

Deconfined criticality in a doped random quantum Heisenberg magnet

2019 Physics Marker Lectures
Penn State University, State College
December 6, 2019

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Talk online: sachdev.physics.harvard.edu

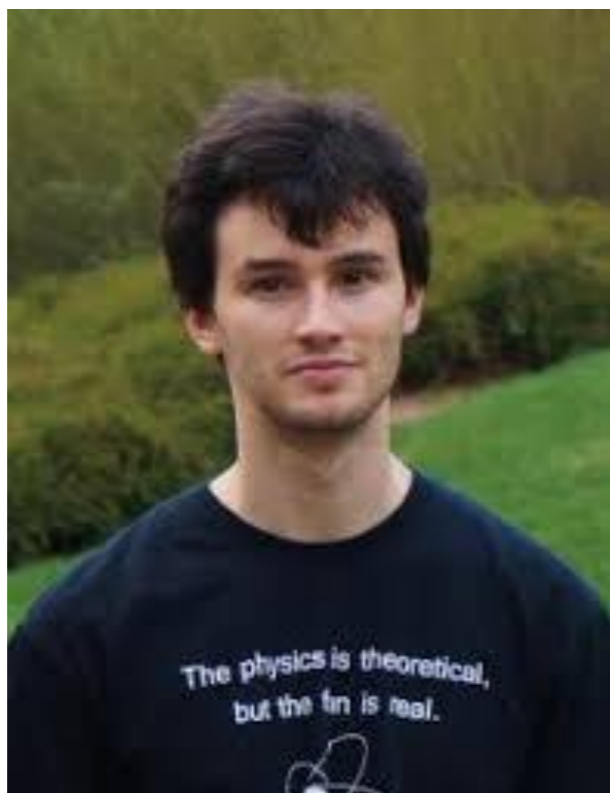




Darshan Joshi



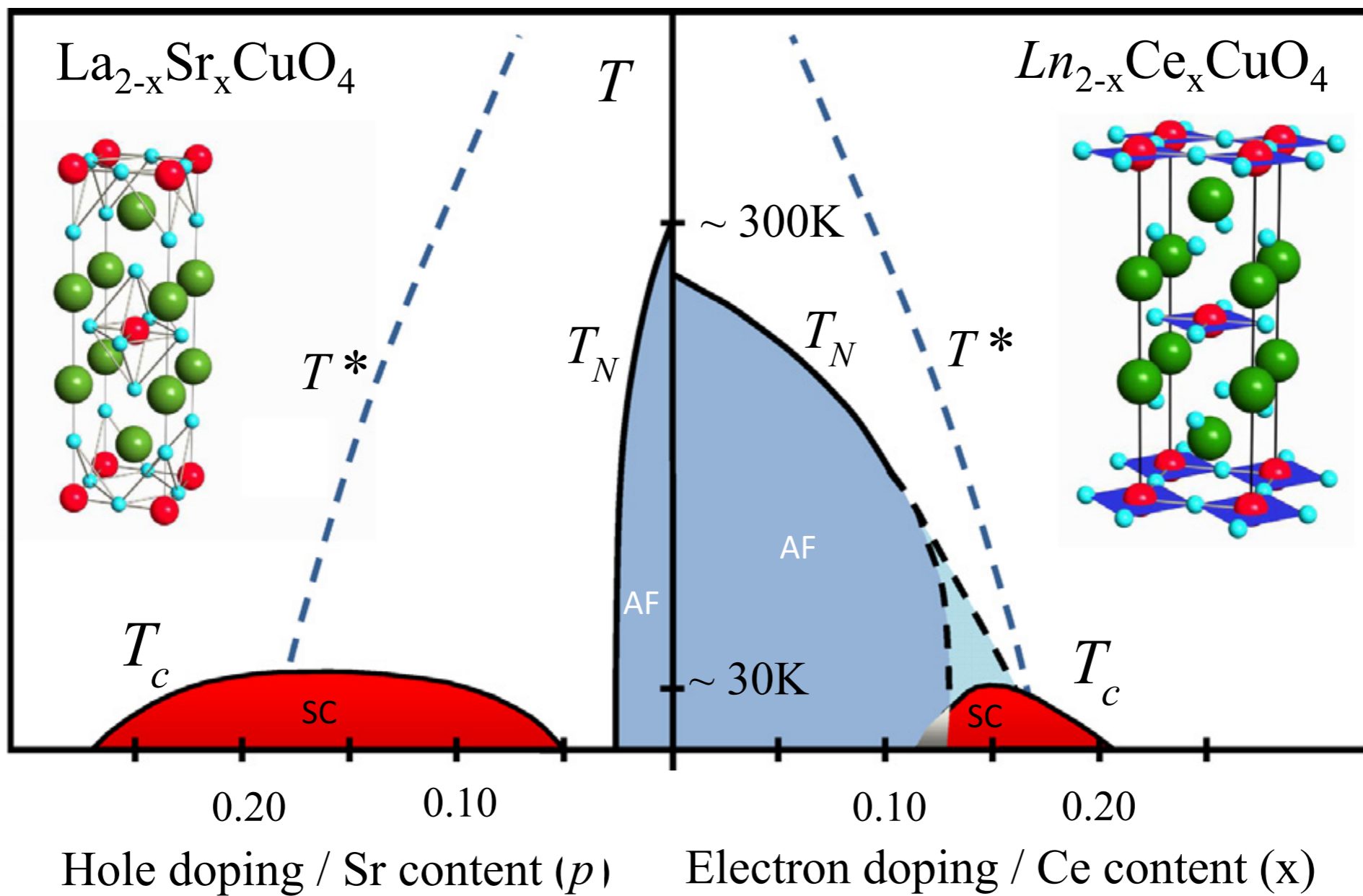
Chenyuan Li

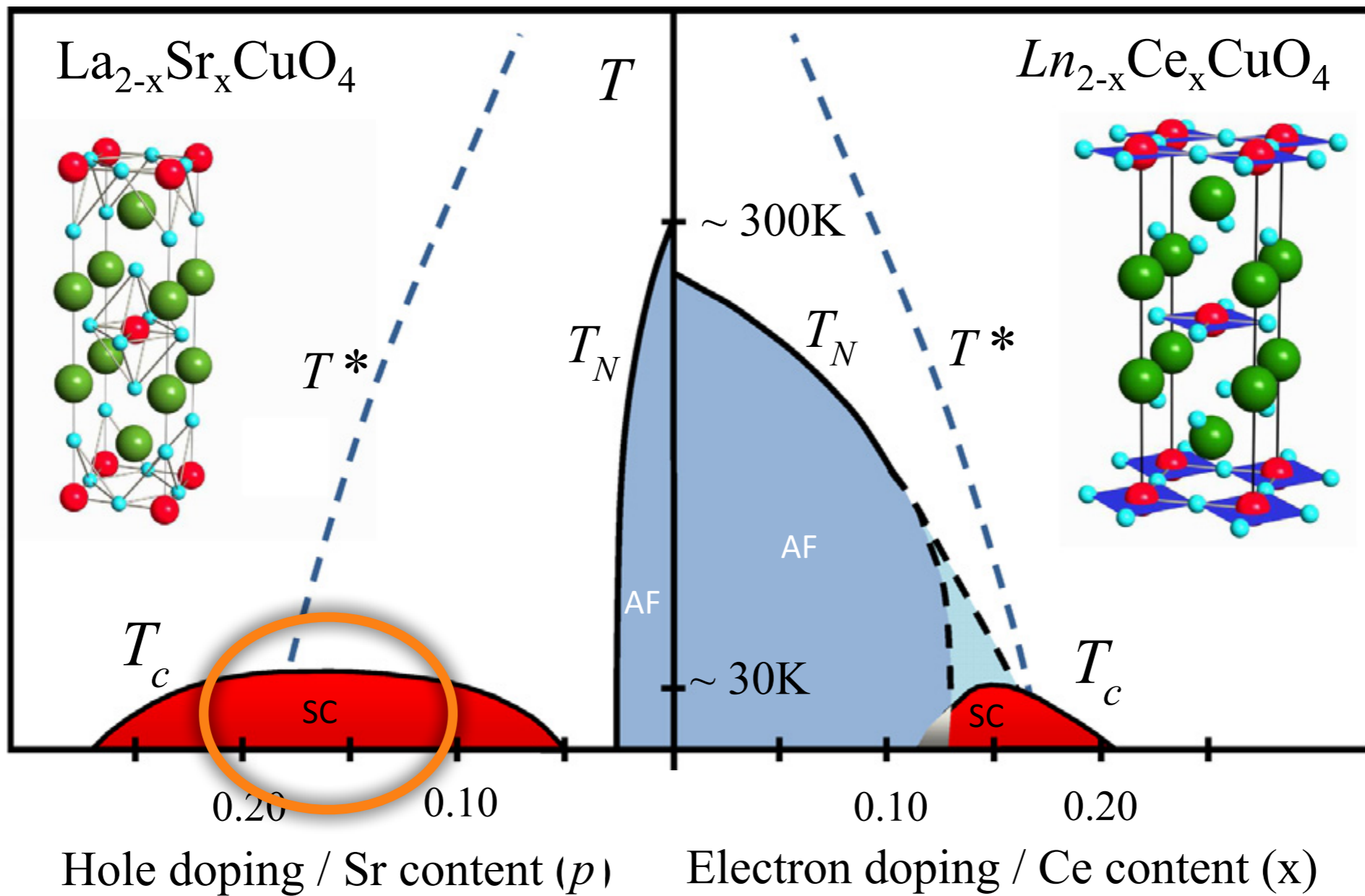


Grigory Tarnopolsky



Antoine Georges

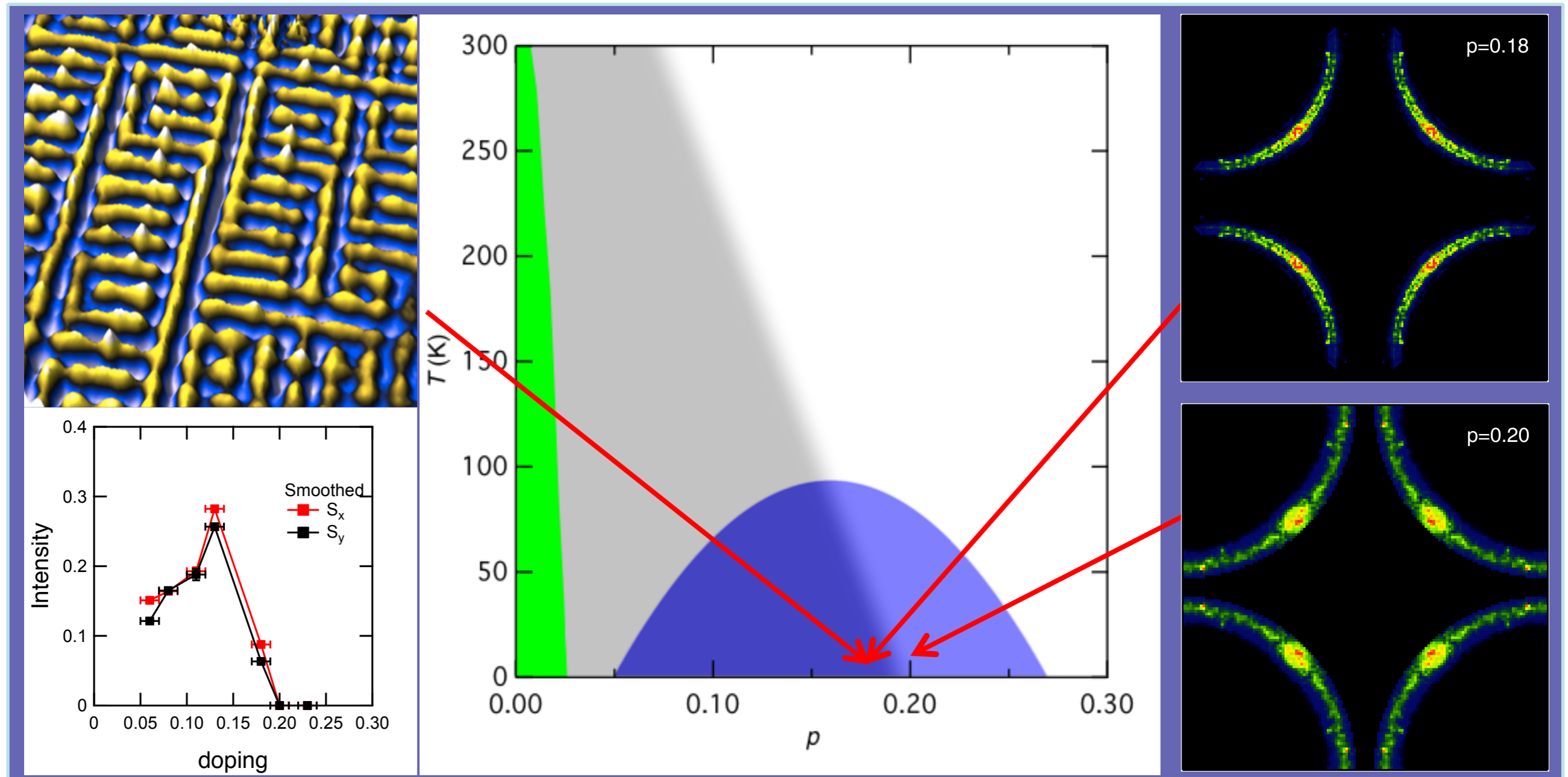




Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, *Science* **344**, 608 (2014)

