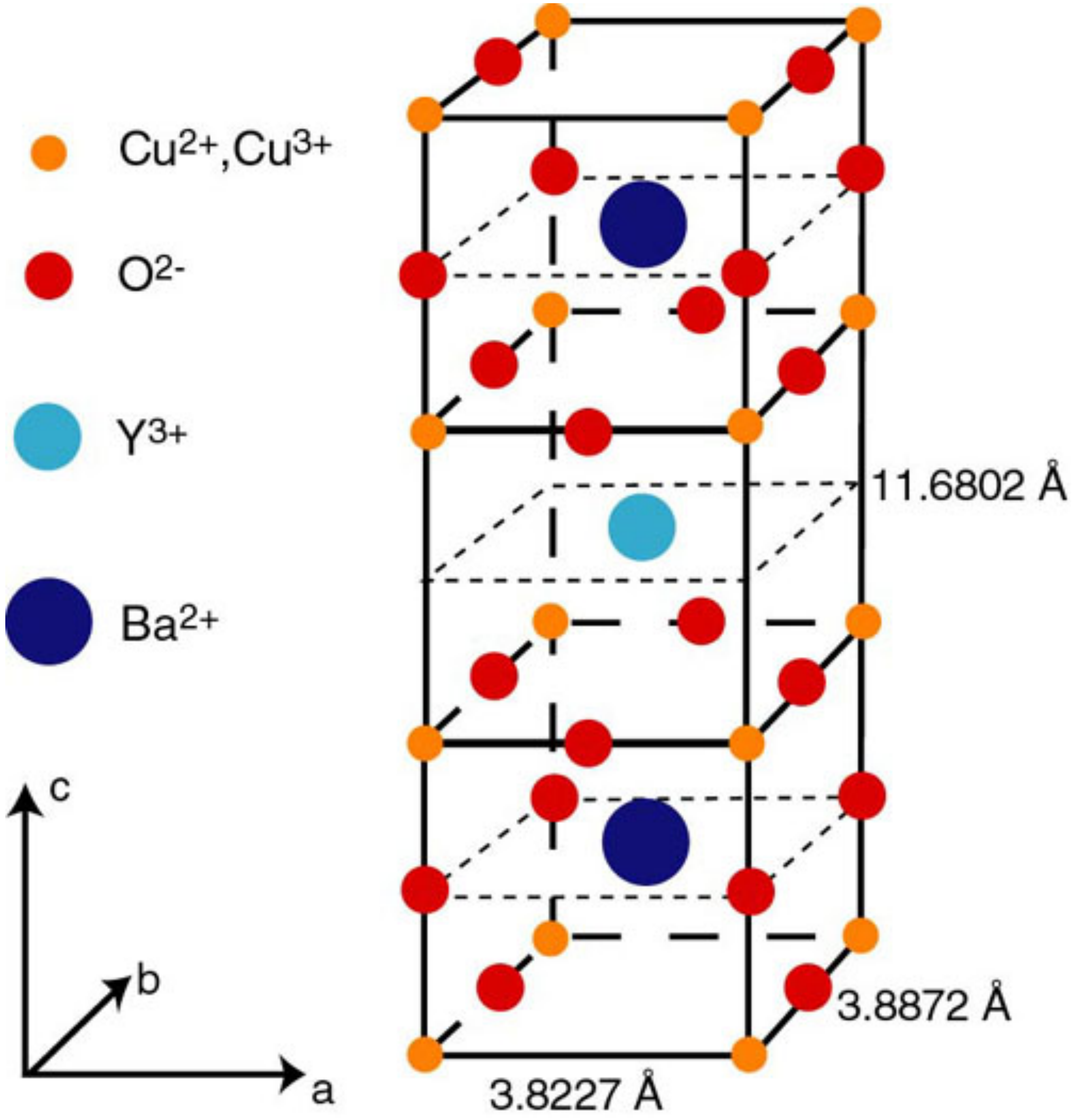
Entanglement in quantum matter: spin liquids and Sachdev-Ye-Kitaev models

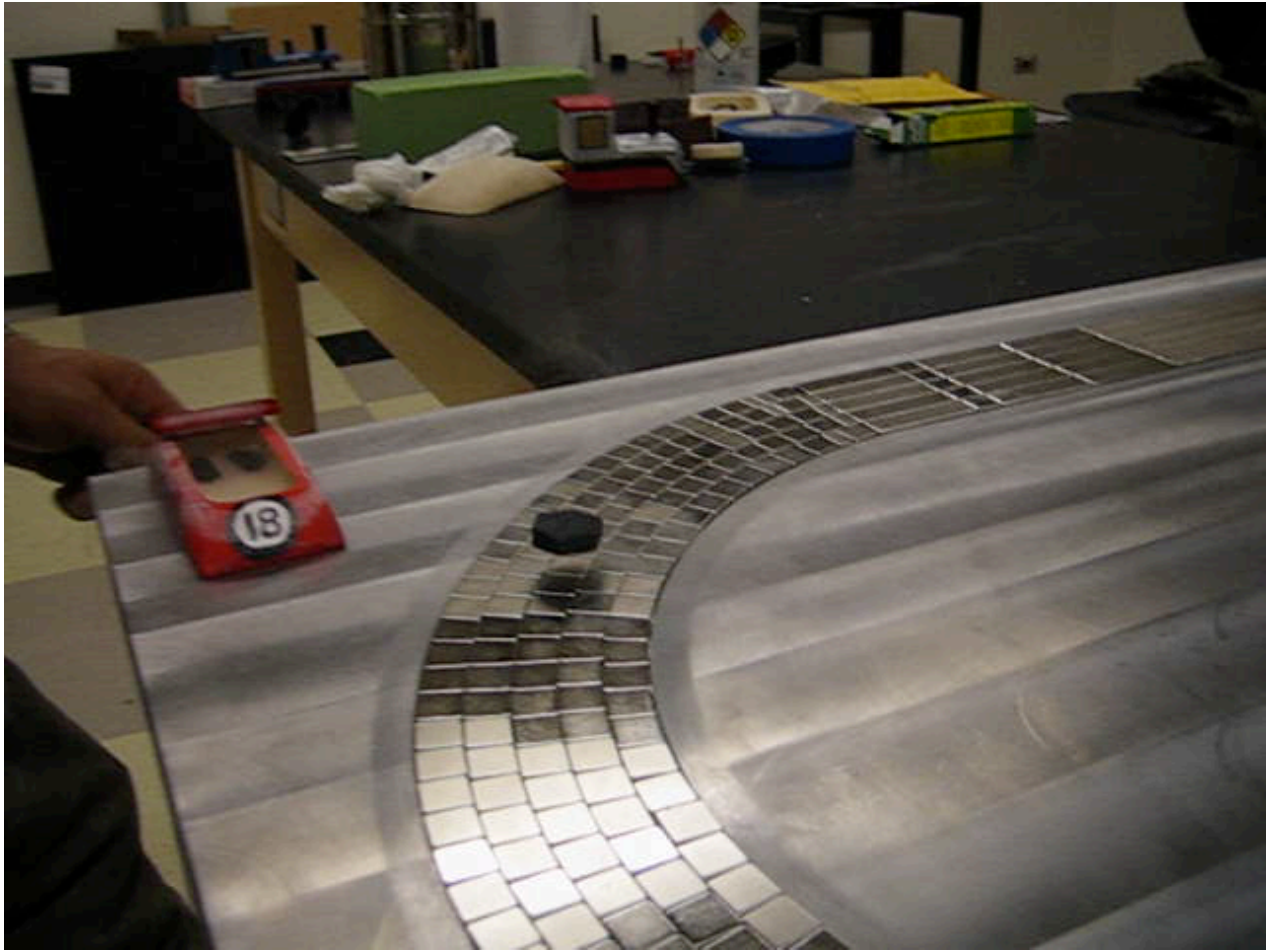
eQMA Distinguished Lecture
Rice University, Houston
February 19, 2025
Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

Cuprate high temperature superconductors



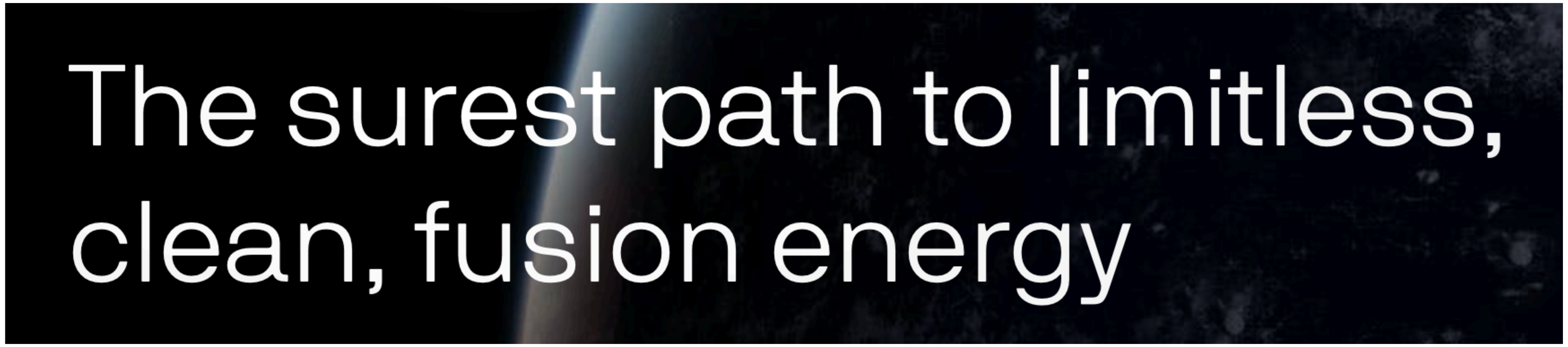
$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

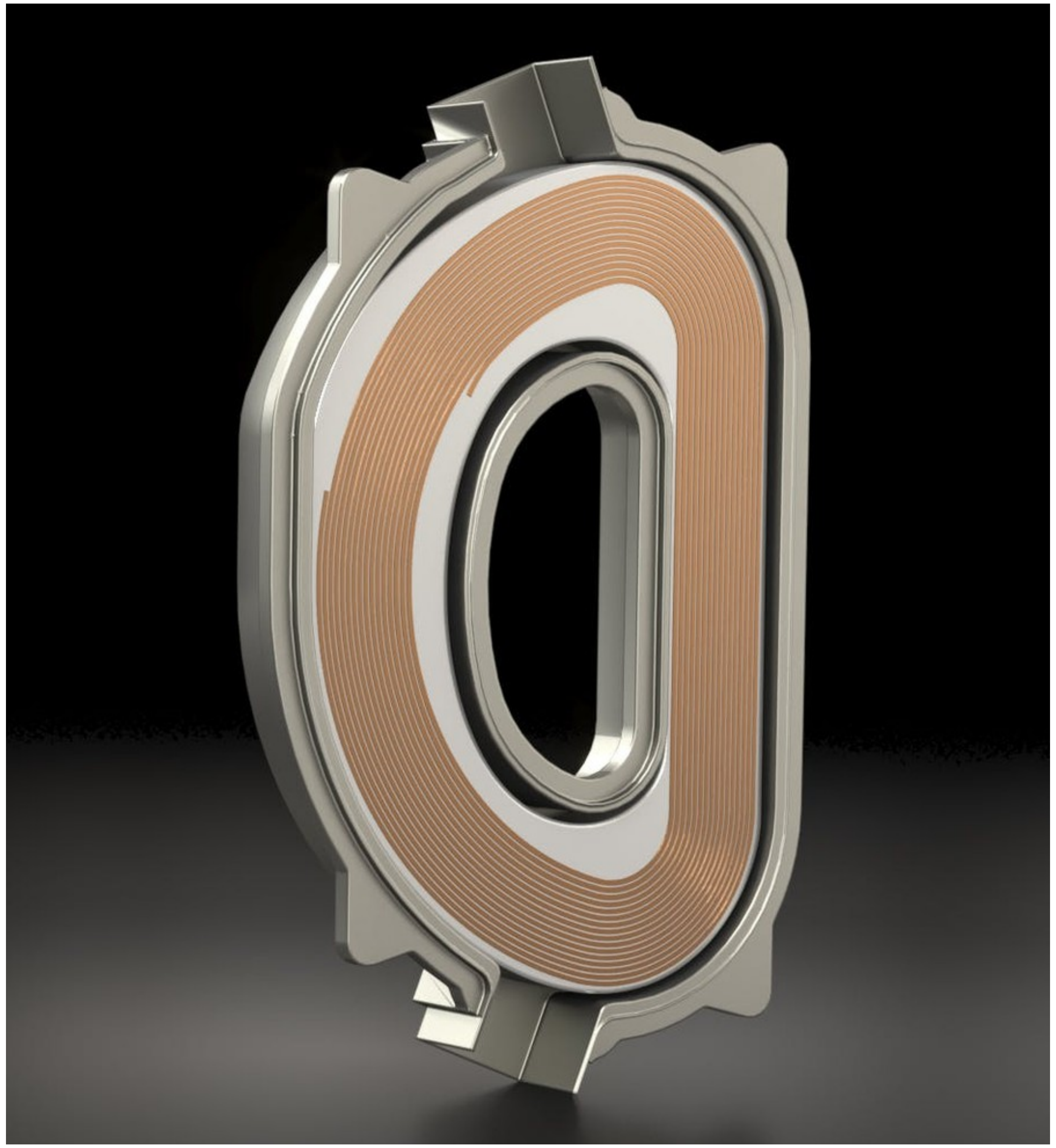
HTS Magnets: Enabling Technology

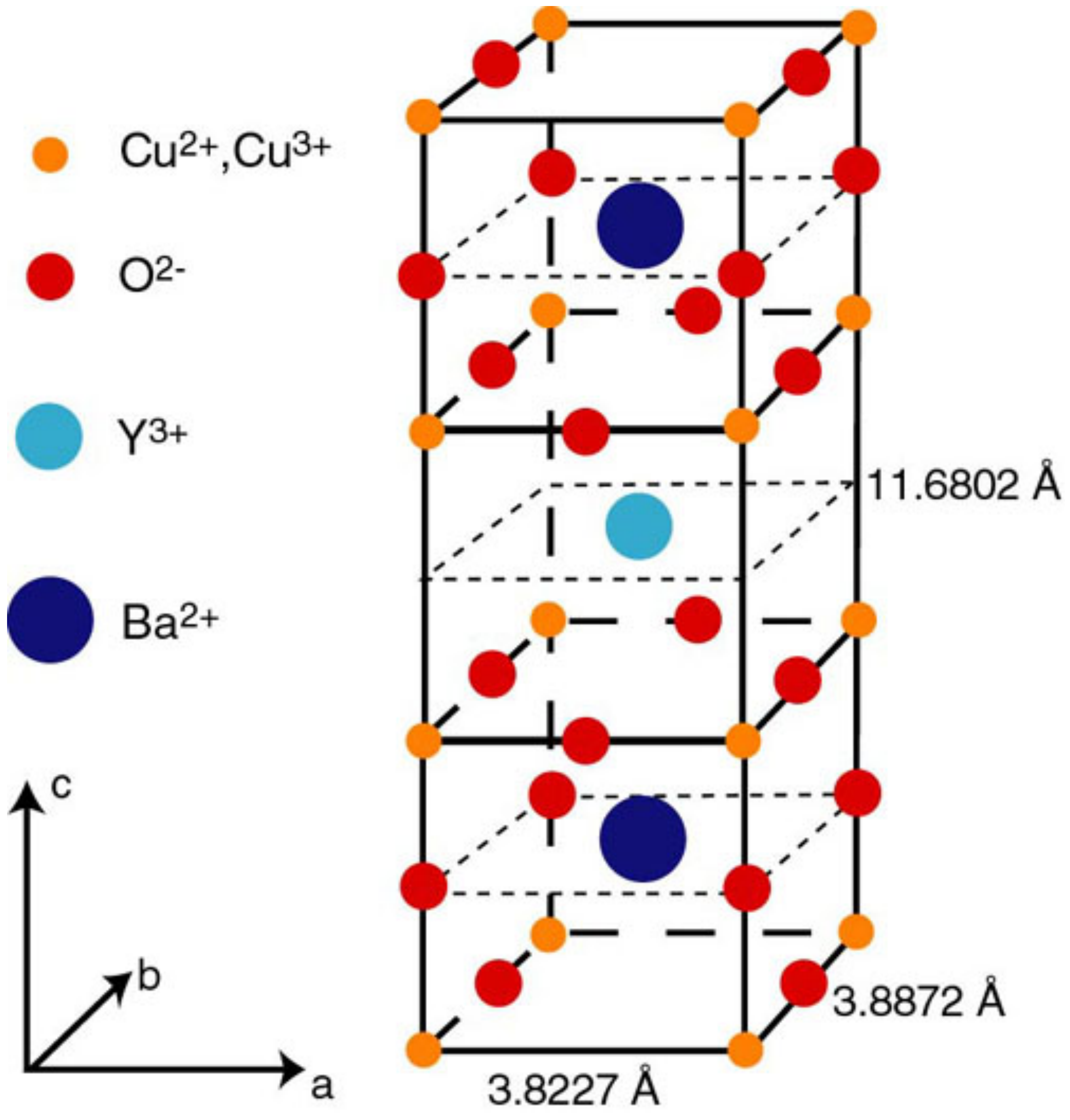


The surest path to limitless, clean, fusion energy

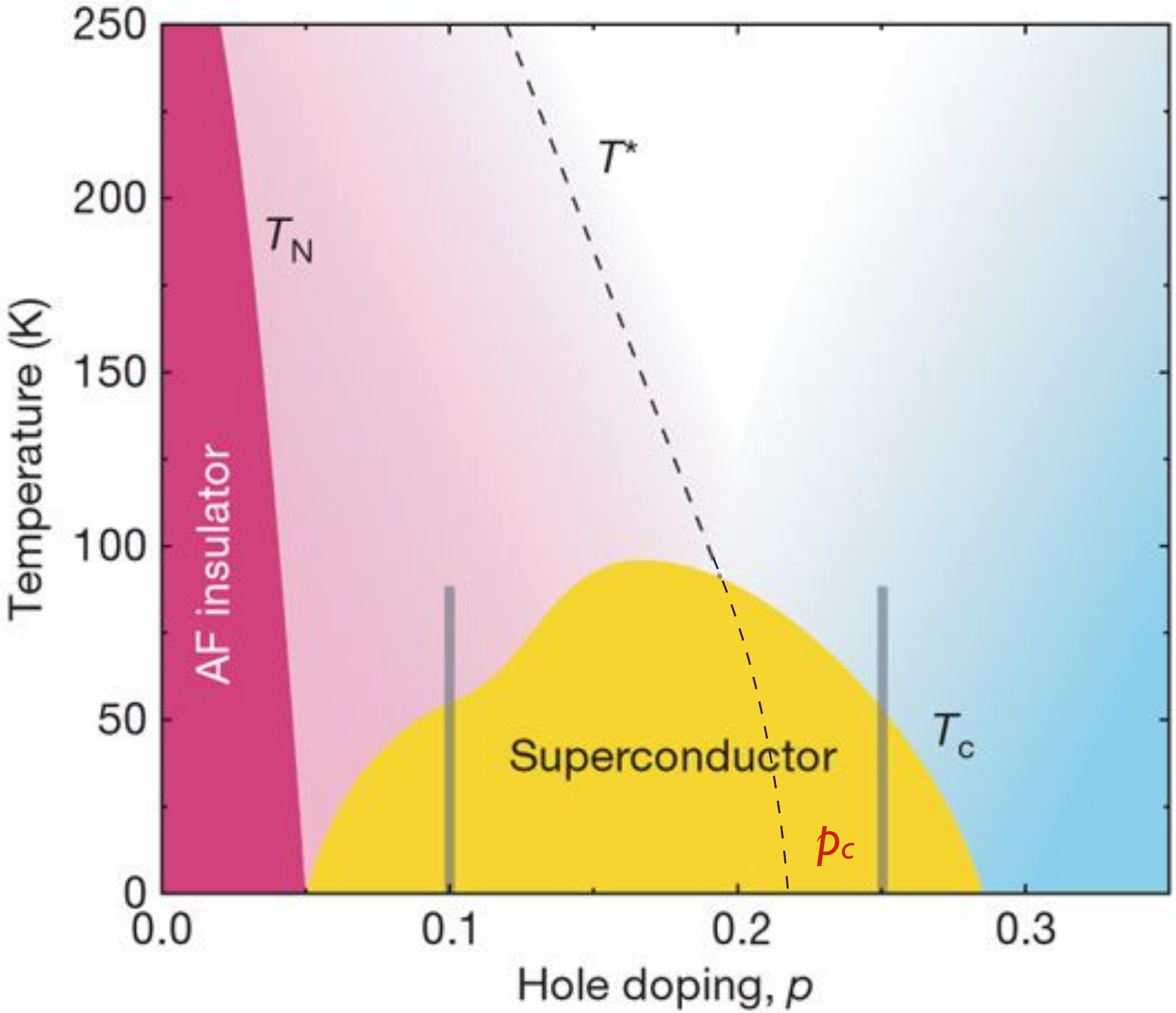
YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

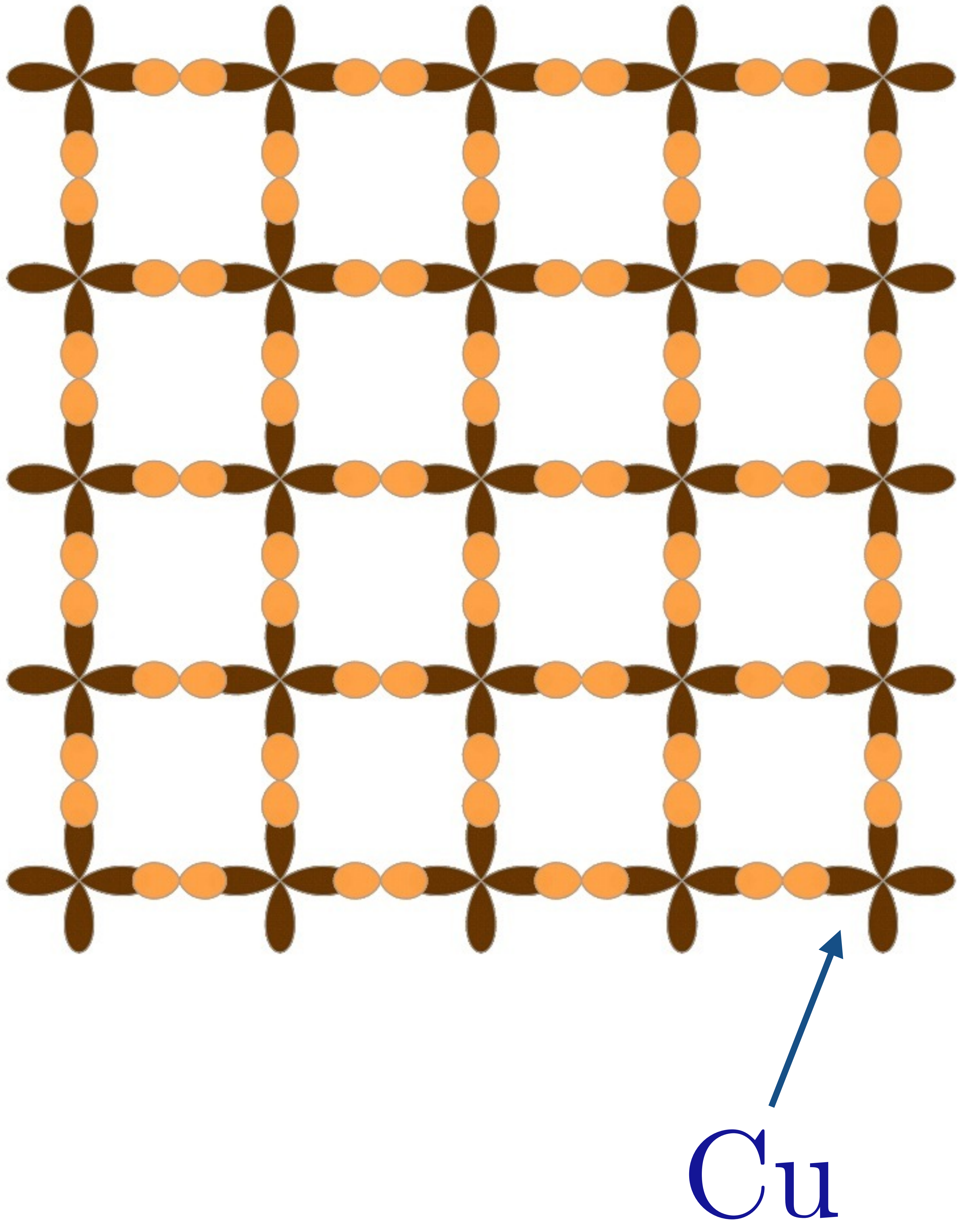
Commonwealth Fusion Systems

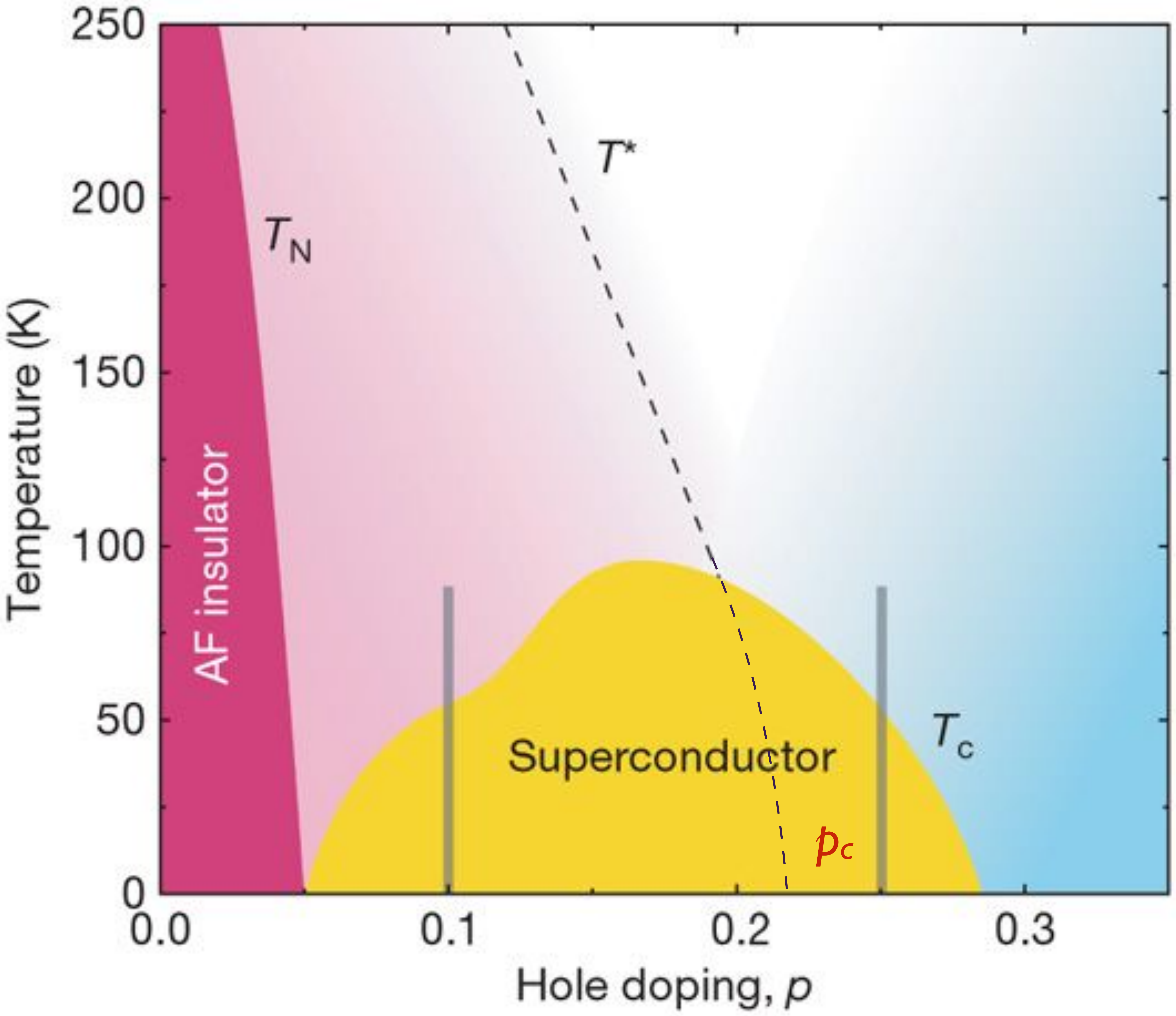




$\mathrm{YBa_2Cu_3O_{6+x}}$




Cu


250
200
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Temperature (K)
0.0
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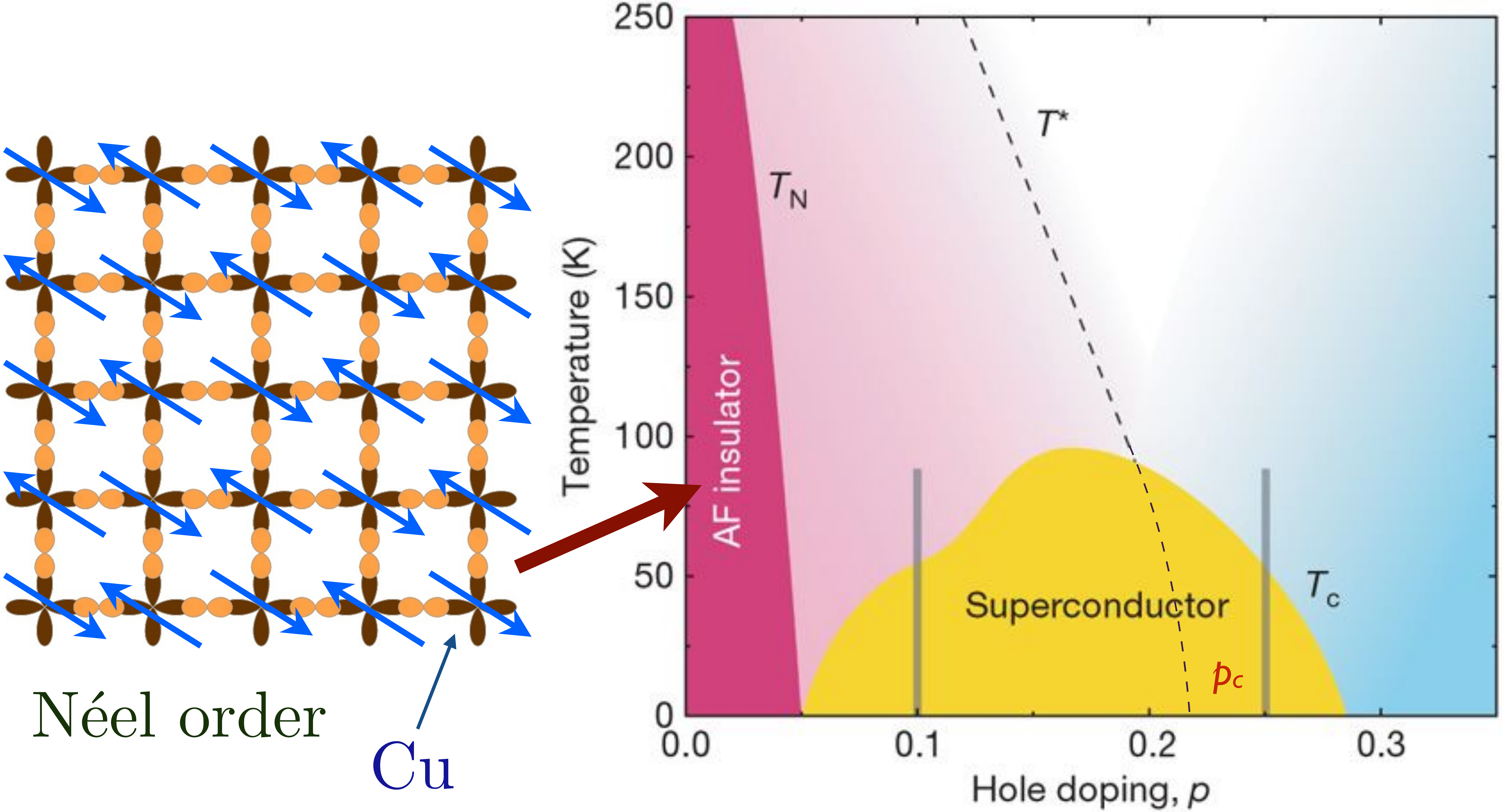
0.2
0.3
Hole doping, p
T_N
T^*
AF insulator
Superconductor
T_c
p_c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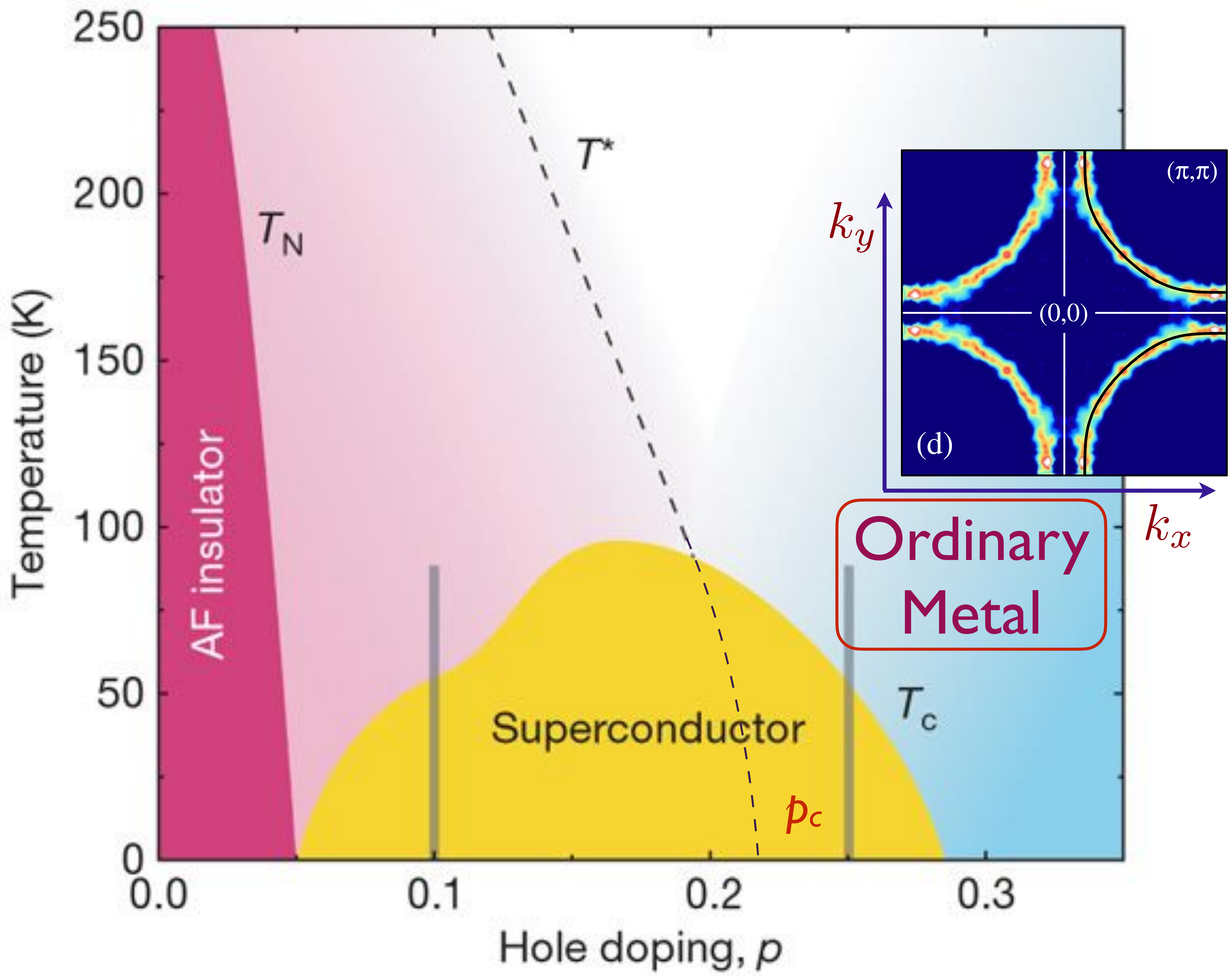
Ordinary metal:

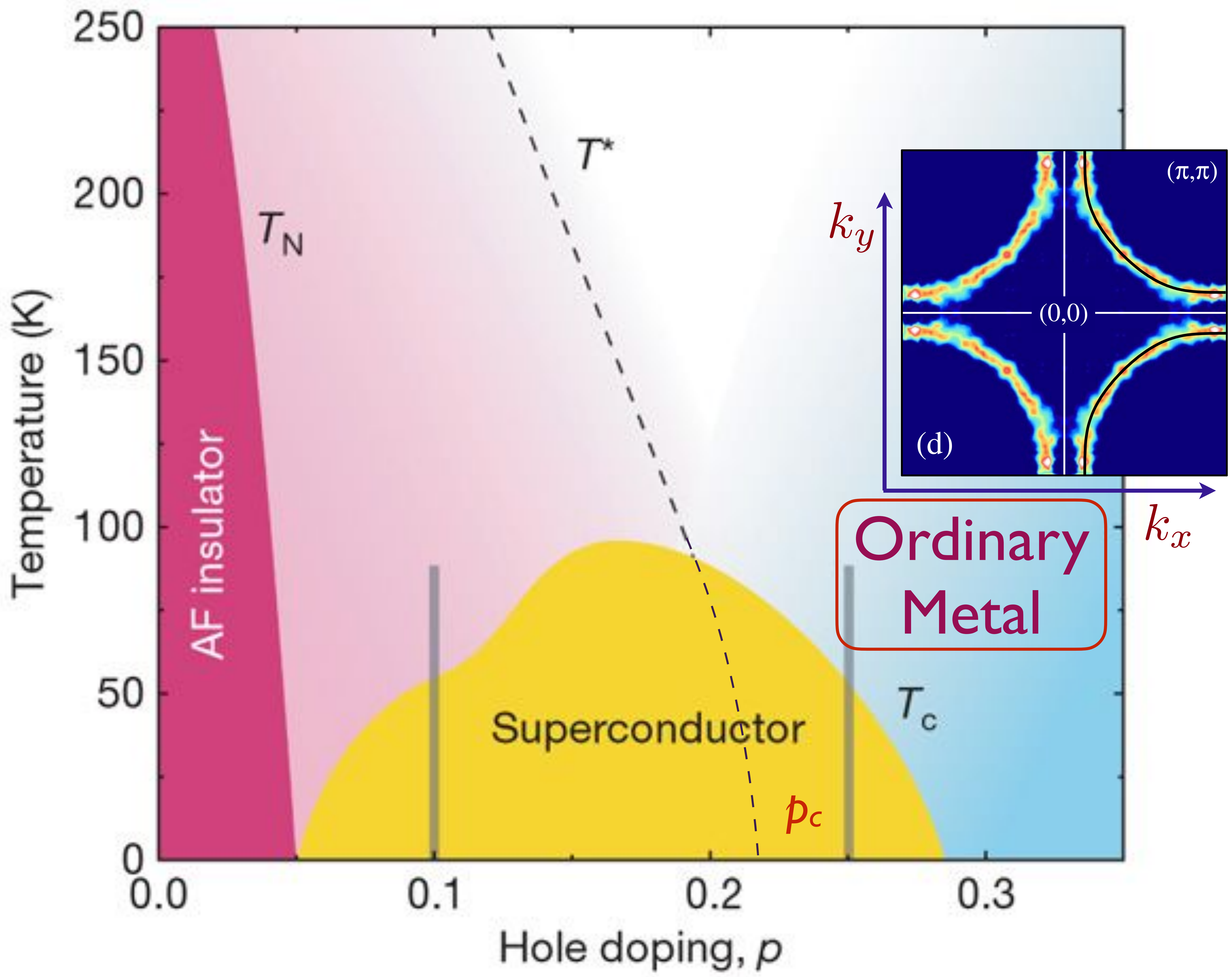
Nearly-free
gas of fermions, with a
Fermi surface between
empty and full states
(Sommerfeld, 1927).



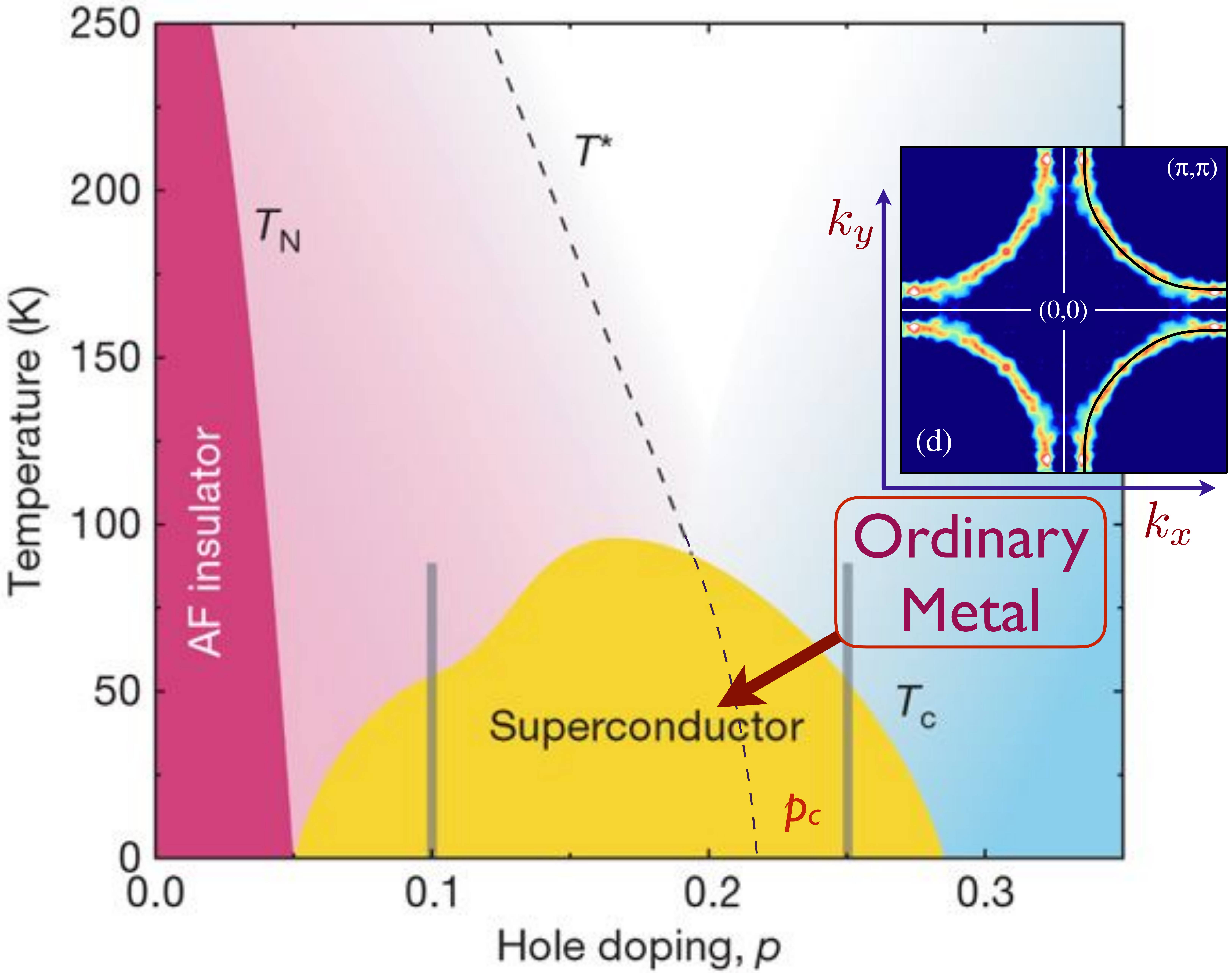
Ordinary metal:

Nearly-free gas of fermions, with a Fermi surface between empty and full states (Sommerfeld, 1927).

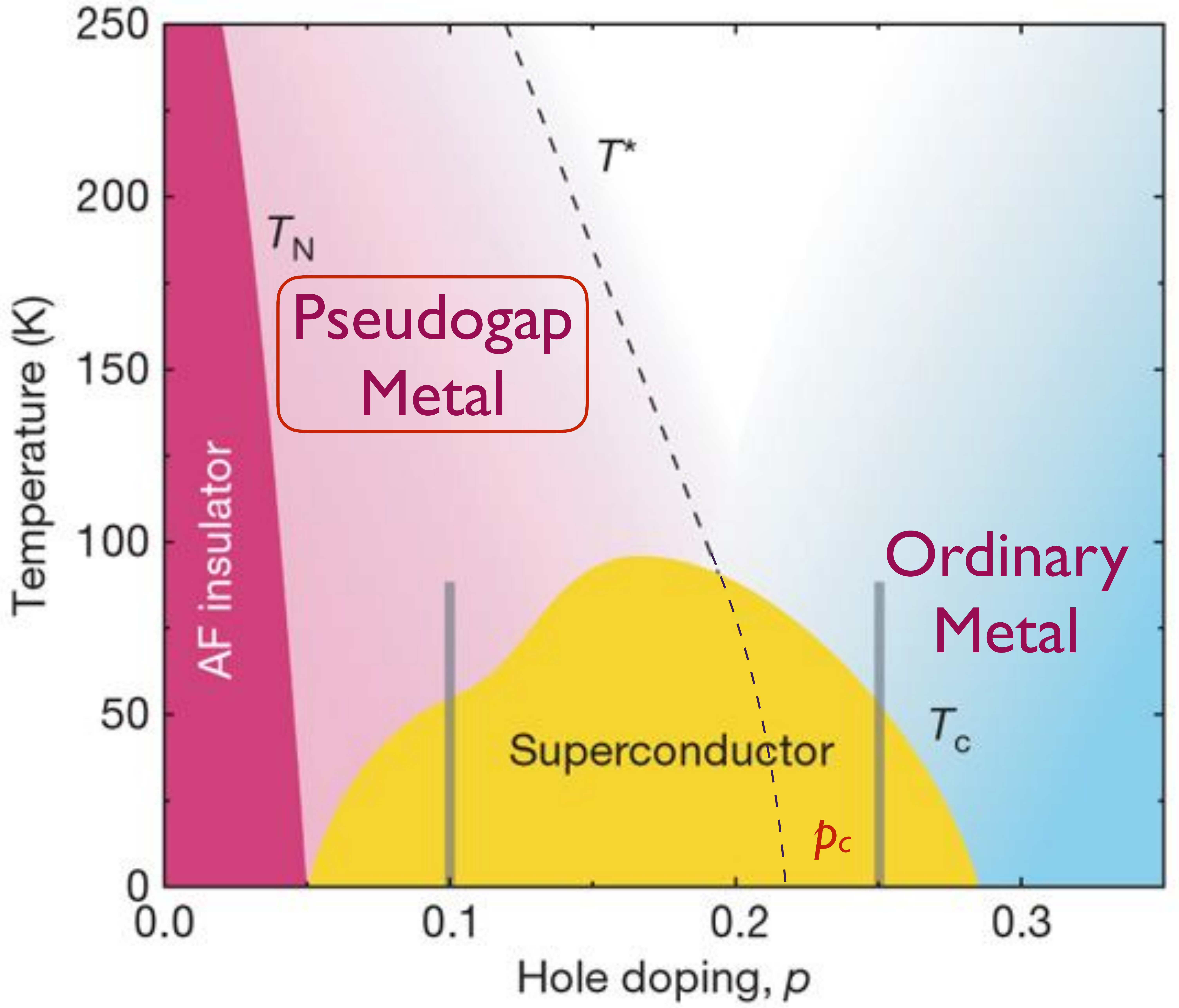
Area enclosed by the Fermi surface is the same as that for free fermions (Luttinger, 1960).



