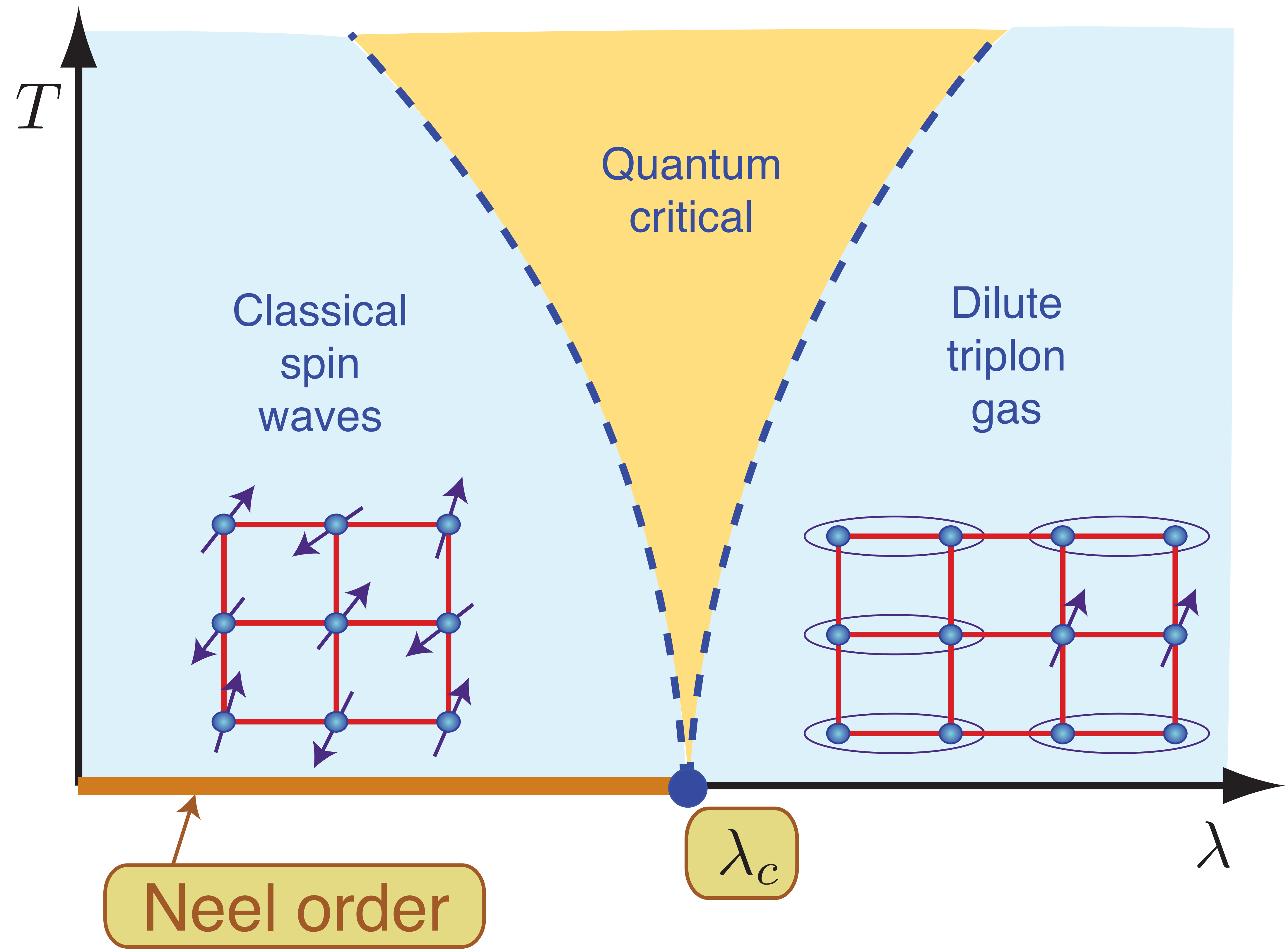


Spin waves

Excitation spectrum in the Néel phase



Spin waves



T
Quantum
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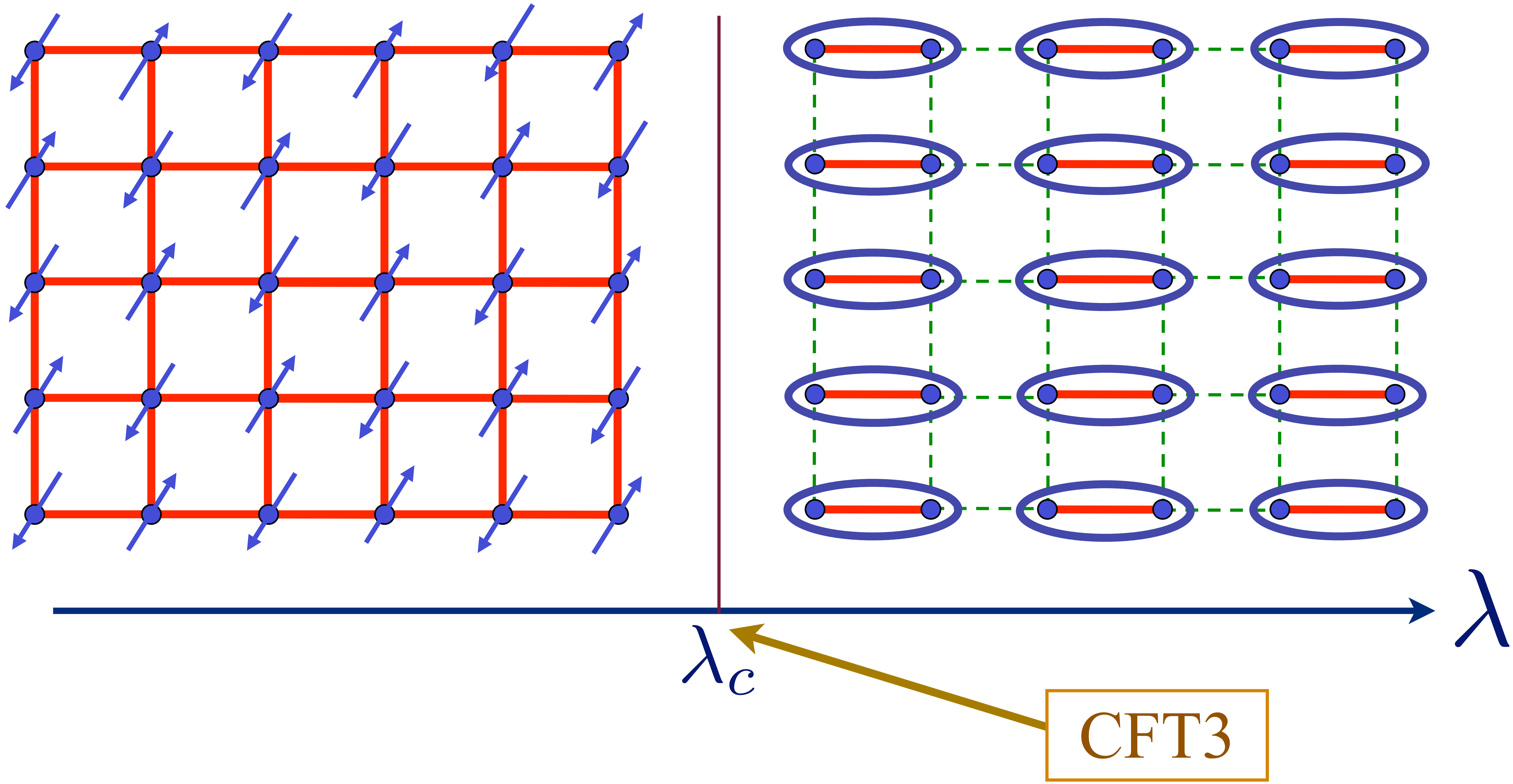
Classical
spin
waves
Dilute
triplon
gas
Neel order
λ_c
λ

Discussion of quantum rotor model

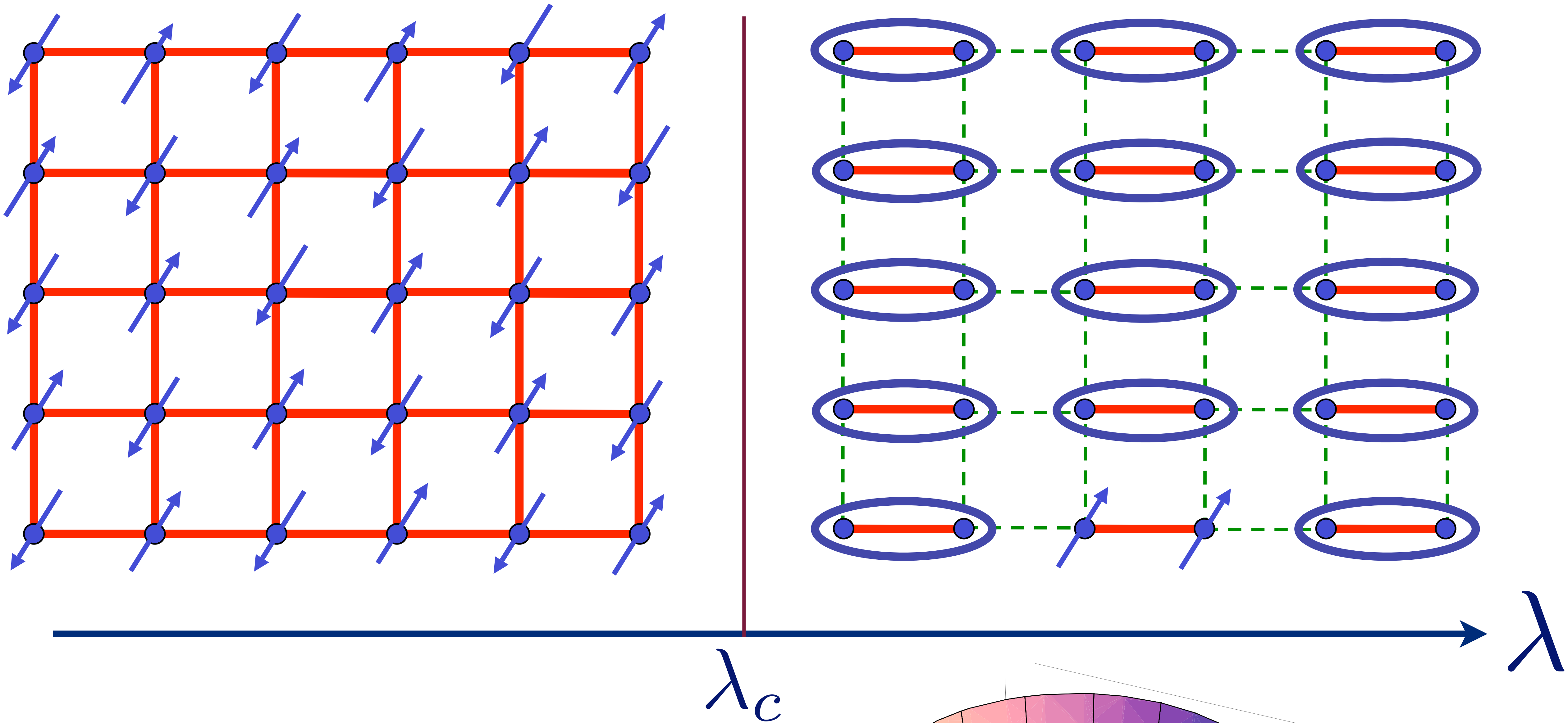
Description using Landau-Ginzburg field theory

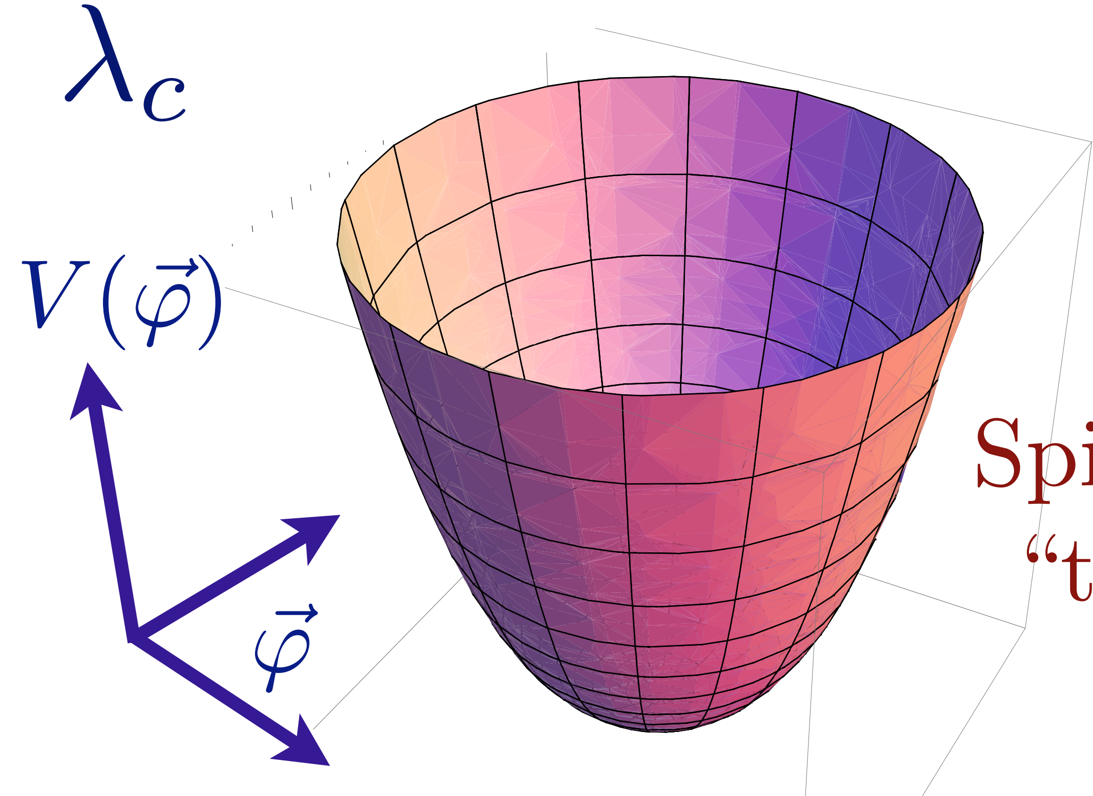


O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c)\vec{\varphi}^2 + u \left(\vec{\varphi}^2\right)^2 \right]$$

Excitation spectrum in the paramagnetic phase



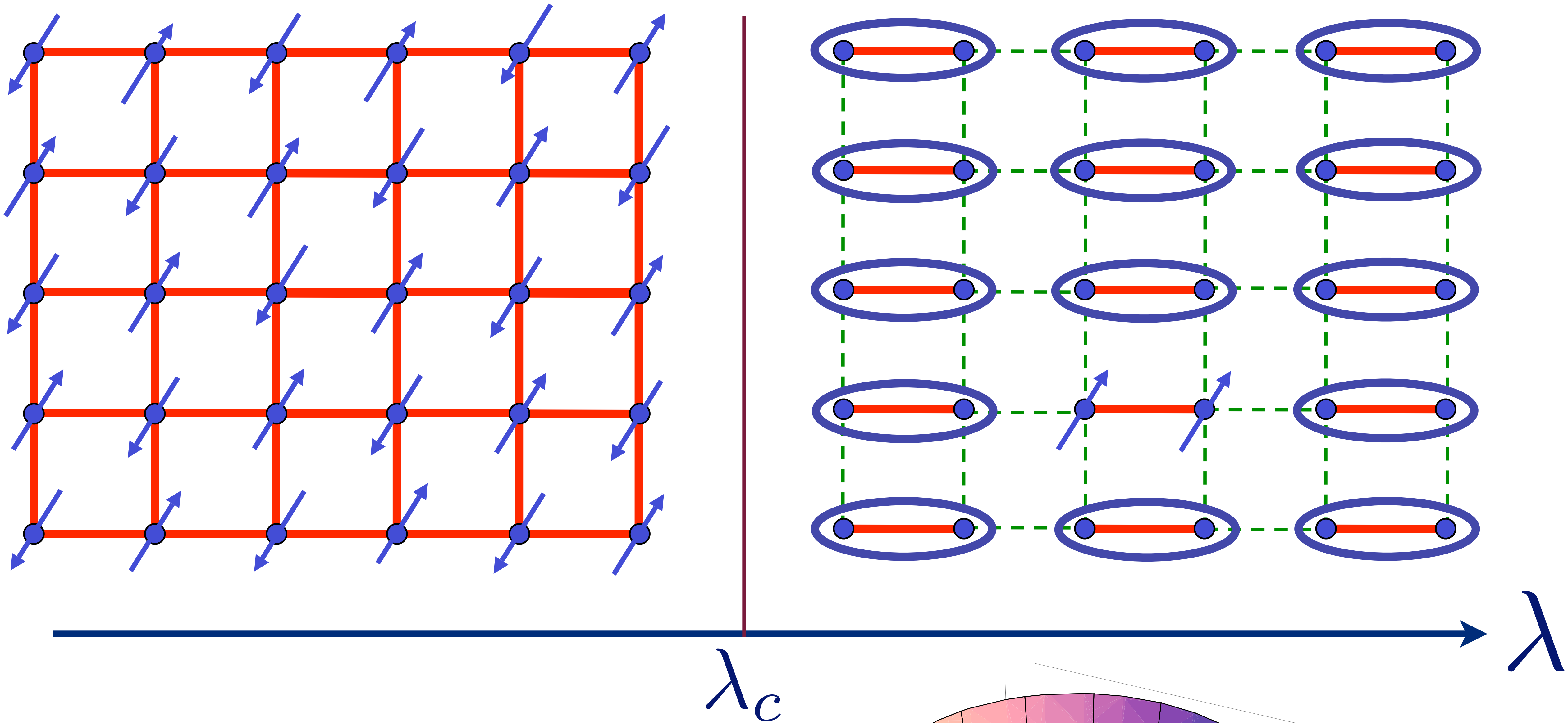


$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$

Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



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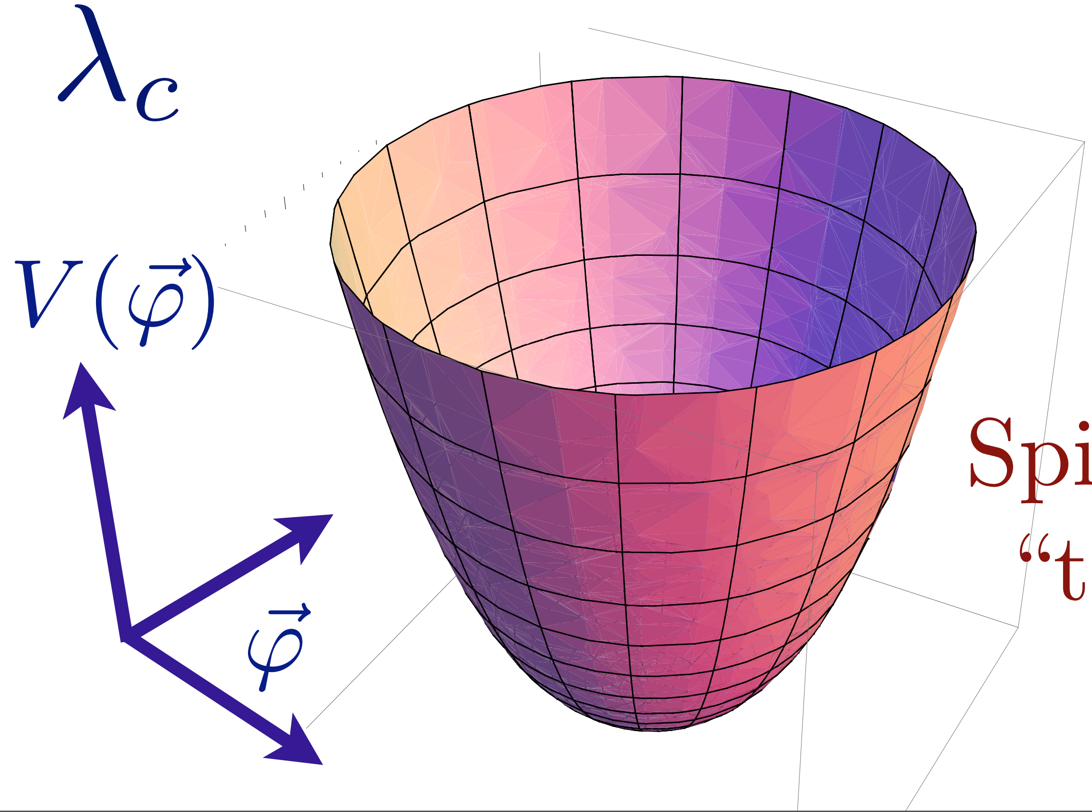

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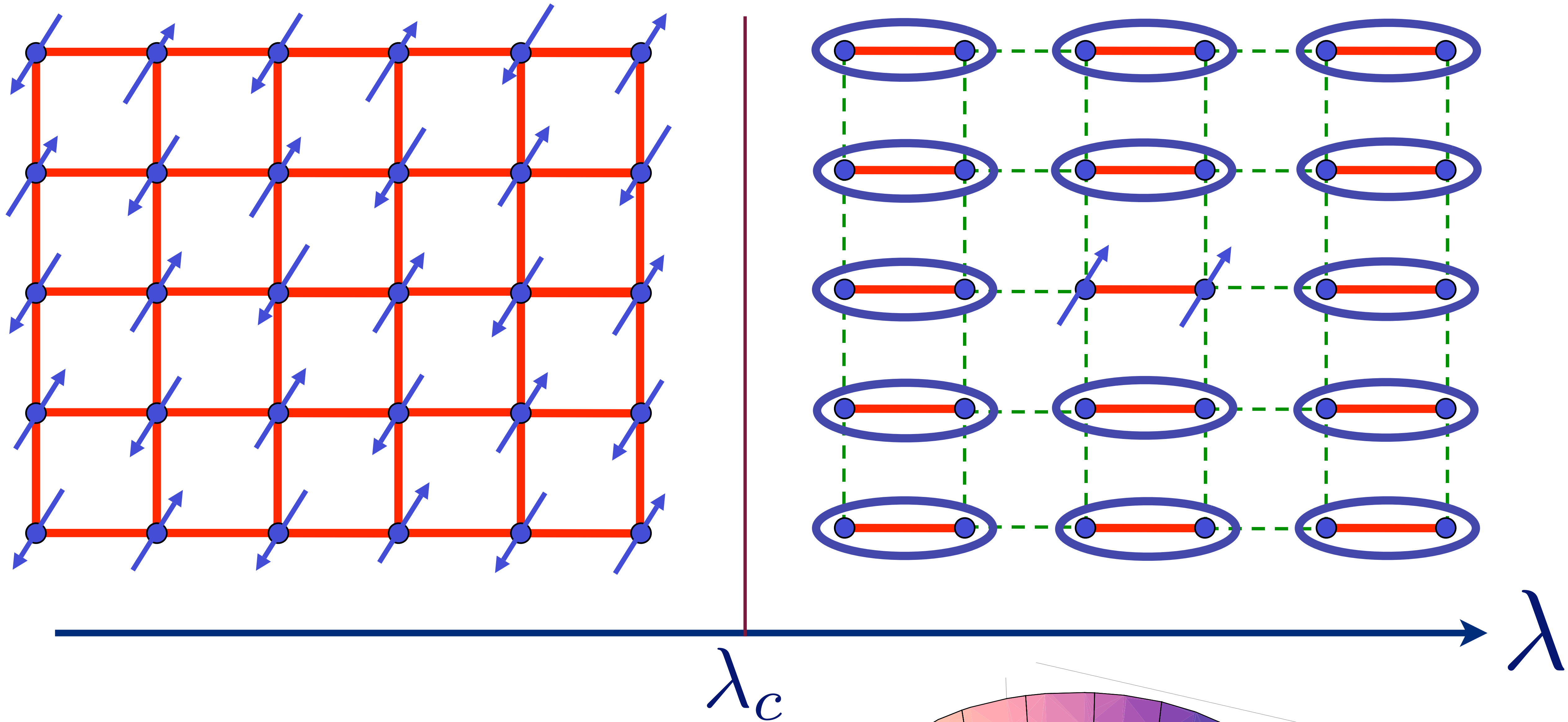

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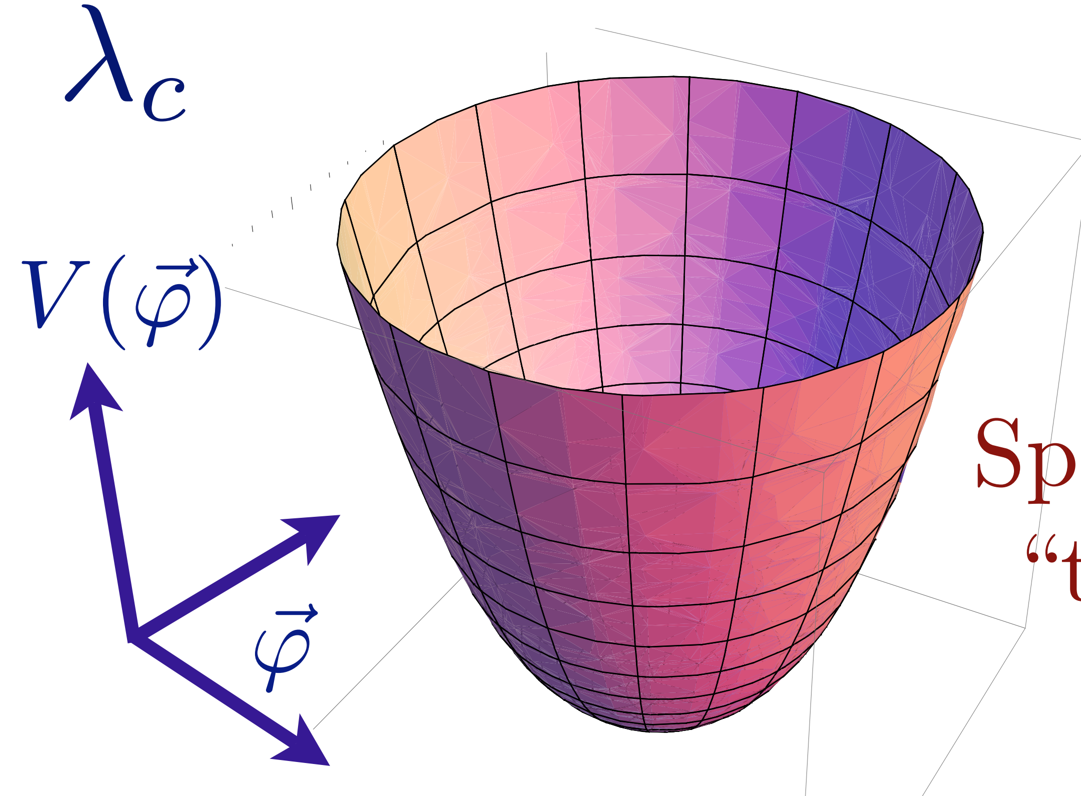

