Planckian metals
and
optimal doping criticality

Ringberg Symposium on
Unconventional Superconductivity and Spin Liquids
Ringberg Castle
October 17, 2019

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

1. Planckian metals

2. Gauge theory for the cuprates near optimal doping

3. Bilocal quantum criticality

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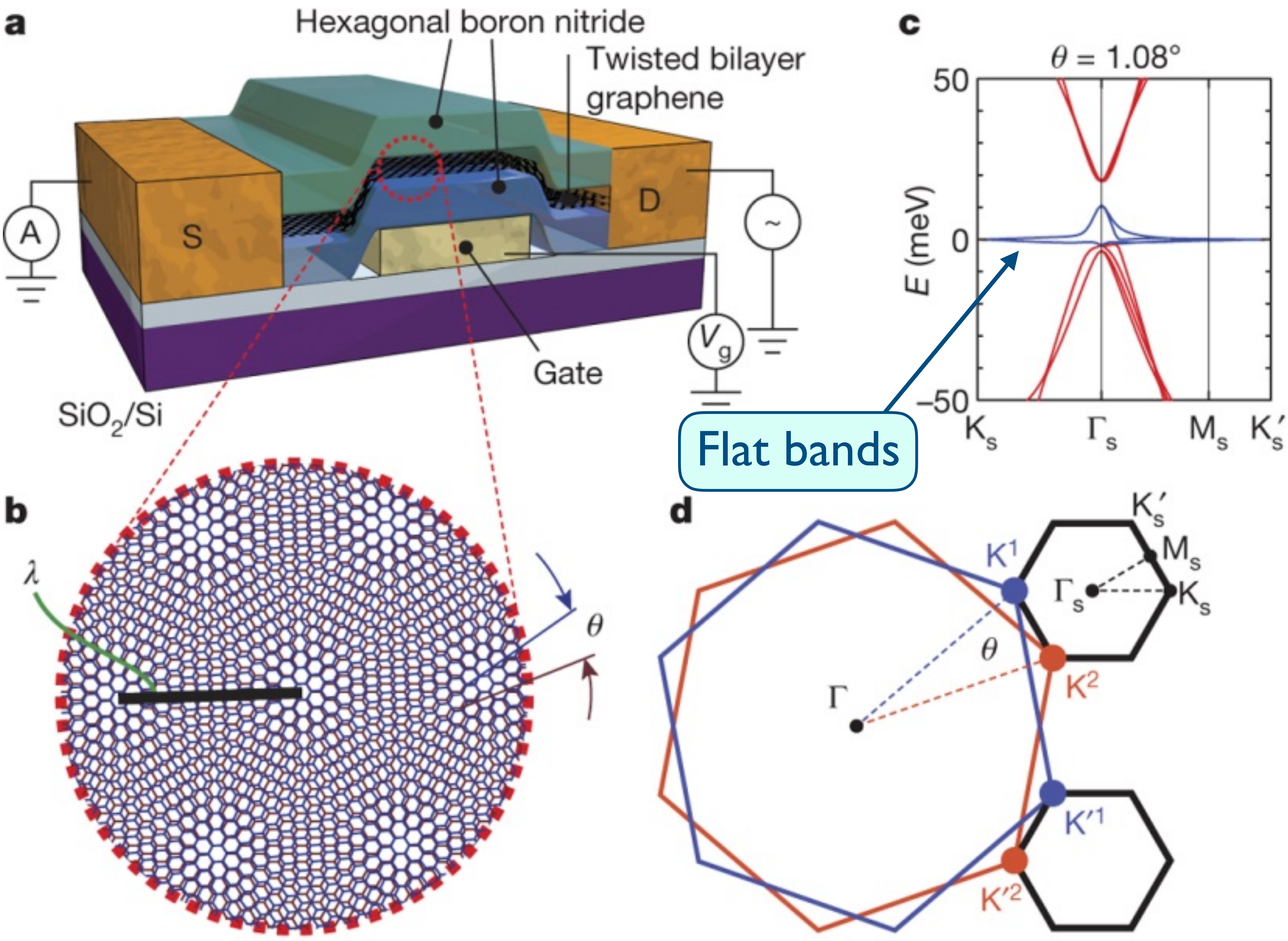
2. Gauge theory for the cuprates near optimal doping

3. Bilocal quantum criticality

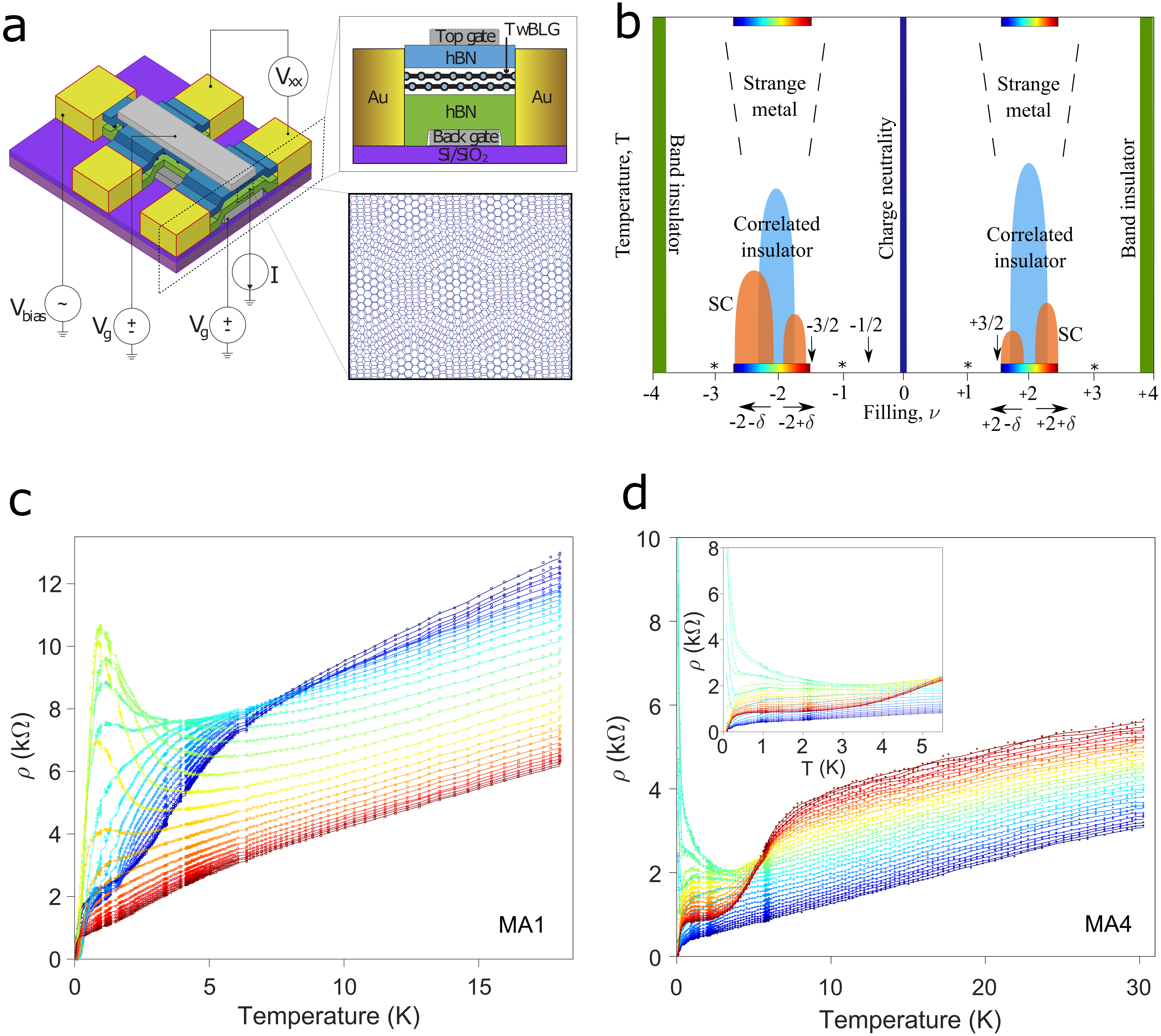
Twisted bilayer graphene



Flat bands

Cao et al., Nature **556**, 80 (2018)

Twisted bilayer graphene



Cao *et al.*, arXiv:1901.03710

Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2}\,\frac{1}{\tau}$$

with a universal scattering rate

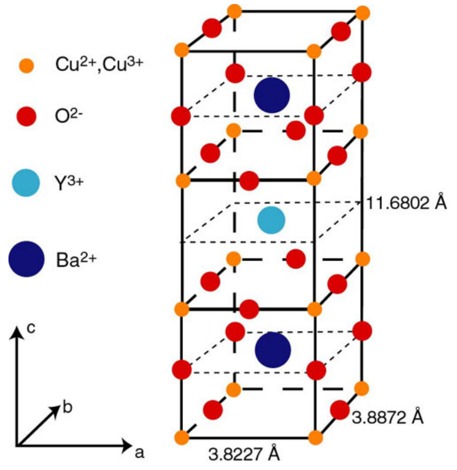
$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar}\,,$$

independent of the strength of interactions!

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \, \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

The complex SYK model

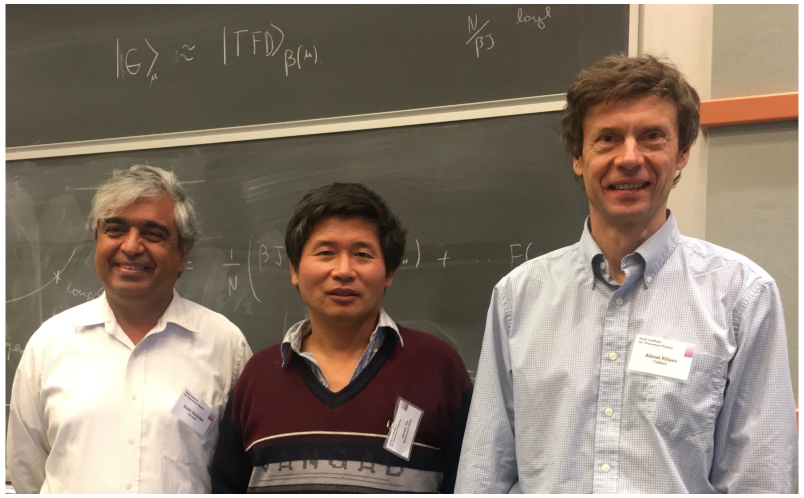
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta + e \sum_\alpha c^\dagger_\alpha c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

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Flat band

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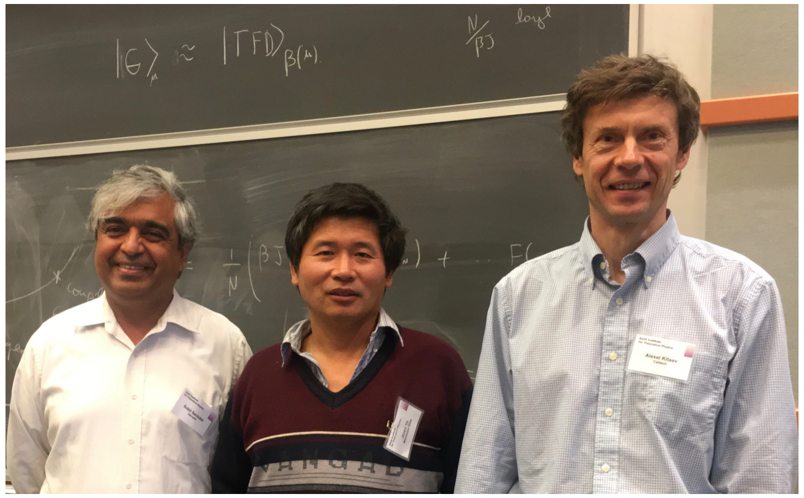

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Random interactions

Flat band

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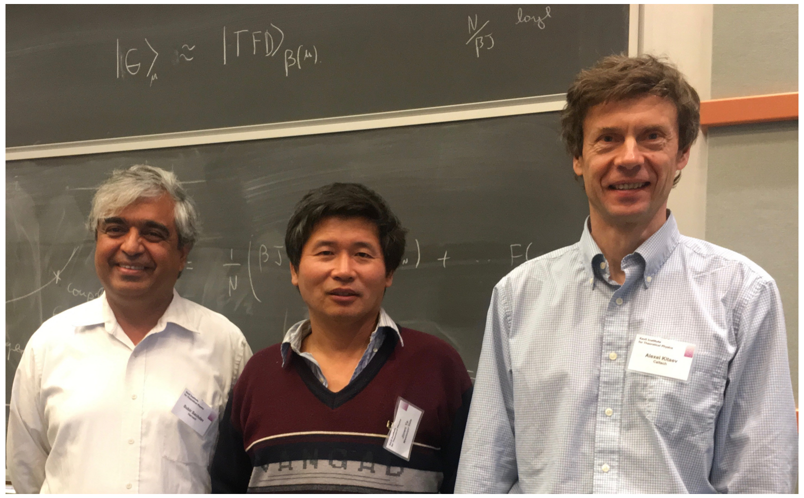

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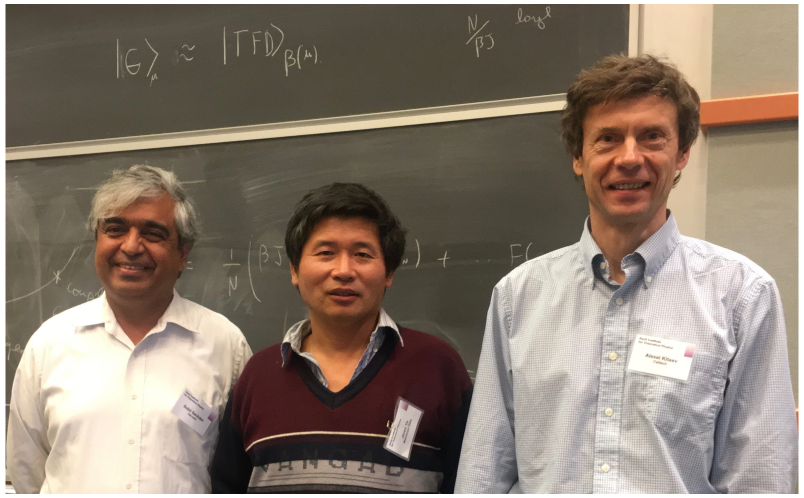
Random interactions

Flat band

Density

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

There is a one-parameter family
of critical solutions with varying e/U,
yielding different $0 < \mathcal{Q} < 1$.

For long (imaginary) times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = A e^{-2\pi \mathcal{E} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

In a Fermi liquid,

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \frac{T}{\sin(\pi T \tau)}$$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

A. Georges and O. Parcollet
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Determines the particle-hole asymmetry, and $\mathcal{E} = \mathbb{C} e/U$, with $\mathbb{C} = 0.41$ from a numerical solution.

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Known dimensionless constant

In a Fermi liquid,

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \frac{T}{\sin(\pi T \tau)}$$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

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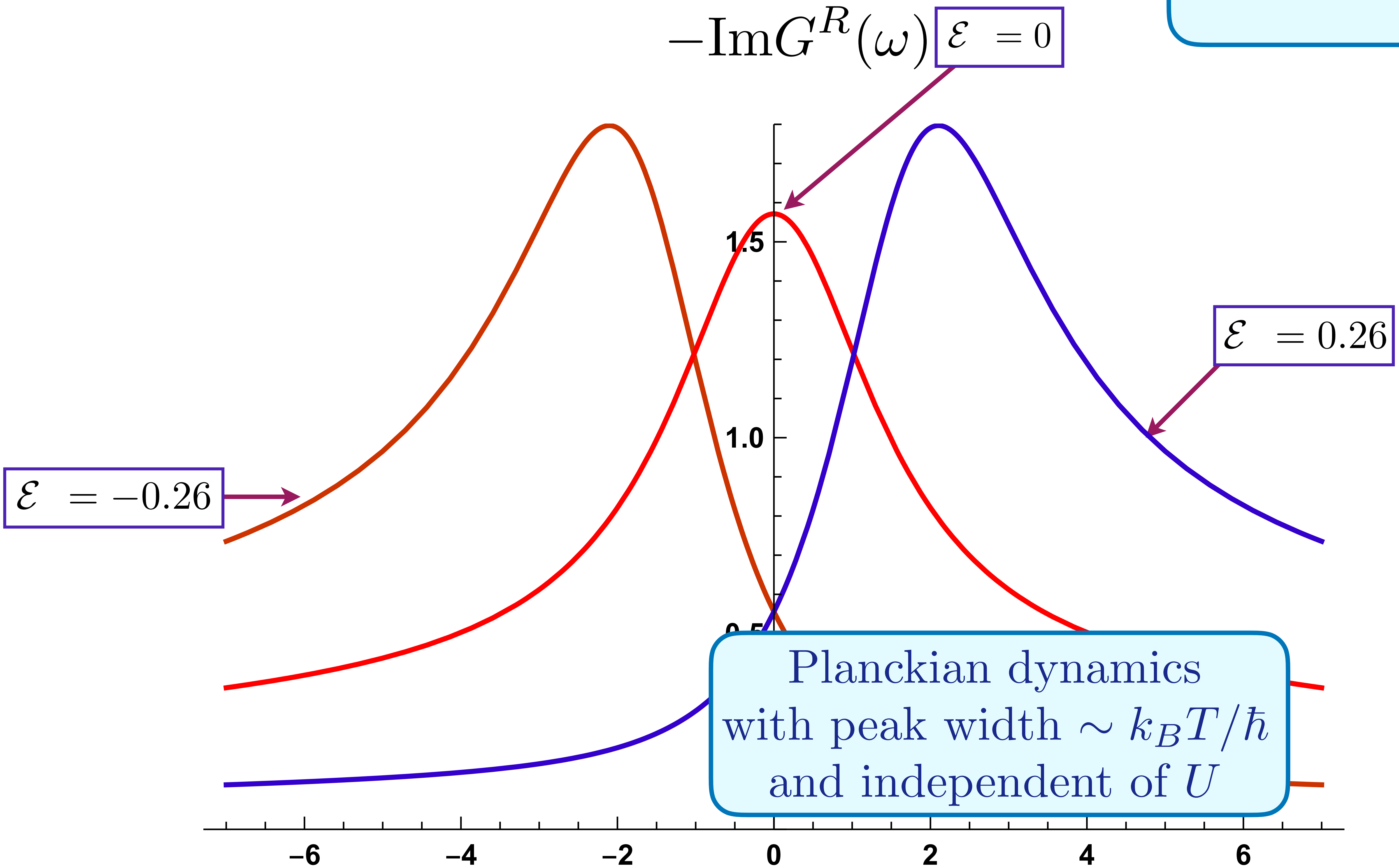
The SYK model