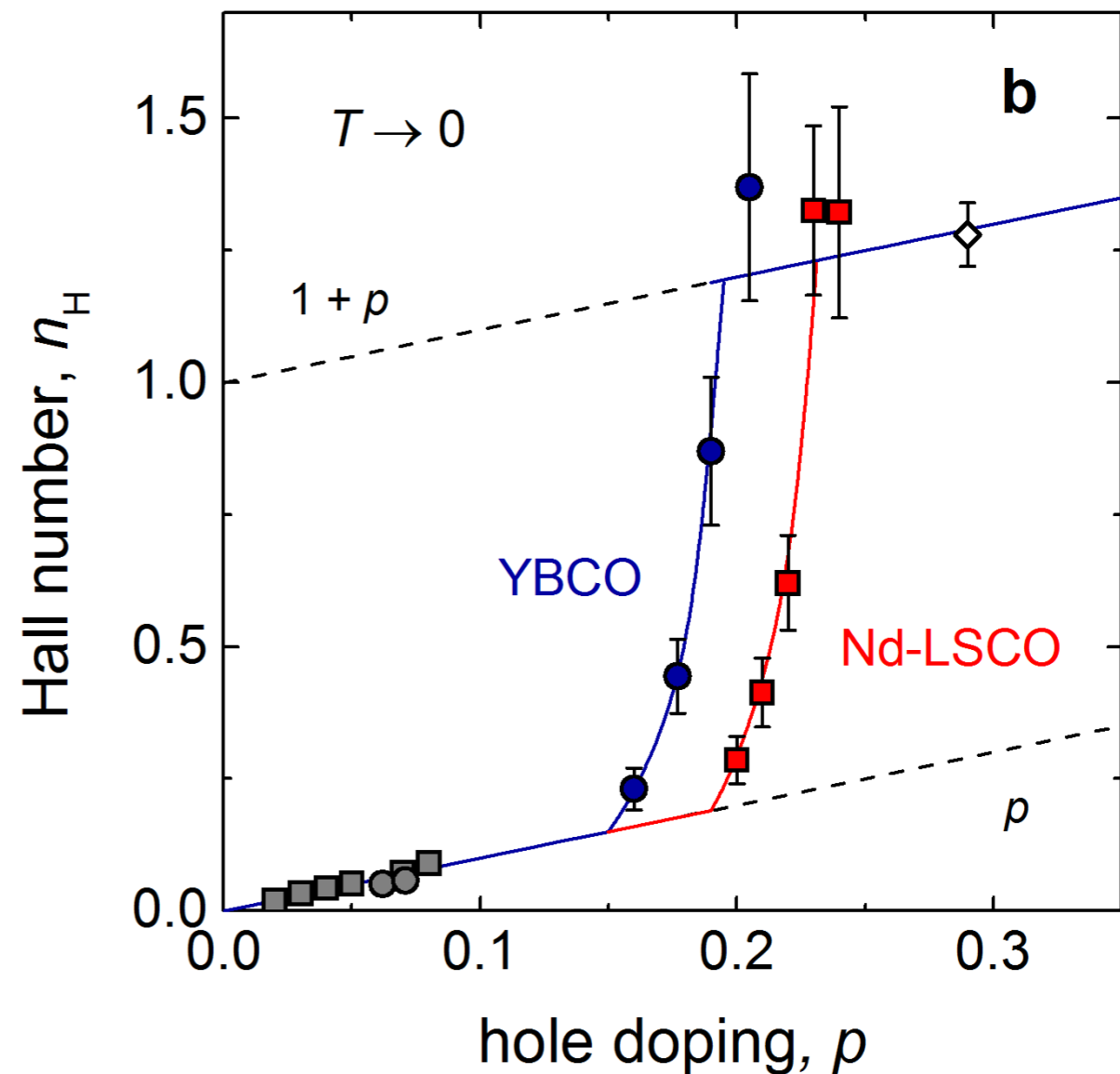
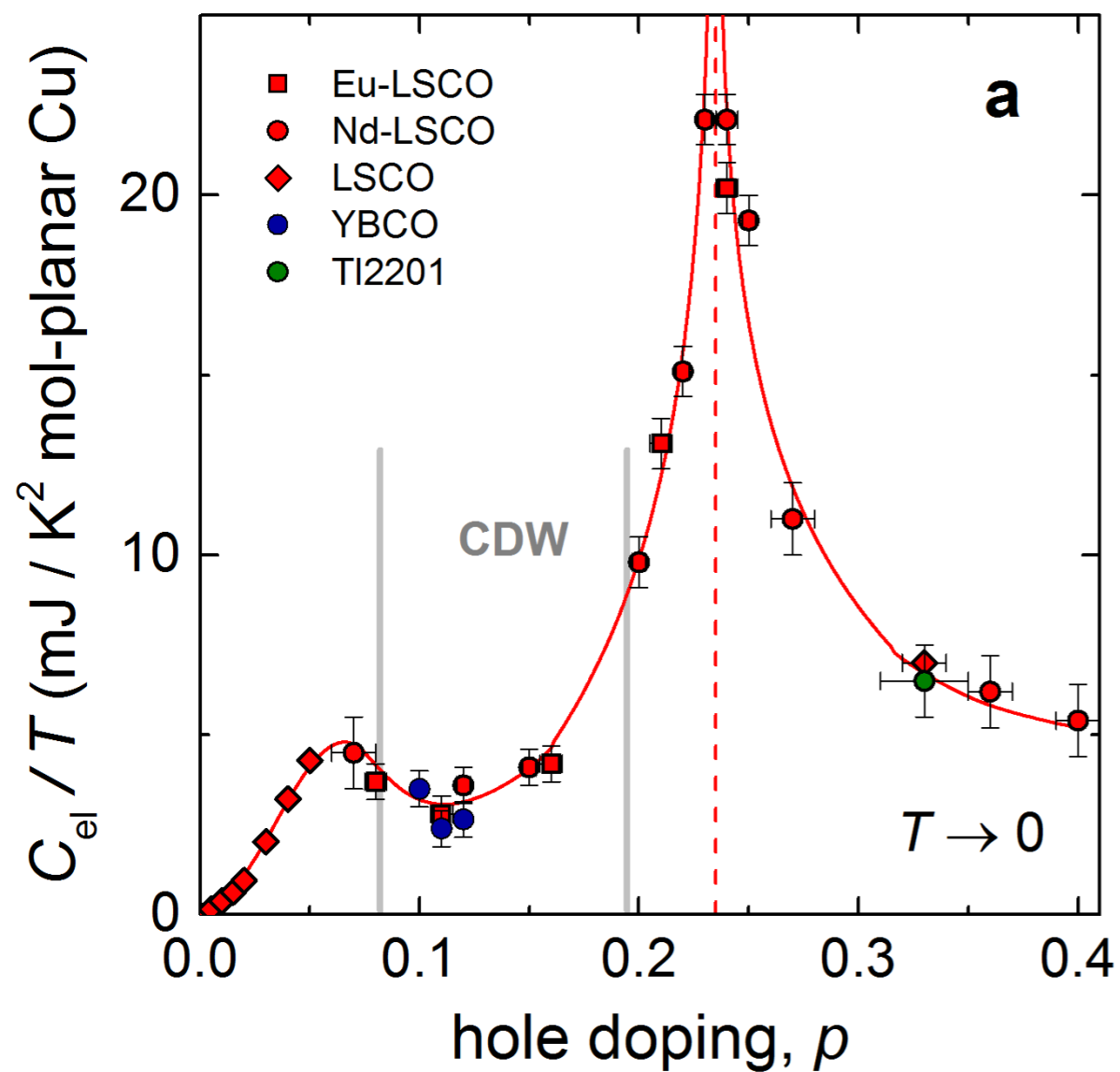
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, *Science* **344**, 612 (2014)



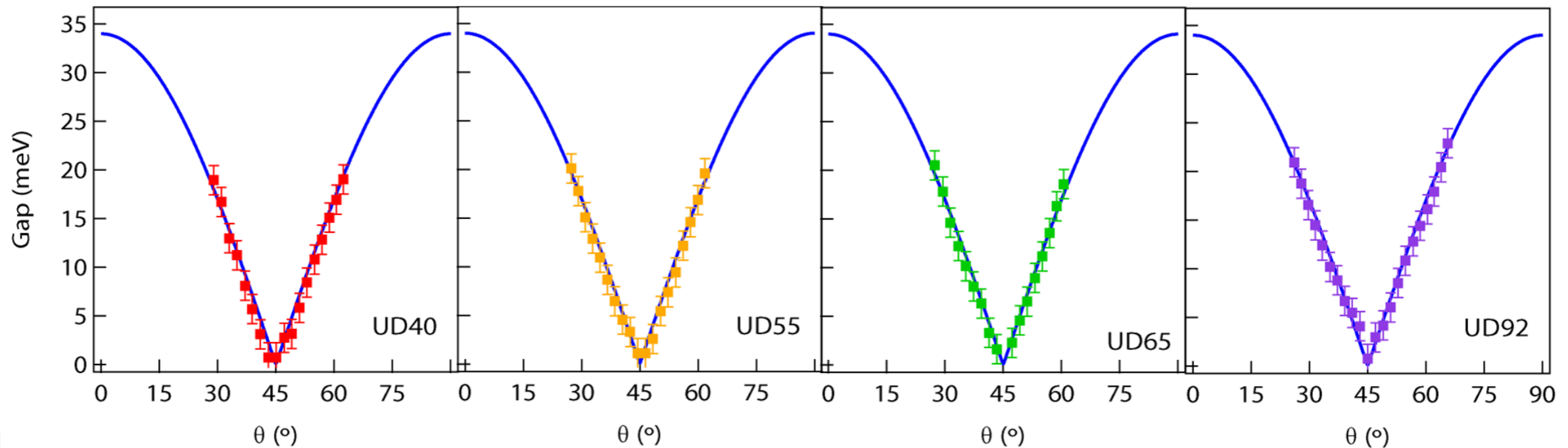
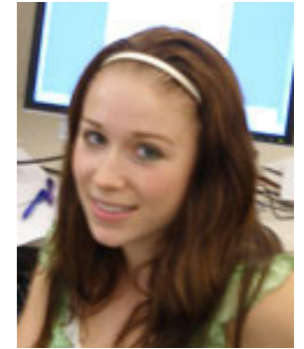
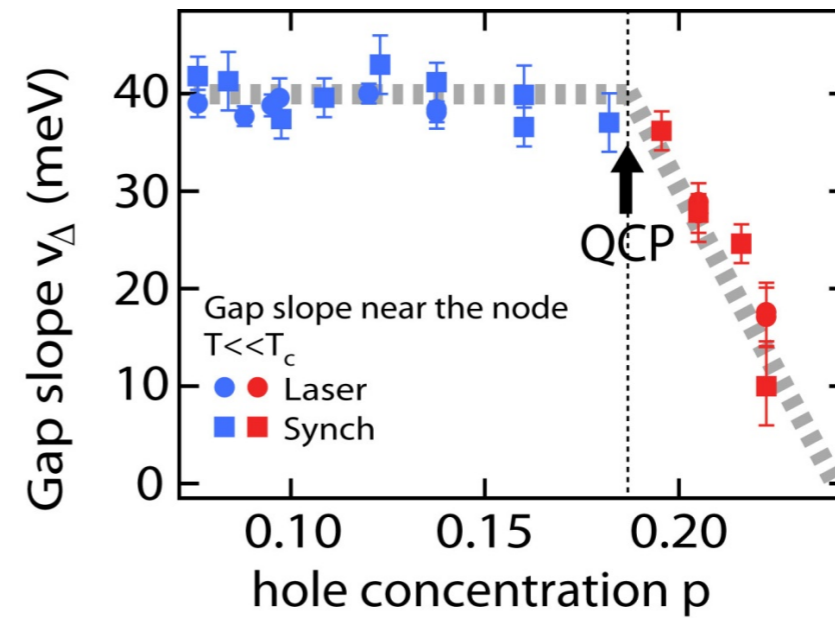
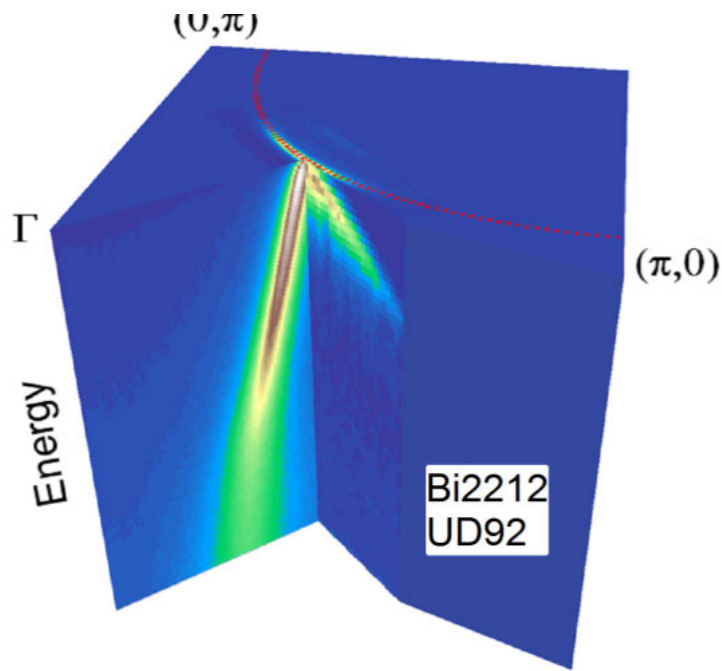
Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507

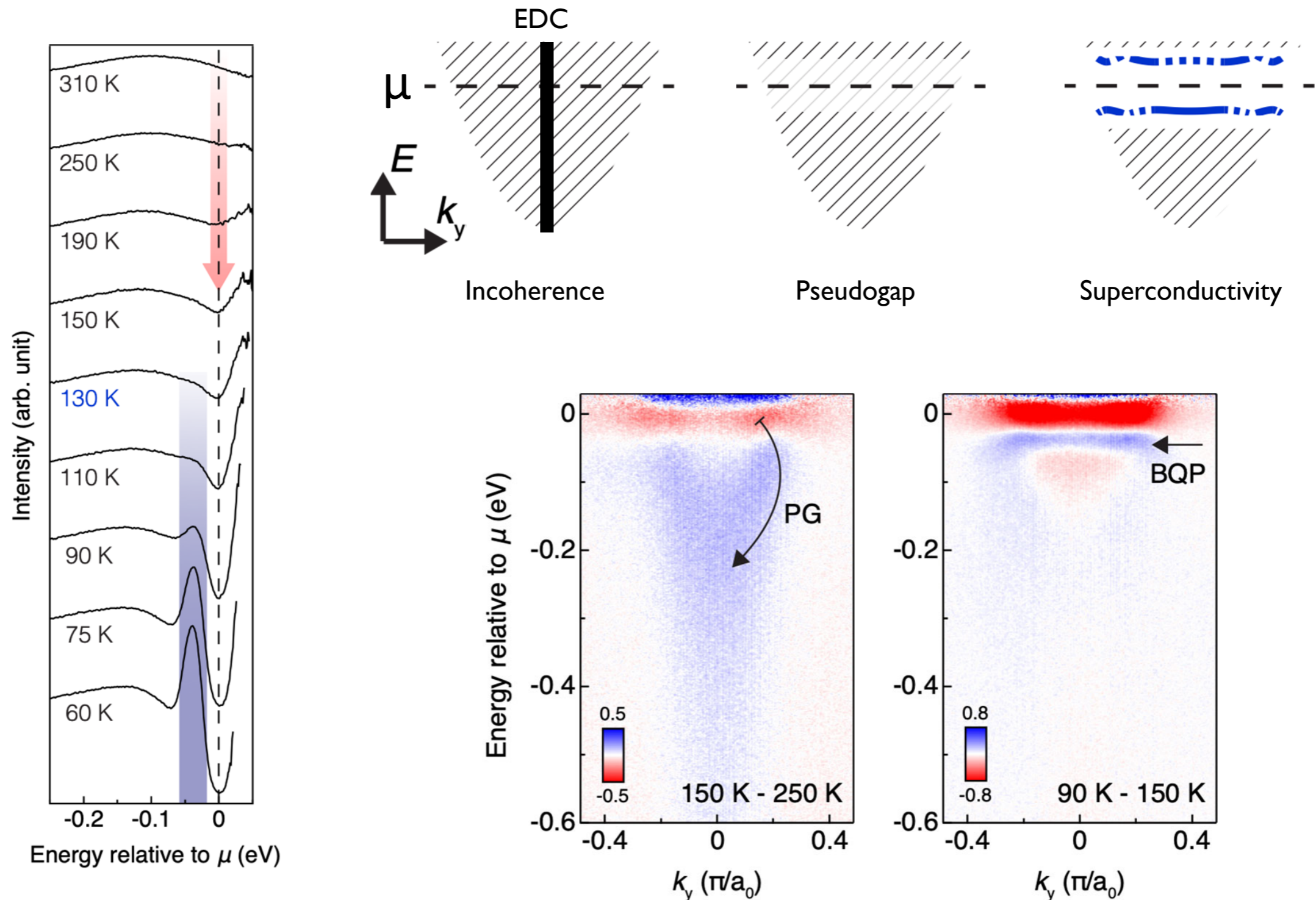


Precision Measurement of the Node



I. M. Vishik, M. Hashimoto, Rui-Hua He, Wei-Sheng Lee, Felix Schmitt, Donghui Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa, S. Uchida, Kazuhiro Fujita, S. Ishida, M. Ishikado, Yoshiyuki Yoshida, Hiroshi Eisaki, Zahid Hussain, Thomas P. Devereaux, and Zhi-Xun Shen, PNAS **109**, 18332 (2012)

