Quantum matter without quasiparticles: SYK models, black holes, and the cuprate strange metal

Workshop on Frontiers of Quantum Materials
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Subir Sachdev

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Talk online: sachdev.physics.harvard.edu

PHYSICS
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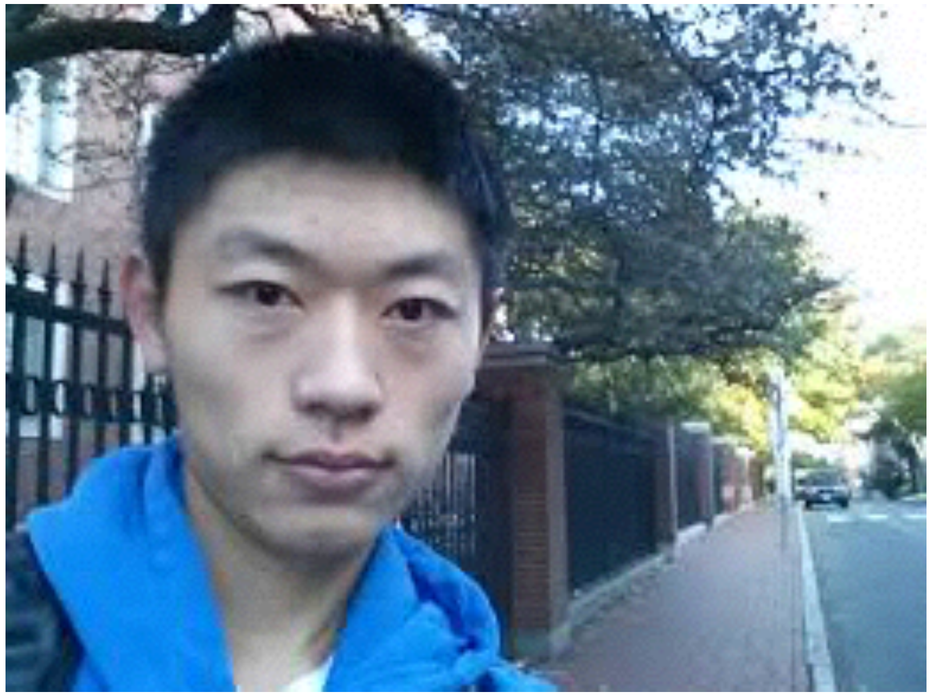
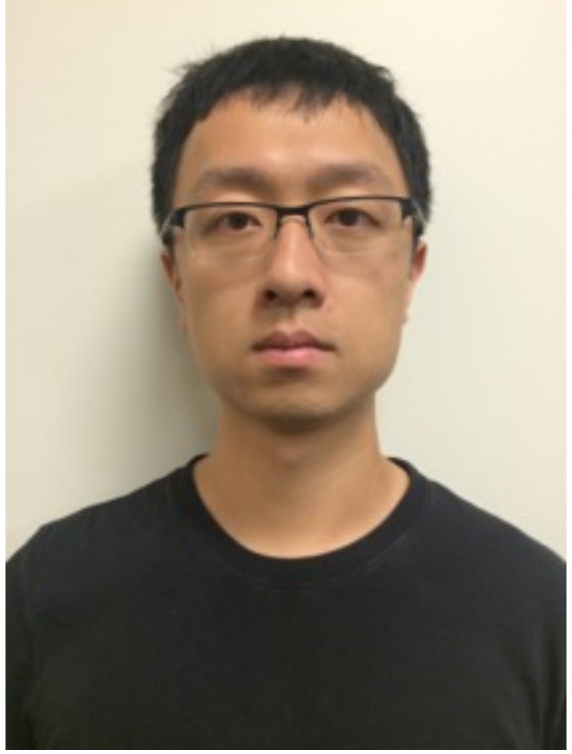
Aavishkar Patel, Harvard



Wenbo Fu, Harvard

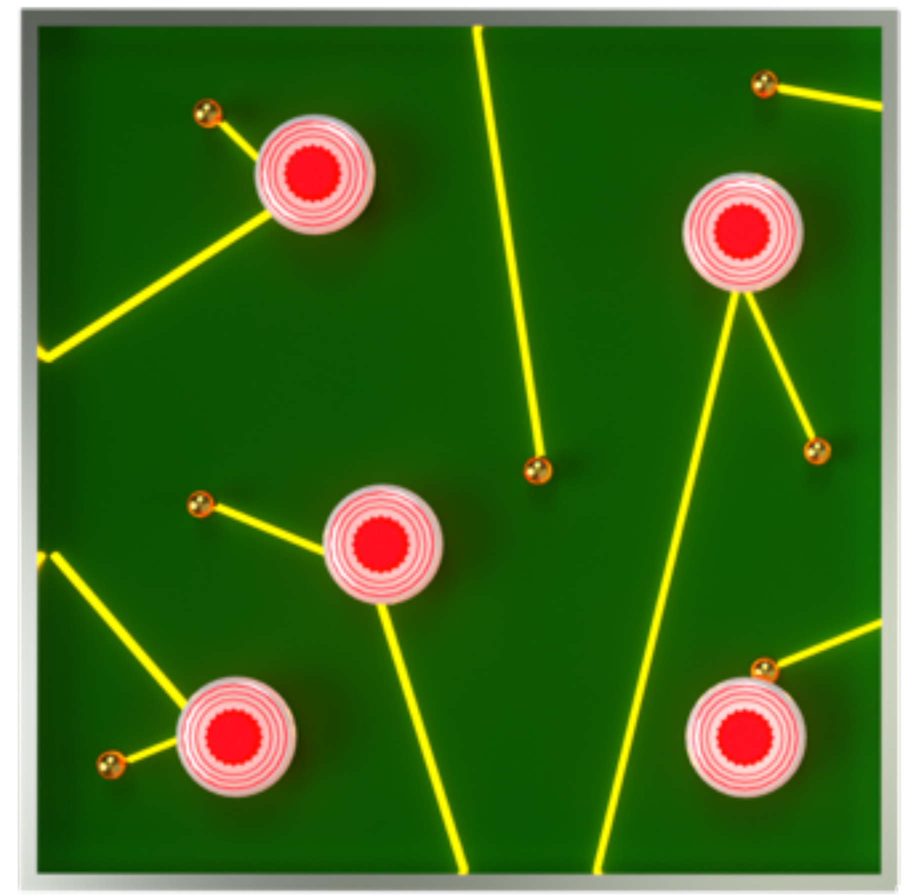


Yingfei Gu, Stanford

Richard Davison, Harvard

Quantum matter with quasiparticles:

- *Magnon*
- *Roton*
- *Plasmon*
- *Polaron*
- *Exciton*
- *Laughlin quasiparticle*
- *Composite fermion*
- *Bogoliubovon*
- *Anderson-Higgs mode*
- *Massless Dirac Fermions*
- *Weyl fermions*
- *....*



Quantum matter <u>with</u> quasiparticles:

Most generally, a quasiparticle is an "additive" excitation:

Quasiparticles can be combined to yield additional excitations, with energy determined by the energies and densities of the constituents. Such a procedure yields all the low-lying excitations. Then we can apply the Boltzmann-Landau theory to make predictions for dynamics.

Quantum matter without quasiparticles:

No quasiparticle structure to excitations.

But how can we be sure that no quasiparticles exist in a given system?
Perhaps there are some exotic quasiparticles inaccessible to current experiments……..

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Consider how rapidly the system loses "phase coherence", reaches local thermal equilibrium, or becomes "chaotic"

Local thermal equilibration or phase coherence time, τ_φ:

- There is an *lower bound* on τ_φ in all many-body quantum systems of order $\hbar/(k_B T)$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

  and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T;
  e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
  and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A bound on quantum chaos:

- The time over which a many-body quantum system becomes "chaotic" is given by $\tau_L = 1/\lambda_L$, where λ_L is the "Lyapunov exponent" determining memory of initial conditions. This LYAPUNOV TIME obeys the rigorous lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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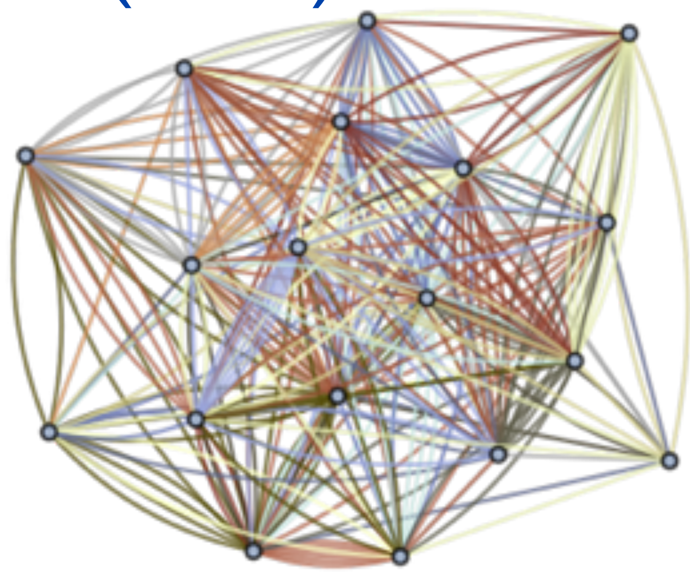
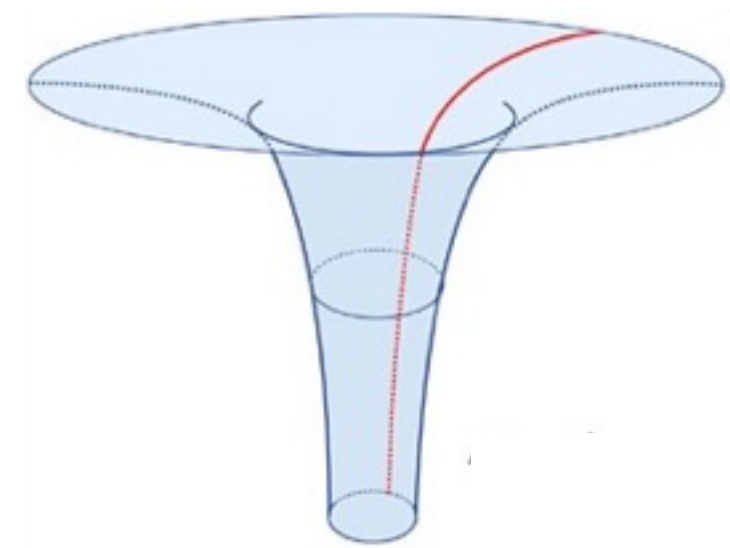
Quantum matter without quasiparticles
$\approx$ fastest possible many-body quantum chaos

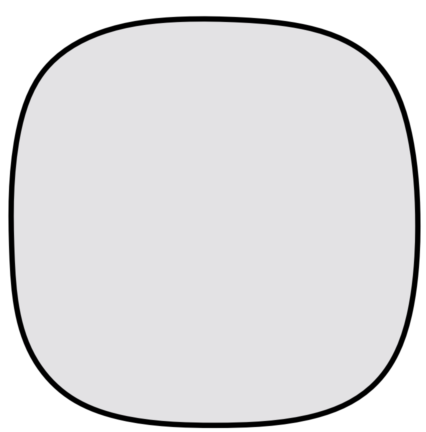
Quantum matter <u>without</u> quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons





Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

Quantum matter without quasiparticles:

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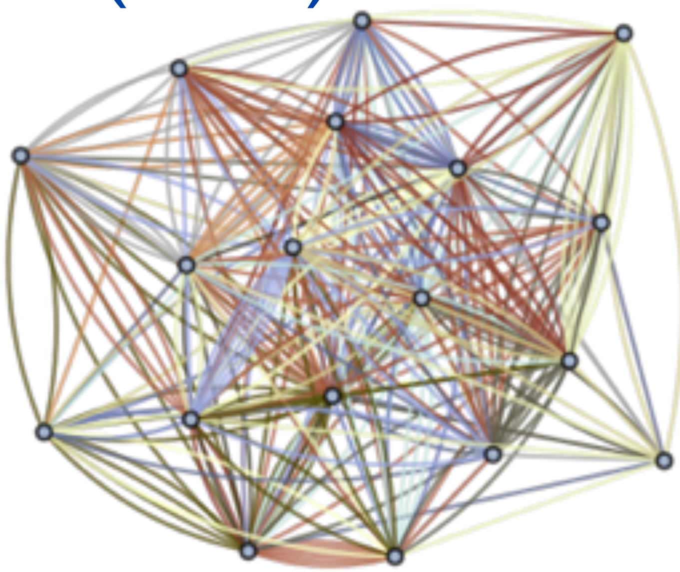
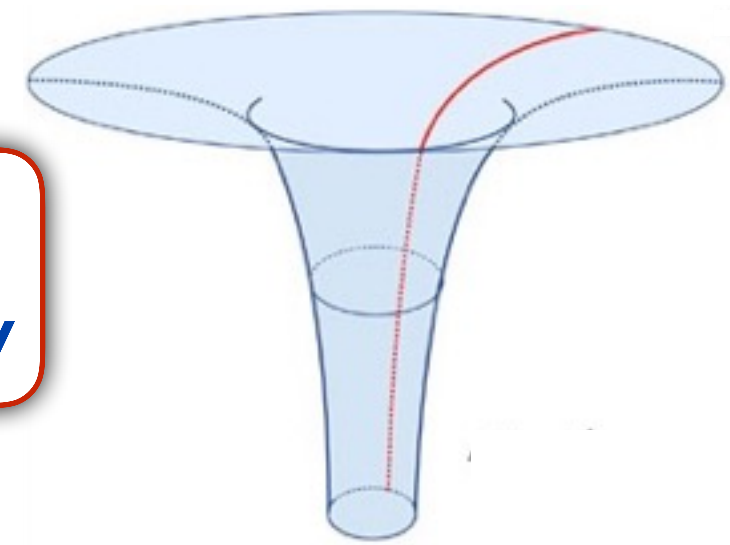
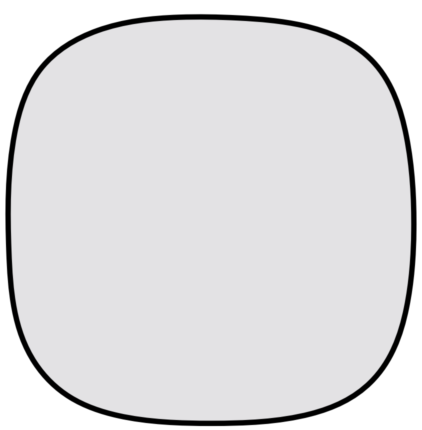