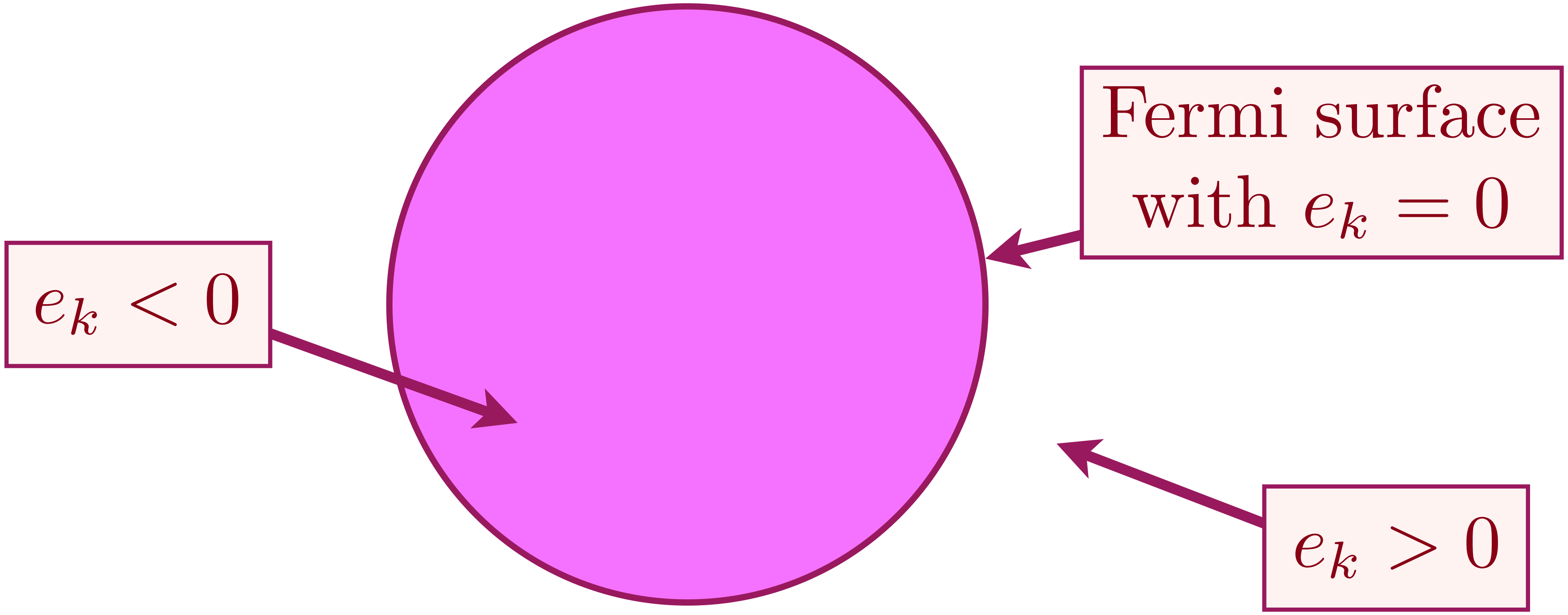
$$\mathcal{E} = \mathbb{C}\frac{e}{U}$$



Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Adding dispersion



$e_k < 0$

Fermi surface with $e_k = 0$

$e_k > 0$

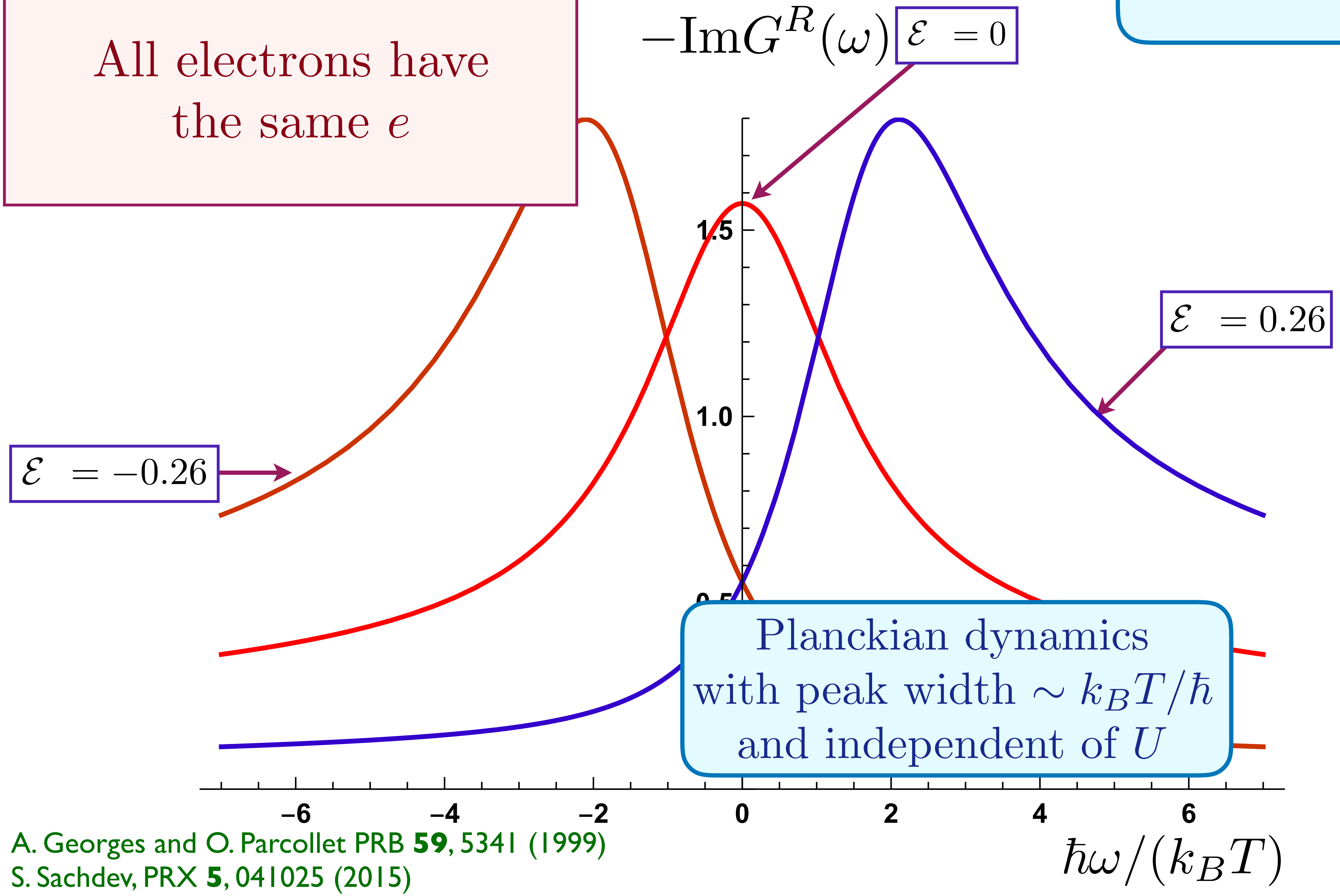
- All electrons in the (flat band) SYK model have the same e

- In a more realistic metal, the electrons have a dispersion e_k (k is momentum), and $e_k = 0$ is the Fermi surface.

Flat band metal

$$\mathcal{E} = \mathbb{C}\,\frac{e}{U}$$

All electrons have the same e



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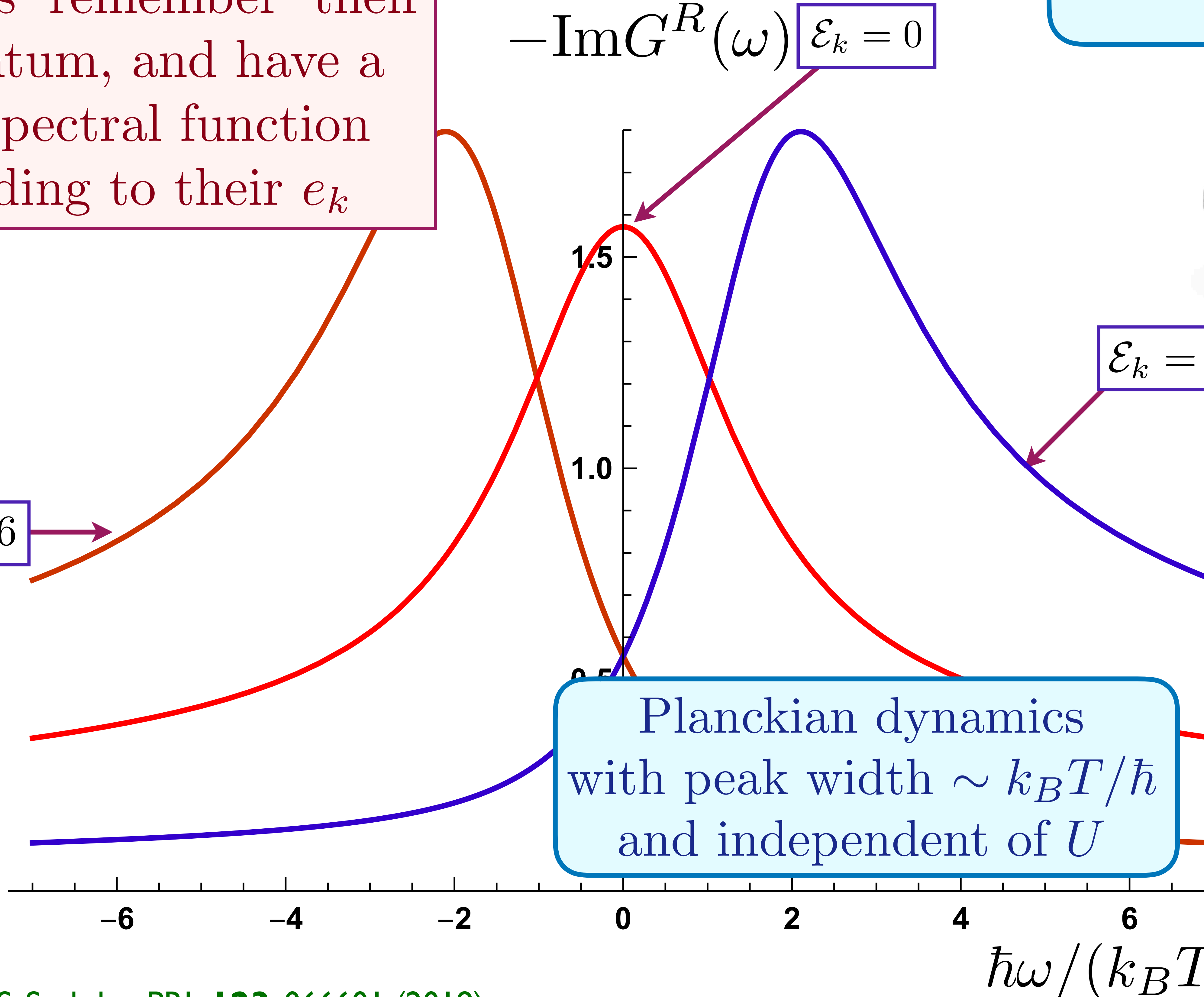
S. Sachdev, PRX **5**, 041025 (2015)

Planckian metal ansatz with dispersion

$$\mathcal{E}_k = \mathbb{C}\,\frac{e_k}{U}$$

Electrons ‘remember’ their momentum, and have a SYK spectral function according to their e_k



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

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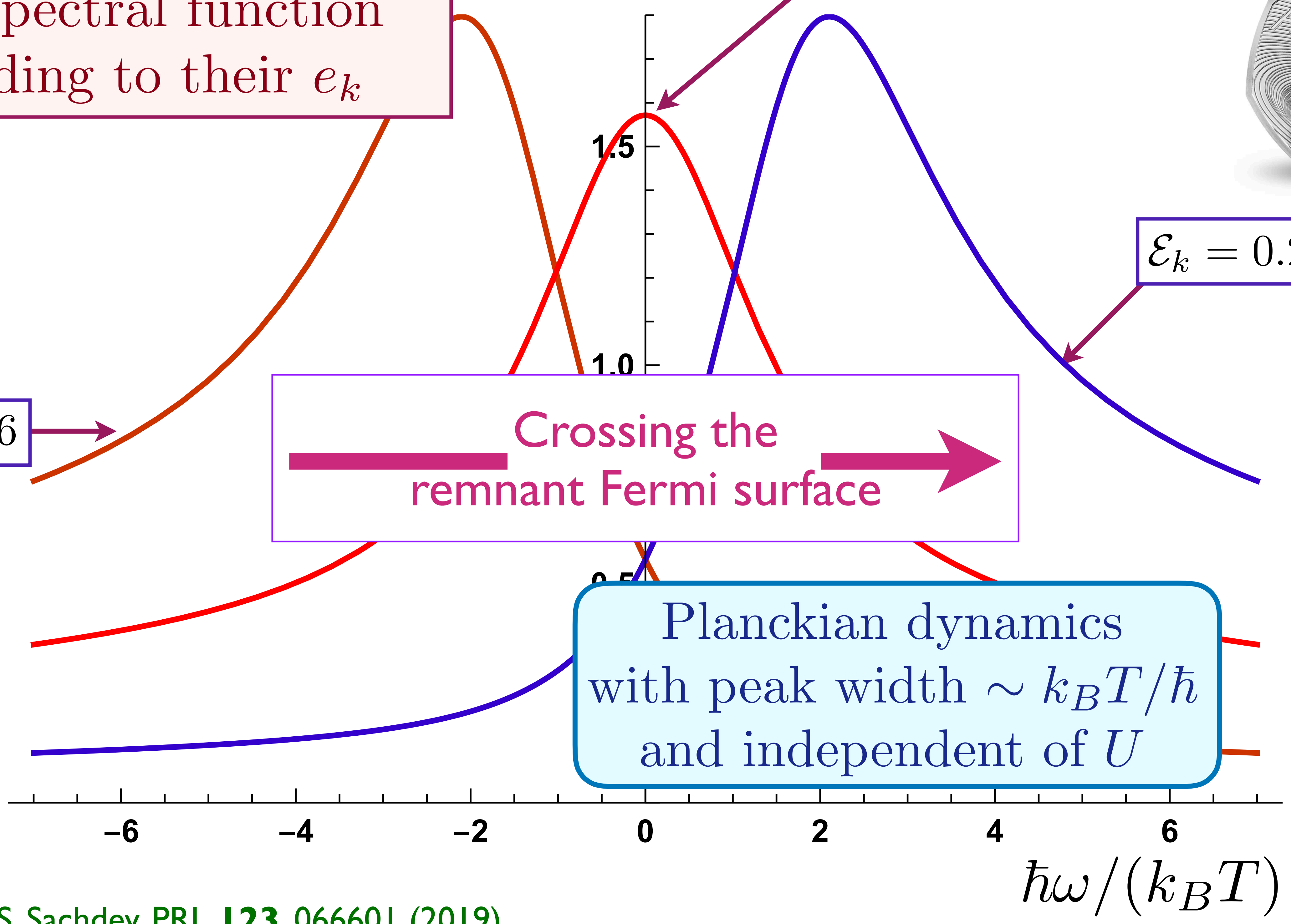


$\mathcal{E}_k = 0$

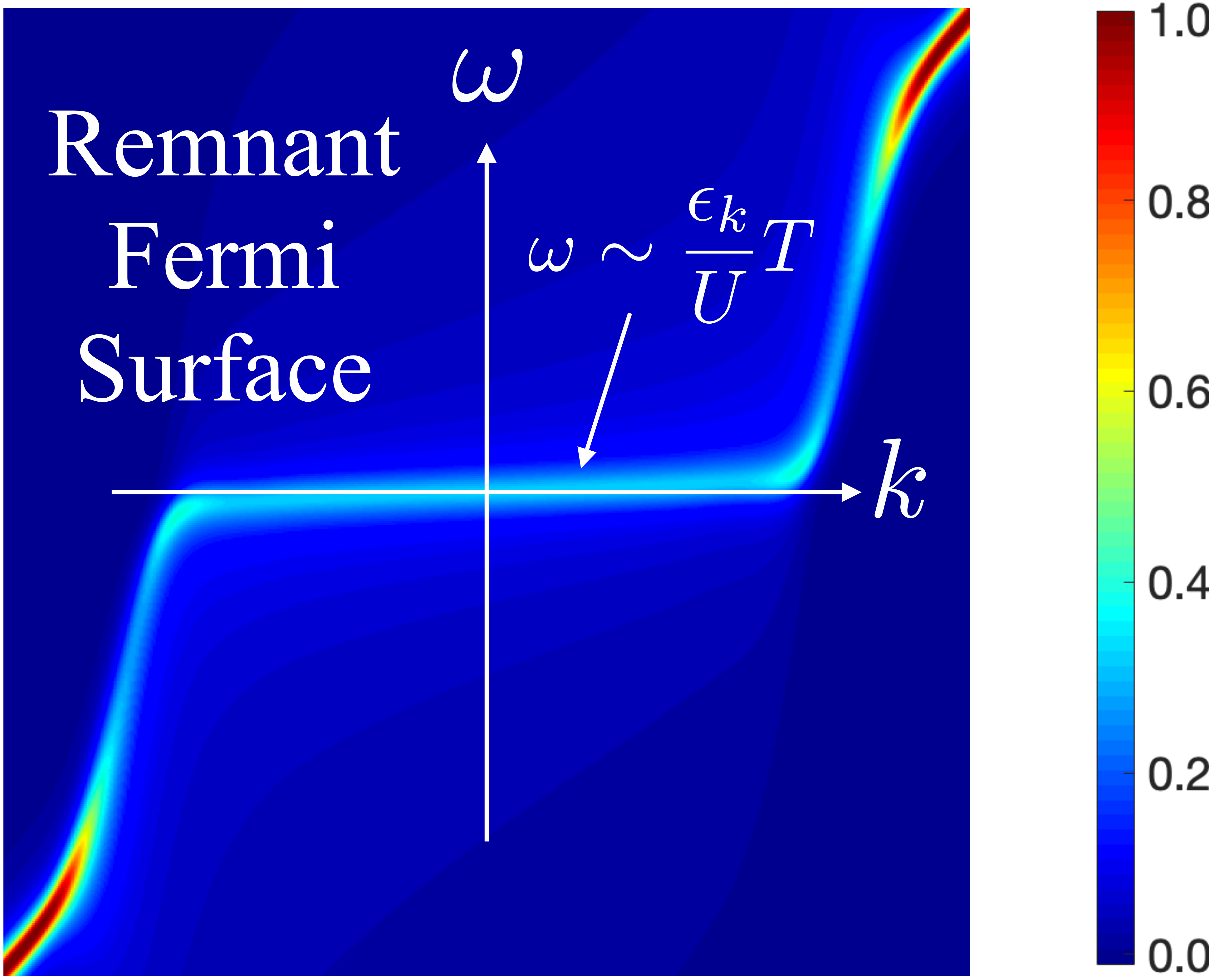
$\mathcal{E}_k = 0.26$

$\mathcal{E}_k = -0.26$

Crossing the remnant Fermi surface

Planckian dynamics with peak width $\sim k_BT/\hbar$ and independent of U

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)



Remnant Fermi Surface

$\omega \sim \frac{\epsilon_k}{U} T$

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Flat band metal

For a dispersionless SYK model

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim e^{-(e/U)2\pi\mathbb{C}T\tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

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Planckian metal ansatz with dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion e_k (k is the momentum)

$$\boxed{\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle \sim e^{-(e_k/U)2\pi \mathbb{C} T\tau} \times \left(\frac{T/U}{\sin(\pi T\tau)} \right)^{1/2}}$$

At $e_k = 0$ we have a 'remnant Fermi surface' with a particle-hole symmetric spectral function.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Planckian metal ansatz with dispersion

For a strongly-interacting metal with underlying quasiparticle dispersion e_k (k is the momentum)

$$\boxed{\left\langle c_k(\tau)\, c_k^\dagger(0) \right\rangle \sim e^{-(e_k/U)2\pi\mathbb{C}T\tau} \times \left(\frac{T/U}{\sin(\pi T\tau)}\right)^{1/2}}$$

No free parameters—everything is determined by the (underlying) quasiparticle dispersion e_k, and the interaction strength U.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Resistivity of a Planckian metal as $T \to 0$

From the Kubo formula,

$$\sigma = \frac{e^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{de}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\text{Im}\, G^R_{\text{SYK}} \left(e, \frac{\omega}{T} \right) \right]^2 \text{sech}^2 \left(\frac{\omega}{2T} \right)$$

where the Fermi surface is defined by $e_k = 0$, $\boldsymbol{v}_F = \nabla_{\boldsymbol{k}} e_k$ on the Fermi surface, and

$$m^* = \frac{d\, V_{FS}}{\oint_{FS} |\boldsymbol{v}_F|} \,,$$

with d the spatial dimensionality, and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^*.

