Quantum Entanglement at all distances

The Racah Memorial Lecture
The Racah Institute of Physics
June 21, 2021

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Subir Sachdev

PHYSICS
HARVARD

Talk online: sachdev.physics.harvard.edu

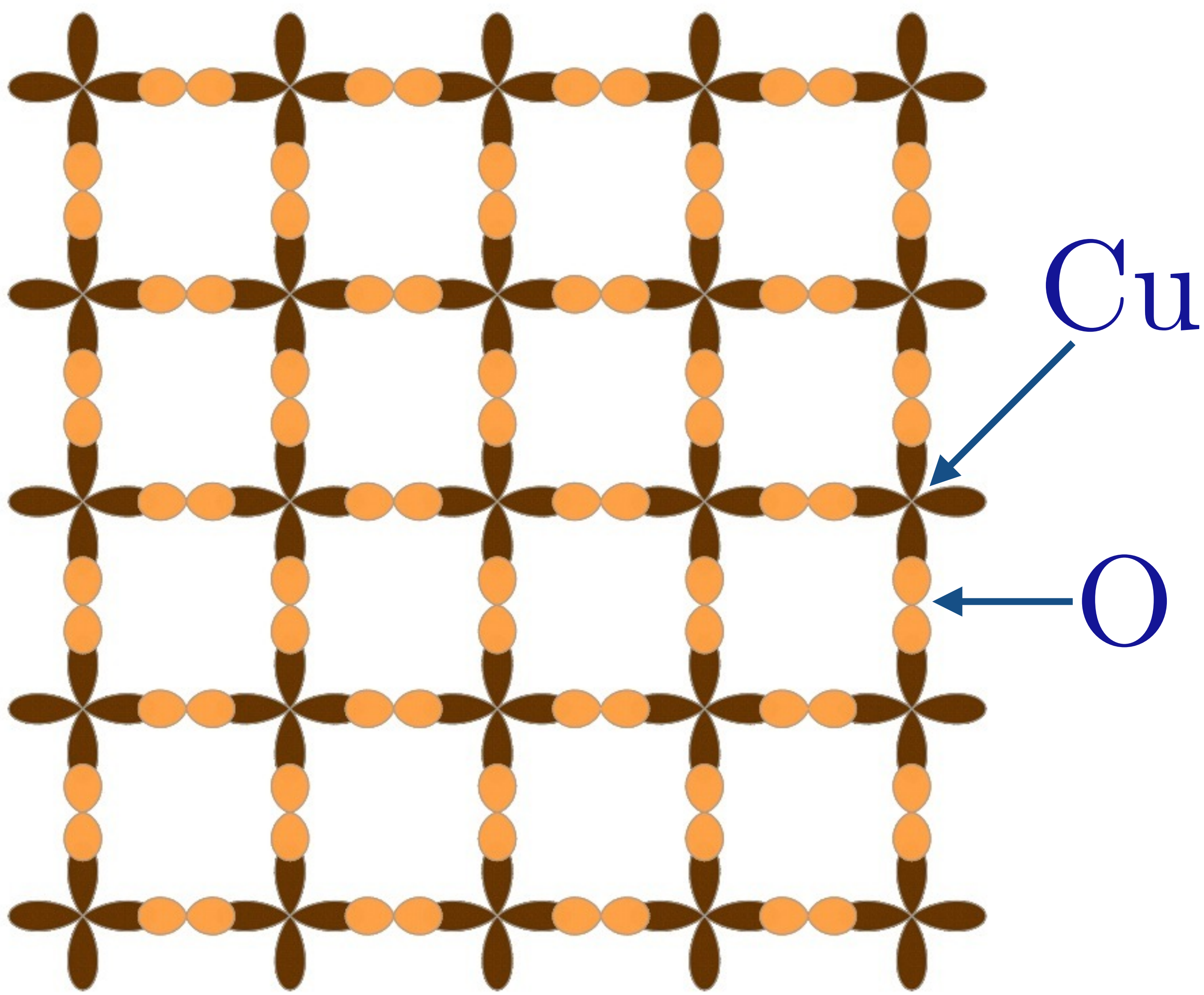
1. Introduction to the cuprates

2. Random matrix and SYK models

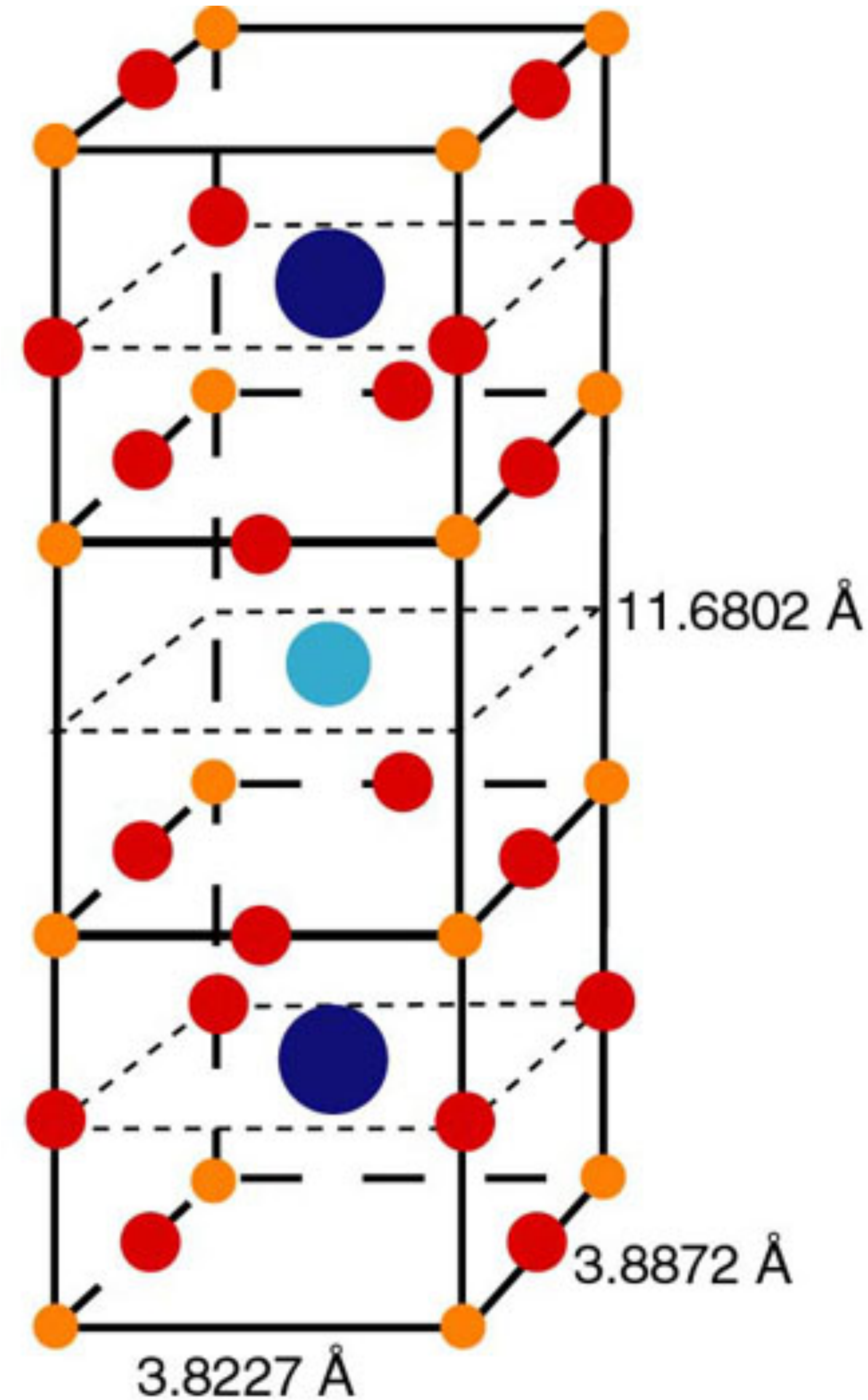
3. Black holes

4. Back to the cuprates

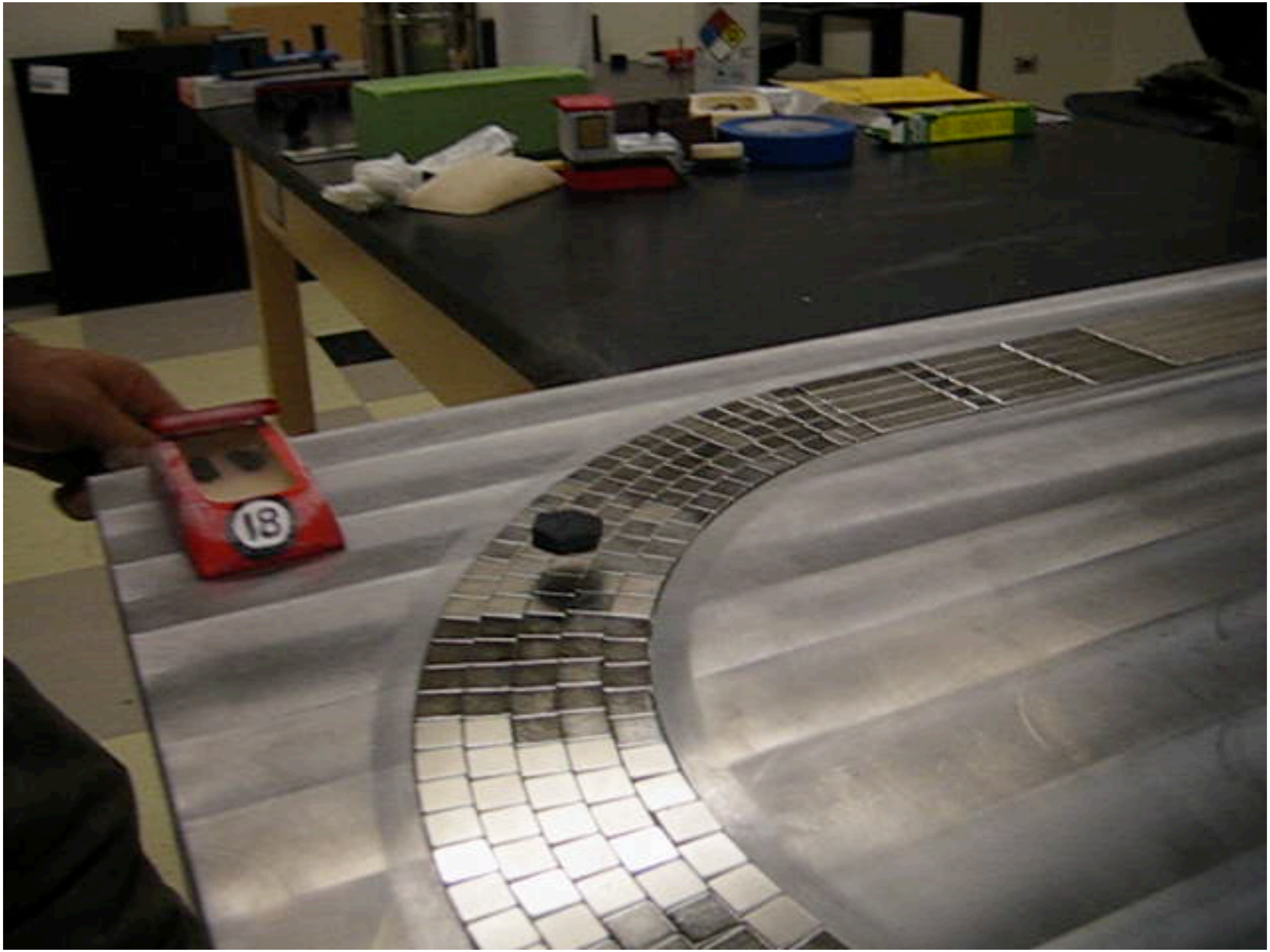
High temperature superconductors



CuO_2 plane

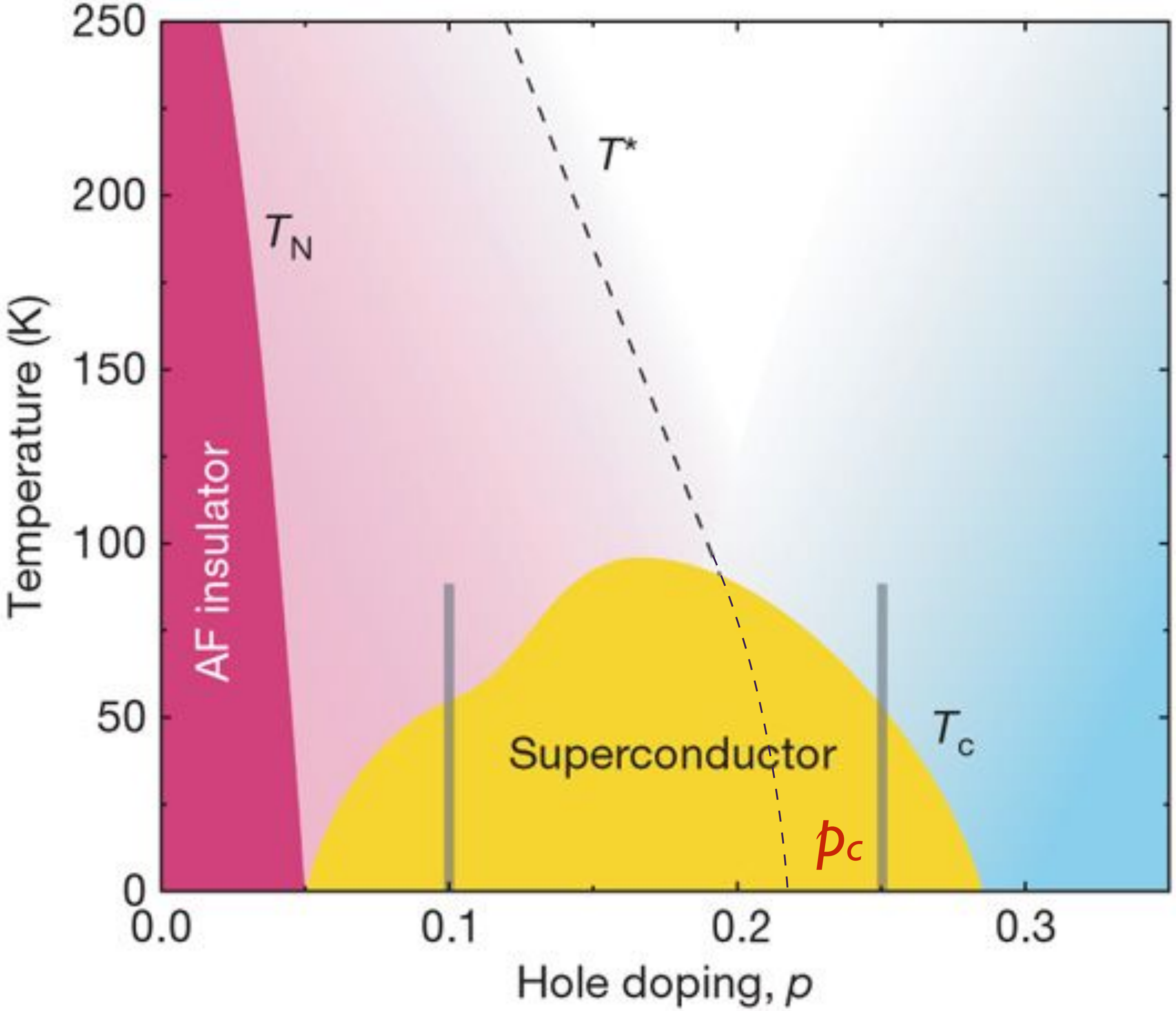


$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University



250
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150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
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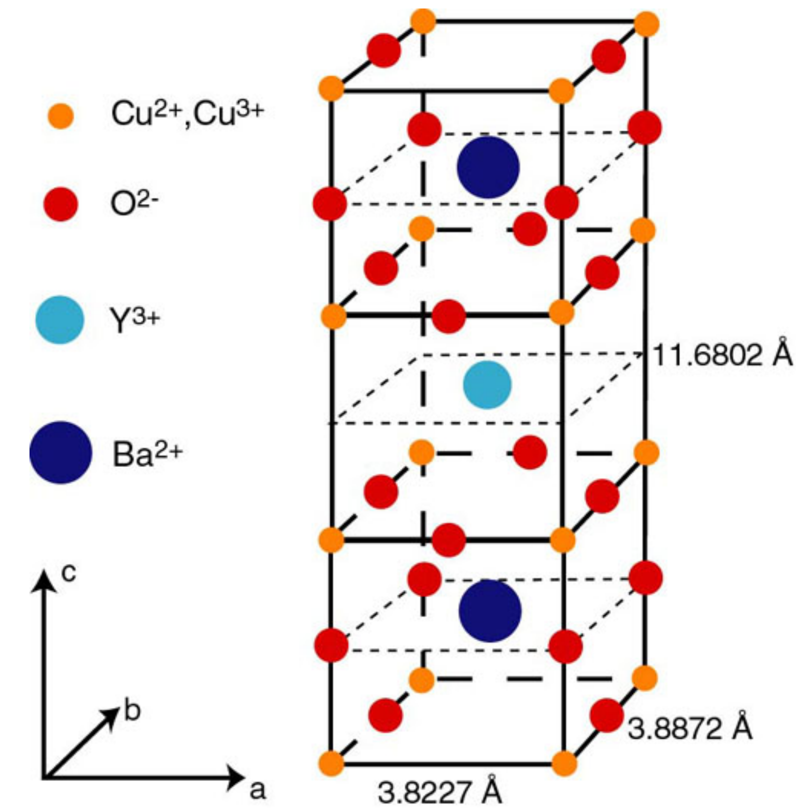
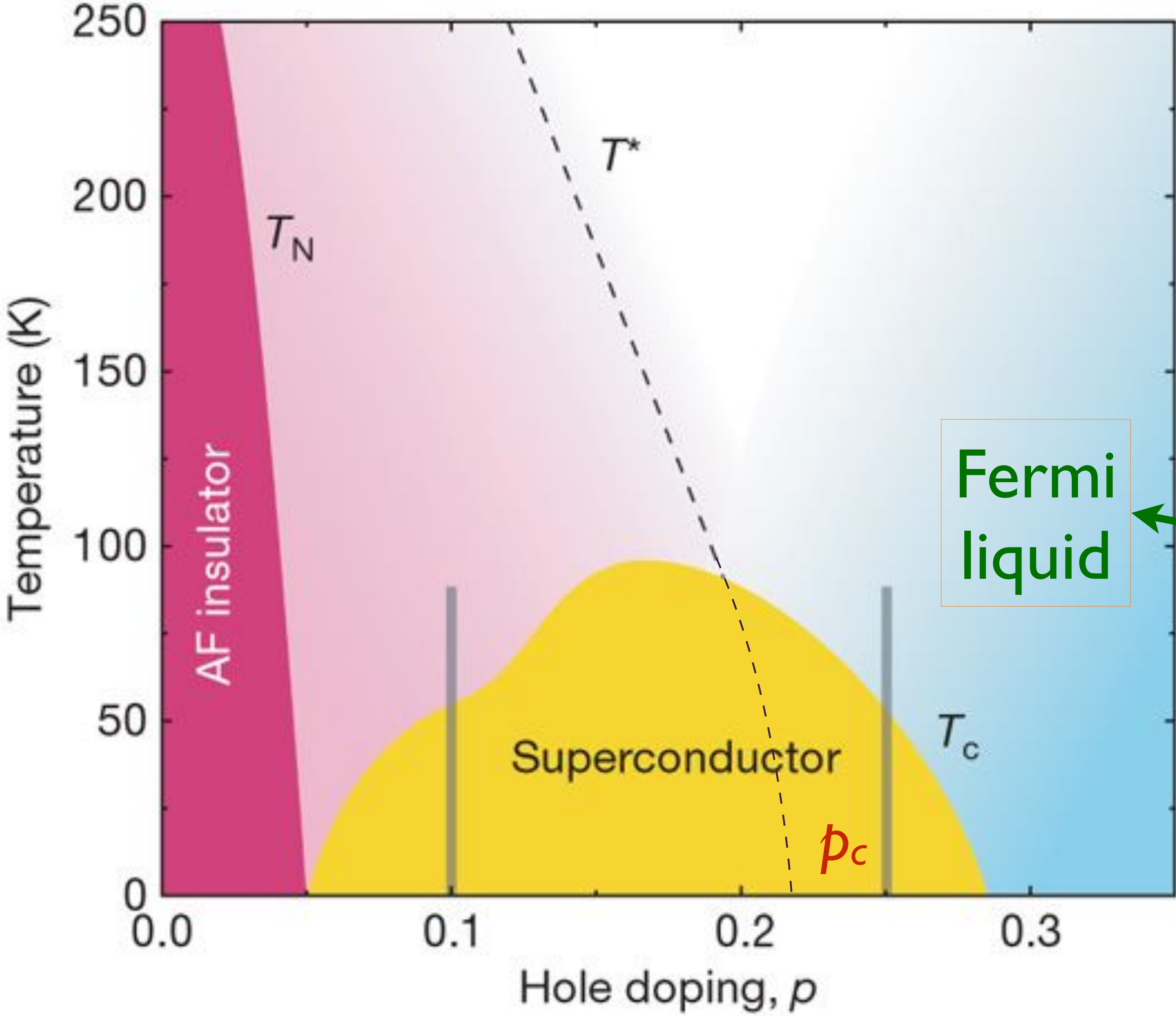
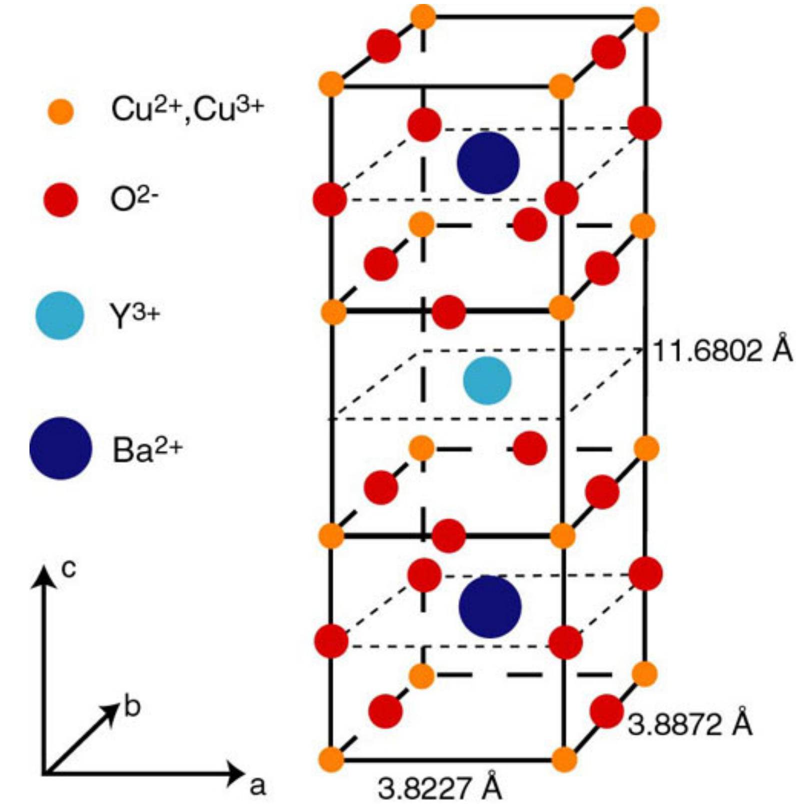
T_c
p_c

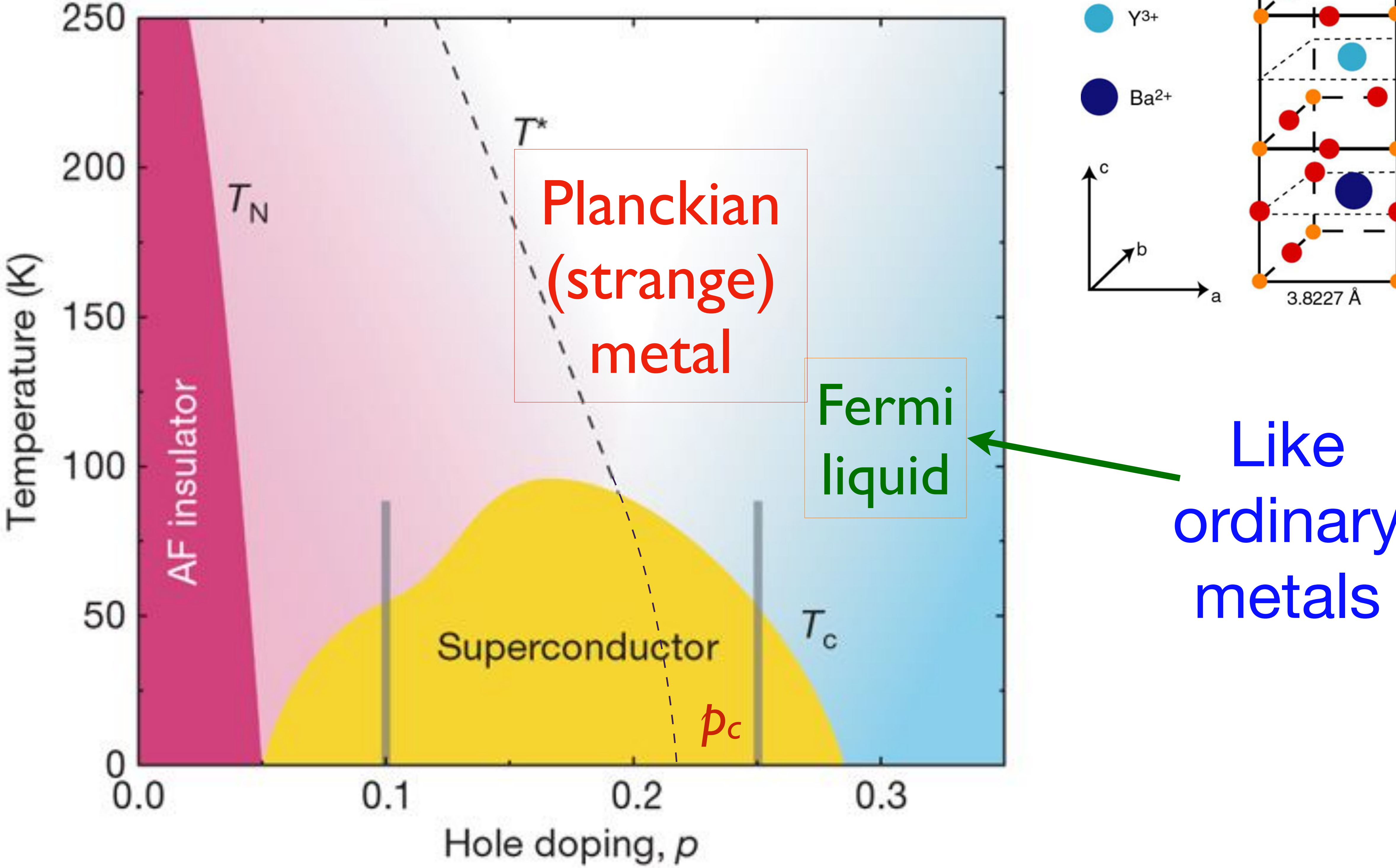


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å





Like ordinary metals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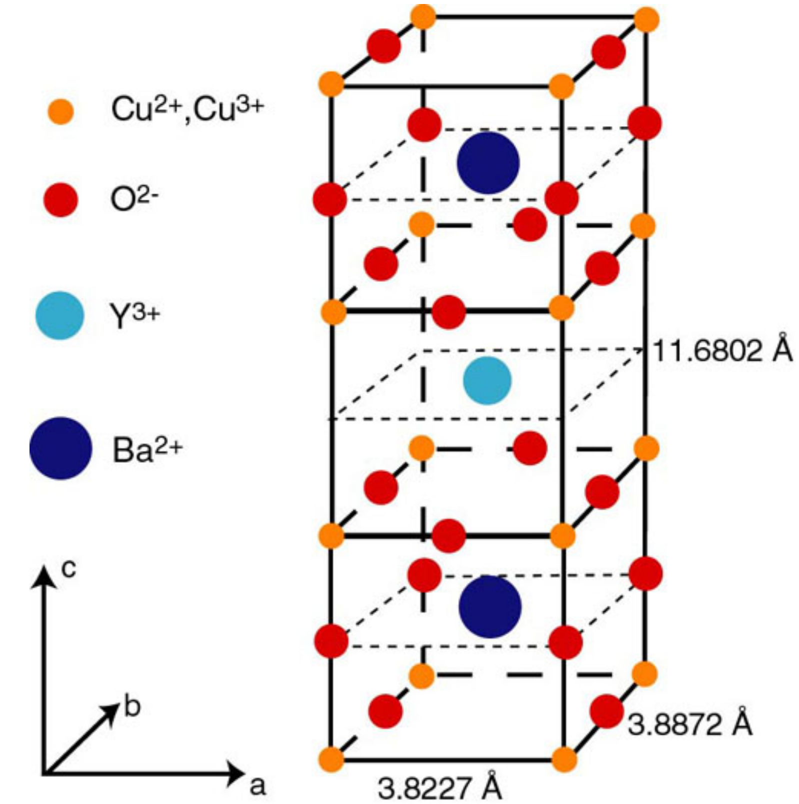
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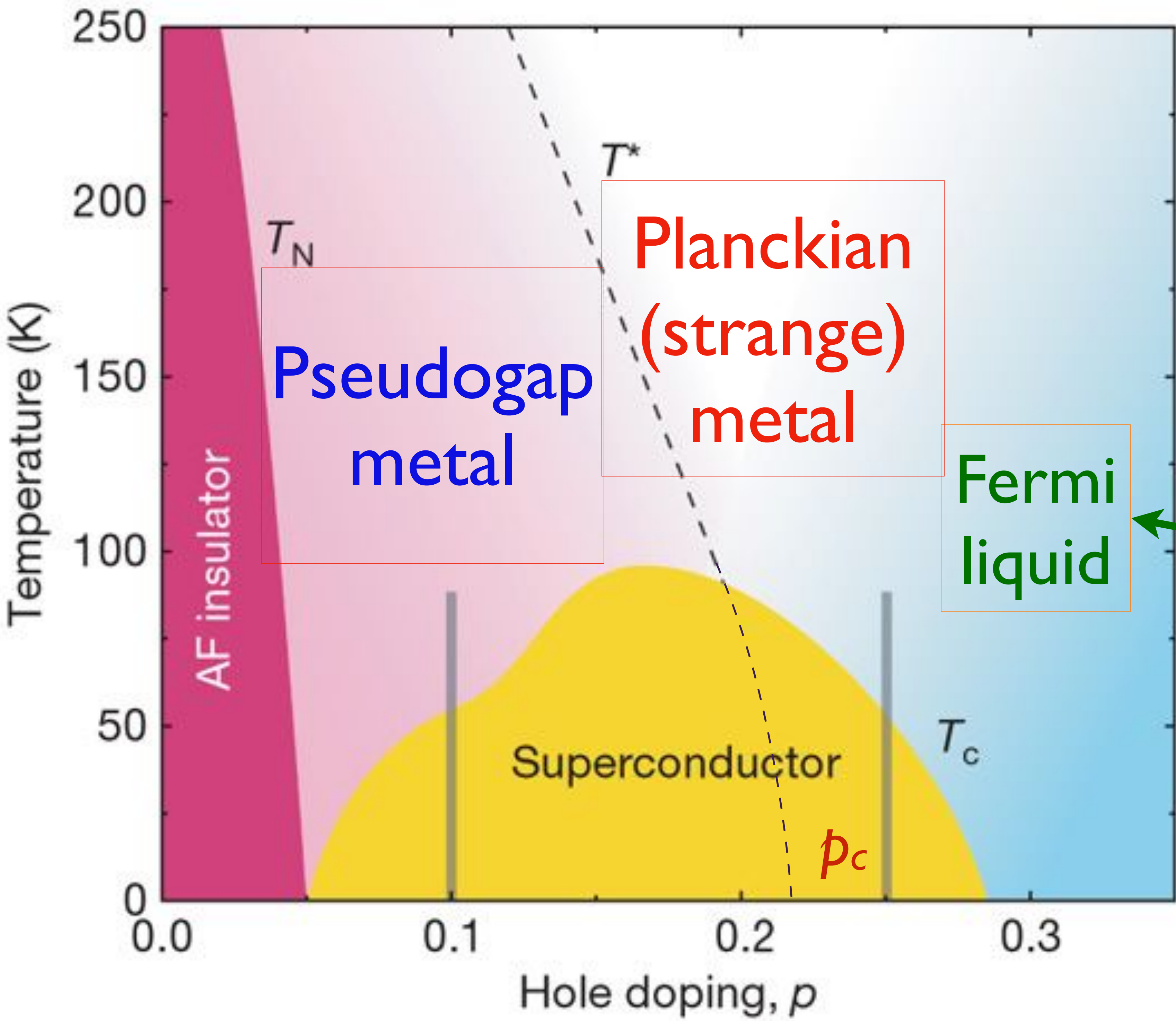