Same low energy theory

Black holes with AdS_2 horizons





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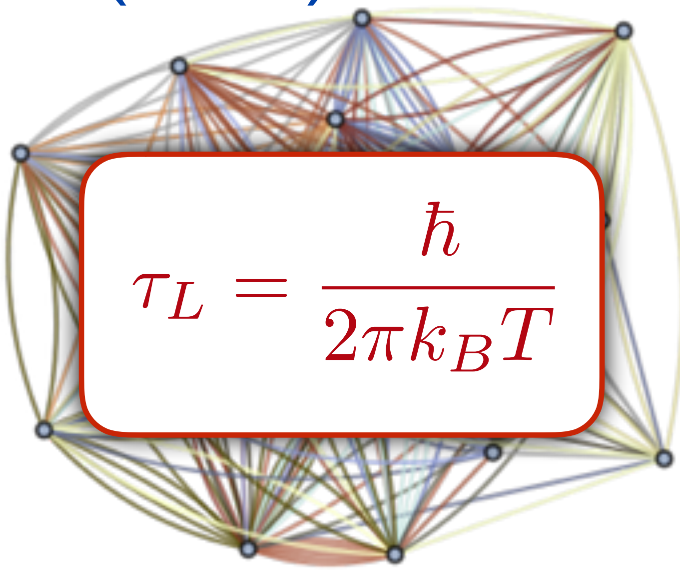
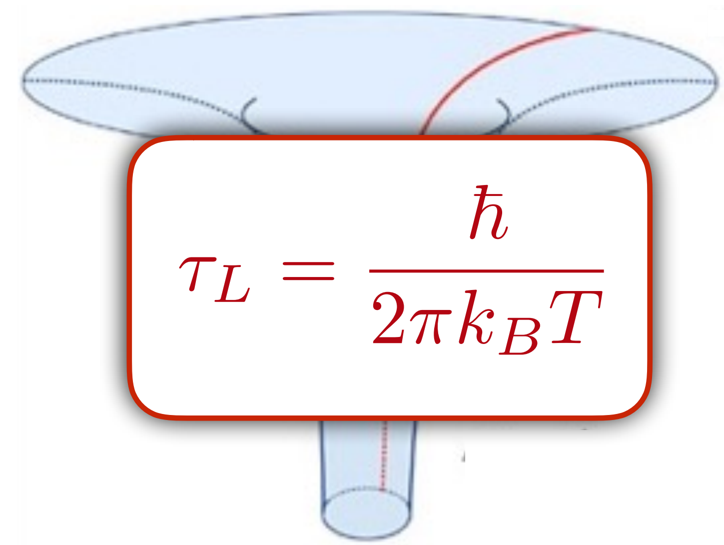


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Black holes with AdS_2 horizons



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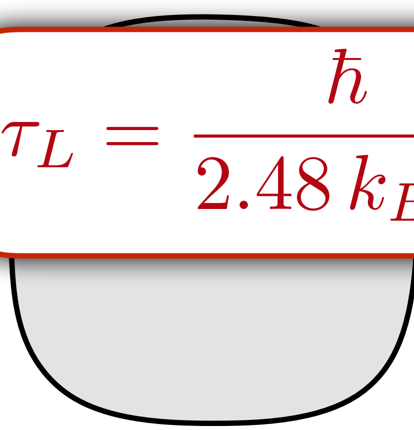

$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

Quantum matter without quasiparticles:

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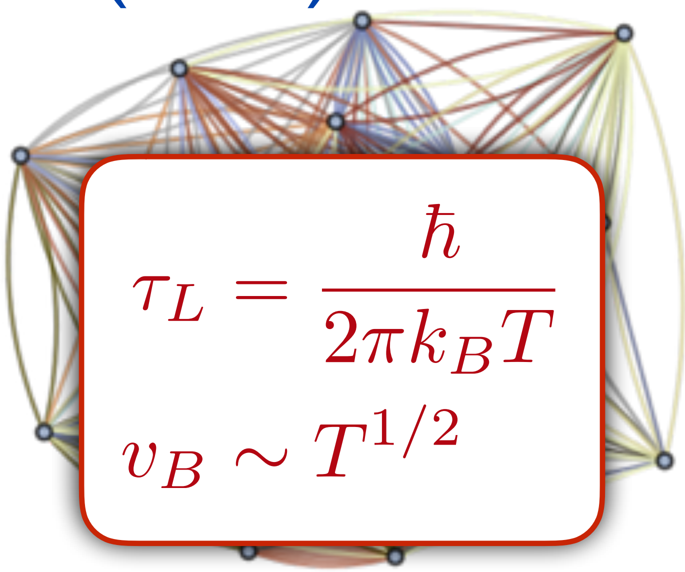

$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

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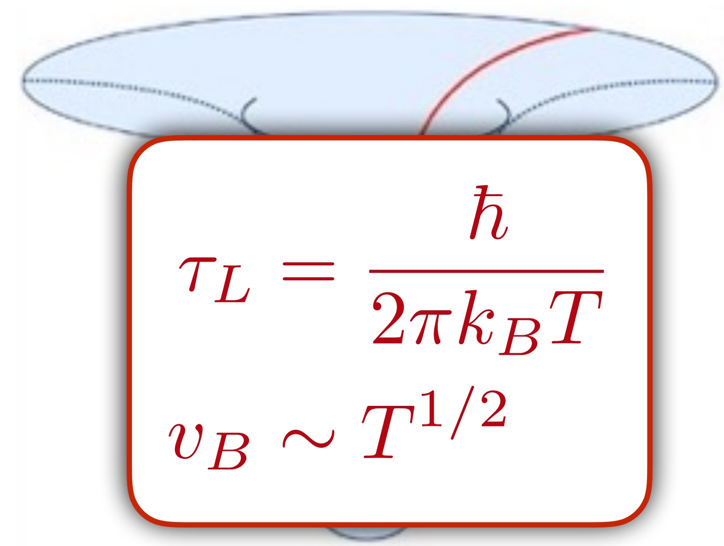

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Fermi surface coupled to a gauge field

$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$v_B \sim \frac{v_F^{5/3}}{e^{4/3}\gamma^{1/3}} T^{1/3}$$

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

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The Sachdev-Ye-Kitaev (SYK) models

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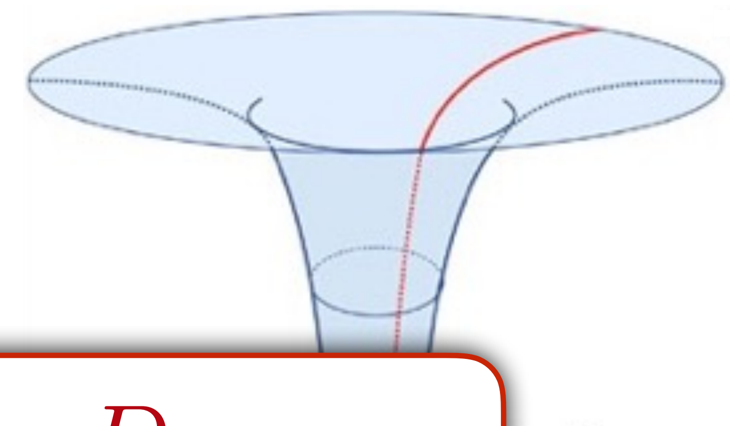
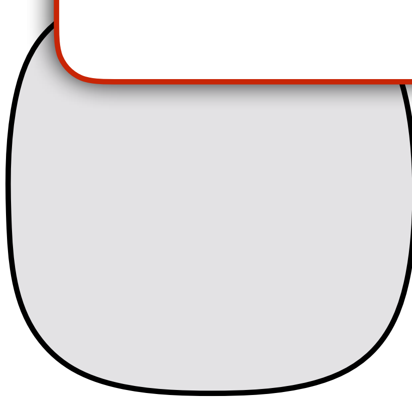


Thermal diffusivity, D_E:

$$D_E = (\text{universal number}) \times v_B^2 \tau_L$$

in all three models



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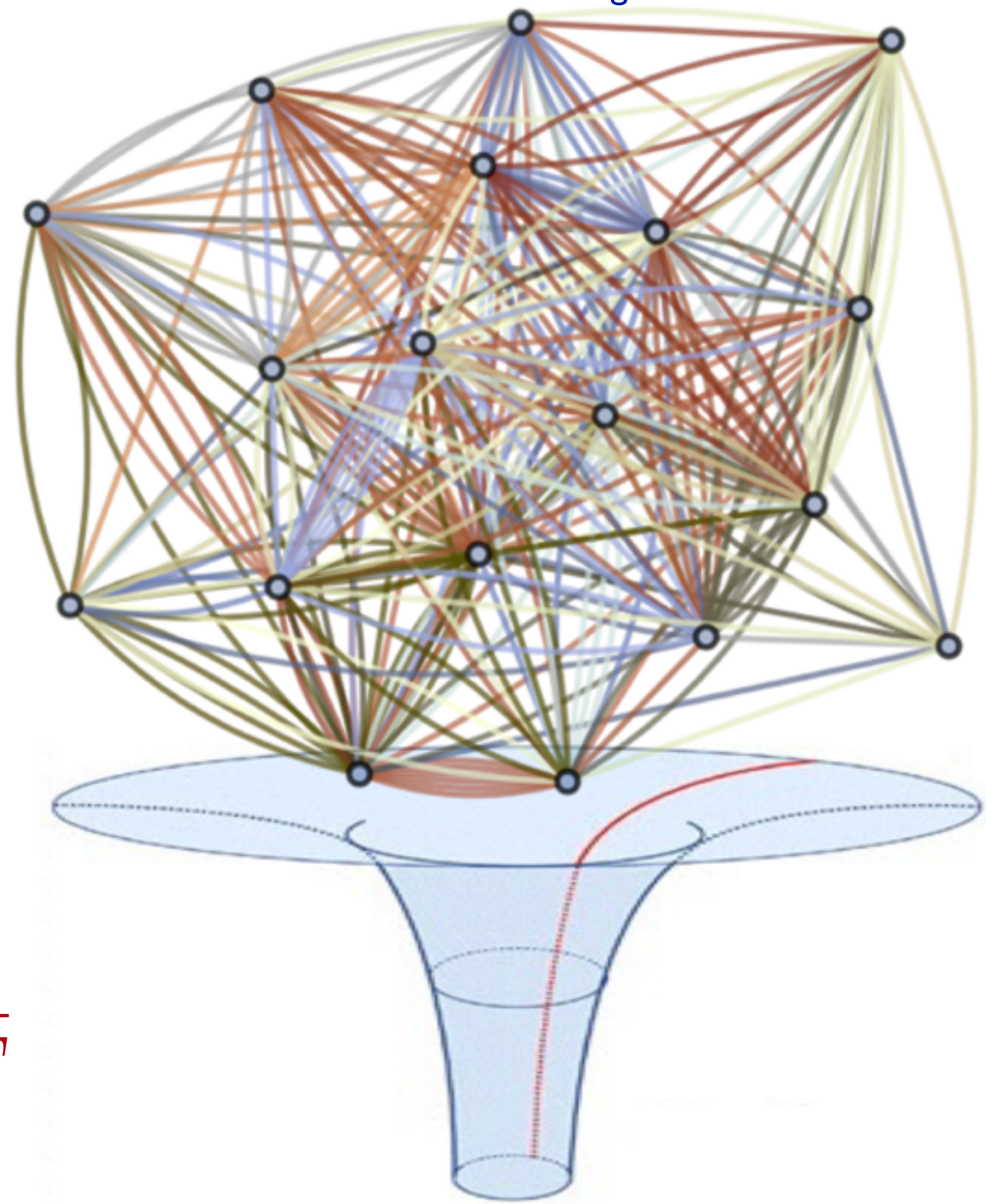
τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS_2
- Fastest possible quantum chaos with $\tau_L = \dfrac{\hbar}{2\pi k_B T}$

