Quantum criticality in the cuprate superconductors

Reviews:
arXiv:0907.0008
arXiv:0810.3005 (with Markus Mueller)

PHYSICS
HARVARD

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PHYSICS
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Outline

1. Quantum-criticality of bosons
   Superfluid-insulator transition

2. The AdS/CFT correspondence
   Exact solutions for quantum critical transport

3. Quantum criticality in the cuprates
   Global phase diagram and the spin density wave transition in metals

4. AdS_4 theory of compressible quantum liquids
   Fermi surfaces and quantum oscillations

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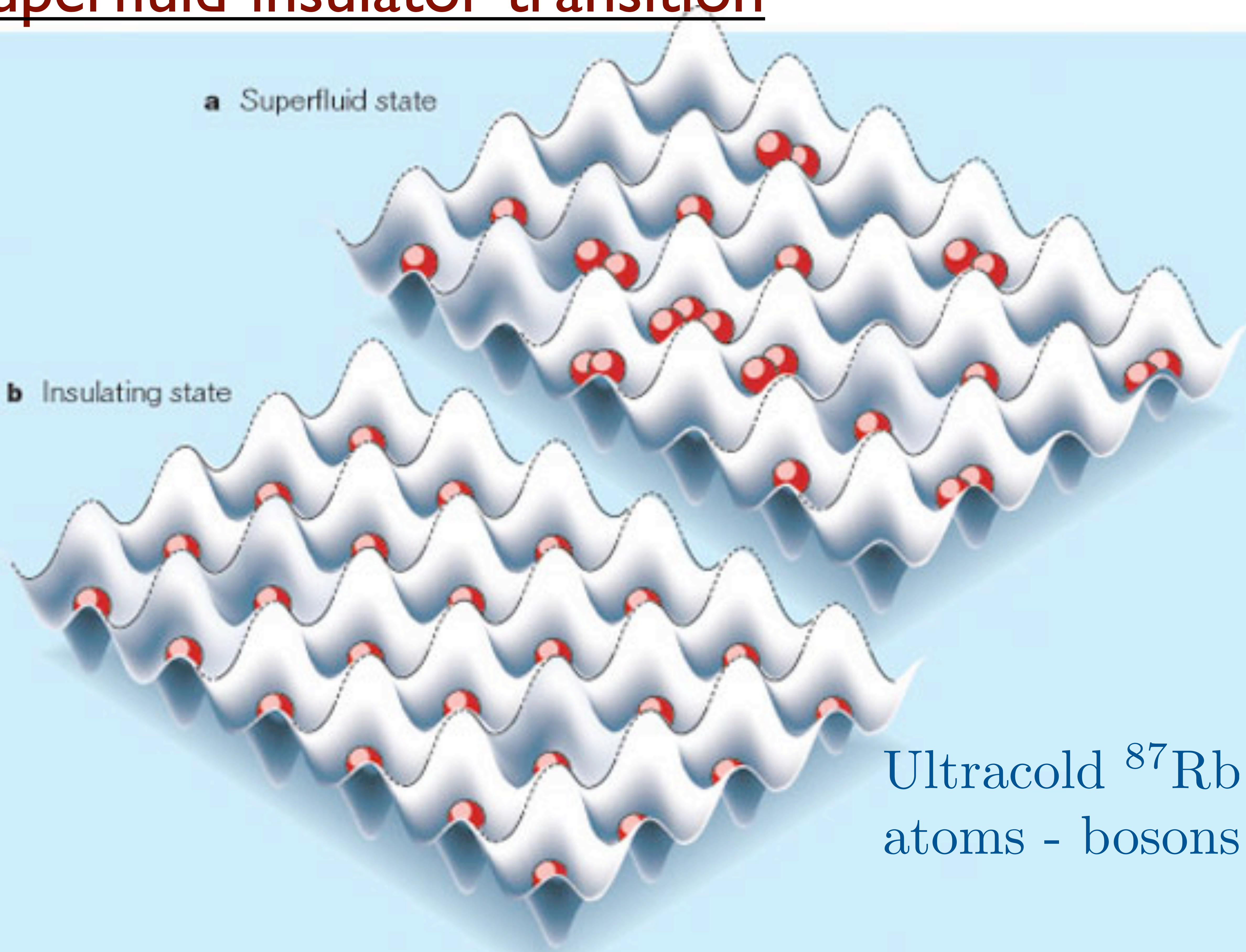
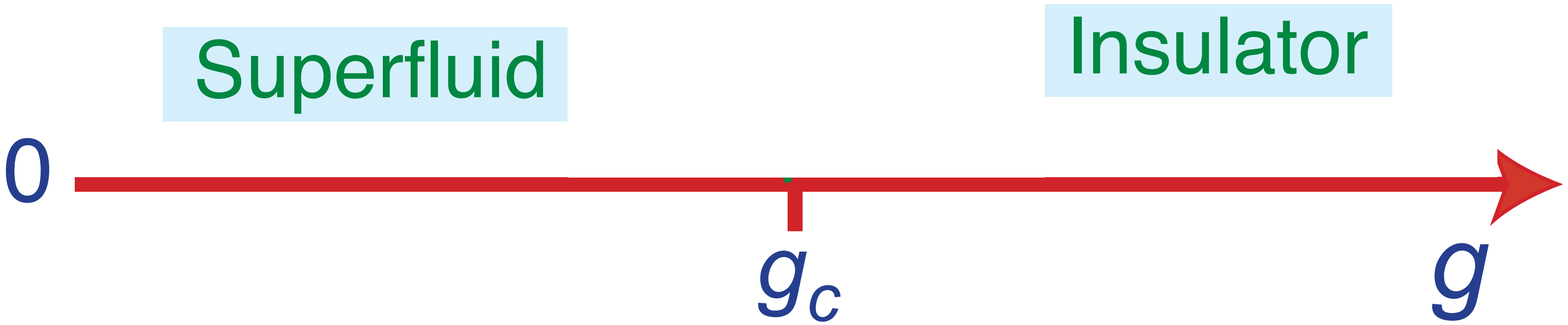
Superfluid-insulator transition



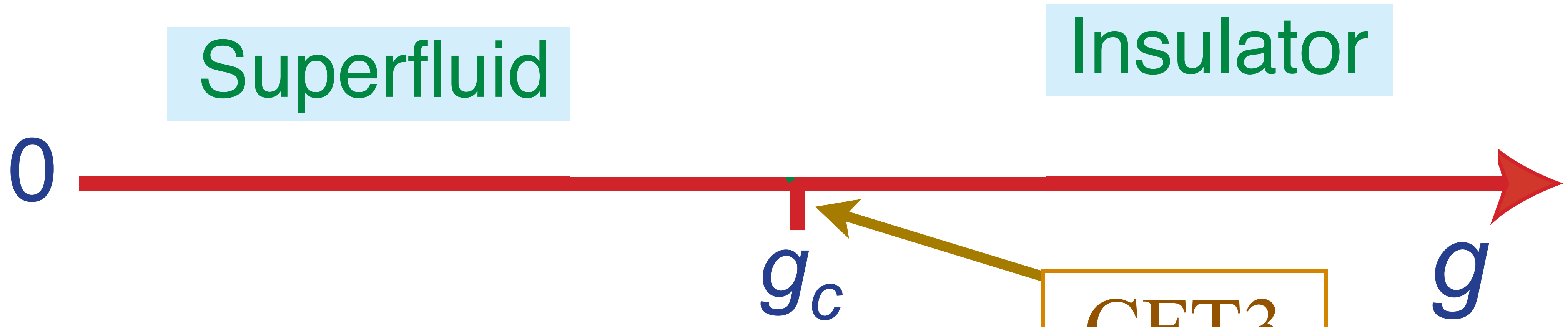
Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

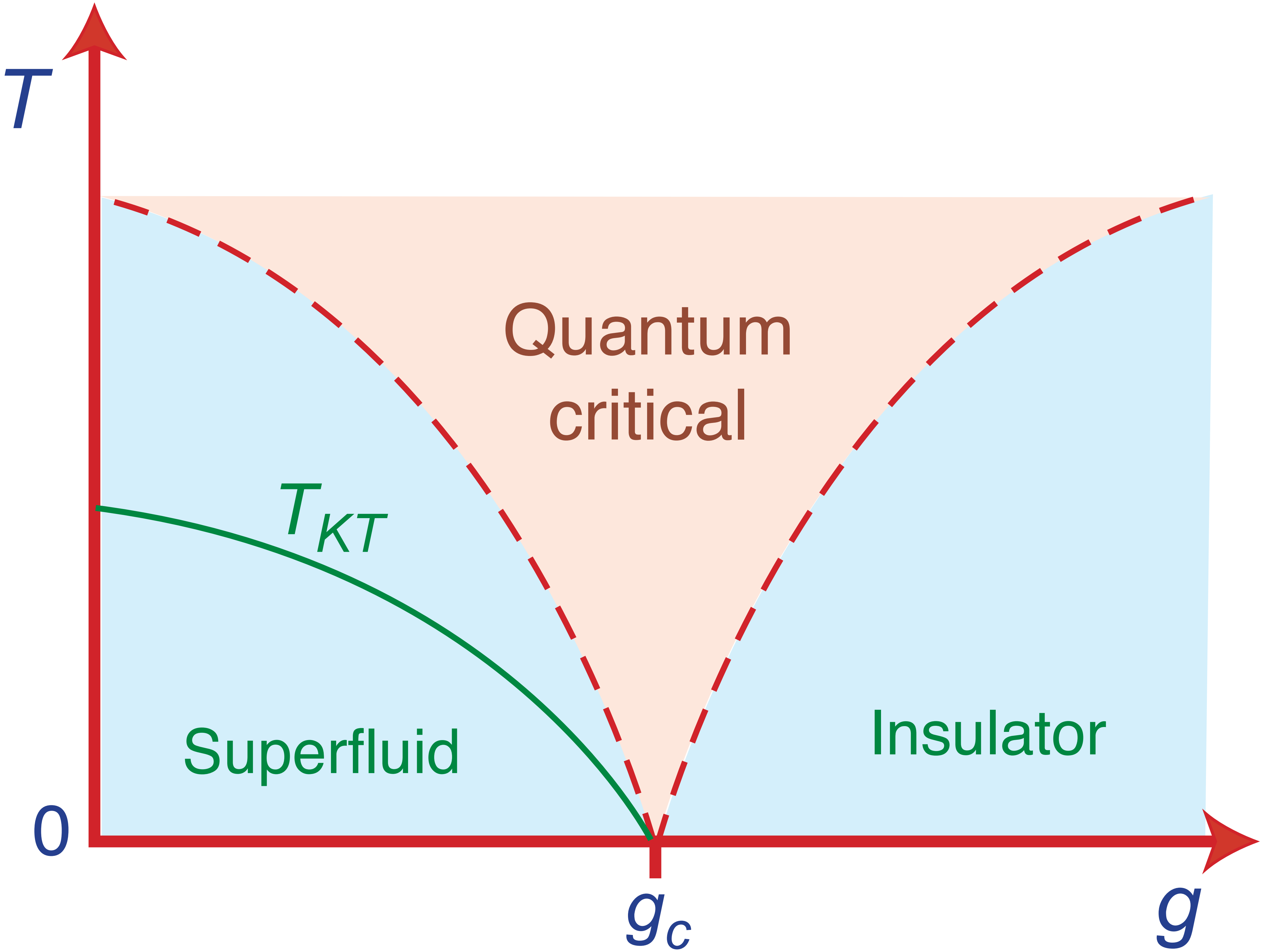

Superfluid
Insulator
0
g_c
g

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

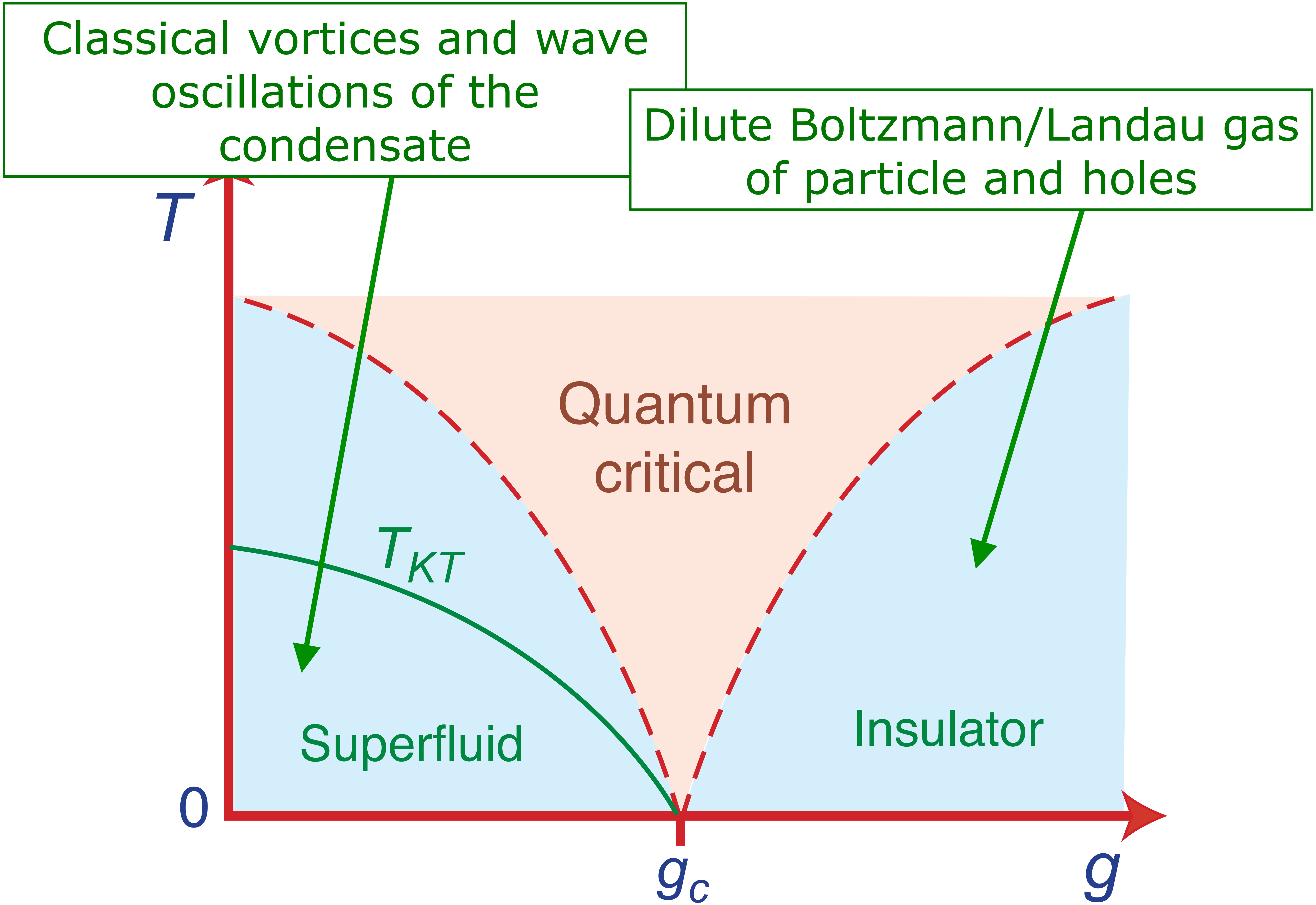
$\langle \psi \rangle \neq 0$ $\langle \psi \rangle = 0$



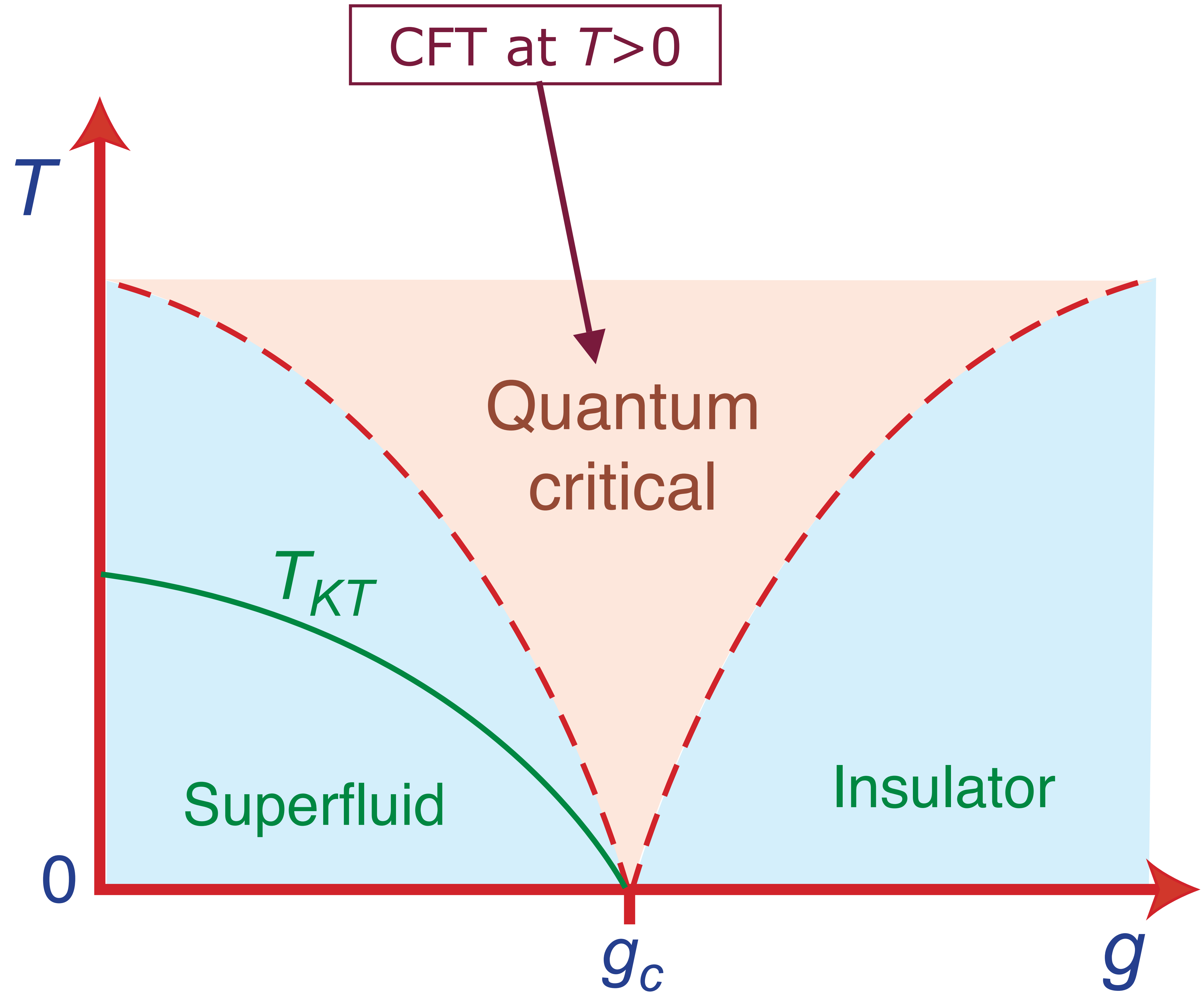
M. P. A. Fisher *et al.*



T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g



Classical vortices and wave oscillations of the condensate
Dilute Boltzmann/Landau gas of particle and holes
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g



CFT at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$
$$\sigma_{\text{Insulator}}(T \to 0) = 0$$
$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

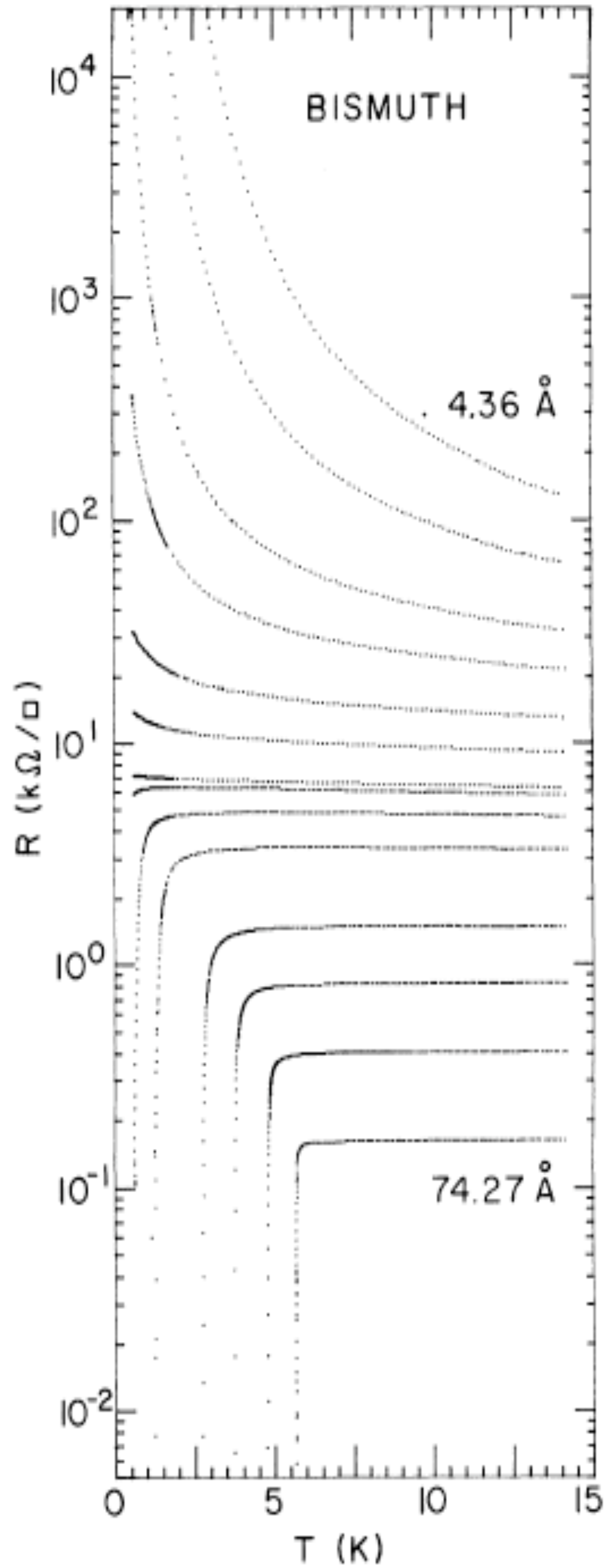


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

Quantum critical transport

Quantum *"perfect fluid"* with shortest possible relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k, \omega) \quad = \quad \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

However, for *all* CFT3s, at $\hbar\omega \ll k_B T$, we have the Einstein relation

$$\chi(k,\omega) = 4e^2\chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D\chi_c = \frac{4e^2}{h}\Theta_1\Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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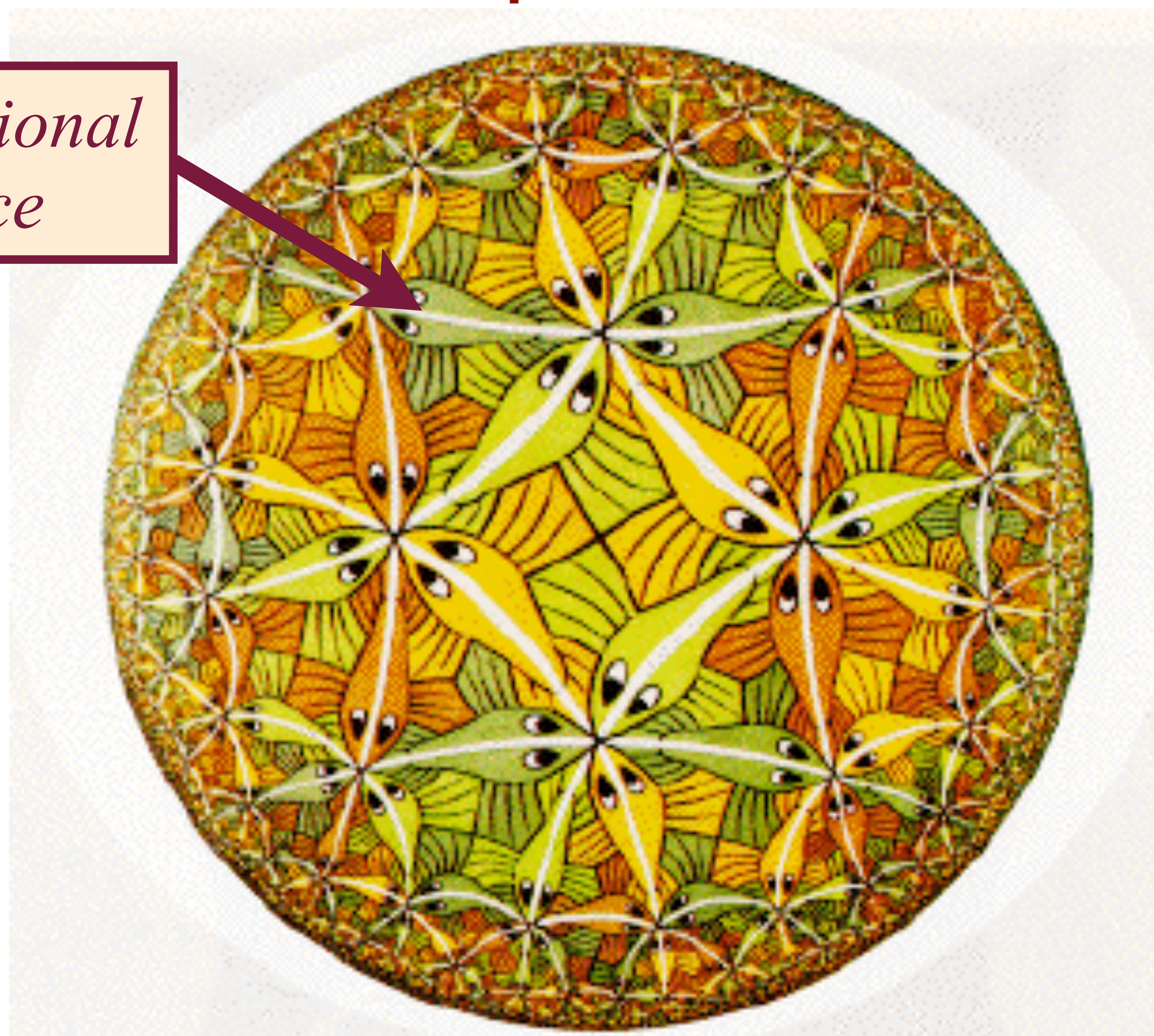
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence

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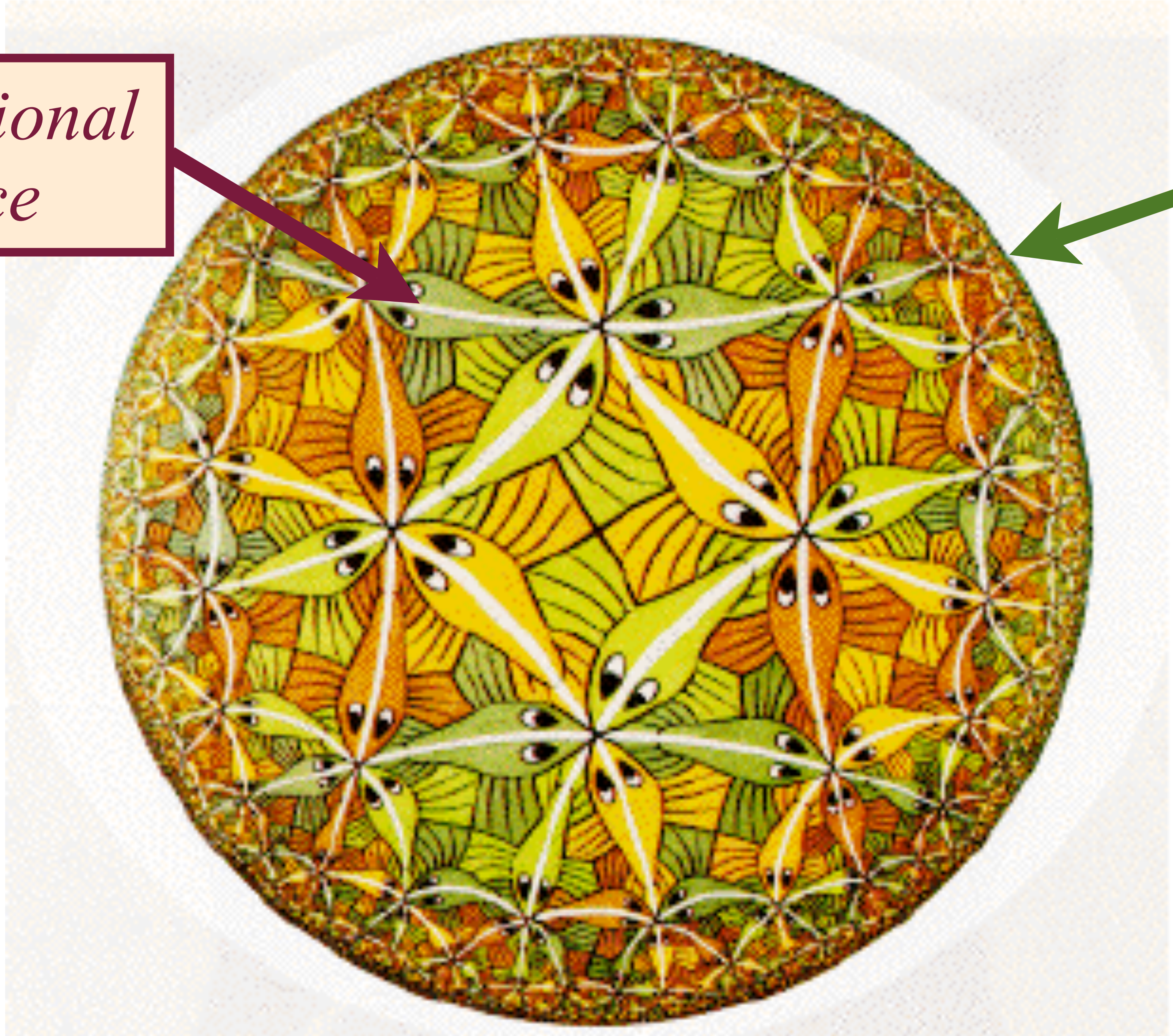
3+1 dimensional AdS space