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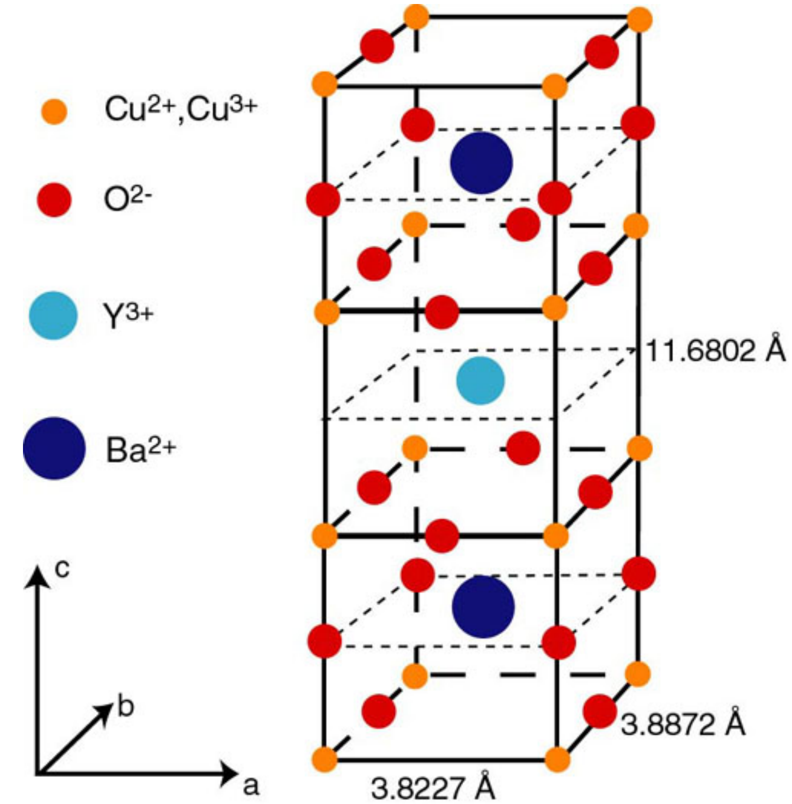
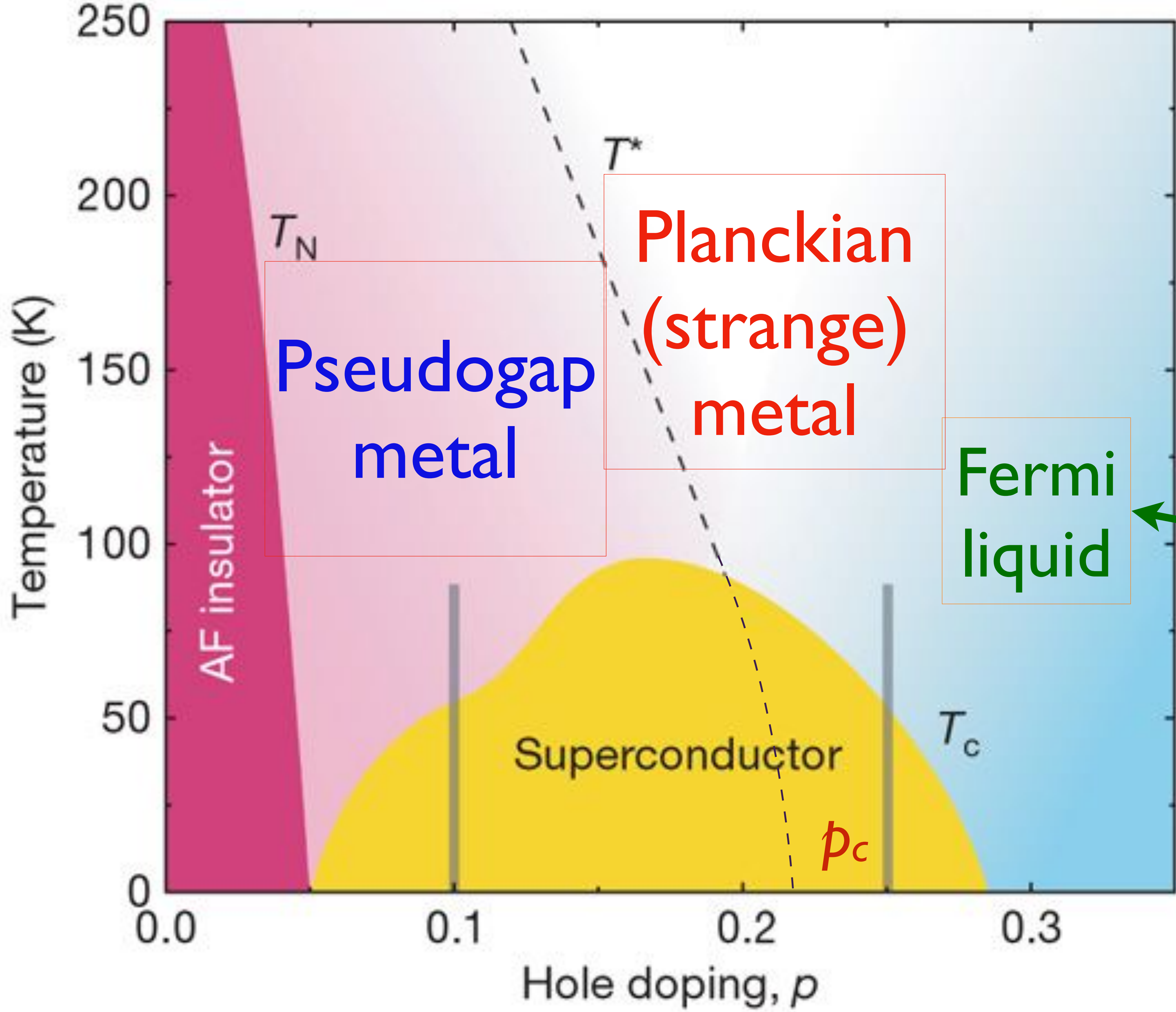
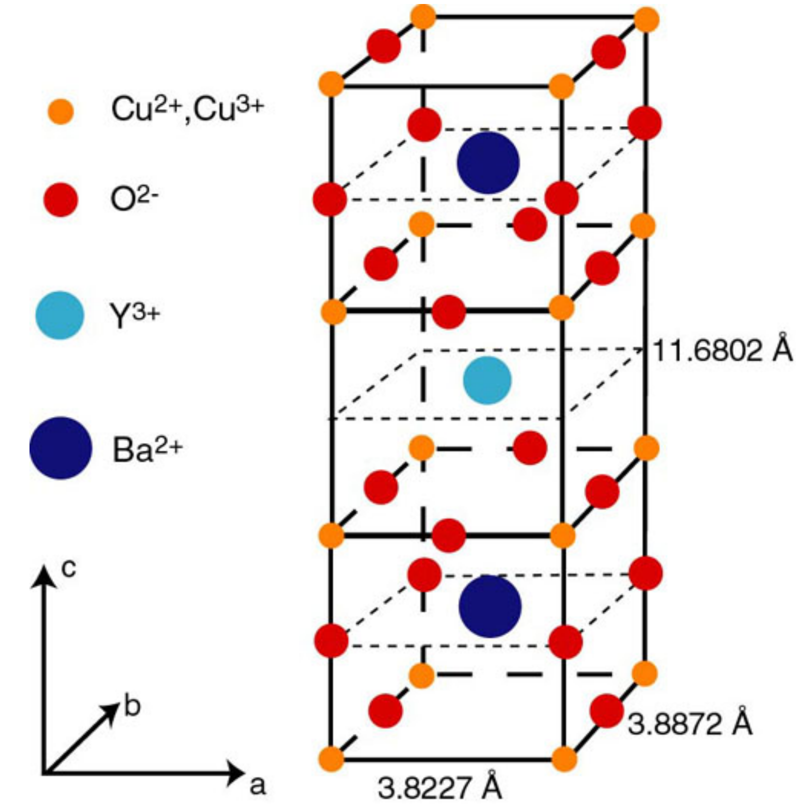


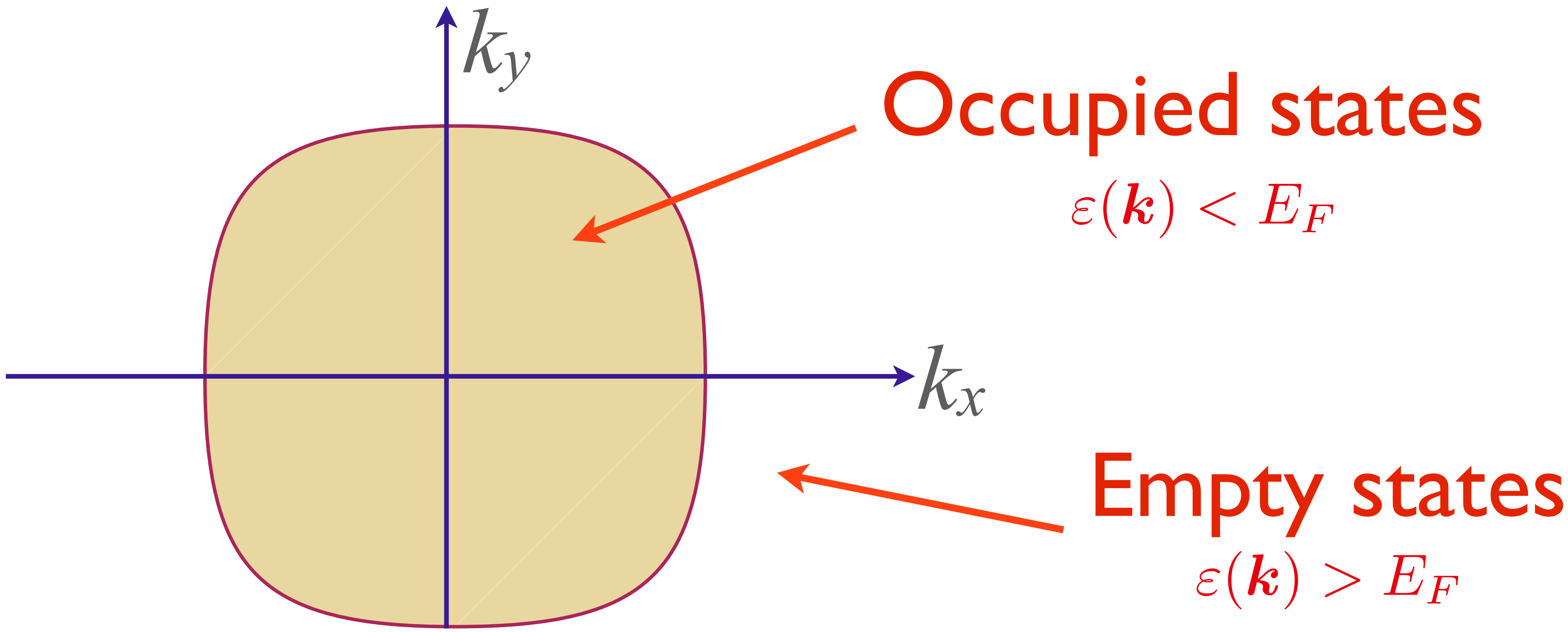




Like ordinary metals

Theory of ordinary metals

Electrons move with momentum $\mathbf{k}$ through the lattice with dispersion $\varepsilon(\boldsymbol{k})$



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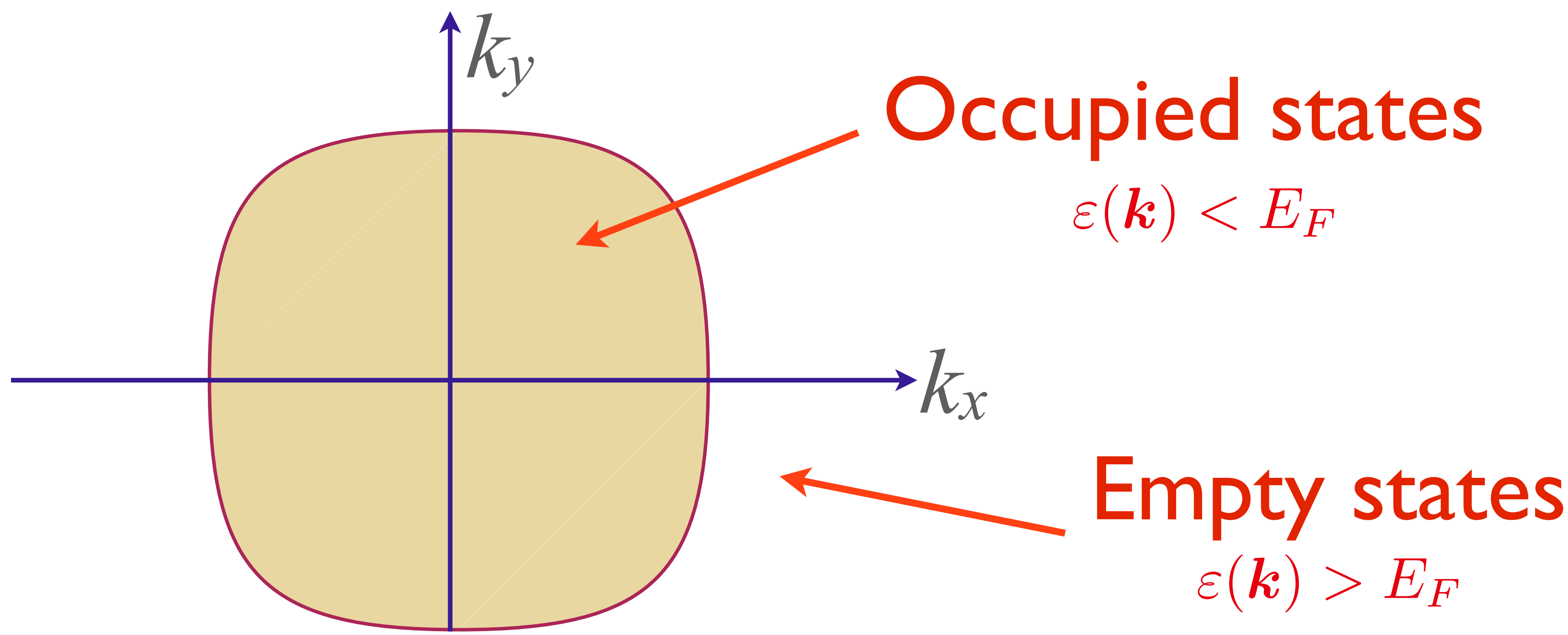

Fundamental principles of Fermi liquid theory:

- Low energy excitations are nearly-free "quasielectrons" near the Fermi surface.

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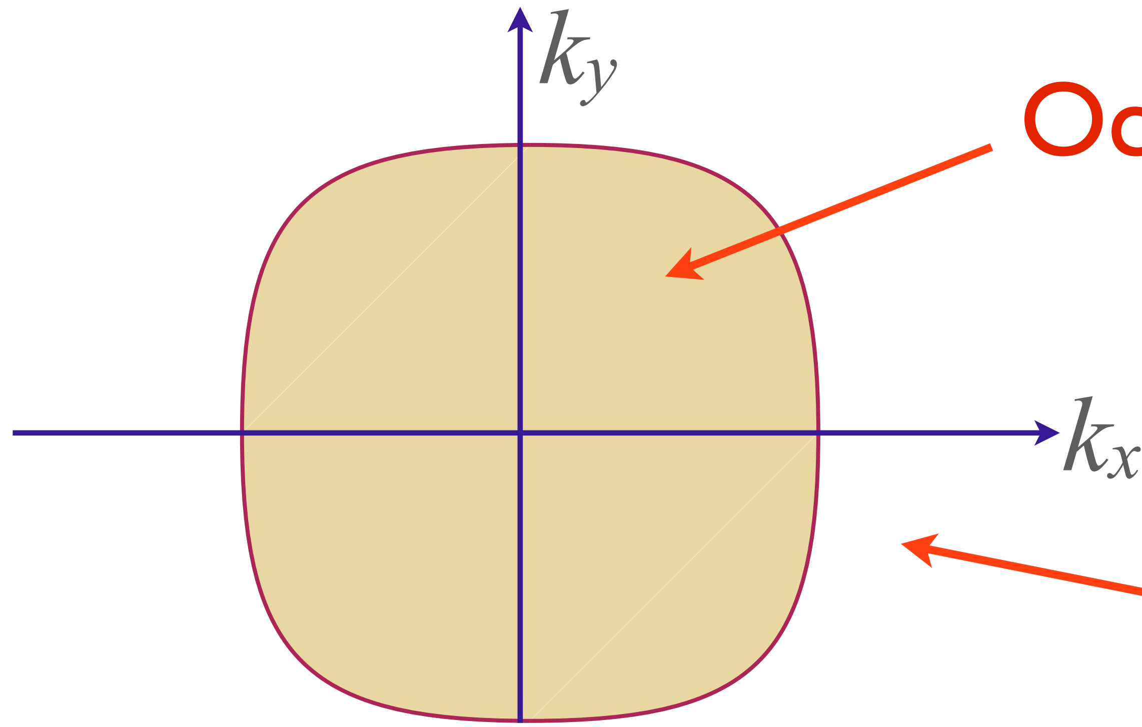

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- The electron-electron scattering rate, $1/\tau \sim U^2 T^2$, much slower than the Planckian bound $k_B T/\hbar$ as $T \to 0$.

S. Sachdev, *Quantum Phase Transitions* (1999)

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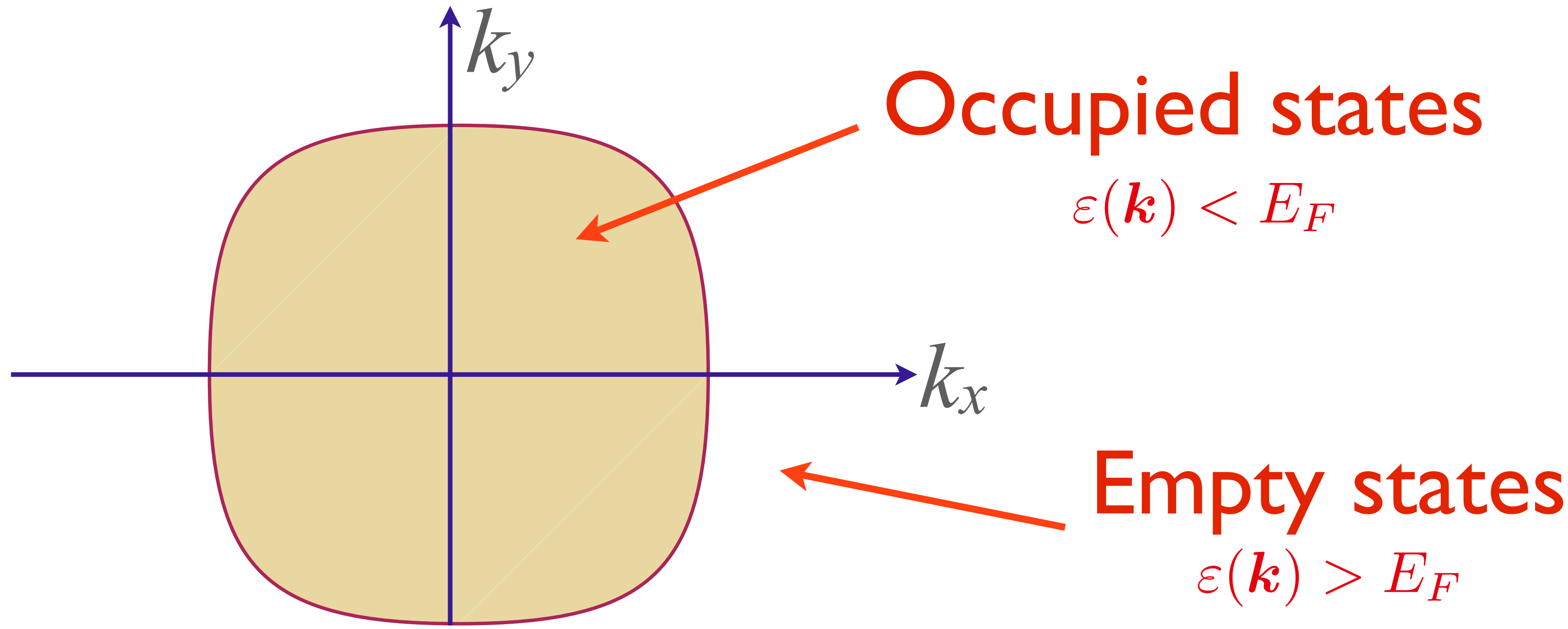

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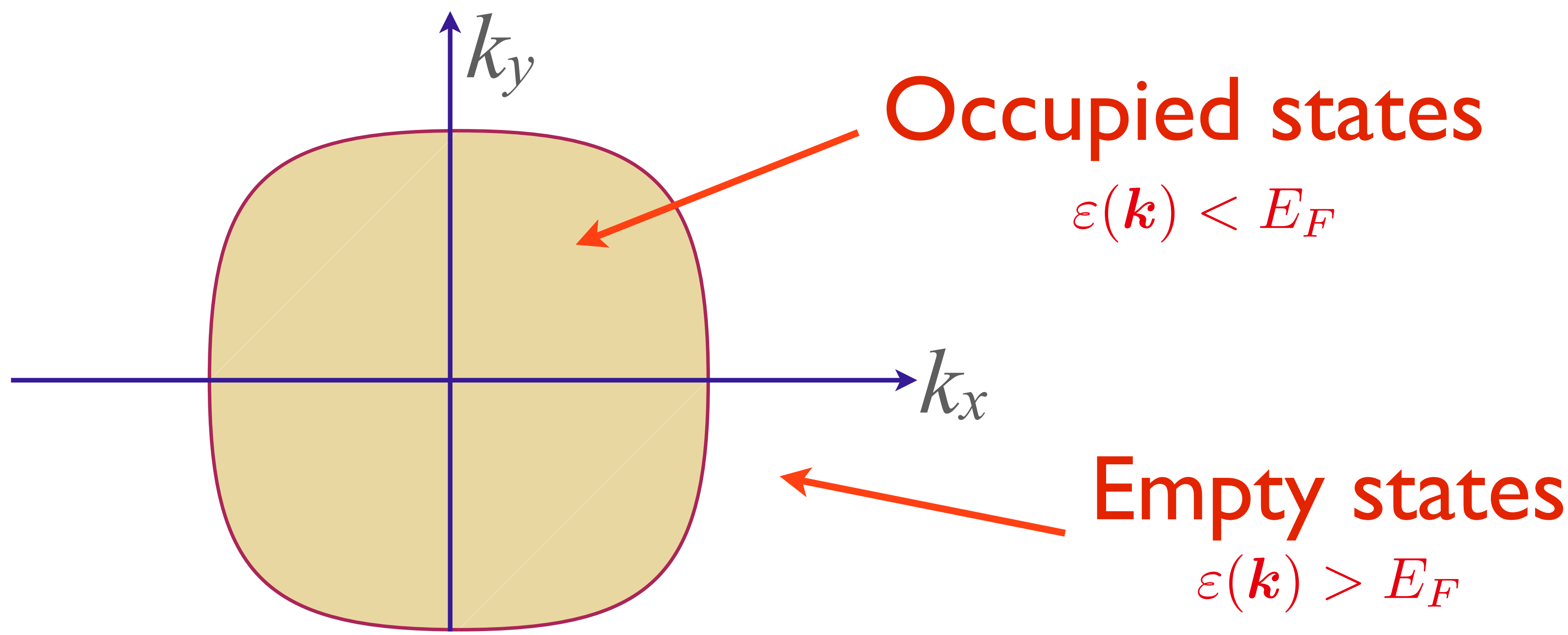

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Violated in the pseudogap metal

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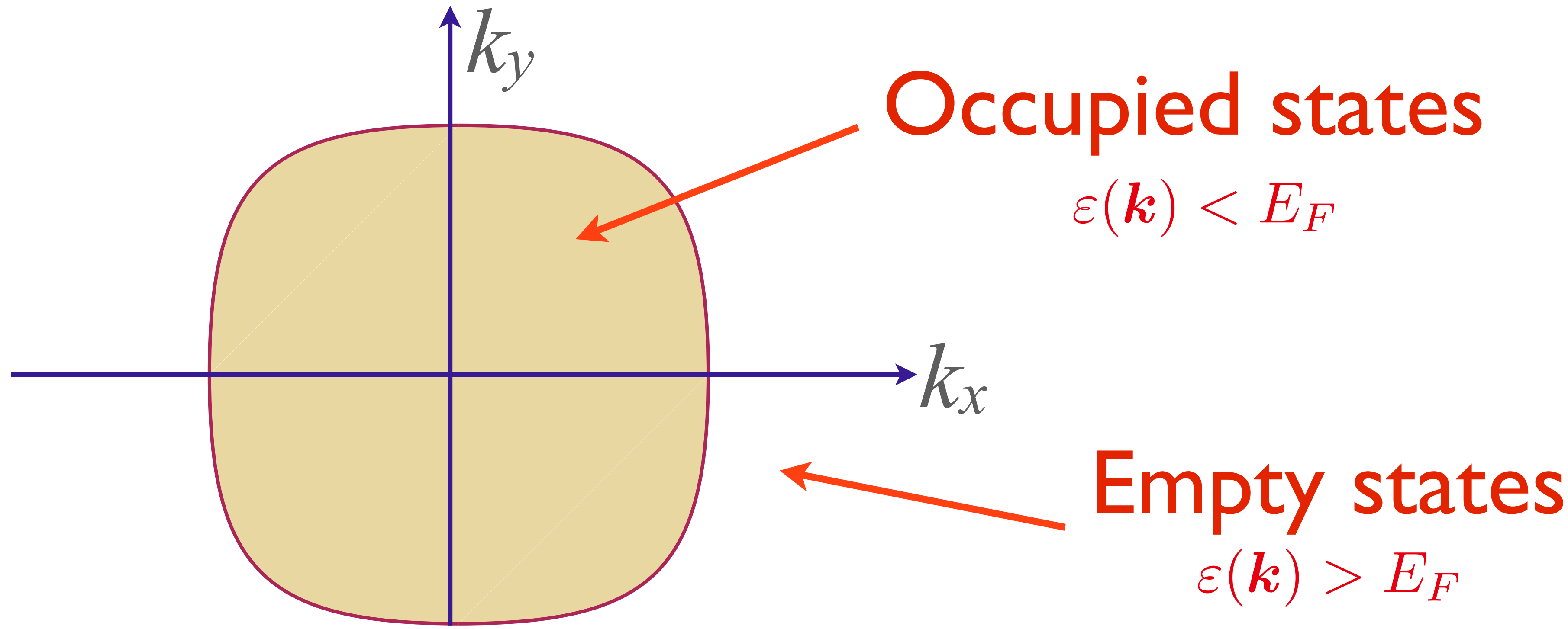

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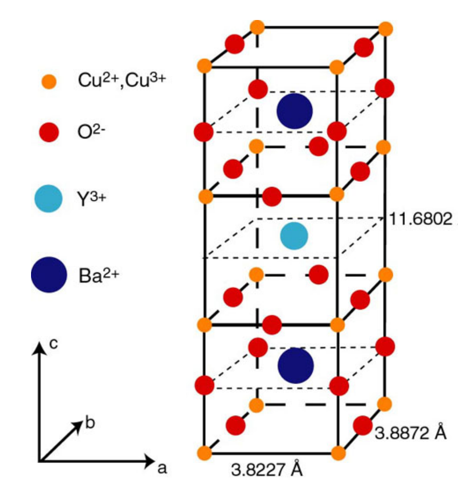
Violated in the Planckian metal

A. Legros et al., Nature Physics **15**, 142 (2019)

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Planckian electron scattering time τ in 6 different cuprates

Linear-T Resistivity $\rho = \frac{m^*}{ne^2\tau} \quad \Rightarrow \quad \frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$



Current flow without quasiparticles

See also: G. Grissonnanche et al., arXiv:2011.13054

Theory of metals

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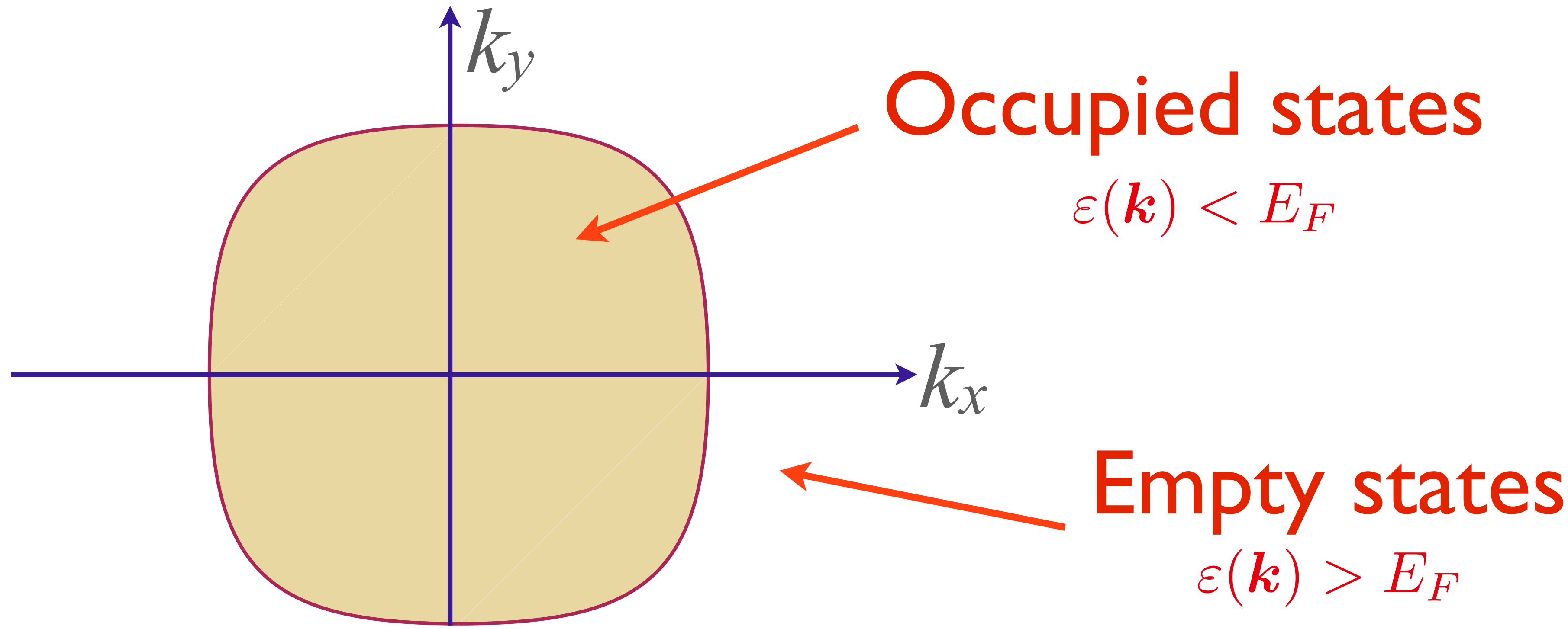

- Needed: a theory of a Planckian metal with $1/\tau \sim k_B T/\hbar$

Missing ingredient:
multi-particle, long-range
quantum entanglement

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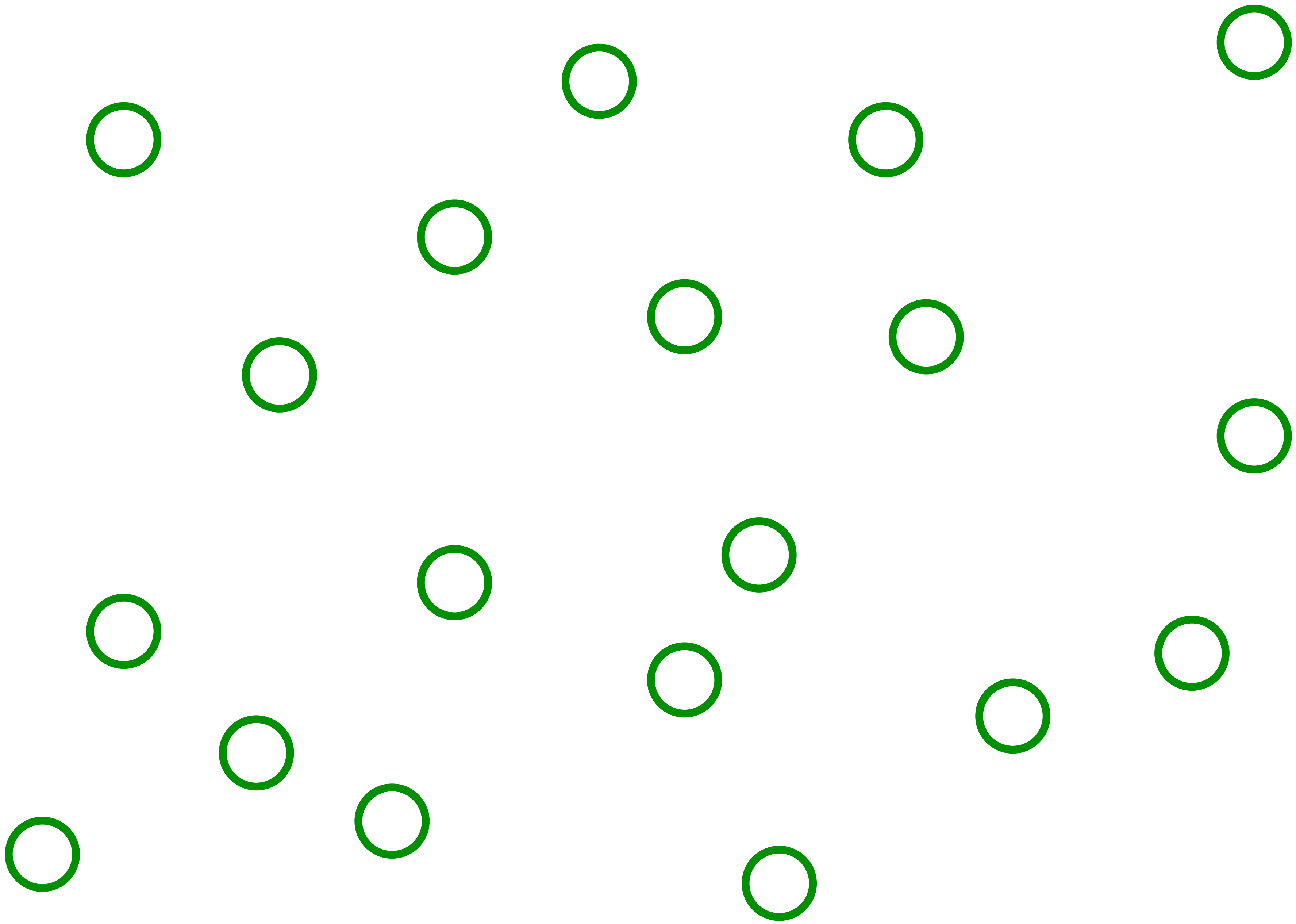
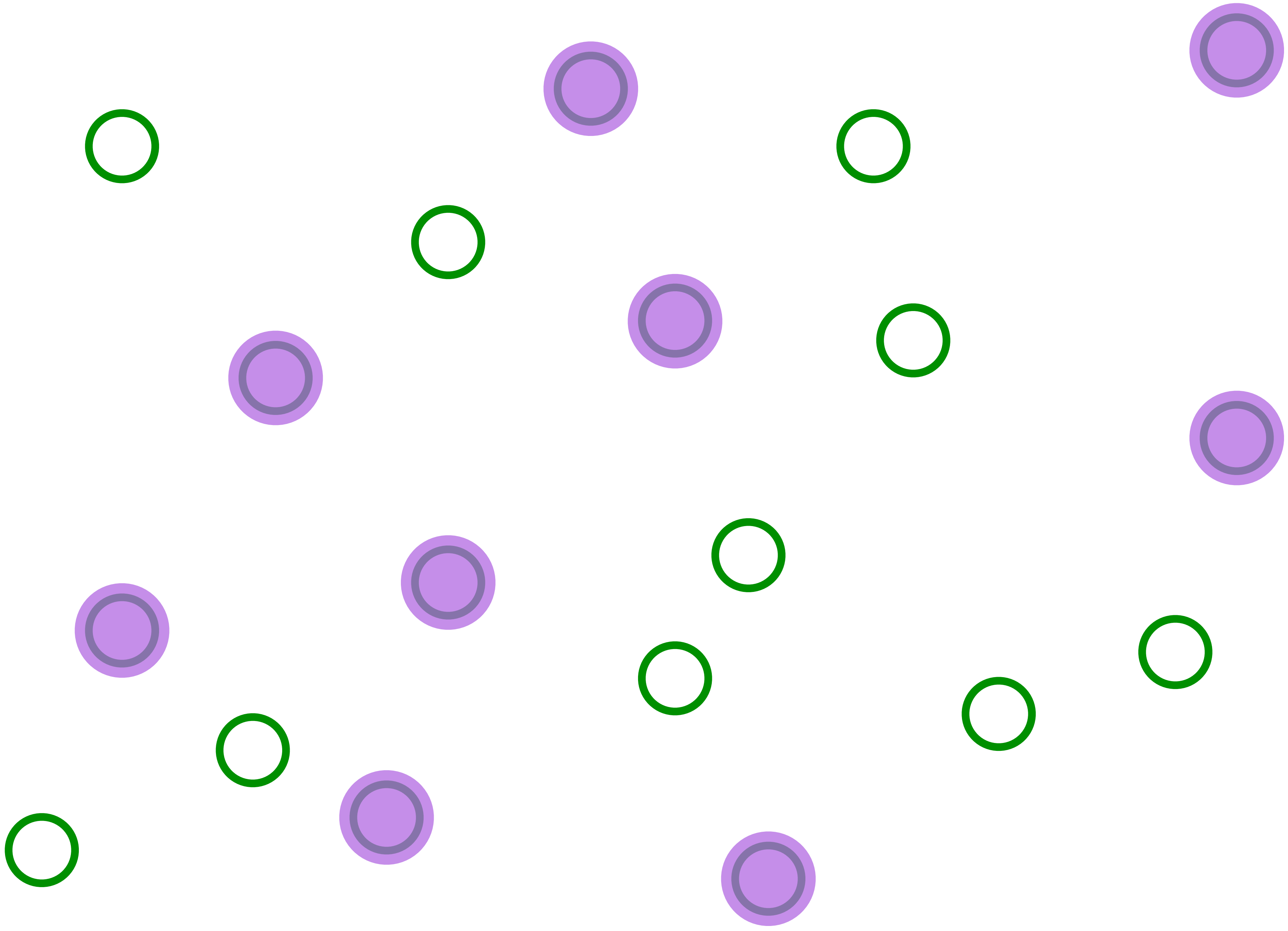
The random matrix model