Figure credit: L. Balents

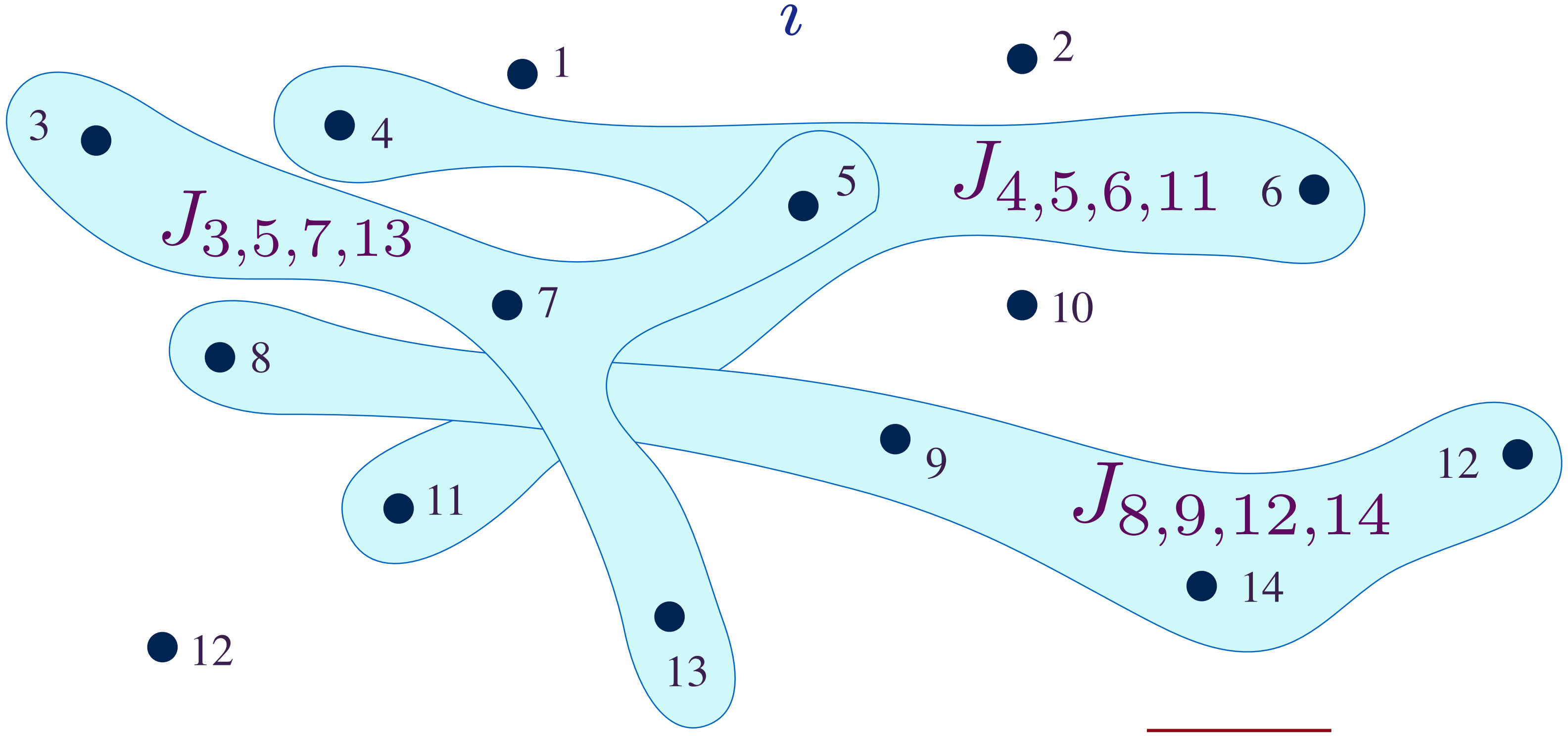


SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK and AdS_2

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$
  Not a ground state degeneracy: due to an exponentially small (in N) many-body level spacing at all energies down to the ground state energy.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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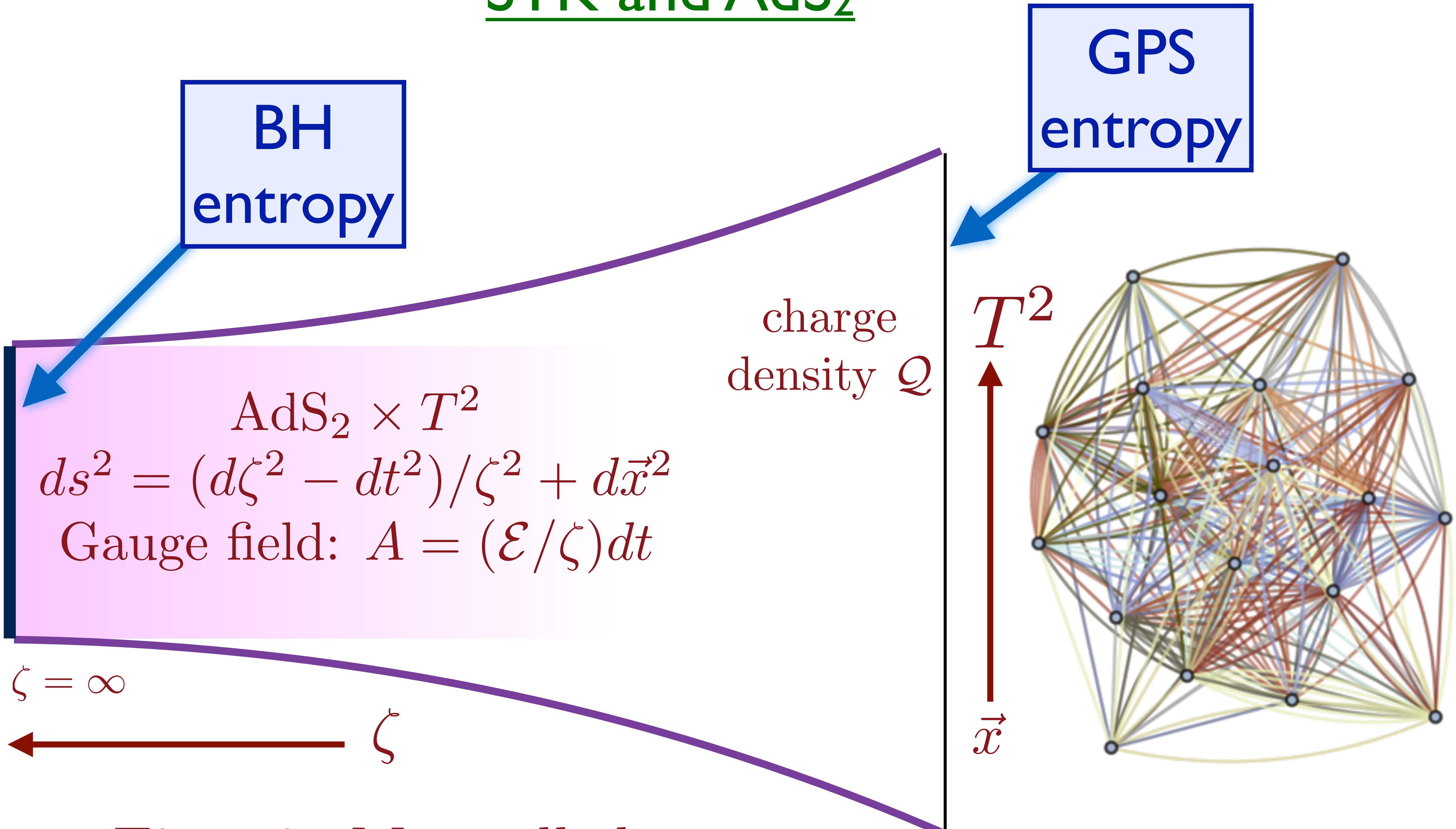
- The correlators and thermodynamics of SYK match those of quantum matter holographically dual to AdS_2 black hole horizons (as computed by T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)). SYK models are "are states of matter at non-zero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nördstrom black holes". The Bekenstein-Hawking entropy is NS_0 (**GPS = BH**).

S. Sachdev, PRL **105**, 151602 (2010)

SYK and AdS_2



Einstein-Maxwell theory
+ cosmological constant

S. Sachdev, PRL **105**, 151602 (2010)

Mapping to SYK applies when temperature $\ll 1/(\text{size of } T^2)$

SYK and AdS_2

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

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SYK and AdS_2

$$G(i\omega) = \frac{1}{\cancel{i\omega} + \cancel{\mu} - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \cancel{\mu} - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished

S. Sachdev, PRX **5**, 041025 (2015)

SYK and AdS_2

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \quad \sim \quad (\tau_1 - \tau_2)^{-1/2}$$
$$\Sigma_s(\tau_1 - \tau_2) \quad \sim \quad (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

A. Kitaev, unpublished

SYK and AdS_2

Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism
- SL(2,R) is the isometry group of AdS_2.

Coupled SYK models

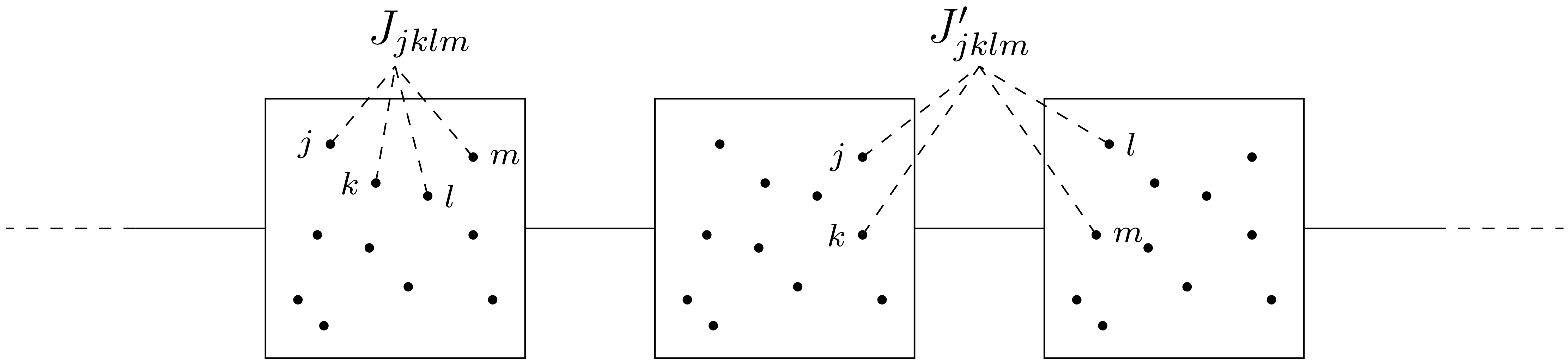

Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832

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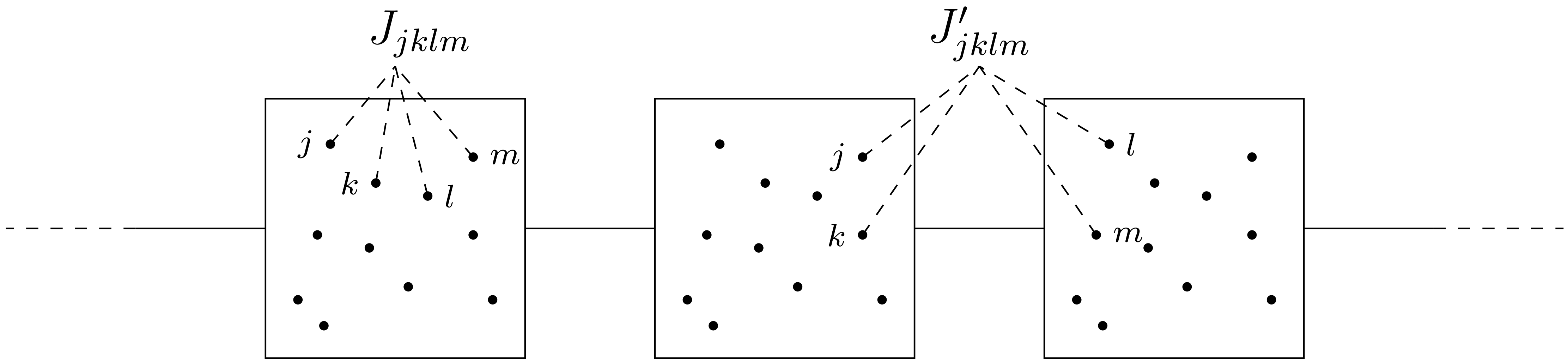

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Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832

Can compute butterfly velocity, and thermo-electric transport correlators, and they precisely match those of momentum-dissipating charged black holes with $\mathrm{AdS}_2 \times R^d$ horizons.

