Quantum entanglement at all distances

PRL का अमृत व्याख्यान

Physical Research Laboratory, Ahmedabad

March 30, 2022

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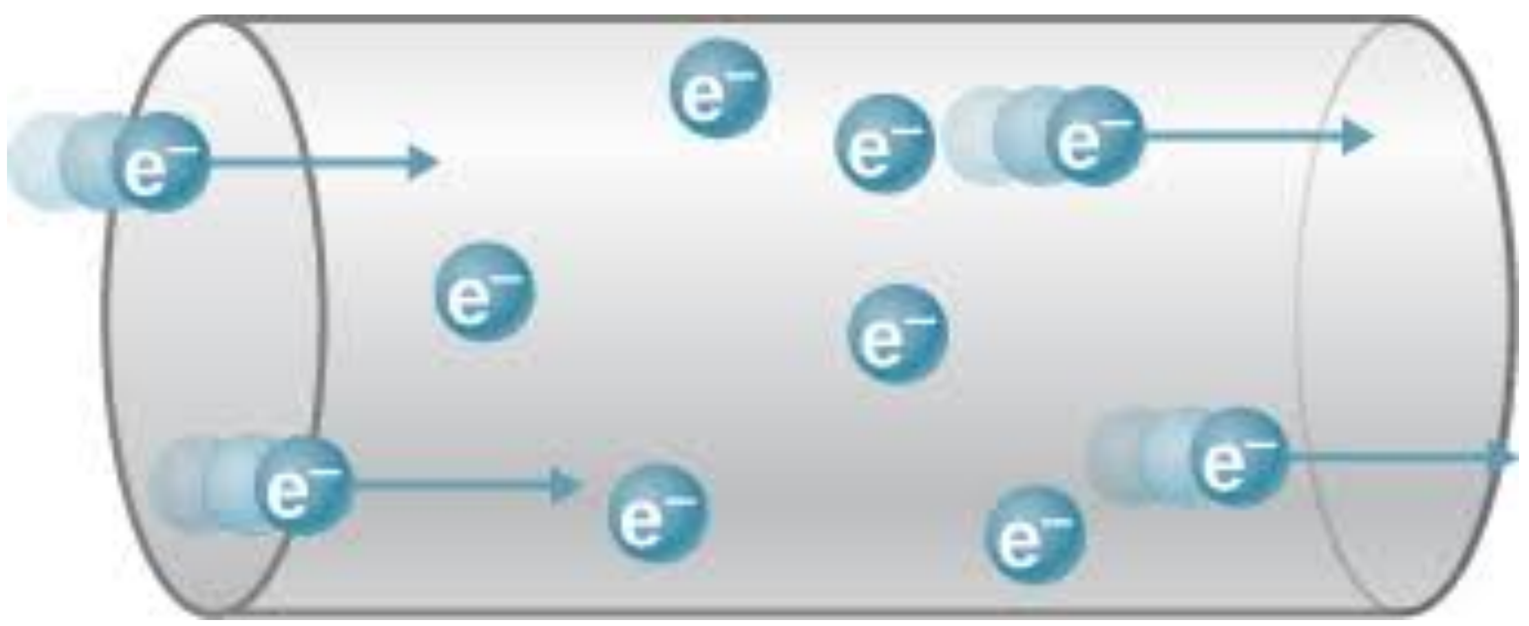
Talk online: sachdev.physics.harvard.edu

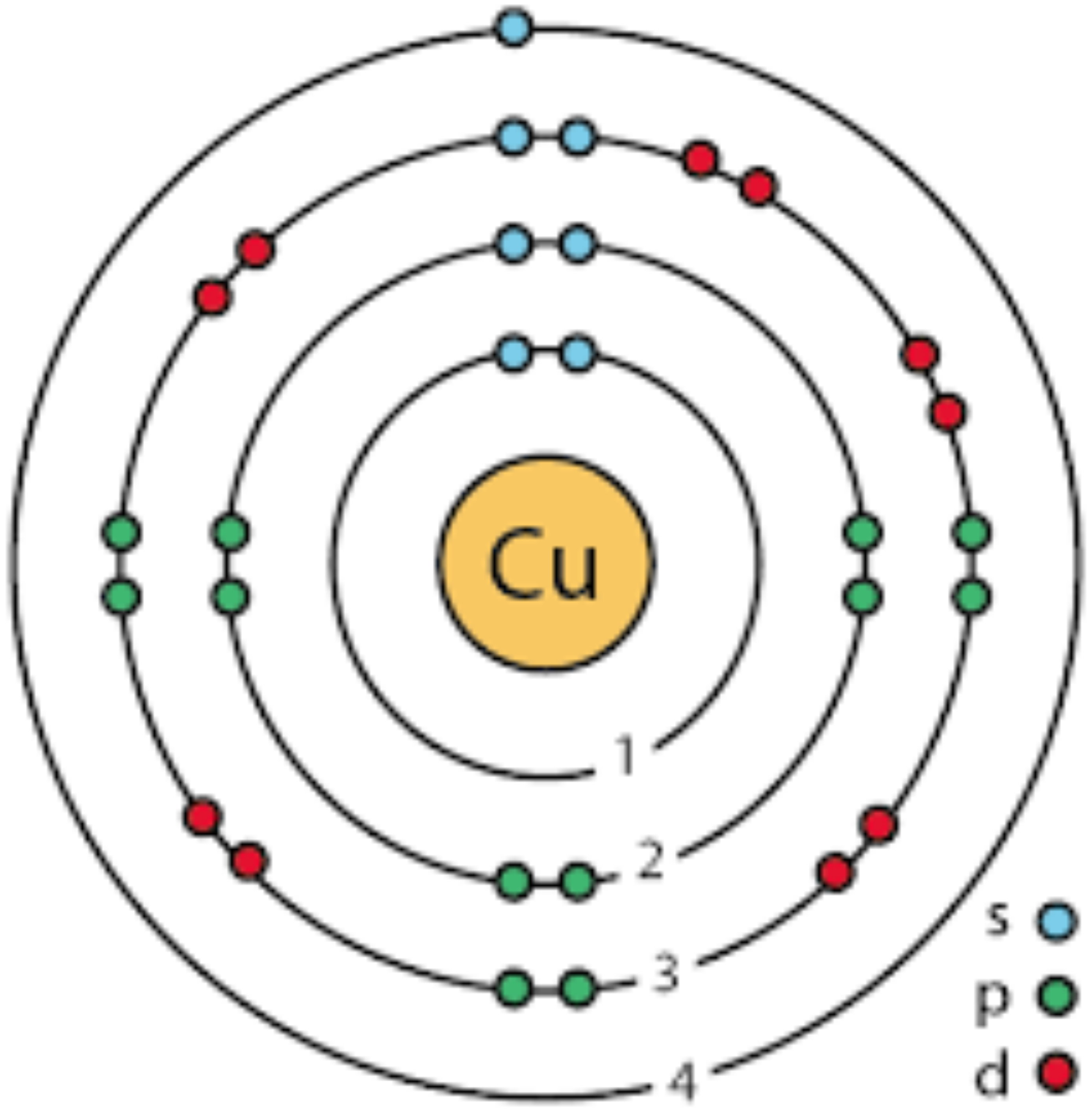
INSTITUTE FOR ADVANCED STUDY

PHYSICS HARVARD

Quantum theory of electrons: ordinary metals and strange metals

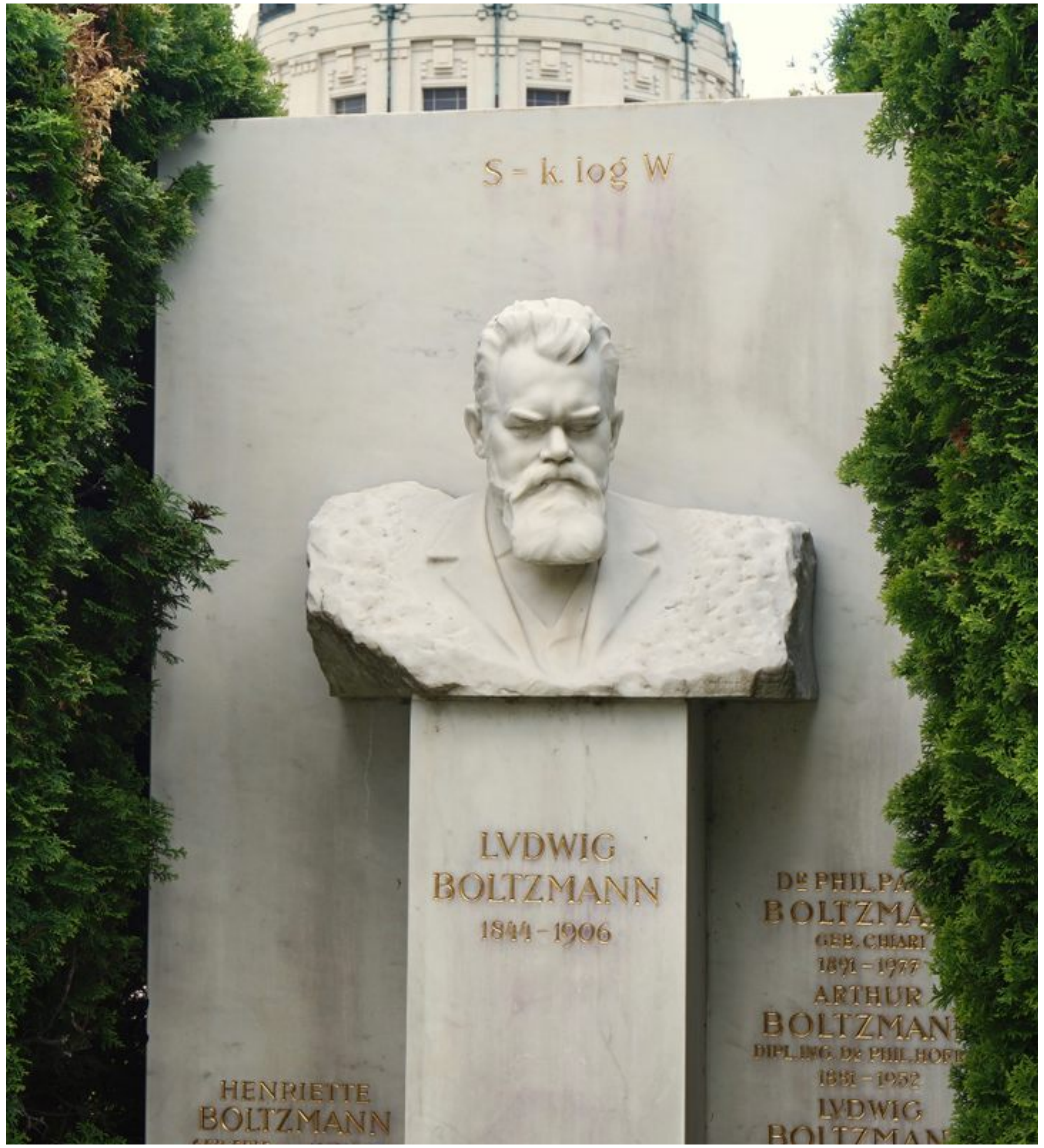
Copper





Each copper atom donates its outermost electron
These electrons move freely throughout the crystal and carry current

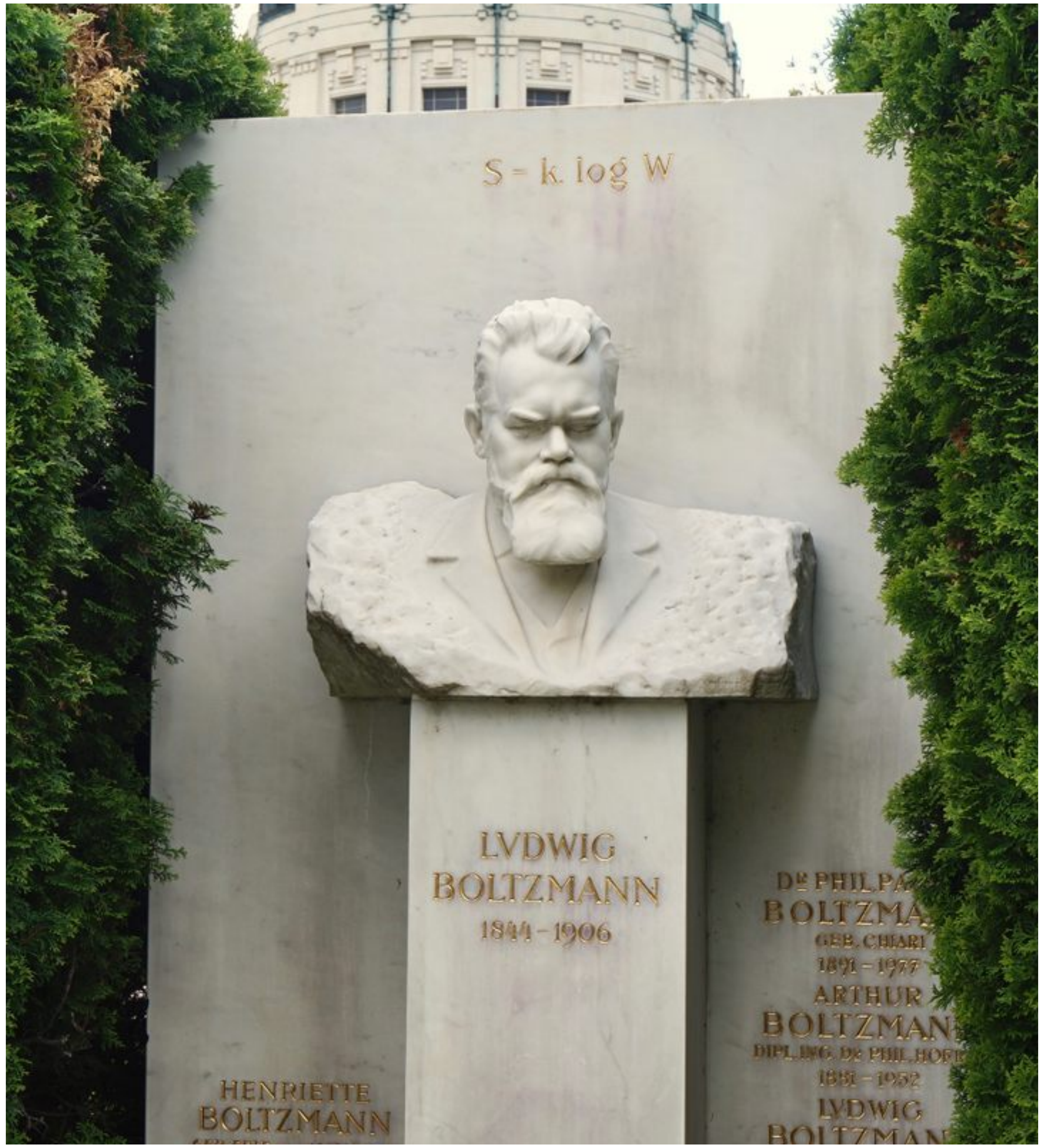
Statistical interpretation of entropy



Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$



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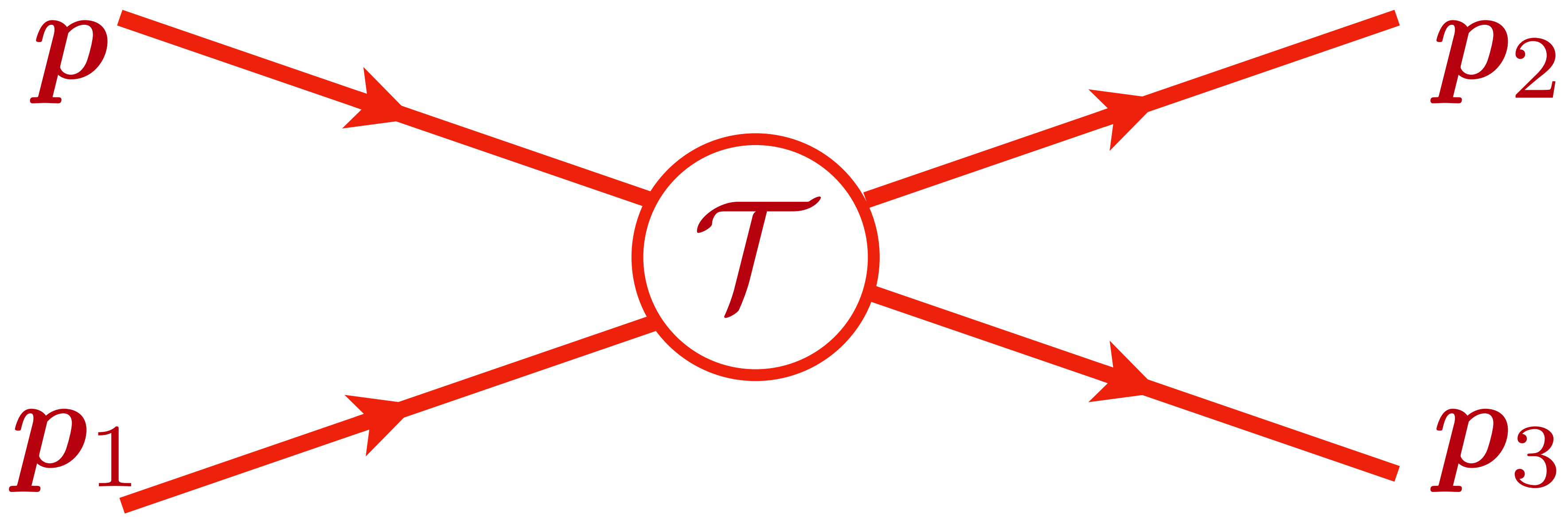
Boltzmann equation (1872)
Dilute classical gas

Molecular chaos: successive collisions are statistically independent

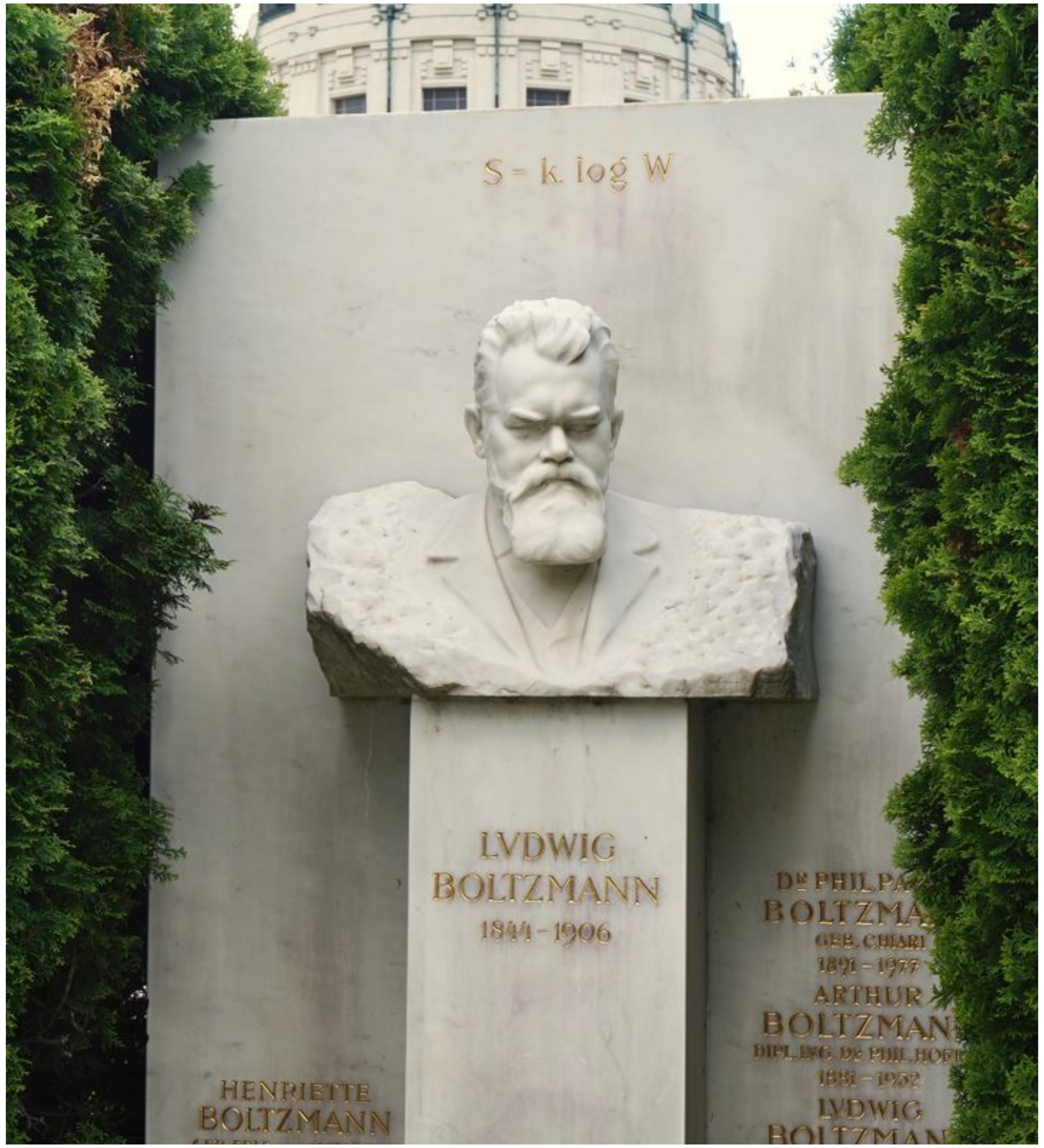
$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$-2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



Quantum Boltzmann equation (Landau)
Dense gas of electrons



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Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
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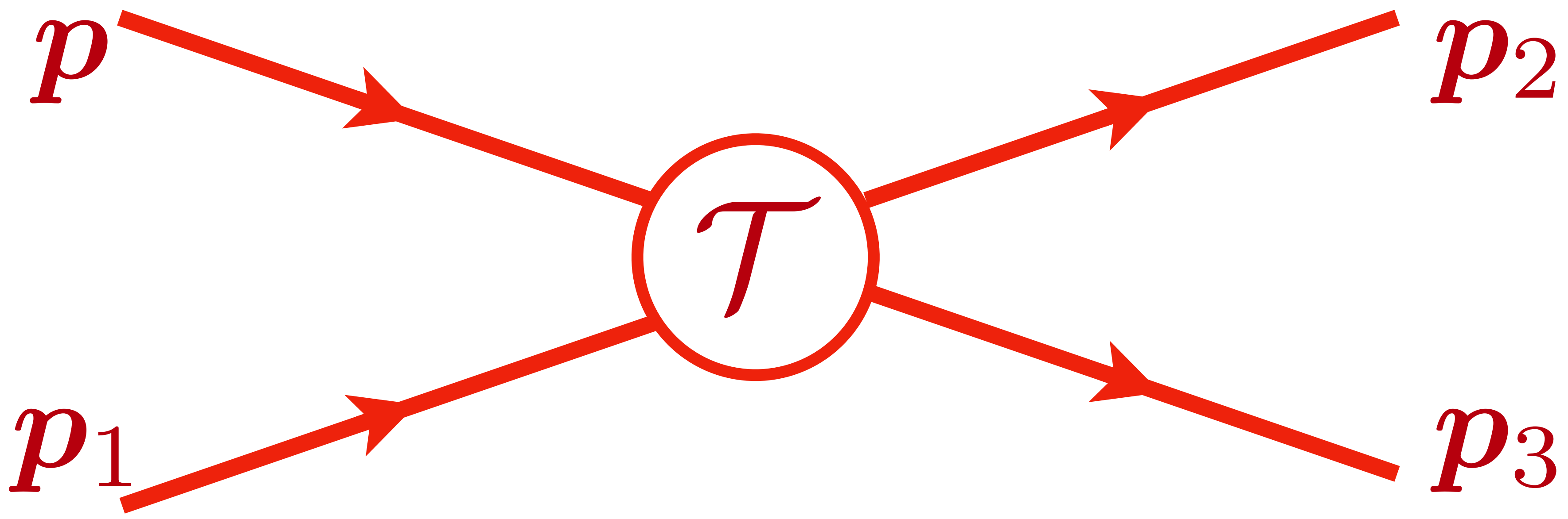
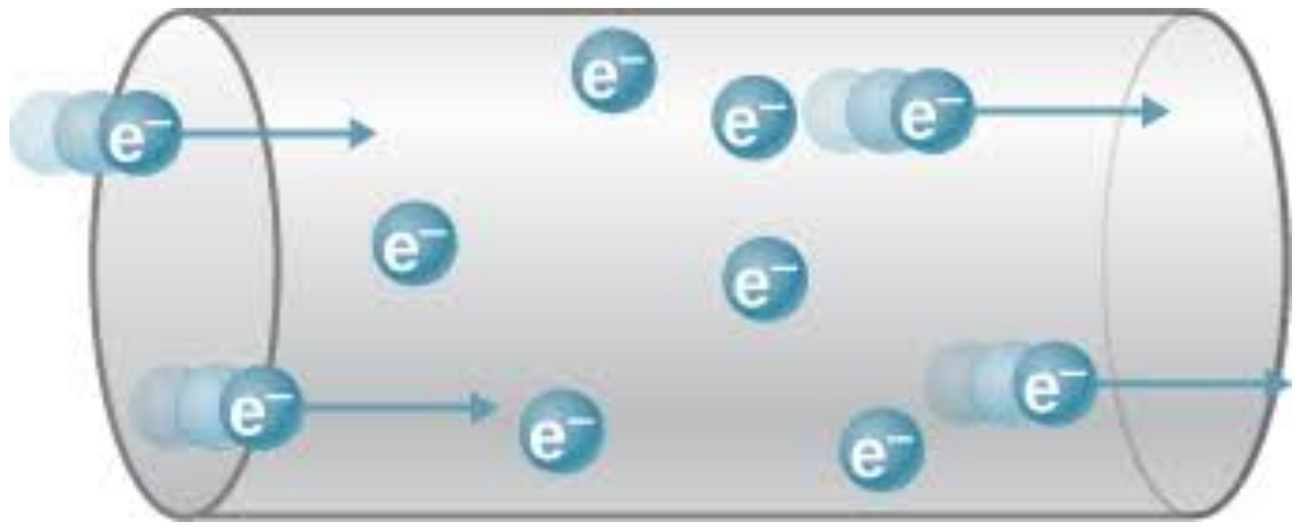
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in Copper

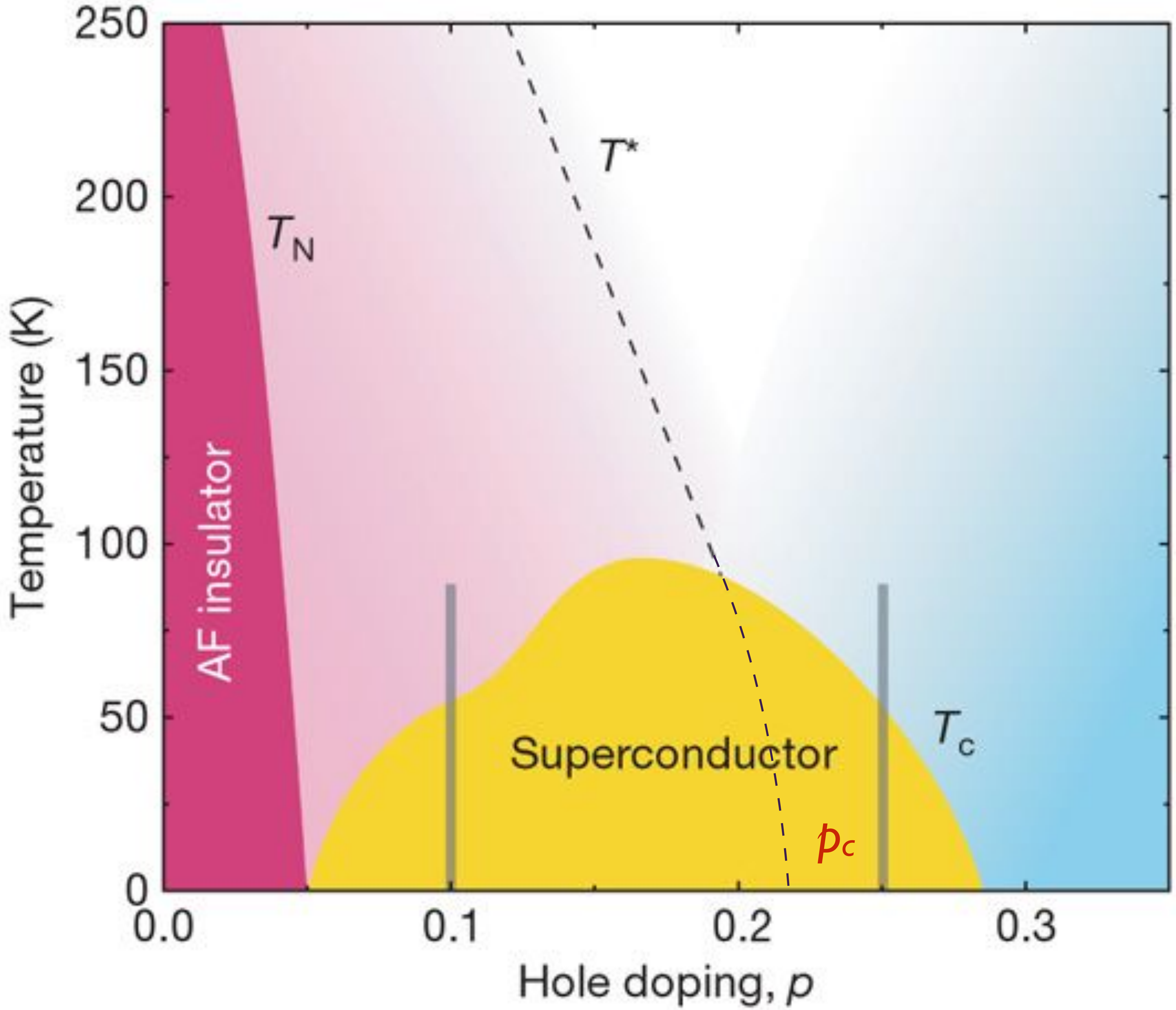


Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

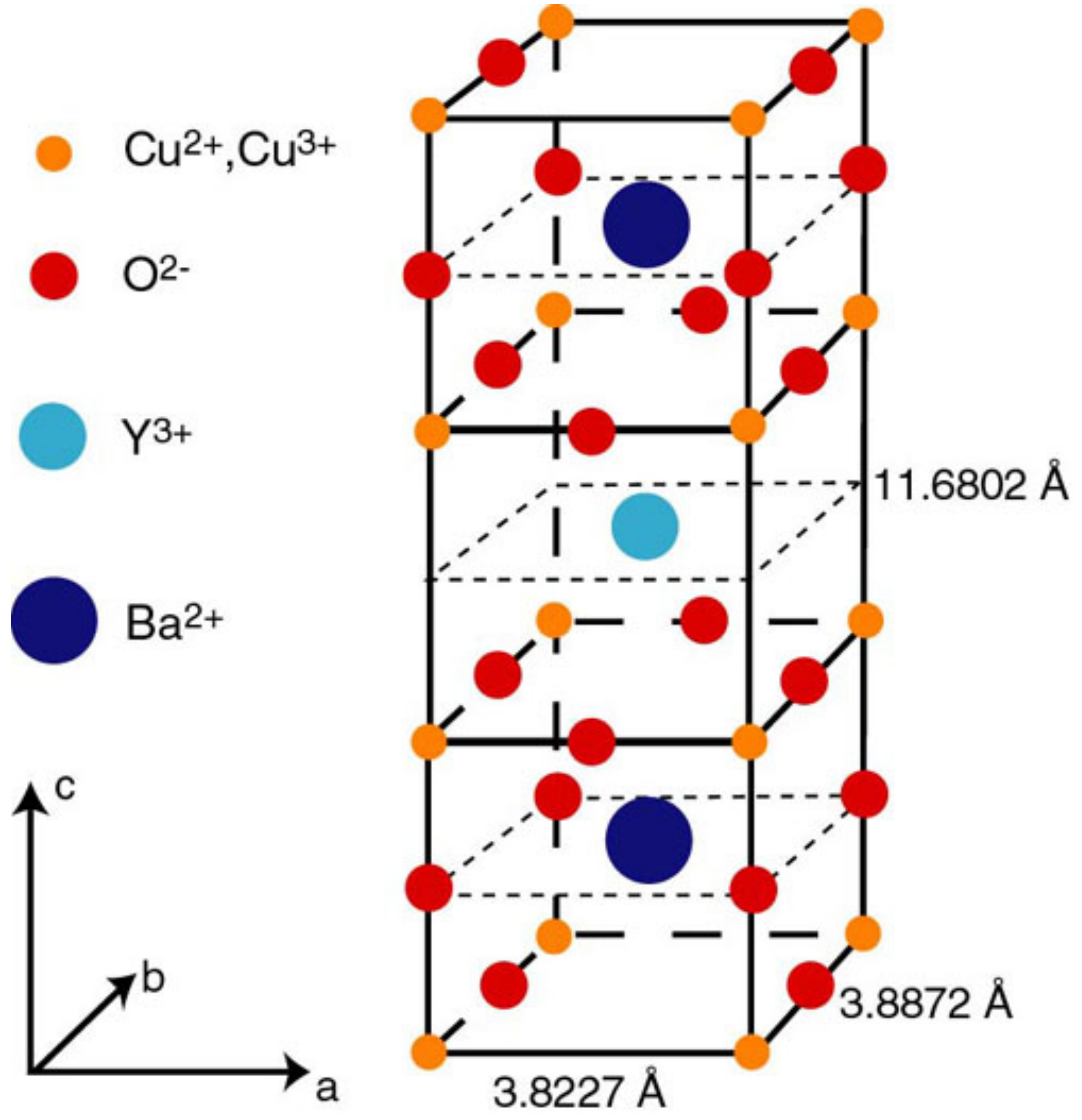
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

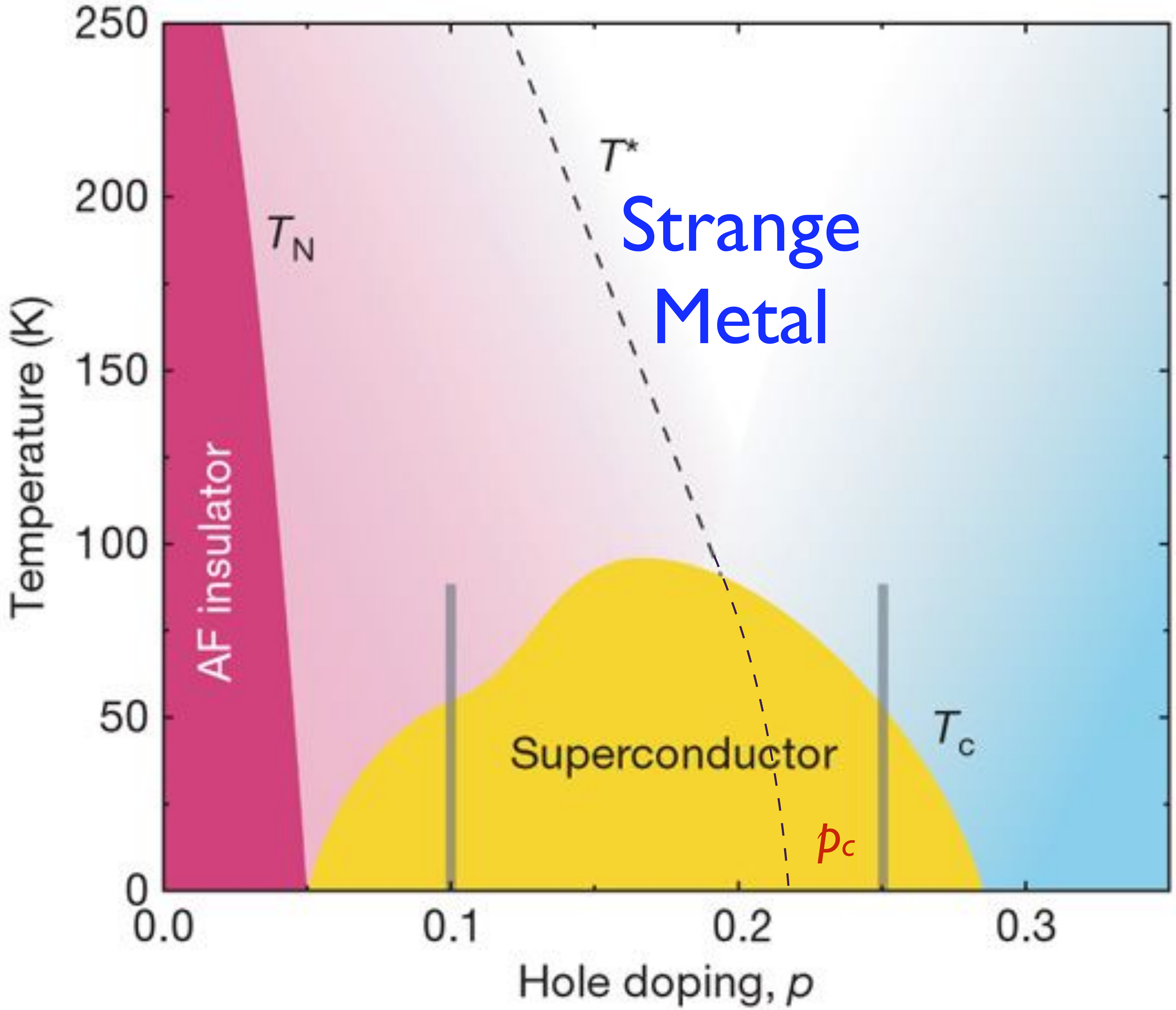
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.



250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å


250
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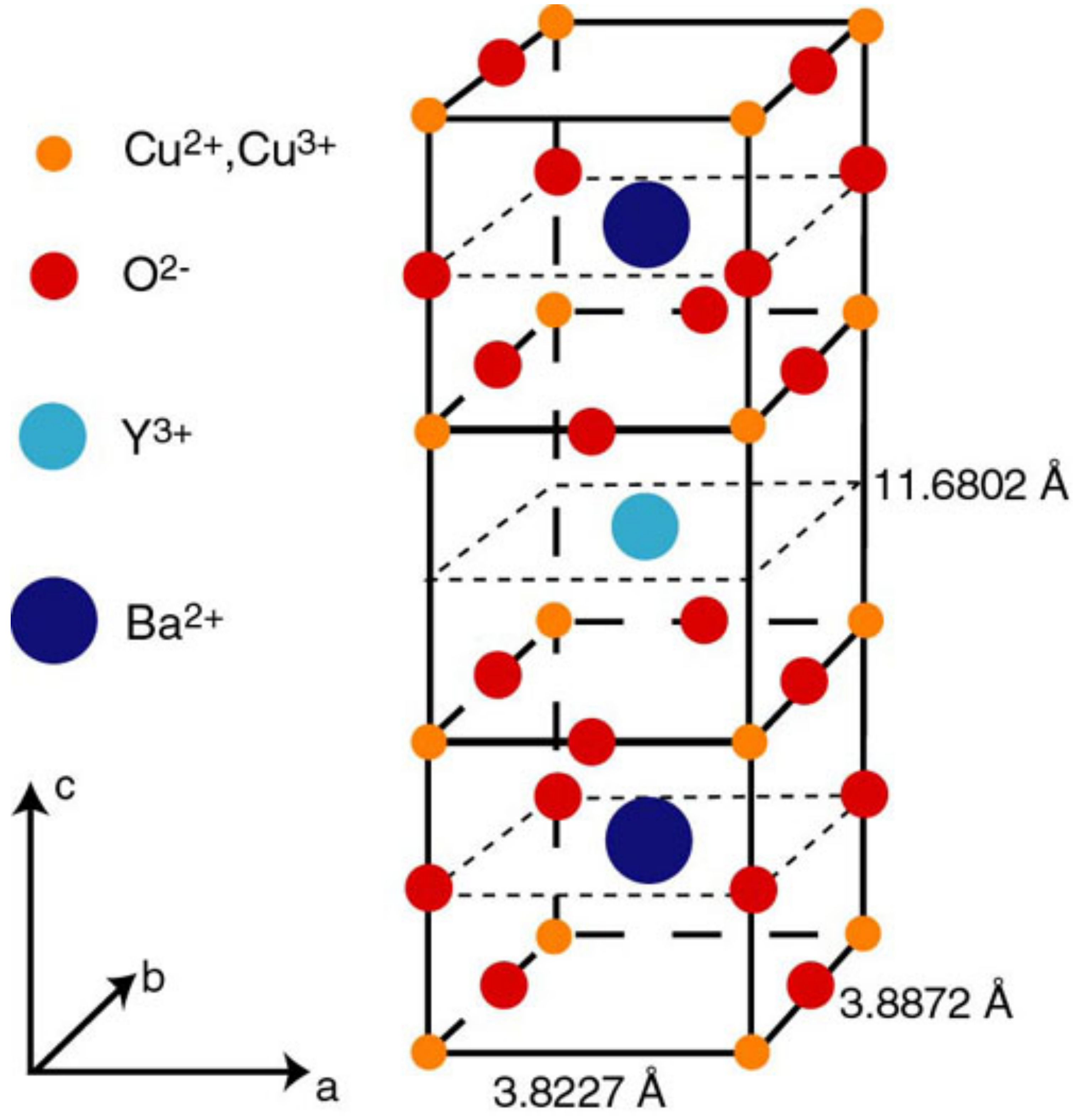
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c
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11.6802 Å
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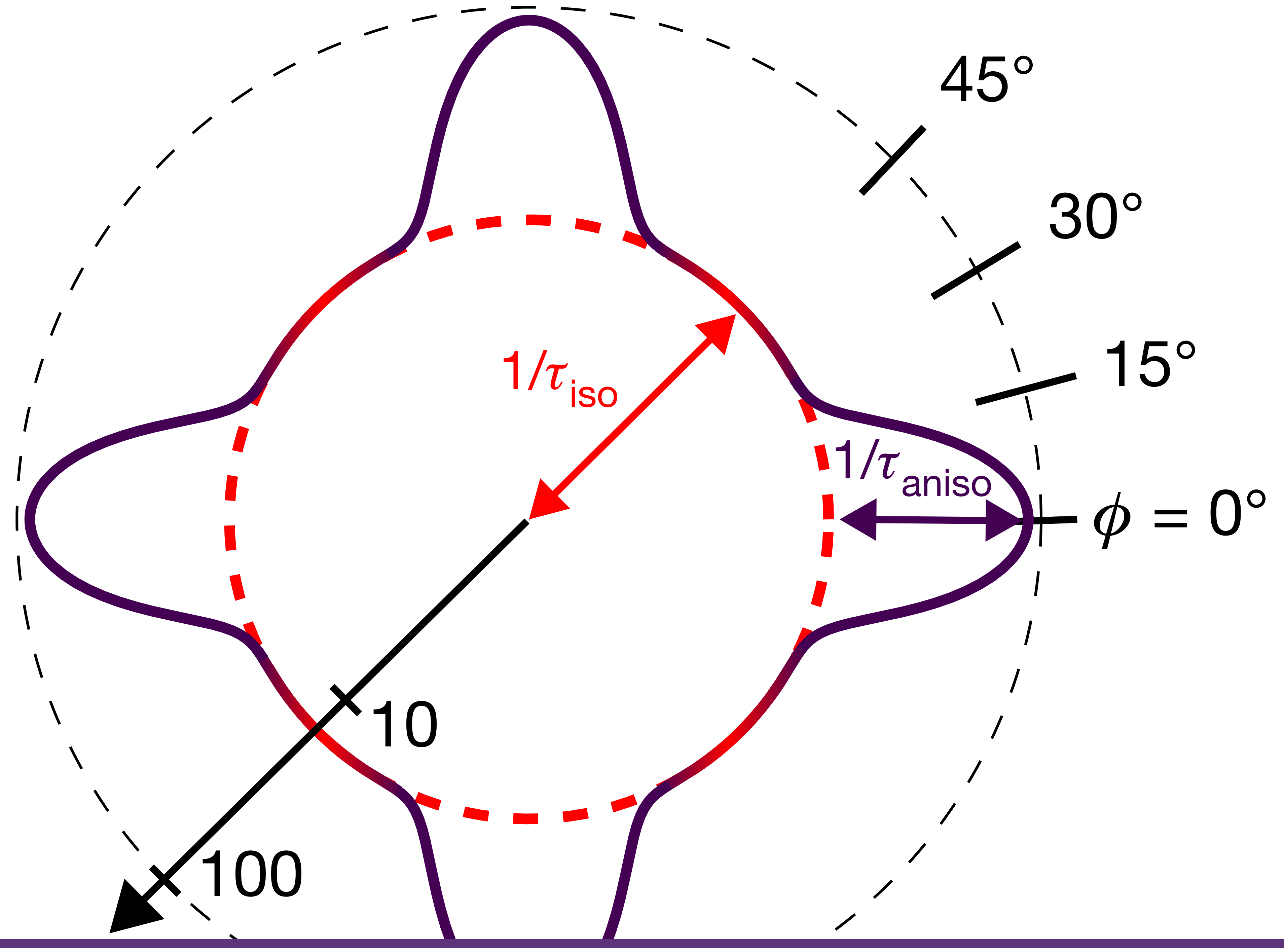
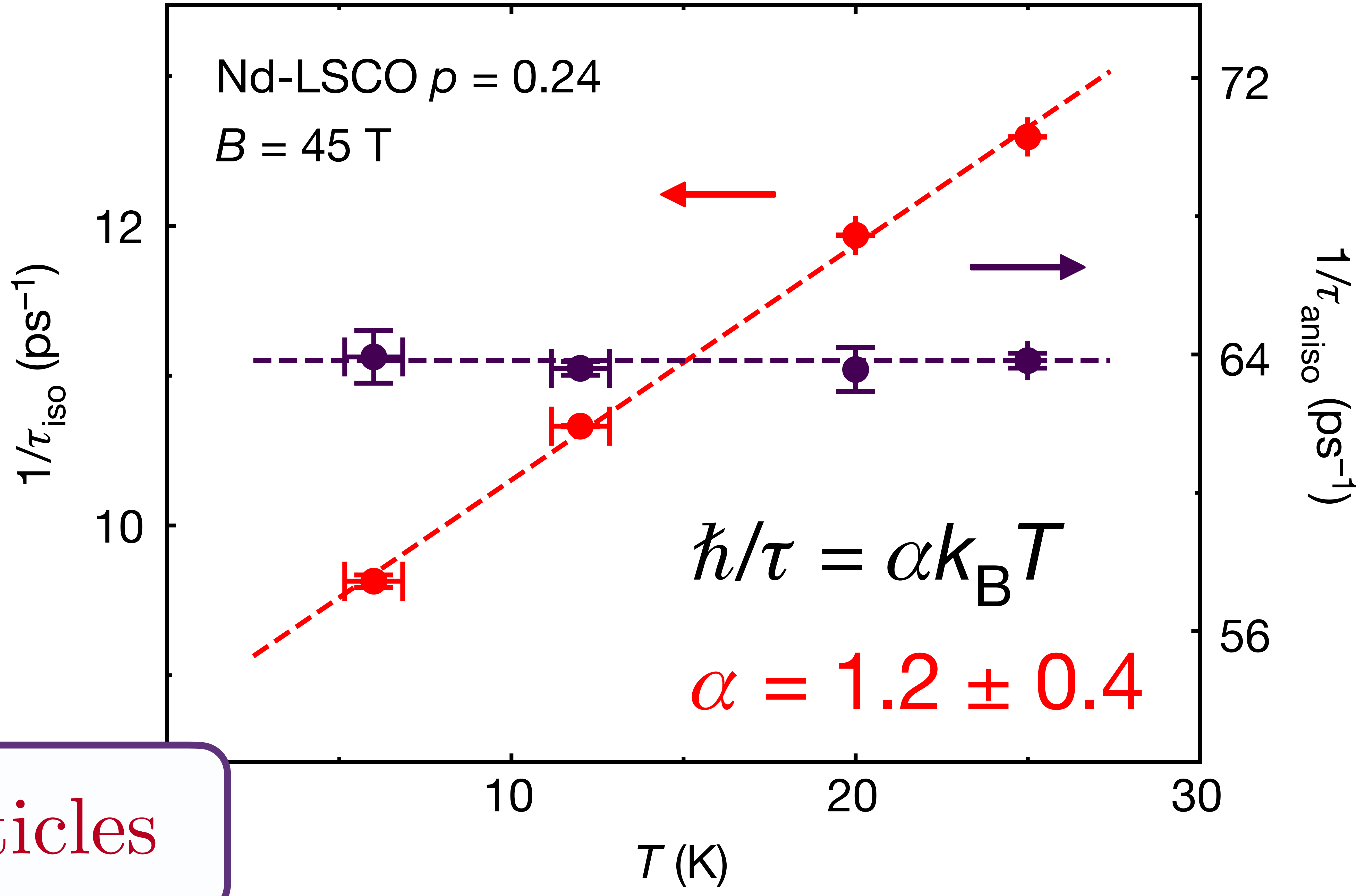
Linear-in temperature resistivity from an isotropic Planckian scattering rate

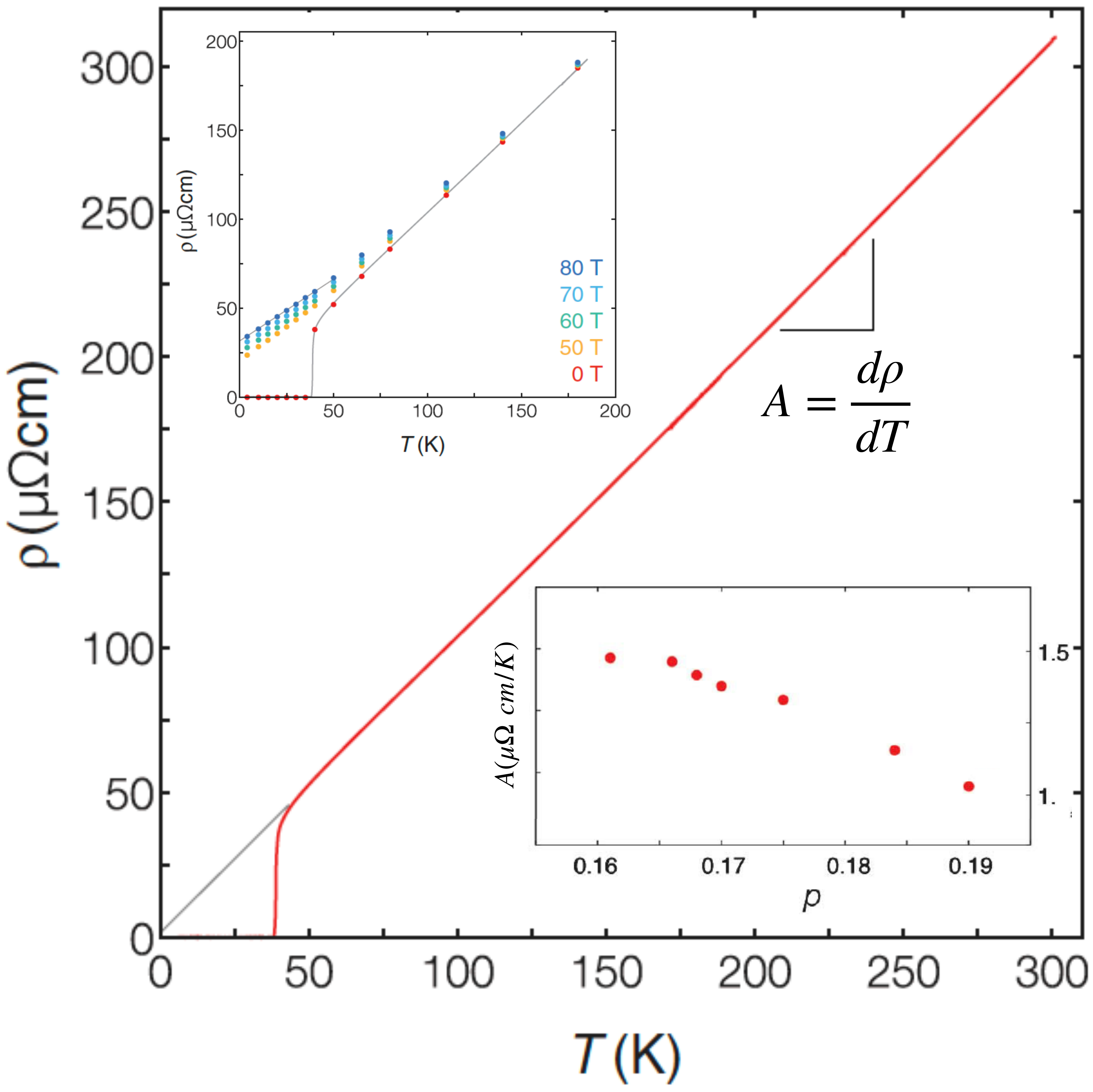
Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw

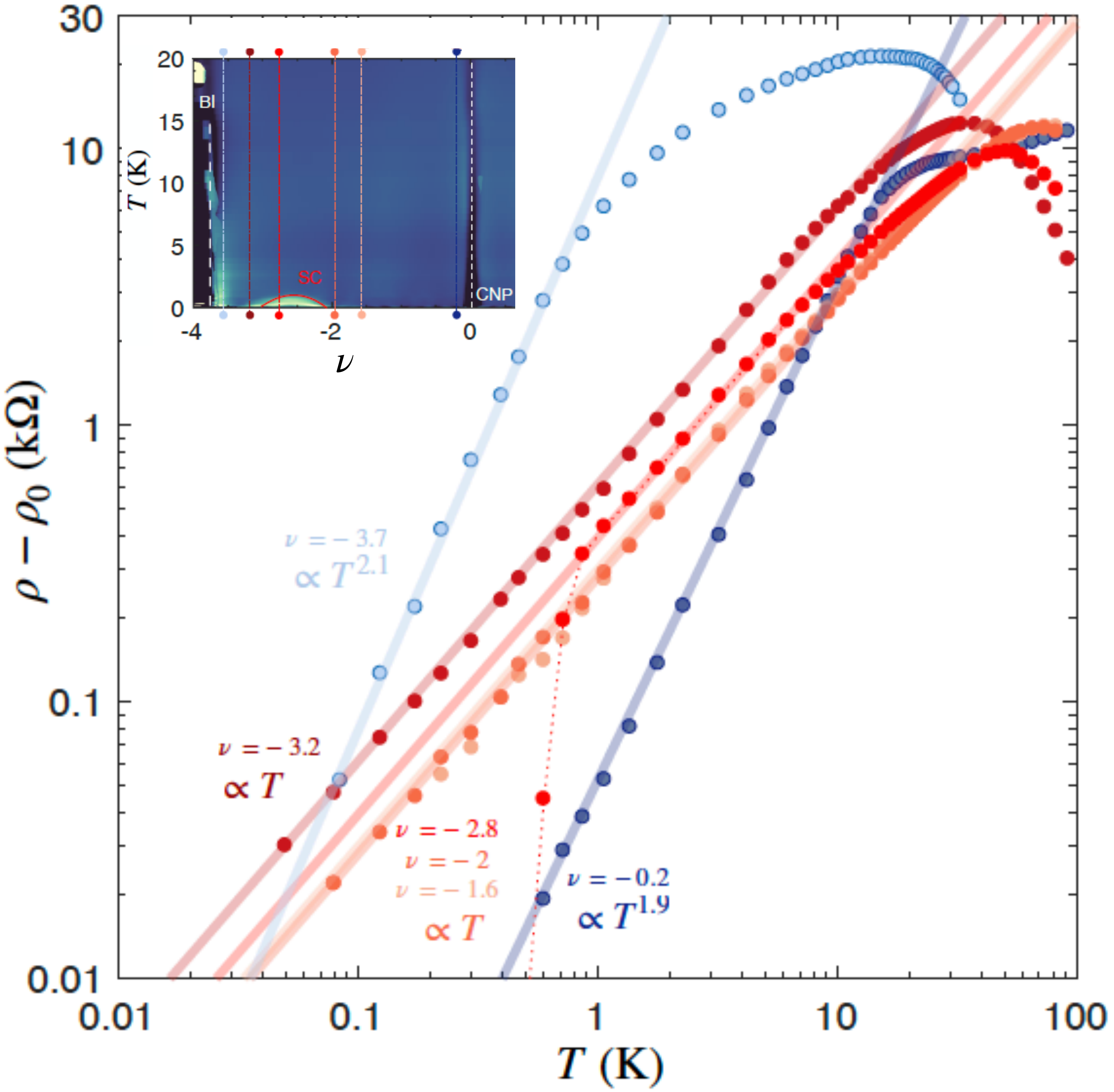




Current flow without quasiparticles



LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Questions

- Needed: A theory for current flow in a 'strange metal' with an entangled soup of electrons.

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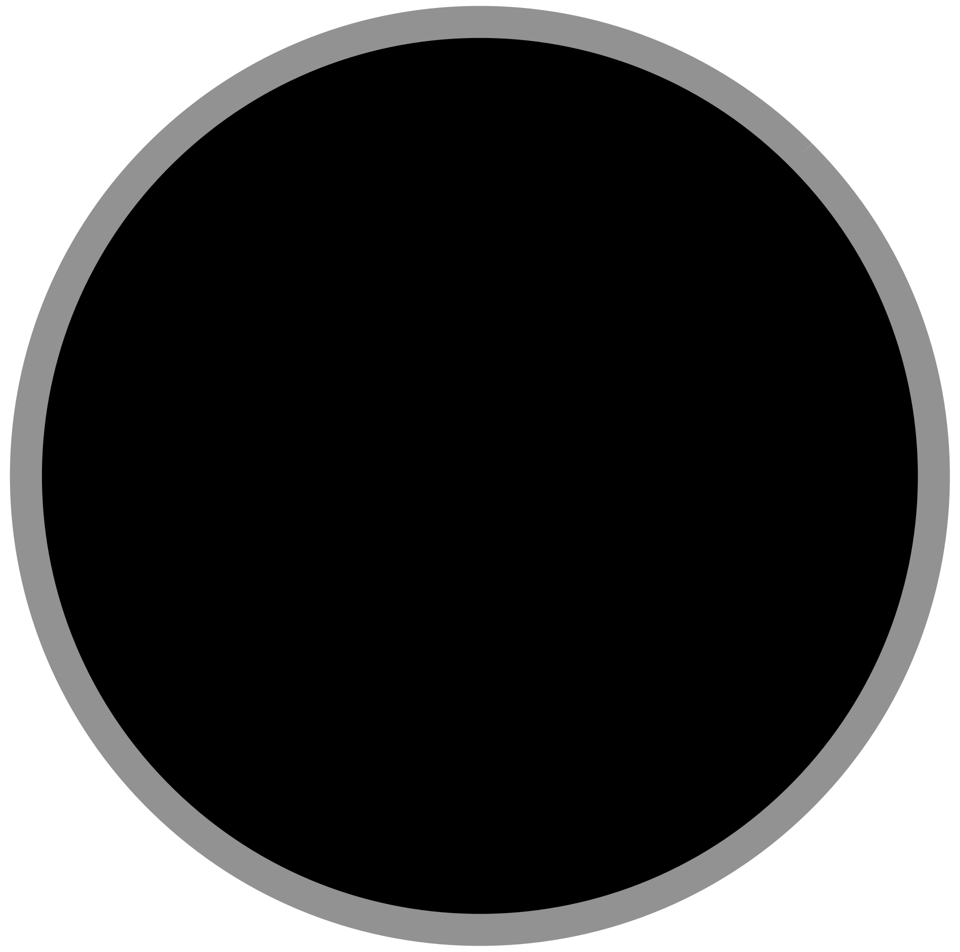
- Needed: theory for collision time in resistivity $\sim \hbar/(k_B T)$.
- Needed: theory for the appearance of superconductivity in such a ‘strange metal’.

Complex
quantum
entanglement
in
black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

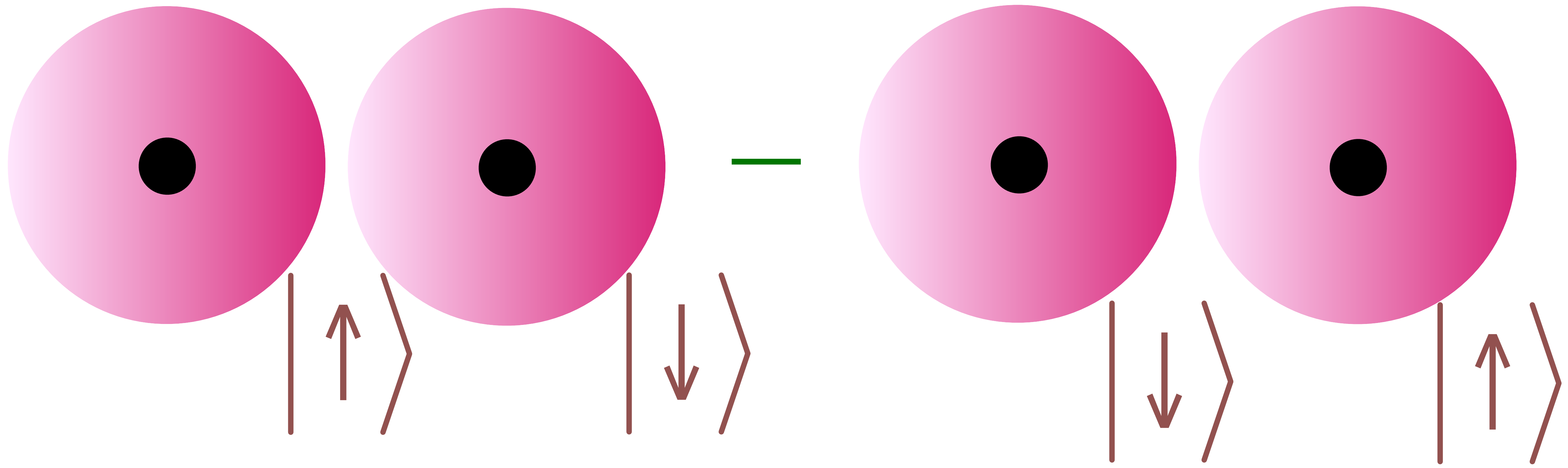
Horizon radius $R = \frac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

Quantum Entanglement across a black hole horizon



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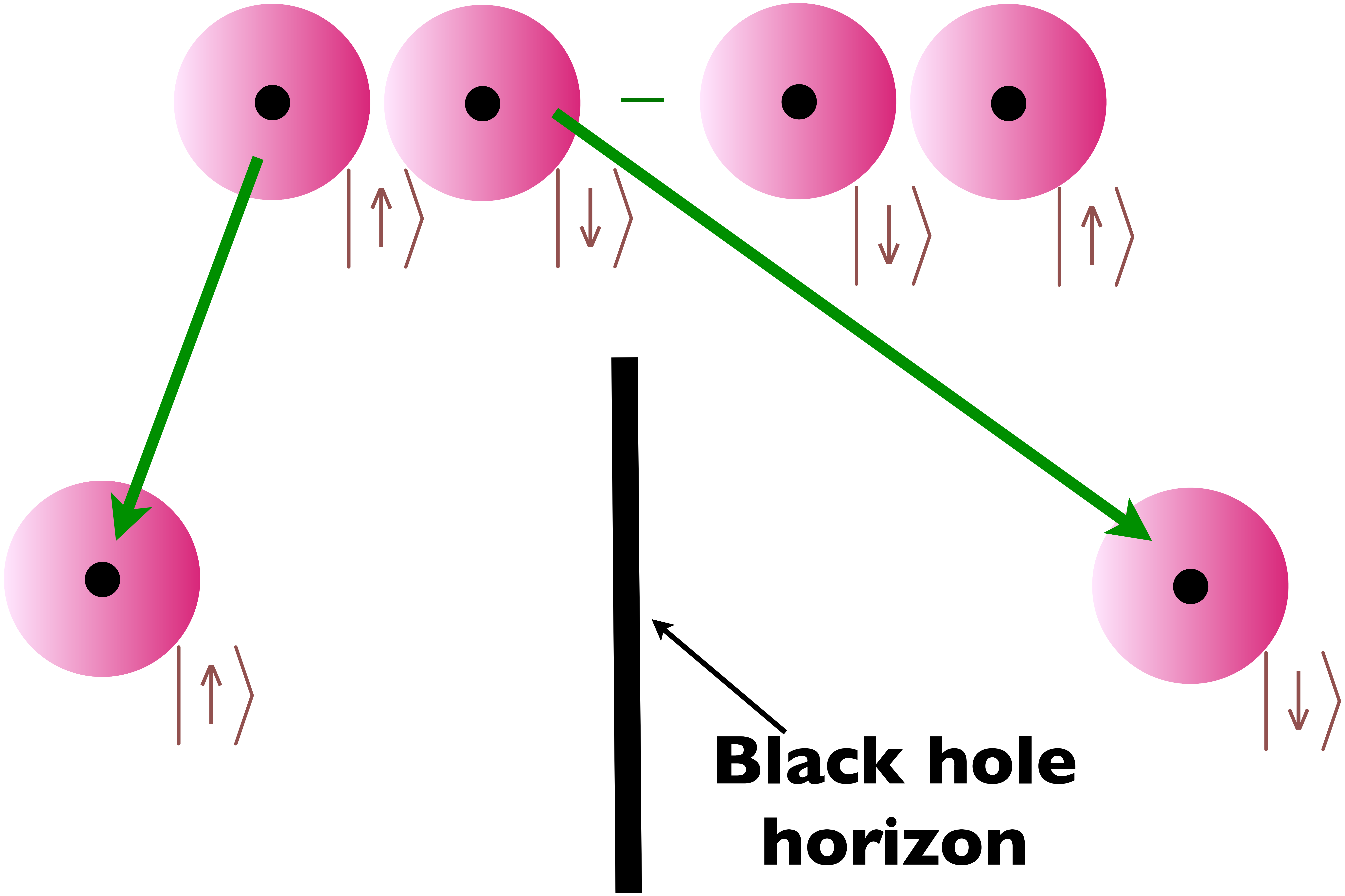

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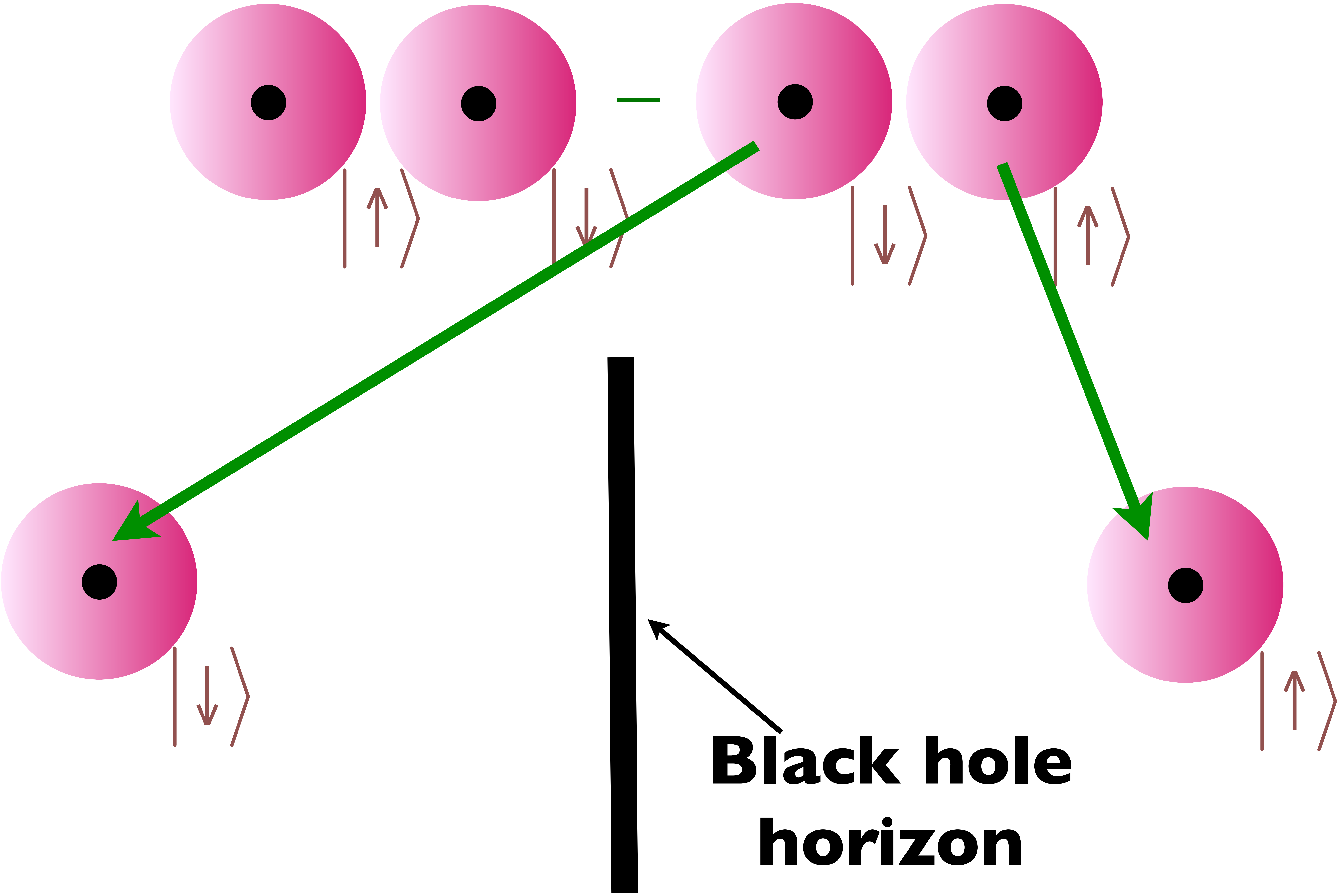
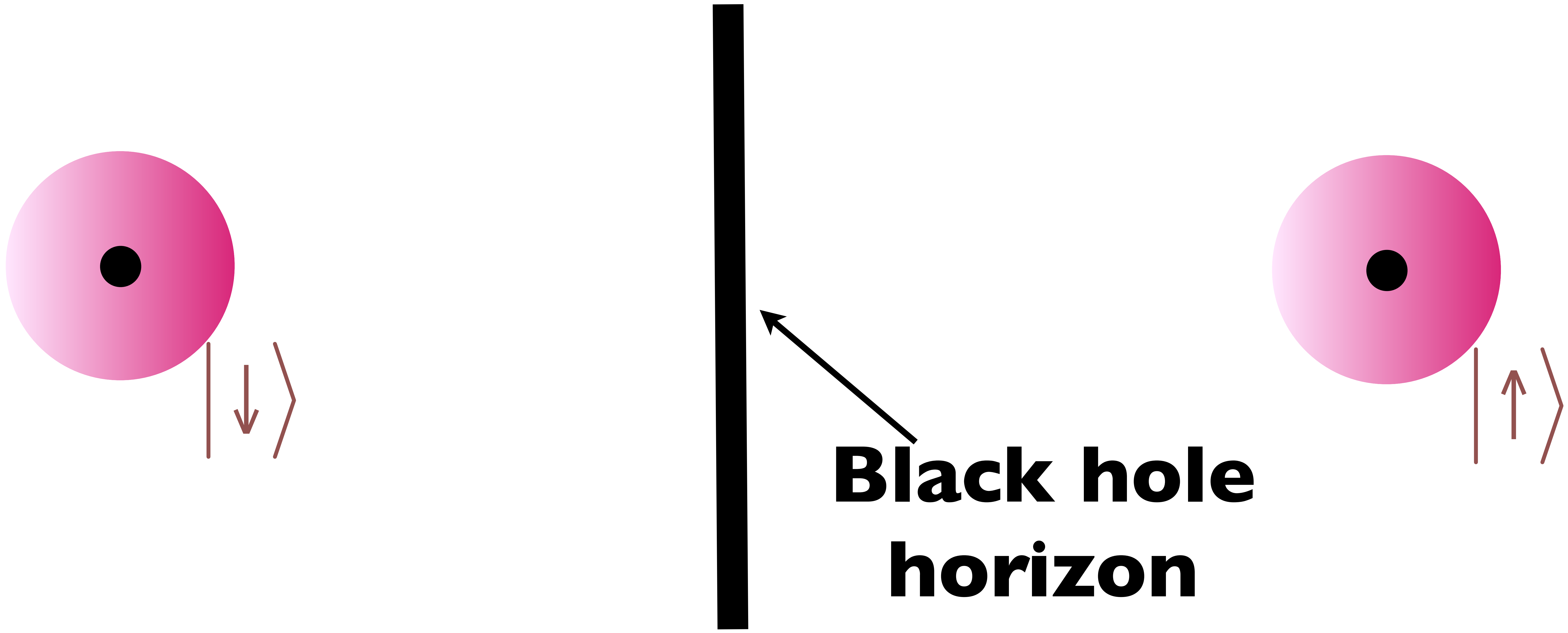


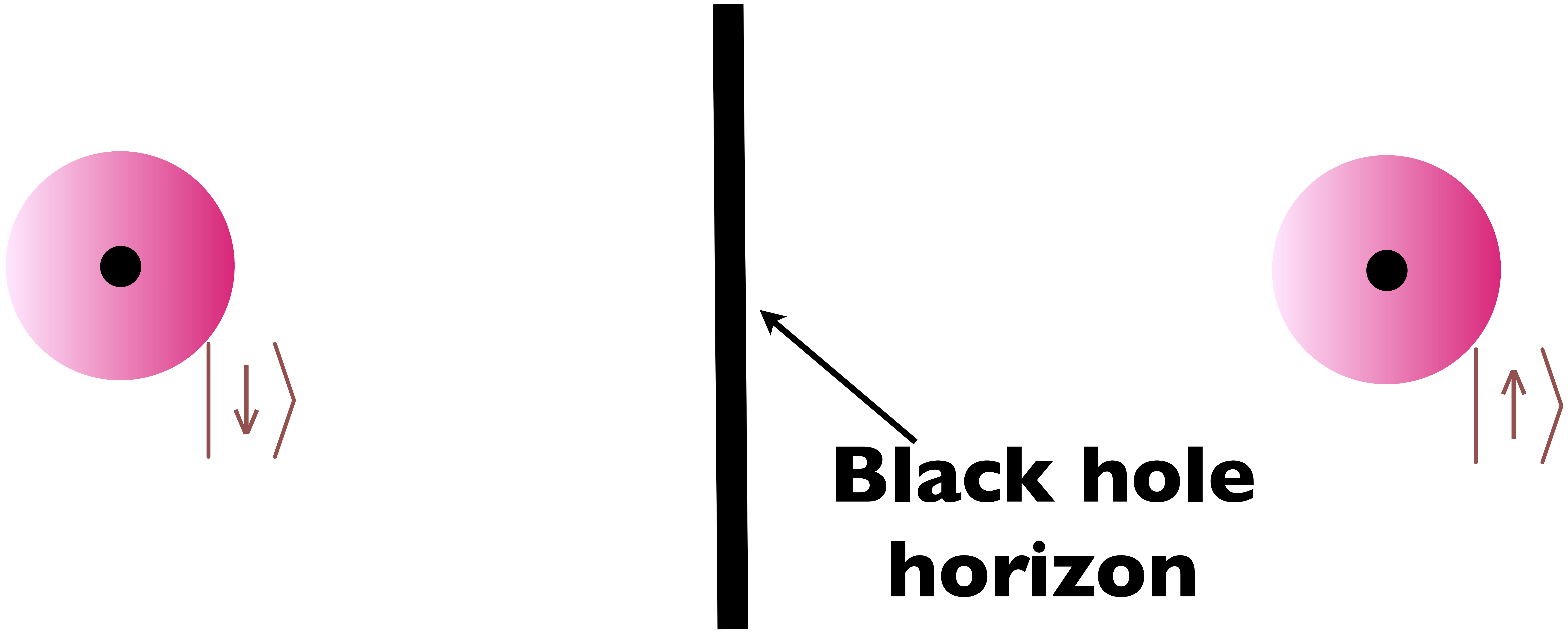
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



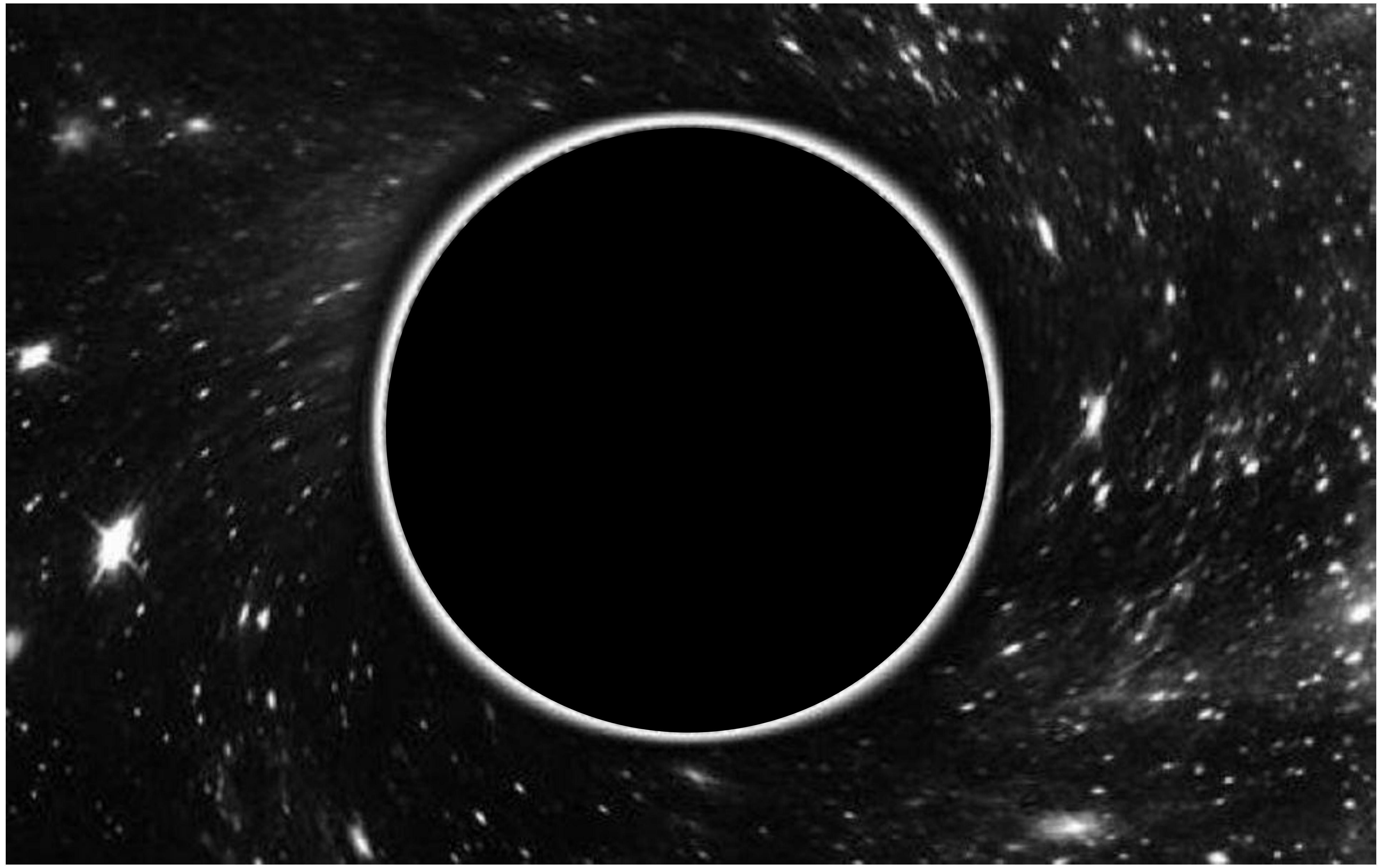
Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)



Quantum Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.



T.B. BAKKER / DR. J.P. VAN DER SCHAAR

J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

Obtained by evaluating the Feynman path integral of Einstein gravity in "imaginary time" using a semiclassical approximation

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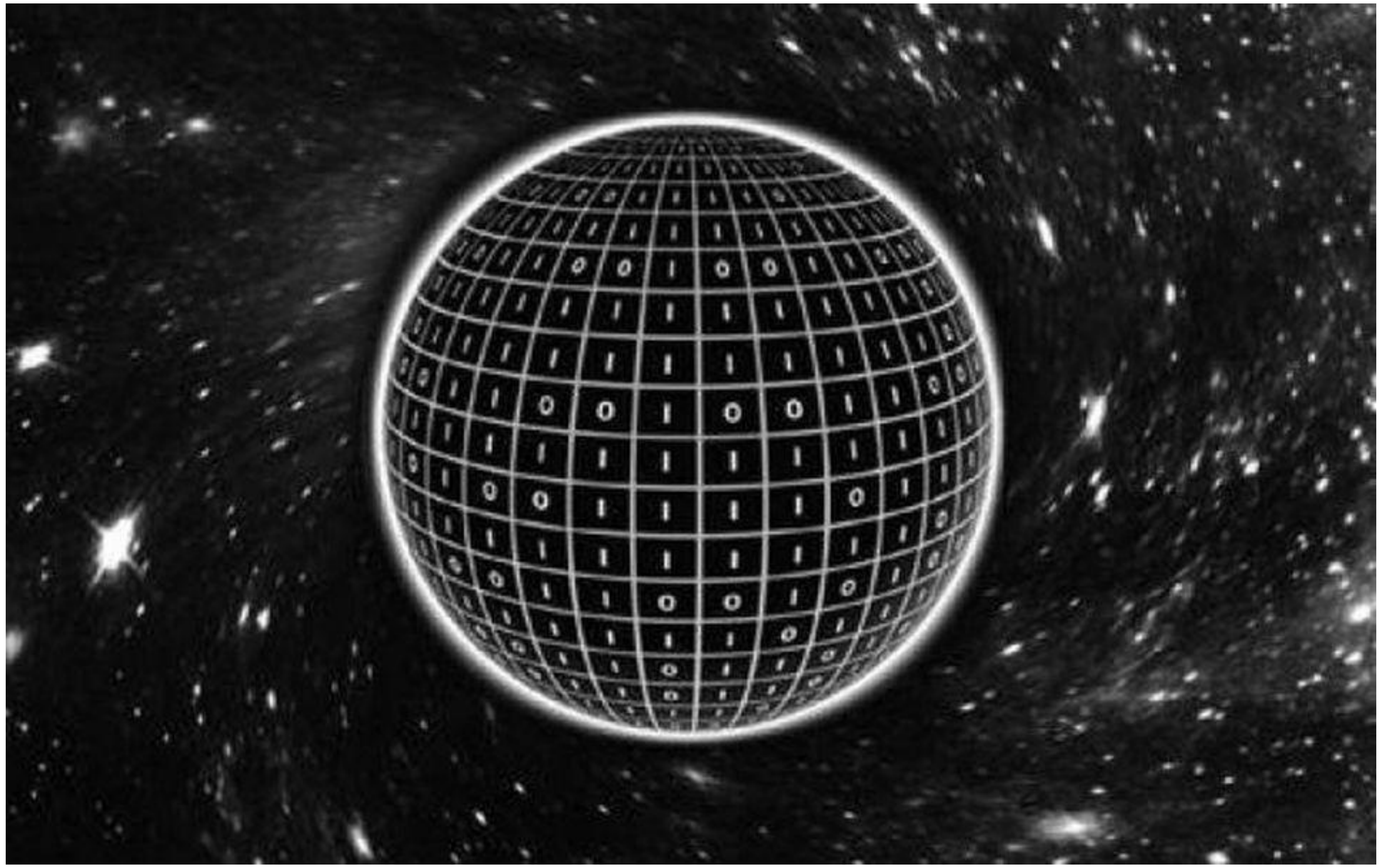

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Remarkable features:

- Entropy is finite.
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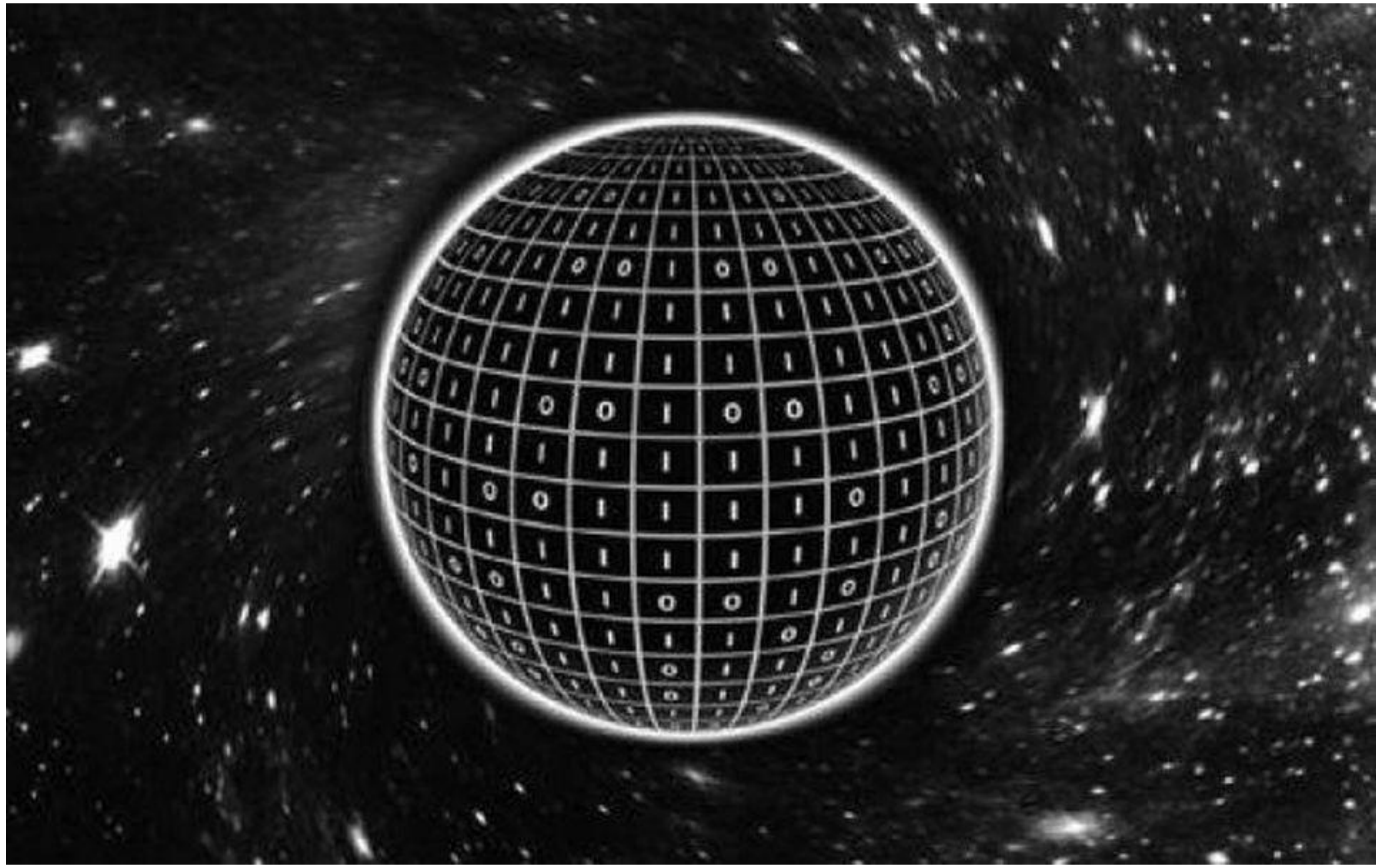

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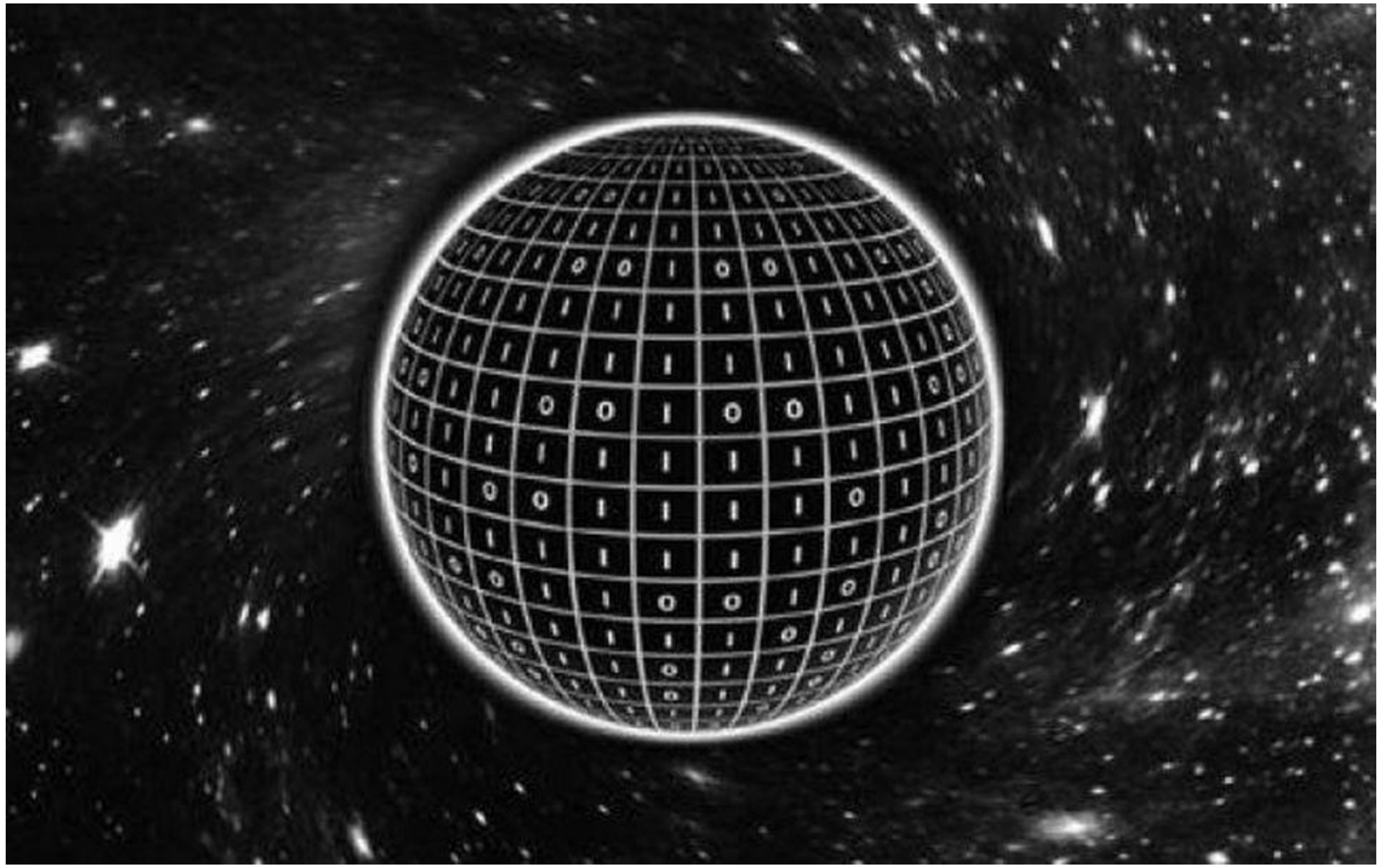
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3 = \hbar/(k_B T_H)$ which is Planckian!



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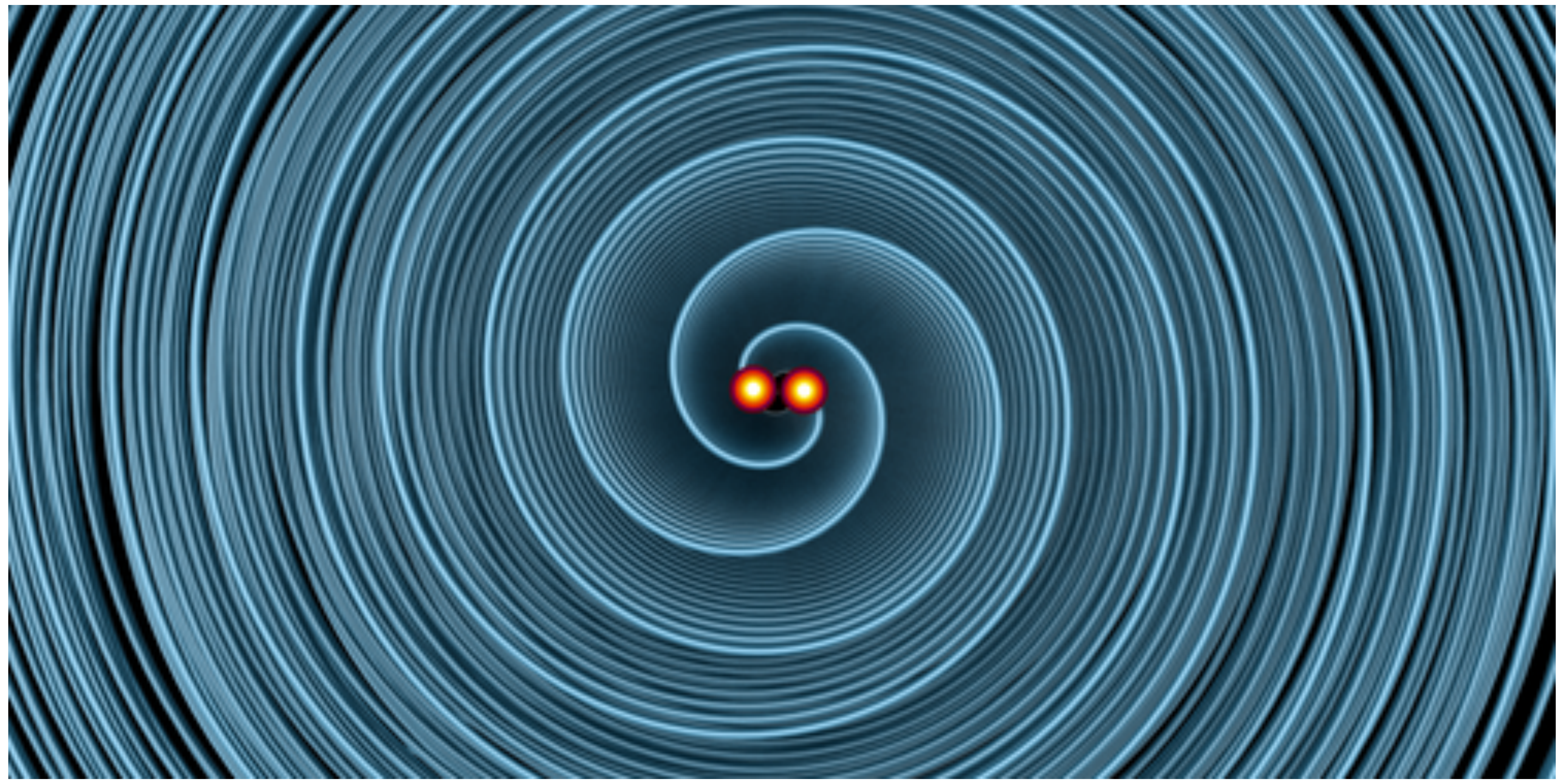
Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

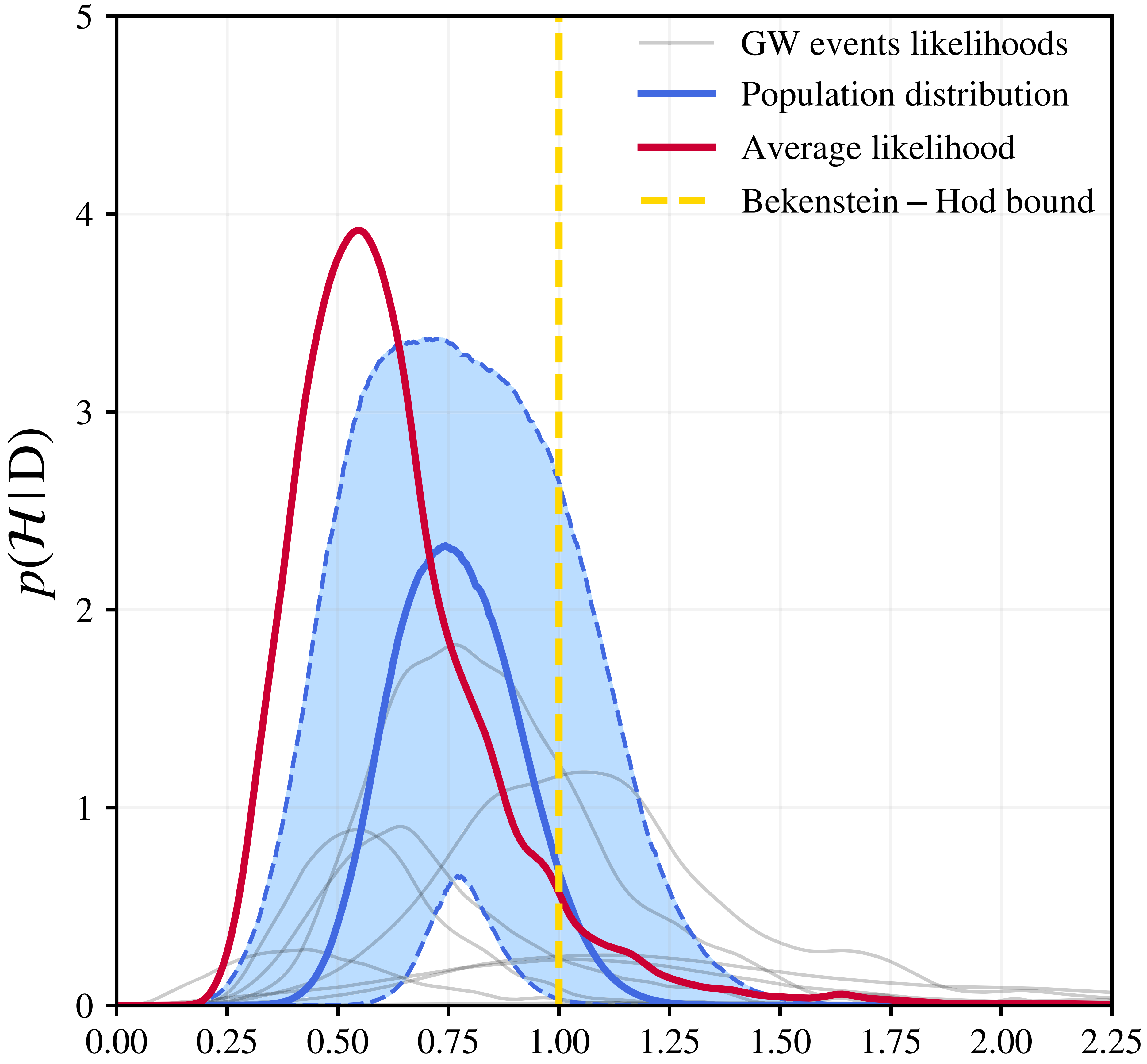
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$



$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

Questions

- Is the semi-classical theory of Hawking meaningful, and can we compute quantum corrections to S_{BH}?

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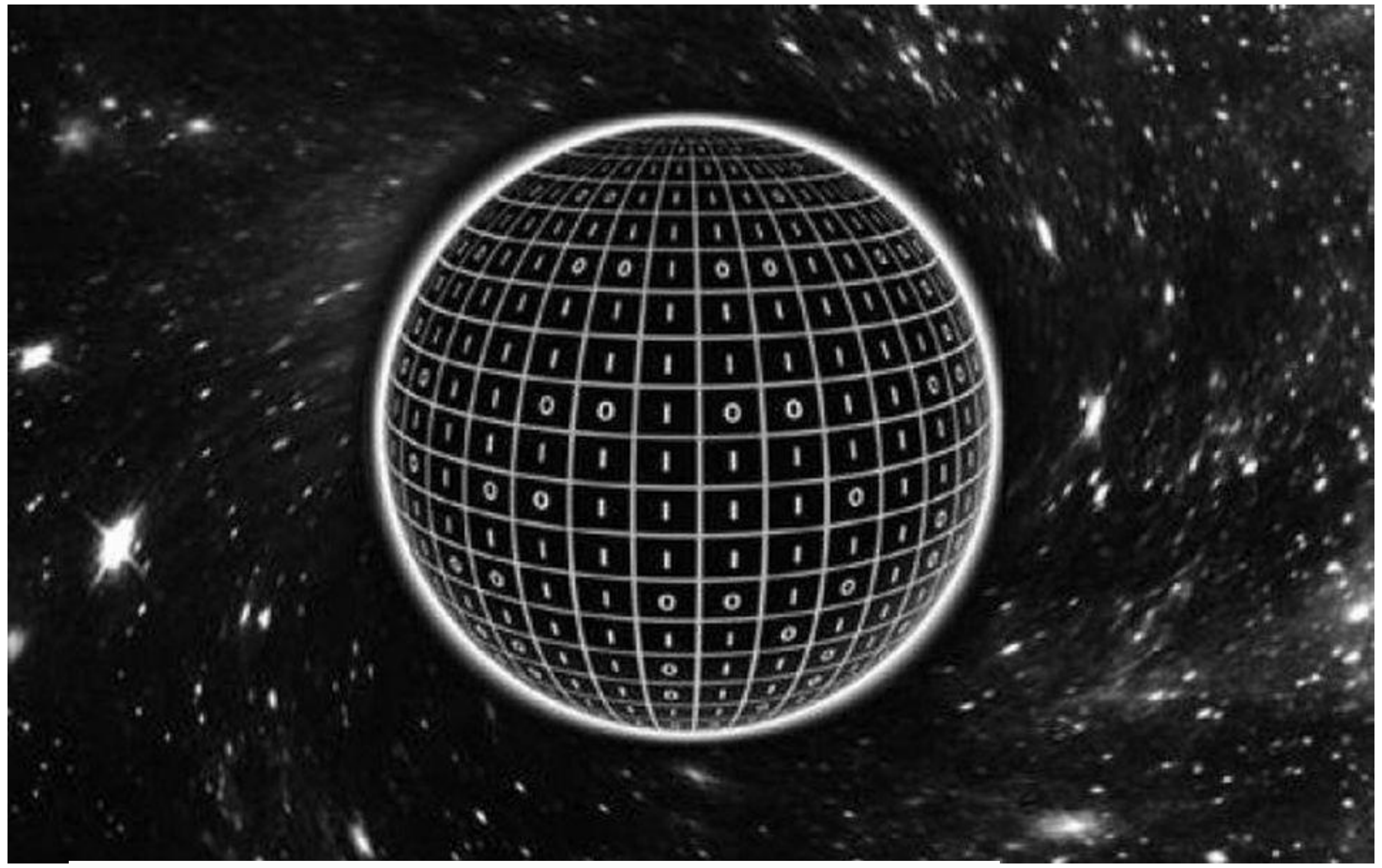

T.B. BAKKER / DR. J.P.VAN DER SCHAAR

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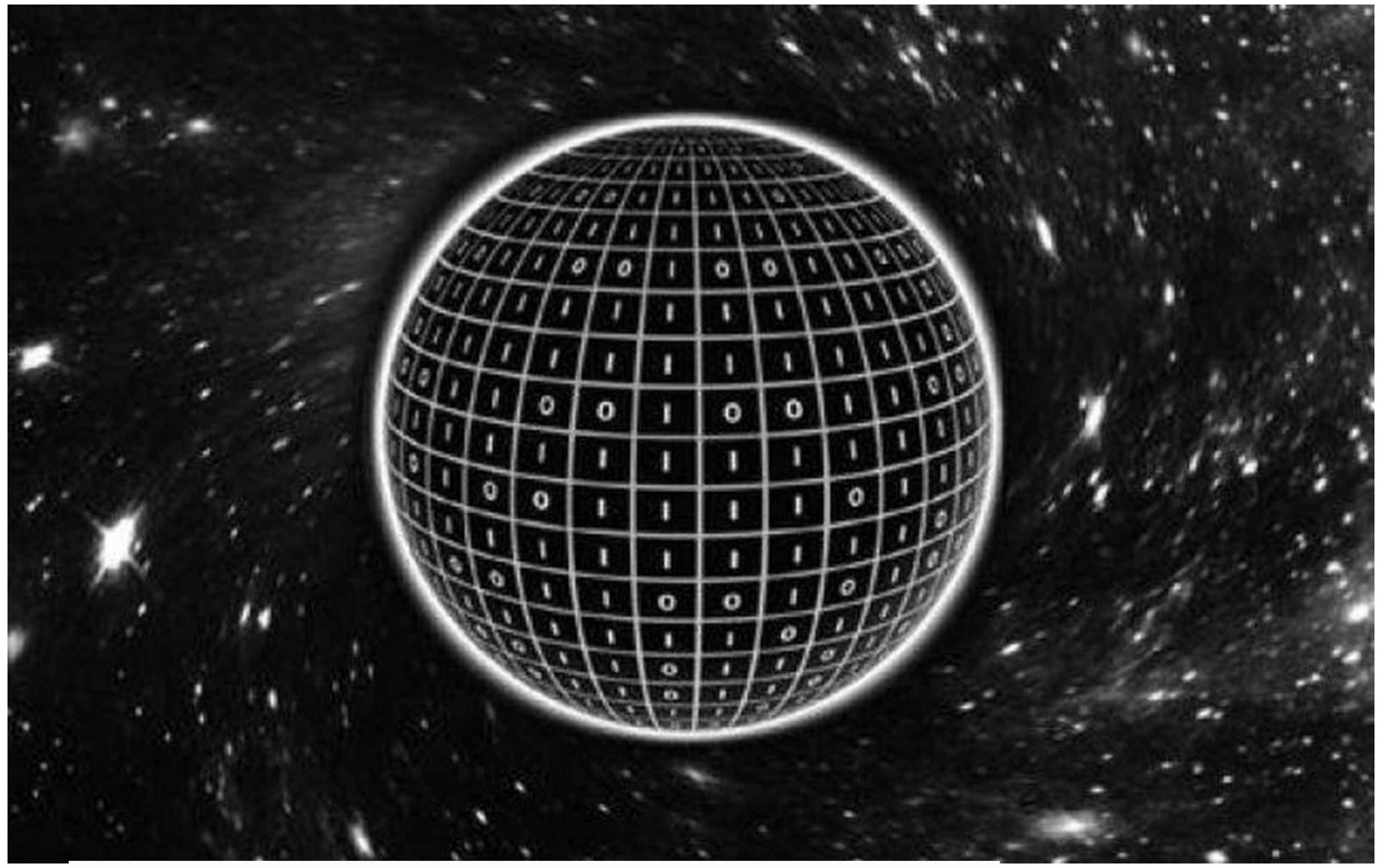
- The unitary quantum system cannot have particle-like excitations if it is to reproduce the rapid Planckian dynamics at the rate $k_B T/\hbar$.



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- Can the resulting entropy be understood 'holographically' as that of a unitary quantum system in one lower spatial dimension with a finite number of states?
- The unitary quantum system cannot have particle-like excitations if it is to reproduce the rapid Planckian dynamics at the rate $k_B T/\hbar$.
- Can we compute the evolution of the entropy as the black hole evaporates? Is it that of an evaporating unitary quantum system?

Sachdev-Ye-Kitaev Model

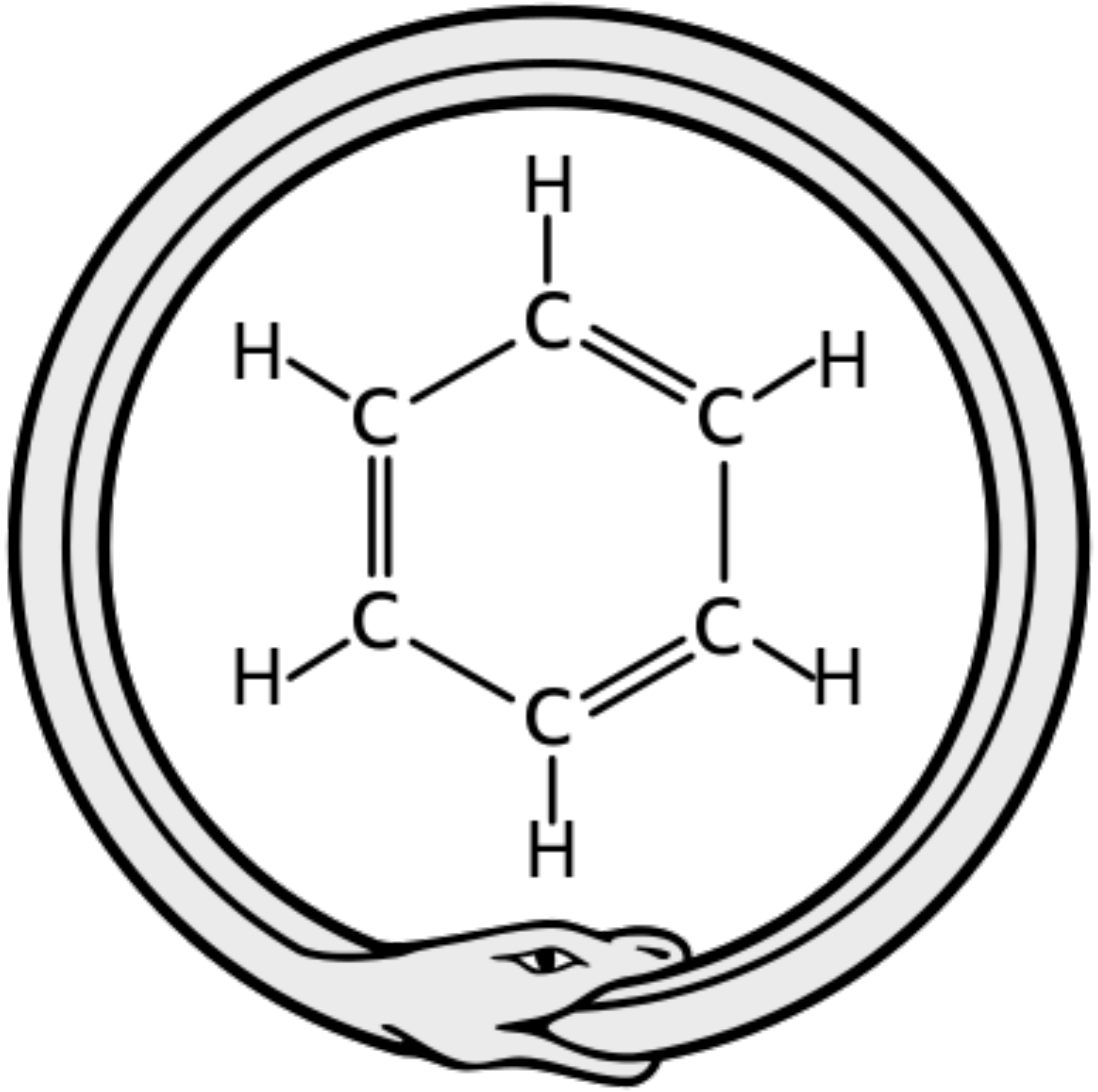
A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

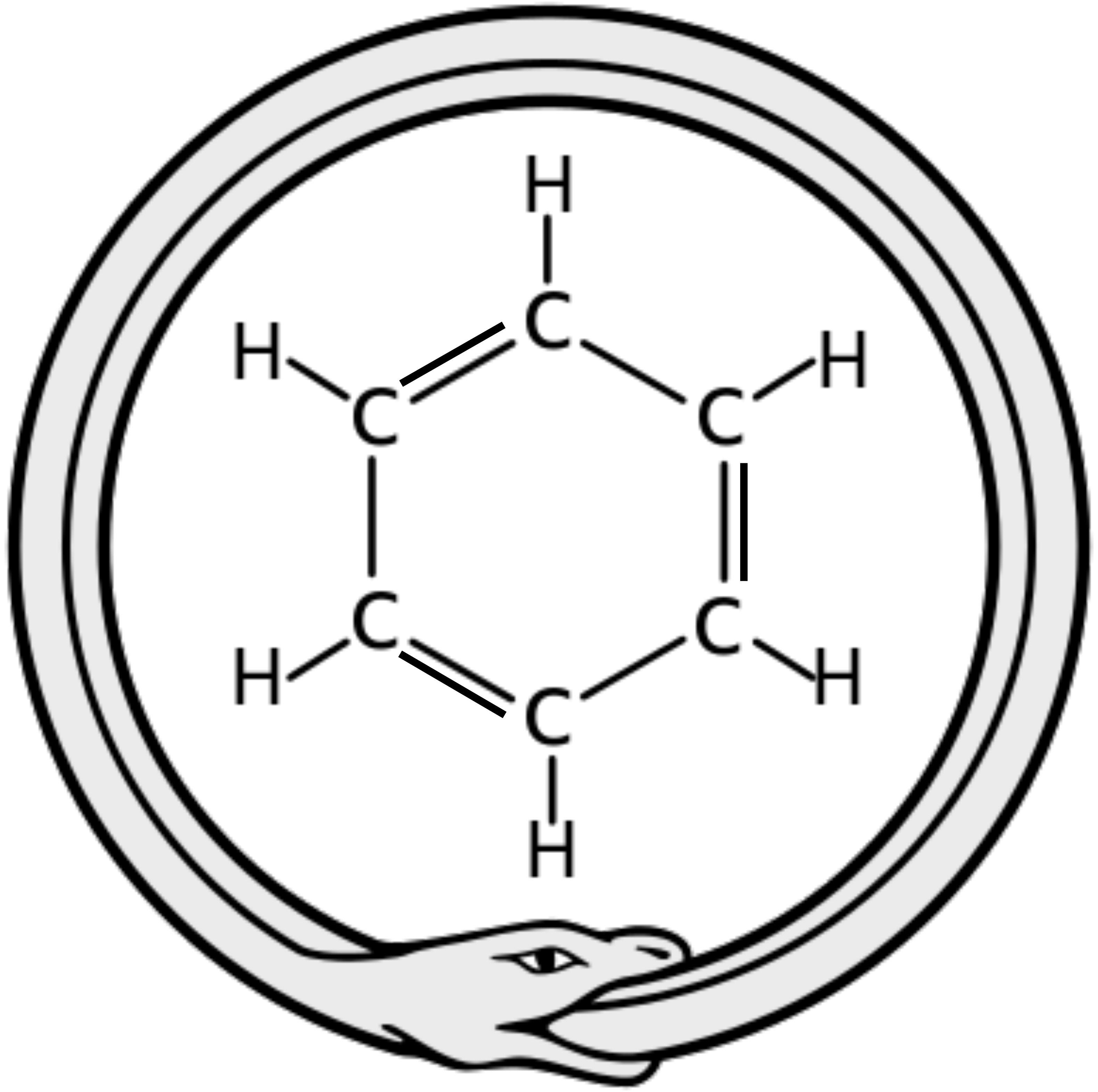
leading to a metal with no particle-like excitations

August Kekule, theory of the benzene molecule, 1865

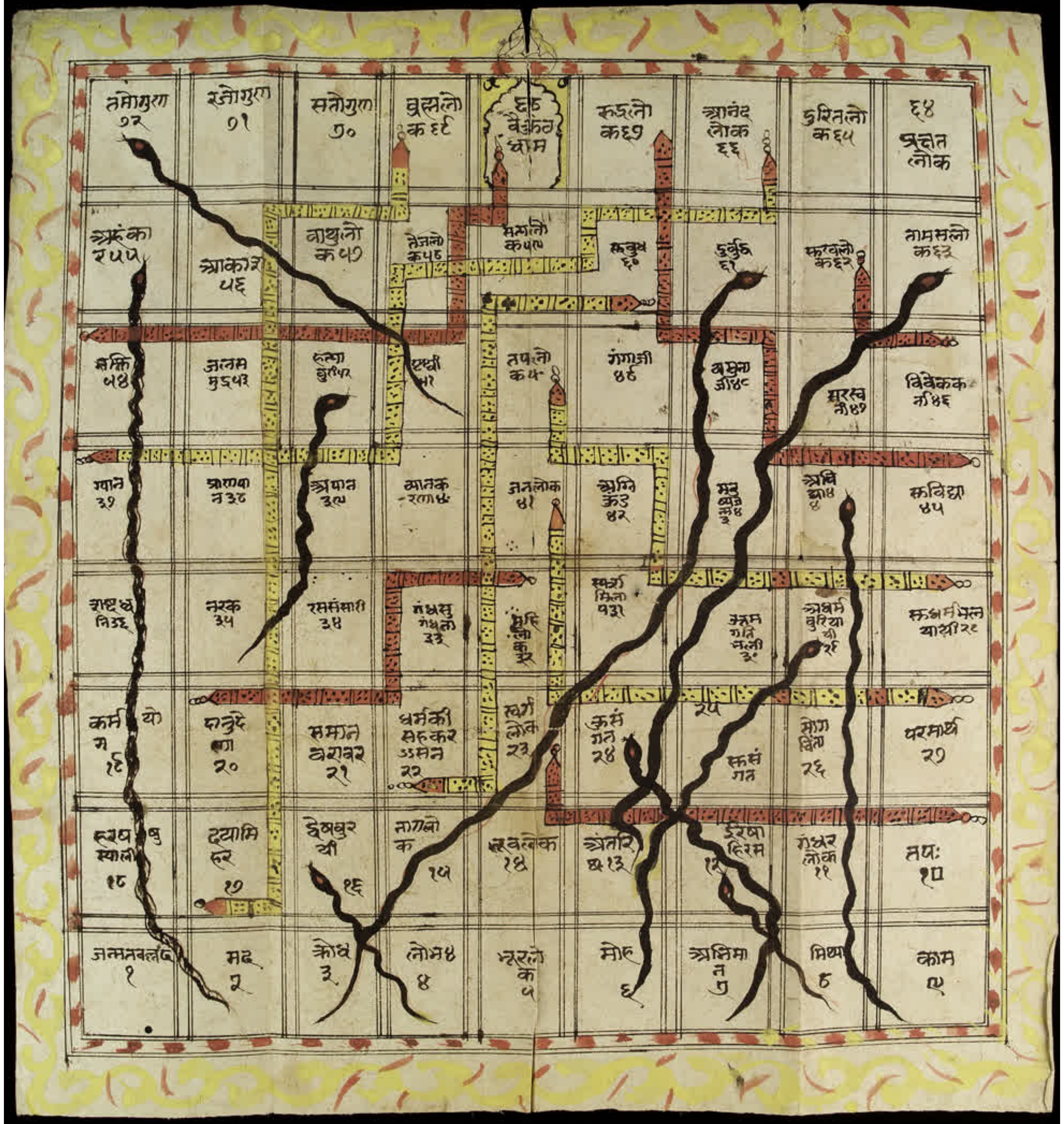
Kekule's dream

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*





*Wikipedia



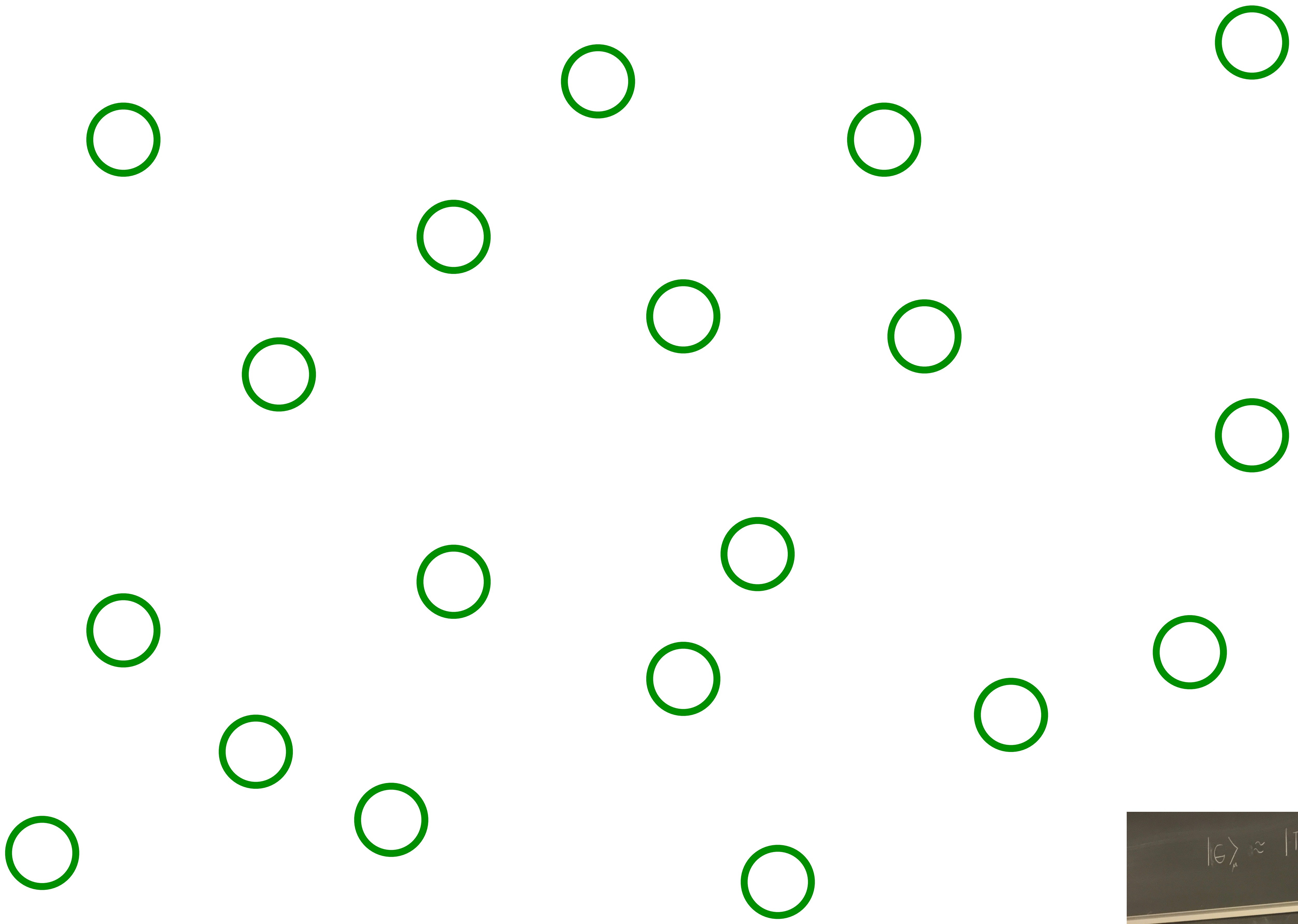
My dream*

Snakes and ladders

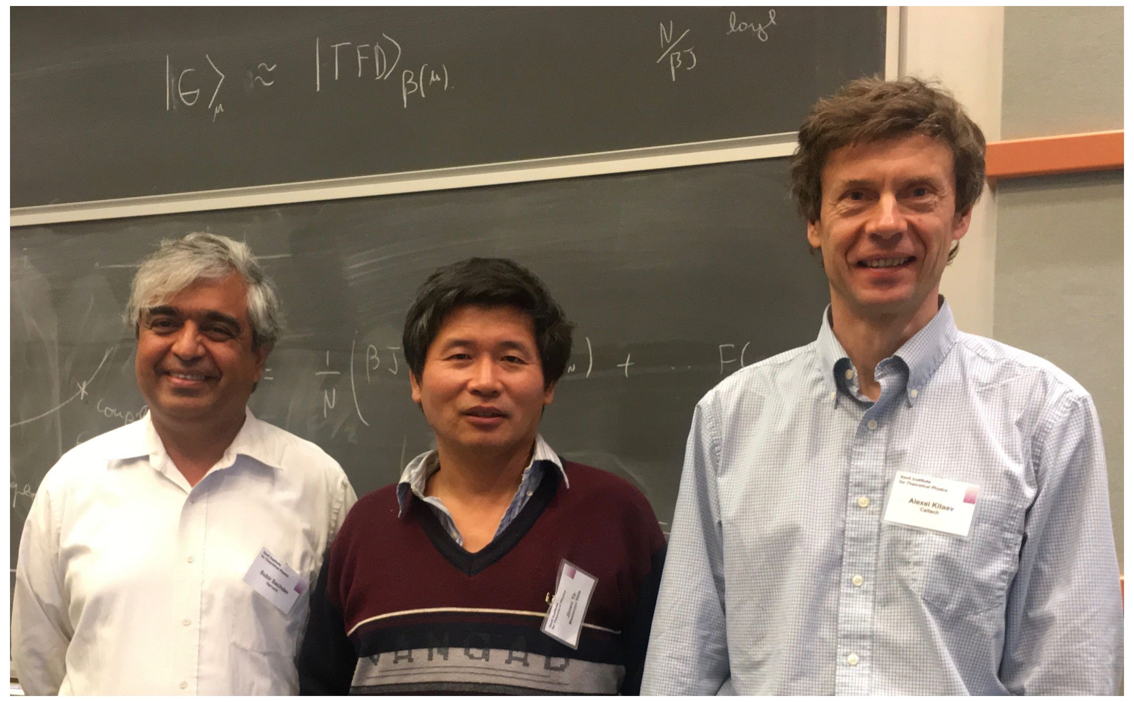
*Not true

The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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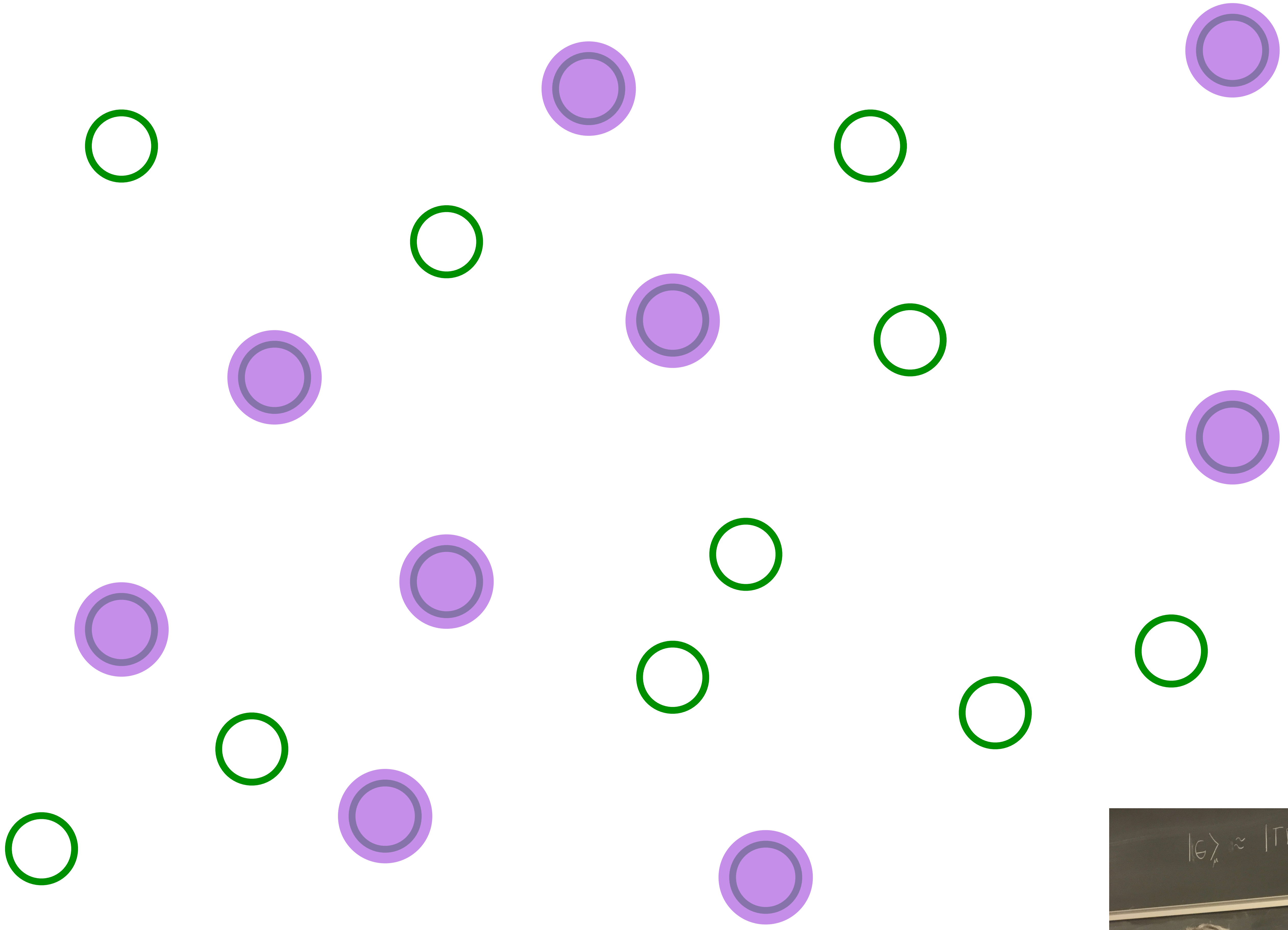
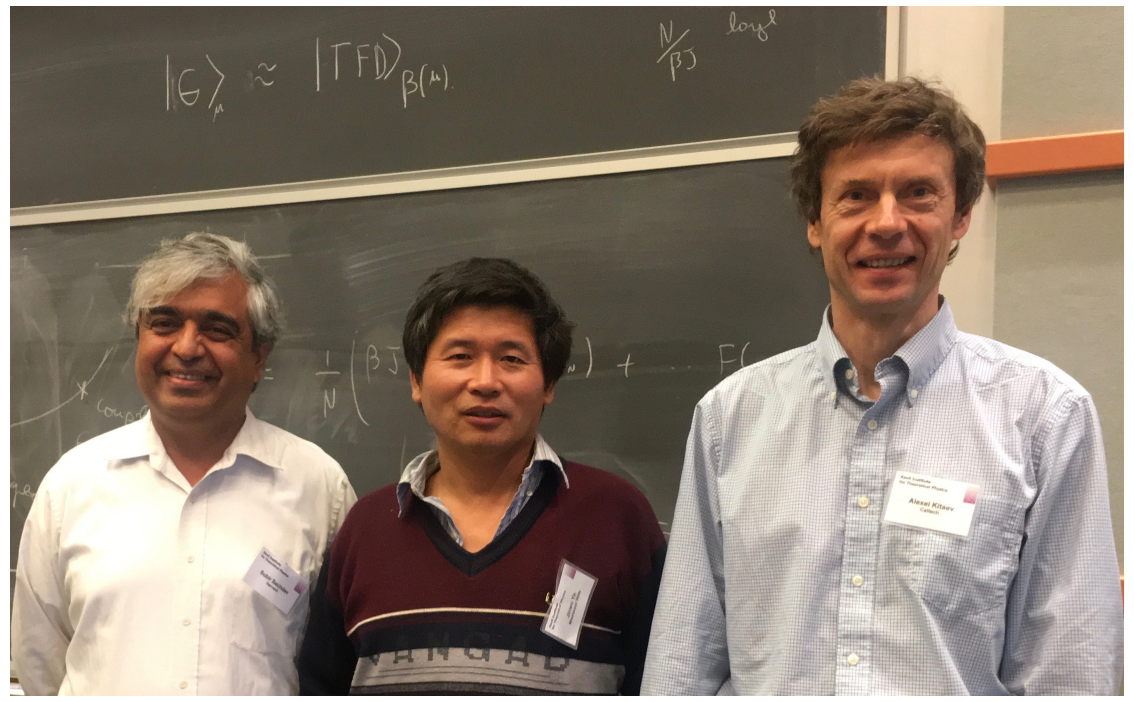


Place electrons randomly on some sites



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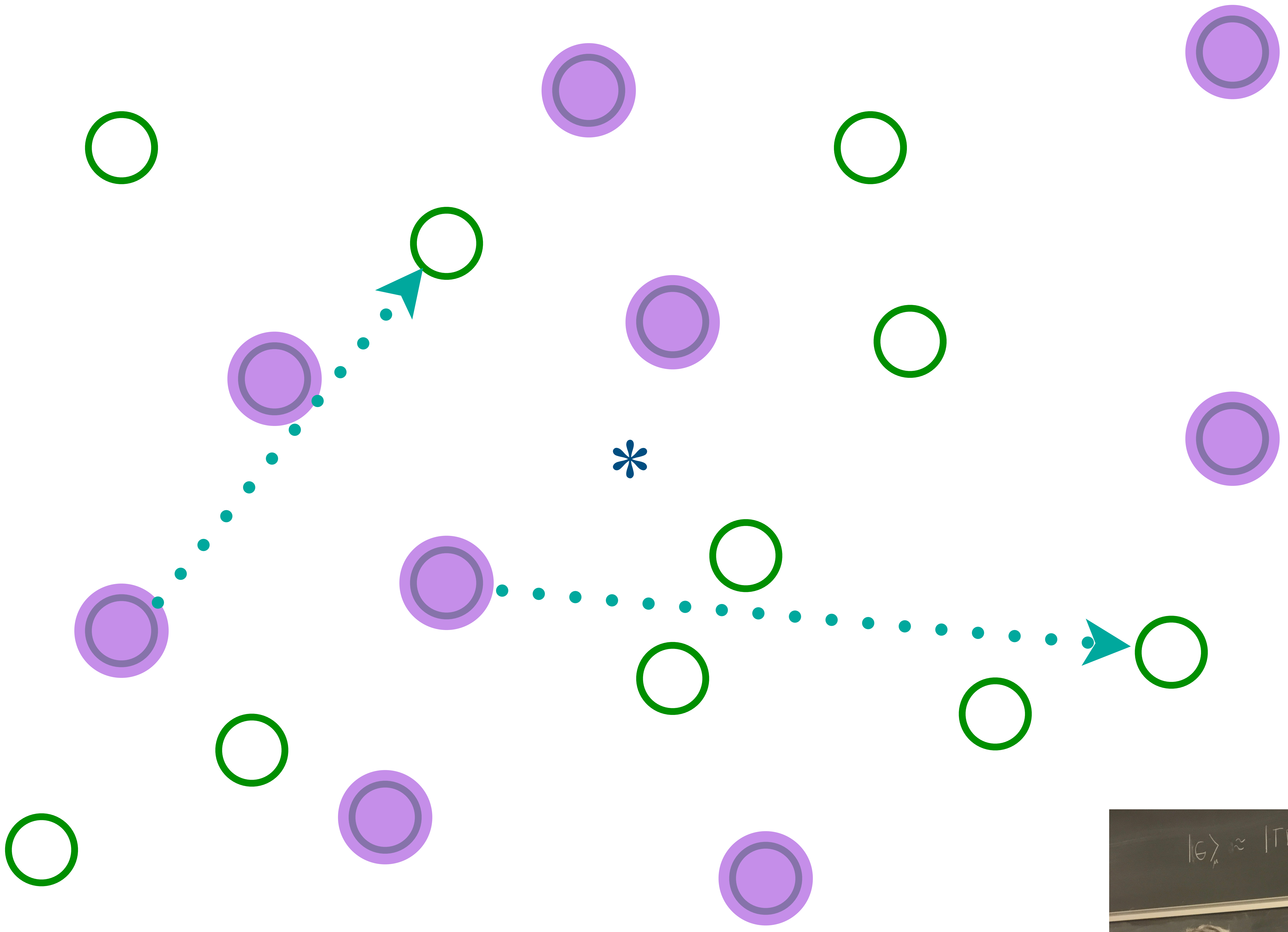

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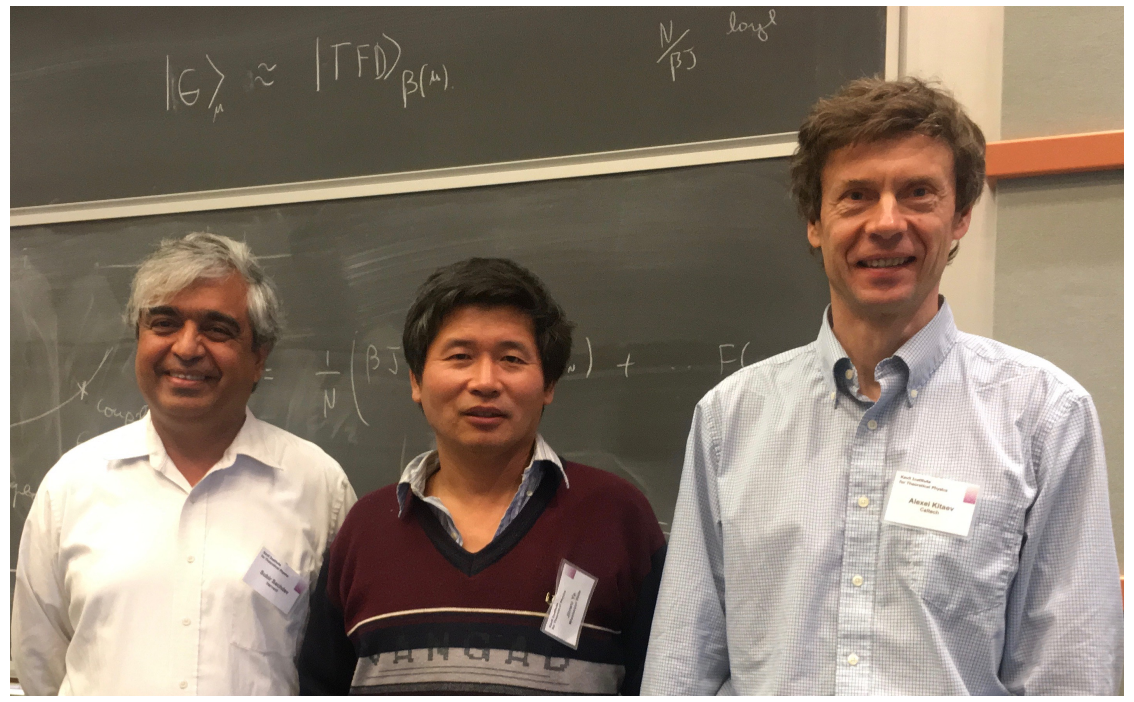

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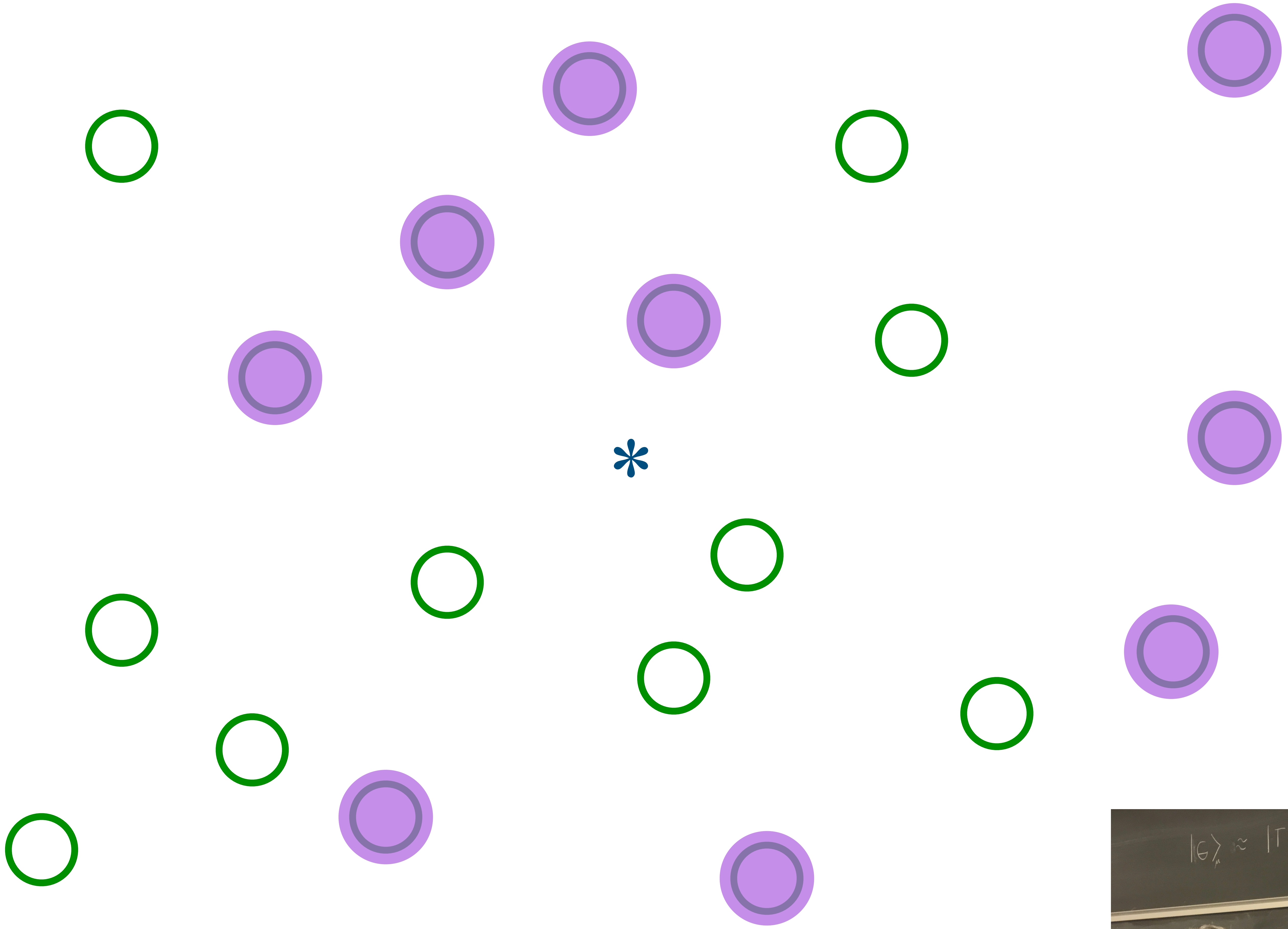
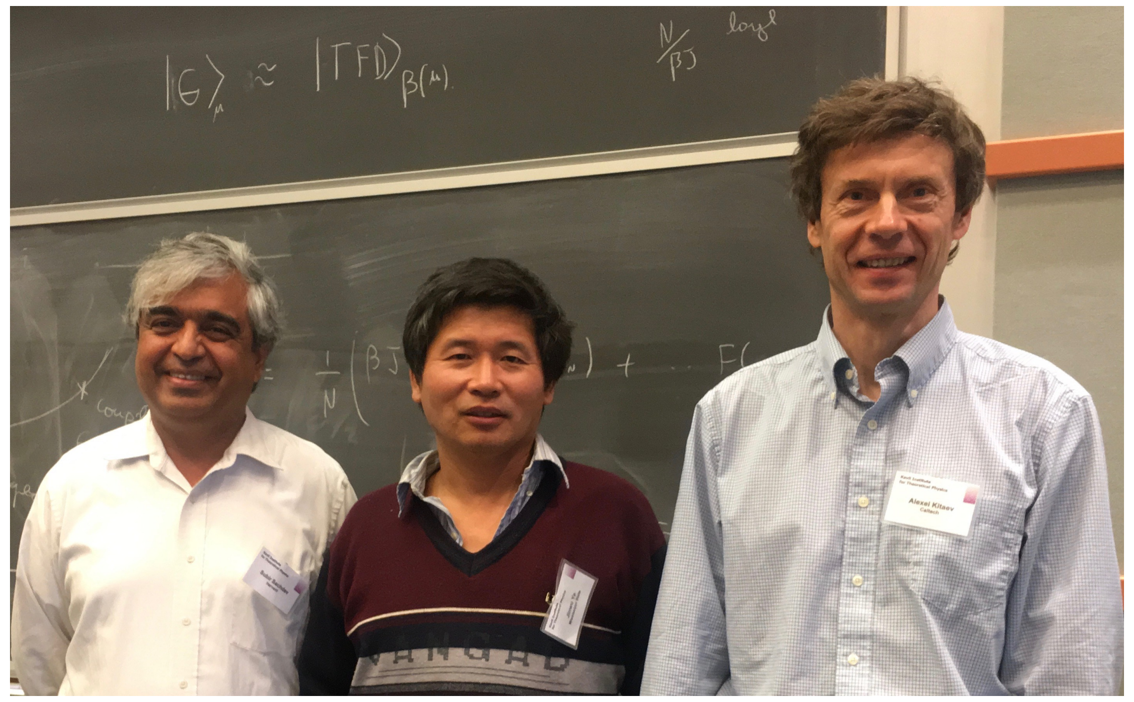


Entangle electrons pairwise randomly



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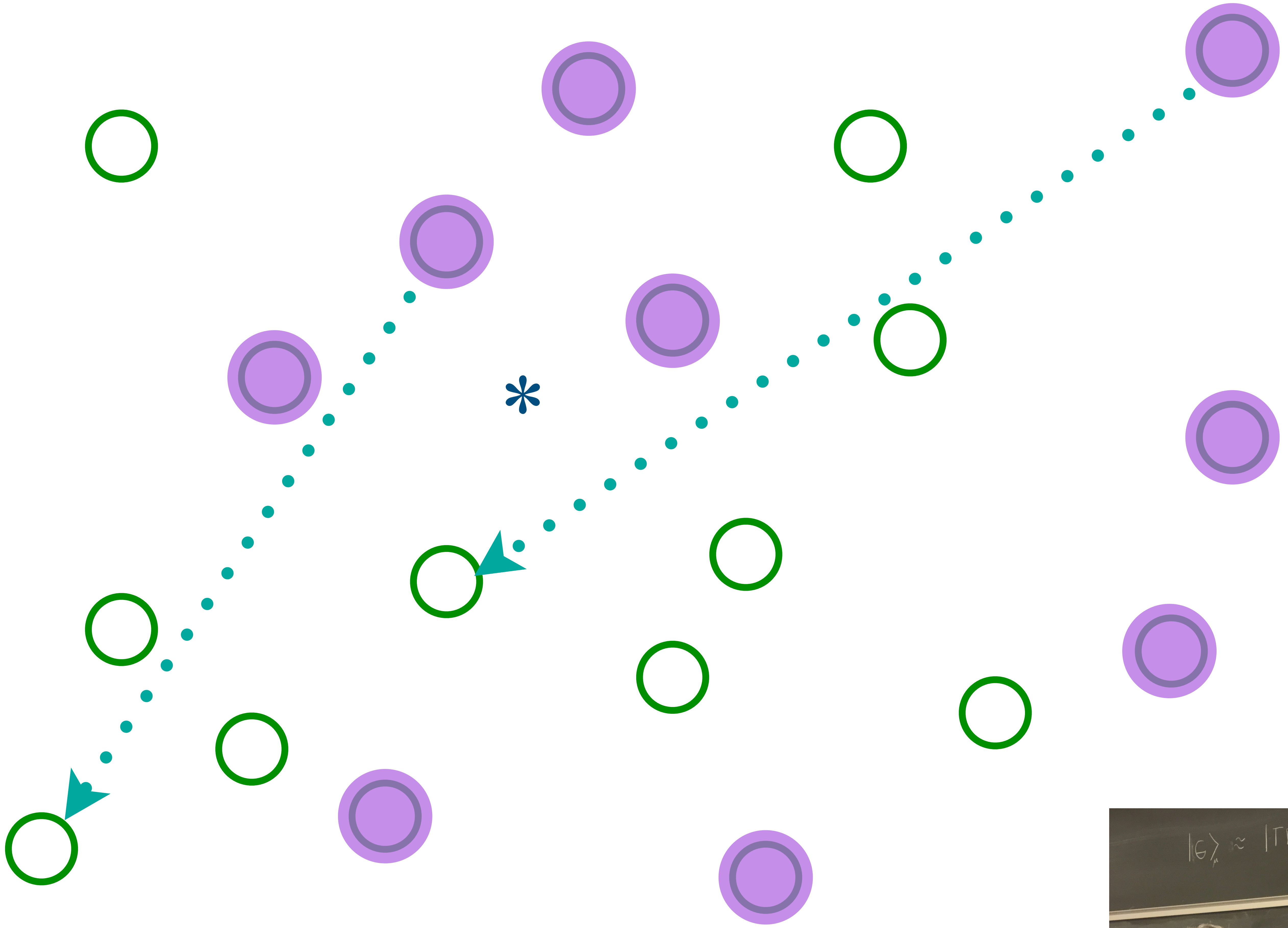

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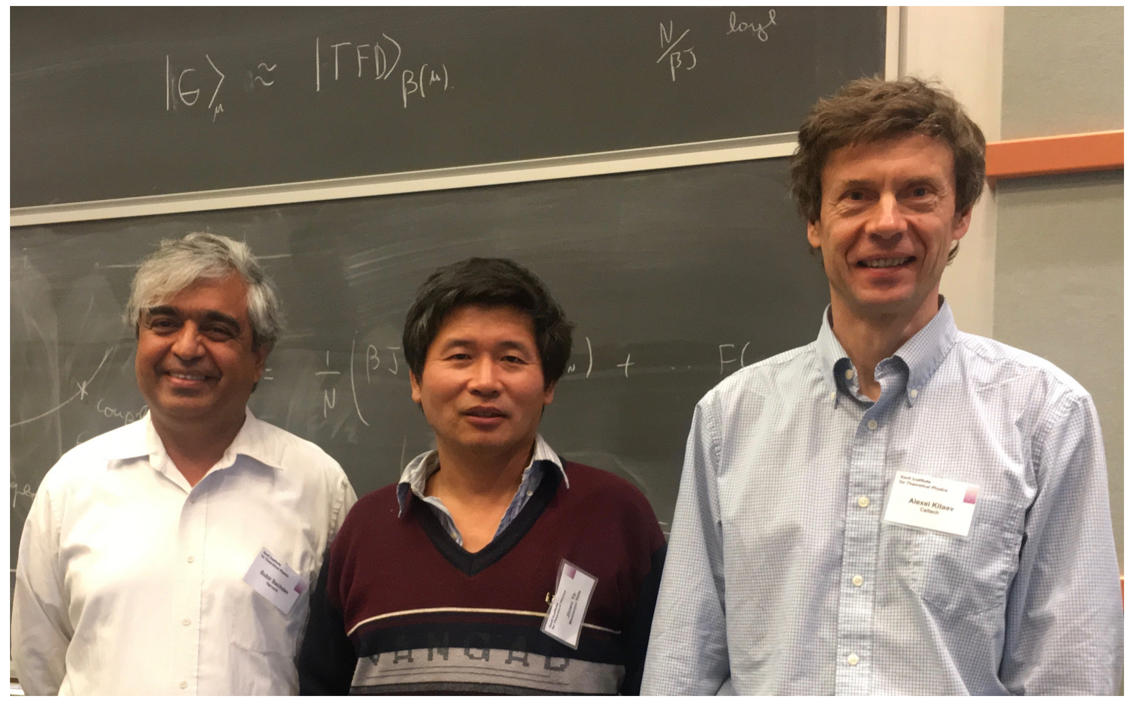

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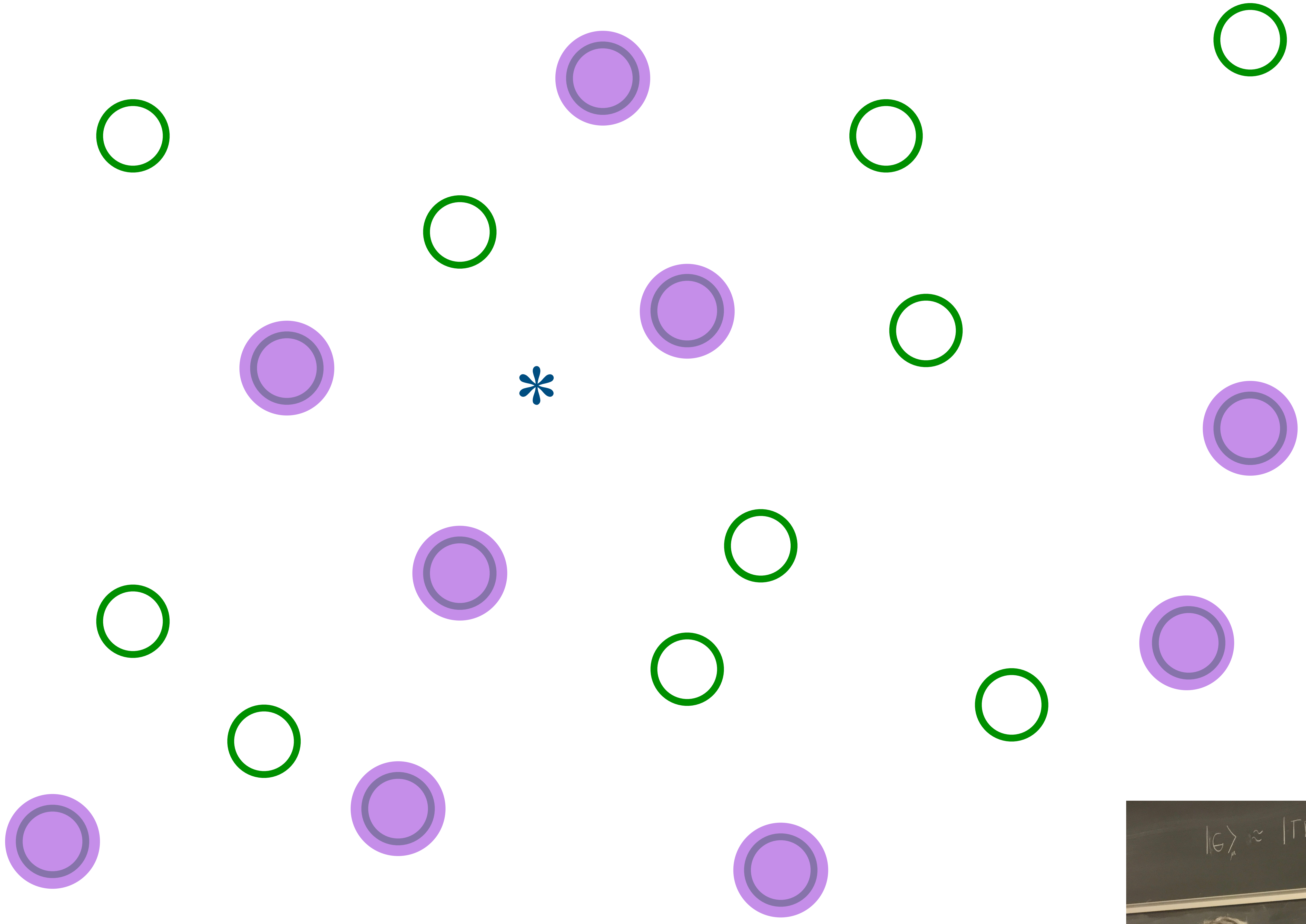

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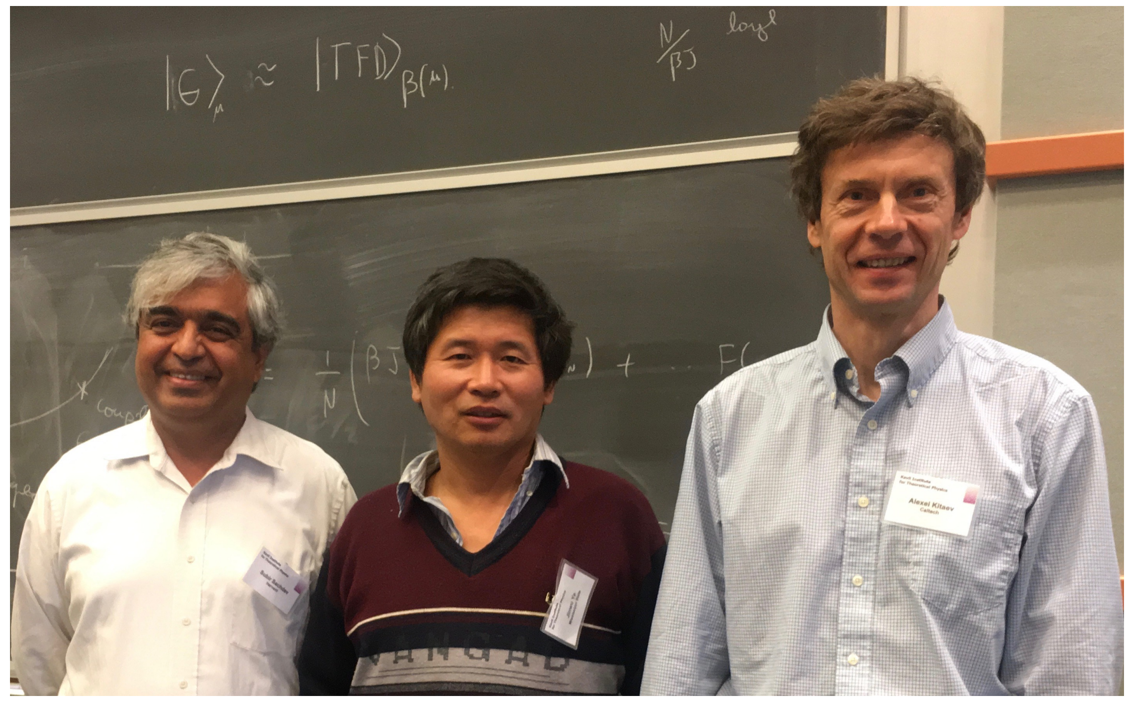

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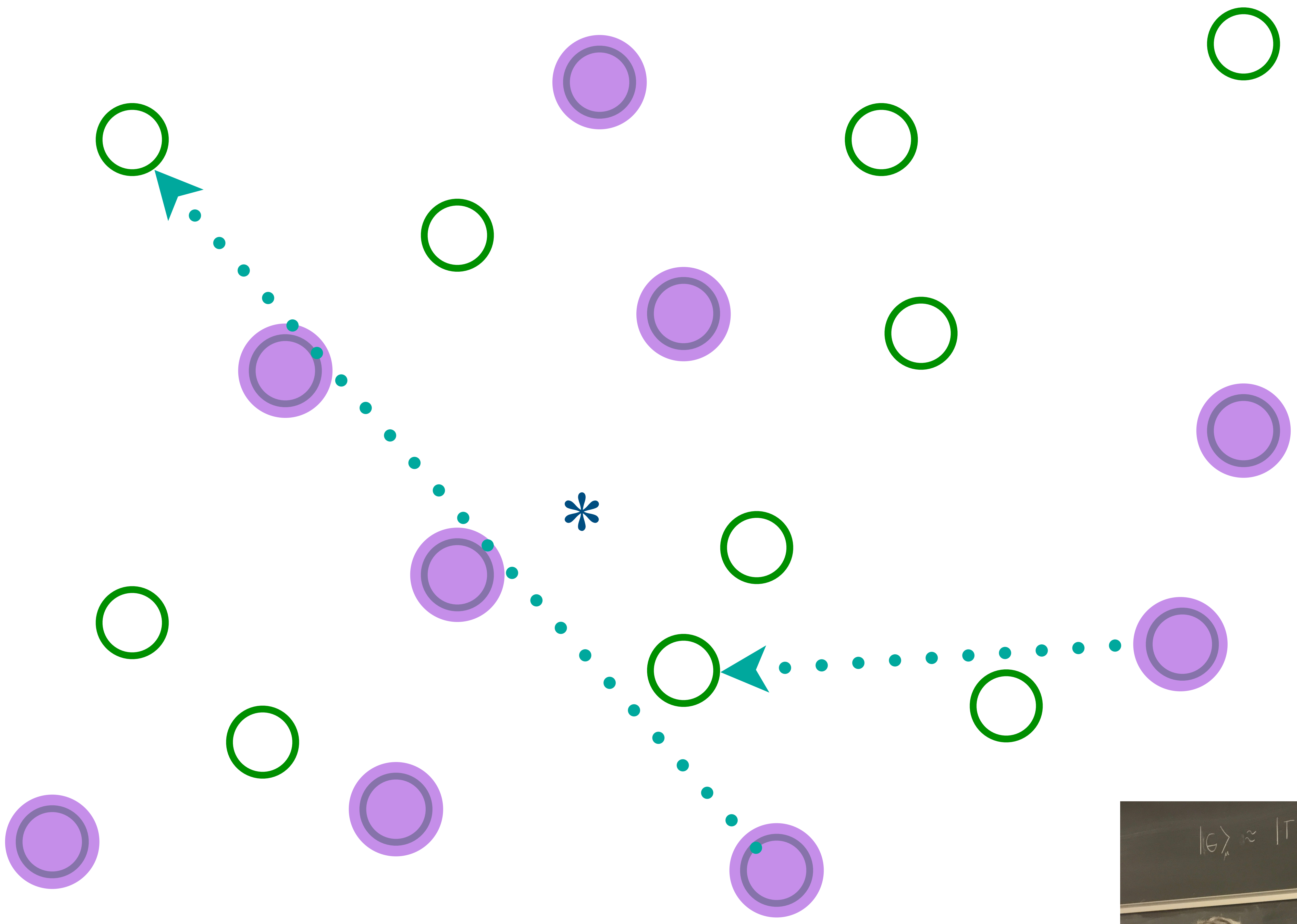

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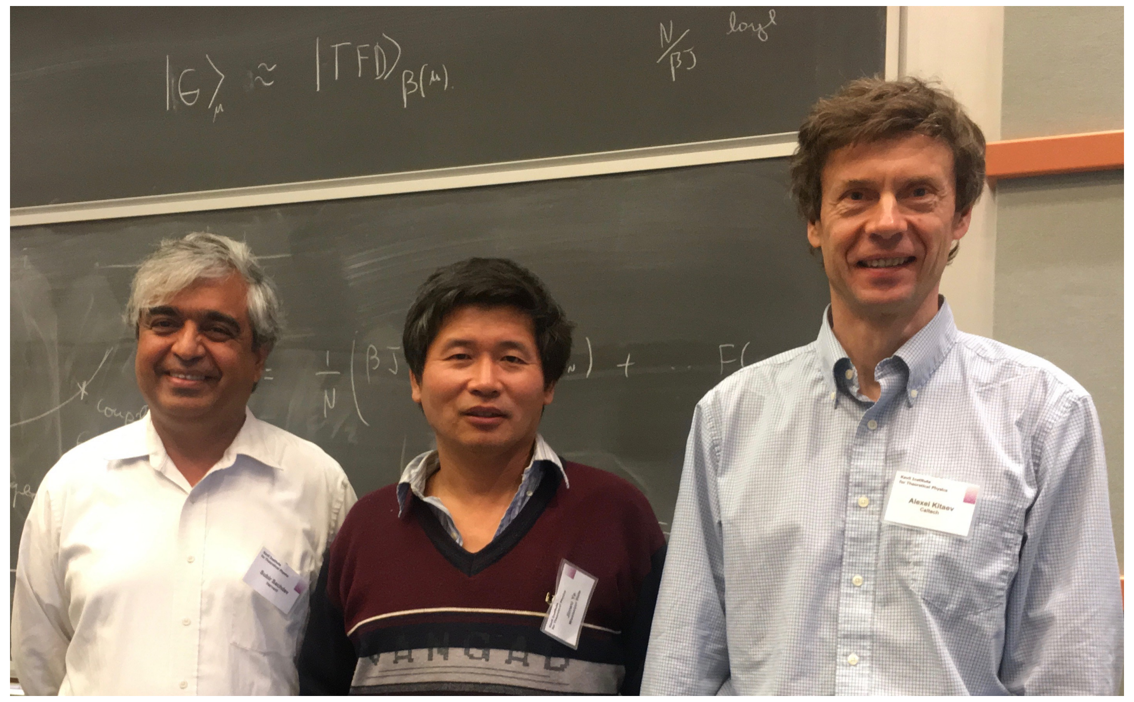

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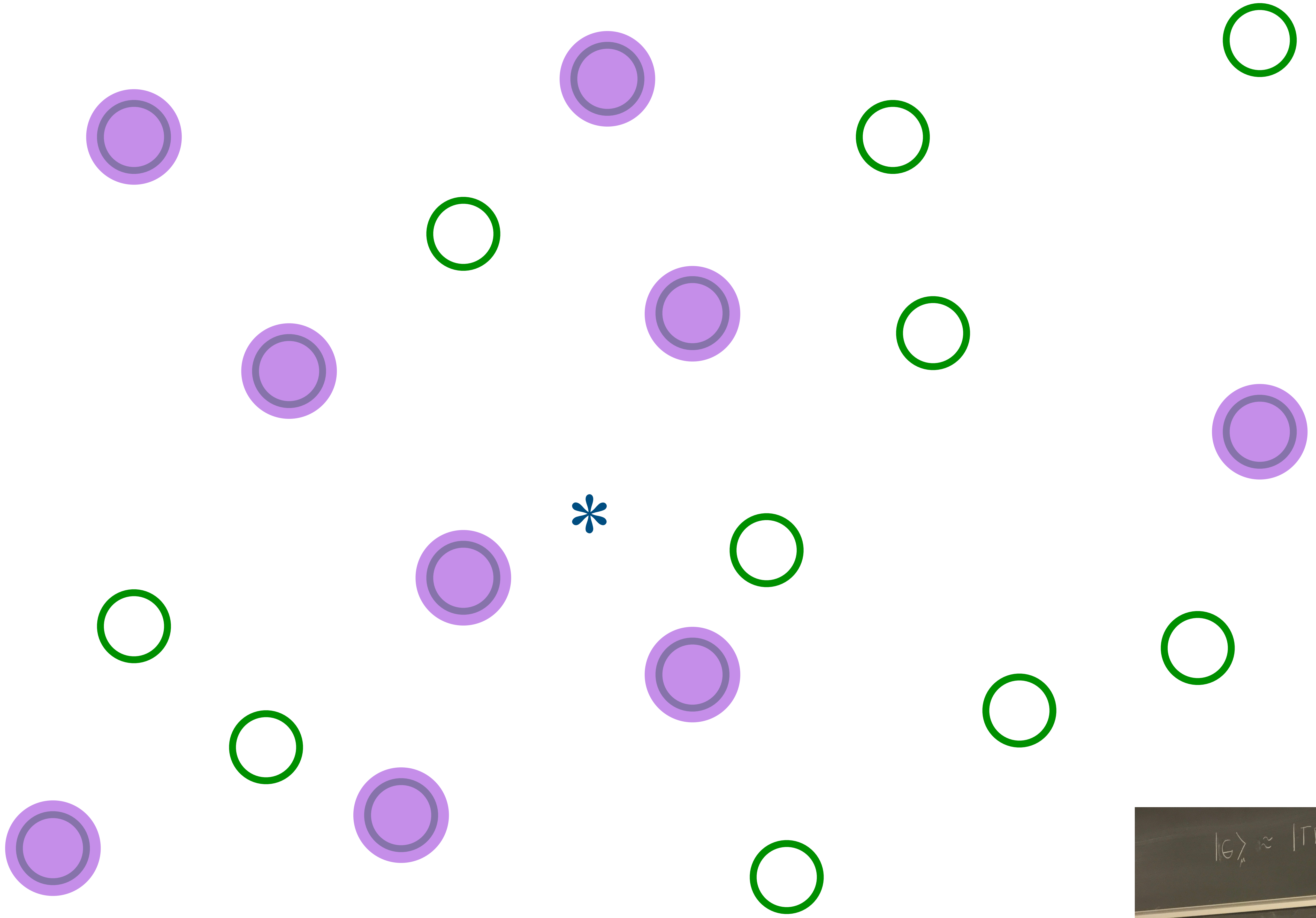
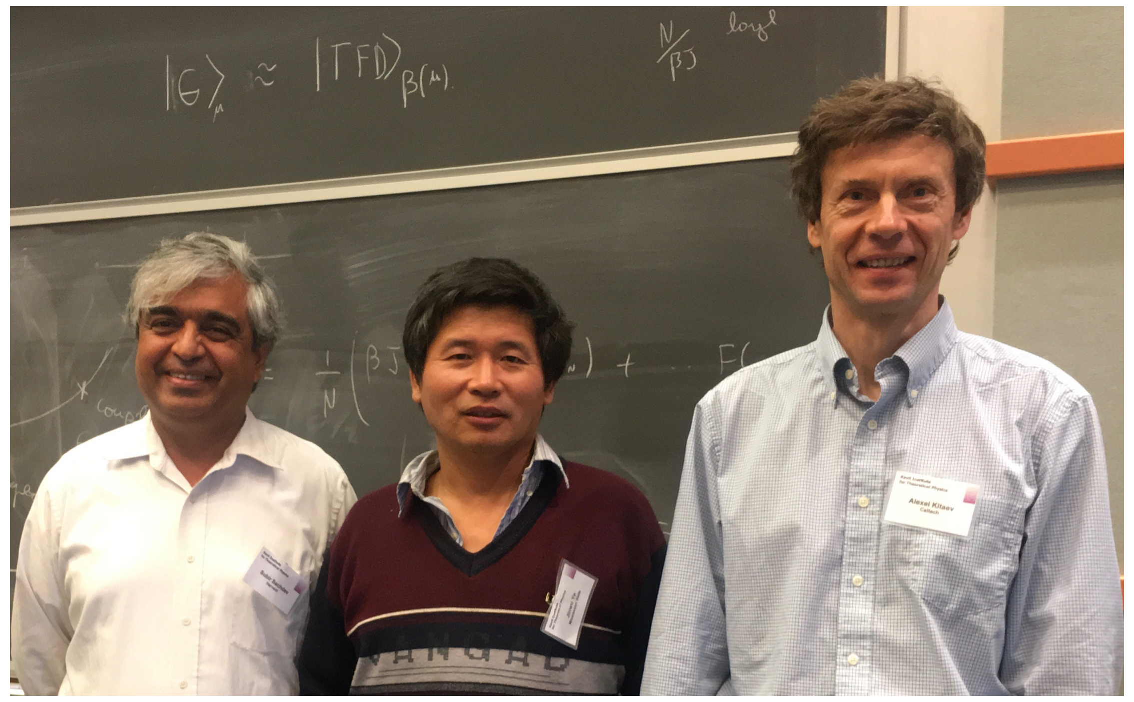


Entangle electrons pairwise randomly



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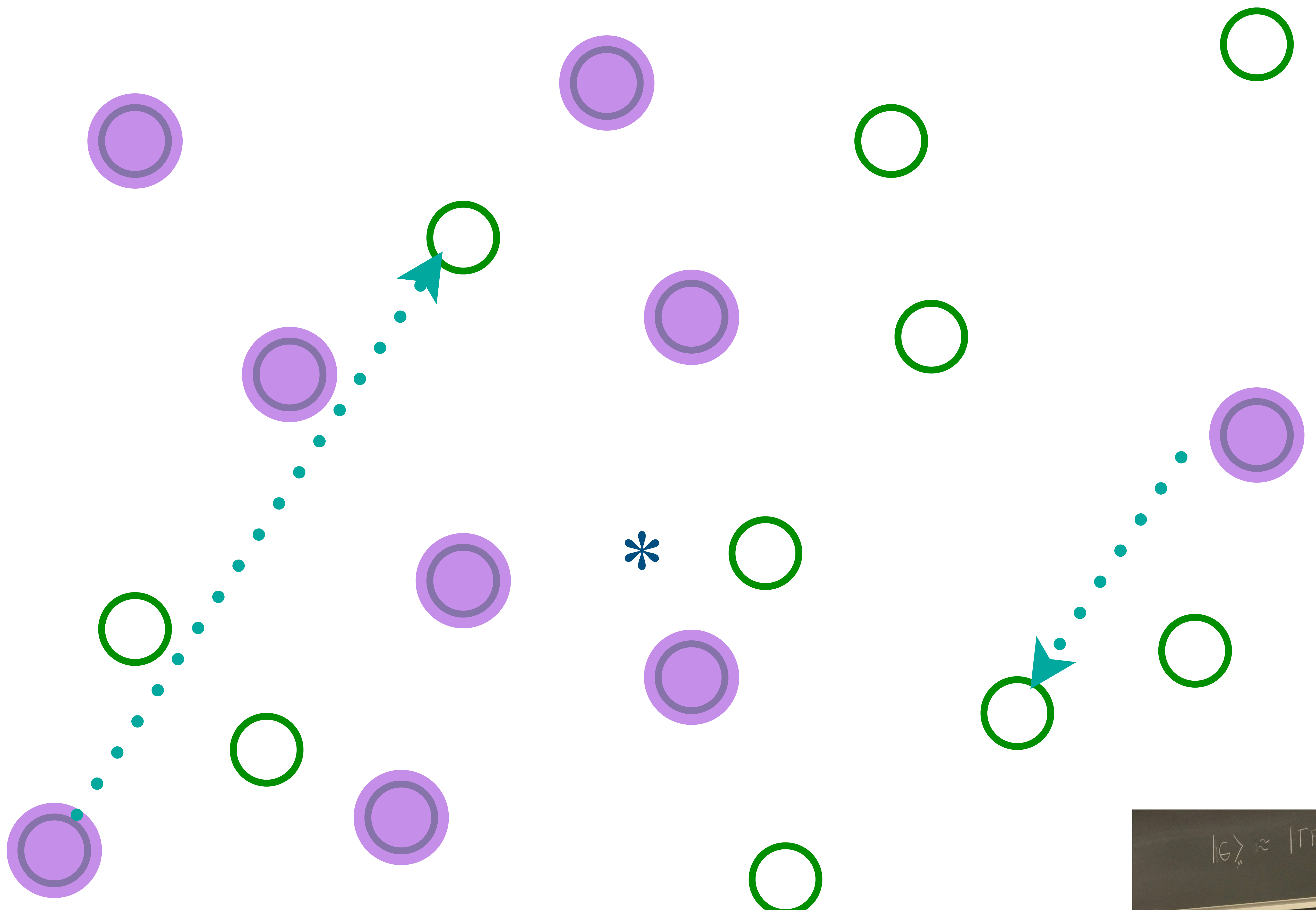

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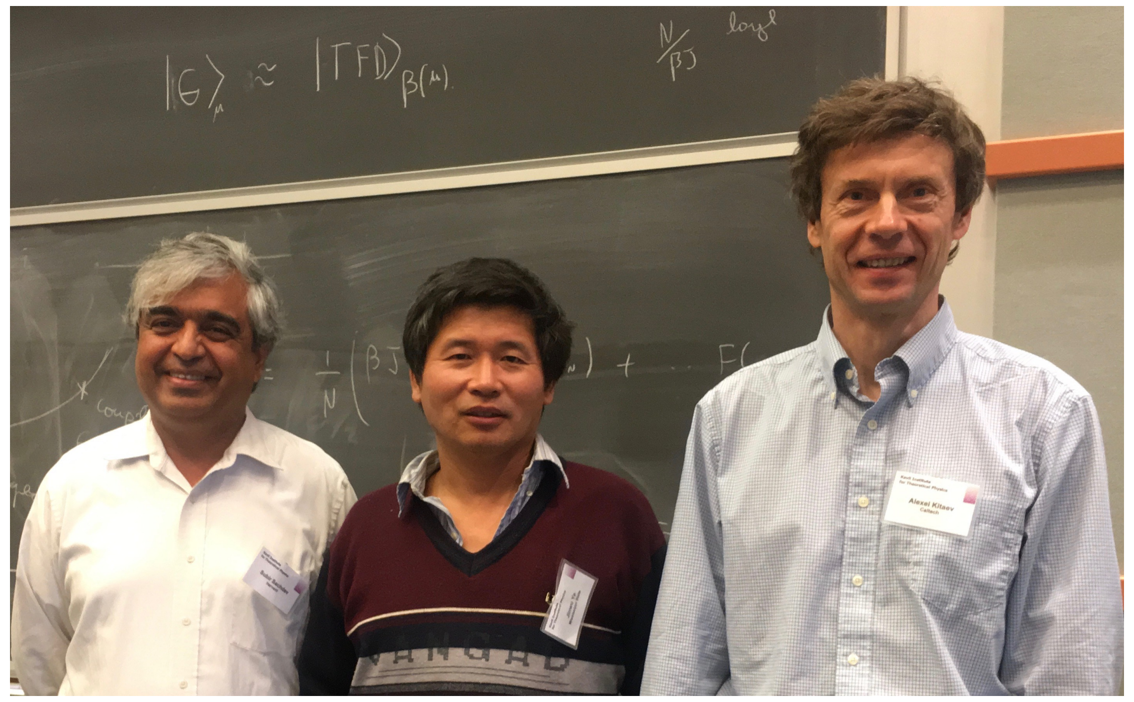

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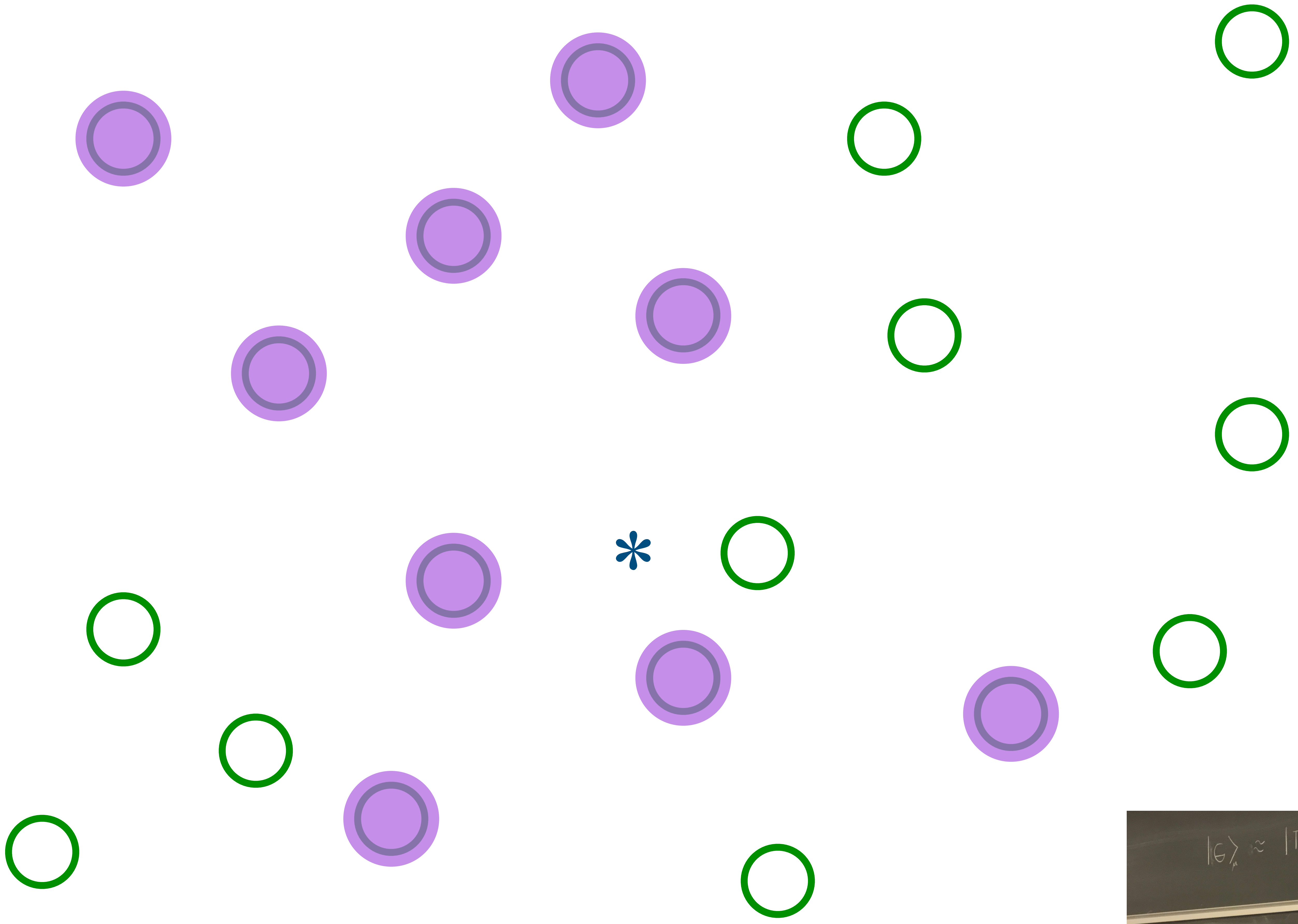

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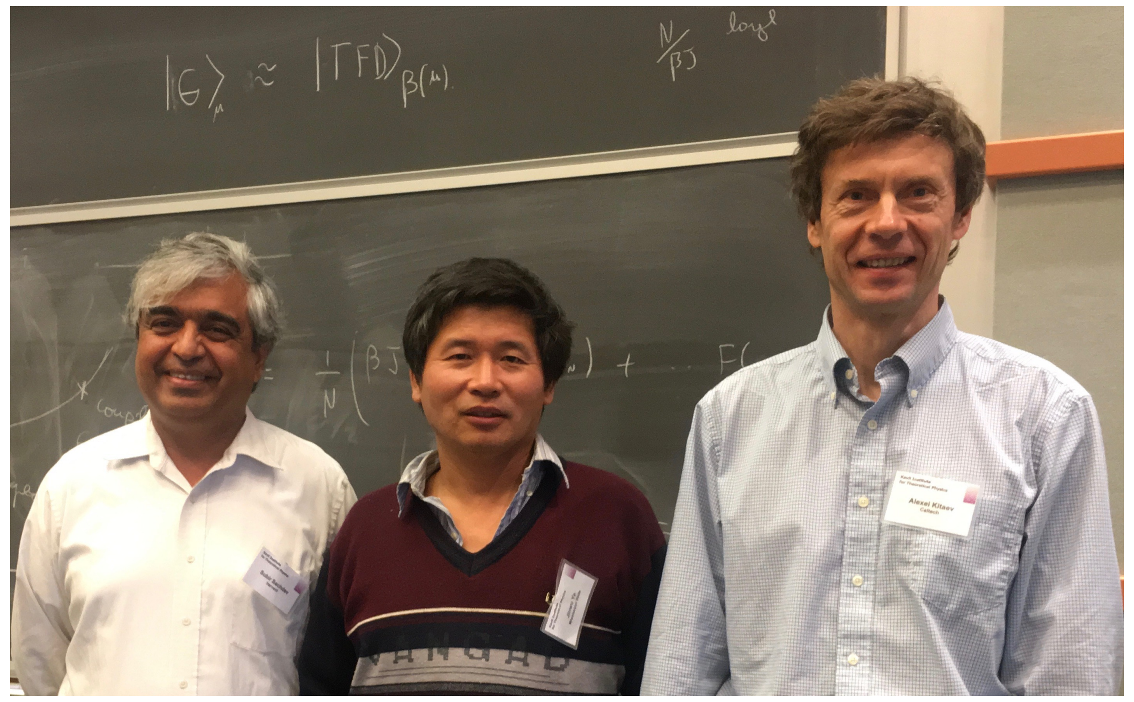

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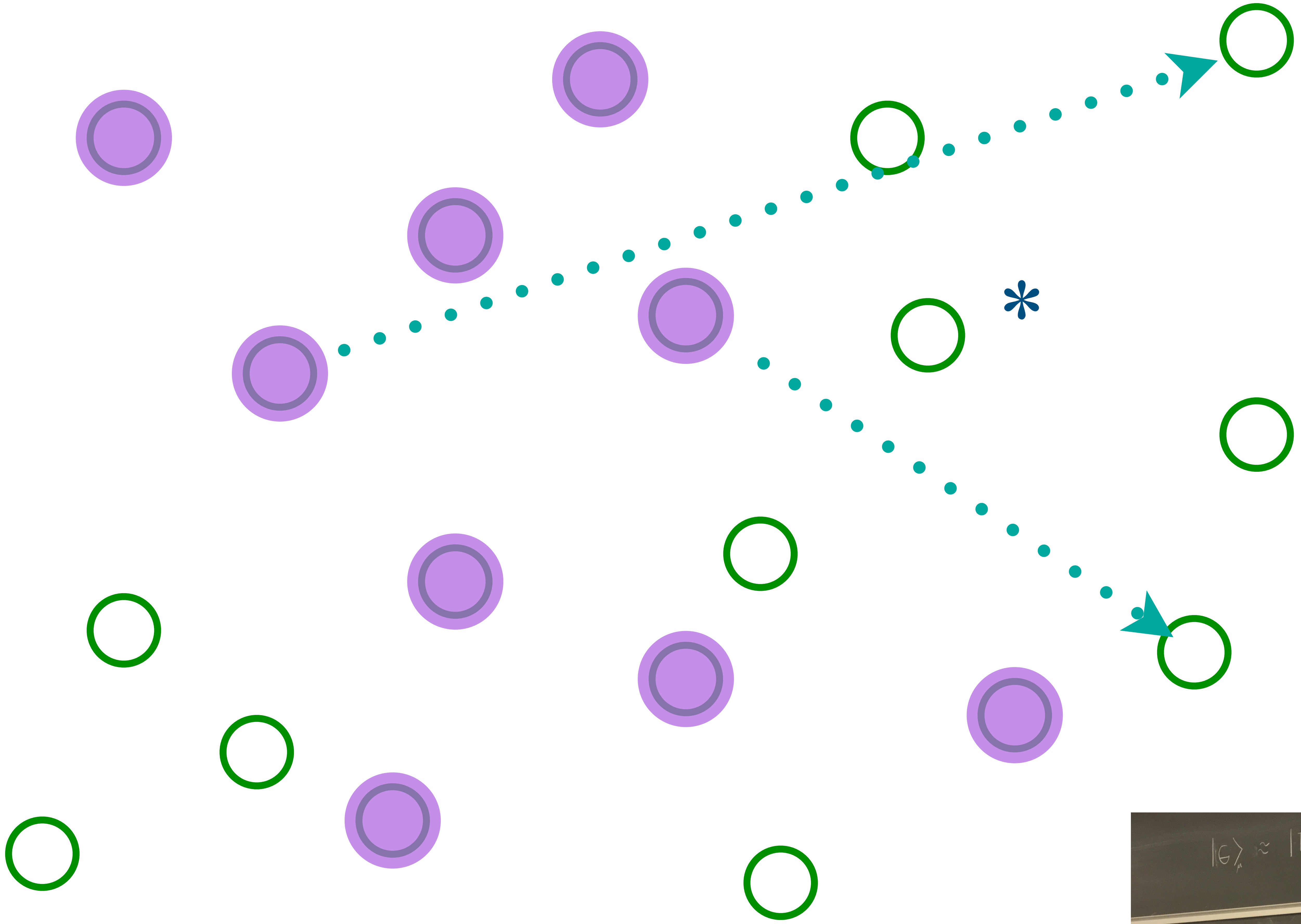

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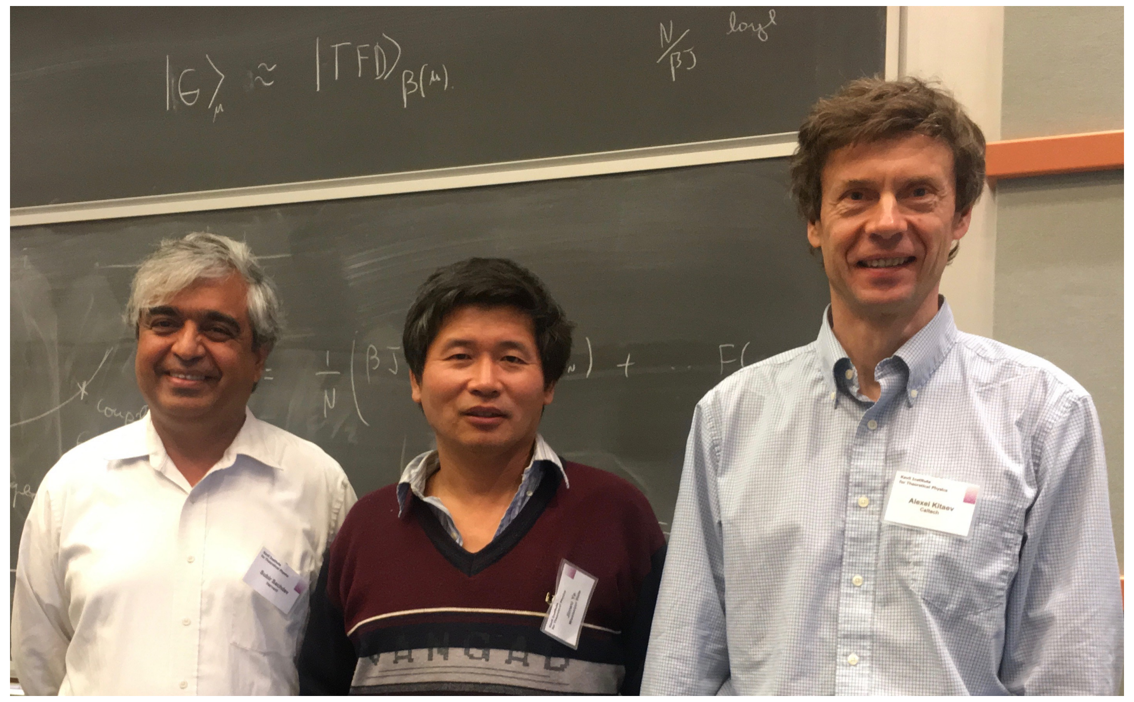

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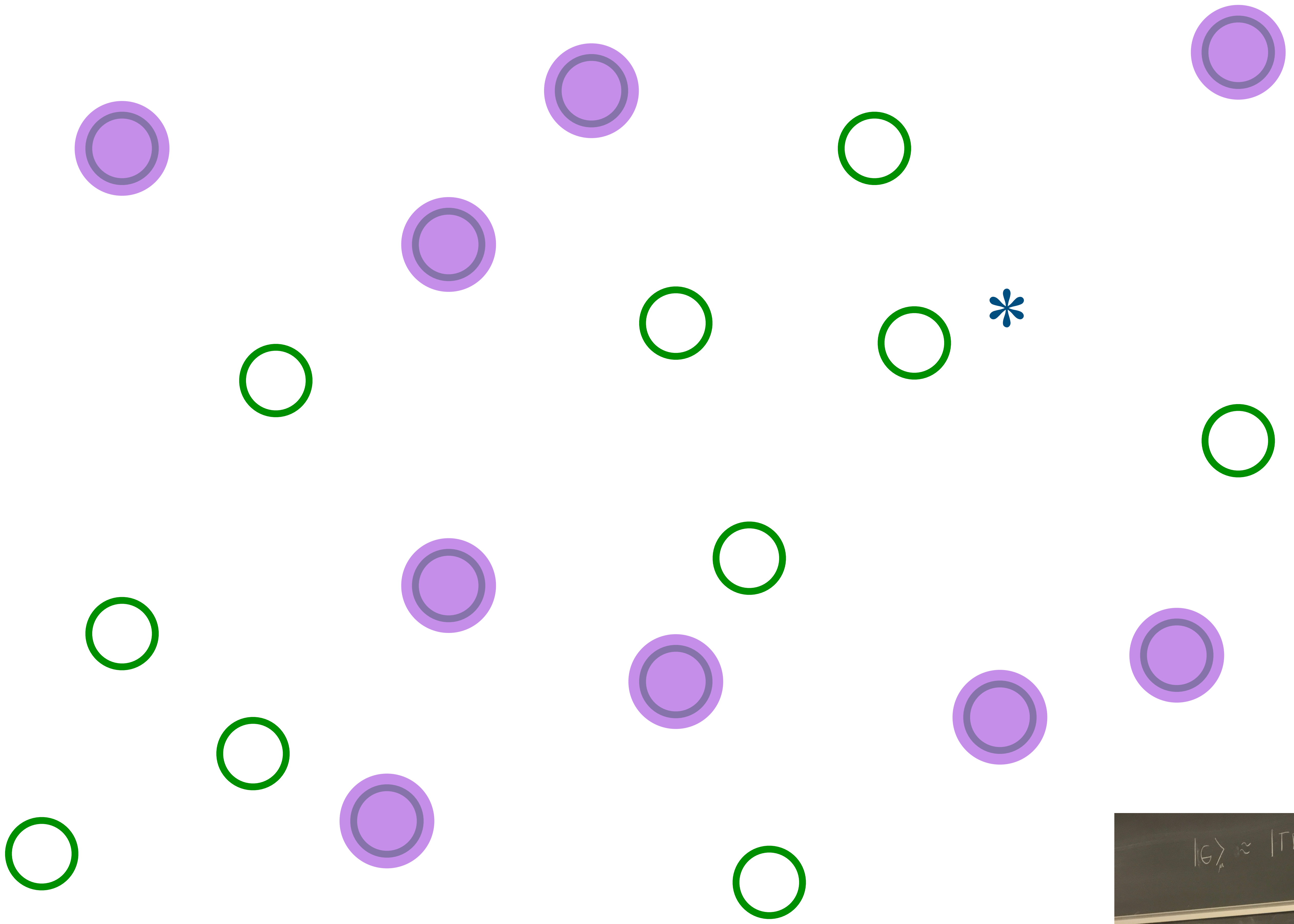

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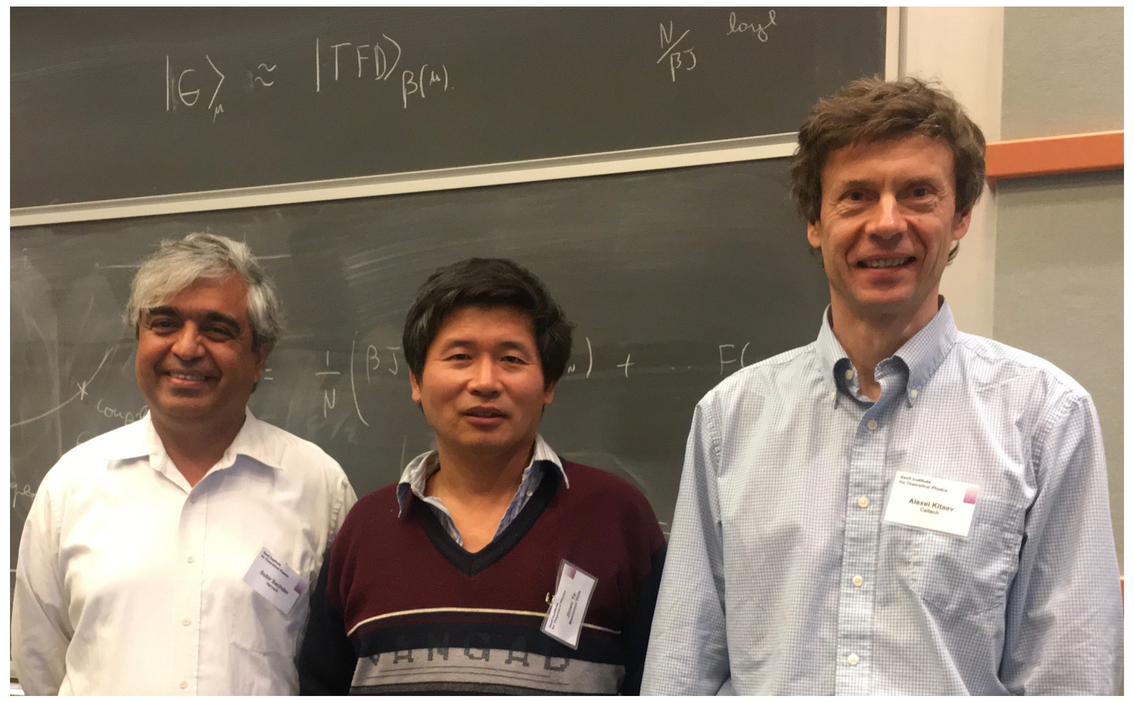

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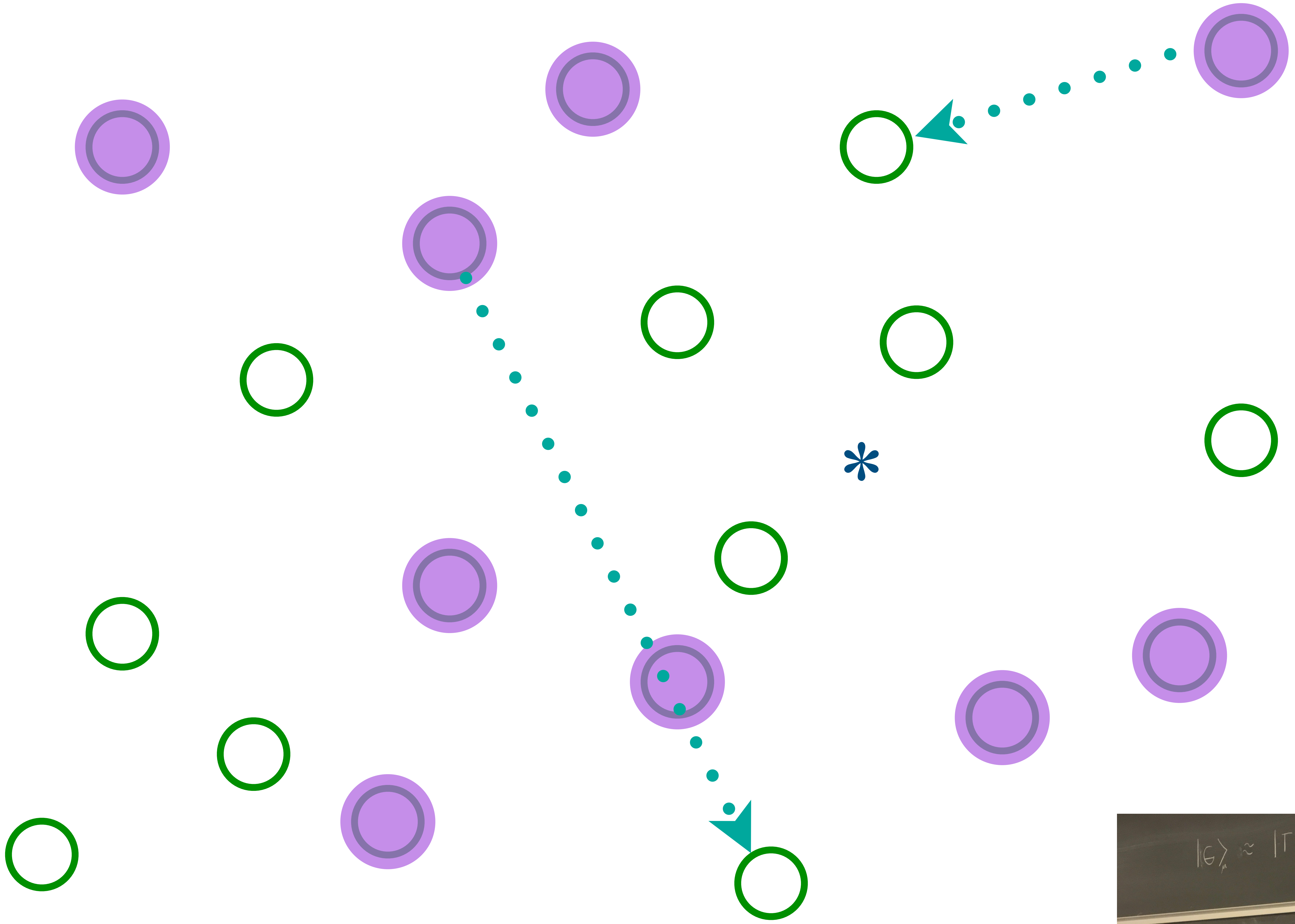

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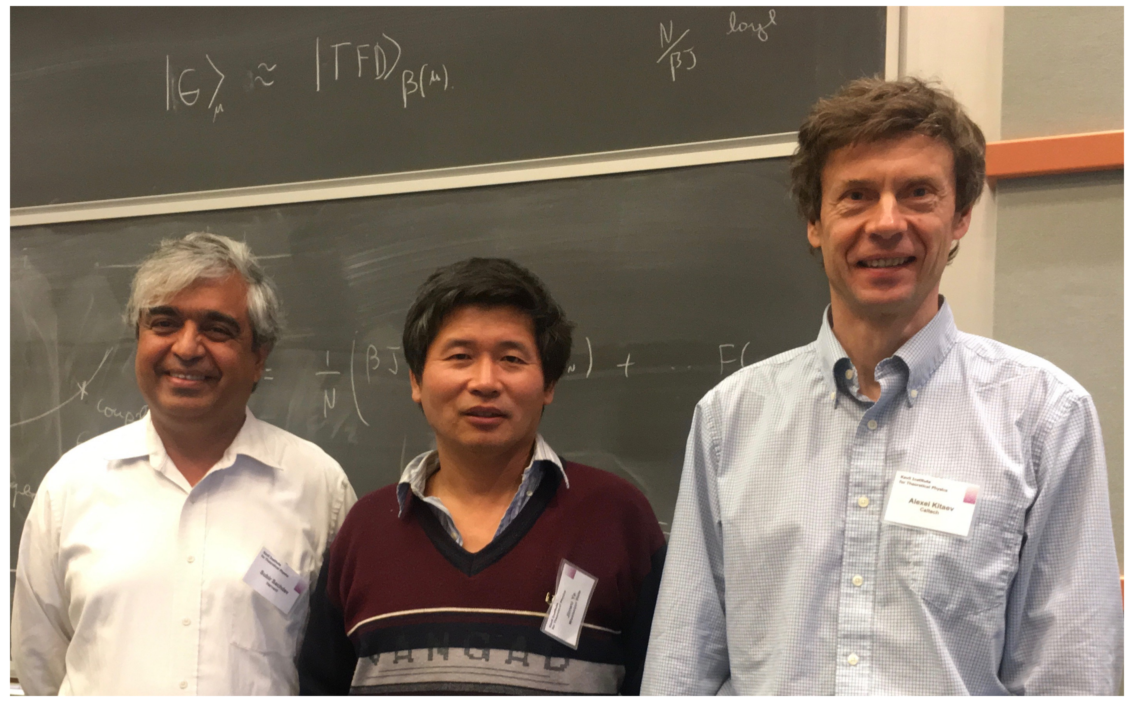

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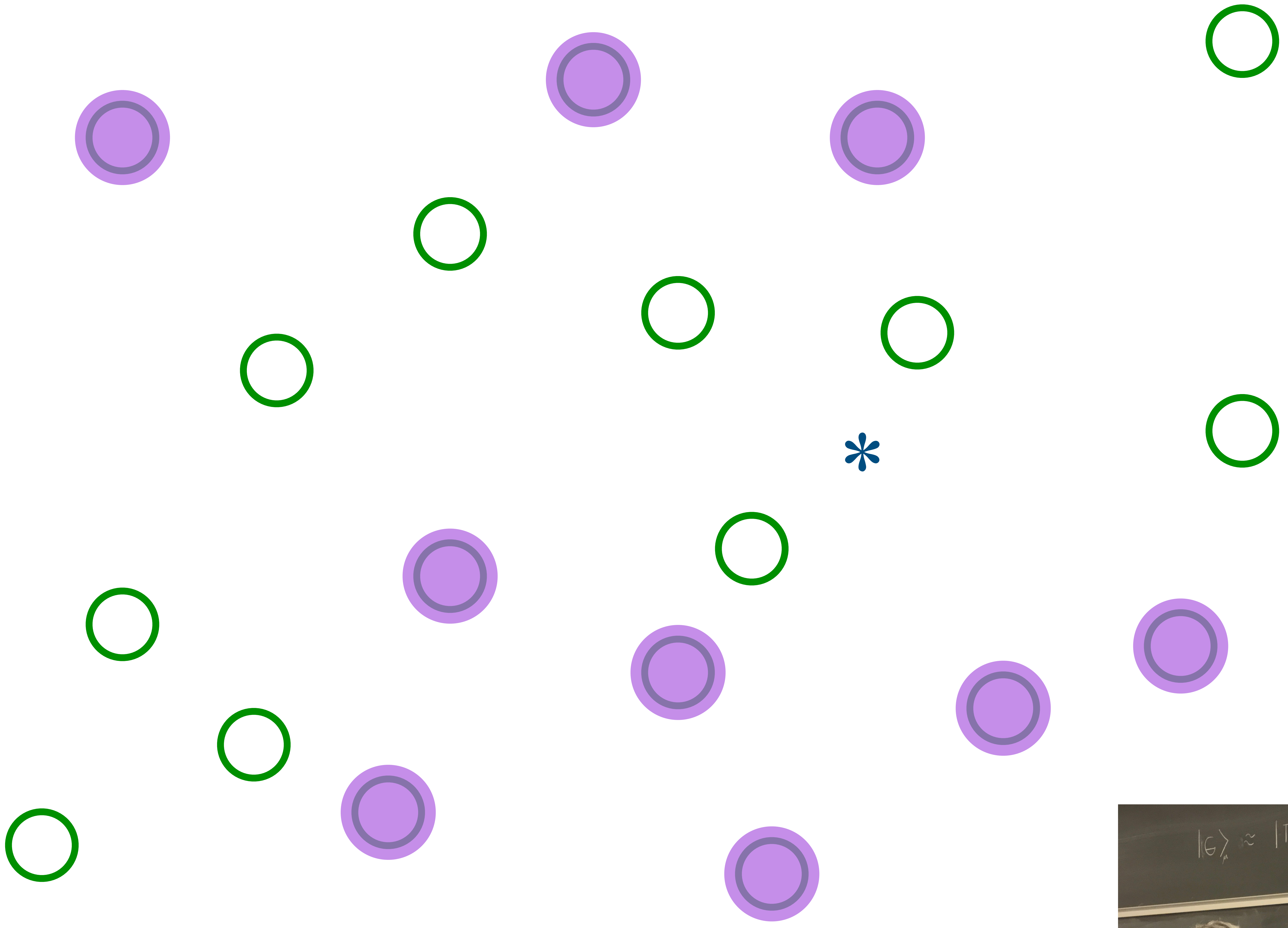

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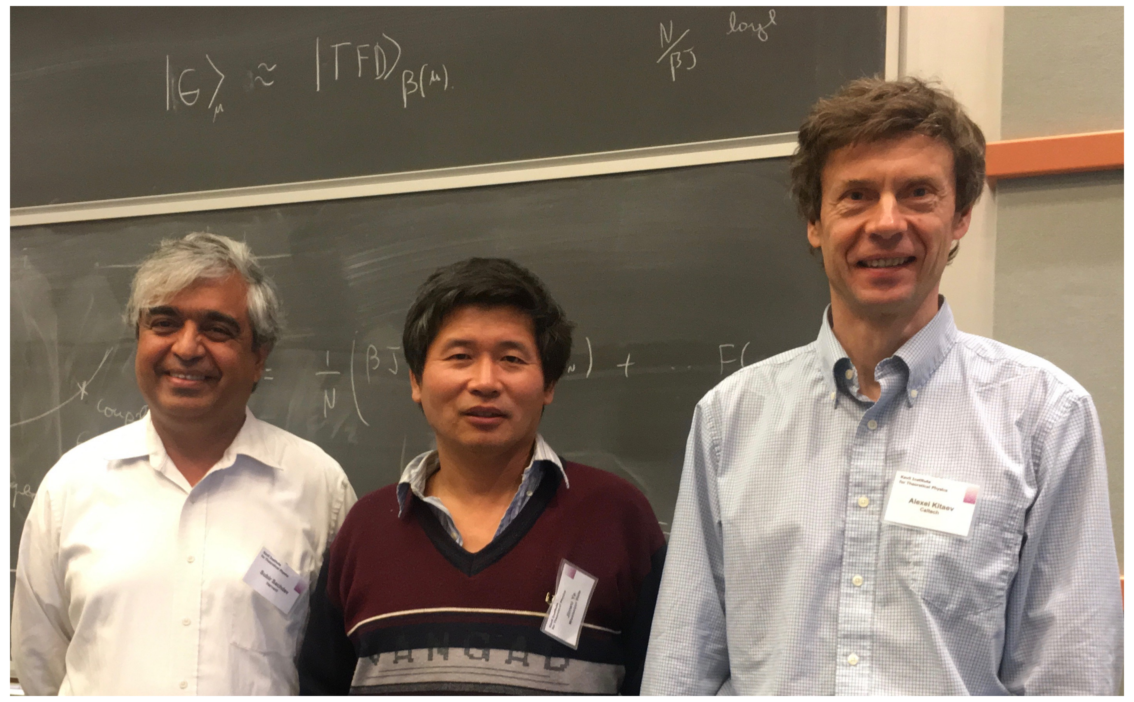


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

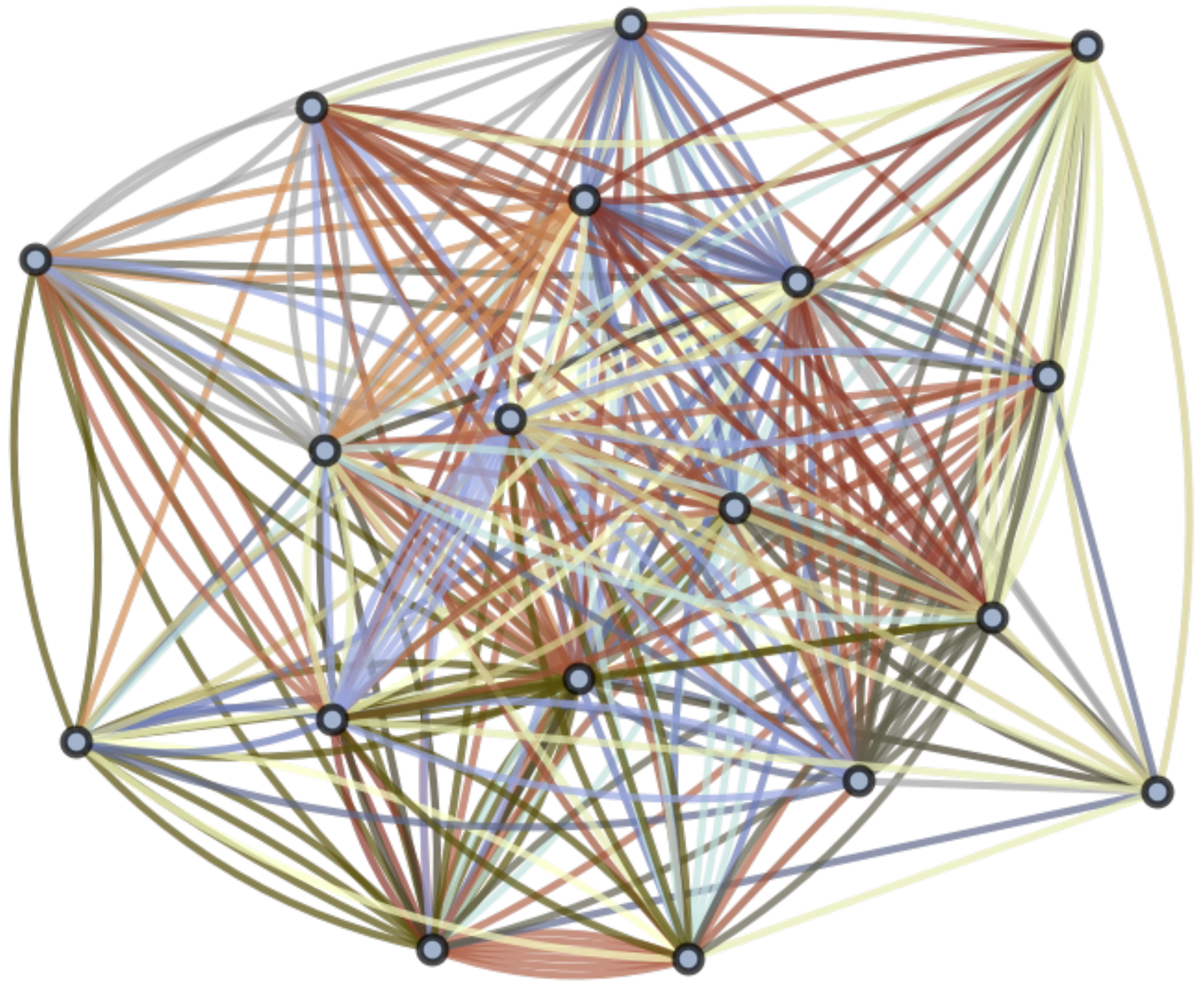
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

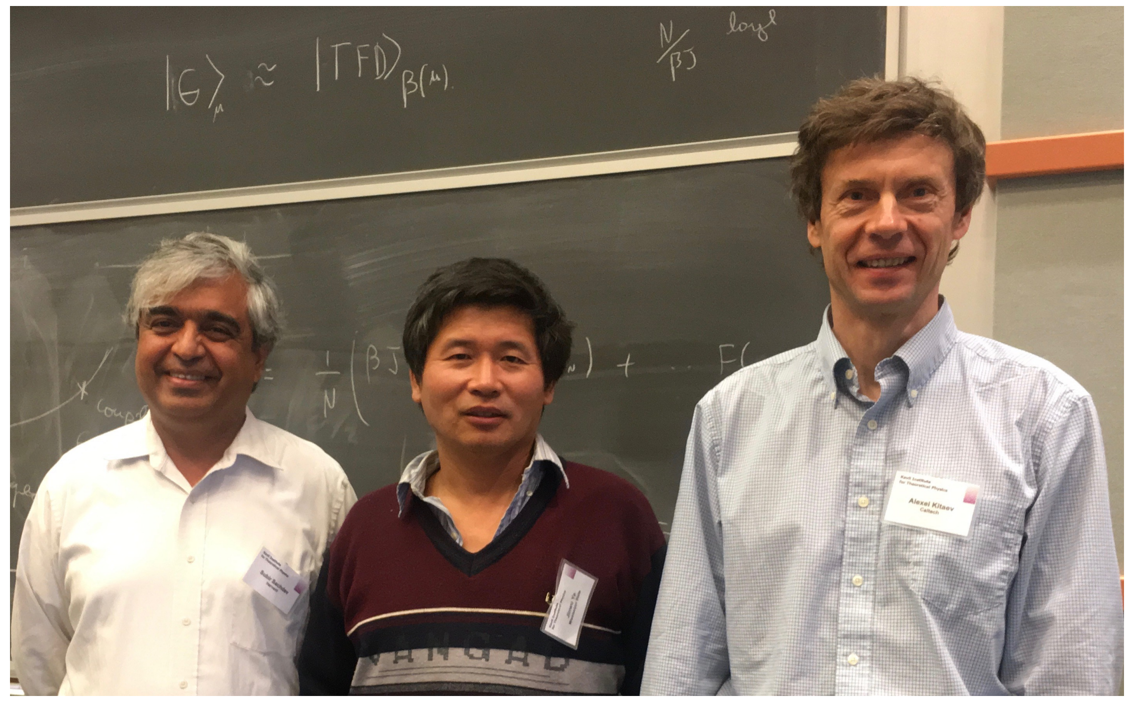
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

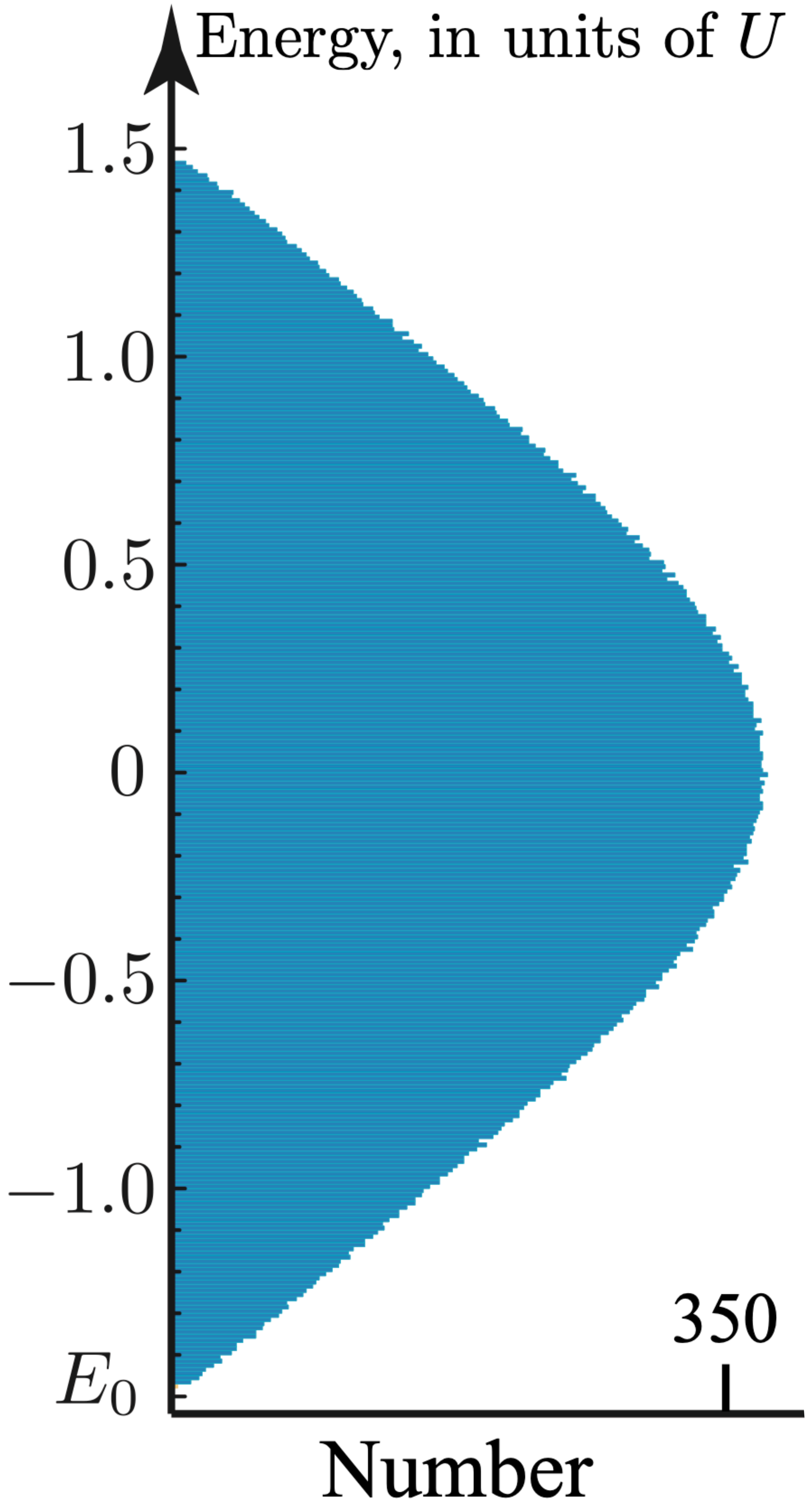
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The SYK model

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Many-body density of states

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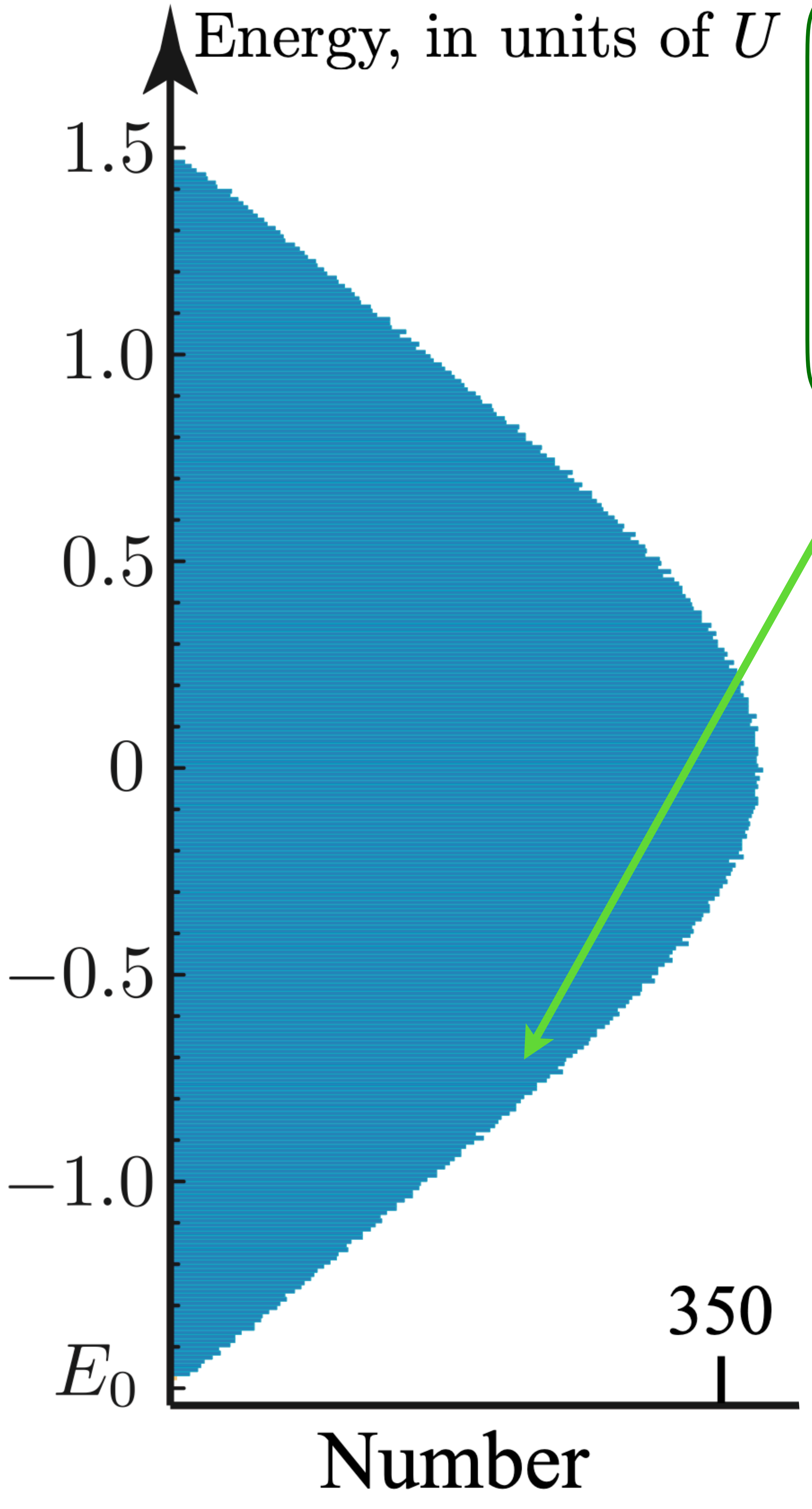

$$D(E) \sim e^{S(E)}$$
$$= e^{Ns_0 + \sqrt{2N\gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

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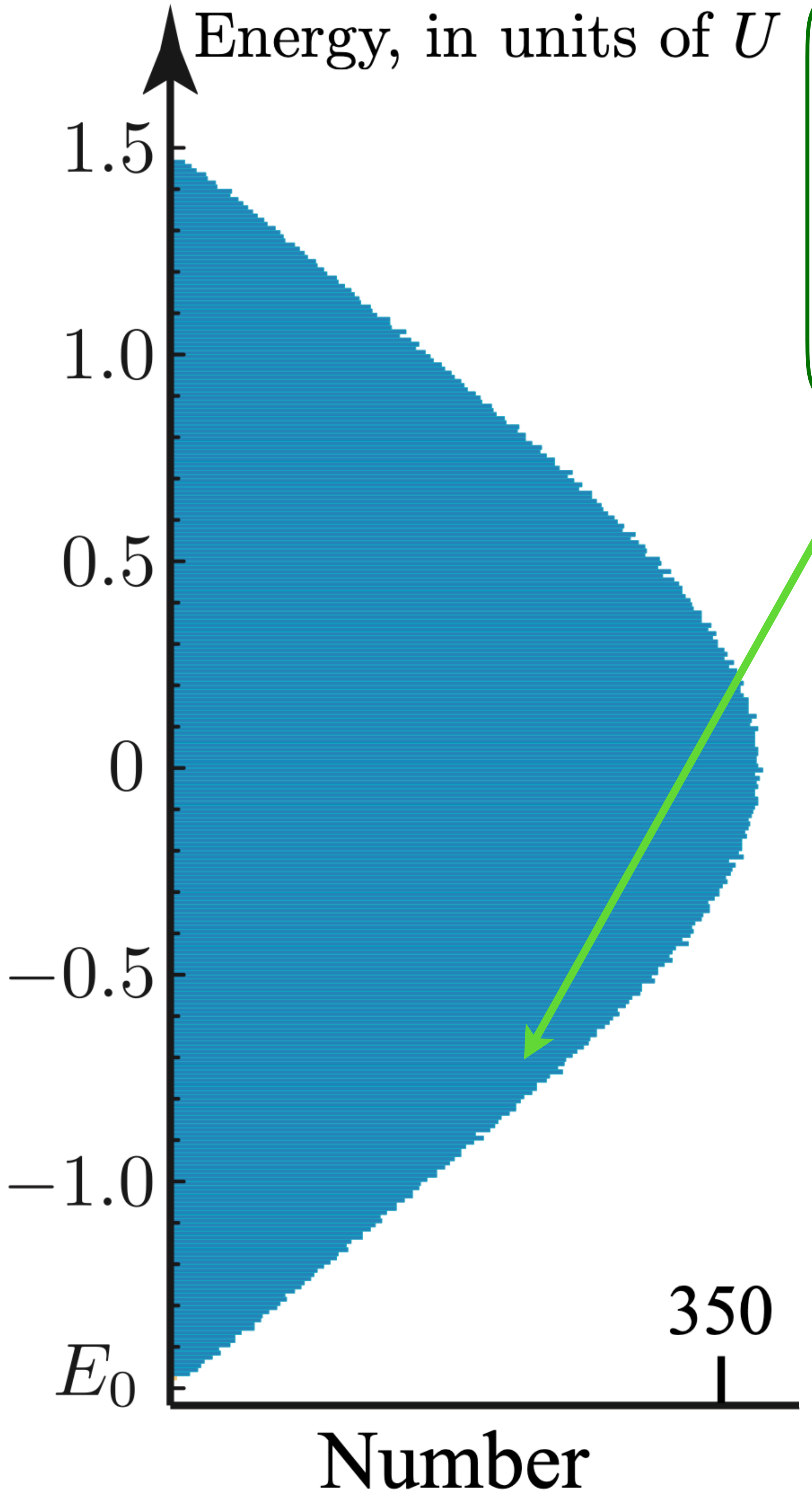

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$s_0 = 0.464848\ldots$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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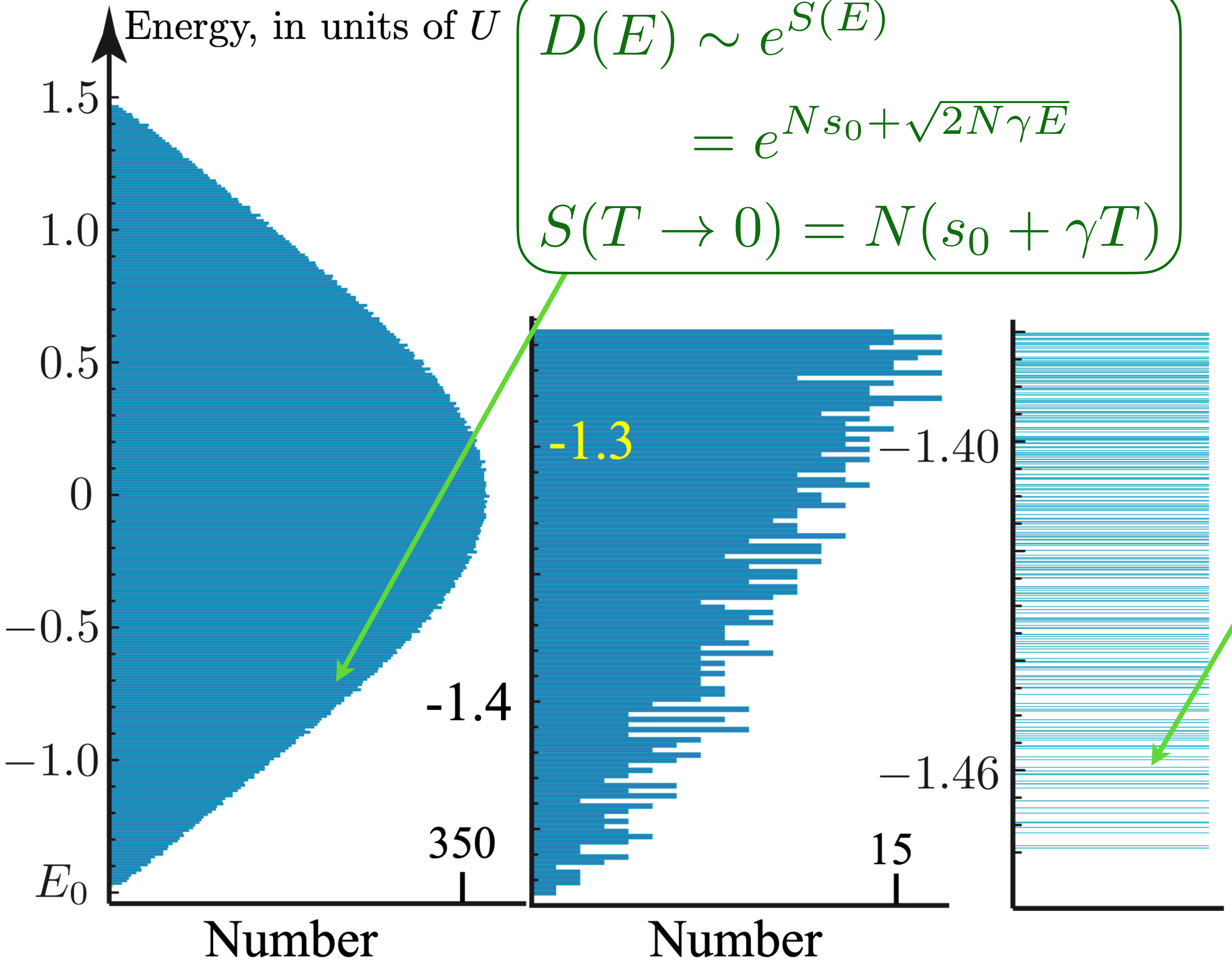

$$D(E) \sim 2\, e^{N s_0} \sqrt{2N\gamma E}$$

No particle-like decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848\ldots$$

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The SYK model

Universal Planckian time dynamics

- Observables have Planckian time scaling
  'Green's function' $G(\omega, T) \sim \omega^{-1/2} F(\hbar\omega / k_B T)$.

- The low energy theory is expressed in terms of a 'time reparameterization mode'.
  Similar to: Einstein's general relativity is expressed in terms of curvature of spacetime
  (a 'spacetime reparameterization mode').

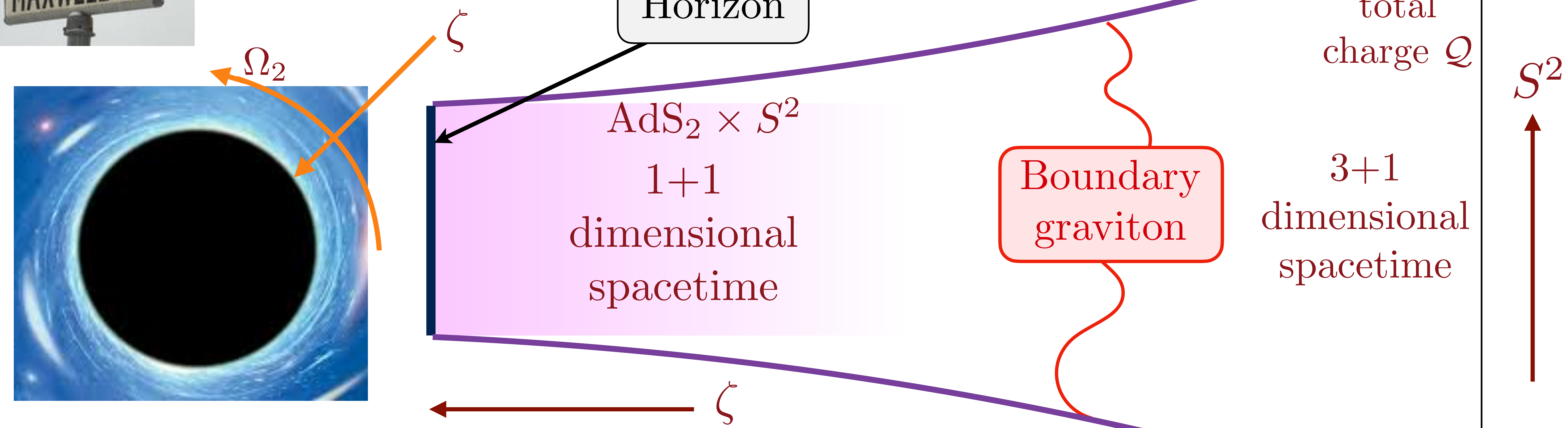
D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

From the SYK model to charged black holes

Reissner-Nordstrom black hole of Einstein-Maxwell theory

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Is there a mapping to a quantum system in 0+1 dimensions?

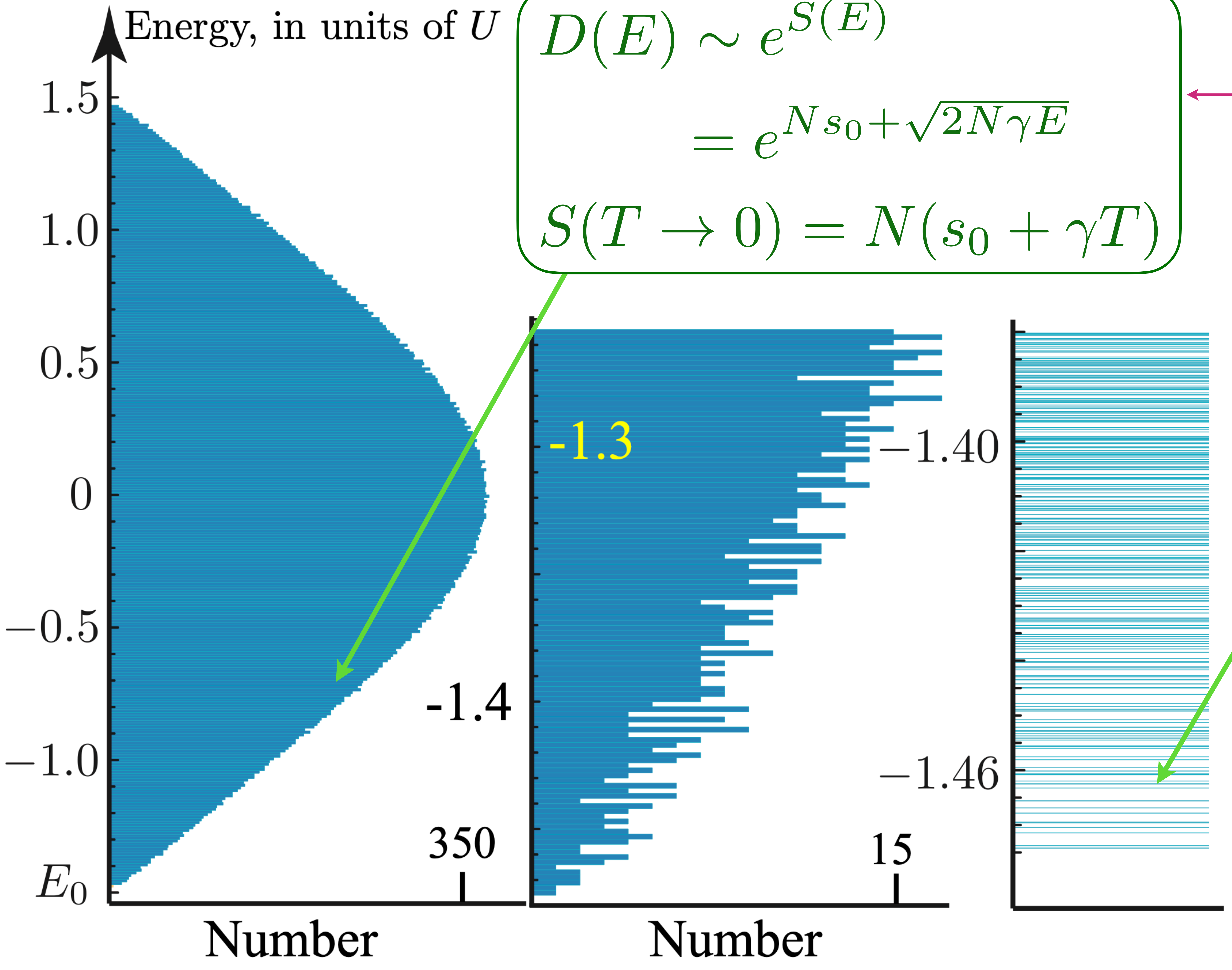
Reissner-Nordstrom black hole of Einstein-Maxwell theory

Yes!

The low energy theory of a charged black hole is *exactly* the low energy theory of time reparameterizations of the SYK model.

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



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$$D(E) \sim 2\, e^{Ns_0} \sinh(\sqrt{2N\gamma E})$$
$$S(T) = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{U}{T}\right)$$

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Complex SYK model

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

Same entropy and (coarse-grained) density of states in a model of interacting (fermionic) qubits with a discrete spectrum!

$$D(E) \sim e^{S(E)} = e^{Ns_0 + \sqrt{2N\gamma E}}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2\, e^{Ns_0} \sinh(\sqrt{2N\gamma E})$$

$$S(T) = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{U}{T}\right)$$

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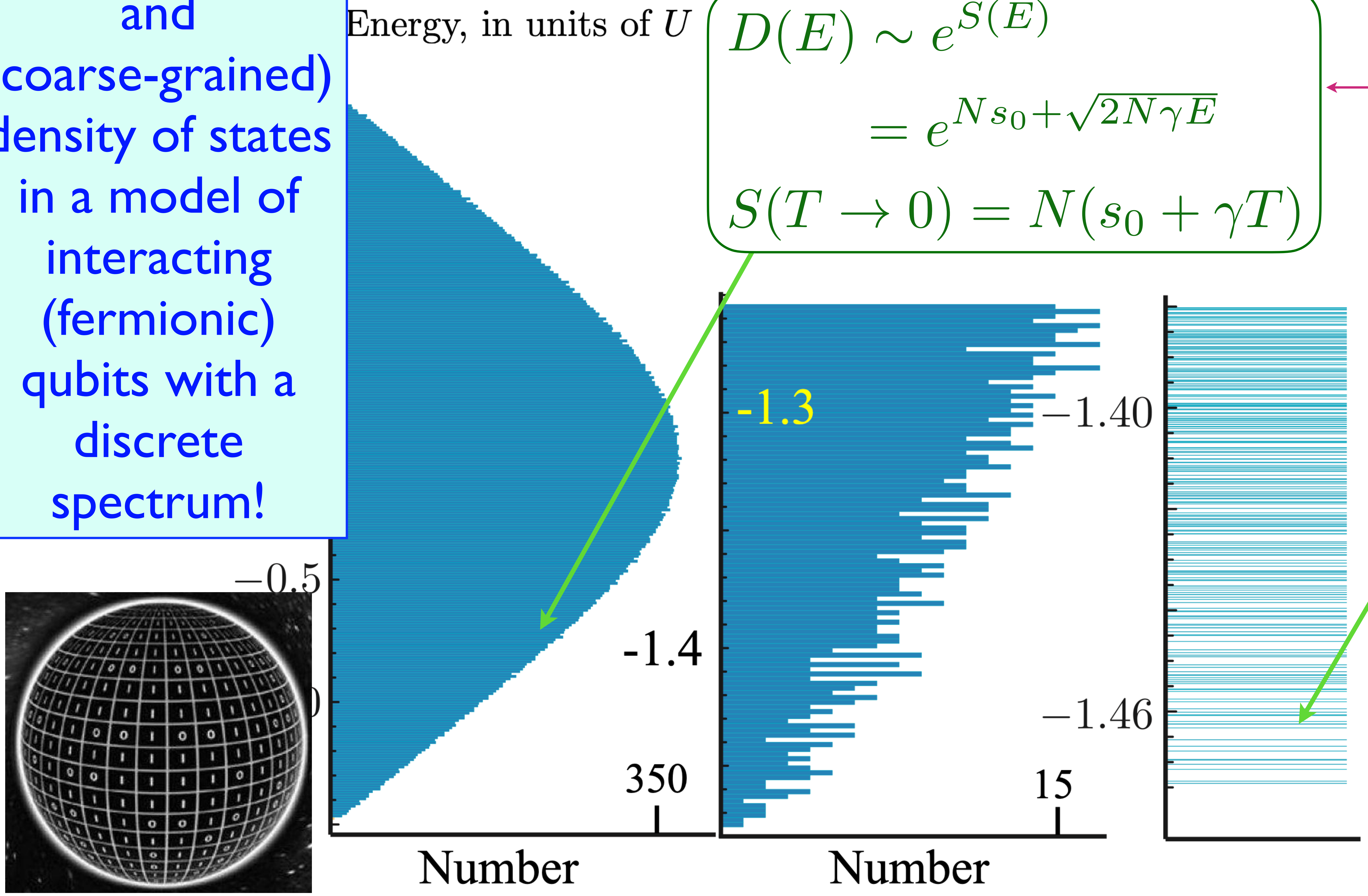

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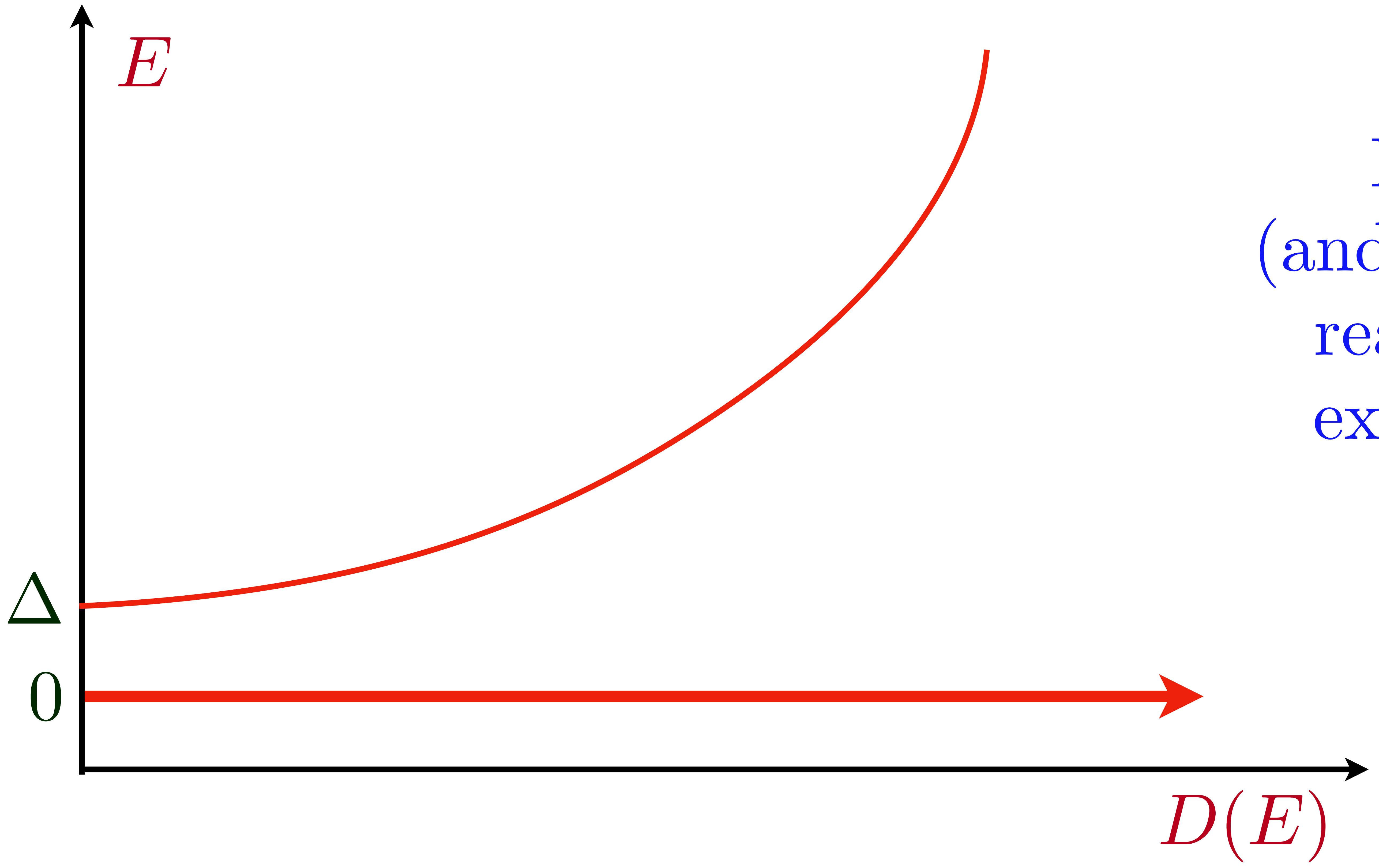


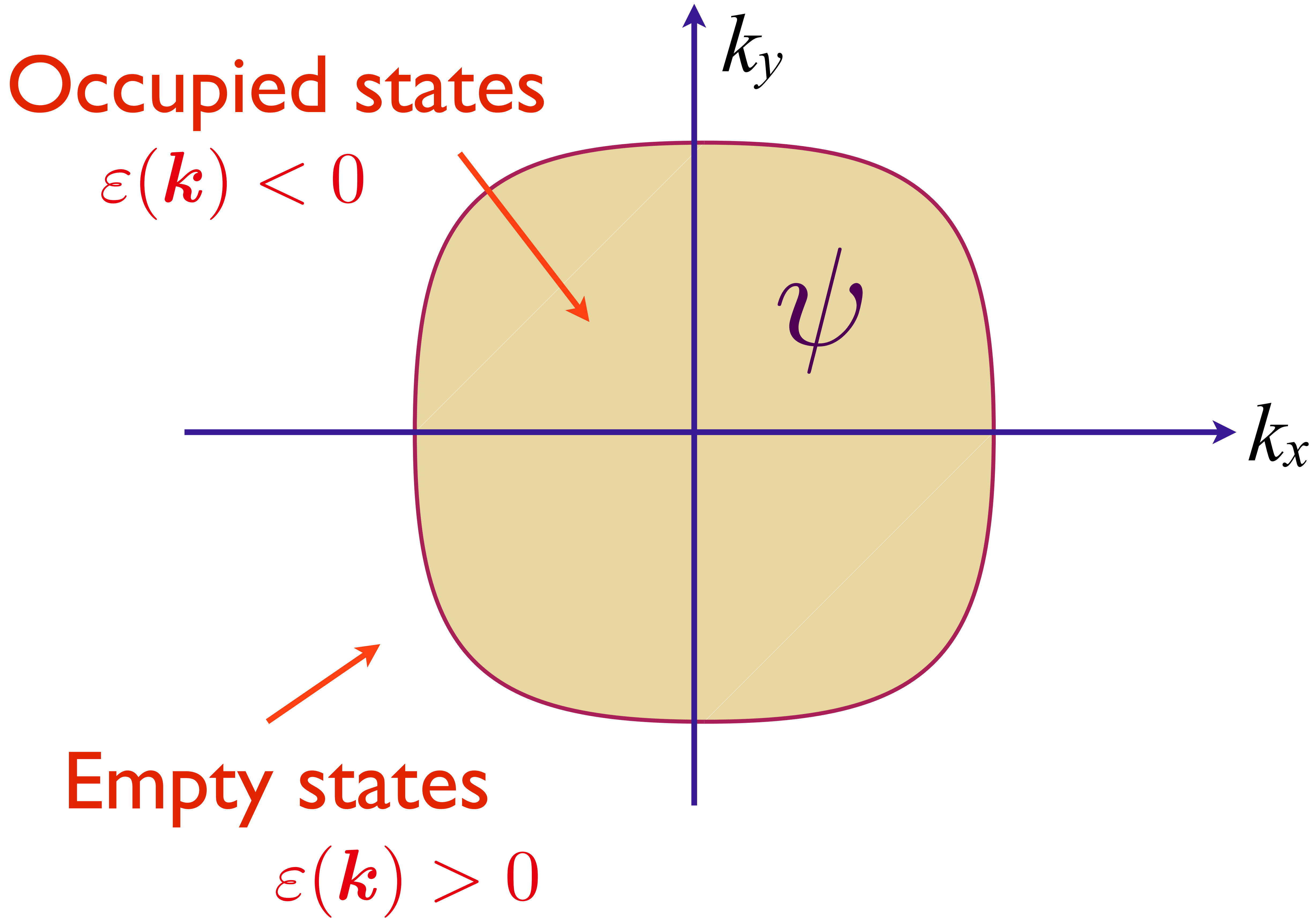
Black holes in string theory
(and supersymmetric SYK models)
realize the entropy as an exact,
exponentially large, degeneracy
of the ground state.

$$D(E) \sim \exp\left(\frac{A_0}{4G} + \ldots\right)\delta(E) + f_{\rm reg}(E-\Delta), \quad \Delta \sim R_h^{-1}$$

Fu, Gaiotto, Maldacena, Sachdev (2017); Stanford, Witten (2017); Heydeman, Iliesiu, Turiaci, Zhao (2020)

From the SYK model to linear-T resistivity

Fermi surface coupled to a critical boson



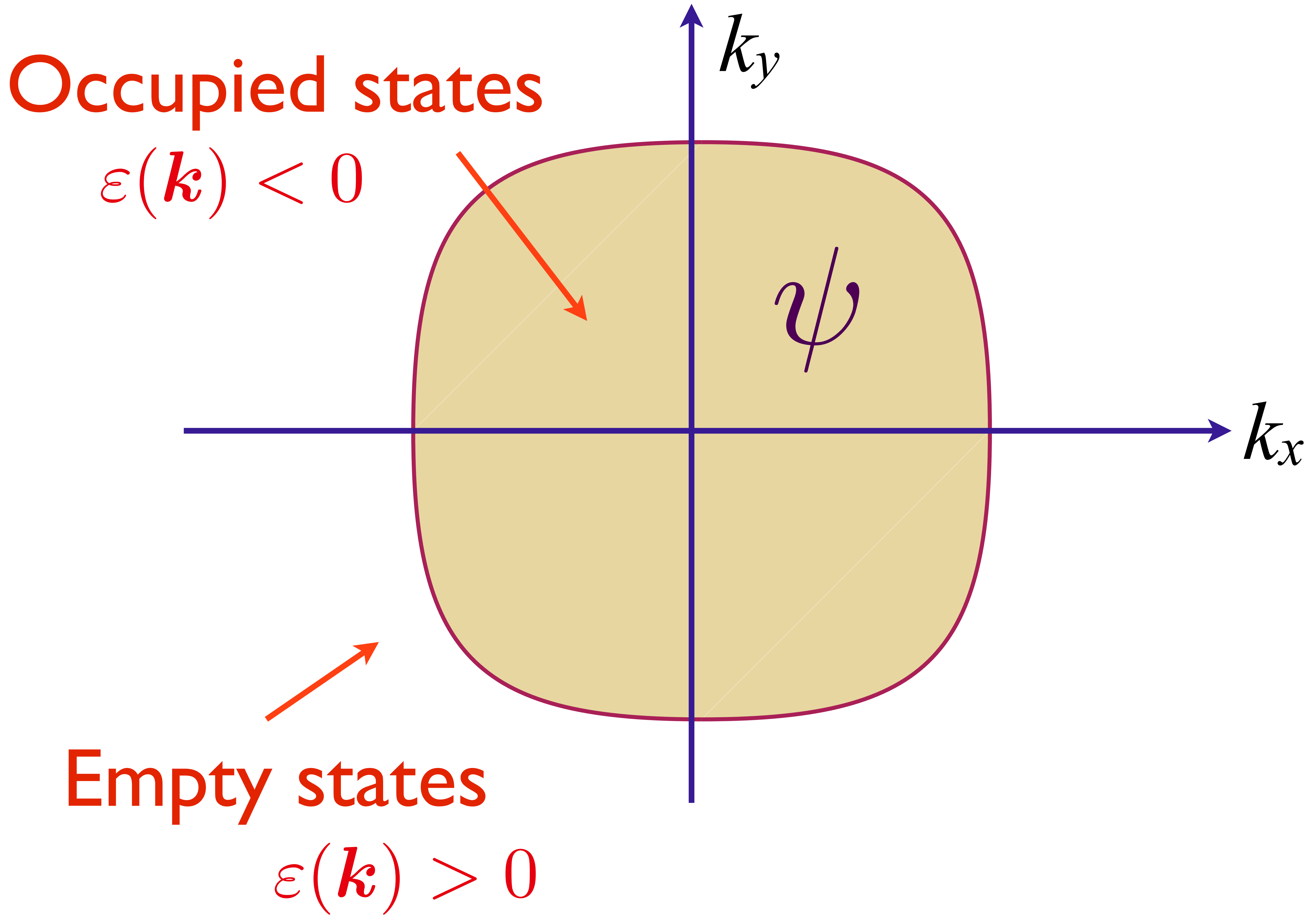
+

a critical boson

ϕ

- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field

Fermi surface coupled to a critical boson



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"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

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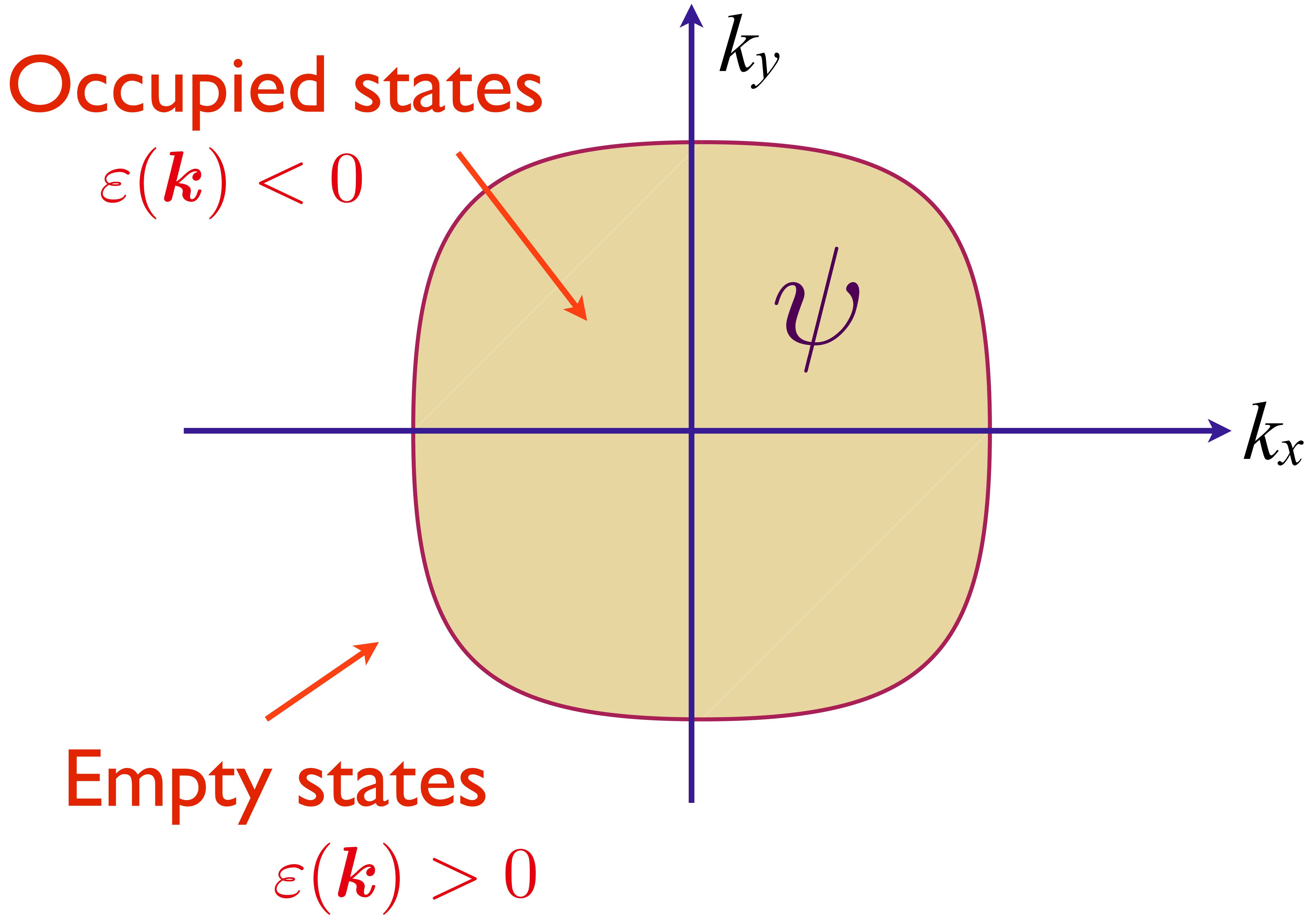

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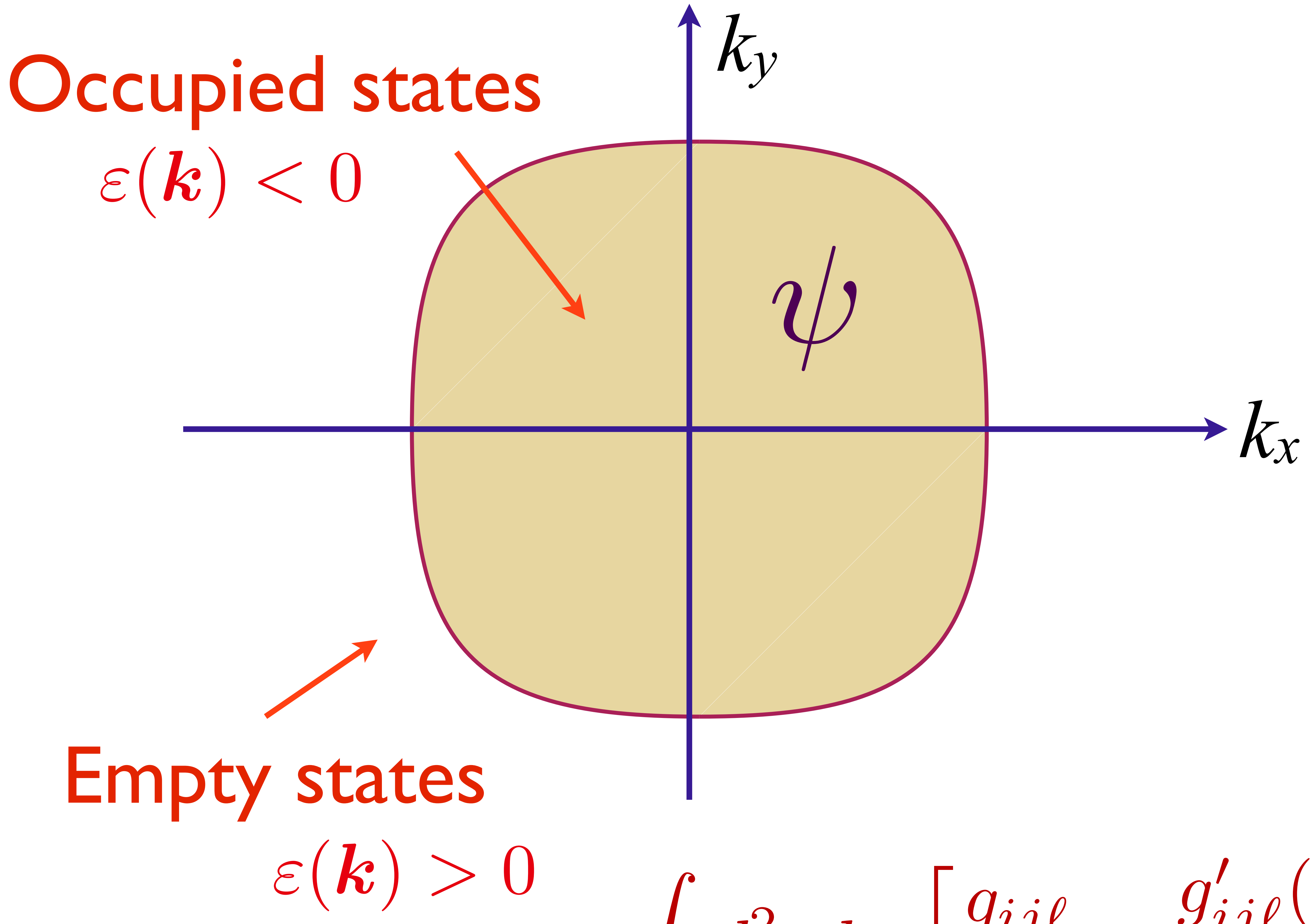
"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N} \int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

Random couplings in flavor space leads to large N theory of a strange metal, with zero resistivity

Fermi surface coupled to a critical boson



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$$\int d^2r d\tau \left[\frac{g_{ij\ell}}{N} + \frac{g'_{ij\ell}(r)}{N} \right] \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

Random couplings in flavor *and* position space leads to large N theory of a strange metal, with linear-T resistivity

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990

Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

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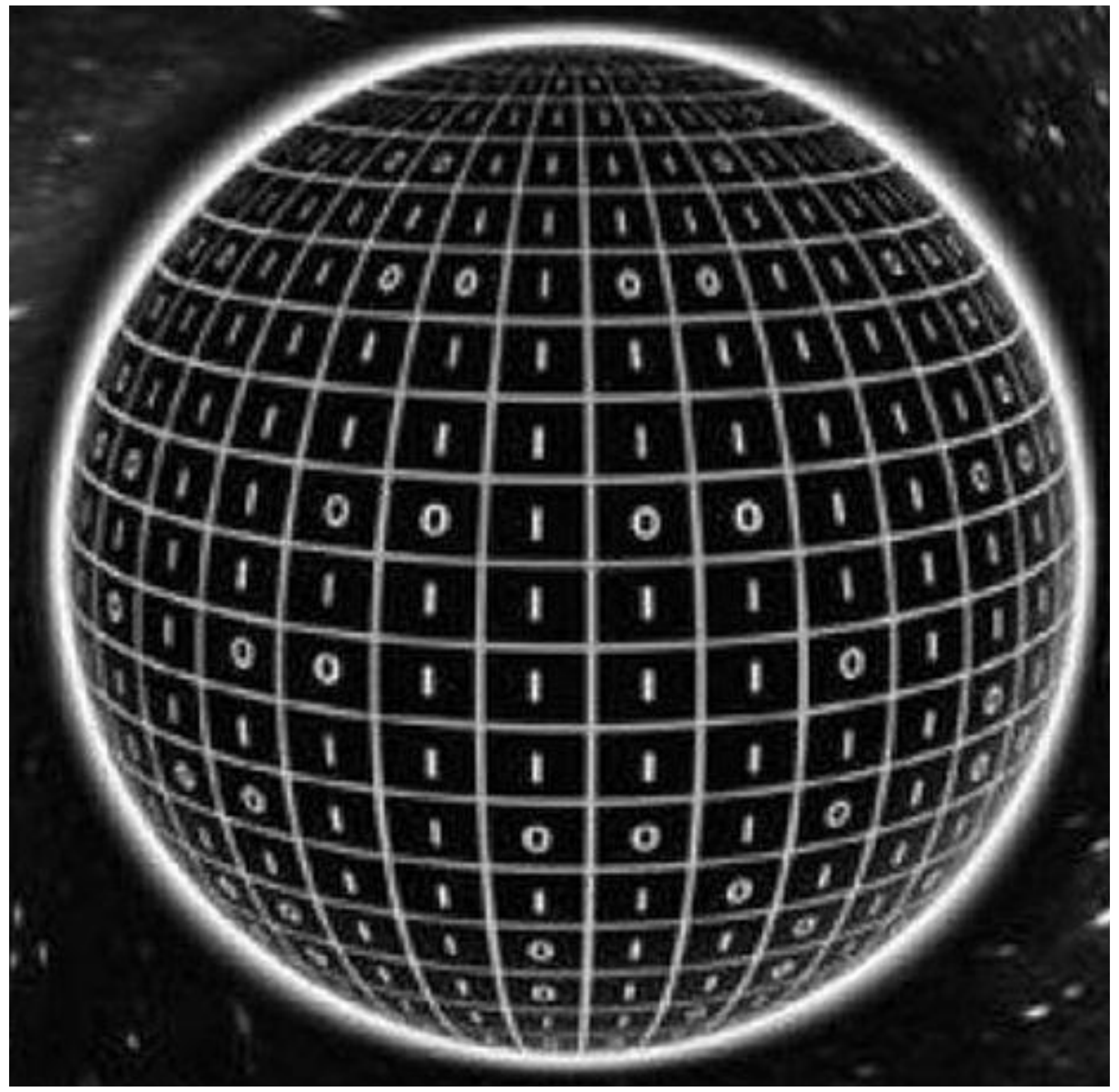
- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Low energy theory of time reparameterizations is the theory of the boundary graviton in 1+1 dimensional quantum gravity on AdS_2.

Summary

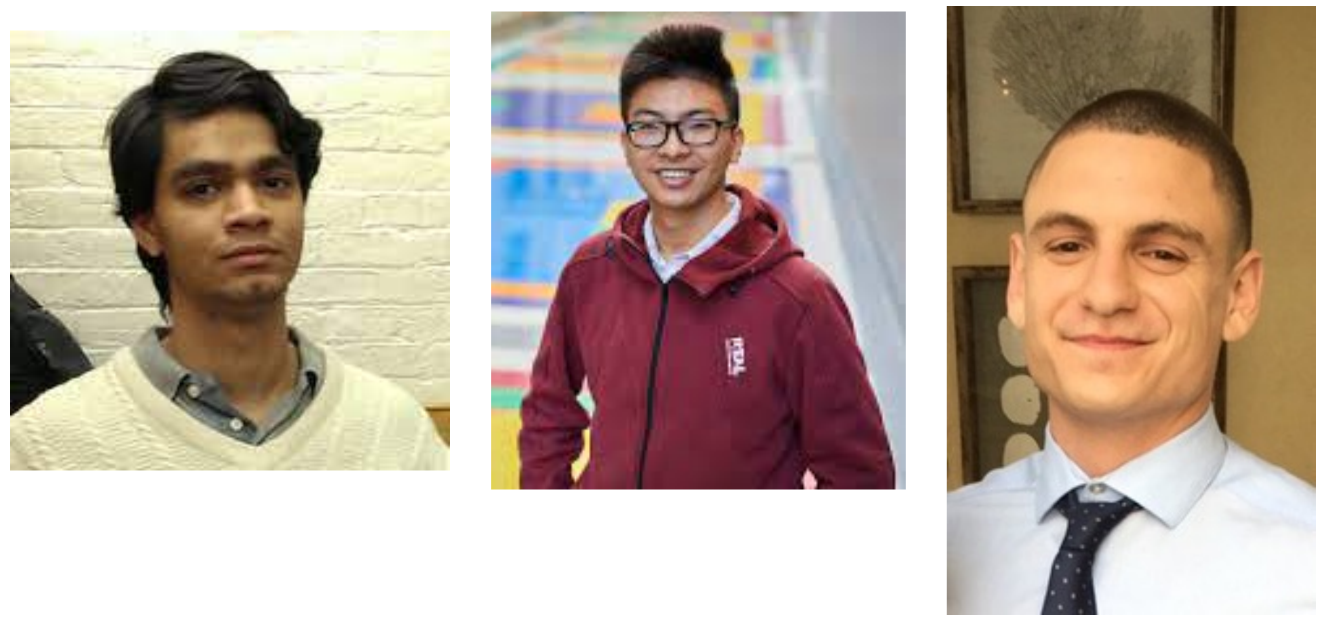
- The semiclassical entropy of Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.



Summary

- The semiclassical entropy of Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.

- Linear-T resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.



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