Statistical mechanics of strange metals and black holes

Purdue University
April 28, 2022

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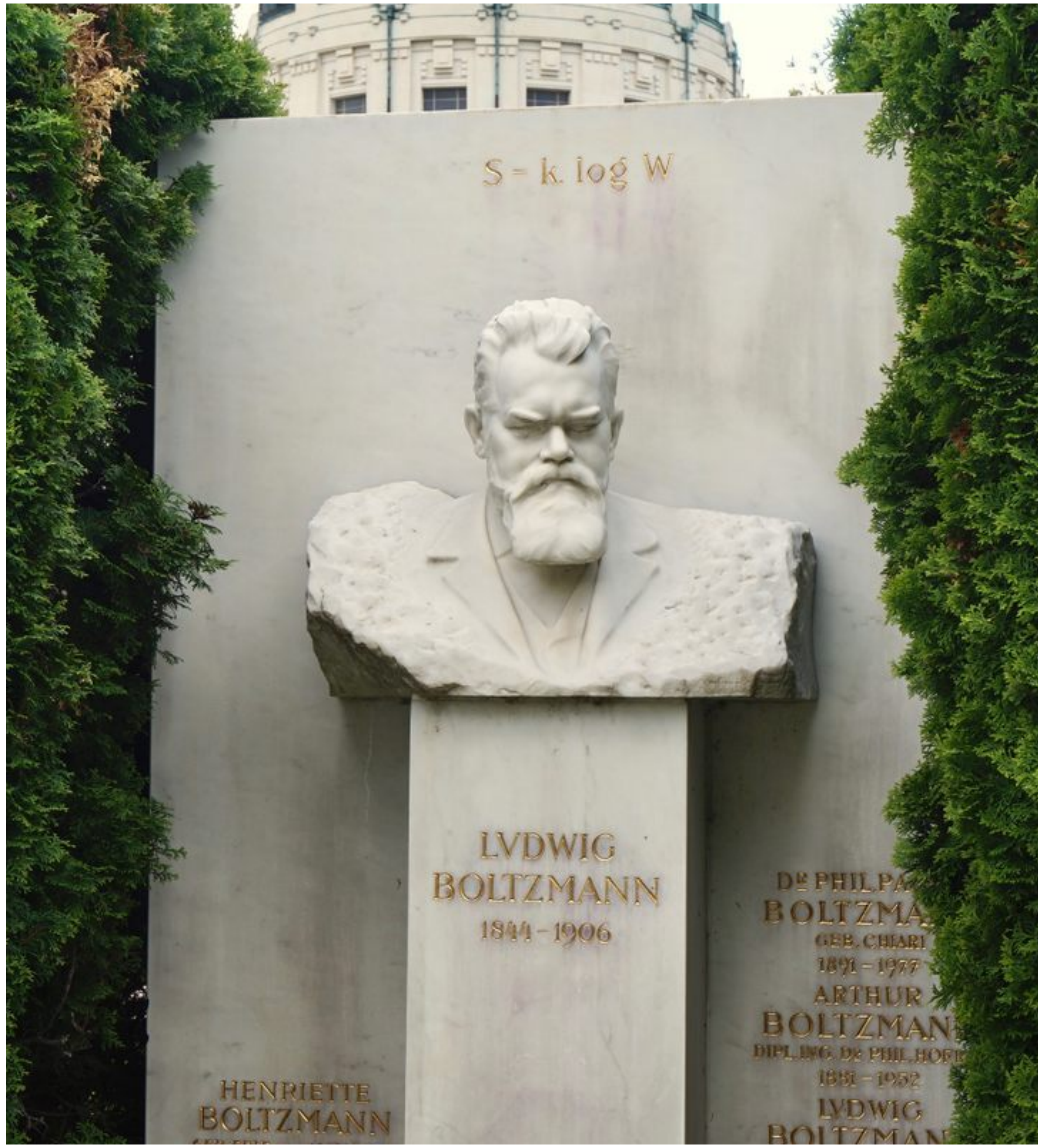
Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

PHYSICS HARVARD

Foundations by Boltzmann

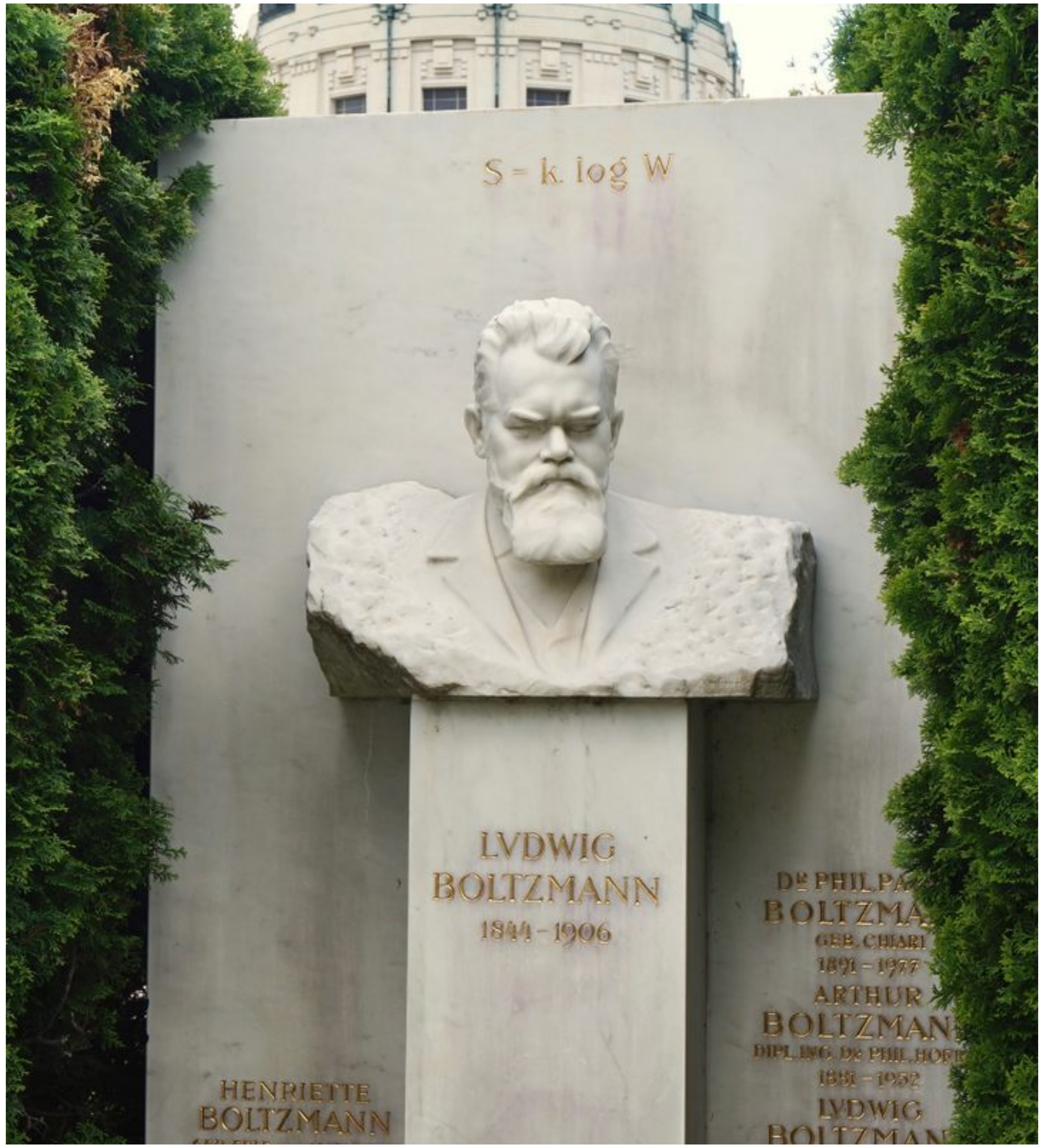


Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$



Ludwig Boltzmann
20 February 1844 - September 5, 1906
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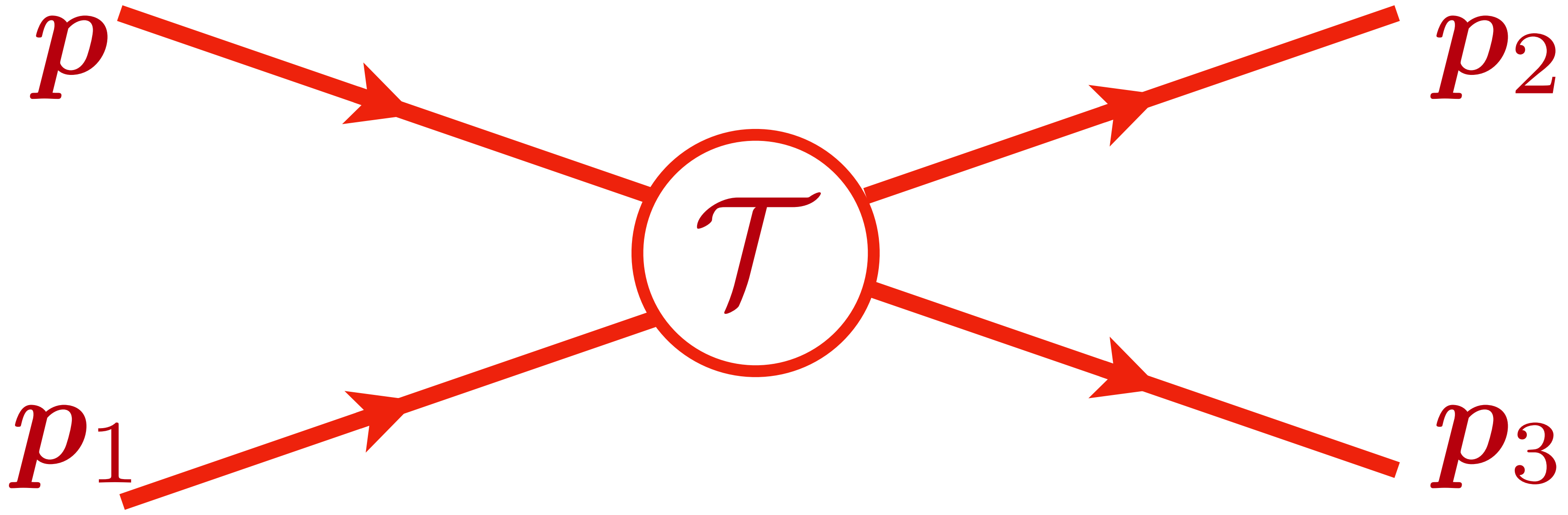
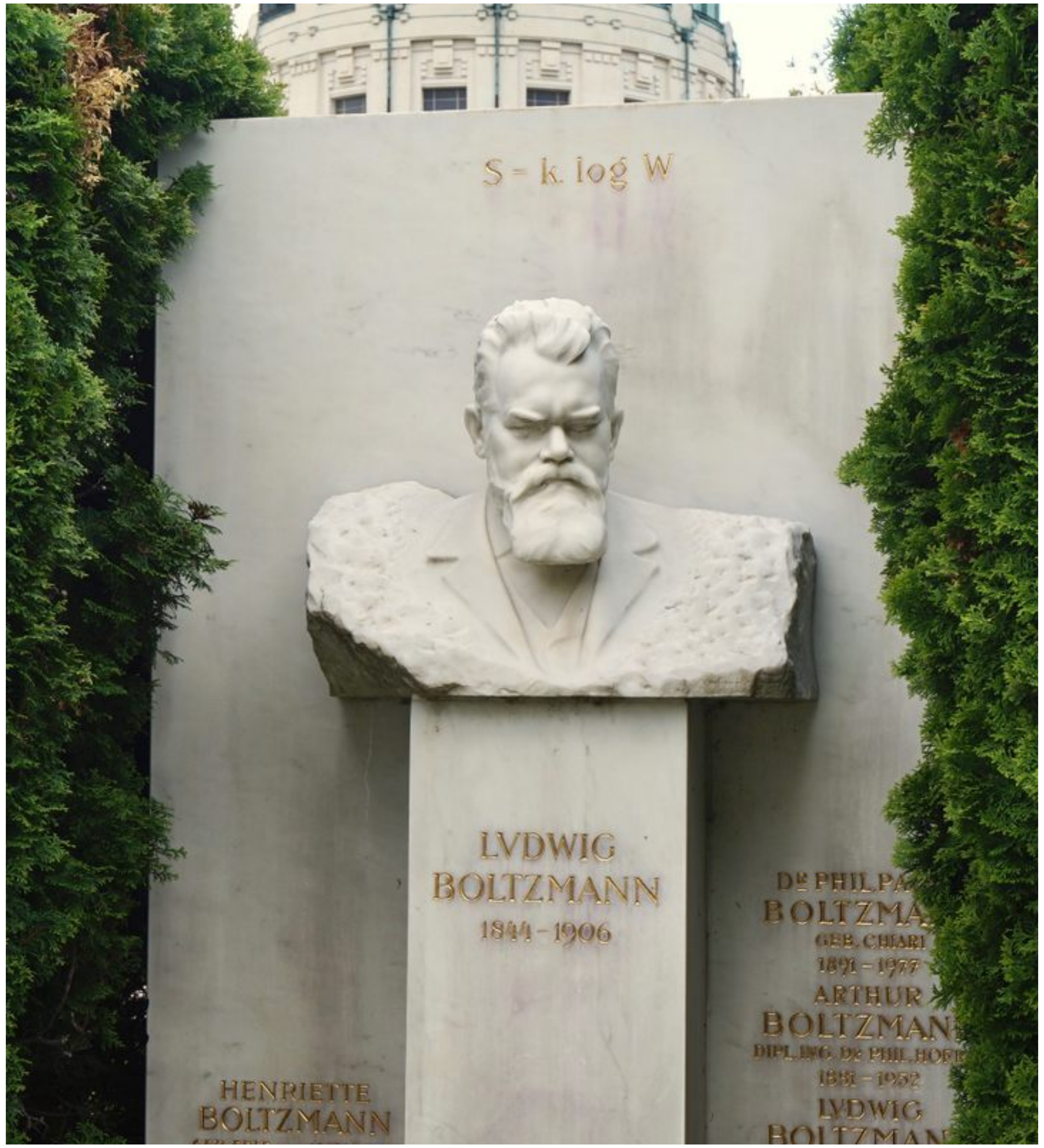
Boltzmann equation (1872)
Dilute classical gas

Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} = \mathcal{C}[f]$$

$$\mathcal{C}[f] \propto \int_{\boldsymbol{p}_{1,2,3}} \cdots [f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3}]$$





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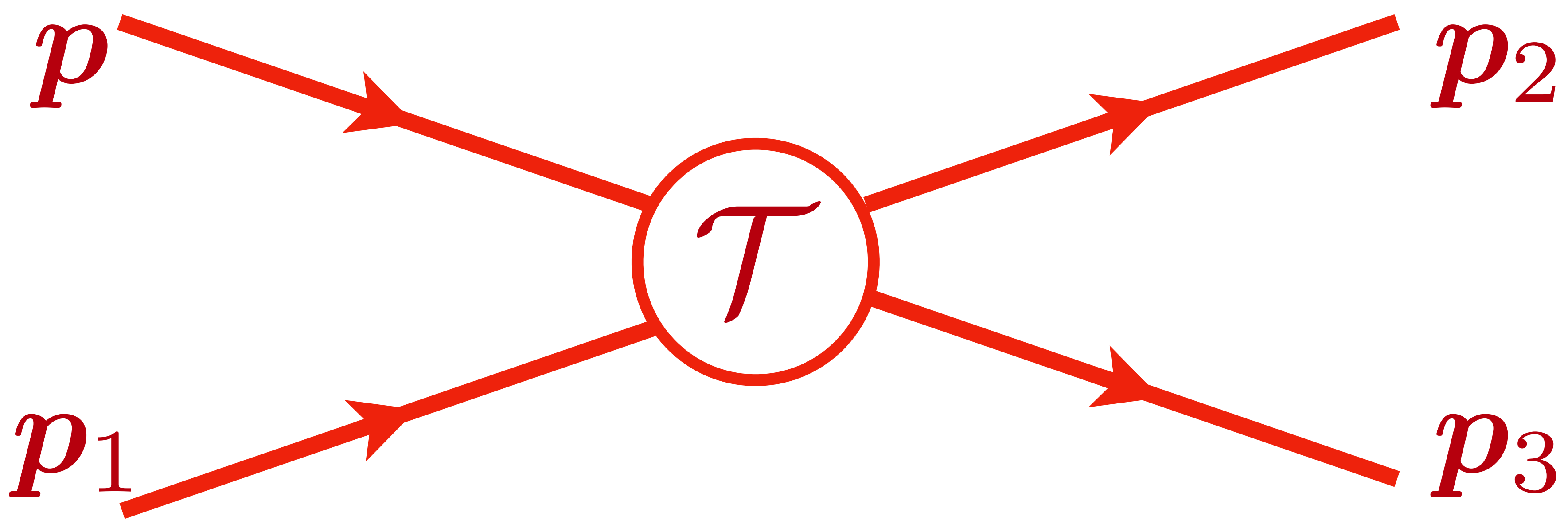
Quantum Boltzmann equation (Landau)
Dense gas of electrons

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} = \mathcal{C}[f]$$

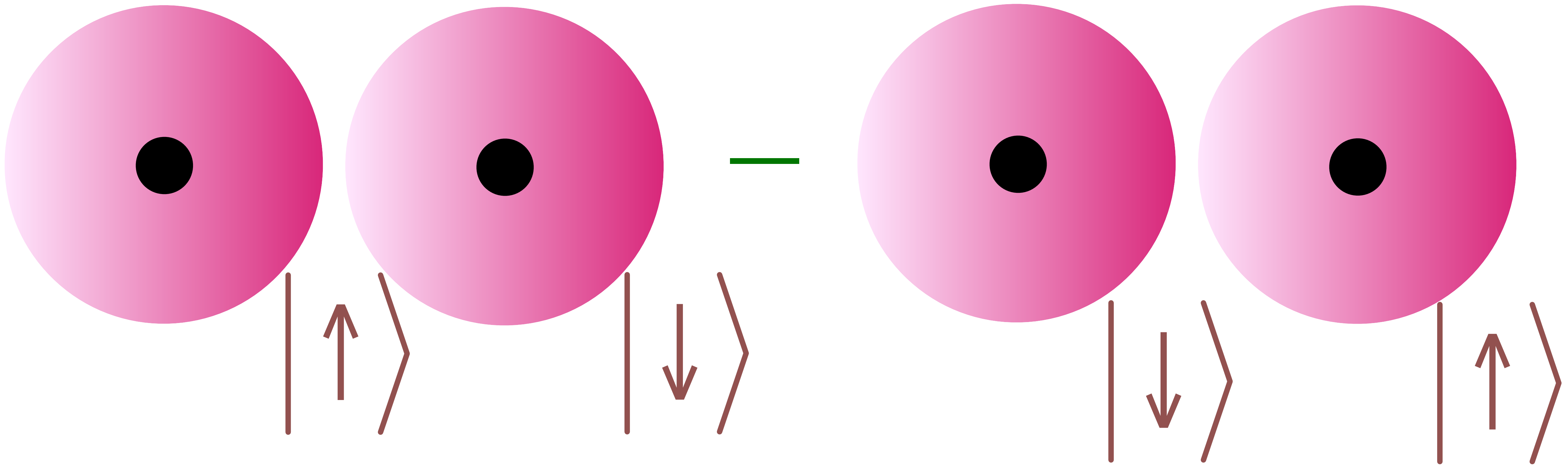
$$\mathcal{C}[f] \propto \int_{\boldsymbol{p}_{1,2,3}} \cdots [f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1})]$$



Quantum Entanglement

Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



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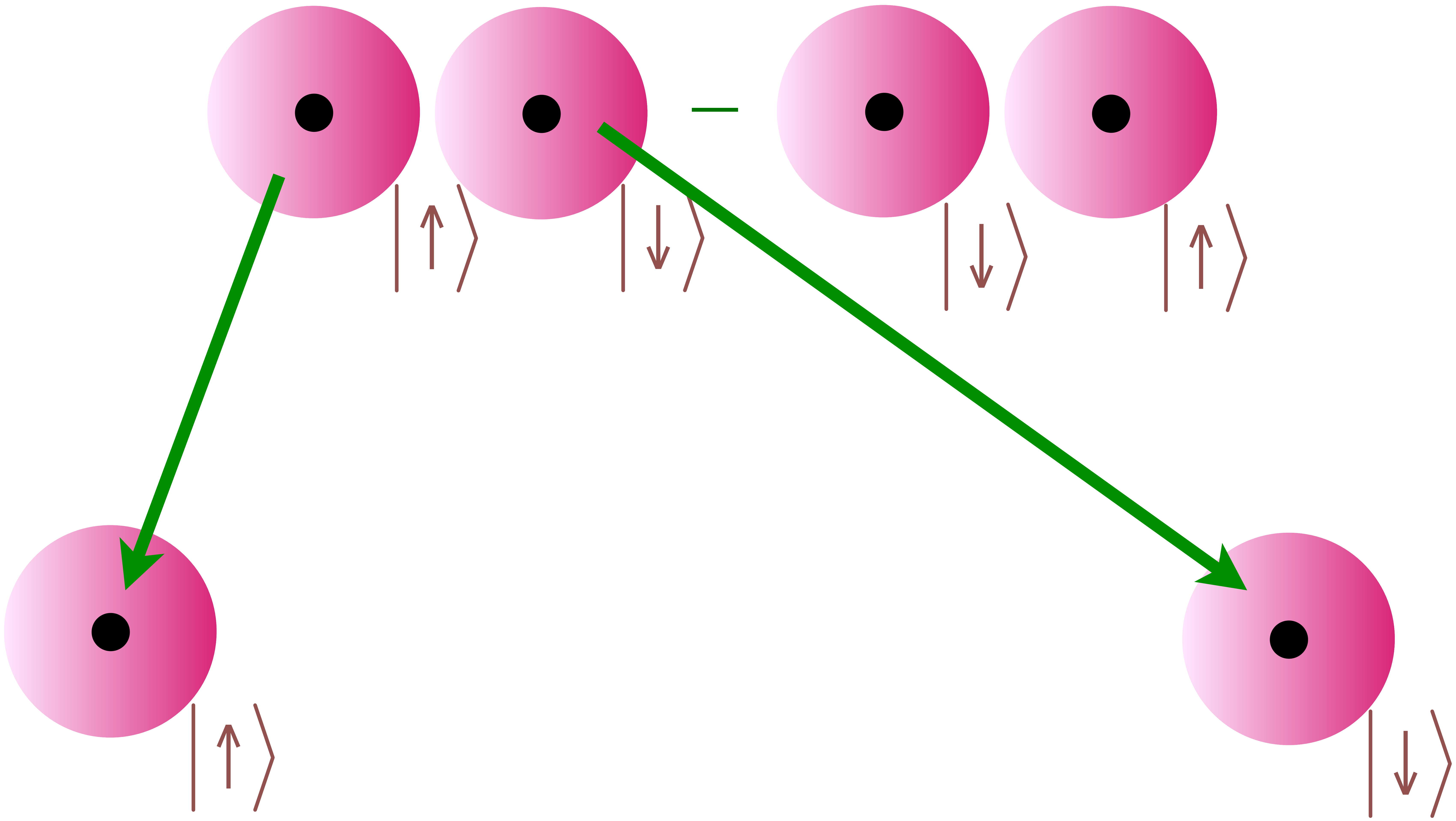

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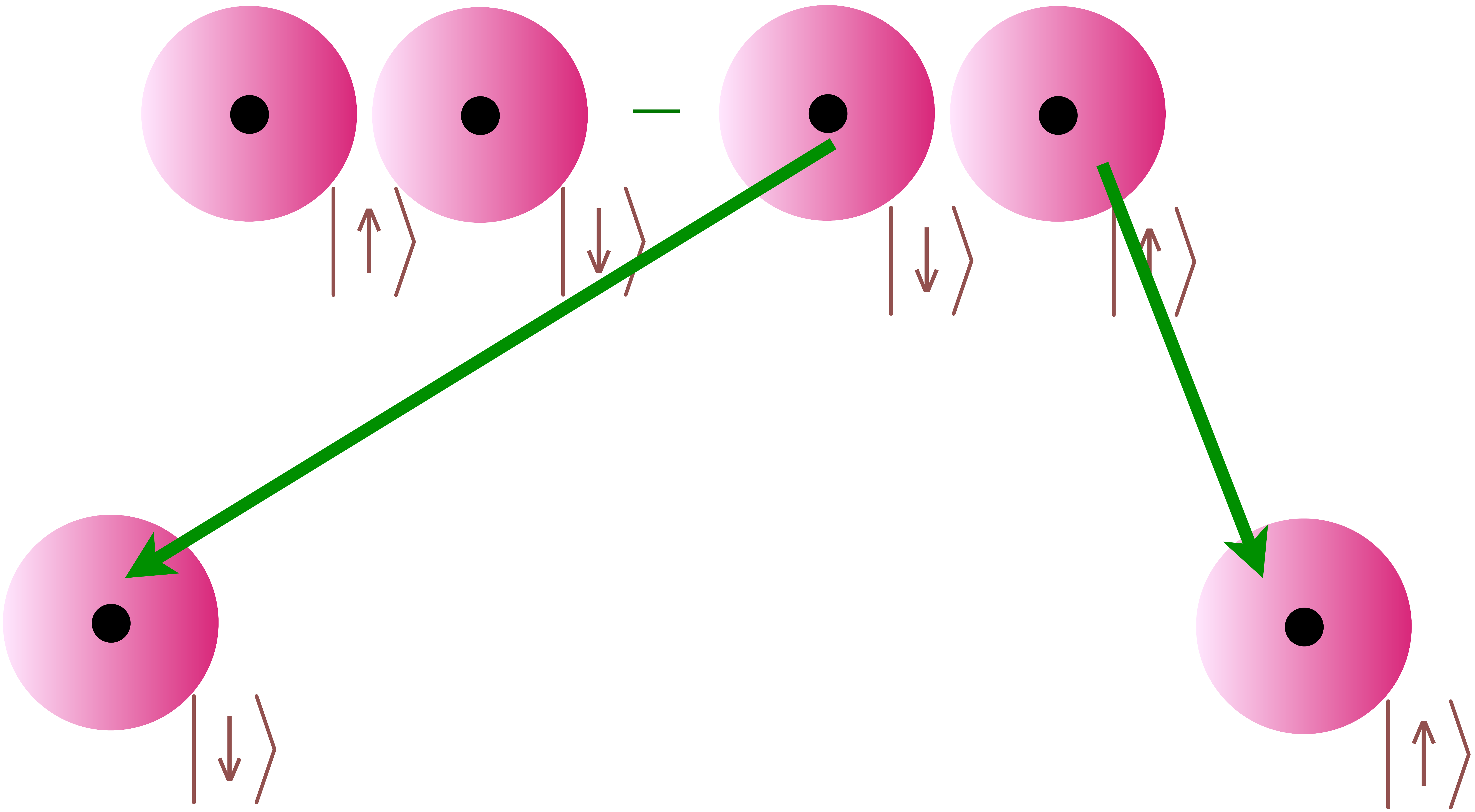

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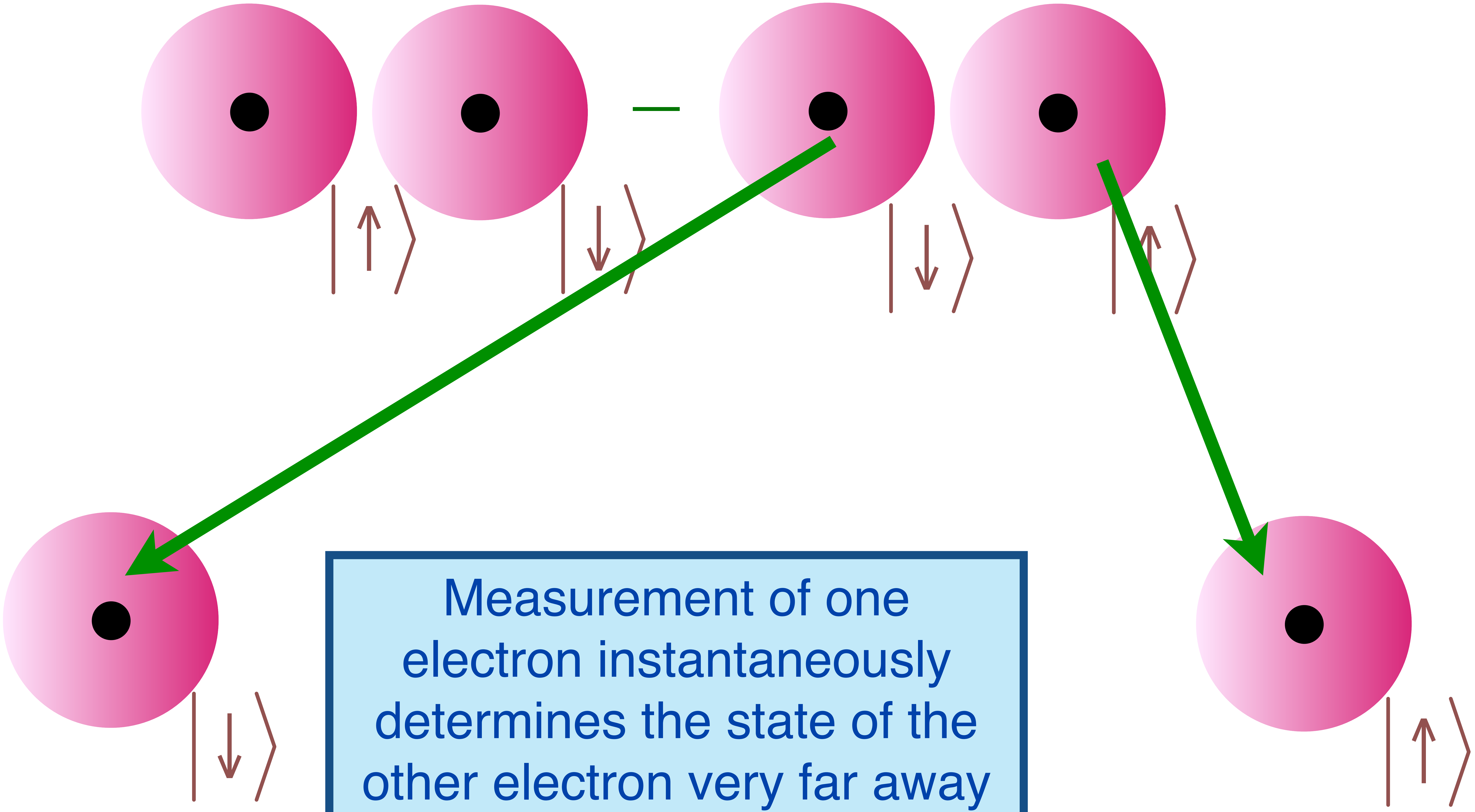

Measurement of one electron instantaneously determines the state of the other electron very far away

Quantum Entanglement

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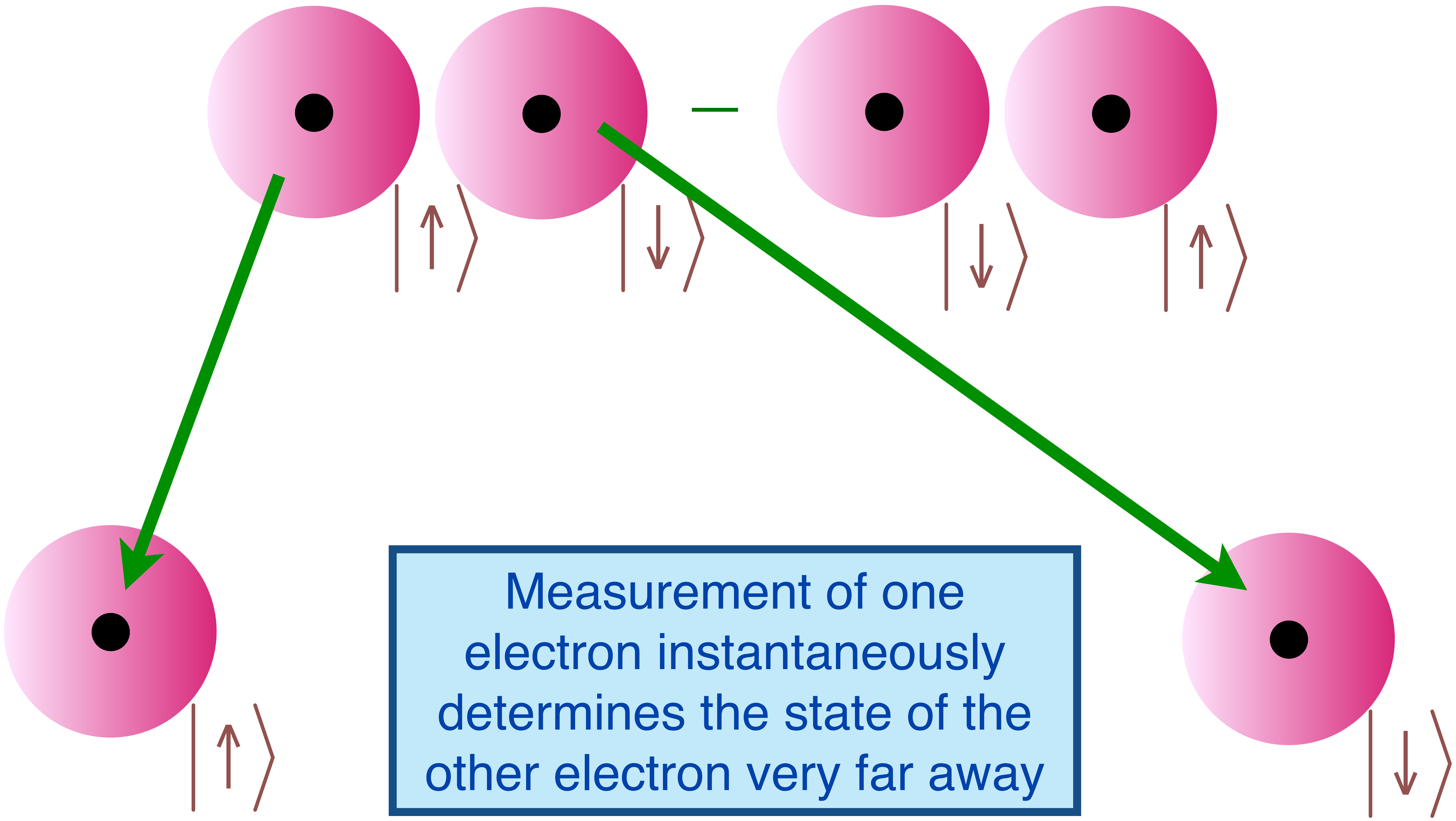

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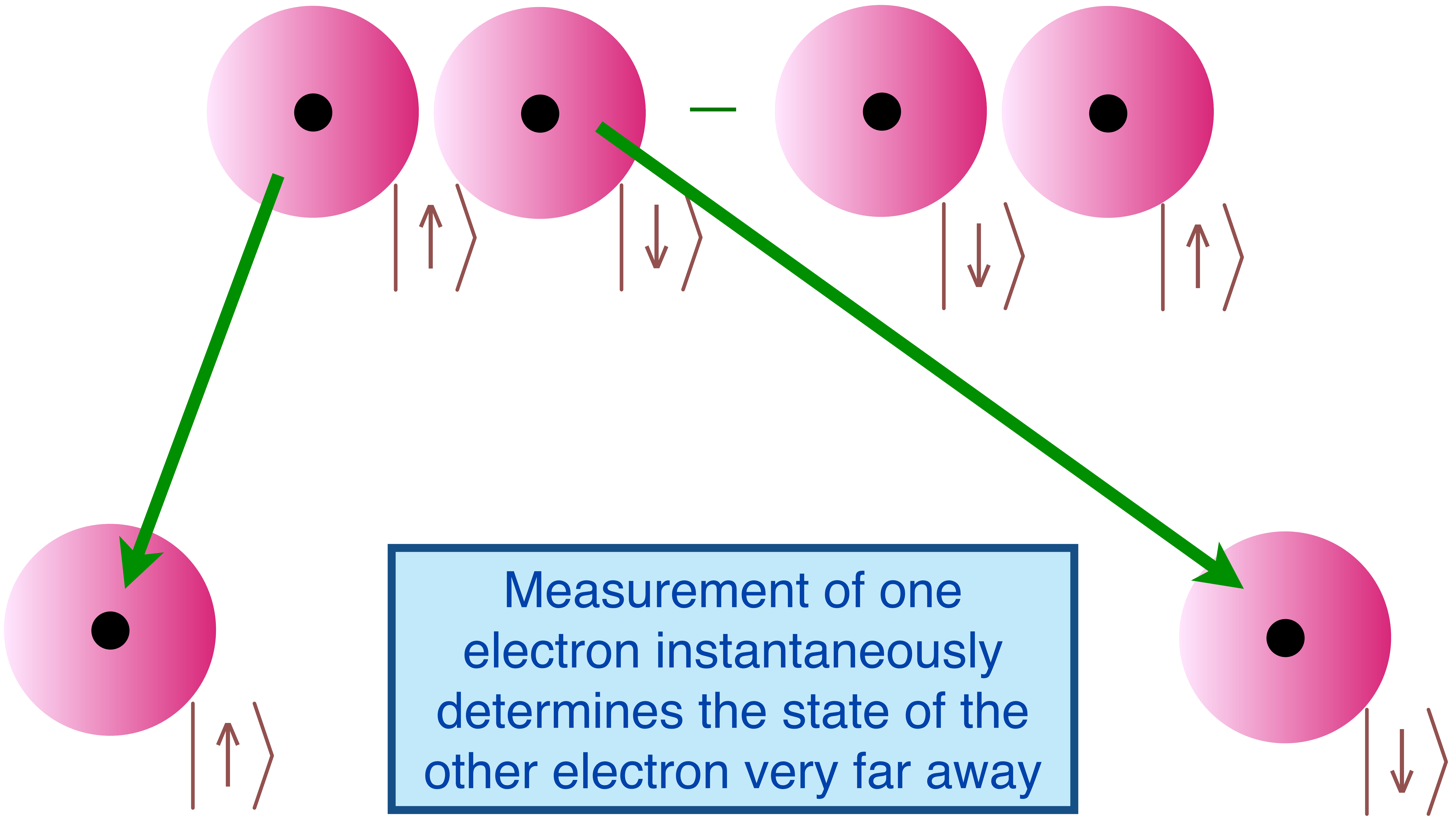

Measurement of one electron instantaneously determines the state of the other electron very far away

Spooky action at a distance !

