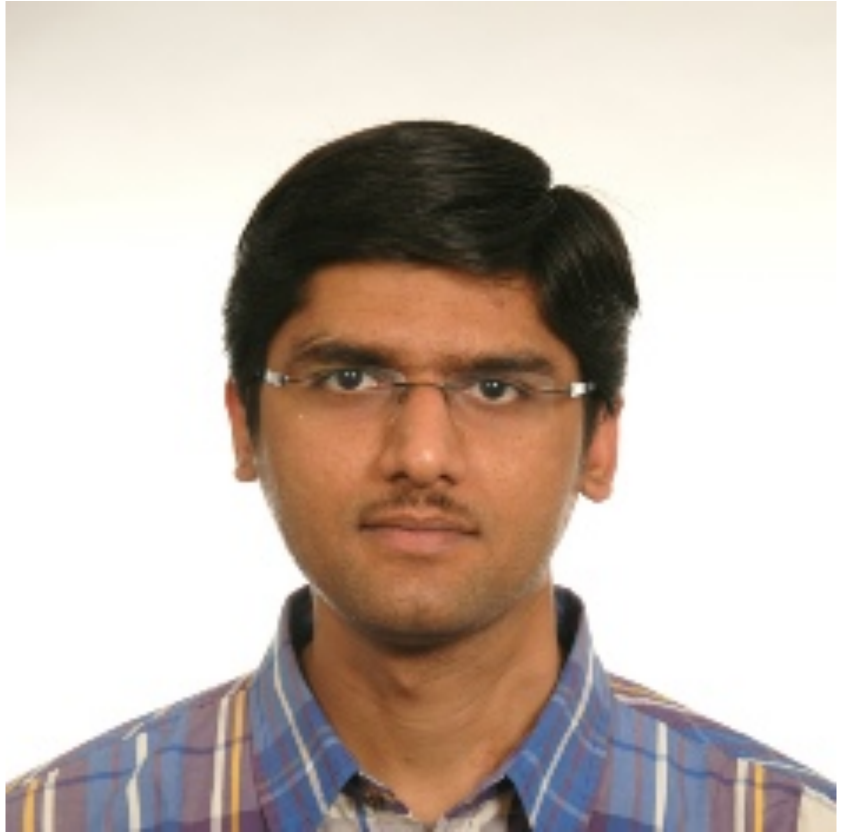
Linear in temperature resistivity in the limit of zero temperature from the time reparameterization soft mode

Quantum Aspects of Space-Time and Matter Seminar
Max Planck Institute for Gravitational Physics
Albert Einstein Institute, Potsdam
May 14, 2020

Subir Sachdev

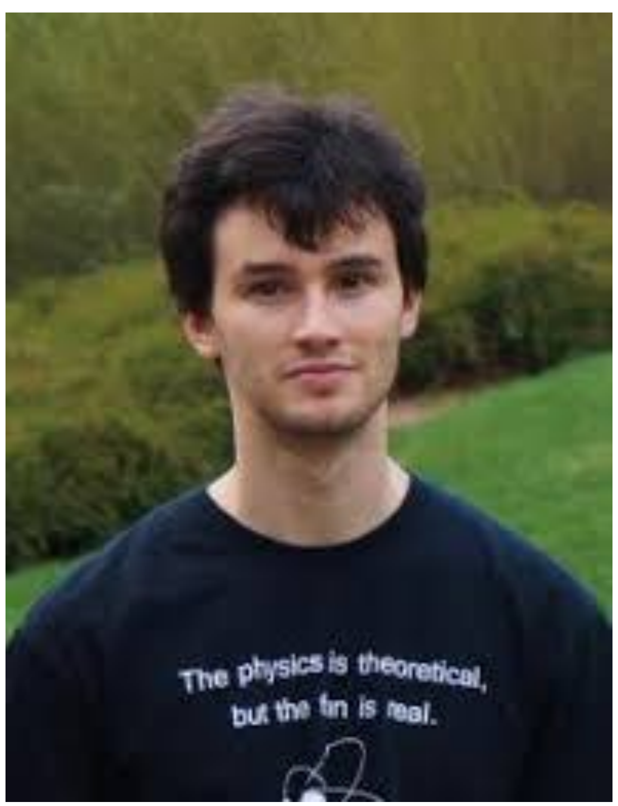
Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD



Darshan Joshi

Chenyuan Li



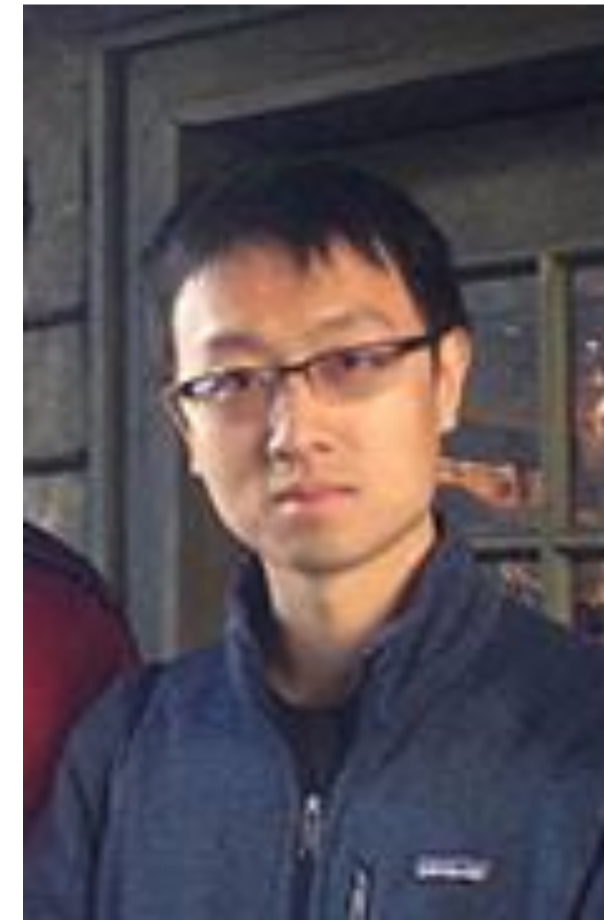
Grigory Tarnopolsky

Physical Review X
10, 021033 (2020)

Antoine Georges

Haoyu Guo

Annals of Physics,
to appear
arXiv:2004.05182



Yingfeu Gu

Remarkable recent observation of
'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is linear in T down to very low T.
Using the Drude formula

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau_D}$$

a universal Drude scattering rate is observed

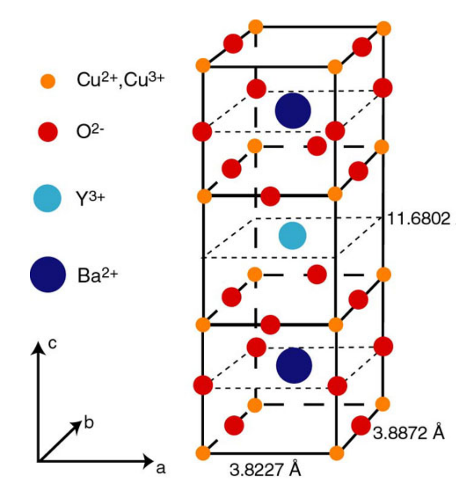
$$\frac{1}{\tau_D} \approx \frac{k_B T}{\hbar},$$

independent of the strength of interactions,
as $T \to 0$

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau_D} = \alpha \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

Challenge for theory:

A model in which

$$\lim_{T\to 0} \frac{d\rho}{dT} \neq 0$$

1. SYK criticality + *time reparameterization soft mode*

2. Charged black holes

3. SYK lattice models

4. *Fractionalization* and SYK criticality in t-J models with random exchange

5. Linear-in-T resistivity down to zero T

1. SYK criticality +
   time reparameterization soft mode

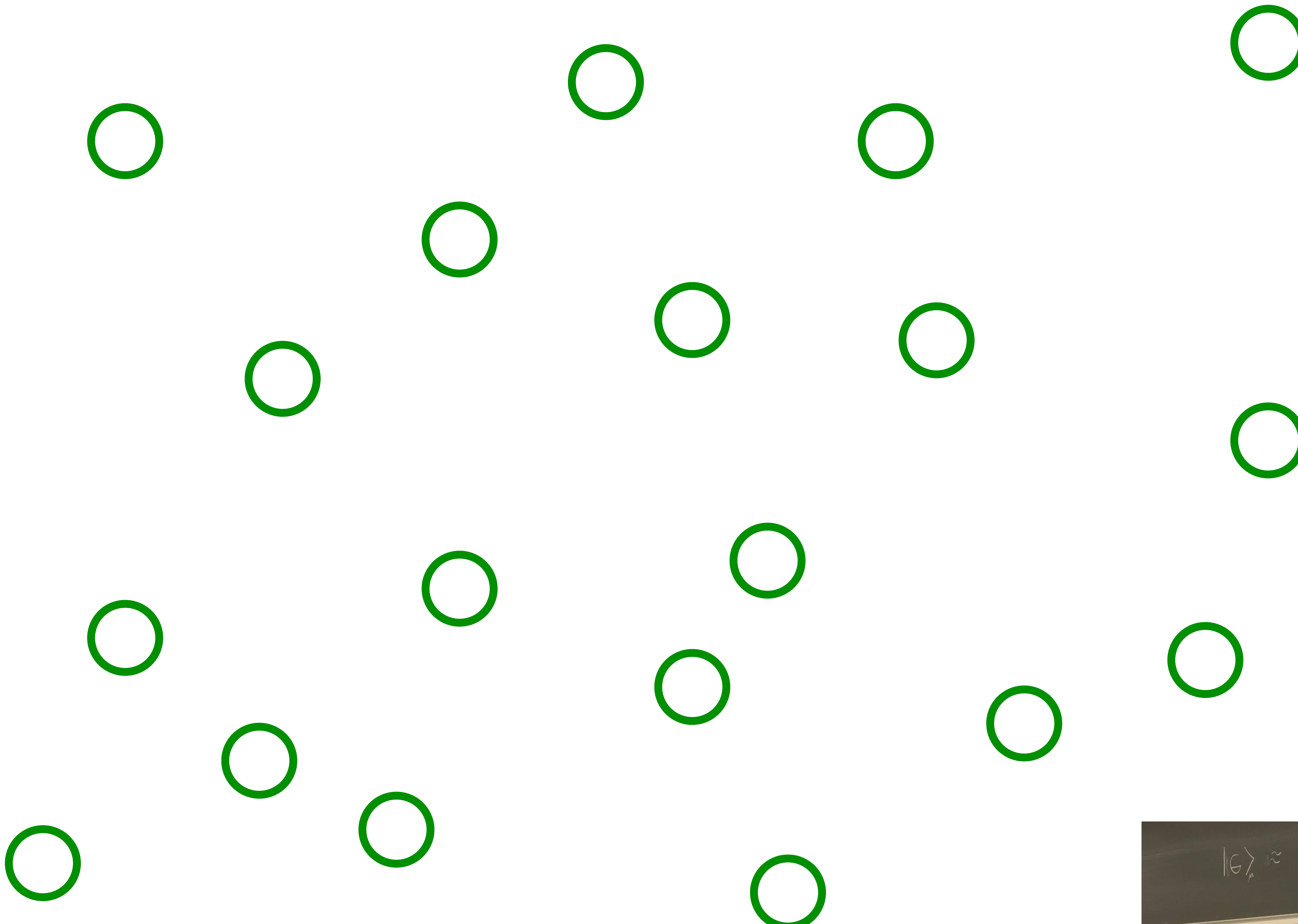
2. Charged black holes

3. SYK lattice models

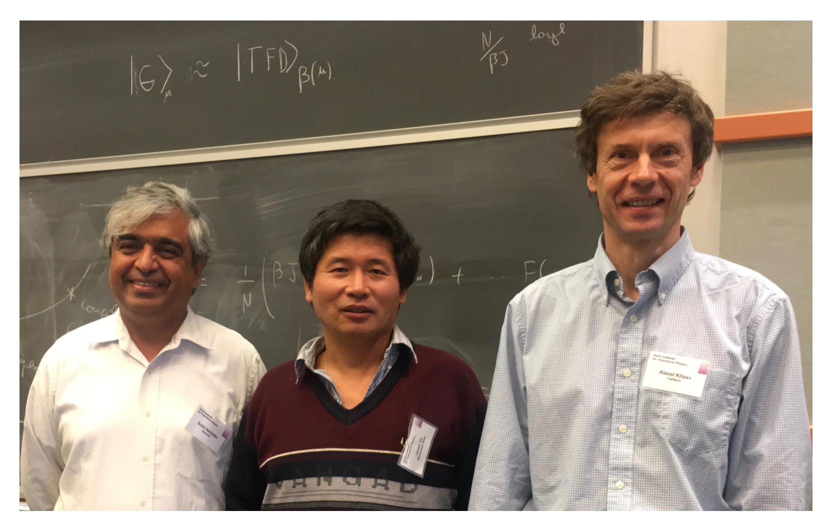
4. *Fractionalization* and SYK criticality
   in t-J models with random exchange

5. Linear-in-T resistivity down to zero T

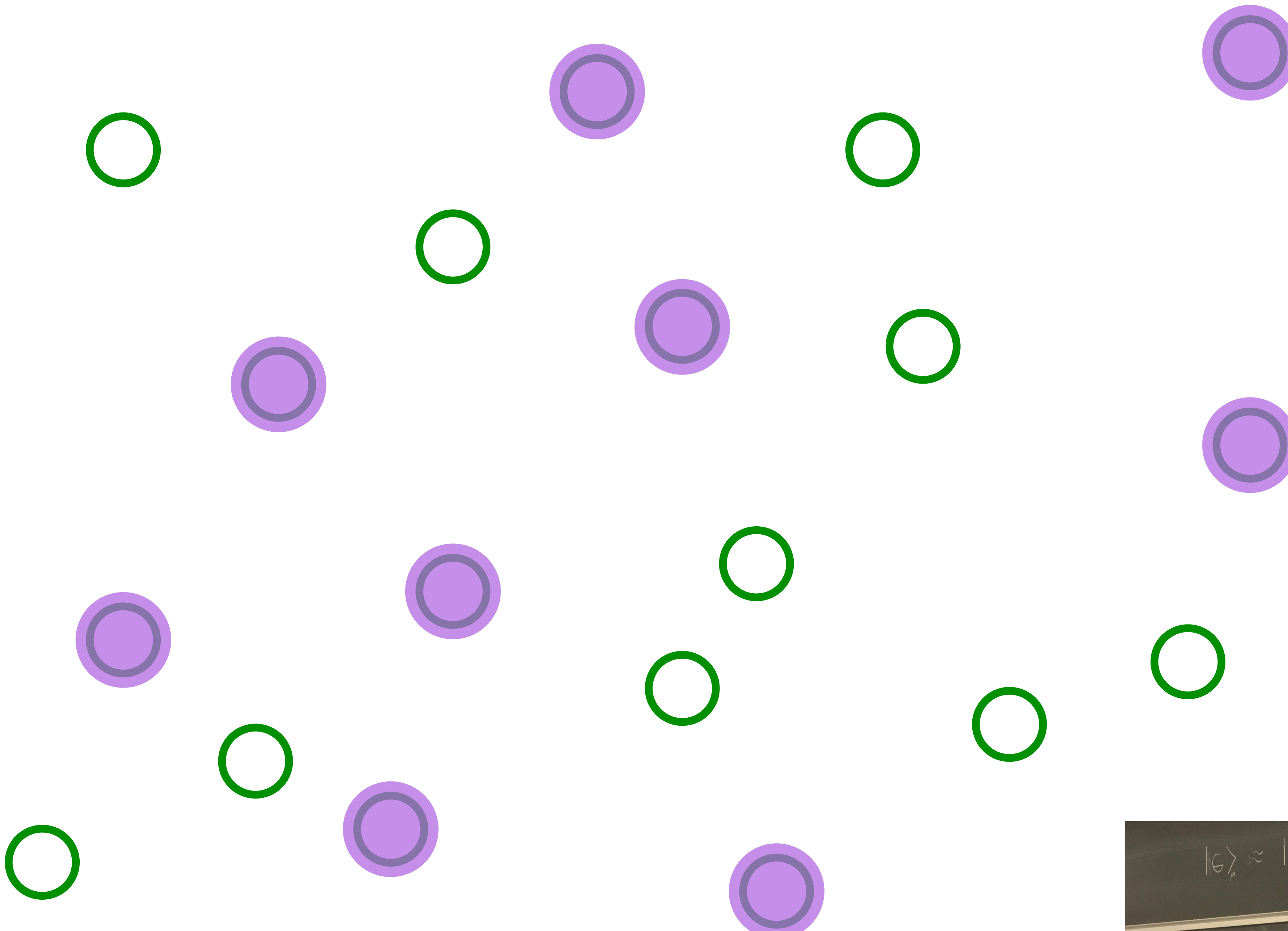
The Sachdev-Ye-Kitaev (SYK) model



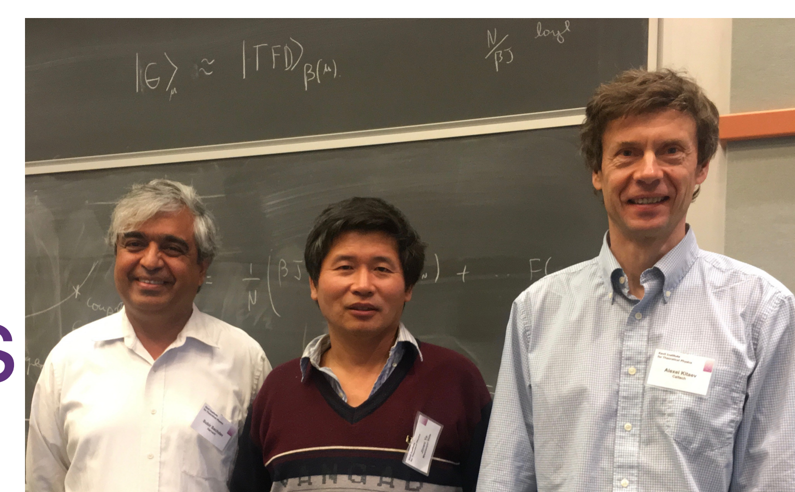
Pick a set of random positions



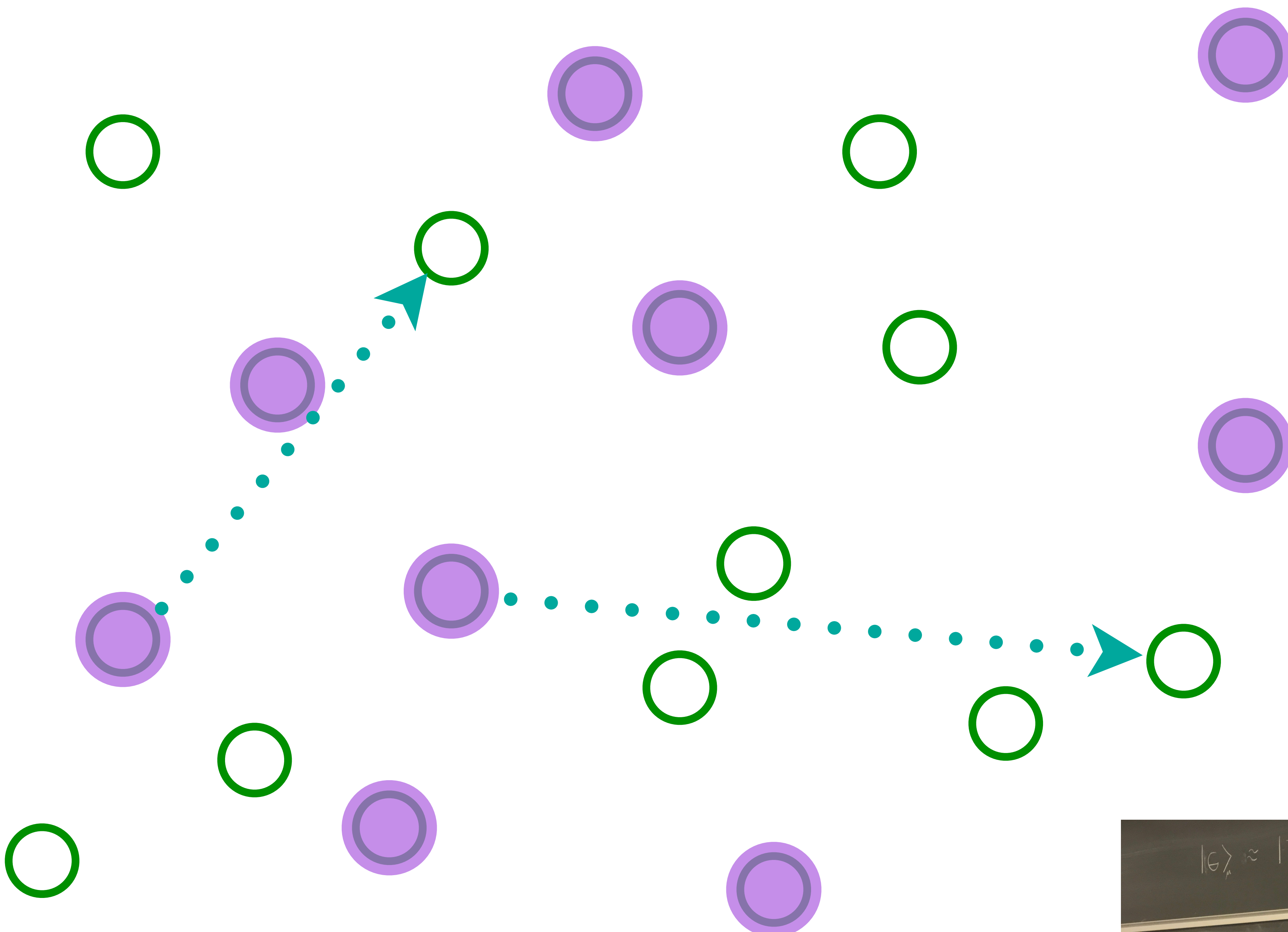
The complex SYK model



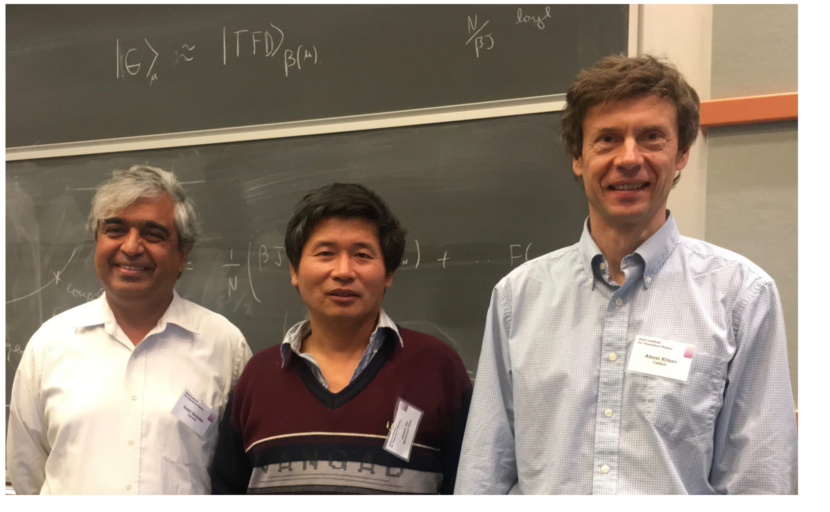
Place electrons randomly on some sites



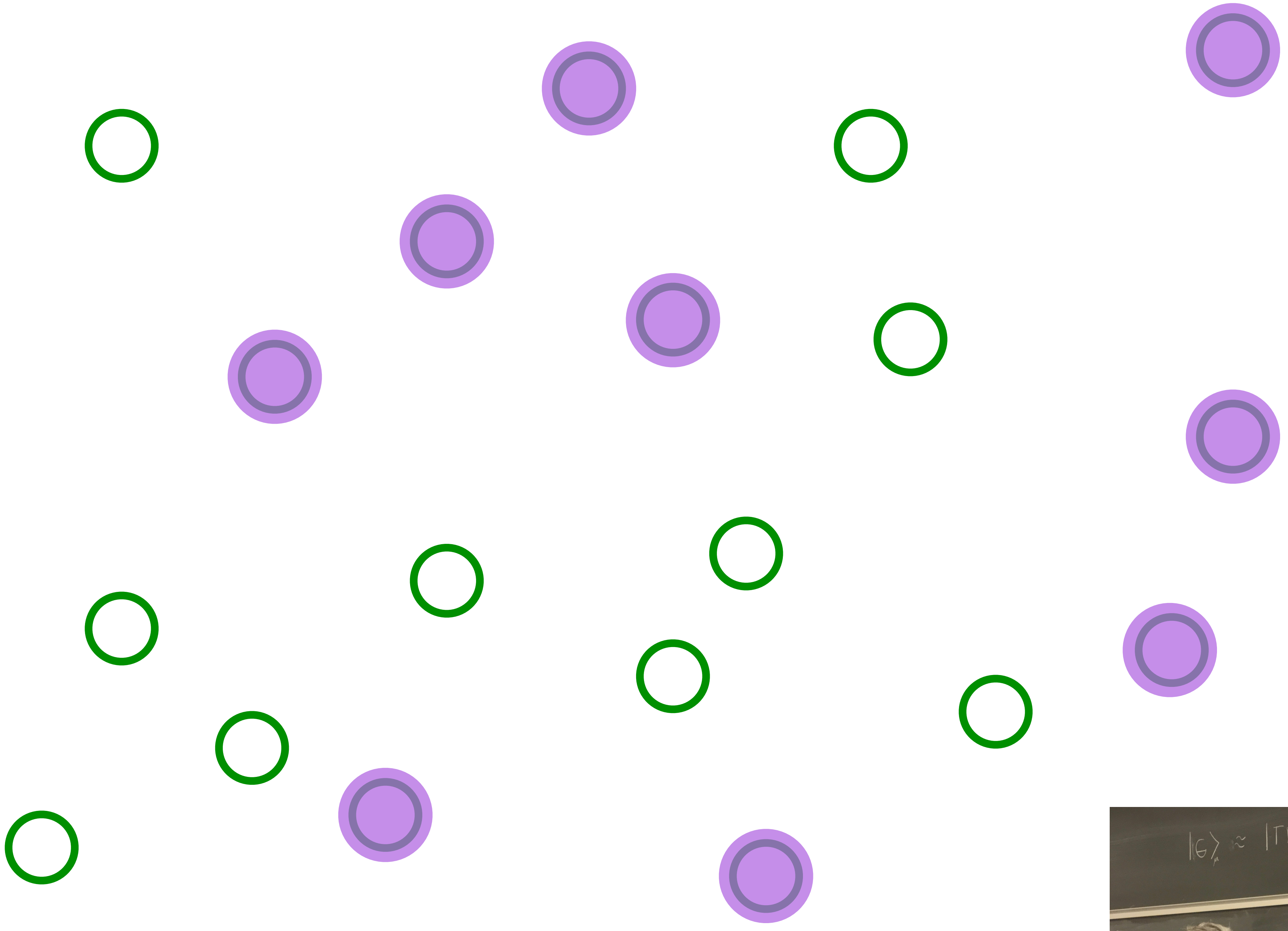
The complex SYK model



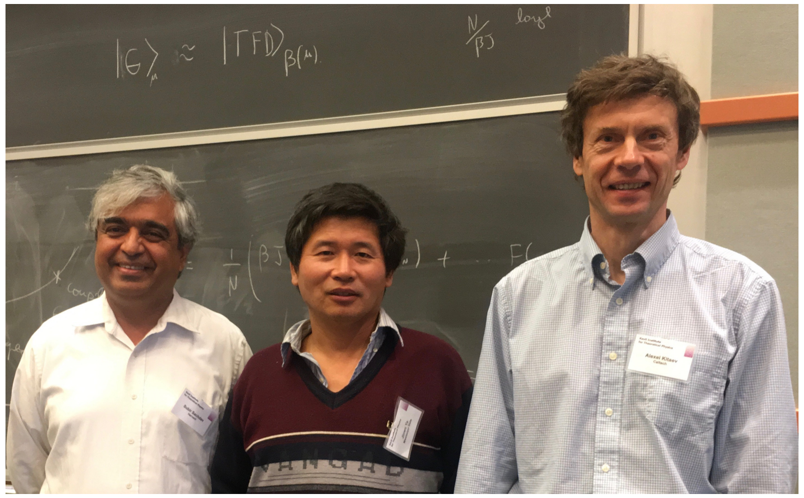
Place electrons randomly on some sites



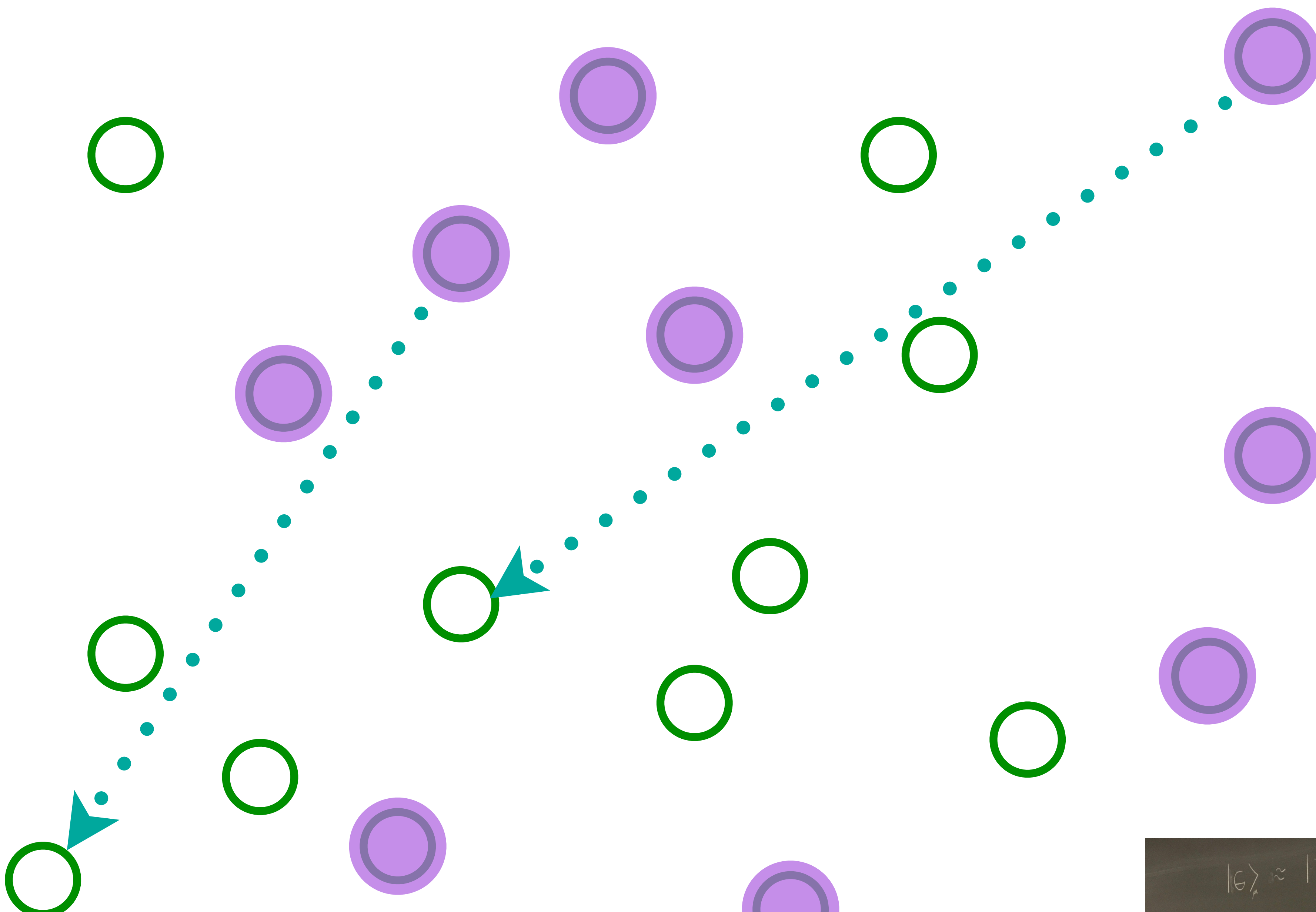
The complex SYK model



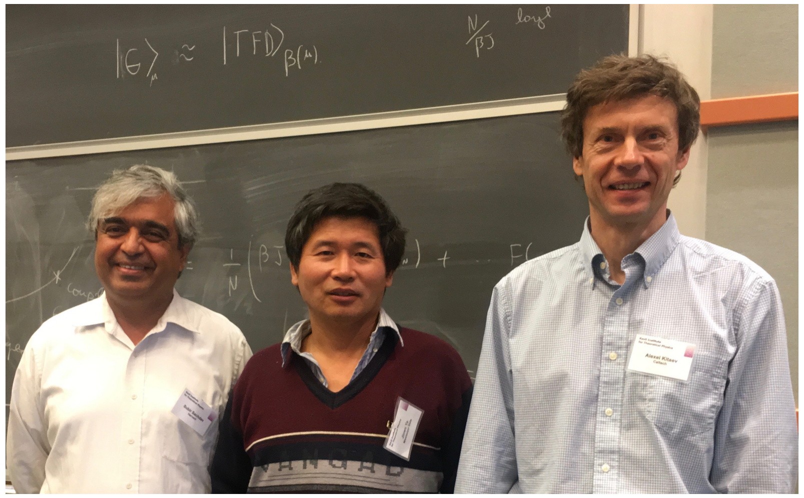
Entangle electrons pairwise randomly



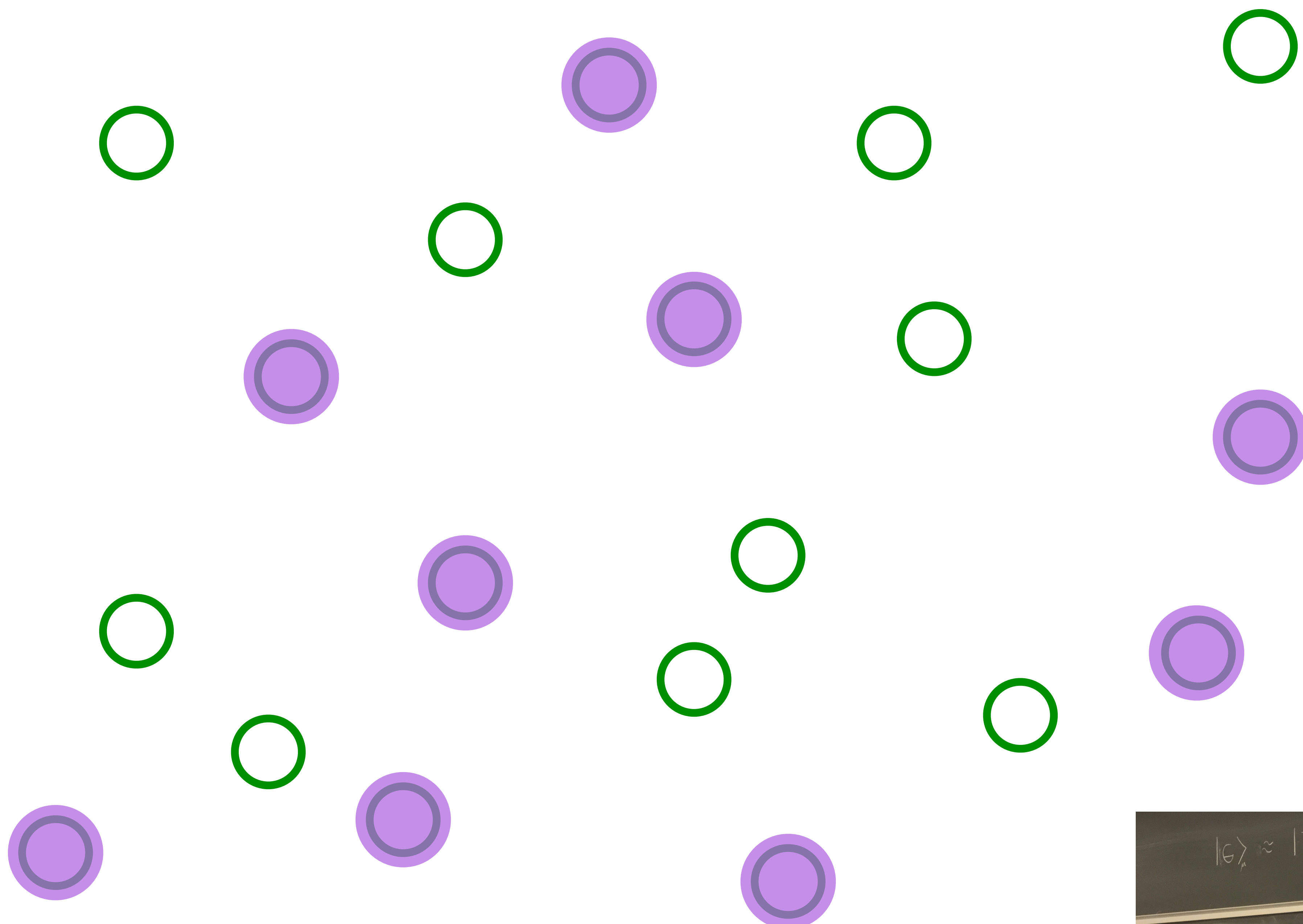
The complex SYK model



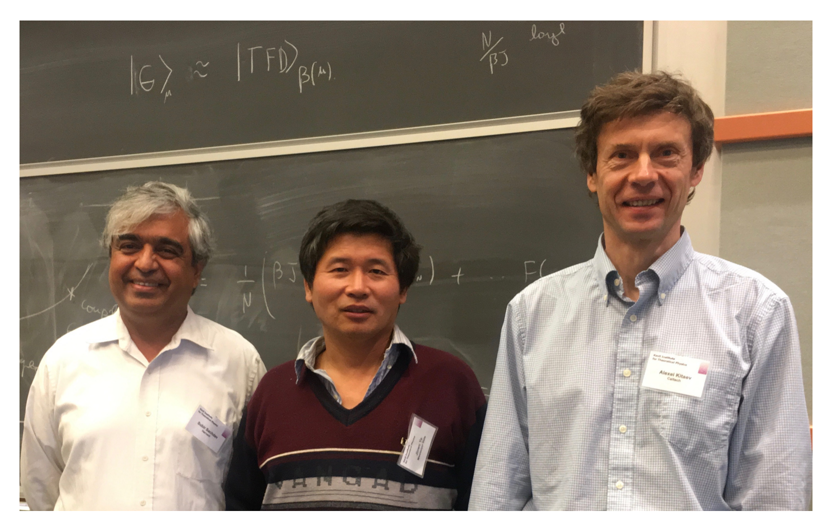
Entangle electrons pairwise randomly



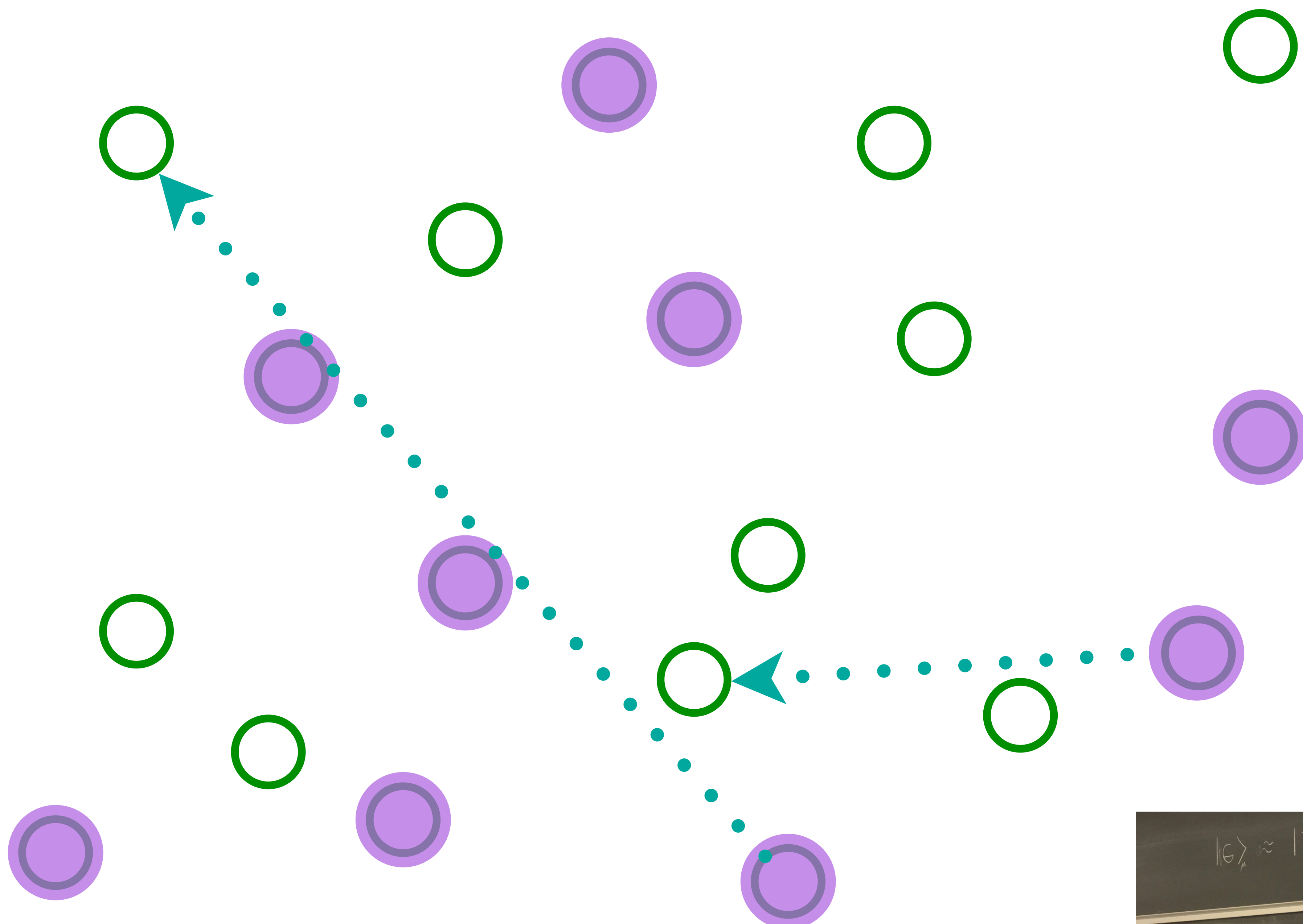
The complex SYK model



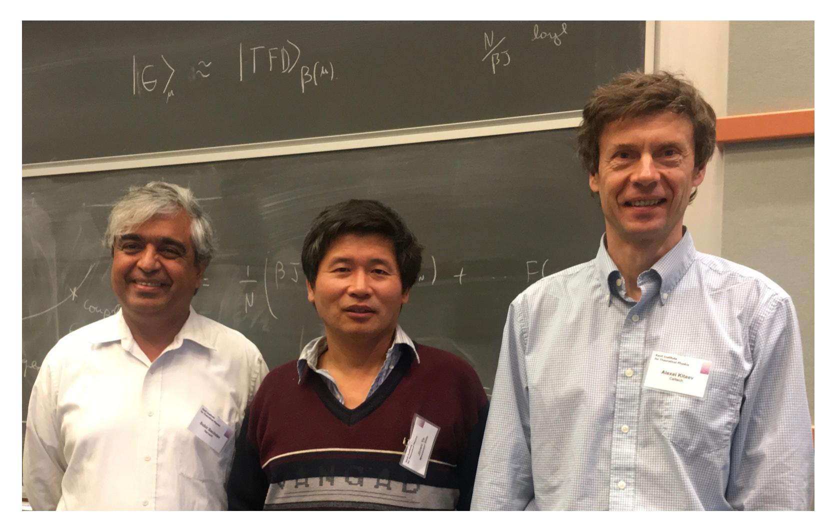
Entangle electrons pairwise randomly



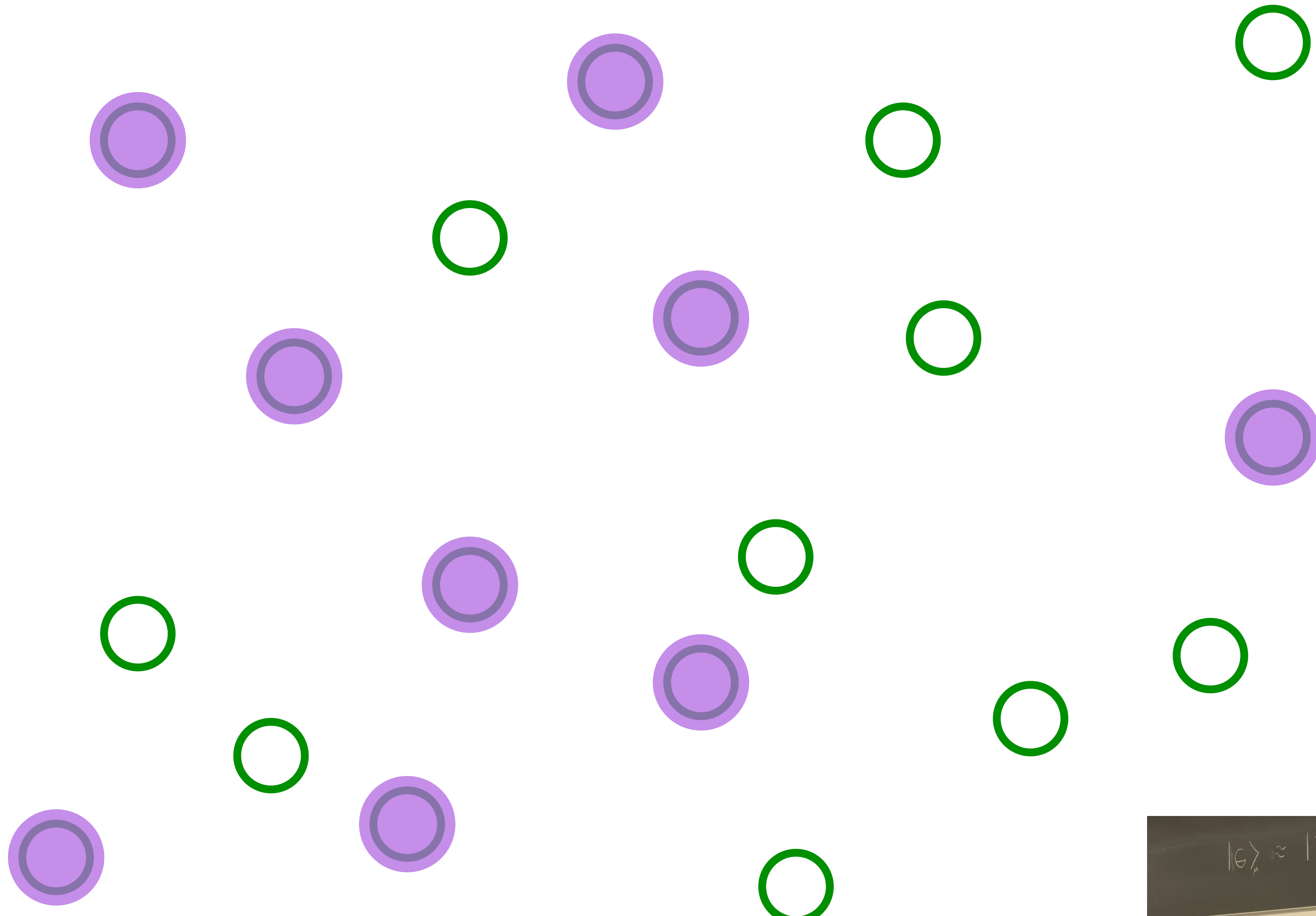
The complex SYK model



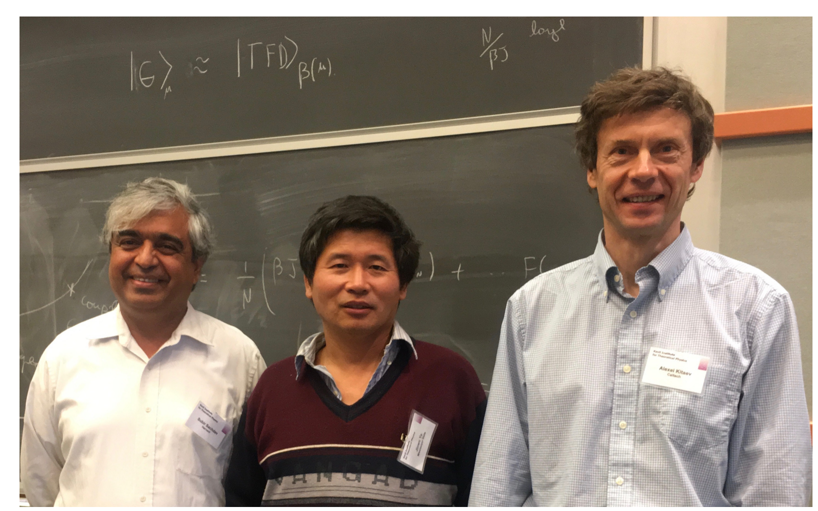
Entangle electrons pairwise randomly



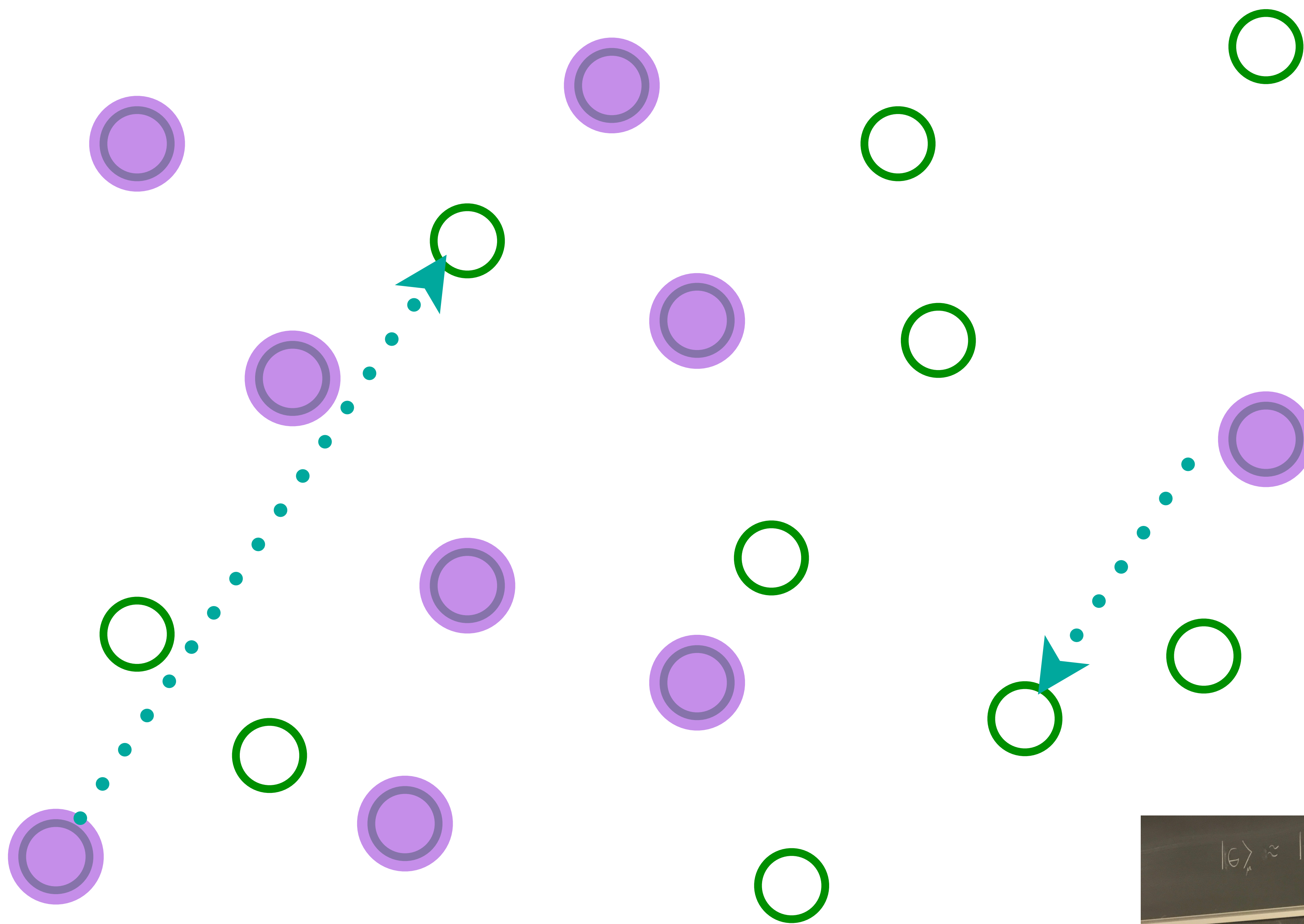
The complex SYK model



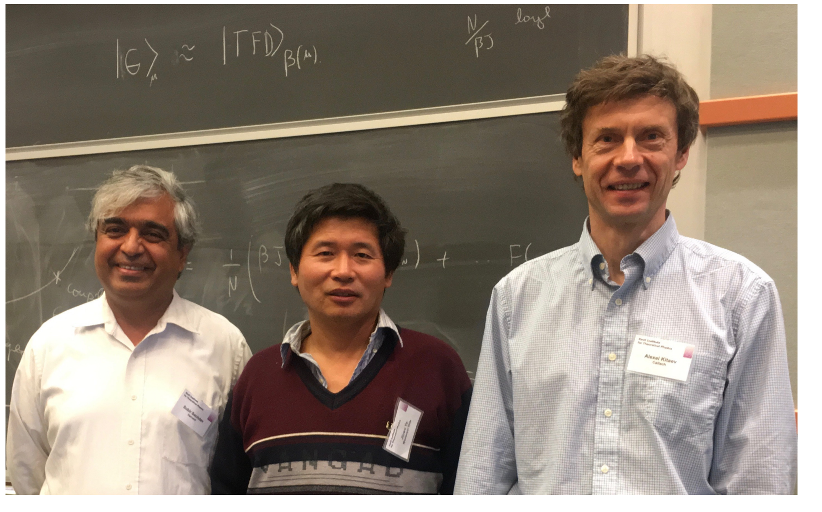
Entangle electrons pairwise randomly



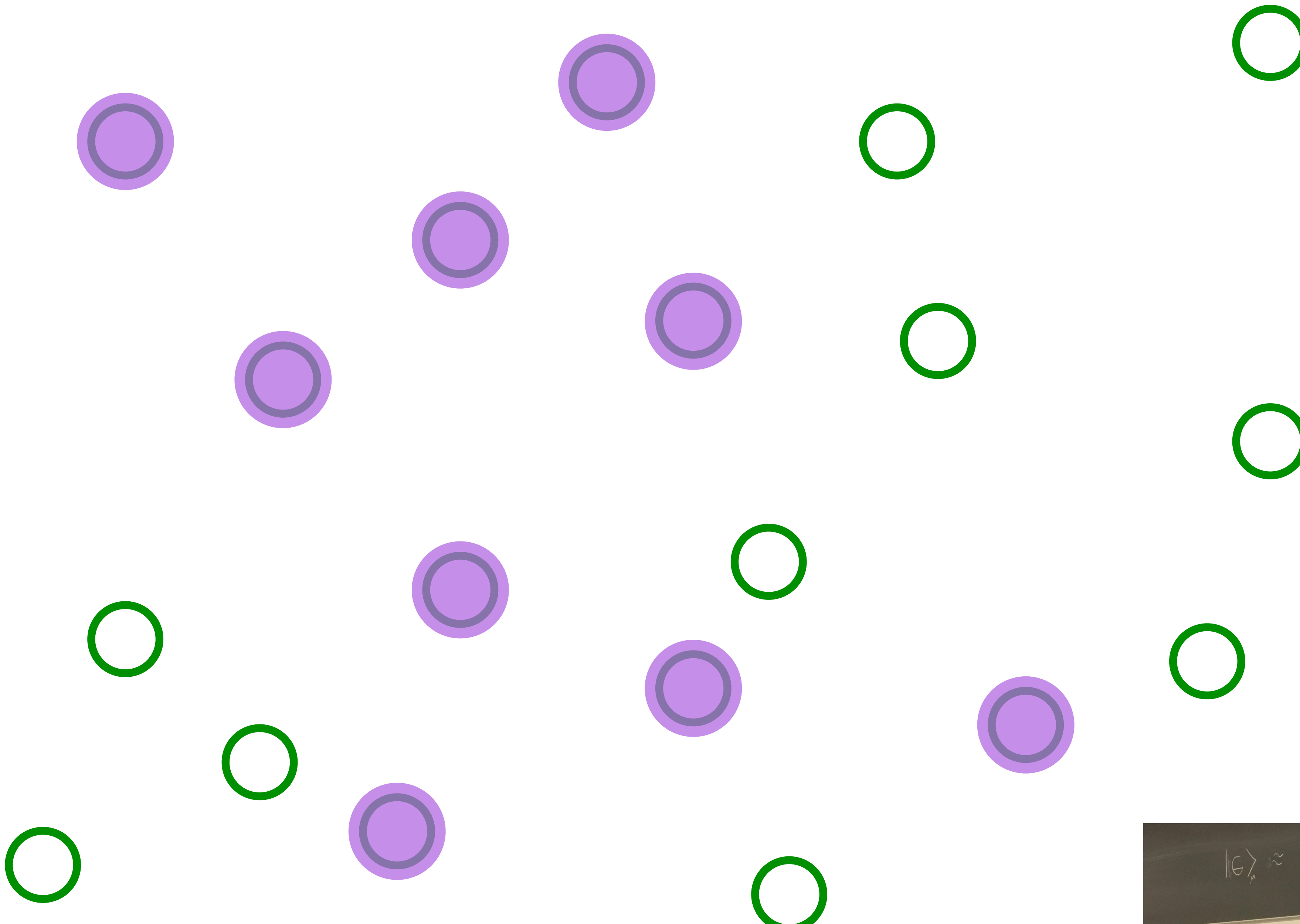
The complex SYK model



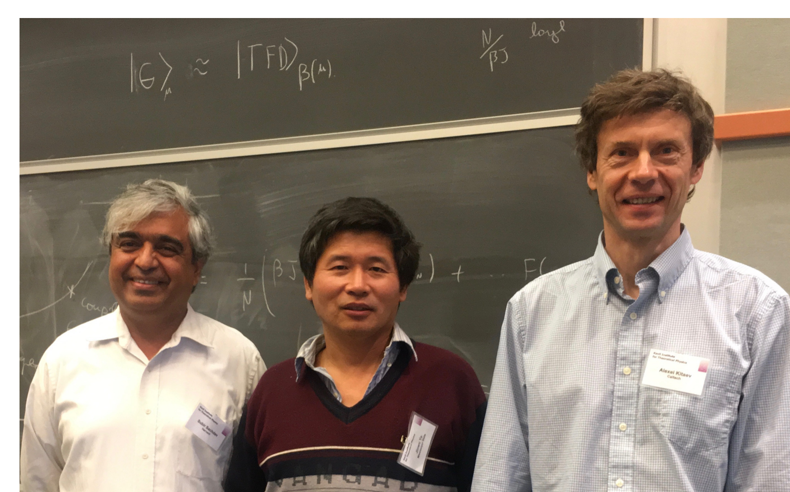
Entangle electrons pairwise randomly



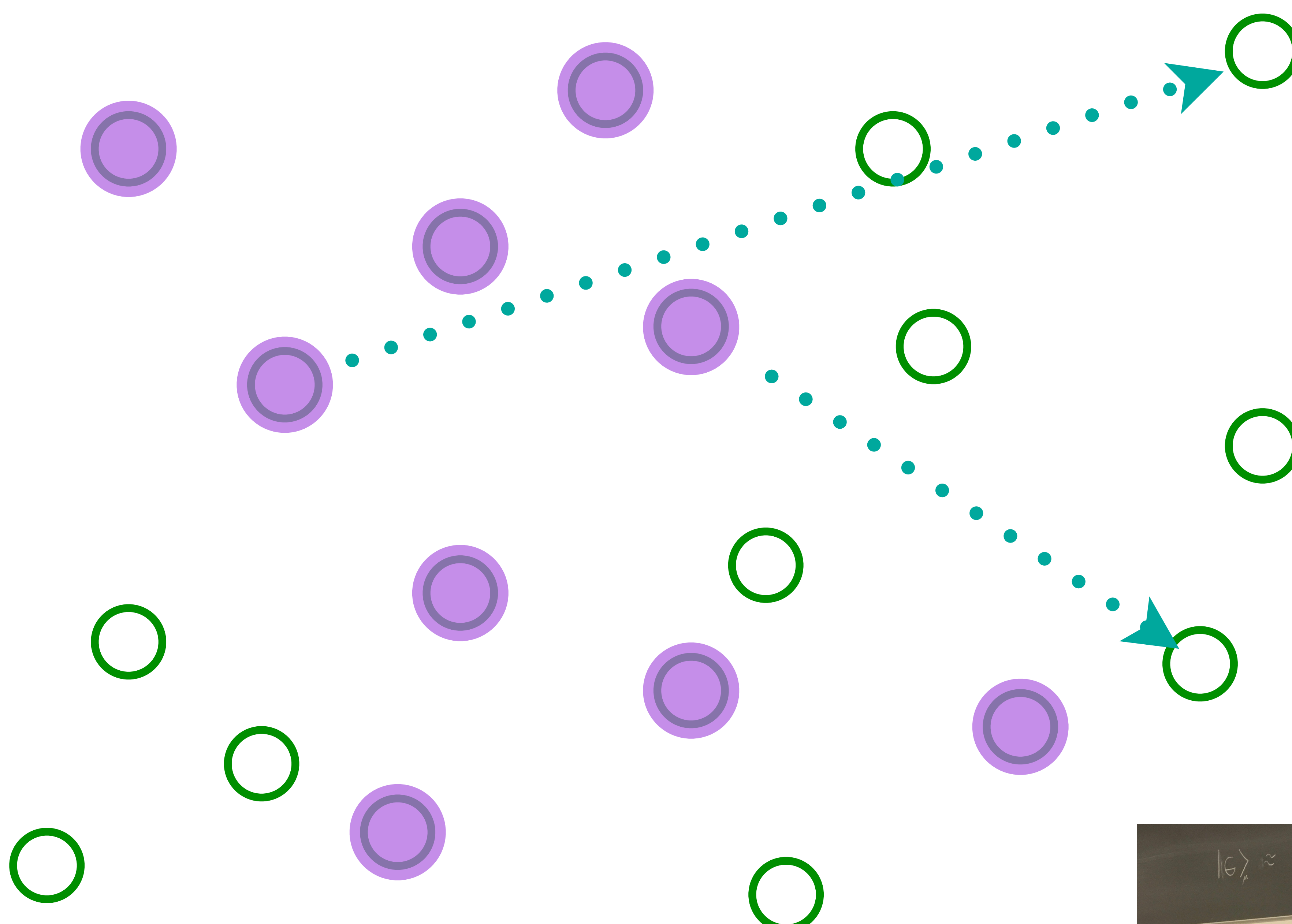
The complex SYK model

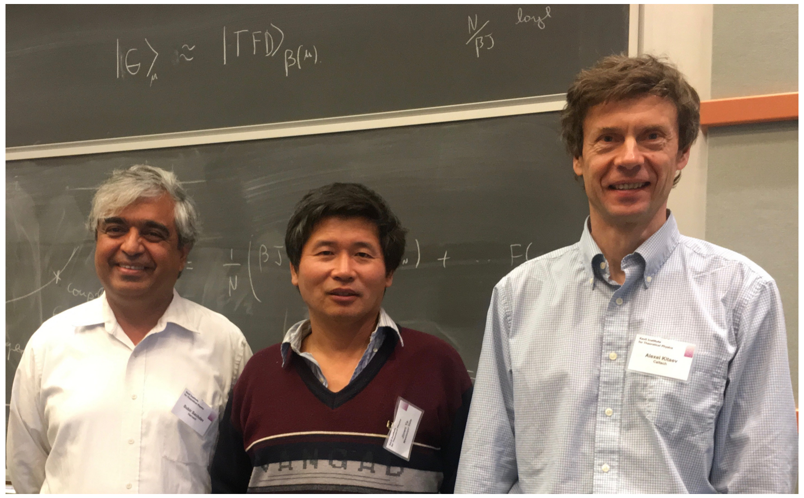


Entangle electrons pairwise randomly



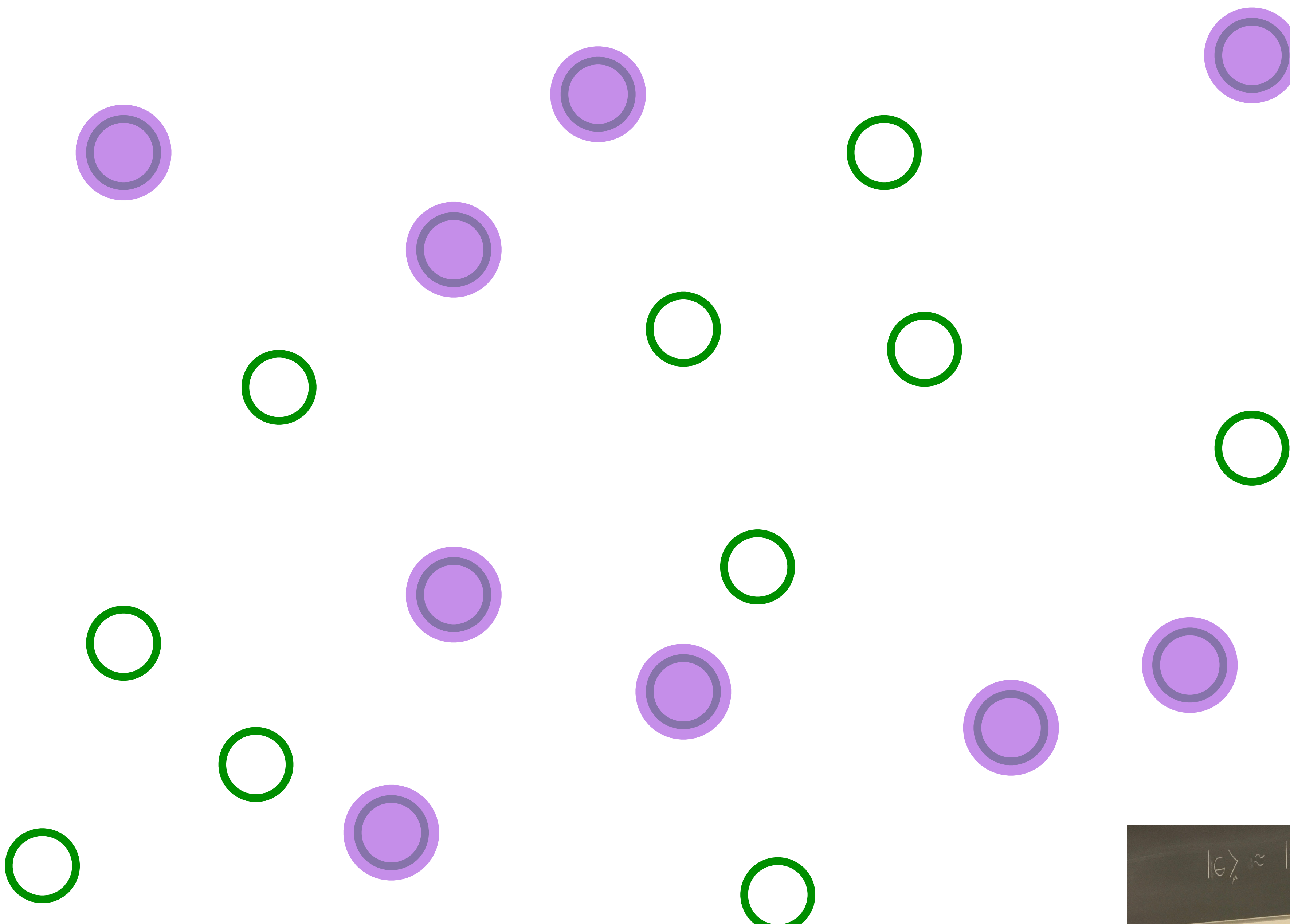
The complex SYK model



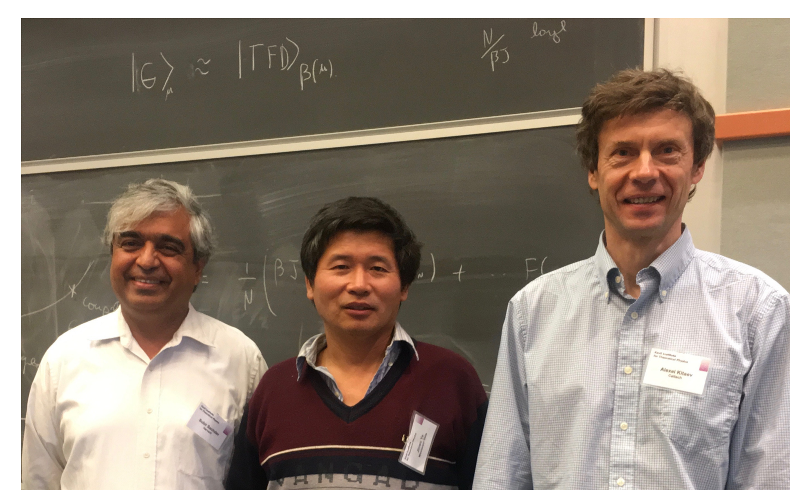


Entangle electrons pairwise randomly

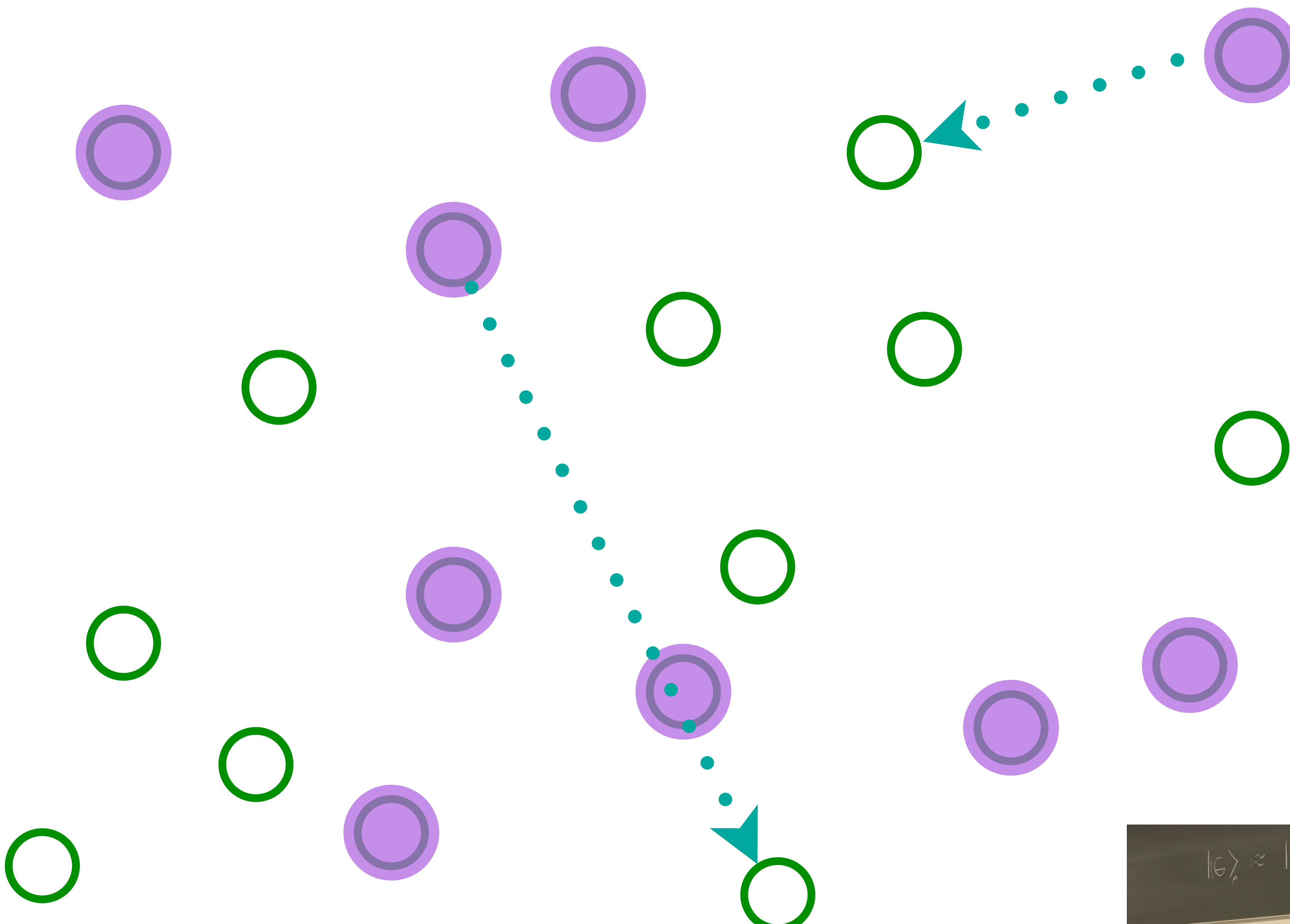
The complex SYK model



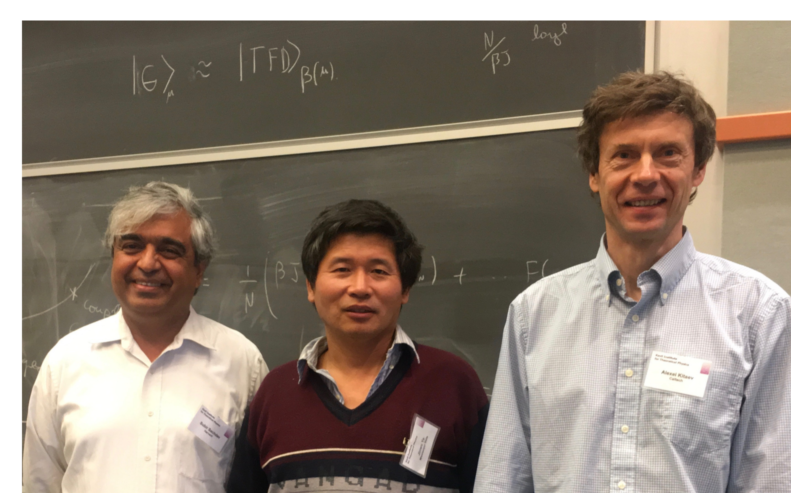
Entangle electrons pairwise randomly



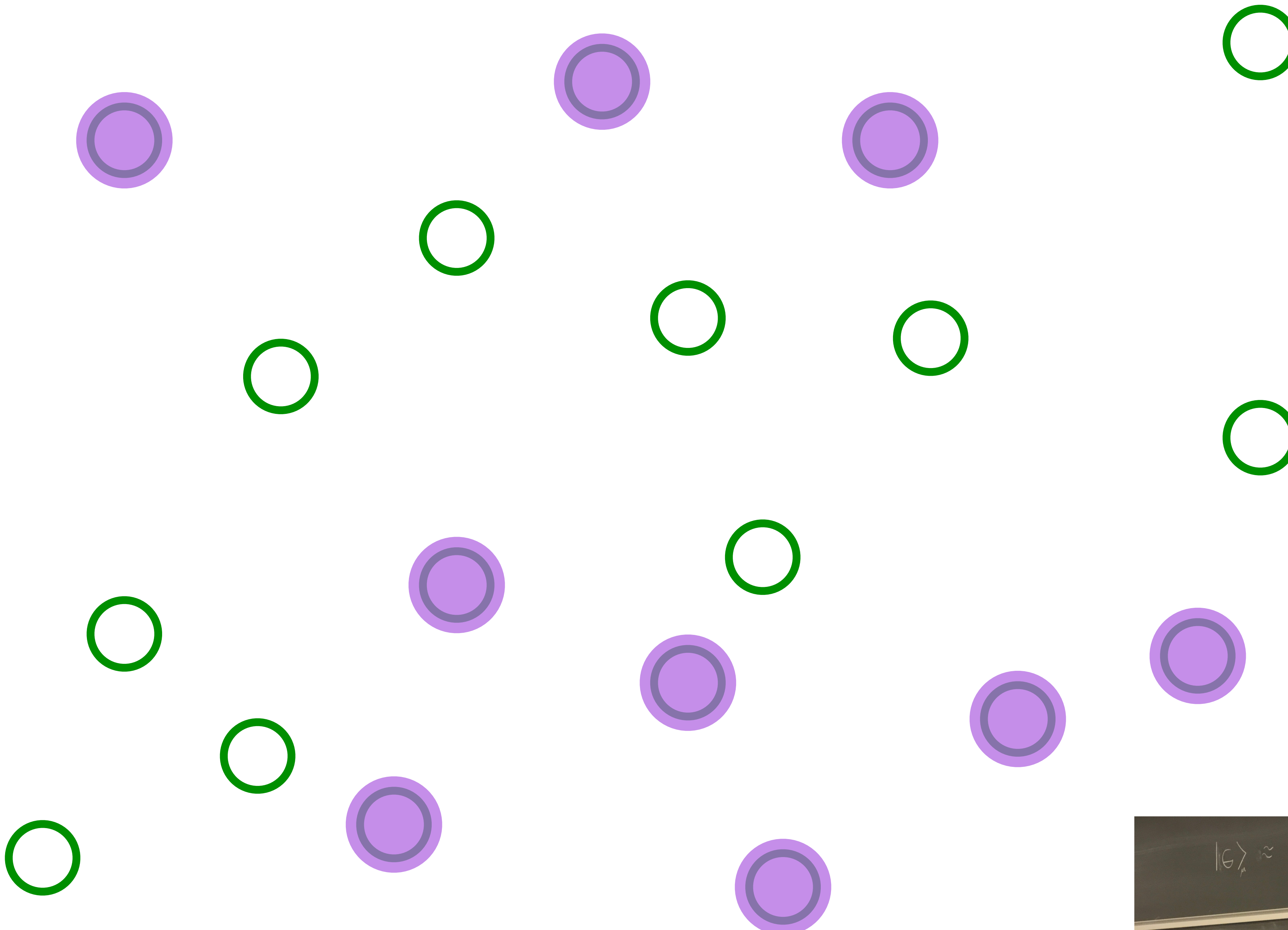
The complex SYK model



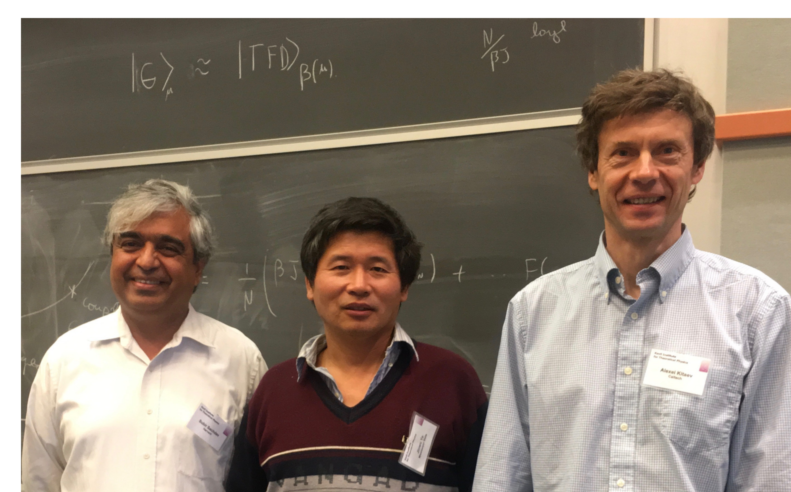
Entangle electrons pairwise randomly



The complex SYK model



Entangle electrons pairwise randomly



The complex SYK model

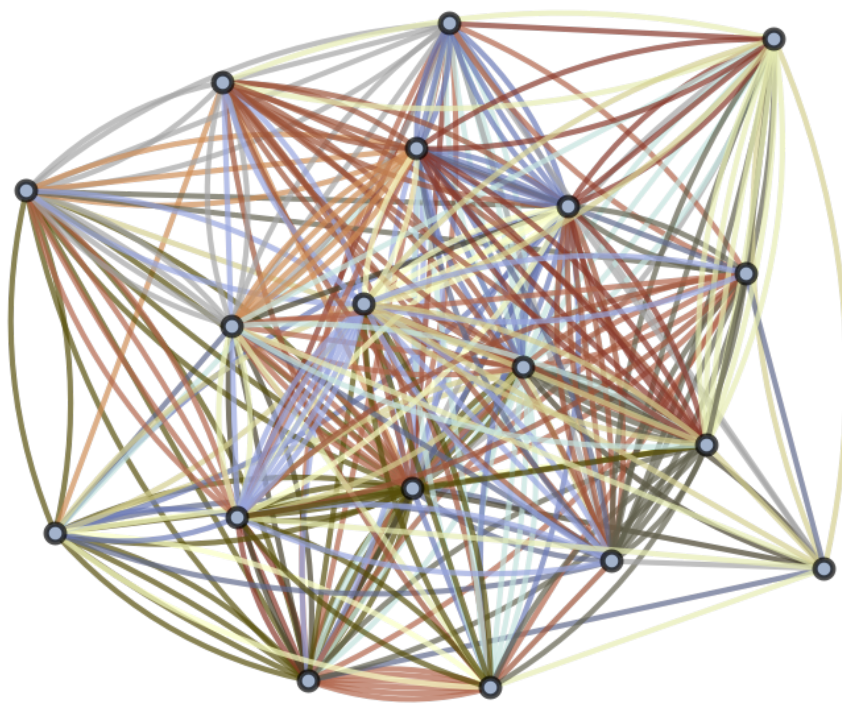
$$H = \frac{1}{(2N)^{3/2}} \sum_{a,b,c,d=1}^{N} U_{ab;cd}\, c_a^\dagger c_b^\dagger c_c c_d - \mu \sum_a c_a^\dagger c_a$$

$$c_a c_b + c_b c_a = 0 \quad , \quad c_a c_b^\dagger + c_b^\dagger c_a = \delta_{ab}$$

$$\mathcal{Q} = \frac{1}{N} \sum_a c_a^\dagger c_a$$

$U_{ab;cd}$ are independent random variables

with $\overline{U_{ab;cd}} = 0$ and $\overline{|U_{ab;cd}|^2} = U^2$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

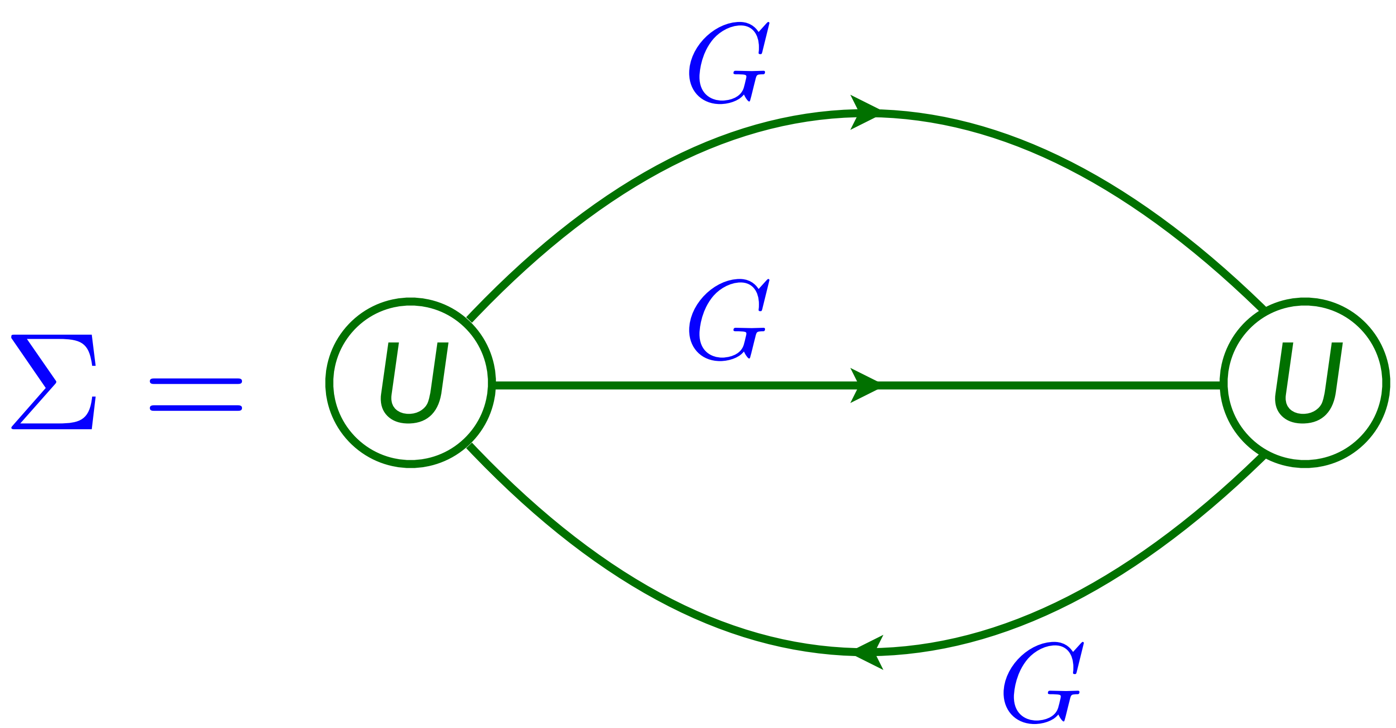
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Feynman graph expansion in $U_{ab;cd}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

The large N limit is given by the sum of "melon" diagrams.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The complex SYK model

Key properties

1. There is a quantum critical state, without quasiparticle excitations, for a range of charge densities around $\mathcal{Q} = 1/2$.

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The complex SYK model

There is a one-parameter family of critical solutions with varying $\mathcal{Q}$, characterized by a dimensionless parameter $\mathcal{E}$.

For long times $\tau > 0$

$$\left\langle c_a(\tau) c_a^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$$\left\langle c_a^\dagger(\tau) c_a(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$\mathcal{E}$ determines the particle-hole asymmetry, and $A(\mathcal{E})$ is a known function. $\mathcal{E}$ is determined by ϵ/U.

In a Fermi liquid,

$$\left\langle c_a(\tau) c_a^\dagger(0) \right\rangle = \left\langle c_a^\dagger(\tau) c_a(0) \right\rangle = \widetilde{A}/\tau$$

The complex SYK model

Key properties

1. There is a quantum critical state, without quasiparticle excitations, for a range of charge densities around $\mathcal{Q} = 1/2$.

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The complex SYK model

Key properties

1. There is a quantum critical state, without quasiparticle excitations, for a range of charge densities around $\mathcal{Q} = 1/2$.

2. There is a non-zero extensive entropy as $T \to 0$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

$$\lim_{T \to 0} \lim_{N \to \infty} \frac{S}{N} = \mathcal{S}_0(\mathcal{Q}) \neq 0$$

This entropy is *not* due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-N\mathcal{S}_0}$ down to the ground state.

The complex SYK model

Key properties

1. There is a quantum critical state, without quasiparticle excitations, for a range of charge densities around $\mathcal{Q} = 1/2$.

2. There is a non-zero extensive entropy as $T \to 0$

$$\lim_{T \to 0} \lim_{N \to \infty} \frac{S}{N} = \mathcal{S}_0(\mathcal{Q}) \neq 0$$

   This entropy is *not* due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-N\mathcal{S}_0}$ down to the ground state.

3. Thermal equilibration in a 'Planckian time' $\sim \hbar/(k_B T)$

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

The complex SYK model

Key properties

4. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. The action for this soft mode is controlled by an emergent SL(2,R) symmetry.

A. Kitaev, KITP talk (2015)

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)

A. Kitaev and J. Suh, JHEP 183 (2018)

The complex SYK model

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

The complex SYK model

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll U$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations.
The singular part of the self-energy and the Green's function obey

$$\int_0^\beta d\tau_2 \, \Sigma_{\rm sing}(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma_{\rm sing}(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

The complex SYK model

$$\int_0^\beta d\tau_2 \, \Sigma_(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

These equations are invariant under

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.
By using $f(\sigma) = \tan(\pi T \sigma)/(\pi T)$ we can
now obtain the $T > 0$ solution from the $T = 0$ solution.

A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

The complex SYK model

Let us write the large N saddle point solutions of S as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

The saddle point will be invariant under a reperamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\widetilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

A. Kitaev

The complex SYK model

Key properties

4. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. The action for this soft mode is controlled by an emergent SL(2,R) symmetry.

A. Kitaev, KITP talk (2015)

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)

A. Kitaev and J. Suh, JHEP 183 (2018)

The complex SYK model

Key properties

4. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. The action for this soft mode is controlled by an emergent SL(2,R) symmetry.

5. Maximal quantum Lyapunov exponent for the out-of-time-order correlator (OTOC):

$$\left\langle c_a^\dagger(t) c_b(0) c_a(t) c_b^\dagger(0) \right\rangle = C_0 + C_1 \left(\frac{e^{\lambda t}}{N} \right) + \ldots$$

with $\lambda = 2\pi k_B T/\hbar$.

A. Kitaev, KITP talk (2015)
J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)
A. Kitaev and J. Suh, JHEP 183 (2018)

The complex SYK model

Key properties

4. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. The action for this soft mode is controlled by an emergent SL(2,R) symmetry.

5. Maximal quantum Lyapunov exponent for the out-of-time-order correlator (OTOC):

$$\left\langle c_a^\dagger(t) c_b(0) c_a(t) c_b^\dagger(0) \right\rangle = C_0 + C_1 \left(\frac{e^{\lambda t}}{N} \right) + \ldots$$

with $\lambda = 2\pi k_B T/\hbar$.

6. For spinful fermions, spin correlations decay as

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

1. SYK criticality +
time reparameterization soft mode

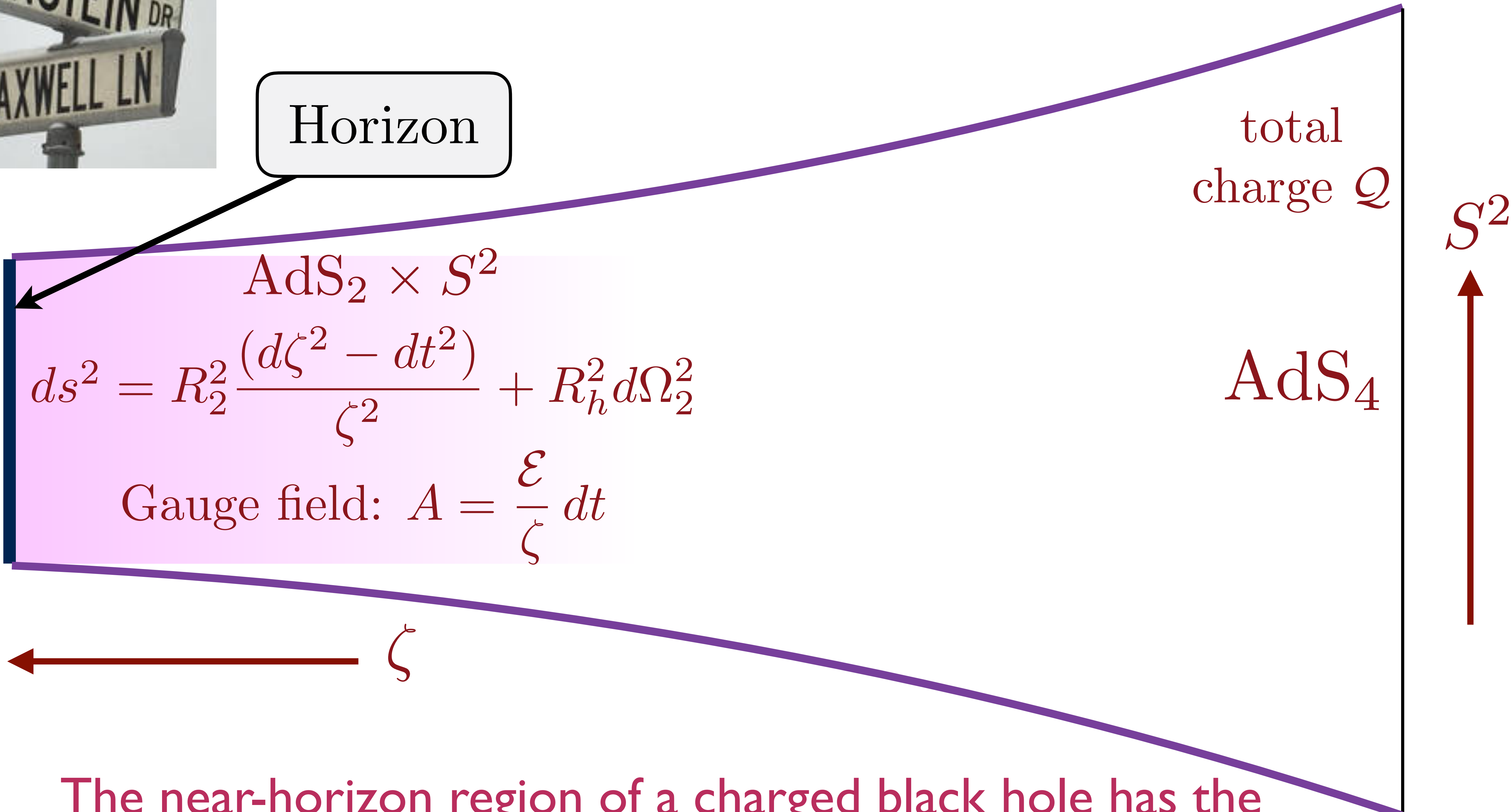
2. Charged black holes

3. SYK lattice models

4. *Fractionalization* and SYK criticality
in t-J models with random exchange

5. Linear-in-T resistivity down to zero T

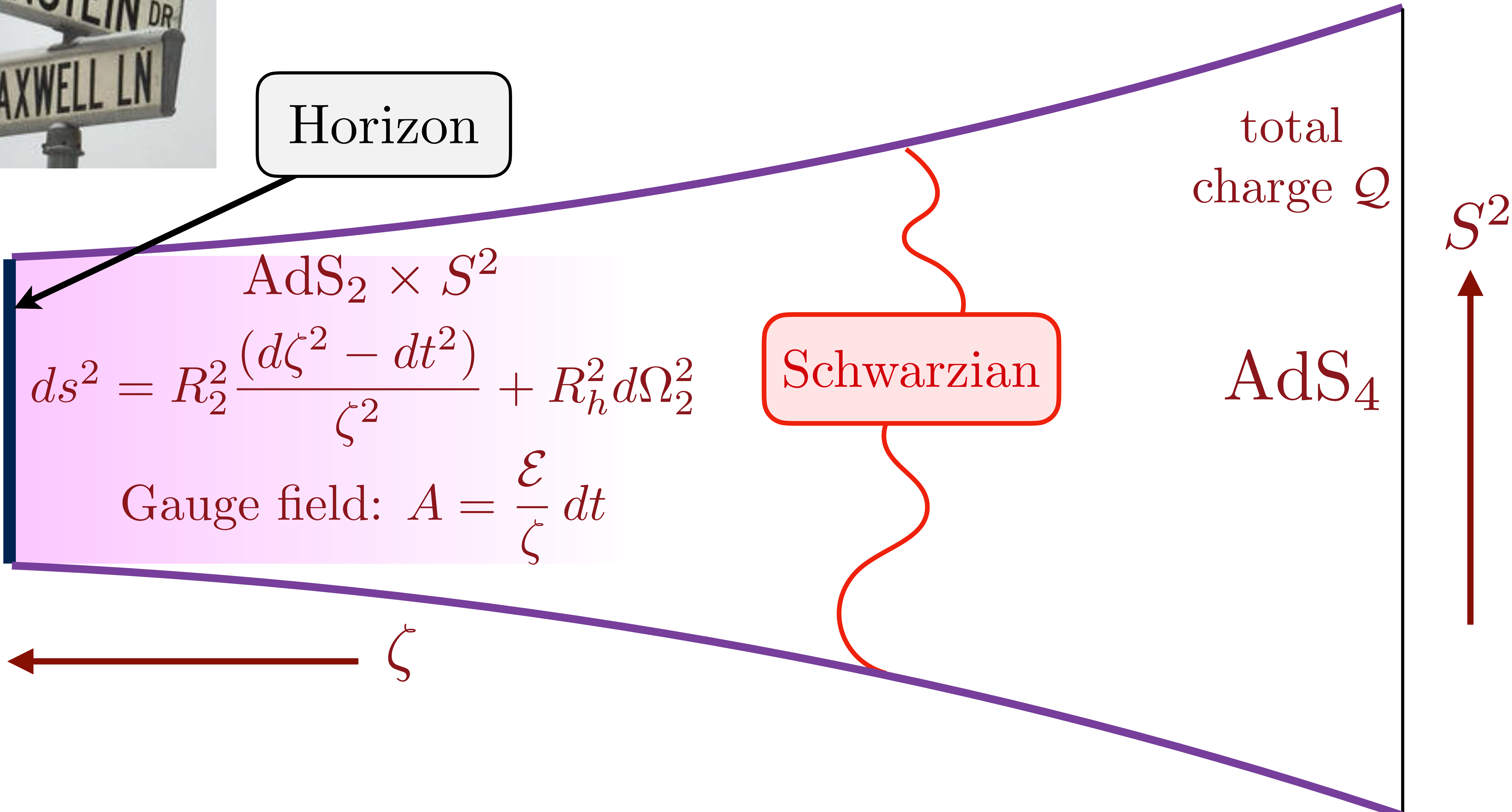
Charged black holes



The near-horizon region of a charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime, which has SL(2,R) symmetry.

There is a non-zero entropy, NS_0, in the limit of zero temperature, given by horizon area in the 4-dimensional spacetime.

Charged black holes



The effective theory for the fluctuations of the one-dimensional boundary of the AdS_2 region, beyond which the geometry is AdS_4, has a time reparameterization soft mode. A Schwarzian soft mode action can be derived from the parent Einstein-Maxwell action in 4 dimensions; the same action can also be obtained from the SYK model.

Main result

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

A. Kitaev (2015)

S. Sachdev, Phys. Rev. X **5**, 041025 (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)

K. Jensen, Phys. Rev. Lett. **117**, 111601 (2016)

J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev,
Phys. Rev. B **95**, 155131 (2017)

A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746 P. Nayak, A. Shukla,

R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062

U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239

S. Sachdev, arXiv:1902.04078

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, arXiv:1910.14099

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number NQ, at temperatures $T \ll U$

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number NQ, at temperatures $T \ll U$

and

Charged black holes in 3+1 dimensions of radius R_h, with total charge NQ, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in $0 + 1$ dimensions

Main result

The common low T path integral is $\mathcal{Z} = \int \mathcal{D}f \, \mathcal{D}\phi \, e^{-I}$. This can be exactly evaluated, and the action is

$$I = -N\mathcal{S}_0 + N \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left(\frac{\partial \phi}{\partial \tau} + i(2\pi\mathcal{E}T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\alpha_S}{J} \,\mathrm{Sch}[\tan(\pi T f(\tau)), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$$f(\tau + 1/T) = f(\tau) + 1/T,$$

ϕ is a phase conjugate to the charge density with

$$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n, \; n \text{ integer},$$

$\mathrm{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.

The couplings are related to the entropy $S(T, \mathcal{Q})$ and the chemical potential μ via

$$\frac{S(T \to 0, \mathcal{Q})}{N} = \mathcal{S}_0(\mathcal{Q}) + \frac{4\pi^2 \alpha_S}{J} T, \quad K = \left(\frac{d\mathcal{Q}}{d\mu} \right)_{T \to 0}, \quad 2\pi\mathcal{E} = \frac{d\mathcal{S}_0(\mathcal{Q})}{d\mathcal{Q}}$$

1. SYK criticality +
   time reparameterization soft mode

2. Charged black holes

3. SYK lattice models

4. *Fractionalization* and SYK criticality
   in t-J models with random exchange

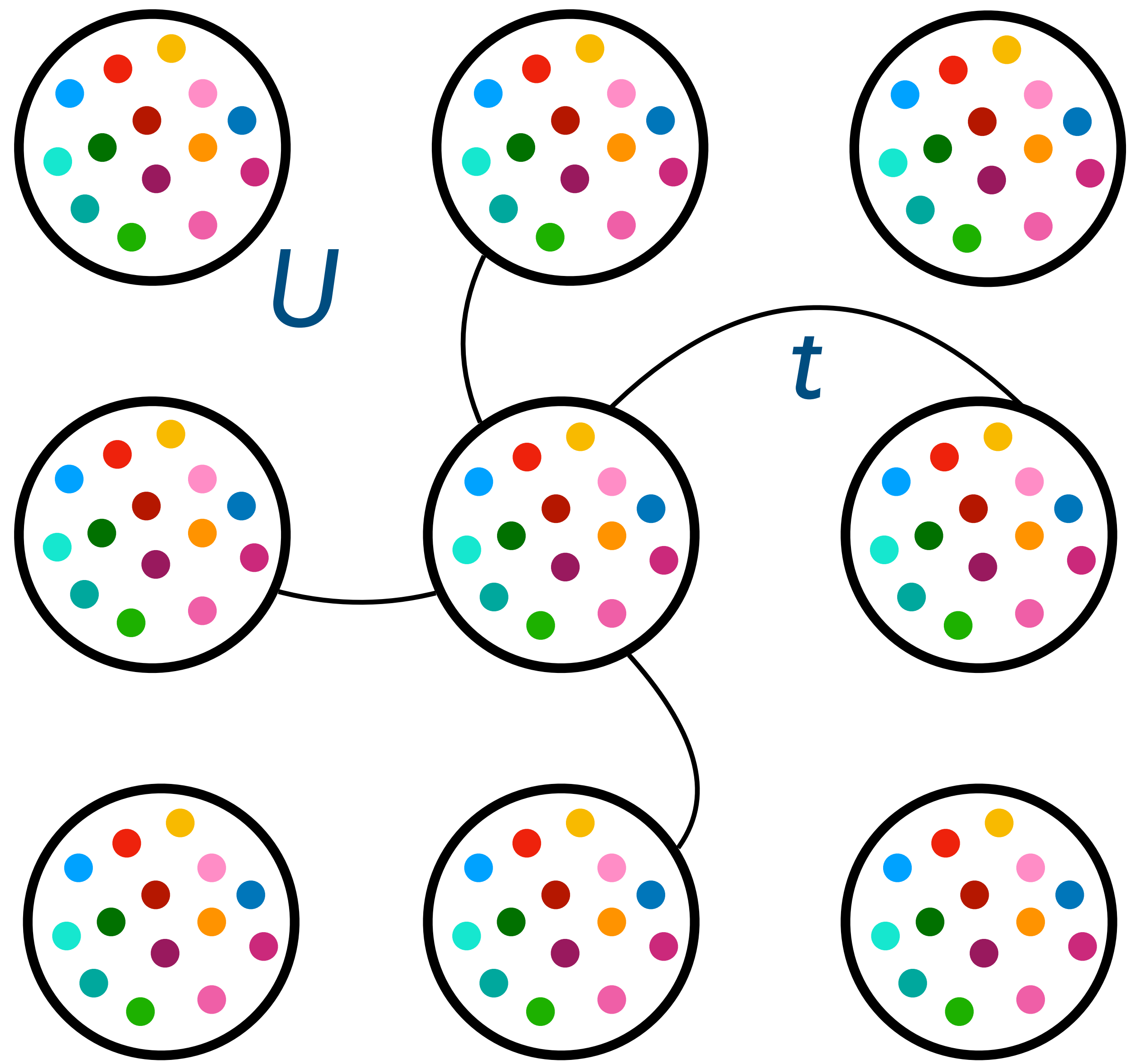
5. Linear-in-T resistivity down to zero T

A strange metal: lattice of SYK islands

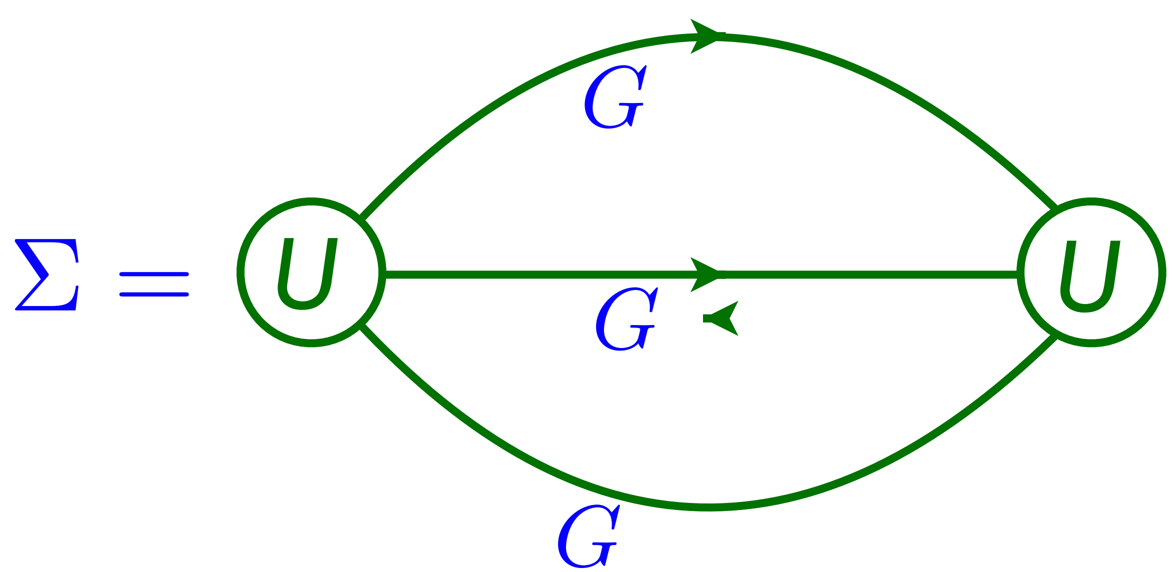
$$H = \frac{1}{(2N)^{3/2}} \sum_{i} \sum_{a,b,c,d=1}^{N} U_{i,ab;cd}\, c_{ia}^\dagger c_{ib}^\dagger c_{ic} c_{id} - t \sum_{\langle ij \rangle} \sum_{a} c_{ia}^\dagger c_{ja}$$

Random interaction within each island U.

Amplitude to hop between islands t.



$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A strange metal: lattice of SYK islands

$$H = \frac{1}{(2N)^{3/2}} \sum_{i} \sum_{a,b,c,d=1}^{N} U_{i,ab;cd}\, c_{ia}^\dagger c_{ib}^\dagger c_{ic} c_{id} - t \sum_{\langle ij \rangle} \sum_{a} c_{ia}^\dagger c_{ja}$$

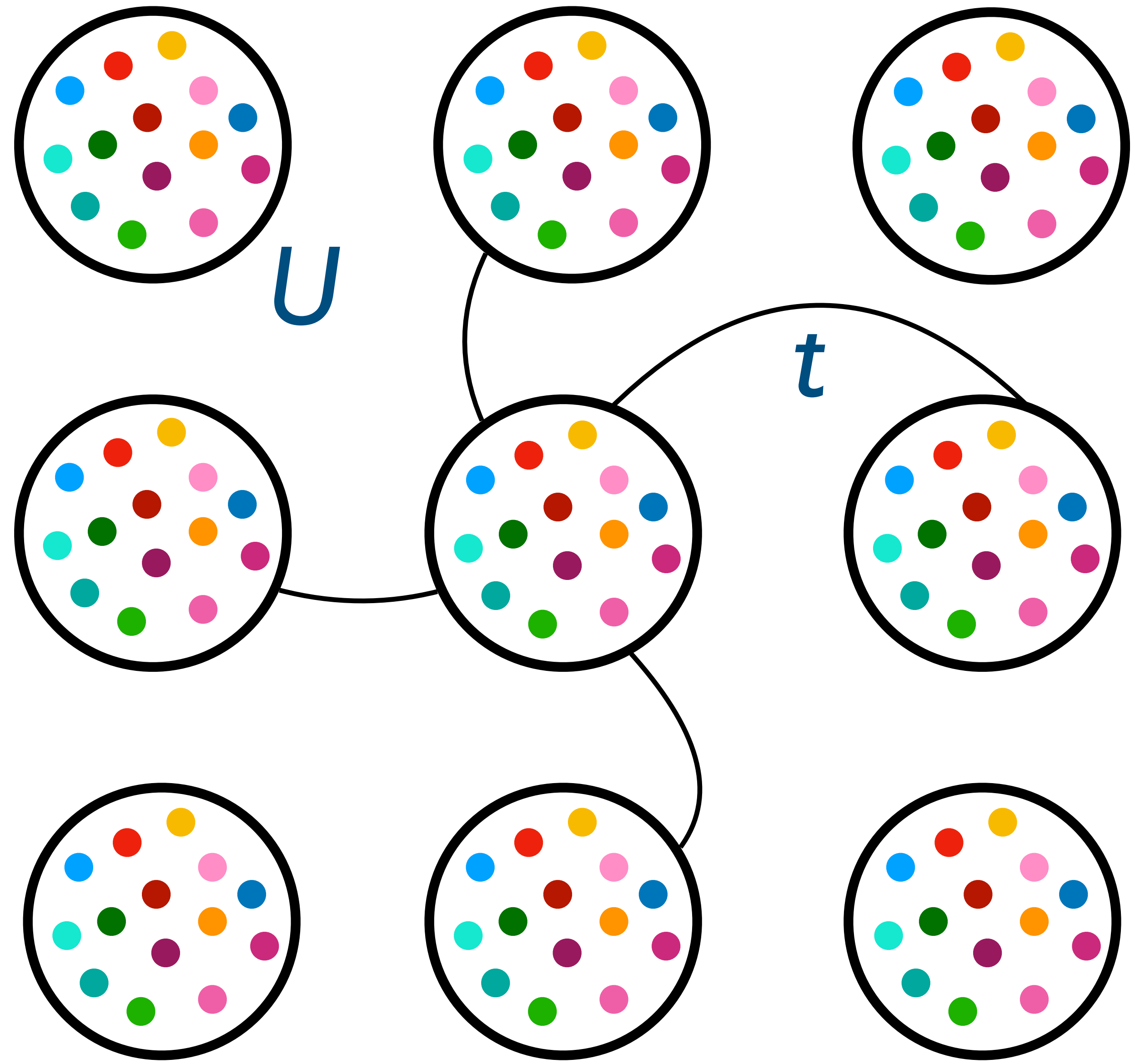
Random interaction within each island U.

Amplitude to hop between islands t.

Model yields SYK criticality and resistivity

$$\rho \sim T$$

for $t^2/U \lesssim T \lesssim U$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A strange metal: lattice of SYK islands

- Linear-in-T resistivity only for $T > t^2/U$.
- Insensitive to electron density; no critical density
- No pseudogap phase.
- No 'Mottness': on-site (Hubbard) repulsion is missing.

1. SYK criticality + *time reparameterization soft mode*

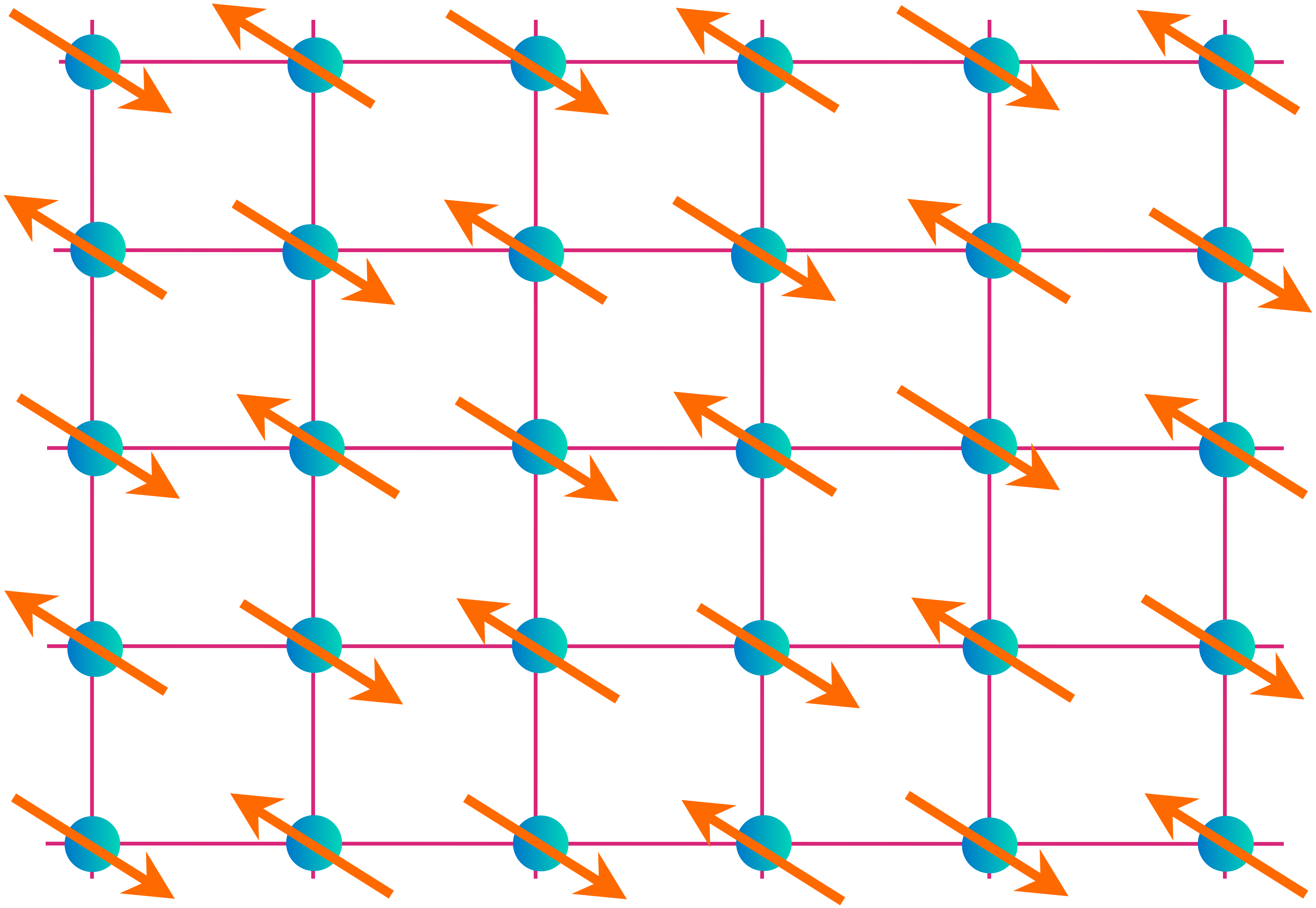
2. Charged black holes

3. SYK lattice models

4. *Fractionalization* and SYK criticality in t-J models with random exchange

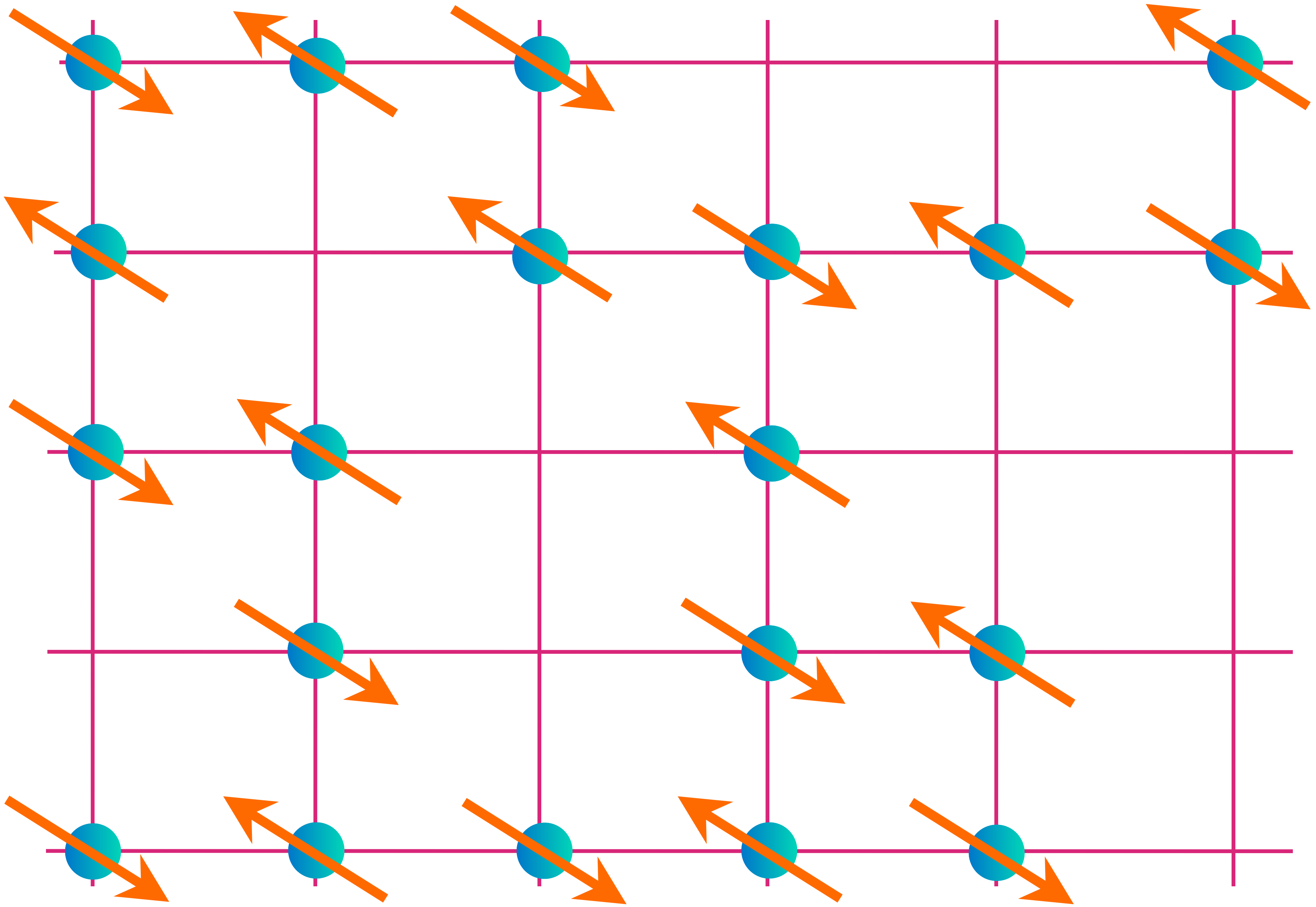
5. Linear-in-T resistivity down to zero T

The t-J model



Square lattice of Cu sites

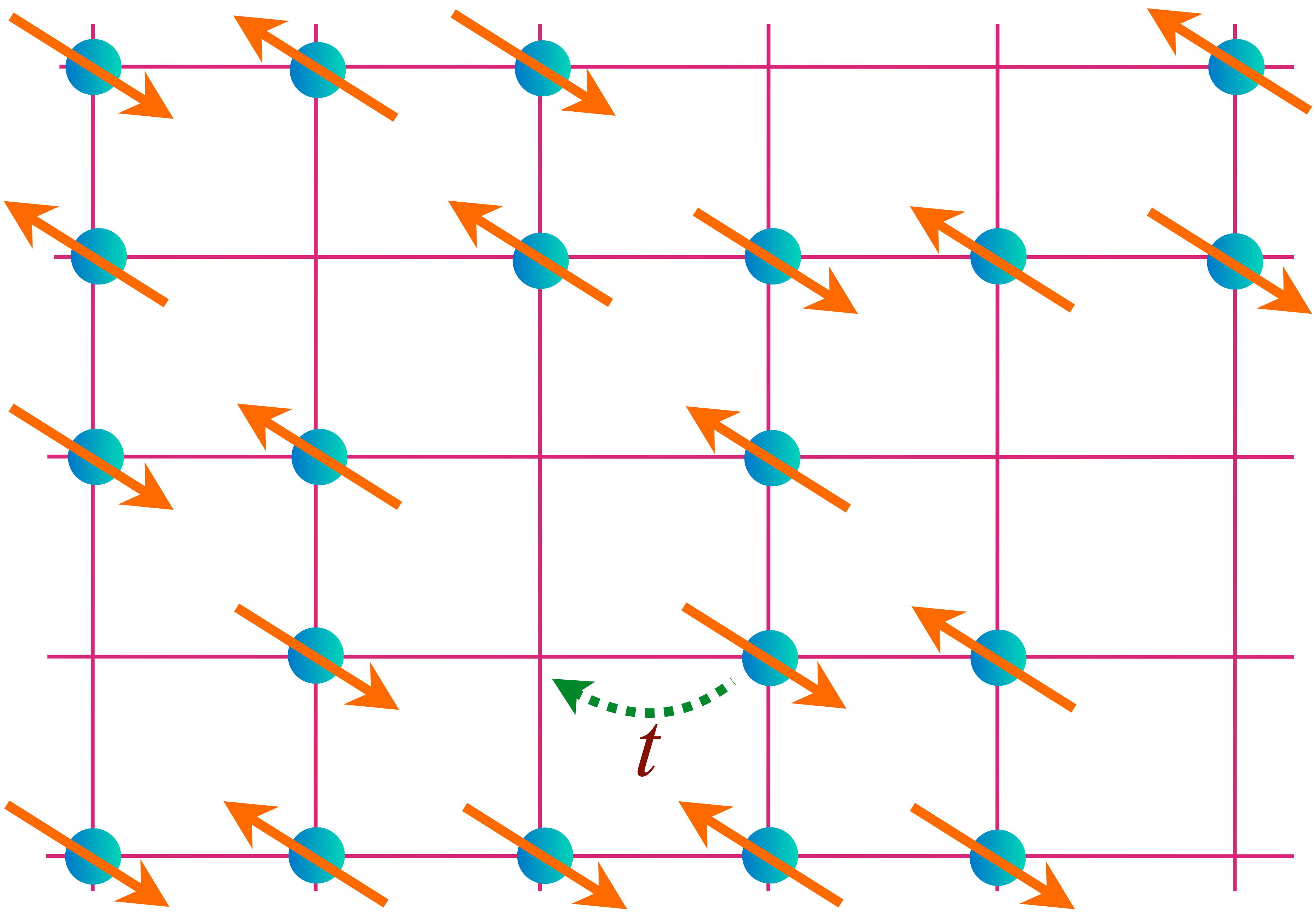
The t-J model



Remove fraction p electrons

Square lattice of Cu sites

The t-J model

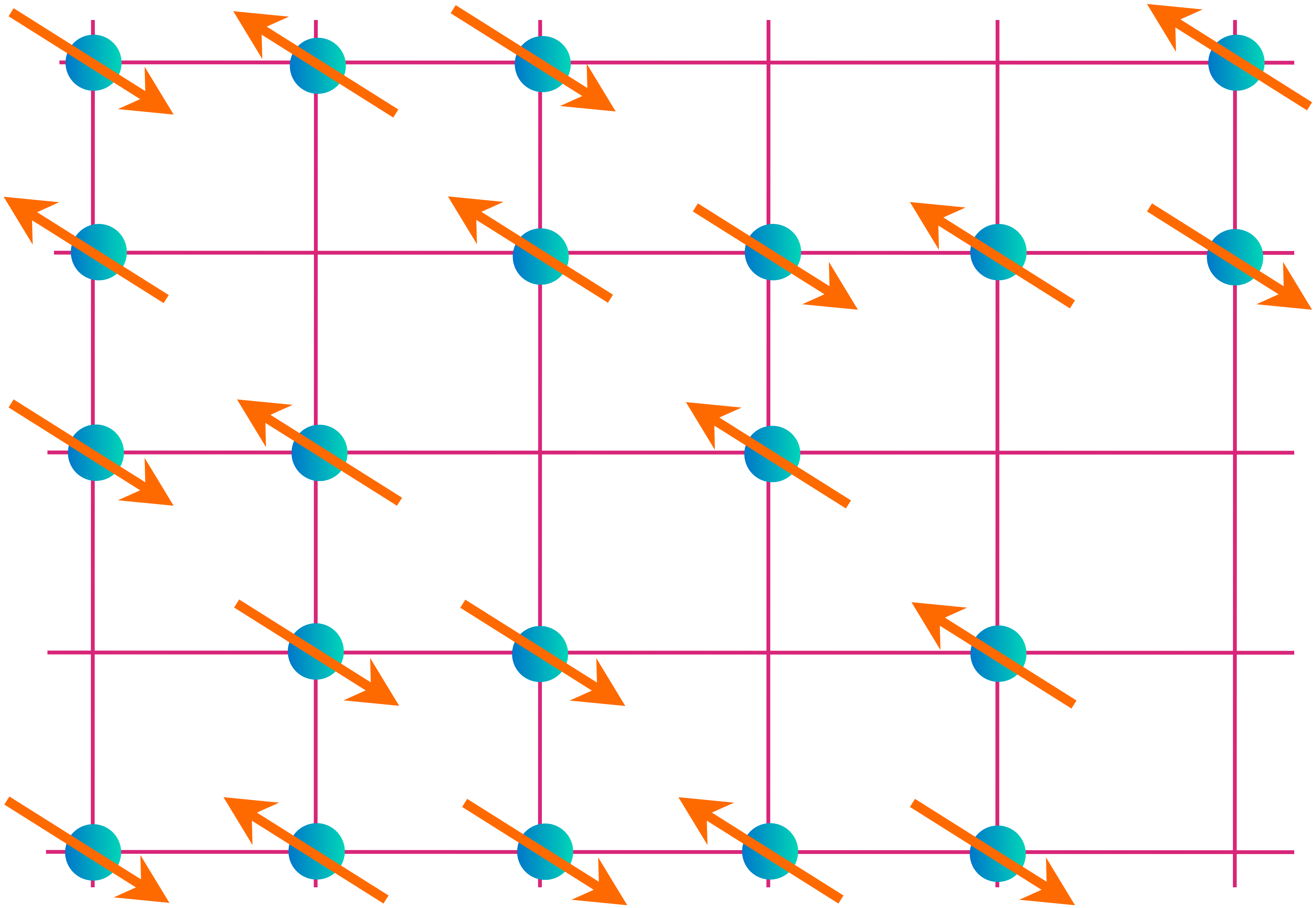


Remove fraction p electrons

Hopping amplitude t

Square lattice of Cu sites

The t-J model

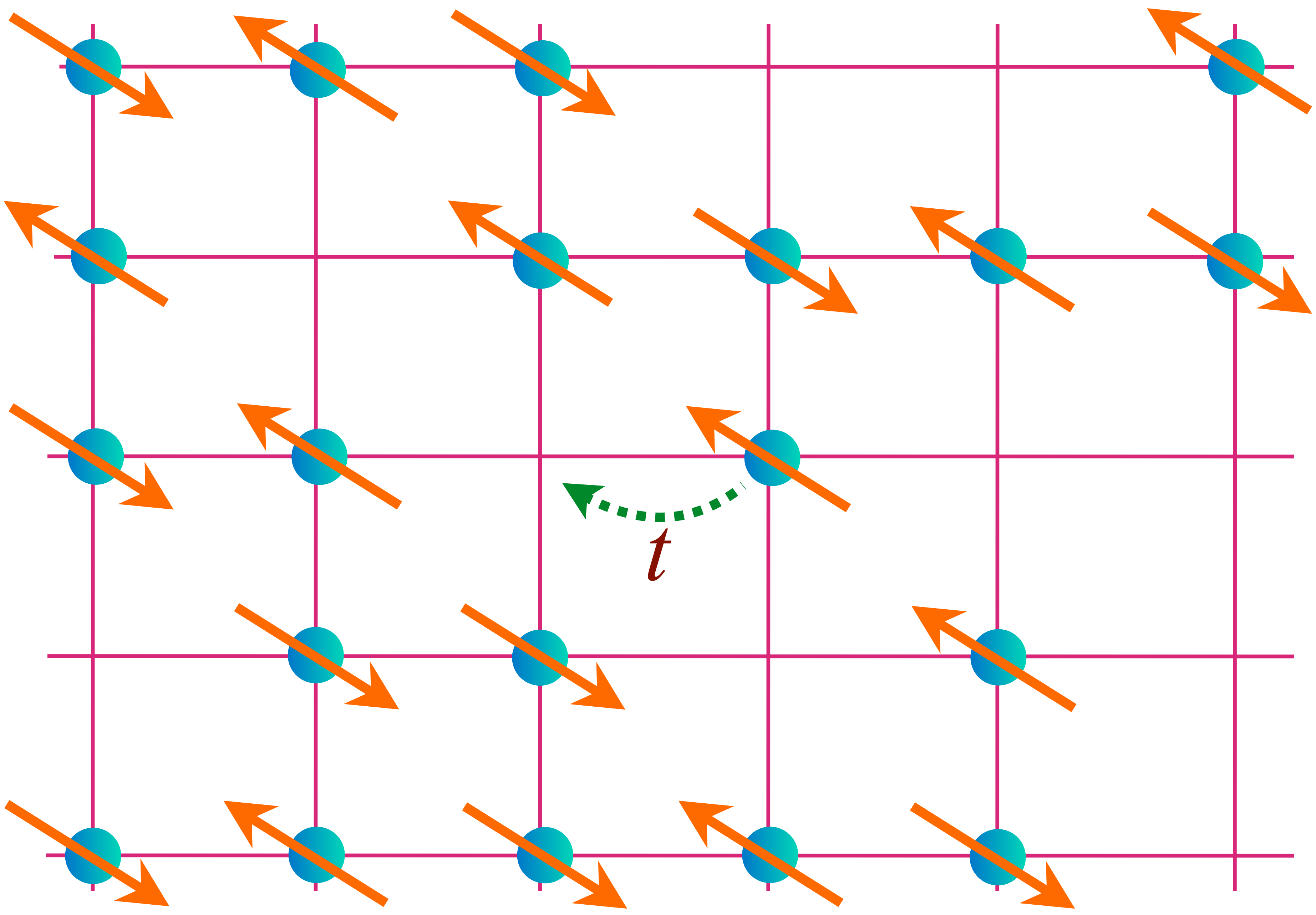


Remove fraction p electrons

Hopping amplitude t

Square lattice of Cu sites

The t-J model

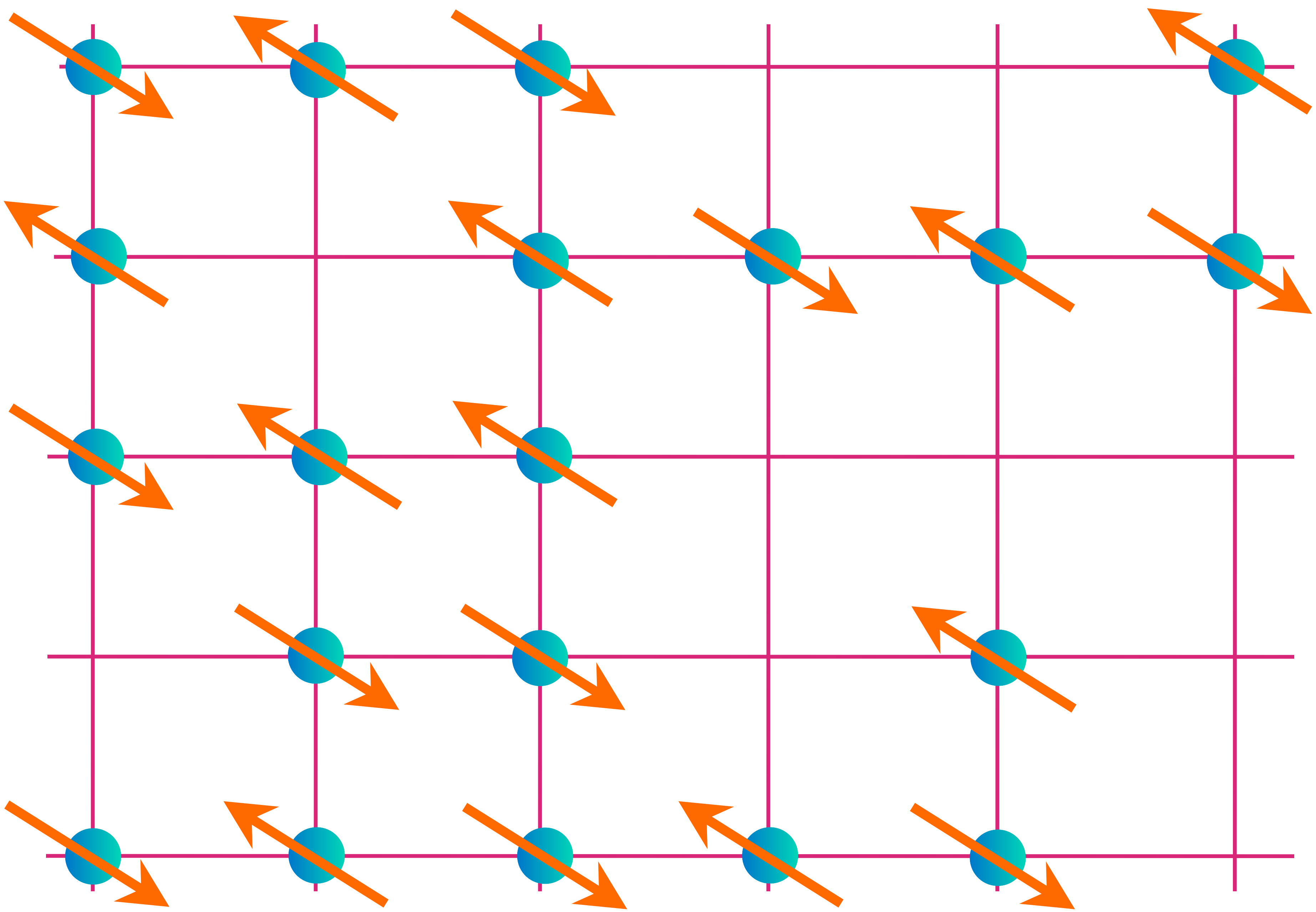


Remove fraction p electrons

Hopping amplitude t

Square lattice of Cu sites

The t-J model

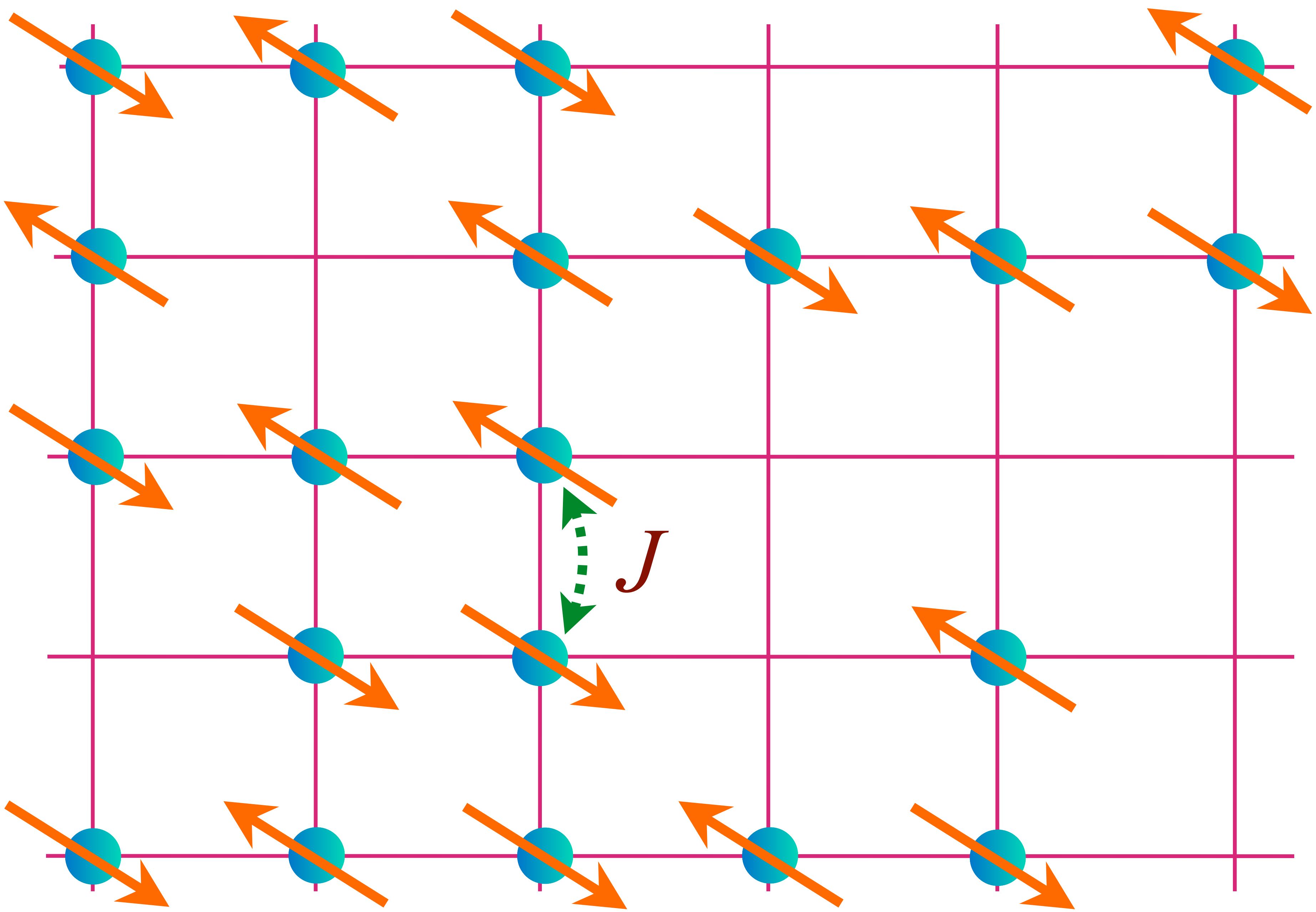


Remove fraction p electrons

Hopping amplitude t

Square lattice of Cu sites

The t-J model



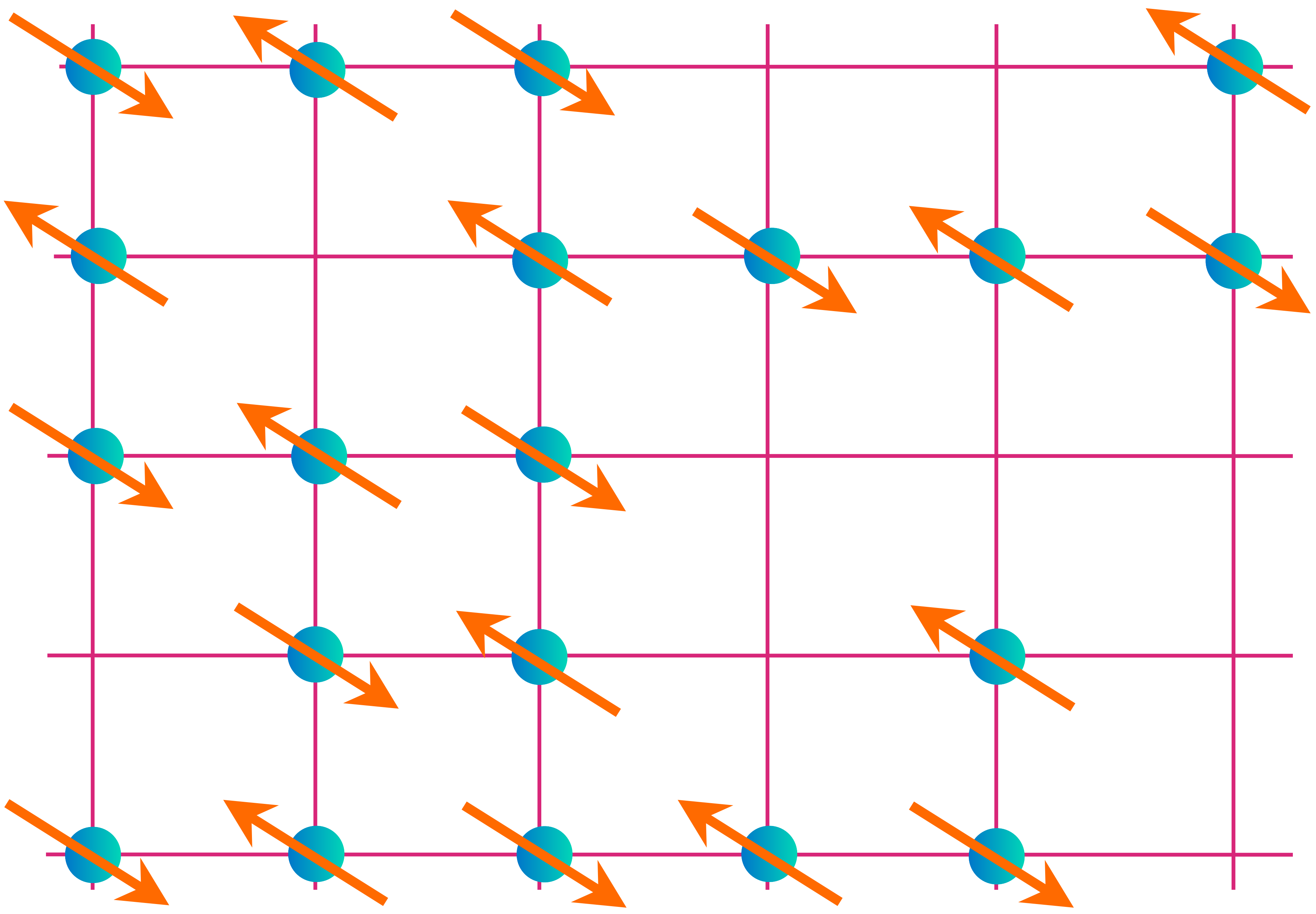
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The t-J model



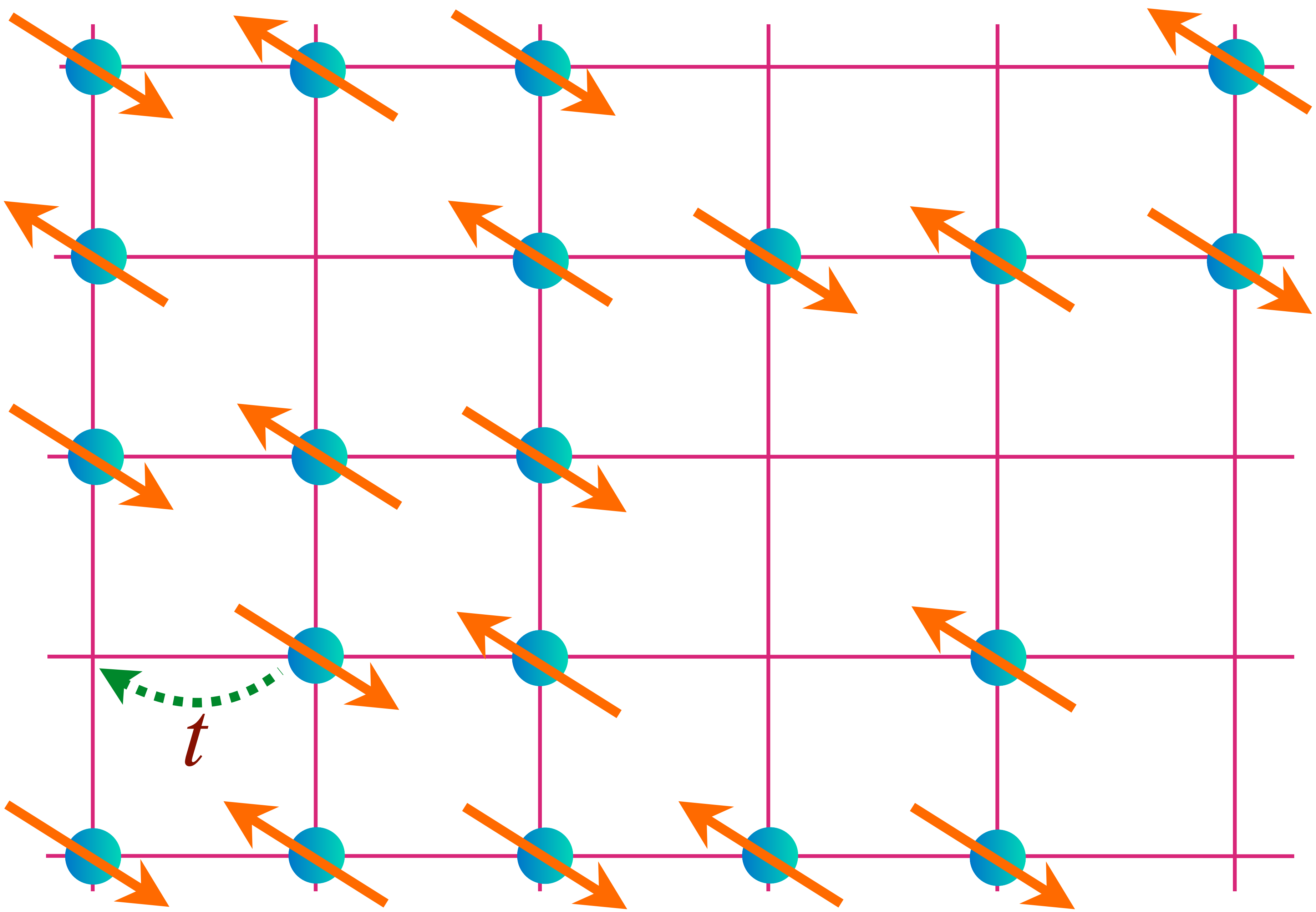
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The t-J model



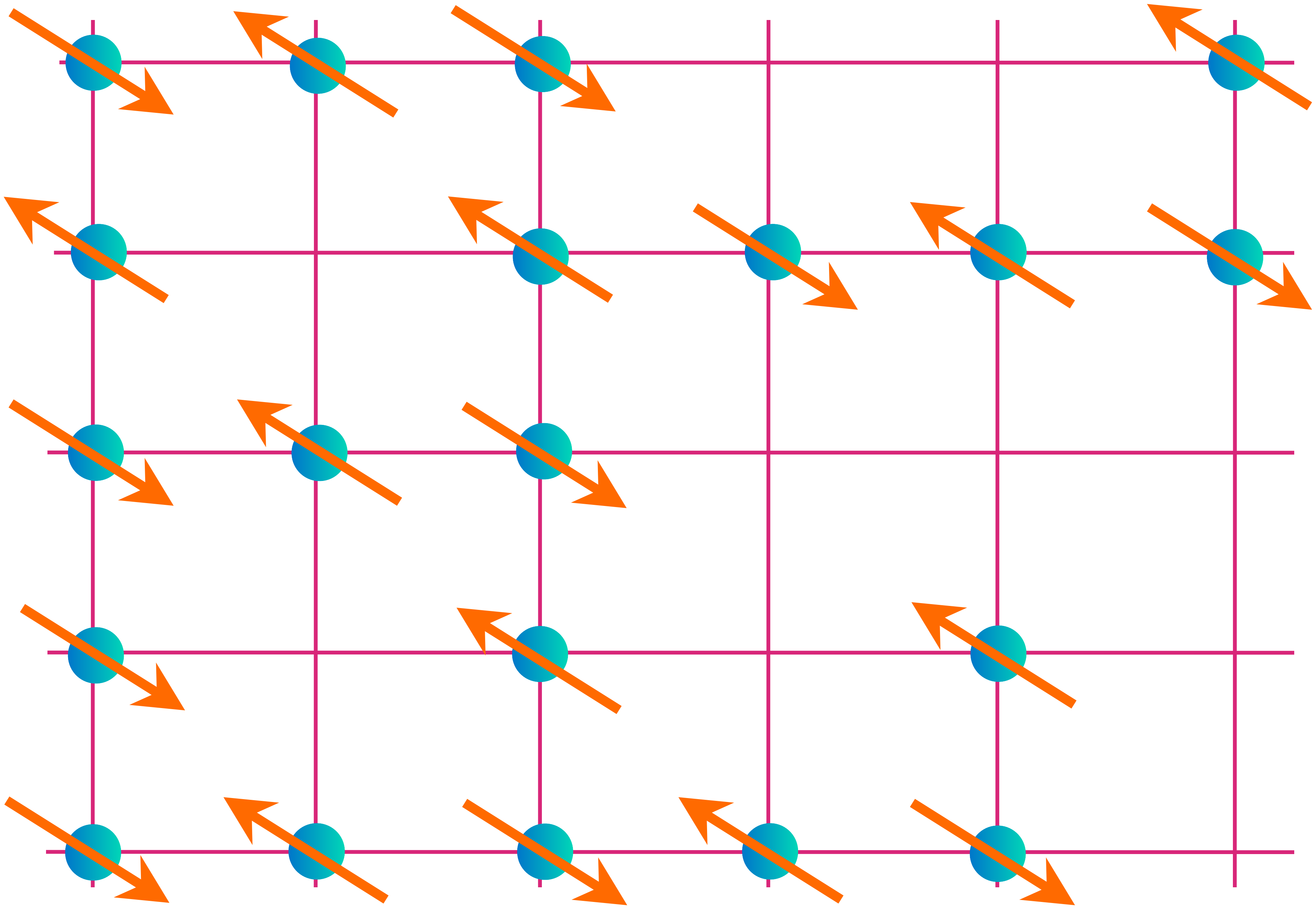
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The *t*-*J* model



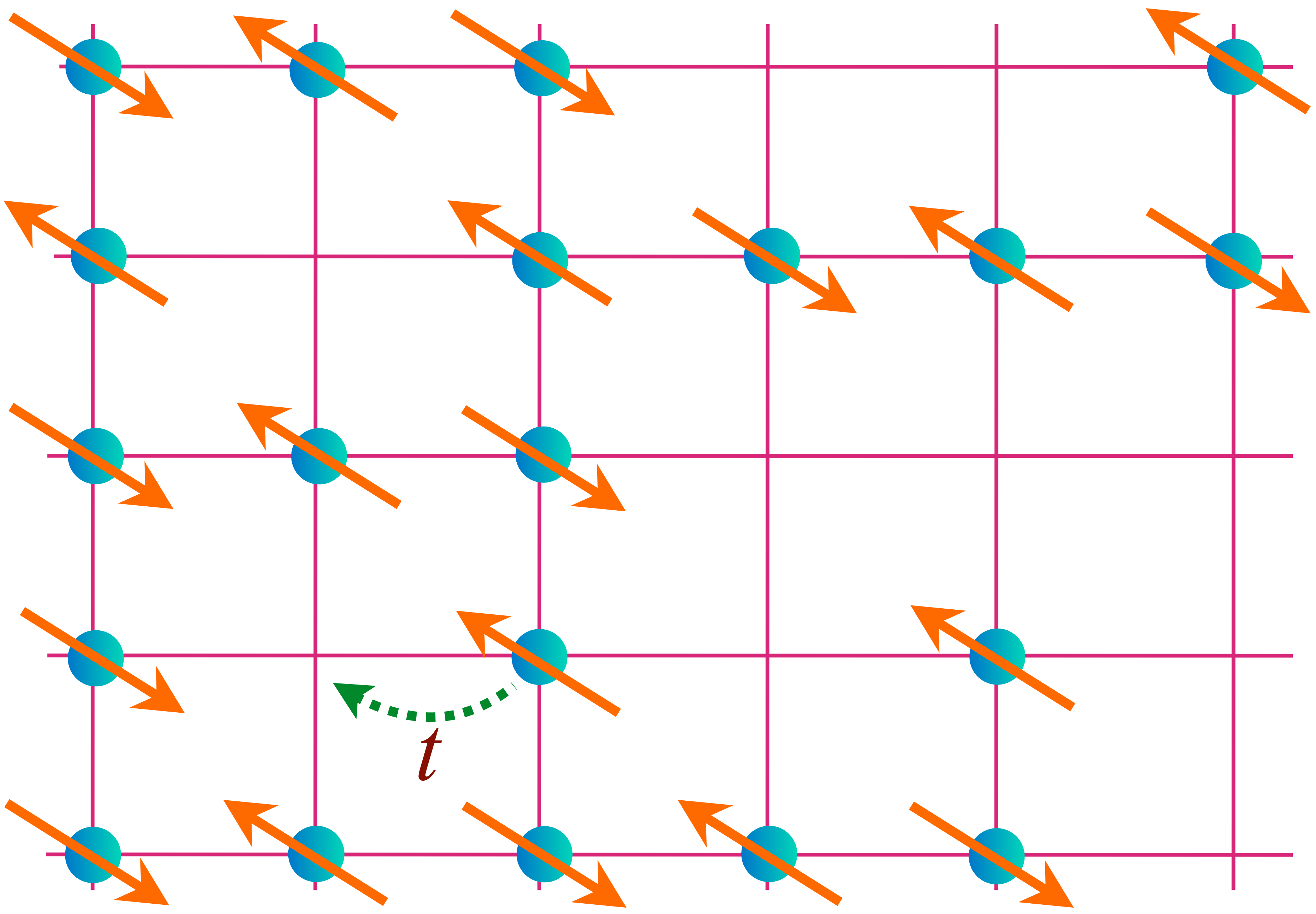
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The t-J model



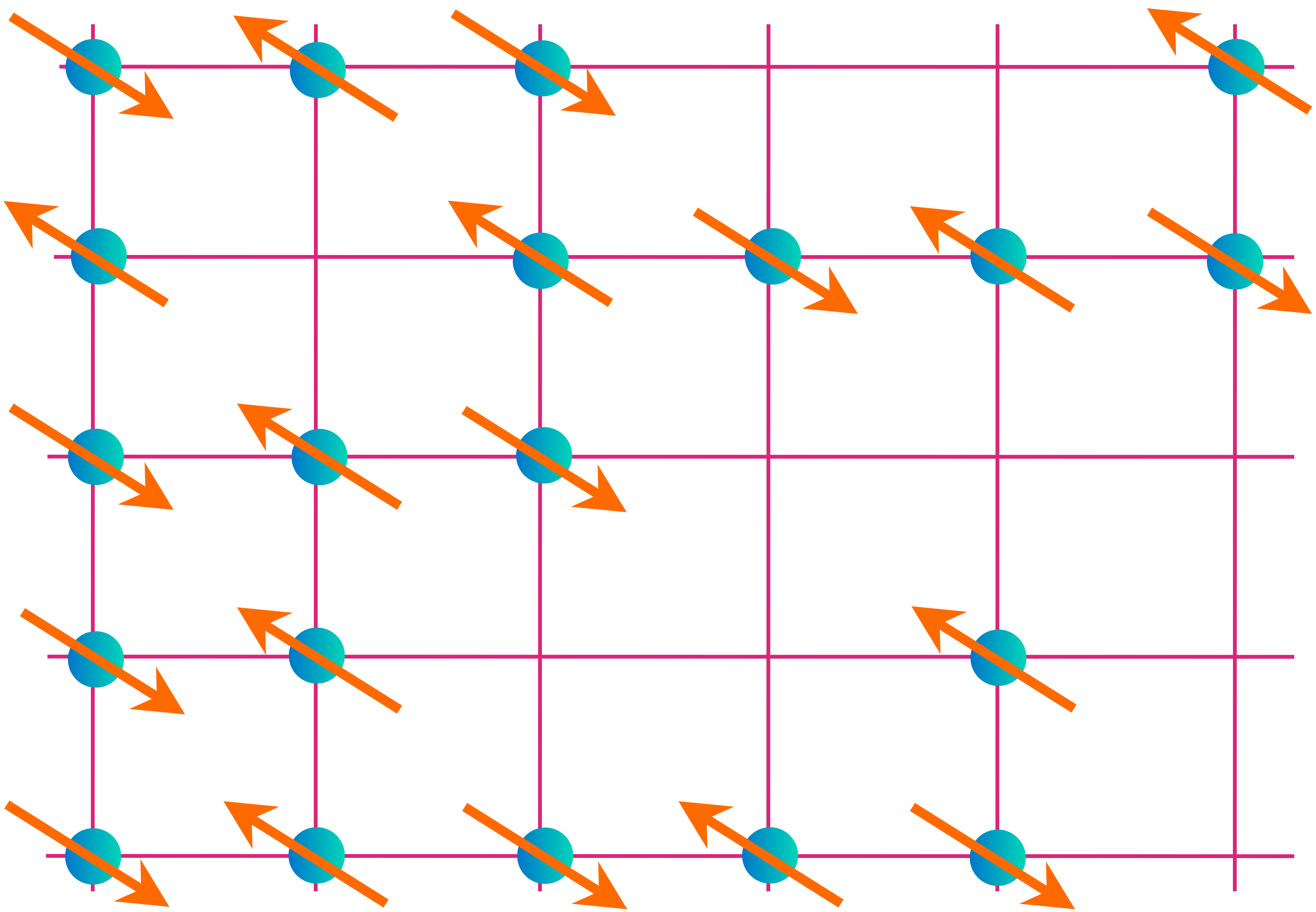
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The *t*-*J* model



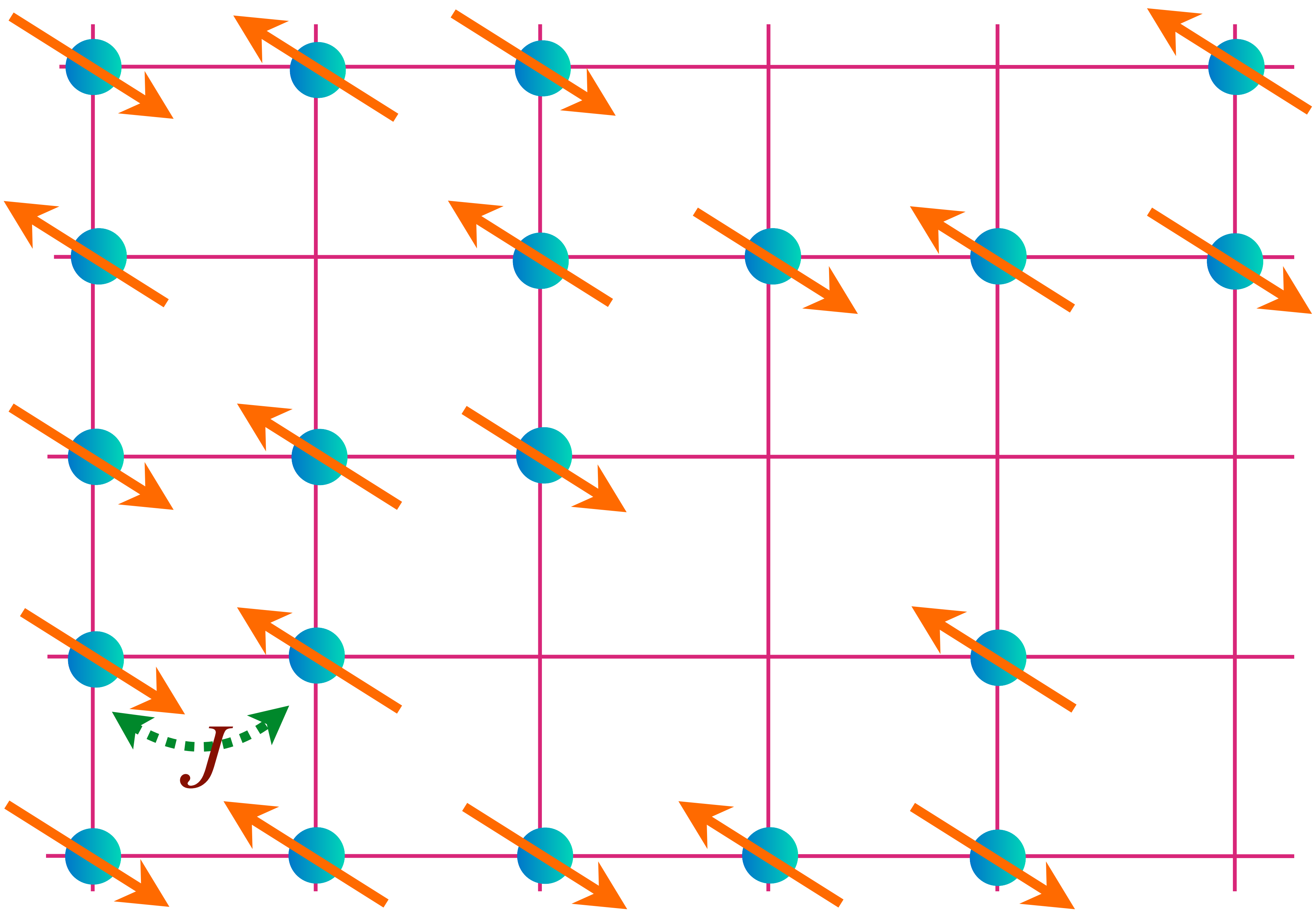
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The t-J model



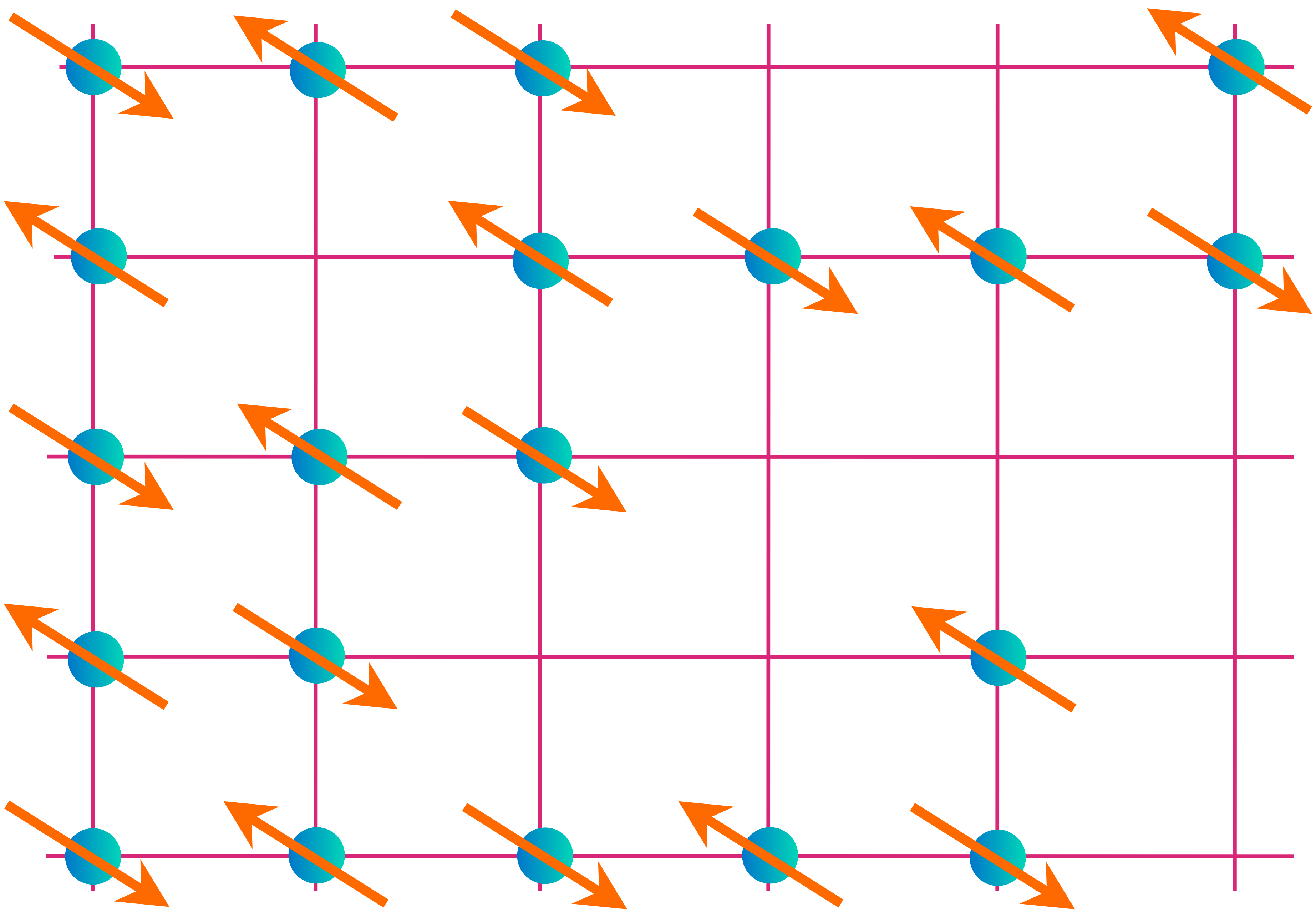
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The t-J model



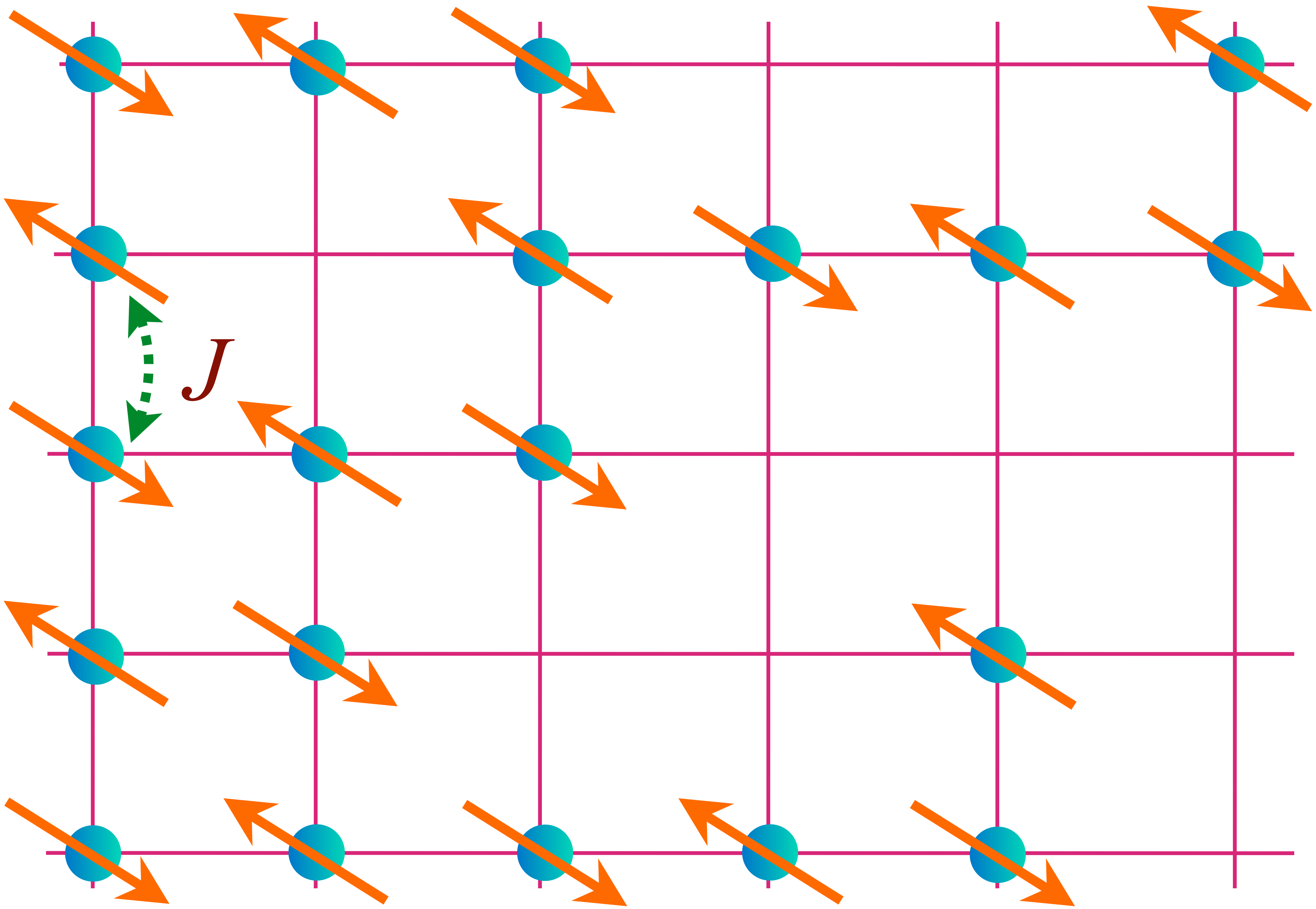
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The t-J model



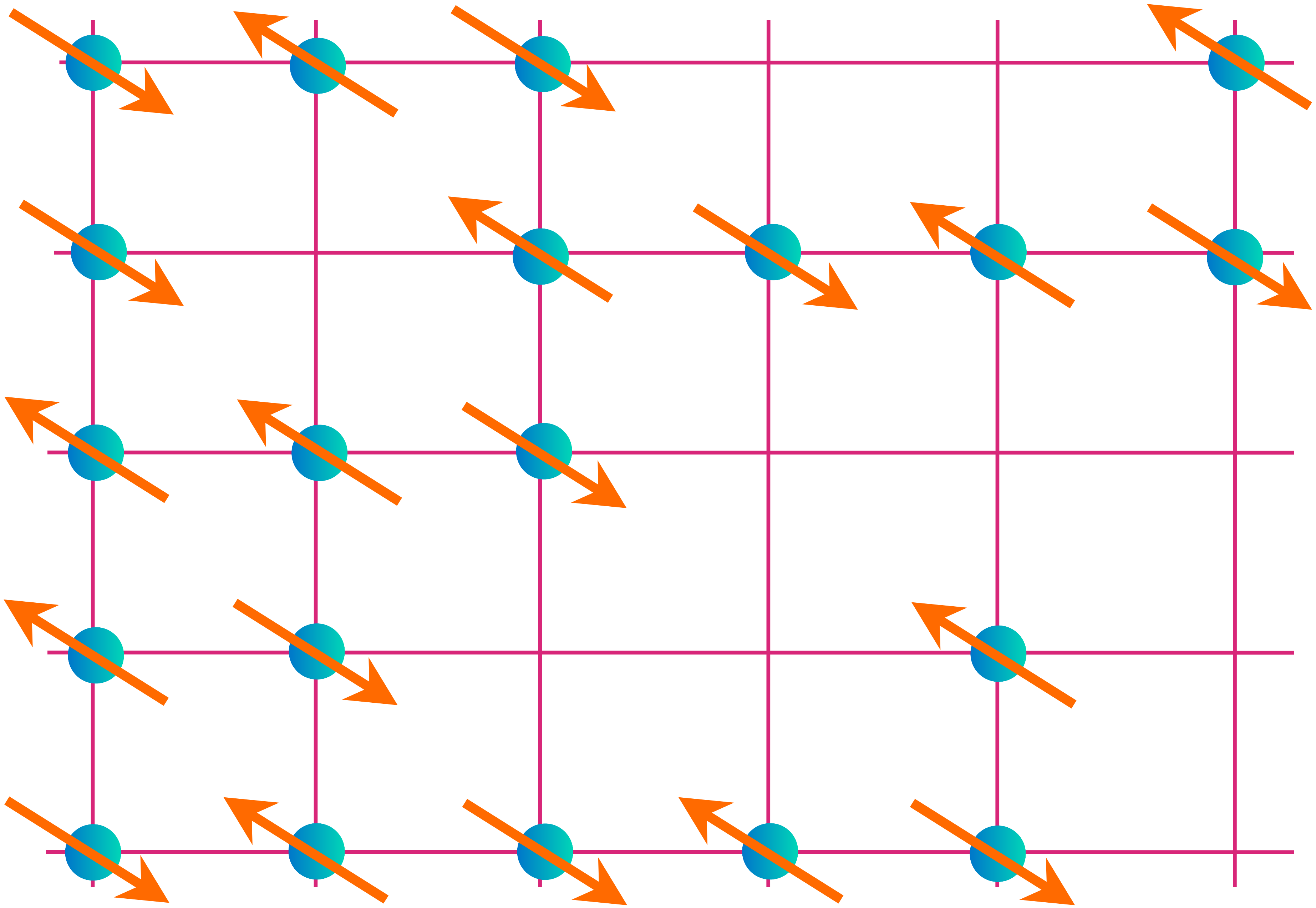
Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

Square lattice of Cu sites

The t-J model



Square lattice of Cu sites

Remove fraction p electrons

Hopping amplitude t

Spin Exchange J

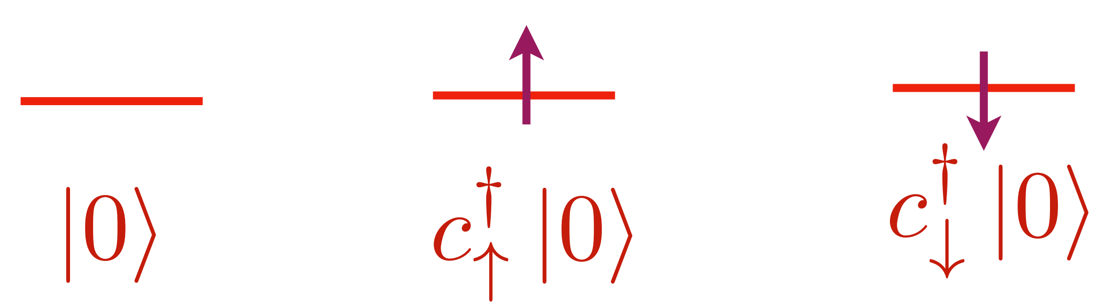
The t-J model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c^\dagger_{j\beta}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_\alpha c^\dagger_{i\alpha} c_{i\alpha} \leq 1, \quad \sum_\alpha \left\langle c^\dagger_{i\alpha} c_{i\alpha} \right\rangle = 1 - p$$



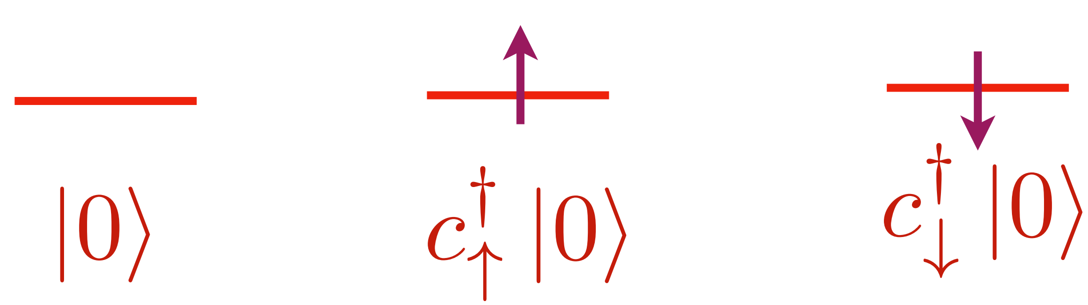
The t-J model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \boxed{\sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1,} \quad \sum_{\alpha} \left\langle c_{i\alpha}^{\dagger} c_{i\alpha} \right\rangle = 1 - p$$



The t-J model

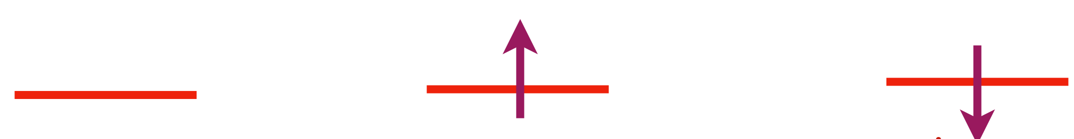
$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c^\dagger_{j\beta}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \boxed{\sum_\alpha c^\dagger_{i\alpha} c_{i\alpha} \leq 1,} \quad \sum_\alpha \left\langle c^\dagger_{i\alpha} c_{i\alpha} \right\rangle = 1 - p$$

We take the large z limit in a lattice with co-ordination number z and J_{ij} random, $\overline{J_{ij}} = 0$, $\overline{J^2_{ij}} = J^2$

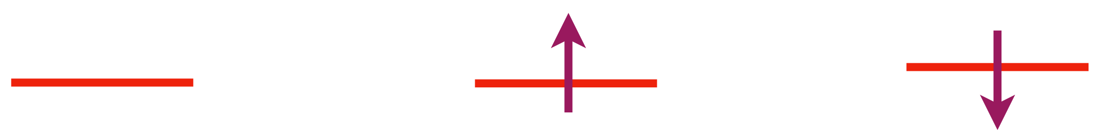


$|0\rangle$ $\quad c^\dagger_\uparrow |0\rangle$ $\quad c^\dagger_\downarrow |0\rangle$

Fractionalization in the *t-J* model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$b^\dagger \left| v \right\rangle \qquad f_\uparrow^\dagger \left| v \right\rangle \qquad f_\downarrow^\dagger \left| v \right\rangle$

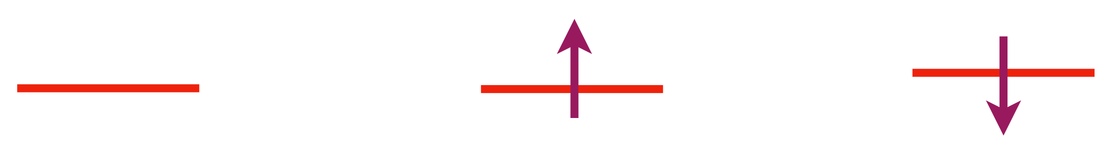
$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad b \to b e^{i\phi}, \quad f_\alpha \to f_\alpha e^{i\phi}$

Fractionalization in the *t*-*J* model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} f_{i\alpha}^{\dagger} f_{j\alpha} b_i b_j^{\dagger} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} \frac{J_{ij}}{4} f_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} f_{i\beta} \cdot f_{j\gamma}^{\dagger} \vec{\sigma}_{\gamma\delta} f_{j\delta}$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$b^{\dagger} |v\rangle \qquad f_{\uparrow}^{\dagger} |v\rangle \qquad f_{\downarrow}^{\dagger} |v\rangle$$

$$\begin{aligned} c_\alpha &= f_\alpha b^{\dagger} \\ \vec{S} &= \frac{1}{2} f_\alpha^{\dagger} \sigma_{\alpha\beta} f_\beta \\ f_\alpha^{\dagger} f_\alpha + b^{\dagger} b &= 1 \end{aligned}$$

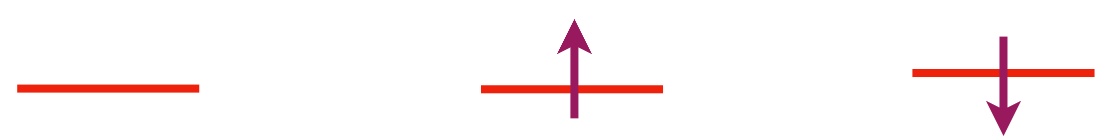
U(1) gauge invariance, $\quad b \to b e^{i\phi}, \quad f_\alpha \to f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space; both t and J terms in H are *quartic* in terms of fractionalized particles.

Fractionalization in the t-J model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} \mathfrak{f}_i \mathfrak{f}_j^\dagger \mathfrak{b}_{i\alpha}^\dagger \mathfrak{b}_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} \frac{J_{ij}}{4} \mathfrak{b}_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \mathfrak{b}_{i\beta} \cdot \mathfrak{b}_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \mathfrak{b}_{j\delta}$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$\mathfrak{f}^\dagger |v\rangle \qquad \mathfrak{b}_\uparrow^\dagger |v\rangle \qquad \mathfrak{b}_\downarrow^\dagger |v\rangle$$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

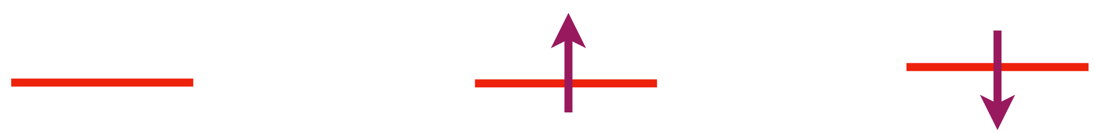
U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space; both t and J terms in H are *quartic* in terms of fractionalized particles.

Fractionalization in the t-J model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} \mathfrak{f}_i \mathfrak{f}_j^\dagger \mathfrak{b}_{i\alpha}^\dagger \mathfrak{b}_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} \frac{J_{ij}}{4} \mathfrak{b}_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \mathfrak{b}_{i\beta} \cdot \mathfrak{b}_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \mathfrak{b}_{j\delta}$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$\mathfrak{f}^\dagger |v\rangle$ $\qquad$ $\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\qquad$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$

U(1) gauge invariance, $\qquad \mathfrak{f} \to \mathfrak{f} e^{i\phi}, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space; both t and J terms in H are *quartic* in terms of fractionalized particles.

Fractionalization in the t-J model

Large M limit of SU$(M'|M)$ theory

Each site has 3 states which we map to the space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \quad , \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$
$$c_\alpha = f_\alpha b^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

To obtain a large M limit, let $\alpha = 1 \ldots M$, endow the boson with an 'orbital' index $a = 1 \ldots M'$ and send $M \to \infty$ at fixed $k = M'/M$. Then

$$c_{a\alpha} = f_\alpha b_a^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b_a^\dagger b_a = \frac{M}{2}$$

Fractionalization in the t-J model

Large M limit of SU$(M'|M)$ theory

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green's functions:

$$
\begin{aligned}
G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\
\Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\
G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\
\Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k\, t^2 G_f(\tau) G_b(\tau) G_b(-\tau)
\end{aligned}
$$

Here μ_f and μ_b are chemical potentials chosen to satisfy

$$
\langle f^\dagger f \rangle = \frac{1}{2} - kp \quad , \quad \langle b^\dagger b \rangle = p \,.
$$

Fractionalization in the t-J model

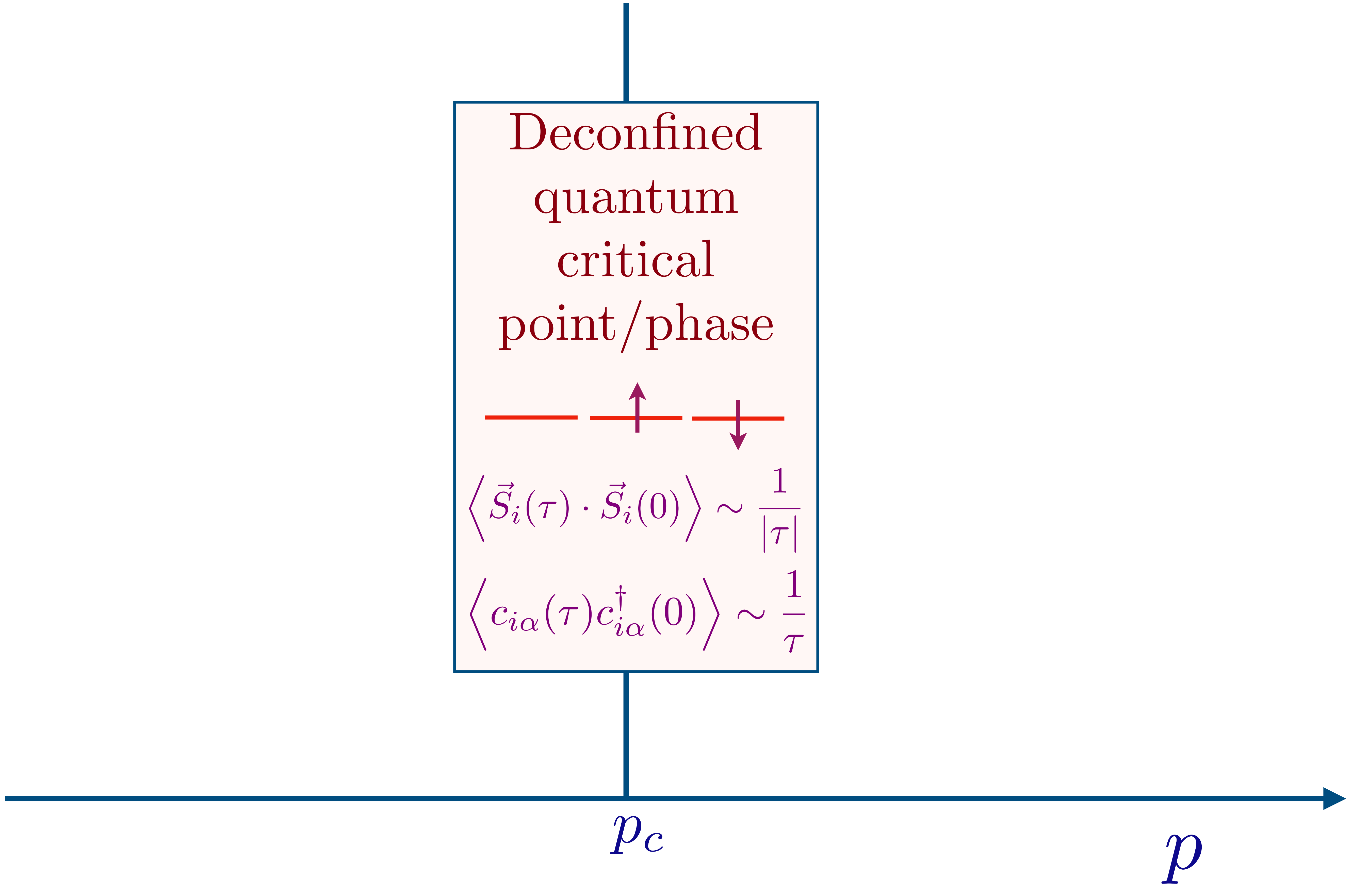
Large M limit of SU$(M'|M)$ theory

The critical solution which is self-consistent in both the t and J terms has $\Delta_b = \Delta_f = 1/4$, implying

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \begin{cases} \dfrac{A_+}{|\tau|} & , \quad \tau > 0 \\ -\dfrac{A_-}{|\tau|} & , \quad \tau < 0 \end{cases} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \, .$$

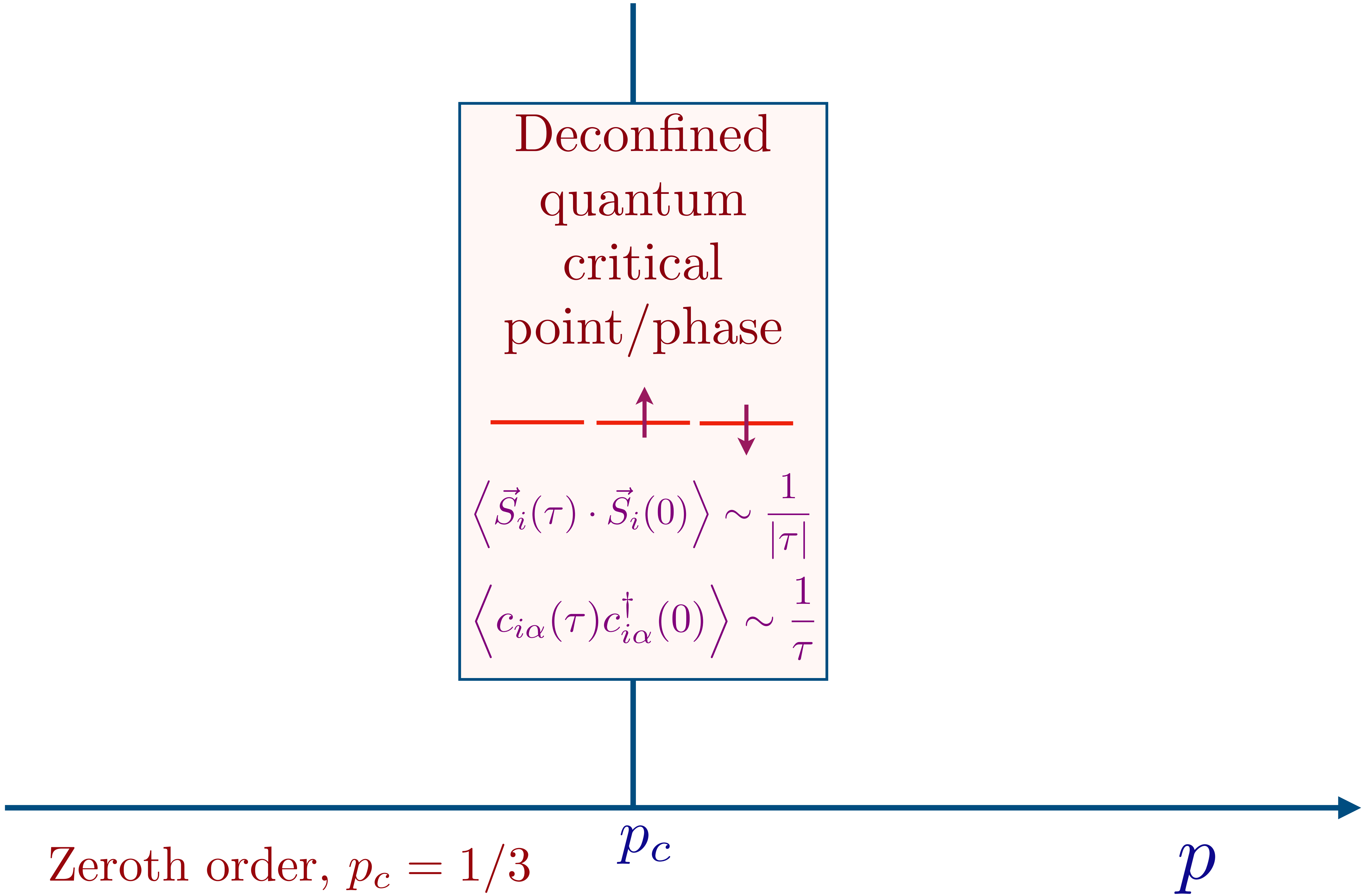
RG computations show that these results for the exponents of gauge-invariant operators are expected to be <u>exact</u> beyond the large M limit.

t-J model phase diagram



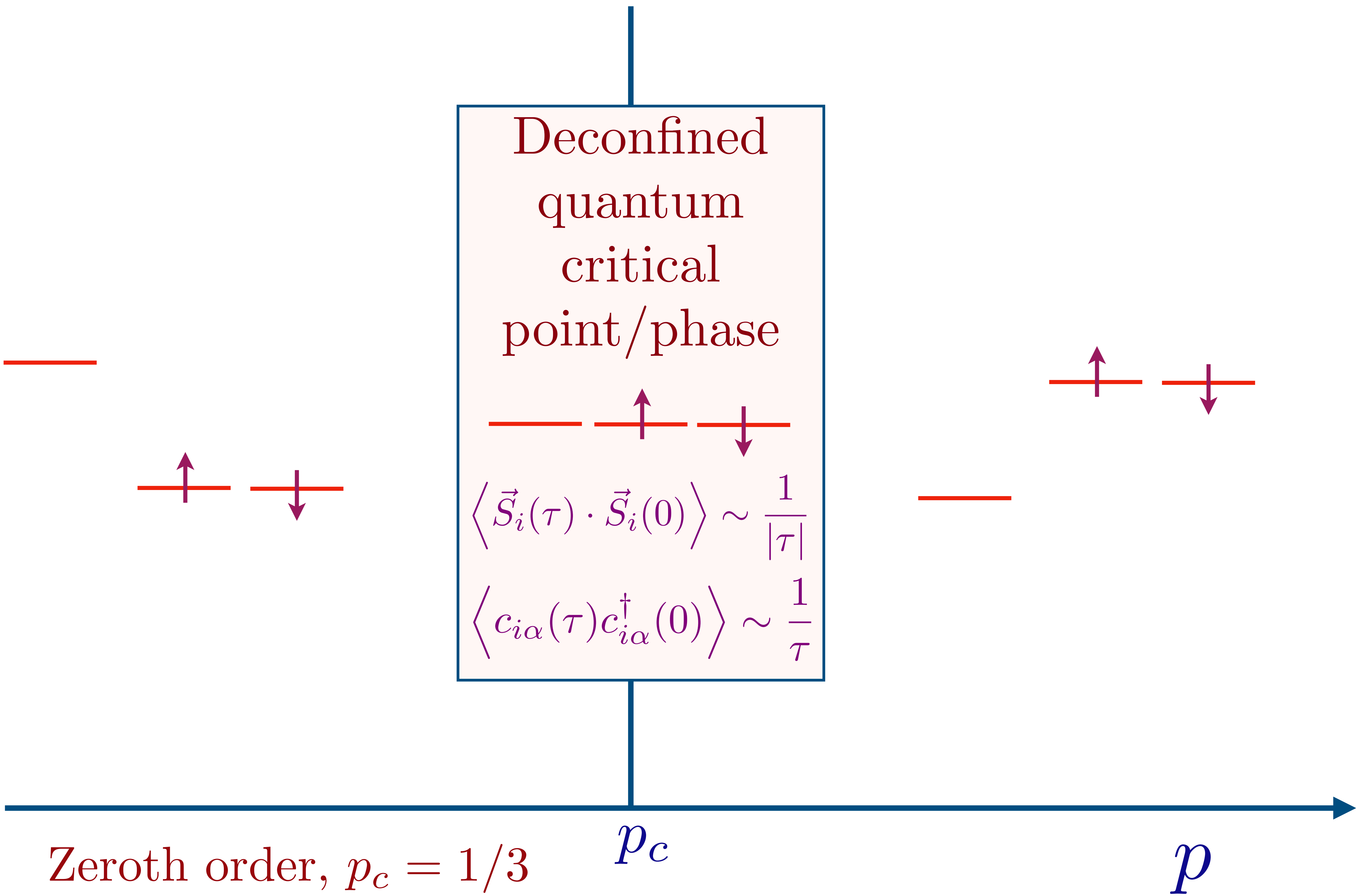
D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX to appear; arXiv:1912.08822

t-J model phase diagram



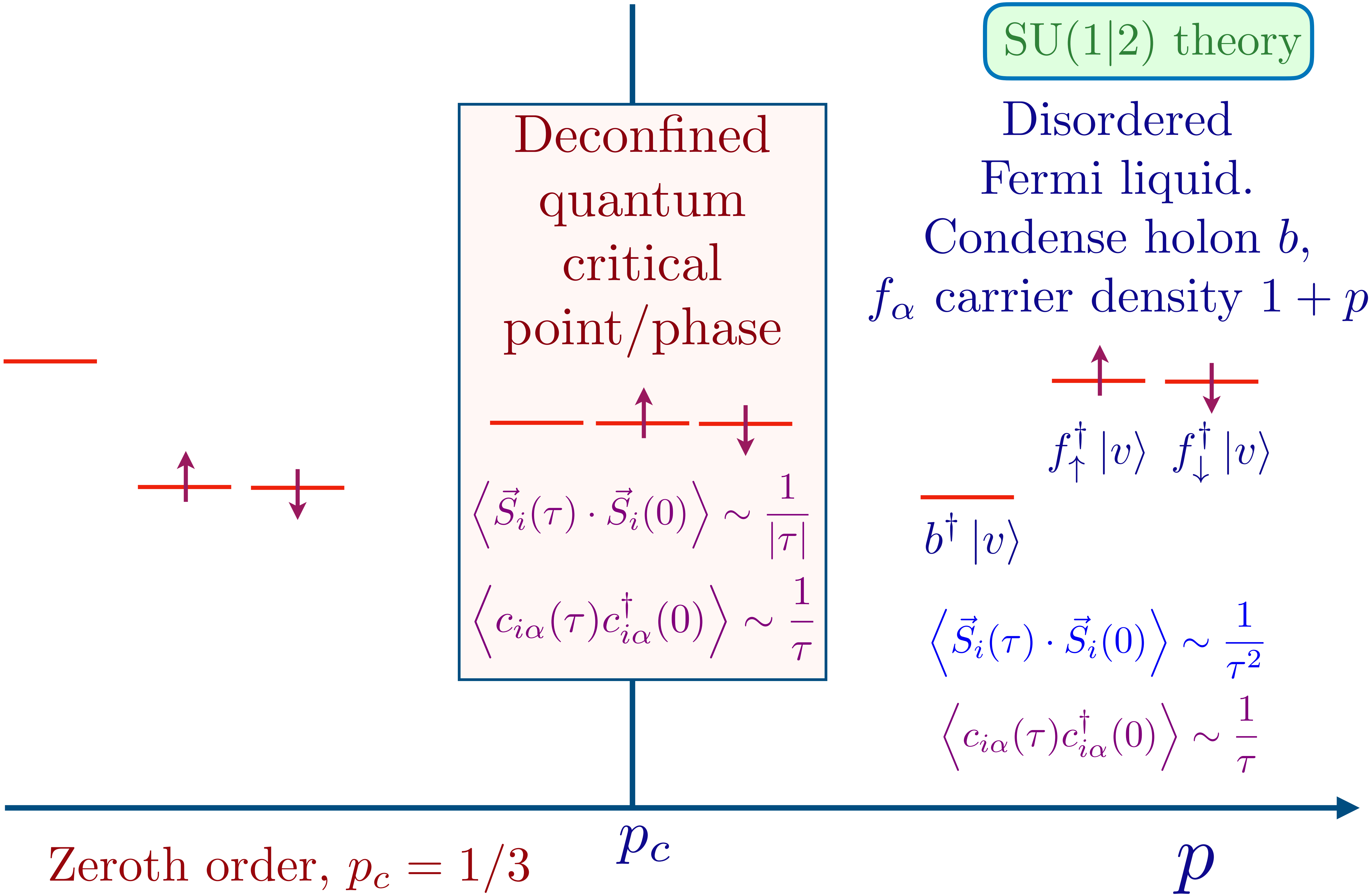
D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX to appear; arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX to appear; arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX to appear; arXiv:1912.08822

t-J model phase diagram

SU(2|1) theory

Metallic spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}^\dagger_\uparrow |v\rangle \quad \mathfrak{b}^\dagger_\downarrow |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

Deconfined quantum critical point/phase

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

SU(1|2) theory

Disordered Fermi liquid.
Condense holon b,
f_α carrier density $1+p$

$f^\dagger_\uparrow |v\rangle \quad f^\dagger_\downarrow |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c^\dagger_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

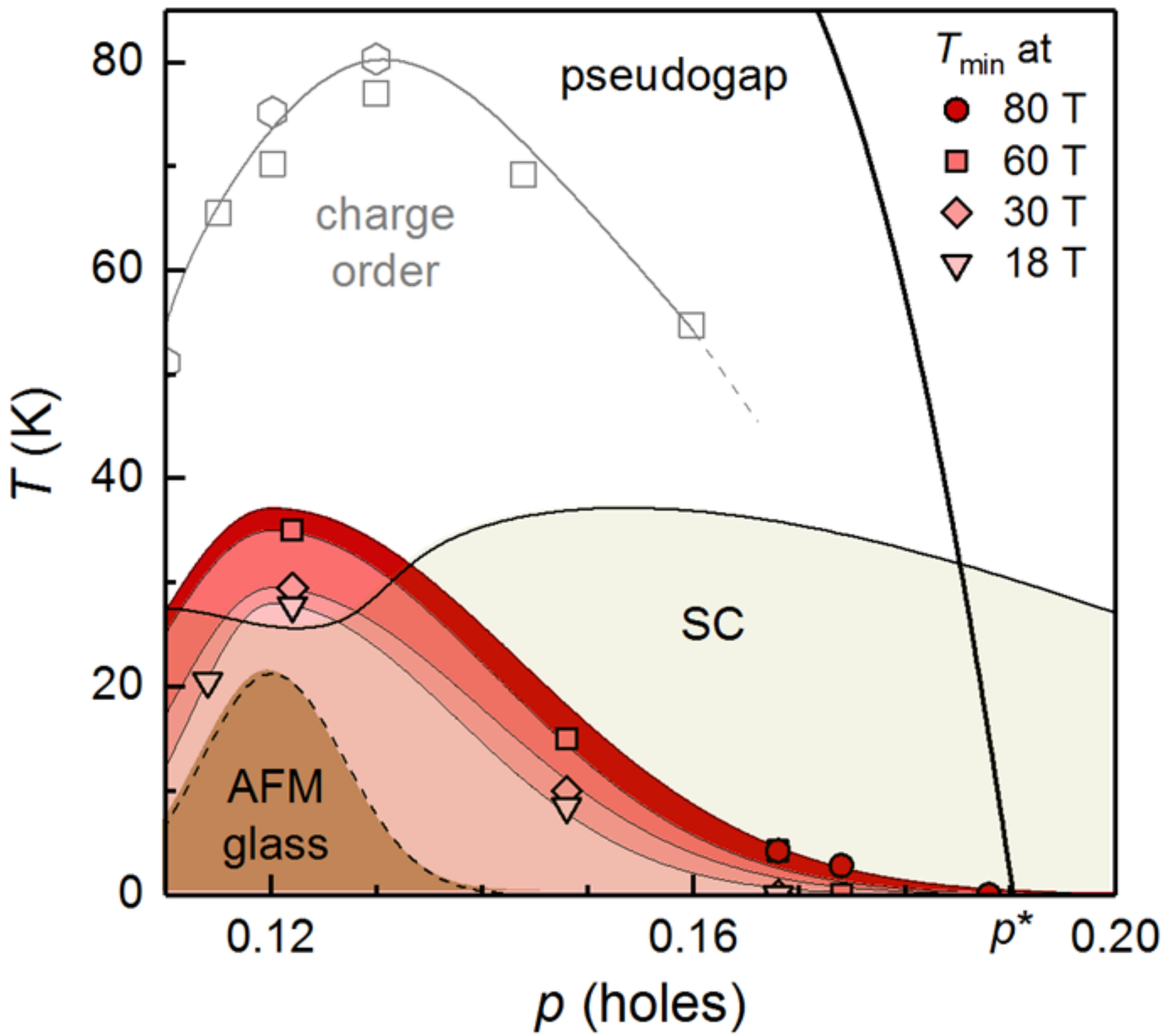
p_c $\qquad$ p

Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX to appear; arXiv:1912.08822

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Mehdi Frachet[1†], Igor Vinograd[1†], Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



arXiv:1909.10258

Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

1. SYK criticality +
   time reparameterization soft mode

2. Charged black holes

3. SYK lattice models

4. *Fractionalization* and SYK criticality
   in t-J models with random exchange

5. Linear-in-T resistivity down to zero T

At the critical point/phase of the t-J model, the Fermi liquid-like behavior of the electron Green's function

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \right\rangle \sim \frac{1}{\tau}$$

leads to a non-zero *residual resitivity*, $\rho(0) \neq 0$.

At the critical point/phase of the t-J model, the Fermi liquid-like behavior of the electron Green's function

$$\left\langle c_{i\alpha}(\tau)c_{i\alpha}^{\dagger}(0)\right\rangle \sim \frac{1}{\tau}$$

leads to a non-zero *residual resitivity*, $\rho(0) \neq 0$.

However, the critical state is *not* a Fermi liquid, as indicated by the slow decay of the spin correlations

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0)\right\rangle \sim \frac{1}{|\tau|}$$

Moreover, in a Fermi liquid, we expect $\rho(T) - \rho(0) \sim T^2$, which also does not hold here.

Time reparameterization soft mode

The leading corrections to the SL(2,R) invariant critical Green's function arise from the time reparameterization soft mode, and these take the form

$$\left\langle c_{i\alpha}(\tau)c^{\dagger}_{i\alpha}(0)\right\rangle \sim \frac{\pi T}{\sin(\pi T\tau)}\left(1+\alpha_G\,\frac{T}{J}\,\Phi_{\rm non-conformal}(T\tau)\right)$$

where $\Phi_{\rm non-conformal}(T\tau)$ is a computable (in the large M limit) scaling function, and α_G is universally proportional to the co-efficient α_S of the Schwarizan action for the time reparameterization mode.

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)
A. Kitaev and J. Suh, JHEP 183 (2018)
Haoyu Guo, Yingfei Guo, S. Sachdev , arXiv: 2004.05182

Time reparameterization soft mode

Finally, computing the resistivity from this Green's function via the Kubo formula, we find

$$\rho(T) = \rho(0)\left(1 + 8\alpha_G \frac{T}{J} + \ldots\right)$$

Haoyu Guo, Yingfei Guo, S. Sachdev , arXiv: 2004.05182

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

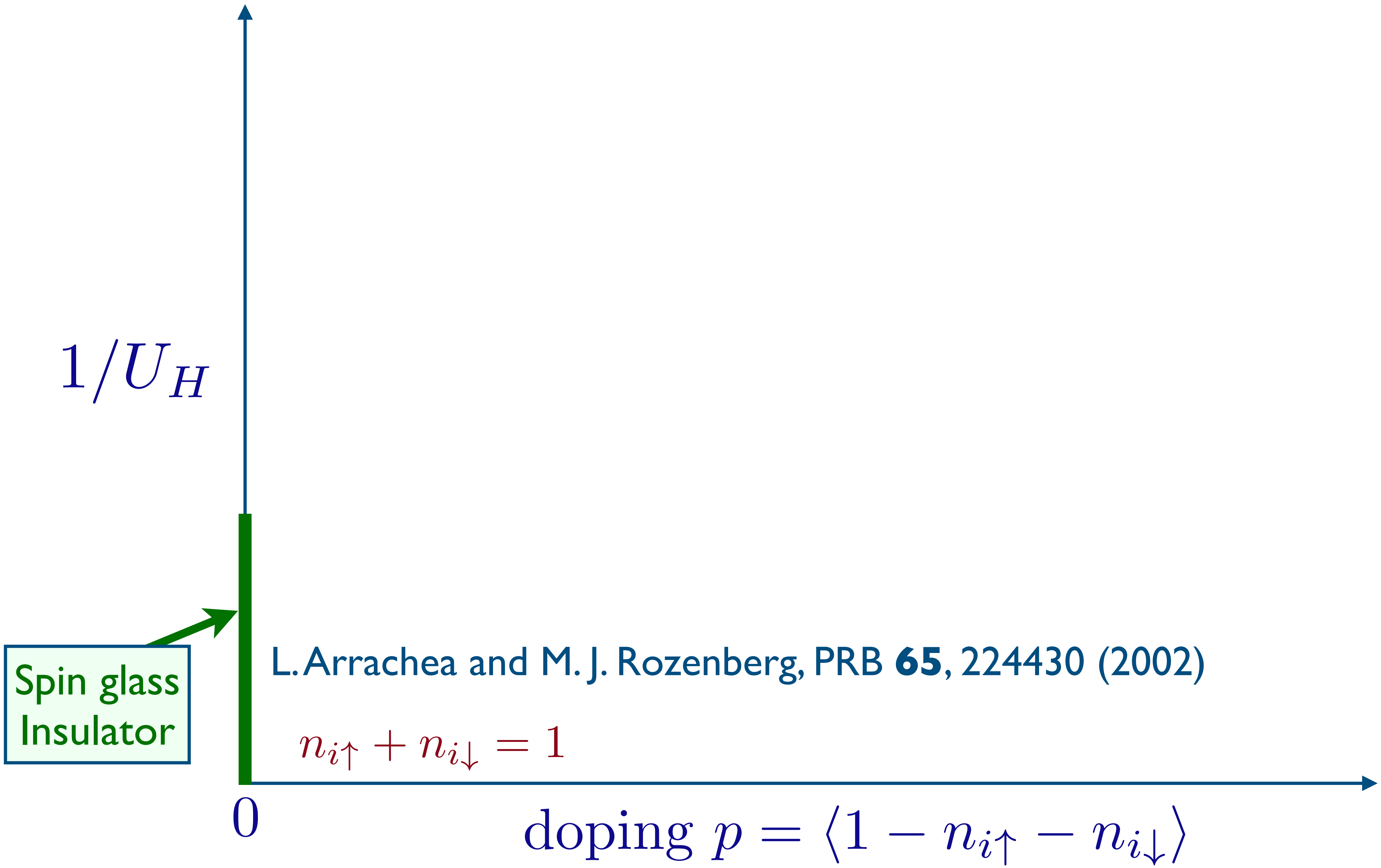
$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha},$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t_{ij}^2} = t^2$$

$$U_H > 0 \text{ non-random}$$

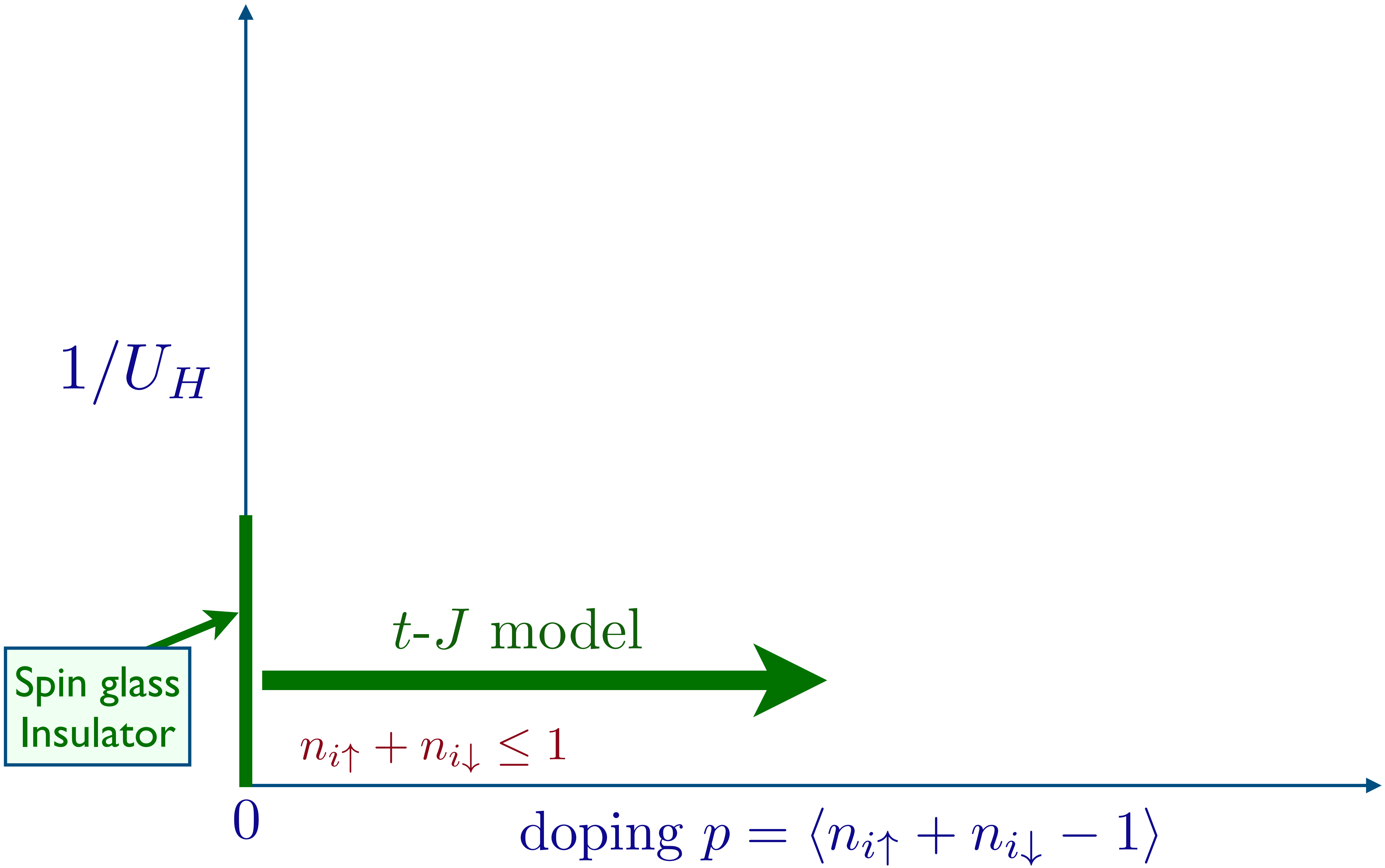
Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



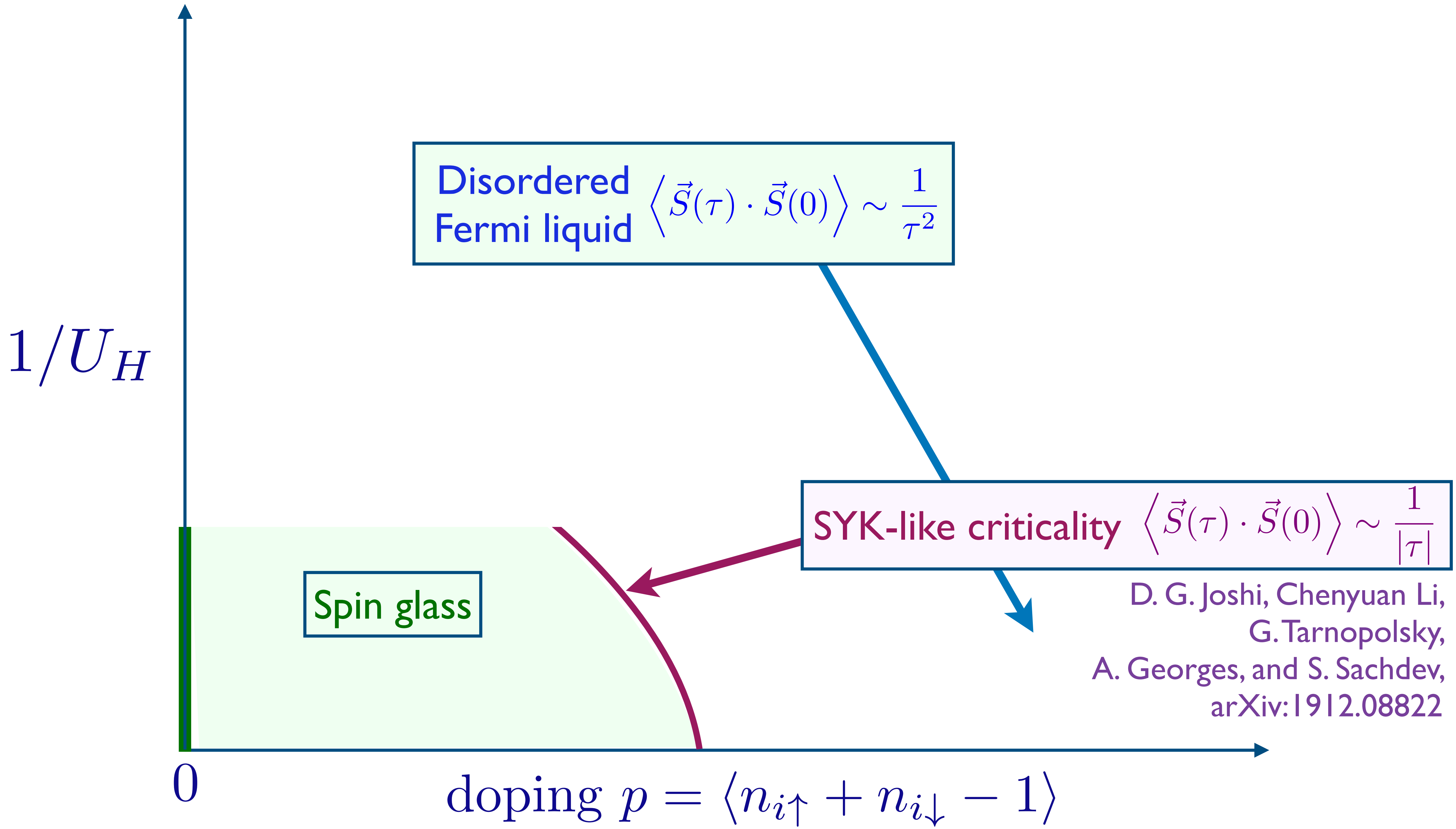
Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



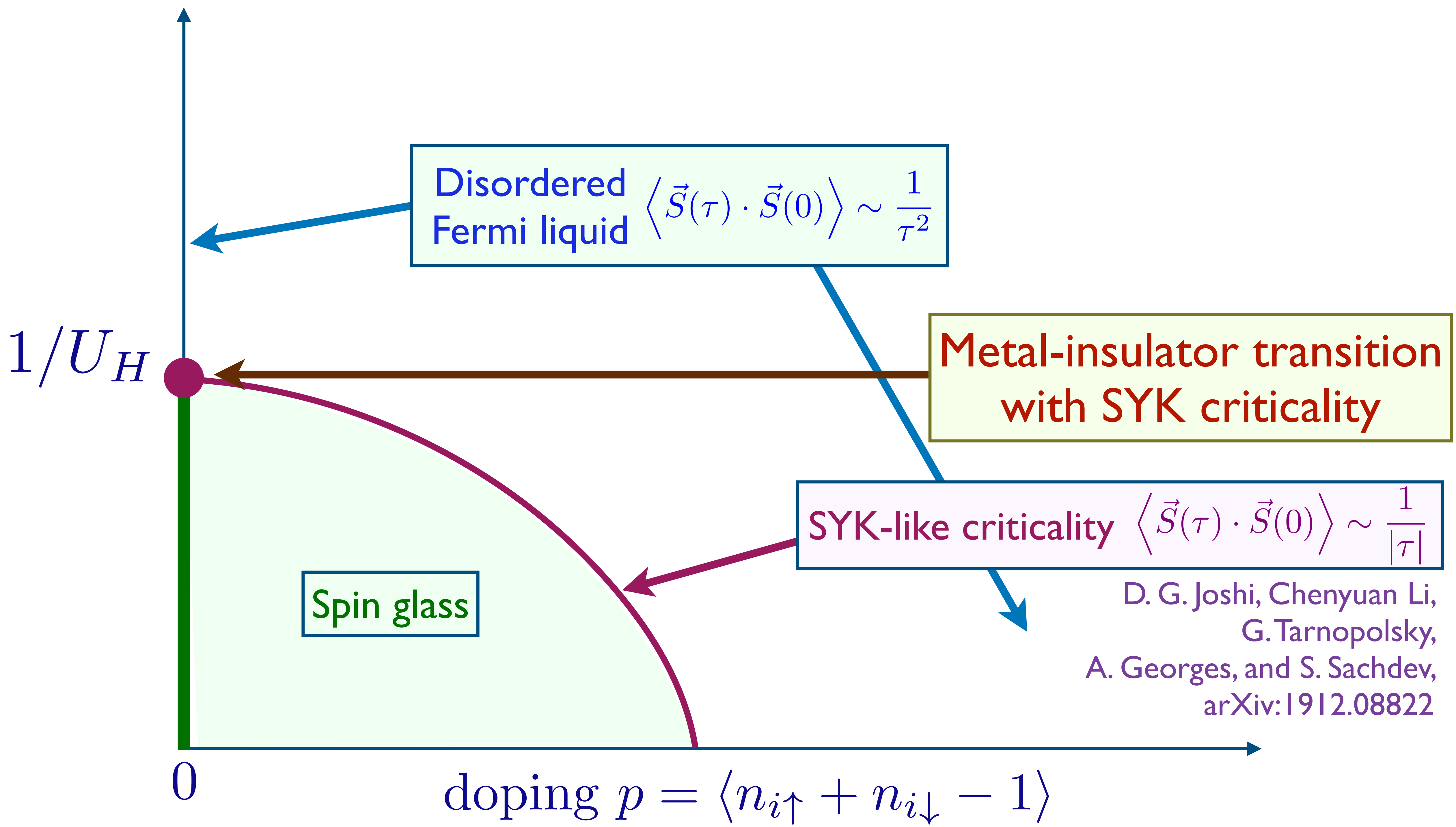
Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



Random t-J-U_H model

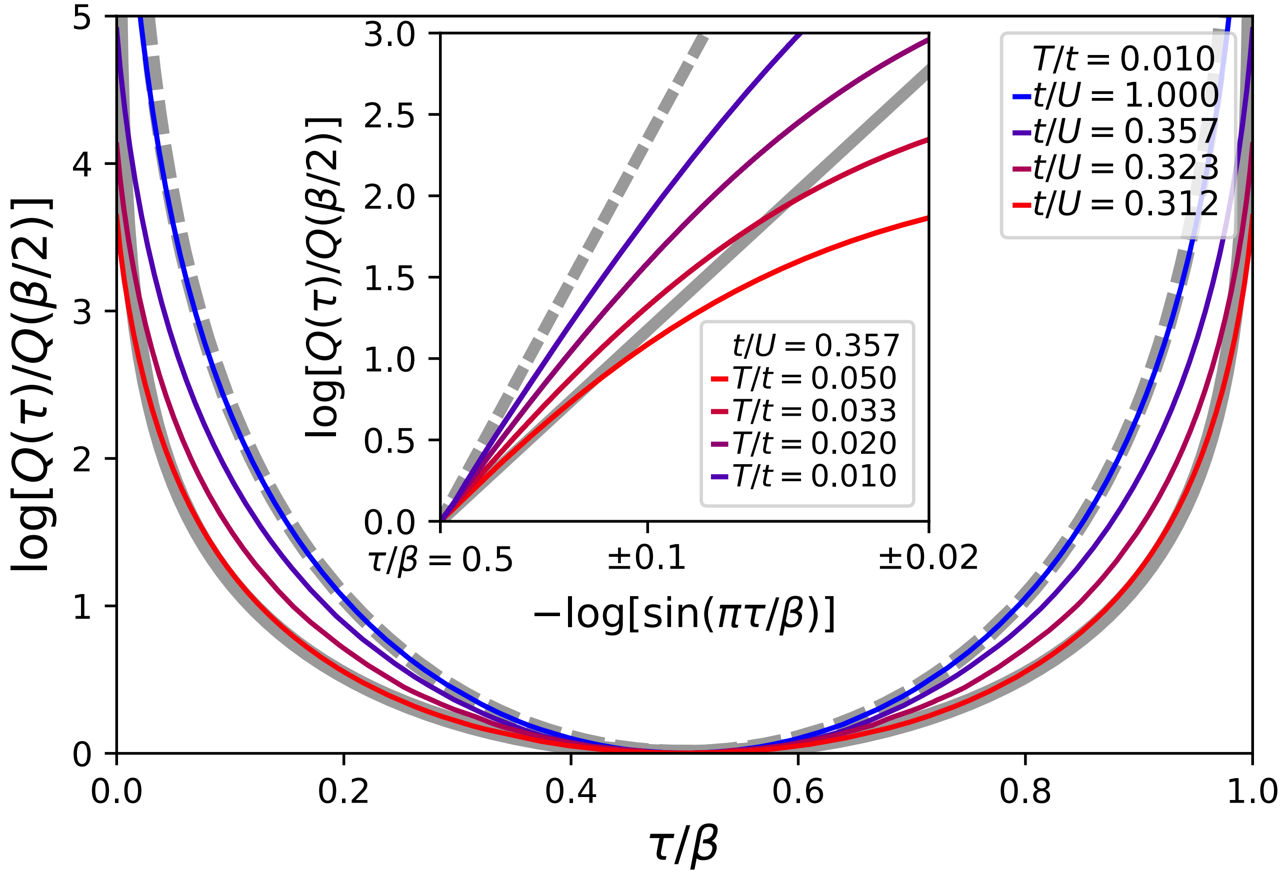
$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

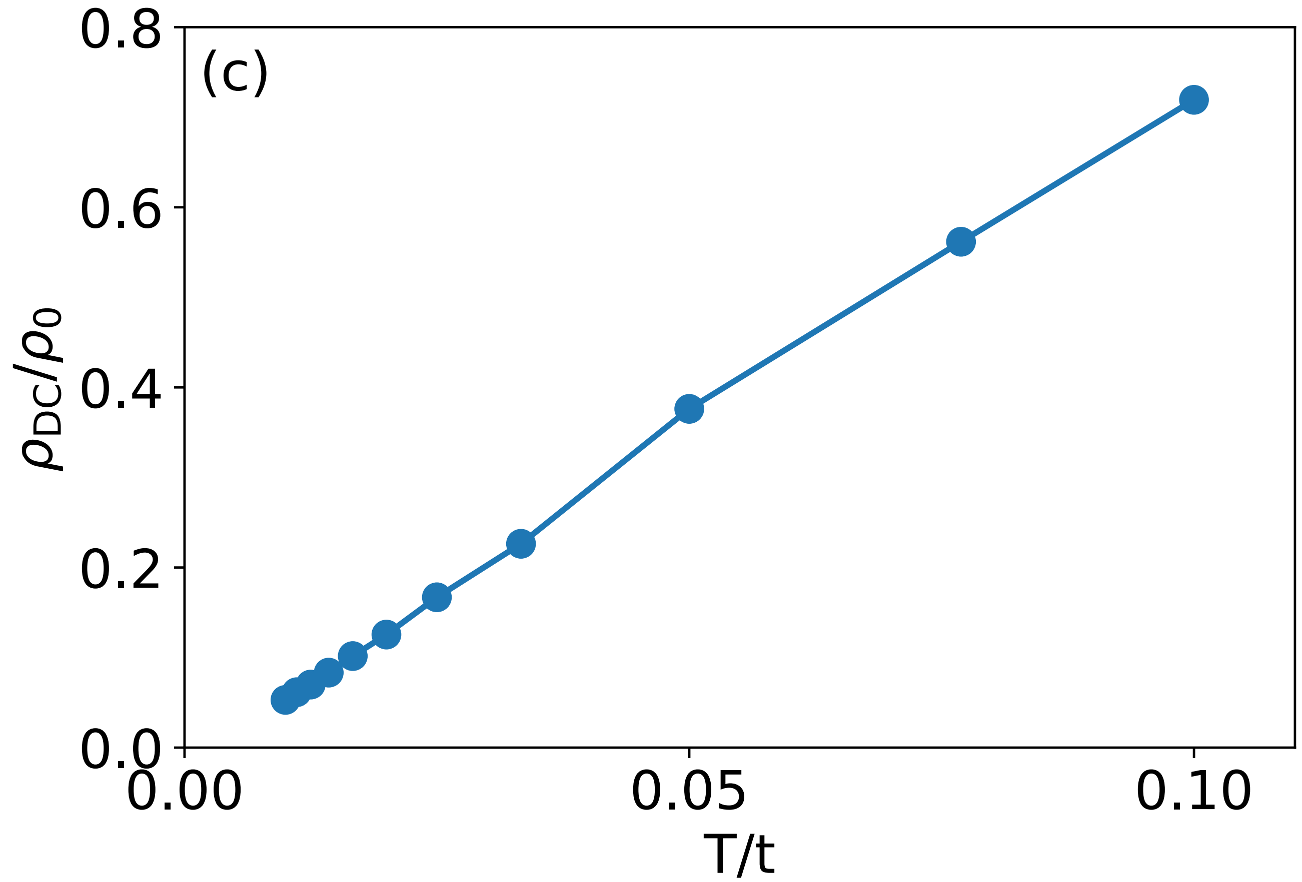


D. G. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, and S. Sachdev, arXiv:1912.08822

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-$1/2$ fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim



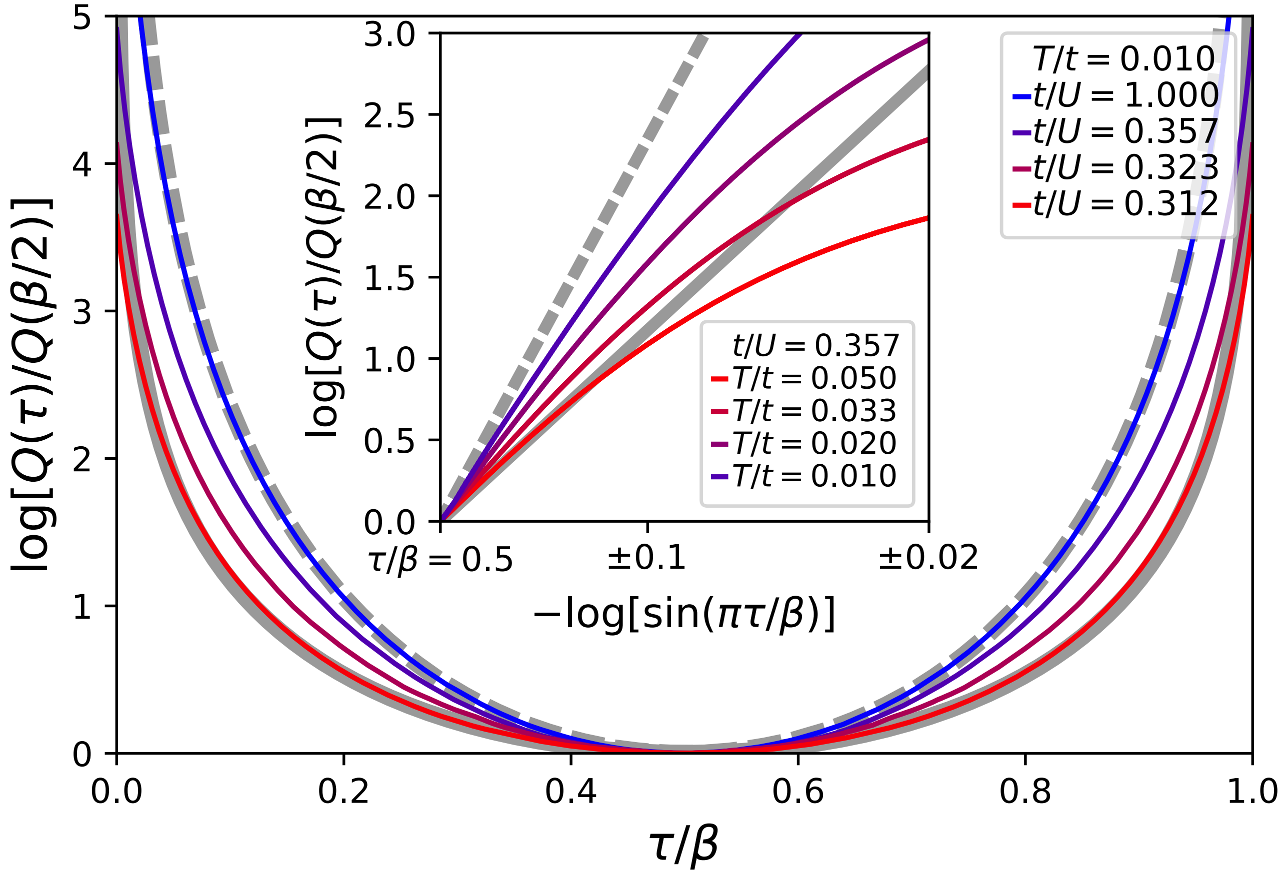


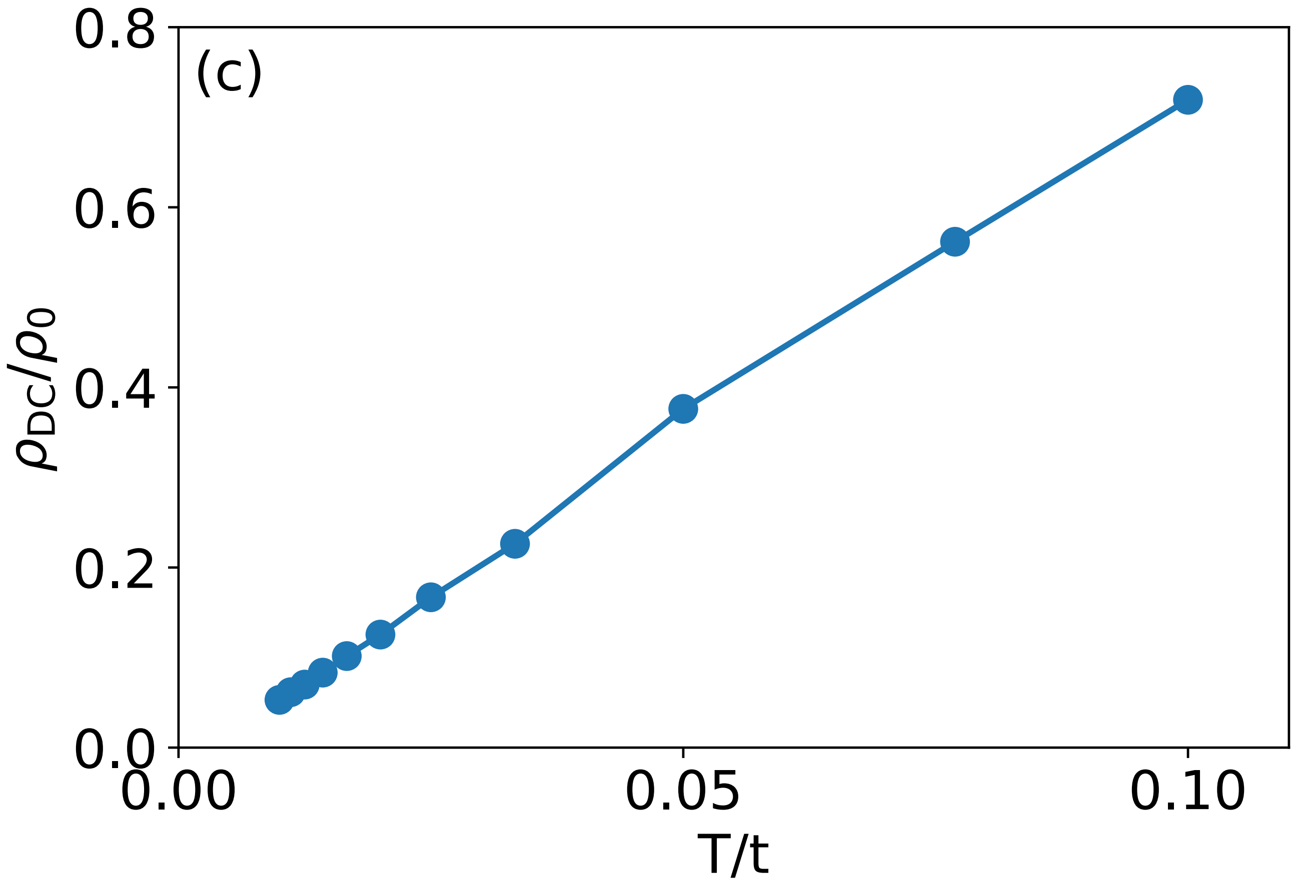
arXiv.org > cond-mat > arXiv:2002.07181

Condensed Matter > Strongly Correlated Electrons

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-$1/2$ fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim





Critical spin correlations:

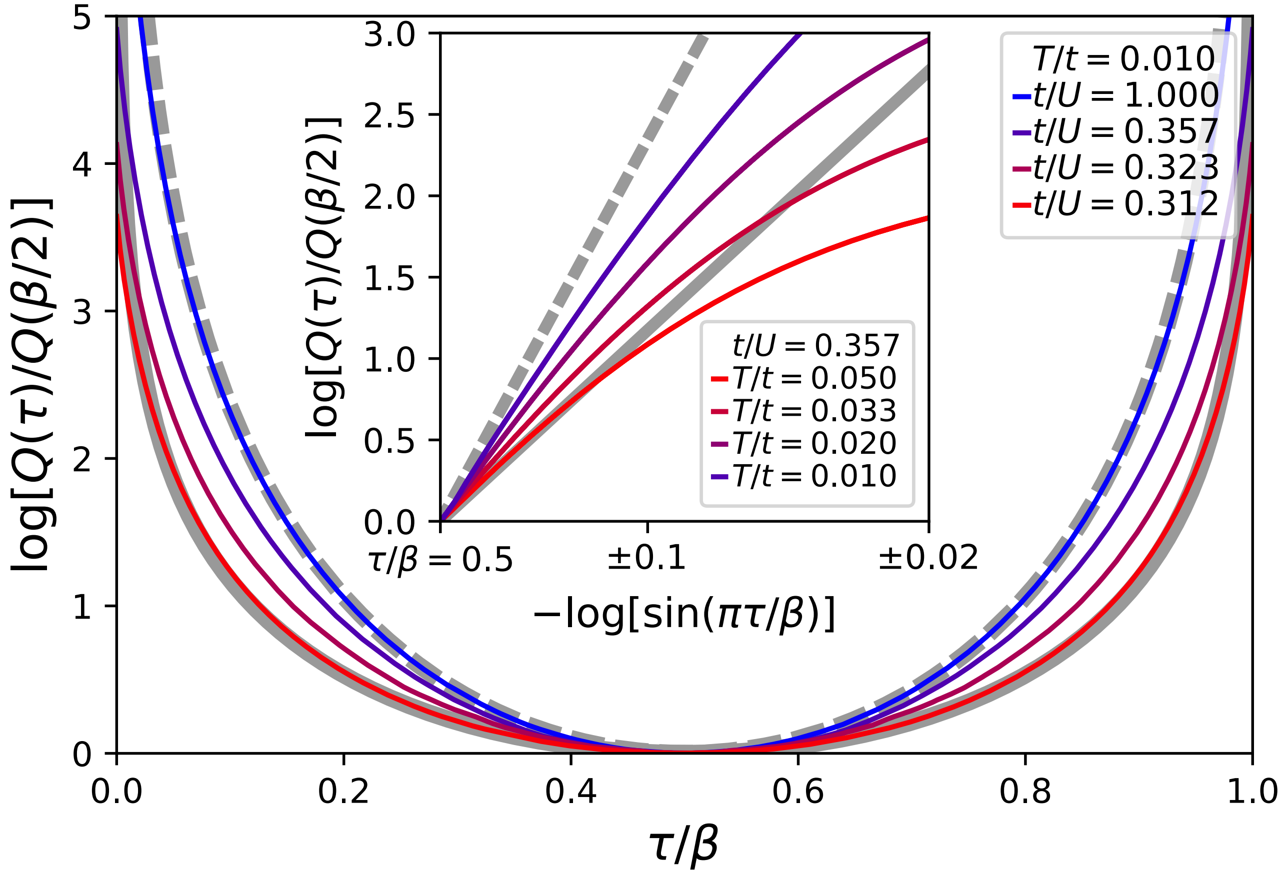
$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

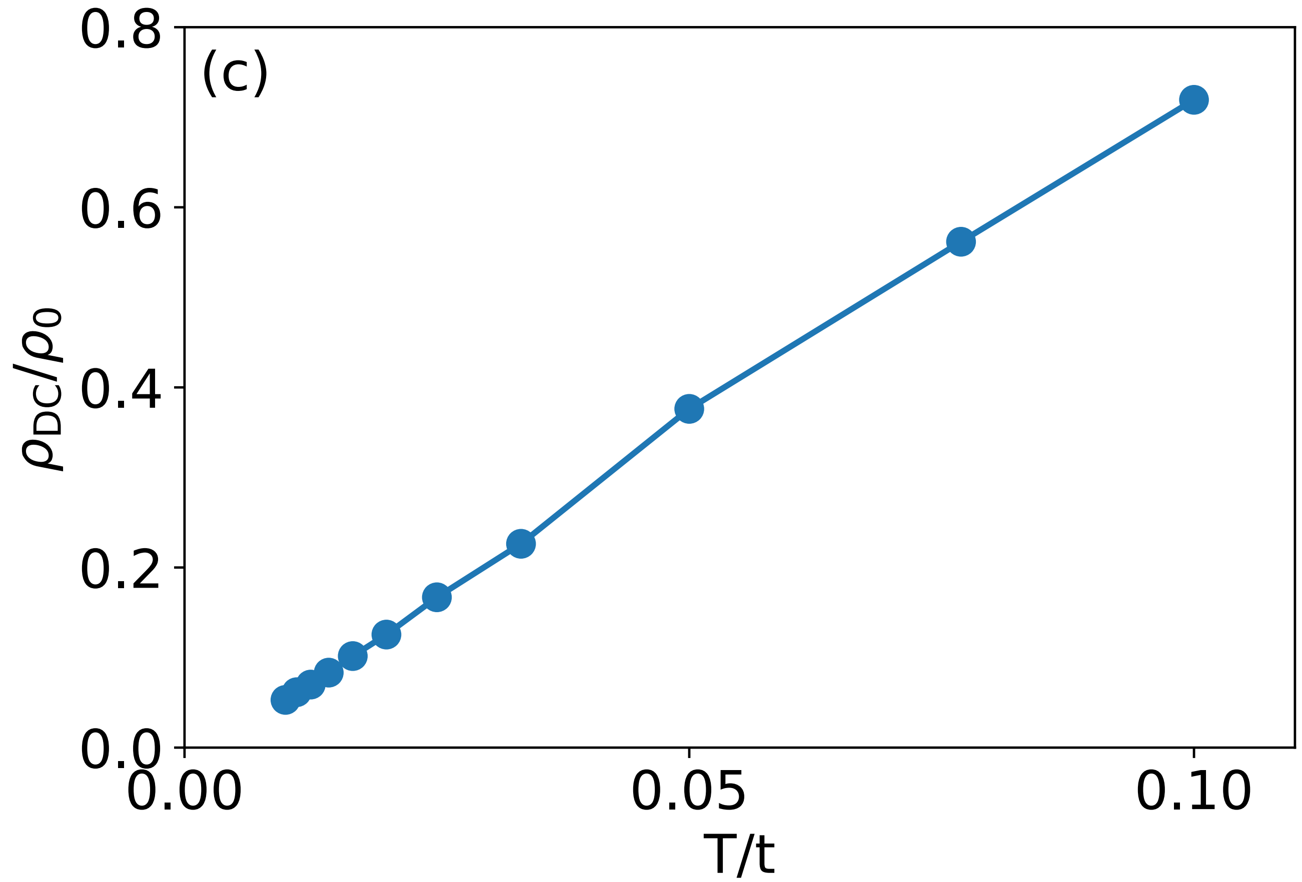
arXiv.org > cond-mat > arXiv:2002.07181

Condensed Matter > Strongly Correlated Electrons

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-$1/2$ fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim





Critical spin correlations: $\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}$

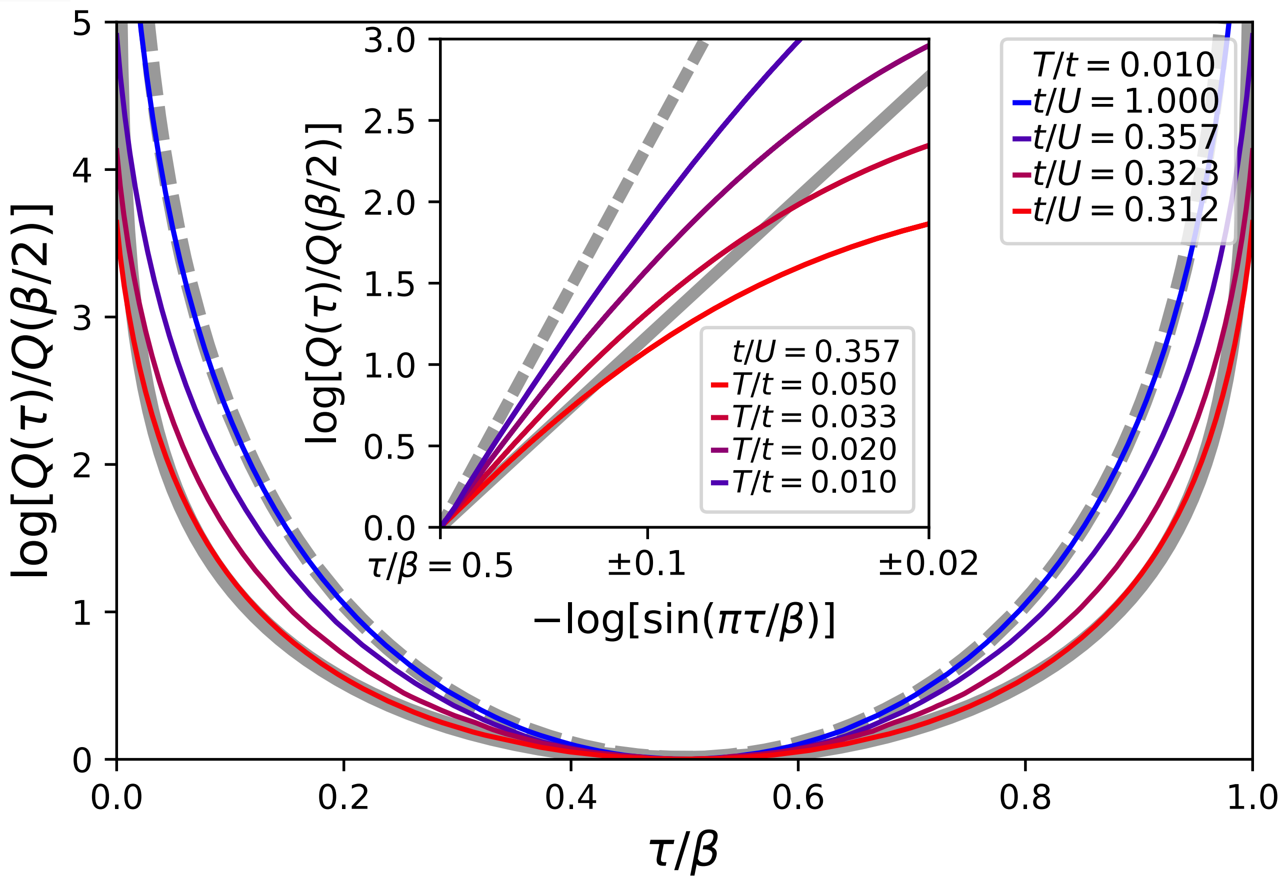
Resisitivity $\rho \sim T$ to the lowest T at the critical point in a large-dimension model

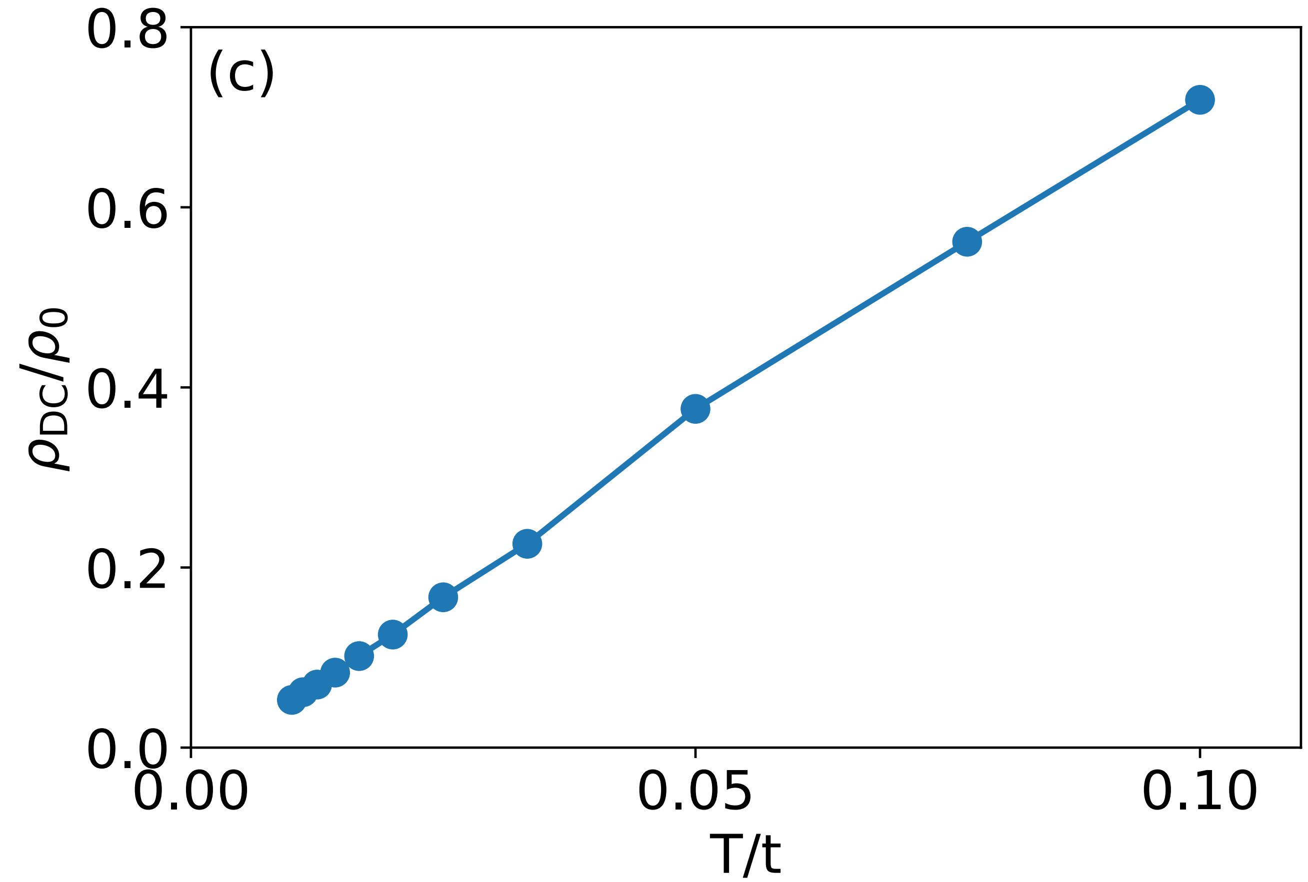
arXiv.org > cond-mat > arXiv:2002.07181

Condensed Matter > Strongly Correlated Electrons

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-$1/2$ fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim





Critical spin correlations:

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

Resisitivity $\rho \sim T$ to the lowest T at the critical point in a large-dimension model

Mapping to SYK criticality in a large M theory (with SU(M) spin symmetry)
G. Tarnopolsky, Chenyuan Li, D.G. Joshi, and S. Sachdev, PRB **101**, 205106 (2020)

1. SYK criticality +
   time reparameterization soft mode

2. Charged black holes

3. SYK lattice models

4. *Fractionalization* and SYK criticality
   in t-J models with random exchange

5. Linear-in-T resistivity down to zero T

Challenge for theory:

A model in which

$$\lim_{T \to 0} \frac{d\rho}{dT} \neq 0$$