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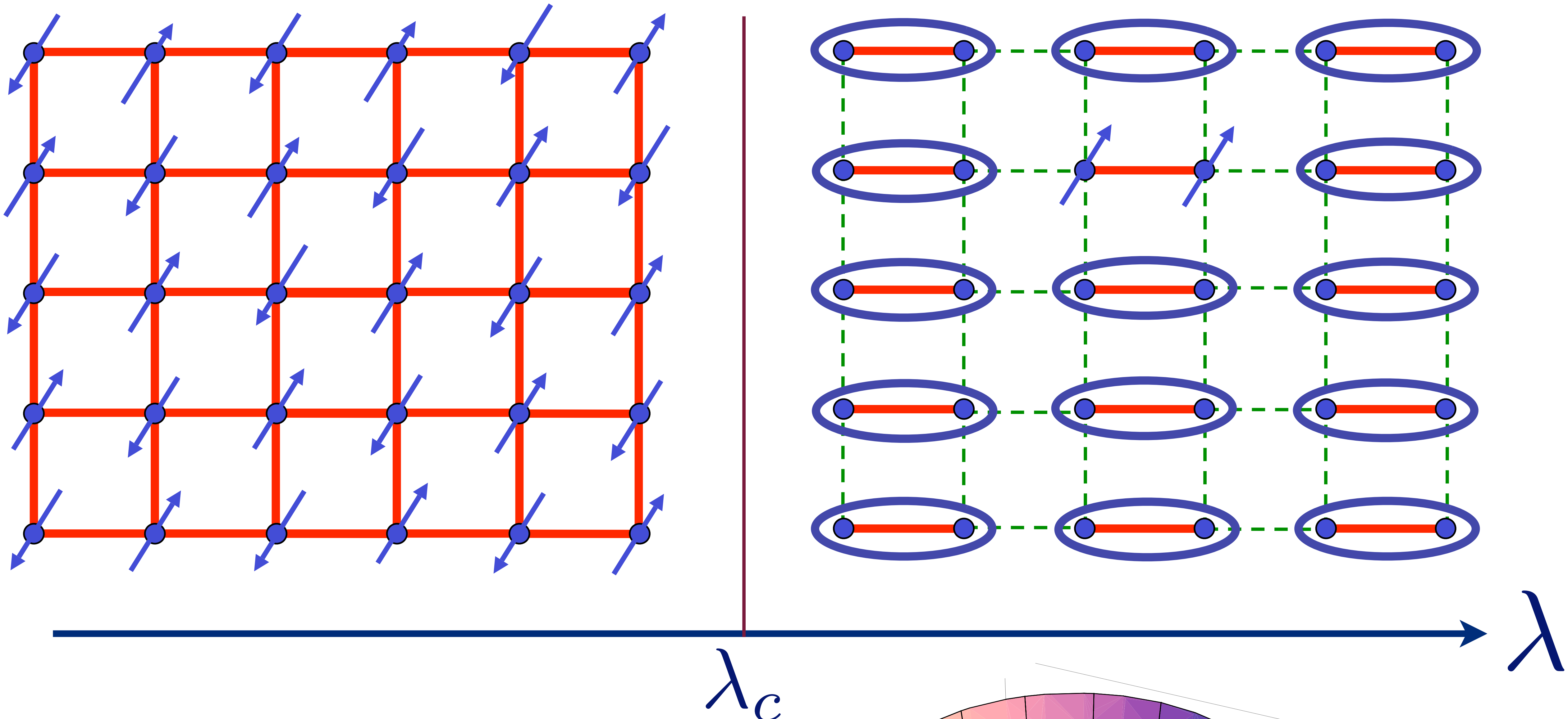

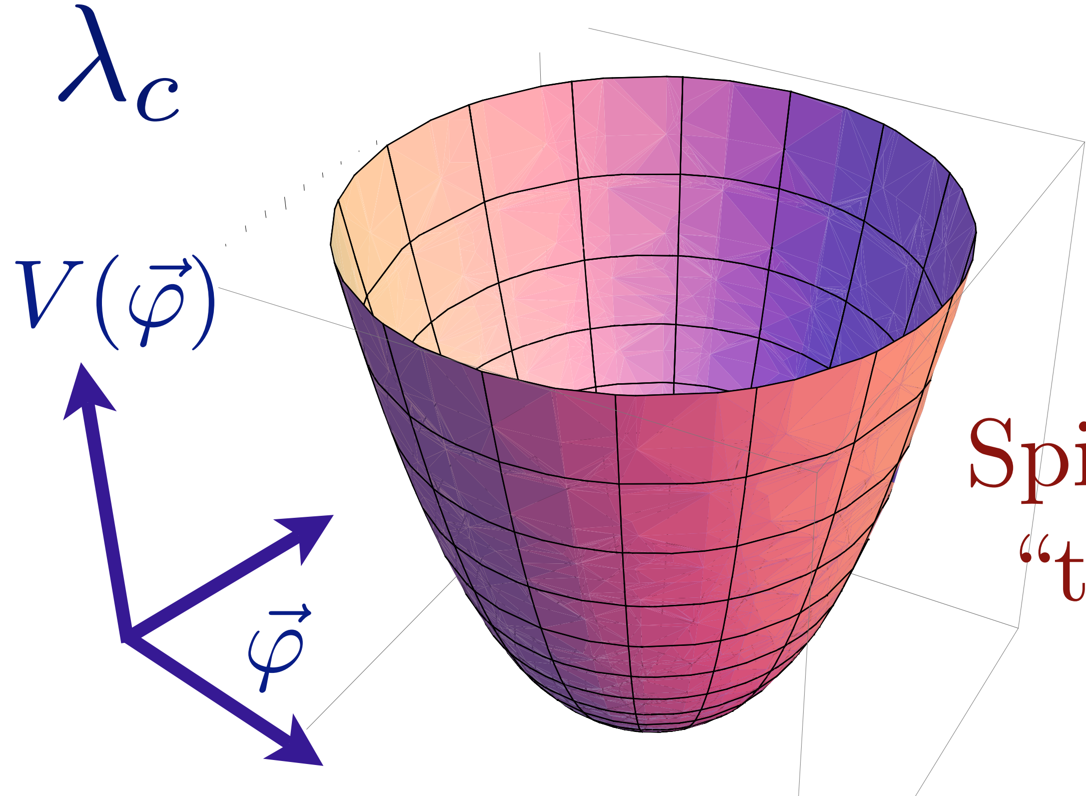

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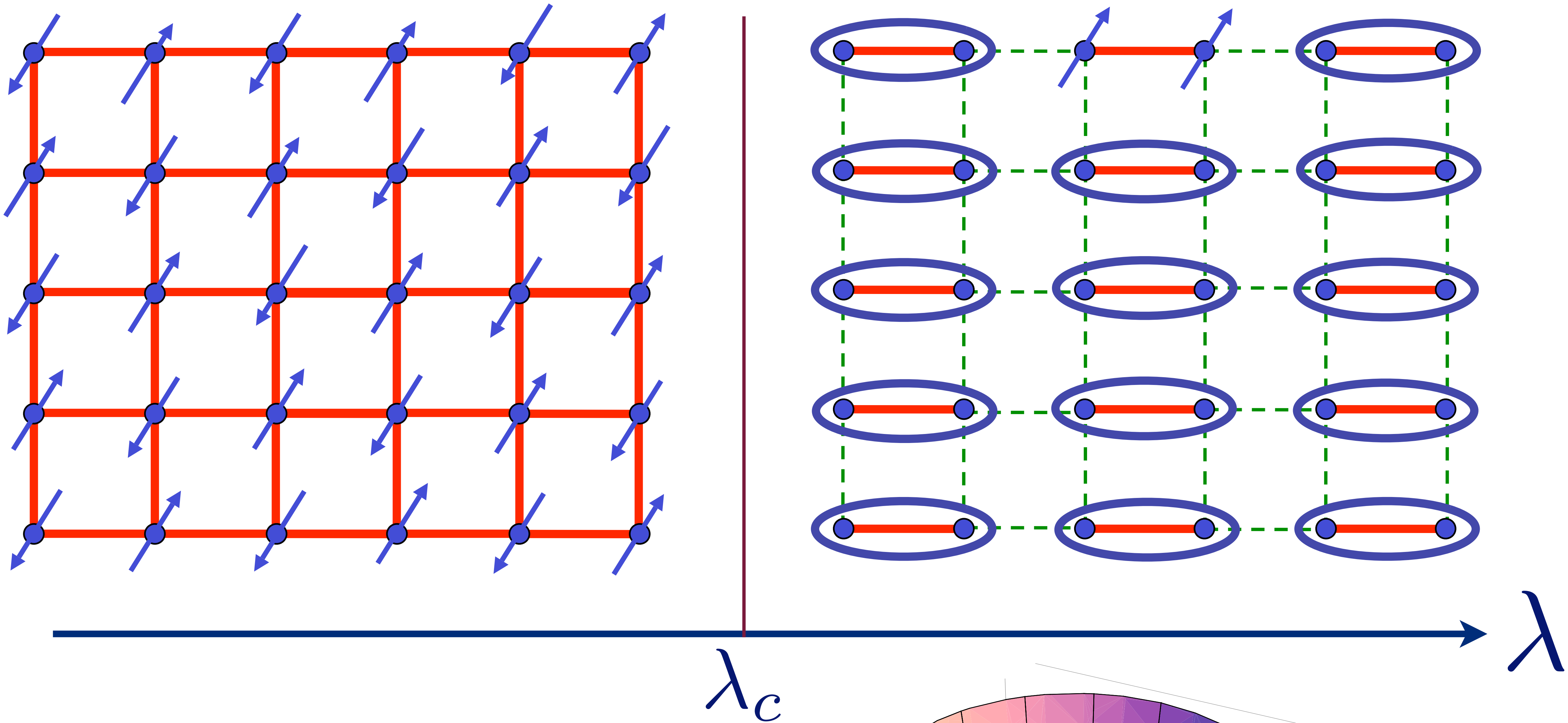

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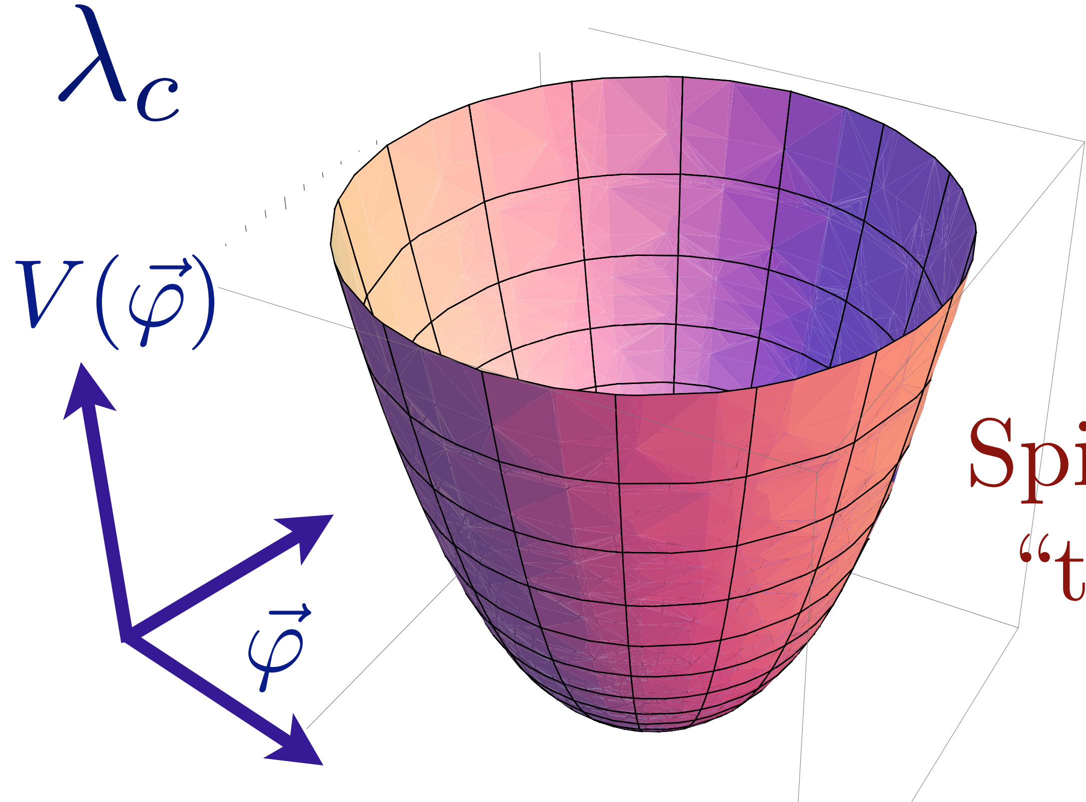

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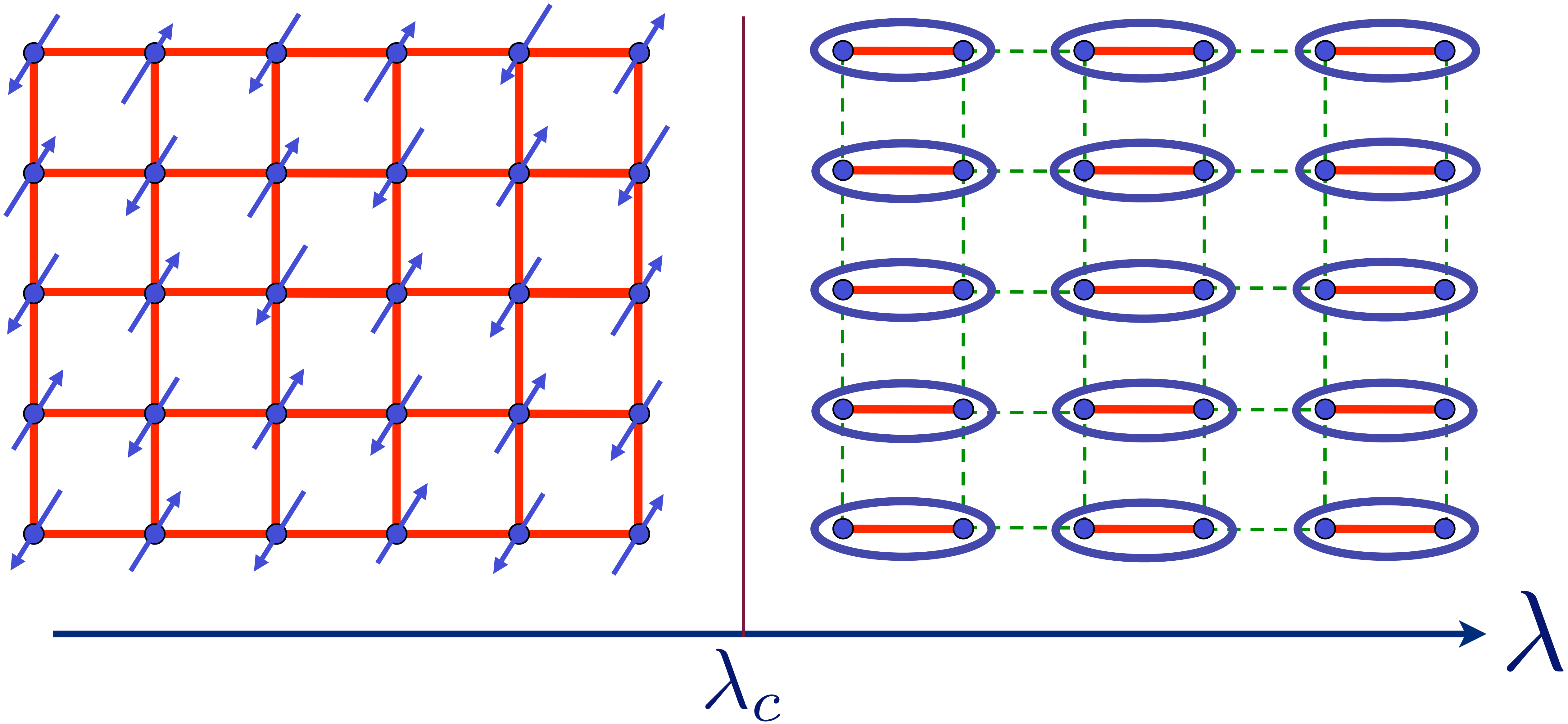

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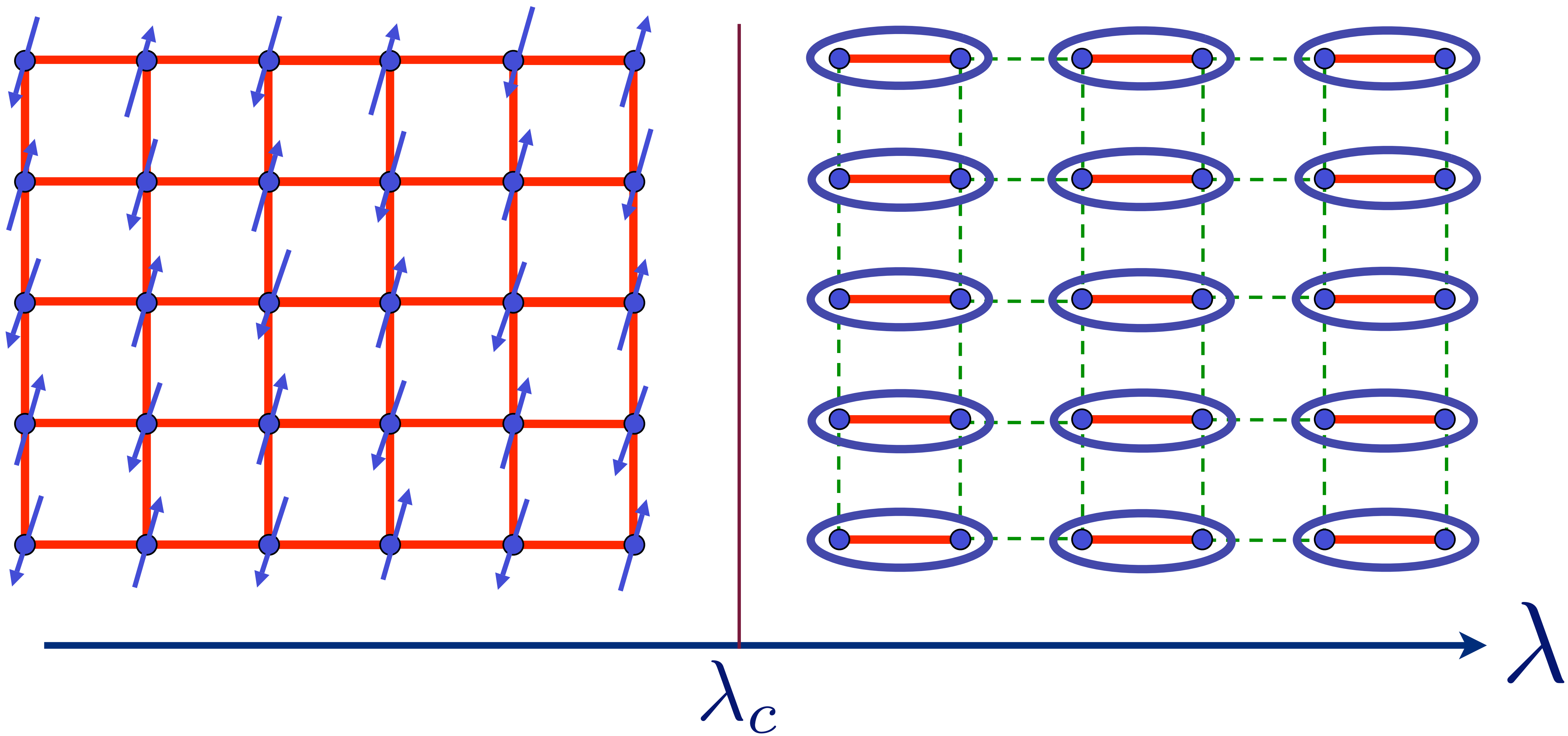

Spin waves

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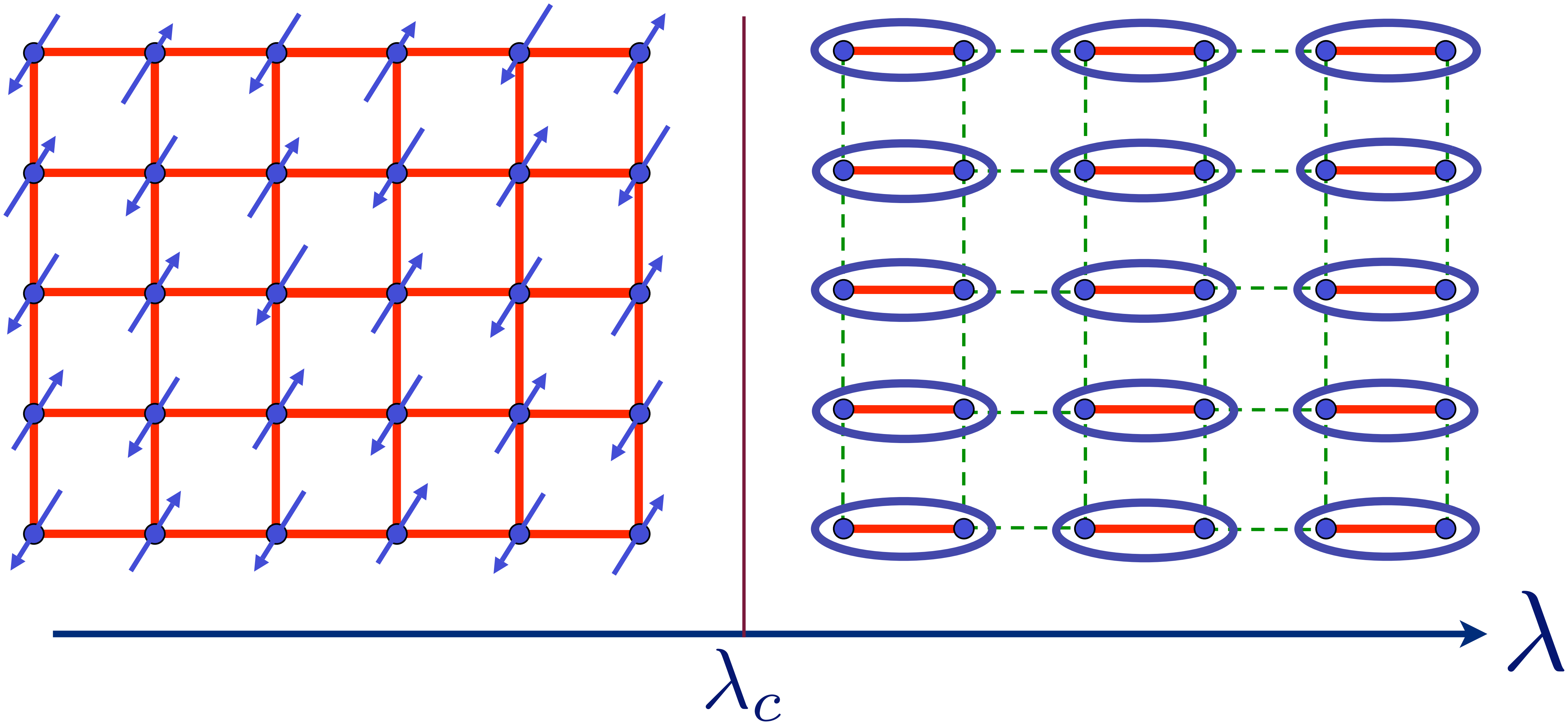

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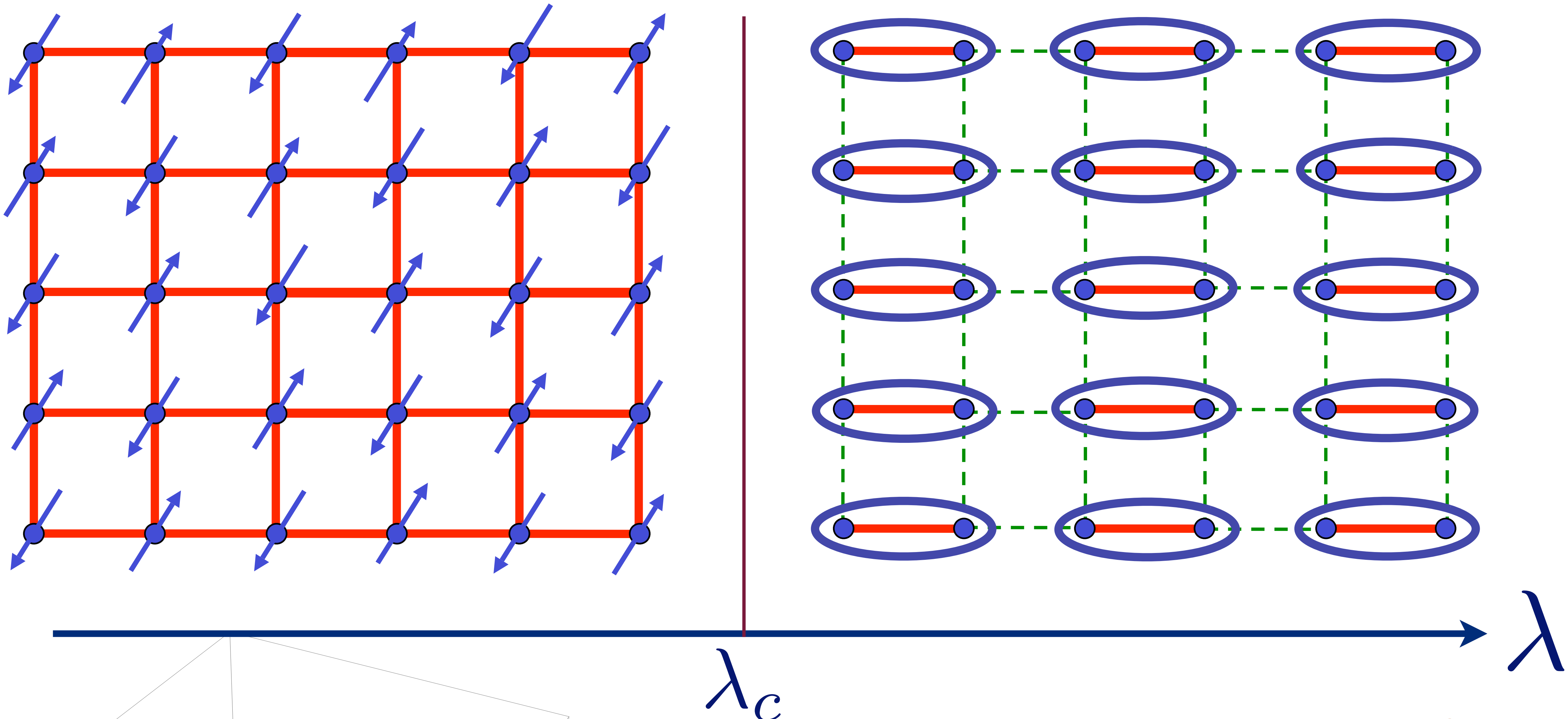