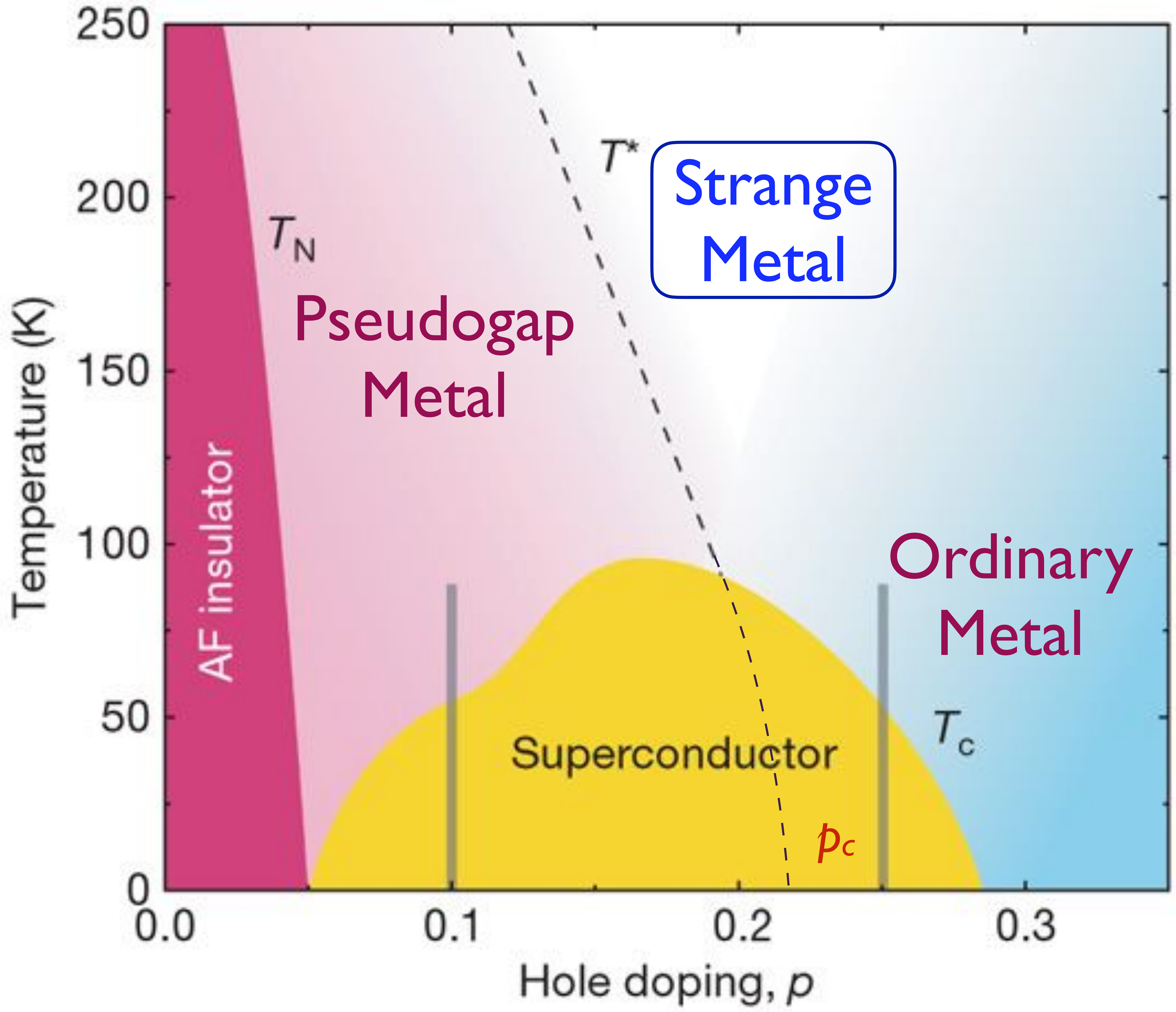
Superconductor:
Bose-Einstein condensation of fermion pairs (Bardeen, Cooper, Schrieffer, 1957)



Pseudogap metal:
many-particle
entanglement
similar to that of
stationary electrons
in a spin liquid



<u>Strange metal</u>:
many-particle
entanglement
similar to that of
mobile electrons in
Sachdev-Ye-Kitaev
models



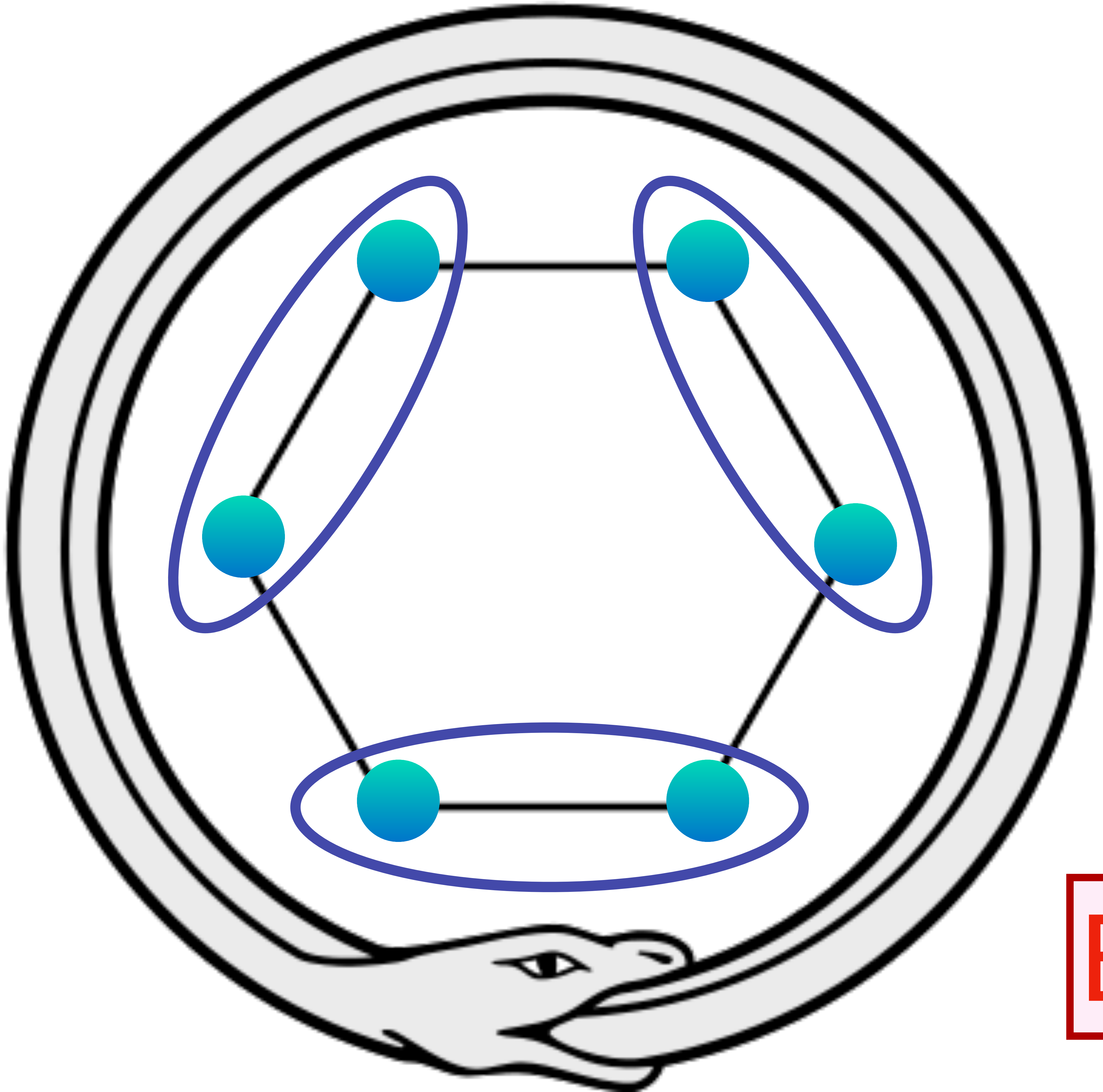
1. Entanglement of stationary electrons: the simplest spin liquid: Z_2
2. Square lattice spin liquid and the cuprate pseudogap metal
3. Entanglement of mobile electrons and metals without quasiparticles: the SYK model
4. From the SYK model to black holes
5. From the SYK model to the universal 2d-YSYK theory of strange metals

Entanglement of stationary electrons.

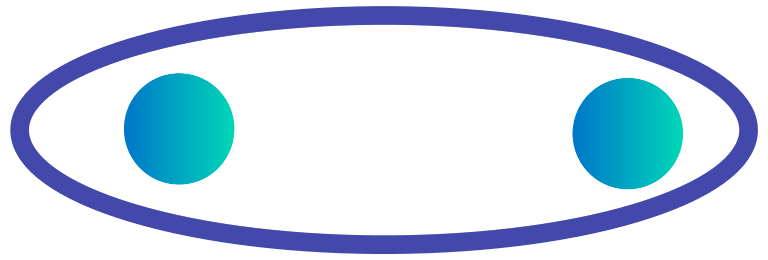
The simplest spin liquid: Z_2

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



Benzene



$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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*Wikipedia

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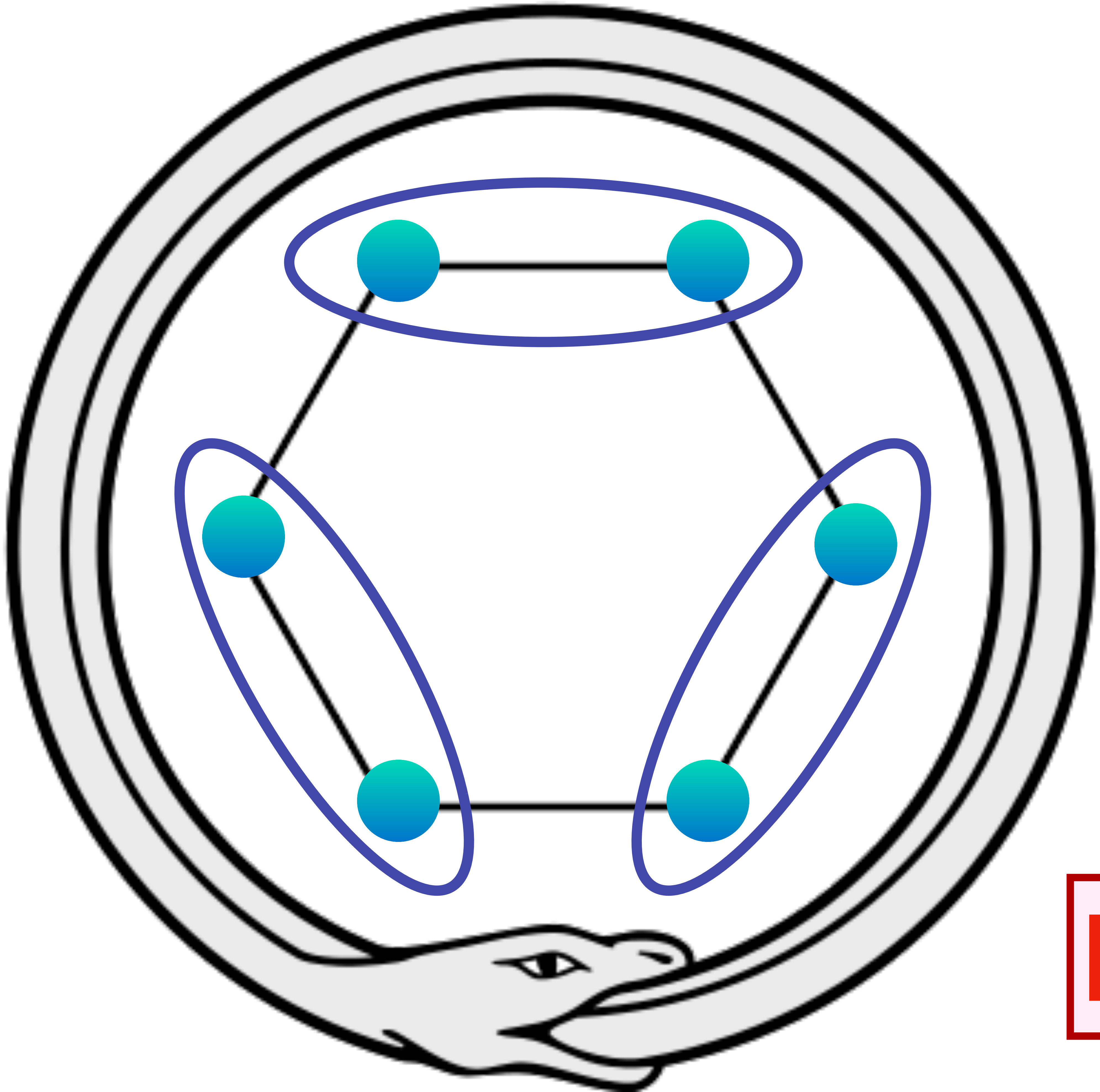

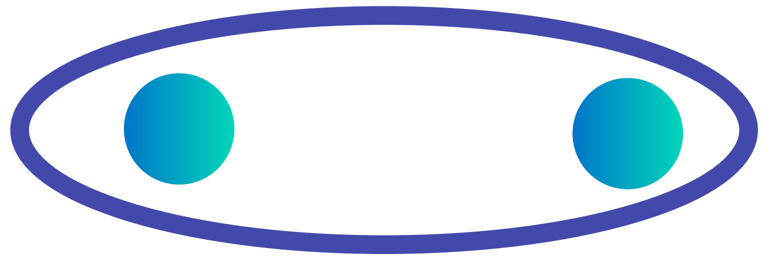

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Benzene

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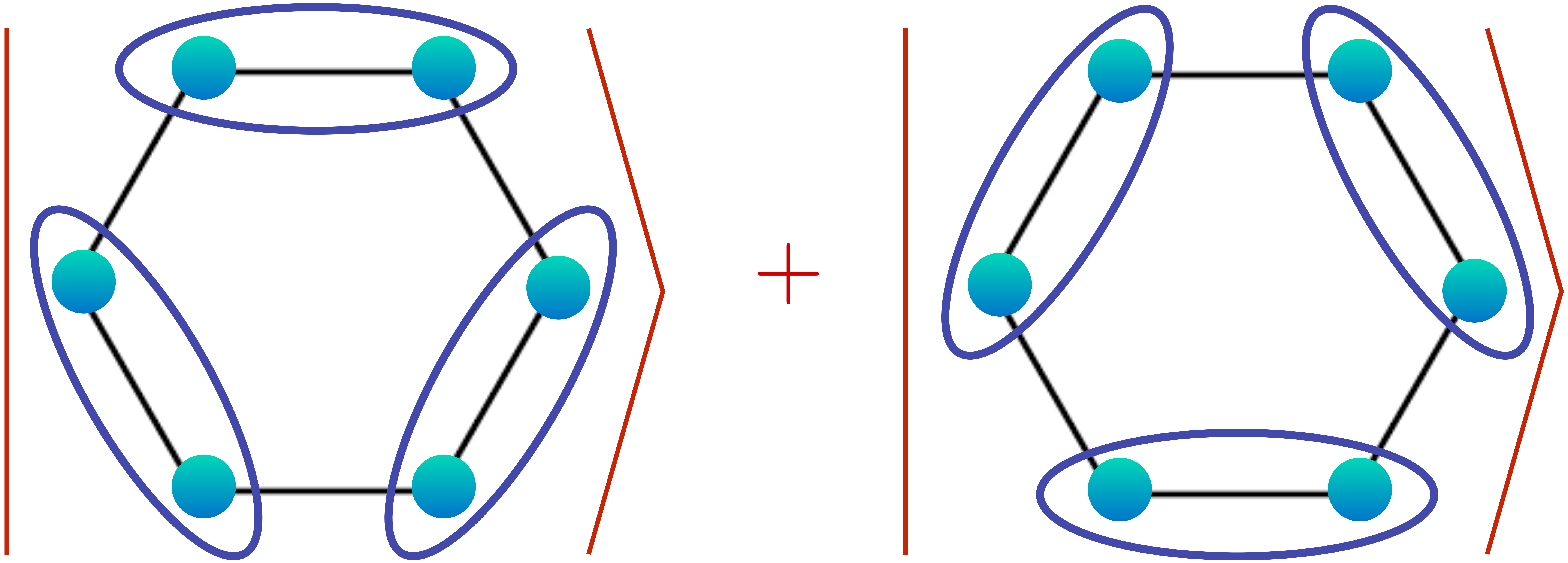
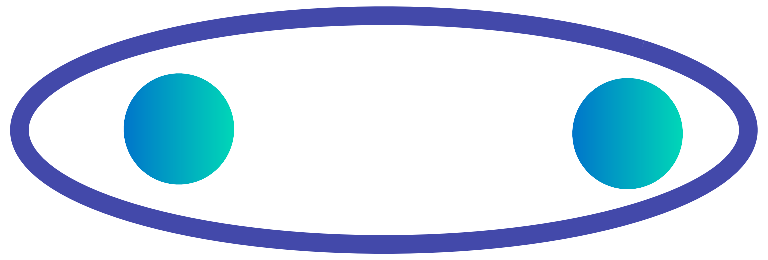
$|\Psi\rangle =$





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Benzene

Triangular lattice antiferromagnet

Spin model with S=1/2 per unit cell

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$$[S_\alpha, S_\beta] = i\epsilon_{\alpha\beta\gamma} S_y$$

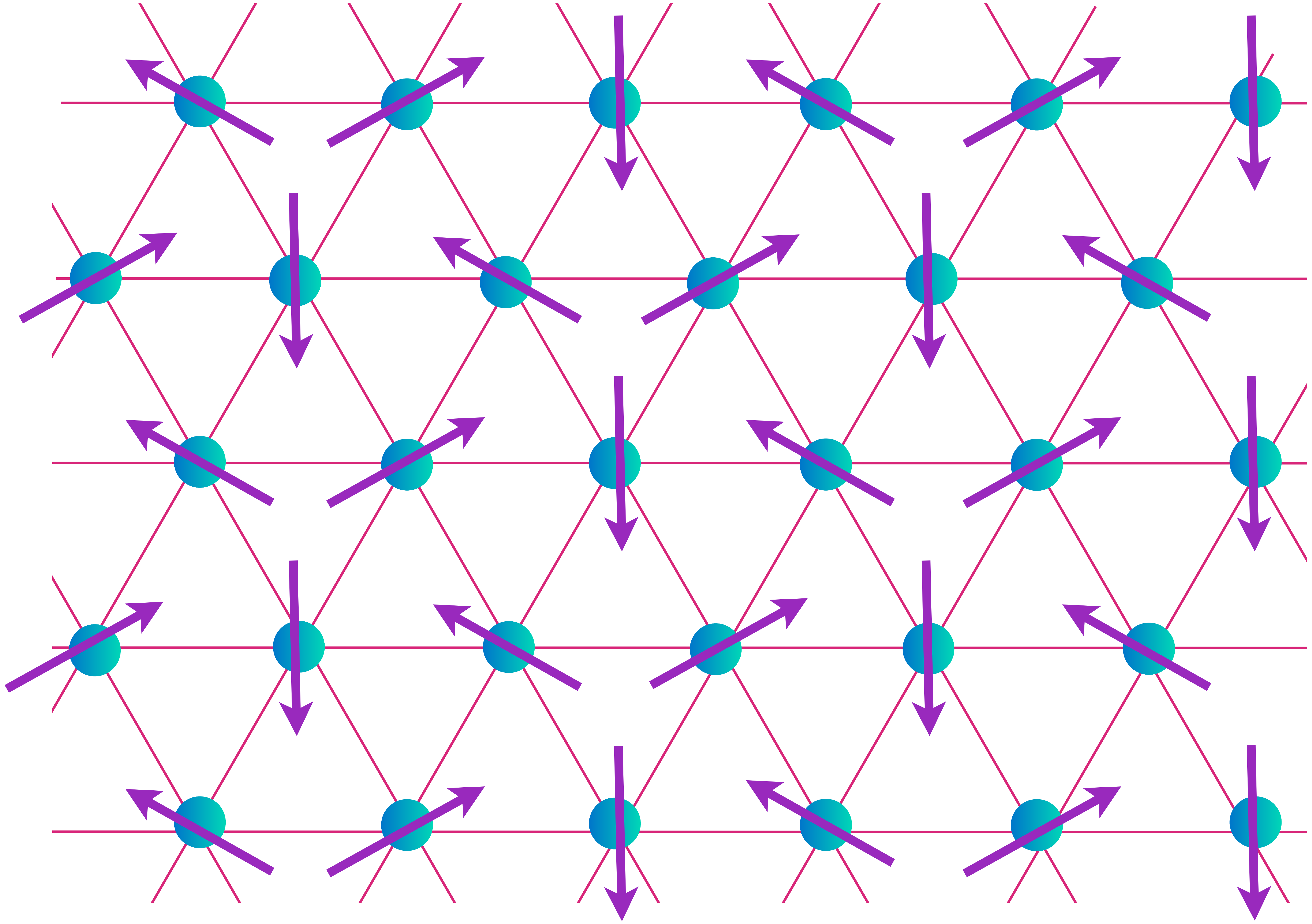
$$S_\alpha^2 = S(S+1);$$

$$S = 1/2$$

$$S_z \left|\uparrow\right\rangle = (1/2) \left|\uparrow\right\rangle$$

$$S_z \left|\downarrow\right\rangle = -(1/2) \left|\downarrow\right\rangle$$

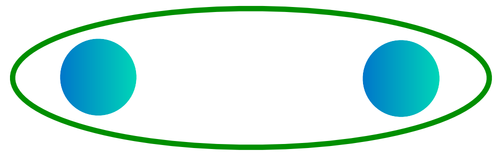
on each site i



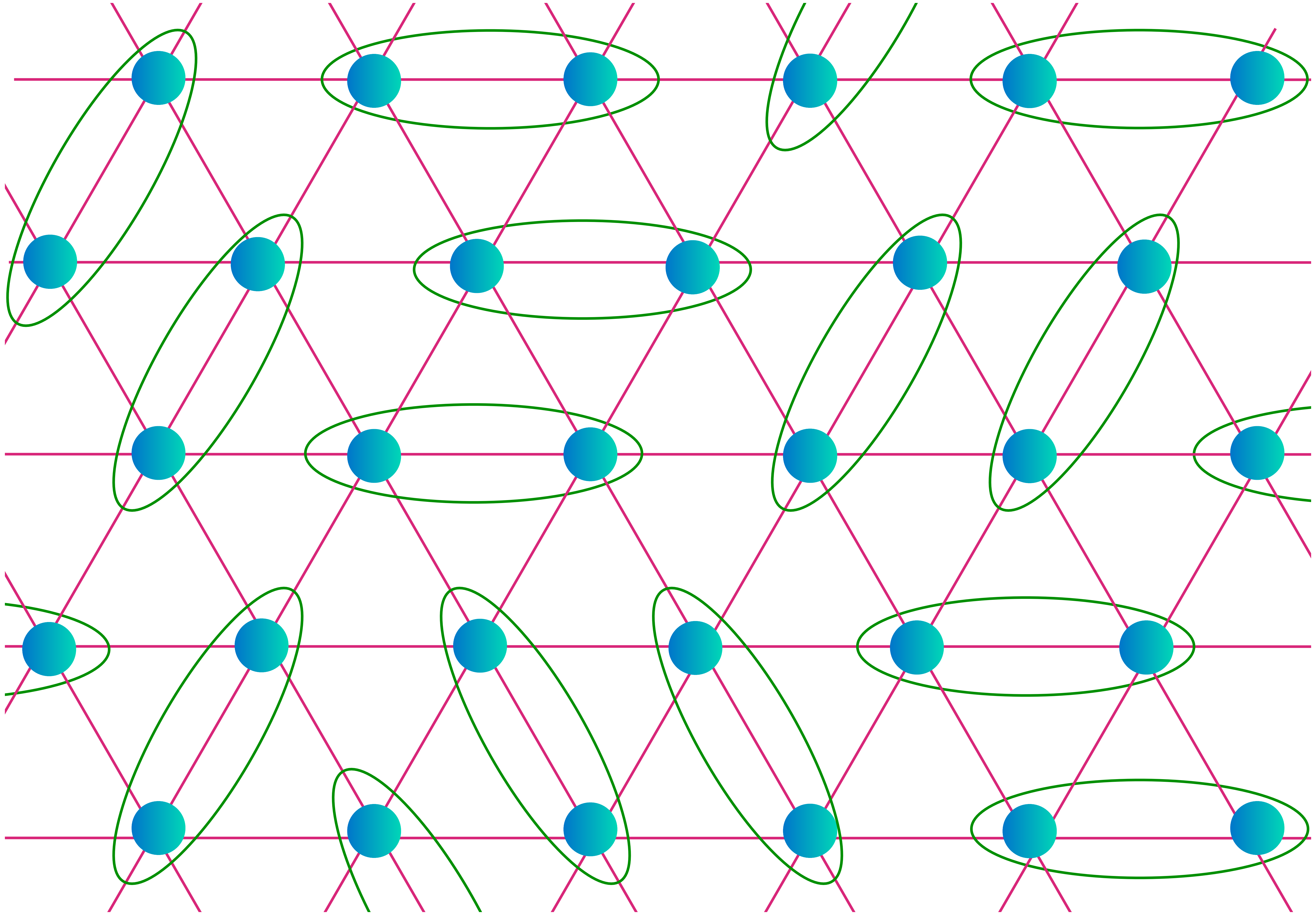
Nearest-neighbor model has non-collinear Neel order

Spin liquid: resonating valence bonds

Spin model with S=1/2 per unit cell



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}}\,|\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Spin liquid: resonating valence bonds

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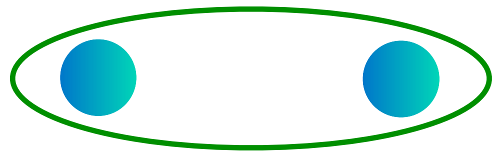

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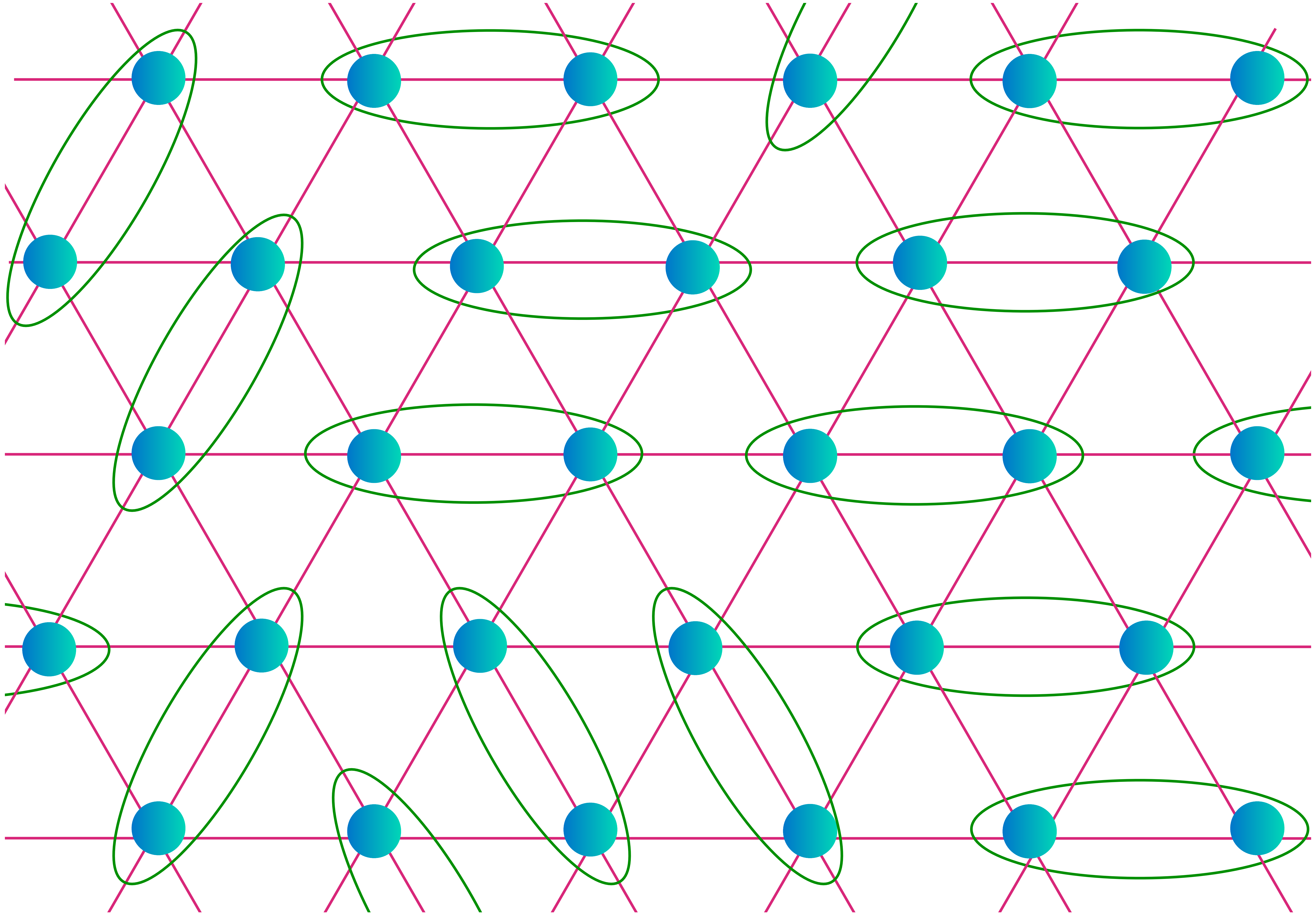

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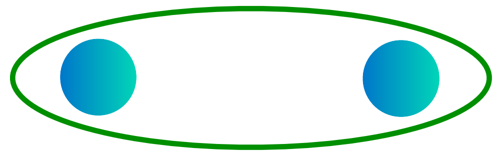

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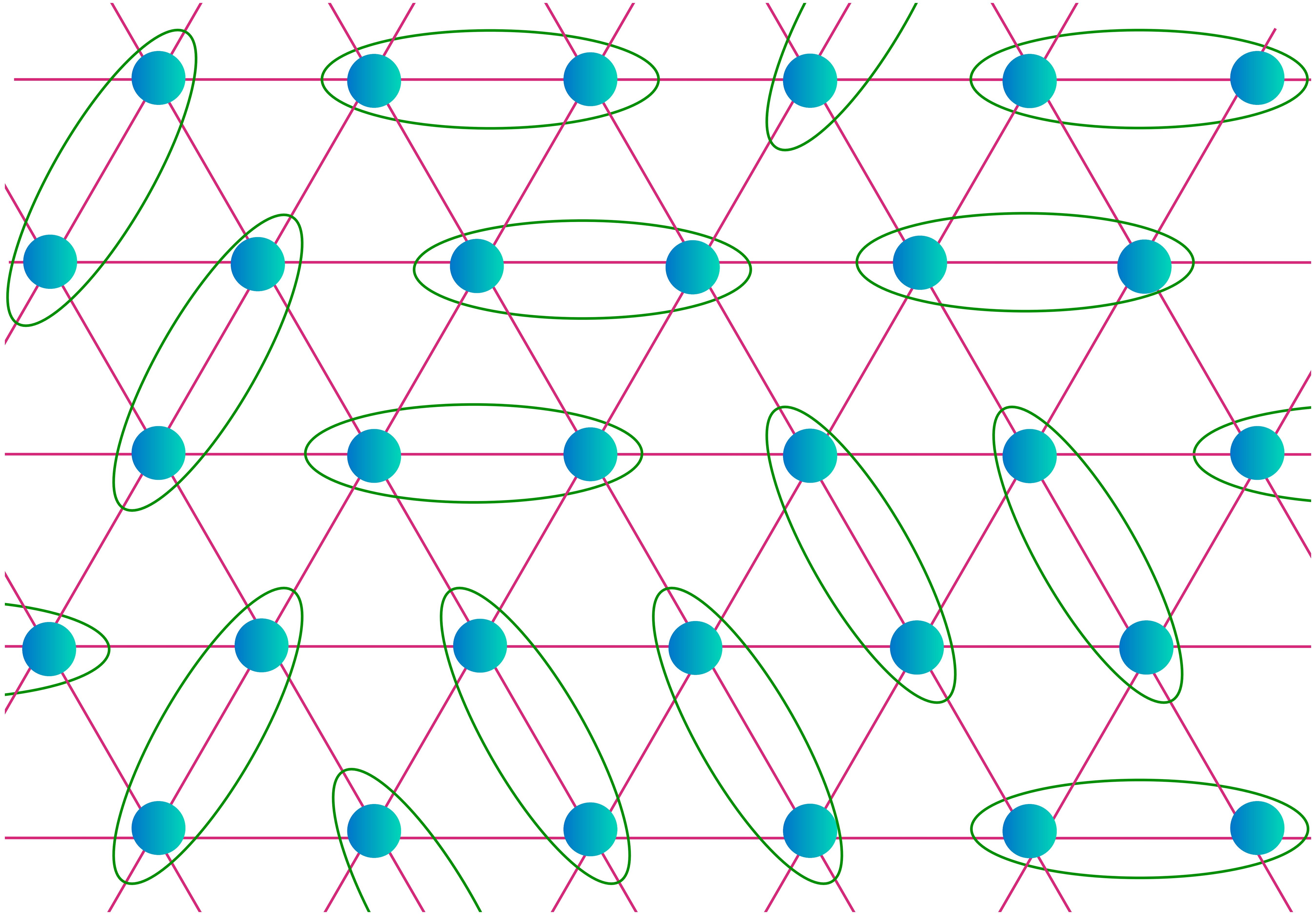

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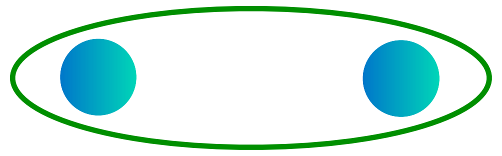

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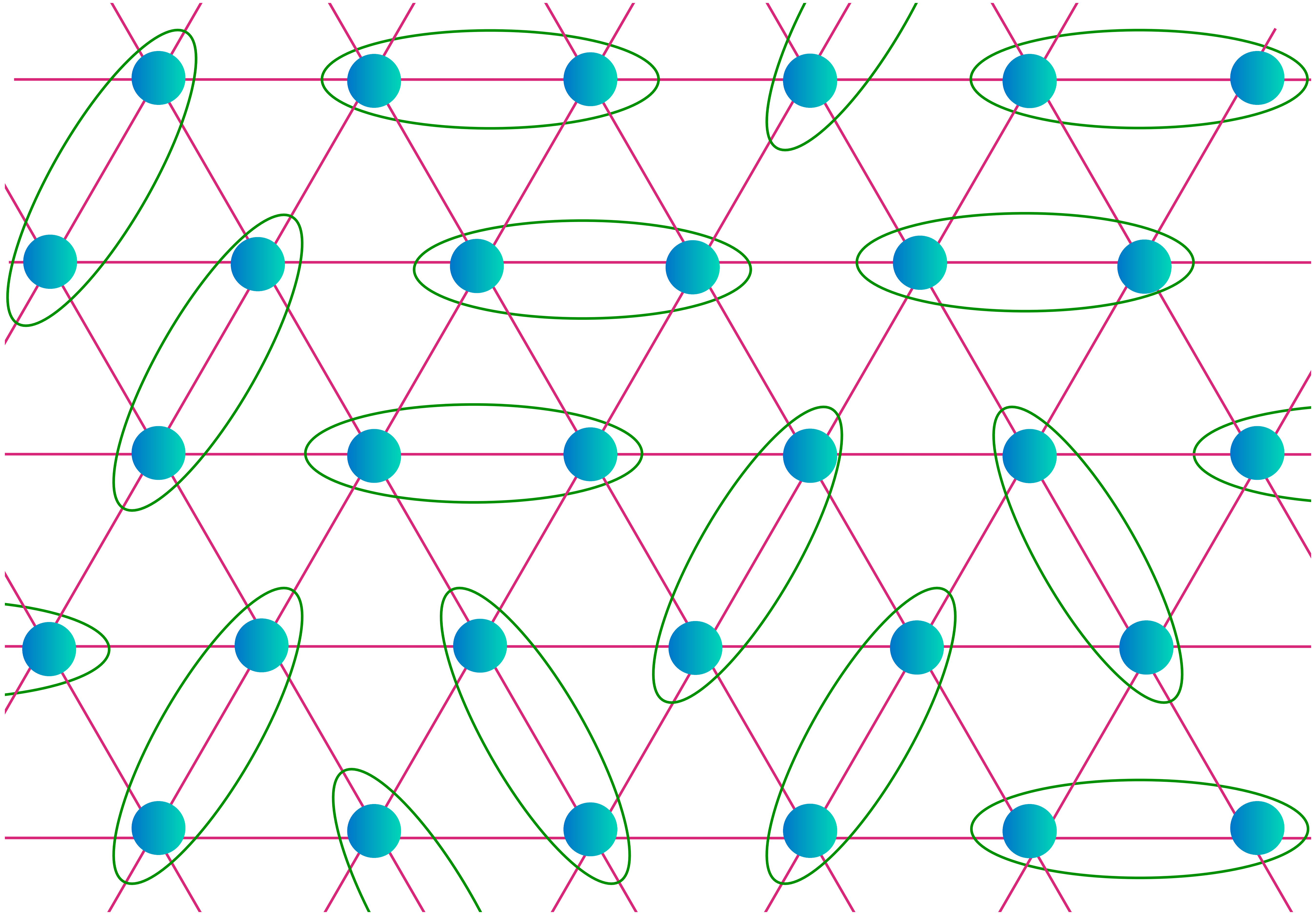

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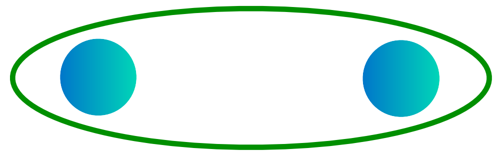

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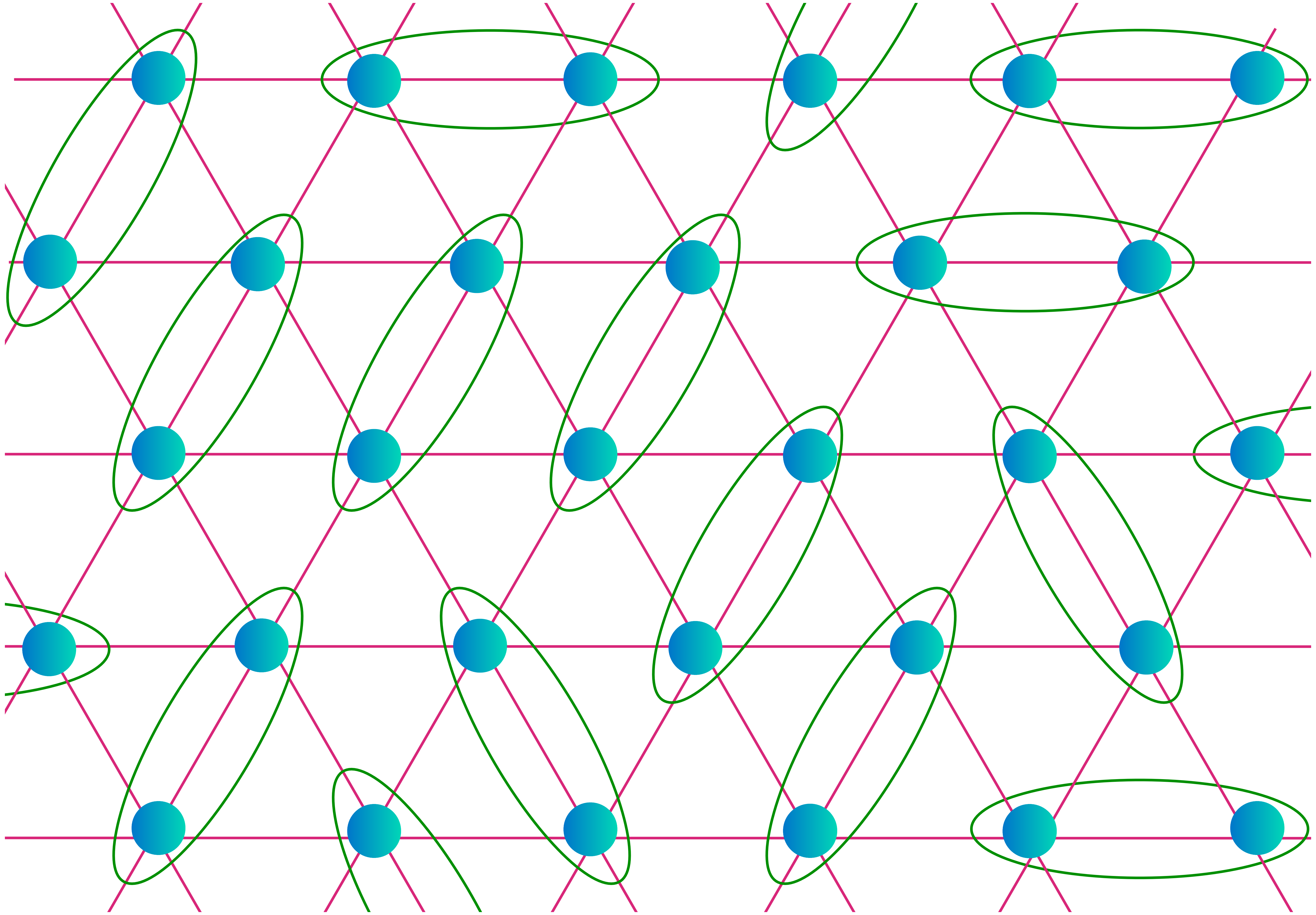

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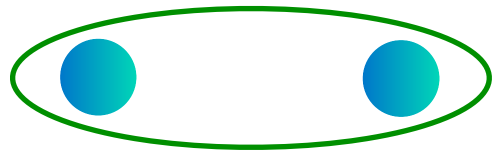

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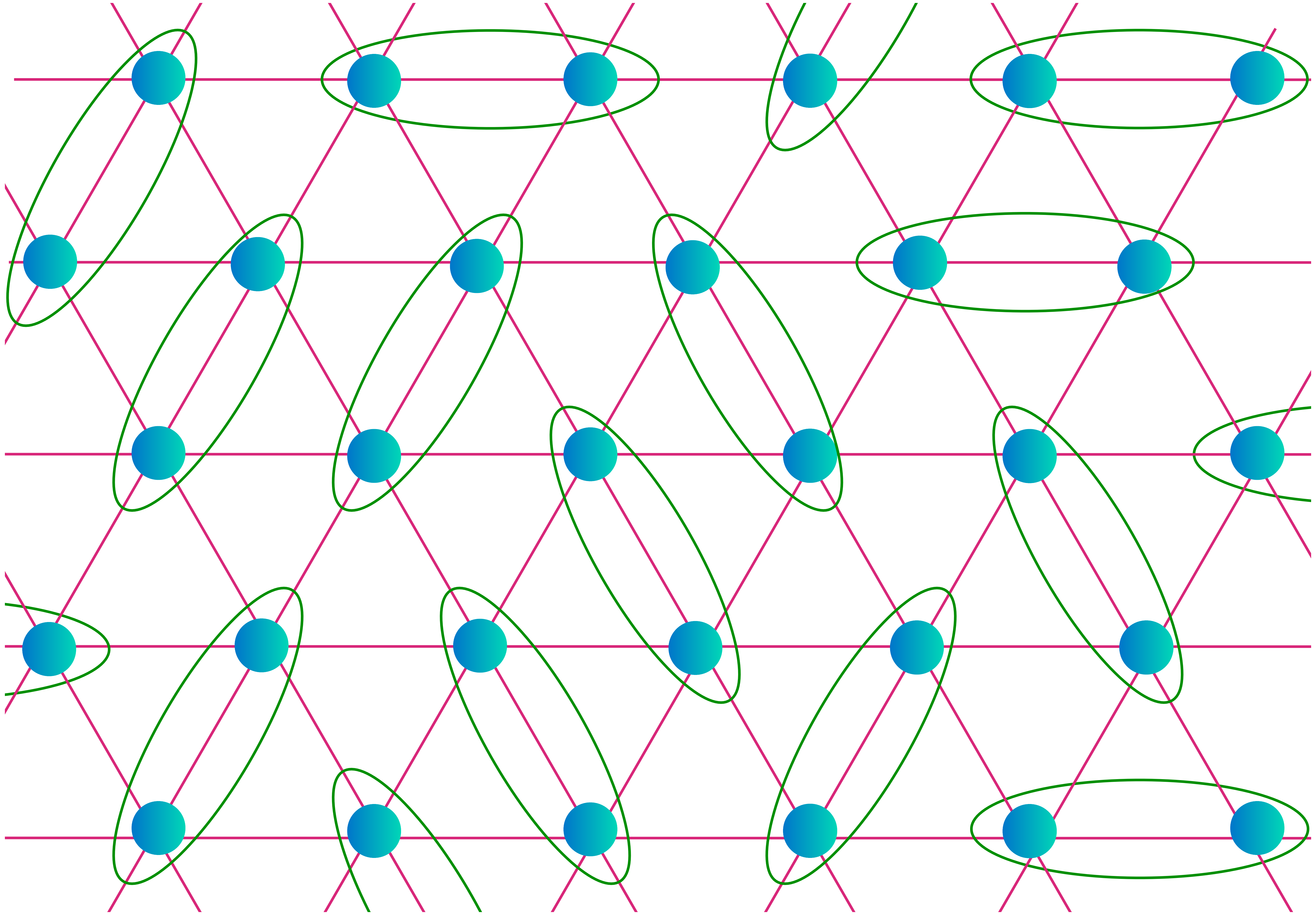

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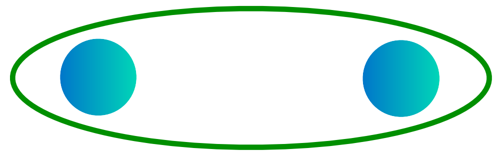
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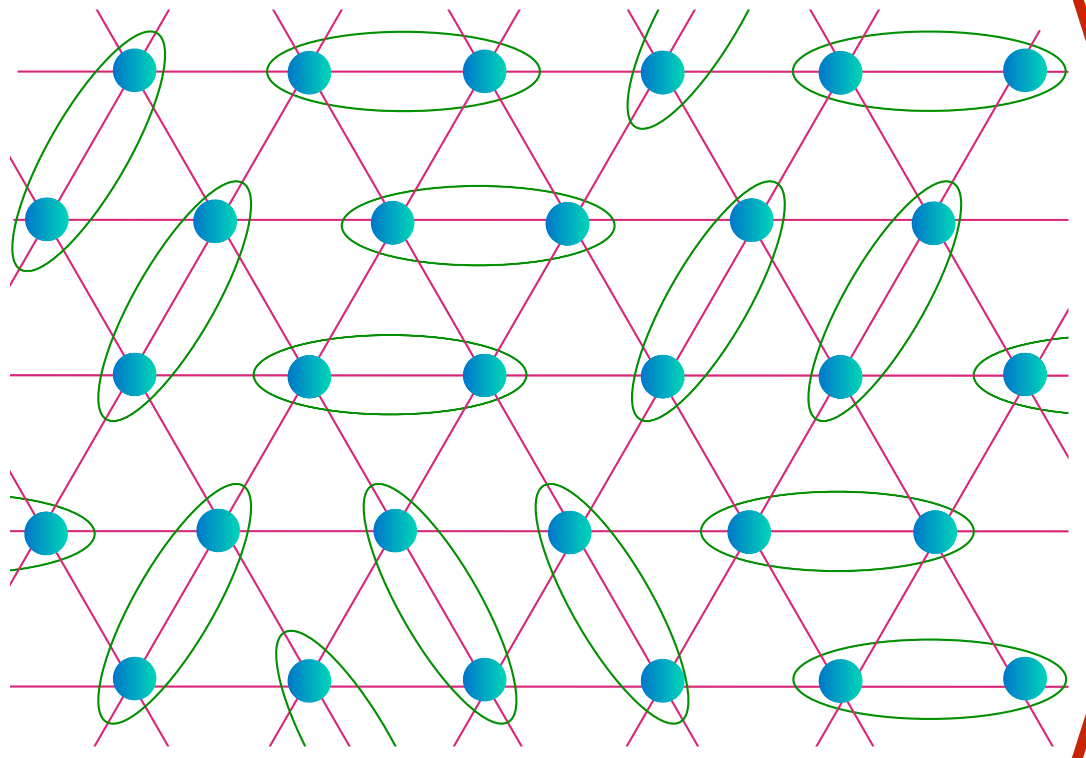
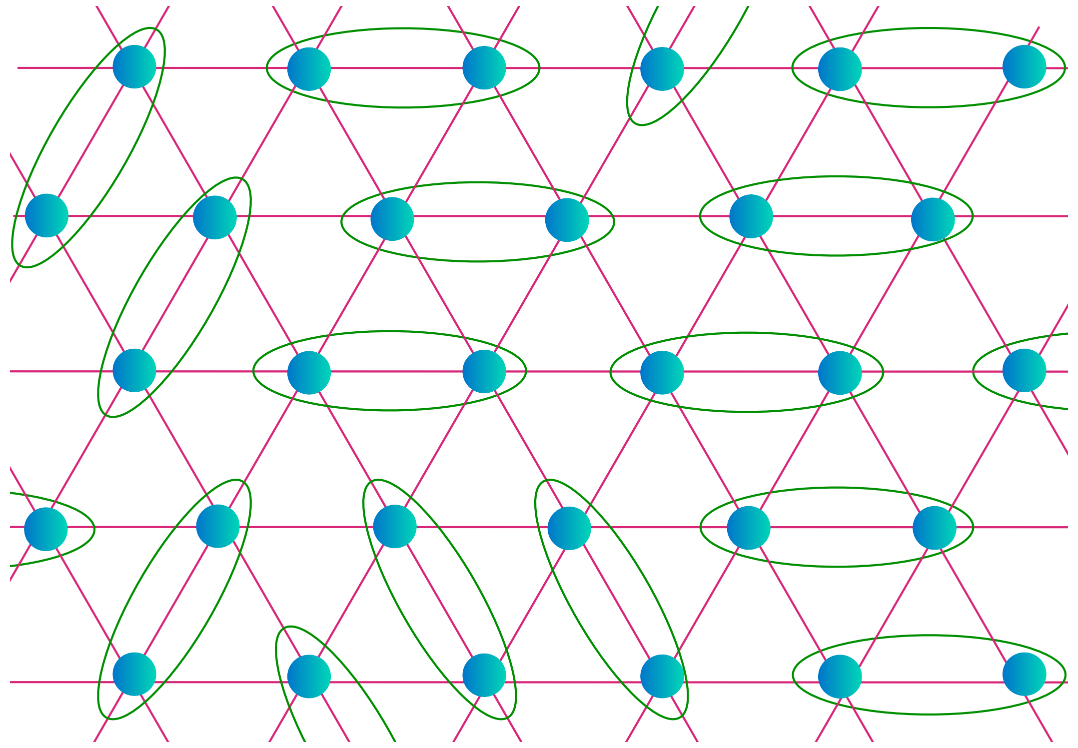
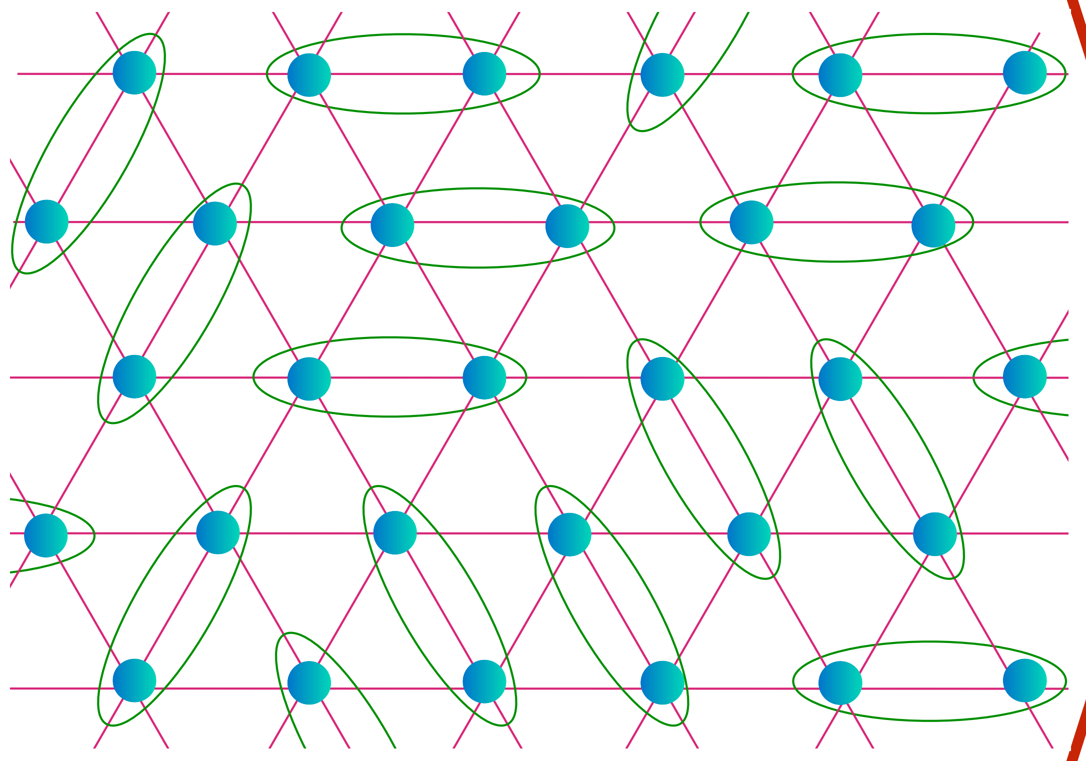
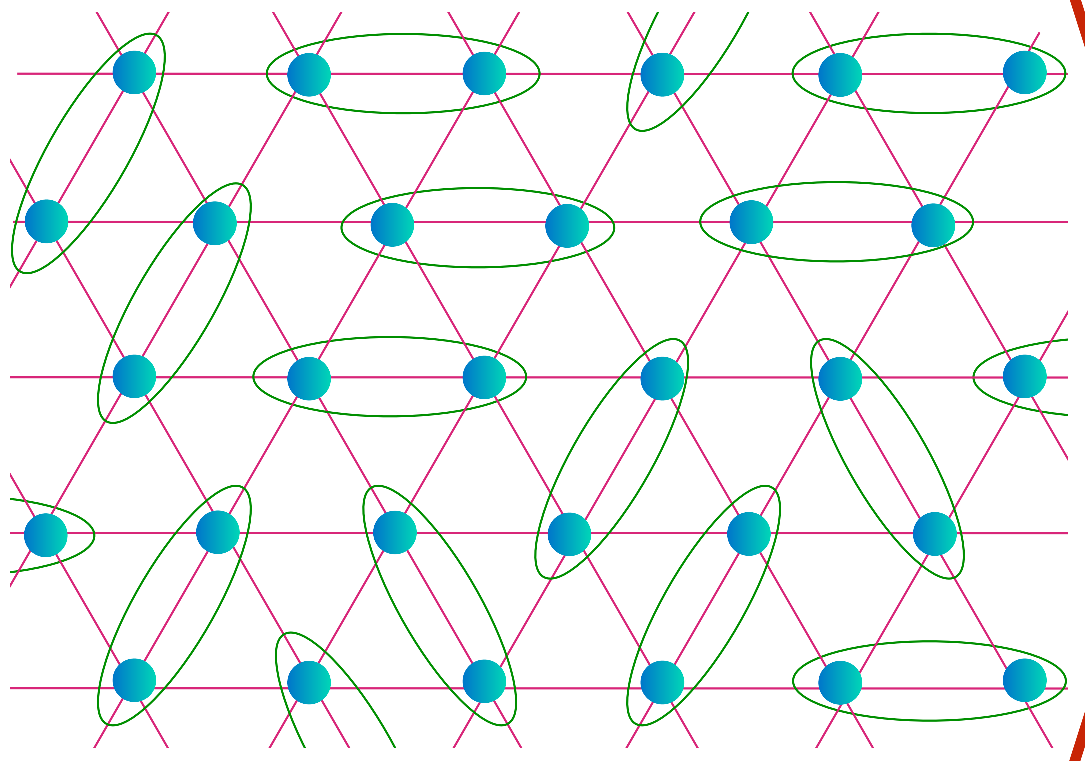
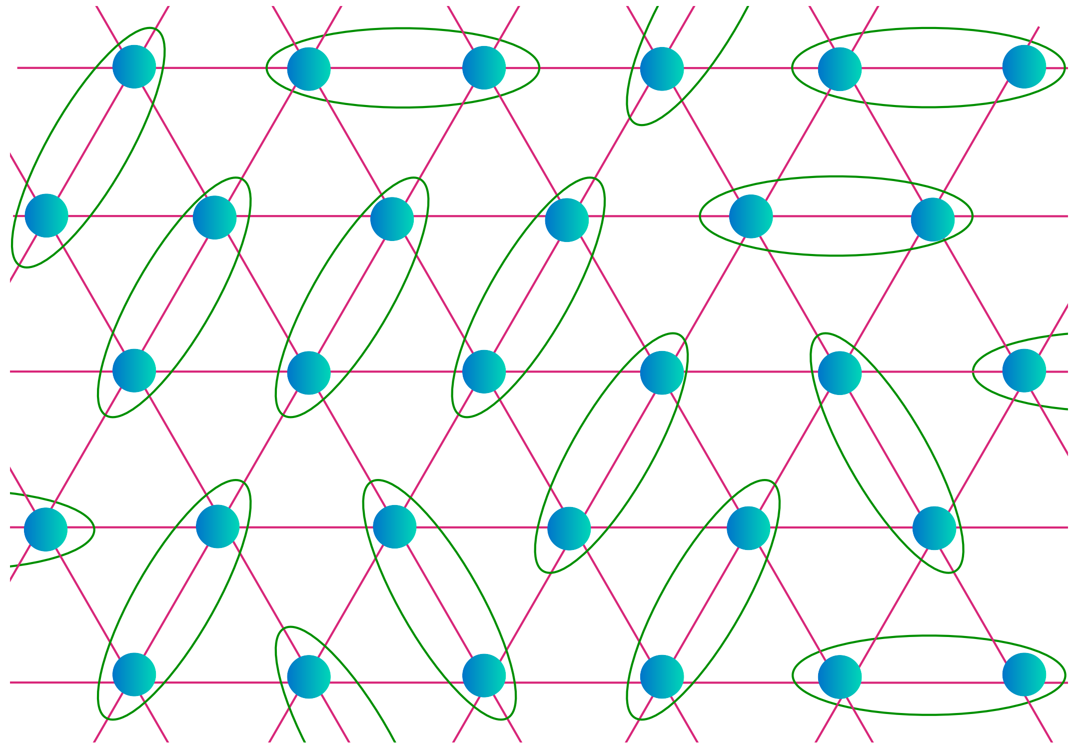
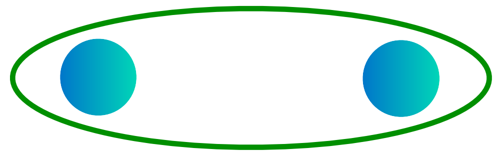
$|\Psi\rangle = C_1\Big|$  $\Big\rangle + C_2\Big|$  $\Big\rangle$

$+\, C_3\Big|$  $\Big\rangle + C_4\Big|$  $\Big\rangle + C_5\Big|$  $\Big\rangle + \ldots$

Spin liquid: resonating valence bonds



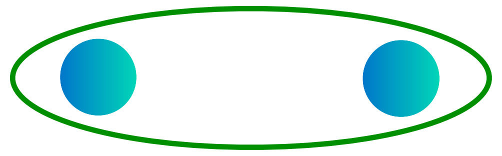
$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Key feature: fractionalization.