Diskussionen; denn Hedi, als Quäkerin, legt Deine Aussprüche oft anders aus, als ich alter Heide (das heißt, ich bin das eigentlich garnicht, sondern recht fromm; nur im Vergleich zu Hedi).

Grüß unsere Freunde in Princeton, Neumann, Ladenburg, Weyl und den bissigen Pauli. In alter Freundschaft

Dein Max Born

Unter den experimentellen Arbeiten meines Mitarbeiters Fürth war nur eine, deren Idee von mir ausgegangen ist, der photoelektrische Fouriertransformer. Er ist später von dem Edinburgher Zweig der Firma Ferranti weiter entwickelt worden, hat sich aber nicht in die Praxis eingeführt.

[84]

3. 3. 47

Lieber Born!

Wenn ich nicht ein ausgepichter alter Gauner wäre mit einem versteinerten schlechten Gewissen, dann hätt' ich es nicht so lange ausgehalten, ohne Dir zu schreiben. Darum, erstens hat Deiner Frau Gedicht über das indische Lebensideal tiefen Eindruck auf mich gemacht, so daß es mich nicht gewundert hätte, wenn es von dem alten Goethe geschrieben wäre; zweitens war ich sehr beeindruckt von dem Beitrag, den Du dem sonderbaren Schulmeister Schilpp für den mir gewidmeten Band gegeben hast. Es ist so viel Wärme darin und ein so deutlicher Beweis dafür, für wie sonderbar und versteinert Du meine Haltung der statistischen Quantentheorie gegenüber ansiehst. Endlich gefiel mir Deine Sorge für den Transport Deines chinesischen Schützlings besonders gut, der Dir – glücklicherweise ohne mein Dazwischentreten – glücklich und schweigsam entglitten ist. Ich hatte mit Weyl über den Fall beraten, und wir waren darin überein gekommen, daß wir das Problem nicht in der von Dir vorgeschlagenen Weise zu lösen imstande gewesen wären, und daß ich mich an den englischen Botschafter wenden

solle, der die Sache schandenhalber zu einem befriedigenden Abschluß bringen würde. Glücklicherweise verbummelte ich diesen Schritt einige Tage, worauf Dein erlösender Brief ankam.

Meine physikalische Haltung kann ich Dir nicht so begründen, daß Du sie irgendwie vernünftig finden würdest. Ich sehe natürlich ein, daß die prinzipiell statistische Behandlungsweise, deren Notwendigkeit im Rahmen des bestehenden Formalismus ja zuerst von Dir klar erkannt wurde, einen bedeutenden Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft daran glauben, weil die Theorie mit dem Grundsatz unvereinbar ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstellen soll, ohne spukhafte Fernwirkungen. Allerdings bin ich nicht fest davon überzeugt, daß es wirklich mit der Theorie eines kontinuierlichen Feldes gemacht werden kann, obwohl ich hierfür eine bisher recht vernünftig erscheinende Möglichkeit gefunden habe. Die rechnerischen Schwierigkeiten sind jedoch so groß, daß ich ins Gras beißen werde, bevor ich selbst eine sichere Überzeugung hierüber erlangt habe. Aber davon bin ich fest überzeugt, daß man schließlich bei einer Theorie landen wird, deren gesetzmäßig verbundene Dinge nicht Wahrscheinlichkeiten sondern gedachte Tatbestände sind, wie man es bis vor kurzem als selbstverständlich betrachtet hat. Zur Begründung dieser Überzeugung kann ich aber nicht logische Gründe, sondern nur meinen kleinen Finger als Zeugen beibringen, also keine Autorität, die außerhalb meiner Hand irgendwelchen Respekt einflößen kann.

Ich freue mich, daß Dein Leben und Deine Arbeit fruchtbar und befriedigend sind. Dies hilft einem hinweg über die Tollheiten der Menschen, die das Schicksal des sogenannten homo sapiens im Großen bestimmen. Es war ja wohl nie besser, aber man hat es in seiner Jämmerlichkeit nicht so deutlich gesehen, und die Folgen der Pfuscherei waren weniger katastrophal als unter den gegenwärtigen Umständen.

Sei mit den Deinen herzlich gegrüßt von Deinem

A. Einstein.

deren Notwendigkeit im Ra[...] erkannt wurde, einen bedeuten[...]
mus ja zuerst von Dir klar erkannt wurde, einen bedeuten-
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[...] überzeugt, daß es wirklich mit der Theorie

amount of validity in the [...] framework [...]
recognise clearly as necessary given the framework [...]
malism. I cannot seriously believe in it because the theory cannot be rec-
onciled with the idea that physics should represent a reality in time and
space, free from spooky actions at a distance. I am, however, not yet
[...]inced that it can really be achieved with a continuous field
[...] this which so

I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

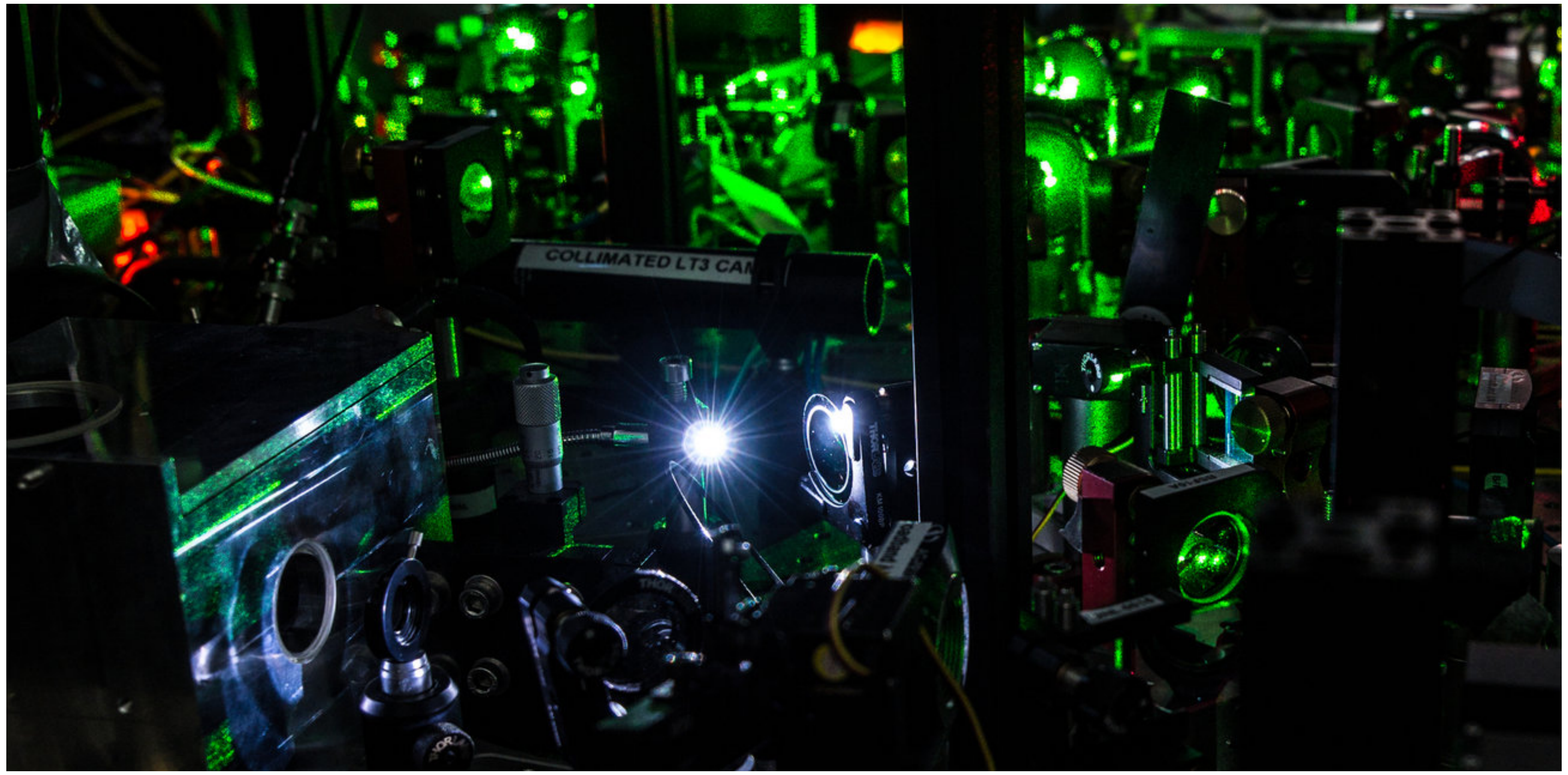
Albert Einstein to Max Born, 3 March 1947

The New York Times

Sorry, Einstein. Quantum Study Suggests 'Spooky Action' Is Real.

By JOHN MARKOFF OCT. 21, 2015

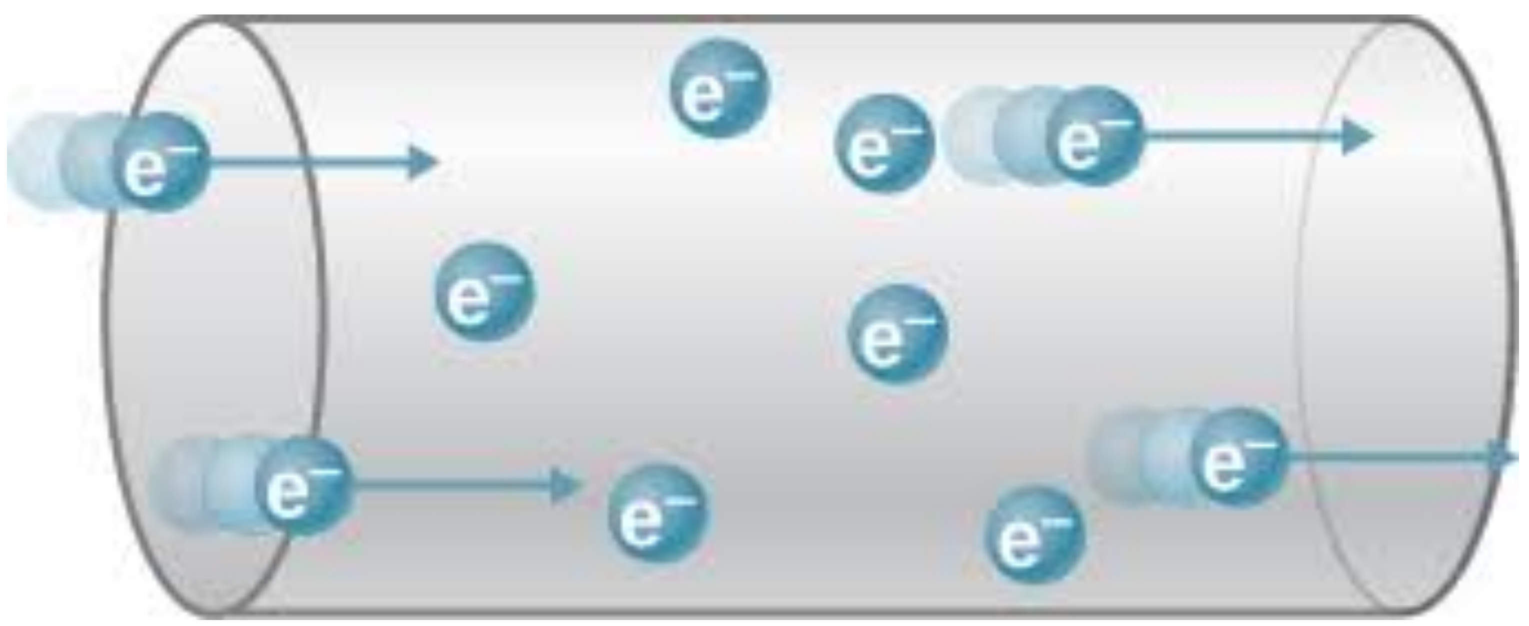
In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other's behavior.

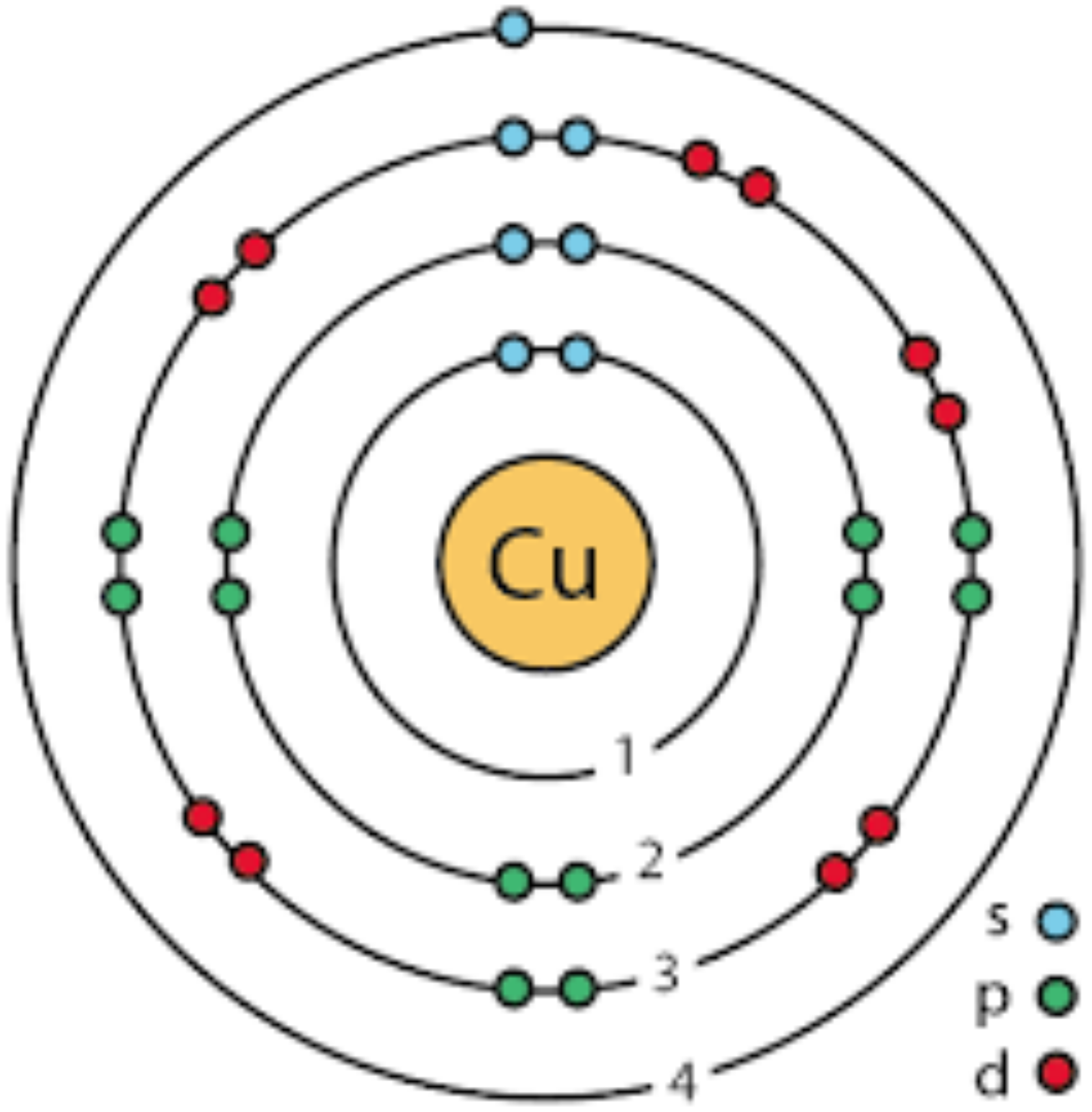


Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Quantum theory of electrons: ordinary metals and strange metals

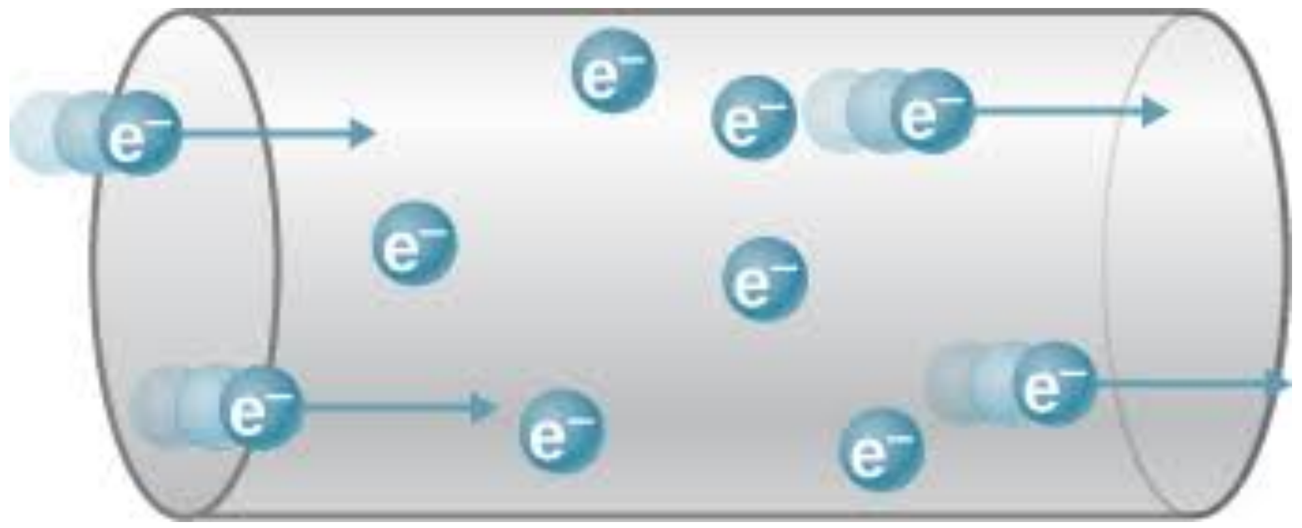
Copper





Each copper atom donates its outermost electron
These electrons move freely throughout the crystal and carry current

Current flow with electrons in Copper

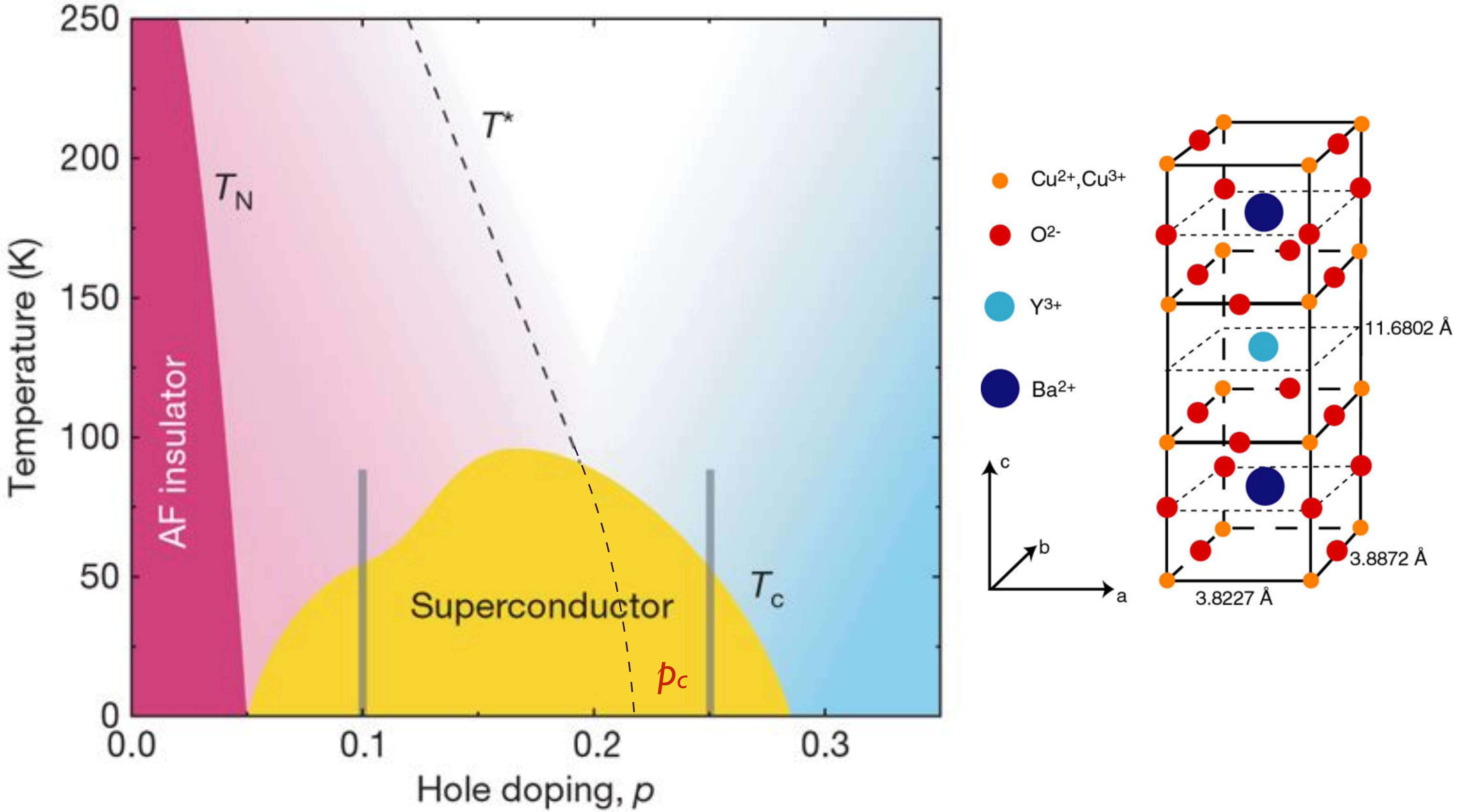


Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

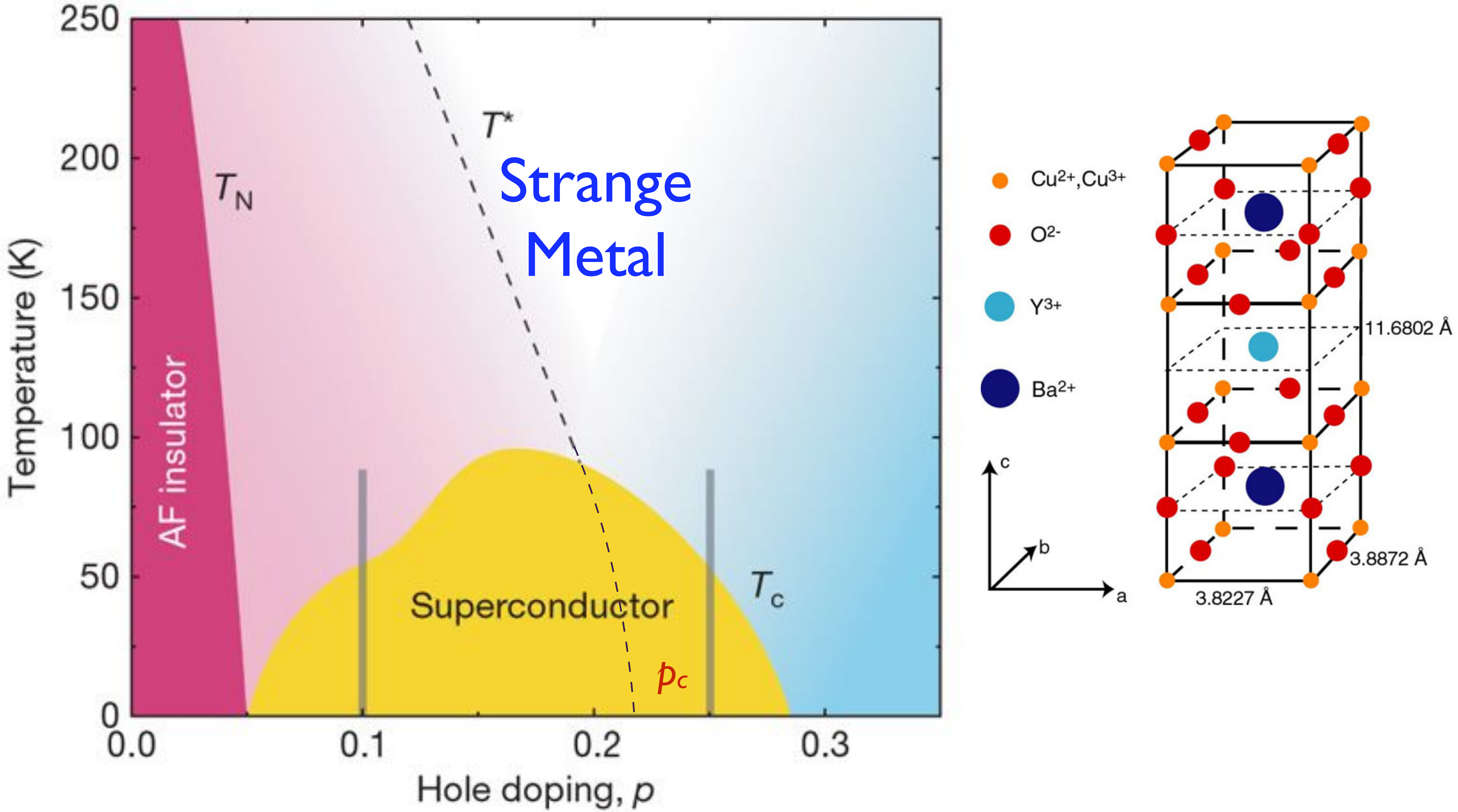
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.



250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

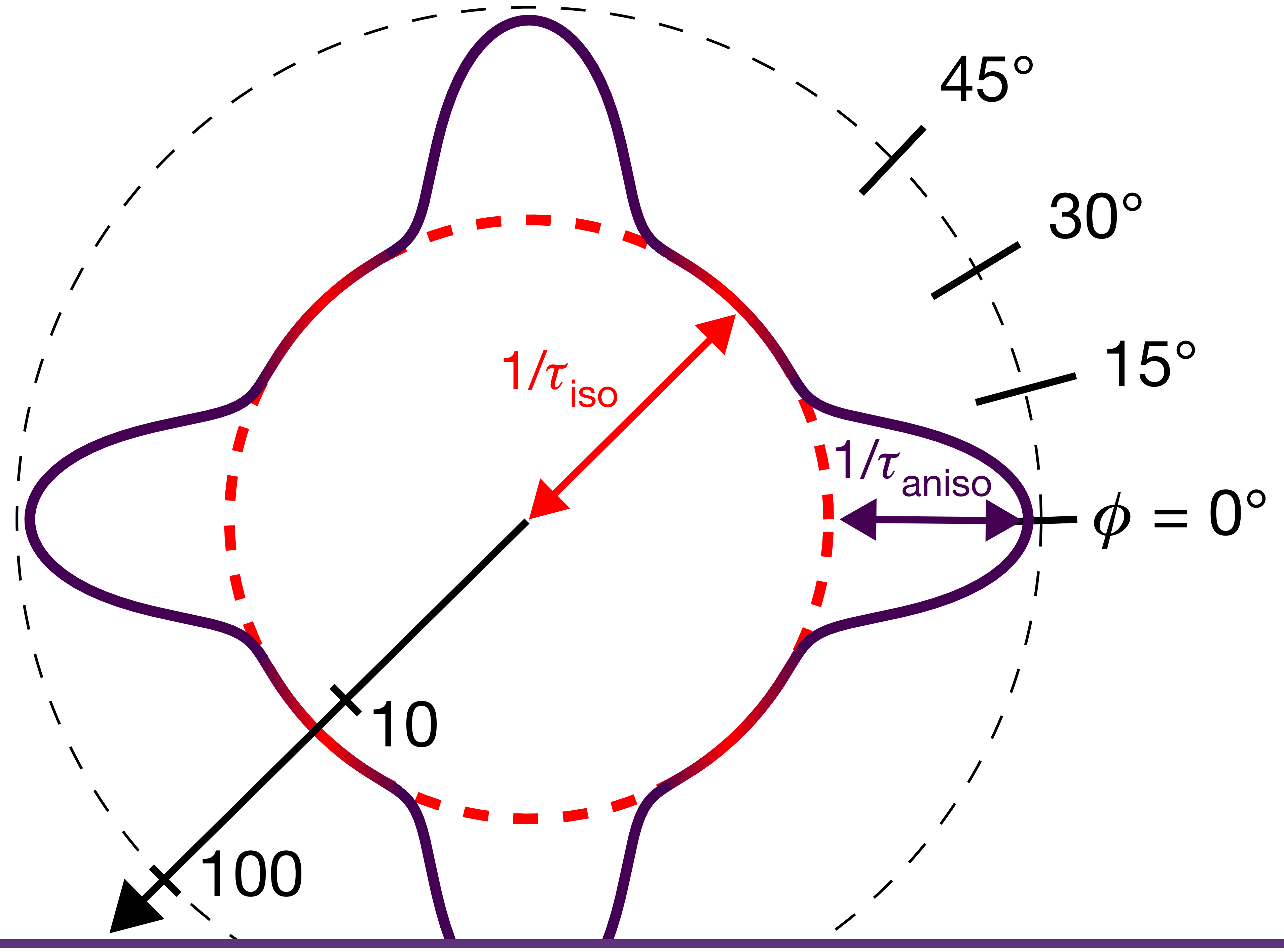


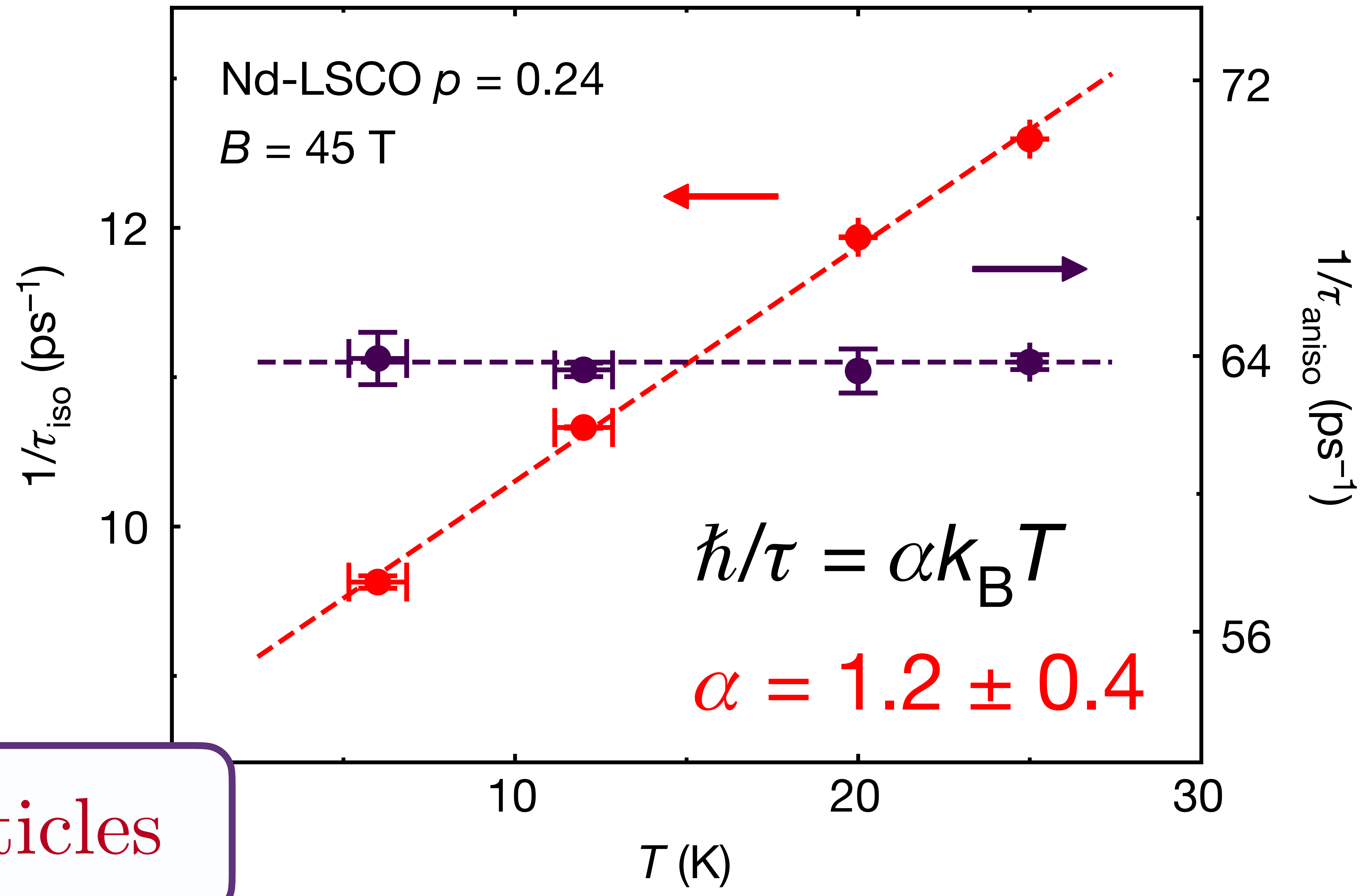
Strange Metal
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
T_c
Superconductor
p_c
Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Linear-in temperature resistivity from an isotropic Planckian scattering rate