R. Davison, Wenbo Fu, Yingfei Gu, K. Jensen. S. Sachdev, unpublished

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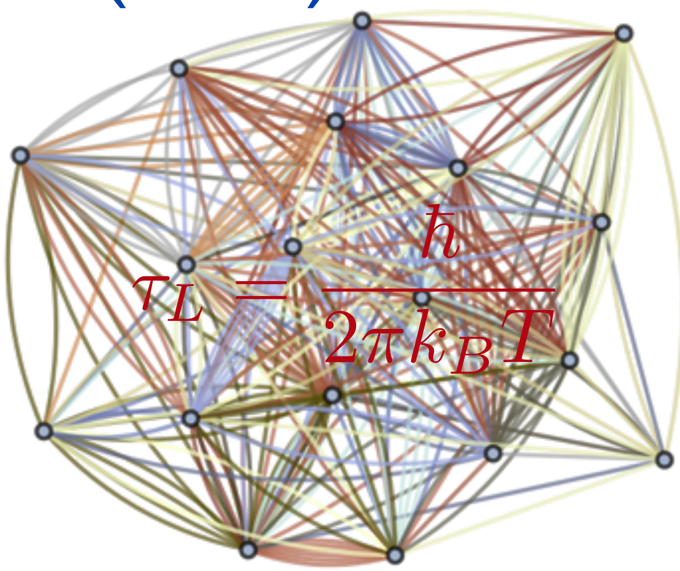
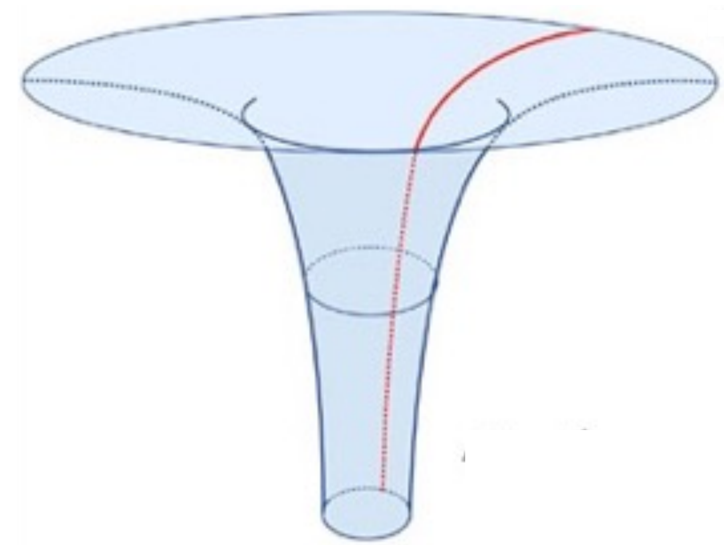
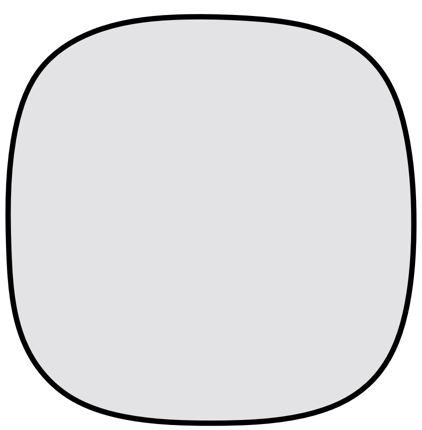


Black holes with AdS_2 horizons





Fermi surface coupled to a gauge field

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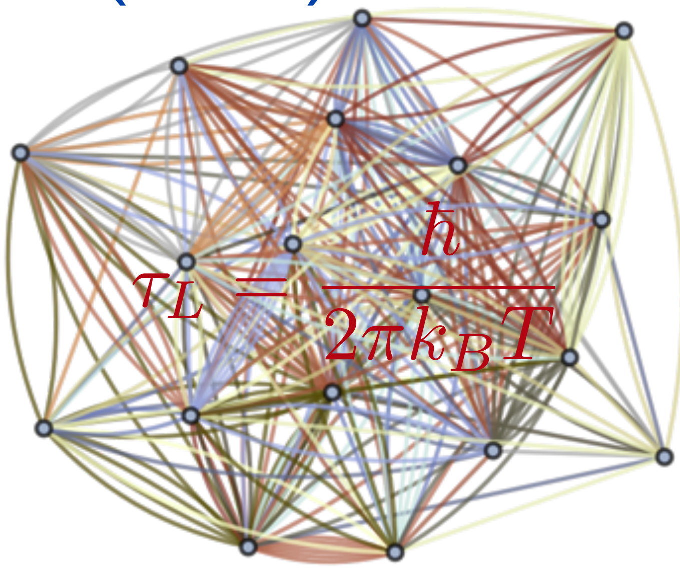

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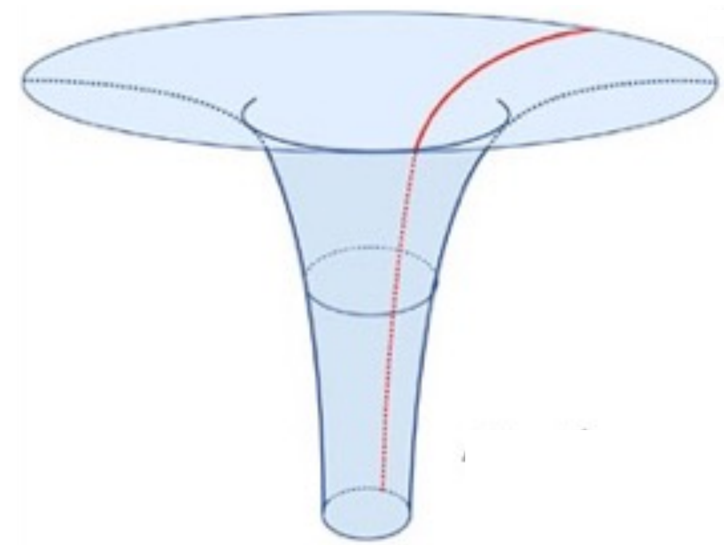

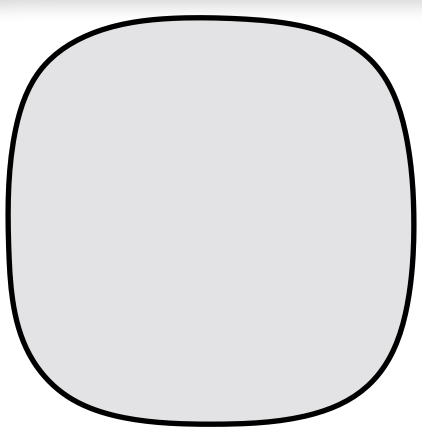

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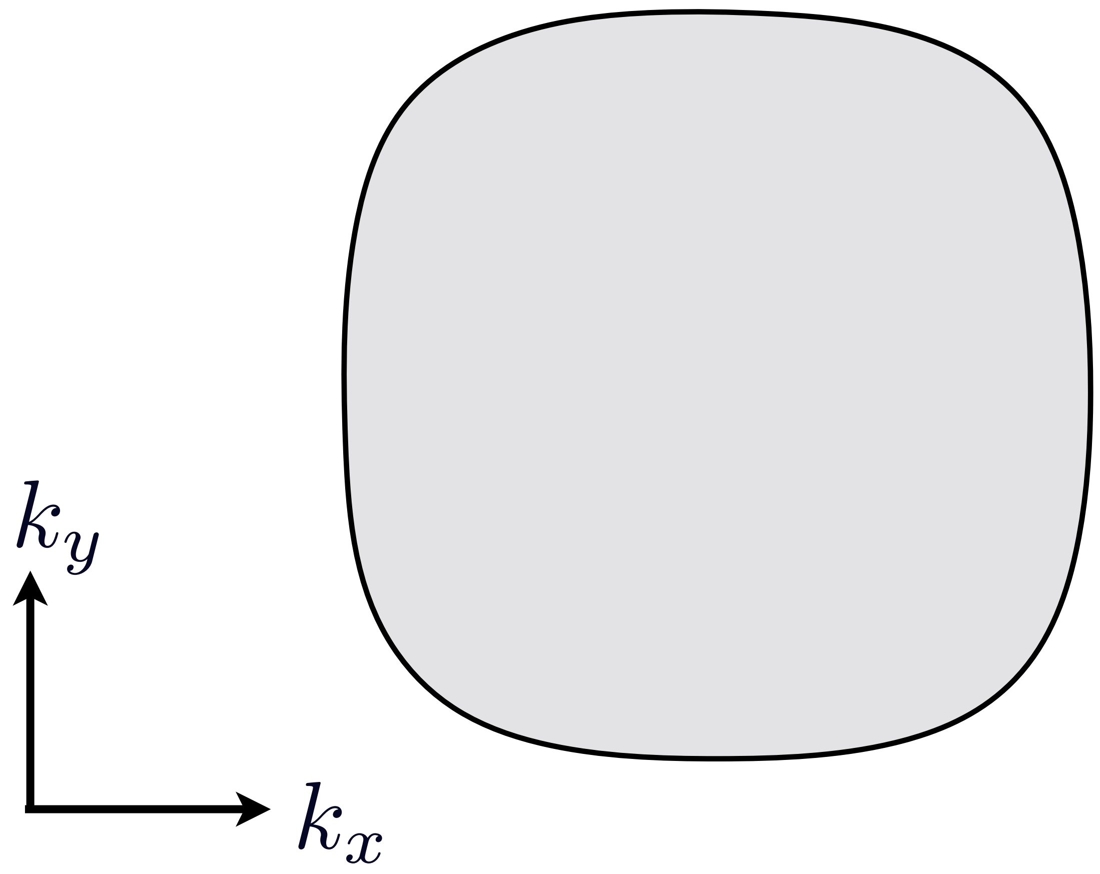

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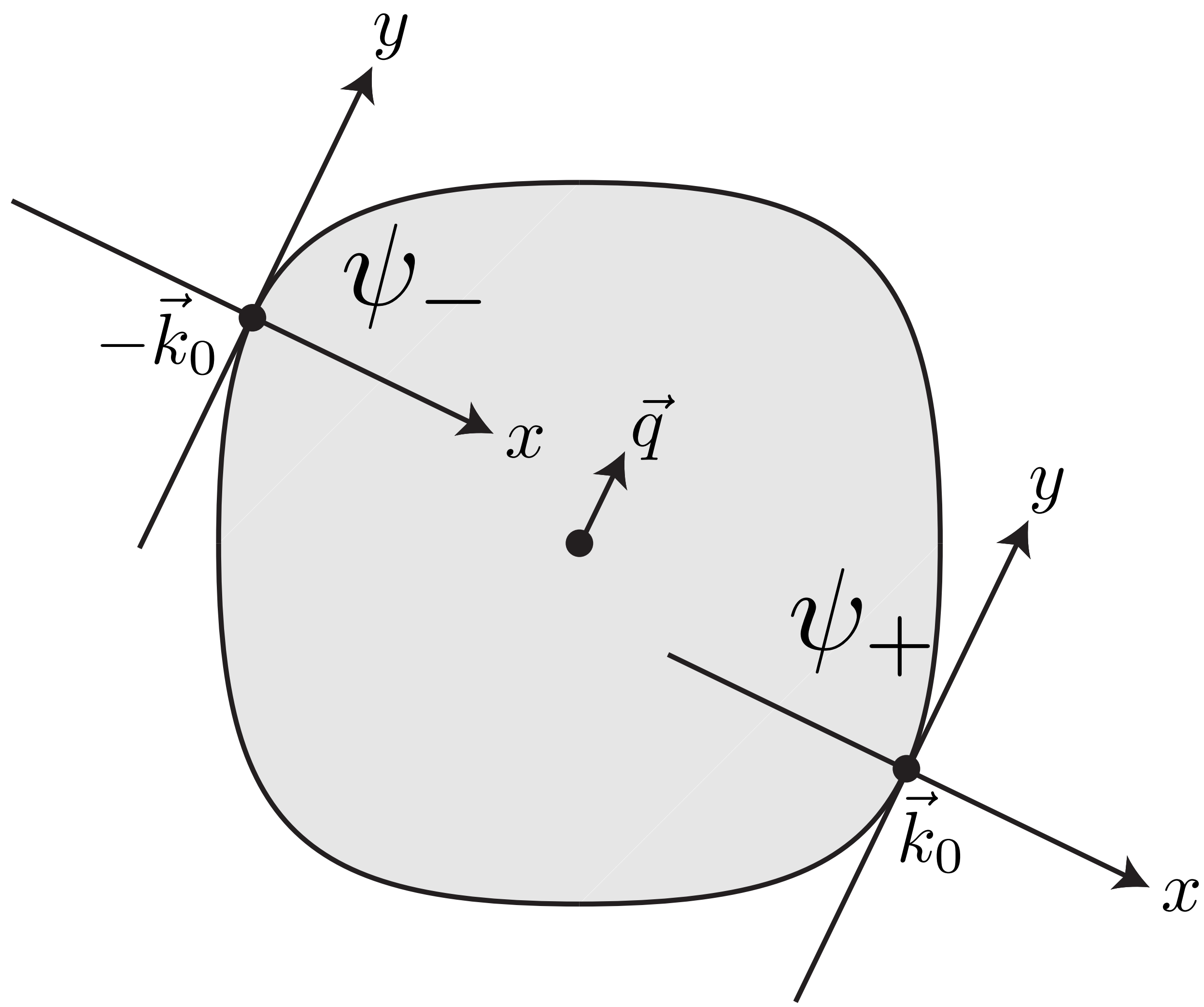


$$\mathcal{L}[\psi_\pm, a] =$$

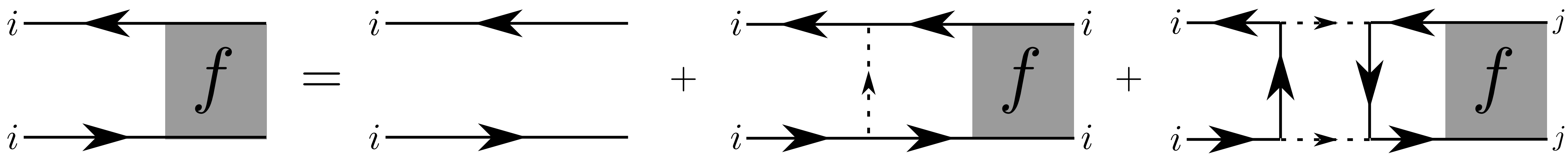
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos



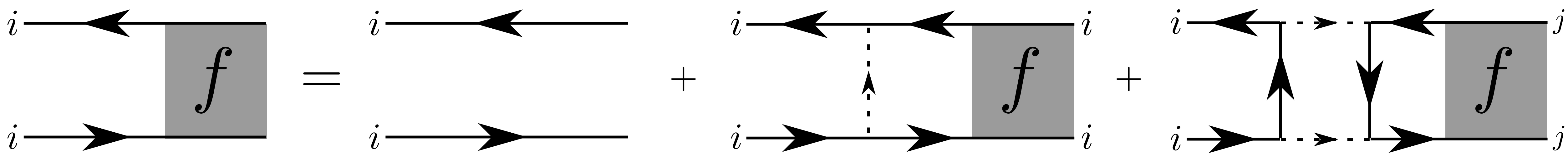
$$f(t) = \frac{1}{N^2}\theta(t) \sum_{i,j=1}^{N} \int d^2x \; \mathrm{Tr}\Big[e^{-\beta H/2} \{\psi_i(x,t), \psi_j^\dagger(0)\} \times e^{-\beta H/2} \{\psi_i(x,t), \psi_j^\dagger(0)\}^\dagger \Big]$$

$$\sim \exp\Big((t - x/v_B)/\tau_L \Big)$$

A.A. Patel and S. Sachdev, arXiv:1611.00003

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos



Strongly-coupled theory with no quasiparticles and fast scrambling:

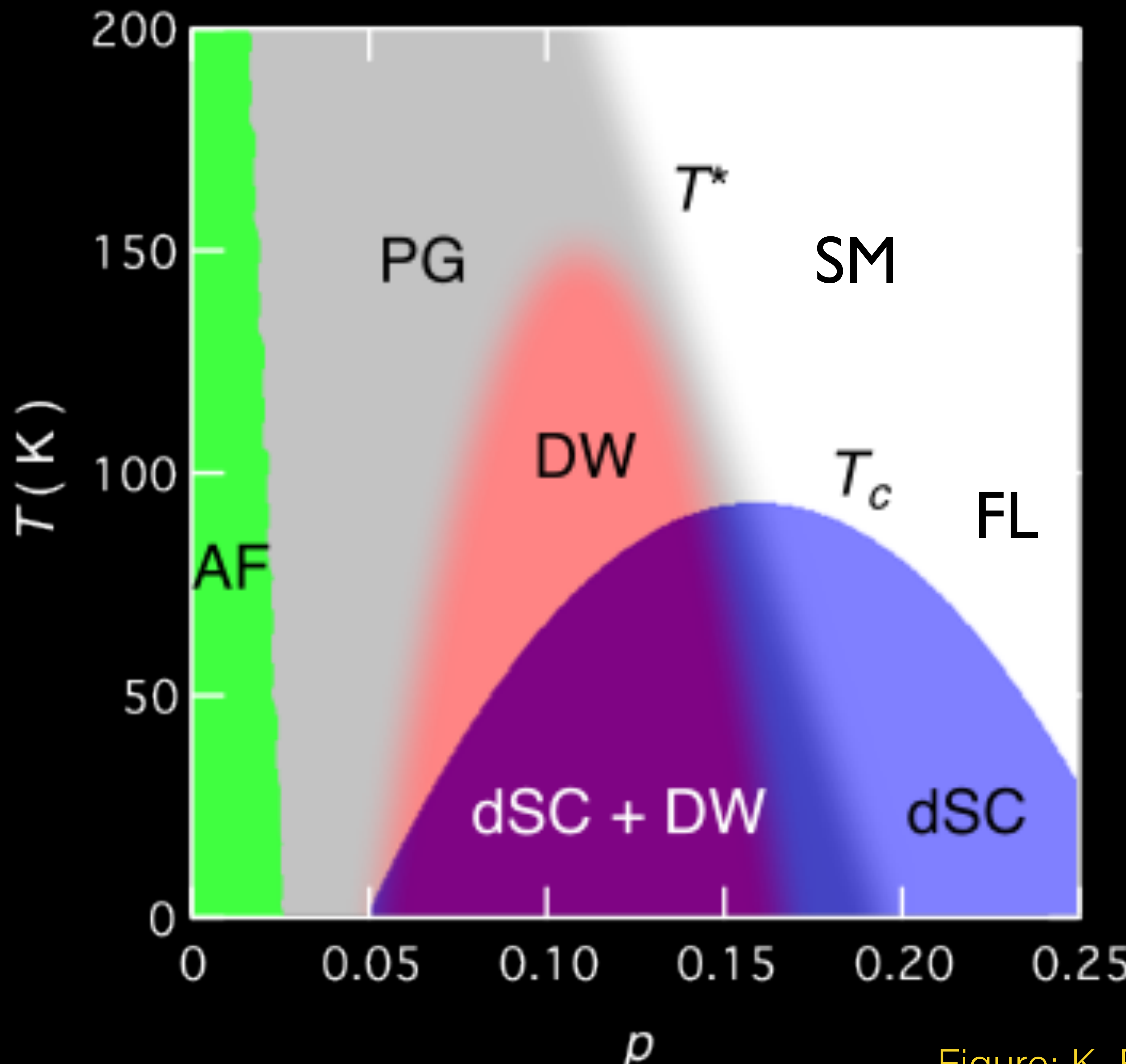
$$\tau_L \approx \frac{\hbar}{2.48\, k_B T}$$

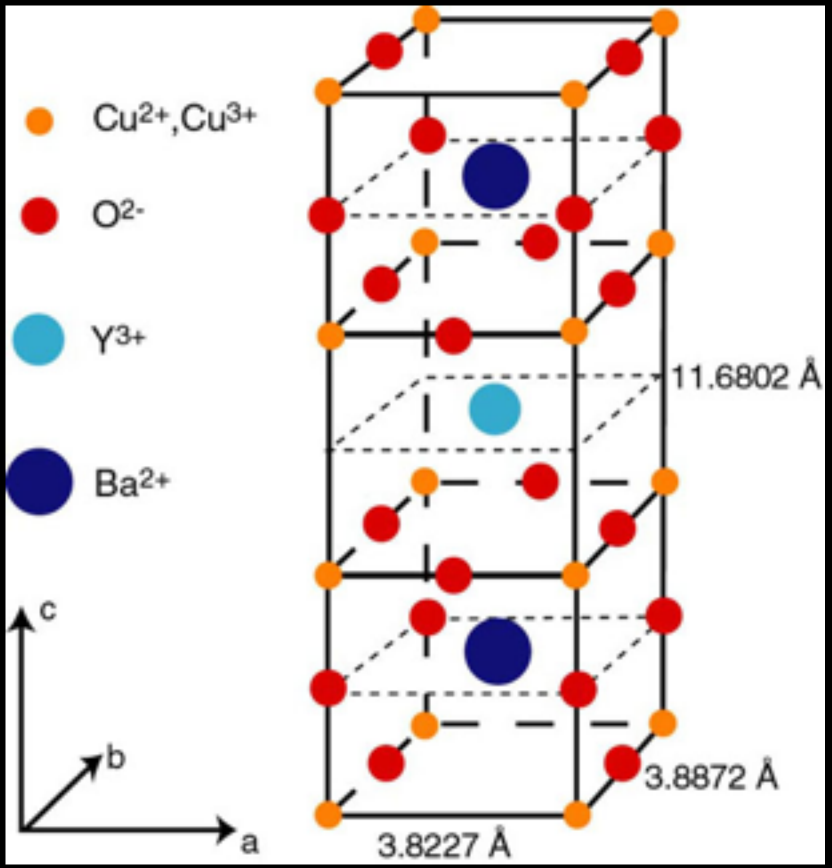
$$v_B \approx 4.1\, \frac{N T^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}}$$

$$D_E \approx 0.42\, v_B^2 \tau_L$$

N is the number of fermion flavors, v_F is the Fermi velocity, γ is the Fermi surface curvature, e is the gauge coupling constant.