Evaluating the integrals, we find

$$\rho = \frac{m^*}{ne^2} \, 2.71 \mathbb{C} \, \frac{k_B T}{\hbar} \quad , \quad \text{using } \mathcal{E} = \mathbb{C} e/U \,,$$

where $n = V_{FS}/(2\pi)^d$ is the density.

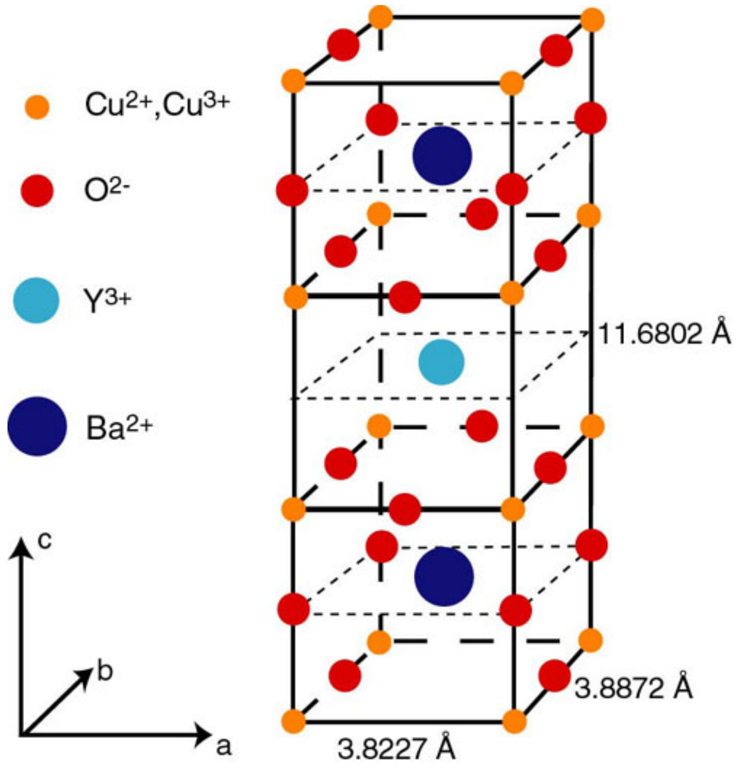
A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Resistivity of a Planckian metal as $T \to 0$

$$\boxed{\rho = \frac{m^*}{ne^2}\, 2.71\mathbb{C}\, \frac{k_B T}{\hbar}}$$

Note that all explicit dependence on U has cancelled out!

Choosing $\mathbb{C} = 0.41$ as in the SYK model, we have the prefactor $2.71\mathbb{C} = 1.11$.



Aavishkar Patel

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

A lattice SYK model

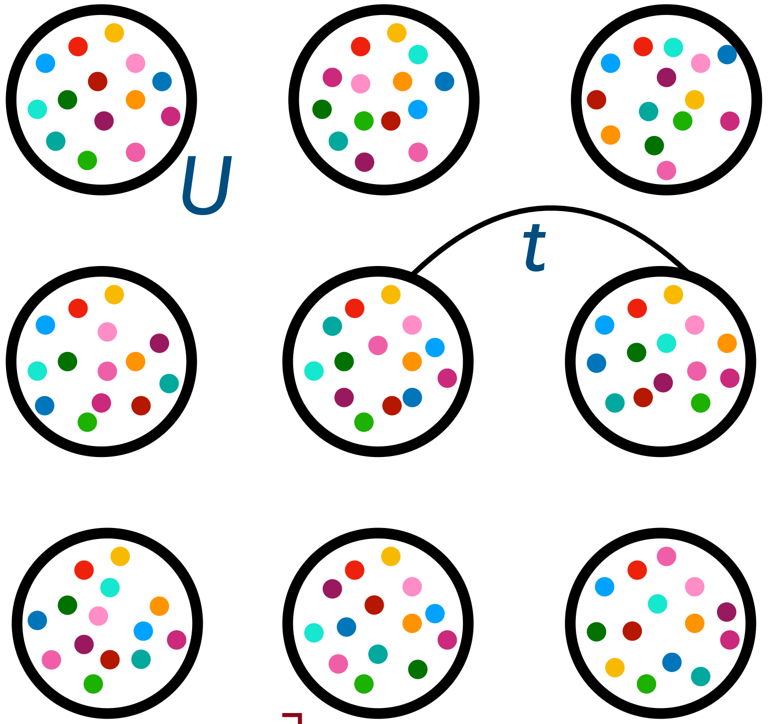
$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

$$+ \sum_{k\alpha} e_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

e_k has a bandwidth W.

- Disordered Fermi liquid for $T < W^2/U$ with resistivity $\sim T^2$.
- Incoherent, bad metal for $W^2/U < T < U$ with linear-in-T resistivity $\sim (h/e^2)(T/(W^2/U))$.



$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} =$$
$$U^2 \Big[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \Big]$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999); Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta}$$
$$+ \sum_{k\alpha} e_k c_{k\alpha}^\dagger c_{k\alpha}$$

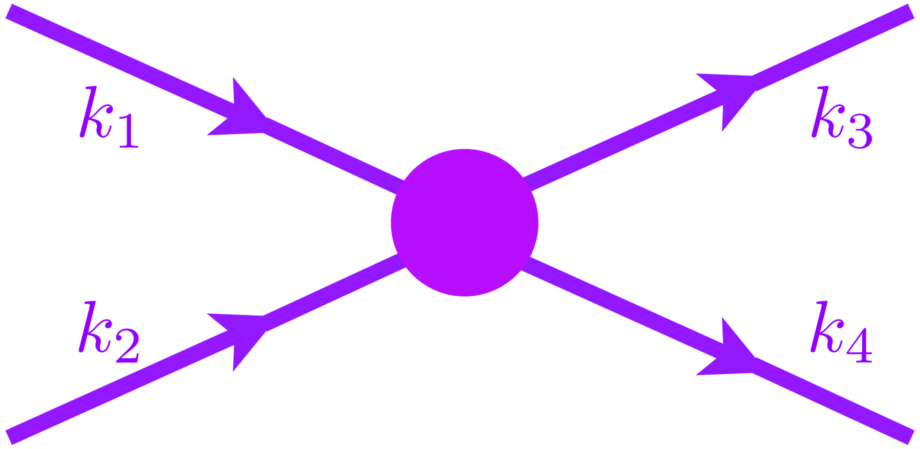
$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$
e_k has a bandwidth W.

We examine a model with weaker $W \lesssim U$, but impose a **resonance condition**.
This leads to a solution which obeys the Planckian ansatz as $T \to 0$.

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2\left[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\right] \times \left[\delta(e_{k_1}+e_{k_2}-e_{k_3}-e_{k_4})+\delta(e_{k_5}+e_{k_6}-e_{k_7}-e_{k_8})\right]$$

This implies off-site interactions with correlations
which decay with a power-law in space.

Resonant SYK model

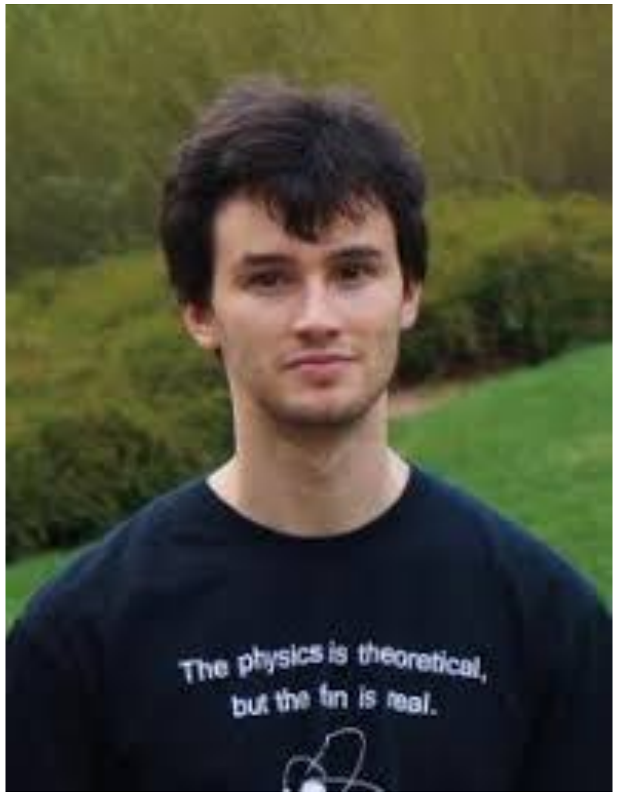


Interactions with $e_{k_1} + e_{k_2} \neq e_{k_3} + e_{k_4}$ are non-resonant: we "integrate these out" in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion e_k, which we have already accounted for.

Keep only the **interactions resonant in the bare quasiparticle energy** with $e_{k_1} + e_{k_2} = e_{k_3} + e_{k_4}$ and account for them with a self-consistent SYK-like analysis.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Mathias Scheurer



Grigory Tarnopolsky

Harley Scammell

Gauge theory for the cuprates near optimal doping

Physical Review B **99**, 054516 (2019)

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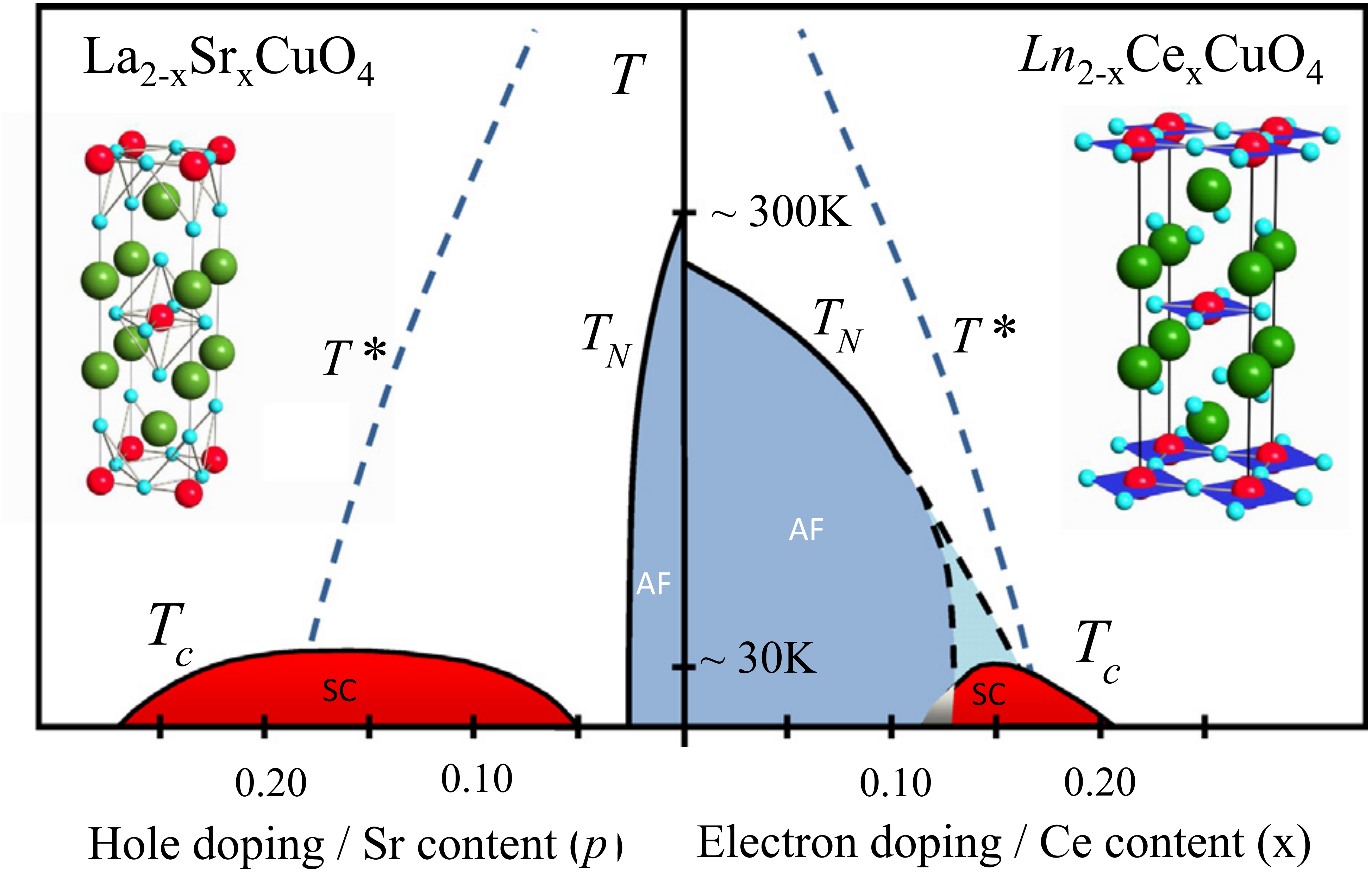
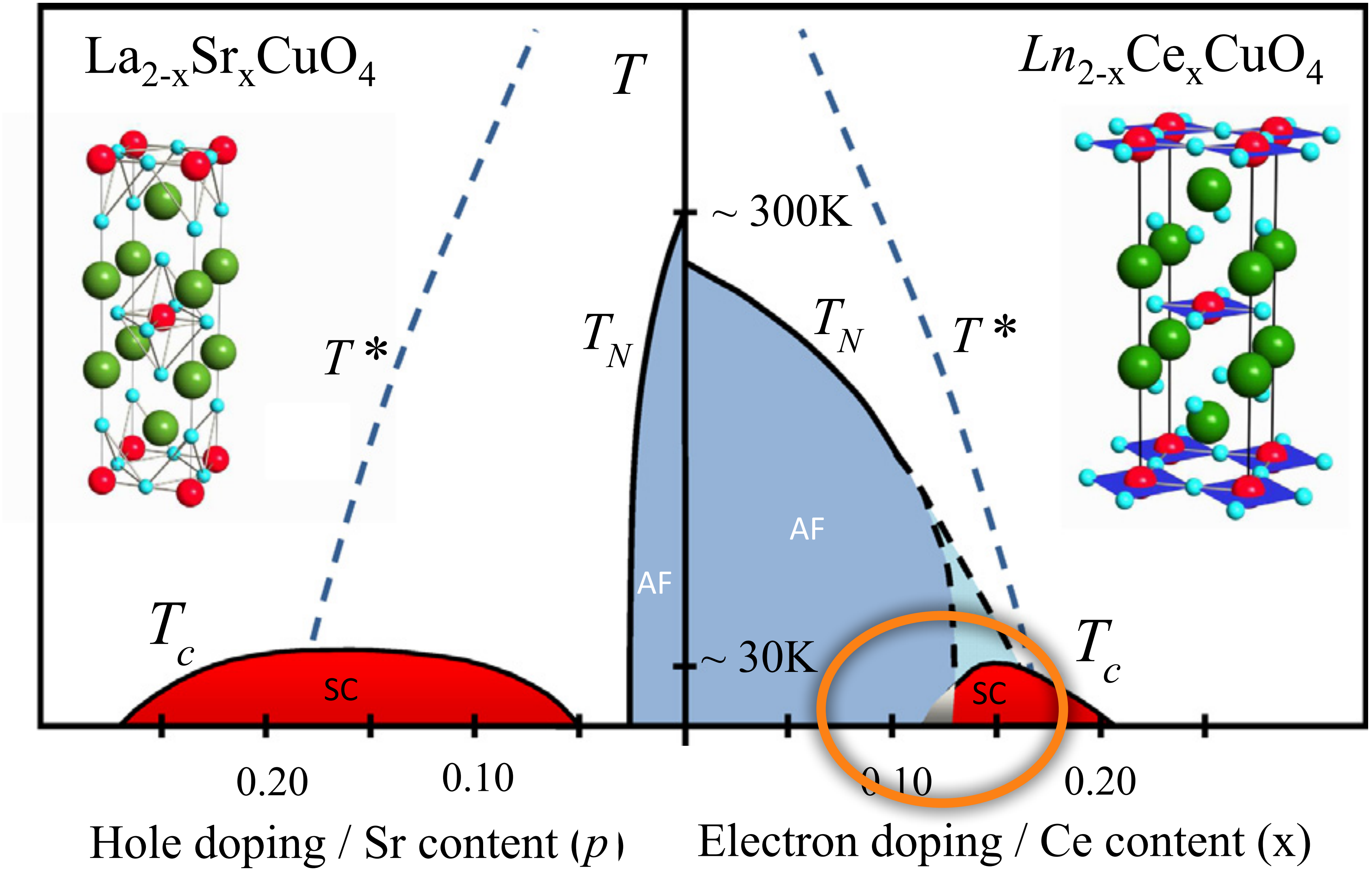

$La_{2-x}Sr_xCuO_4$
T
$Ln_{2-x}Ce_xCuO_4$
~ 300K
T^*
T_N
T_N
T^*
AF
AF
T_c
~ 30K
T_c
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