Excitations are particle-like,
but cannot be created
by local operators.
The excitations are classified
under distinct
superselection/anyon sectors.

RVB: Z_2 spin liquid

Fractionalized excitations: a "spinon" with spin S=1/2



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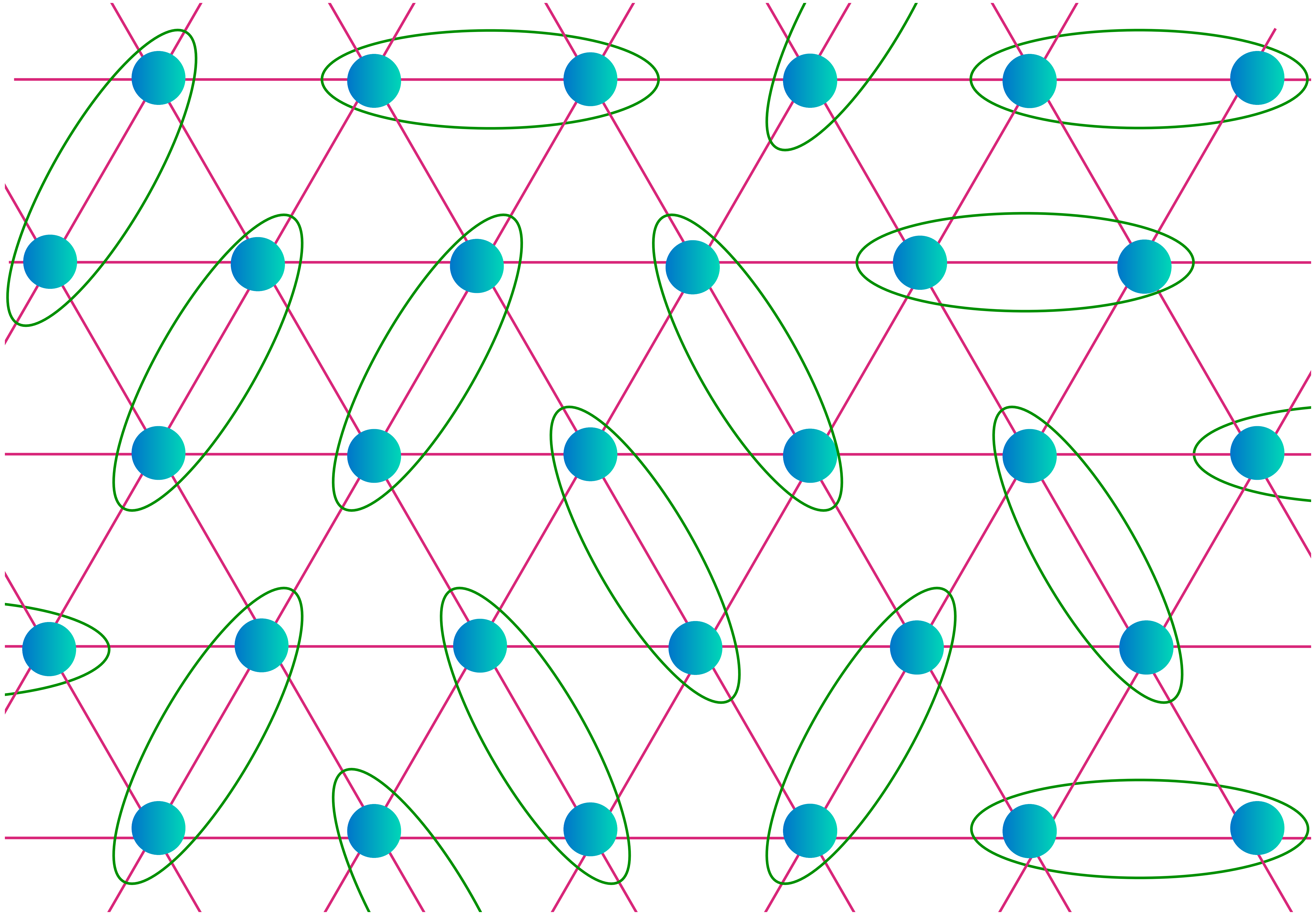

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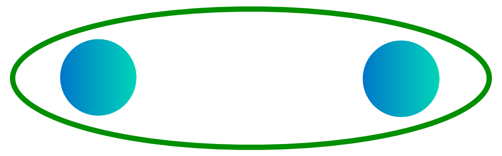

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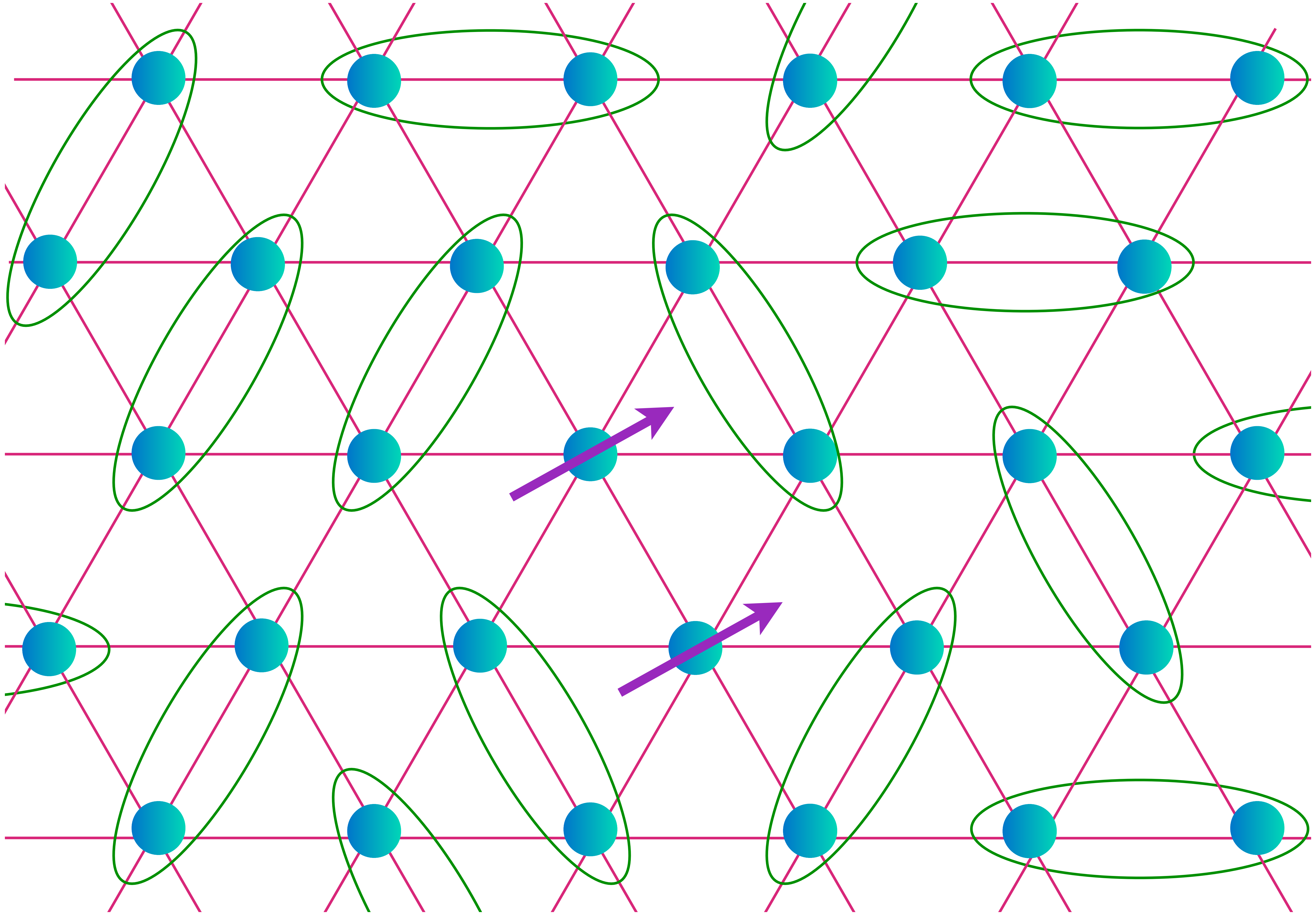

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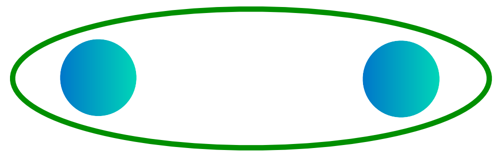

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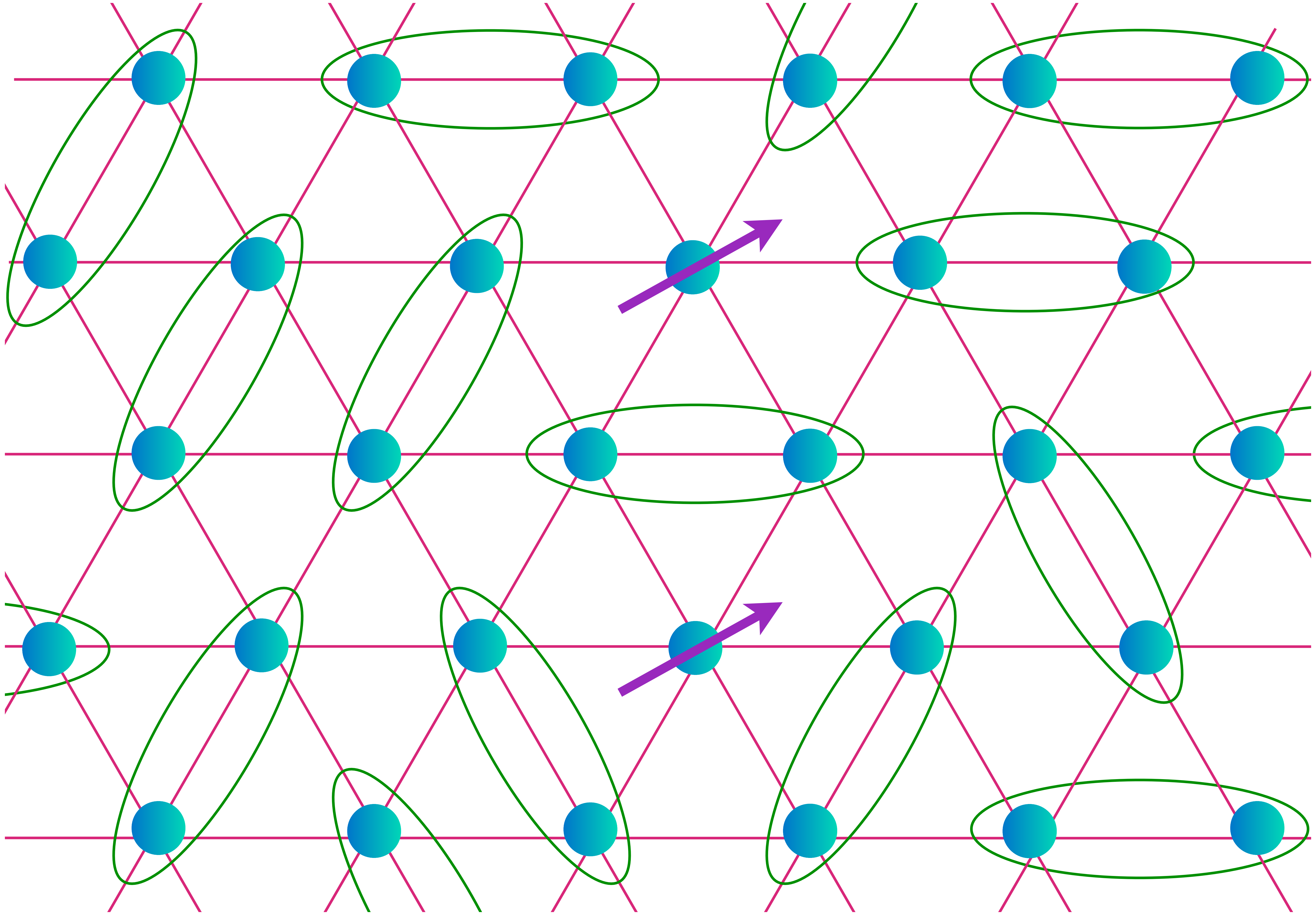

Kivelson, Baskaran…..

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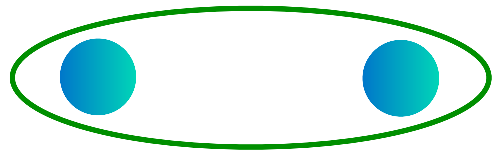

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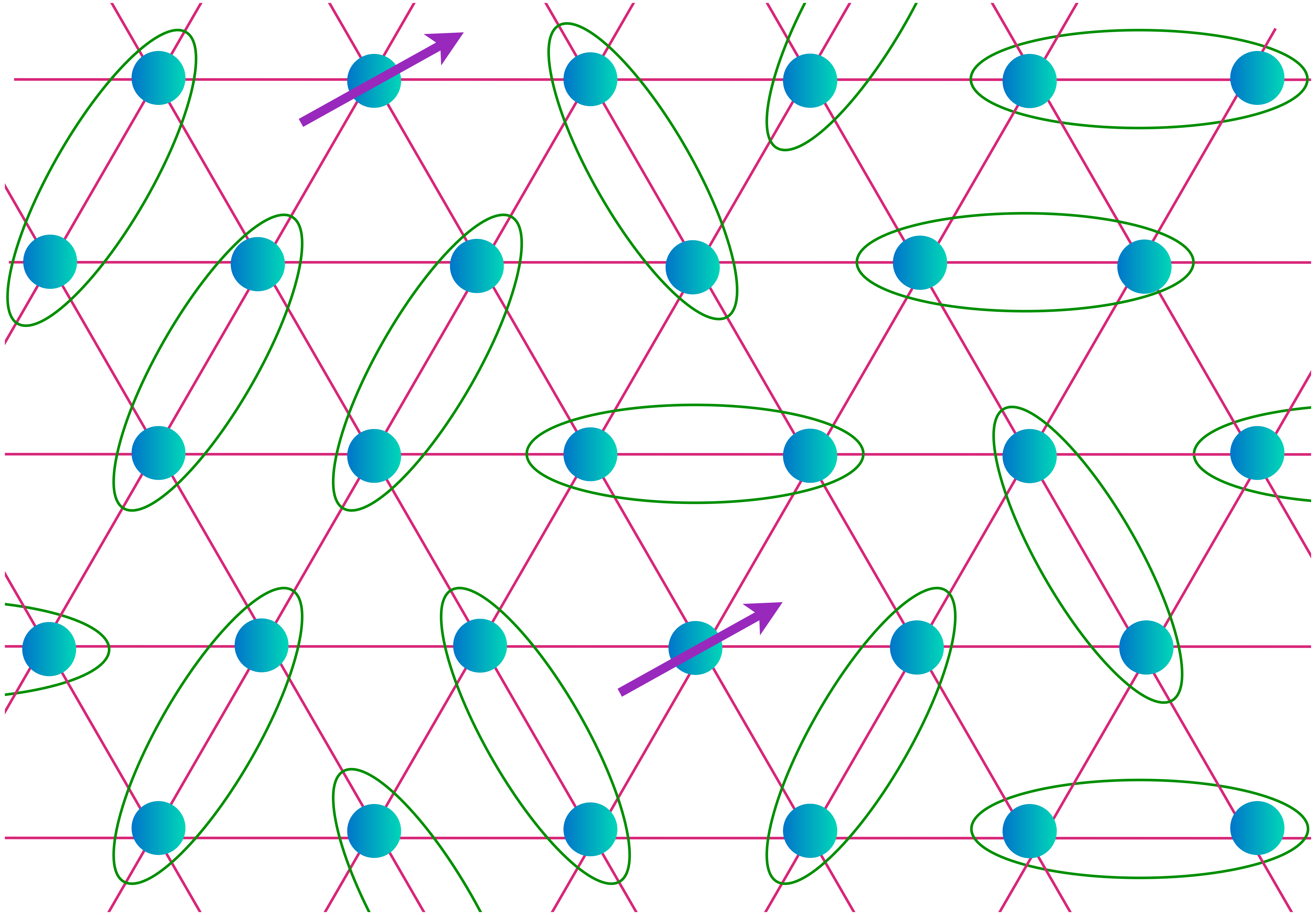

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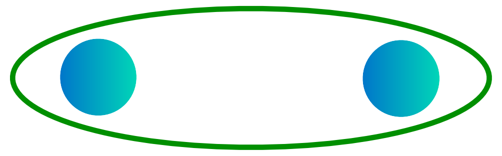

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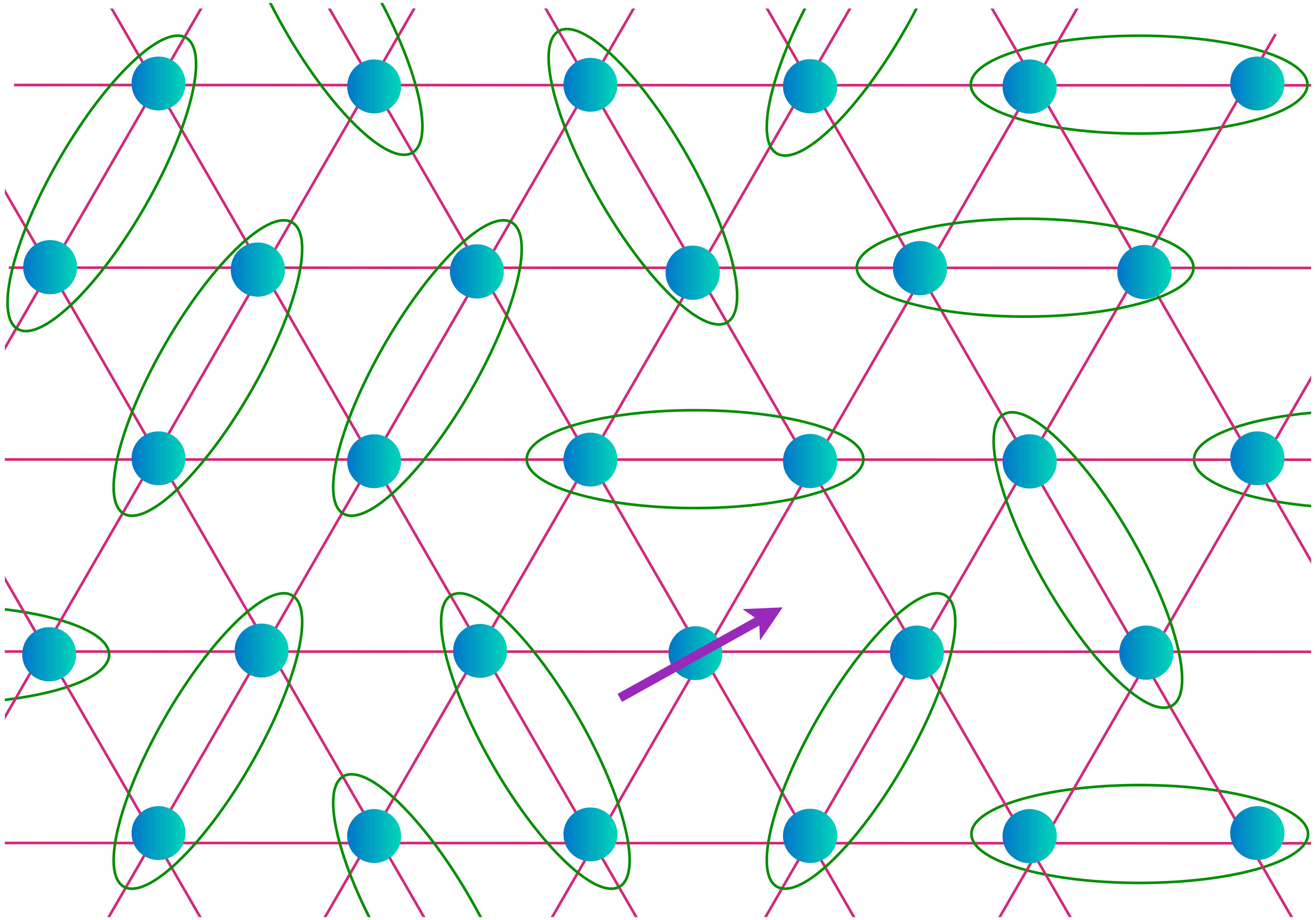
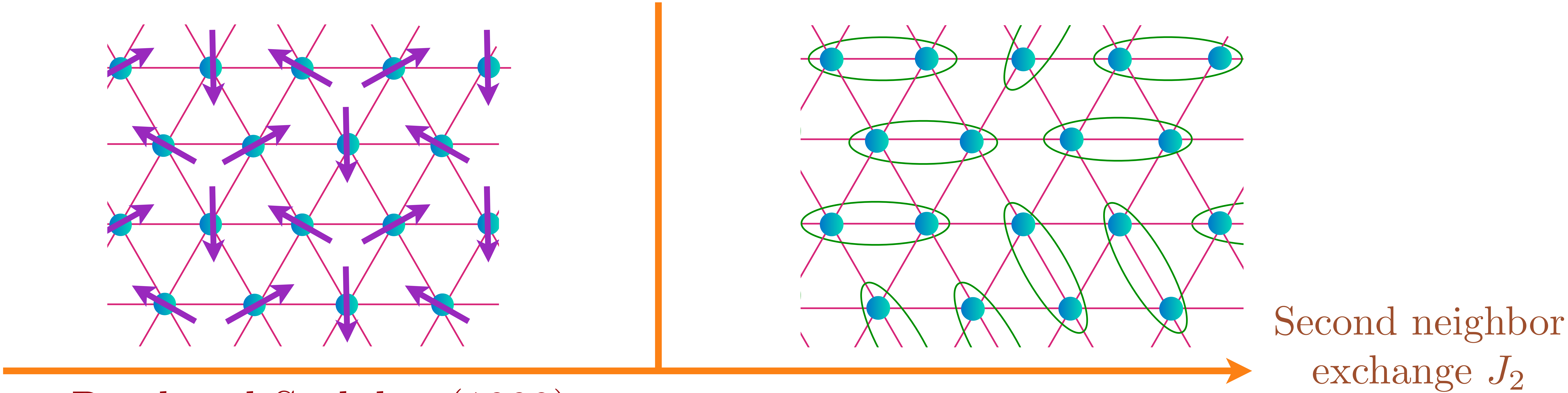


Kivelson, Baskaran…..

Quantum phase transition from ordered antiferromagnet to Z_2 spin liquid



Read and Sachdev (1990):

The RVB spin liquid has an emergent $\mathbb{Z}_2$ spin liquid.

The spinons carry electric $\mathbb{Z}_2$ gauge charges.

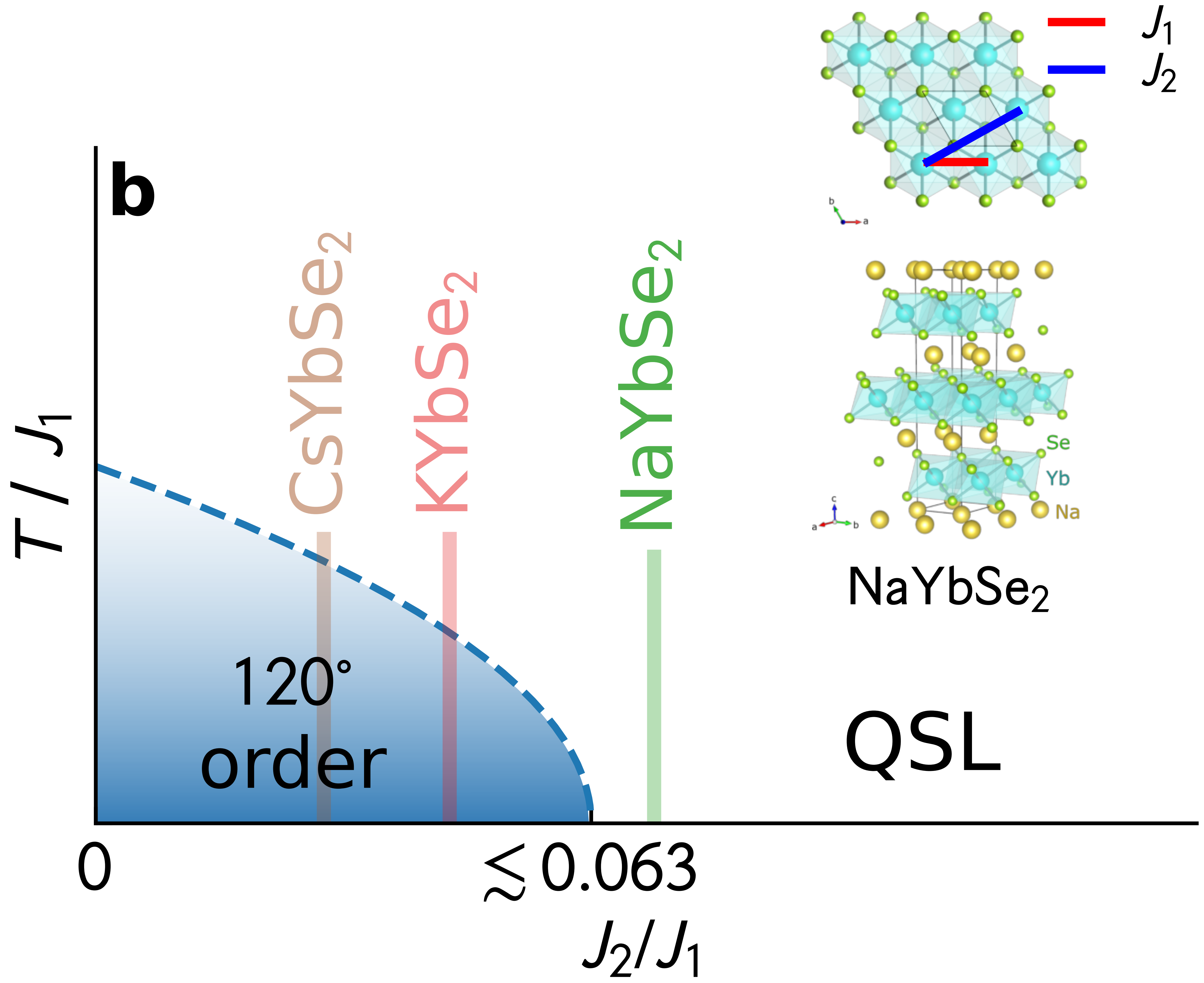
There are also emergent particles ('visons' or 'm') which carry magnetic $\mathbb{Z}_2$ gauge charges.

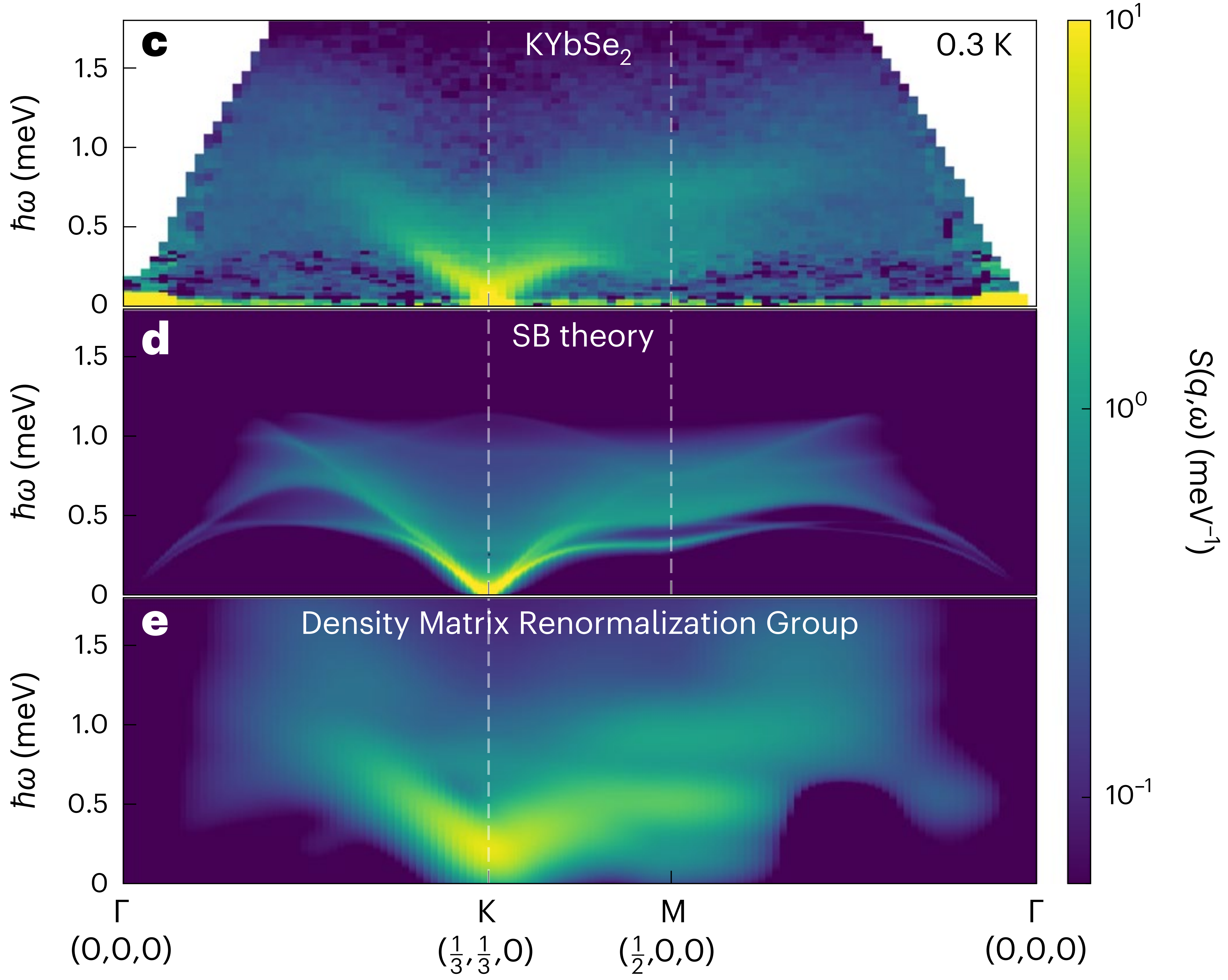
This structure ("unitary modular tensor category") is the same as that found in Kitaev's toric code (1997).

Proximate spin liquid and fractionalization in the triangular antiferromagnet $KYbSe_2$

A. O. Scheie[1], E. A. Ghioldi[2,3], J. Xing[4], J. A. M. Paddison[4], N. E. Sherman[5,6], M. Dupont[5,6], L. D. Sanjeewa[7,8], Sangyun Lee[9], A. J. Woods[9], D. Abernathy[1], D. M. Pajerowski[1], T. J. Williams[1], Shang-Shun Zhang[10], L. O. Manuel[3], A. E. Trumper[3], C. D. Pemmaraju[11], A. S. Sefat[4], D. S. Parker[4], T. P. Devereaux[11,12], R. Movshovich[9], J. E. Moore[5,6], C. D. Batista[2,13] & D. A. Tennant[2,13]

Nature Physics **20**, 74 (2024)

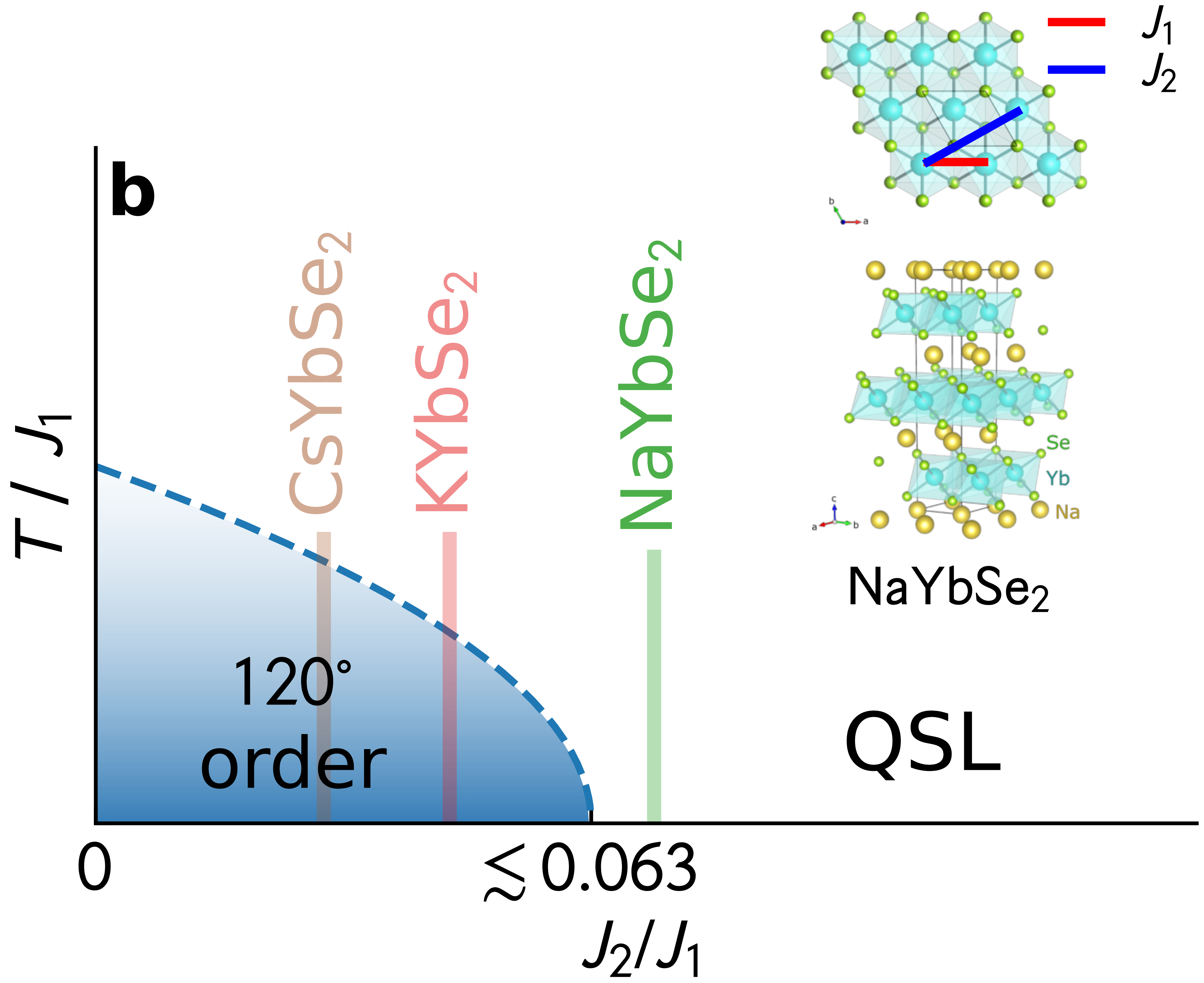




Spectrum and low-energy gap in triangular quantum spin liquid $NaYbSe_2$

A. O. Scheie,[1,*] Minseong Lee,[2,†] Kevin Wang,[3] P. Laurell,[4] E. S. Choi,[5] D. Pajerowski,[6] Qingming Zhang,[7] Jie Ma,[8] H. D. Zhou,[4] Sangyun Lee,[2] S. M. Thomas,[1] M. O. Ajeesh,[1] P. F. S. Rosa,[1] Ao Chen,[9] Vivien S. Zapf,[2] M. Heyl,[9] C. D. Batista,[4,6] E. Dagotto,[4,10] J. E. Moore,[3,‡] and D. Alan Tennant[4,11,§]

arXiv:2406.17773



We observe a continuum of (neutron) scattering, which is reproduced by matrix product simulations, and no phase transition is detected in any bulk measurements …

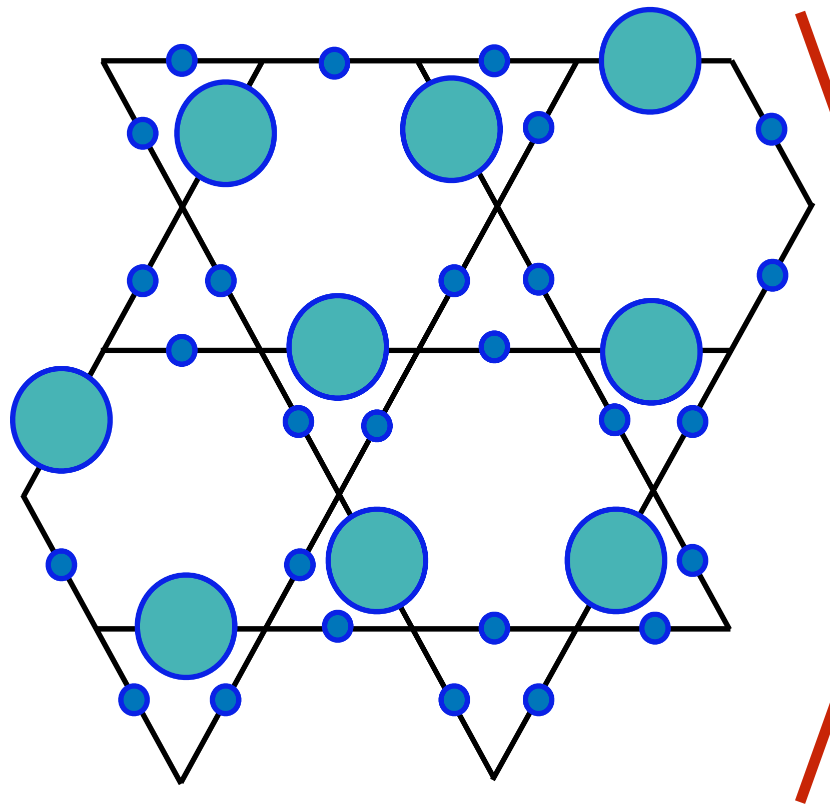
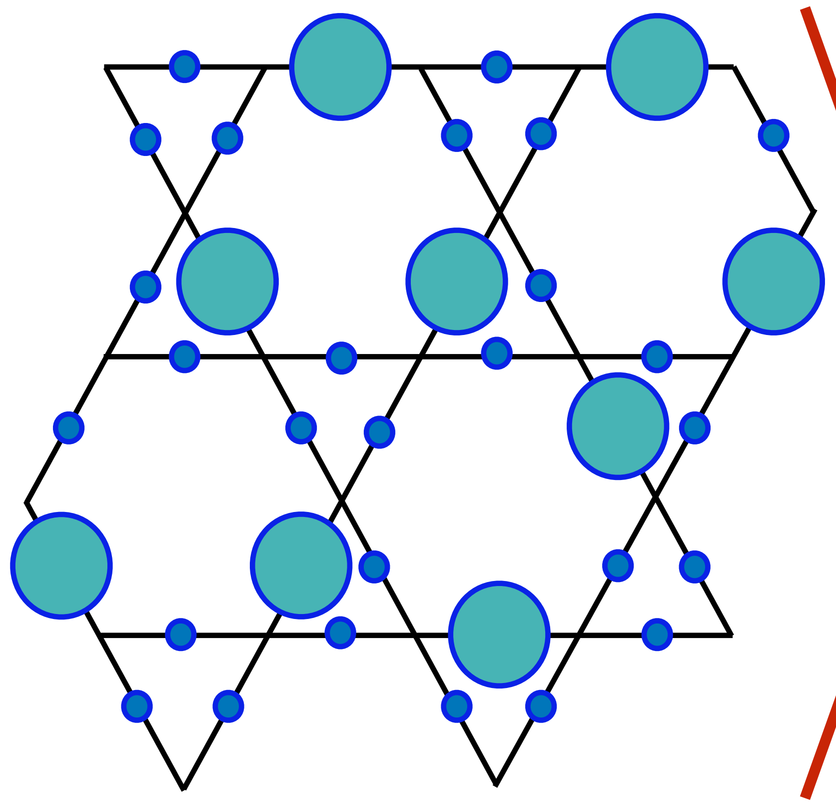
AC susceptibility shows a significant 23 mK downturn, indicating a gap in the magnetic spectrum …

$NaYbSe_2$ is within the quantum spin liquid phase … with a gapped $\mathbb{Z}_2$ liquid the most natural explanation.

Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, Science **374**, 1242 (2021).

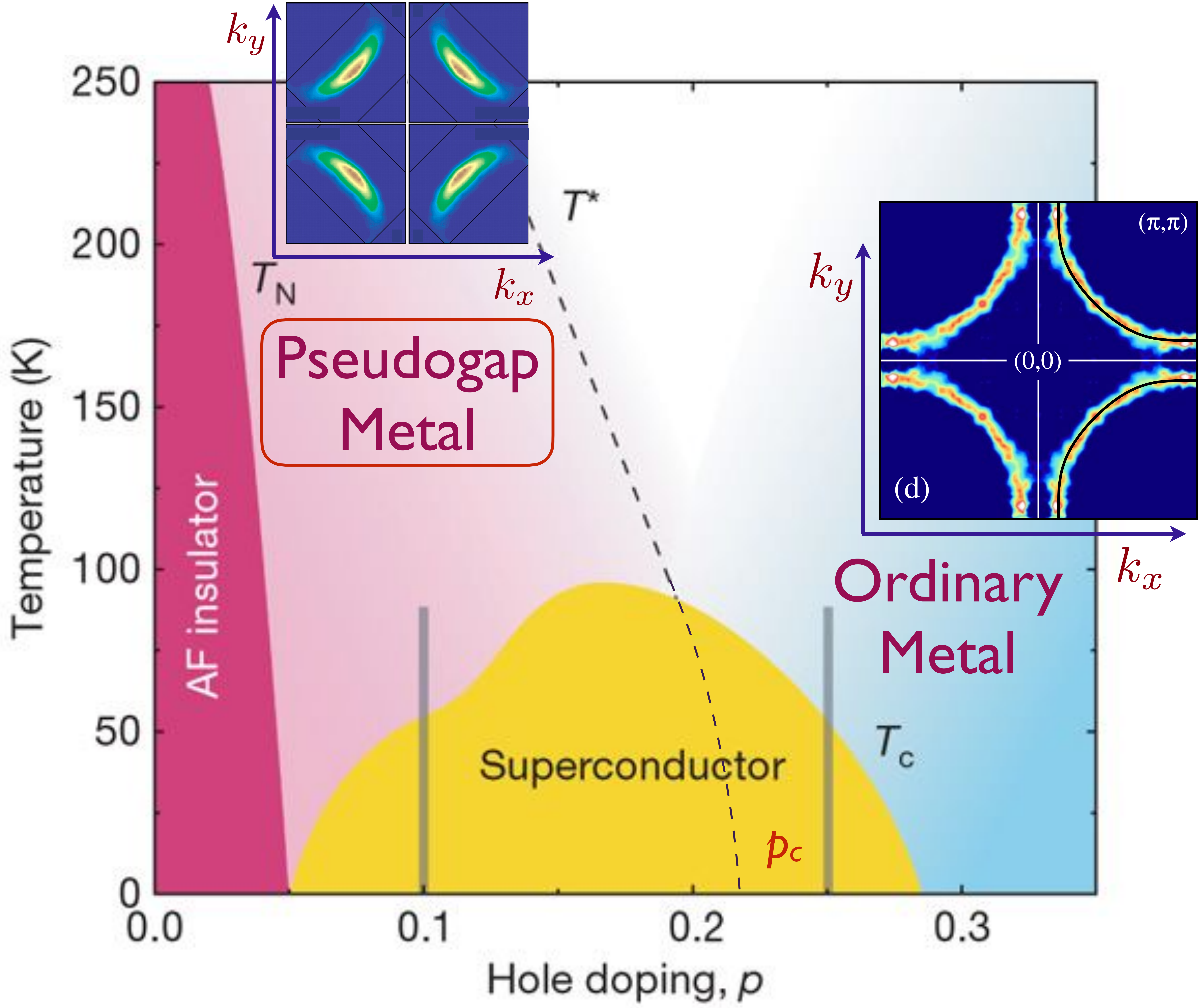
Rydberg atoms on the link-kagome lattice.