Pick a set of random positions

The random matrix model



Place electrons randomly on some sites

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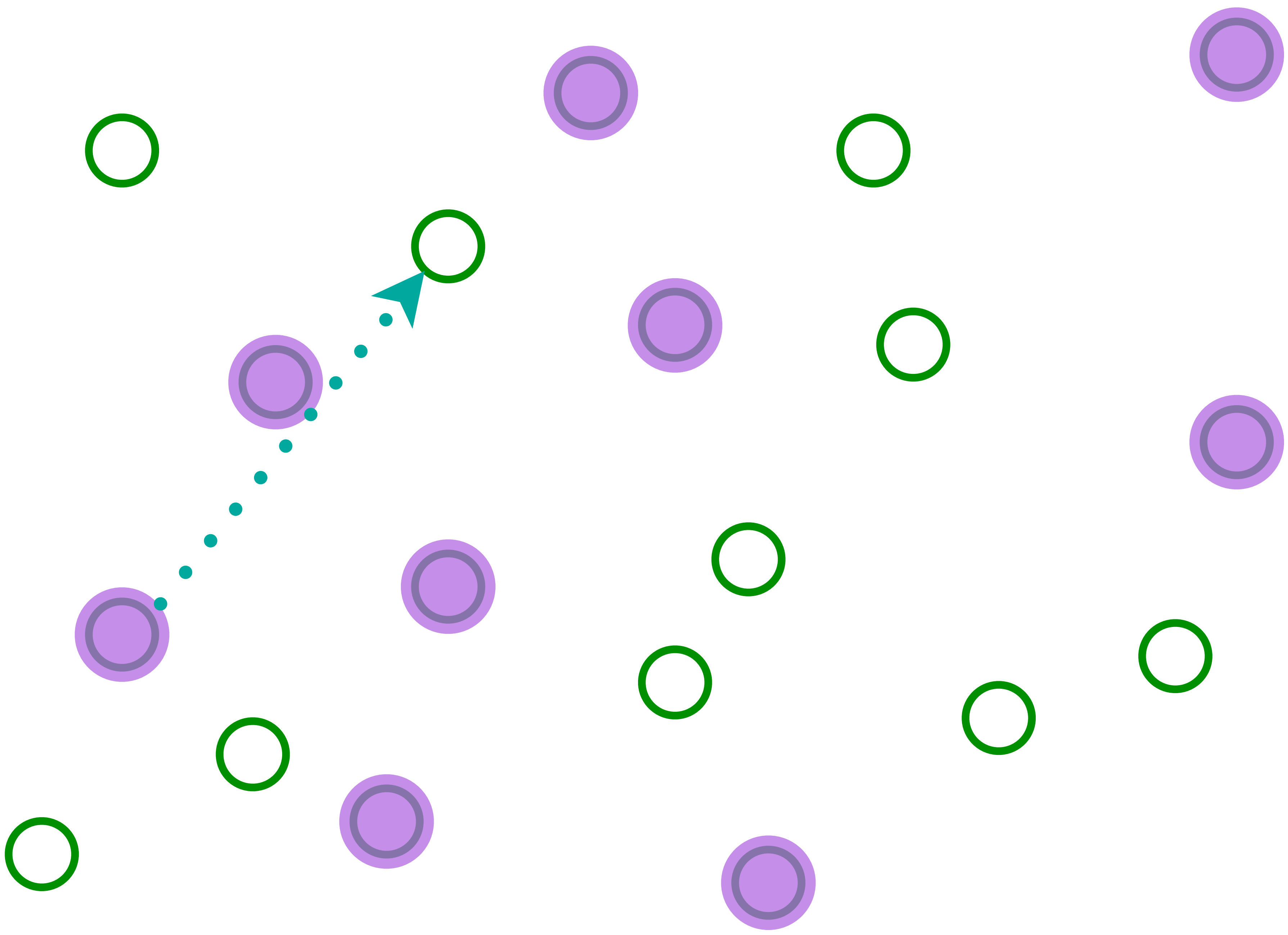

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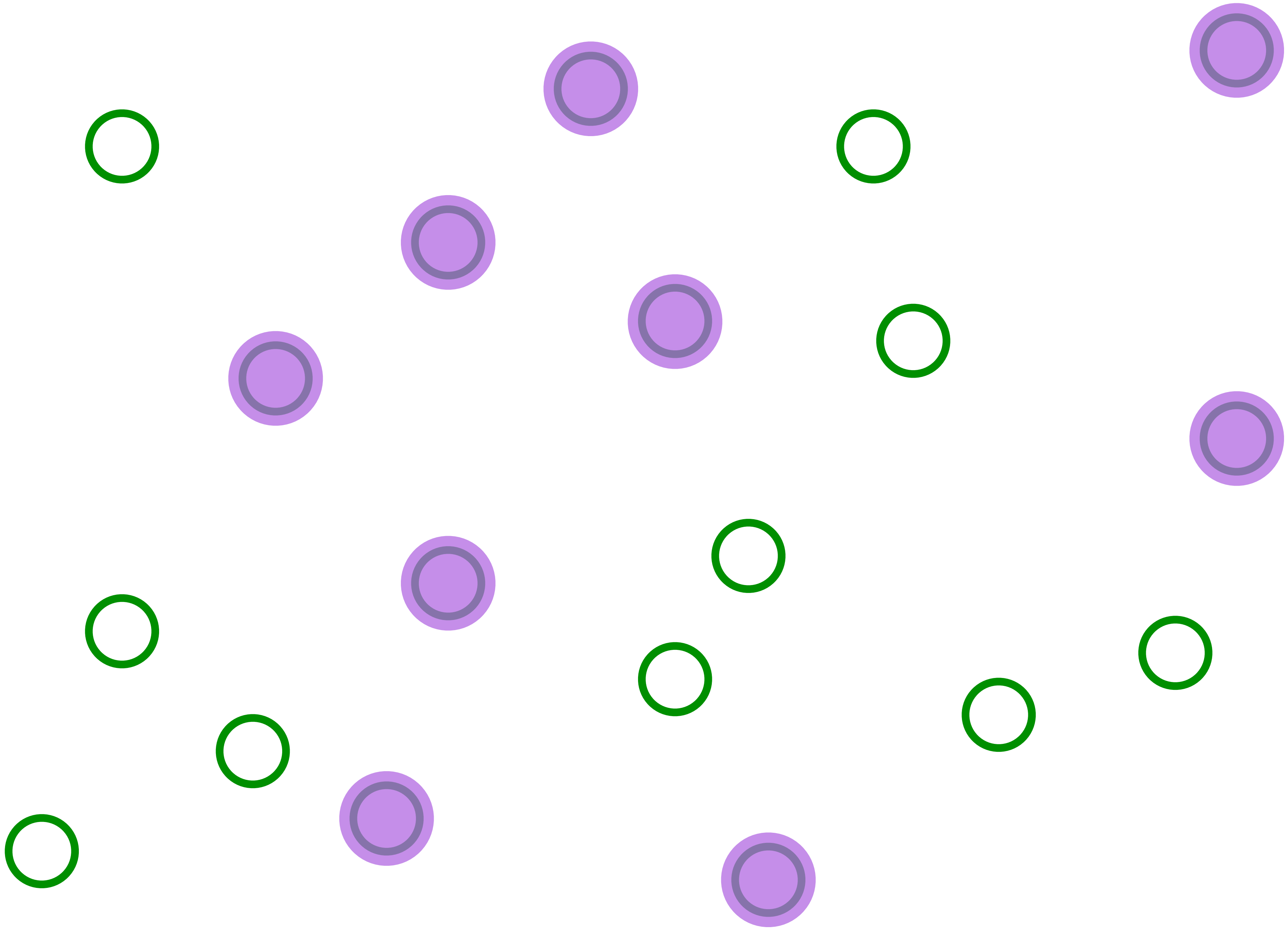

Electrons can hop anywhere with a random amplitude

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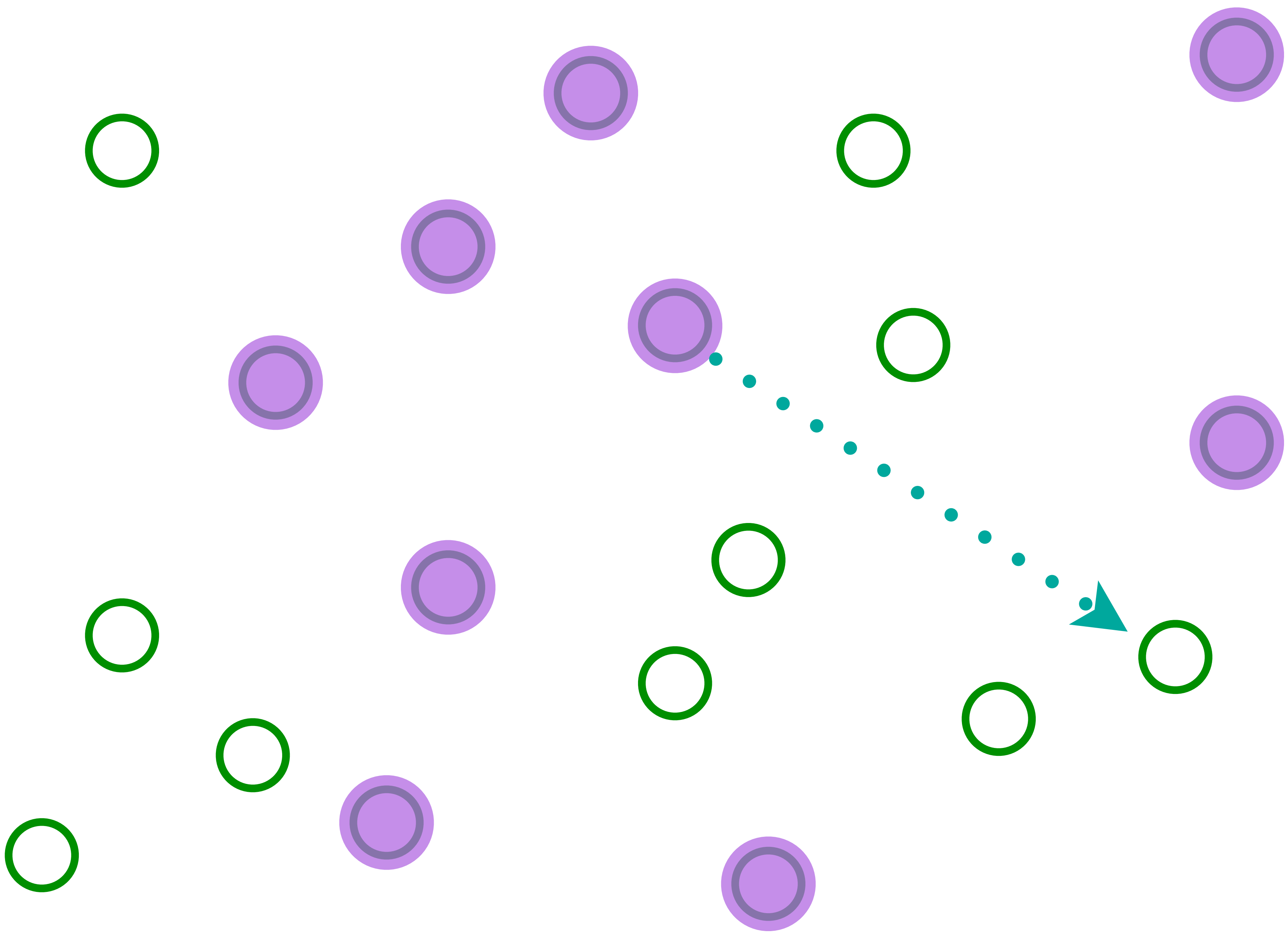

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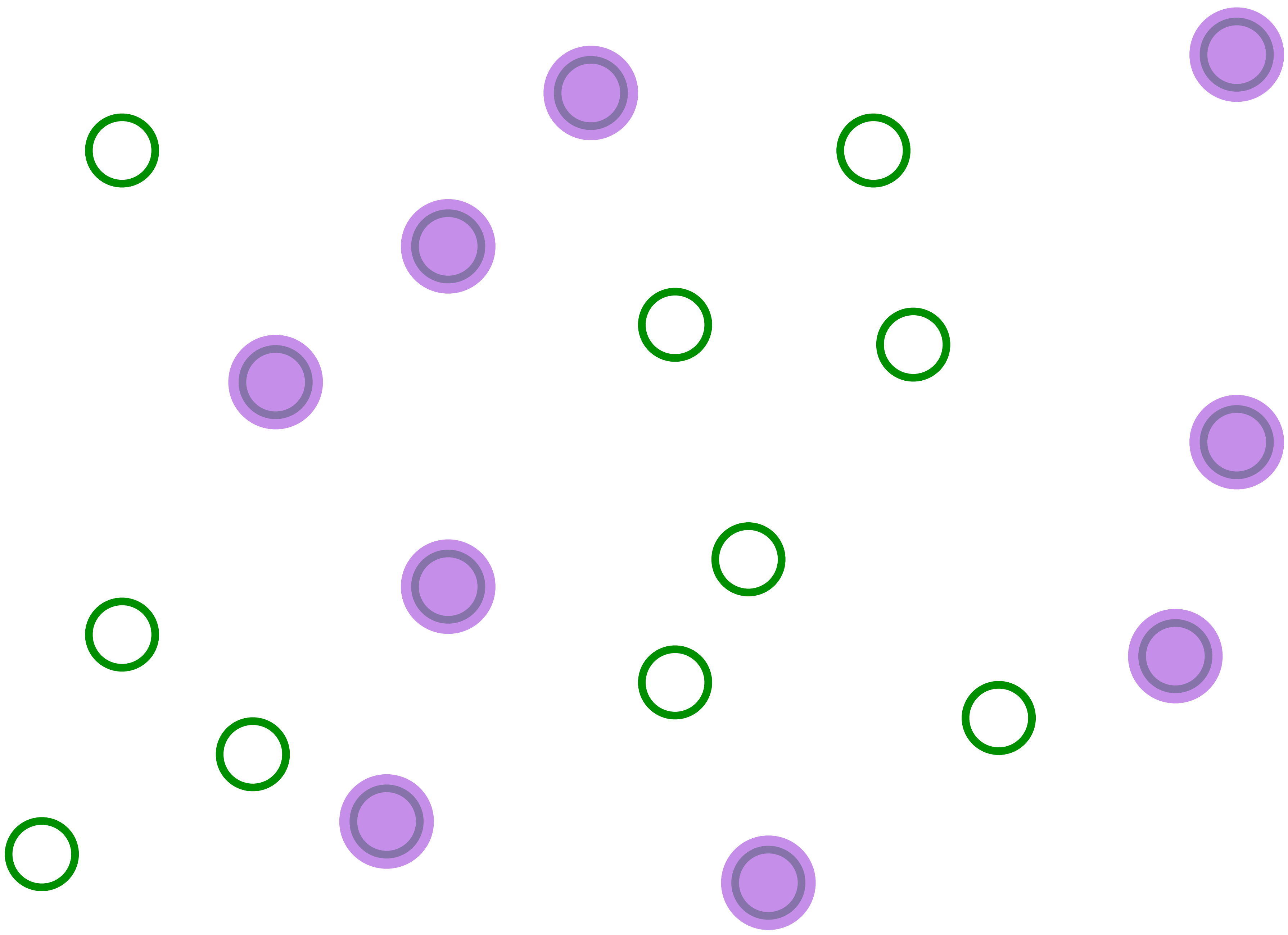

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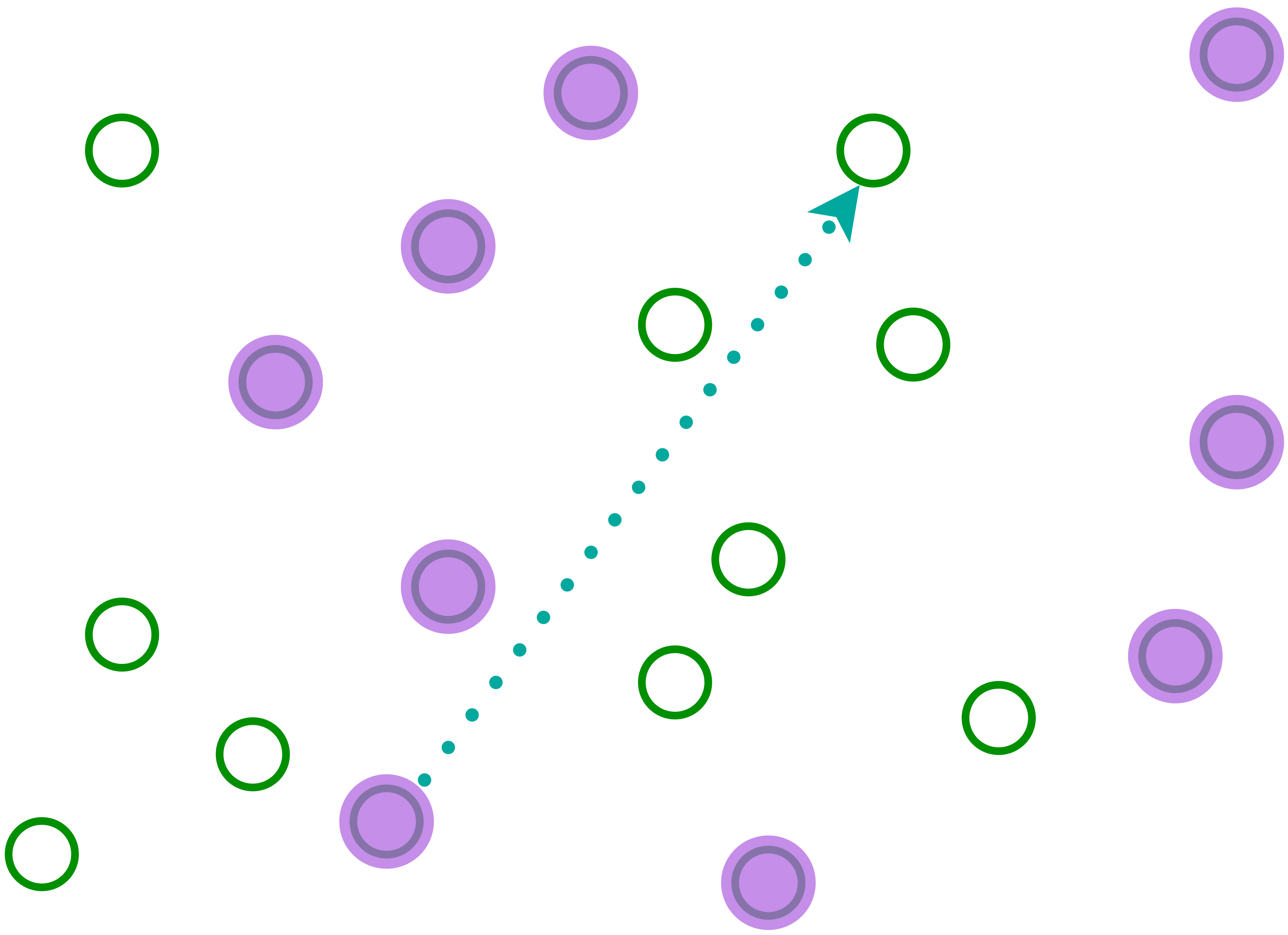

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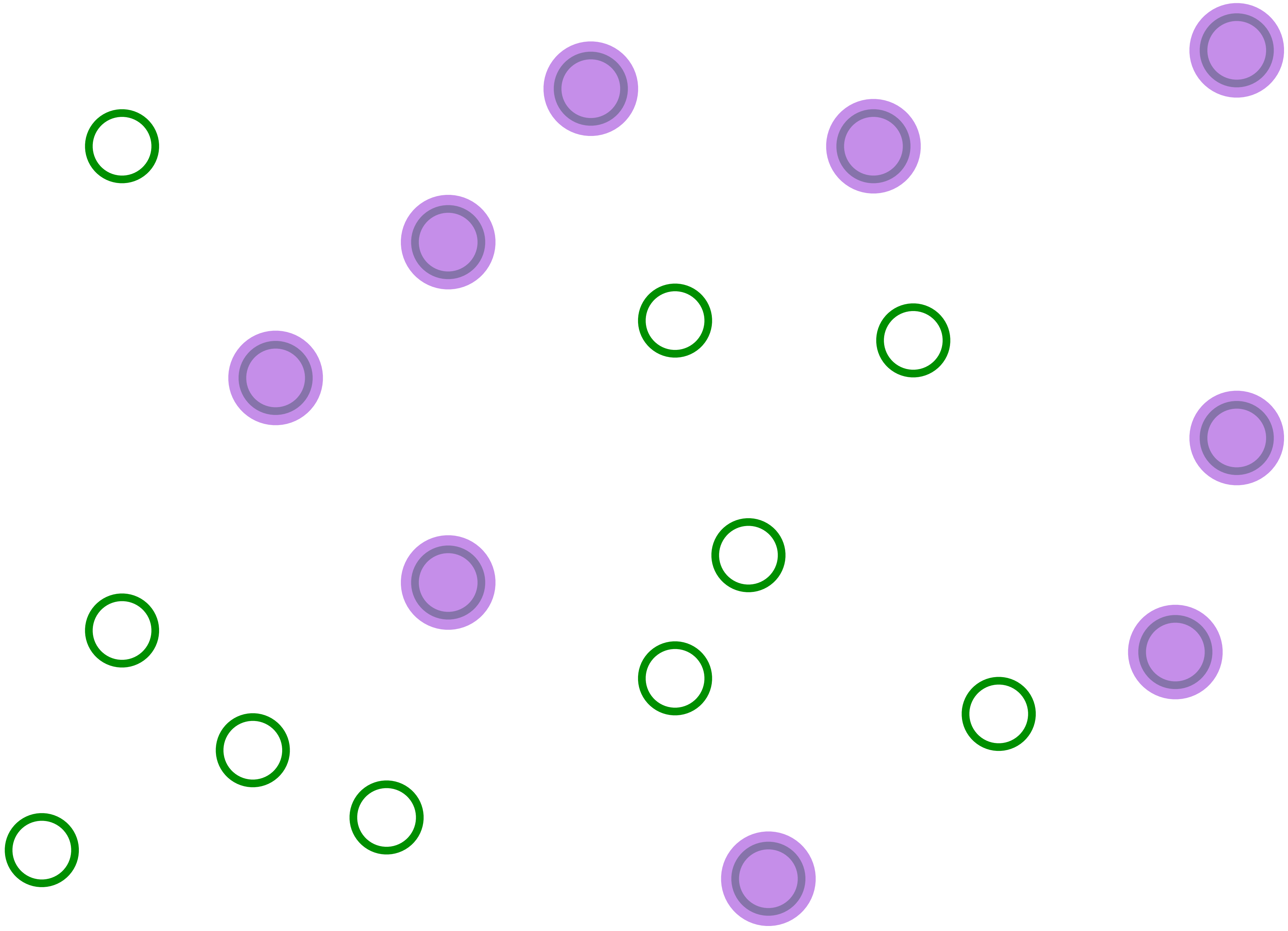

Electrons can hop anywhere with a random amplitude

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

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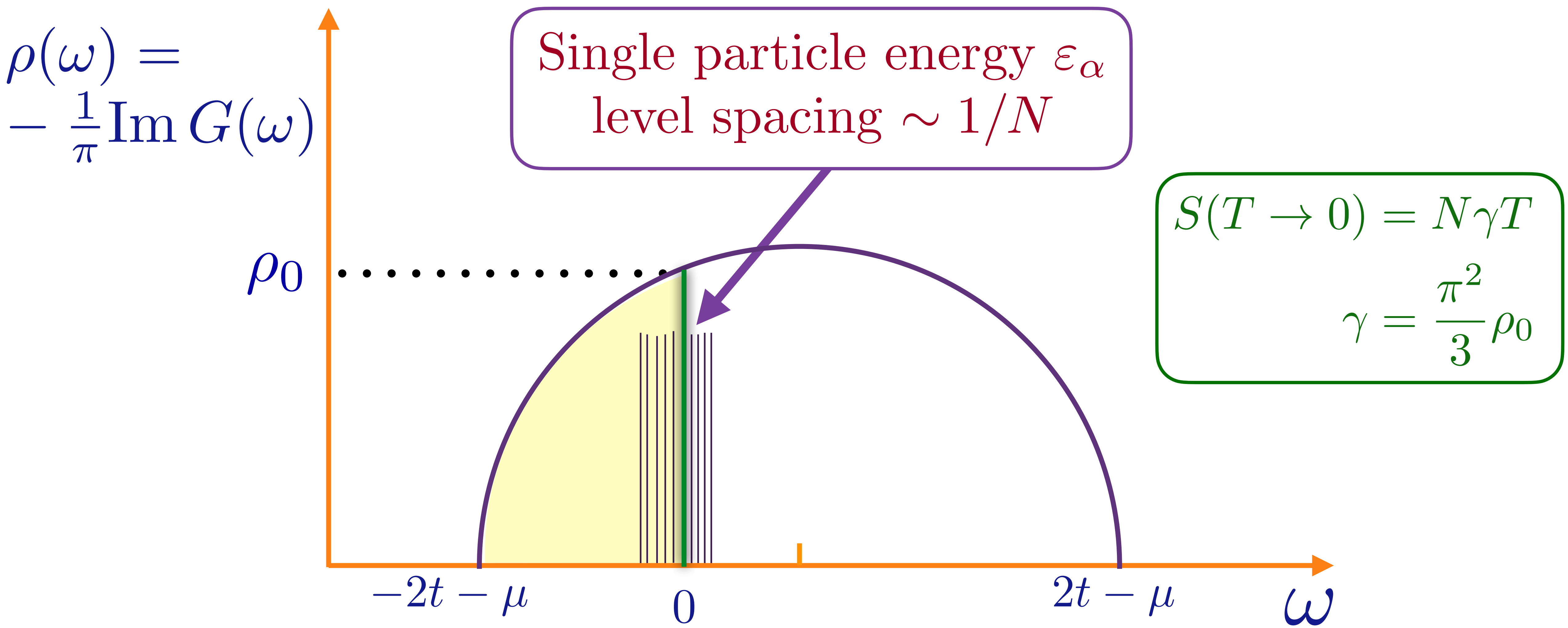

Many-body density of states

$$D(E) = \sum_a \delta(E - E_a); \quad E_0 + E_a \Rightarrow \text{Many body eigenvalue}$$

For random matrix model: $E_0 + E_a = \sum_\alpha n_\alpha \varepsilon_\alpha$

$n_\alpha = 0, 1$, occupation number

Random matrix model

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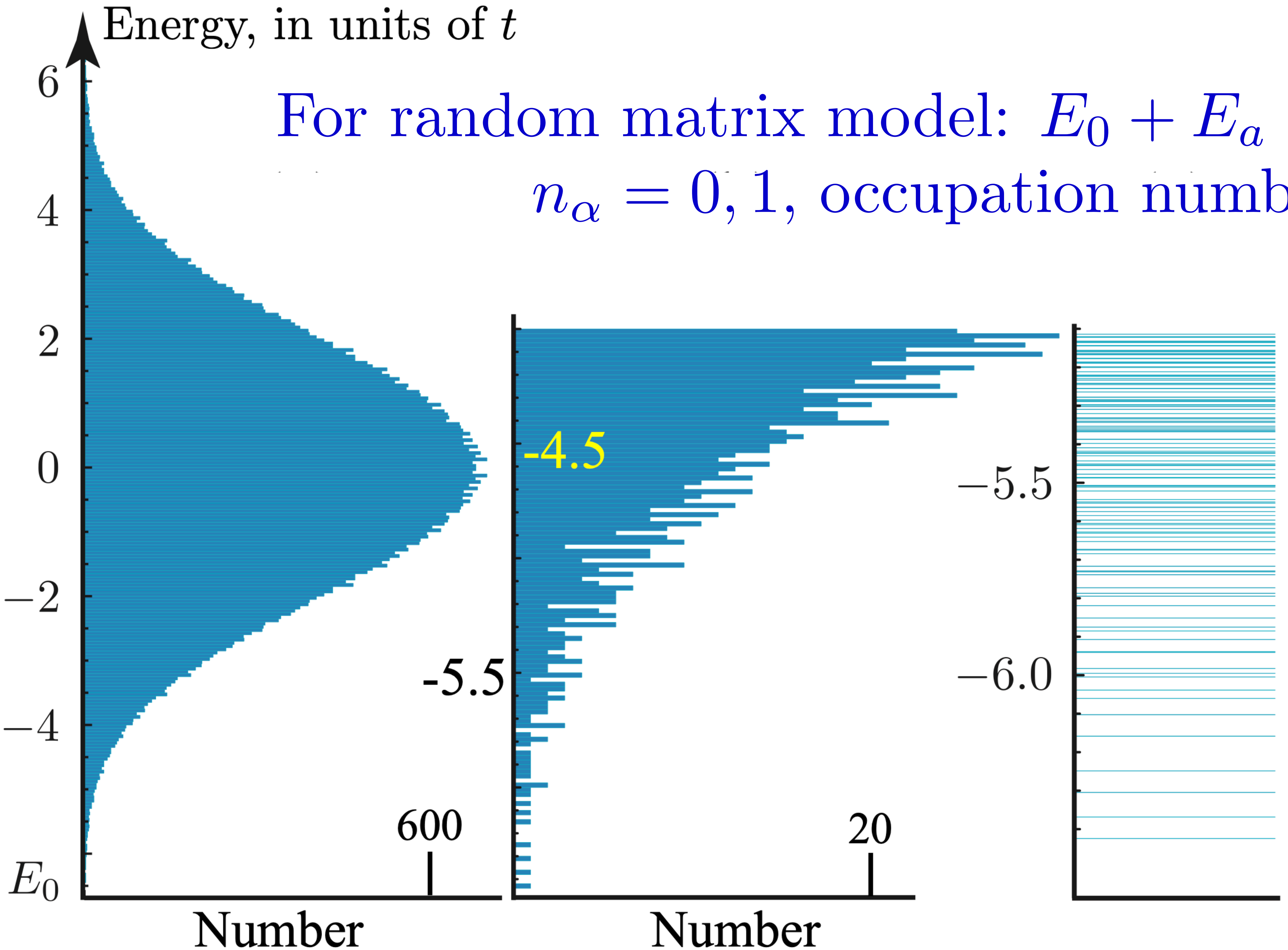

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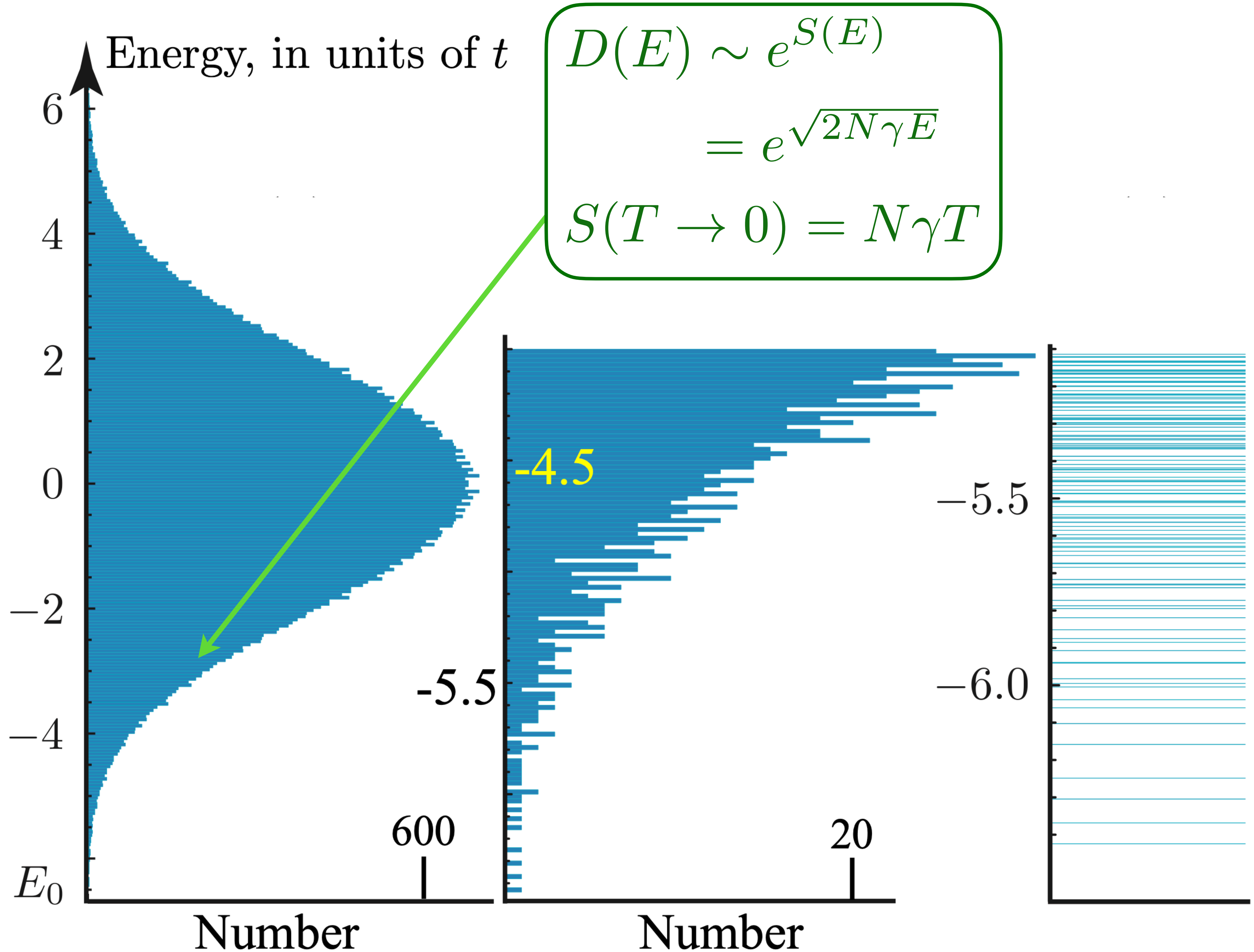

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$$D(E) \sim e^{S(E)}$$
$$= e^{\sqrt{2N\gamma E}}$$
$$S(T \to 0) = N\gamma T$$