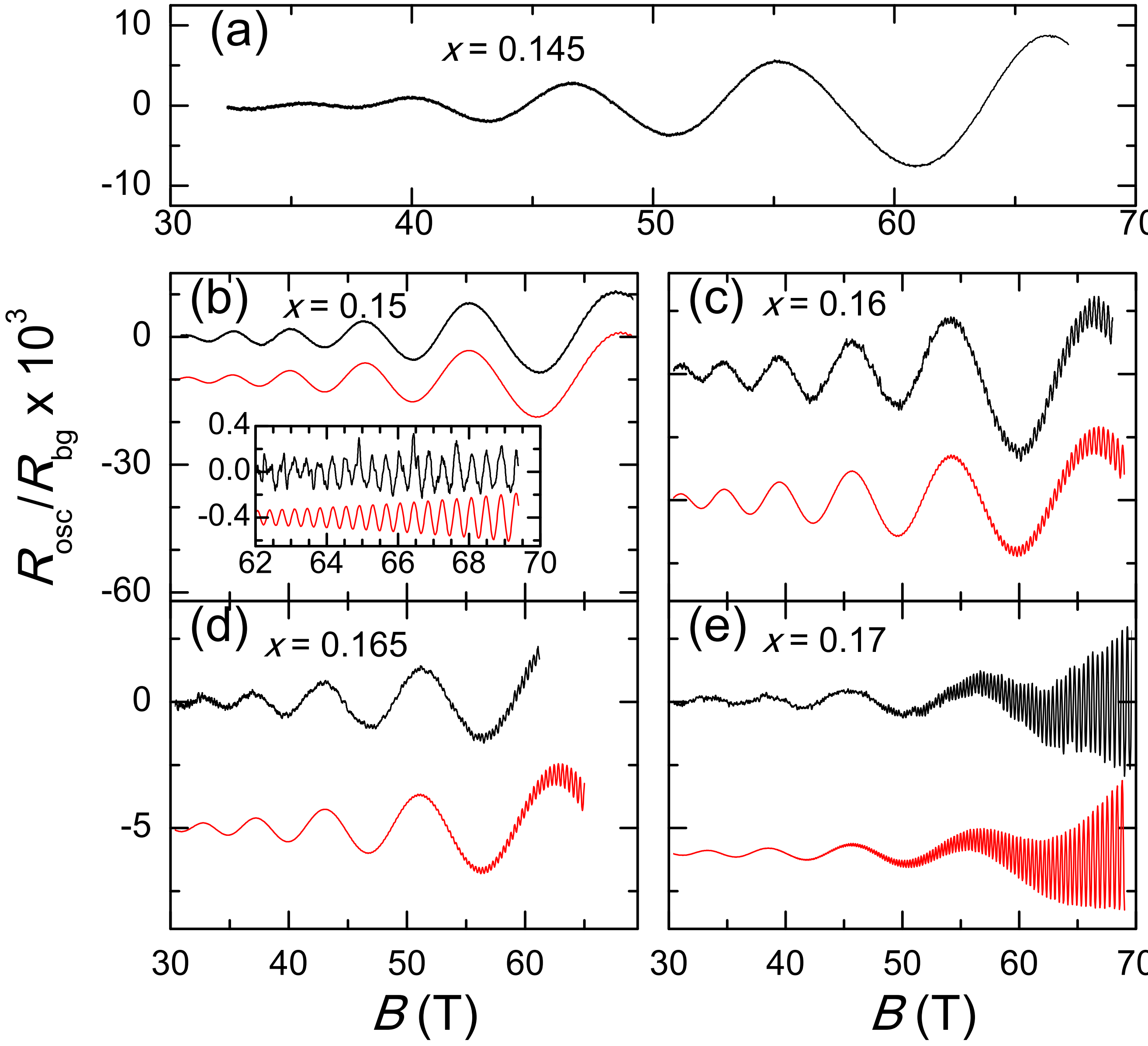

La2-xSrxCuO4
T
Ln2-xCexCuO4
~ 300K
T*
TN
TN
T*
AF
AF
Tc
SC
~ 30K
SC
Tc
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

PHYSICAL REVIEW B **92**, 094501 (2015)

Correlation between Fermi surface transformations and superconductivity in the electron-doped high-T_c superconductor $Nd_{2-x}Ce_xCuO_4$

T. Helm,[1,*] M. V. Kartsovnik,[1,†] C. Proust,[2] B. Vignolle,[2] C. Putzke,[3,‡] E. Kampert,[3] I. Sheikin,[4] E.-S. Choi,[5] J. S. Brooks,[5]
N. Bittner,[1,§] W. Biberacher,[1] A. Erb,[1,6] J. Wosnitza,[3] and R. Gross[1,6,‖]

- Quantum oscillations show the presence of small hole pockets up to a doping $x = 0.175$



Although antiferromagnetic order disappears near $x = 0.14$, perhaps there is field-induced antiferromagnetic order up to $x = 0.175$?

Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen, PNAS **116**, 3449 (2019)

- New photoemission measurements in NCCO at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.
- The energy gap between the electron and hole pockets collapses near $x = 0.17$ like an order parameter.

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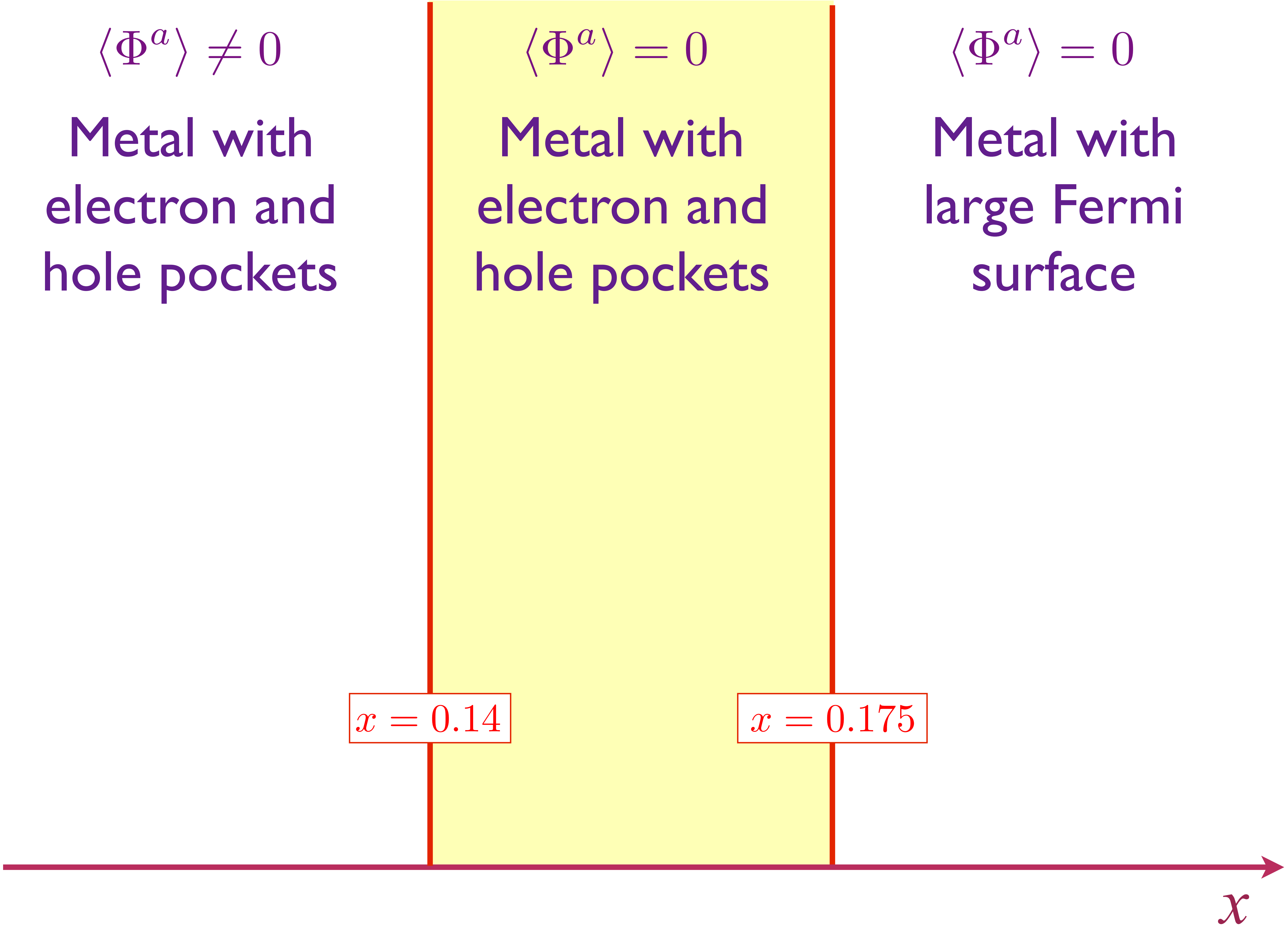
- The energy gap between the electron and hole pockets collapses near $x = 0.17$ like an order parameter.

- “The totality of the data points to a mysterious order between $x = 0.14$ and $x = 0.17$, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin.”

$\Phi^a \Rightarrow$ Antiferromagnetism



$\langle \Phi^a \rangle \neq 0$

Metal with electron and hole pockets

$\langle \Phi^a \rangle = 0$

Metal with electron and hole pockets

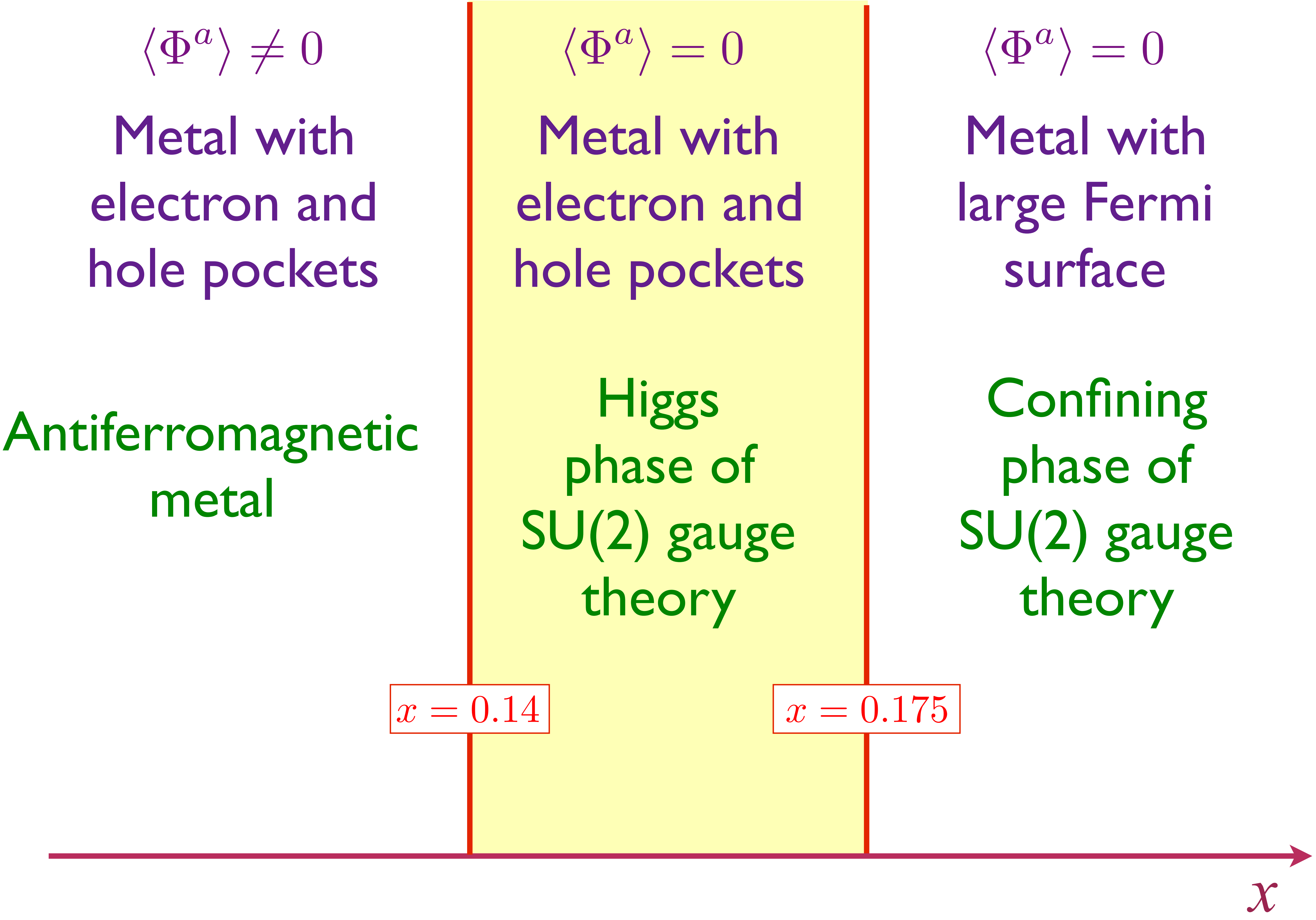
$\langle \Phi^a \rangle = 0$

Metal with large Fermi surface

$x = 0.14$

$x = 0.175$

$\Phi^a \Rightarrow$ Antiferromagnetism



$\langle \Phi^a \rangle \neq 0$

Metal with electron and hole pockets

Antiferromagnetic metal

$\langle \Phi^a \rangle = 0$

Metal with electron and hole pockets

Higgs phase of SU(2) gauge theory

$\langle \Phi^a \rangle = 0$

Metal with large Fermi surface

Confining phase of SU(2) gauge theory

$x = 0.14$

$x = 0.175$

Transforming to a rotating reference frame

We can (exactly) transform the Hubbard model to the "spin-fermion" model:
electrons $c_{i\alpha}$ on the square lattice with dispersion

$$\begin{aligned} \mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c_{i,\alpha}^\dagger c_{i+\boldsymbol{v}_\rho,\alpha} + c_{i+\boldsymbol{v}_\rho,\alpha}^\dagger c_{i,\alpha} \right) \\ &\quad -\mu \sum_i c_{i,\alpha}^\dagger c_{i,\alpha} + \mathcal{H}_{\rm int} \end{aligned}$$

are coupled to a magnetic moment order parameter $\Phi^a(i)$, $a = x, y, z$

$$\begin{aligned} \mathcal{H}_{\rm int} &= -\lambda \sum_i \Phi^a(i) c_{i,\alpha}^\dagger \sigma_{\alpha\beta}^a c_{i,\beta} + V(\Phi^a) \\ V(\Phi^a) &= s\Phi^a\Phi^a + u\Phi^a\Phi^a\Phi^b\Phi^b \end{aligned}$$

Transforming to a rotating reference frame

For fluctuating antiferromagnetism (spin density waves (SDW)), we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^a \Phi^a(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Gauge theory of fluctuating SDW

Field	Symbol	Statistics	$SU(2)_{\rm gauge}$	$SU(2)_{\rm spin}$	$U(1)_{\rm e.m.charge}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

Note that the transformation to a rotating reference frame is ambiguous up to a SU(2) gauge transformation, V_i

$$\left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right) \rightarrow V_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right)$$

$$R_i \rightarrow R_i V_i^\dagger$$

$$\sigma^a H^a(i) \rightarrow V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

$\Phi^a \Rightarrow$ Antiferromagnetism

$\langle \Phi^a \rangle \neq 0$	$\langle \Phi^a \rangle = 0$	$\langle \Phi^a \rangle = 0$
Metal with electron and hole pockets	Metal with electron and hole pockets	Metal with large Fermi surface
Antiferromagnetic metal	Higgs phase of SU(2) gauge theory	Confining phase of SU(2) gauge theory
$\langle H \rangle \neq 0$ $\langle R \rangle \neq 0$	$\langle H \rangle \neq 0$ $\langle R \rangle = 0$	$\langle H \rangle = 0$ $\langle R \rangle \neq 0$

$x = 0.14$

$x = 0.175$

x

Mean field theory for the pseudogap metal

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the SDW order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma^a_{ss'} \psi_{i,s'} + V(H^a)$$

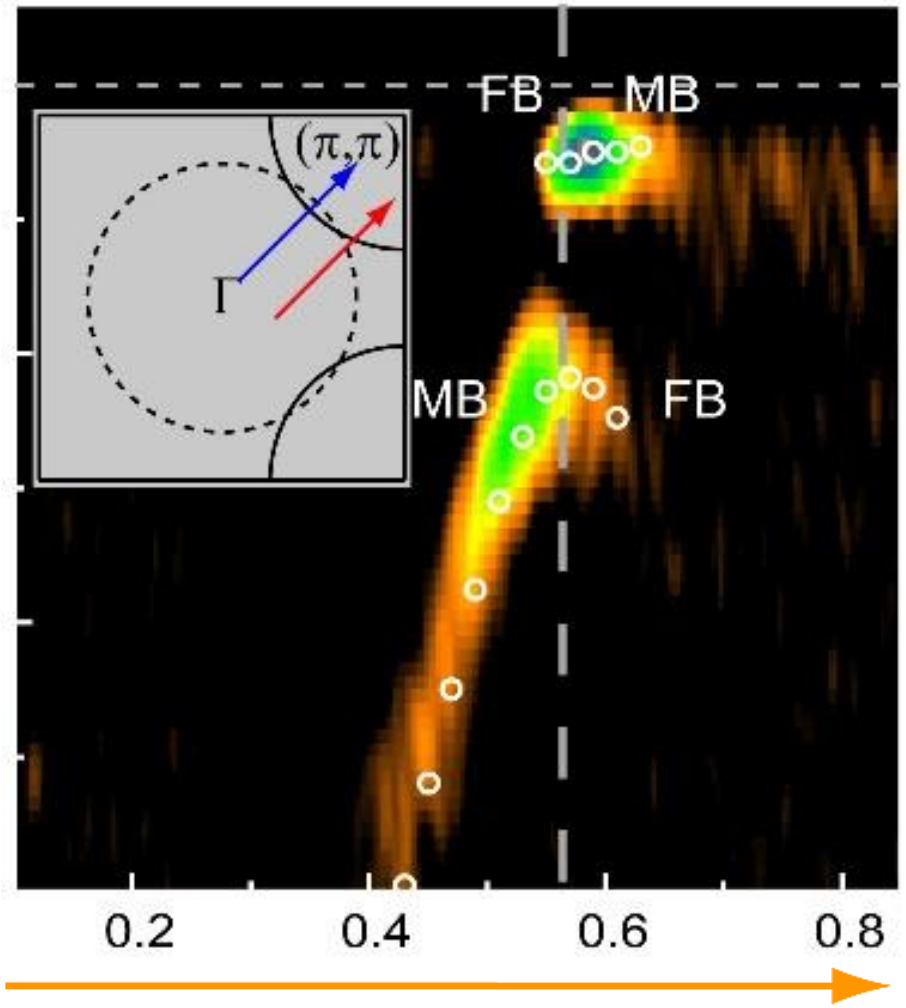
IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of (fluctuating) SDW SRO will inherit the small Fermi surfaces of the electrons in the presence of SDW LRO.