$|\Psi\rangle = \Bigg|$  $\Bigg\rangle + \Bigg|$  $\Bigg\rangle + \ldots$

Evidence for
$\mathbb{Z}_2$ spin liquid
correlations

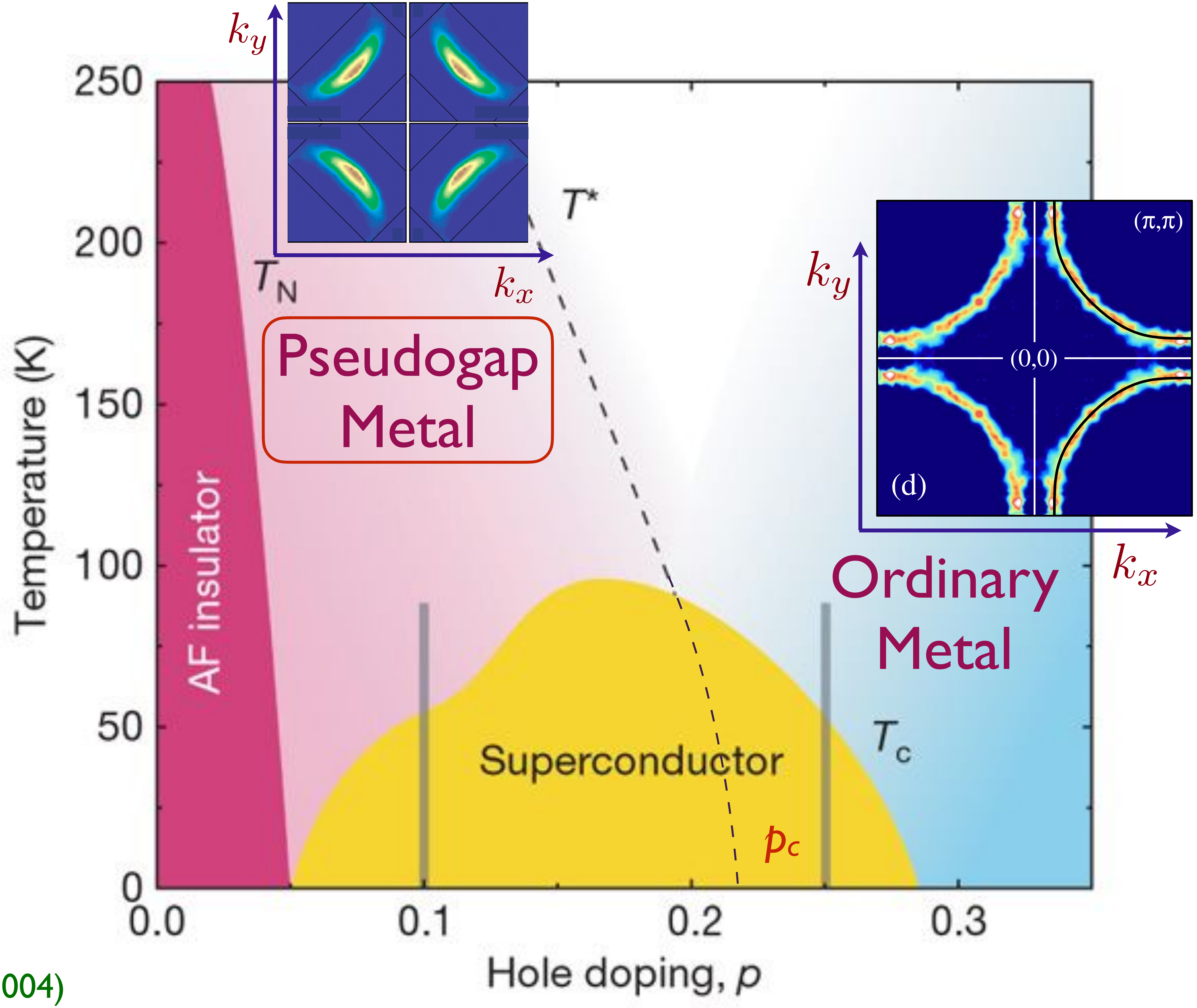
Square lattice spin liquid
and the
cuprate pseudogap metal

“Pseudogap metal”
Fermi surface
modified by
electron-electron
interactions



“Pseudogap metal”
Fermi surface
modified by
electron-electron
interactions

Non-Luttinger area
requires
fractionalized
spin liquid
background



T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)

Anisotropic damping and wave vector dependent susceptibility of the spin fluctuations in $La_{2-x}Sr_xCuO_4$ studied by resonant inelastic x-ray scattering

H. C. Robarts, M. Barthélemy, K. Kummer, M. García-Fernández, J. Li, A. Nag, A. C. Walters, K. J. Zhou, and S. M. Hayden

PHYSICAL REVIEW B **100**, 214510 (2019)

- Spin waves in insulator broaden into a continuum:

  Most natural interpretation is a spinon continuum, similar to that observed on the triangular lattice in $KYbSe_2$

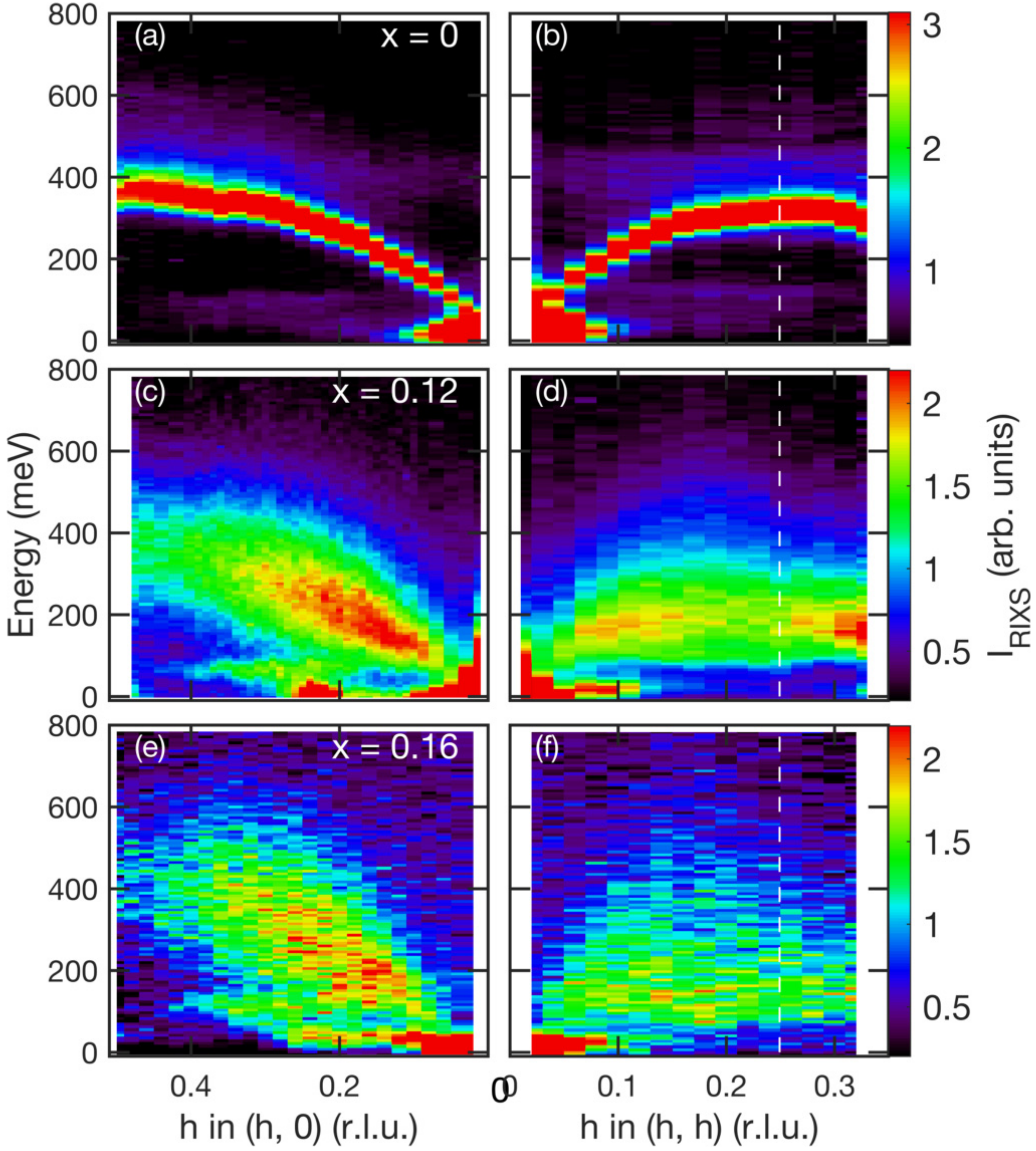
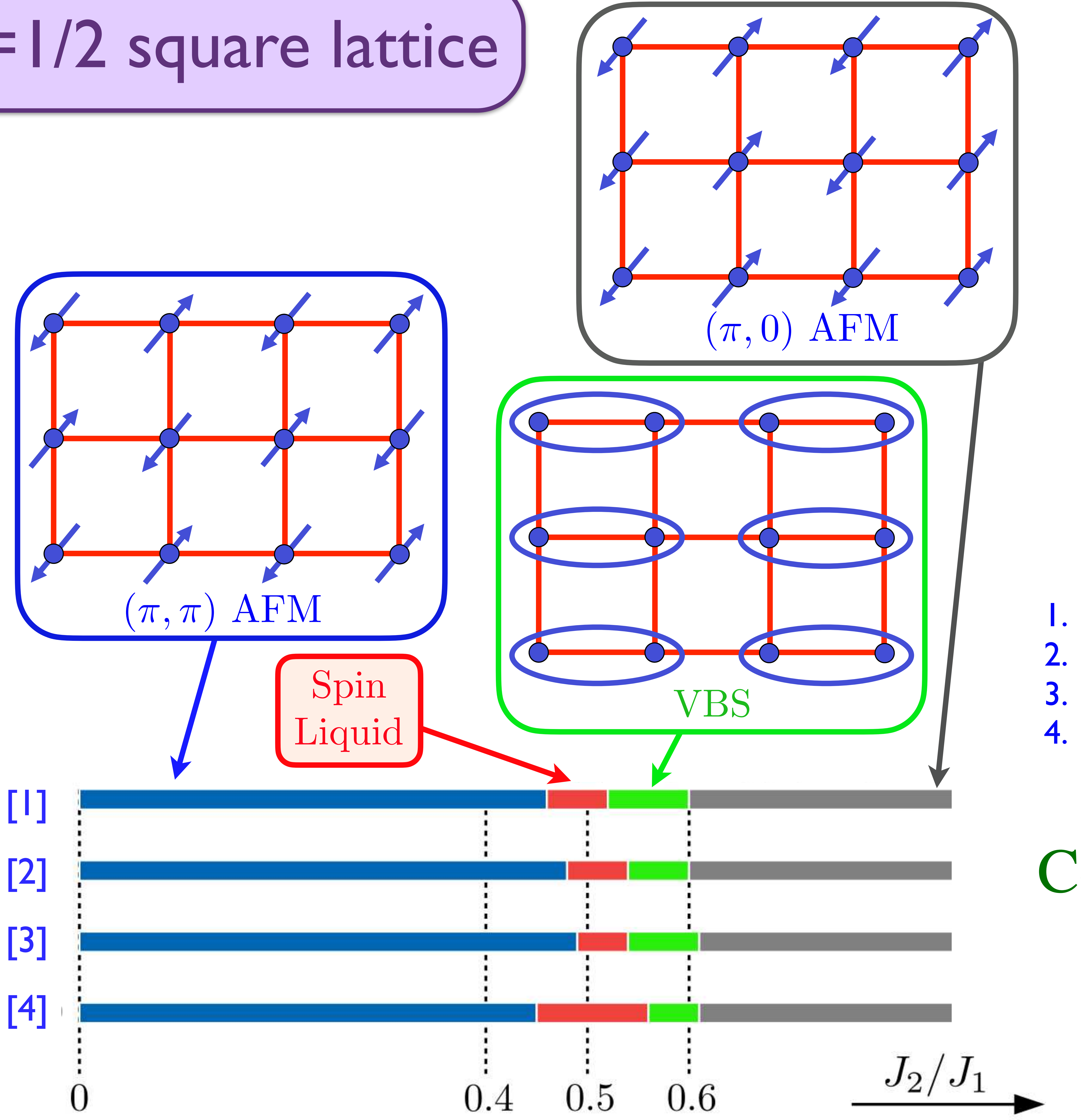


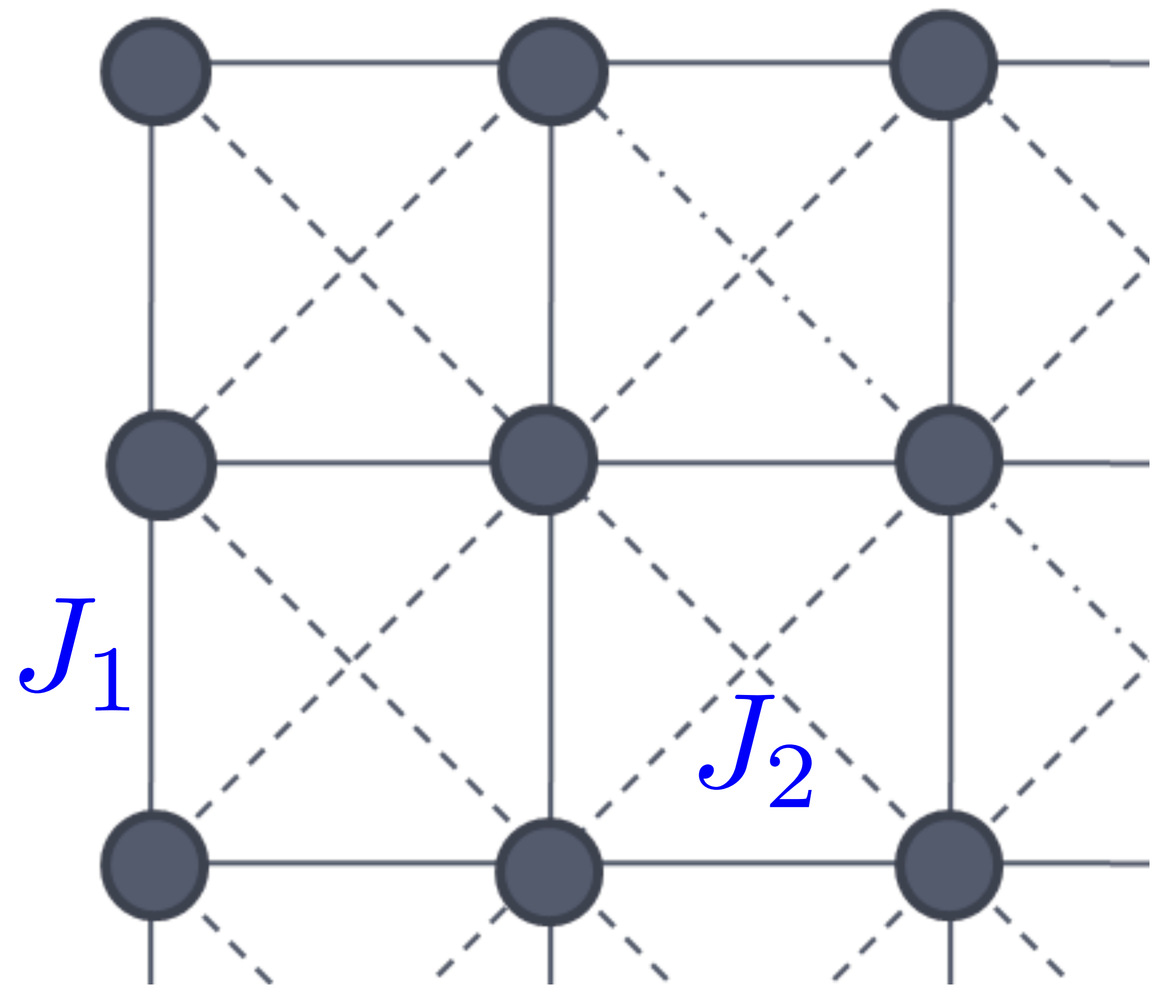
FIG. 2. I_{RIXS} intensity maps as a function of **Q** in LSCO $x = 0$ ($T \approx 20$ K), 0.12, and 0.16 ($T \approx 30$ K).

Pietro Bonetti, Maine Christos and S.S. (BCS), PNAS **121**, e2418633121 (2024)

S=1/2 square lattice



$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



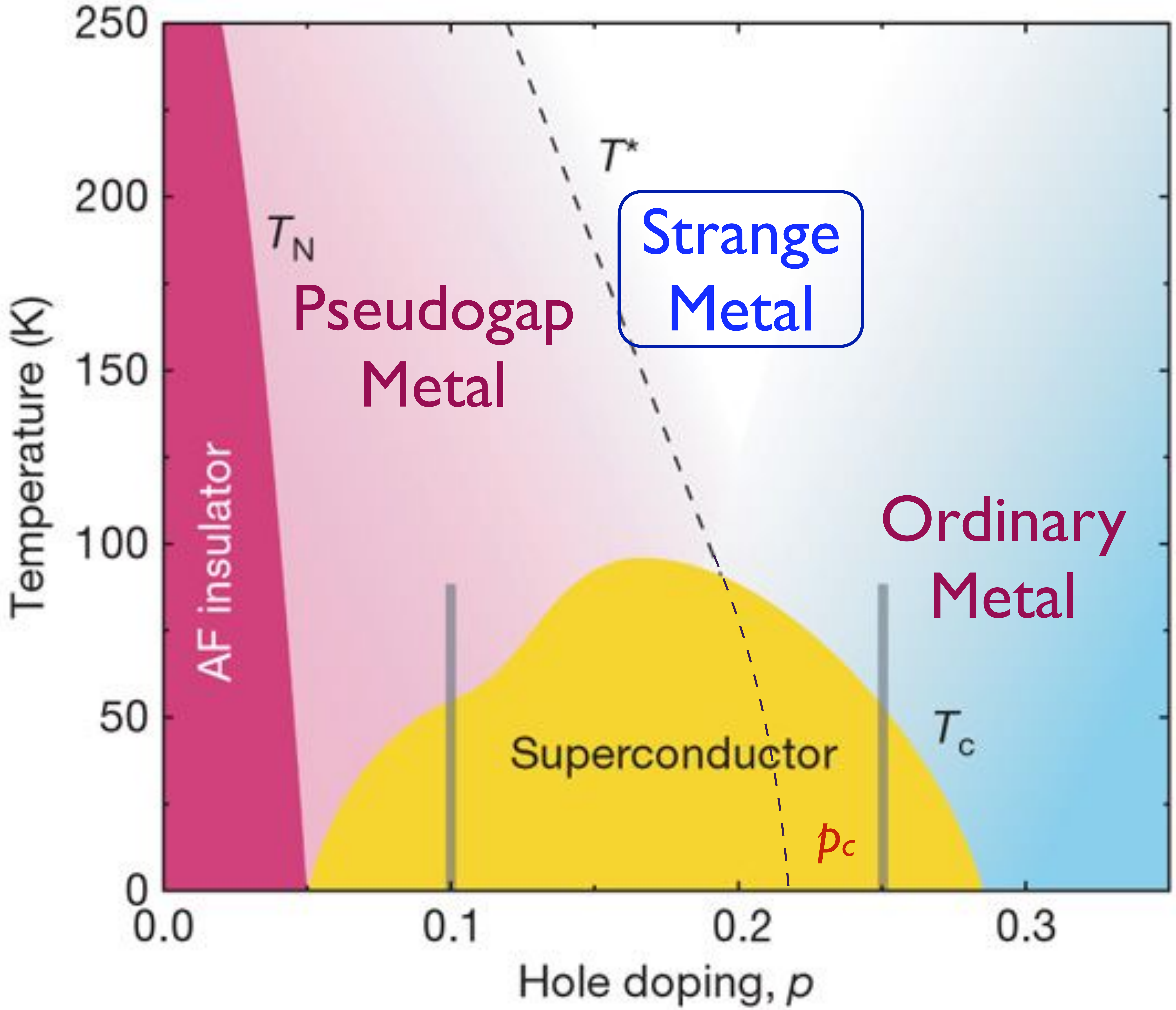
1. L. Wang and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67**, 1034 (2022)

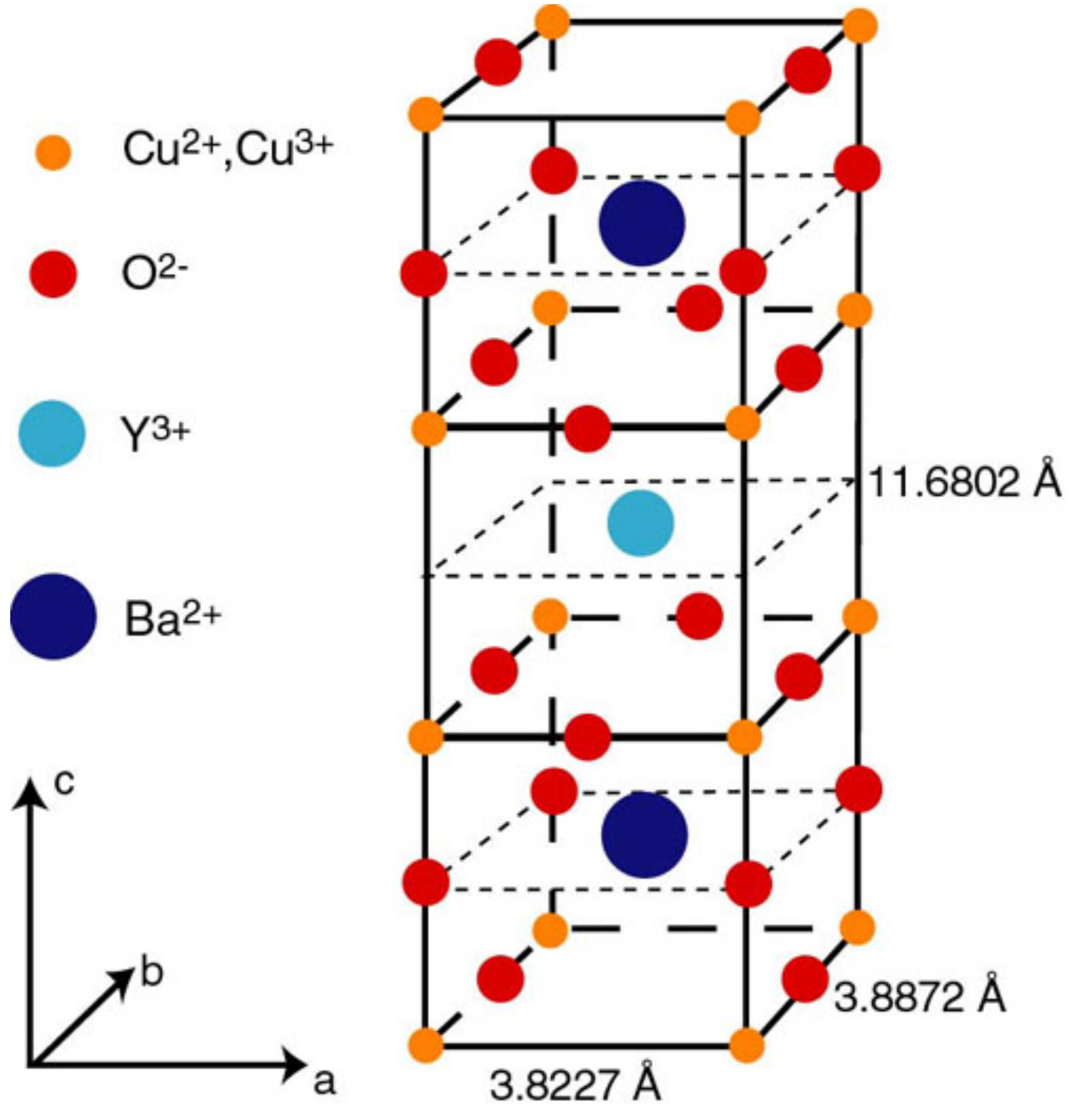
Consistent with U(1) gauge theory of spinons and monopoles

N. Read and S. S., PRL **62**, 1694 (1989)

Entanglement of mobile electrons.

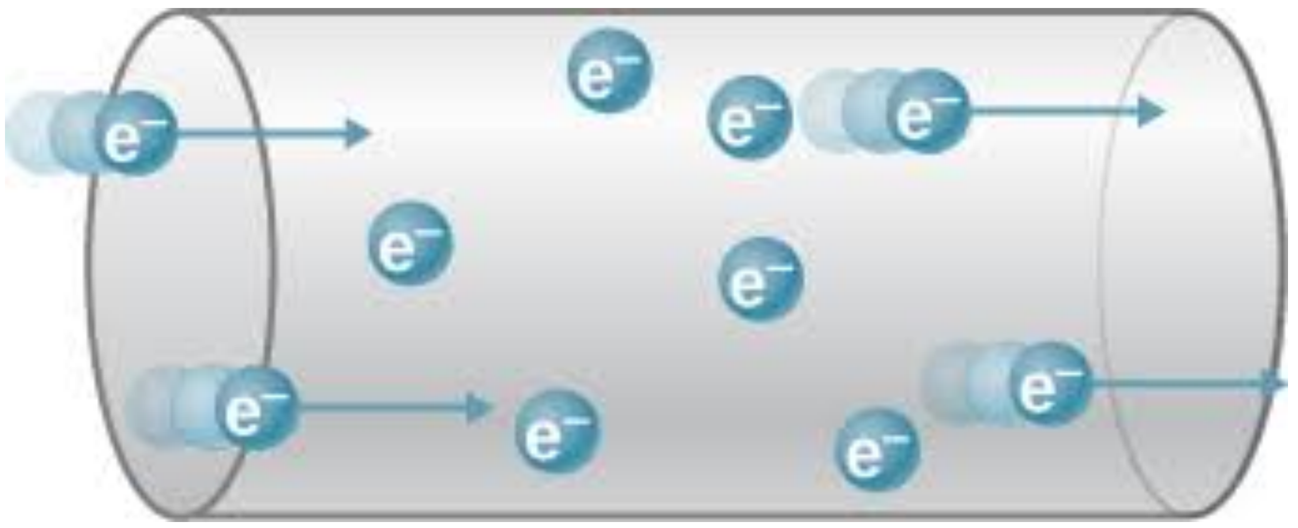
Metals without quasiparticles: the SYK model


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
Pseudogap Metal
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/(UT)^2$ (U is the strength of interactions),
resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

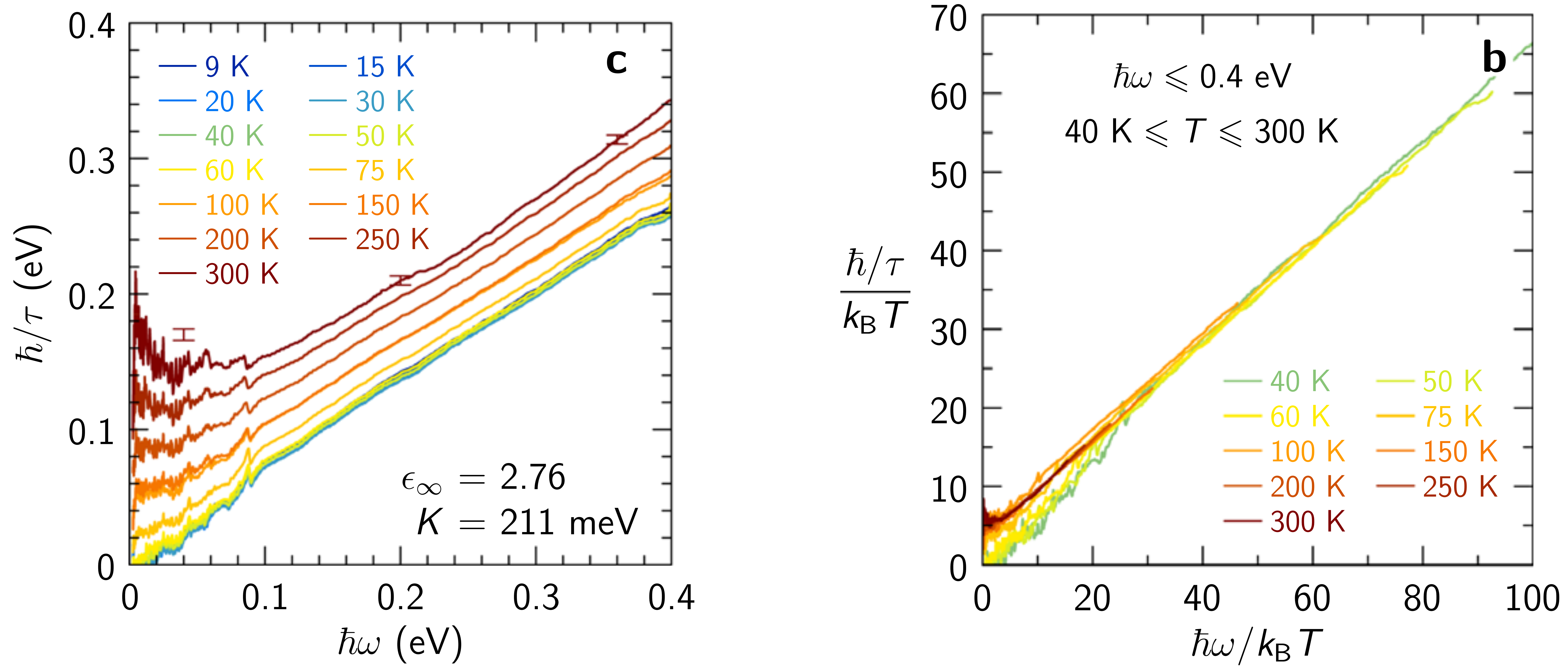
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!

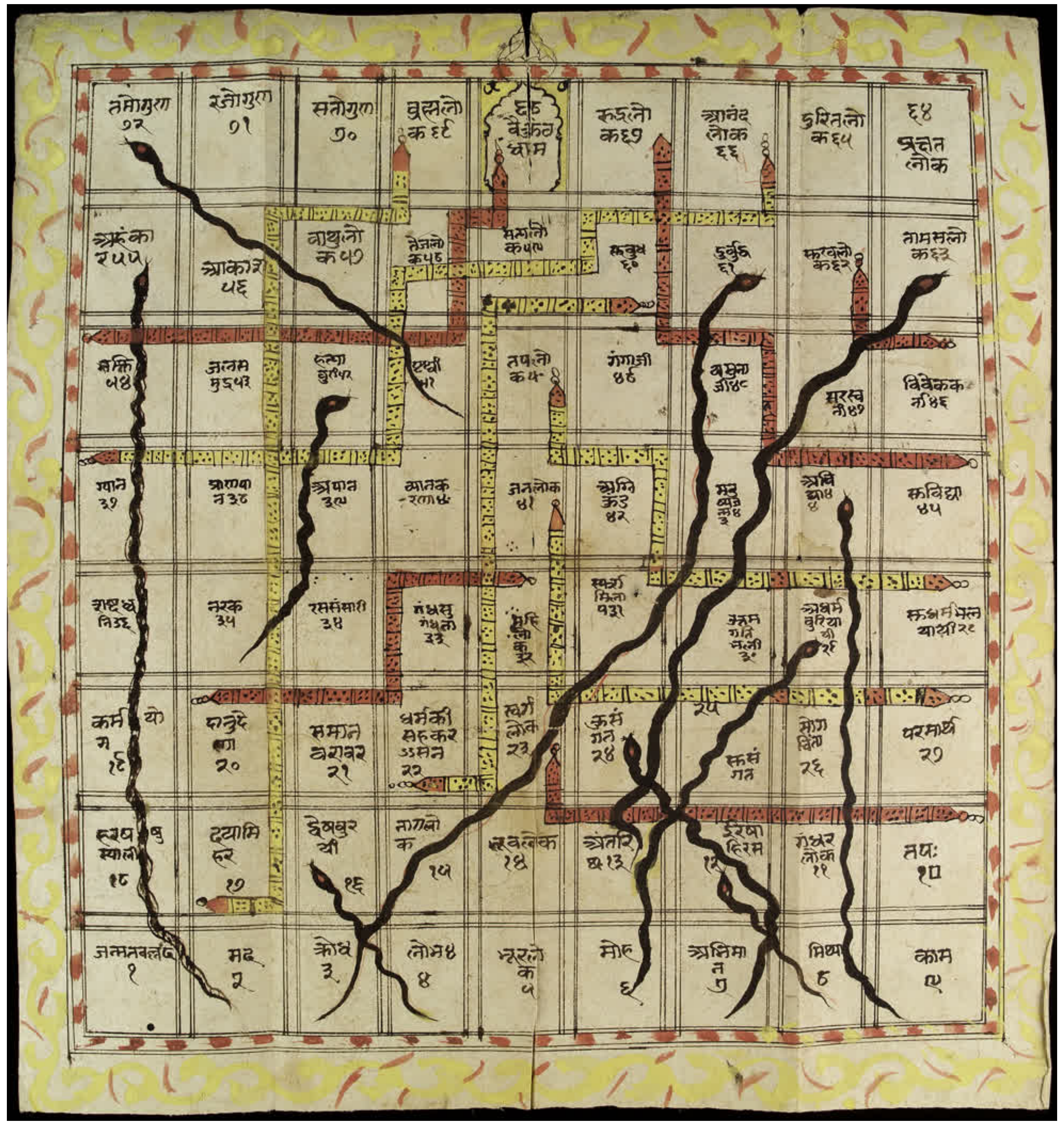
$$\tau(\omega) = \frac{\hbar}{k_BT}F\left(\frac{\hbar\omega}{k_BT}\right)$$



Needed:
a solvable model of quantum entanglement in a metal

The Sachdev-Ye-Kitaev model of many-particle entanglement

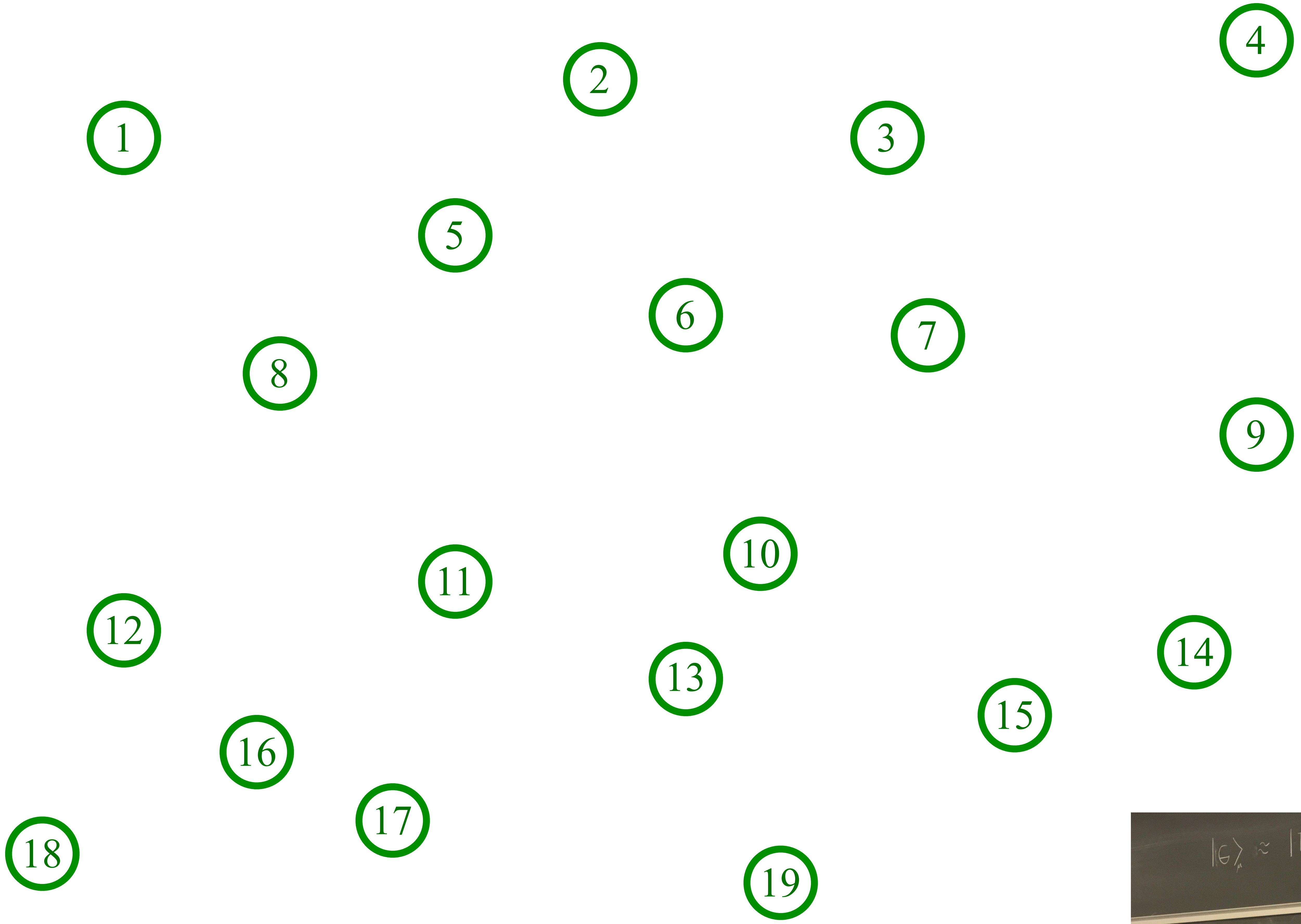
My spooky dream (1992)*

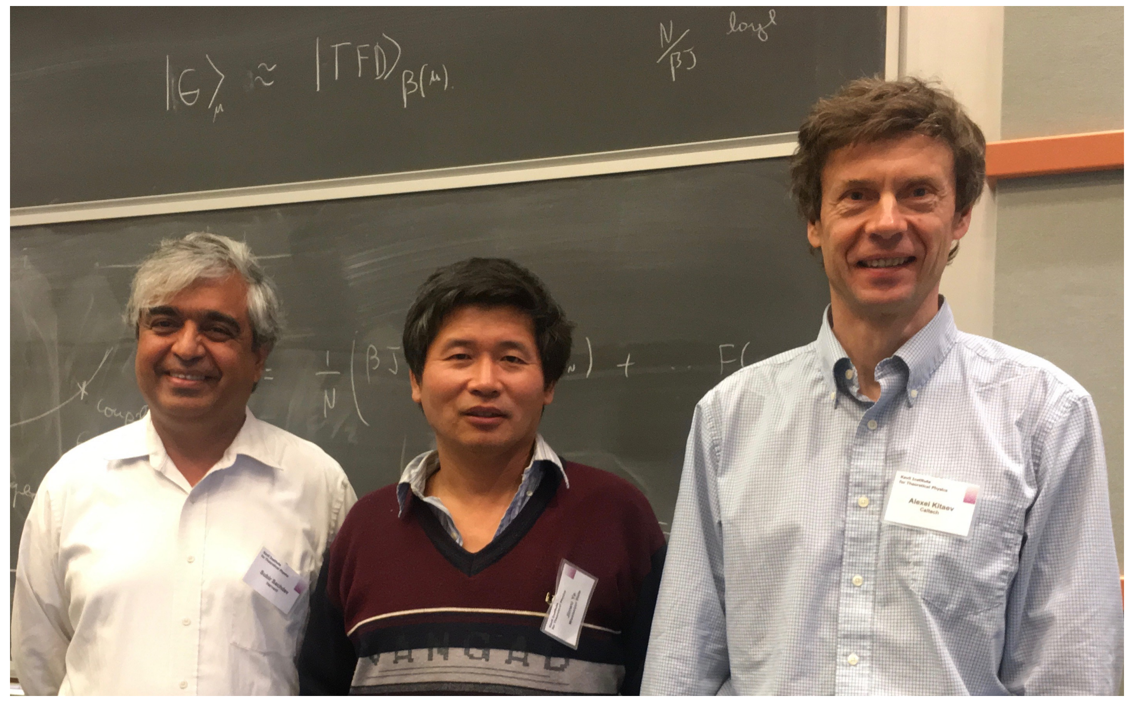


*not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

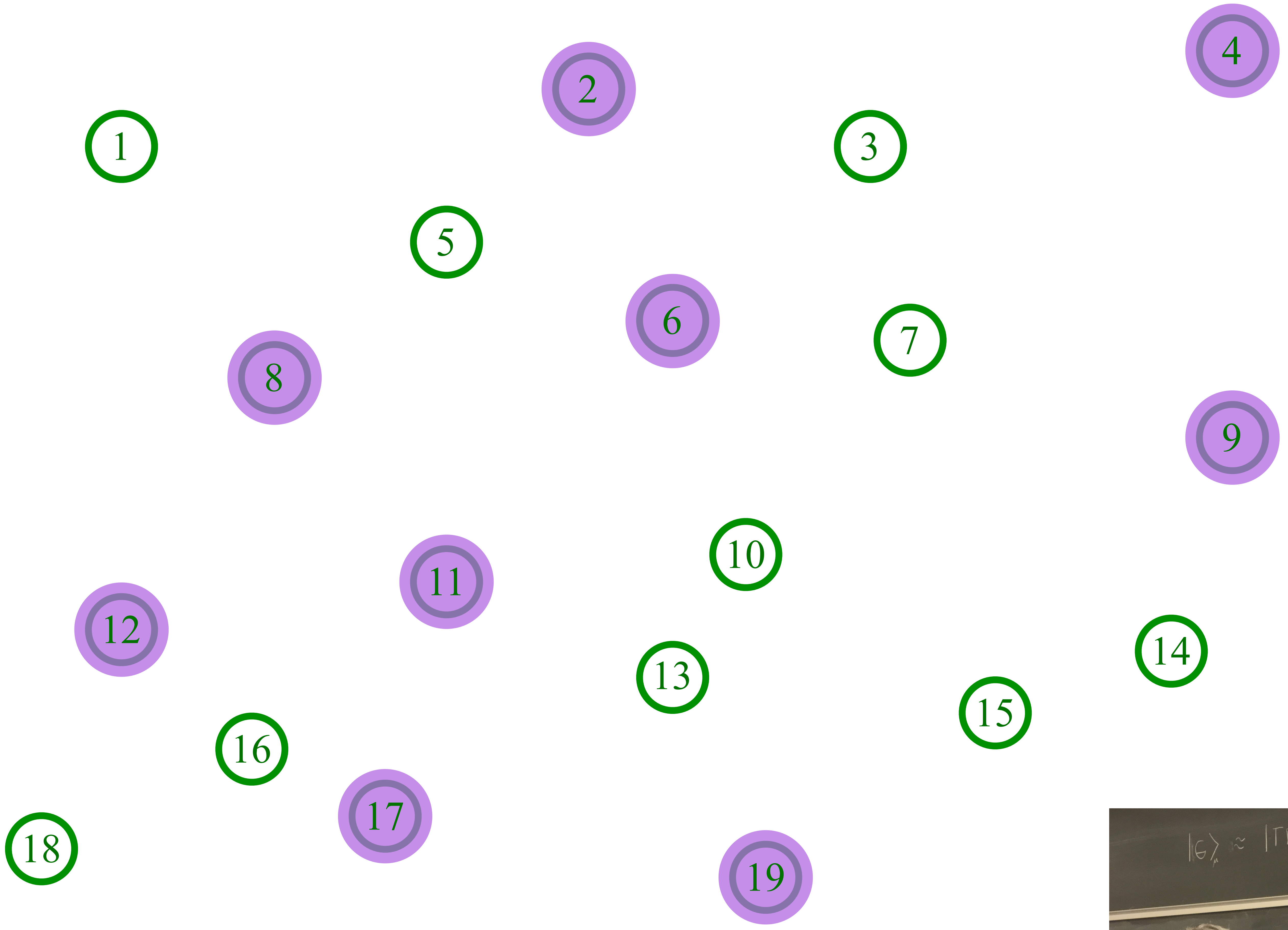


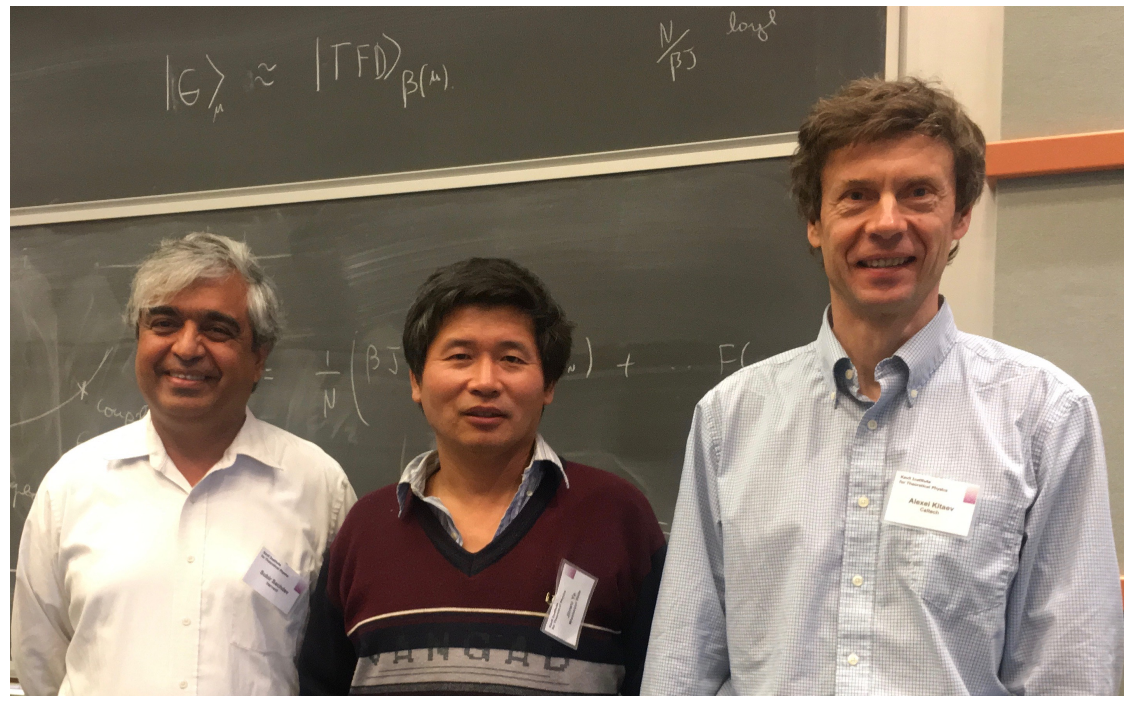


Pick a set of random positions

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



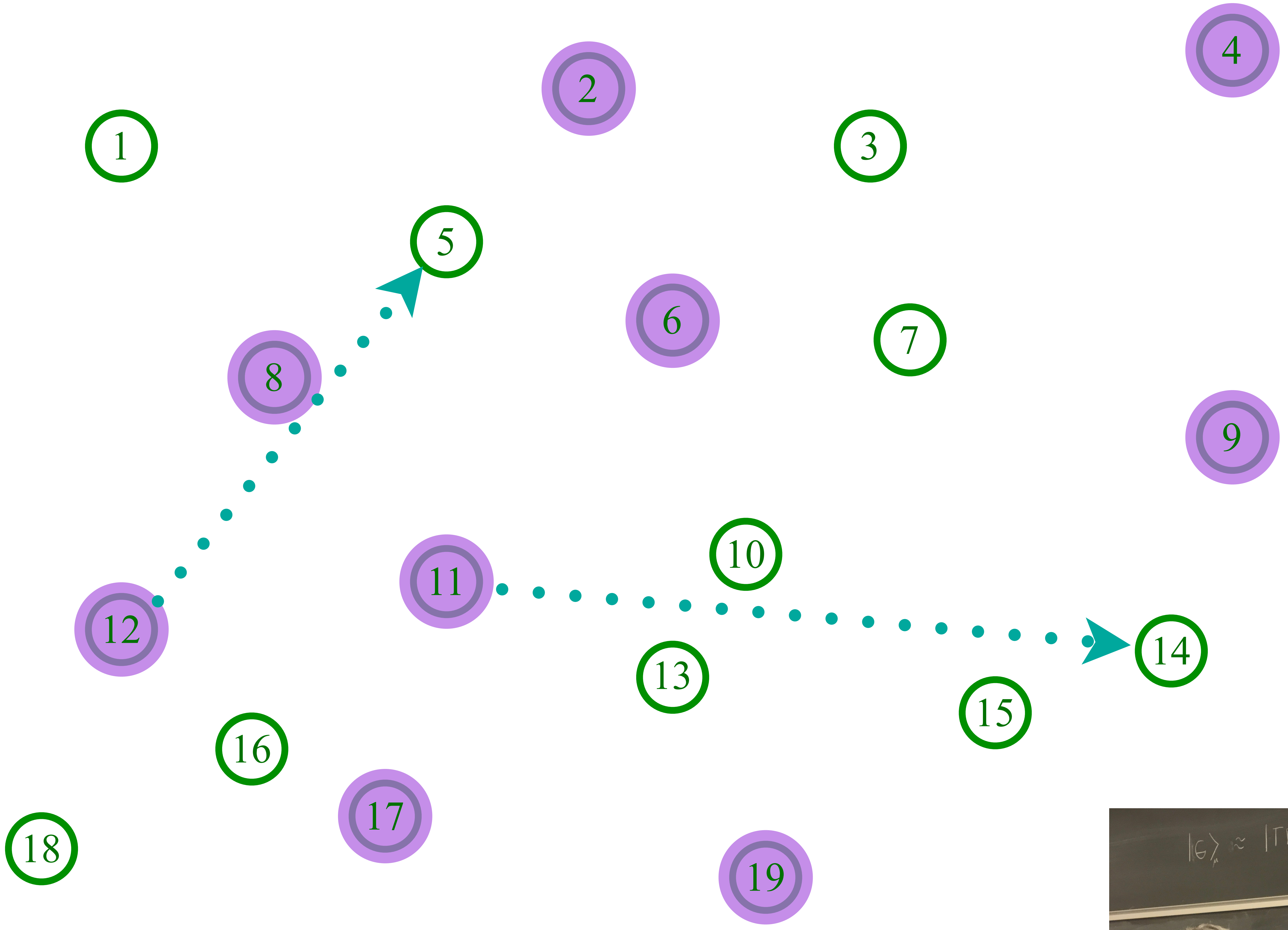


Place electrons randomly on some sites

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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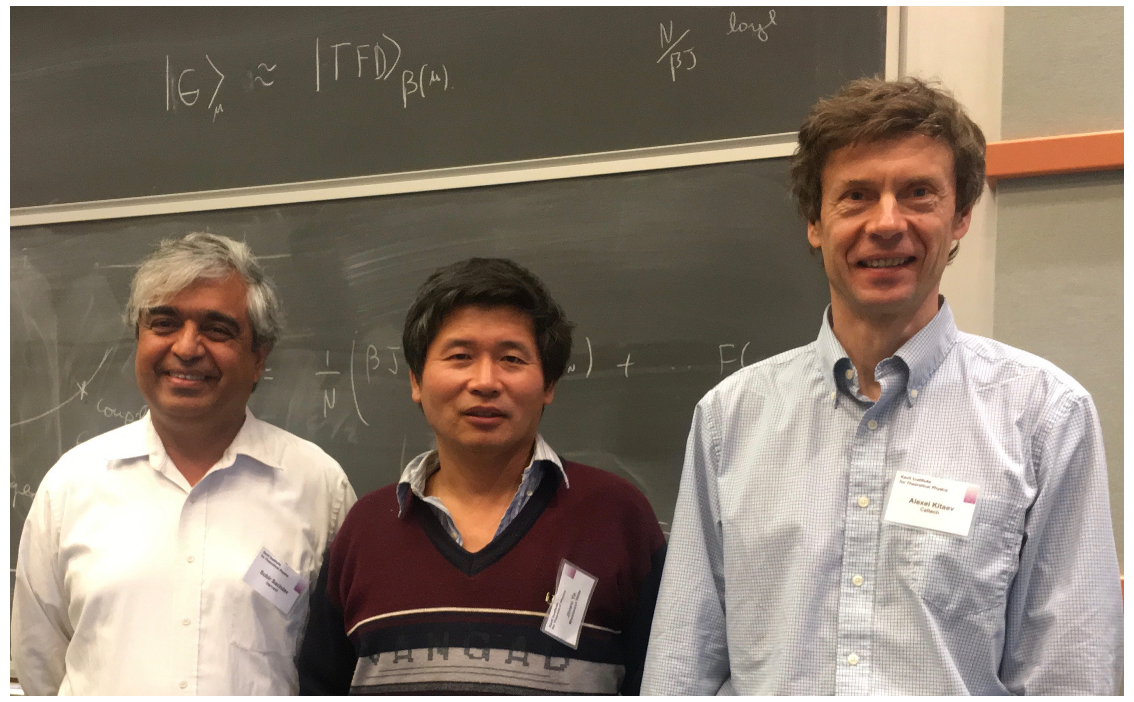