A.A. Patel and S. Sachdev, arXiv:1611.00003

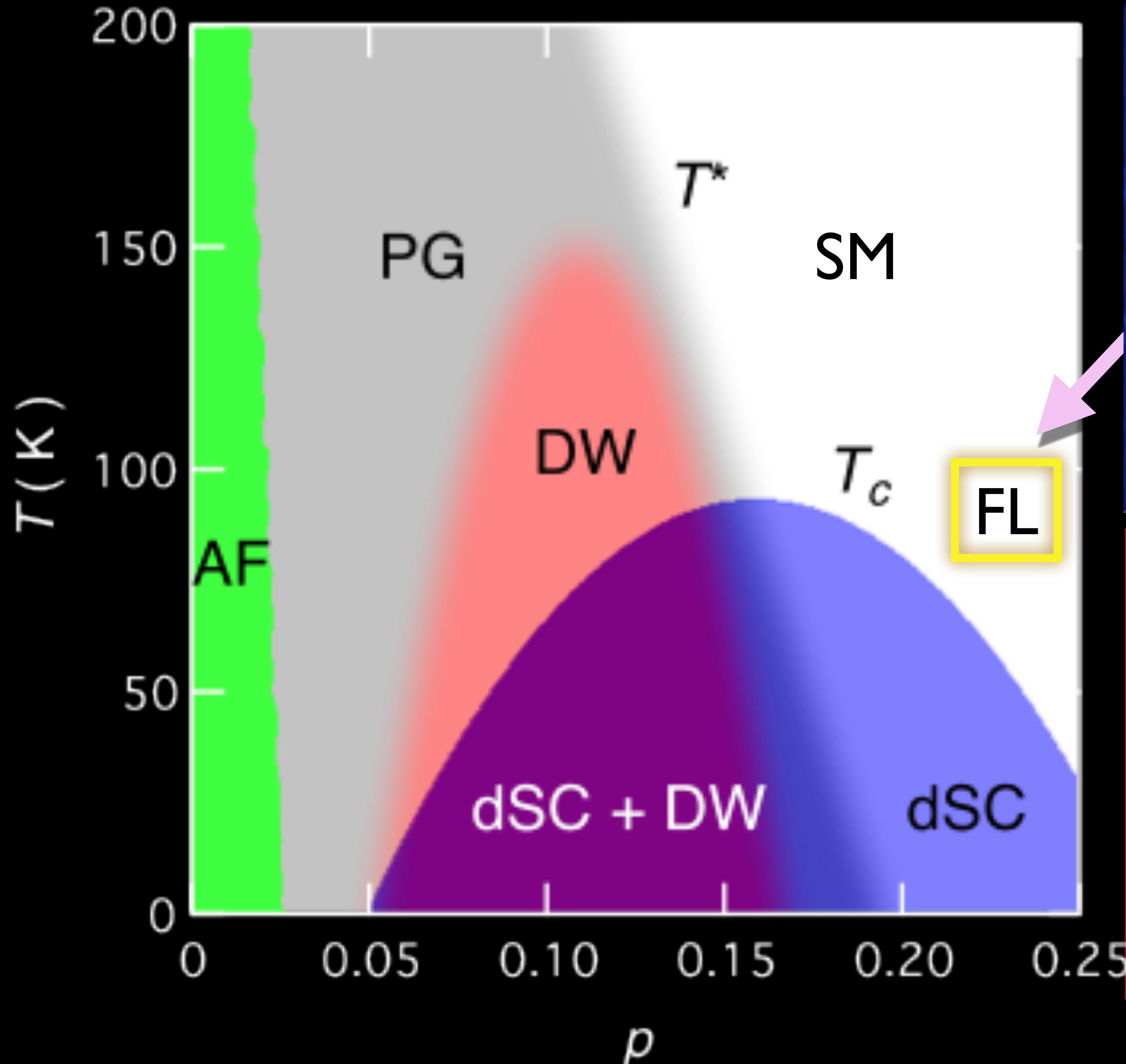


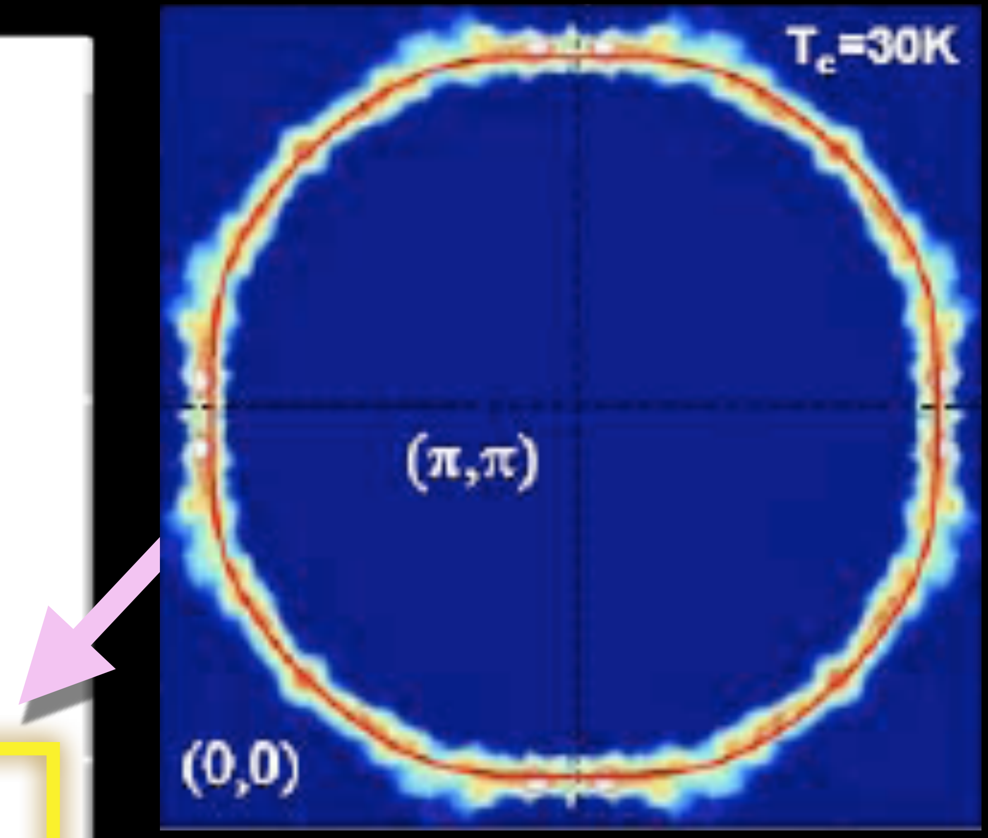


$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

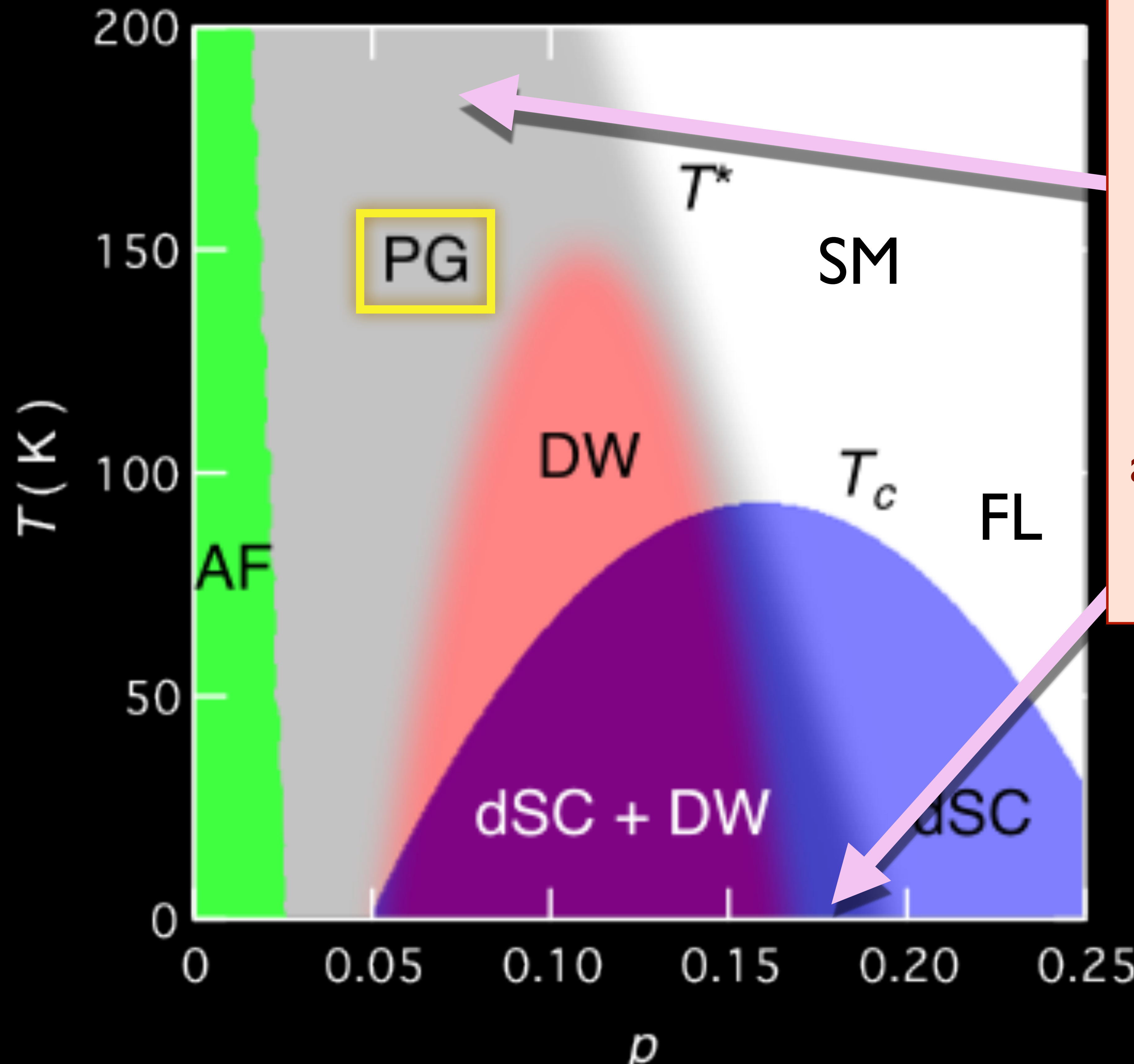
M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





A conventional metal:
the Fermi liquid with Fermi surface of size *1+p*

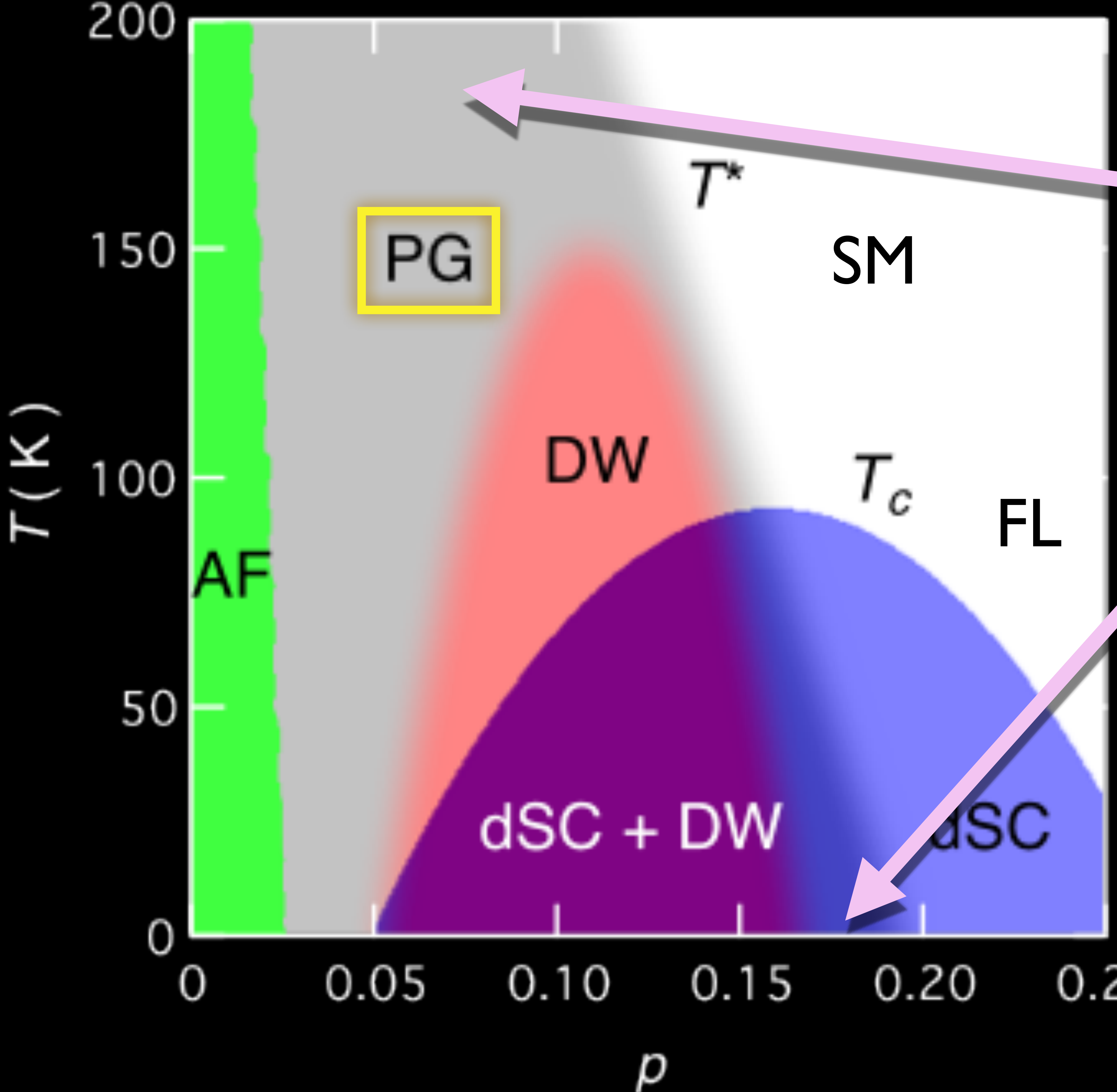
S. Badoux, W. Tabis, F. Laliberté, G. Grissonnanche, B. Vignolle, D. Vignolles, J. Béard, D.A. Bonn, W.N. Hardy, R. Liang, N. Doiron-Leyraud, L. Taillefer, and C. Proust, Nature **531**, 210 (2016).



Pseudogap metal at low p

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

T. Senthil, M. Vojta and S. Sachdev, PRB **69**, 035111 (2004)

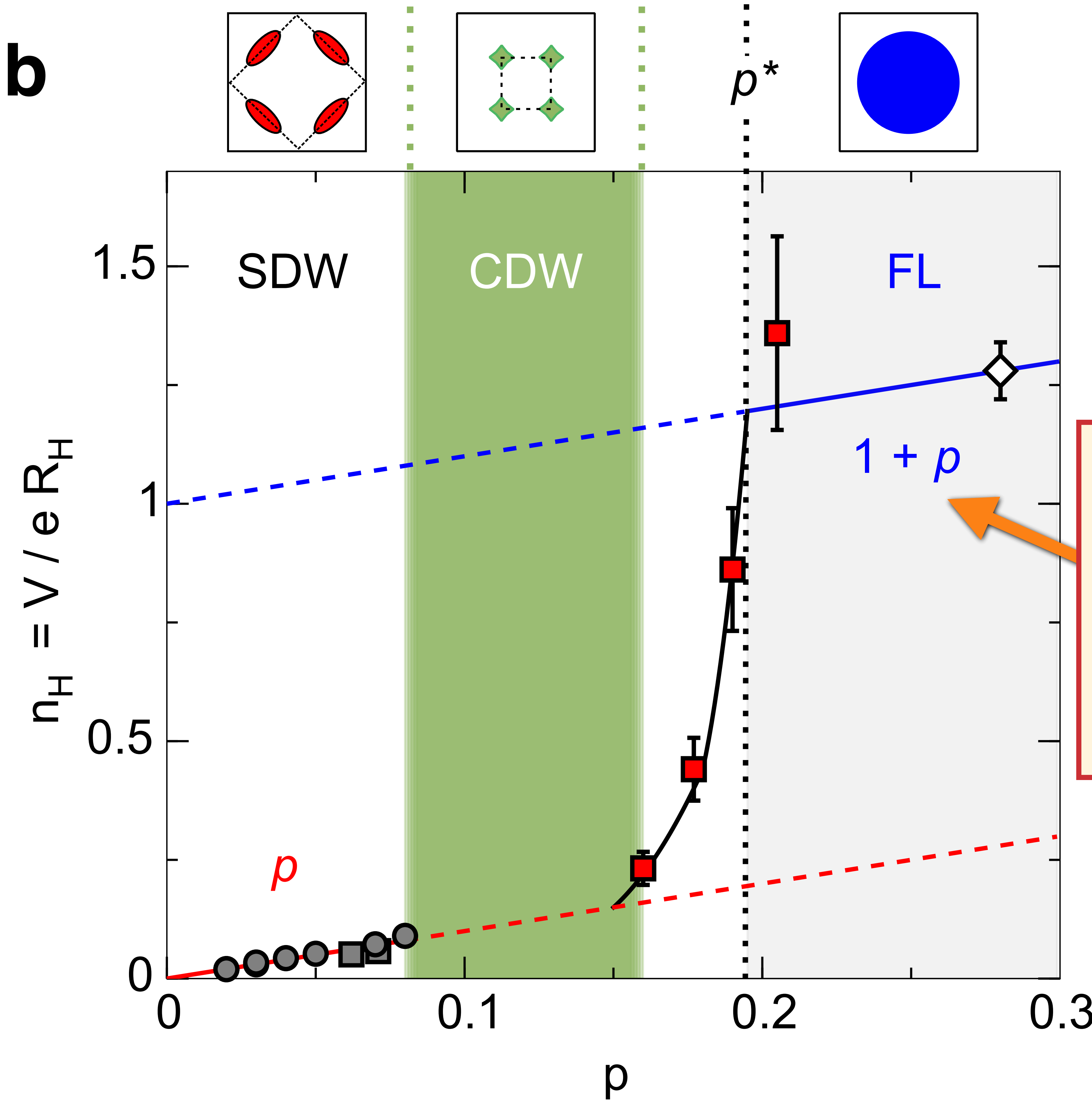


<u>Pseudogap metal</u> at low *p*

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size *p* and *not 1+p*.