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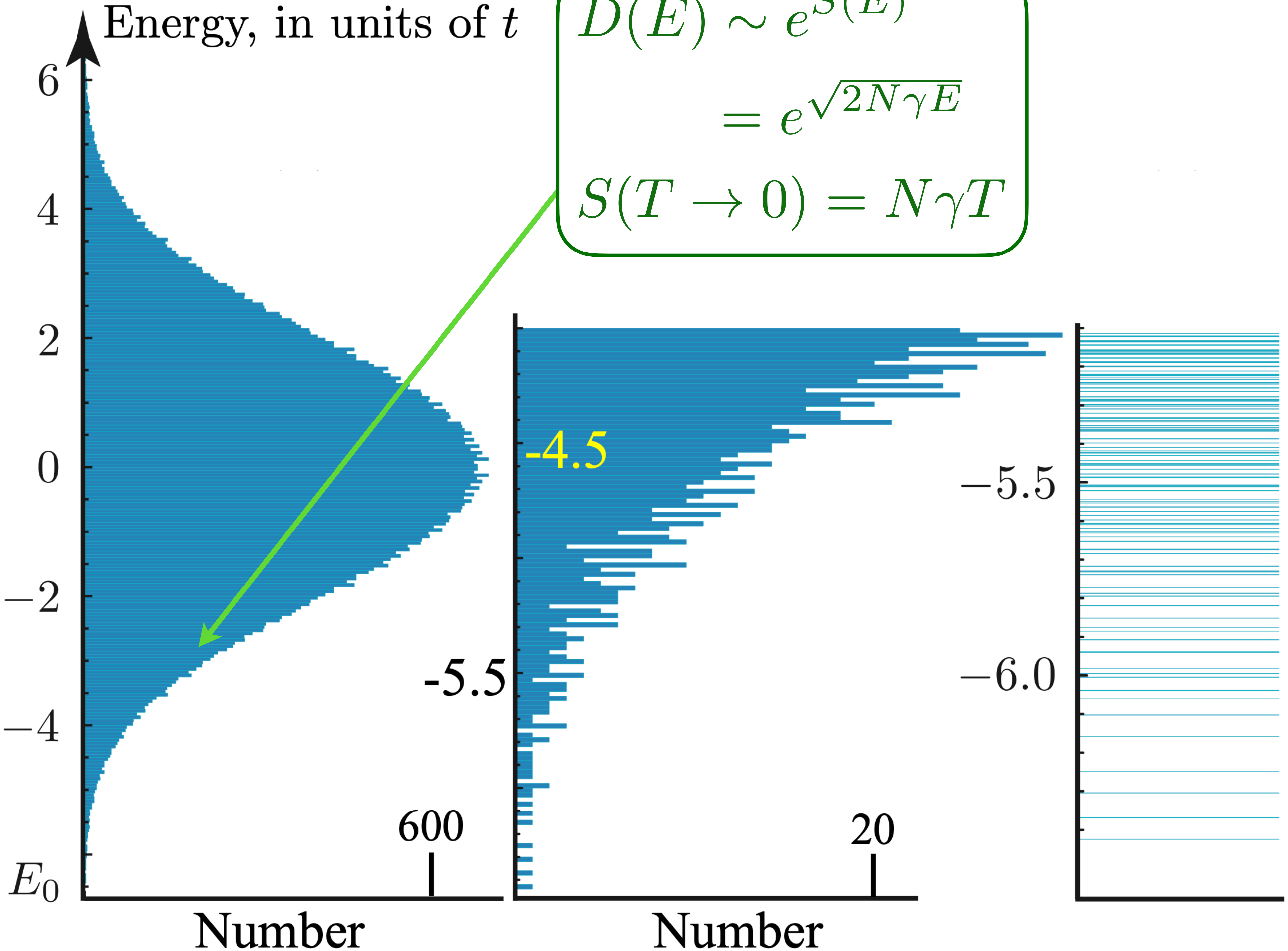
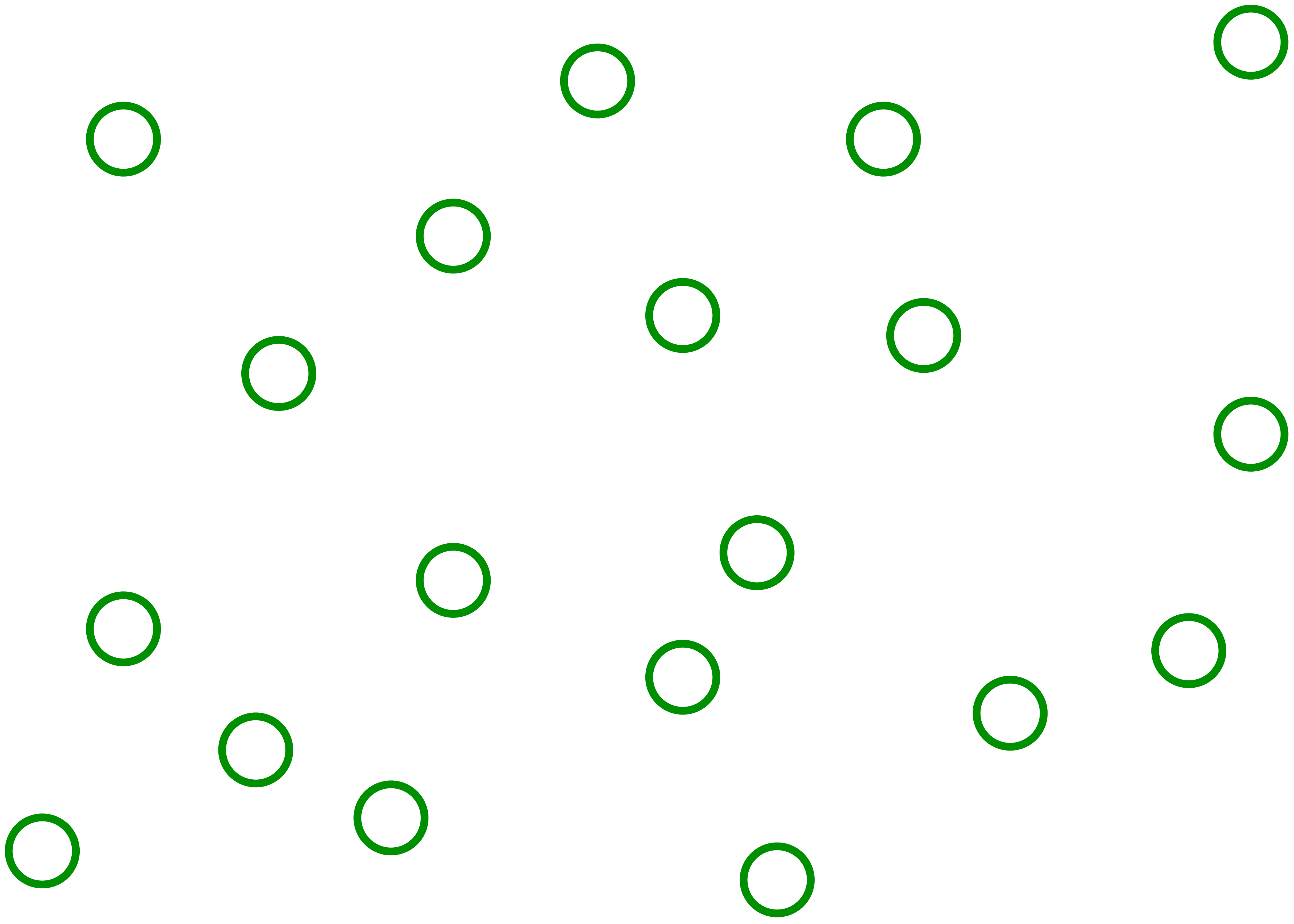


$$D(E) \sim N$$

Random matrix model

The SYK model



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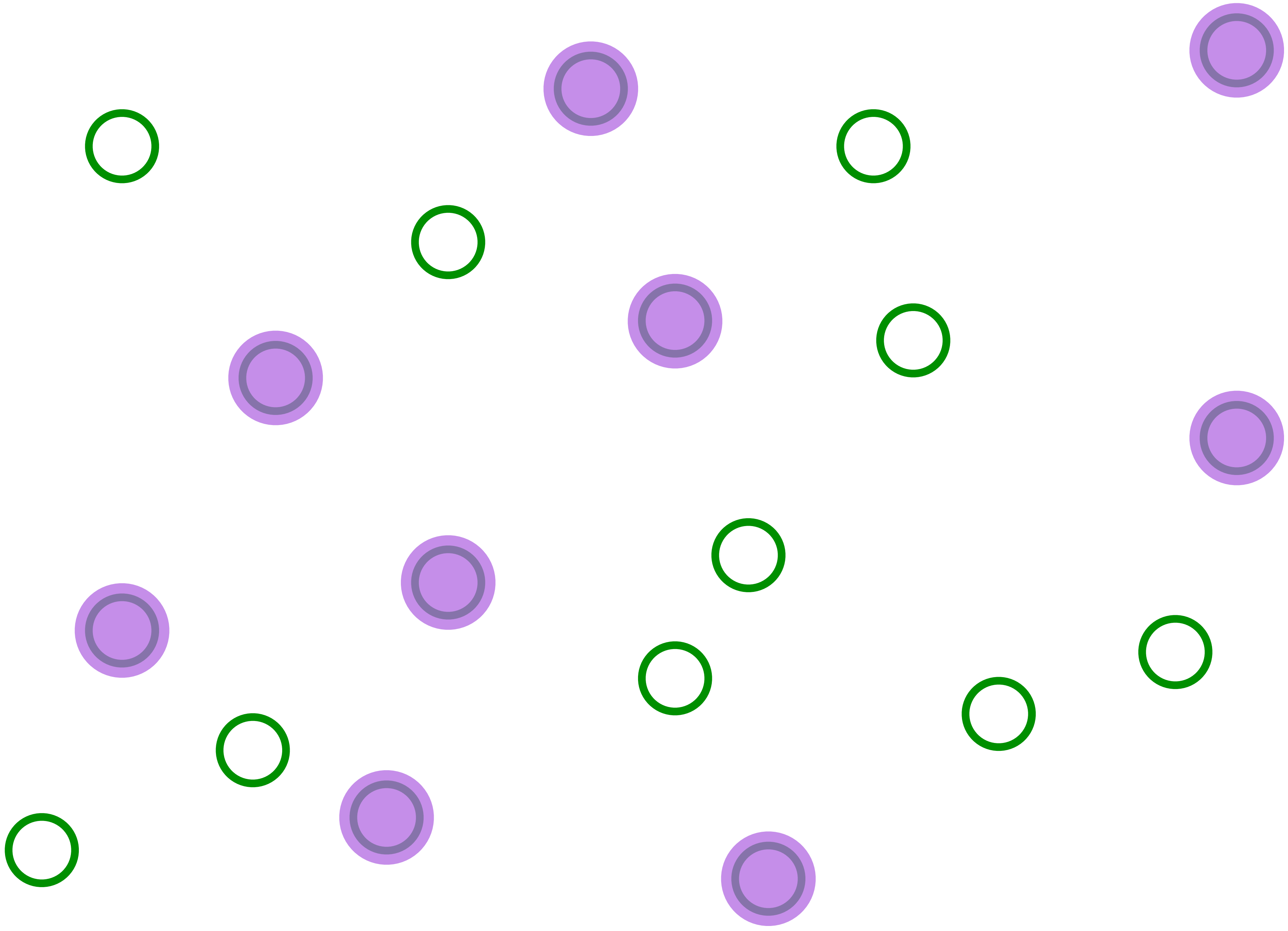

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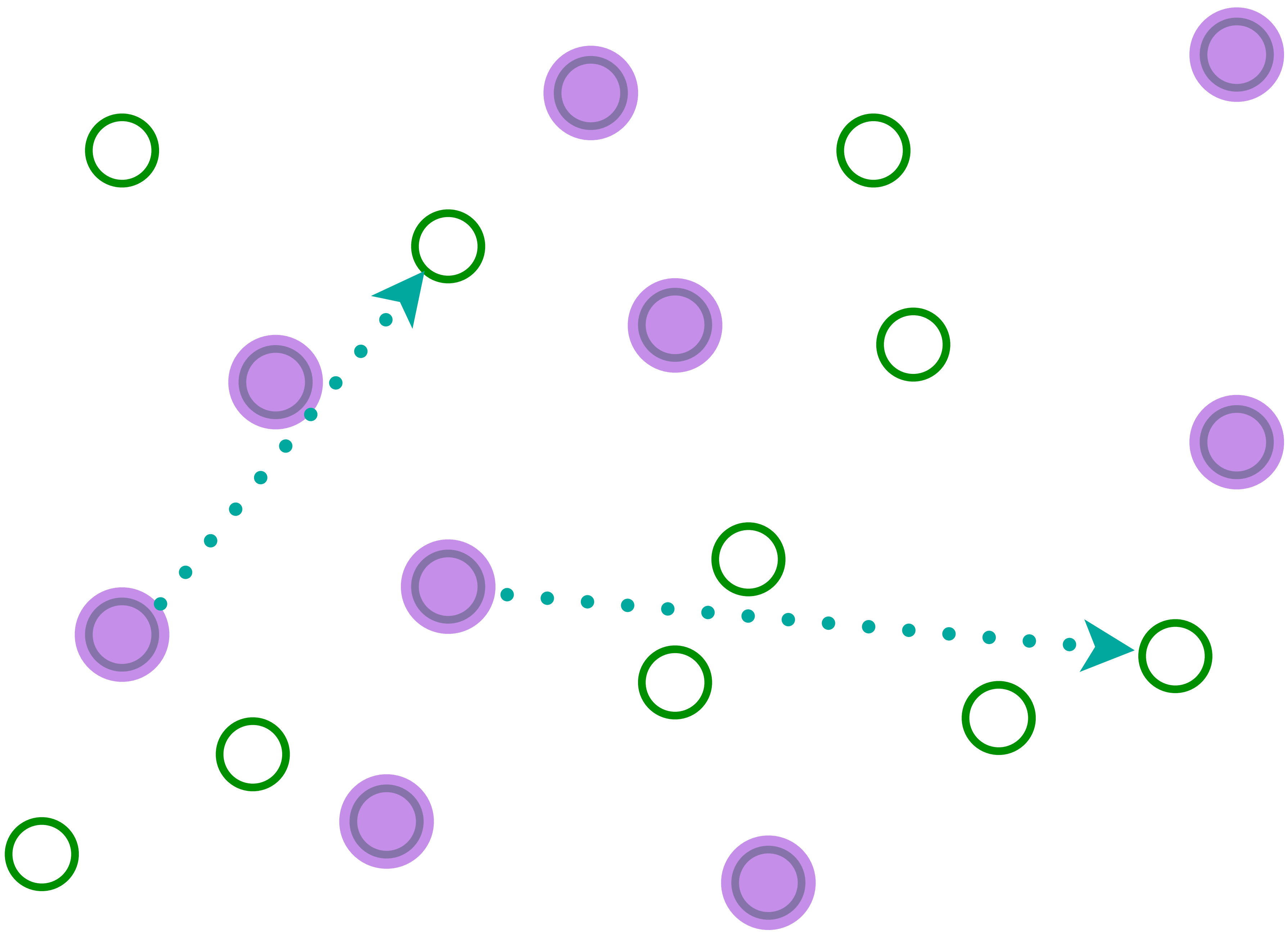

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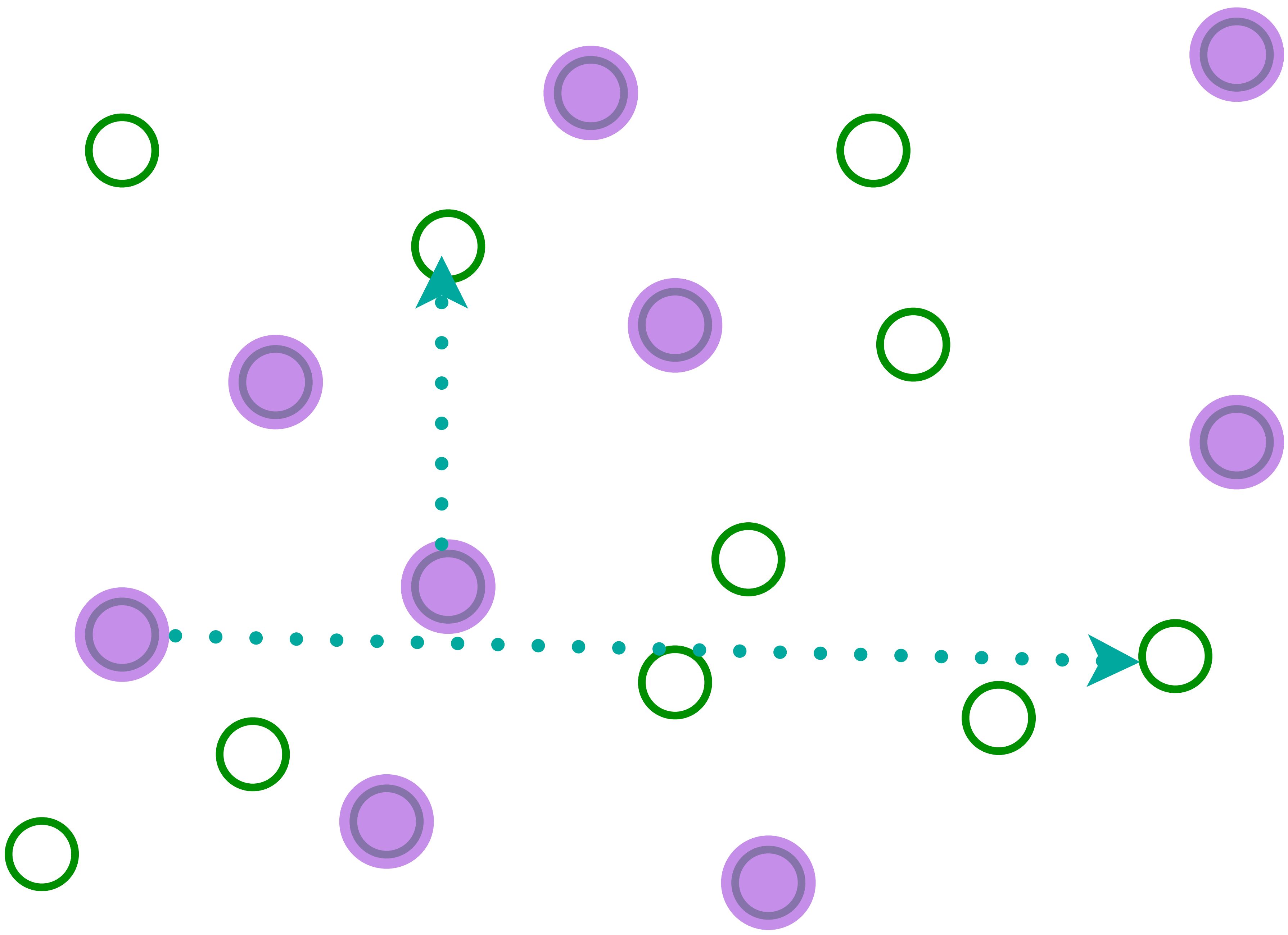

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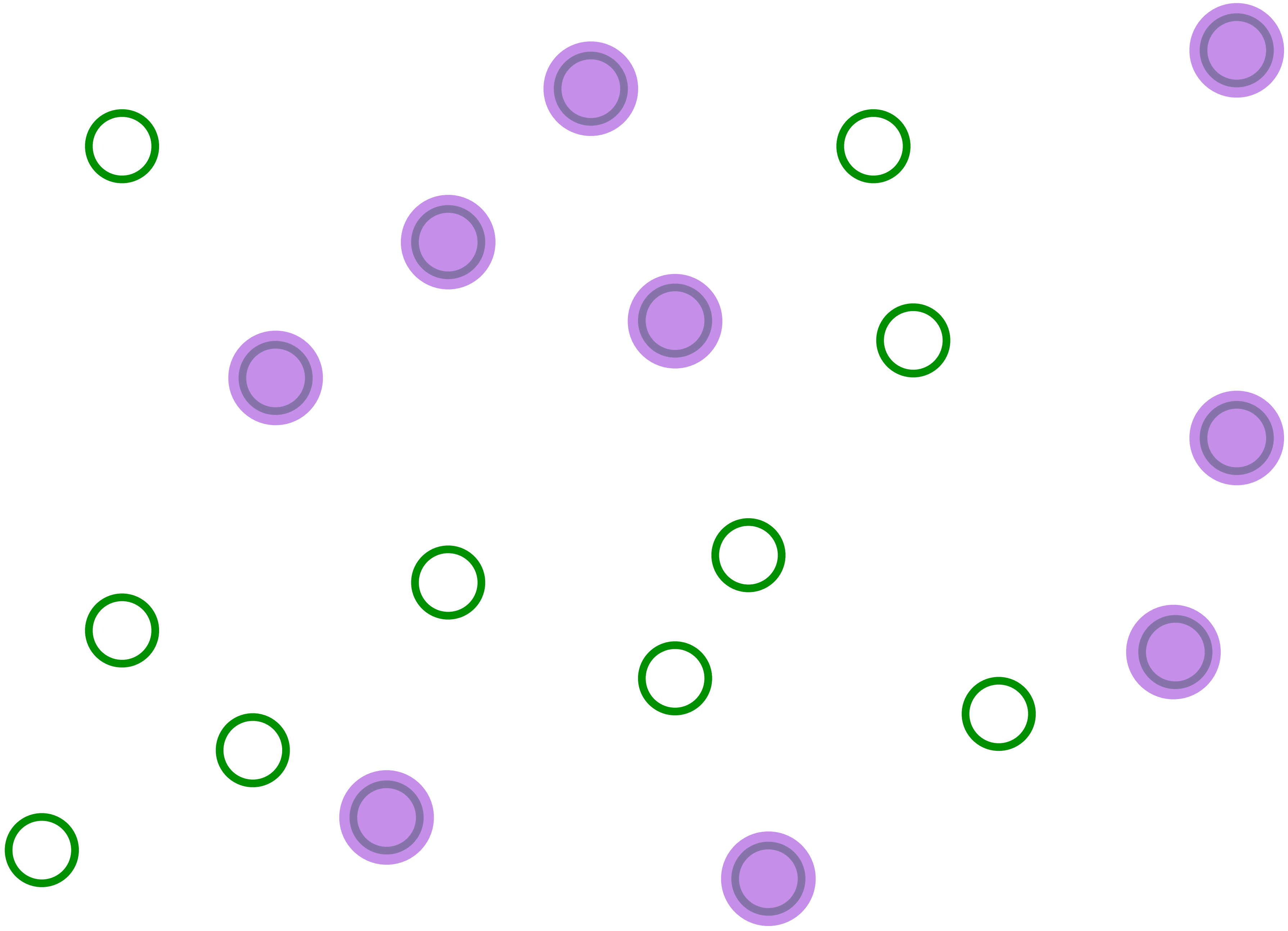

Entangle electrons pairwise randomly

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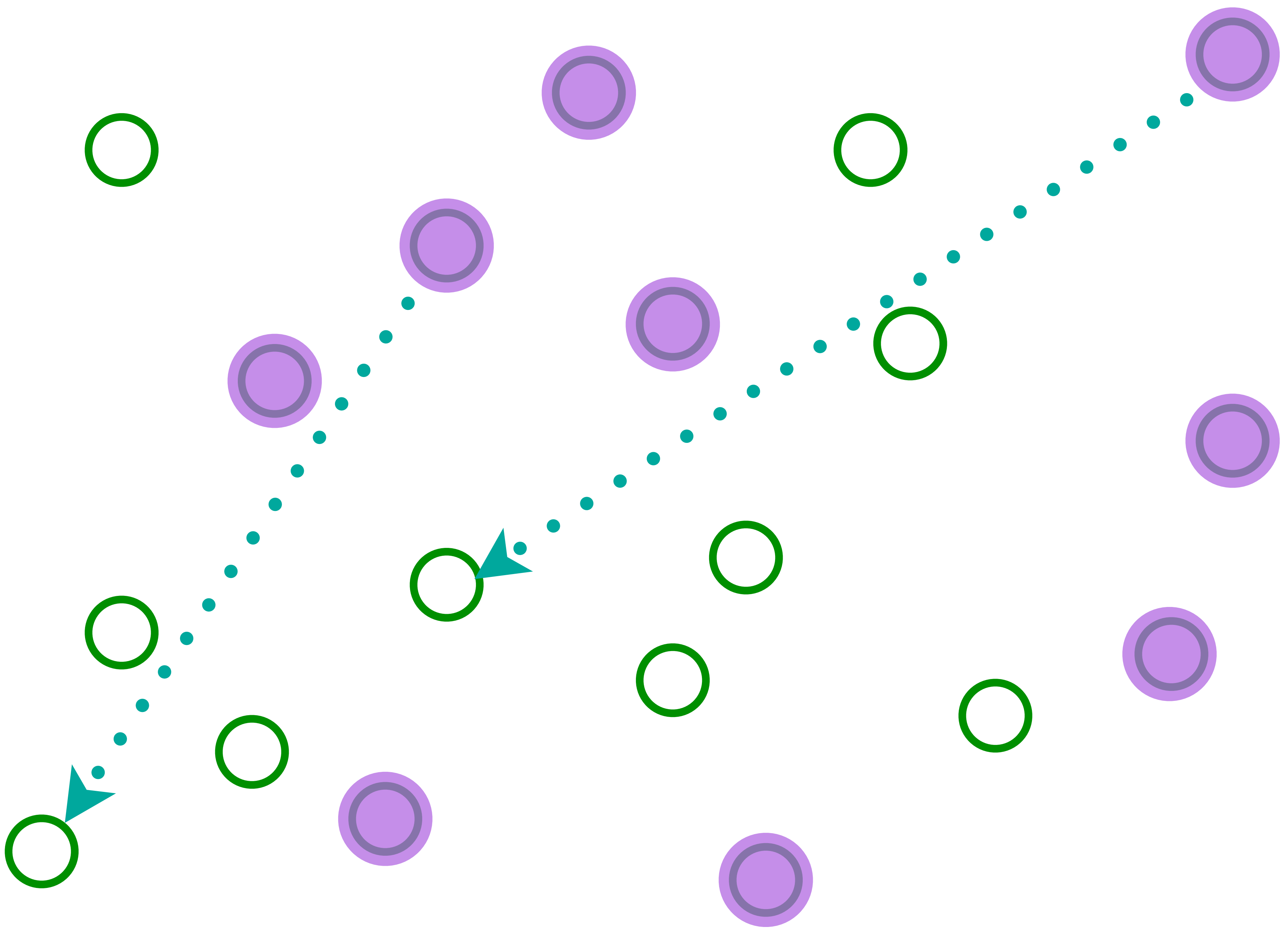

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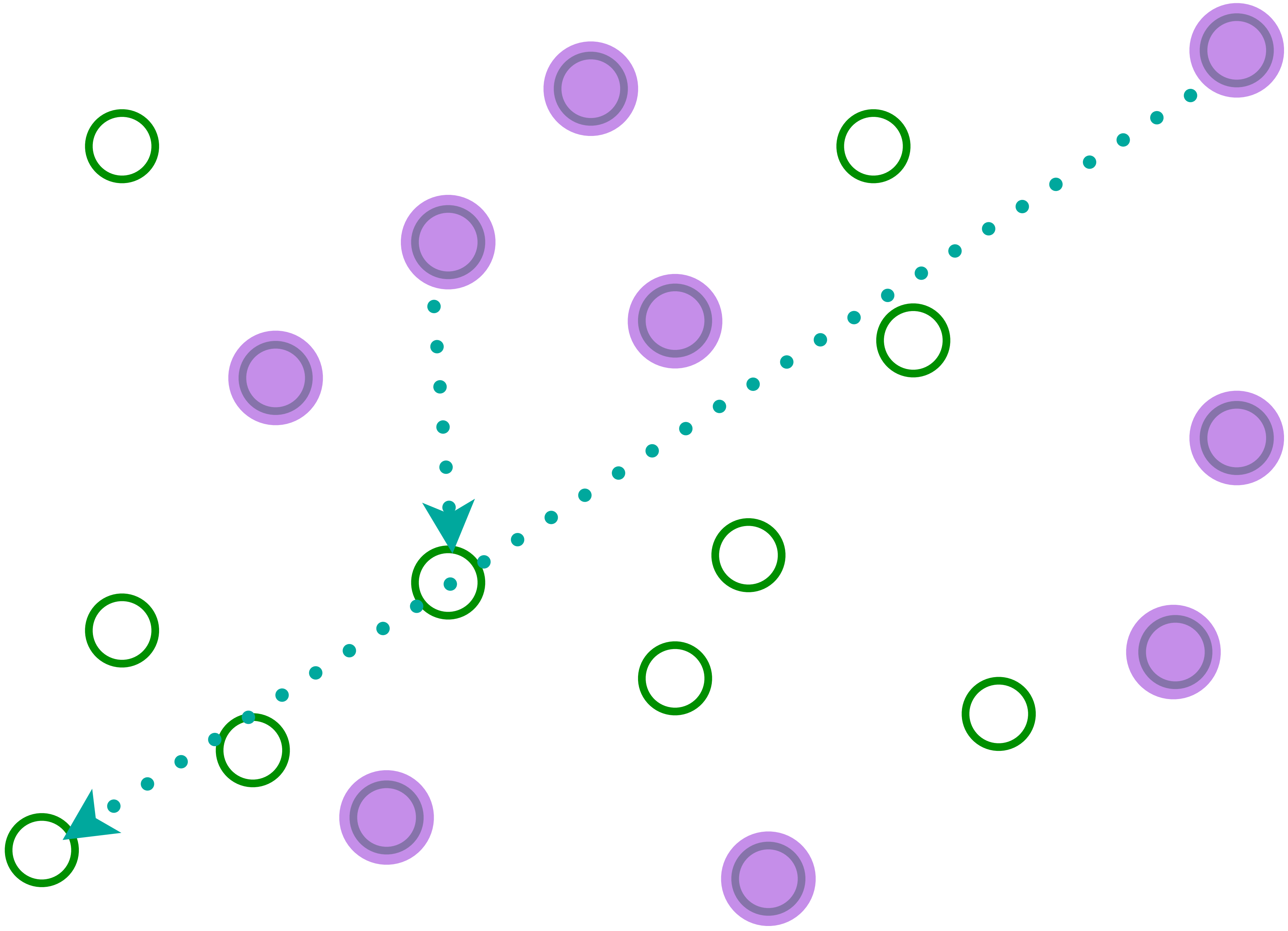

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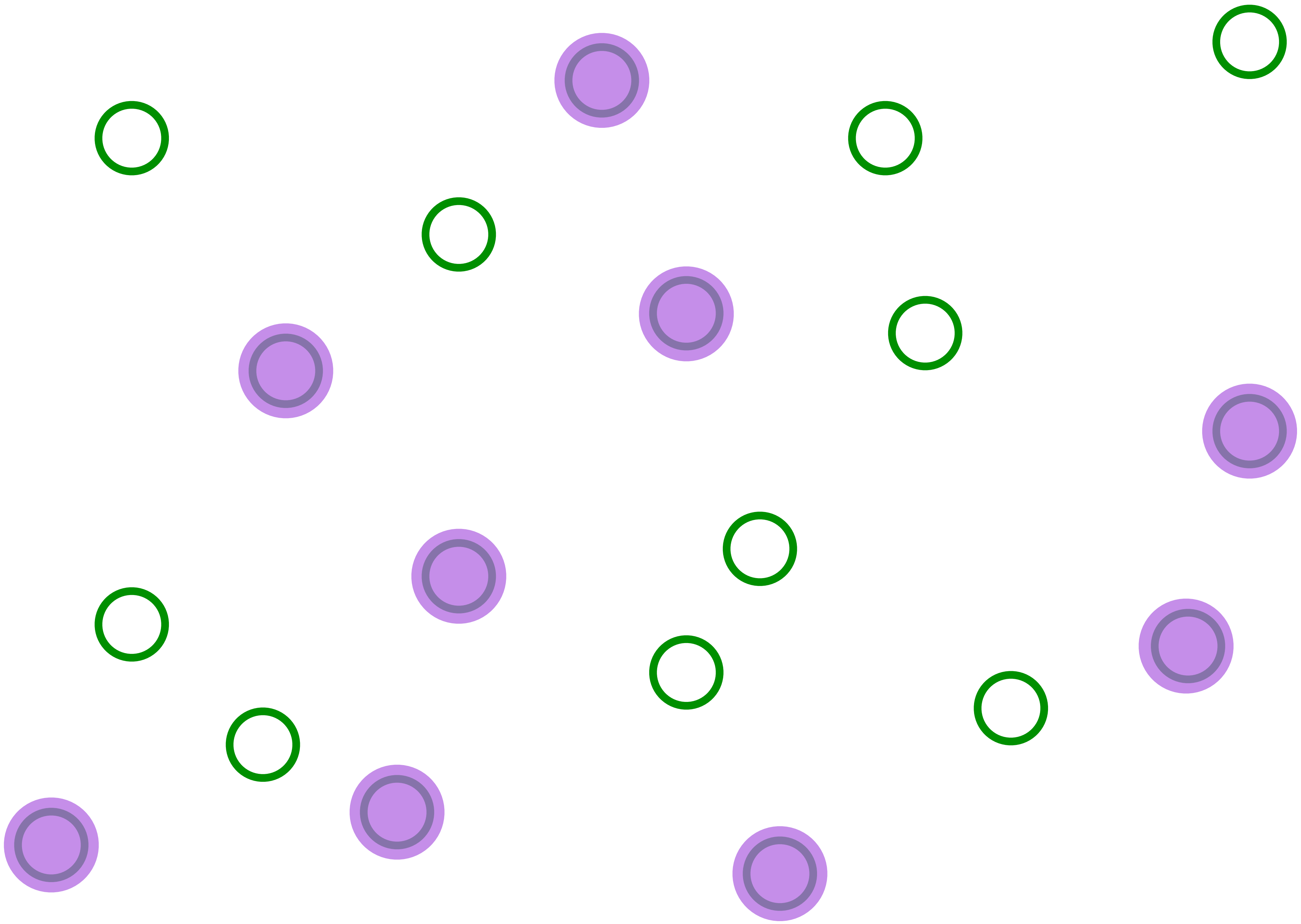

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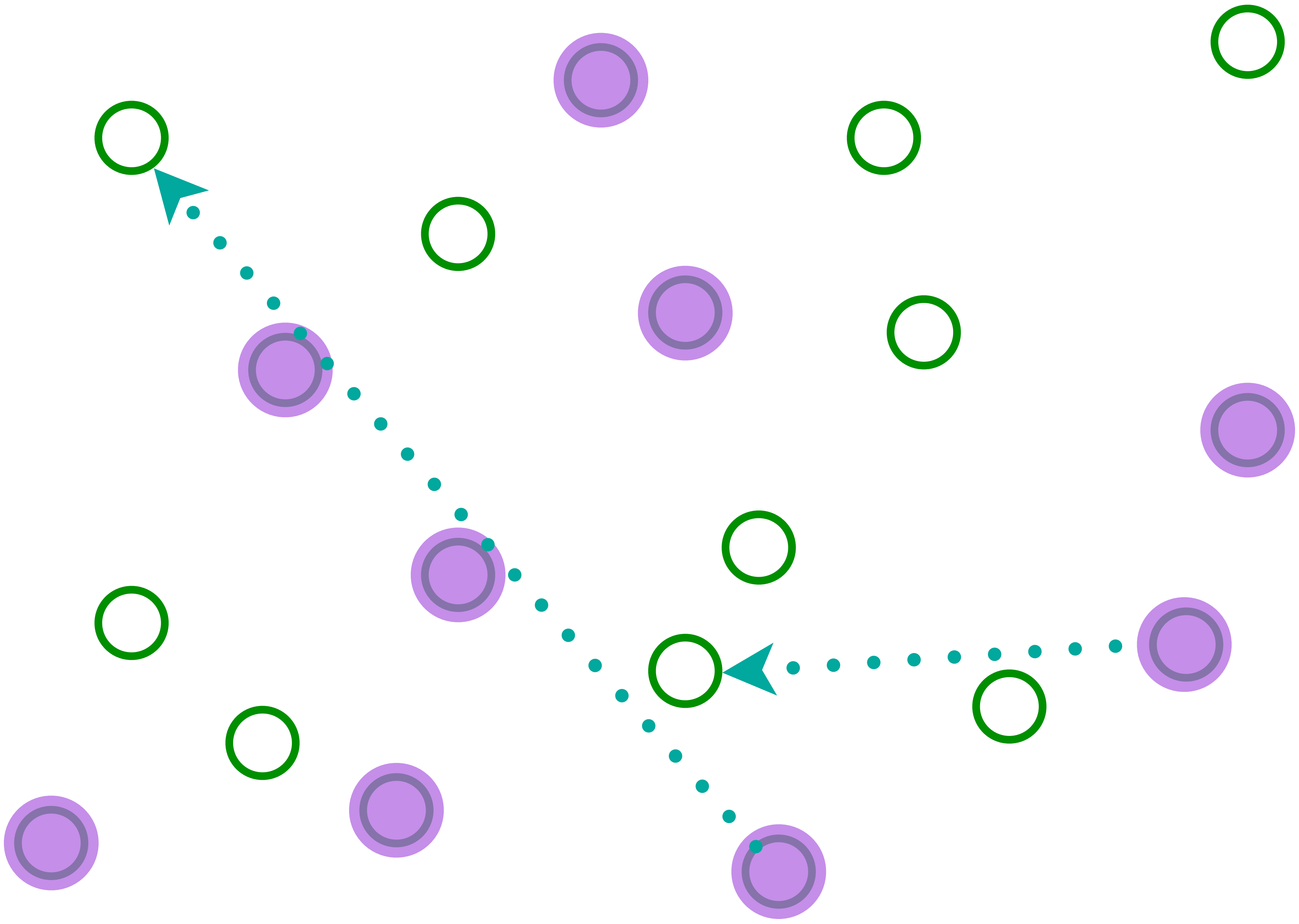