The Sachdev-Ye-Kitaev (SYK) model

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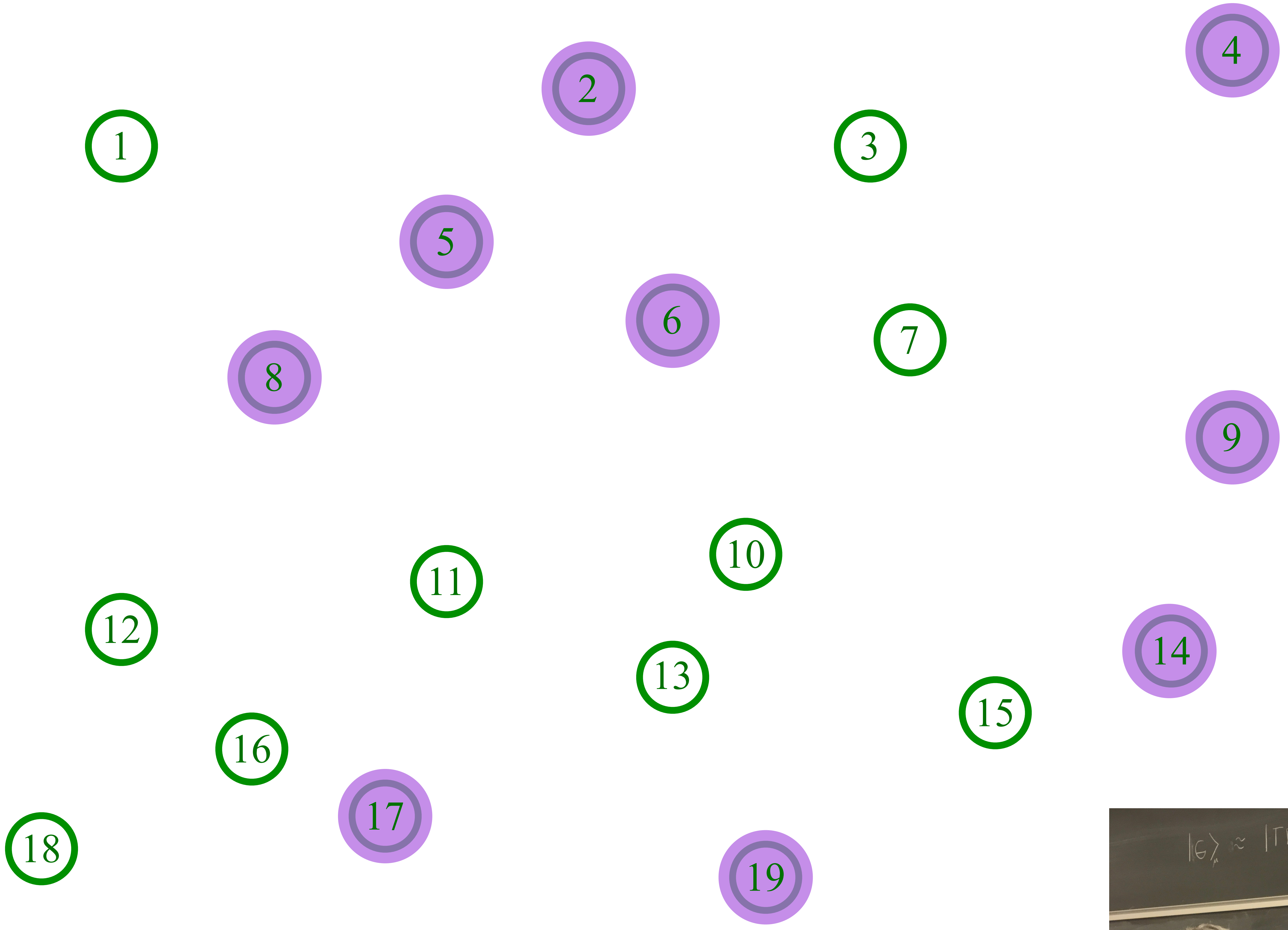
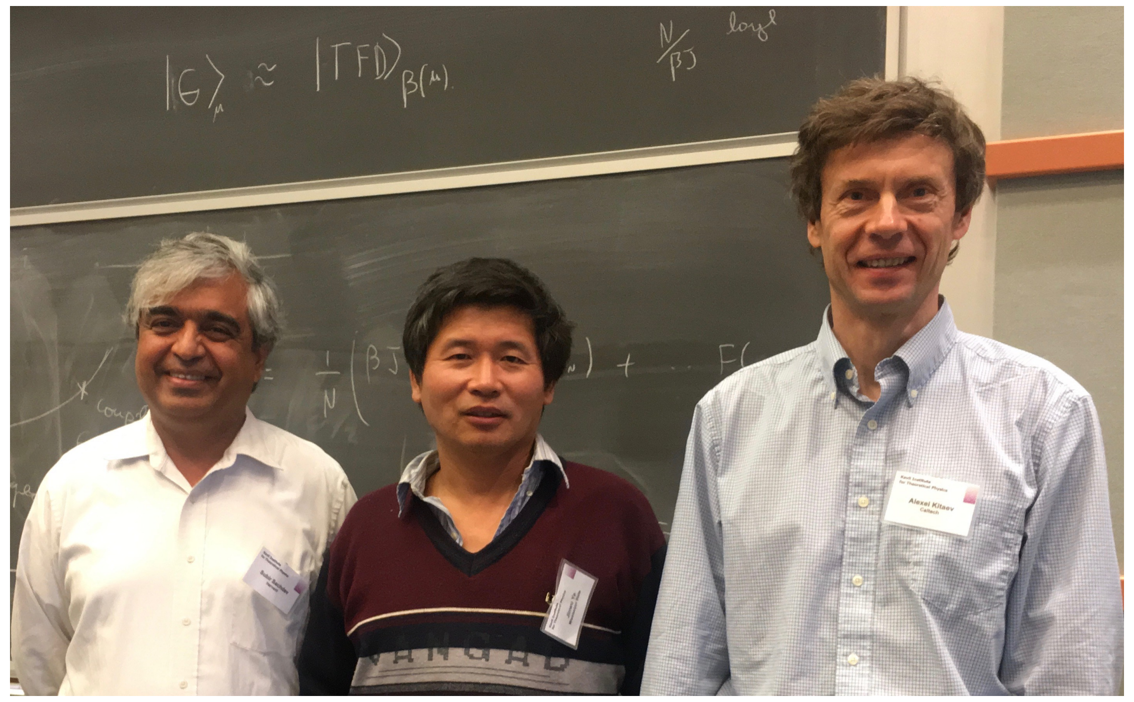


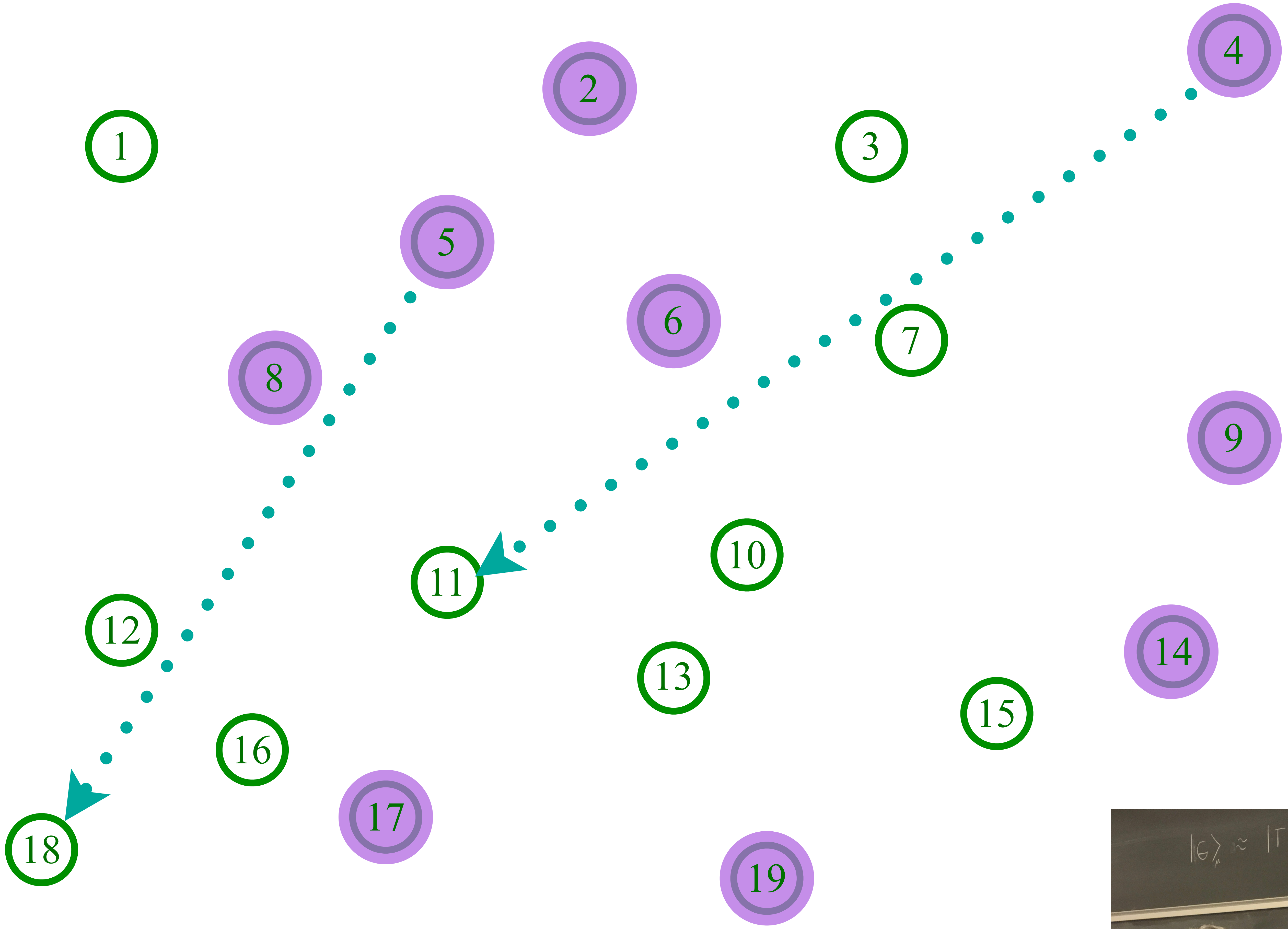
Entangle electrons pairwise randomly



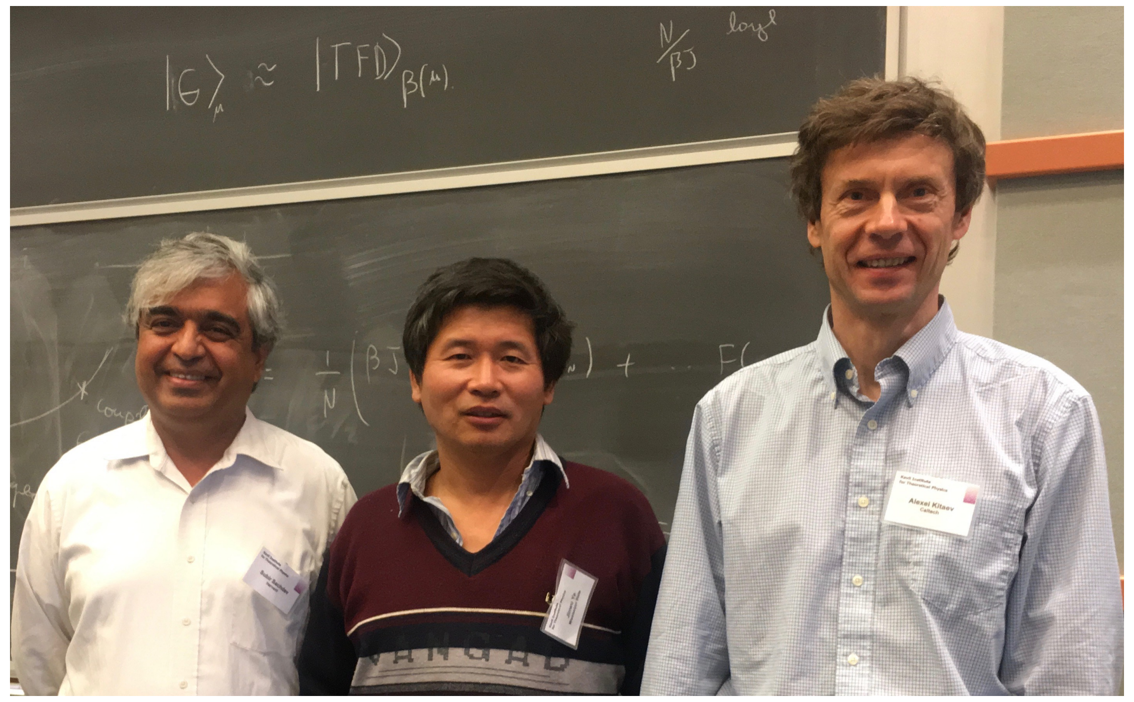
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$

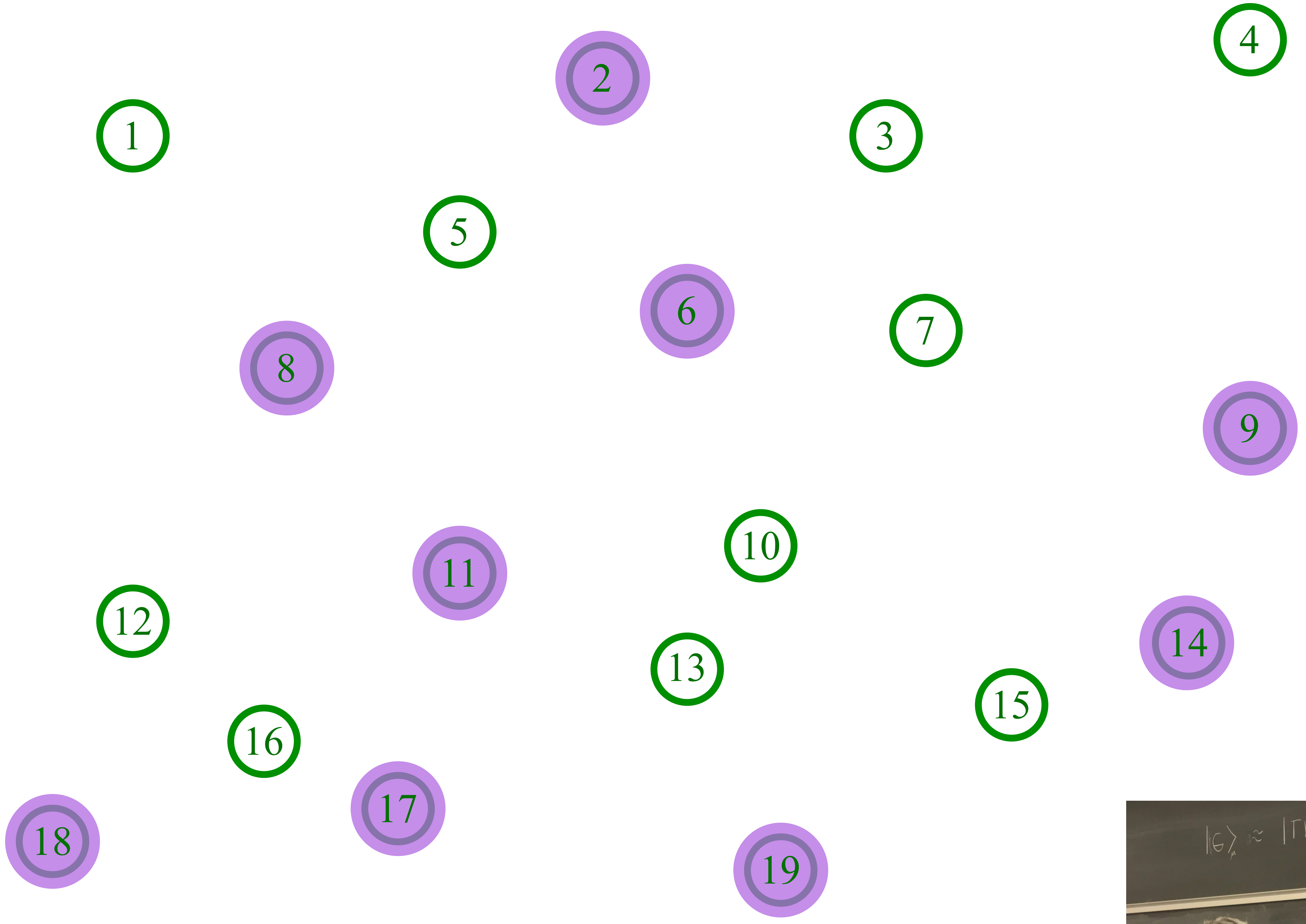


Entangle electrons pairwise randomly



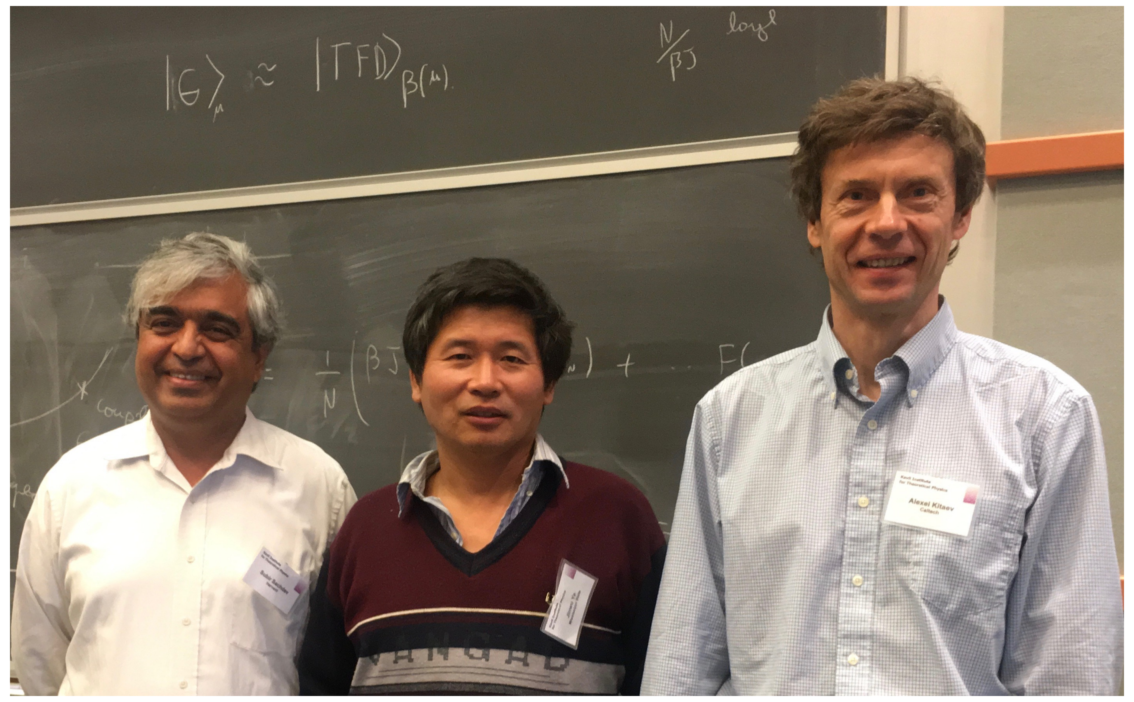
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



$$U_{4,5;11,18}$$

Entangle electrons pairwise randomly



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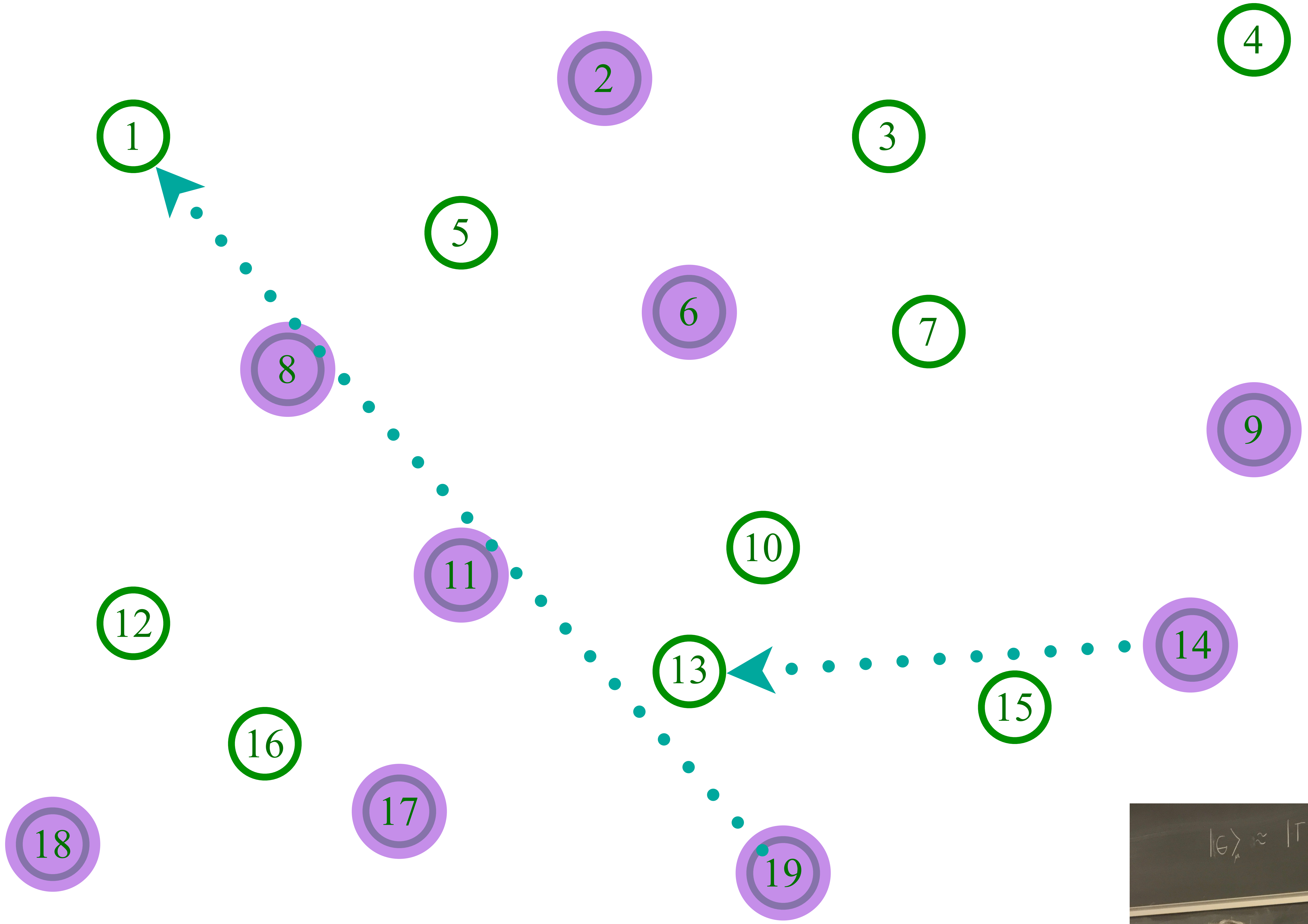
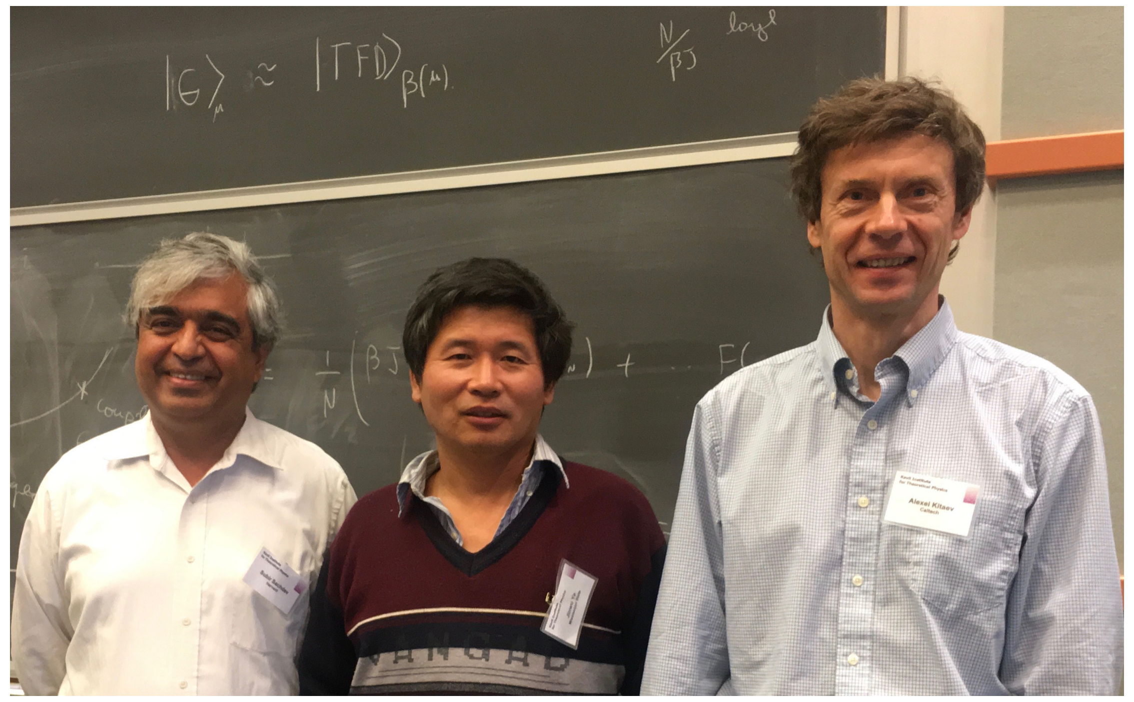
Sachdev, Ye (1993); Kitaev (2015)



$U_{14,19;1,13}$

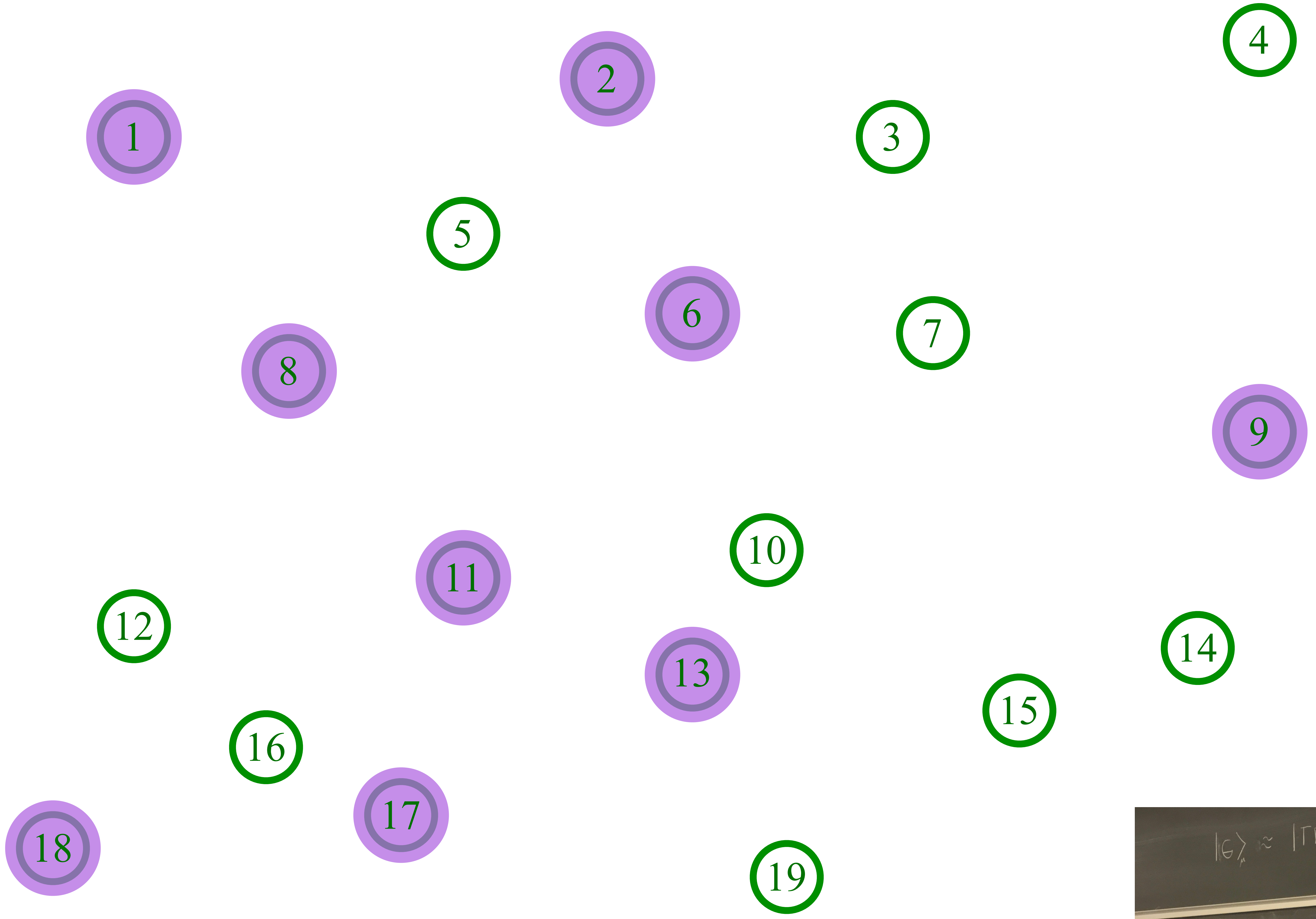
Entangle electrons pairwise randomly



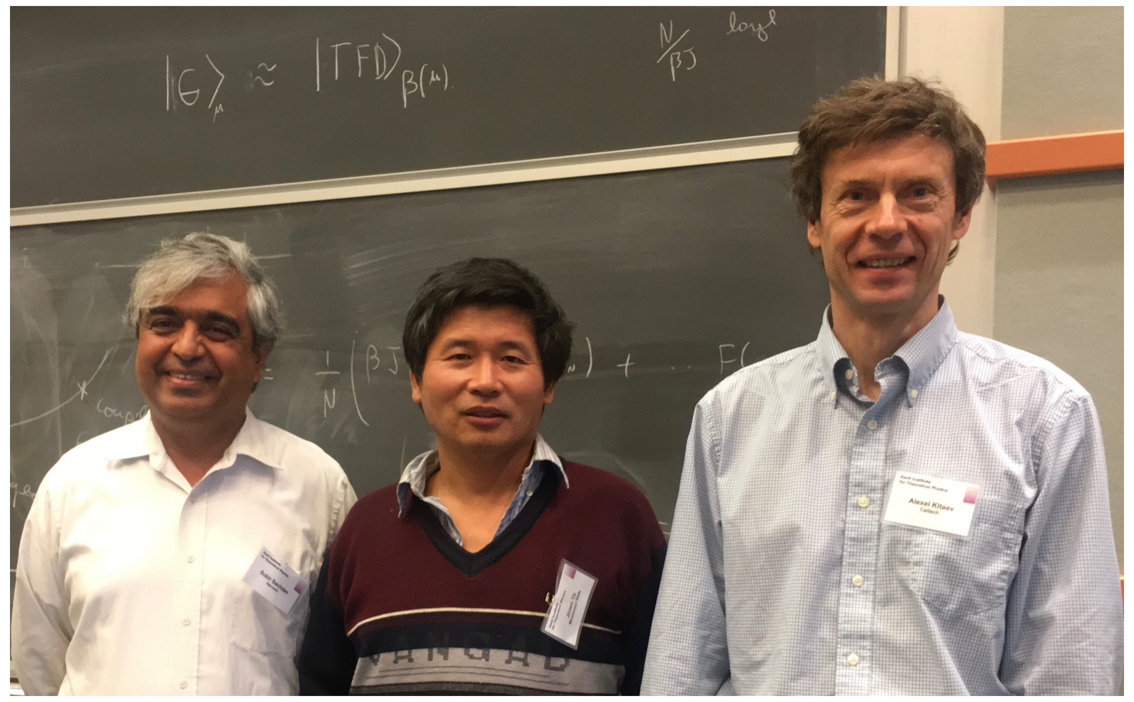
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



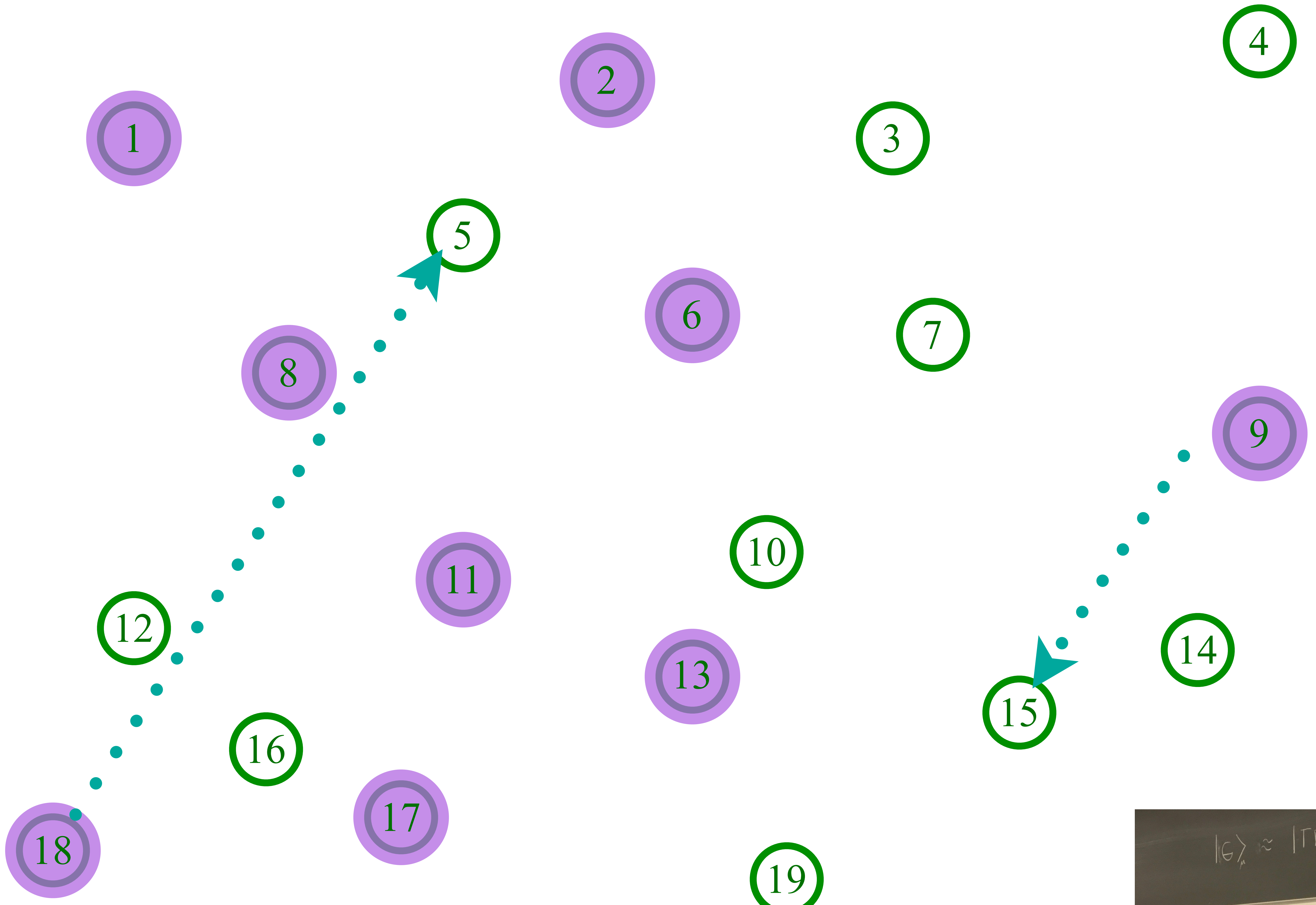
Entangle electrons pairwise randomly



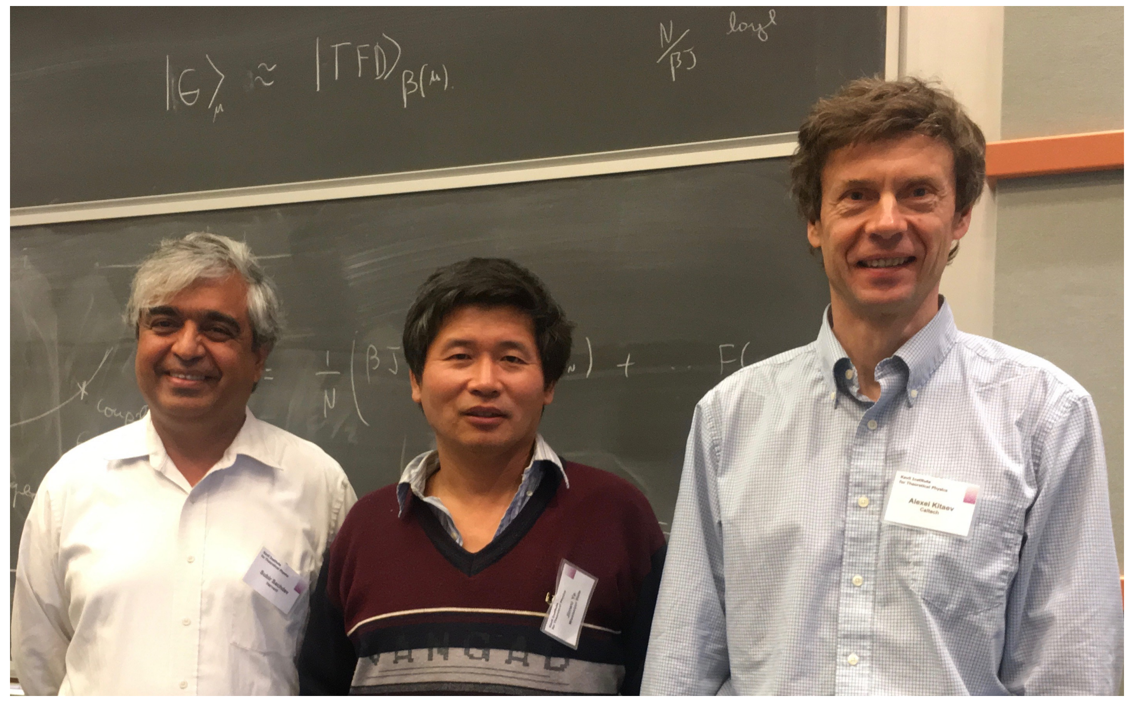
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



Entangle electrons pairwise randomly



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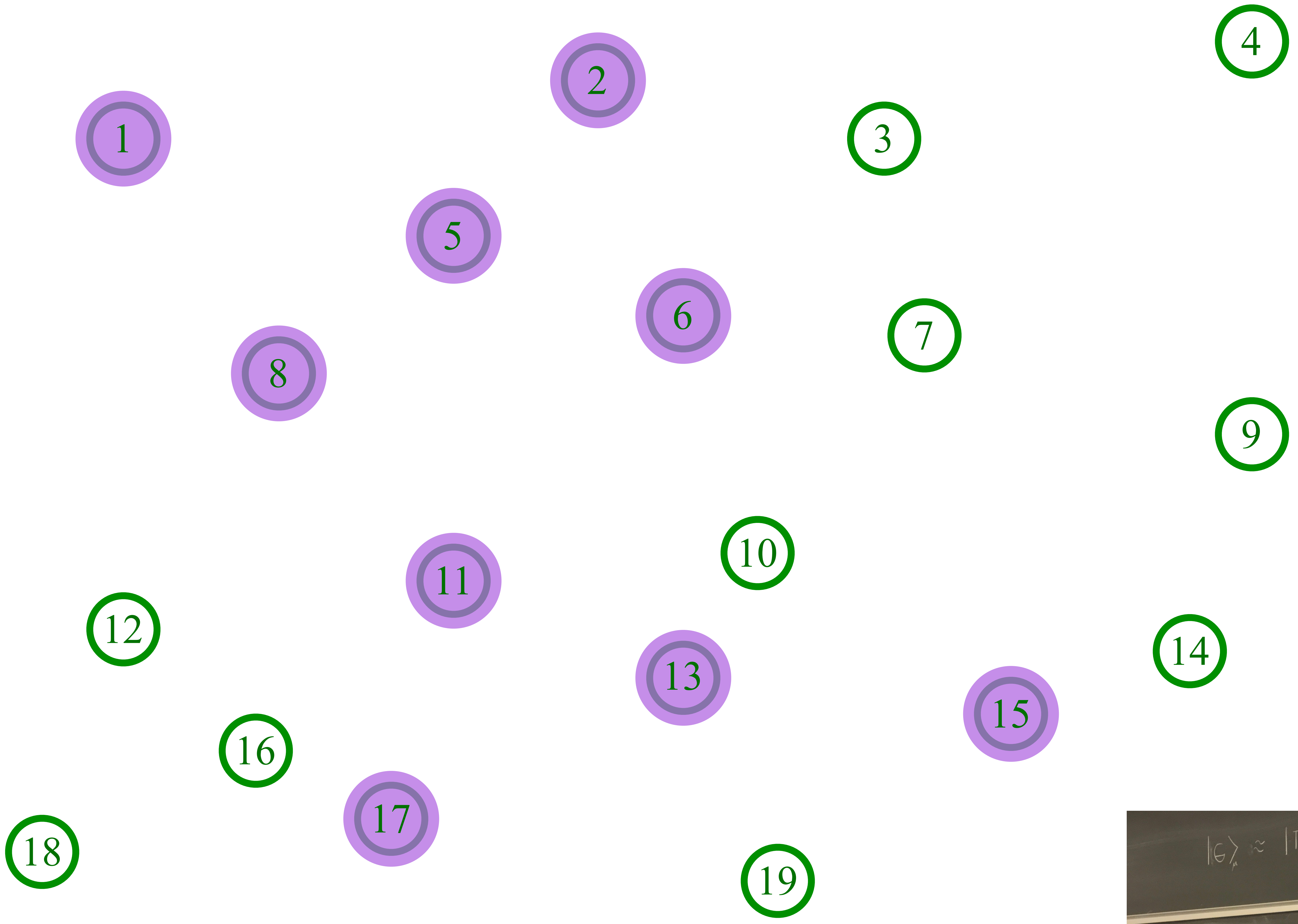
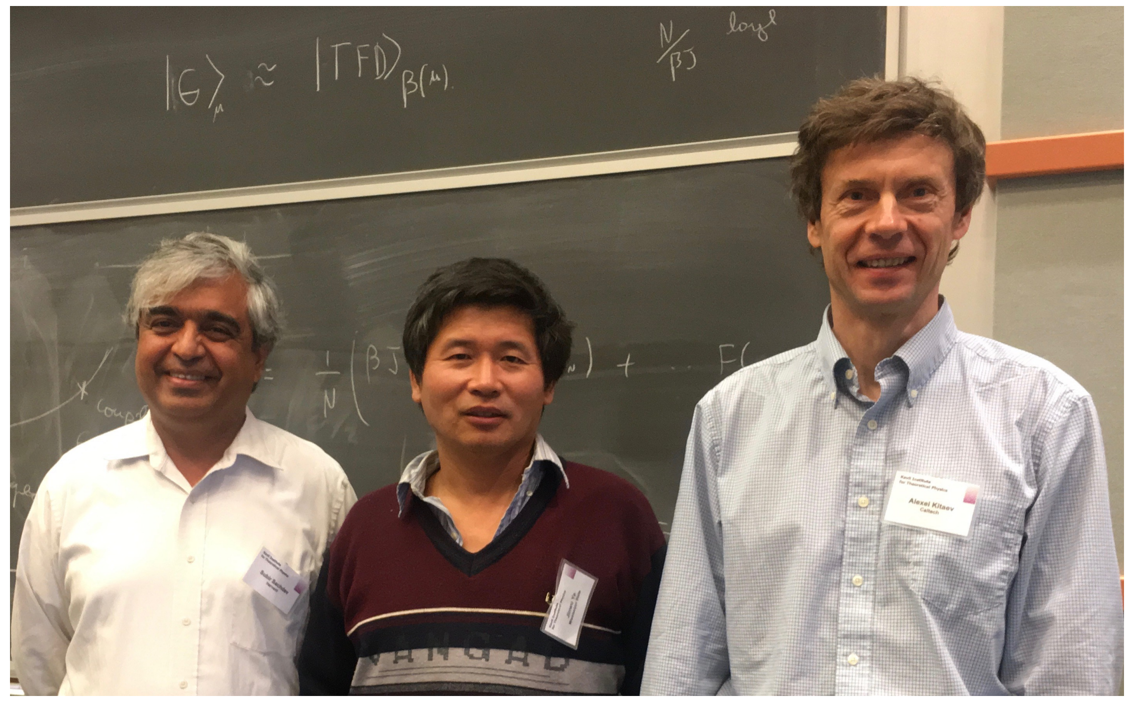


$$U_{9,18;5,15}$$

Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

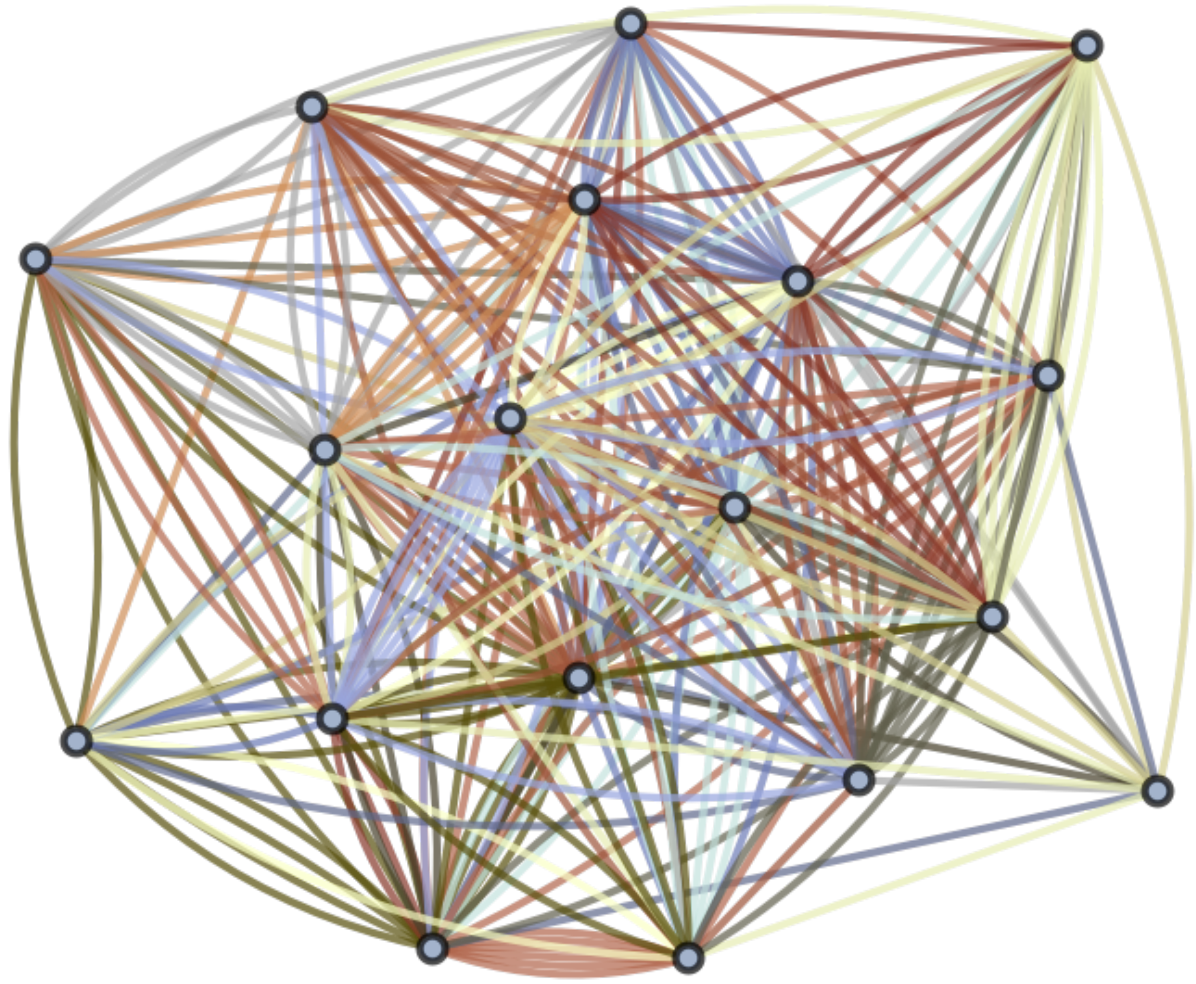
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

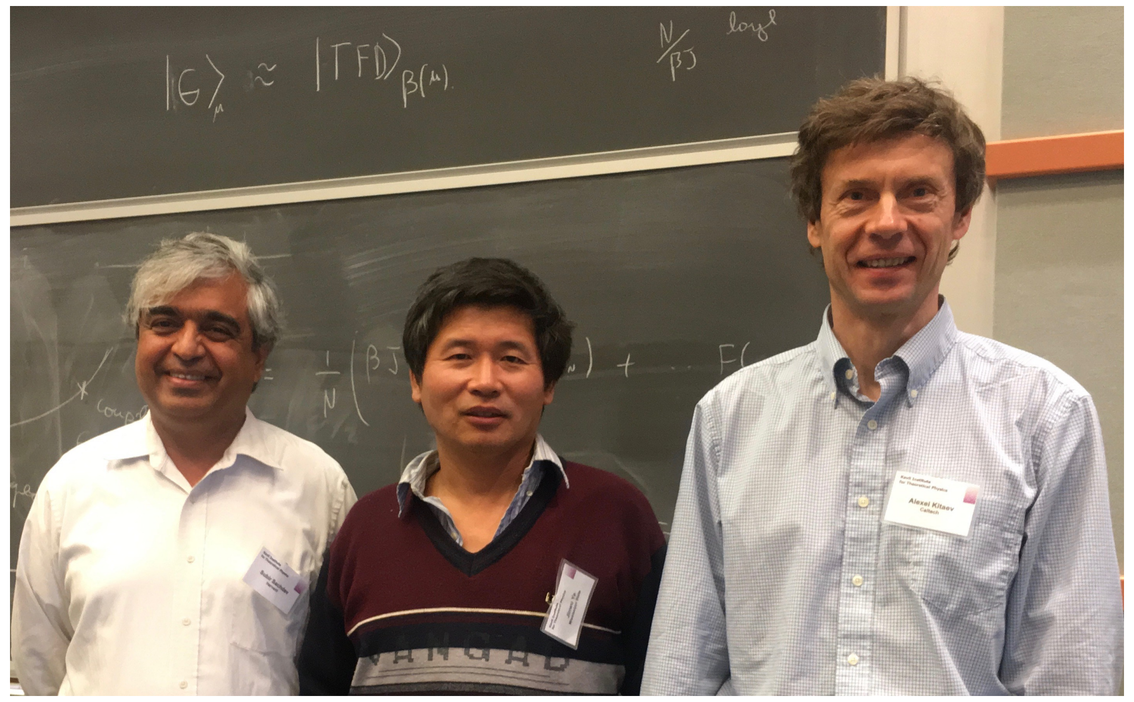
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \,; \quad [\mathcal{H}, \mathcal{Q}] = 0\,; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

No quasiparticles: yields a metal in which
current is carried
not by individual electrons,
but by an entangled "quantum soup"

The SYK model

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right) \text{ independent of } U.$$

No bosons, fermions, anyons …

S. Sachdev and J. Ye, PRL **70**, 3339 (1993); A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The SYK model

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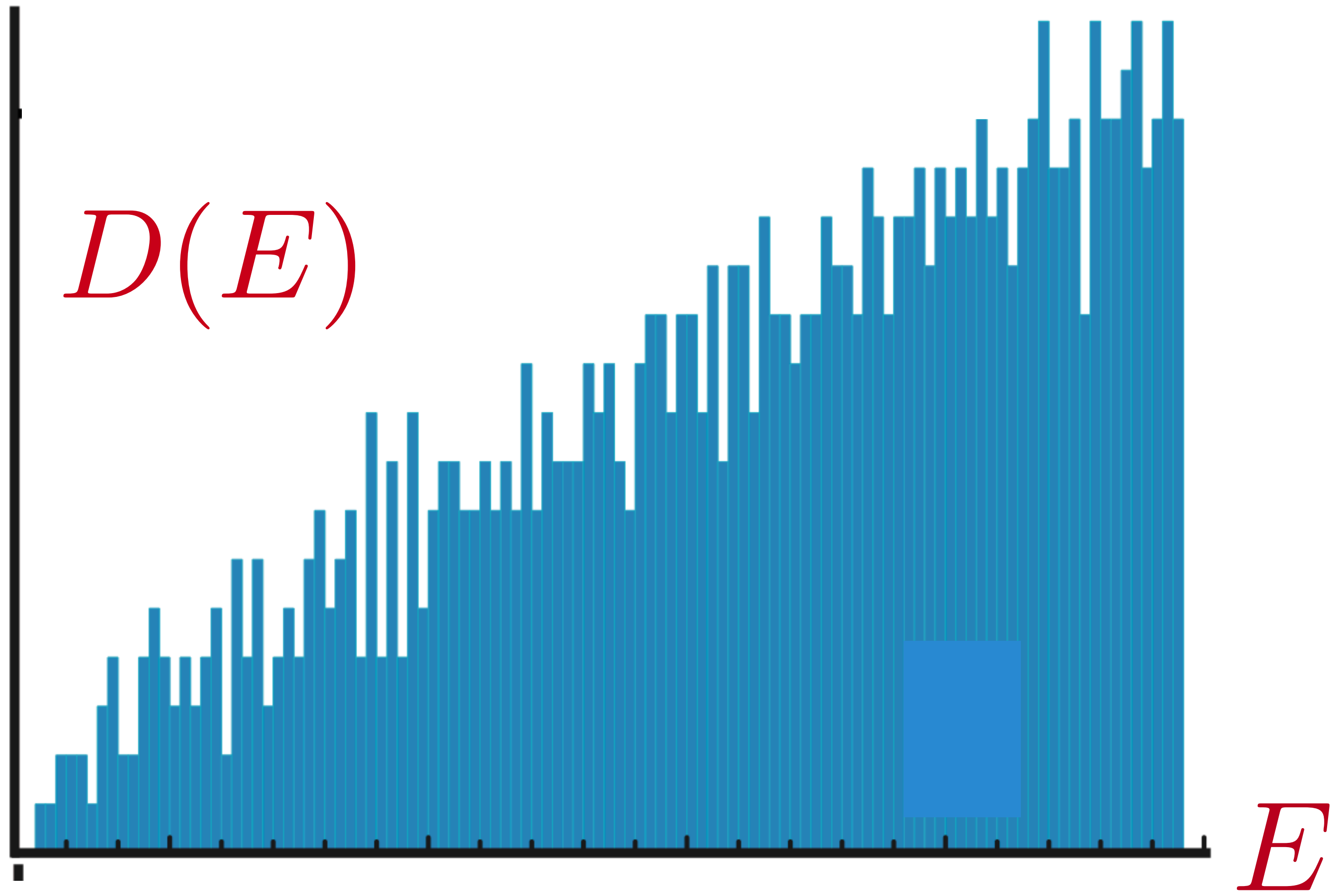
No bosons, fermions, anyons …