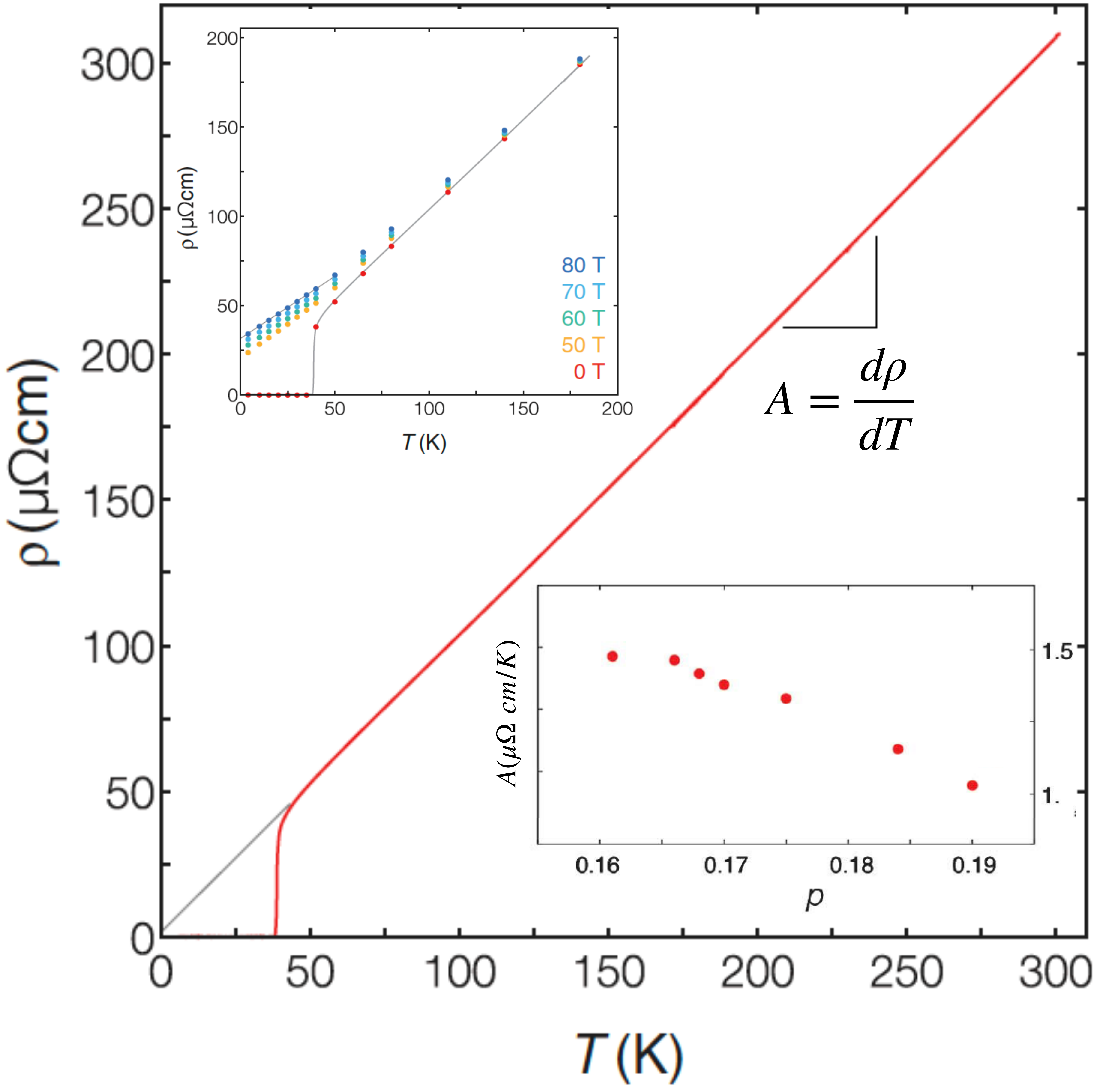
Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw

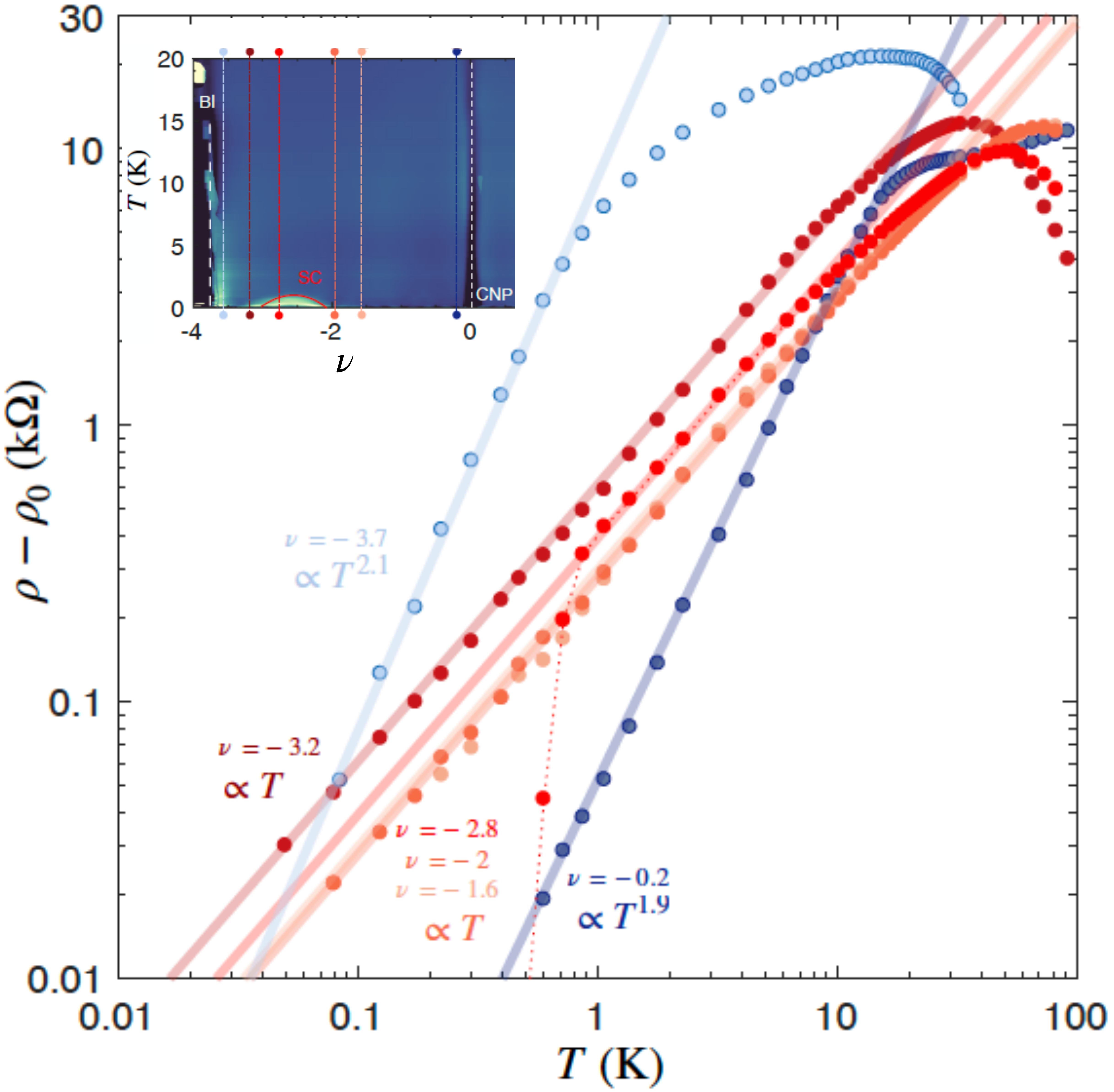




Current flow without quasiparticles



LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Questions

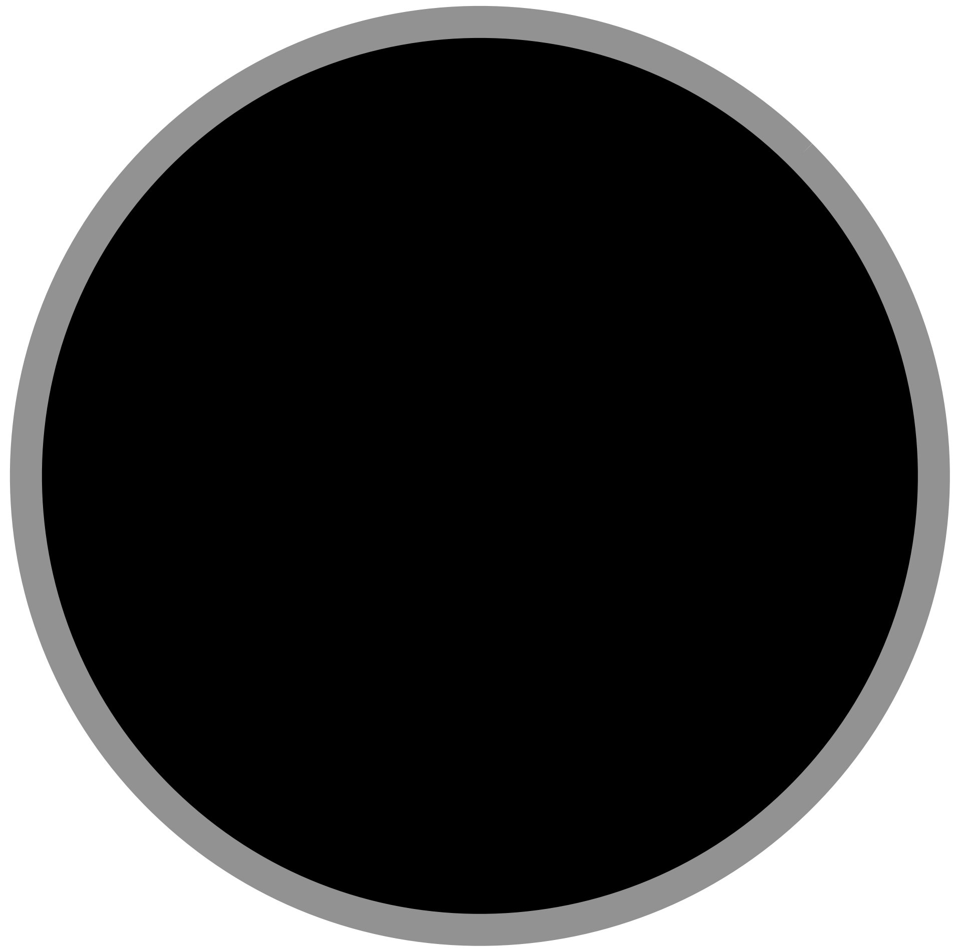
- Needed: A theory for current flow in a 'strange metal' with an entangled soup of electrons.
- Needed: theory for collision time in resistivity $\sim \hbar/(k_B T)$.
- Needed: theory for the appearance of superconductivity in such a 'strange metal'.

Quantum black holes and holography

Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \dfrac{2GM}{c^2}$



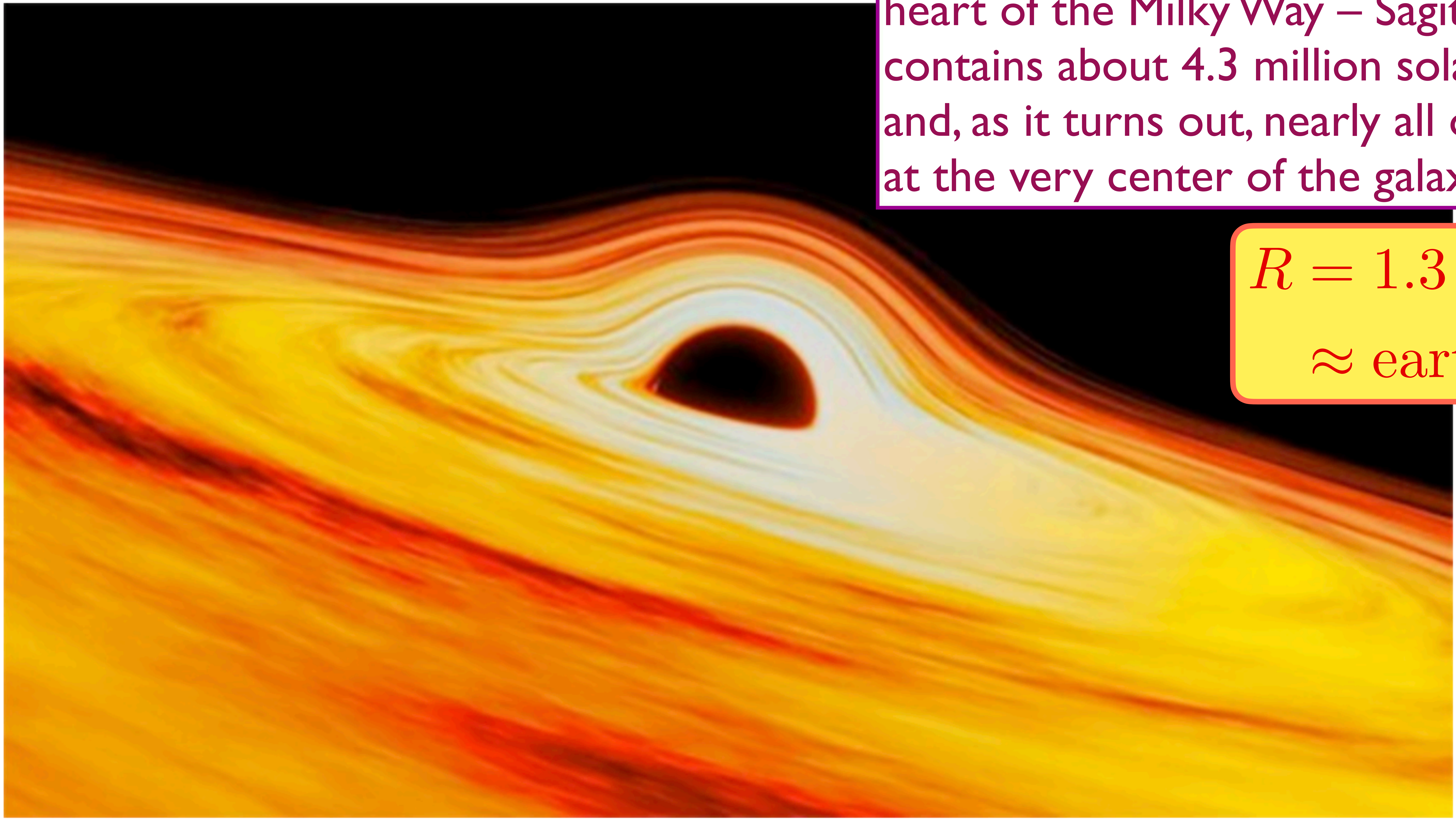
G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A* contains about 4.3 million solar masses, and, as it turns out, nearly all of the mass at the very center of the galaxy.

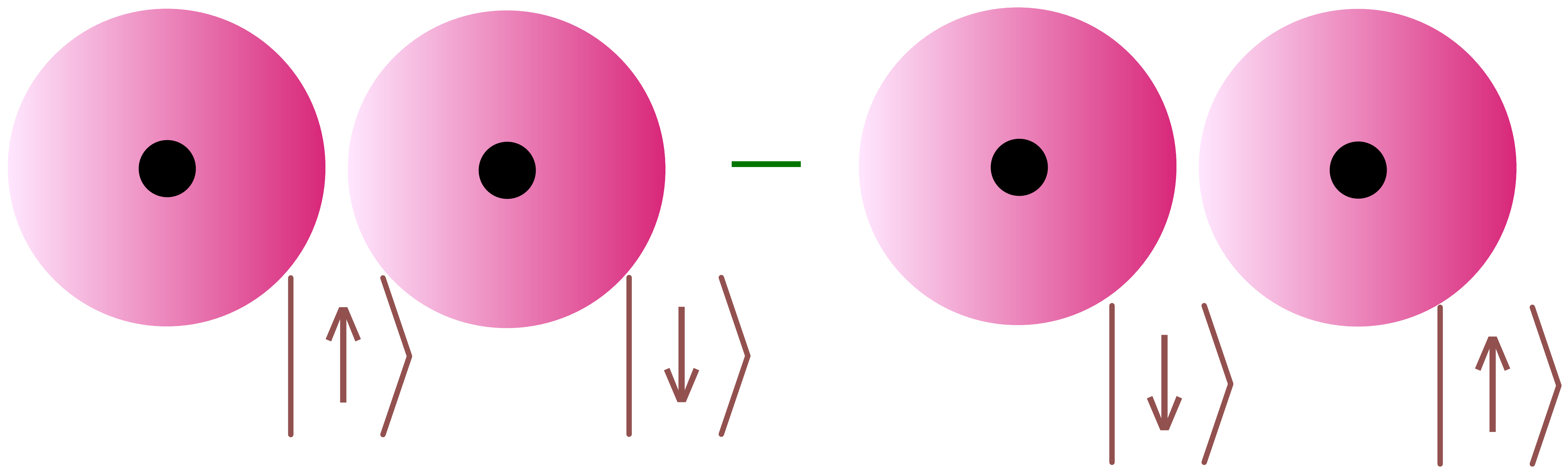
$$R = 1.3 \times 10^{11}\mathrm{m}$$

$$\approx \text{earth's orbit}$$



An artist's impression of Sagittarius A*, the supermassive black hole at the heart of the Milky Way. Image: International Gemini Observatory/NOIRLab/NSF/AURA/J. da Silva/(Spaceengine); M. Zamani (NSF's NOIRLab)

Quantum Entanglement across a black hole horizon



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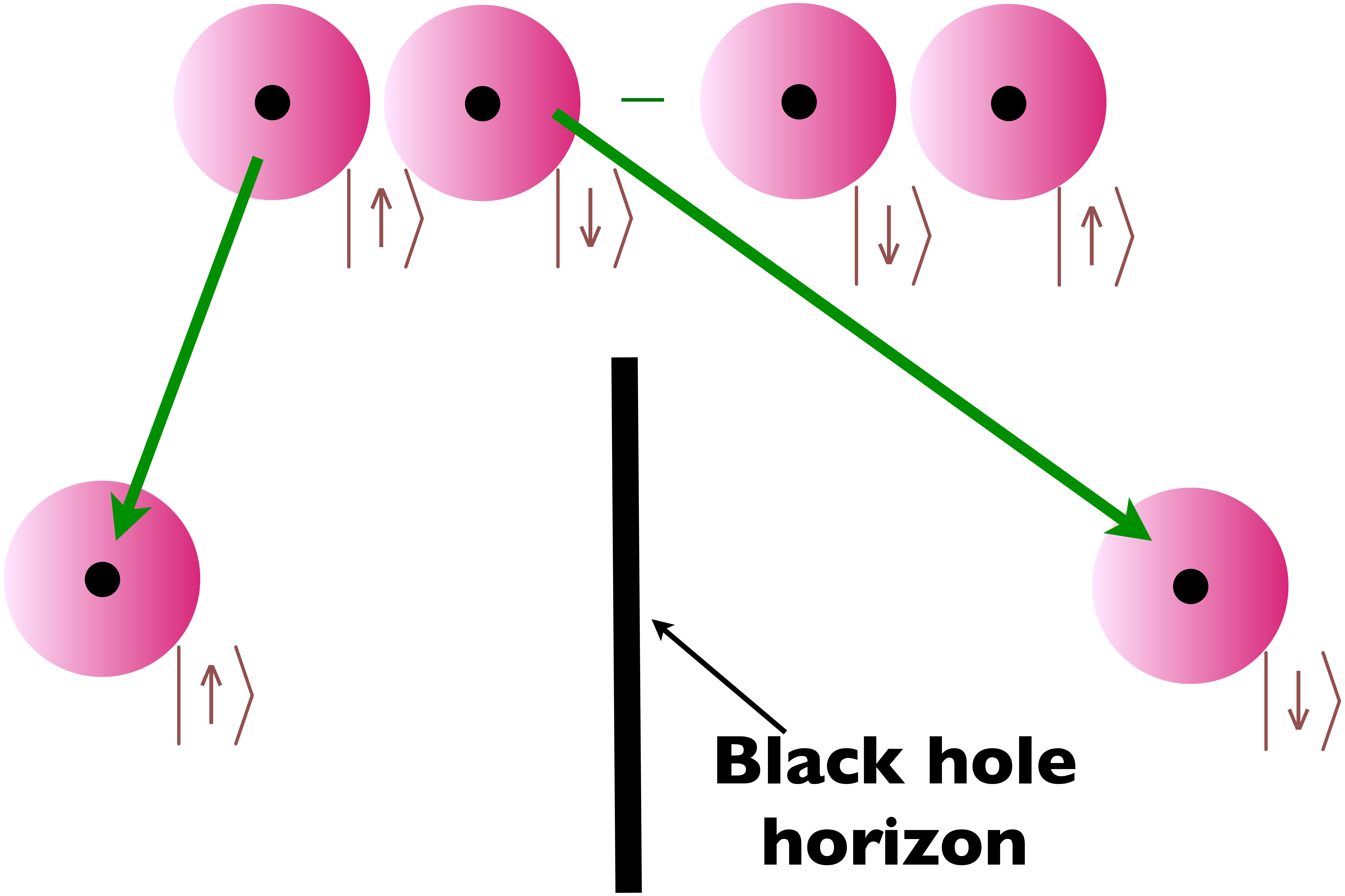

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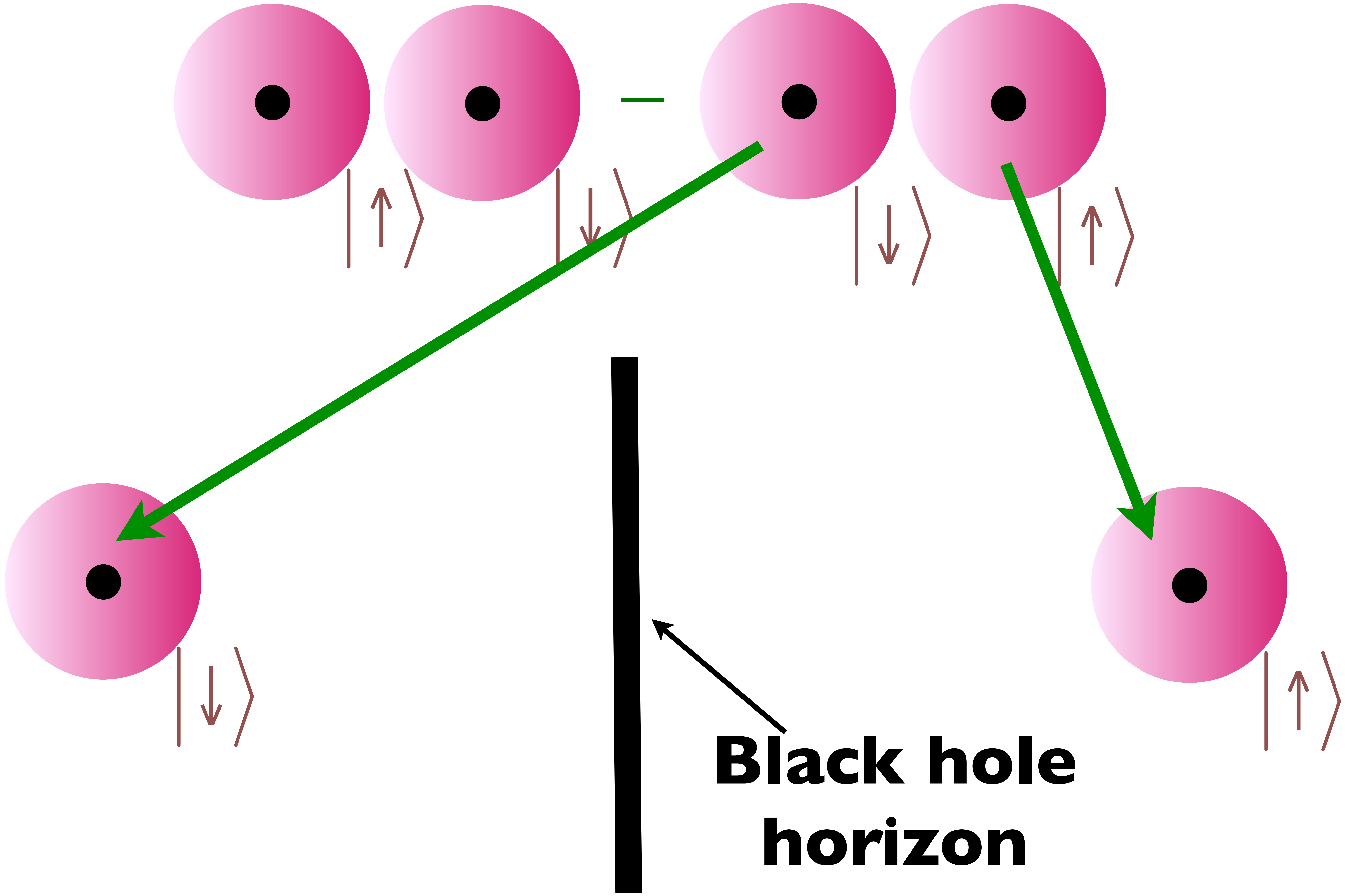
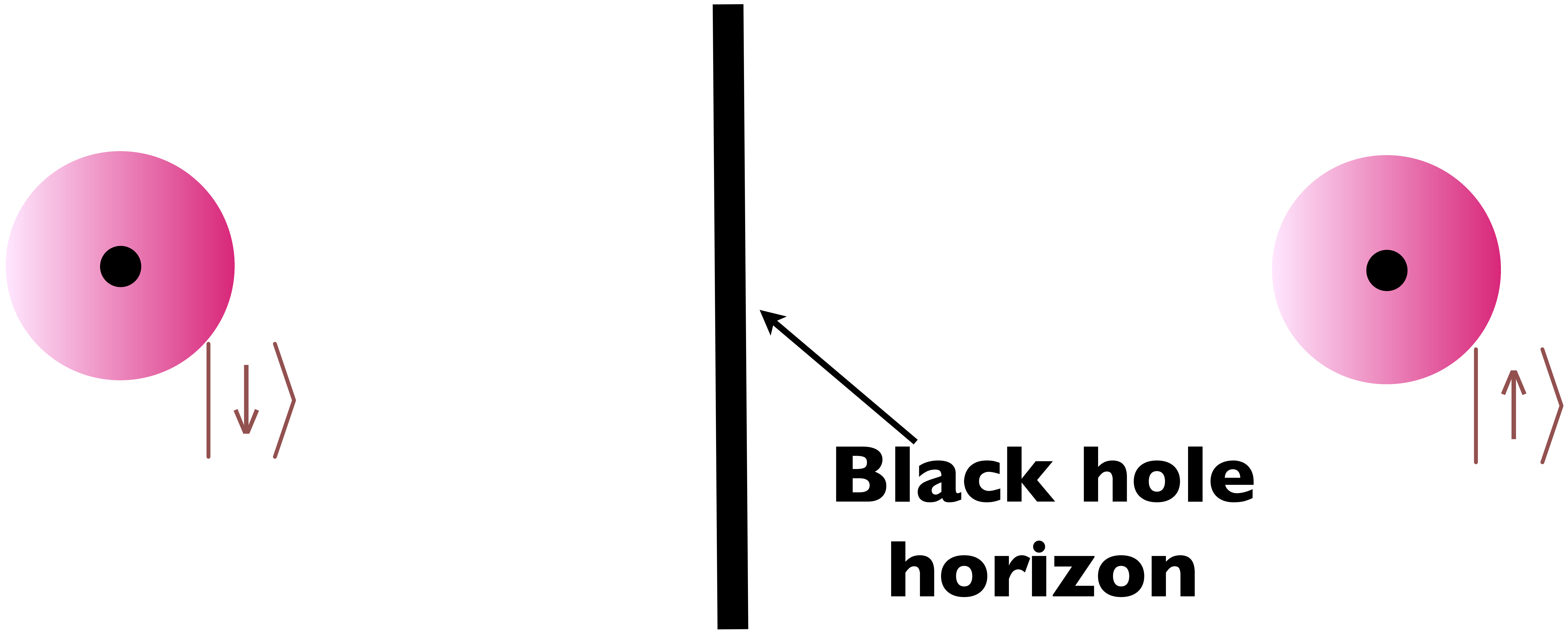


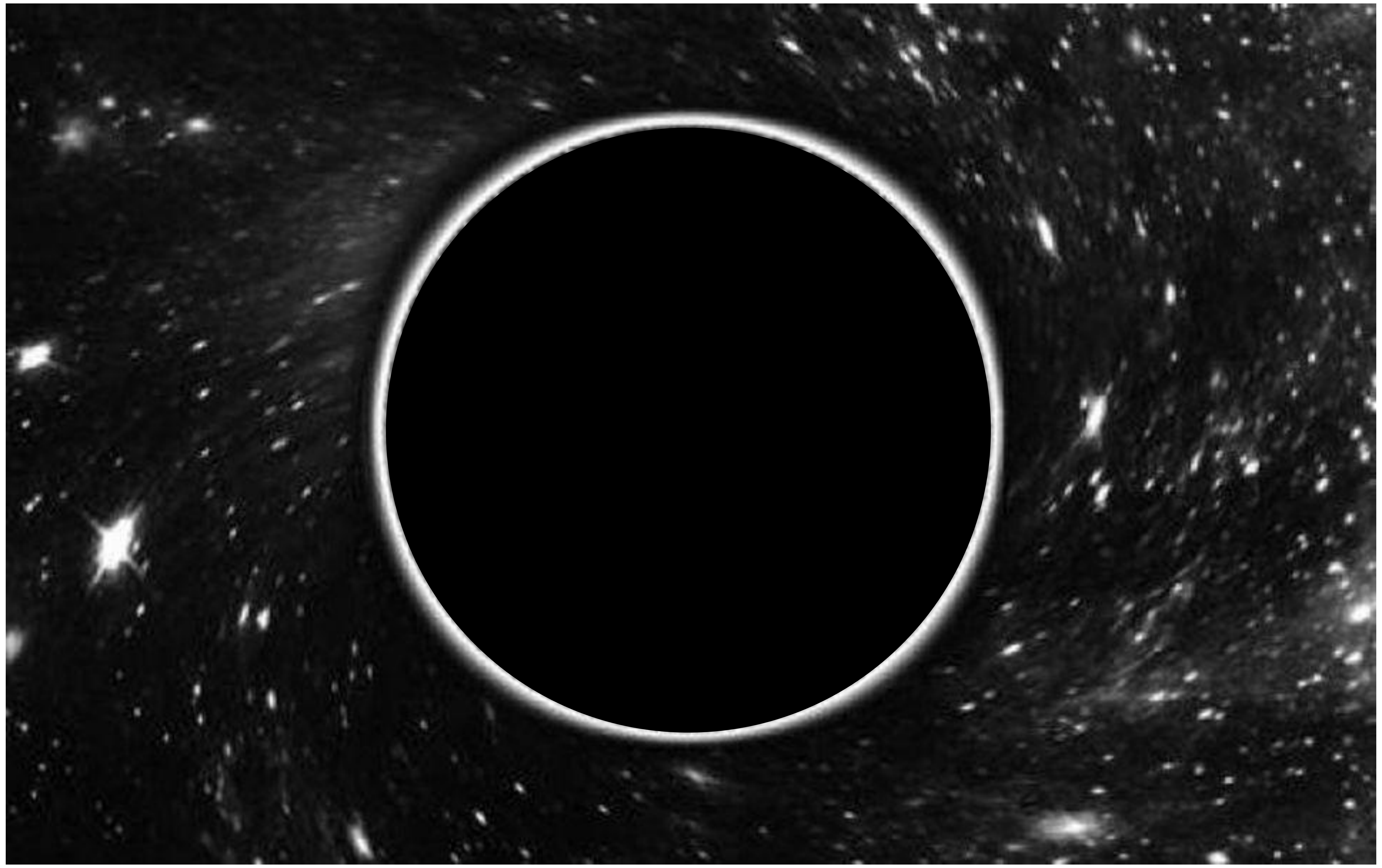
Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)



Quantum Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.



T.B. BAKKER / DR. J.P. VAN DER SCHAAR

J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

Quantum Black holes

Bohr-Sommerfeld semiclassical quantum theory of a black hole in d spatial dimensions

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_{\mu} \exp\left(-\frac{1}{\hbar}\int d^d x \int_0^{\hbar/(k_B T)} d\tau \sqrt{g}\, \mathcal{L}_d[g_{\mu\nu}, a_{\mu}]\right)$$

$g_{\mu\nu} \Rightarrow$ spacetime metric, $\quad g = \det(g_{\mu\nu})$

$a_{\mu} \Rightarrow$ Electromagnetic gauge field

$\mathcal{L}_d \Rightarrow$ *Classical* Einstein-Maxwell action

$$\mathcal{U}(t) = \exp\left(-i\mathcal{H}t/\hbar\right) \quad \Leftrightarrow \quad \mathcal{Z} = \mathrm{Tr}\exp\left(-\mathcal{H}/(k_B T)\right)$$

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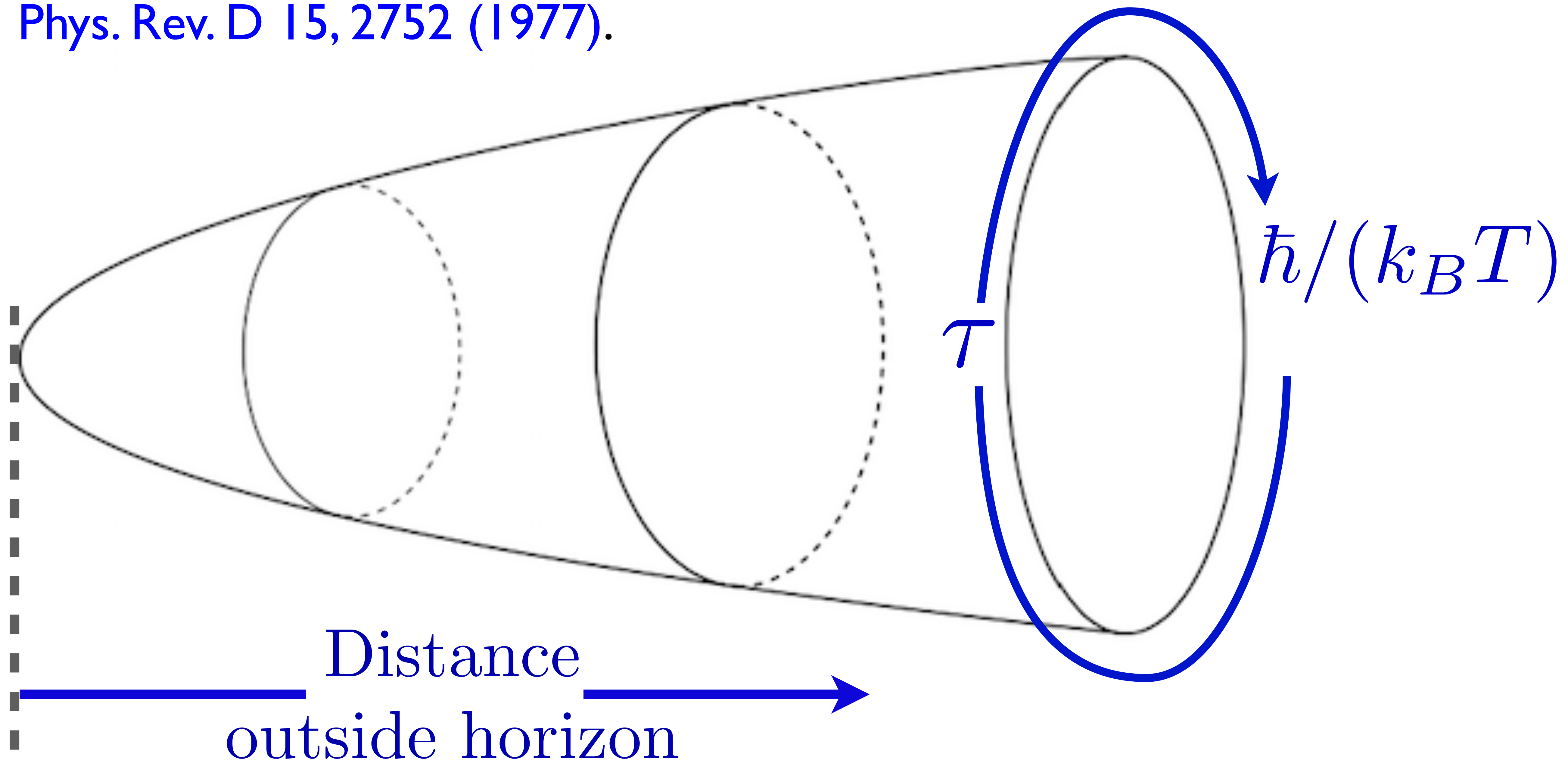
G.W. Gibbons and S.W. Hawking, Phys. Rev. D 15, 2752 (1977).



Spacetime geometry of a black hole
in imaginary time τ

Evaluate path integral
at black hole saddle point

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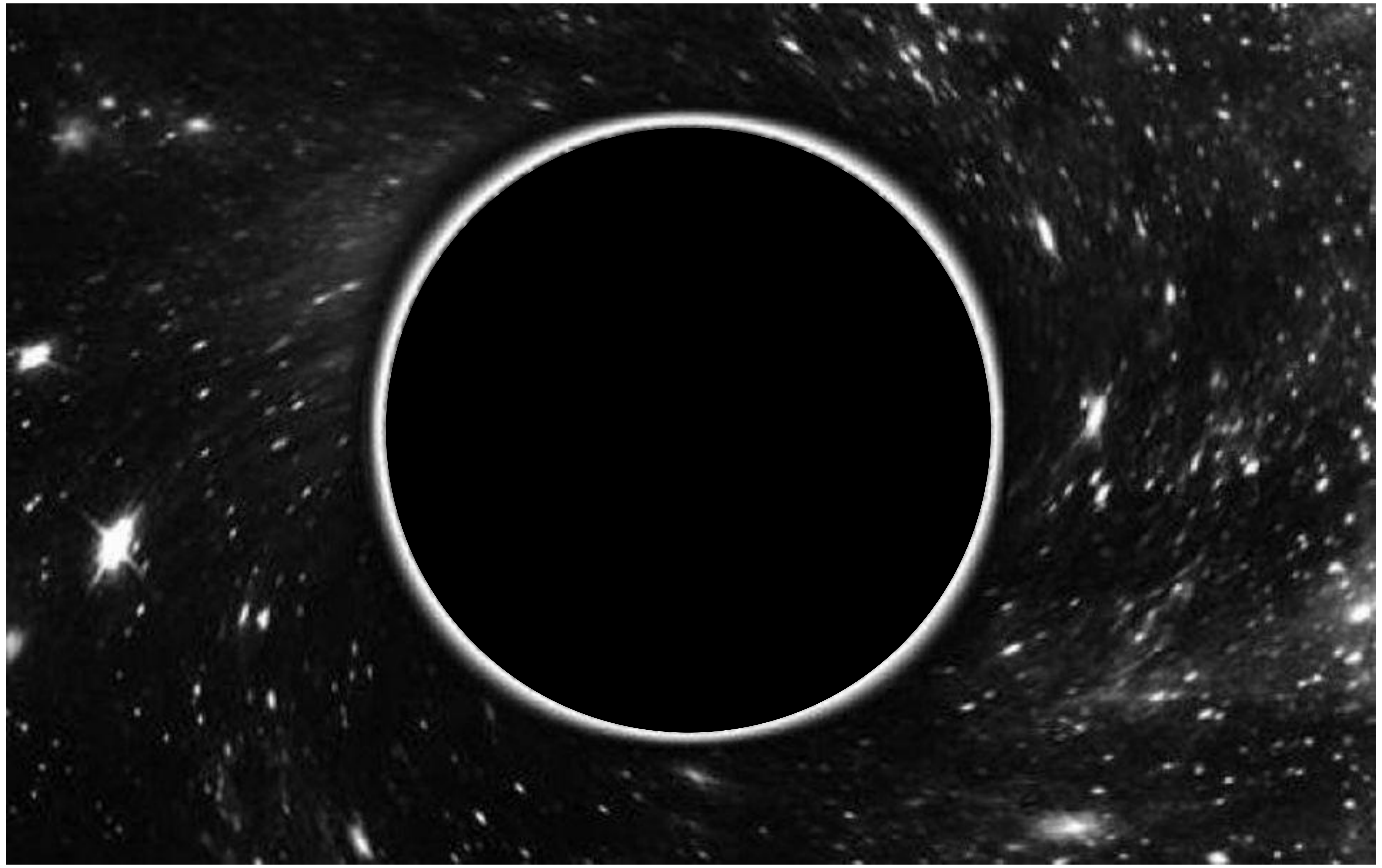

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Remarkable features:

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- Entropy is not proportional to volume

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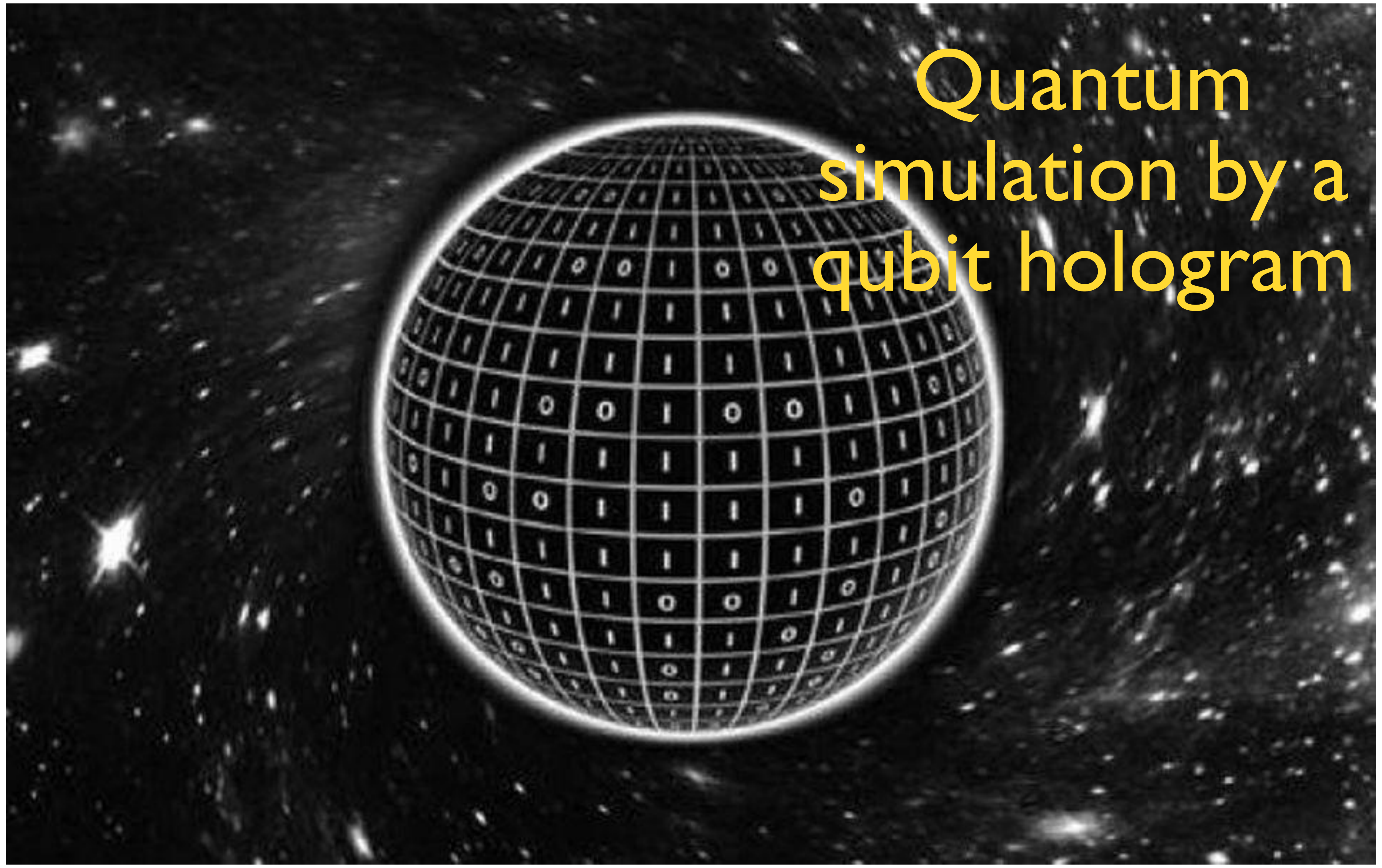

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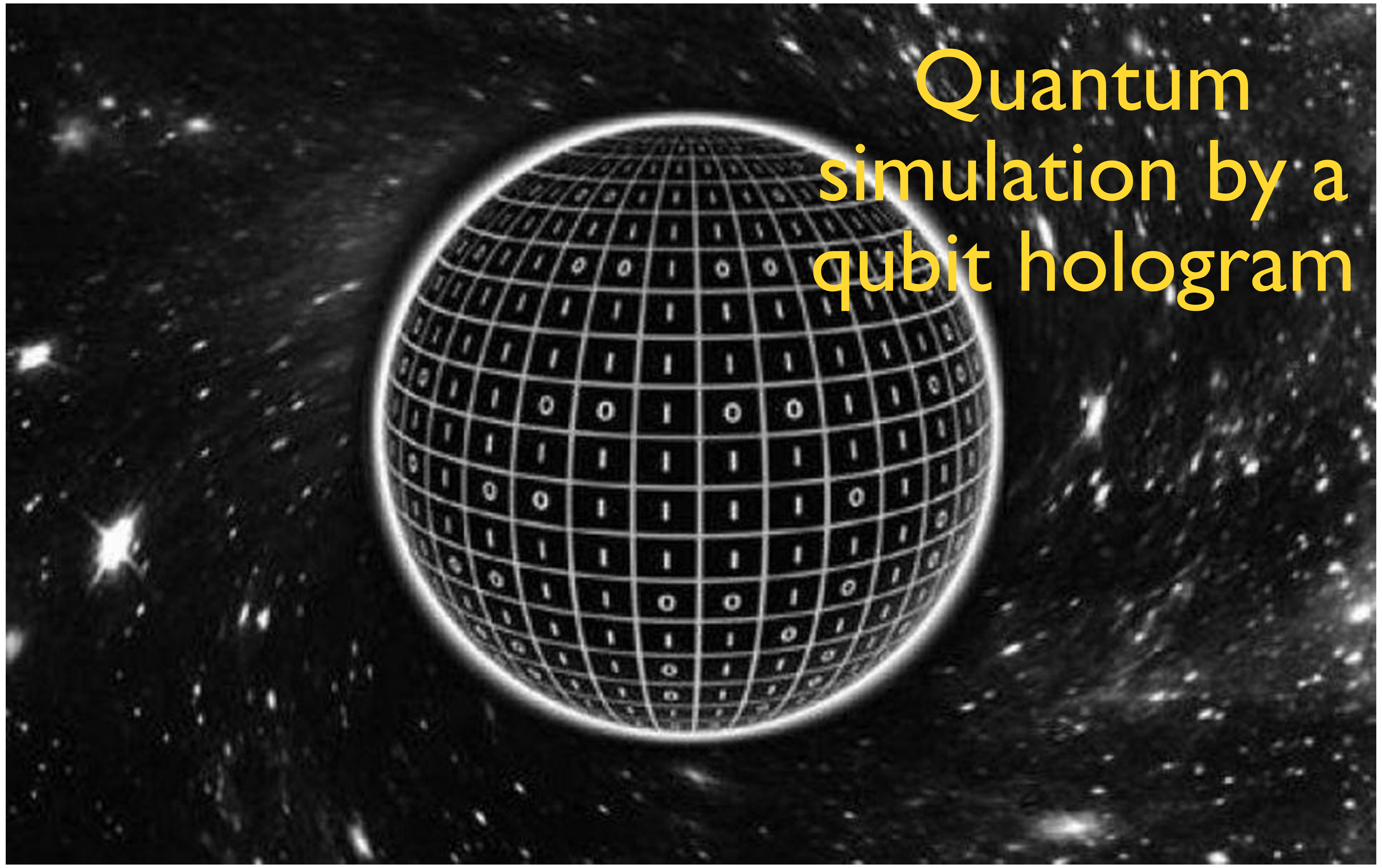
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3$



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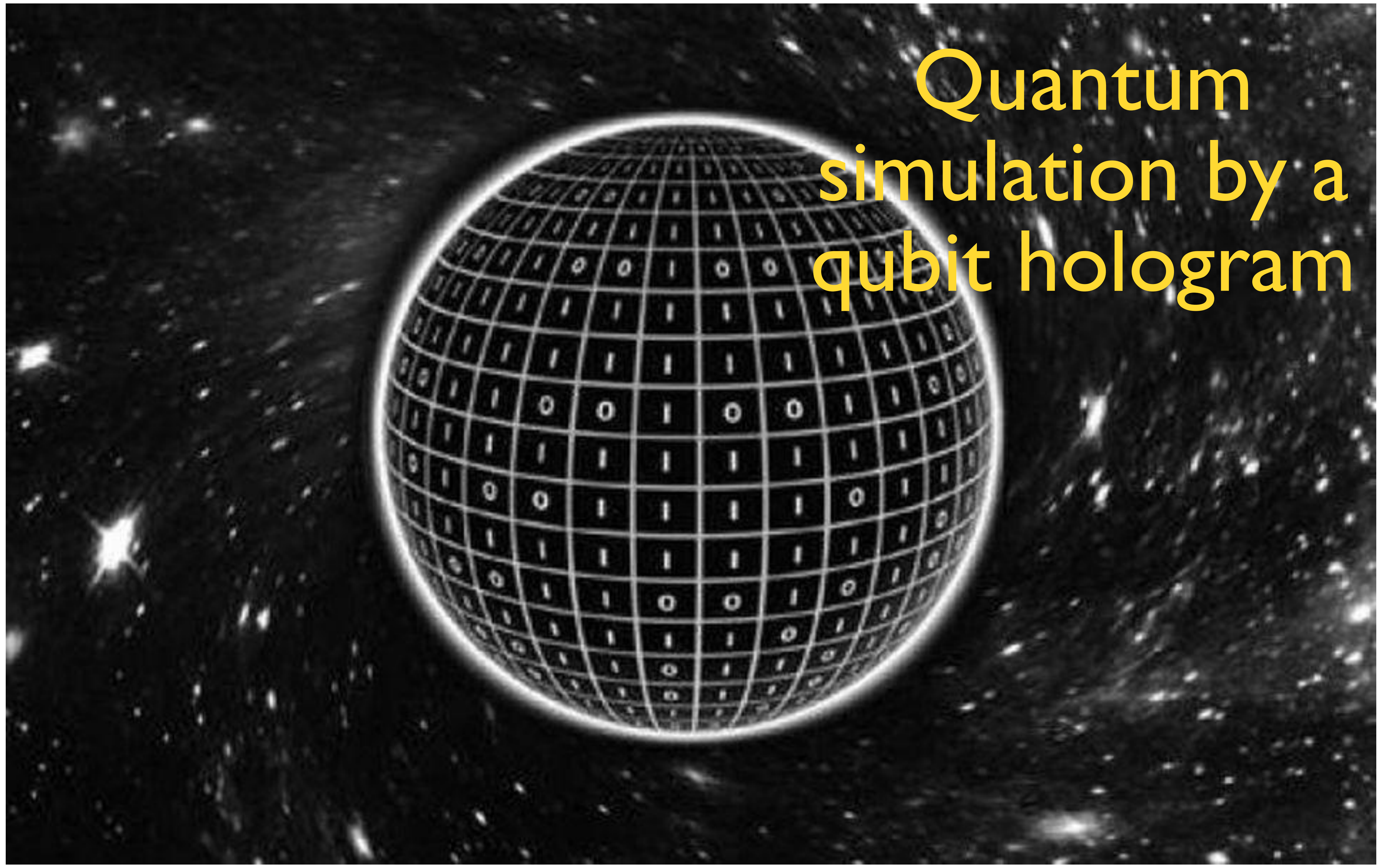
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3 = \hbar/(k_B T_H)$ which is Planckian!



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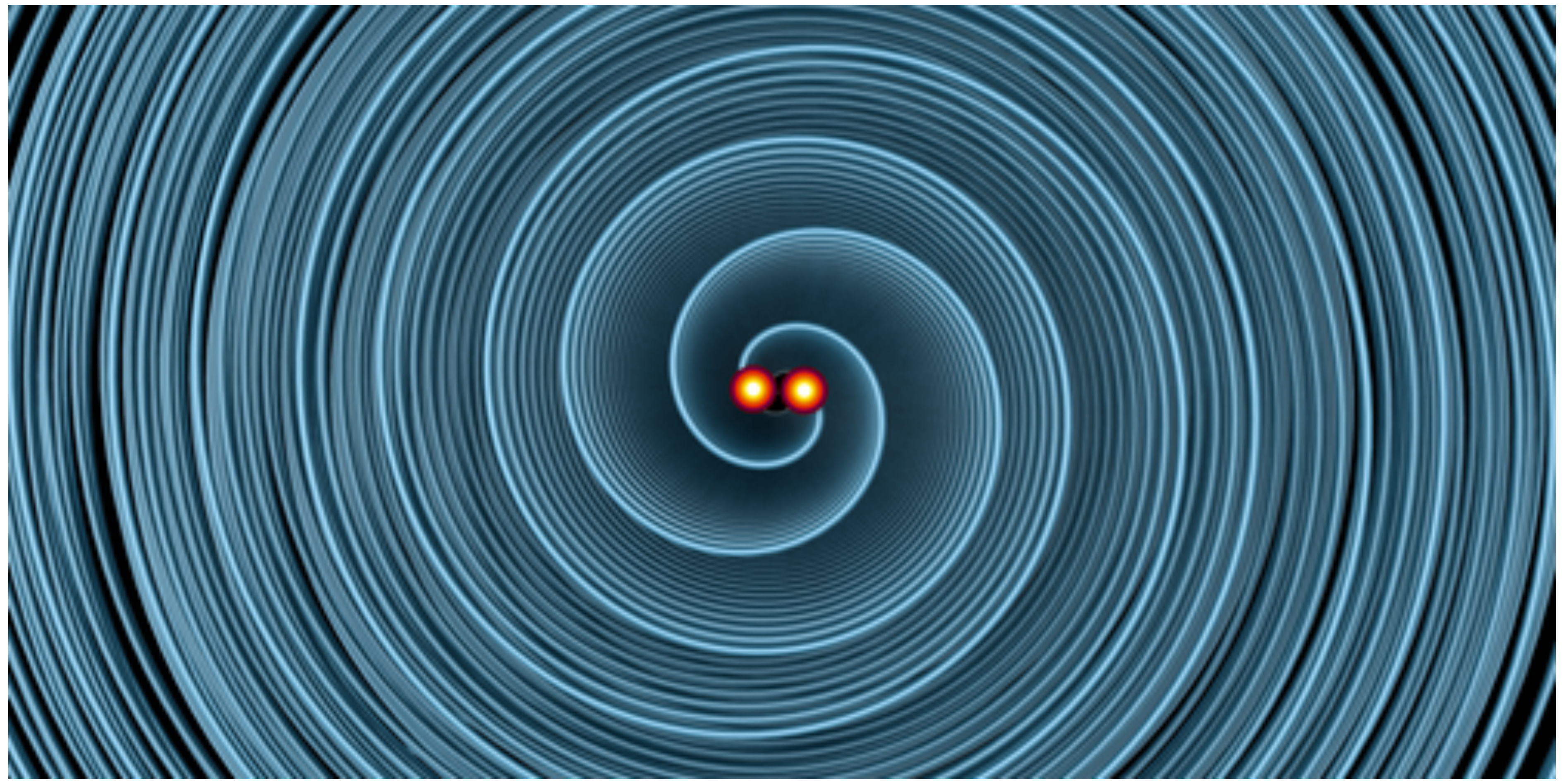
Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

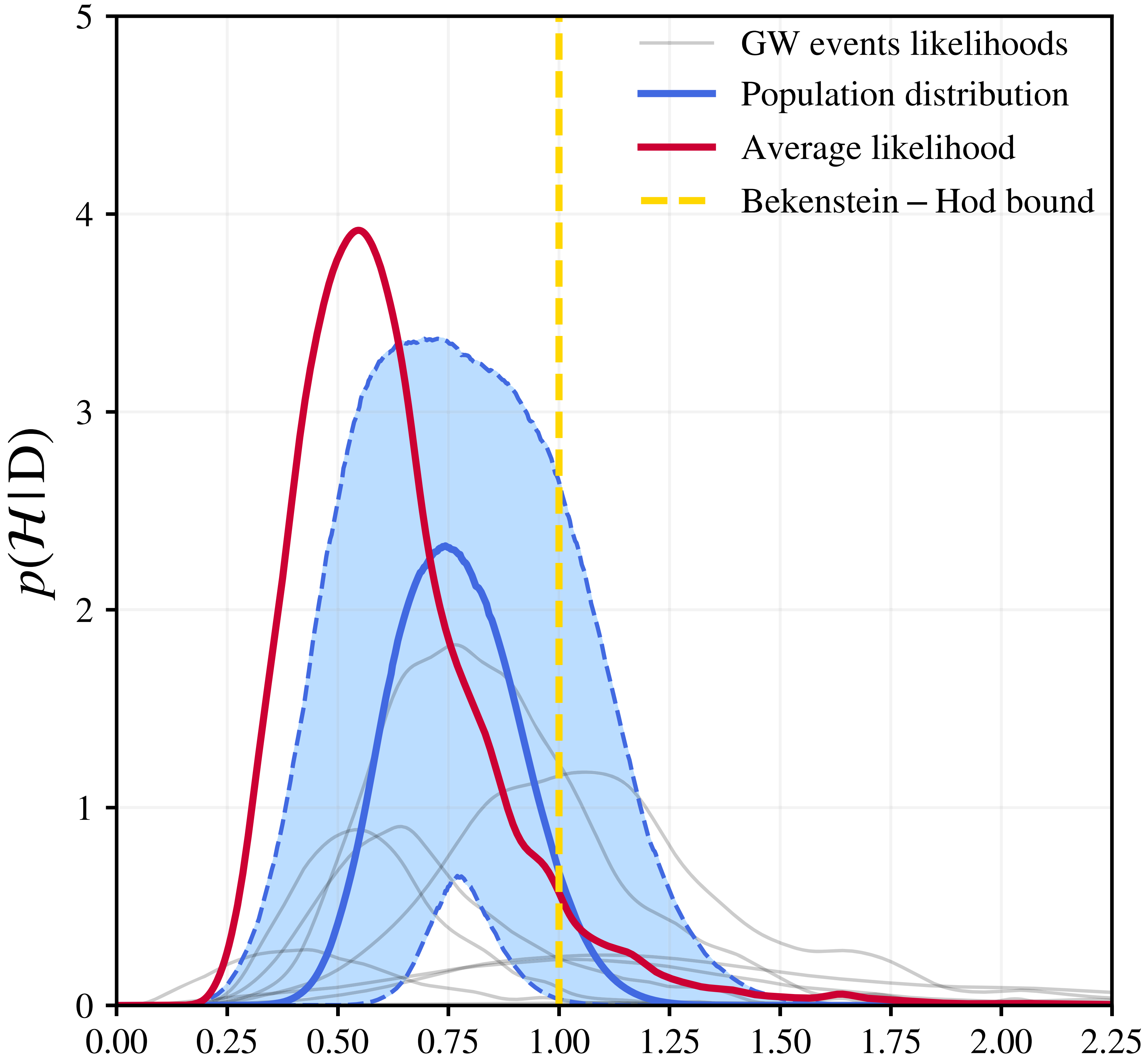
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$



$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

Questions

- Can Hawking's Bohr-Sommerfield theory of black hole entropy be interpreted in terms of a $D(E)$ of a Schrödinger-Heisenberg quantum system?

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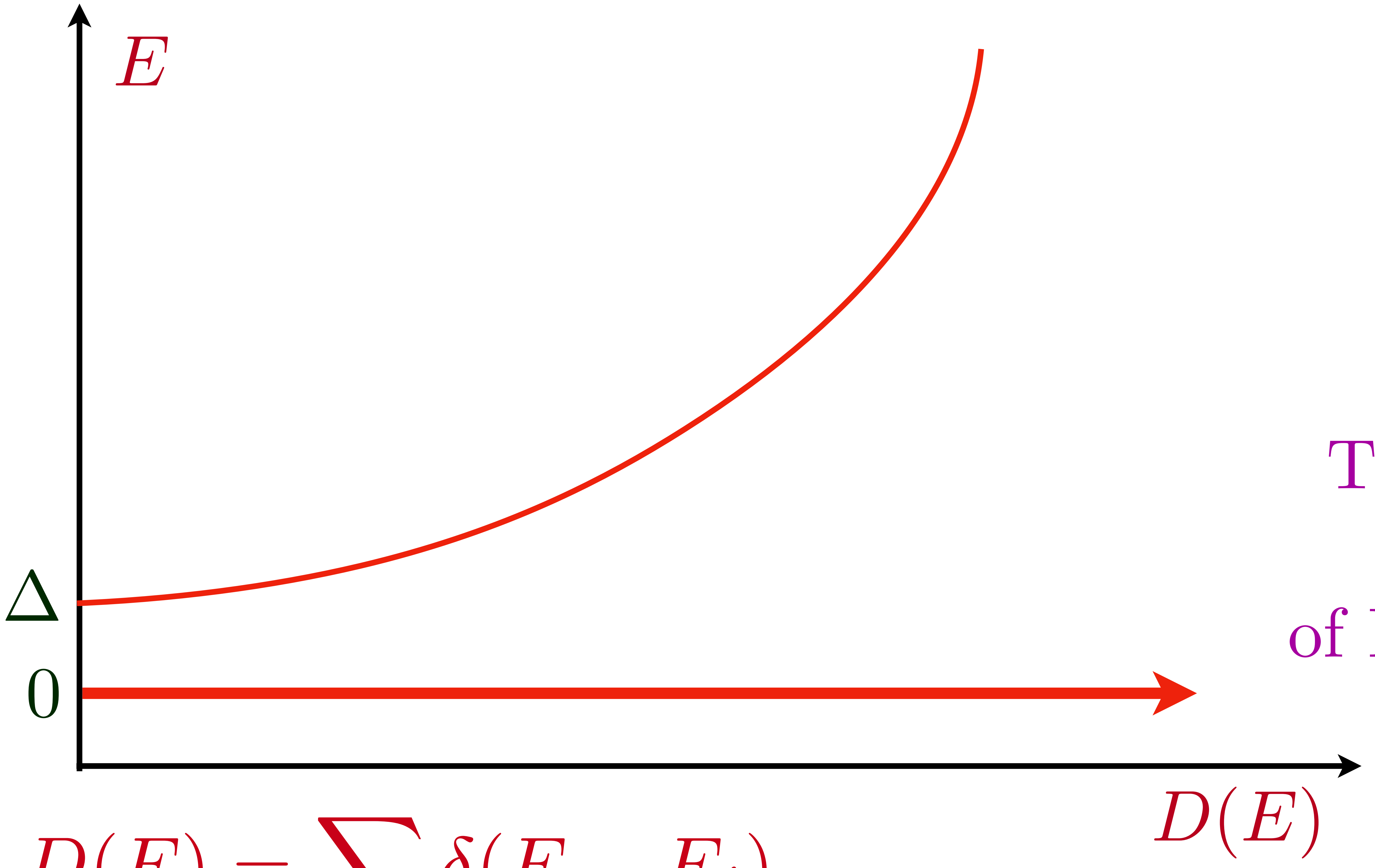


$$D(E) = \sum_i \delta(E - E_i)$$
$$= \exp\left(S/k_B\right)\delta(E) + \ldots$$

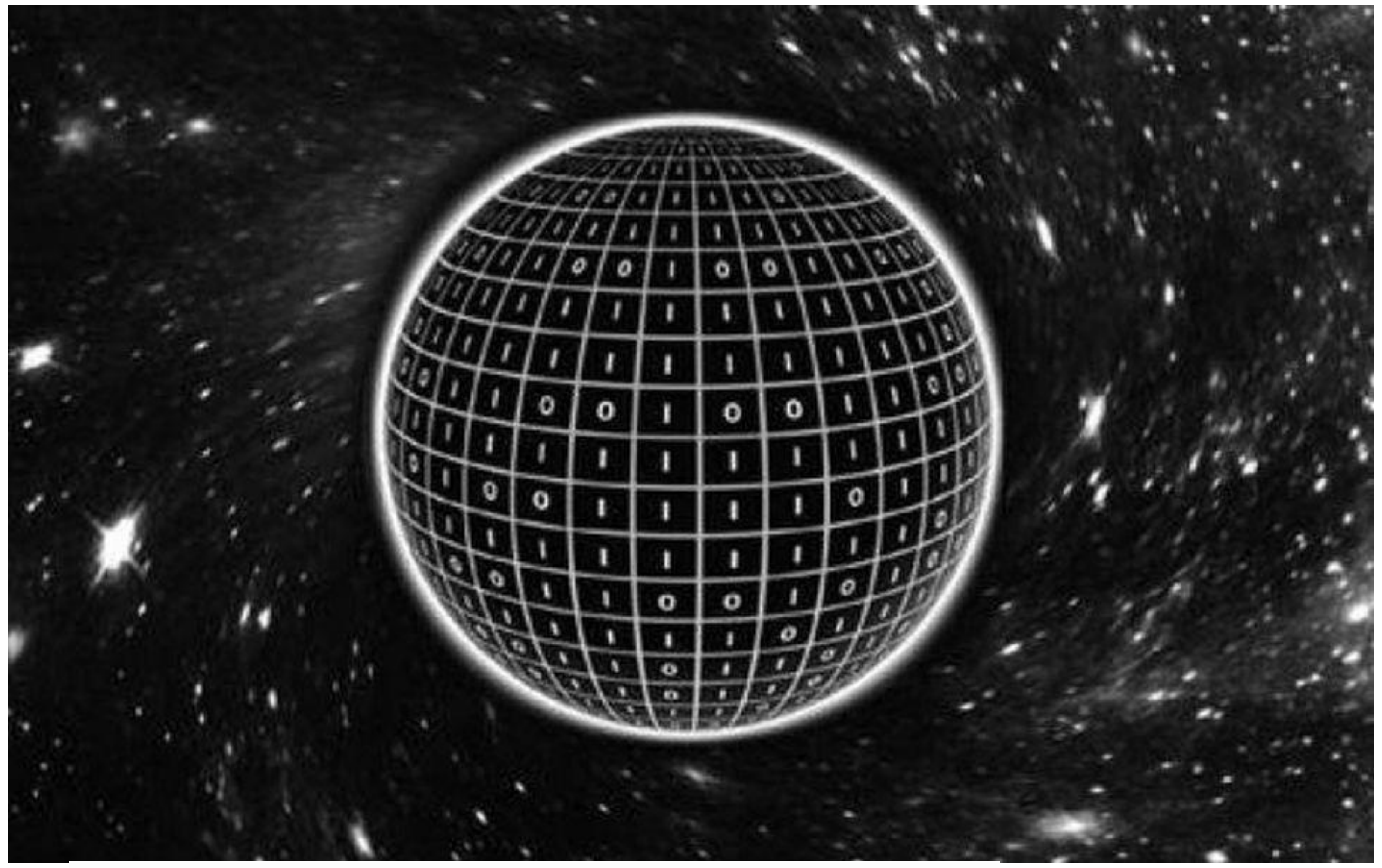
Black holes in string theory realize the entropy as an exact, exponentially large, degeneracy of the ground state. This is not the generic behavior of the semiclassical path integral of Einstein gravity, as we will see …

Questions

- Can Hawking's Bohr-Sommerfield theory of black hole entropy be interpreted in terms of a $D(E)$ of a Schrödinger-Heisenberg quantum system?

- Can black hole entropy be understood 'holographically', as that of a unitary quantum system in one lower spatial dimension with a finite number of states?

G. 'tHooft (1993); L. Susskind (1993); Maldacena (1998)



T.B. BAKKER / DR. J.P. VAN DER SCHAAR

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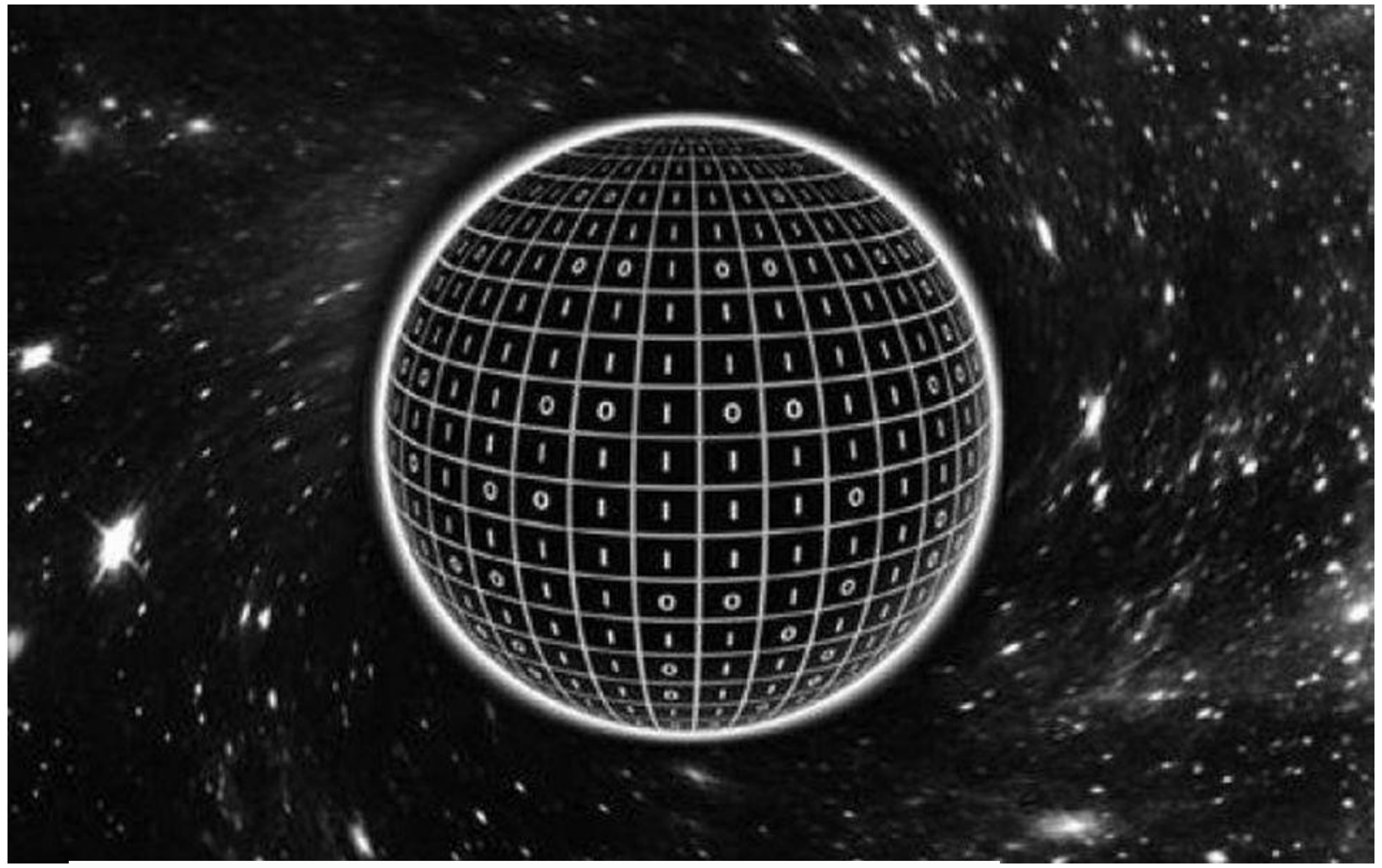

T.B. BAKKER / DR. J.P. VAN DER SCHAAR

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- The unitary quantum system cannot have particle-like excitations if it is to reproduce the rapid Planckian dynamics at the rate $k_B T/\hbar$.

- Can we compute the evolution of the entropy as the black hole evaporates? Is it that of an evaporating unitary quantum system?

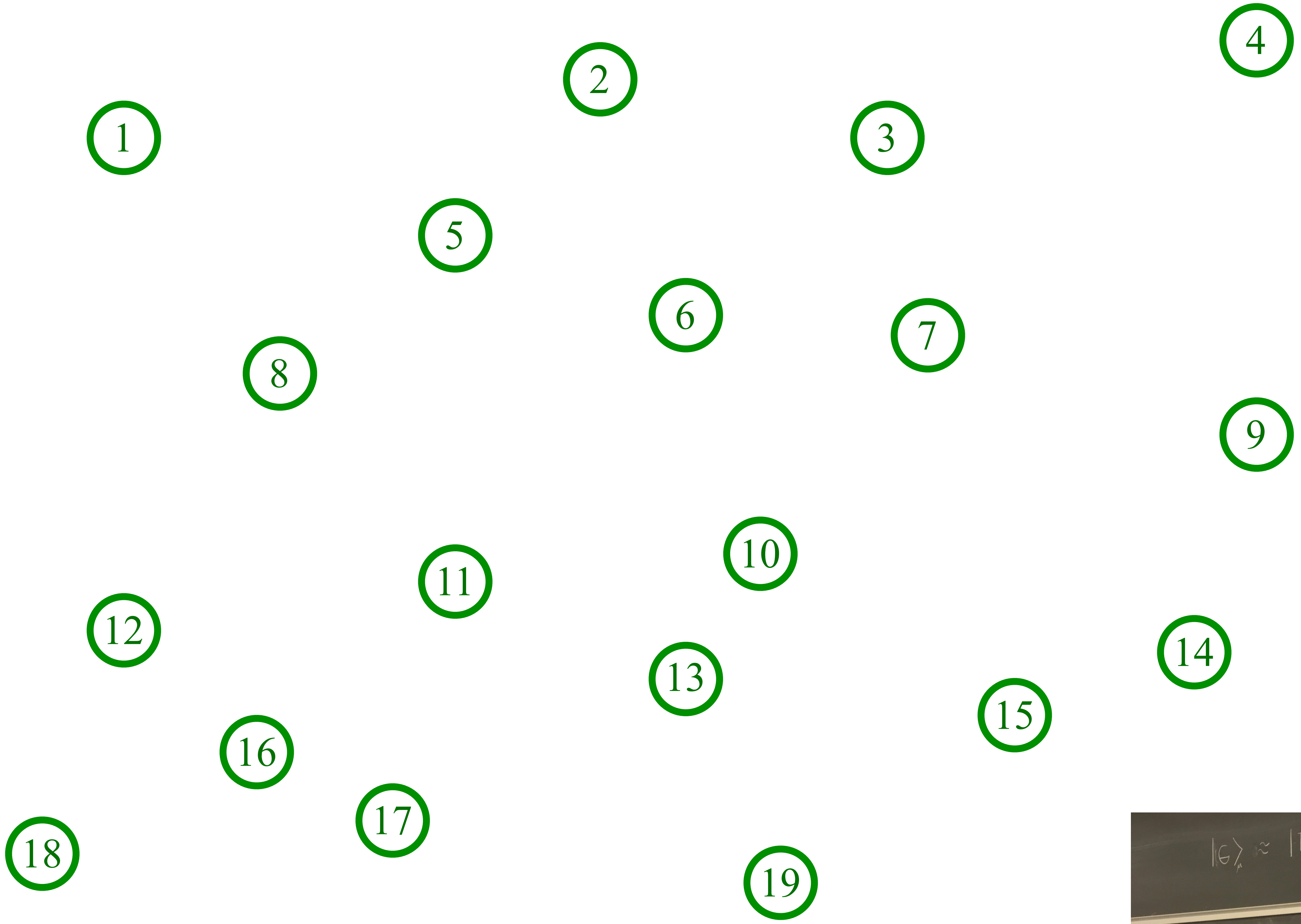
Sachdev-Ye-Kitaev Model

A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

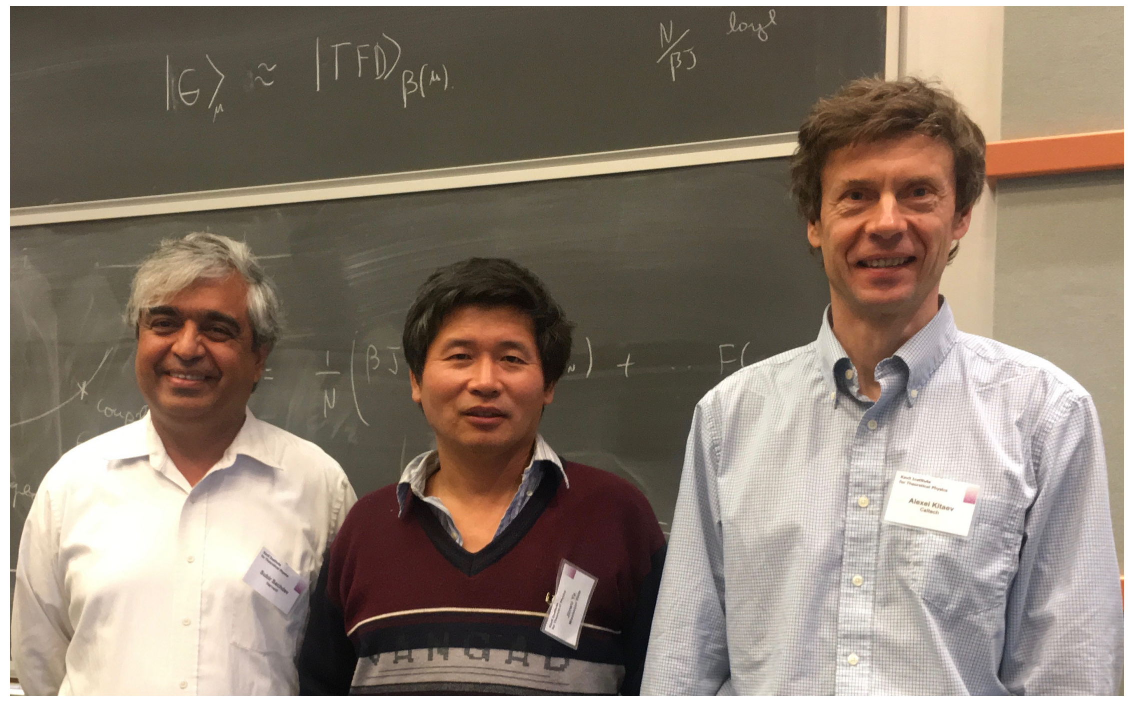
leading to a metal with no particle-like excitations

The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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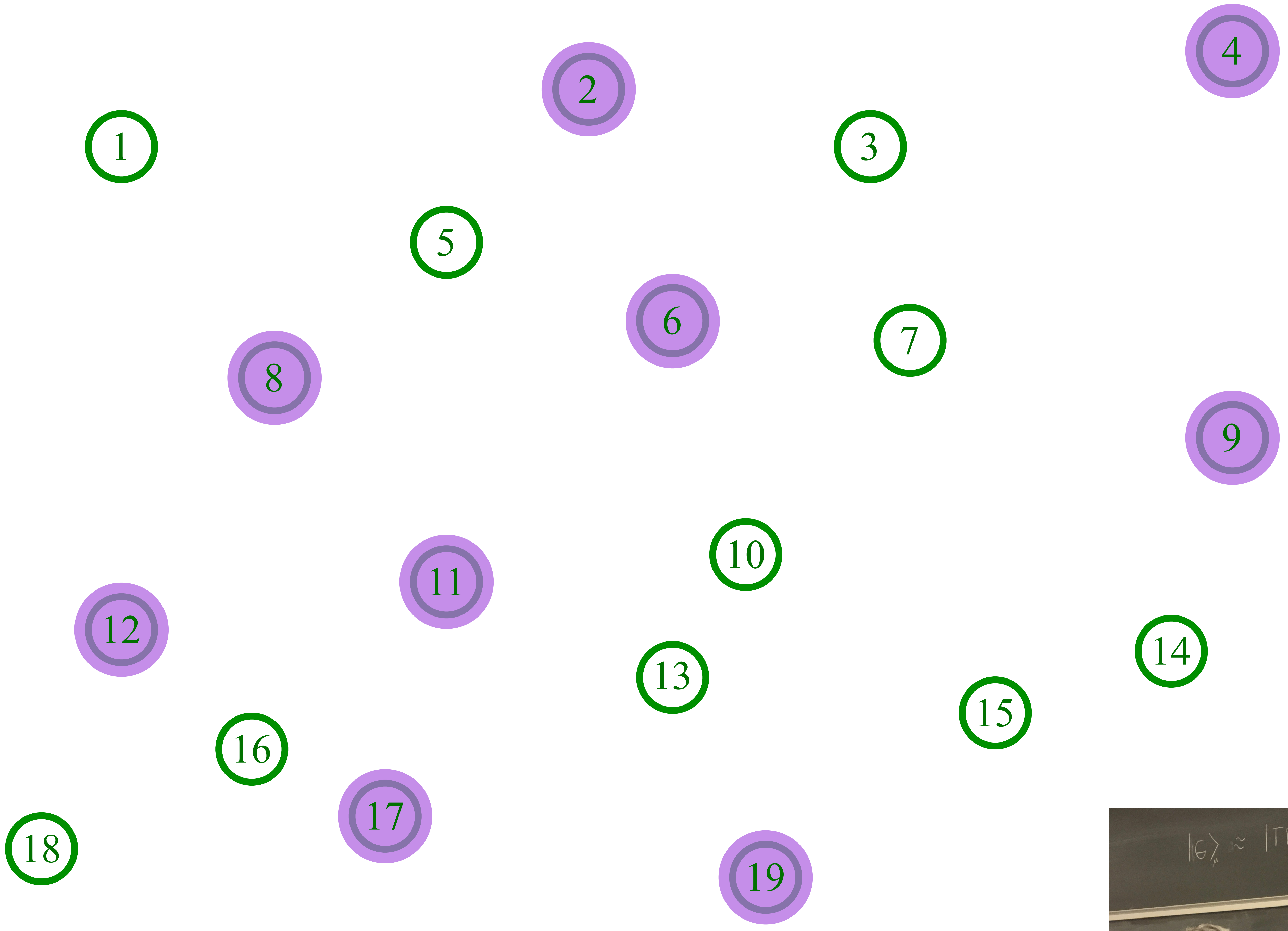
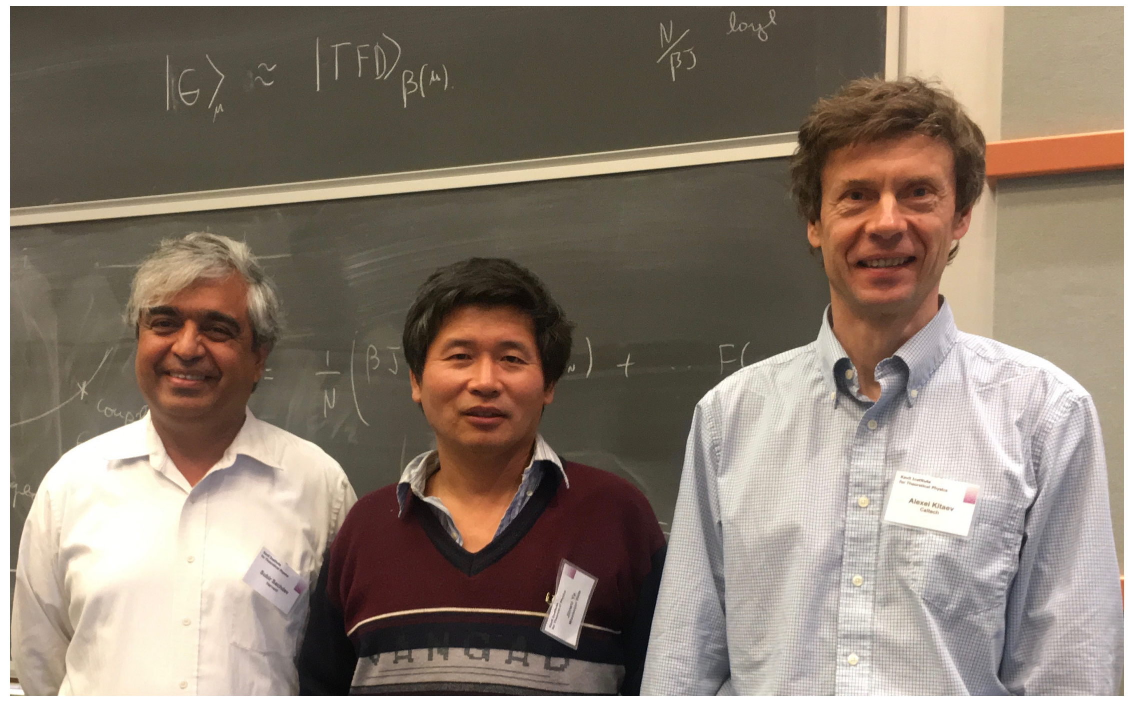


Place electrons randomly on some sites



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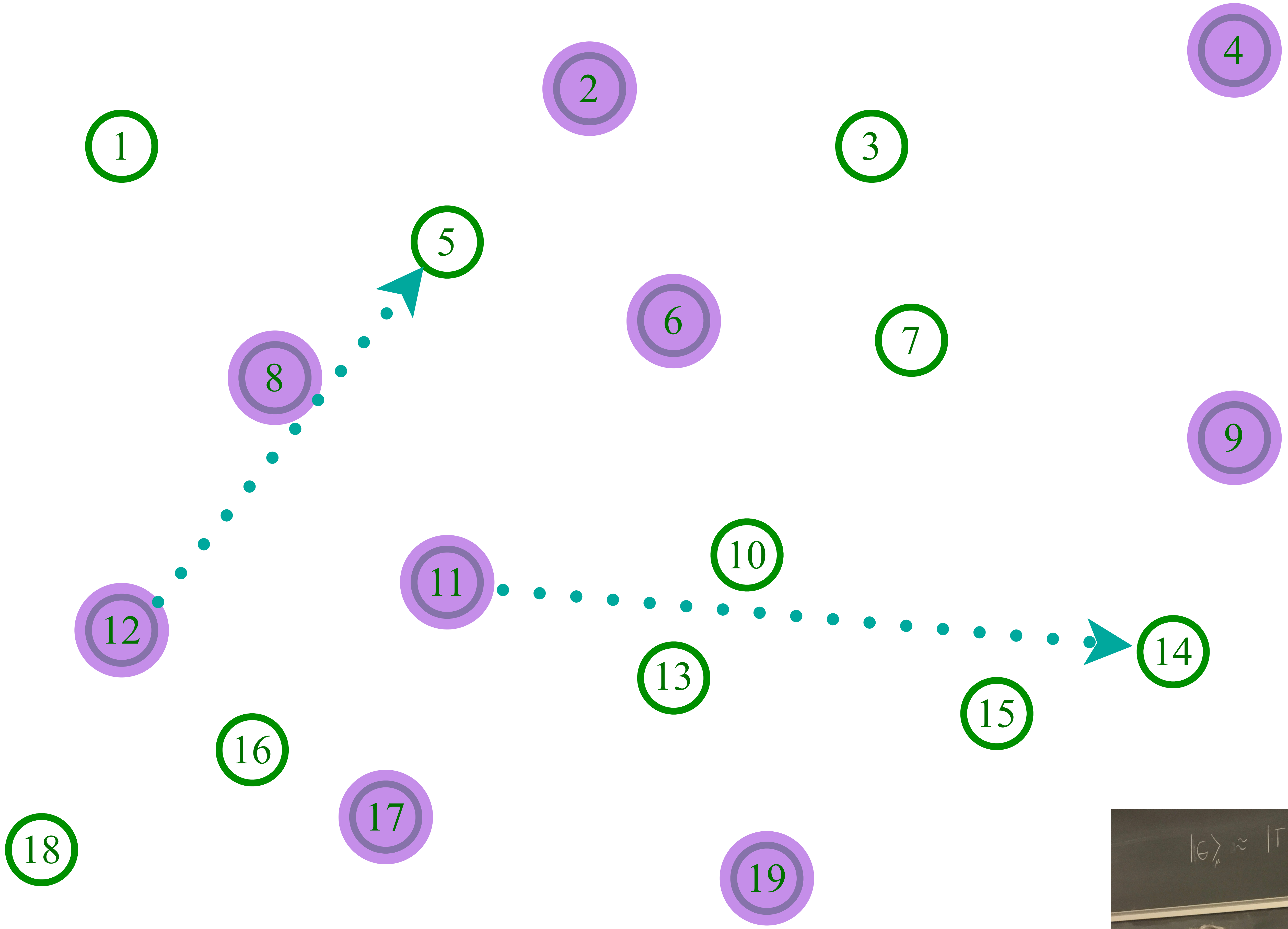
$$U_{11,12;5,14}$$



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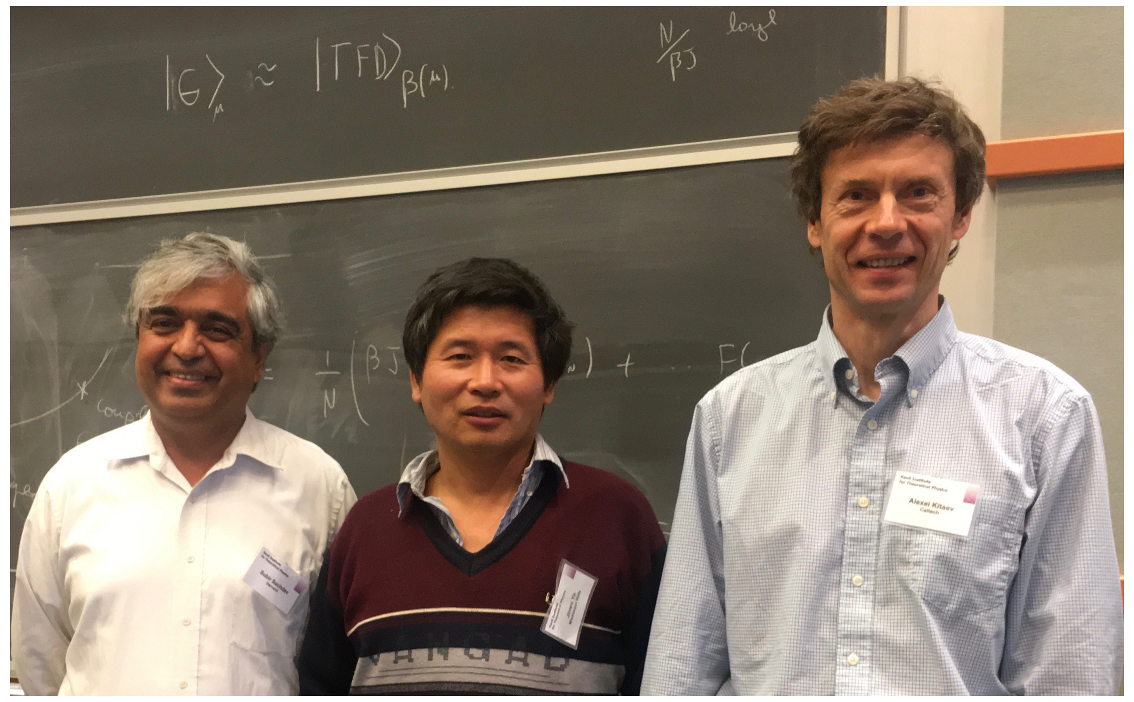

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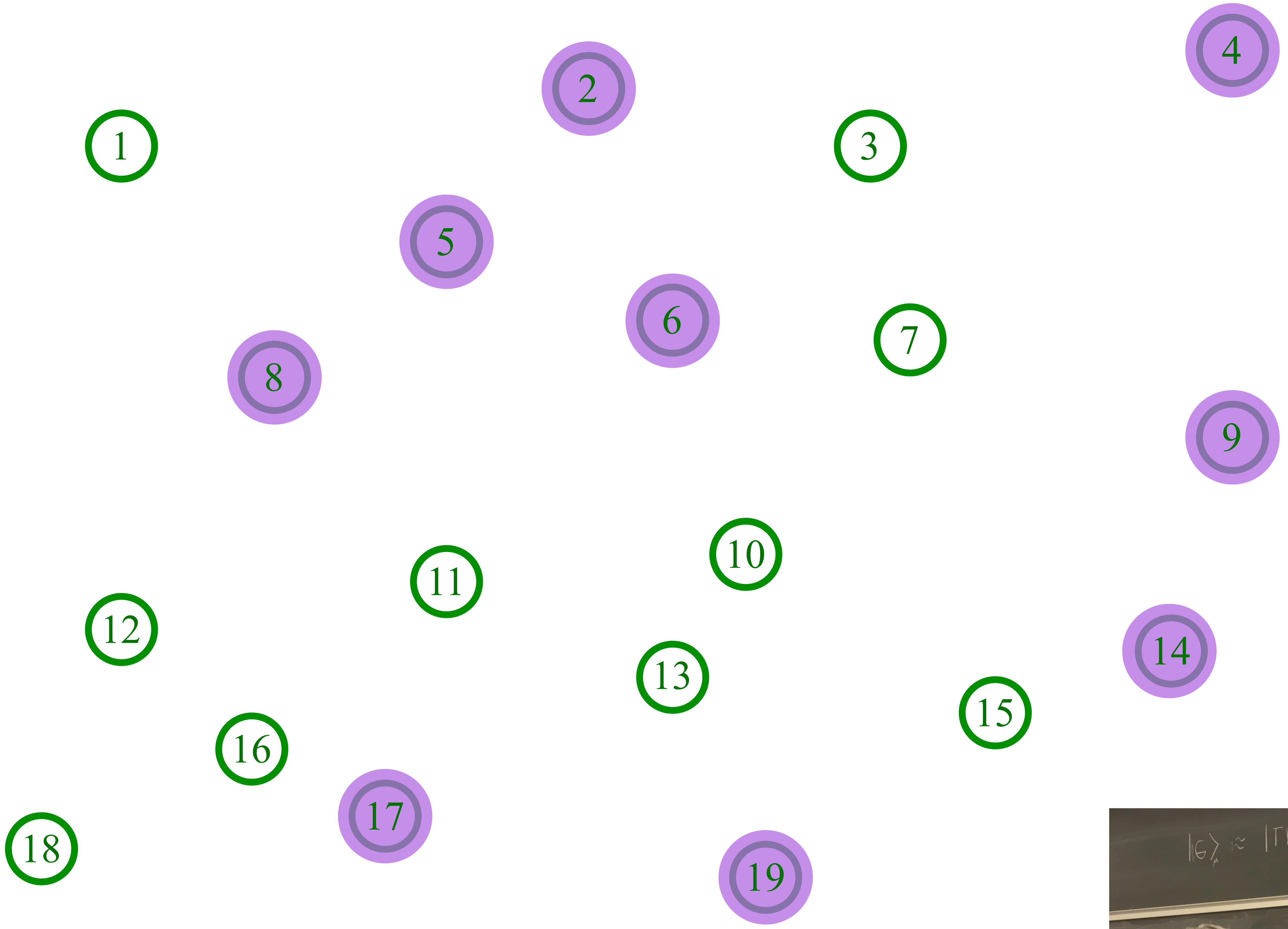
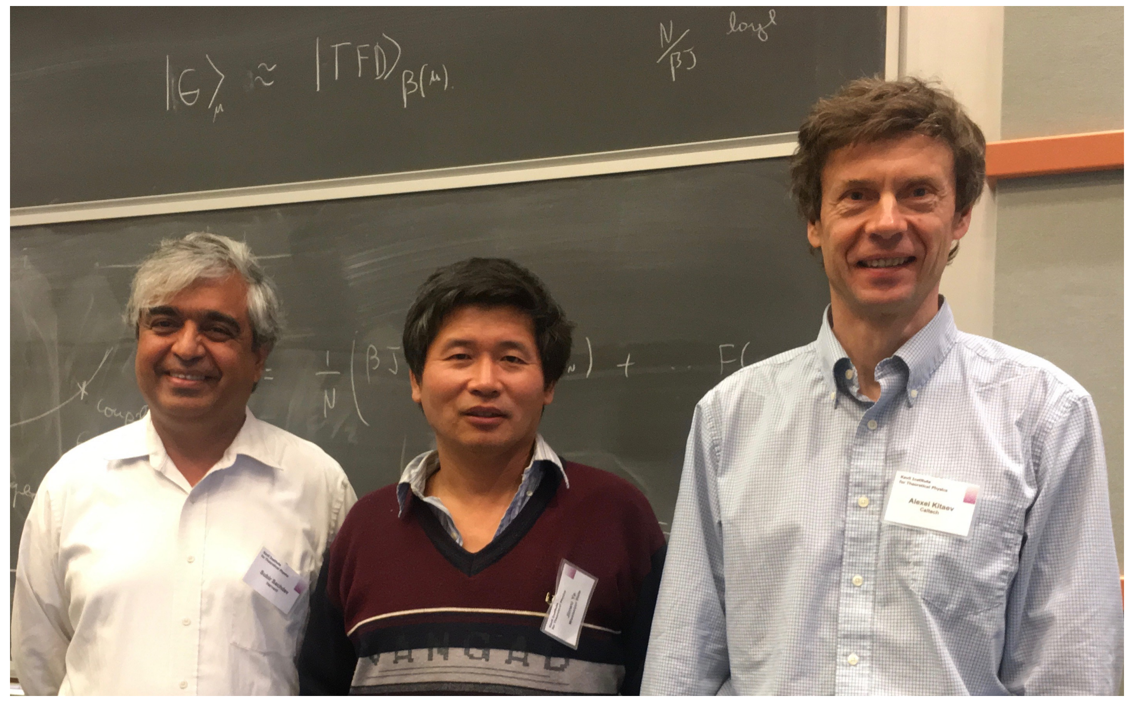


Entangle electrons pairwise randomly



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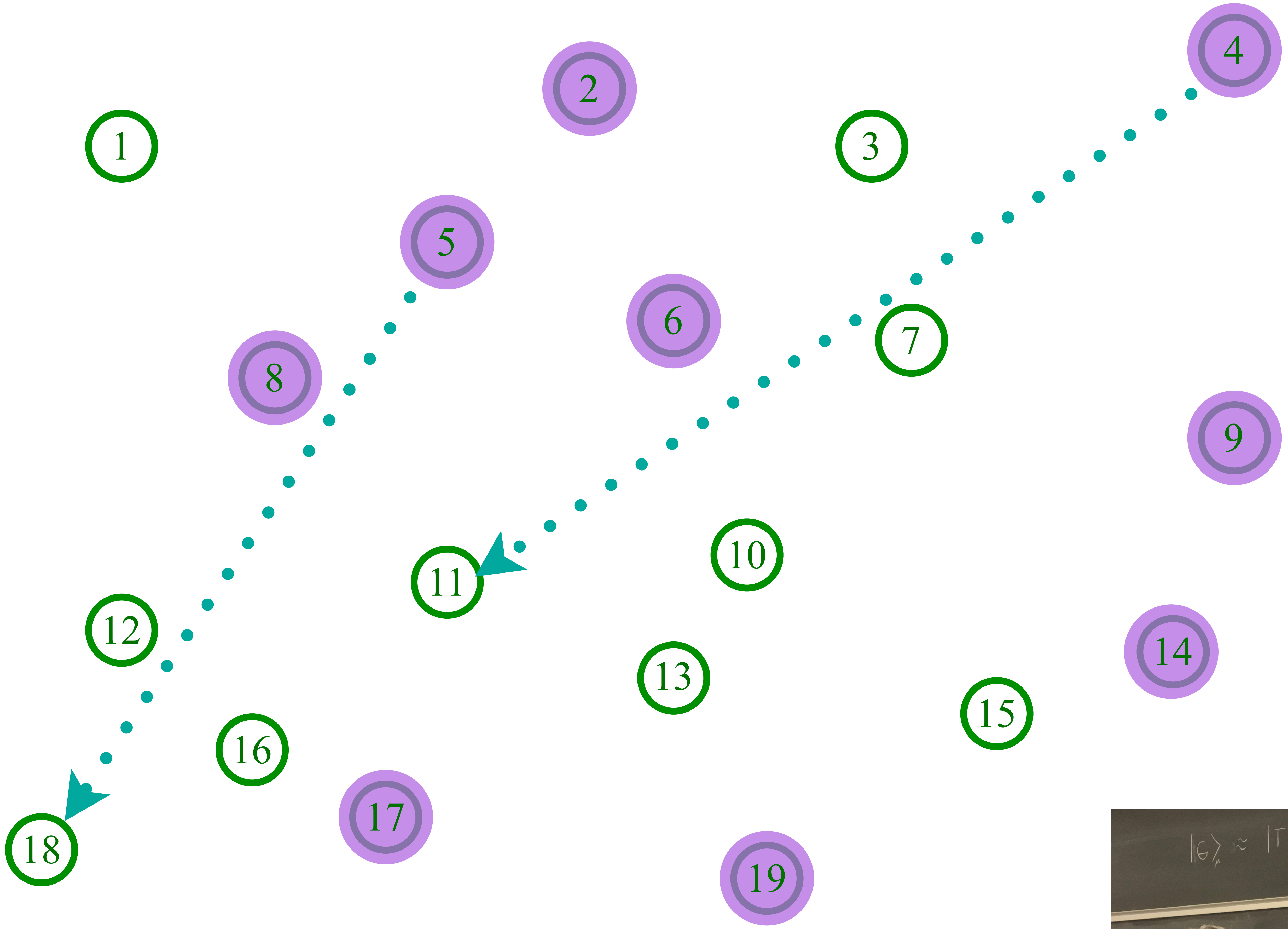
Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



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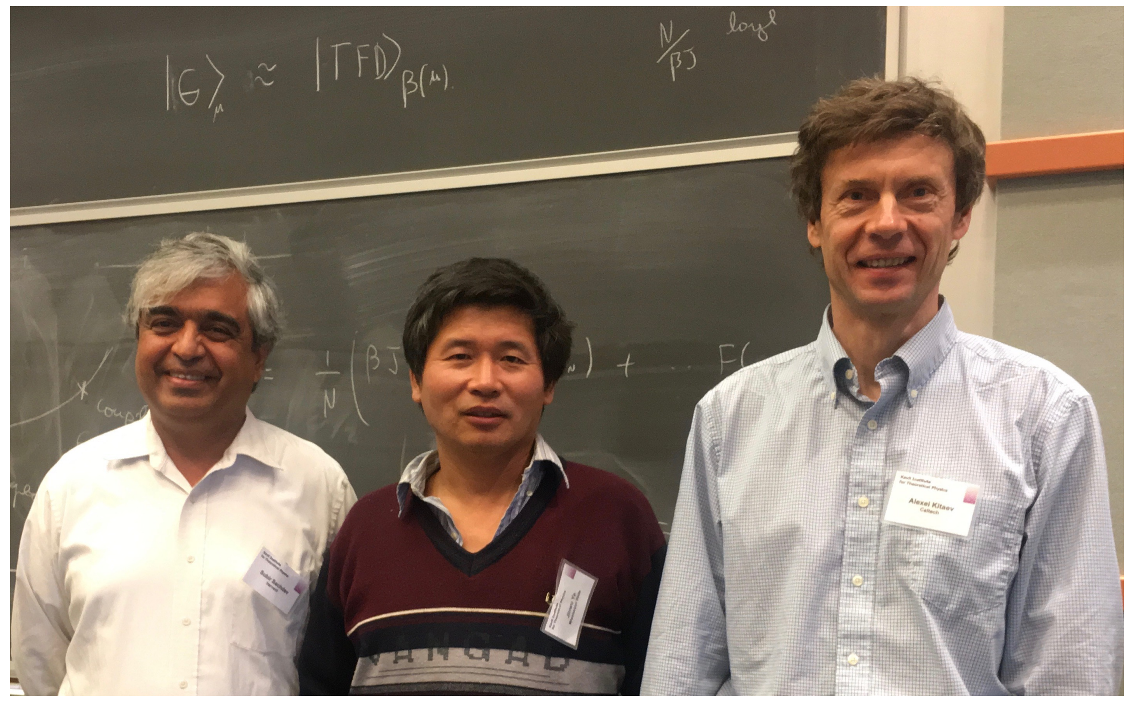

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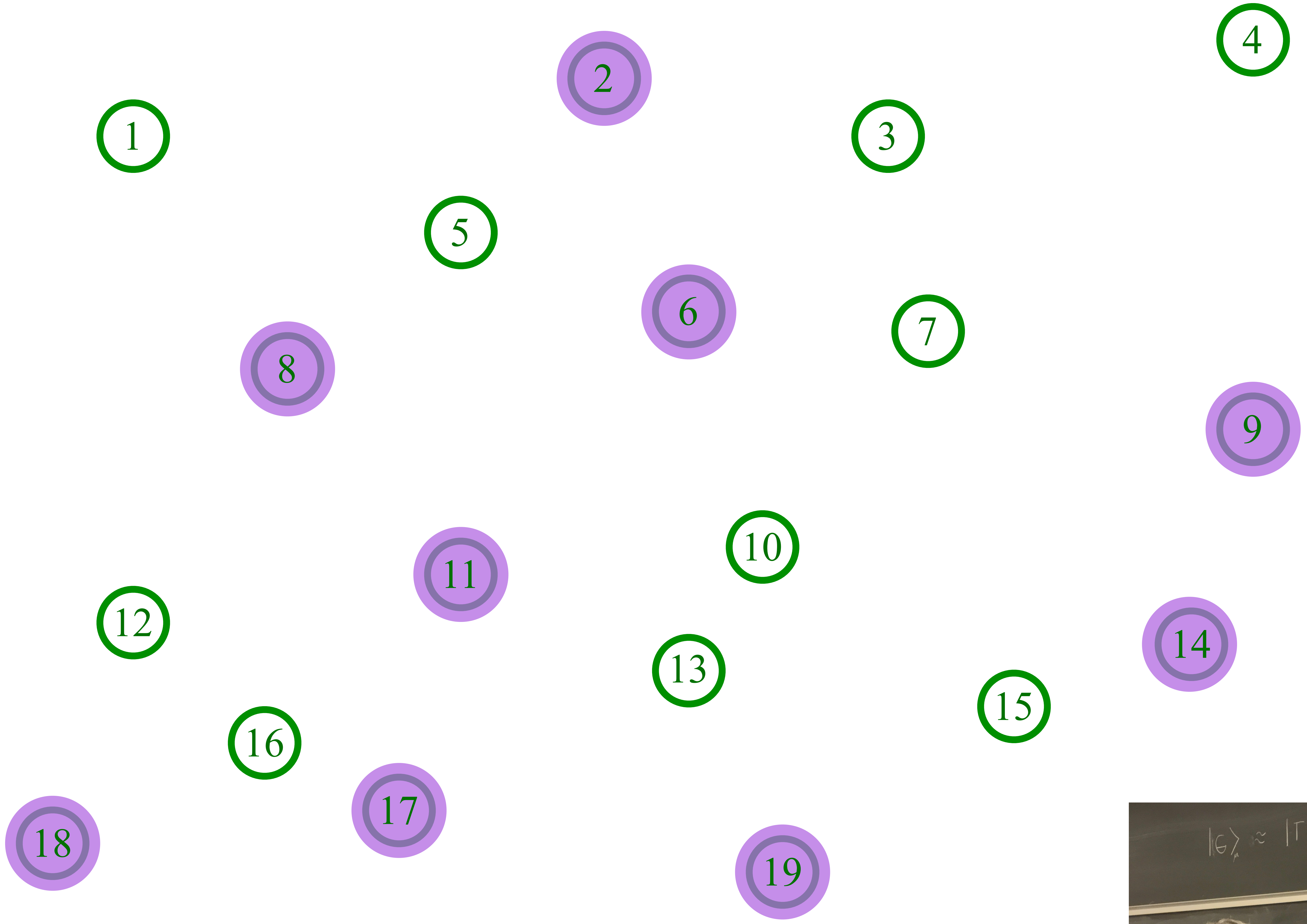

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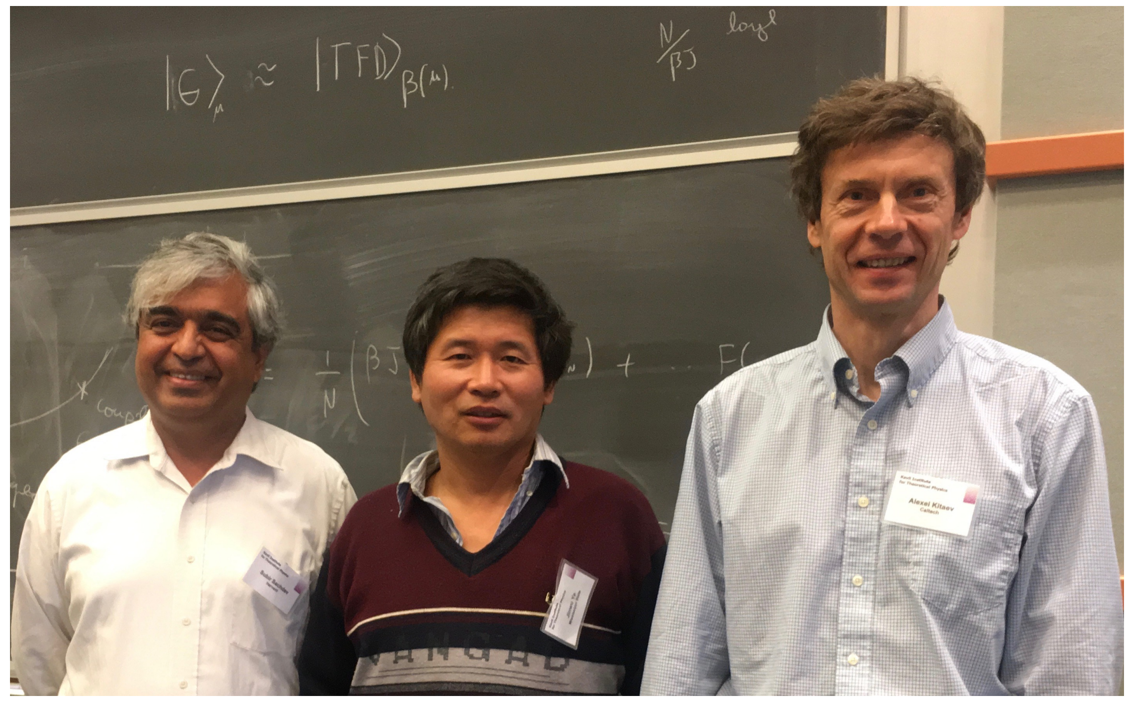

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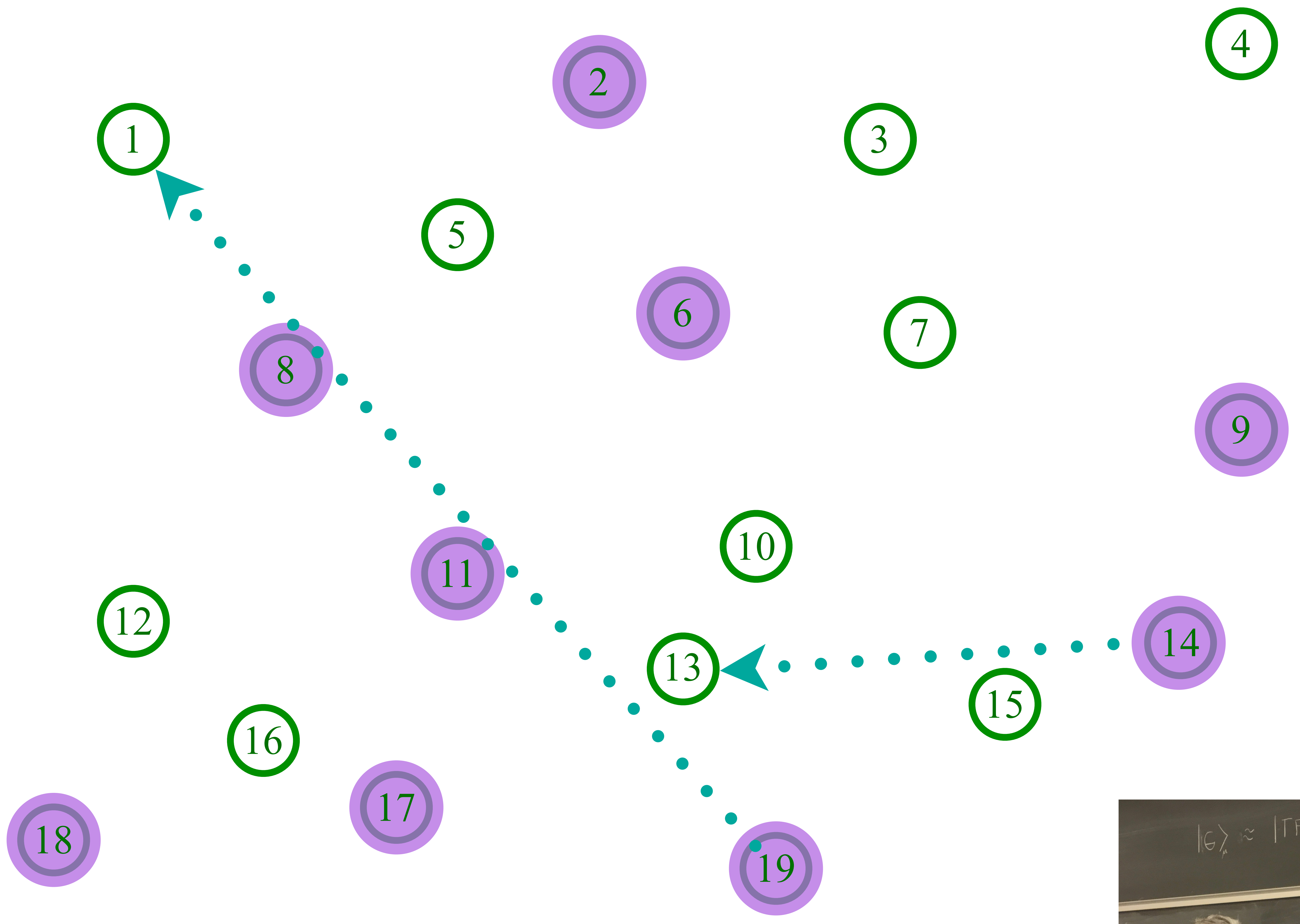

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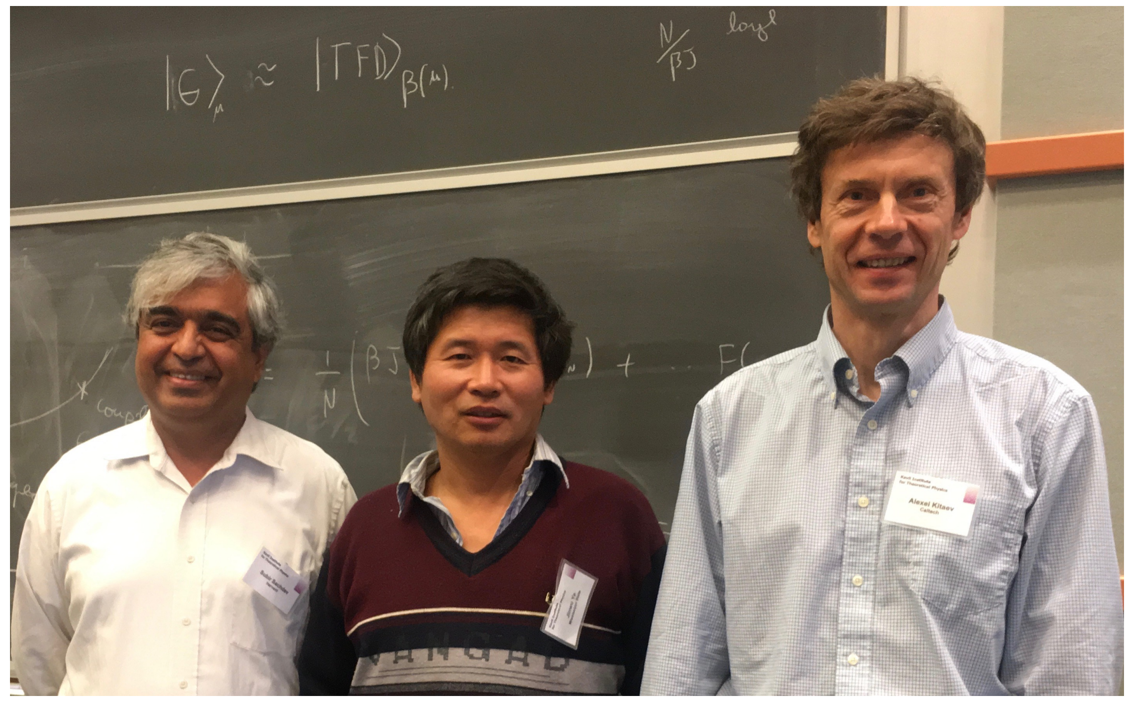

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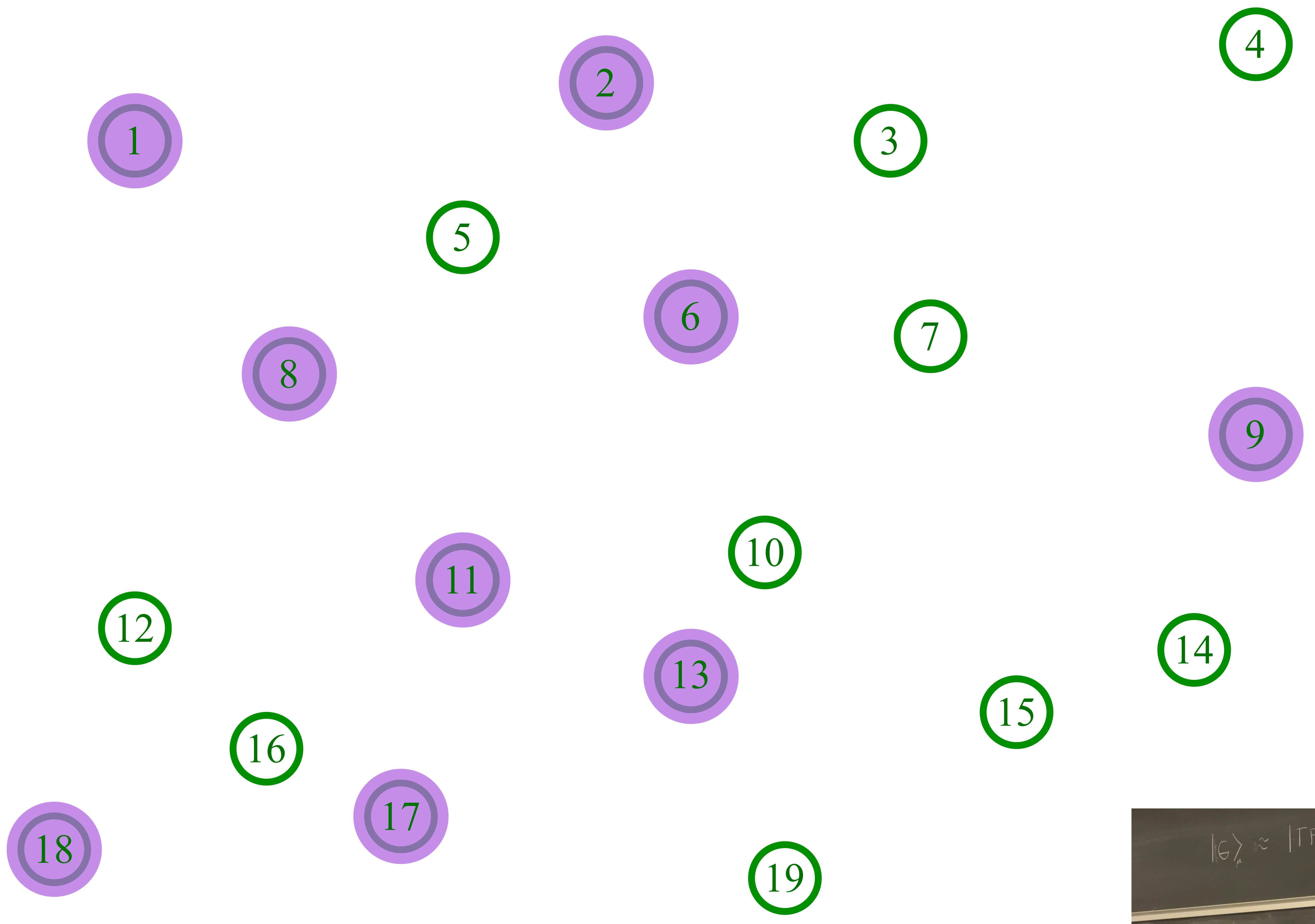

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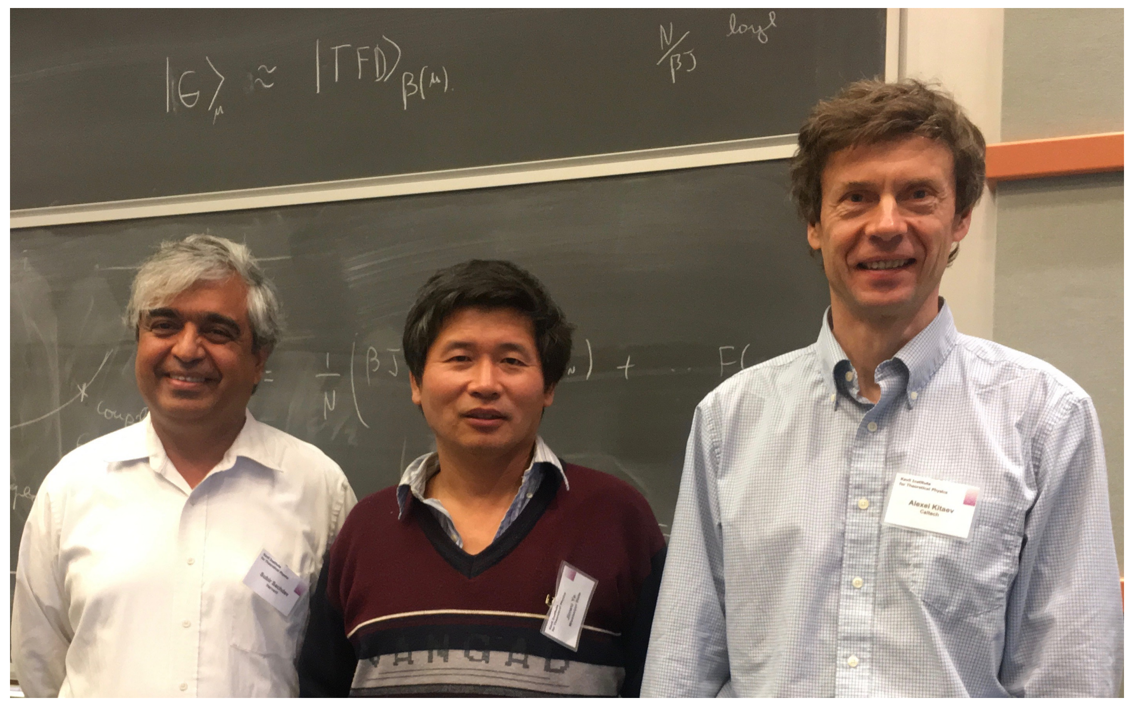

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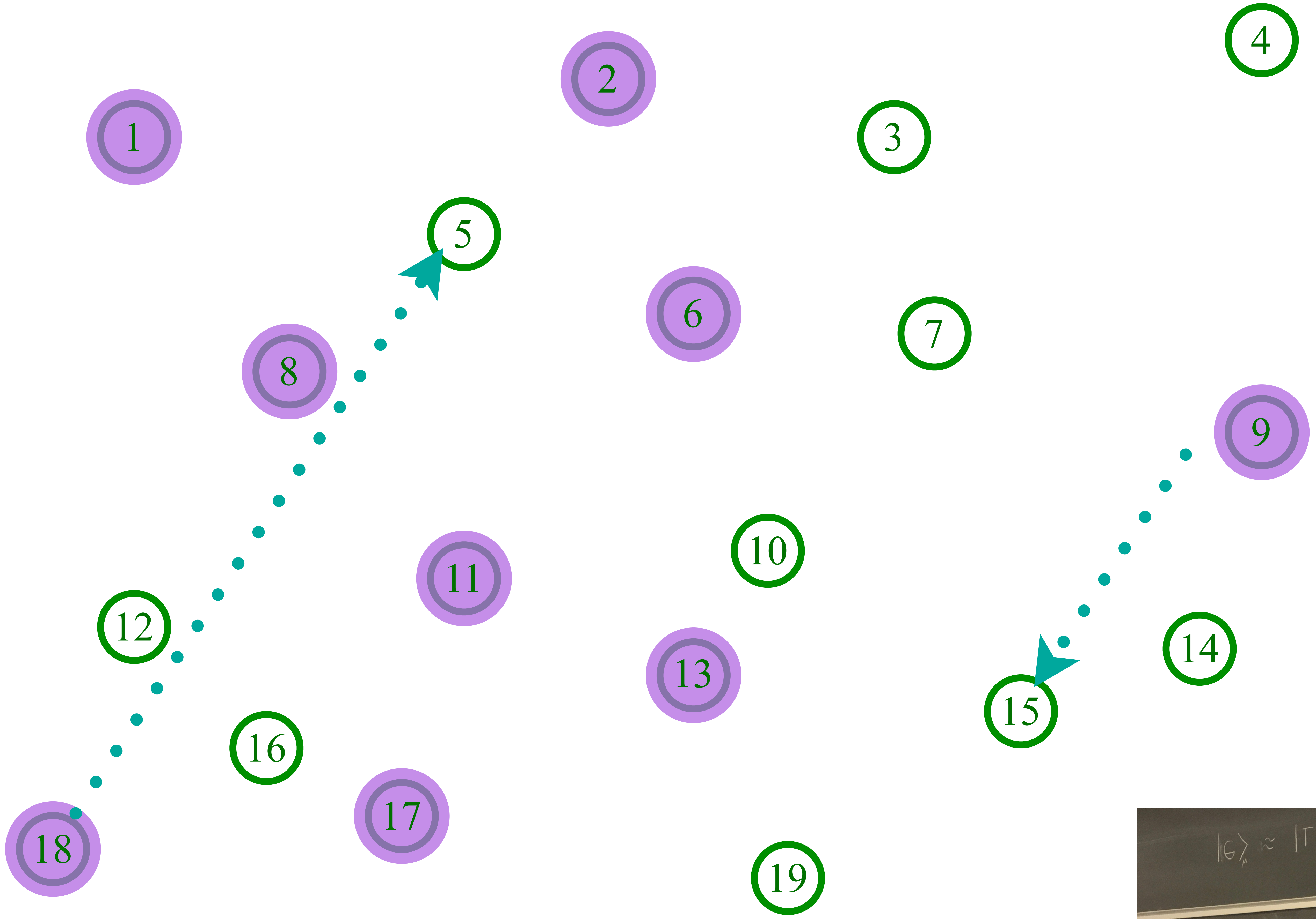

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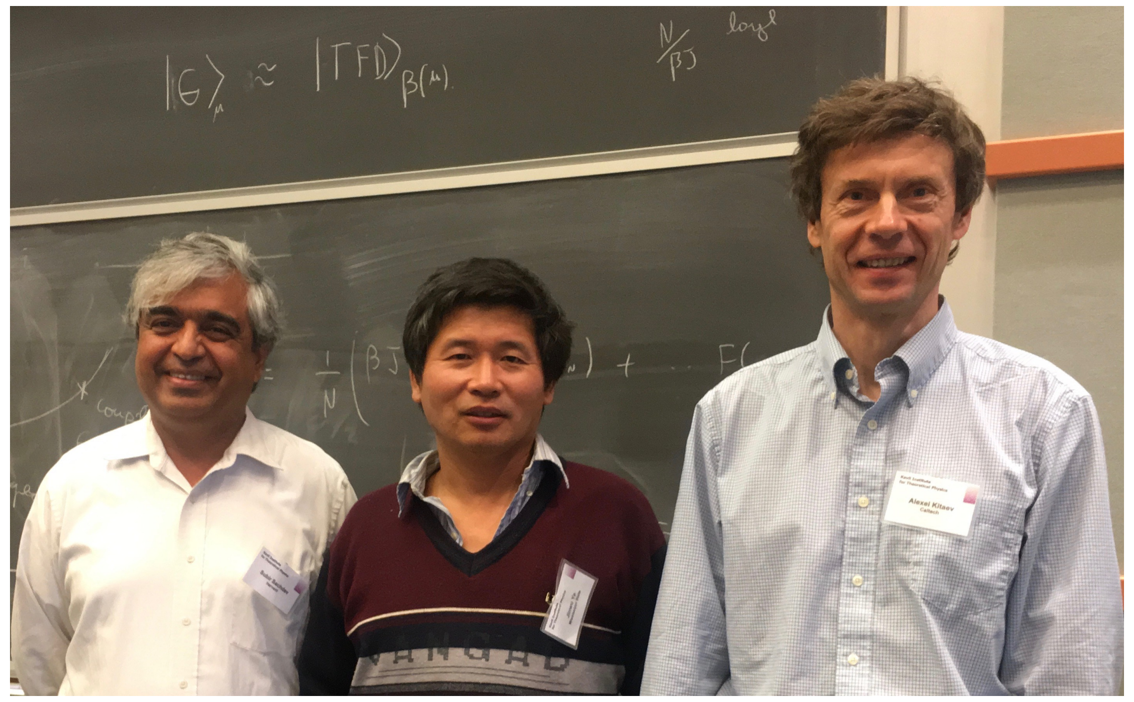

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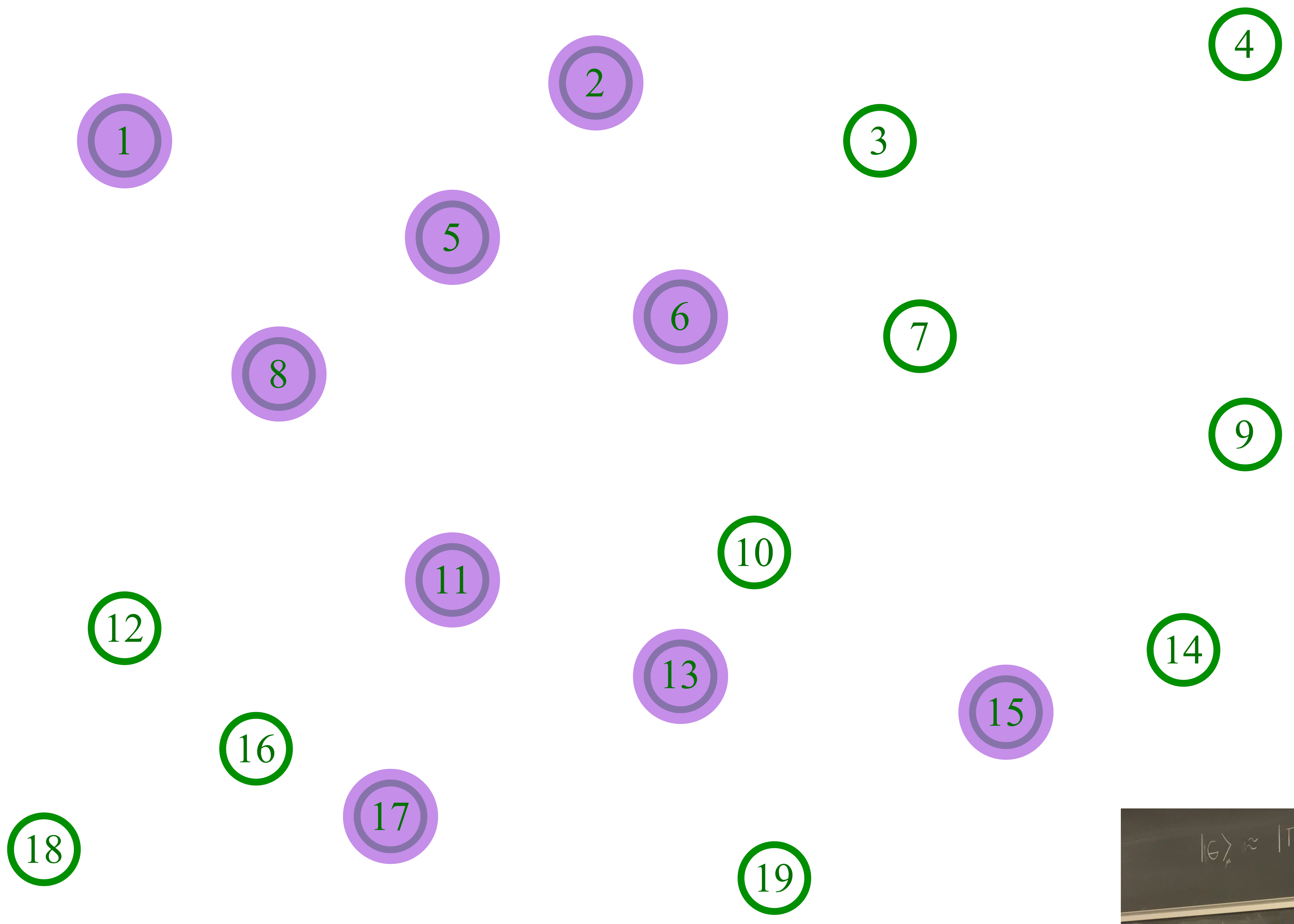

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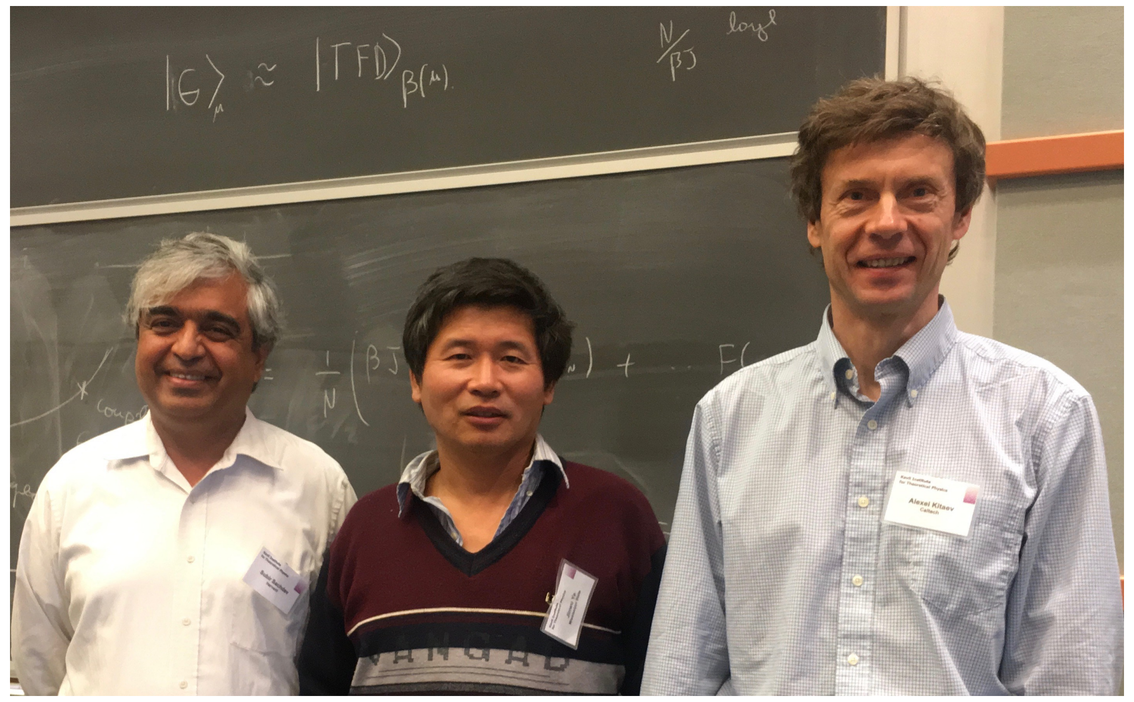
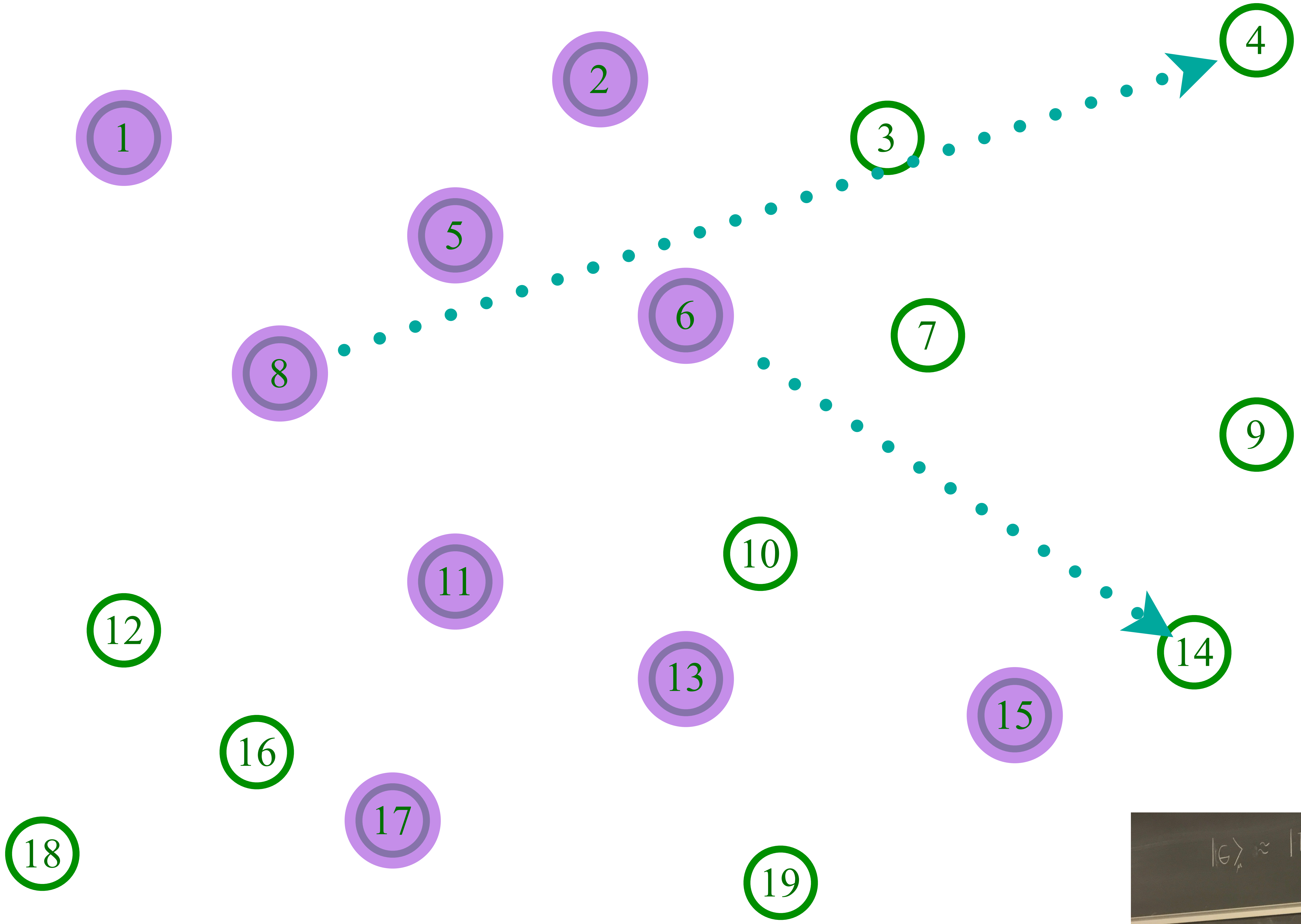
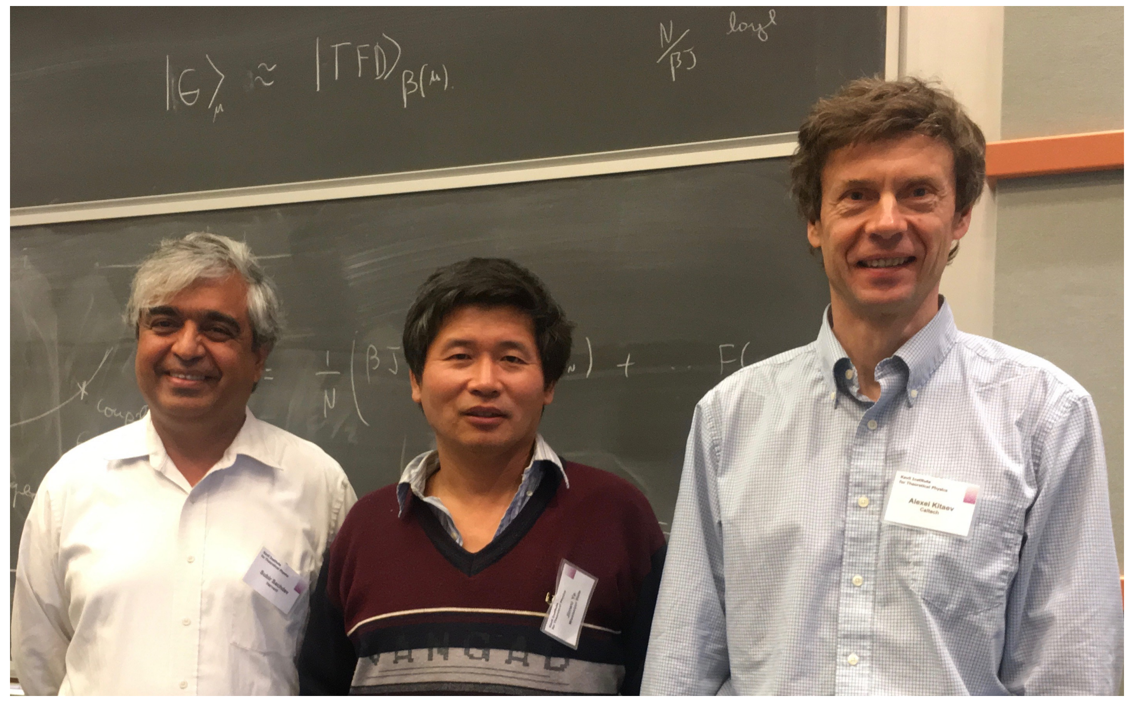


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$





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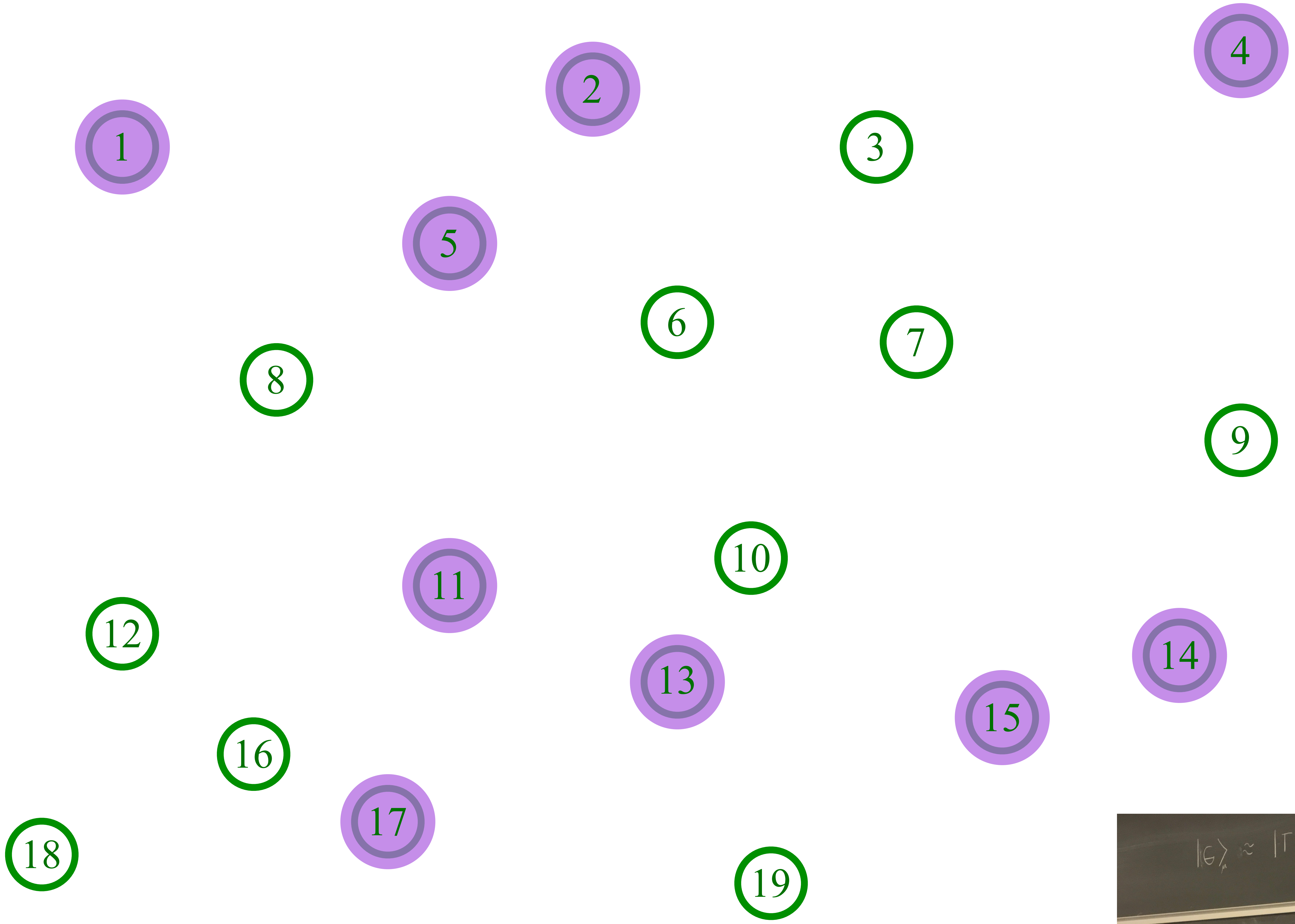

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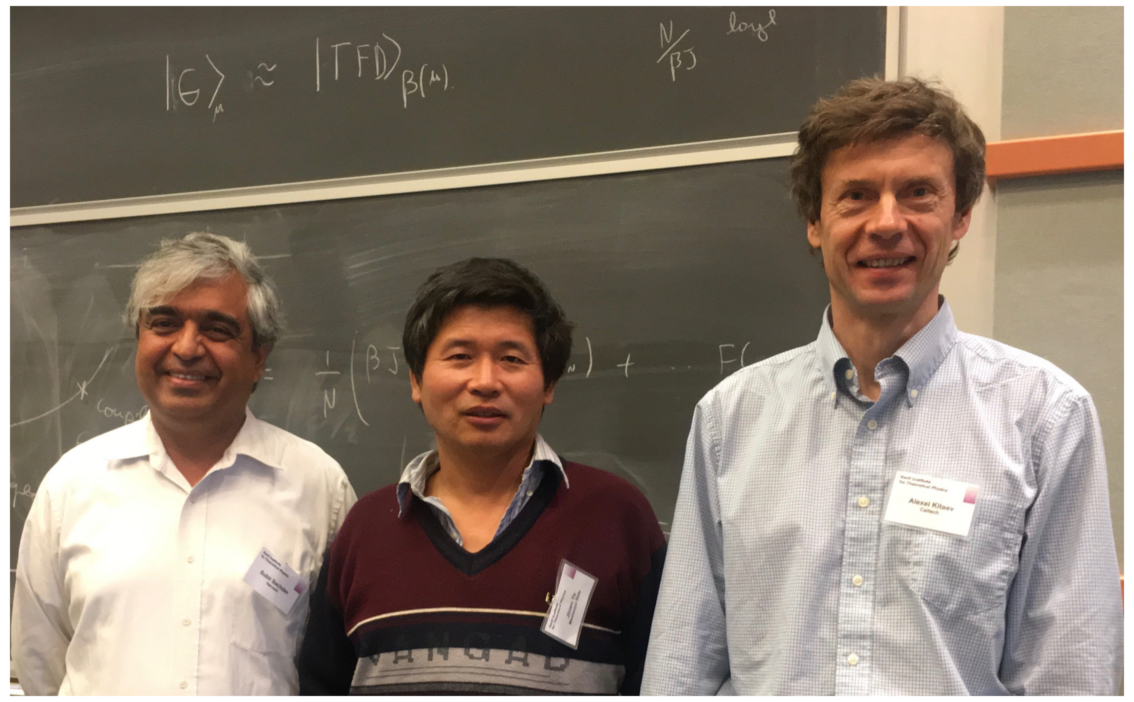


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

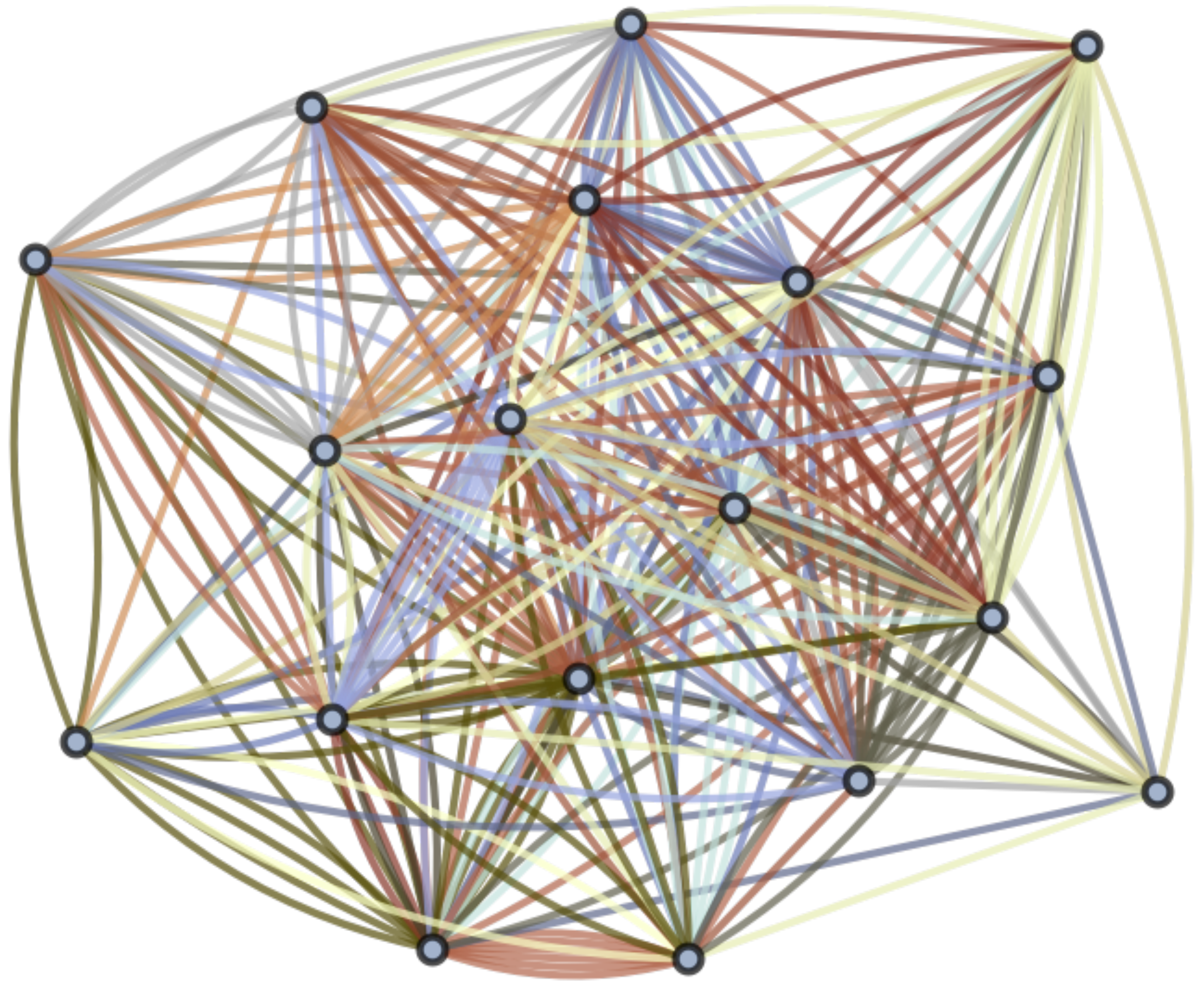
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

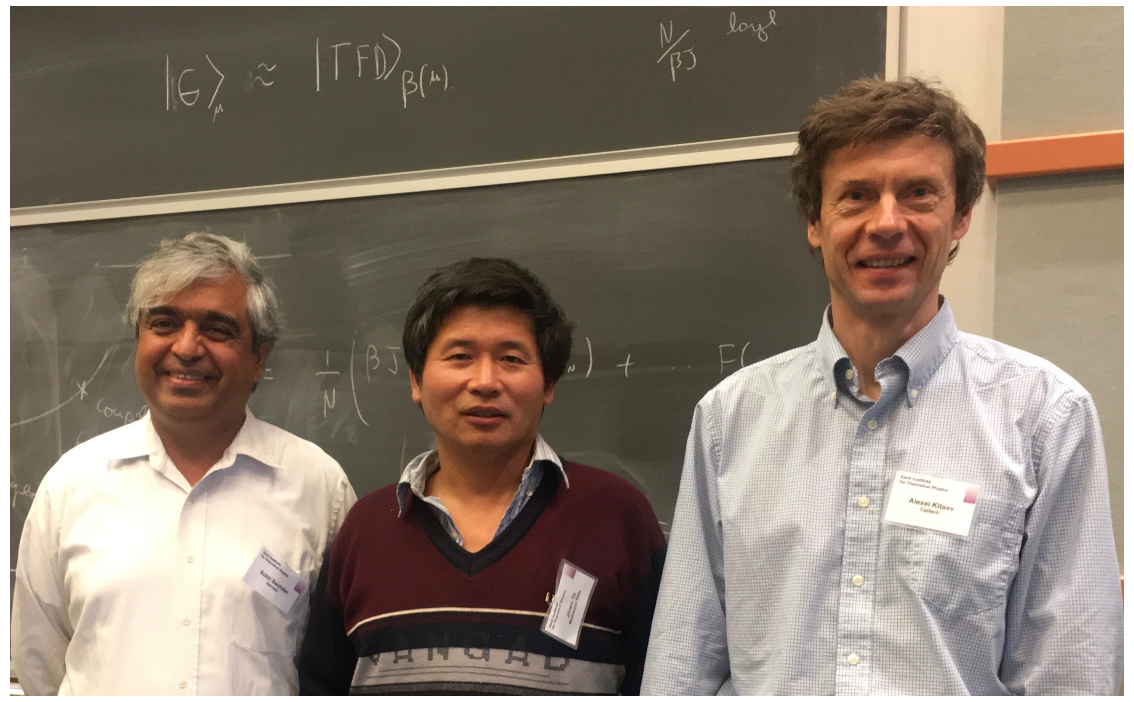
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





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- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

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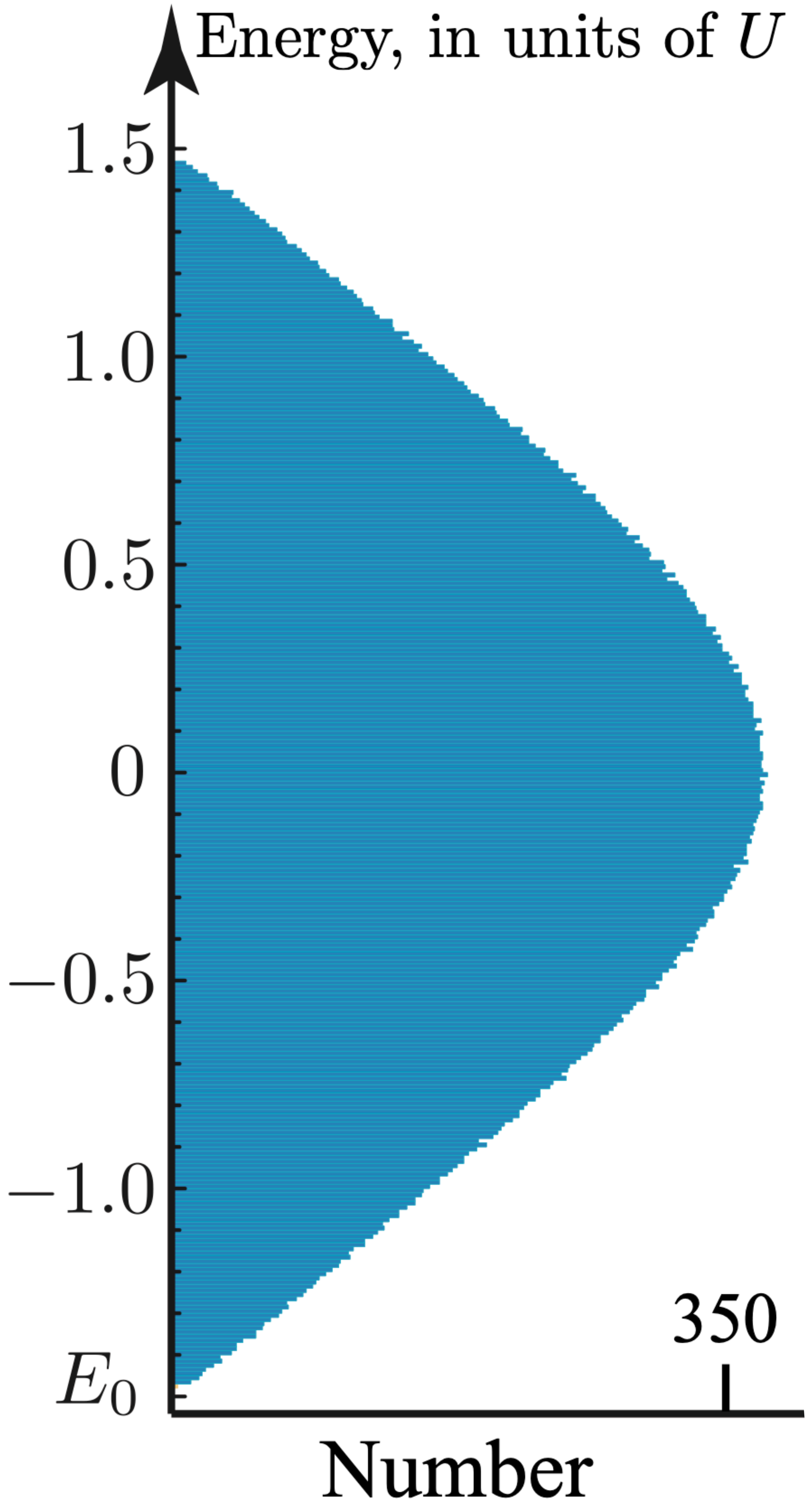
$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Many-body density of states

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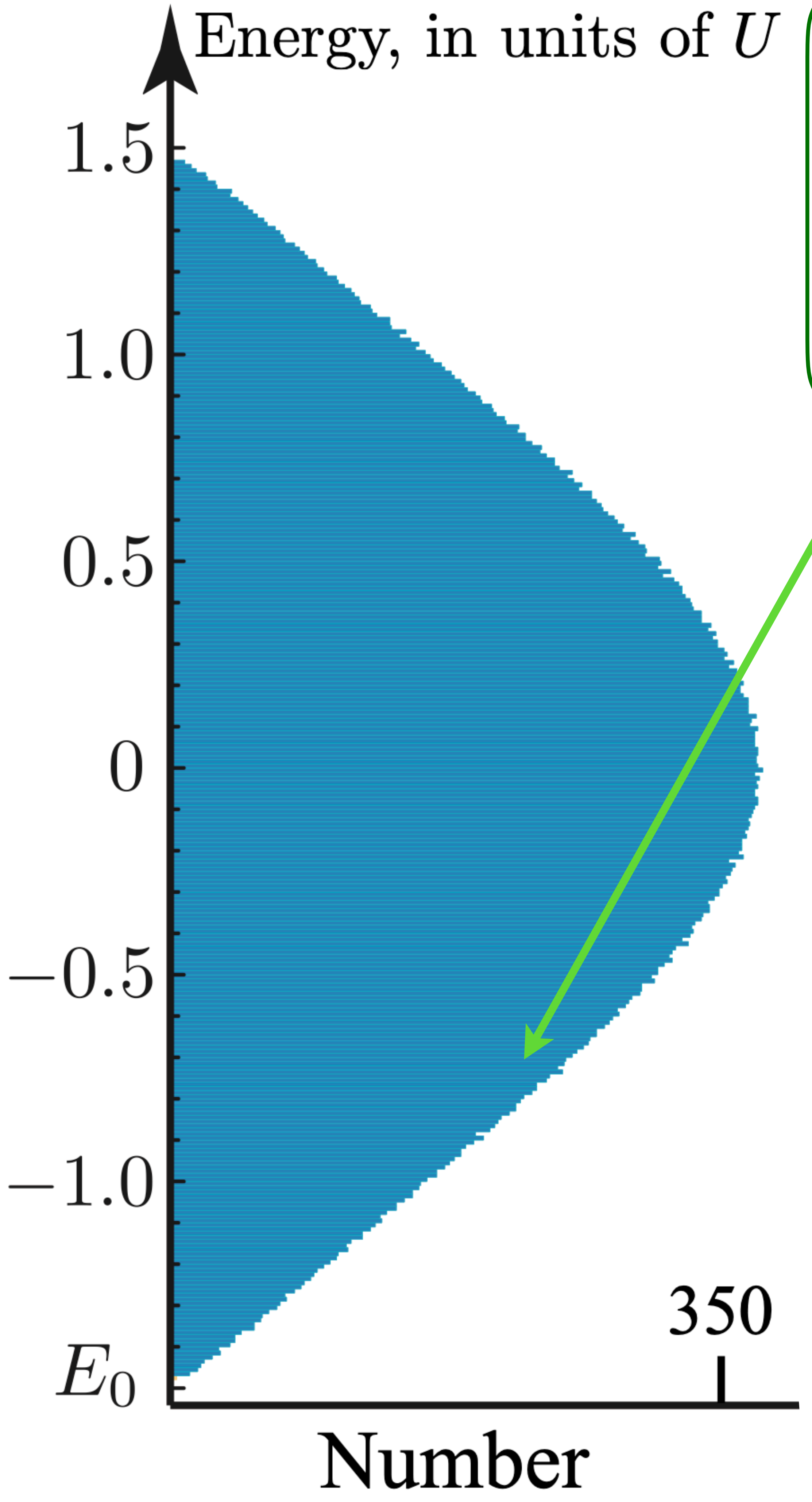

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$$= e^{N s_0 + \sqrt{2N\gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

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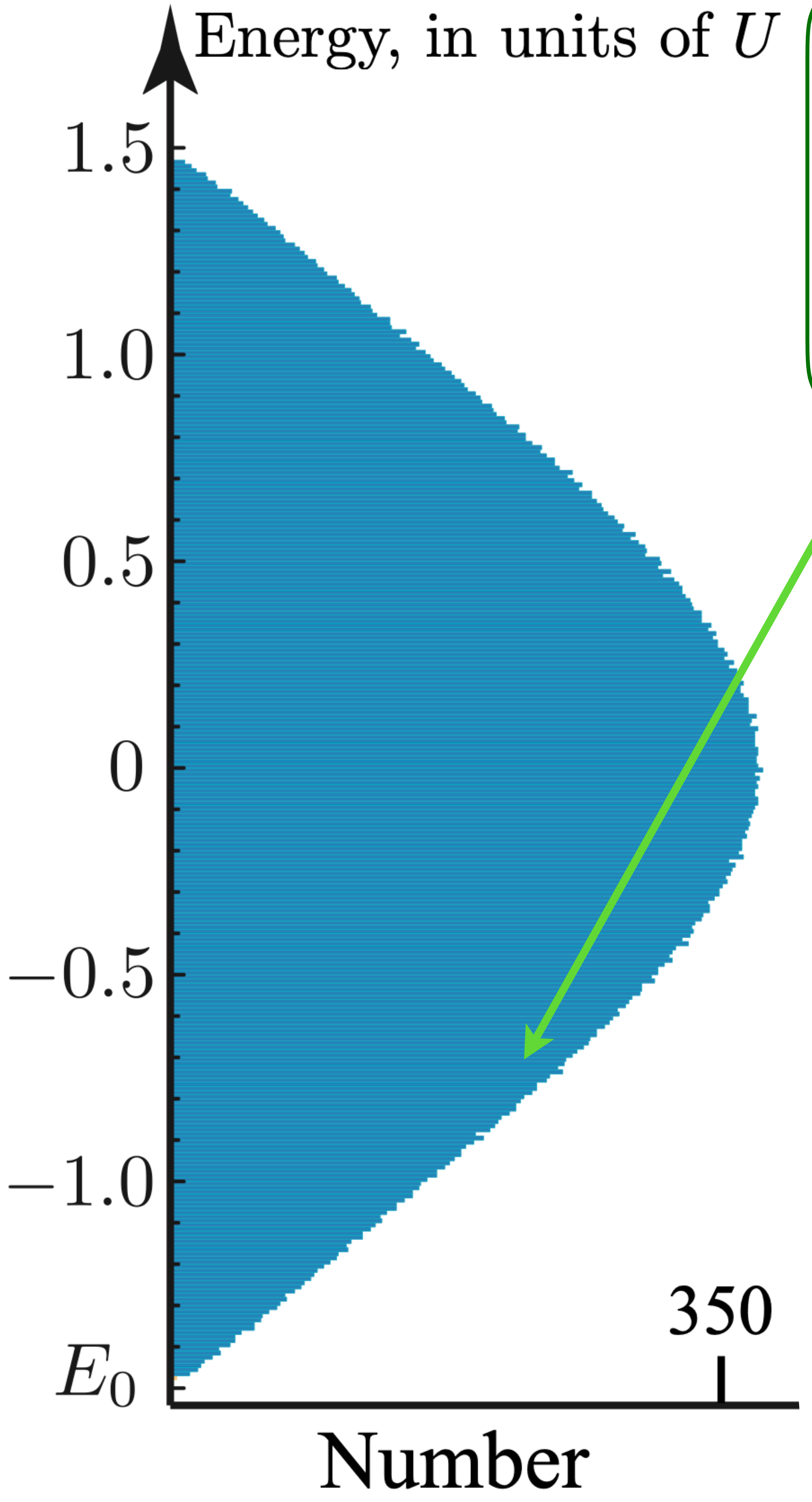

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$s_0 = 0.464848\ldots$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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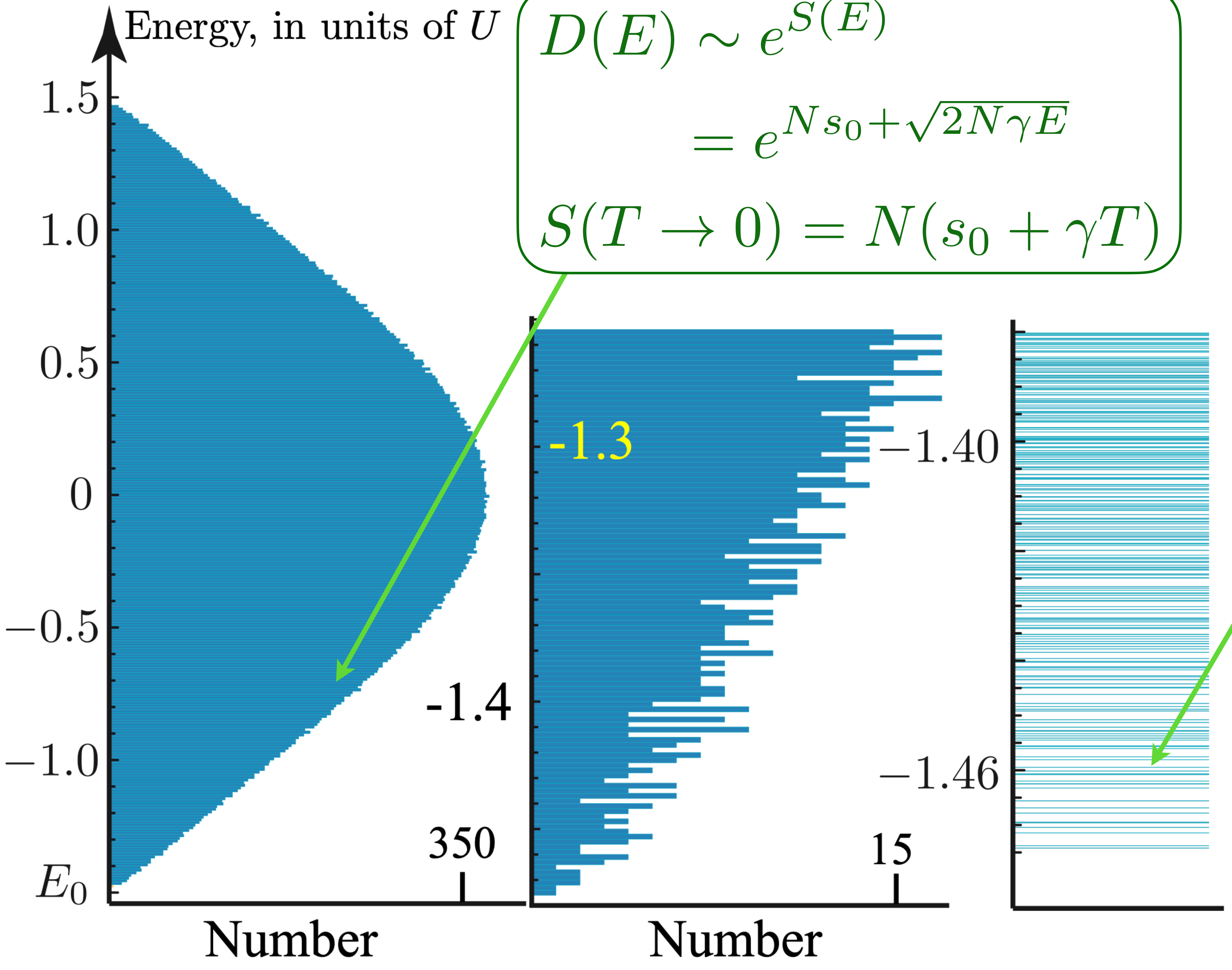

$$D(E) \sim 2\, e^{N s_0} \sqrt{2N\gamma E}$$

No particle-like decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Maldacena, Stanford (2016); Bagrets *et al.* (2017); Cotler *et al.* (2017); Kitaev, Suh (2017); Stanford, Witten (2017)

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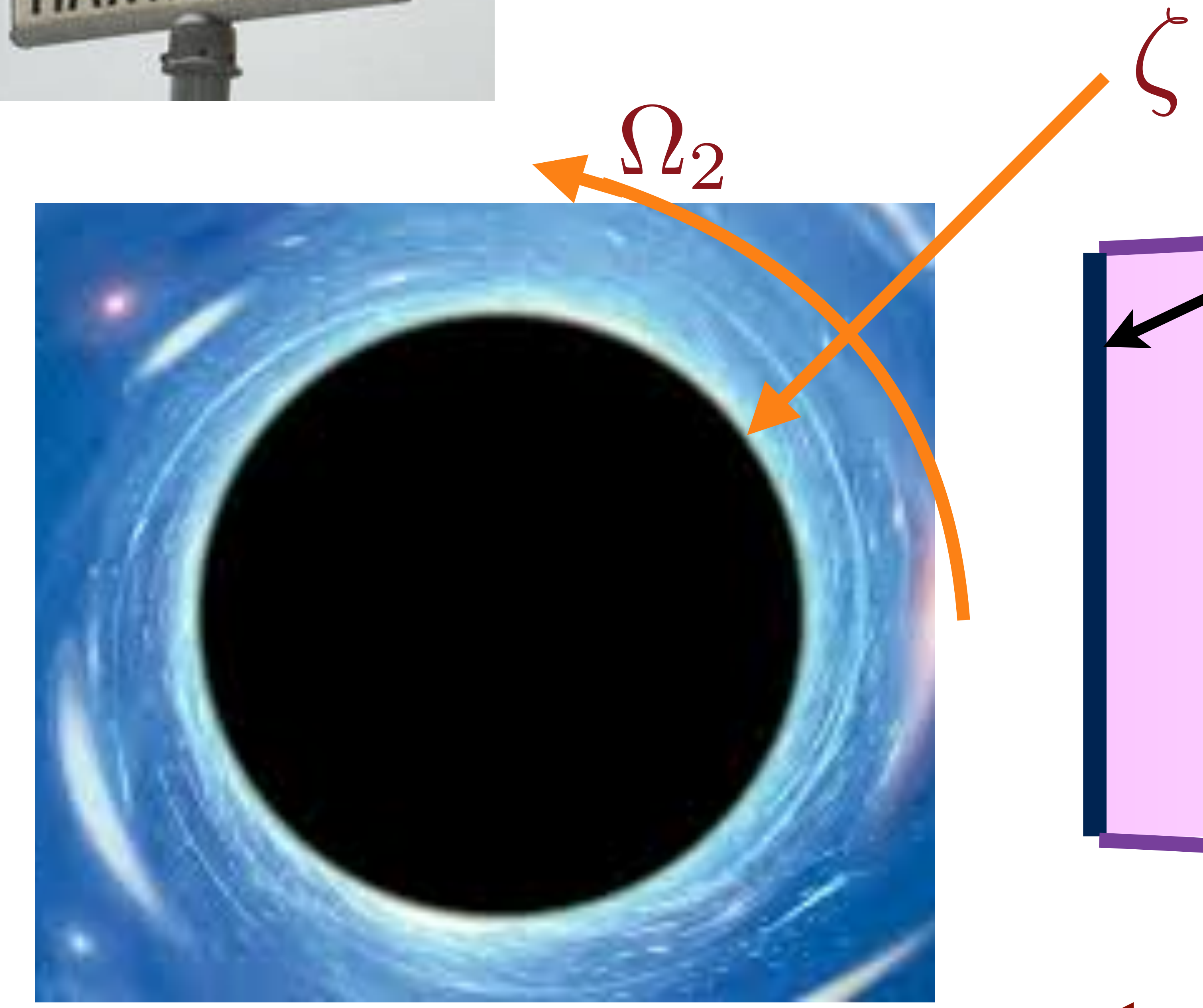
- The $D(E)$ is determined by a time-reparameterization $\tau \to f(\tau)$ mode (similar to the graviton being fluctuations of the spacetime metric), and a phase mode $\phi(\tau)$:

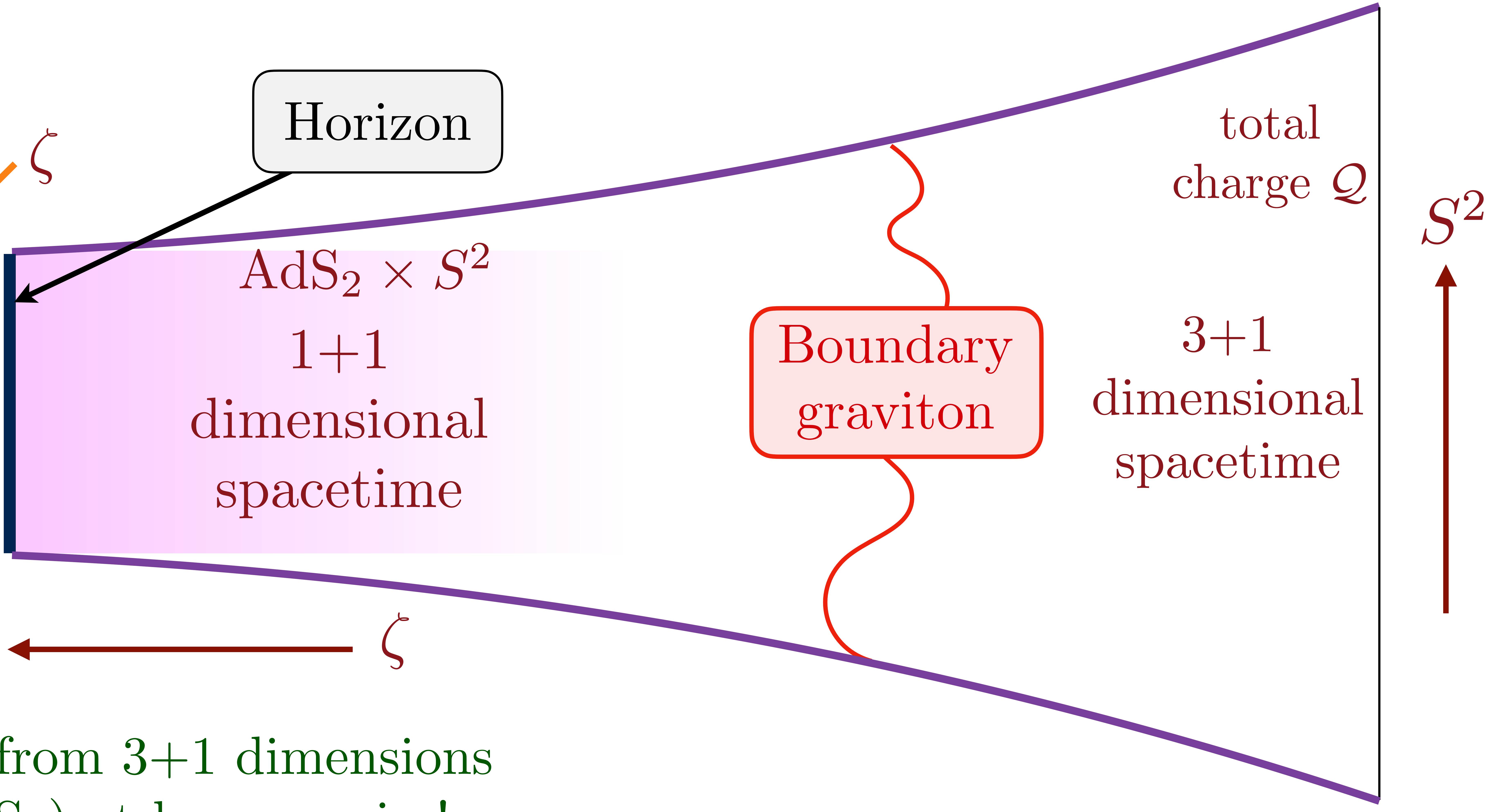
$$\mathcal{Z}_{SYK} = e^{N s_0} \int \mathcal{D}f \mathcal{D}\phi \exp\left(-\frac{1}{\hbar} \int_0^{\hbar/(k_B T)} d\tau \, \mathcal{L}_{SYK}[f, \phi] \right)$$

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

From the SYK model to charged black holes

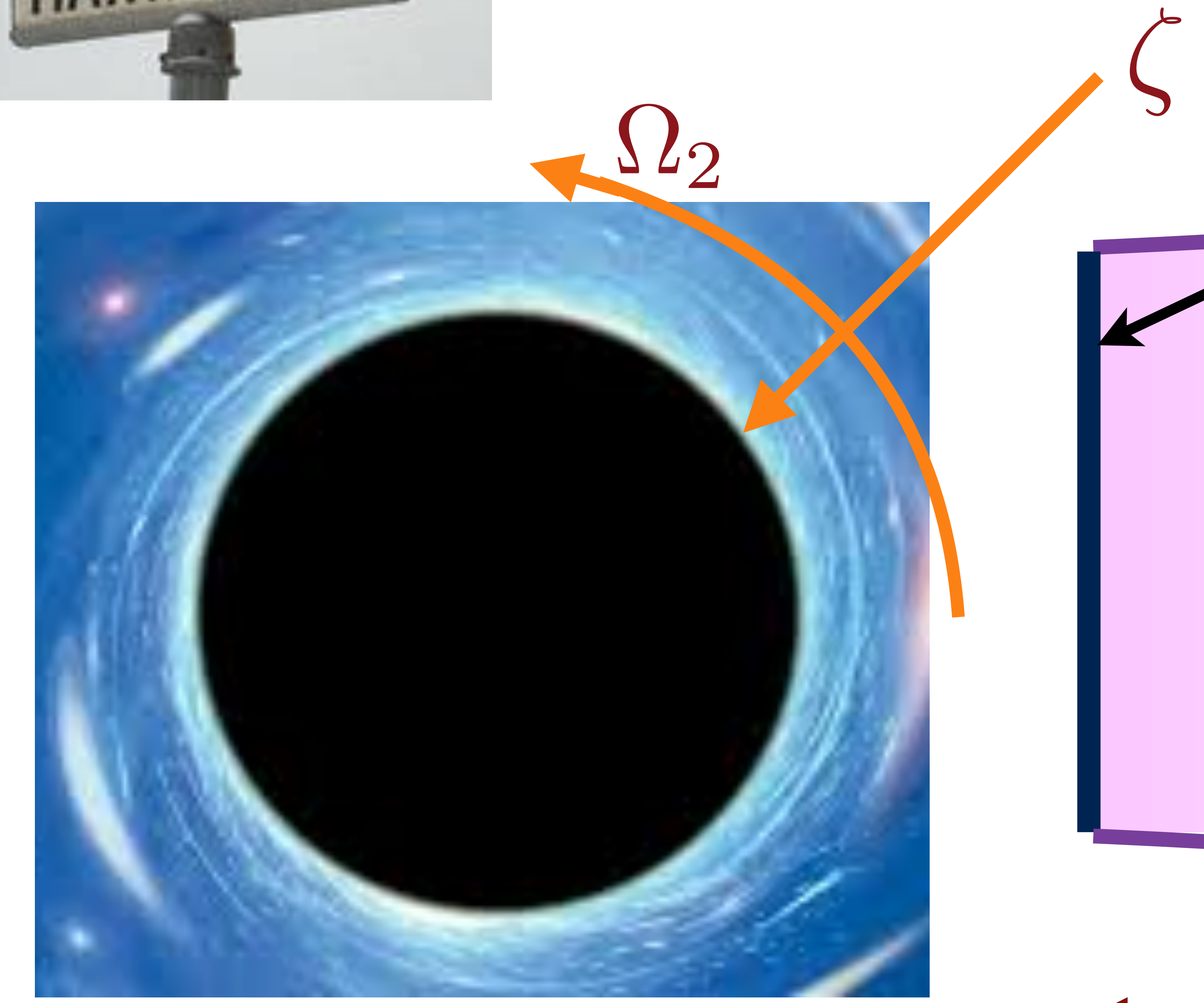
Reissner-Nordstrom black hole of Einstein-Maxwell theory

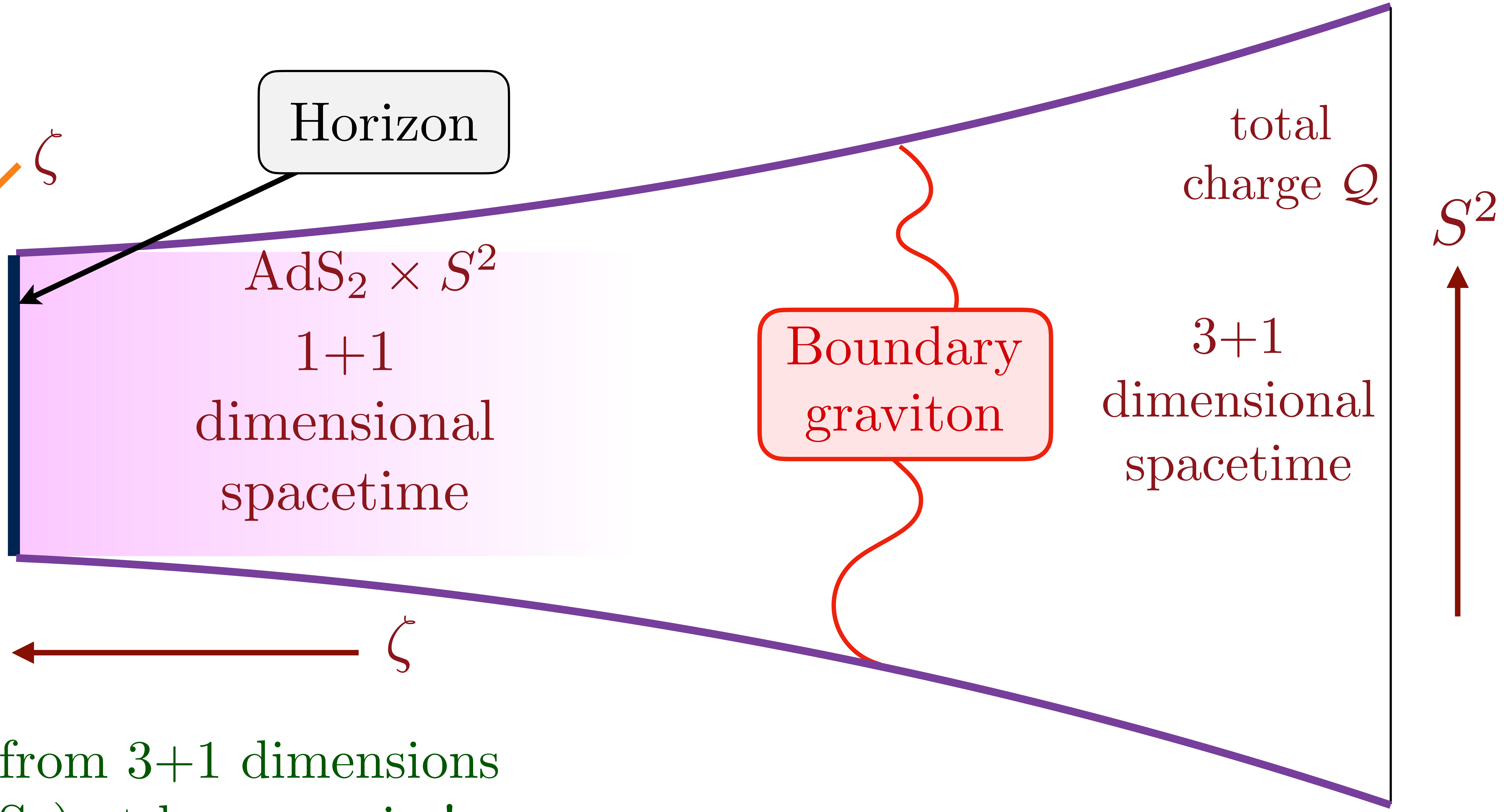




Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Reissner-Nordstrom black hole of Einstein-Maxwell theory

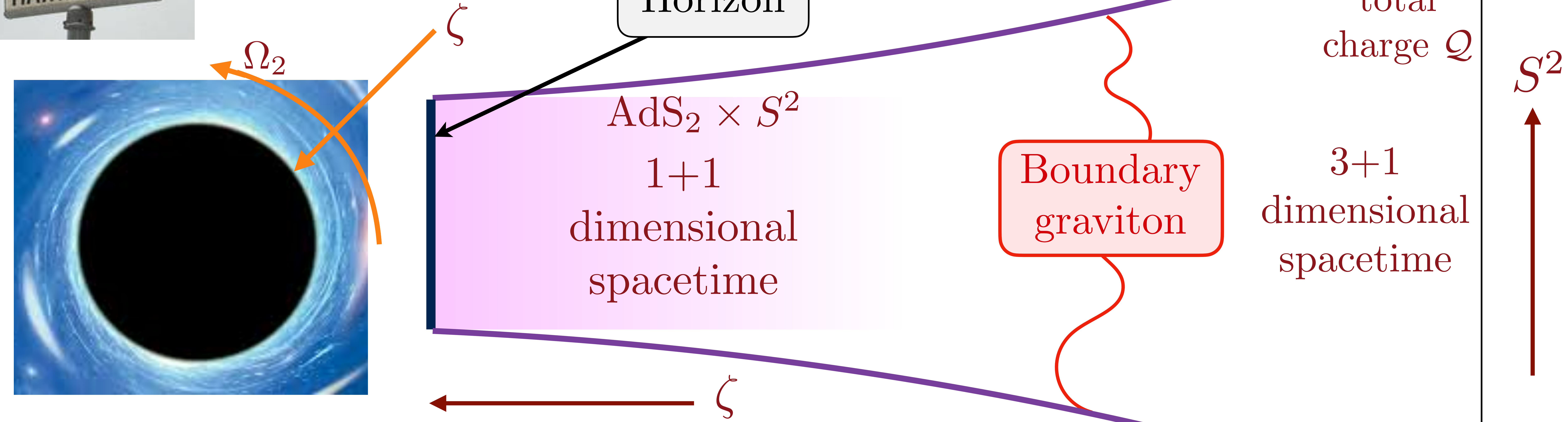




Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Is there a mapping to a quantum system with Planckian dynamics in 0+1 dimensions?

Reissner-Nordstrom black hole of Einstein-Maxwell theory



Yes!

The low energy theory of a charged black hole is *exactly* the low energy theory of time reparameterizations of the SYK model.

Quantum theory of charged black holes

The near-horizon 1+1 dimensional theory of a charged black hole

$$\mathcal{Z}_{\mathcal{Q}} = \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_{\mu} \exp\left(-\frac{1}{\hbar}\int d\zeta \int_0^{\hbar/(k_B T)} d\tau \sqrt{g}\,\mathcal{L}_1[g_{\mu\nu}, a_{\mu}]\right)$$

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$$\Updownarrow$$

$$\mathcal{Z}_{SYK} = e^{N s_0} \int \mathcal{D}f \mathcal{D}\phi \exp\left(-\frac{1}{\hbar}\int_0^{\hbar/(k_B T)} d\tau\, \mathcal{L}_{SYK}[f, \phi]\right)$$

after relating the boundary component of $g_{\mu\nu}$ to f,
and the boundary value of a_τ to ϕ.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Iliesiu, Turaci (2020)

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$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{A_0 c^3}{4} + \frac{\sqrt{\pi} A_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right)$$

Gibbons Hawking

where A_0 is the area of the black hole horizon at $T = 0$.

Quantum theory of charged black holes

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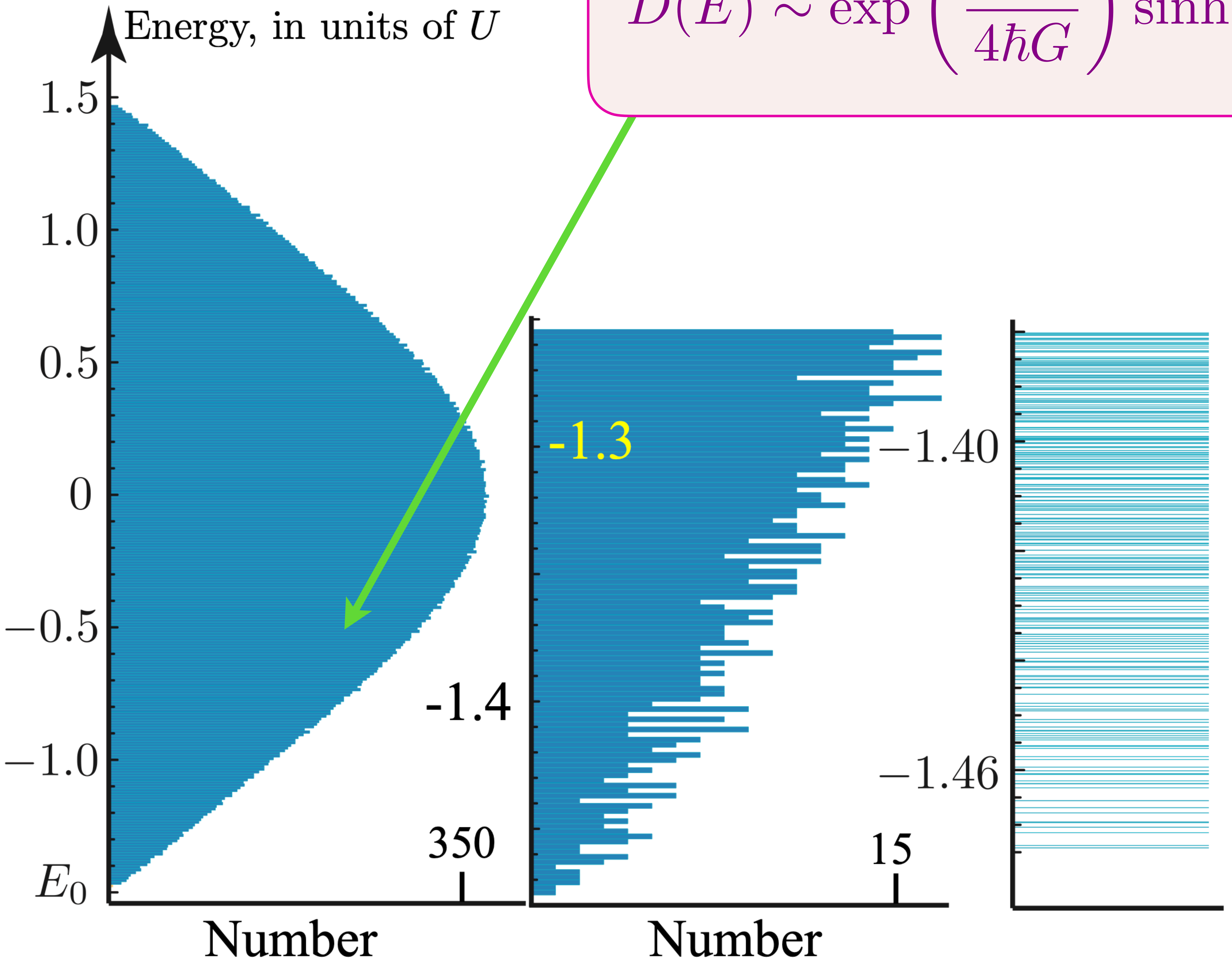
SYK

$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{A_0 c^3}{4} + \frac{\sqrt{\pi} A_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right) - \frac{3}{2}\ln\left(\frac{\sqrt{c^5/\hbar G}}{k_B T/\hbar}\right) + \ldots .$$

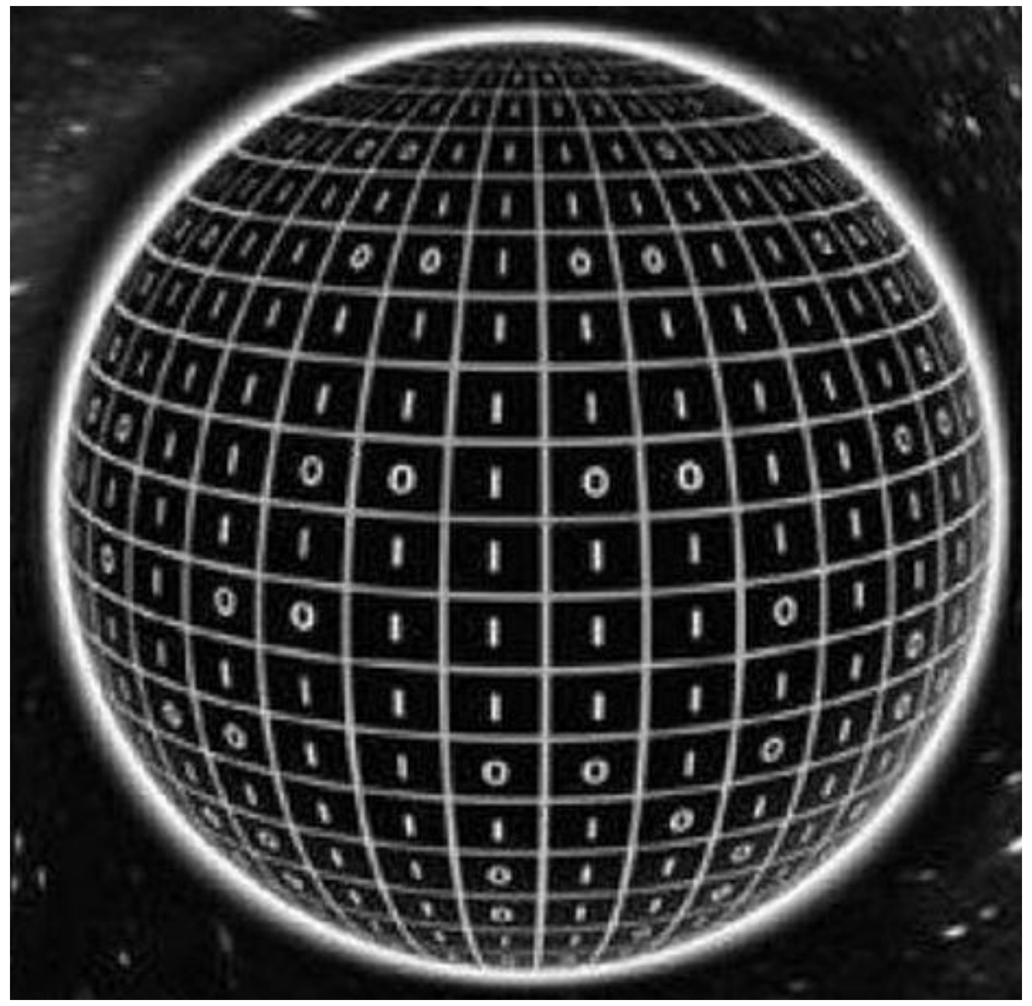
where A_0 is the area of the black hole horizon at $T = 0$.

Quantum theory of charged black holes

$$D(E) \sim \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

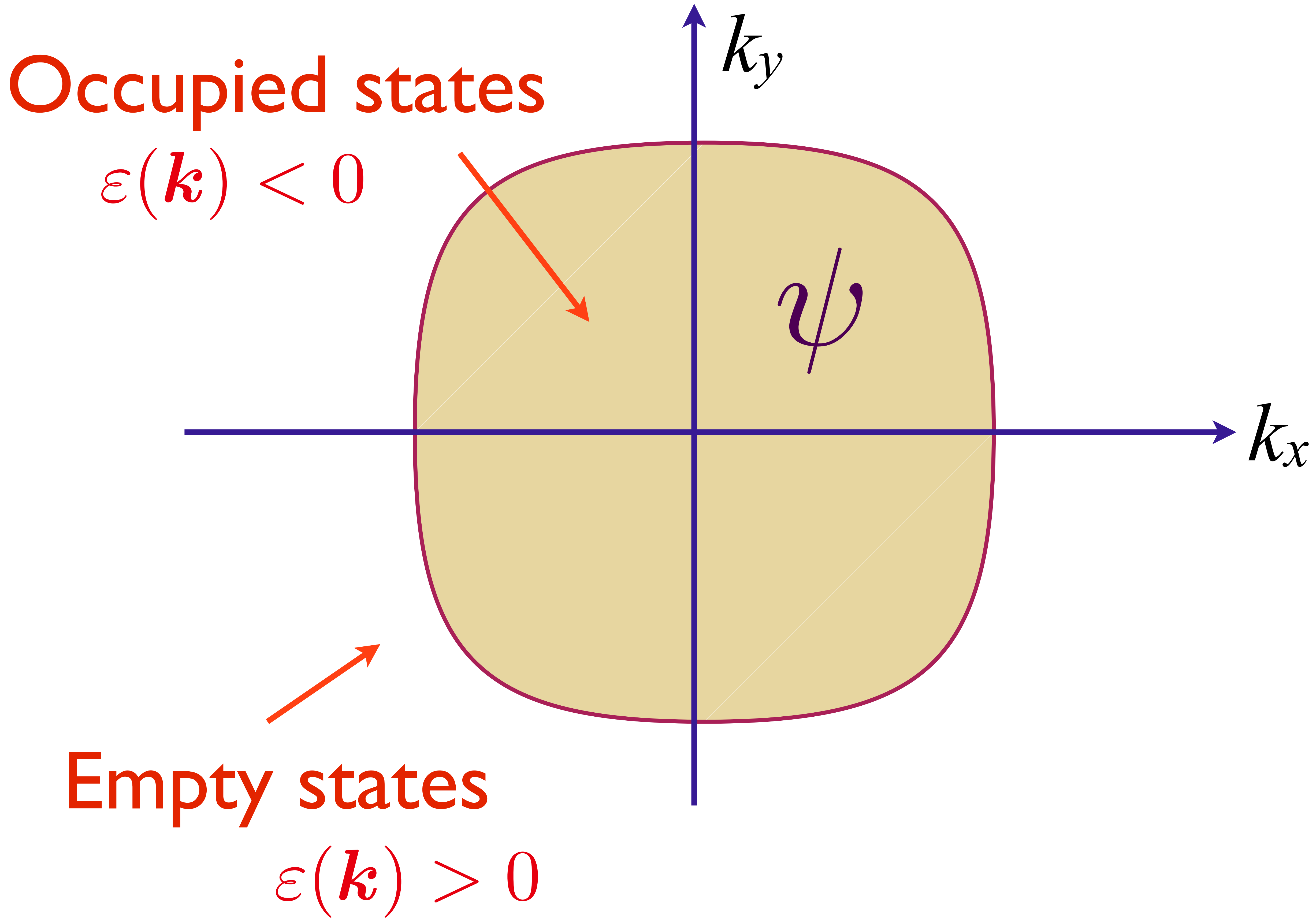


Same lower energy coarse-grained density of states in a model of interacting (fermionic) qubits with a discrete spectrum!



From the SYK model to linear-T resistivity

Fermi surface coupled to a critical boson



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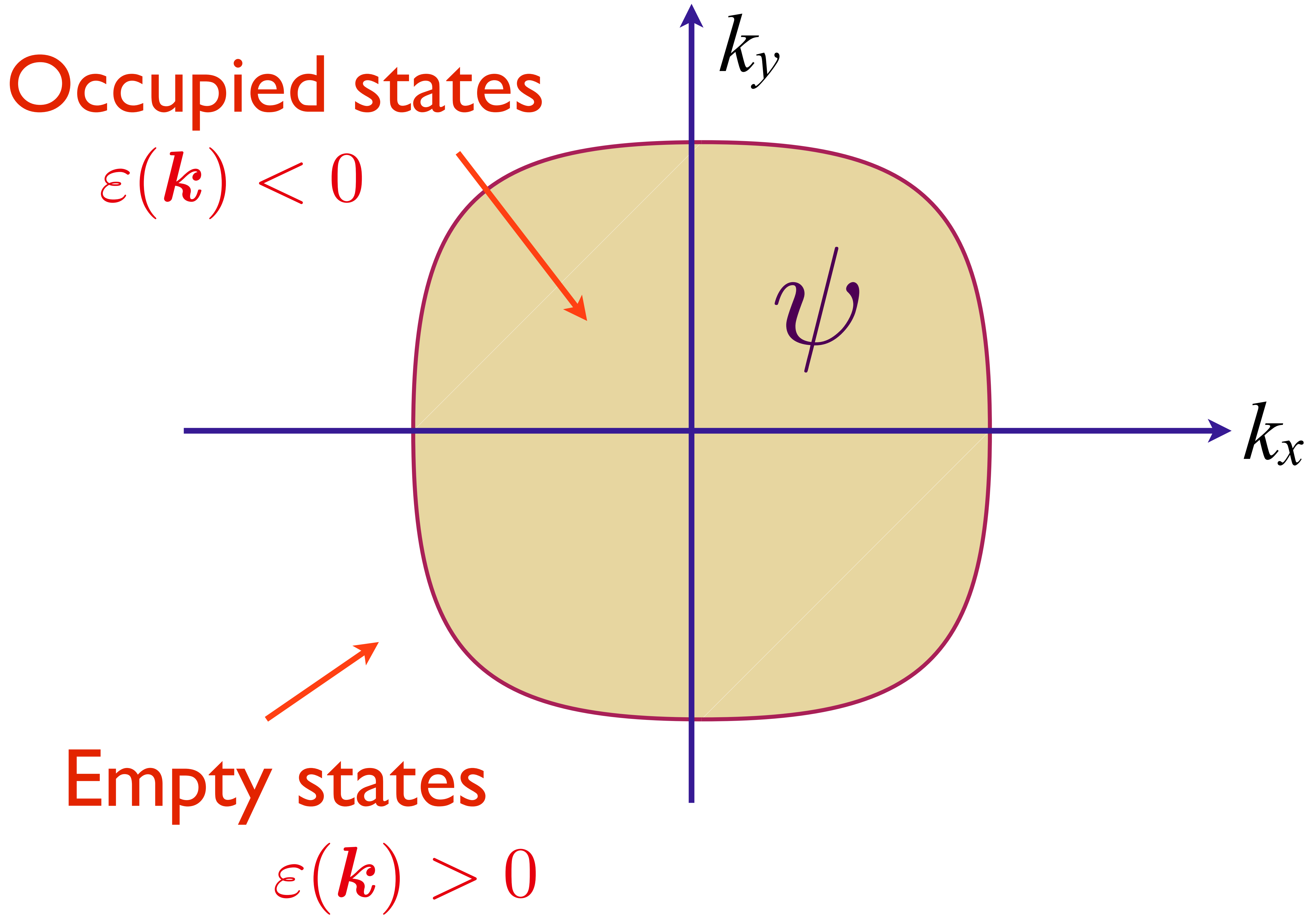
a critical boson

ϕ

- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field

Fermi surface coupled to a critical boson



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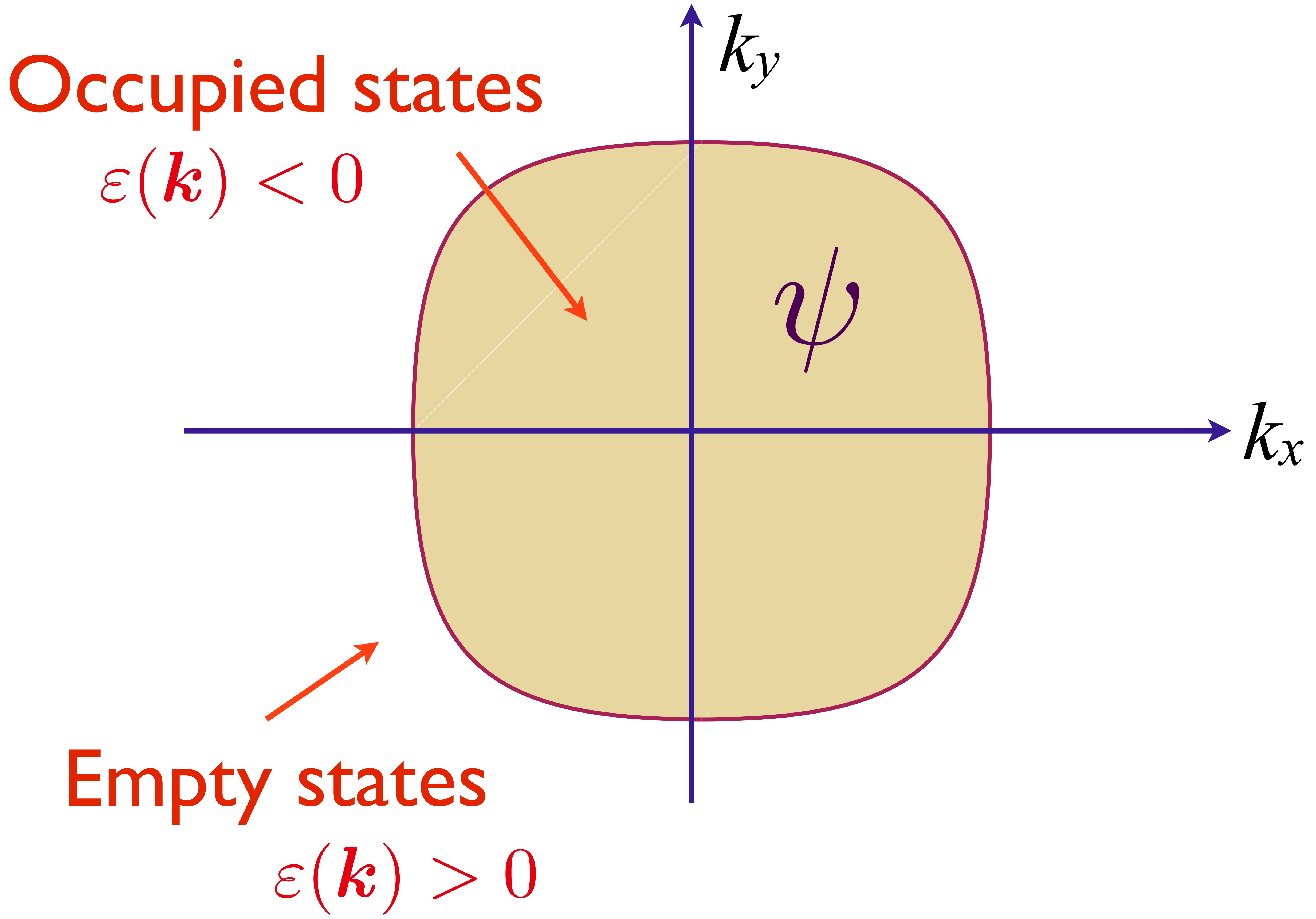
"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

Fermi surface coupled to a critical boson



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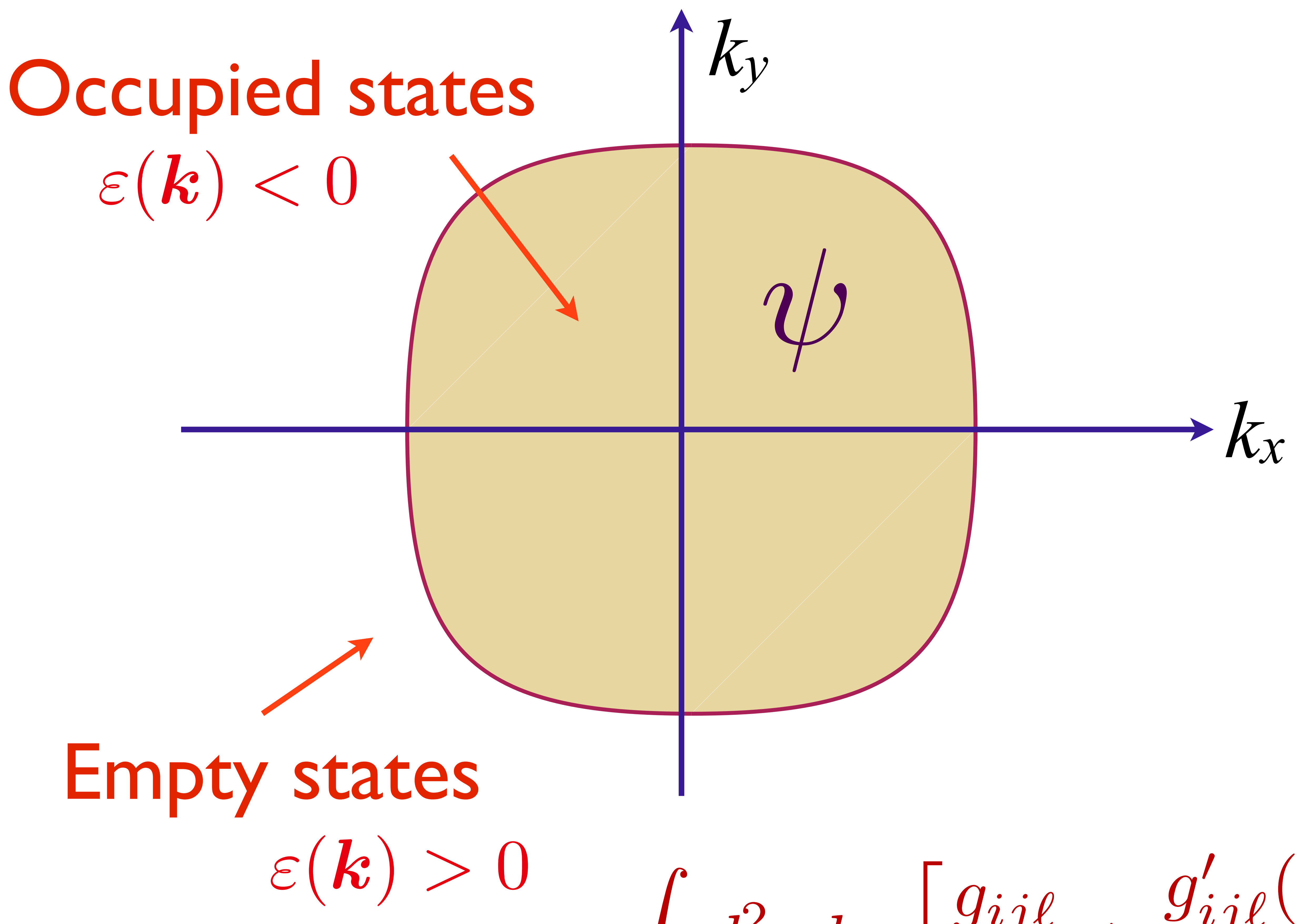
"Yukawa" coupling: $\frac{g_{ij\ell}}{N} \int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

Random couplings in flavor space leads to large N theory of a strange metal, with zero resistivity

Fermi surface coupled to a critical boson



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$$\int d^2 r d\tau \left[\frac{g_{ij\ell}}{N} + \frac{g'_{ij\ell}(r)}{N} \right] \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

Random couplings in flavor *and* position space leads to large N theory of a strange metal, with linear-T resistivity

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990

Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

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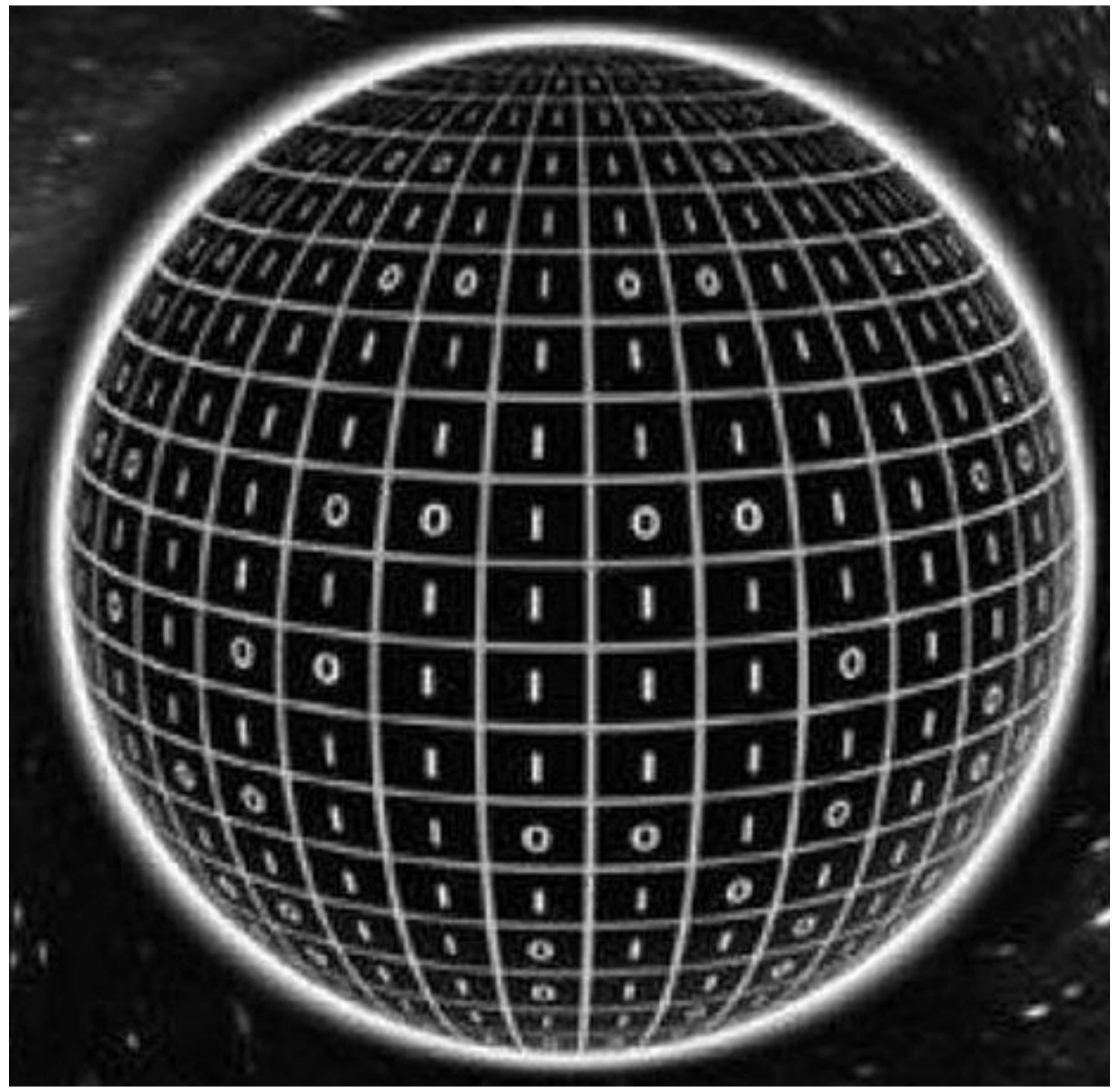
- Low energy theory of time reparameterizations is the theory of the boundary graviton in 1+1 dimensional quantum gravity on AdS_2.

Summary

- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.

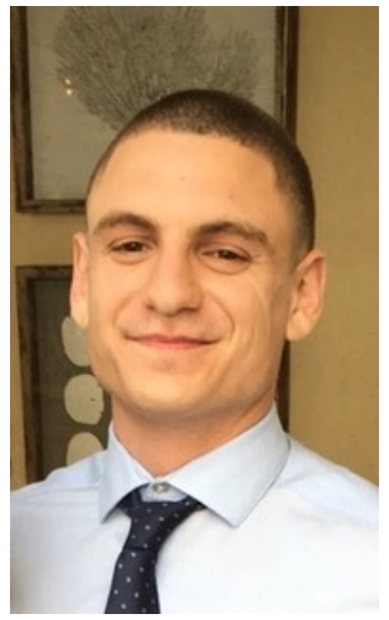


Summary

- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.

- Linear-T resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.





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