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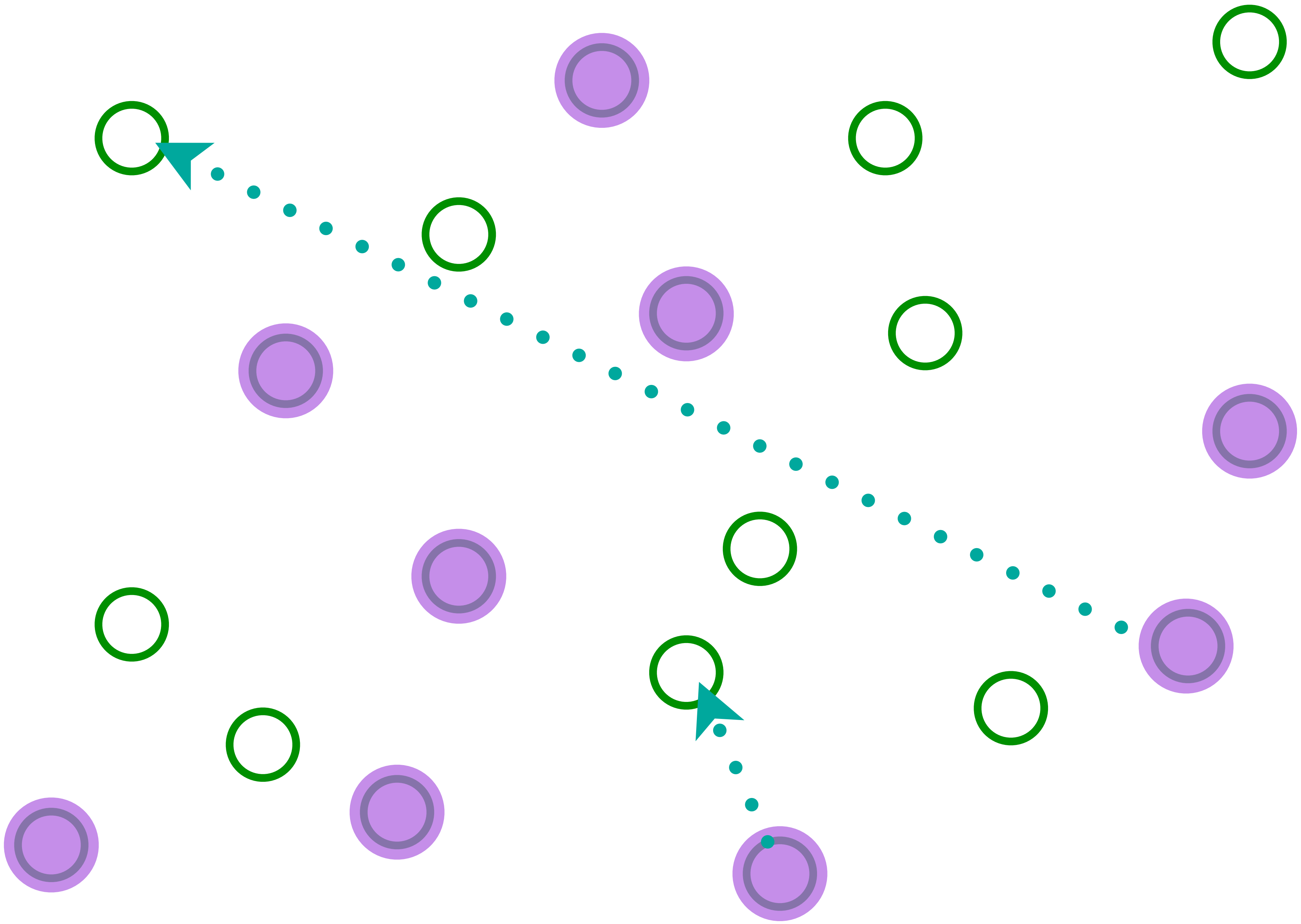

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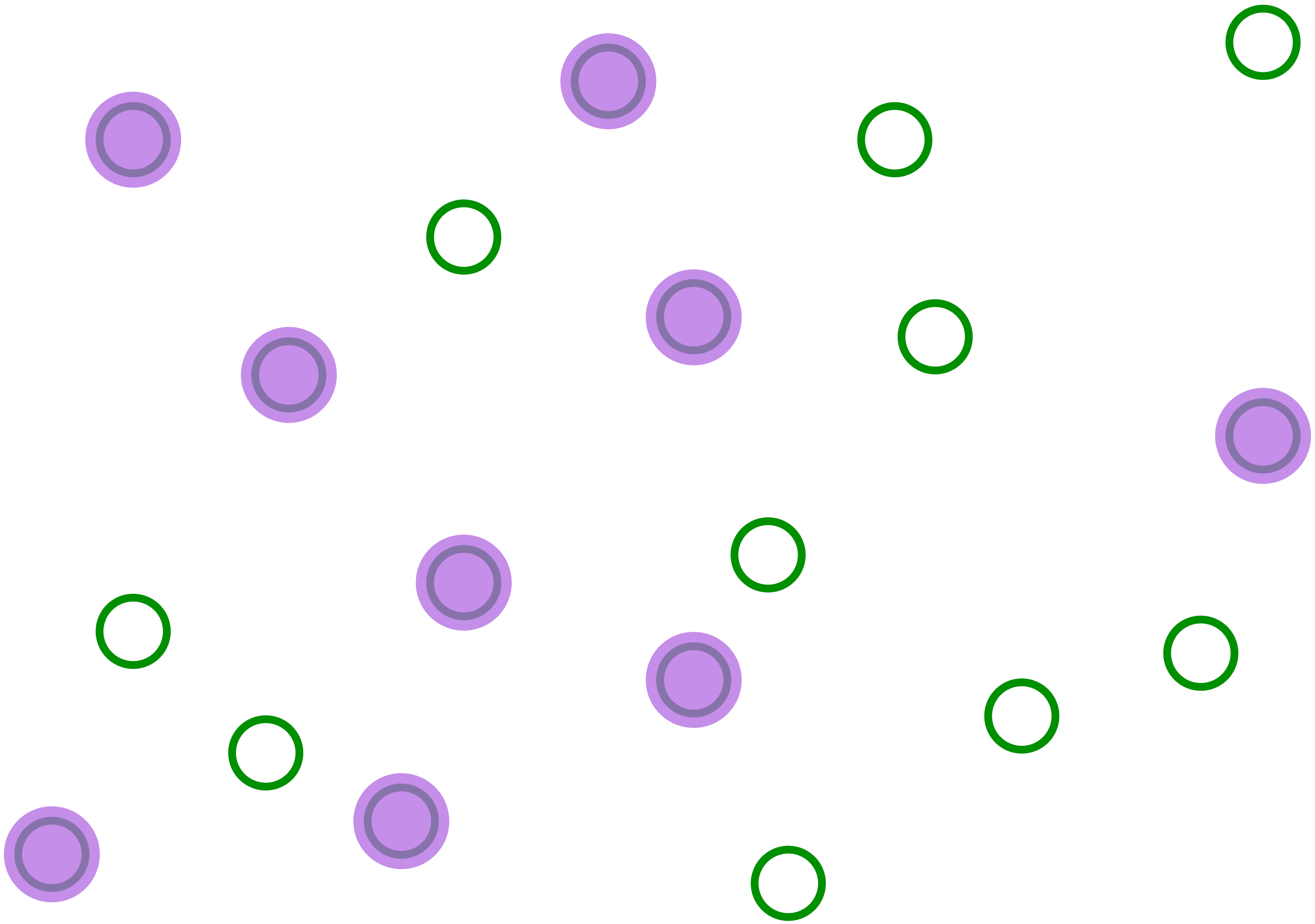

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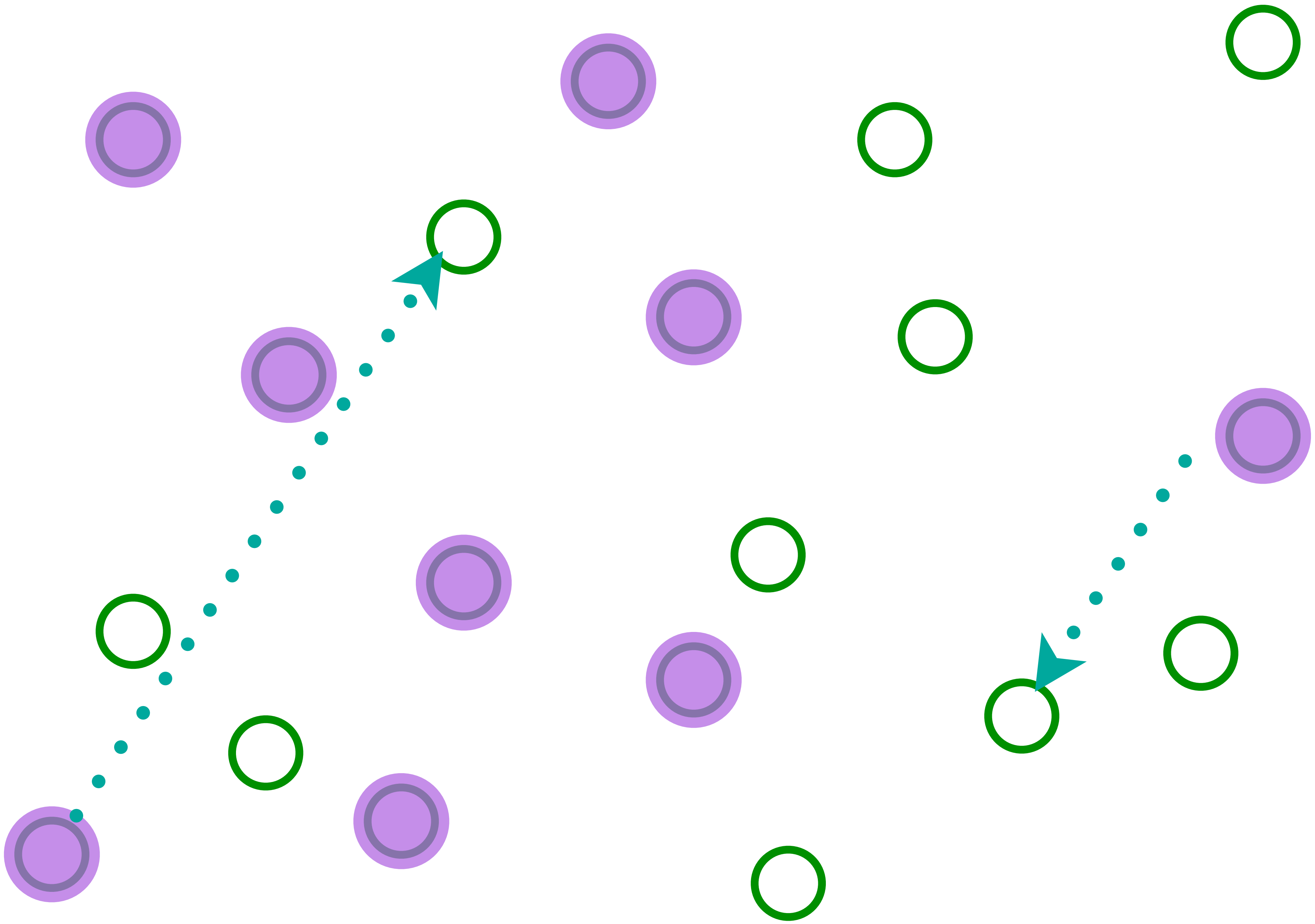

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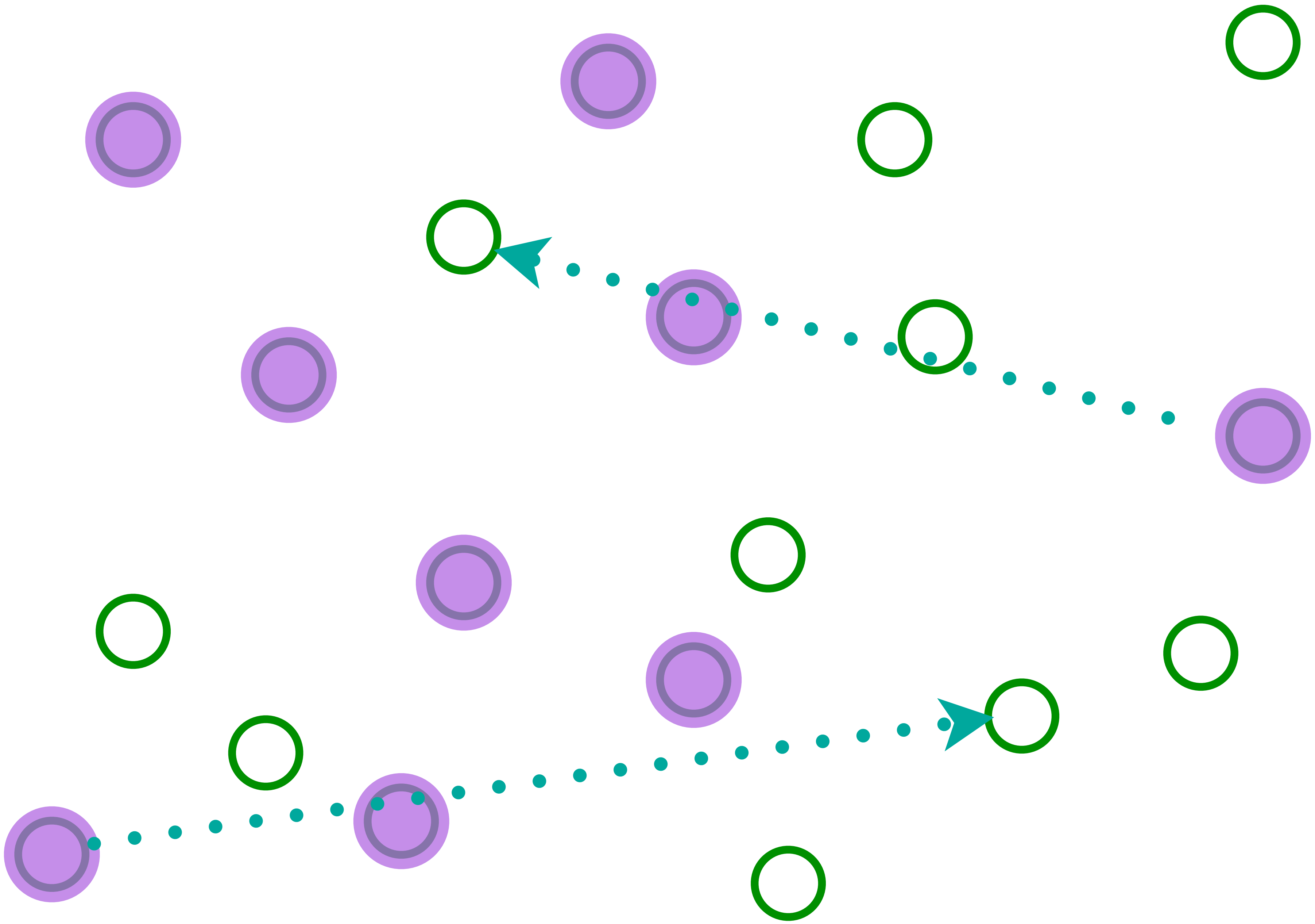

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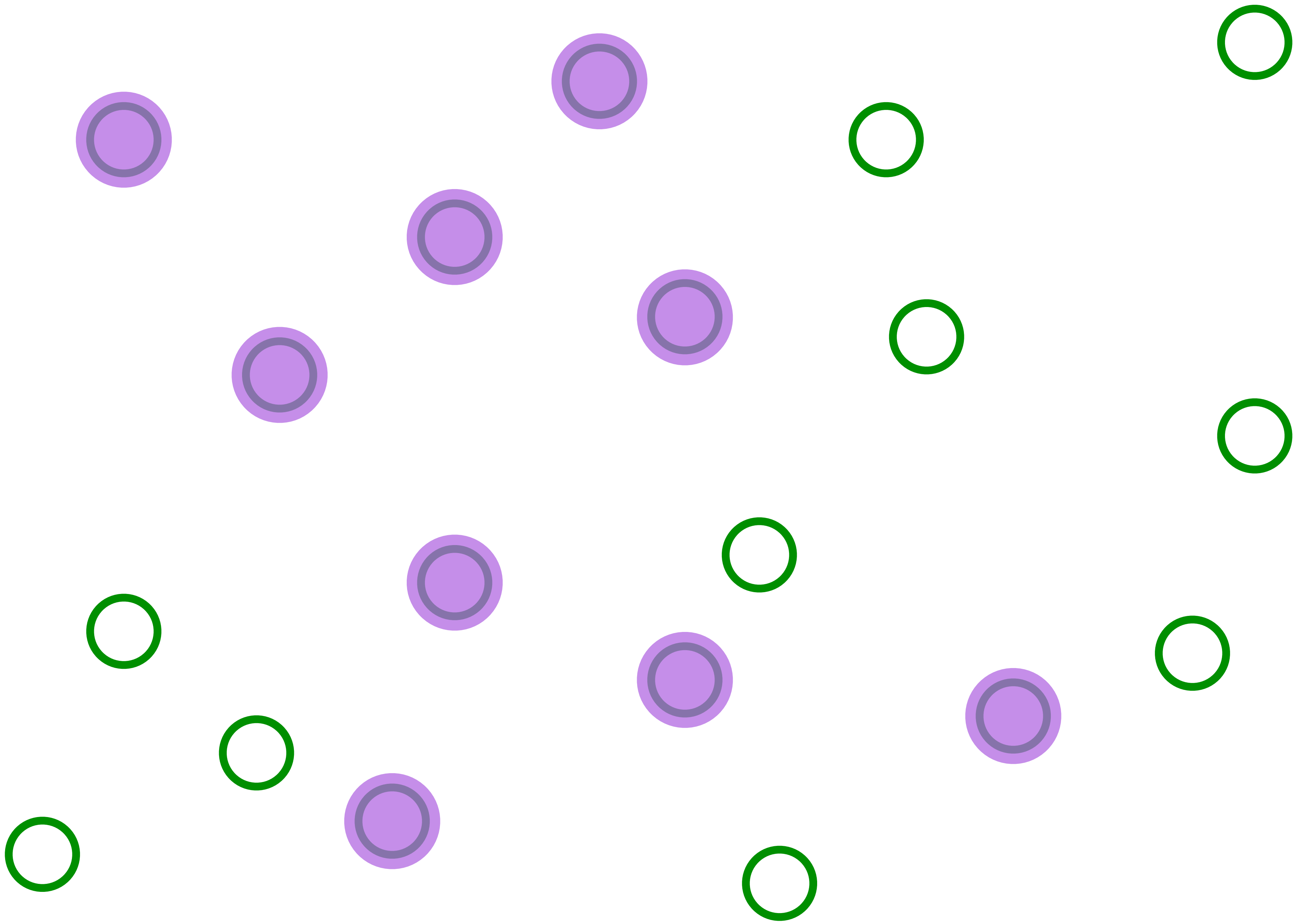


Entangle electrons pairwise randomly

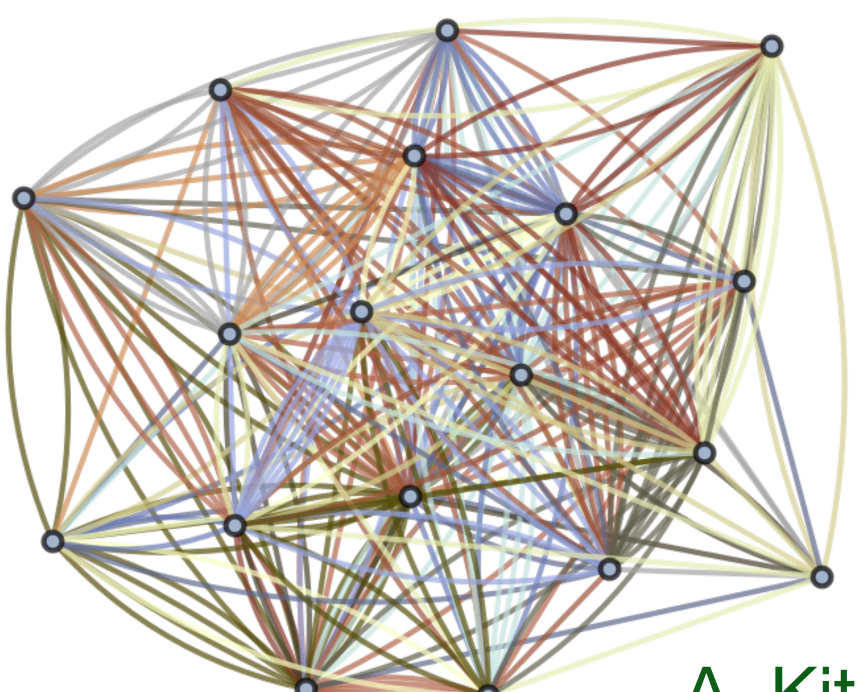
The Sachdev-Ye-Kitaev (SYK) model

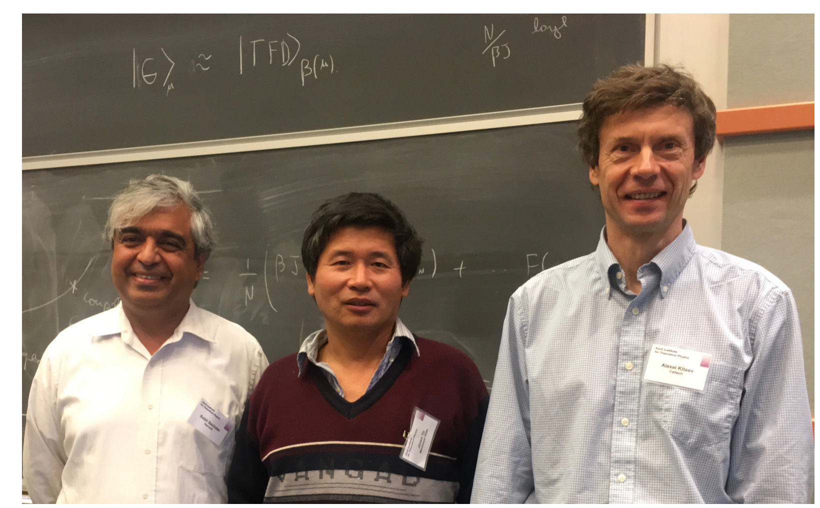
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,\gamma,\delta=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields Planckian metal.



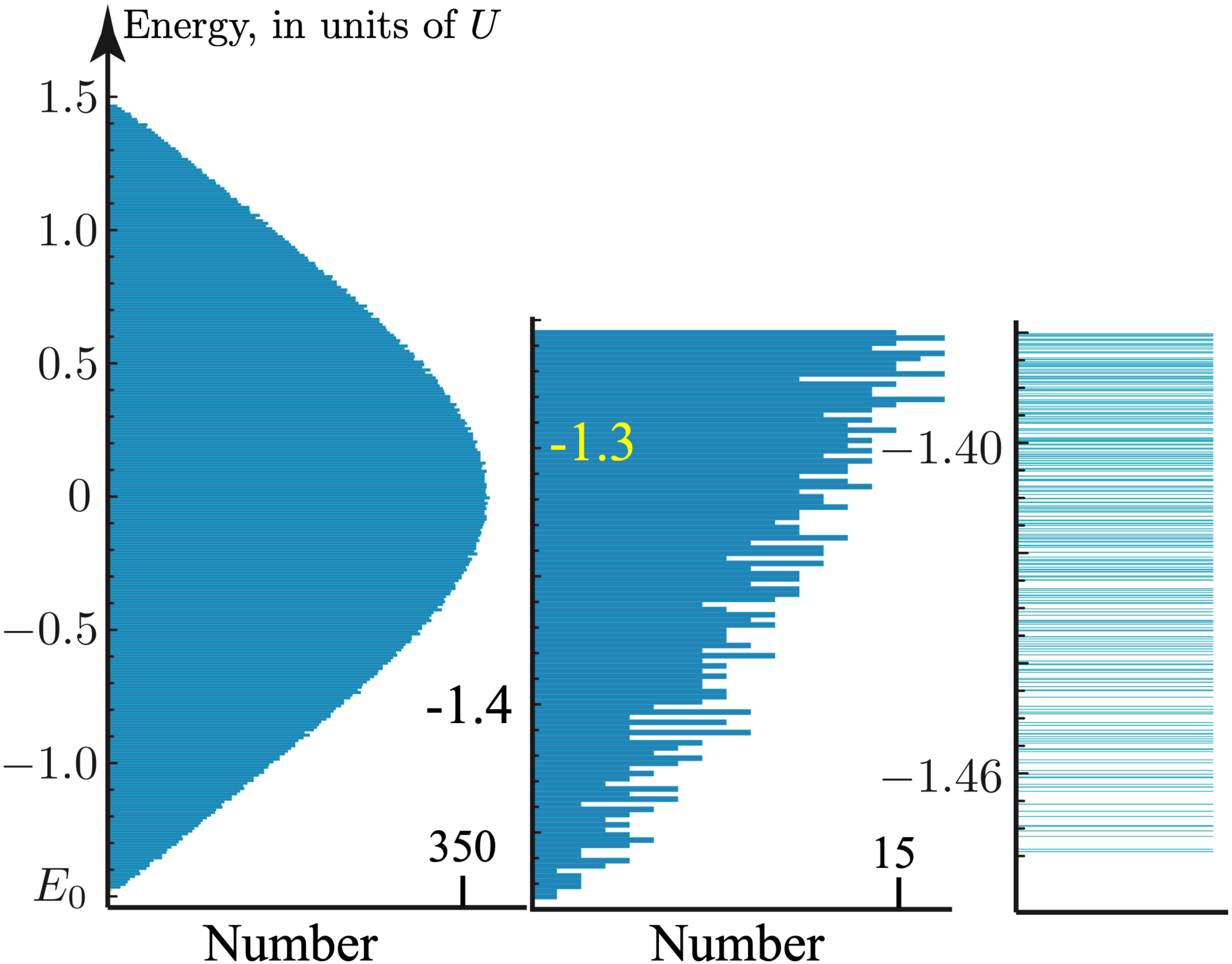


S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (20

Many-body density of states

$$D(E) = \sum_a \delta(E - E_a); \quad E_0 + E_a \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

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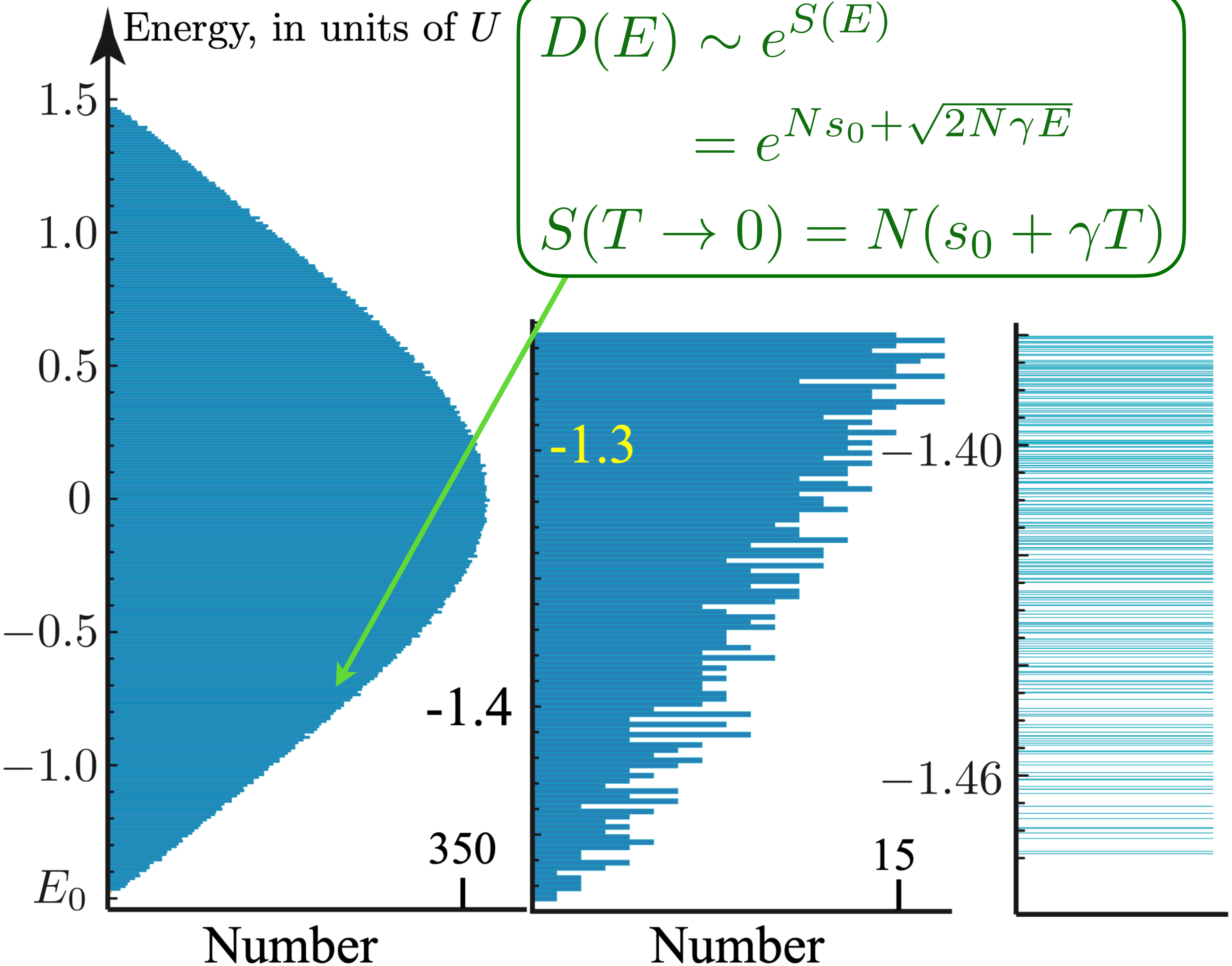

$$D(E) \sim e^{S(E)}$$
$$= e^{Ns_0 + \sqrt{2N\gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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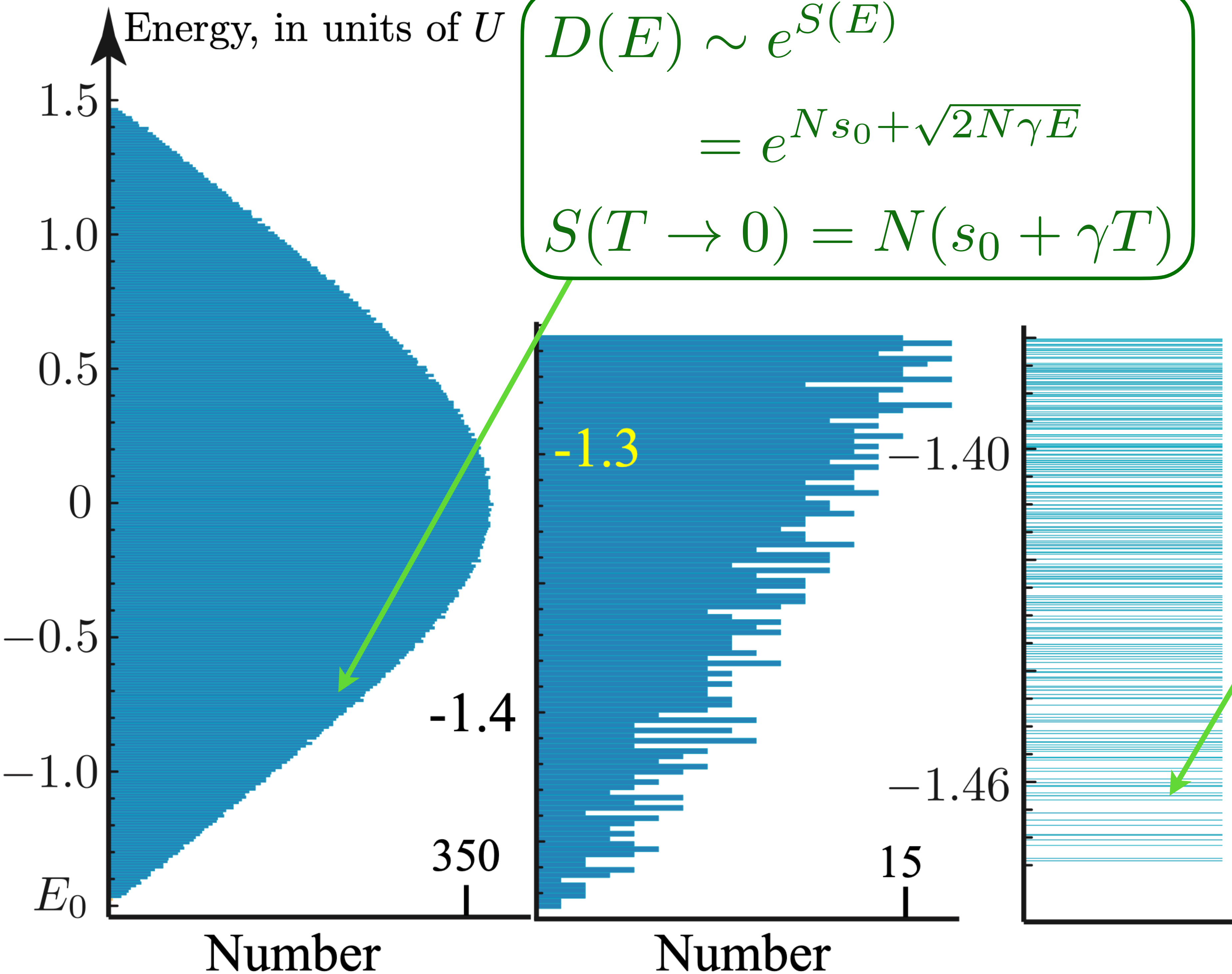

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No quasiparticle decomposition of many-body states

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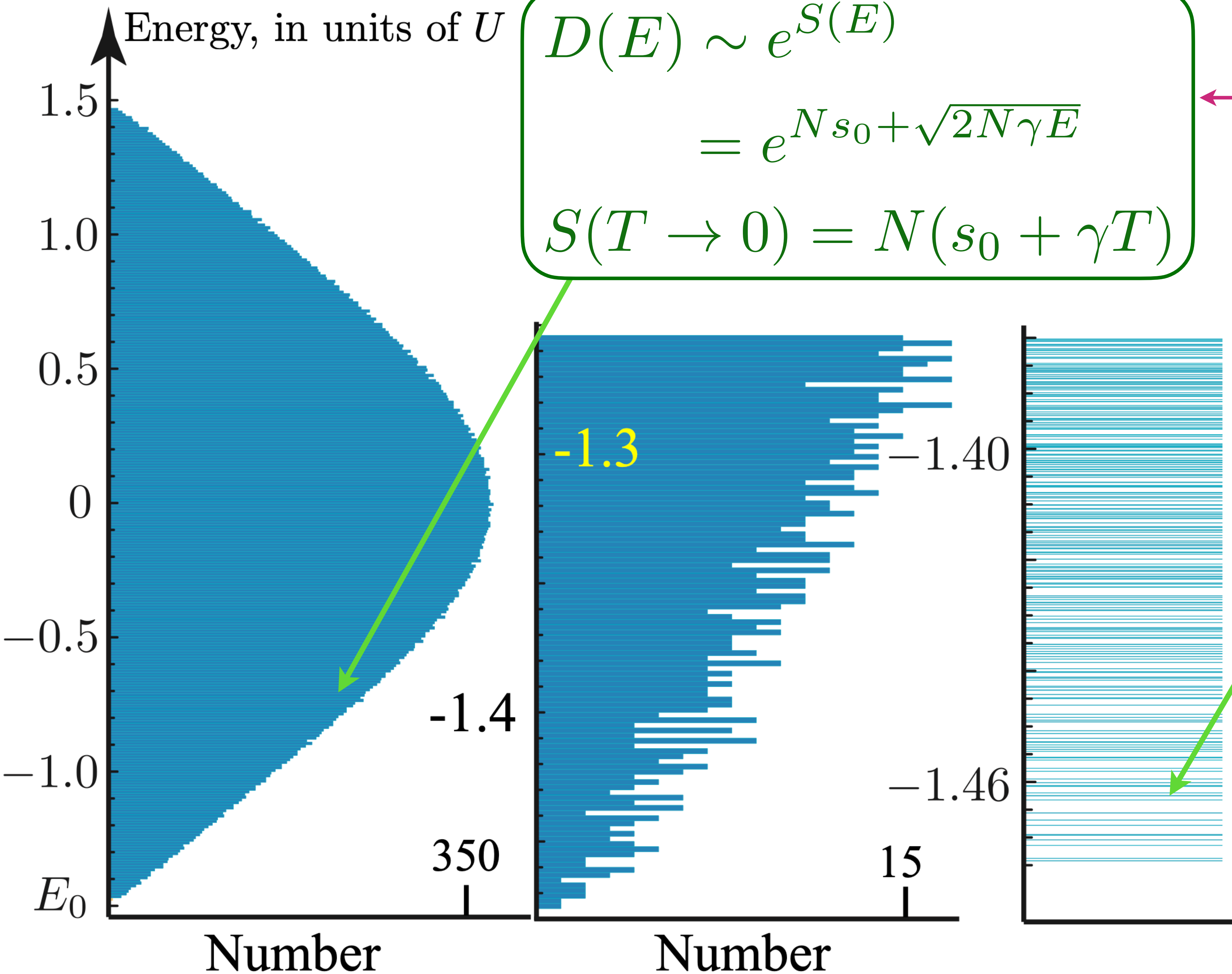