2. Zero temperature entropy without exponential ground state degeneracy!



$$\lim_{T\to 0}\lim_{N\to\infty}\frac{1}{N}S(T) = s_0 \quad , \quad D(E \to 0) = e^{N s_0}\sinh(\sqrt{2N\gamma E})$$

$$s_0 = 0.4648476991708051074 9.... \text{ for } \mathcal{Q} = 1/2.$$

A. Georges, O. Parcollet, and S. Sachdev (**GPS**), Physical Review B **63**, 134406 (2001)

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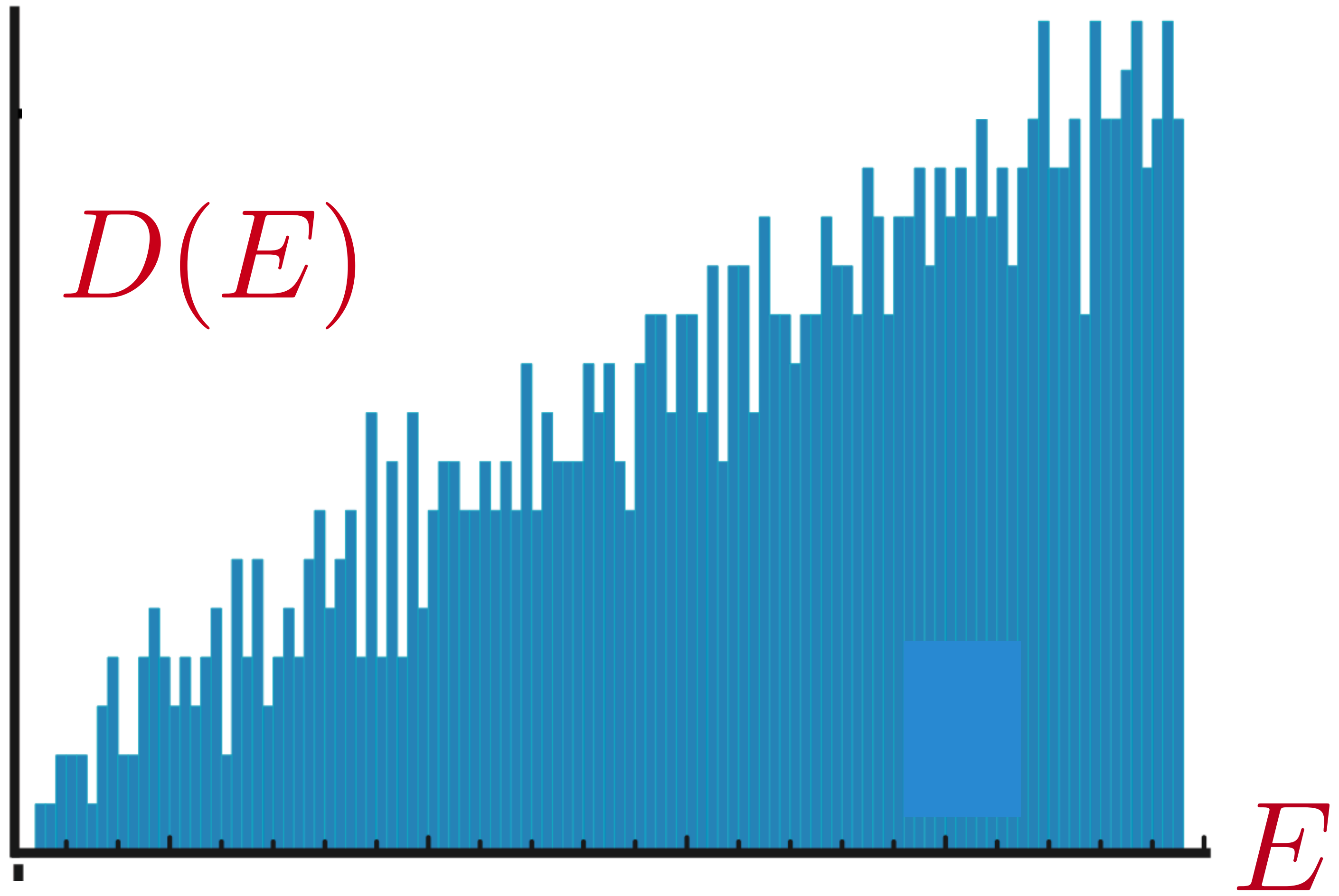

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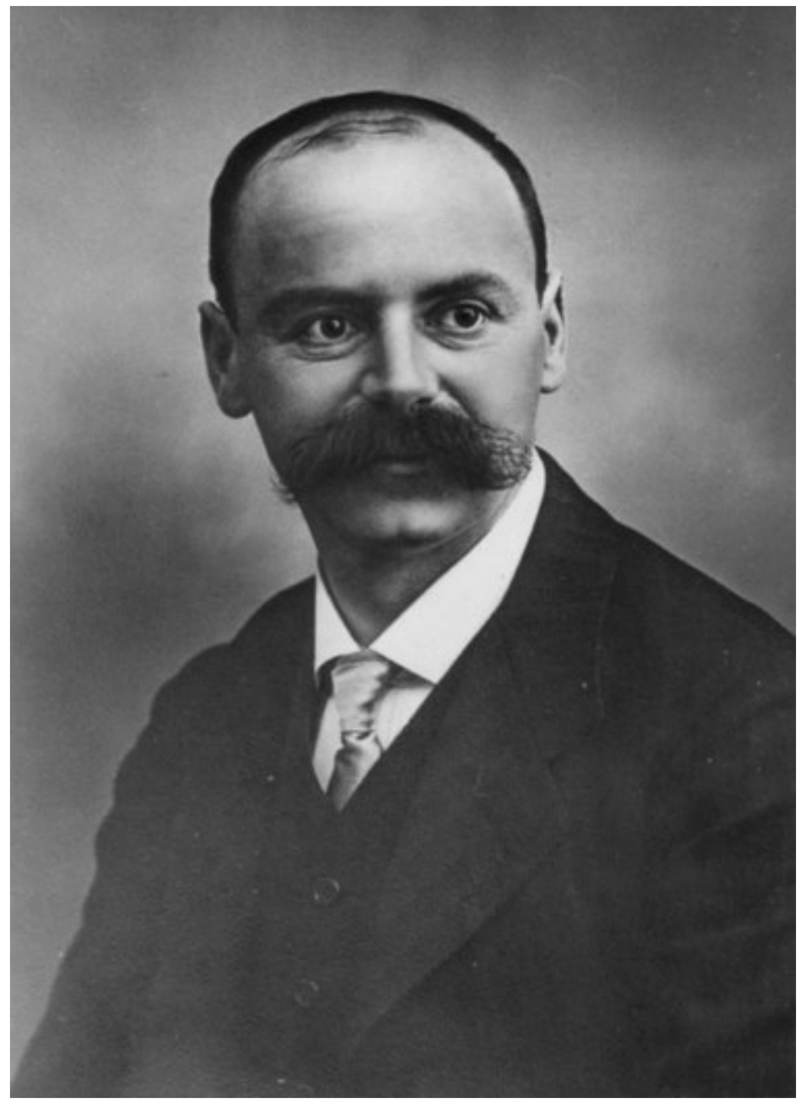
A. Georges, O. Parcollet, and S. Sachdev (**GPS**), Physical Review B **63**, 134406 (2001)

Important development: EDMFT of quantum criticality in Kondo lattice models by Qimiao Si

From
the SYK model
to
black holes

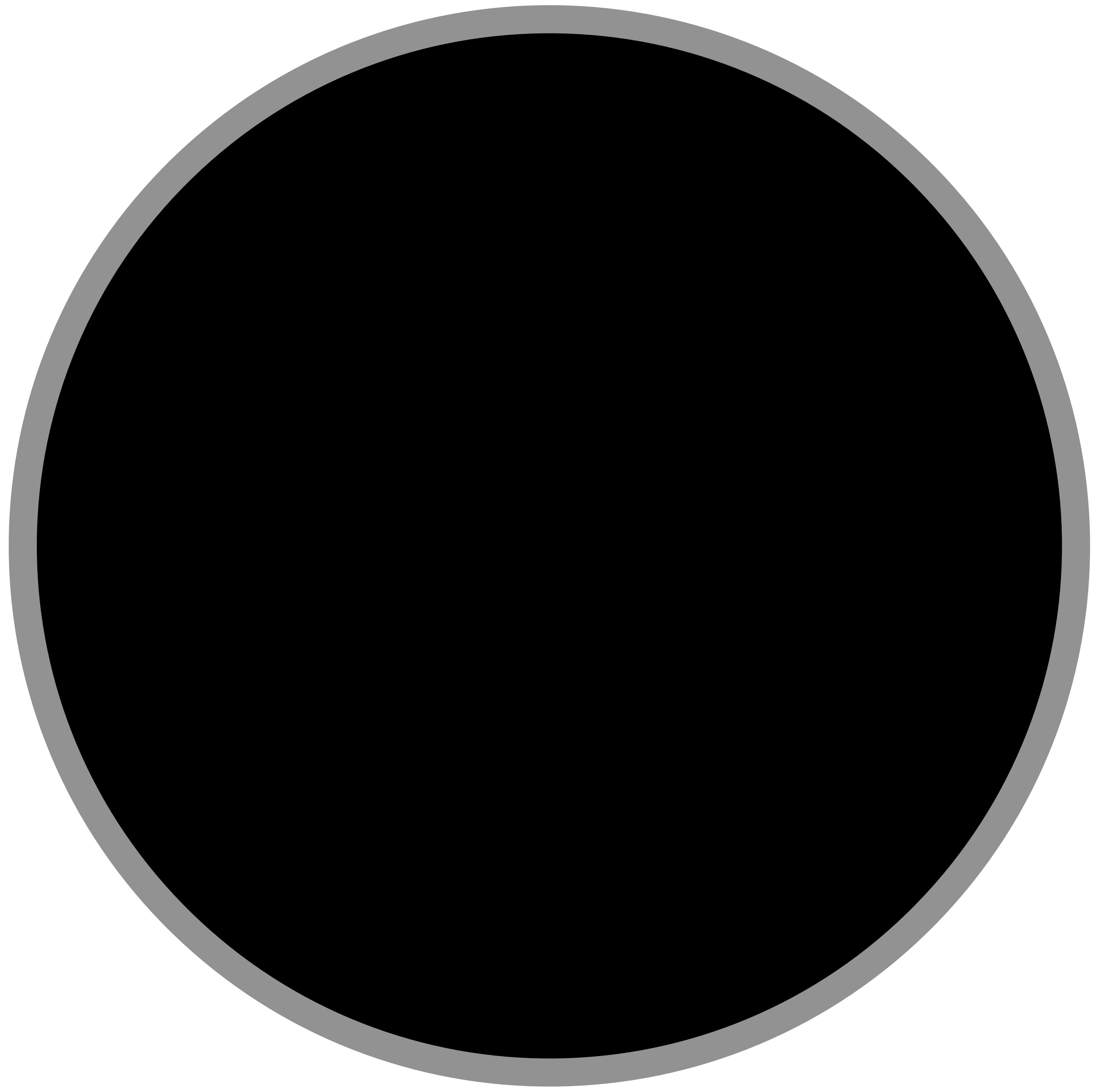
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

Event Horizon Telescope

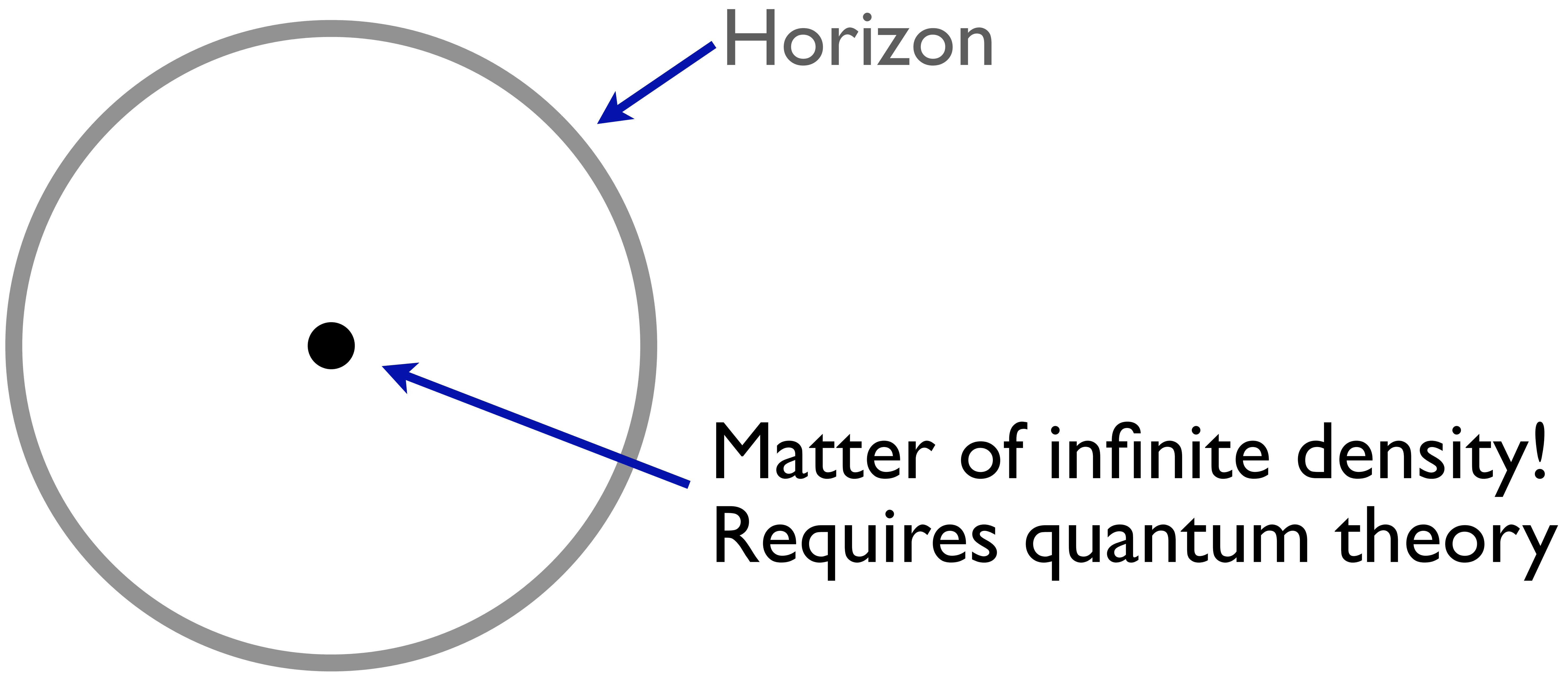
The supermassive black hole at the center of the M87 galaxy contains about 6.5 billion solar masses.

It is rotating at about 90% of the maximal spin

$$R = 1.8 \times 10^{13}\mathrm{m}$$
$$\approx \text{solar system size}$$

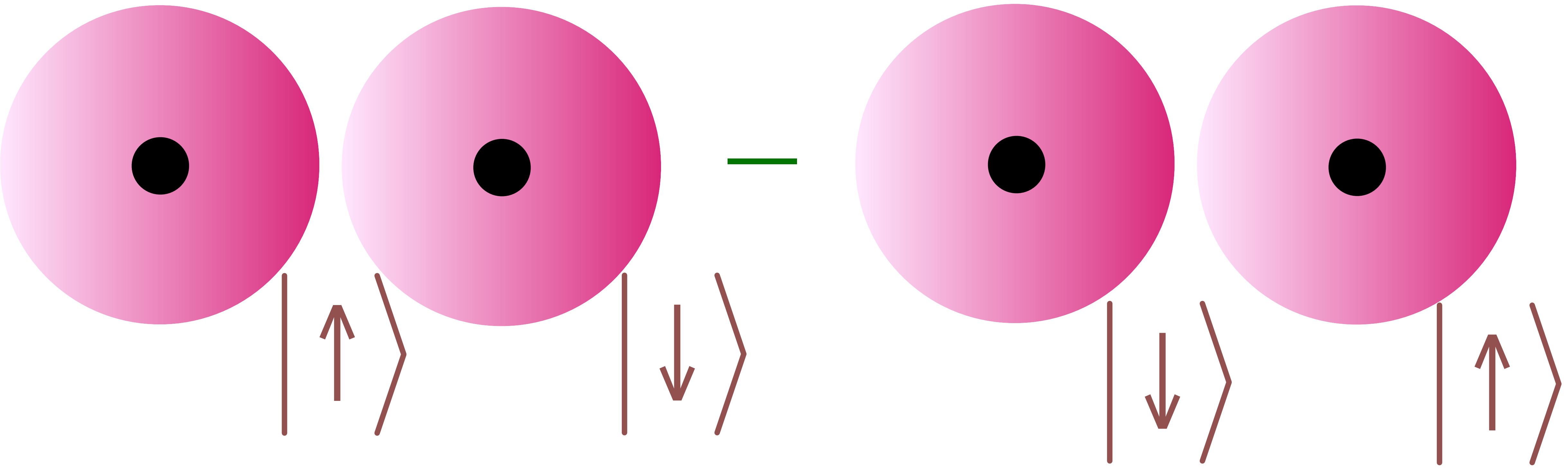
What is inside a black hole ???

In Einstein’s theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



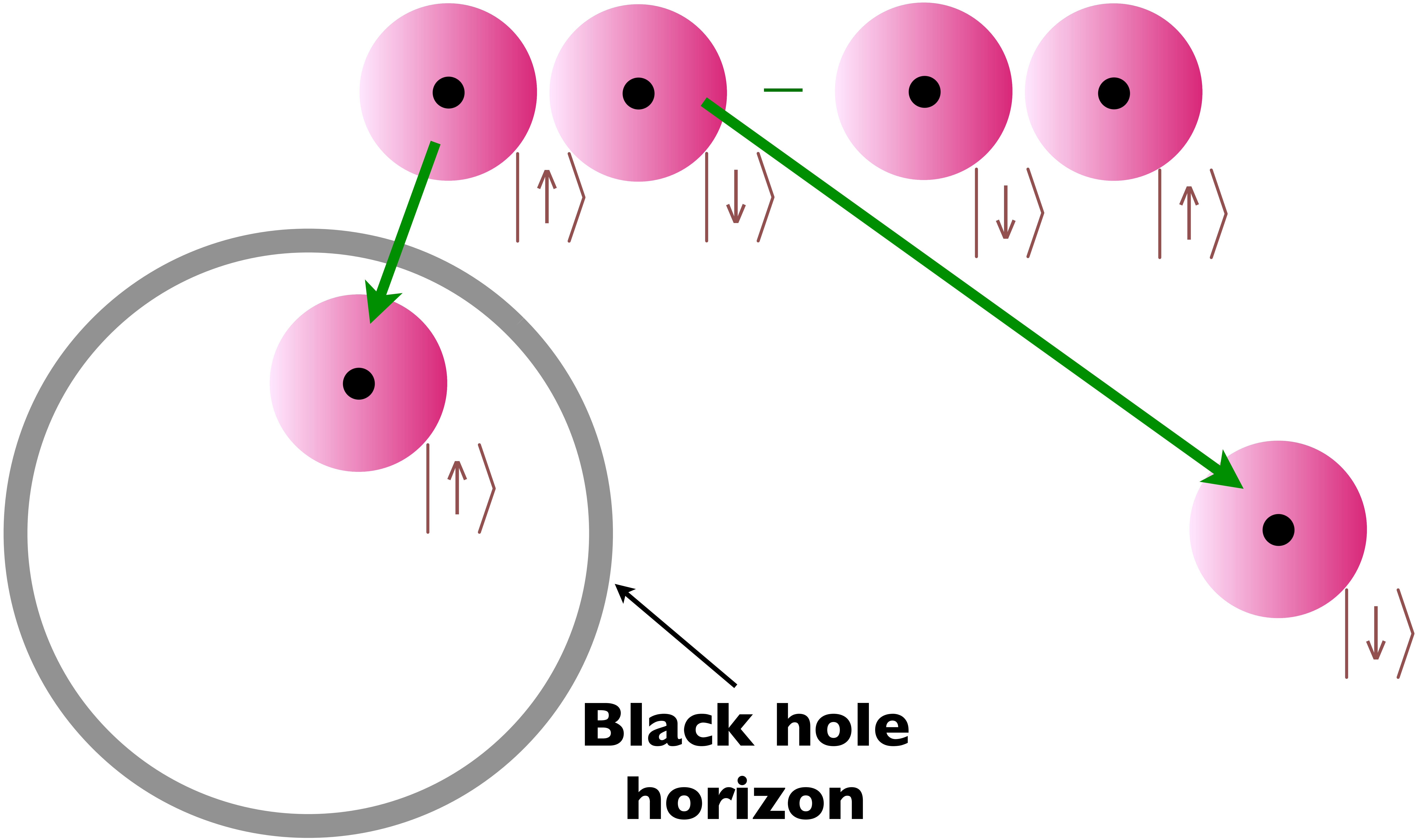
Matter of infinite density!
Requires quantum theory

Quantum Entanglement across a black hole horizon



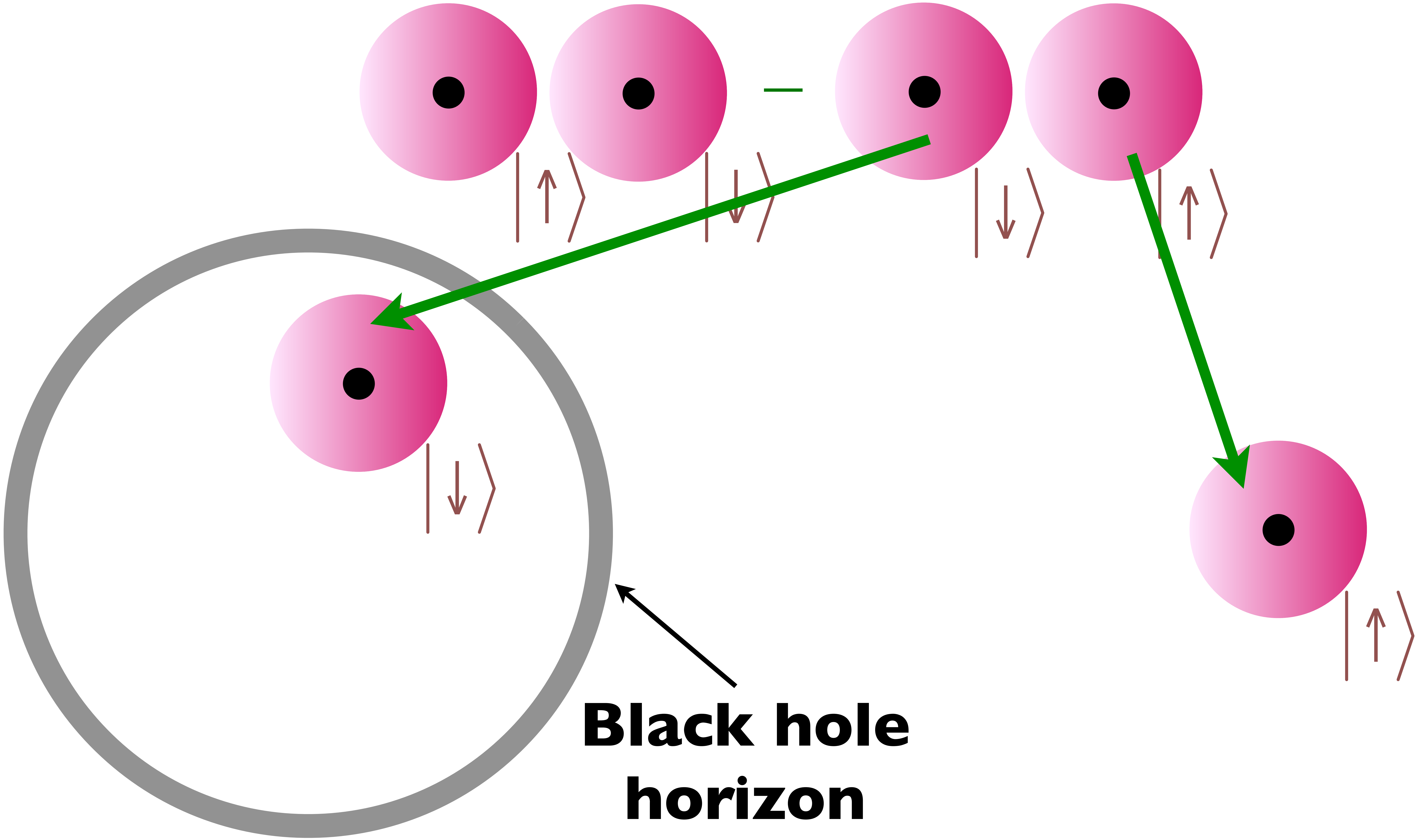
$$|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle$$

Quantum Entanglement across a black hole horizon



Black hole horizon

Quantum Entanglement across a black hole horizon

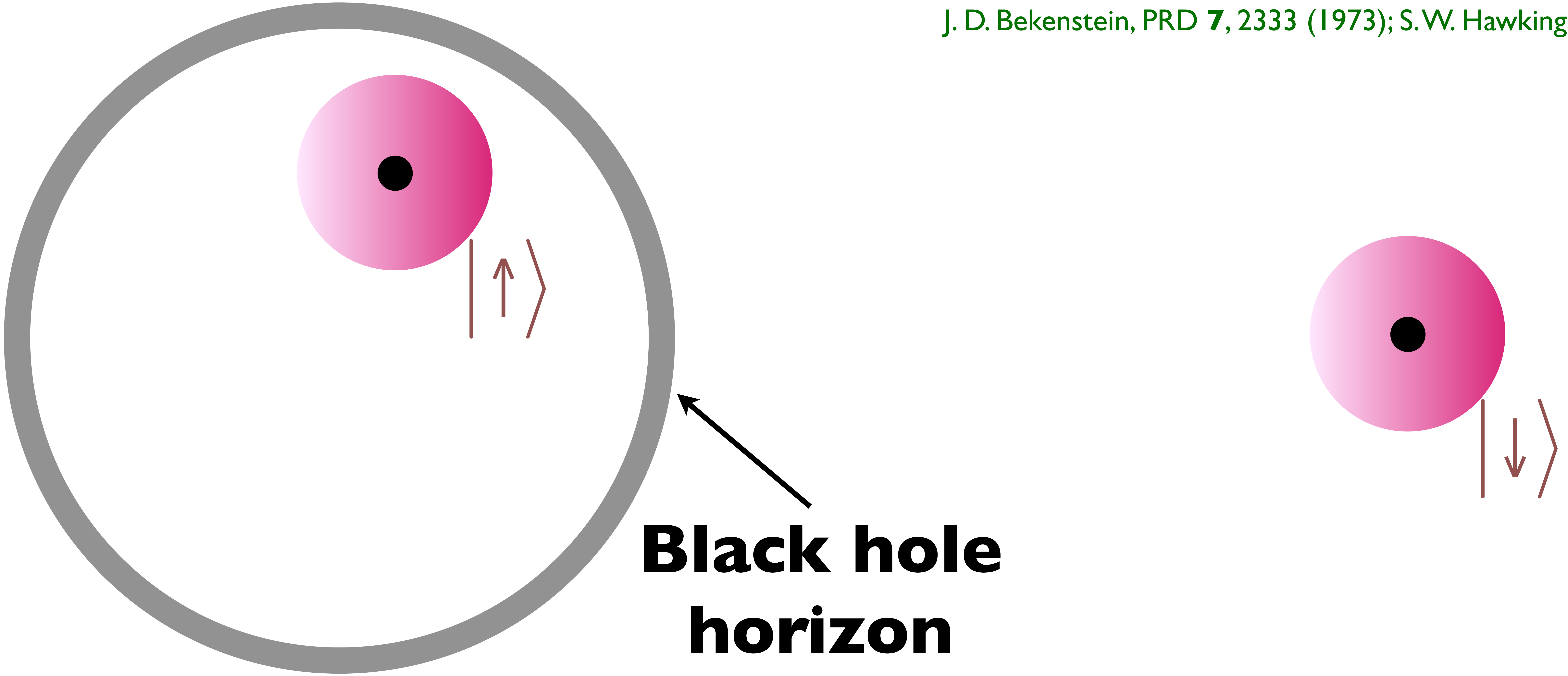


Quantum Entanglement across a black hole horizon

Bekenstein, Hawking: Black holes have a temperature and an entropy!

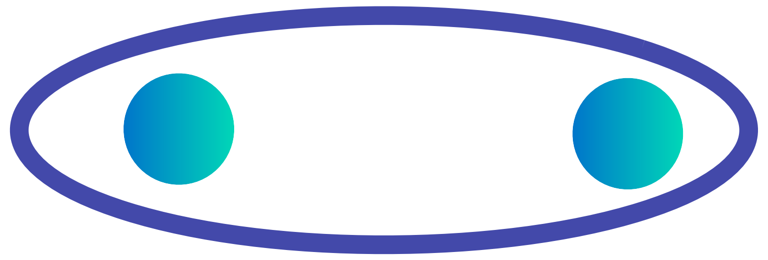
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.

J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974)

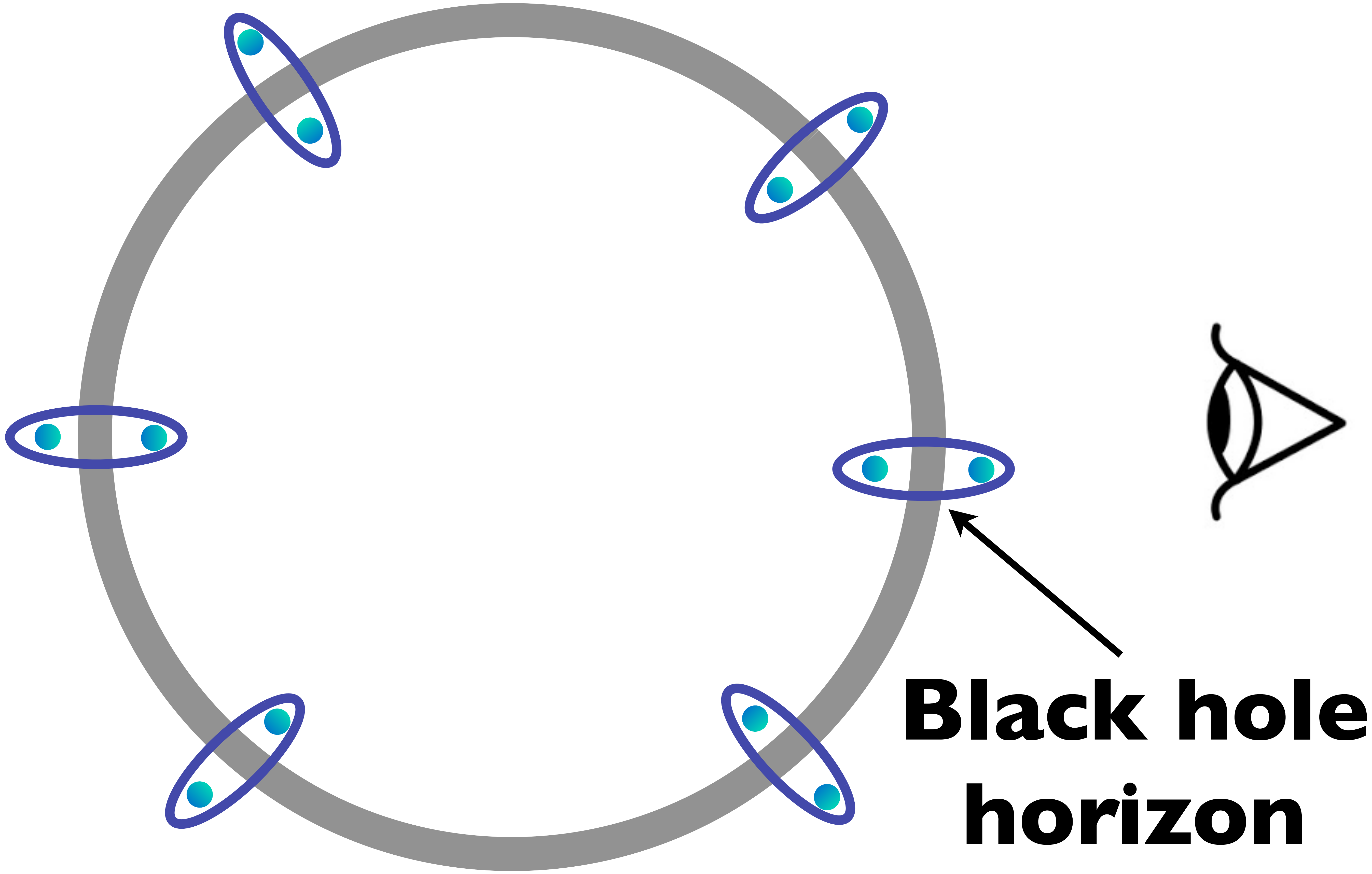


Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Bekenstein-Hawking obtained

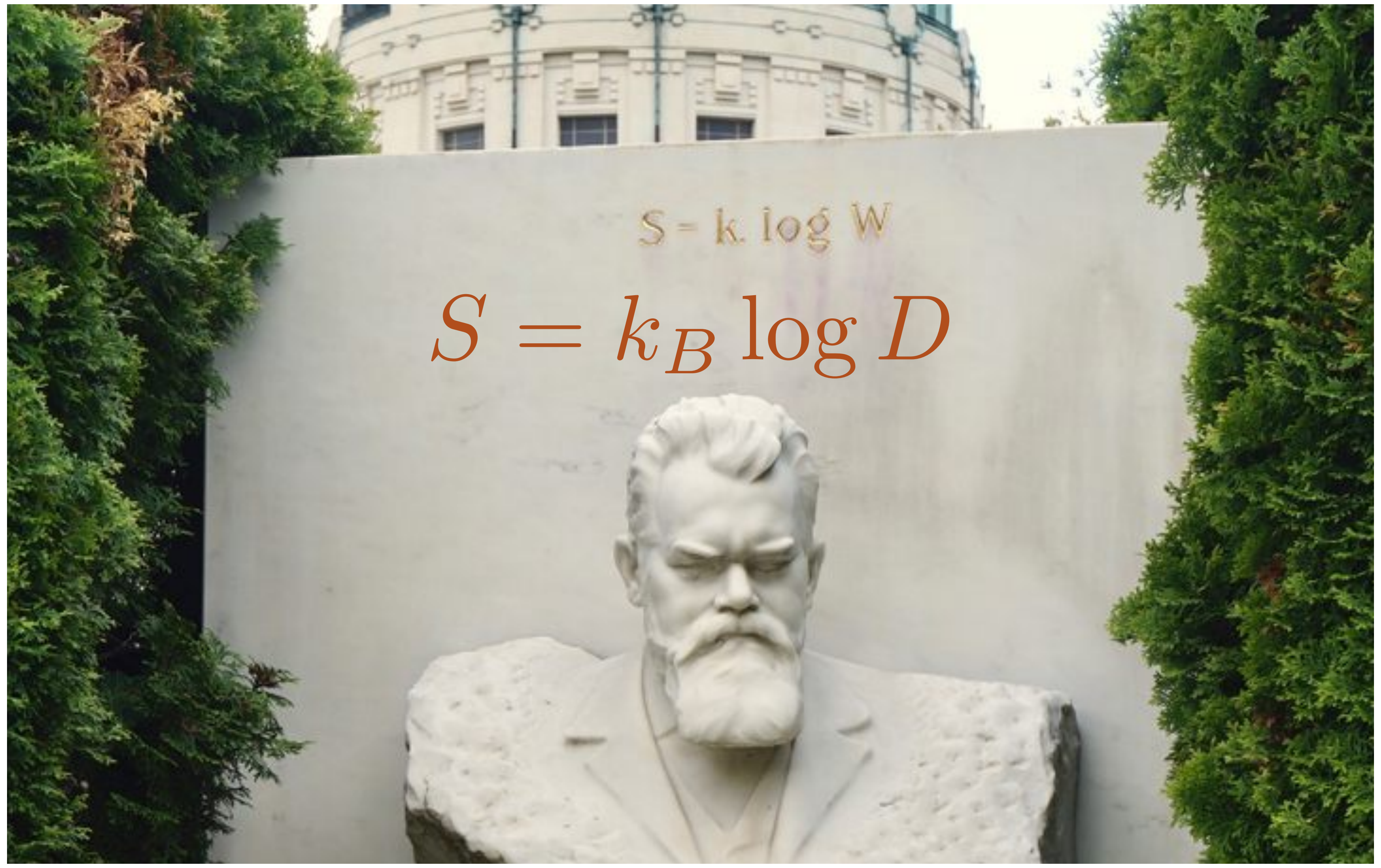
$$S = \frac{k_B A c^3}{4G\hbar}$$

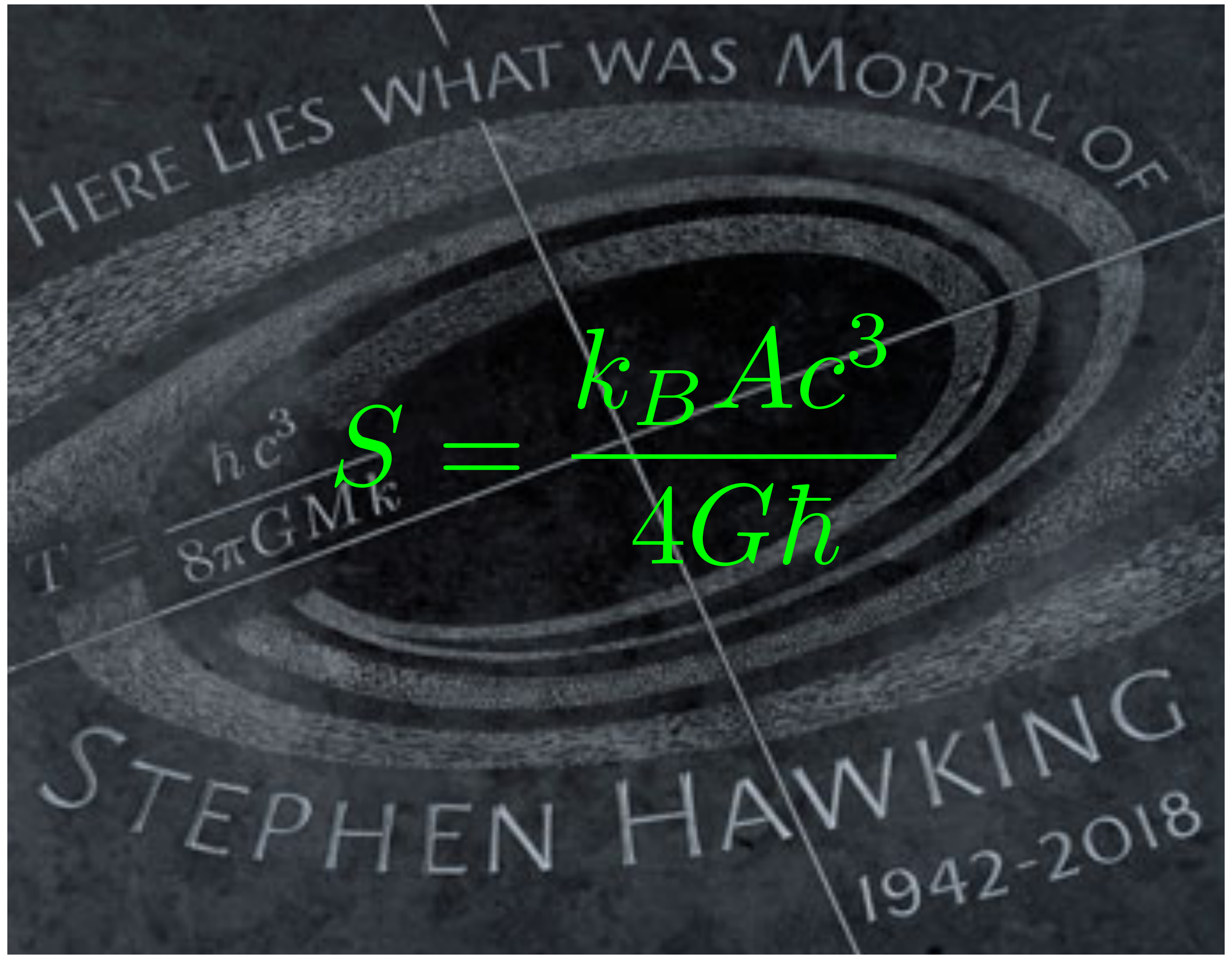
where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?





Connections between the SYK model and black holes

- Black hole 'ring-down' or 'quasinormal mode damping' or 'chaos' times are Planckian $\sim \hbar/(k_B T)$

C.V. Vishveshwara, Nature **227**, 936 (1970)

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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- Charged black holes have a non-zero Bekenstein-Hawking entropy in the limit $T \to 0$:

  $S_{BH} = A_0 c^3/(4\hbar G)$ where $A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

  Also applies to rotating neutral black holes.

U. Moitra, S.K. Sake, S.P. Trivedi and V. Vishal, JHEP 11 (2019) 047.
D. Kapec, A. Sheta, A. Strominger and C. Toldo, PRL 133 (2024) 021601
M. Kolanowski, D. Marolf, I. Rakic, M. Rangamani and G.J. Turiaci, arXiv:2409.16248

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

The SYK model

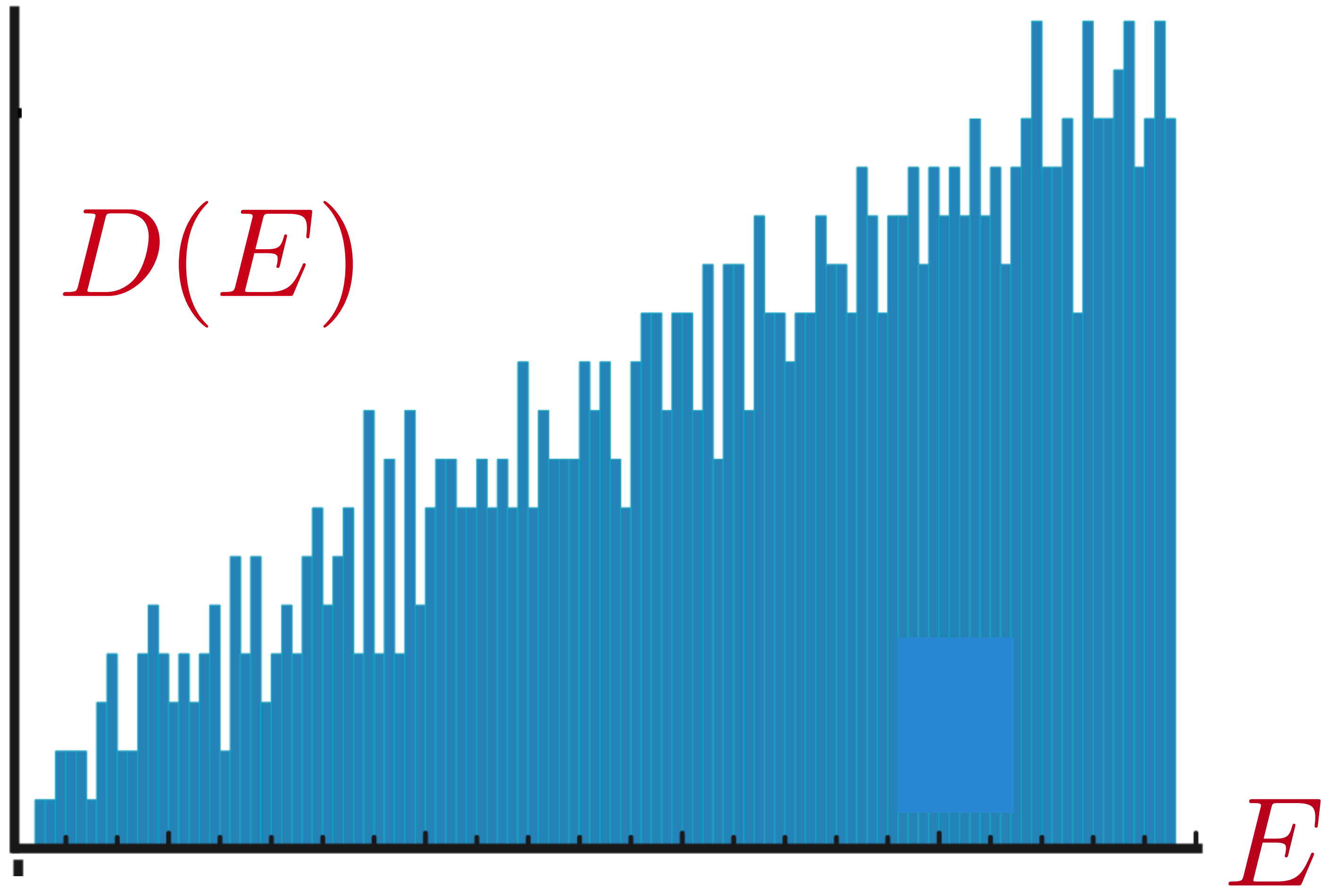
Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right) \text{ independent of } U.$$

No bosons, fermions, anyons ...

2. Zero temperature entropy without exponential ground state degeneracy!



$$\lim_{T\to 0}\lim_{N\to\infty}\frac{1}{N}S(T) = s_0 \quad , \quad D(E\to 0) = e^{N s_0}\sinh(\sqrt{2N\gamma E})$$

$$s_0 = 0.4648476991708051074\ldots \text{ for } \mathcal{Q} = 1/2.$$

A. Georges, O. Parcollet, and S. Sachdev (**GPS**), Physical Review B **63**, 134406 (2001)

Important development: EDMFT of quantum criticality in Kondo lattice models by Qimiao Si

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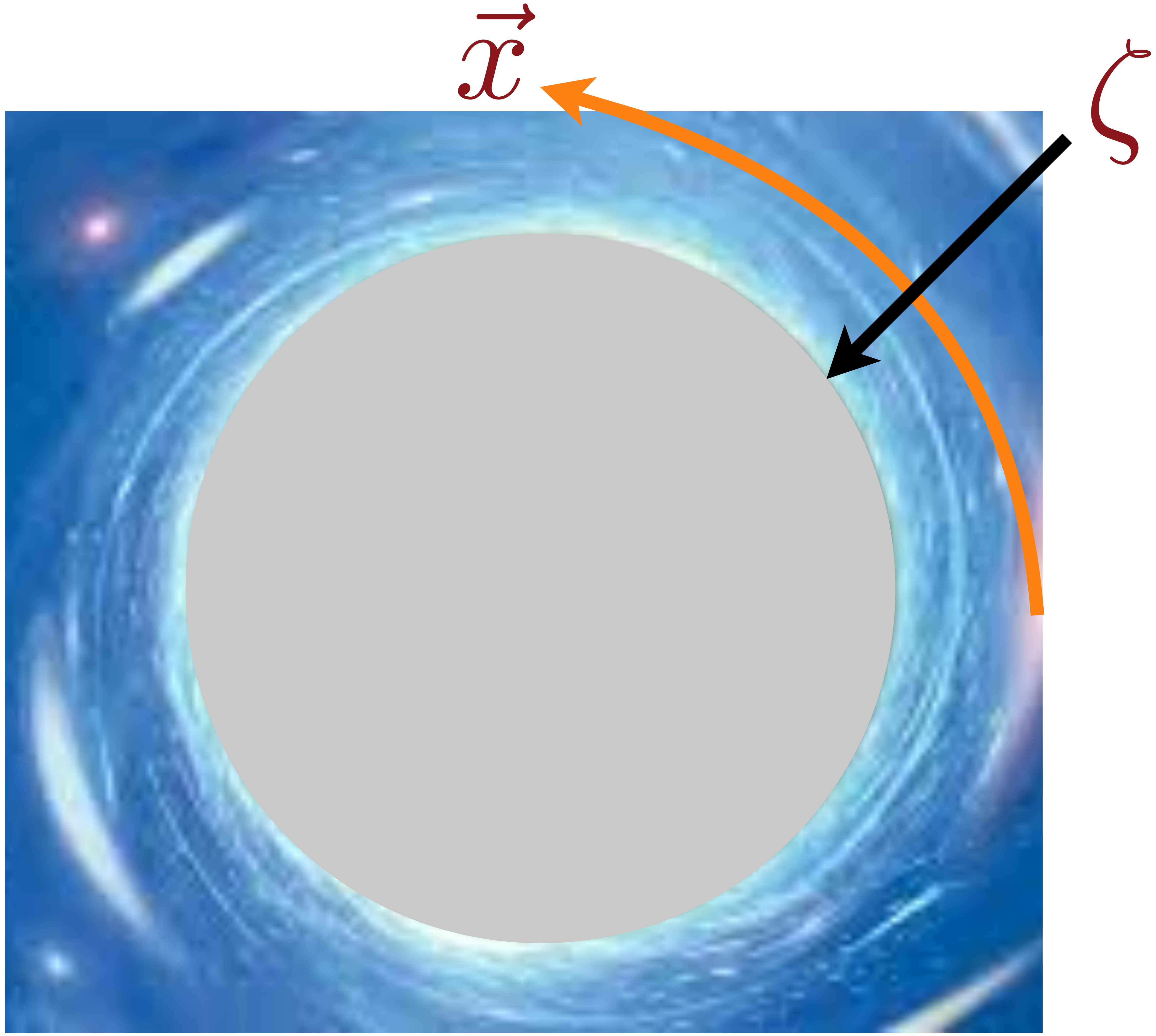
  Also applies to rotating neutral black holes.