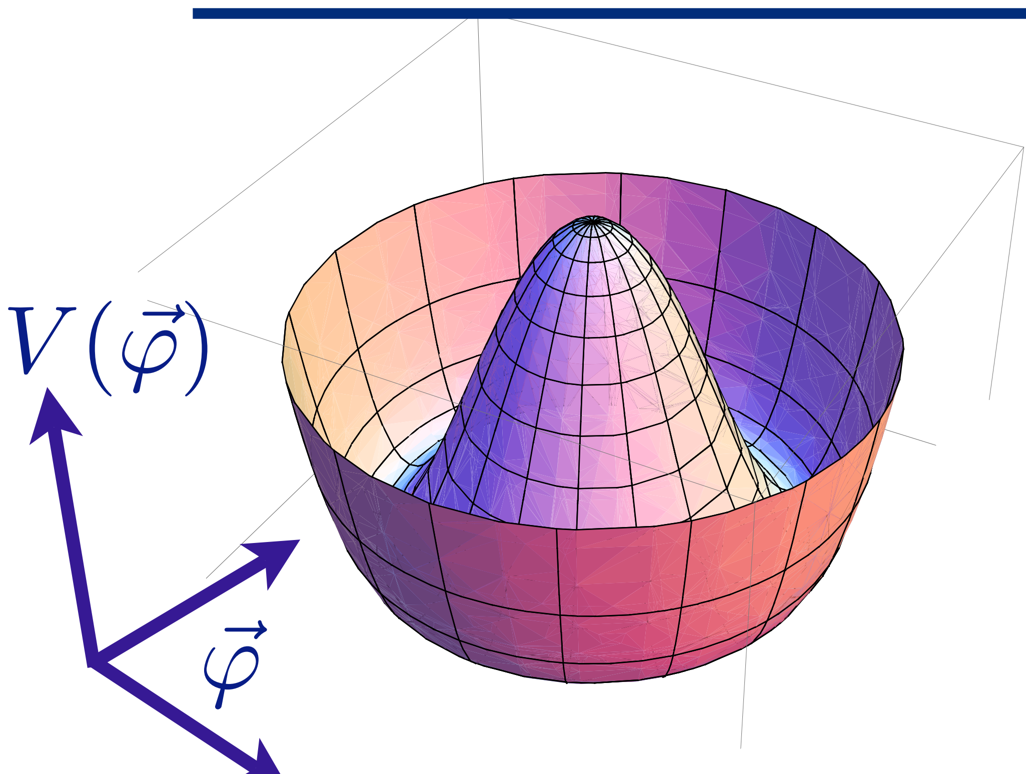

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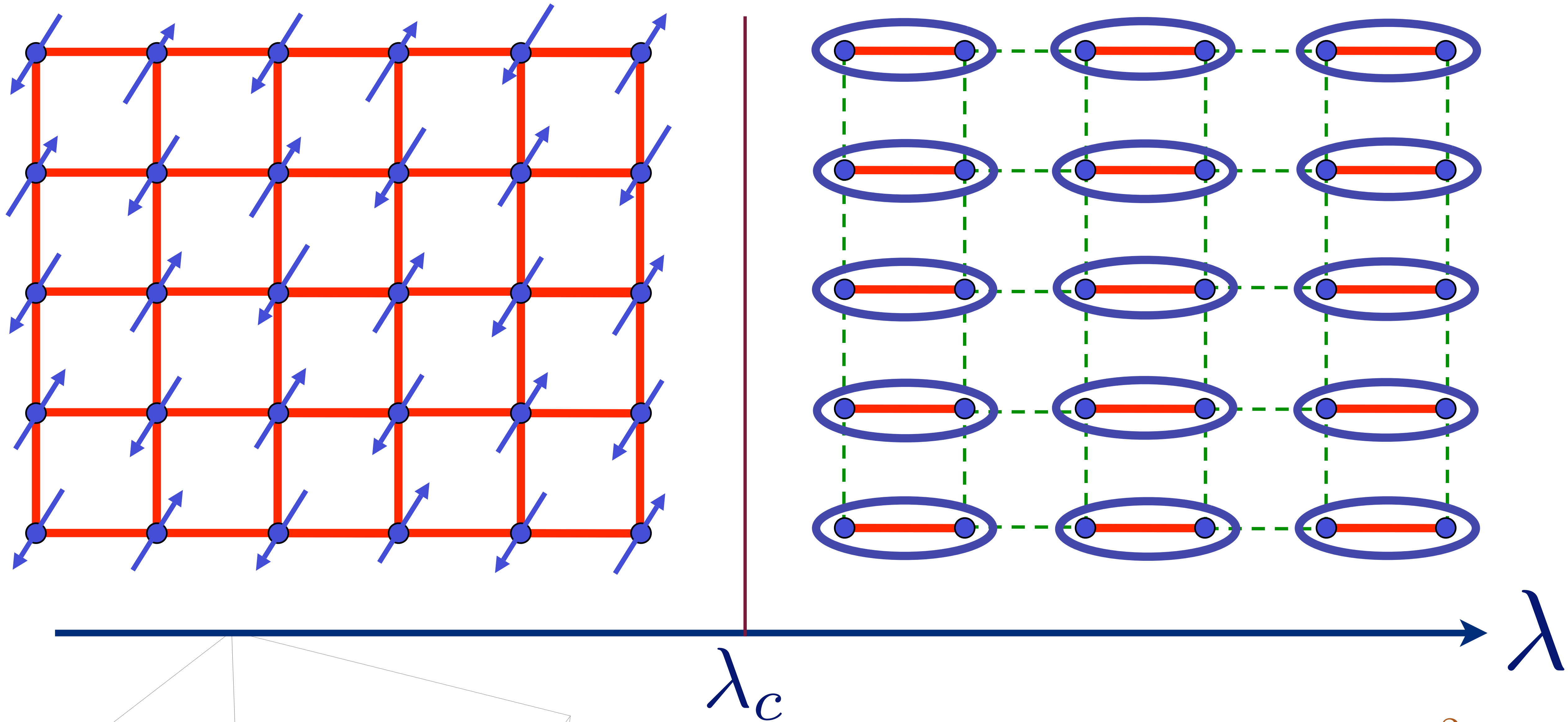

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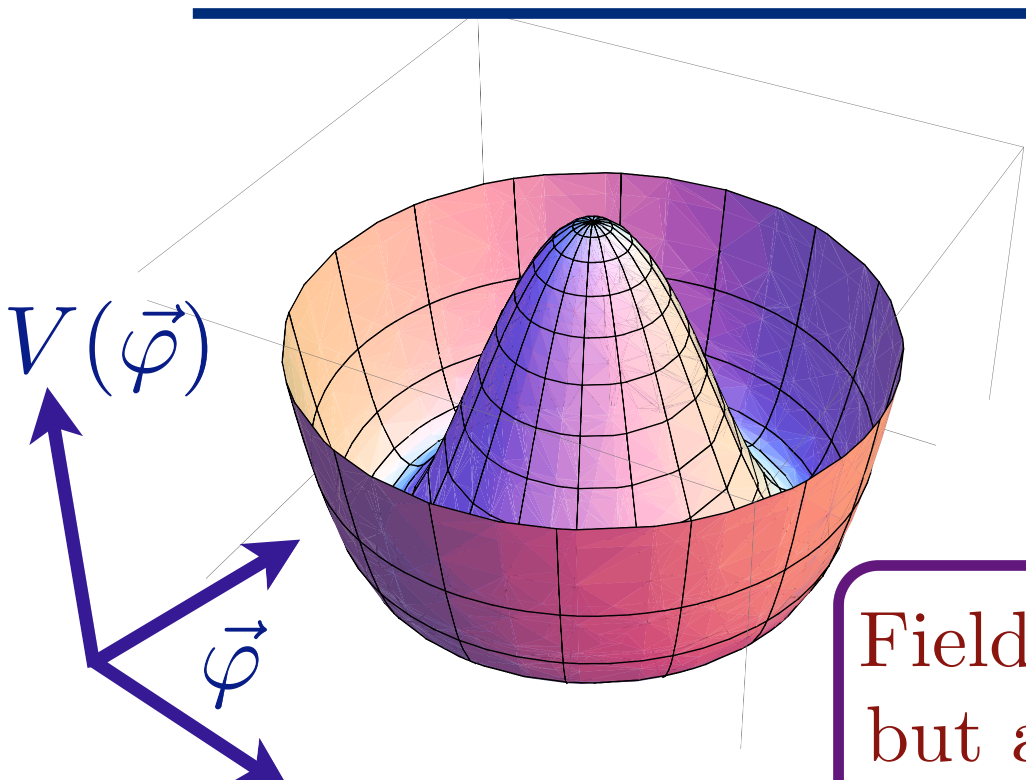
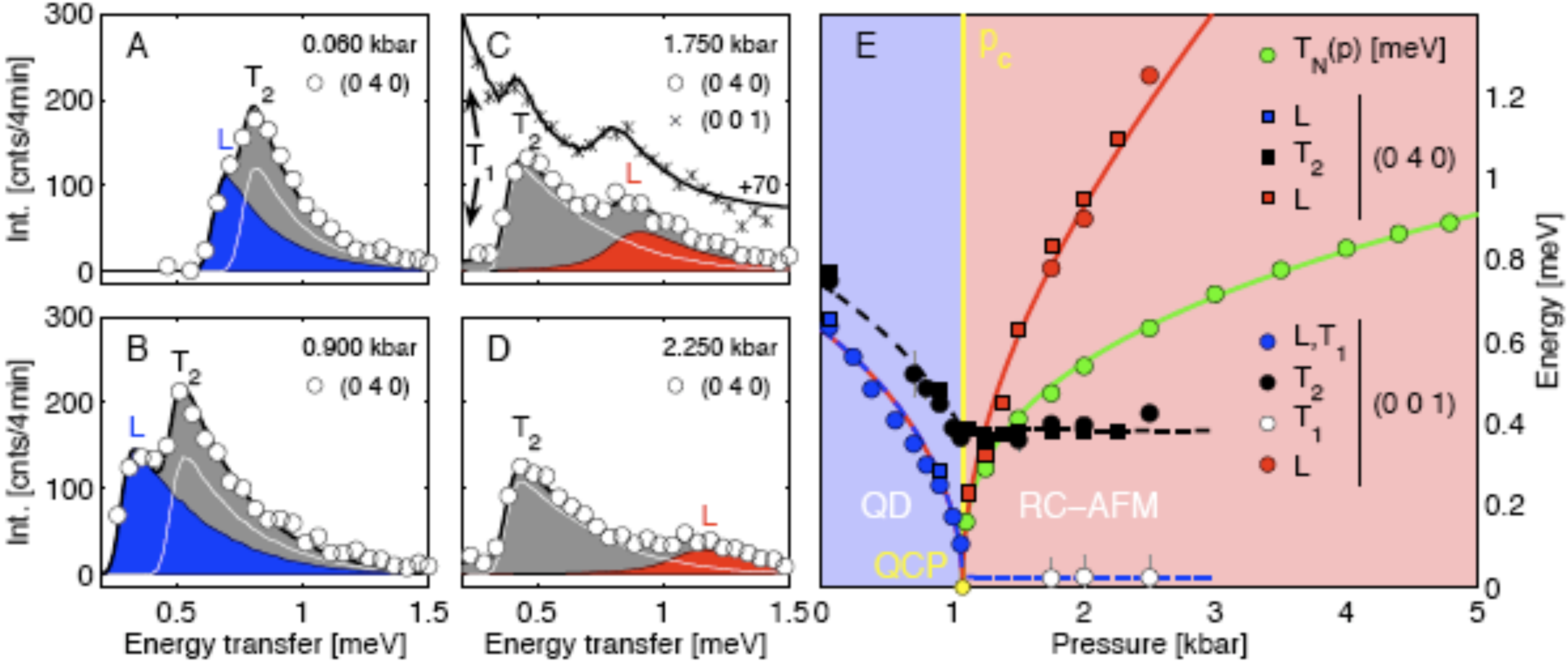


Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle

$TlCuCl_3$ with varying pressure



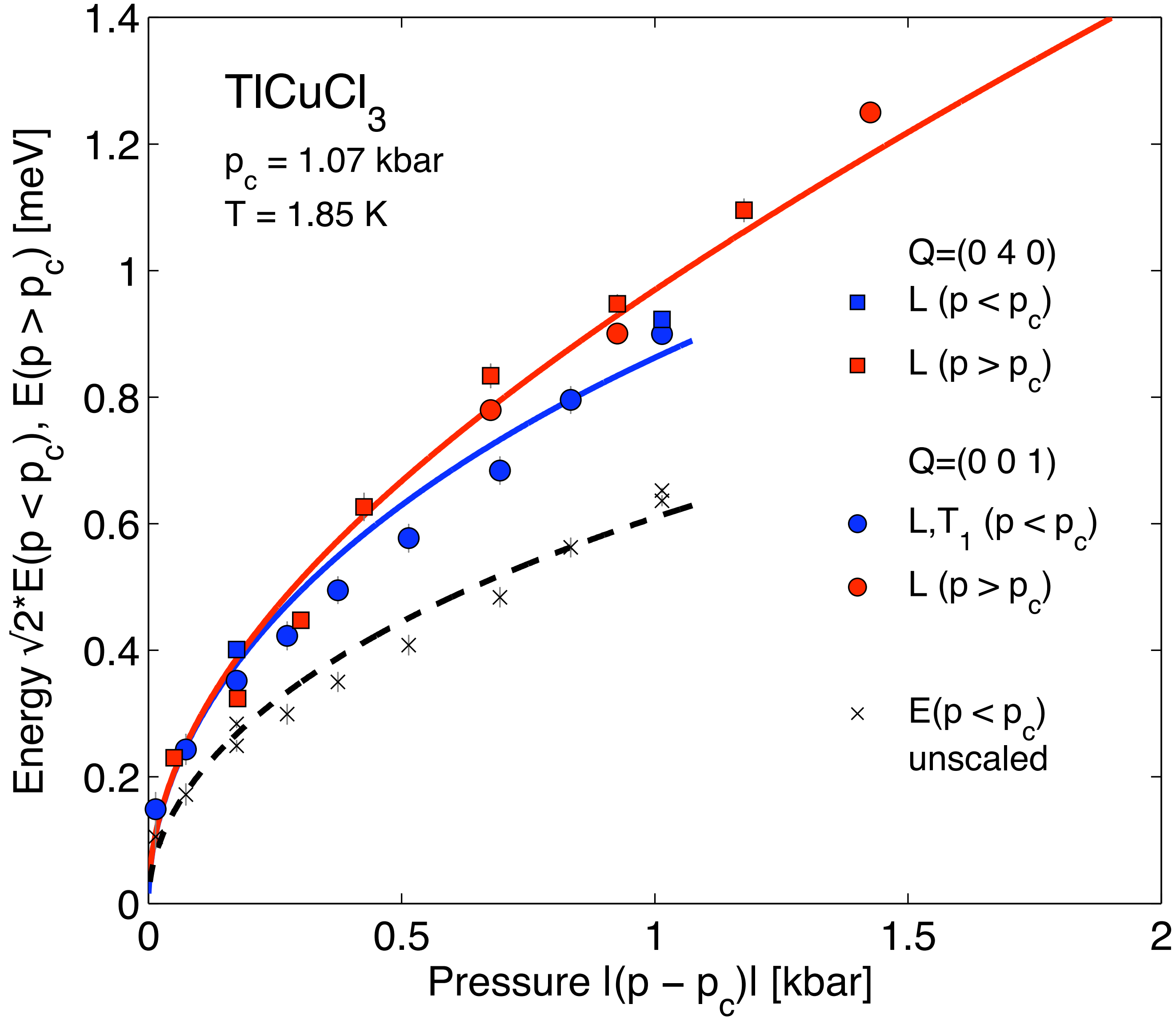
Observation of 3 → 2 low energy modes,
emergence of new Higgs particle in the Néel phase,
and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

$$\frac{\text{Energy of "Higgs" particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$



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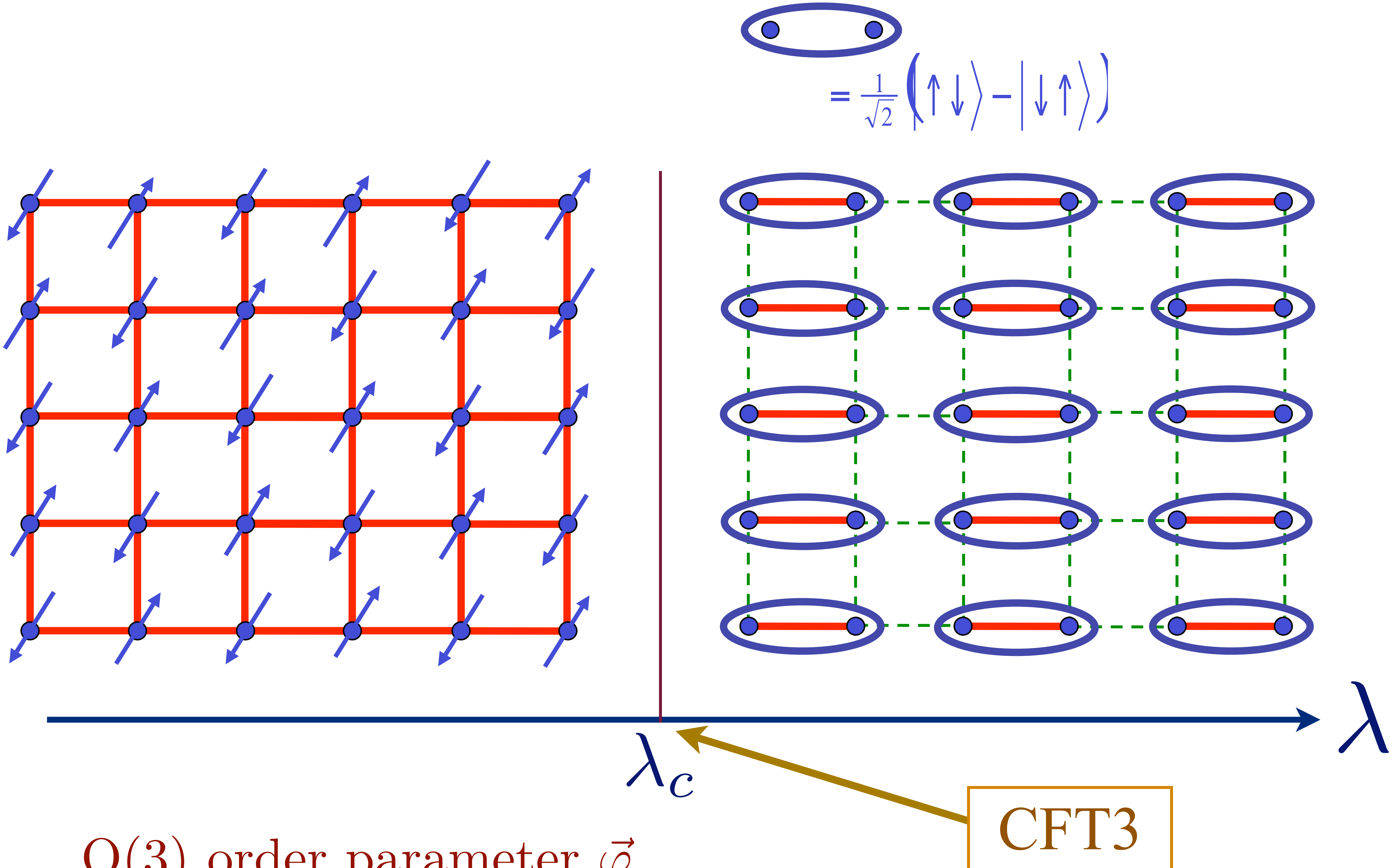

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Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [...] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096-\sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096+\sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107[d]	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230-\sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
$1.8230+\sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

S. Wenzel and W. Janke, arXiv:0808.1418

M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

Field-theoretic RG of CFT3
E. Vicari *et al.*

S. Wenzel and W. Janke, arXiv:0808.1418
M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

Outline

1. Coupled dimer antiferromagnets

   Landau-Ginzburg quantum criticality

2. Spin density waves in metals

   Paramagnon quantum criticality

3. Spin liquids and valence bond solids

   Schwinger-boson mean-field theory and U(1) gauge theory

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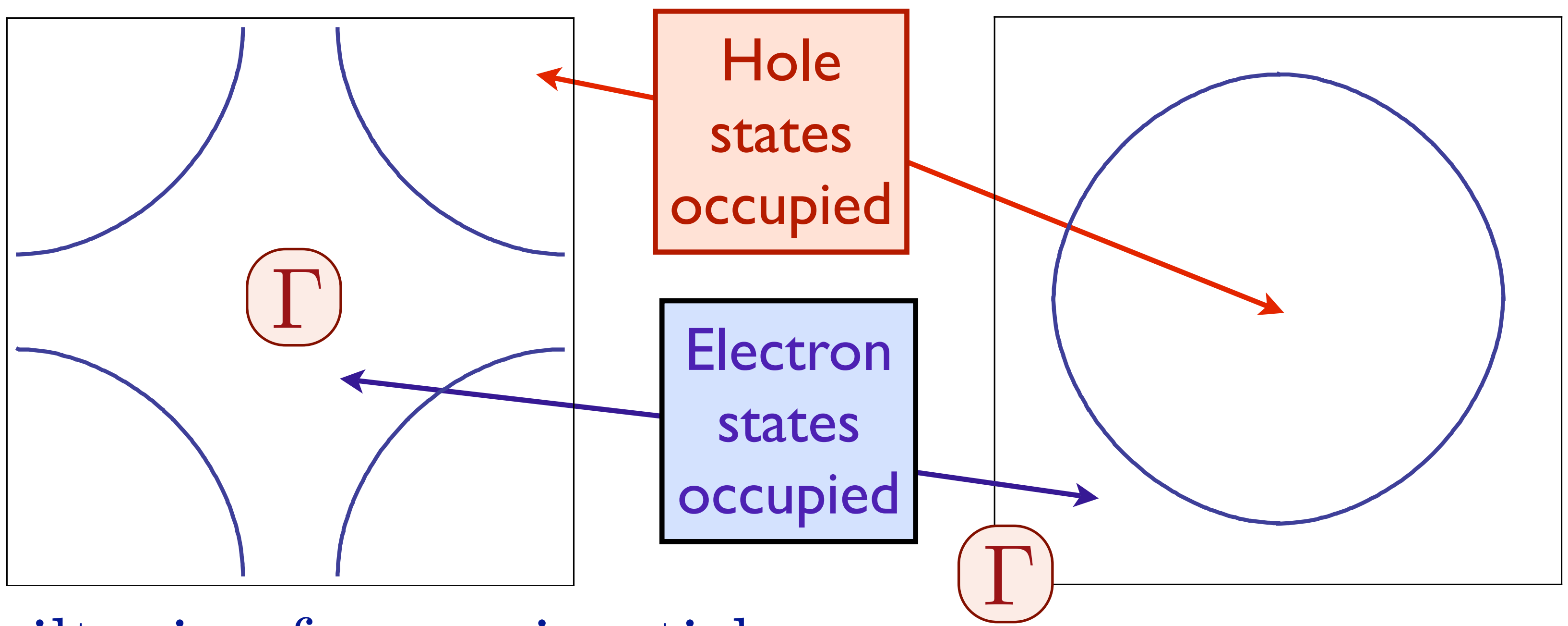
Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-p) & \text{for hole-doping } p \\ 2\pi^2(1+x) & \text{for electron-doping } x \end{cases}$$

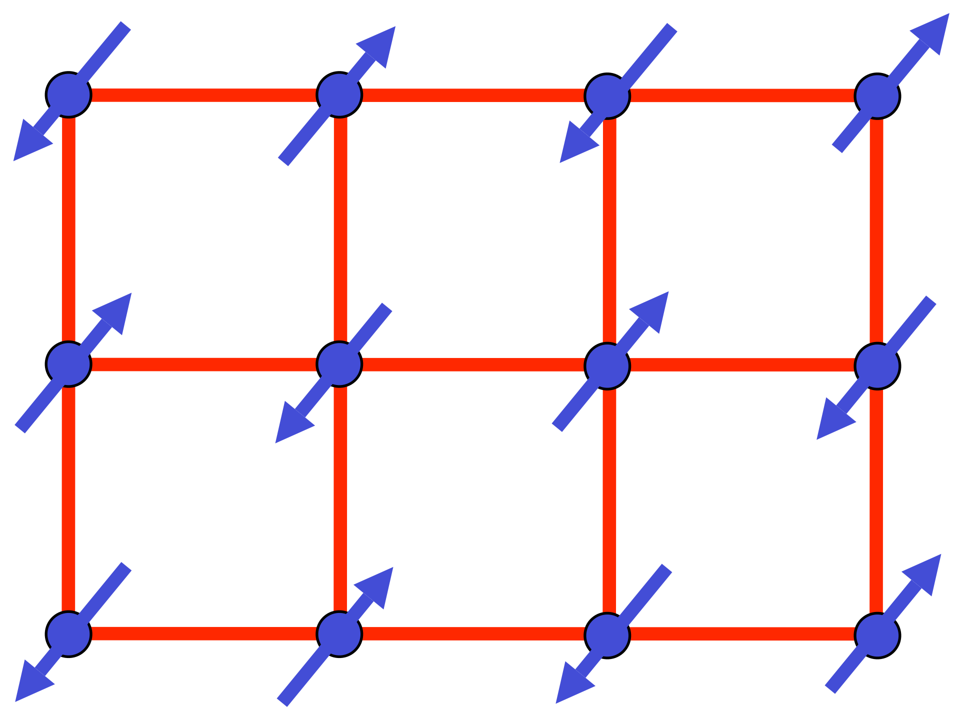
The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

A spin density wave (SDW) is the spontaneous appearance of an oscillatory spin polarization. The electron spin polarization is written as

$$\vec{S}(\mathbf{r}, \tau) = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the SDW order parameter, and $\mathbf{K}$ is a fixed ordering wavevector. For simplicity we will consider the case of $\mathbf{K} = (\pi, \pi)$, but our treatment applies to general $\mathbf{K}$.



Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

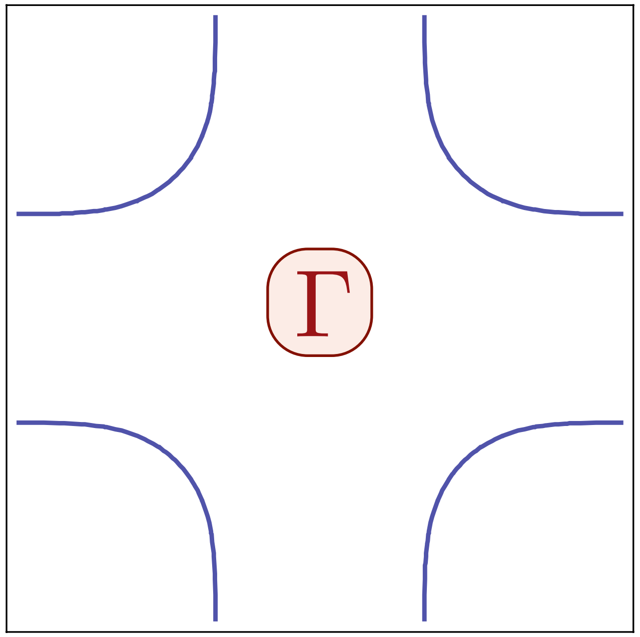
$$H_{\text{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\text{sdw}}$ for $\vec{\varphi} \propto (0, 0, 1)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

This leads to the Fermi surfaces shown in the following slides for electron and hole doping.

