Fermi surfaces large and small: unifying theories of the Kondo lattice and Hubbard models

Quantum Many-Body Days
UNC, Chapel Hill
Sep 28, 2021

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

PHYSICS
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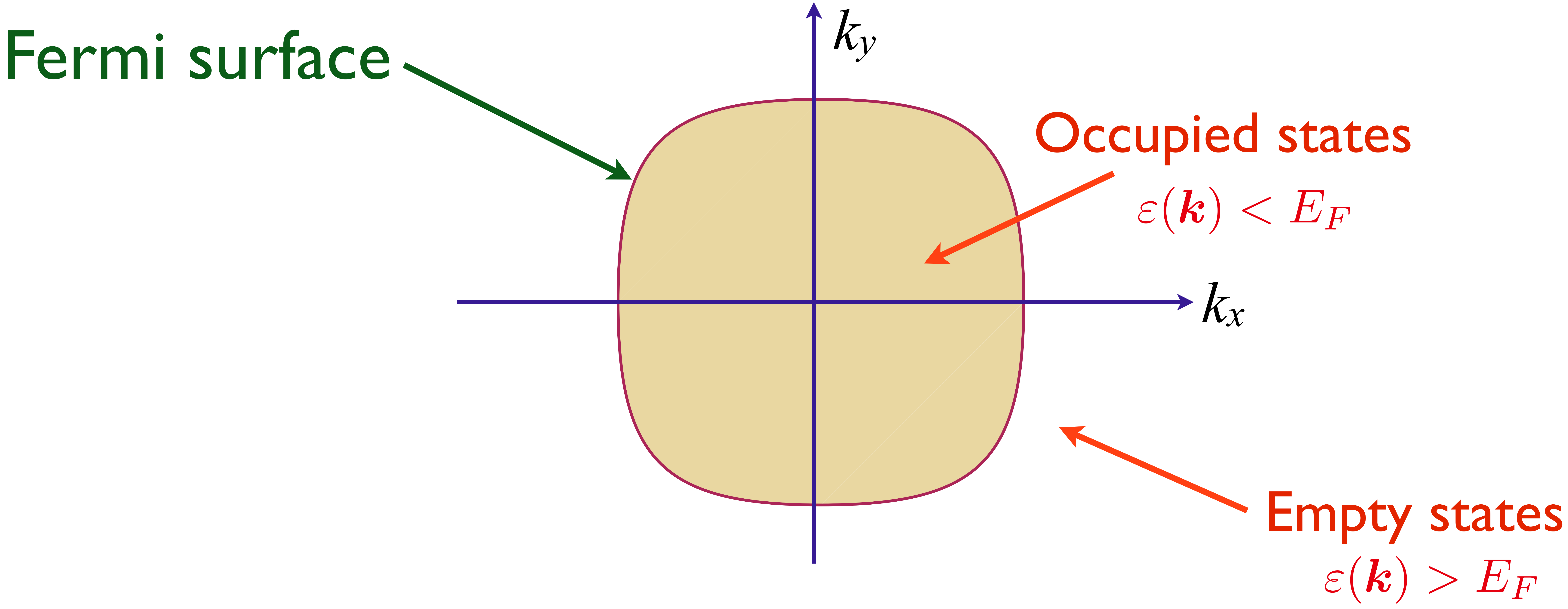
Luttinger theorem

Electrons move with momentum $\mathbf{k}$ through the lattice with dispersion $\varepsilon(\boldsymbol{k})$

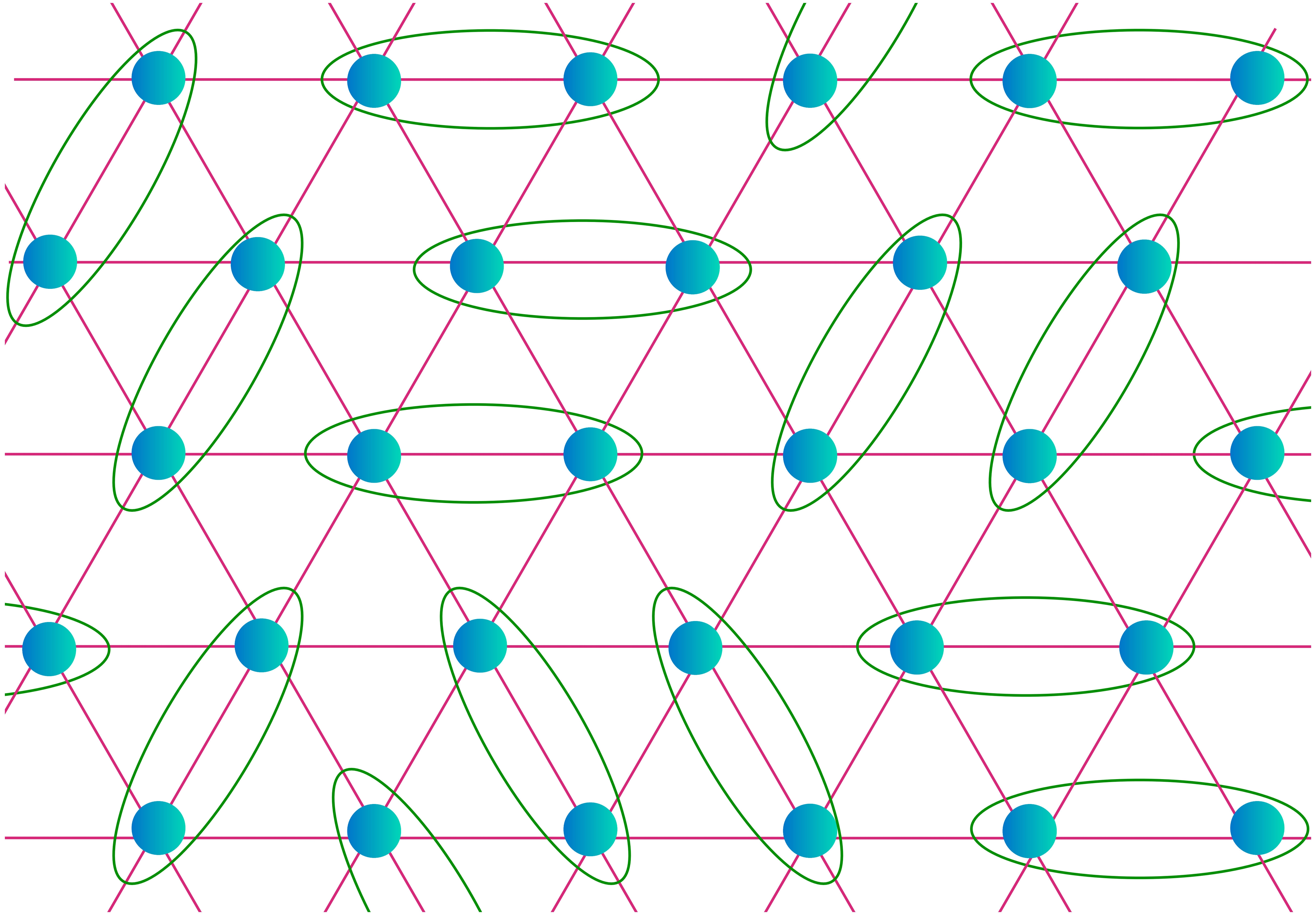


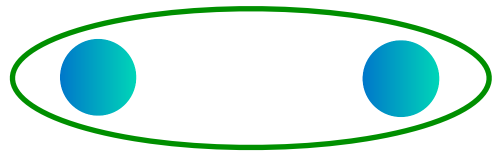
$$2 \times \frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons (mod 2)}$$

1. Spin liquids and violations of the Luttinger theorem: FL* and HFL phases of the Kondo lattice model.
2. Hubbard model - the vanilla FL phase
3. Hubbard model - the FL* phase: fractionalizing the paramagnon

Mott insulator: Triangular lattice antiferromagnet

Resonating valence bonds:
Z_2 spin liquid





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

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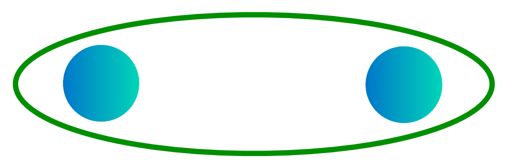

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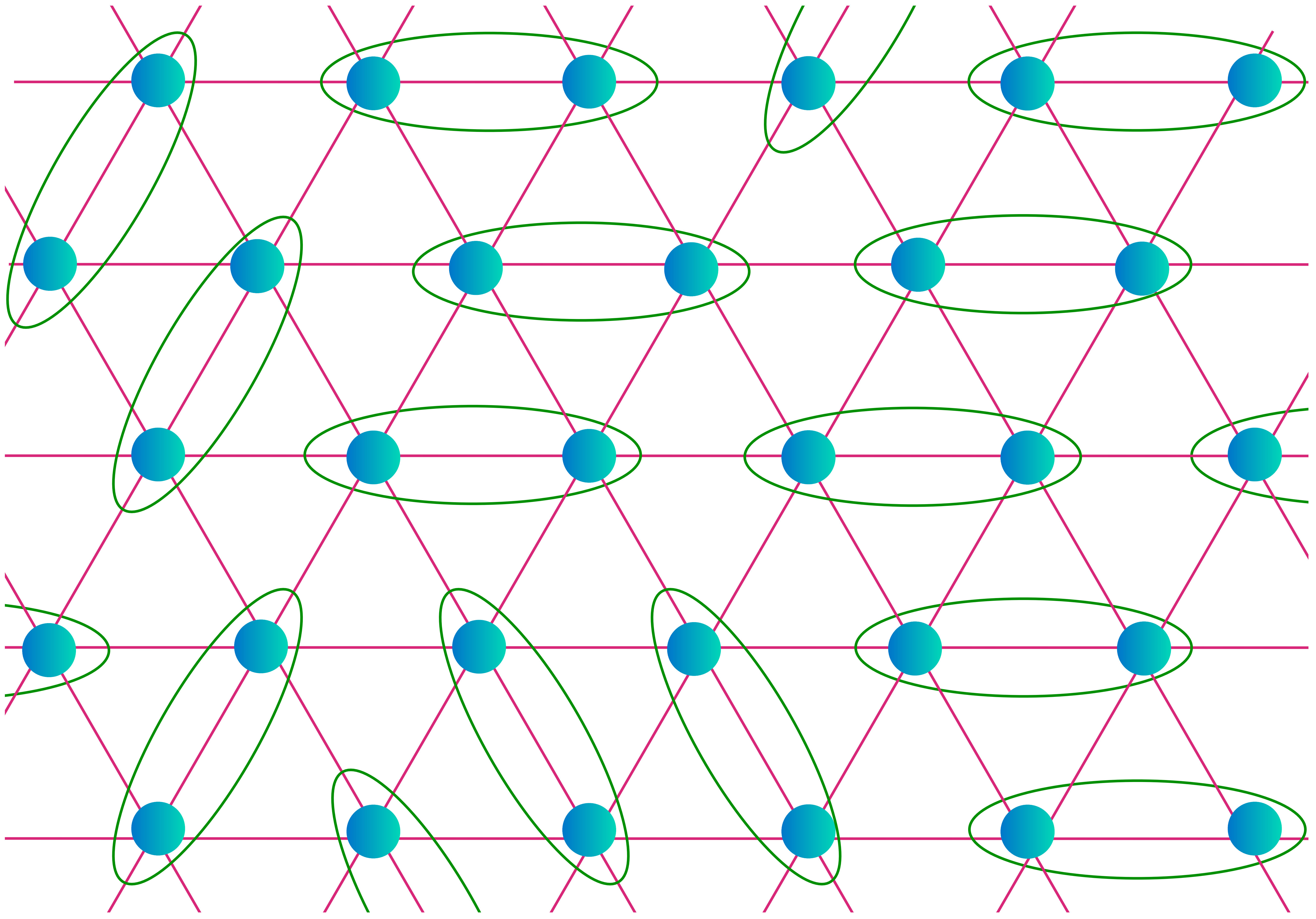

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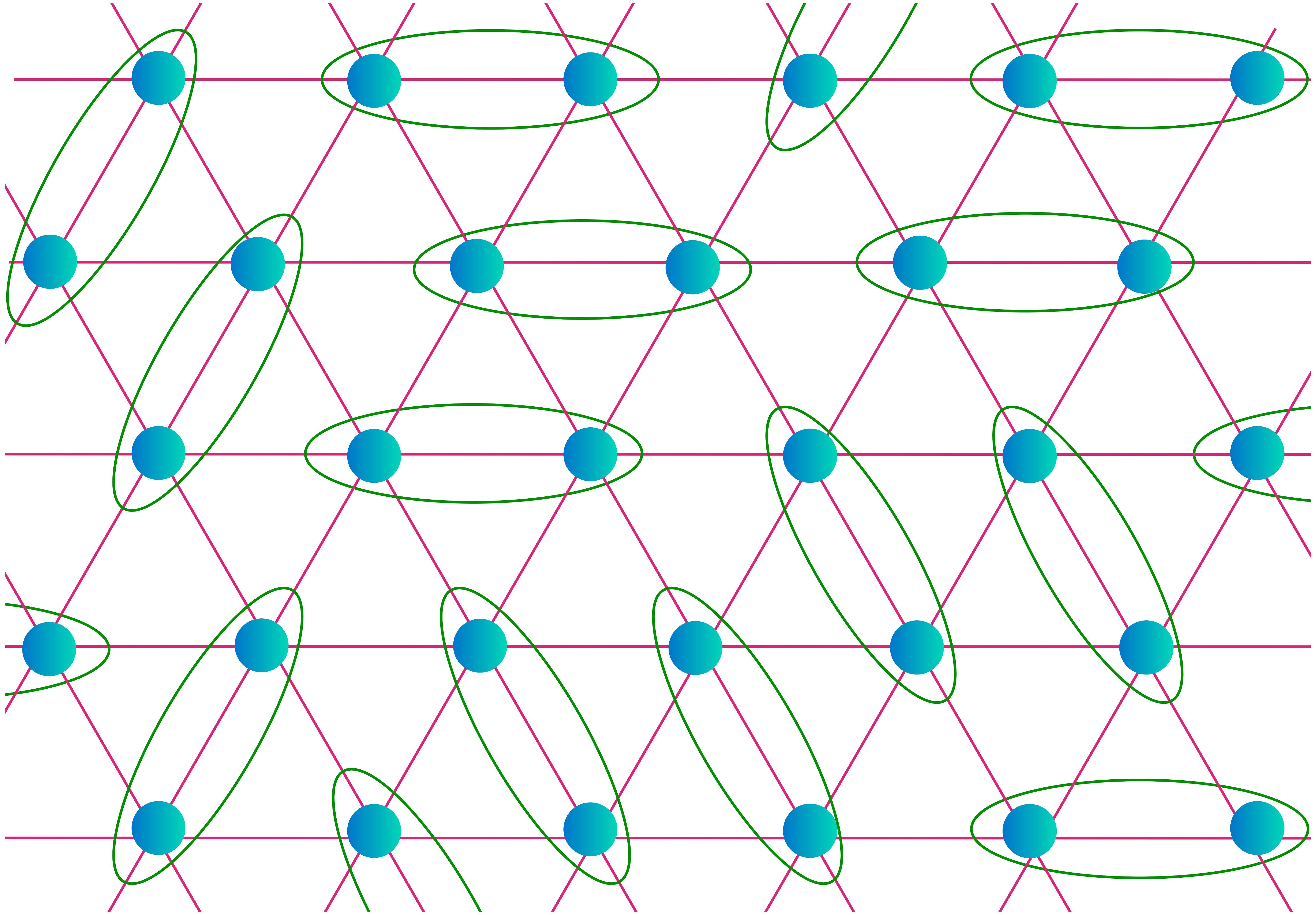

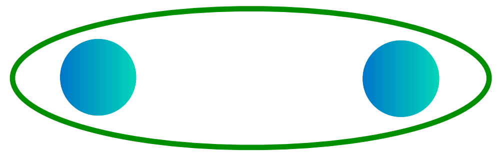

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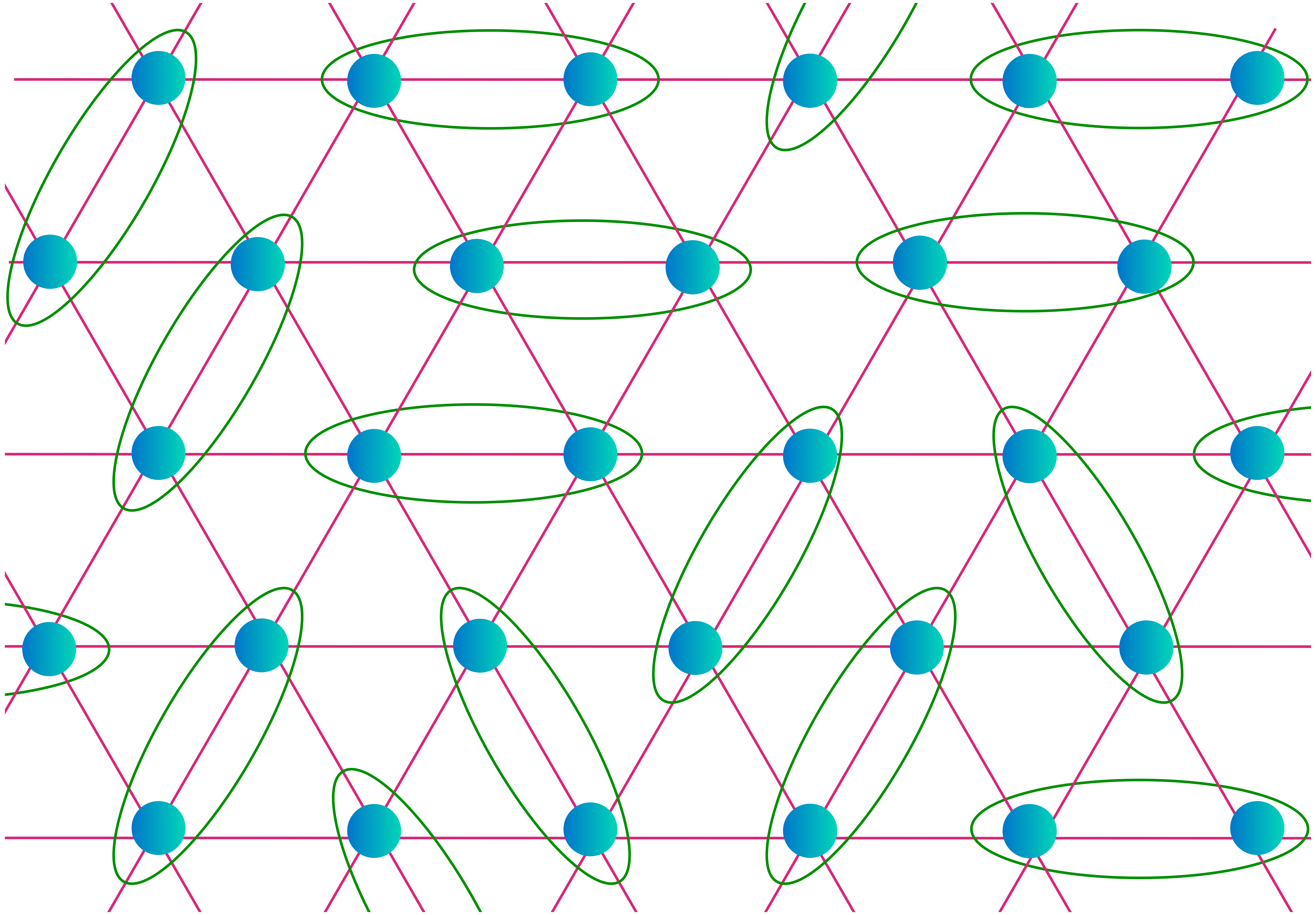

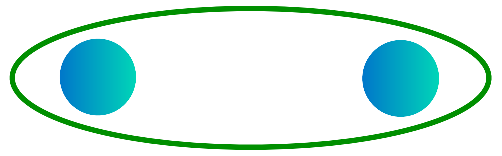

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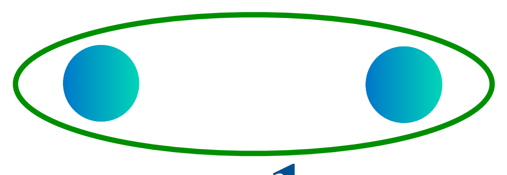

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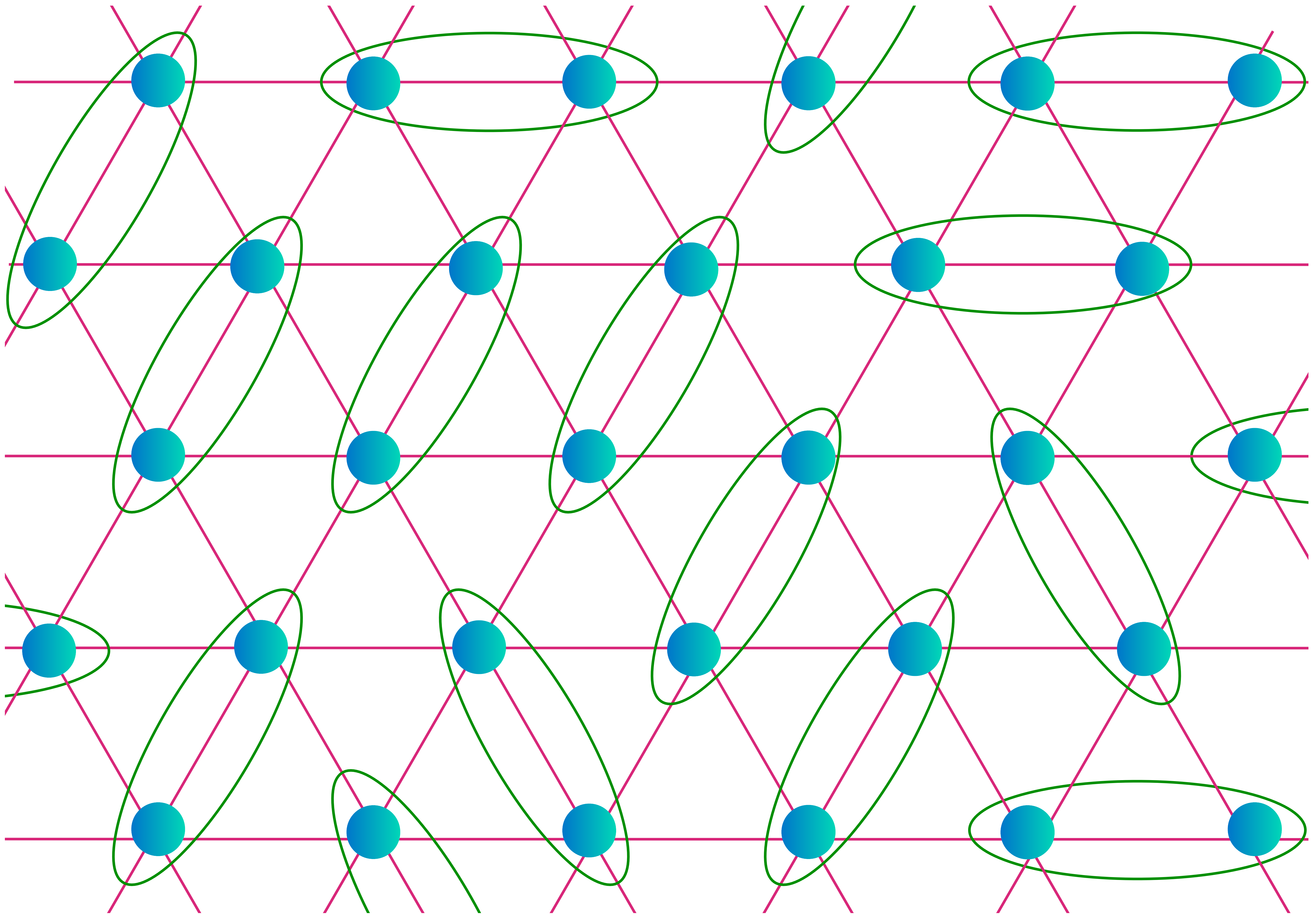

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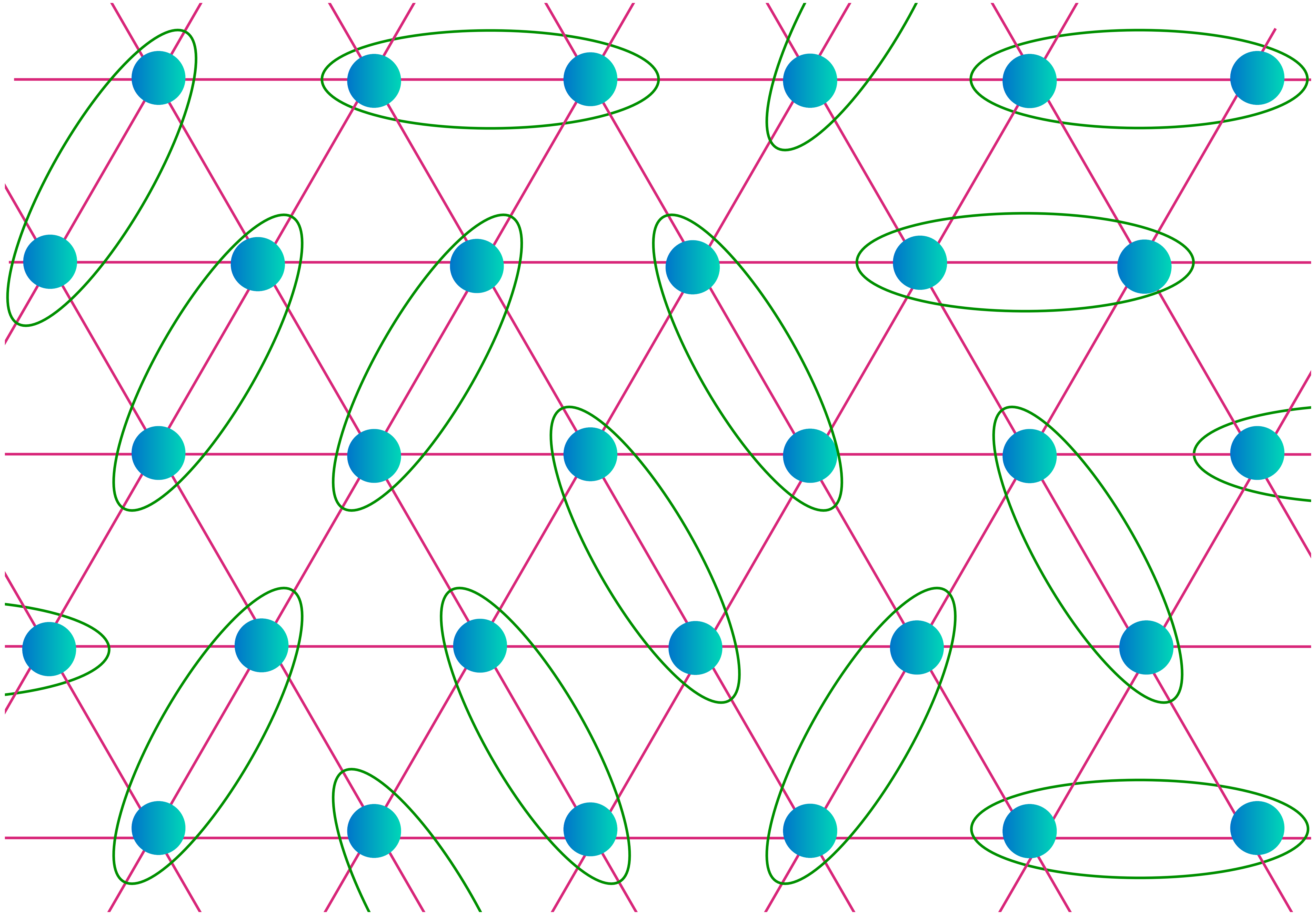

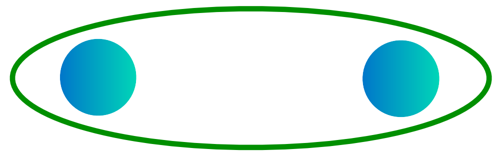

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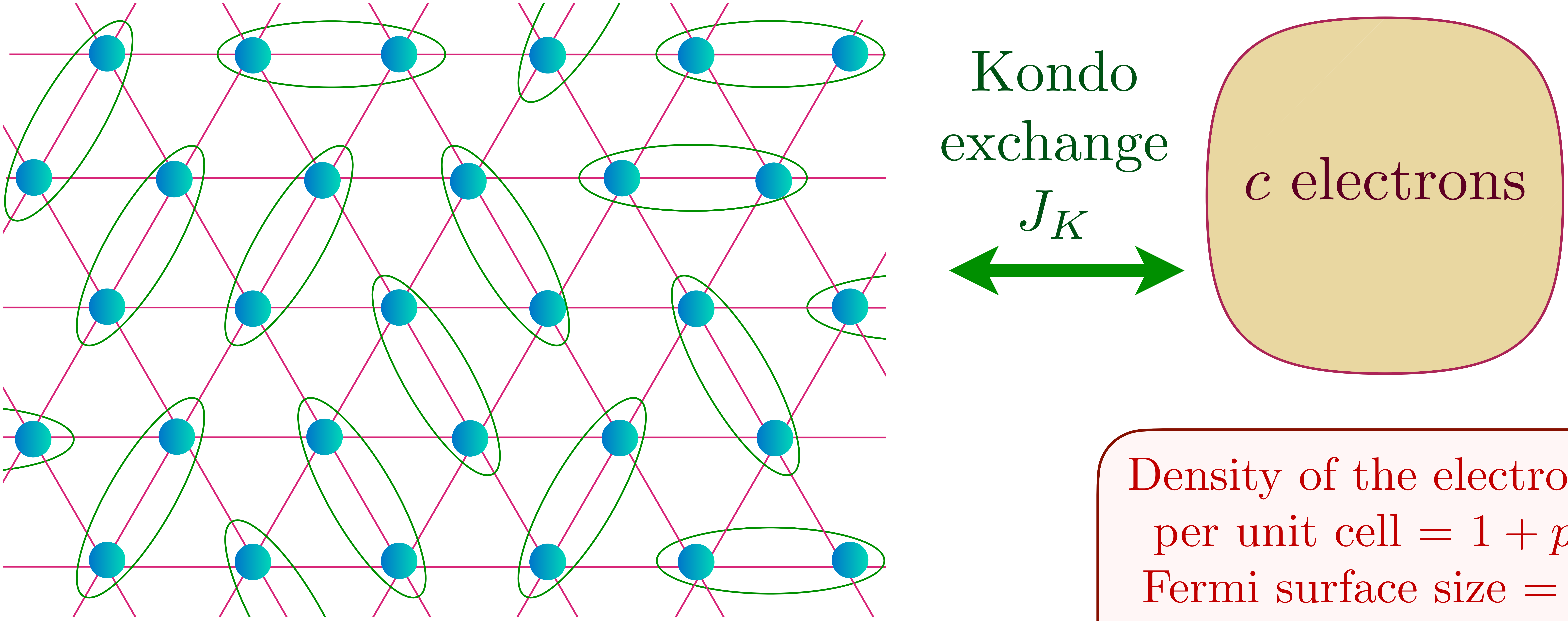
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

Density of the electrons per unit cell = 1, but no Fermi surface
$\Rightarrow$ Violation of Luttinger theorem

Kondo lattice: FL* phase

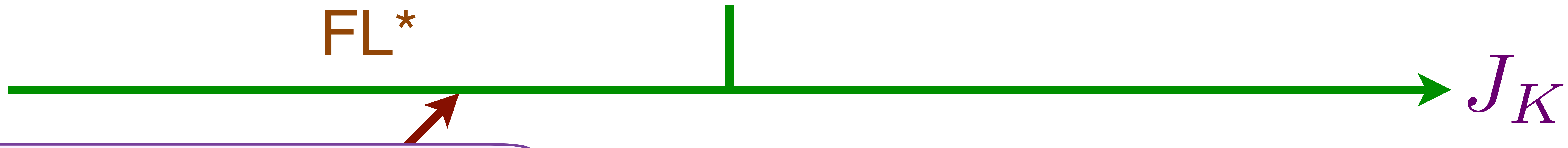


f electrons

Density of the electrons
per unit cell $= 1 + p$,
Fermi surface size $= p$
$\Rightarrow$ Violation of
Luttinger theorem

Kondo lattice

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_{i} J_K \, c^{\dagger}_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



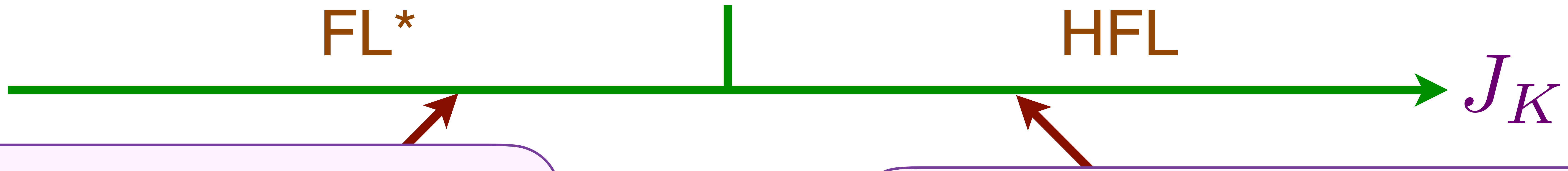
Small Fermi surface of size p

$$\begin{aligned} |\mathrm{FL}*\rangle &= [\text{Projection onto one } f \text{ per site}] \\ &\quad \bowtie |\text{Slater determinant of } f\rangle \\ &\quad \otimes |\text{Slater determinant of } c\rangle \end{aligned}$$

S. Burdin, D. R. Grempel, and A. Georges, PRB **66**, 045111 (2002)
T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)
A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

Kondo lattice

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



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Large Fermi surface of size $1 + p$

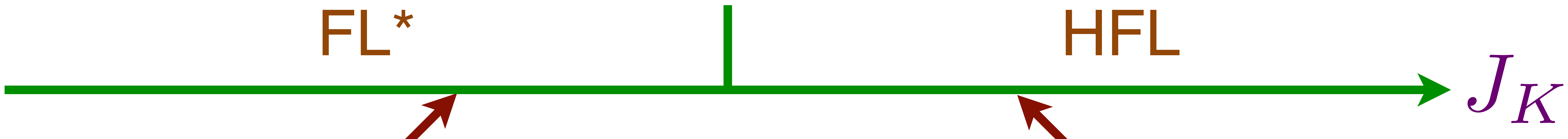
$$|\mathrm{HFL}\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } (c, f)\rangle$$

Coleman, Read, Newns, Millis, P.A. Lee, Auerbach, Levin

Kondo lattice

- Use fermionic spinons $\boldsymbol{S}_i = \frac{1}{2} f_{i\sigma}^\dagger \boldsymbol{\tau}_{\sigma\sigma'} f_{i\sigma'}$, $\sum_\sigma f_{i\sigma}^\dagger f_{i\sigma} = 1$. This introduces a $\mathrm{U}(1)_{\mathrm{gauge}}$ gauge symmetry $f_{i\sigma} \to f_{i\sigma} e^{-i\vartheta_i}$ and the total symmetry is $\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{em}}$.

- In the FL* state, the $\mathrm{U}(1)_{\mathrm{gauge}} \times \mathrm{U}(1)_{\mathrm{em}}$ symmetry is unbroken.



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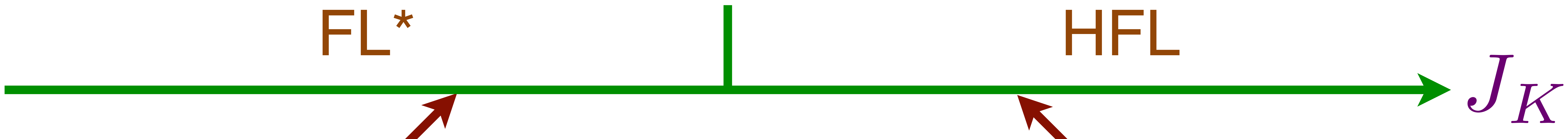
- In the HFL state, the U(1)$_{\rm gauge}\times$ U(1)$_{\rm em}$ symmetry is 'Higgsed' by the condensation of the hybridization boson $B_i \sim f_{i\sigma}^\dagger c_{i\sigma}$ to a diagonal U(1)$_{\rm diag}$ symmetry. The Luttinger theorem arguments can only be applied to the unbroken U(1)$_{\rm diag}$ symmetry, which counts *both* c and f fermions, and so the Fermi surface is *large*.



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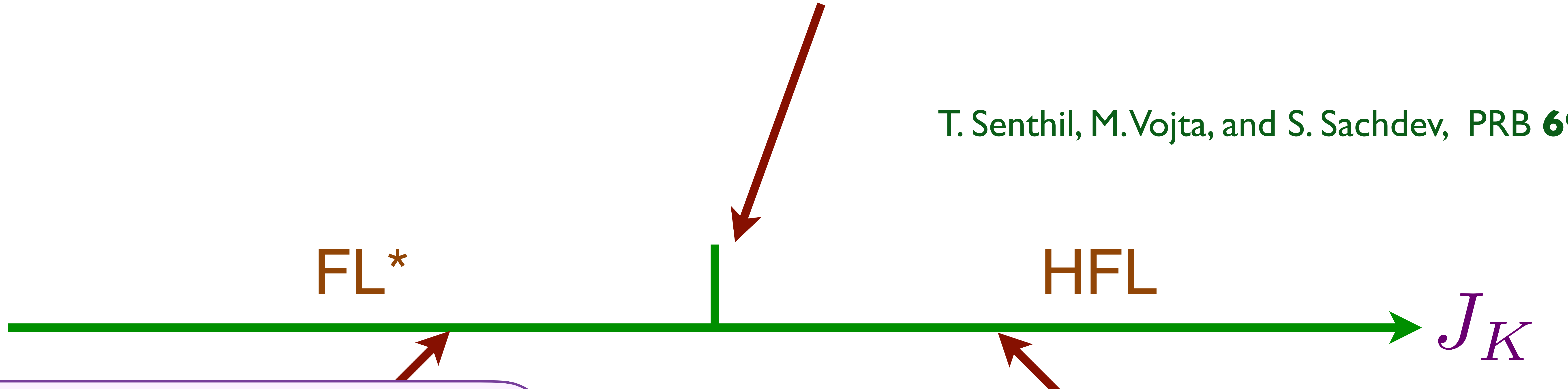
Kondo lattice

Deconfined criticality of a U(1) gauge theory with a
Higgs field, spinons, and a small Fermi surface of electrons.
(FL* can be replaced by a confining phase with AFM or VBS order).

T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)



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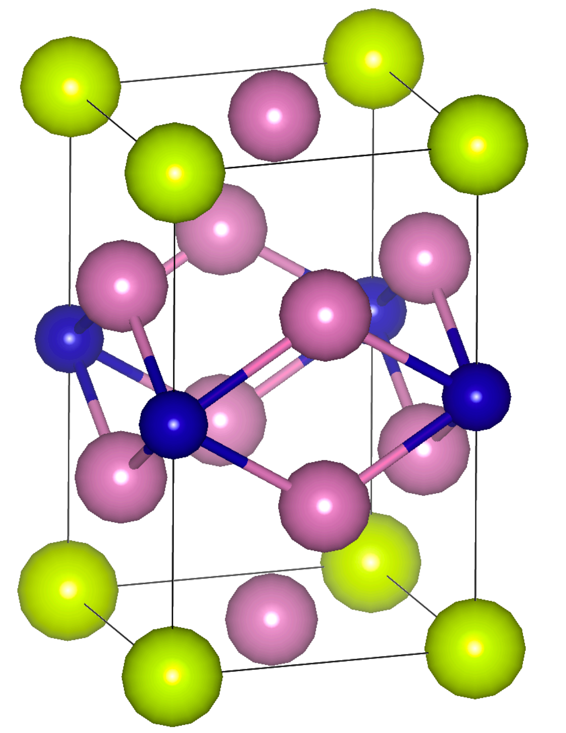
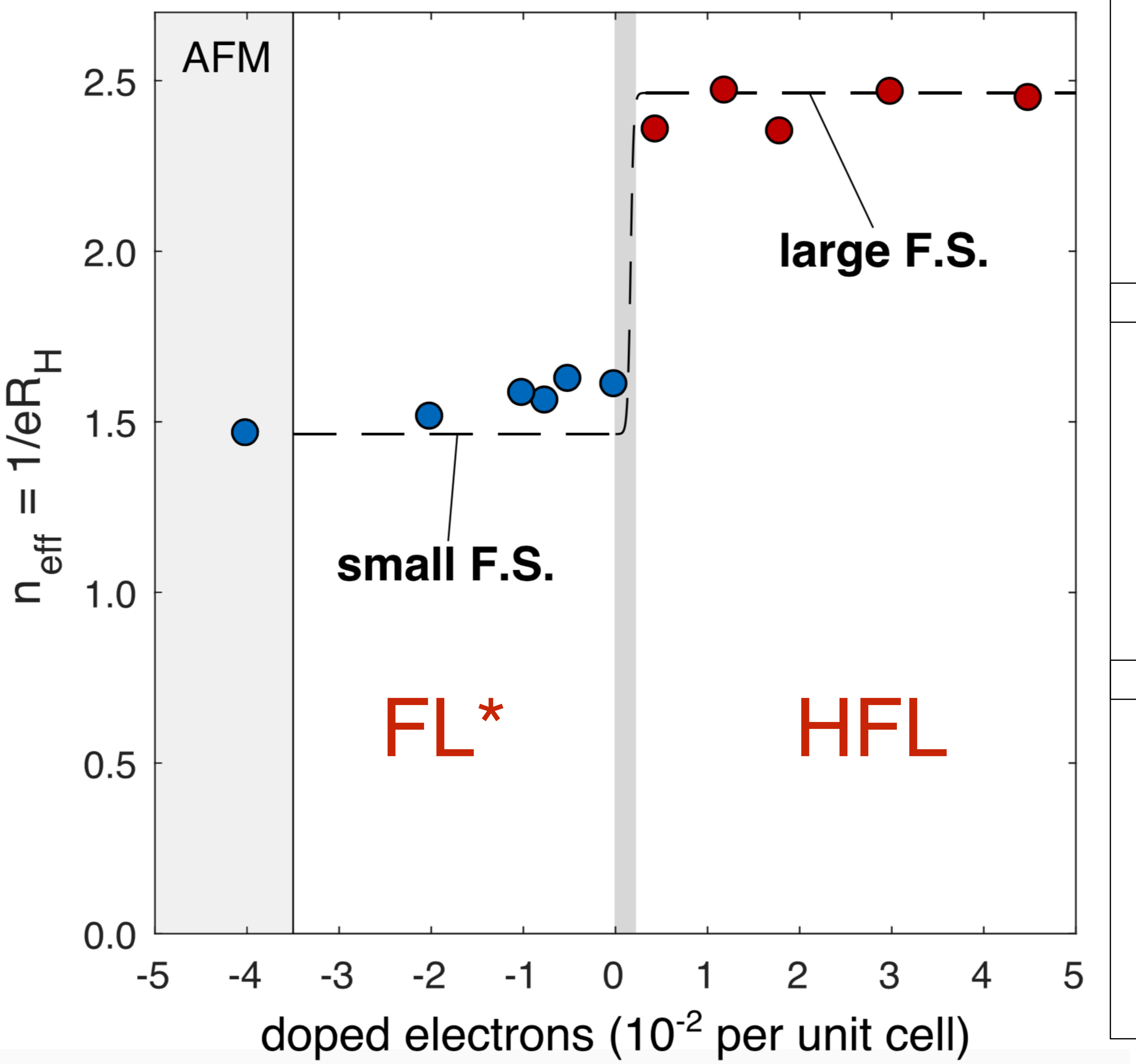
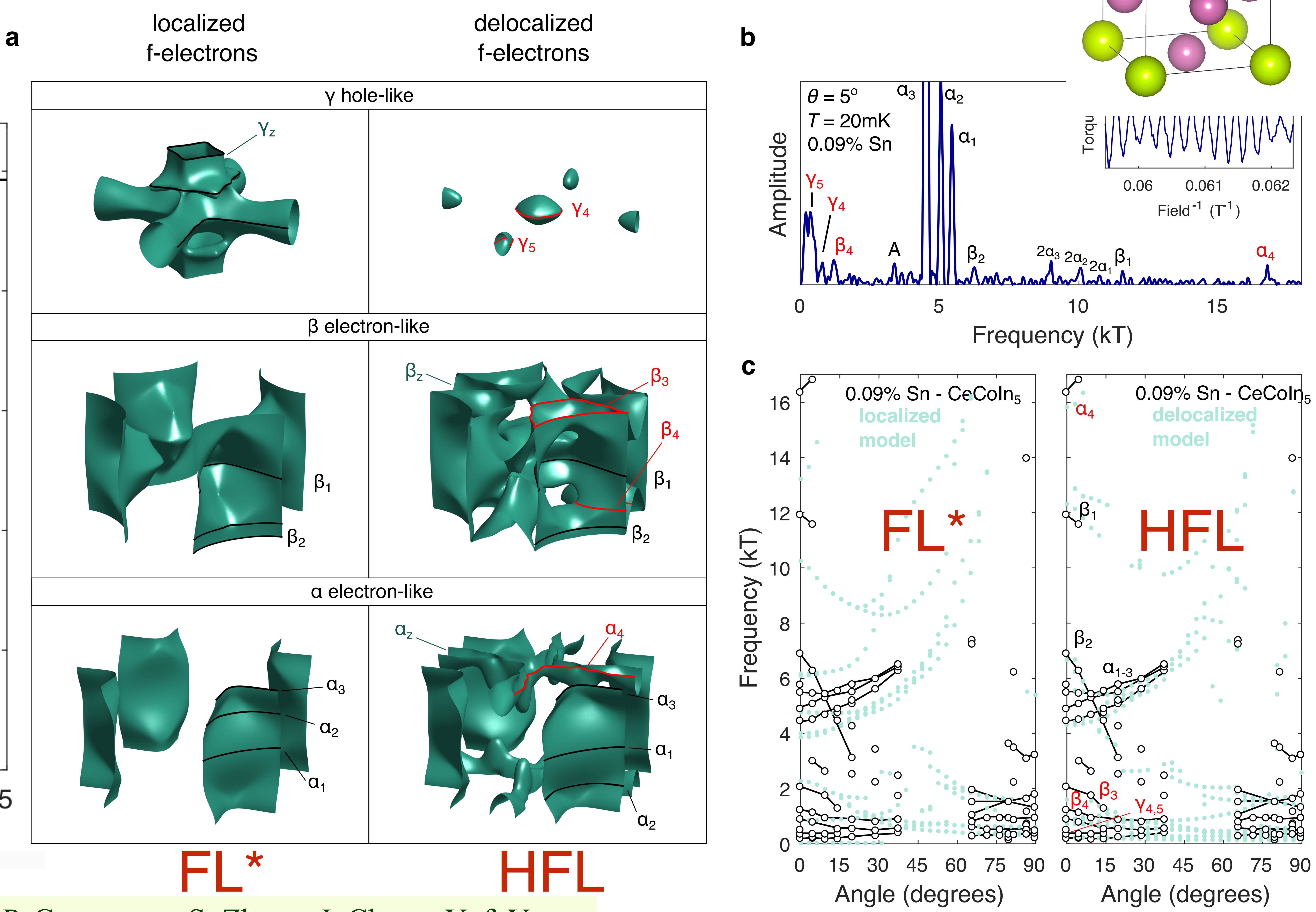
Evidence for freezing of charge degrees of freedom across a critical point in $CeCoIn_5$

Nikola Maksimovic, Taylor Cookmeyer, Jan Rusz, Vikram Nagarajan, Amanda Gong, Fanghui Wan,
Stefano Faubel, Ian M. Hayes, Sooyoung Jang, Yochai Werman, Peter M. Oppeneer, Ehud Altman, James G. Analytis



arXiv:2011.12951





See also H. Zhao, J. Zhang, M. Lyu, S. Bachus, Y. Tokiwa, P. Gegenwart, S. Zhang, J. Cheng, Y.-f. Yang, G. Chen, Y. Isikawa, Q. Si, F. Steglich, and P. Sun, Nature Physics 15, 1261 (2019) for CePdAl

1. Spin liquids and violations of the Luttinger theorem: FL* and HFL phases of the Kondo lattice model.
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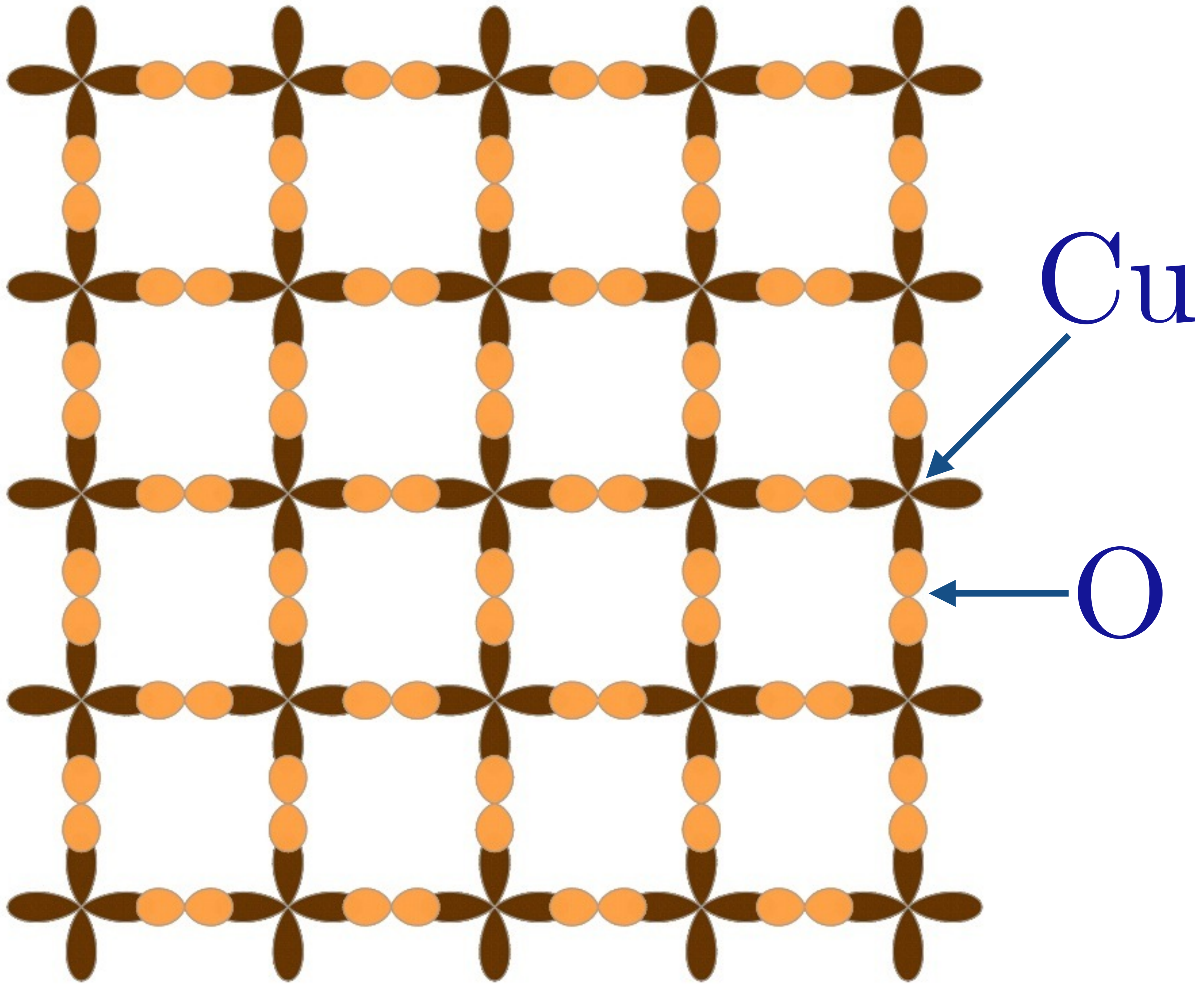
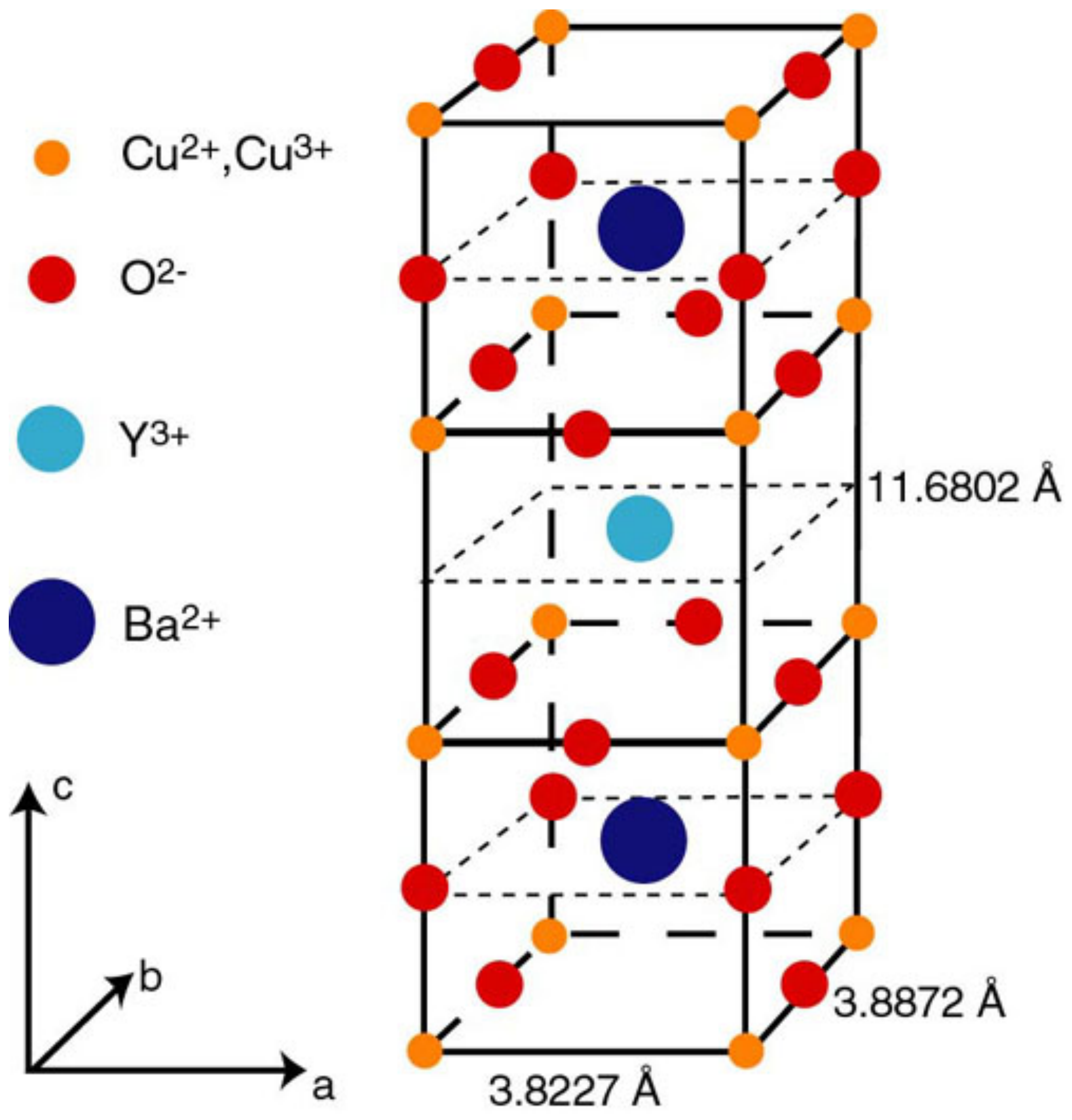
$$\mathcal{H}_H = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

High temperature superconductors



CuO_2 plane



$YBa_2Cu_3O_{6+x}$

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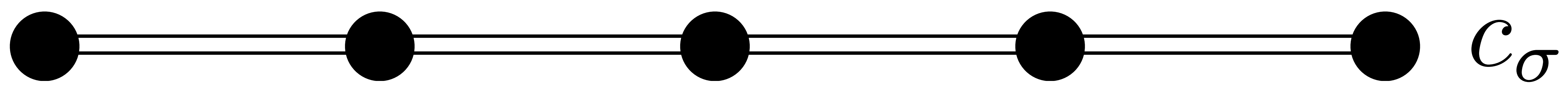
- The Luttinger theorem implies a FL phase with 'large' Fermi surface of size $1 + p$ holes (or $1 - p$ electrons) for all U and all p.



density
1+p

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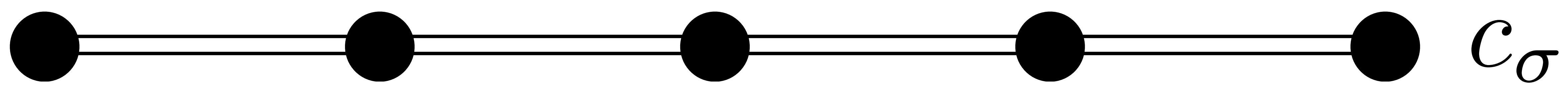

density
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P.W. Anderson, P.A. Lee, M. Randeria, T. M. Rice, N. Trivedi, and F. C. Zhang, Journal of Physics Condensed Matter **16**, R755 (2004)

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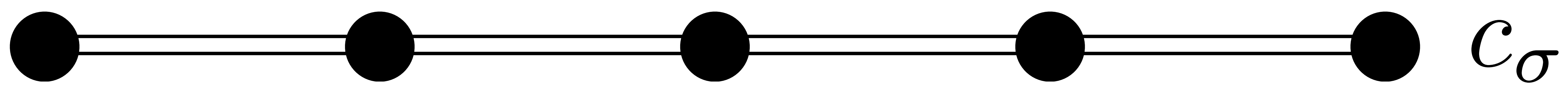
- The main effect of the projection is a 'Brinkman-Rice' enhancement of the quasiparticle mass as $p \to 0$, with $m^*/m \sim 1/p$.



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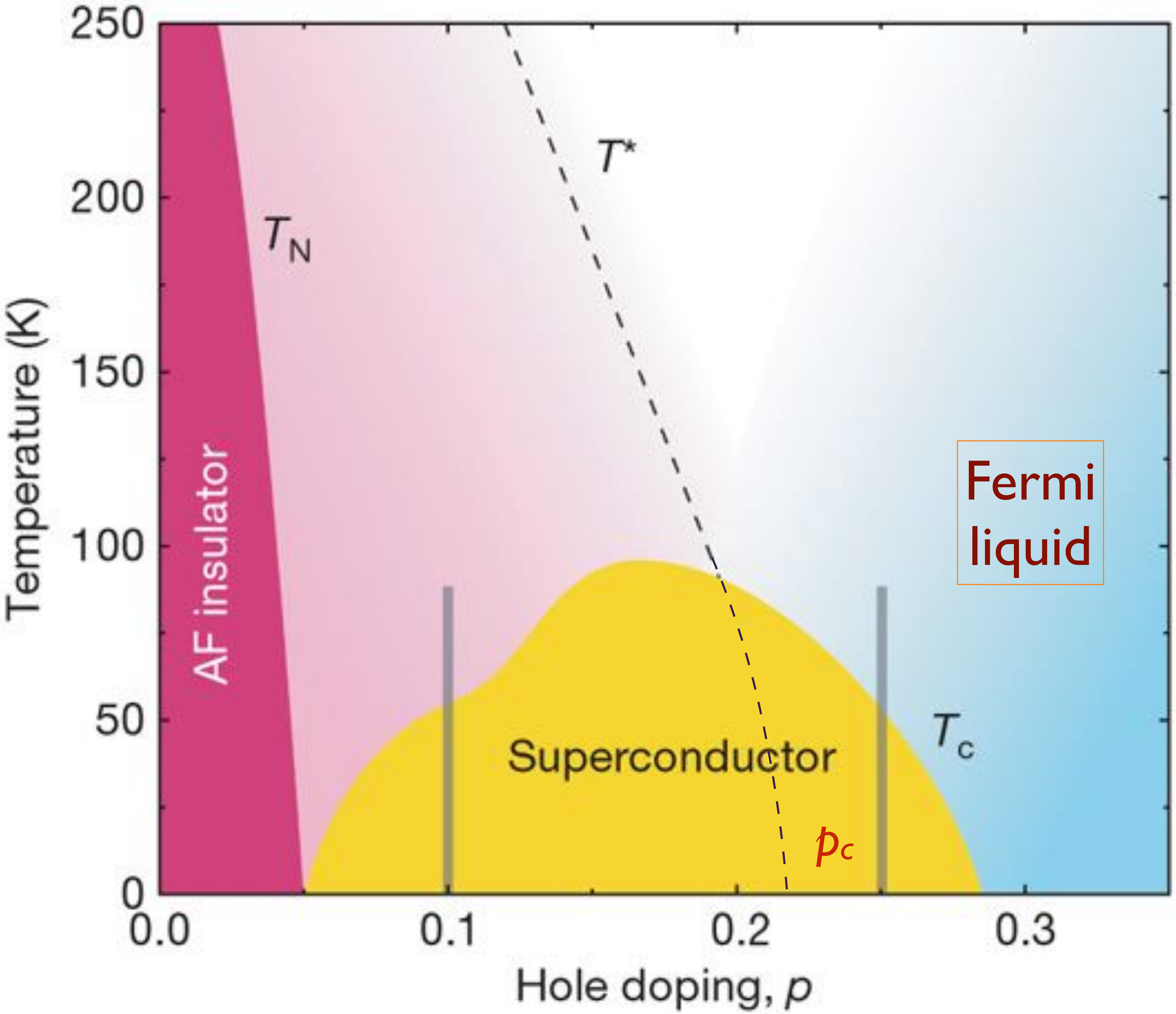
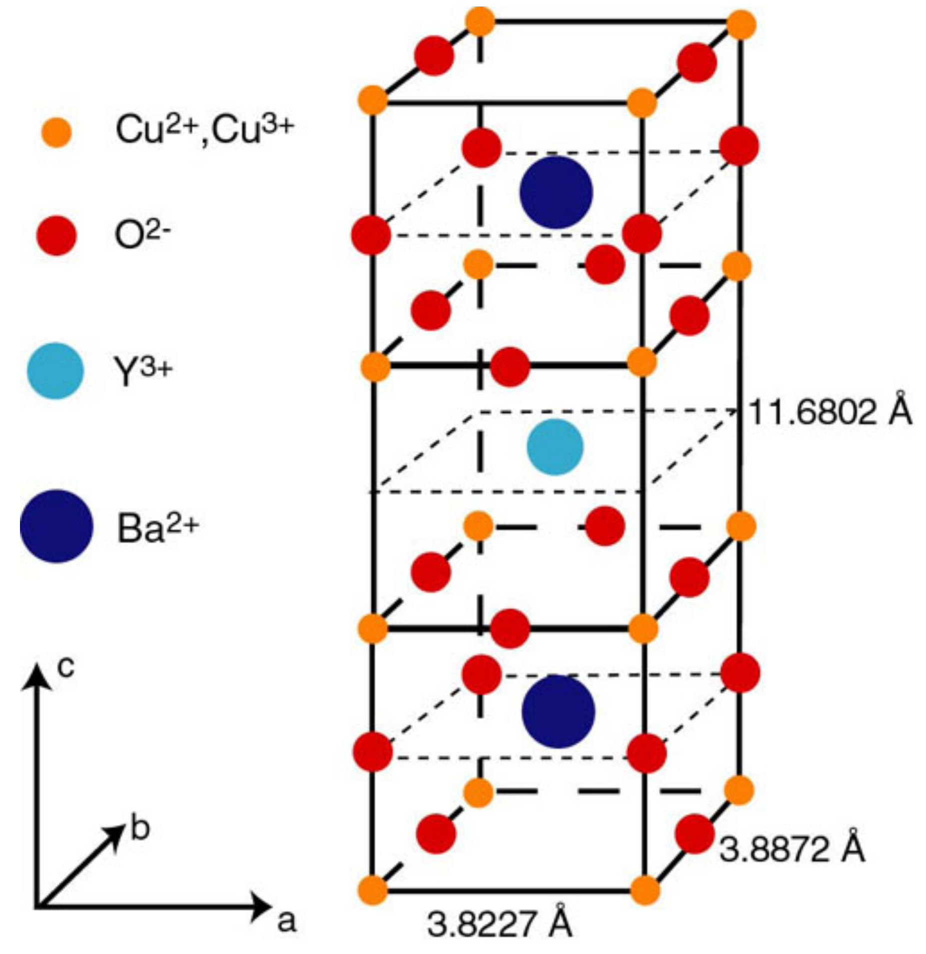
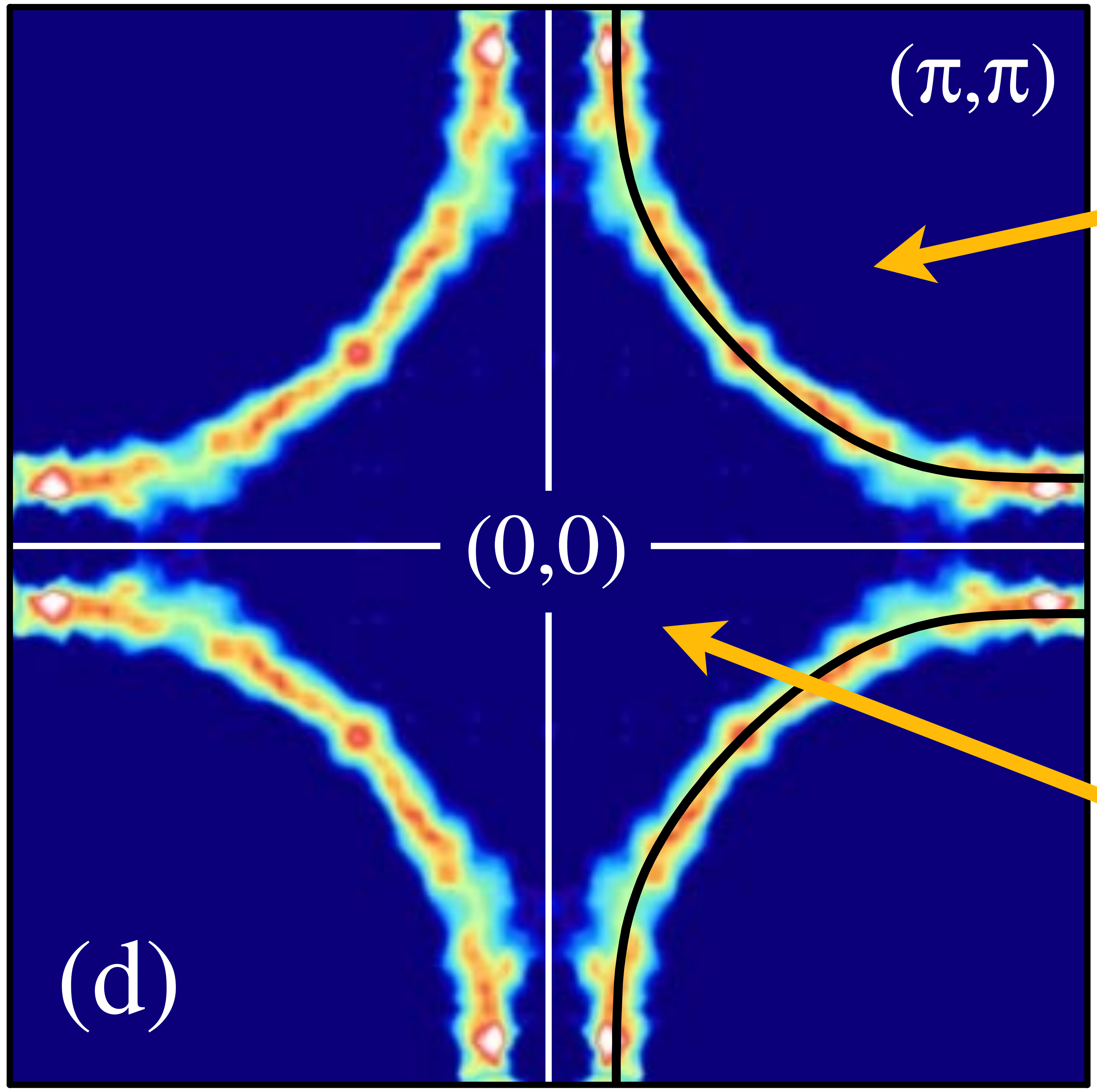

250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c
Fermi liquid


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Photoemission at large p



$1+p$ holes

1-p electrons

Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

$1+p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

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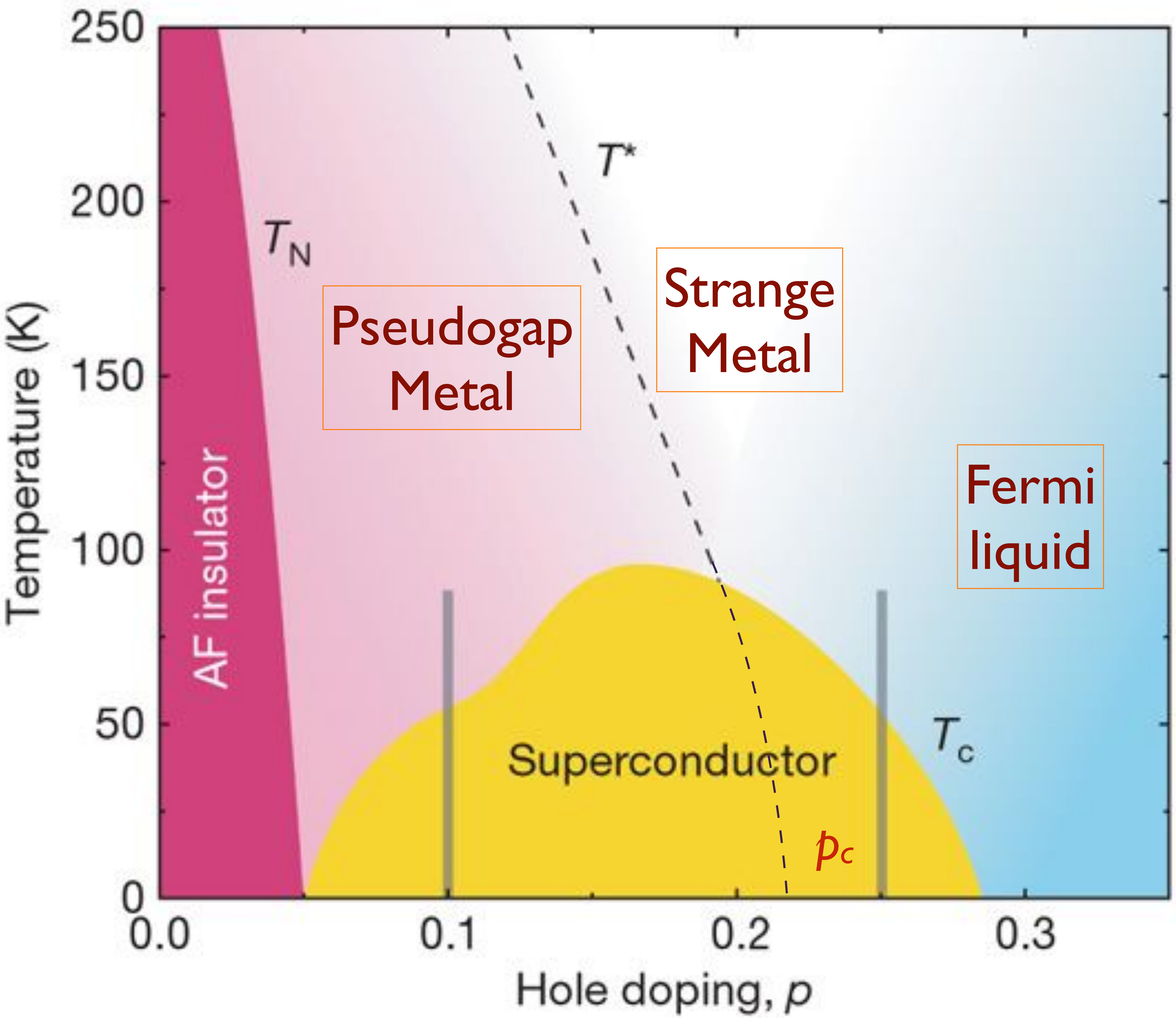

250
200
150
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0.0
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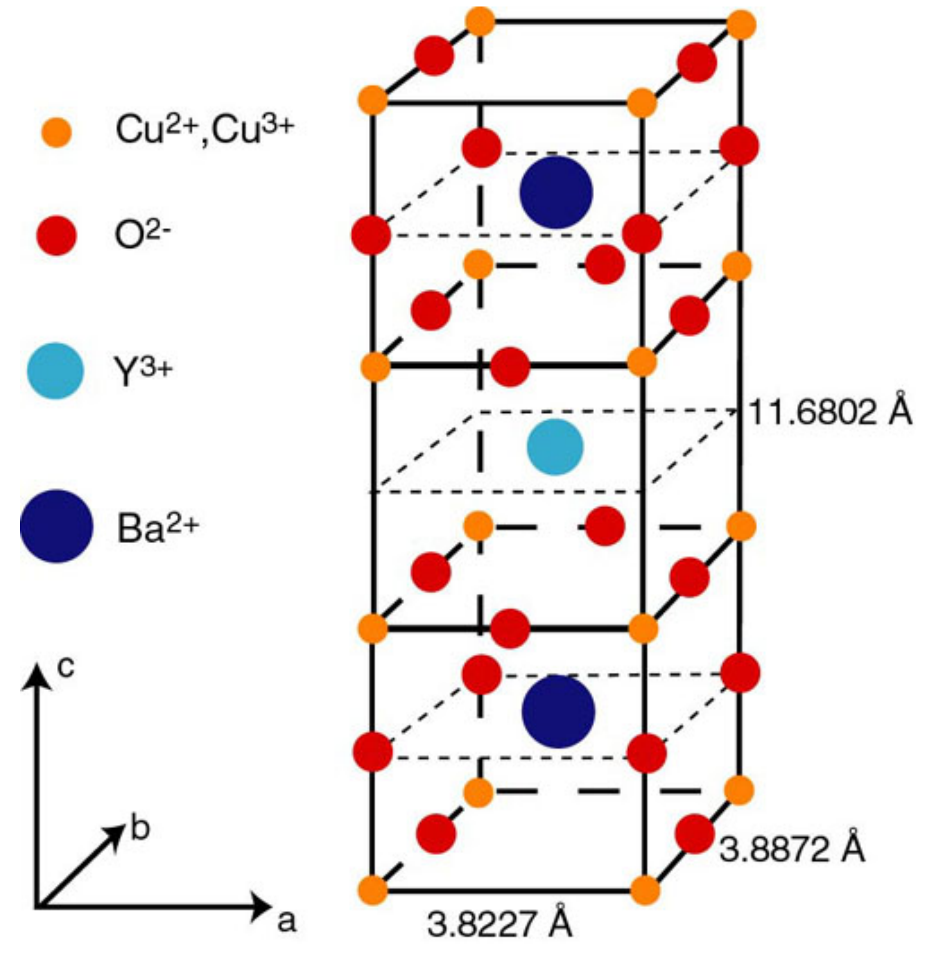
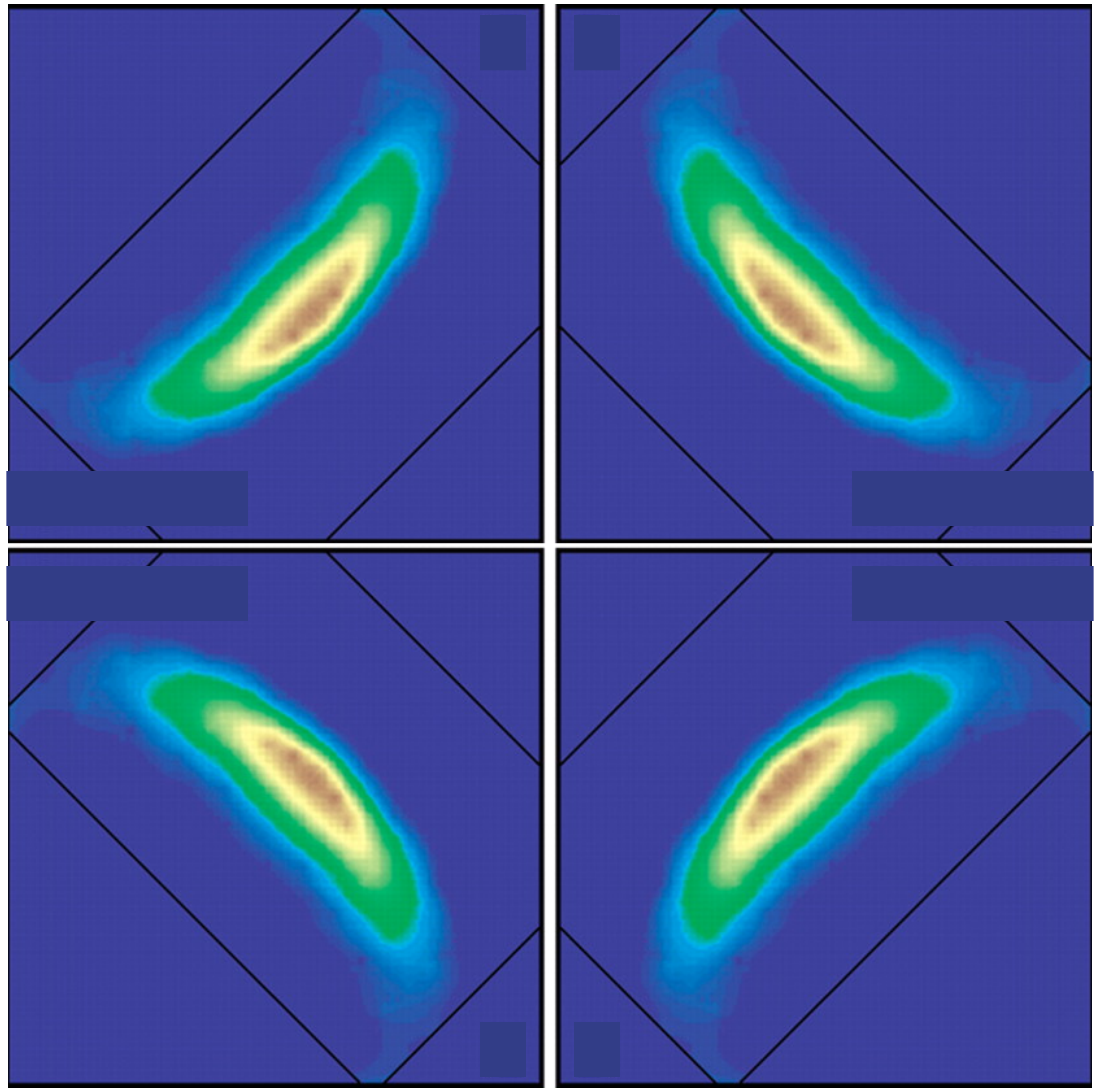
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Photoemission at small p



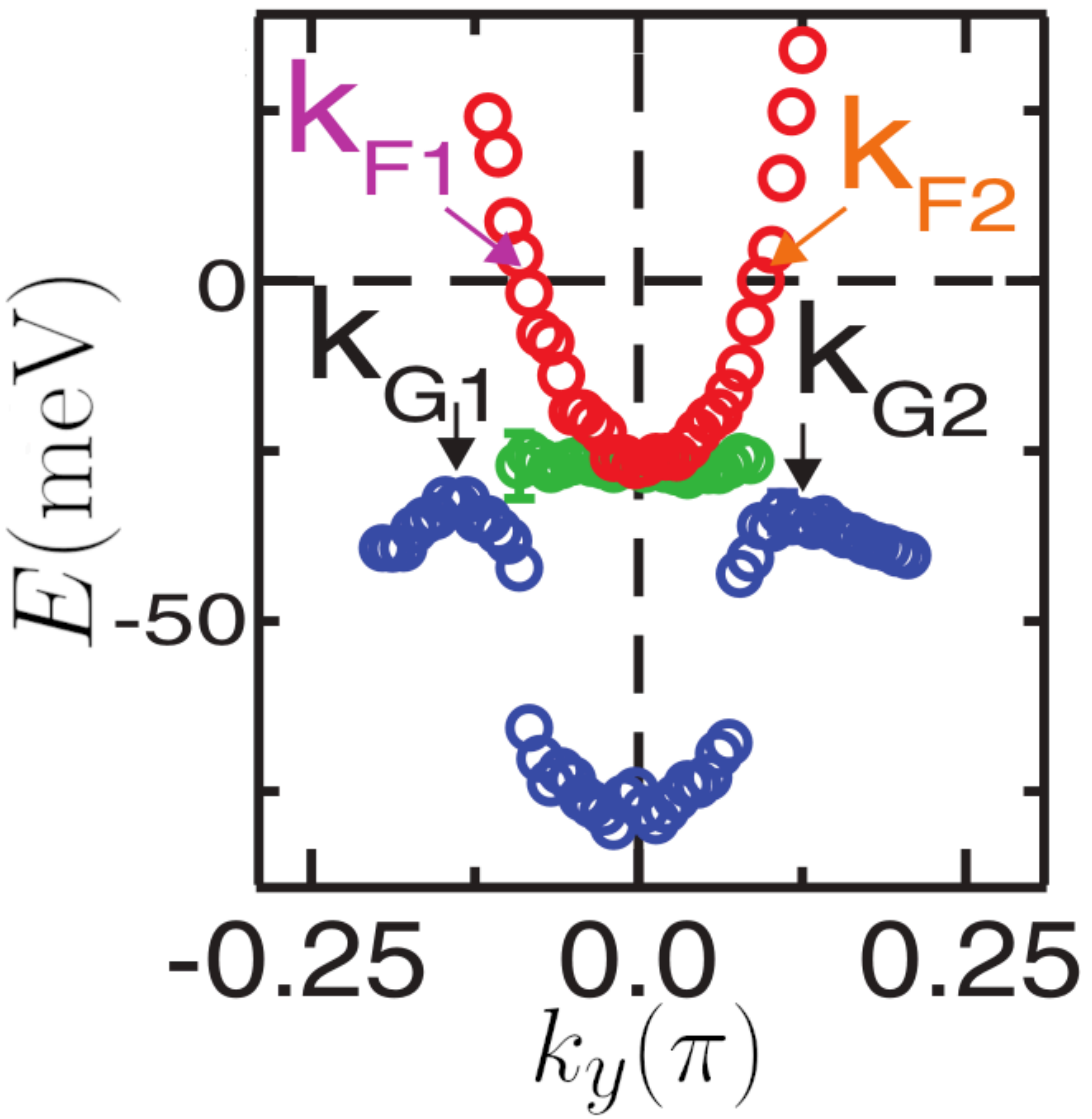
$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

“Fermi arcs”

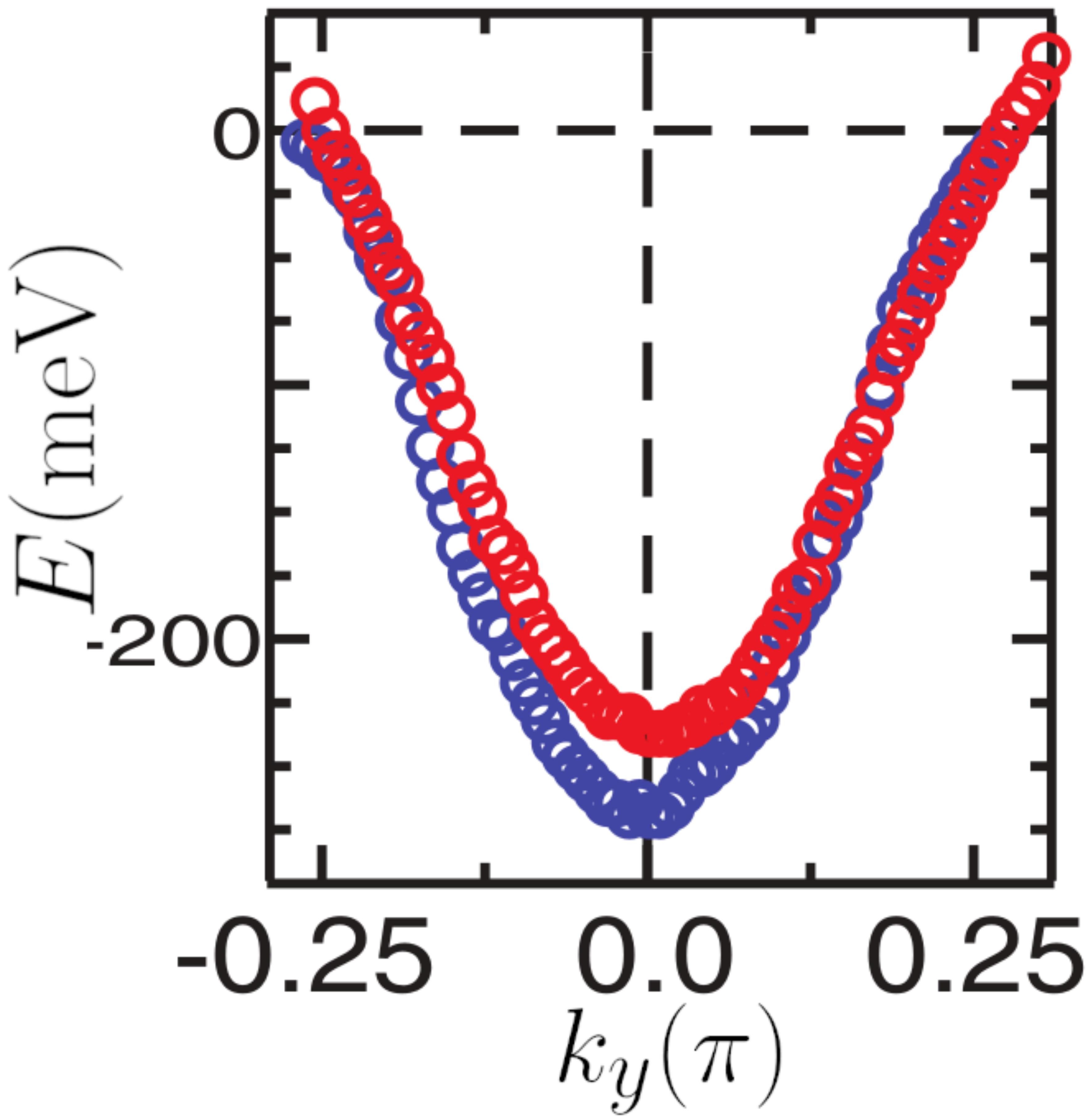
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana,
Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission at small and large p

Anti-node: $k_x = \pi$



Node: $k_x = 2$



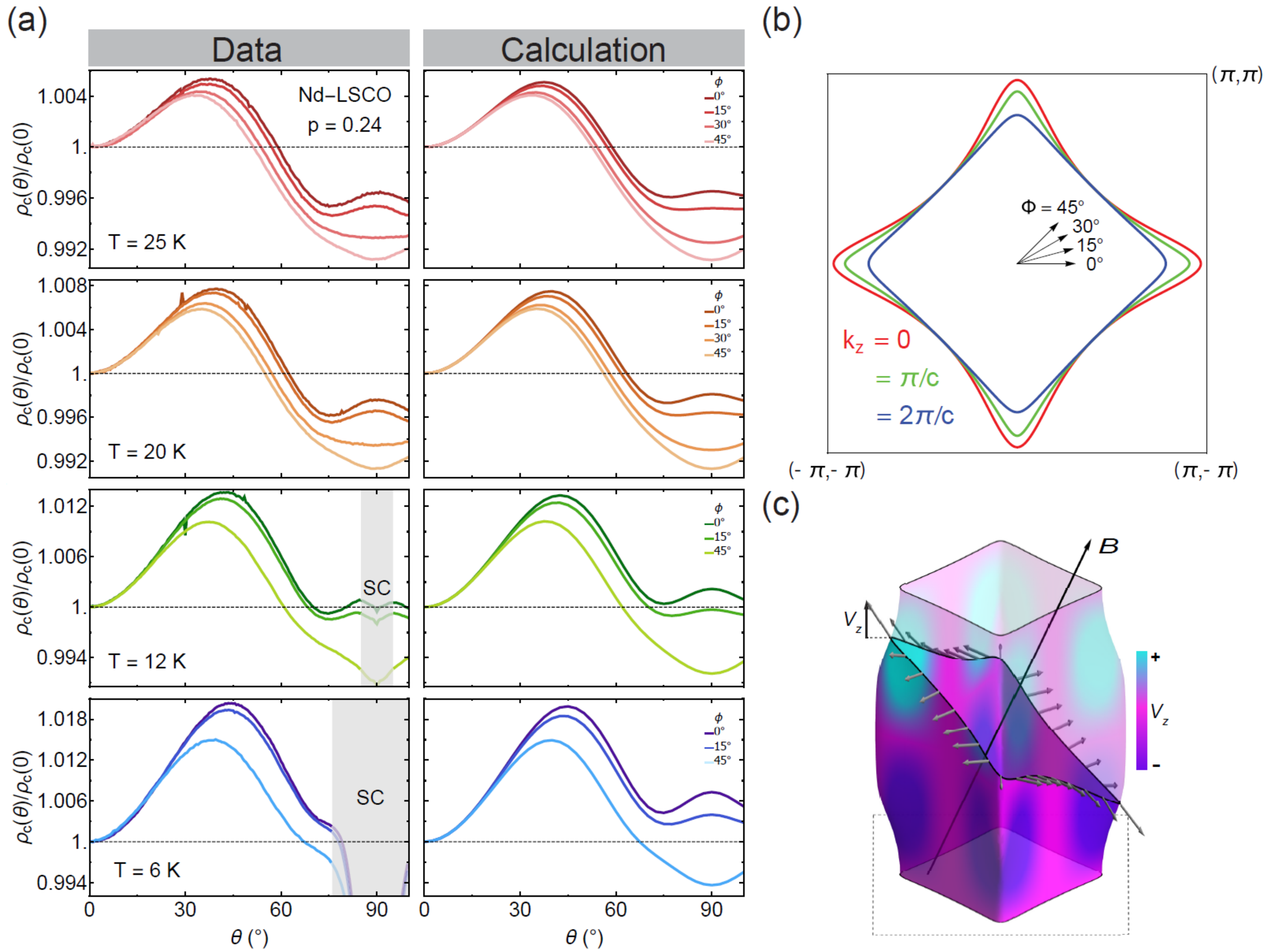
Electronic dispersion in the pseudogap metal

He *et al.*, *Science* **331**, 1579 (2011)

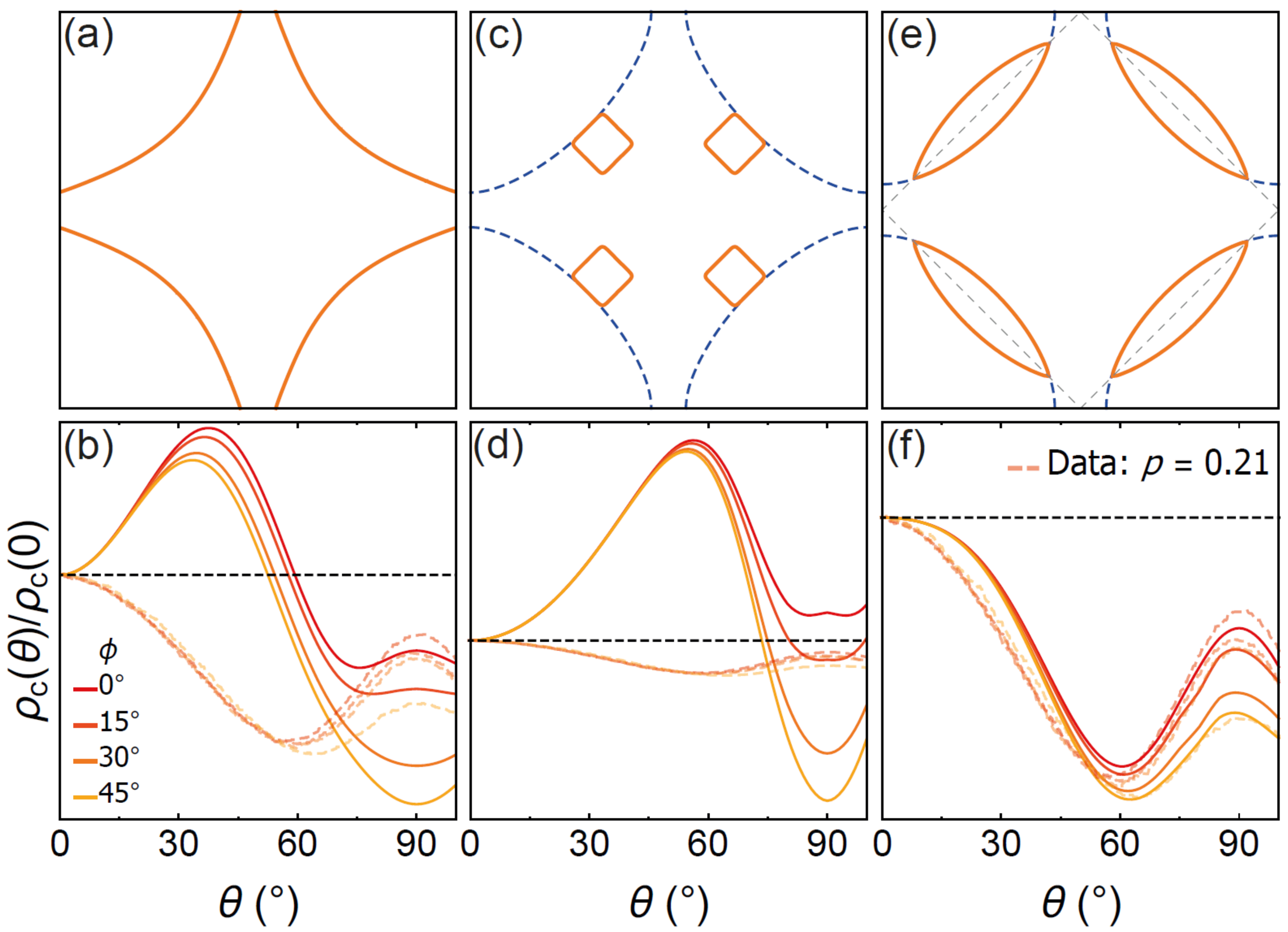
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, arXiv:2004.01725

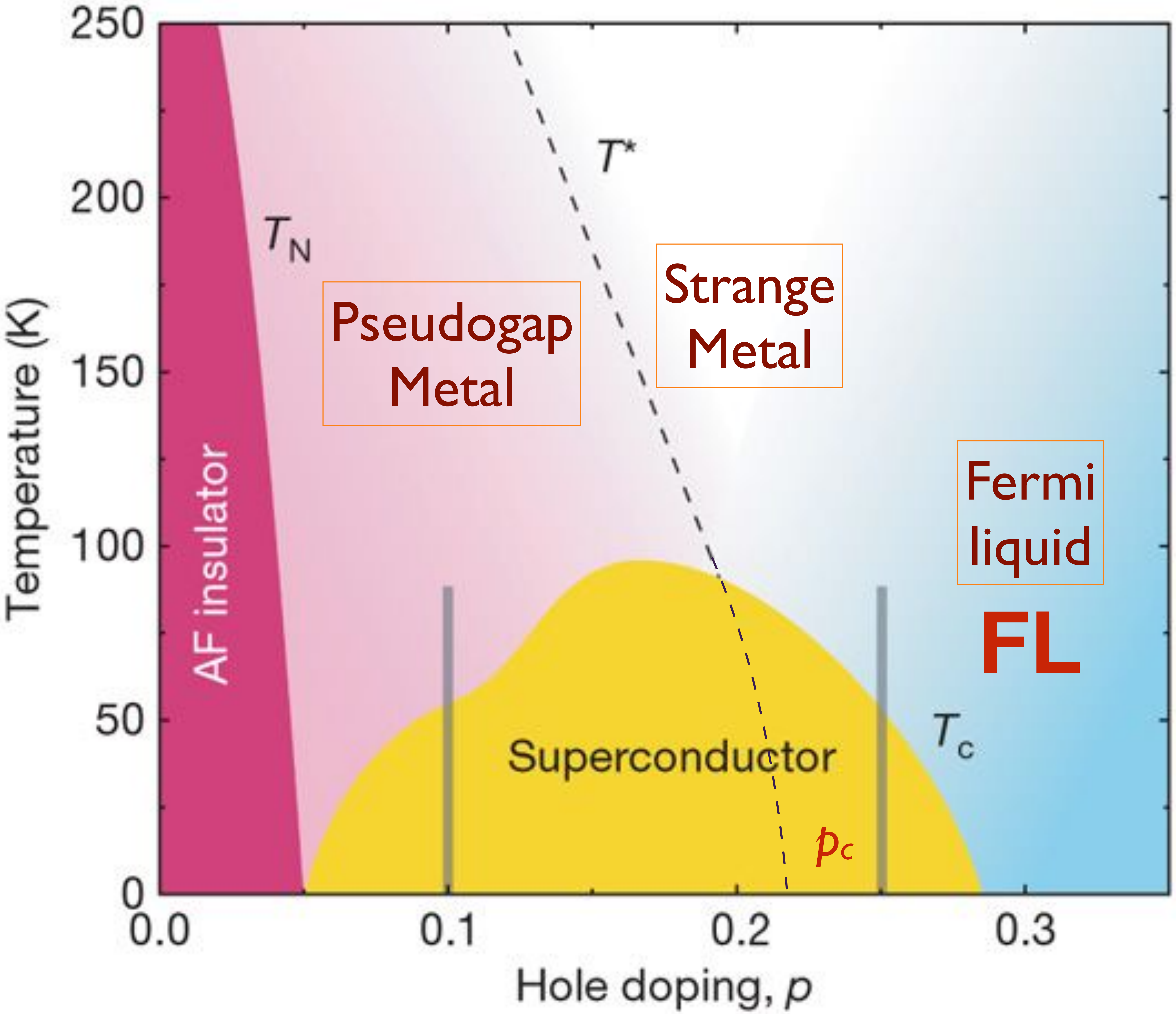
We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$. Above the critical doping p*—outside of the pseudogap phase—we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below p*, however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a Q = (π,π) wavevector. While static Q = (π,π) antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.

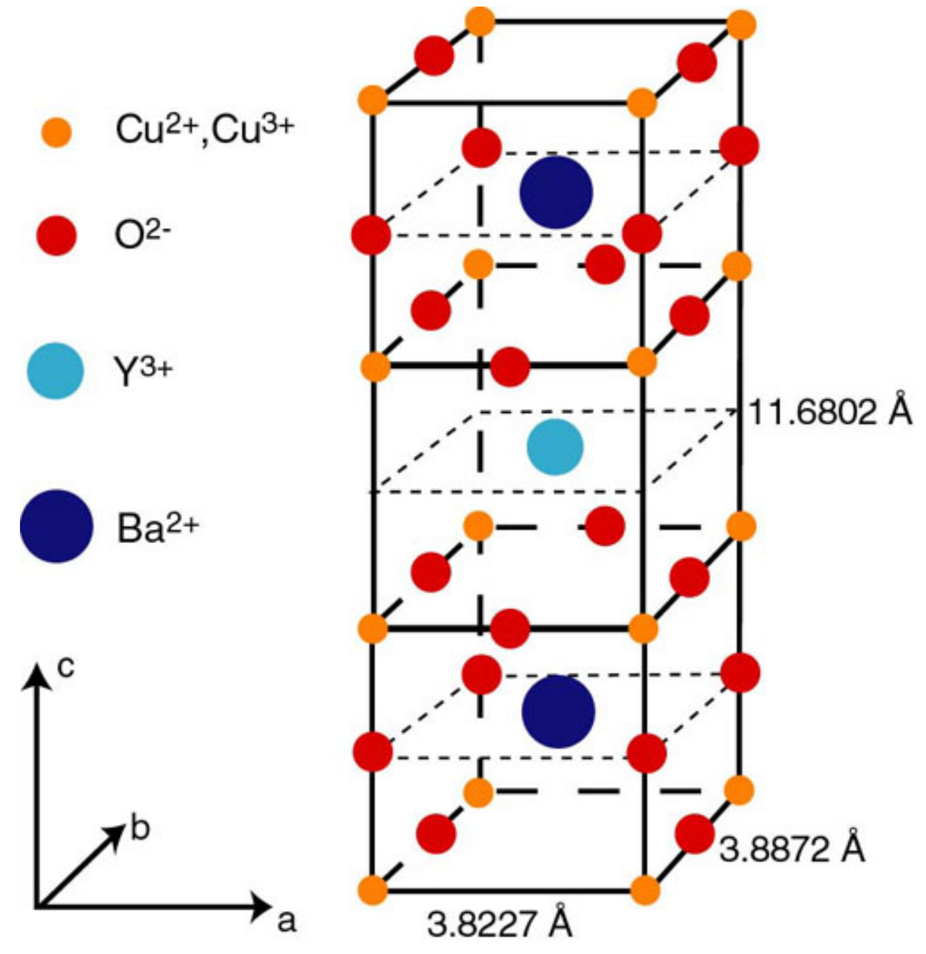


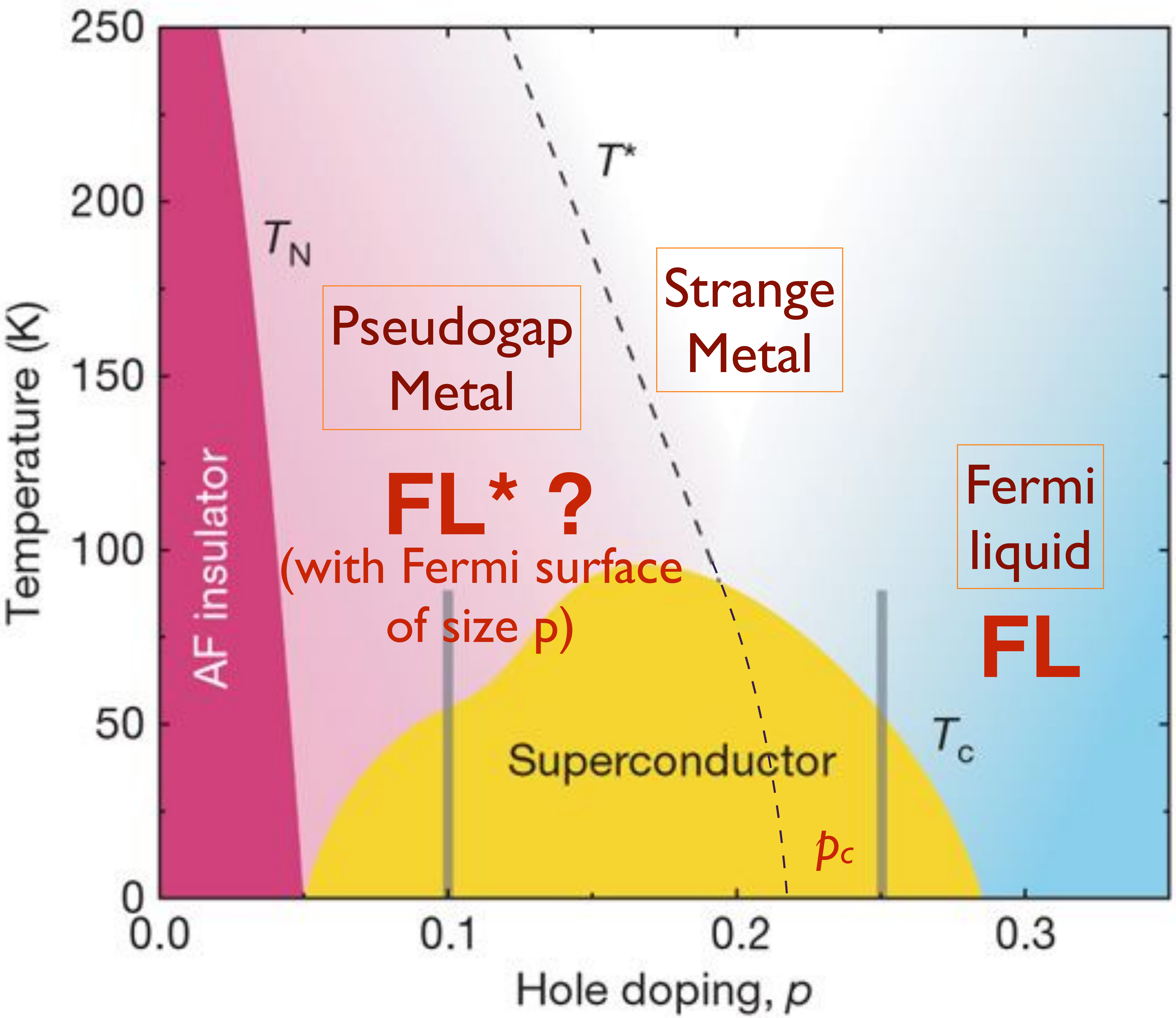
$p > p_c$ Large Fermi surface

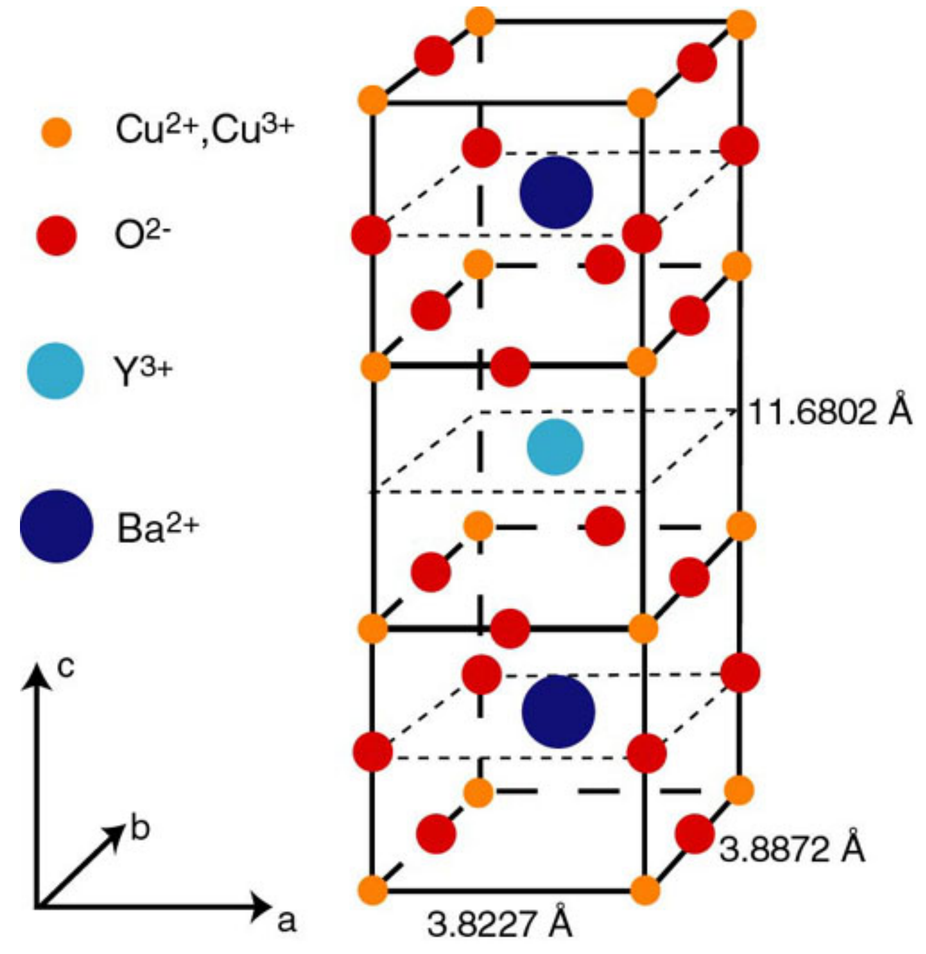


$p < p_c$ Reconstructed Fermi surface


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
Pseudogap Metal
T*
Strange Metal
Fermi liquid
FL
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Strange Metal
FL* ?
(with Fermi surface of size p)
Fermi liquid
FL
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

The pseudogap metal ≈ FL* (these papers fractionalize the mobile electron)

X.-G. Wen and P. A. Lee, "Theory of Underdoped Cuprates," Phys. Rev. Lett. **76**, 503 (1996), arXiv:cond-mat/9506065 [cond-mat].

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K.-Y. Yang, T. M. Rice, and F.-C. Zhang, "Phenomenological theory of the pseudogap state," Phys. Rev. B **73**, 174501 (2006), arXiv:cond-mat/0602164 [cond-mat.supr-con].

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J. Feldmeier, S. Huber, and M. Punk, "Exact solution of a two-species quantum dimer model for pseudogap metals," Phys. Rev. Lett. **120**, 187001 (2018), arXiv:1712.01854 [cond-mat.str-el].

B. Verheijden, Y. Zhao, and M. Punk, "Solvable lattice models for metals with Z_2 topological order," SciPost Physics **7**, 074 (2019), arXiv:1908.00103 [cond-mat.str-el].

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Quantum phases of the Shraiman-Siggia model by S. Sachdev Physical Review B **49**, 6770 (1994).

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M. Punk and S. Sachdev, "Fermi surface reconstruction in hole-doped t-J models without long-range antiferromagnetic order," Phys. Rev. B **85**, 195123 (2012), arXiv:1202.4023 [cond-mat.str-el].

M. Punk, A. Allais, and S. Sachdev, "A quantum dimer model for the pseudogap metal," Proc. Nat. Acad. Sci. **112**, 9552 (2015), arXiv:1501.00978 [cond-mat.str-el].

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S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, "Gauge theory for the cuprates near optimal doping," Phys. Rev. B **99**, 054516 (2019), arXiv:1811.04930 [cond-mat.str-el].

S. Sachdev, "Topological order, emergent gauge fields, and Fermi surface reconstruction," Rep. Prog. Phys. **82**, 014001 (2019), arXiv:1801.01125 [cond-mat.str-el].

Holon metal

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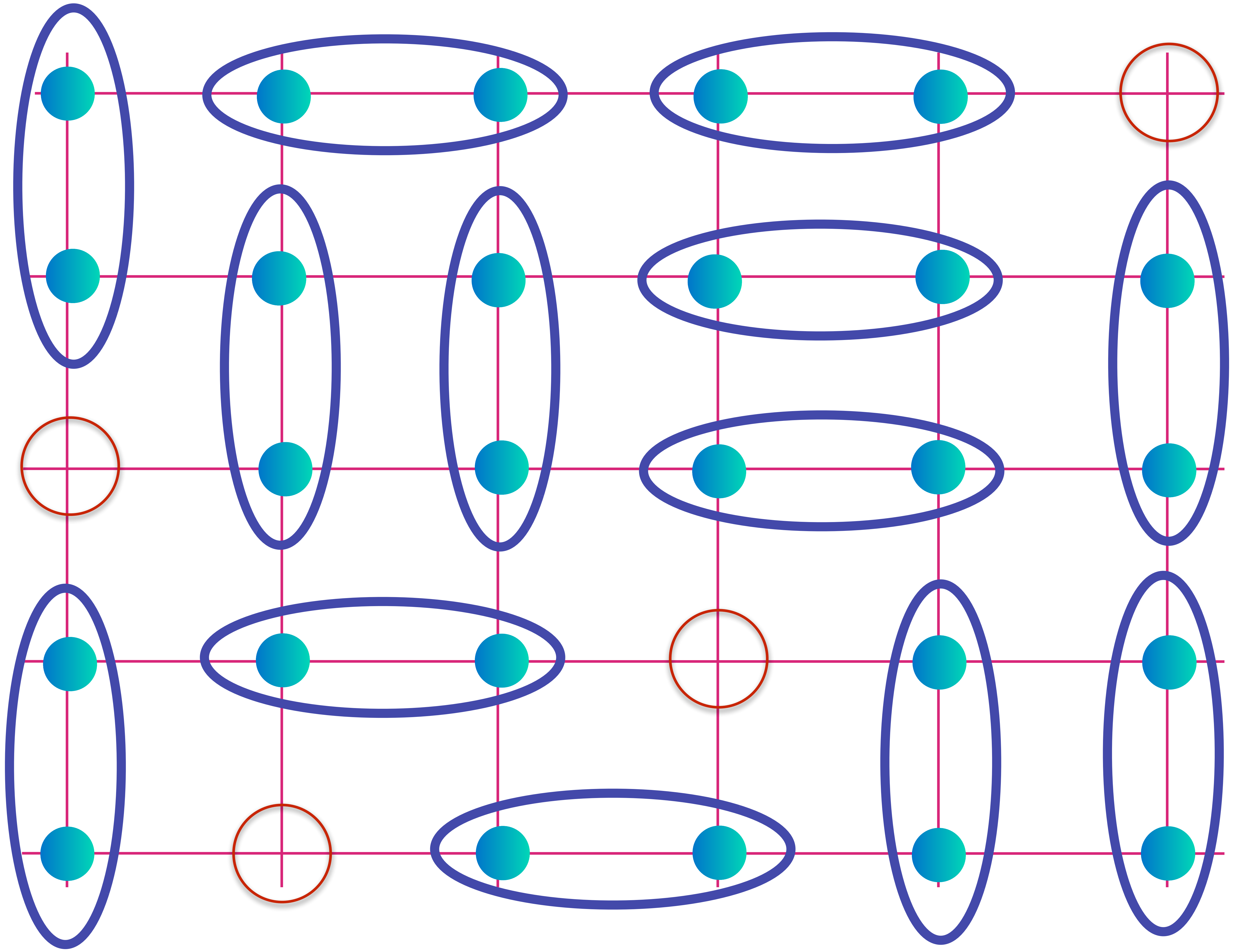
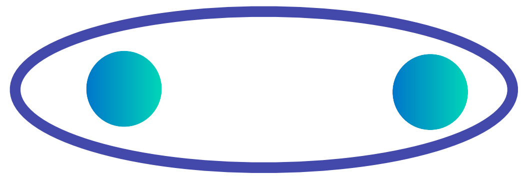


Spin liquid with density p of spinless, charge +e "holons".

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

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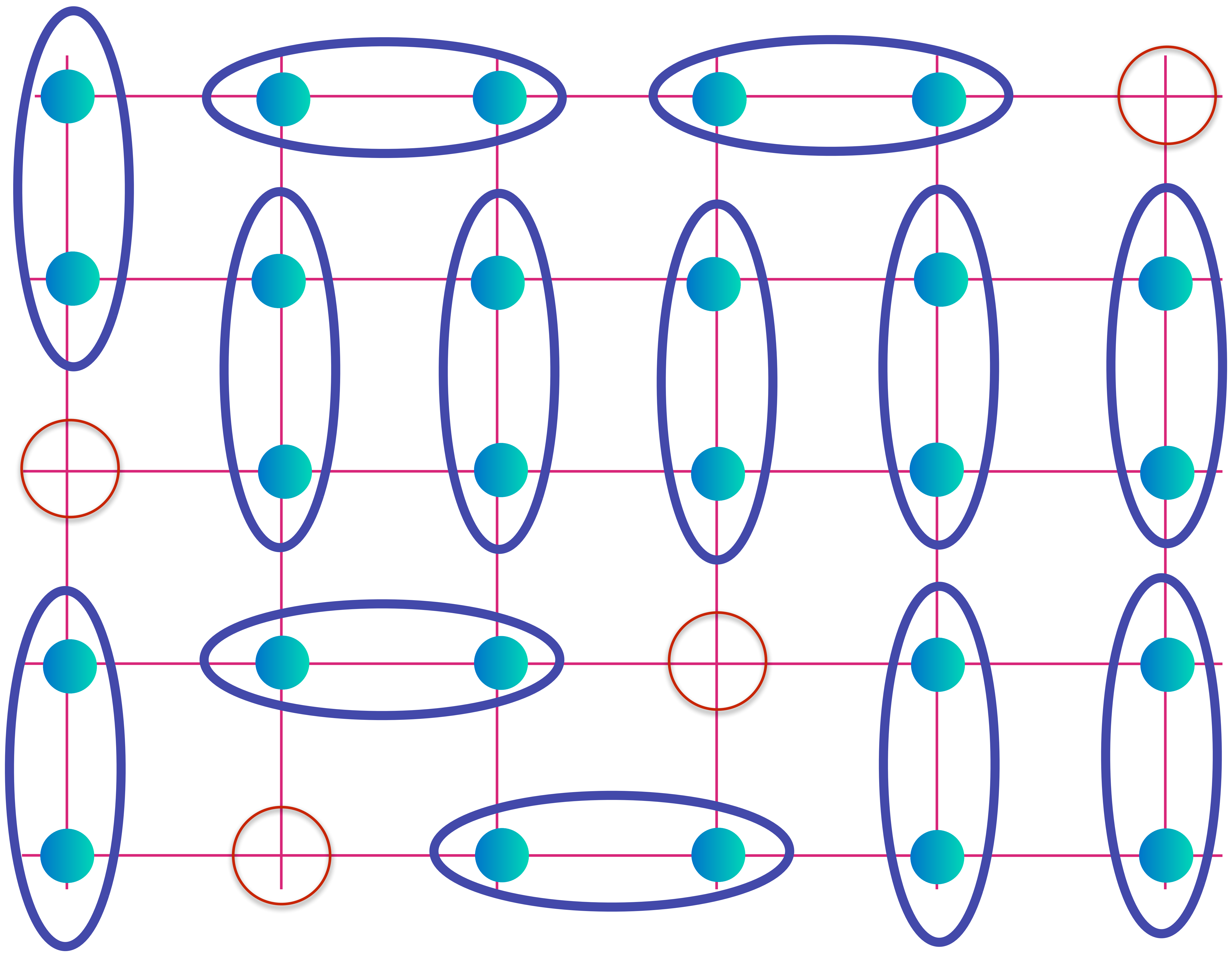

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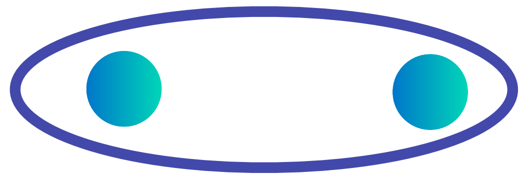

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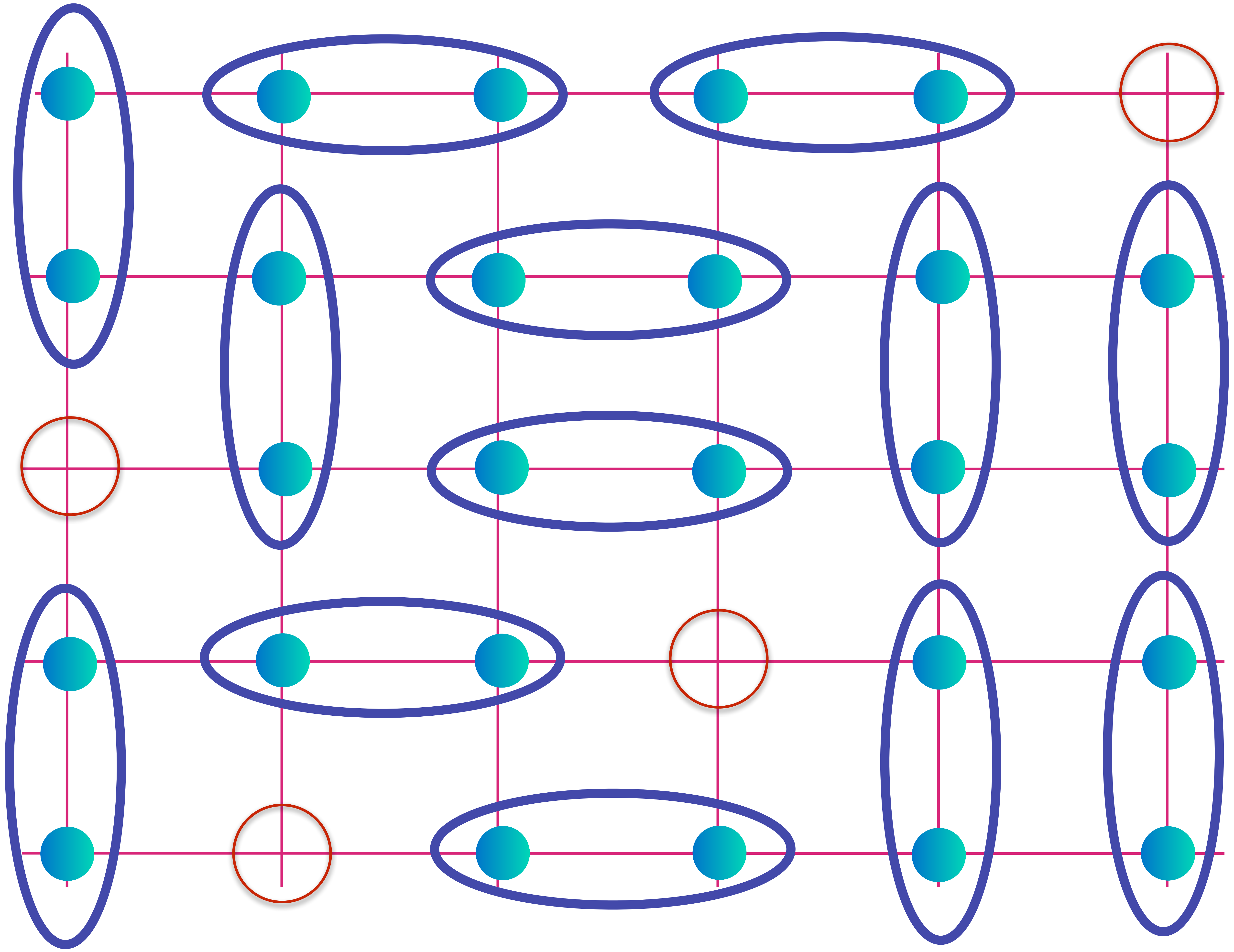

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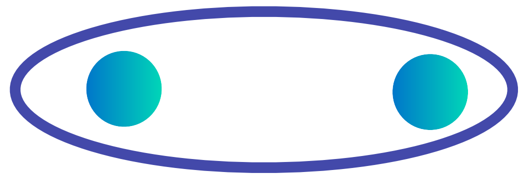 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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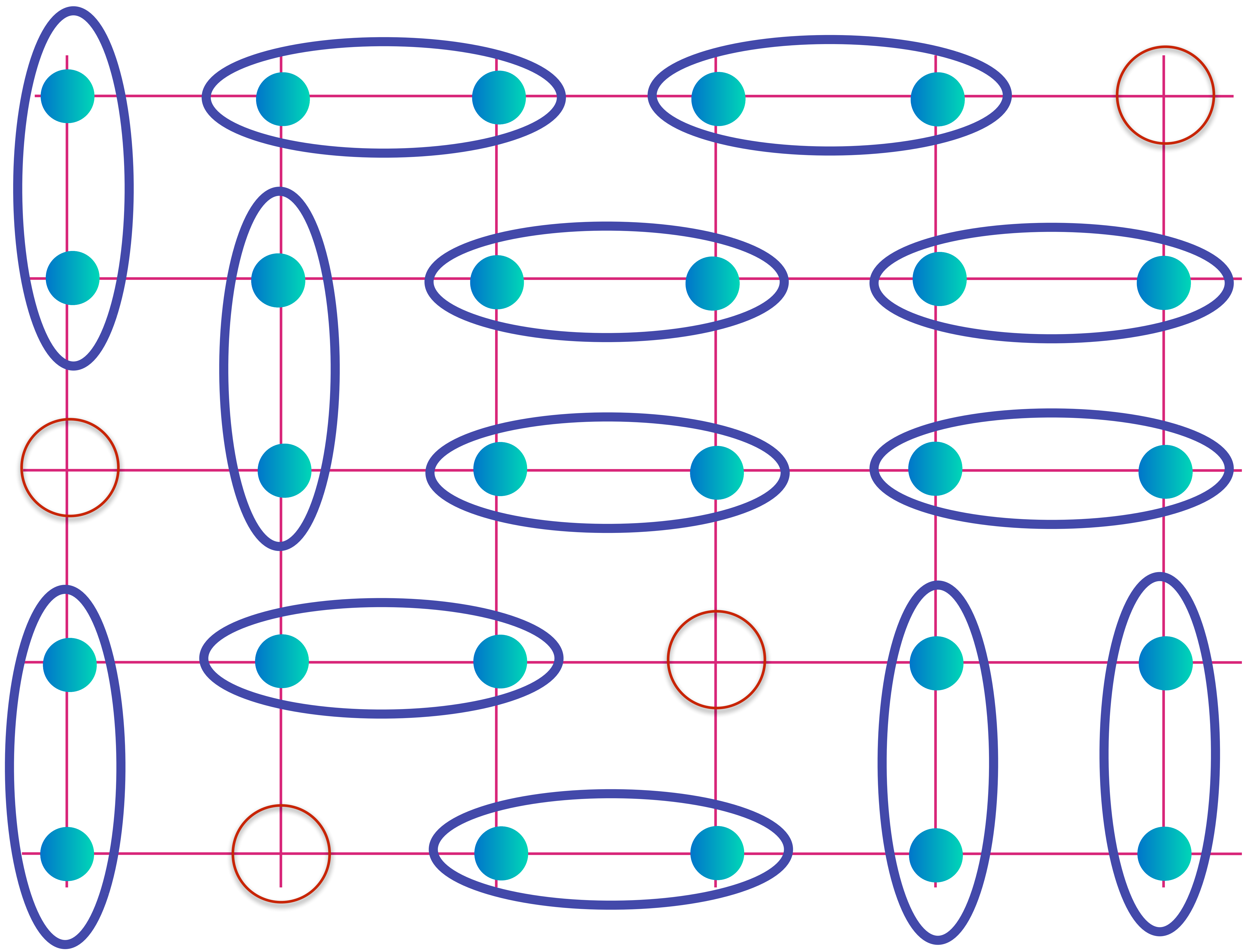

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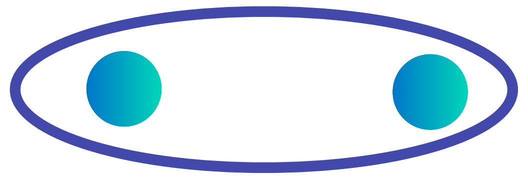 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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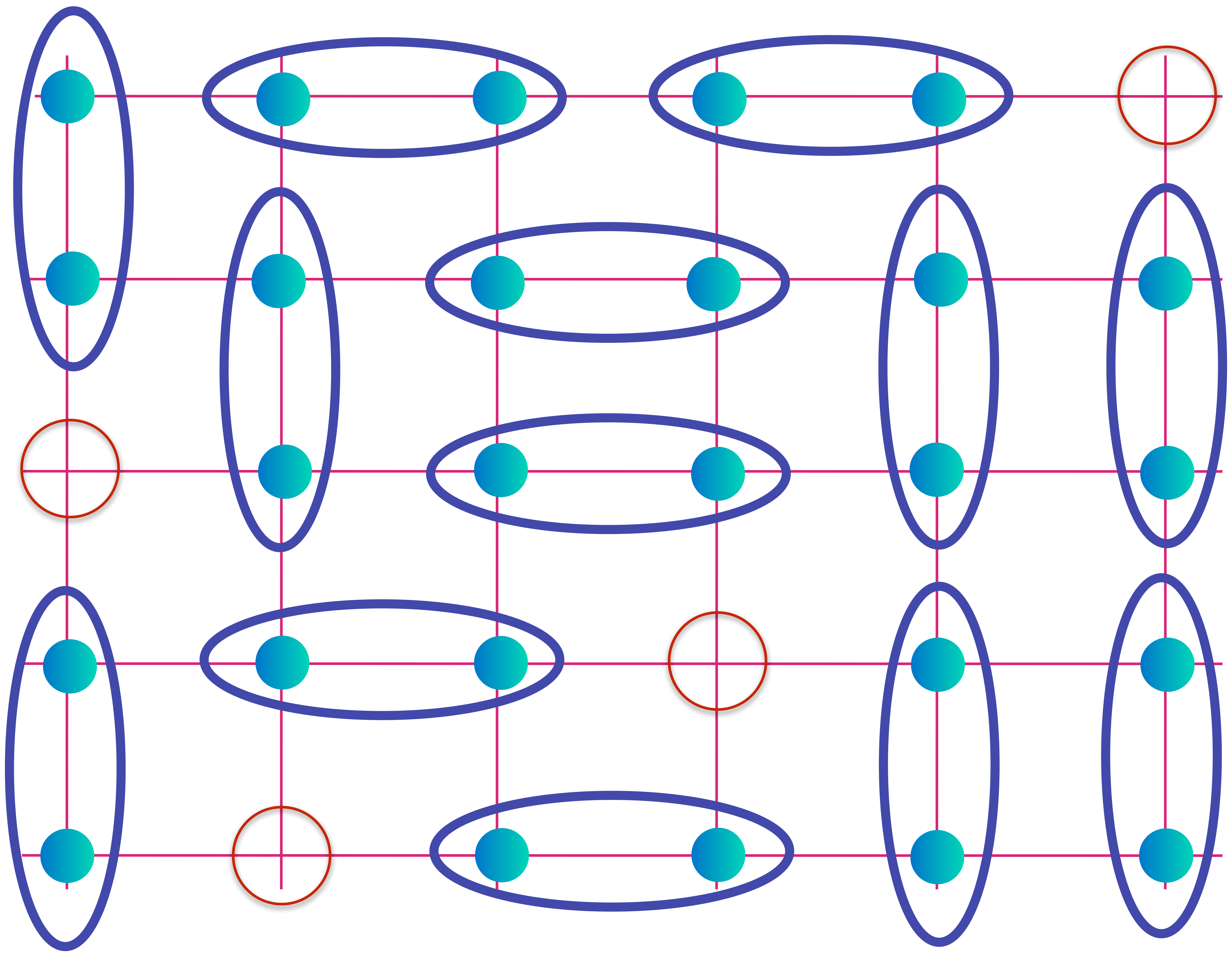

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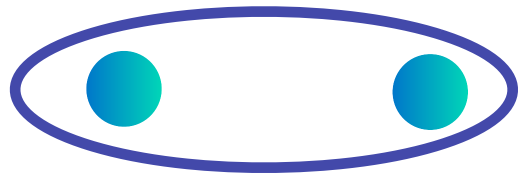

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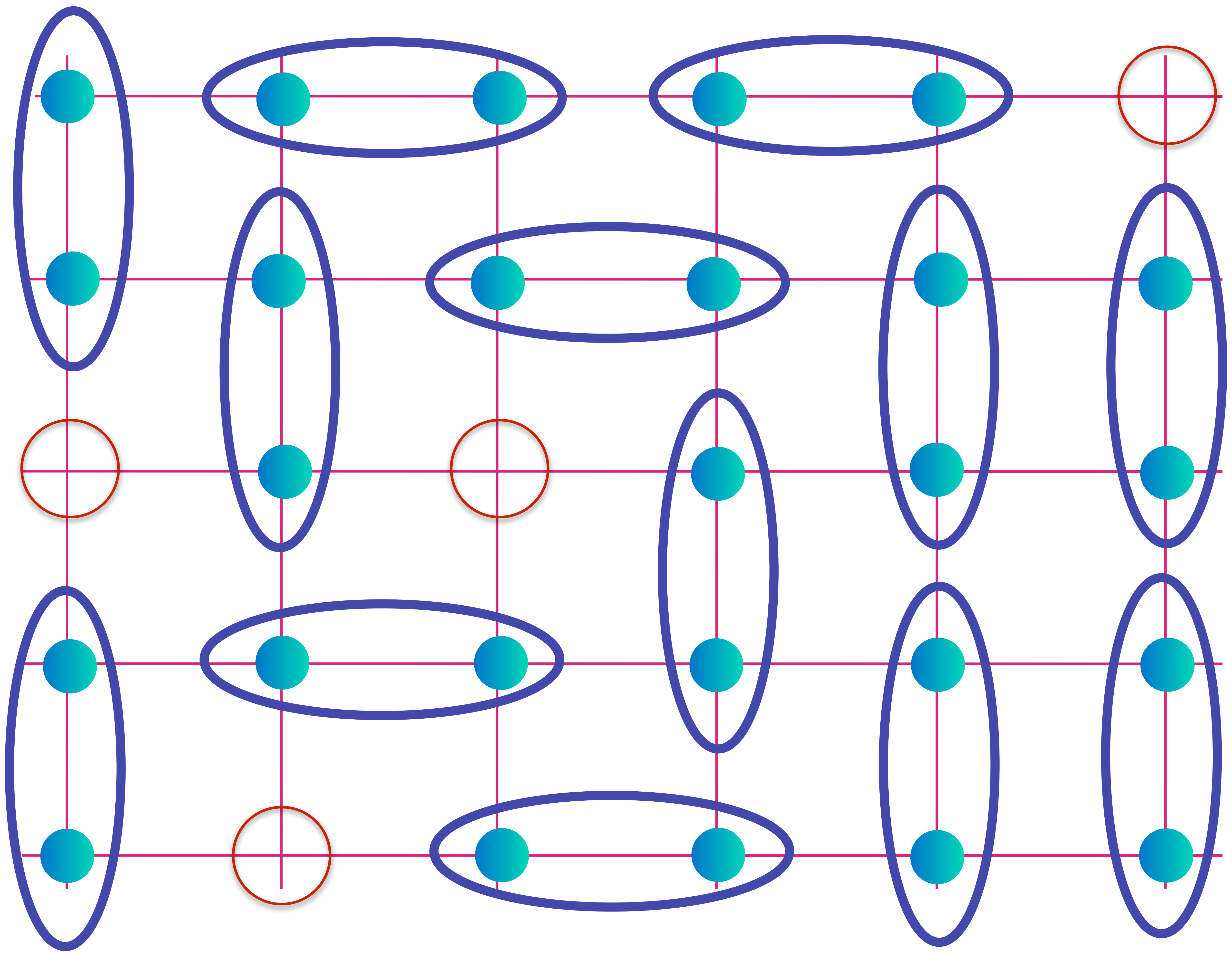

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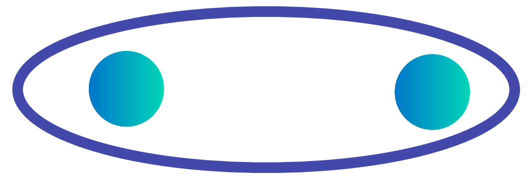

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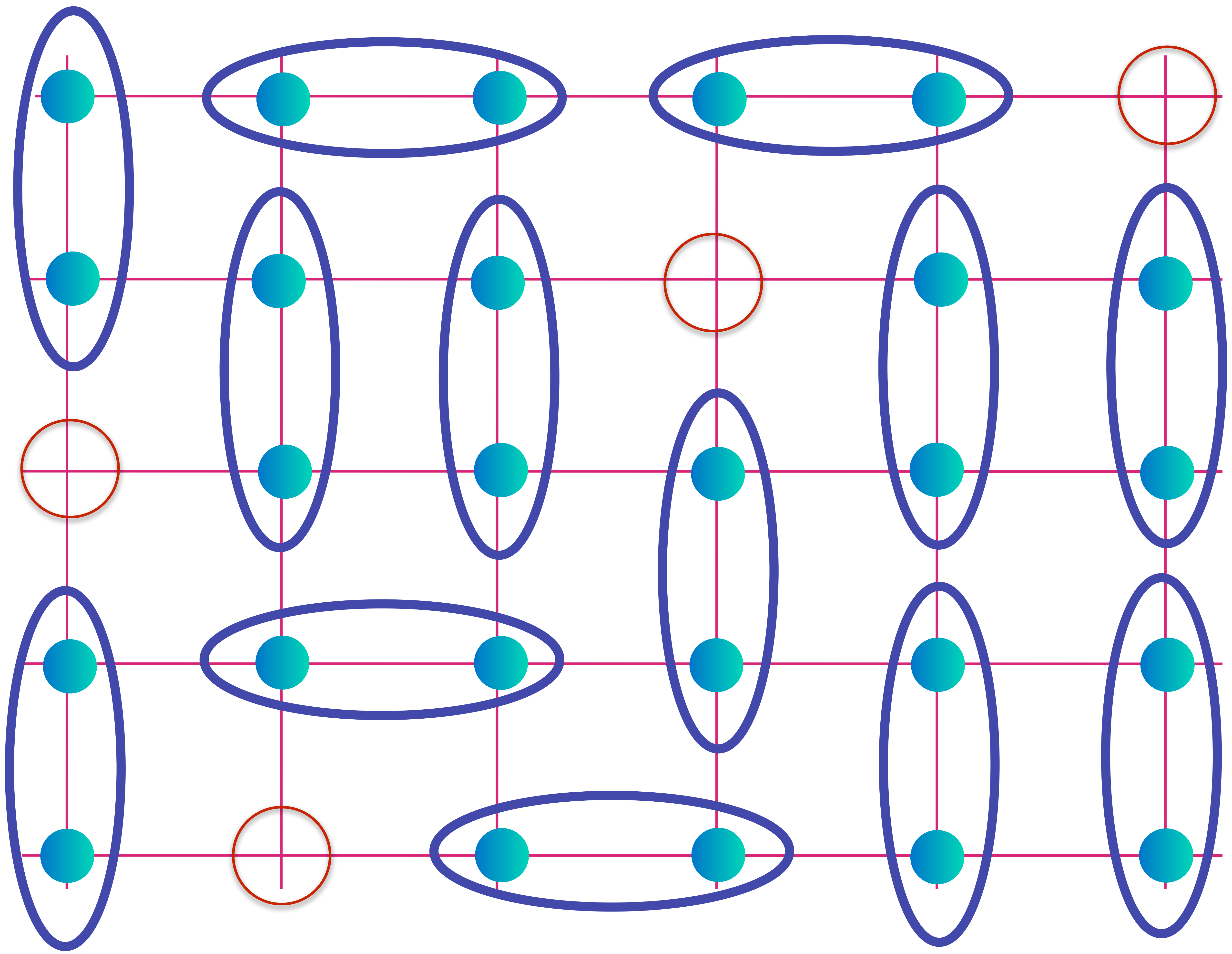

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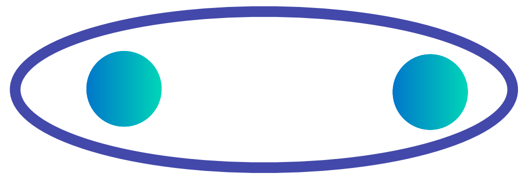 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

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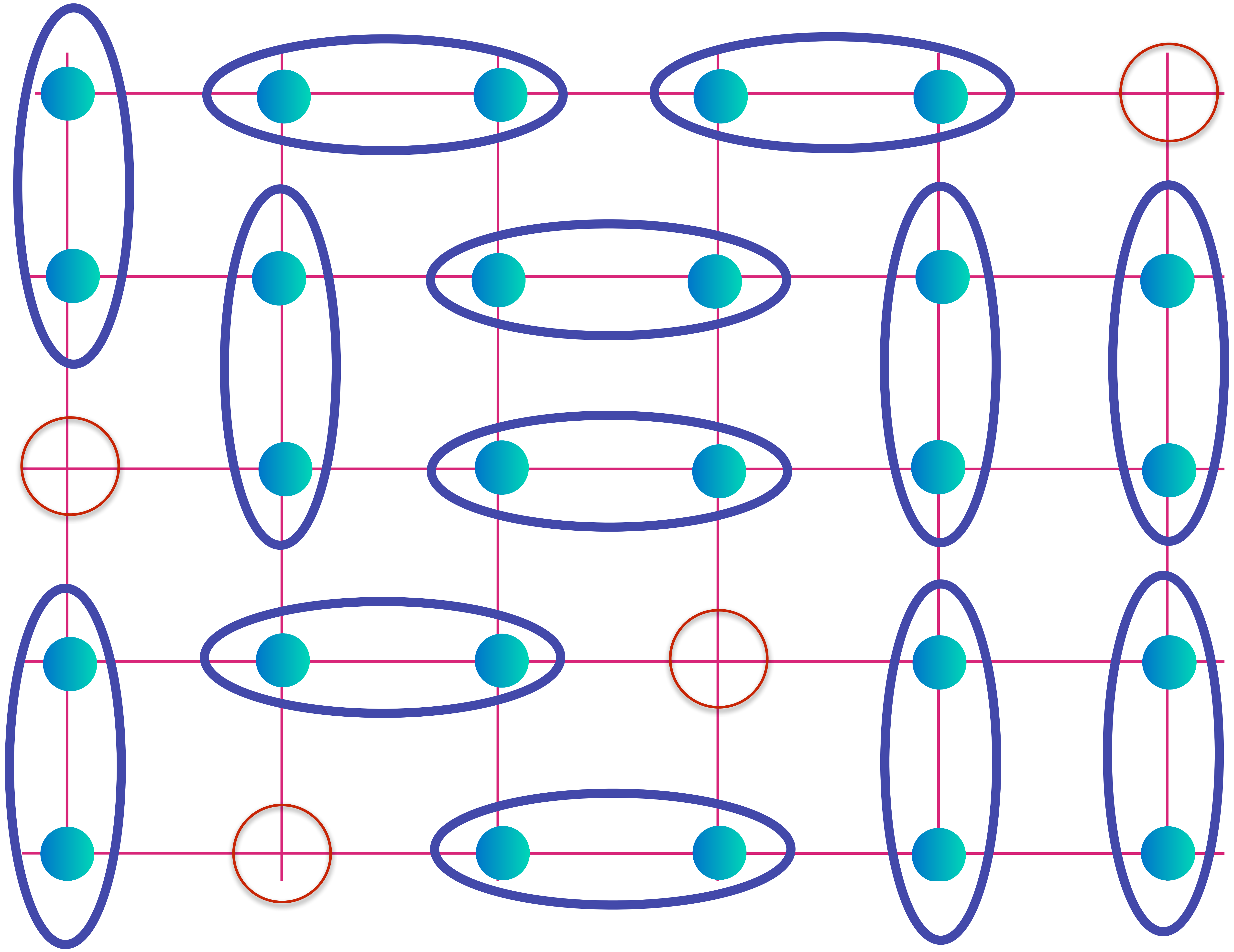

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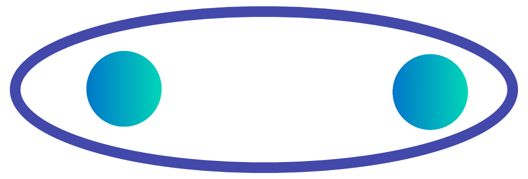

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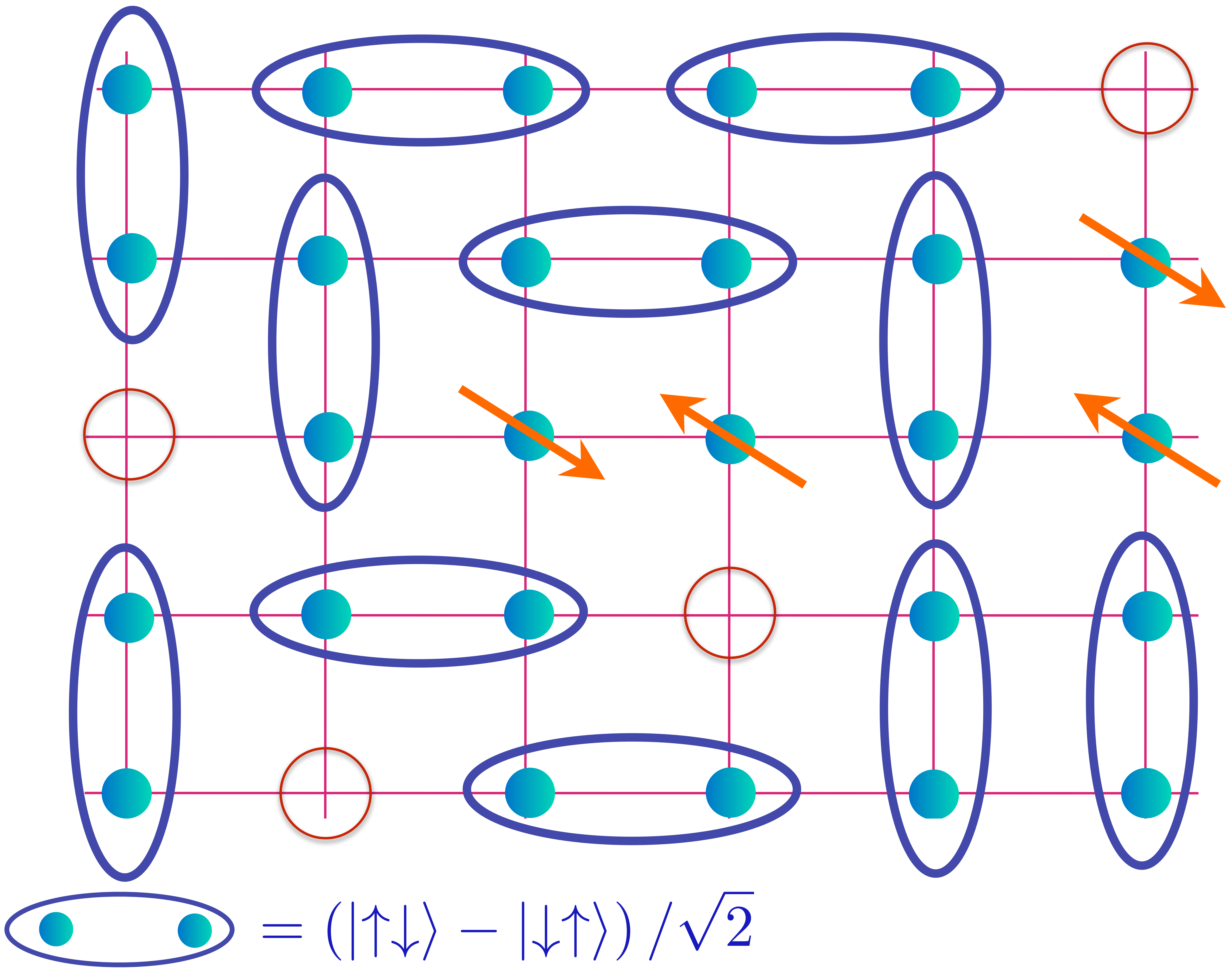

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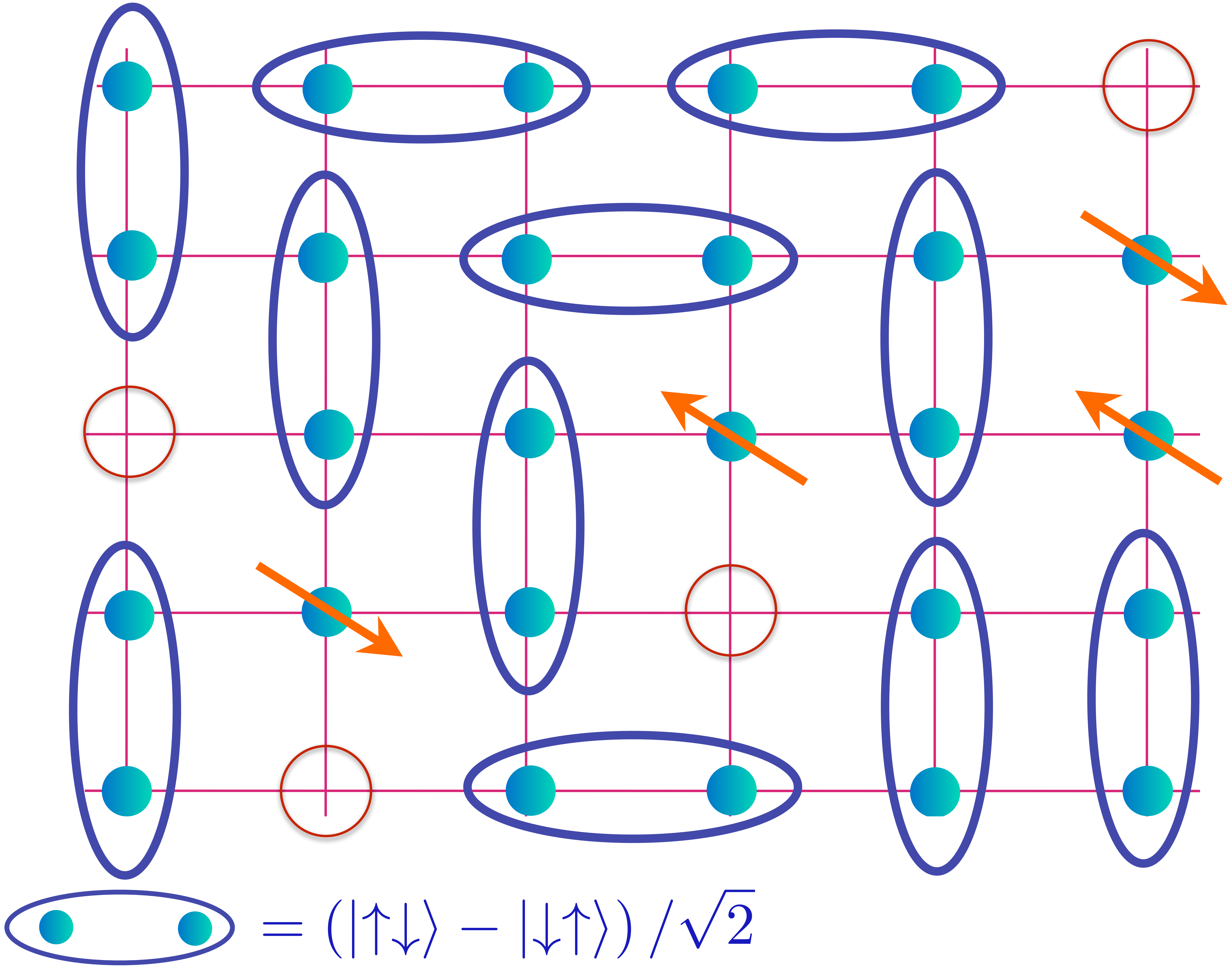

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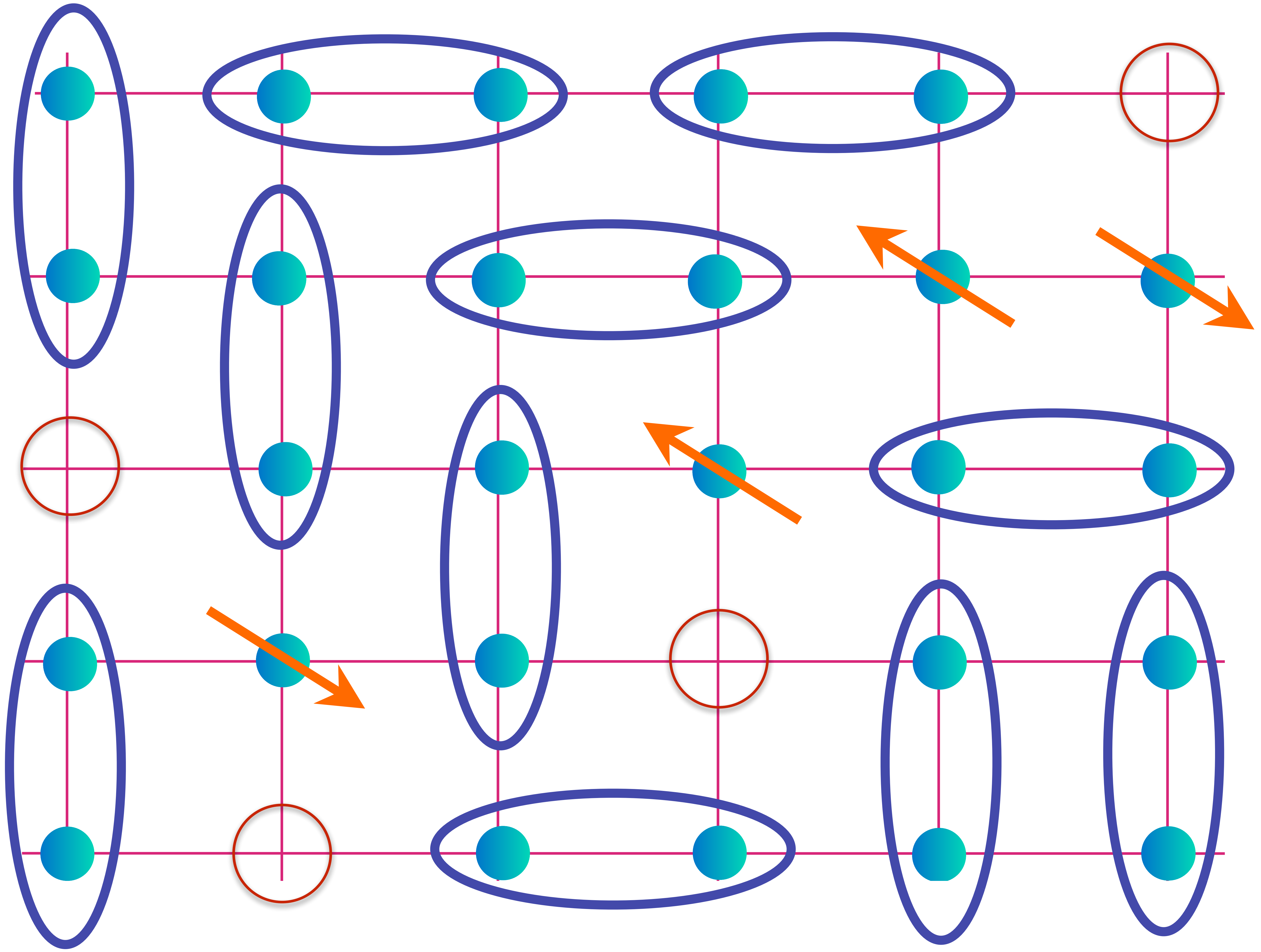

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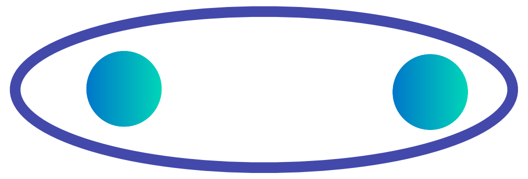

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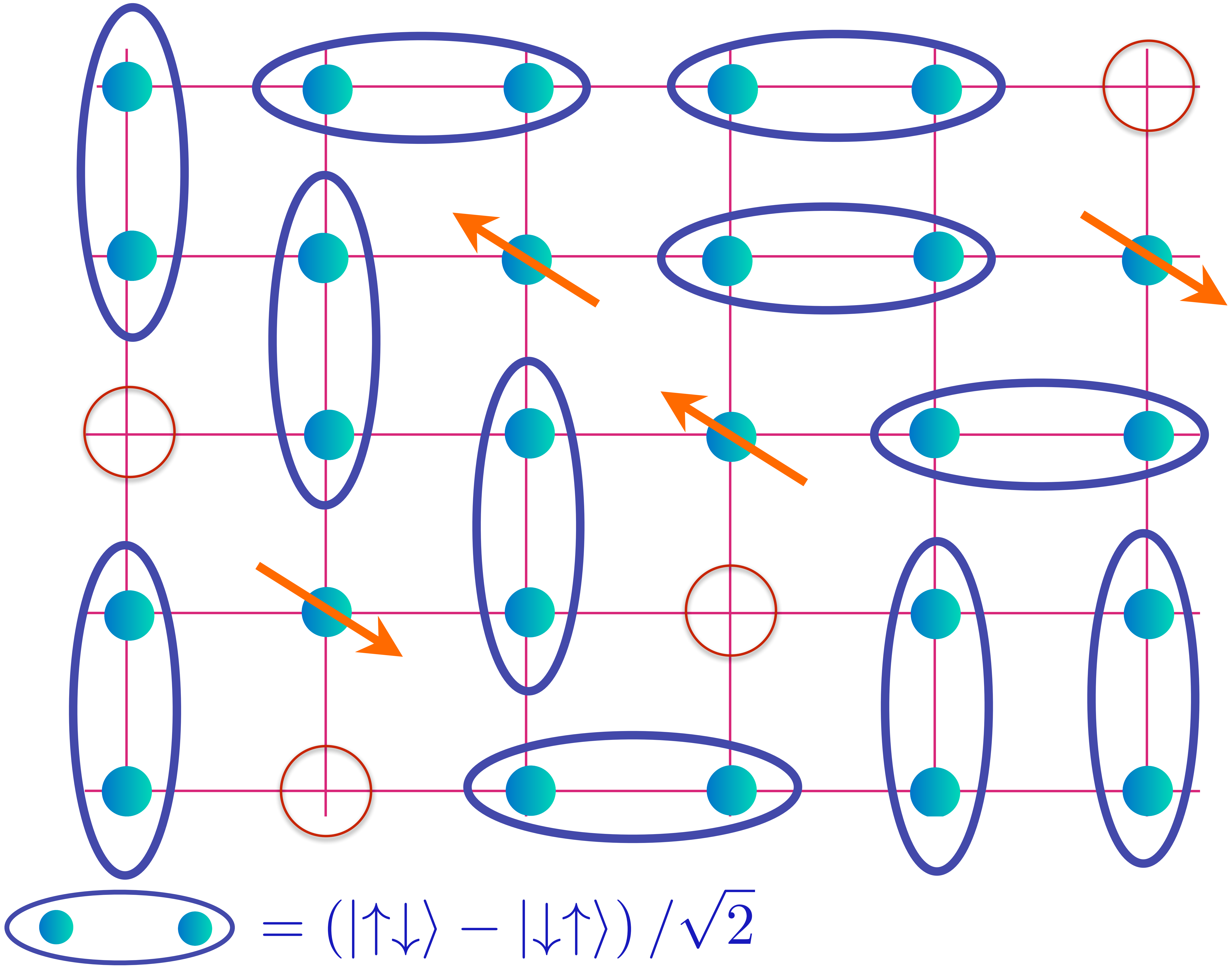

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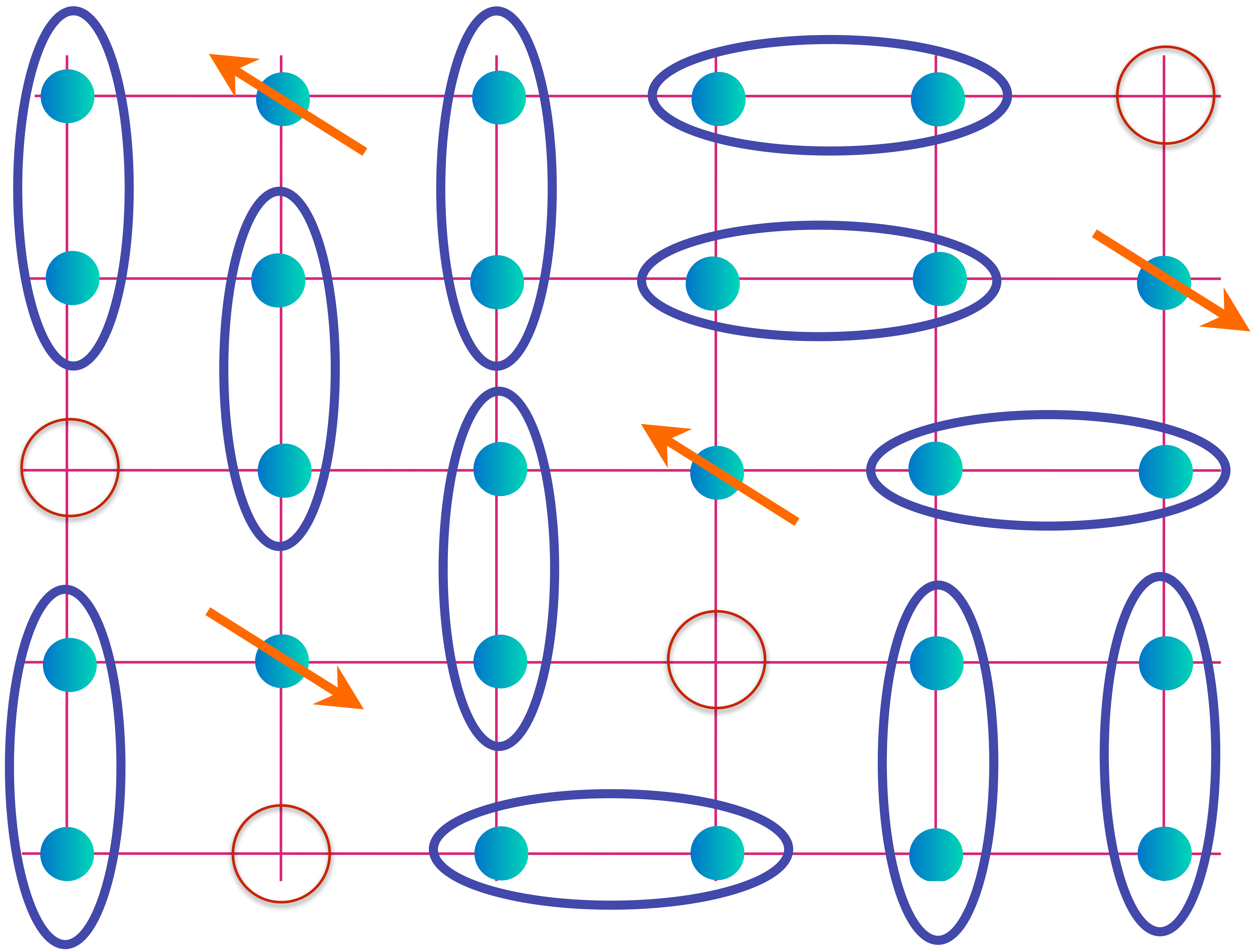

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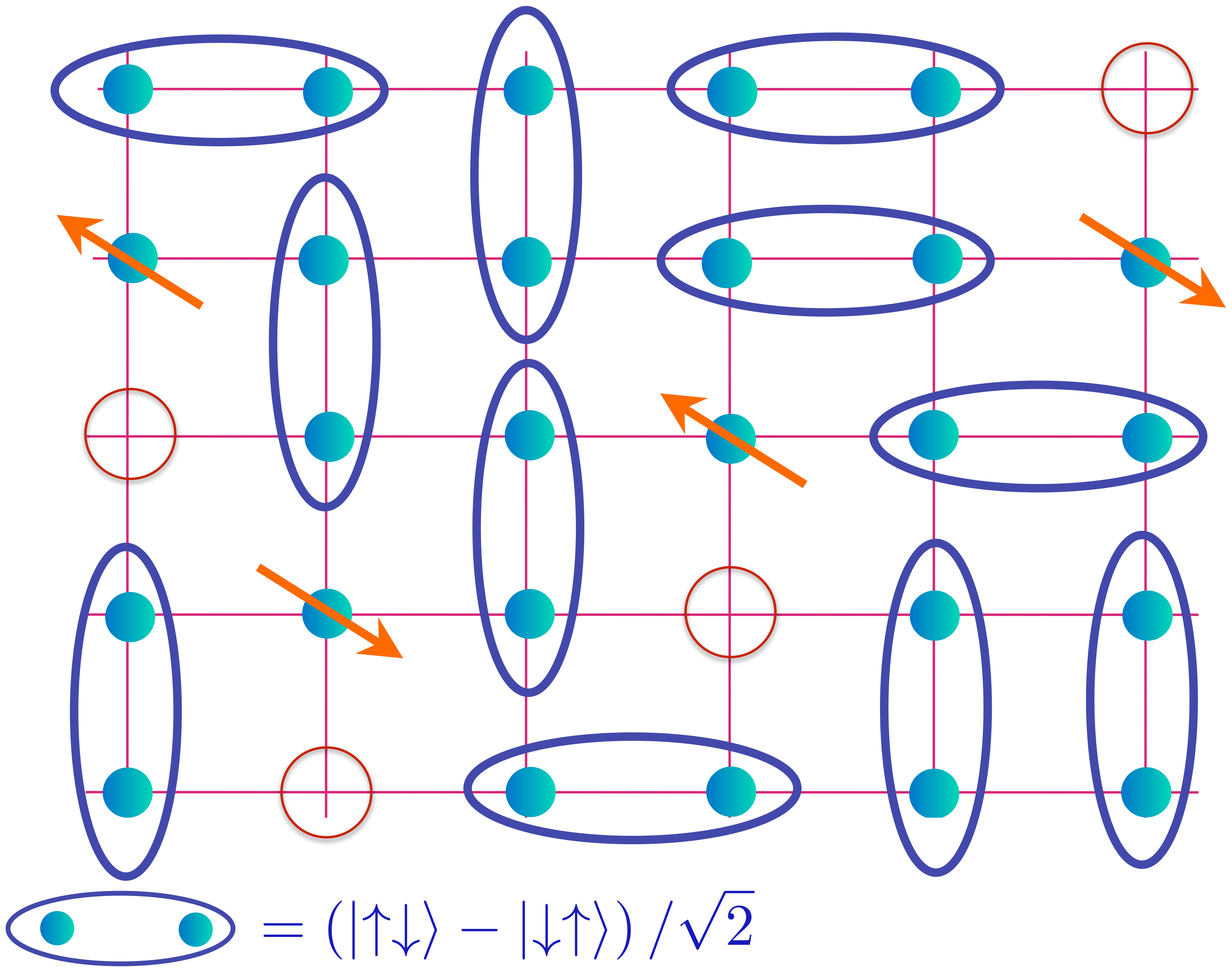

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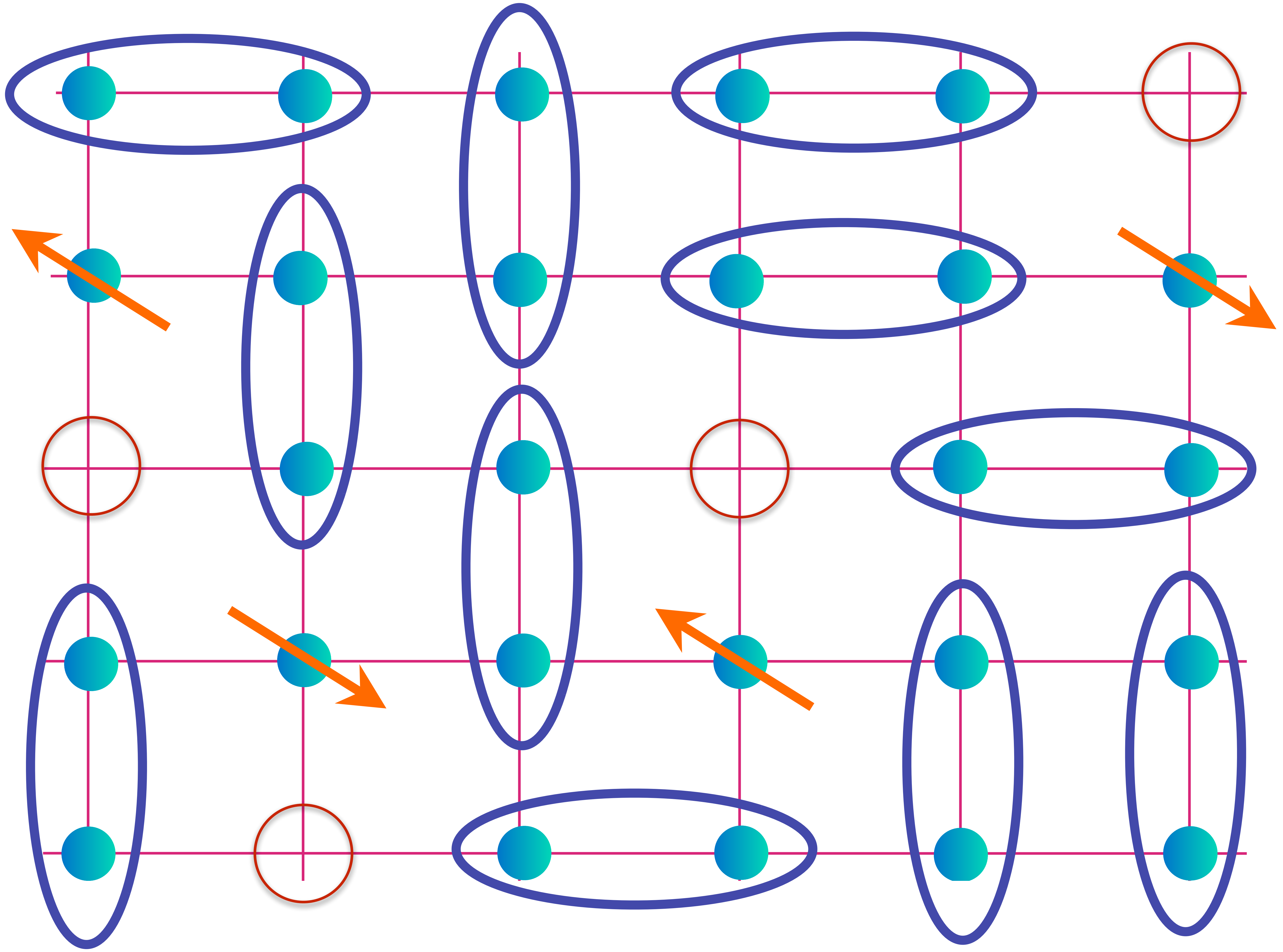

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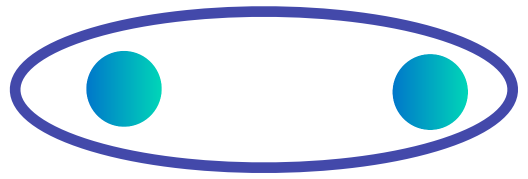

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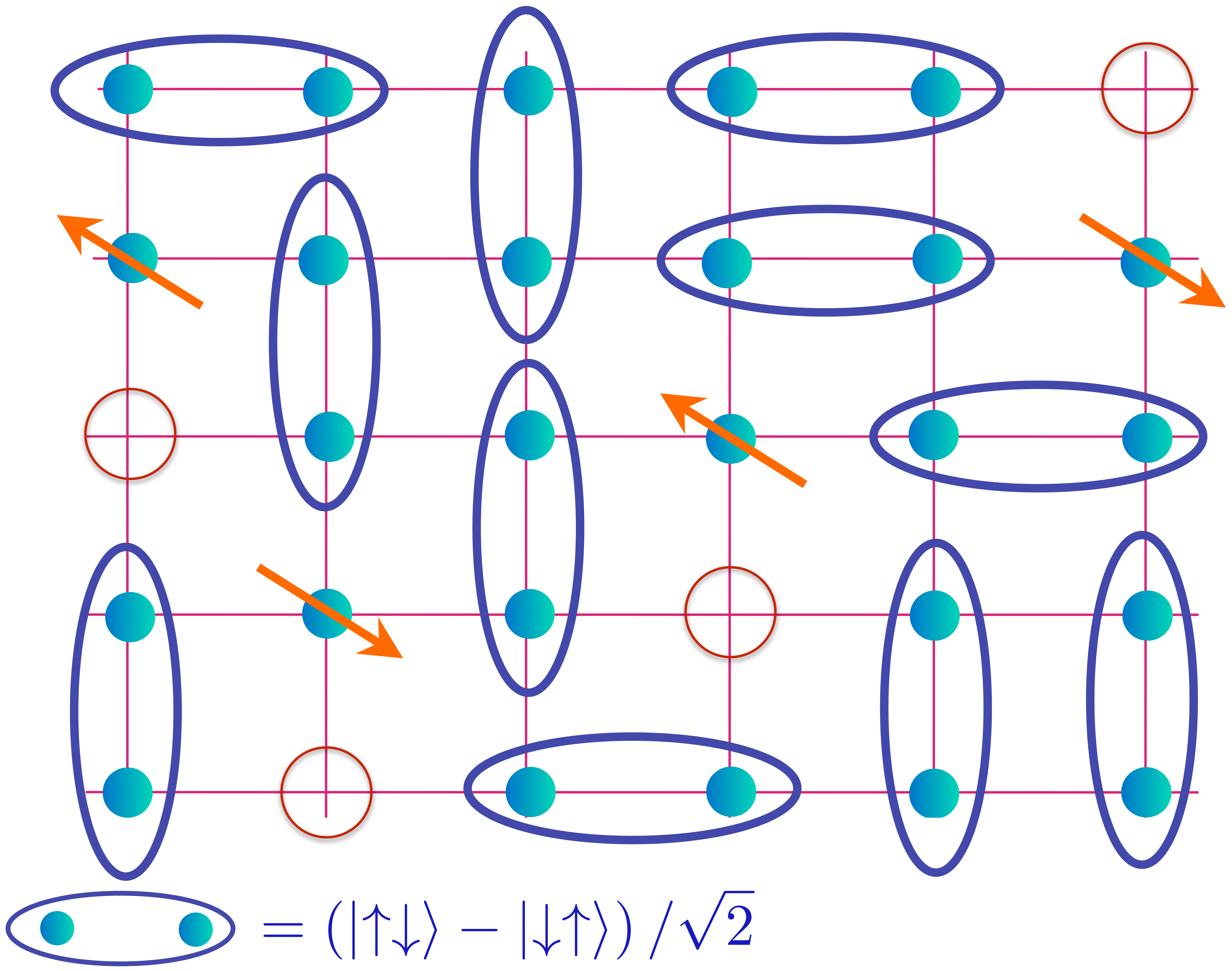

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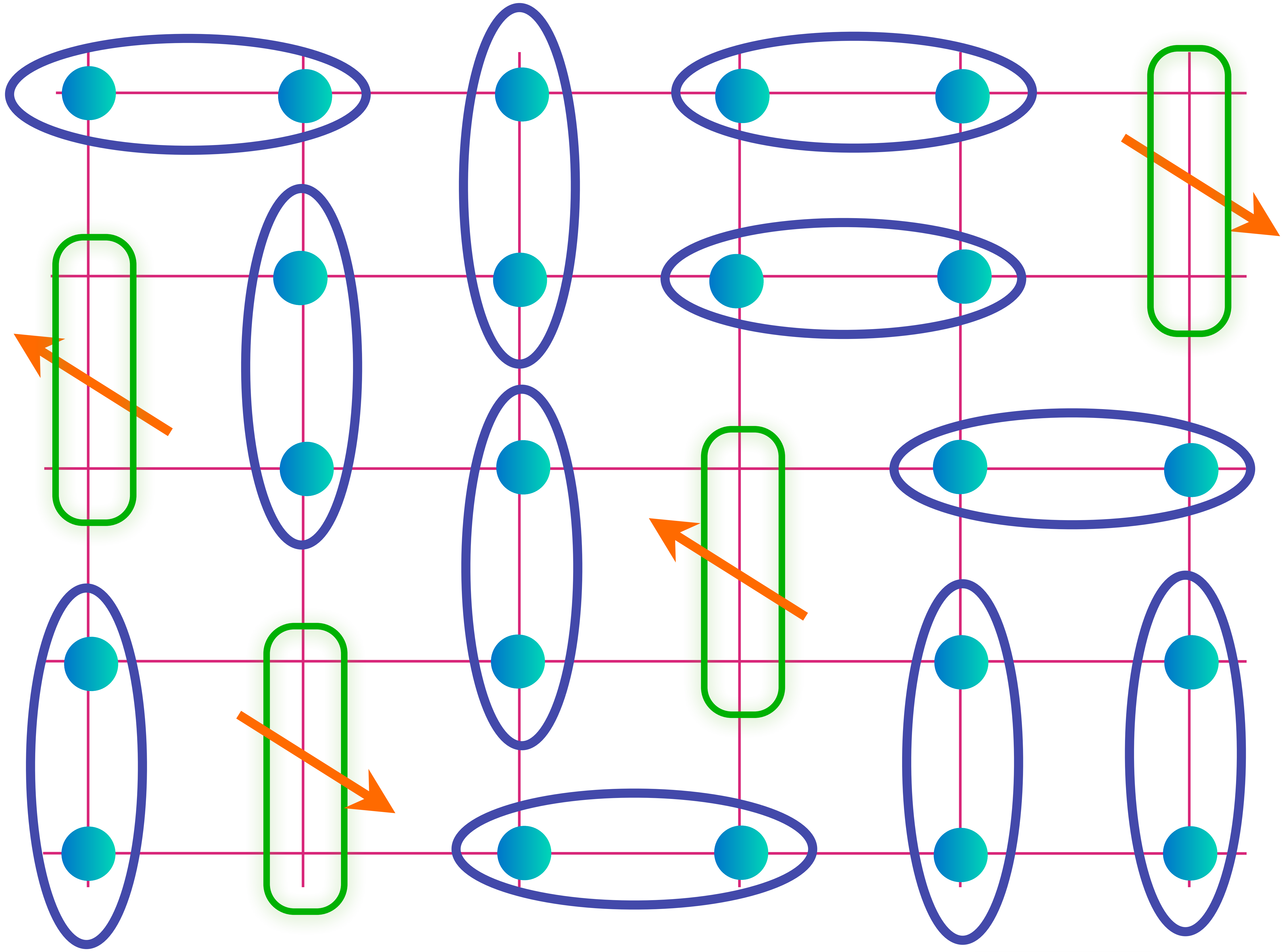
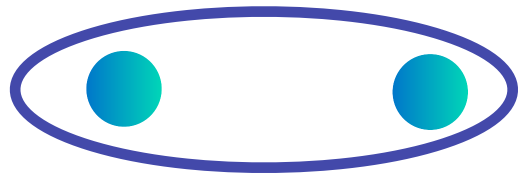


Each green "dimer" is a "bound state" of a vacancy and a free spin



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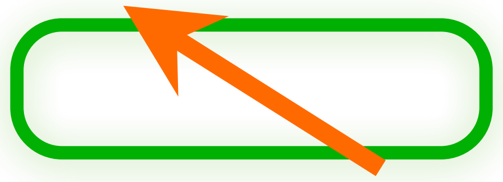

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$$

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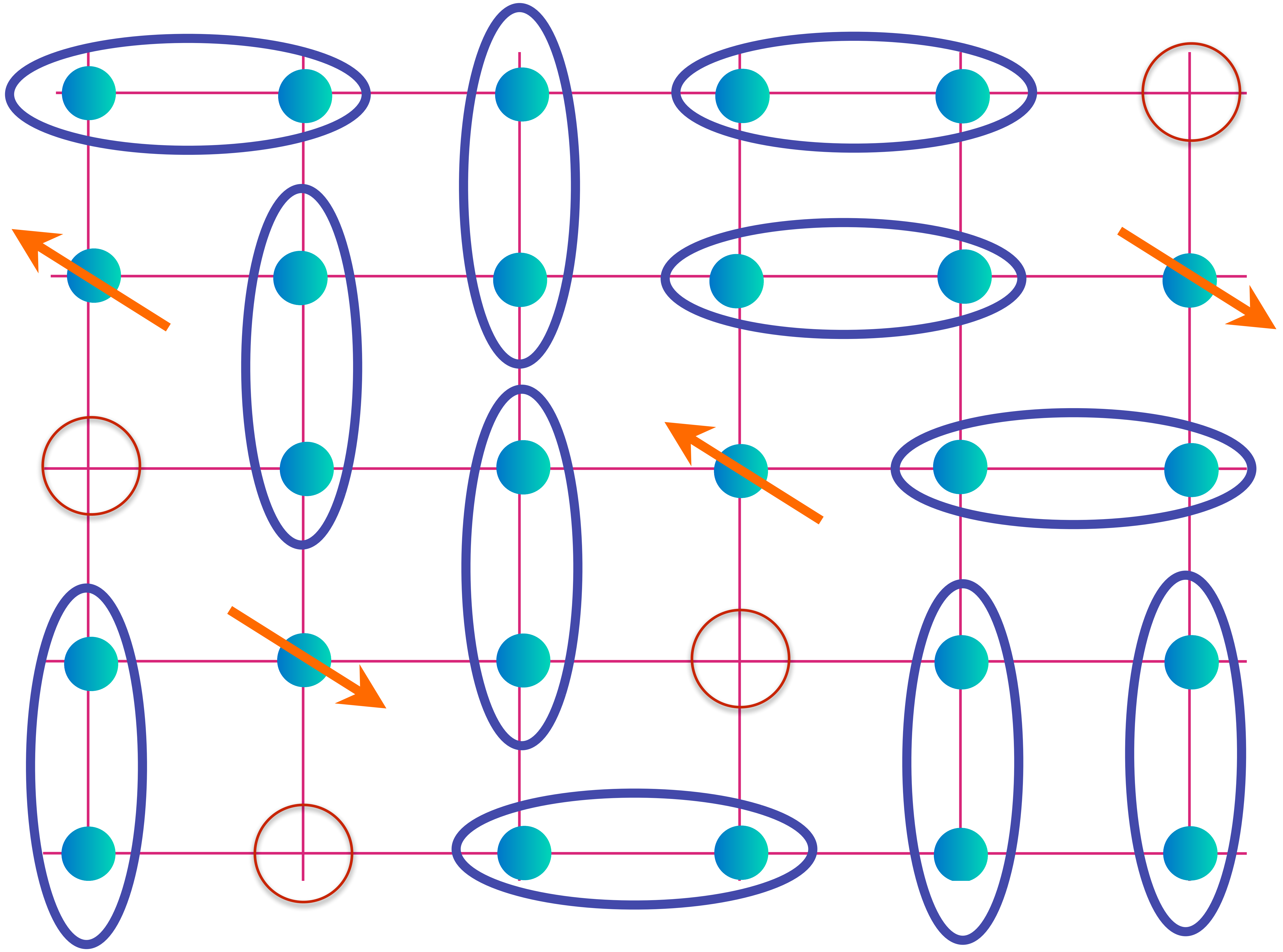

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$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

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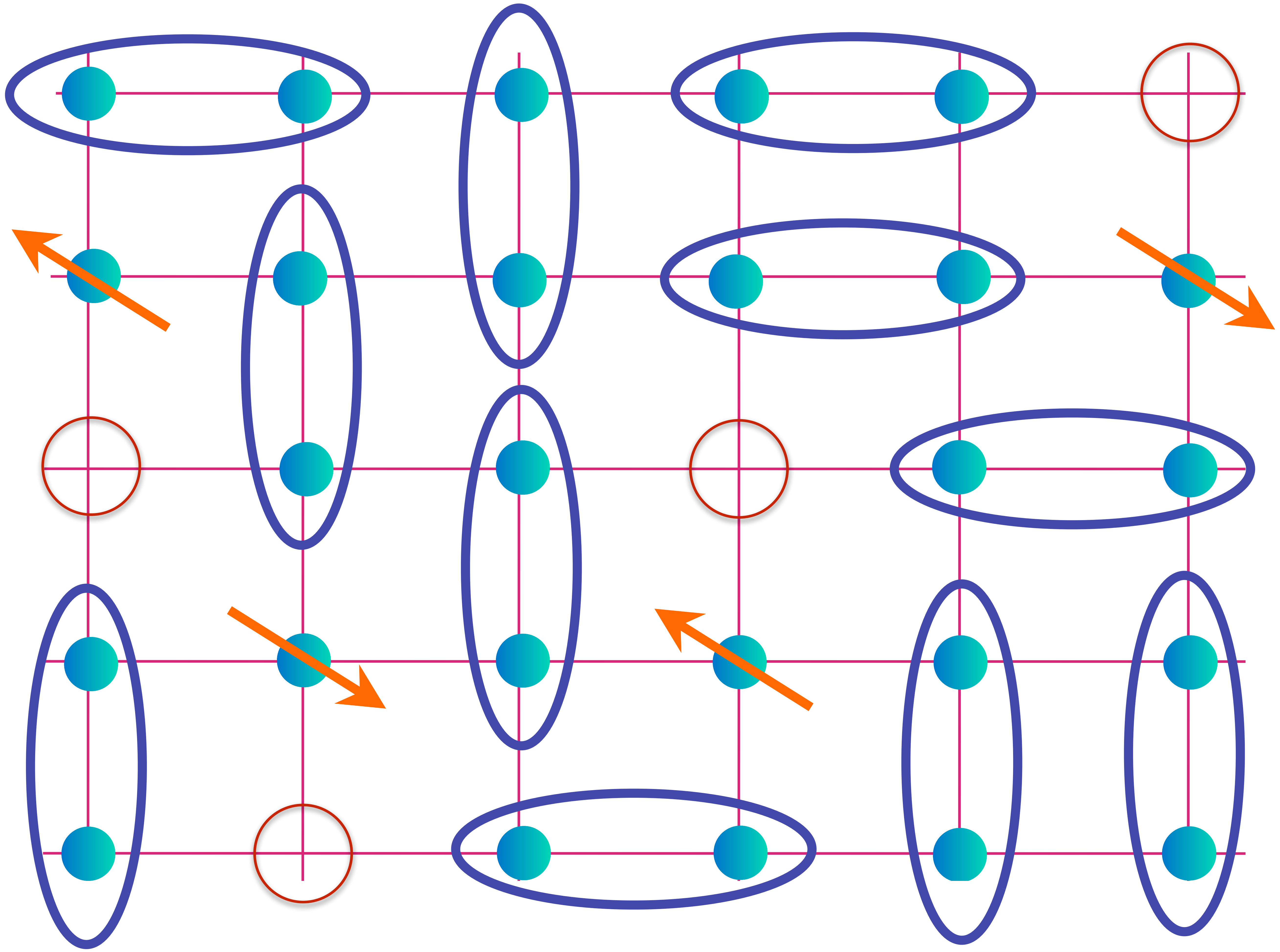

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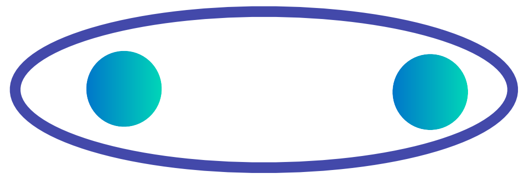
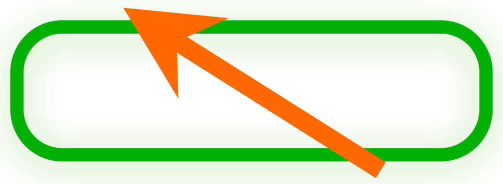
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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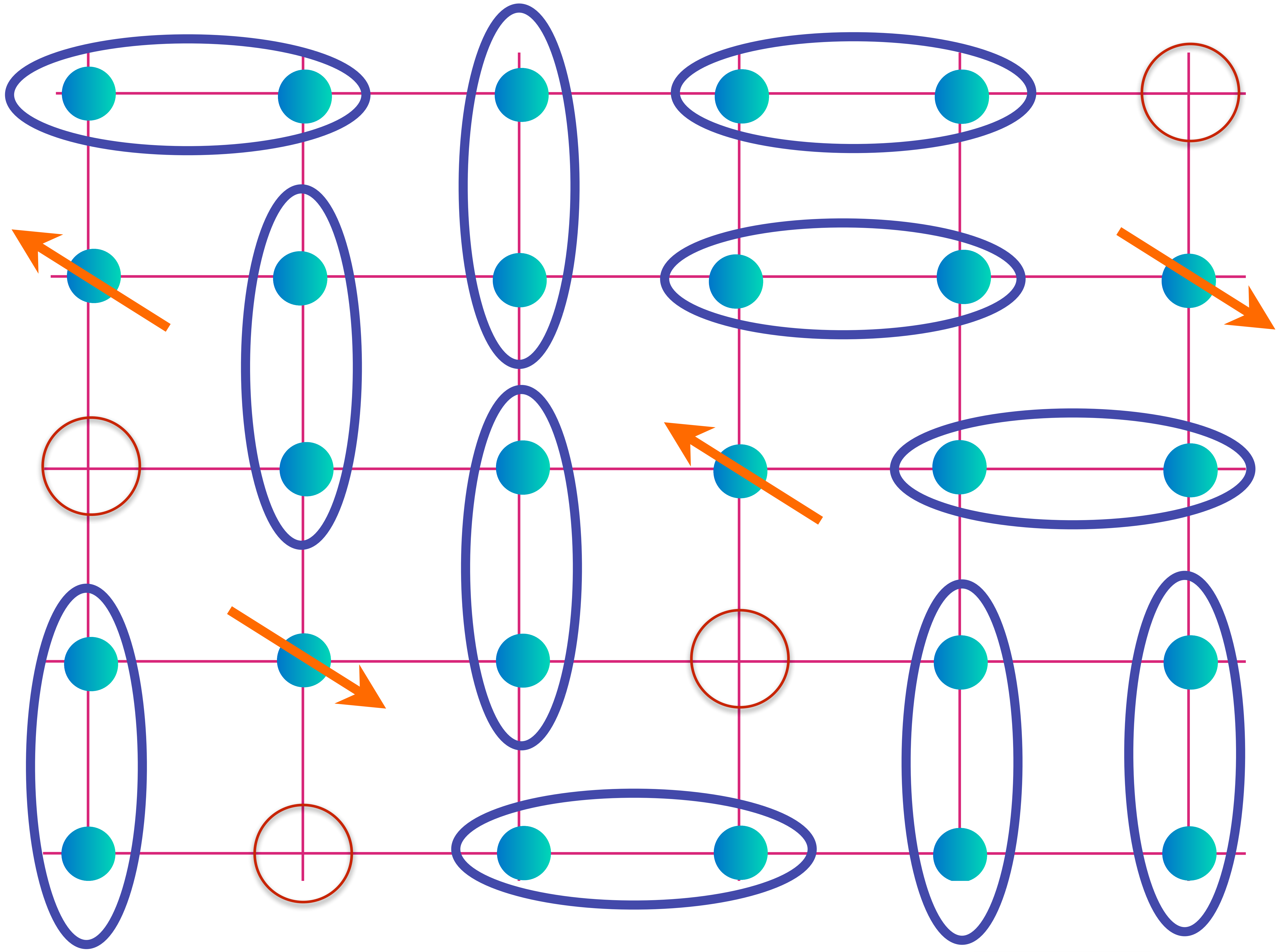

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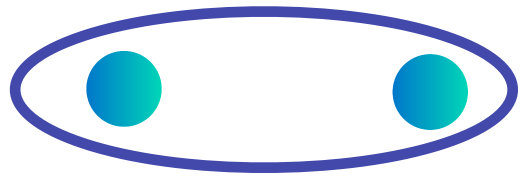

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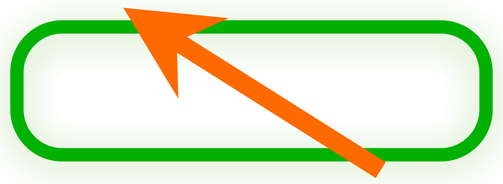

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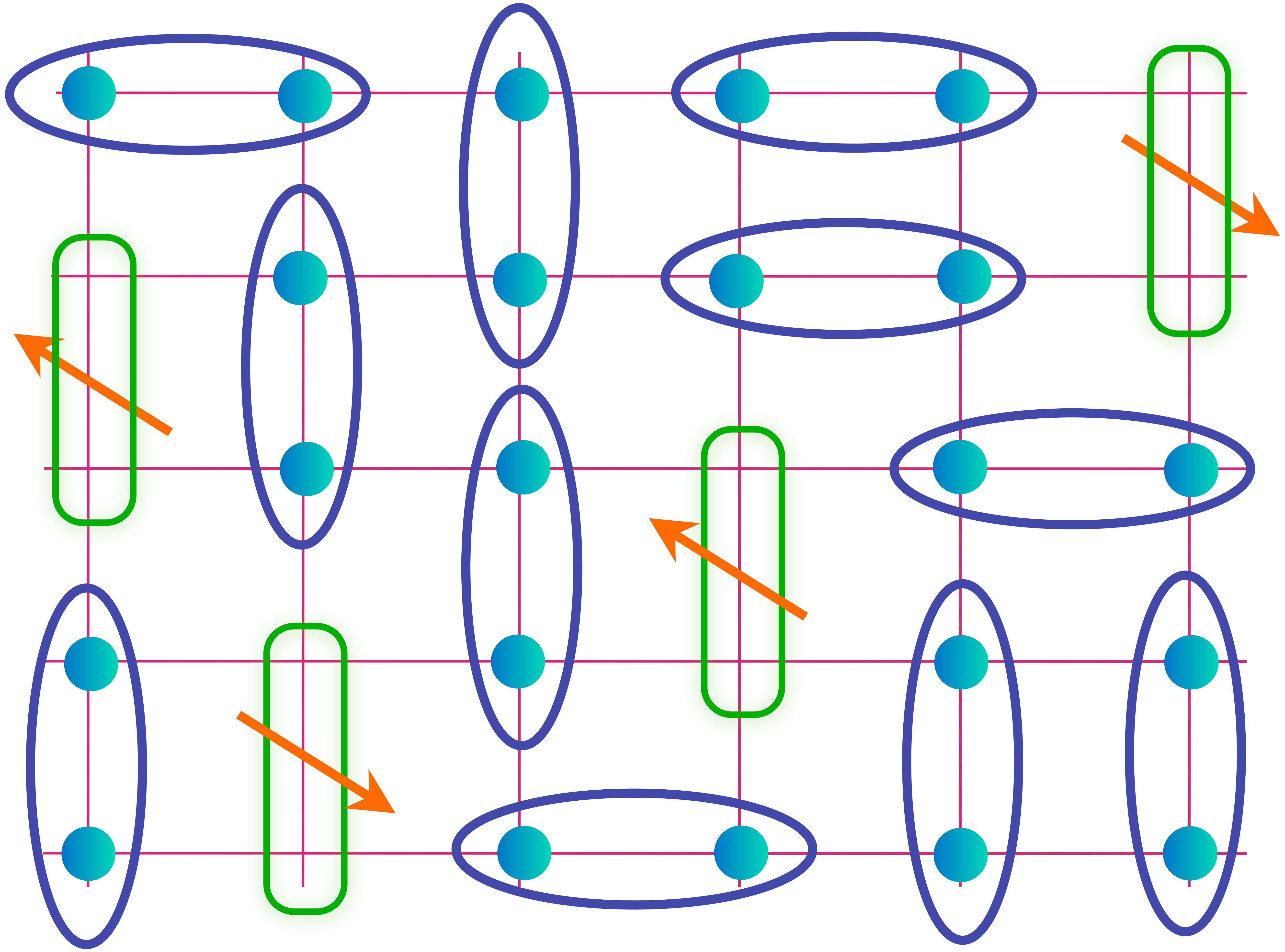
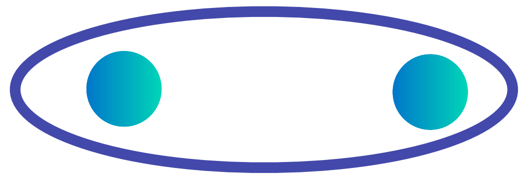


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



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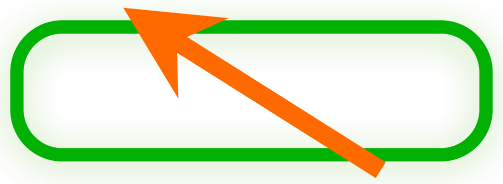

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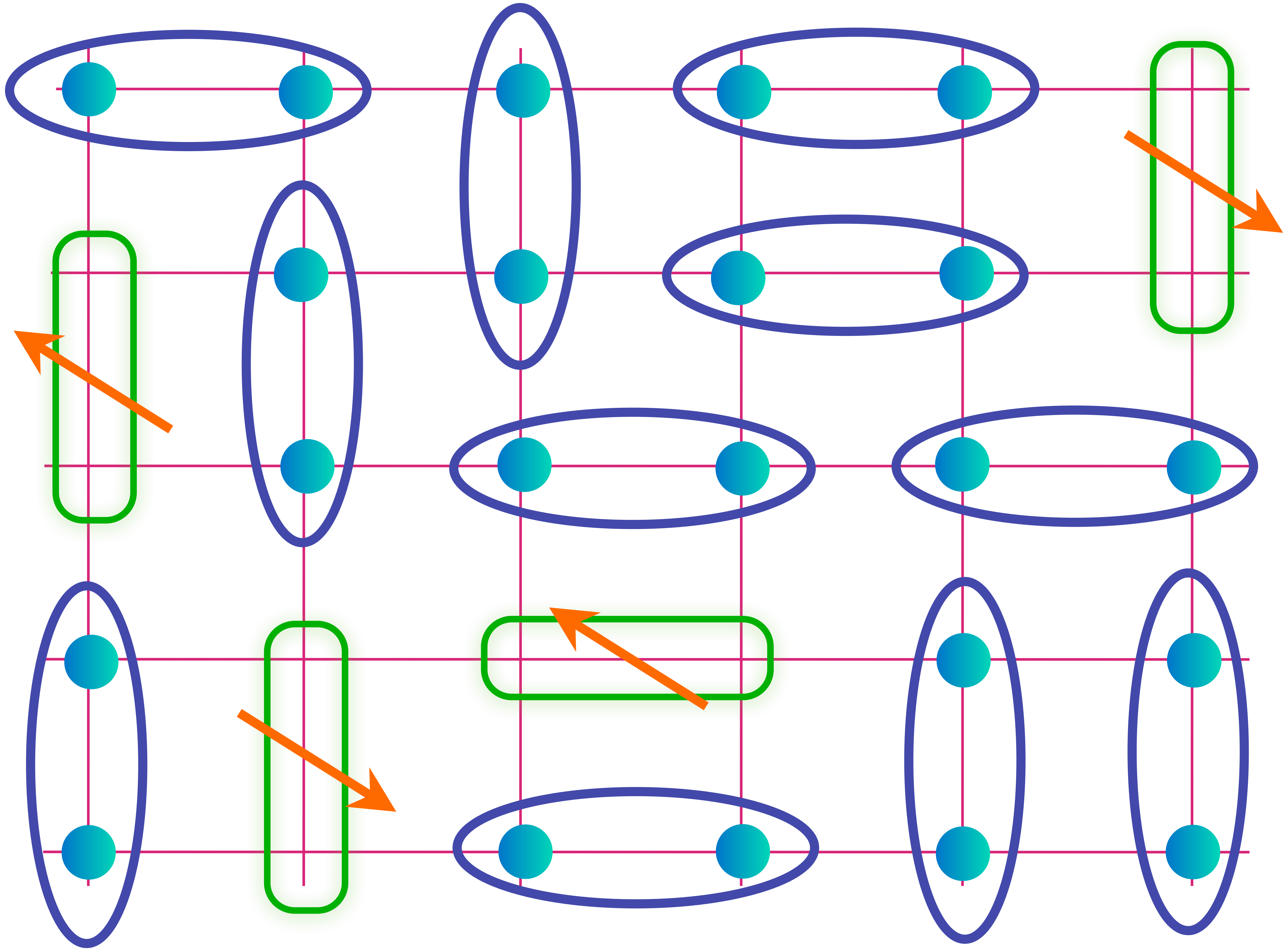

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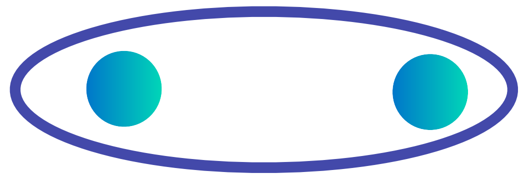
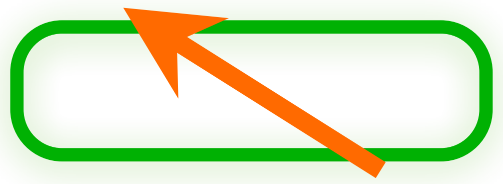
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$

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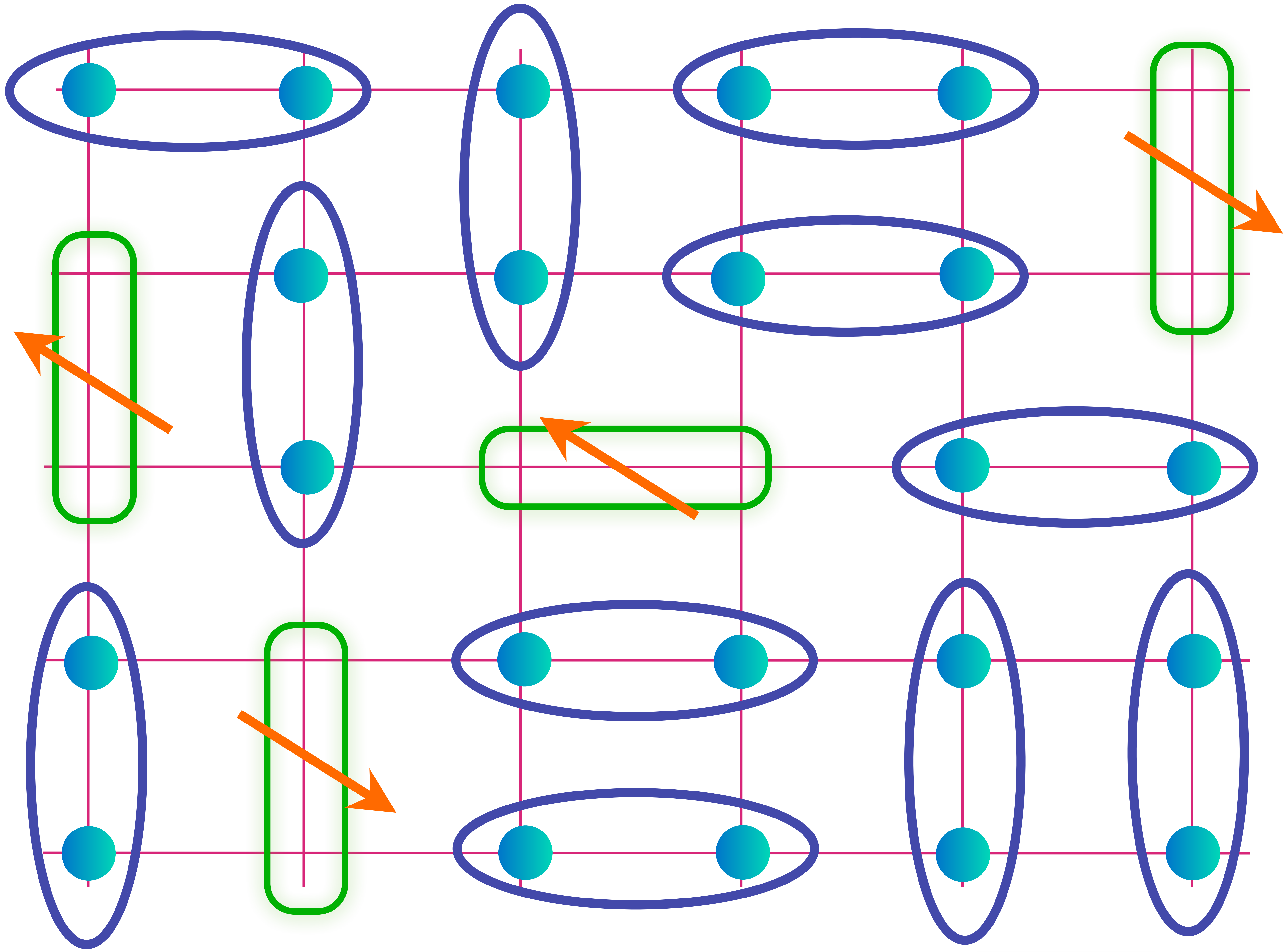

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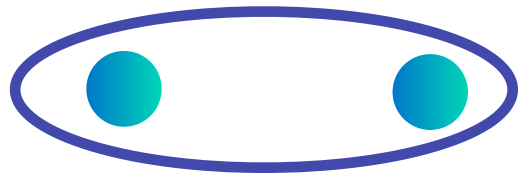

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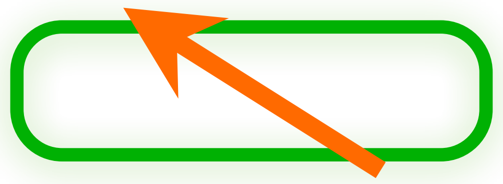

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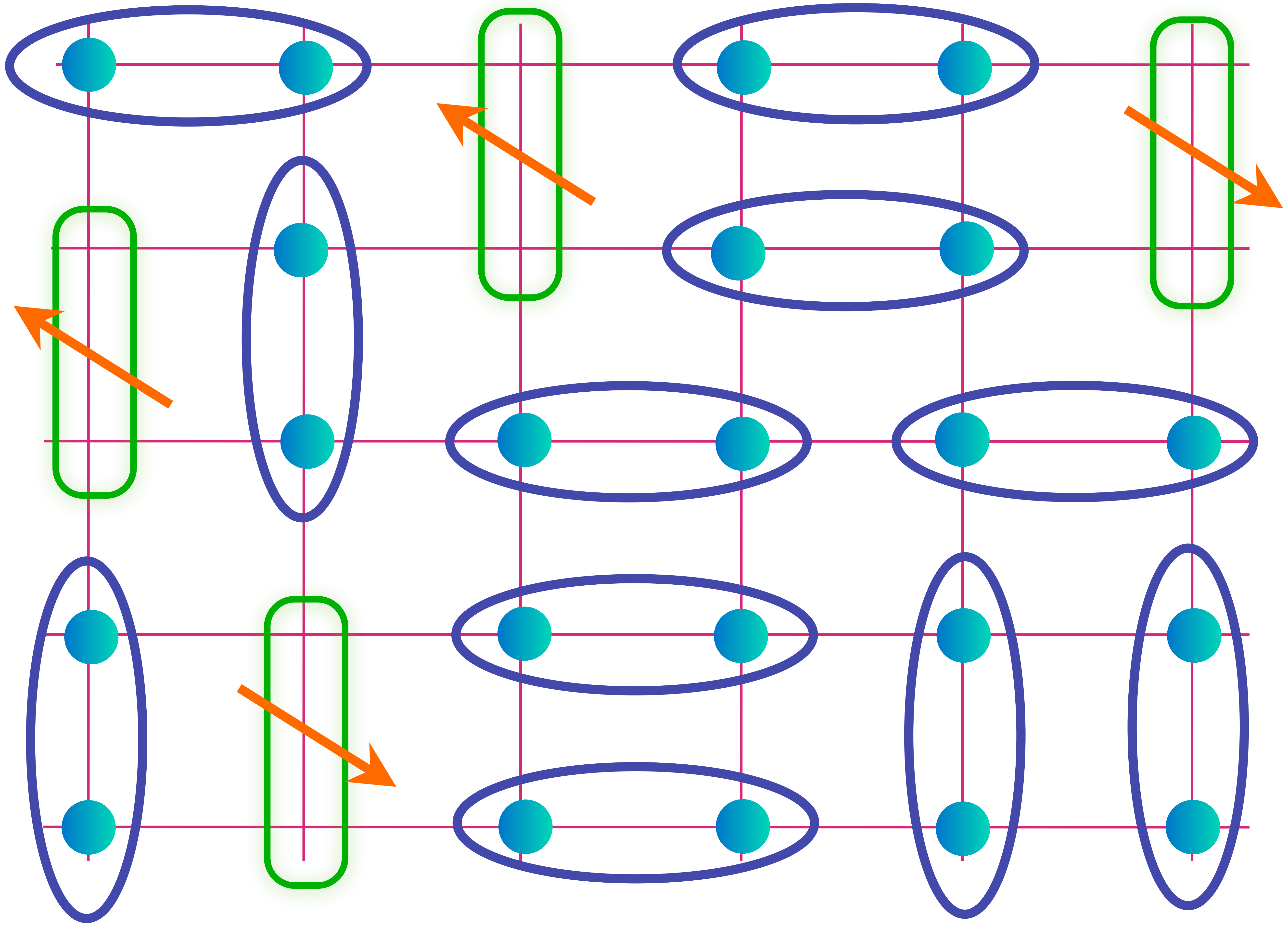

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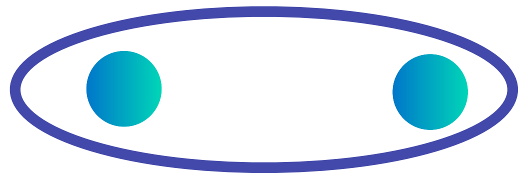

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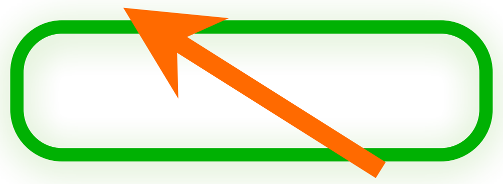

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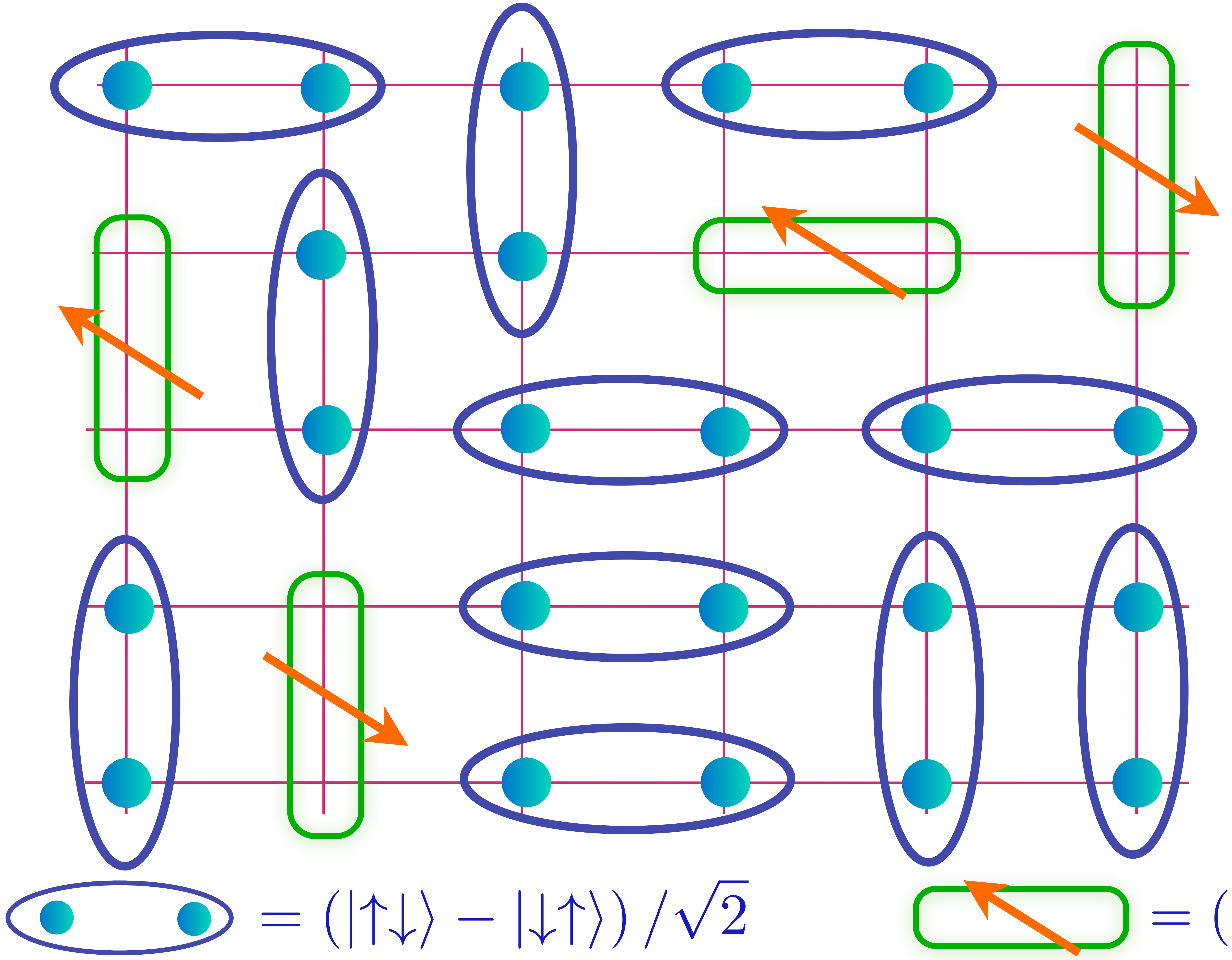

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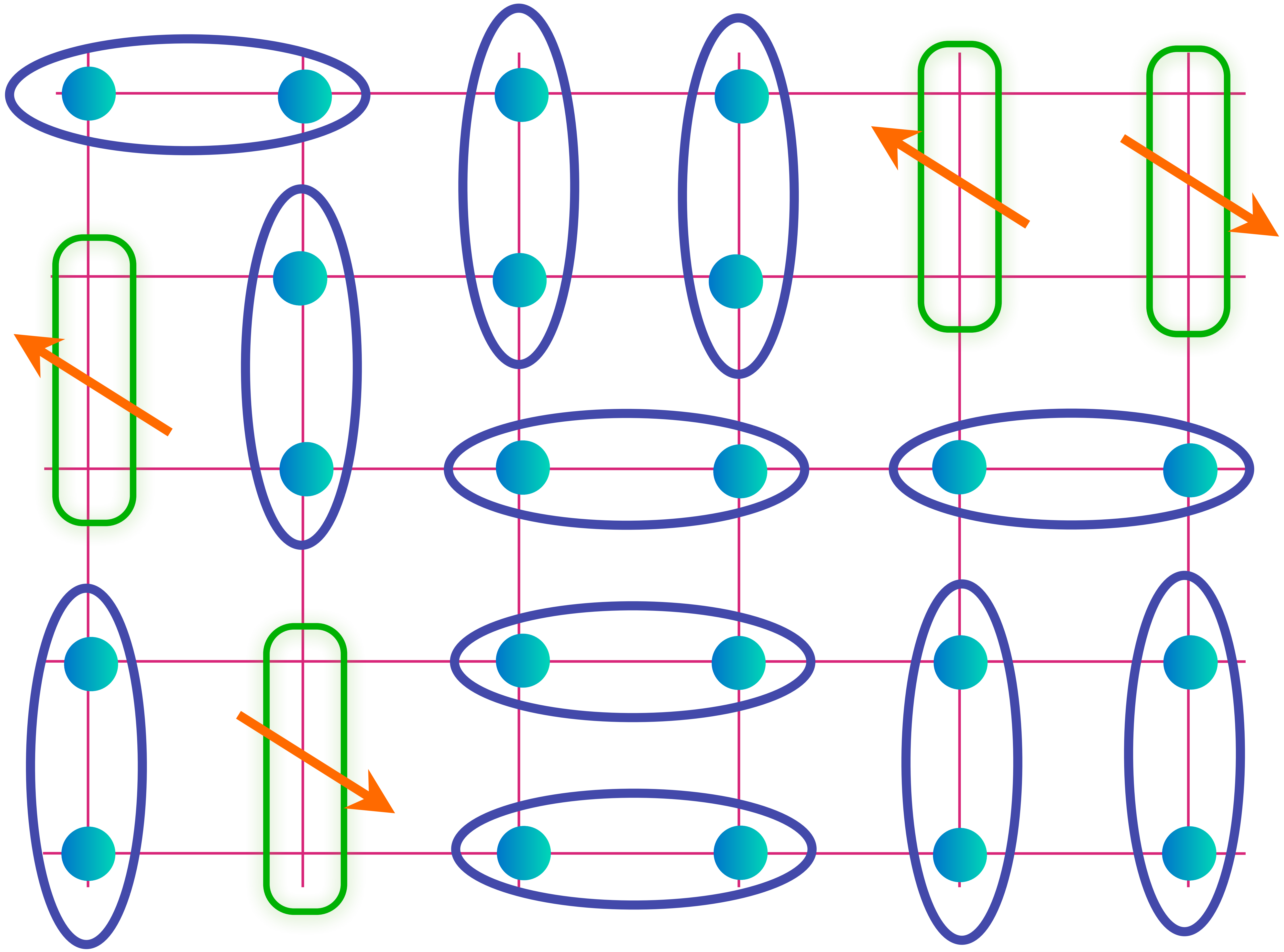

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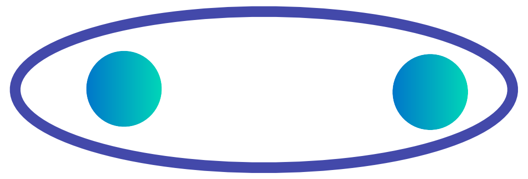

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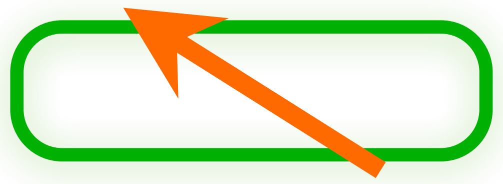

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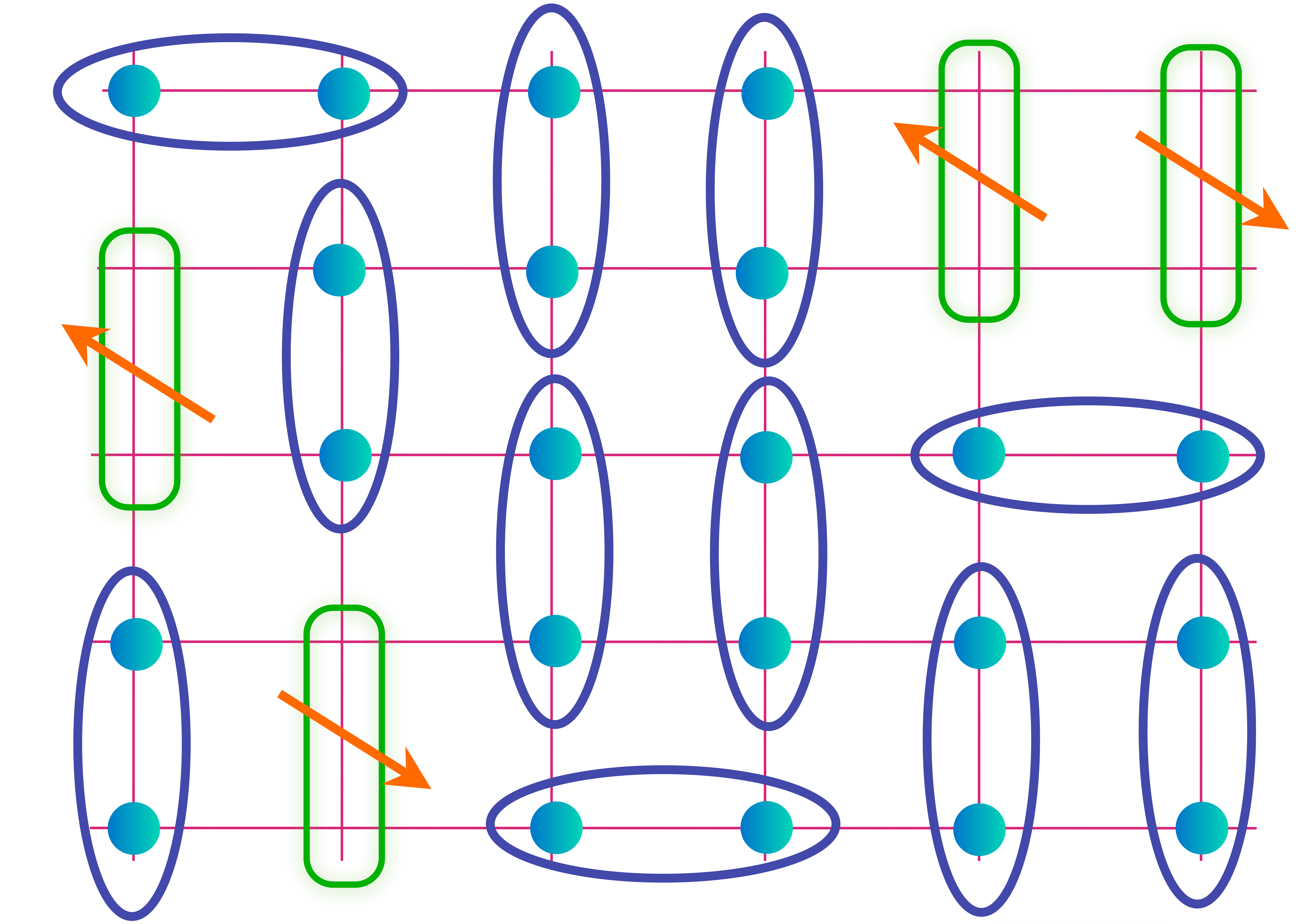

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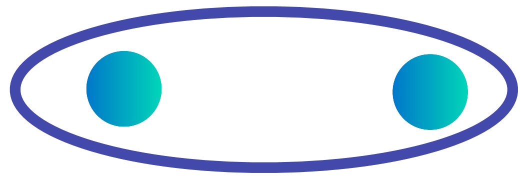
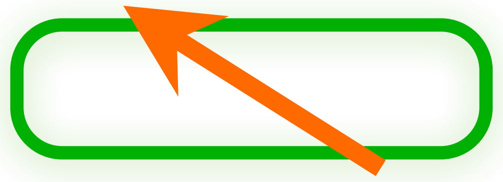
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

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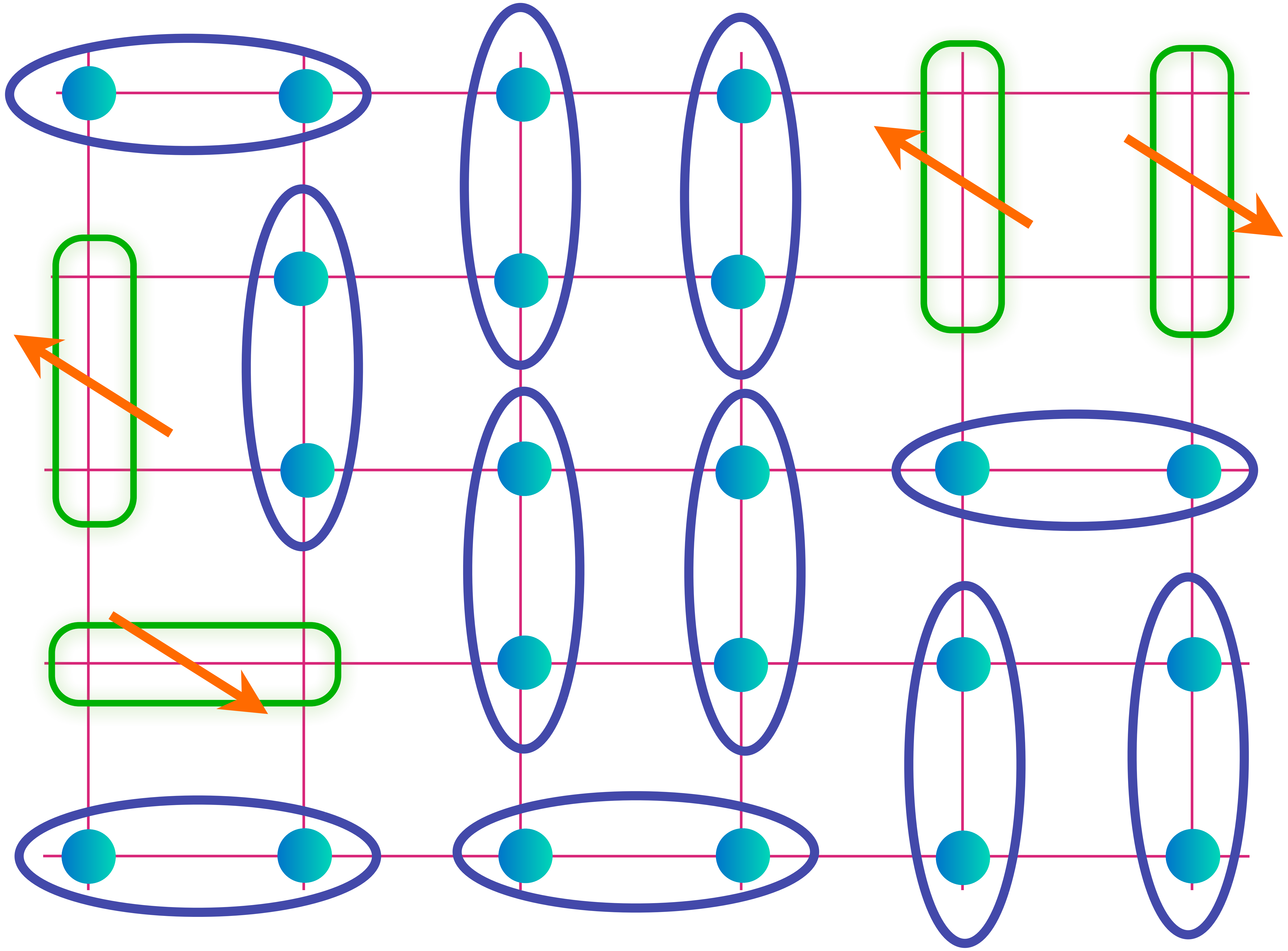

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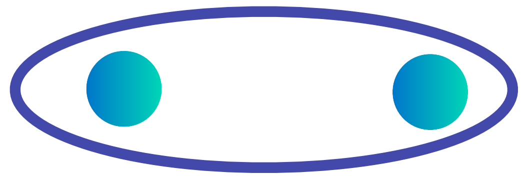

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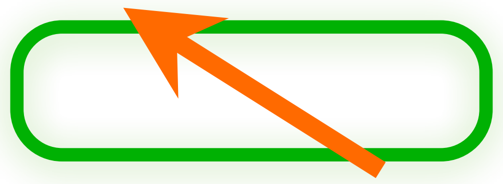

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The pseudogap metal = FL*

Main lesson from the Kondo lattice

Do not fractionalize the mobile electron, $c_{i\sigma} \neq f_{i\sigma} b^\dagger$.

The pseudogap metal = FL*

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Do not fractionalize the mobile electron, $c_{i\sigma} \neq f_{i\sigma} b^\dagger$.

We avoid this particular fractionalization because the excitations around the small Fermi surface carry spin-1/2 and charge e, just like the bare electron: so it is cumbersome to fractionalize all the electrons (which occurs on energy scale J), and then undo the fractionalization for a small density of them by forming bound states of spinons and holons (which occurs on an energy scale t). In particular, no complete theory of this bound state formation has yet been presented.

Yahui Zhang

arXiv: 2001.09159
arXiv: 2006.01140
arXiv: 2103.05009

Maria Tikhanovskaya

Alexander Nikolaenko

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

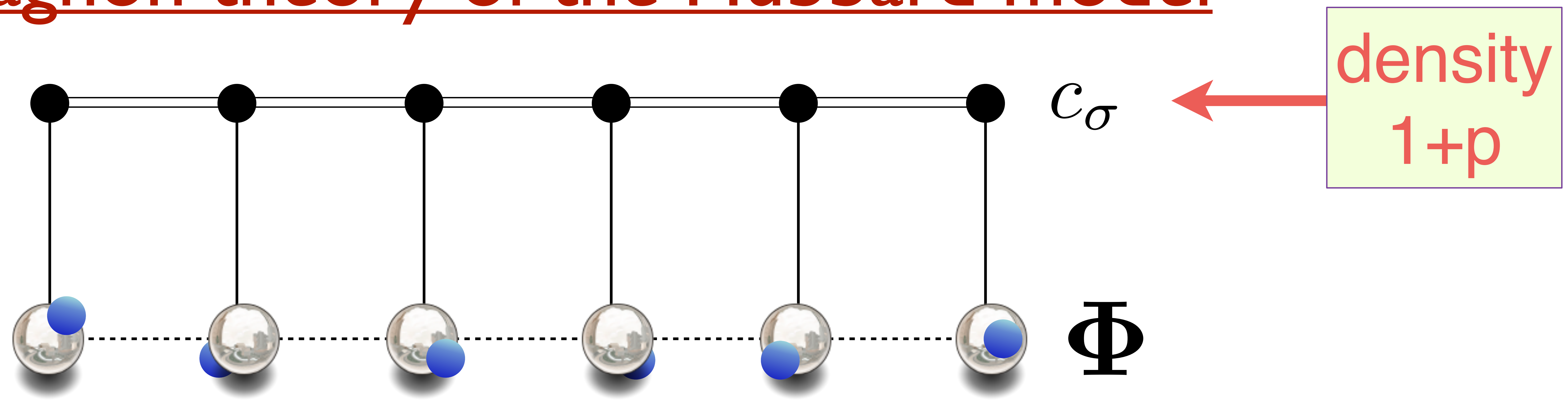
$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

This yields the 'Hertz-Millis' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model



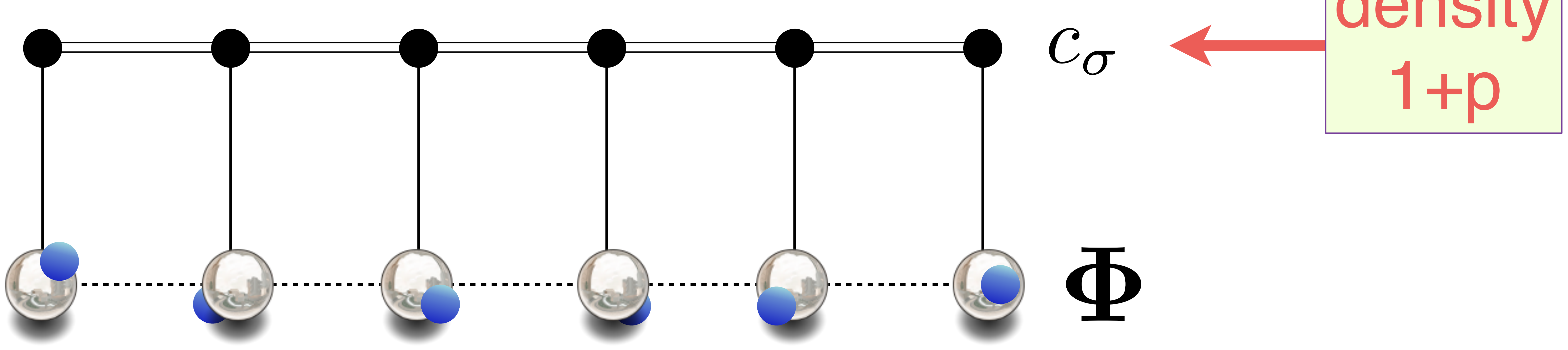
Quantum rotors

$$|\mathbf{\Phi}_i| = 1$$

$$\mathcal{H}_{\rm rotor} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \frac{g}{2} \boldsymbol{L}_i^2 - \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{\Phi}_i$$

Paramagnon theory of the Hubbard model

Quantum rotors $|\mathbf{\Phi}_i| = 1$



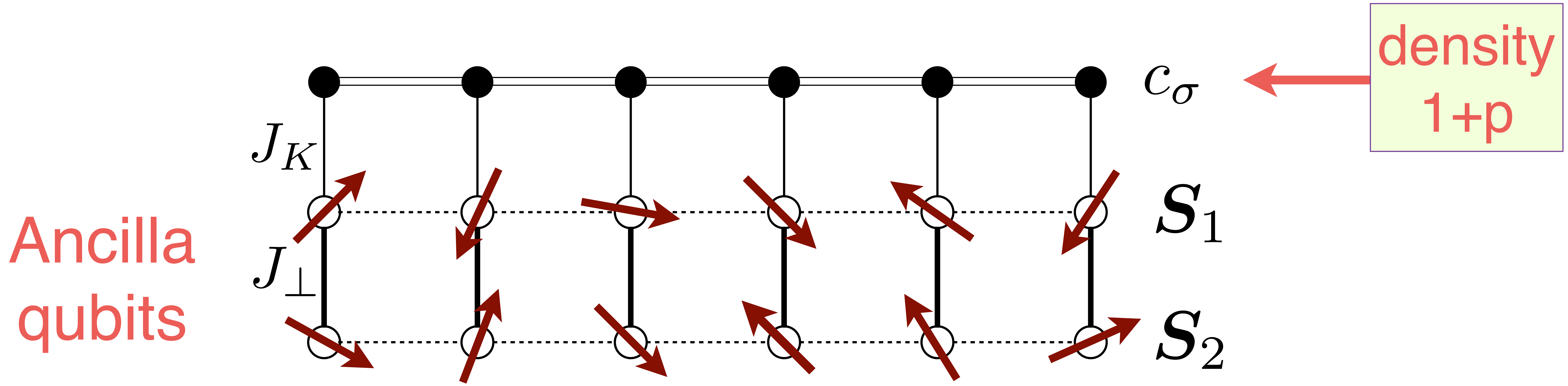
$$\mathcal{H}_{\text{rotor}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i \frac{g}{2} \boldsymbol{L}_i^2 - \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{\Phi}_i$$

Each rotor has eigenvalues $g\ell(\ell+1)/2$, degeneracy $2\ell+1$, $\ell = 0, 1, 2 \ldots$
Restrict to the $\ell = 0, 1$ states, and represent each rotor by 2 "ancilla qubits", $S = 1/2$ spins $\boldsymbol{S}_{1i}$ and $\boldsymbol{S}_{2i}$, with an antiferromagnetic coupling $J_\perp = g$

$$\boldsymbol{L}_i = \boldsymbol{S}_{1i} + \boldsymbol{S}_{2i}$$

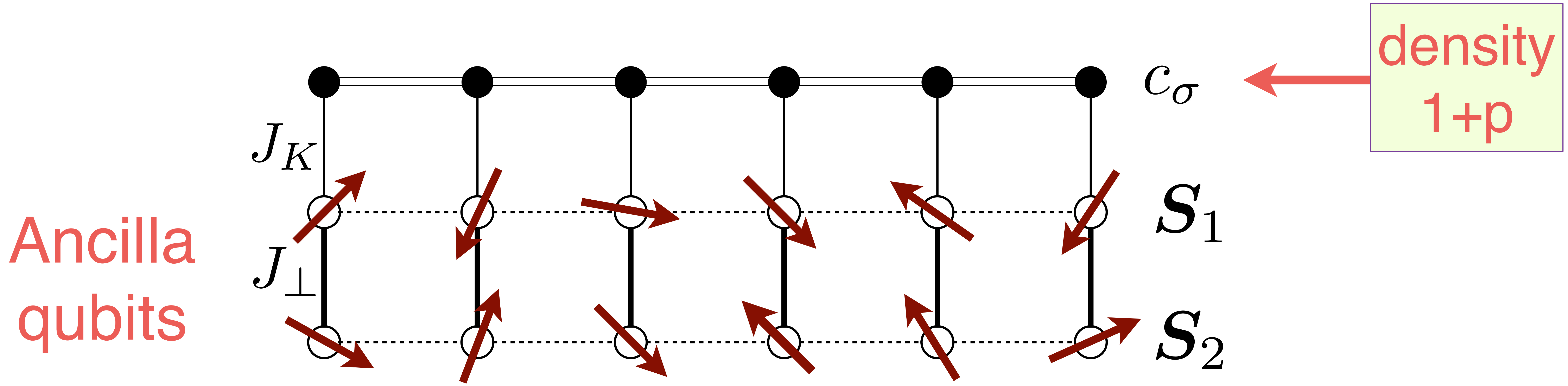
$$\mathbf{\Phi}_i = \frac{1}{\sqrt{3}} \left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i} \right)$$

Ancilla theory of the Hubbard model



$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \left[J_K \, c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i} \right] + \sum_{\langle ij \rangle} \left[J_1 \, \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{1j} + J_2 \, \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2j} \right]$$

Ancilla theory of the Hubbard model



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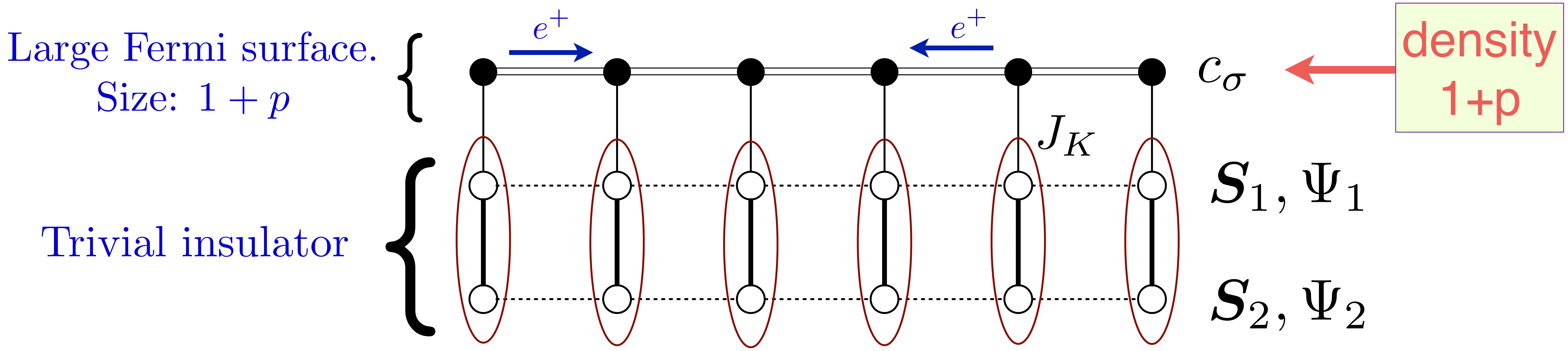
Performing a Schrieffer-Wolff transformation in powers of $1/J_\perp$, we obtain

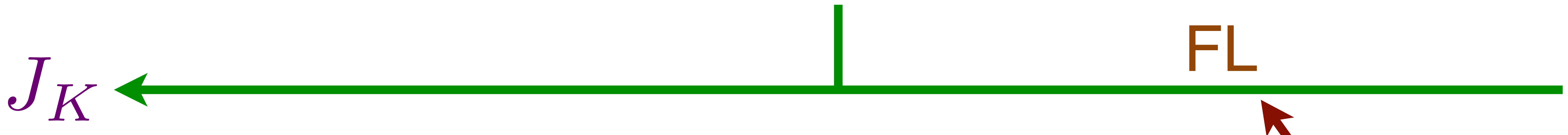
$$\mathcal{H} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + U \sum_i \left[c_{i\uparrow}^\dagger c_{i\uparrow} \right] \left[c_{i\downarrow}^\dagger c_{i\downarrow} \right] + J \sum_{\langle ij \rangle} \left[c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \cdot \left[c_{j\rho}^\dagger \frac{\boldsymbol{\tau}_{\rho\rho'}}{2} c_{j\rho'} \right]$$

i.e. we recover a Hubbard-Heisenberg model with *no ancillas* and

$$U = \frac{3J_K^2}{8J_\perp} + \frac{3J_K^3}{16J_\perp^2} + \ldots \quad , \quad J = \frac{J_K^2 (J_1 + J_2)}{4J_\perp^2}$$

Ancilla theory of the Hubbard model

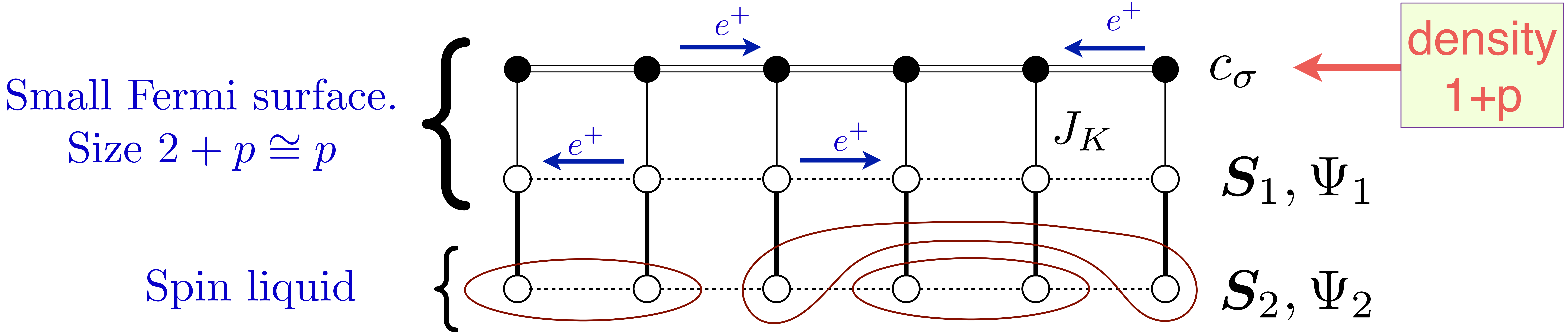


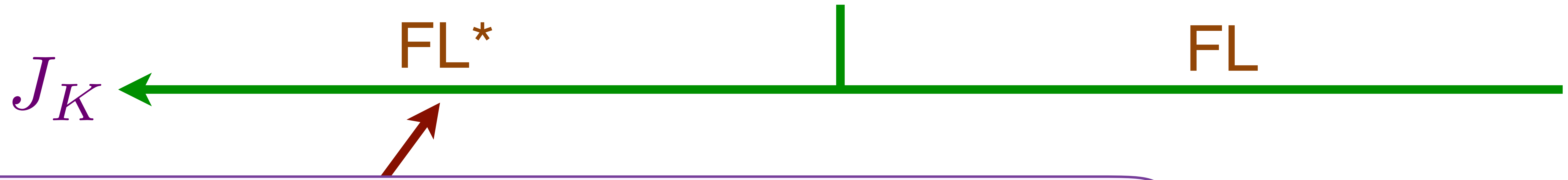


Large Fermi surface of size $1+p$

$$|\mathrm{FL}\rangle = |\text{Rung singlets of } \Psi_1, \Psi_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

Ancilla theory of the Hubbard model

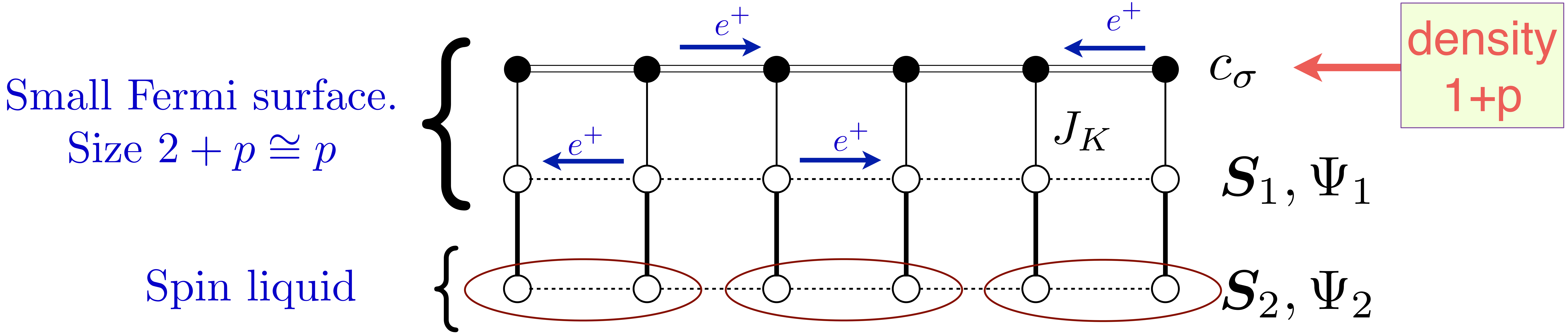


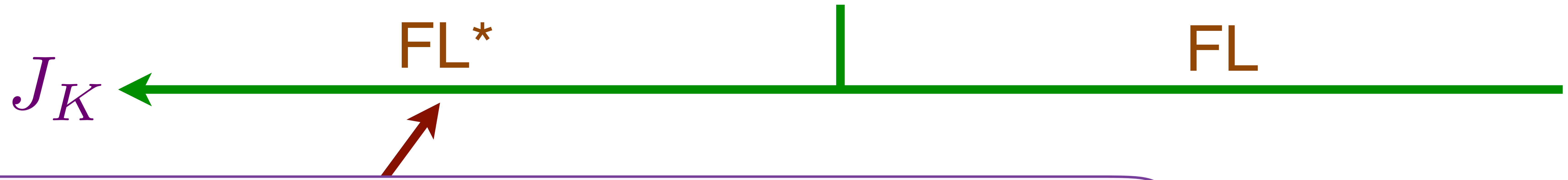


Small Fermi surface of size p

$$
\begin{aligned}
|\mathrm{FL}*\rangle = & \left[\text{Projection onto rung singlets of } \Psi_1, \Psi_2\right] \\
& \bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle \\
& \otimes |\text{Slater determinant of } \Psi_2\rangle
\end{aligned}
$$

Ancilla theory of the Hubbard model





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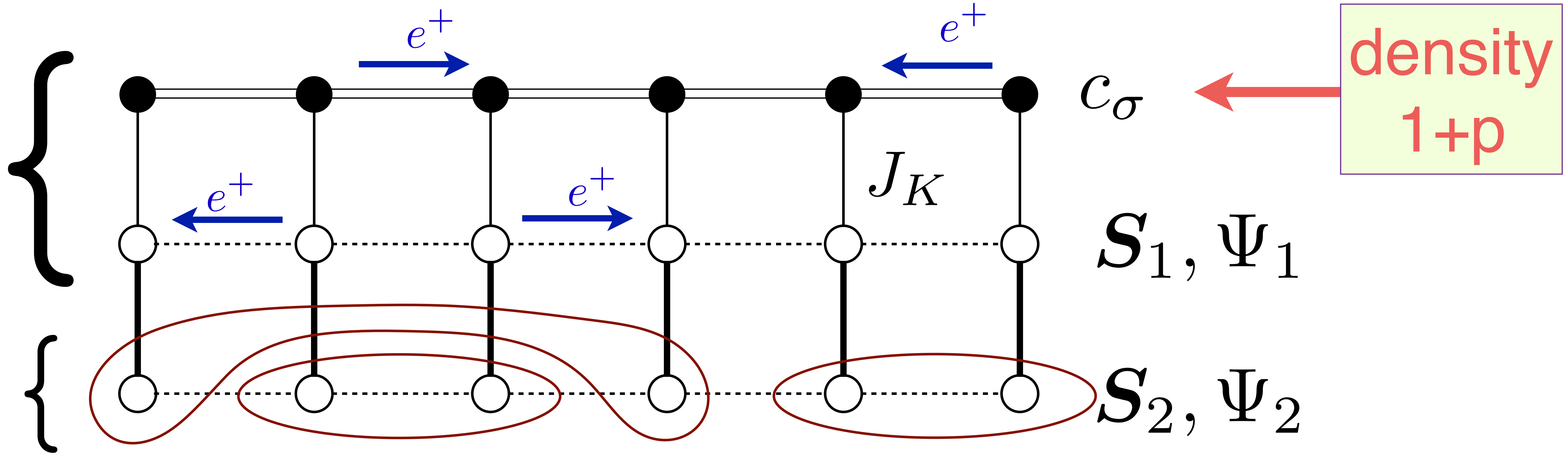
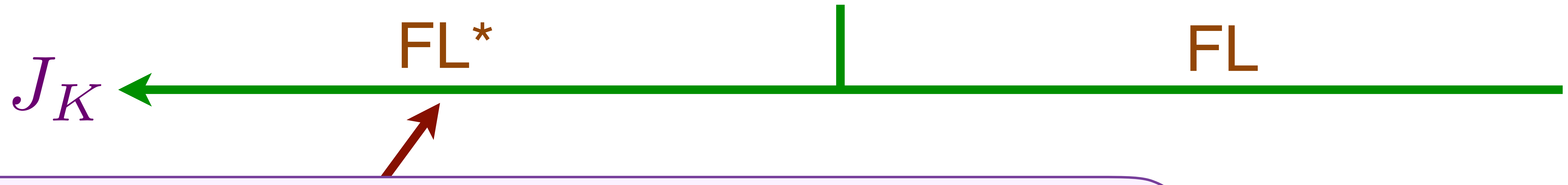
Ancilla theory of the Hubbard model

Small Fermi surface.
Size $2 + p \cong p$

Spin liquid





Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } \Psi_1, \Psi_2]$$
$$\bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle$$
$$\otimes |\text{Slater determinant of } \Psi_2\rangle$$

$(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2$ gauge theory of **one-band** model

Write fermion operators as 2×2 matrices

$$\mathbf{\Psi} = \begin{pmatrix} \Psi_\uparrow & -\Psi_\downarrow^\dagger \\ \Psi_\downarrow & \Psi_\uparrow^\dagger \end{pmatrix} \quad , \quad \widetilde{\mathbf{\Psi}} = \begin{pmatrix} \widetilde{\Psi}_\uparrow & -\widetilde{\Psi}_\downarrow^\dagger \\ \widetilde{\Psi}_\downarrow & \widetilde{\Psi}_\uparrow^\dagger \end{pmatrix}$$

Constraints $\Psi_\alpha^\dagger \Psi_\alpha = 1$ and $\widetilde{\Psi}_\alpha^\dagger \widetilde{\Psi}_\alpha = 1$ lead to:

P.A. Lee, N. Nagaosa, and X.-G. Wen, RMP **78**, 17 (2006)

$$\mathrm{SU}(2)_1 : \quad \mathbf{\Psi} \to \mathbf{\Psi} U_1 \quad , \quad \widetilde{\mathbf{\Psi}} \to \widetilde{\mathbf{\Psi}}$$

$$\mathrm{SU}(2)_2 : \quad \mathbf{\Psi} \to \mathbf{\Psi} \quad , \quad \widetilde{\mathbf{\Psi}} \to \widetilde{\mathbf{\Psi}} U_2$$

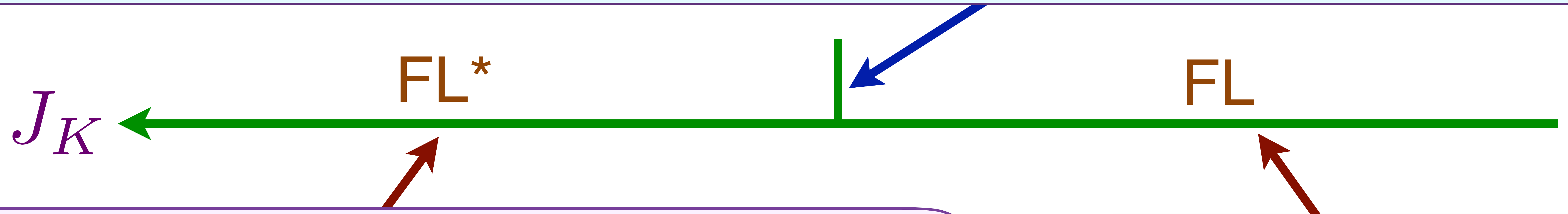
S. Sachdev, M.A. Metlitski, Yang Qi, and Cenke Xu, PRB **80**, 155129 (2009)
S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, PRB **99**, 054516 (2019)

Rung singlet formation $\boldsymbol{S}_1 + \boldsymbol{S}_2 \approx 0$ leads to:

$$\mathrm{SU}(2)_S : \quad \mathbf{\Psi} \to U_S \mathbf{\Psi} \quad , \quad \widetilde{\mathbf{\Psi}} \to U_S \widetilde{\mathbf{\Psi}}$$

Ancilla theory of the Hubbard model

- Deconfined criticality of a $(\mathrm{SU}(2)_S \times \mathrm{U}(1)_1)/Z_2$ gauge theory.
- 'Hybridization-Higgs' boson $\sim C_\sigma^\dagger \Psi_a$ which condenses on the FL* side (in Kondo lattice, Higgs boson was condensed on the FL side).
- Gauge-charged 'ghost' Fermi surface of Ψ_1 fermions.
- Large Fermi surface of c_σ gauge-neutral electrons.



Small Fermi surface of size p

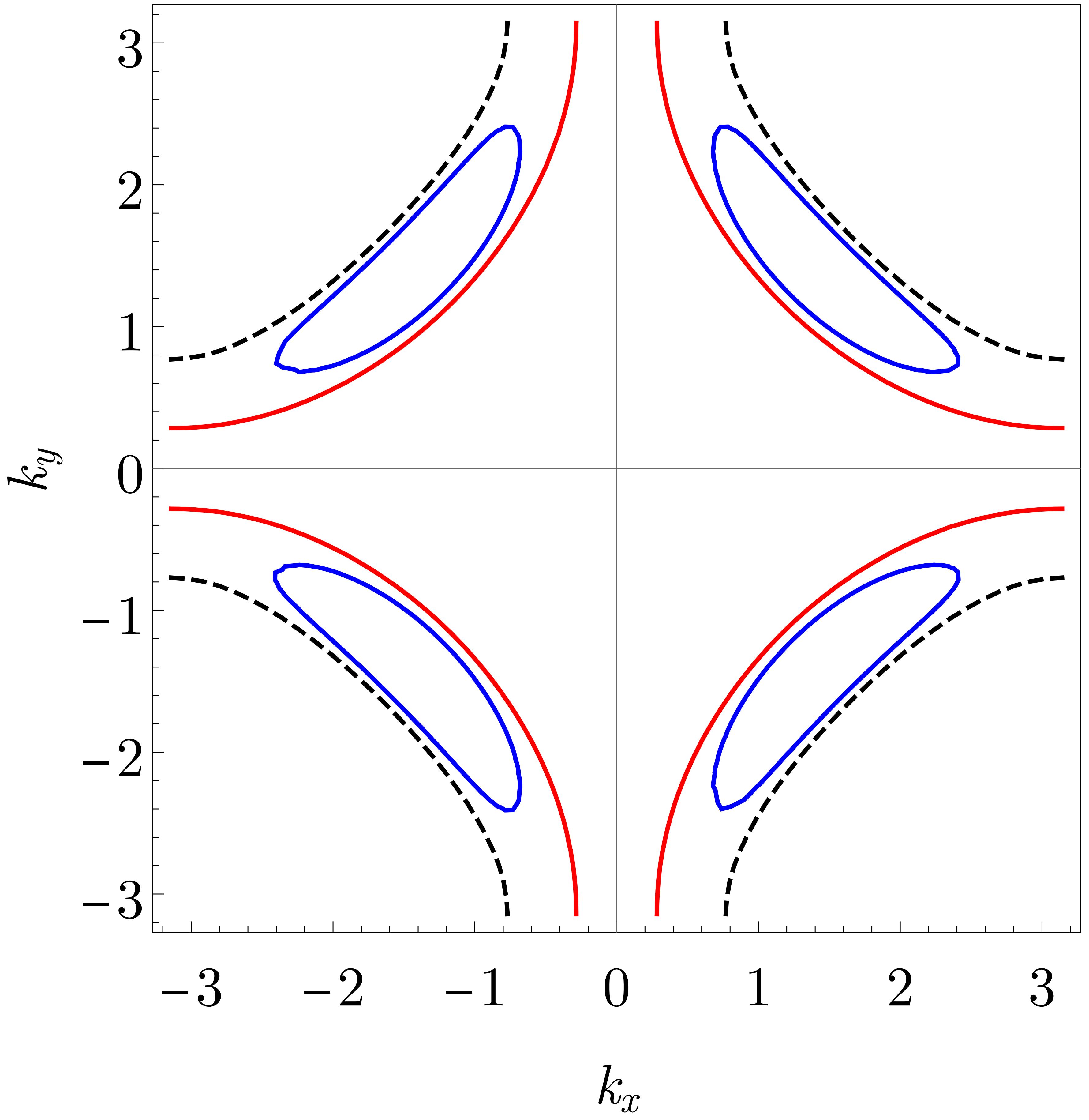
$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } \Psi_1, \Psi_2] \bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle \otimes |\text{Slater determinant of } \Psi_2\rangle$$

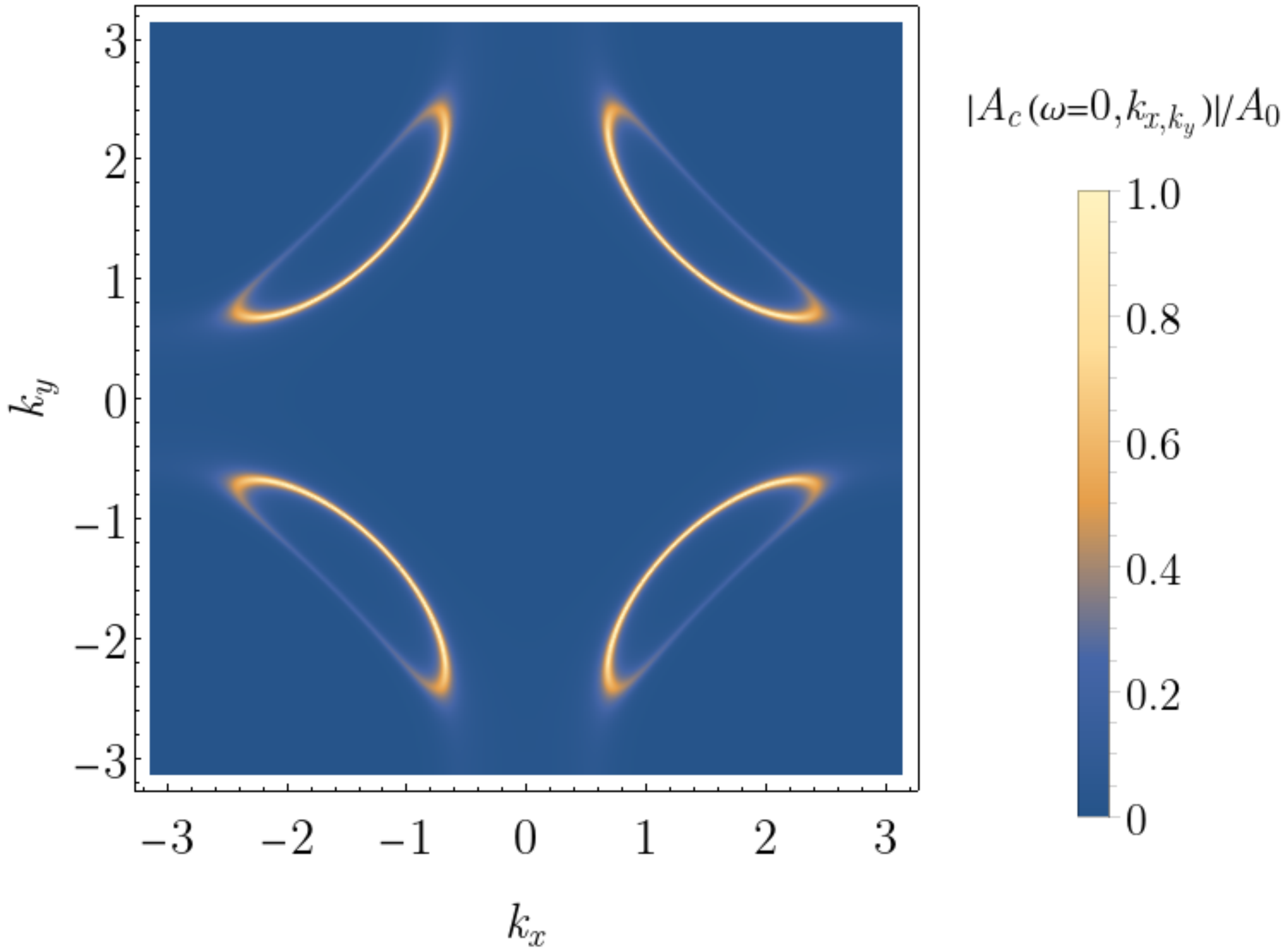
Large Fermi surface of size $1 + p$

$$|\mathrm{FL}\rangle = |\text{Rung singlets of } \Psi_1, \Psi_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

FL* in a *one-band* model

"Fermi arc" spectral functions

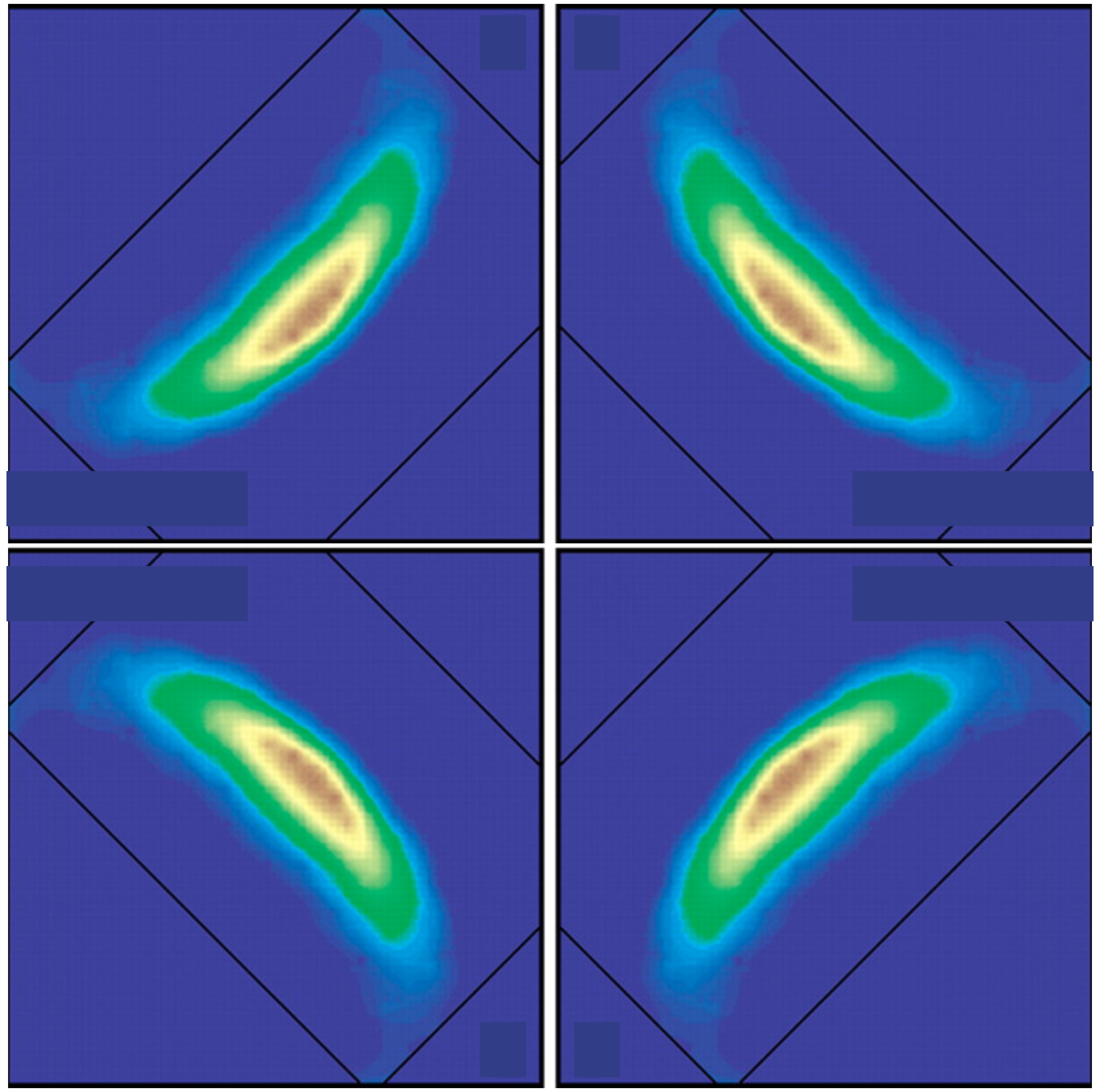




FL* Hamiltonian: $[(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2] \times \mathrm{U}(1)_{\mathrm{em}}$ is broken to $\mathrm{U}(1)_{\mathrm{diag}}$ by Higgs condensate Φ:

$$H = -\sum_{i,j} t_{ij} c^\dagger_{i;\alpha} c_{j;\alpha} + \sum_{i,j} t_{1,ij} \Psi^\dagger_{i;\alpha} \Psi_{j;\alpha} + \sum_i \Phi_i (c^\dagger_{i;\alpha} \Psi_{i;\alpha} + \Psi^\dagger_{i;\alpha} c_{i;\alpha})$$

Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

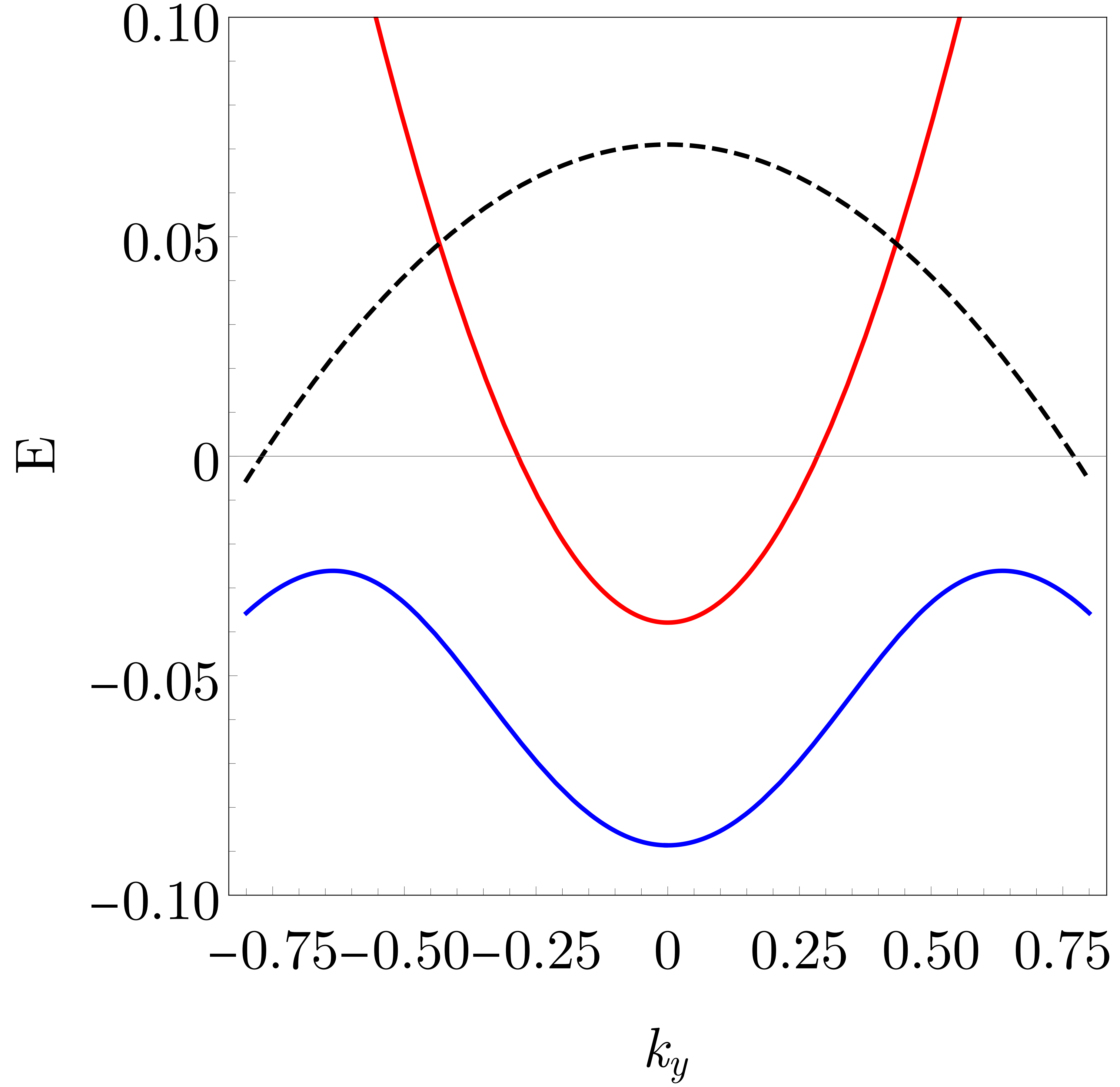
“Fermi arcs”

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

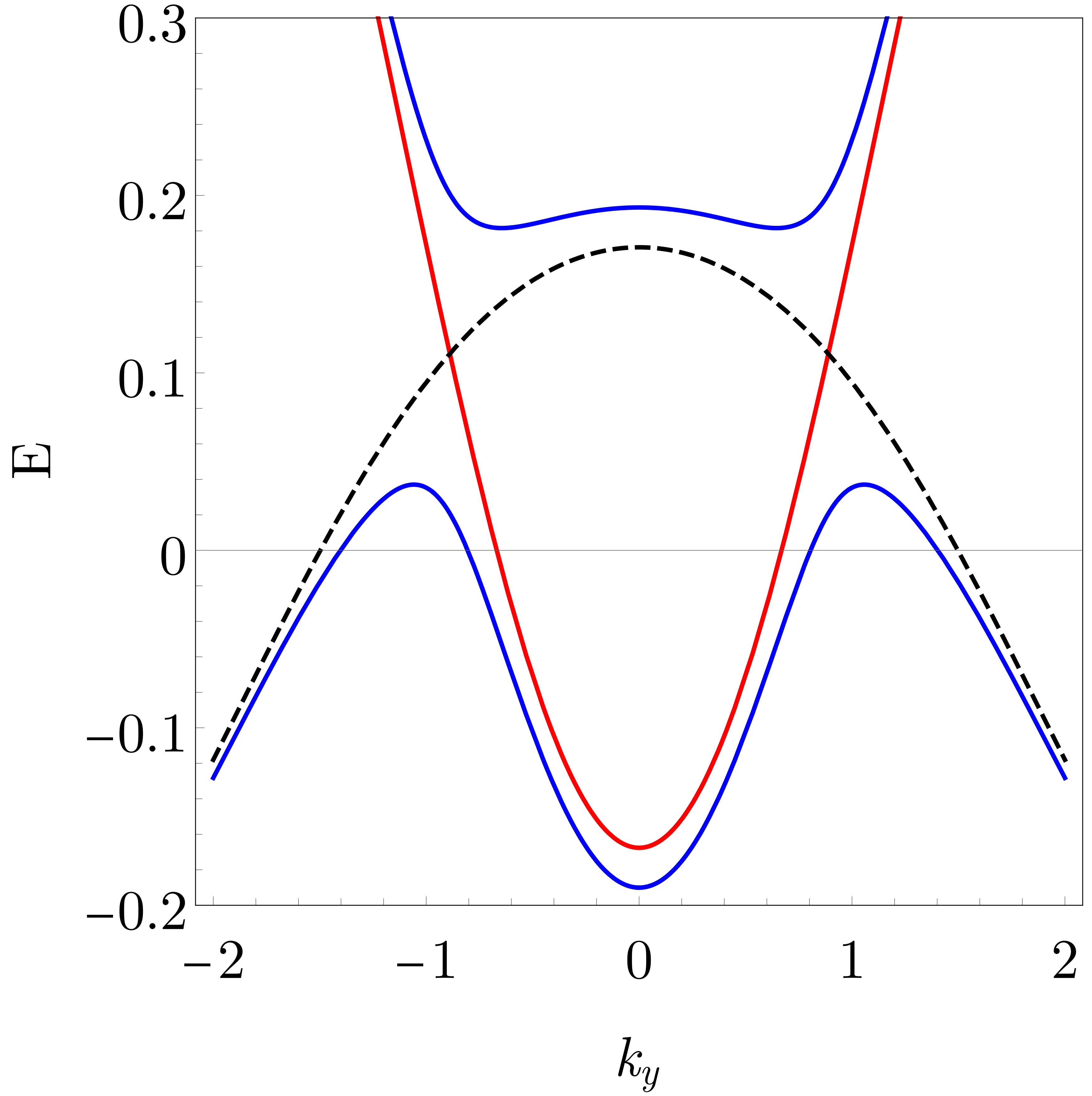
FL* in a ***one-band*** model

Electronic dispersion

Anti-node: $k_x = \pi$



Node: $k_x = 2$



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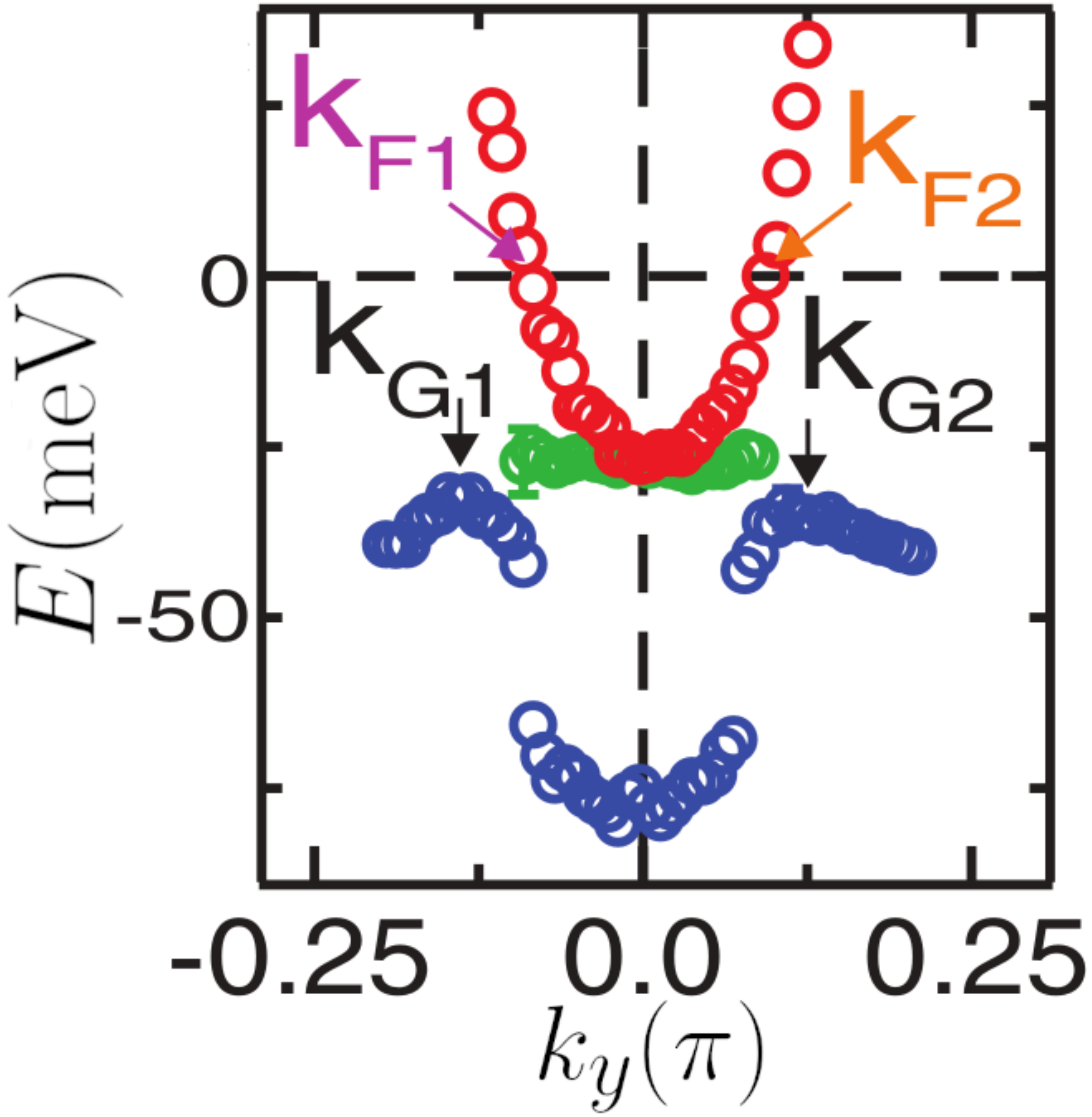
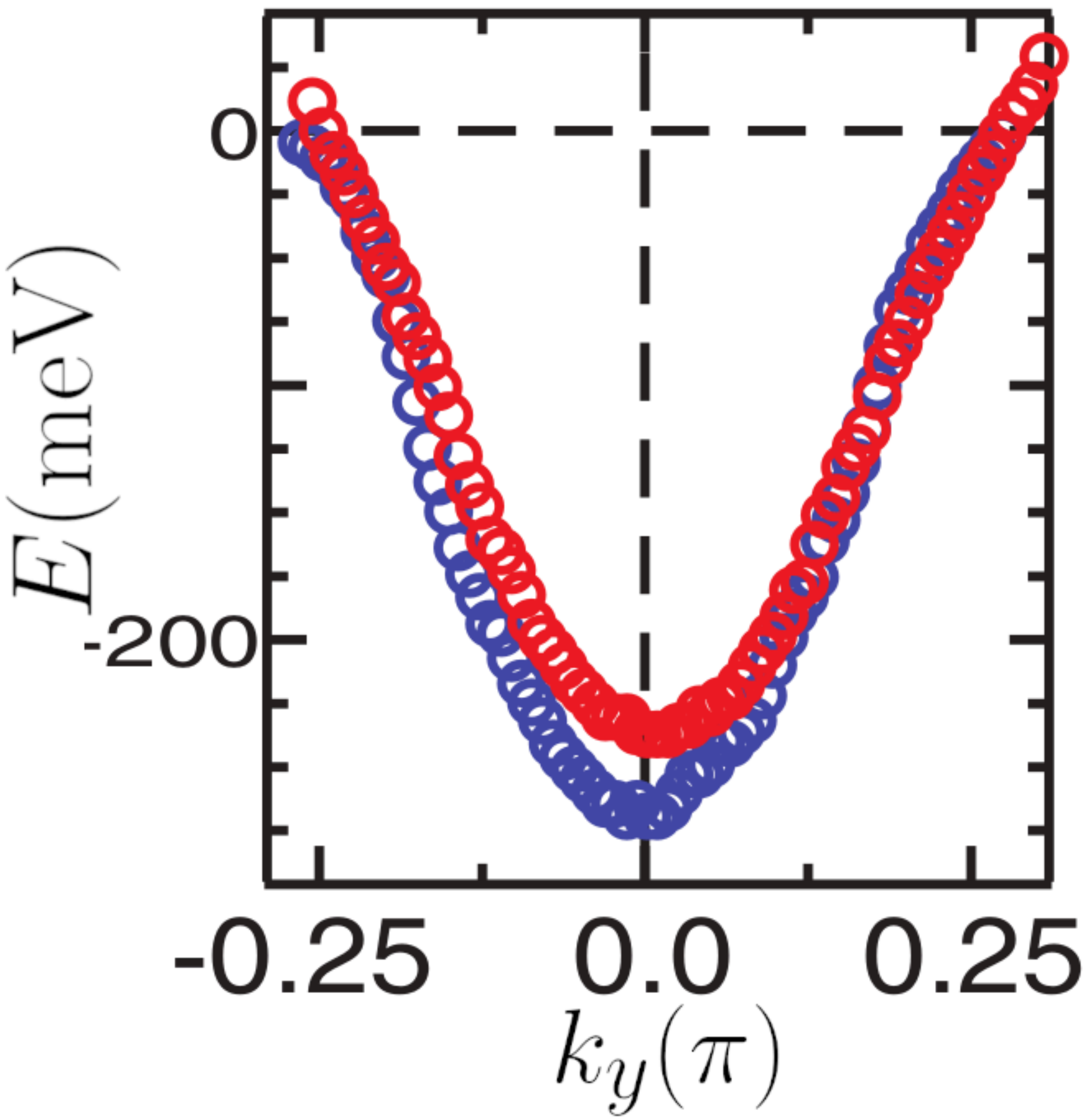
ARPES experiment

He *et al.*, *Science* **331**, 1579 (2011)

Anti-node: $k_x = \pi$

Node: $k_x = 2$



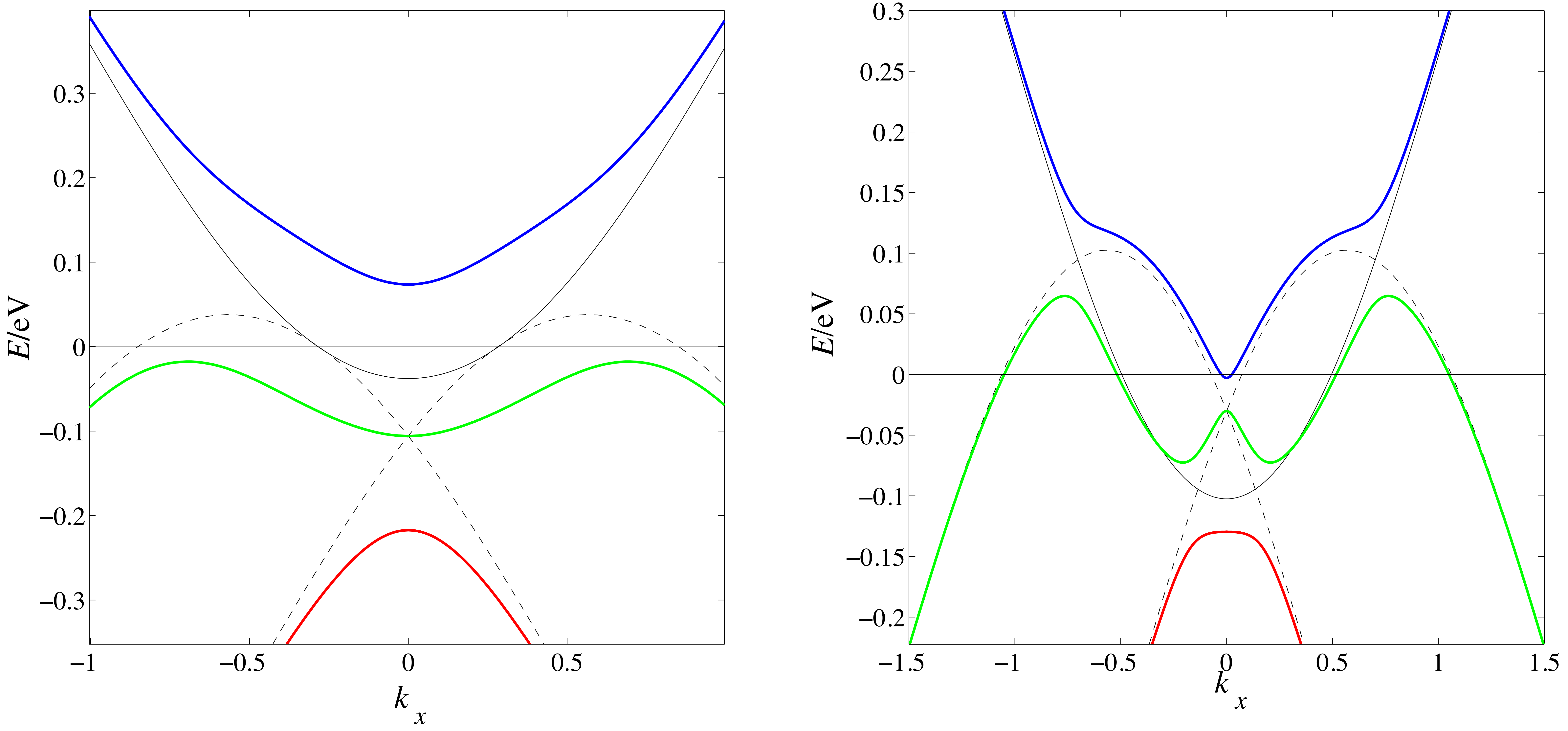


Electronic dispersion in the pseudogap metal

Pair Density Wave

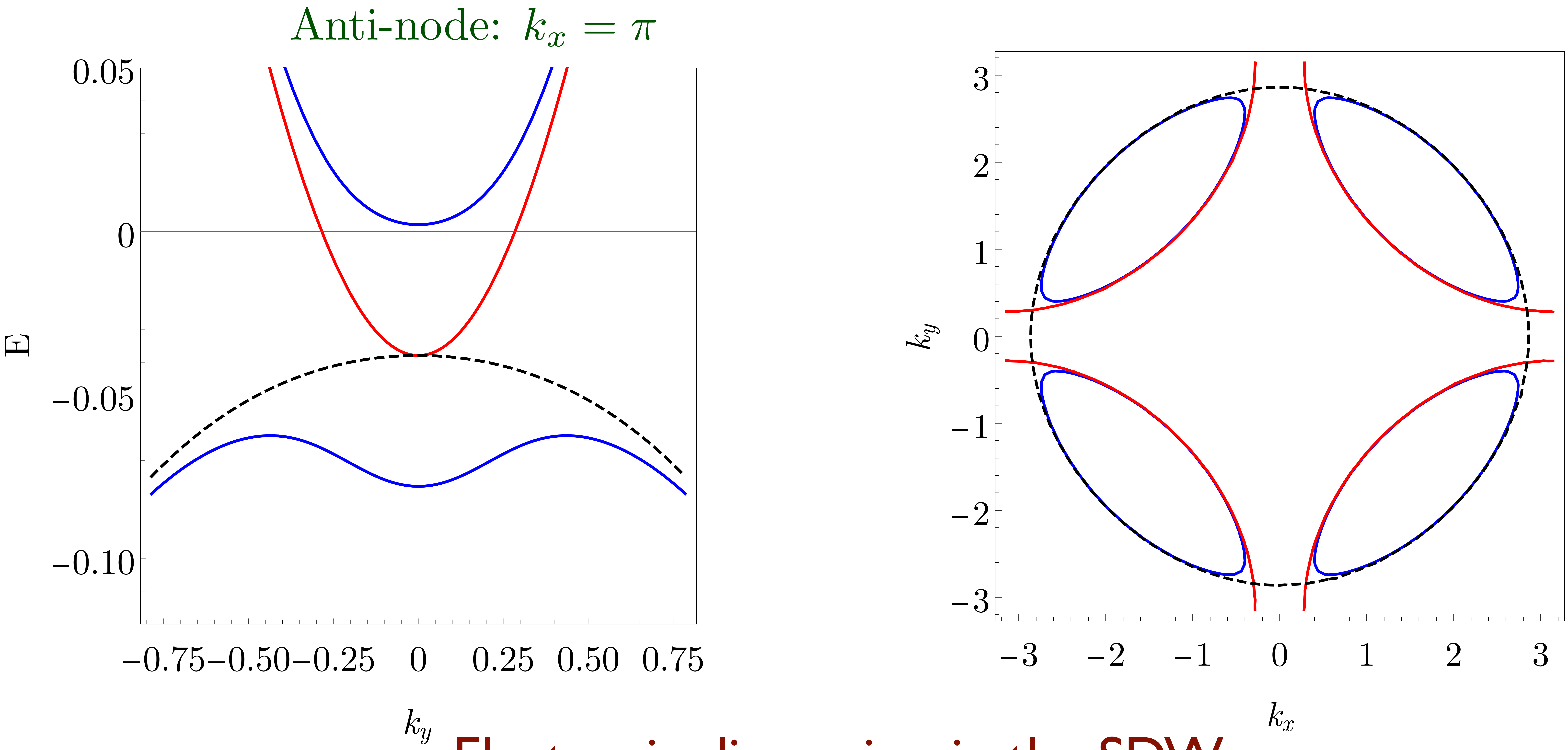
P.A. Lee, PRX **4**, 031017 (2014)

Anti-node: $k_y = \pi$ Node: $k_y = \pi - 1$



Electronic dispersion in the PDW

Spin Density Wave



Electronic dispersion in the SDW

FL*

- Recent evidence for a FL* phase in a Kondo lattice: $CeCoIn_5$ (Maksimovic *et al.*, arXiv:2011.12951, and in CePdAl, Zhao *et al.*, Nature Physics **15**, 1261 (2019). And perhaps YbB_{12} (Liu *et al.* arXiv:2102.09545)?

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- Ancilla theory of FL* for the pseudogap metal of the cuprates: Don't fractionalize the mobile electron, but fractionalize the 'paramagnon rotor' into 'ancilla qubits'. Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.

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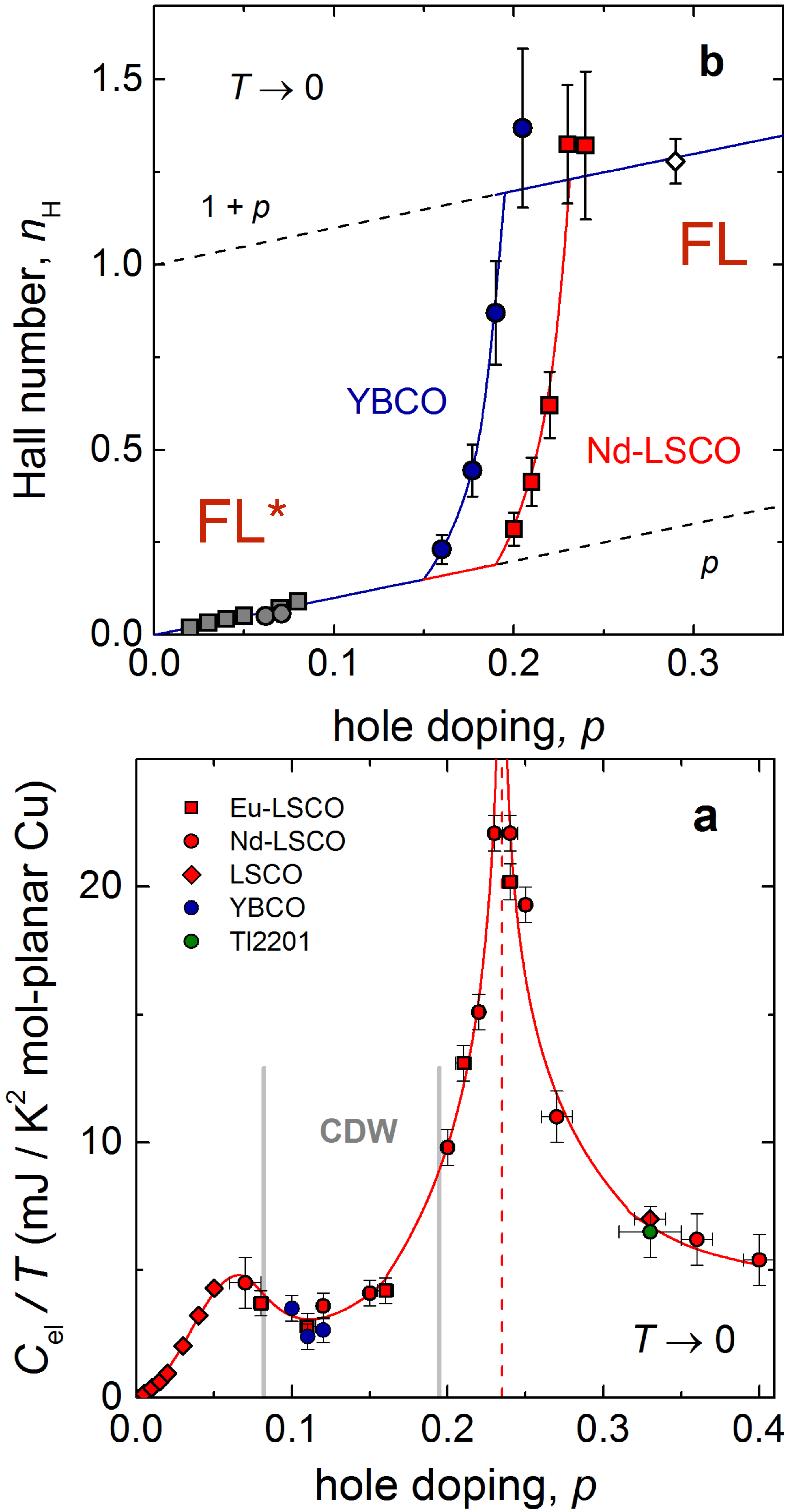
- Theory of FL-FL* transition on a single band Hubbard model: $(SU(2)\times U(1))/\mathbb{Z}_2$ gauge theory coupled to hybridization boson, a gauge-neutral *large* Fermi surface of electrons, and a 'ghost' Fermi surface.
  Prediction: critical 'ghost' Fermi surfaces near the transition.

Cuprates



Evidence for
ghost Fermi surfaces
in the
FL*-FL transition
in a
single-band model ?

$CeCoIn_5$