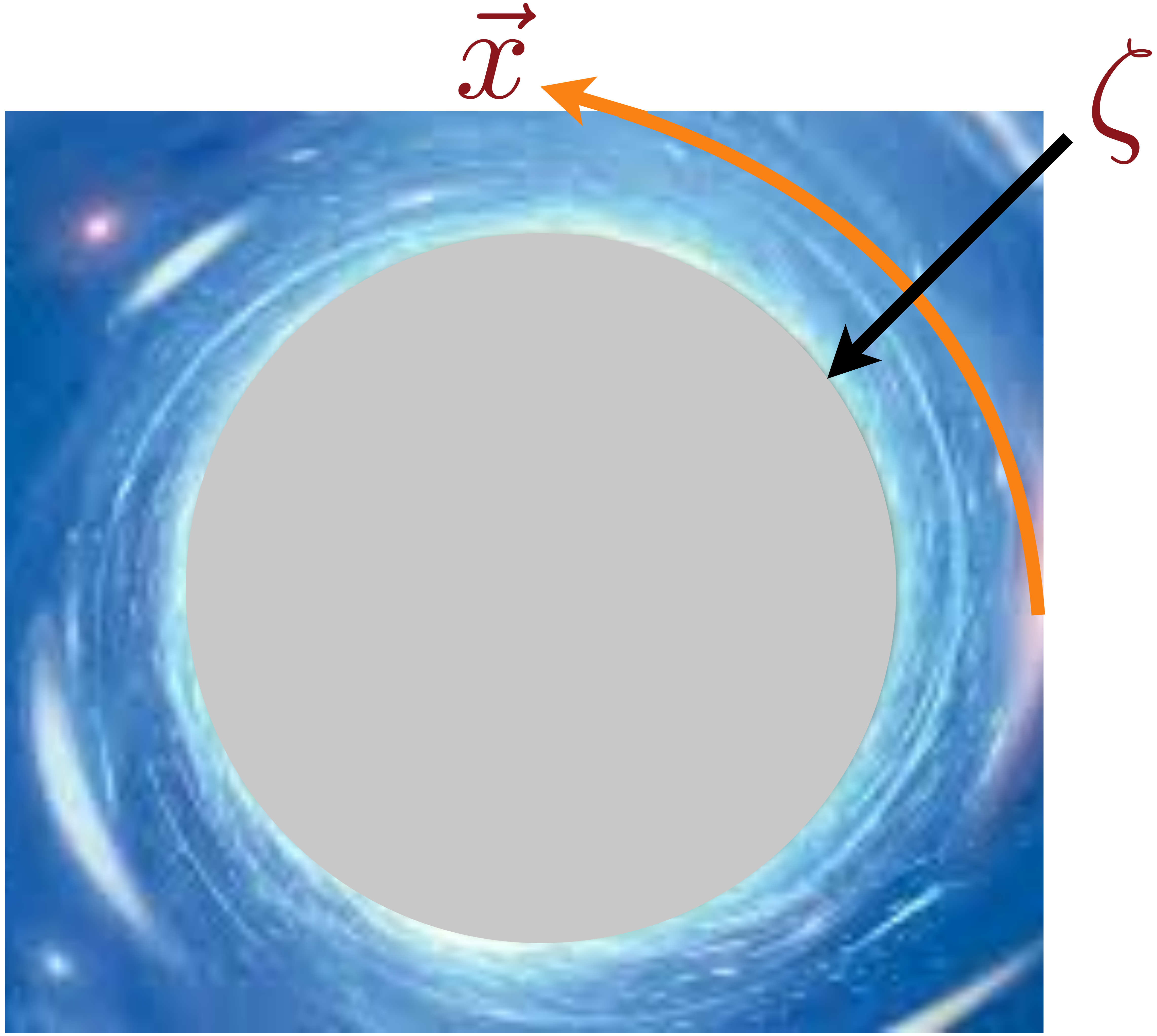
- The example of the SYK model implies that S_{BH} is *not* realized by an exponentially large ground state degeneracy (as is the case in all earlier string-theoretic computations).

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

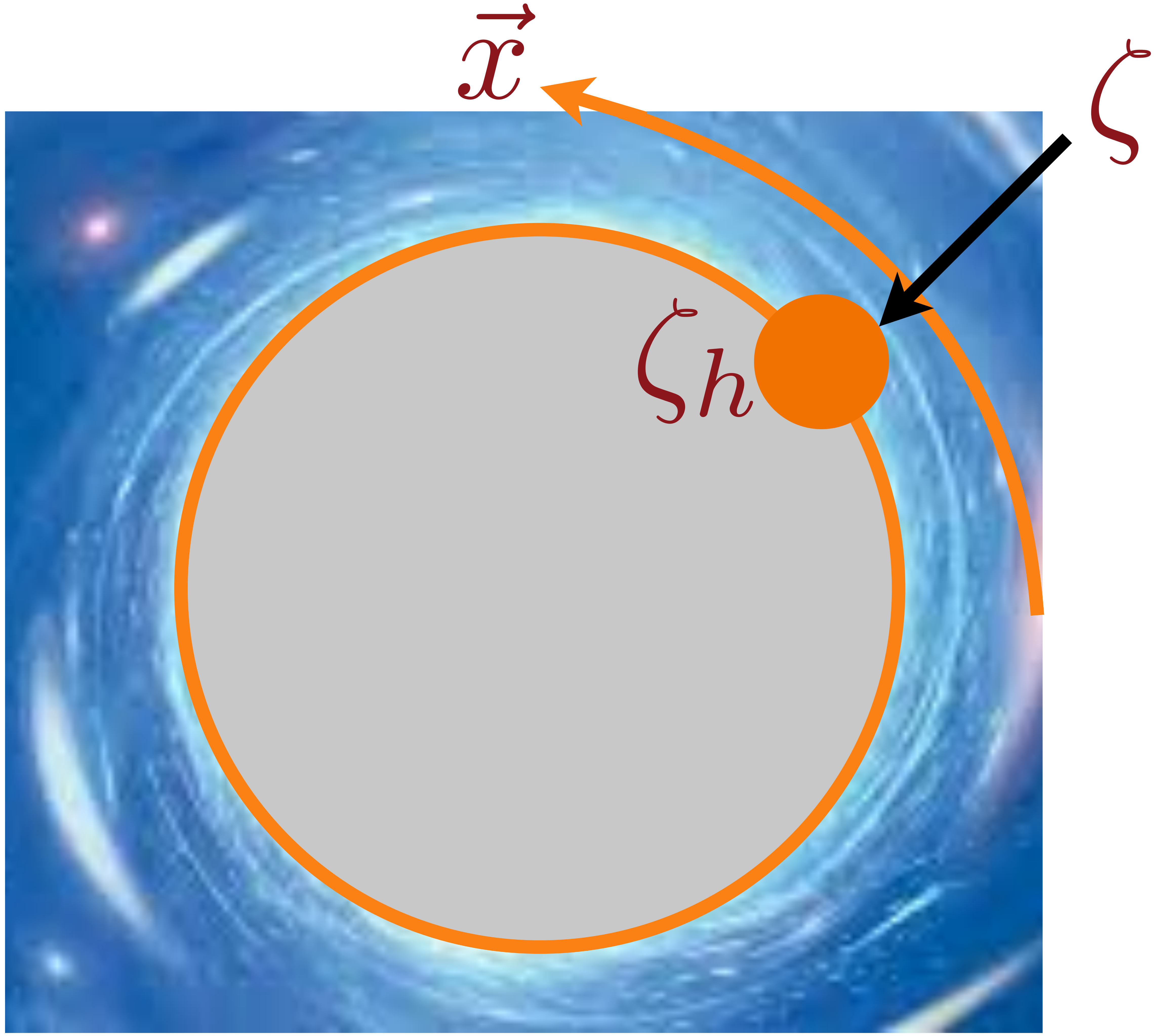


Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



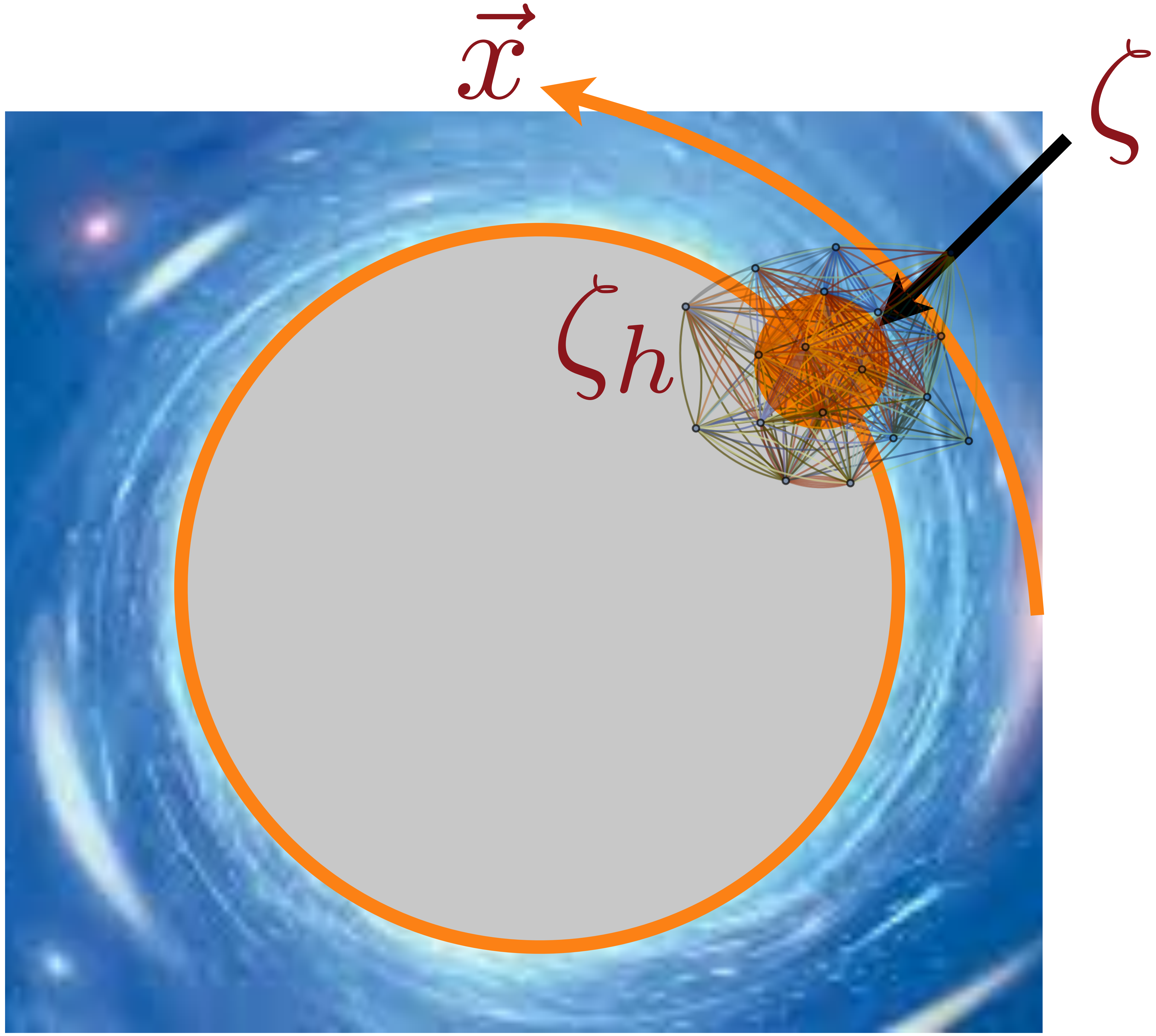
Zooming into the
near-horizon region
of a charged black hole
at low temperature,
yields a theory
in one space (ζ) and
one time dimension

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



So we need only consider complex entanglement at one spatial "point" on the horizon ($\zeta = \zeta_h$), just as is described by the SYK model

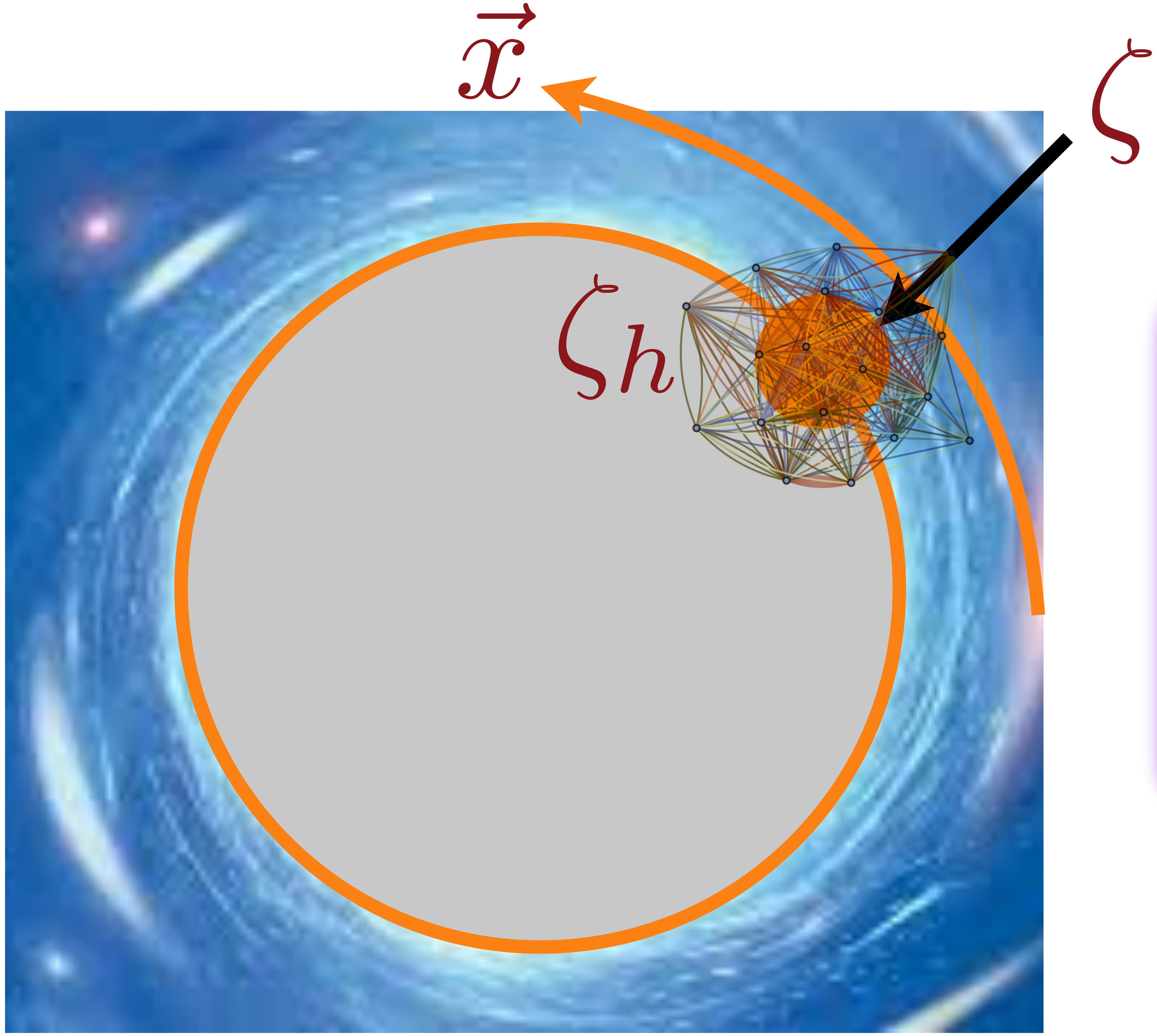
Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



The quantum versions of Maxwell's and Einstein's equations in this two-dimensional spacetime are also the equations describing electron entanglement in the SYK model!

Maldacena, Stanford Yang; Kitaev Suh

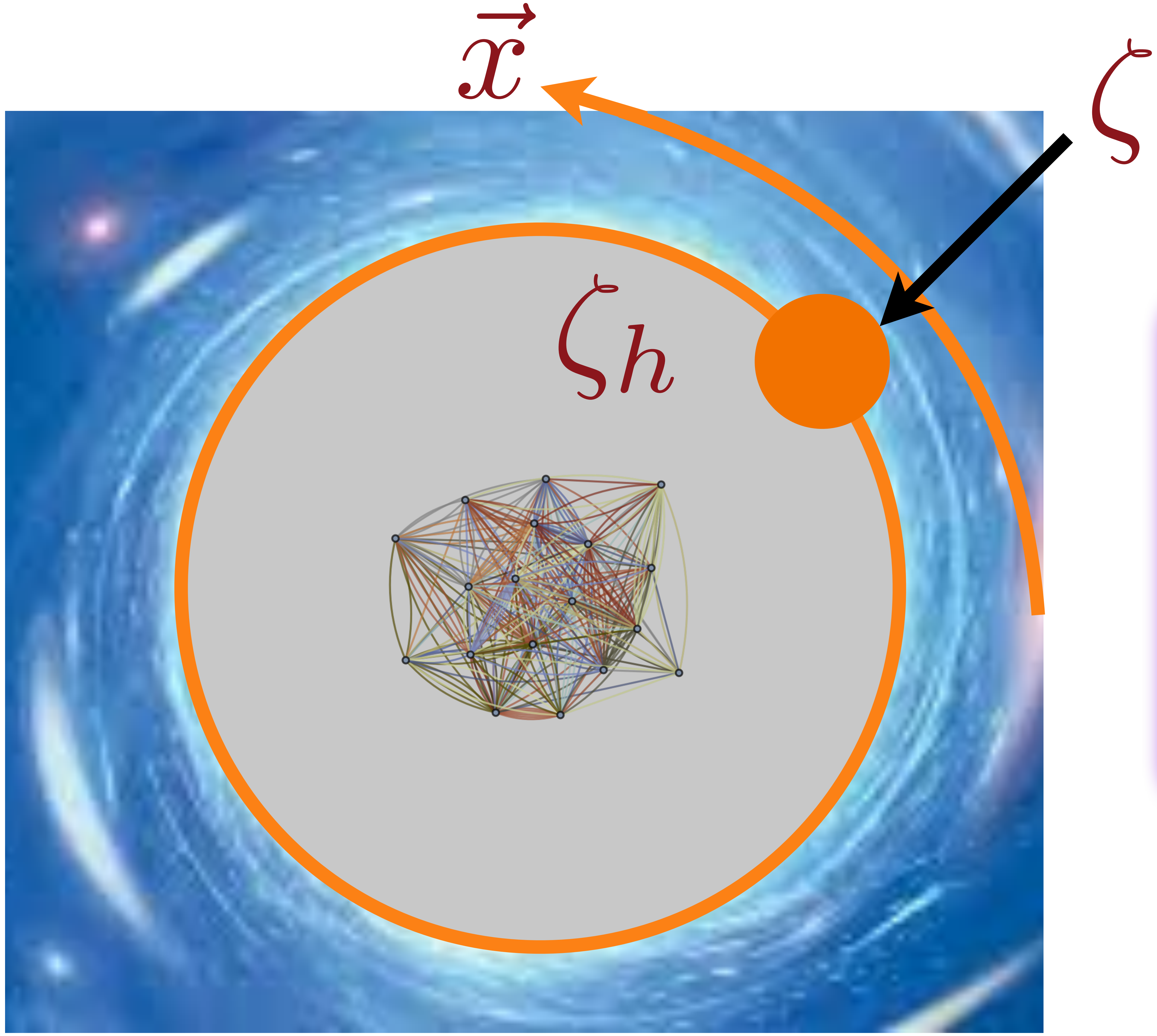
Quantum simulation of charged black holes by the SYK model



The SYK model simulates the interior of the black hole to an outside observer

Maldacena, Stanford Yang; Kitaev Suh

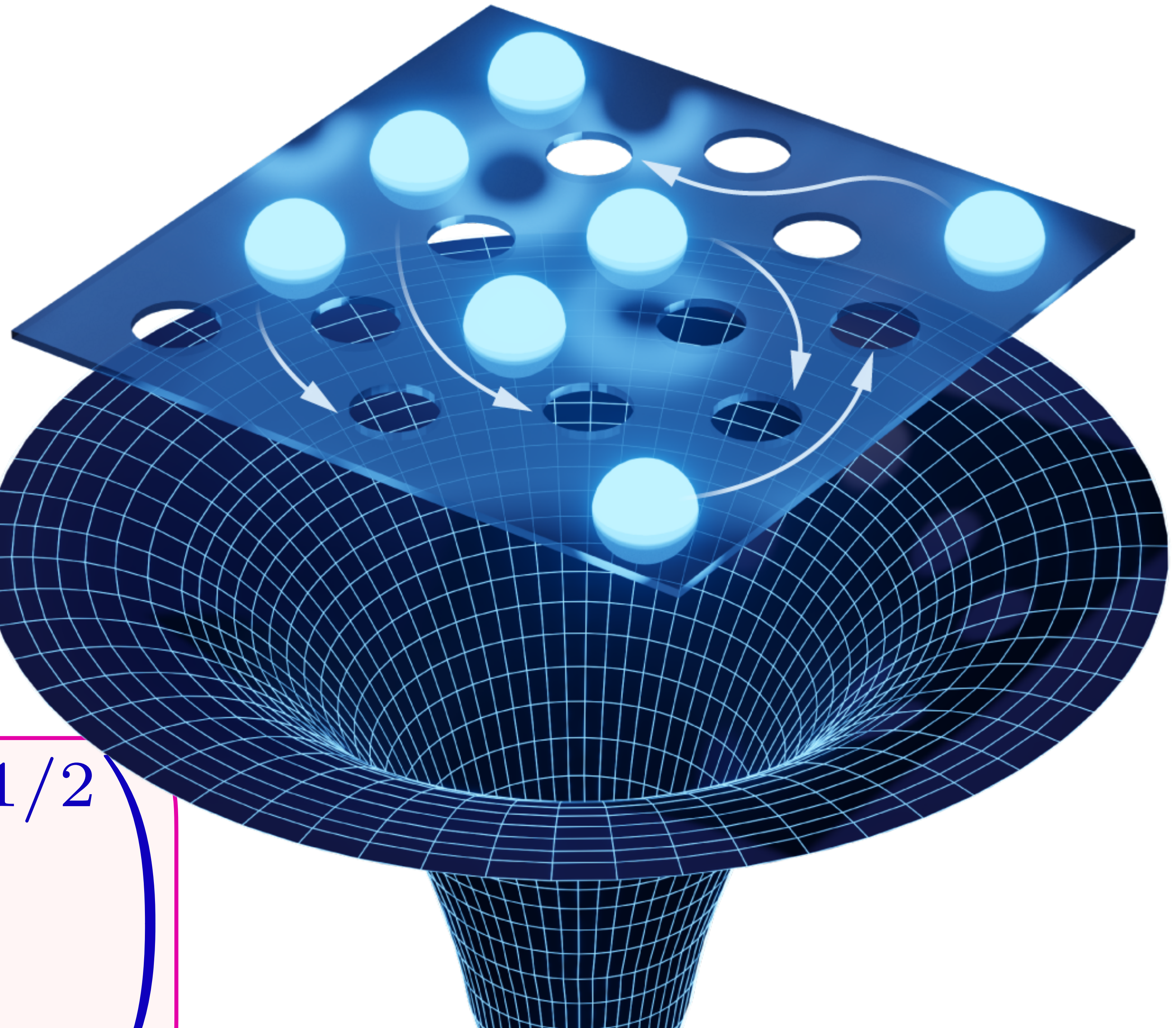
Quantum simulation of charged black holes by the SYK model



The SYK model simulates the interior of the black hole to an outside observer

Maldacena, Stanford Yang; Kitaev Suh

D(E) of charged black holes from the SYK model



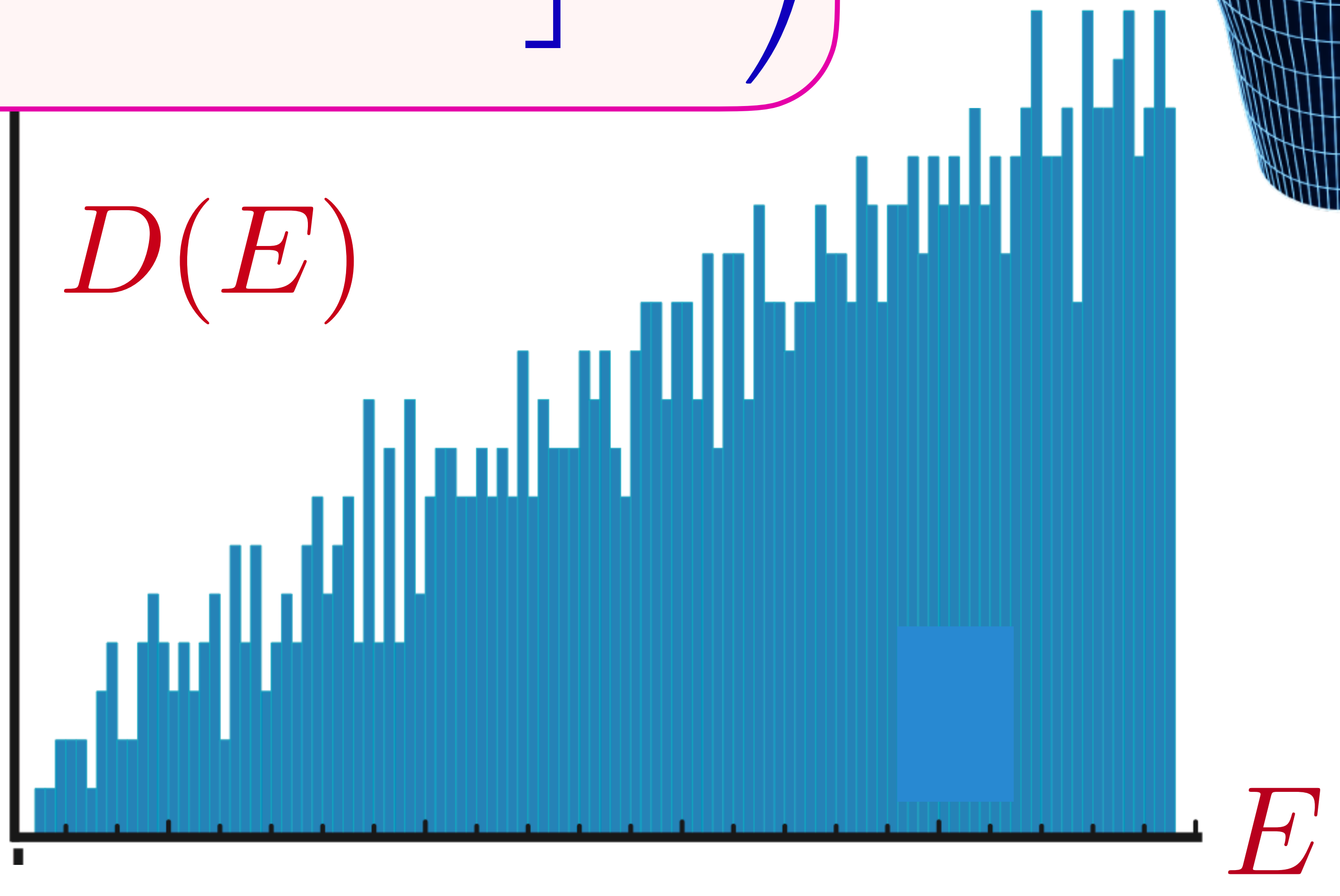
- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}{A_0}^{3/2}c^2}{\hbar^2 G}E\right]^{1/2}\right)$$

Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

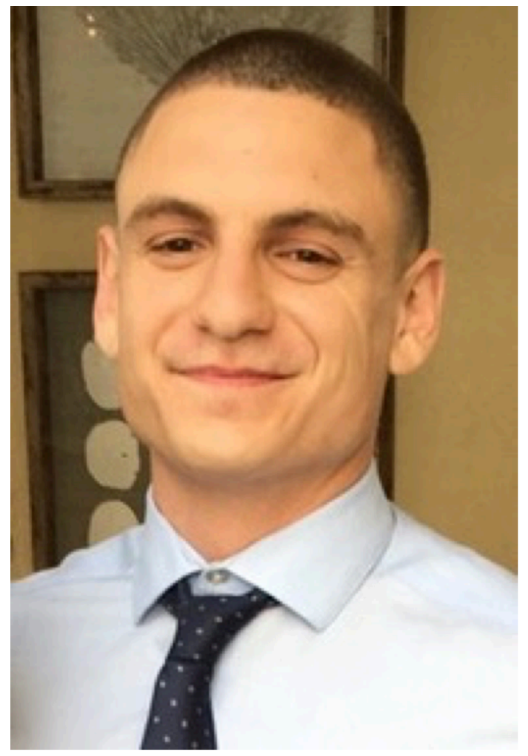
From the SYK model to the universal 2d-YSYK theory of strange metals

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, *PRL* **133**, 186502 (2024)


Aavishkar Patel, Flatiron

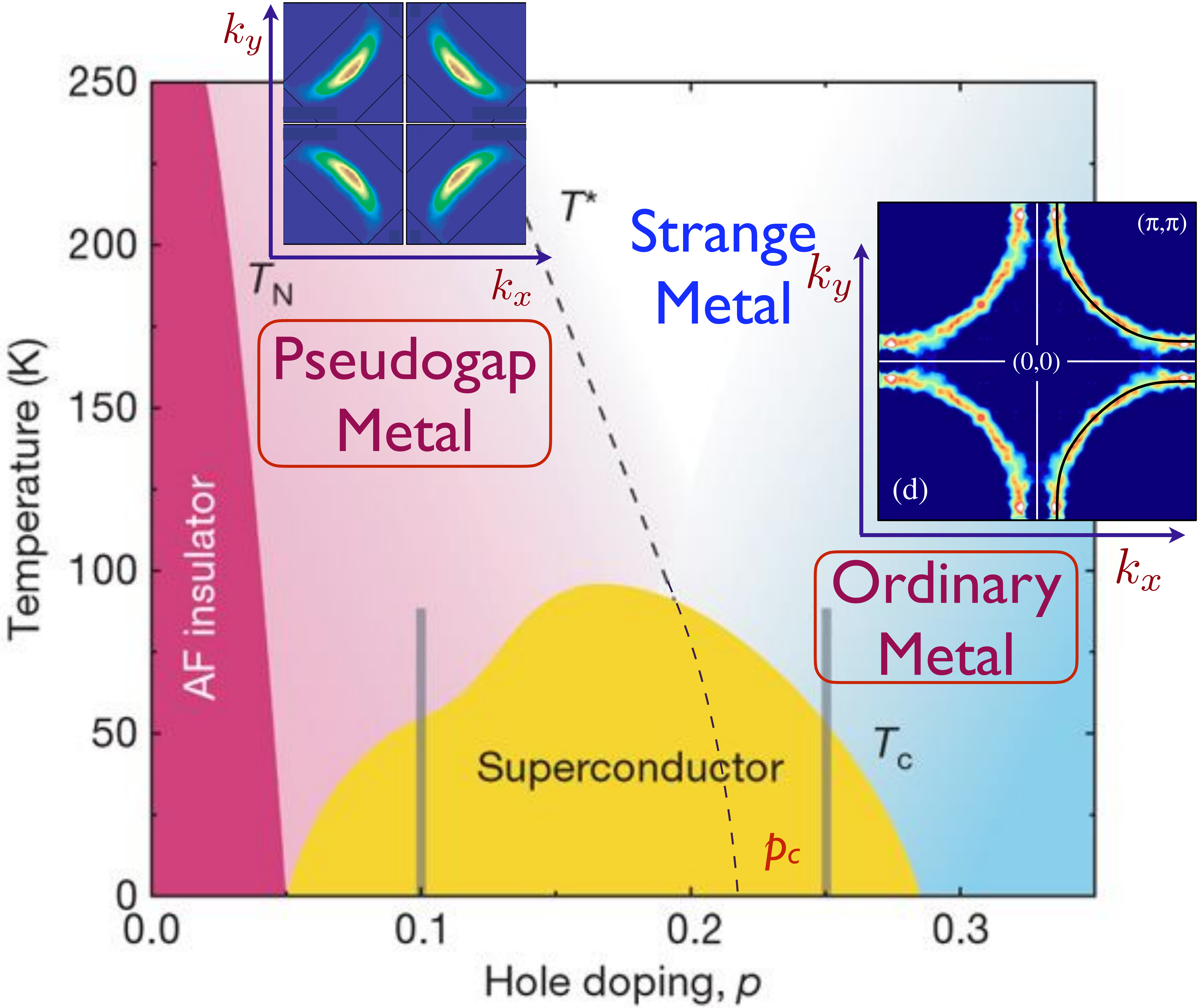

Chenyuan Li, Rice


Haoyu Guo, Cornell

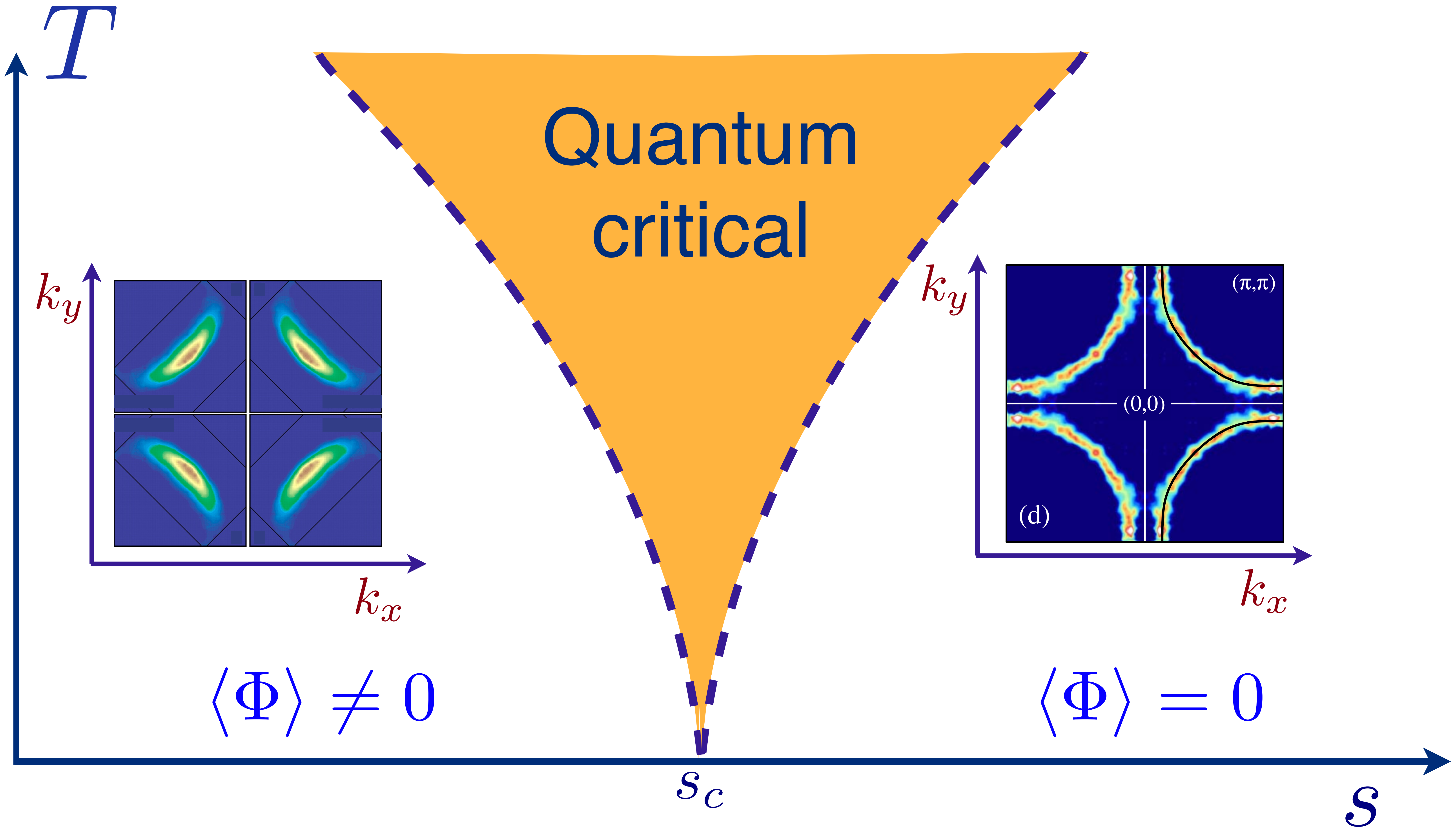

Ilya Esterlis, Wisconsin

See also Tsz Chun Wu, Yunxiang Liao, and Matthew S. Foster, PRB **106**, 155108 (2022)

Quantum phase transition from the Ordinary Metal to the Pseudogap Metal



Quantum phase transition of Fermi surface change



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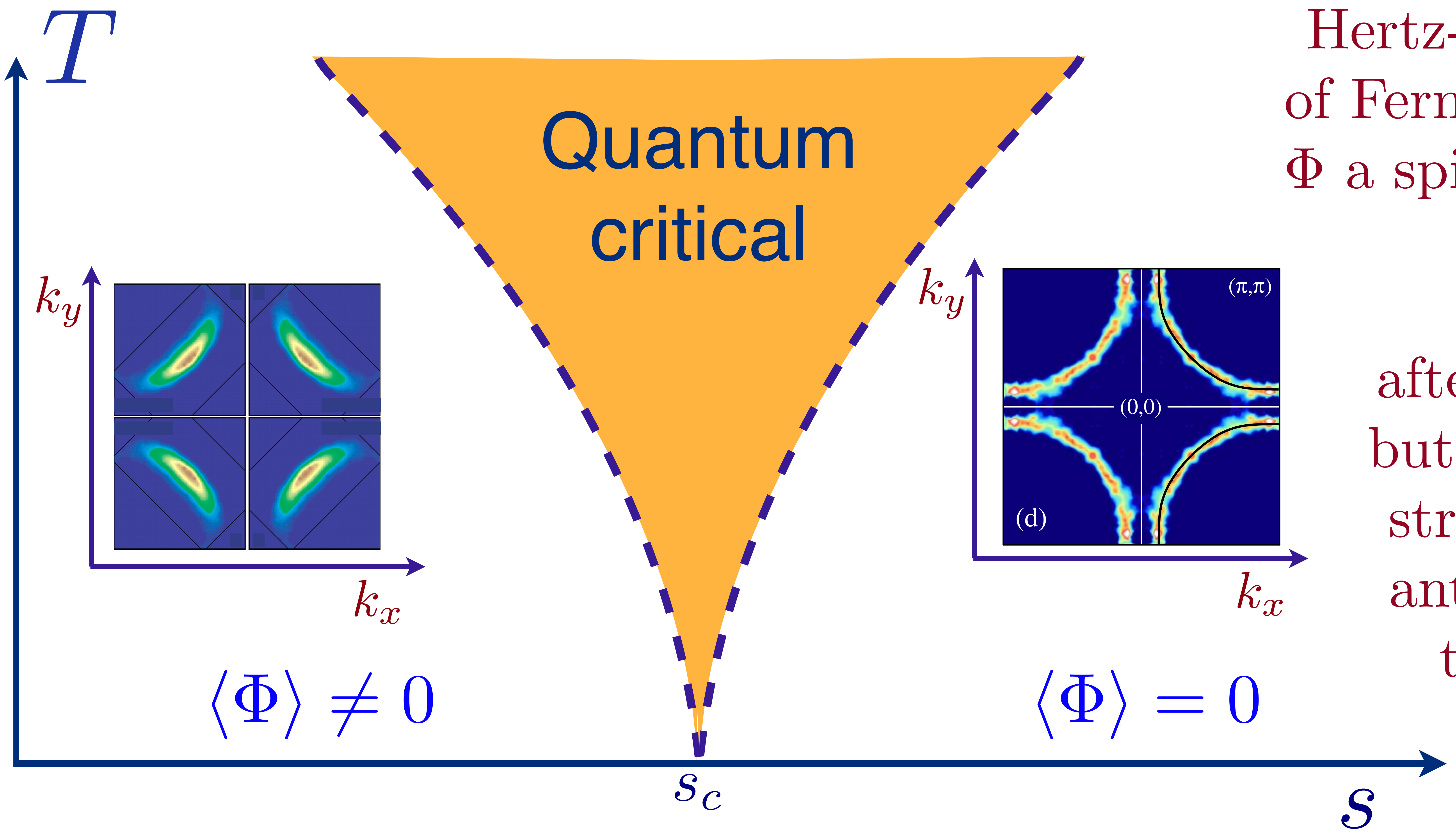
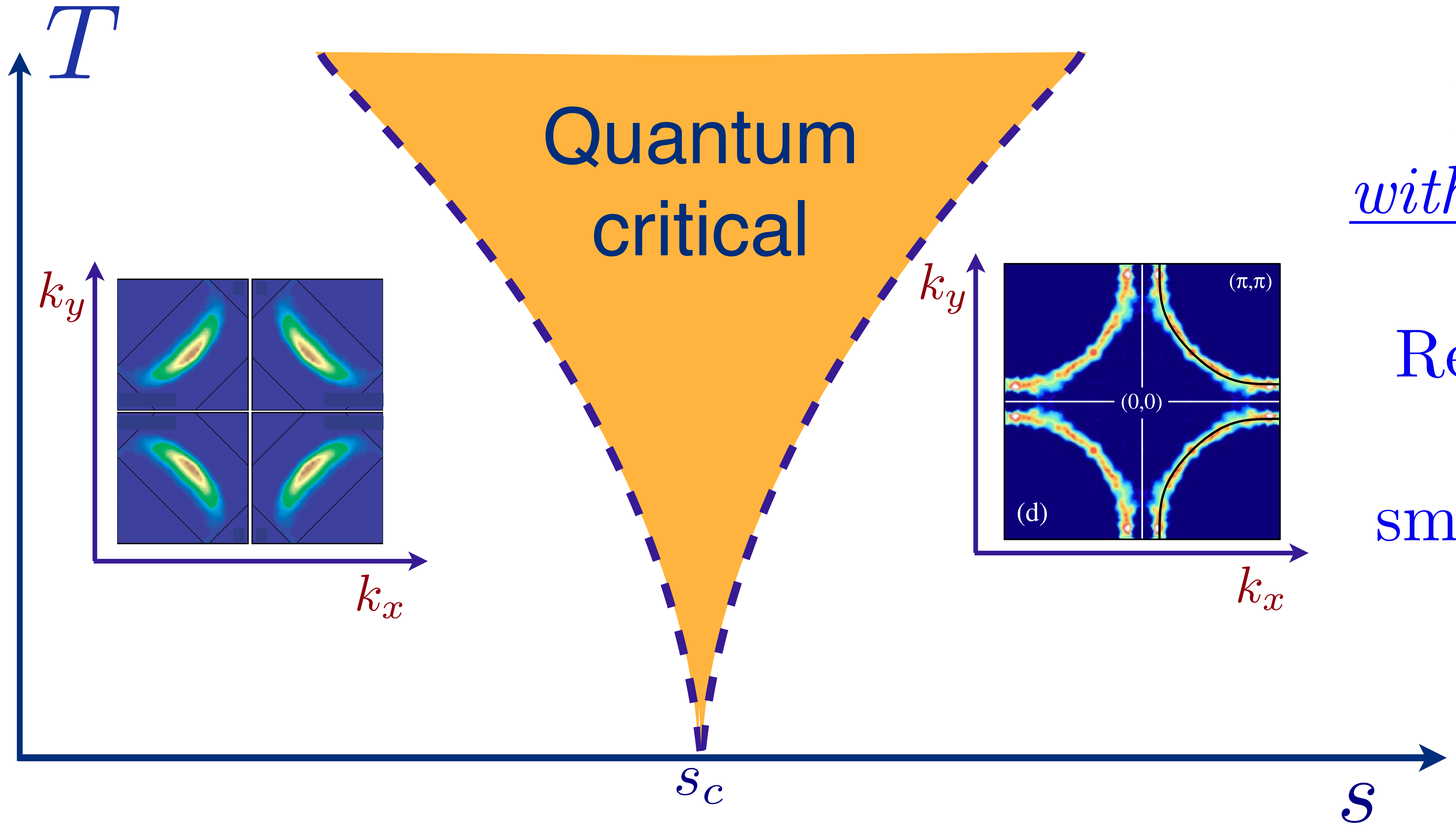


Hertz-Millis (and EDMFT) theory of Fermi surface reconstruction with Φ a spin density wave (SDW) order.

Can apply at lower T after accounting for disorder, but SDW correlations are not strong enough to induce the anti-nodal gap in the higher temperature pseudogap.

Quantum phase transition of Fermi surface change

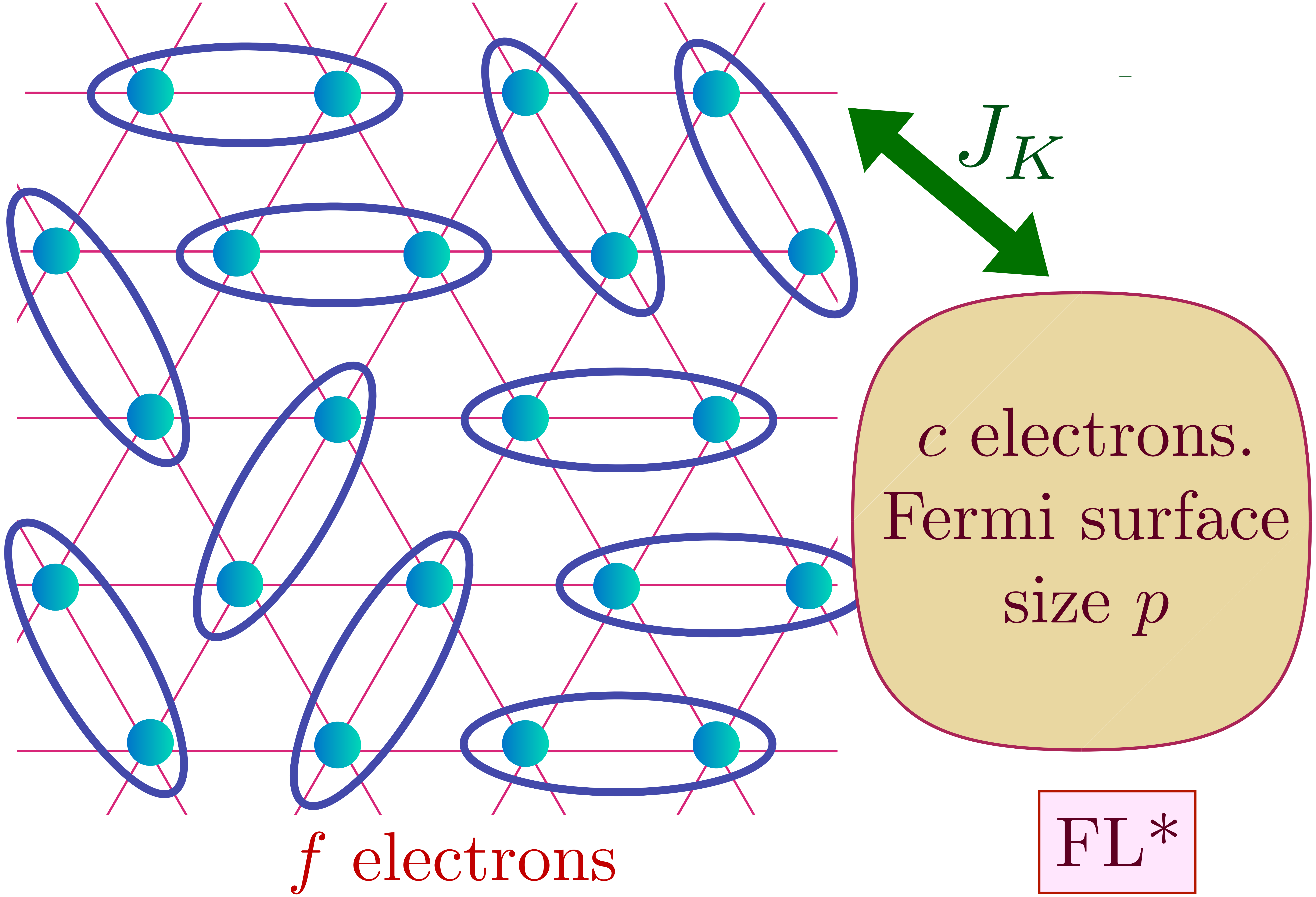


Fermi volume change *without* symmetry breaking.

Requires a “background” spin liquid in the small Fermi surface phase.

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Luttinger volume violation in the Kondo lattice



J_K

c electrons. Fermi surface size p

f electrons

FL*

c electron Fermi surface size does not change (to all orders) after Kondo coupling, J_K, to f electron spin liquid.