Spin density wave theory in electron-doped cuprates



S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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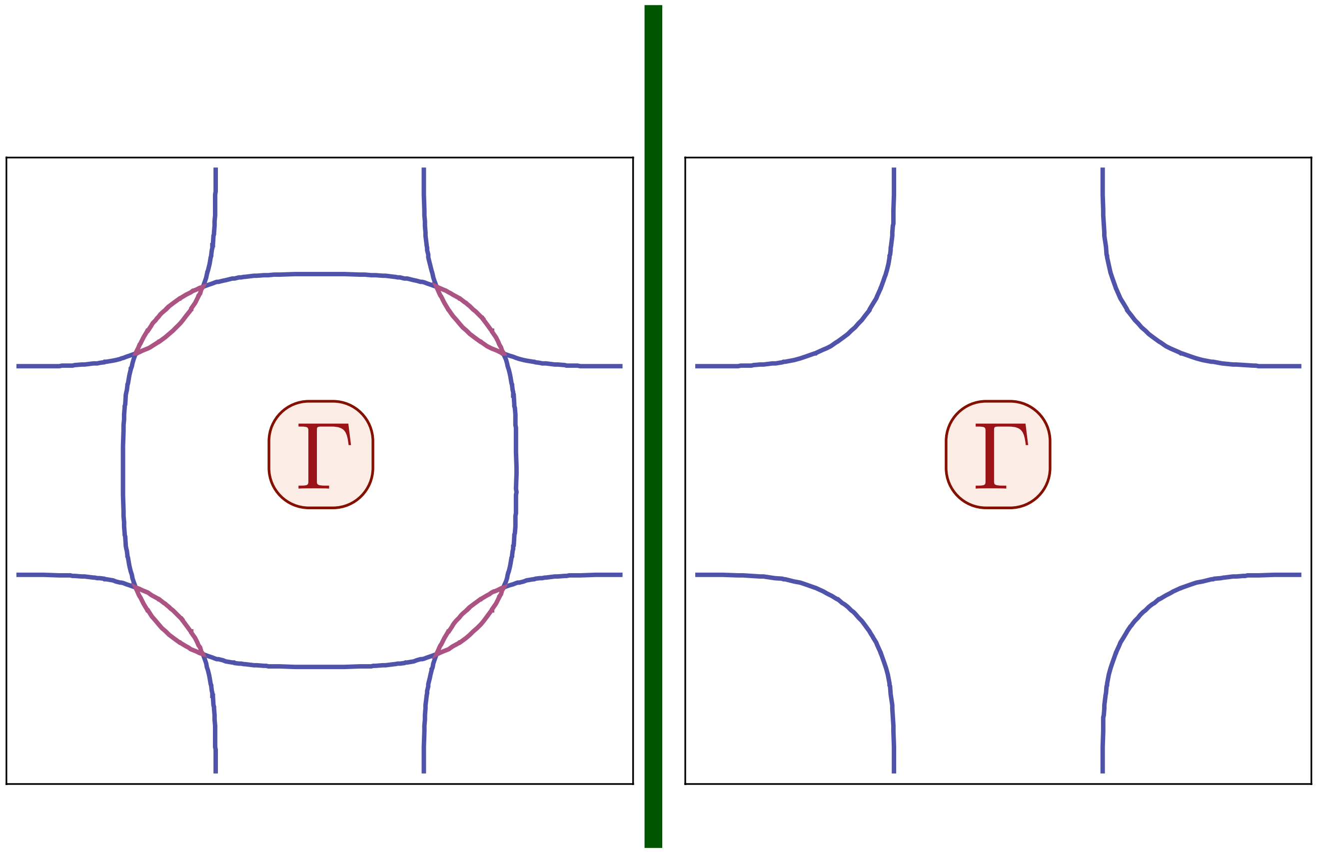

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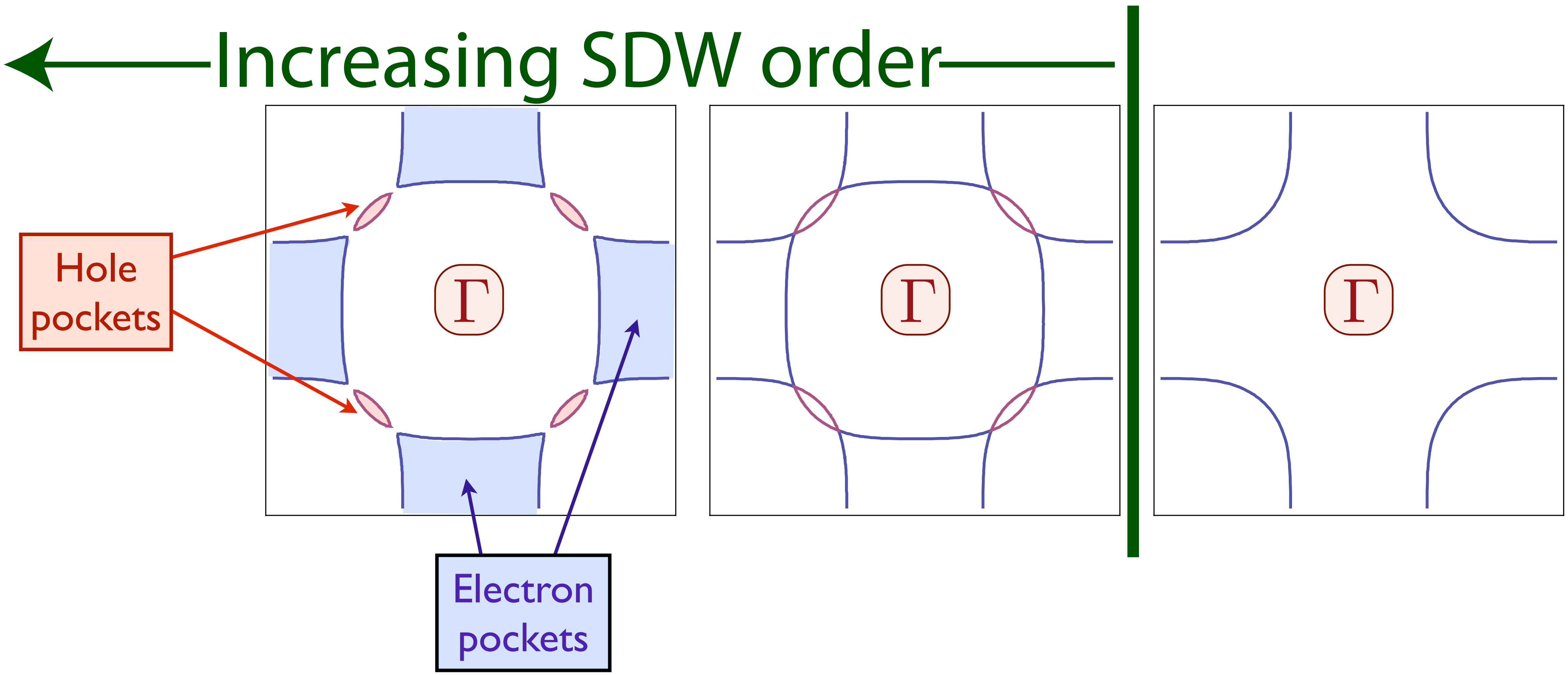

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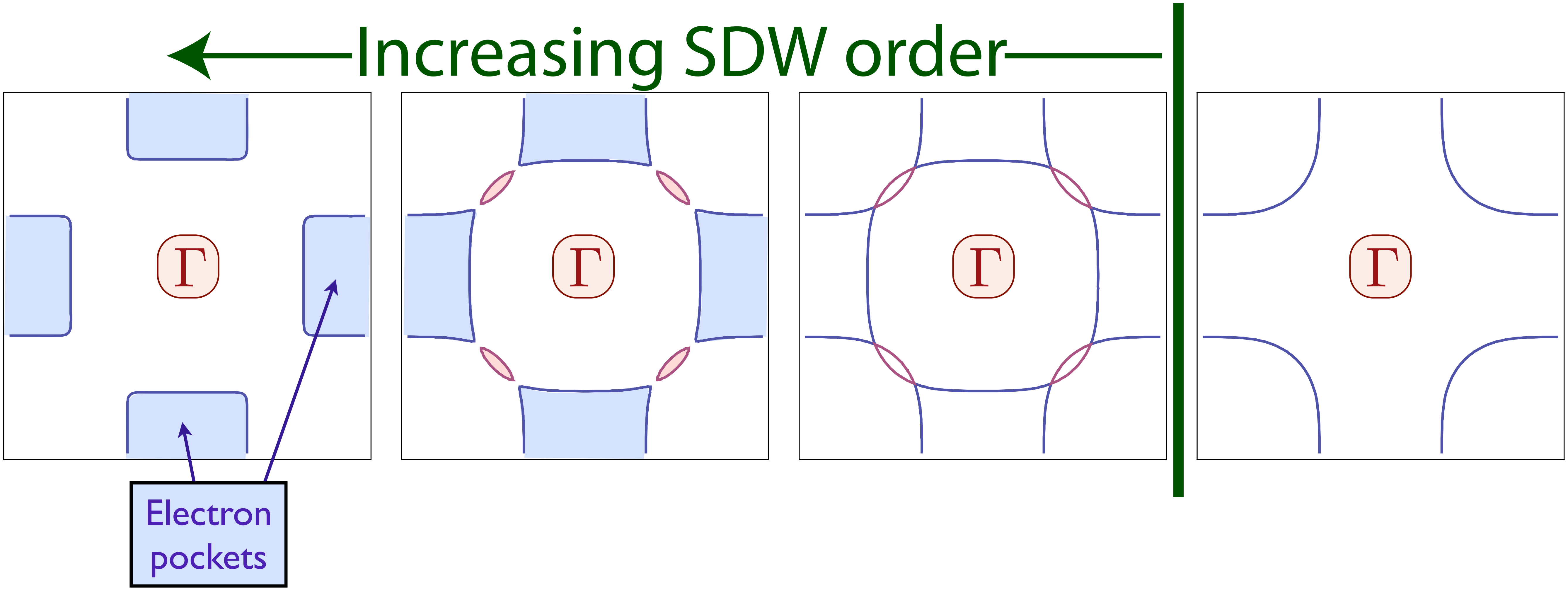

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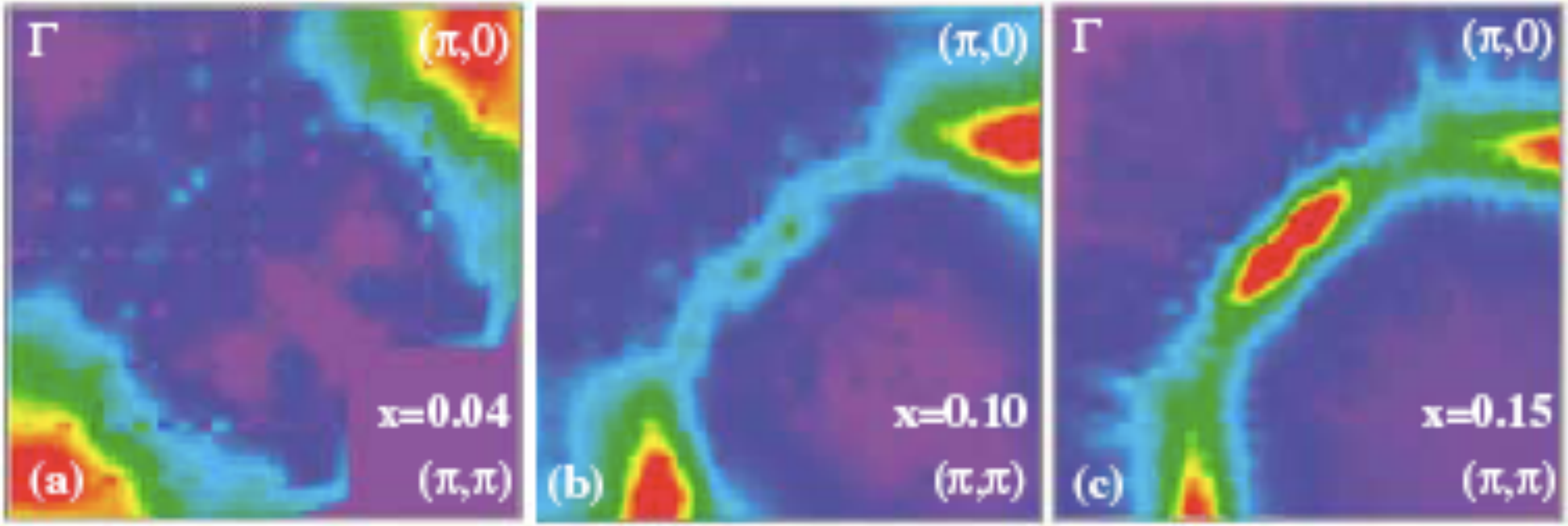
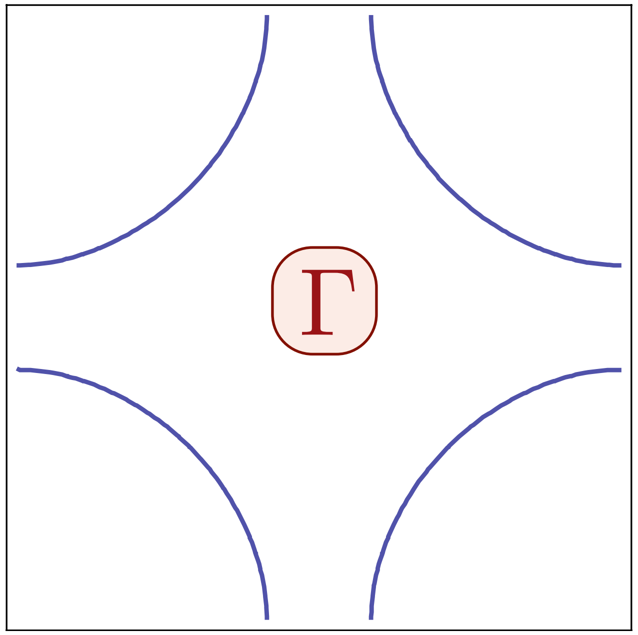
Photoemission in NCCO



N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Spin density wave theory in hole-doped cuprates



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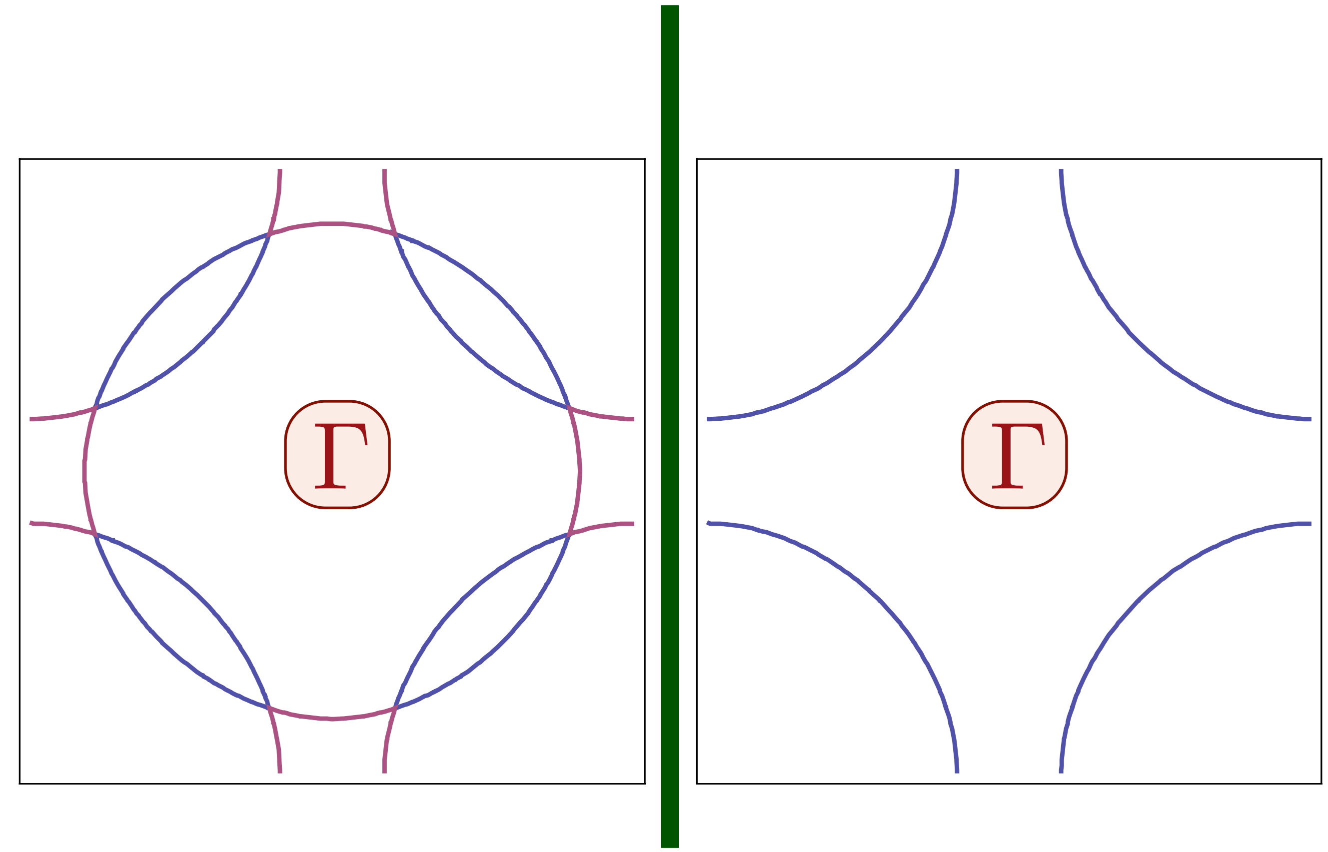

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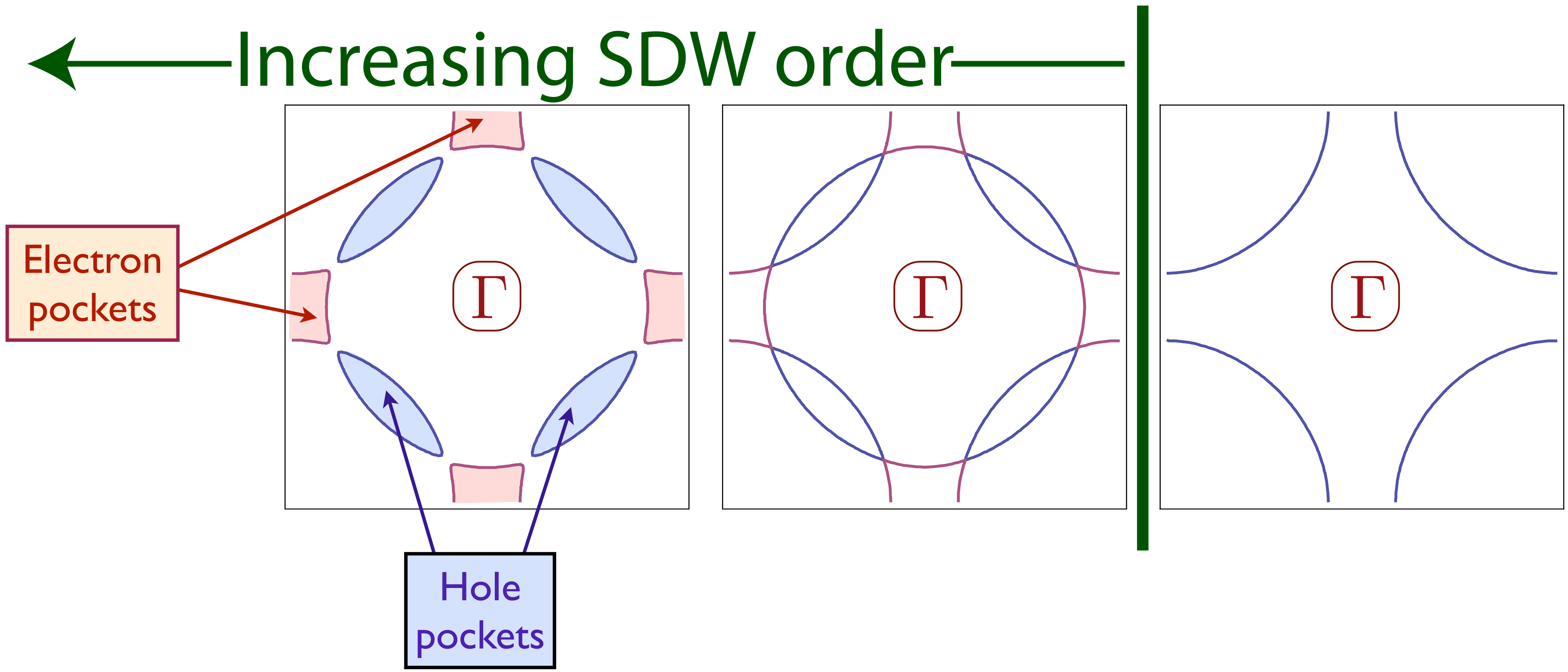

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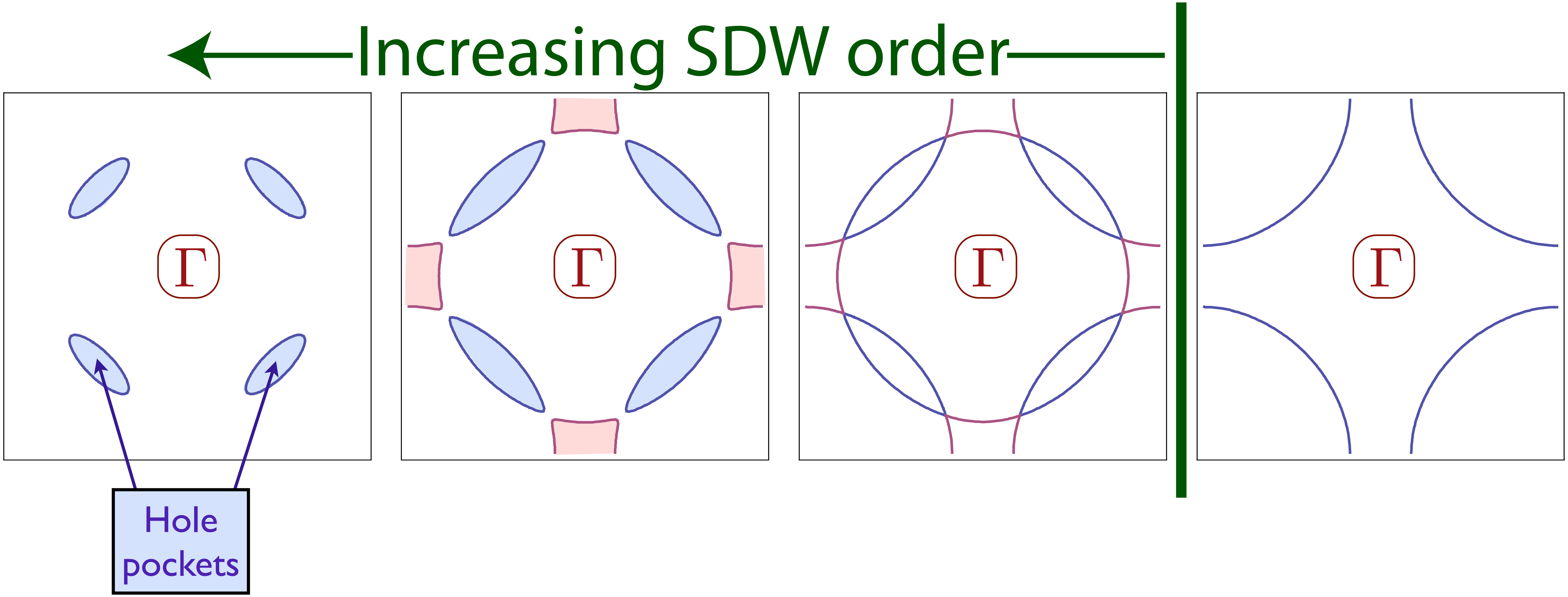

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$$H_{\rm sdw} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices. At the quantum critical point for the onset of SDW order, we integrate out the fermions and derive an effective action functional for $\vec{\varphi}$.

Spin density wave theory

This functional has the form

$$\mathcal{S} = \int \frac{d^2 q}{4\pi^2} \int \frac{d\omega}{2\pi} |\vec{\varphi}(\mathbf{q},\omega)|^2 \Big[r + q^2 + \chi(\mathbf{K},\omega) \Big] + u \int d^2 x d\tau (\vec{\varphi}^2(x,\tau))^2 + \ldots$$

The susceptibility, χ, has a non-analytic dependence on ω because of Landau damping:

$$\chi(\mathbf{K},\omega) = \chi_0 + \chi_1 |\omega| + \ldots$$

This leads to a critical point with dynamic critical exponent $z = 2$, and upper-critical dimension $d = 2$.