If present at $T=0$, a metal with a size *p* Fermi surface (and translational symmetry preserved) *<u>must</u>* have *<u>topological order</u>*

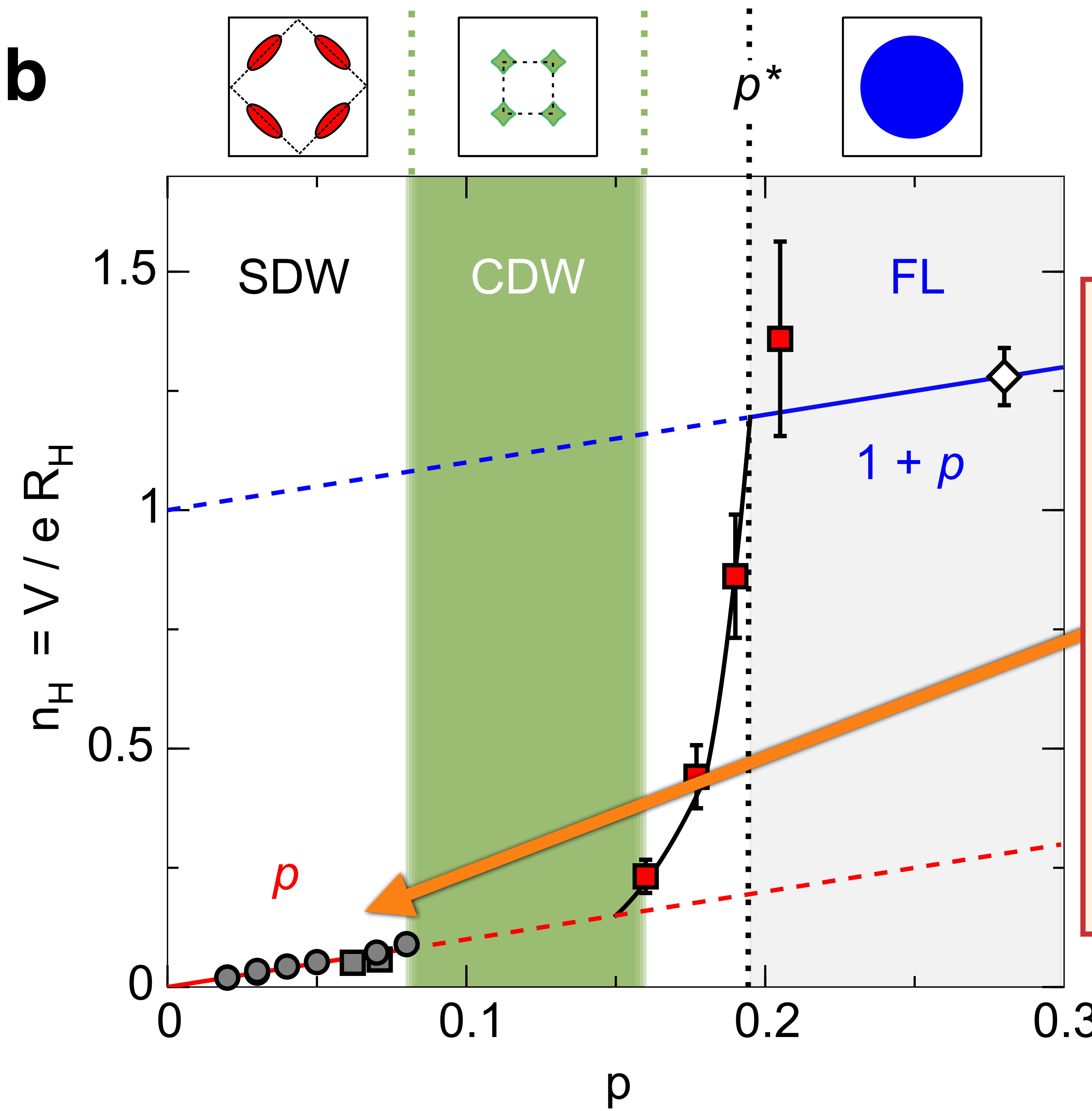
Hall effect measurements in YBCO



Fermi liquid (FL) with carrier density $1+p$

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

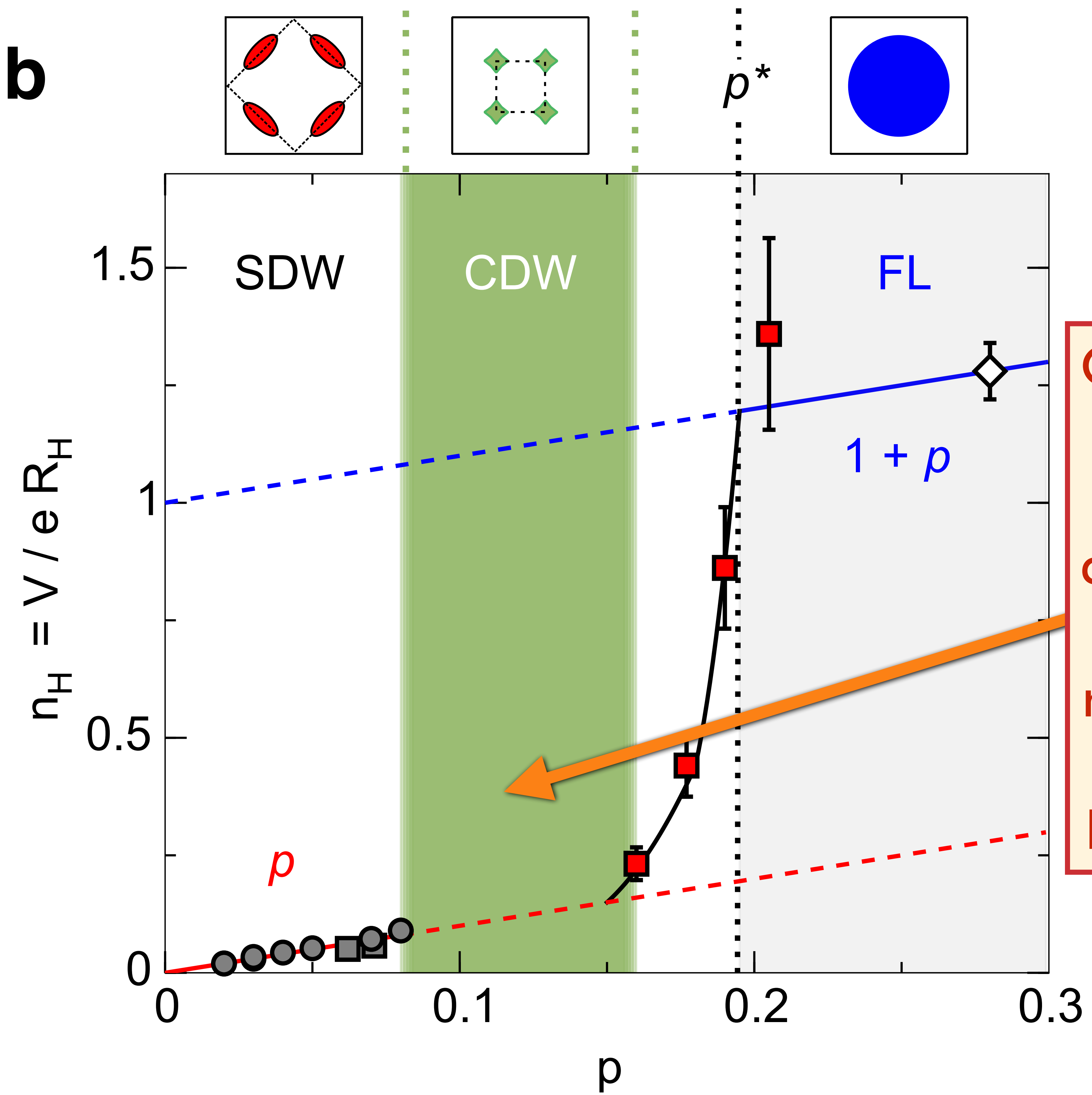
Hall effect measurements in YBCO



Spin density wave (SDW) breaks translational invariance, and the Fermi liquid then has carrier density *p*

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

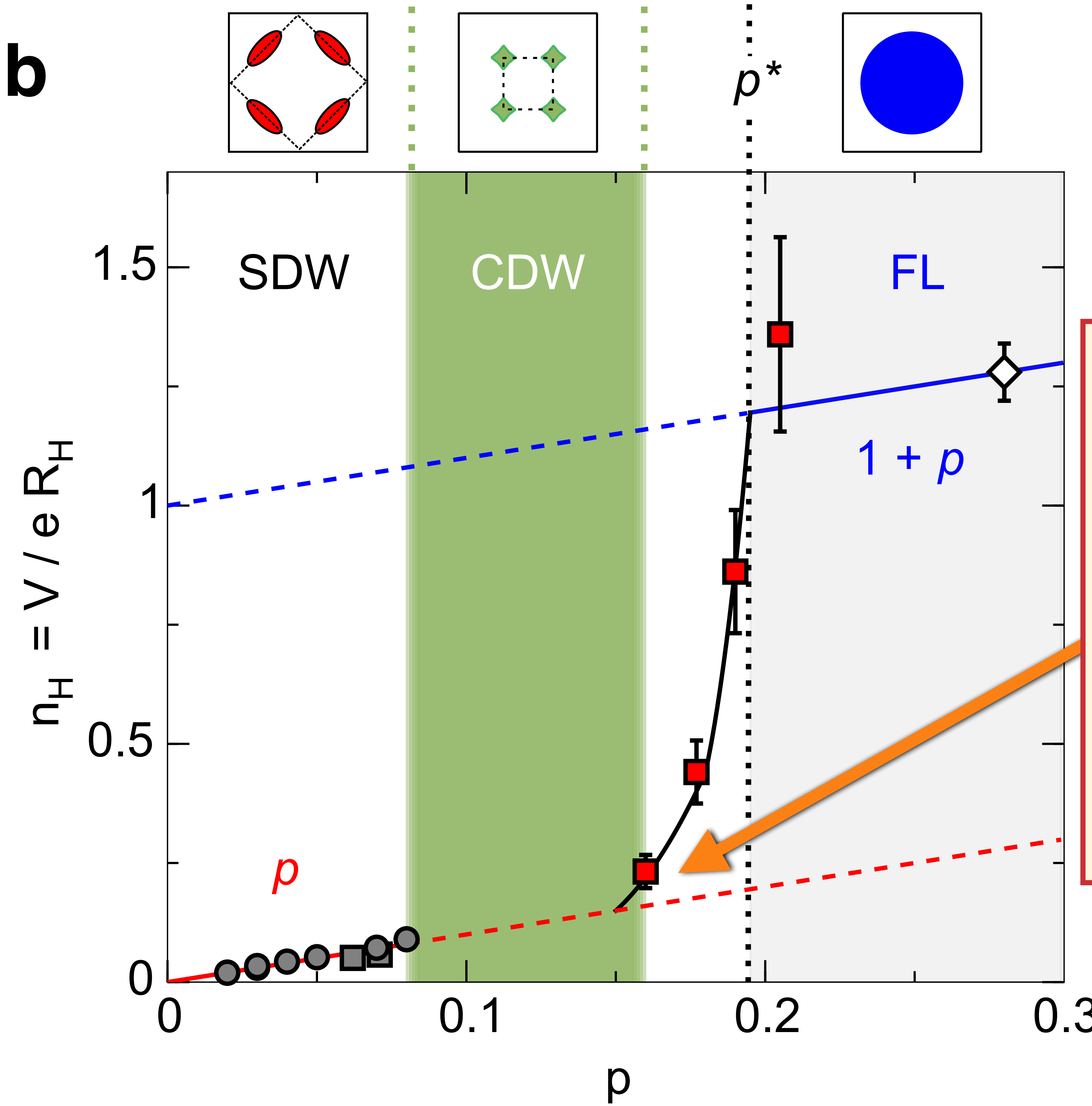
Hall effect measurements in YBCO



Charge density wave (CDW) leads to complex Fermi surface reconstruction and negative Hall resistance