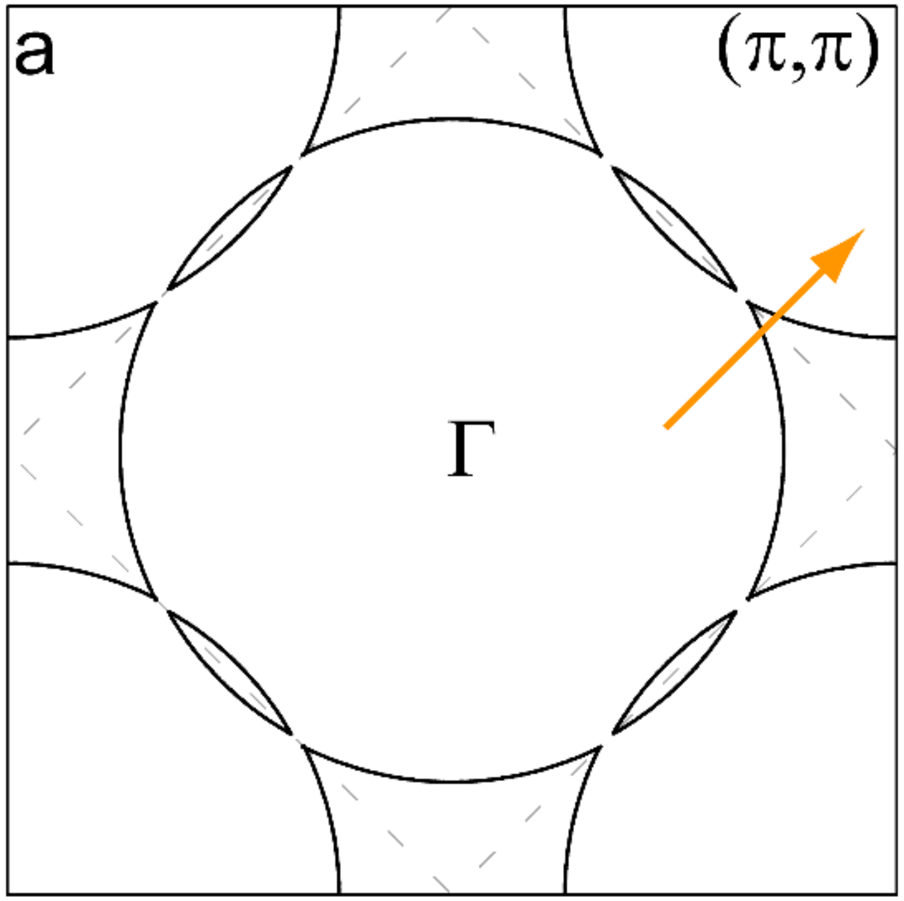
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

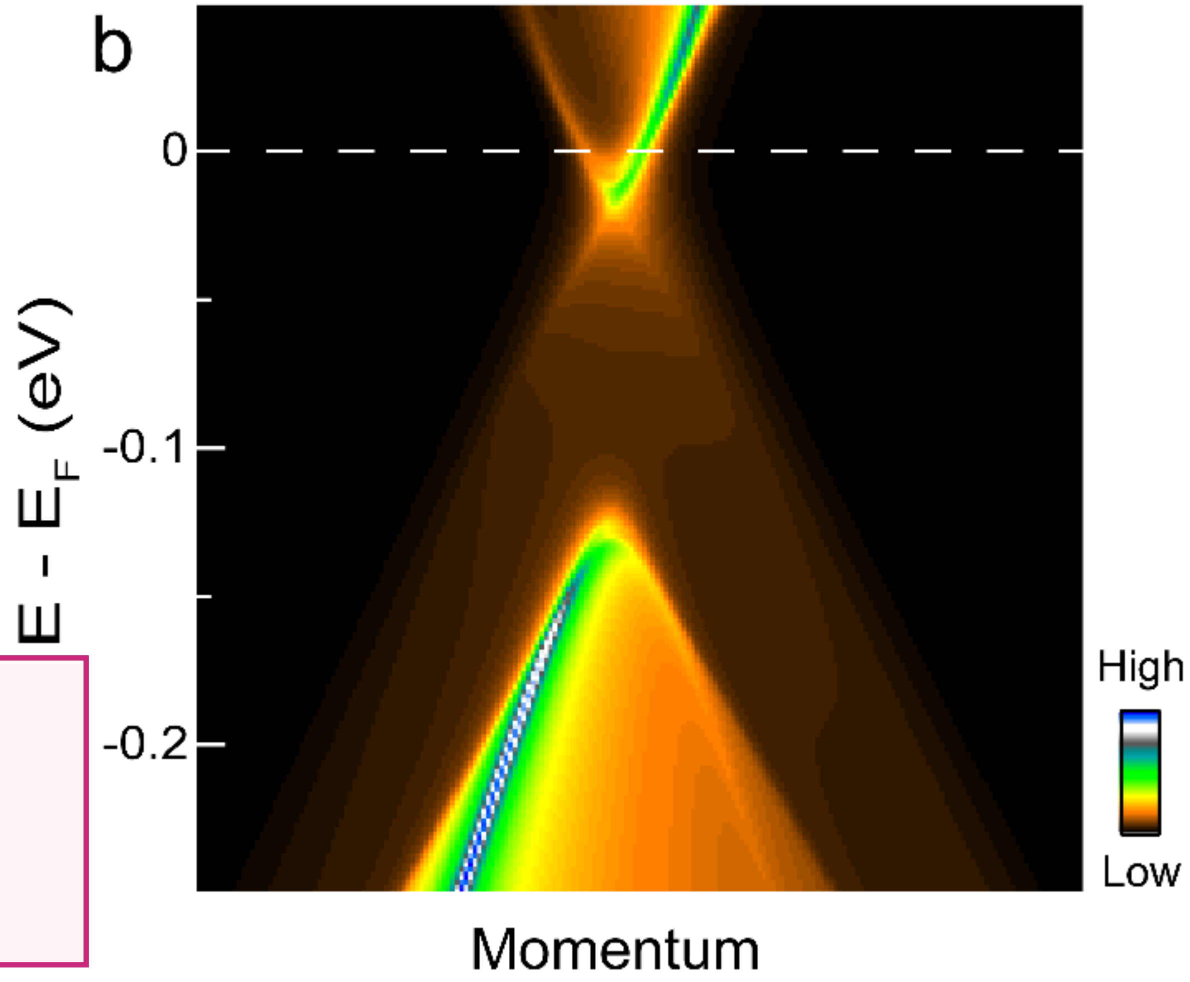
Mathias Scheurer

M. S. Scheurer, S. Chatterjee, Wei Wu,
M. Ferrero, A. Georges, and S. Sachdev,
PNAS **115**, E3665 (2018)

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto,
K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu,
Y. S. Lee, T. P. Devereaux and Z.-X. Shen, PNAS **116**, 3449 (2019)

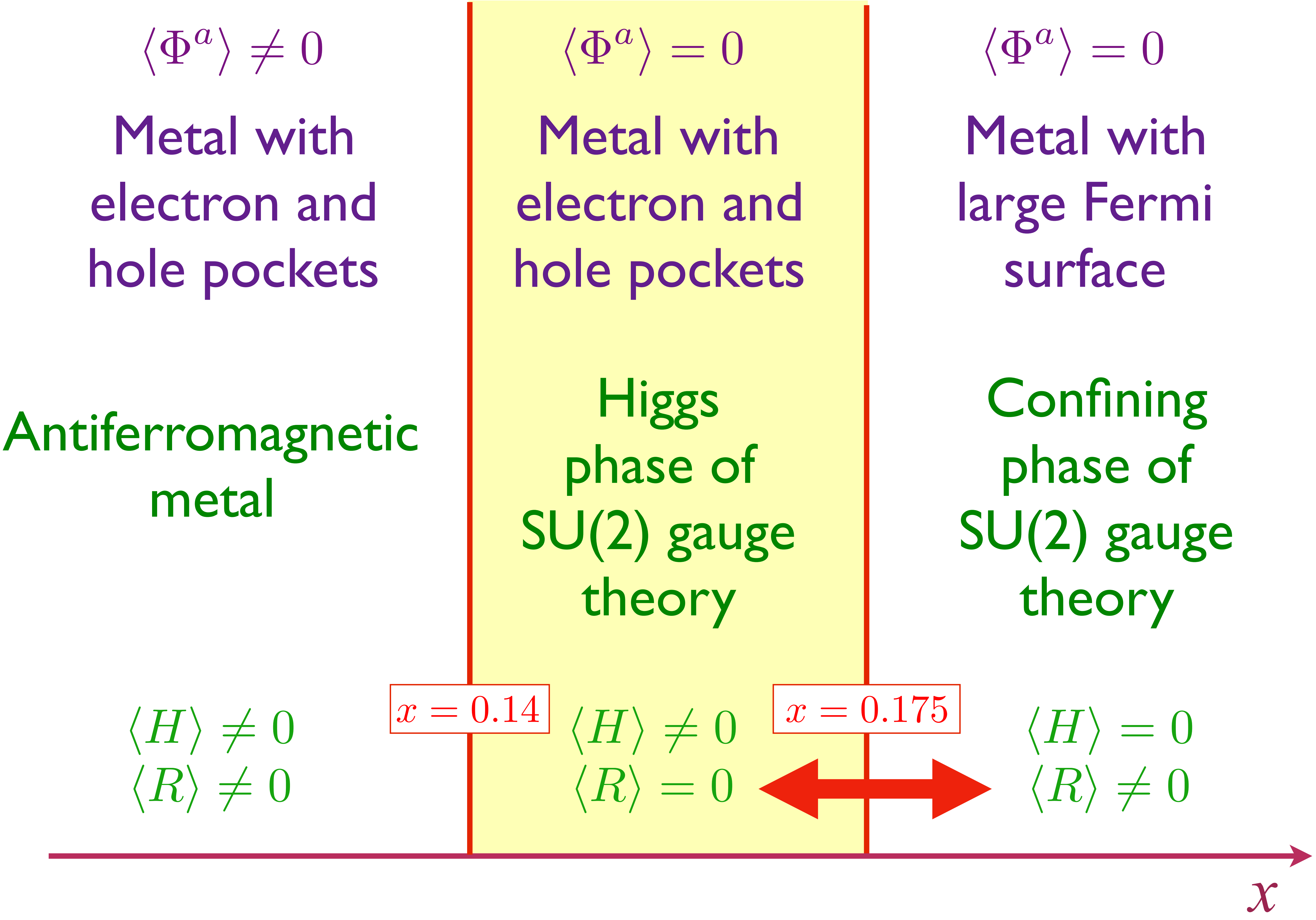






SU(2) gauge theory of fluctuating SDW
compared with experiments on NCCO,
and with numerics on the Hubbard model

$\Phi^a \Rightarrow$ Antiferromagnetism



$\langle \Phi^a \rangle \neq 0$	$\langle \Phi^a \rangle = 0$	$\langle \Phi^a \rangle = 0$
Metal with electron and hole pockets	Metal with electron and hole pockets	Metal with large Fermi surface
Antiferromagnetic metal	Higgs phase of SU(2) gauge theory	Confining phase of SU(2) gauge theory
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Gauge theory of fluctuating SDW

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Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

SU(2) gauge theory for FL*: Assume the Ψ and R form an electron-like bound state, and develop an effective theory for the Higgs fields and the electrons to describe also the crossover/transition to the Fermi liquid state on the overdoped side.

Gauge theory of fluctuating SDW

Taking the continuum limit for the Higgs field:

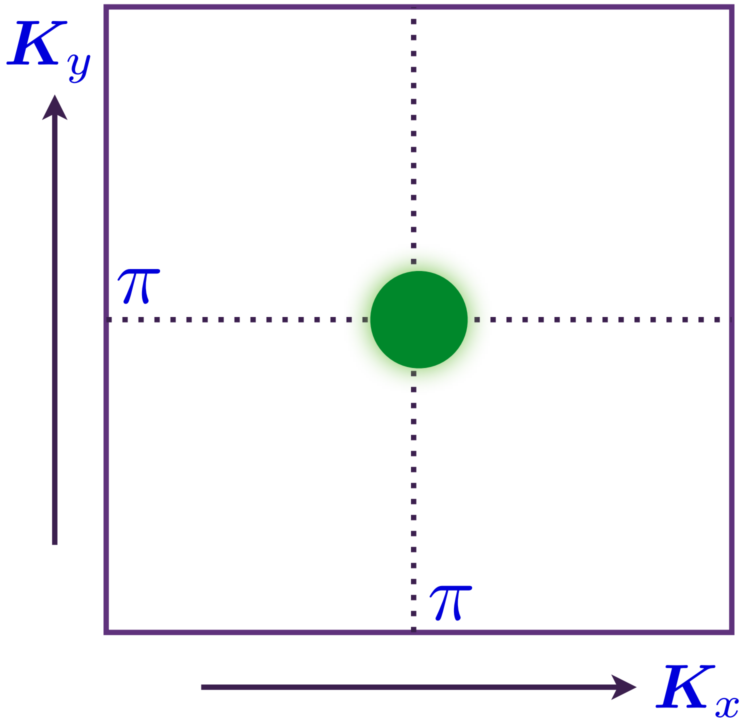
We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (electron doped cuprates):

$N_h = 1$,

$\boldsymbol{K} = (\pi, \pi)$,

$H^a(i) = H_1^a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$



Optimal doping for electron-doped cuprates

SU(2) gauge theory

SU(2) gauge theory with $N_h = 1$ adjoint real Higgs fields $\mathcal{H}^a$ $(a = 1, 2, 3)$, and gauge-invariant, electron-like fermions c_α with a large Fermi surface.

$$\mathcal{L} \;=\; \frac{1}{2}\left(\partial_\mu \mathcal{H}^a - \epsilon_{abc} A^b_\mu \mathcal{H}^c\right)^2 + \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + V(\mathcal{H}^a)$$

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon_{abc} A^b_\mu A^c_\nu \\ V(H^a) &= s\, H^a H^a + u_0\, H^a H^a H^b H^b \end{aligned}$$

Optimal doping for electron-doped cuprates

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$$\begin{aligned}\mathcal{L} \quad &= \quad \frac{1}{2}\left(\partial_\mu \mathcal{H}^a - \epsilon_{abc} A_\mu^b \mathcal{H}^c\right)^2 + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + V(\mathcal{H}^a) \\ &\quad - \sum_{j,\rho} t_\rho \left(c_{j,\alpha}^\dagger c_{j+\boldsymbol{v}_\rho,\alpha} + c_{j+\boldsymbol{v}_\rho,\alpha}^\dagger c_{j,\alpha}\right) - \mu \sum_j c_{j,\alpha}^\dagger c_{j,\alpha}\end{aligned}$$

$$\begin{aligned}F_{\mu\nu}^a \quad &= \quad \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\ V(H^a) \quad &= \quad s\, H^a H^a + u_0\, H^a H^a H^b H^b\end{aligned}$$

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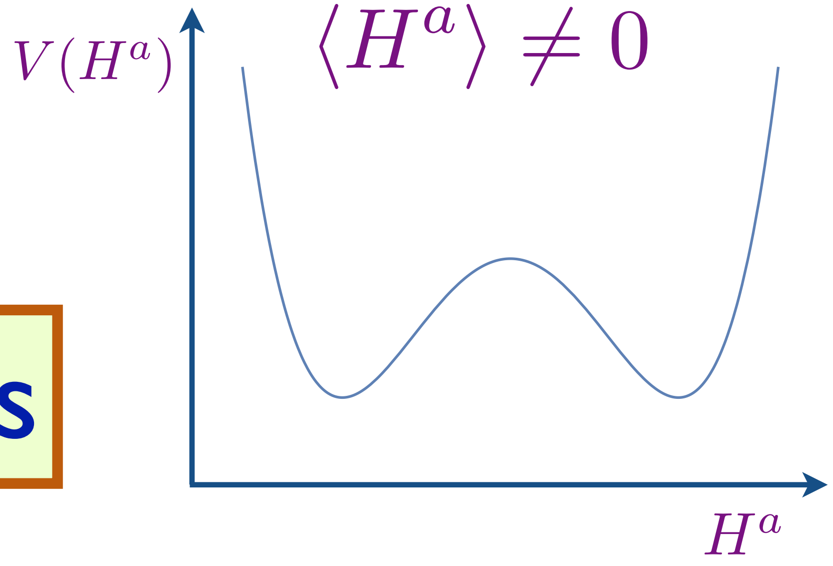
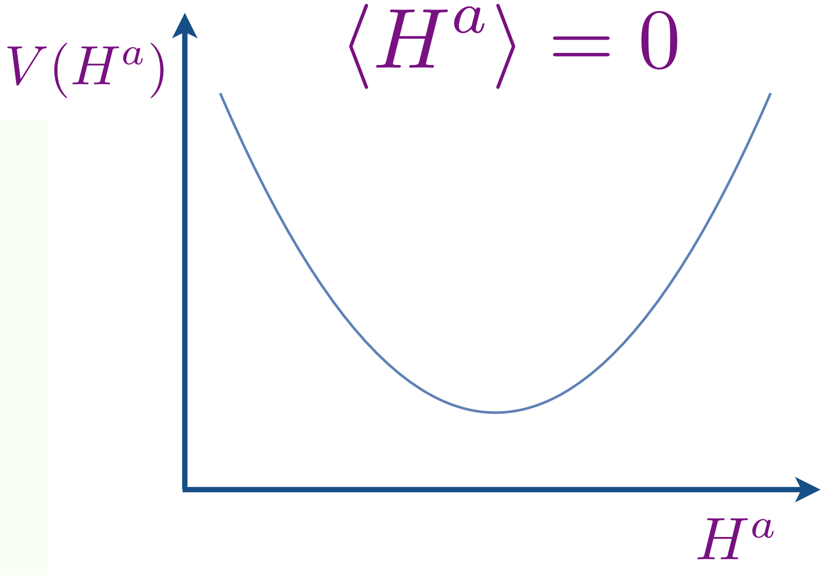
\mathcal{L} &= \frac{1}{2}\left(\partial_\mu \mathcal{H}^a - \epsilon_{abc} A^b_\mu \mathcal{H}^c\right)^2 + \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + V(\mathcal{H}^a) \\
&\quad - \sum_{j,\rho} t_\rho \left(c^\dagger_{j,\alpha} c_{j+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{j+\boldsymbol{v}_\rho,\alpha} c_{j,\alpha} \right) - \mu \sum_j c^\dagger_{j,\alpha} c_{j,\alpha} \\
&\quad + \lambda \sum_j c^\dagger_{j,\alpha} c_{j,\alpha} H^a(j) H^a(j) \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon_{abc} A^b_\mu A^c_\nu \\
V(H^a) &= s\, H^a H^a + u_0\, H^a H^a H^b H^b
\end{aligned}$$

The fermions do not have Yukawa coupling to the Higgs fields, or a minimal coupling to the gauge fields: both are prohibited by gauge invariance.

$N_h = 1$

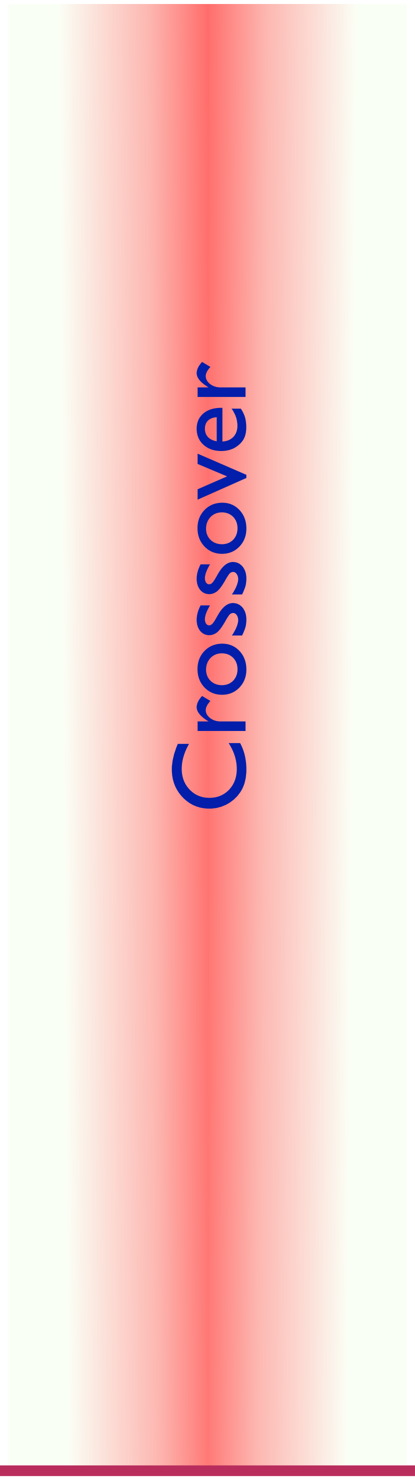
Phase diagrams of SU(2) gauge theory





Higgs

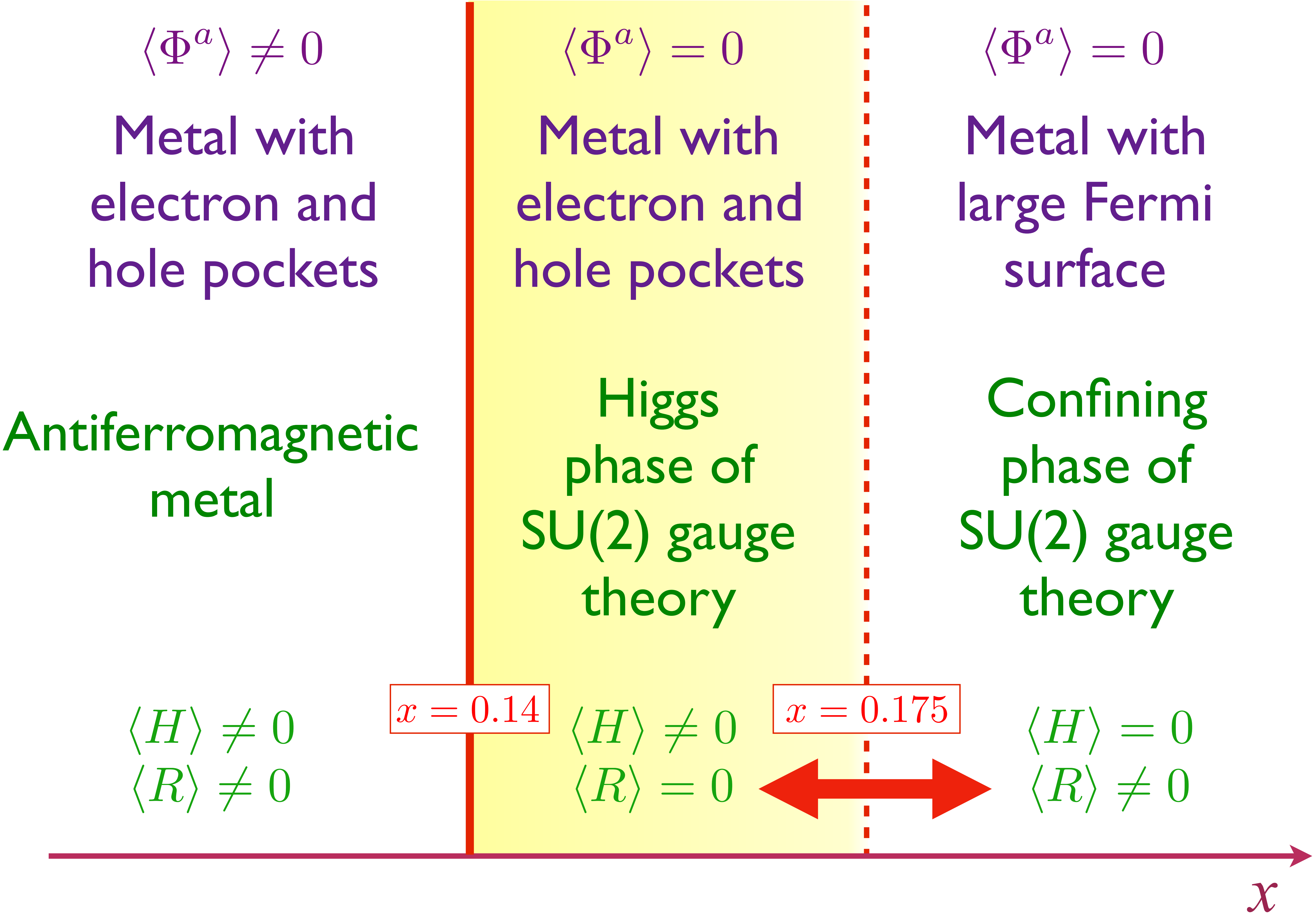
- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles
- Monopole action $\sim \sqrt{-s}$, leading to an exponentially large confinement scale



Confinement

A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

$\Phi^a \Rightarrow$ Antiferromagnetism



$\langle \Phi^a \rangle \neq 0$

Metal with electron and hole pockets

Antiferromagnetic metal

$\langle H \rangle \neq 0$
$\langle R \rangle \neq 0$

$x = 0.14$

$\langle \Phi^a \rangle = 0$

Metal with electron and hole pockets

Higgs phase of SU(2) gauge theory

$\langle H \rangle \neq 0$
$\langle R \rangle = 0$

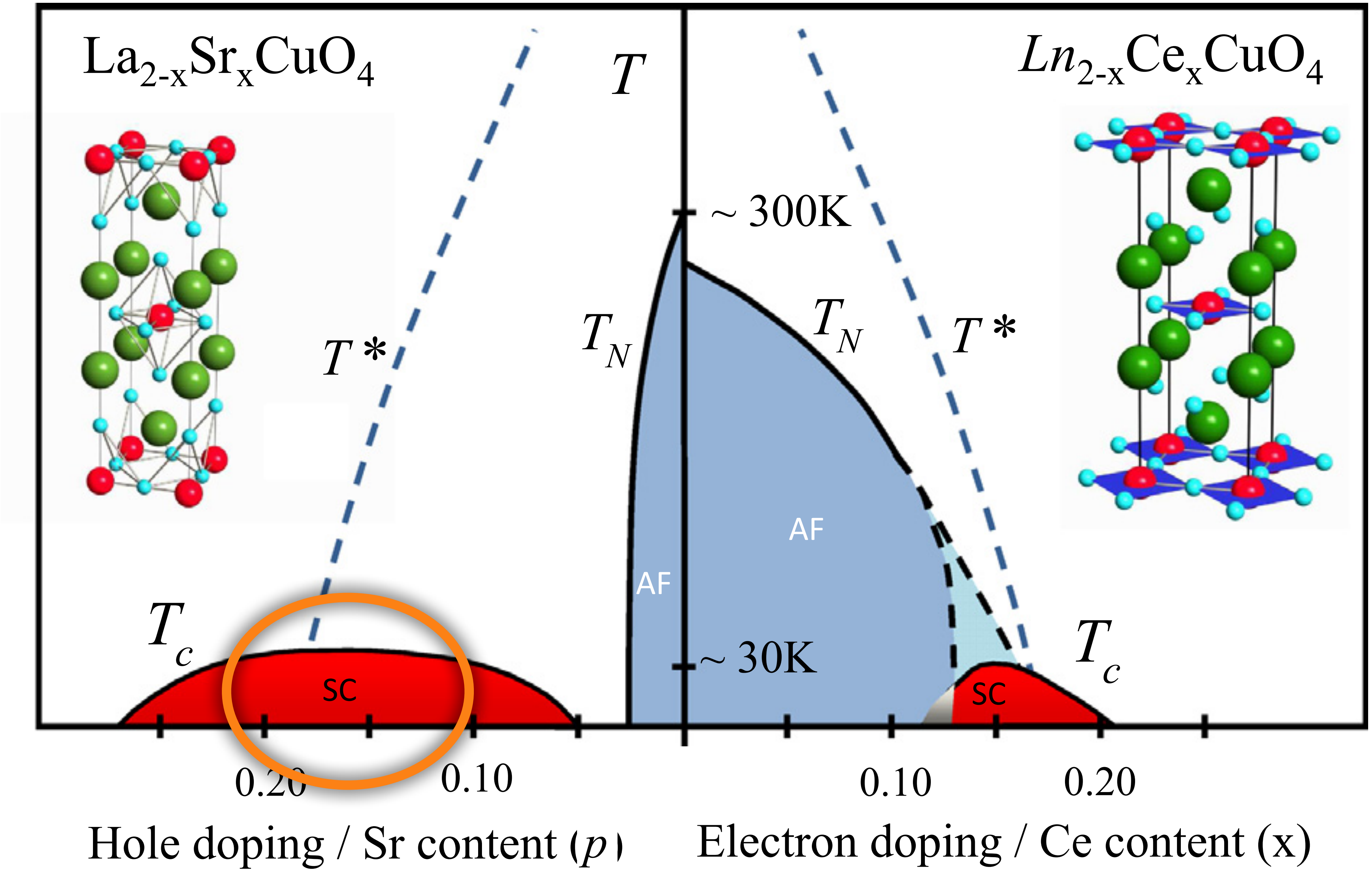
$x = 0.175$

$\langle \Phi^a \rangle = 0$

Metal with large Fermi surface

Confining phase of SU(2) gauge theory

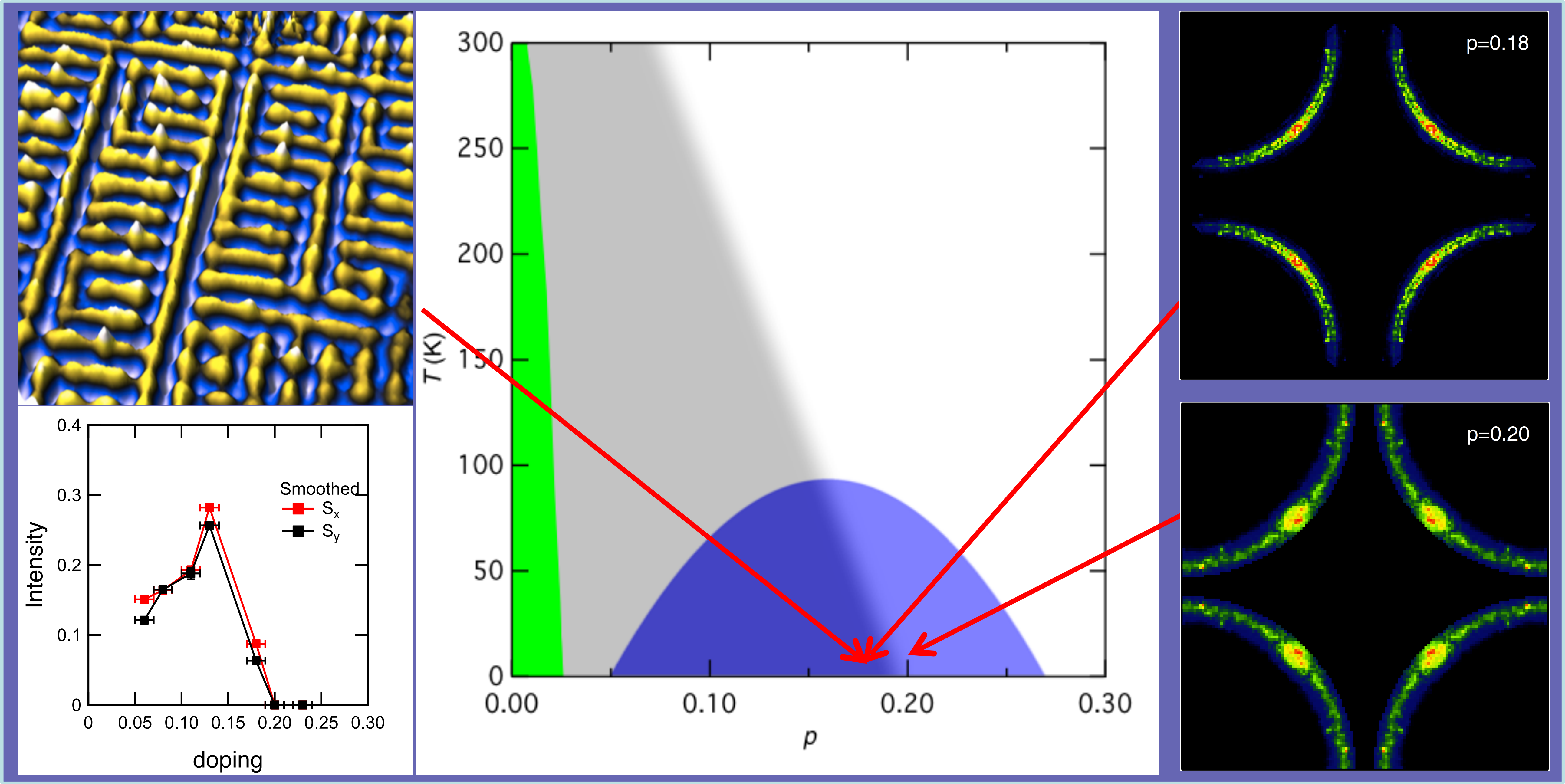
$\langle H \rangle = 0$
$\langle R \rangle \neq 0$


La2-xSrxCuO4
T
Ln2-xCexCuO4
~ 300K
T*
TN
TN
T*
AF
AF
Tc
SC
~ 30K
SC
Tc
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

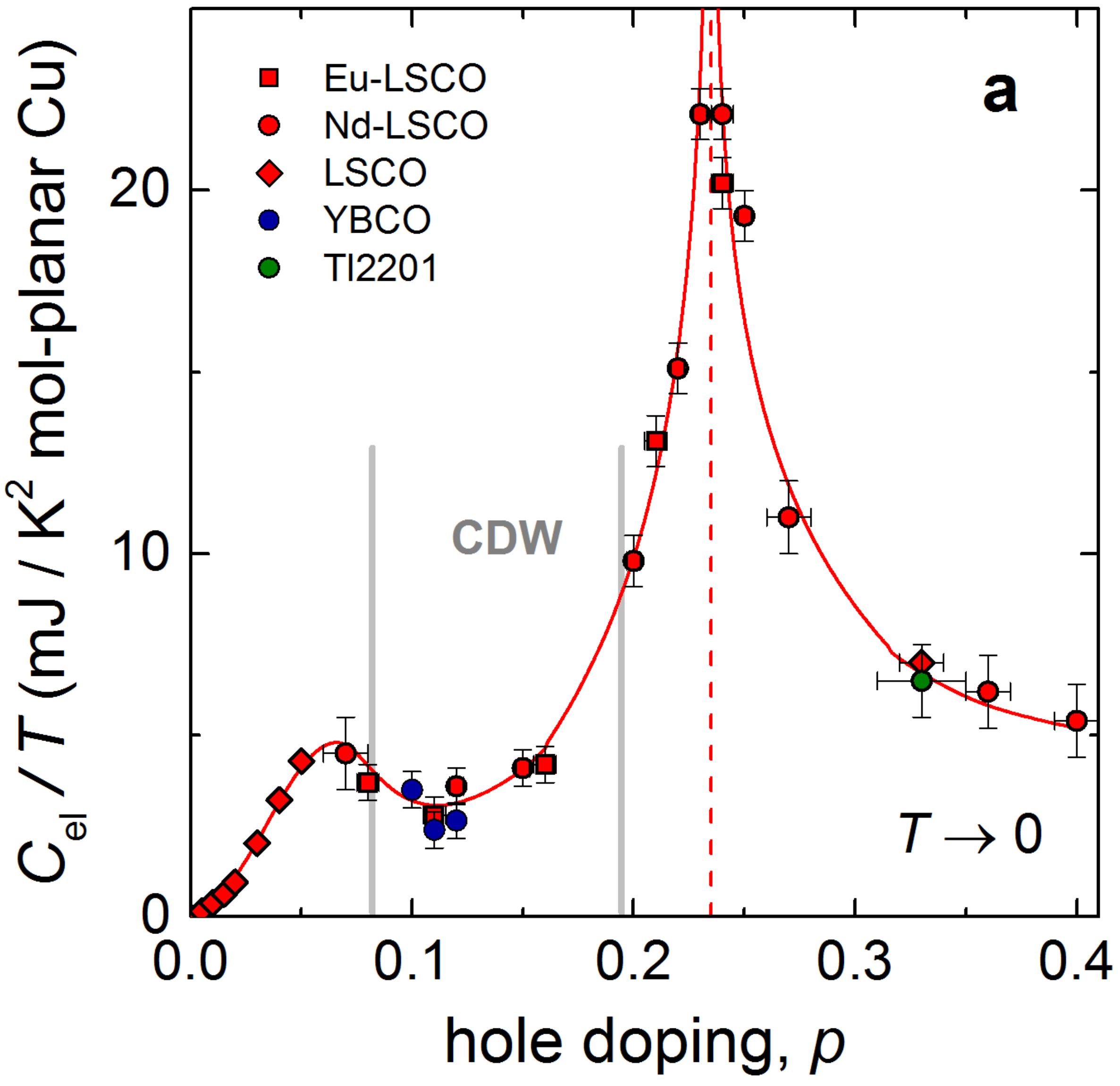
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)