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fermi-volume-changing QPT in the Kondo lattice

$\langle \Phi \rangle = 0$

$\langle \Phi \rangle \neq 0$

Higgs boson
$\Phi \sim f_\sigma^\dagger c_\sigma$

J_K

c electrons.
Fermi surface
size p

$f + c$ electrons.
Fermi surface
size $1 + p$

FL*

FL

f electrons

J_K

J_c

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fermi-volume-changing QPT in the Kondo lattice

Crucial to include spatial disorder in the value of J_K

$\langle \Phi \rangle = 0$

$\langle \Phi \rangle \neq 0$

Higgs boson
$\Phi \sim f_\sigma^\dagger c_\sigma$

J_K

c electrons.
Fermi surface
size p

$f + c$ electrons.
Fermi surface
size $1 + p$

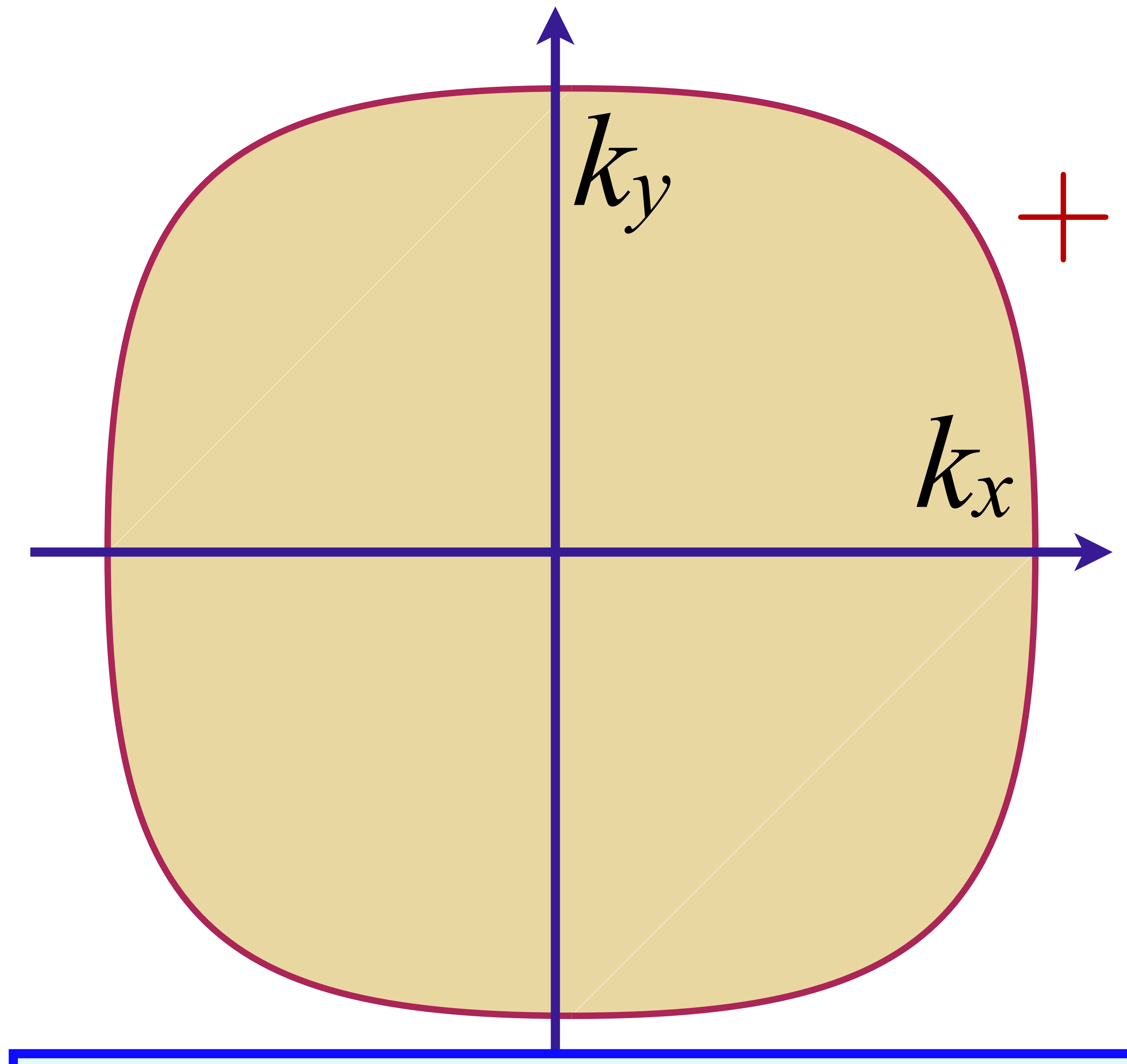
FL*

FL

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Kondo lattice + critical boson with potential and mass disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})]\,[\Phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

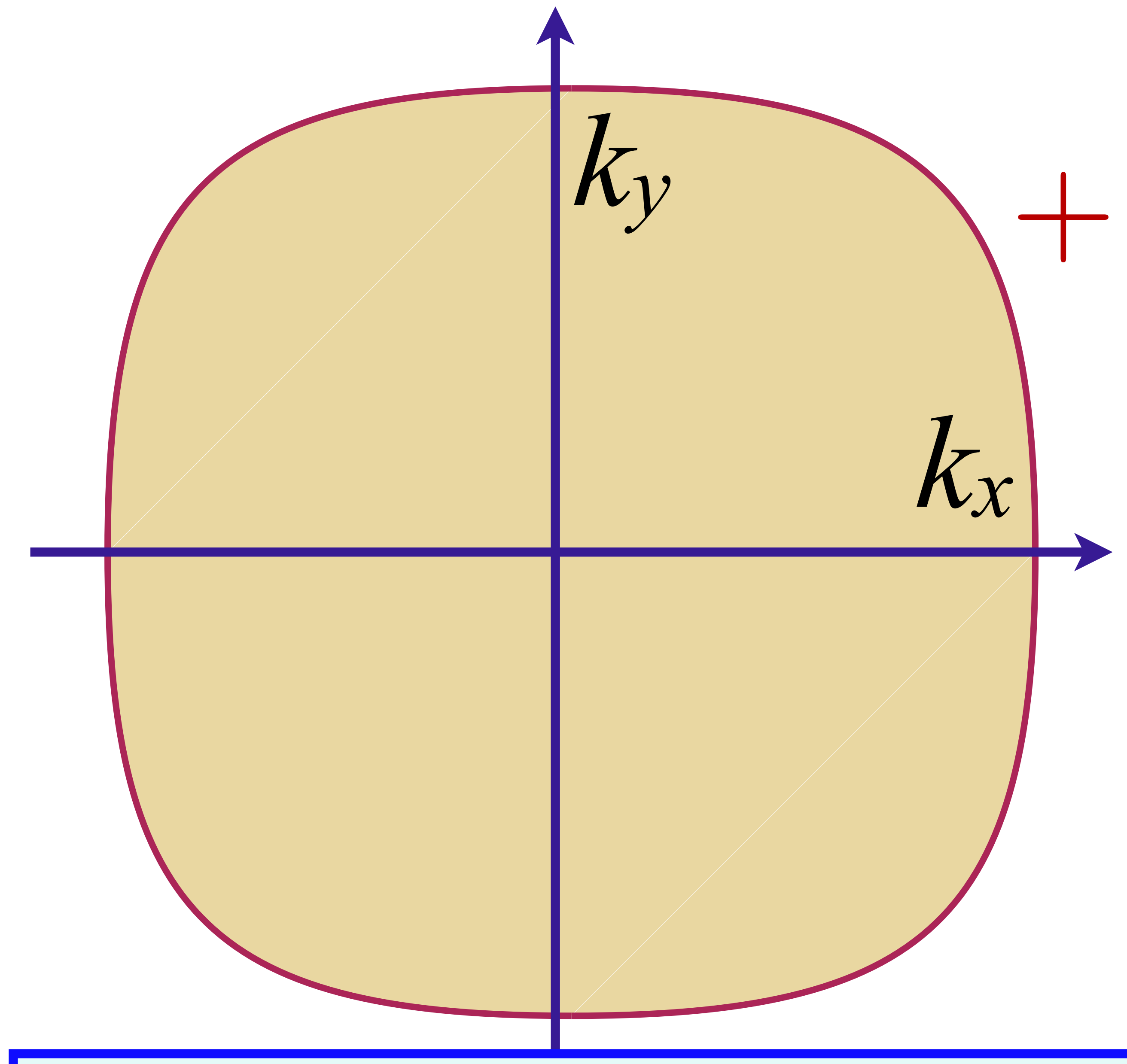
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$v(\boldsymbol{r})$ leads to elastic scattering of c_σ and ‘Altshuler-Aronov’ corrections;
Harris disorder $\delta s(\boldsymbol{r})$ is strongly relevant—cannot use perturbative methods.

Kondo lattice + critical boson with potential and mass disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})]\,[\Phi(\boldsymbol{r})]^2 \xrightarrow{\text{Rescale }\Phi} +g\,c^\dagger_\sigma(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

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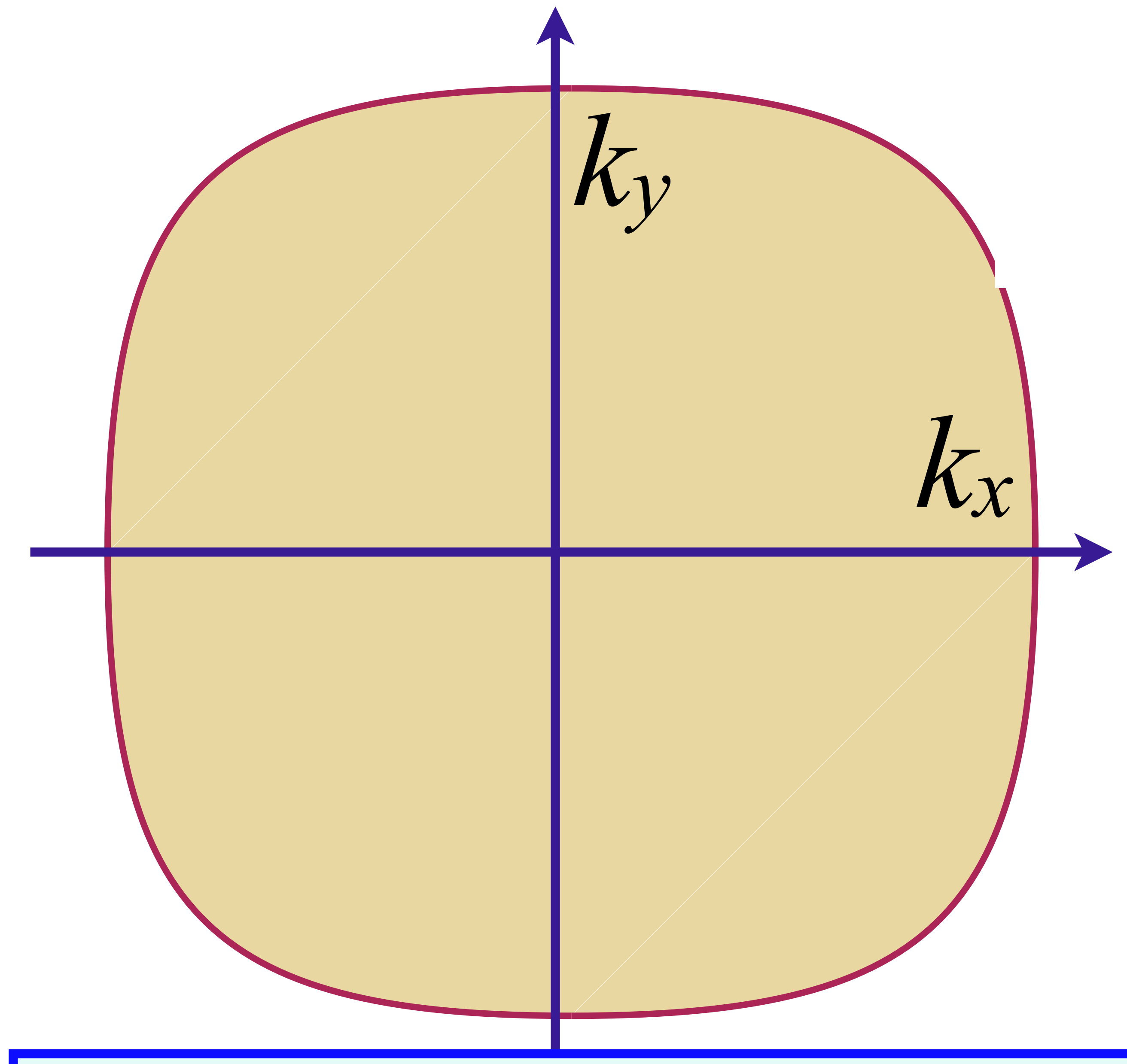

$$+s\,[\Phi(\boldsymbol{r})]^2+[g+g'(\boldsymbol{r})]\,c_{\sigma}^{\dagger}(\boldsymbol{r})f_{\sigma}(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c_{\sigma}^{\dagger}(\boldsymbol{r})c_{\sigma}(\boldsymbol{r})$$

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Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})}=0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')}=g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

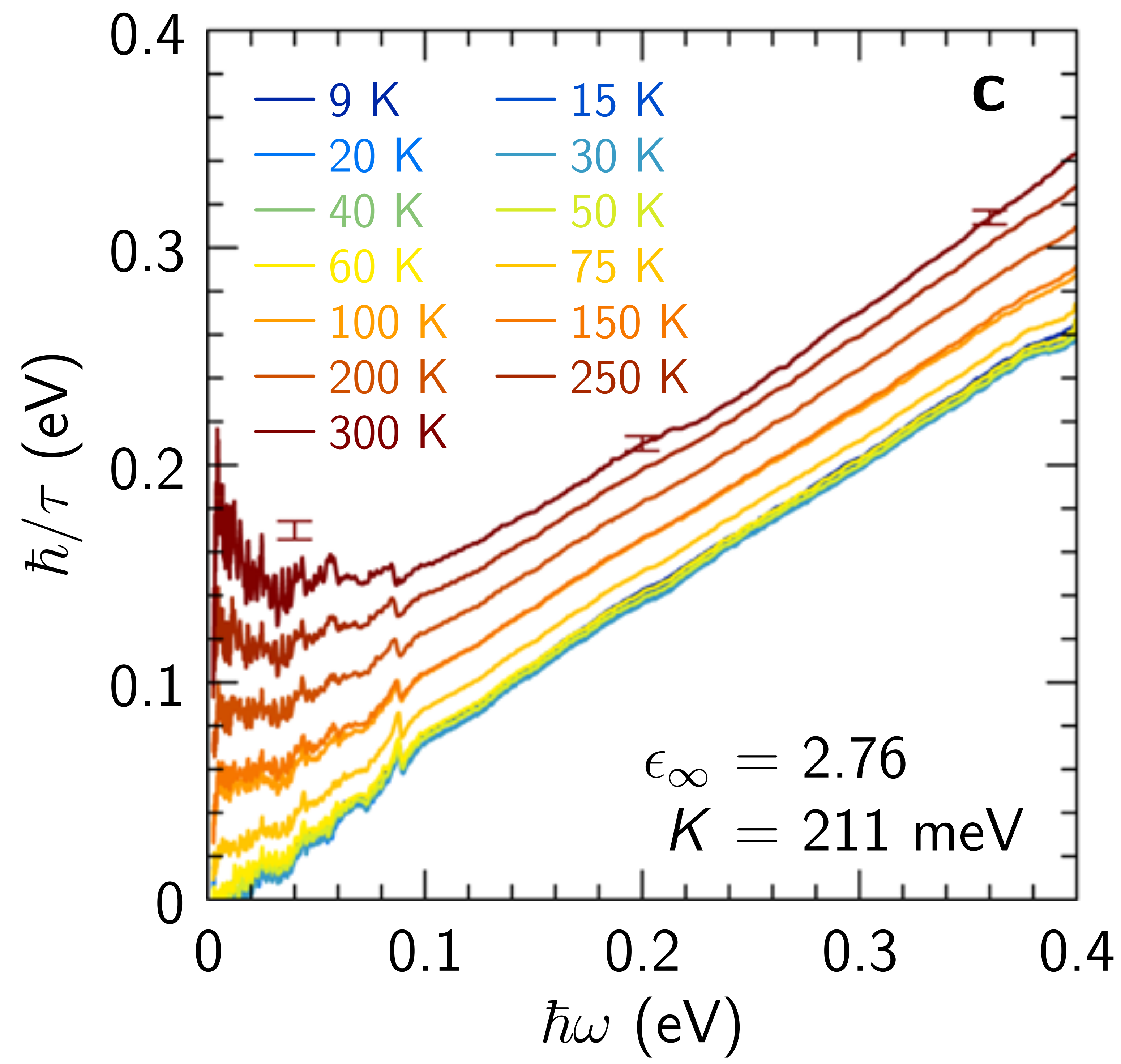
The 2d-YSYK model: can apply self-consistent SYK methods!
Should be applicable as long as eigenmodes of $\Phi(\boldsymbol{r})$ are extended.

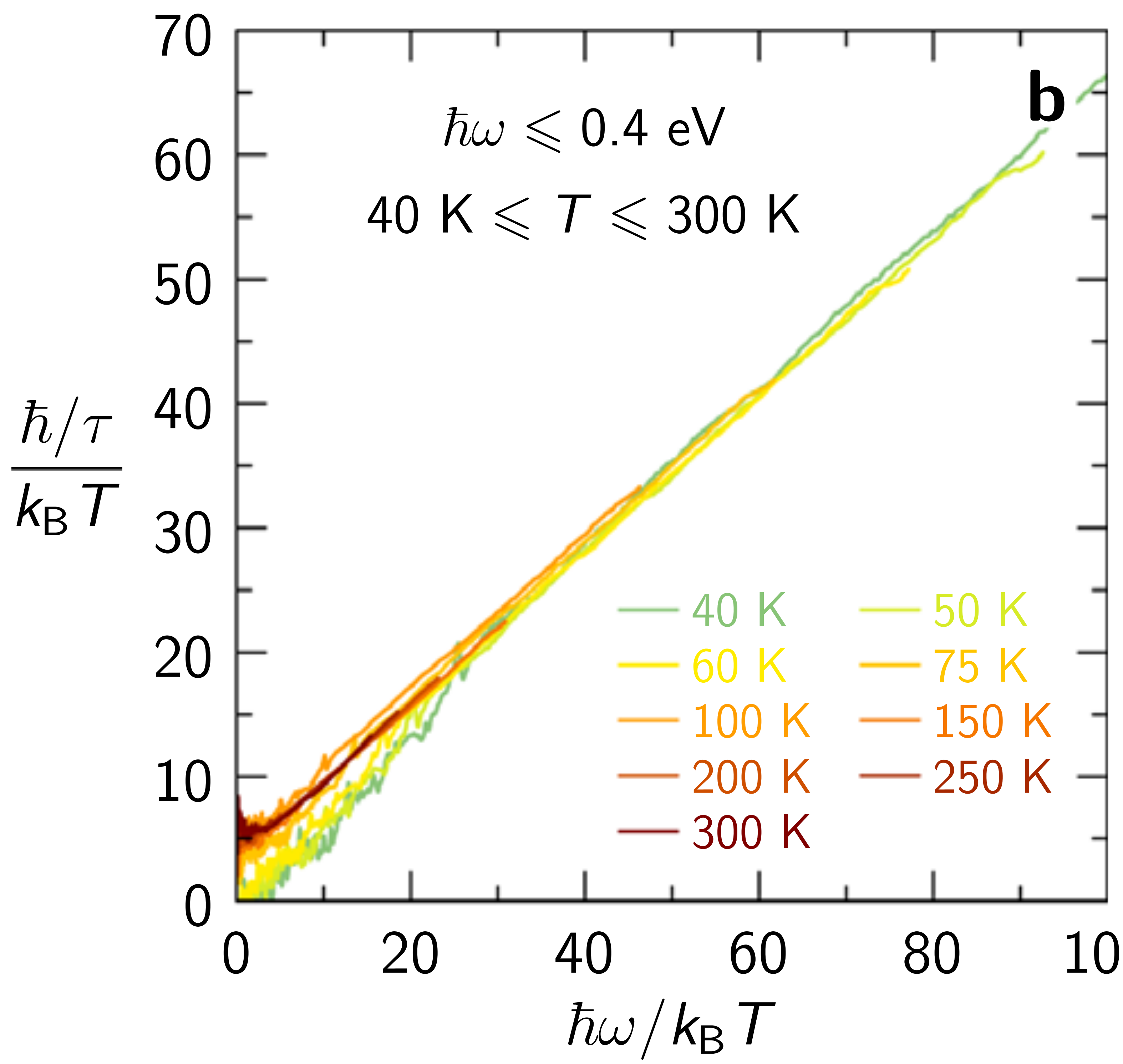
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

$S(T \to 0) \sim T\ln(1/T)$.

$La_{2-x}Sr_xCuO_4$

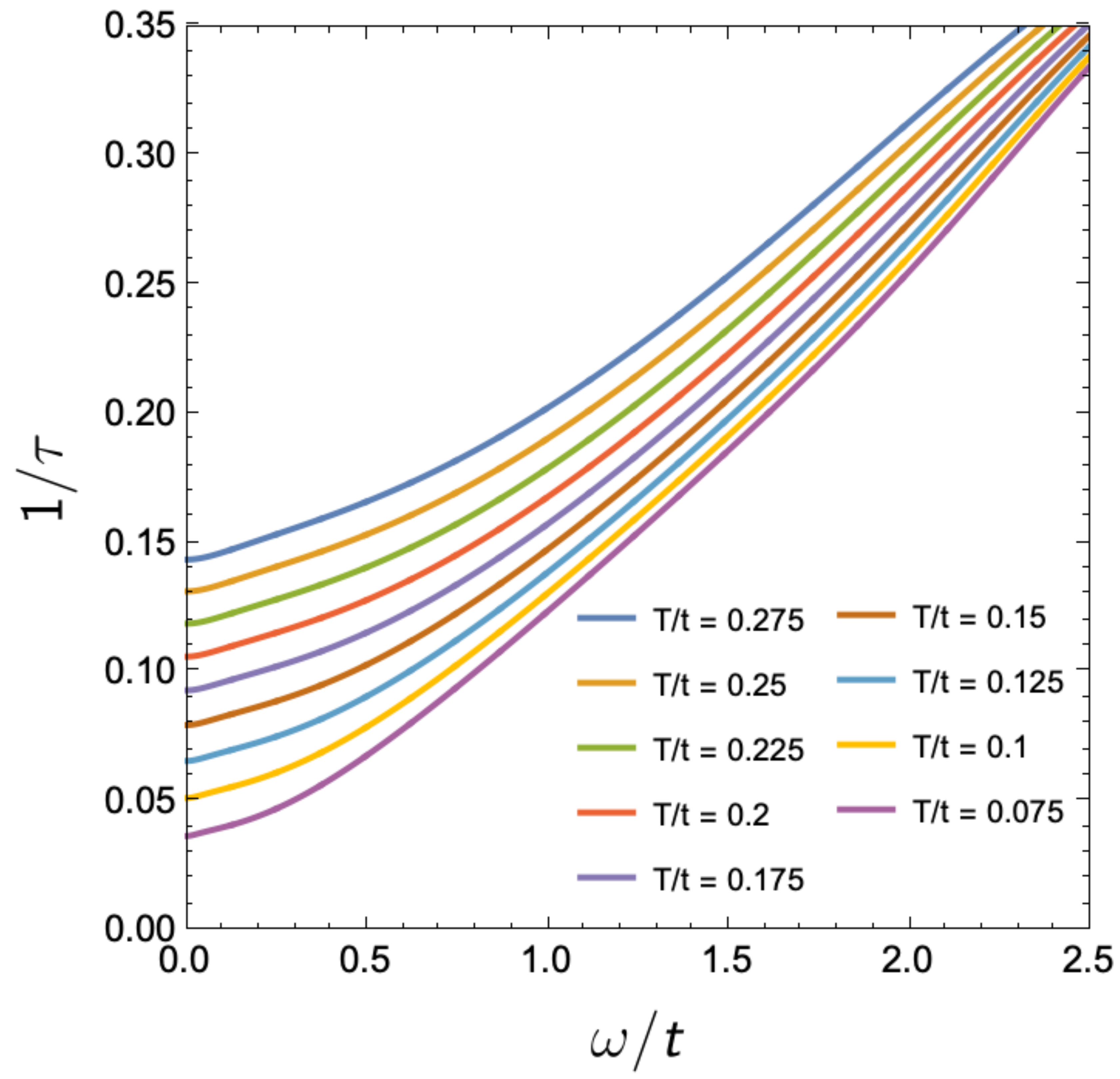
$p = 0.24$

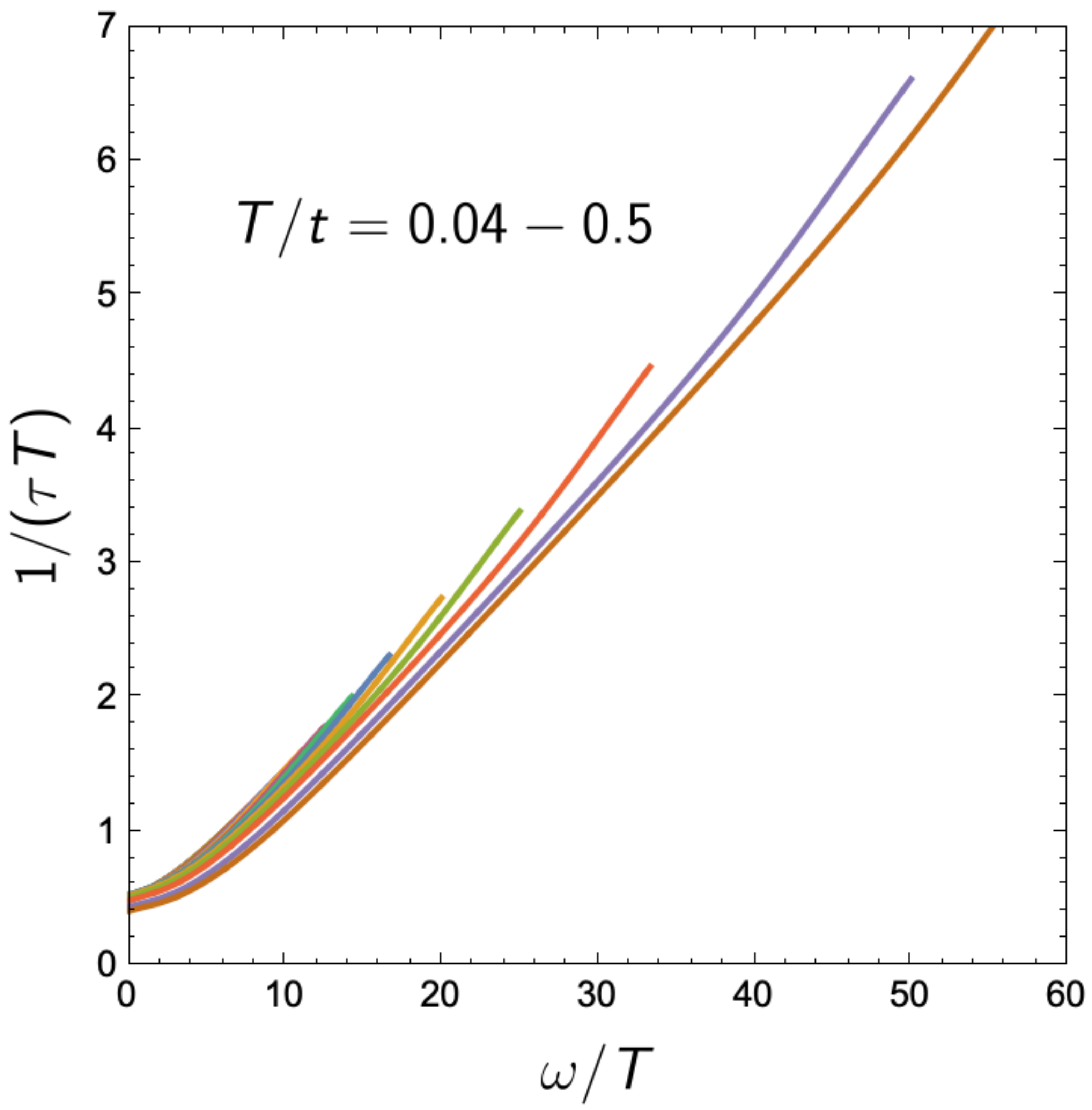
$T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

$$\sigma(\omega) = i\,\frac{e^2 K/(\hbar d_c)}{\hbar\omega\,\dfrac{m^*(\omega)}{m} + i\,\dfrac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

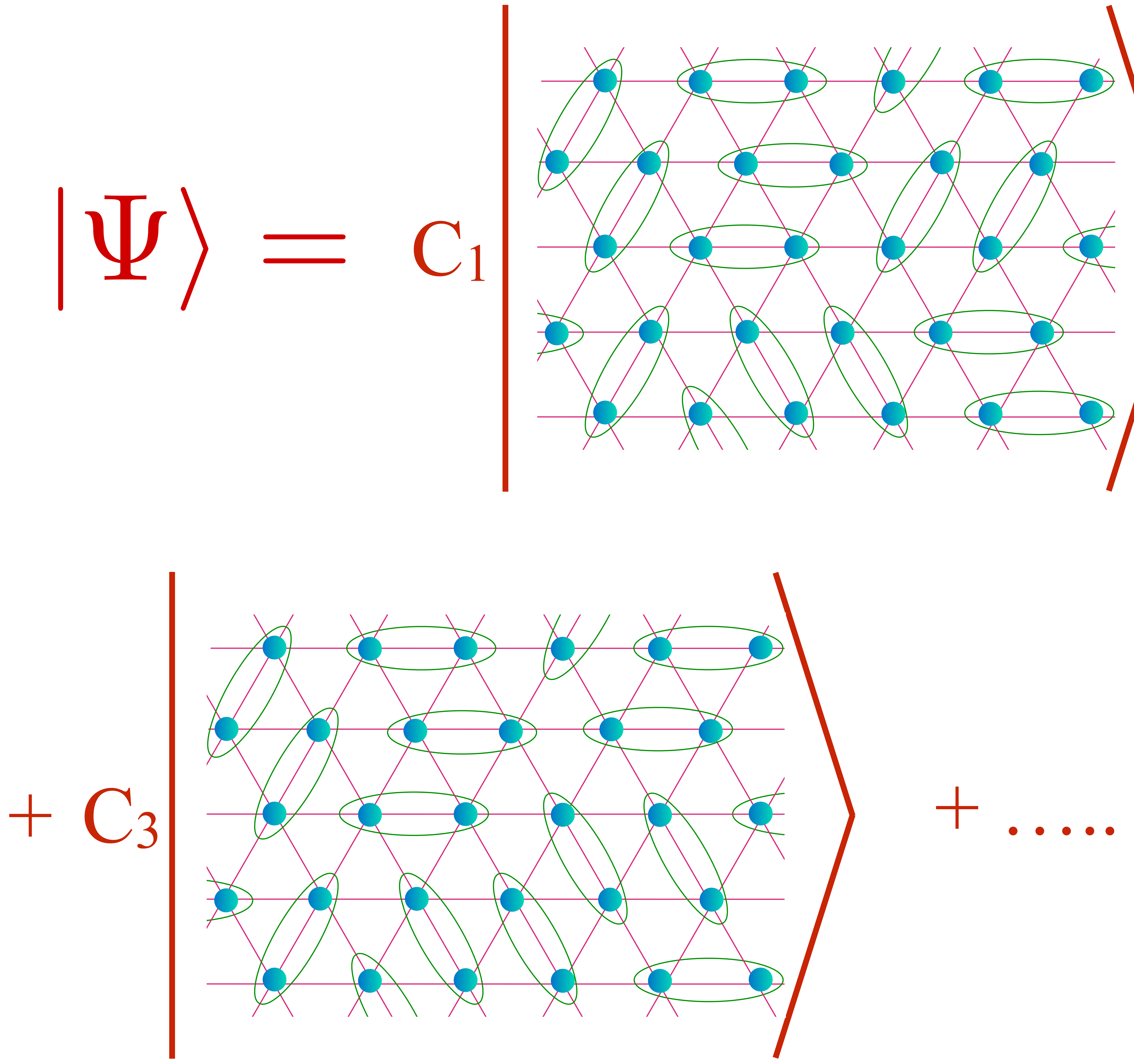
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

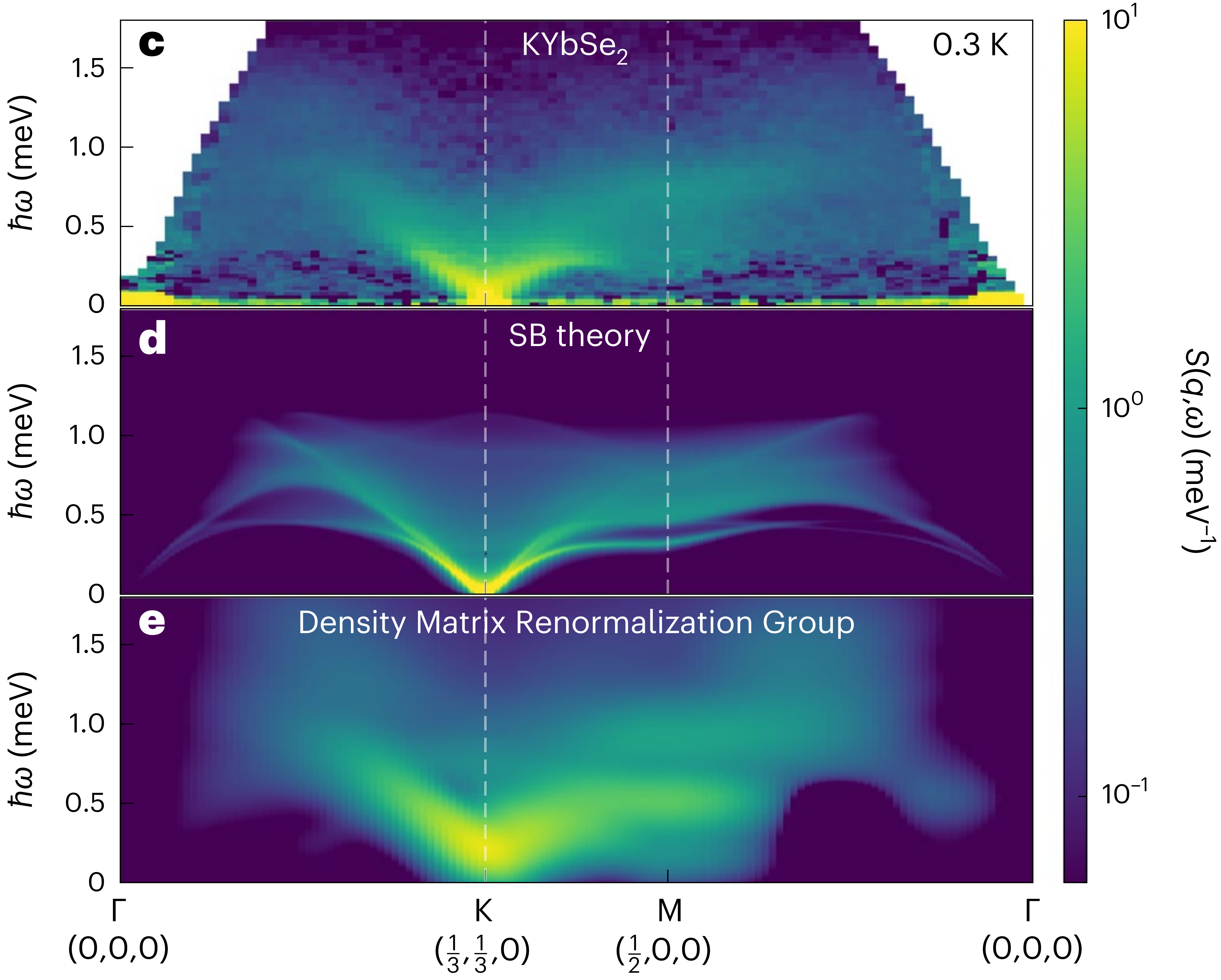
and entropy $S(T \to 0) \sim T \ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Recap

Entanglement of stationary electrons

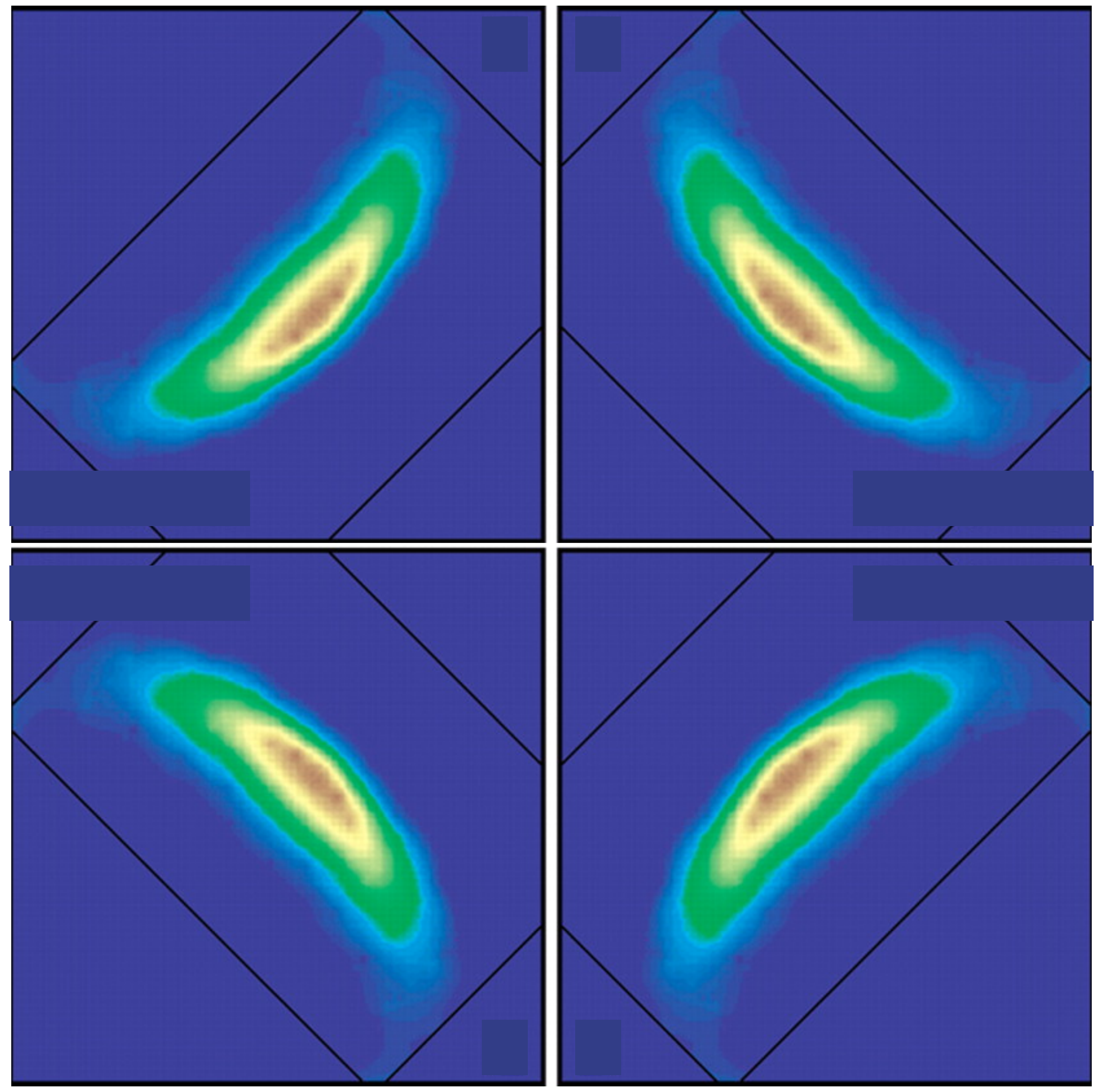
Theory and experiments on Z_2 spin liquids





Entanglement of stationary electrons

Gapless U(1) spin liquid of the pseudogap metal in the cuprates



$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

H. C. Robarts…..S. M. Hayden
PRB **100**, 214510 (2019)

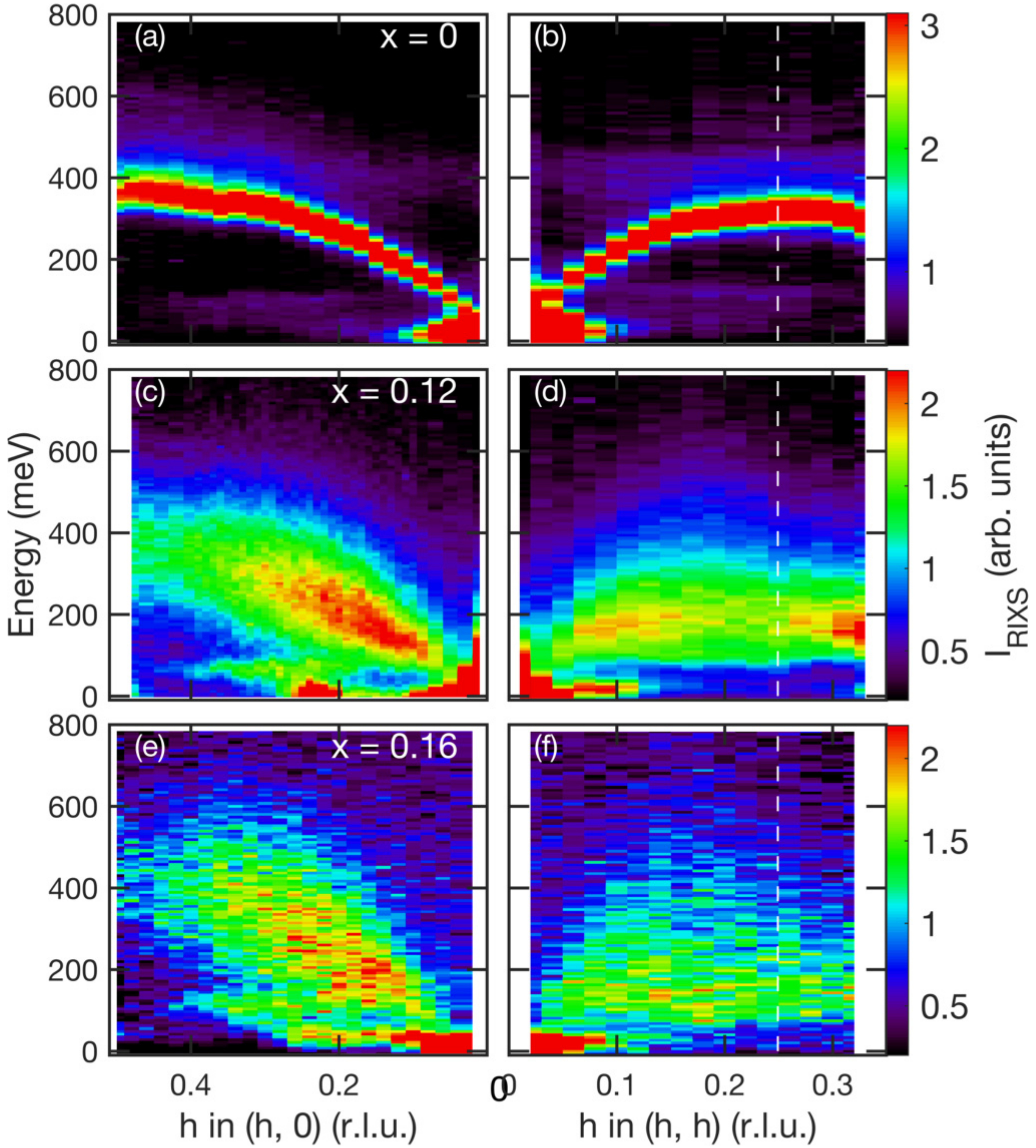
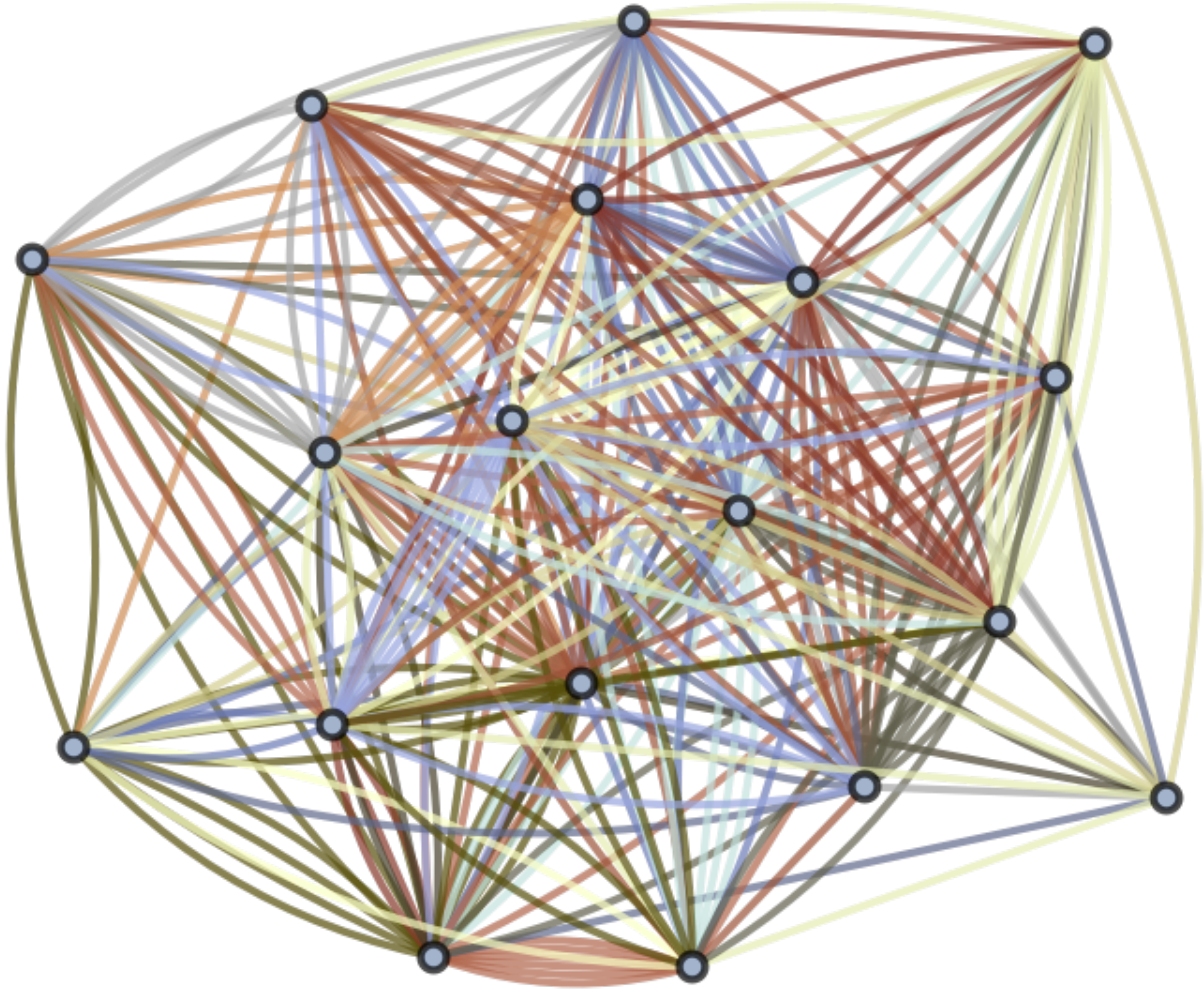


FIG. 2. I_{RIXS} intensity maps as a function of **Q** in LSCO $x = 0$ ($T \approx 20$ K), 0.12, and 0.16 ($T \approx 30$ K).

Entanglement of mobile electrons

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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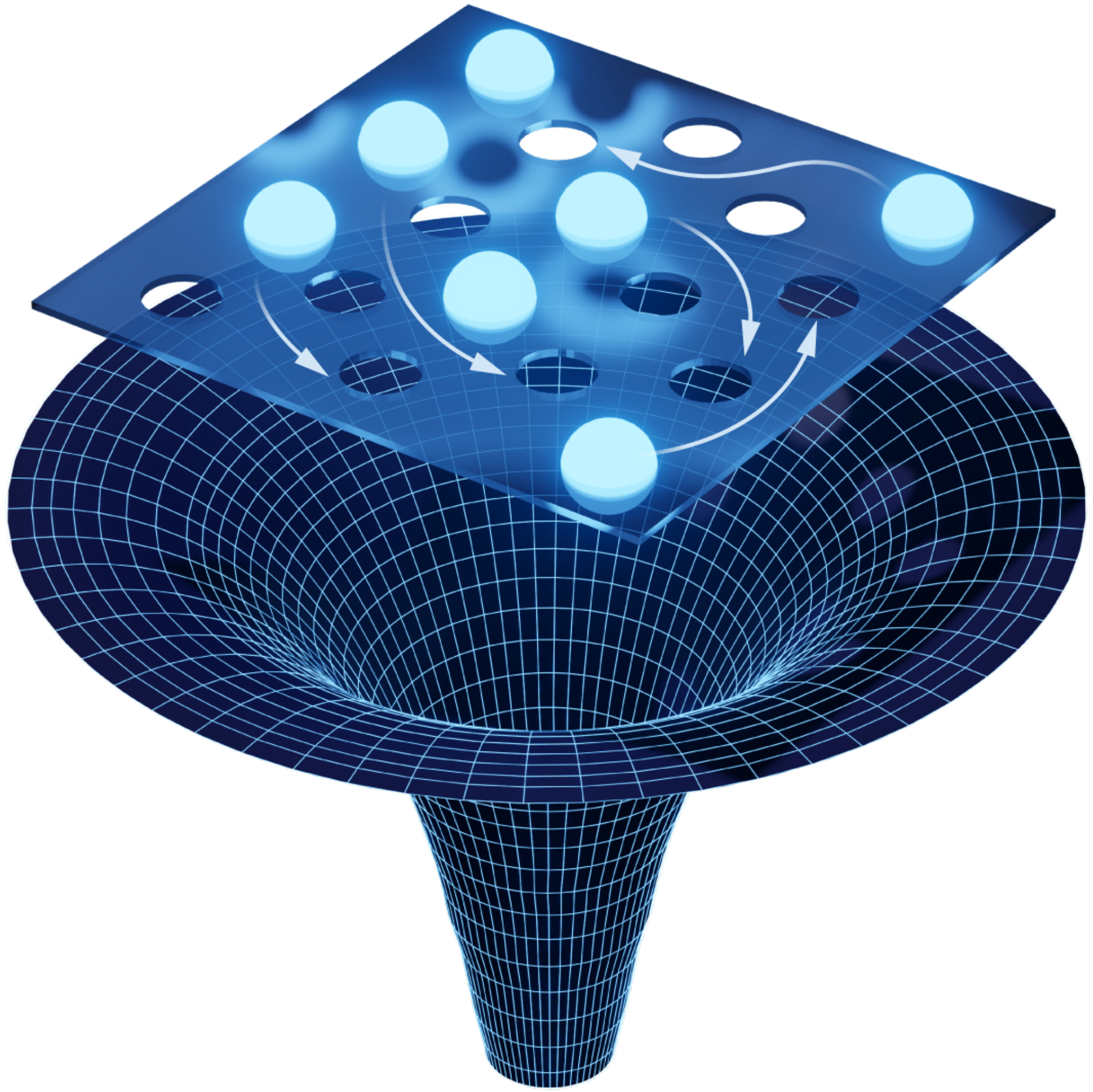

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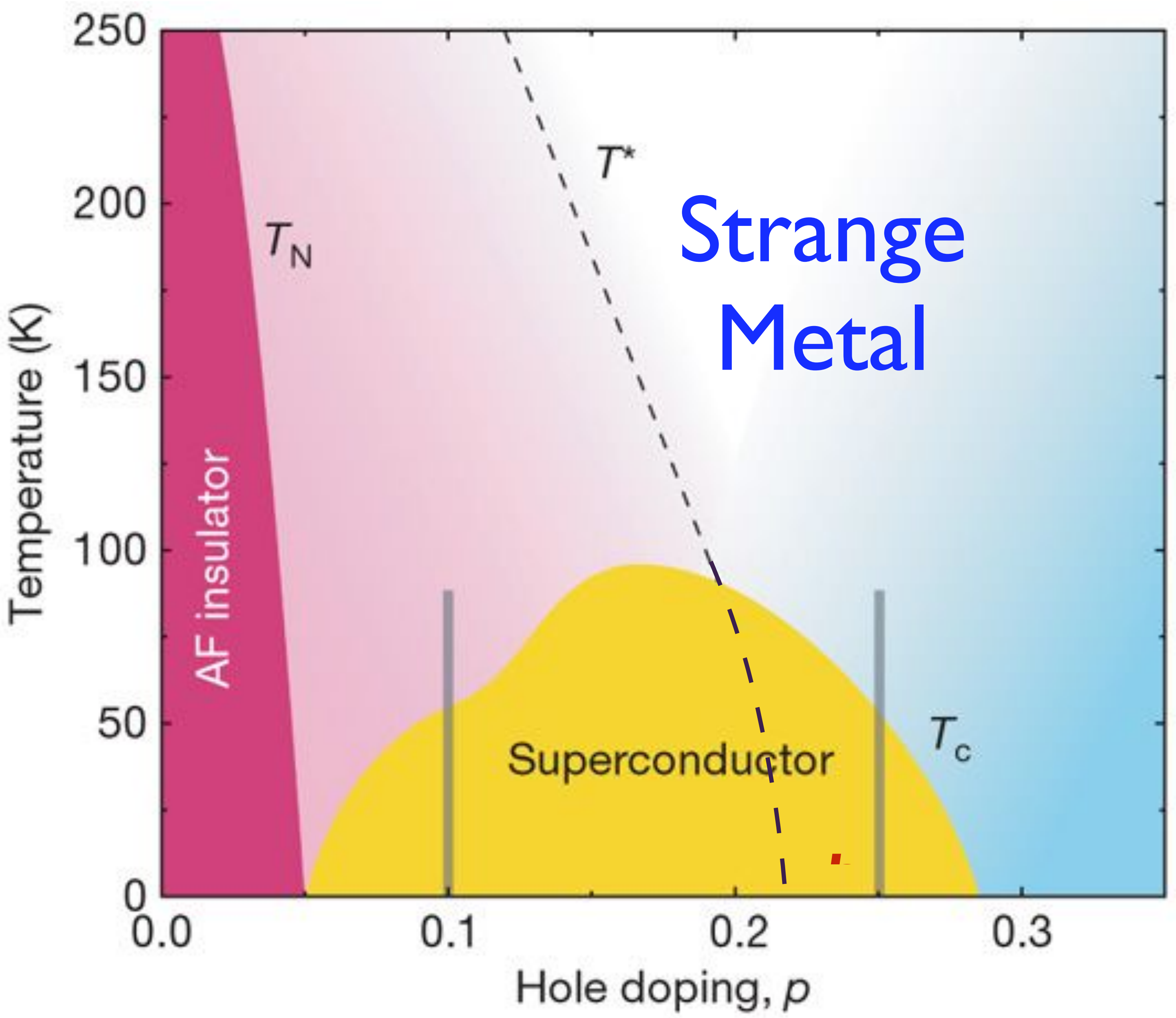

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A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials