Quantum criticality in the high temperature superconductors

Paul Scherrer Institute, Switzerland
December 19, 2013

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Talk online: sachdev.physics.harvard.edu

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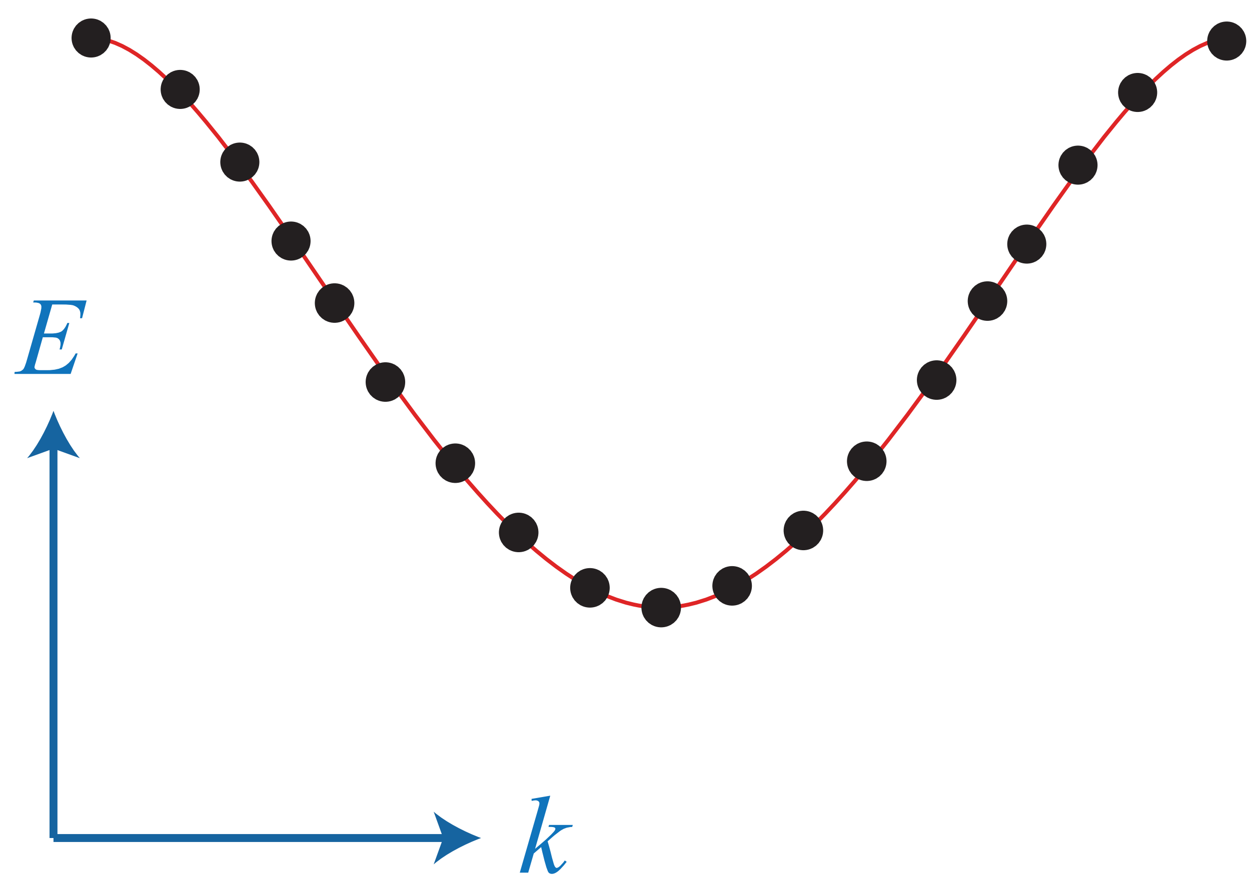
Sommerfeld-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

Band insulators

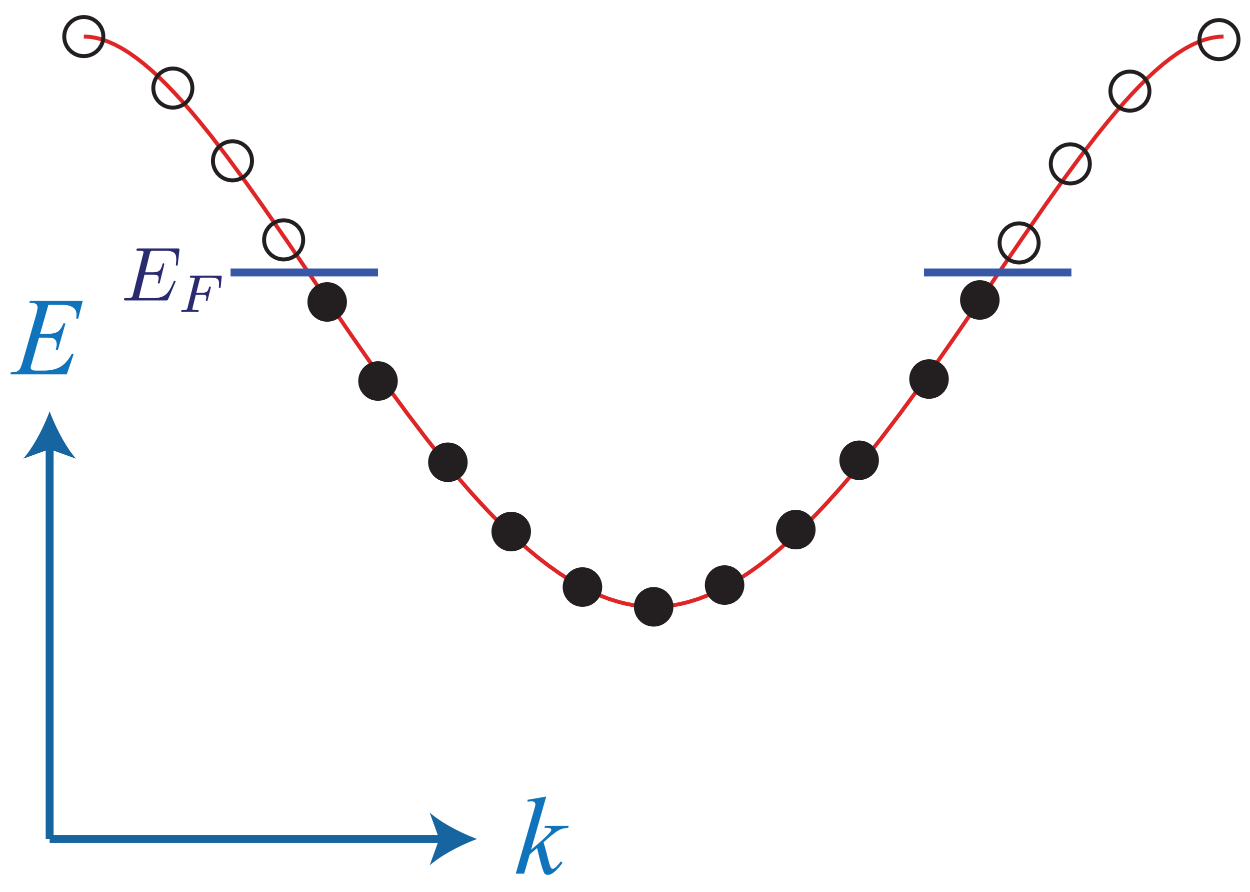


An even number of electrons per unit cell

Sommerfeld-Bloch theory of metals, insulators, and superconductors:

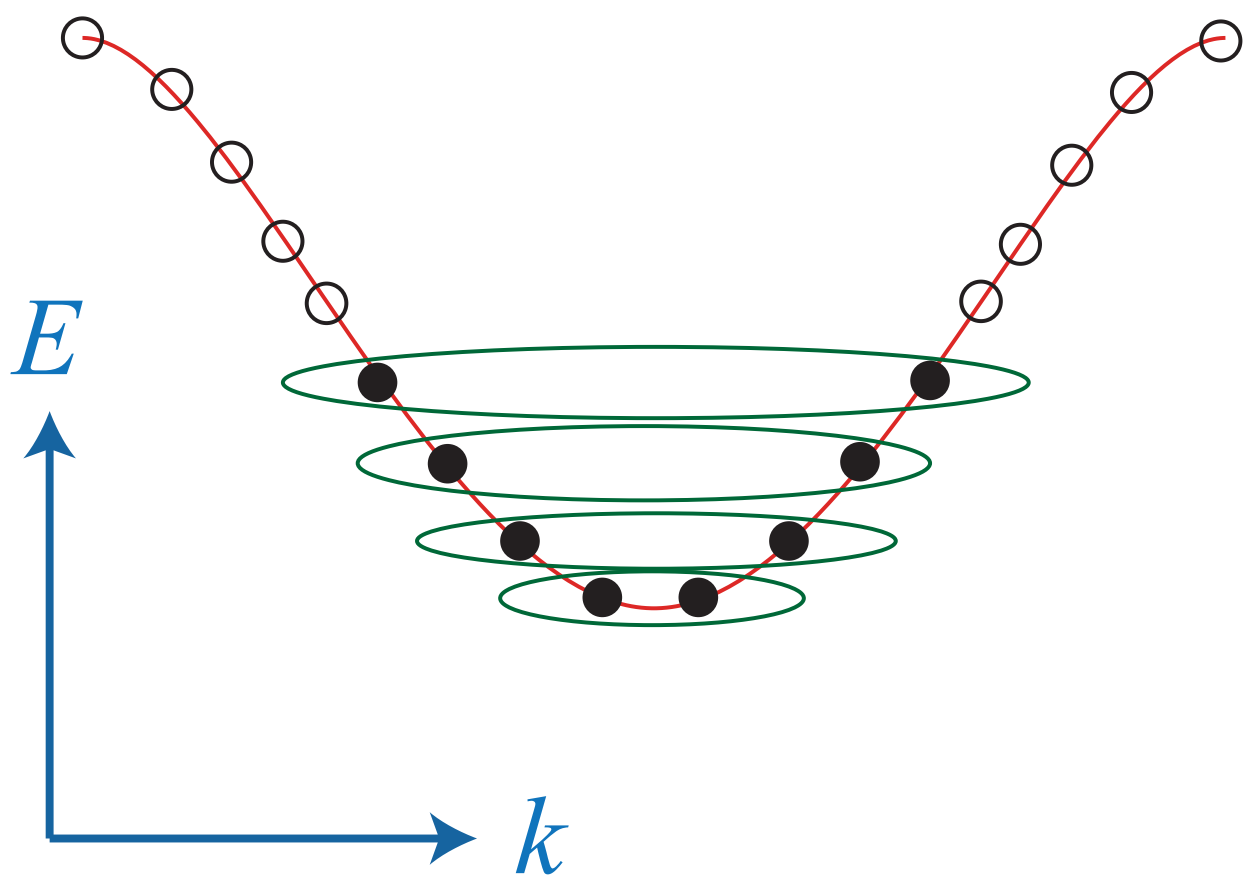
many-electron quantum states are adiabatically connected to independent electron states

Metals



Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

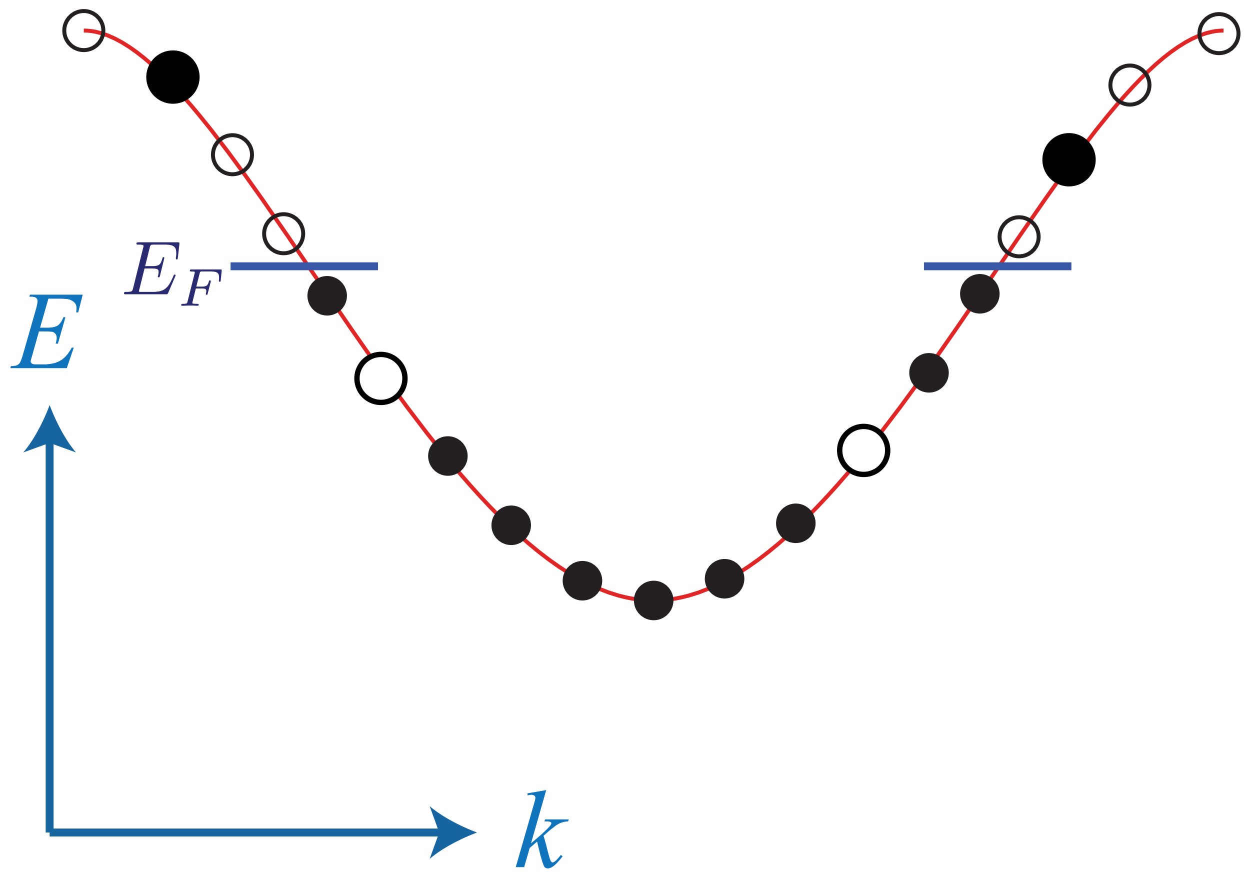
Superconductors



Boltzmann-Landau theory of dynamics of metals:

Long-lived ***quasiparticles*** (and ***quasiholes***) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Modern phases of quantum matter

Not adiabatically connected to independent electron states:

many-particle

quantum entanglement,

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter

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Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

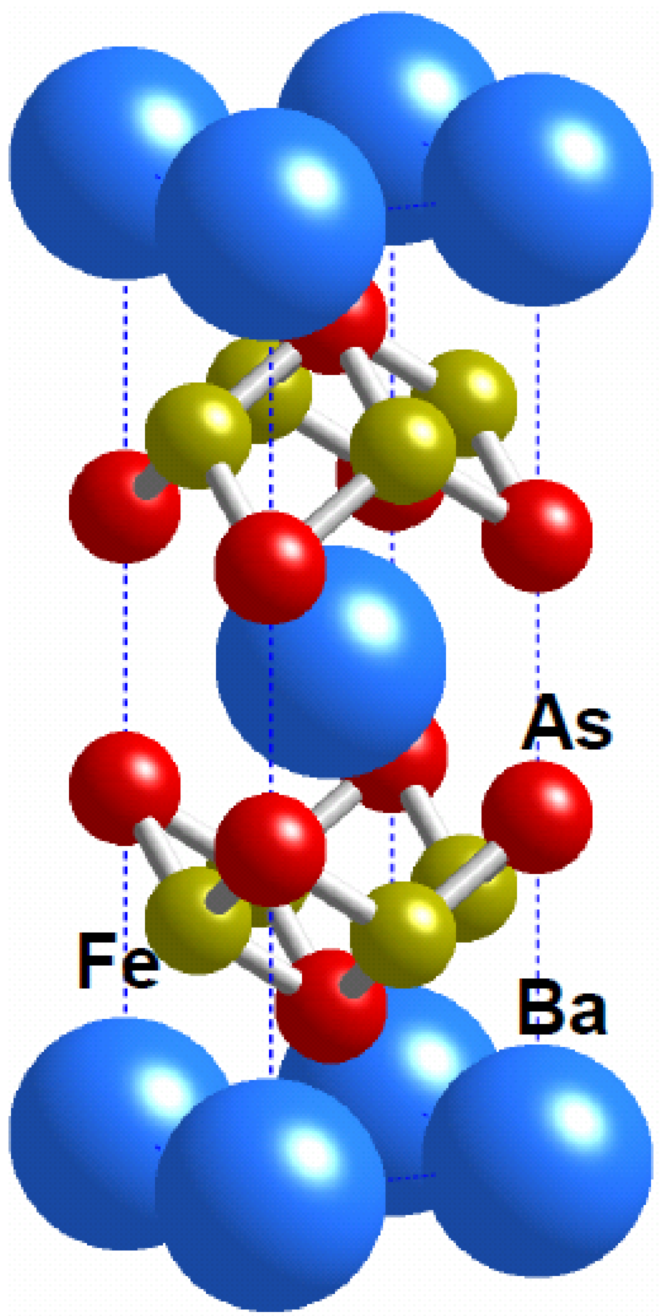
many-particle

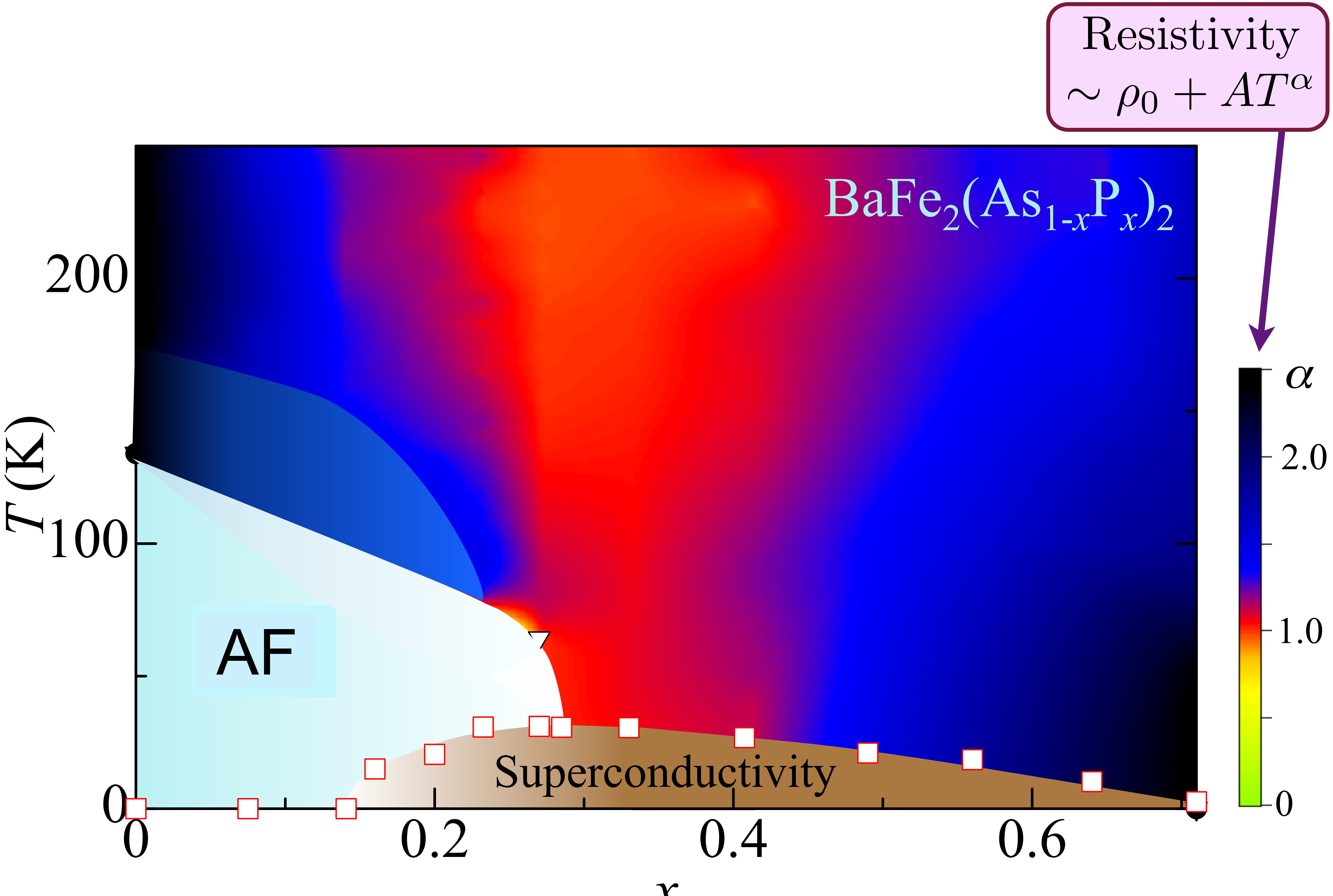
quantum entanglement,

and no quasiparticles

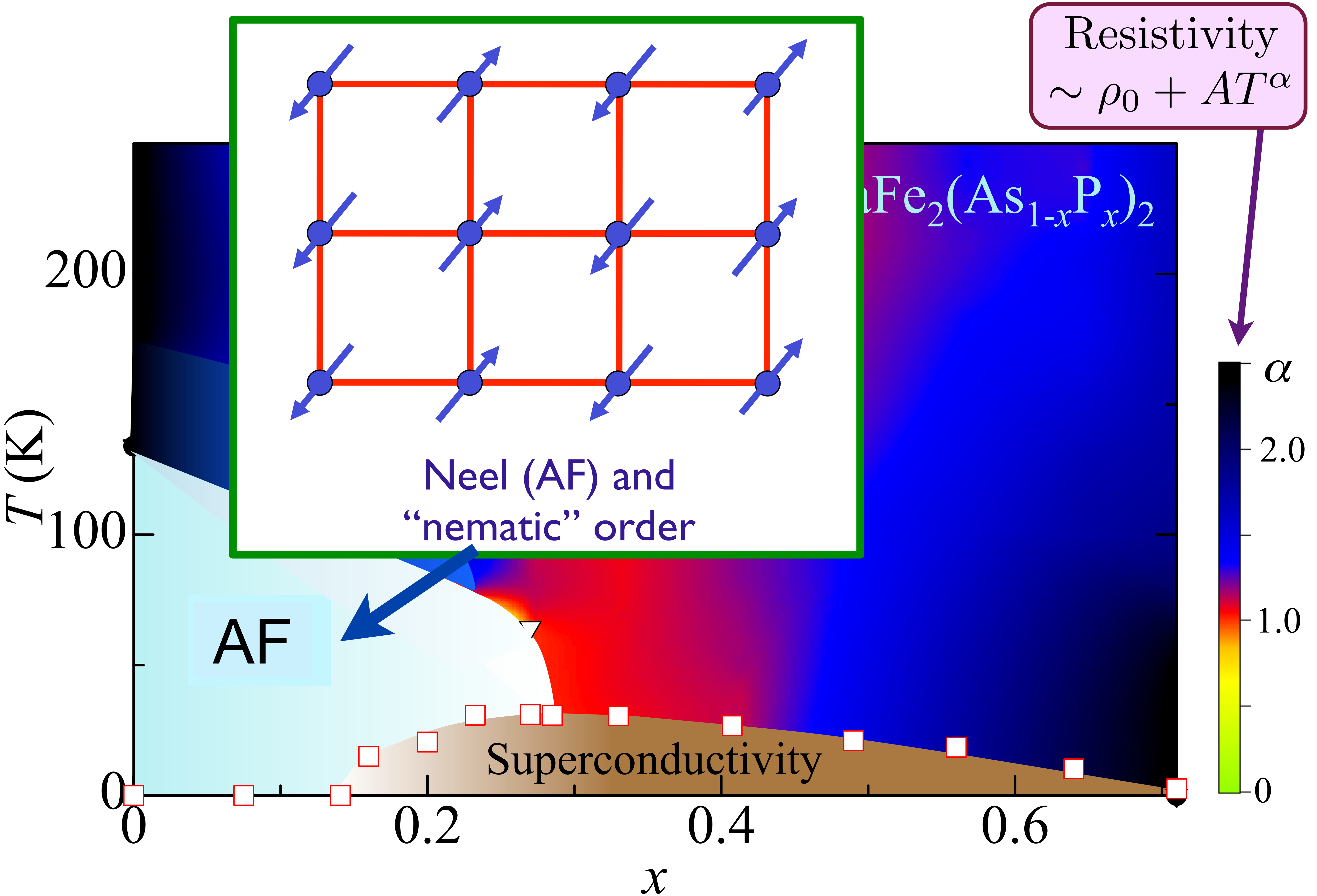
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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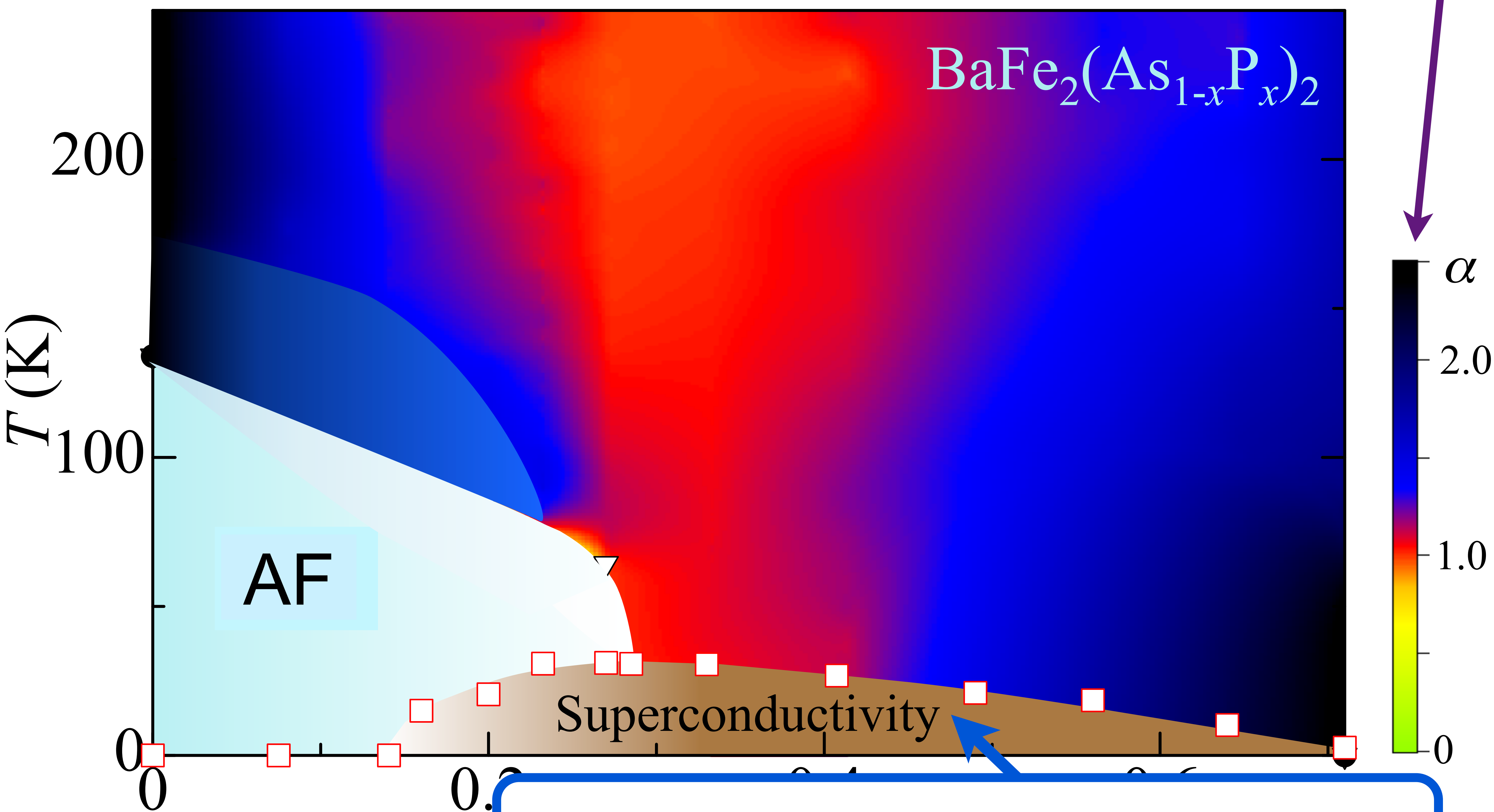
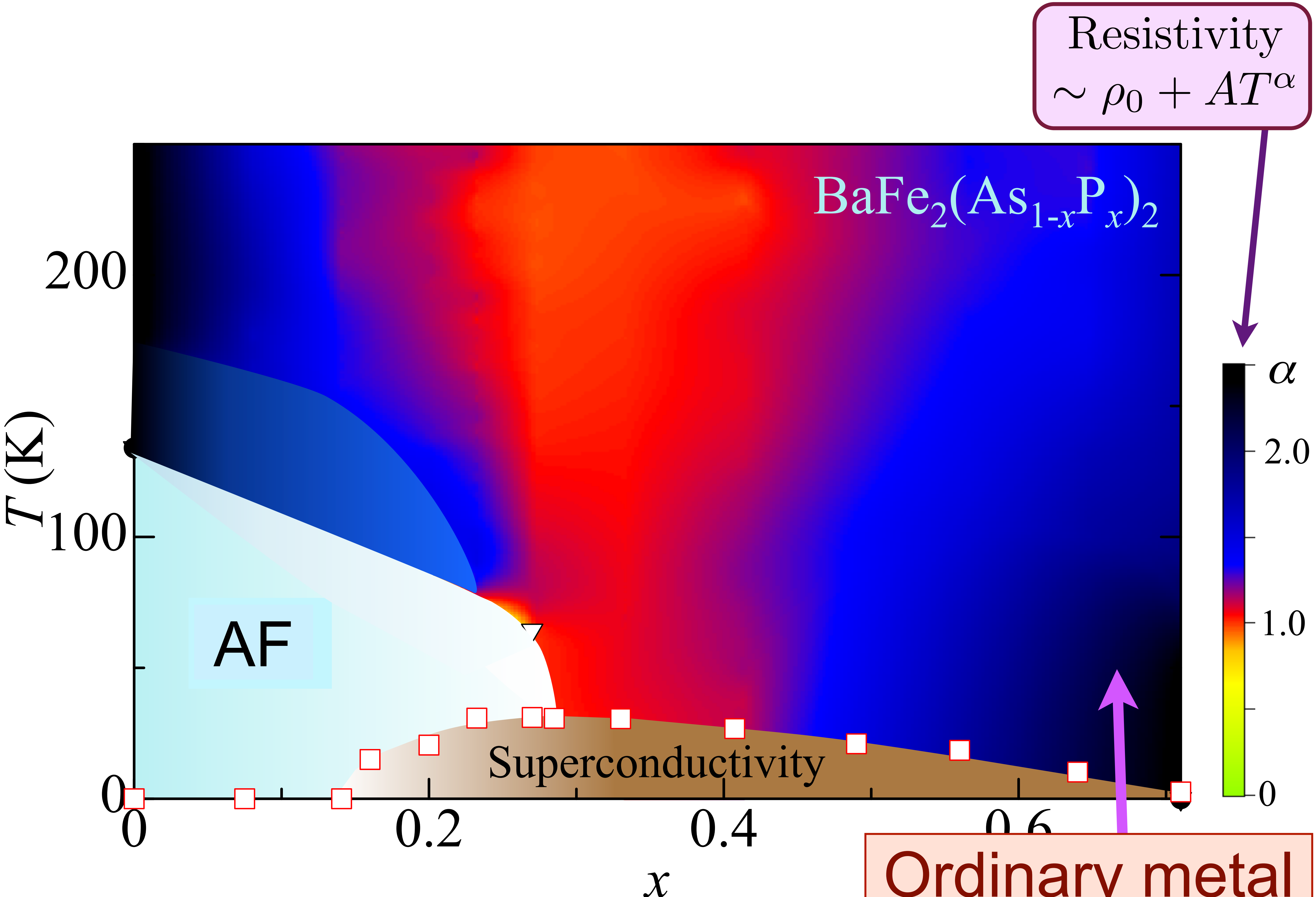
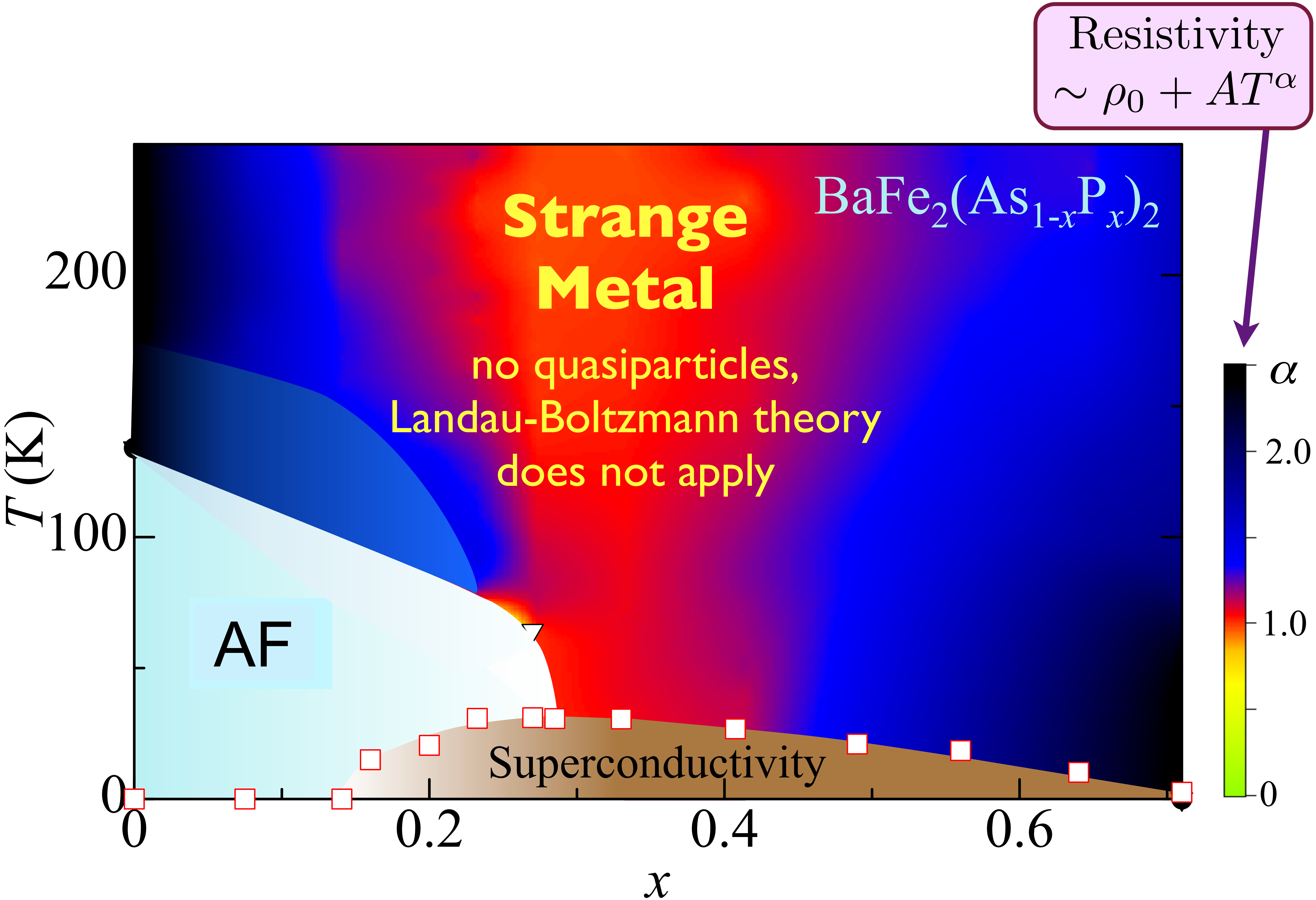


Superconductor
Bose condensate of pairs of electrons

S. Kasahara, T. Shiba
H. Ike



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O...
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Ma...
Physical Review B **81**, 184519 (2010)



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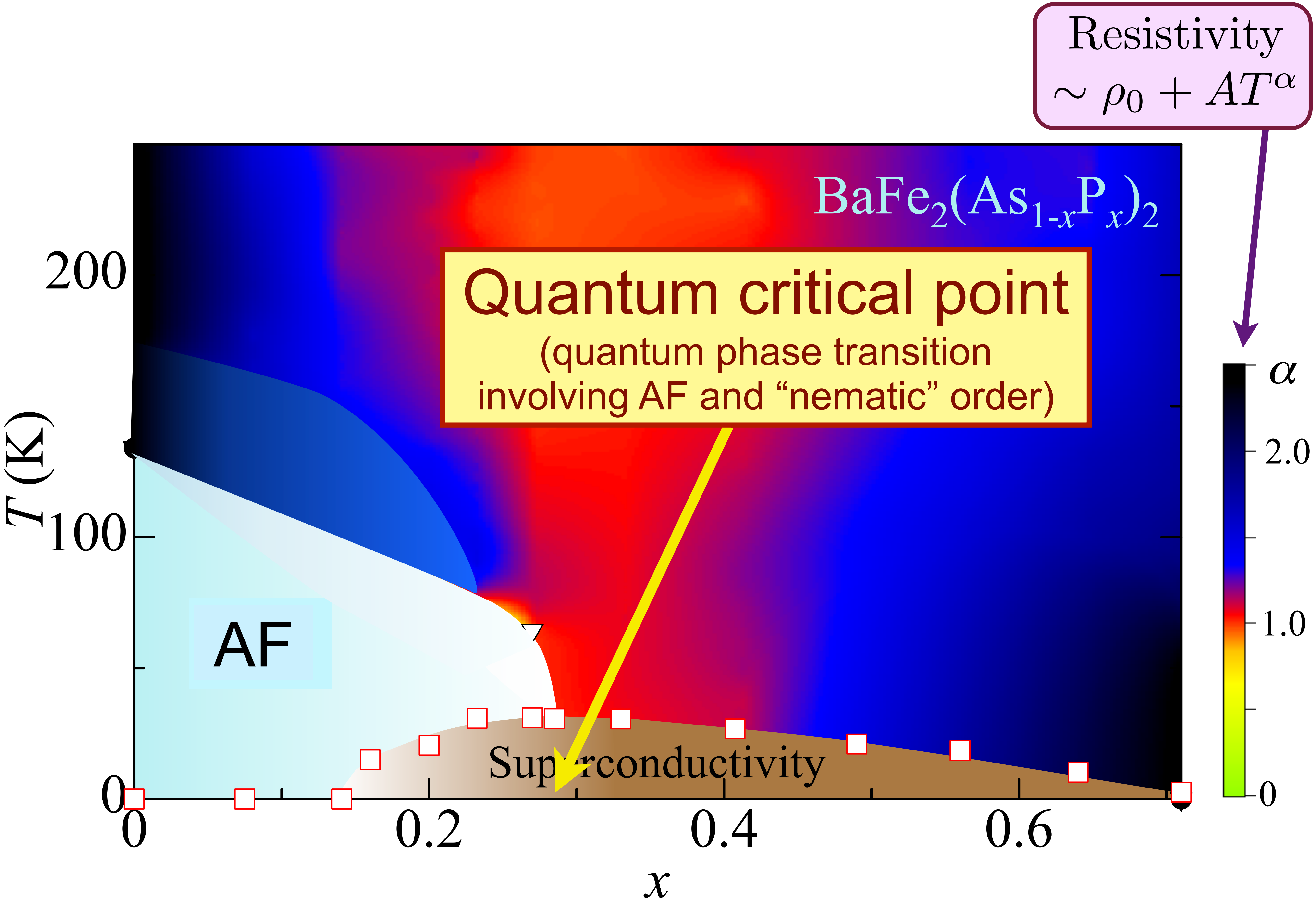

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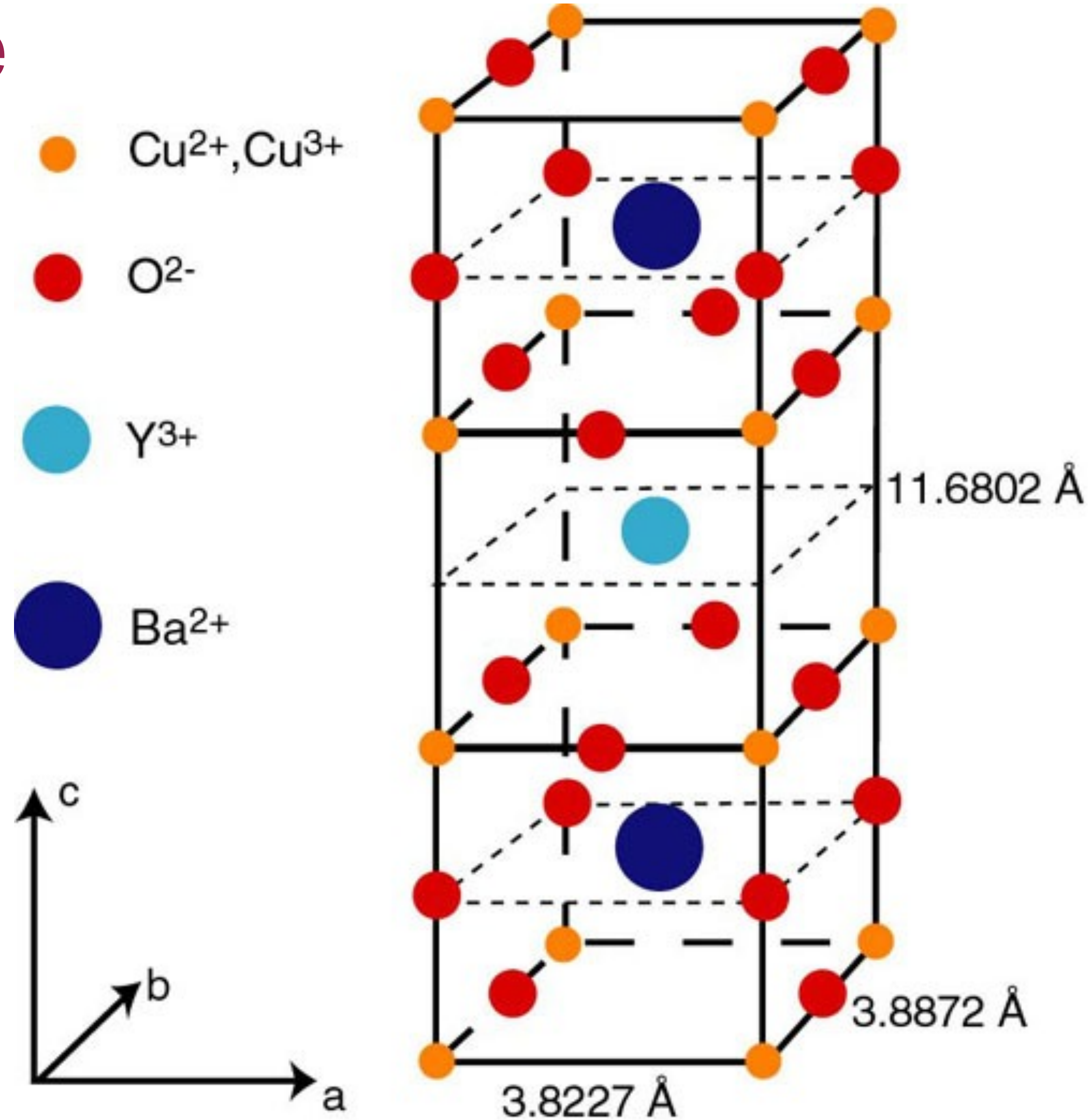
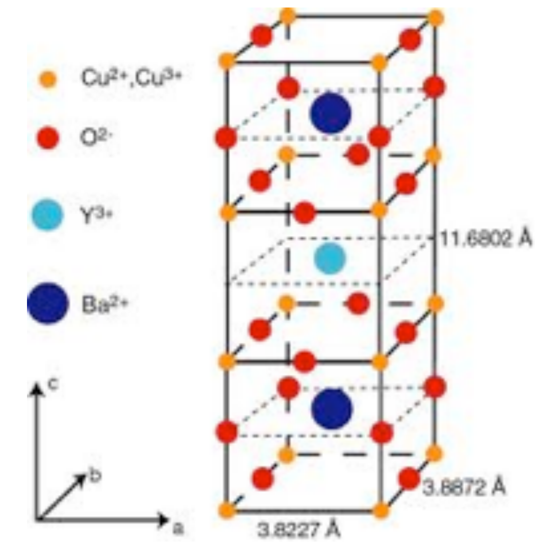
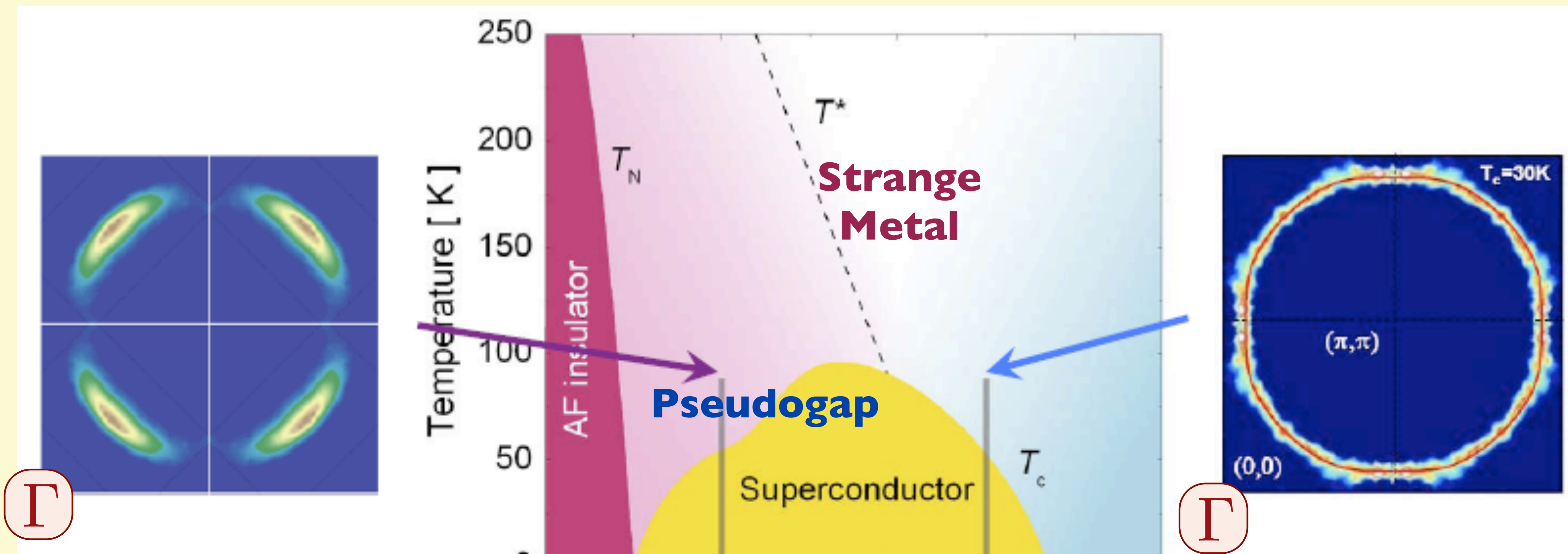
High temperature superconductors



$YBa_2Cu_3O_{6+x}$





K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

Smaller hole Fermi-pockets

Large hole Fermi surface

Outline

1. Quantum critical point in an insulator

   Non-quasiparticle dynamics

2. Quantum critical point in a metal

   The iron-based superconductors

3. The pseudogap regime of the hole-doped cuprate superconductors

   Angular fluctuations of a multicomponent order

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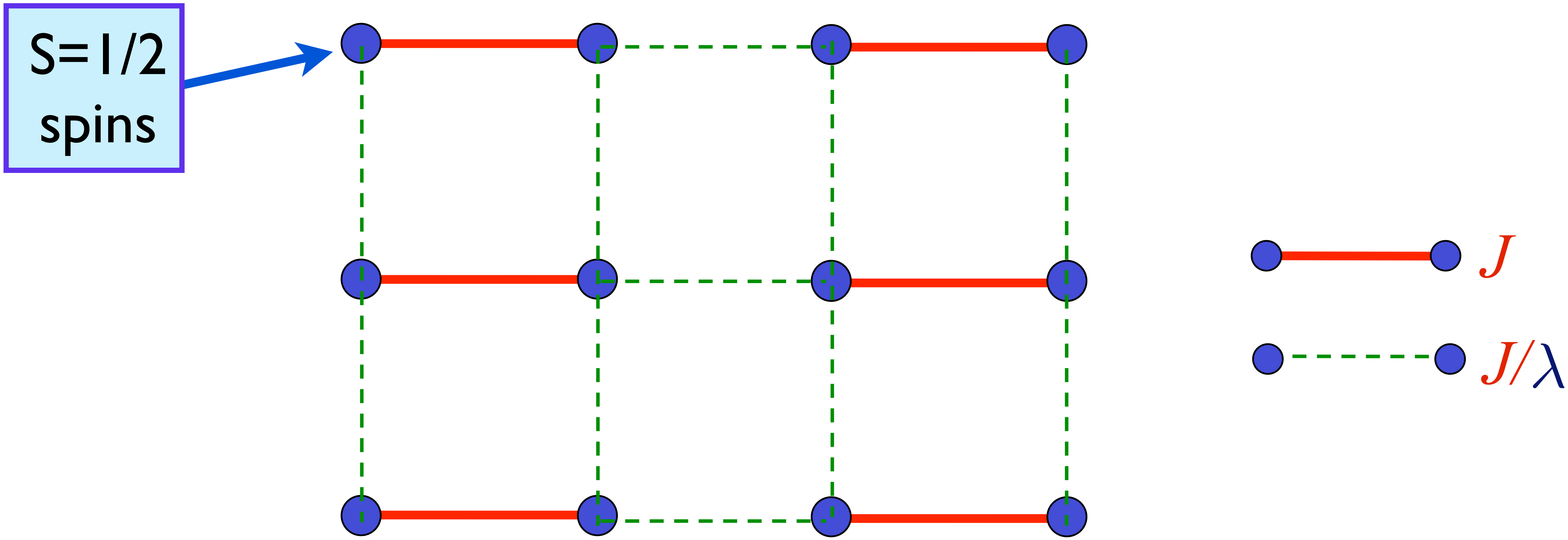
Square lattice antiferromagnet

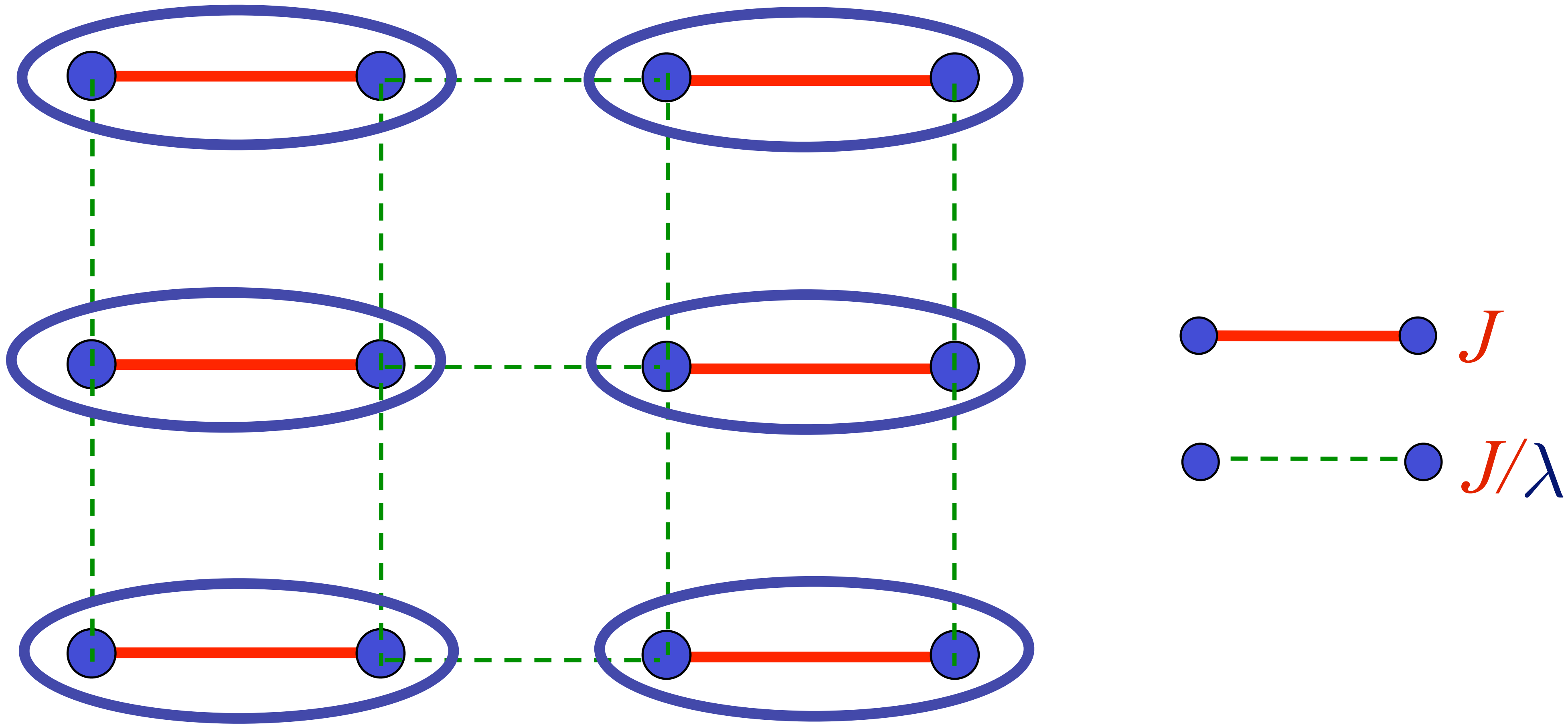
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

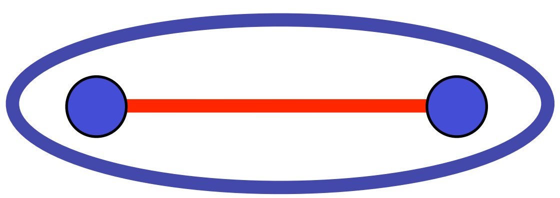


Examine ground state as a function of λ

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



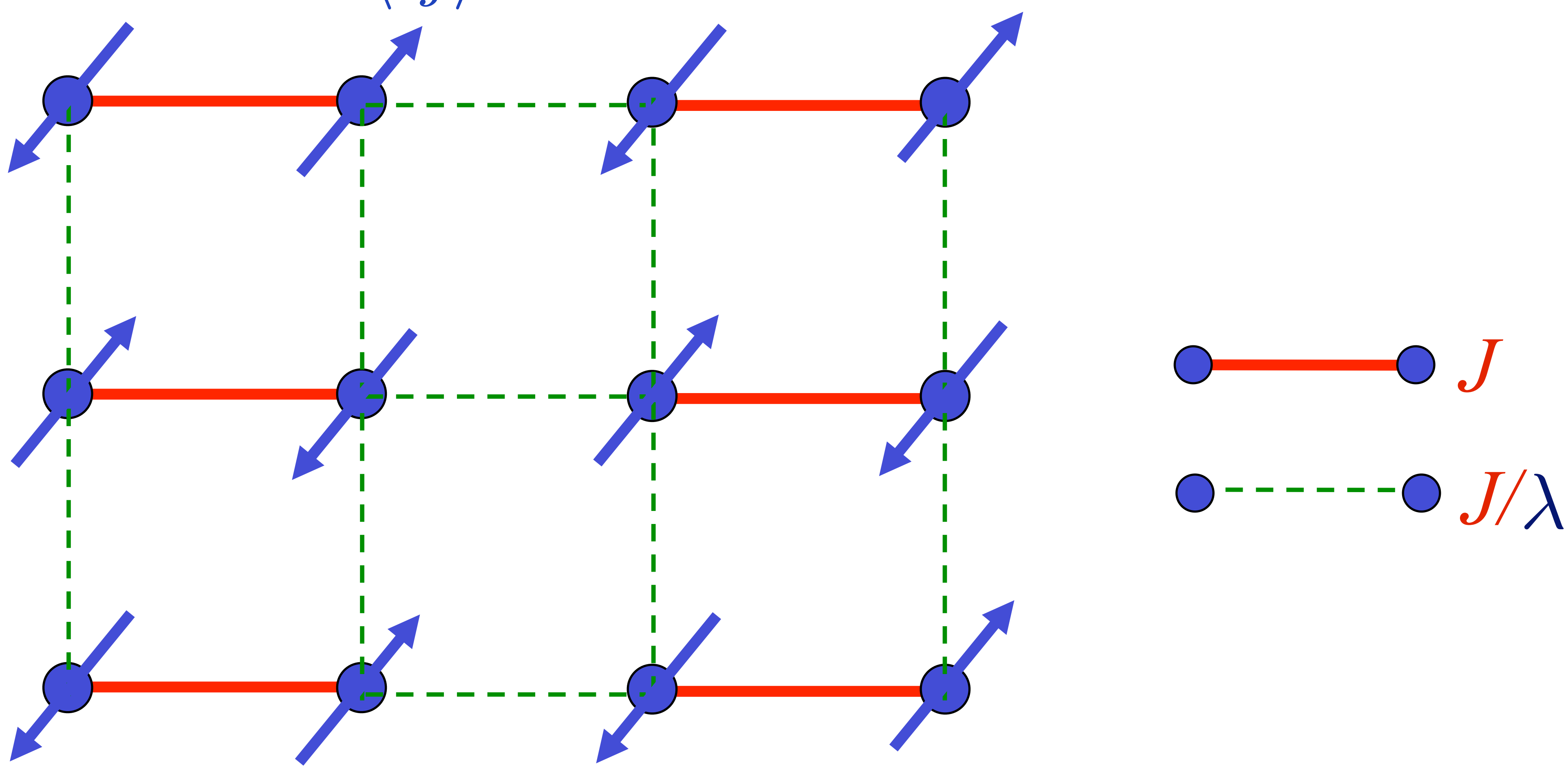


$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

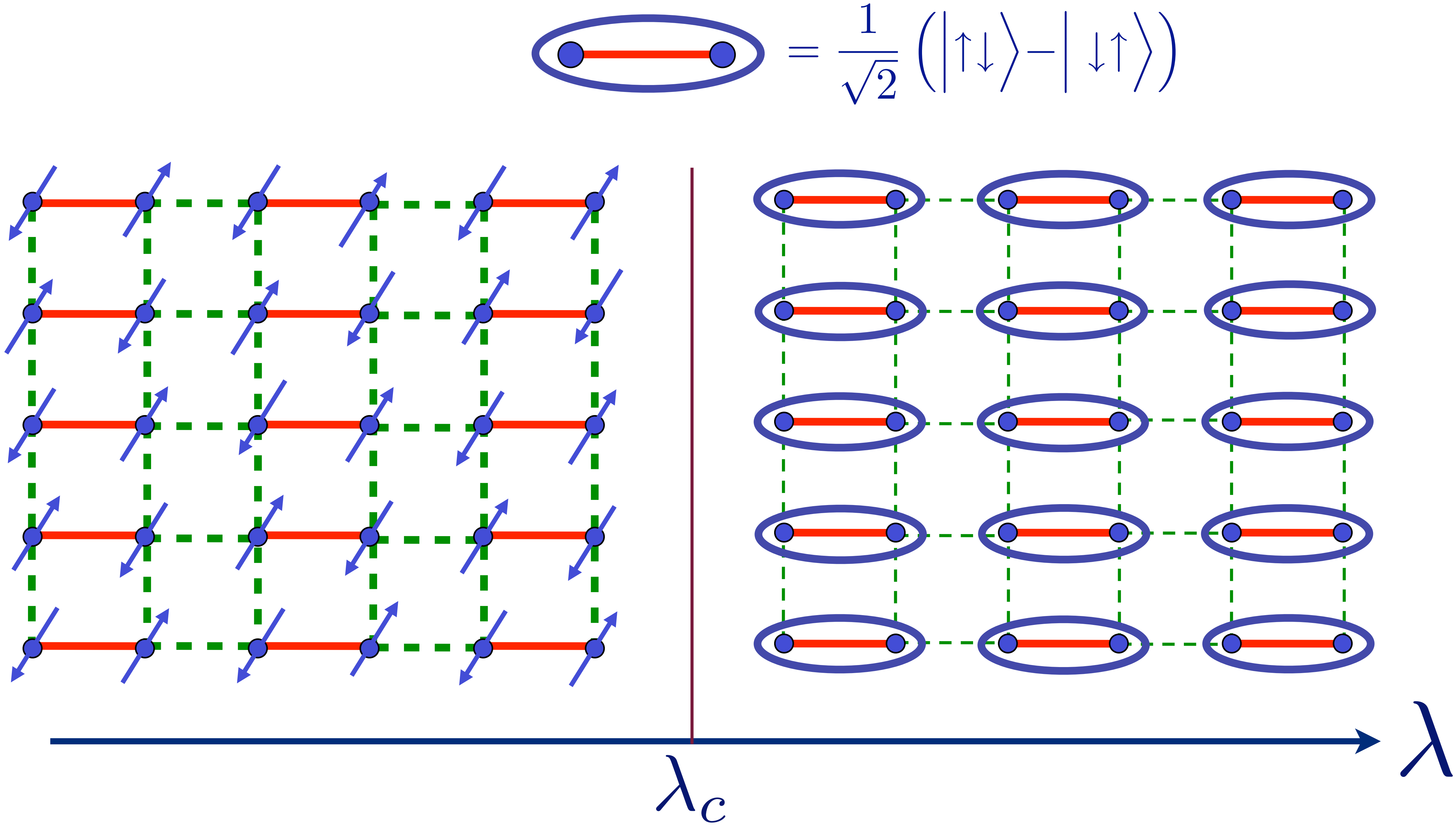


For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

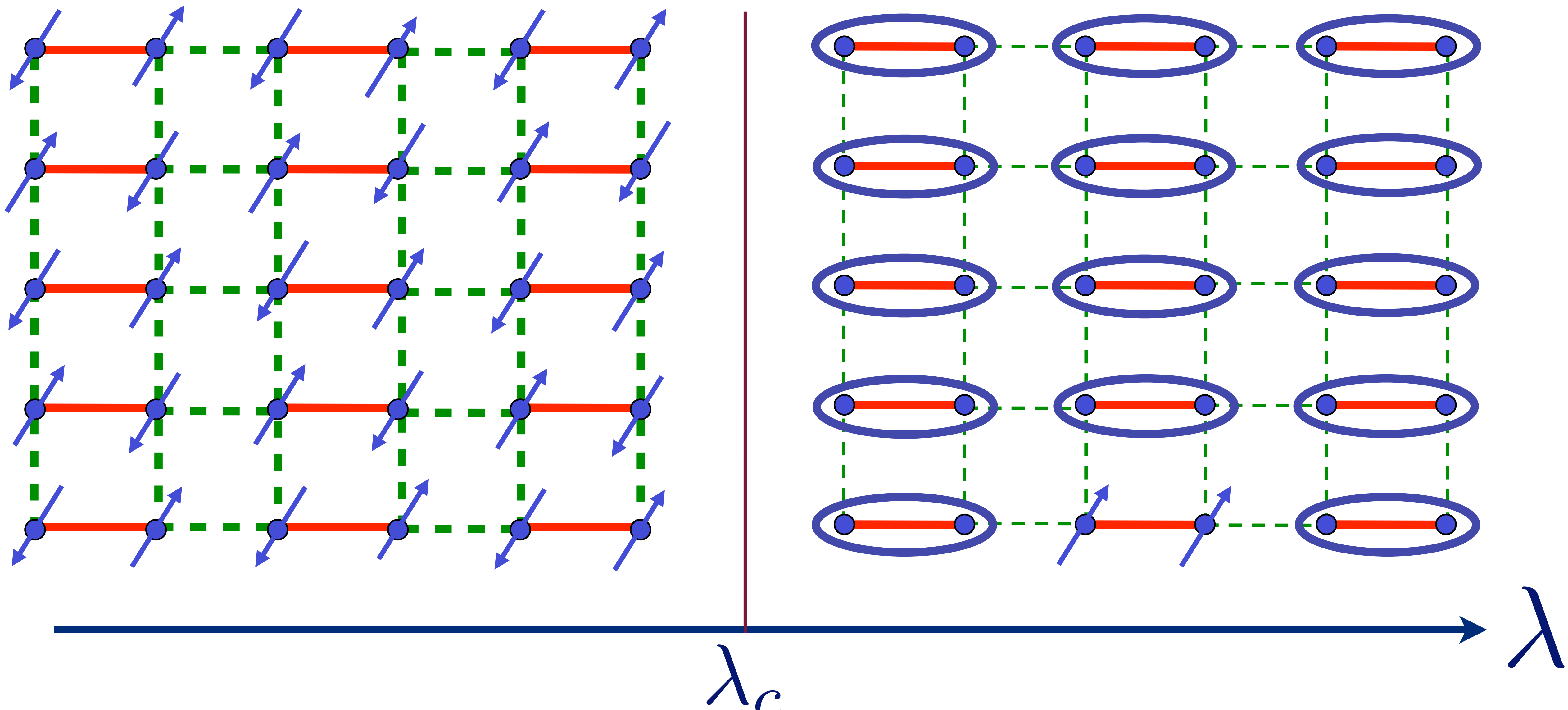
$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.



$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$
λ
λ_c

Excitation spectrum in the paramagnetic phase



Spin $S = 1$
"triplon"

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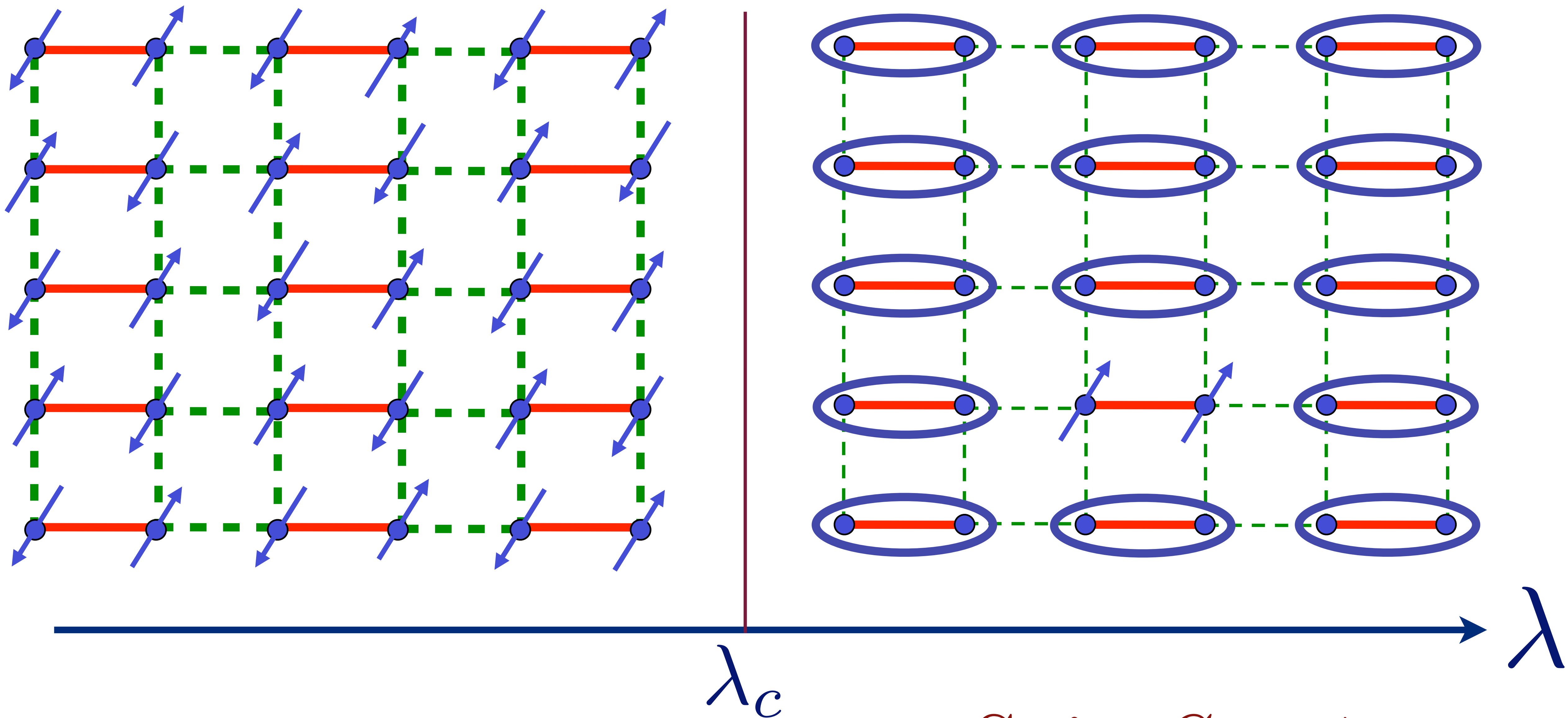

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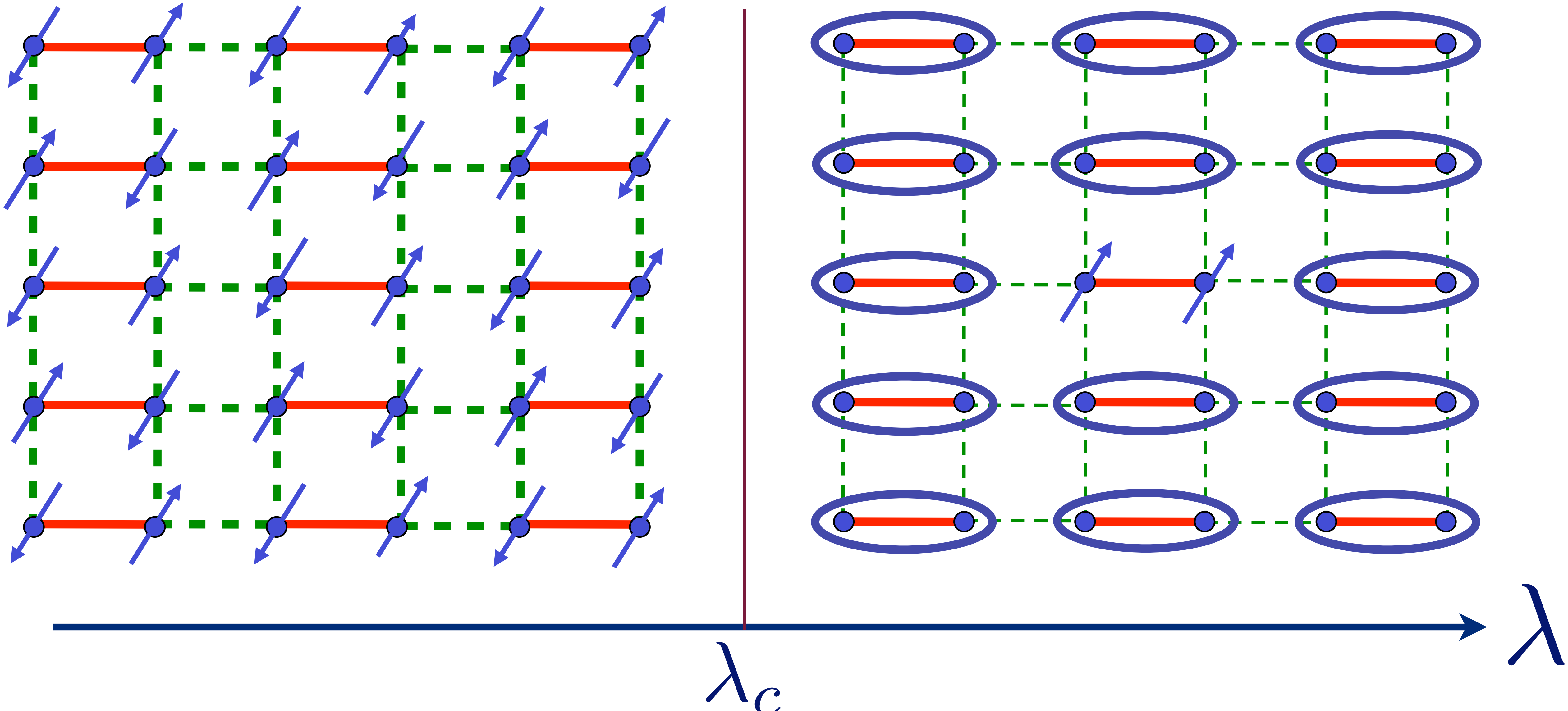

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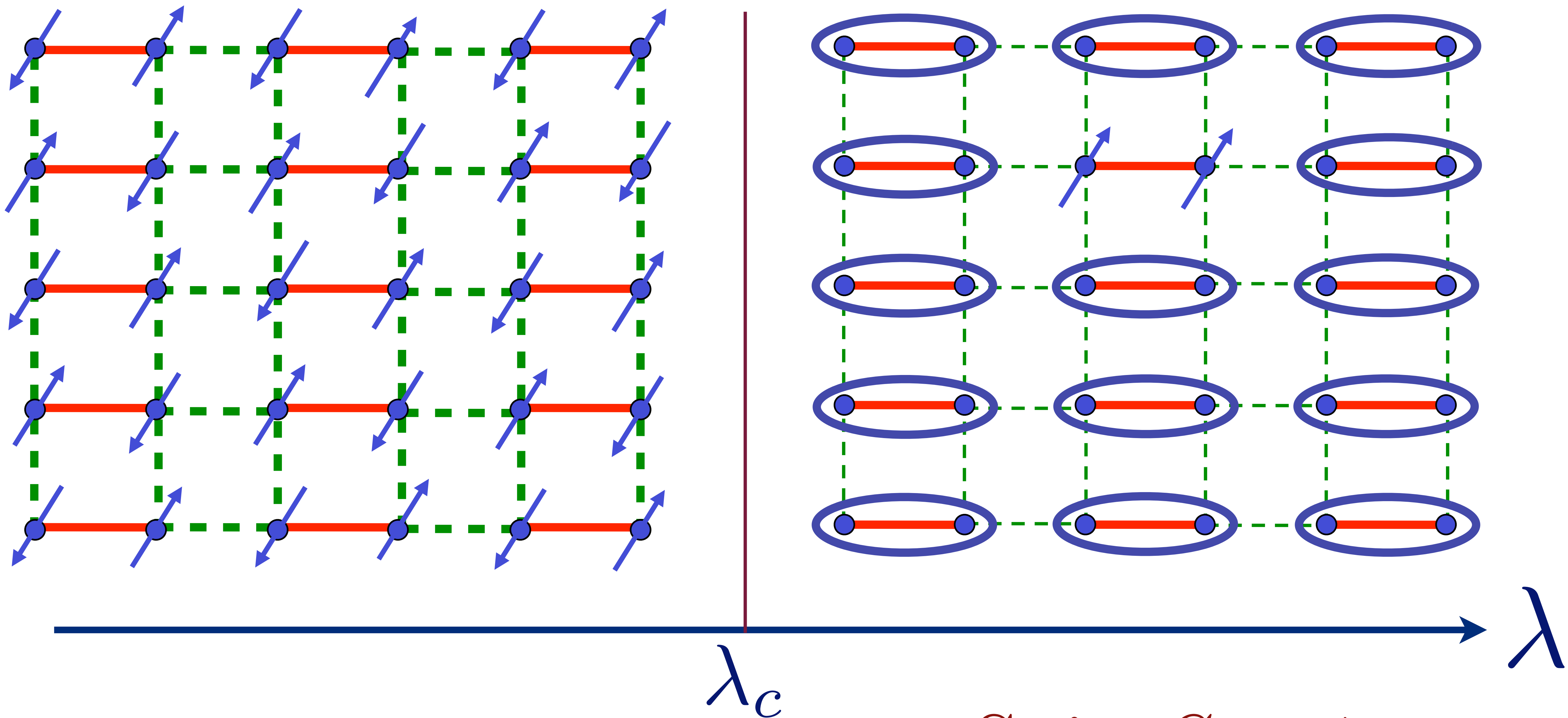

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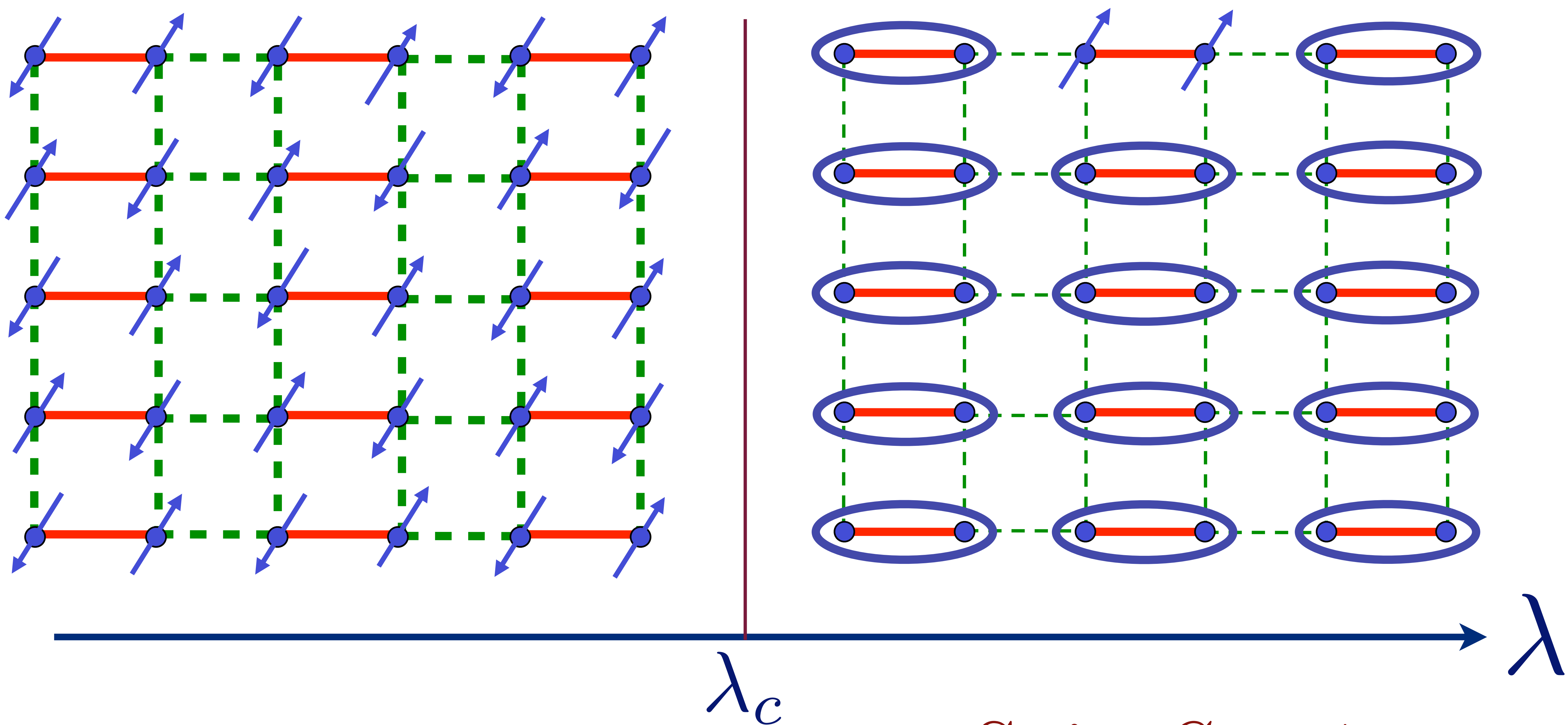

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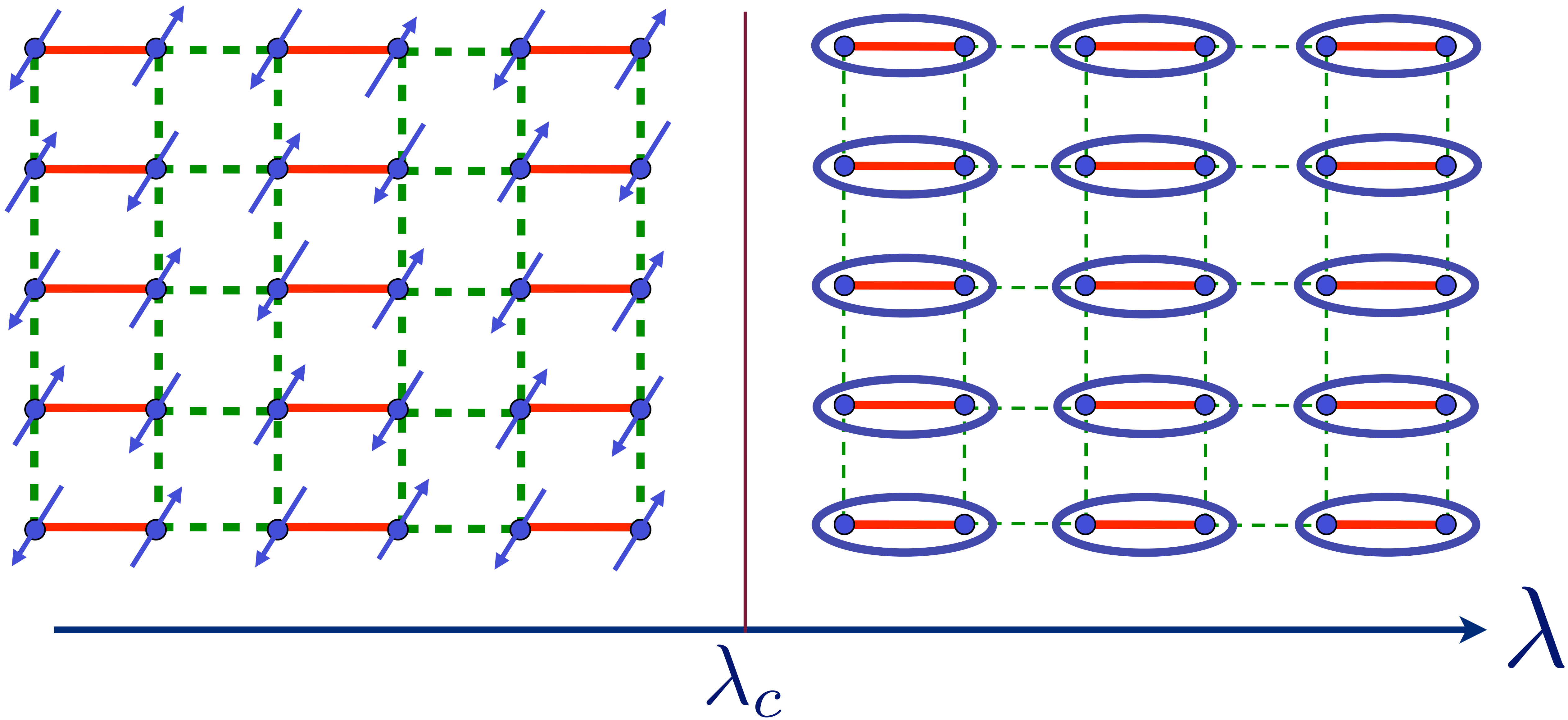
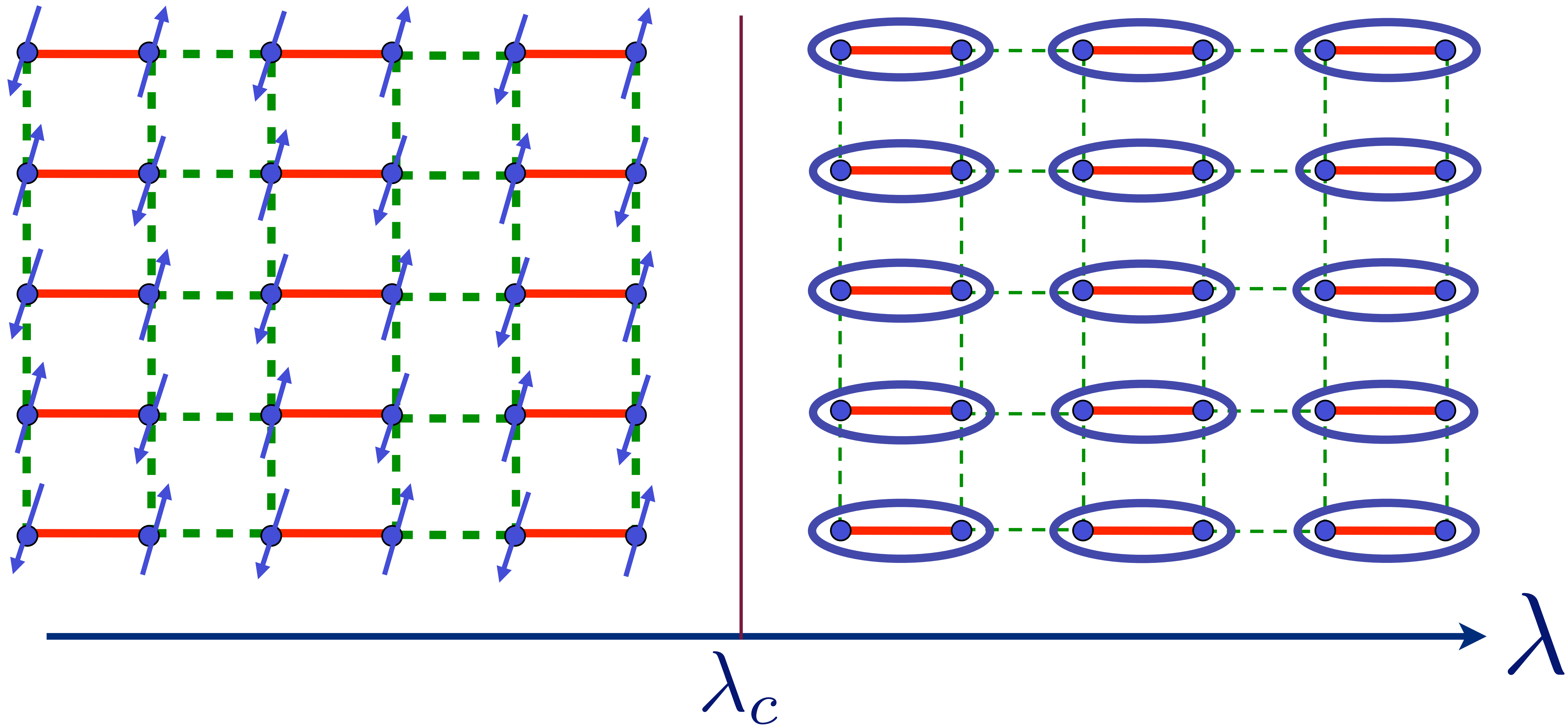


Spin waves

Excitation spectrum in the Néel phase



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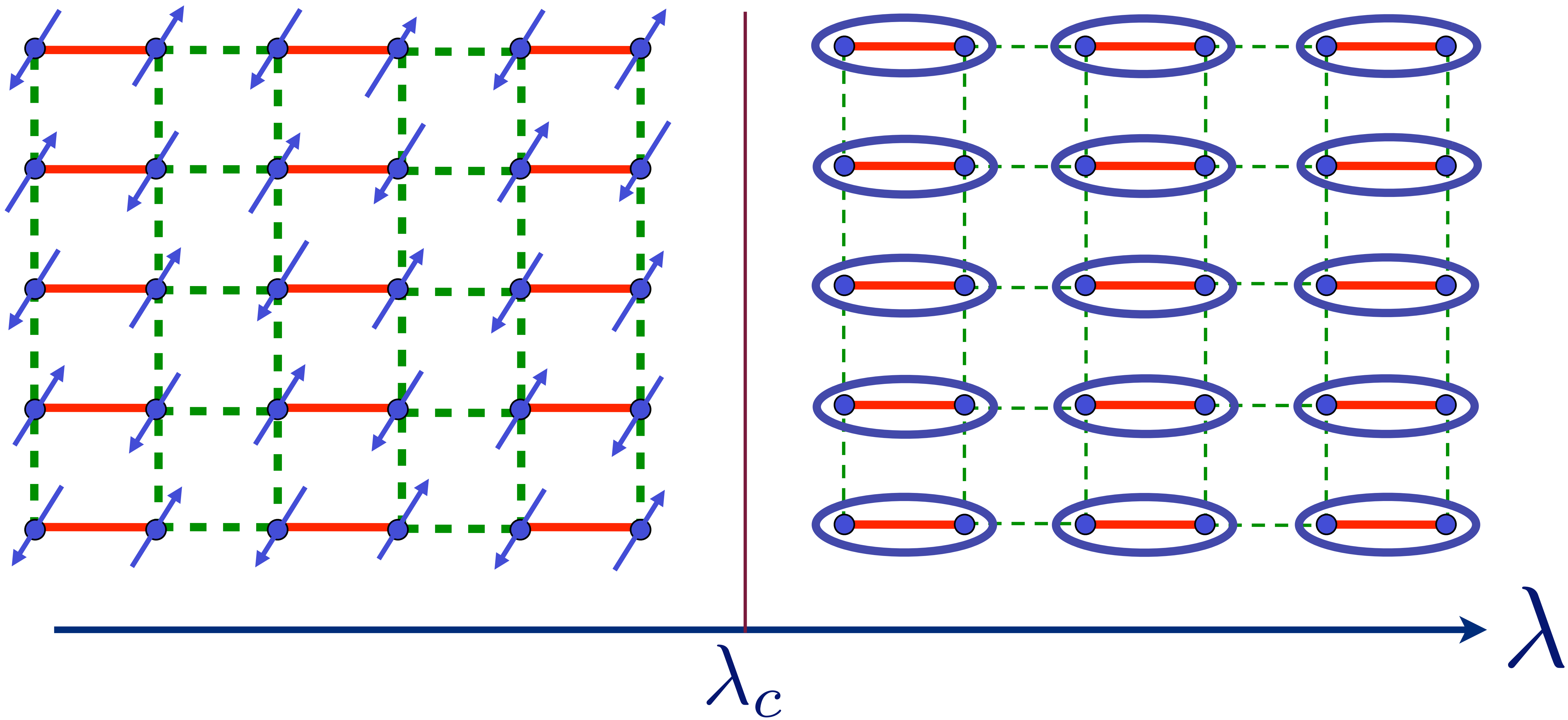
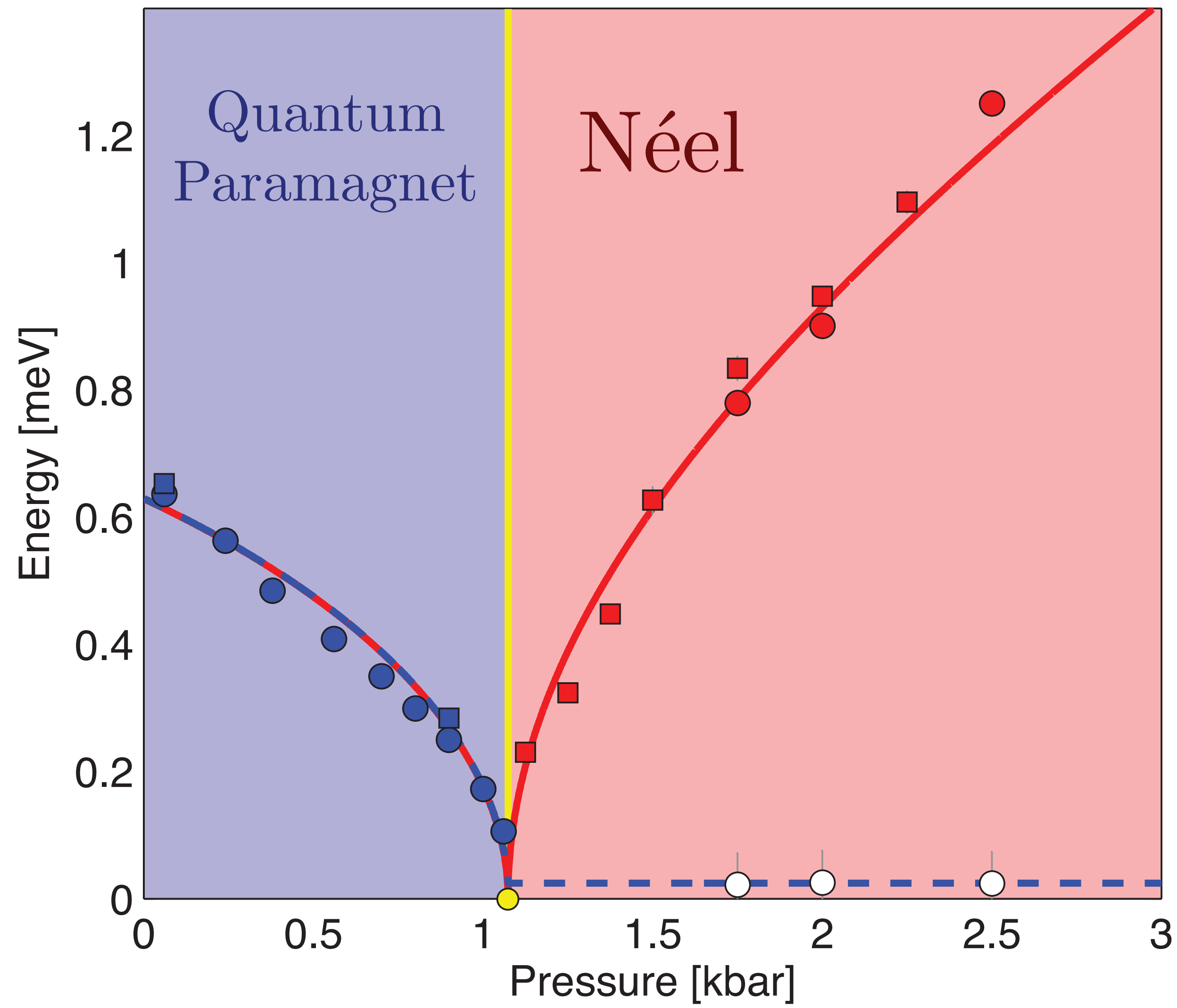


Spin waves

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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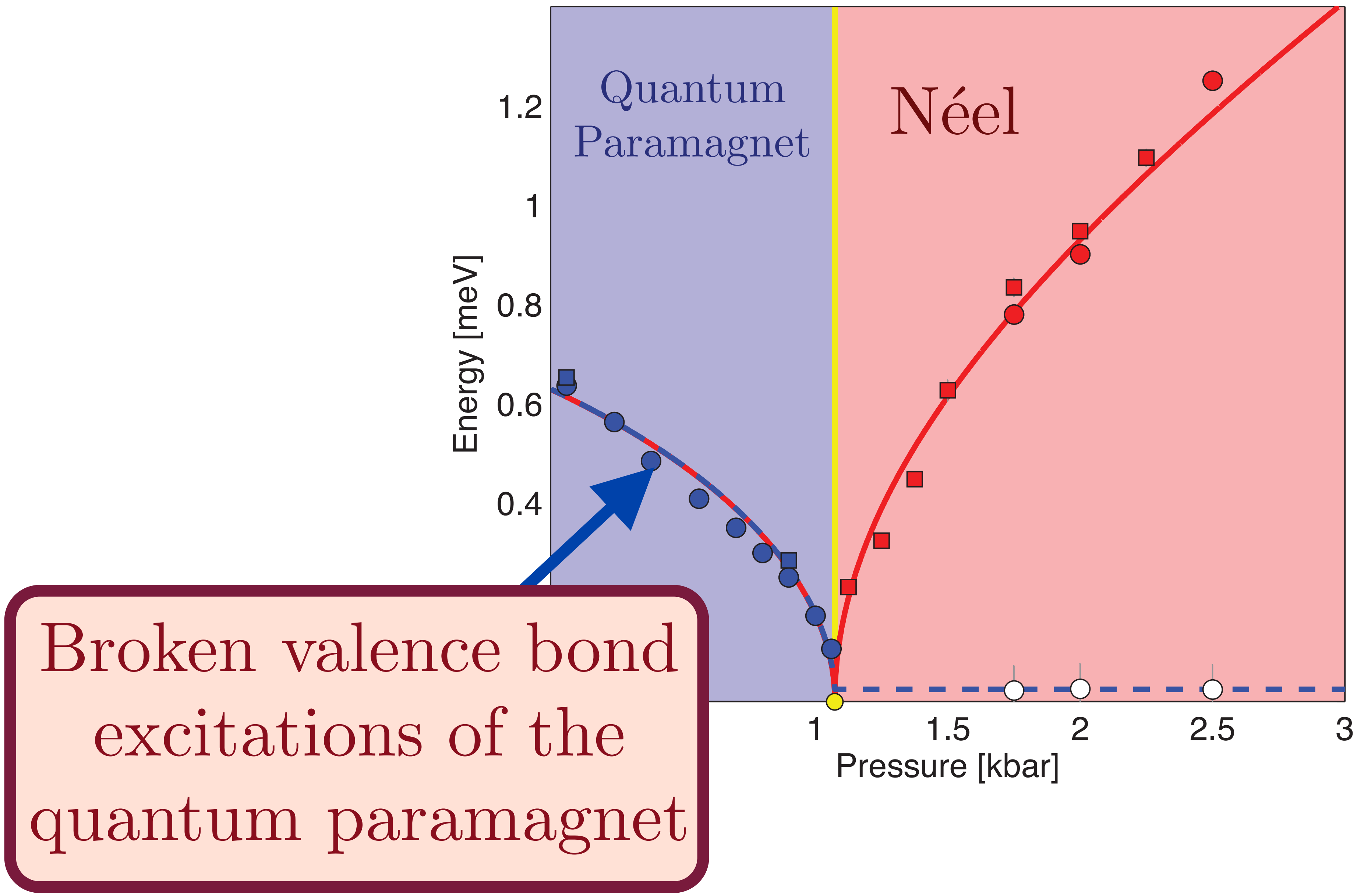

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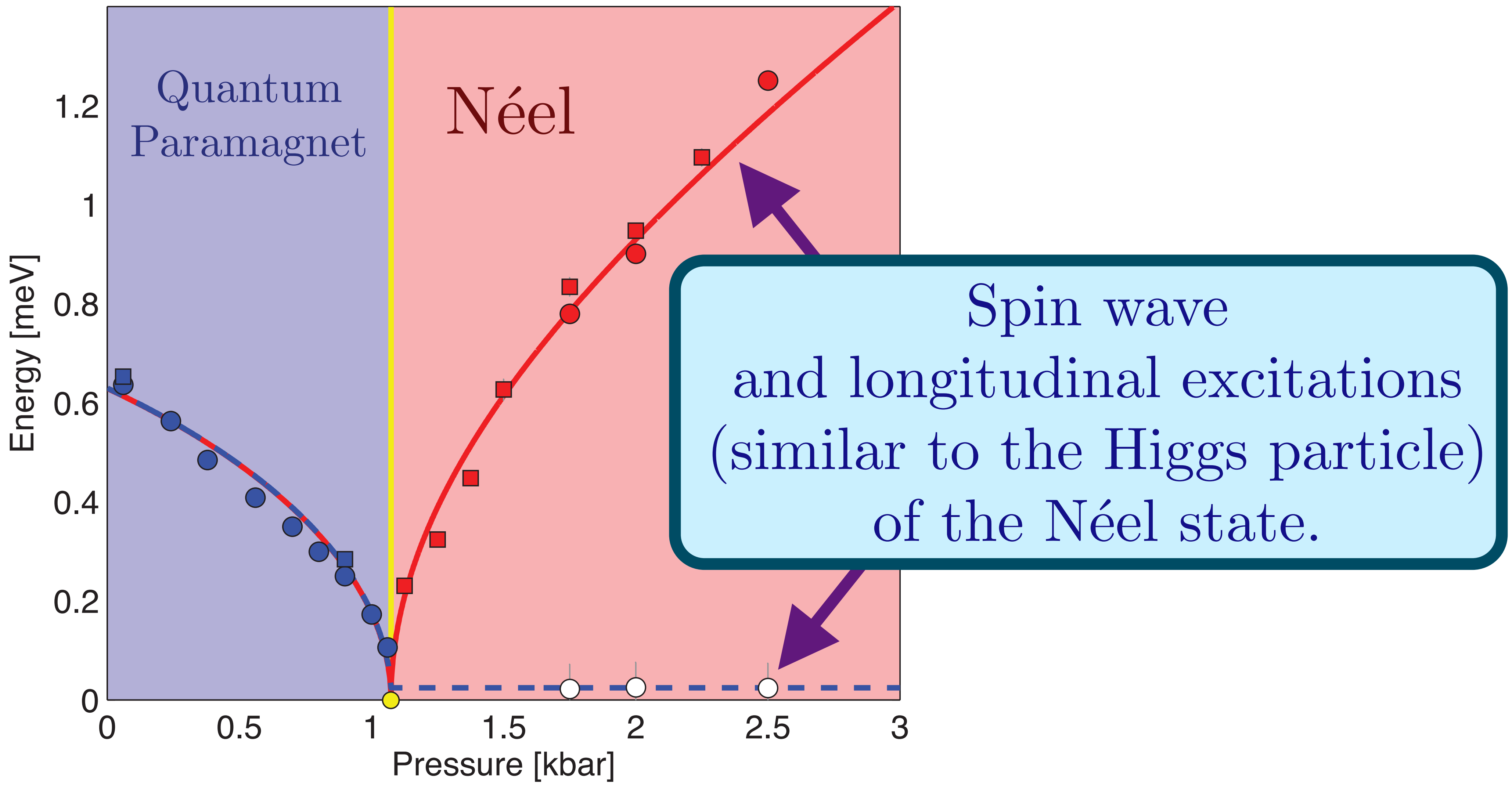
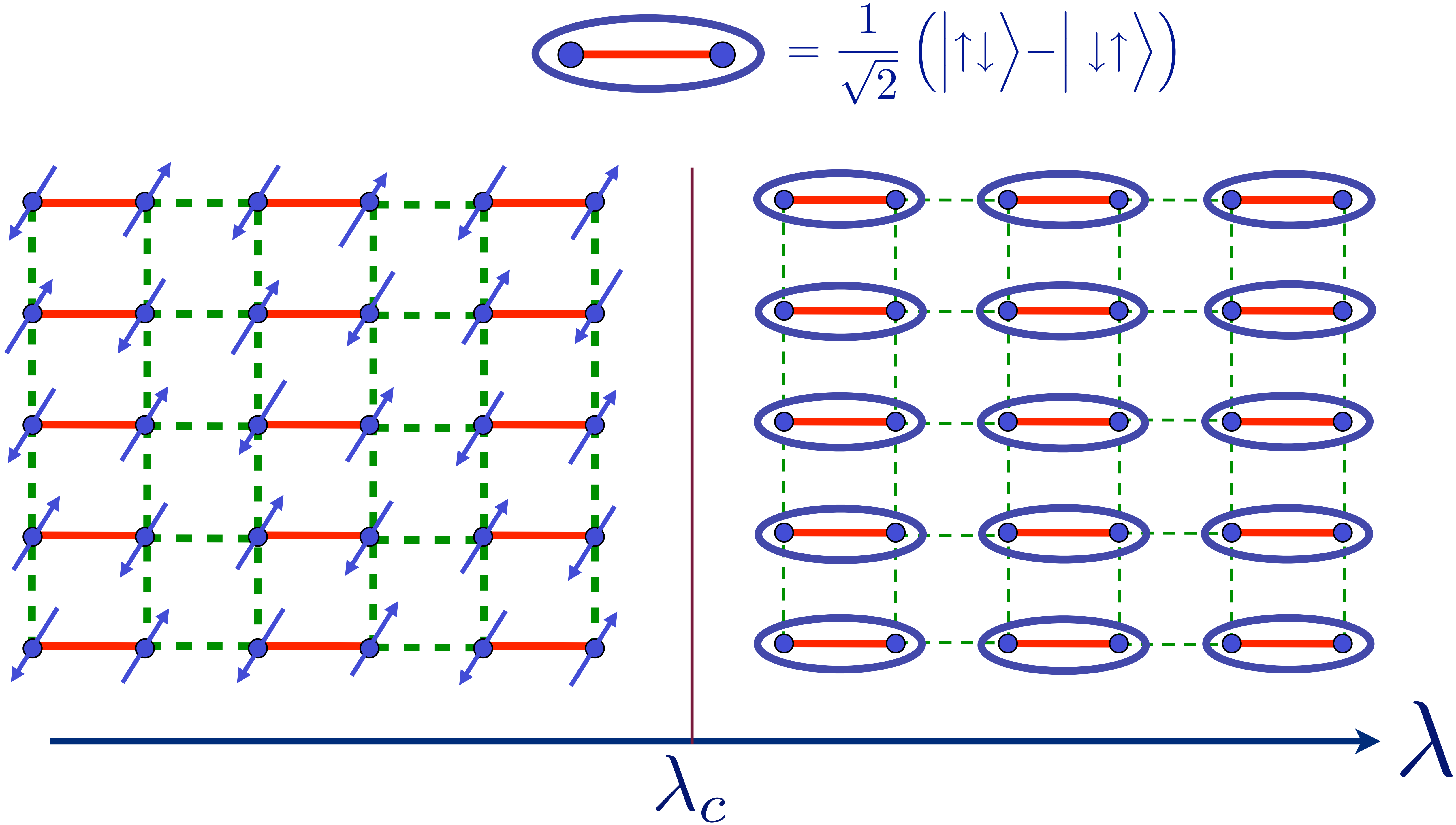


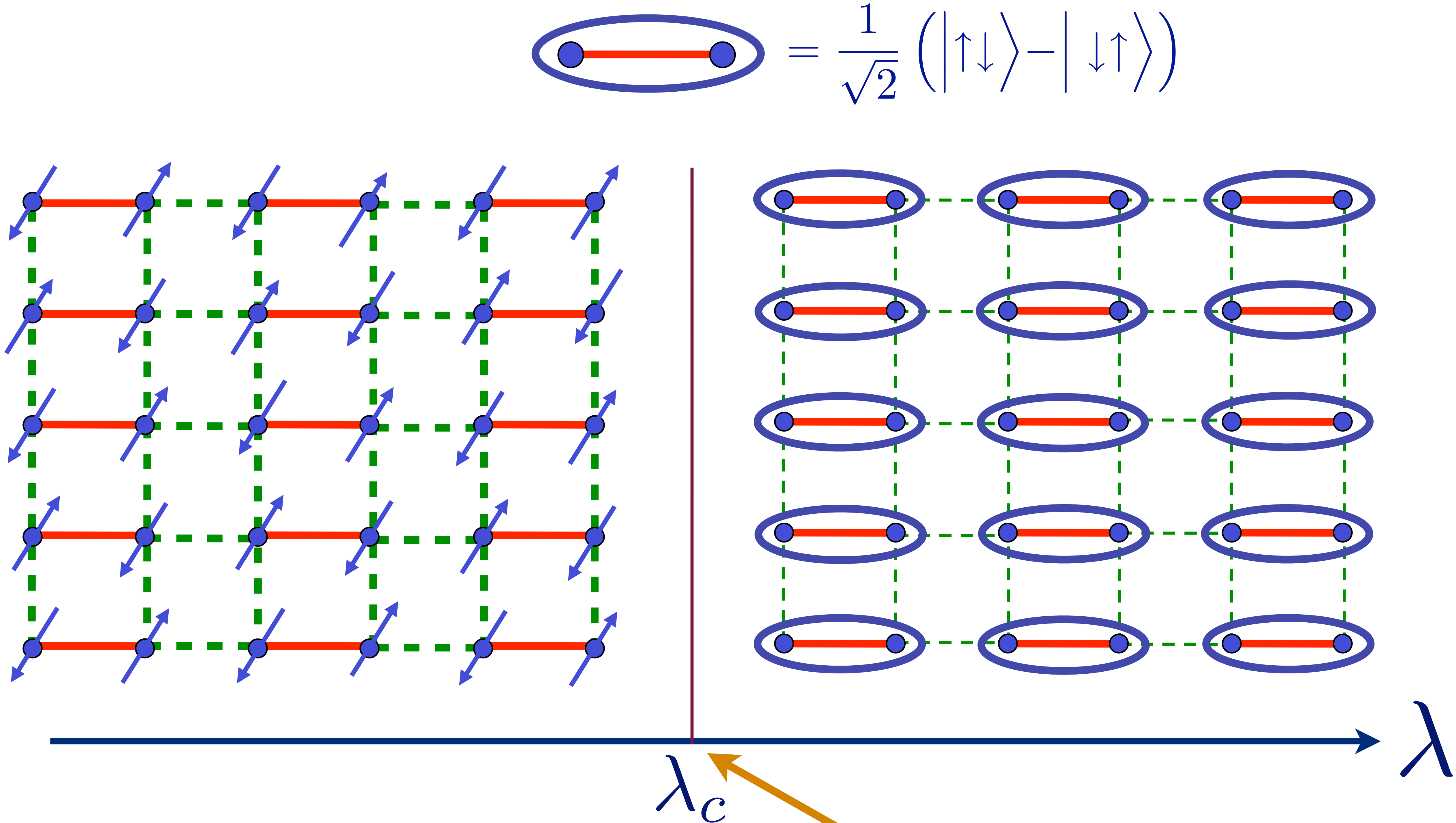
"Higgs" particle appears at theoretically predicted energy

S. Sachdev, arXiv:0901.4103

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

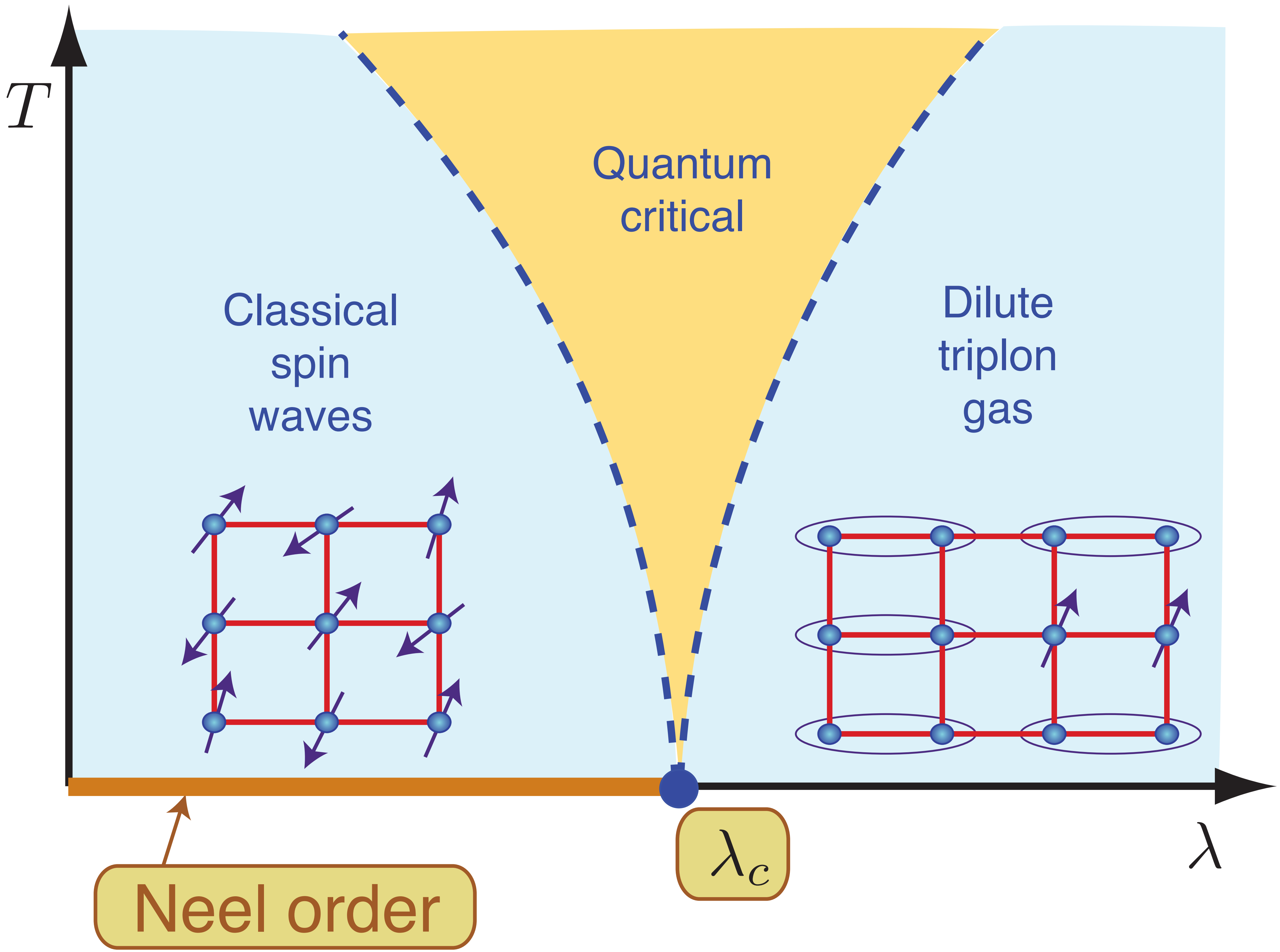


$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$
λ_c
λ



Quantum critical point:
A new state of matter
with long-range quantum entanglement
and no quasiparticles

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).



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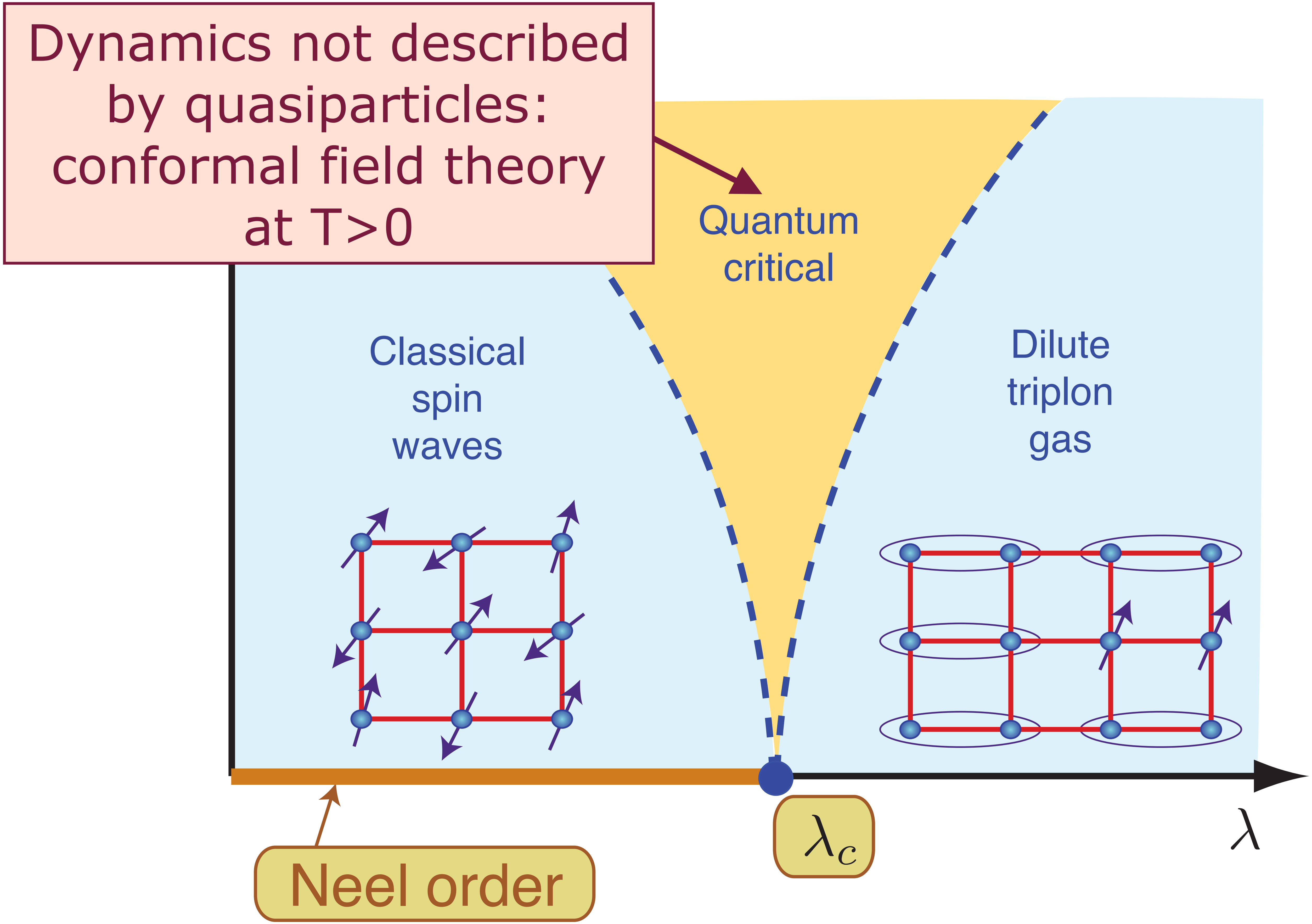
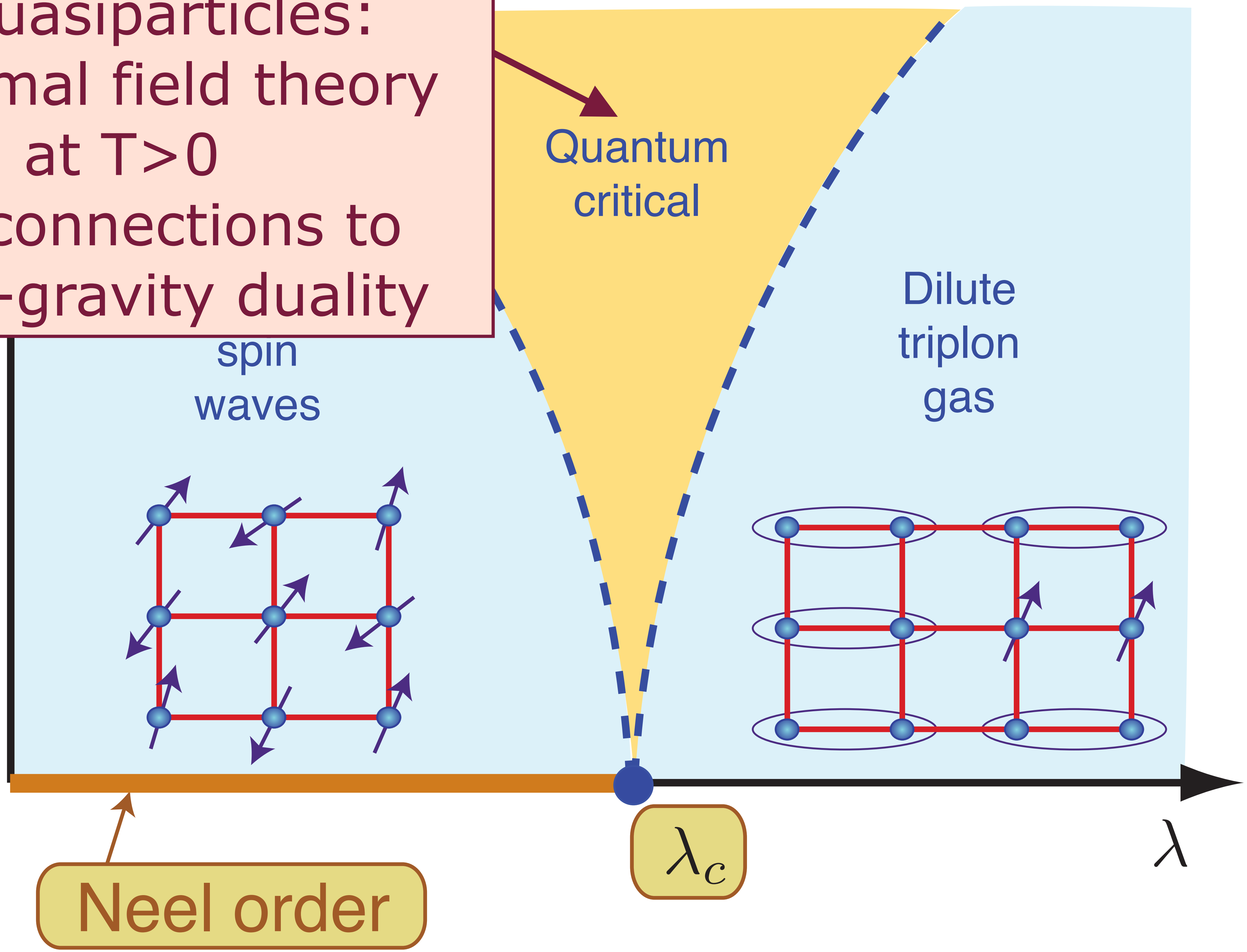


S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

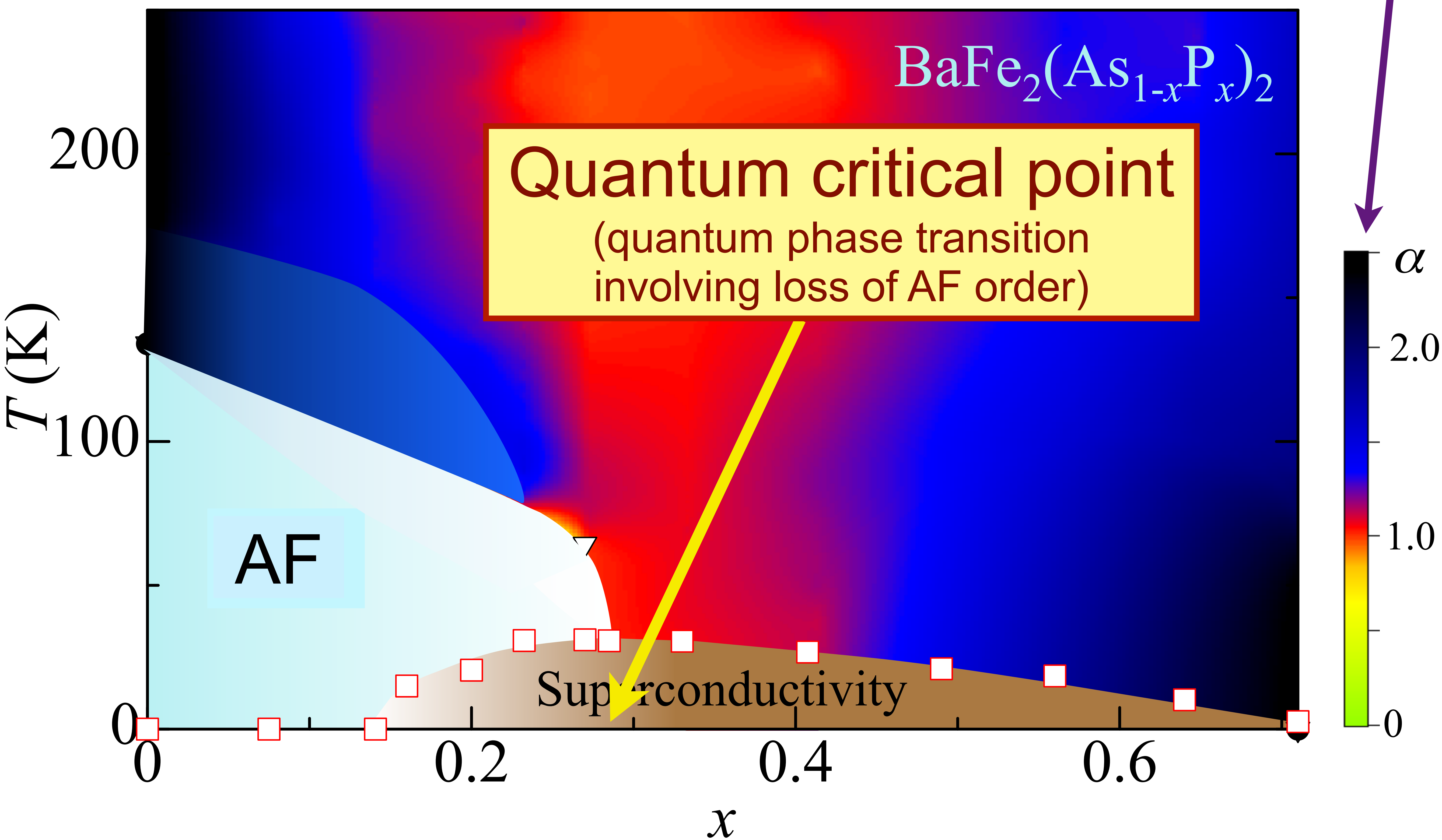
Dynamics not described by quasiparticles: conformal field theory at T>0 with connections to gauge-gravity duality



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

Similar QCP, but in a metal

Resistivity $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Outline

1. Quantum critical point in an insulator
   Non-quasiparticle dynamics

2. Quantum critical point in a metal
   The iron-based superconductors

3. The pseudogap regime of the hole-doped cuprate superconductors
   Angular fluctuations of a multicomponent order

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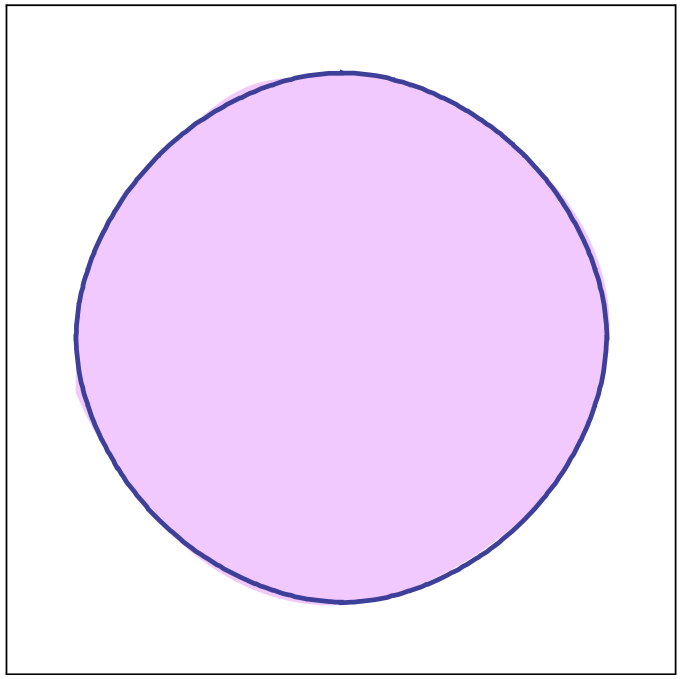
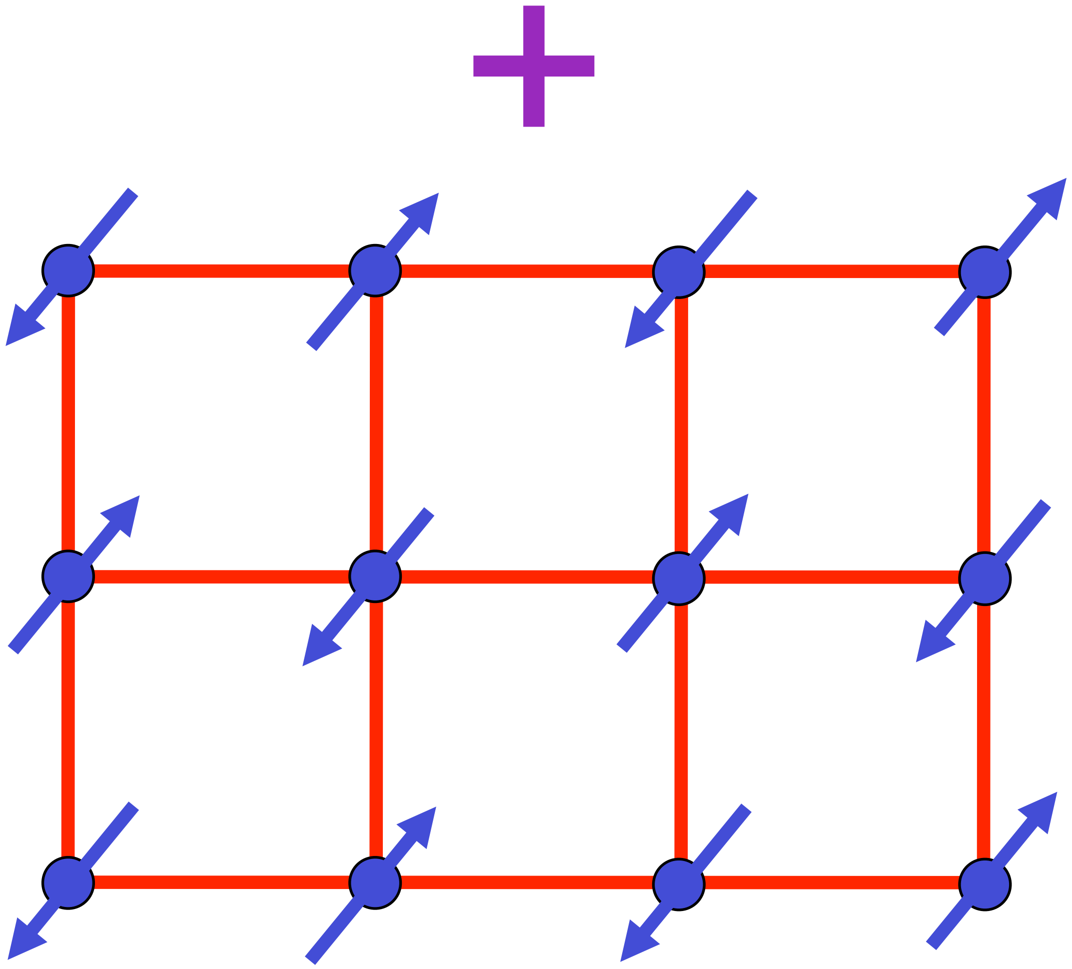
Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



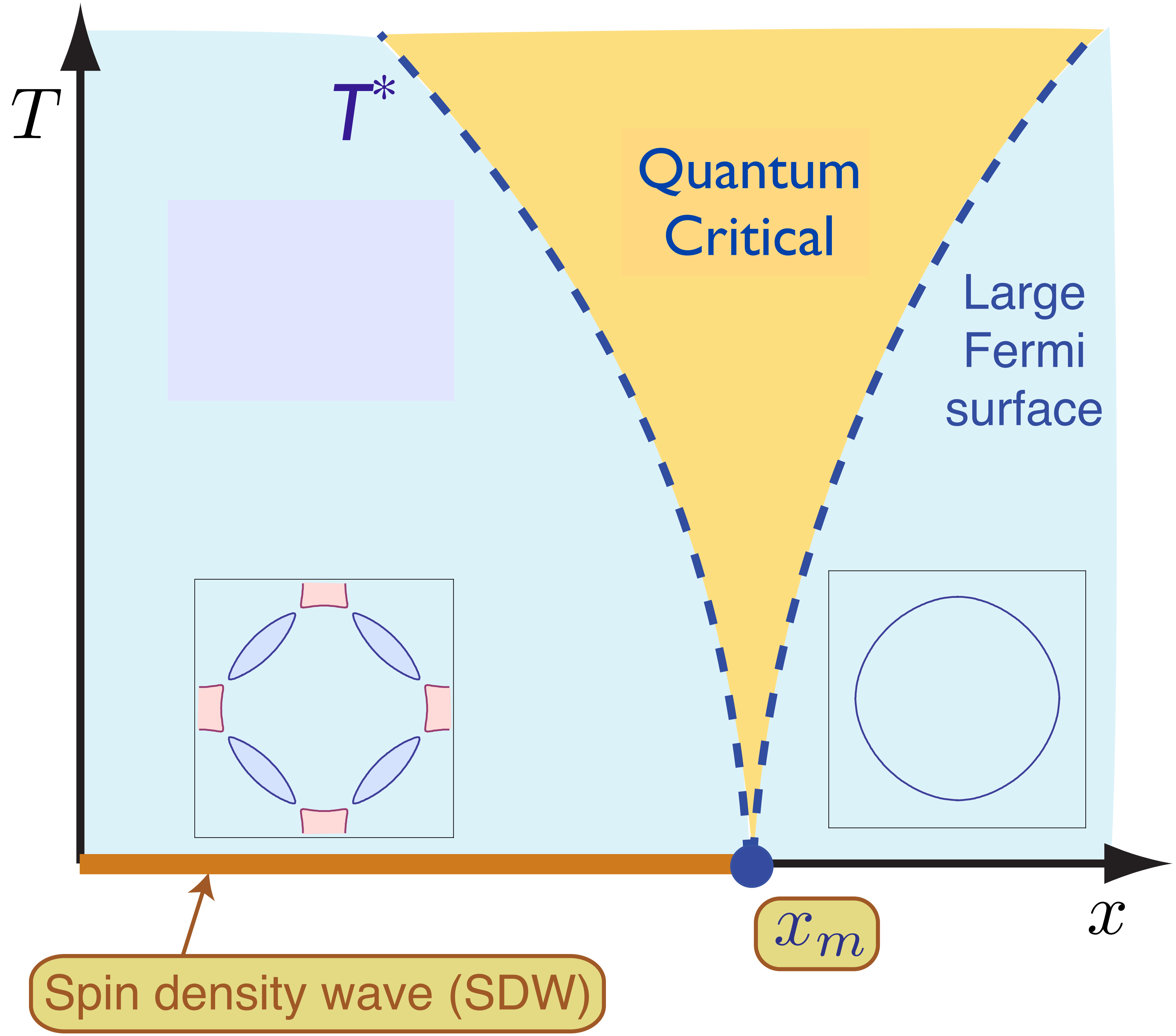


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

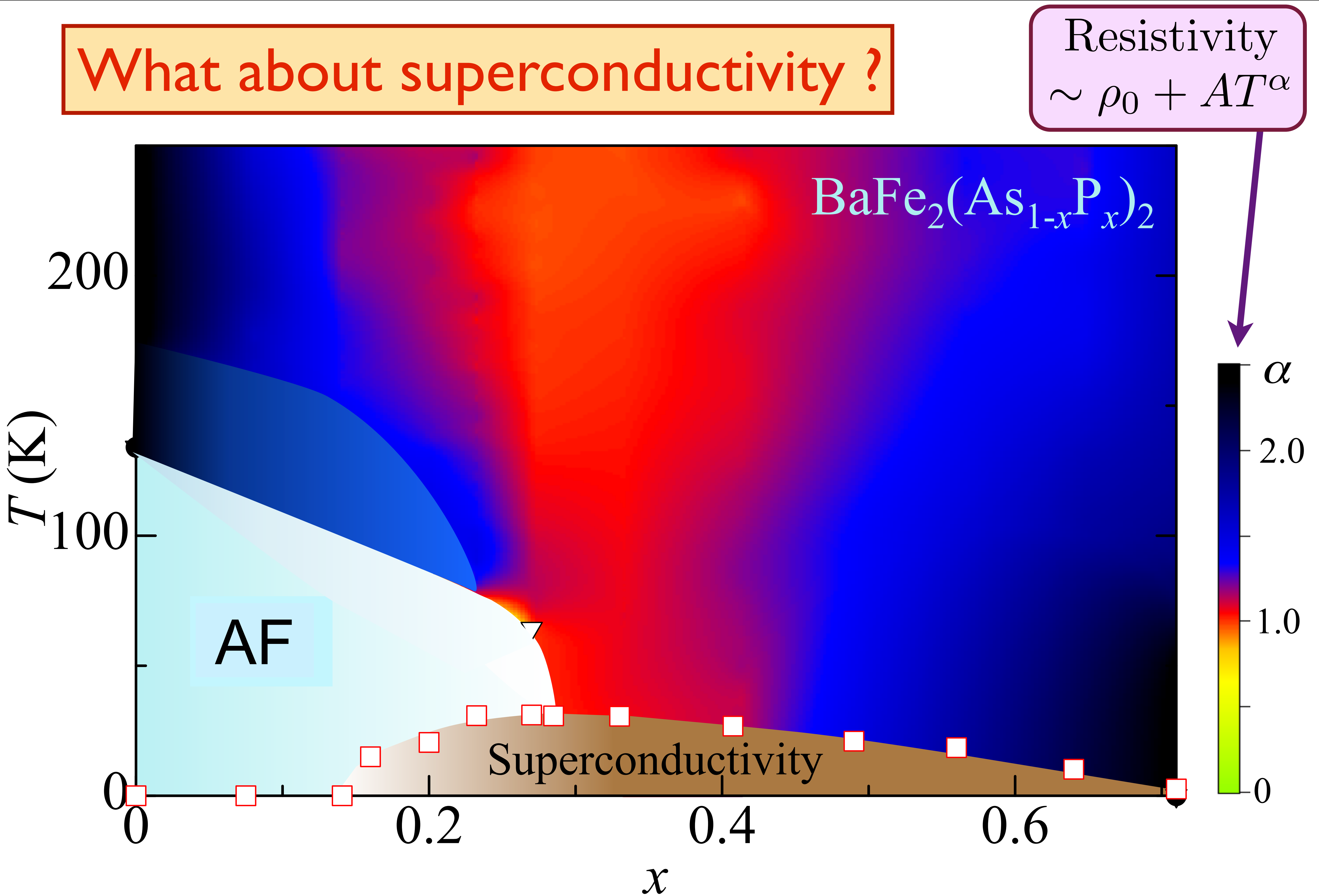
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

What about superconductivity ?



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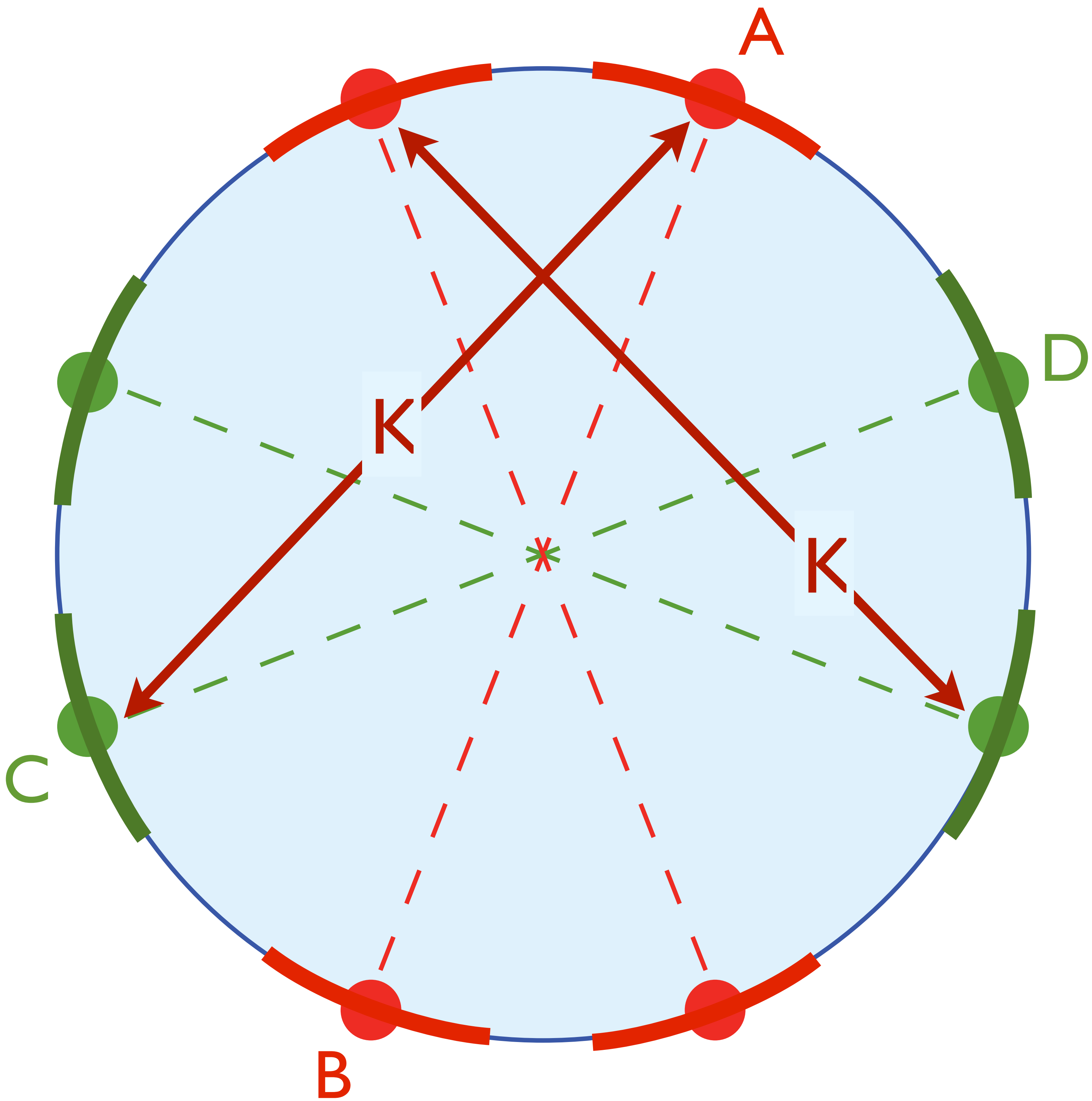
Origin of superconductivity

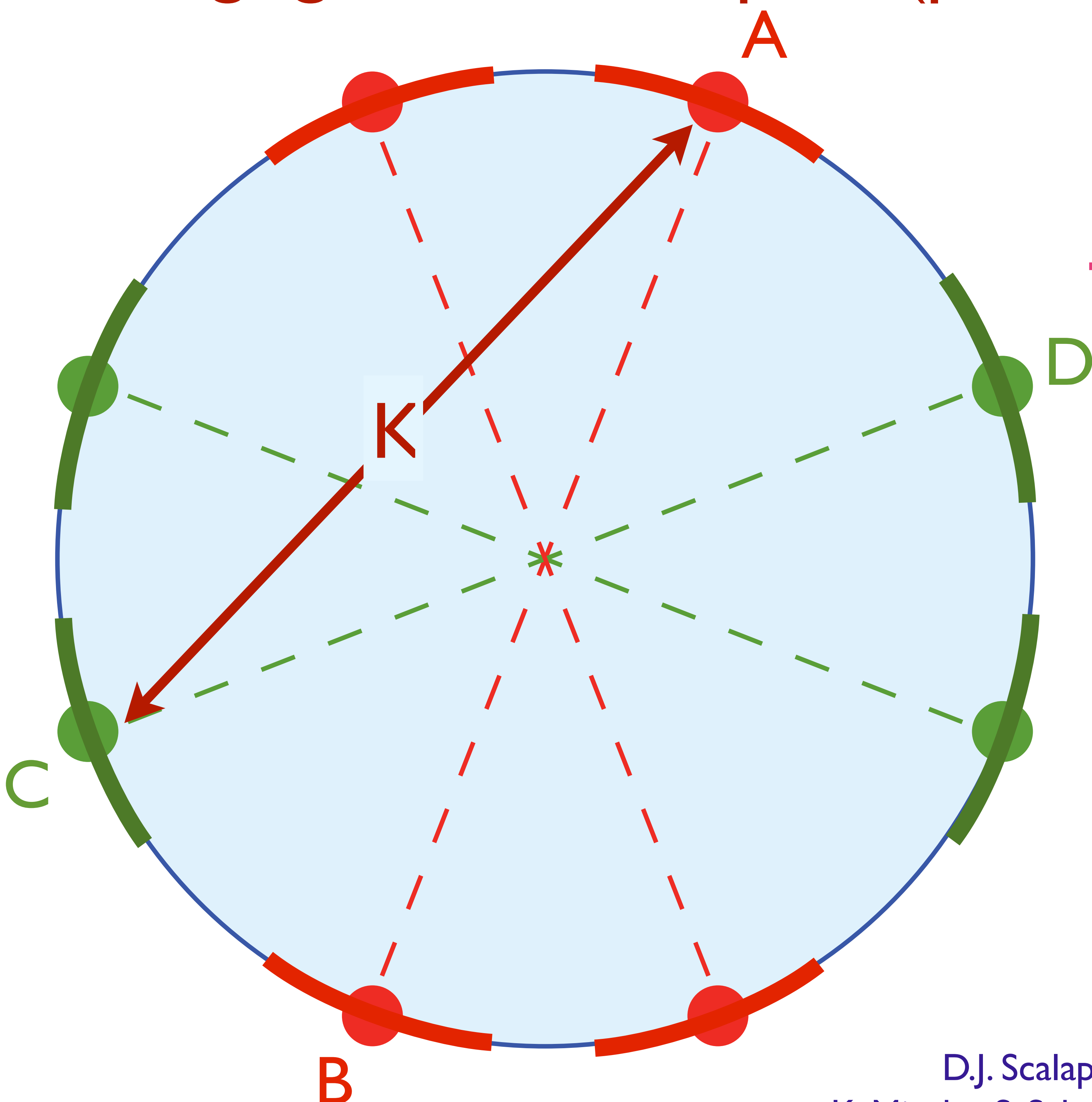
Focus on points on the Fermi surface separated by **K**

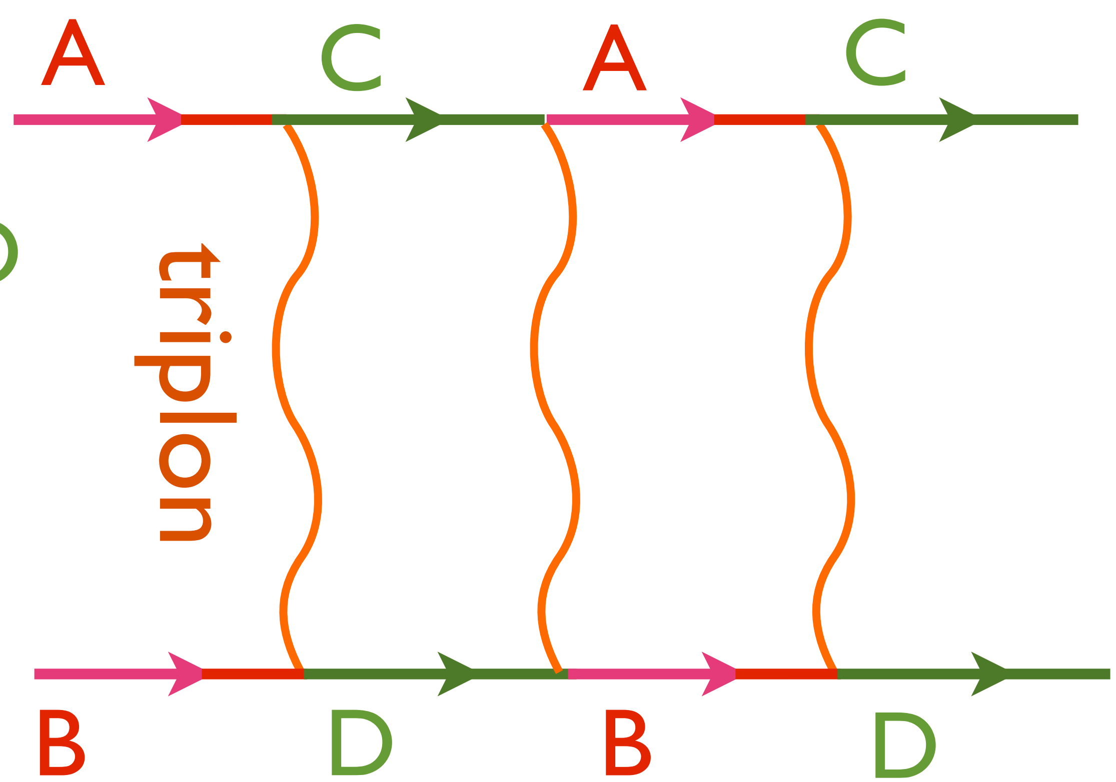


Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

Origin of superconductivity

Pairing "glue" from triplon (paramagnon) exchange





V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
Ar. Abanov, A.V. Chubukov, and A.M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

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- Pairing glue becomes stronger.

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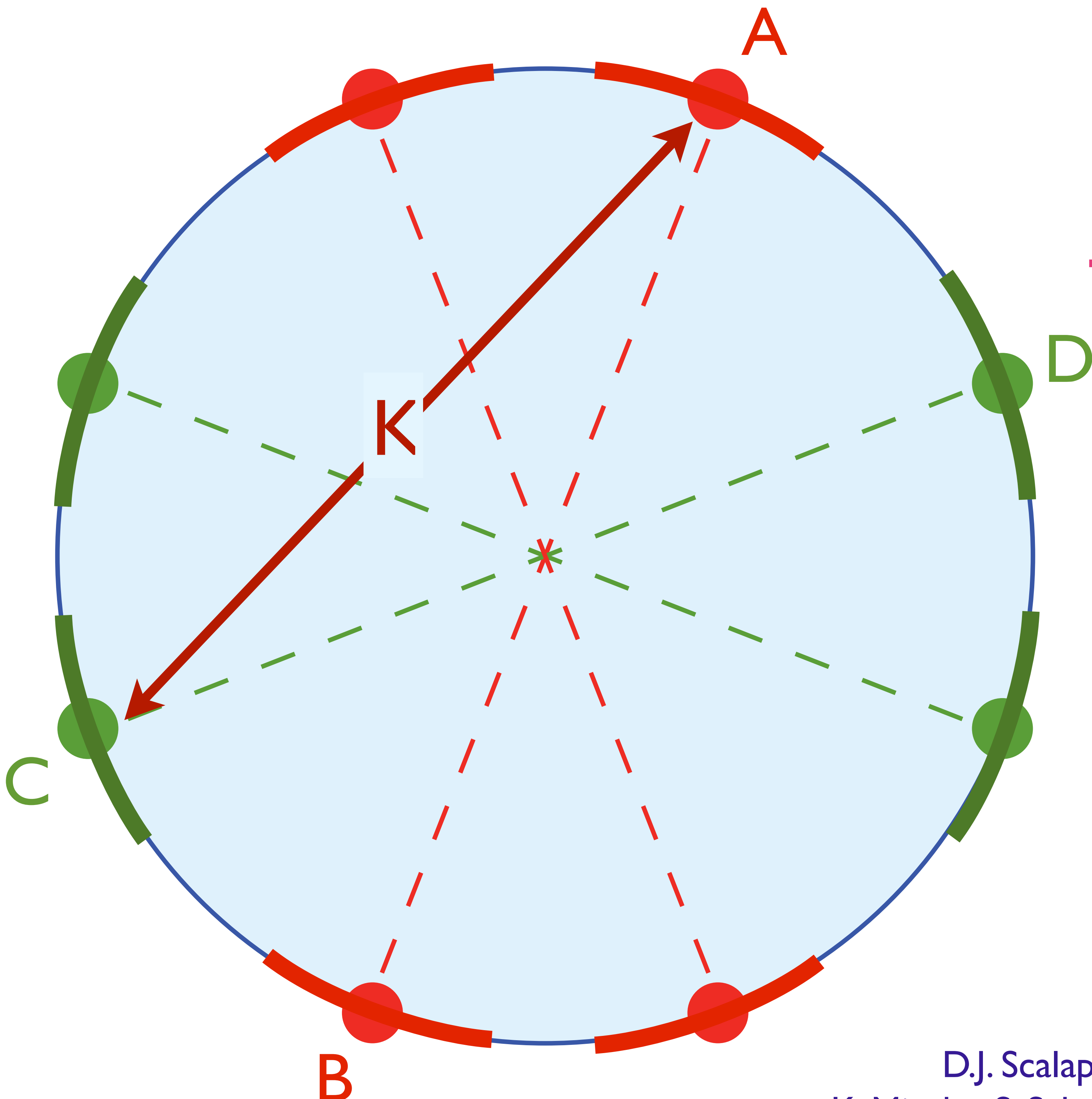
- Pairing glue becomes stronger.

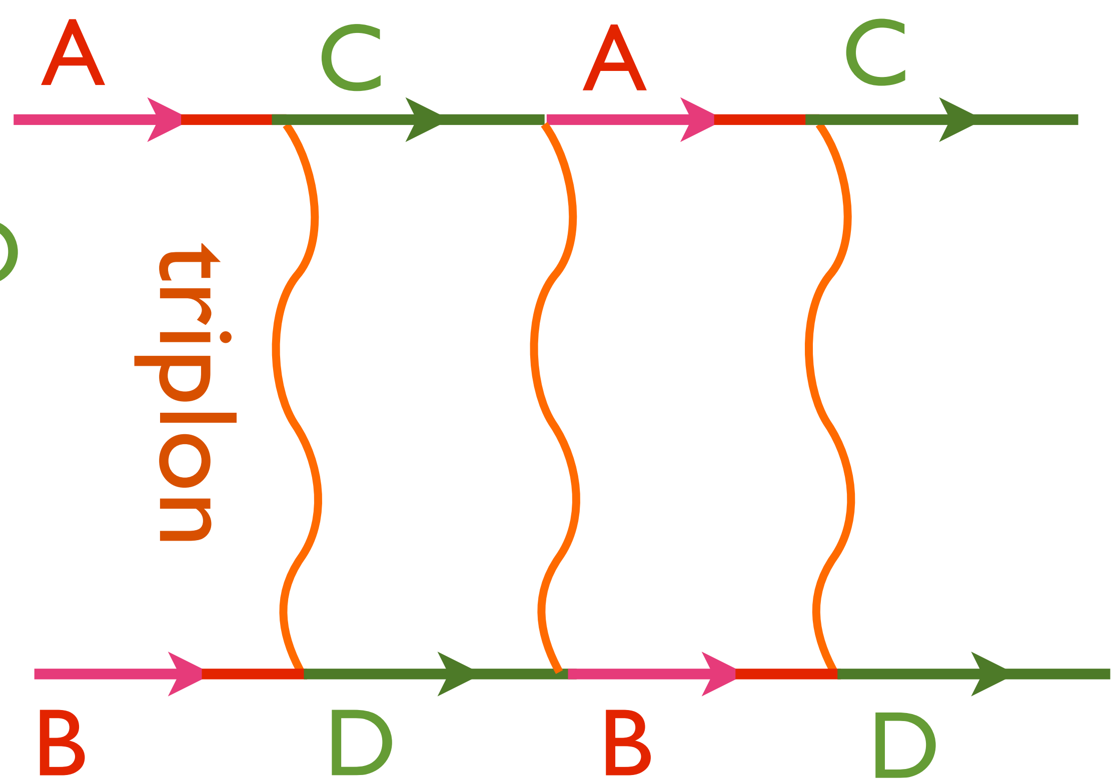
- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.

- An instability to charge-density-wave/bond order can become nearly degenerate with super-conductivity if the Fermi-surface is not too curved.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Pairing "glue" from triplon (paramagnon) exchange





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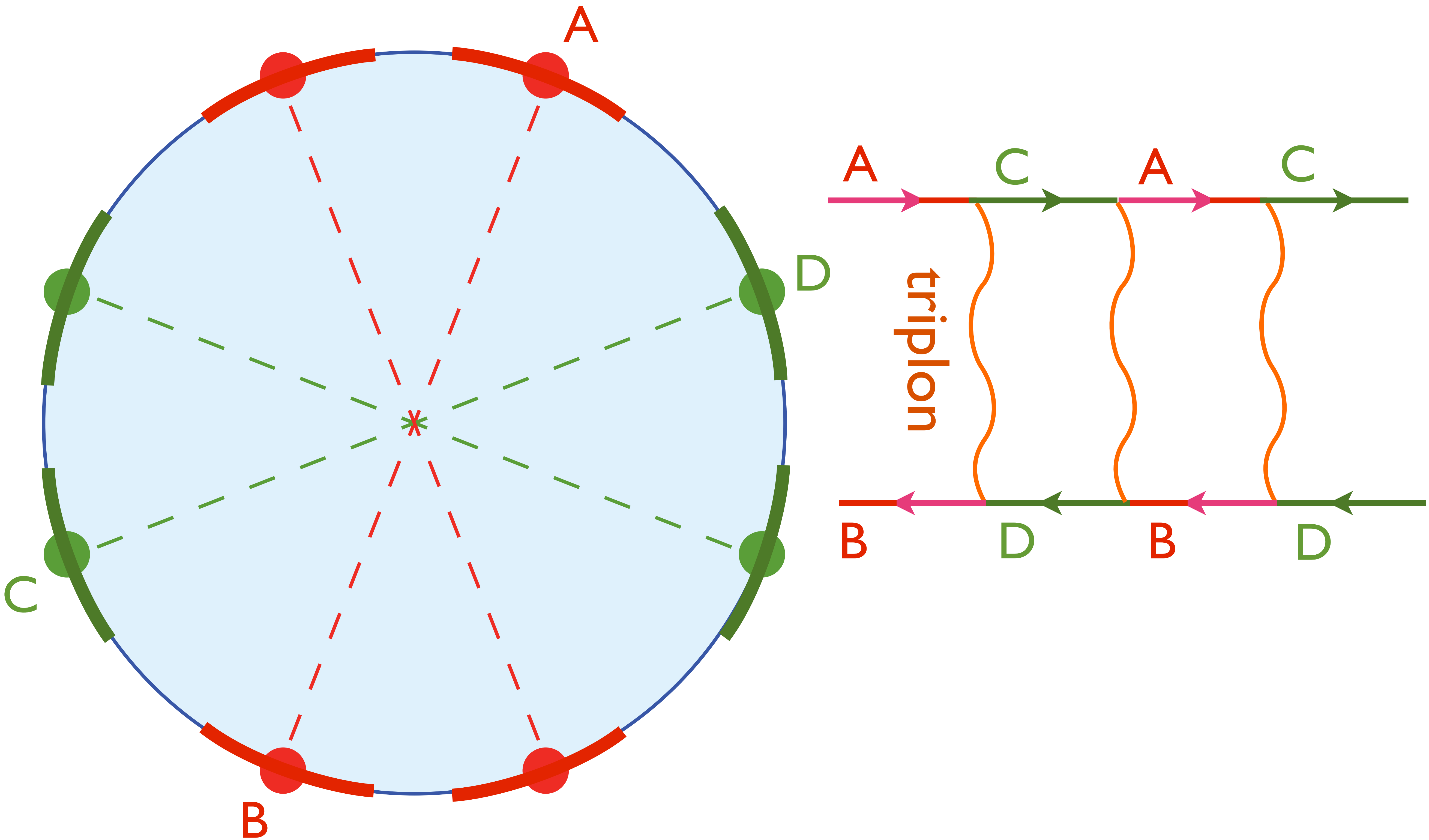
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Same "glue" leads to bond/charge order !



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

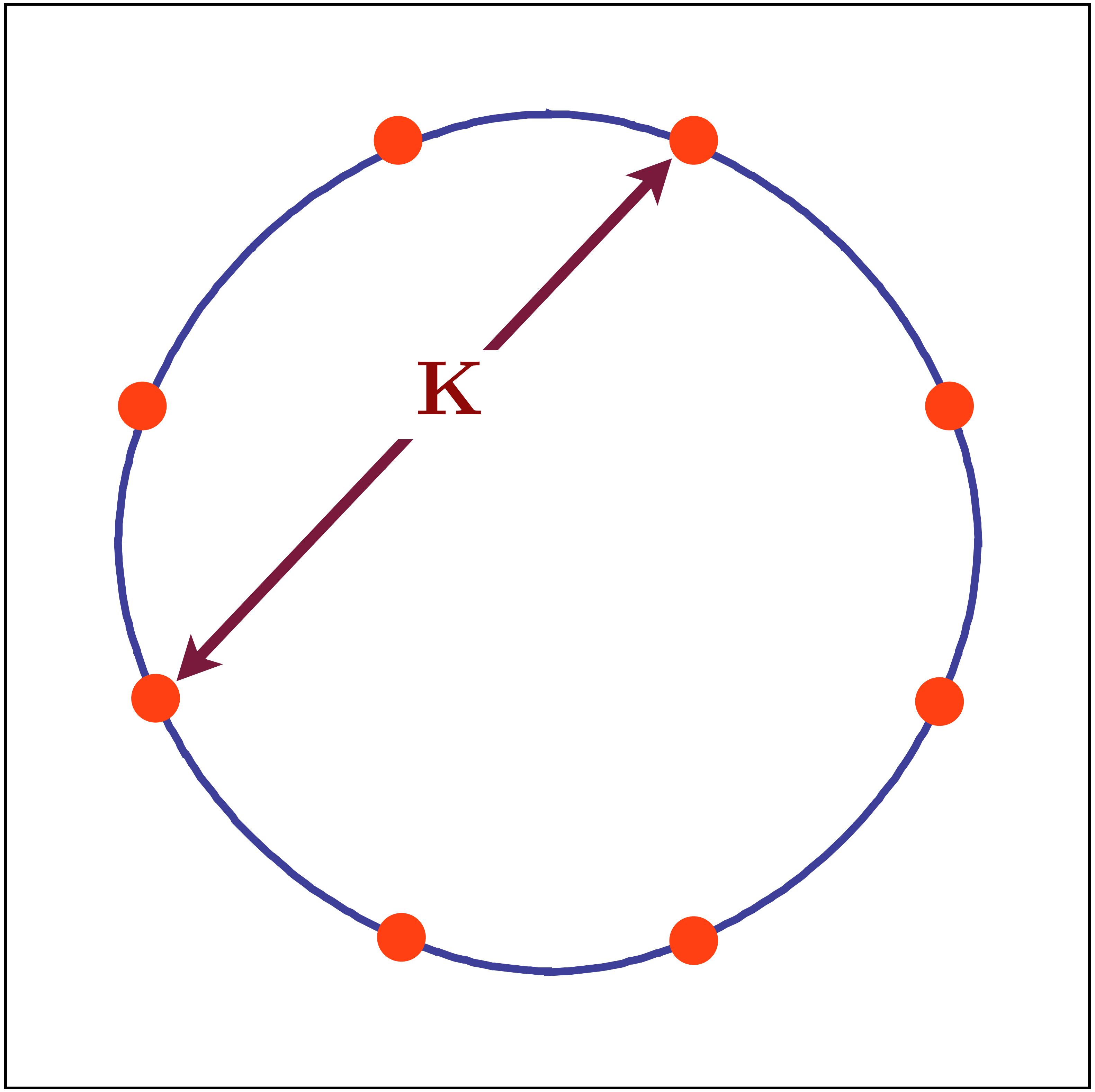
- Pairing glue becomes stronger.

- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.

- An instability to charge-density-wave/bond order can become nearly degenerate with super-conductivity if the Fermi-surface is not too curved.

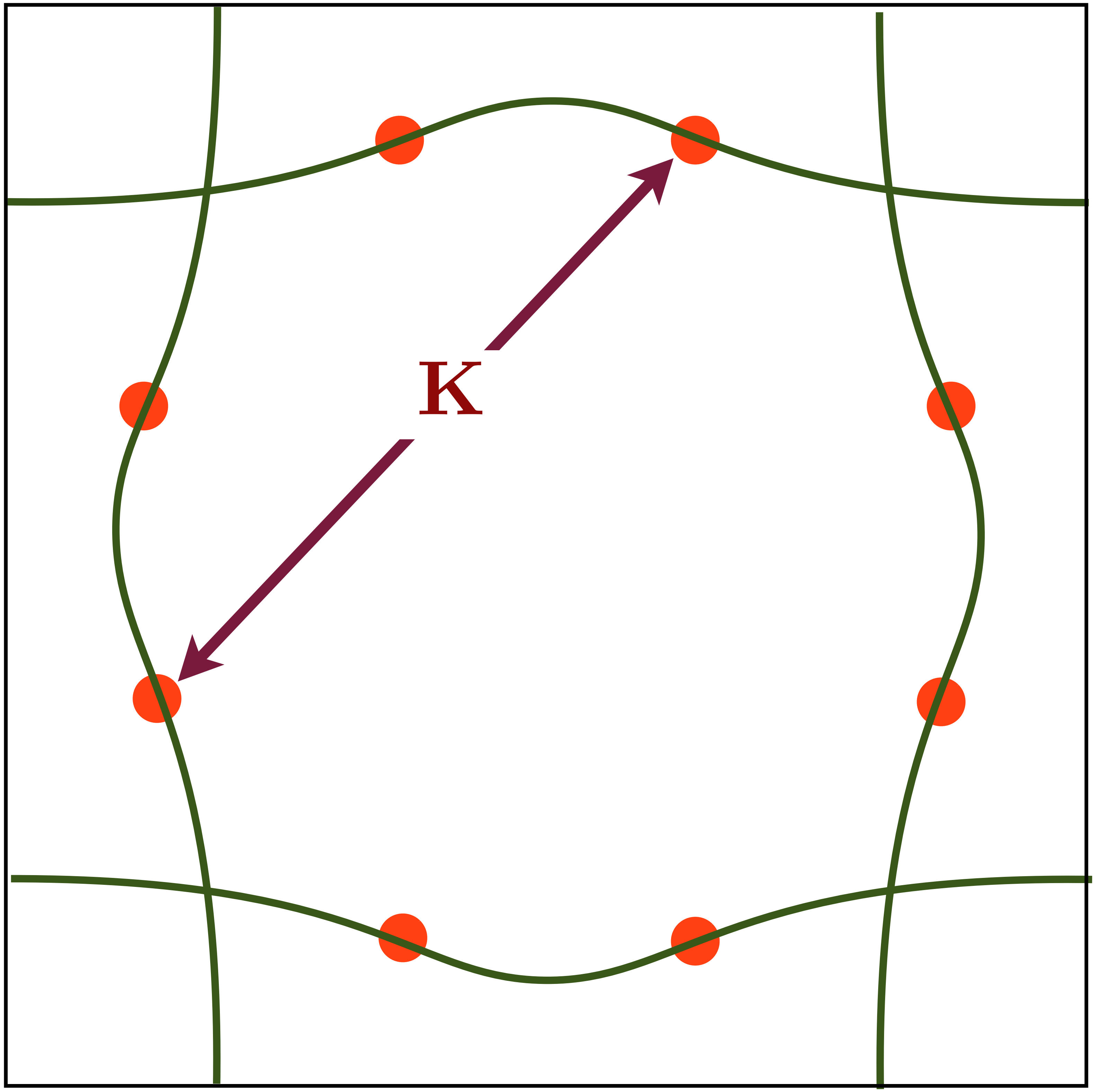
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

QMC for the onset of antiferromagnetism



Hot spots in a single band model

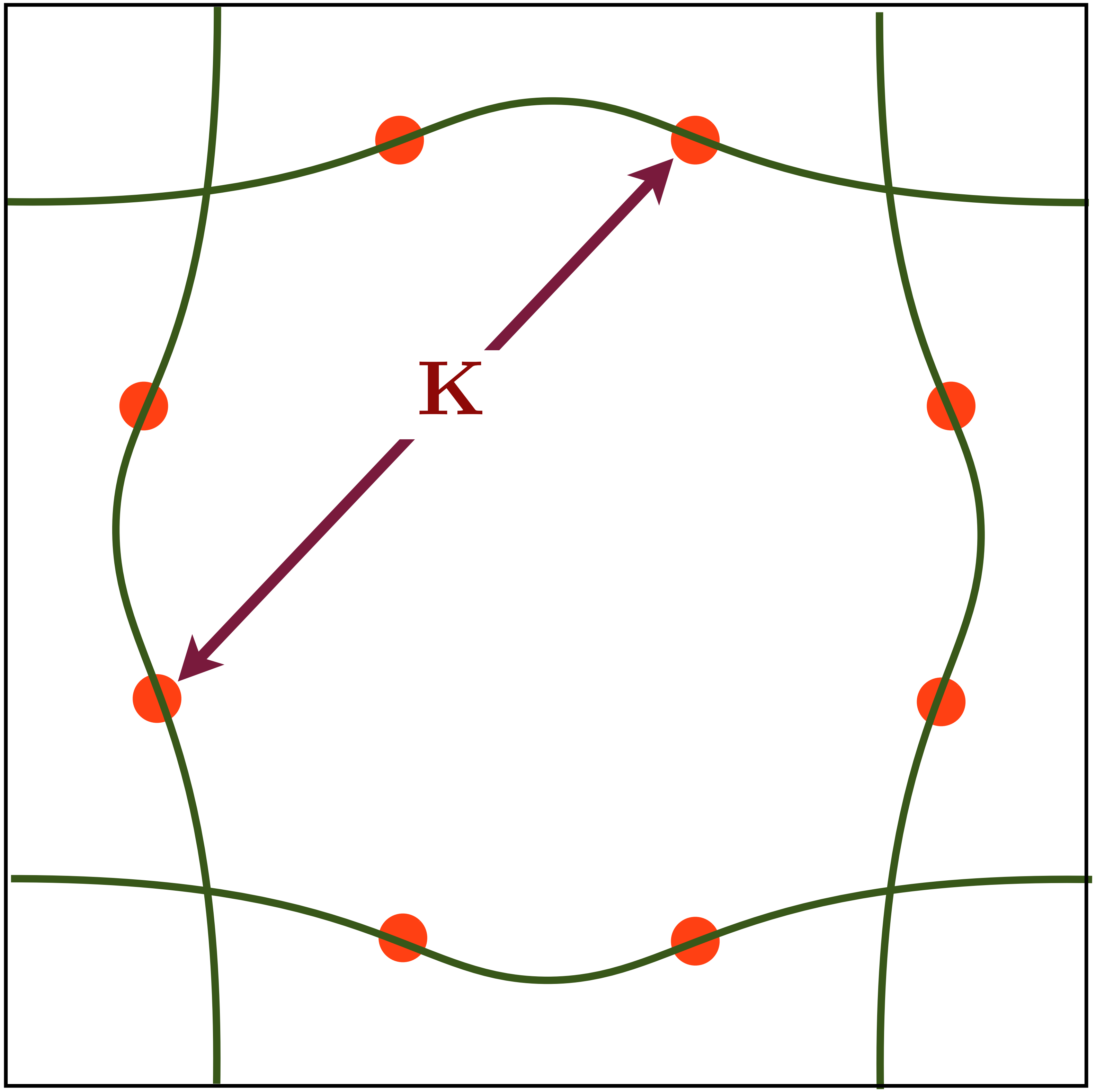
QMC for the onset of antiferromagnetism



Hot spots in a two band model

E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
(2012).

QMC for the onset of antiferromagnetism



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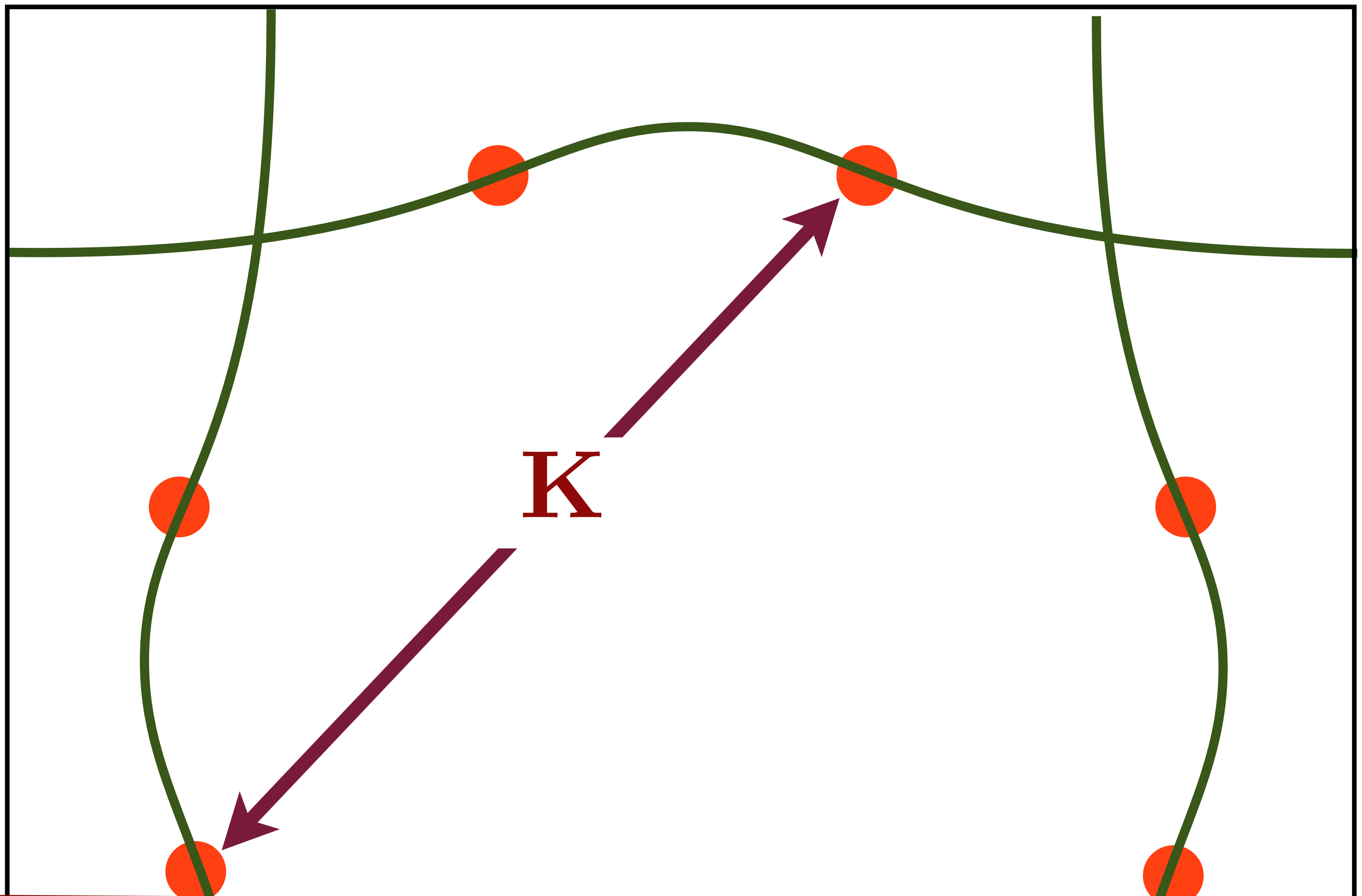


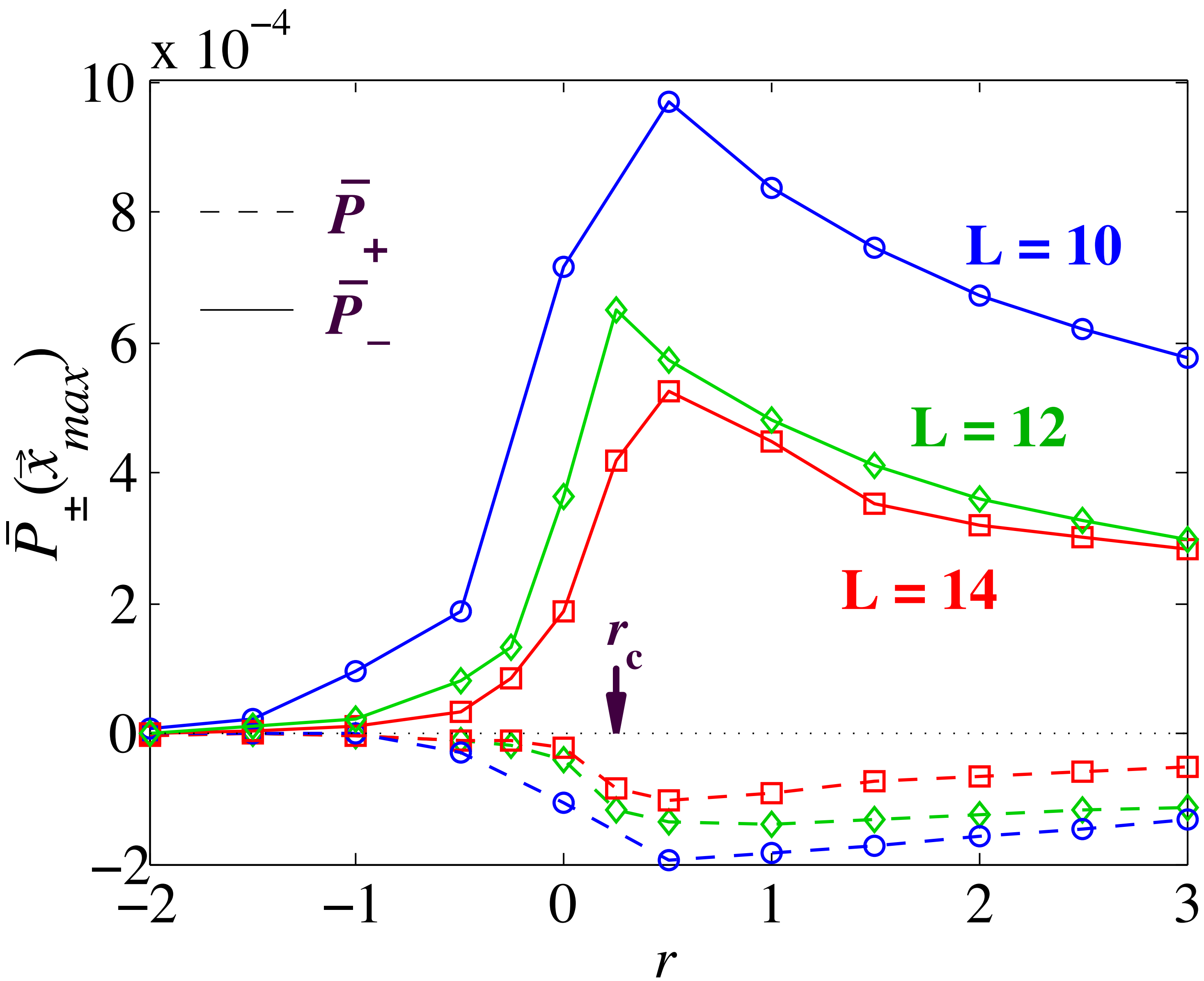
E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
(2012).

No sign problem in
fermion determinant Monte Carlo !

Determinant is positive because of Kramer's degeneracy, and no additional symmetries are needed; holds for arbitrary band structure and band filling, provided ***K*** only connects hot spots in distinct bands

Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals

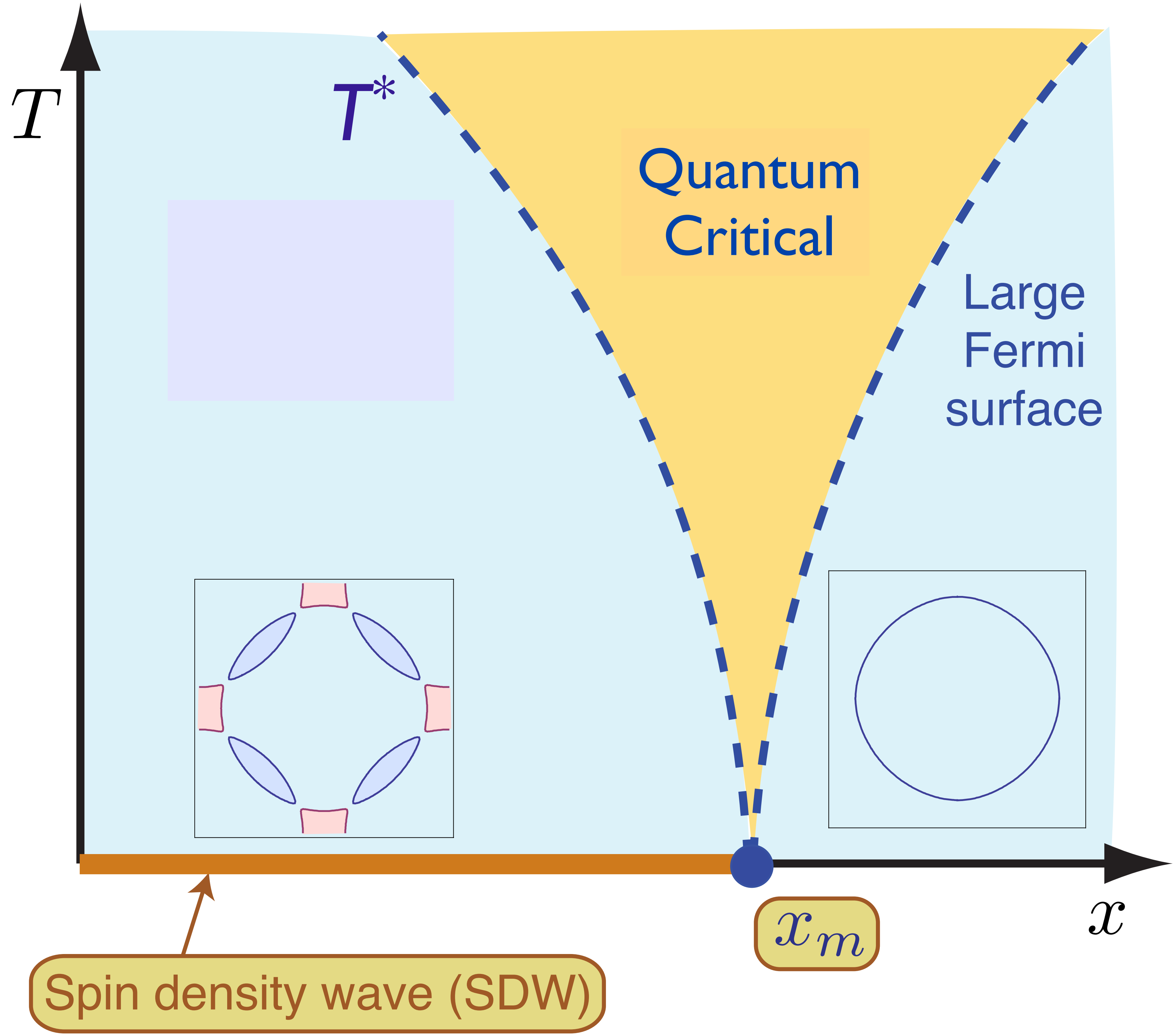
d-wave superconducting survives in the strong-coupling region across the quantum critical point



s/d pairing amplitudes P_+/P_- as a function of the tuning parameter r

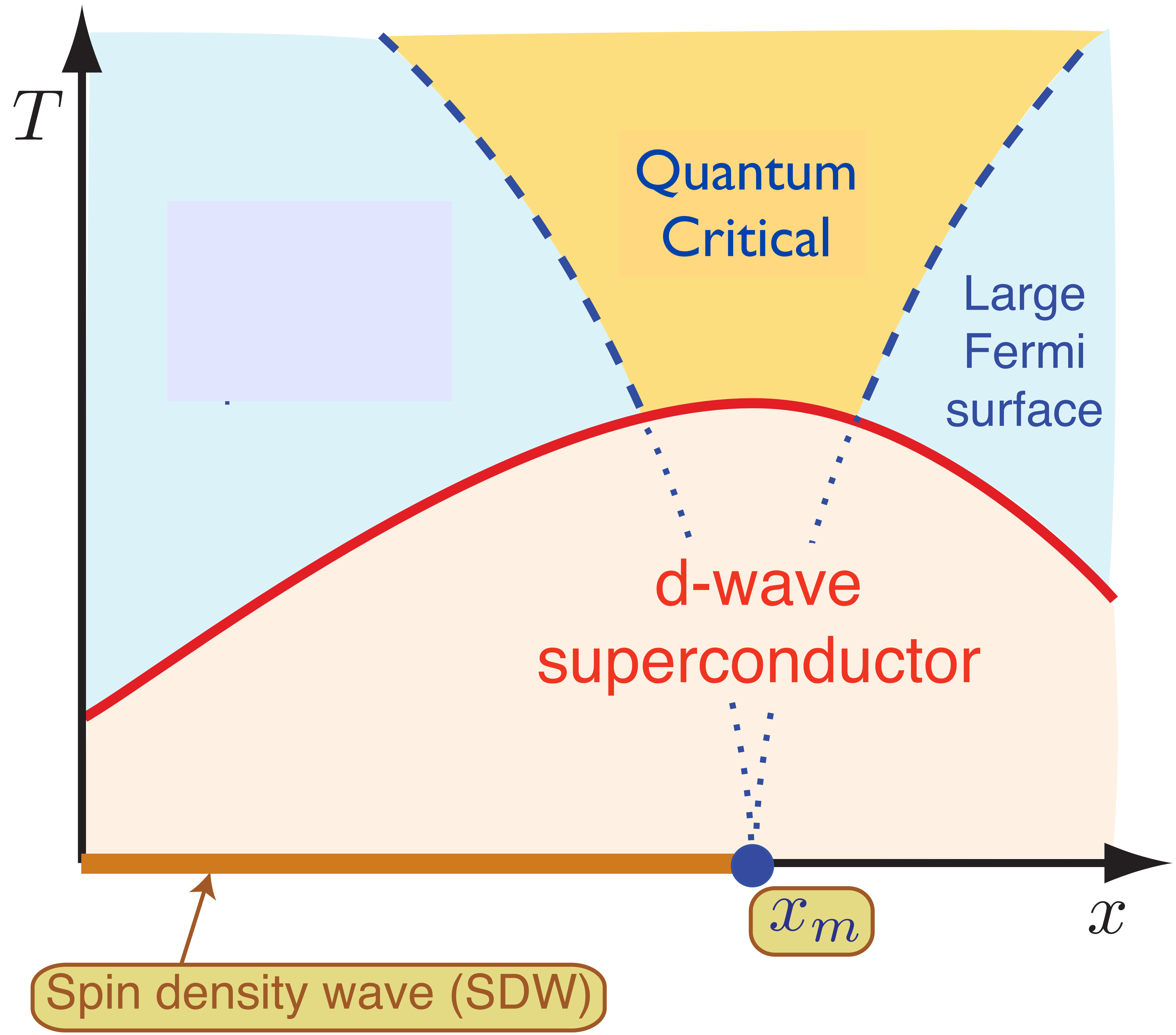
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Fermi surface+antiferromagnetism

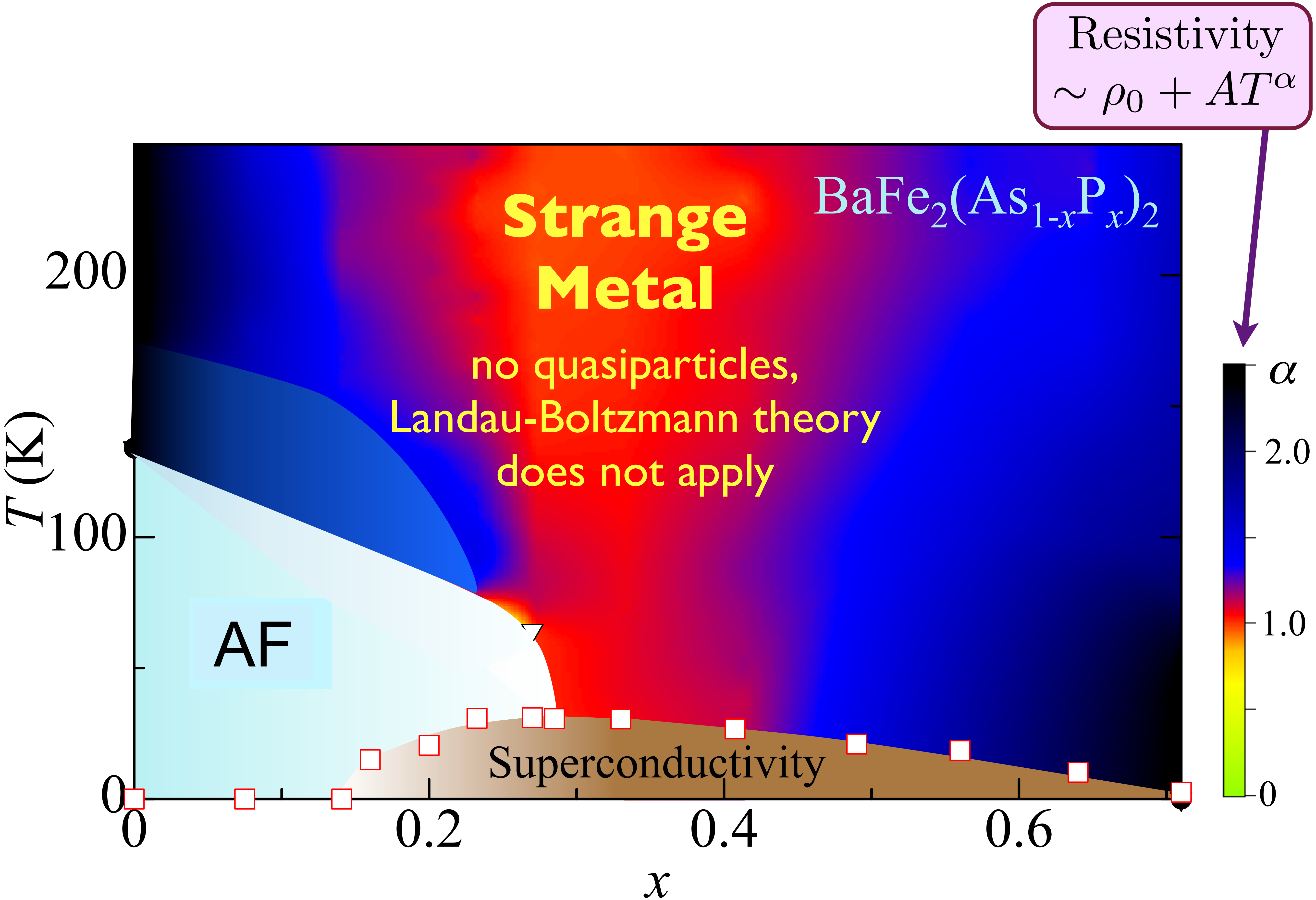


Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Outline

1. Quantum critical point in an insulator

   Non-quasiparticle dynamics

2. Quantum critical point in a metal

   The iron-based superconductors

3. The pseudogap regime of the hole-doped cuprate superconductors

   Angular fluctuations of a multicomponent order

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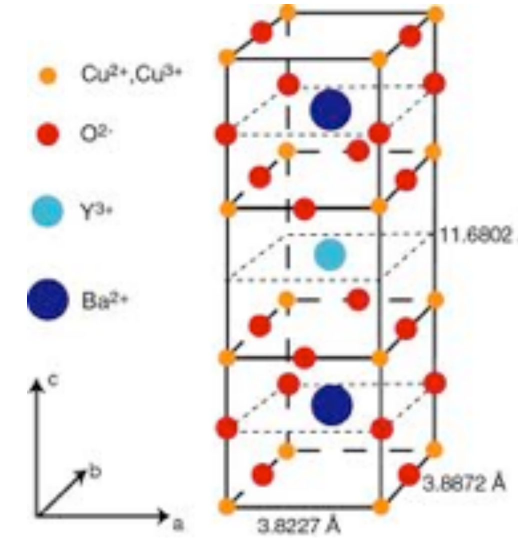
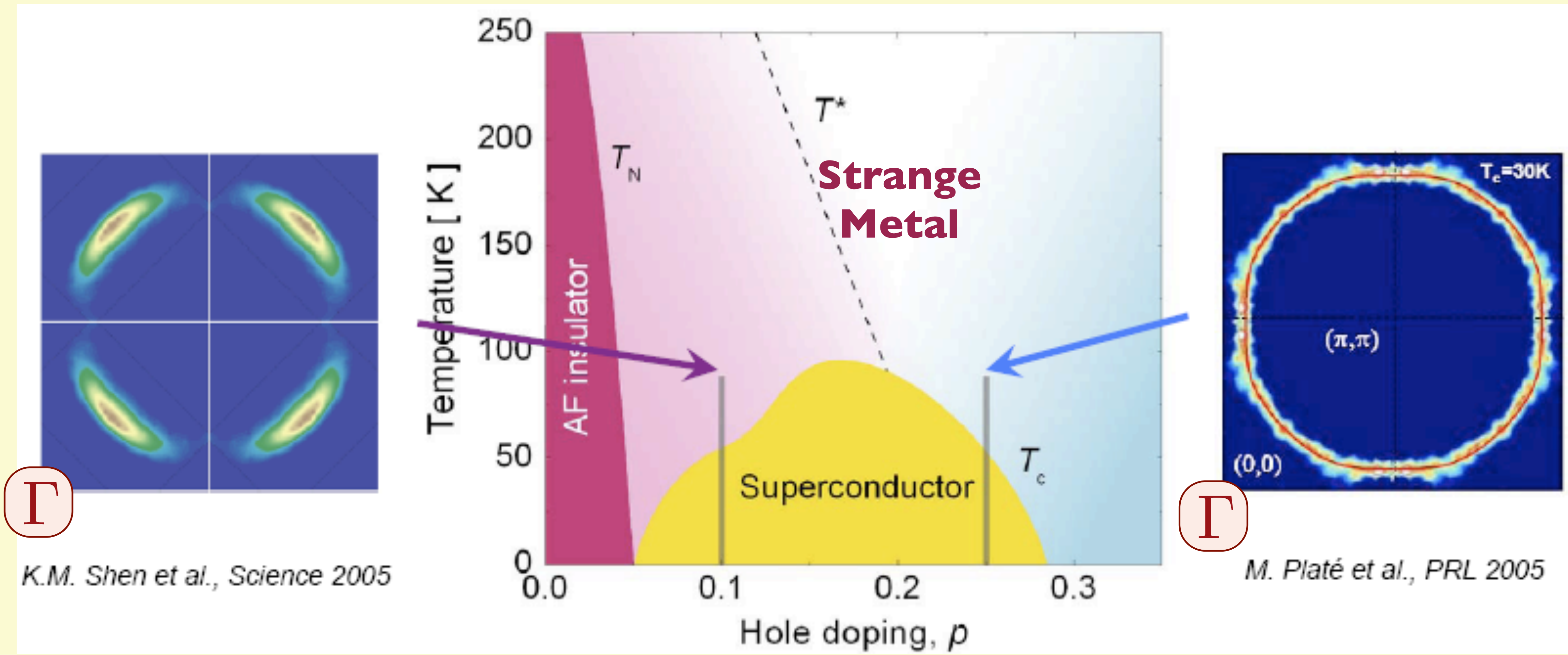
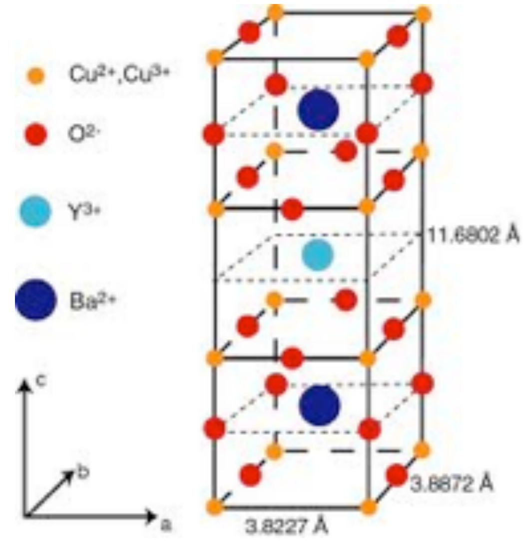


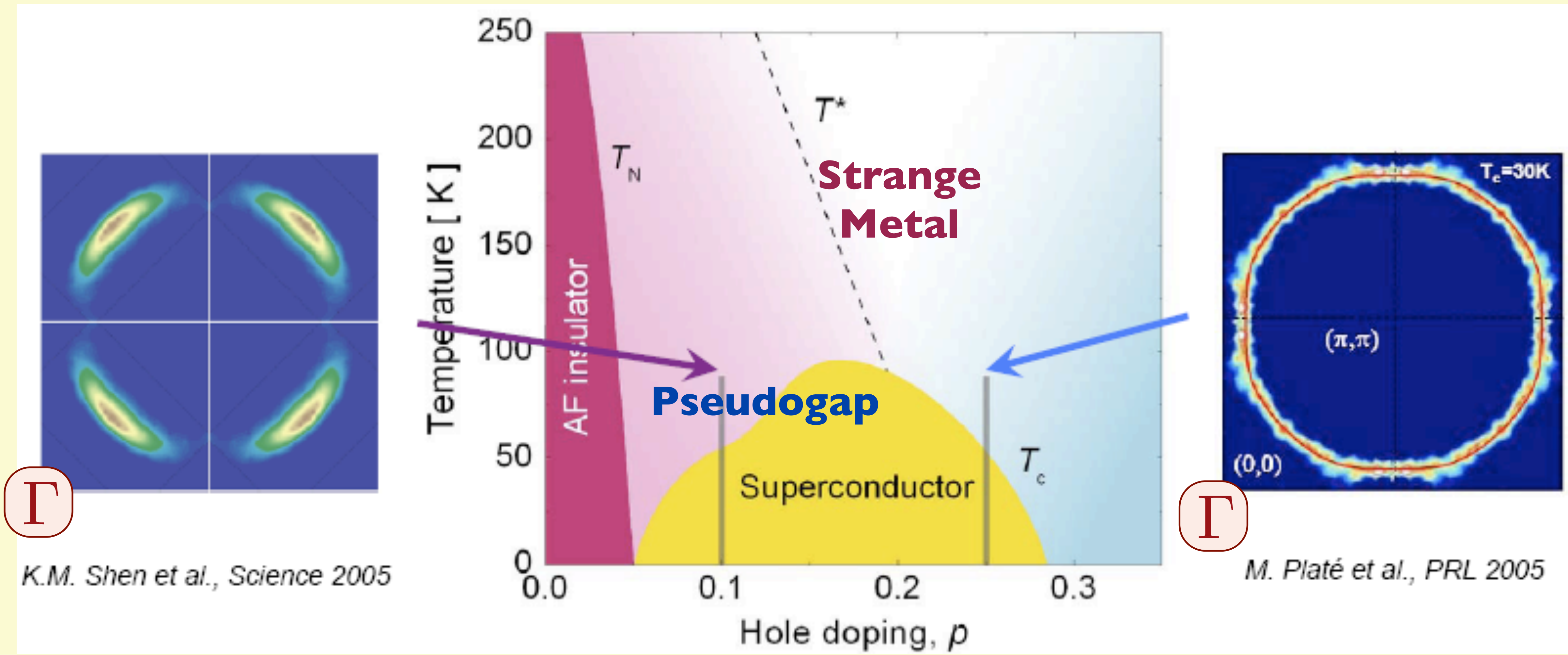


Smaller hole Fermi-pockets

Large hole Fermi surface

What about the pseudogap ?





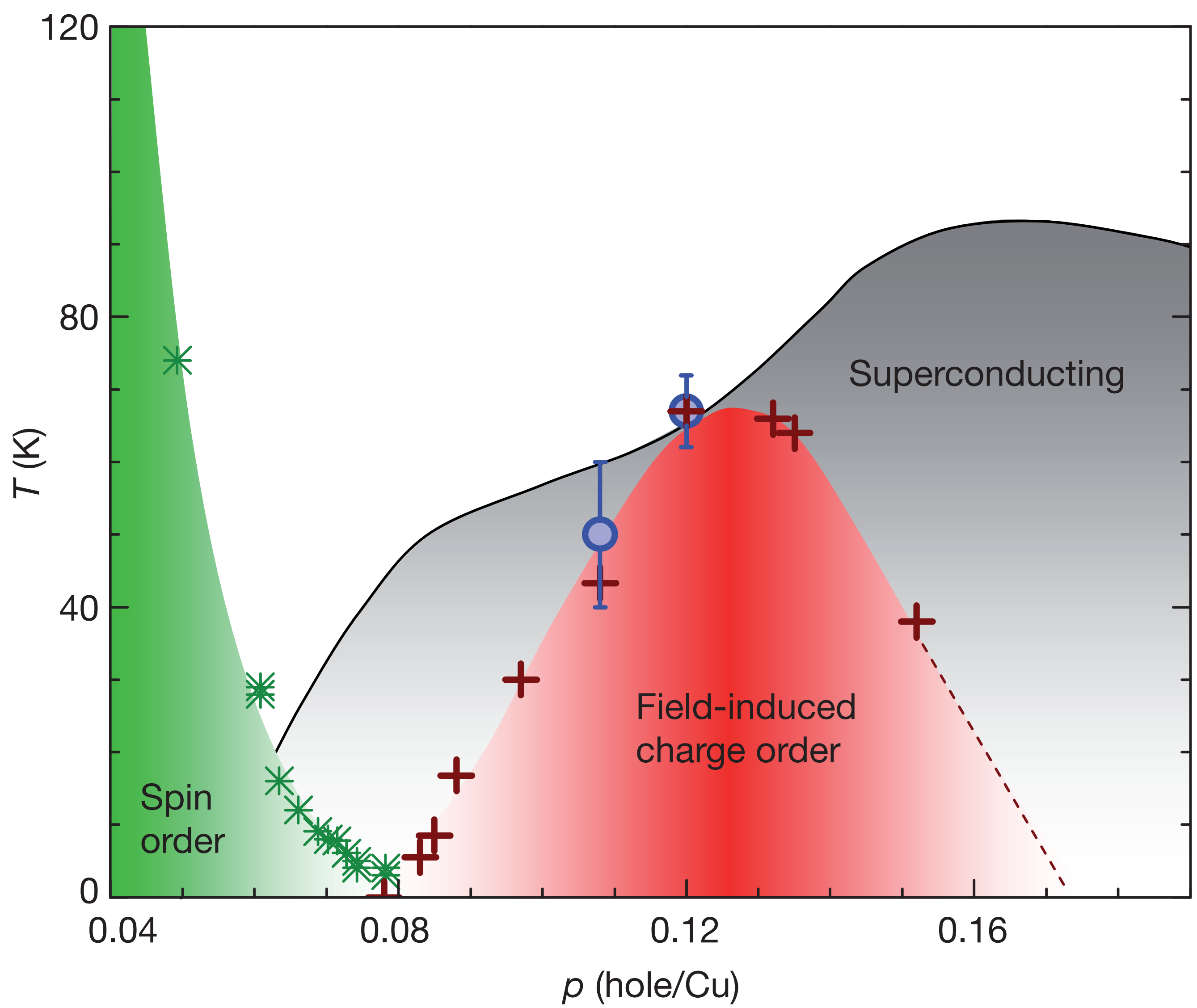
Smaller hole Fermi-pockets

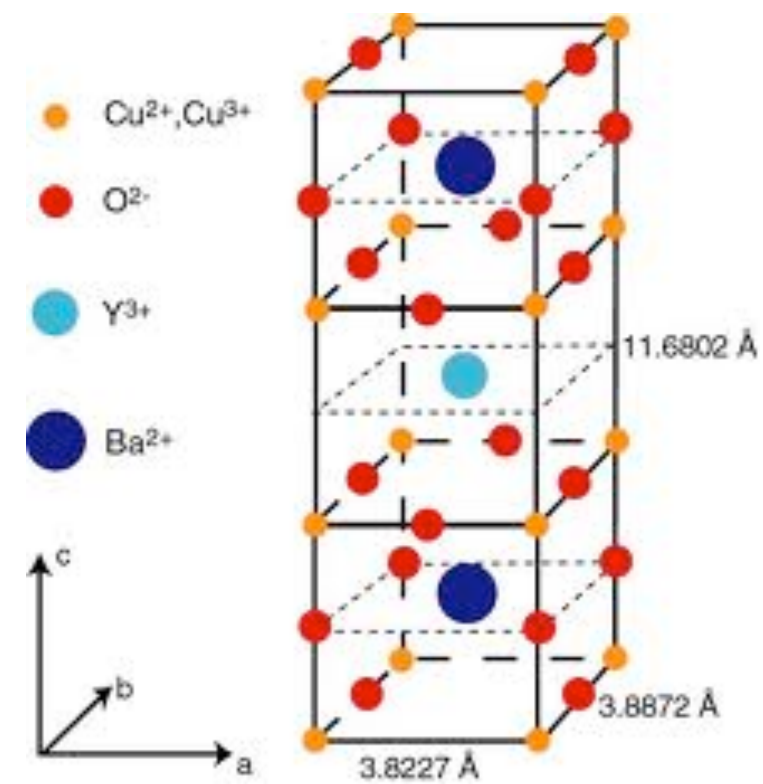
Large hole Fermi surface

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

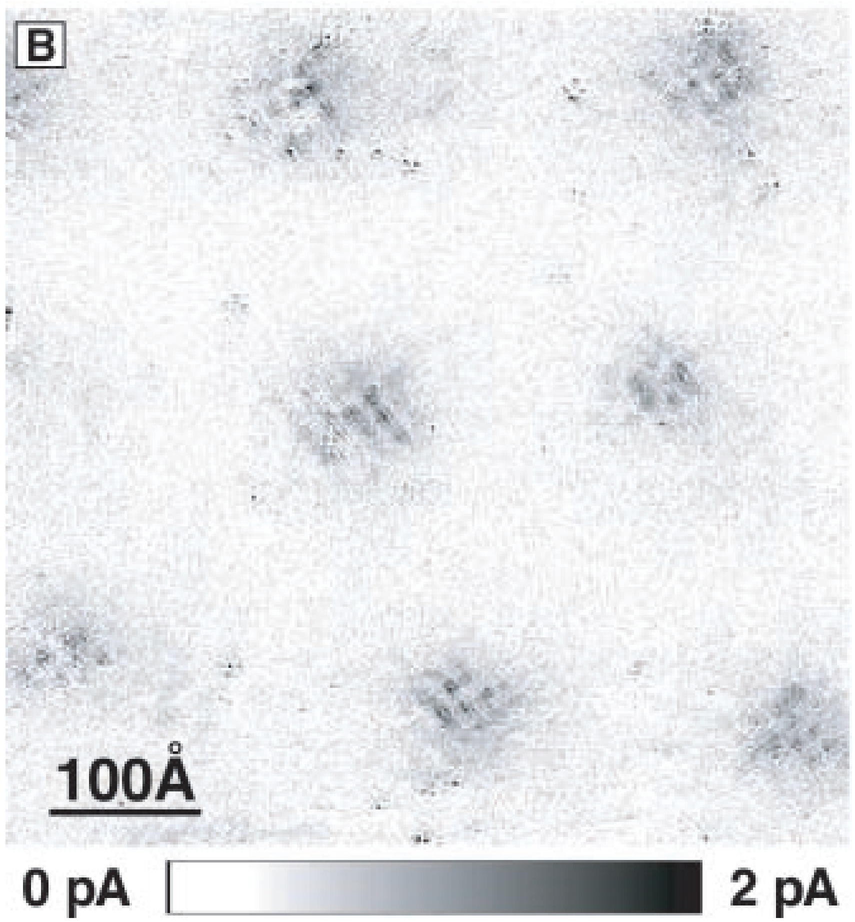
Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191





- Charge order was originally observed around vortex cores, indicating its competition with superconductivity.



A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

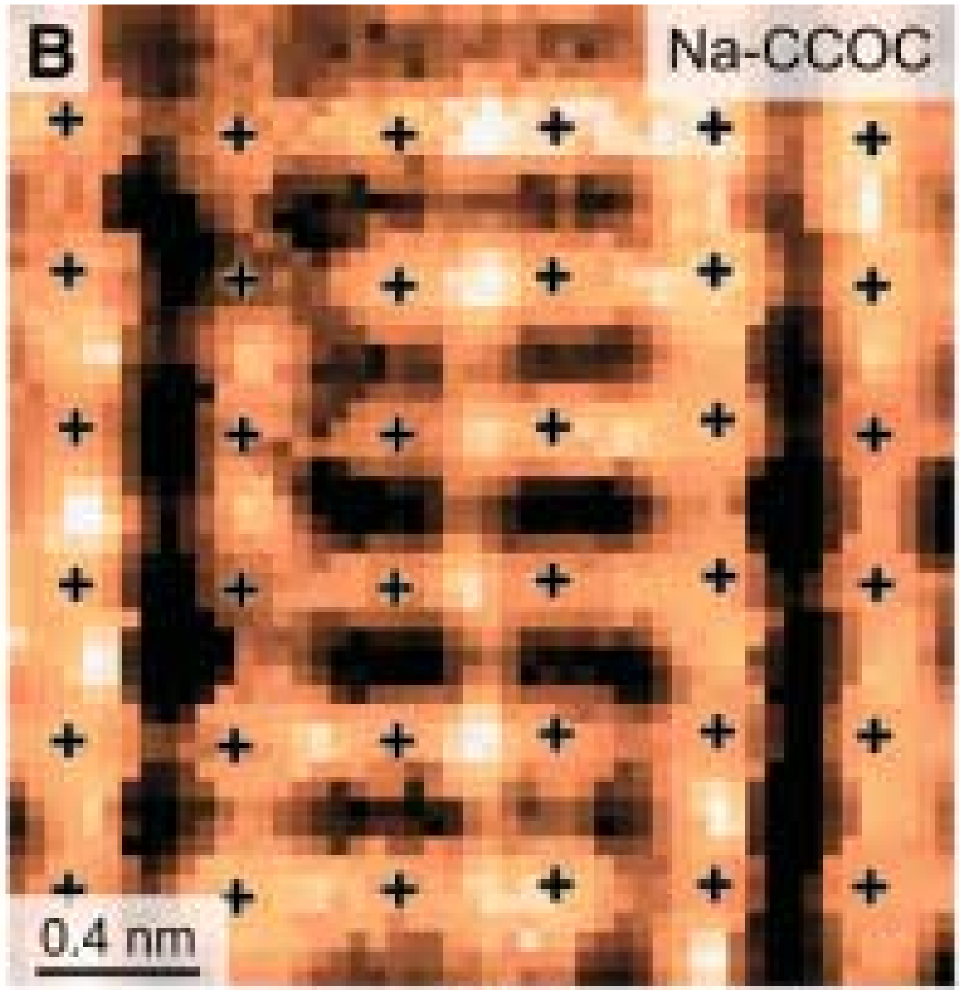
J. E. Hoffman,[1] E. W. Hudson,[1,2]* K. M. Lang,[1] V. Madhavan,[1] H. Eisaki,[3]† S. Uchida,[3] J. C. Davis[1,2]‡

SCIENCE VOL 295 18 JANUARY 2002

An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,[1] C. Taylor,[1] K. Fujita,[1,2] A. Schmidt,[1] C. Lupien,[3] T. Hanaguri,[4] M. Azuma,[5] M. Takano,[5] H. Eisaki,[6] H. Takagi,[2,4] S. Uchida,[2,7] J. C. Davis[1,8]*

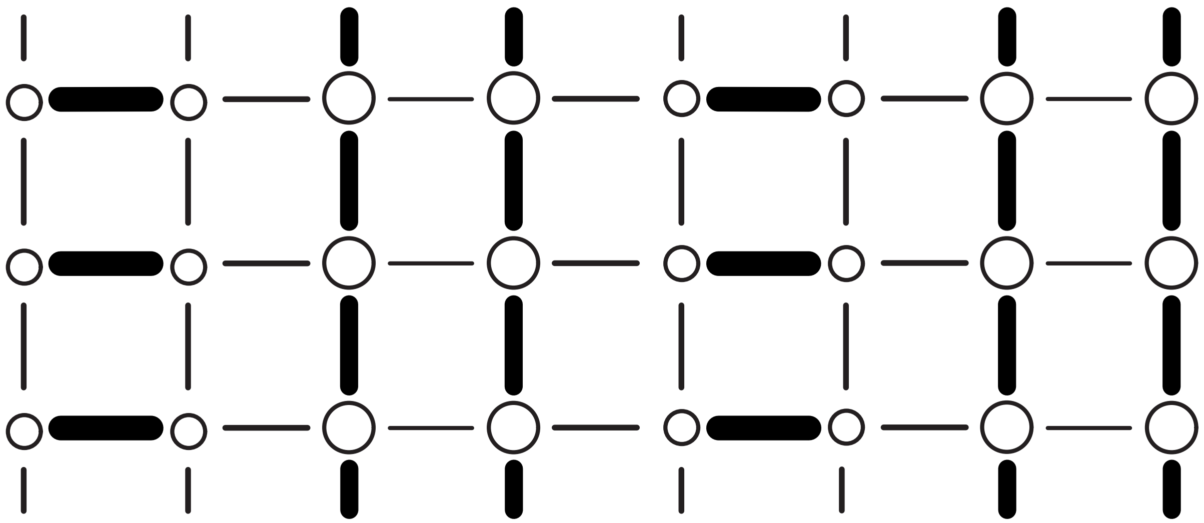
9 MARCH 2007 VOL 315 **SCIENCE**



PHYSICAL REVIEW B **77**, 094504 (2008)

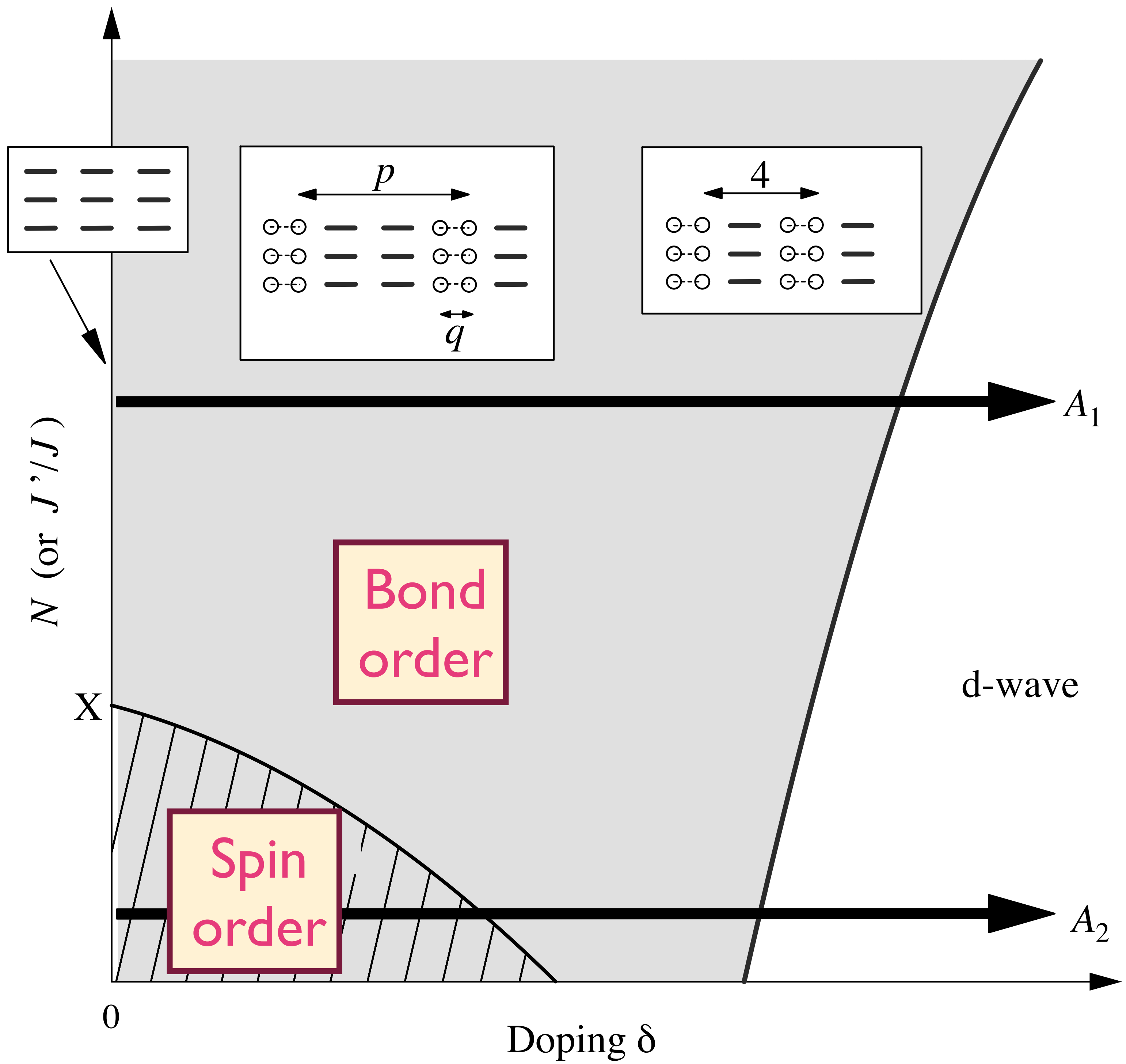
Superconducting *d*-wave stripes in cuprates: Valence bond order coexisting with nodal quasiparticles

Matthias Vojta and Oliver Rösch



We point out that unidirectional bond-centered charge-density-wave states in cuprates involve electronic order in both *s*- and *d*-wave channels, with nonlocal Coulomb repulsion suppressing the *s*-wave component.

Phase diagram of doped antiferromagnets



S. Sachdev and R. La Placa, Physical Review Letters **111**, 027202 (2013)
M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B 77, 094504 (2008)
S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991)

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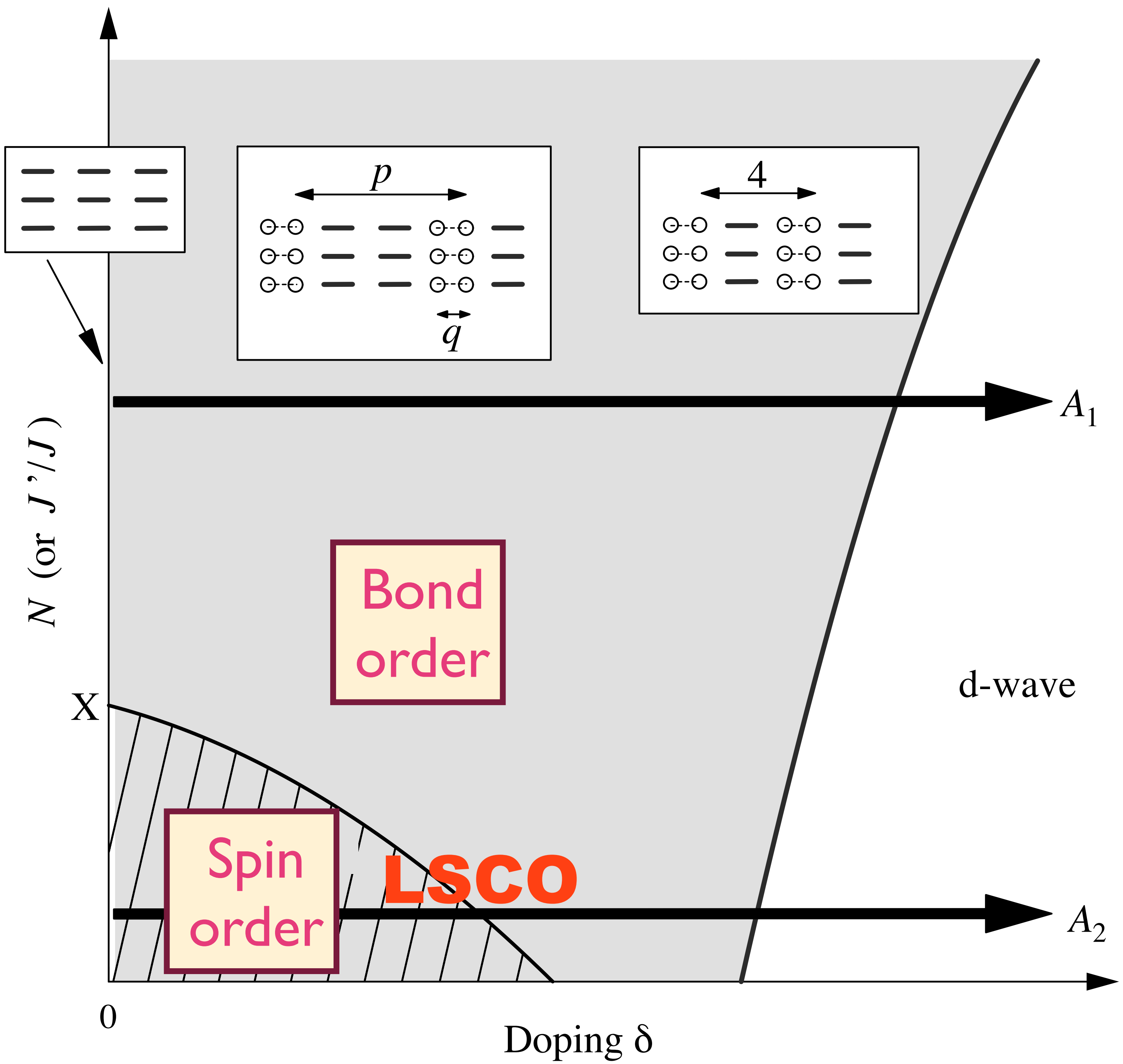

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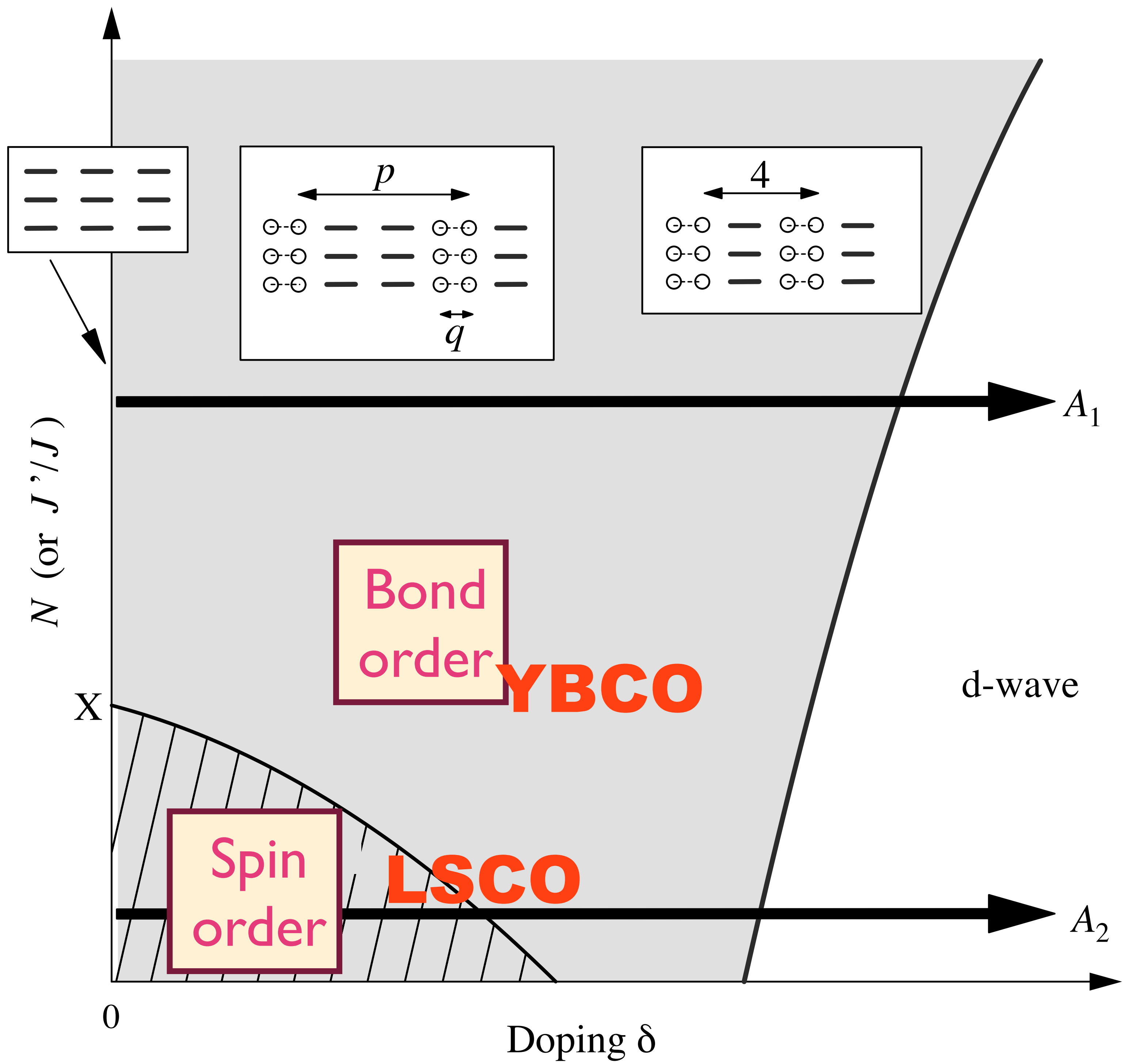

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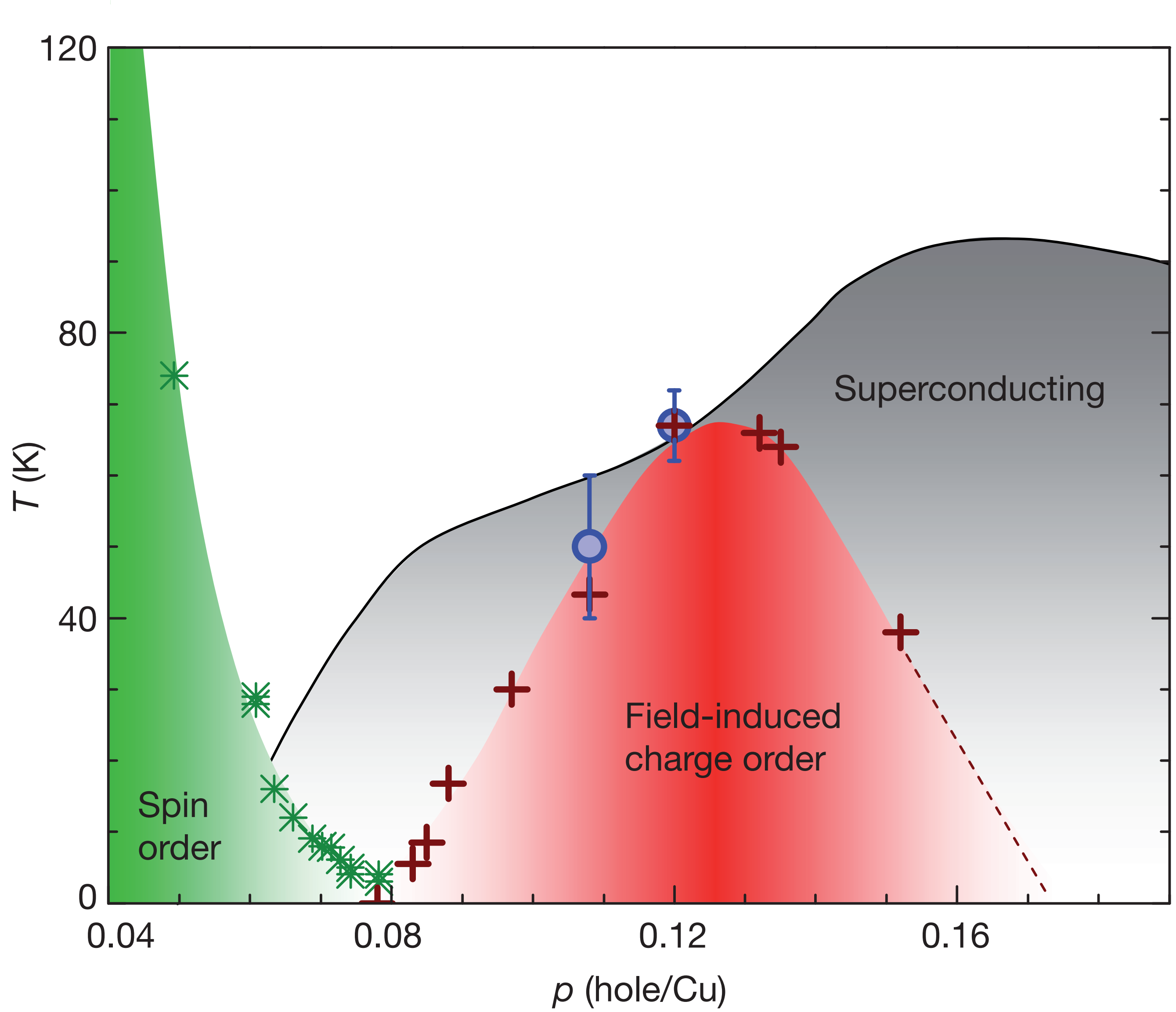
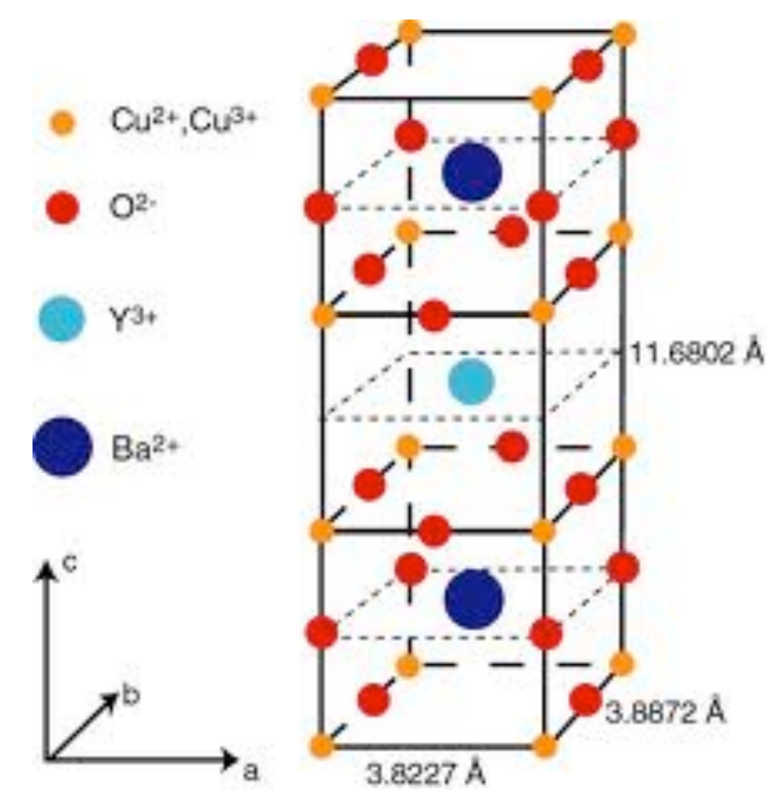
8 SEPTEMBER 2011 | VOL 477 | NATURE | 191



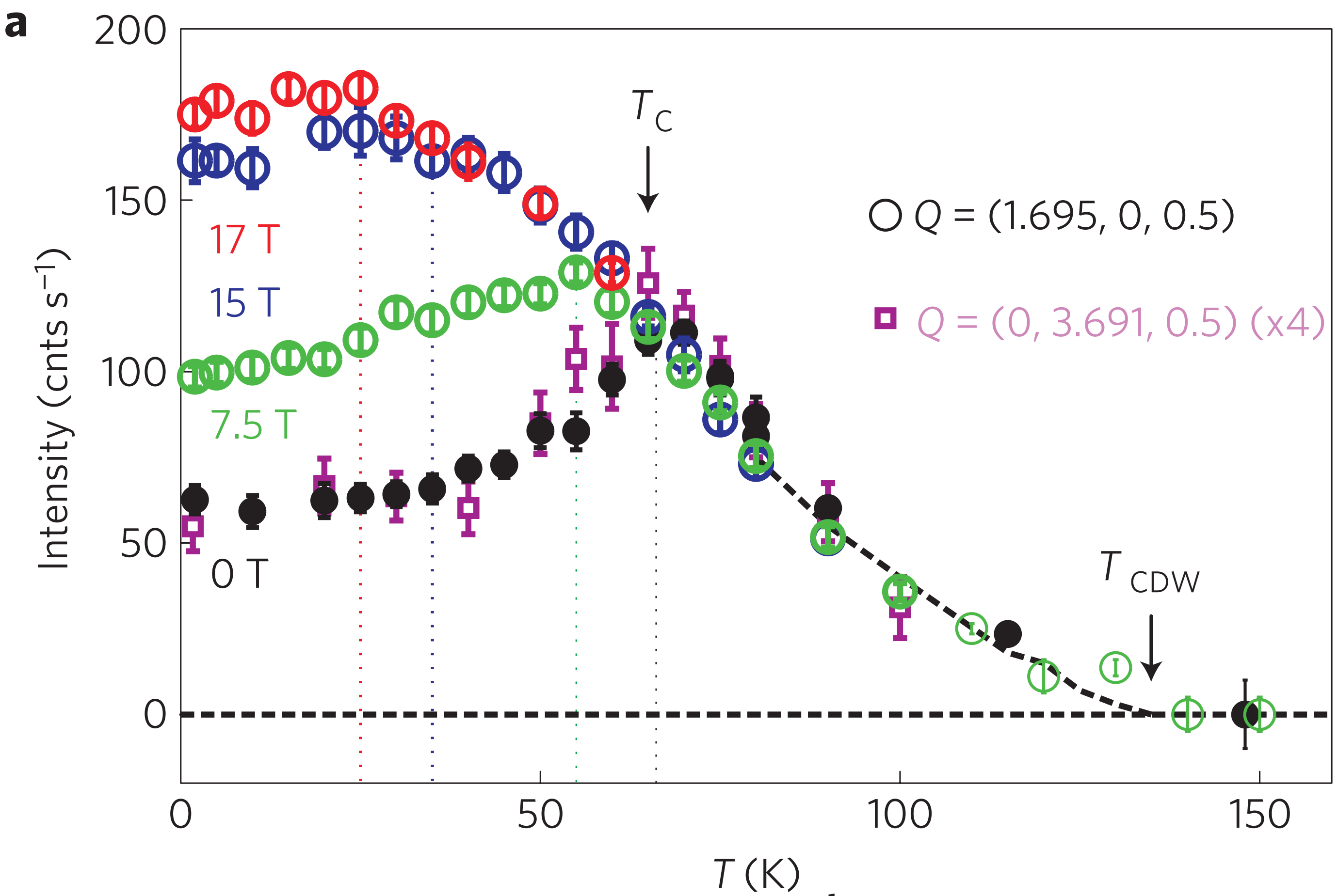


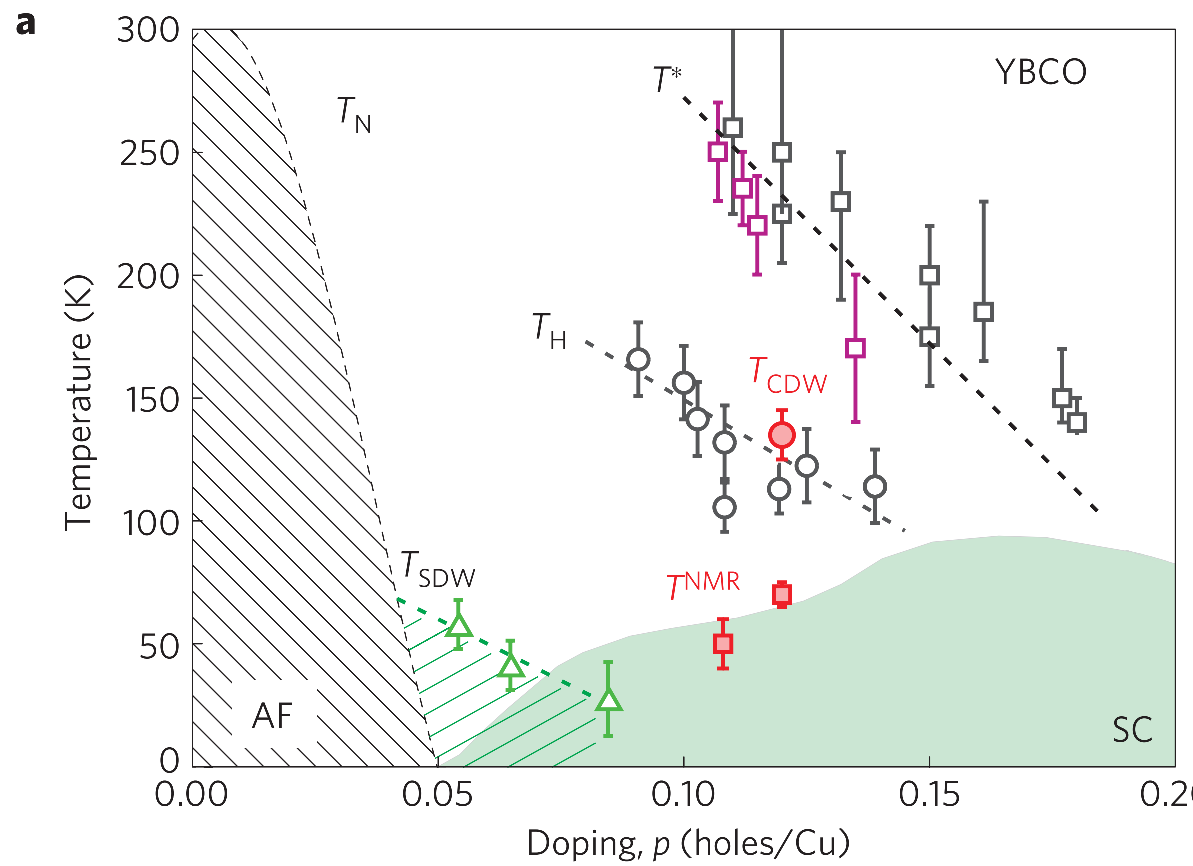
Friday, December 20, 13

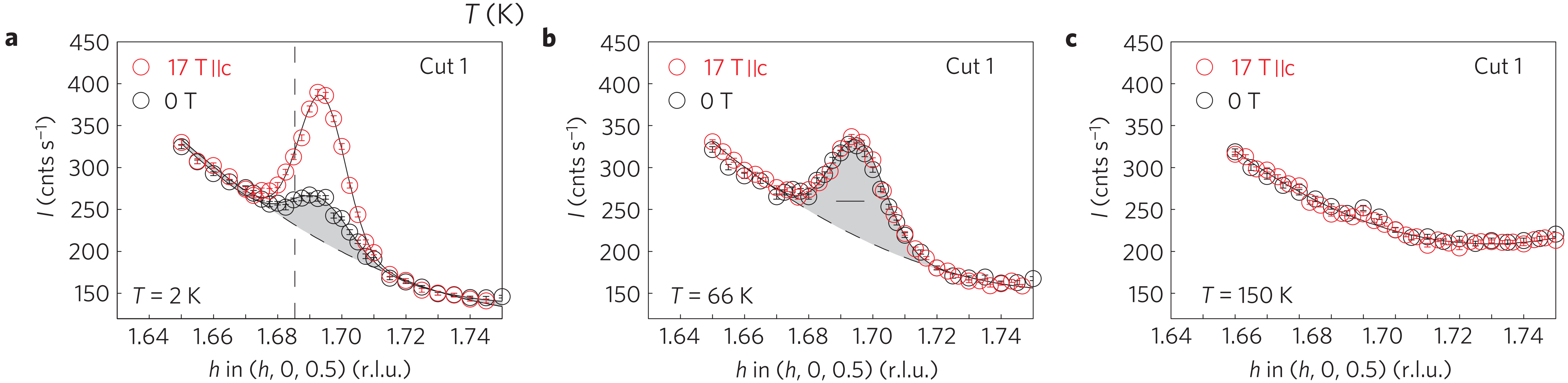
Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]







A.J. Achkar et al., to appear

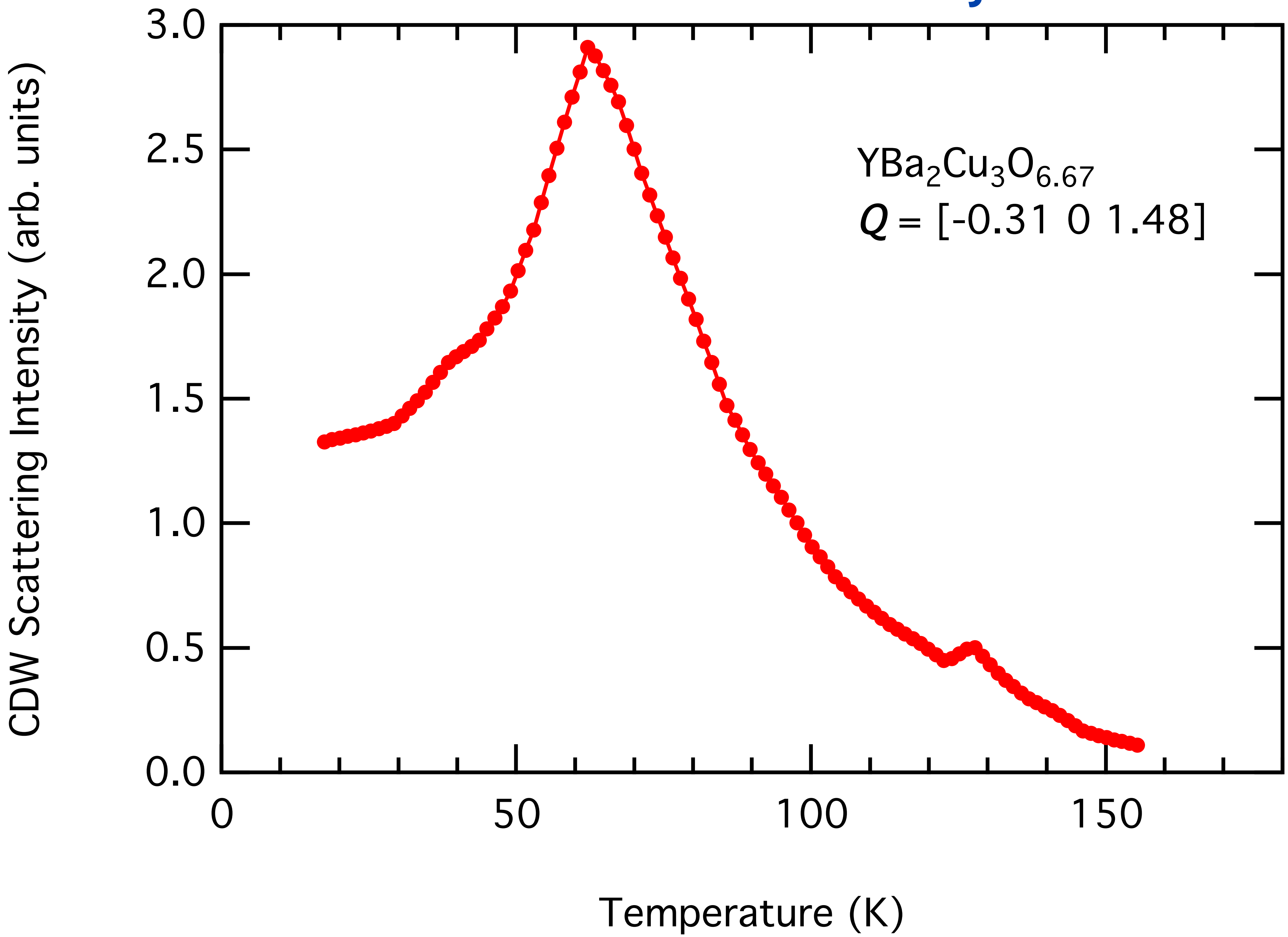
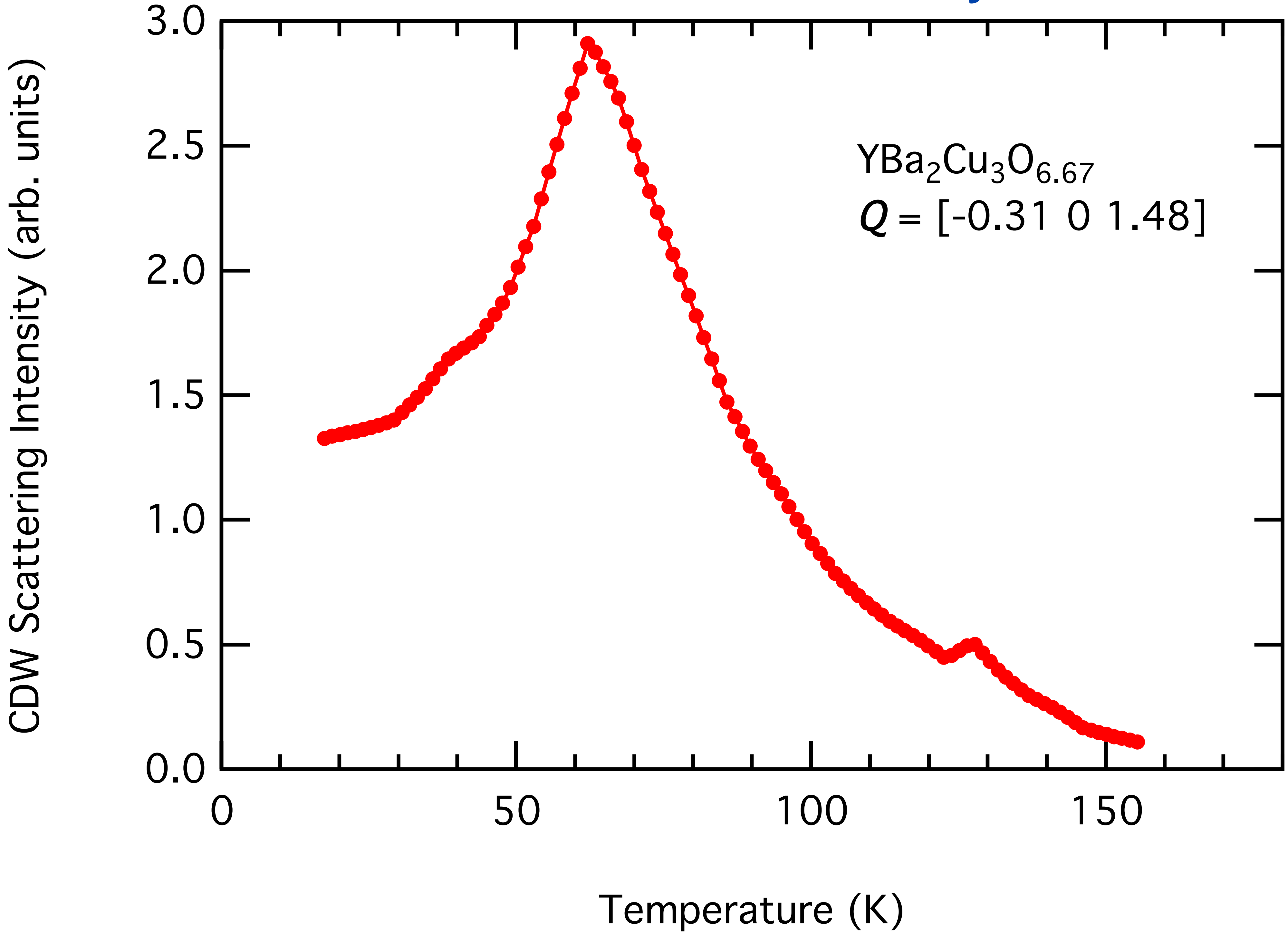


FIG. 1: The temperature dependence of the CDW scattering intensity at **Q** = [-0.31 0 1.48] in $YBa_2Cu_3O_{6.67}$ measured by resonant x-ray scattering in Ref. [4]. This sample has $T_c \approx 65.5$K.



Onset is unlike an arrested ordering transition, or precursor critical fluctuations

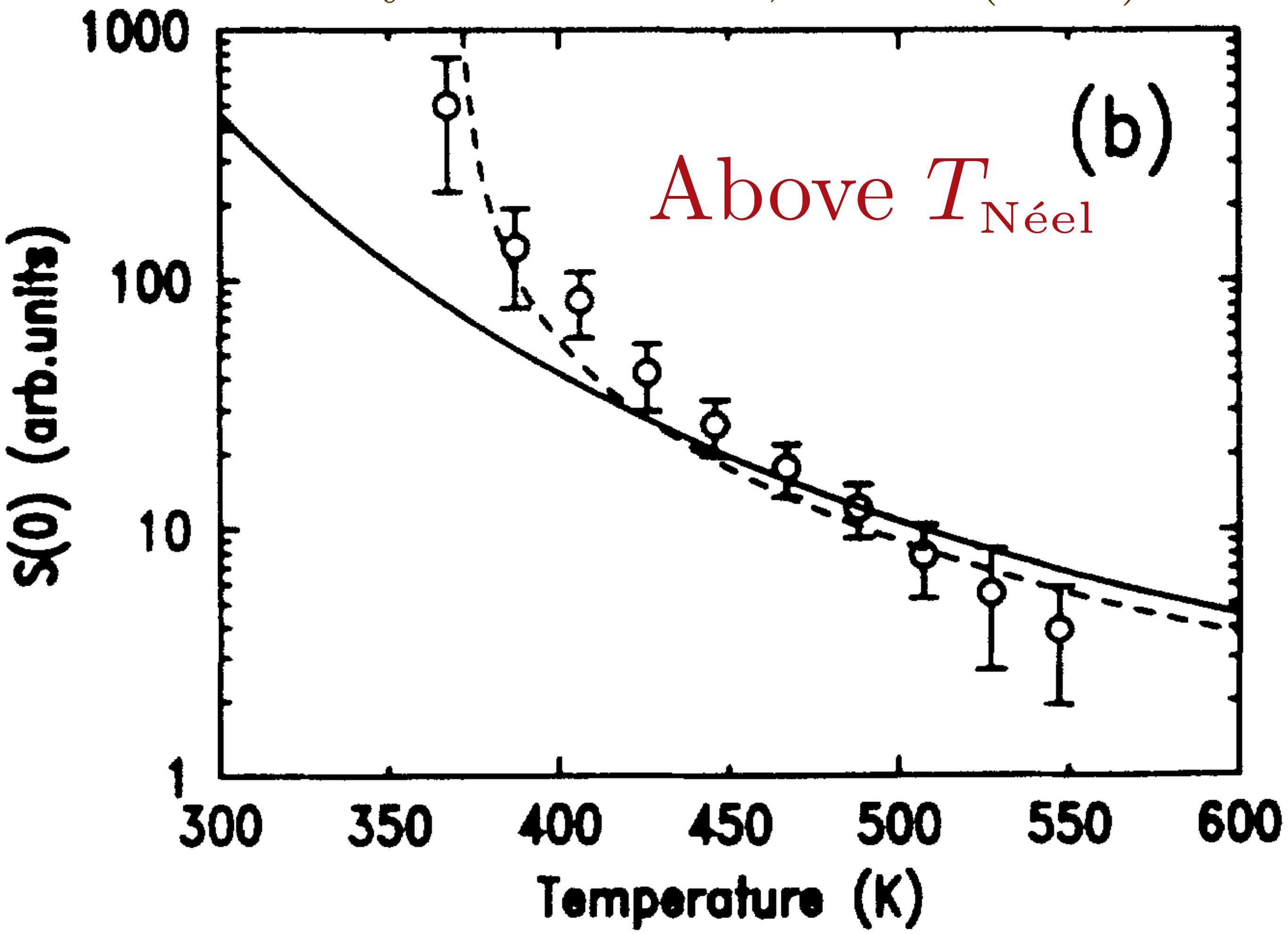
Key idea: analogy with the onset of antiferromagnetism in the *insulator* La_2CuO_4

Gradual onset of intensity over a wide range of T is a consequence of *angular thermal* fluctuations of an order parameter with 3 or more components in 2 spatial dimensions

Polyakov, 1975
Chakravarty, Halperin, Nelson 1989

$$T_{\text{Néel}} = 325\text{K}$$

B. Keimer *et al.*,
Phys. Rev. B **46**, 14034 (1992).



Multi-component order parameter for the pseudogap

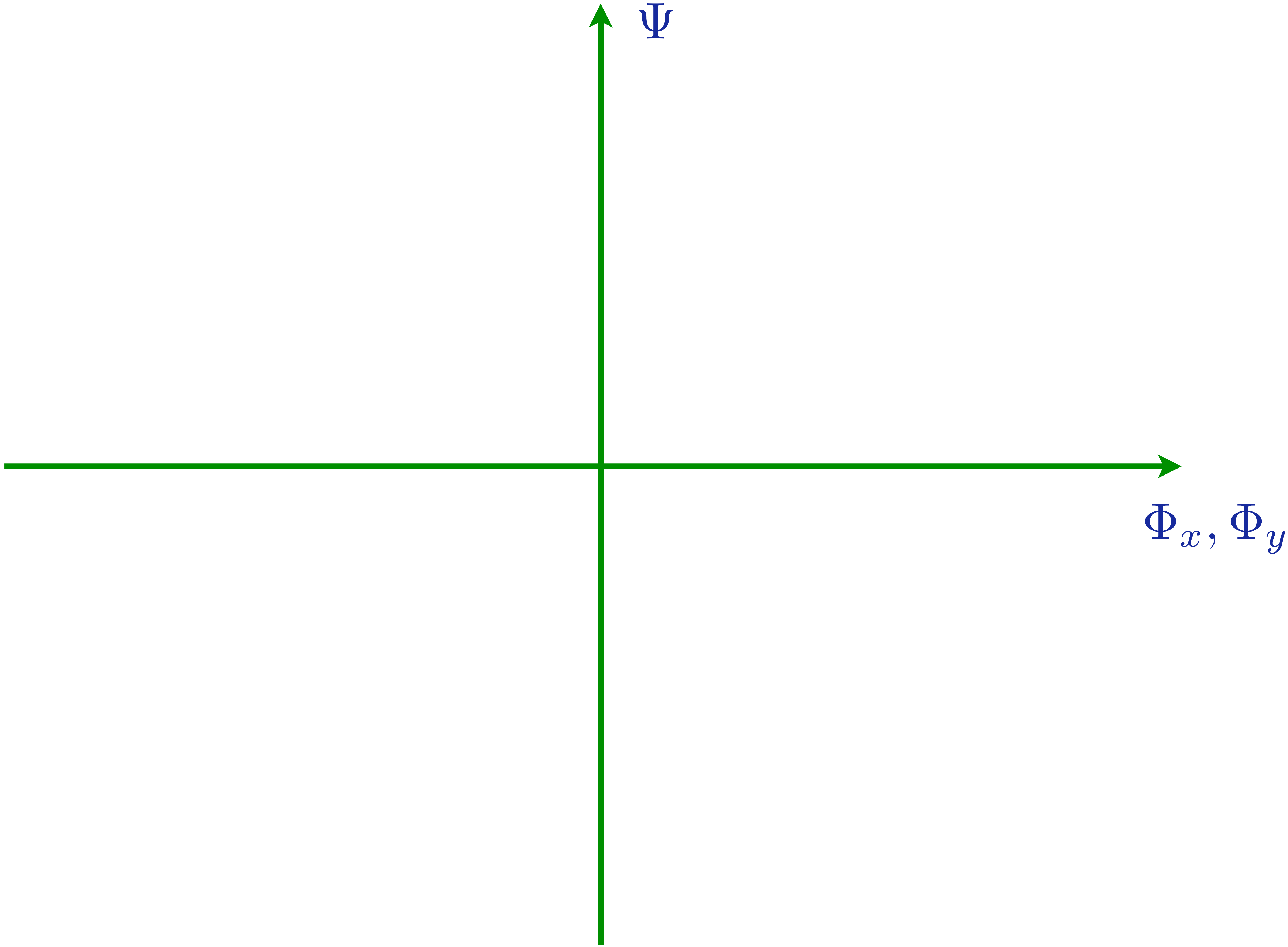
Superconducting order $\Psi(\mathbf{r})$:

$$\left\langle c_{i\alpha}^\dagger c_{j\beta}^\dagger \right\rangle = \varepsilon_{\alpha\beta} \left[\sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

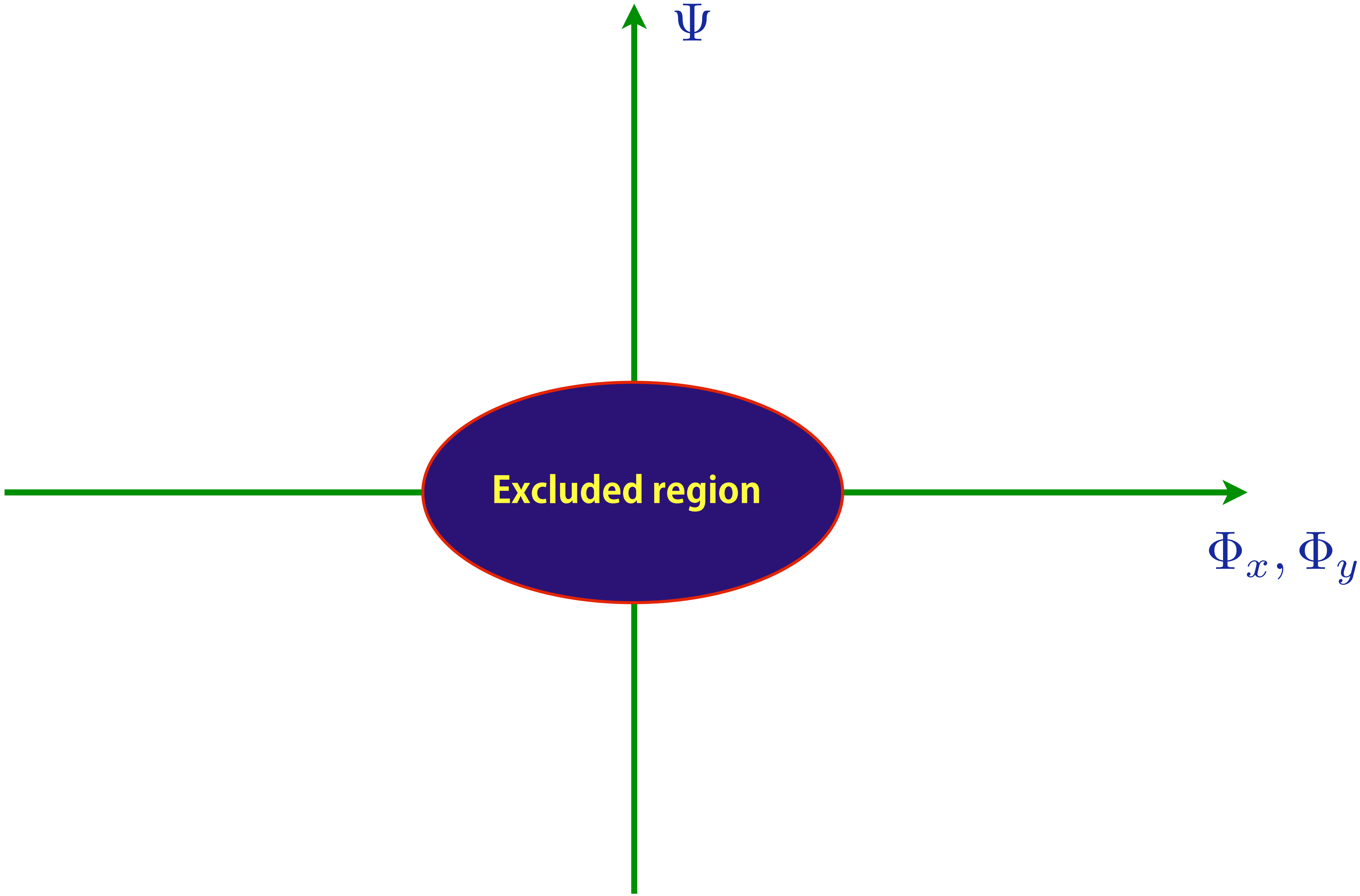
Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\begin{aligned}\left\langle c_{i\alpha}^\dagger c_{j\beta} \right\rangle = {} & \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_x}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right) \\ & + \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_y}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_y\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)\end{aligned}$$

Multi-component order parameter



Multi-component order parameter



Support from theory of antiferromagnetic quantum criticality

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

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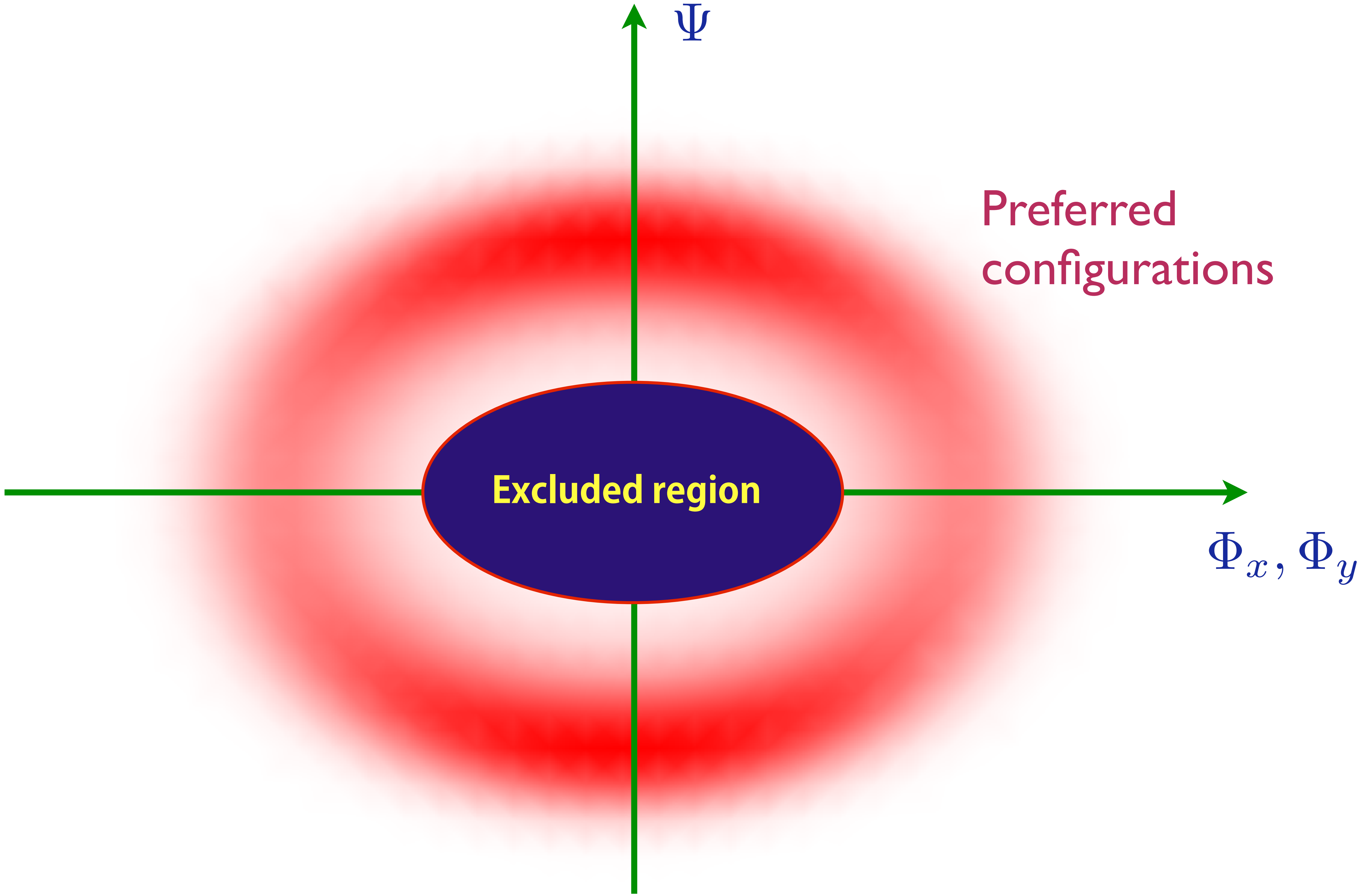

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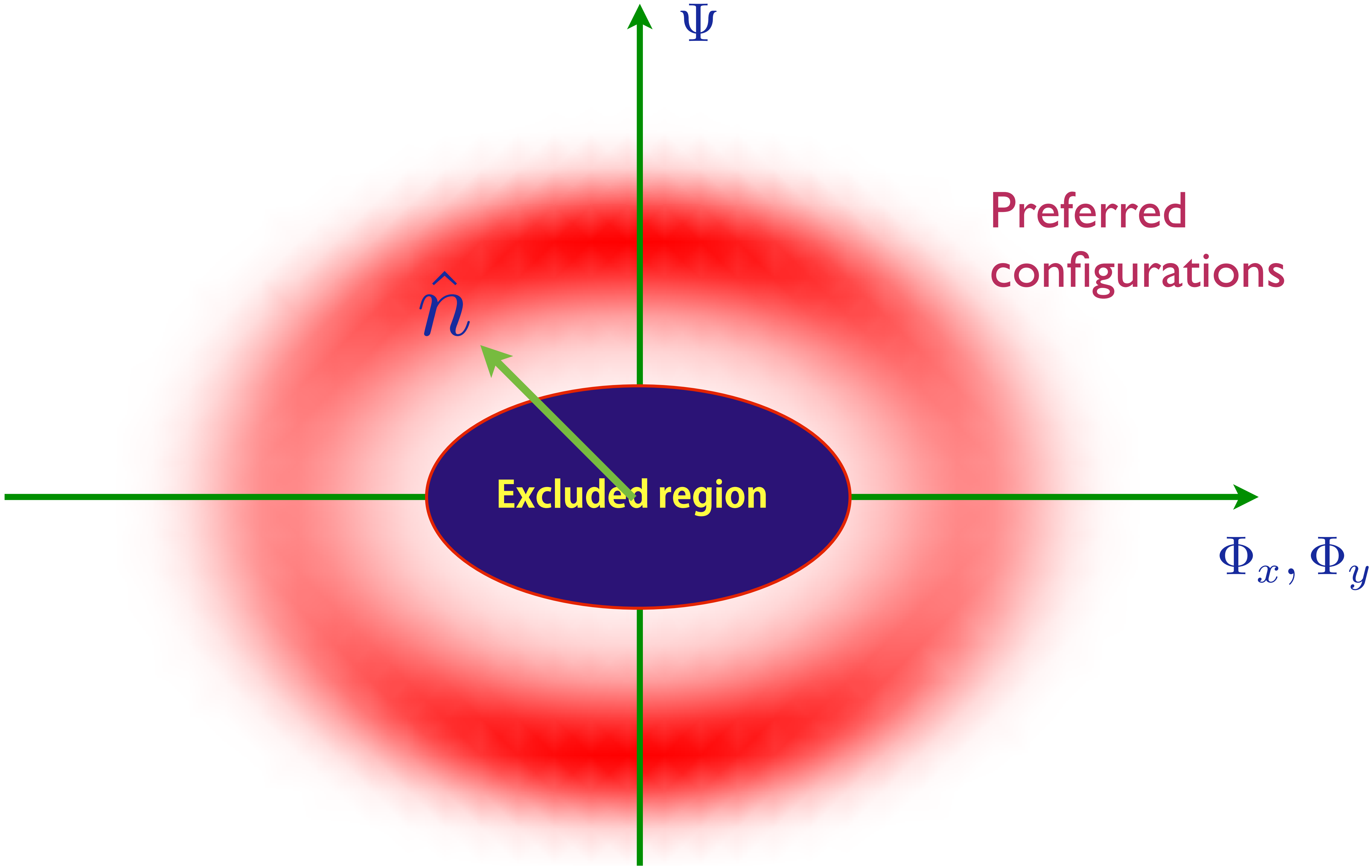


Label order parameter by a
6-component unit vector n_α with $\sum_\alpha n_\alpha^2 = 1$

O(6) non-linear sigma model

$$\mathcal{Z} = \int \mathcal{D}n_\alpha(\mathbf{r})\, \delta\left(\sum_{\alpha=1}^{6} n_\alpha^2(\mathbf{r}) - 1\right) \exp\Bigg(-\frac{\rho_s}{2T}\int d^2r \Bigg[\sum_{\alpha=1}^{2}(\nabla n_\alpha)^2 + \lambda \sum_{\alpha=3}^{6}(\nabla n_\alpha)^2 + g\sum_{\alpha=3}^{6} n_\alpha^2 + w\left[\left(n_3^2+n_4^2\right)^2 + \left(n_5^2+n_6^2\right)^2\right]\Bigg]\Bigg).$$

where $\Psi \propto n_1 + in_2$, $\Phi_x \propto n_3 + in_4$, $\Phi_y \propto n_5 + in_6$.

Describes $O(6) \Rightarrow O(2)\times O(2)\times O(2)\times \mathbb{Z}_2$. The coupling g determines the anisotropy between superconductivity and charge order.

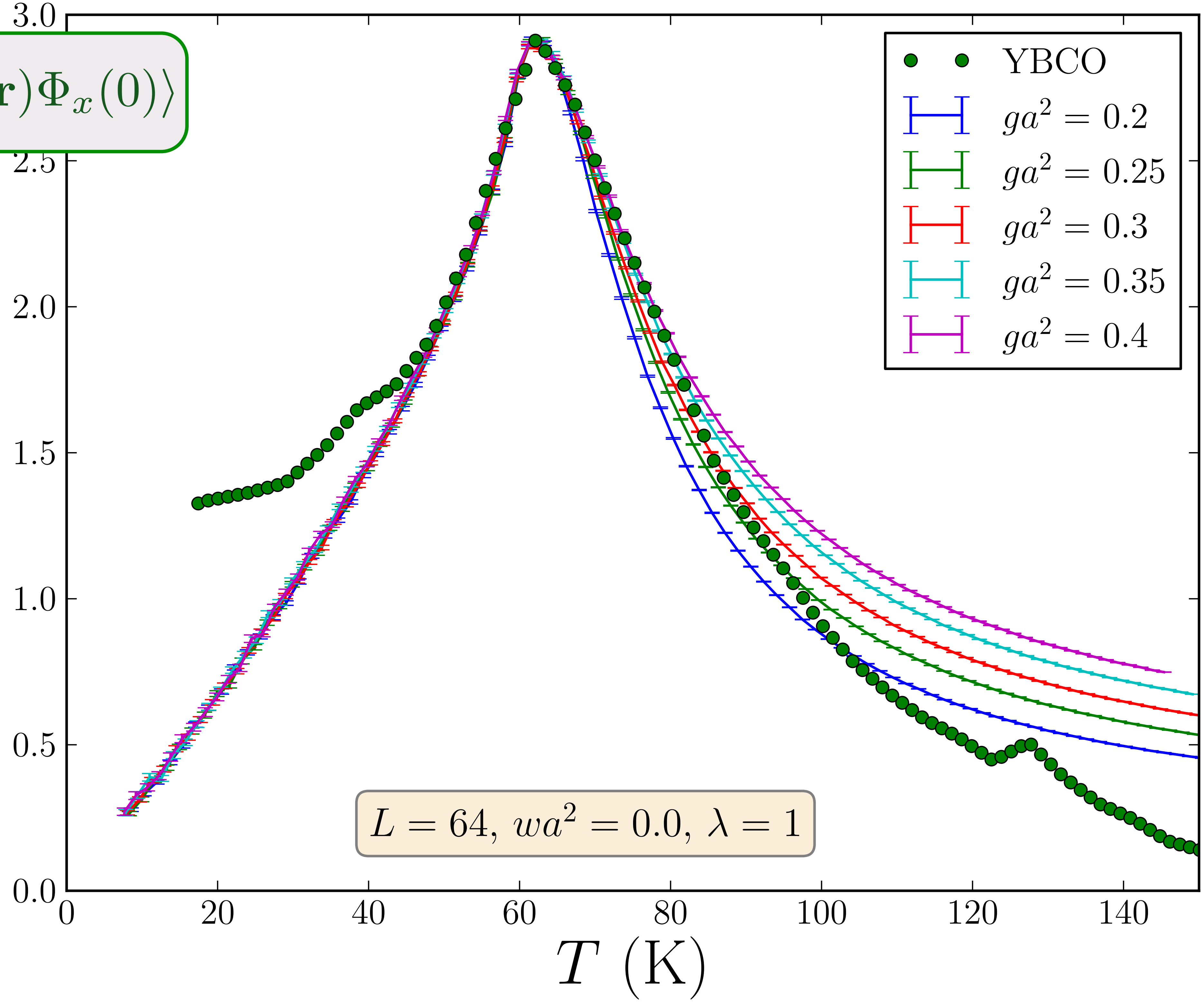
Solve by cluster Monte Carlo and $1/N$ expansion.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

Comparison of Monte Carlo with experiments

$$S_{\Phi_x} = \int d^2 r \, \langle \Phi_x(\mathbf{r}) \Phi_x(0) \rangle$$

Charge order structure factor S_{Φ_x}



For $ga^2 = 0.30$ and $wa^2 = 0.0$ we have $\rho_s = 160$K.
The height was also rescaled to make the peak heights match.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

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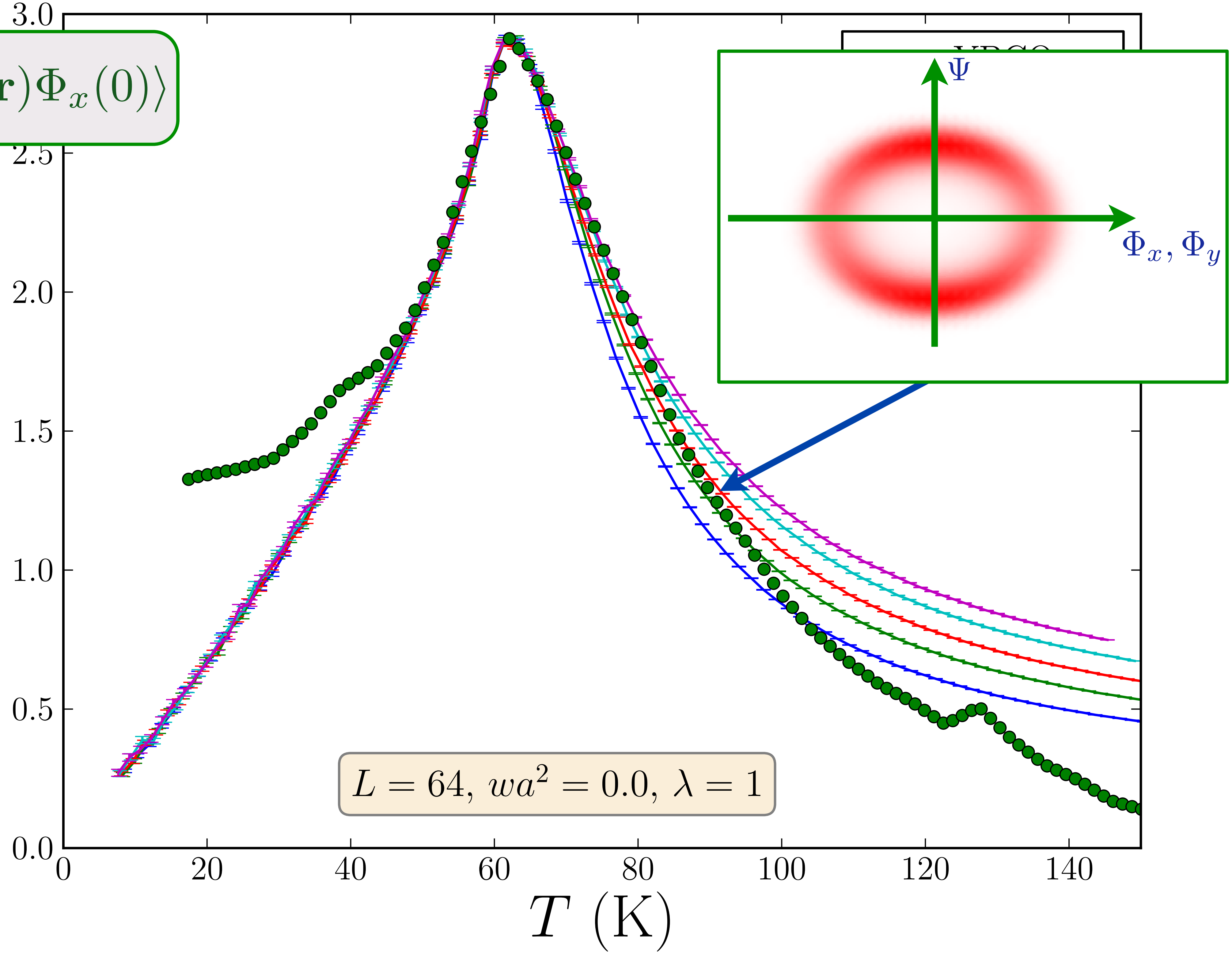

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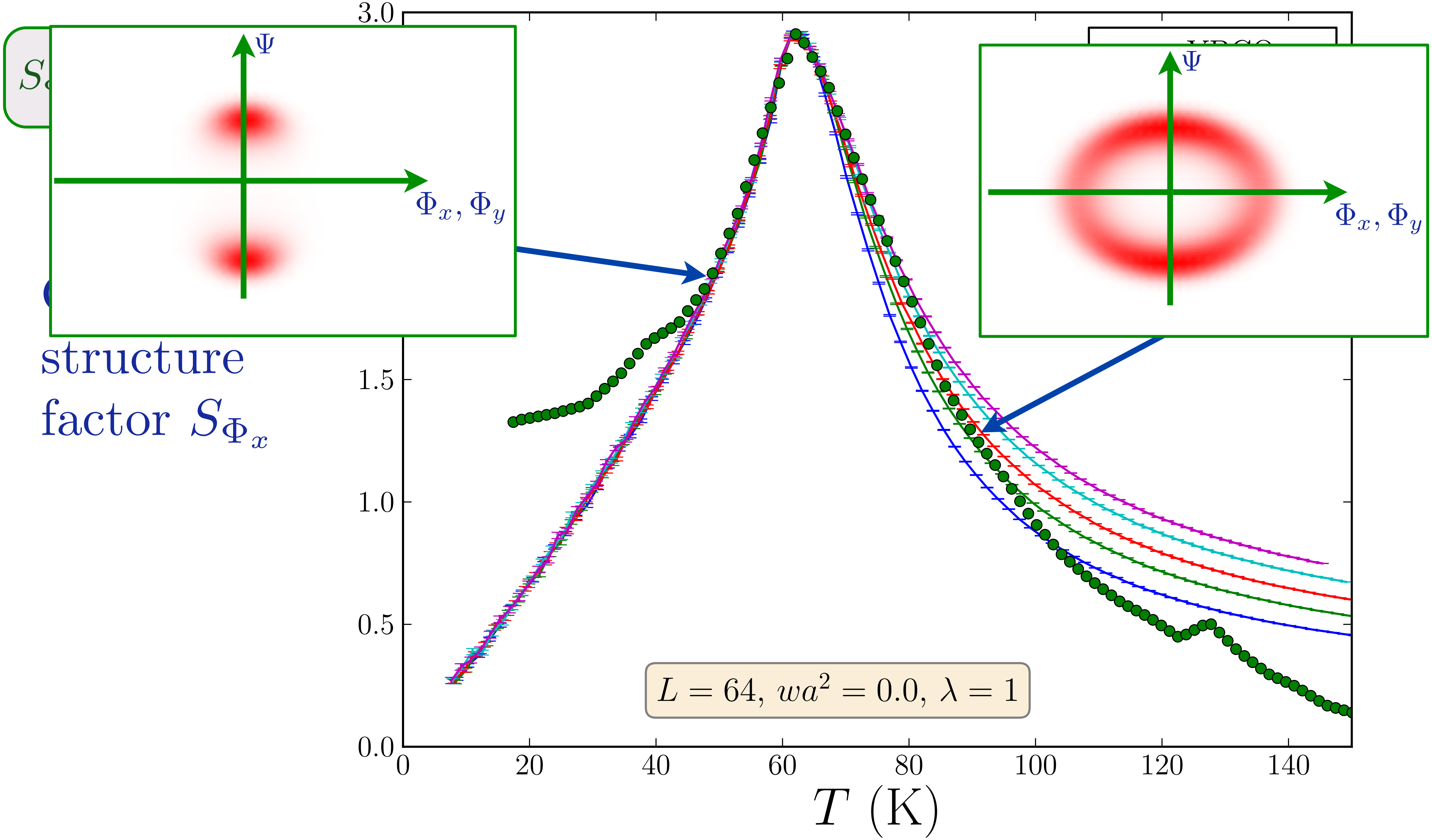

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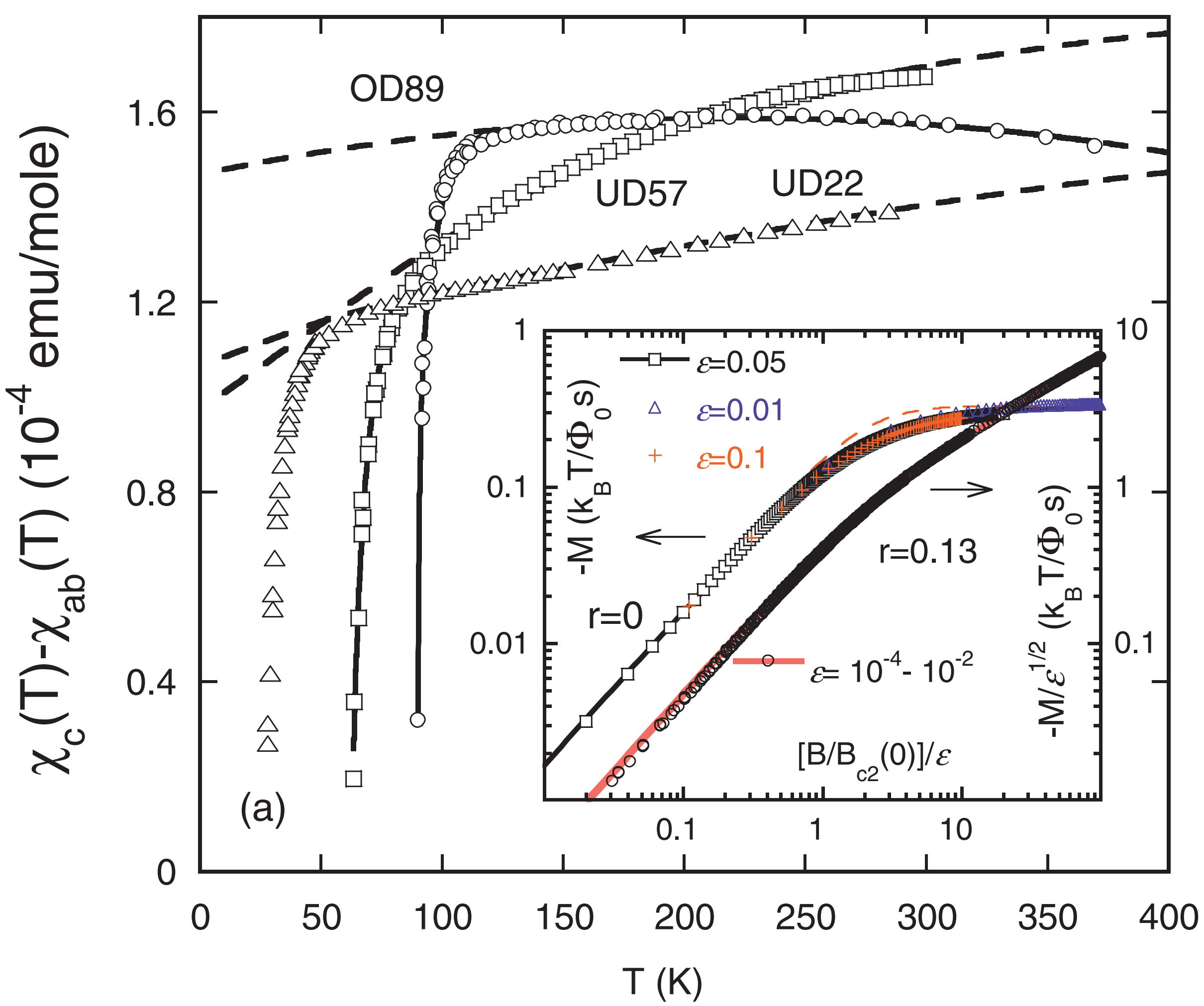
Other experiments in the pseudogap

- The *same* set of parameters used to describe X-ray scattering, also predict the strength of superconducting fluctuations above T_c. Indeed $YBa_2Cu_3O_{6+x}$ shows significant fluctuation diamagnetism over the same range of temperatures.



PHYSICAL REVIEW B **88**, 060505(R) (2013)

I. Kokanović,[1,2,*] D. J. Hills,[1] M. L. Sutherland,[1] R. Liang,[3] and J. R. Cooper[1]

Diamagnetism of $YBa_2Cu_3O_{6+x}$ crystals above T_c: Evidence for Gaussian fluctuations

- Strongly-coupled quantum criticality leads to a novel regime of quantum dynamics without quasiparticles.

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- Antiferromagnetic interactions induce *d*-wave superconductivity in metals. This has now been established in the strong-coupling region near an antiferromagnetic critical point

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- Antiferromagnetic interactions induce *d*-wave superconductivity in metals. This has now been established in the strong-coupling region near an antiferromagnetic critical point

- The "pseudogap" phase is described by the angular fluctuations of a 6-component vector representing the superconducting and bond orders