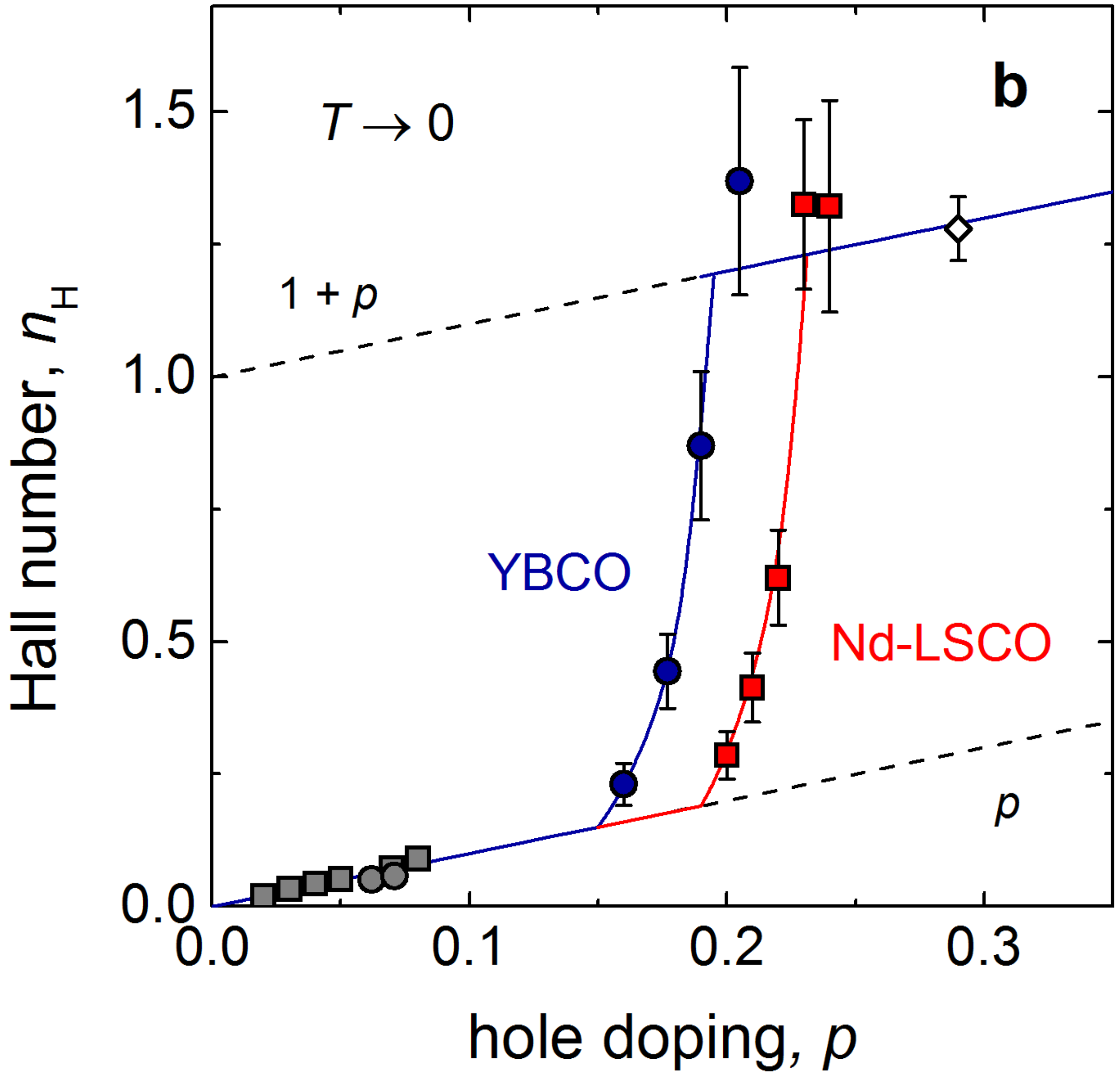


Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507





Gauge theory of fluctuating SDW

Taking the continuum limit for the Higgs field:

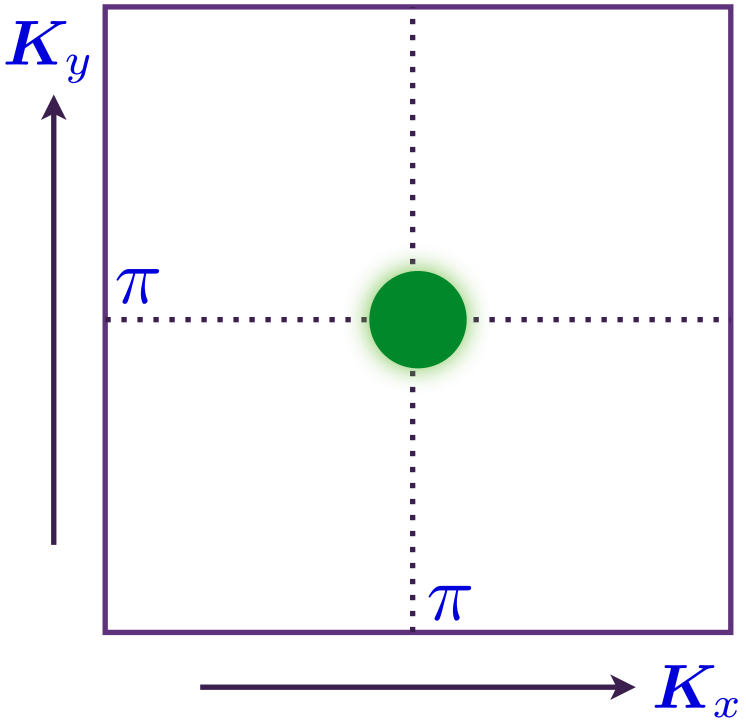
We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (electron doped cuprates):

$N_h = 1$,

$\boldsymbol{K} = (\pi, \pi)$,

$H^a(i) = H_1^a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$



Gauge theory of fluctuating SDW

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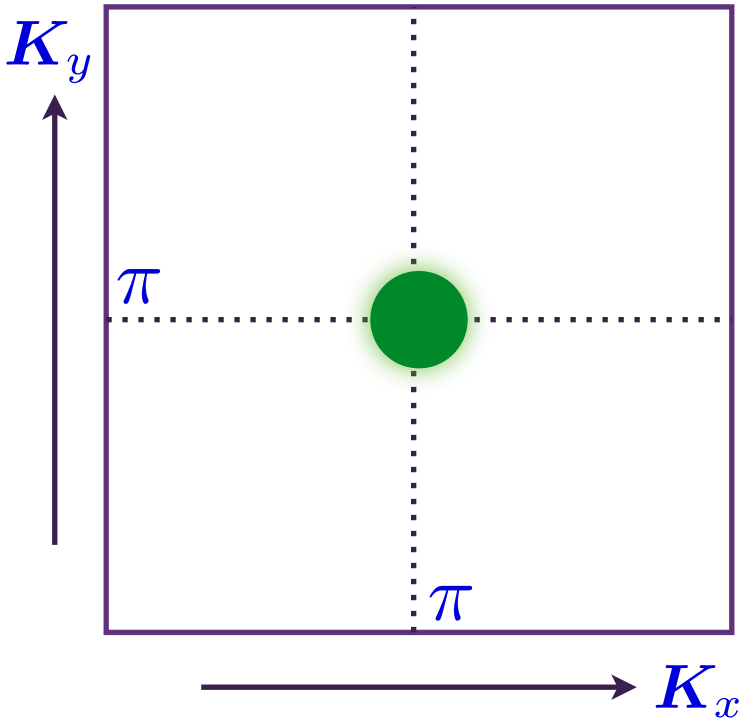
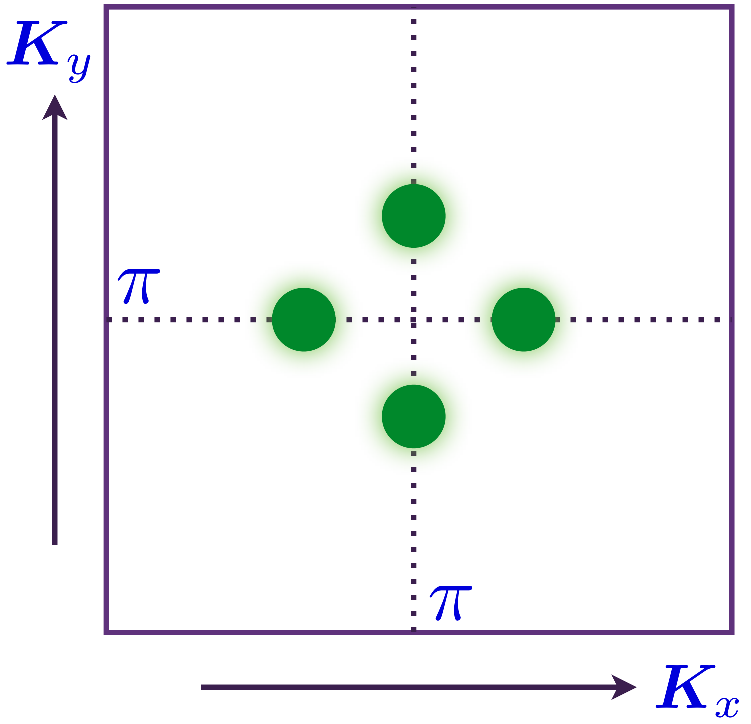




Bidirectional incommensurate correlations (hole doped cuprates):

$N_h = 4$,

$\boldsymbol{K}_y = (\pi, \pi - \delta)$, $\boldsymbol{K}_x = (\pi - \delta, \pi)$,

$$H^a(i) = \mathrm{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H_3^a(\boldsymbol{r}) + iH_4^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right\}$$

Optimal doping for hole-doped cuprates

SU(2) gauge theory

For the hole-doped cuprates, $N_h = 4$, we define complex Higgs fields

$$\mathcal{H}_x^a = H_1^a + i H_2^a \quad , \quad \mathcal{H}_y^a = H_3^a + i H_4^a \, .$$

The SU(2) gauge theory is

$$\mathcal{L} = \frac{1}{2} \left| \partial_\mu \mathcal{H}_x^a - \epsilon_{abc} A_\mu^b \mathcal{H}_x^c \right|^2 + \frac{1}{2} \left| \partial_\mu \mathcal{H}_y^a - \epsilon_{abc} A_\mu^b \mathcal{H}_y^c \right|^2 + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a$$
$$+ V(\mathcal{H}_{x,y}^a) + \text{coupling to electrons with large Fermi surface}$$

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\ V(\mathcal{H}_{x,y}^a) &= s \left(\mathcal{H}_x^{a*} \mathcal{H}_x^a + \mathcal{H}_y^{a*} \mathcal{H}_y^a \right) + u_0 \left(\mathcal{H}_x^{a*} \mathcal{H}_x^a + \mathcal{H}_y^{a*} \mathcal{H}_y^a \right)^2 \\ & \frac{u_1}{4} \left(\mathcal{H}_x^{a*} \mathcal{H}_x^a - \mathcal{H}_y^{a*} \mathcal{H}_y^a \right)^2 + \frac{u_2}{2} \left[\left| \mathcal{H}_x^a \mathcal{H}_x^a \right|^2 + \left| \mathcal{H}_y^a \mathcal{H}_y^a \right|^2 \right] \\ & + u_3 \left(\left| \mathcal{H}_x^a \mathcal{H}_y^a \right|^2 + \left| \mathcal{H}_x^a \mathcal{H}_y^{a*} \right|^2 \right) \, . \end{aligned}$$

Optimal doping for hole-doped cuprates

There are multiple gauge-invariant order parameters for broken symmetries
(Note: there is no SDW order in the Higgs phase)

- Ising nematic order

$$\phi = \mathcal{H}_x^{a*}\mathcal{H}_x^a - \mathcal{H}_y^{a*}\mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $2\boldsymbol{K}_{x,y}$

$$\Phi_x = \mathcal{H}_x^a\mathcal{H}_x^a \quad , \quad \Phi_y = \mathcal{H}_y^a\mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $\boldsymbol{K}_x \pm \boldsymbol{K}_y$

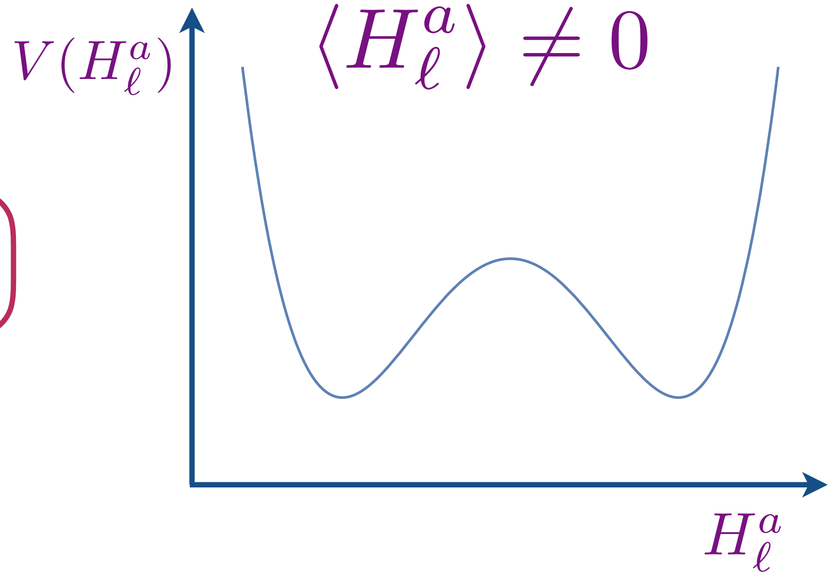
$$\Phi_+ = \mathcal{H}_x^a\mathcal{H}_y^a \quad , \quad \Phi_- = \mathcal{H}_x^a\mathcal{H}_y^{a*}$$

- (Modulated) scalar spin chirality

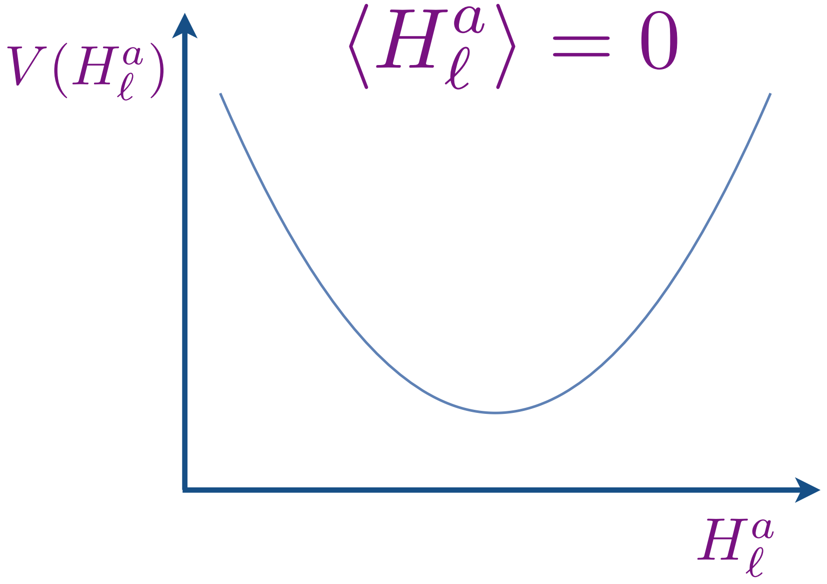
$$\chi_{ijk} = \epsilon_{abc} H^a(\boldsymbol{r}_i) H^b(\boldsymbol{r}_j) H^c(\boldsymbol{r}_k)$$

Phase diagram of SU(2) gauge theory for hole-doped cuprates

$\boxed{N_h = 4}$



$\langle H_\ell^a \rangle \neq 0$



$\langle H_\ell^a \rangle = 0$

Higgs
U(1) confinement
or Z_2 deconfined

One or more of Ising-nematic, CDW, scalar spin chirality, and $\mathbb{Z}_2$ topological orders