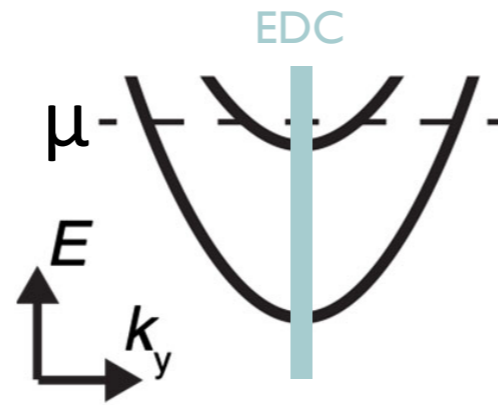
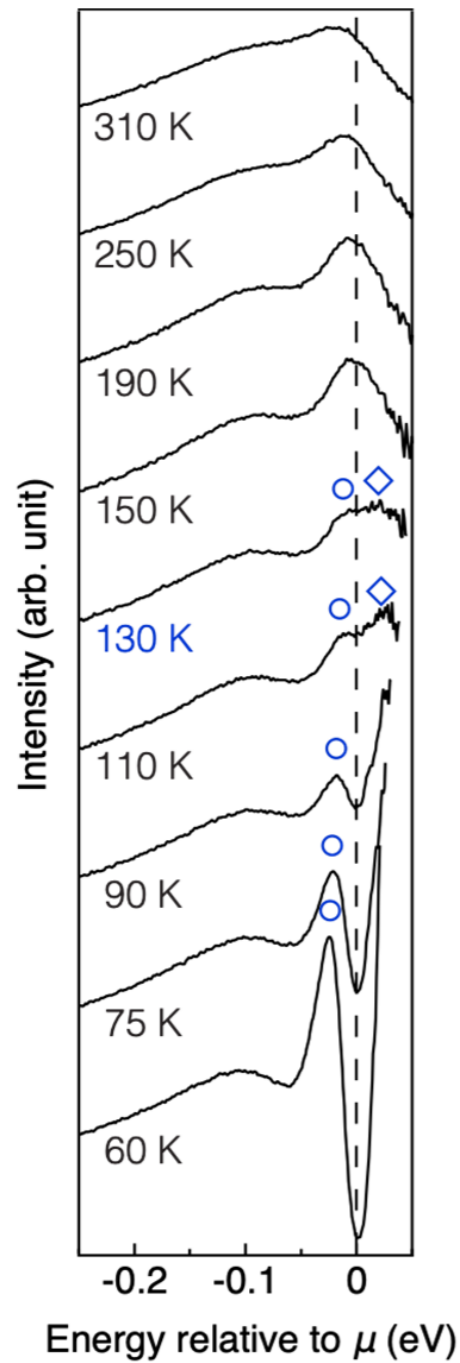
Two “gaps” for $p < 0.19$ ($T_c \sim 86$ K)



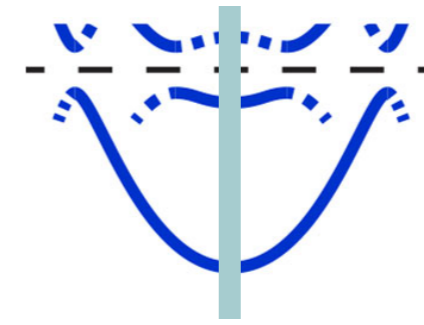
Su-Di Chen, Makoto Hashimoto, Yu He, Dongjoon Song, Ke-Jun Xu, Jun-Feng He, T. P. Devereaux, Hiroshi Eisaki, Dong-Hui Lu, J. Zaanen, Zhi-Xun Shen, *Science* **366**, 6469 (2019)



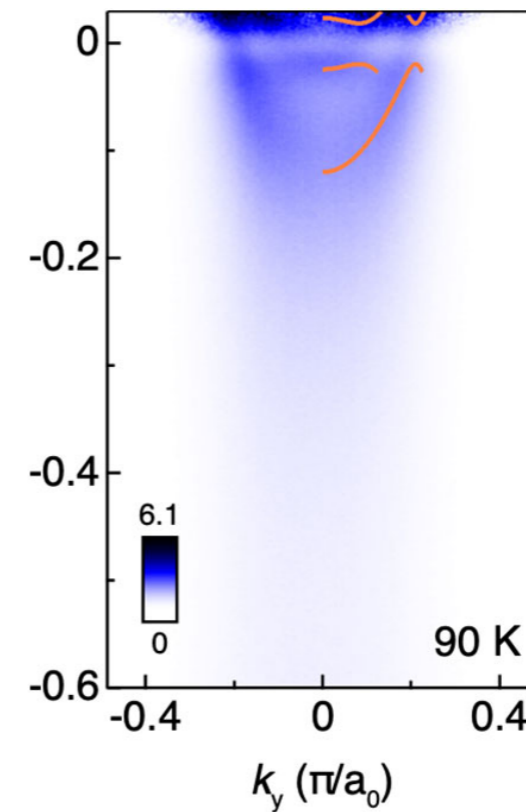
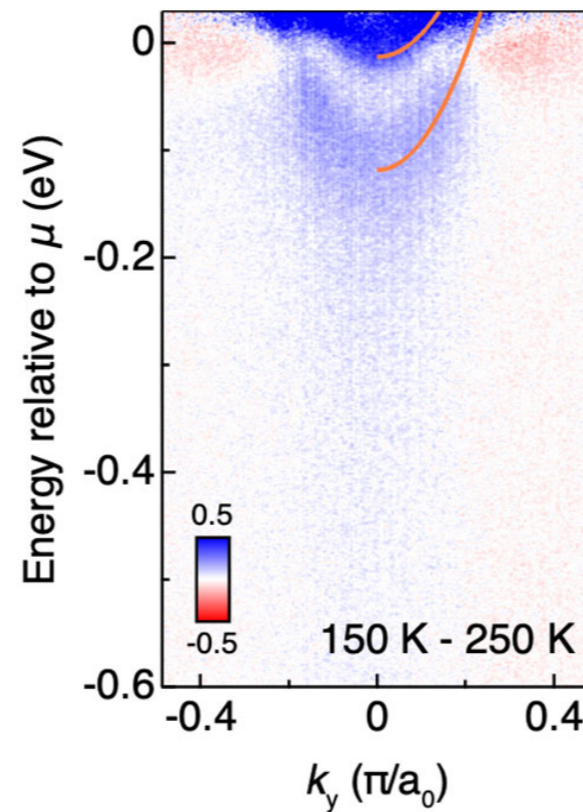
One gap for $p > 0.19$ ($T_c \sim 81$ K)



Normal state



Superconducting gap present



t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1$$

$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

$$t_{ij} \text{ random, } \overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$$

$$\text{---} \\ |0\rangle$$

$$\text{---} \uparrow \\ c_{\uparrow}^\dagger |0\rangle$$

$$\text{---} \downarrow \\ c_{\downarrow}^\dagger |0\rangle$$

t-J model

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$$|0\rangle \Rightarrow b^\dagger |v\rangle \quad , \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$

$$c_\alpha = f_\alpha b^\dagger$$

$$\vec{S} = \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta$$

$$f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

$$\text{U(1) gauge invariance,} \quad b \rightarrow b e^{i\phi}, \quad f_\alpha \rightarrow f_\alpha e^{i\phi}$$

The physical electron (c_α) and spin (\vec{S}) operators are rotations in this $\text{SU}(1|2)$ superspin space.

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$$\text{U(1) gauge invariance,} \quad f \rightarrow f e^{i\phi}, \quad \mathbf{b}_\alpha \rightarrow \mathbf{b}_\alpha e^{i\phi}$$

The physical electron (c_α) and spin (\vec{S}) operators are rotations in this $\text{SU}(2|1)$ superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

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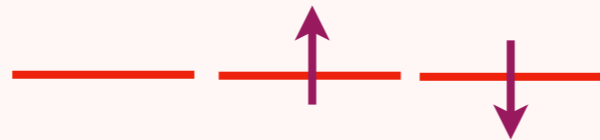
$$\text{SU}(1|2) \equiv \text{SU}(2|1)$$

$$\text{U}(1) \text{ gauge invariance,} \quad f \rightarrow f e^{i\phi}, \quad \mathbf{b}_\alpha \rightarrow \mathbf{b}_\alpha e^{i\phi}$$

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t - J model phase diagram

Deconfined
quantum
critical
point



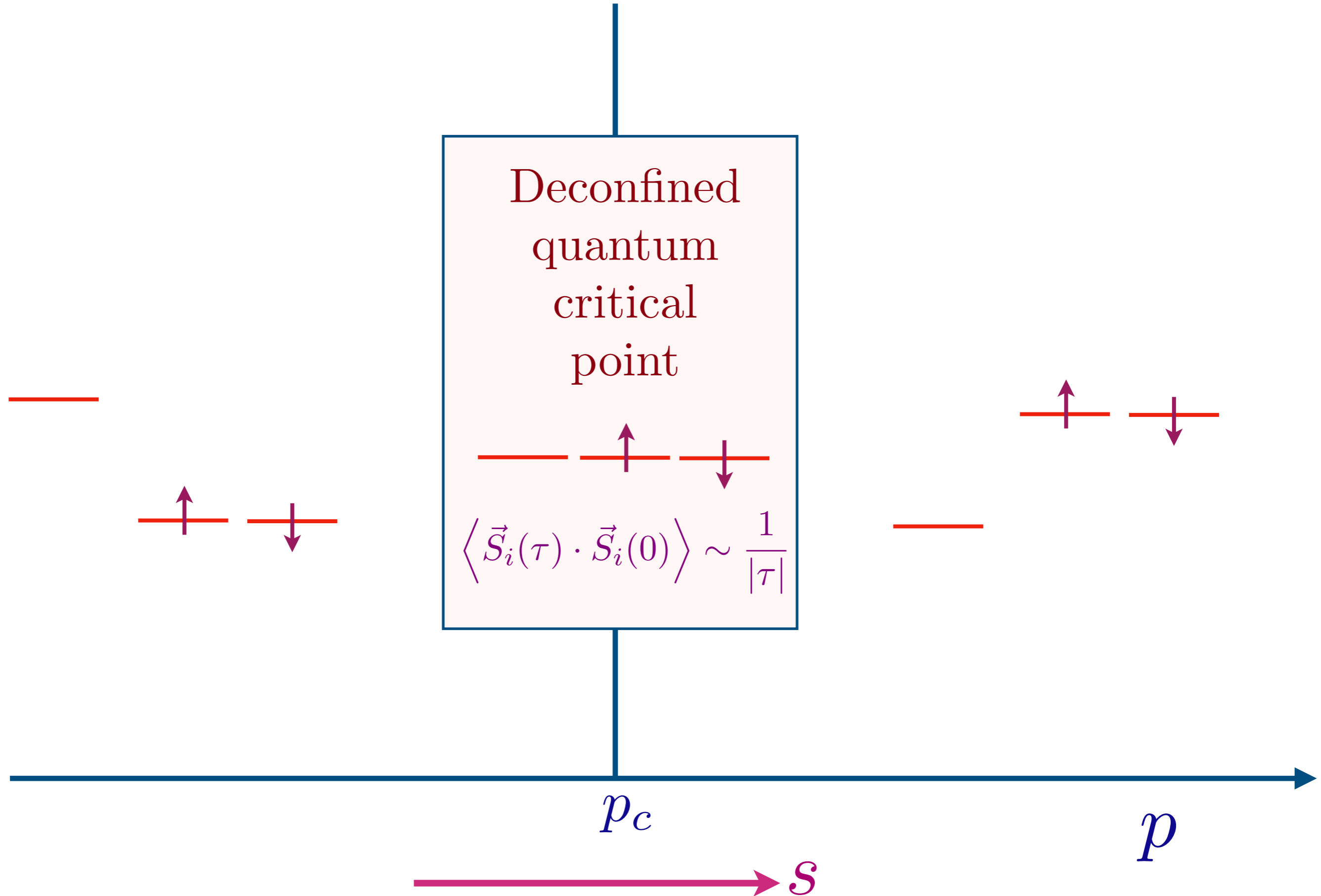
$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$$

p_c

p



t - J model phase diagram



t - J model phase diagram

SU(1|2) theory

Disordered
Fermi liquid.
Condense holon b ,
 f_α carrier density $1 + p$

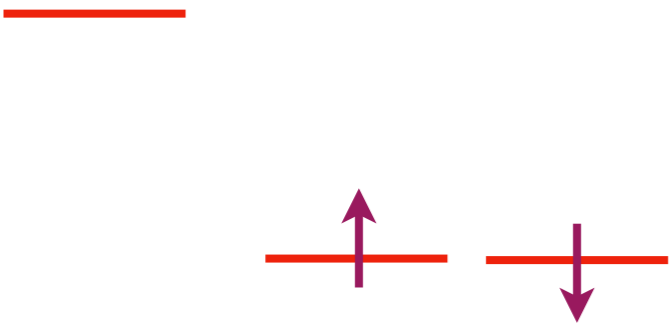
Deconfined
quantum
critical
point

$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$

$b^\dagger |v\rangle$

$f_\uparrow^\dagger |v\rangle$ $f_\downarrow^\dagger |v\rangle$

$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$



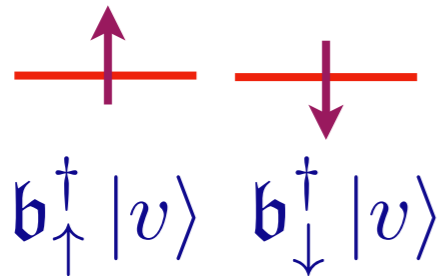
t - J model phase diagram

SU(2|1) theory

Metallic spin glass.

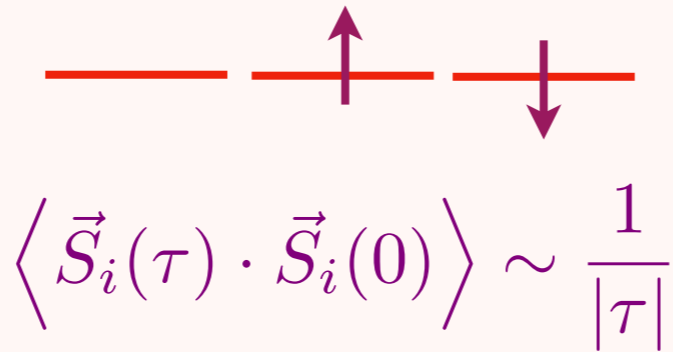
Condense spinon \mathbf{b}_α ,
 f carrier density p

$f^\dagger |v\rangle$



$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \text{constant}$$

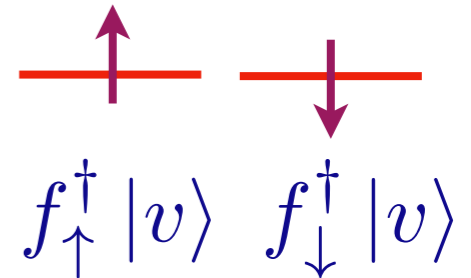
Deconfined quantum critical point



SU(1|2) theory

Disordered Fermi liquid.

