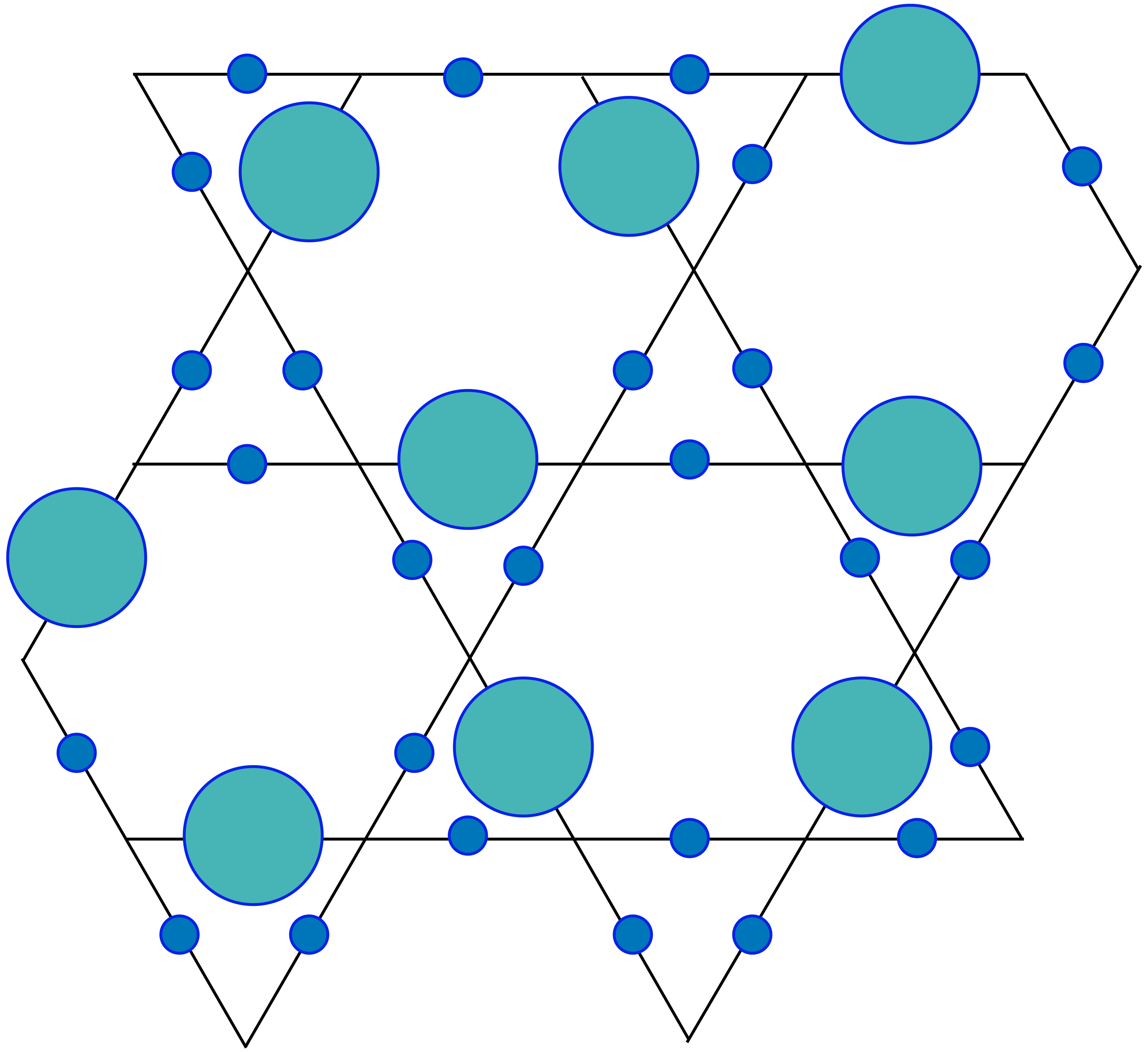
Harnessing Multi-Electron Entanglement in Modern Quantum Materials



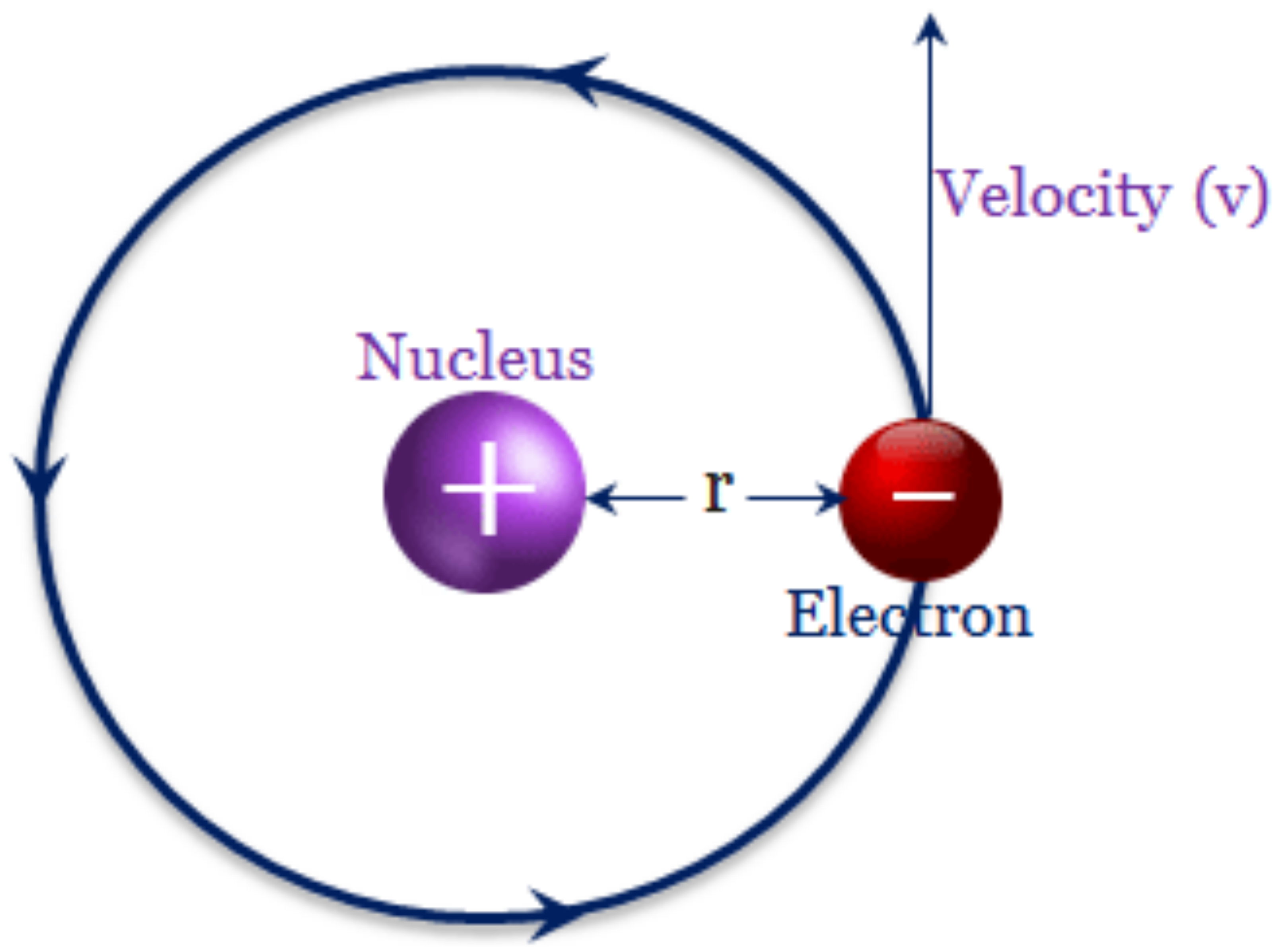
Quantum India Bengaluru
July 31, 2025

Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

Quantum mechanics
to
quantum materials: the first 100 years

Hydrogen atom

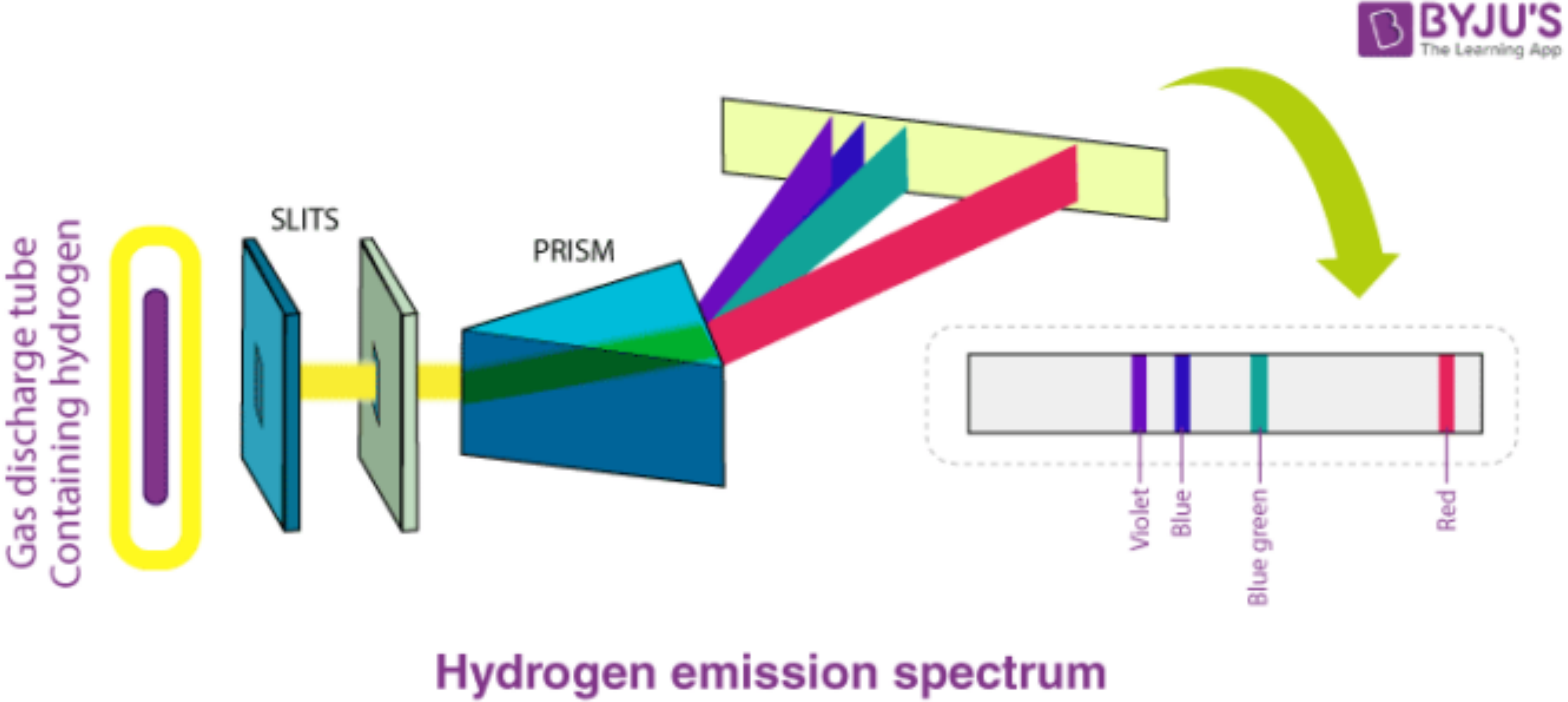


$\Rightarrow 10^{-10}$ meters $\Leftarrow$

The motion of the electron around the proton is *not* described by the same theory as the motion of the planets around the sun.

It is described by the quantum theory of Schrödinger and Heisenberg (1925).

- Schrödinger, Heisenberg (1925): Discovery of the equation obeyed by a single electron, replacing Newton's laws of motion. These equations precisely described the light emission spectrum of a single hydrogen atom.



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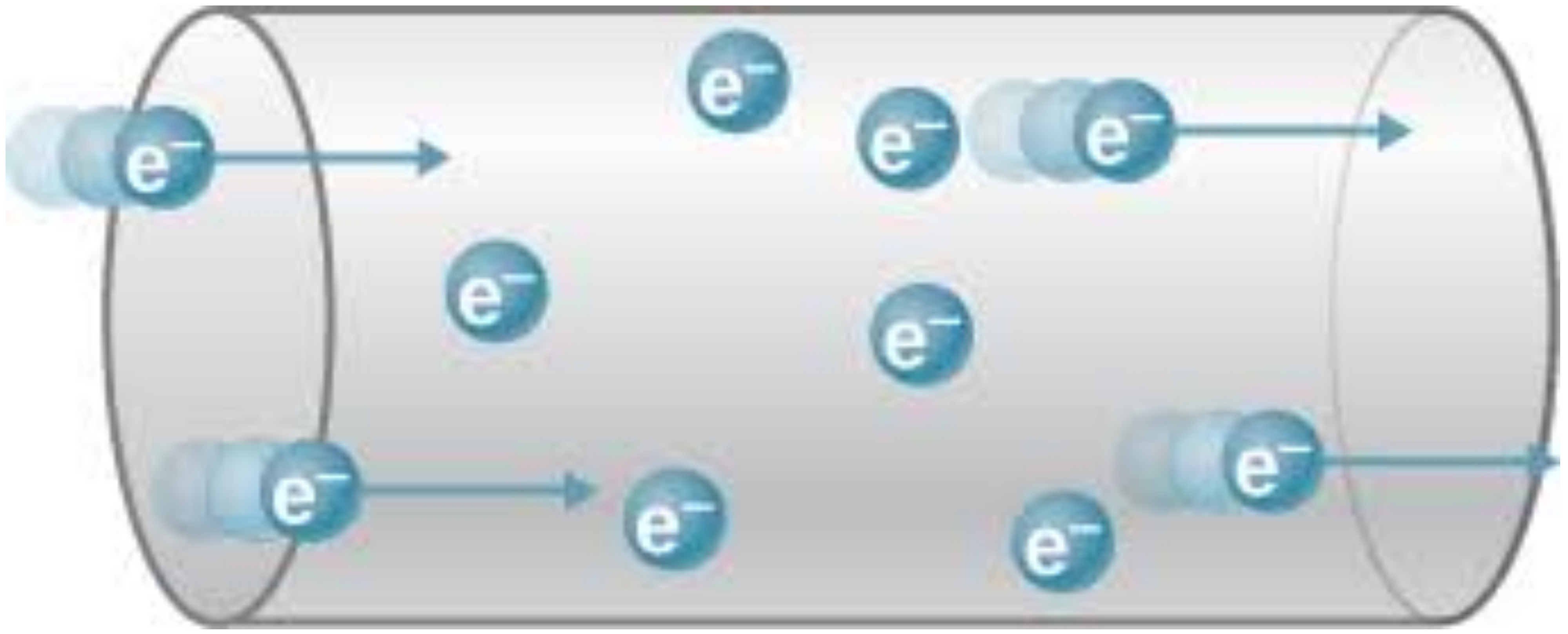

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- Bardeen, Cooper, Schrieffer (1957): Pairs of electrons behave like **bosons**, and this is the explanation for superconductivity.

- Today: Many particles exhibit many *emergent phenomena*, related to **quantum entanglement**. These are crucial to understanding modern quantum materials, such as the high temperature superconductors.

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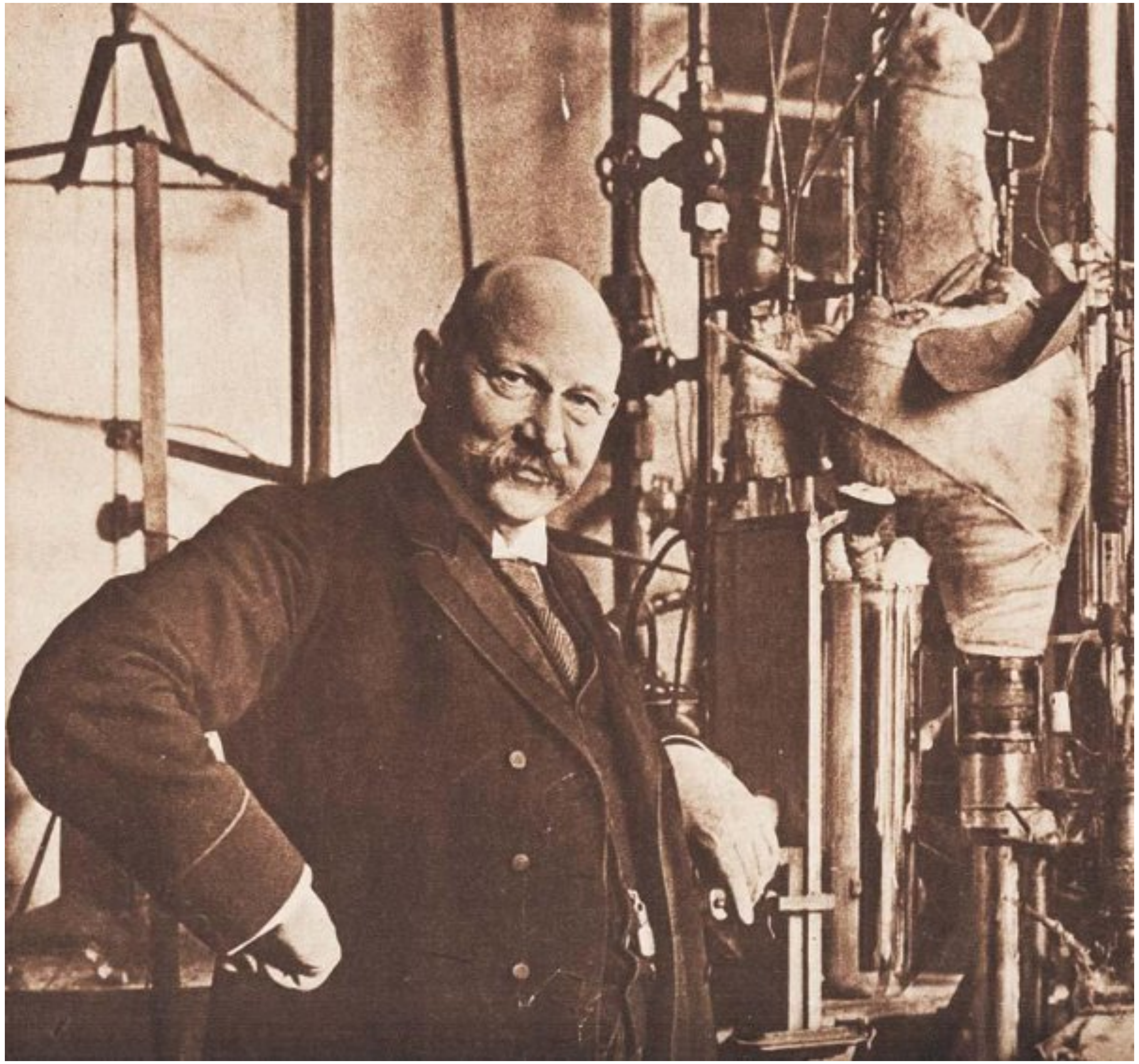
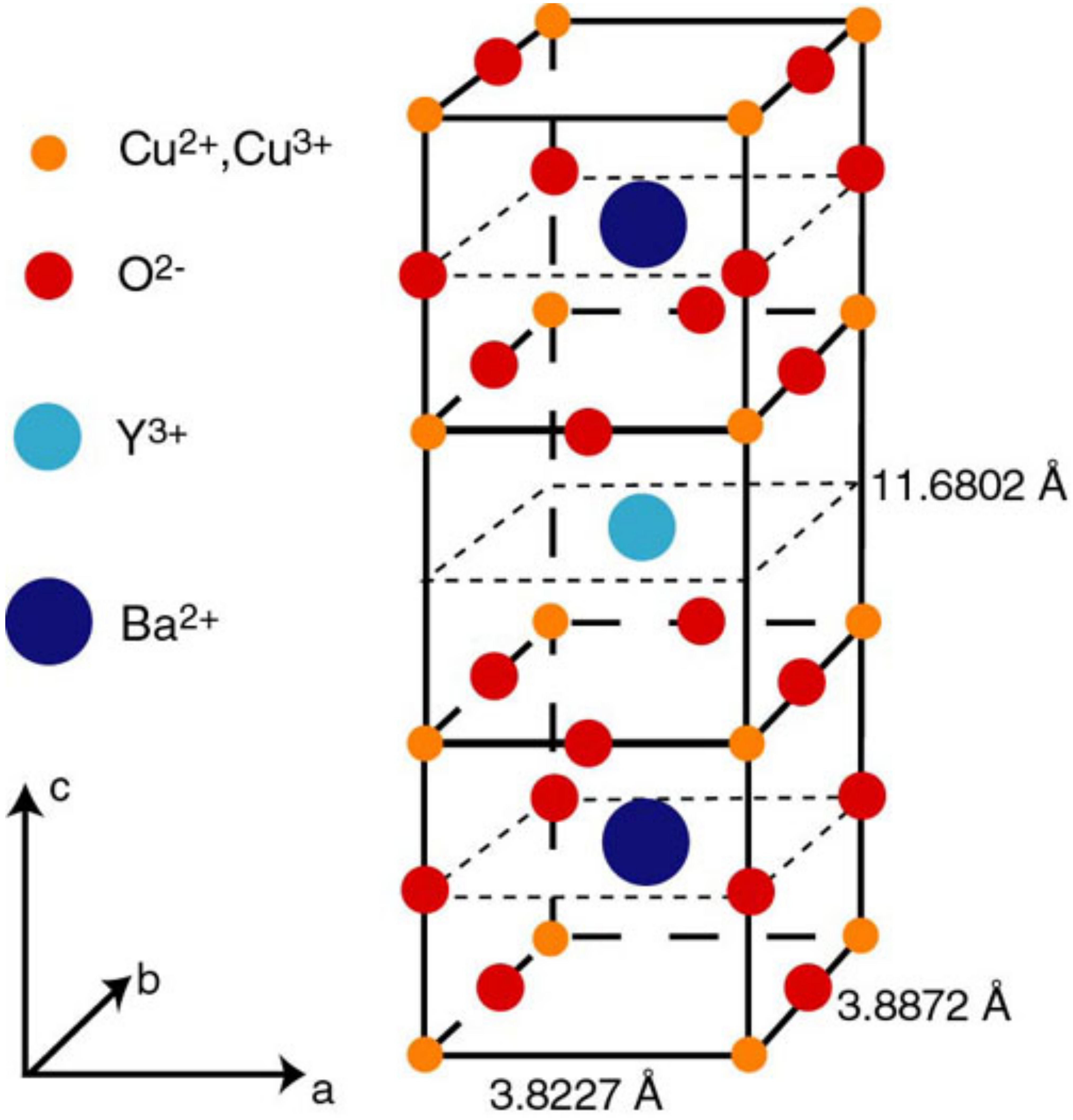
- Ideas on multi-particle entanglement in quantum materials have strongly influenced quantum computing, especially quantum error correction (and vice versa).

Superconductors

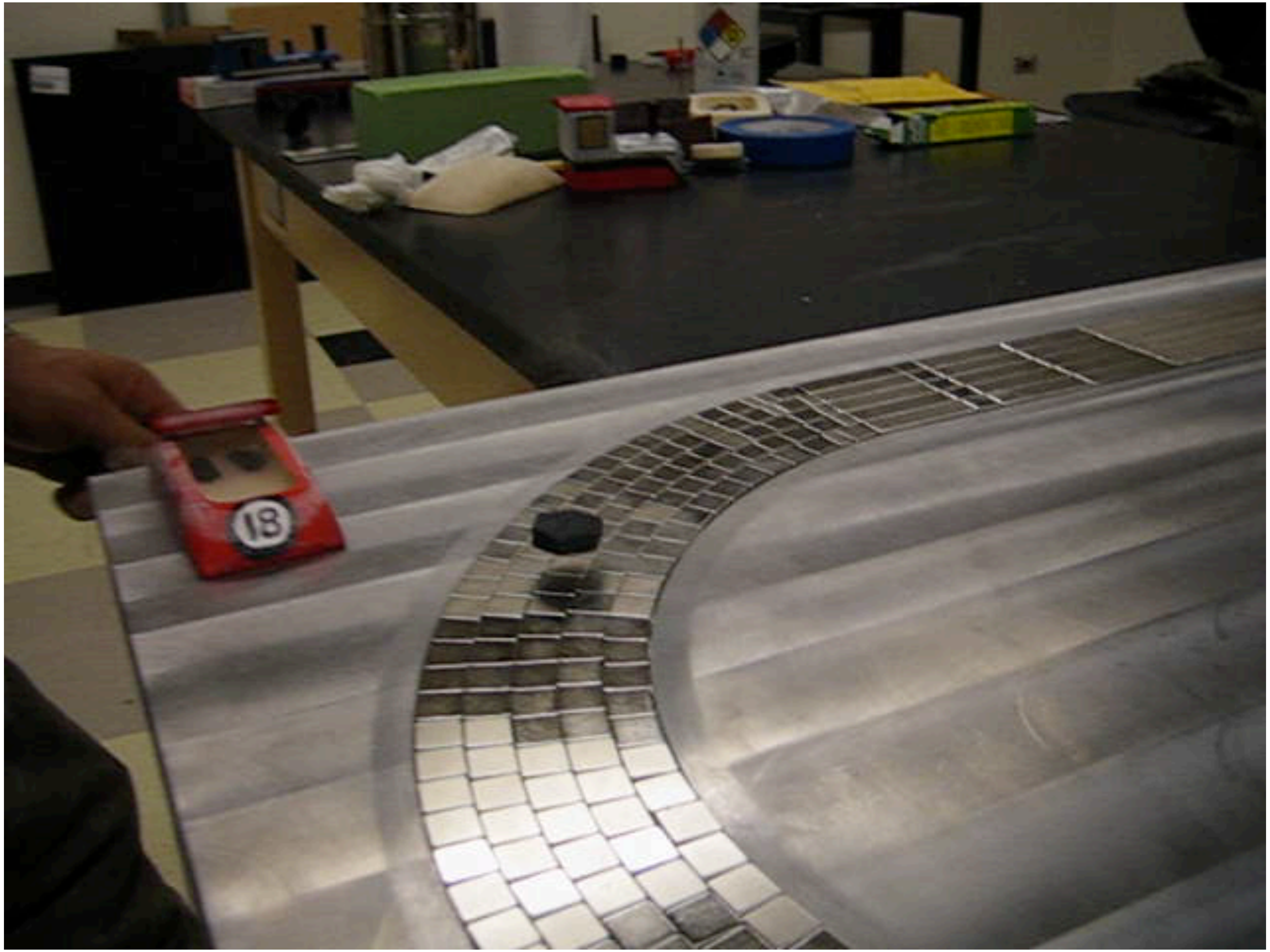


Kamerlingh Onnes 1911:
Mercury is a superconductor below -269 °C

Cuprate high temperature superconductors



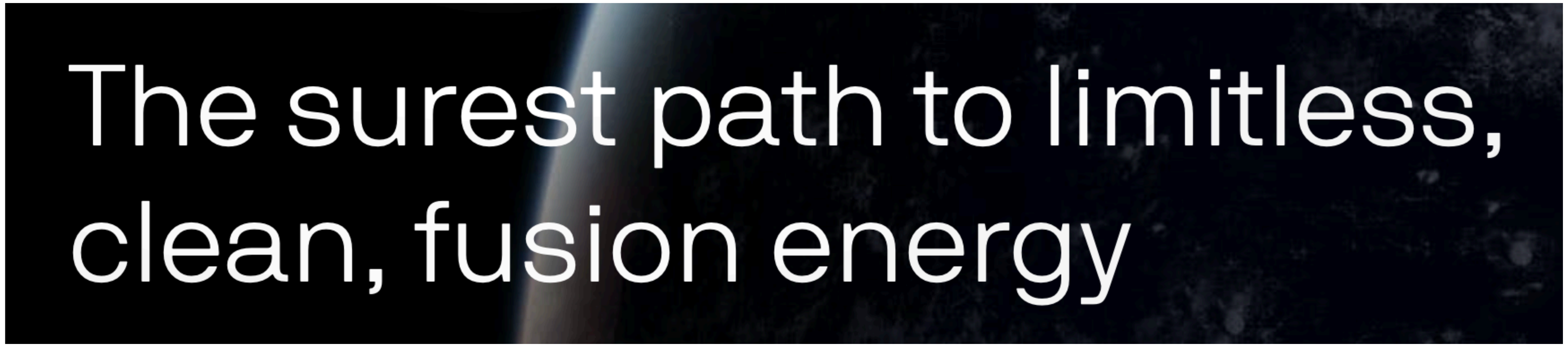
$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

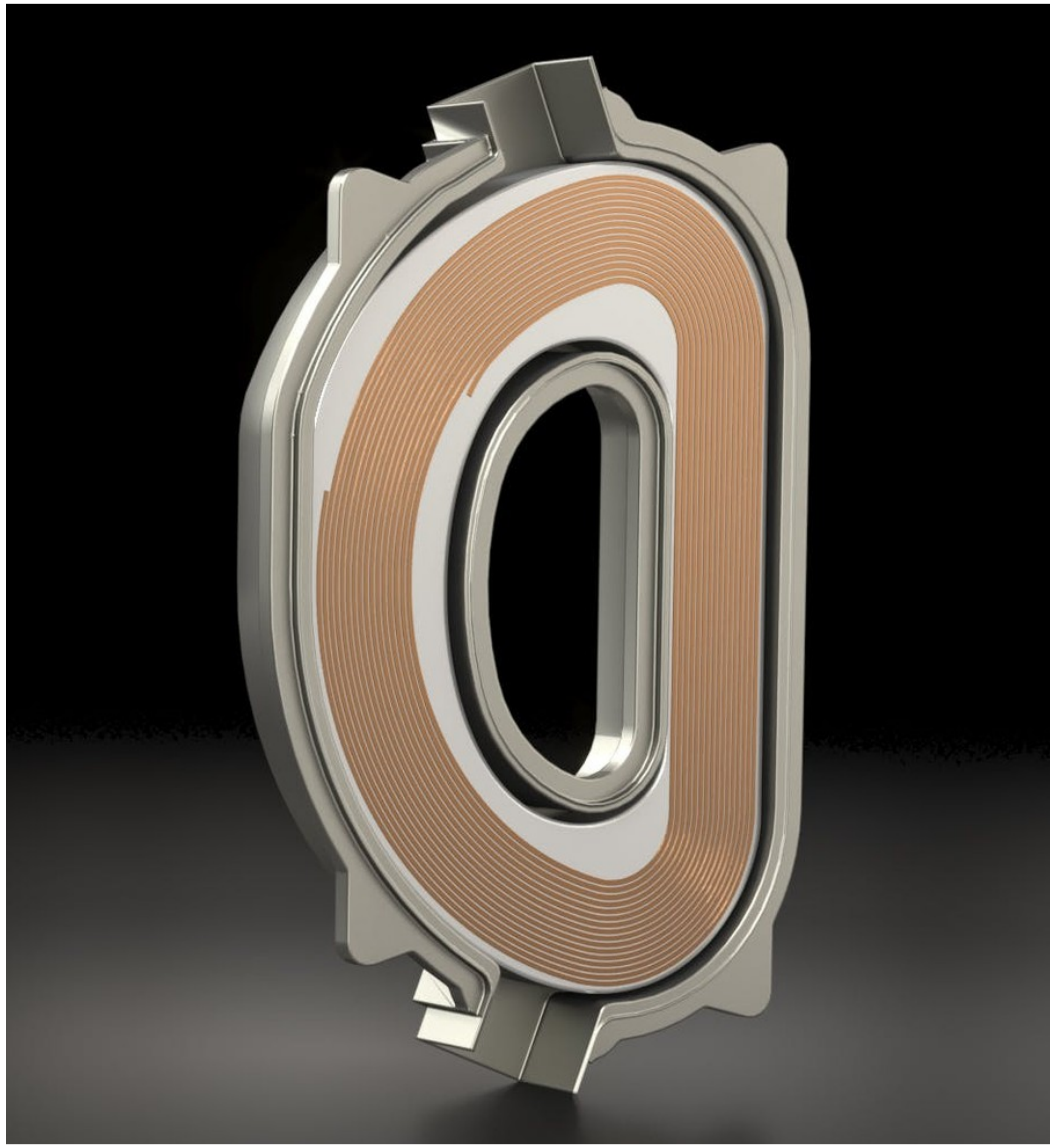
Julian Hetel and Nandini Trivedi, Ohio State University

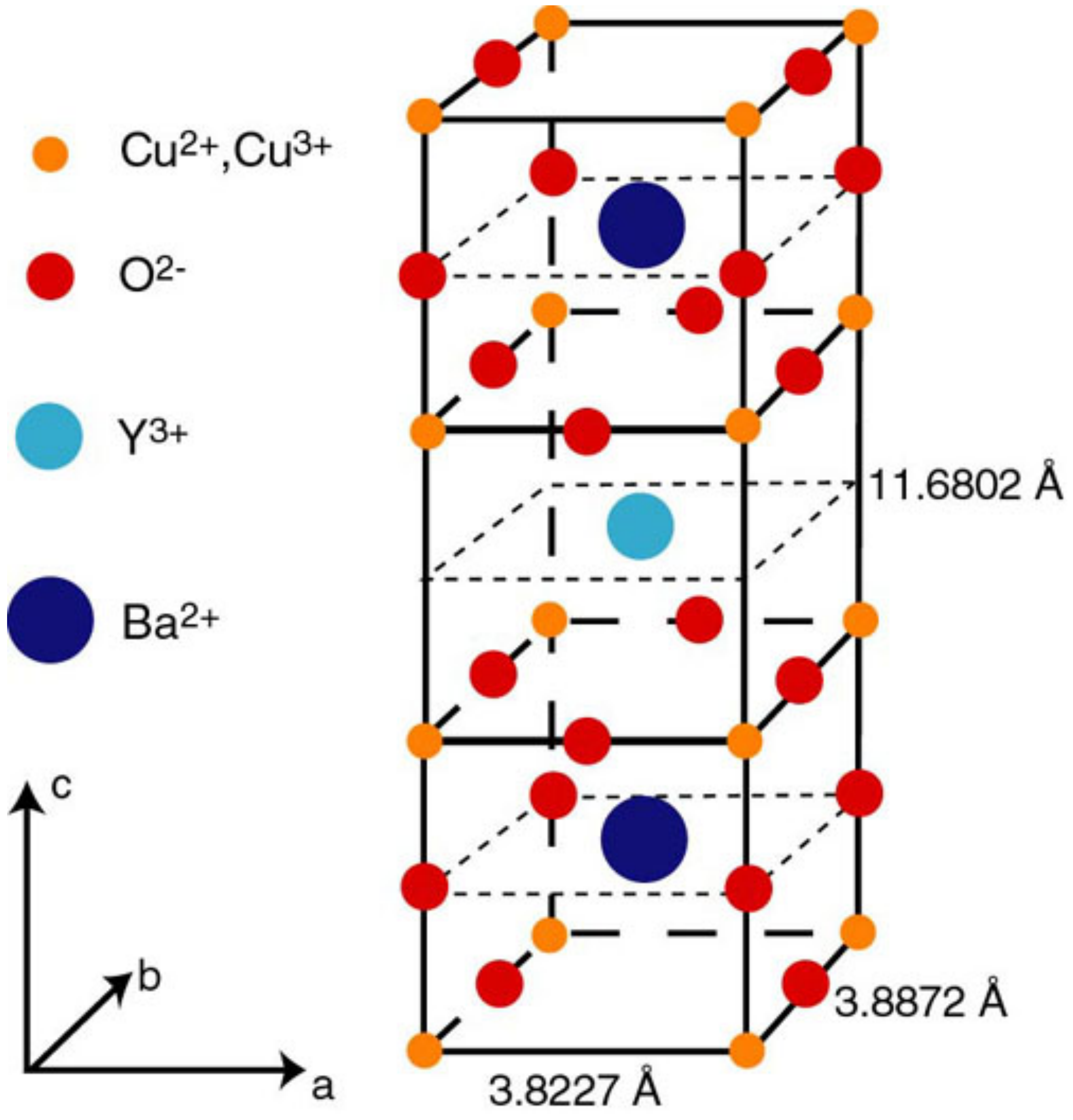
HTS Magnets: Enabling Technology


The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

Commonwealth Fusion Systems





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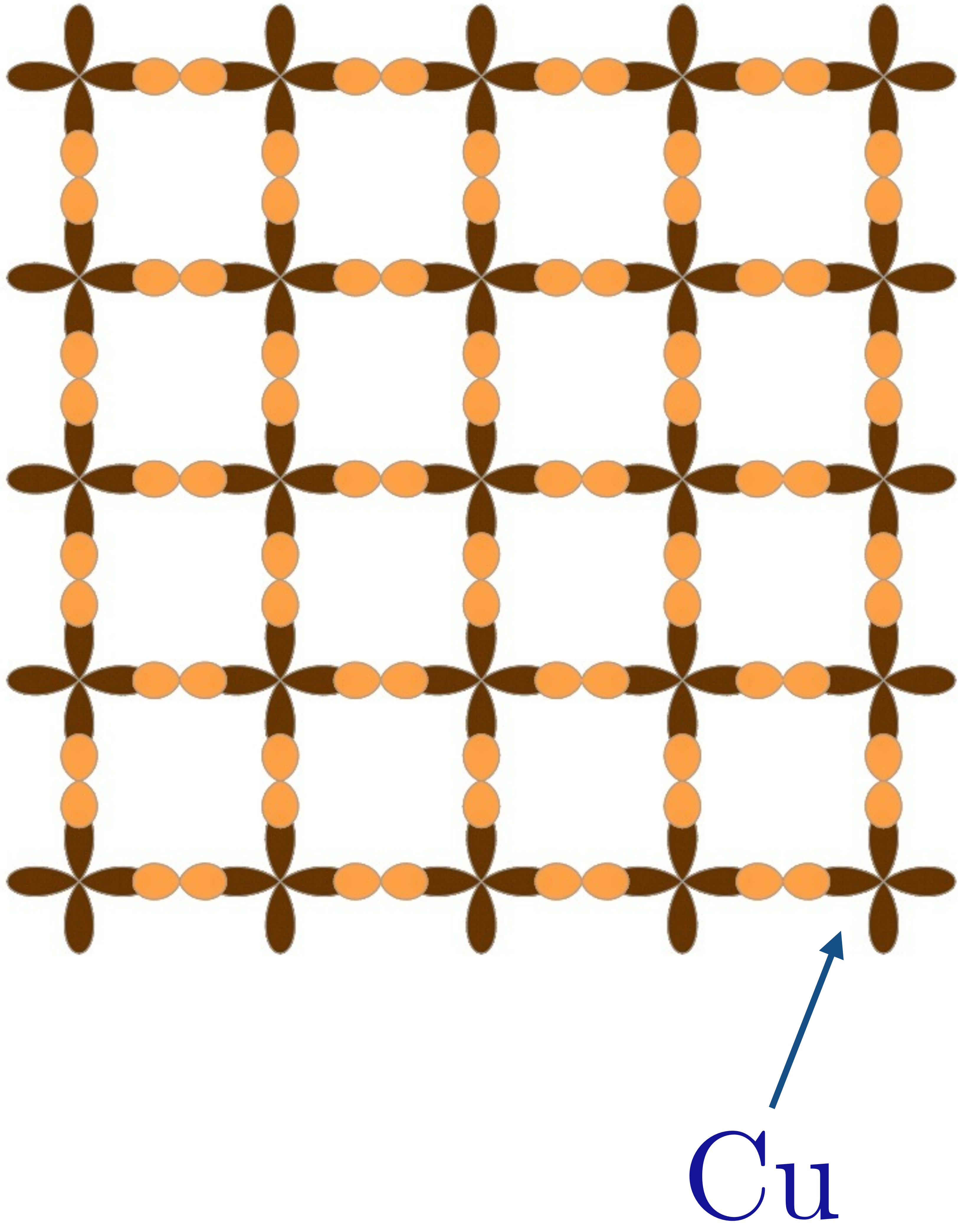

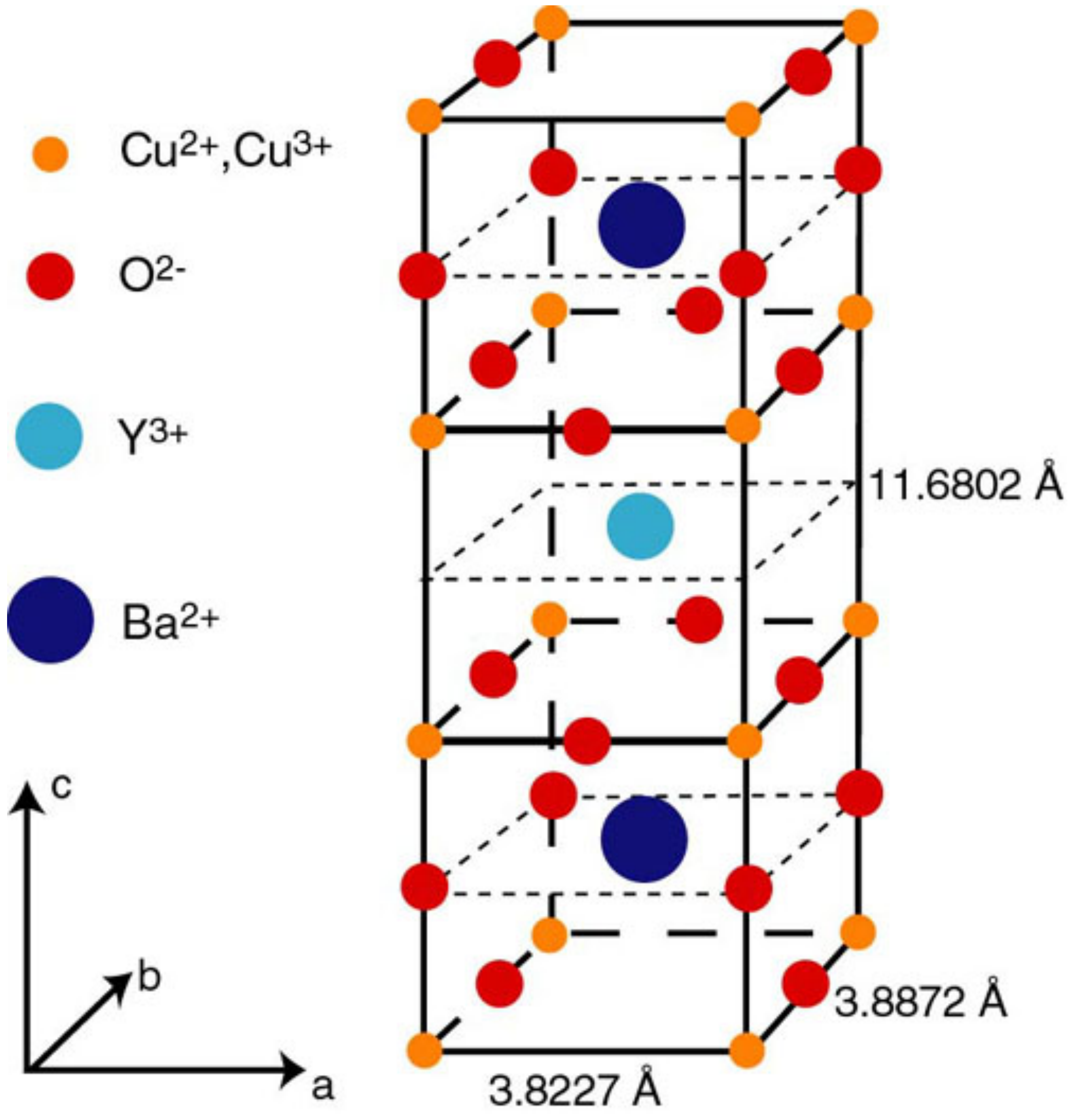

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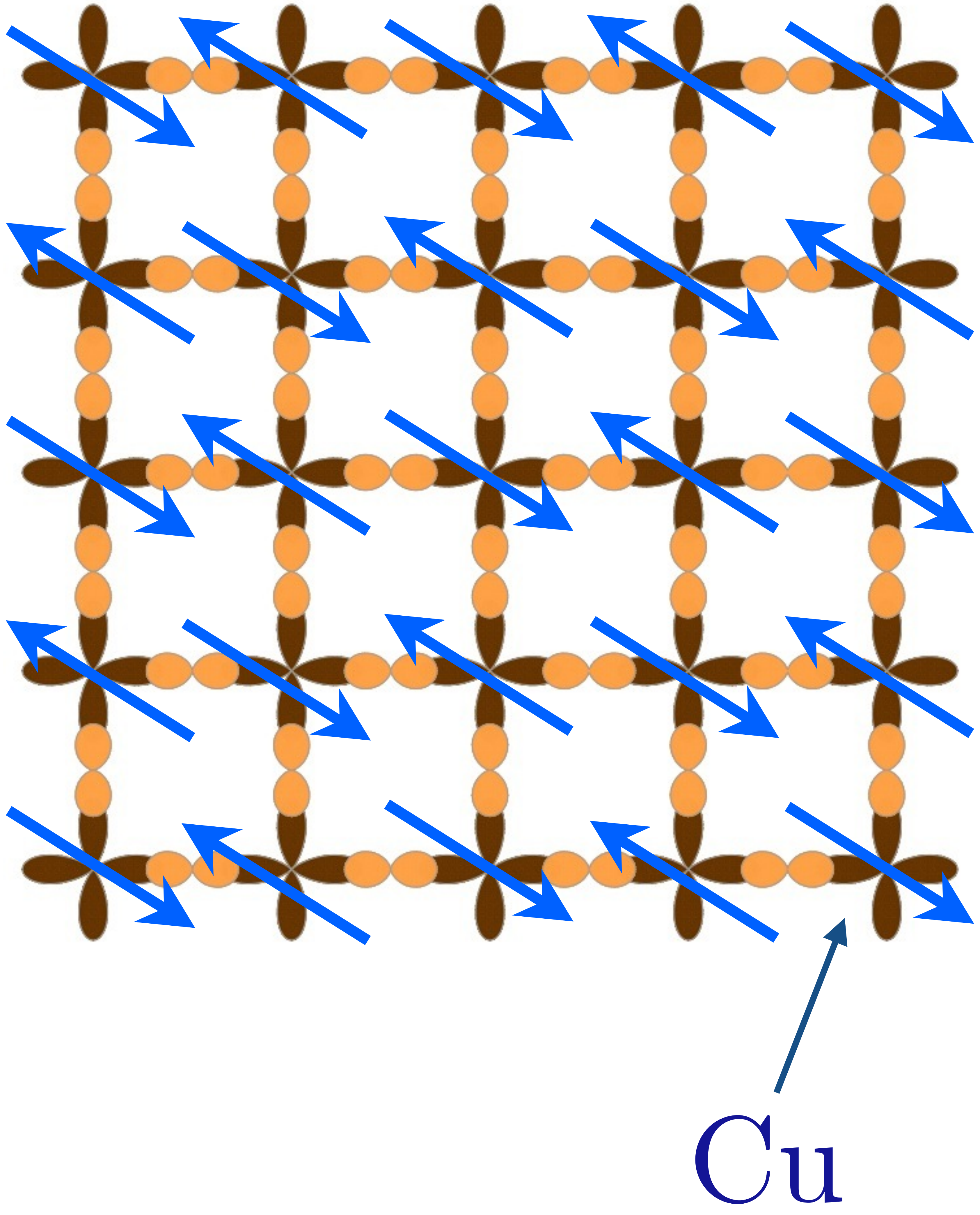
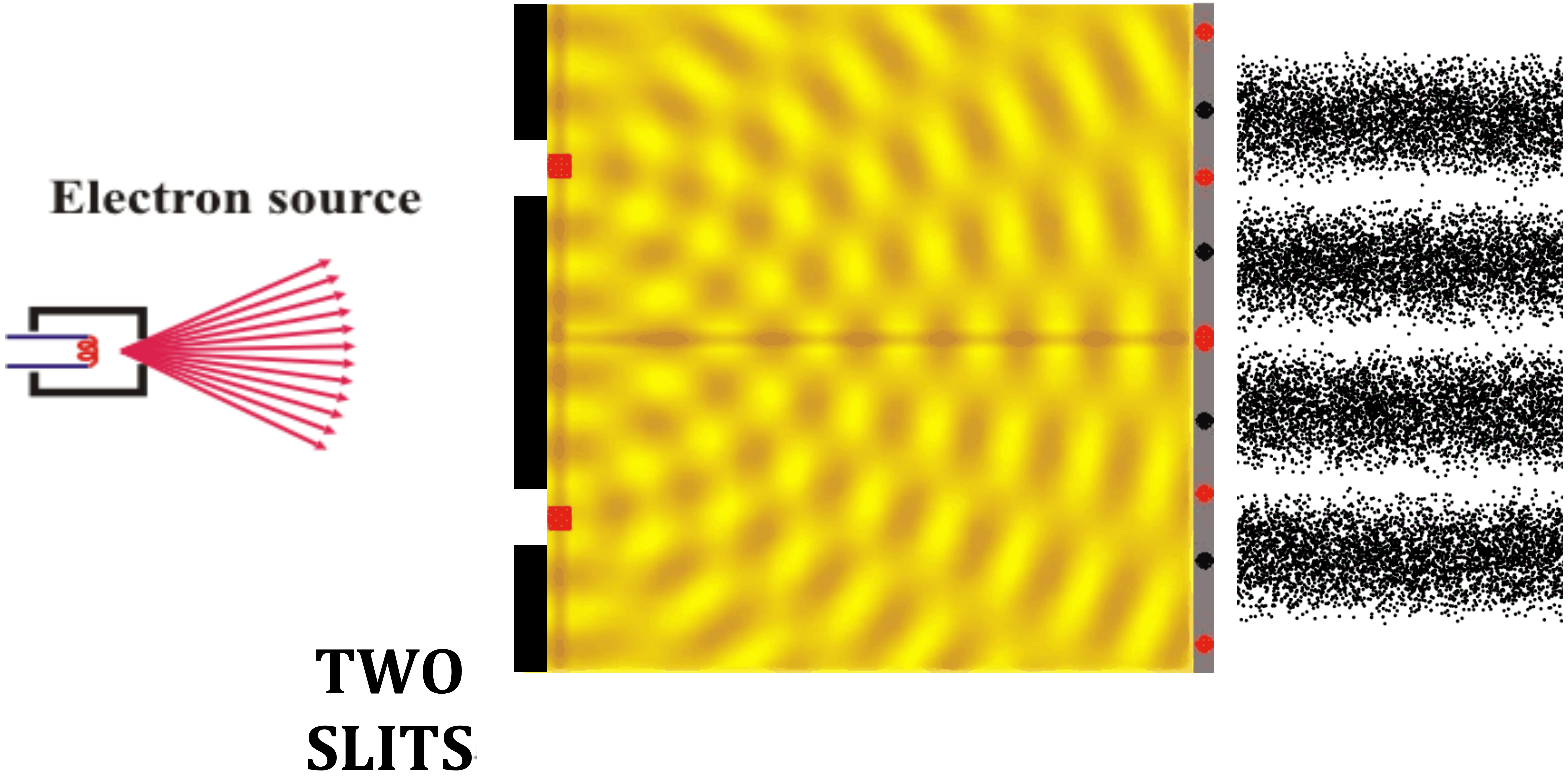


P.W. Anderson and G. Baskaran (1988): The key to high temperature superconductivity is the formation of a "resonating valence bond state" (a type of **quantum spin liquid**) which entangles the electrons on Cu

Quantum entanglement

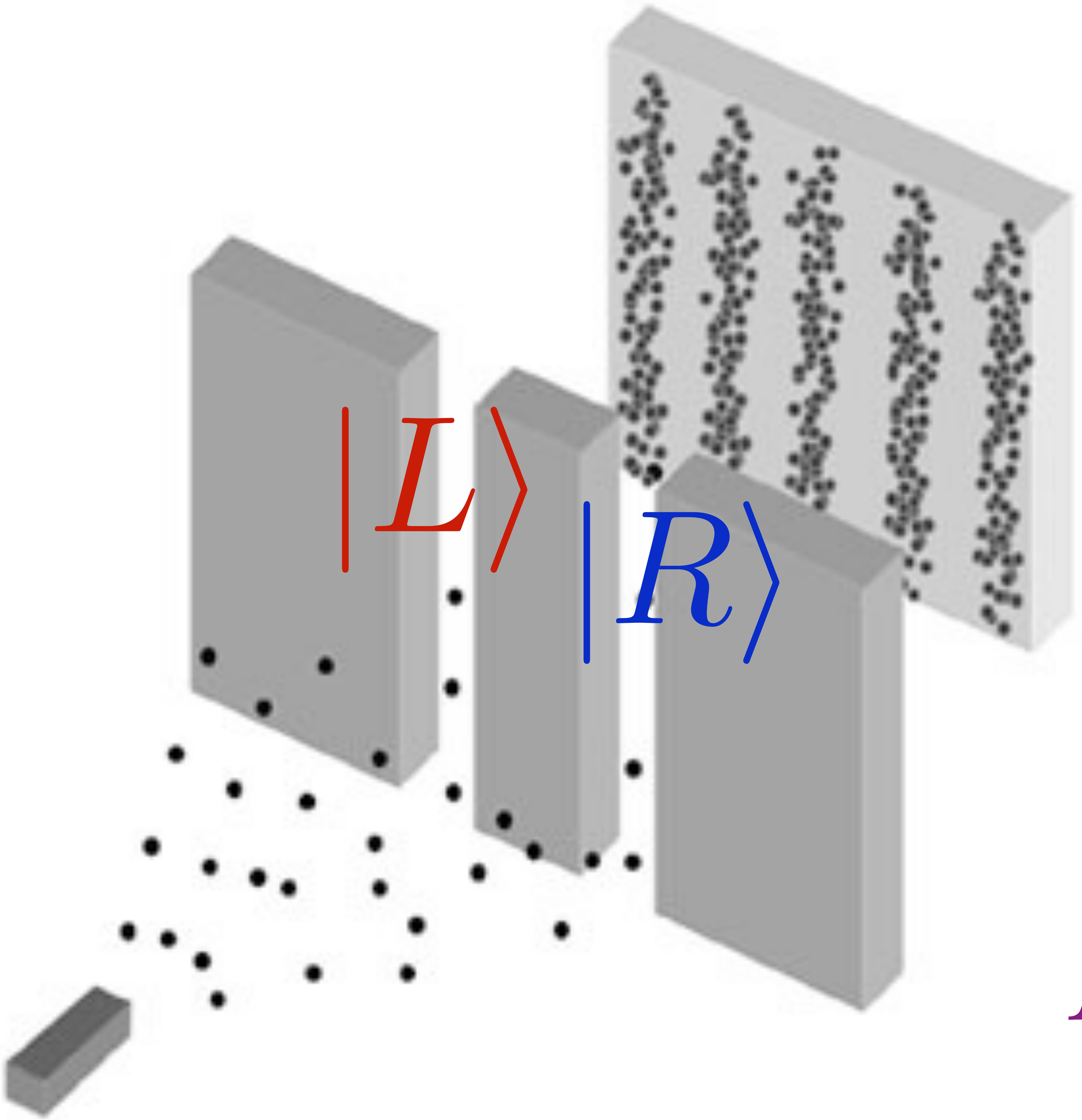
The double slit experiment



Unlike water waves, electrons arrive one-by-one (so is it like a particle ?)

Interference of electrons

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

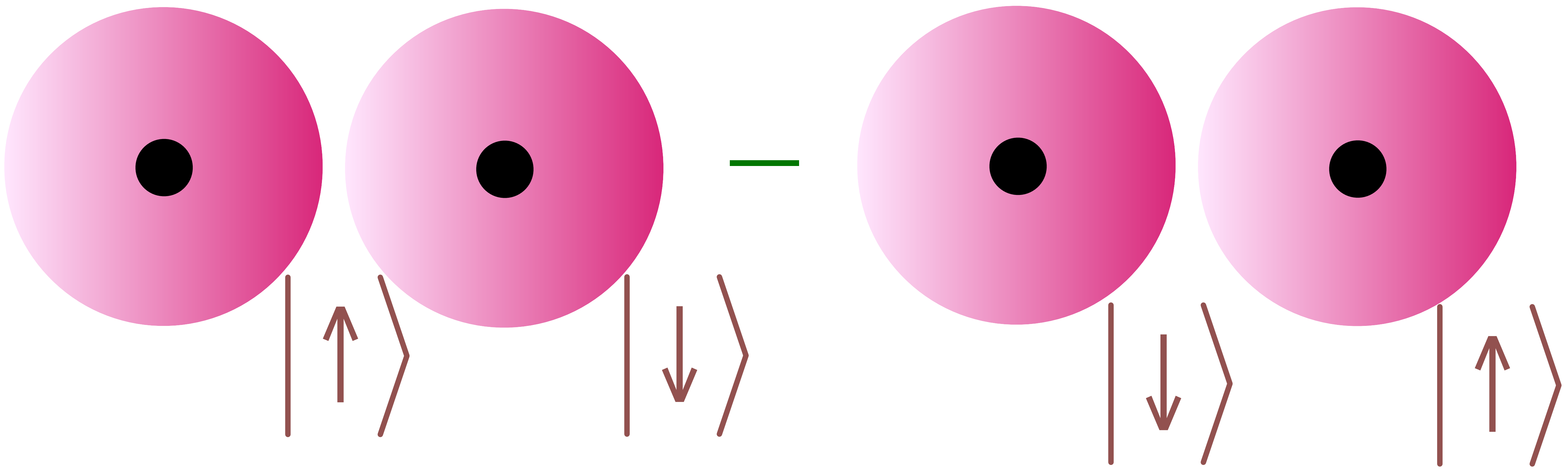
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



$$|\uparrow\rangle\,|\downarrow\rangle - |\downarrow\rangle\,|\uparrow\rangle$$

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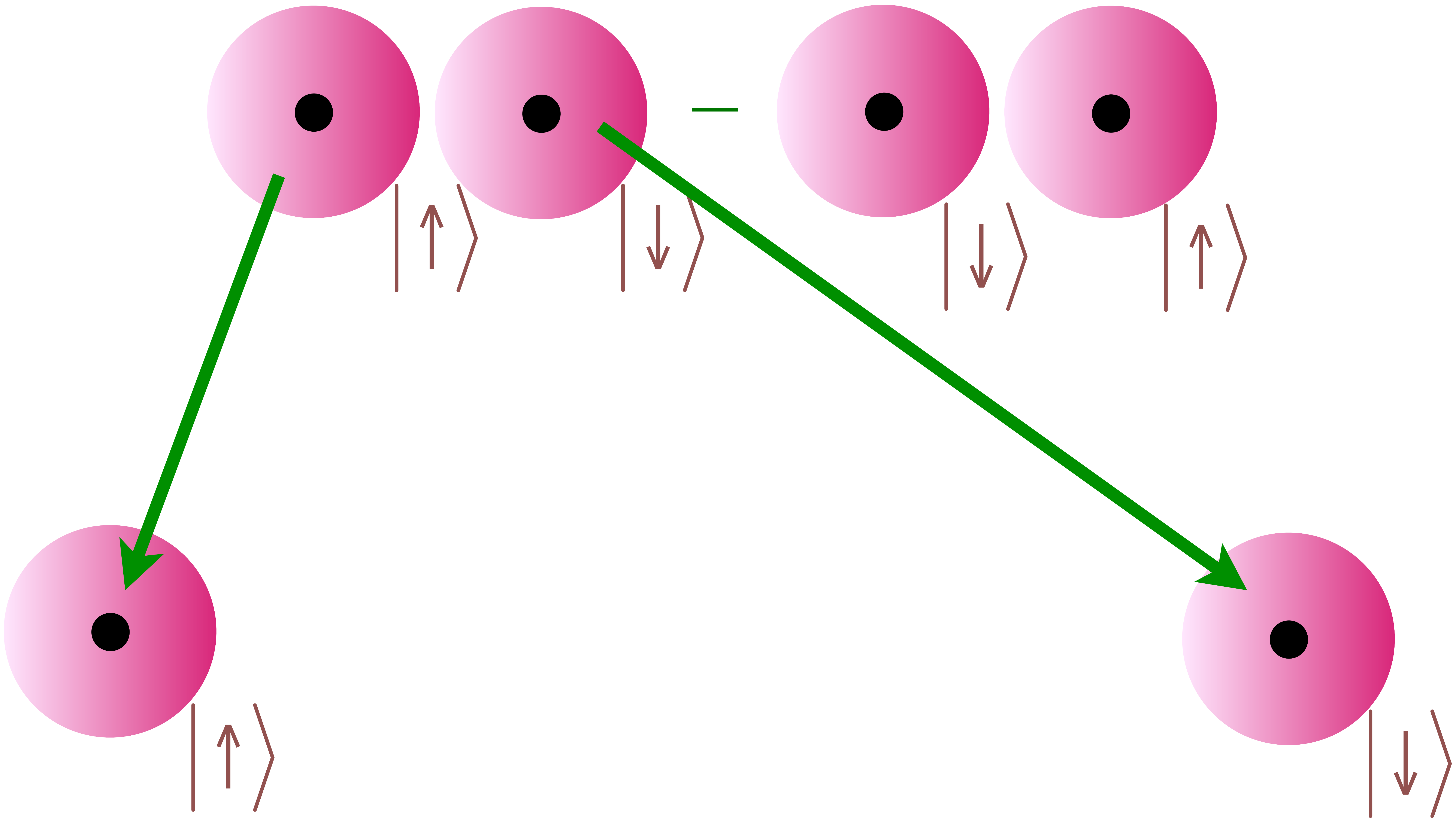

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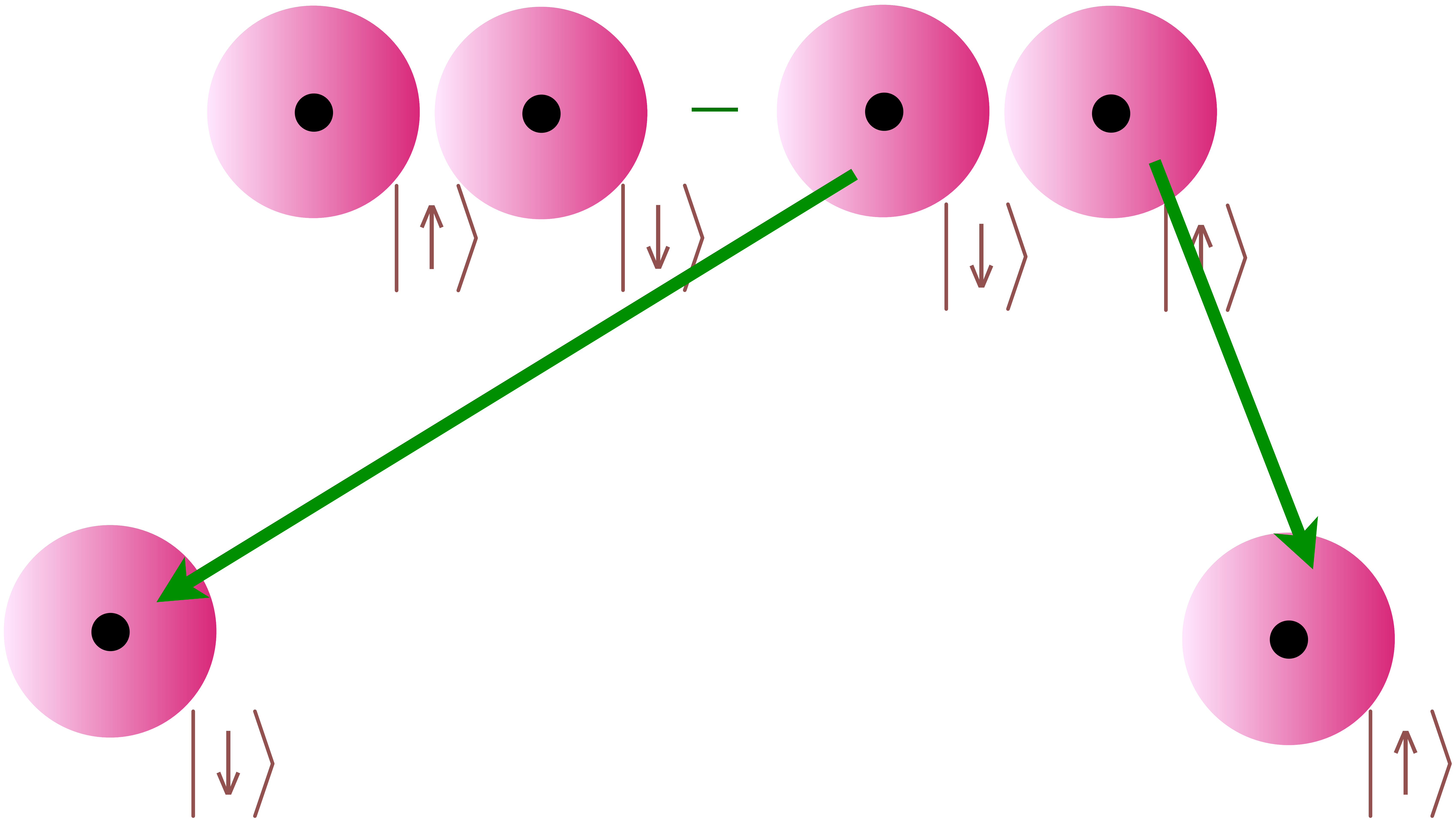

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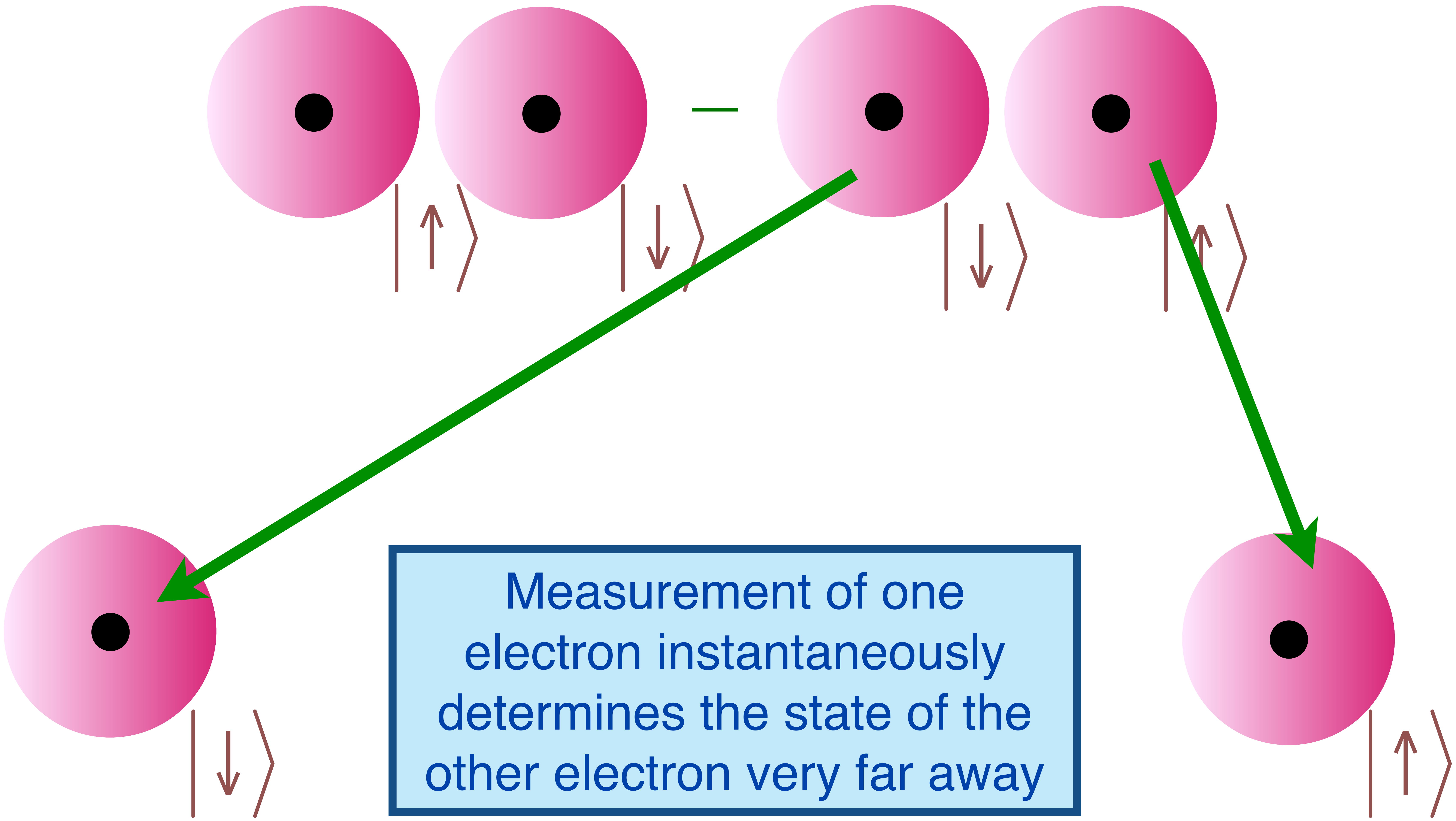

Measurement of one electron instantaneously determines the state of the other electron very far away

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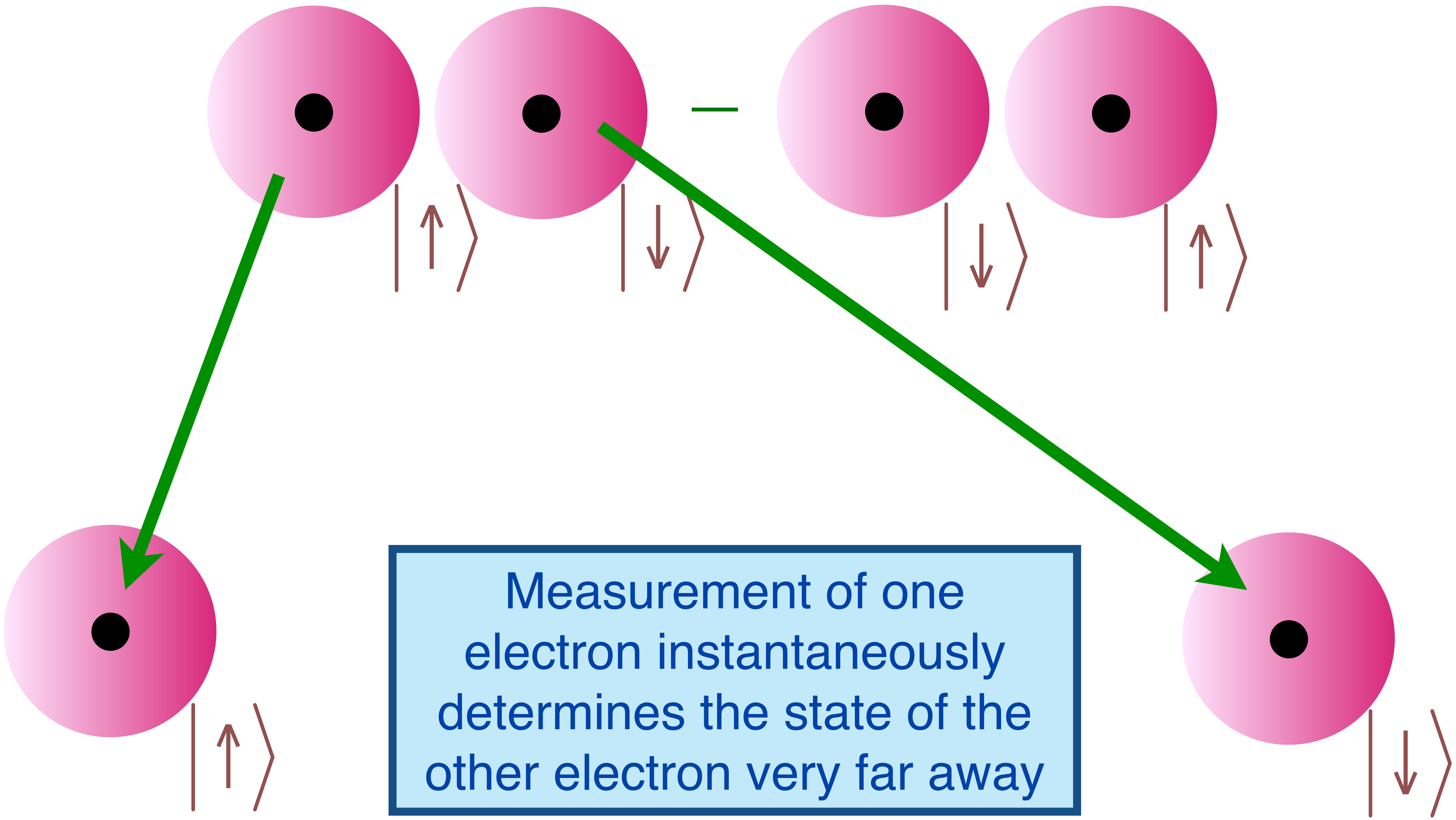

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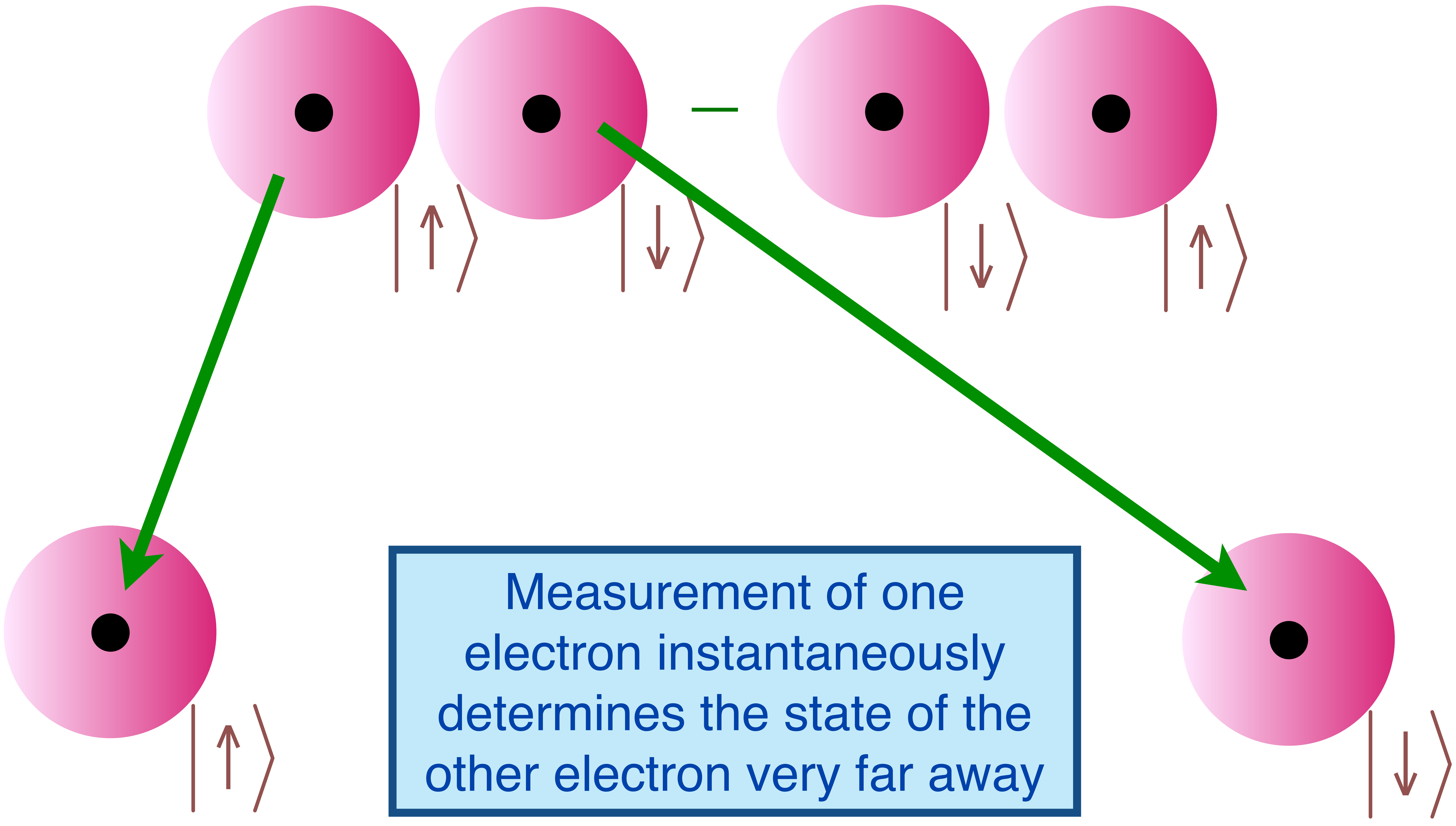

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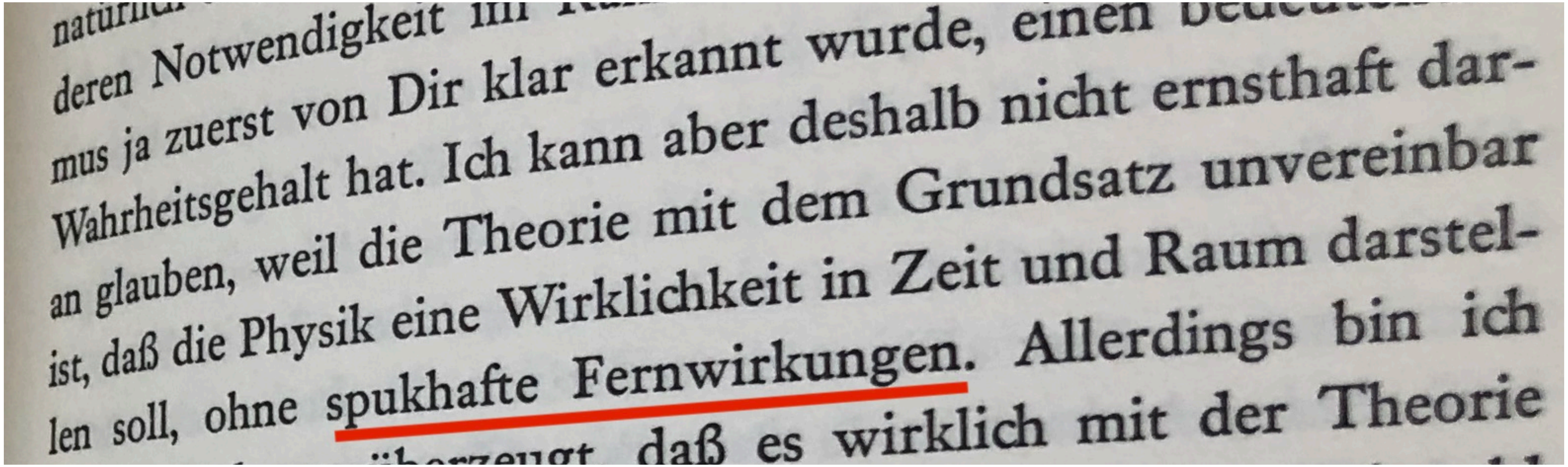
Spooky action at a distance !



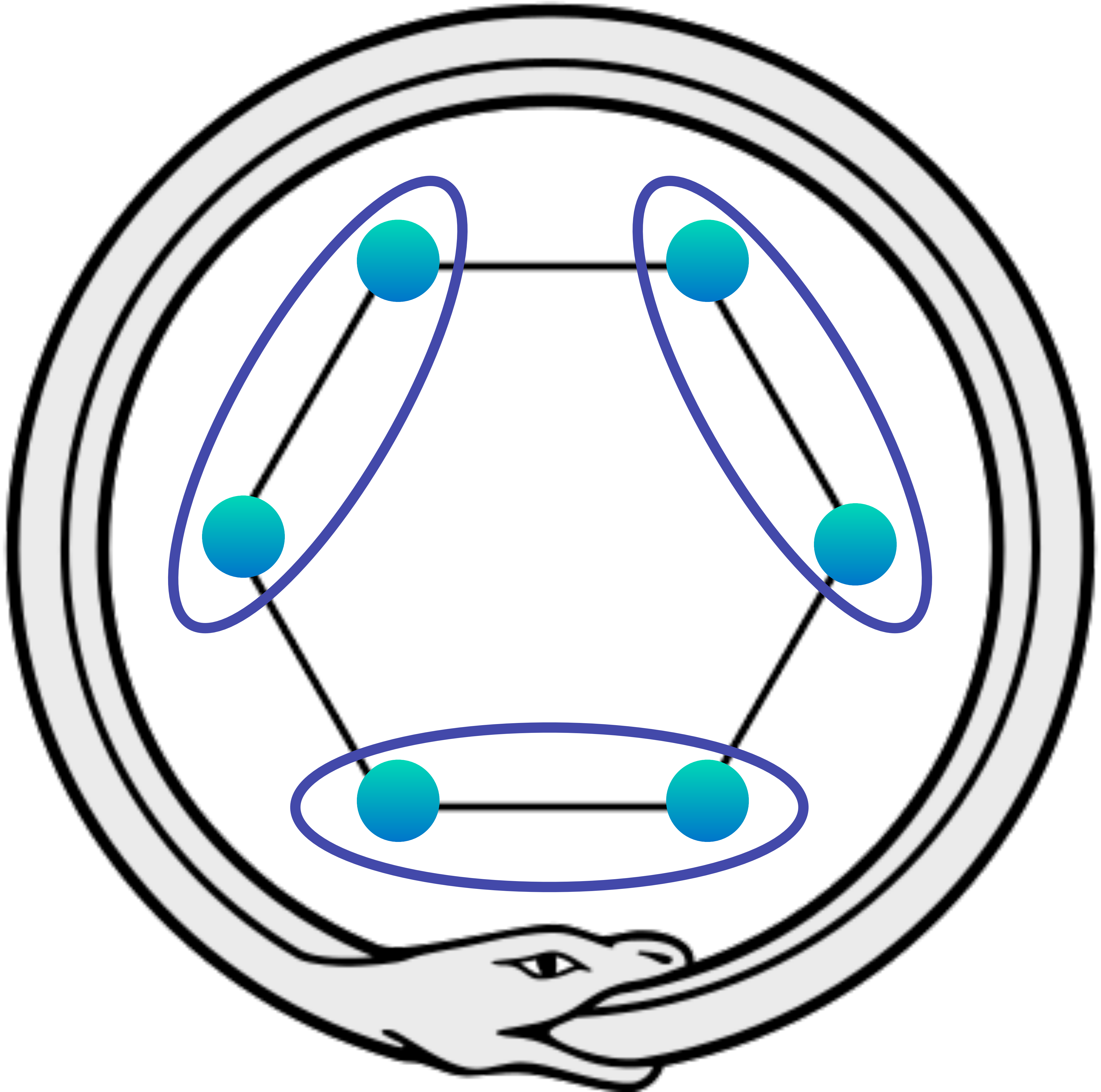
deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen bedeut
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft daran glauben, weil die Theorie mit dem Grundsatz unvereinbar ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstellen soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt, daß es wirklich mit der Theorie

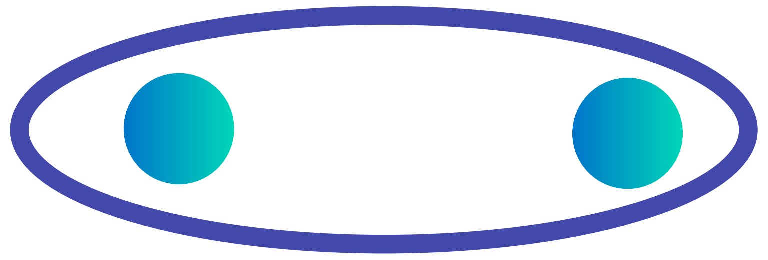
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail[*]





$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

[*] Wikipedia

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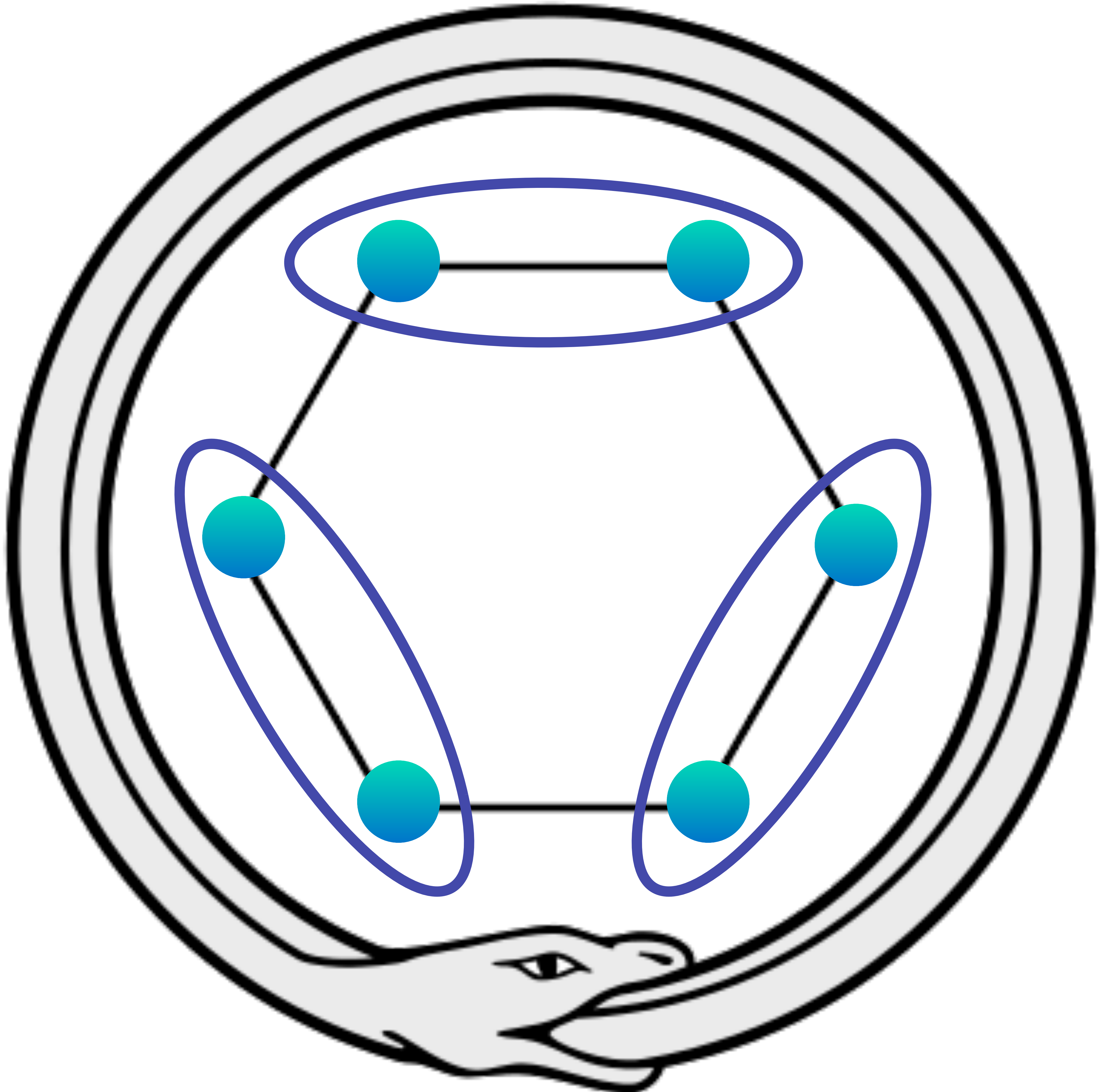

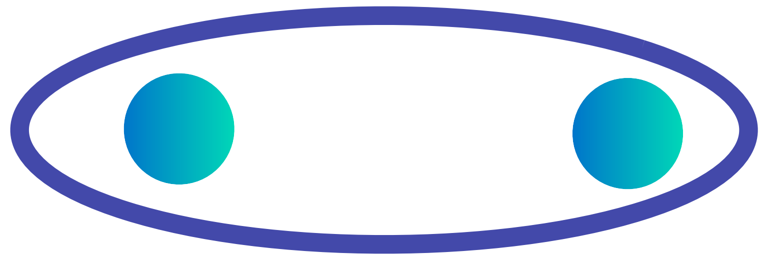

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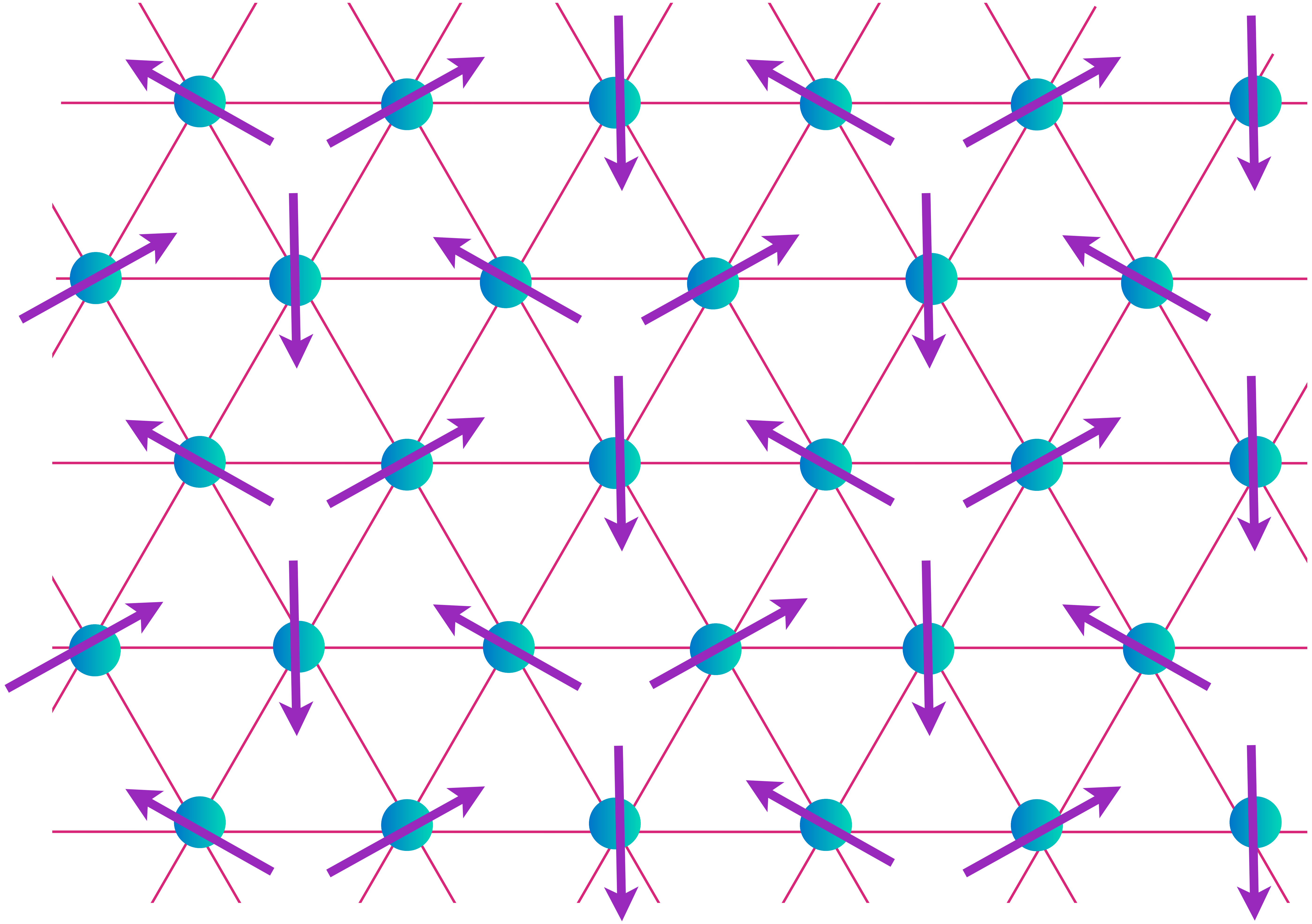
Benzene

*Wikipedia

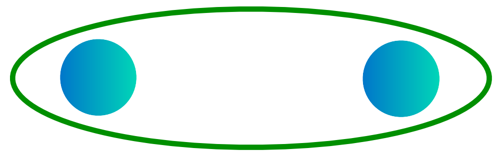
Quantum spin liquids and quantum error correction

Triangular lattice antiferromagnet



Nearest-neighbor model has non-collinear Neel order

Spin liquid: resonating valence bonds



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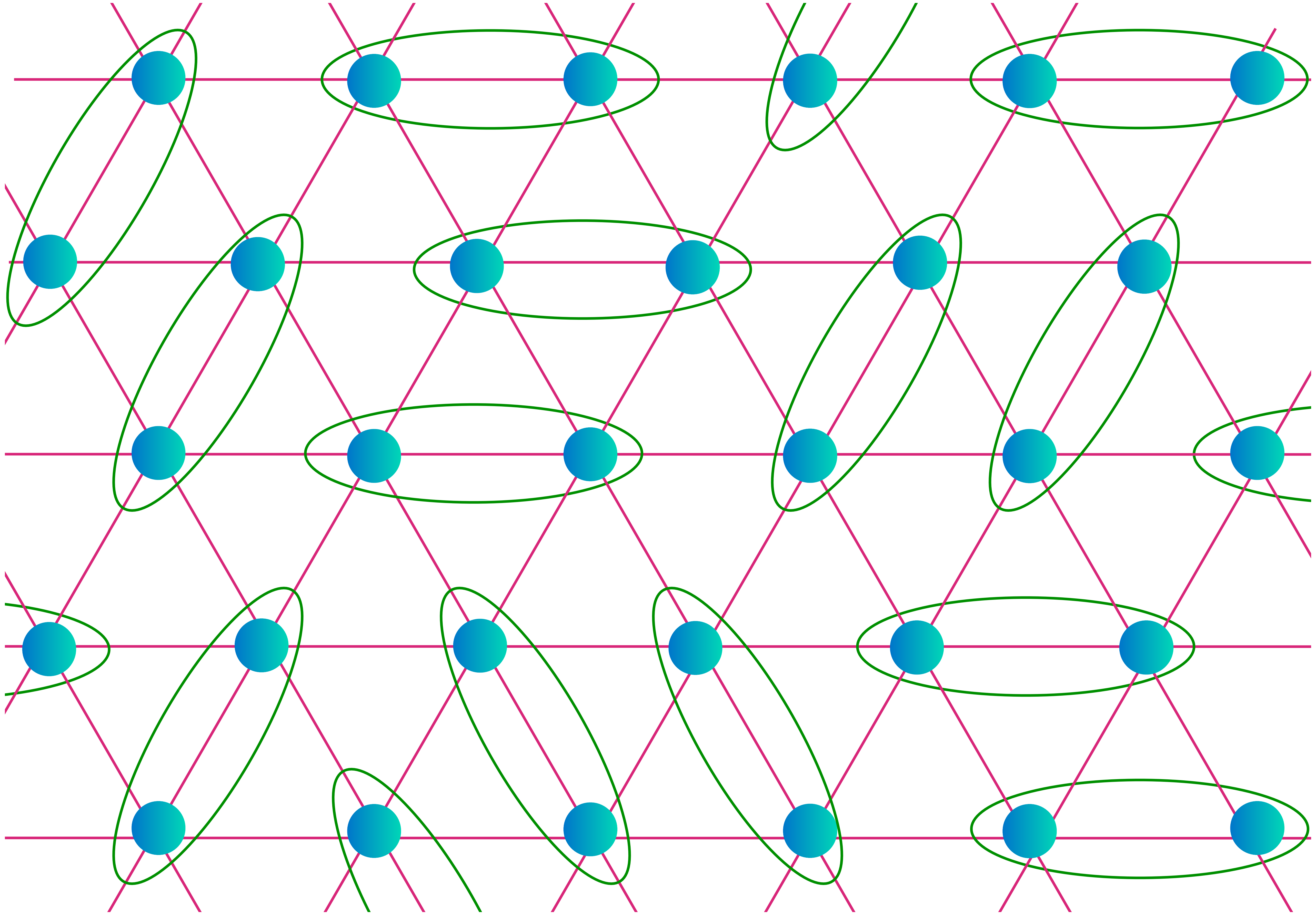
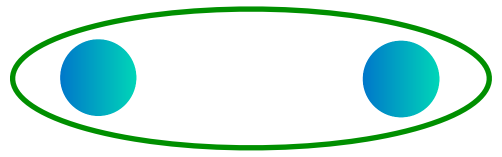


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Spin liquid: resonating valence bonds



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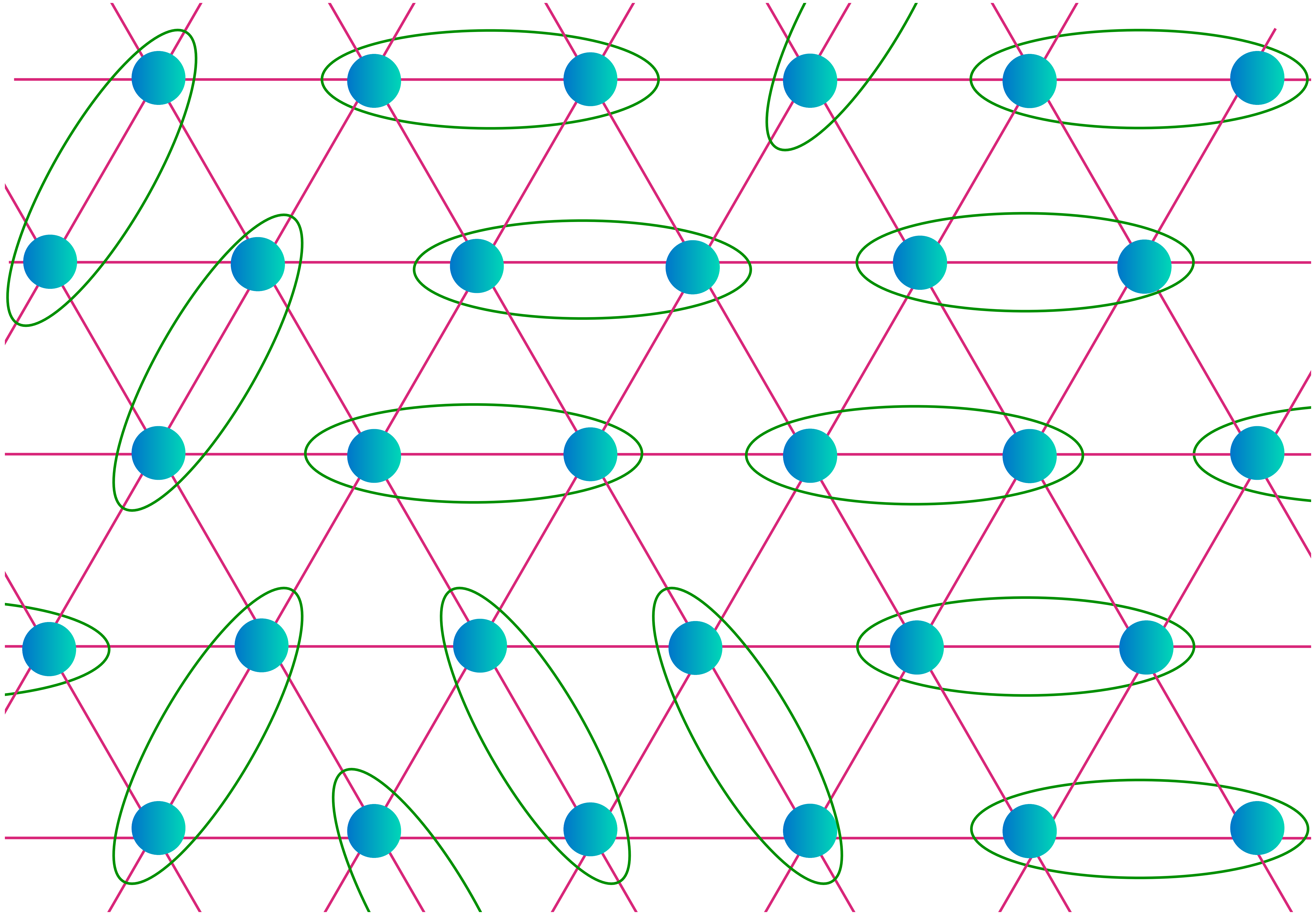

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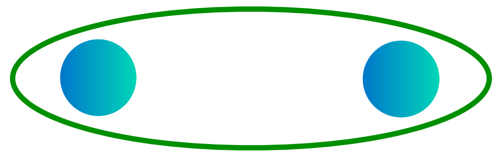

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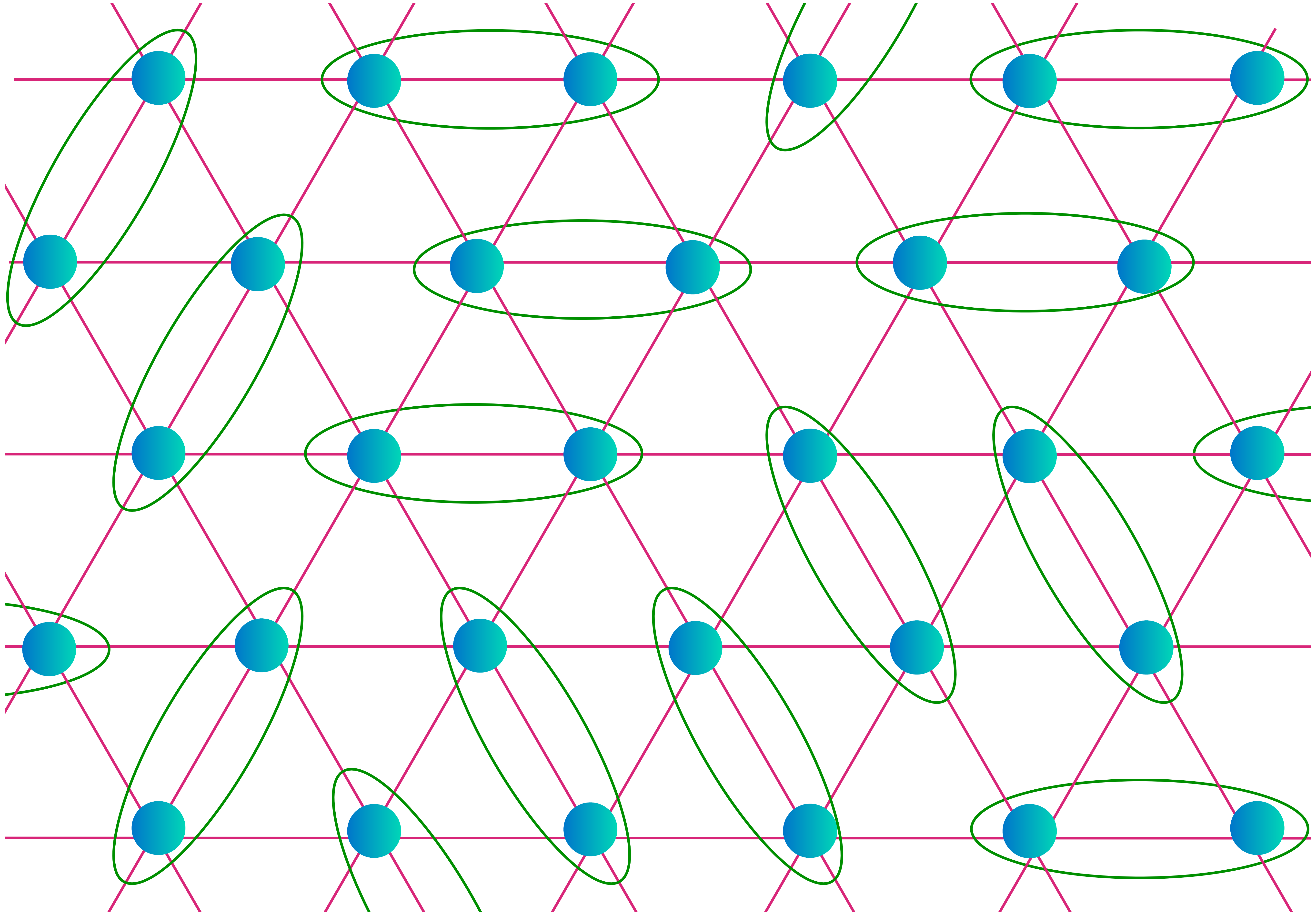

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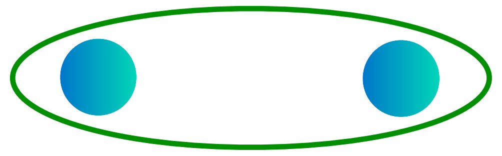

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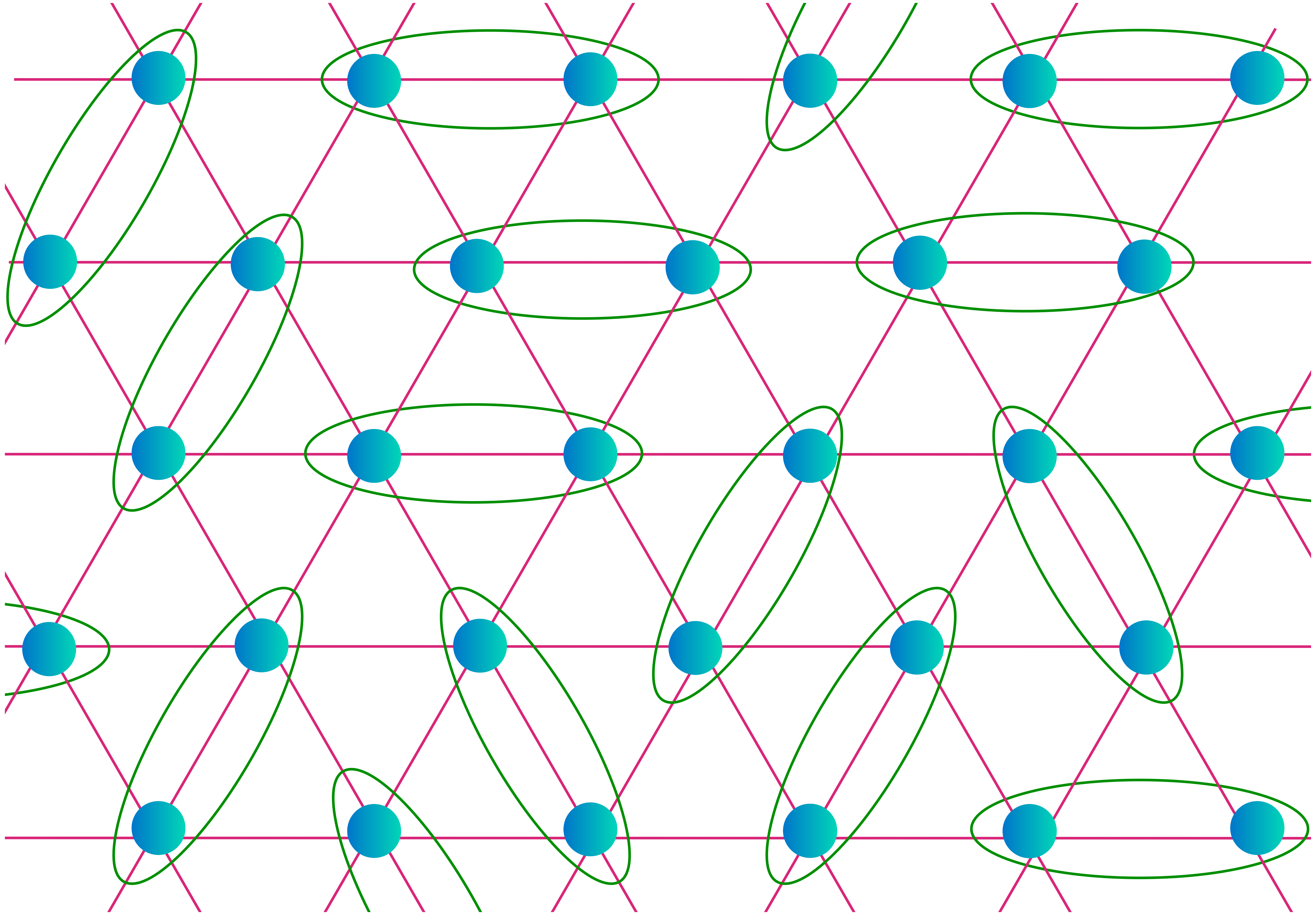

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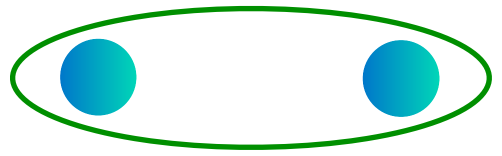

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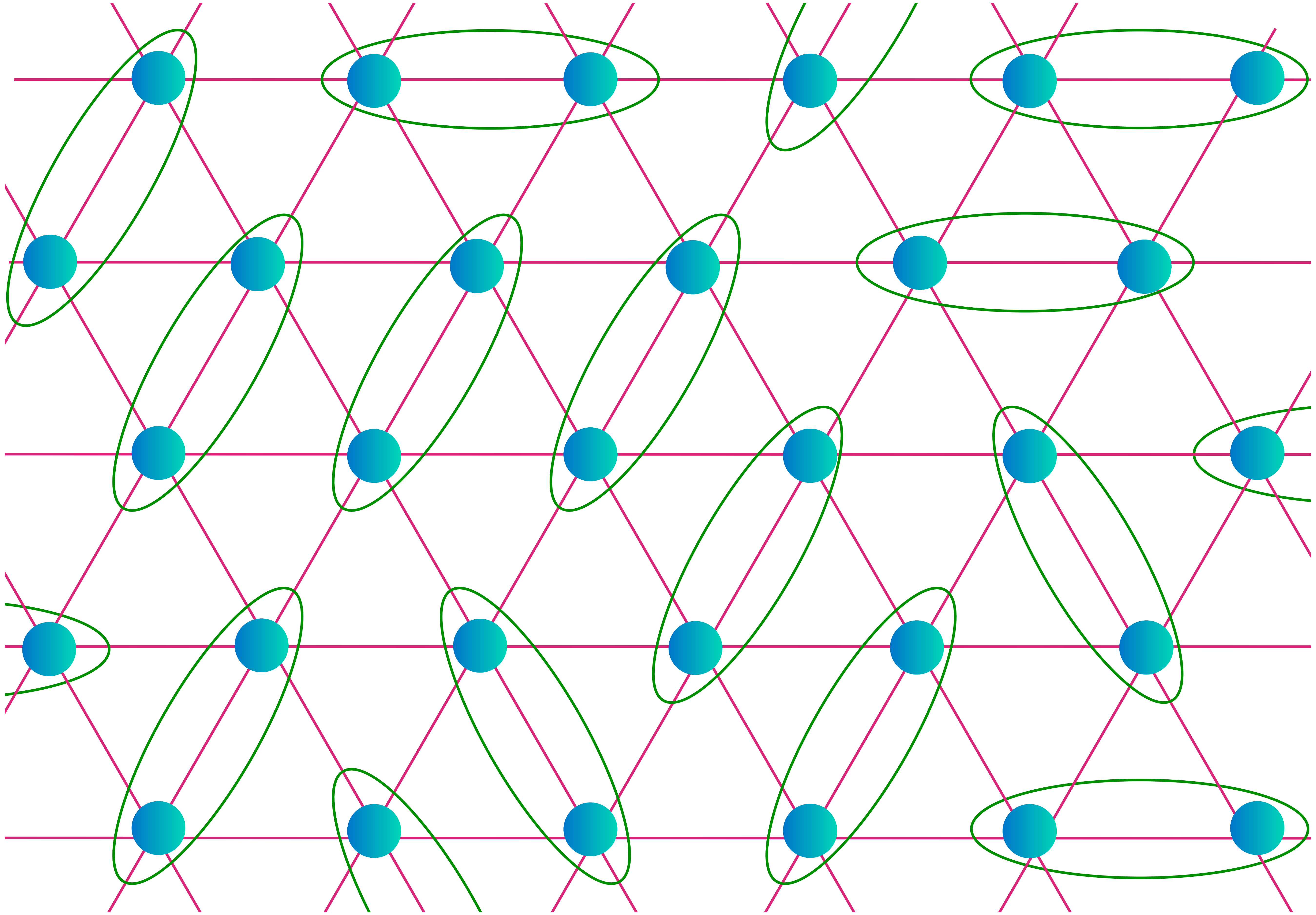

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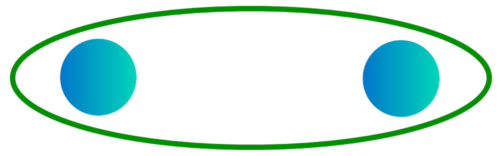

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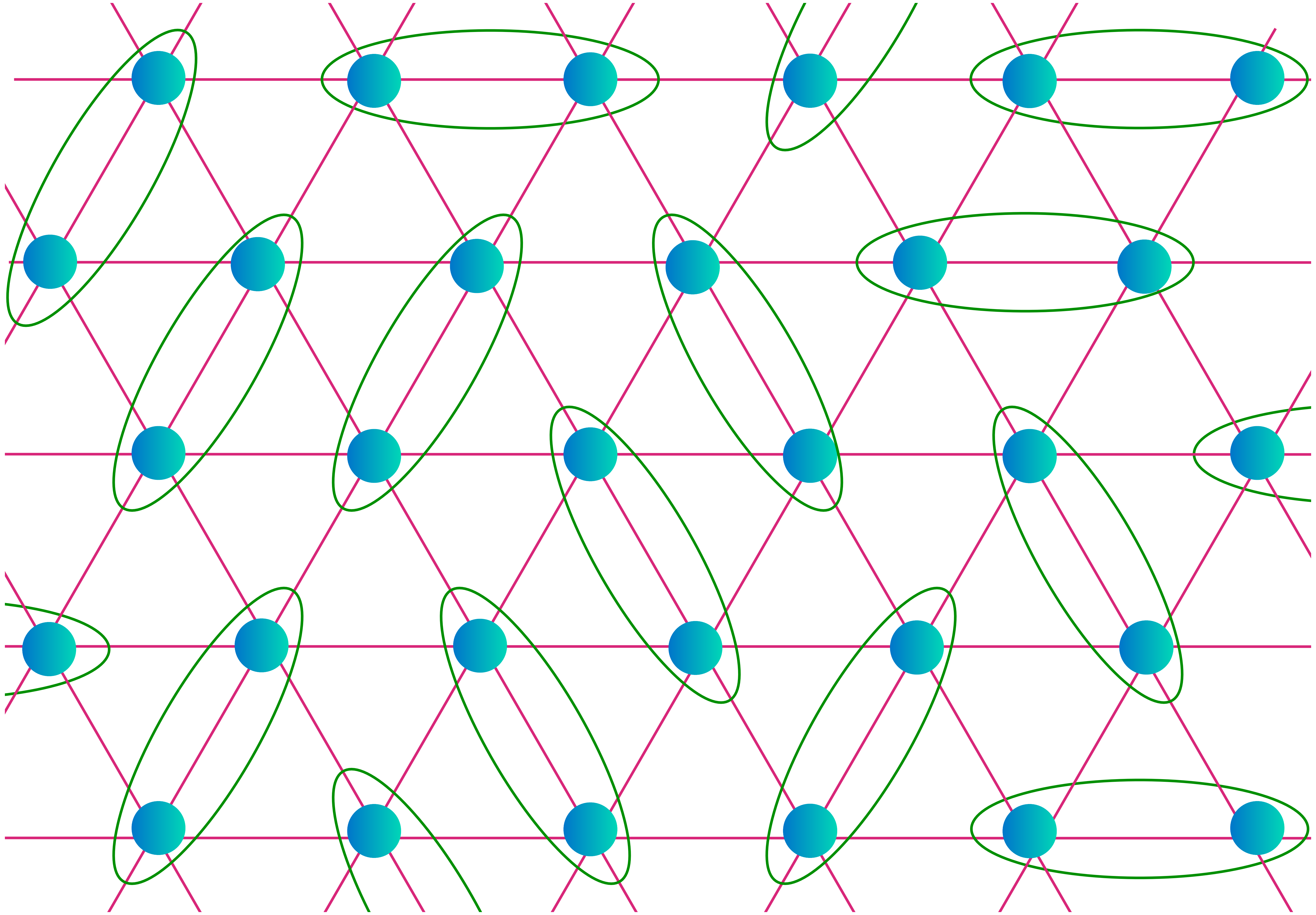

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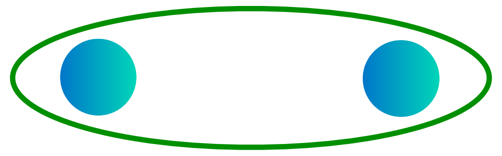

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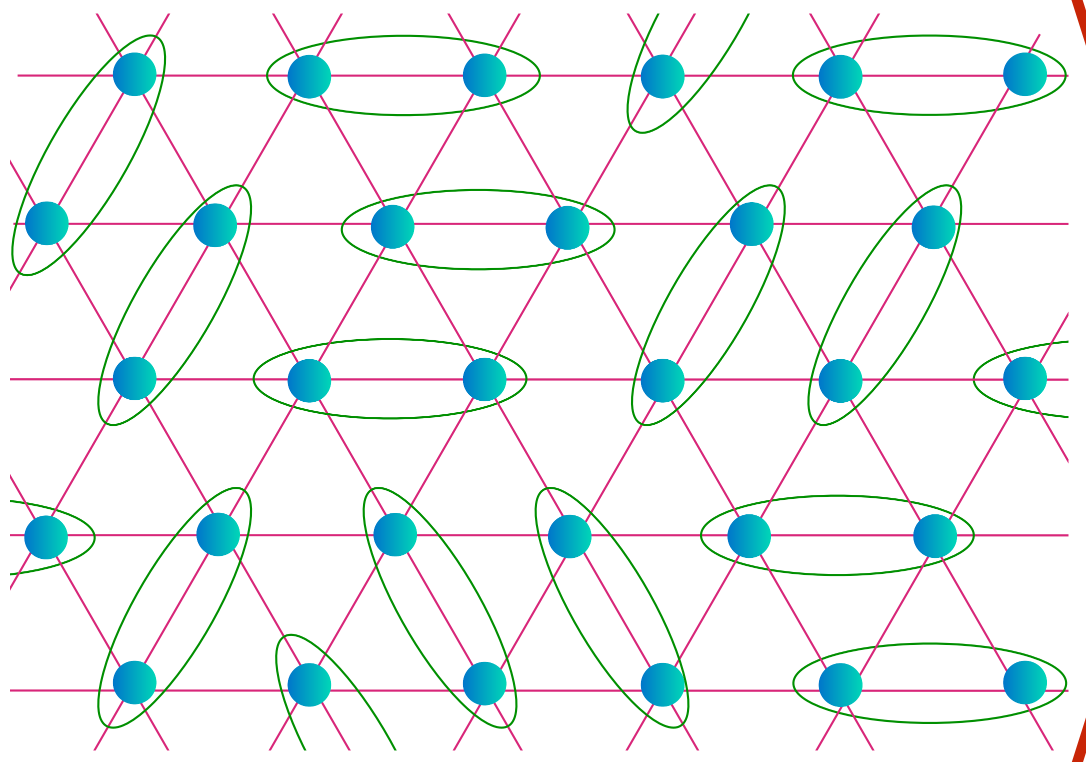
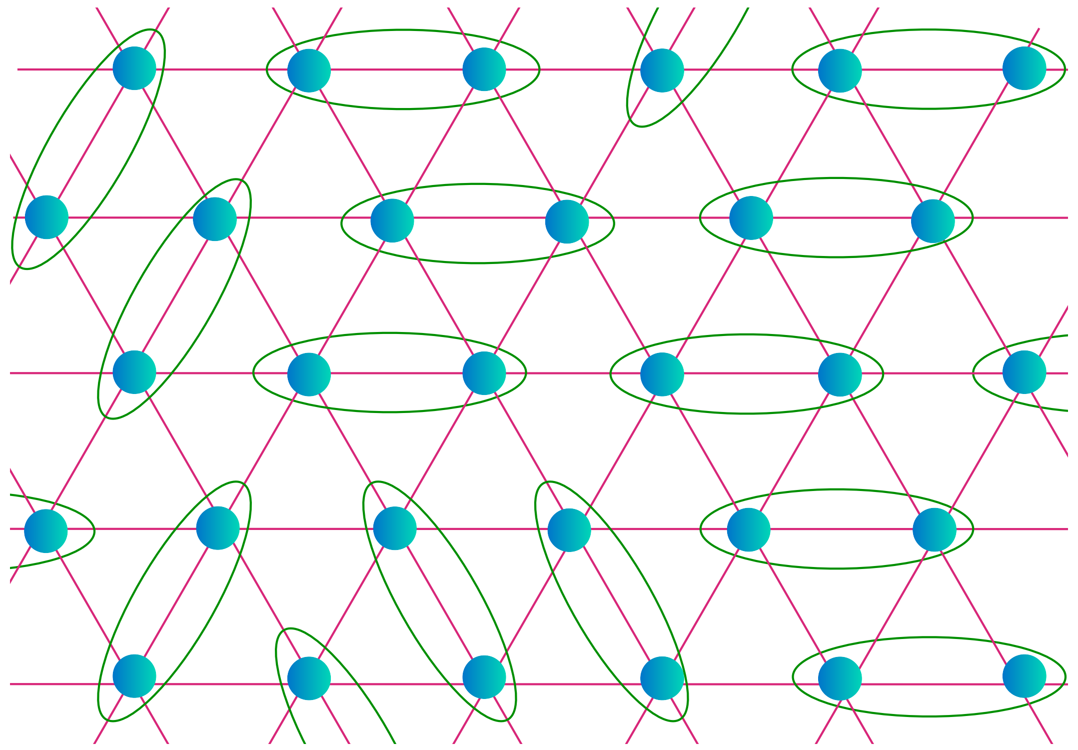
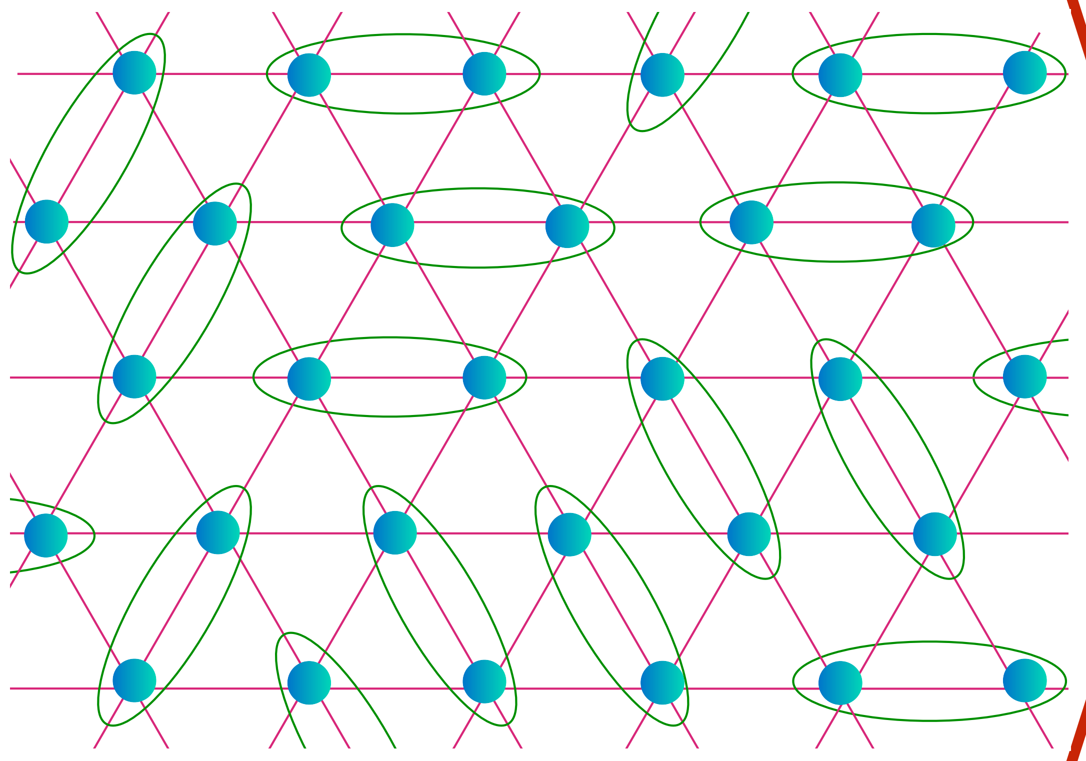
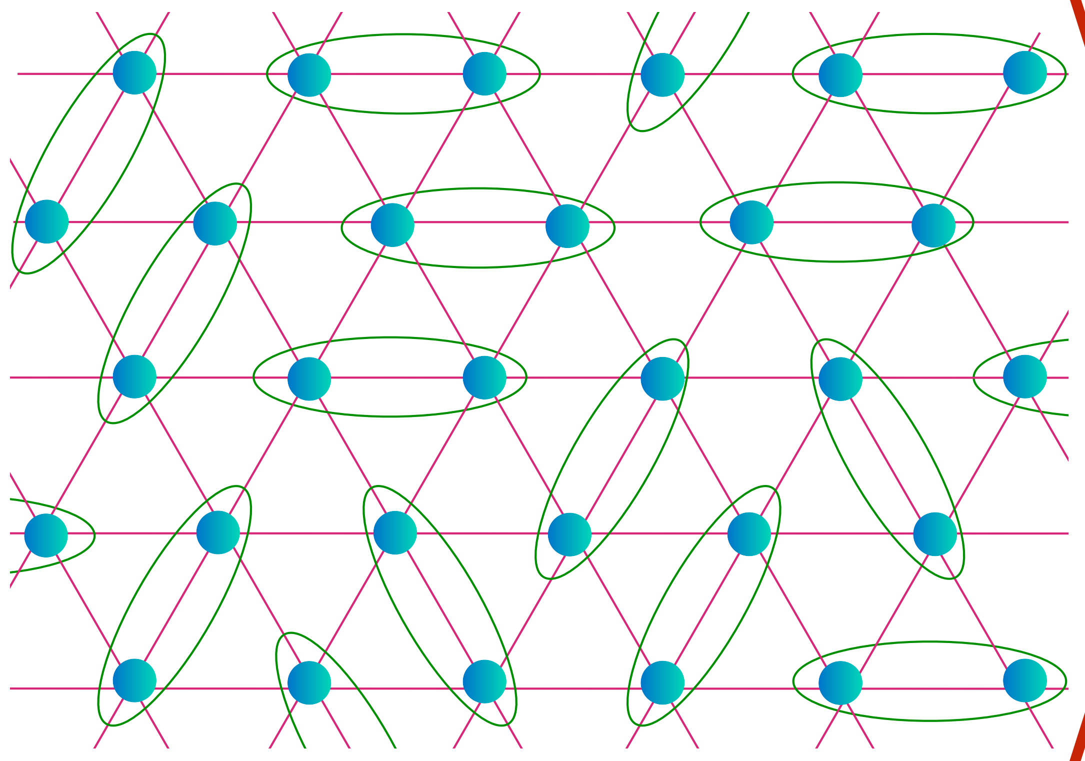
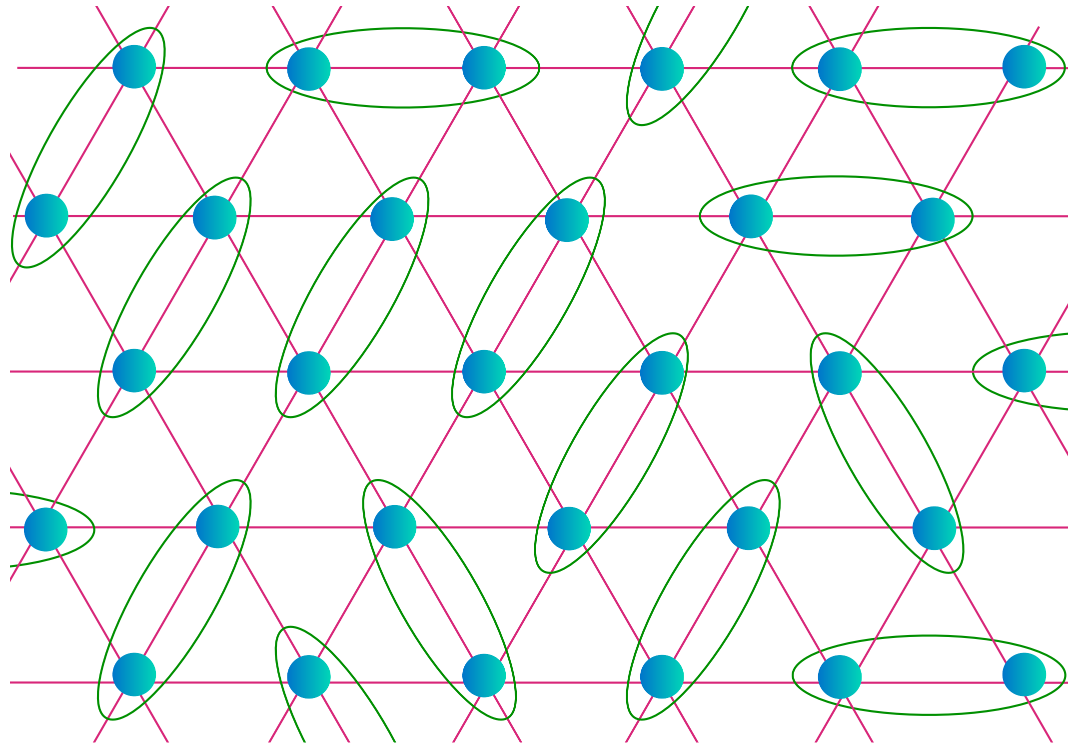
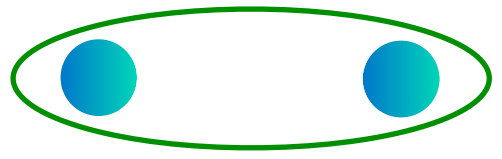
$|\Psi\rangle = C_1 \Big|$  $\Big\rangle + C_2 \Big|$  $\Big\rangle$

$+\, C_3 \Big|$  $\Big\rangle + C_4 \Big|$  $\Big\rangle + C_5 \Big|$  $\Big\rangle + \ldots$

Spin liquid: resonating valence bonds



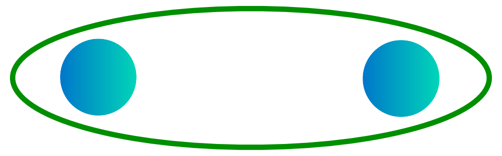
$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Key feature: fractionalization.

Excitations are particle-like,
but cannot be created
by local operators.
The excitations are classified
under distinct
anyon sectors.

Spin liquid: resonating valence bonds

Fractionalized excitations: a "spinon" with spin S=1/2



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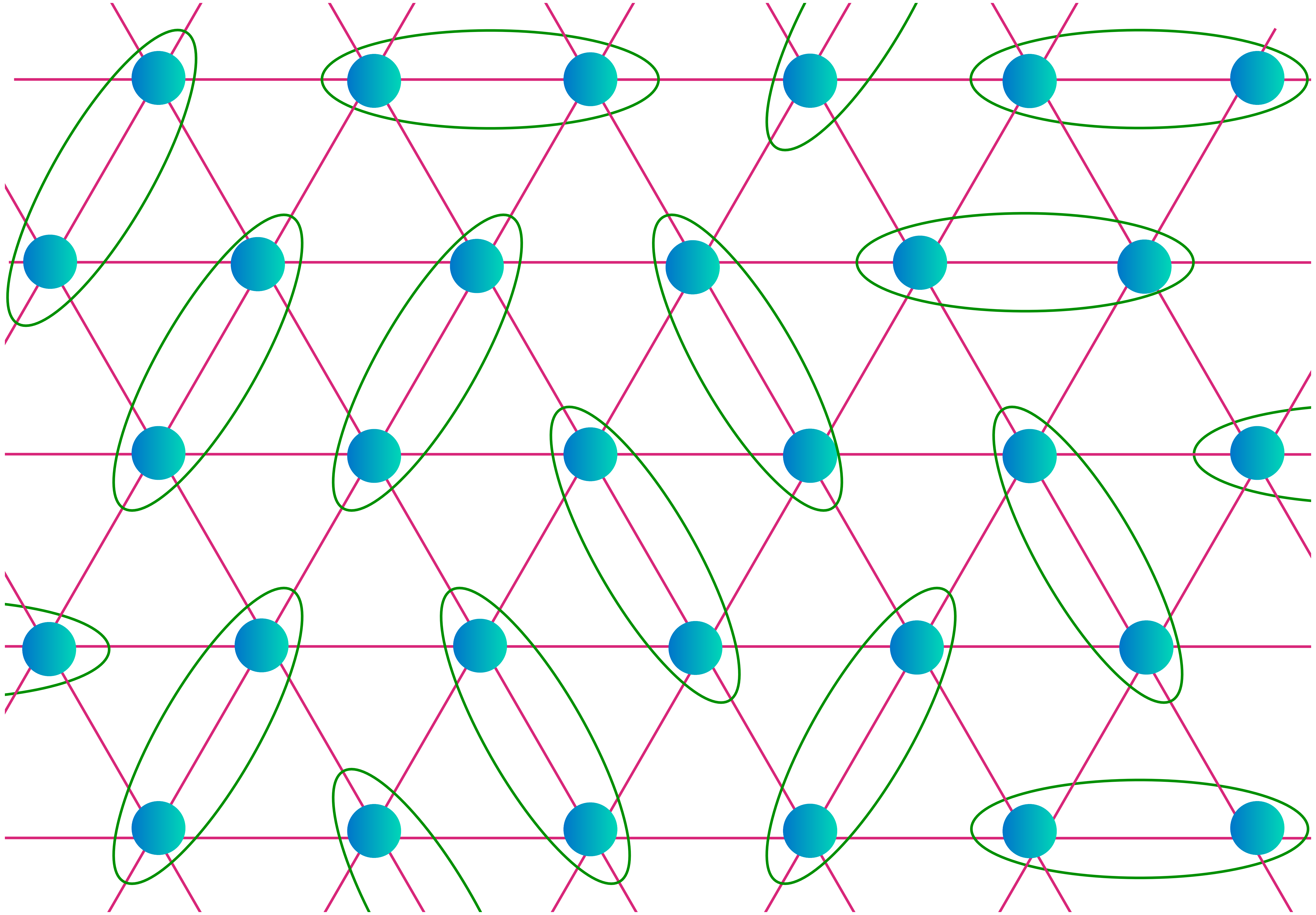
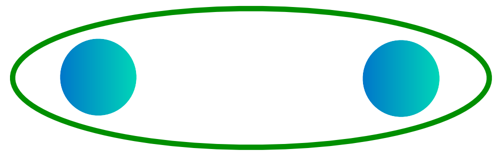


Spin liquid: resonating valence bonds

Fractionalized excitations: a "spinon"



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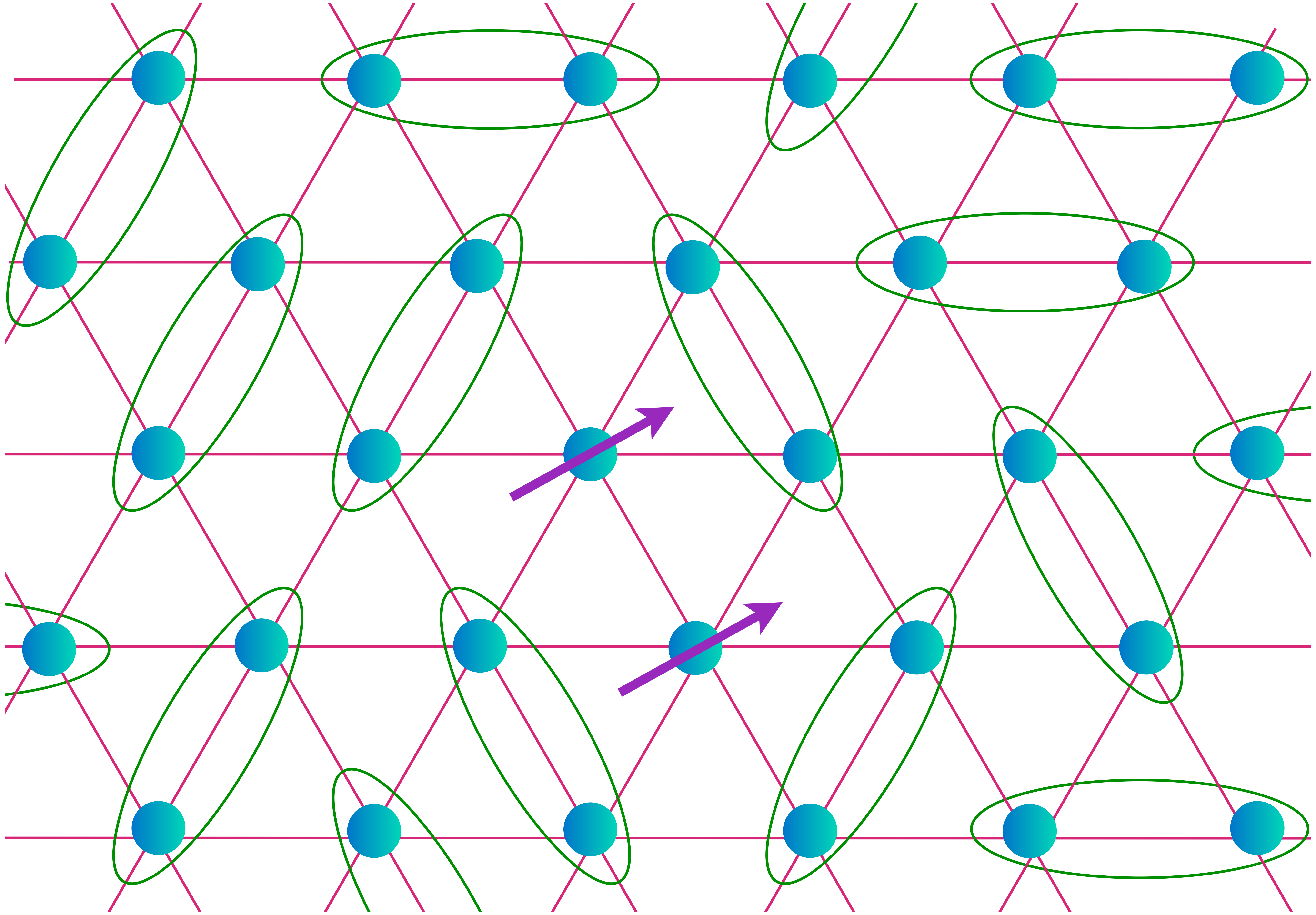
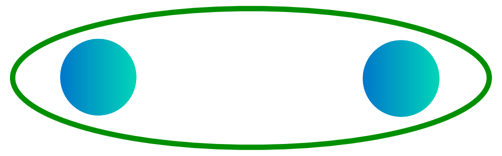


Spin liquid: resonating valence bonds

Fractionalized excitations: a “spinon”



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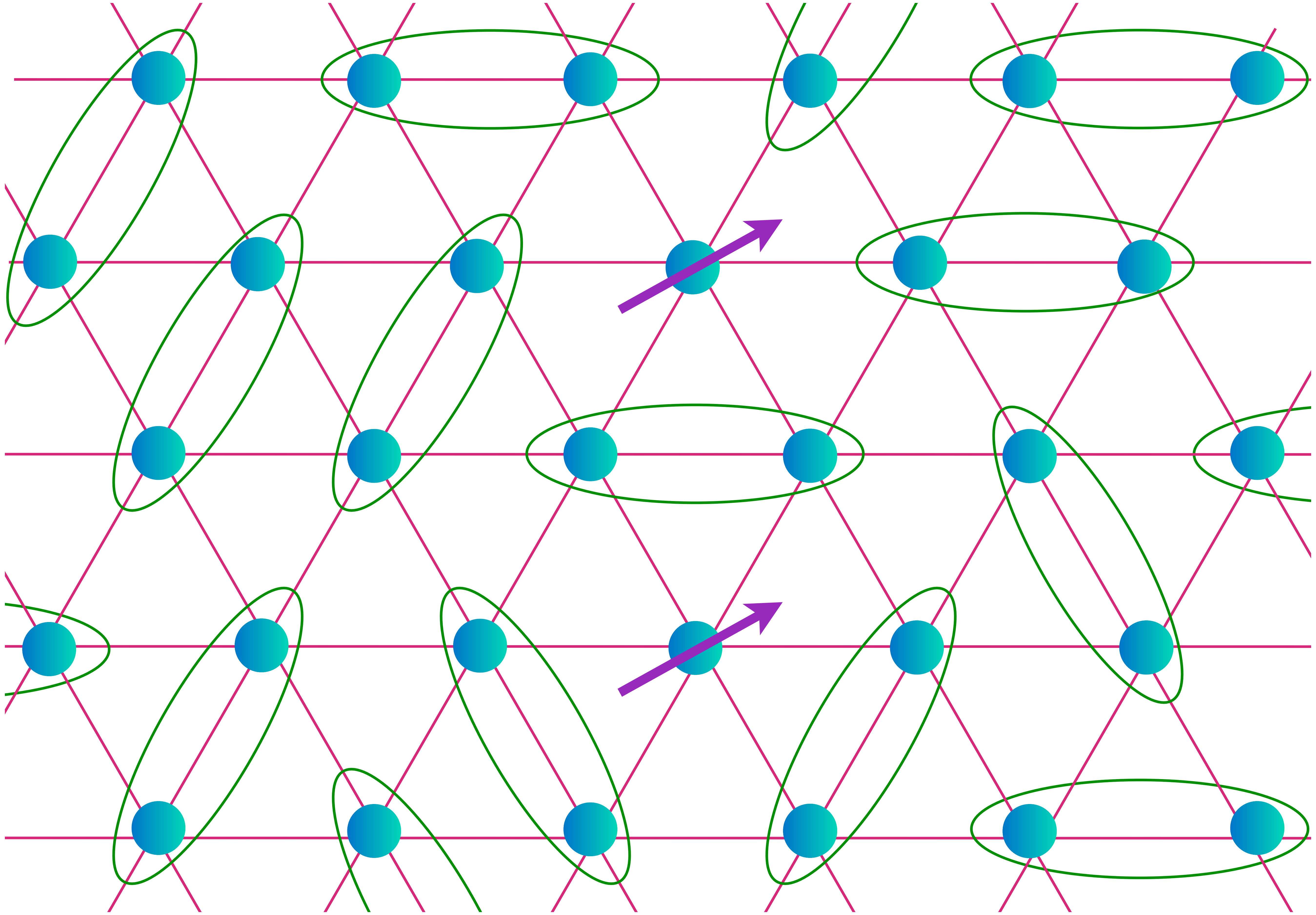
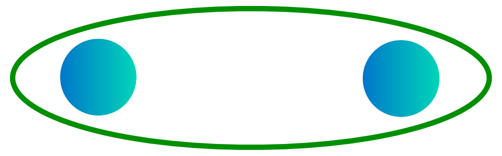
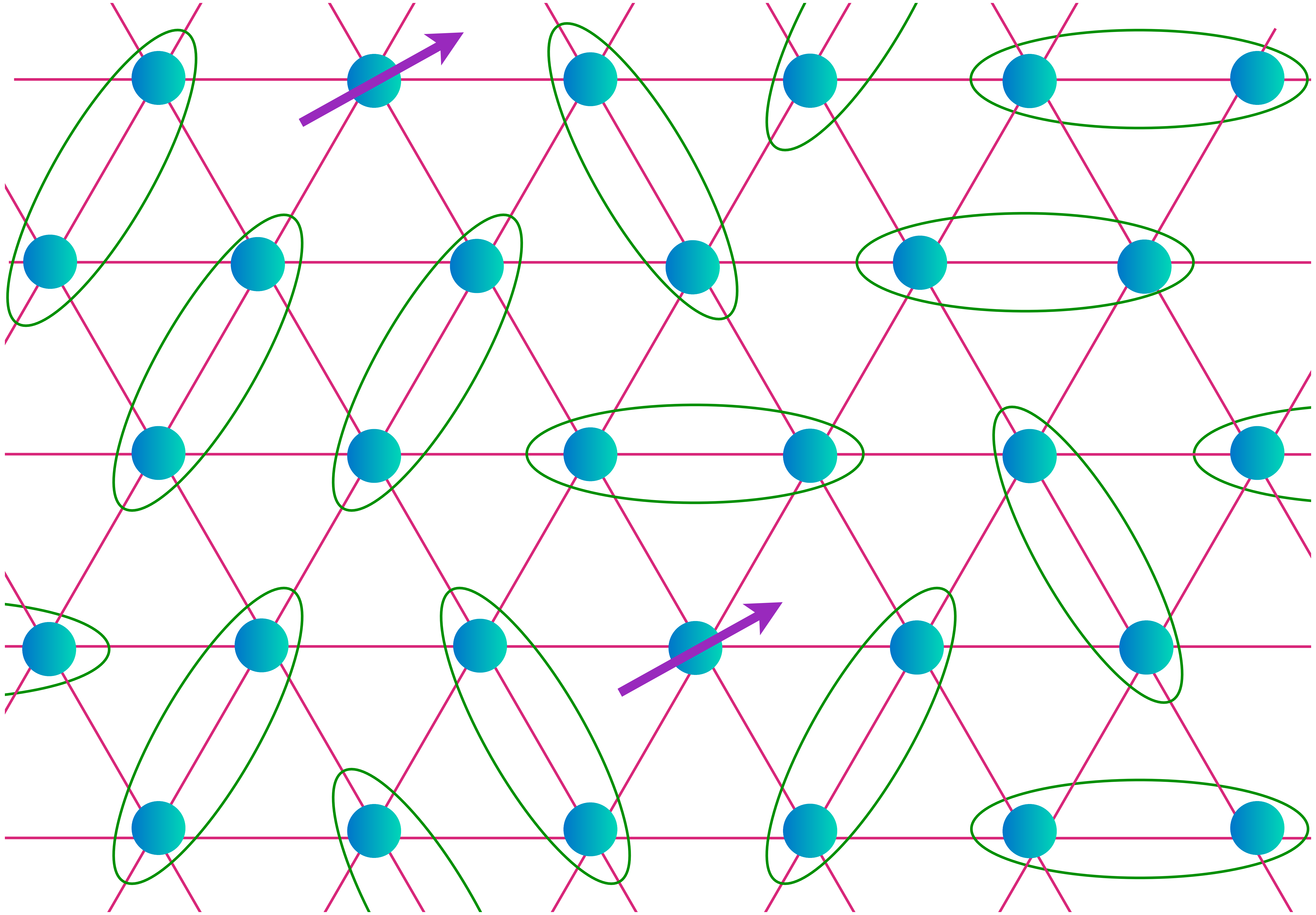


Spin liquid: resonating valence bonds

Fractionalized excitations: a "spinon"



$$= \frac{1}{\sqrt{2}} \left(| \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle \right)$$



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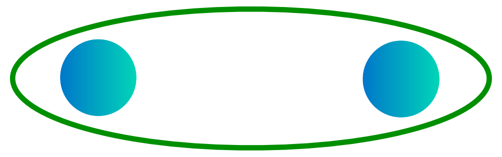

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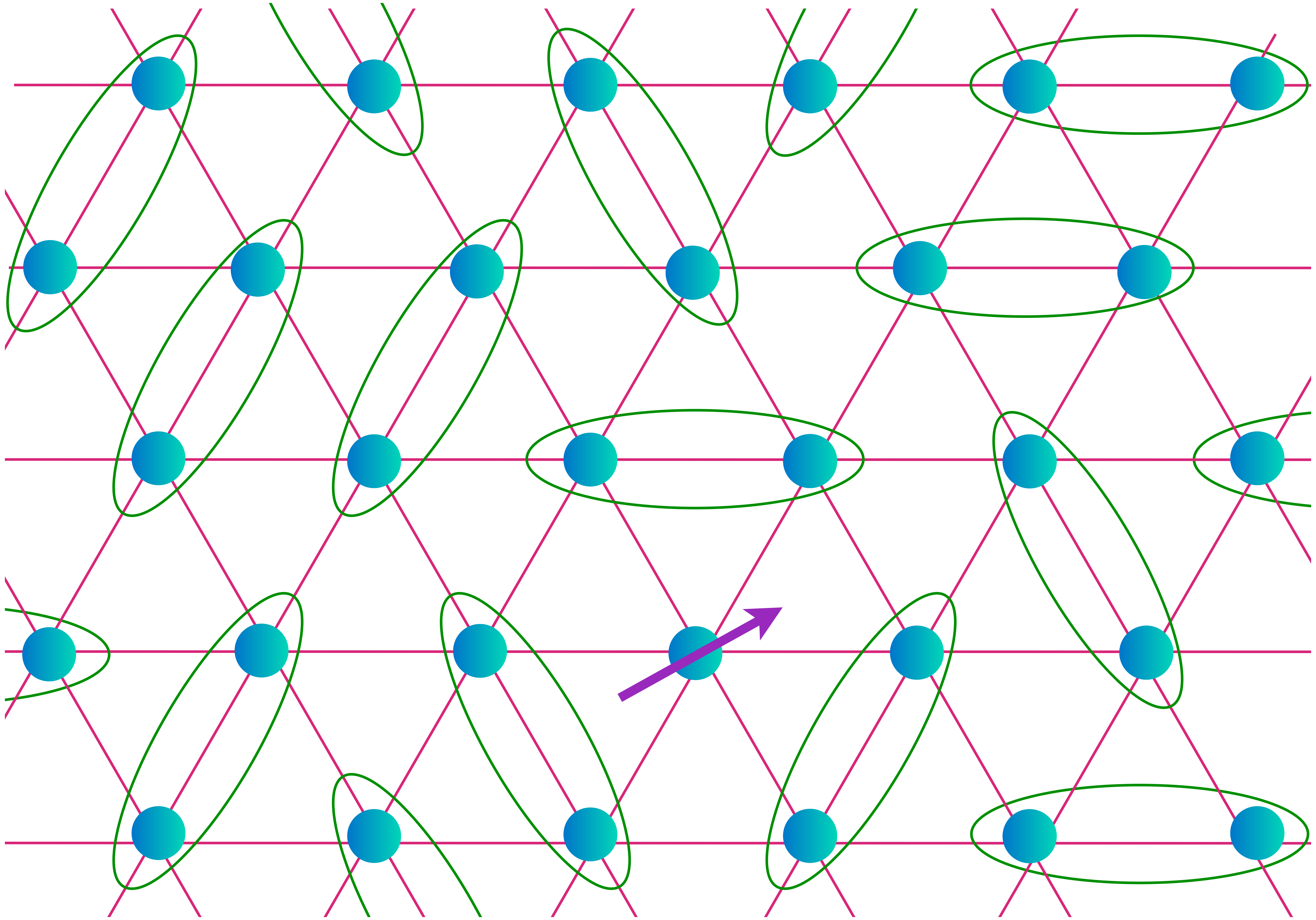

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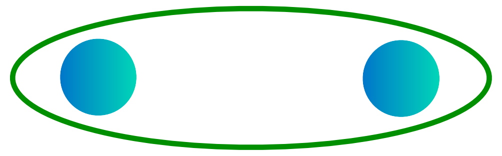

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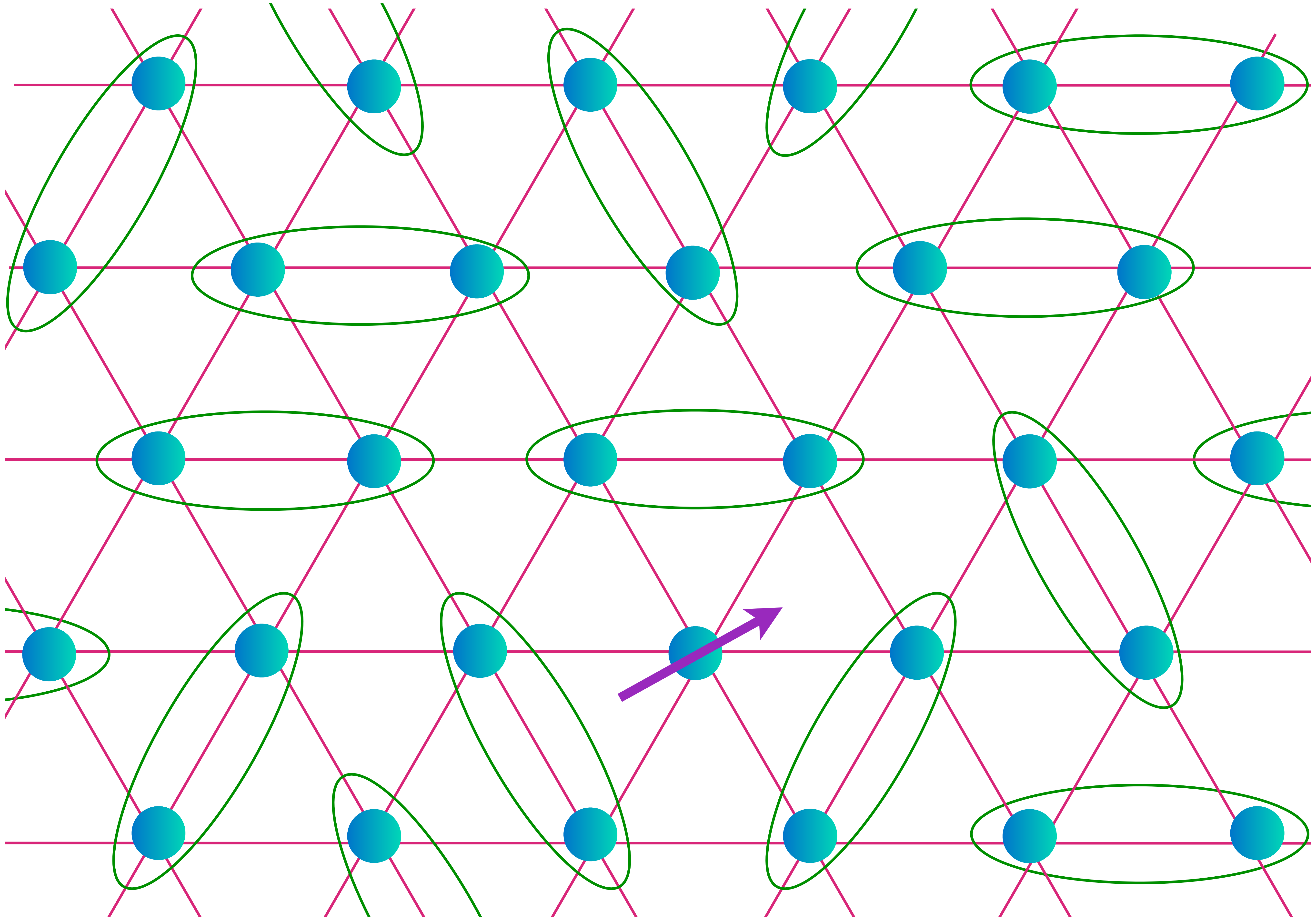

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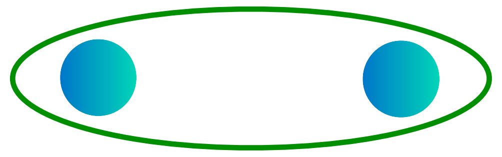

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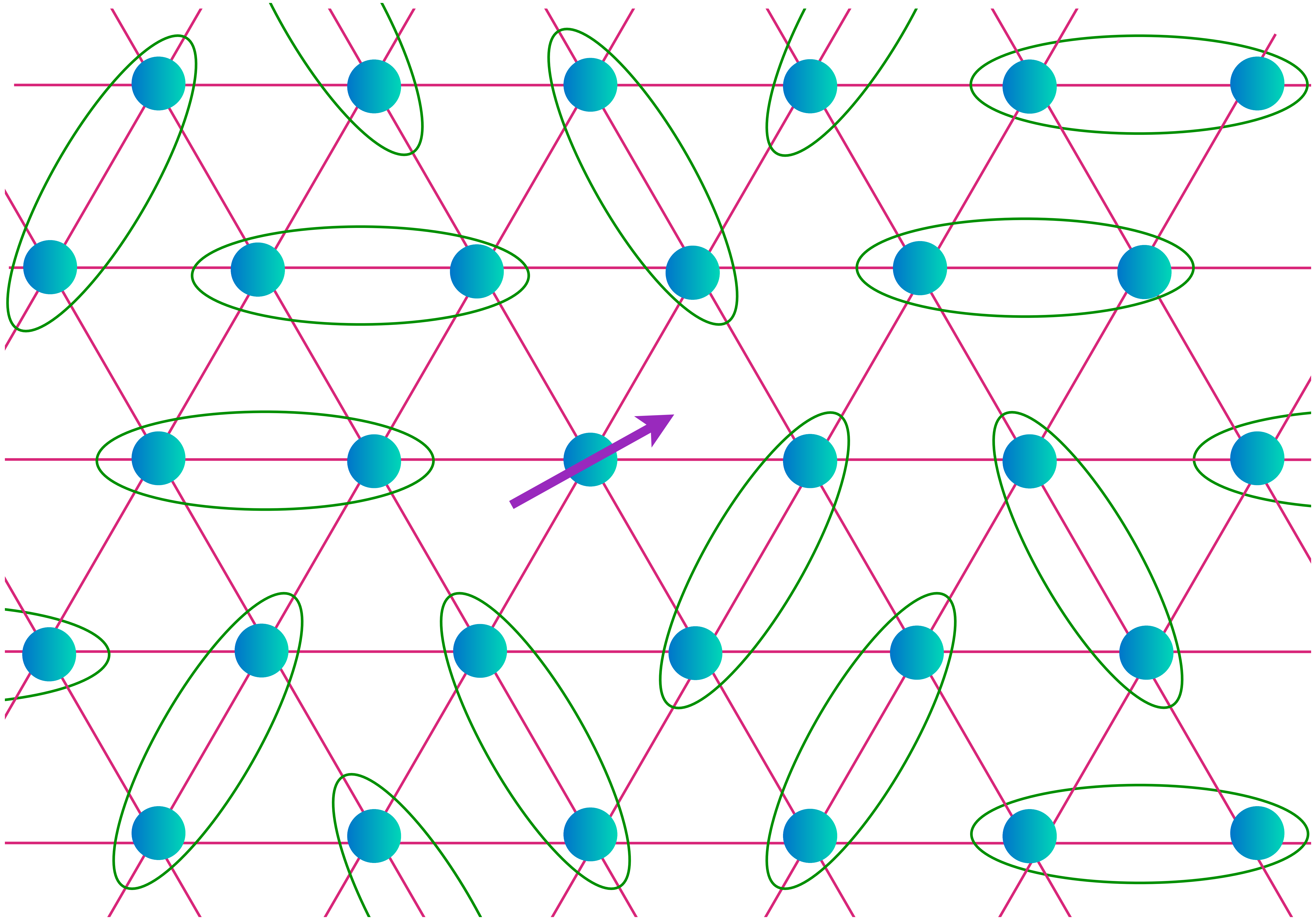

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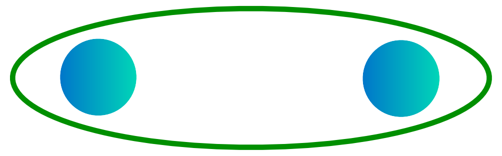

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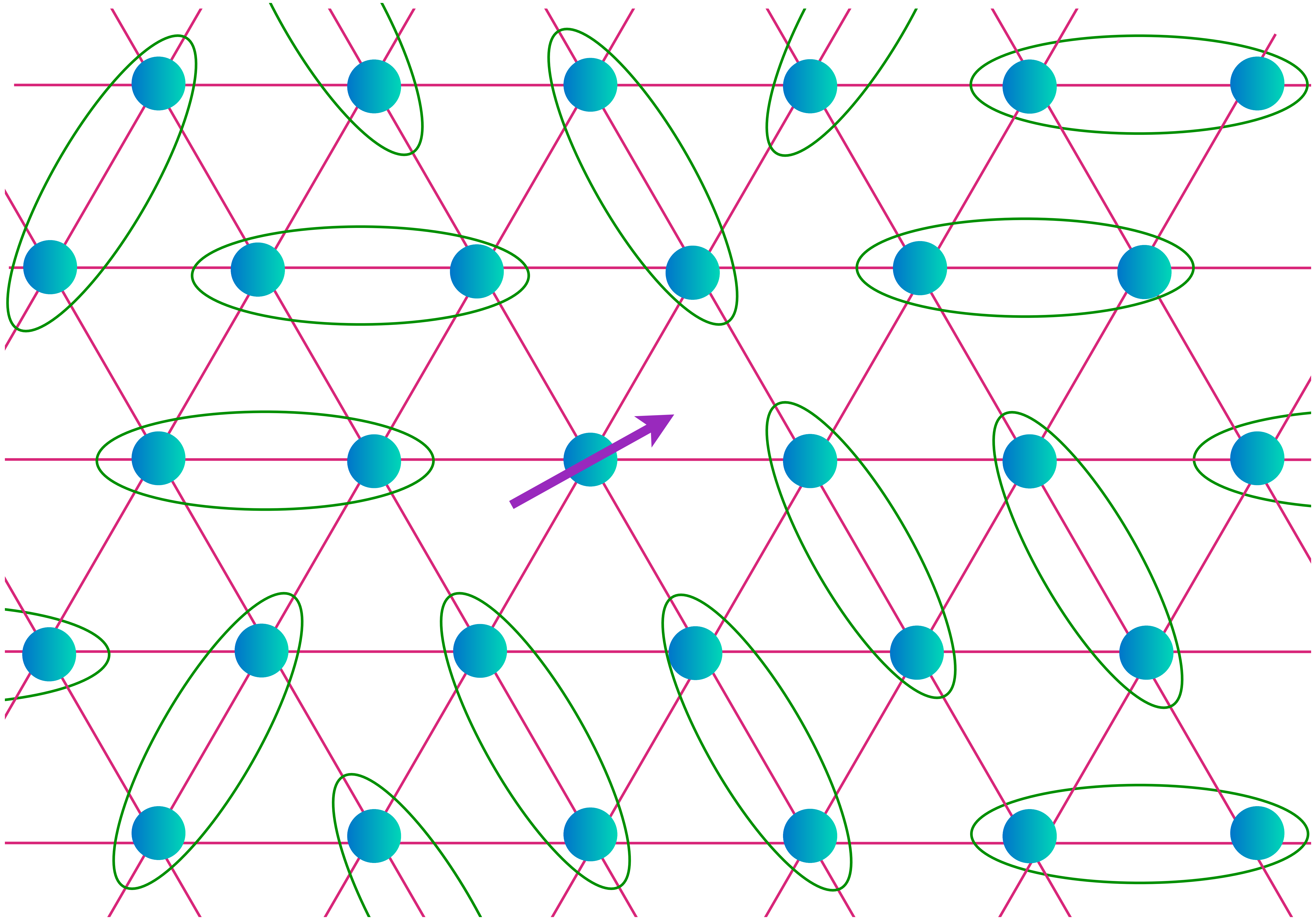

Spin liquid: resonating valence bonds

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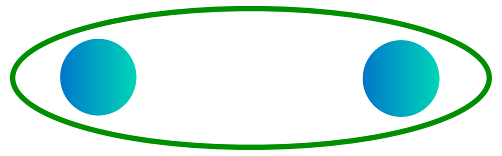

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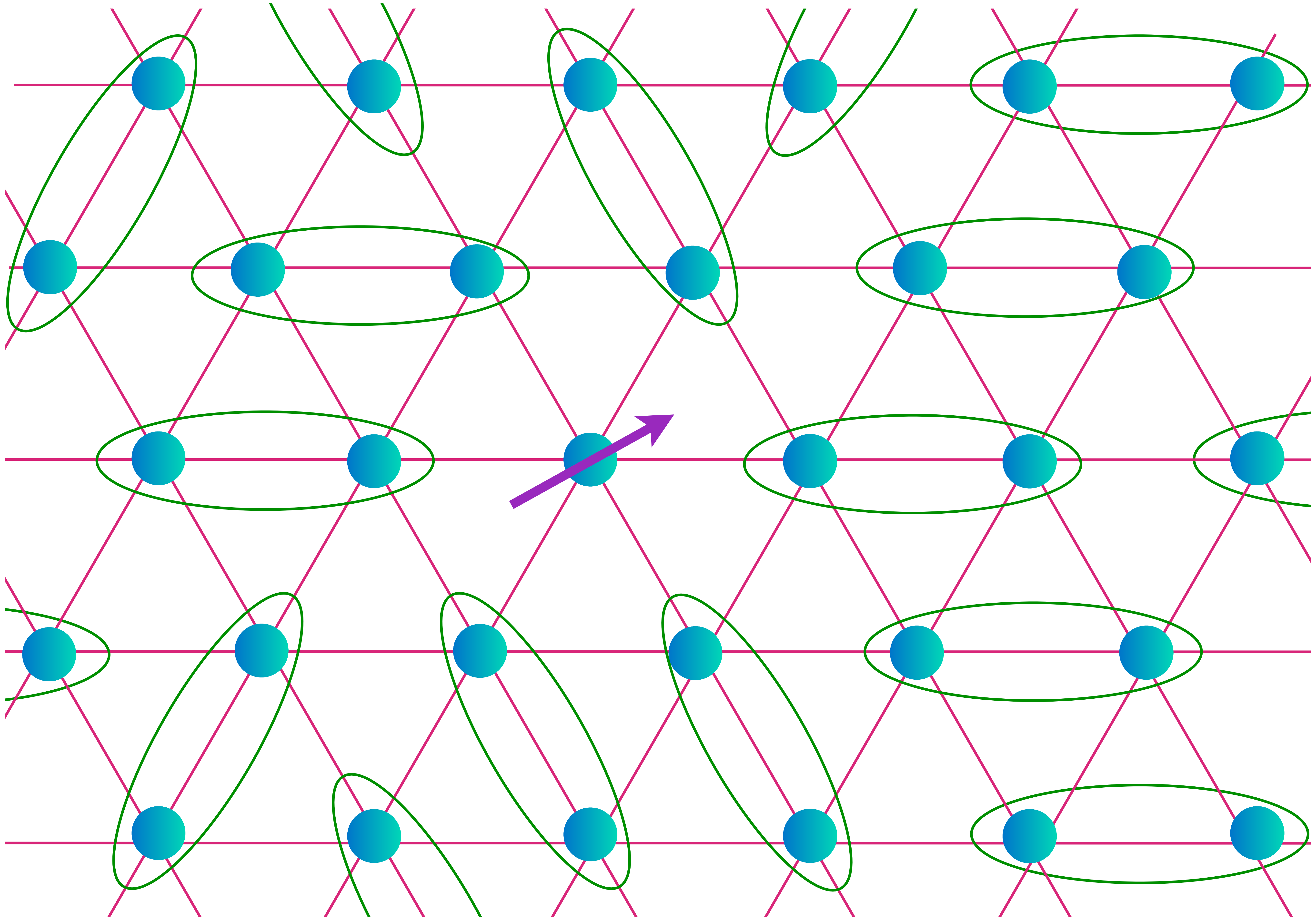

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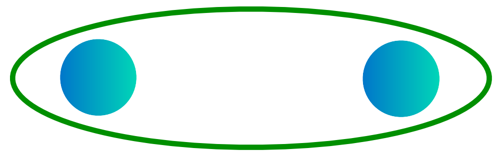
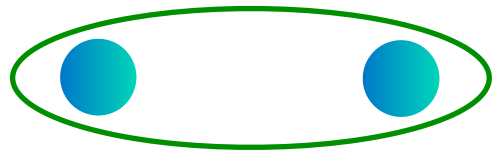


$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$$|\Psi\rangle = C_1|\;\rangle + C_2|\;\rangle + C_3|\;\rangle + C_4|\;\rangle + C_5|\;\rangle + \dots$$

Another anyon—involves subtle changes in sign of superposition



$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

$$C_n \;\Rightarrow\; C_n\,(-1)^{\text{Number of dimers across green line}}$$

Read and Sachdev (1990):
Determined the anyon structure of the resonating valence bond spin liquid.

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The anyon structure of this spin liquid is the same as Kitaev's *toric code* (1997).

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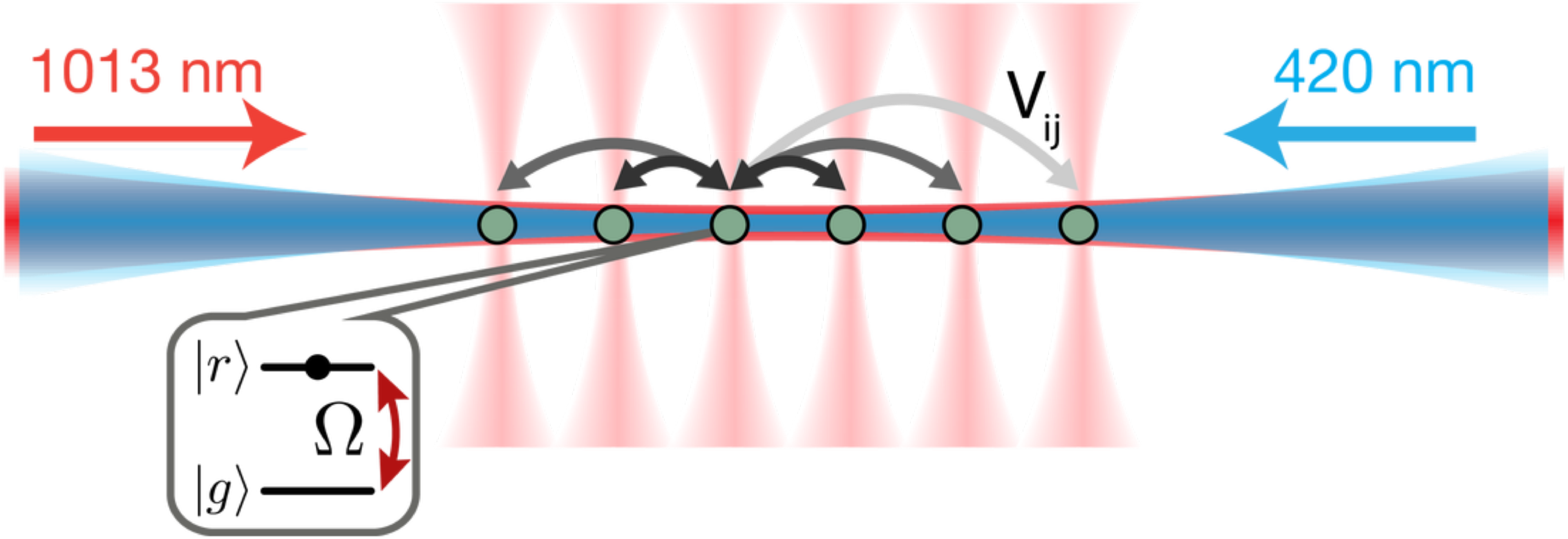
Kitaev (1997): Place the quantum information in a superposition
of anyon sectors for fault-tolerant quantum computation.

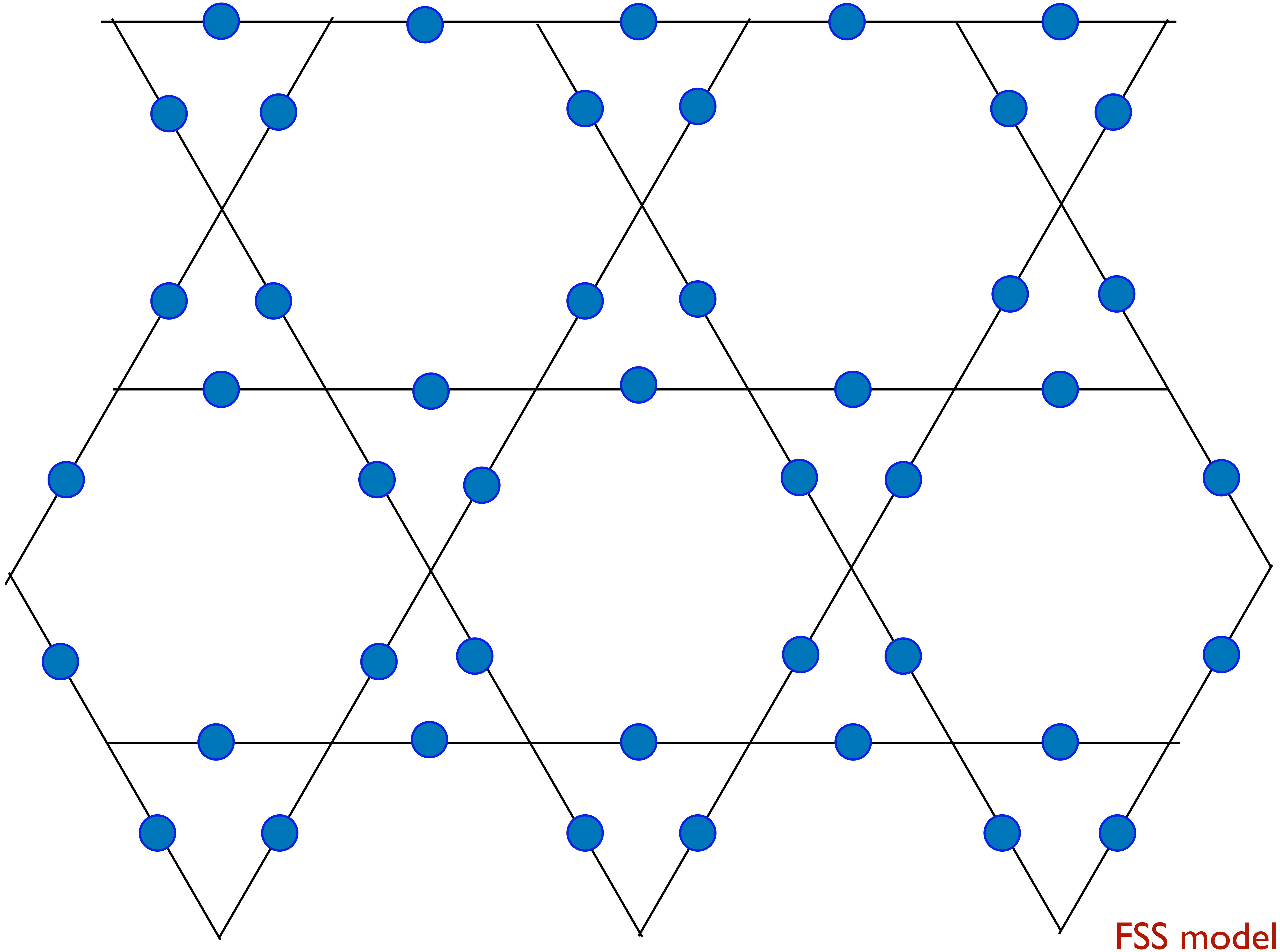
Qubits using Rydberg atoms in optical tweezers



Realized at Raman Research Institute by Prof. Sanjukta Roy

S. Ebadi, Tout T. Wang, H. Levine, A. Keesling, G. Semeghini, A. Omran, D. Bluvstein, R. Samajdar, H. Pichler, Wen Wei Ho, Soonwon Choi, S. Sachdev, M. Greiner, V. Vuletić, and M. D. Lukin, Nature **595**, 227 (2021)

Entangling Rydberg atom qubits



FSS model:
S. Sachdev, K. Sengupta, and S.M. Girvin, PRB **66**, 075128 (2002)
P. Fendley, K. Sengupta, S. Sachdev, PRB **69**, 075106 (2004)

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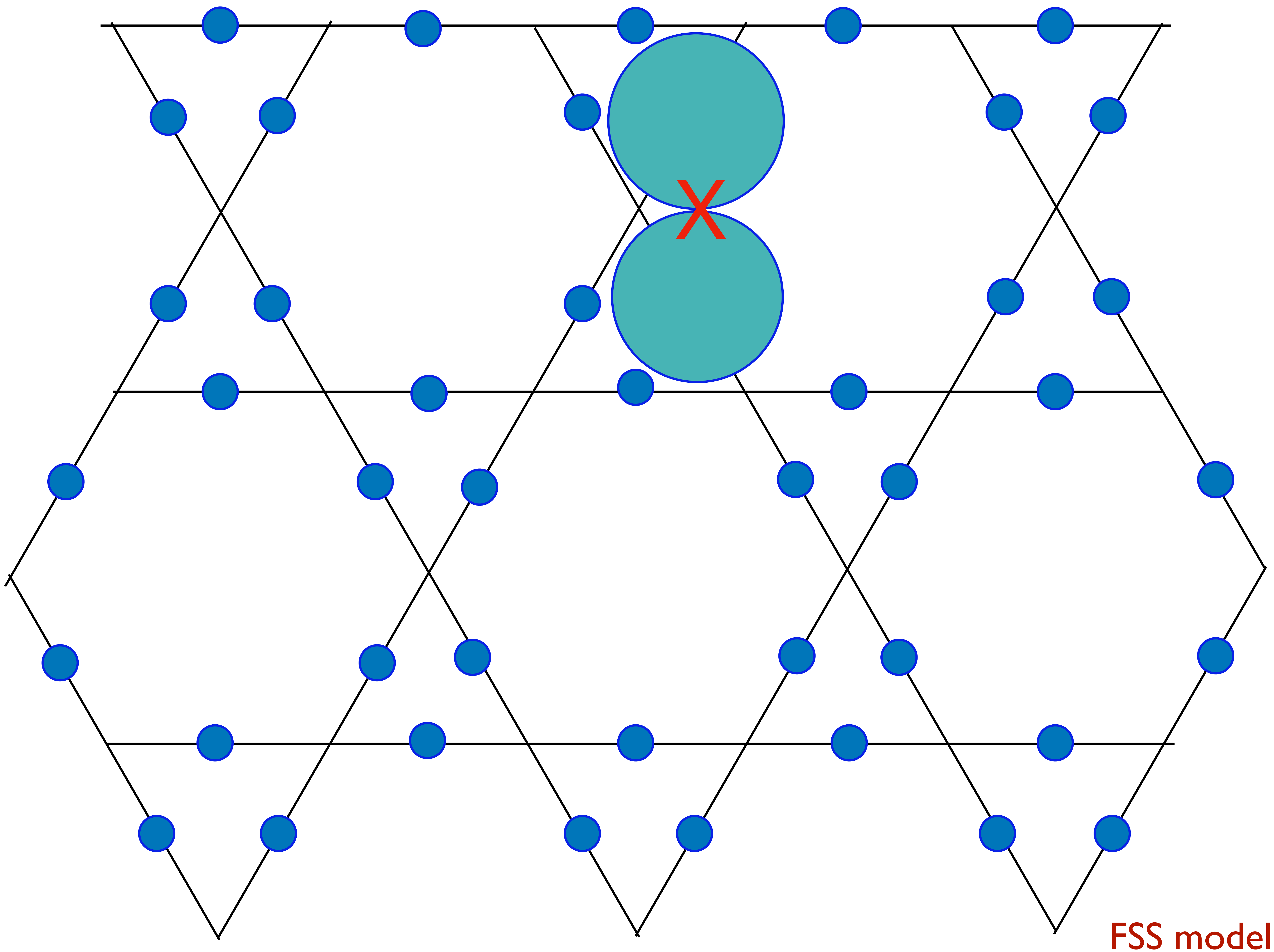

NOT
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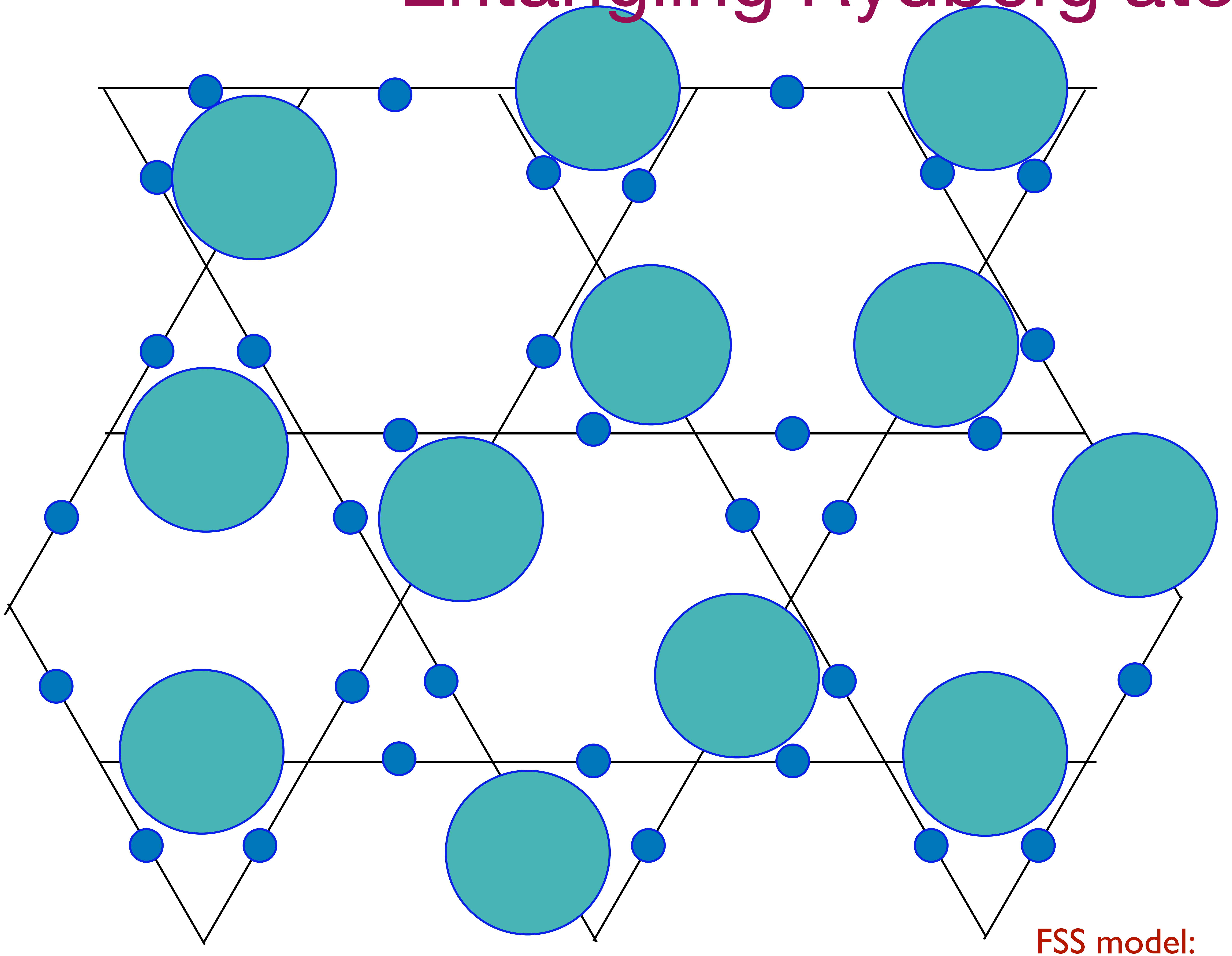

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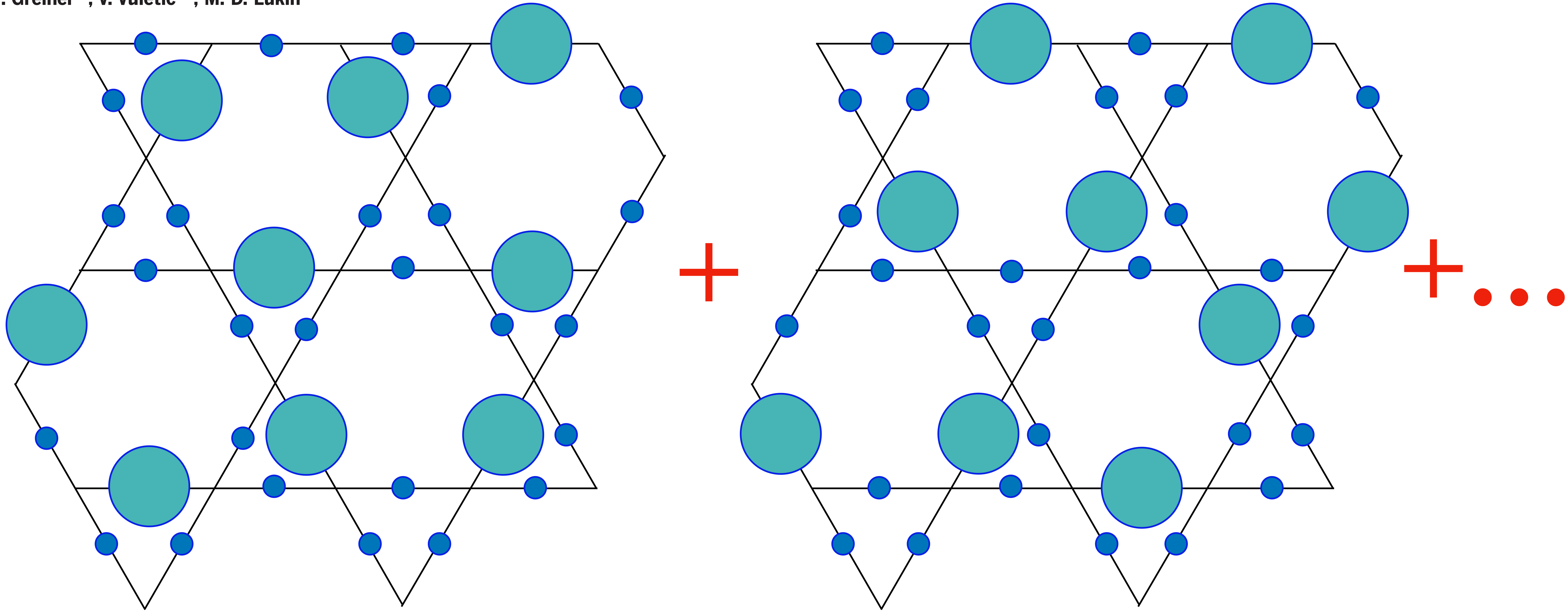
Probing topological spin liquids on a programmable quantum simulator

Science **374**, 1242–1247 (2021)

G. Semeghini[1], H. Levine[1], A. Keesling[1,2], S. Ebadi[1], T. T. Wang[1], D. Bluvstein[1], R. Verresen[1], H. Pichler[3,4], M. Kalinowski[1], R. Samajdar[1], A. Omran[1,2], S. Sachdev[1,5], A. Vishwanath[1*], M. Greiner[1*], V. Vuletić[6*], M. D. Lukin[1*]

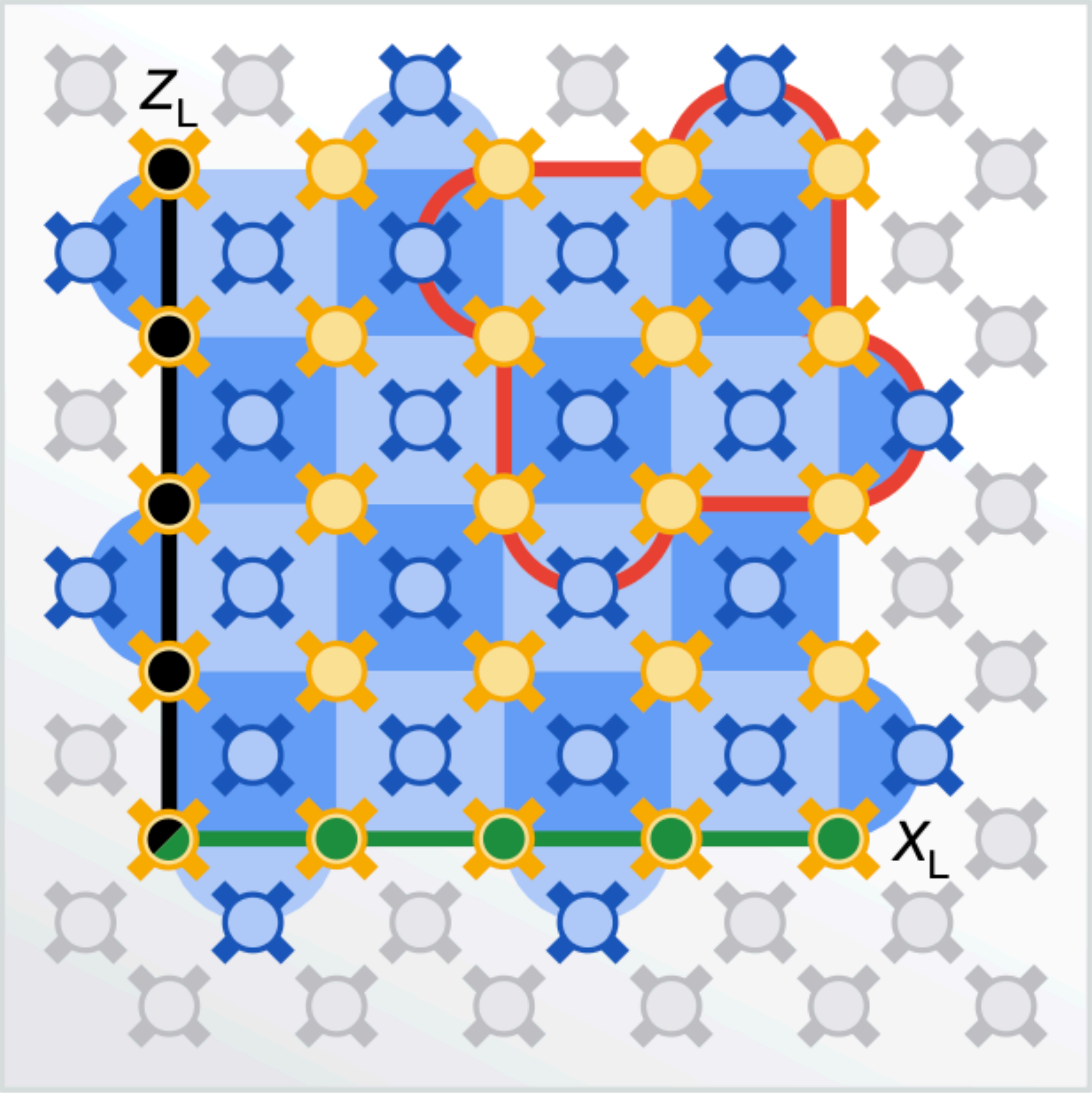


Entanglement similar to that in a RVB spin liquid

Suppressing quantum errors by scaling a surface code logical qubit

Google Quantum AI*

676 | Nature | Vol 614 | 23 February 2023



"Surface code" is closely related to the "toric code"

Although it has had a dramatic impact on theory/experiments on fault-tolerant quantum computations,
the theory of spin liquids with well-defined anyons
does *not* solve the problem of
high temperature superconductivity in the cuprates

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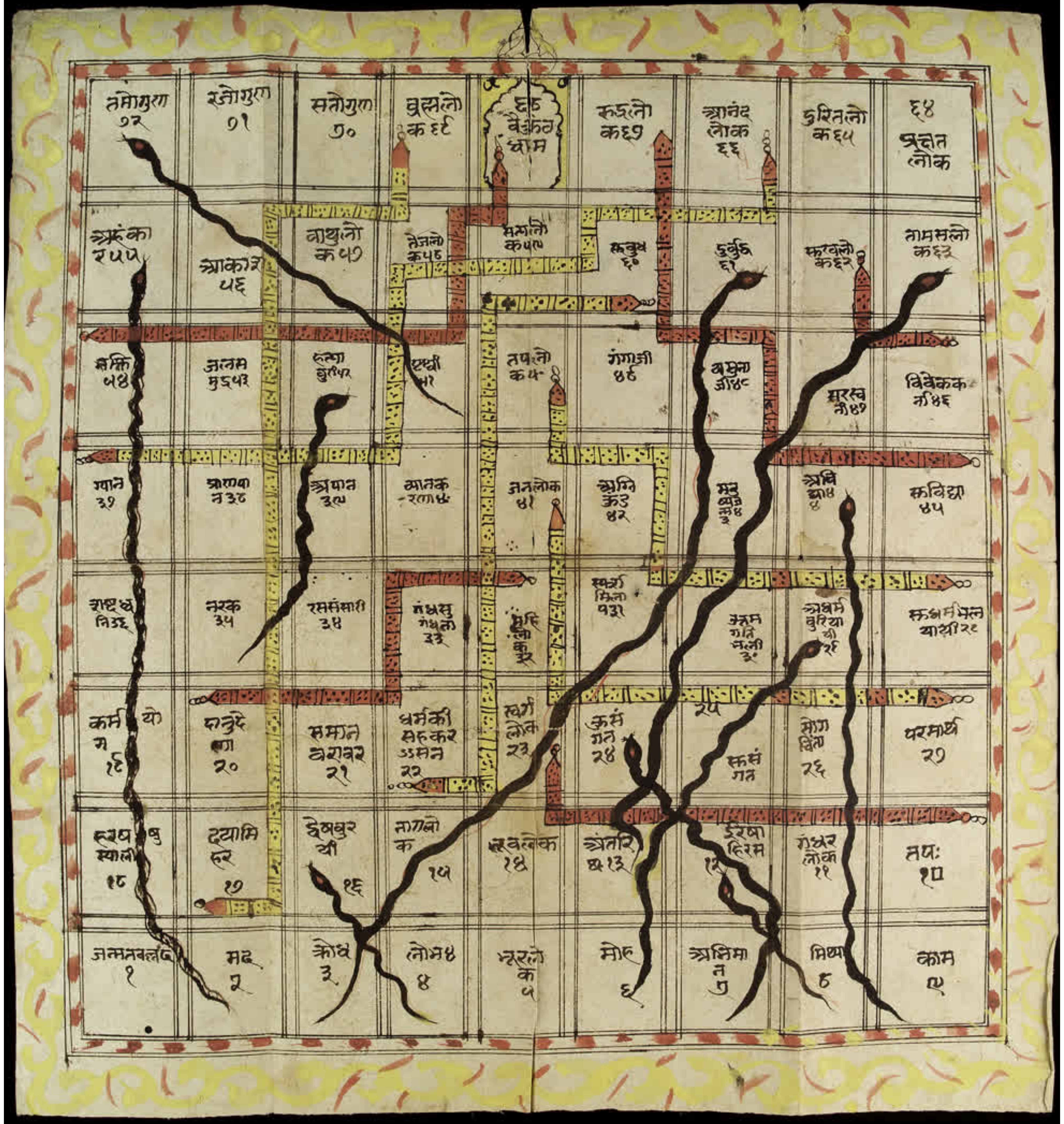
The Sachdev-Ye-Kitaev model
of entanglement of mobile fermions



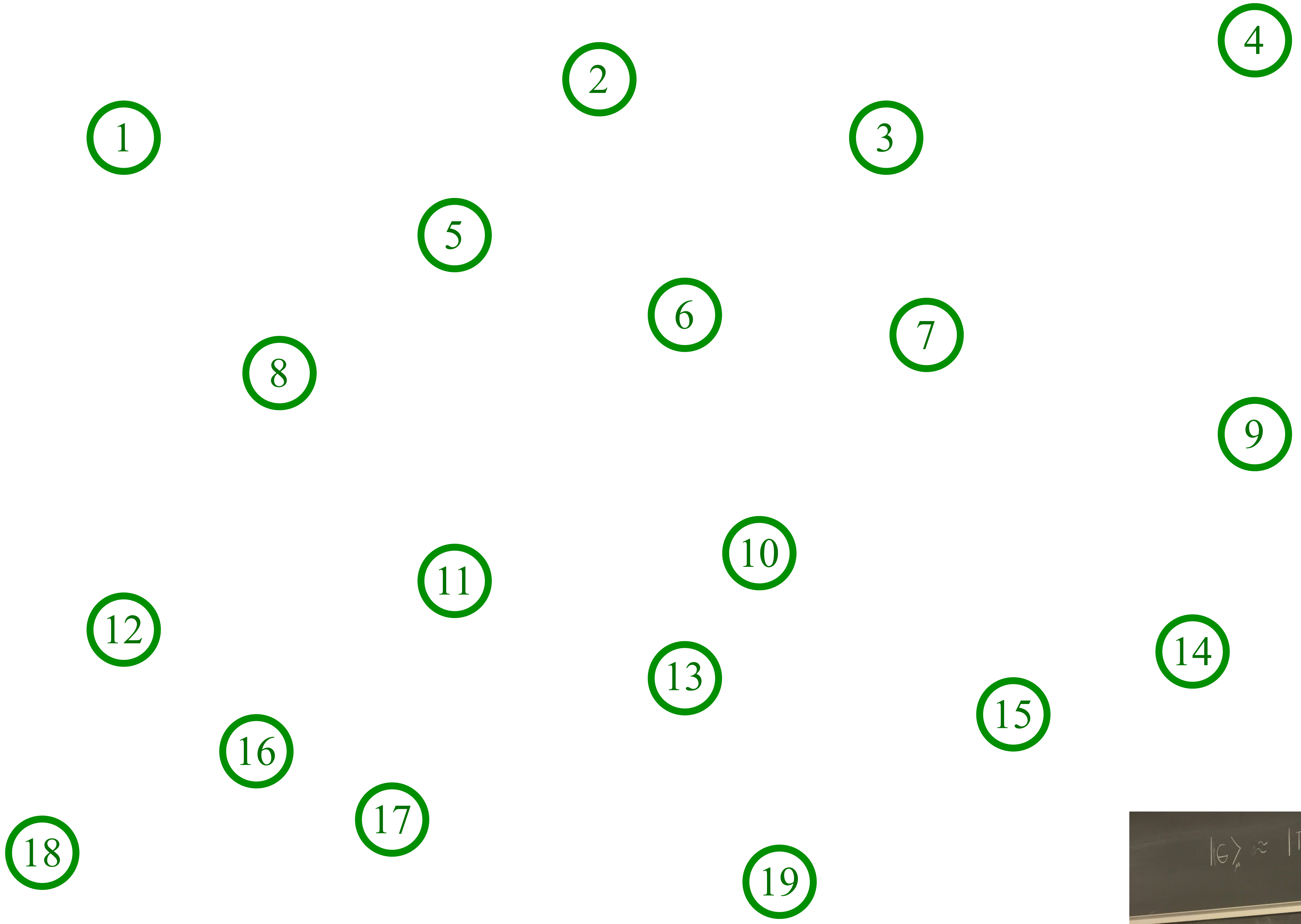
My spooky dream*

Ancient Indian game of Snakes and Ladders

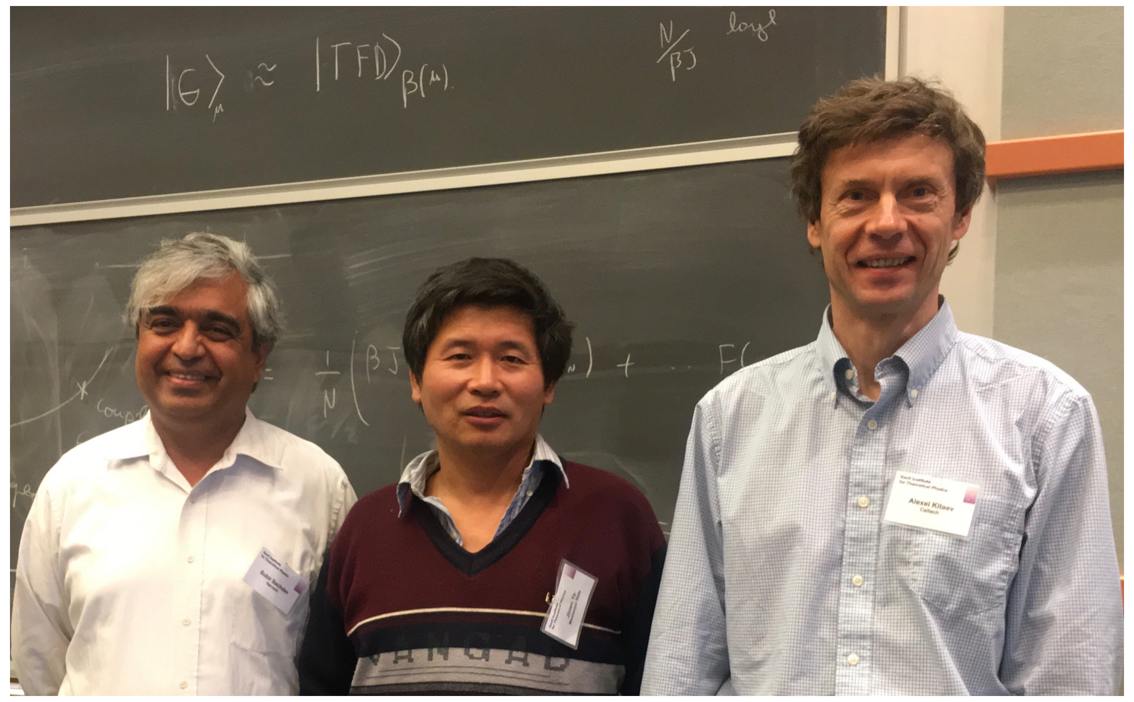
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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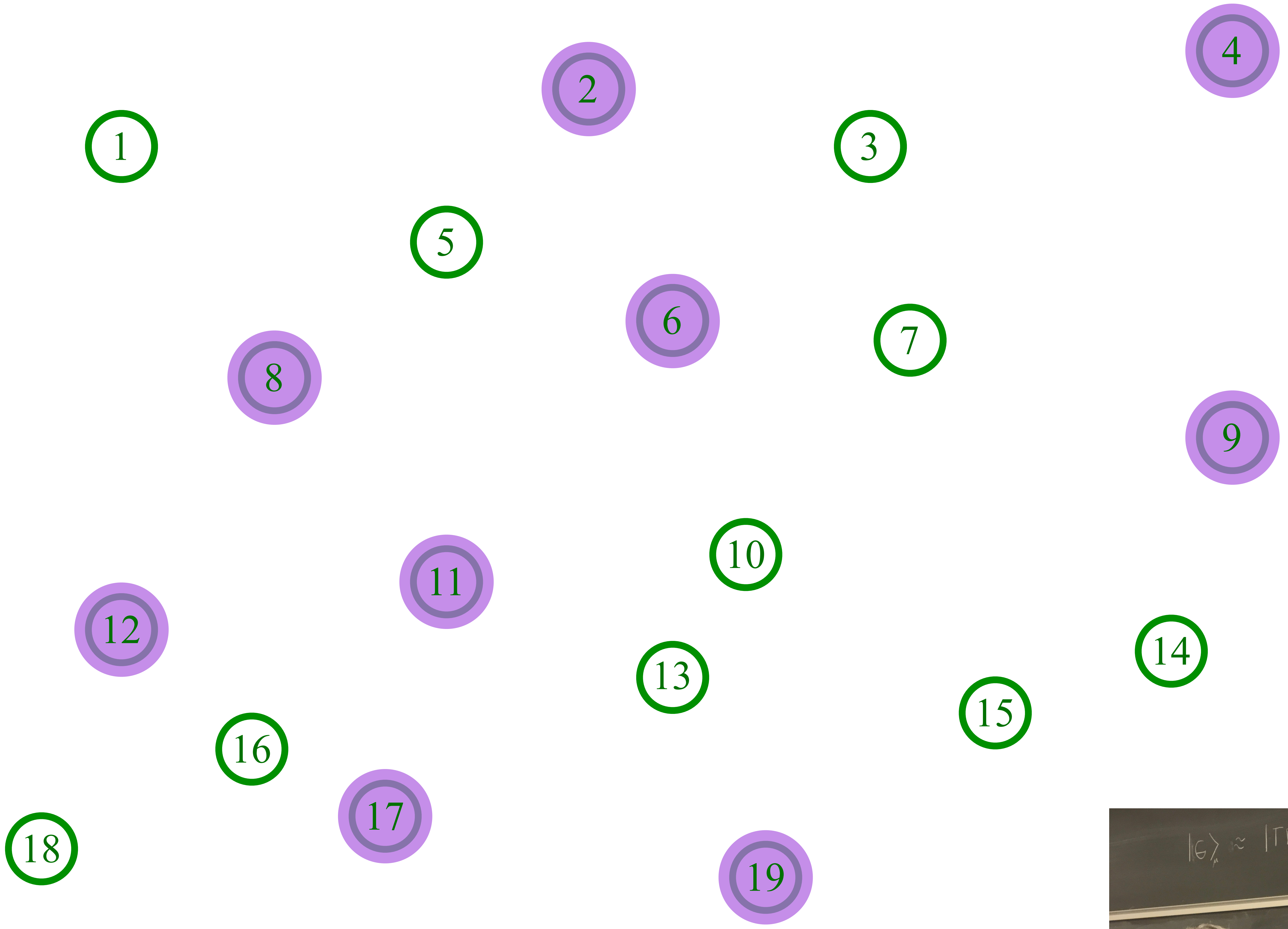
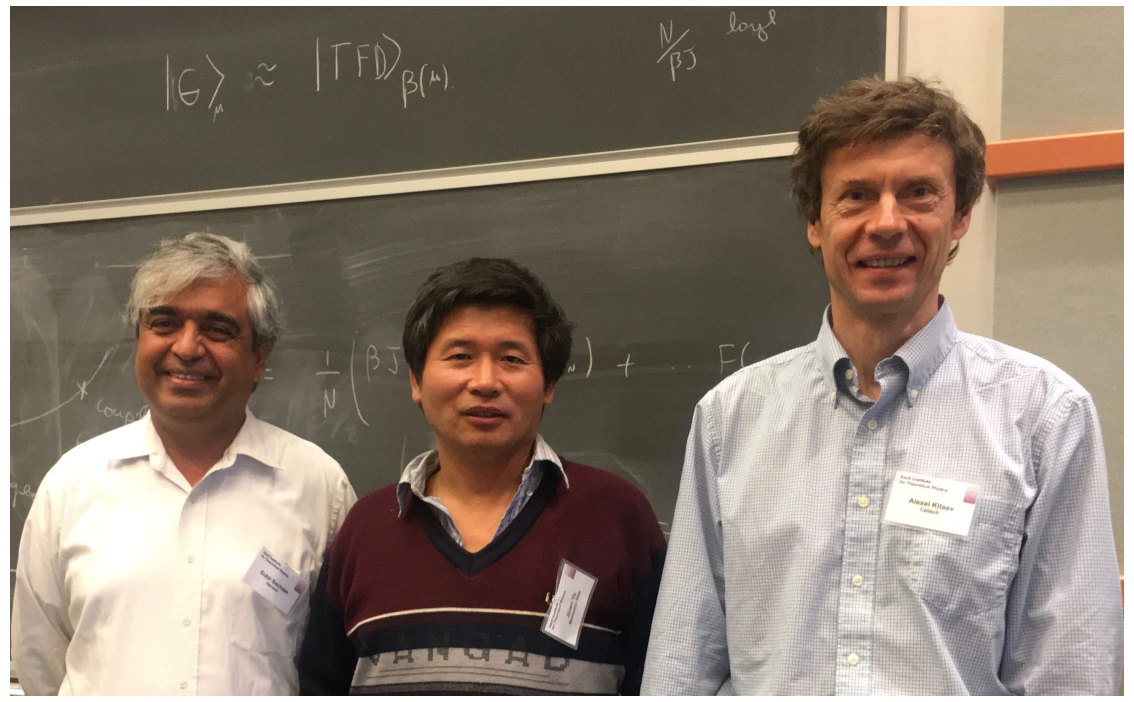


Place electrons randomly on some sites



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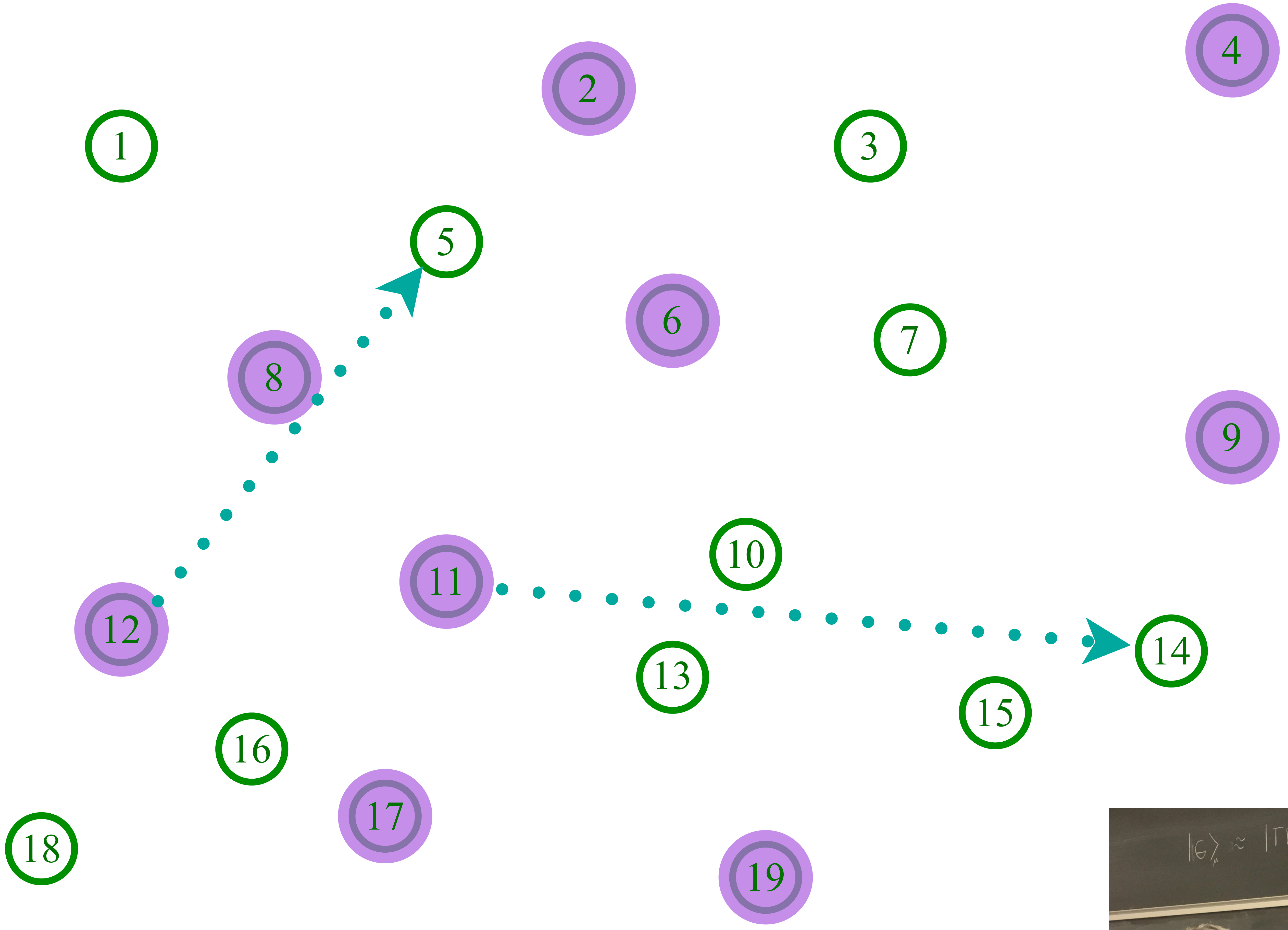
Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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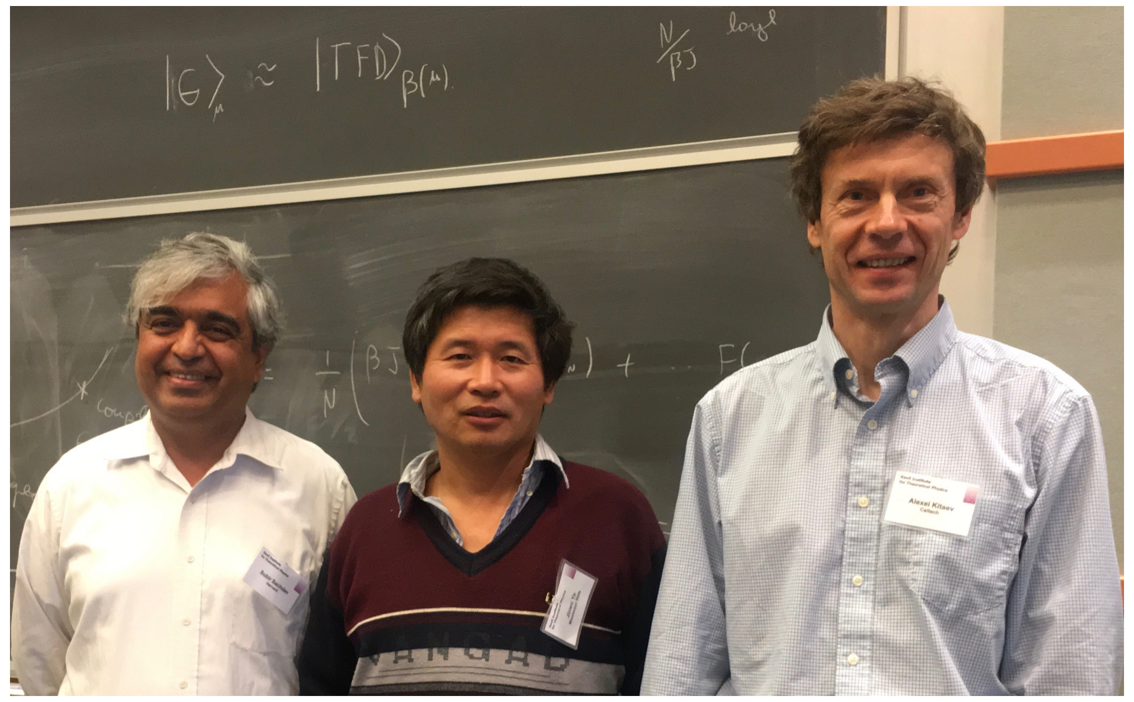

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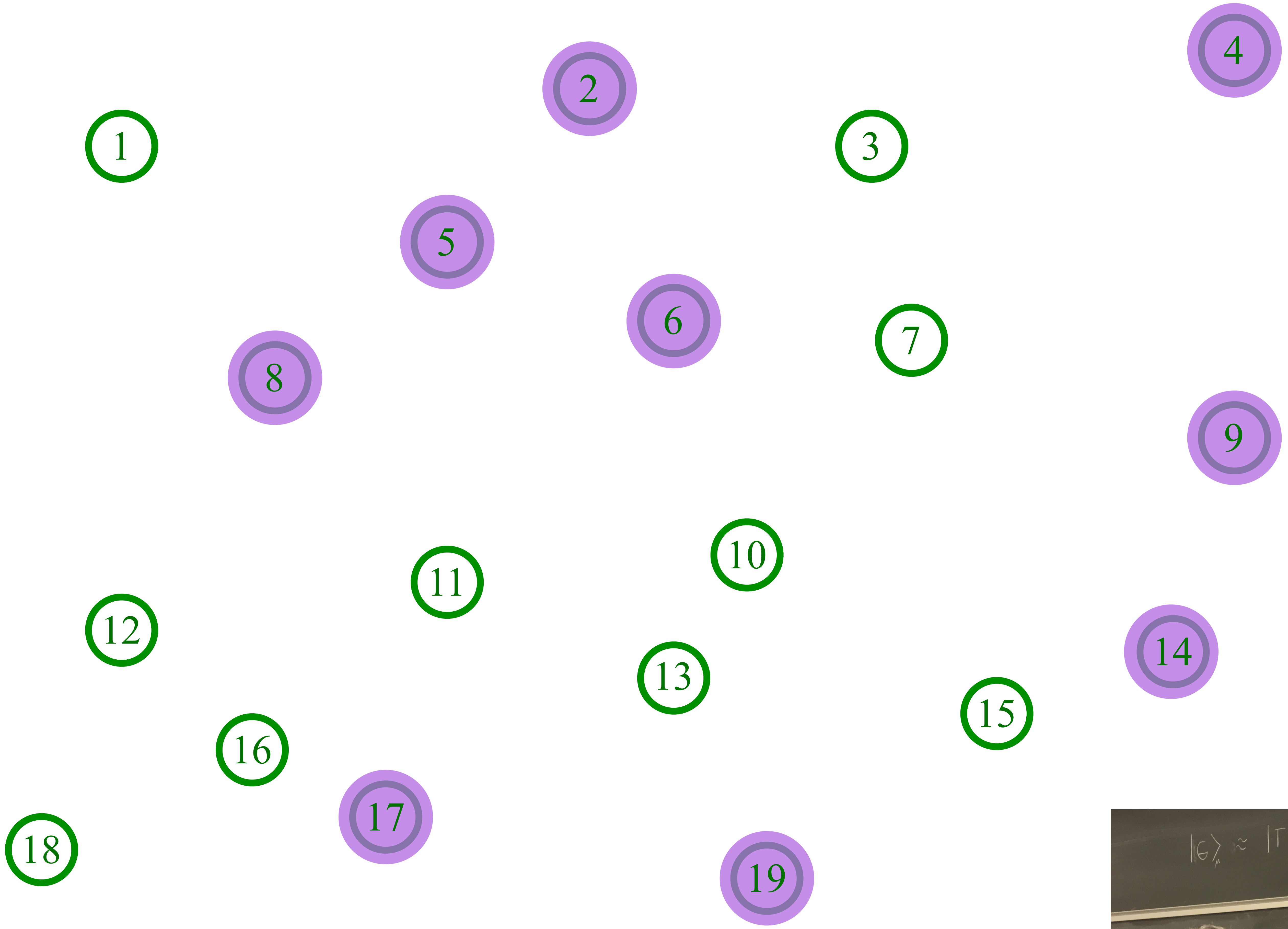
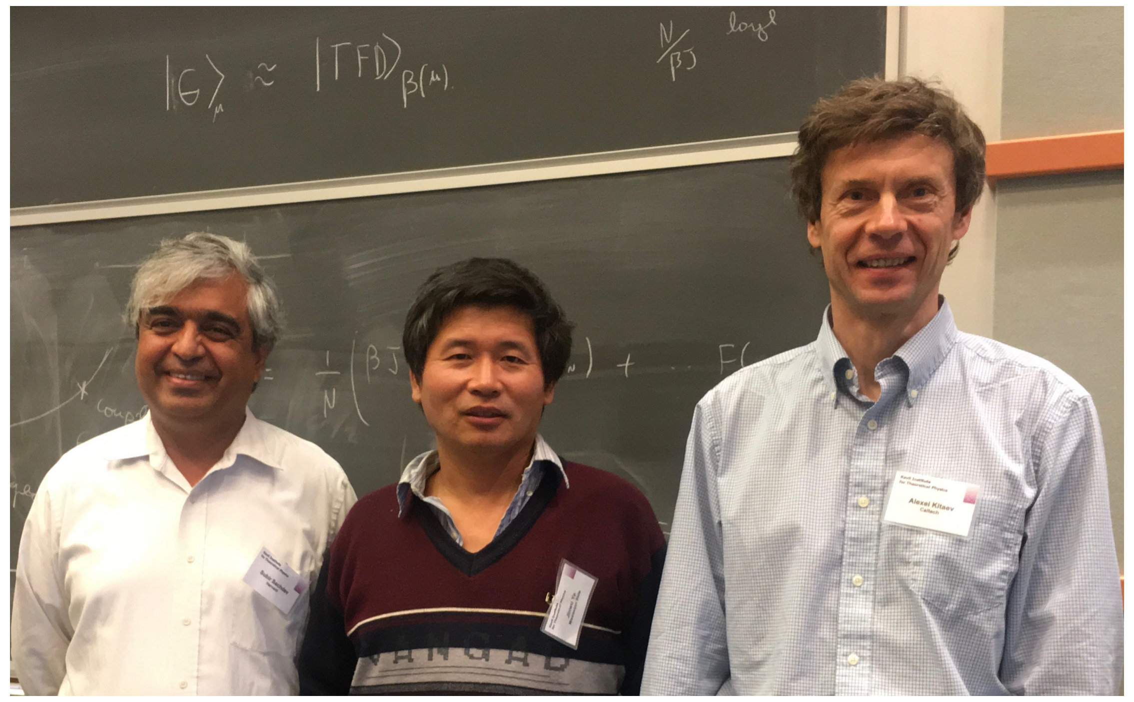


Entangle electrons pairwise randomly



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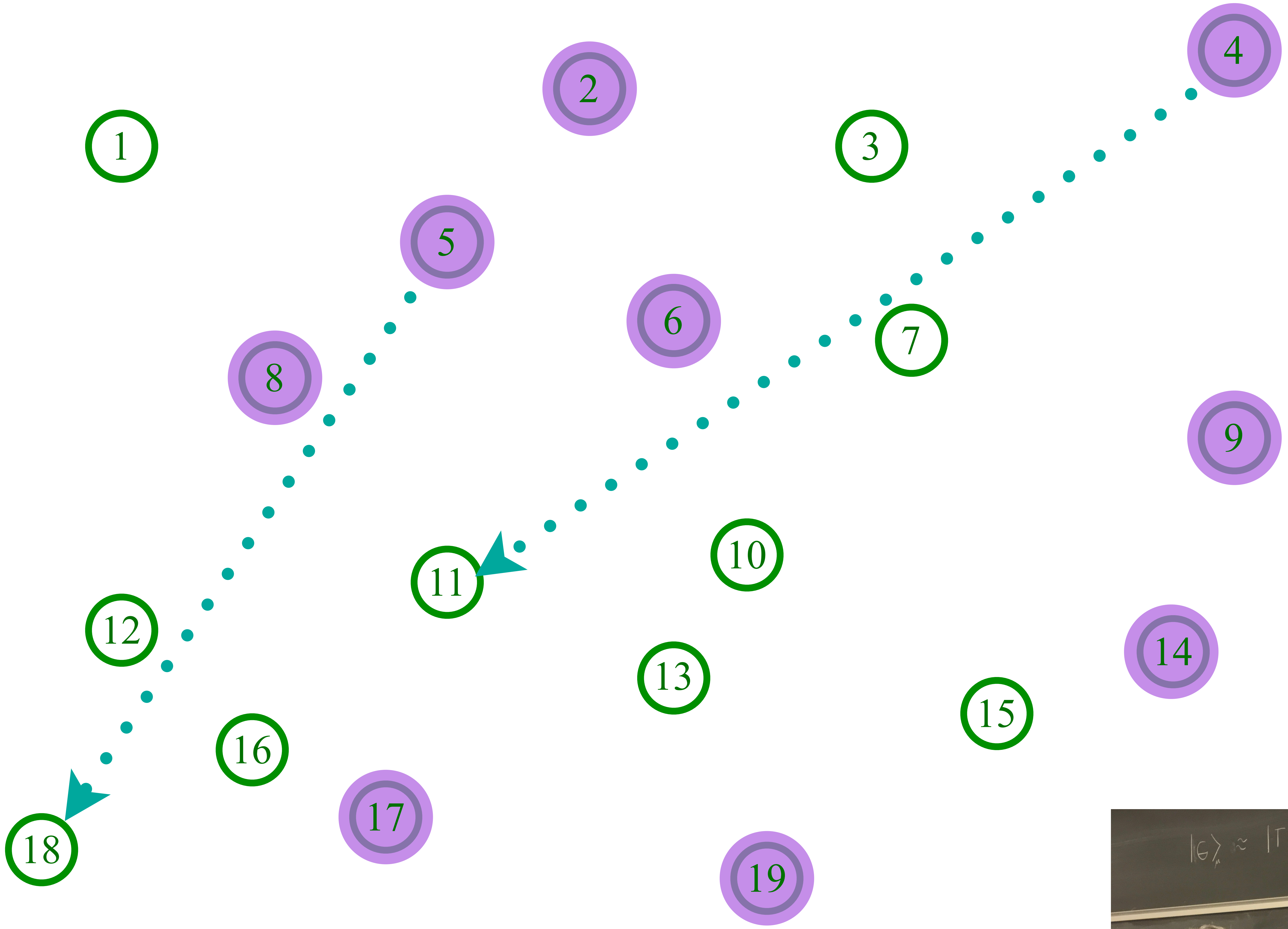
$$U_{4,5;11,18}$$



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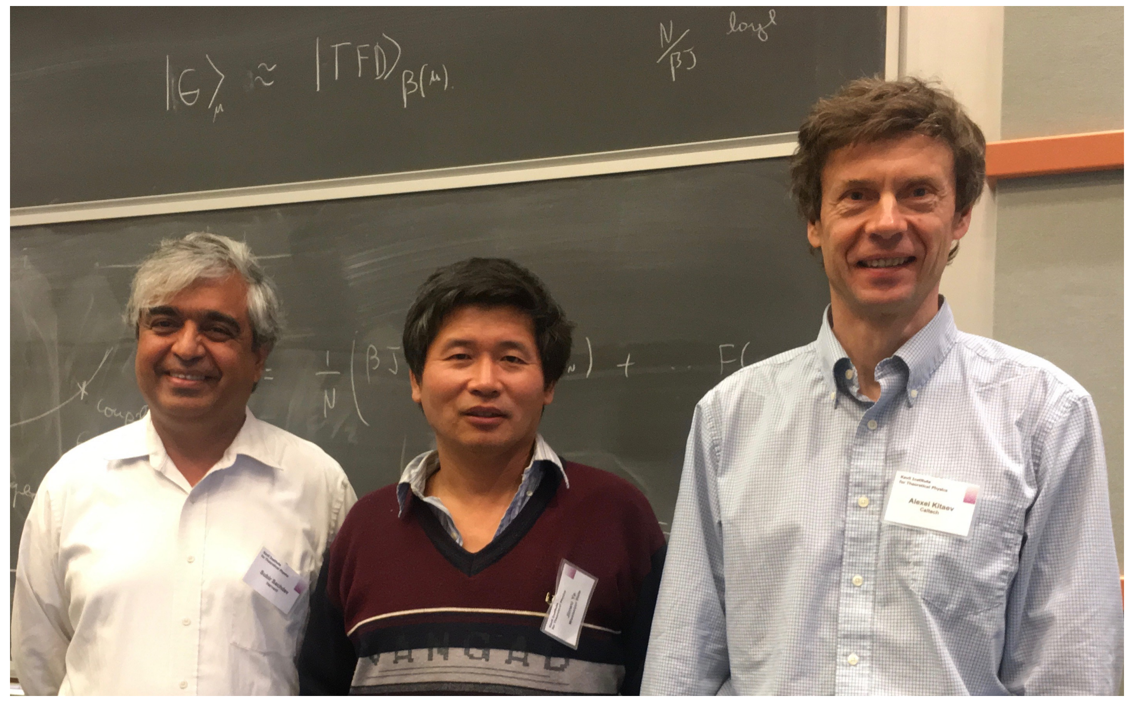

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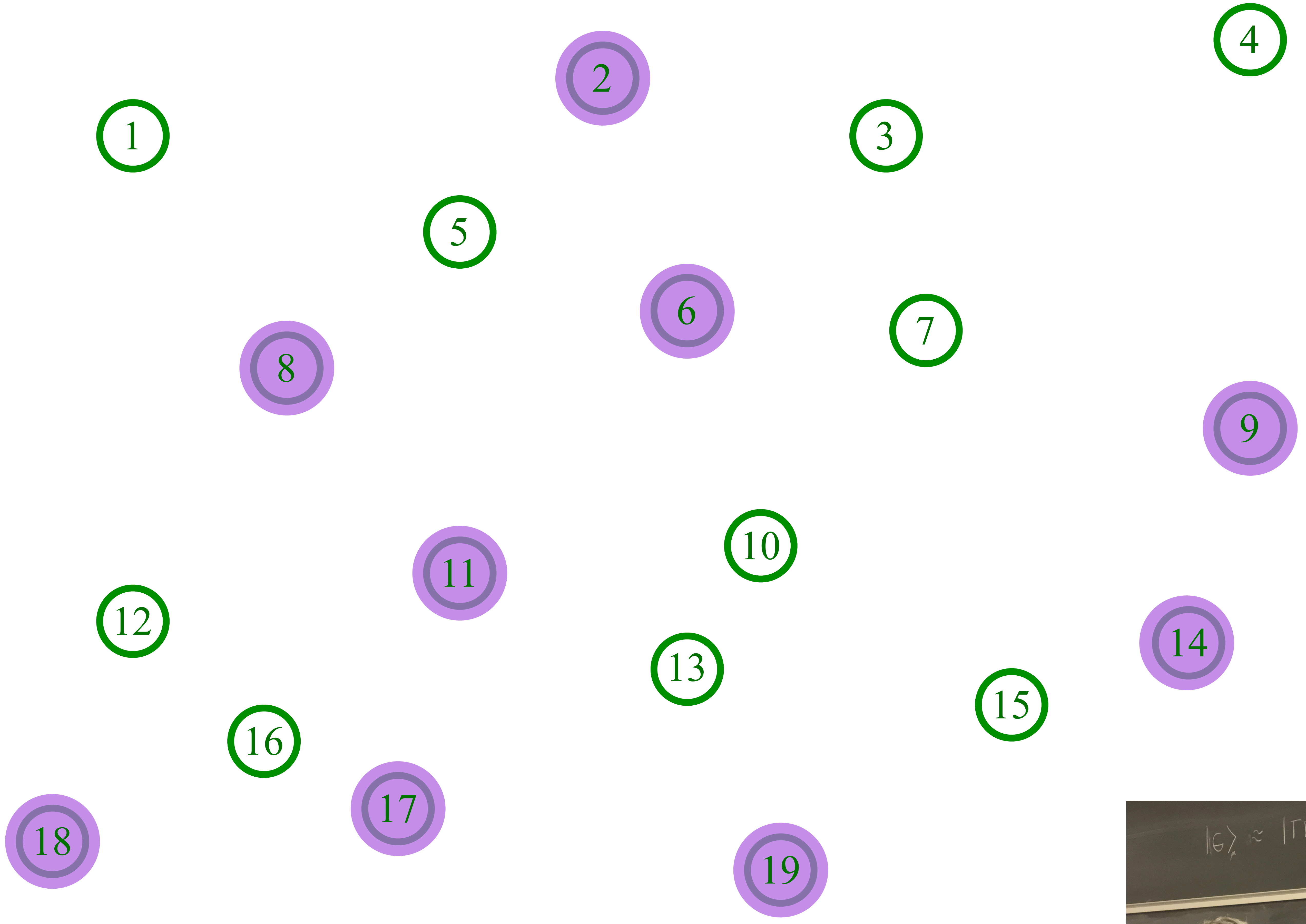

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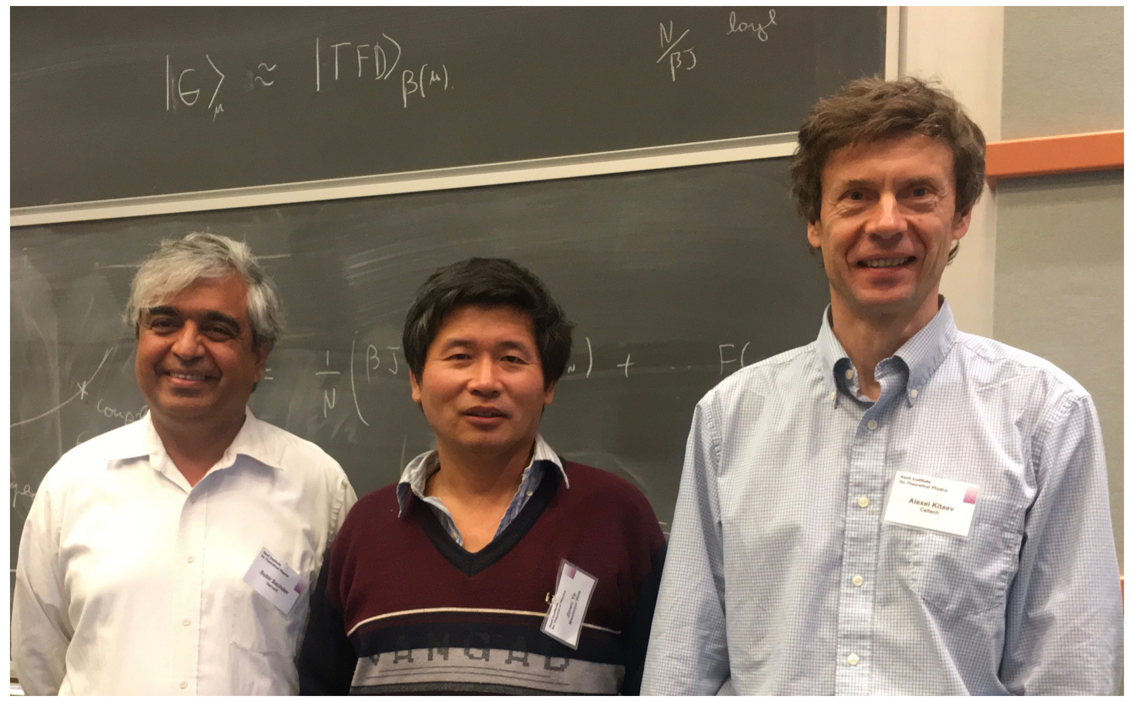

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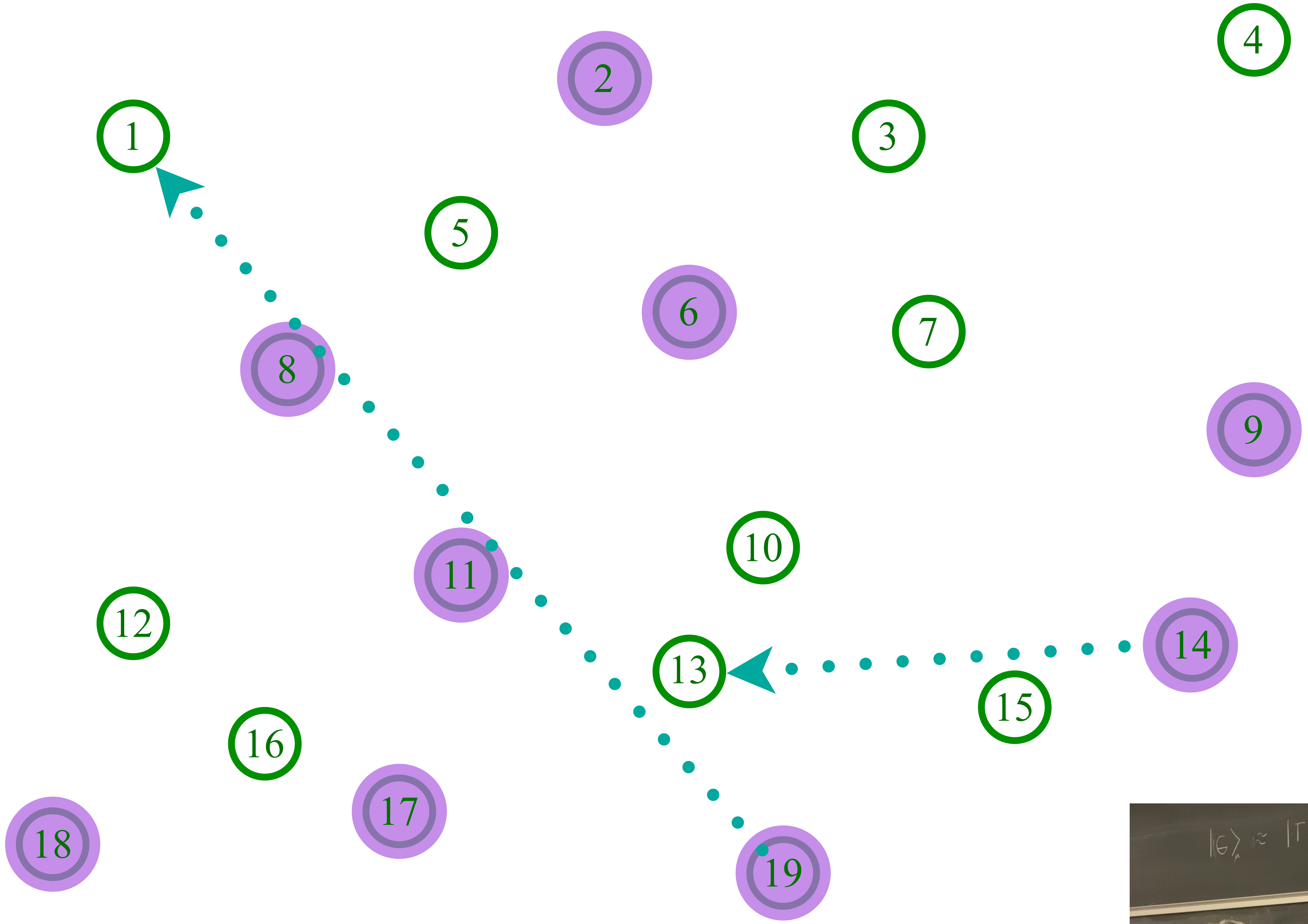

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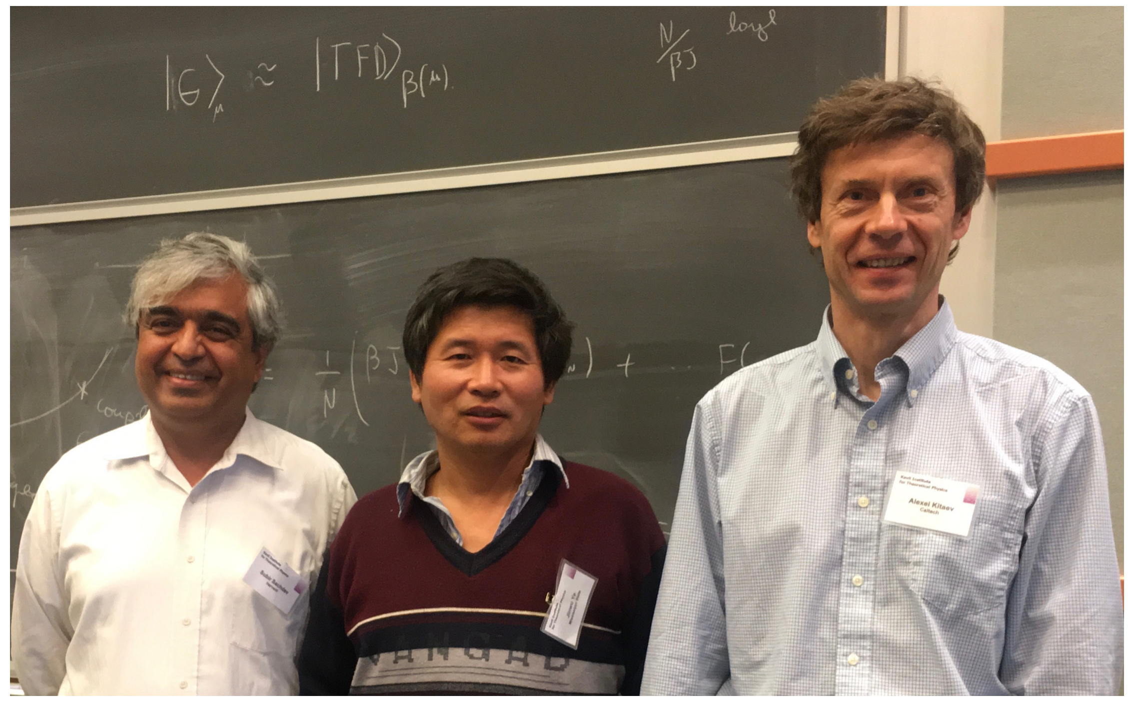

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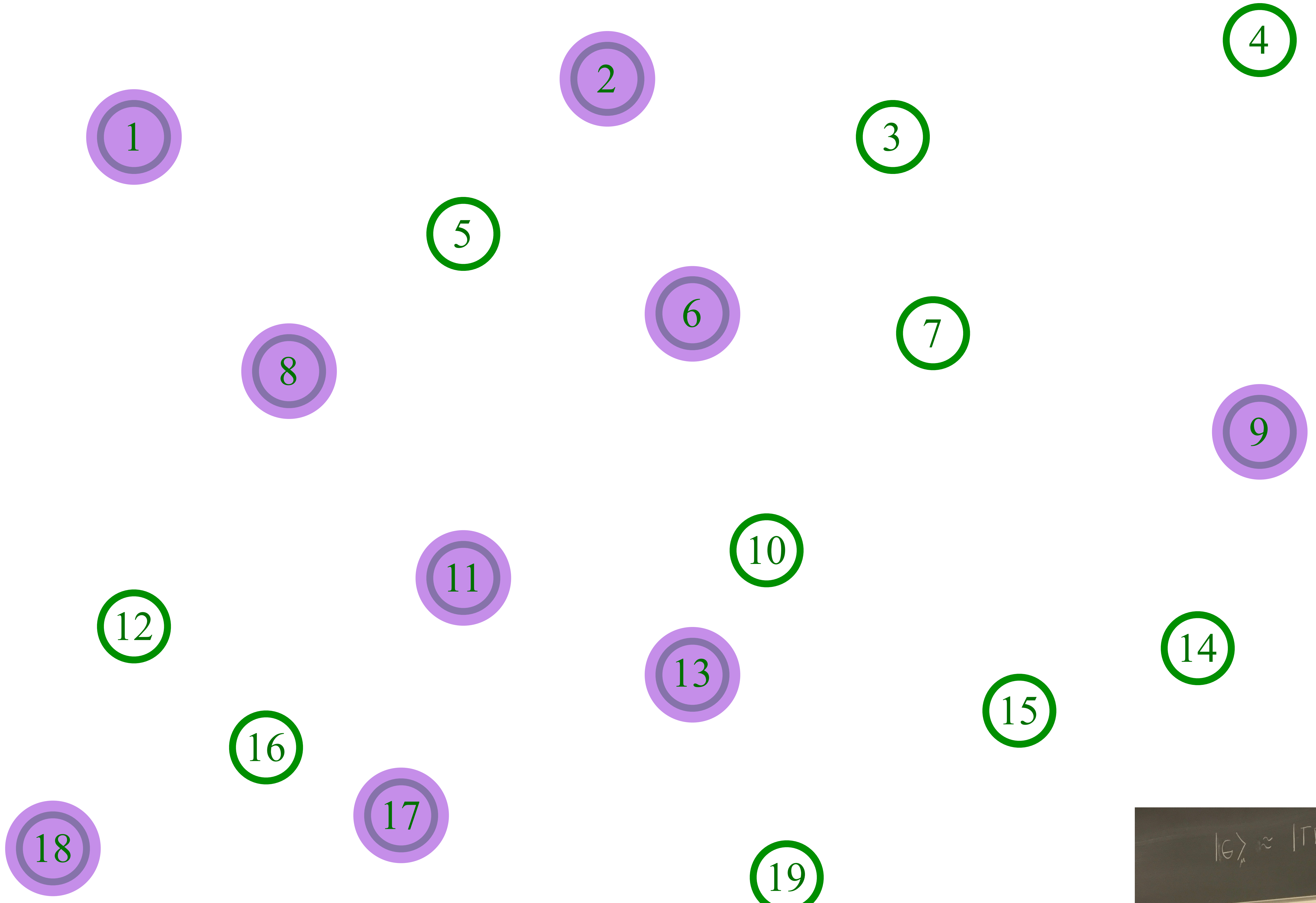

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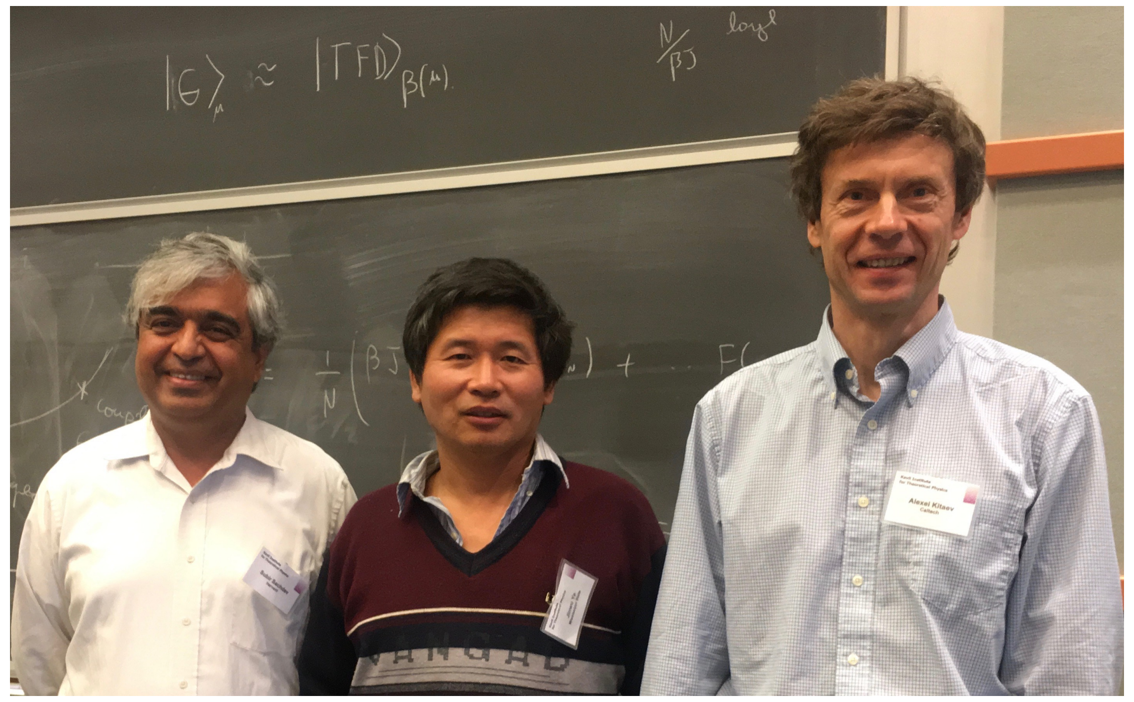

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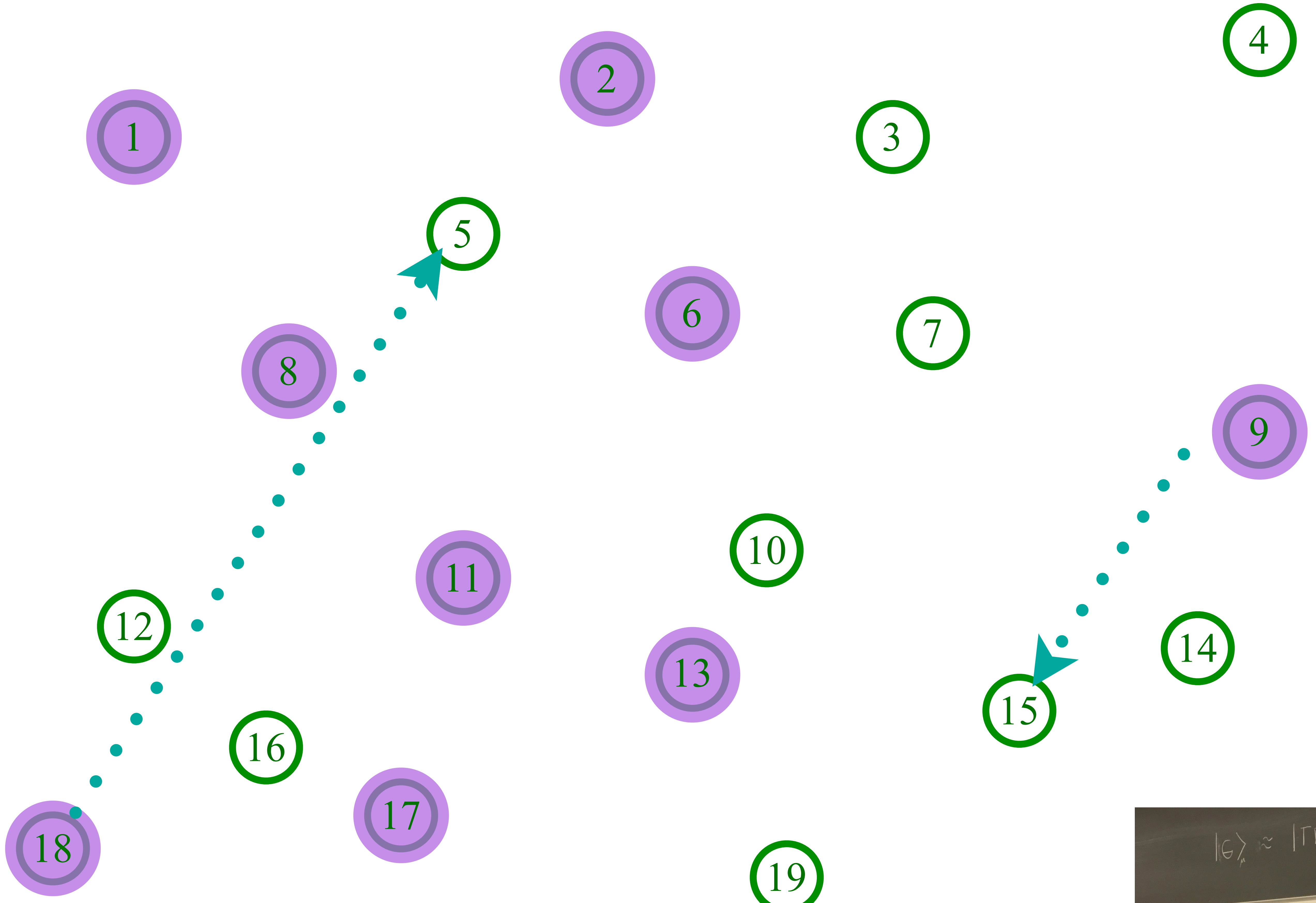

$$U_{9,18;5,15}$$

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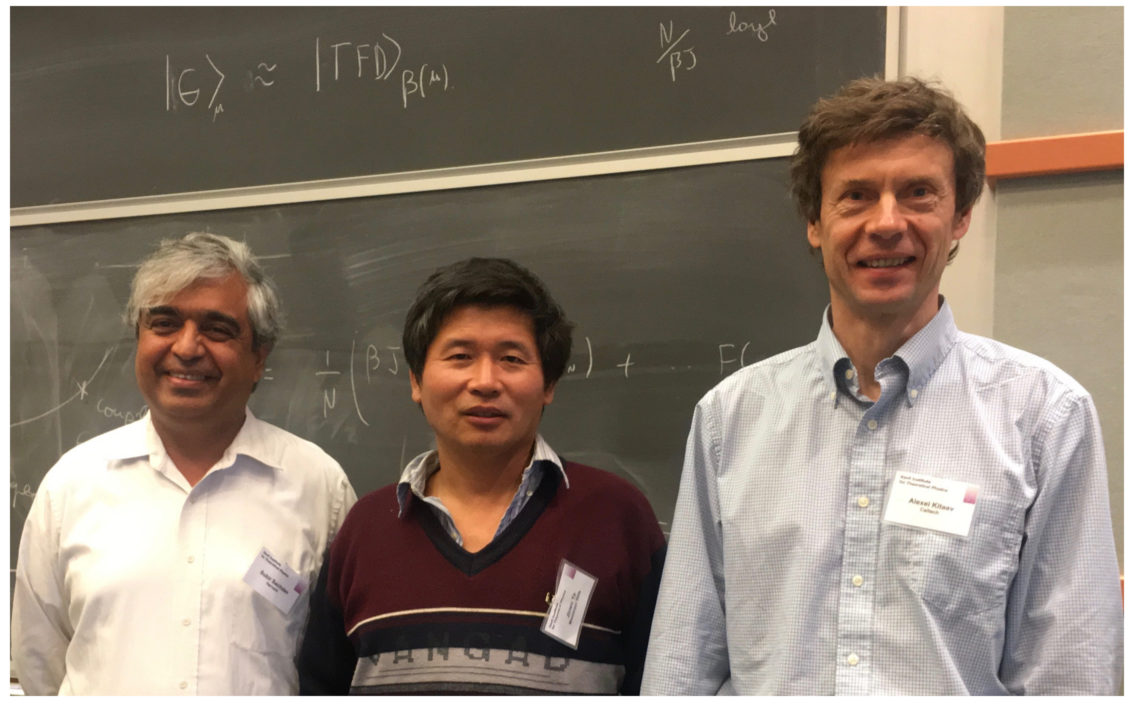

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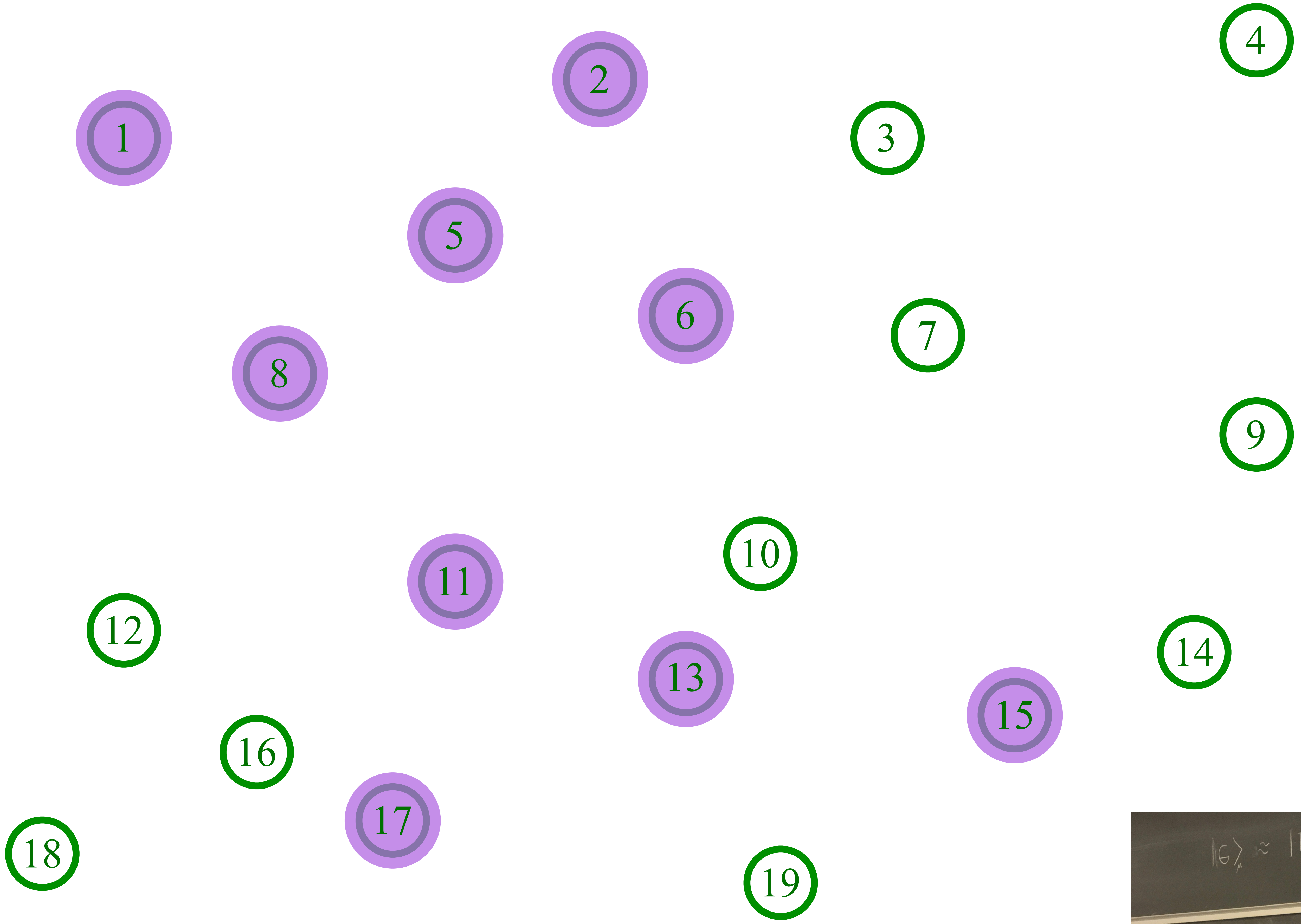

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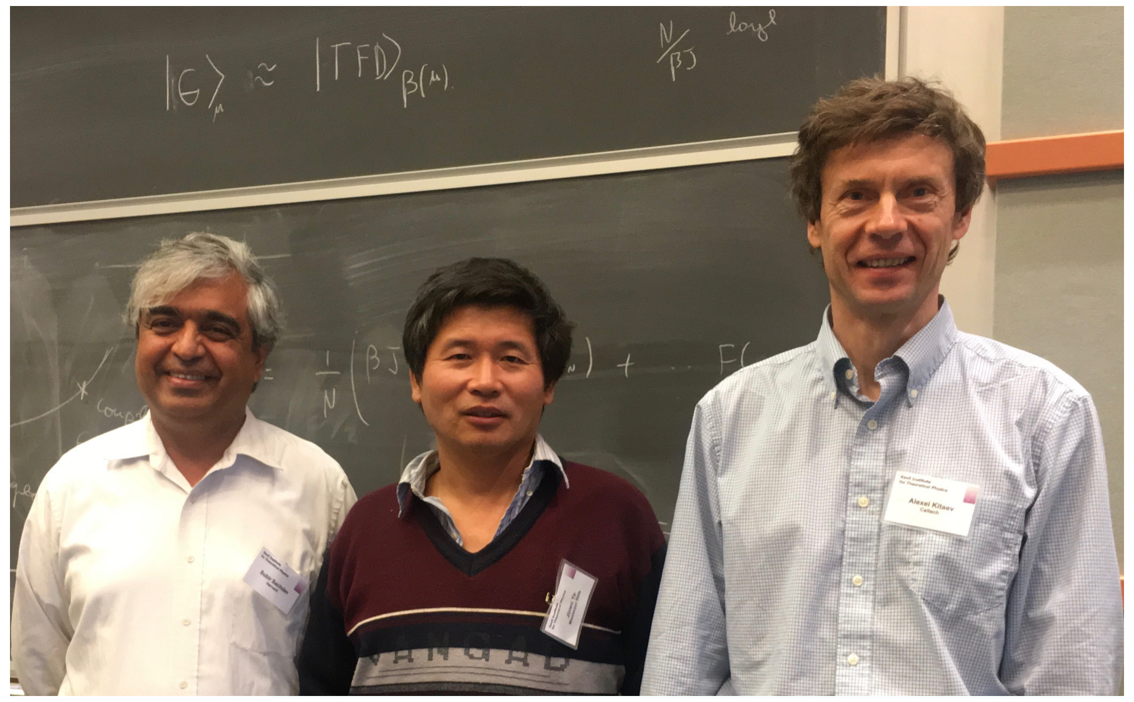

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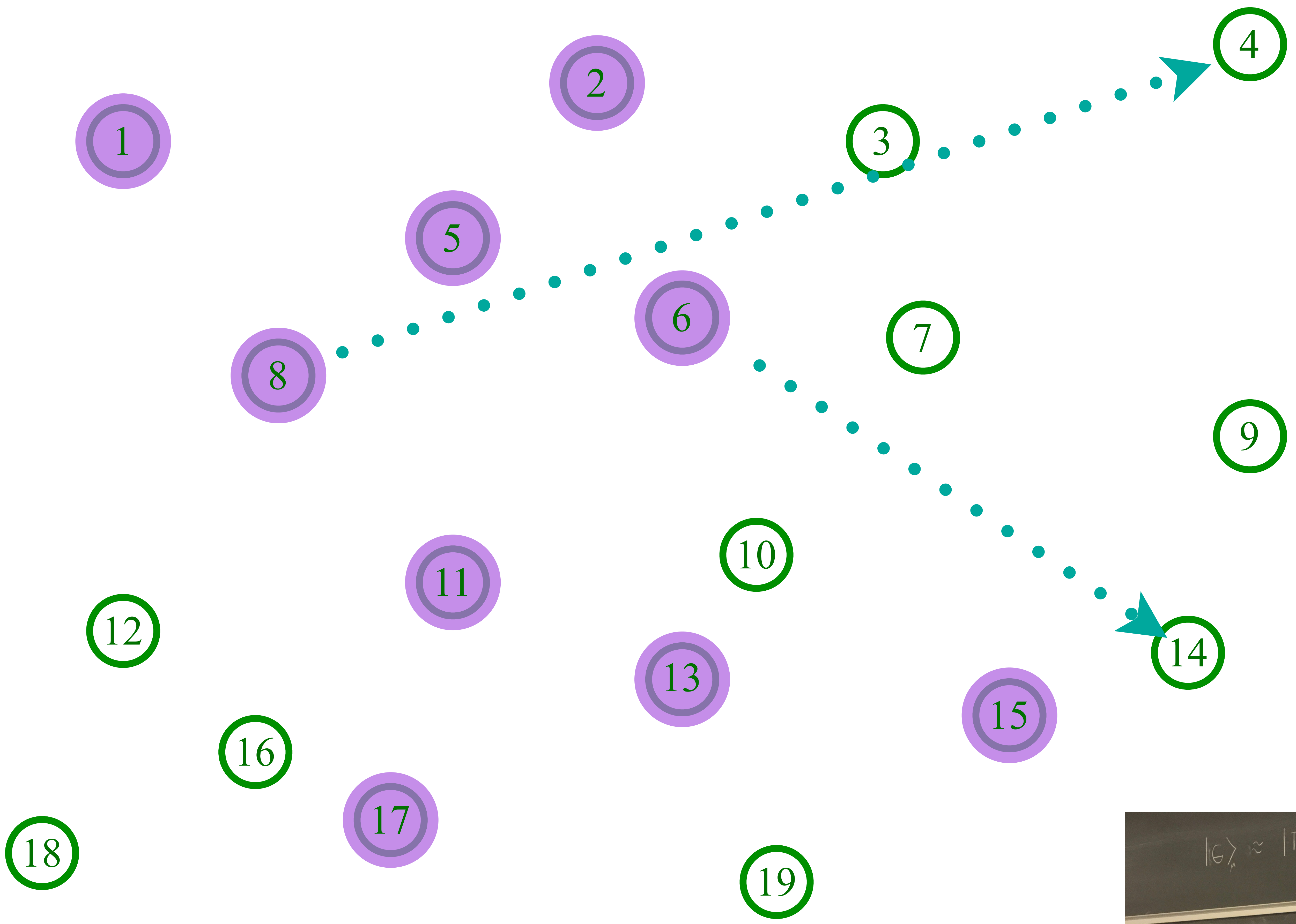
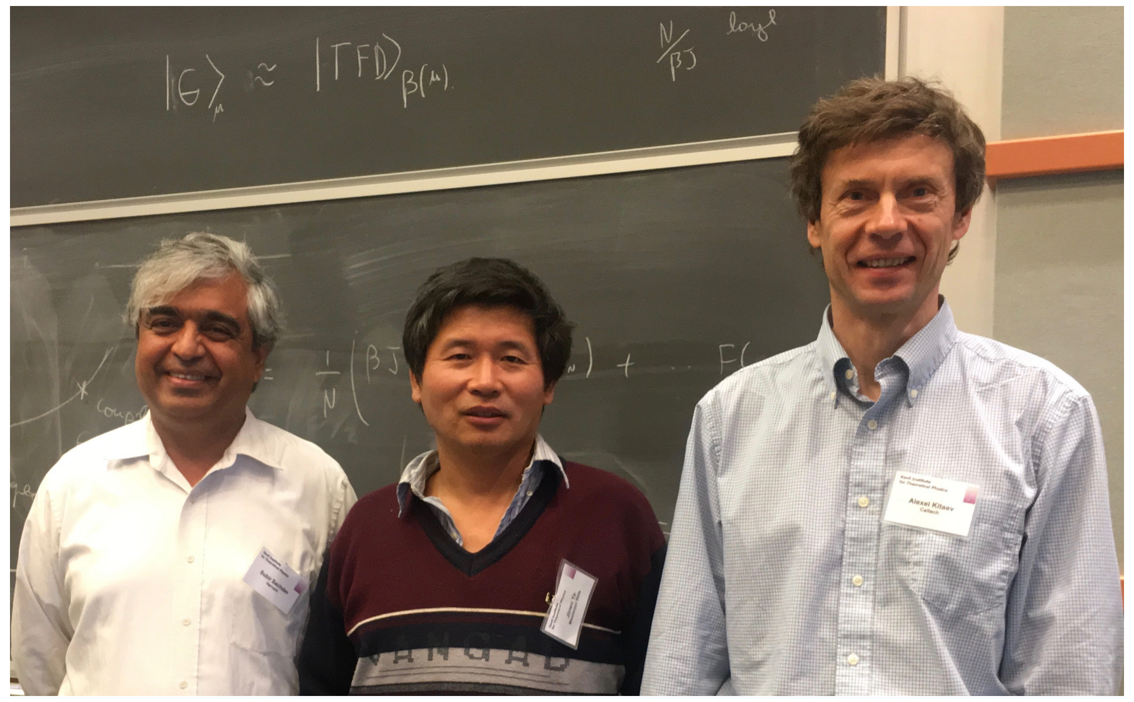


$U_{6,8;4,14}$

Entangle electrons pairwise randomly



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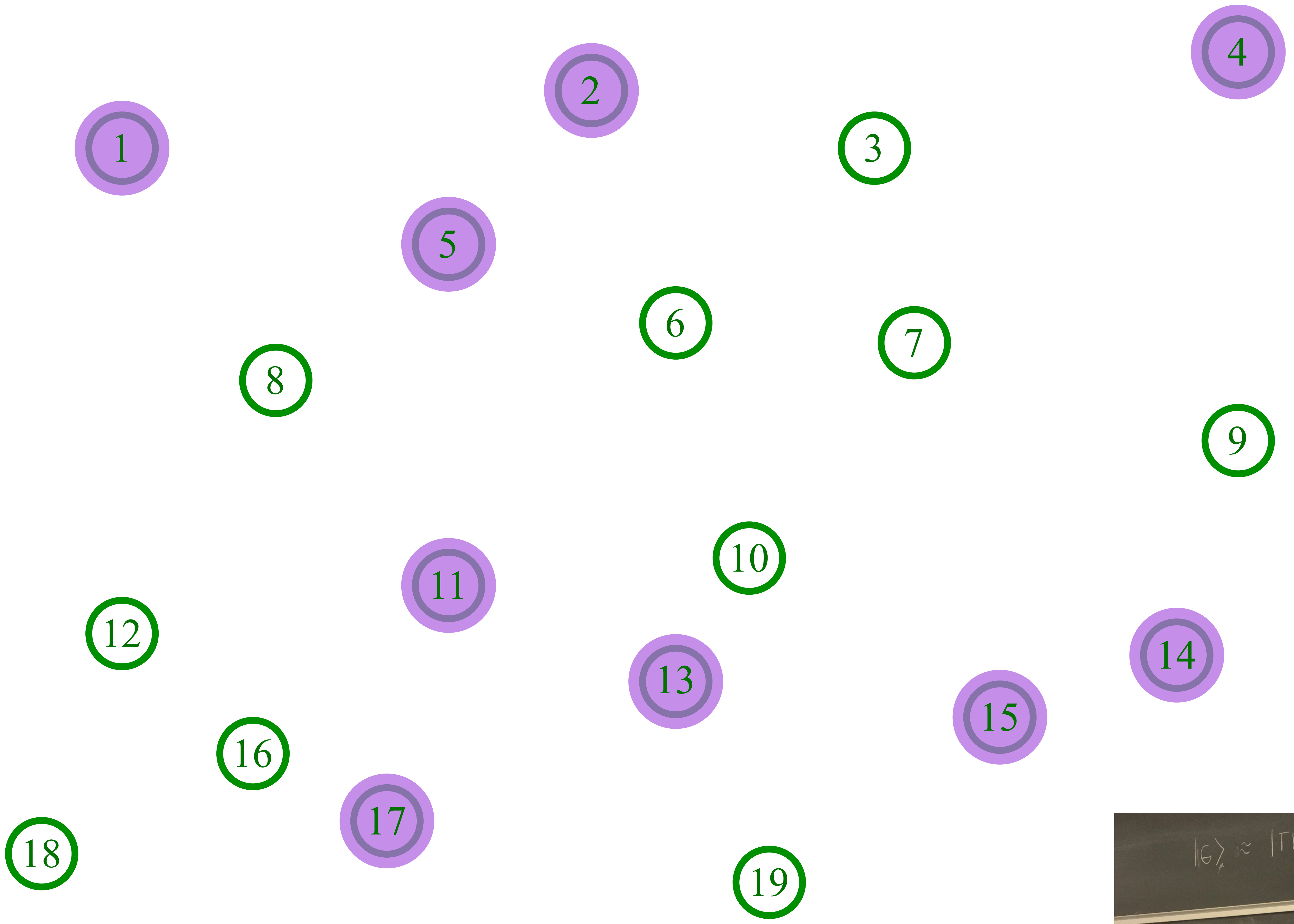

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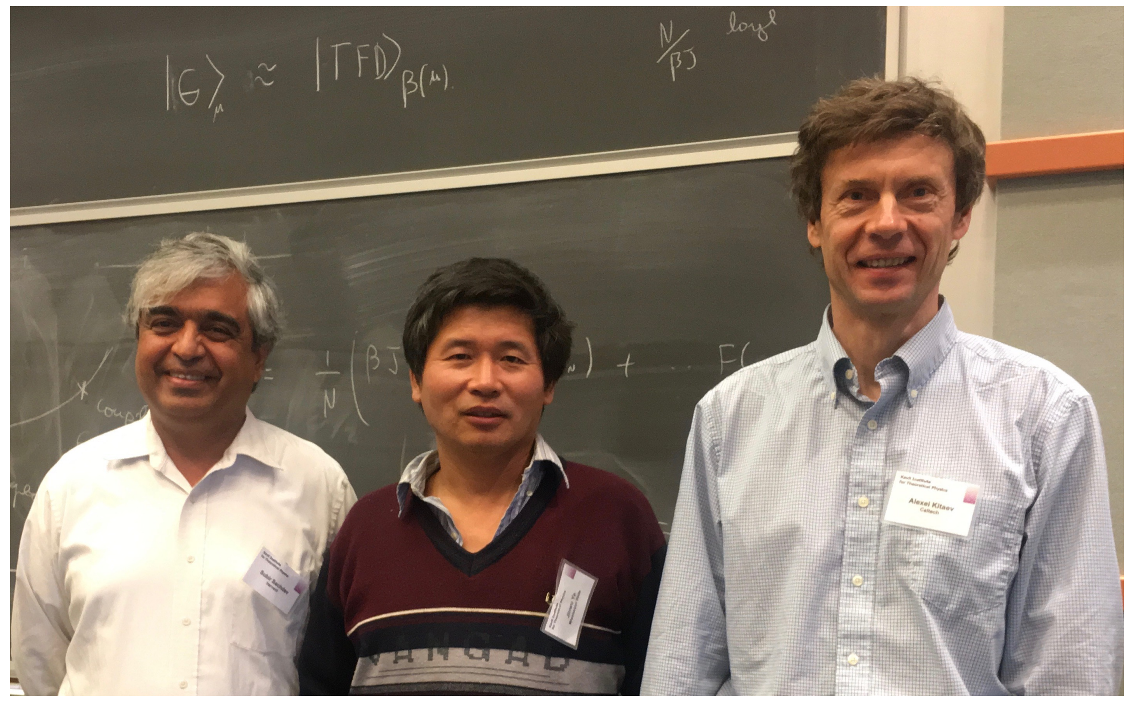

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

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Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

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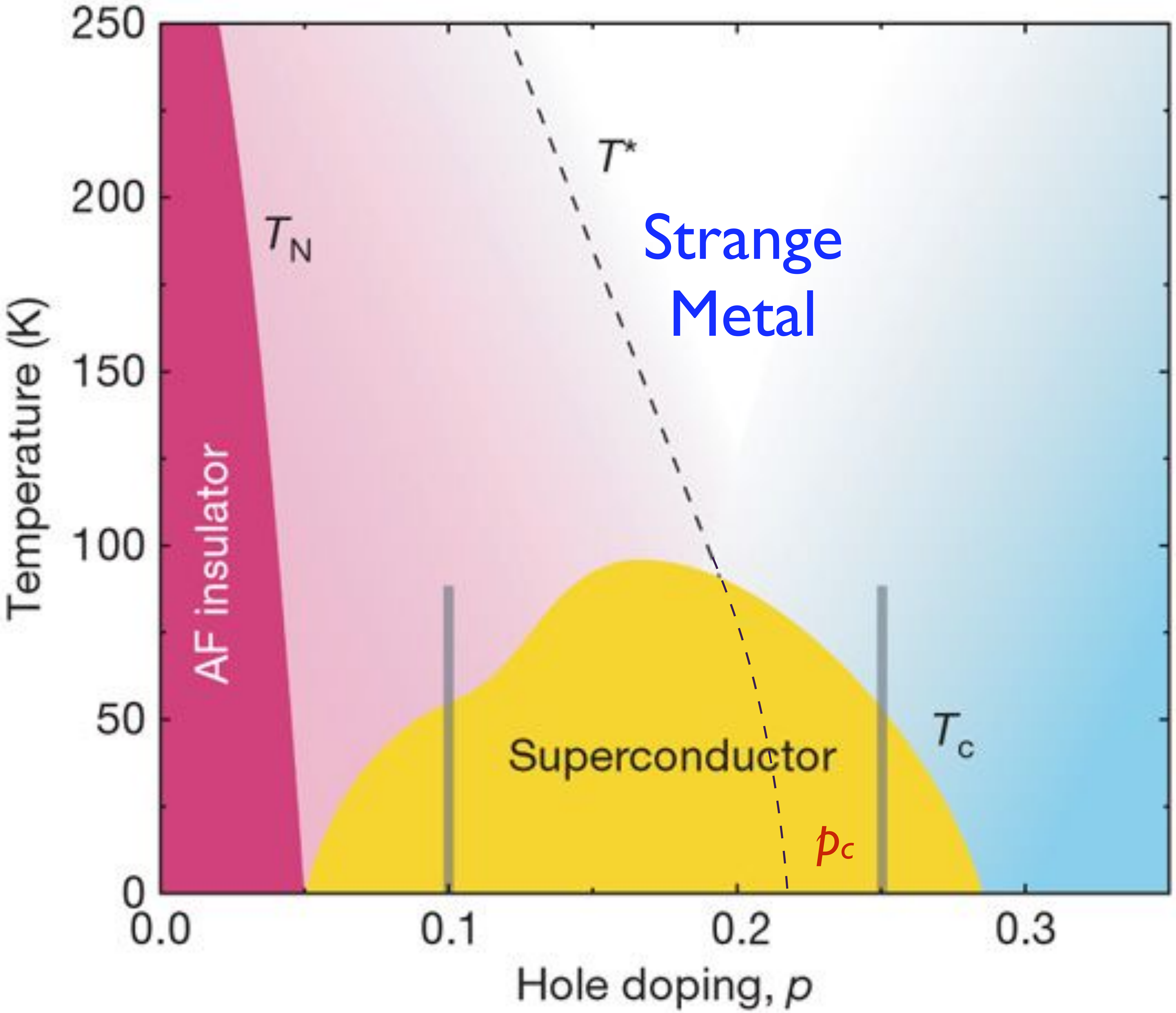
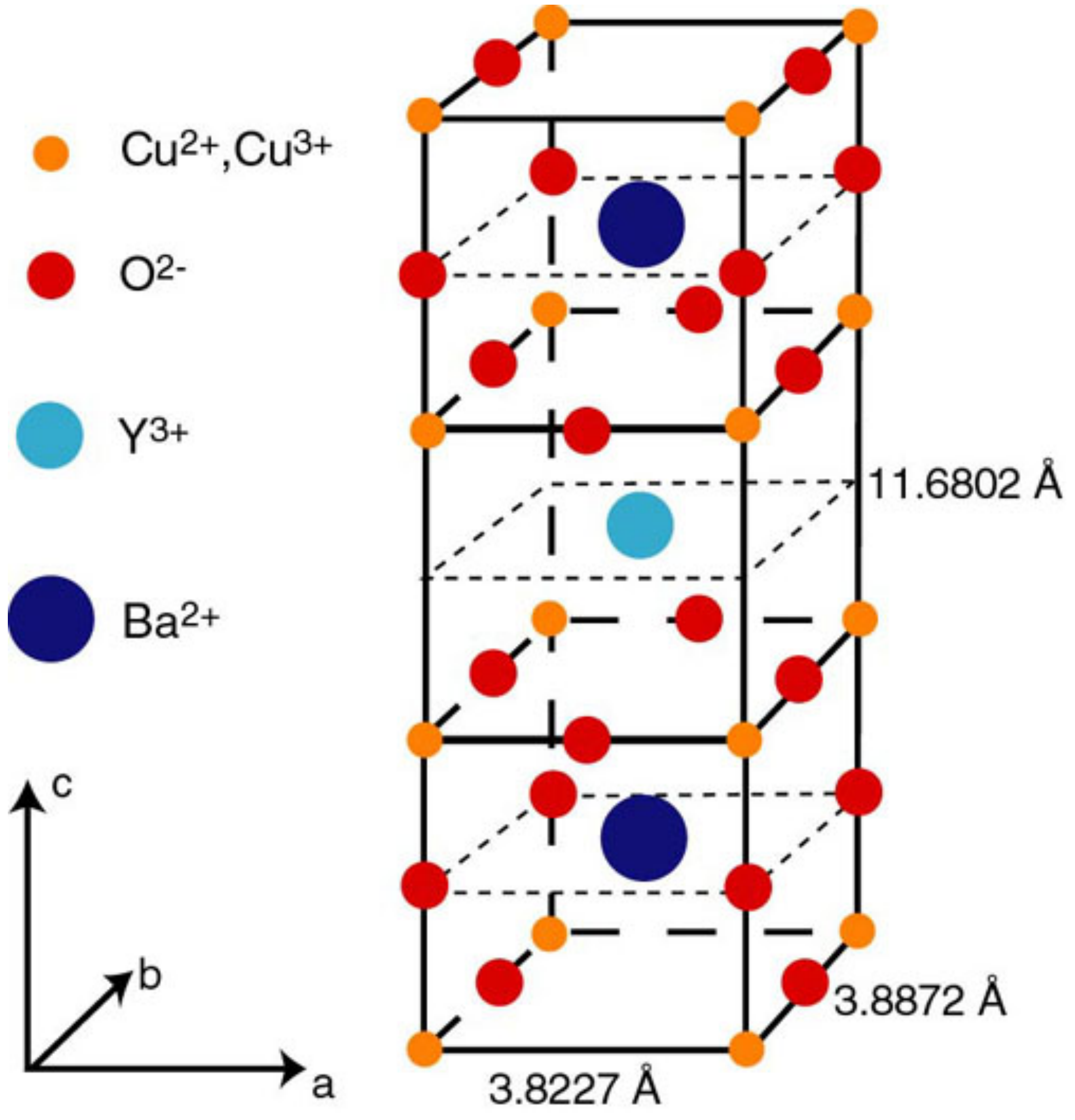
Current is carried by an "entangled quantum soup"





$YBa_2Cu_3O_{6+x}$

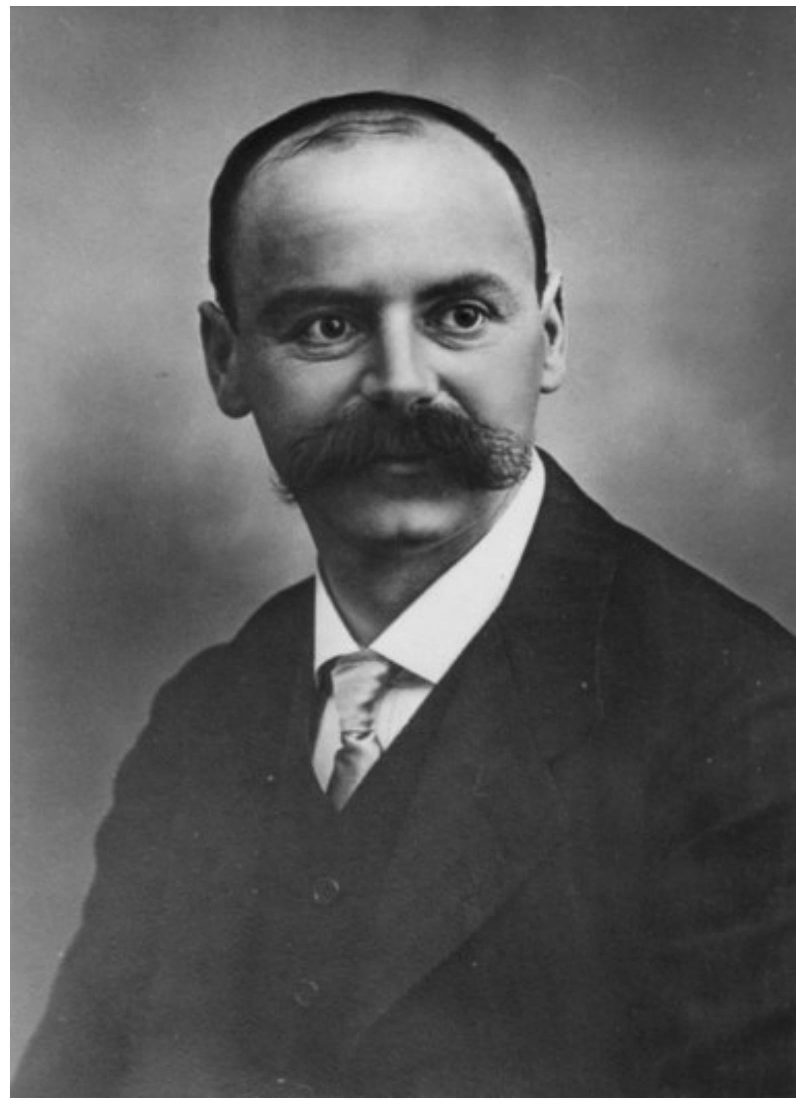
The "strange metal" has no particle-like/anyon excitations
and is described by a SYK-type theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Quantum entanglement,
the SYK model,
and black holes
holes

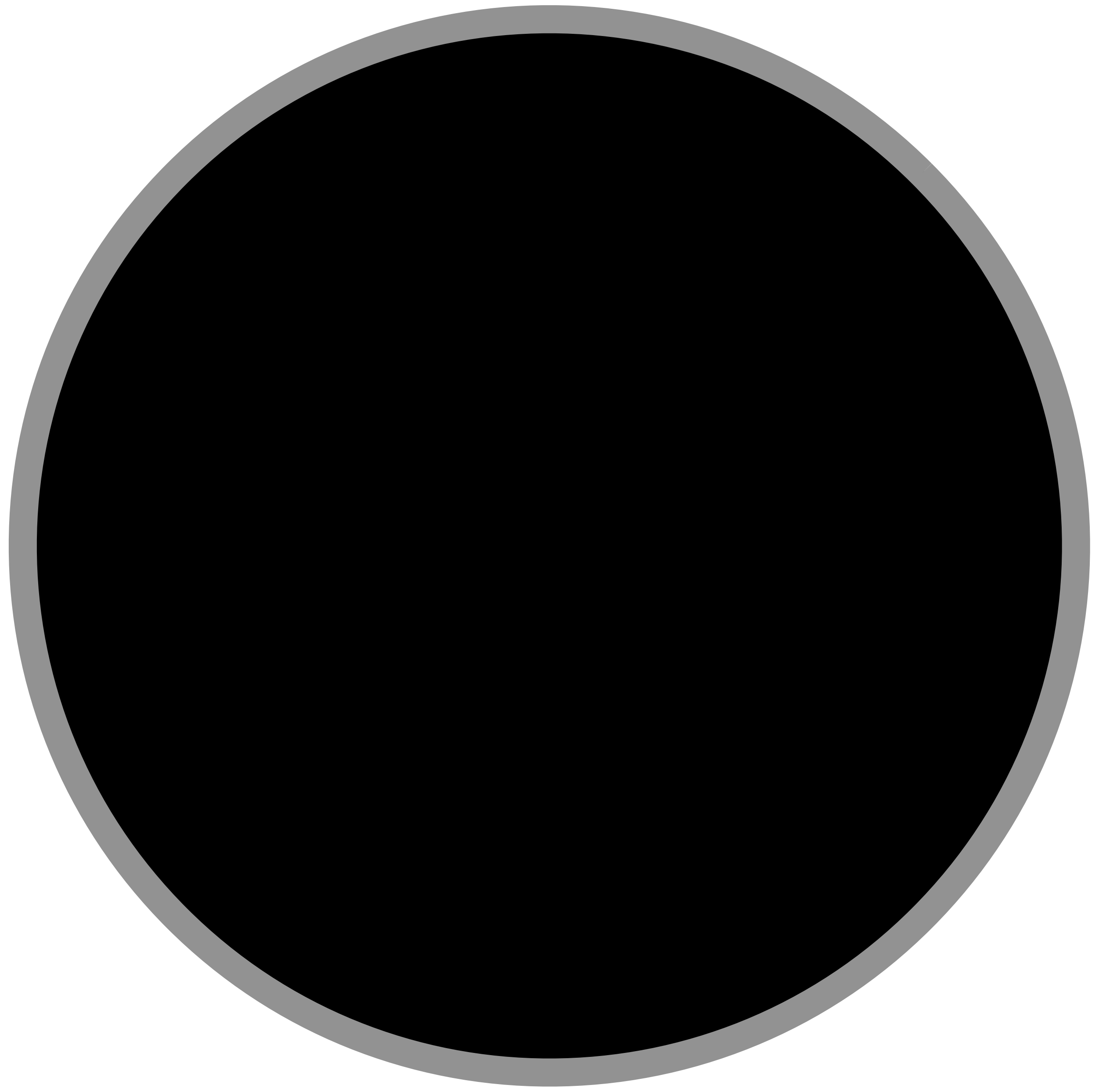
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

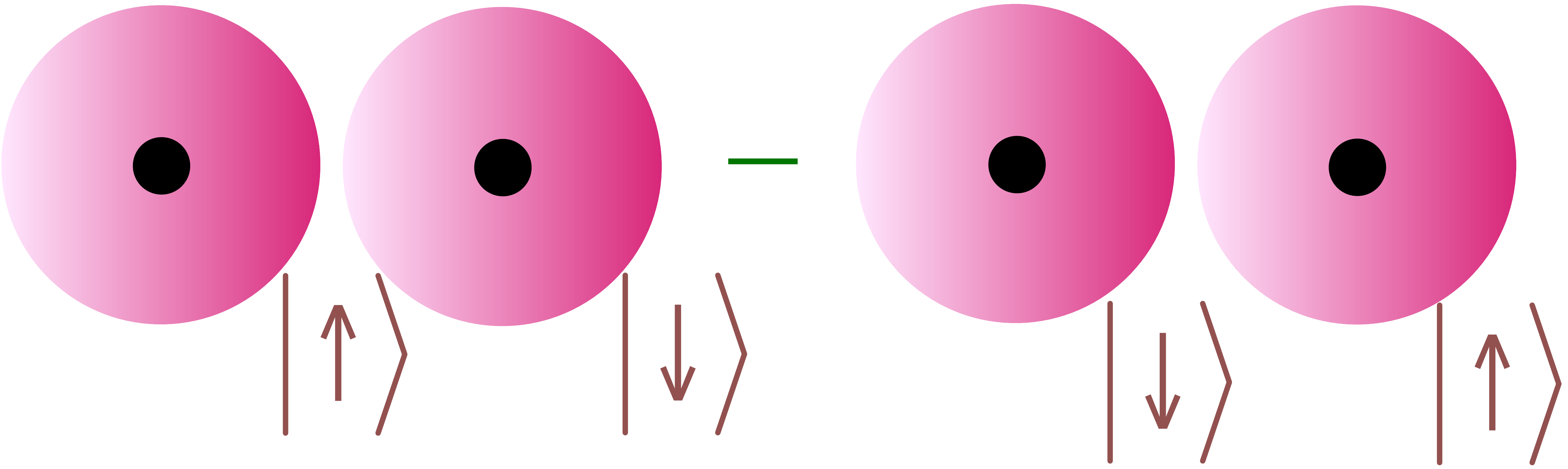
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



G Newton's constant, c velocity of light, M mass of black hole

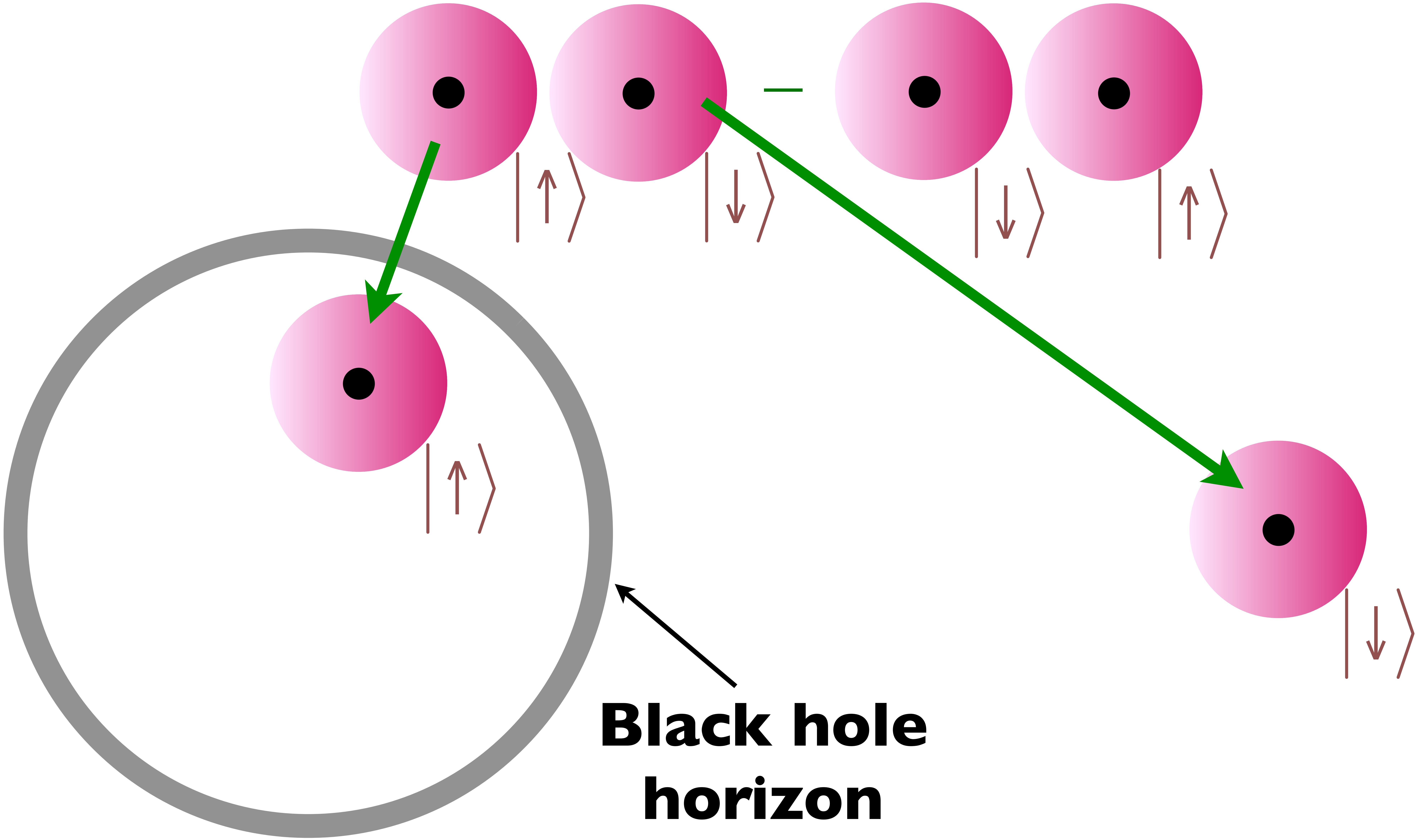
For $M =$ earth's mass, $R \approx 9\,mm$!

Quantum Entanglement across a black hole horizon



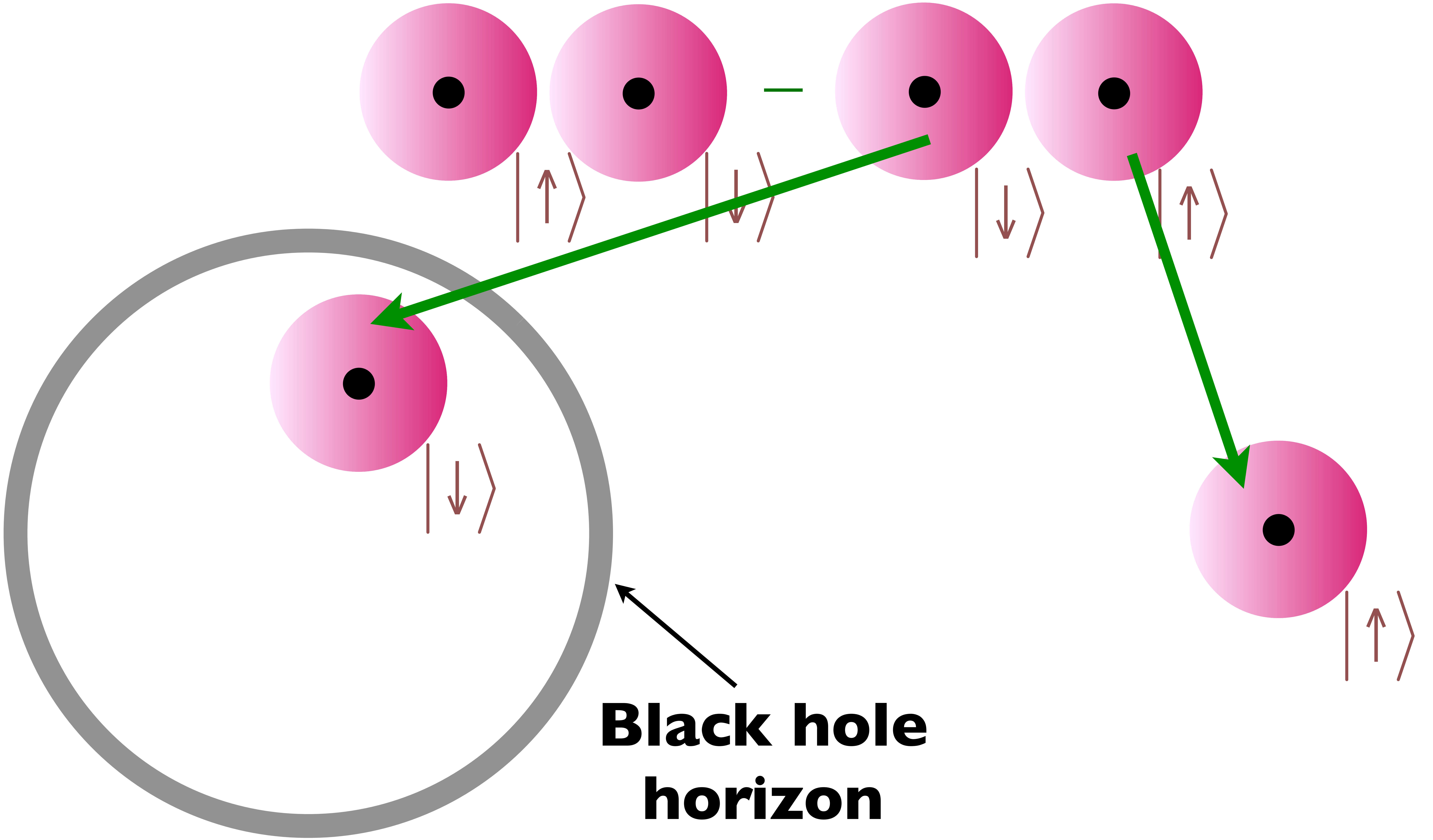
$$|\uparrow\rangle \, |\downarrow\rangle - |\downarrow\rangle \, |\uparrow\rangle$$

Quantum Entanglement across a black hole horizon



Black hole horizon

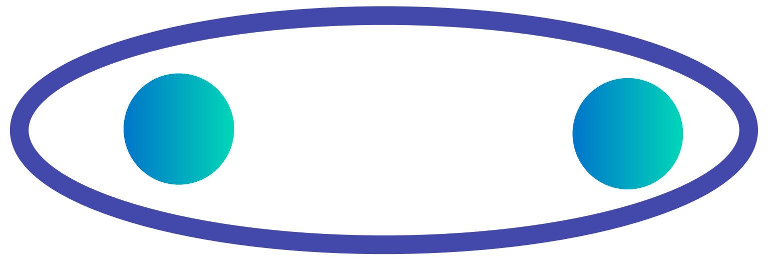
Quantum Entanglement across a black hole horizon



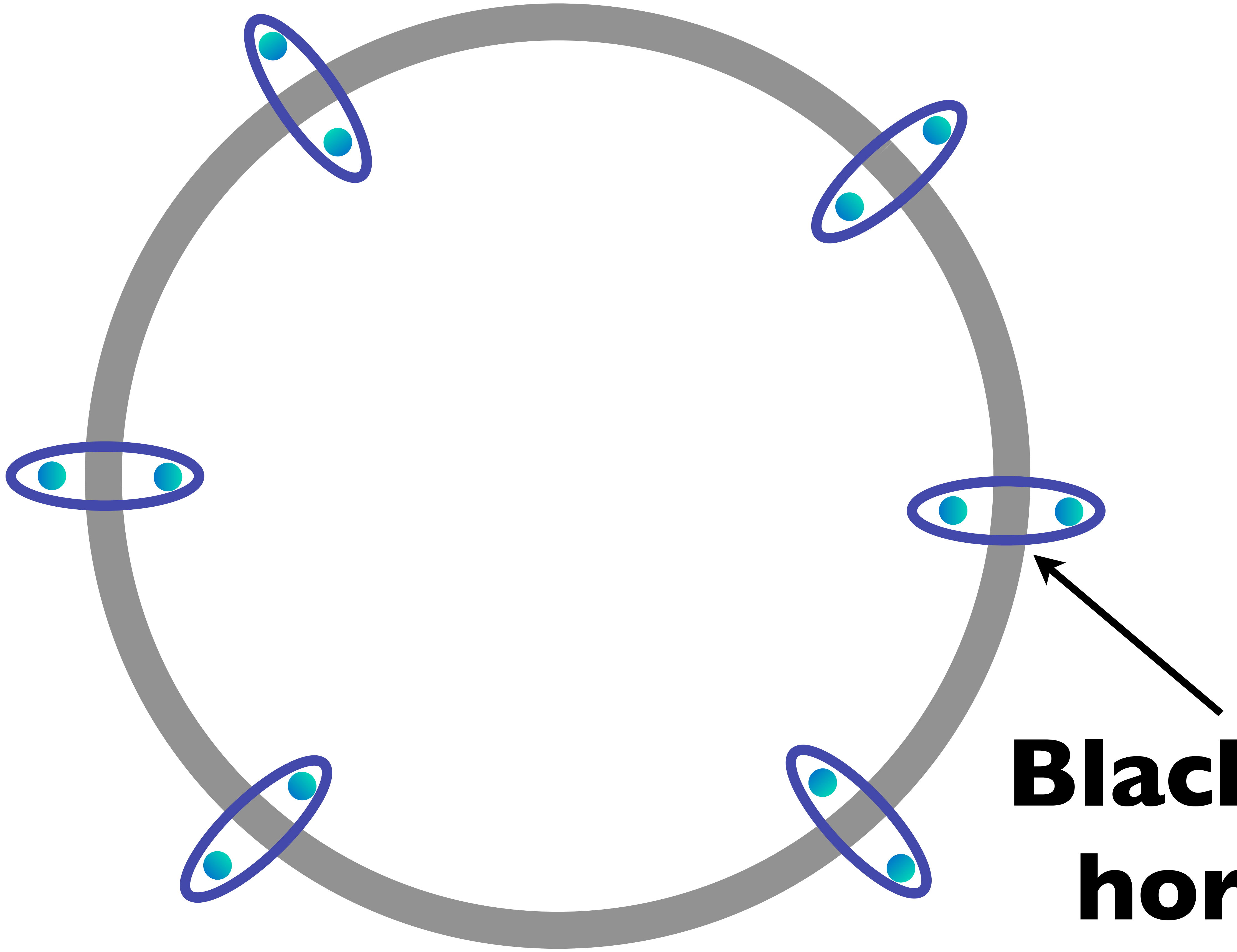
Black hole horizon

Quantum Entanglement across a black hole horizon

Quantum entanglement
on the surface



$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



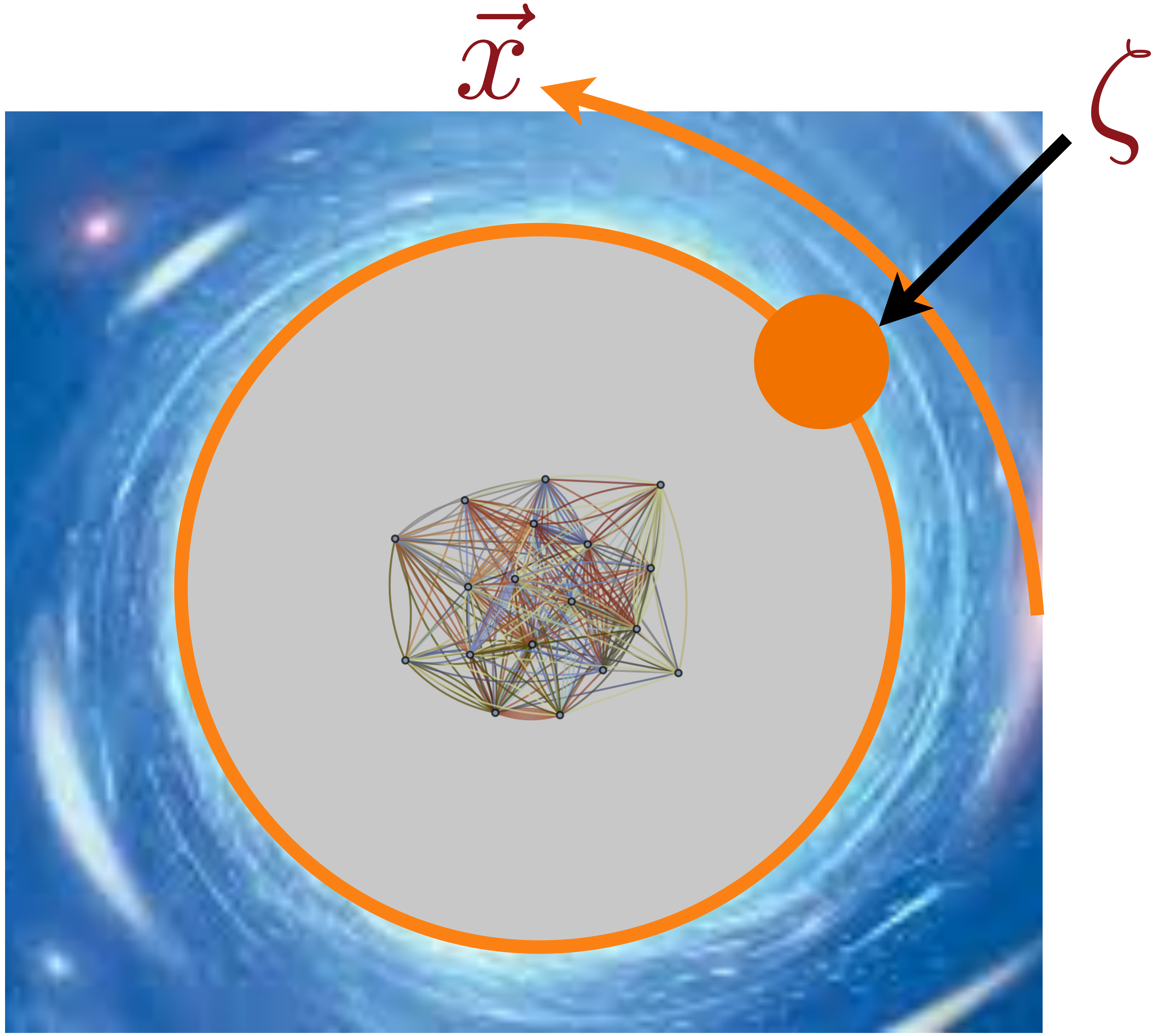
By computations *outside* the black hole, Hawking obtained the black hole entropy

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

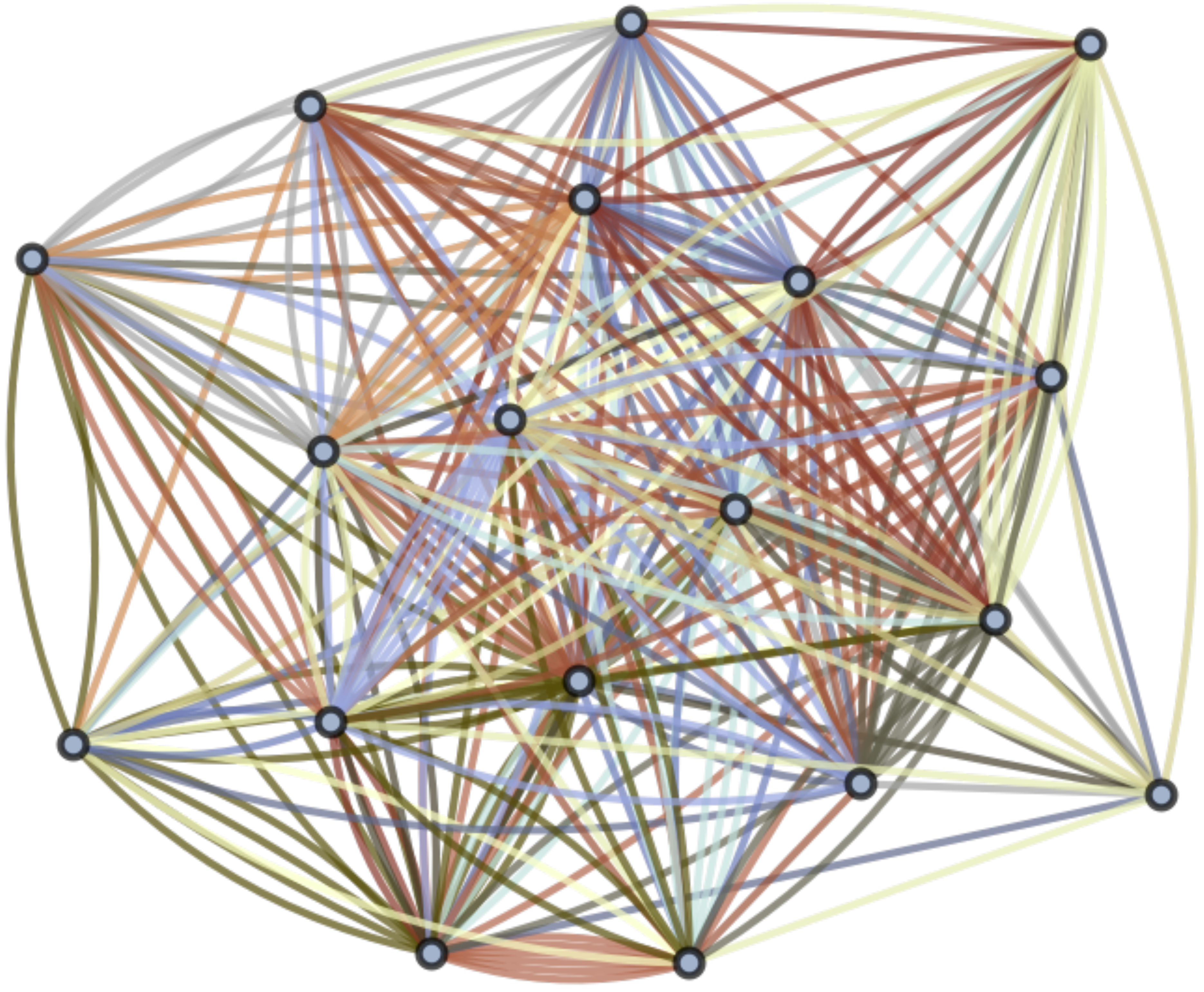
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge



The quantum versions of Maxwell’s and Einstein’s equations in ζ space and time are also the equations describing electron entanglement in the SYK model!

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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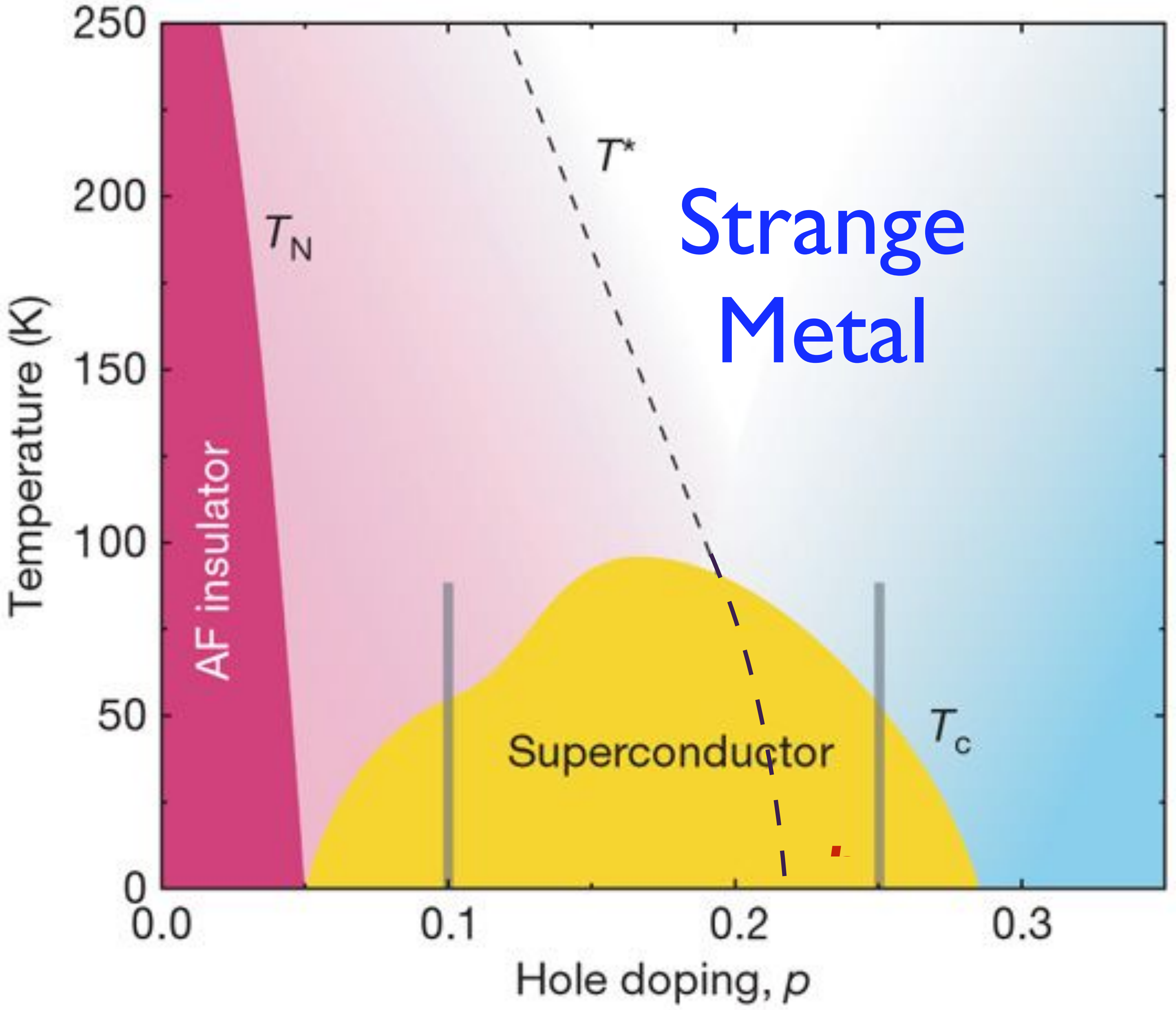
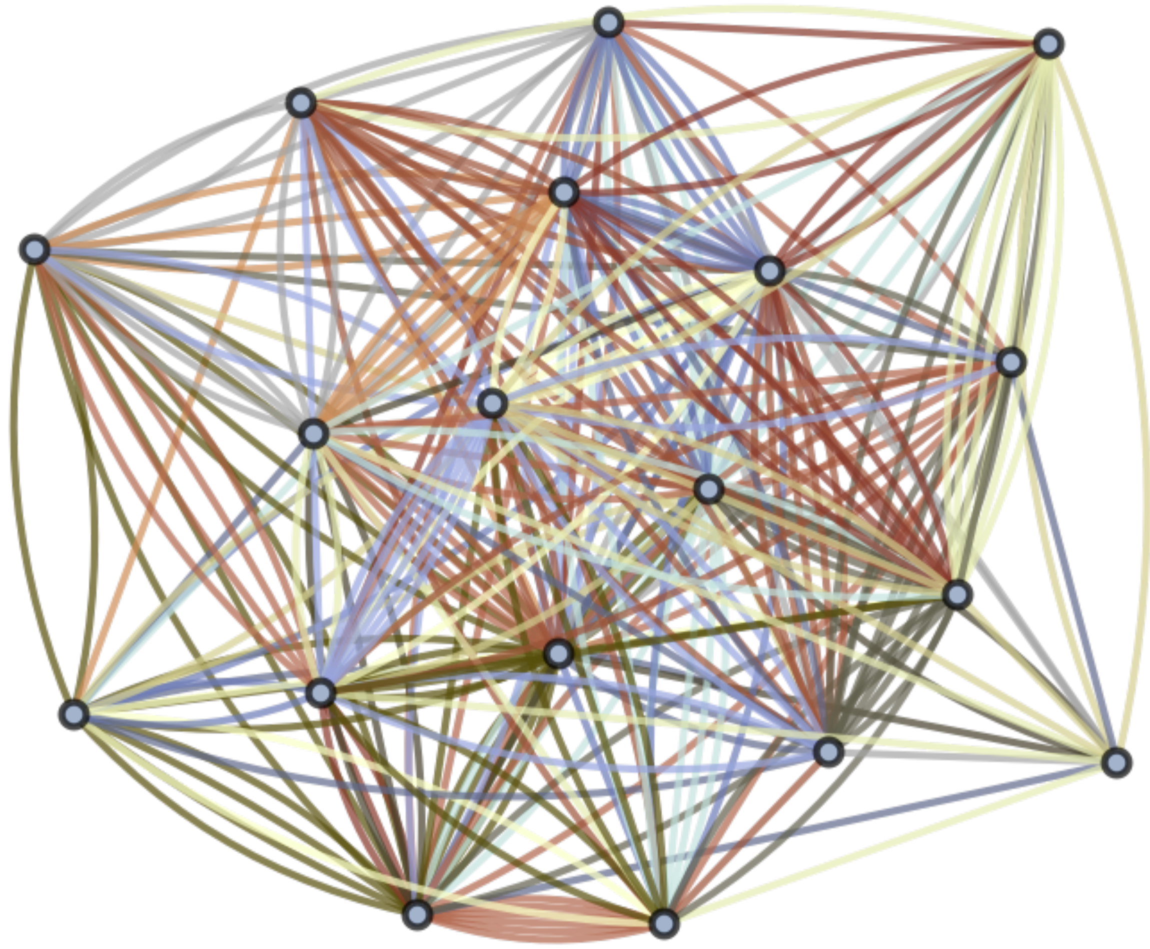


In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



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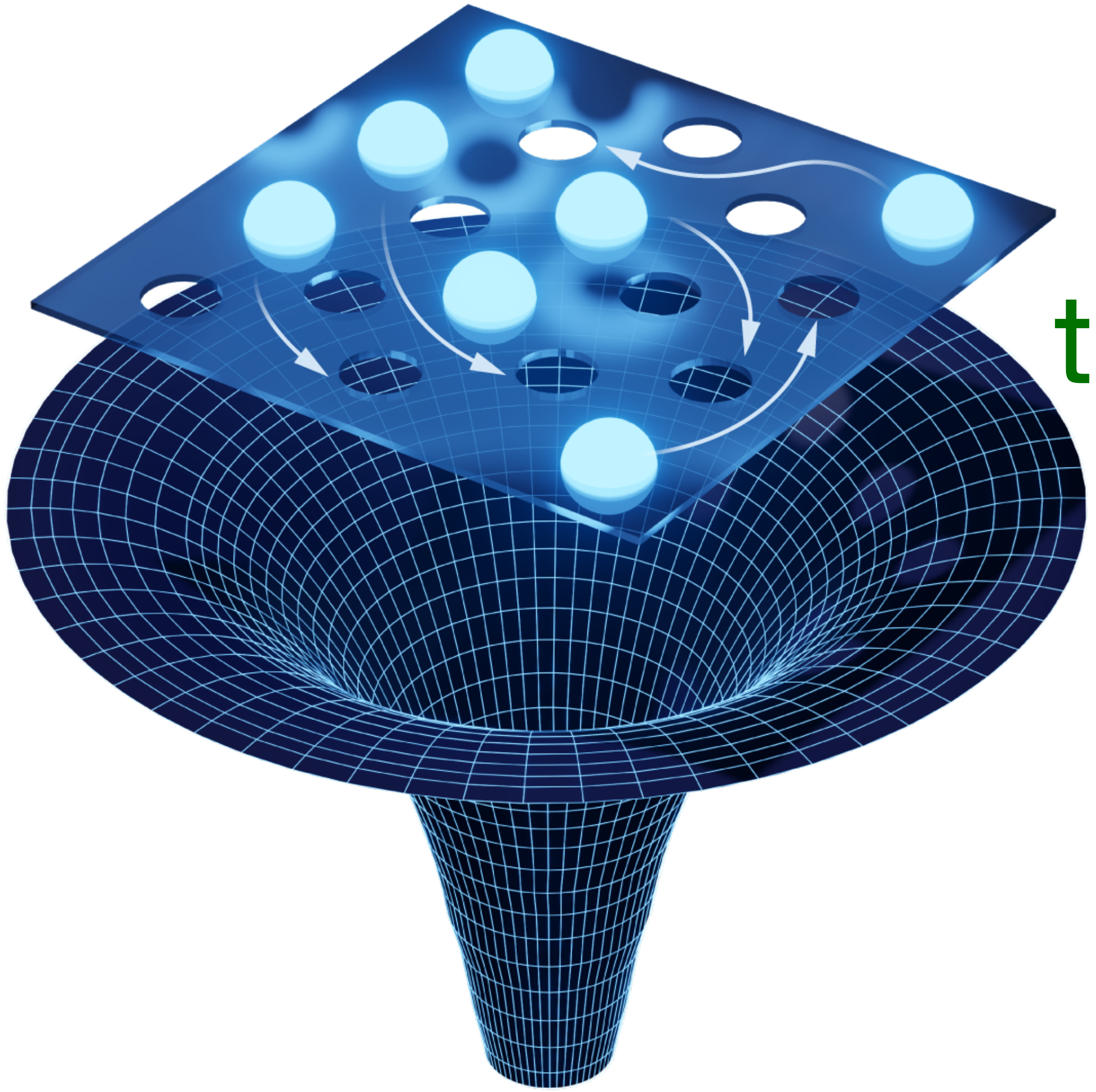


In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

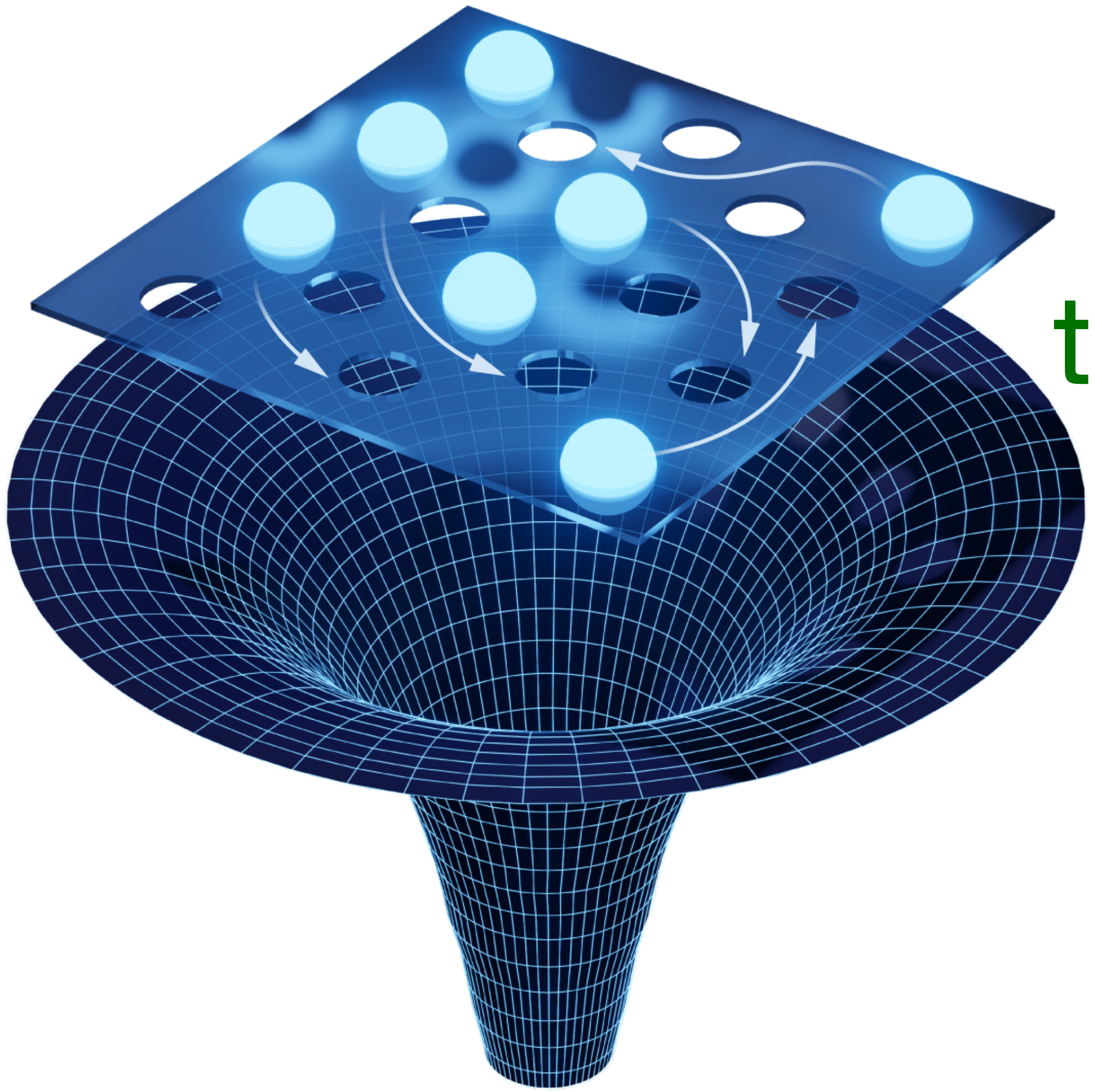
Sachdev, Ye (1993)

In a *dual* set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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the interior of *charged black holes*

Can the complex quantum entanglement in the SYK model lead to a quantum advantage?

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)