A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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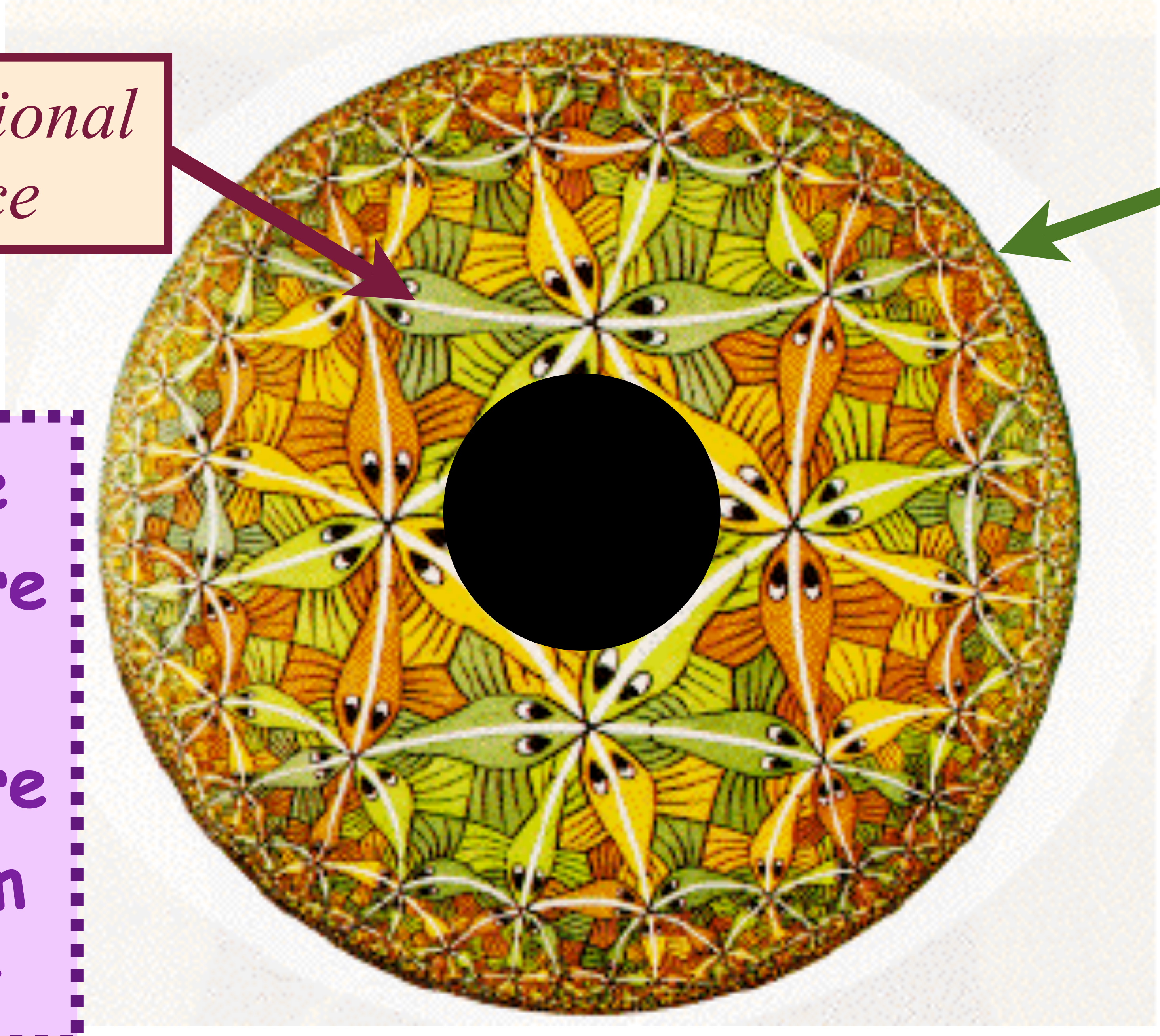

Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten

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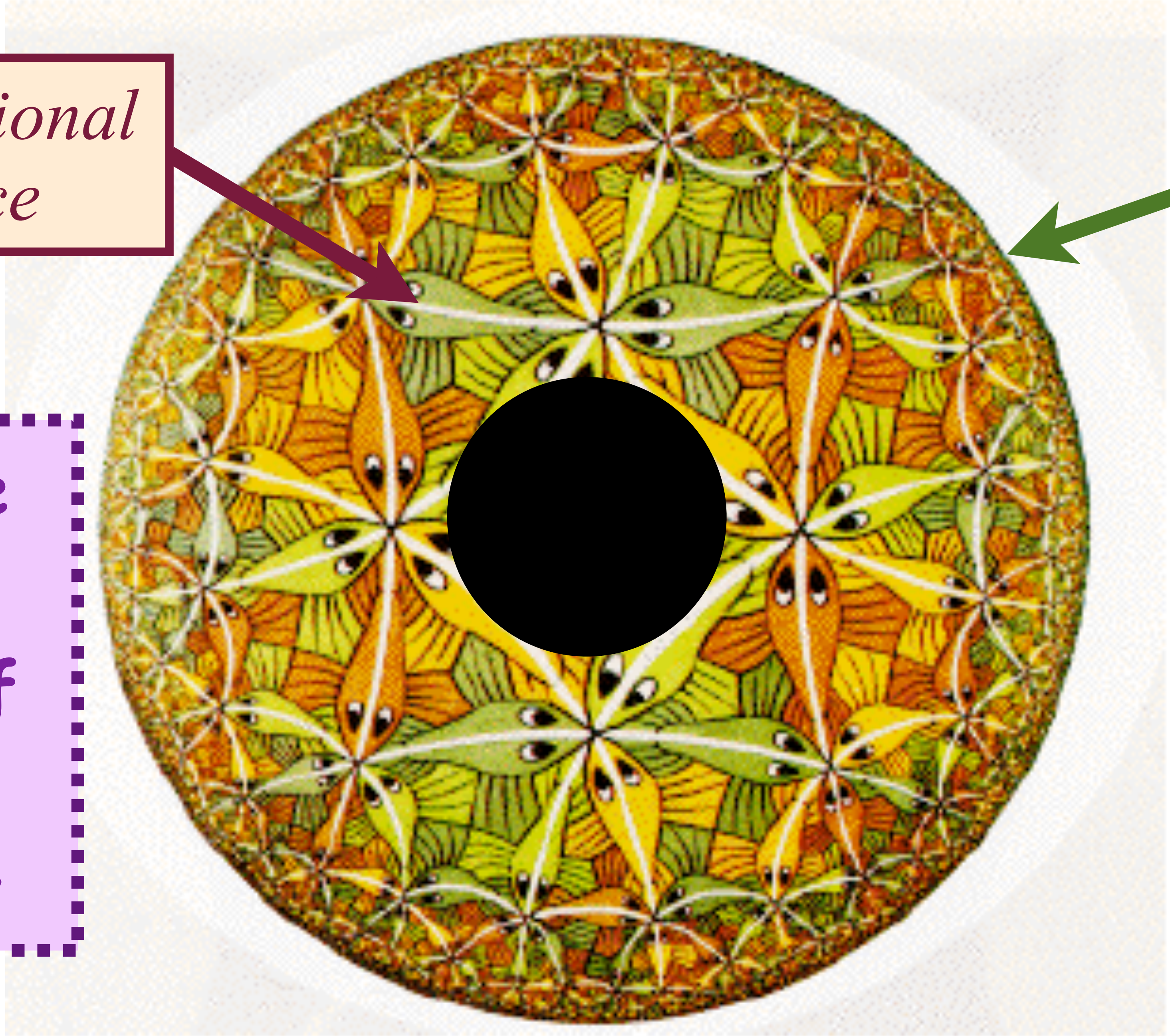

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

AdS/CFT correspondence

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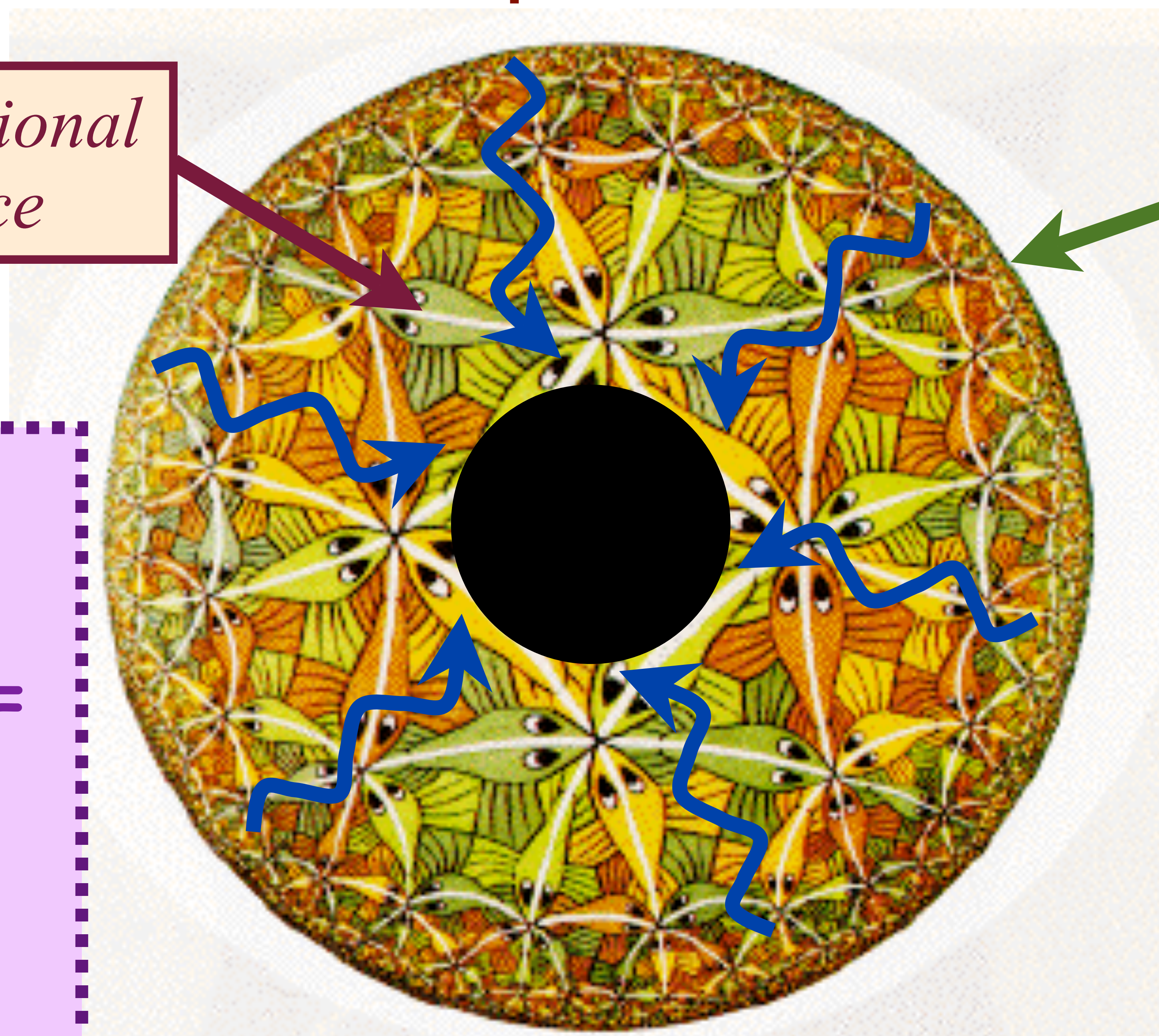
3+1 dimensional AdS space



Quantum criticality in 2+1 dimensions

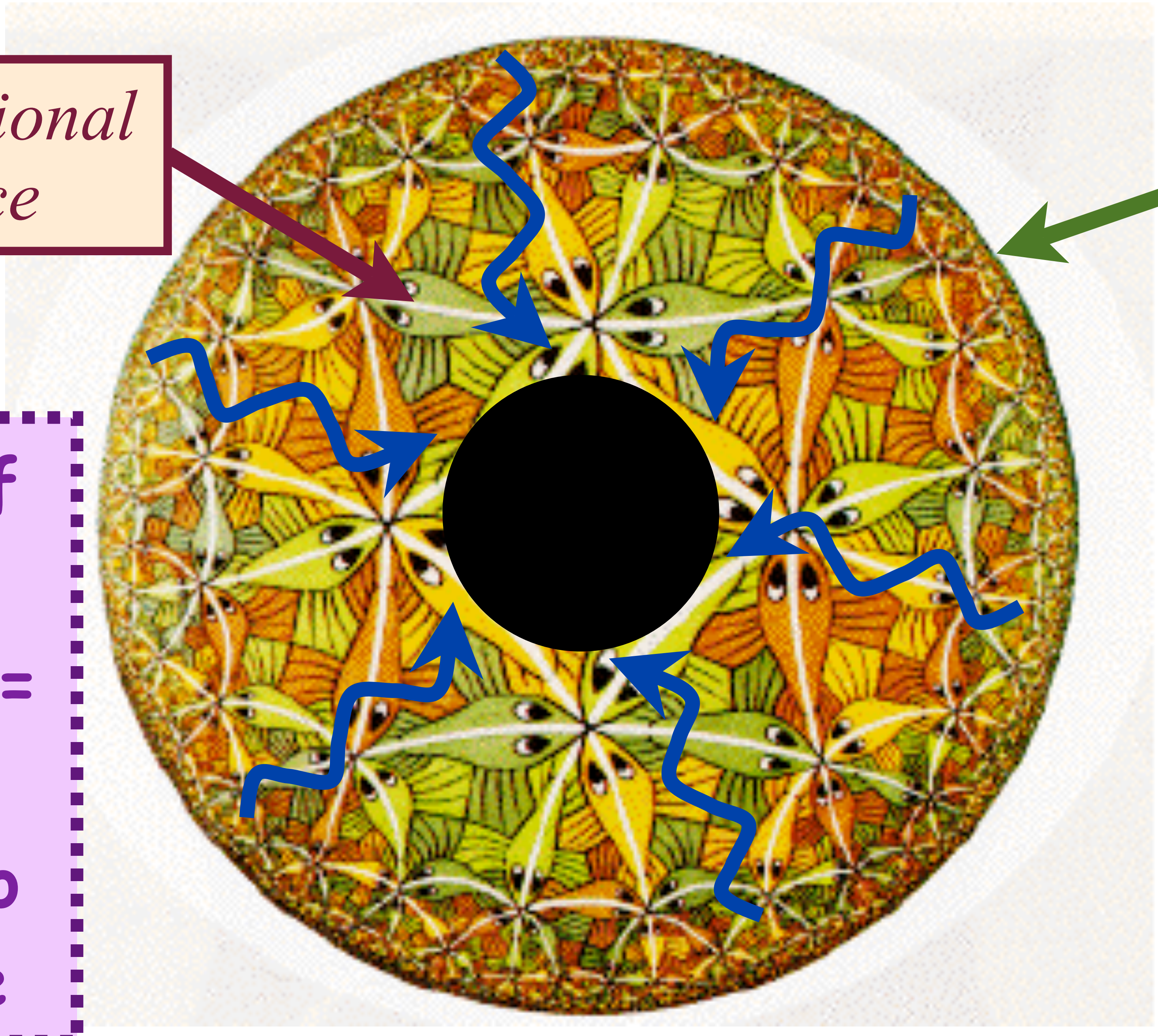
Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

3+1 dimensional AdS space

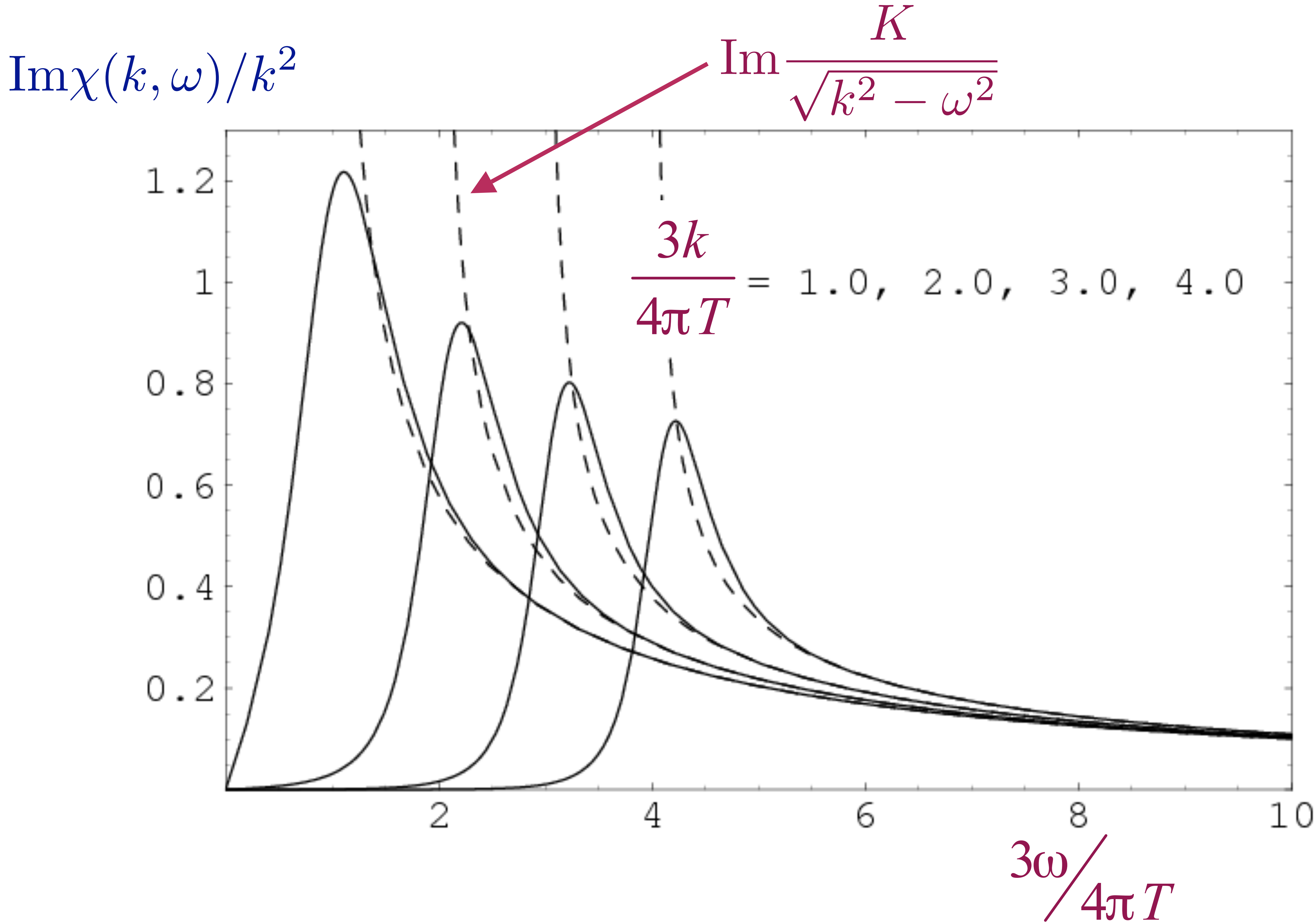


Quantum criticality in 2+1 dimensions

Friction of quantum criticality = waves falling into black hole

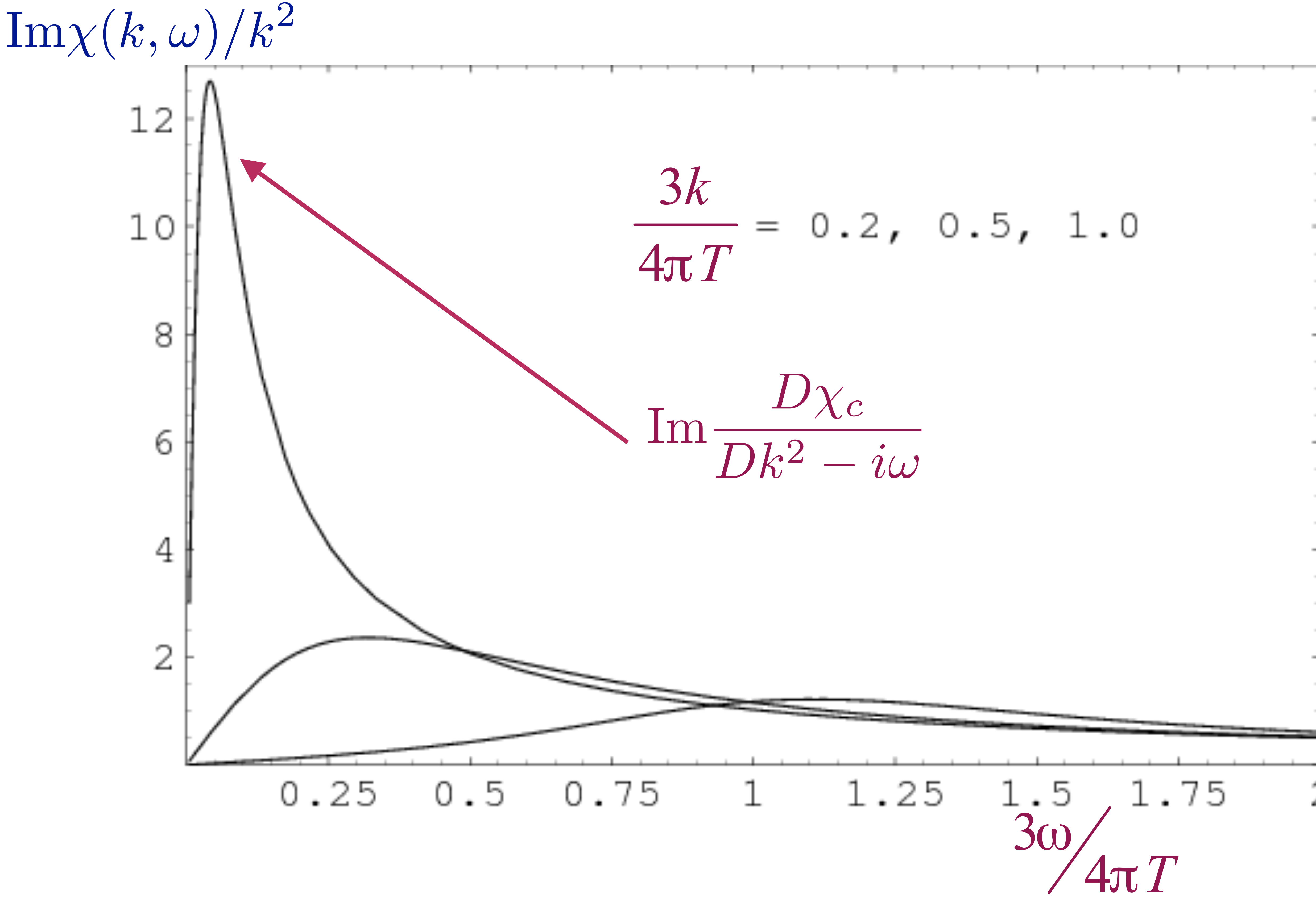
Kovtun, Policastro, Son

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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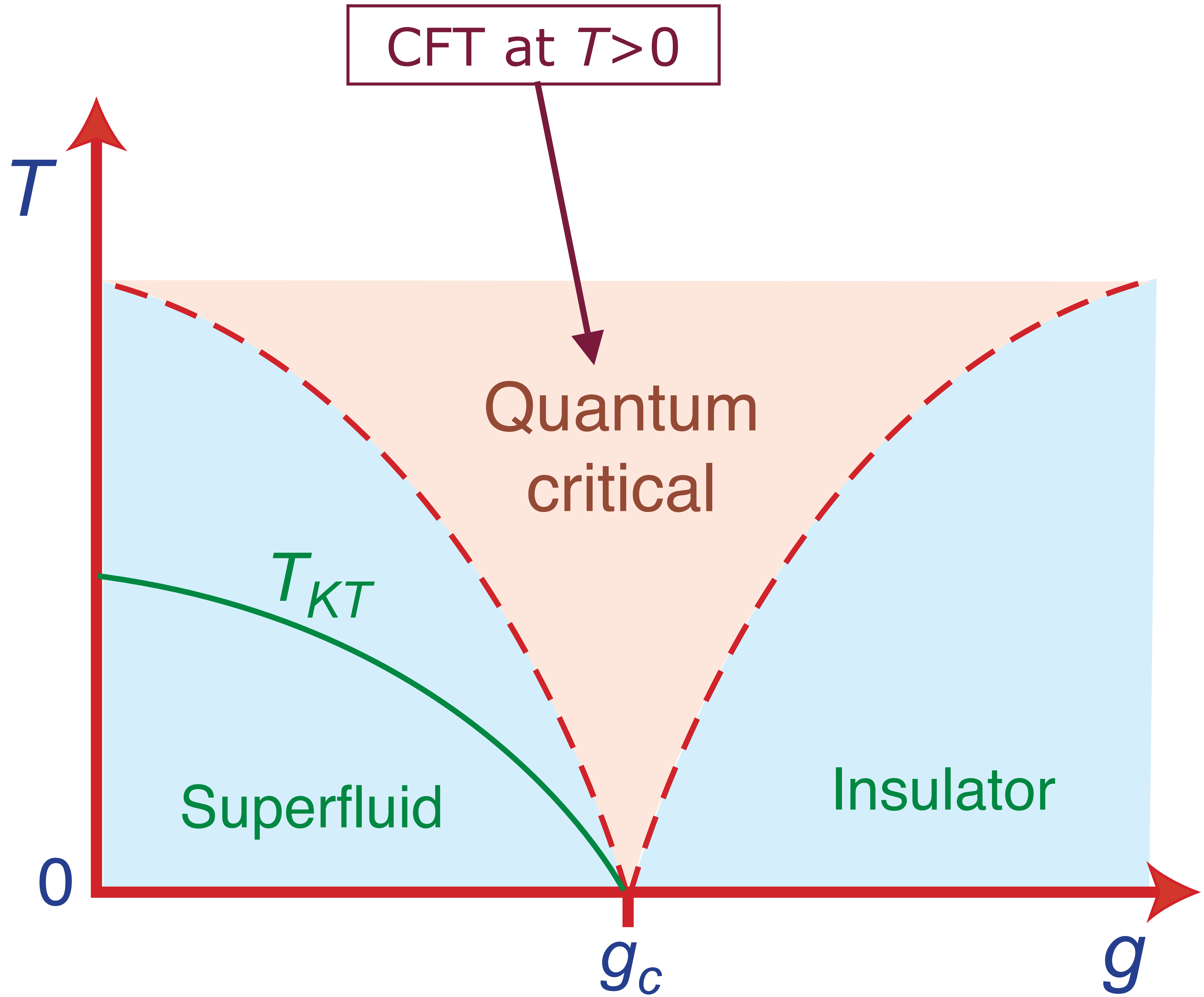
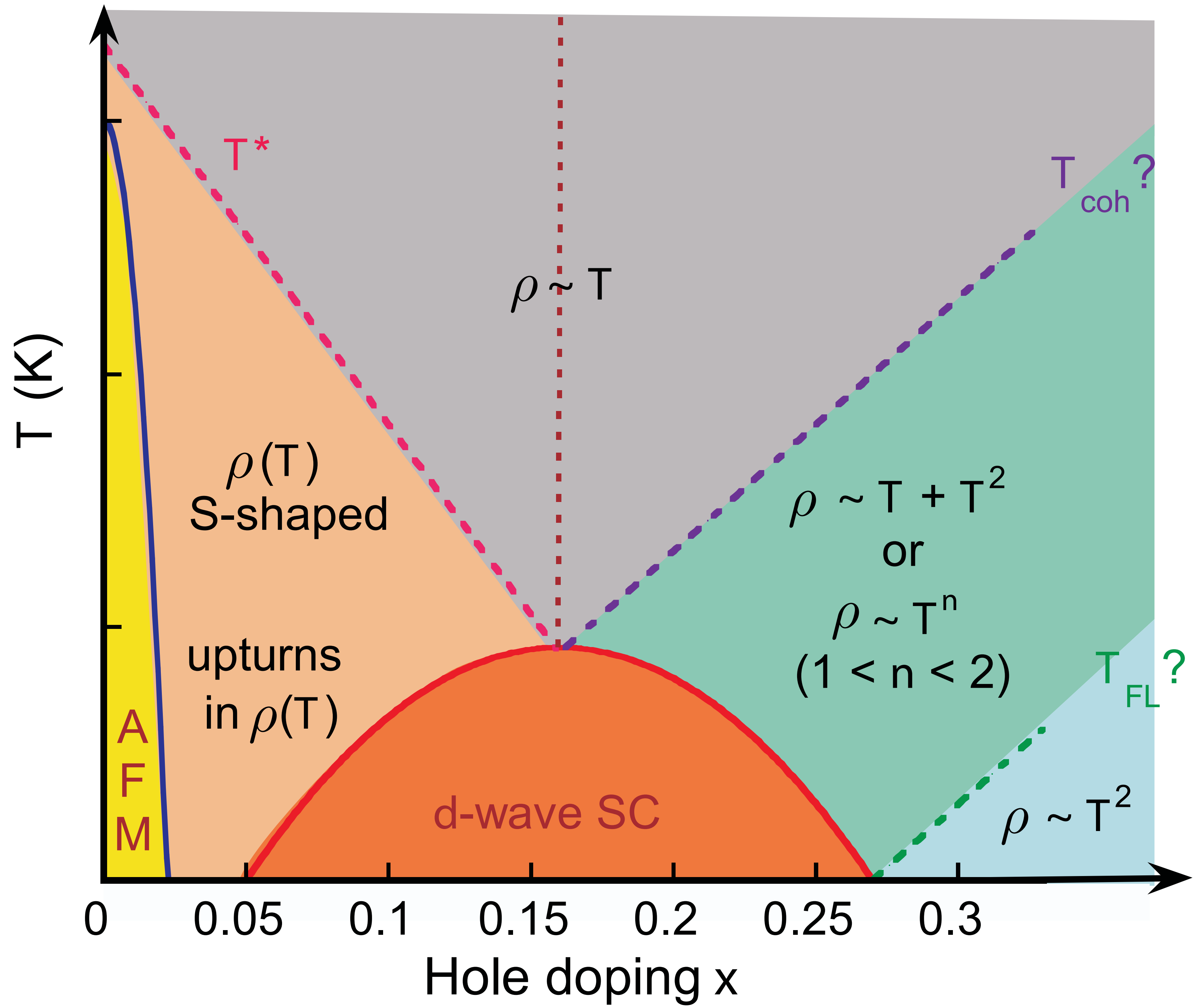