Condense holon b ,
 f_α carrier density $1 + p$



$b^\dagger |v\rangle$

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$$

p_c

p

s

1. Insulating random magnet
2. Deconfined criticality at non-zero doping
3. Phase diagram of disordered Hubbard model

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2. Deconfined criticality at
non-zero doping

3. Phase diagram of disordered
Hubbard model

Insulating J model

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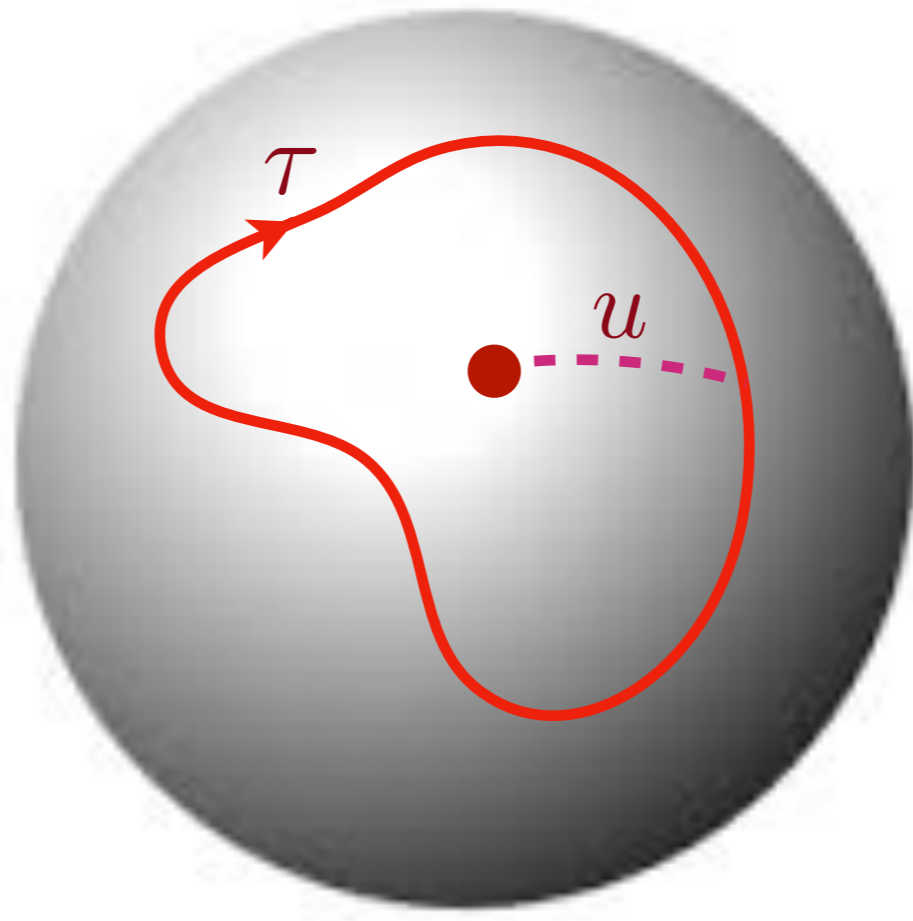
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$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

Insulating J model

$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau) \delta(\vec{S}^2 - 1) e^{-\mathcal{S}}$$

$$\mathcal{S} = \frac{i}{2} \int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right) - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau')$$



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From this action we compute

$$\bar{Q}(\tau - \tau') = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$Q(\tau) = \bar{Q}(\tau).$$

Insulating J model: large M limit

Express the spin operator in terms of fermions $\vec{S} = (1/2) f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1 \dots M$. The fermions obey the constraint

$$\sum_{\alpha=1}^M f_\alpha^\dagger f_\alpha = \frac{M}{2}$$

In the large M limit we obtain for the fermion Green's function G and self energy Σ (same as the SYK equations)

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution is

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}} \quad , \quad \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$$

Insulating J model

$$H = \frac{1}{\sqrt{N}} \sum_{i < j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Numerical studies for SU(2) spin-1/2 show spin-glass order!

L. Arrachea and M. J. Rozenberg, PRB **65**, 224430 (2002)

Insulating J model: RG

We assume a power-law decay

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}}.$$

Ignore the self-consistency condition for now. We decouple the $\vec{S}(\tau) \cdot \vec{S}(0)$ interaction by introducing a bosonic ($\phi_a, a = 1 \dots 3$) bath.

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$$H_{\text{imp}} = \gamma_0 f_\alpha^\dagger \frac{\sigma_{\alpha\beta}^a}{2} f_\beta \phi_a(0) + \frac{1}{2} \int d^d x [\pi_a^2 + (\partial_x \phi_a)^2]$$

where π_a is canonically conjugate to the field ϕ_a , $\phi_a(0) \equiv \phi_a(x=0)$, and we have the constraint

$$f_\alpha^\dagger f_\alpha = 1.$$

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We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and it can be verified that this correlator decays as above.

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Schwinger fermions

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M.Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)

S. Sachdev, Physica C **357**, 78 (2001)

Insulating J model: RG

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Schwinger bosons

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We can perform a RG analysis in a $\epsilon = 3 - d$ expansion, while imposing the fermion constraint *exactly*. The two-loop β function is

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 - \gamma^5 + \dots$$

This has a stable fixed point at $\gamma^{*2} = \epsilon/2 + \epsilon^2/4 + \dots$

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The scaling dimension of the spin operator is $\dim[\vec{S}] = \epsilon/2$, exact to *all* orders in ϵ . This implies the correlator

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Finally, we impose the self-consistency condition $Q(\tau) = \overline{Q}(\tau)$, and obtain the same self-consistent result as in the large M expansion

$$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}.$$

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SU(2|1) theory

$$\mathbf{b}_\alpha^\dagger \mathbf{b}_\alpha + f^\dagger f = 1$$

U(1) gauge invariance,

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t-J model

$$\mathcal{Z} = \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}}$$

$$\mathcal{S} = i \int d\tau \text{Tr} (\mathcal{P} \partial_\tau \mathcal{P} \partial_u \mathcal{P})$$

P. B. Wiegmann, PRL **60**, 821 (1988)

$$+ \int d\tau d\tau' \text{Tr} (\mathcal{P}(\tau) \mathcal{Q}(\tau - \tau') \mathcal{P}(\tau')) .$$

Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$.

t-J model

$$\mathcal{Z} = \int \mathcal{D}f_\alpha(\tau) \mathcal{D}b(\tau) \mathcal{D}\lambda(\tau) e^{-\mathcal{S}}$$

$$\begin{aligned} \mathcal{S} = & \int d\tau \left[f_\alpha^\dagger(\tau) \left(\frac{\partial}{\partial\tau} + s_0 + i\lambda \right) f_\alpha(\tau) + b^\dagger(\tau) \left(\frac{\partial}{\partial\tau} + i\lambda \right) b(\tau) - i\lambda \right] \\ & - t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') + \text{H.c.} \\ & - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau'). \end{aligned}$$

SU(1|2) theory

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SU(1|2) theory

From this action we determined the correlators

$$\bar{R}(\tau - \tau') = -\frac{1}{2} \langle c_\alpha(\tau) c_\alpha^\dagger(\tau') \rangle_{\mathcal{Z}}$$

$$\bar{Q}(\tau - \tau') = \frac{1}{3} \langle \vec{S}(\tau) \cdot \vec{S}(\tau') \rangle_{\mathcal{Z}}$$

and finally impose the self-consistency conditions

$$R(\tau) = \bar{R}(\tau) \quad , \quad Q(\tau) = \bar{Q}(\tau).$$

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SU(2|1) theory

From this action we determined the correlators

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$$\bar{Q}(\tau - \tau') = \frac{1}{3} \langle \vec{S}(\tau) \cdot \vec{S}(\tau') \rangle_{\mathcal{Z}}$$

and finally impose the self-consistency conditions

$$R(\tau) = \bar{R}(\tau) \quad , \quad Q(\tau) = \bar{Q}(\tau).$$

t - J model RG

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} .$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a , $a = 1 \dots 3$) and fermionic (ψ_α) baths.

t-J model RG

SU(1|2) theory

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$$\begin{aligned} H &= (s_0 + \lambda) f_\alpha^\dagger f_\alpha + \lambda b^\dagger b + g_0 (f_\alpha^\dagger b \psi_\alpha(0) + \text{H.c.}) + \gamma_0 f_\alpha^\dagger \frac{\sigma_{\alpha\beta}^a}{2} f_\beta \phi_a(0) \\ &+ \int |k|^r dk k \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x [\pi_a^2 + (\partial_x \phi_a)^2] \end{aligned}$$

where $a = (x, y, z)$, σ^a are Pauli matrices, π_a is canonically conjugate to the field ϕ_a , and $\phi_a(0) \equiv \phi_a(x=0)$, $\psi_\alpha(0) \equiv \int |k|^r dk \psi_{k\alpha}$.

t-J model RG

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S. Sachdev, Physica C **357**, 78 (2001)

M. Vojta and L. Fritz, PRB **70**, 094502 (2004)

t-J model RG

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The impurity superspin is coupled to a fermionic bath by g_0 , and to a bosonic bath by γ_0 , and s_0 acts as a local field on the superspin - a superKondo problem!

t-J model RG

SU(2|1) theory

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} .$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a , $a = 1 \dots 3$) and fermionic (ψ_α) baths. Then the problem reduces to the Hamiltonian

$$\begin{aligned} H &= (s_0 + \lambda) \mathbf{b}_\alpha^\dagger \mathbf{b}_\alpha + \lambda \mathbf{f}^\dagger \mathbf{f} + g_0 (\mathbf{b}_\alpha^\dagger \mathbf{f} \psi_\alpha(0) + \text{H.c.}) + \gamma_0 \mathbf{b}_\alpha^\dagger \frac{\sigma_{\alpha\beta}^a}{2} \mathbf{b}_\beta \phi_a(0) \\ &\quad + \int |k|^r dk k \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x [\pi_a^2 + (\partial_x \phi_a)^2] \end{aligned}$$

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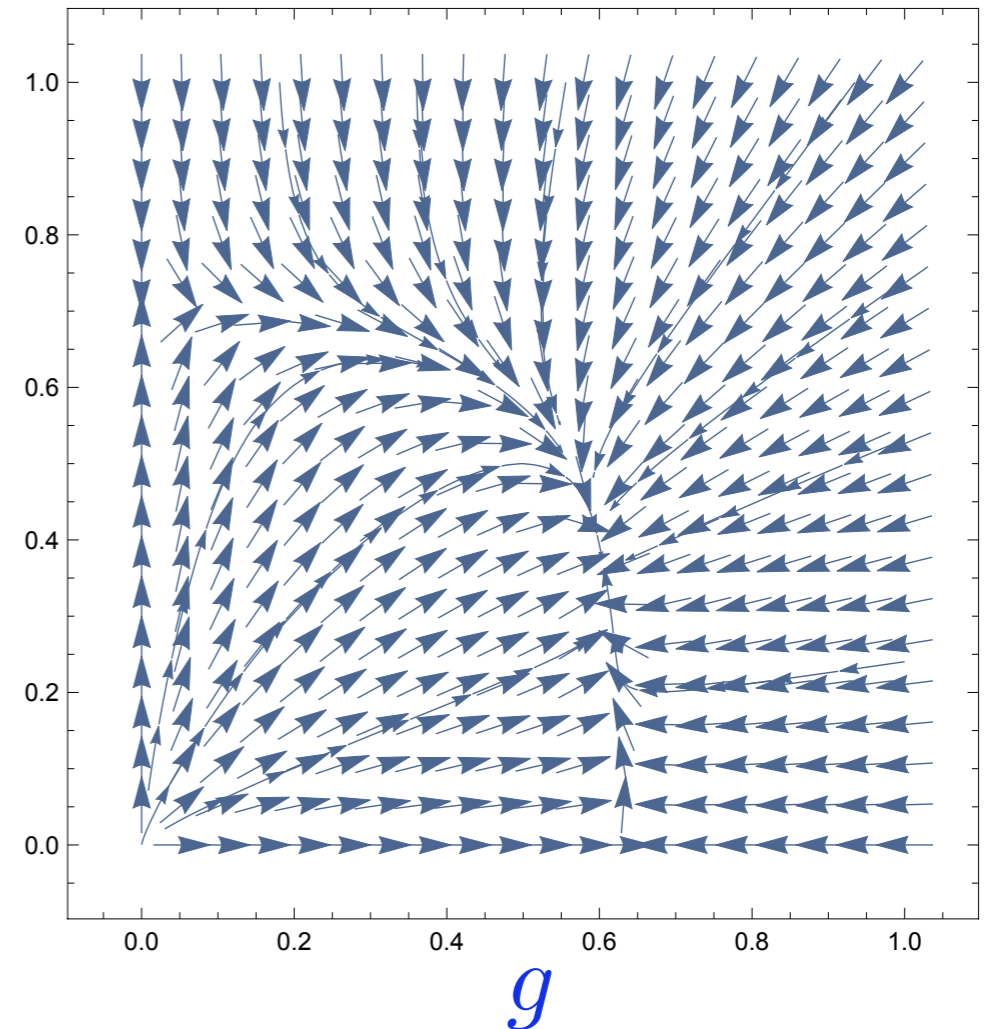
t-J model RG

We can perform a RG analysis for small $\epsilon = 3 - d$ and $\bar{r} = (1 - r)/2$, while imposing the local constraint *exactly*. The one-loop β functions are

$$\beta(g) = -\bar{r}g + \frac{3}{2}g^3 + \frac{3}{8}g\gamma^2,$$

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 + g^2\gamma.$$

$$\beta(s) = -s + 3g^2s - g^2 + \frac{3}{4}\gamma^2. \quad \gamma$$



These equations have a fixed point with $s \approx 0$ with only one relevant direction, corresponding to the flow of s to $\pm\infty$.

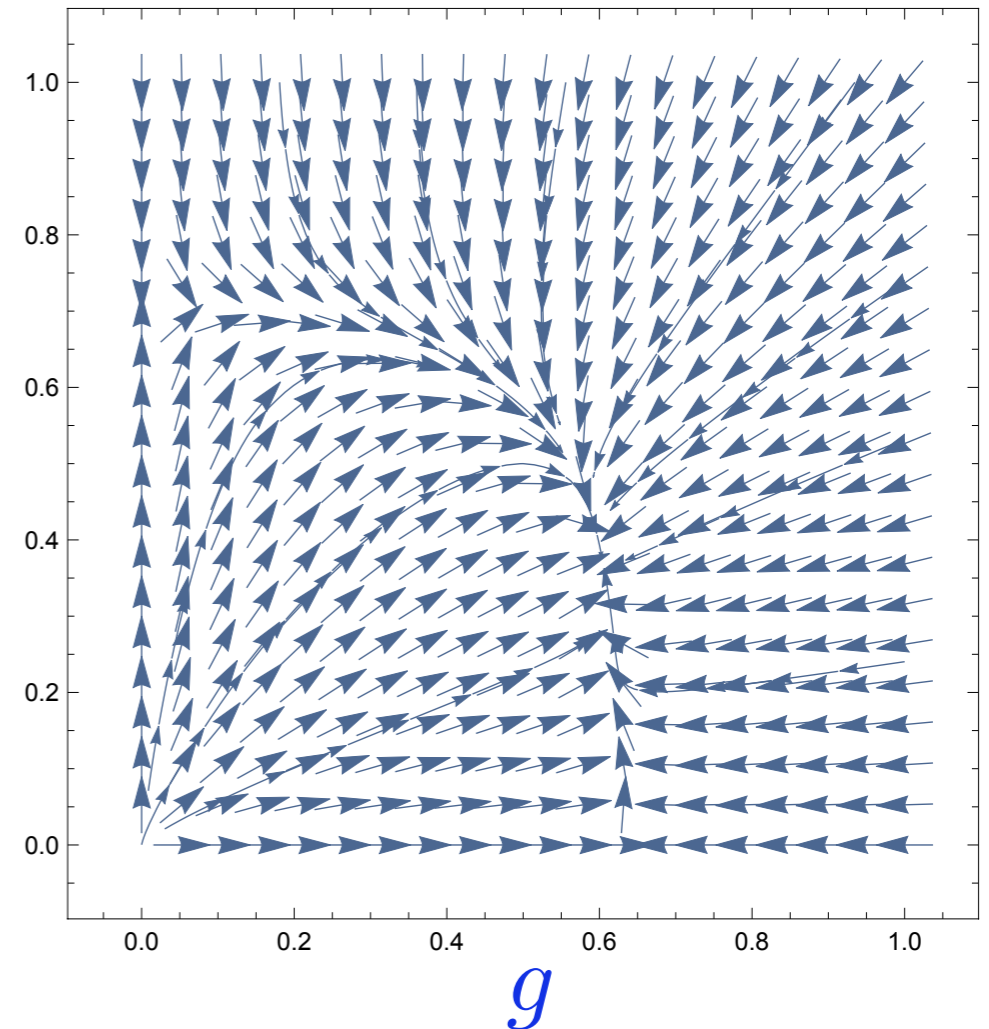
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These equations have a fixed point with $s \approx 0$ with only one relevant direction, corresponding to the flow of s to $\pm\infty$. The 3 states of the superspin are nearly degenerate at the fixed point, and the flows away from the fixed point correspond to different orientations of the field on the superspin: one side (overdoped) favors the holon, and the other side (underdoped) favors the spinon.

t-J model RG

The scaling dimensions of the electron and spin operators can be determined to all orders in ϵ and \bar{r} and these imply

$$\bar{R}(\tau) = -\frac{1}{2} \langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim \frac{\text{sgn}(\tau)}{|\tau|^{1-r}} \quad , \quad \bar{Q}(\tau) = \frac{1}{3} \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|^{3-d}} .$$

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Finally, we impose the self-consistency conditions $R(\tau) = \bar{R}(\tau)$, $Q(\tau) = \bar{Q}(\tau)$ and obtain $r = 0$ ($\bar{r} = 1/2$) and $d = 2$ ($\epsilon = 1$), so that at the critical point we have

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim \frac{\text{sgn}(\tau)}{|\tau|} \quad , \quad \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|} .$$

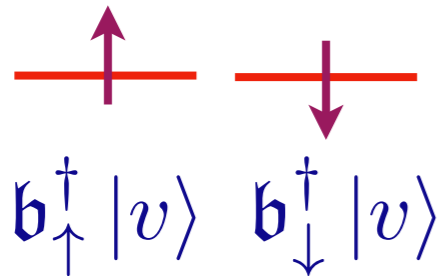
t - J model phase diagram

SU(2|1) theory

Metallic spin glass.

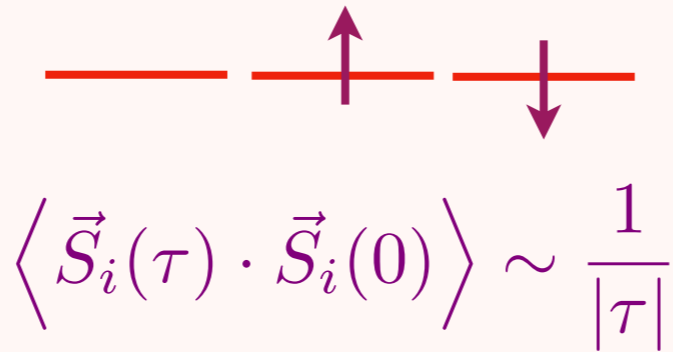
Condense spinon \mathbf{b}_α ,
 f carrier density p

$f^\dagger |v\rangle$



$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \text{constant}$$

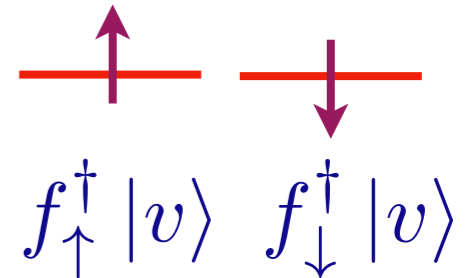
Deconfined quantum critical point



SU(1|2) theory

Disordered Fermi liquid.

Condense holon b ,
 f_α carrier density $1 + p$



$b^\dagger |v\rangle$

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$$

p_c

p

s

t - J model large M

Each site has 3 states which we map to the space of a boson b (the holon) and a fermion f_α (the spinon):

$$\begin{aligned} |0\rangle &\Rightarrow b^\dagger |v\rangle & , & & c_\alpha^\dagger |0\rangle &\Rightarrow f_\alpha^\dagger |v\rangle \\ c_\alpha &= f_\alpha b^\dagger & , & & f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

To obtain a large M limit, let $\alpha = 1 \dots M$, endow the boson with an ‘orbital’ index $a = 1 \dots M'$ and send $M \rightarrow \infty$ at fixed $k = M'/M$. Then

$$c_{a\alpha} = f_\alpha b_a^\dagger \quad , \quad f_\alpha^\dagger f_\alpha + b_a^\dagger b_a = \frac{M}{2}$$

t-J model large M

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green's functions:

$$\begin{aligned}G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\ \Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\ G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\ \Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k t^2 G_f(\tau) G_b(\tau) G_b(-\tau)\end{aligned}$$

Here μ_f and μ_b are chemical potentials chosen to satisfy

$$\langle f^\dagger f \rangle = \frac{1}{2} - kp \quad , \quad \langle b^\dagger b \rangle = p .$$

t-J model large M

These equations have critical solutions with

$$G_f(z) = C_f \frac{e^{-i(\pi\Delta_f + \theta_f)}}{z^{1-2\Delta_f}}, \quad G_b(z) = C_b \frac{e^{-i(\pi\Delta_b + \theta_b)}}{z^{1-2\Delta_b}}, \quad \text{Im}(z) > 0$$

$$\Delta_f + \Delta_b = \frac{1}{2}$$

$$\frac{\theta_f}{\pi} + \left(\frac{1}{2} - \Delta_f \right) \frac{\sin(2\theta_f)}{\sin(2\pi\Delta_f)} = kp$$

$$\frac{\theta_b}{\pi} + \left(\frac{1}{2} - \Delta_b \right) \frac{\sin(2\theta_b)}{\sin(2\pi\Delta_b)} = \frac{1}{2} + p.$$

The last two are analogs of ‘Luttinger’ relations, which follow from an anomaly matching argument (Yingei Gu, A. Kitaev, S. Sachdev, G. Tarnopolsky arXiv:1910.14099).

t - J model large M

The critical solution which is self-consistent in both the t and J terms has $\Delta_b = \Delta_f = 1/2$, implying

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim \begin{cases} \frac{A_+}{|\tau|} & , \quad \tau > 0 \\ -\frac{A_-}{|\tau|} & , \quad \tau < 0 \end{cases} , \quad \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|} .$$

The same exponents are obtained to all orders in the ϵ, \bar{r} expansion, but with $A_+ = A_-$.

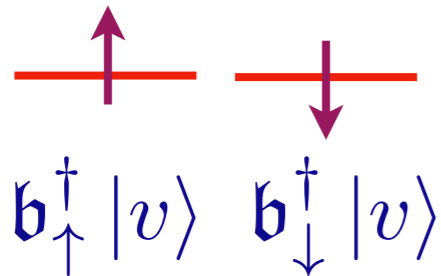
t - J model phase diagram

SU(2|1) theory

Metallic spin glass.

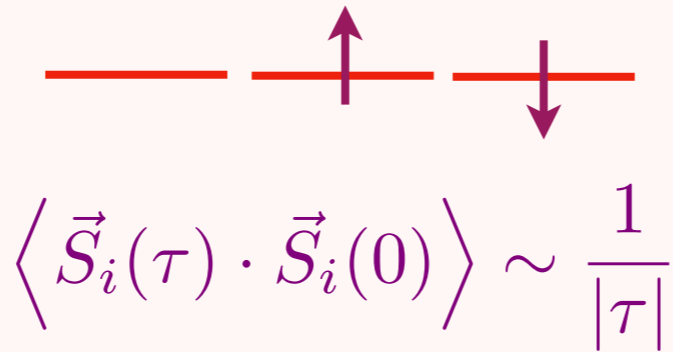
Condense spinon \mathbf{b}_α ,
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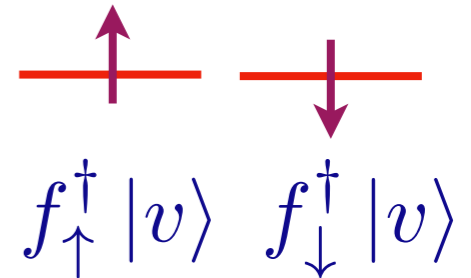
Deconfined quantum critical point



SU(1|2) theory

Disordered Fermi liquid.

Condense holon b ,
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$b^\dagger |v\rangle$

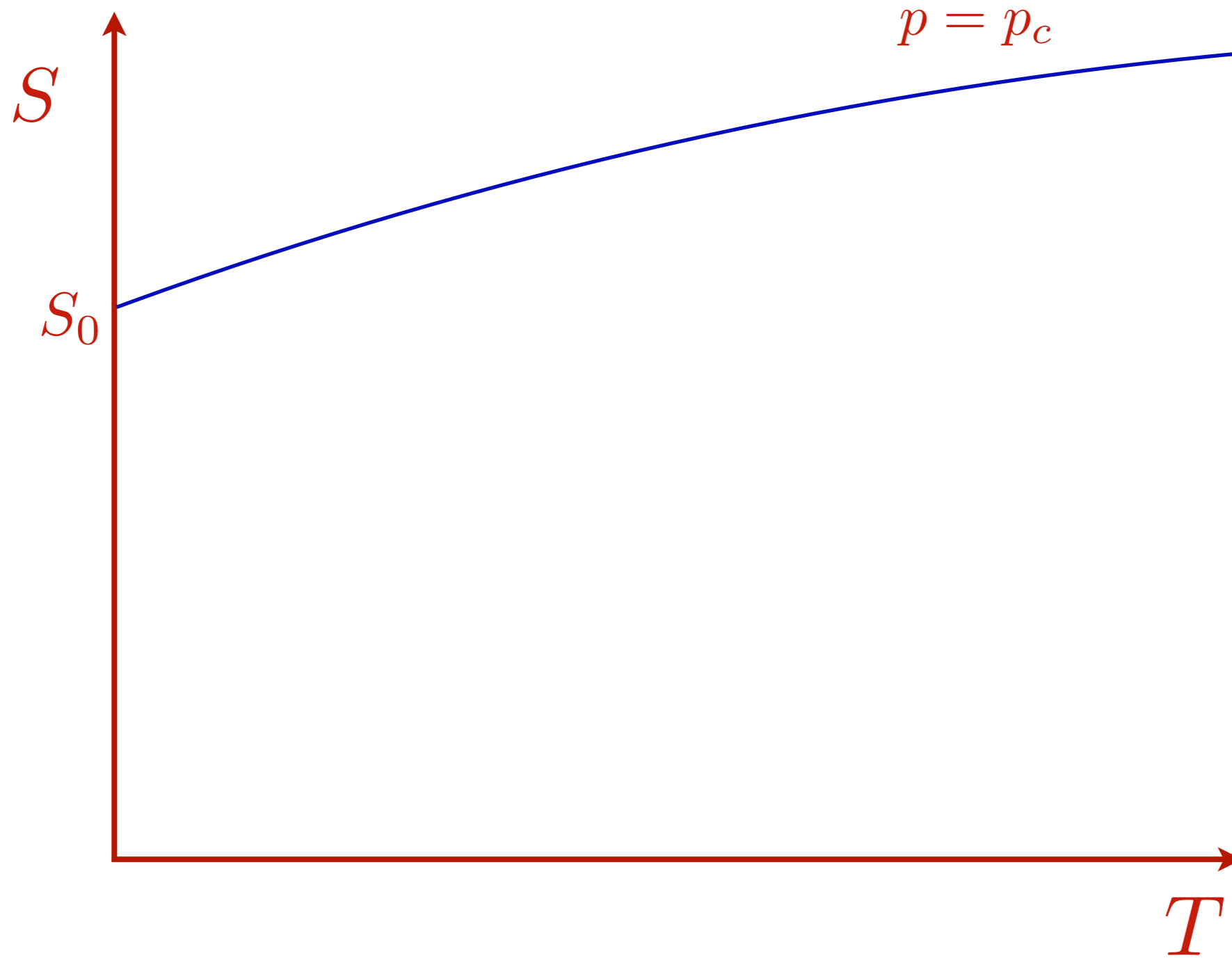
$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$$

p_c

p

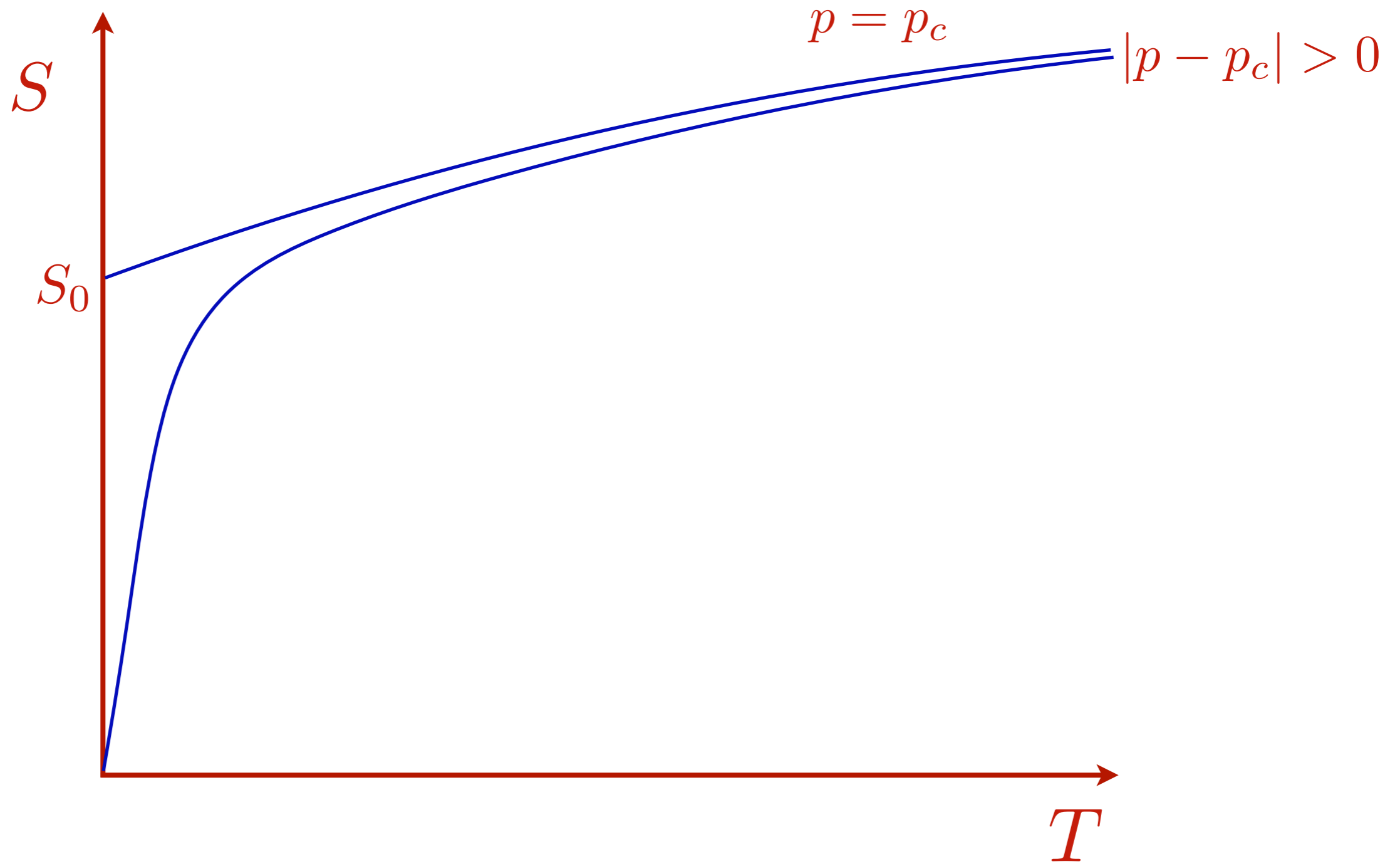
s

t-j model entropy



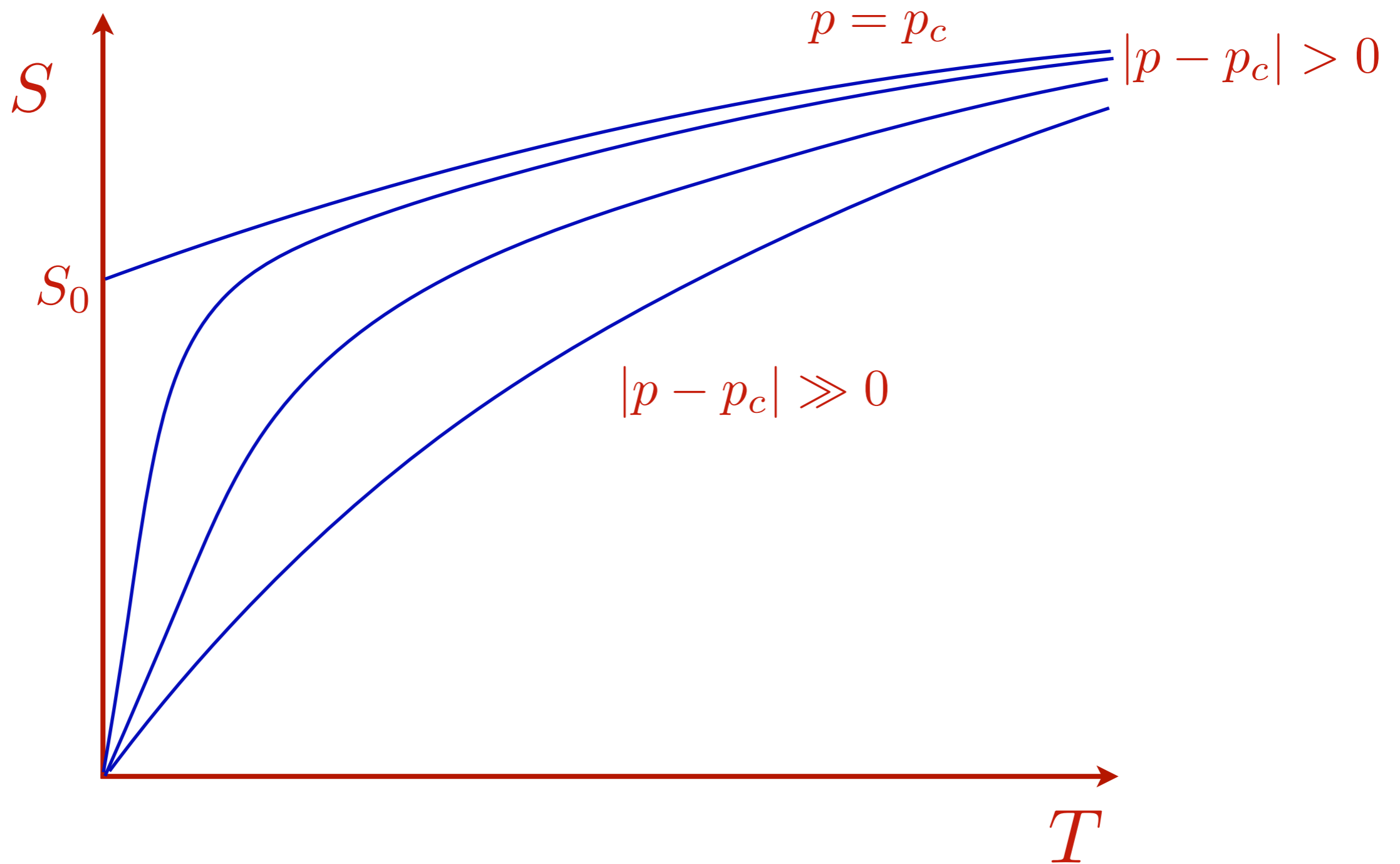
$$\frac{C}{T} = \frac{dS}{dT}$$

t - J model entropy



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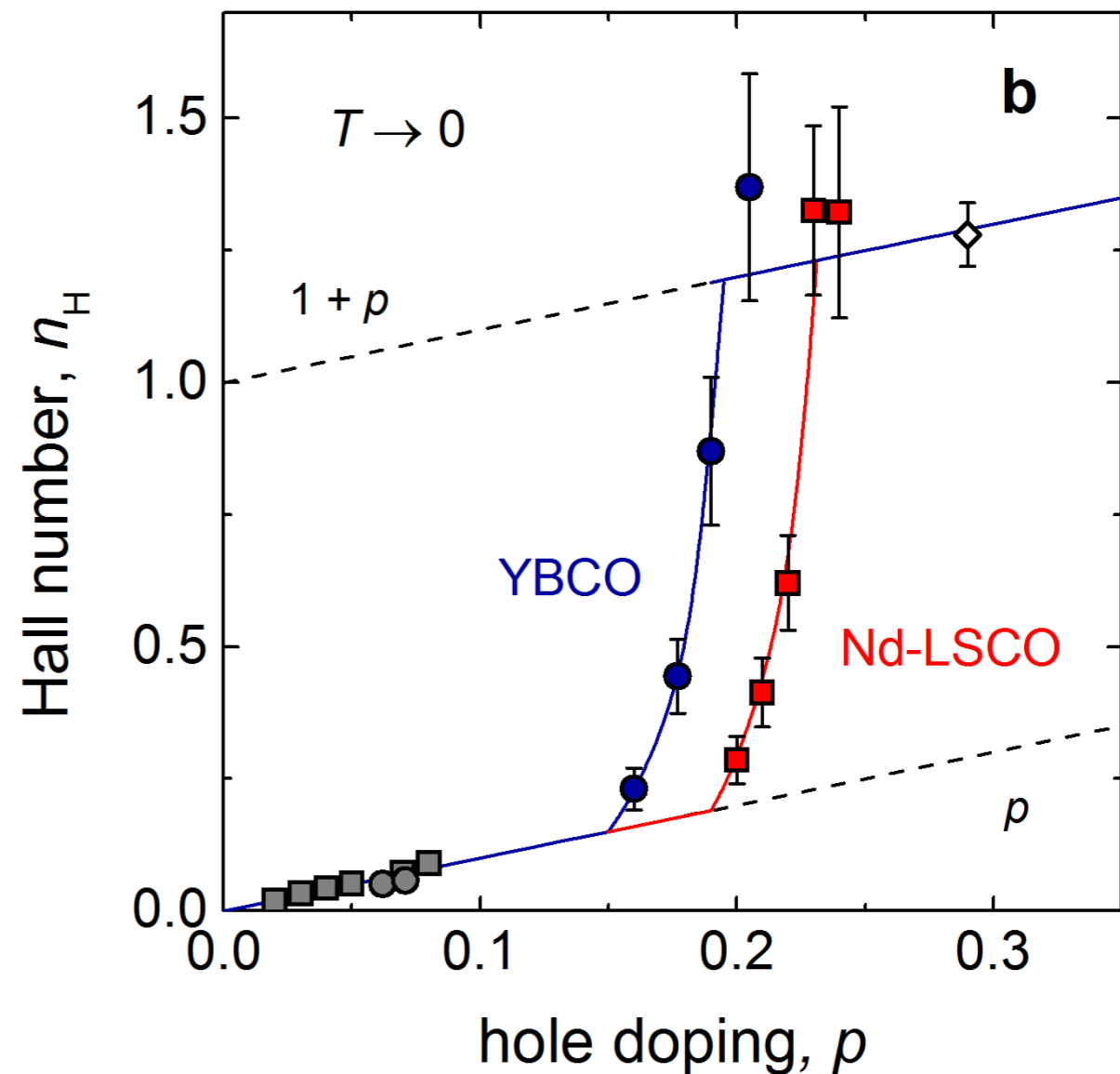
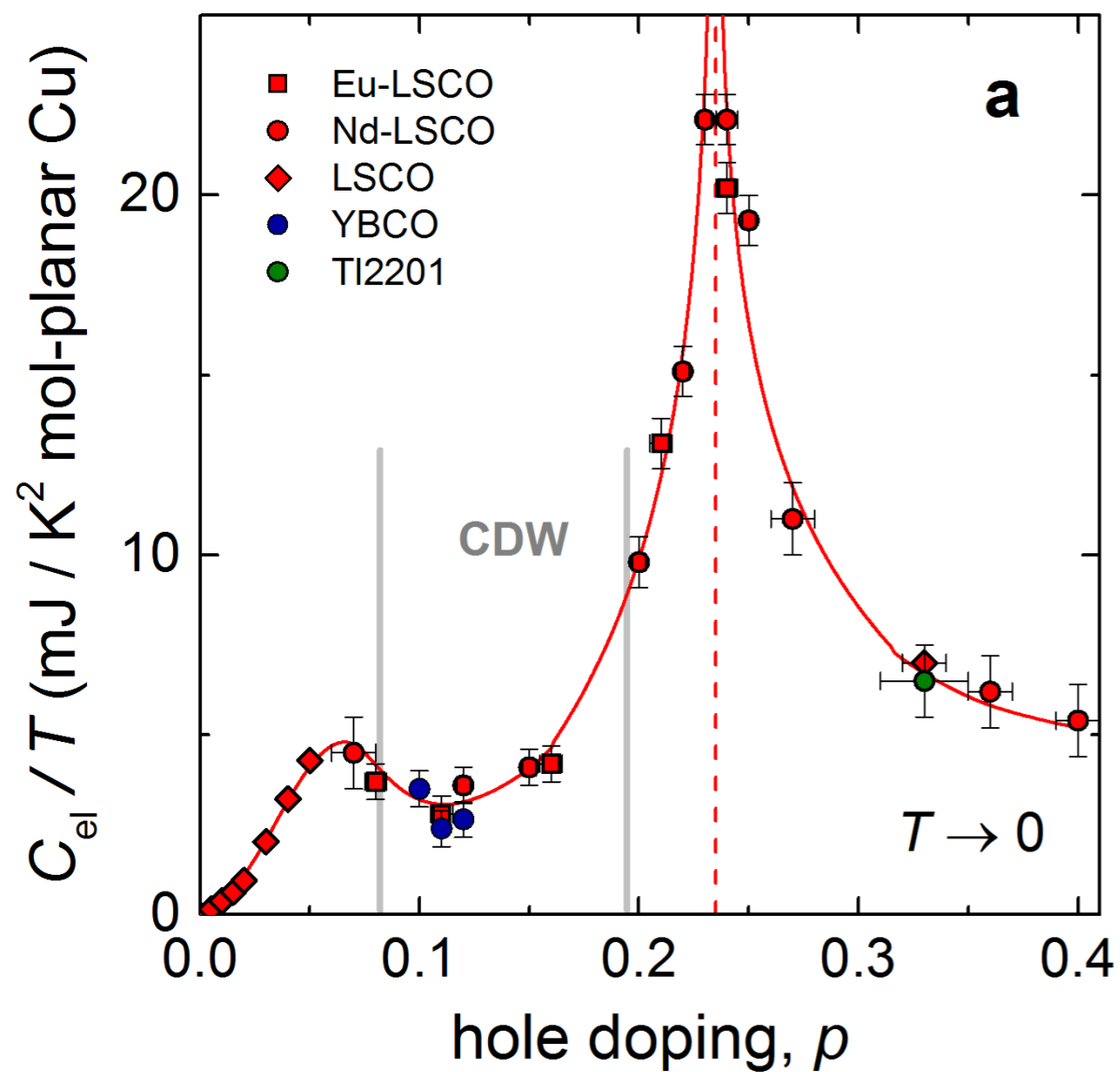


$$\frac{C}{T} = \frac{dS}{dT}$$

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507



1. Insulating random magnet

2. Deconfined criticality at
non-zero doping

3. Phase diagram of disordered
Hubbard model

Random t - J - U model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$

$$t_{ij}, J_{ij} \text{ random}, \quad U > 0$$

Large N limit

$$\mathcal{Z} = \int \mathcal{D}c_\alpha(\tau) e^{-\mathcal{S}}$$

$$\begin{aligned} \mathcal{S} = \int d\tau & \left[c_\alpha^\dagger(\tau) \left(\frac{\partial}{\partial \tau} - \mu \right) c_\alpha(\tau) + U n_\uparrow(\tau) n_\downarrow(\tau) \right] \\ & - t^2 \int d\tau d\tau' R^*(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') \\ & - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau'). \end{aligned}$$

From this action we determine the correlators

$$\begin{aligned} \bar{R}(\tau - \tau') &= -\frac{1}{2} \langle c_\alpha^\dagger(\tau) c_\alpha(\tau') \rangle_{\mathcal{Z}} \\ \bar{Q}(\tau - \tau') &= \frac{1}{3} \langle \vec{S}(\tau) \cdot \vec{S}(\tau') \rangle_{\mathcal{Z}} \end{aligned}$$

and finally impose the self-consistency conditions

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$$t_{ij}, J_{ij} \text{ random}, \quad U > 0$$

$1/U$

0

doping $p = \langle n_{i\uparrow} + n_{i\downarrow} - 1 \rangle$

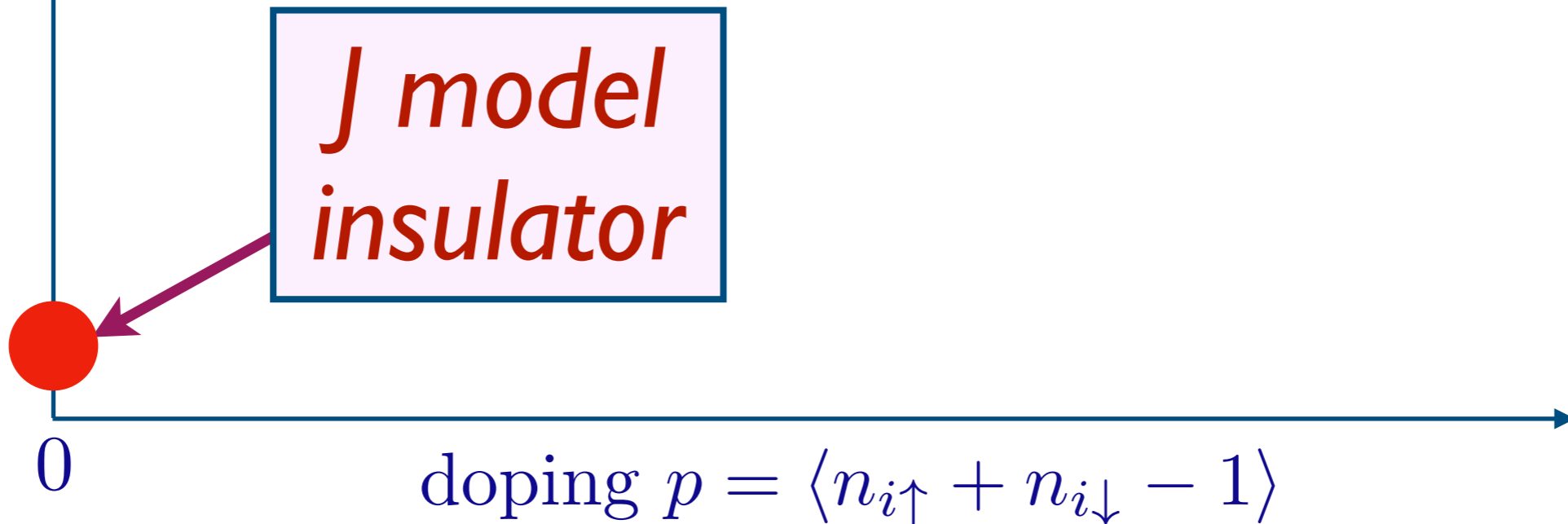
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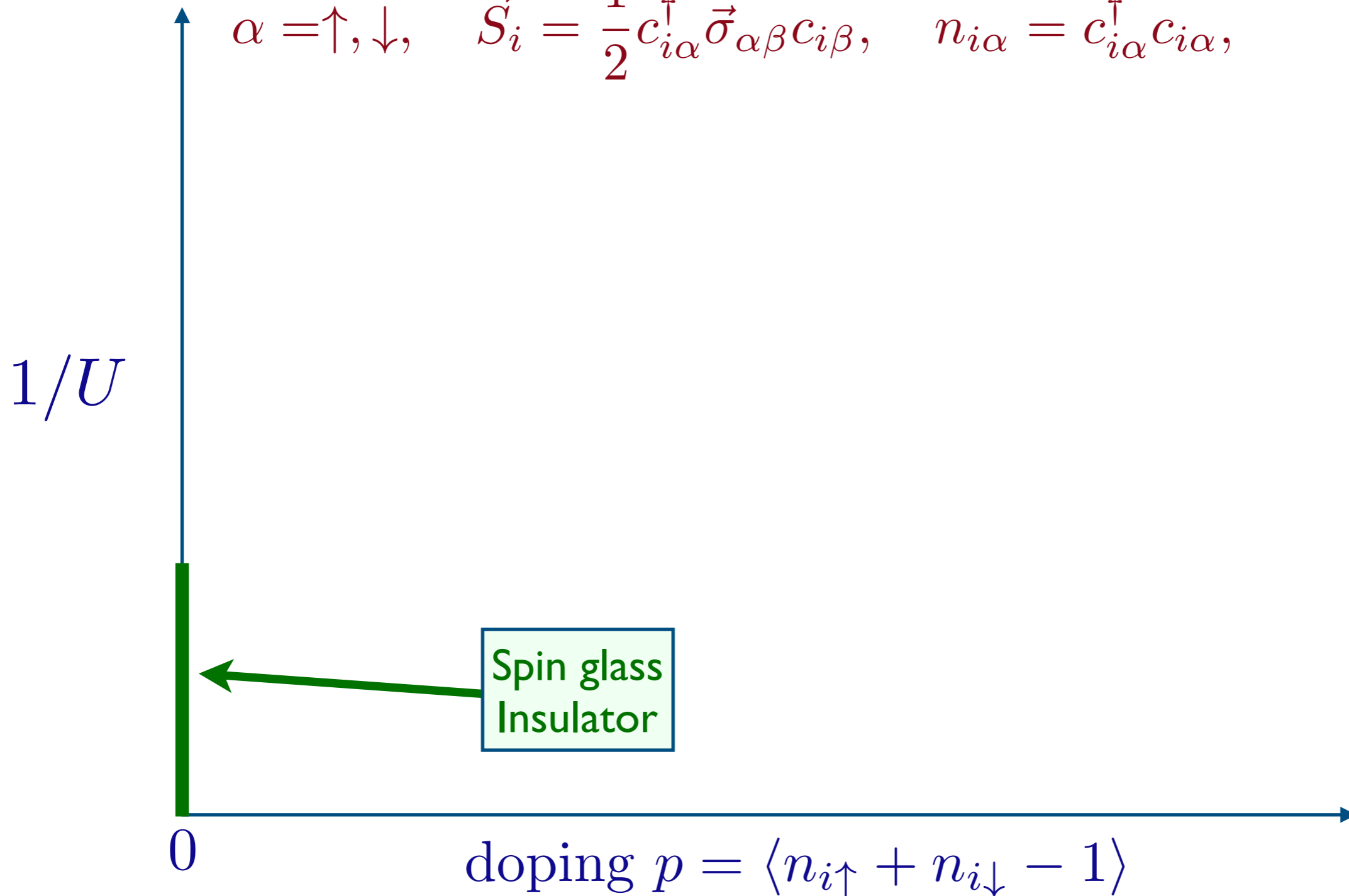
$1/U$



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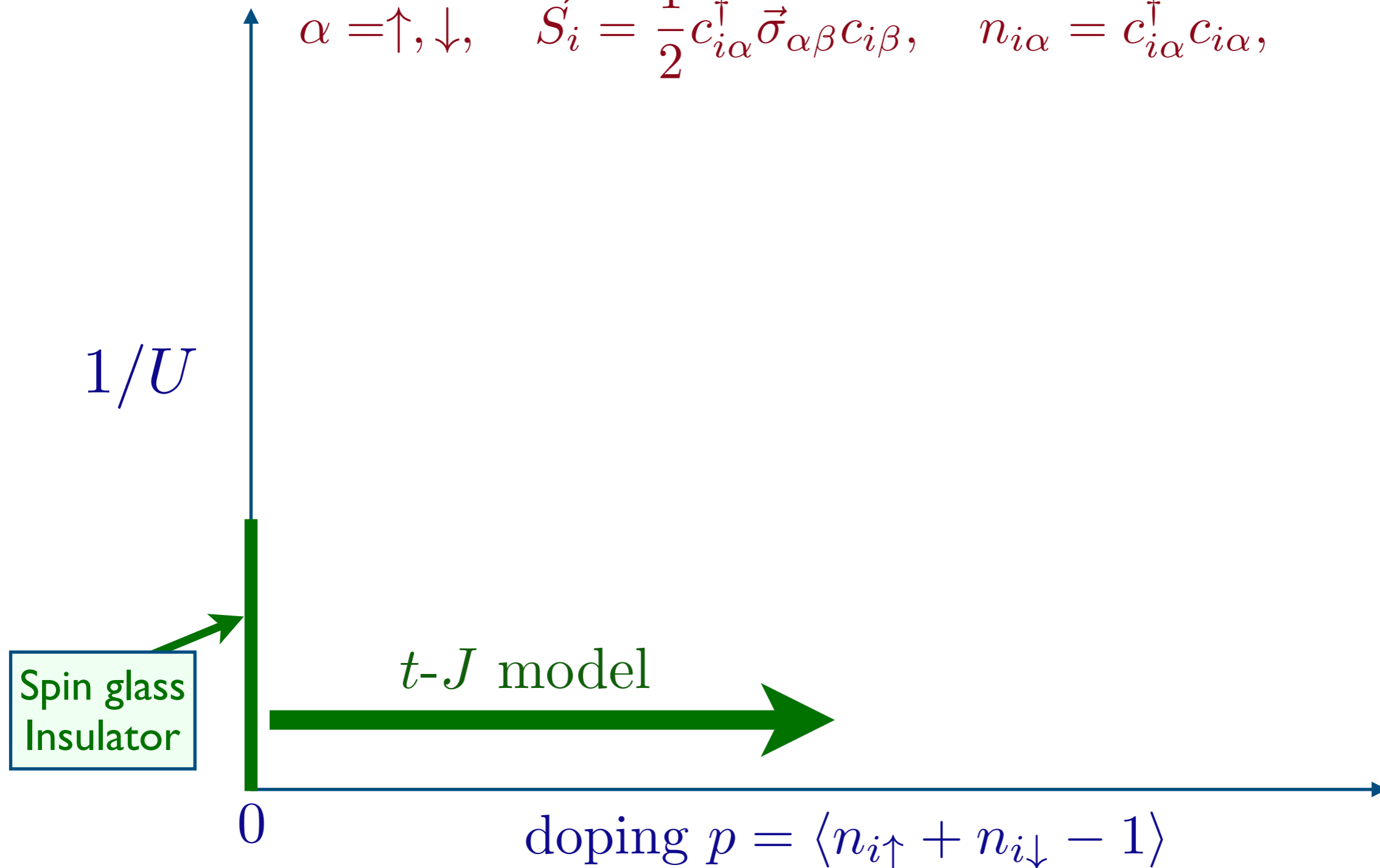
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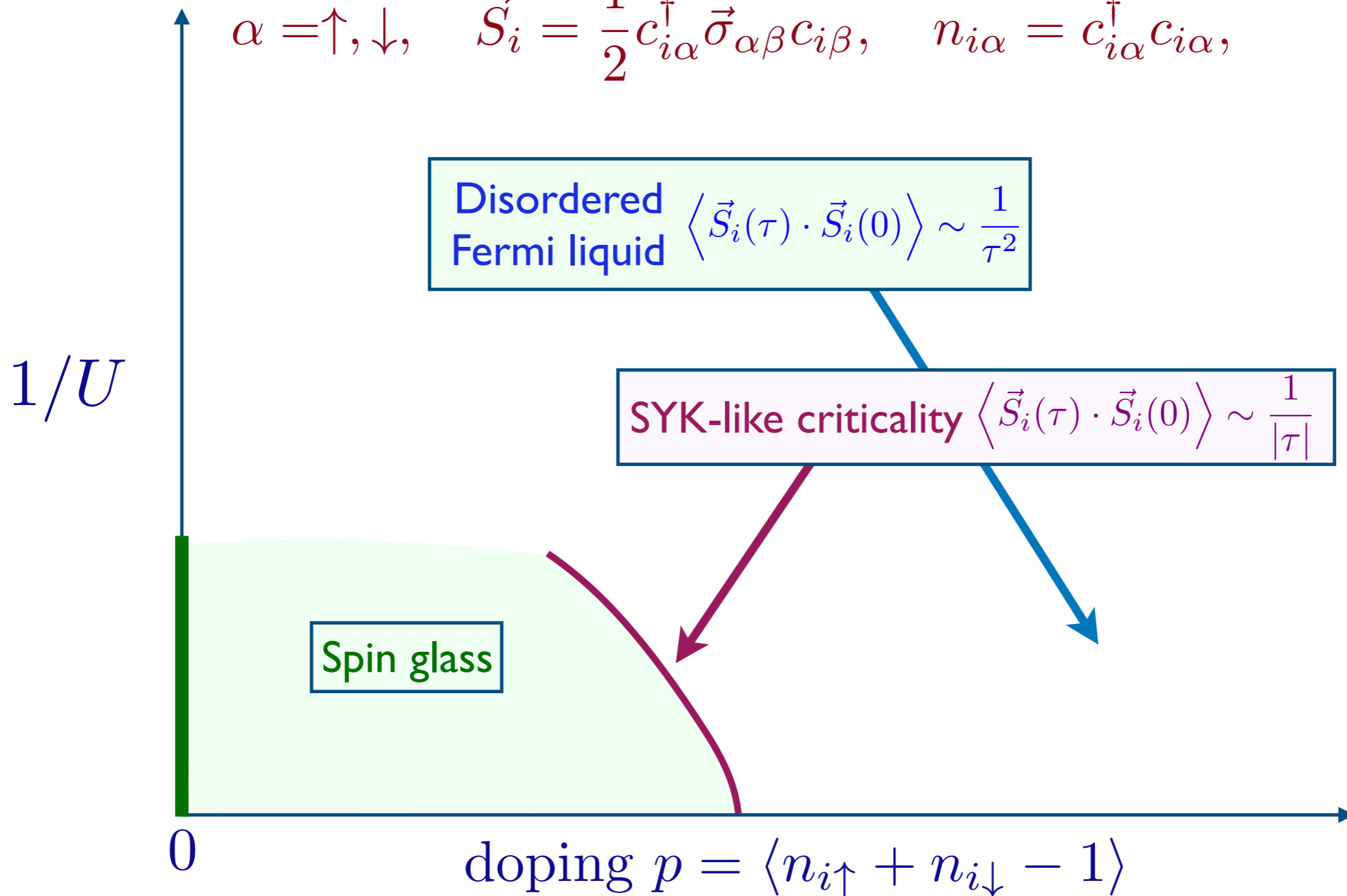
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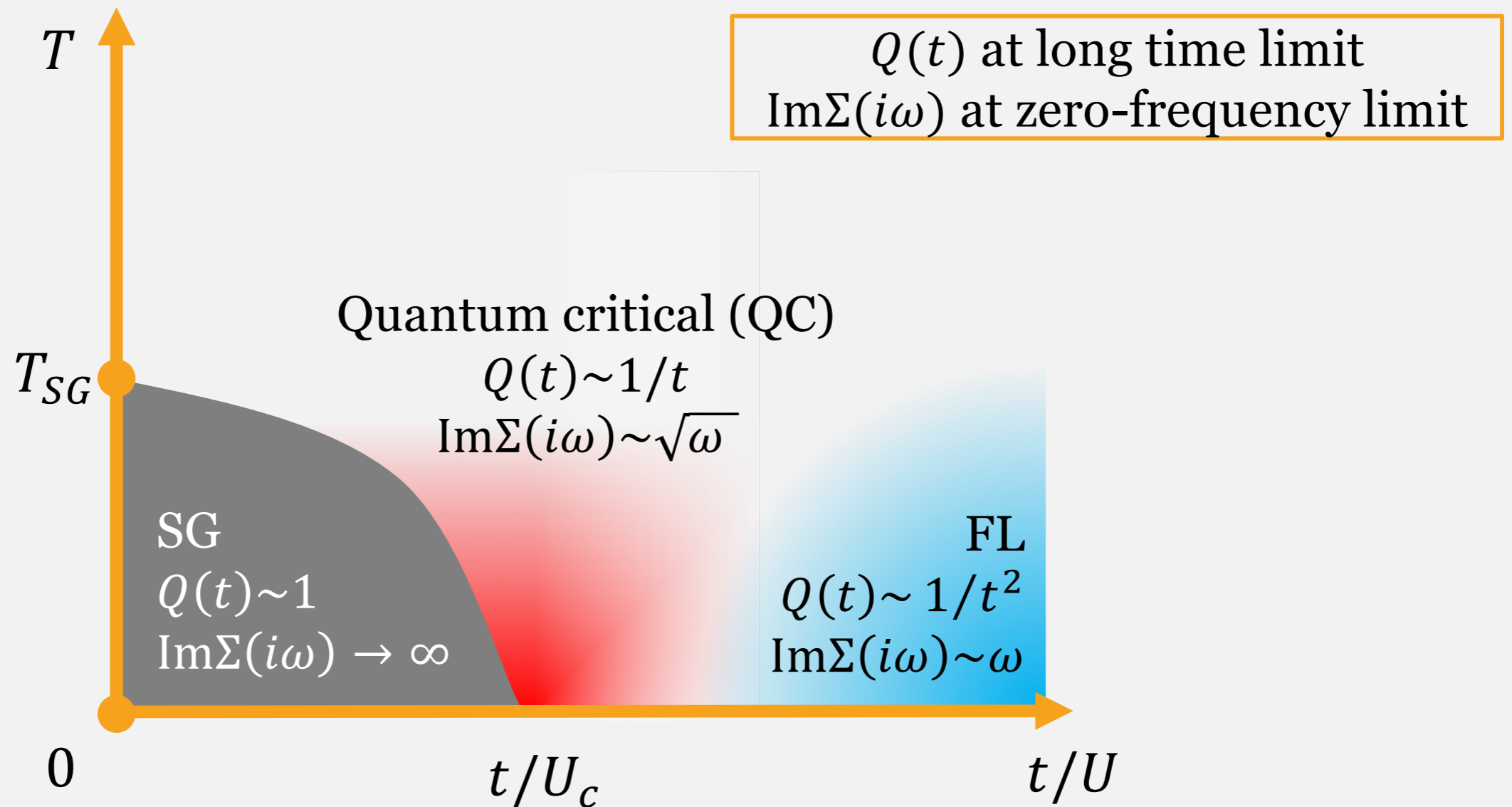


Disordered Fermi liquid $\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$

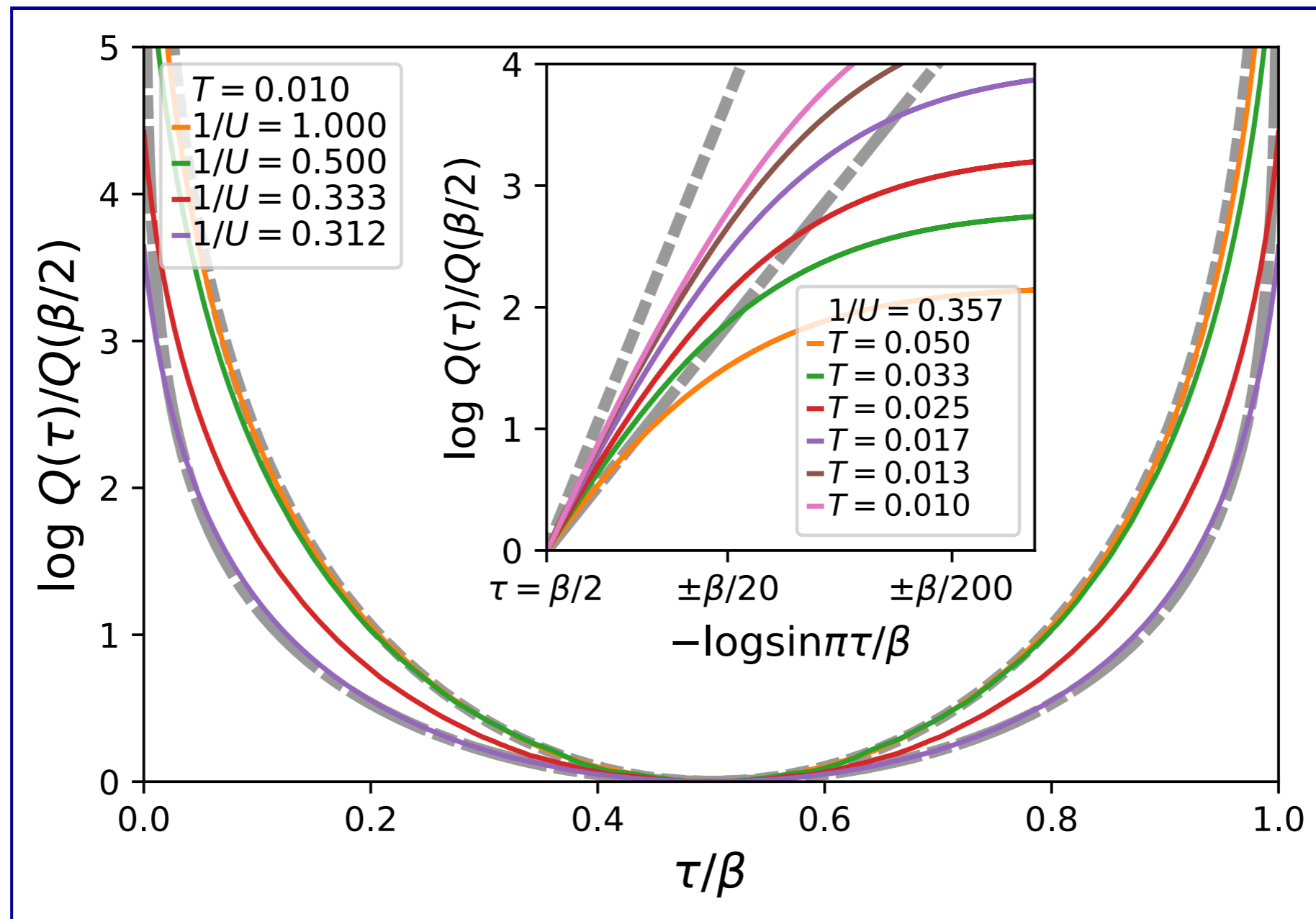
SYK-like criticality $\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$

Spin glass

Quantum Critical Point between a FL Metal and a Spin Glass Mott Insulator



Quantum Critical Point between a FL Metal and a Spin Glass Mott Insulator

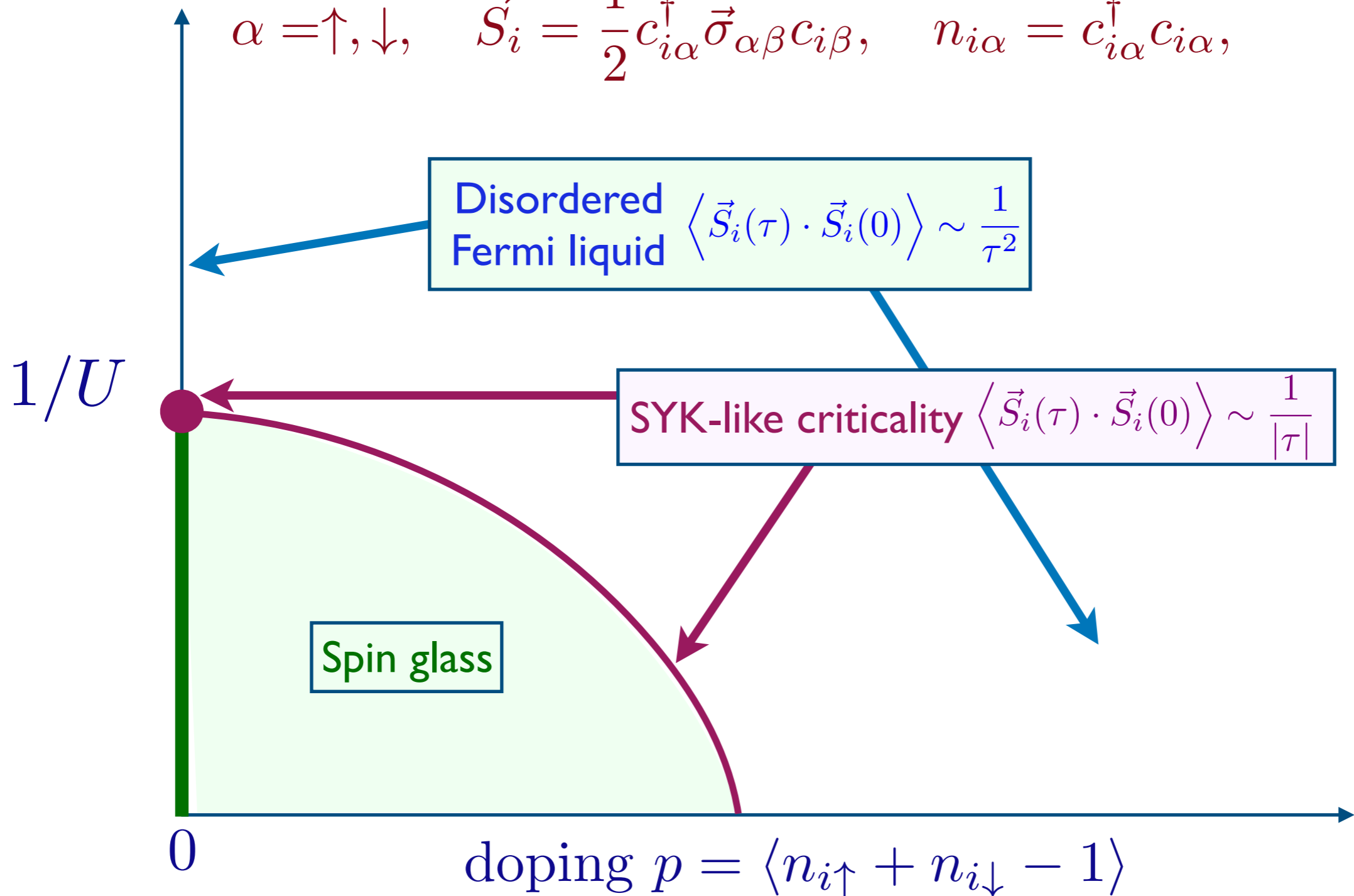


Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-ah Kim,
APS March meeting 2019

Random t - J - U model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

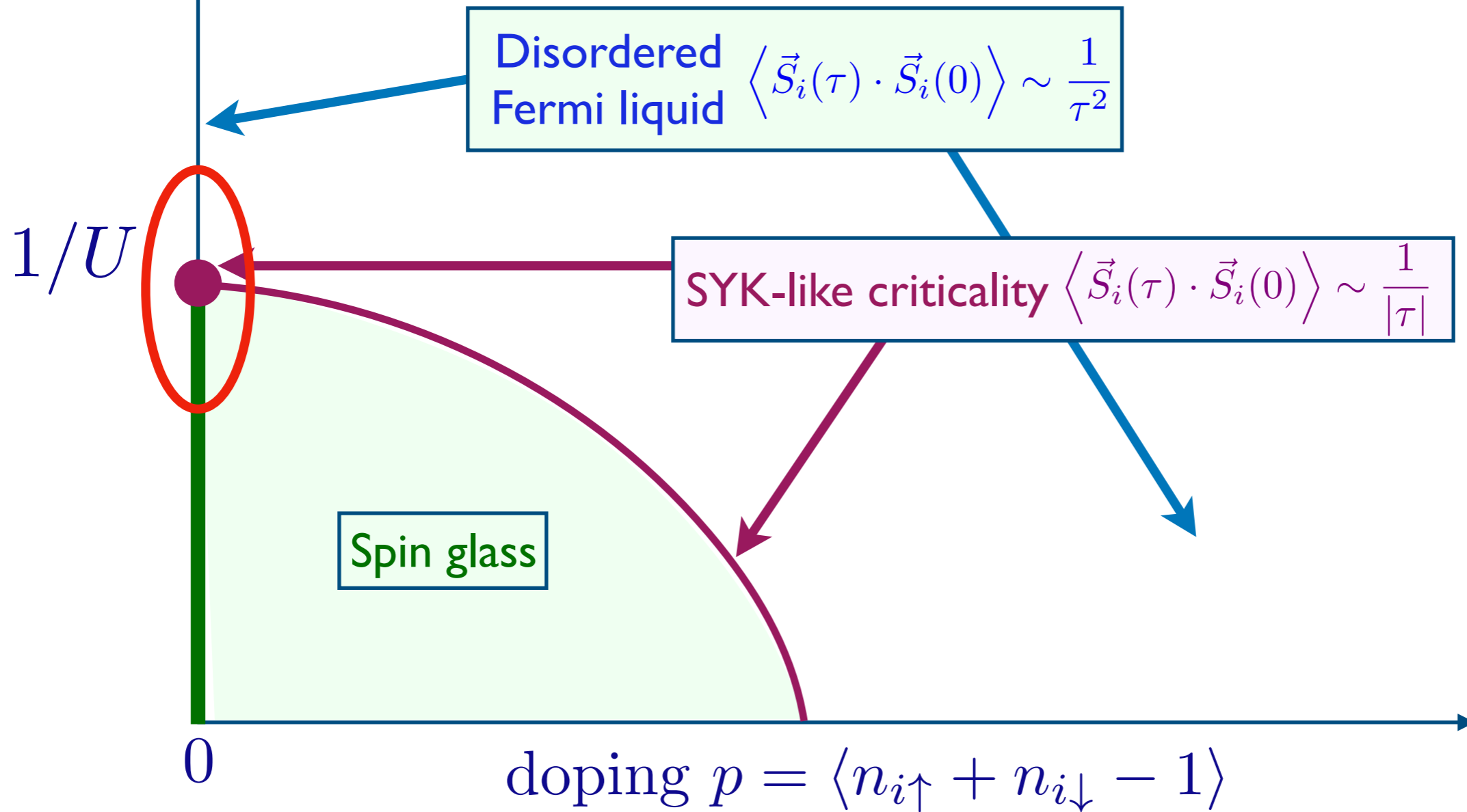
$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$



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Half-filled t - J - U model RG

Let us focus on the critical point at half-filling, and assume a power-law decay

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} .$$

We ignore the self-consistency condition for now. We also decouple the last two terms by introducing bosonic (ϕ_a , $a = 1 \dots 3$) and fermionic (ψ_α) baths. Then the problem reduces to the Hamiltonian

$$\begin{aligned} H_{\text{imp}} &= \frac{U}{2} (c_\alpha^\dagger c_\alpha - 1)^2 + V_0 c_\alpha^\dagger \psi_\alpha(0) + \text{H.c.} + \zeta [\phi_\alpha(0)]^2 \\ &+ \gamma_0 c_\alpha^\dagger \frac{\sigma_{\alpha\beta}^a}{2} c_\beta \phi_a(0) + \int |k|^r dk k \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x [\pi_a^2 + (\partial_x \phi_a)^2] \end{aligned}$$

where $a = (x, y, z)$, σ^a are Pauli matrices, π_a is canonically conjugate to the field ϕ_a , and $\phi_a(0) \equiv \phi_a(x=0)$, $\psi_\alpha(0) \equiv \int |k|^r dk \psi_{k\alpha}$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and $R(\tau)$ with the temporal correlator of $\psi_\alpha(0)$, and it can be verified that these correlators decay as above.

L. Fritz and M. Vojta, PRB **70**, 214427 (2004)

J. H. Pixley, S. Kirchner, K. Ingersent, and Qimiao Si, PRB **88**, 245111 (2013)

Half-filled t - J - U model RG

Now a RG analysis is possible for $d \approx 2$ and $r \approx 1/2$, which is perturbative in U . The one-loop equations are

$$\beta_\gamma = \frac{(1 - 2r)}{2}\gamma + \frac{(d - 2)}{2}\gamma + \frac{\gamma U}{\pi(iA_0)^2} + \frac{\zeta\gamma}{2\pi},$$

$$\beta_U = (1 - 2r)U - \frac{\gamma^2}{8\pi},$$

$$\beta_\zeta = (d - 2)\zeta + \frac{\zeta^2}{2\pi} + \frac{\gamma^2}{2\pi A_0^2}.$$

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The electron operator c_α does not receive any wavefunction renormalization. This implies the correlator

$$\overline{R}(\tau) = -\frac{1}{2} \langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim \frac{\text{sgn}(\tau)}{|\tau|^{1-r}}.$$

The self-consistent value of r is therefore $r = 0$.

There is a non-trivial renormalization of the spin operator \vec{S} at a fixed point with $\zeta \neq 0$. Determining the self-consistent value of d requires computation ...