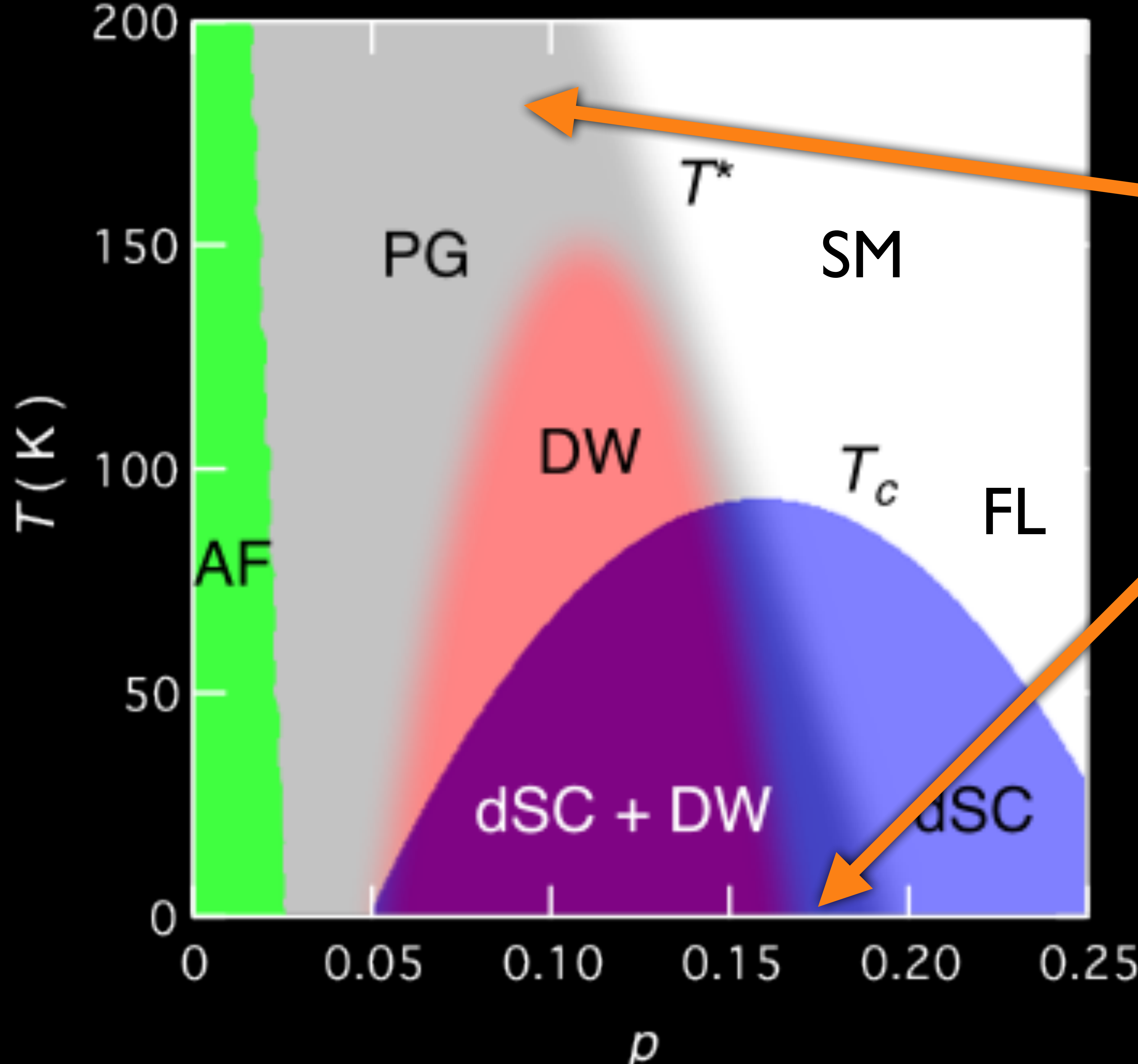
Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

Hall effect measurements in YBCO



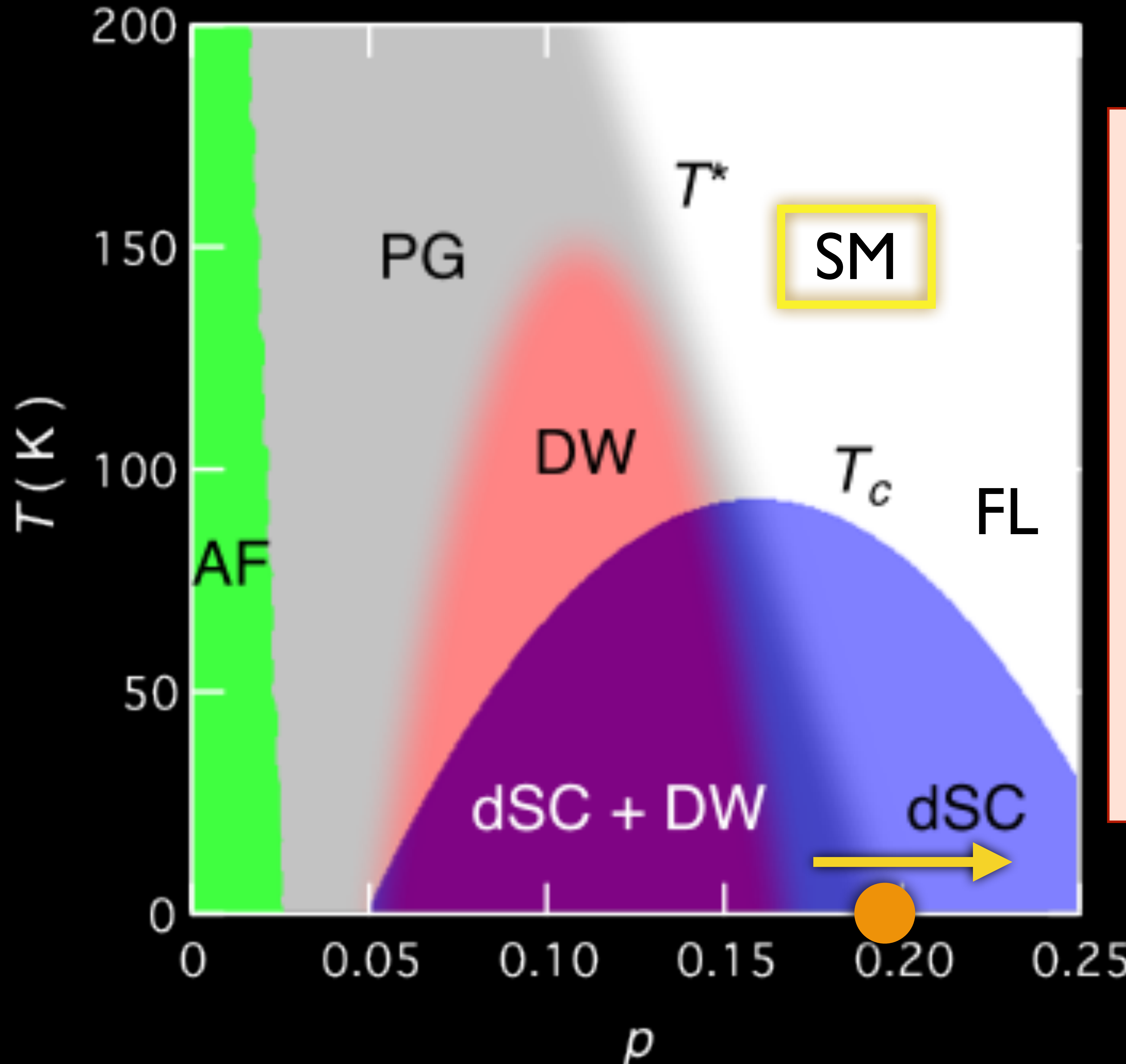
Evidence for a metal with topological order: Fermi surface of size p !

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)



Metal with topological order ?

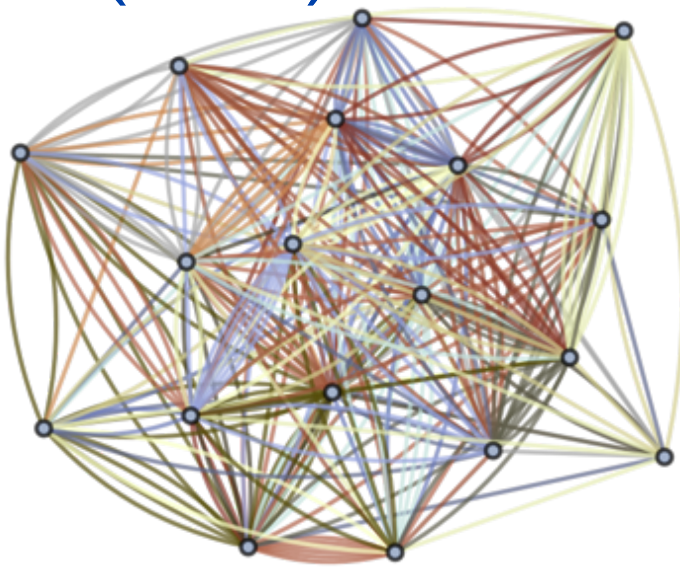
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009); D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579.



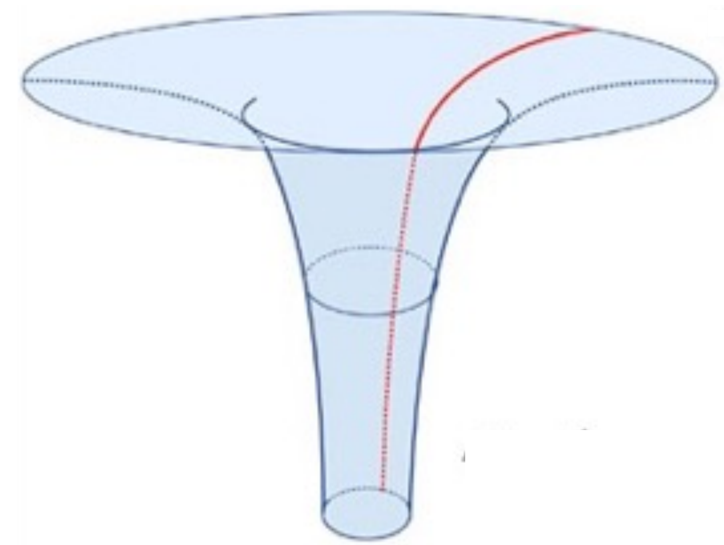
Gauge theory for a topological phase transition, and for the strange metal (SM)

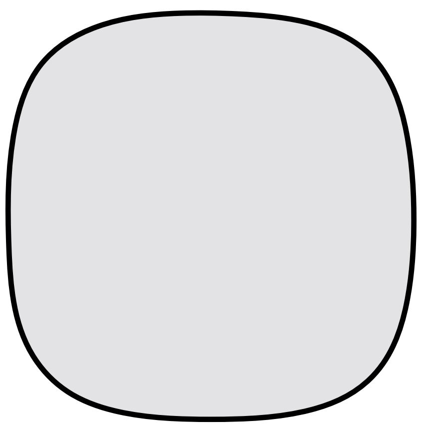
Quantum matter <u>without</u> quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons





Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

Quantum matter without quasiparticles:

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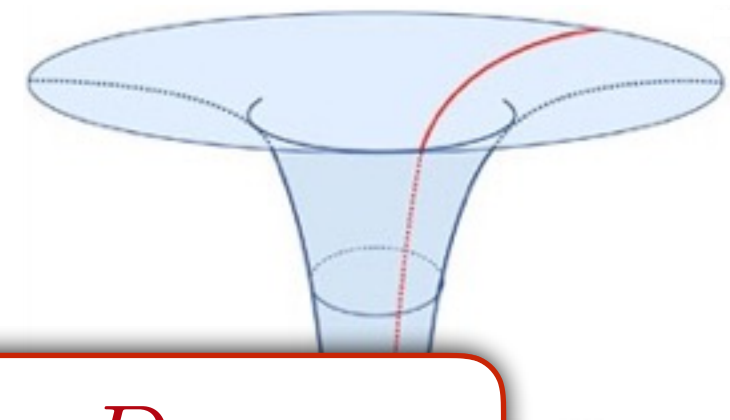
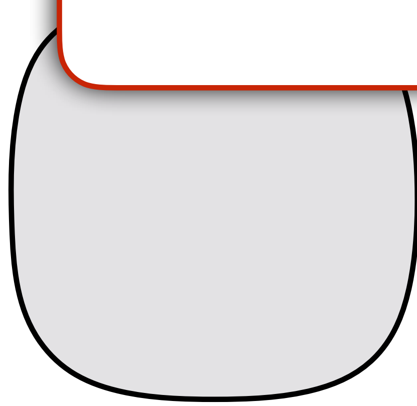
Black holes with AdS_2 horizons



Thermal diffusivity, D_E:

$$D_E = (\text{universal number}) \times v_B^2 \tau_L$$

in all three models



Fermi surface coupled to a gauge field

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τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos