# Where is the quantum critical point in the cuprate superconductors ?

arXiv:0907.0008

PHYSICS
HARVARD

Hole dynamics in an antiferromagnet across a deconfined quantum critical point,
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,
*Phys. Rev. B* **75**, 235122 (2007).

Algebraic charge liquids
R. K. Kaul, Yong-Baek Kim, S. Sachdev, and T. Senthil,
*Nature Physics* **4**, 28 (2008).

Destruction of Neel order in the cuprates by electron doping,
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu,
*Physical Review B* **78**, 045110 (2008).

Paired electron pockets in the underdoped cuprates,
V. Galitski and S. Sachdev,
*Physical Review B* **79**, 134512 (2009).

Competition between spin density wave order and superconductivity in the underdoped cuprates,
Eun Gook Moon and S. Sachdev,
*Physical Review B* **80**, 035117 (2009).

Fluctuating spin density waves in metals
S. Sachdev, M. Metlitski, Yang Qi, and Cenke Xu
arXiv:0907.3732

PHYSICS
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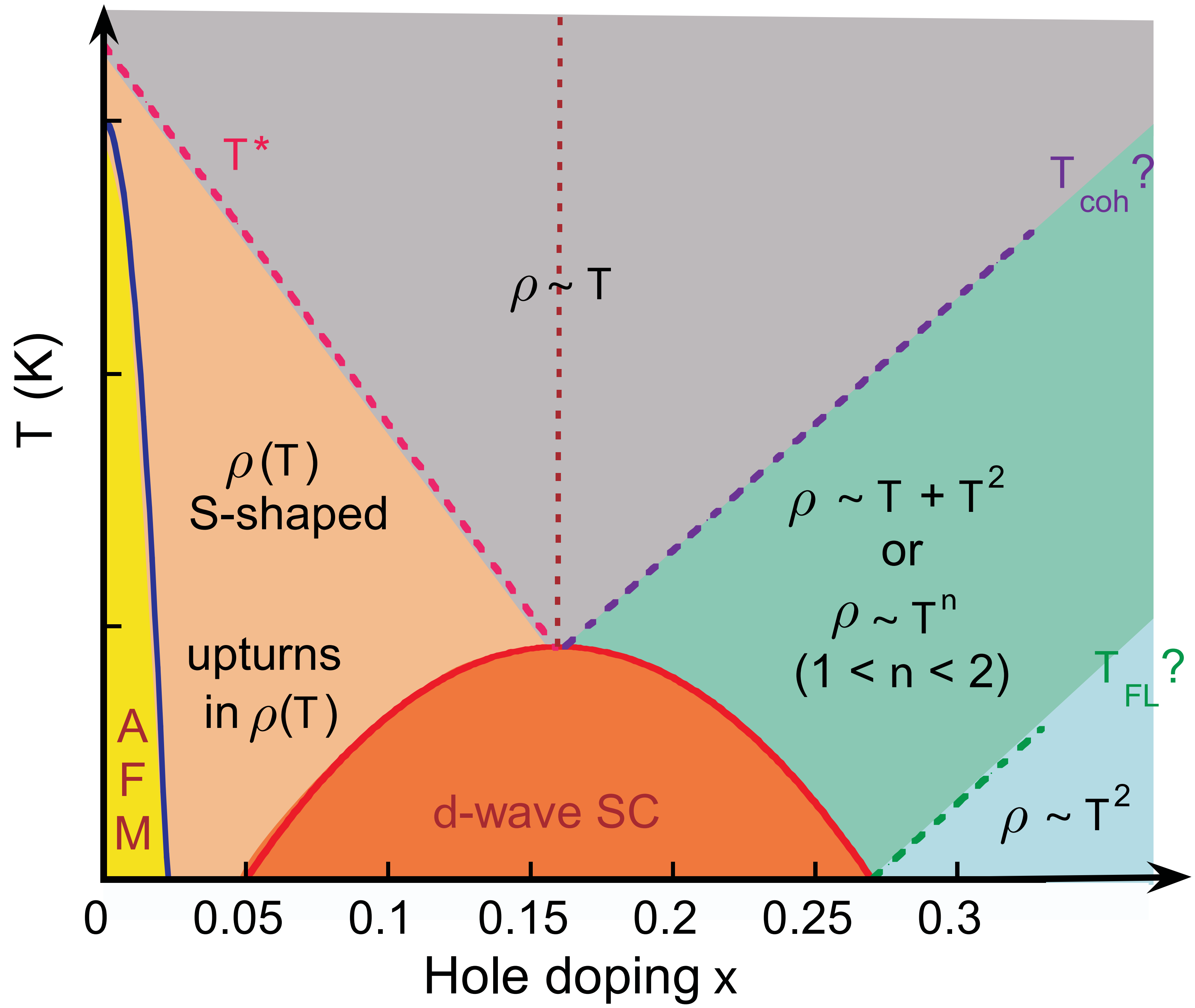
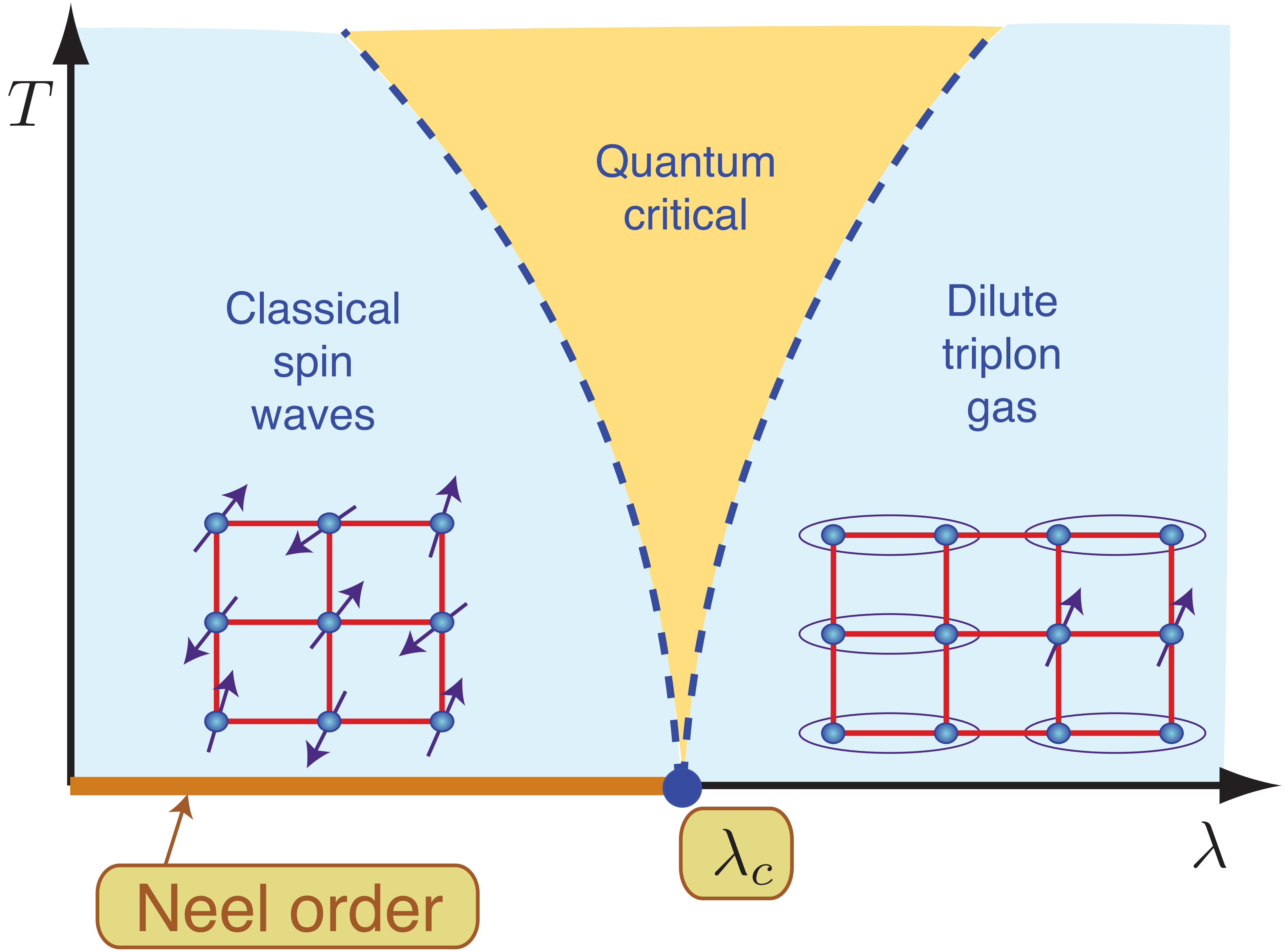
# Crossovers in transport properties of hole-doped cuprates




N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)




Pressure in $TlCuCl_3$

Christian Ruegg et al. , *Phys. Rev. Lett.* **100**, 205701 (2008)

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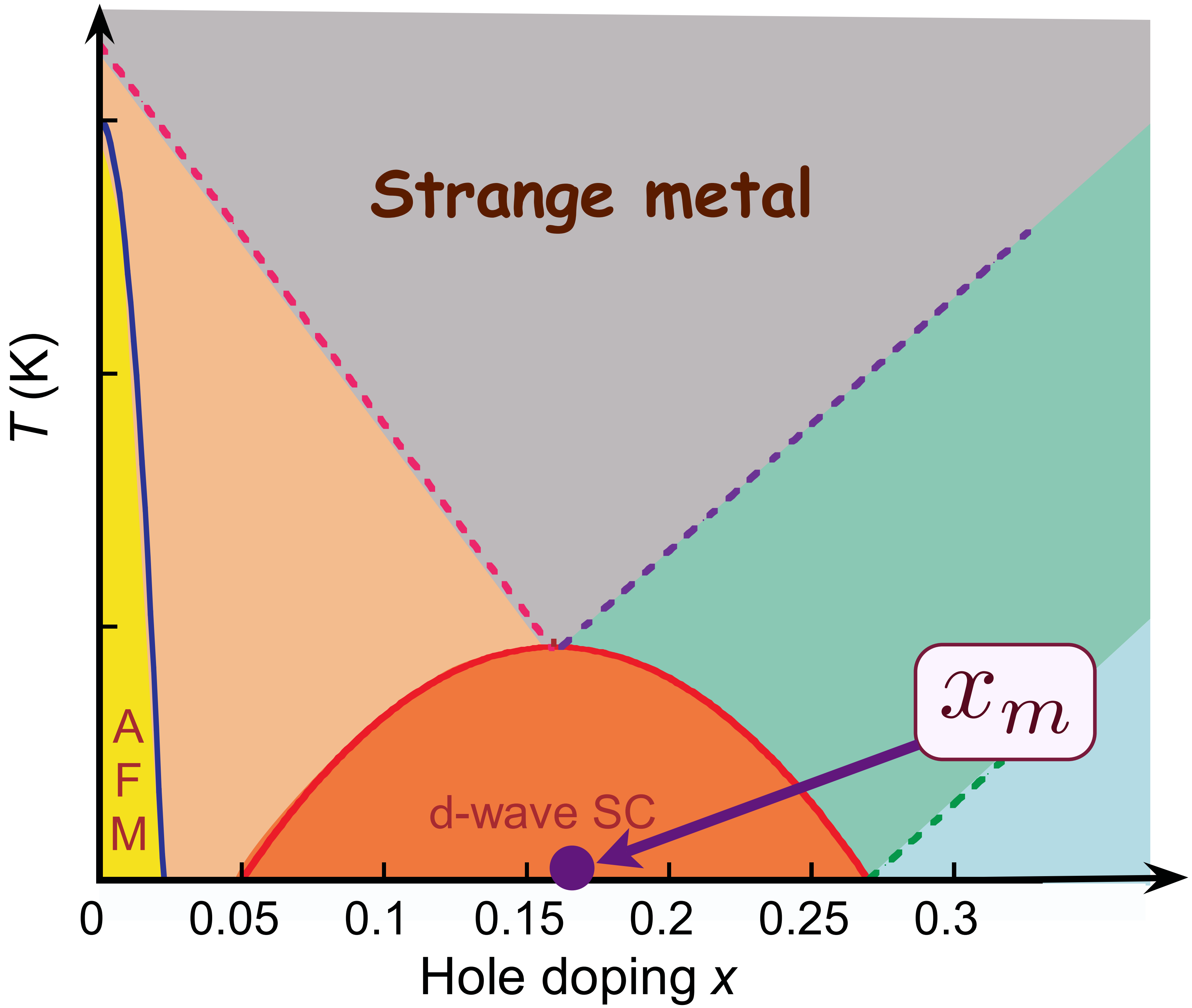
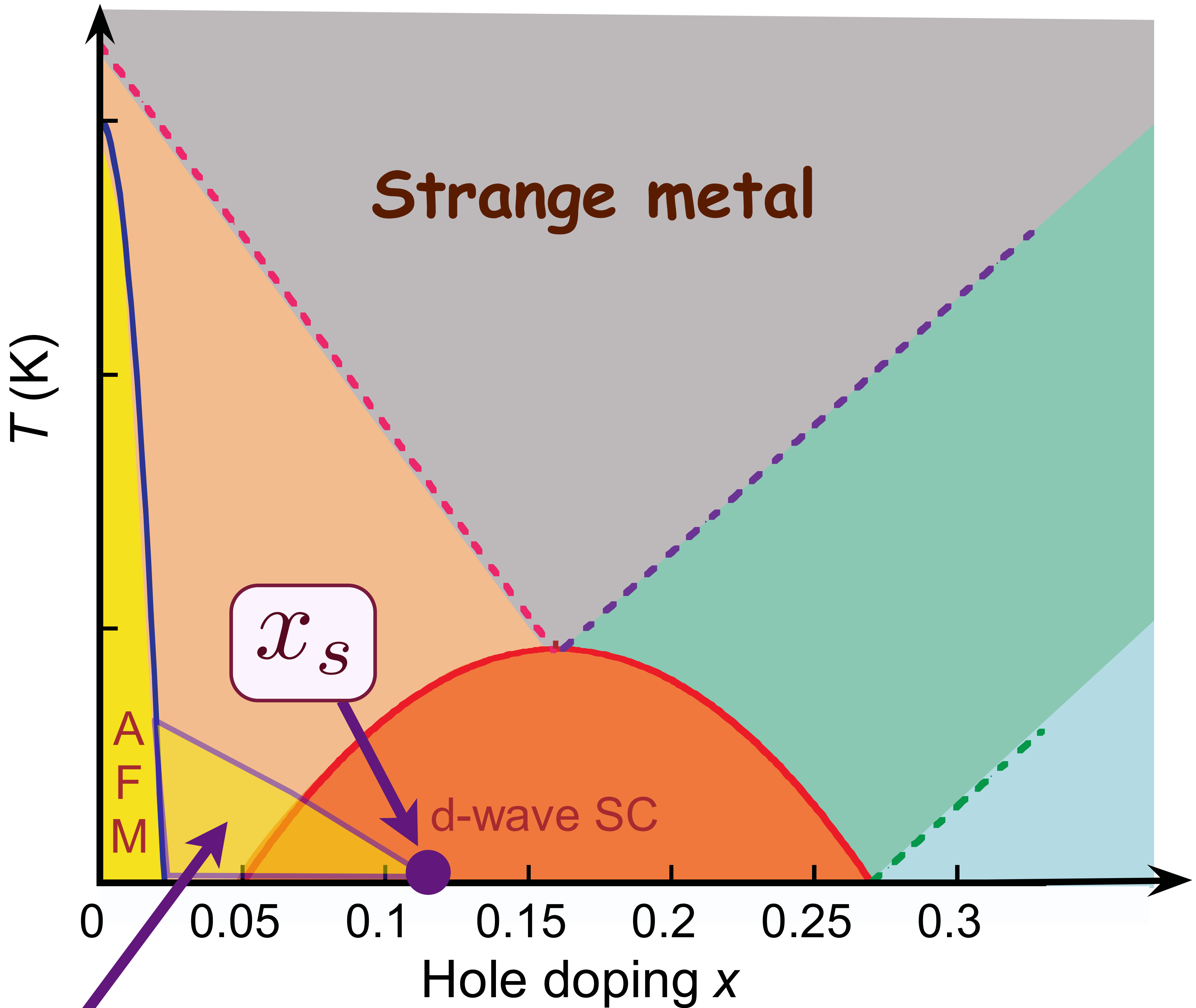



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

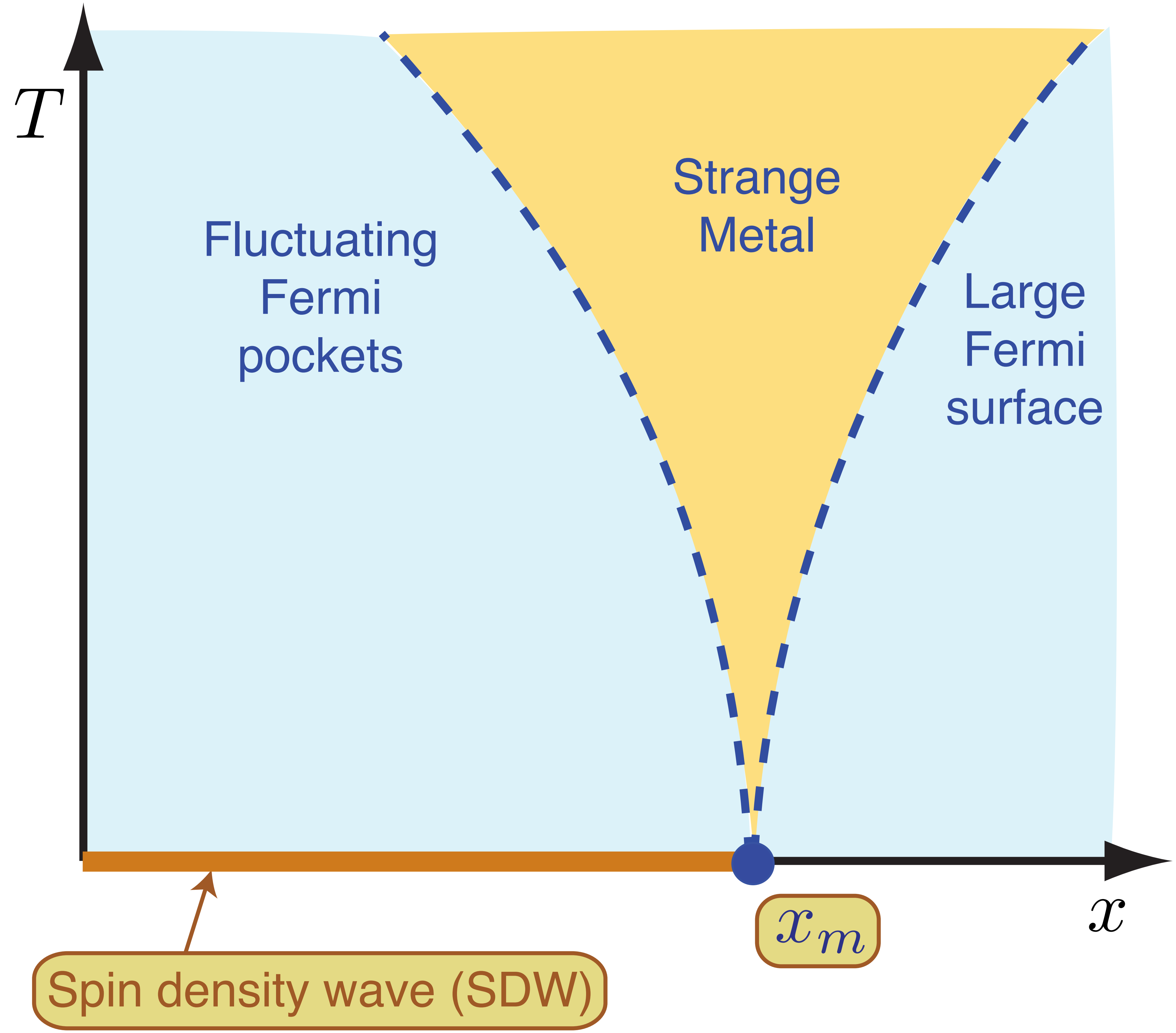
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$ ?

# Only candidate quantum critical point observed at low $T$



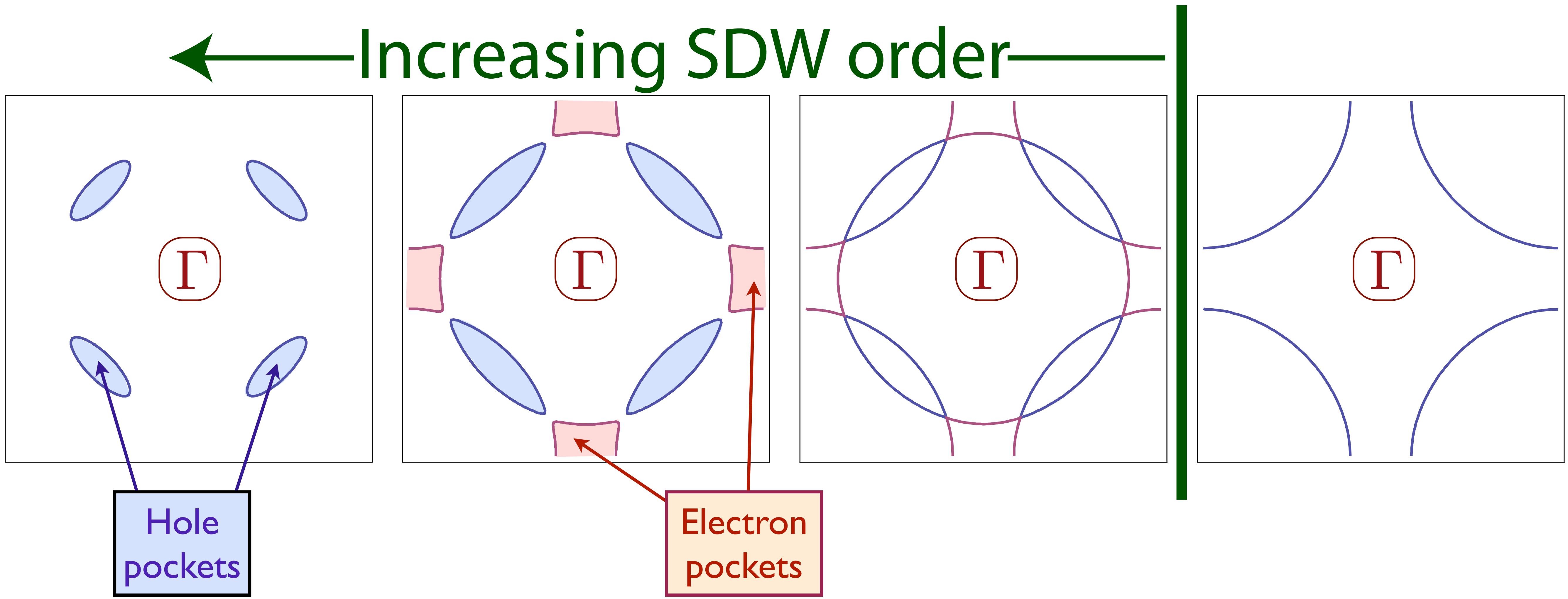

Spin and charge density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

# Theory of quantum criticality in the cuprates




Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

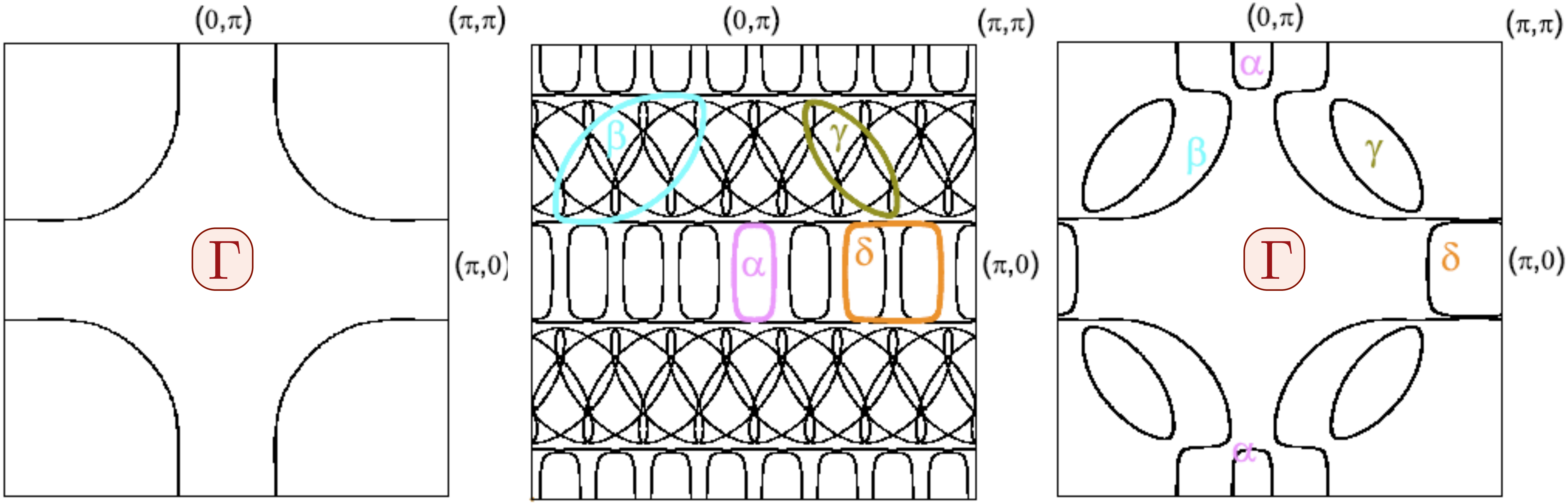
# Spin density wave theory in hole-doped cuprates




The amplitude of the SDW order parameter $\vec{\varphi}$ vanishes at the transition to the Fermi liquid.

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

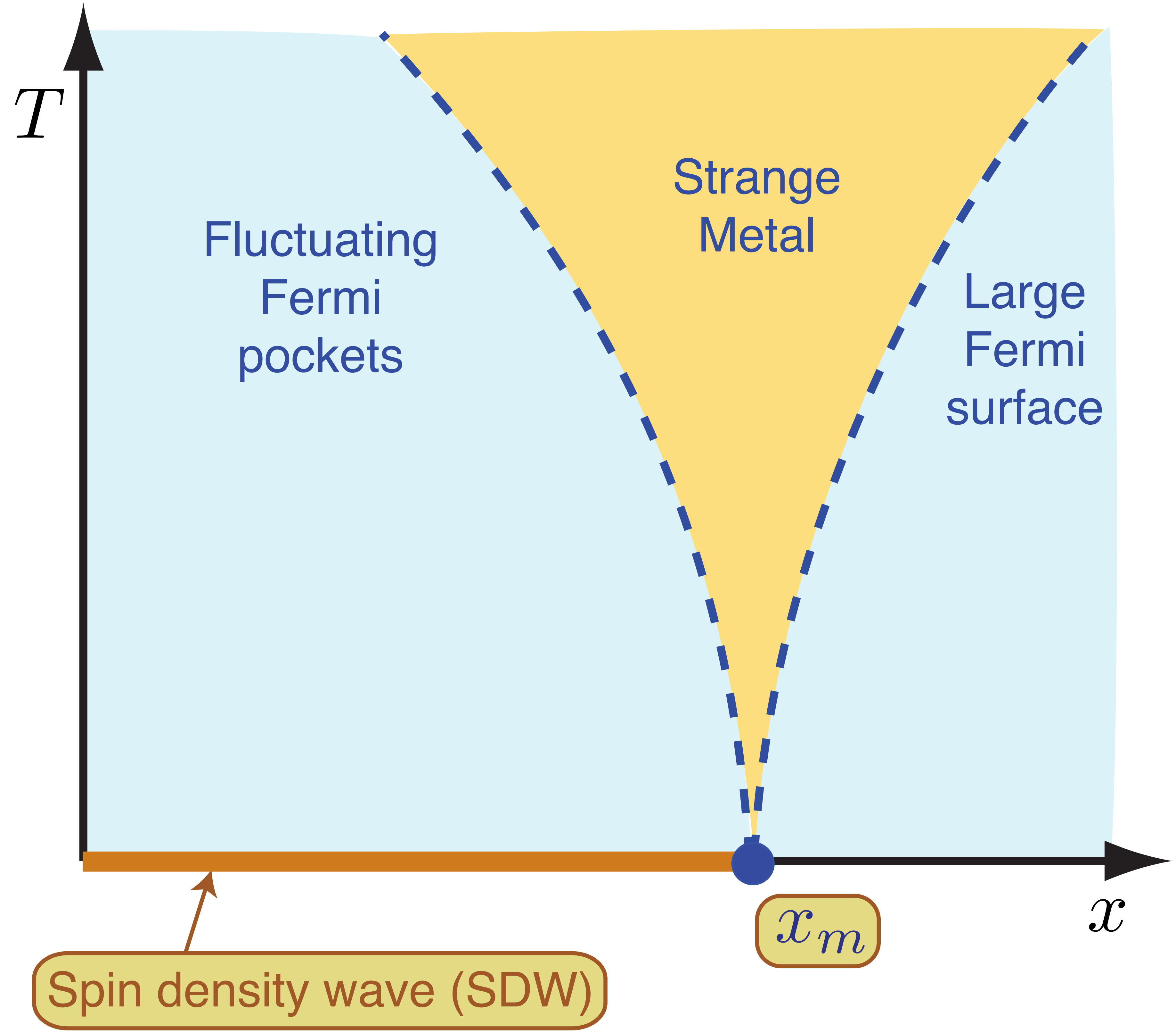
# Spin density wave theory in hole-doped cuprates




## Incommensurate order in $YBa_2Cu_3O_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

# Theory of quantum criticality in the cuprates




Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

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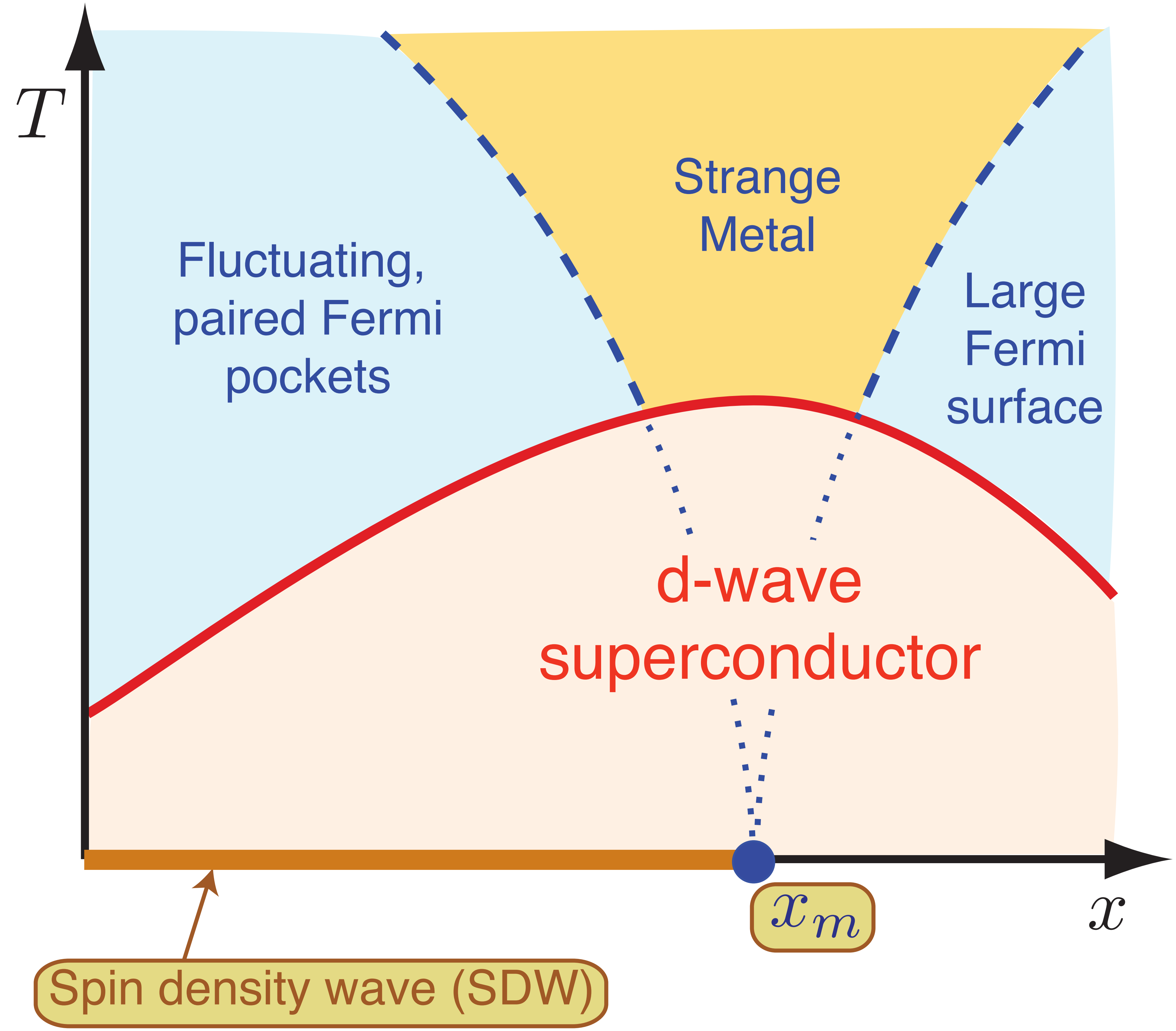
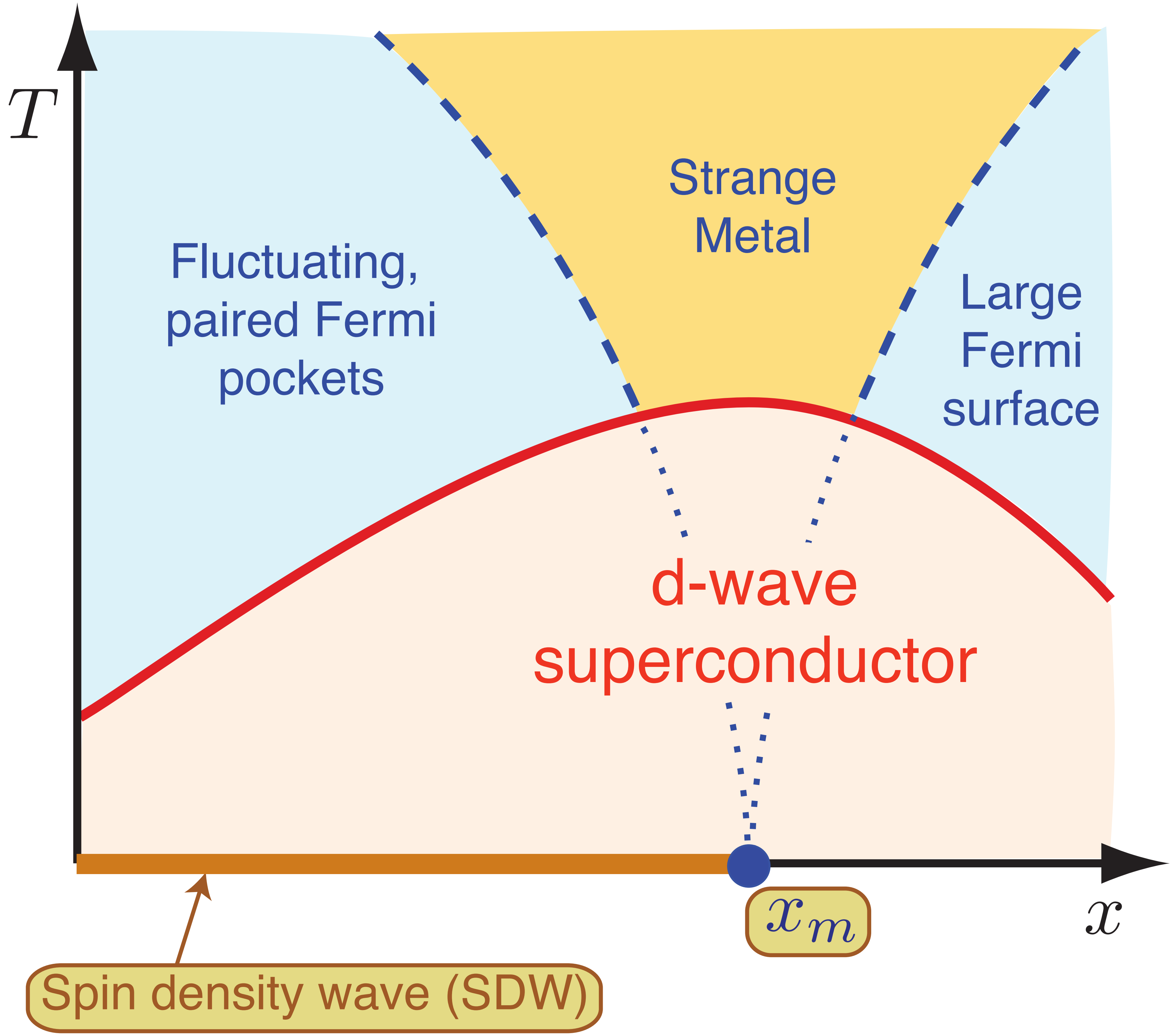


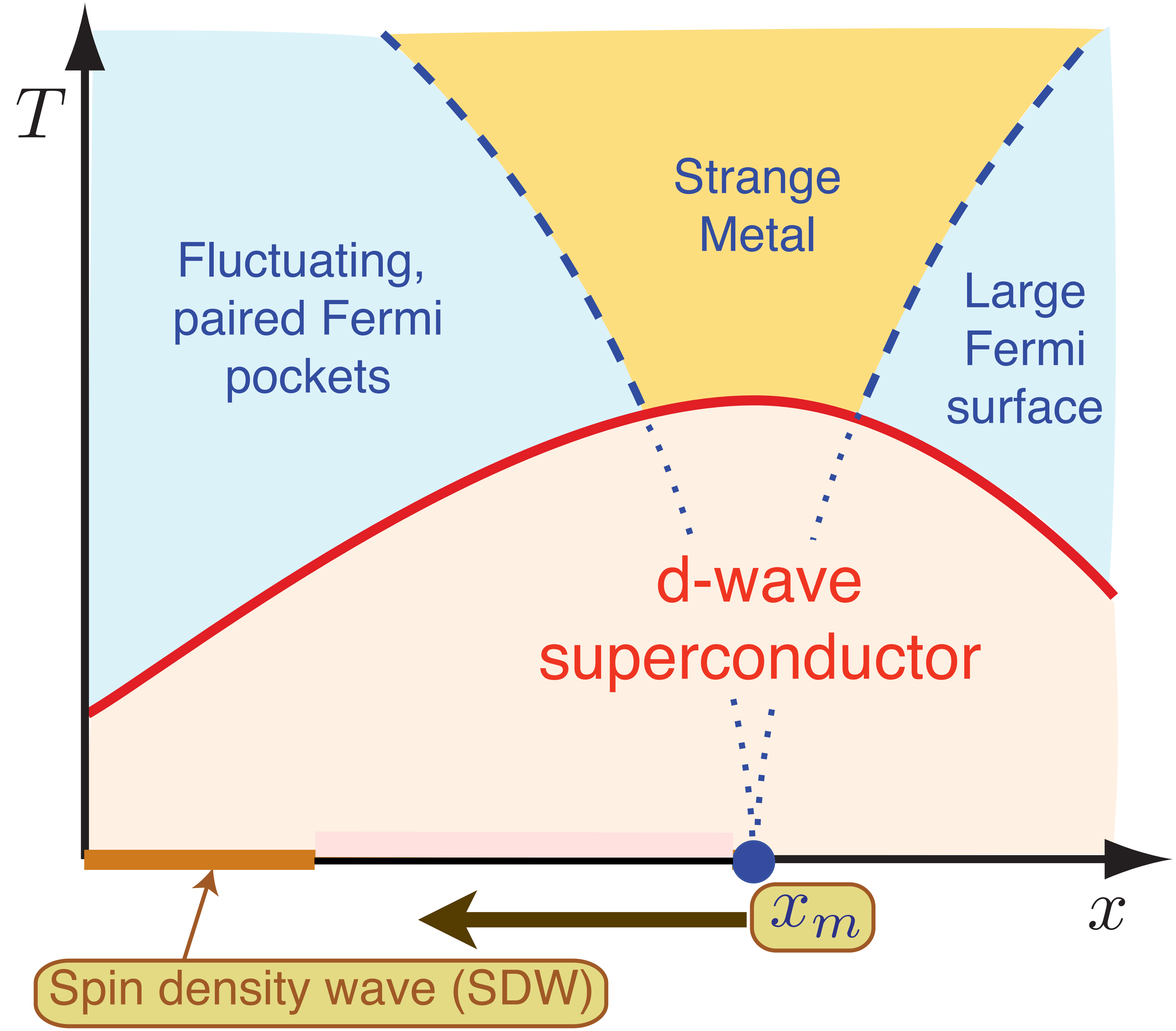

Onset of $d$-wave superconductivity hides the critical point $x = x_m$

# Theory of quantum criticality in the cuprates




Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

# Theory of quantum criticality in the cuprates

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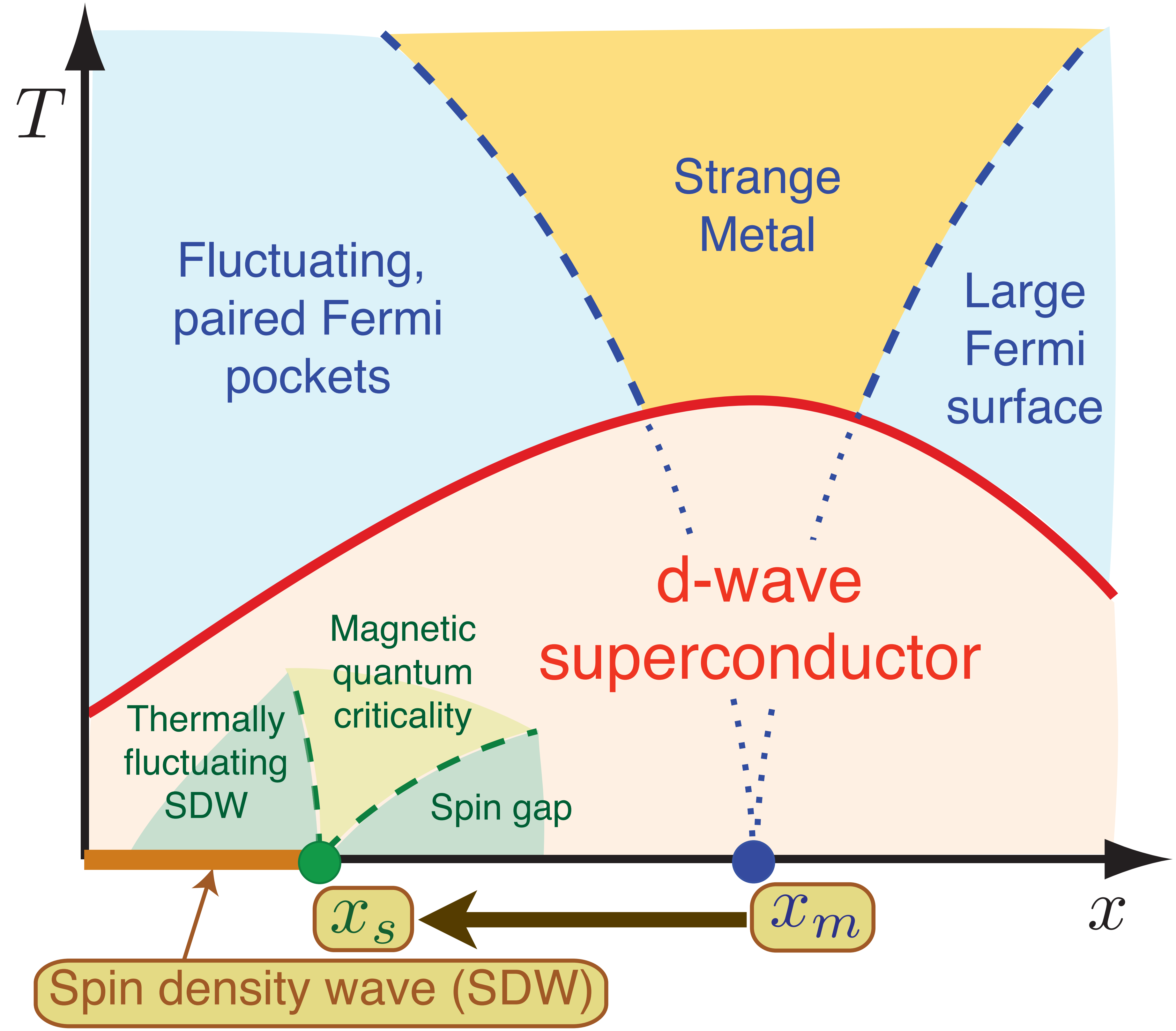


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# Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order
   *Survey of recent experiments*

2. Superconductivity in the overdoped regime
   *BCS pairing by spin fluctuation exchange*

3. Superconductivity in the underdoped regime
   *U(1) gauge theory of fluctuating SDW order*

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# Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity ($\psi$):

$$\mathcal{S} = \int d^2r d\tau \left[ \frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 \right.$$
$$\left. + \kappa\, \vec{\varphi}^2\, |\psi|^2 \right]$$
$$+ \int d^2r \left[ |(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
See also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004);
S. A. Kivelson, D.-H. Lee, E. Fradkin, and V. Oganesyan, *Phys. Rev. B* **66**, 144516 (2002).

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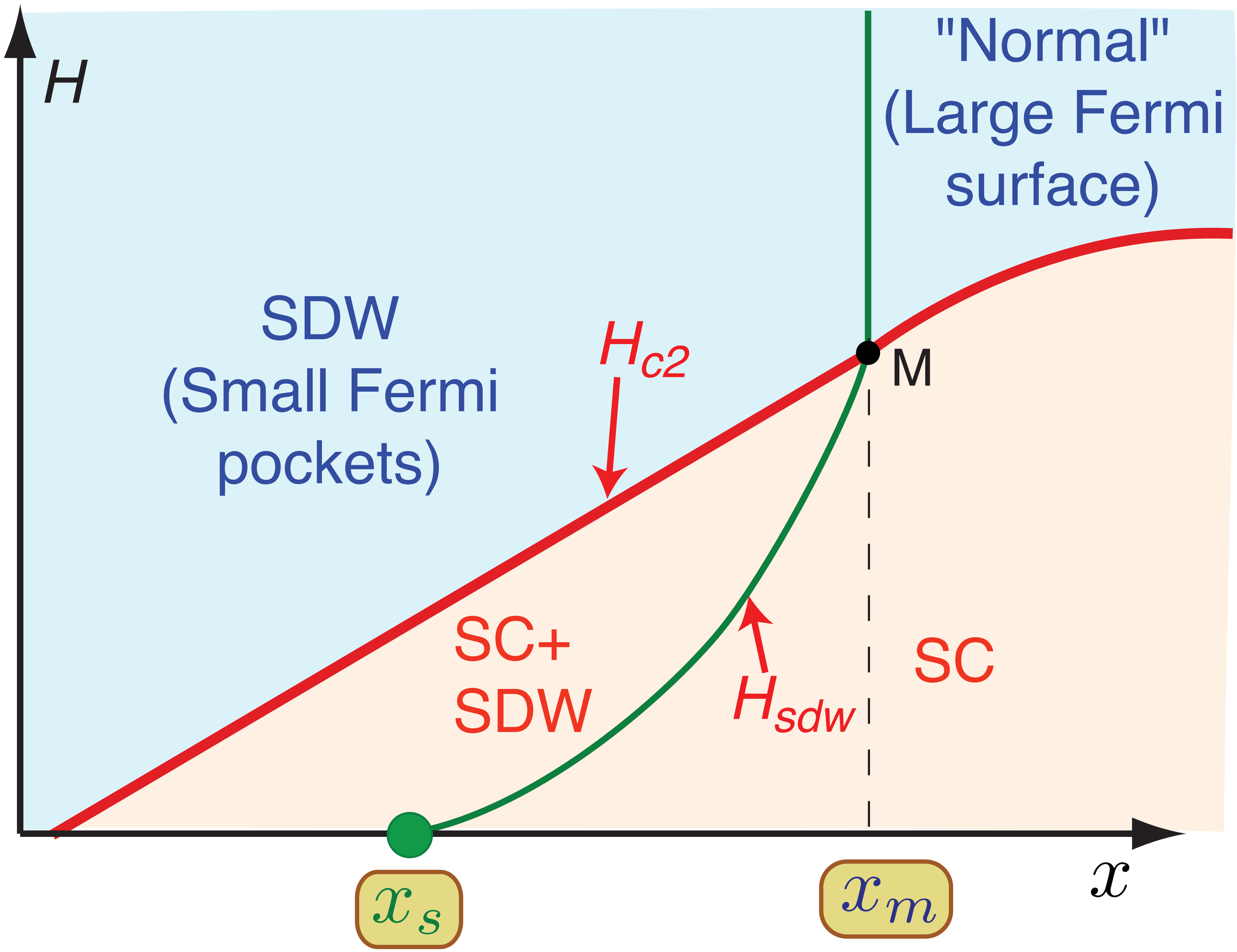


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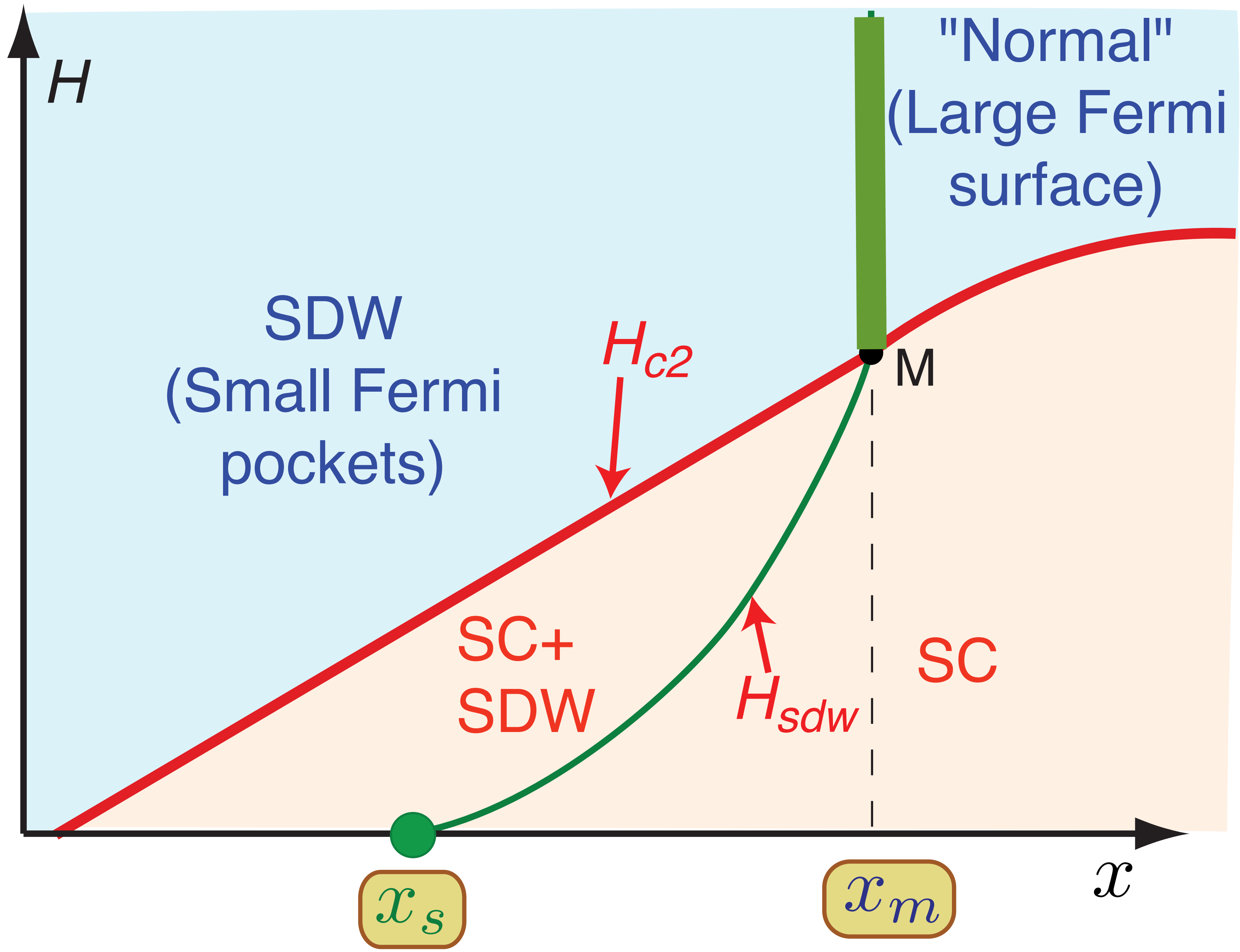


- SDW order is more stable in the metal than in the superconductor: $x_m > x_s$.

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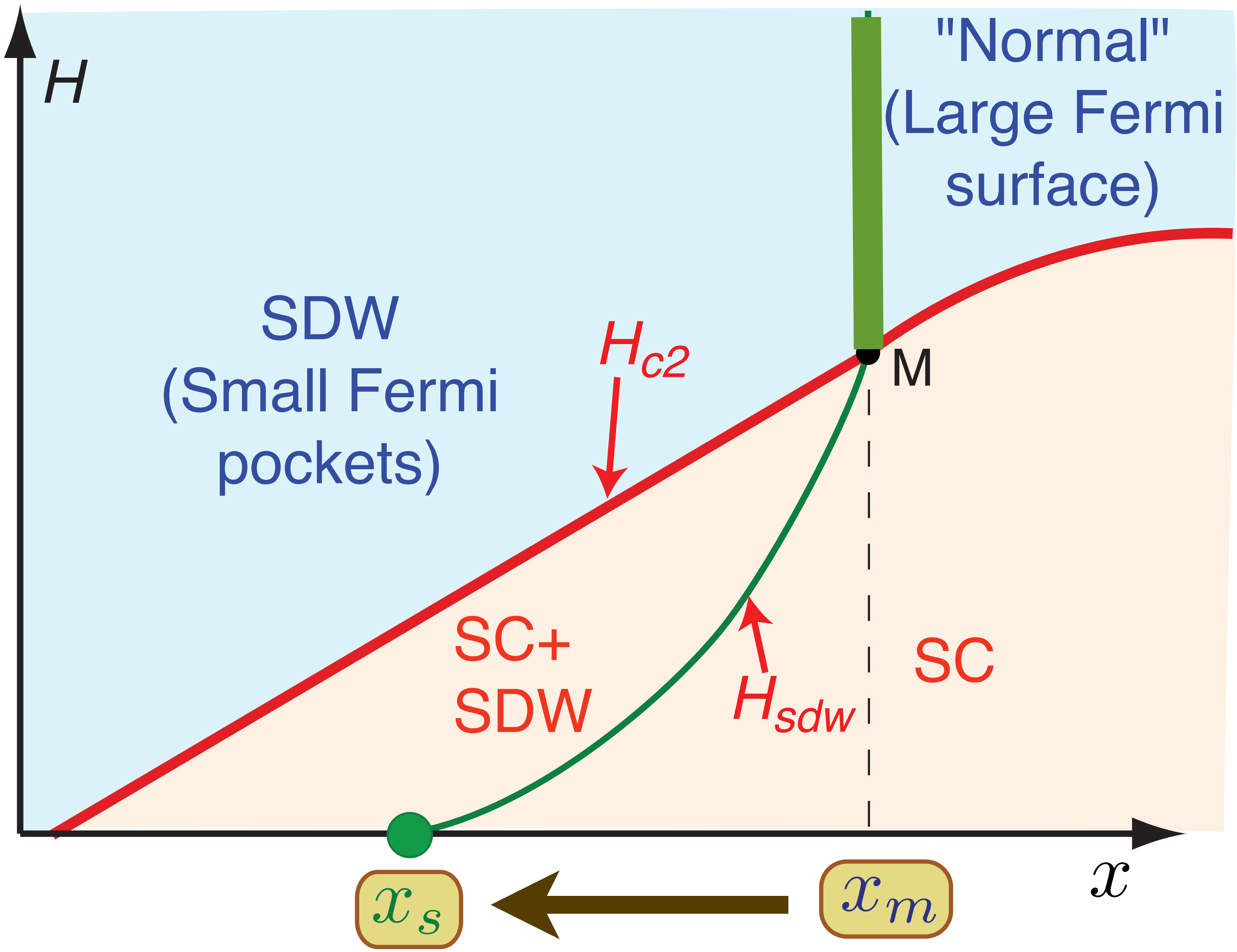


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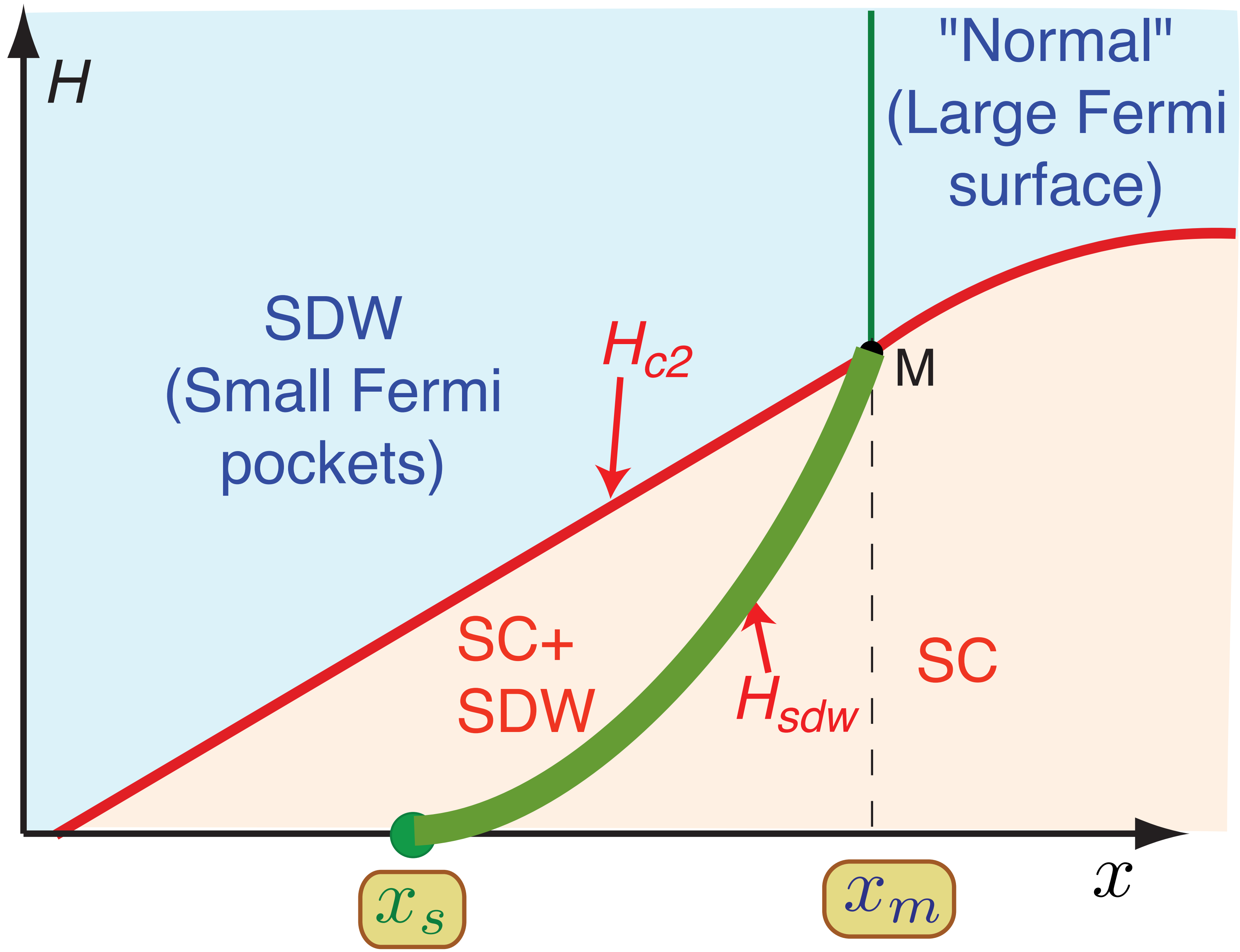


- For doping with $x_s < x < x_m$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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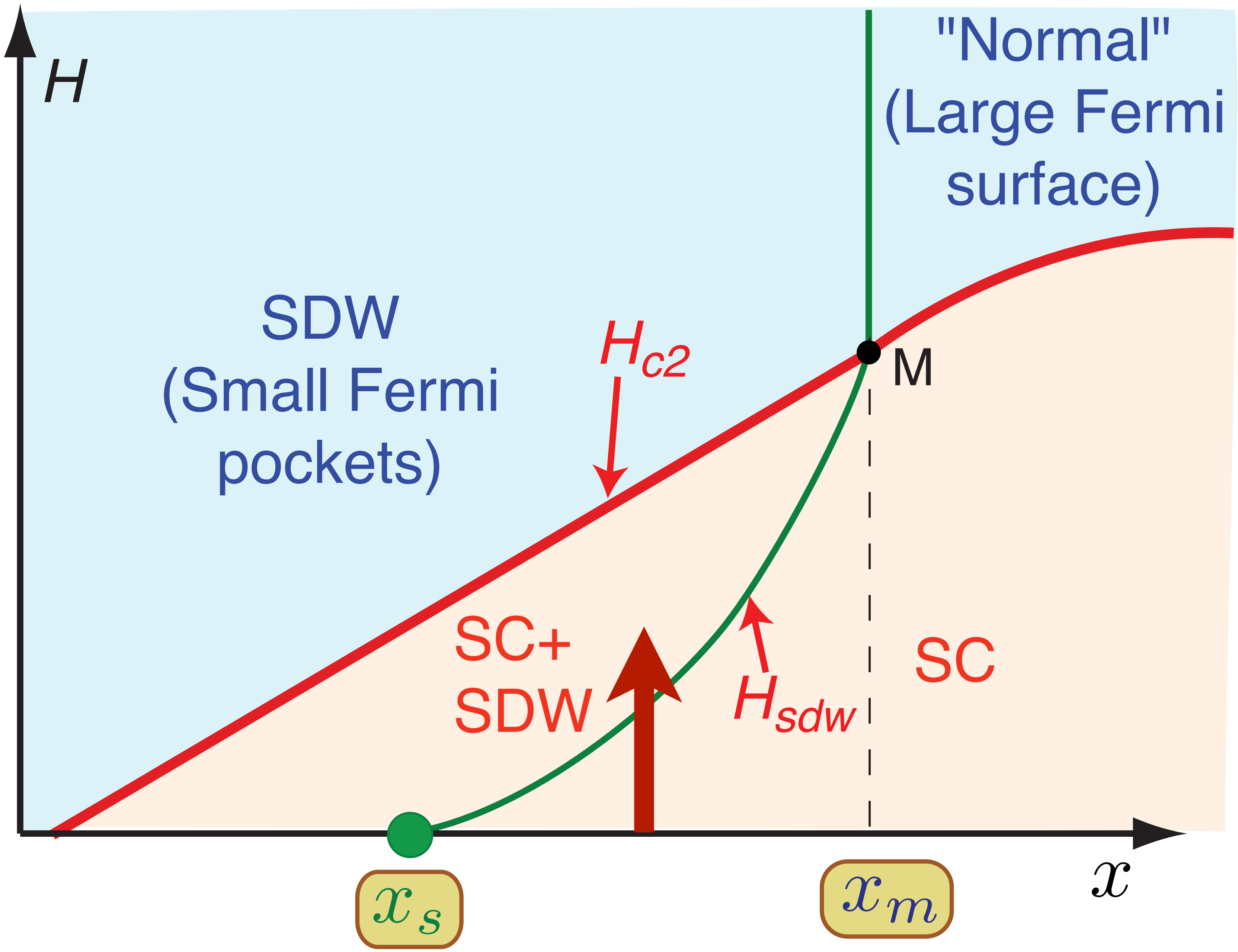


Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
J. Chang *et al.*, *Phys. Rev. Lett.* **102**, 177006 (2009).

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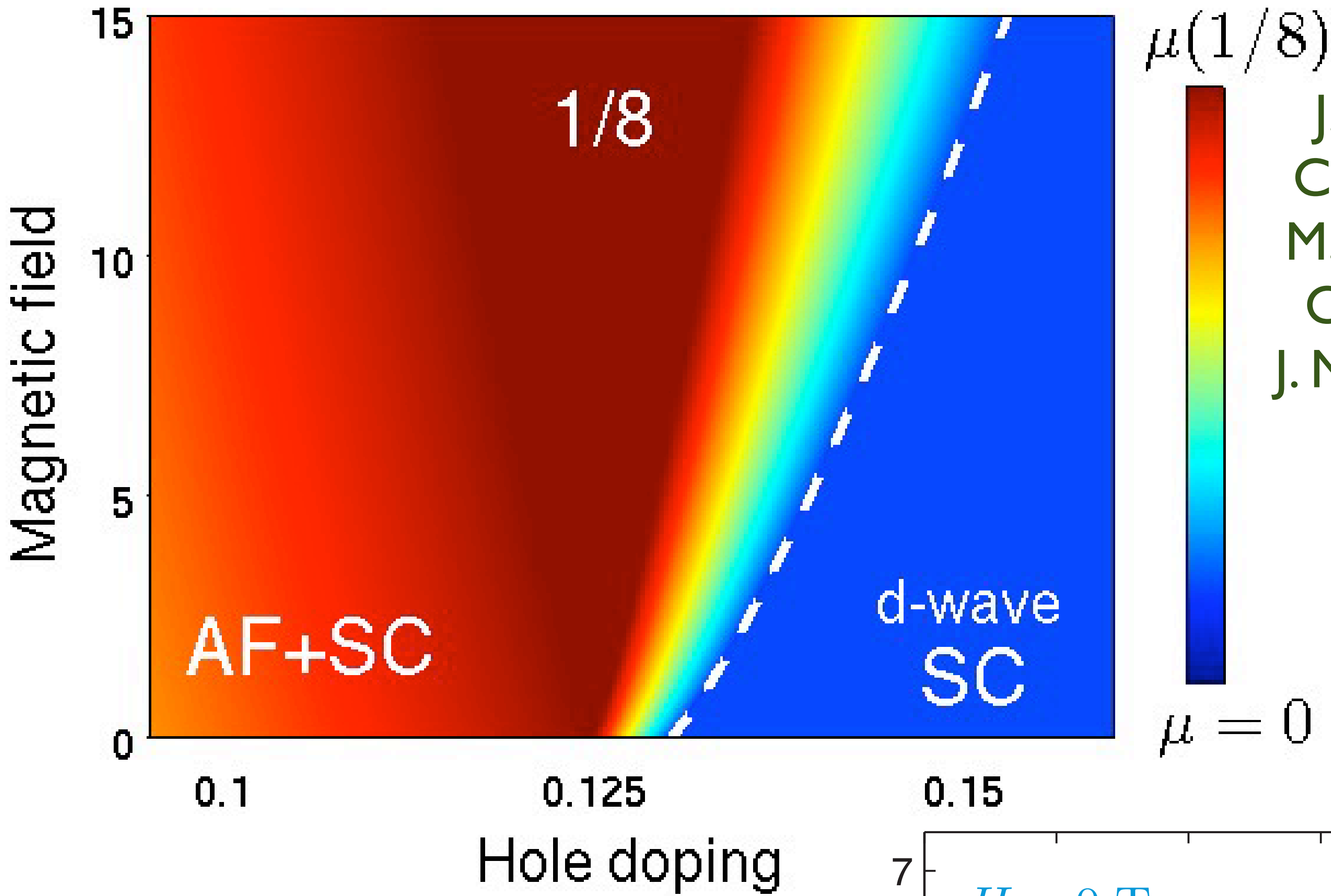
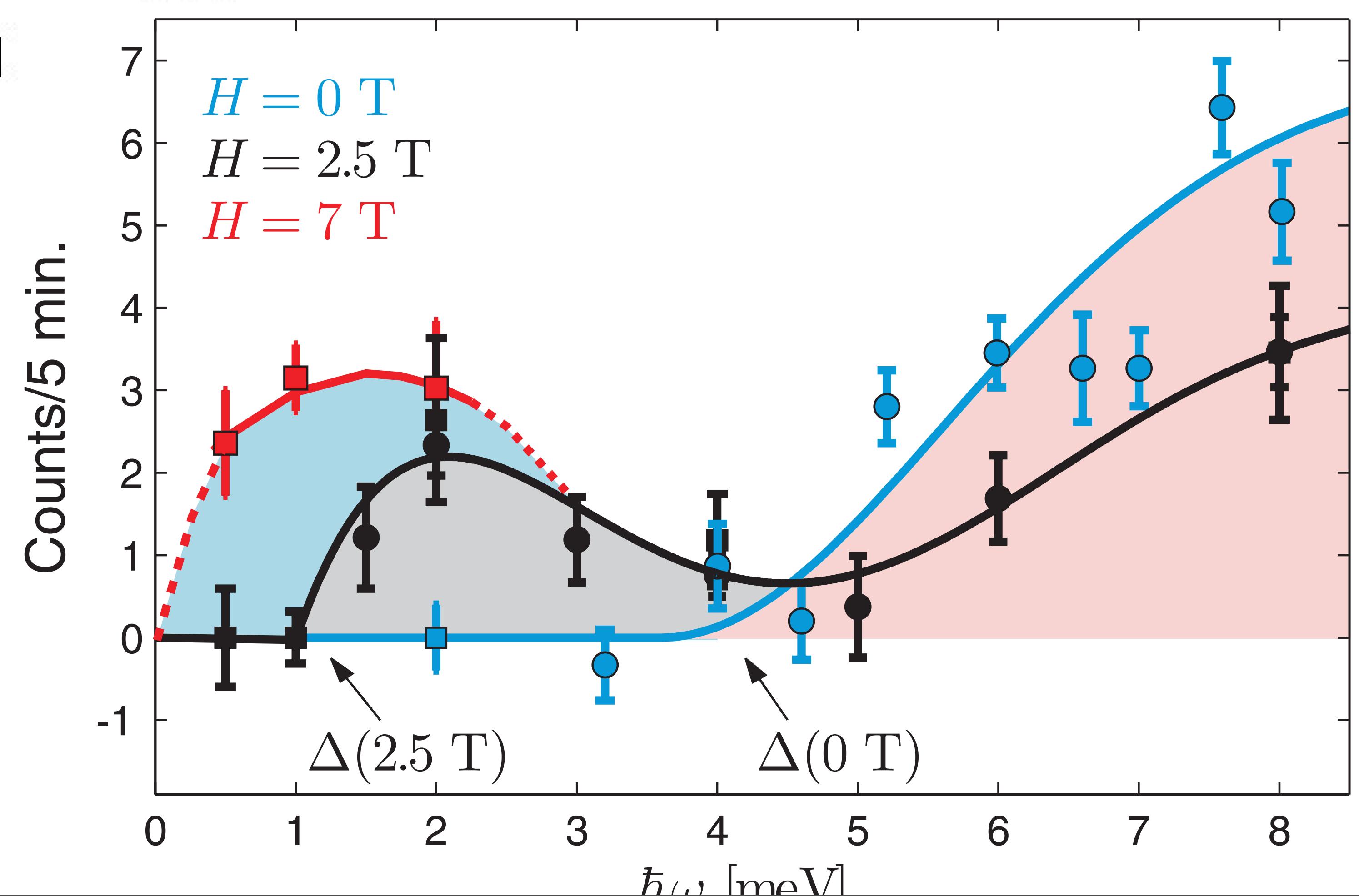



J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).




J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).

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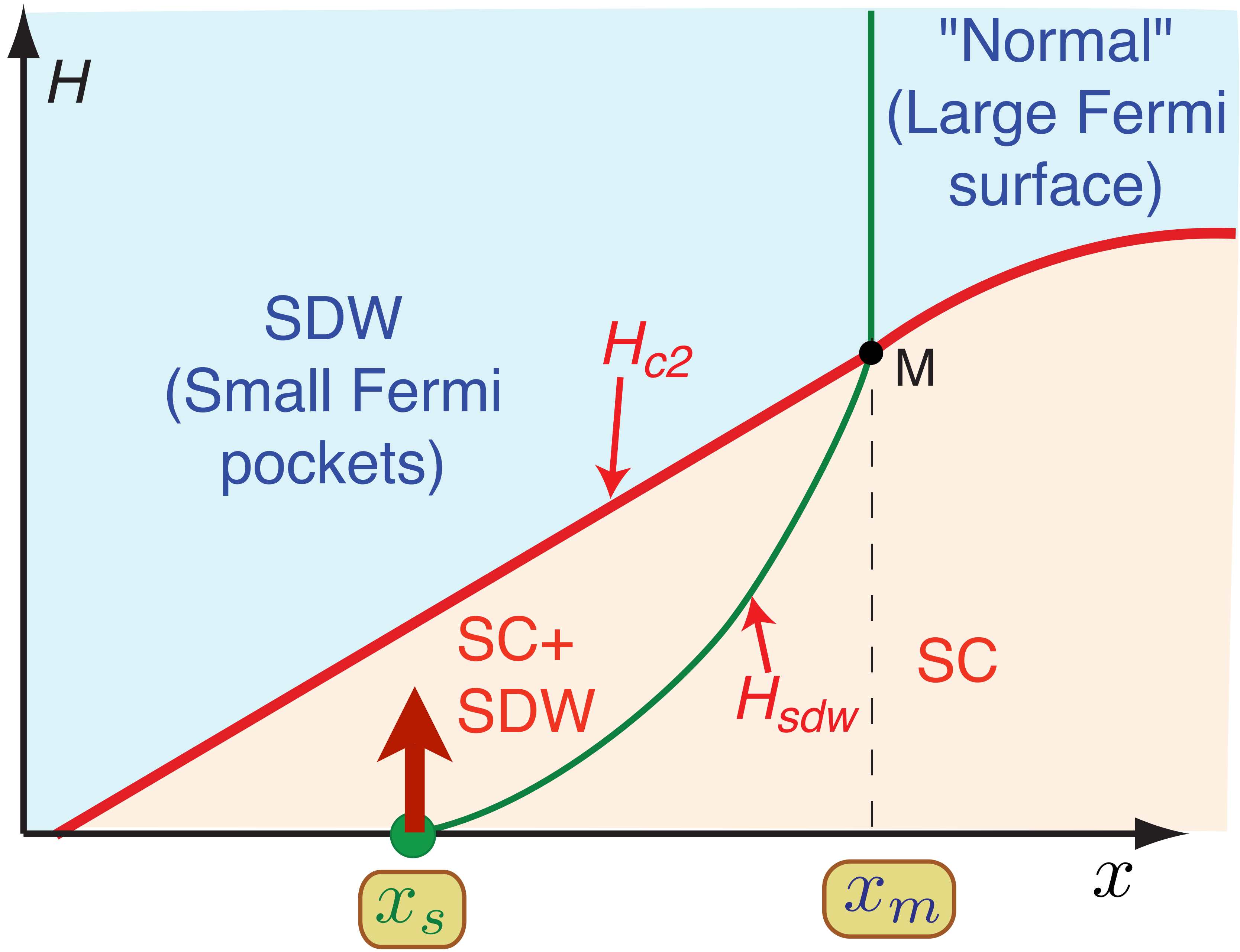


Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, *Phys. Rev. Lett.* **103**, 017001 (2009).

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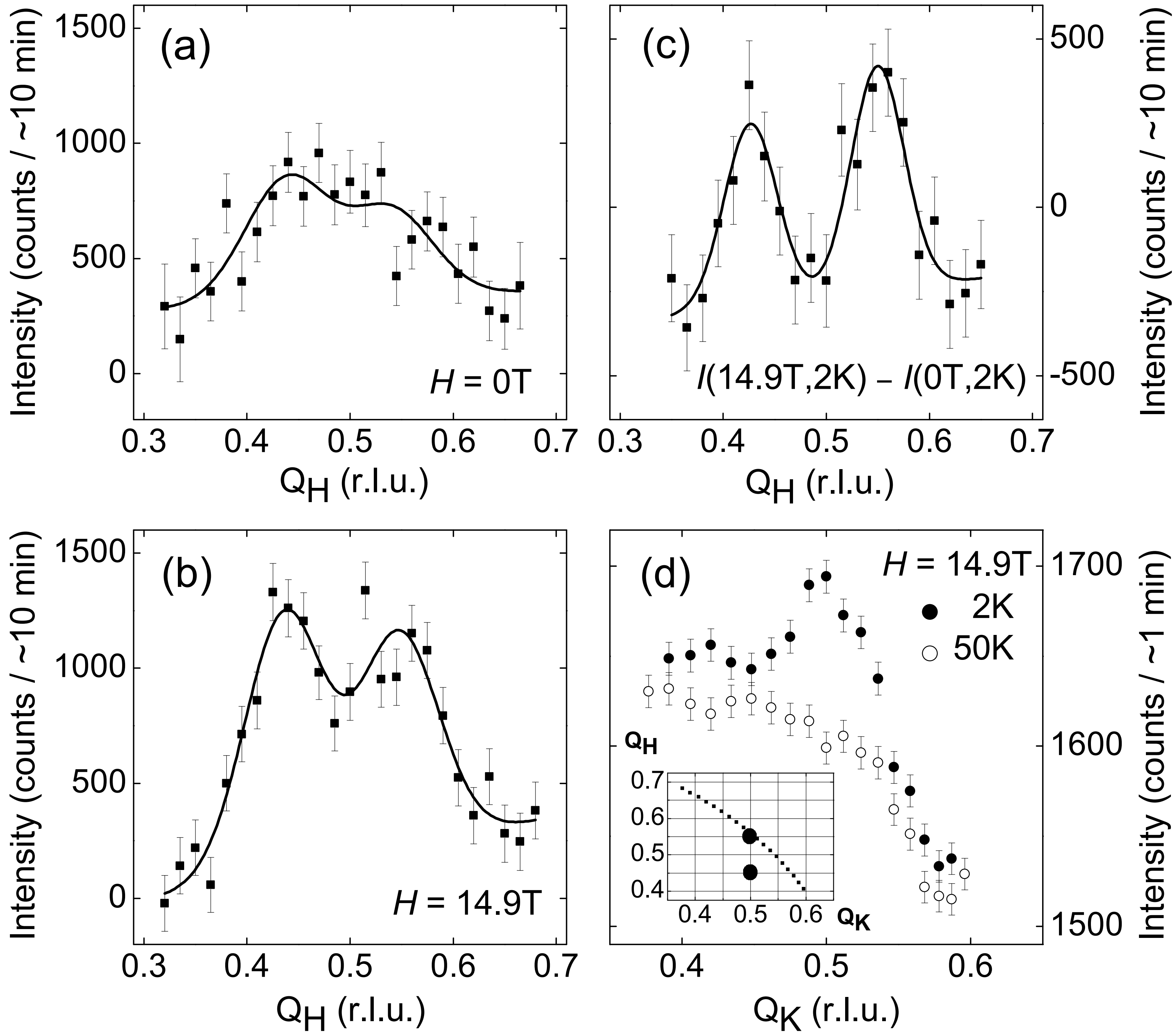


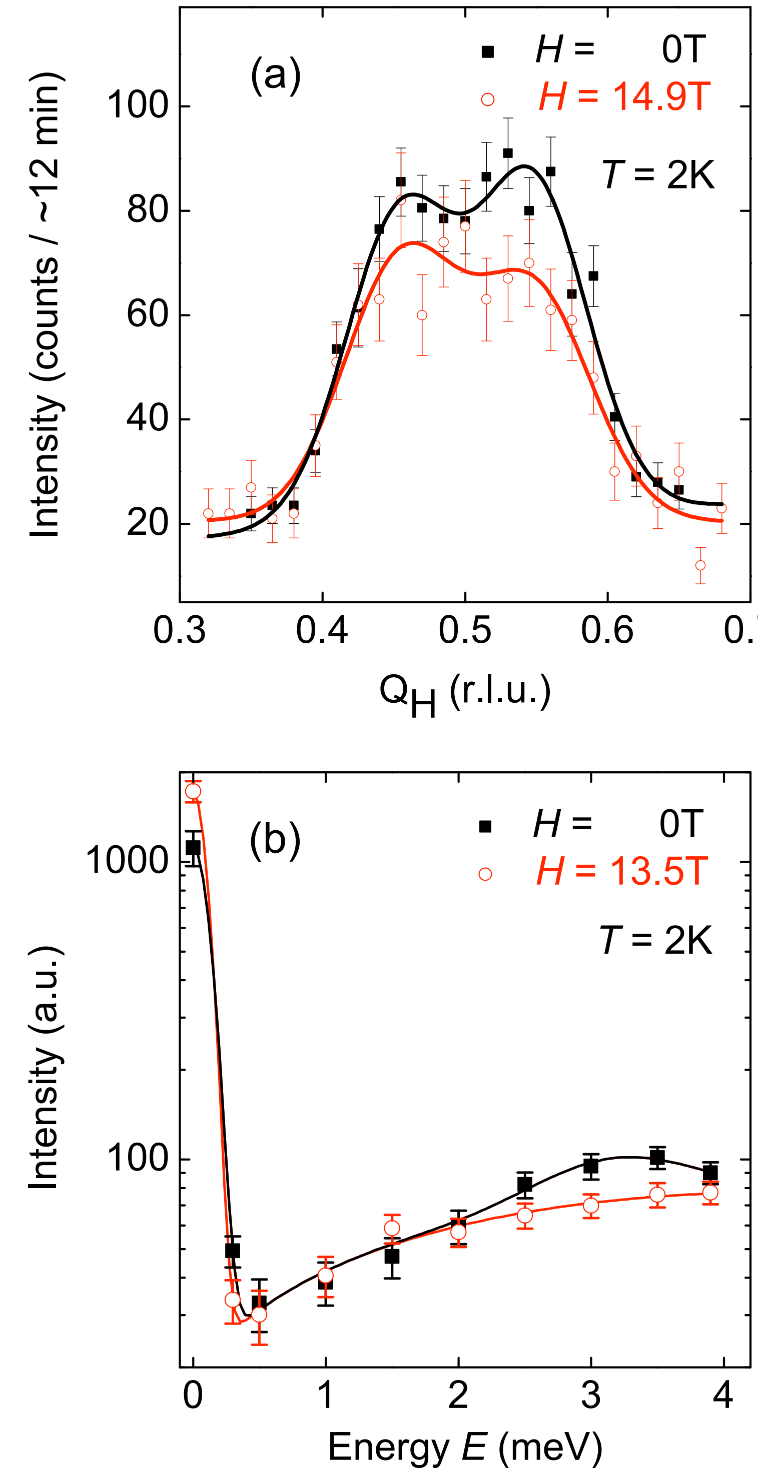


D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)

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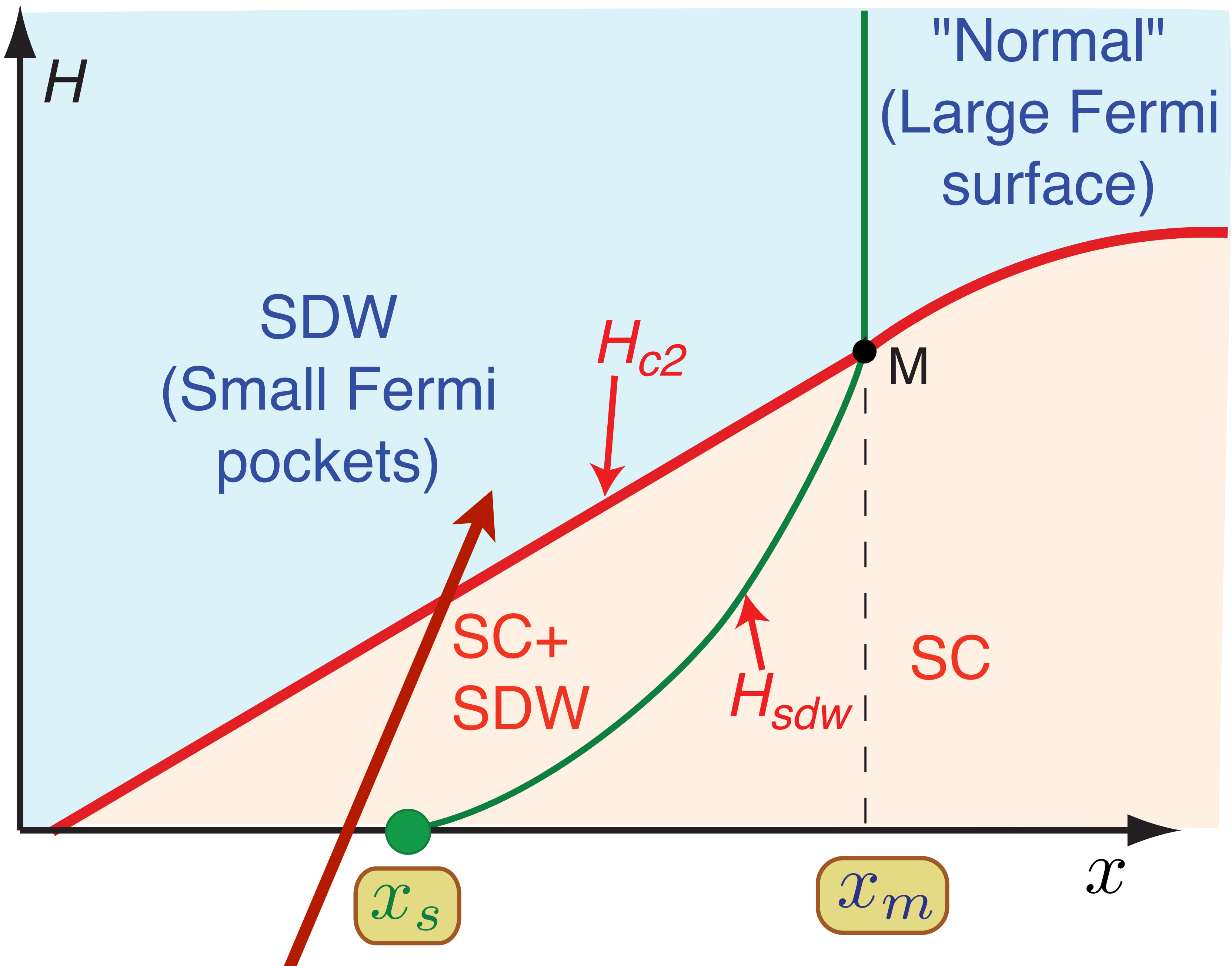



Quantum oscillations without Zeeman splitting

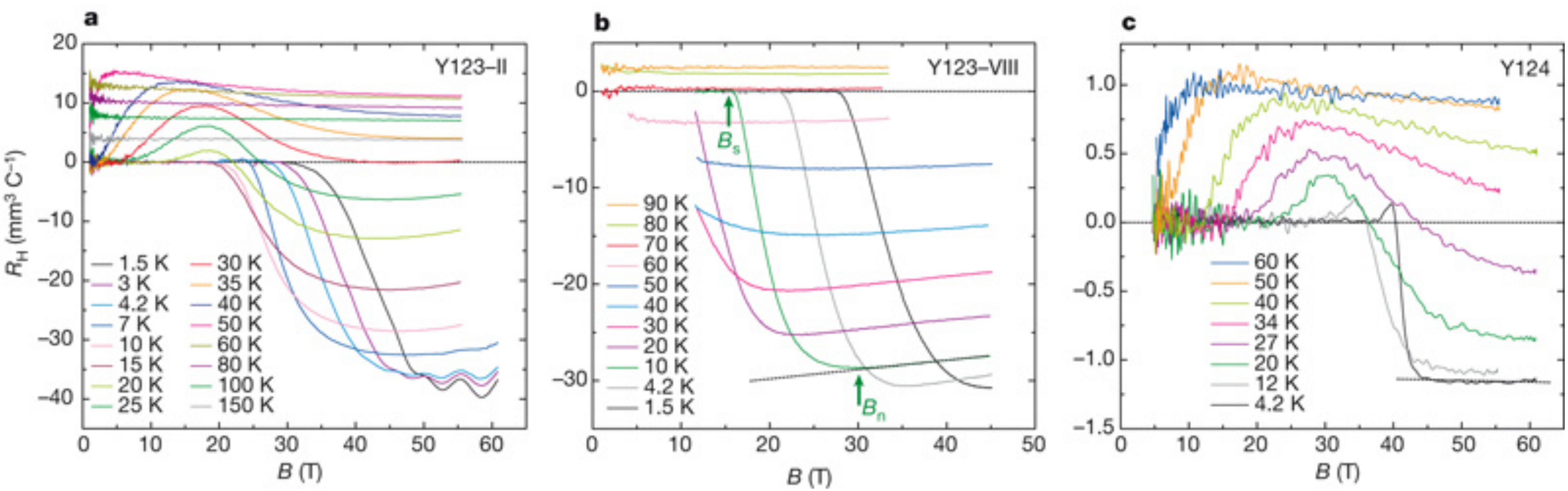

N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007).
S. E. Sebastian, N. Harrison, C. H. Mielke, Ruixing Liang, D. A. Bonn, W.~N.~Hardy, and G. G. Lonzarich, arXiv:0907.2958

# Quantum oscillations

## Electron pockets in the Fermi surface of hole-doped high-$T_c$ superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

*Nature* **450**, 533 (2007)



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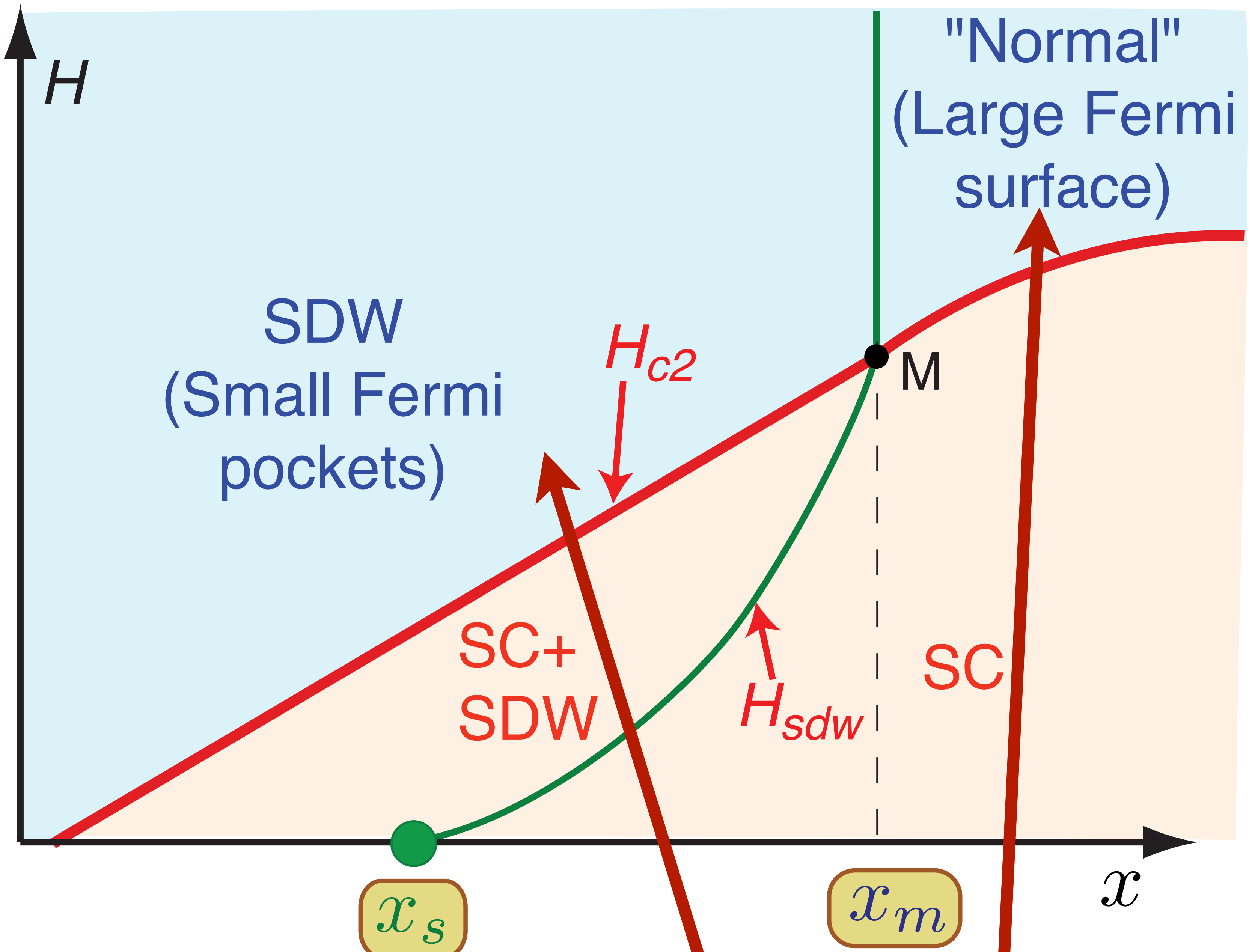
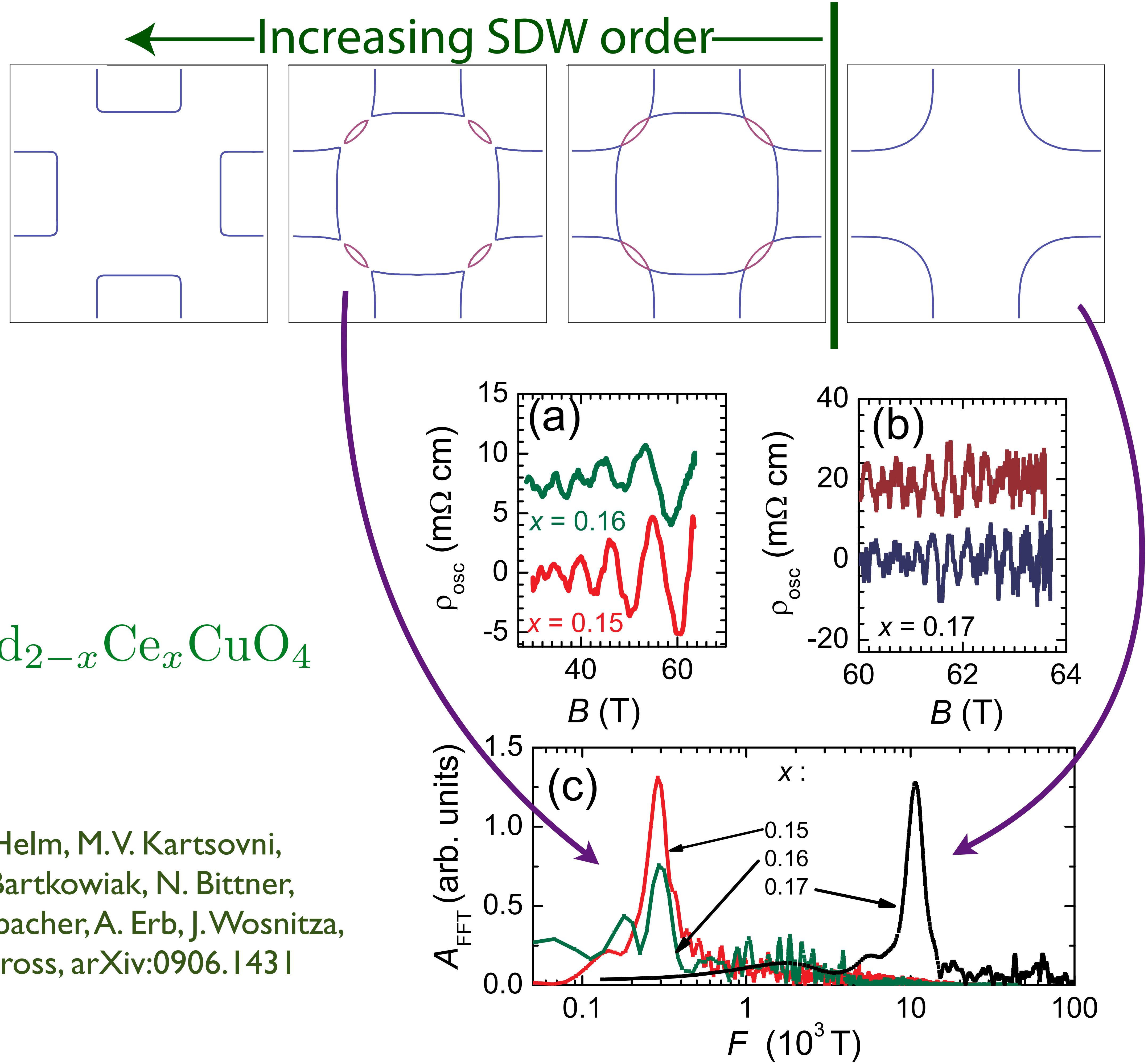



Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$




$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovni,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
R. Gross, arXiv:0906.1431

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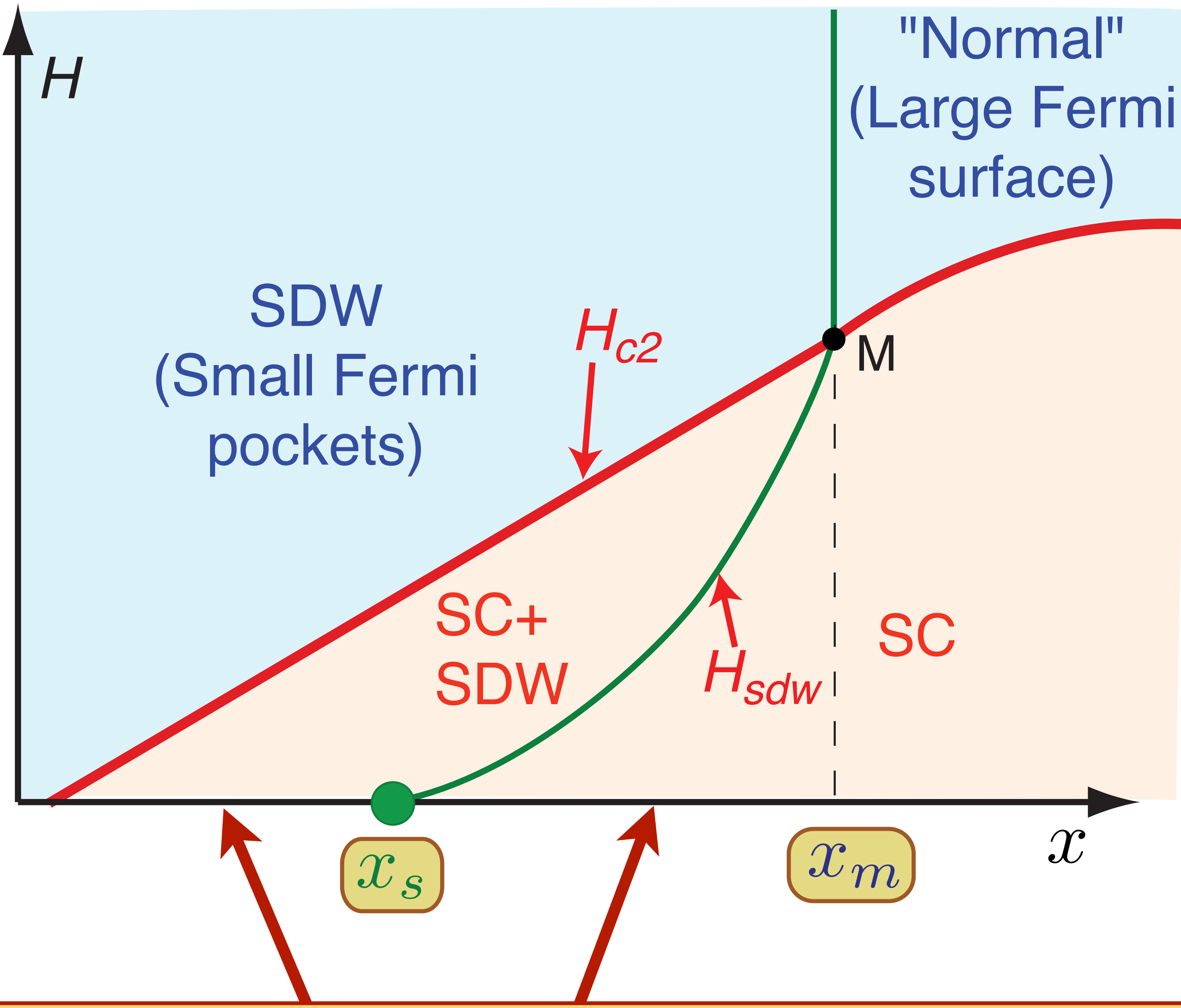
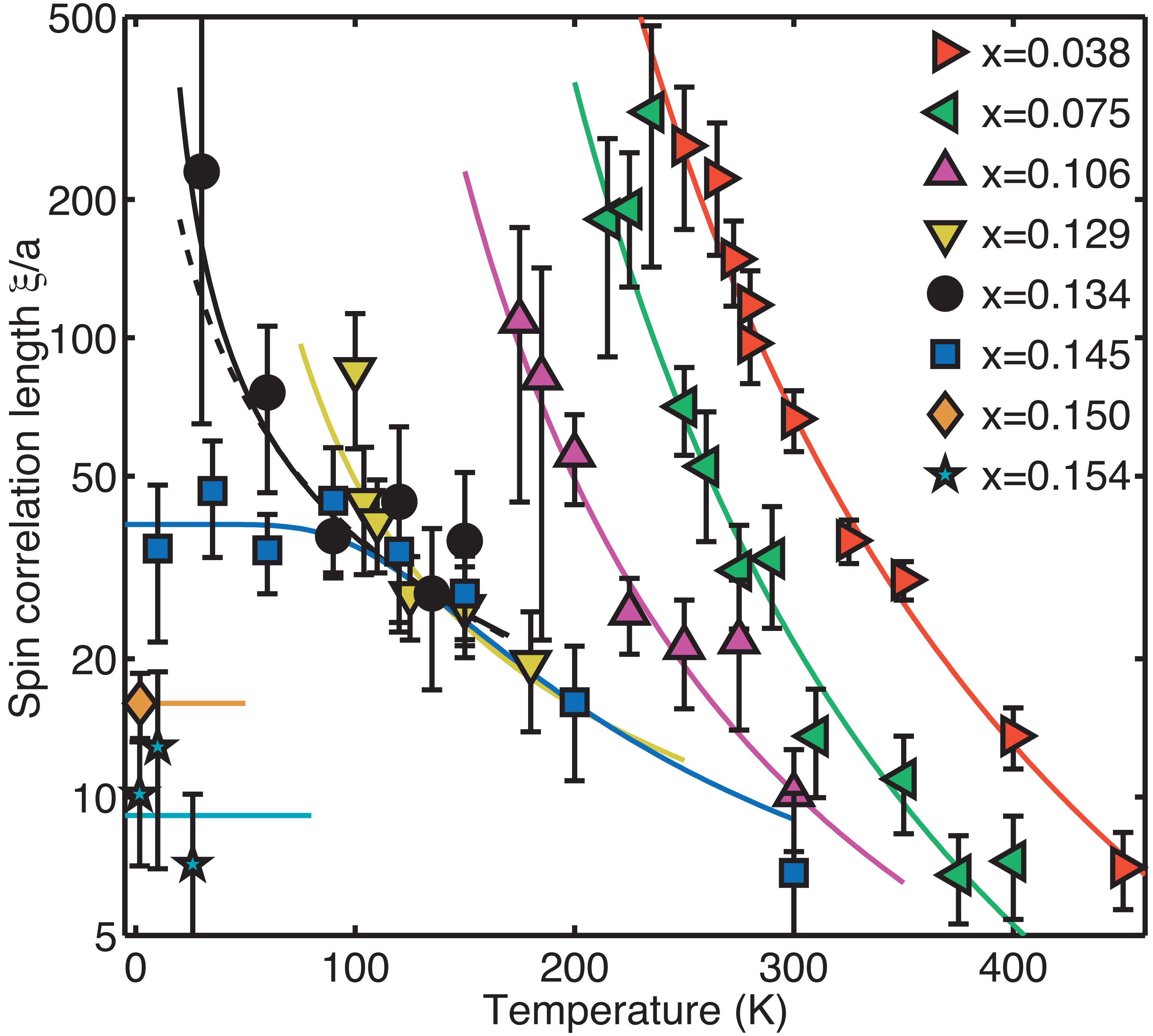
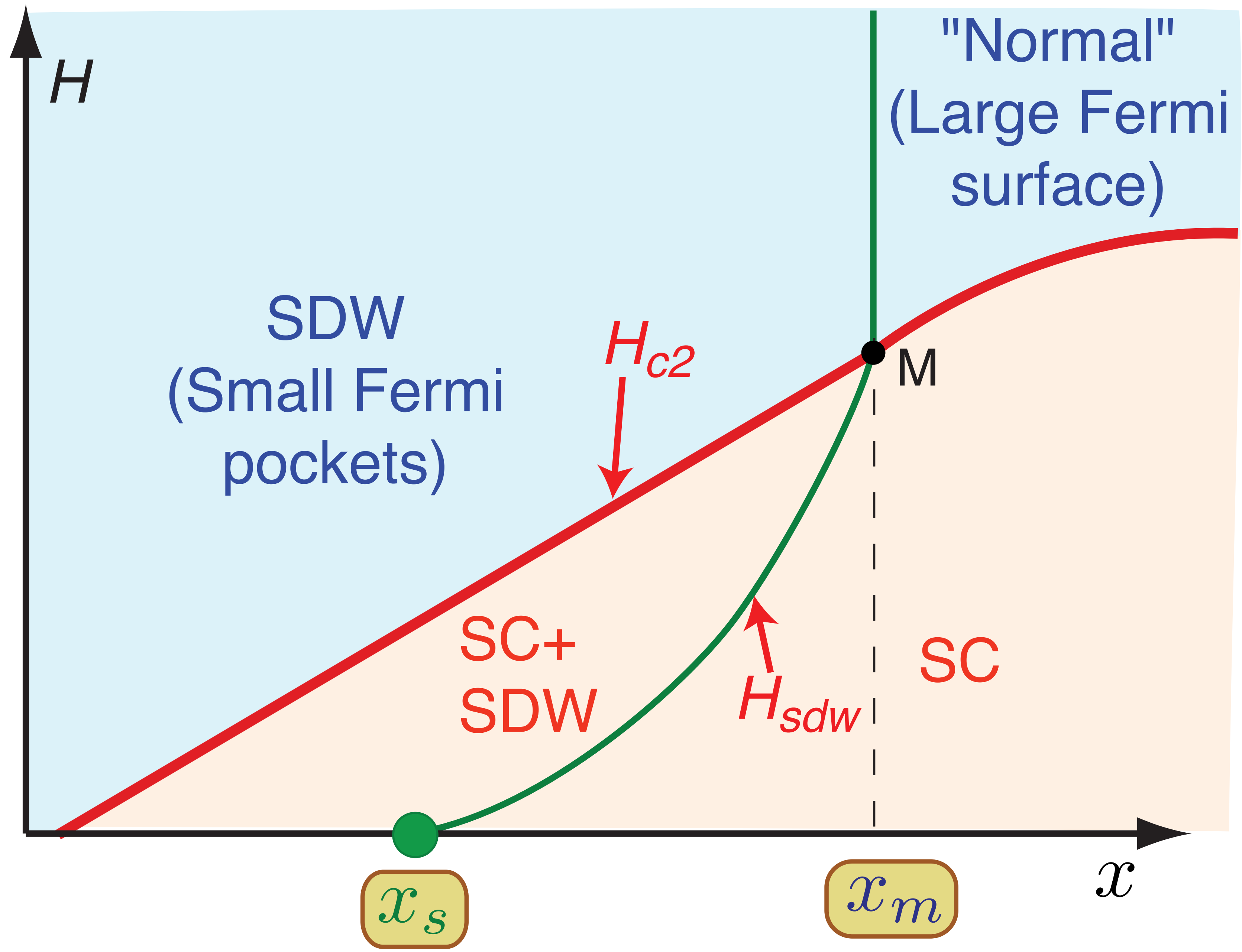



Neutron scattering at $H=0$ in **same** material identifies $x_s = 0.14 < x_m$

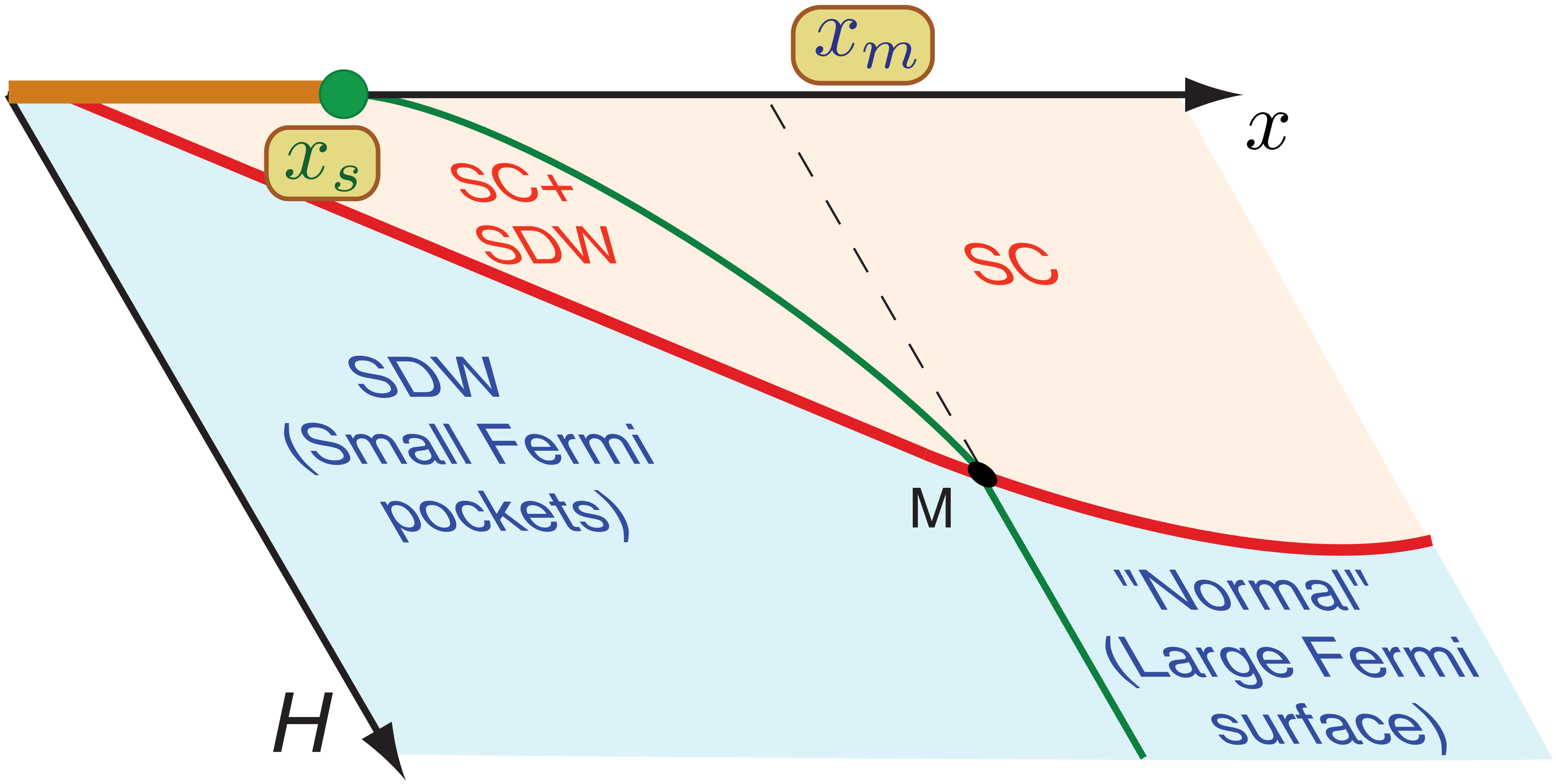
# $Nd_{2-x}Ce_xCuO_4$




E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).

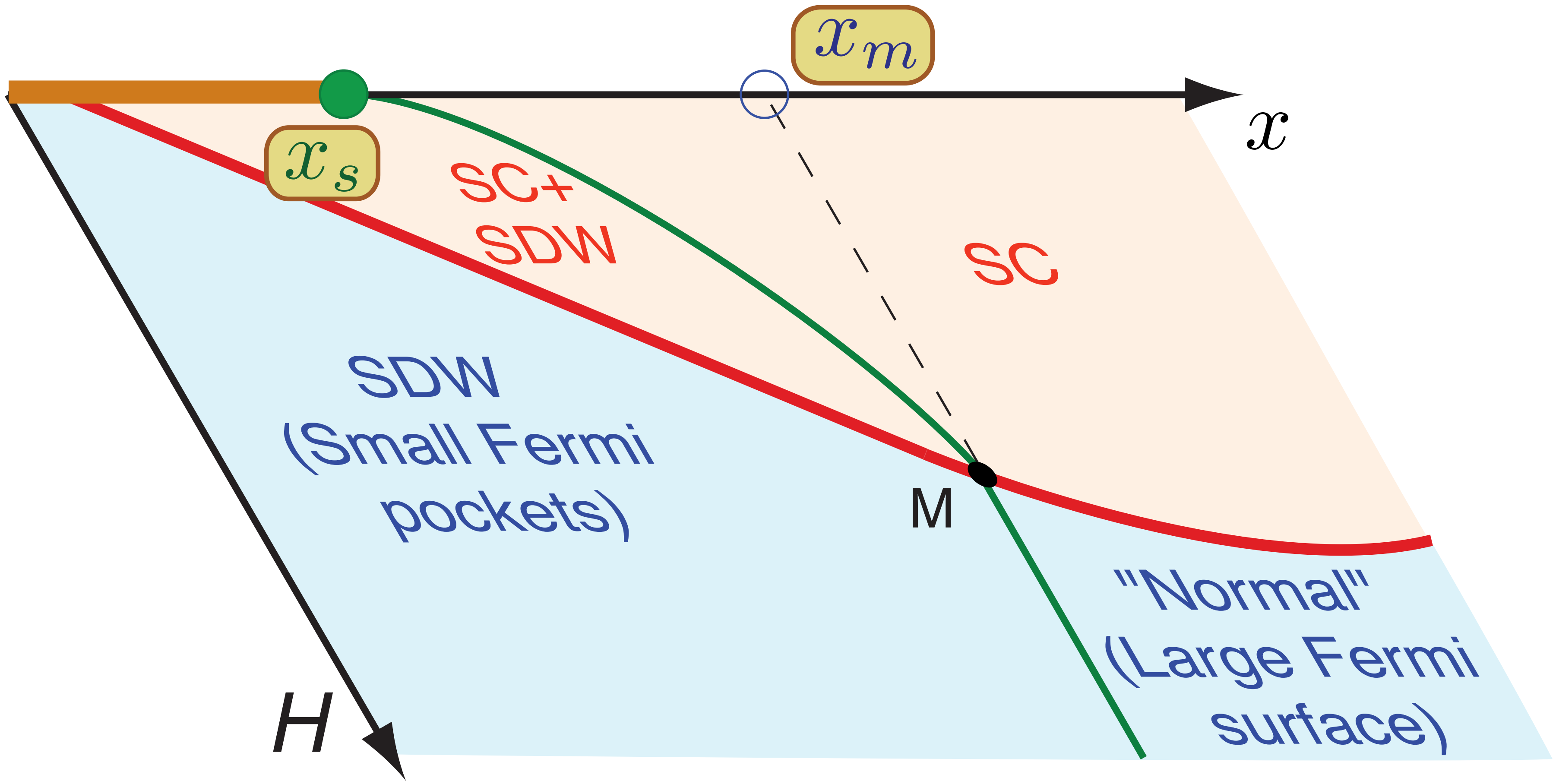


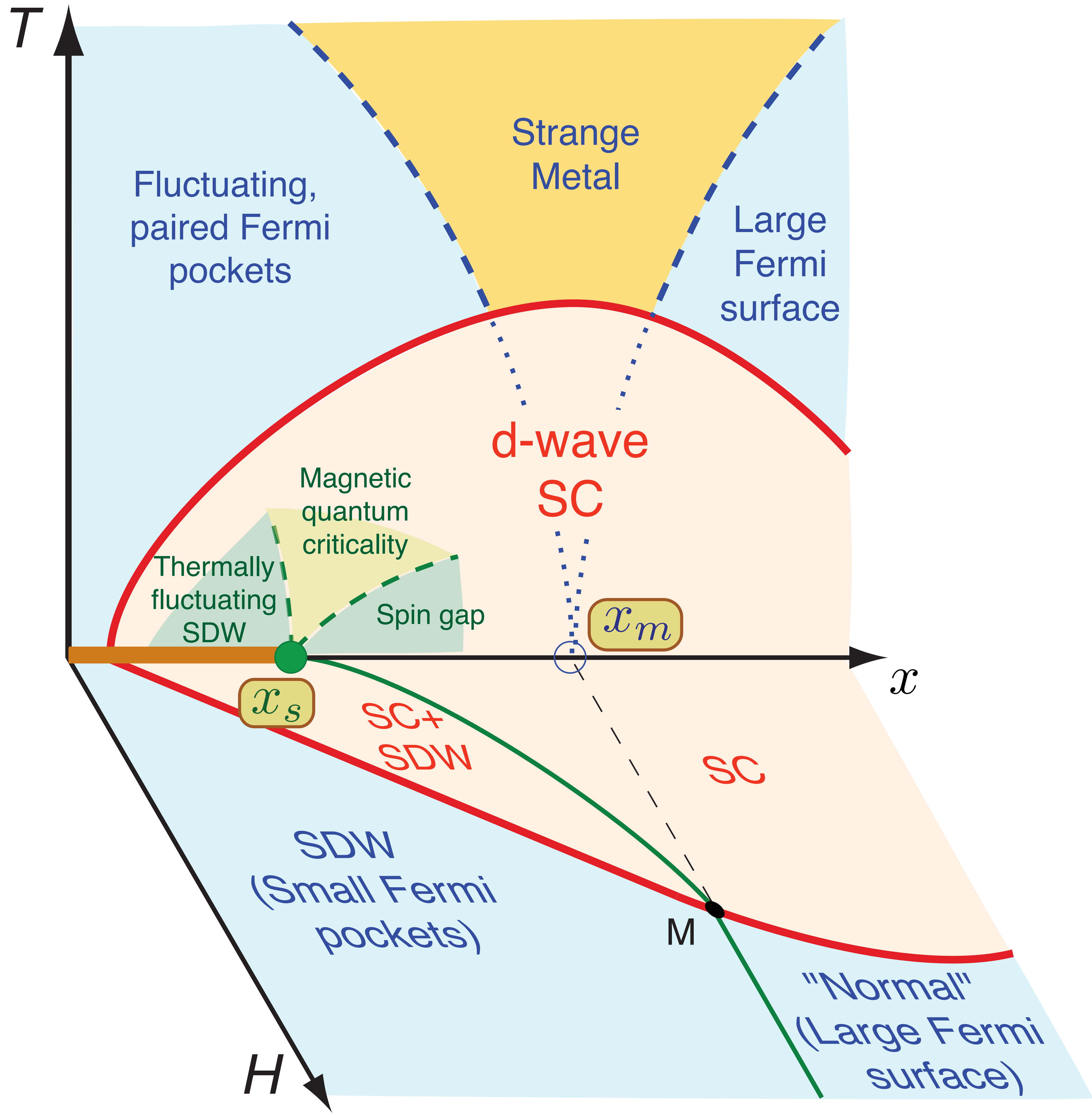
H
"Normal"
(Large Fermi
surface)
SDW
(Small Fermi
pockets)
$H_{c2}$
M
SC+
SDW
$H_{sdw}$
SC
$x$
$x_s$
$x_m$



$x_m$
$x$
$x_s$
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
$H$

Fluctuating,
paired Fermi
pockets




T
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
$x_m$
$x$
$x_s$
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

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1. Phenomenological quantum theory of competition between superconductivity and SDW order
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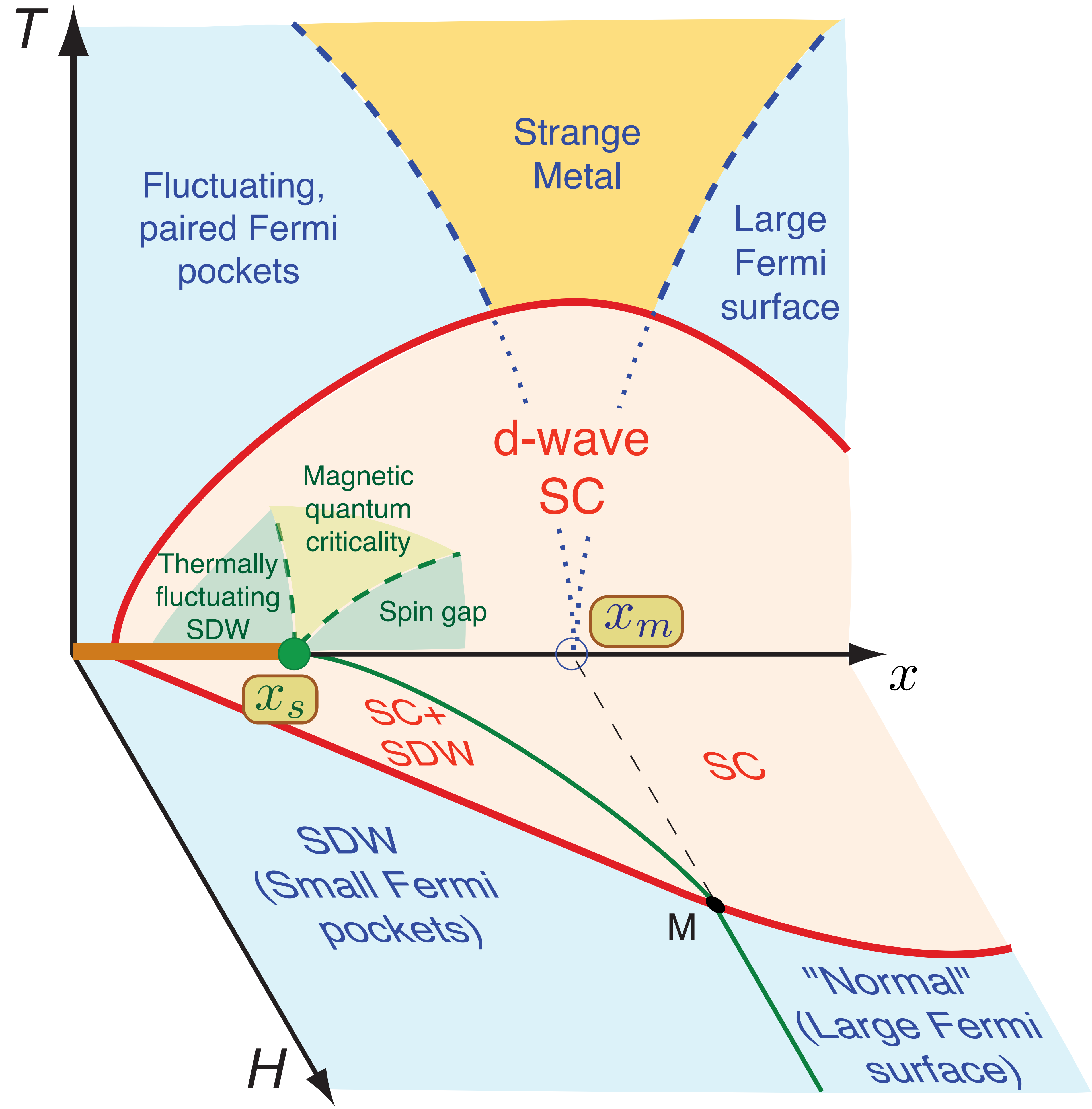
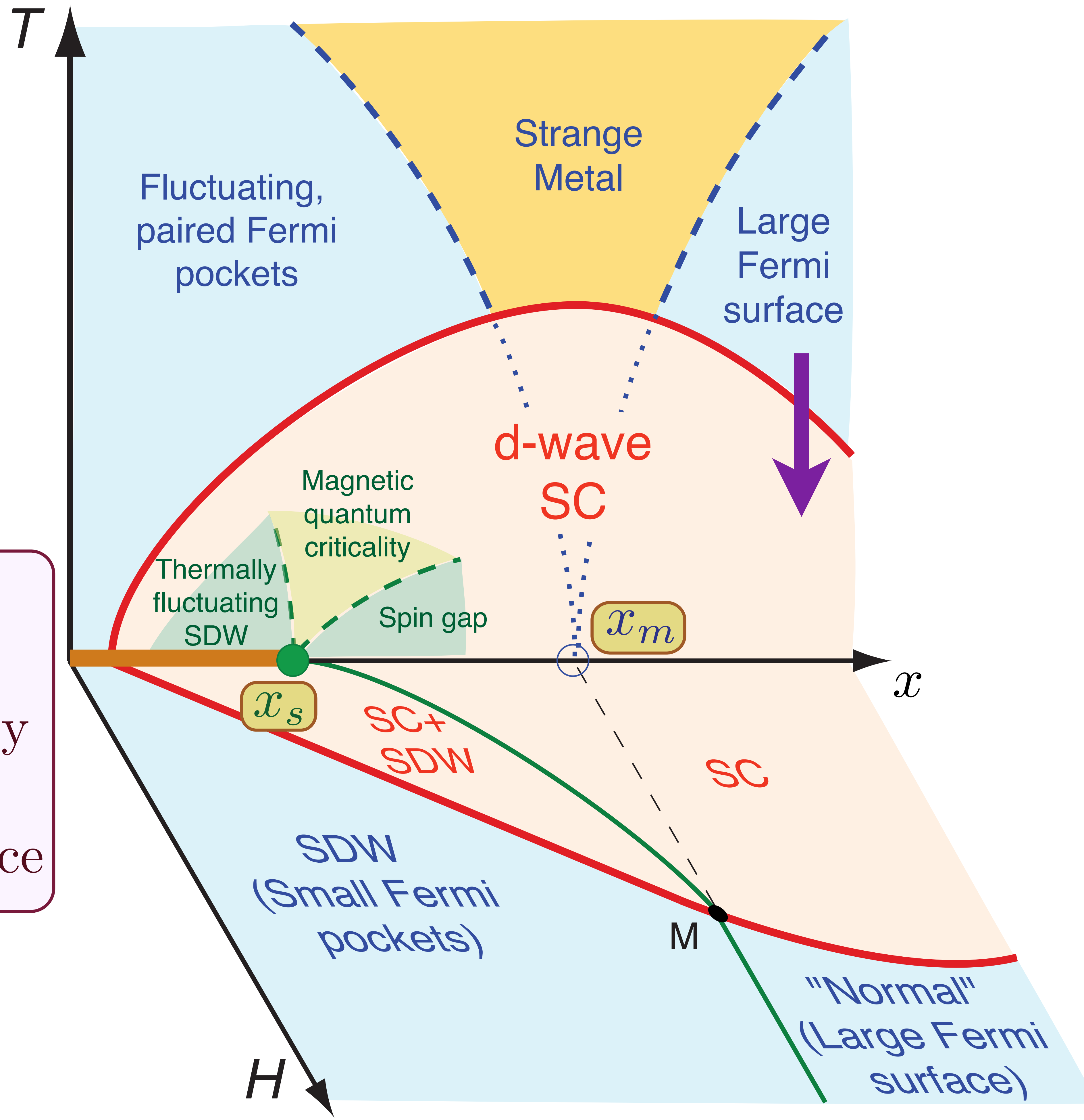
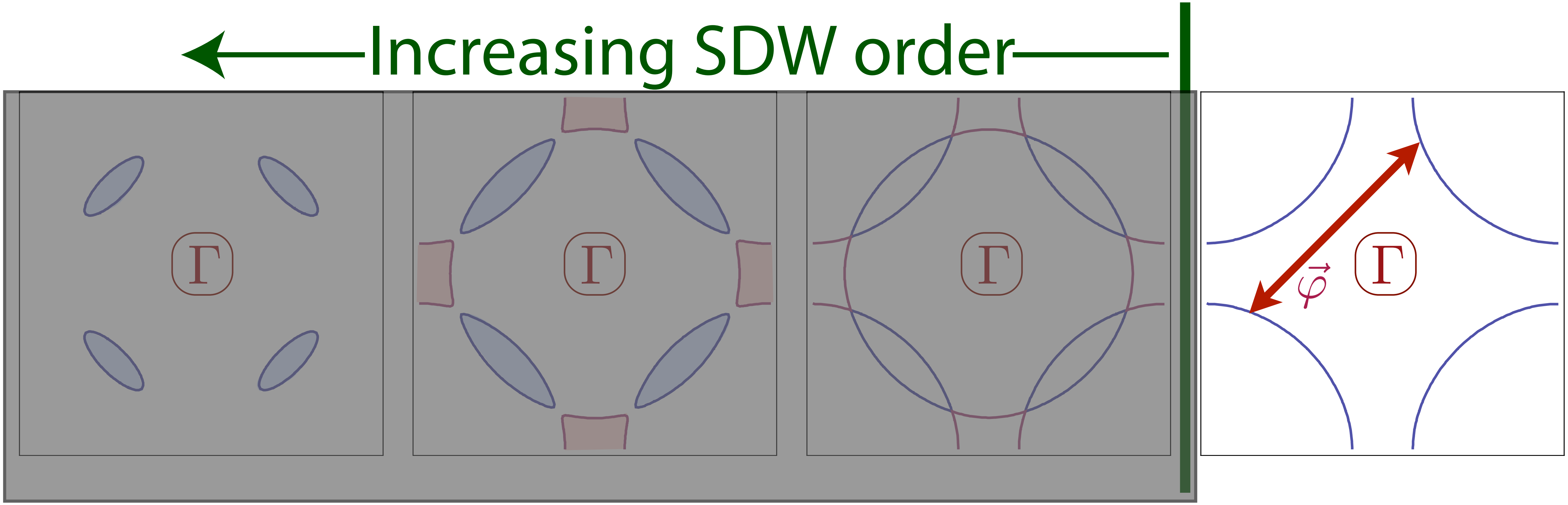


T
Fluctuating,
paired Fermi
pockets
Strange
Metal
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
$x_m$
$x$
$x_s$
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

Theory of the onset of $d$-wave superconductivity from a large Fermi surface



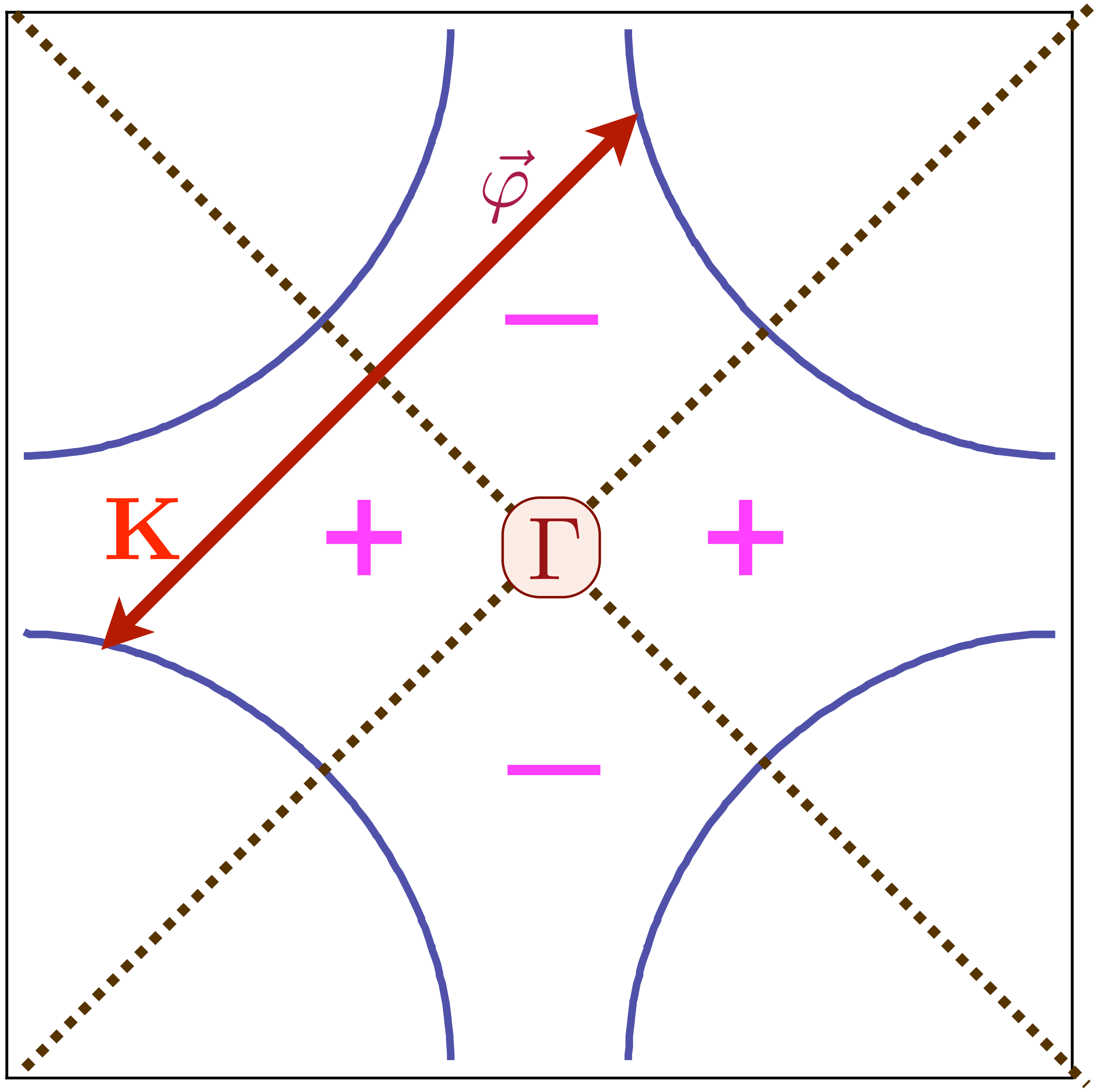
# Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates




Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.


D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

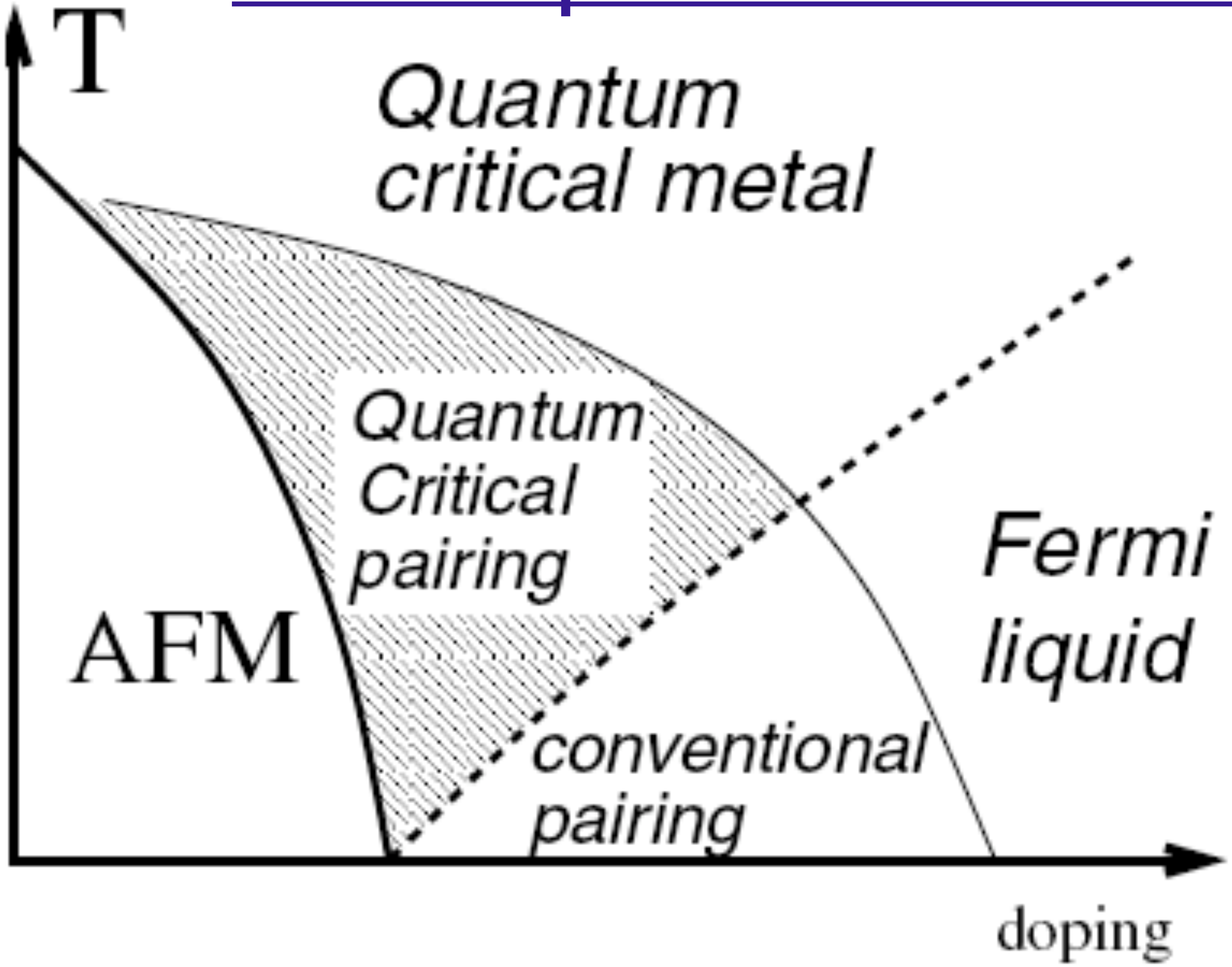
# $d$-wave pairing of the large Fermi surface




$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

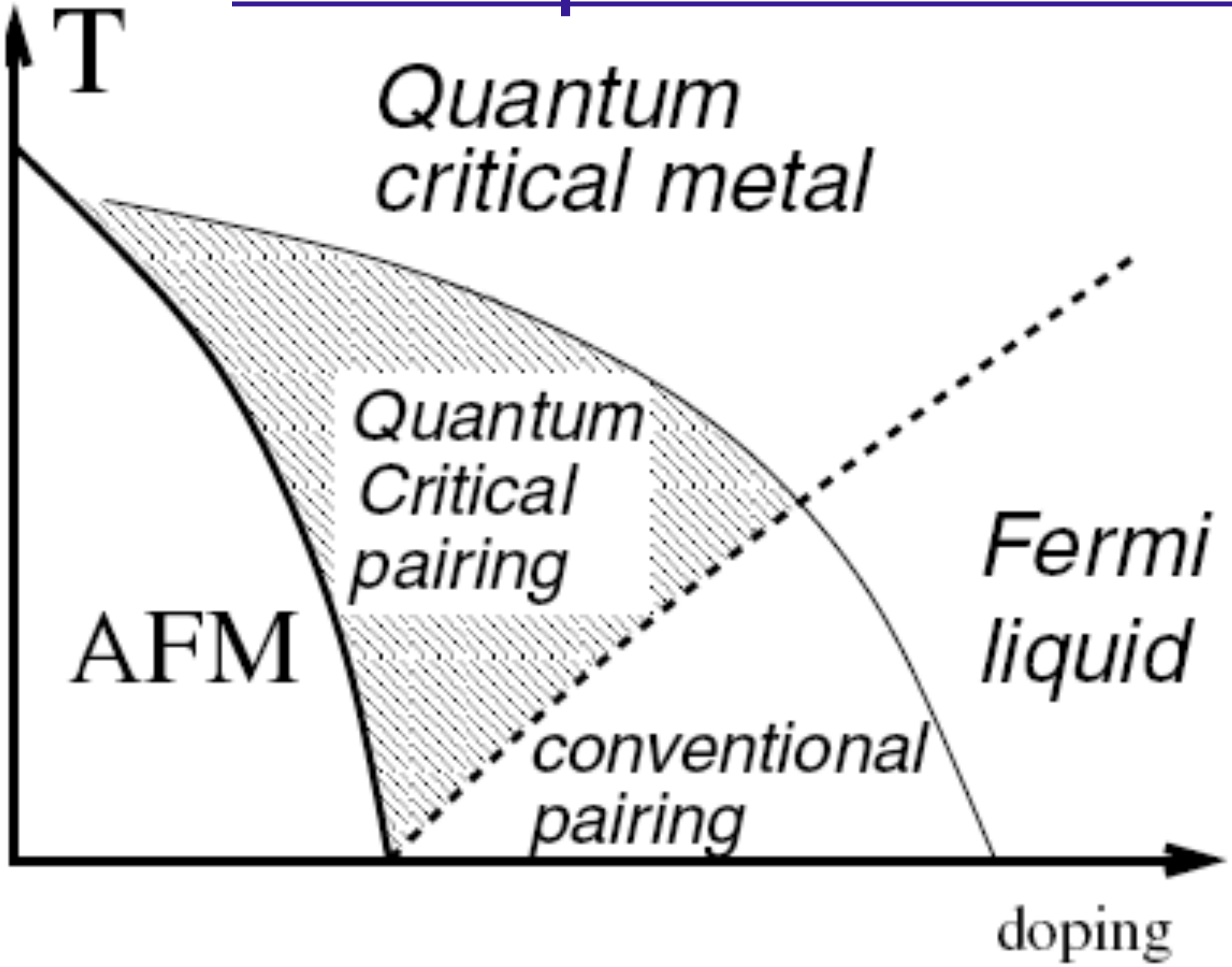

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

# Approaching the onset of antiferromagnetism in the spin-fluctuation theory




Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

# Approaching the onset of antiferromagnetism in the spin-fluctuation theory




- $T_c$ *increases* upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.

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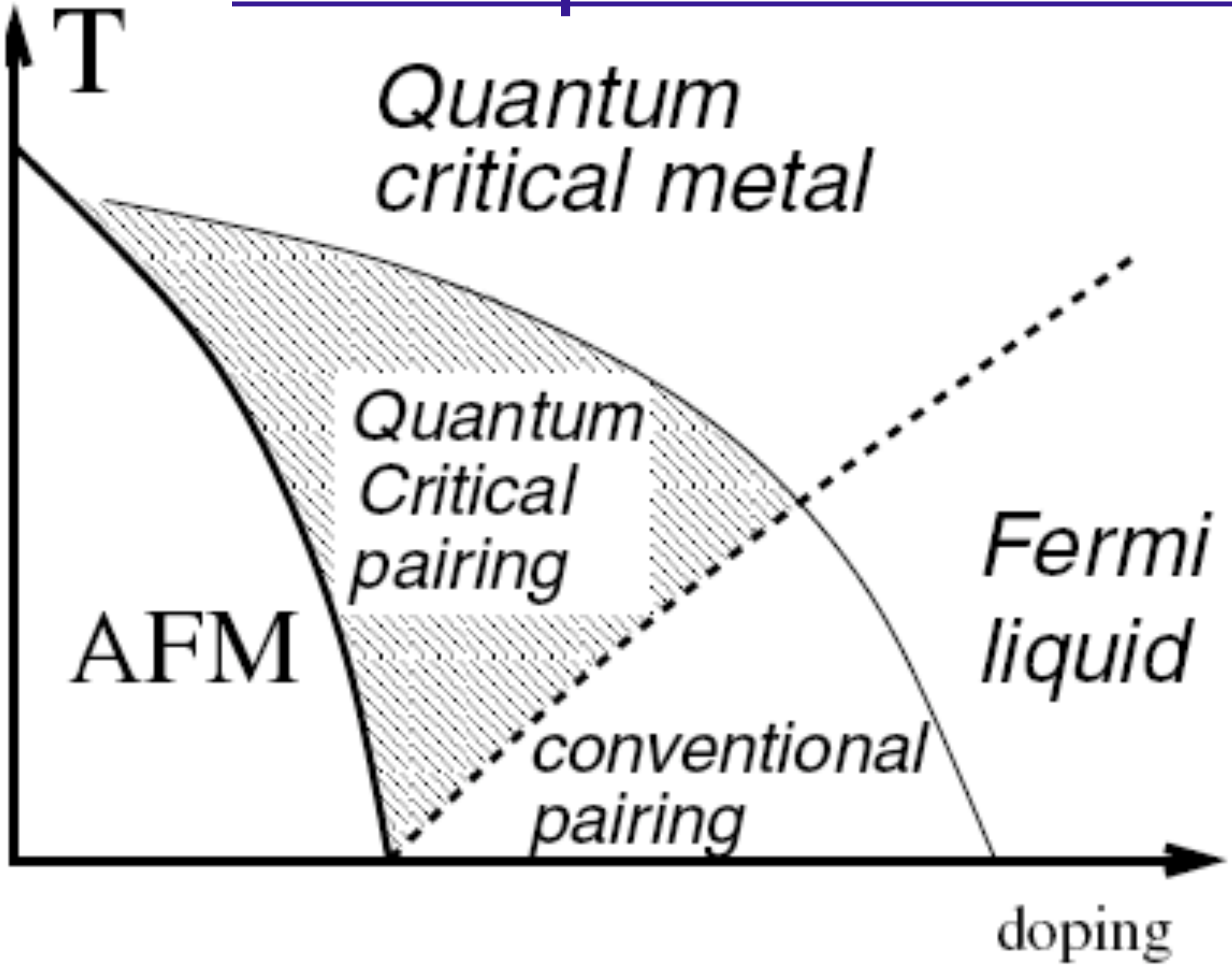
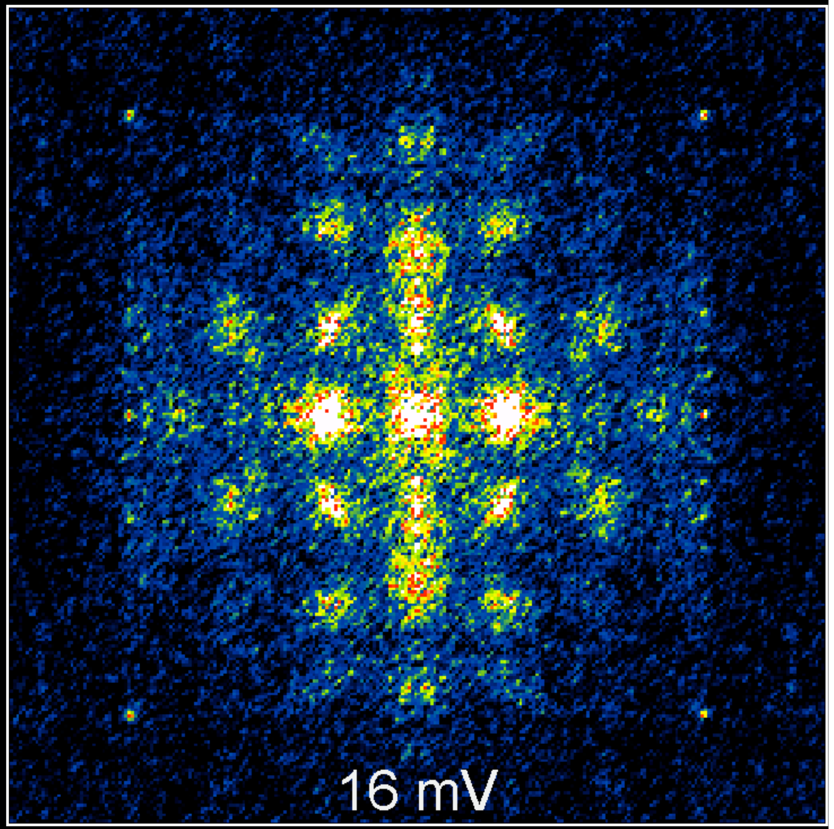
# Approaching the onset of antiferromagnetism in the spin-fluctuation theory

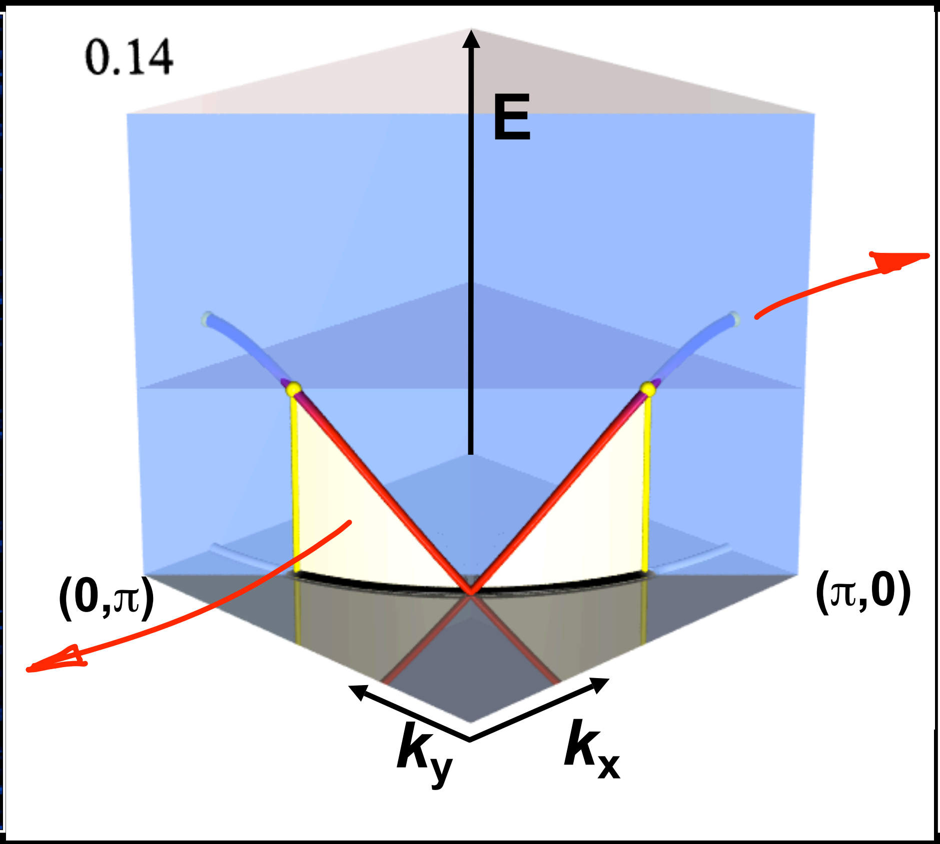


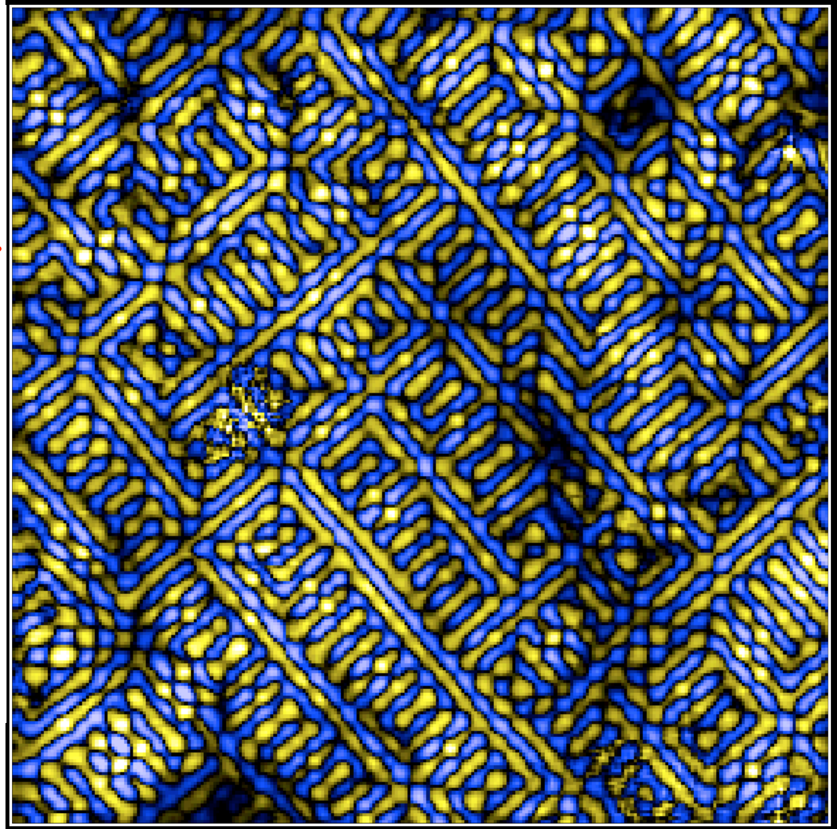

- $T_c$ *increases* upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.
- No simple mechanism for nodal-anti-nodal dichotomy.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

# “Nodal/anti-nodal dichotomy” in STM measurements



J.C. Davis and collaborators

# Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order
   *Survey of recent experiments*

2. Superconductivity in the overdoped regime
   *BCS pairing by spin fluctuation exchange*

3. Superconductivity in the underdoped regime
   *U(1) gauge theory of fluctuating SDW order*

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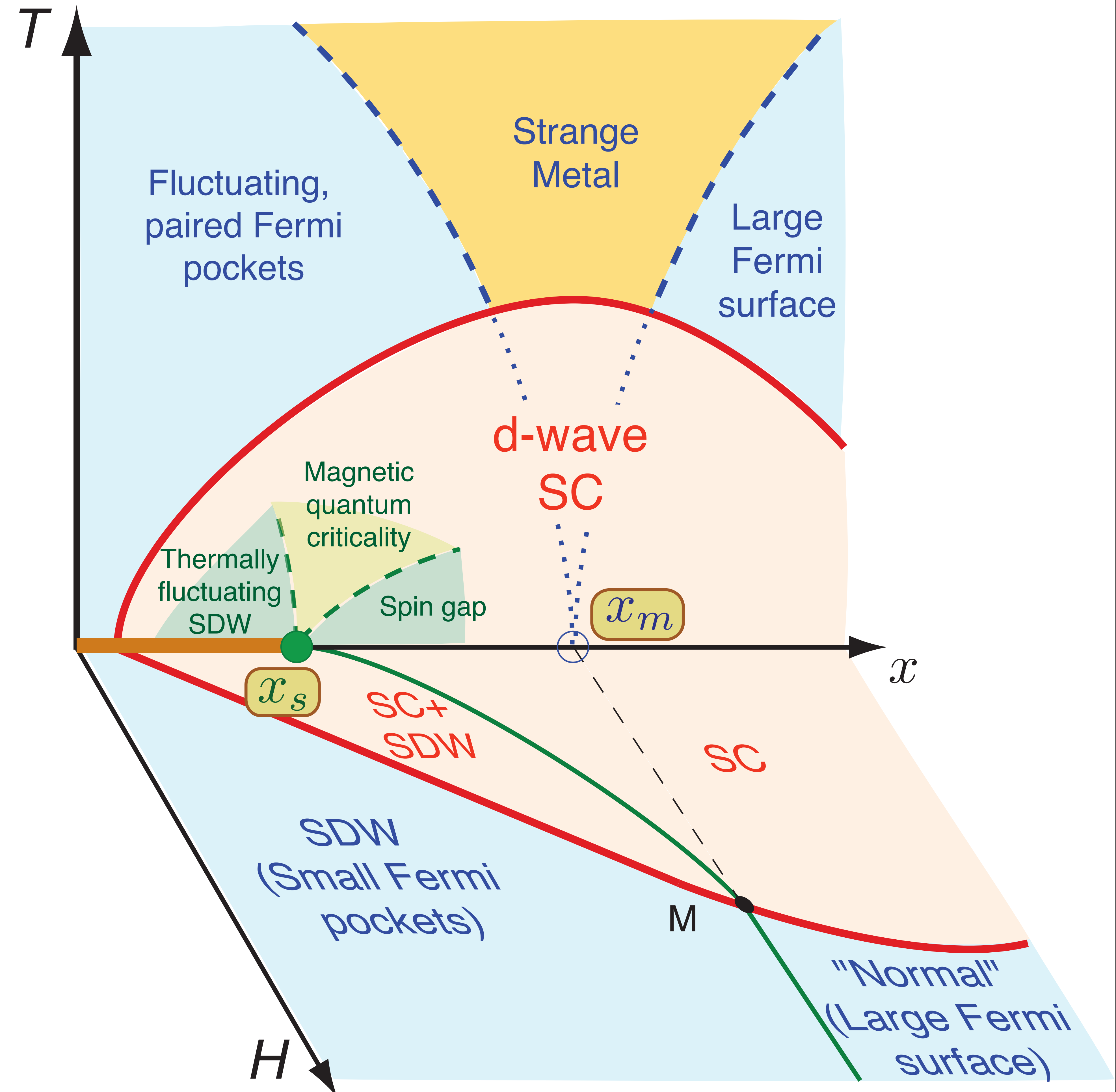
T
Fluctuating, paired Fermi pockets
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Large Fermi surface
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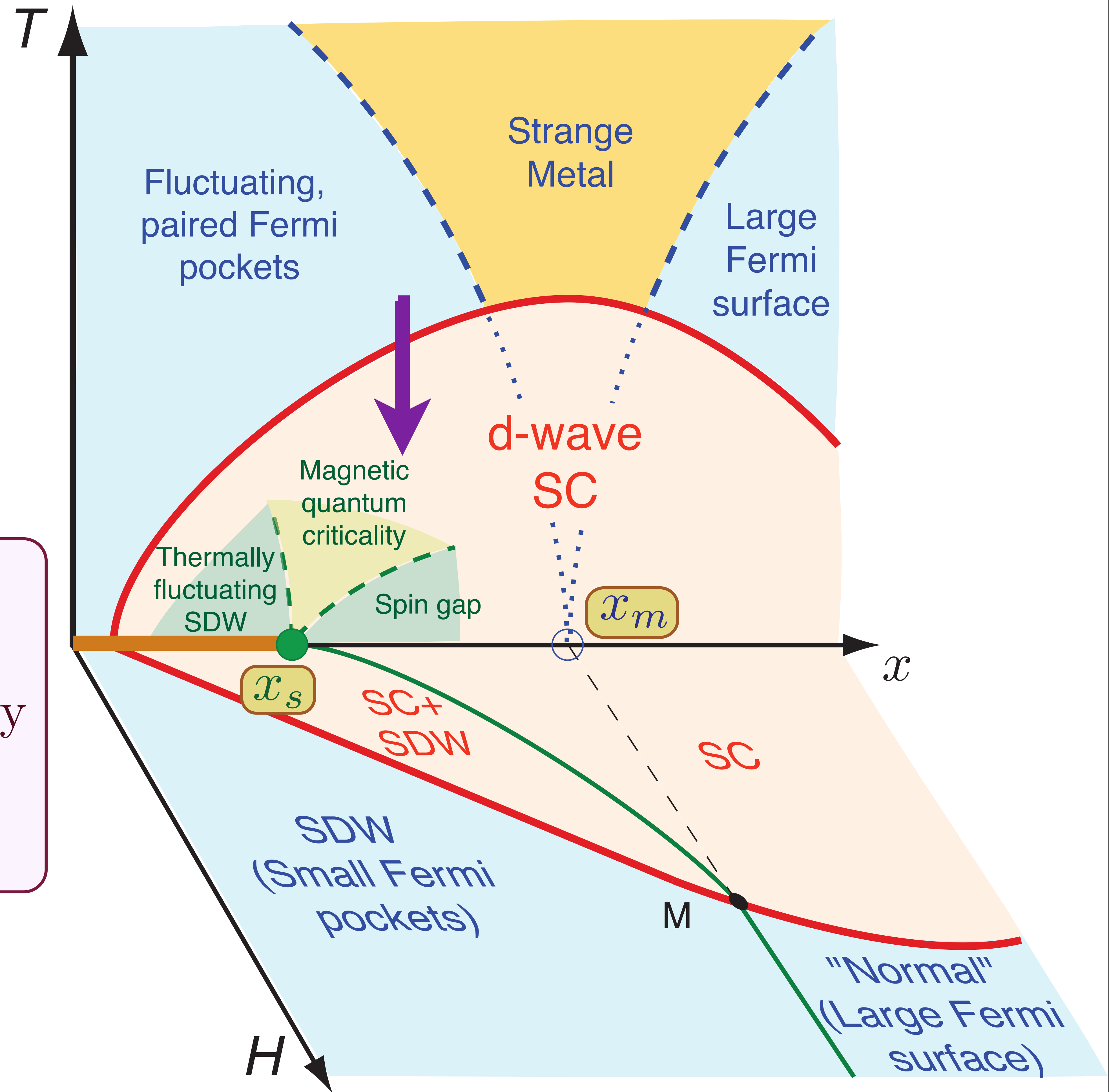
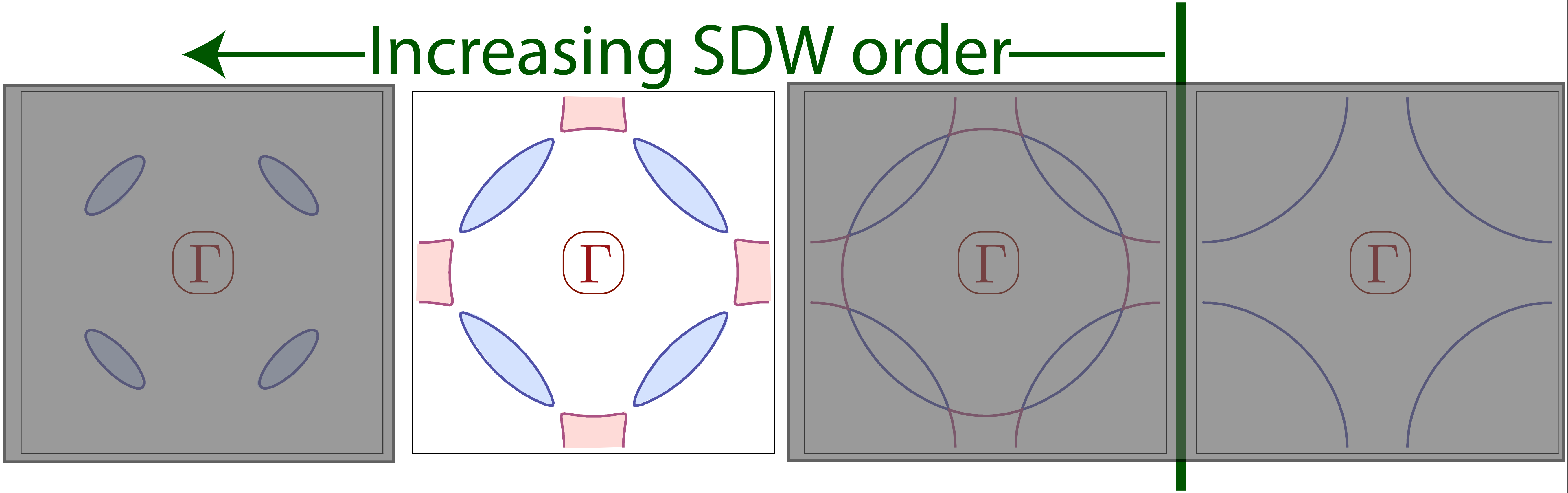
Thermally fluctuating SDW
Spin gap
$x_m$
$x_s$
$x$
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Theory of the onset of $d$-wave superconductivity from small Fermi pockets



# Theory of underdoped cuprates




Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

# Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}$$

the theory is invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha \quad ; \quad \psi_+ \rightarrow e^{-i\theta} \psi_+ \quad ; \quad \psi_- \rightarrow e^{i\theta} \psi_-$$

and the SDW order is given by

$$\hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

# Theory of underdoped cuprates

Starting from the "SDW-fermion" model with Lagrangian

$$\mathcal{L} = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left( \frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} - E_{sdw} \sum_i c^\dagger_{i\alpha} \hat{\vec{\varphi}}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} + \frac{1}{2t} \left( \partial_\mu \hat{\vec{\varphi}} \right)^2$$

# Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions $\psi_p$ with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,

$$\mathcal{L} = \sum_{\mathbf{k}, p=\pm} \Bigg[ \psi^\dagger_{\mathbf{k}p} \left( \frac{\partial}{\partial \tau} - ipA_\tau + \varepsilon_{\mathbf{k} - p\mathbf{A}} \right) \psi_{\mathbf{k}p} \\ - E_{sdw} \psi^\dagger_{\mathbf{k}p} p \psi_{\mathbf{k}+\mathbf{K},p} \Bigg]$$

# Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions $\psi_p$ with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,
- relativistic complex scalars $z_\alpha$ with charge 1, representing the orientational fluctuations of the SDW order

$$\mathcal{L} = \sum_{\mathbf{k},p=\pm} \left[ \psi^\dagger_{\mathbf{k}p} \left( \frac{\partial}{\partial \tau} - ipA_\tau + \varepsilon_{\mathbf{k}-p\mathbf{A}} \right) \psi_{\mathbf{k}p} \right.$$

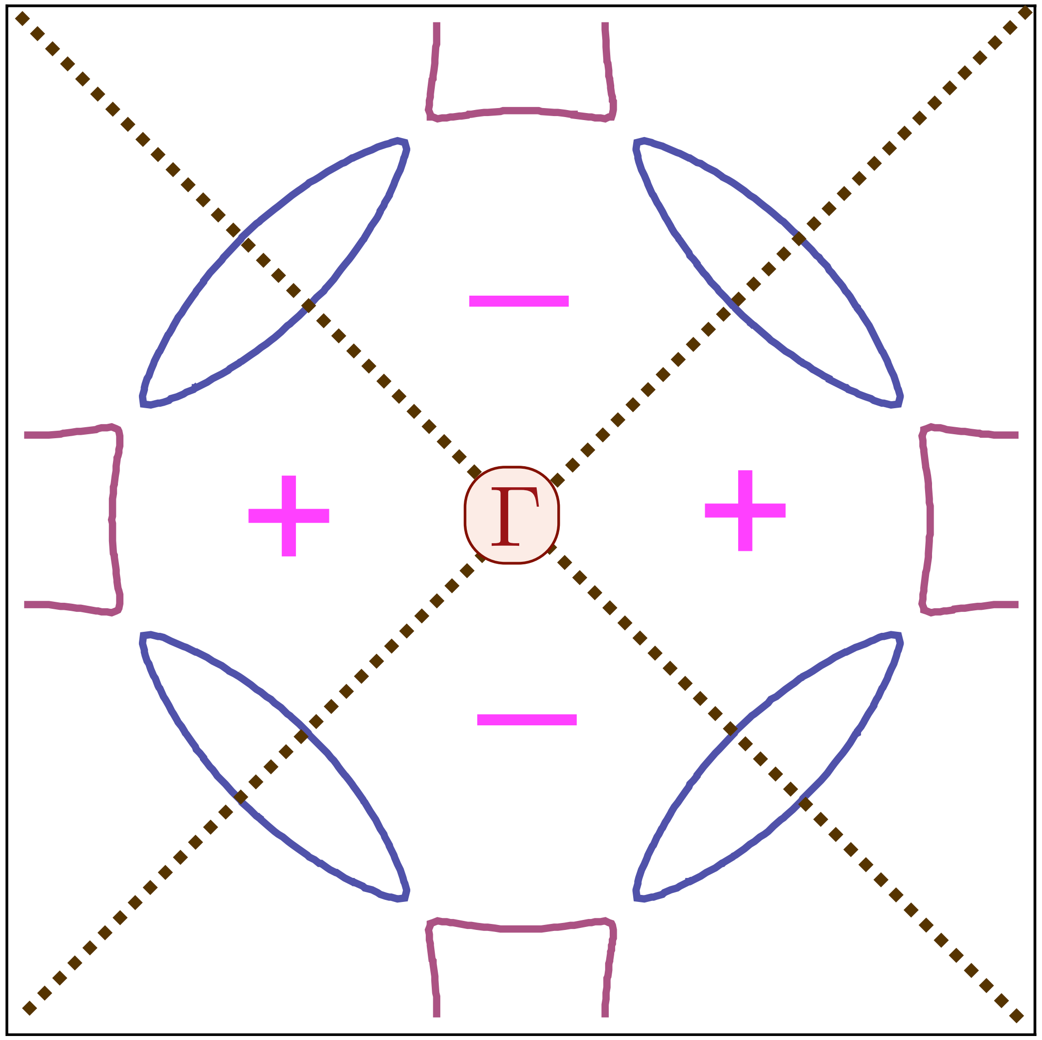
$$\left. - E_{sdw} \psi^\dagger_{\mathbf{k}p} p \psi_{\mathbf{k}+\mathbf{K},p} \right]$$

$$+ \frac{1}{t} \left[ |(\partial_\tau - iA_\tau) z_\alpha|^2 + v^2 |\nabla - i\mathbf{A}) z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1) \right]$$

# Features of theory

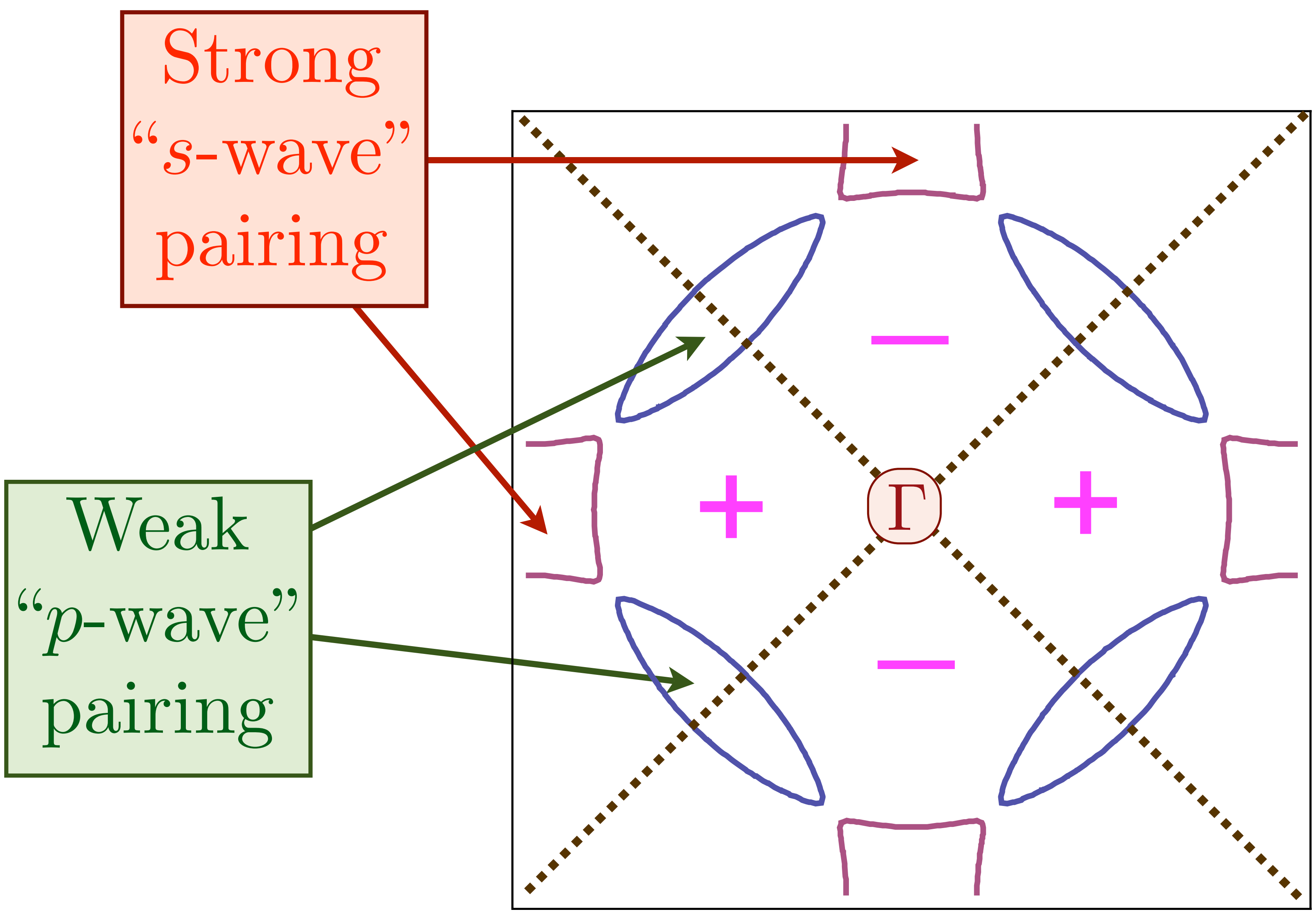
## Features of theory

- $d$-wave superconductivity.

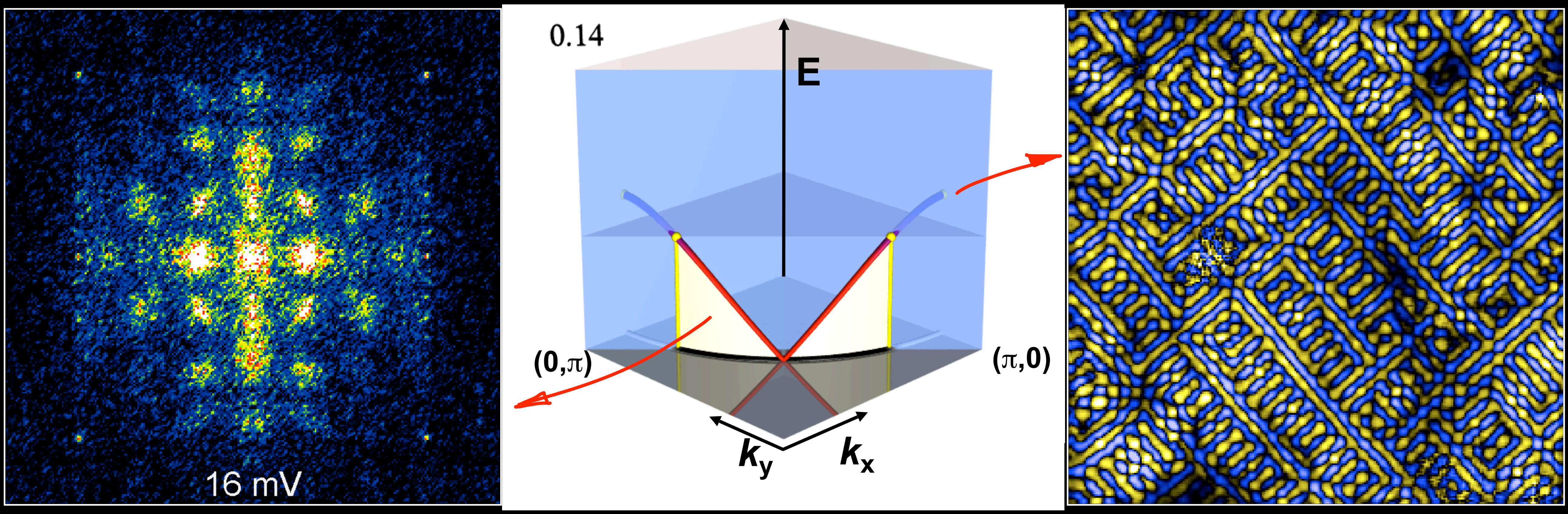


# Features of theory

- $d$-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.



# “Nodal/anti-nodal dichotomy” in STM measurements




J.C. Davis and collaborators

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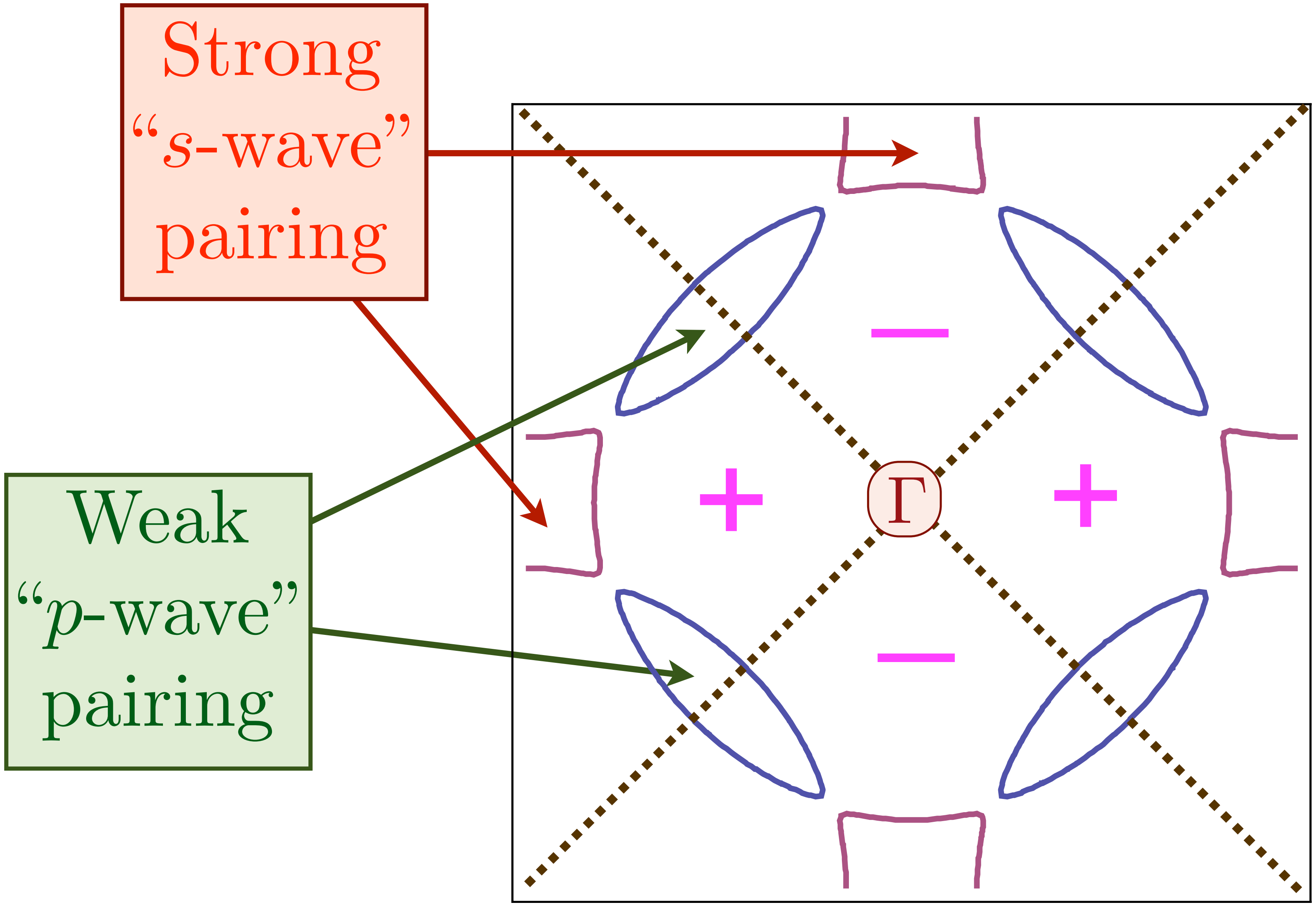
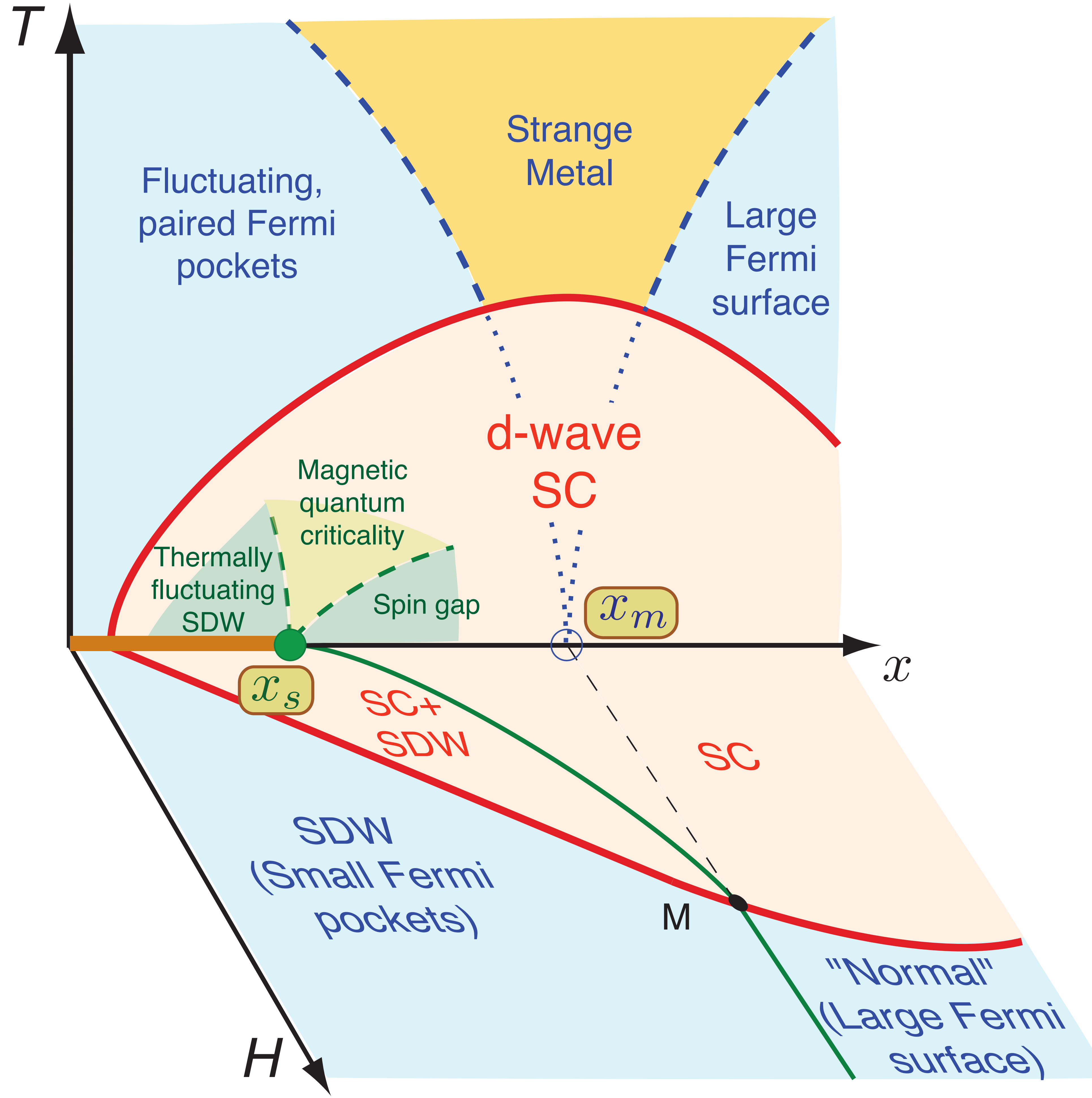


## Features of theory

- $d$-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.
- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor. Onset of confinement, large Fermi surface, and possible charge order.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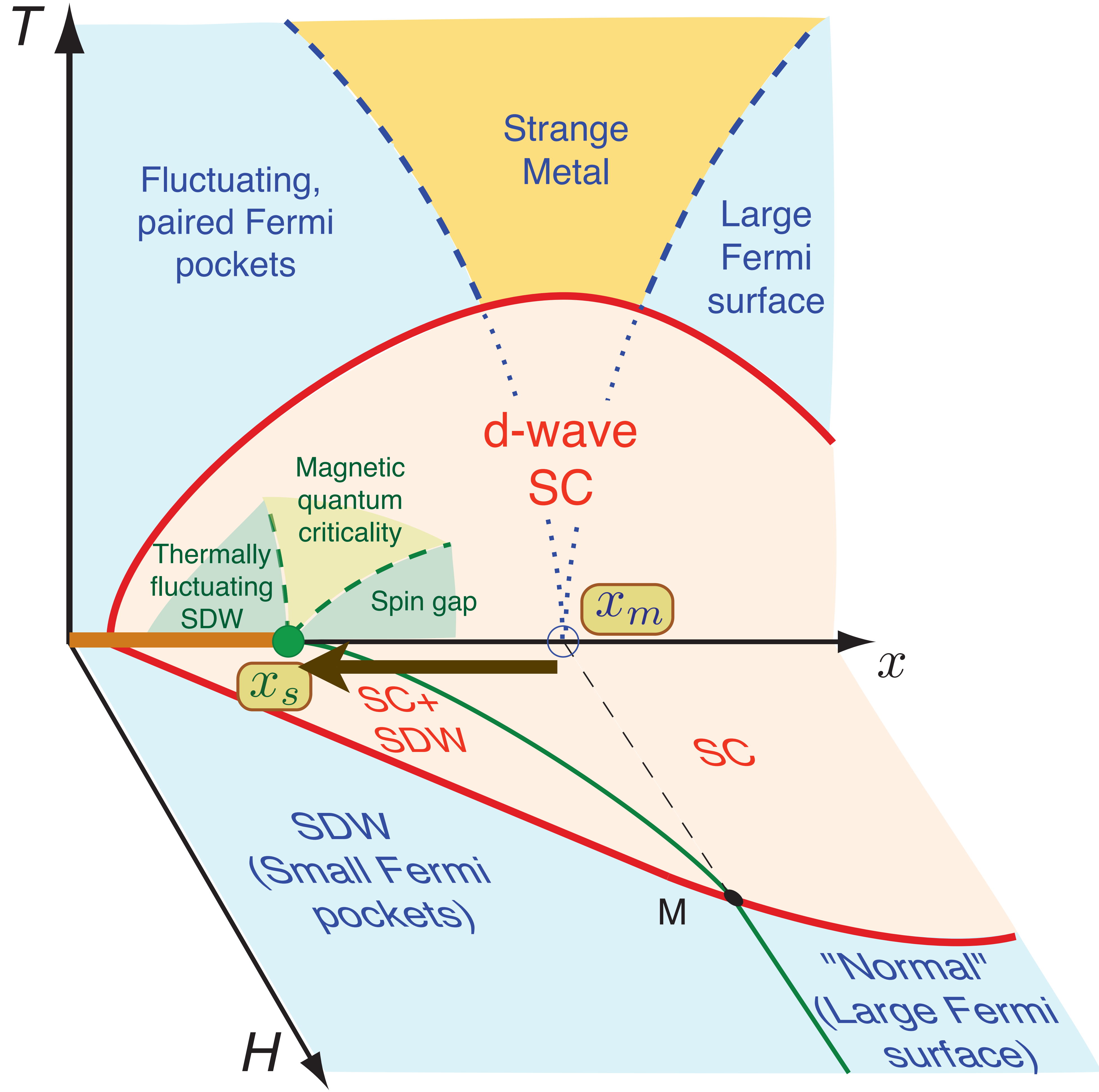
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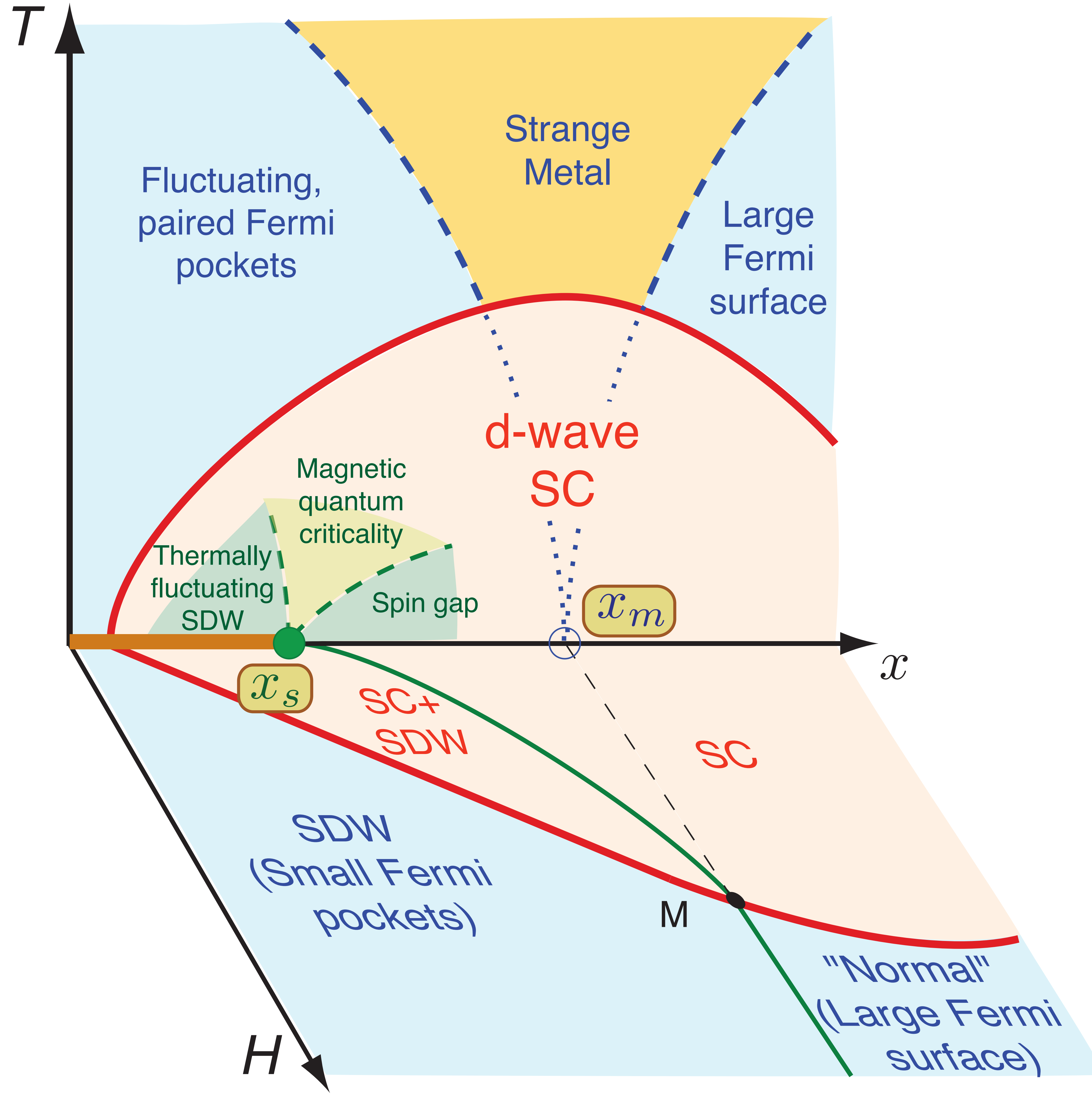

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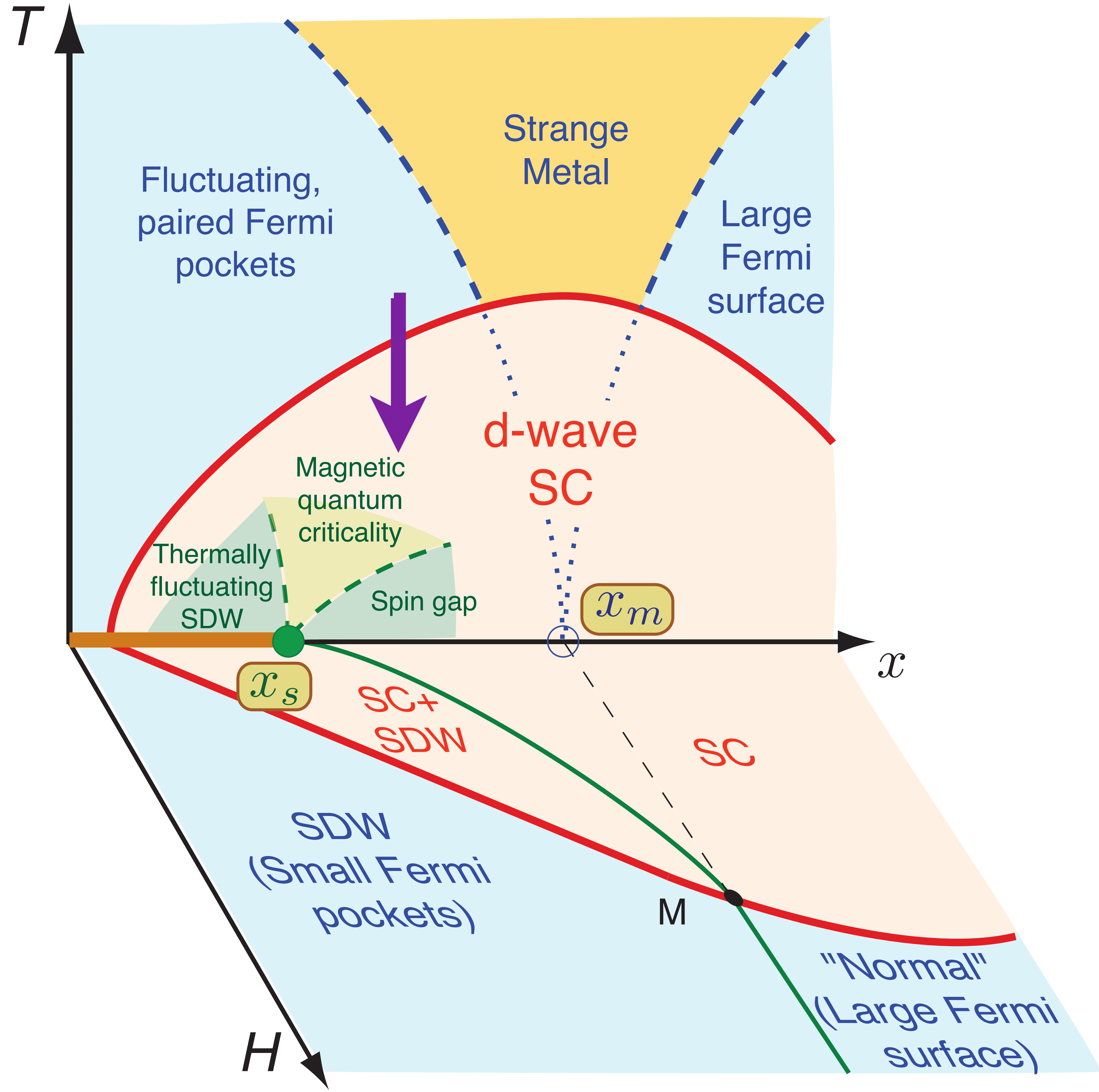

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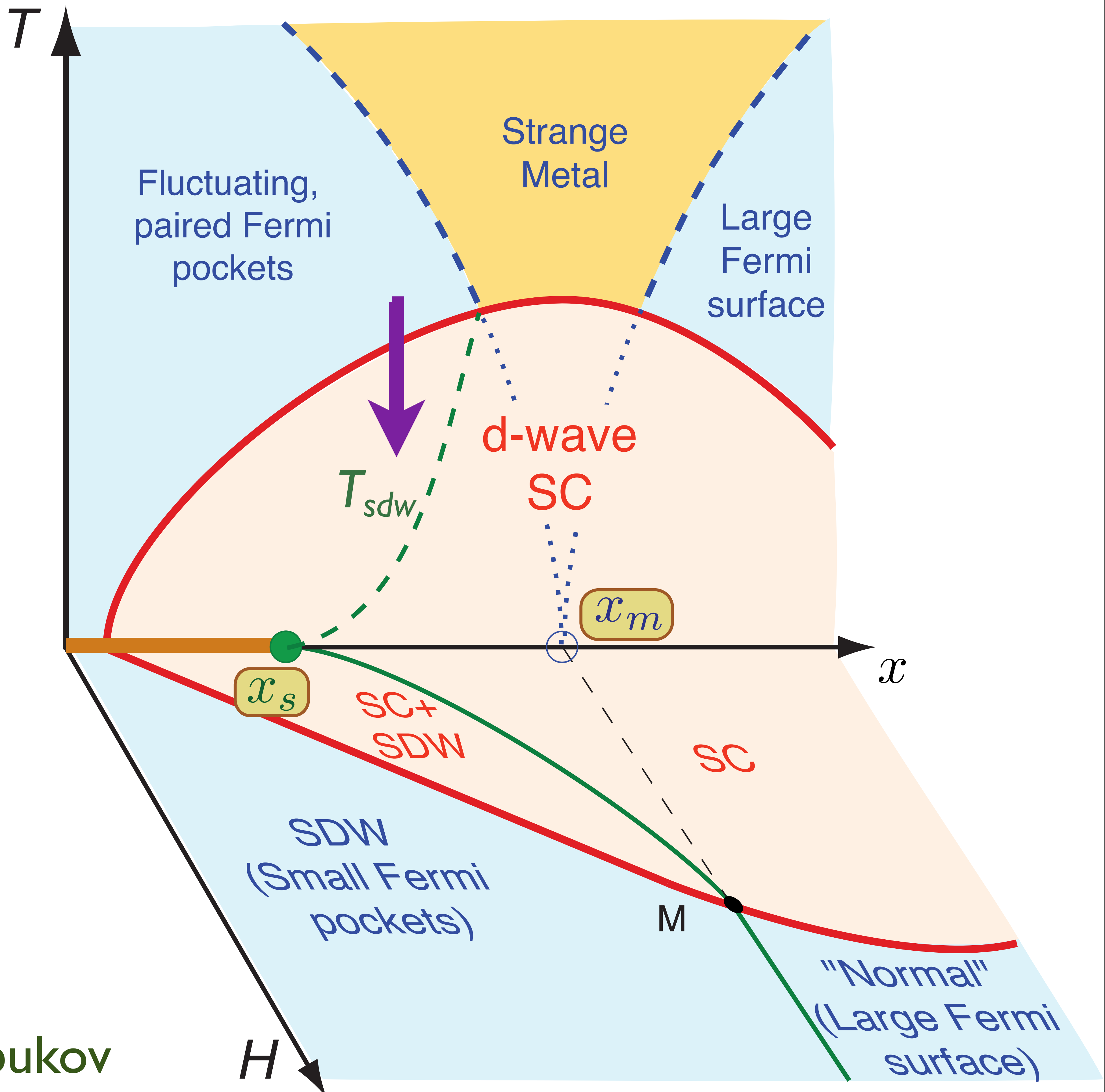
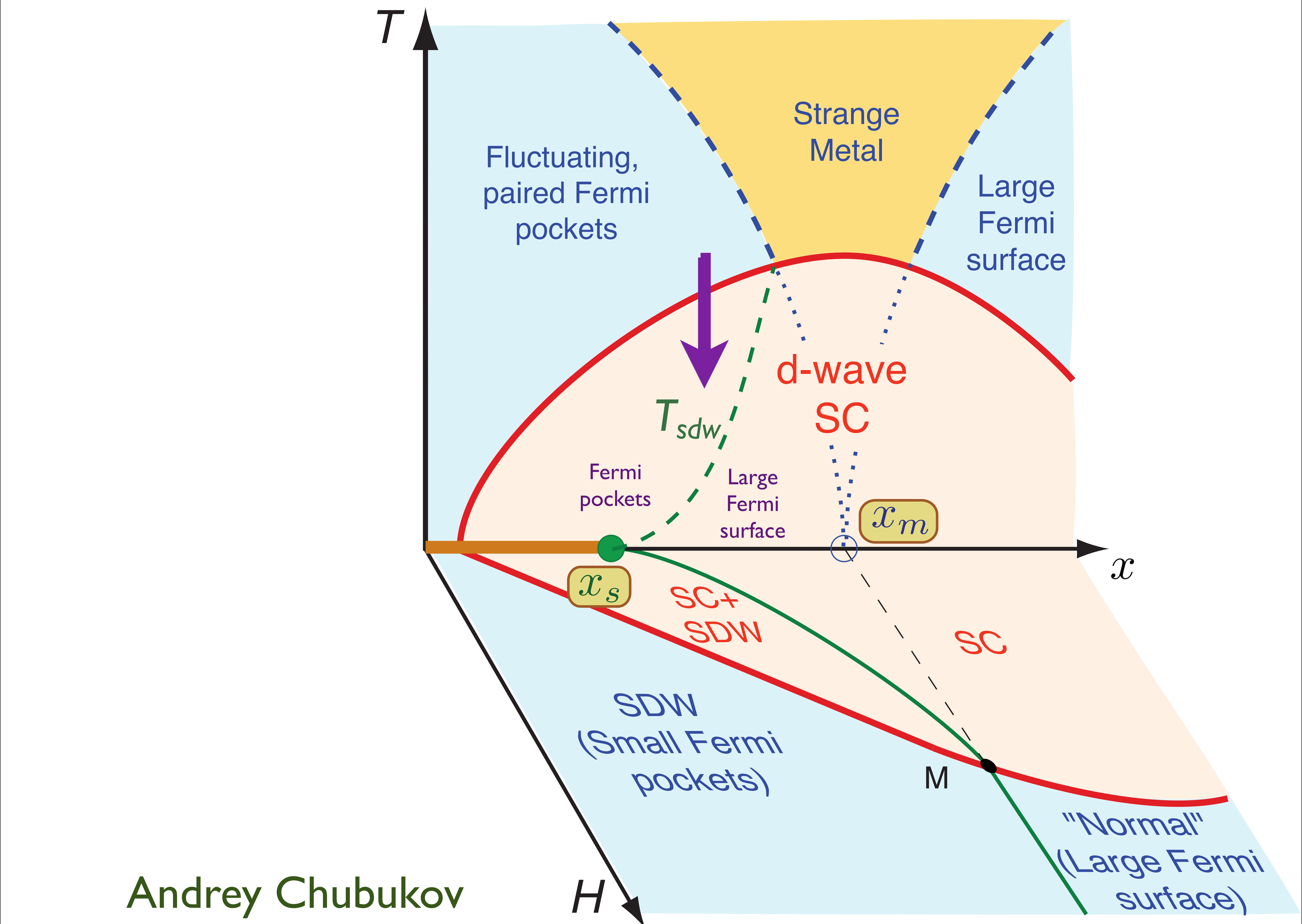



Andrey Chubukov



T
Strange
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Fluctuating,
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$T_{sdw}$
Fermi
pockets
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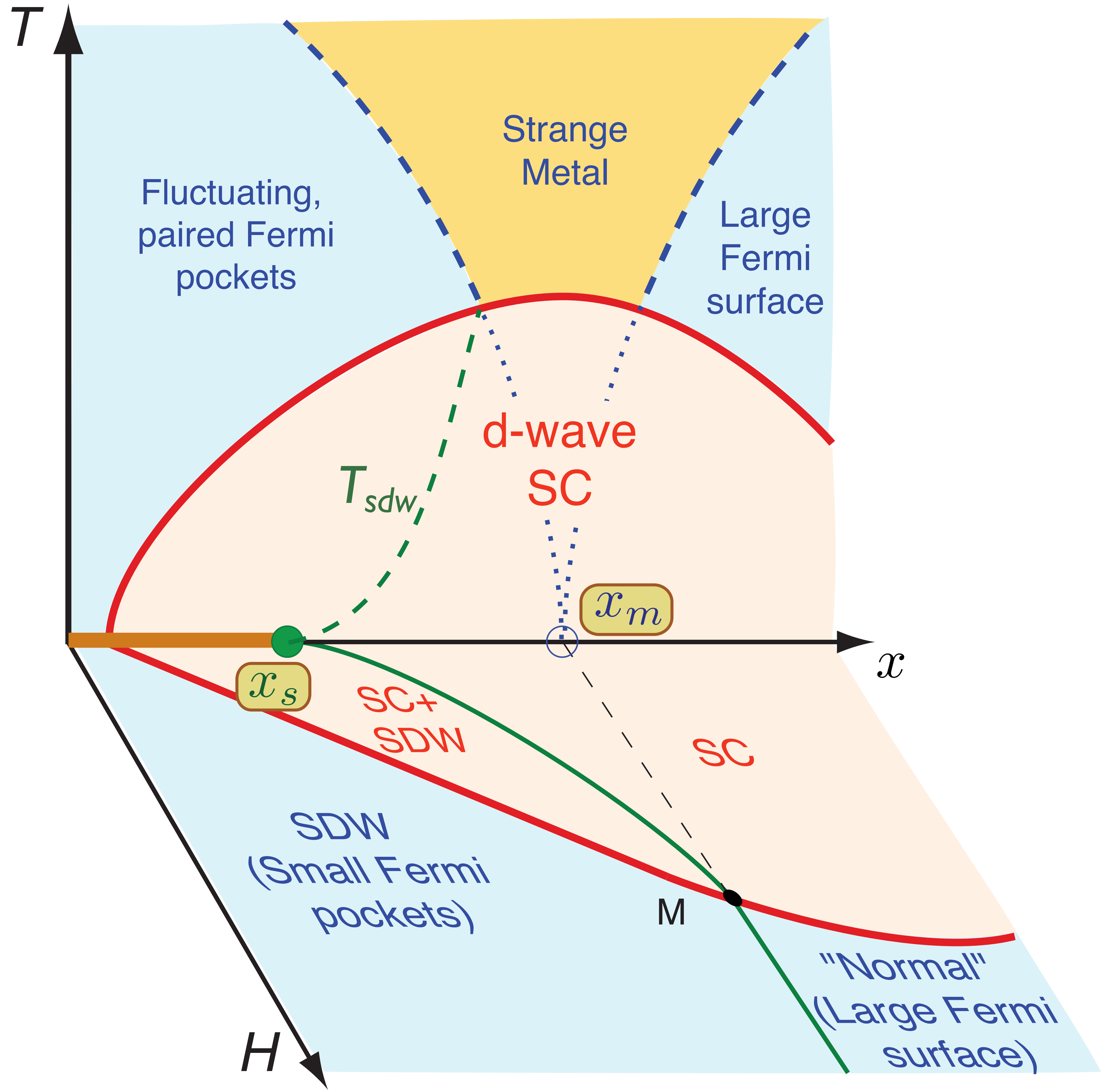

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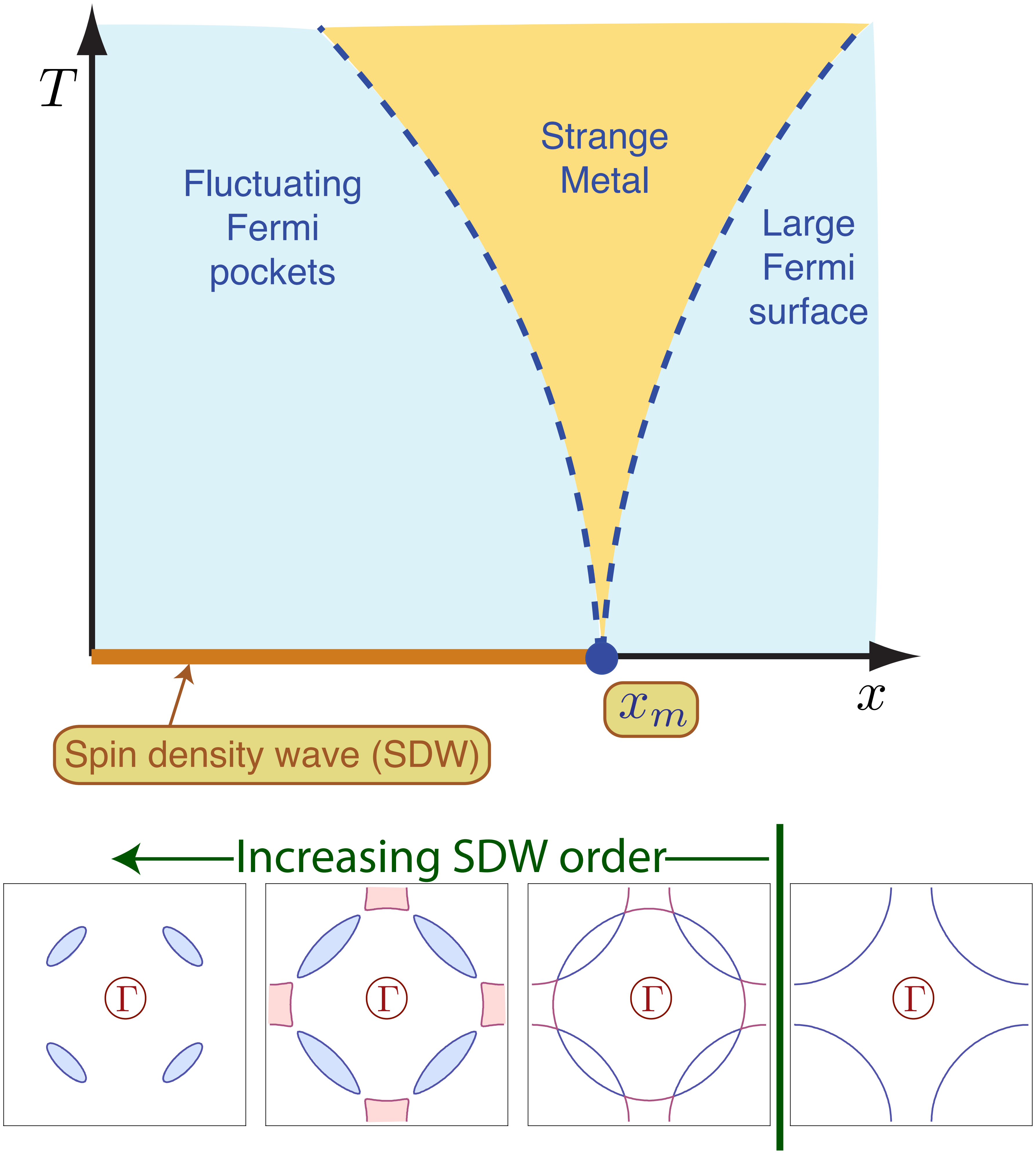
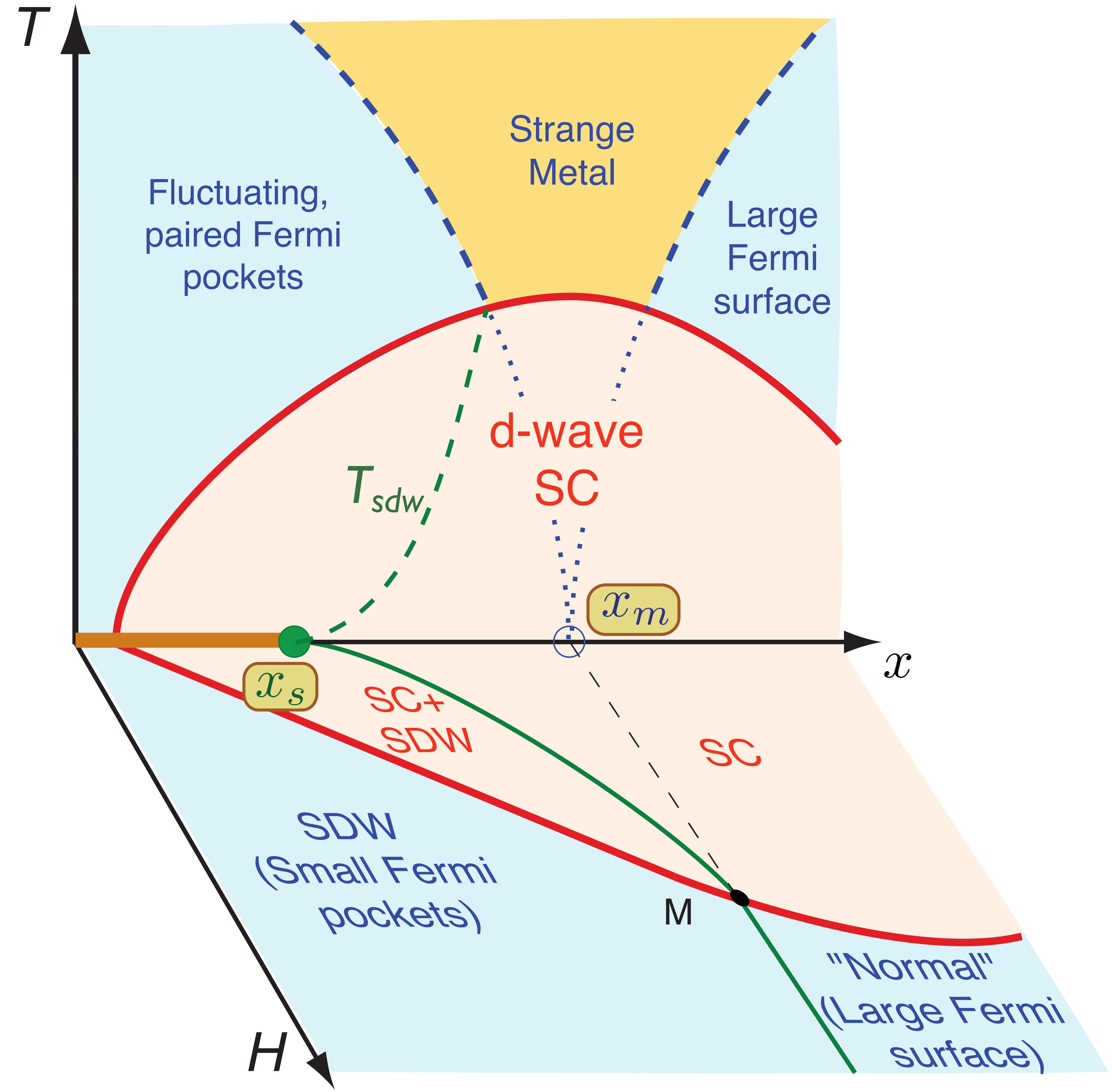


$T$
Strange Metal
Fluctuating Fermi pockets
Large Fermi surface
$x$
$x_m$
Spin density wave (SDW)
Increasing SDW order
Γ
Γ
Γ
Γ



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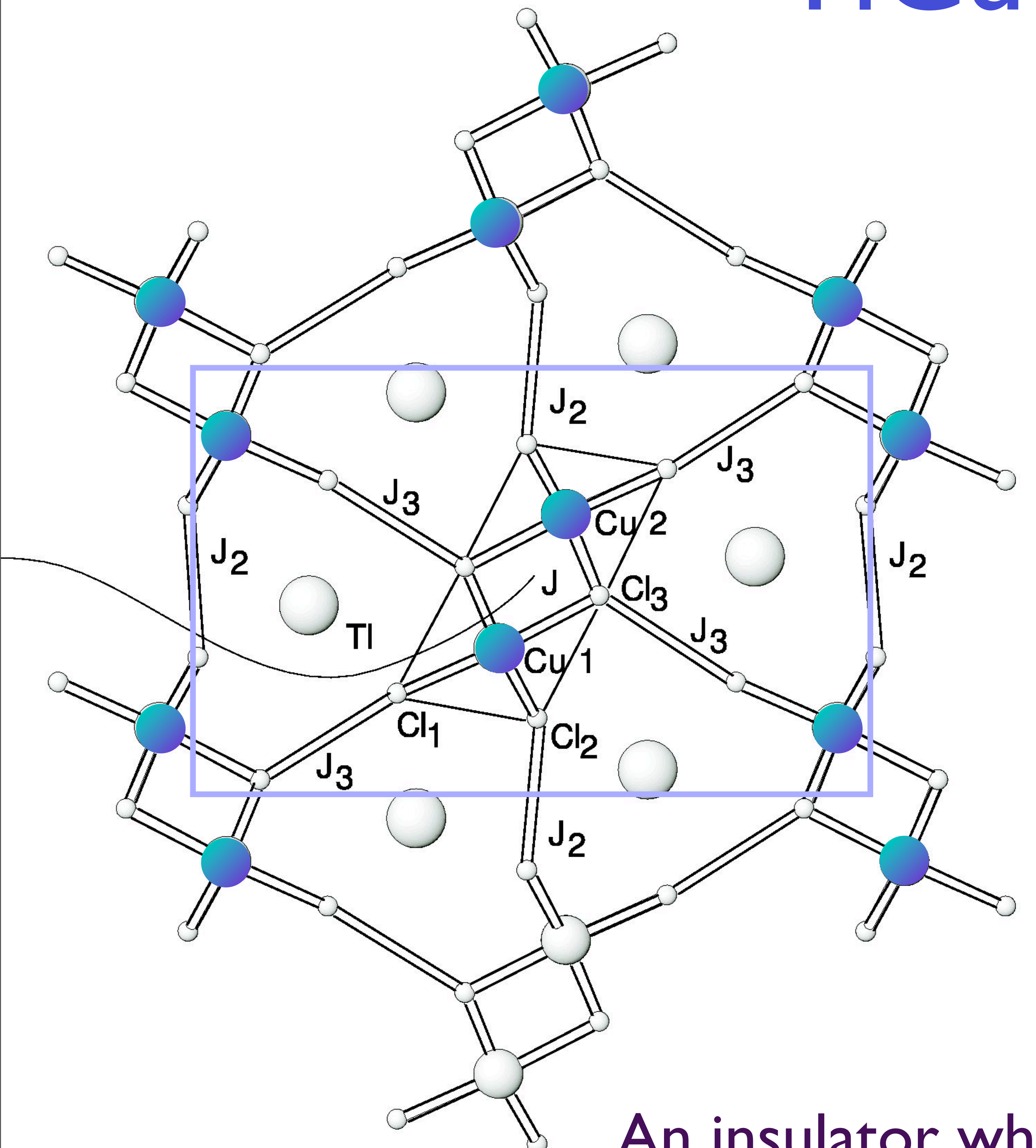
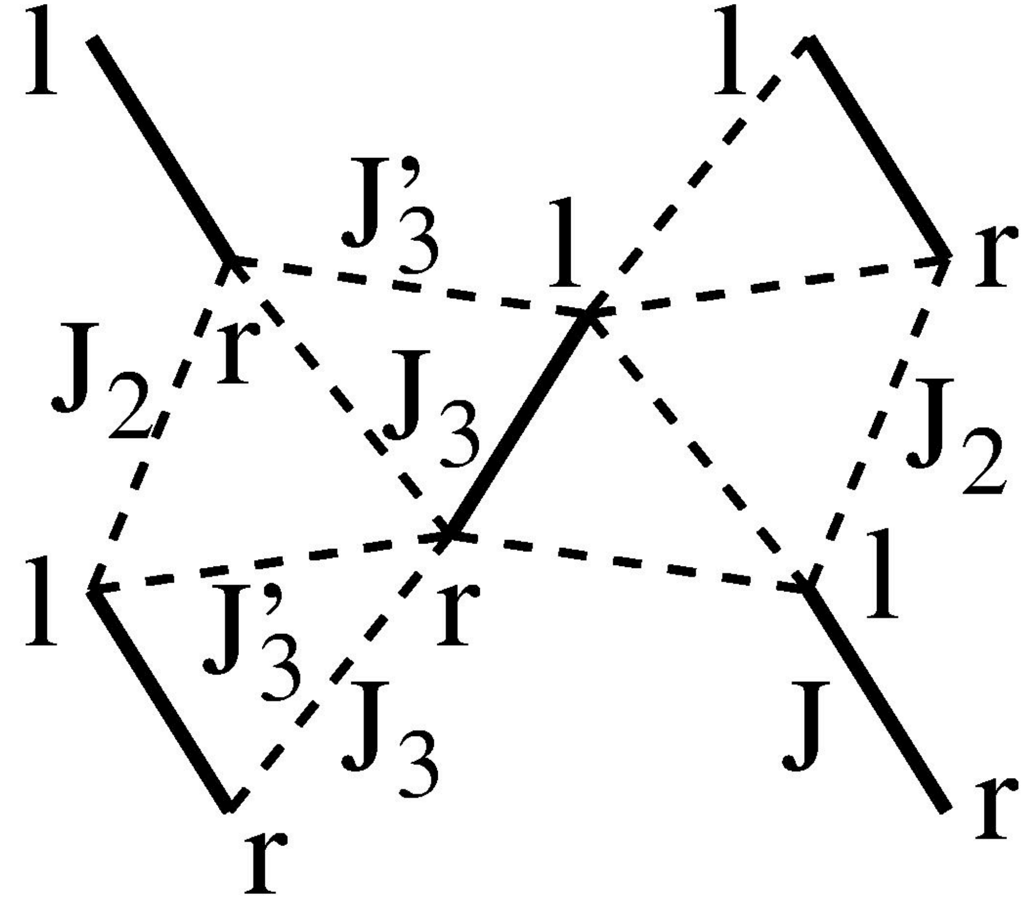
Elusive optimal doping quantum critical point has been “hiding in plain sight”.

It is shifted to lower doping by the onset of superconductivity

# TlCuCl$_3$

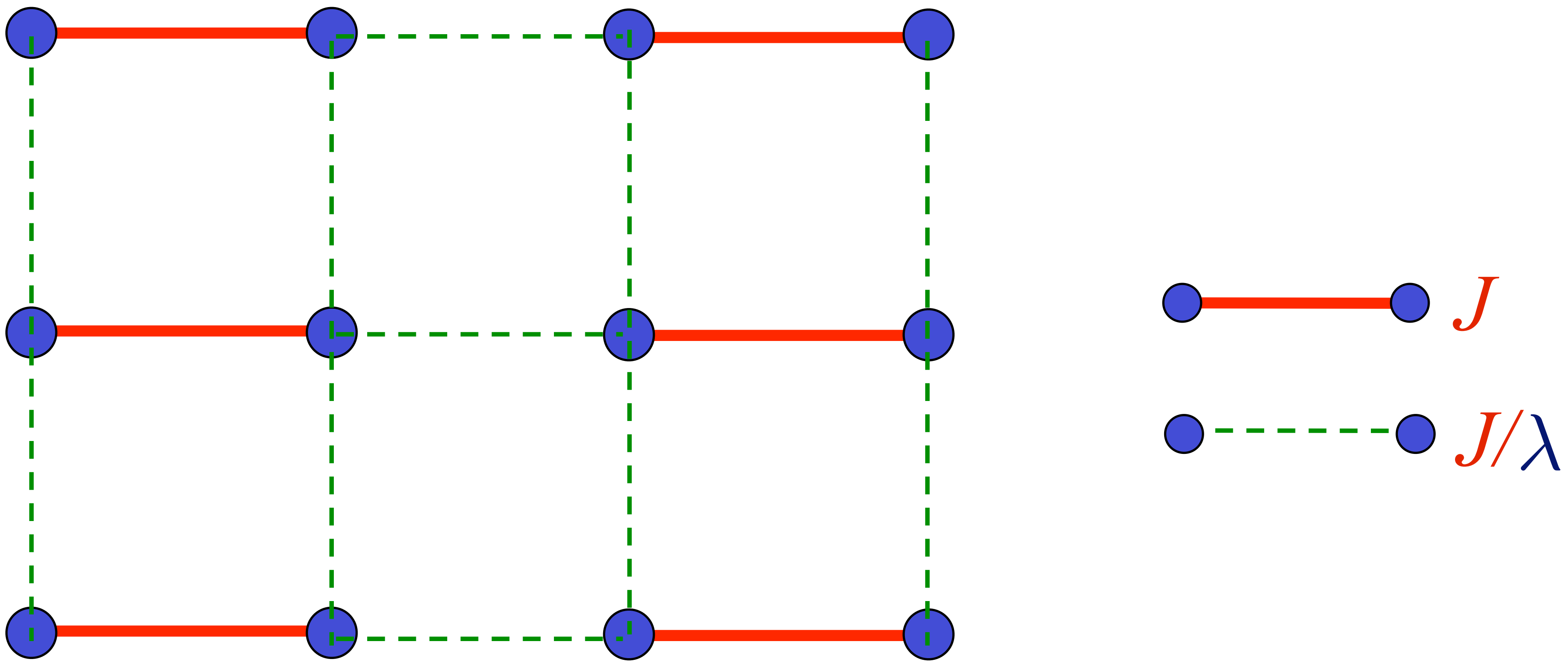



An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

# Square lattice antiferromagnet

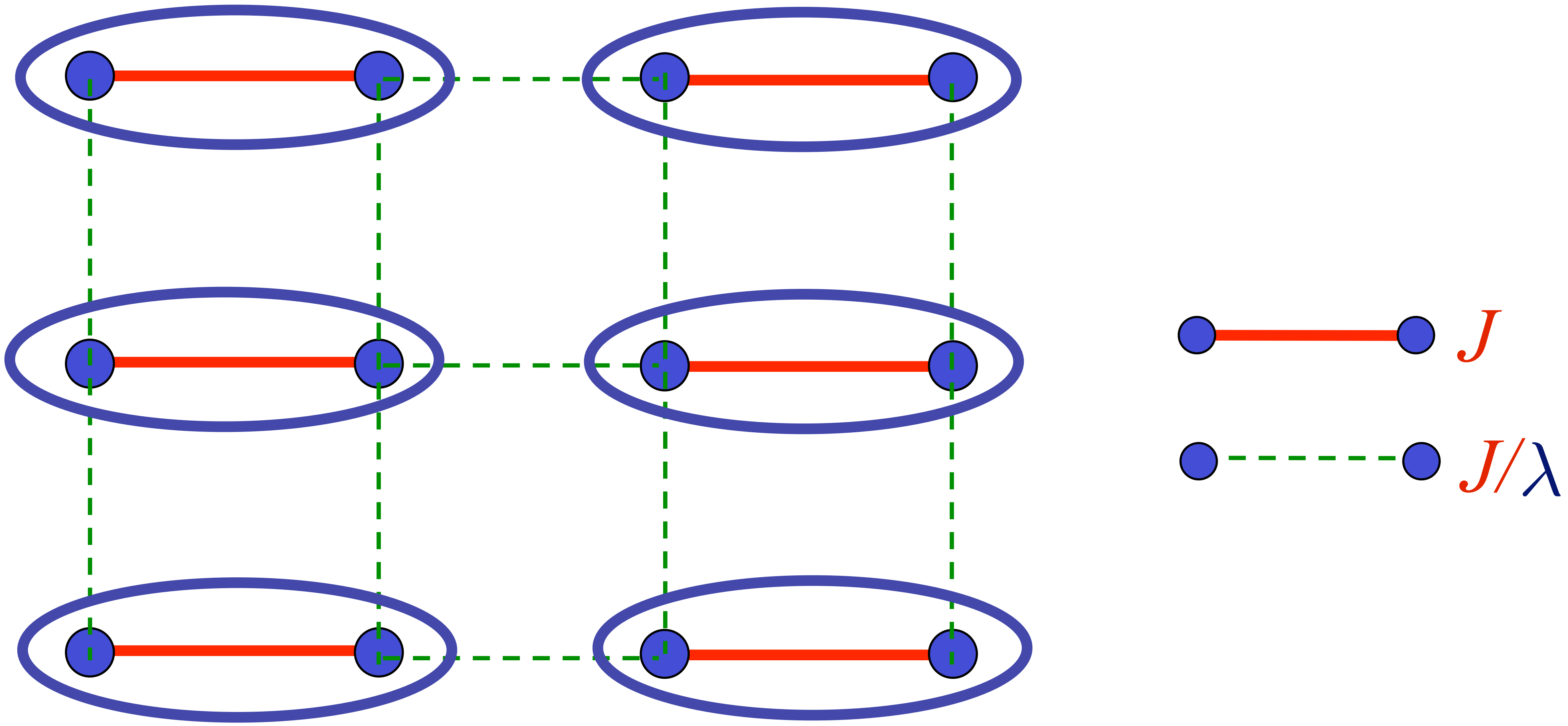
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



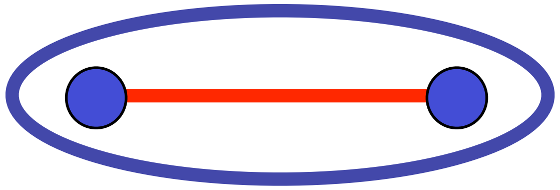

Weaken some bonds to induce spin entanglement in a new quantum phase

# Square lattice antiferromagnet

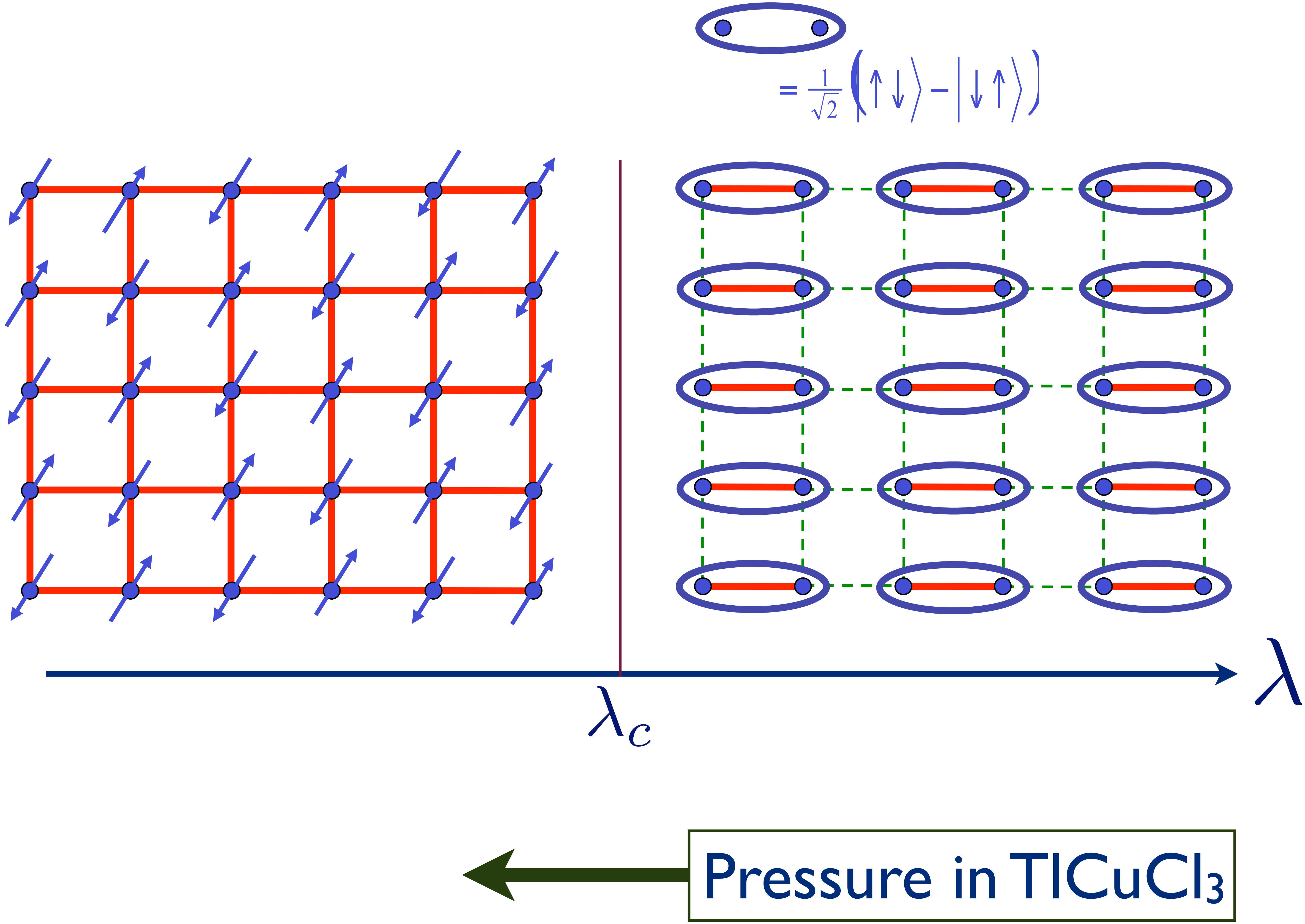
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$




Ground state is a "quantum paramagnet" with spins locked in valence bond singlets

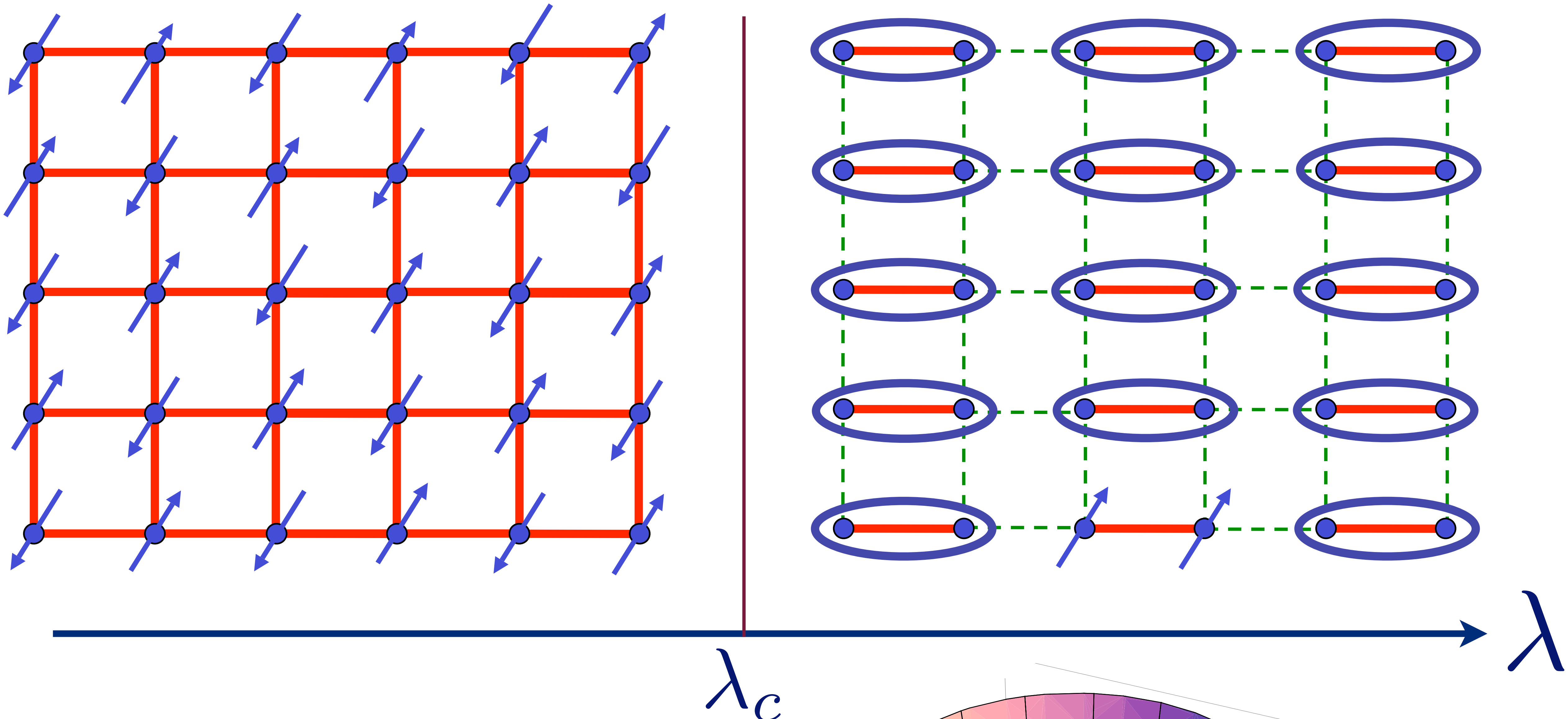


$$= \frac{1}{\sqrt{2}}\Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle\Big)$$



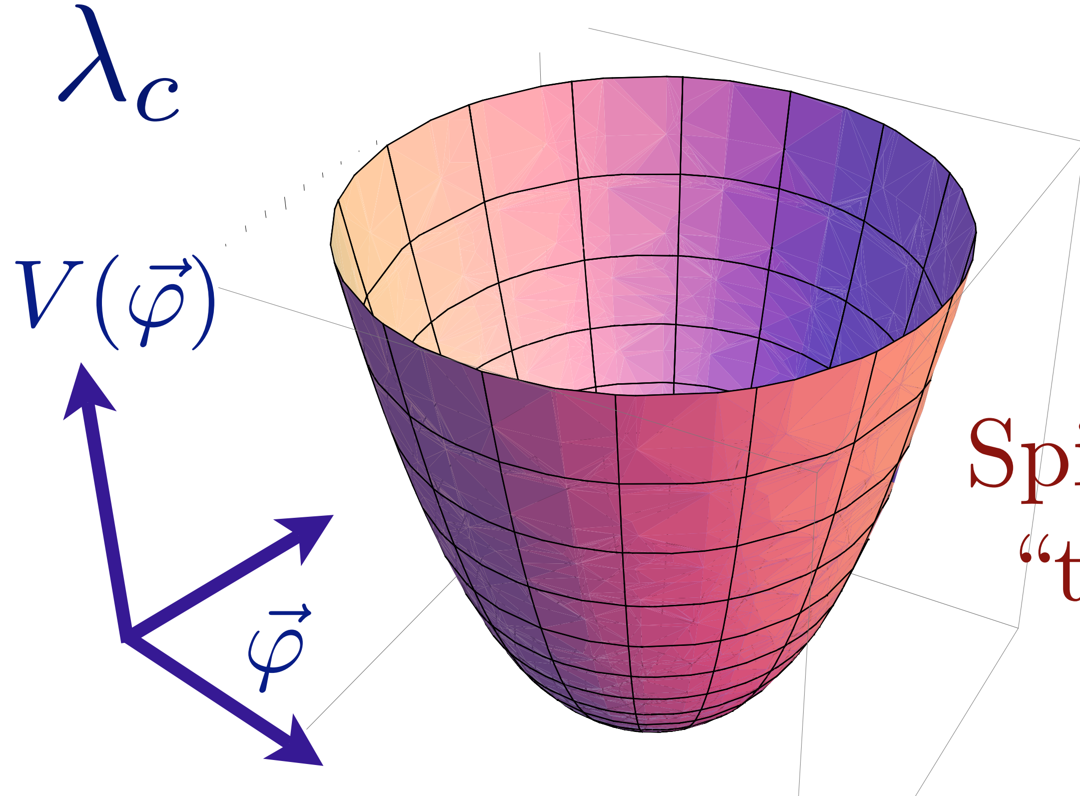
$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
$\lambda$
$\lambda_c$
Pressure in $TlCuCl_3$

# Excitation spectrum in the paramagnetic phase



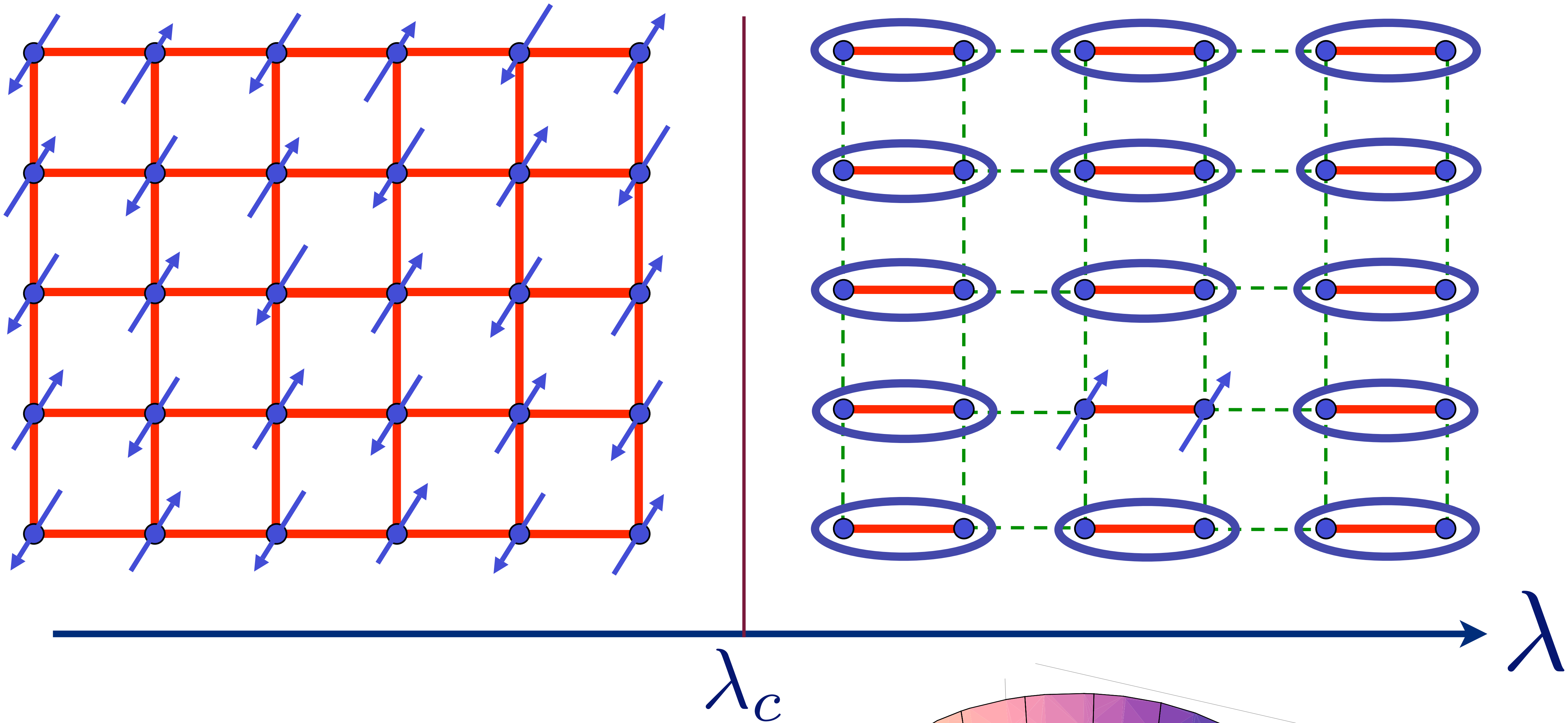

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$




Spin $S = 1$
"triplon"

# Excitation spectrum in the paramagnetic phase

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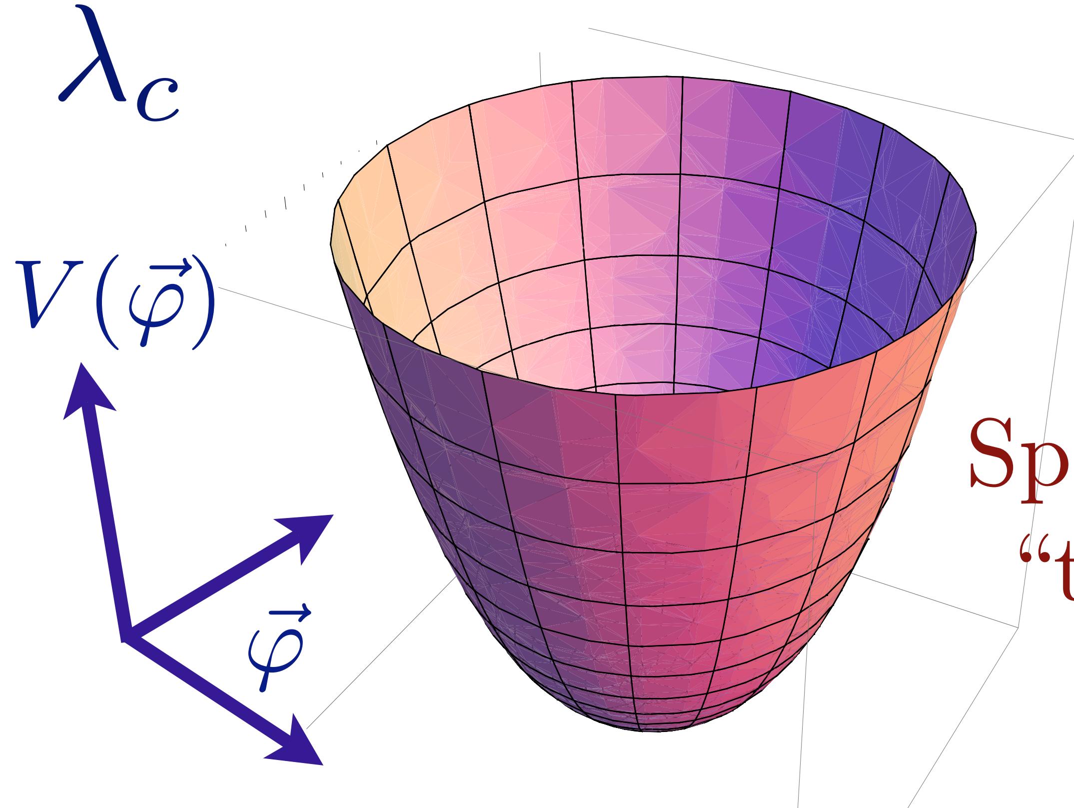


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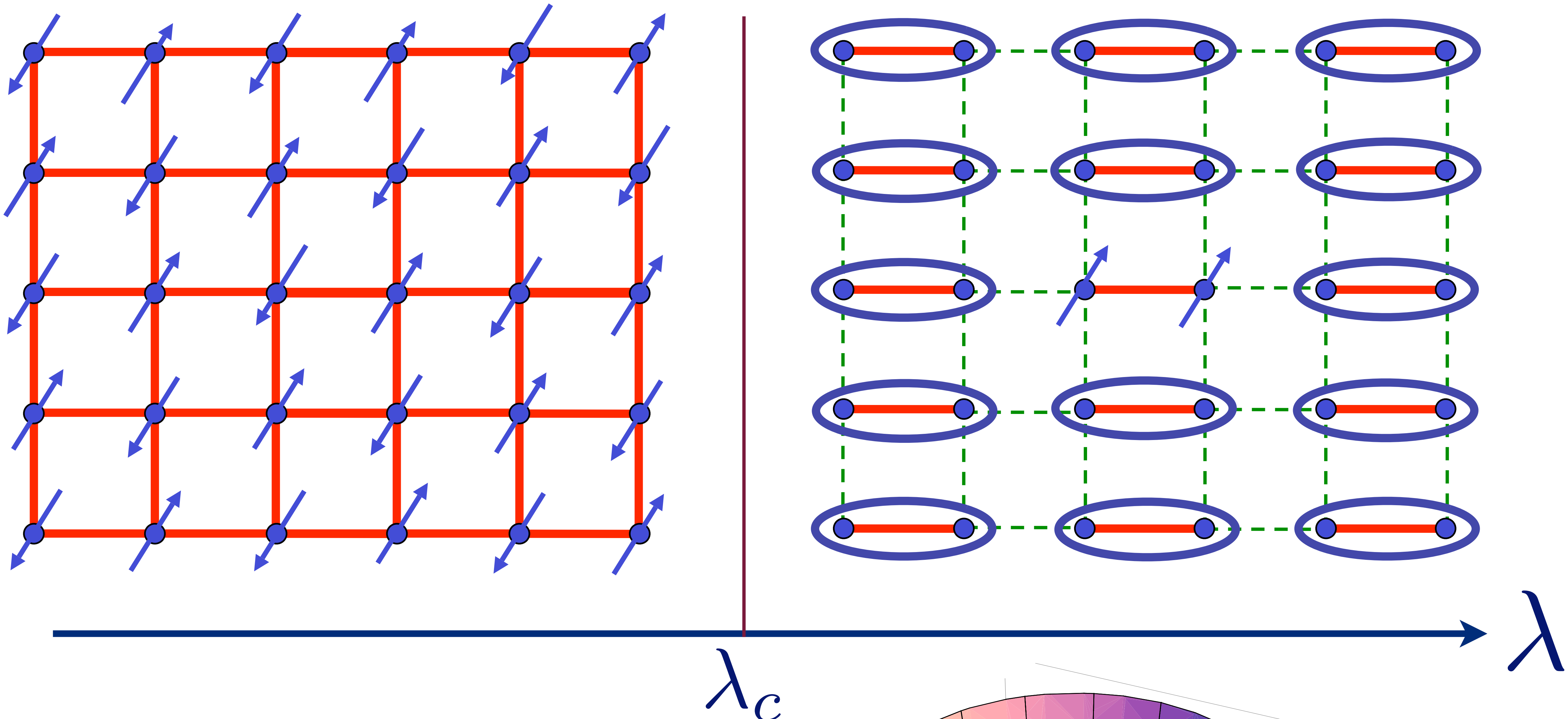


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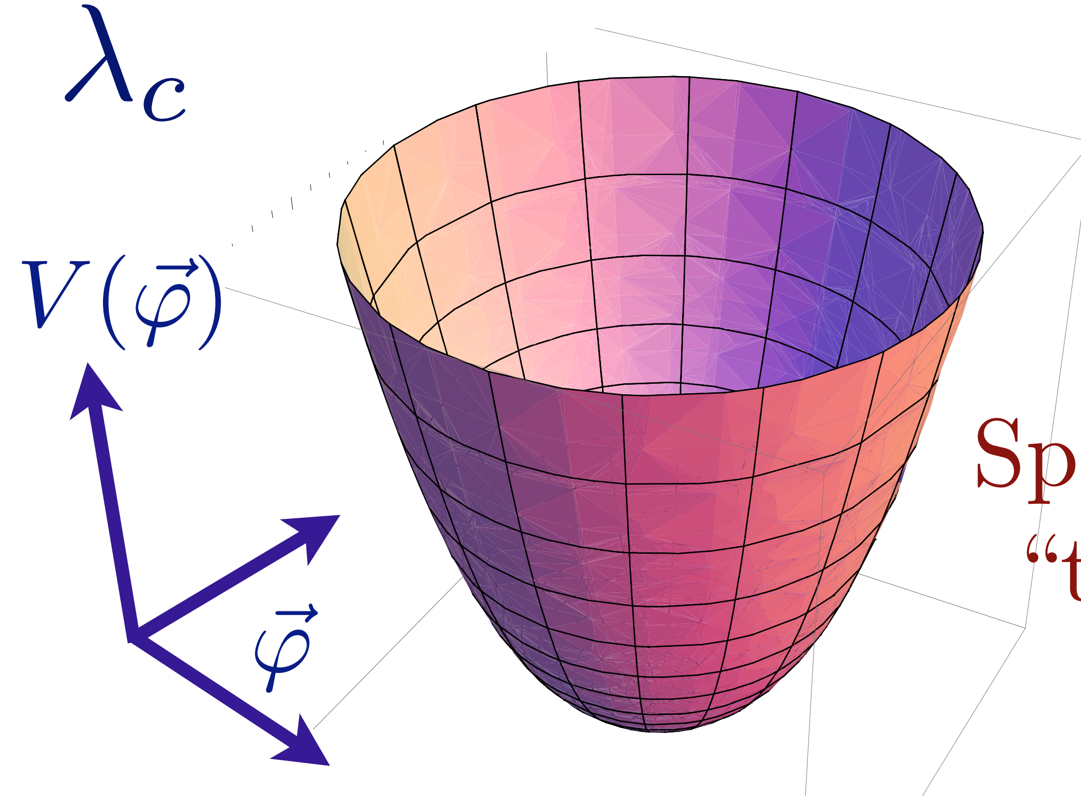


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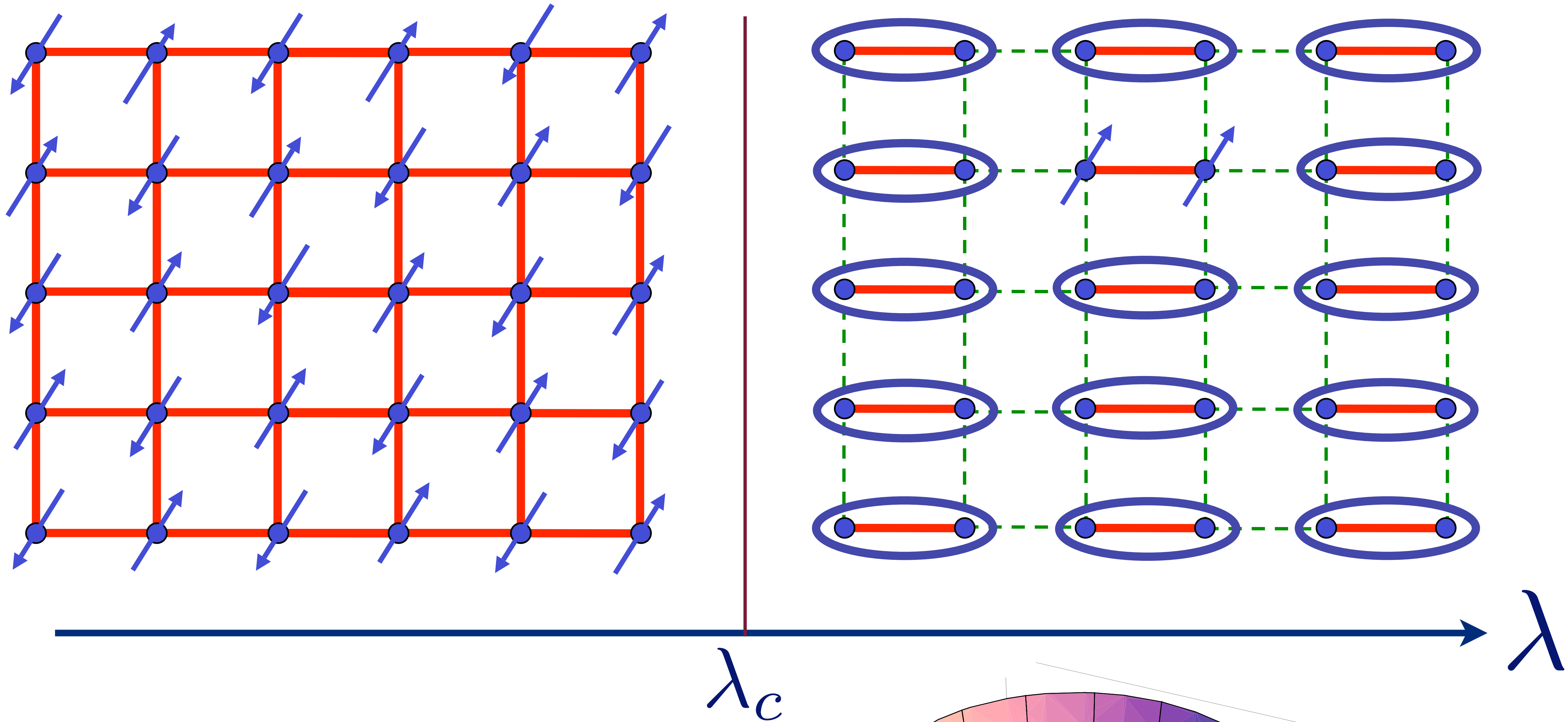
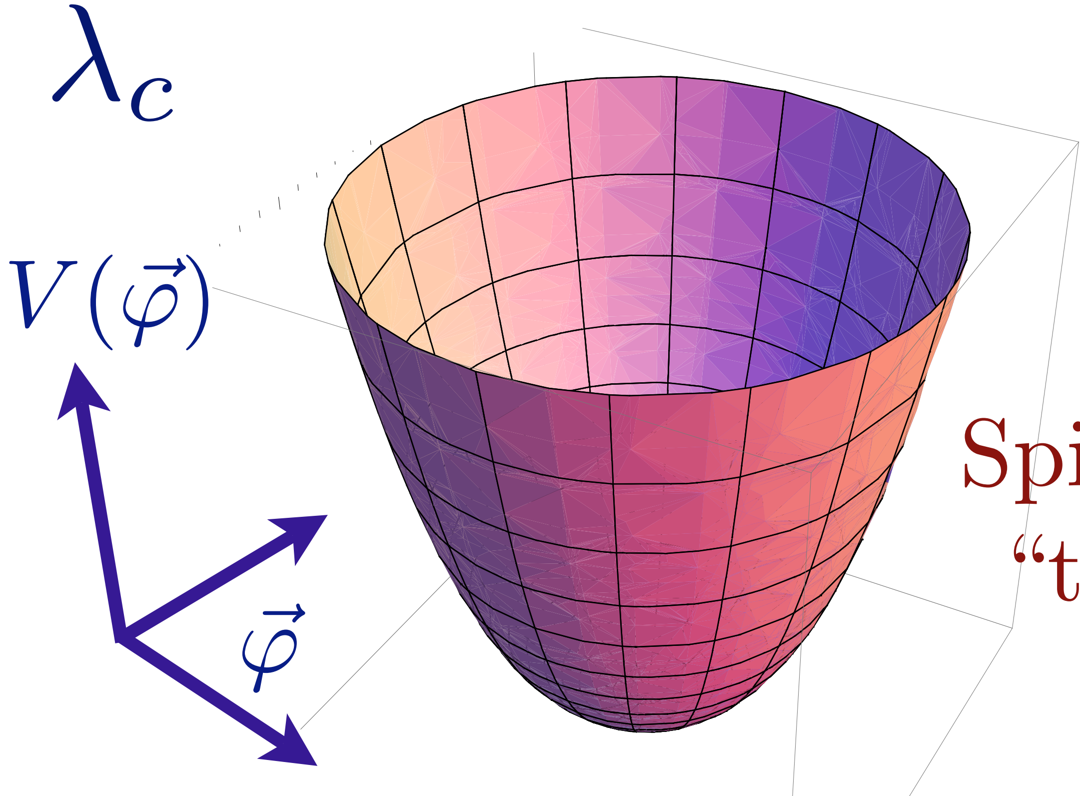
# Excitation spectrum in the paramagnetic phase