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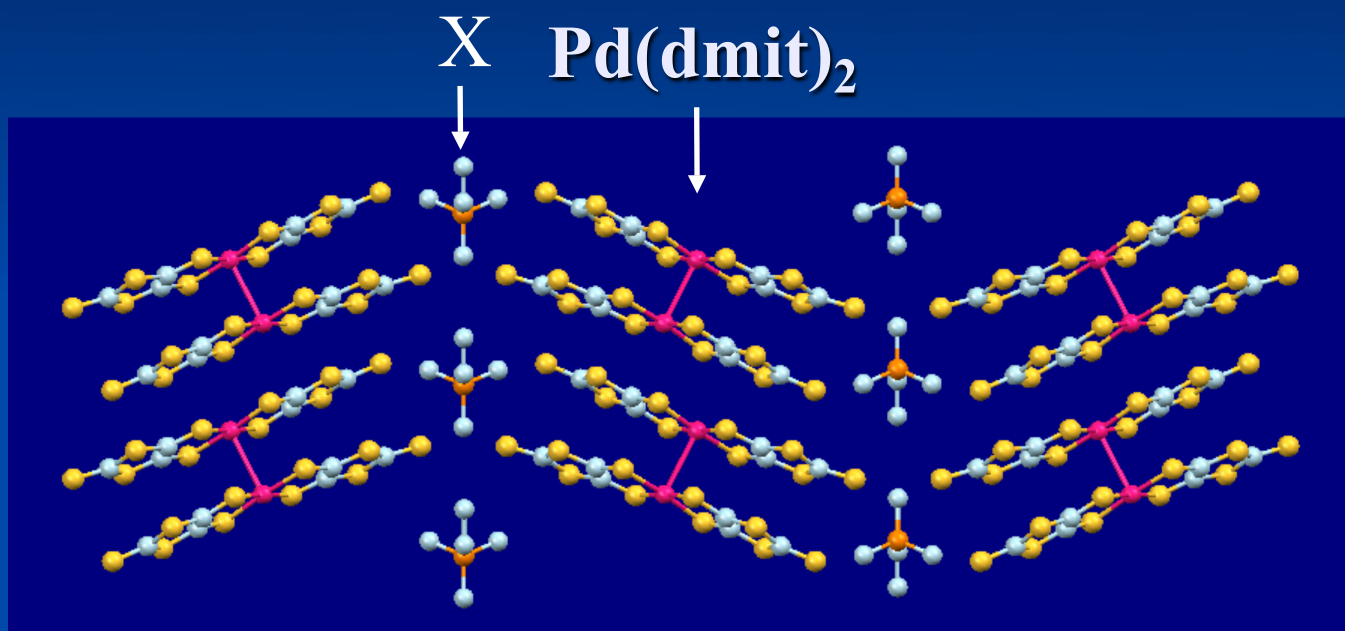
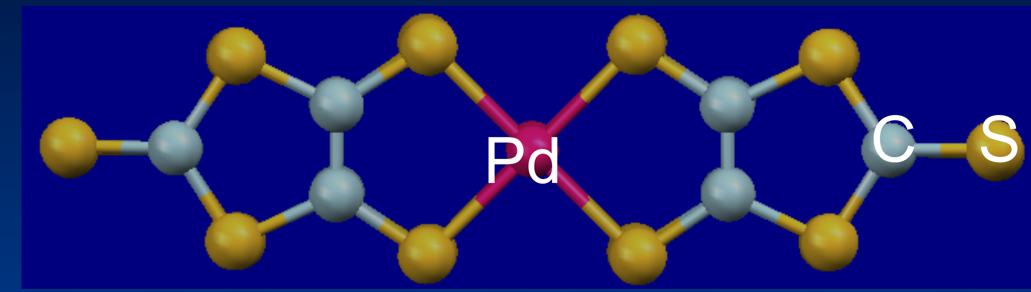
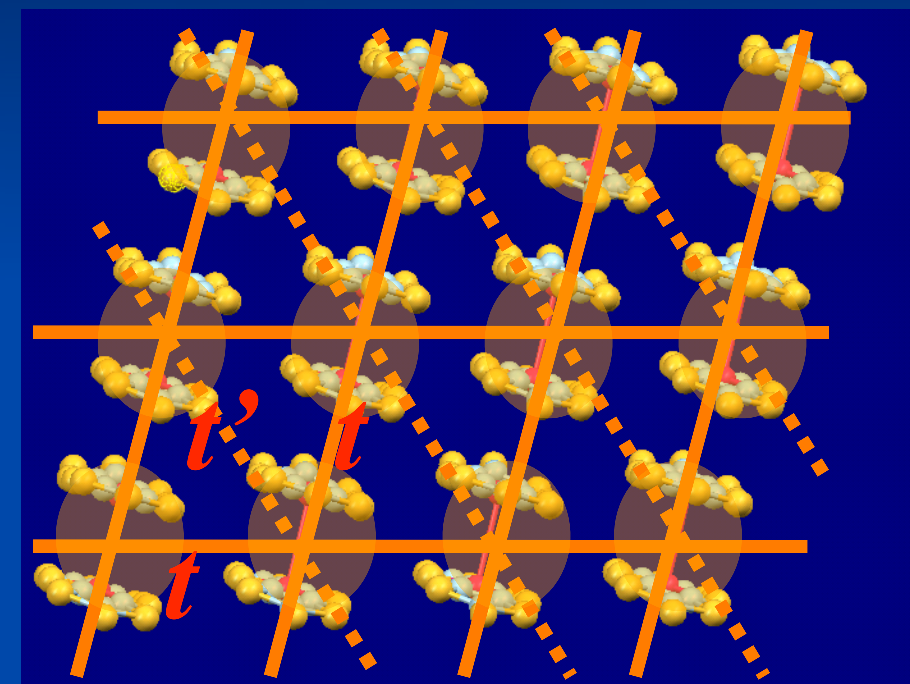
$X[Pd(dmit)_2]_2$







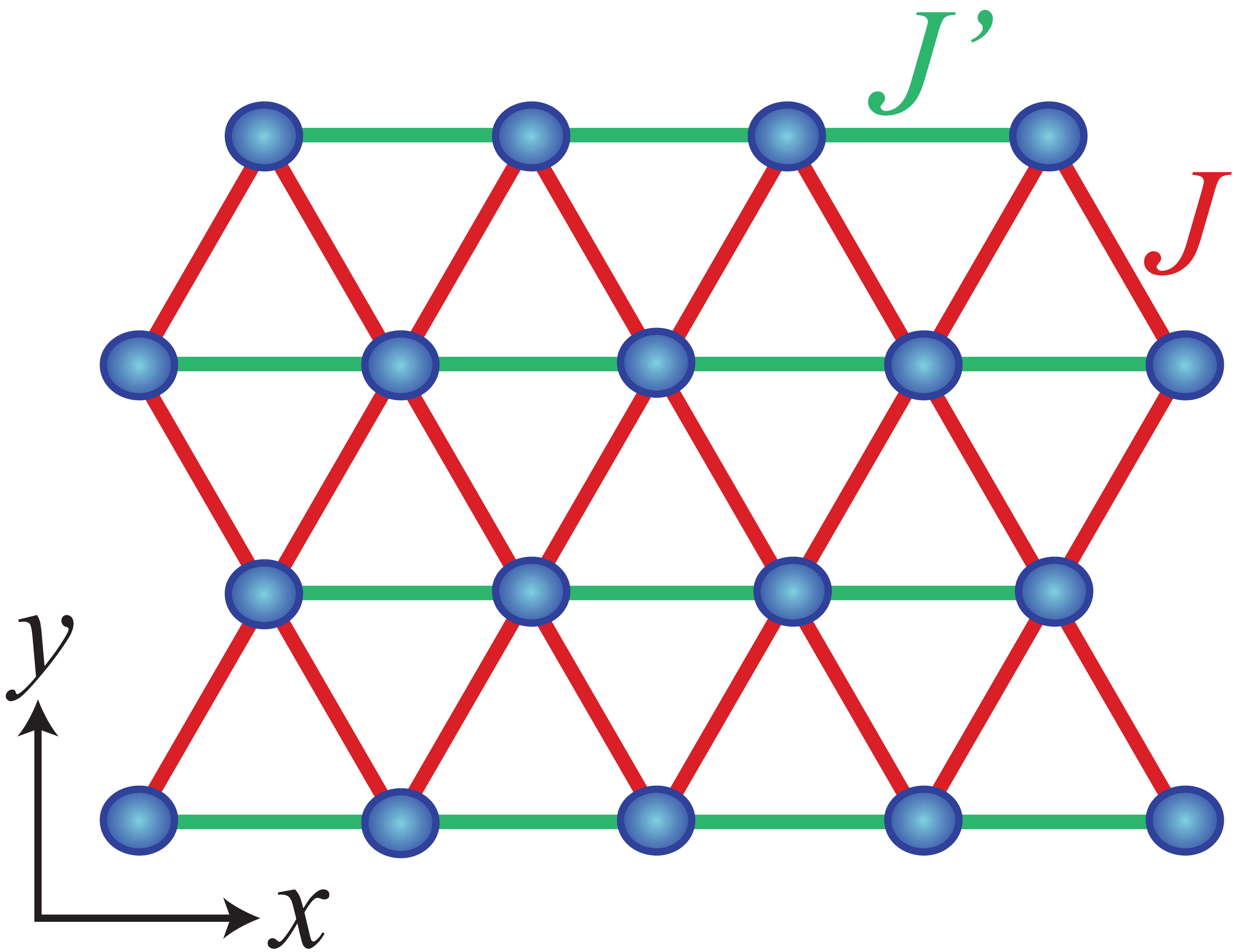
Half-filled band → Mott insulator with spin $S = 1/2$

Triangular lattice of $[Pd(dmit)_2]_2$
→ frustrated quantum spin system

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

$\vec{S}_i \;\Rightarrow\;$ spin operator with $S = 1/2$



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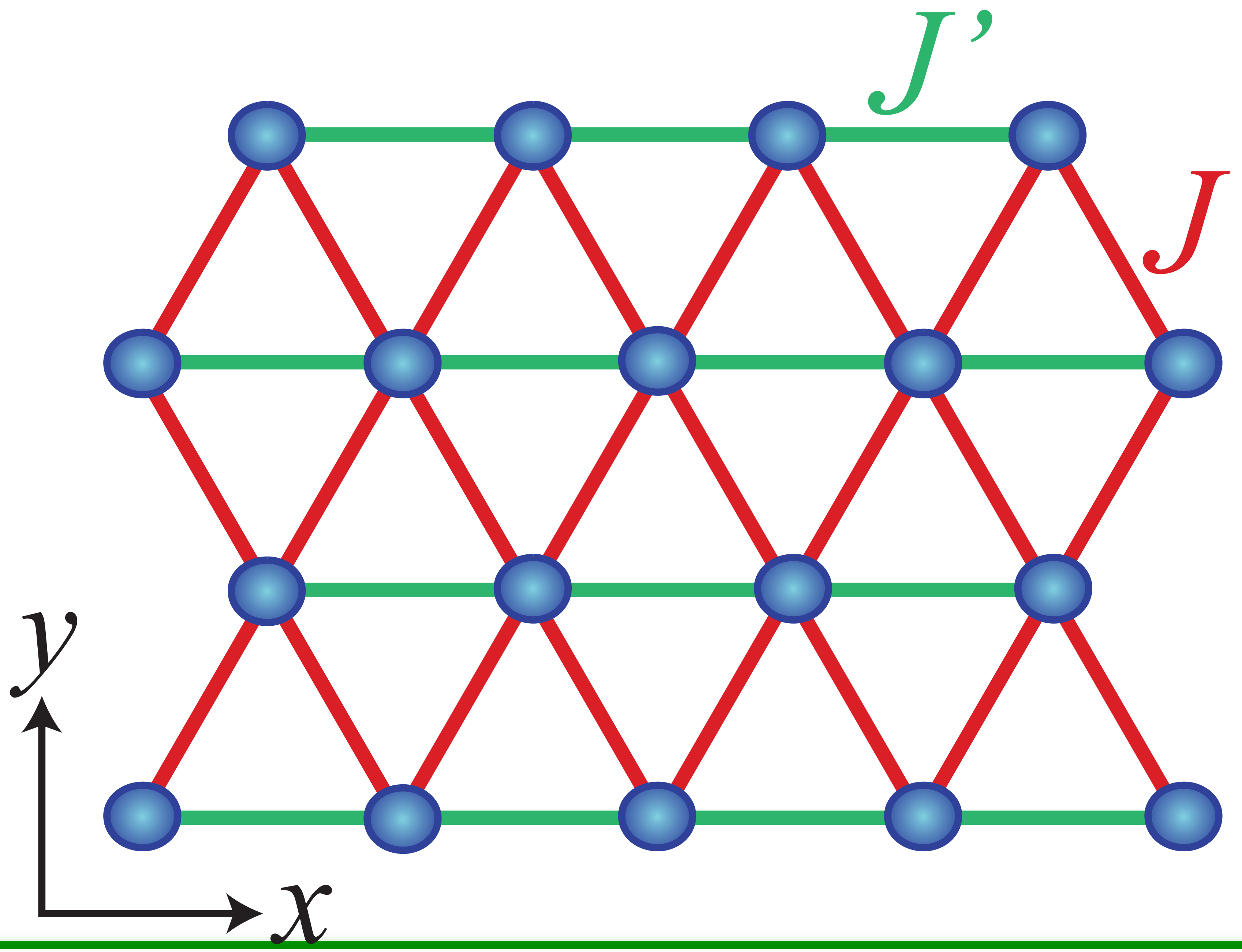
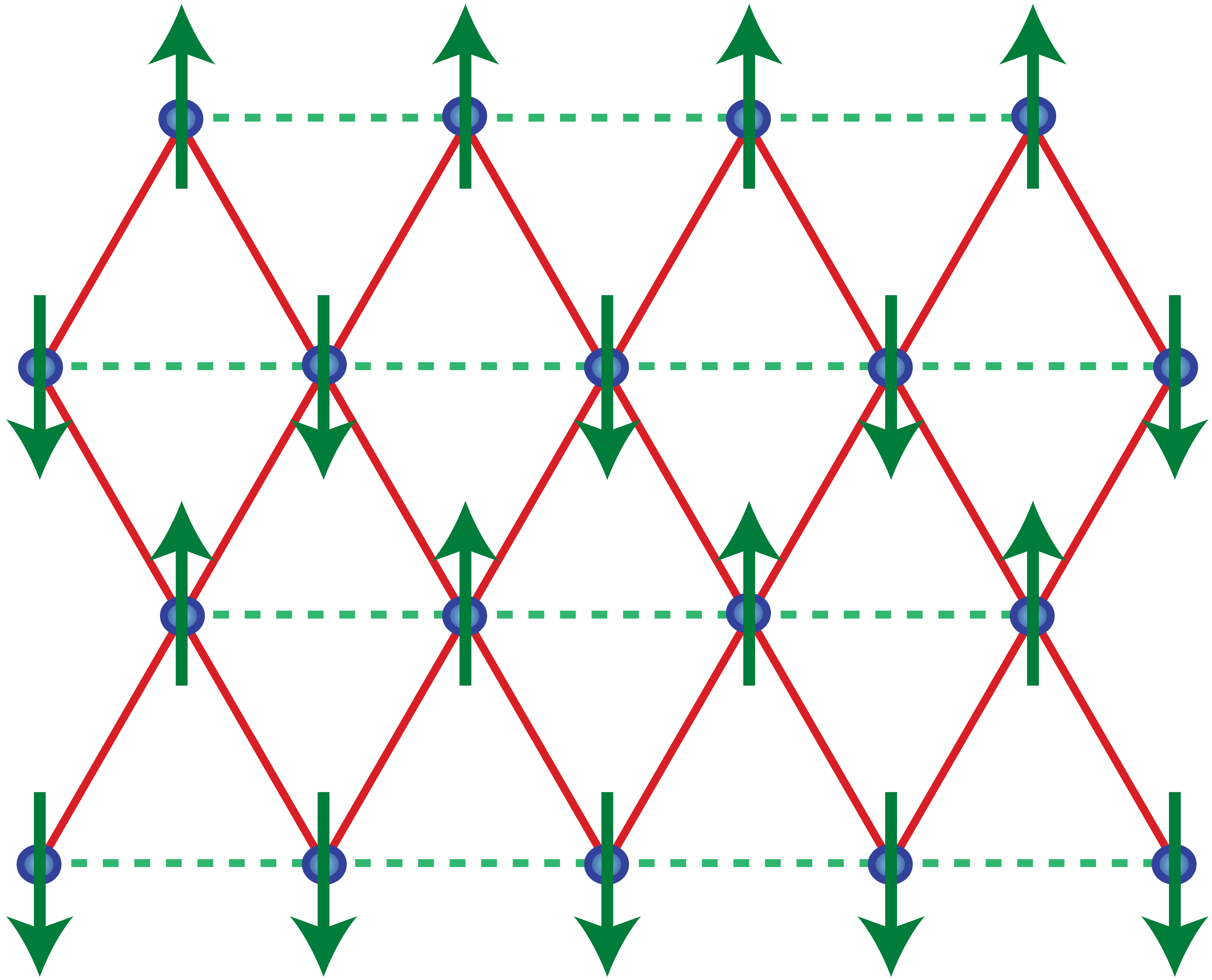


What is the ground state as a function of J'/J ?

Anisotropic triangular lattice antiferromagnet

Broken spin rotation symmetry



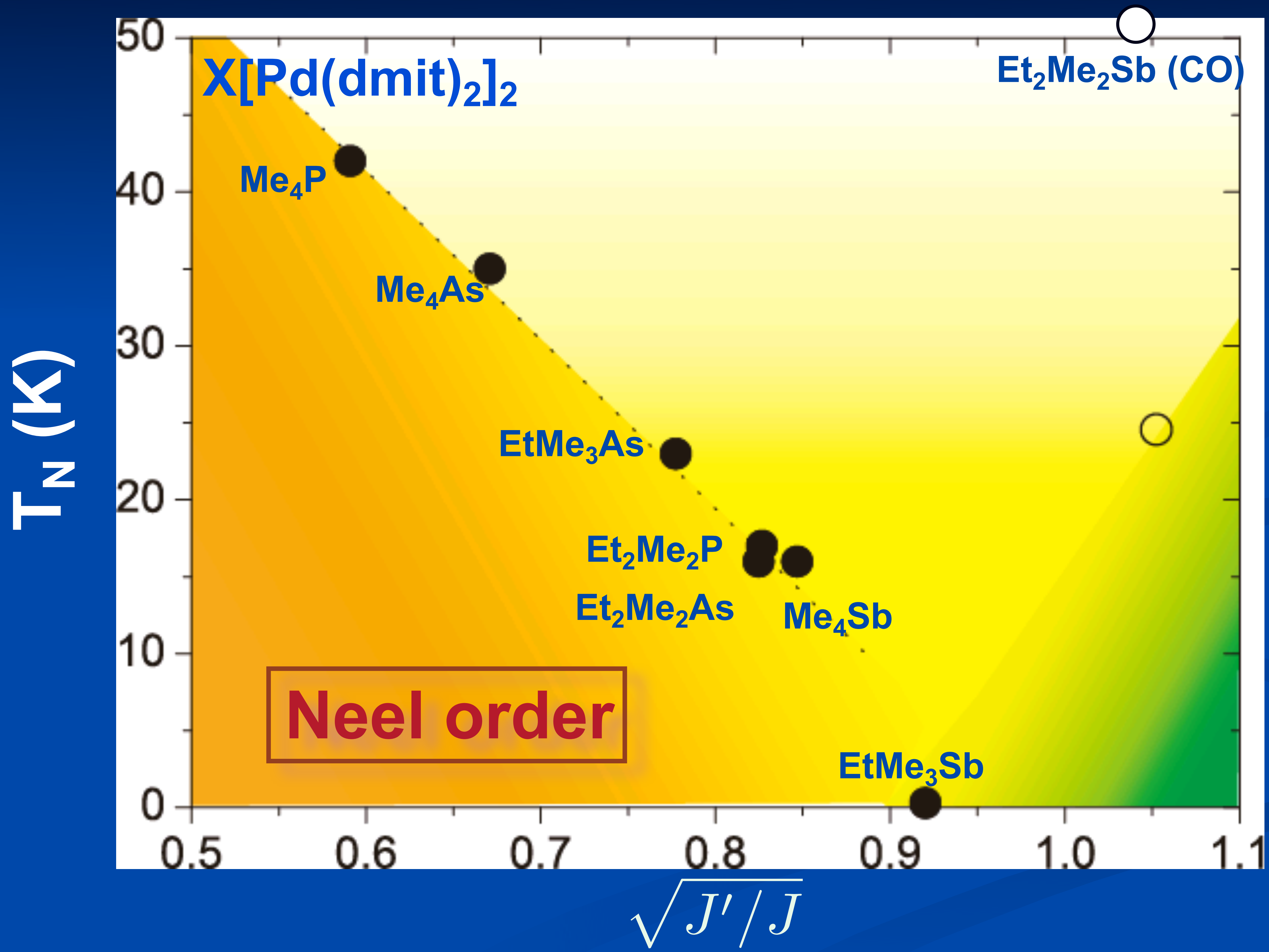
Neel ground state for small J'/J

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO

Magnetic Criticality



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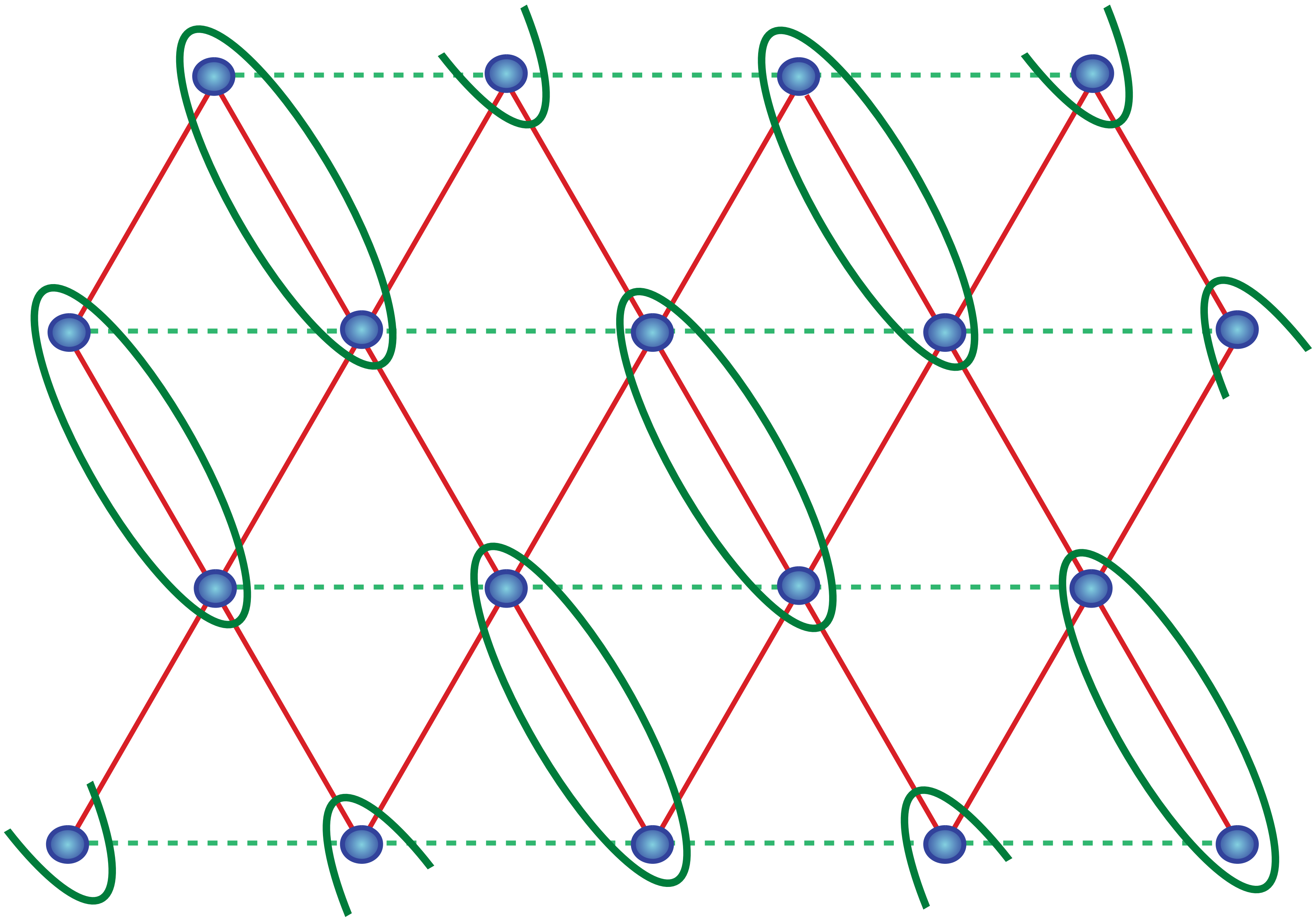

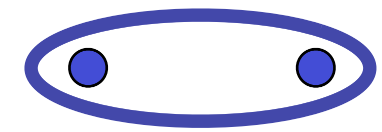

$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

Possible ground state for intermediate *J'/J*

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)

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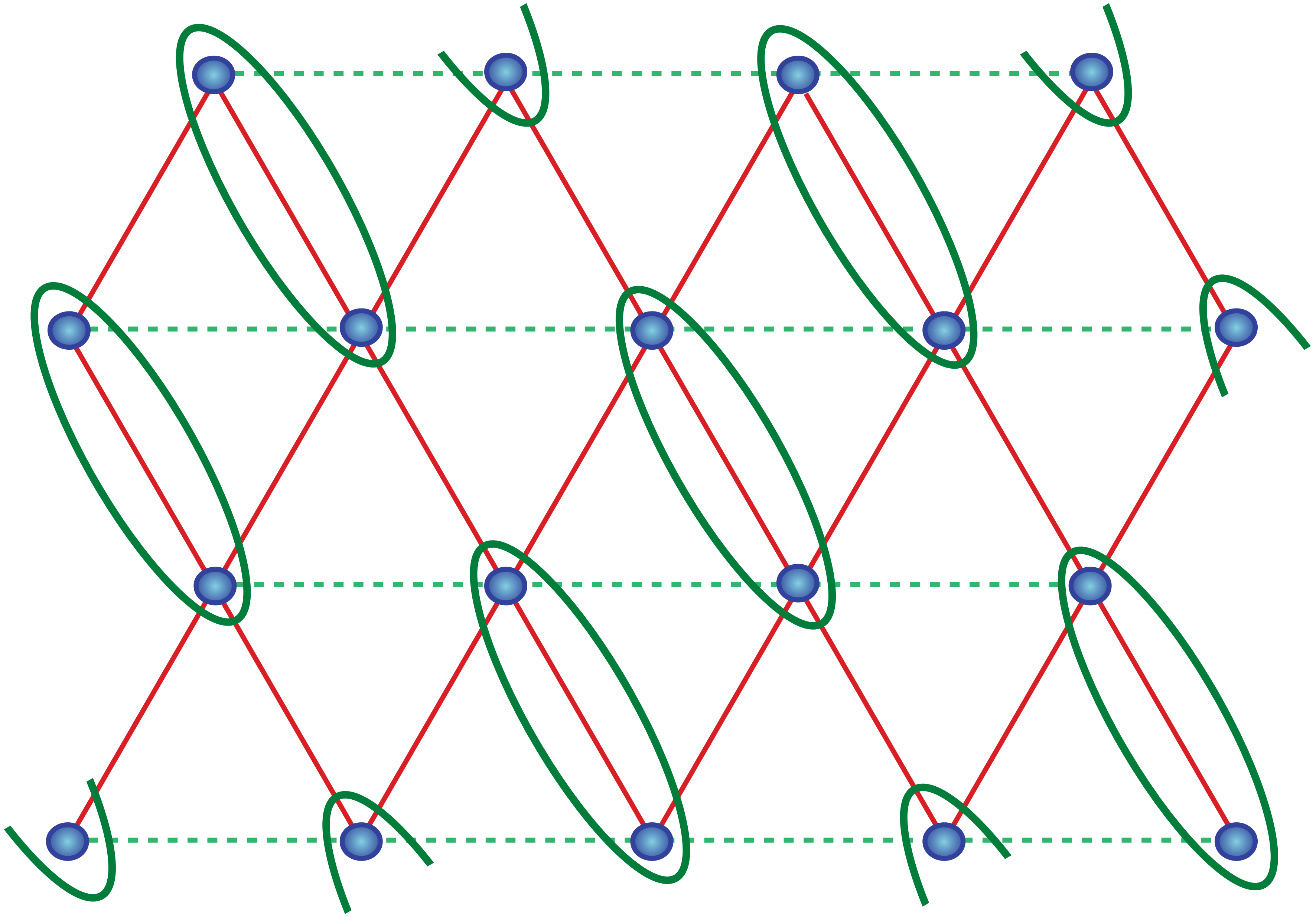
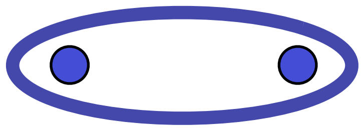
Broken lattice space group symmetry