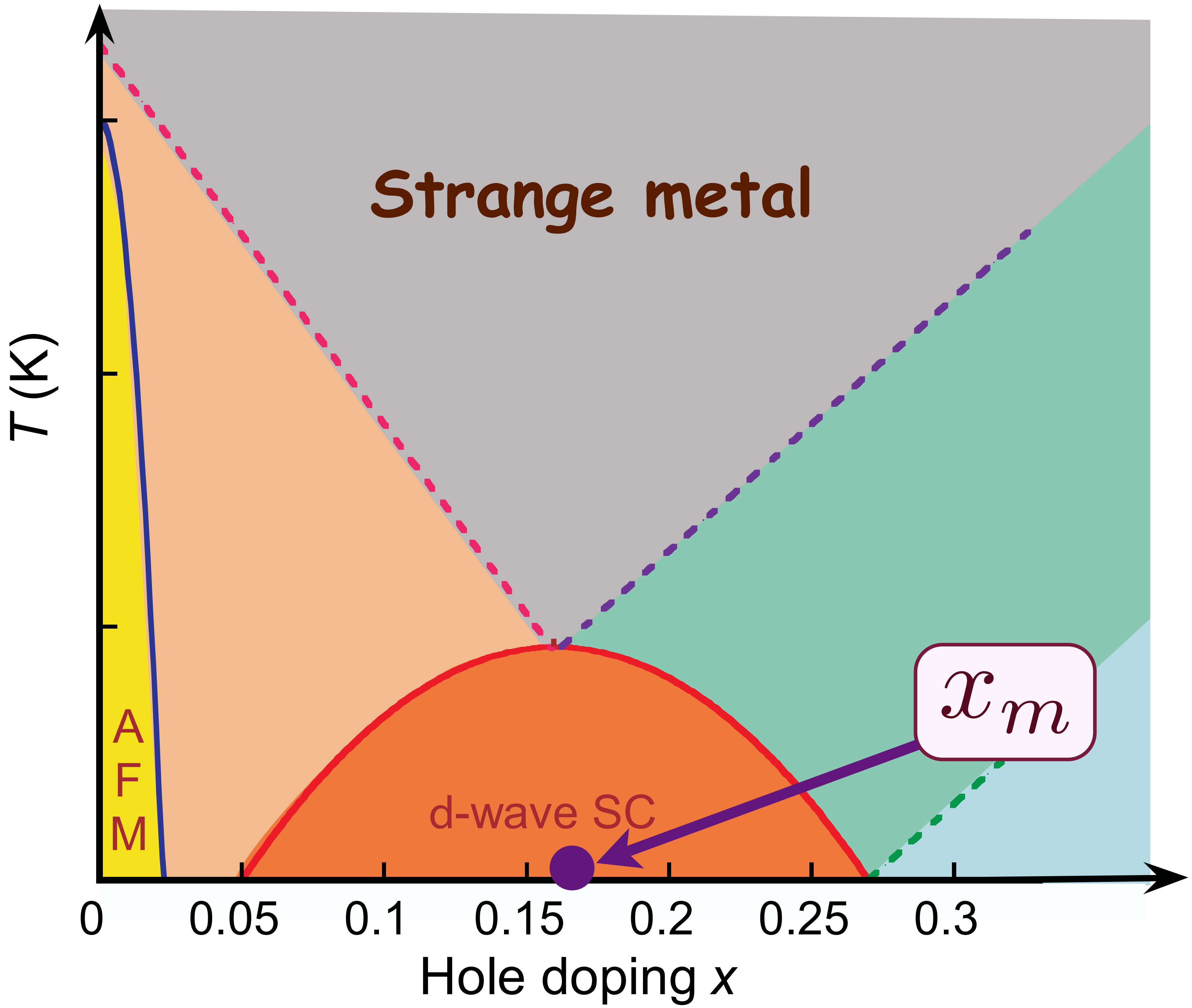
CFT at T>0
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Crossovers in transport properties of hole-doped cuprates



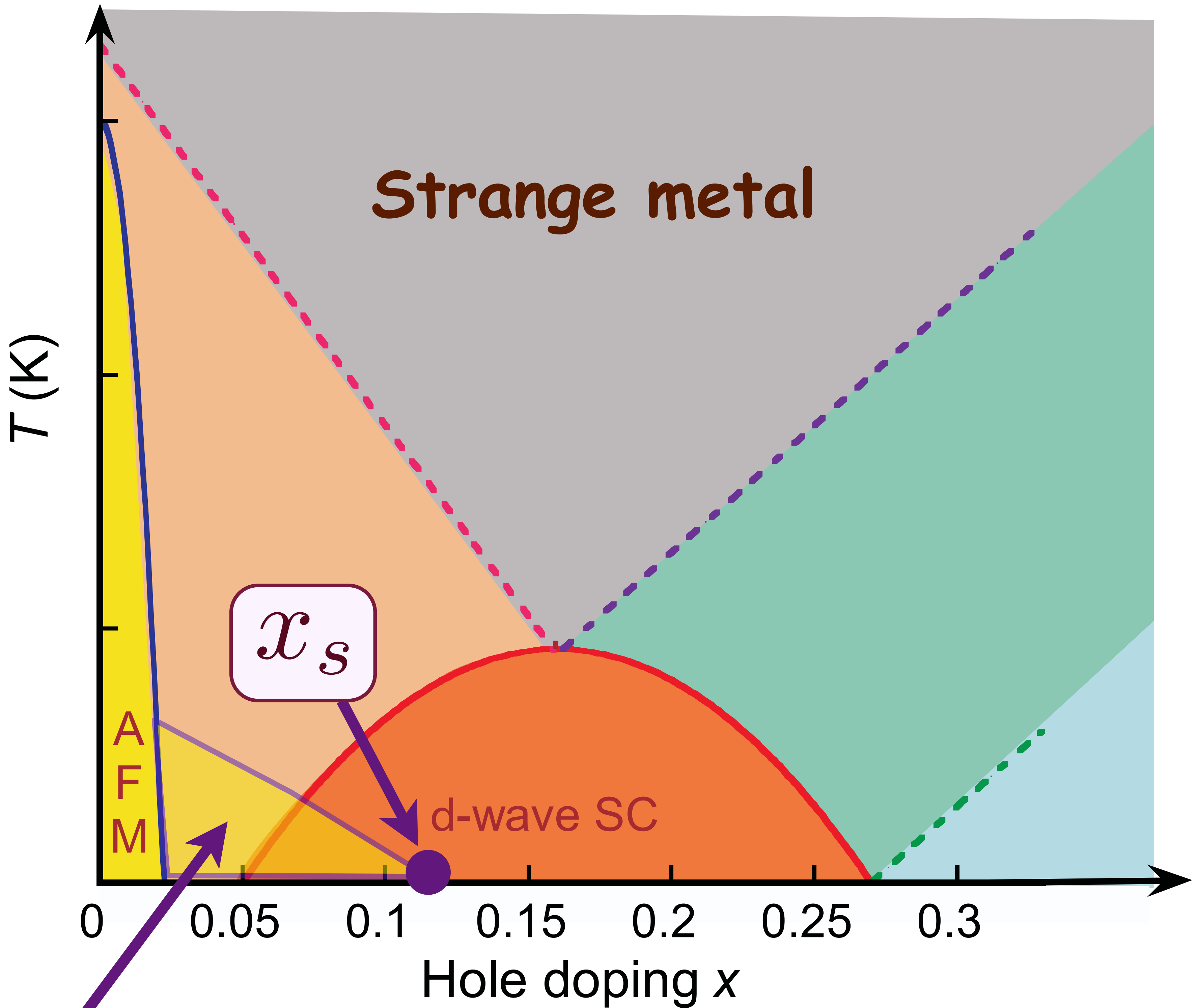
N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



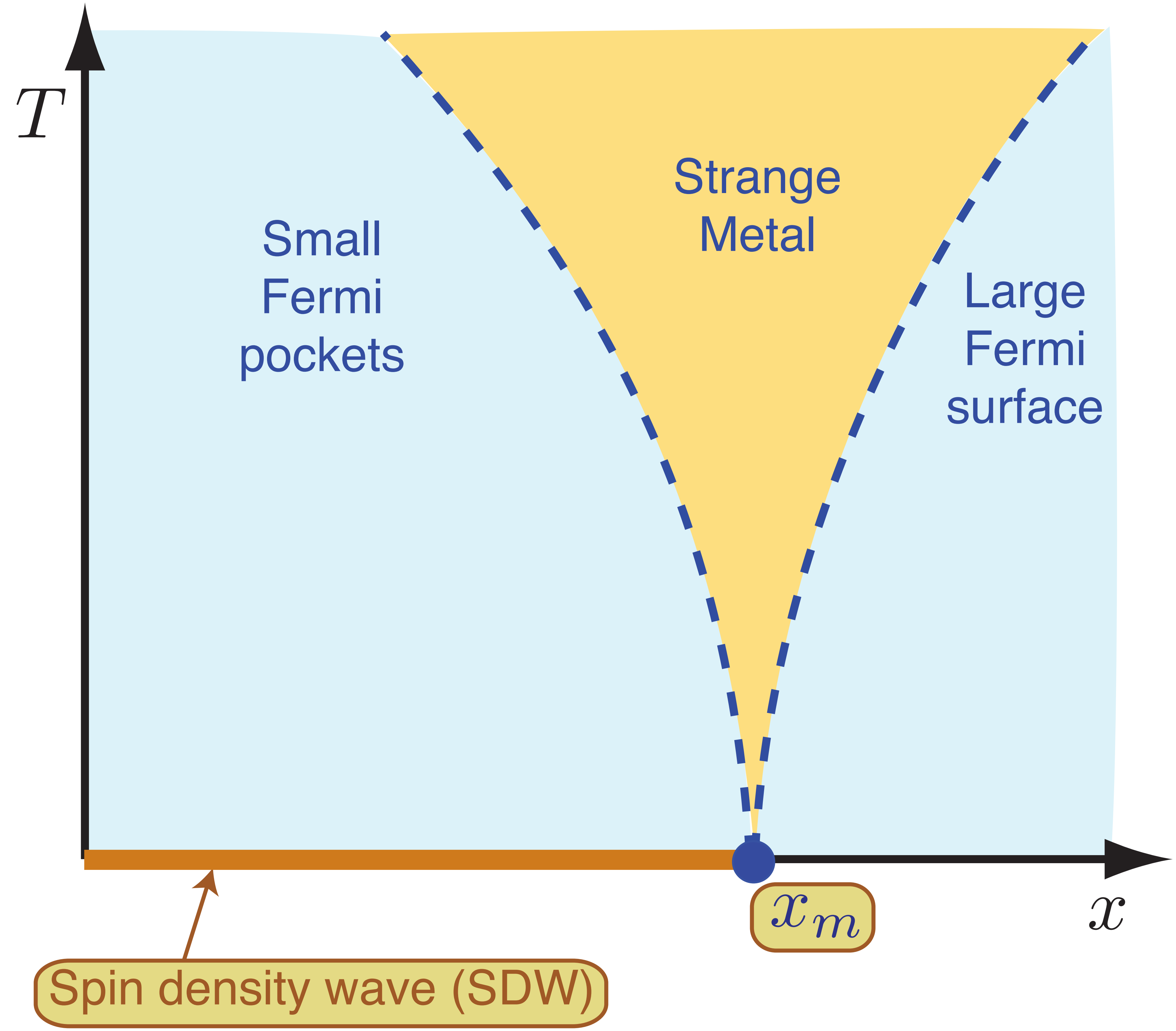
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



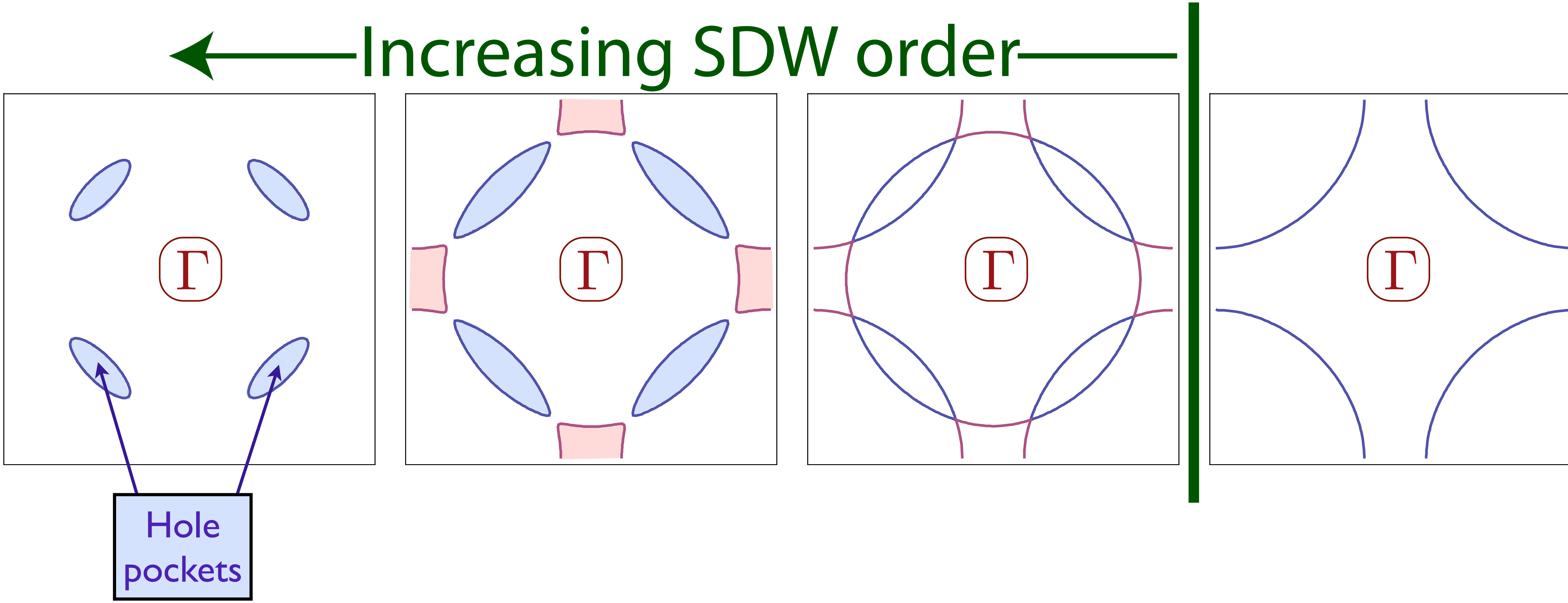
Spin and charge density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

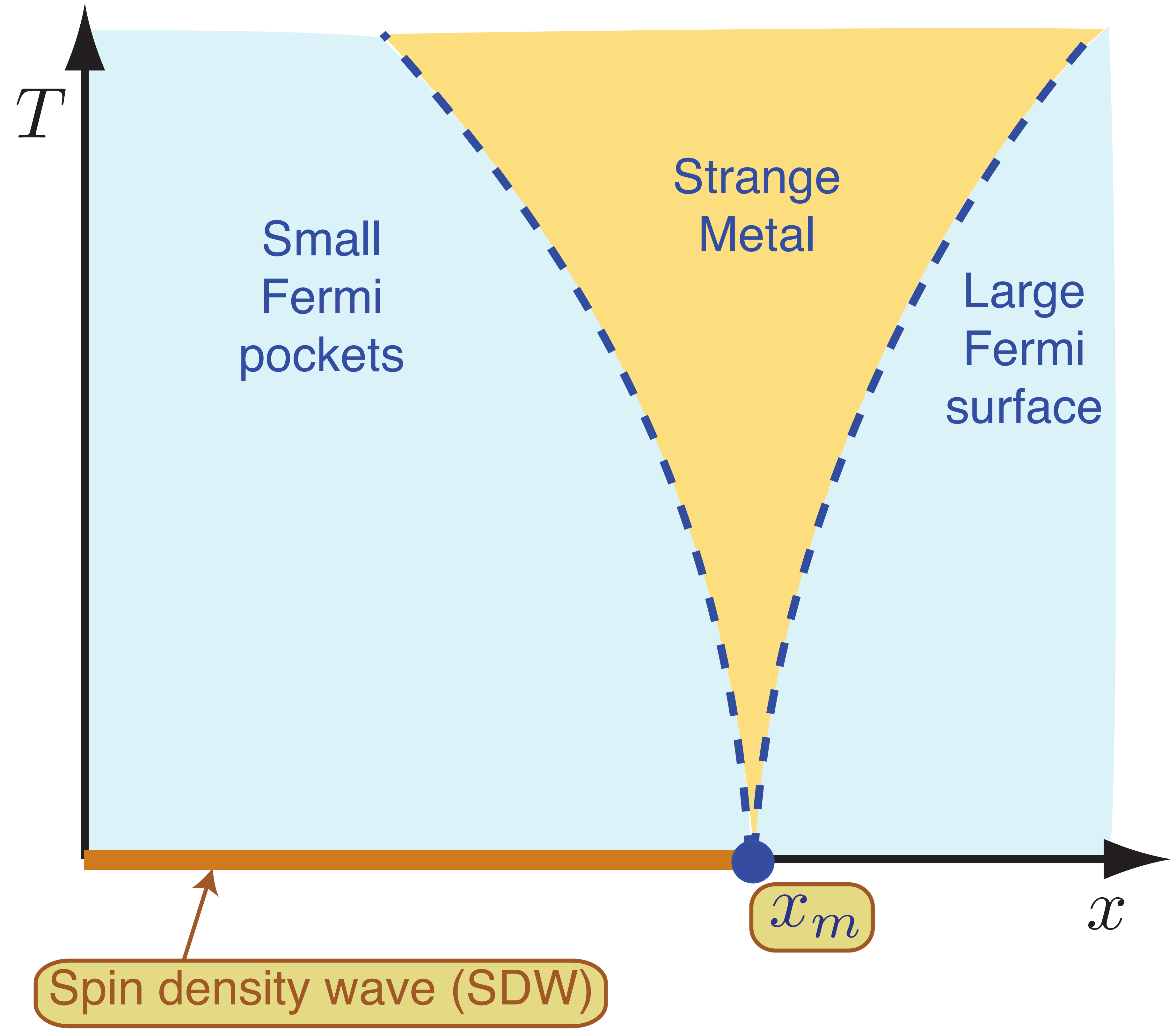
Spin density wave theory in hole-doped cuprates



SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory of quantum criticality in the cuprates



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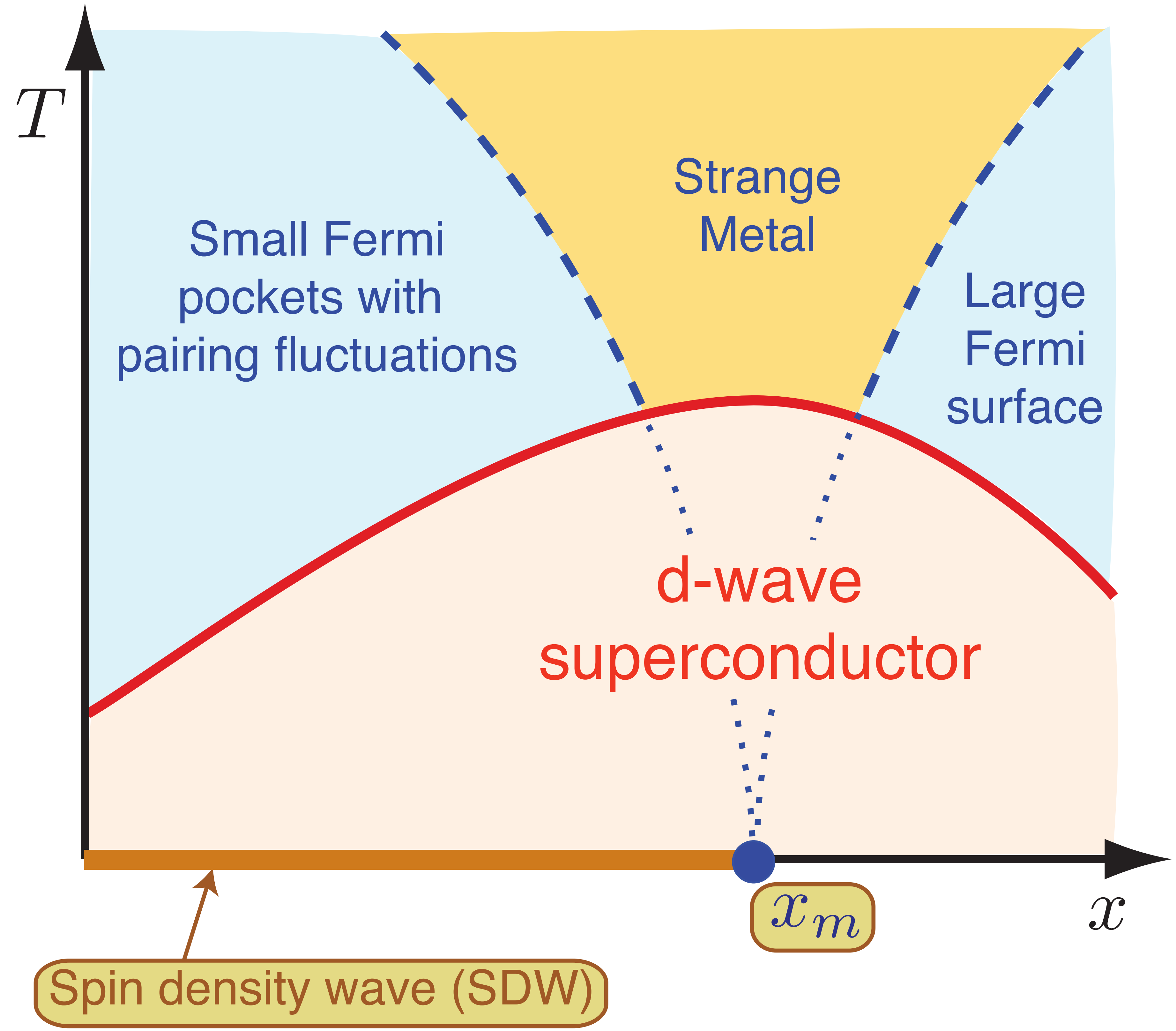
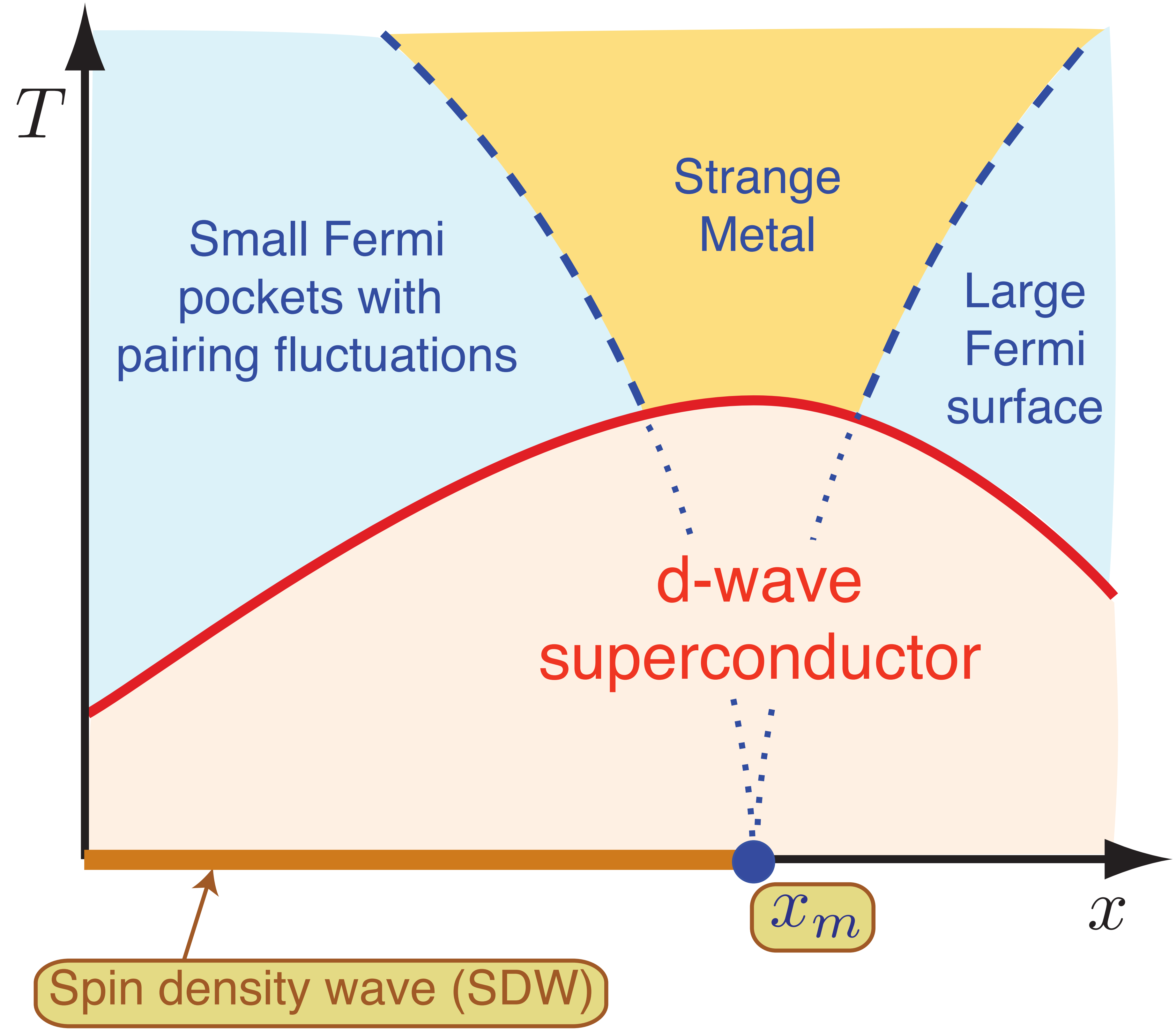


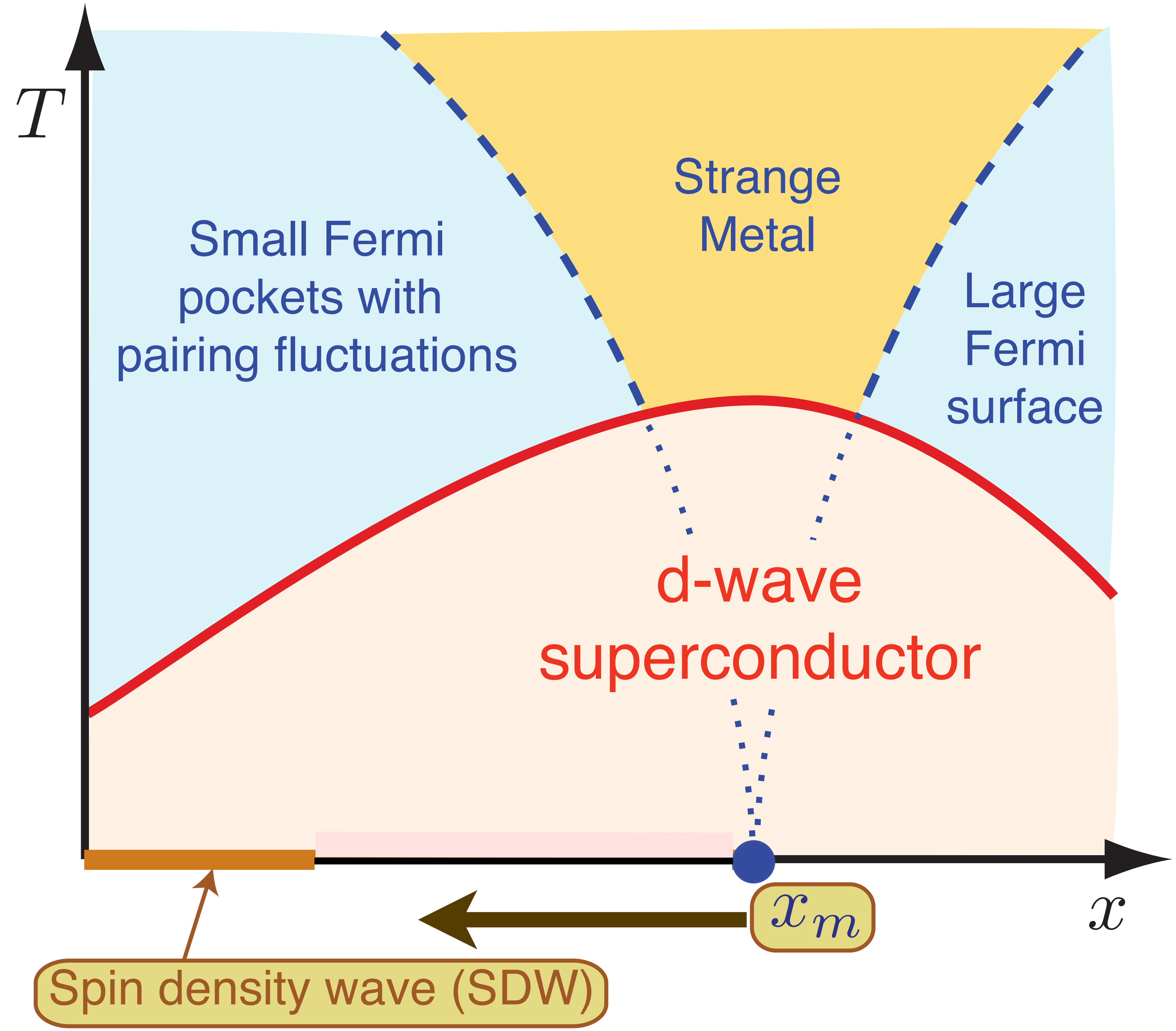
Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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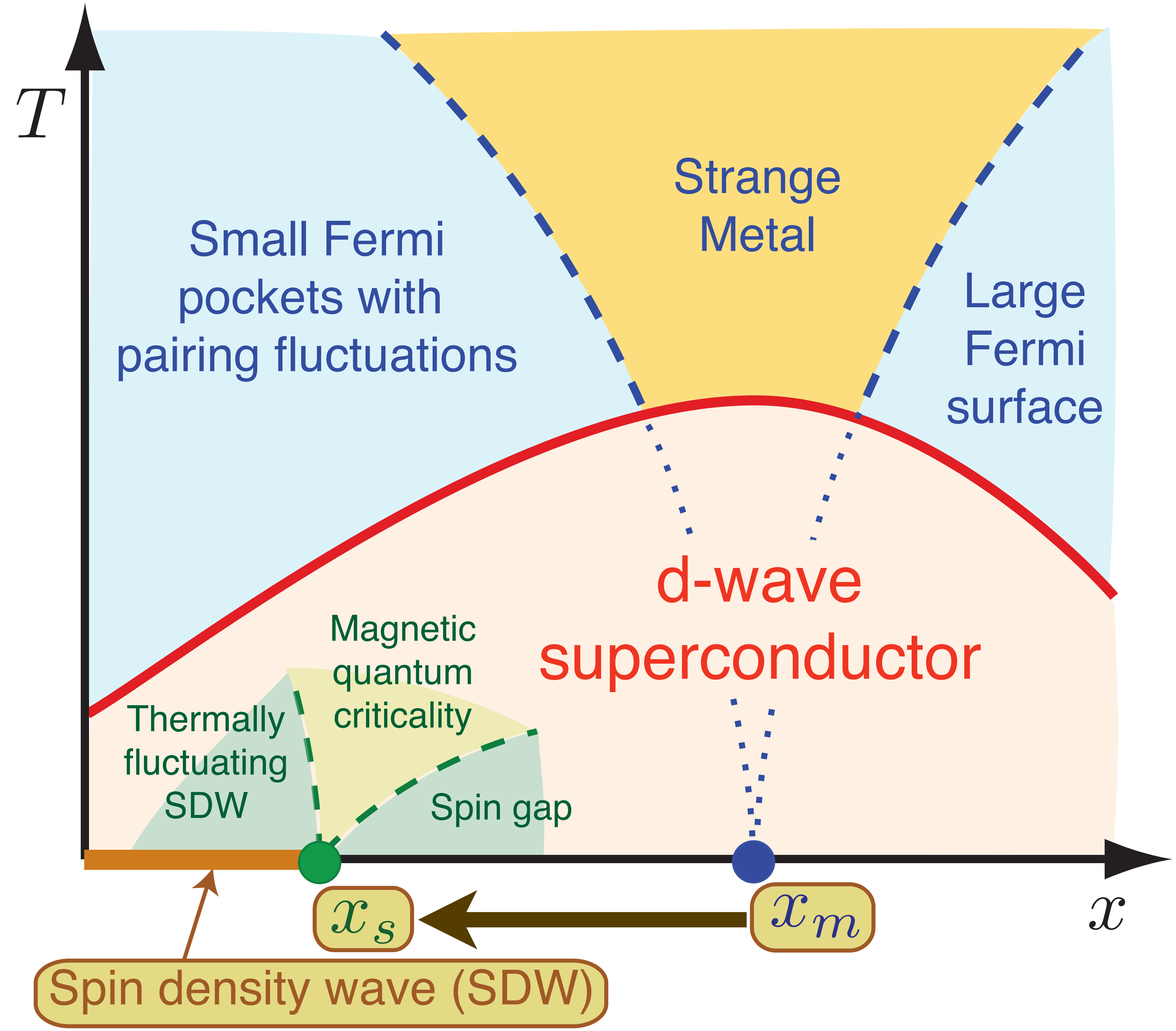

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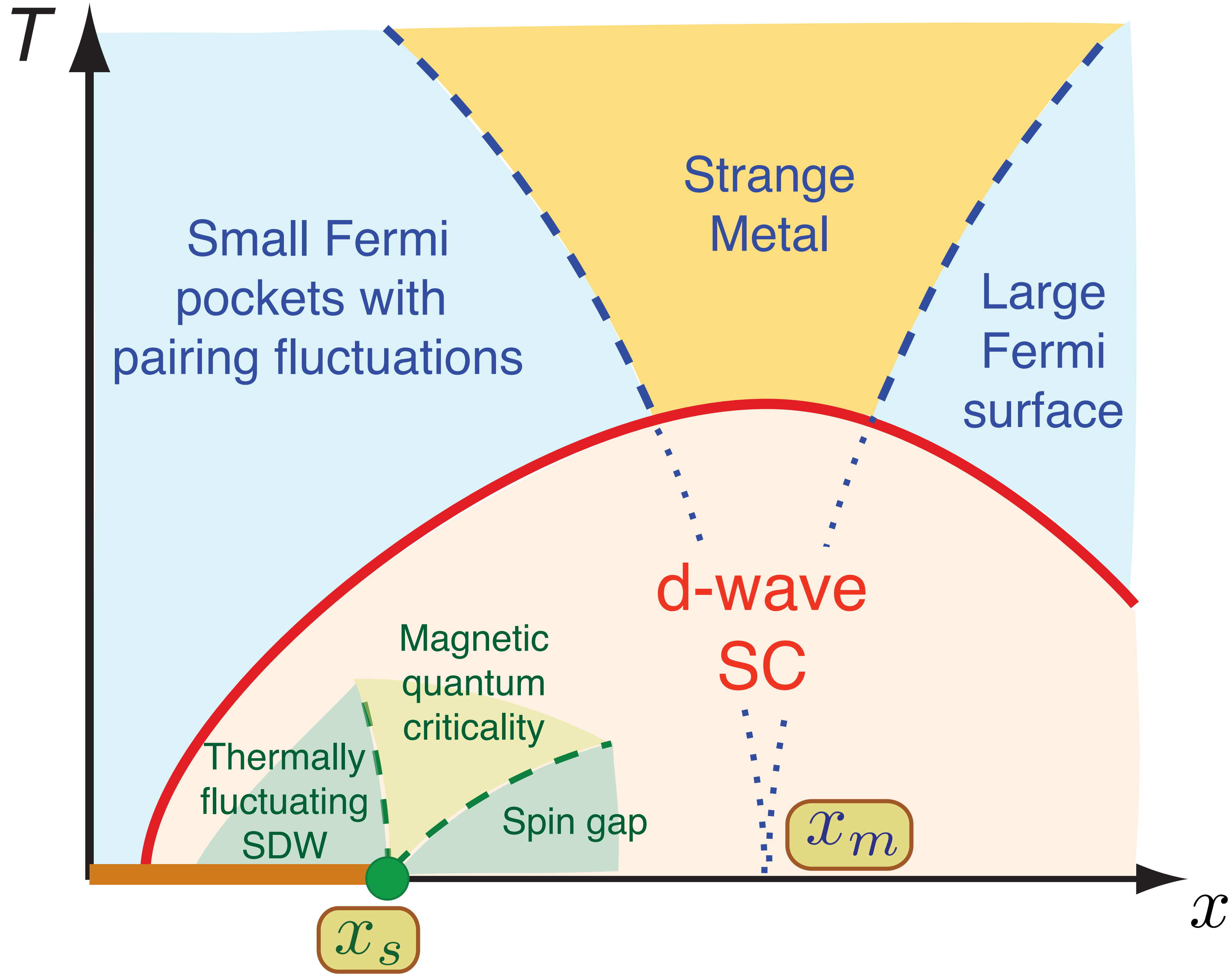
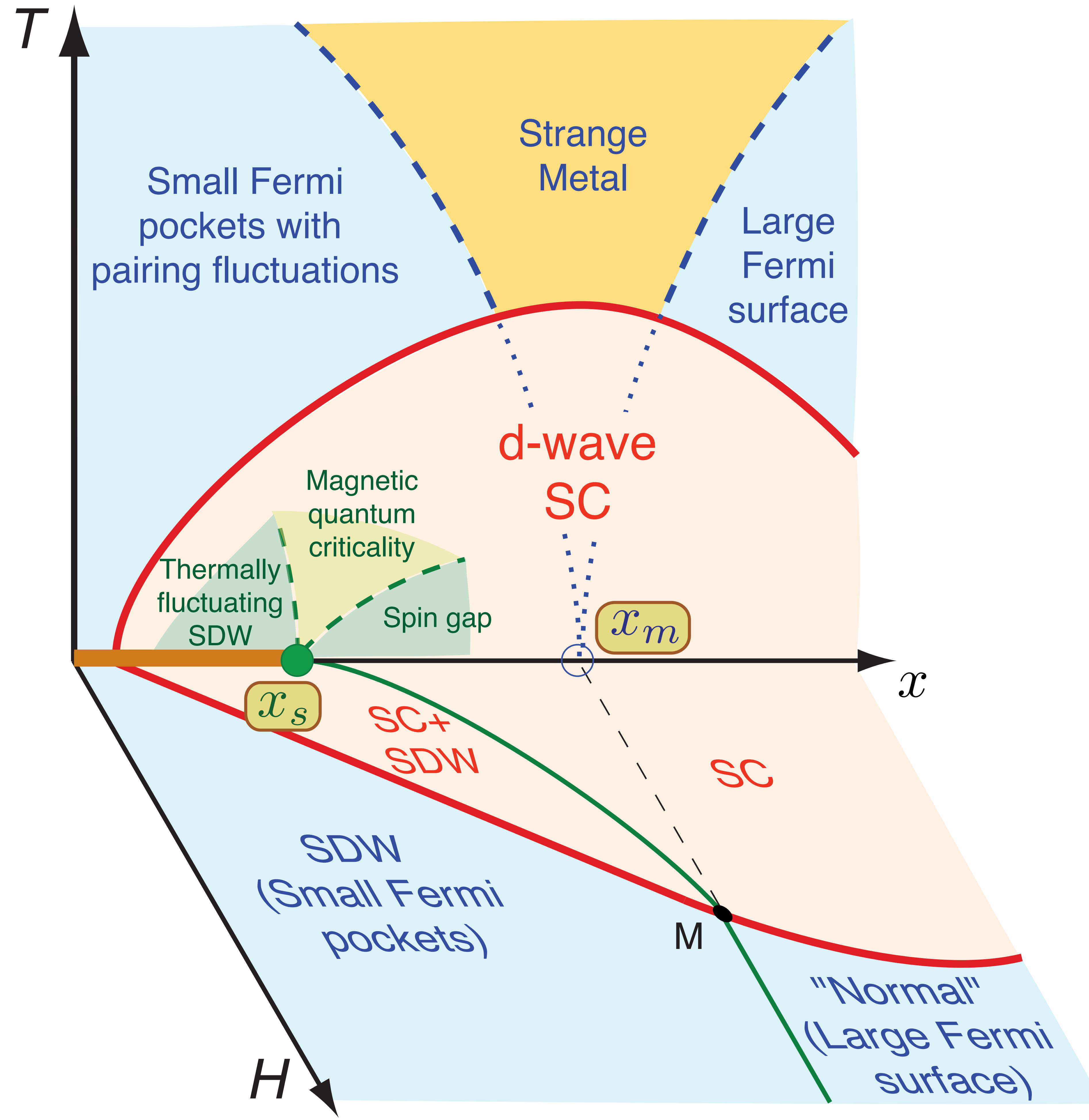
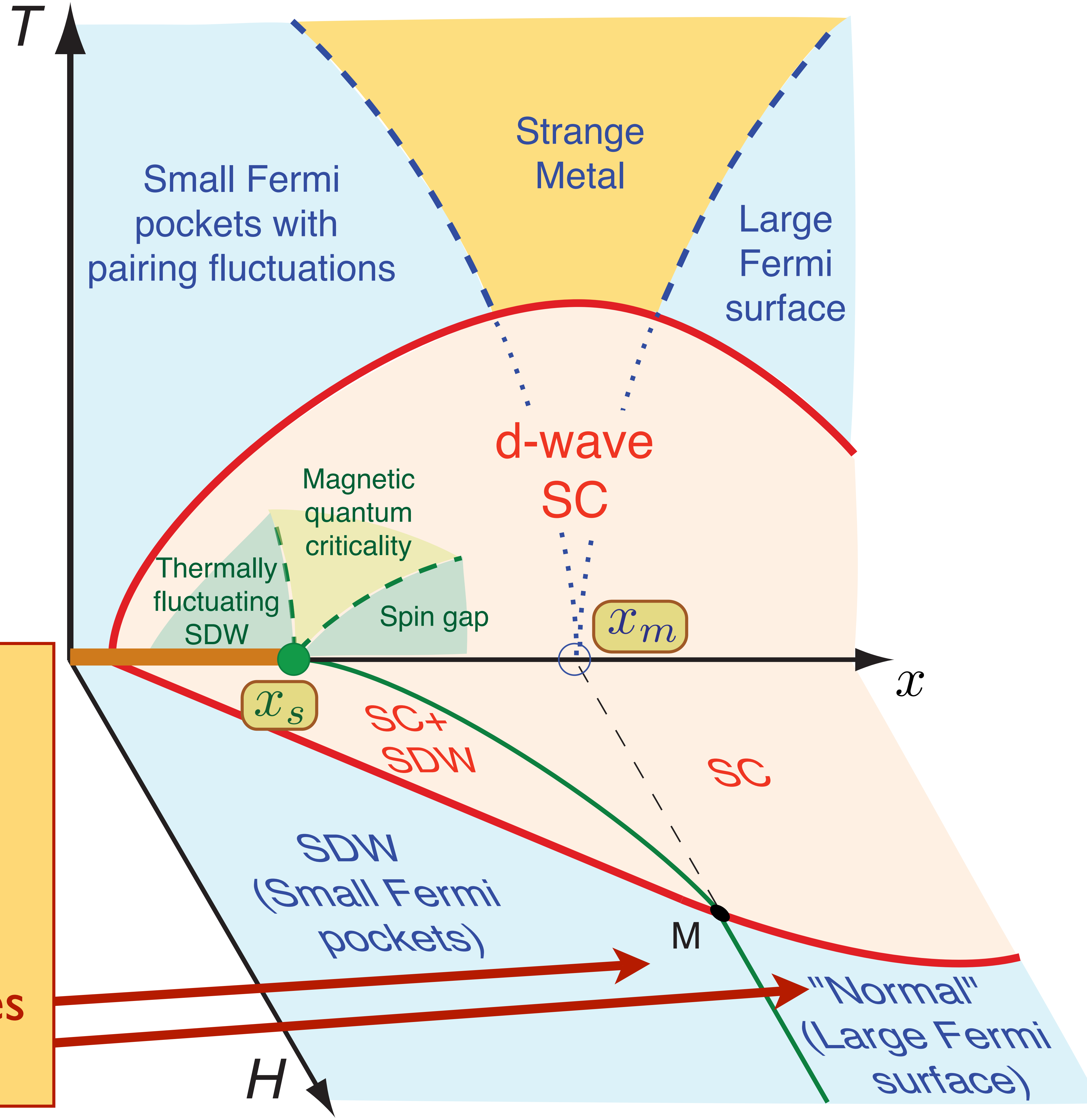


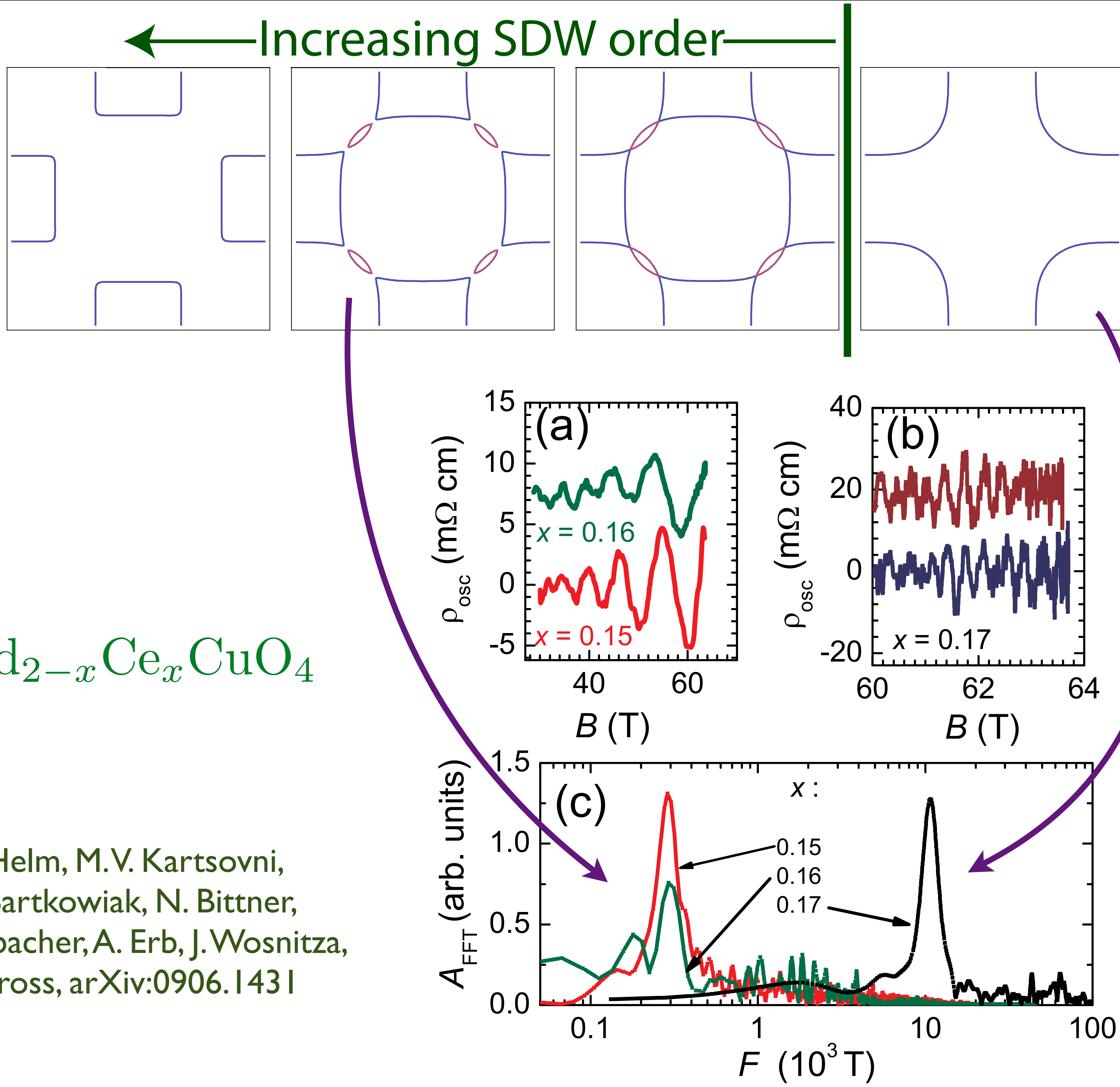
T
Small Fermi
pockets with
pairing fluctuations
Strange
Metal
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x_s
x



T
Small Fermi pockets with pairing fluctuations
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

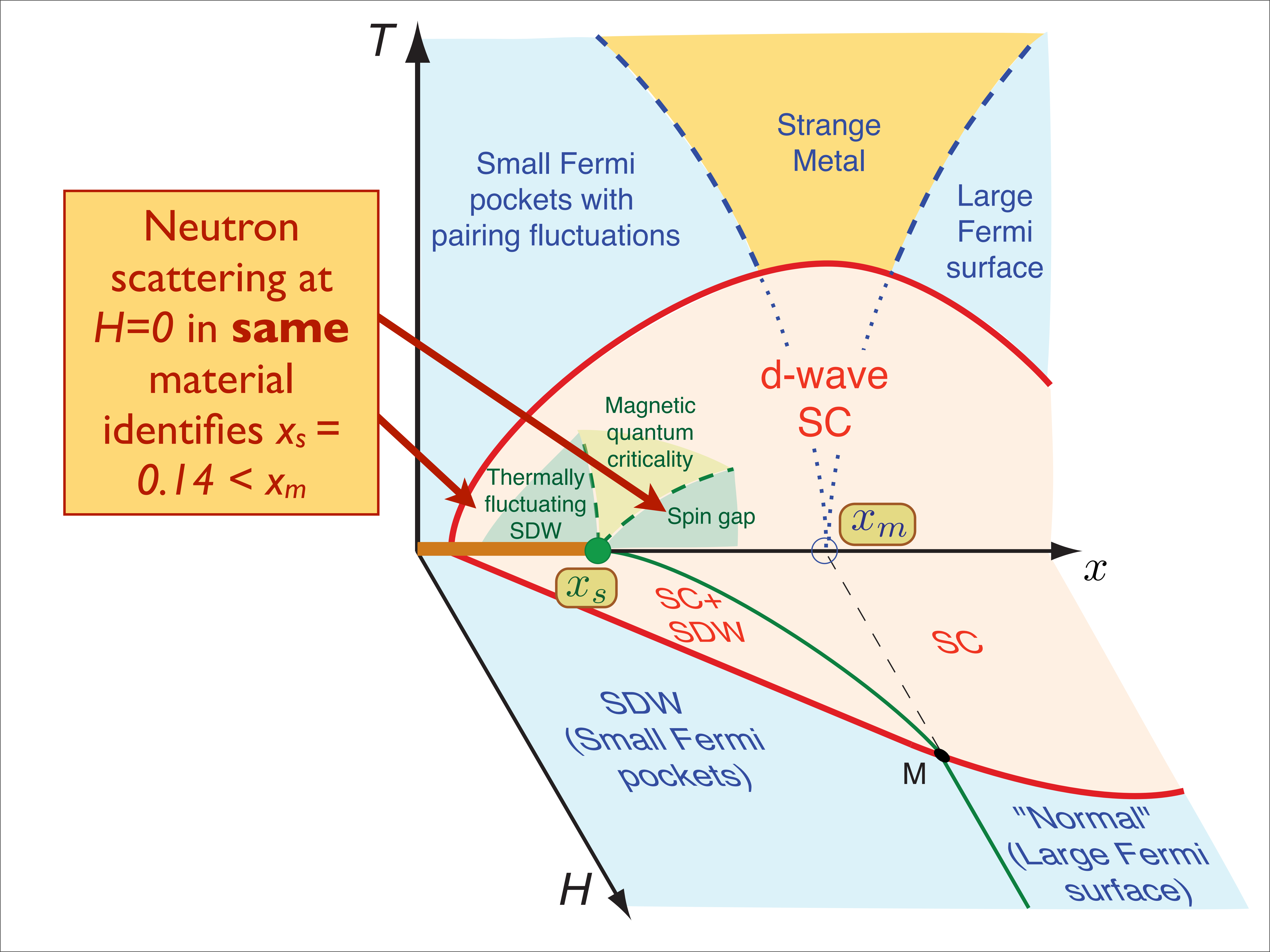


Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$



$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovni,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
R. Gross, arXiv:0906.1431



Neutron scattering at H=0 in **same** material identifies x_s = 0.14 < x_m
T
x
H
Small Fermi pockets with pairing fluctuations
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
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SC+ SDW
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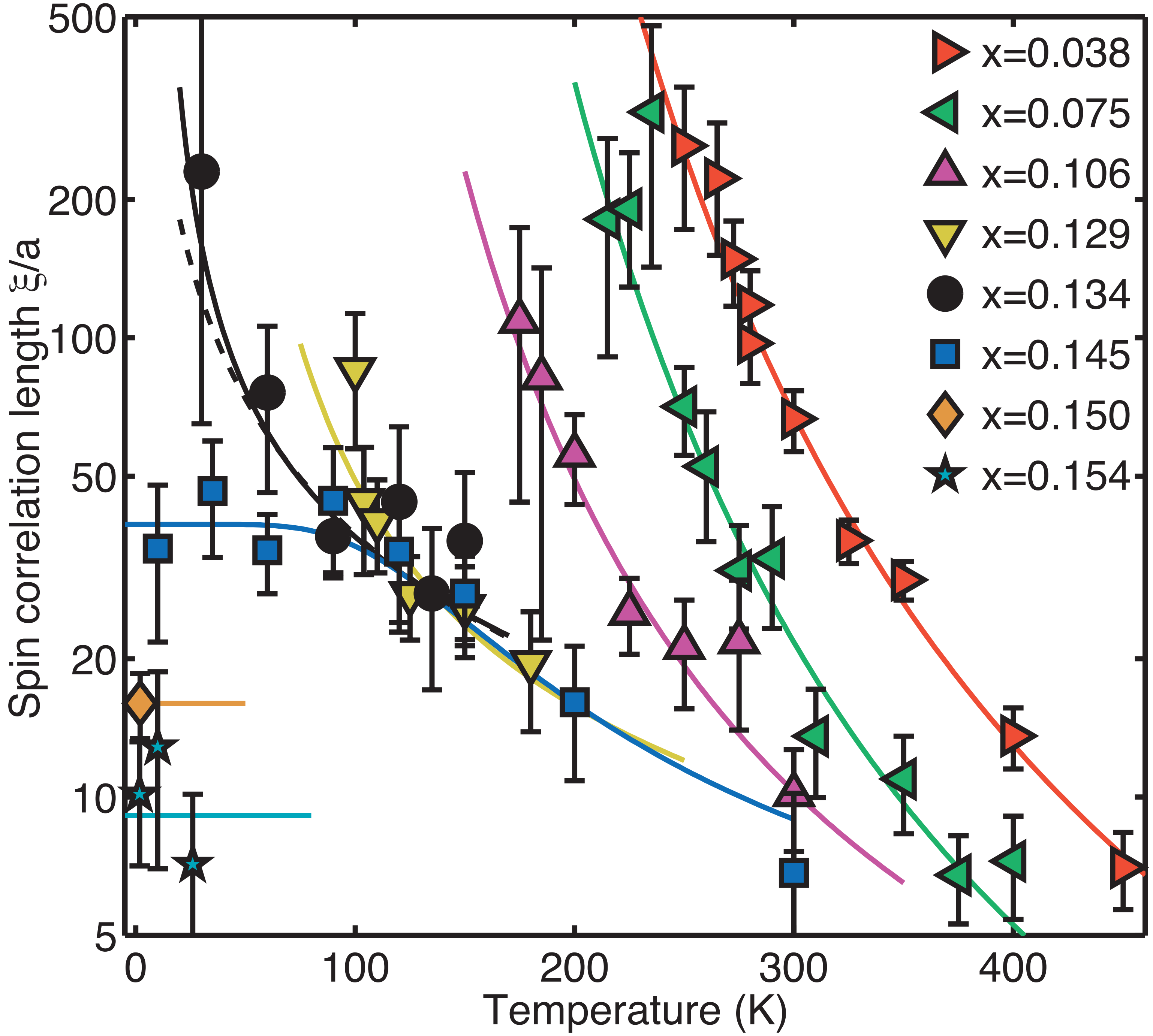
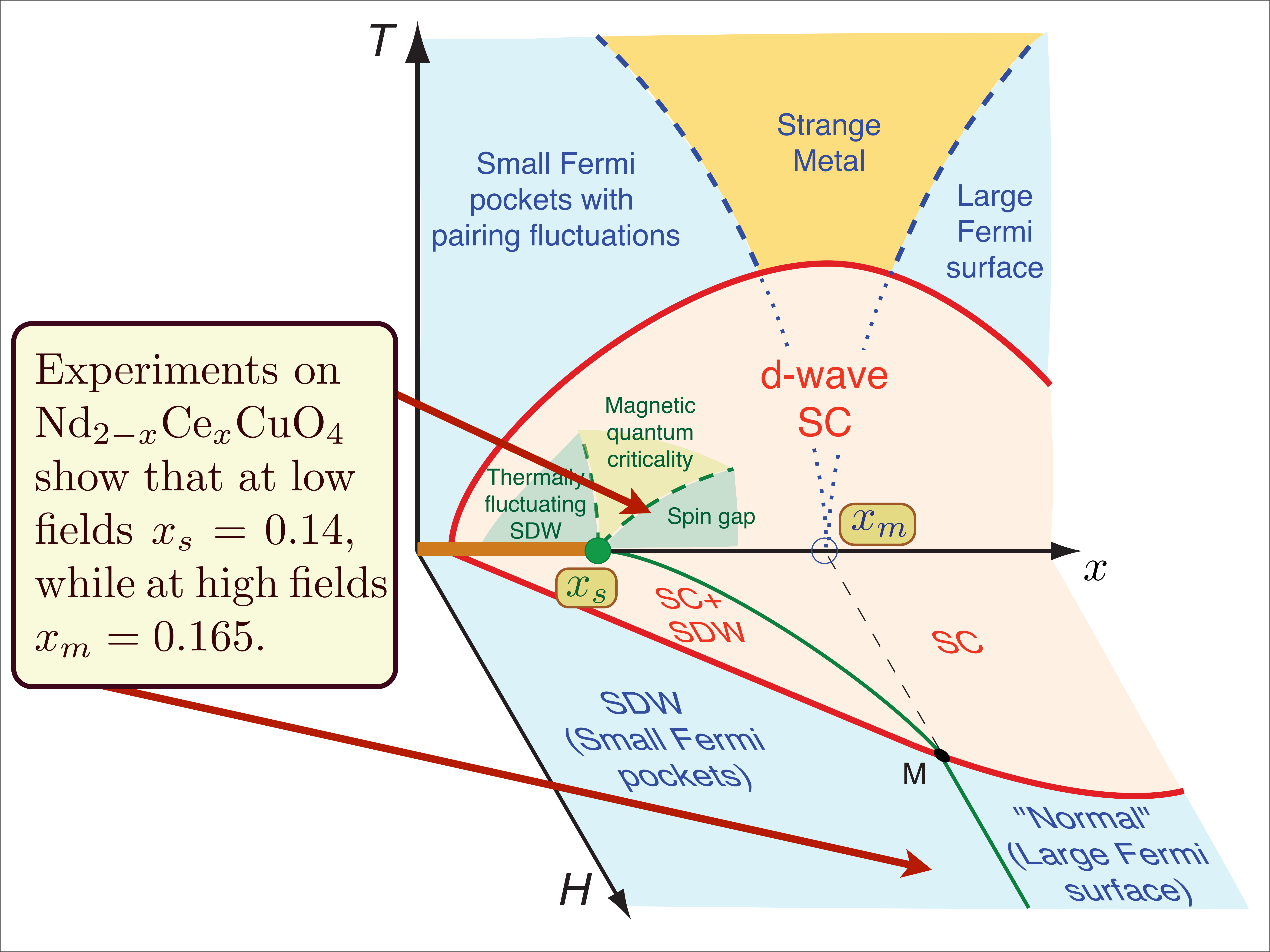
"Normal" (Large Fermi surface)

$Nd_{2-x}Ce_xCuO_4$



E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven,
Nature **445**, 186 (2007).


Experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while at high fields $x_m = 0.165$.
T
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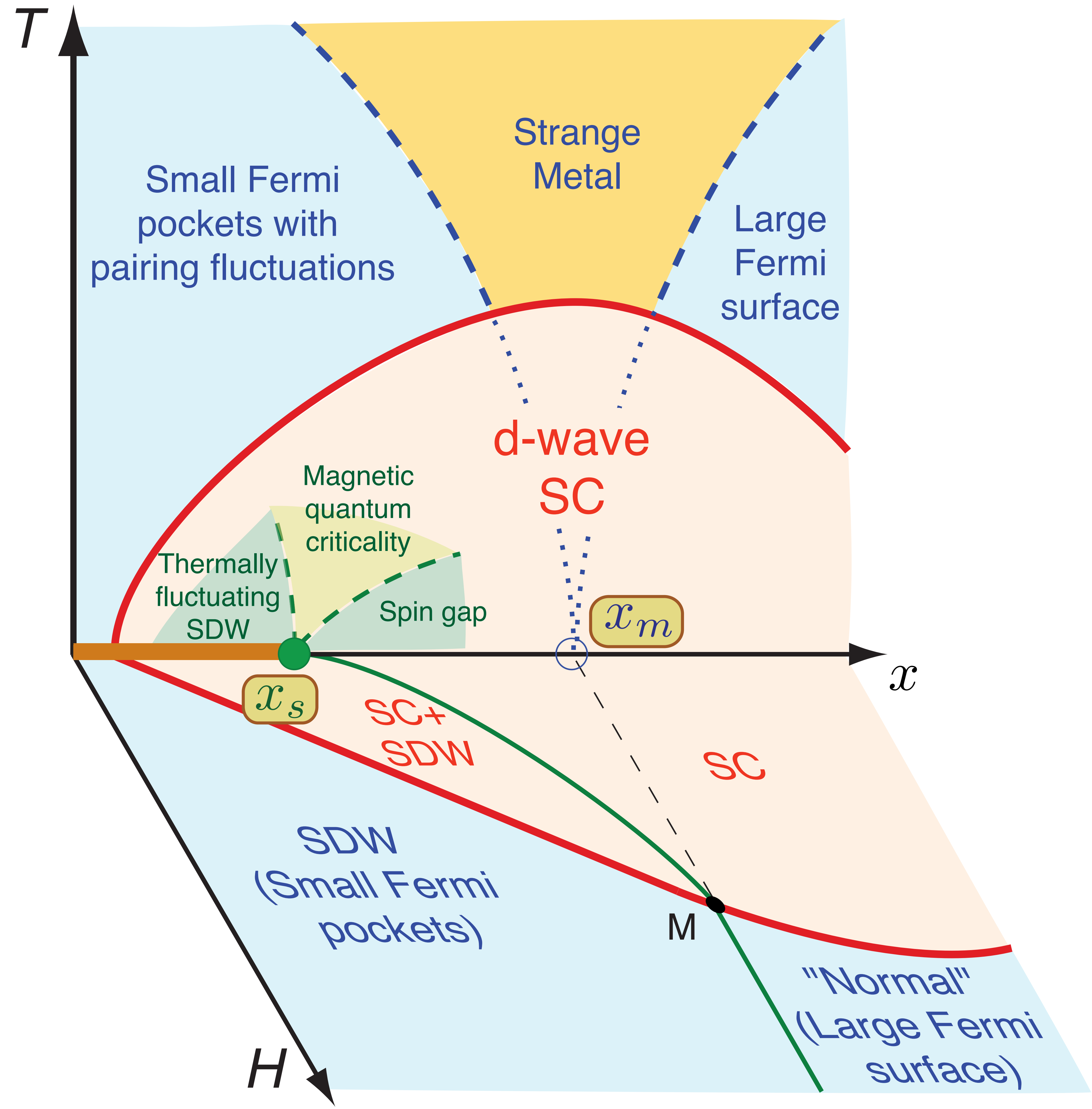
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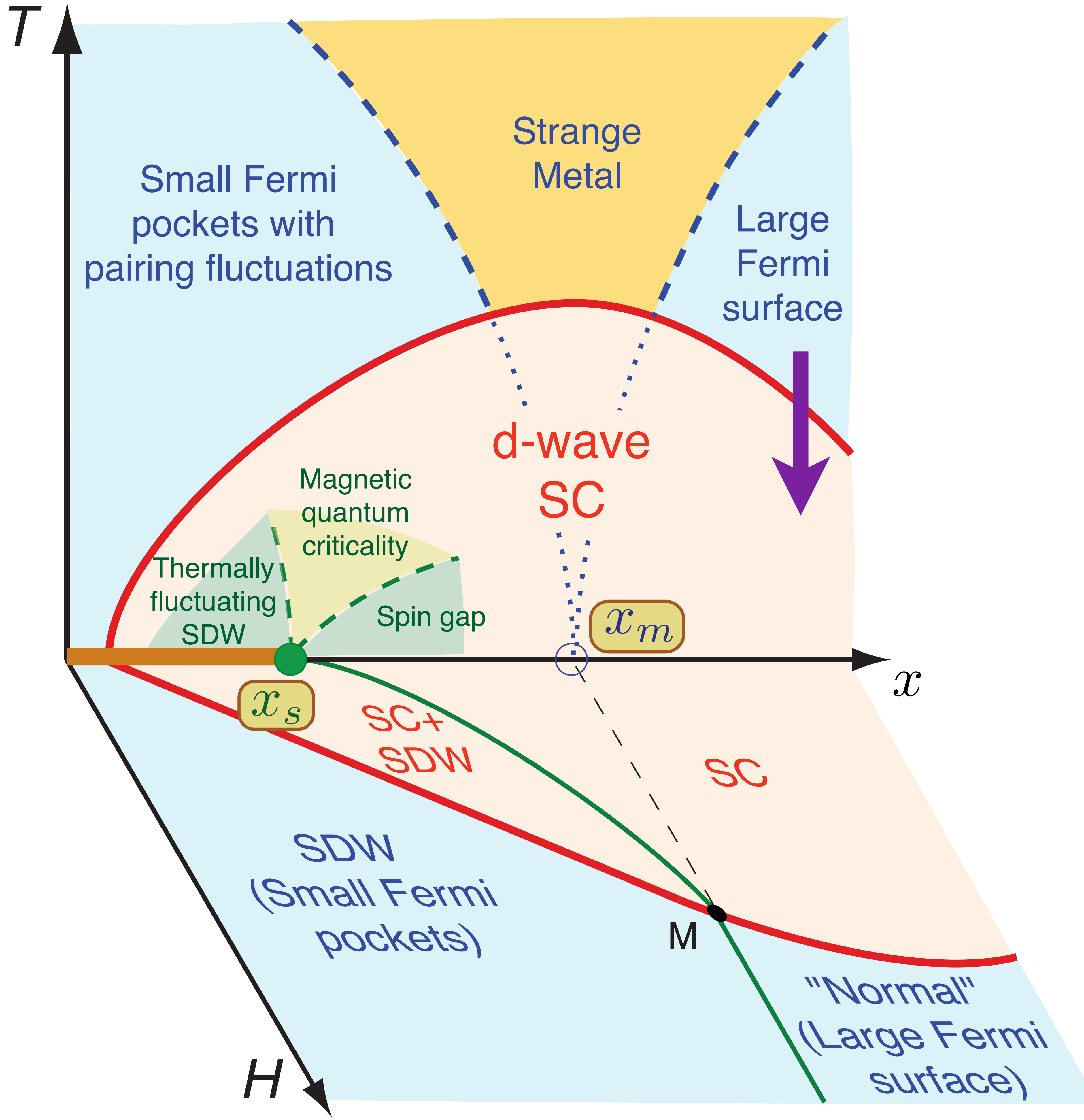

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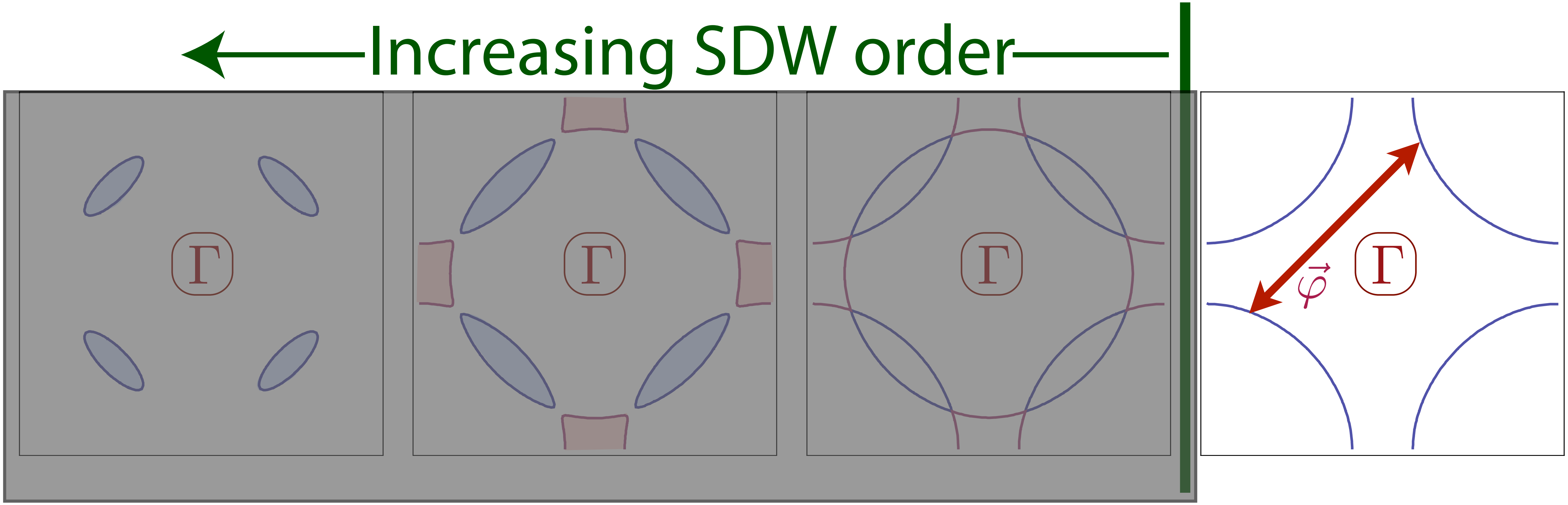
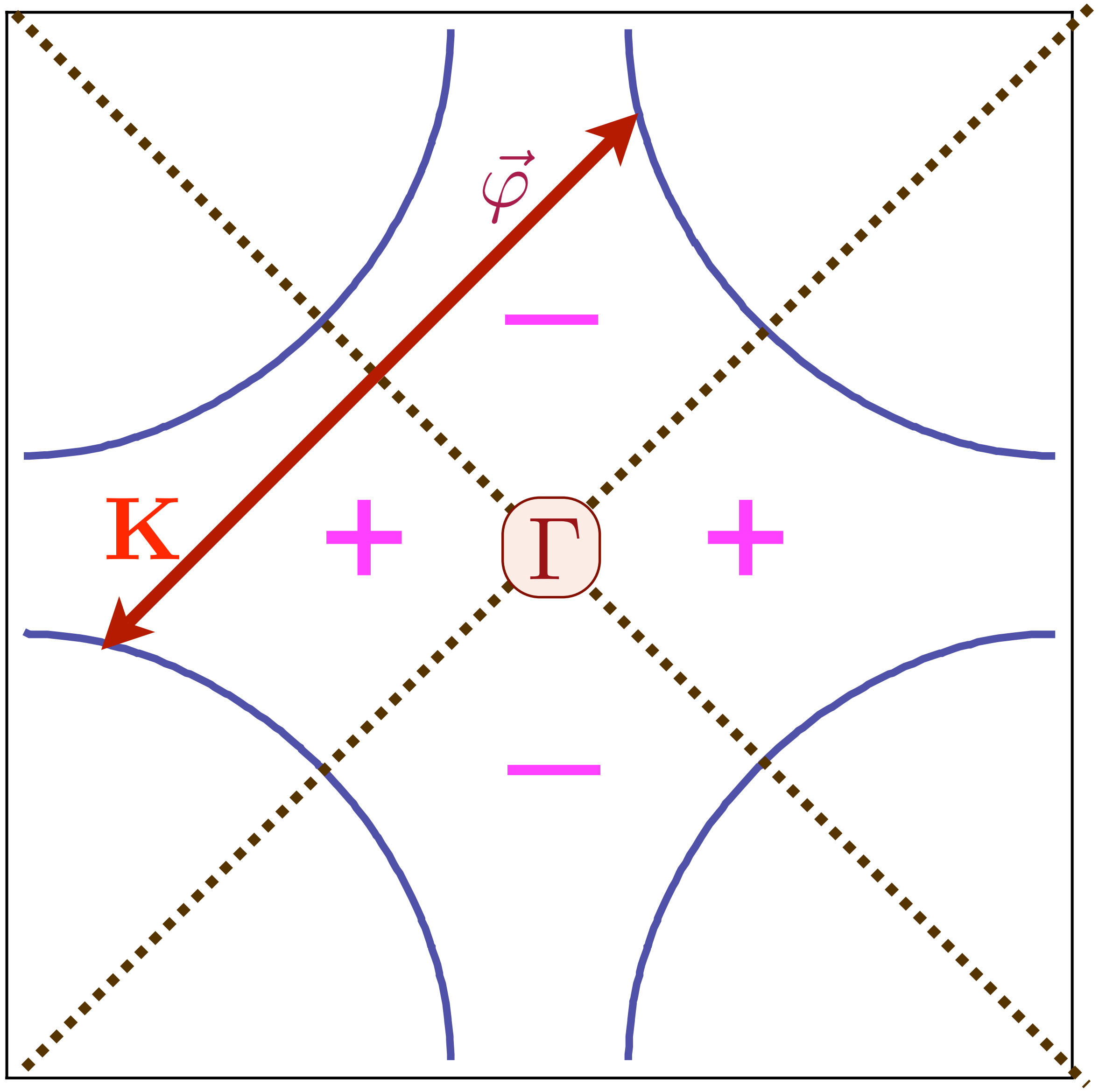
Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

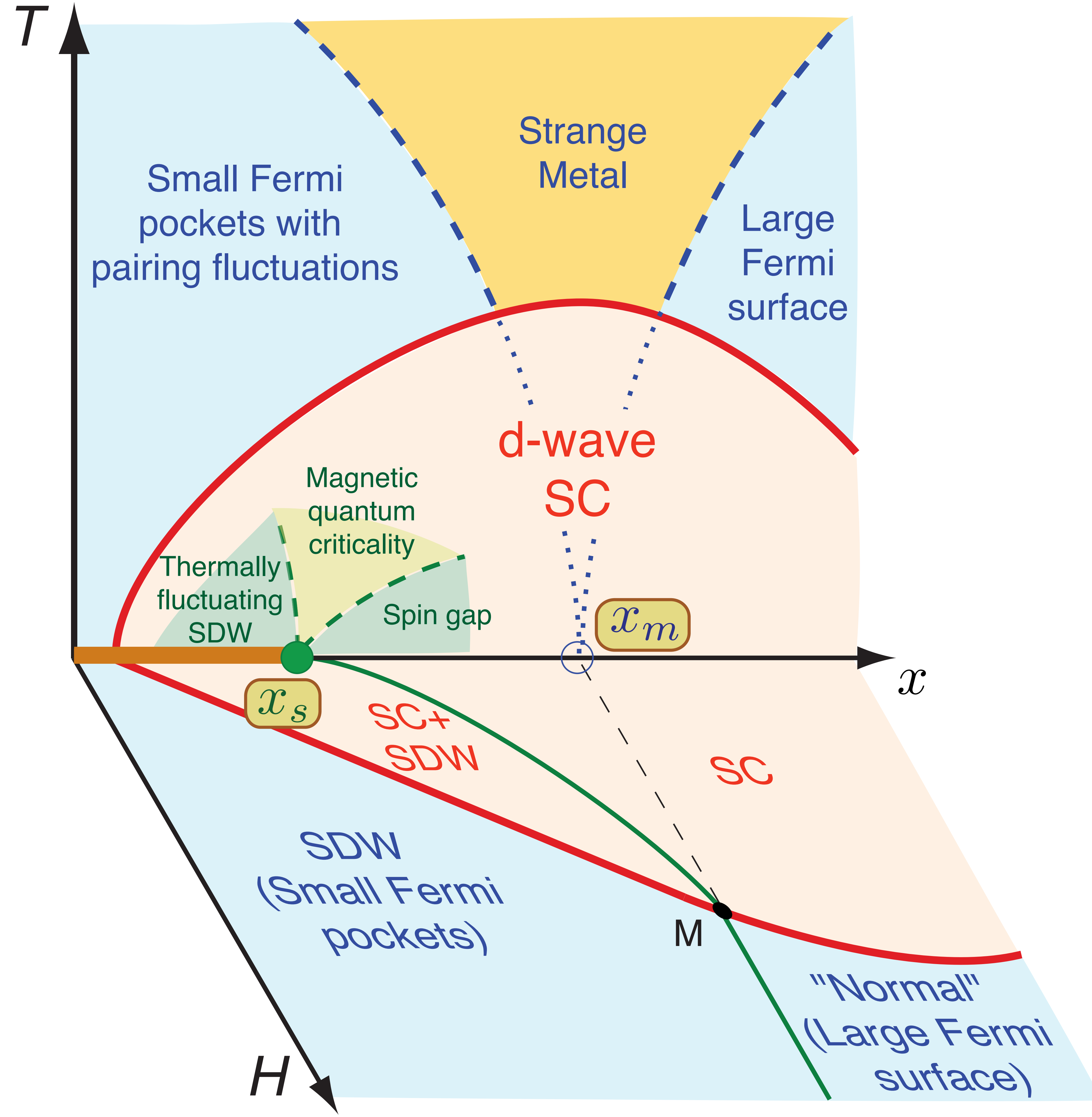
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

d-wave pairing of the large Fermi surface



$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)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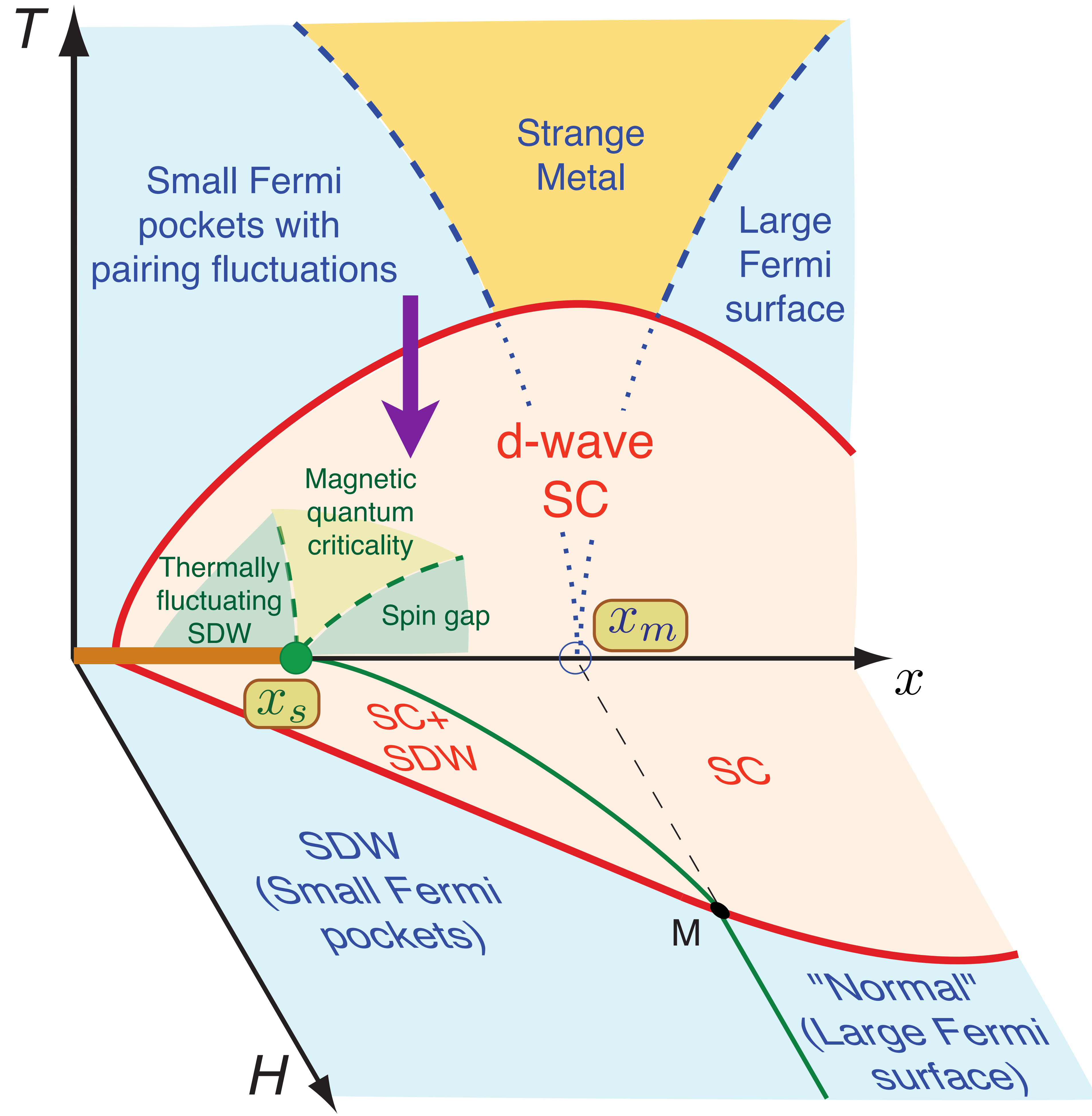
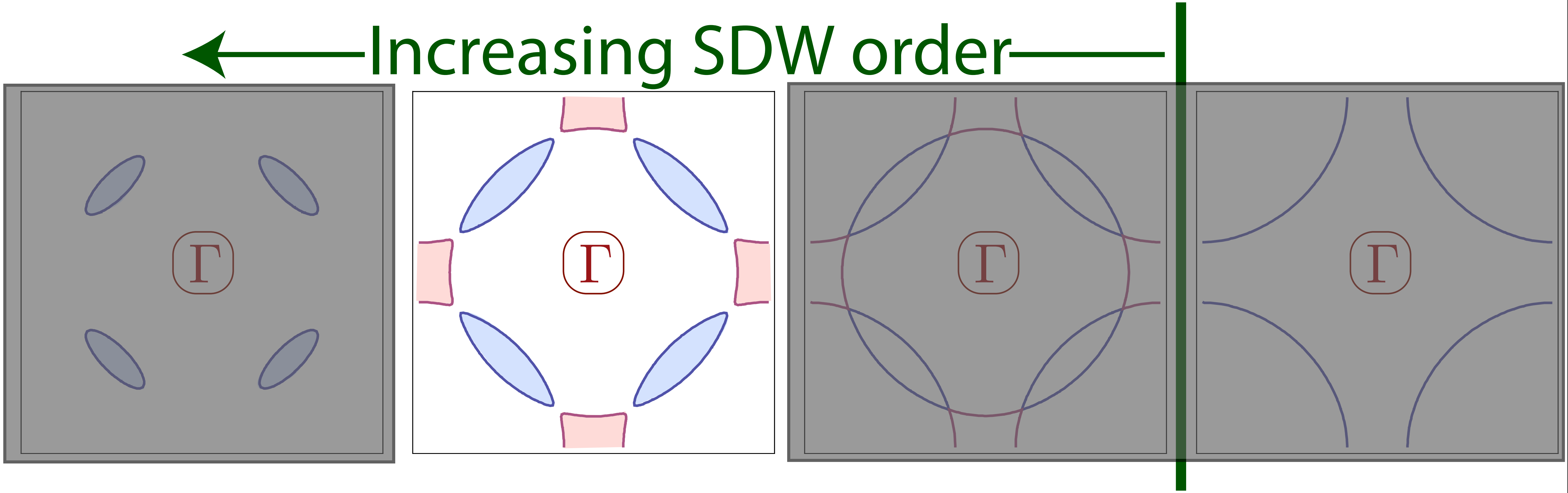
"Normal" (Large Fermi surface)
H



T
Strange Metal
Small Fermi pockets with pairing fluctuations
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Theory of underdoped cuprates



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}$$

the theory is invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha \quad ; \quad \psi_+ \rightarrow e^{-i\theta} \psi_+ \quad ; \quad \psi_- \rightarrow e^{i\theta} \psi_-$$

and the SDW order is given by

$$\hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

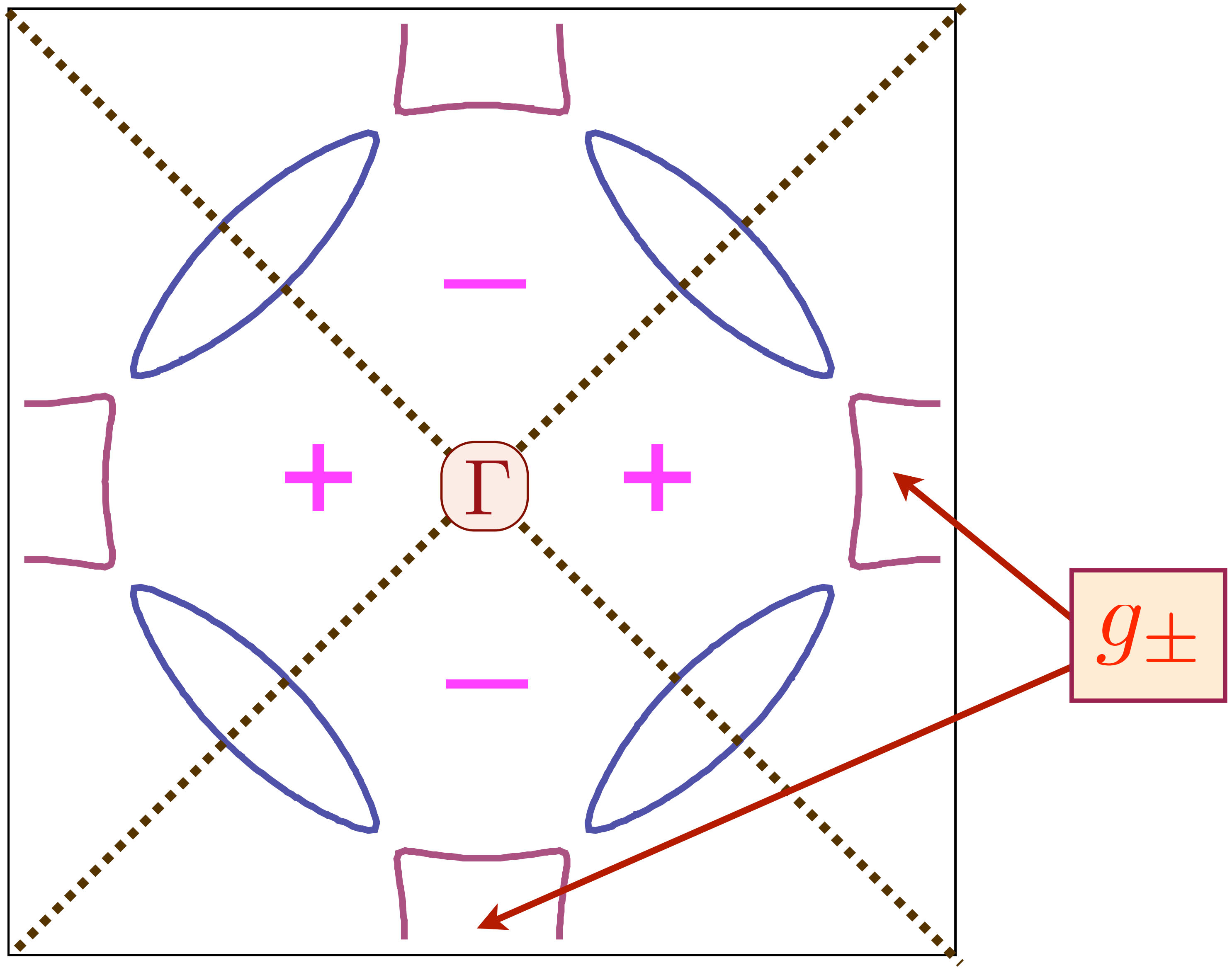
Theory of underdoped cuprates

We obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,

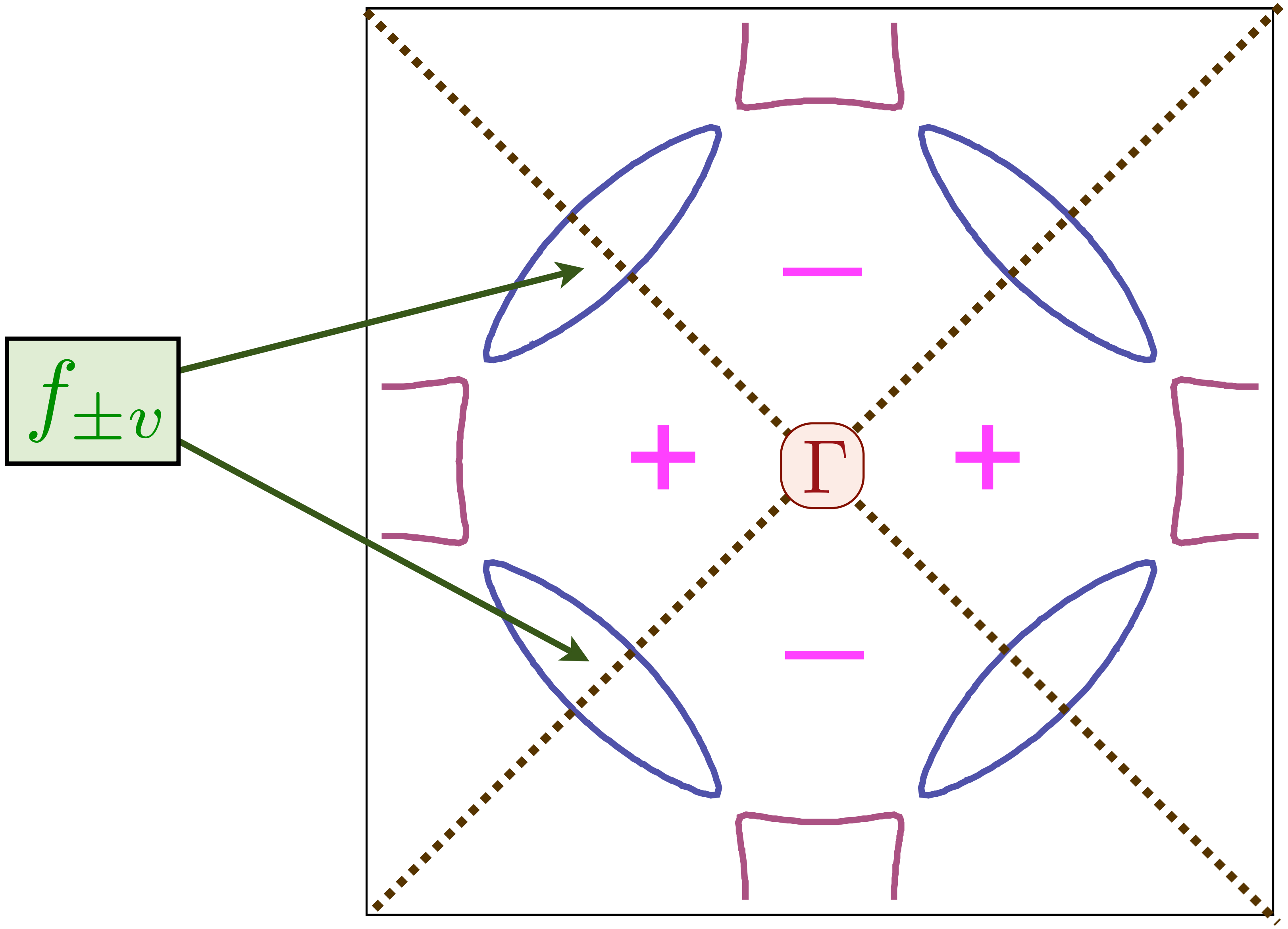
- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order

Strong s-wave pairing of the $g_\pm$ electron pockets



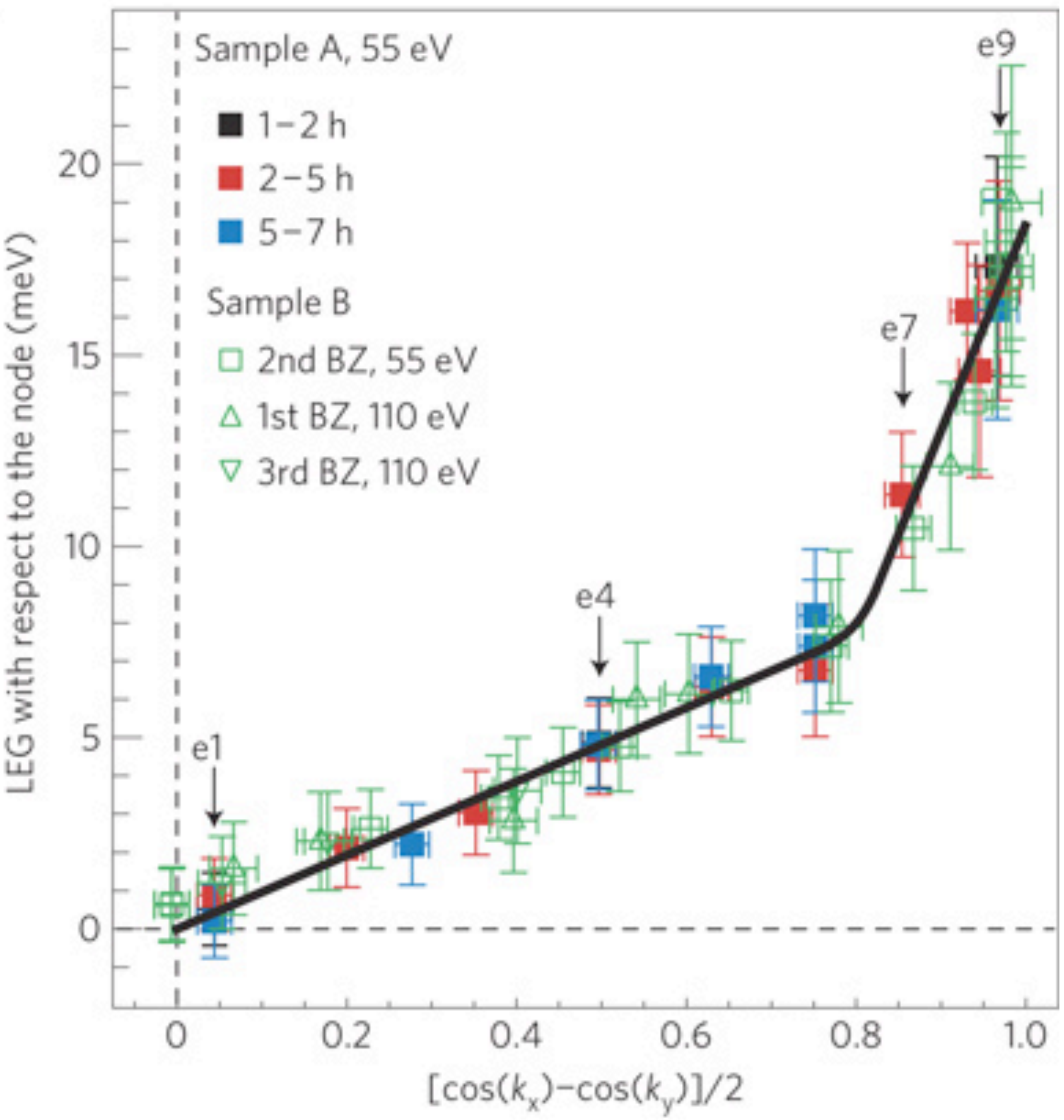
$$\langle g_+ g_- \rangle = \Delta$$

Weak p-wave pairing of the $f_\pm$ hole pockets



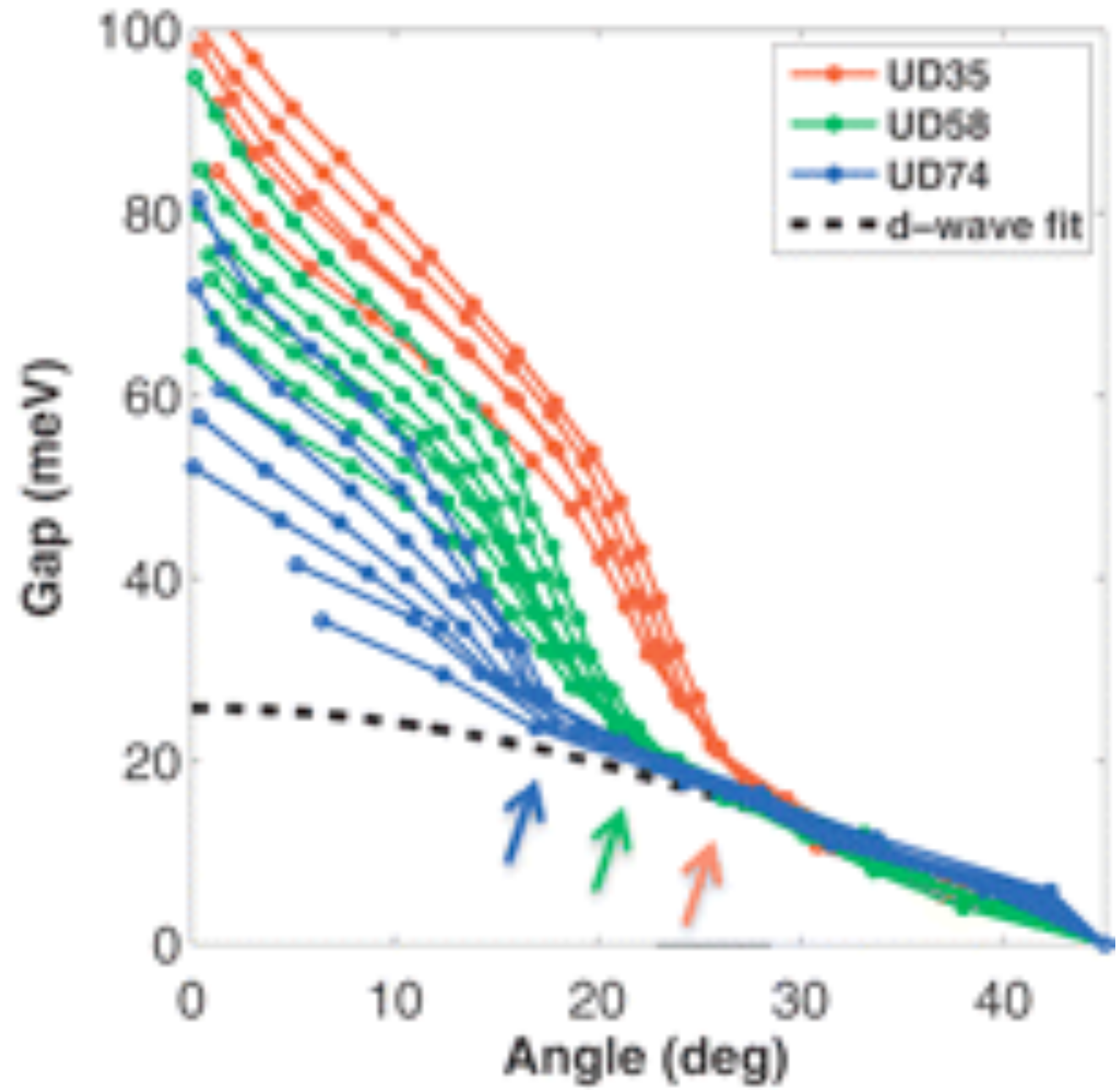
$$\begin{aligned}\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle &\sim (k_x - k_y) J \langle g_+ g_-\rangle;\\ \langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &\sim (k_x + k_y) J \langle g_+ g_-\rangle;\\ \langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &= 0,\end{aligned}$$

Photoemission in LBCO

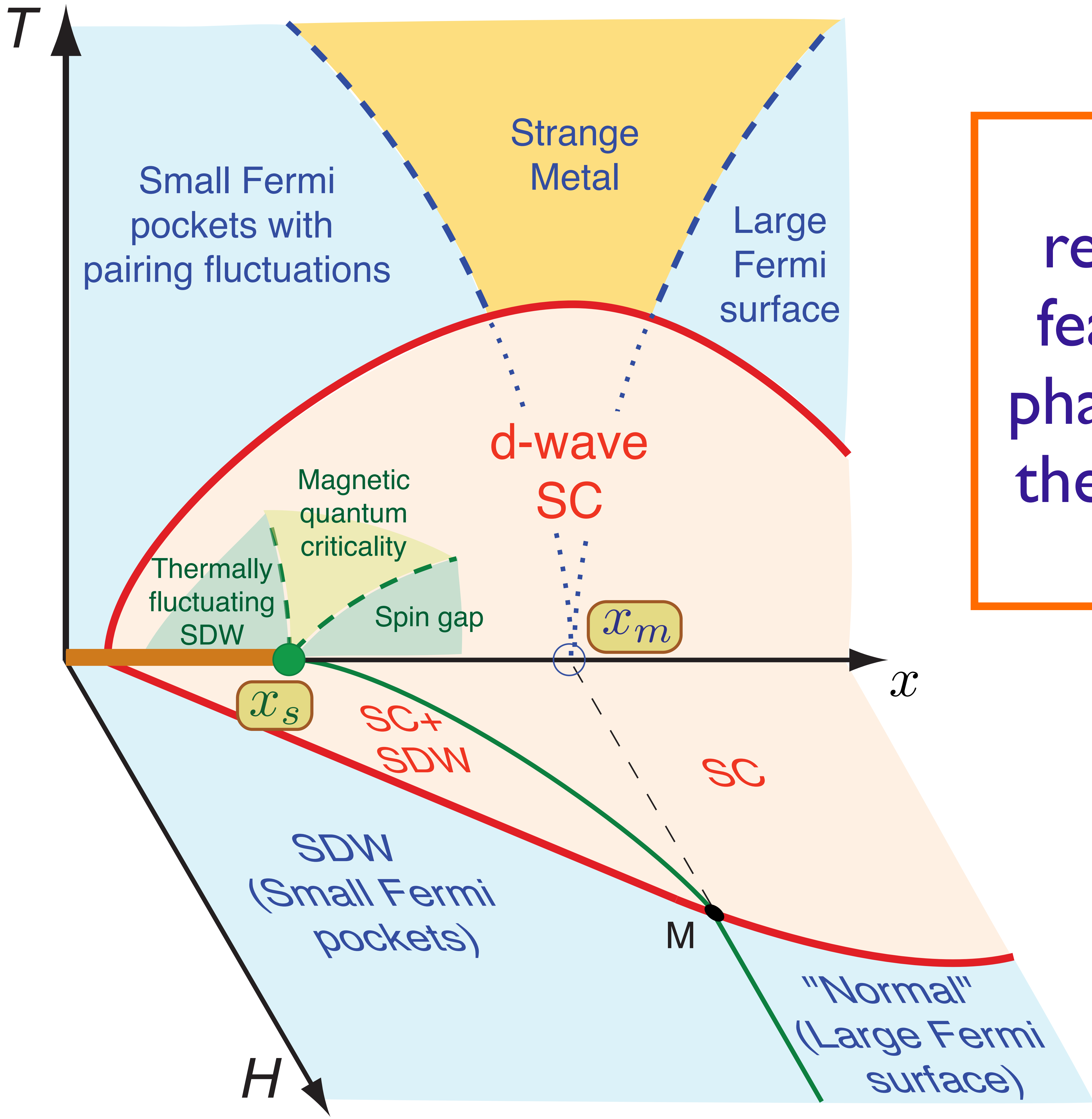


R.-H. He, K. Tanaka, S.-K. Mo, T. Sasagawa, M. Fujita, T. Adachi, N. Mannella, K. Yamada, Y. Koike, Z. Hussain and Z.-X. Shen, *Nature Physics* **5**, 119 (2008)

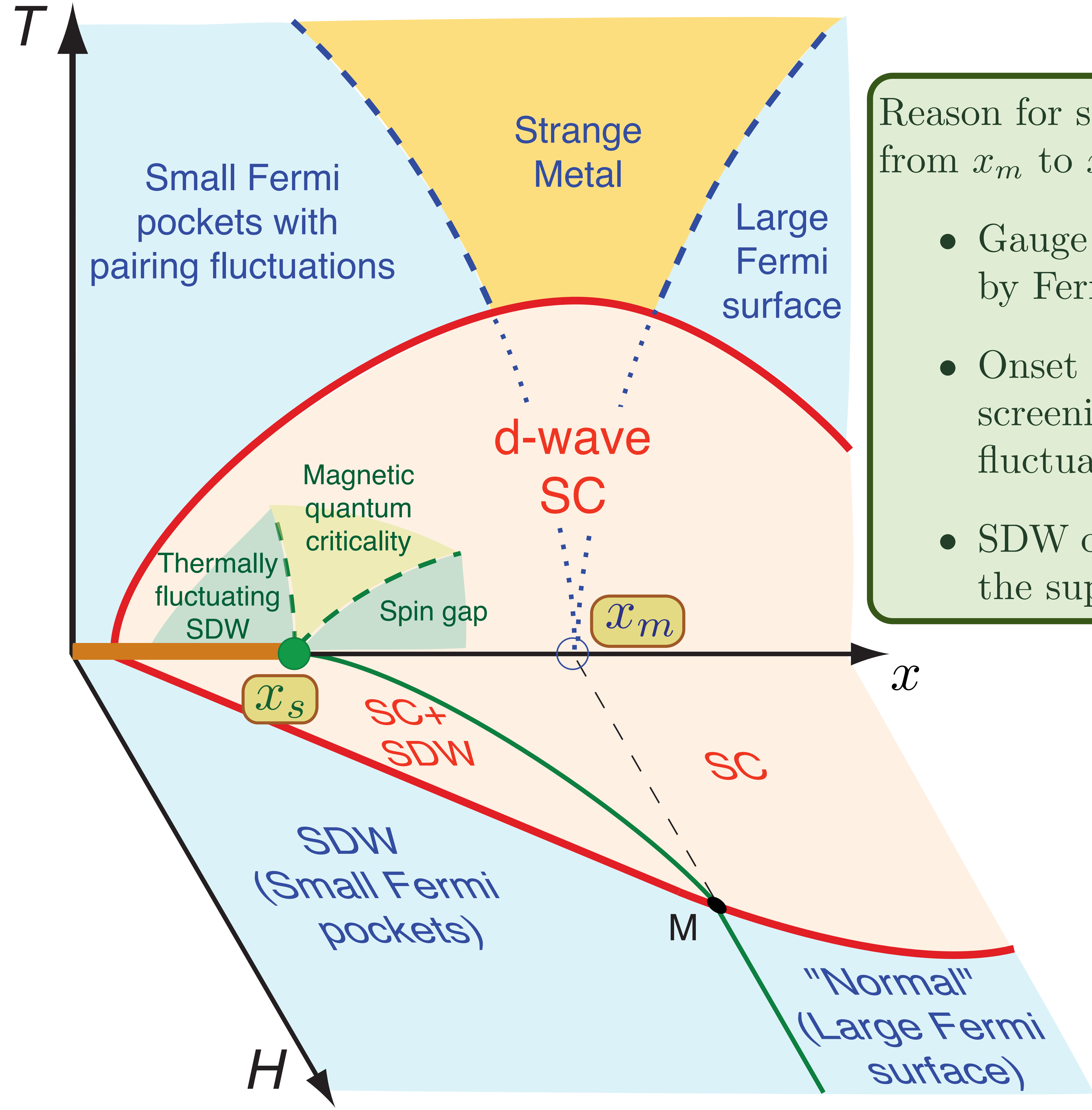
STM in BSCCO



A. Pushp, C.V. Parker, A. N. Pasupathy, K. K. Gomes, S. Ono, J. Wen, Z. Xu, G. Gu, and A. Yazdani, *Science* **324**, 1689 (2009)



Theory reproduces all features of the phase diagram in the underdoped regime



Reason for shift in onset of SDW from x_m to x_s:

- Gauge fluctuations are screened by Fermi surface in metal
- Onset of pairing suppresses screening, and enhances gauge fluctuations
- SDW order is suppressed in the superconductor

Outline

1. Quantum-criticality of bosons
   Superfluid-insulator transition

2. The AdS/CFT correspondence
   Exact solutions for quantum critical transport

3. Quantum criticality in the cuprates
   Global phase diagram and the spin density wave transition in metals

4. AdS_4 theory of compressible quantum liquids
   Fermi surfaces and quantum oscillations

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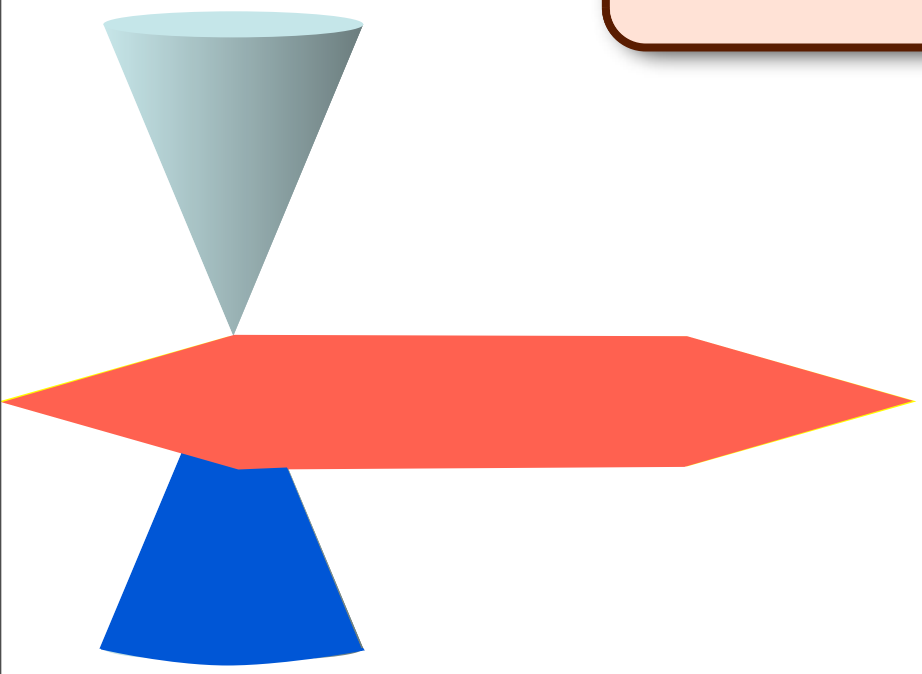
   Fermi surfaces and quantum oscillations

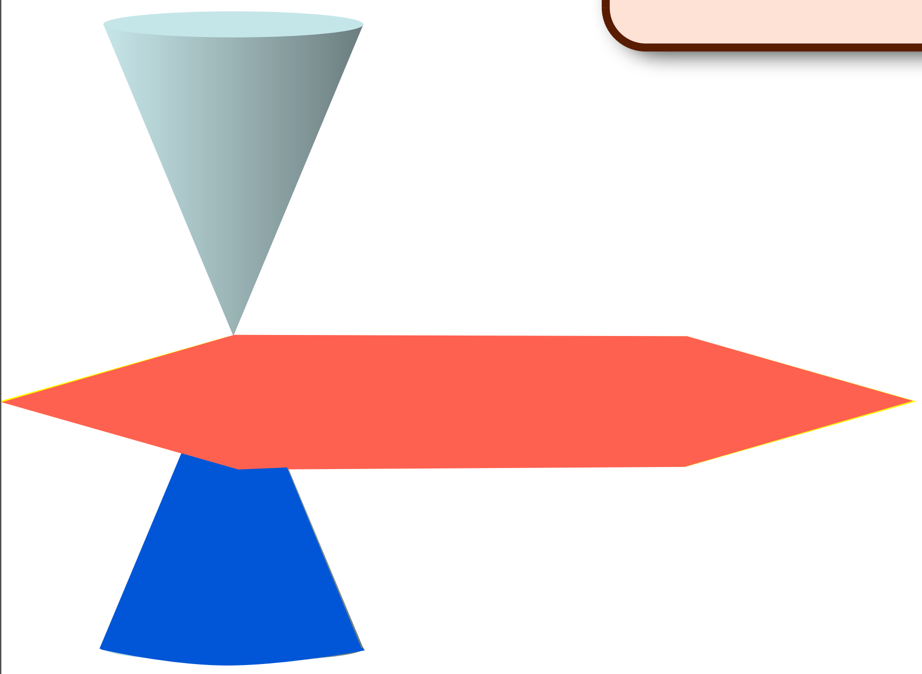
Conformal field theory in 2+1 dimensions at $T = 0$



$\Updownarrow$

Einstein gravity on AdS_4

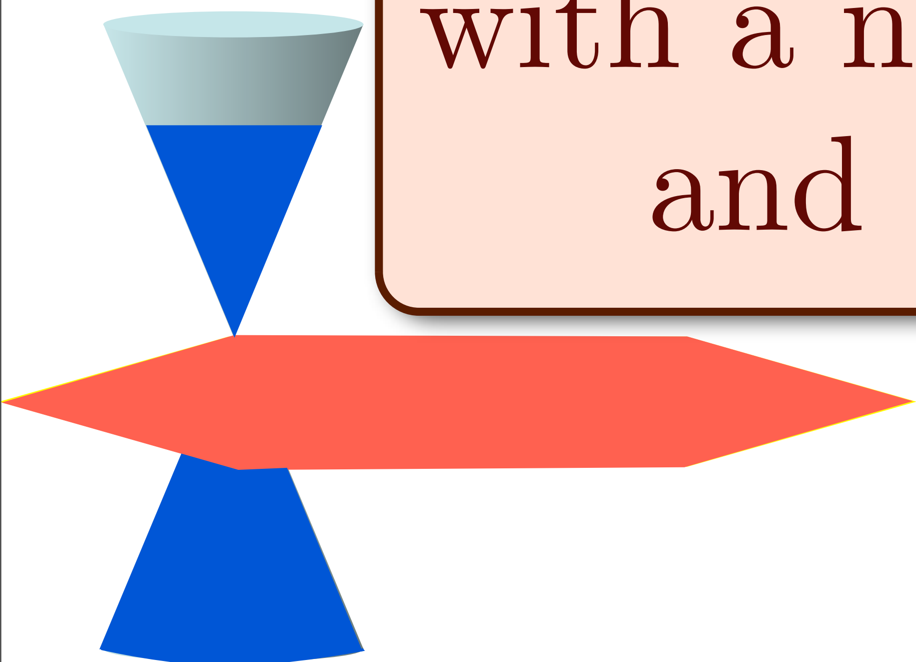
Conformal field theory in 2+1 dimensions at $T > 0$



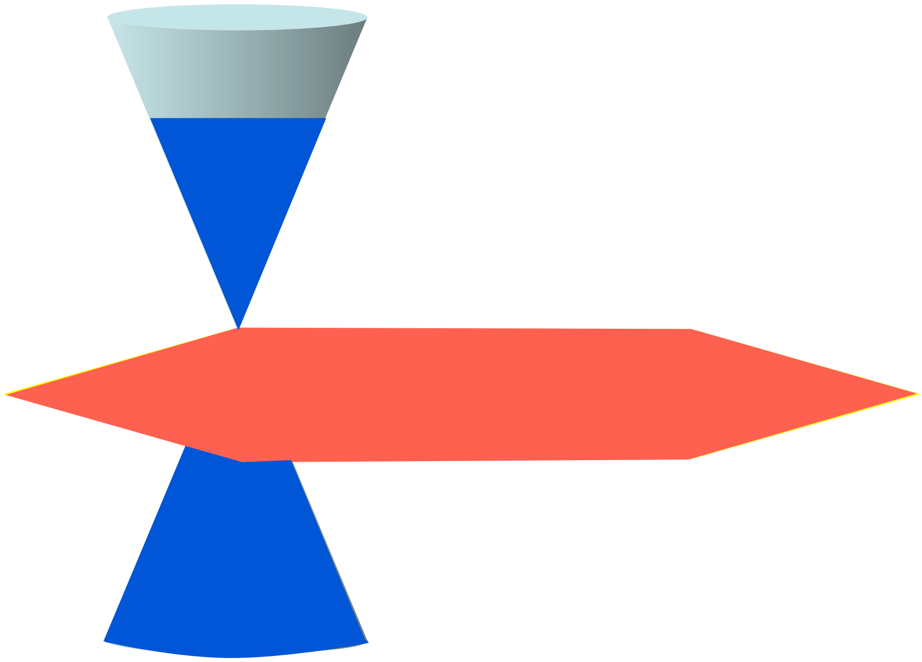
$\updownarrow$

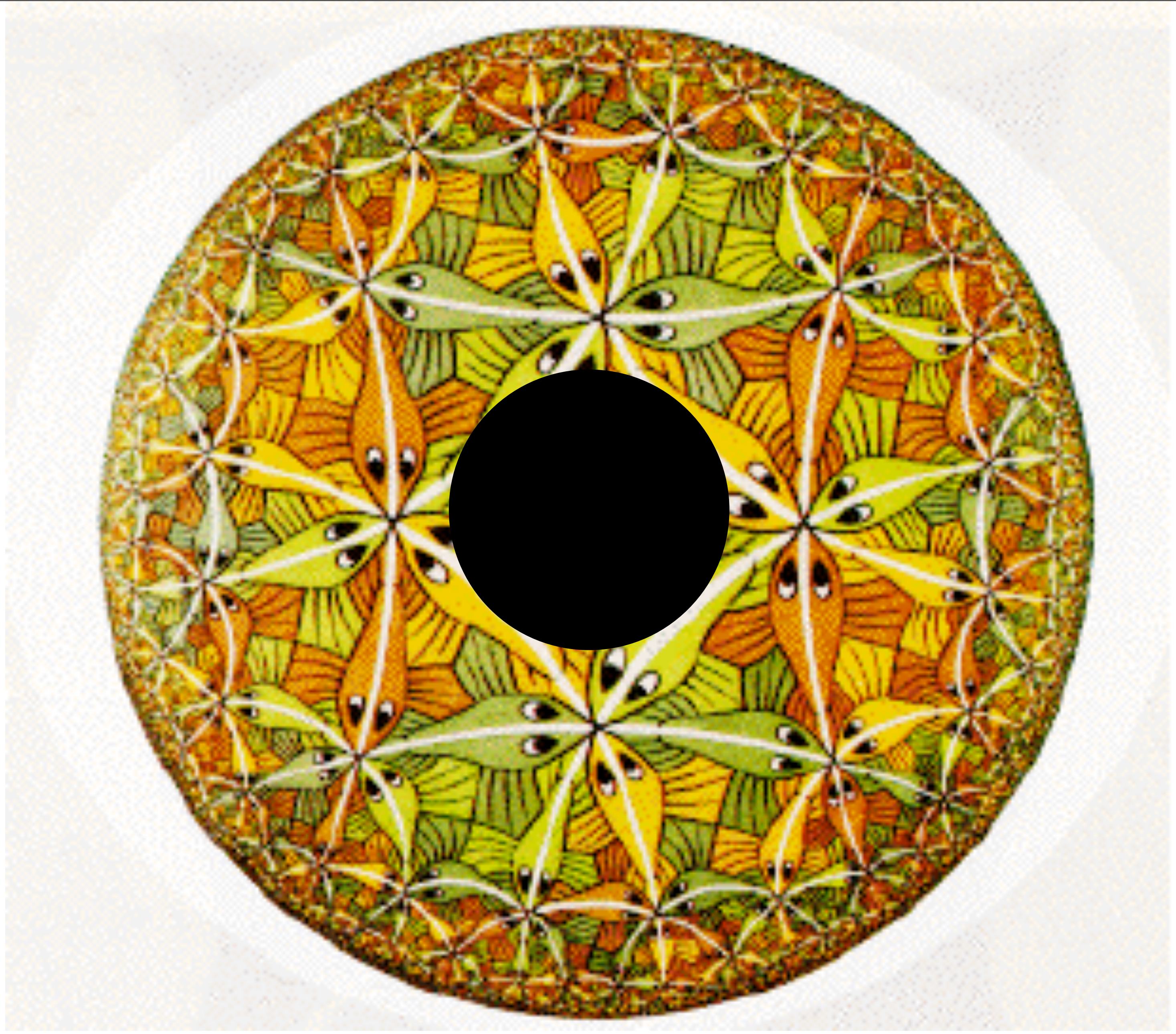
Einstein gravity on AdS_4 with a Schwarzschild black hole

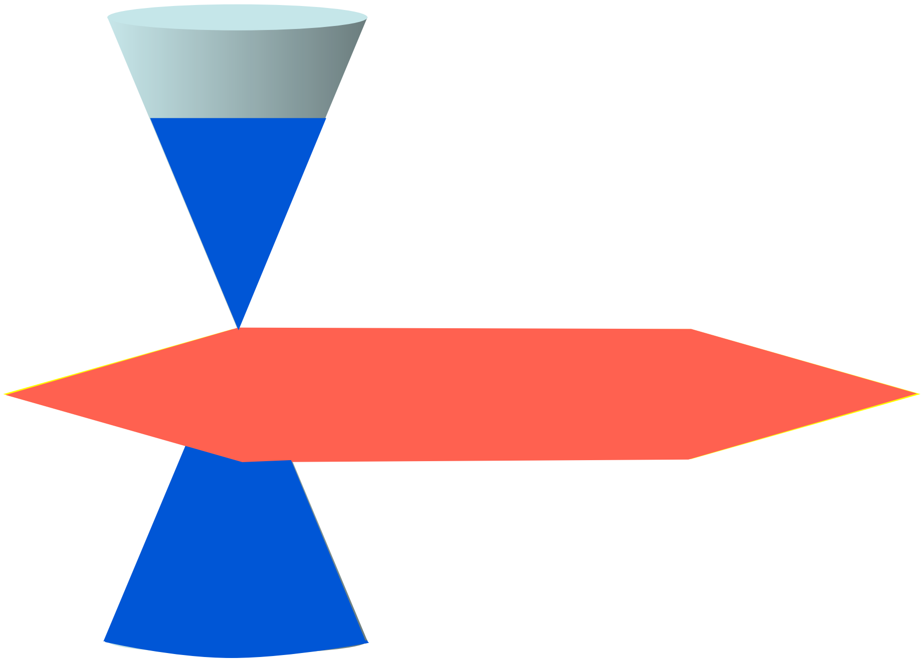
Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B

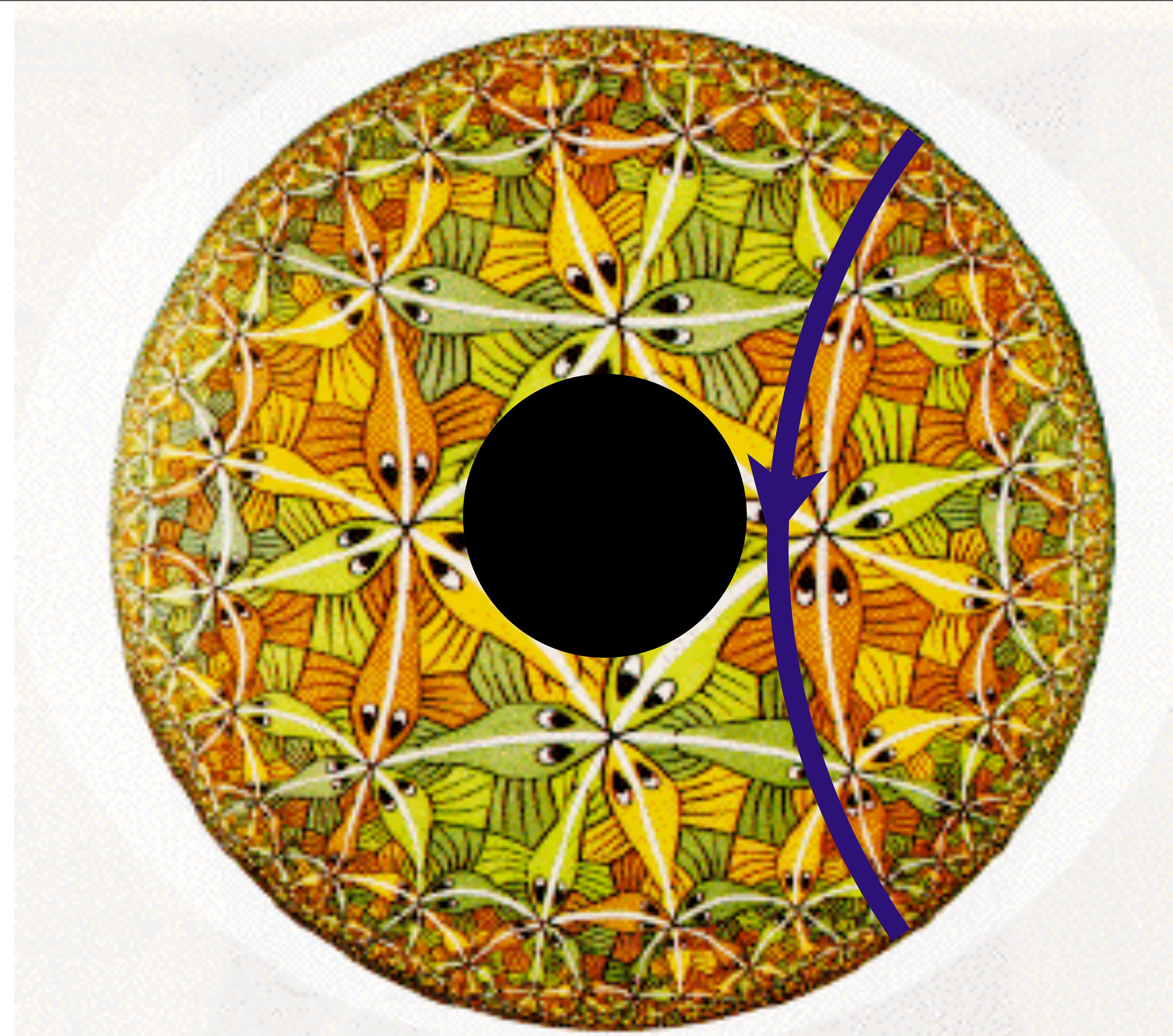


Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges



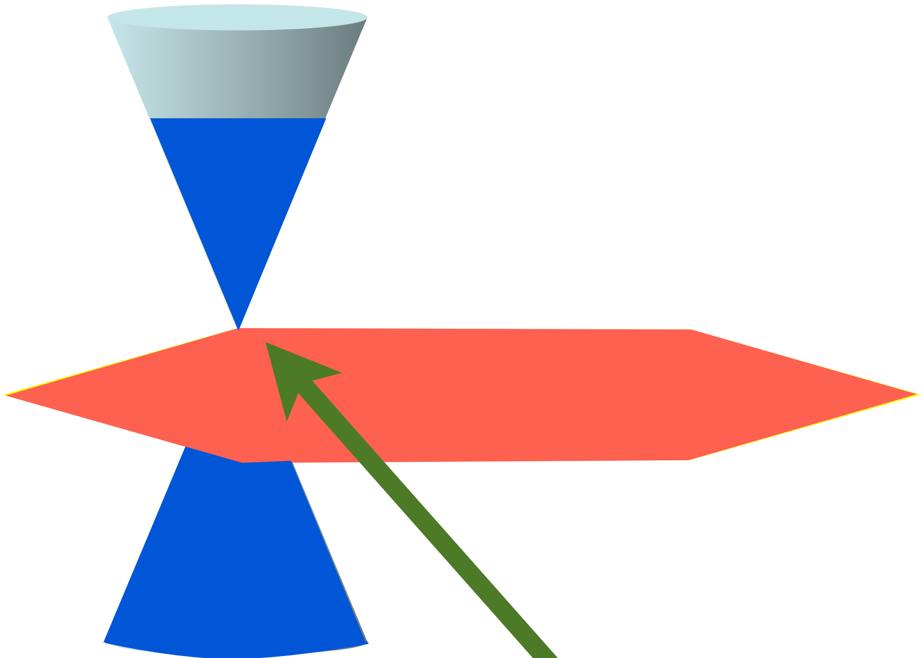


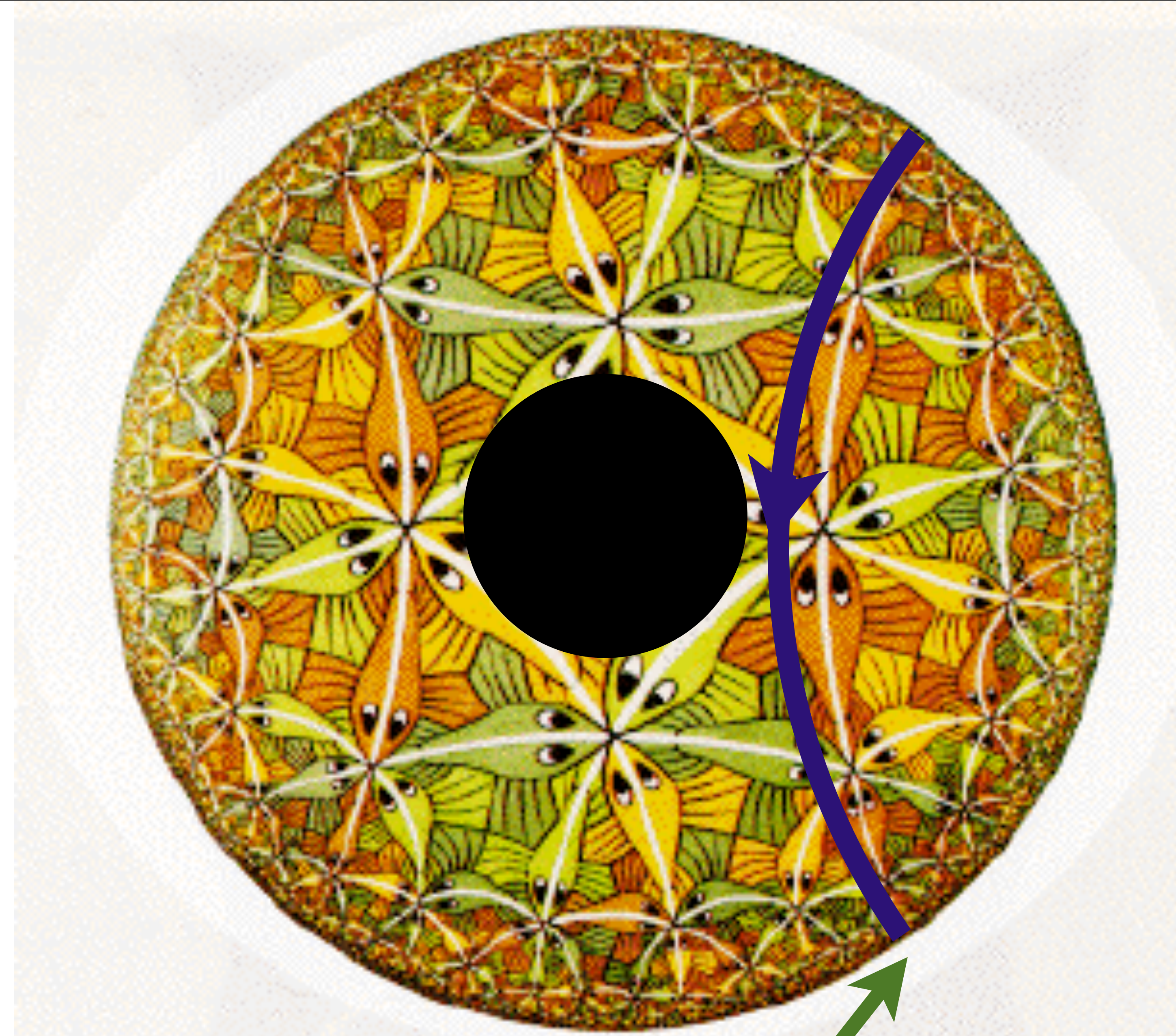




Examine free energy and Green's function of a probe particle

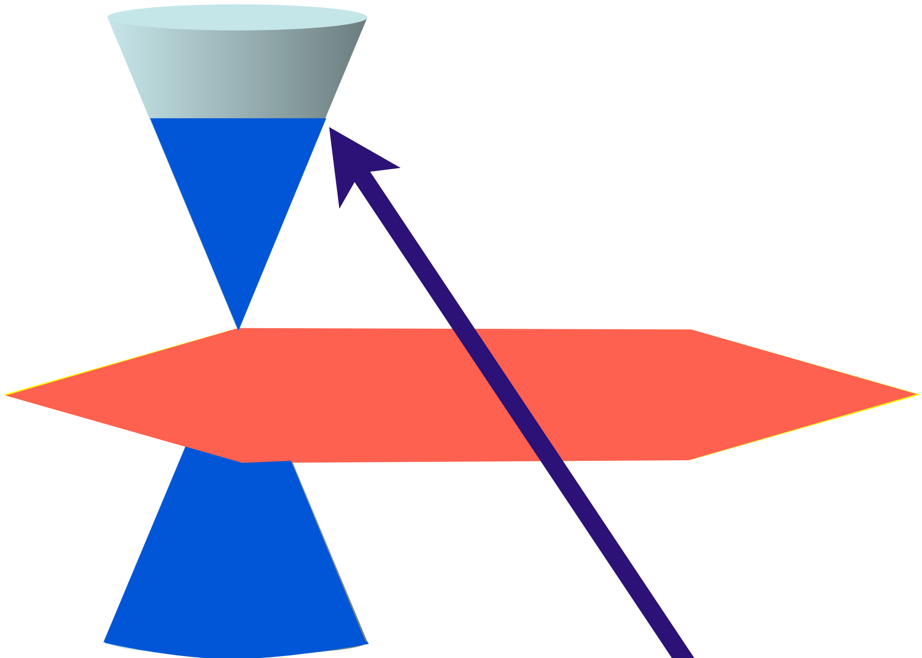
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, to appear

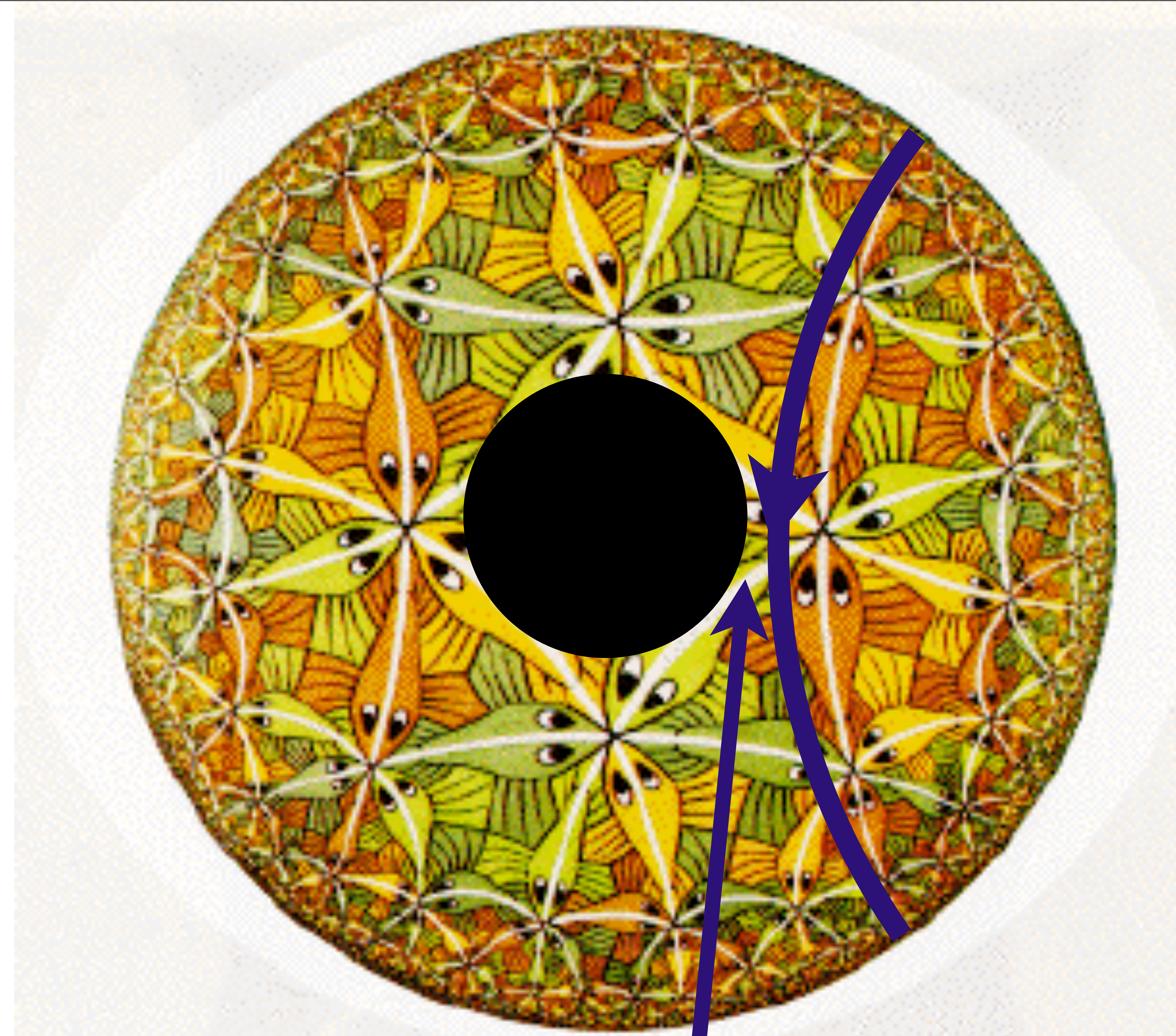




Short time behavior depends upon conformal AdS_4 geometry near boundary

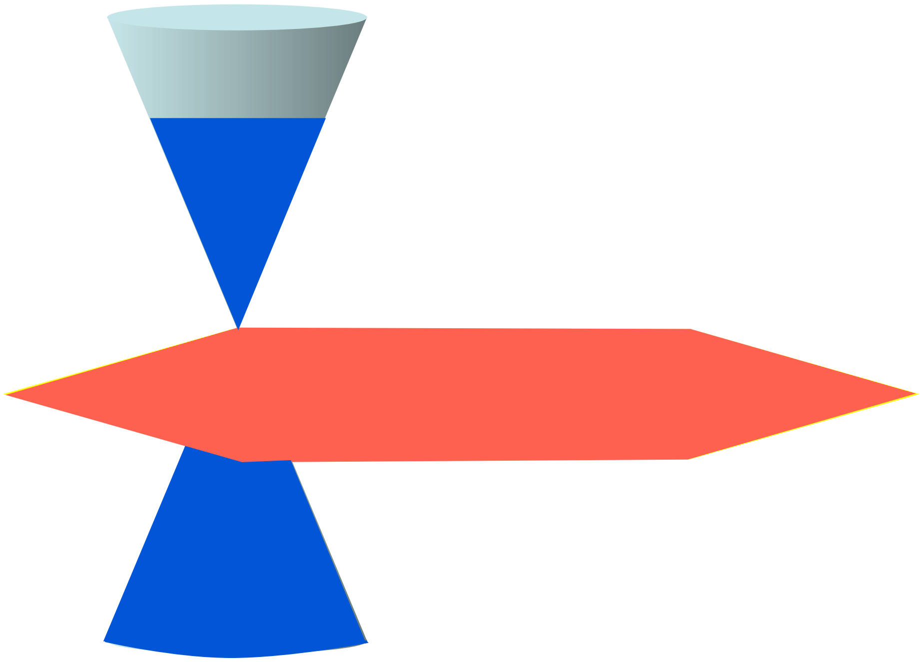
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, to appear

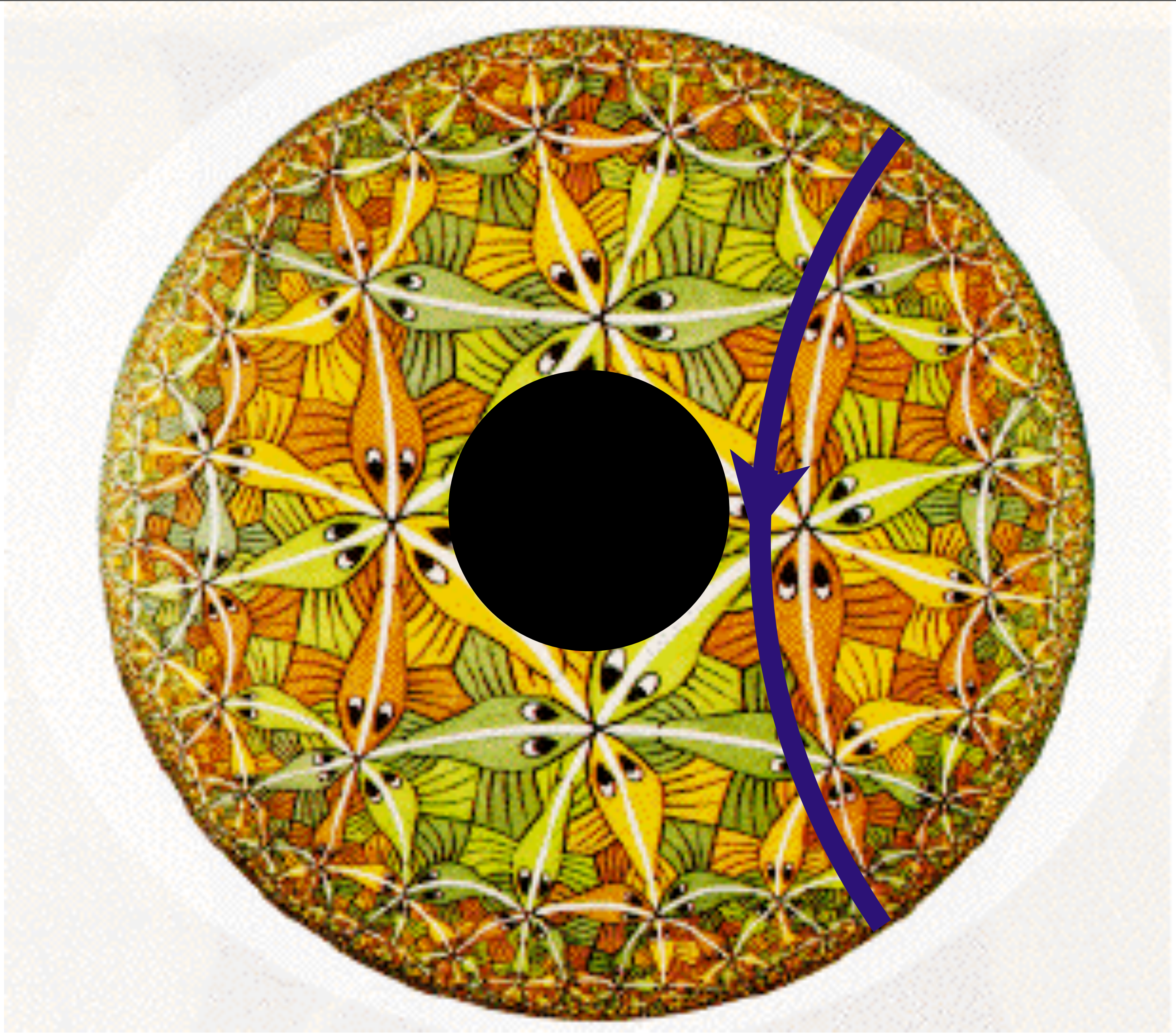




Long time behavior depends upon near-horizon geometry of black hole

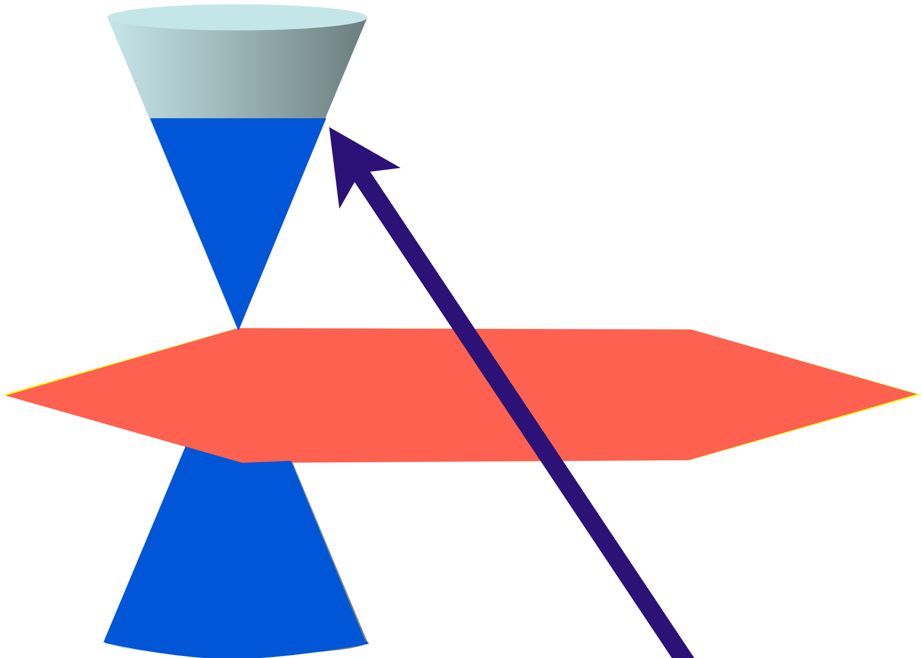
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, to appear

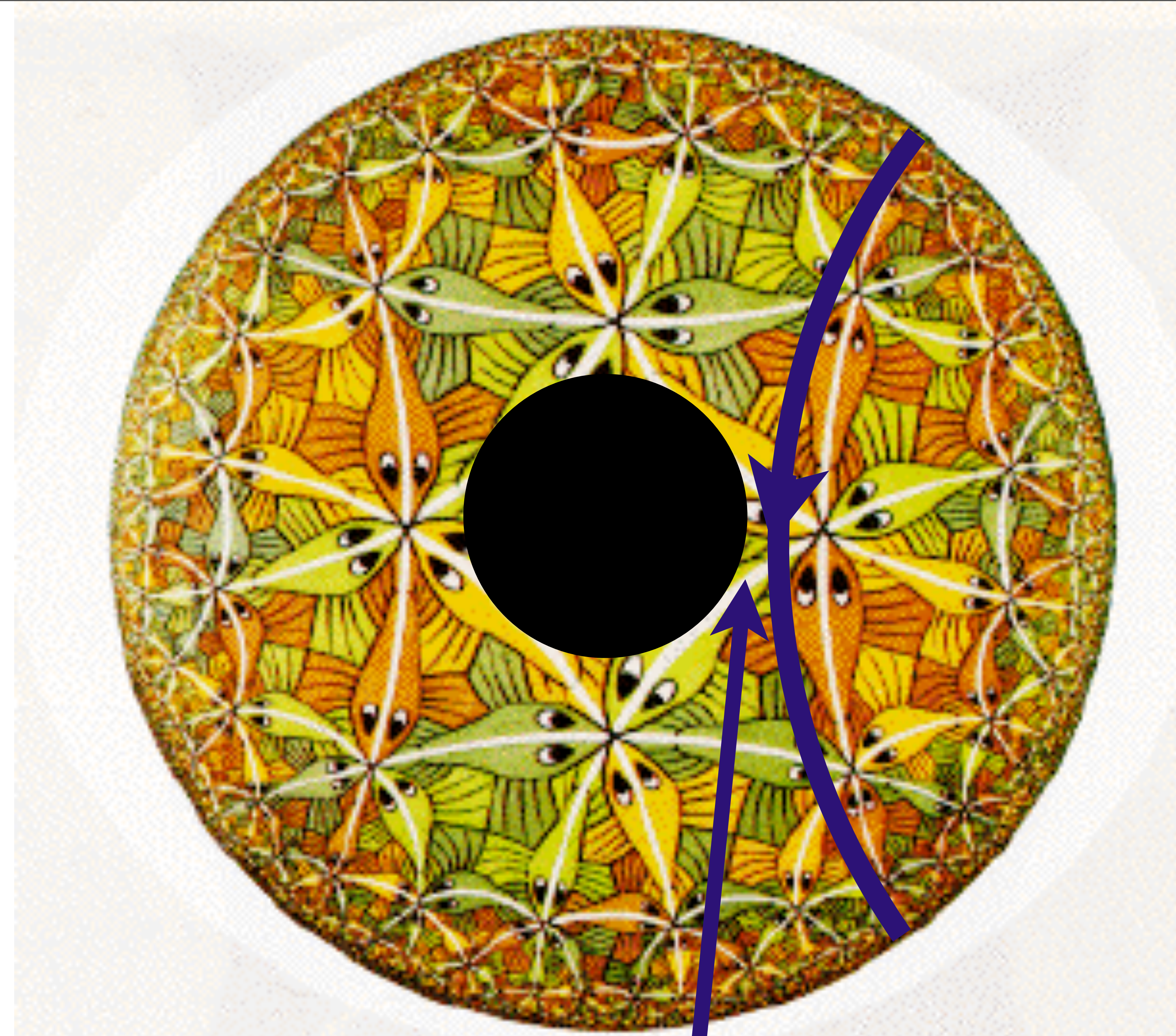




Radial direction of gravity theory is measure of energy scale in CFT

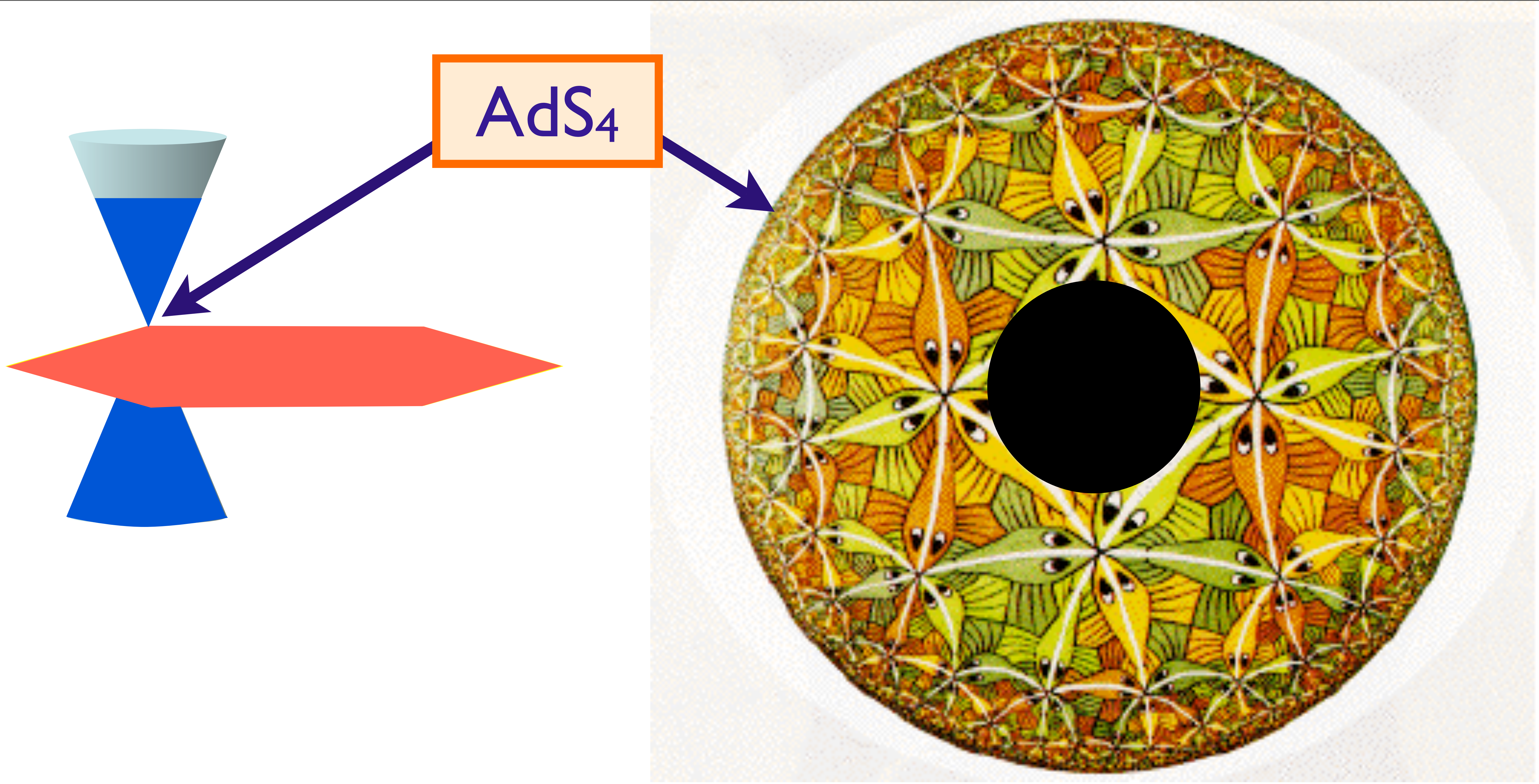
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, to appear



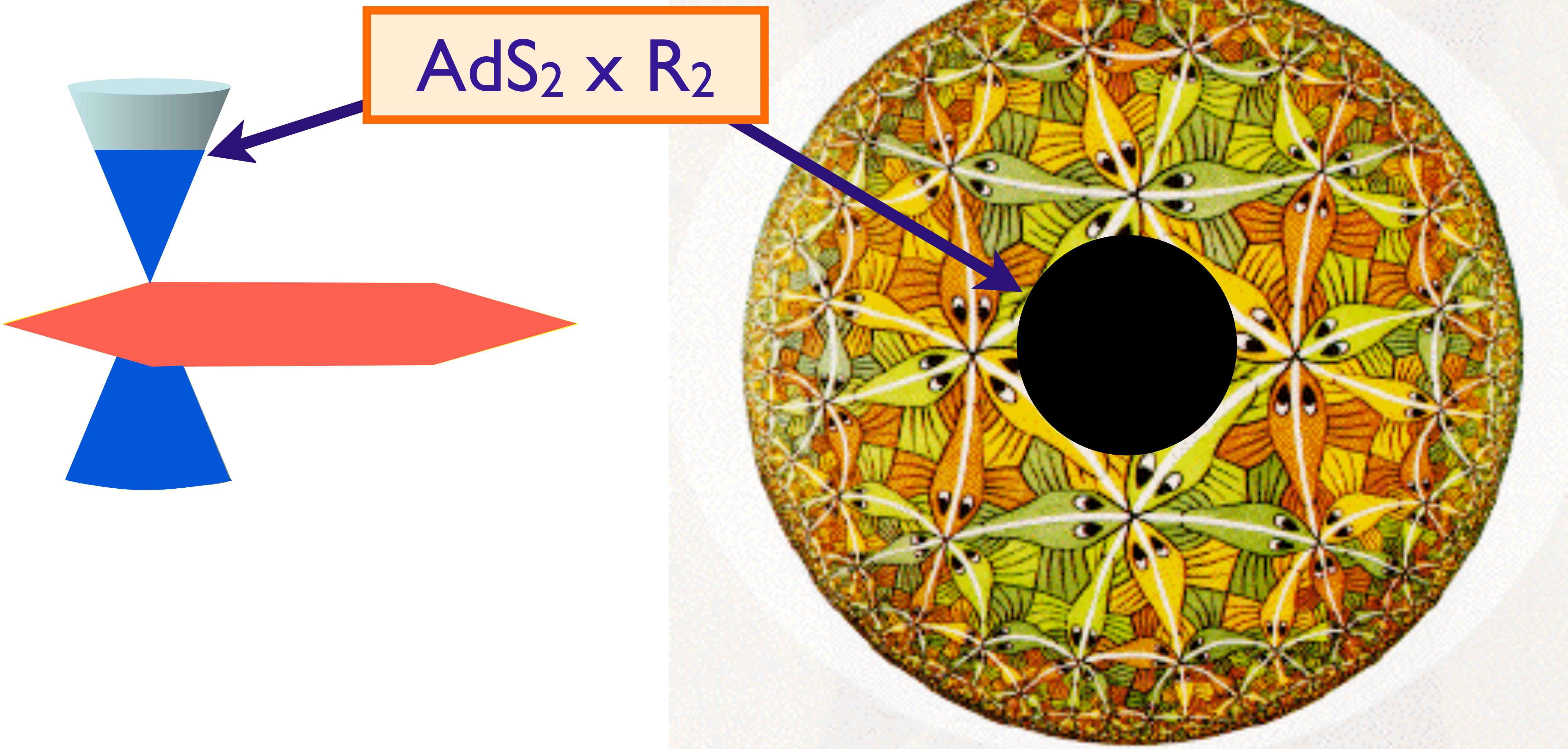


Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

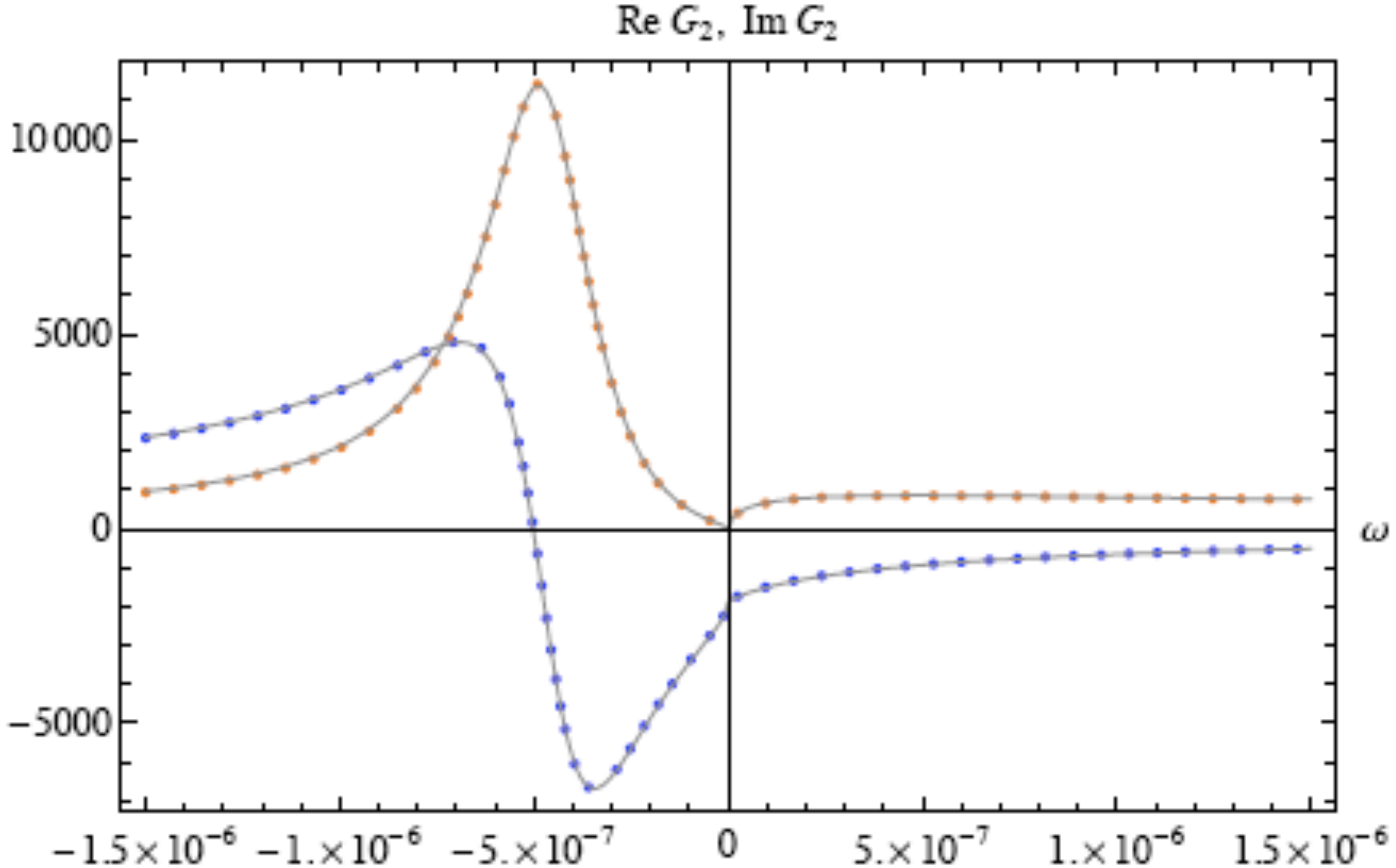


T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

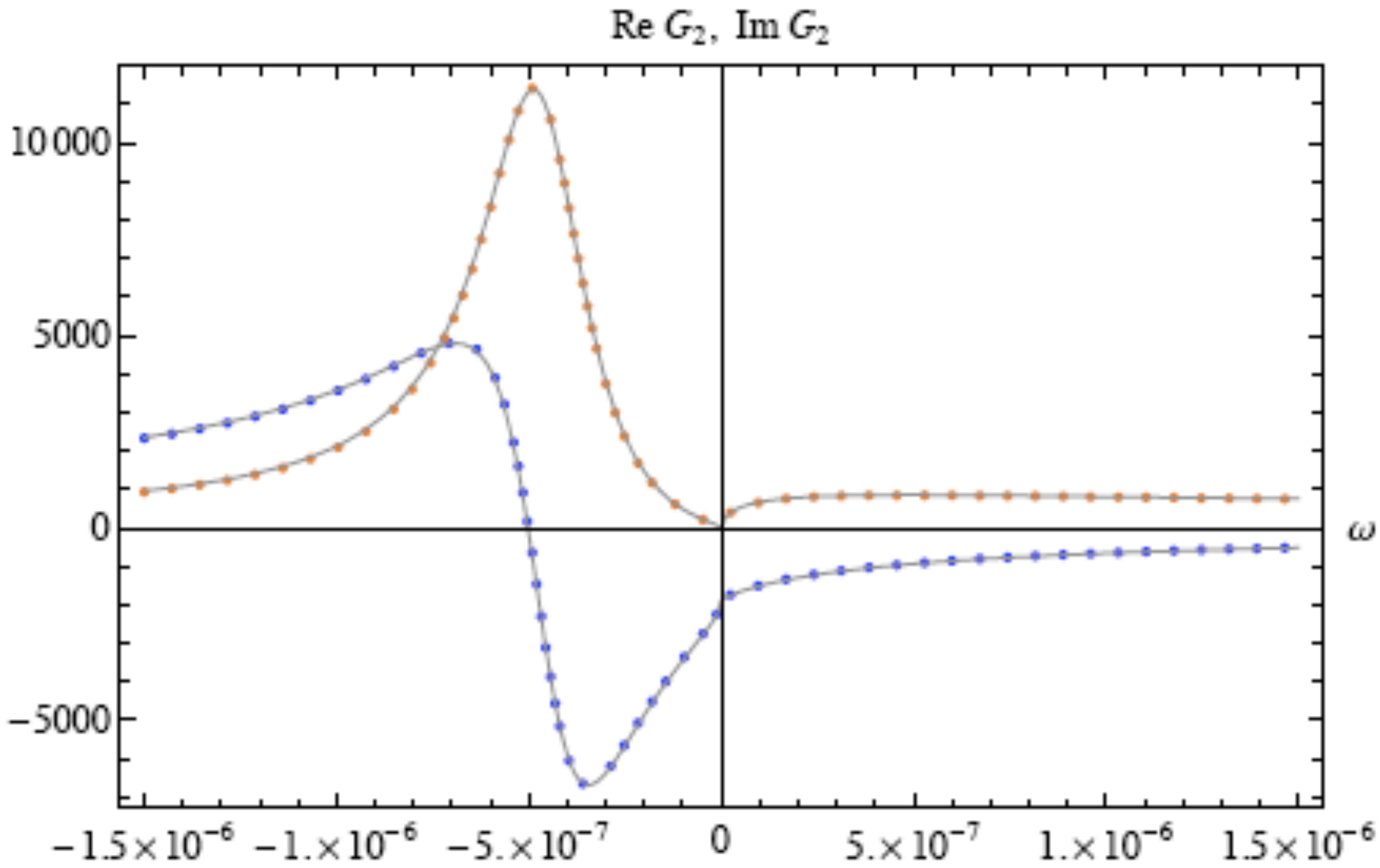
Green's function of a fermion



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

Green's function of a fermion



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

Similar to non-Fermi liquid theories of Fermi surfaces coupled to gauge fields, and at quantum critical points

Free energy from gravity theory

The free energy is expressed as a sum over the "quasinormal frequencies", z_ℓ, of the black hole. Here ℓ represents any set of quantum numbers:

$$
\begin{aligned}
\mathcal{F}_{\text{boson}} &= -T \sum_\ell \ln \left(\frac{|z_\ell|}{2\pi T} \left| \Gamma \left(\frac{i z_\ell}{2\pi T} \right) \right|^2 \right) \\
\mathcal{F}_{\text{fermion}} &= T \sum_\ell \ln \left(\left| \Gamma \left(\frac{i z_\ell}{2\pi T} + \frac{1}{2} \right) \right|^2 \right)
\end{aligned}
$$

Application of this formula shows that the fermions exhibit the dHvA quantum oscillations with expected period ($2\pi/$(Fermi surface ares)) in $1/B$, but with an amplitude corrected from the Fermi liquid formula of Lifshitz-Kosevich.

F. Denef, S. Hartnoll, and S. Sachdev, to appear

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density