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Spin $S = 1$
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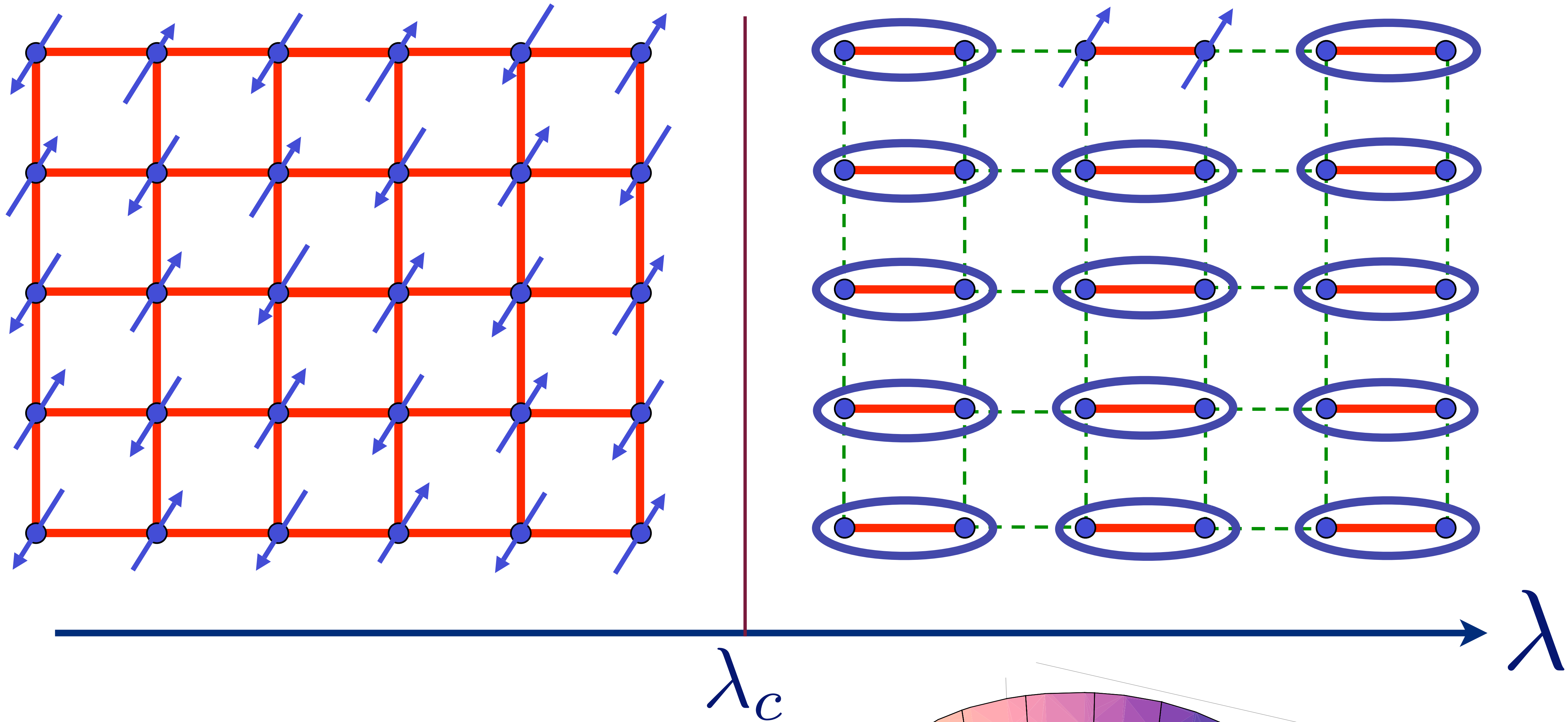
# Excitation spectrum in the paramagnetic phase

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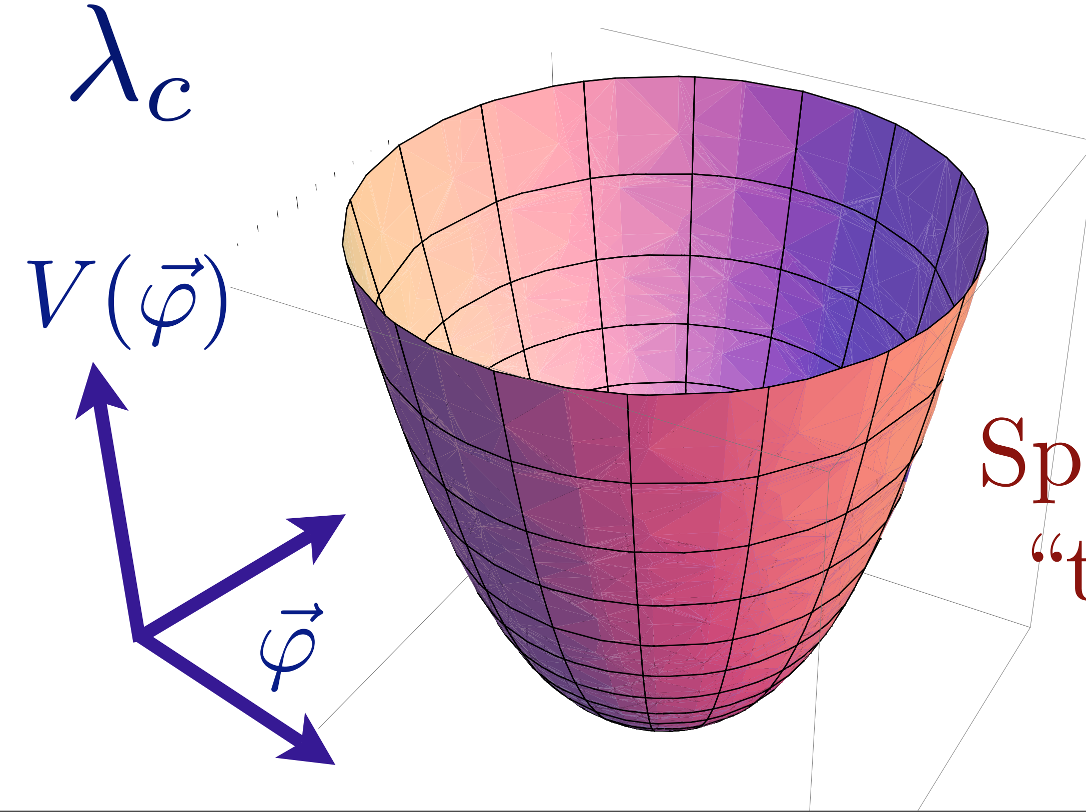
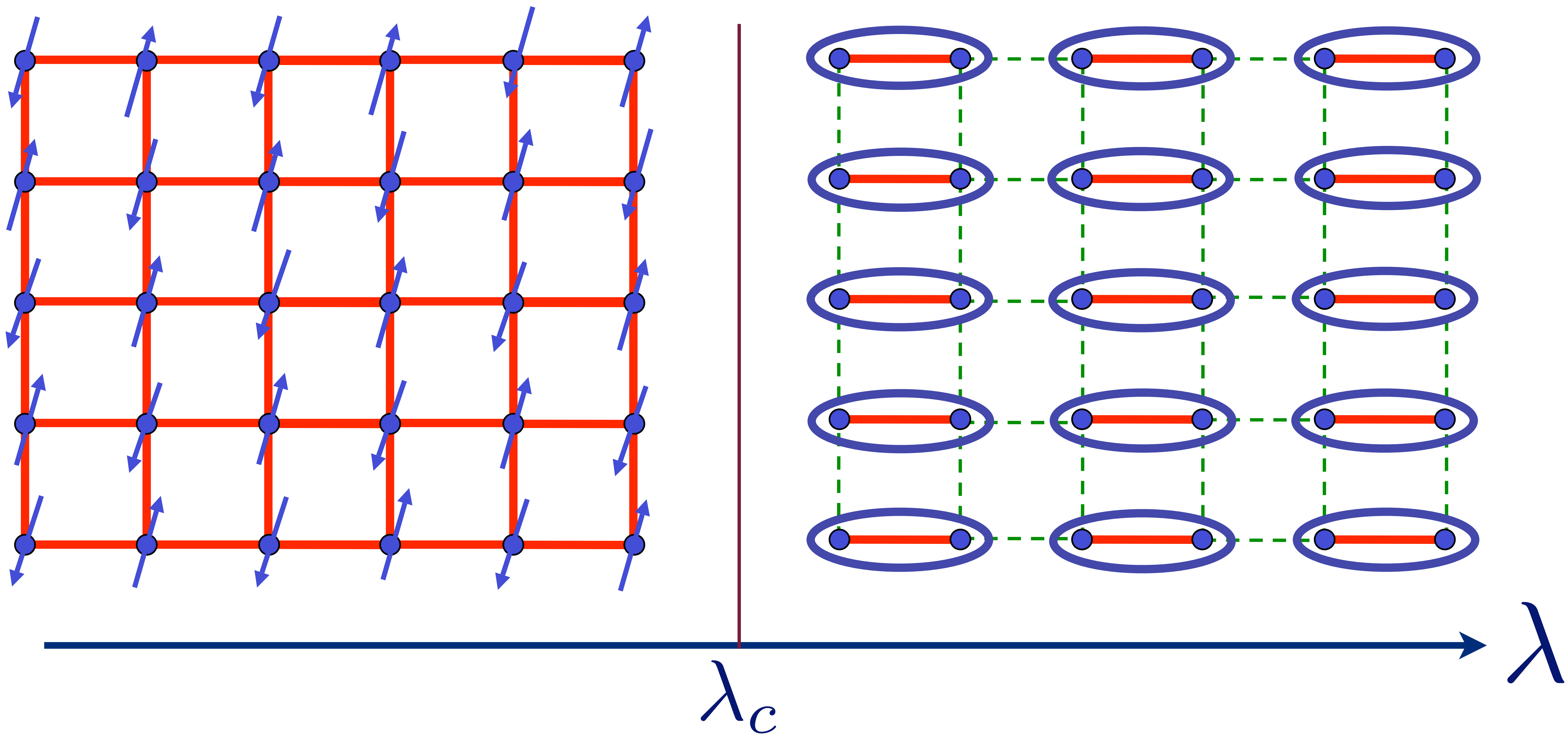


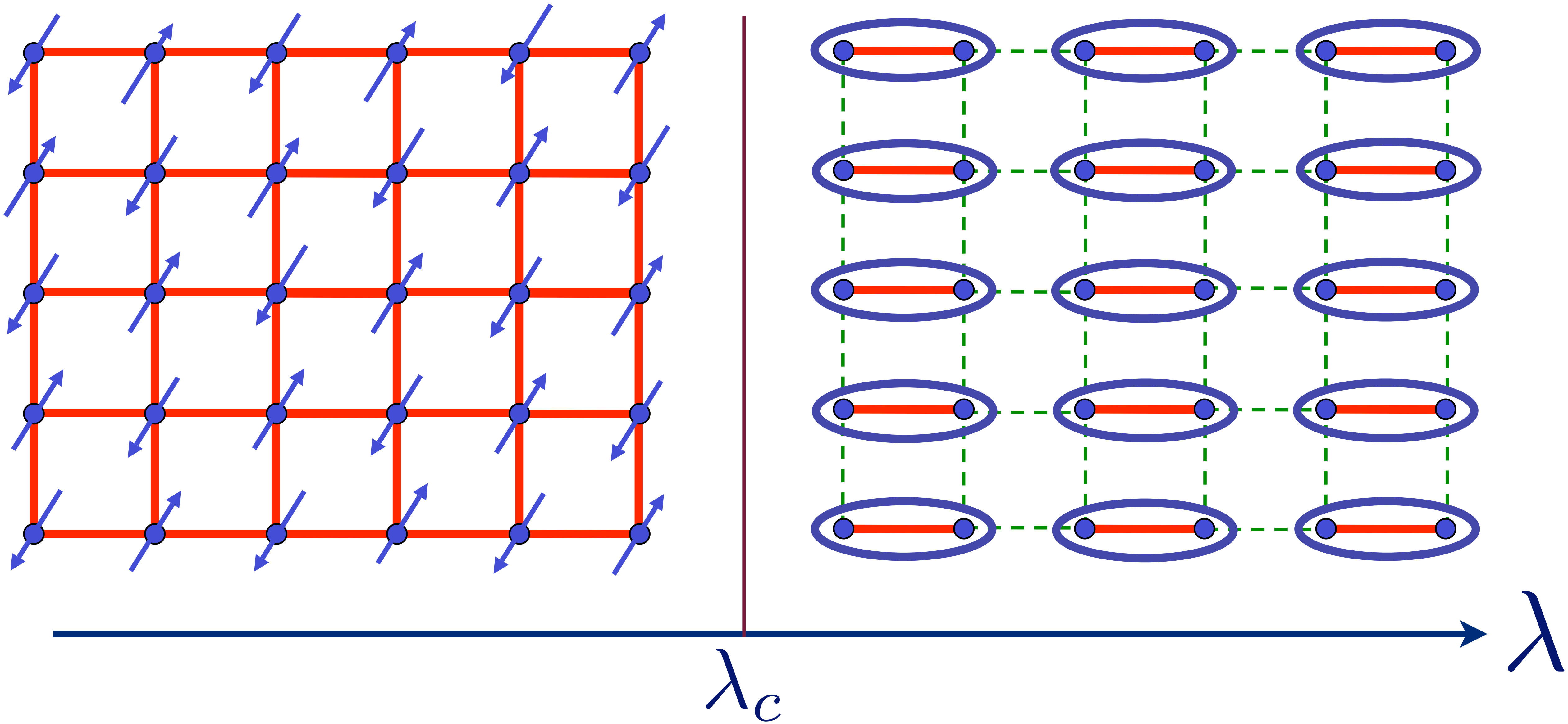

Spin $S = 1$
"triplon"

# Excitation spectrum in the Néel phase



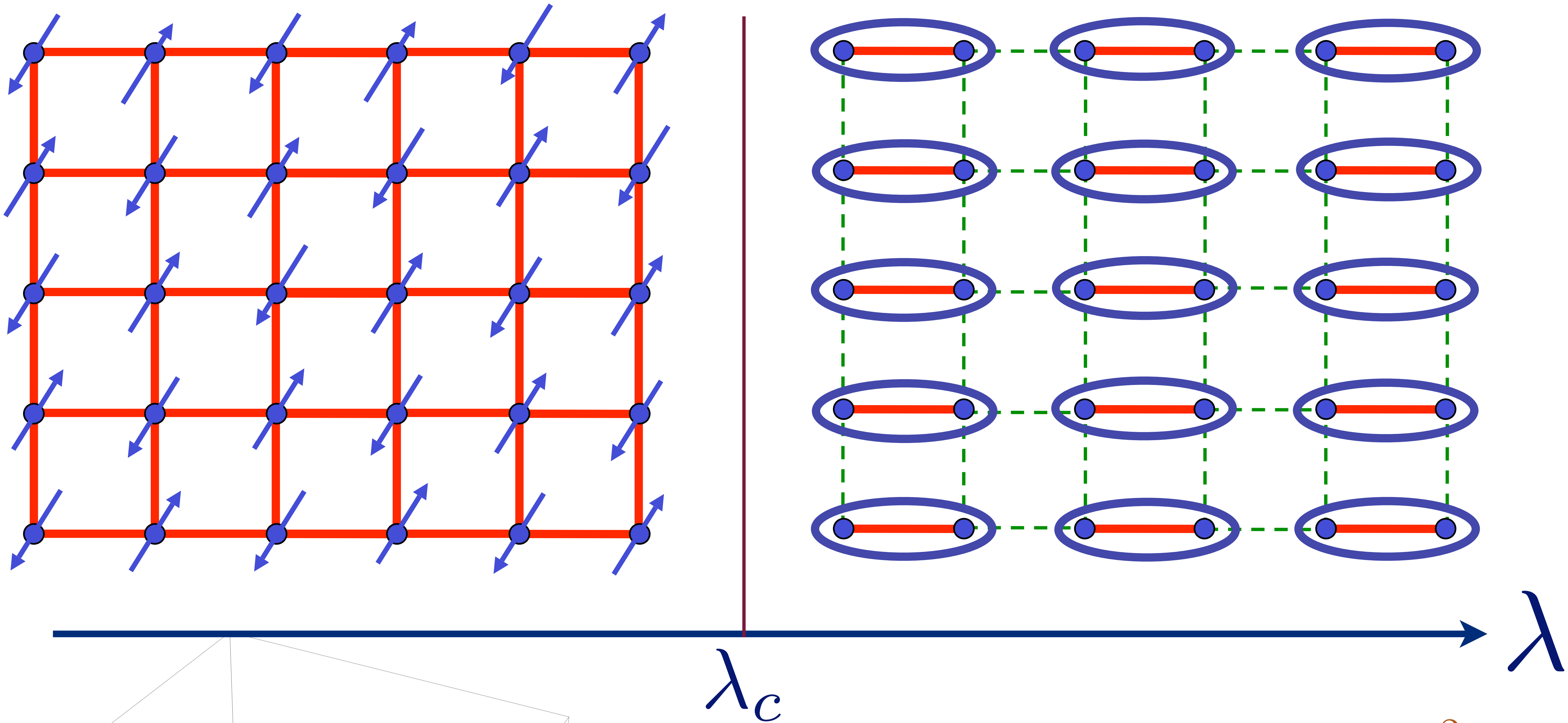

Spin waves

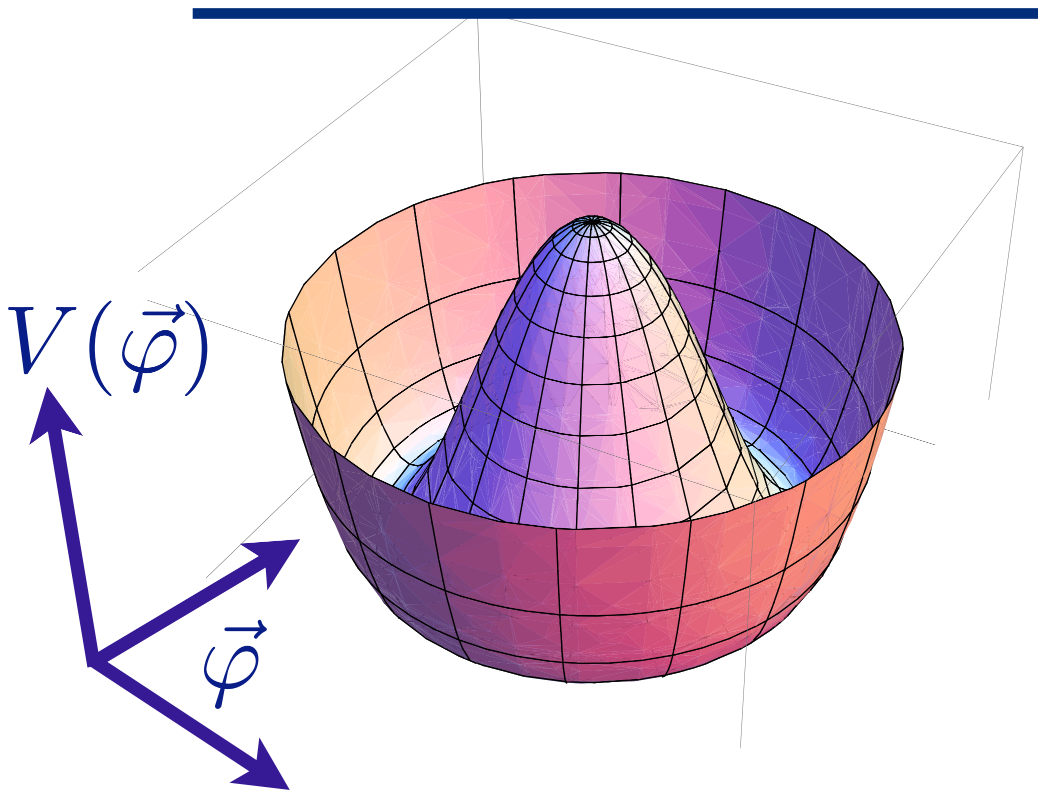
# Excitation spectrum in the Néel phase




Spin waves

# Excitation spectrum in the Néel phase

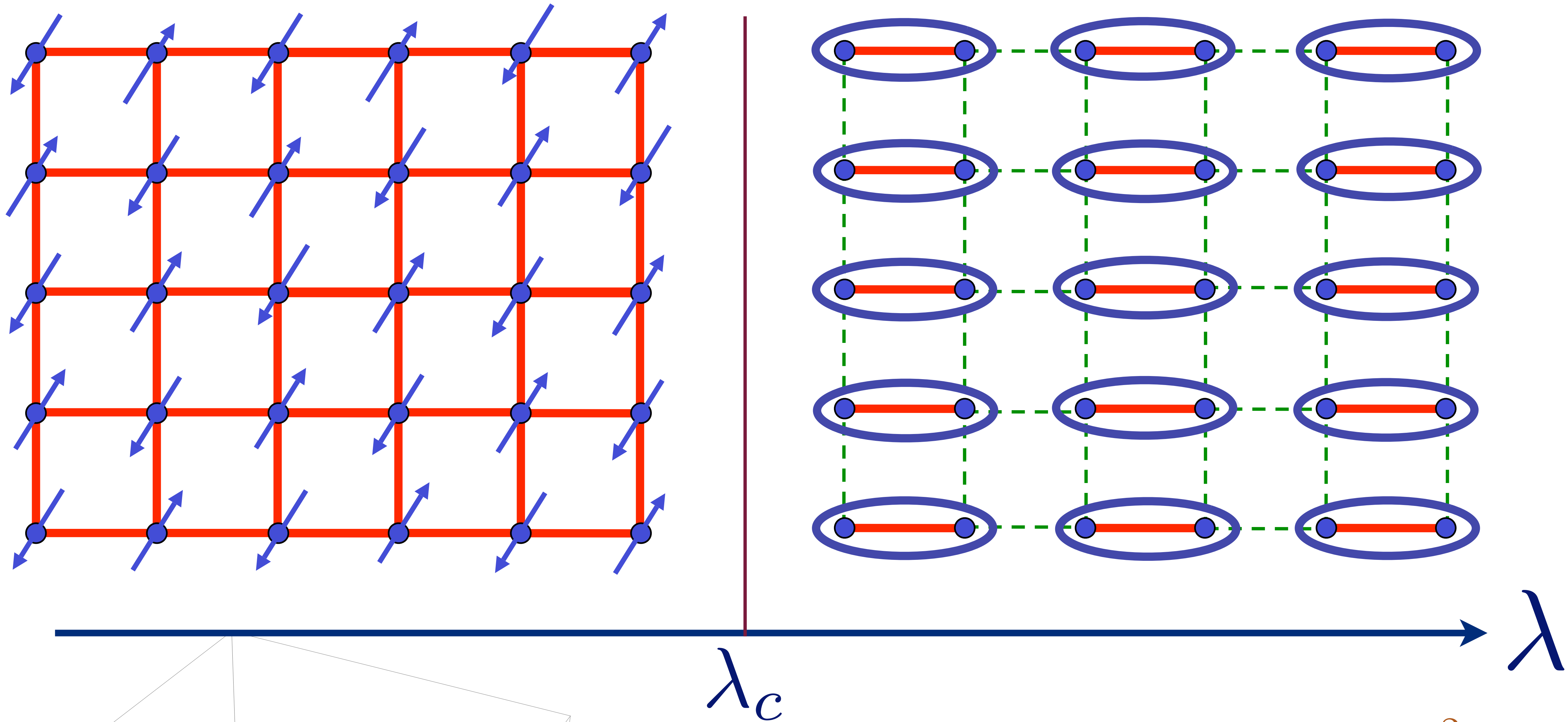


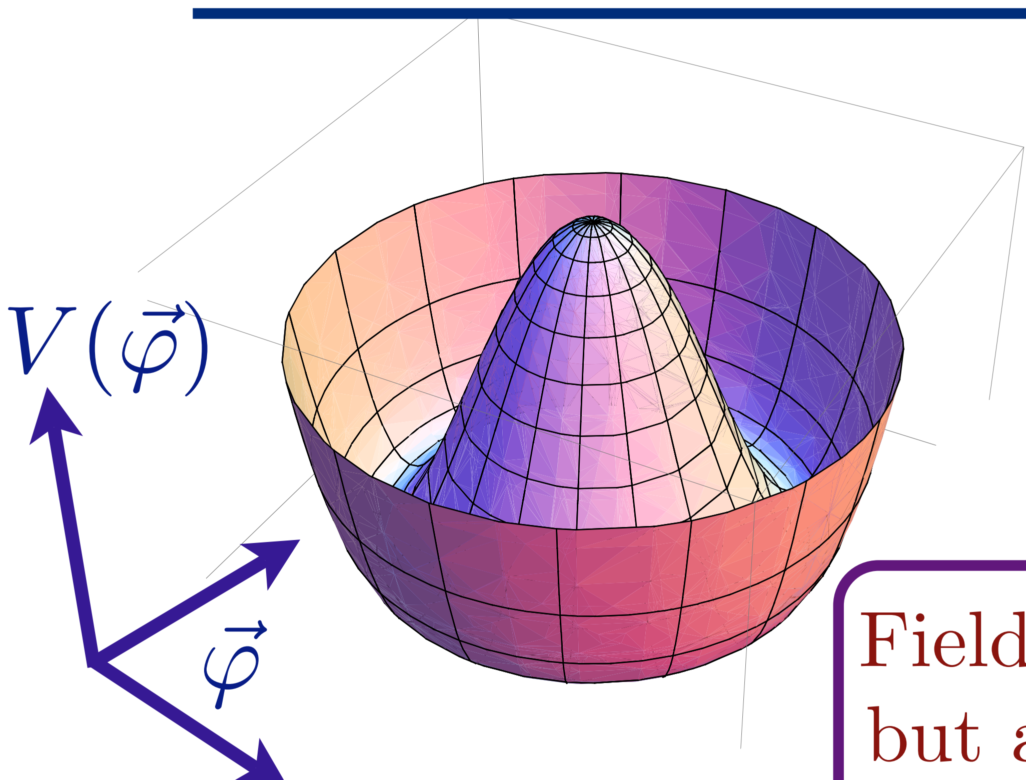




$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda < \lambda_c$$

# Excitation spectrum in the Néel phase



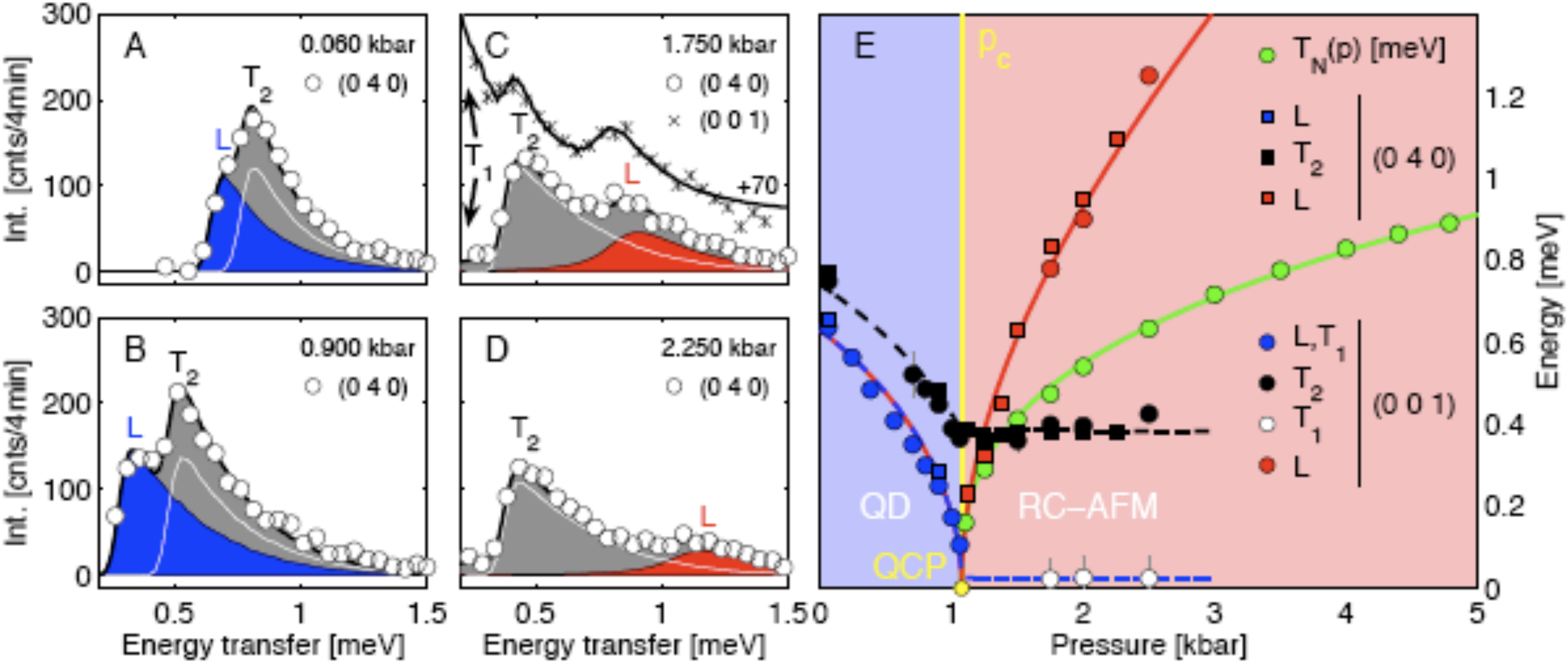




$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda < \lambda_c$$

Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle

# $TlCuCl_3$ with varying pressure



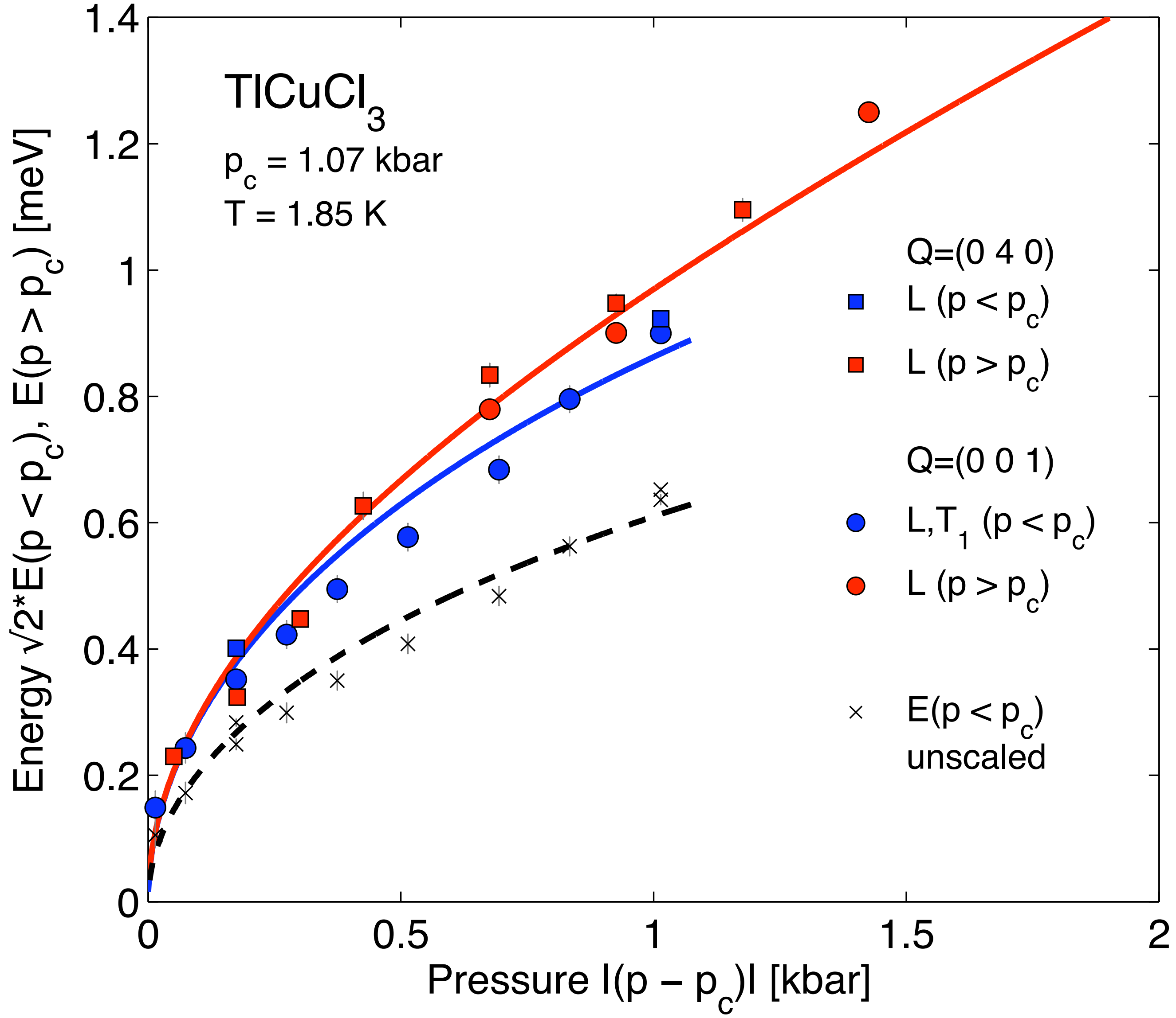

Observation of 3 → 2 low energy modes,
emergence of new Higgs particle in the Néel phase,
and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

# *Prediction of quantum field theory*

$$\frac{\text{Energy of "Higgs" particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

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