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Valence bond solid (VBS)

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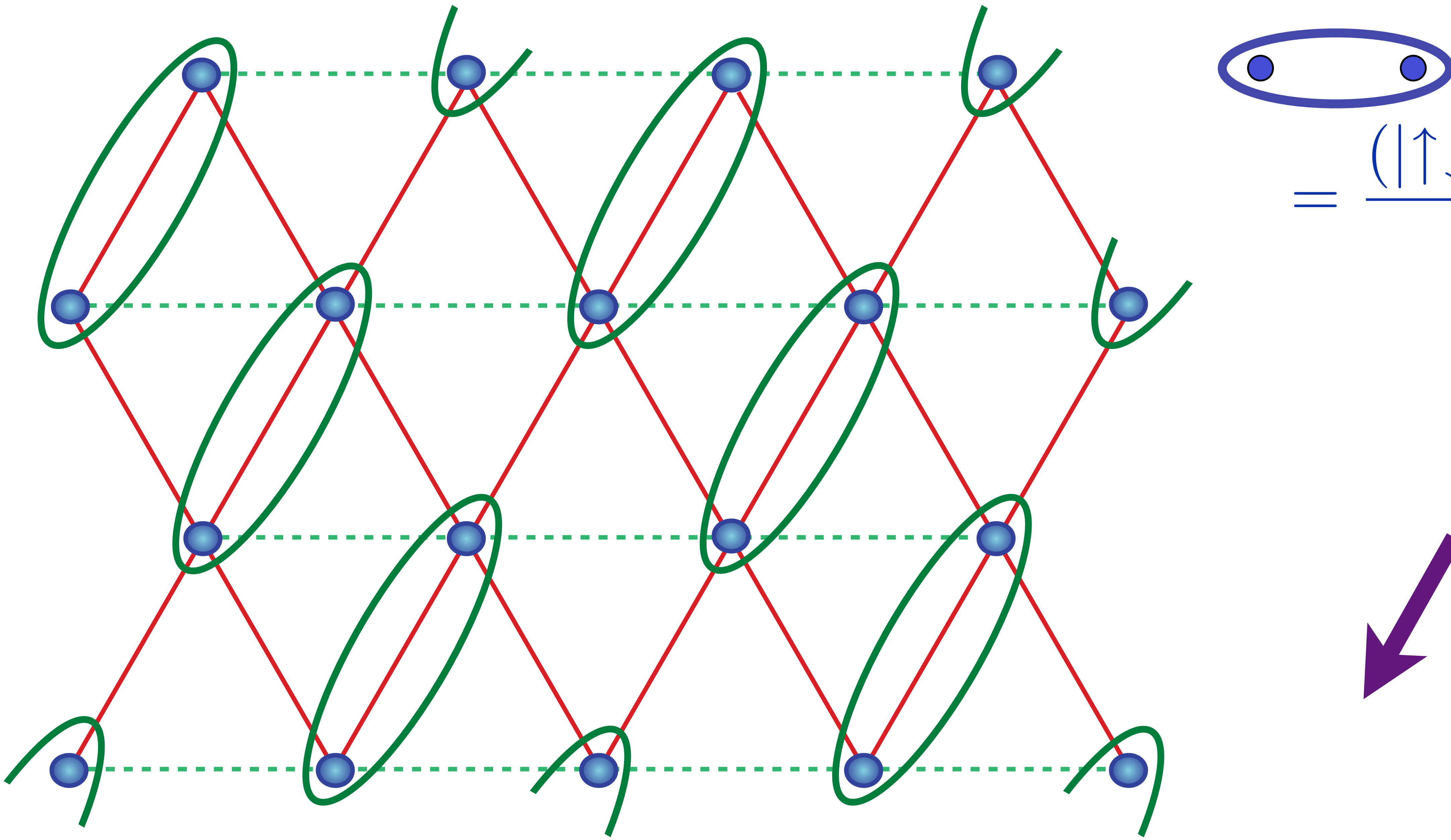

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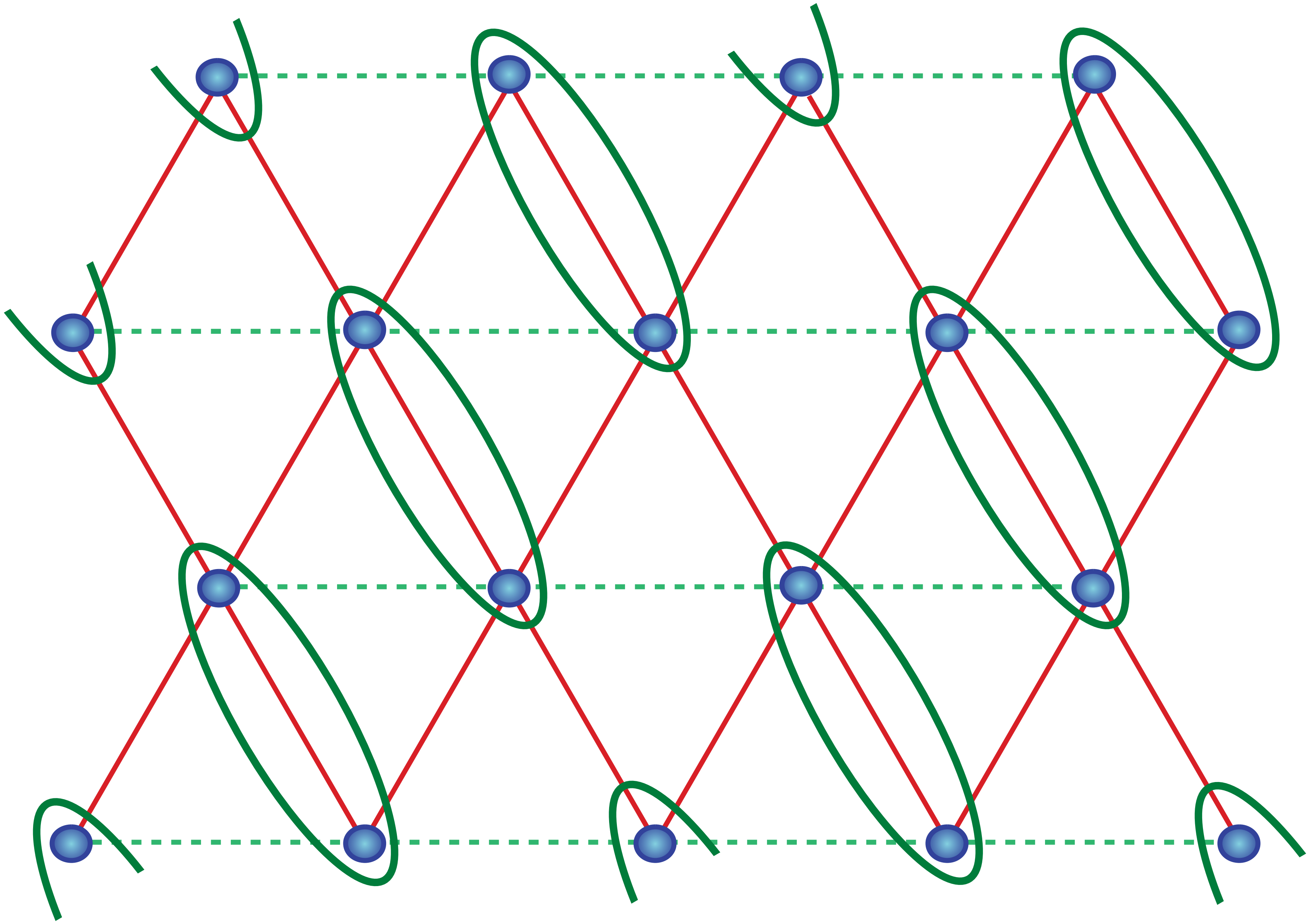

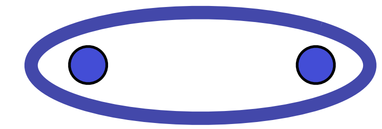

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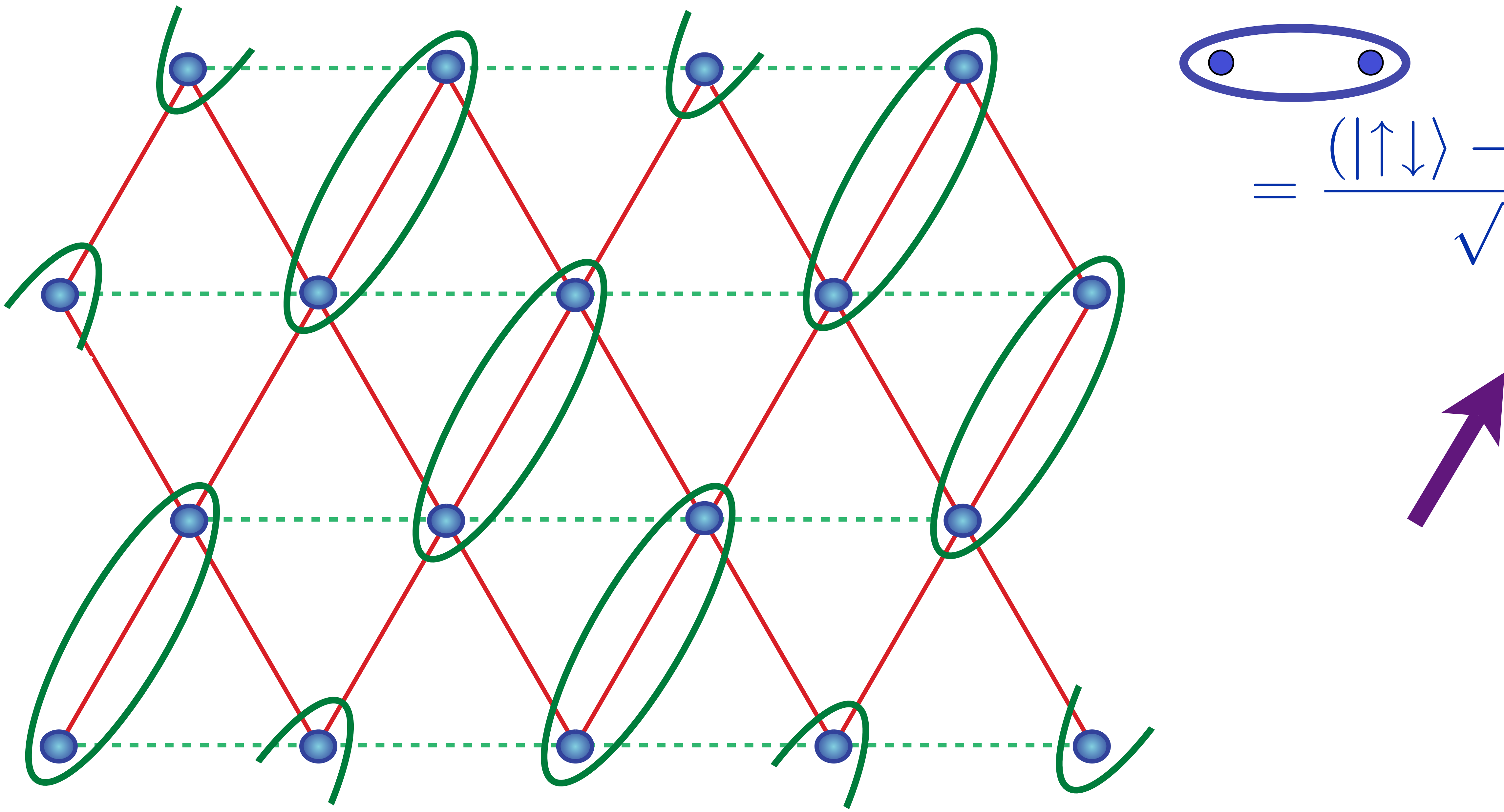

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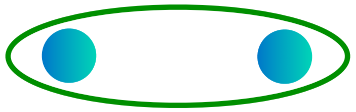
Triangular lattice antiferromagnet

Z_2 spin liquid



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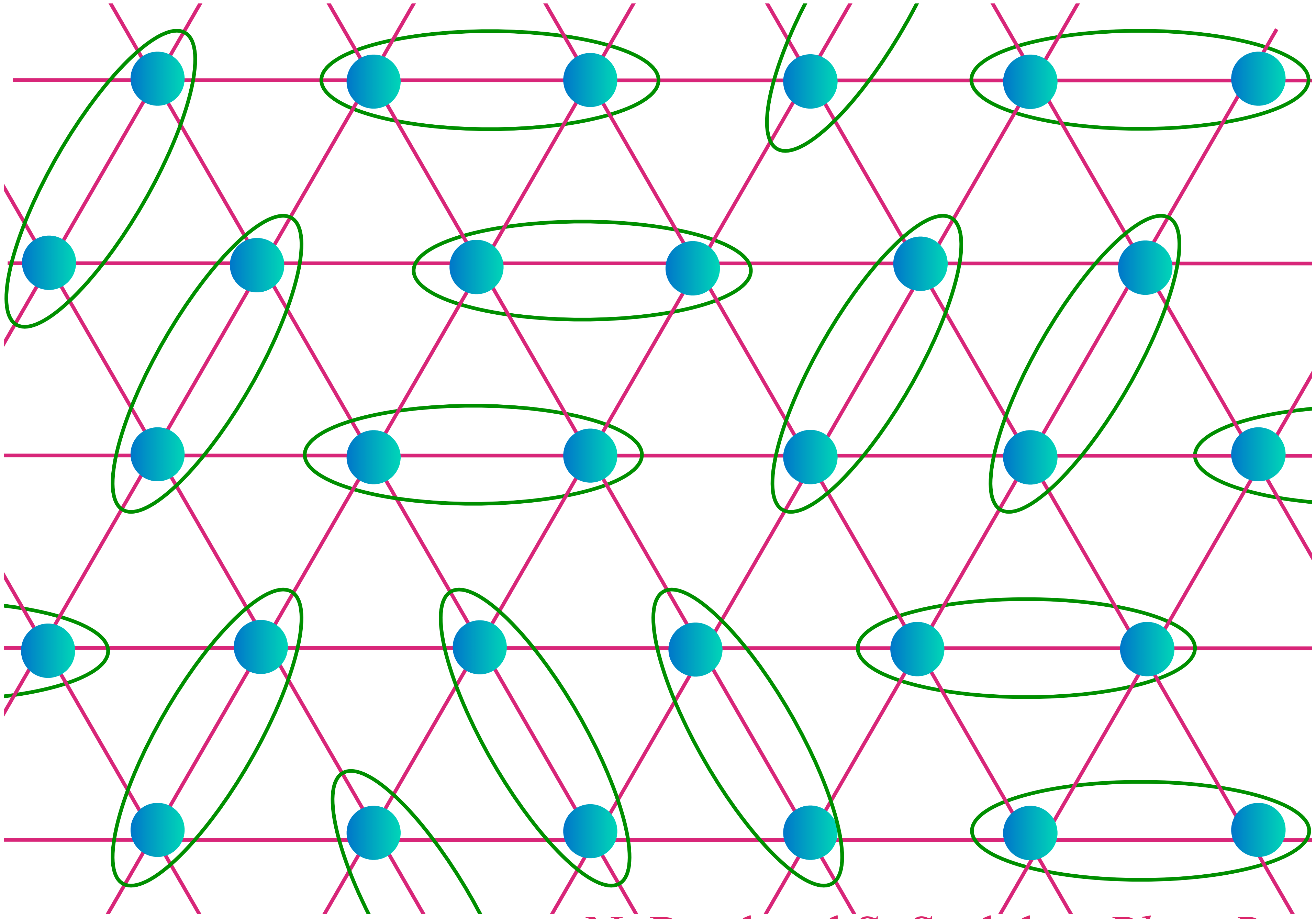

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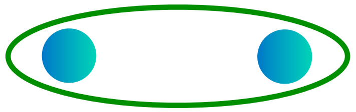

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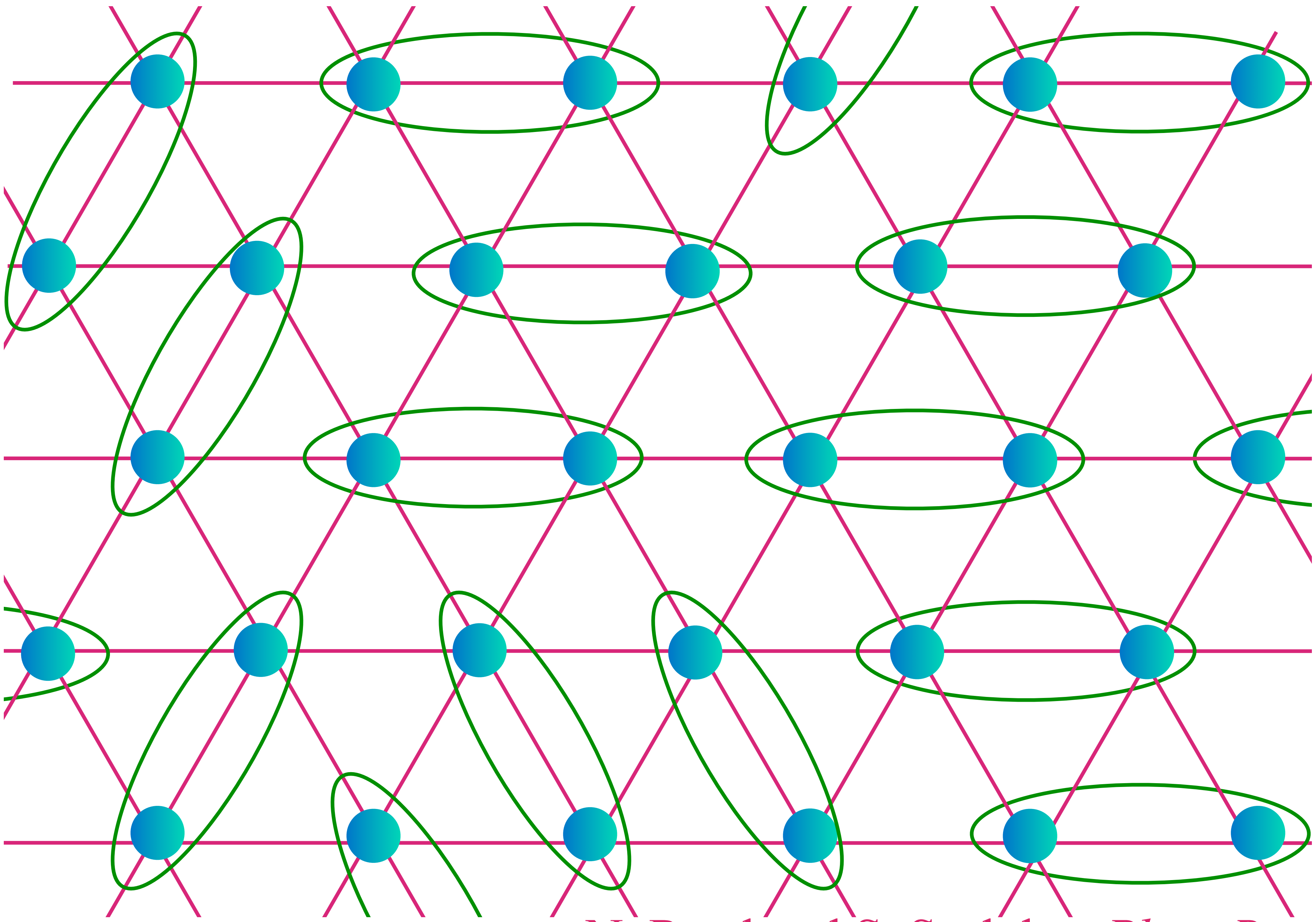

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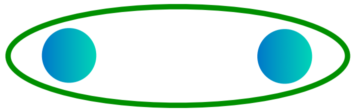

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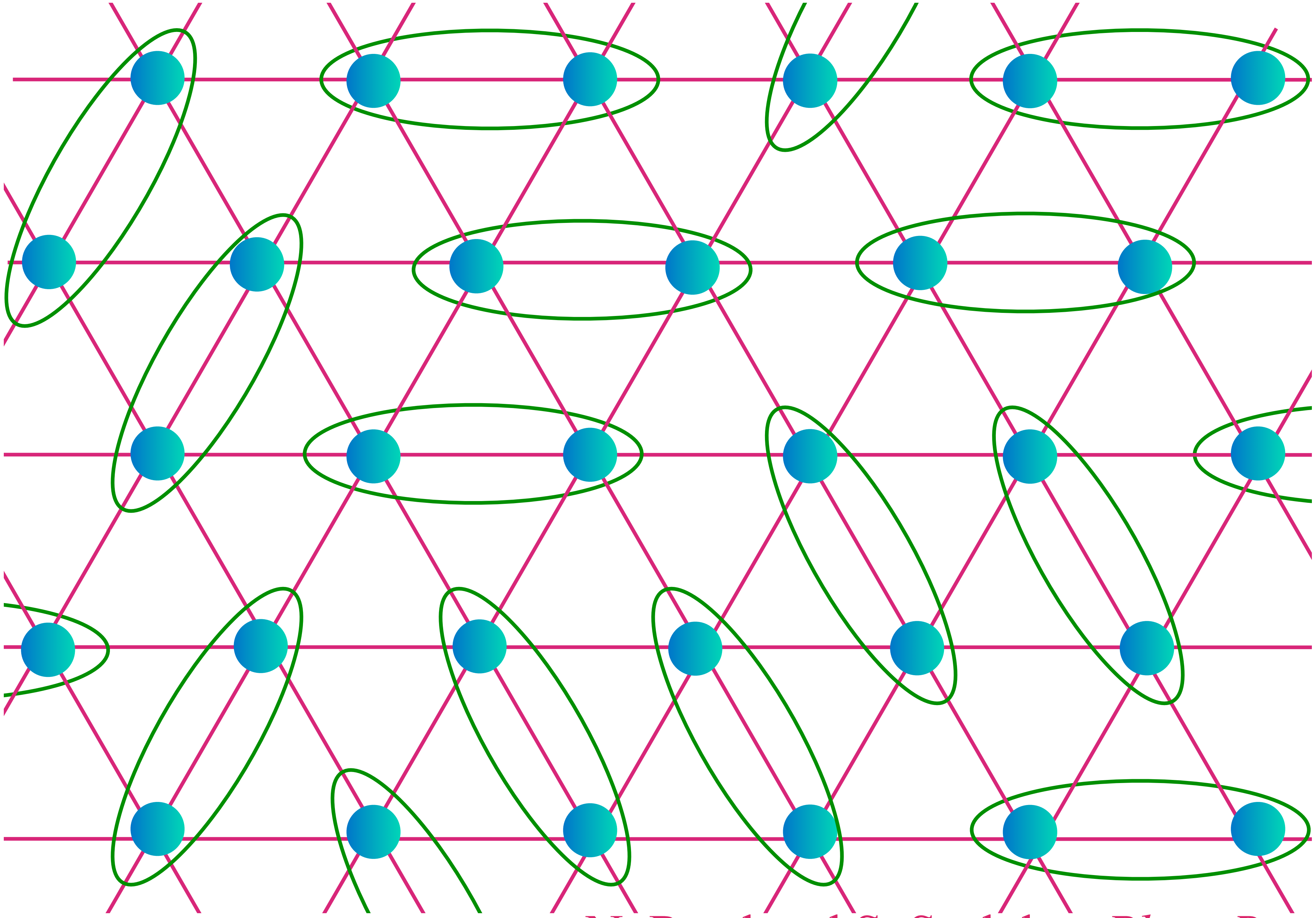

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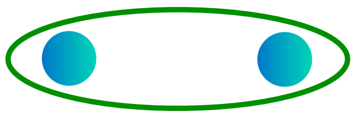

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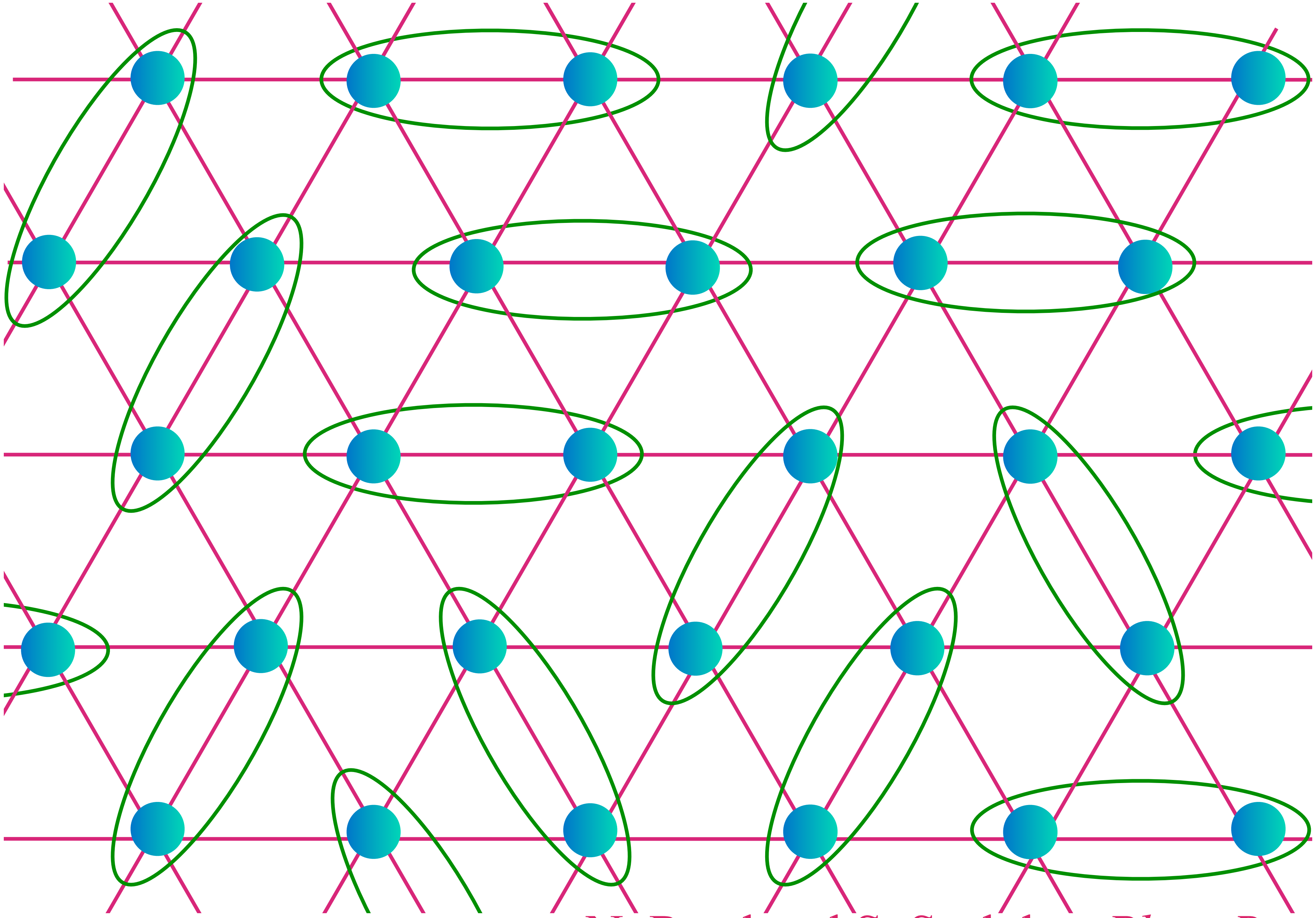