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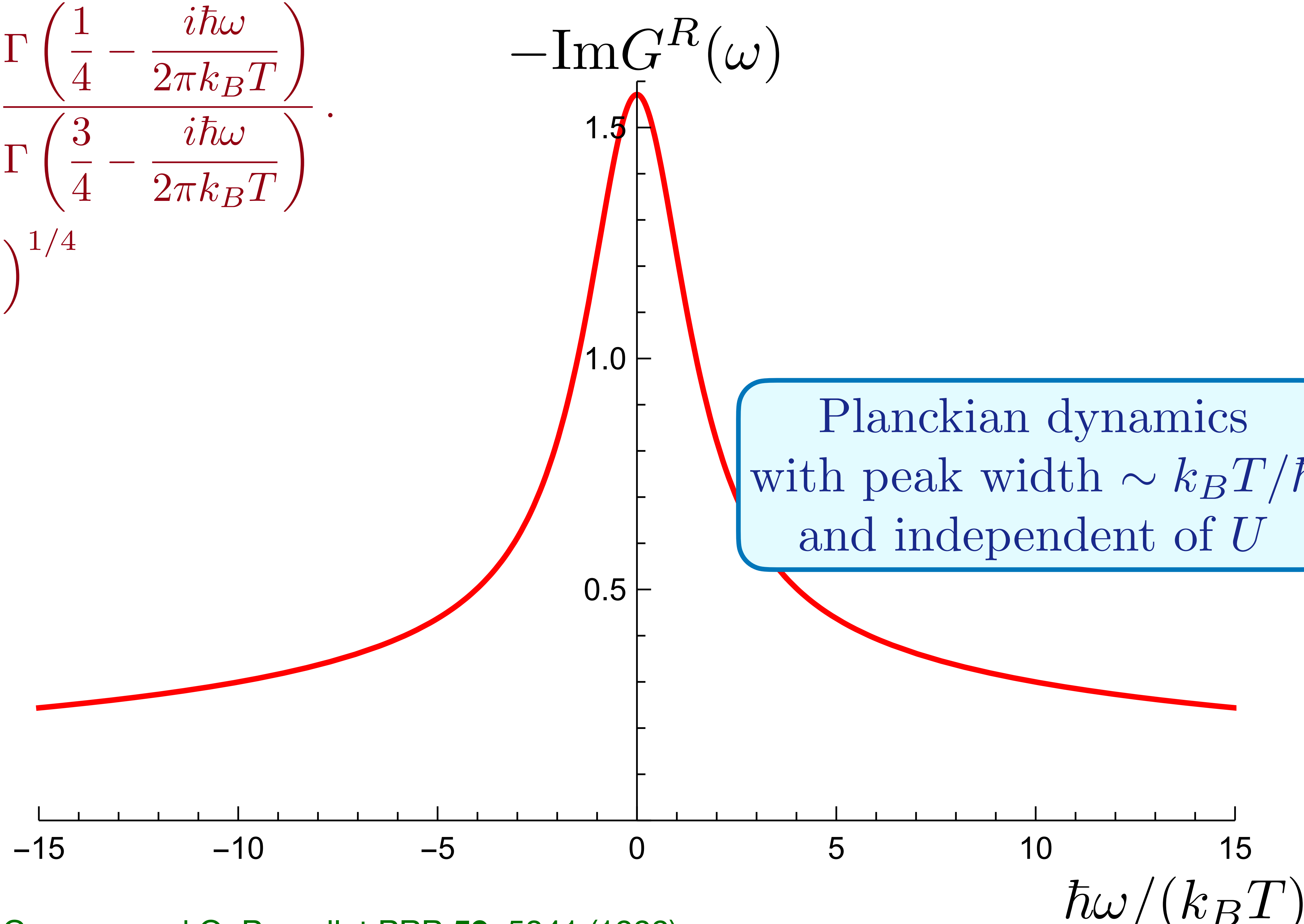
Complex SYK model

The SYK model

$$G^R_{\rm SYK}(\omega) = \frac{-iC}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T}\right)}.$$

$$C = \left(\frac{\pi}{U^2}\right)^{1/4}$$



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

- Low energy theory is scale invariant: characterized by relaxation at the Planckian scale $k_B T/\hbar$ and independent of U.

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  – Log correction to entropy
  $S(T \to 0) = N s_0 - 3/2 \ln(1/T)$.

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  – Linear-in-T resistivity in the random t-J model (after including *fractionalization*)

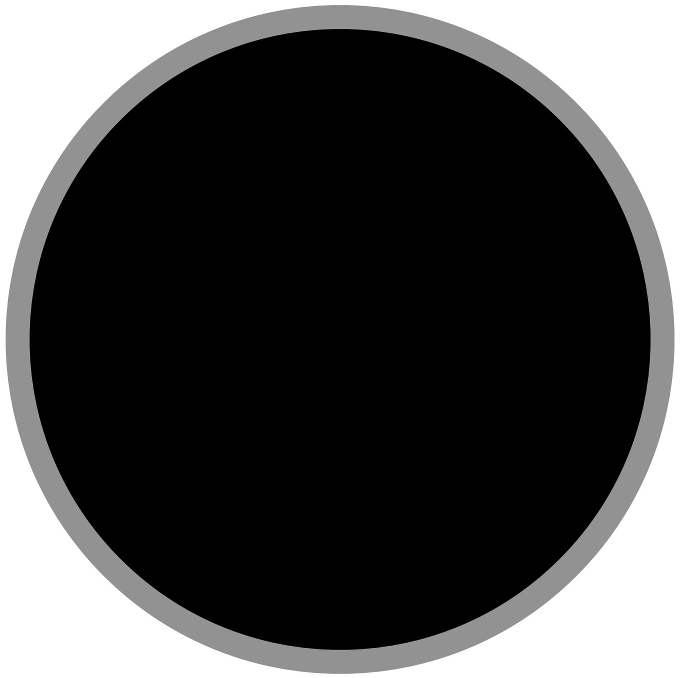
Haoyu Guo, Yingfei Guo, S. Sachdev, (2020)

1. Introduction to the cuprates
2. Random matrix and SYK models
3. Black holes
4. Back to the cuprates

Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

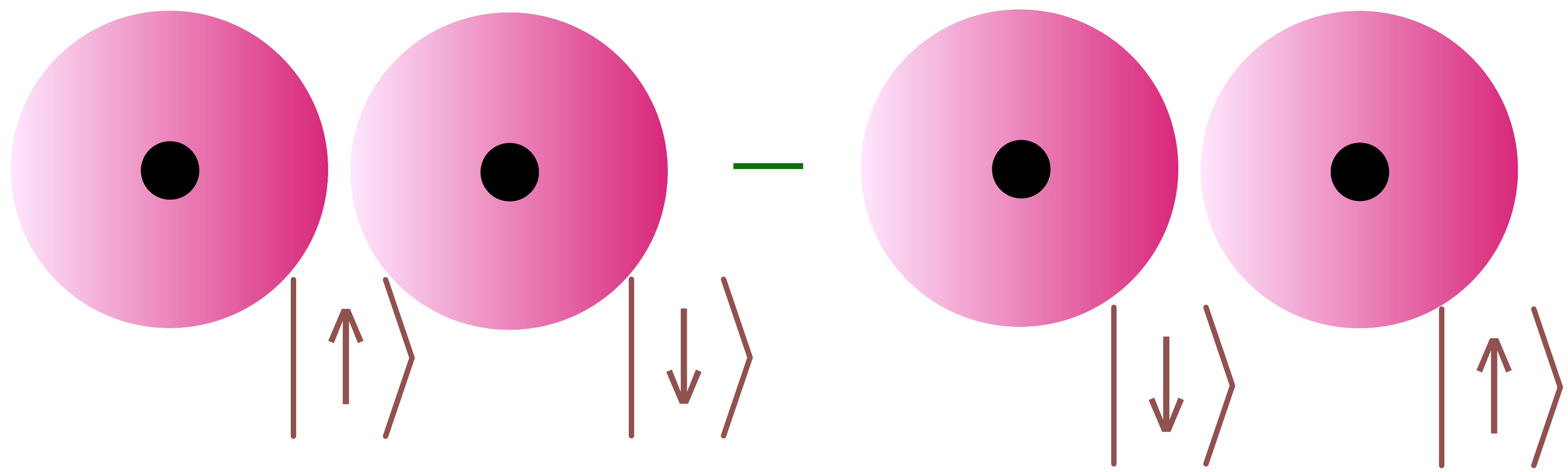


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

Quantum Entanglement across a black hole horizon



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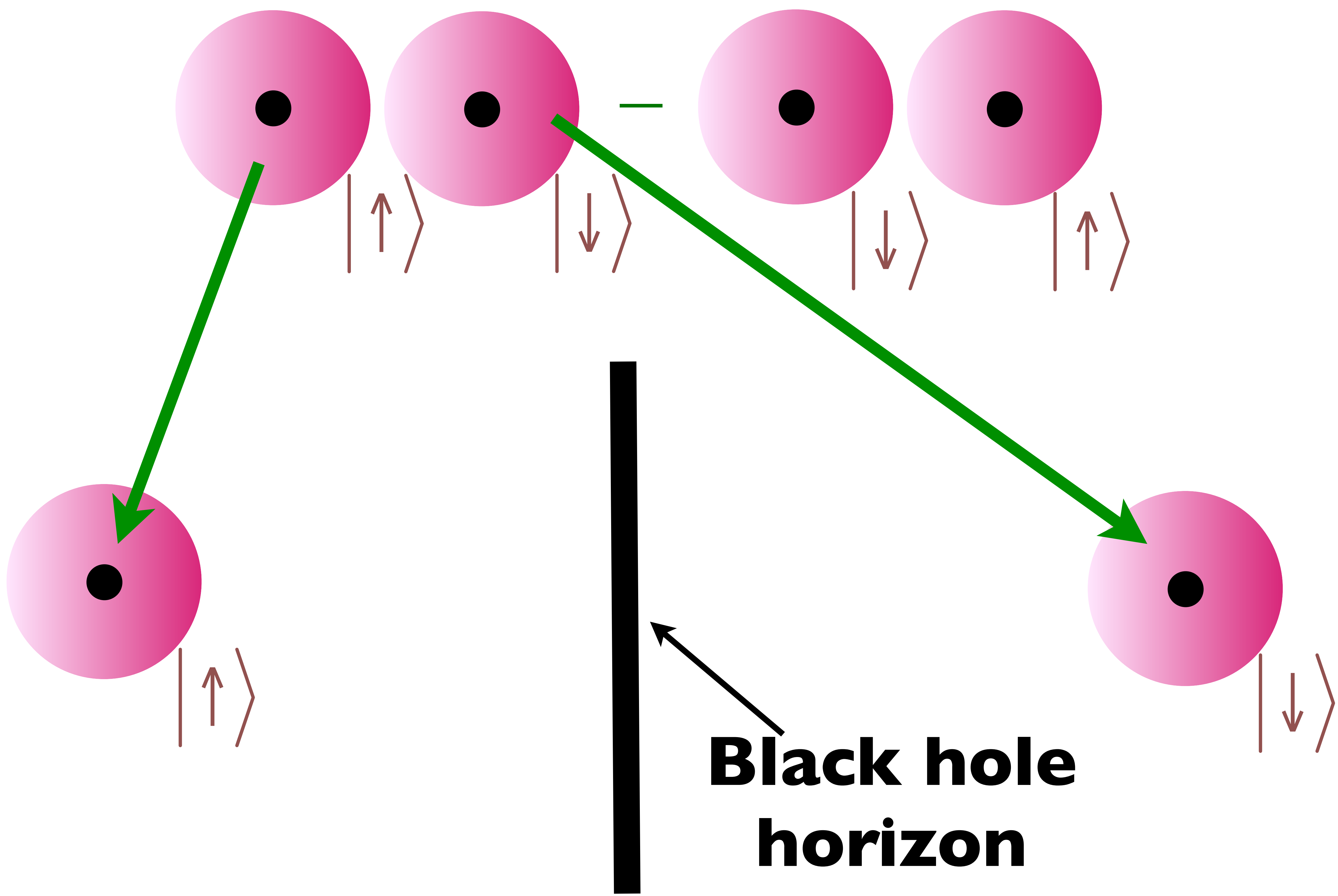

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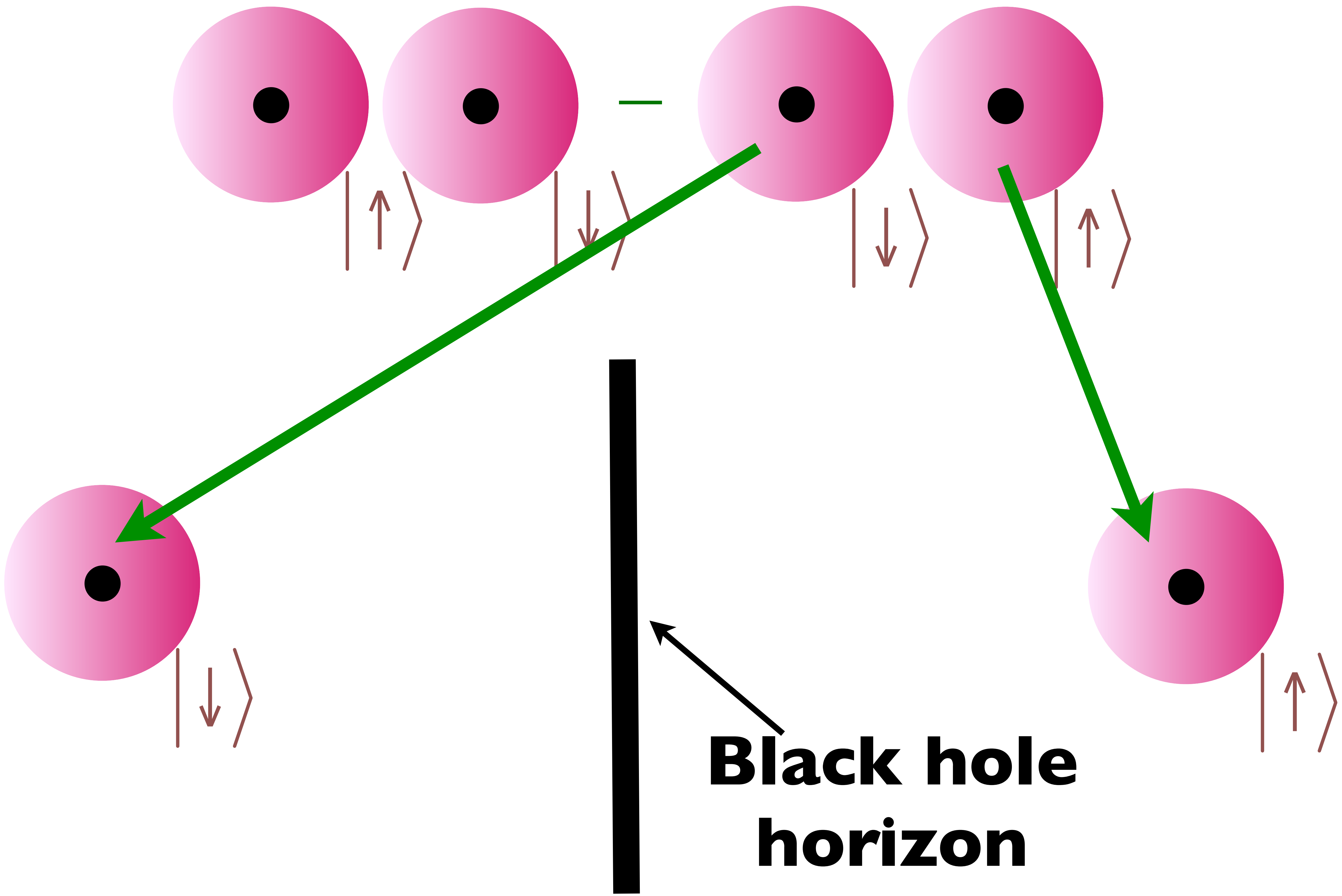
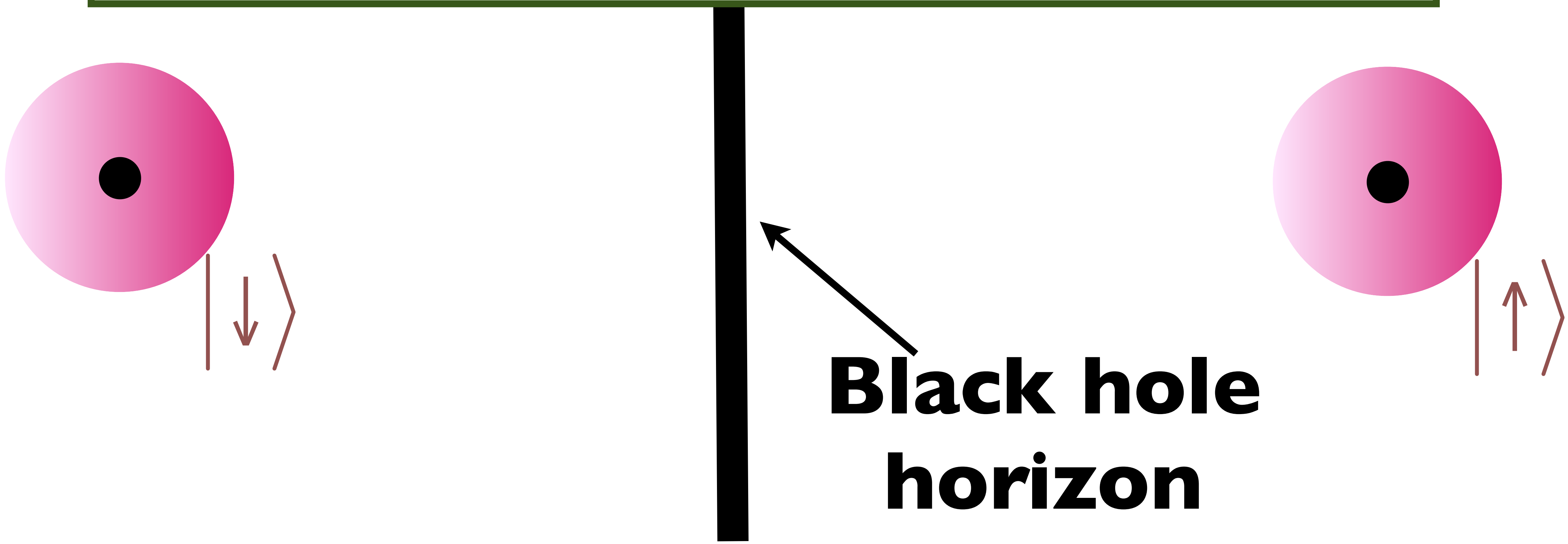


Quantum Entanglement across a black hole horizon

To an outside observer, the state of the electron inside the black hole is an unknown: black holes have an entropy and a temperature



Quantum Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy, S_{BH} is proportional to their surface area.

Bekenstein (1973)
Hawking (1974)

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- What are the microscopic degrees of freedom that lead to black hole entropy (strings ?)

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Bekenstein (1973)
Hawking (1974)

- What are the microscopic degrees of freedom that lead to black hole entropy (strings ?)
- How do we keep track of the quantum information carried by additional matter added to the black hole, and what happens to it when the black hole evaporates ?

Quantum Black holes

- Black holes have an entropy and a temperature, T_H.
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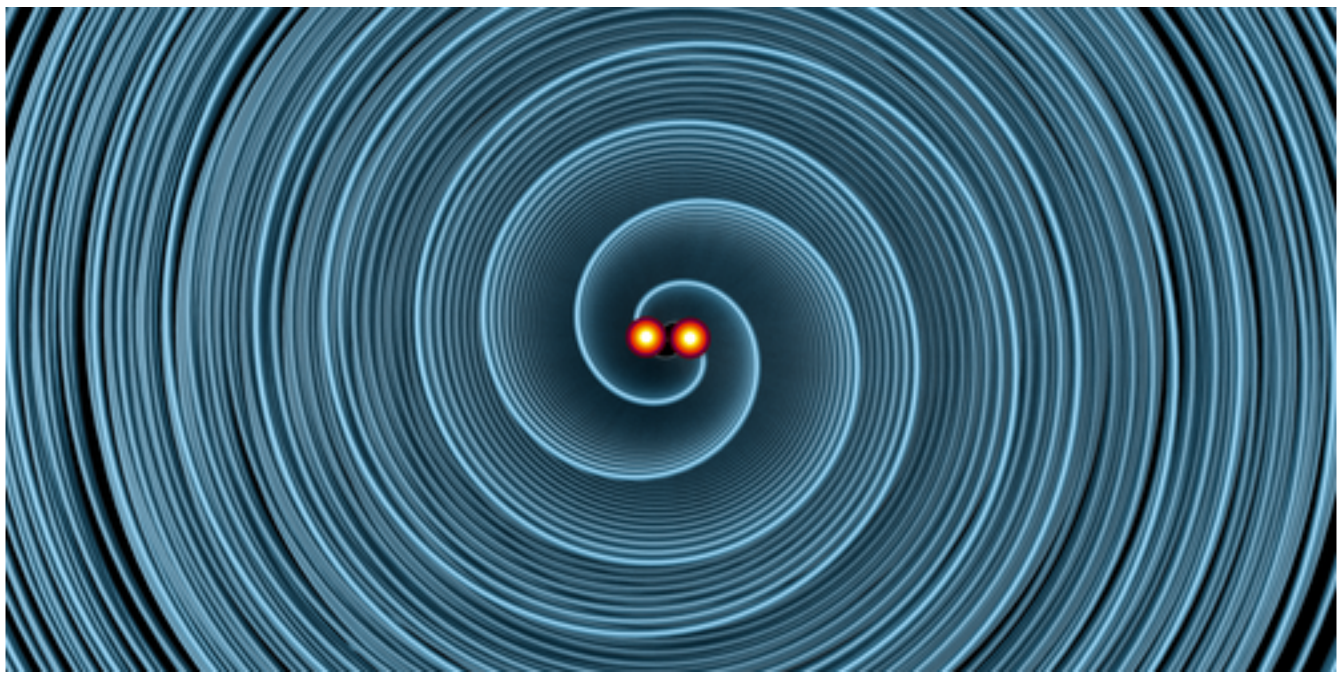
Recent developments: Many aspects of black hole entropy and quantum information are *independent* of microscopic (*i.e.* UV) details, and can be understood in a semiclassical theory of Einstein gravity.

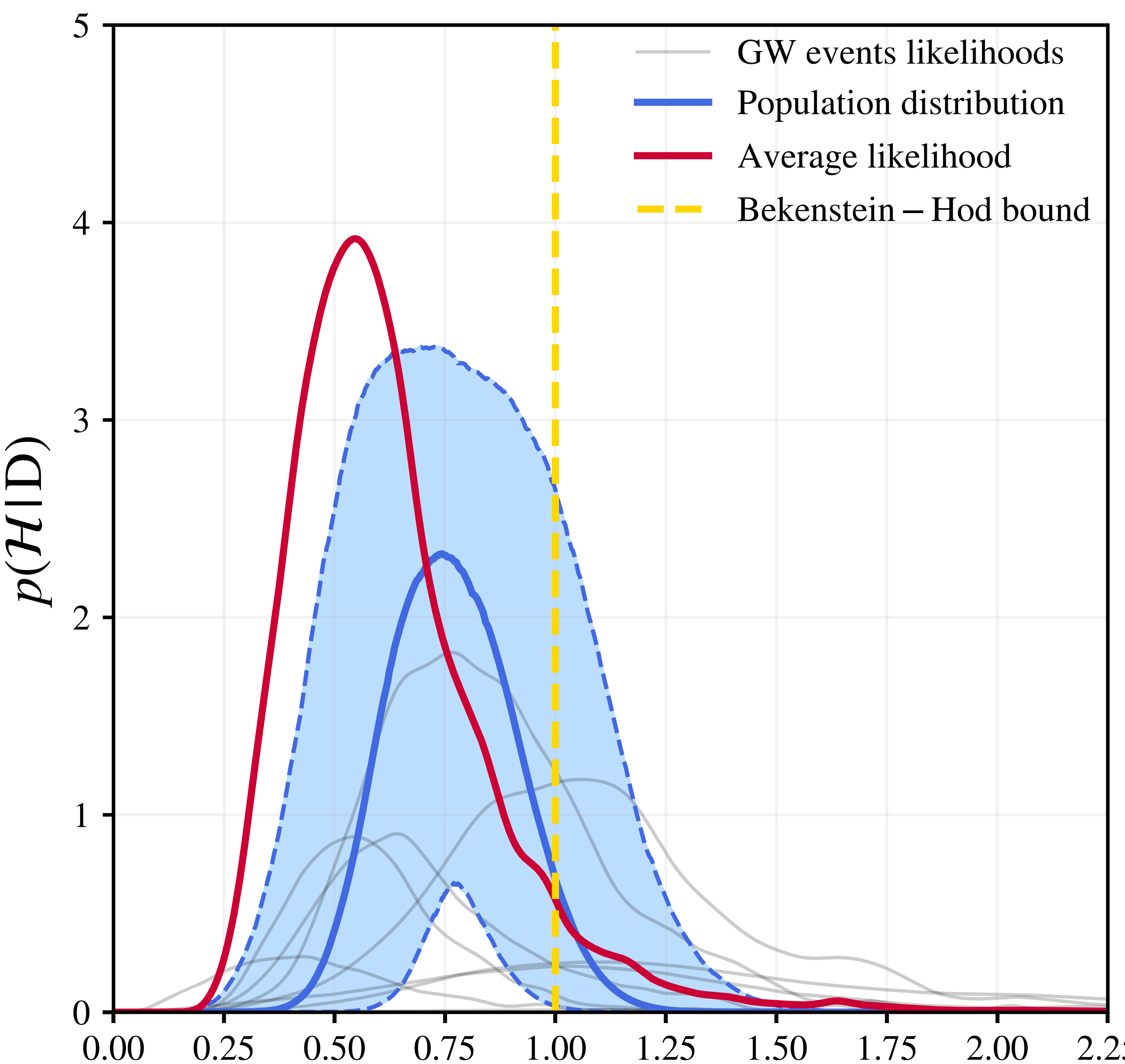
April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, PRL **126**, 161102 (2021);
S. Hod, PRD **75**, 064013 (2007)



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T_H}$$



$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T_H}$$

Quantum Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy, S_{BH} is proportional to their surface area.
- They relax to thermal equilibrium with a Planckian rate $k_B T_H/\hbar$, implying connection to non-quasiparticle dynamics.

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PRL **105**, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.

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String theorists (2010): Models with random couplings ????

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String theorists (2010): Models with random couplings ????

Current understanding: Many key properties of suitable random Hamiltonians self-average, and allow easier access to the physics of strongly entangled and chaotic quantum matter

Thermodynamics of quantum black holes:

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar}\, \mathcal{S}^{(3+1)}_{\text{Einstein gravity}}[g_{\mu\nu}]\right)$$

Metric of spacetime

In general, this summation is not well defined, because to the uncontrollably large number of spacetime configurations.

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$$= \exp\Big(S_{BH}\Big) \times \Bigg(\quad ....????... \quad \Bigg)$$

Metric of spacetime

Gibbons, Hawking (1977)

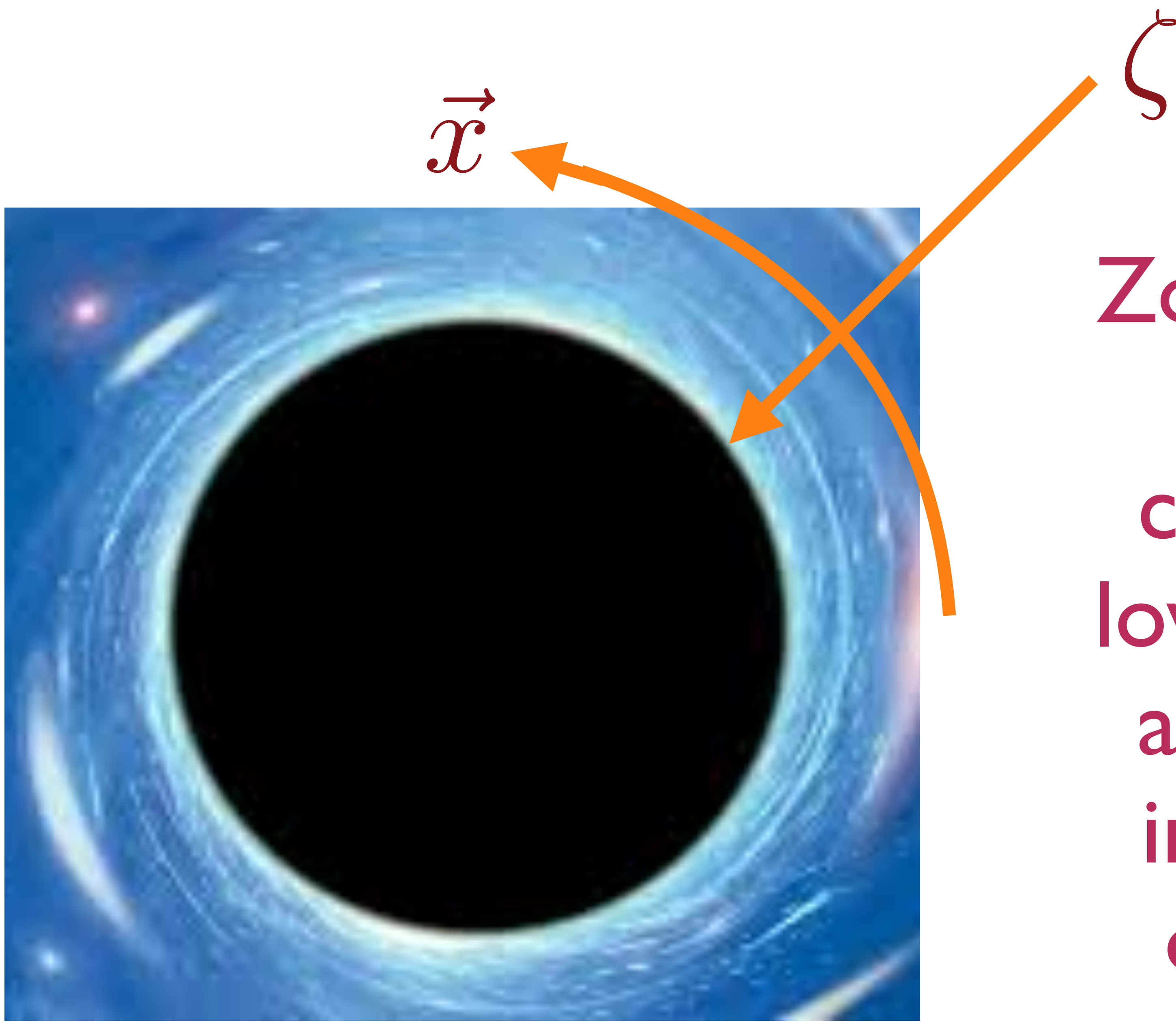
$$S_{BH} = \frac{A(T_H)c^3}{4G\hbar}$$

With $\mathcal{Z} = \mathrm{Tr} e^{-\mathcal{H}/(k_B T_H)}$,
$\hbar/(k_B T_H)$ is the length of the Euclidean time circle.
$A(T_H)$ is the area of the black hole horizon at a temperature T_H.

Interpretation: Black hole entropy is
entanglement entropy across the horizon.