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in the U(1) cases

Possible Deconfined critical SU(2) gauge theory

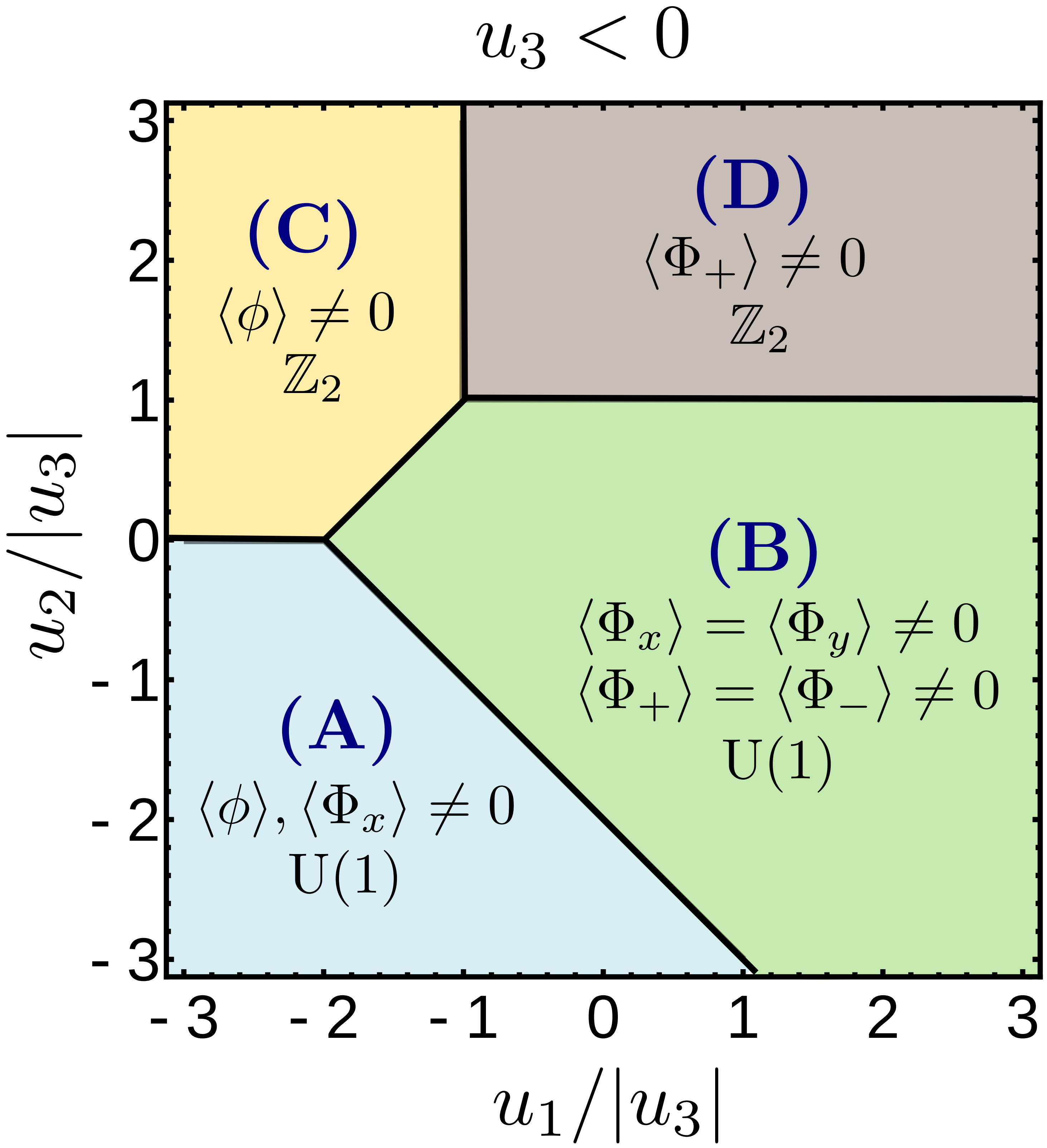
Confinement

Fermi liquid with large Fermi surface

S

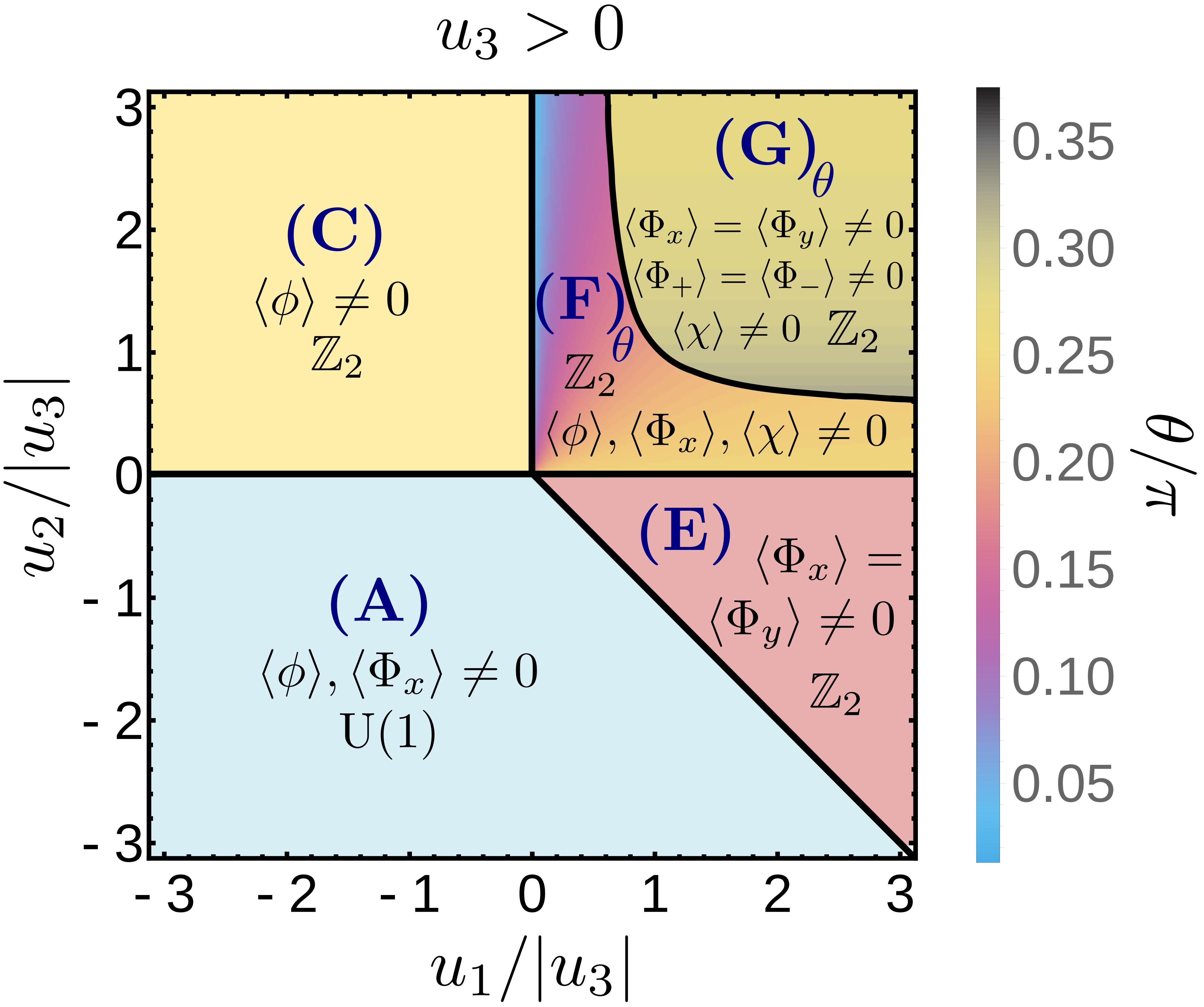
Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



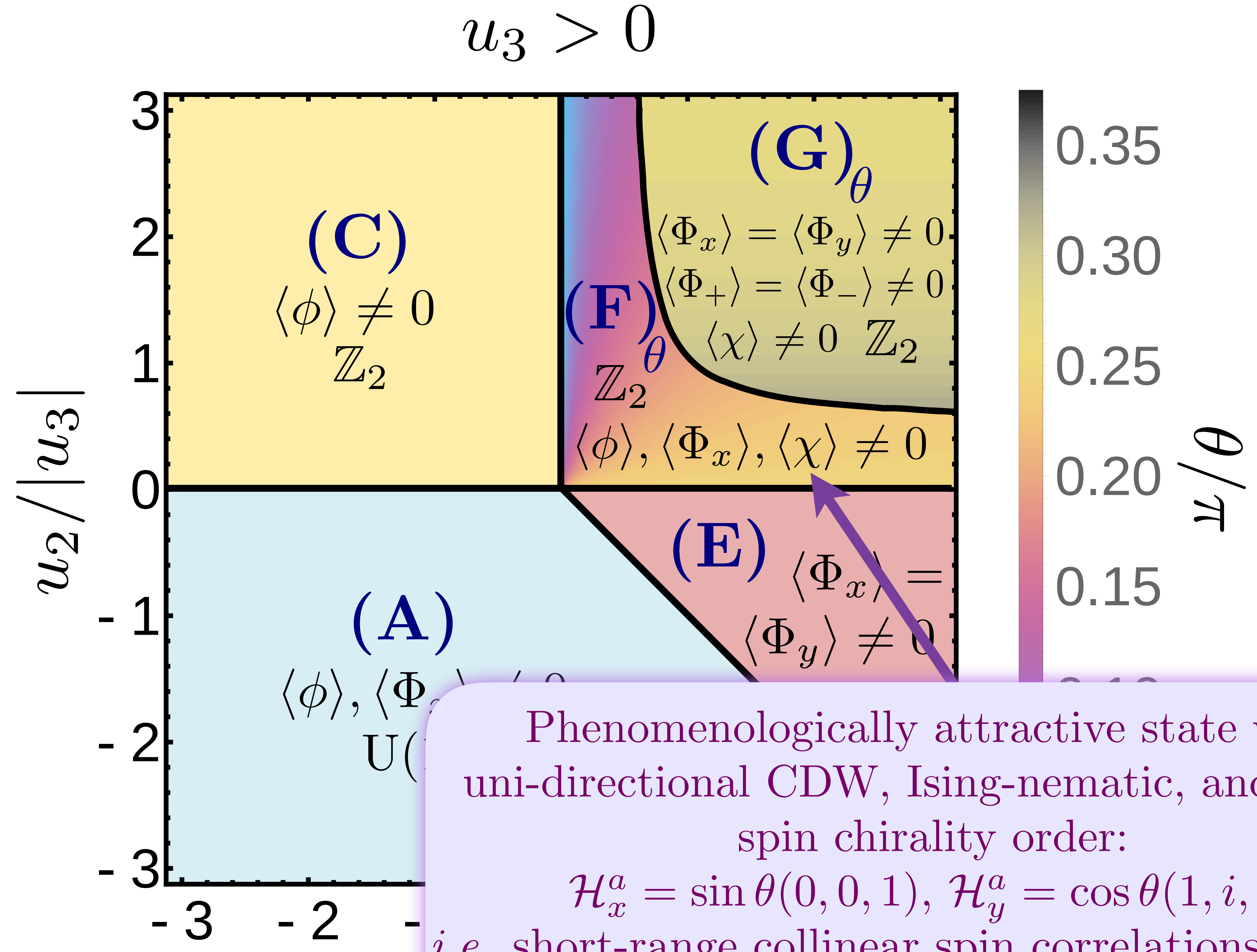
Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase

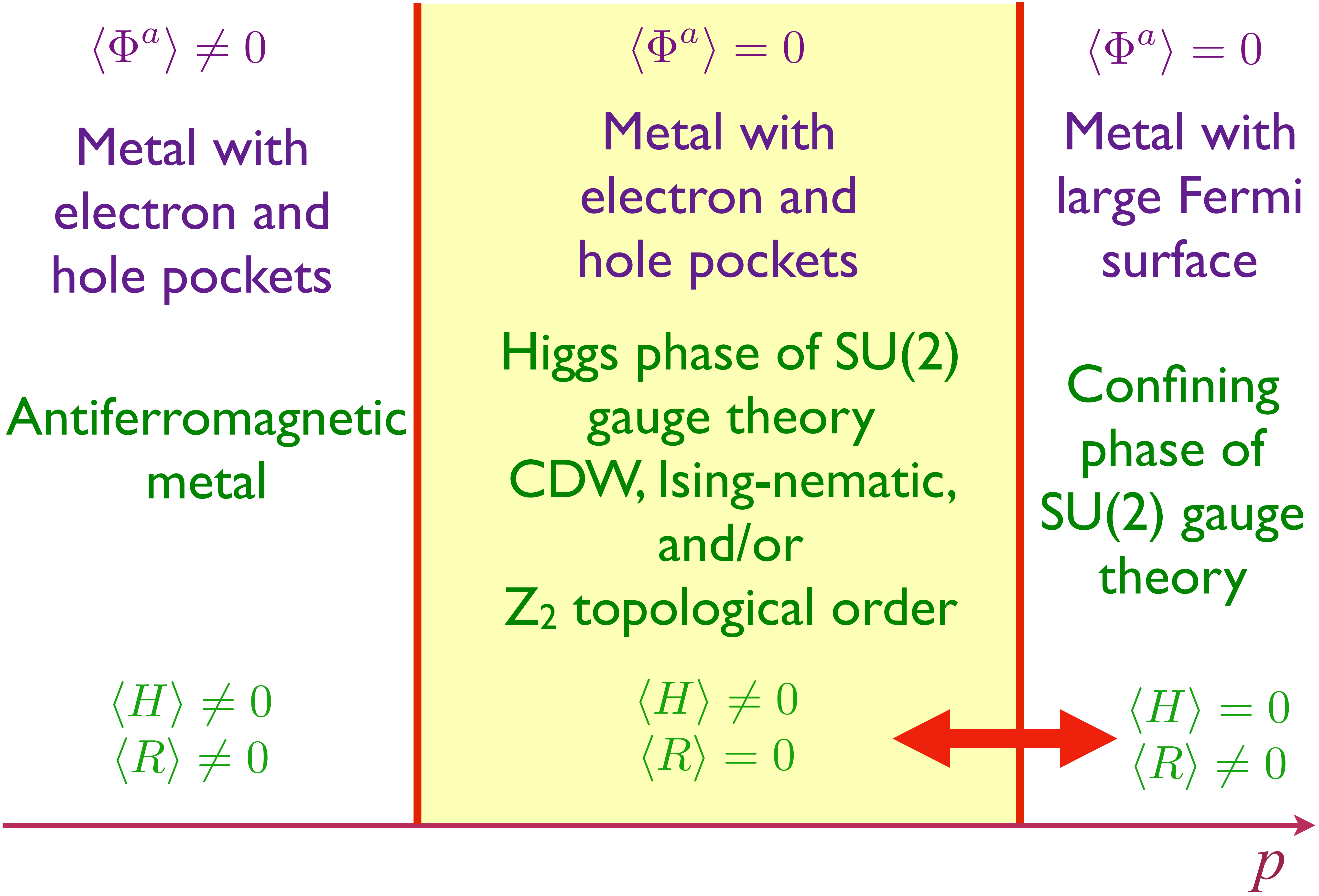


Phenomenologically attractive state with uni-directional CDW, Ising-nematic, and scalar spin chirality order:

$\mathcal{H}^a_x = \sin\theta(0,0,1)$, $\mathcal{H}^a_y = \cos\theta(1,i,0)$

i.e. short-range collinear spin correlations along x, and short-range spiral spin correlations along y.

$\Phi^a \Rightarrow$ Incommensurate antiferromagnetism



$\langle \Phi^a \rangle \neq 0$

Metal with electron and hole pockets

Antiferromagnetic metal

$\langle H \rangle \neq 0$
$\langle R \rangle \neq 0$

$\langle \Phi^a \rangle = 0$

Metal with electron and hole pockets

Higgs phase of SU(2) gauge theory
CDW, Ising-nematic, and/or
Z_2 topological order

$\langle H \rangle \neq 0$
$\langle R \rangle = 0$

$\langle \Phi^a \rangle = 0$

Metal with large Fermi surface

Confining phase of SU(2) gauge theory

$\langle H \rangle = 0$
$\langle R \rangle \neq 0$

1. Planckian metals

2. Gauge theory for the cuprates near optimal doping

3. Bilocal quantum criticality

The path integral for the SYK model is a bilocal field theory

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1)\mathcal{D}\Sigma(\tau_1, \tau_2)e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. The saddle point, $G_s(\tau_1 - \tau_2)$, $\Sigma_s(\tau_1 - \tau_2)$, depends only on time differences, and obeys Planckian ω/T scaling. The fluctuations

$$\begin{aligned} G(\tau_1, \tau_2) &= G_s(\tau_1 - \tau_2) + \delta G(\tau_1, \tau_2) \\ \Sigma(\tau_1, \tau_2) &= \Sigma_s(\tau_1 - \tau_2) + \delta\Sigma(\tau_1, \tau_2) \end{aligned}$$

require bilocal fields.

Similar remarks apply to other random quantum systems, and to DMFT.

Gauge theory of fluctuating SDW coupled to large Fermi surface

SU(2) gauge theory for Higgs field $\mathcal{H}_\ell^a$
$a = 1, 2, 3$ is a SU(2) gauge index
$\ell = 1 \ldots N_h$ is a flavor index.

$$\mathcal{S} \;=\; \int d^2x d\tau \left[\frac{1}{2} \left(\partial_\mu \mathcal{H}_\ell^a - \epsilon_{abc} A_\mu^b \mathcal{H}_\ell^c \right)^2 + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a + V(\mathcal{H}_\ell^a) \right]$$

$+\mathcal{S}_f$–coupling to electrons with large Fermi surface

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$+\mathcal{S}_f$–coupling to electrons with large Fermi surface

$$\mathcal{S}_f = -\frac{1}{2N_h} \int d^2x d\tau d\tau' \mathcal{H}_\ell^a(x,\tau) \mathcal{H}_m^a(x,\tau) J_f(\tau - \tau') \mathcal{H}_\ell^b(x,\tau') \mathcal{H}_m^b(x,\tau')$$

Gauge-invariant order parameters at x, τ couple to gauge invariant order parameters at x, τ' with $J_f(\tau) \sim 1/\tau^2$ via Fermi surface excitations.

This coupling is <u>irrelevant</u> by naive power-counting.

Gauge theory of fluctuating SDW coupled to large Fermi surface

In the large N_h limit, we are required to decouple $\mathcal{S}_f$ by introducing a bilocal field $C_{ab}(x,\tau,\tau')$

$$\mathcal{S}_f = \int d^2x d\tau d\tau' \left[\frac{N_h}{2} \frac{\left[C_{ab}(x,\tau,\tau')\right]^2}{J_f(\tau-\tau')} - C_{ab}(x,\tau,\tau')\mathcal{H}_\ell^a(x,\tau)\mathcal{H}_\ell^b(x,\tau') \right]$$

At the large N_h saddle point, we have $C_{ab}(x,\tau,\tau') = \delta_{ab}C(\tau-\tau')$. Saddle point equations show that C_{ab} displays strong scaling, and $\mathcal{S}_f$ is not irrelevant....

- Resonant SYK models are compressible and dispersive quantum systems with $\hbar\omega/(k_B T)$ scaling as $T \to 0$.

- The resonance is a single ‘fine-tuning’ condition designed to obtain $\hbar\omega/(k_B T)$ scaling as $T \to 0$. However, then many other nice features follow: we obtain a Planckian metal with remnant large Fermi surface at $e_k = 0$, and an effective mass m^* defined by the dispersion of e_k, with a resistivity $\rho \sim (m^*/(ne^2))k_B T/\hbar$ independent of the strength of interactions.

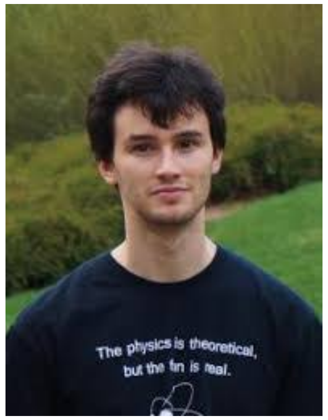
Aavishkar Patel (Harvard ⟶ Miller Fellow at Berkeley)

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

- SU(2) gauge theory of fluctuating SDW order predicts:

  (i) Electron-doped cuprates: a crossover from a Higgs-U(1)-photon regime to a confining Fermi liquid for the electron-doped cuprates.

Mathias Scheurer



Grigory Tarnopolsky

Harley Scammell

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  (ii) Hole-doped cuprates: sharp transition (in the absence of disorder) from a Higgs state with one or more of Ising-nematic, CDW, scalar spin chirality, and $\mathbb{Z}_2$ topological orders, to a confining Fermi liquid (for the case of $\mathbb{Z}_2$ topological order, sharp transition survives in the presence of disorder).


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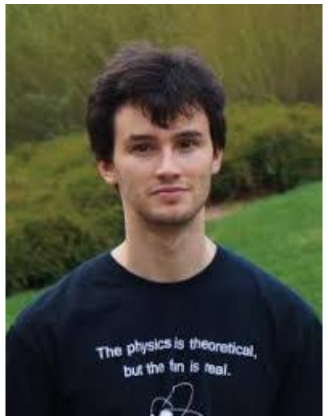

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- SYK model and SU(2) gauge theories (with large Fermi surface) both realize bilocal quantum criticality.


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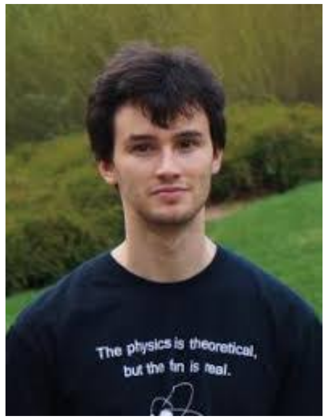

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