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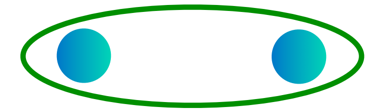

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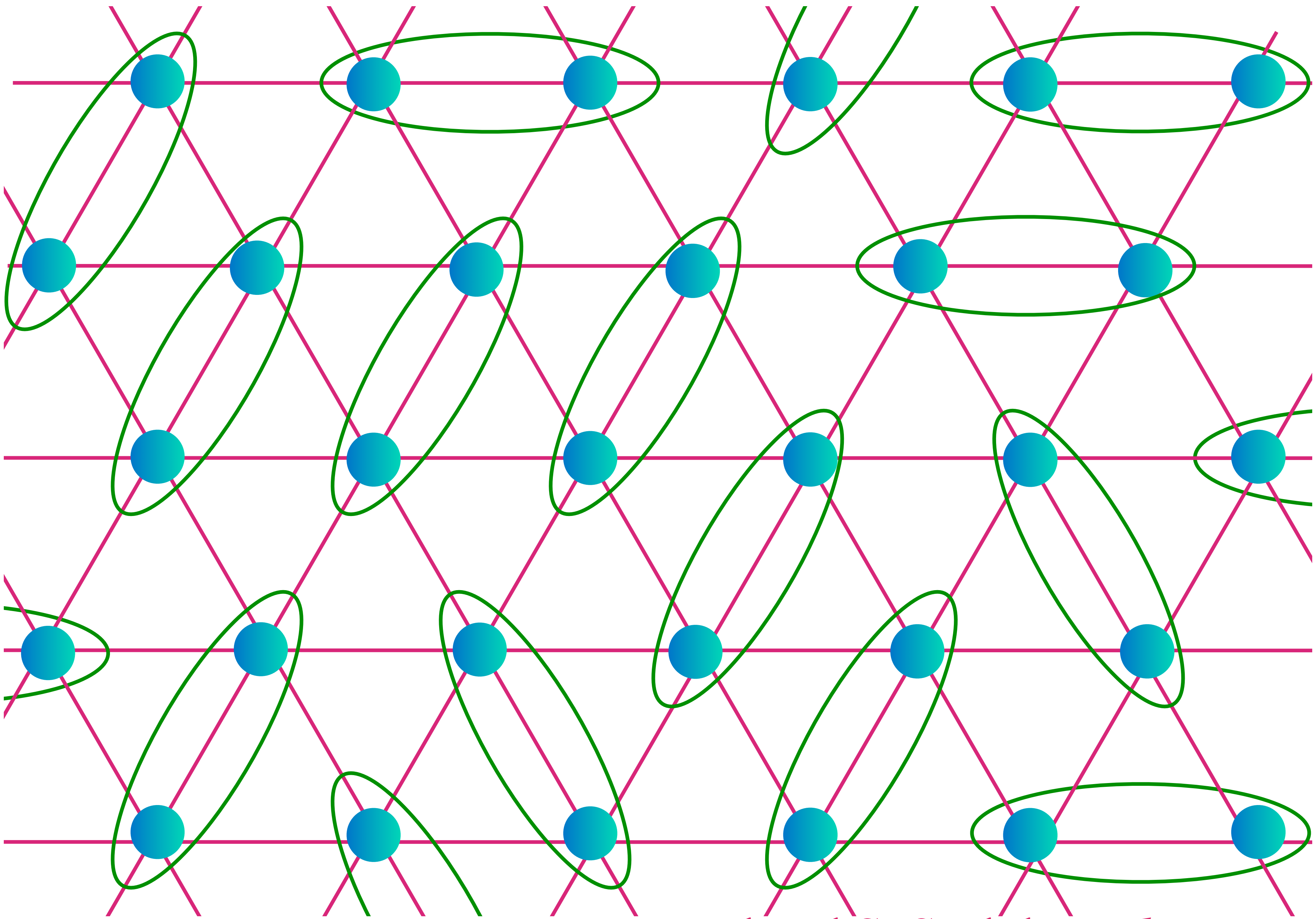

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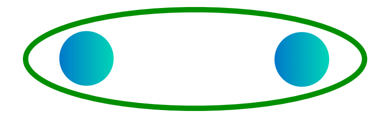

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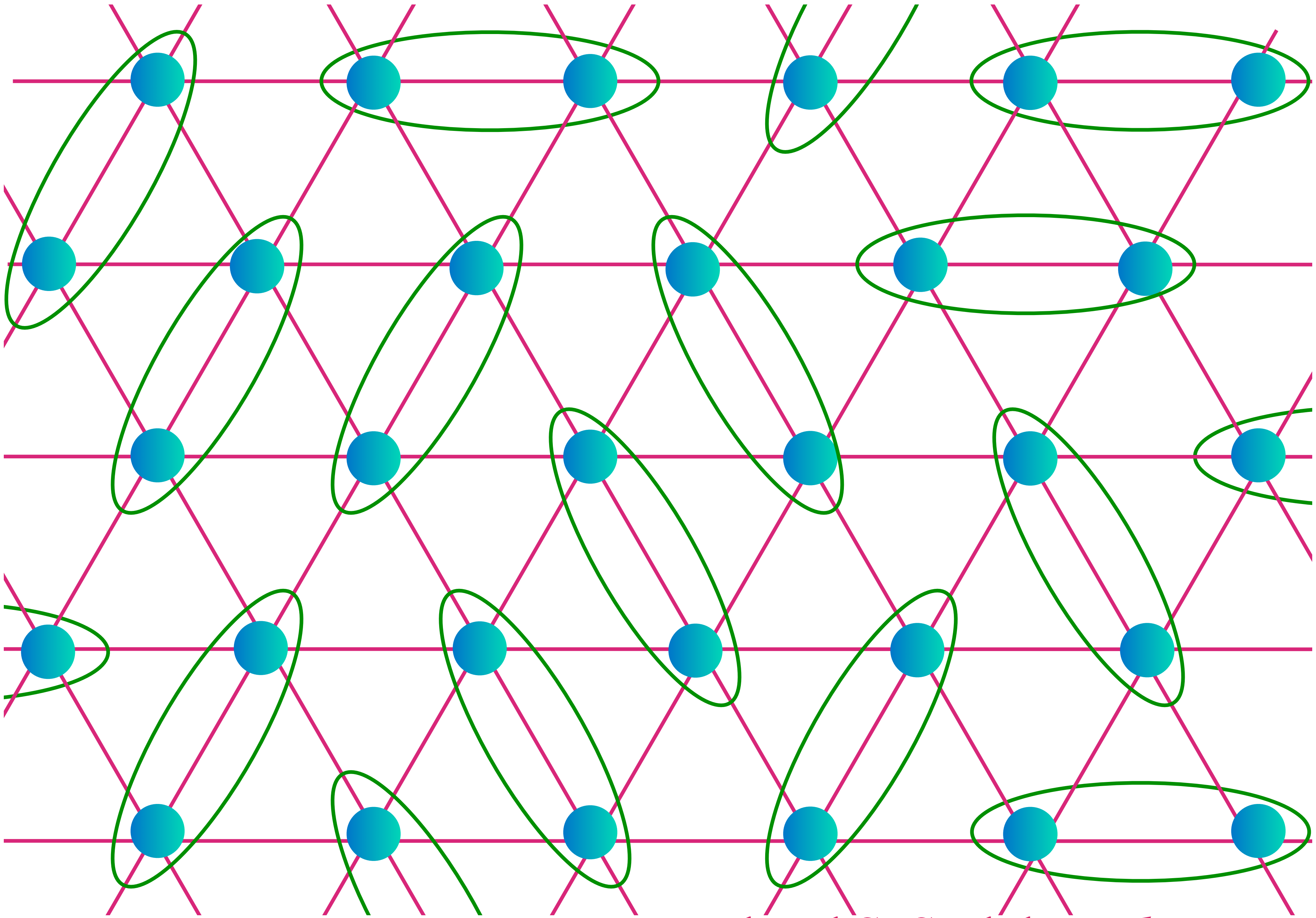

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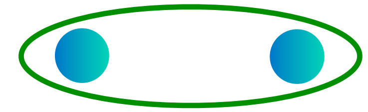
Excitations of the Z_2 Spin liquid

A spinon



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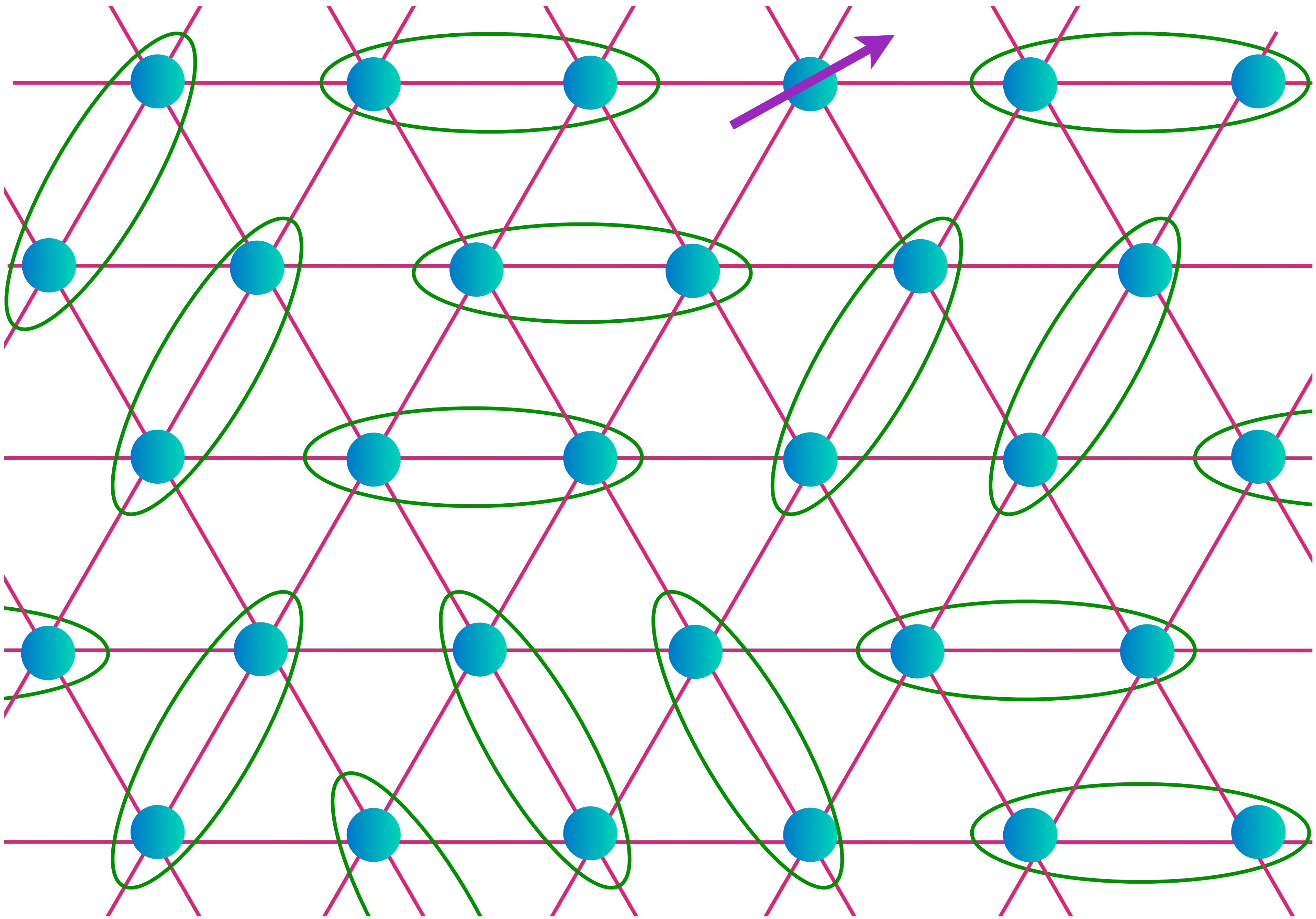

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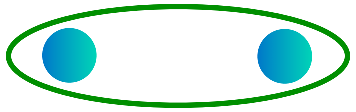

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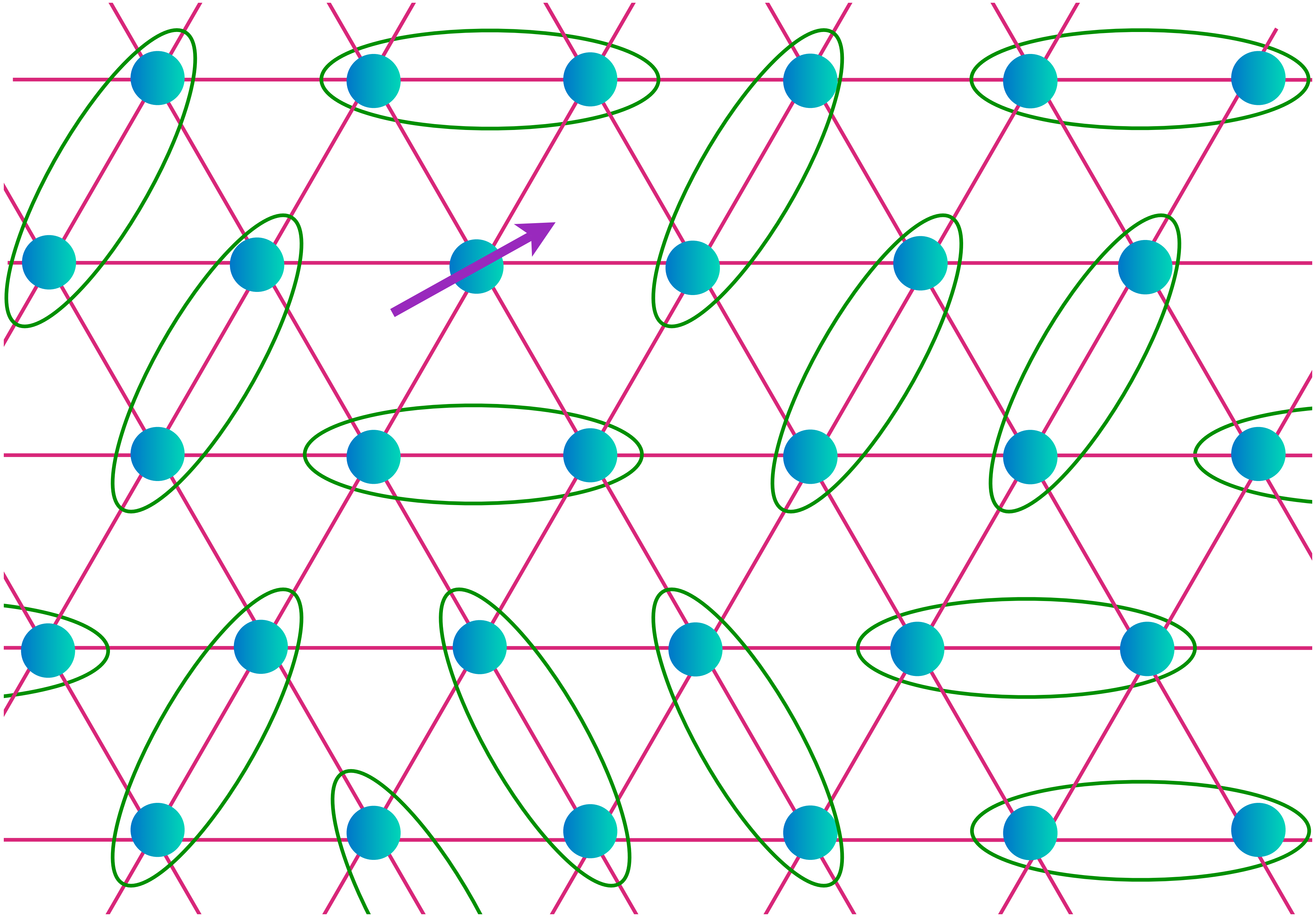

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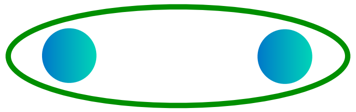

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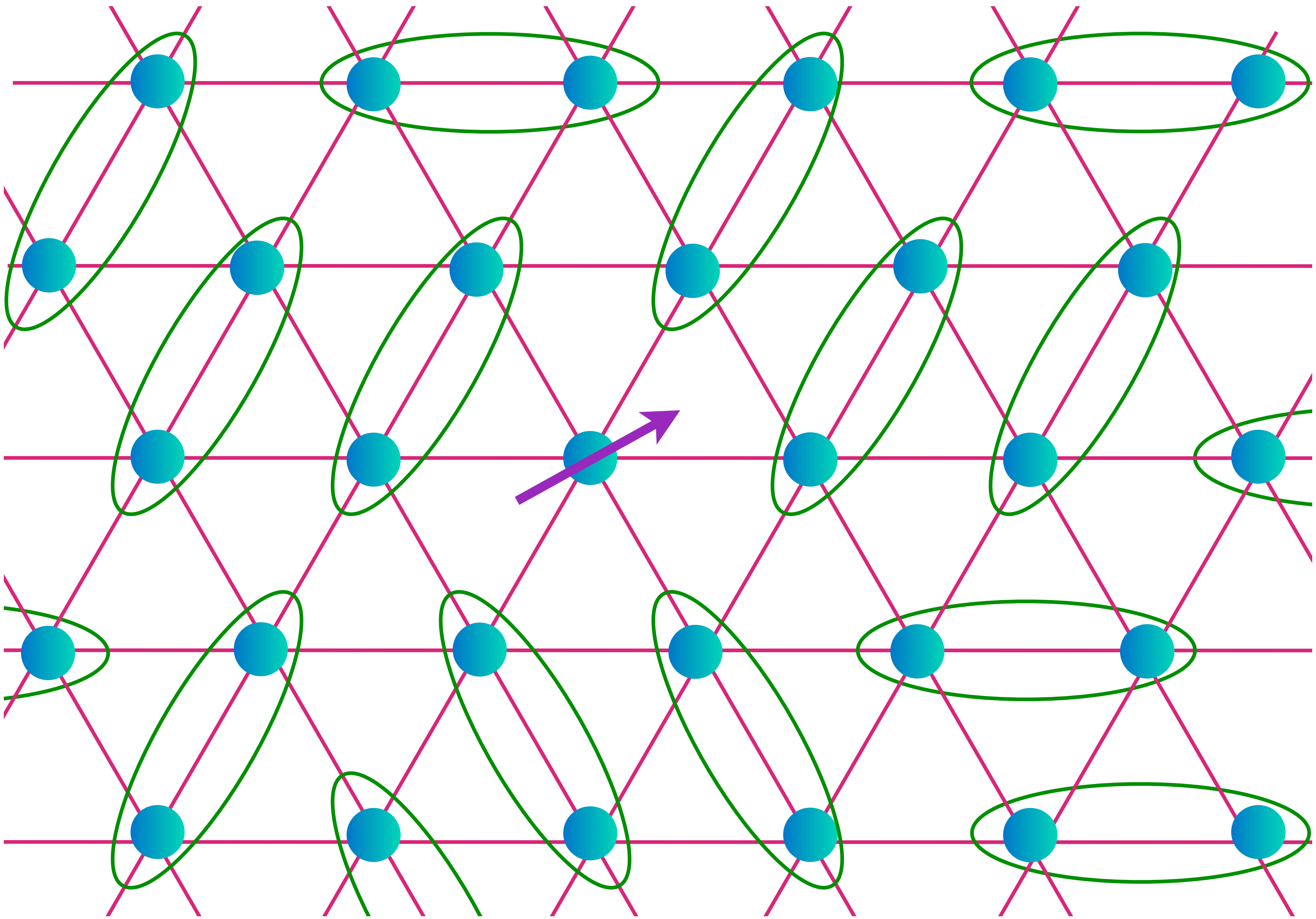
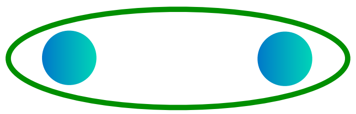
A spinon

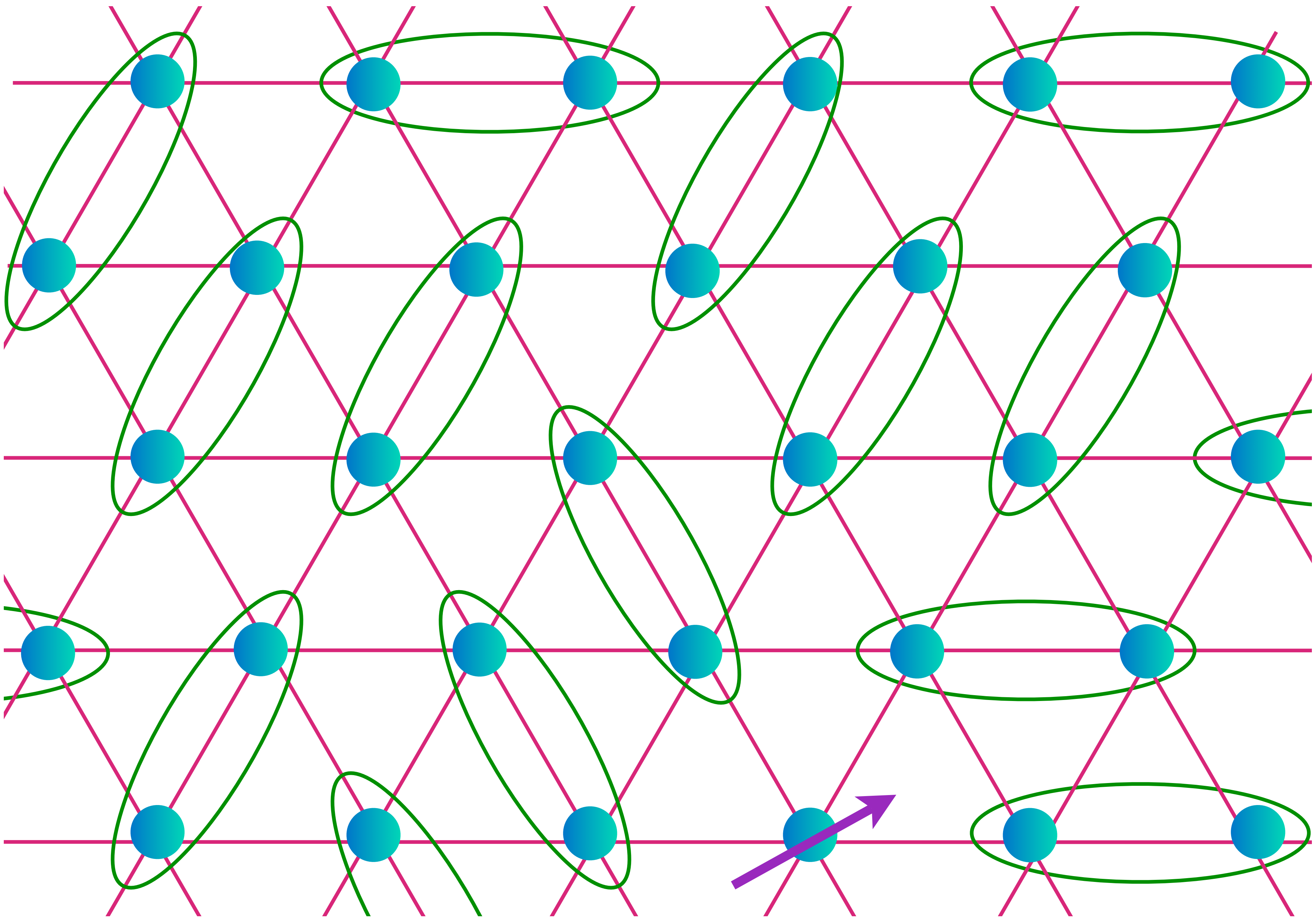


Excitations of the Z_2 Spin liquid



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

A spinon

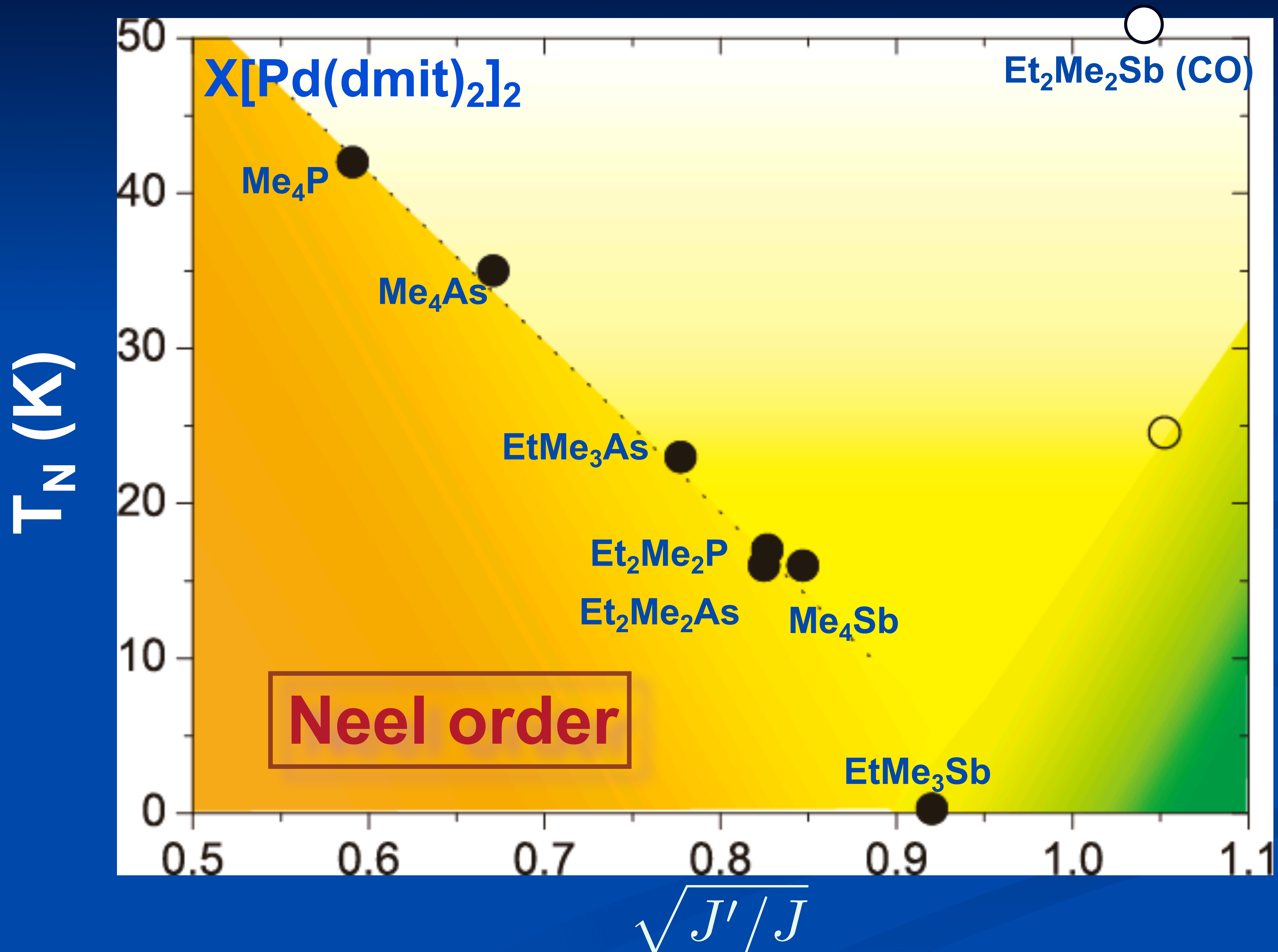


Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

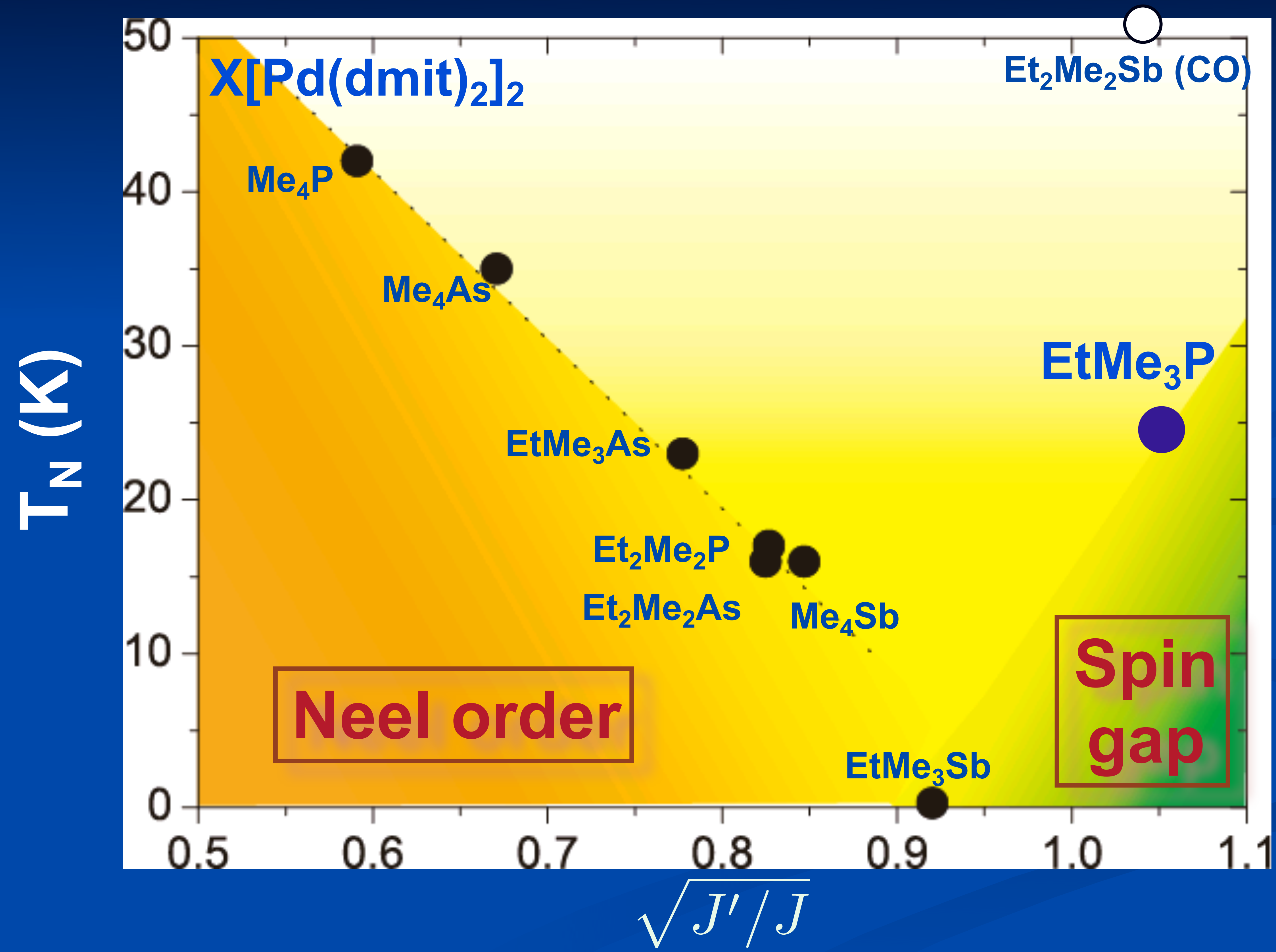
- Néel antiferromagnetic LRO
- Valence bond solid
- Z_2 spin liquid

Magnetic Criticality



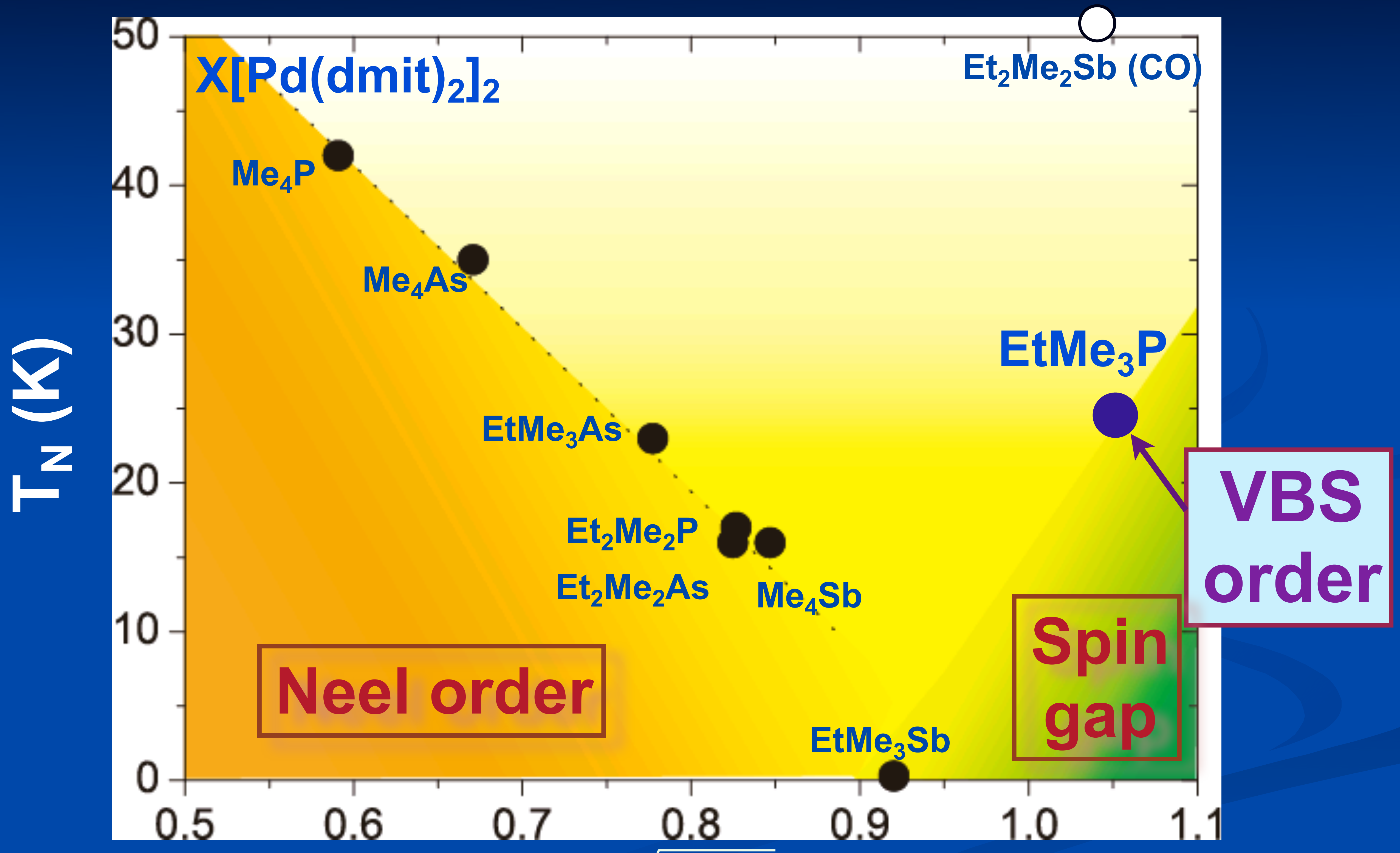
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

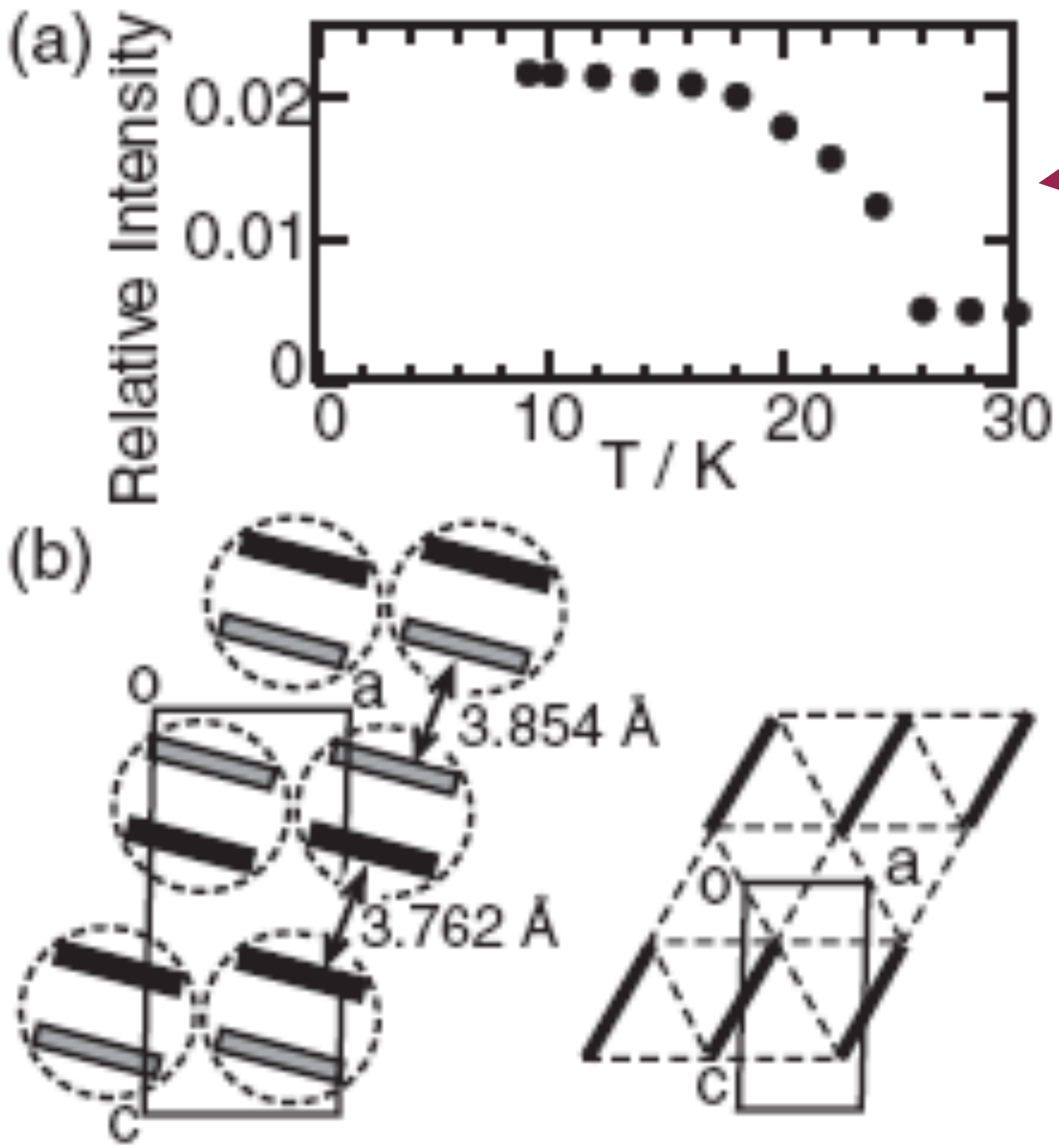
Magnetic Criticality



$\sqrt{J'/J}$

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Observation of a valence bond solid (VBS) in $ETMe_3P[Pd(dmit)_2]_2$



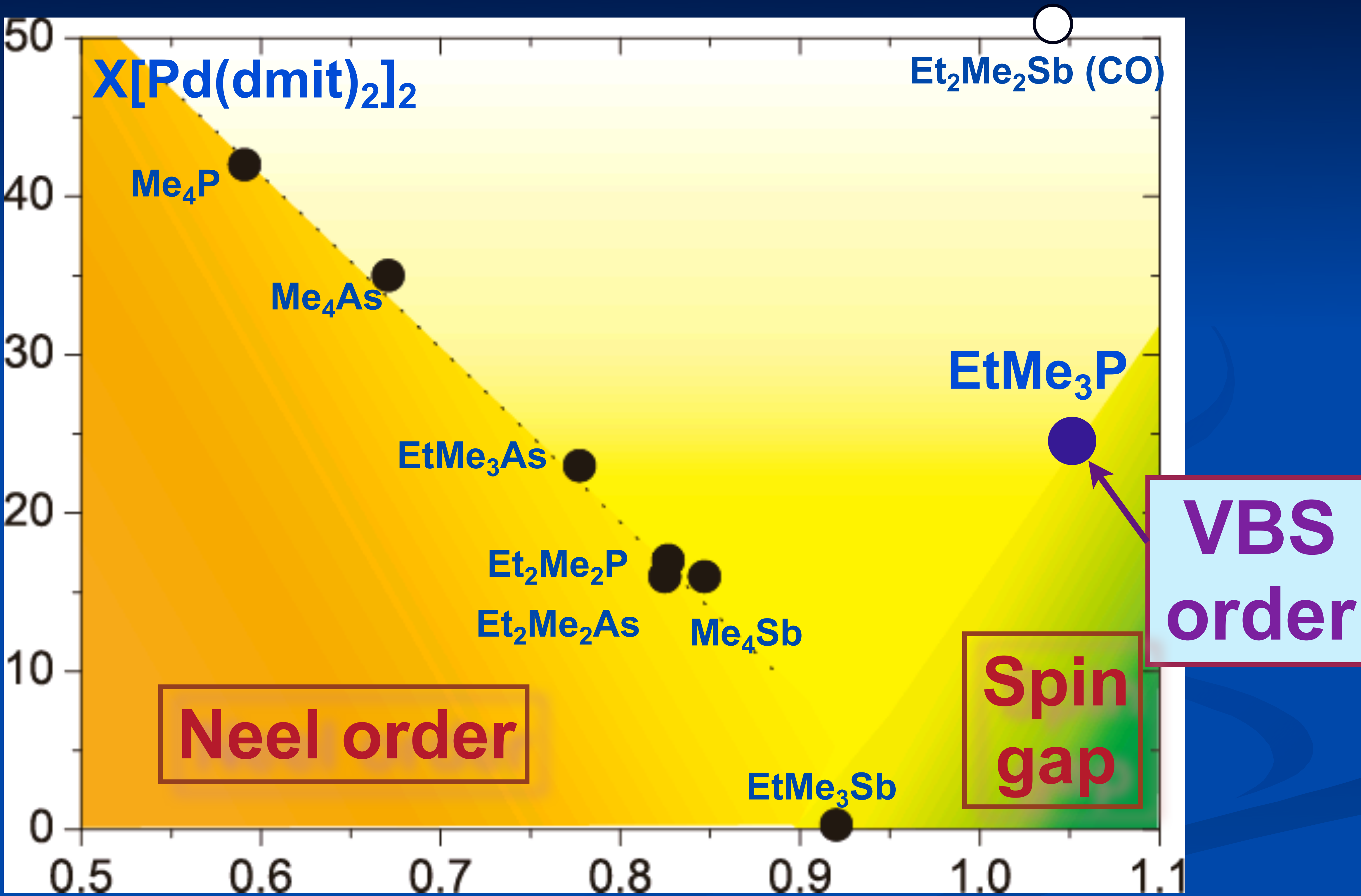
X-ray scattering

Spin gap ~ 40 K

J ~ 250 K

M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

Magnetic Criticality



T_N (K)

$\sqrt{J'/J}$

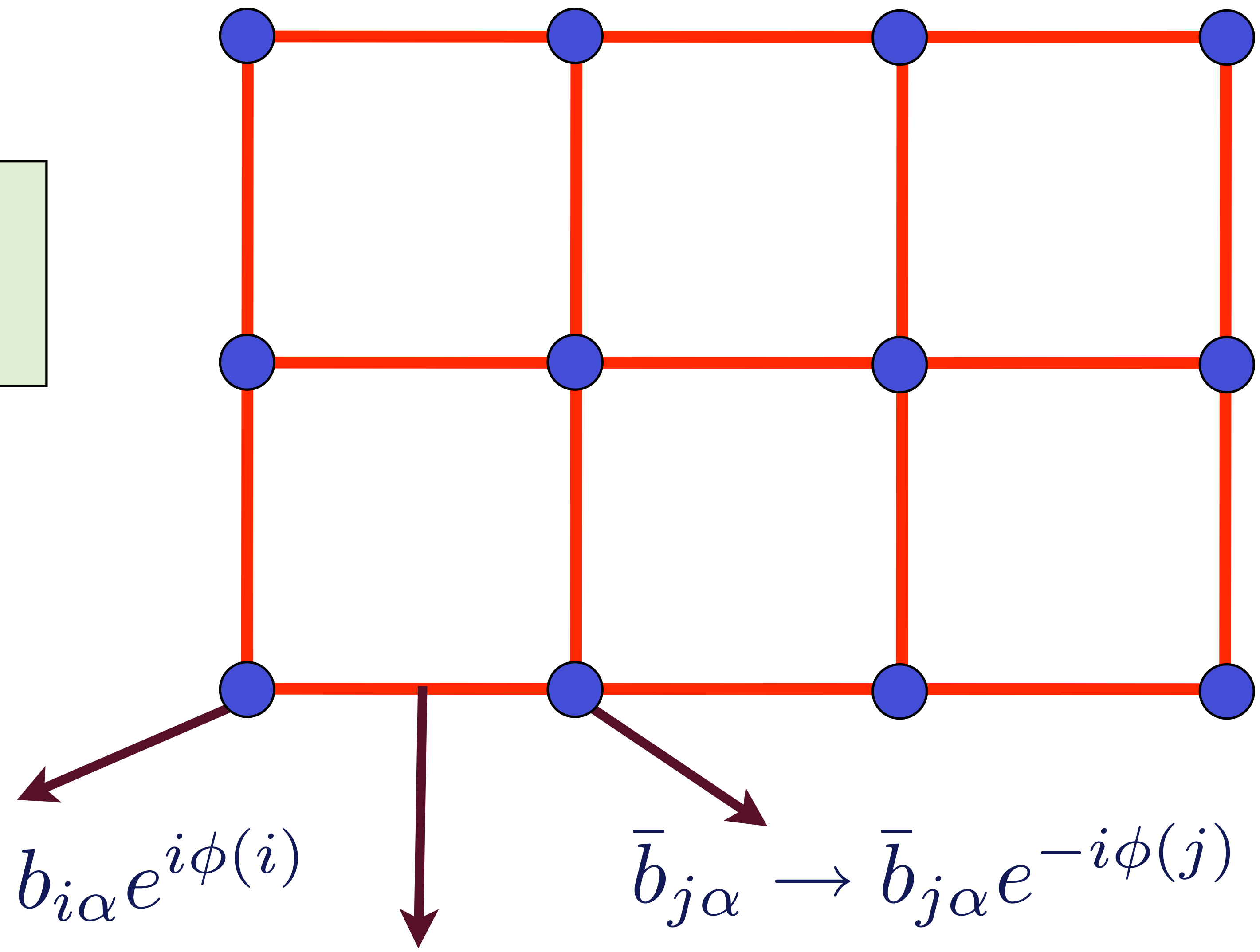
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Discussion of Schwinger bosons on the square lattice and U(1) gauge theory

http://qpt.physics.harvard.edu/leshouches/schwinger_bosons.pdf

Schwinger boson mean field theory on the square lattice and perturbative fluctuations

Origin of gauge invariance



$$b_{i\alpha} \to b_{i\alpha} e^{i\phi(i)}$$

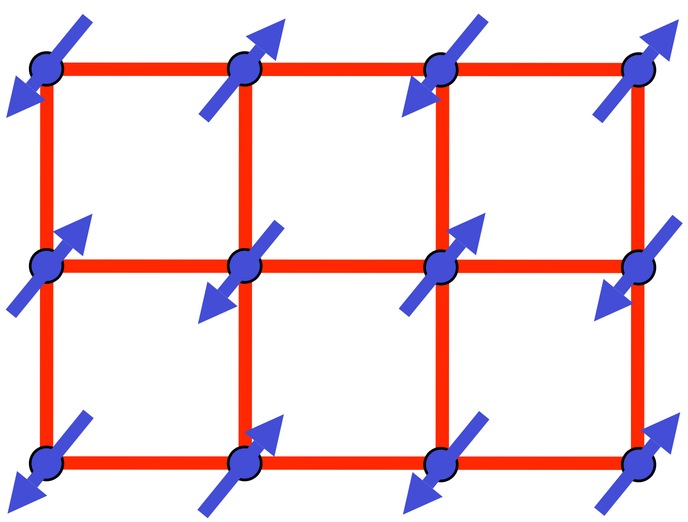
$$\bar{b}_{j\alpha} \to \bar{b}_{j\alpha} e^{-i\phi(j)}$$

$$Q_{ij} \to Q_{ij} e^{i(\phi(i)-\phi(j))}$$

$\to$ with $Q_{ij} = |Q_{ij}| e^{iA_{ij}}$, we have $A_{ij} \to A_{ij} + \phi(i) - \phi(j)$
or $A_{i,i+\hat{\mu}} \to A_{i,i+\hat{\mu}} + \partial_\mu \phi$

Schwinger boson mean field theory on the square lattice and perturbative fluctuations

$$\mathcal{S} = \int d^2x d\tau \Big[|(\partial_\tau - iA_\tau)z_\alpha|^2 + c^2|(\partial_x - iA_x)z_\alpha|^2 + s|z_\alpha|^2 + u\left(|z_\alpha|^2\right)^2 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$



$\langle z_\alpha \rangle \neq 0$

Néel state

Spin liquid state with stable $S = 1/2$ z_α spinons, and a gapless U(1) photon A_μ representing the topological order.

$\langle z_\alpha \rangle = 0$

s_c s

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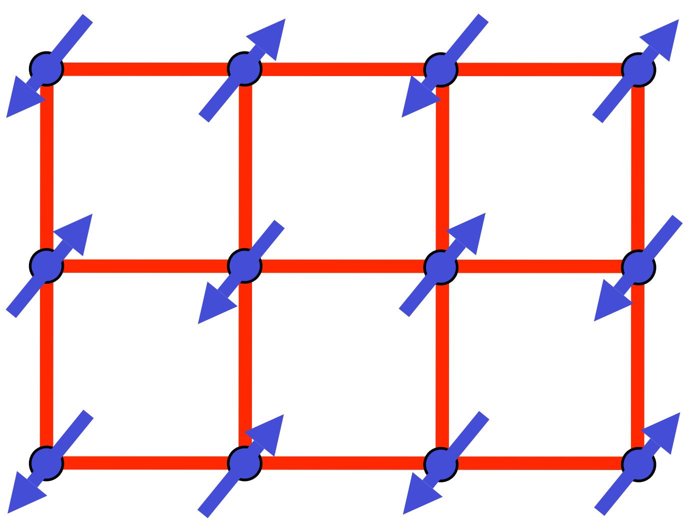

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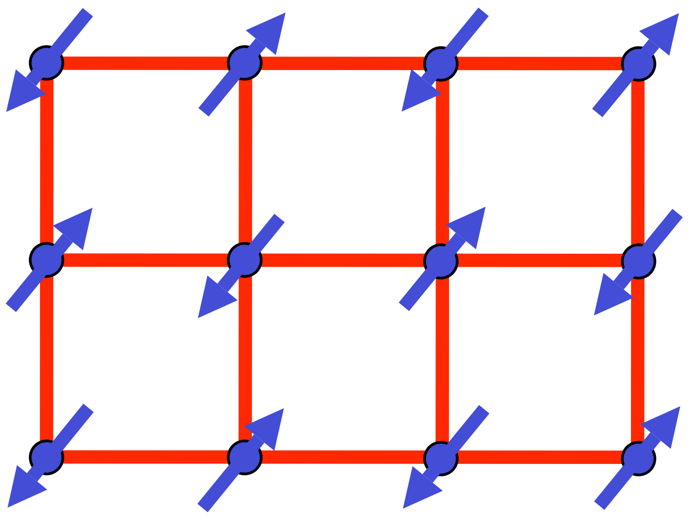

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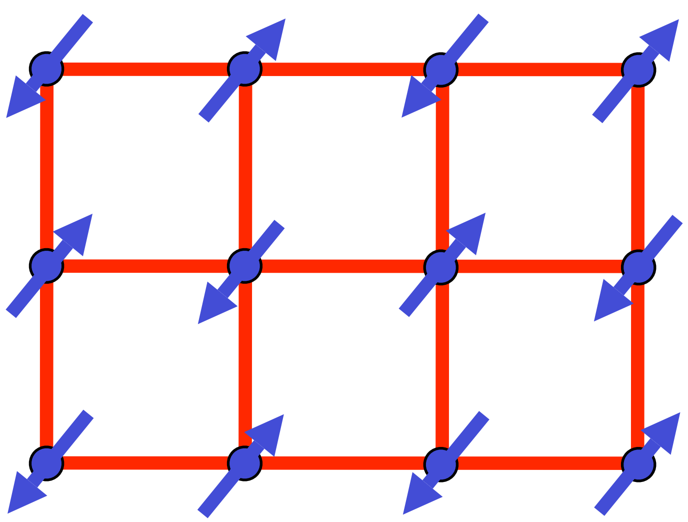

$\langle z_\alpha \rangle \neq 0$

Néel state

Nonperturbative effects lead to a gap in A_μ,
confinement of z_α,
and valence bond solid (VBS) order

$\langle z_\alpha \rangle = 0$

s_c

s