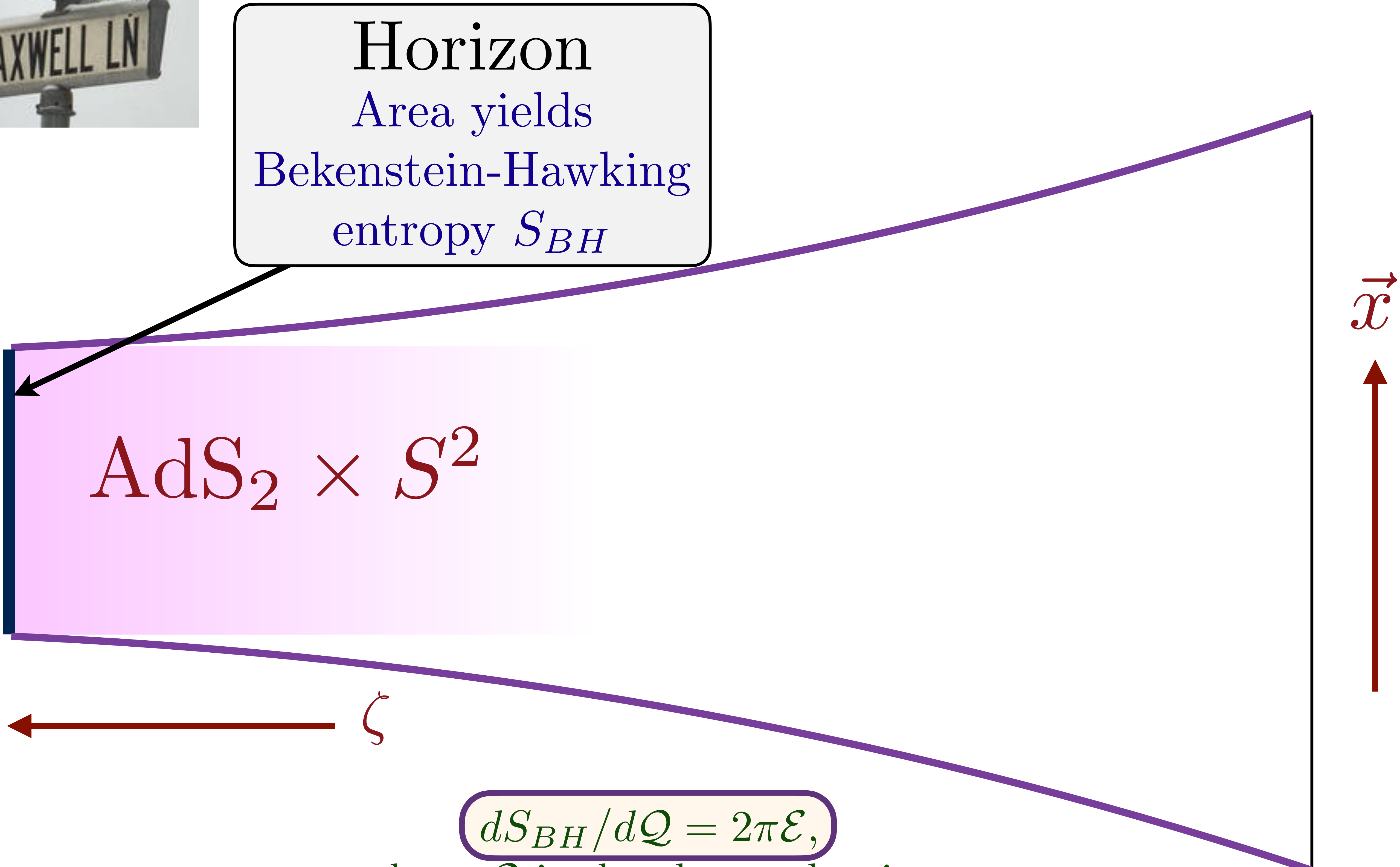
Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space (ζ) and one time dimension

SYK model and charged black holes

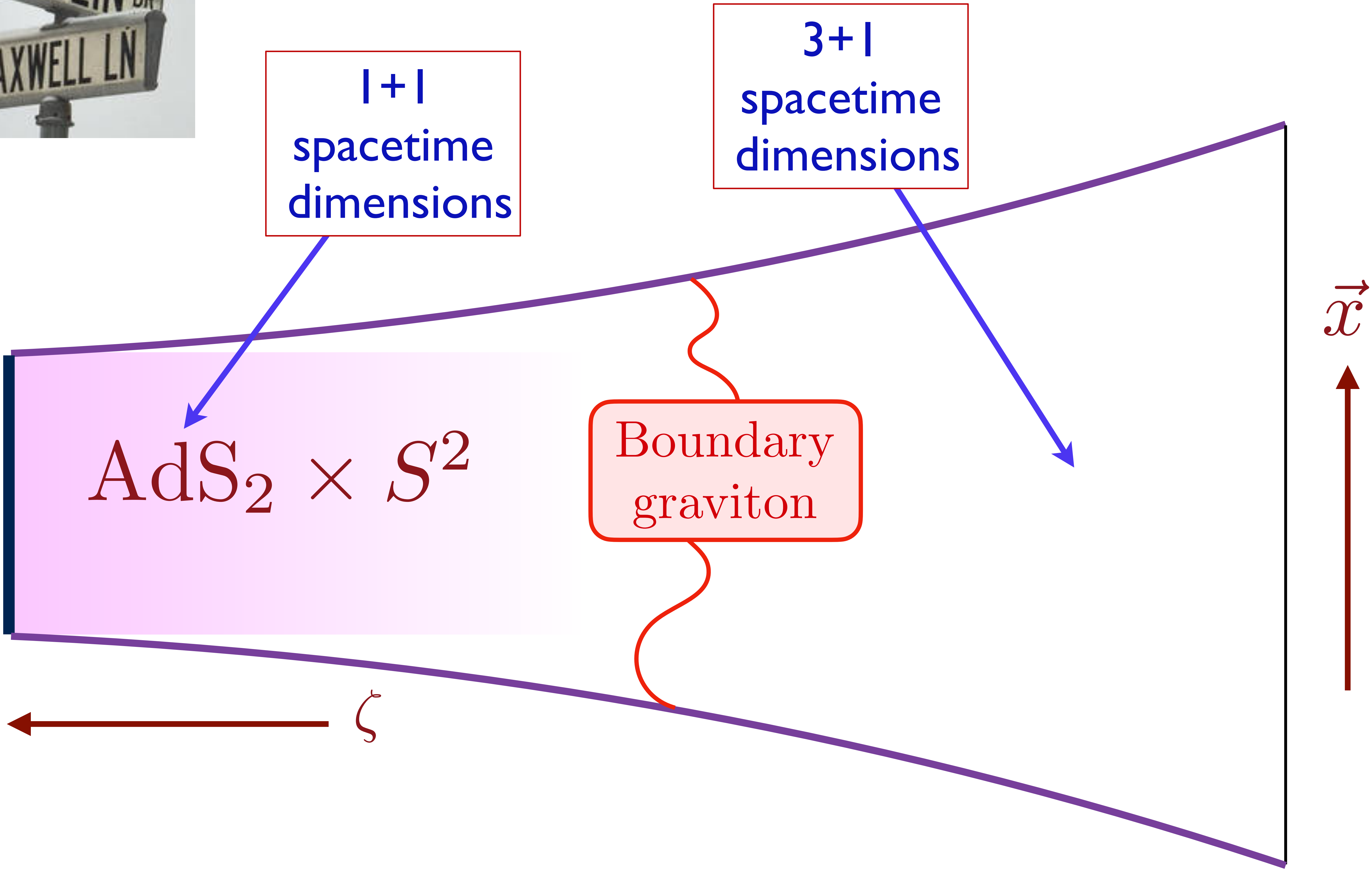


$$dS_{BH}/d\mathcal{Q} = 2\pi\mathcal{E},$$

where $\mathcal{Q}$ is the charge density,
and $\mathcal{E}$ is the electric field on the horizon.

Sen (2005); Sachdev (2015)

SYK model and charged black holes



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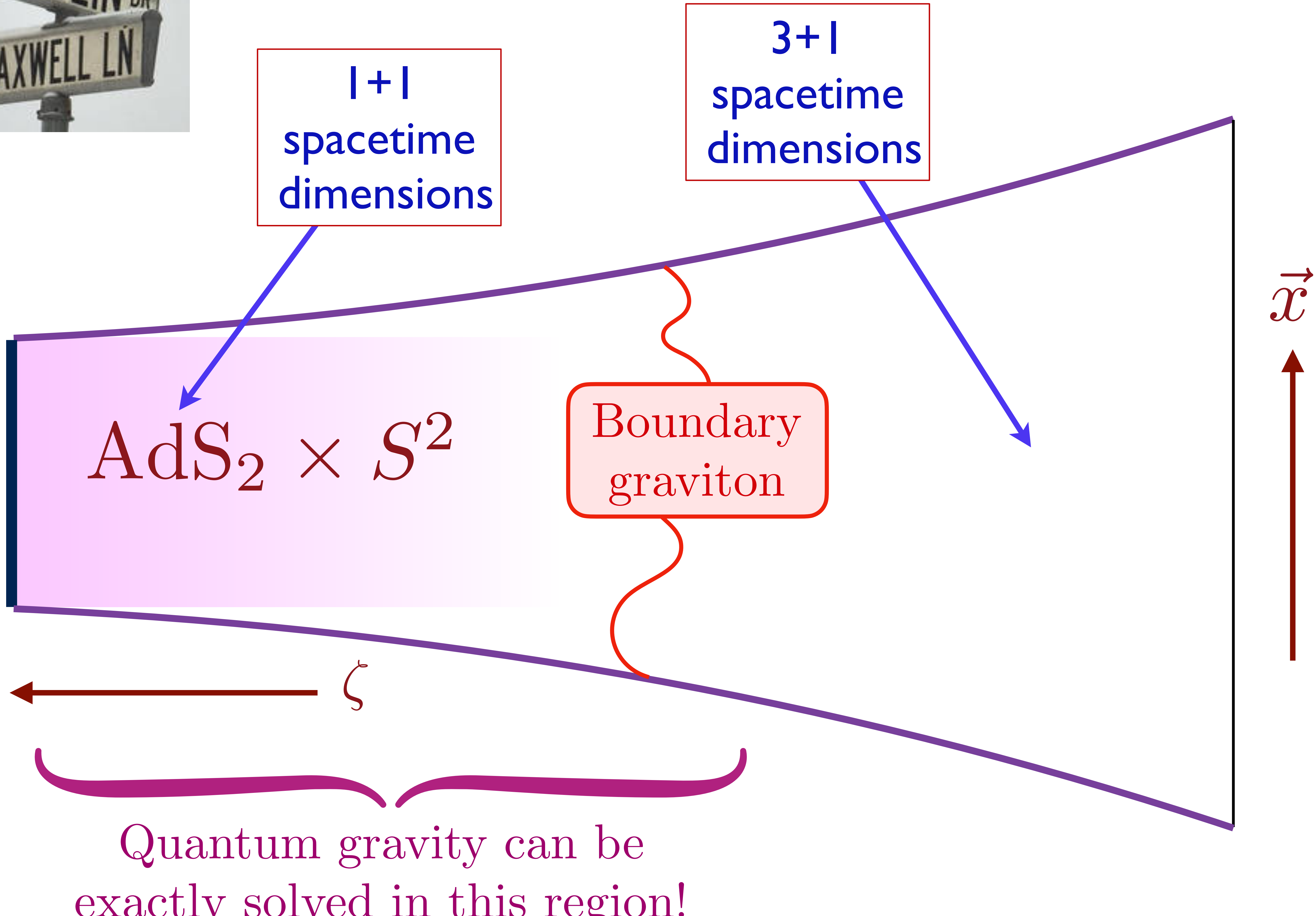

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_{\mu} \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_{\mu}]\right)$$

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Gibbons, Hawking (1977)

$$S_{BH}(T_H \to 0, \mathcal{Q}) = \frac{A(T_H)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2} T_H}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T_H = 0$.

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$$\approx e^{S_{BH}} \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(1+1)}_{\text{JT gravity of AdS}_2\text{ and boundary}}[g_{\mu\nu}, A_\mu]\right)$$

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$$= e^{S_{BH}} \int \mathcal{D}f(\tau) \exp\Big(-\text{Boundary graviton action } [f]$$
$$= \text{SYK low energy action}[f]!\Big)$$

$$S(T_H \to 0, \mathcal{Q}) = S_{BH} - \frac{3}{4}\ln\left(\frac{\hbar c^5}{G T_H^2}\right)$$

$$S_{BH} = \frac{A(T_H)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2} T_H}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T_H = 0$. The $\ln(1/T_H)$ term is the contribution of the boundary graviton, and is also independent of UV details!

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)

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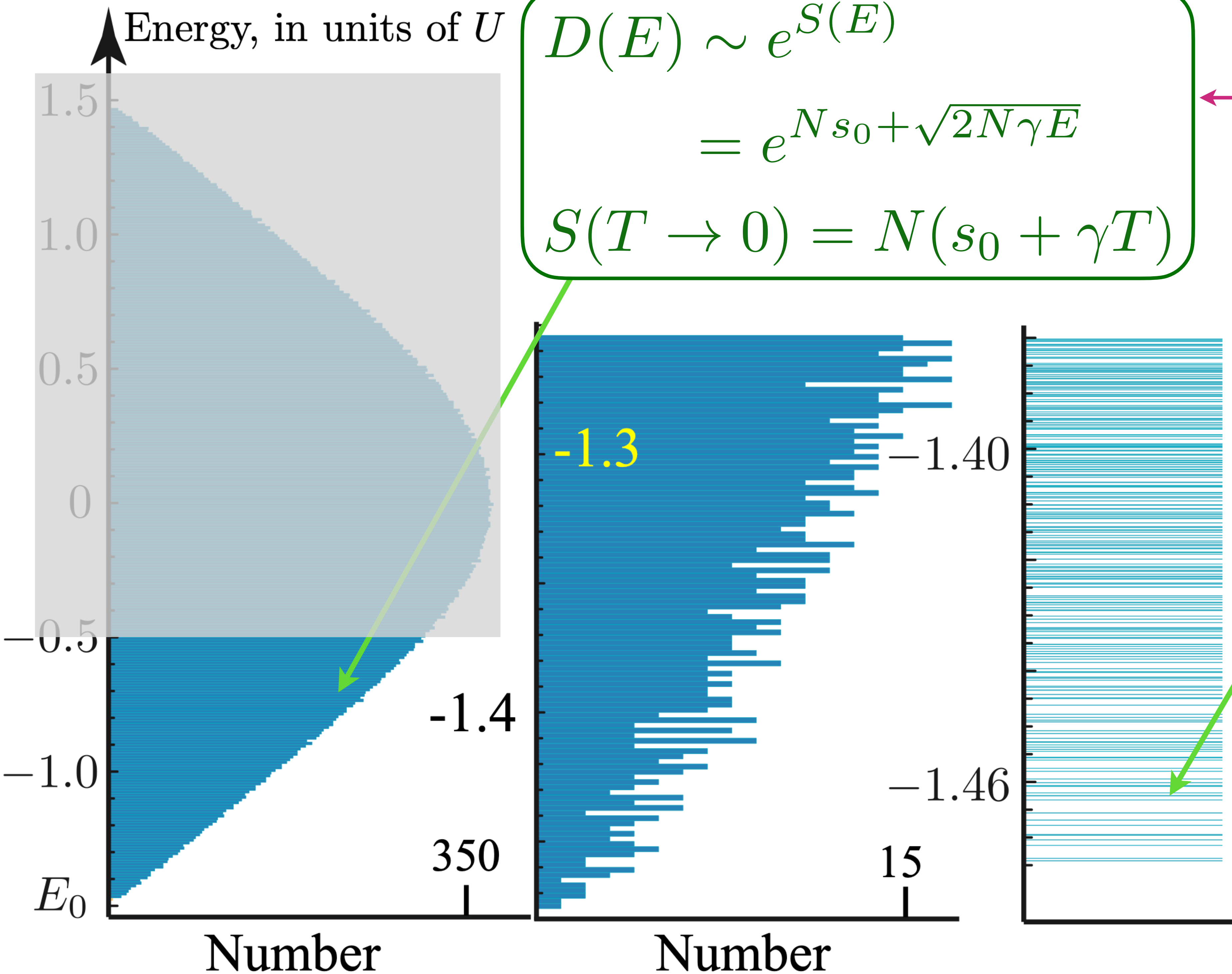

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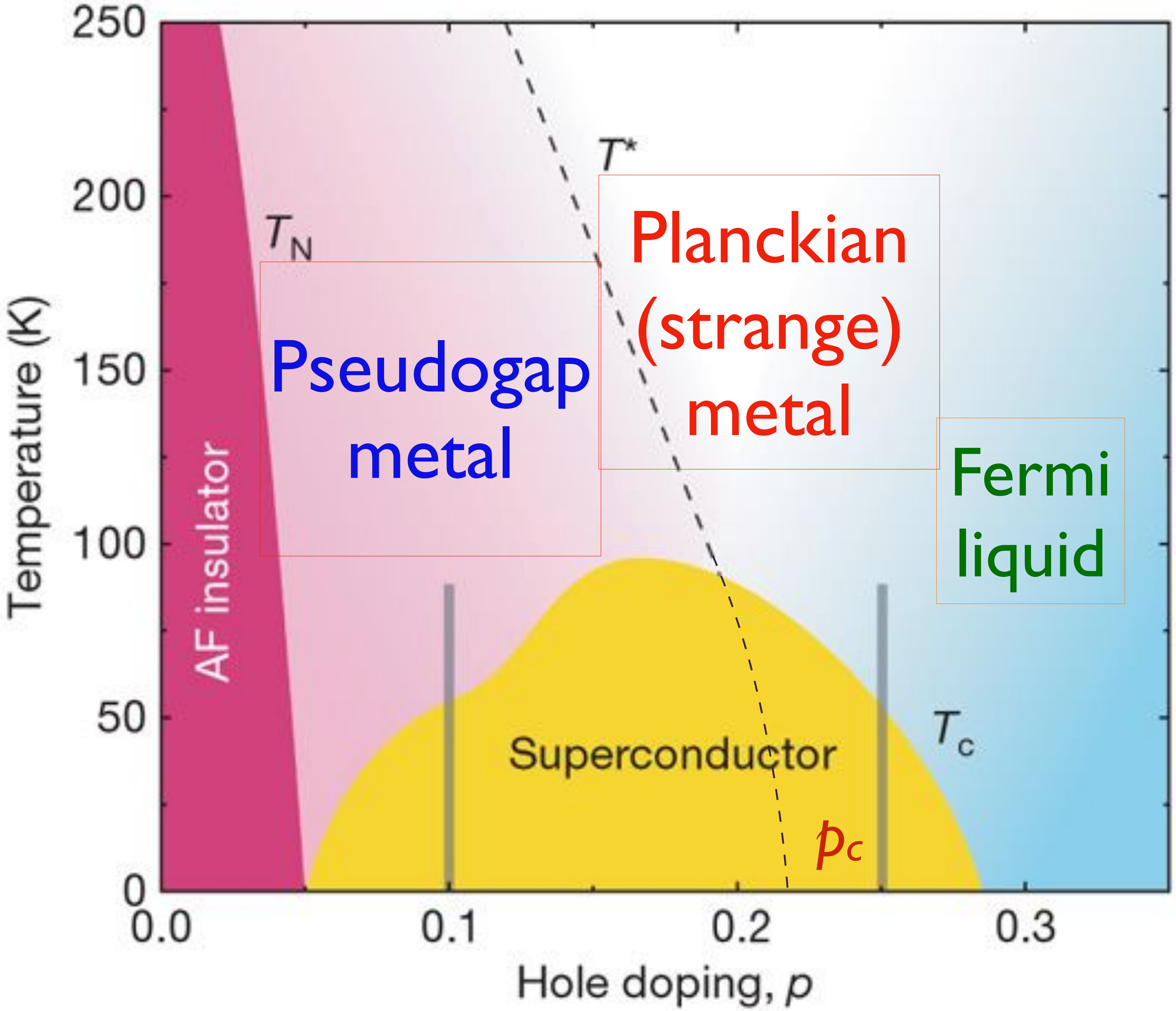
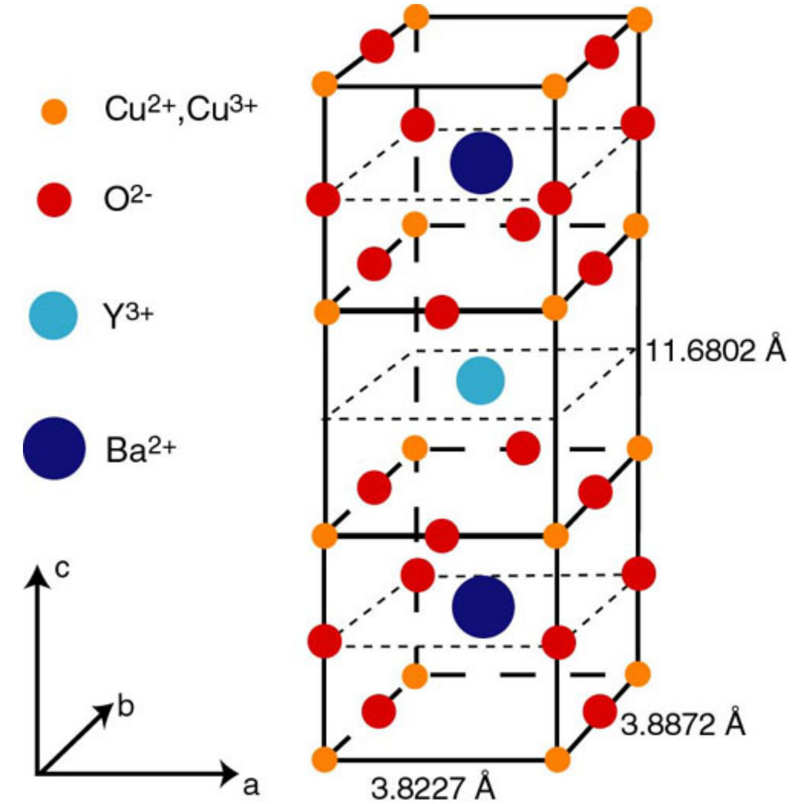
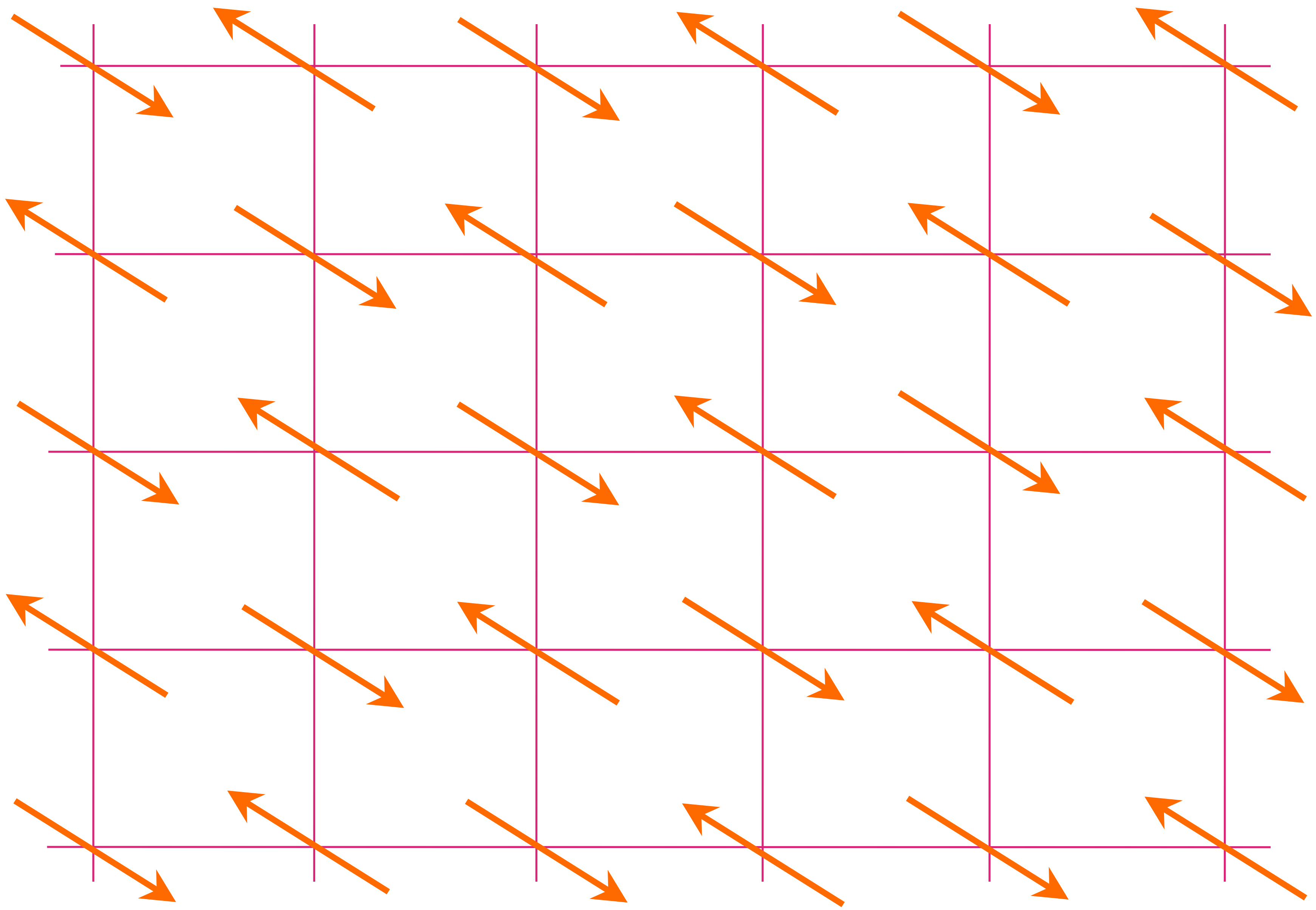


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap metal
Planckian (strange) metal
Fermi liquid
Superconductor
T_c
p_c



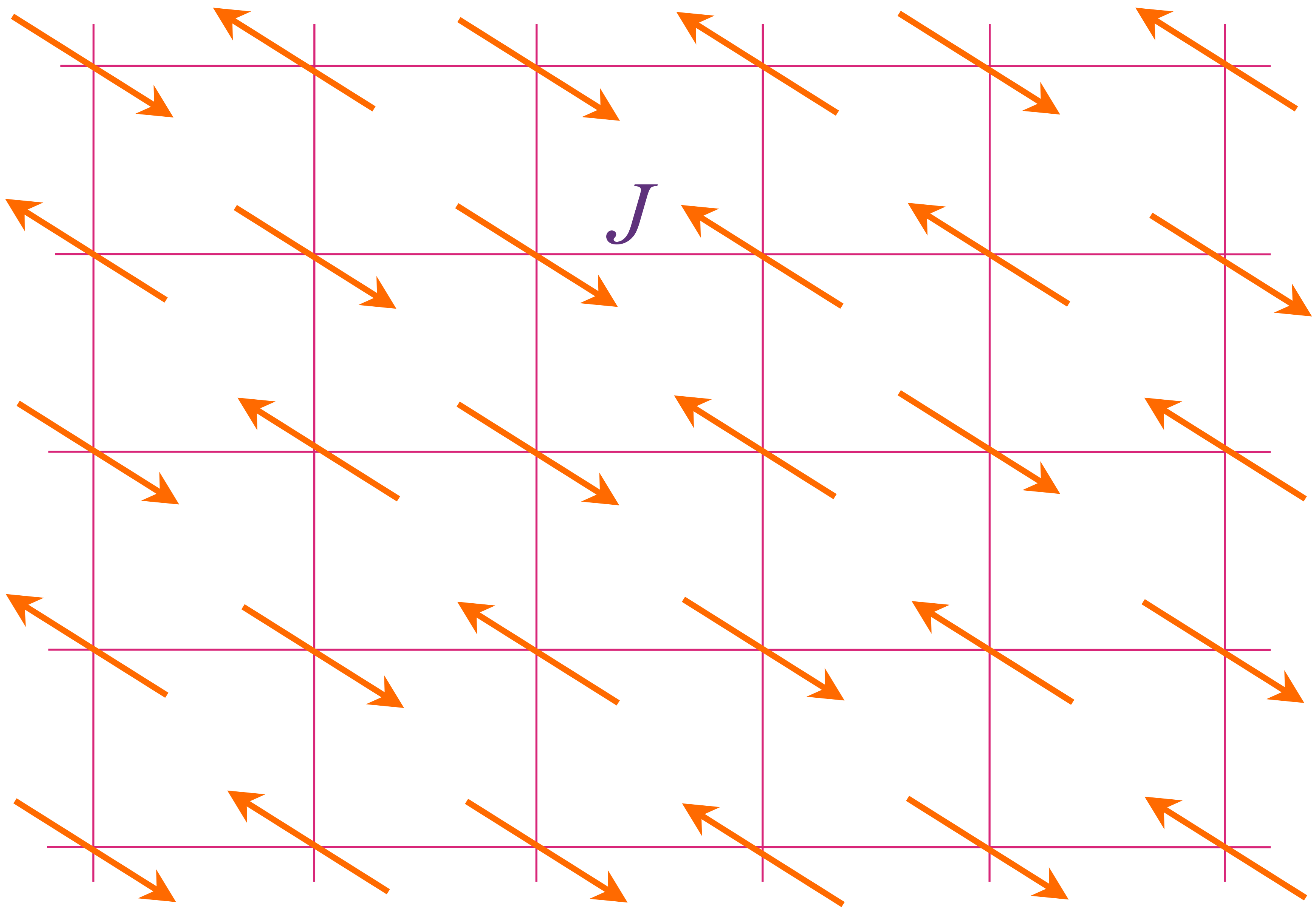
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Insulating antiferromagnet



$p = 0$

Insulating antiferromagnet



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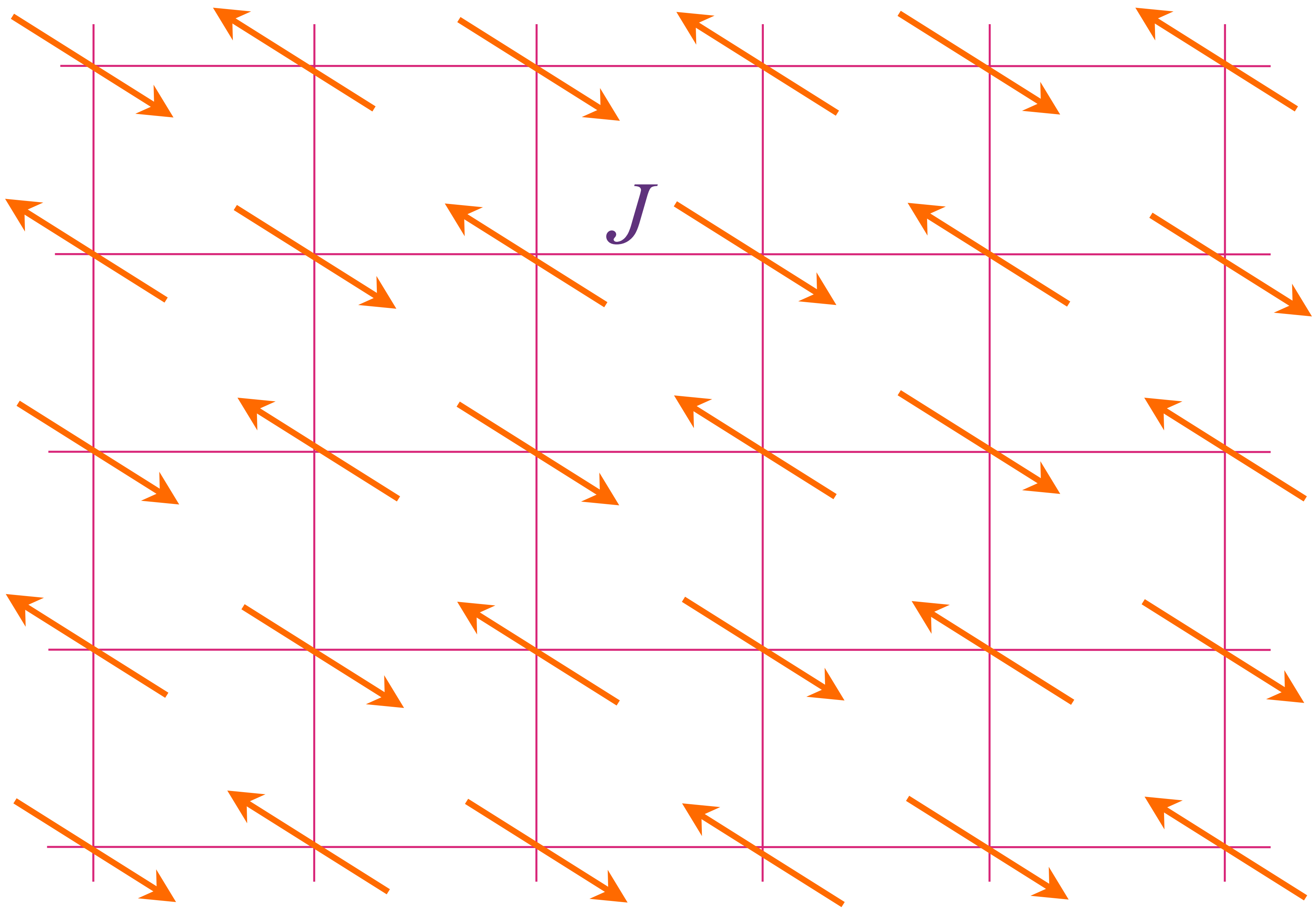
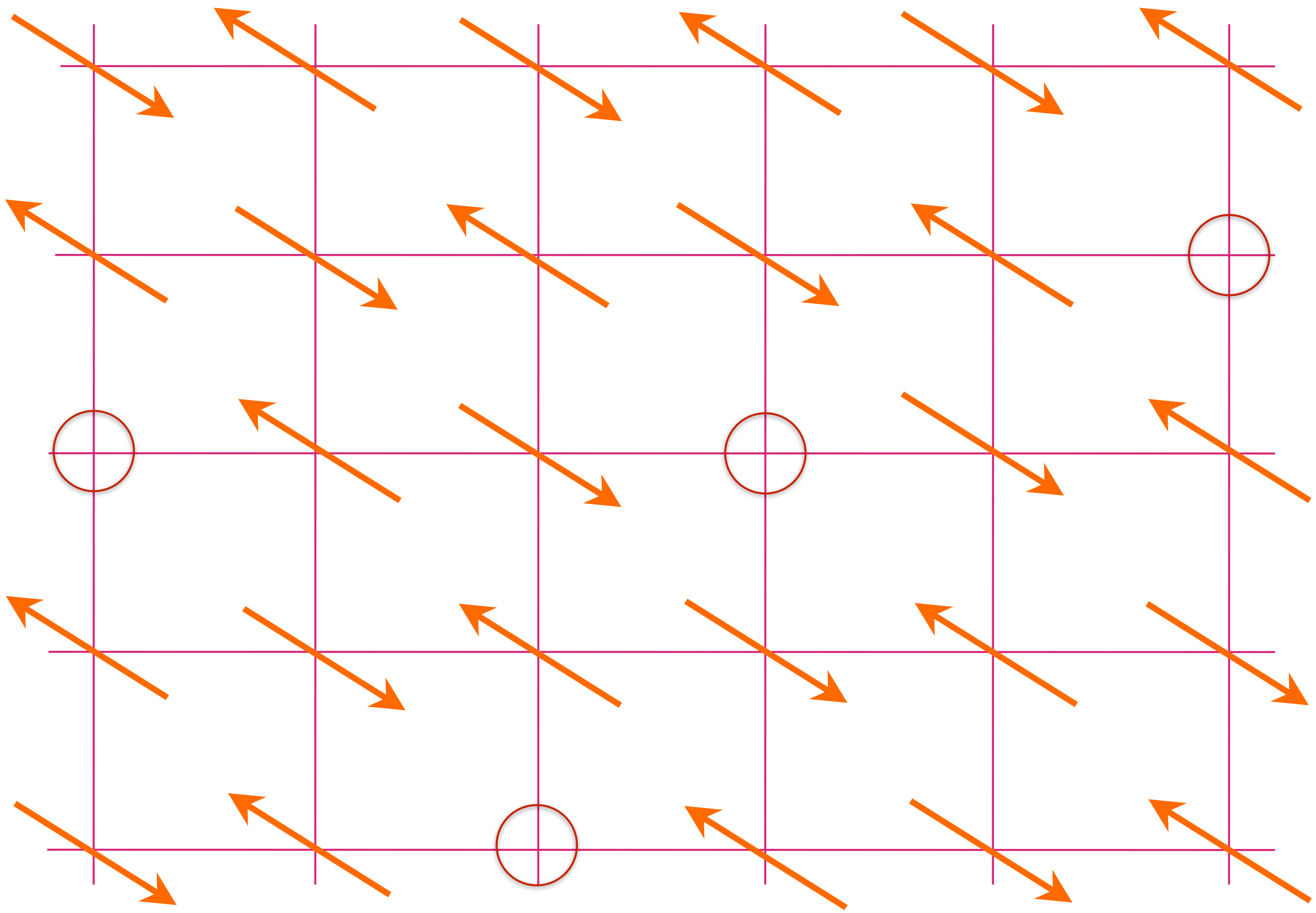


$p=0$

Antiferromagnet doped with hole density p



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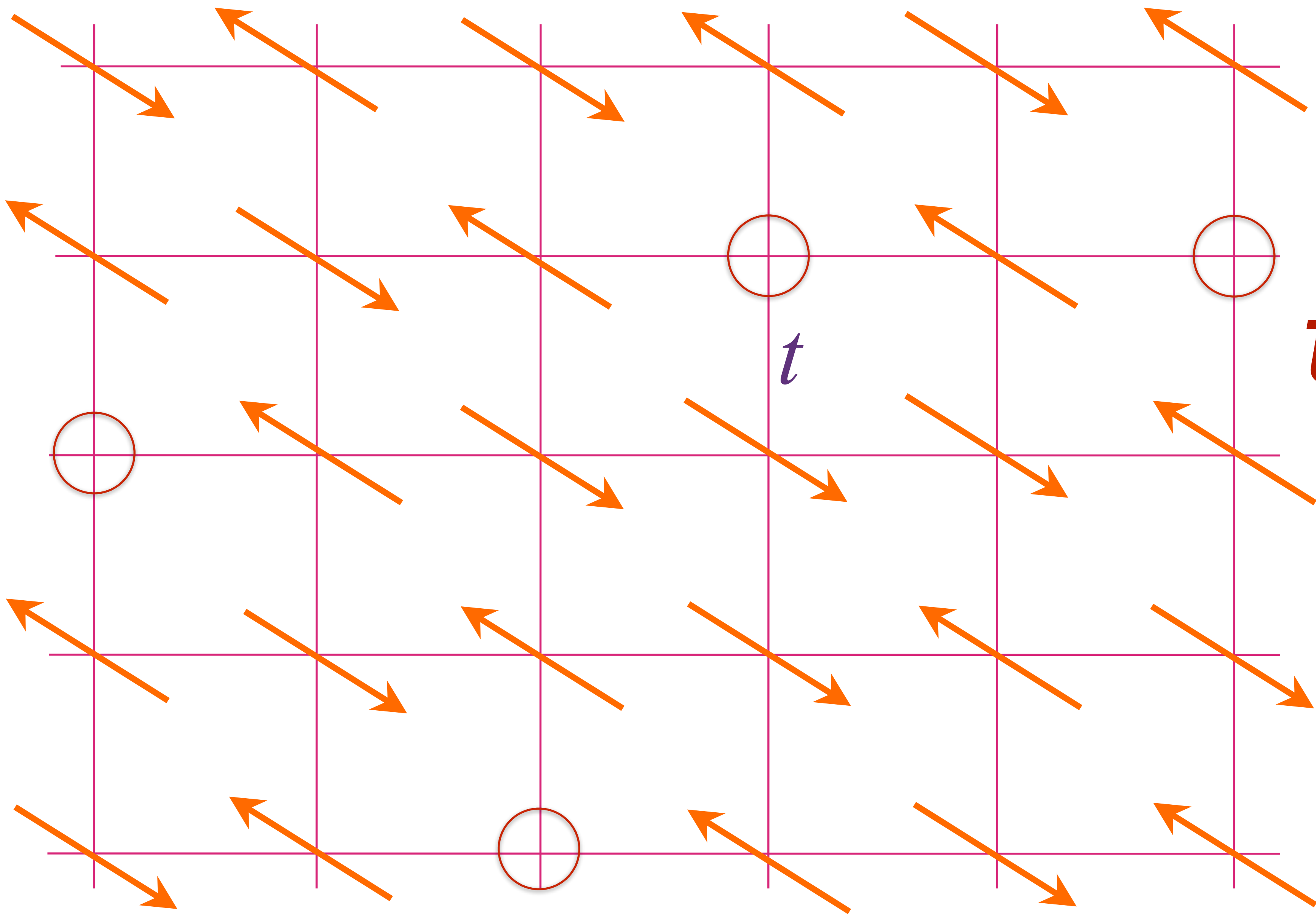
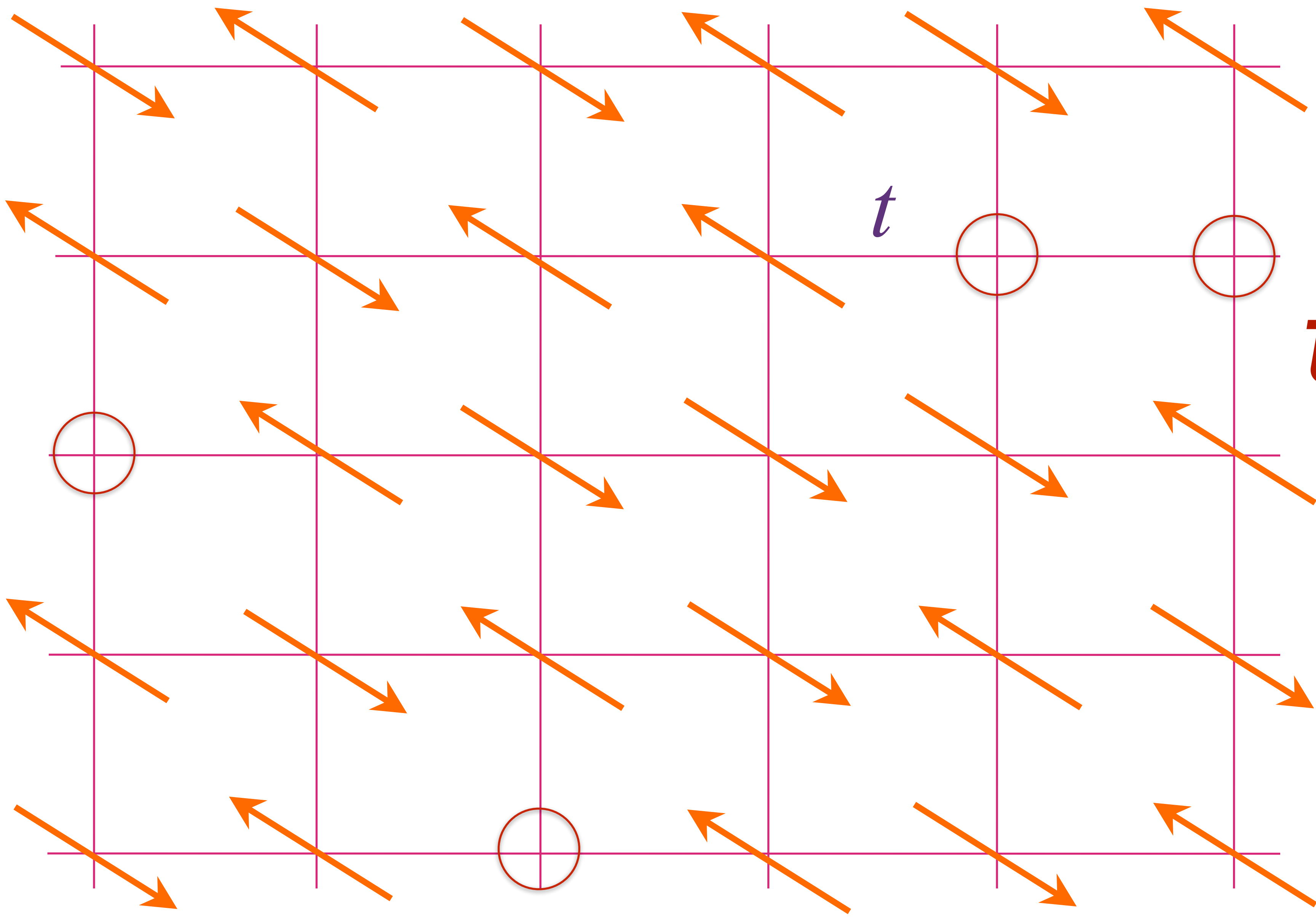


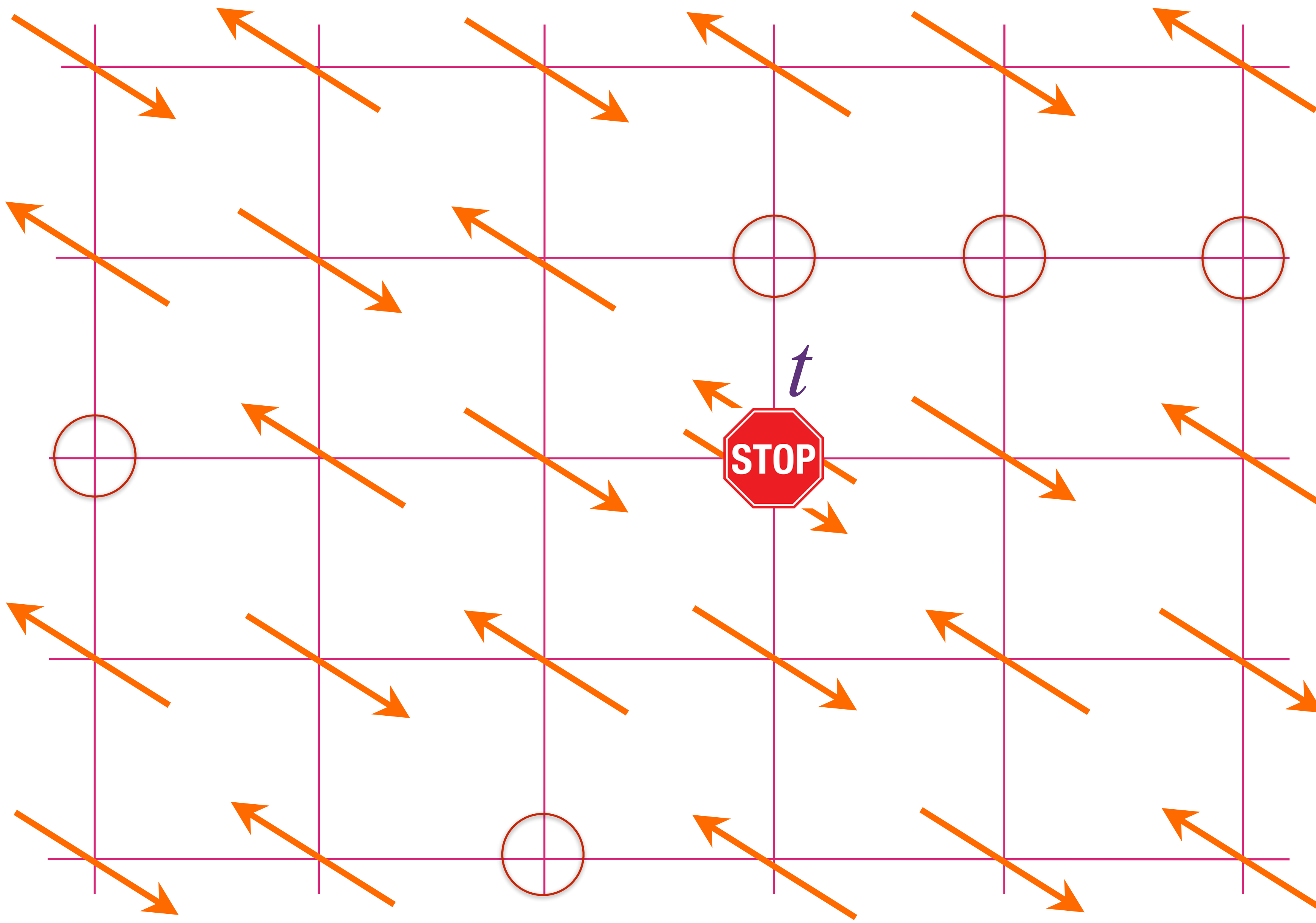
t-J model

Antiferromagnet doped with hole density p



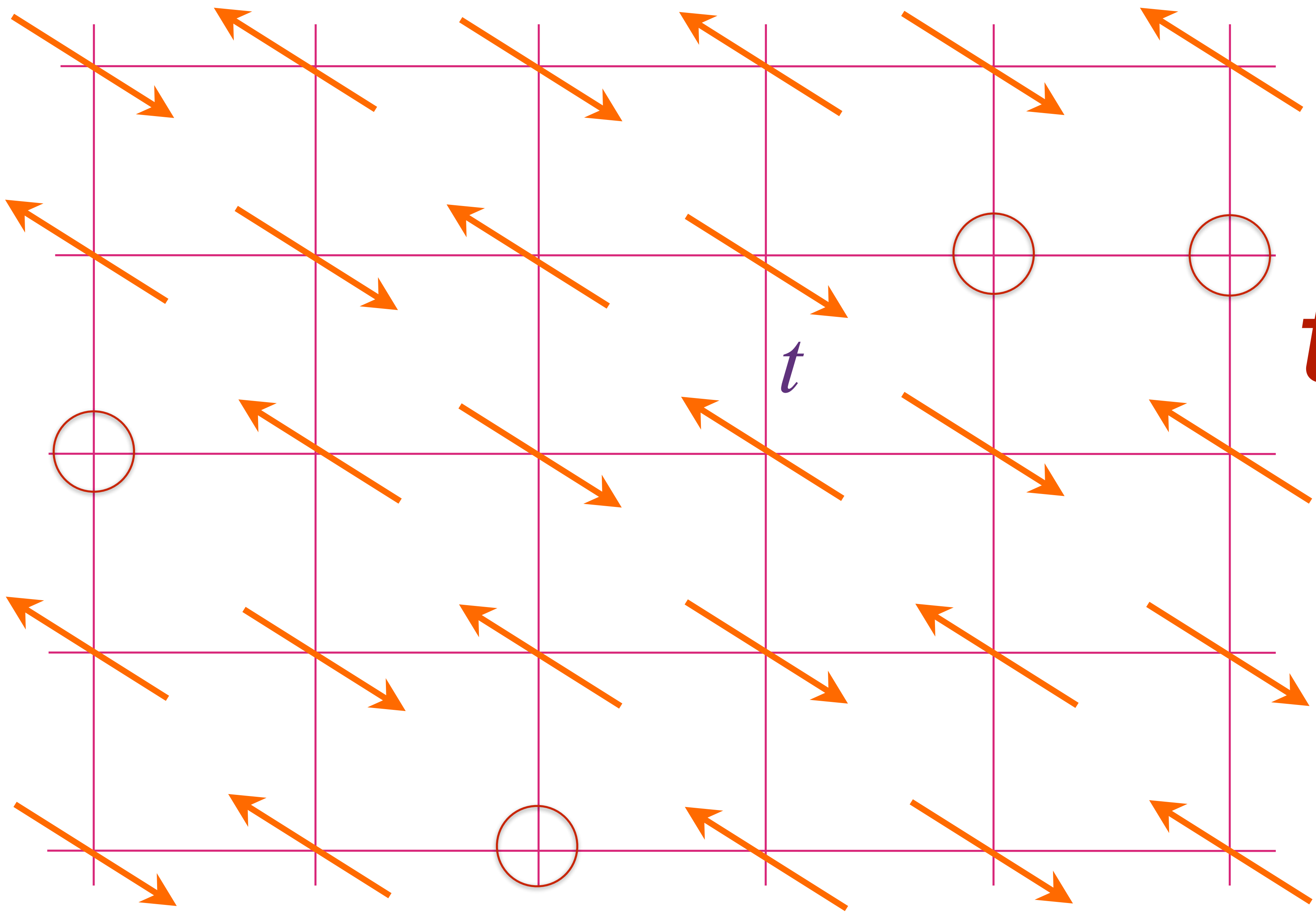
t-*J* model

Antiferromagnet doped with hole density p



t-J model

Antiferromagnet doped with hole density p



t-*J* model

Antiferromagnet doped with hole density p

$$H = -t \sum_{\langle ij \rangle} \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

t-J model

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

$\mathcal{P}_d$ projects out doubly-occupied sites.

Random t-J model doped with hole density p

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i = \tfrac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

$\mathcal{P}_d$ projects out doubly-occupied sites.

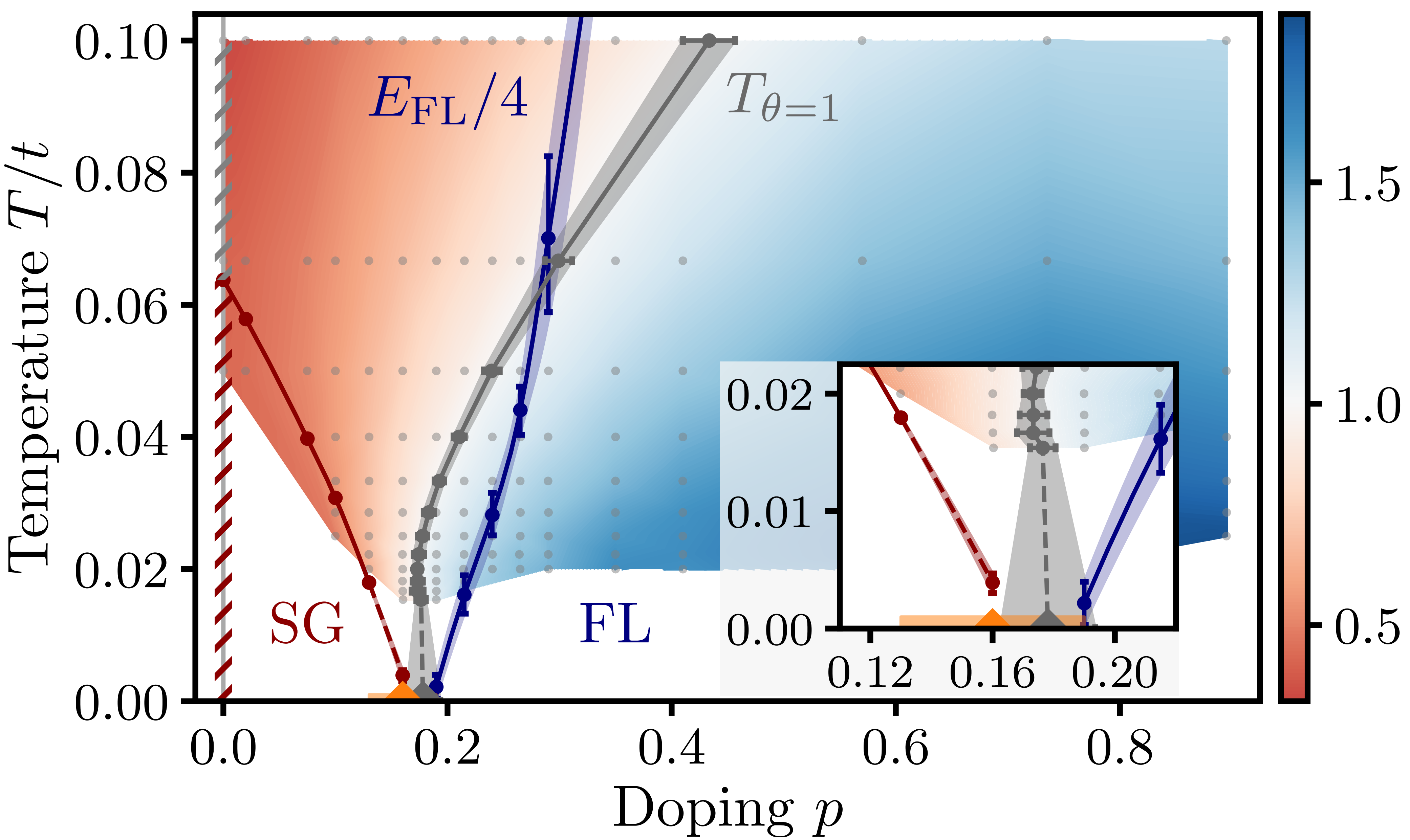
J_{ij} random, $\overline{J_{ij}} = 0$, $\overline{J_{ij}^2} = J^2$

t_{ij} random, $\overline{t_{ij}} = 0$, $\overline{t_{ij}^2} = t^2$

$J \quad \Rightarrow \quad$ two-particle interaction, as in SYK

$t \quad \Rightarrow \quad$ one-particle hopping, as in random matrices

Numerical solution of t-J model on a fully-connected cluster with all-to-all and random t_{ij} and J_{ij}



P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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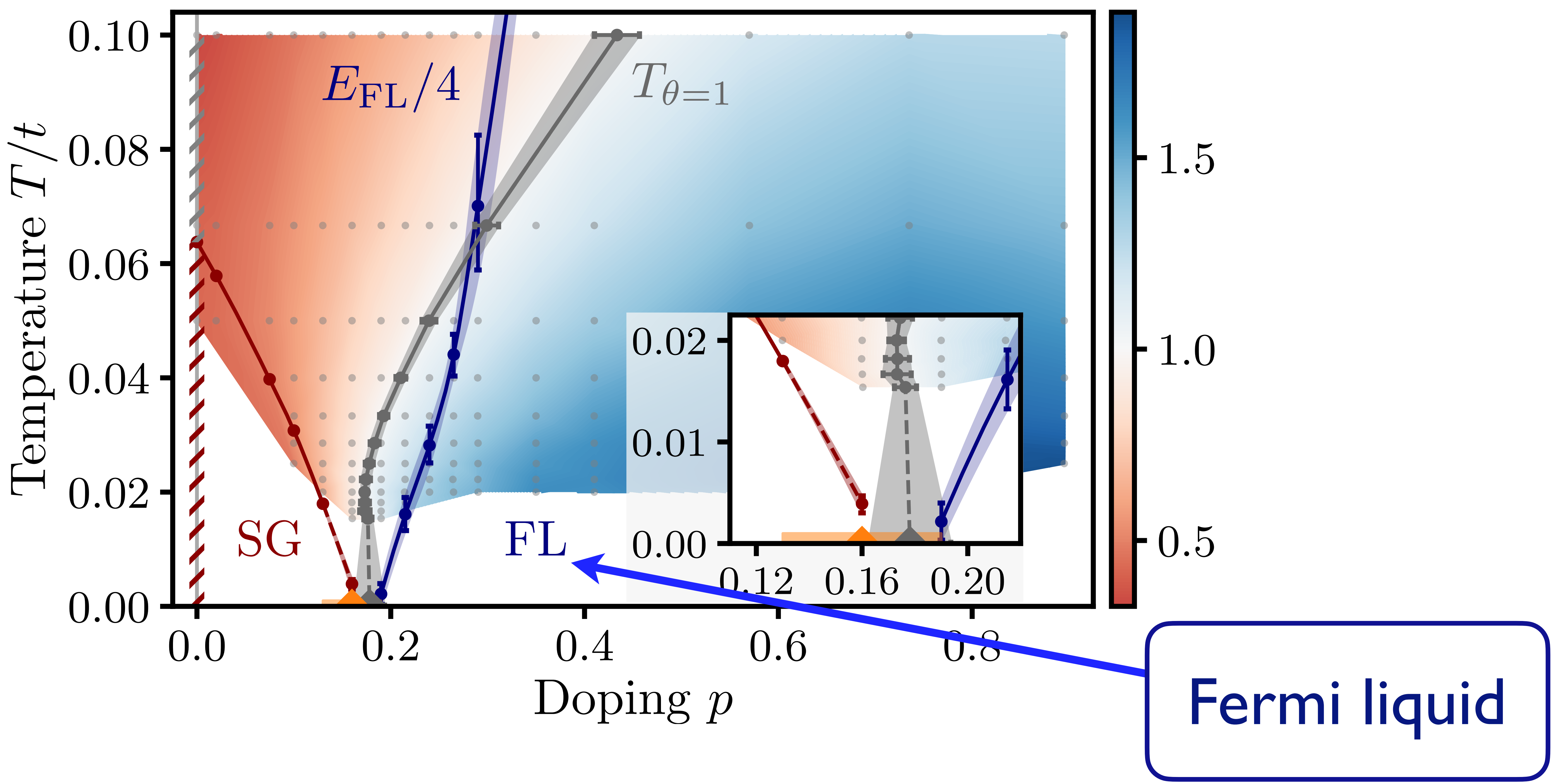

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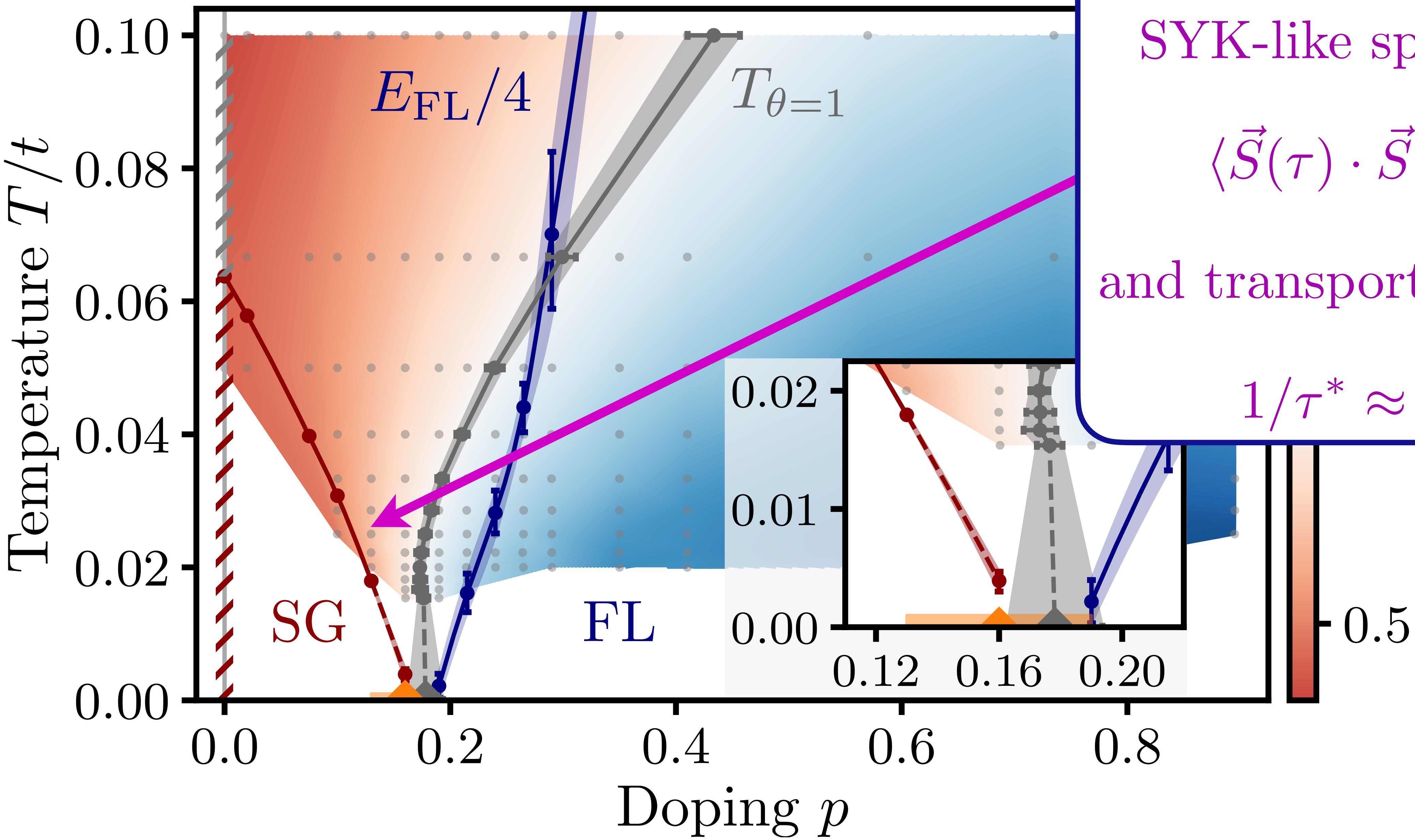

Planckian metal with SYK-like spin correlations

$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim |\tau|^{-1}$

and transport scattering time

$1/\tau^* \approx 1.4\, k_B T/\hbar$

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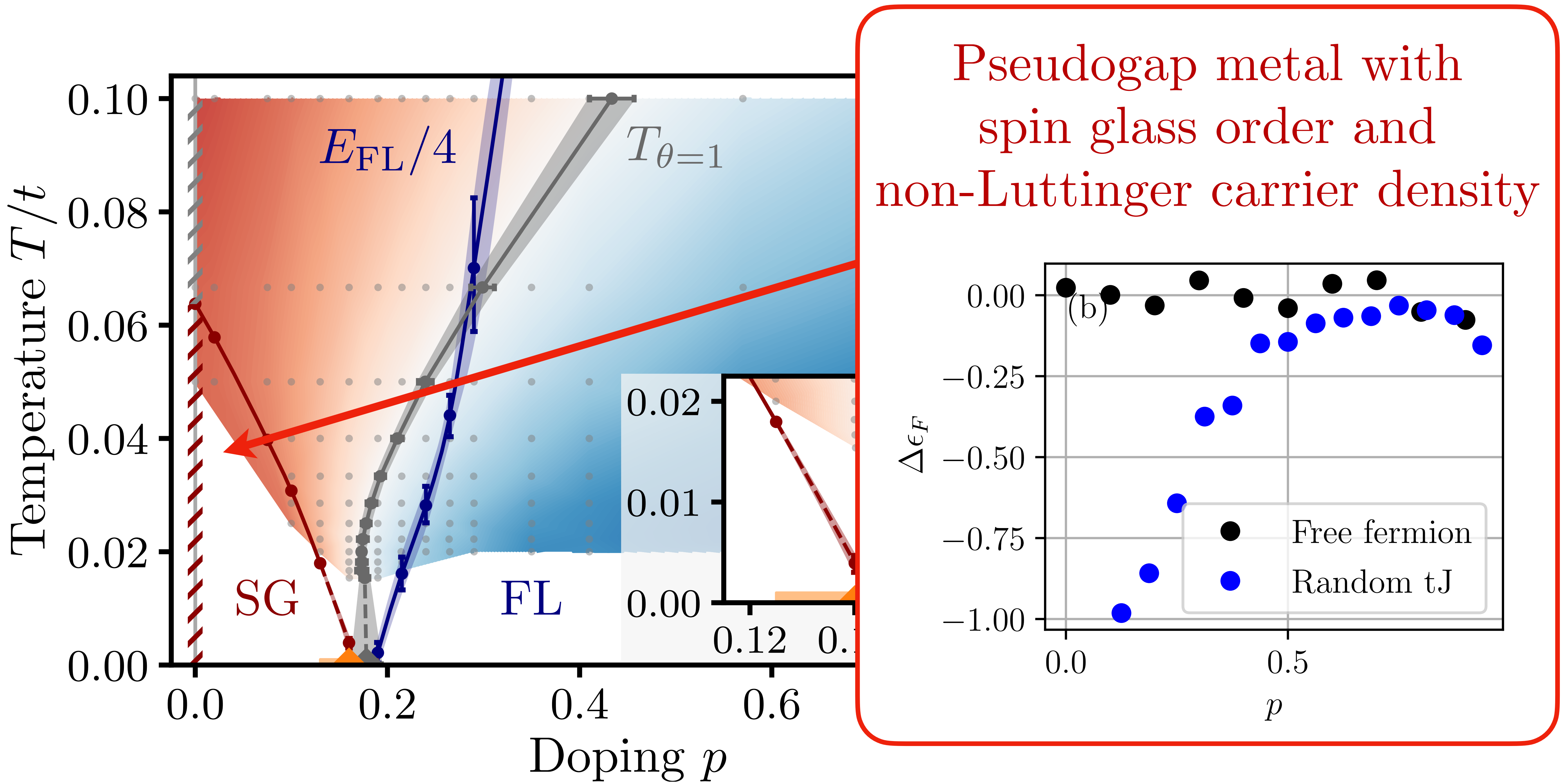

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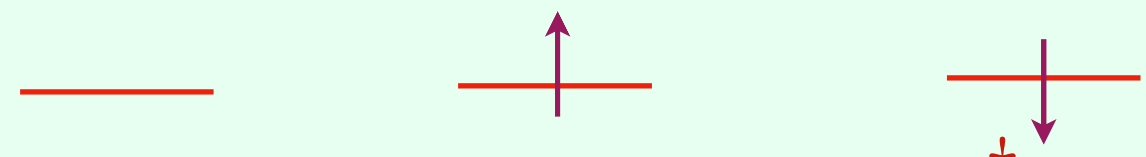
Fractionalization!

t-J model



$|0\rangle \qquad c_\uparrow^\dagger |0\rangle \qquad c_\downarrow^\dagger |0\rangle$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \boxed{\sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} \leq 1,} \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

t-*J* model

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Fractionalization

$$b^{\dagger}|v\rangle \qquad f_{\uparrow}^{\dagger}|v\rangle \qquad f_{\downarrow}^{\dagger}|v\rangle$$

$$c_{\alpha} = f_{\alpha} b^{\dagger} \quad ; \quad \vec{S} = \frac{1}{2} f_{\alpha}^{\dagger} \sigma_{\alpha\beta} f_{\beta}$$

$$f_{\alpha}^{\dagger} f_{\alpha} + b^{\dagger} b = 1$$

U(1) gauge invariance: $b \to b e^{i\phi}, \quad f_{\alpha} \to f_{\alpha} e^{i\phi}$

t-*J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$\mathcal{P}_d$ projects out doubly-occupied sites.

Fractionalized form:

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, f_{i\alpha}^\dagger f_{j\alpha} b_j^\dagger b_i + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} \frac{J_{ij}}{4} \, f_{i\alpha}^\dagger \sigma_{\alpha\beta} f_{i\beta} \cdot f_{j\gamma}^\dagger \sigma_{\gamma\delta} f_{j\delta}$$

Both t_{ij} and J_{ij} are 4-parton terms, and this allows SYK criticality in an intermediate regime.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

t-J model

Fractionalization and emergent gauge fields leading to non-Luttinger volume phases in the pseudogap regime

PHYSICAL REVIEW X **10,** 041057 (2020)

Fermi Surface Reconstruction without Symmetry Breaking

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Summary

- SYK: a solvable model without quasiparticle excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

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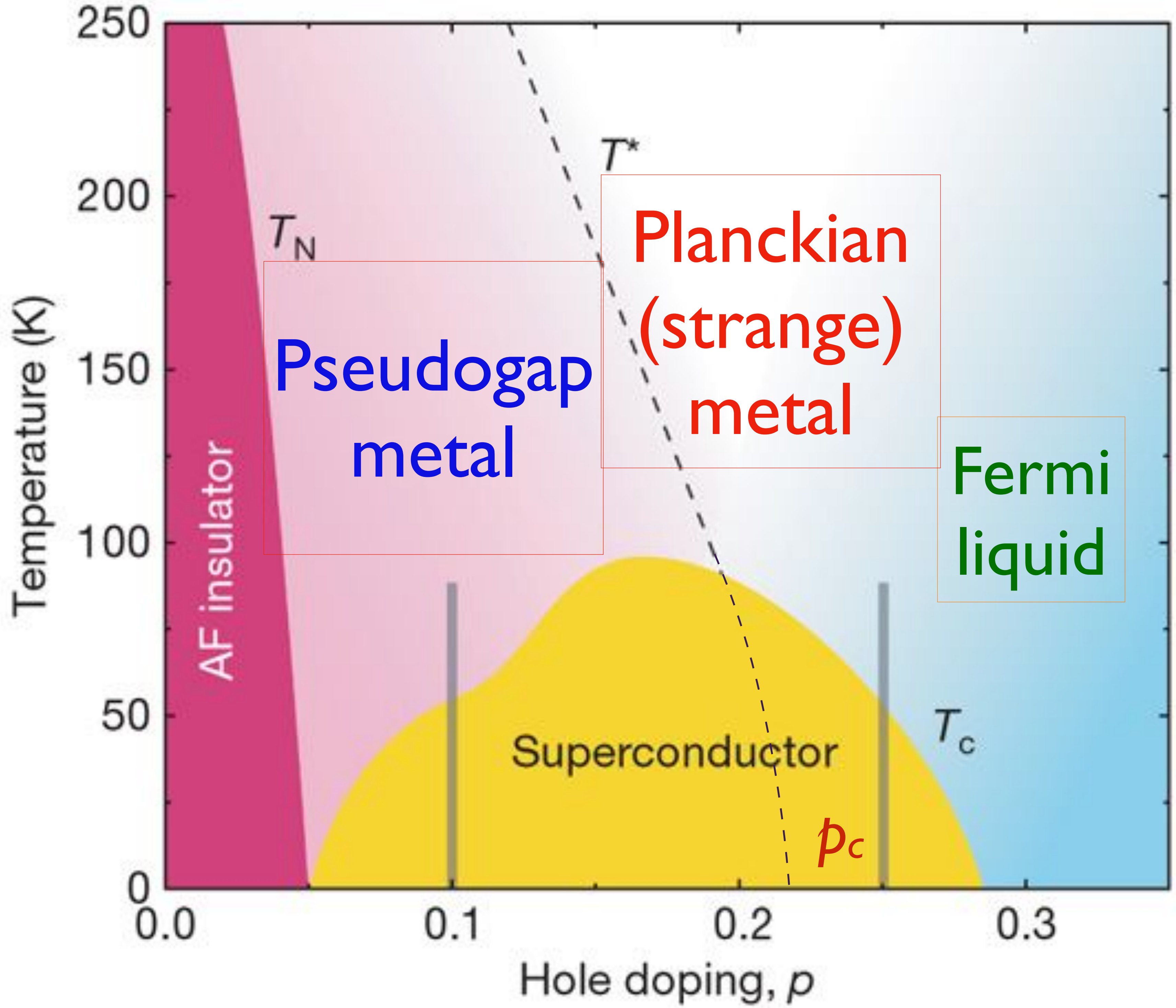
- Similar corrections to the entropy in 'wormholes' have recently lead to progress in understanding the flow of quantum information in evaporating black holes.

Summary

- The random t-J model captures key aspects of the cuprate metals:



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  - Linear-in-T resistivity in the Planckian metal from the time reparameterization soft-mode.

    (the same mode that yields the leading correction to the BH entropy of charged black holes, and the contribution of 'wormholes')