Quantum phases of matter with Rydberg atoms

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Princeton University
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Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
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INSTITUTE FOR ADVANCED STUDY

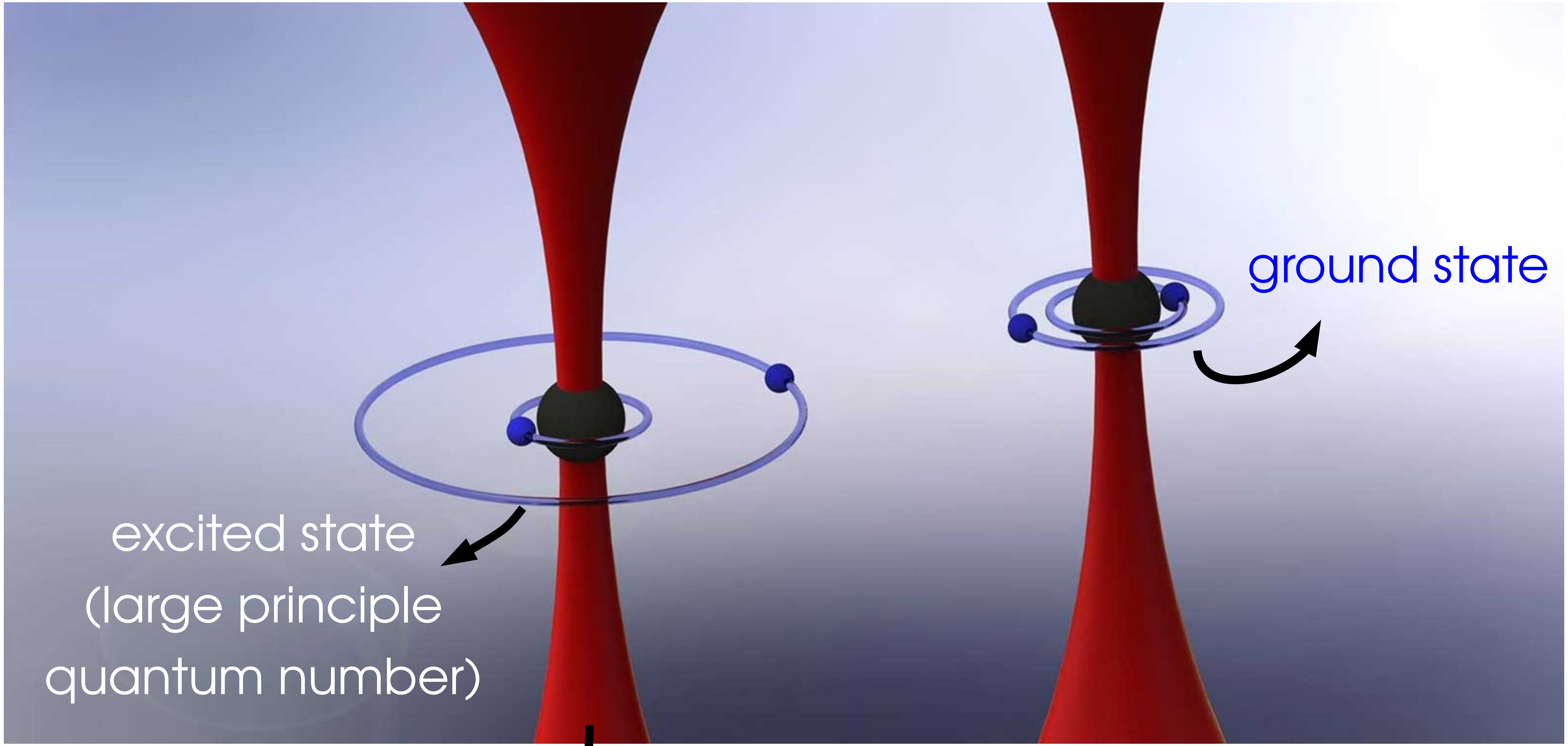
Rhine Samajdar

Seth Whitsitt

Wen Wei Ho

Hannes Pichler

Mikhail Lukin



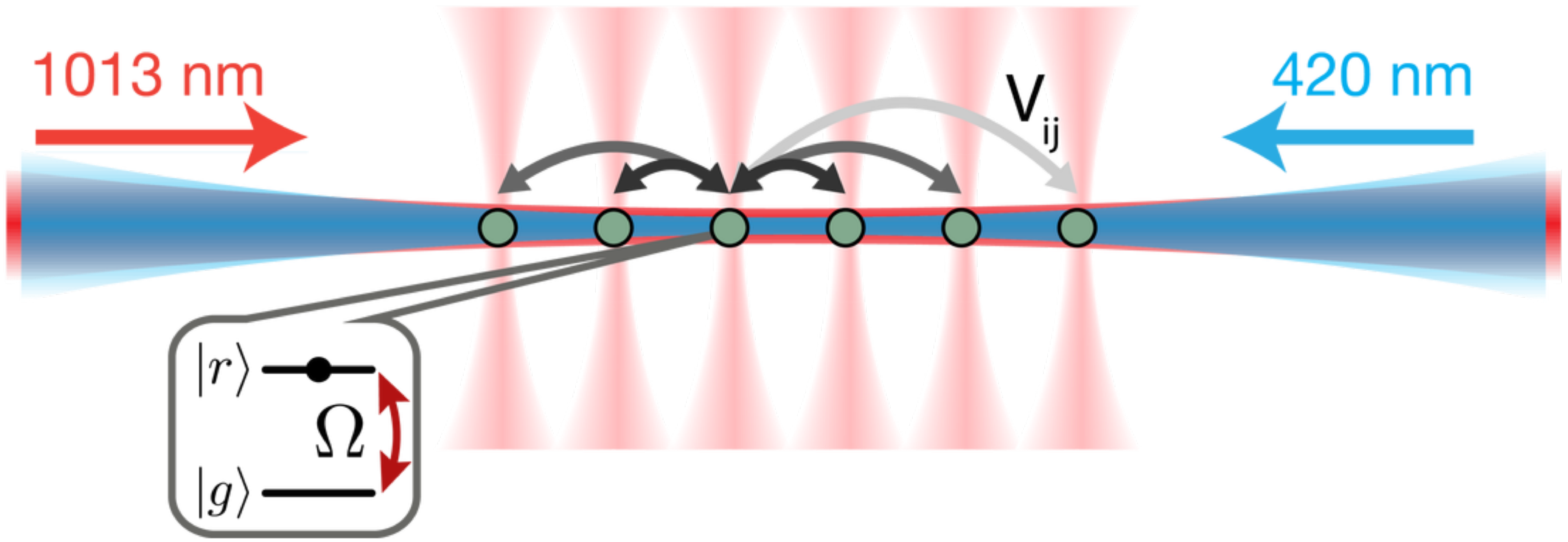
optical tweezer (traps atom)

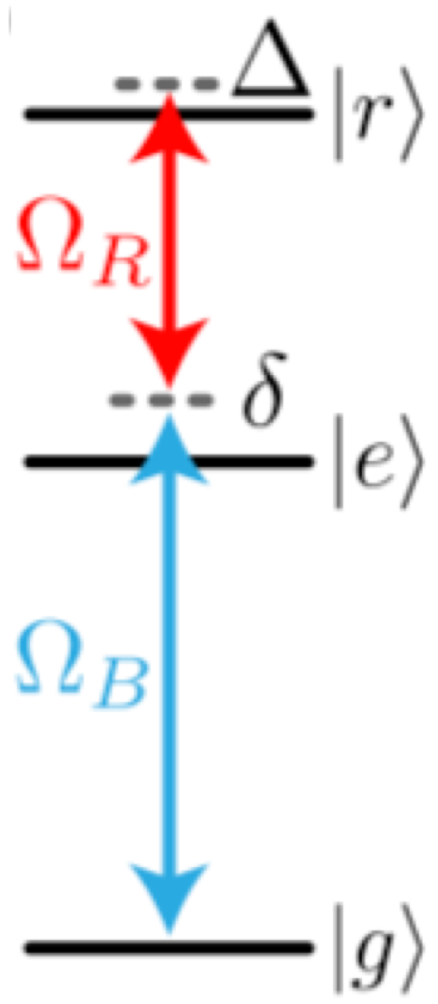
$$V_{|i-j|} \sim |i-j|^{-6}$$

Fig: https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms

$$H_{\mathrm{Ryd}} = \sum_i \left[\frac{\Omega}{2} \left(|g\rangle\langle r| + |r\rangle\langle g| \right)_i - \Delta |r\rangle\langle r| \right] + \sum_{(i,j)} V_{|i-j|} \Big(|r\rangle\langle r|_i \otimes |r\rangle\langle r|_j \Big)$$

QPTs in a Rydberg quantum simulator





$$H_{\text{Ryd}} = \sum_{i=1}^{N} \frac{\Omega}{2} \Big(|g\rangle\langle r| + |r\rangle\langle g| \Big)_i - \Delta \sum_{i=1}^{N} |r\rangle\langle r|_i + \sum_{i<j} V_{|i-j|} \Big(|r\rangle\langle r|_i \otimes |r\rangle\langle r|_j \Big)$$

$$V_{|i-j|} \sim \frac{1}{|r_i - r_j|^6}$$

Bernien et. al., Nature **551**, 579 (2017)
Keesling et. al. Nature **568**, 207 (2019)

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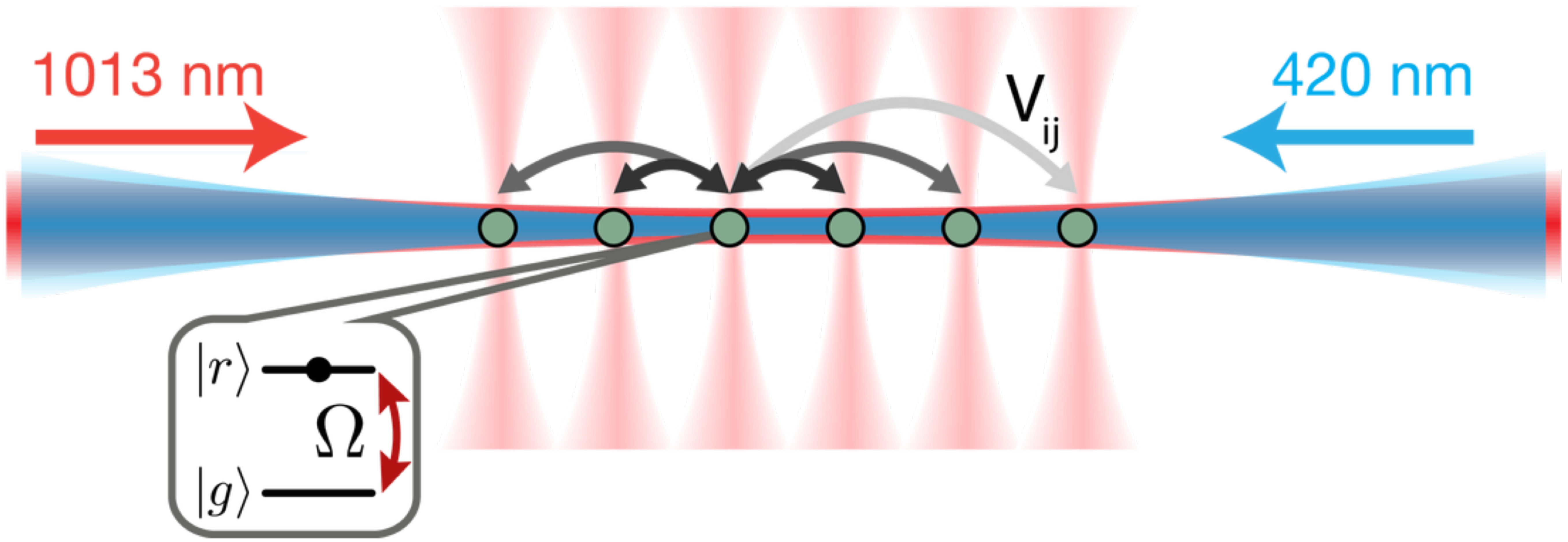

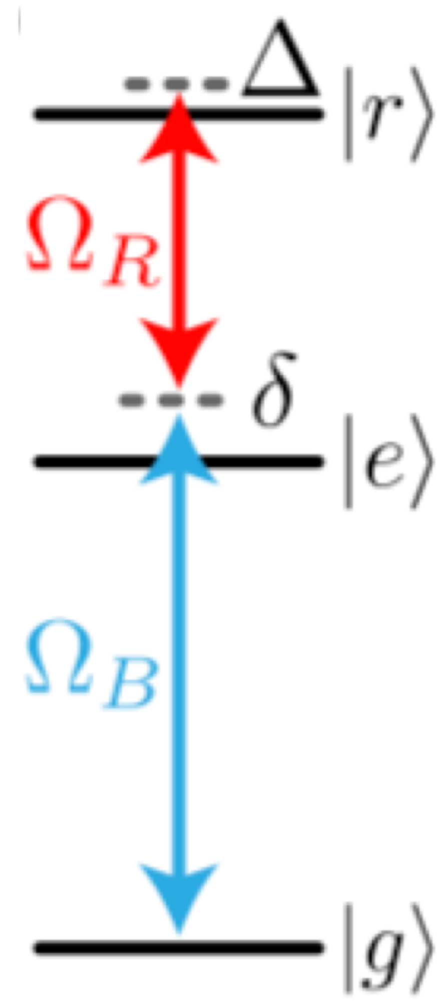

$$\mathcal{H} = \sum_j \left[\frac{\Omega}{2} \left(b_j + b_j^\dagger \right) - \Delta n_j \right] + \sum_{i<j} V_{|i-j|} n_i n_j \qquad V_{|i-j|} \sim \frac{1}{|r_i - r_j|^6}$$

$$n_j \equiv b_j^\dagger b_j$$

$$n_j = 0, 1 \quad \text{‘hard core’ bosons}$$

S. Sachdev, K. Sengupta, and S.M. Girvin, PRB **66**, 075128 (2002)
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The PXP model: ($V_1 = \infty, V_{j\geq 2} = 0$, $\mathcal{P}$ projects out nearest-neighbor boson states)

$$\mathcal{H}_{\rm PXP} = \sum_j \left[\frac{\Omega}{2} \mathcal{P} \left(b_j + b_j^\dagger \right) \mathcal{P} - \Delta b_j^\dagger b_j \right]$$

These models were motivated by 'tilted lattices' of bosonic atoms, and predicted a $\mathbb{Z}_2$ quantum transition which was observed in J. Simon, W. S. Bakr, Ruichao Ma, M. Eric Tai, P. M. Preiss, M. Greiner, Nature **472**, 307 (2011).

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4. Kagome symmetry lattices

   Probing topological spin liquids

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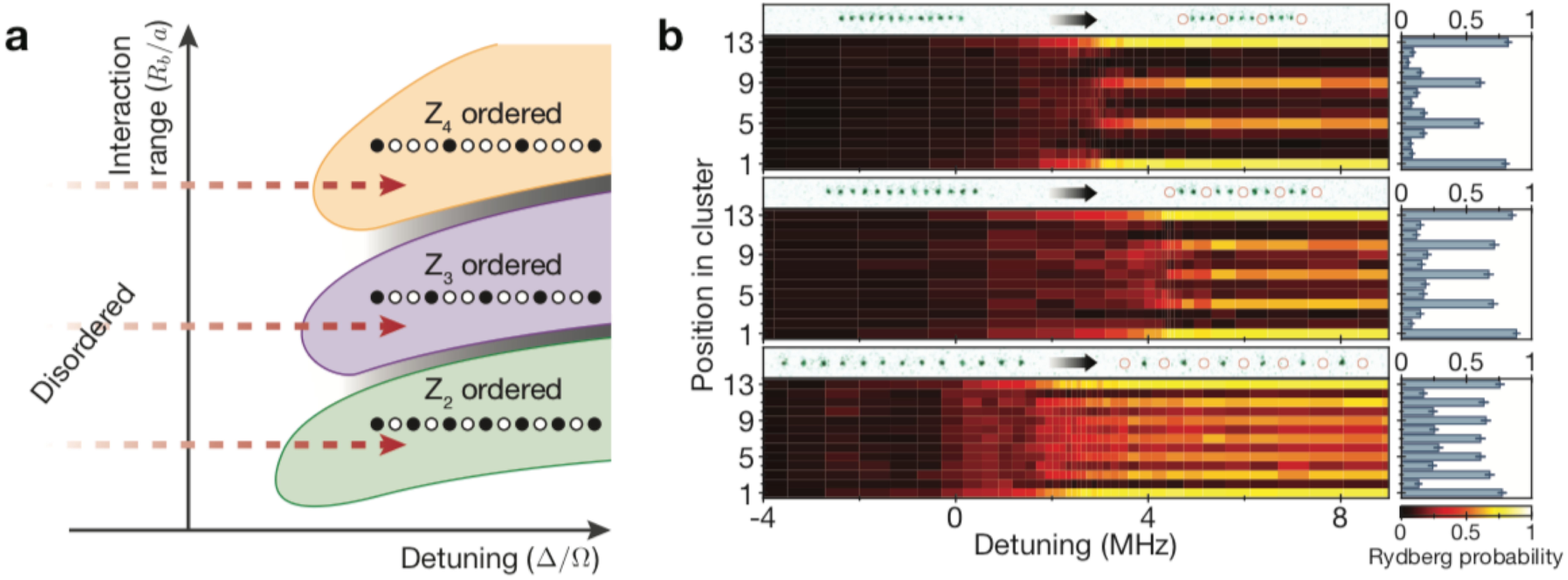

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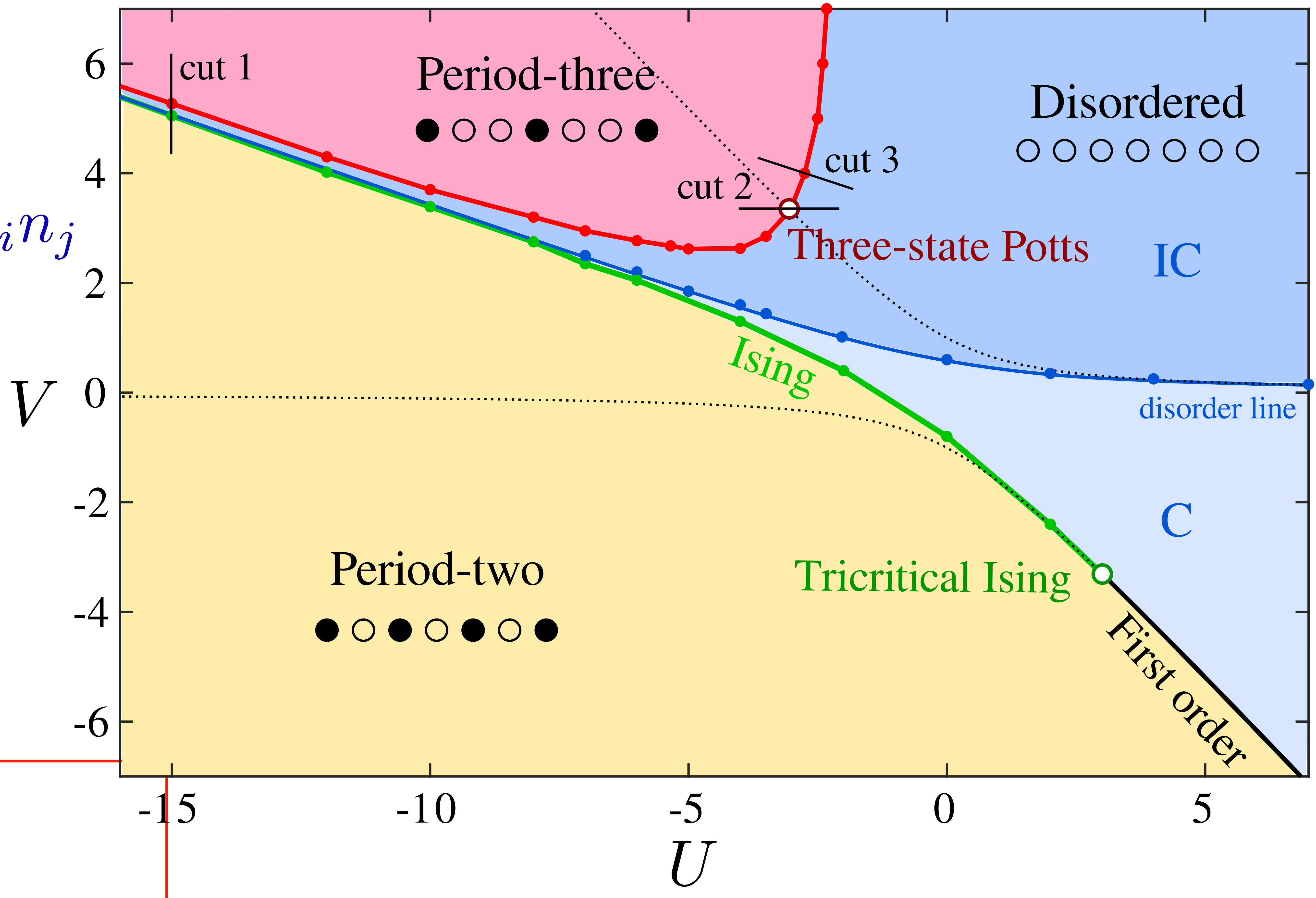
$$V_1 = \infty, \quad V_2 = V, \quad V_{i>2} = 0$$

$$w = \Omega/2, \quad U = -\Delta$$



Nature of Z_3 ordering transition

Direct:

Ostlund, PRB **24**, 398 (1981).
Huse and ME Fisher, PRL **49**, 793 (1982).
Huse, PRB **24**, 5180 (1981).
Selke and Yeomans, Z. Phys. B **46**, 311 (1982).
Howes, Kadanoff, den Nijs, Nucl. Phys. B **215**, 169 (1983)
Huse, Szpilka, Fisher, Physica A **121**, 363 (1983).

IC:

Haldane, Bak, Bohr, PRB **28**, 2743 (1983).
Schulz, PRB **28**, 2746 (1983)
Von Gehlen and Rittenberg, Nucl. Phys. B, 473 (1984)

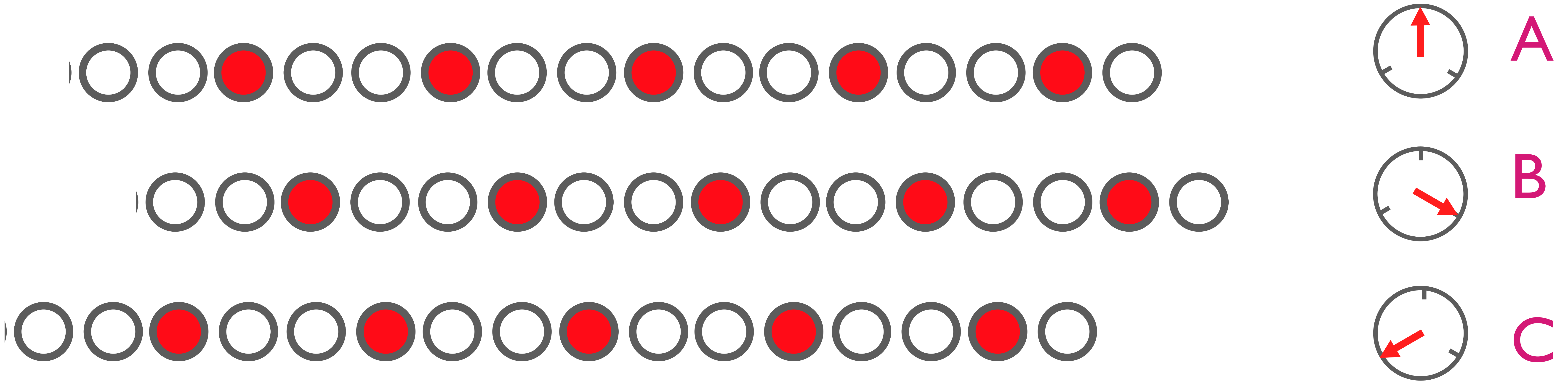
N. Chepiga and F. Mila, PRL **122**, 017205 (2019)
P. Fendley, K. Sengupta, S. Sachdev, PRB **69**, 075106 (2004)

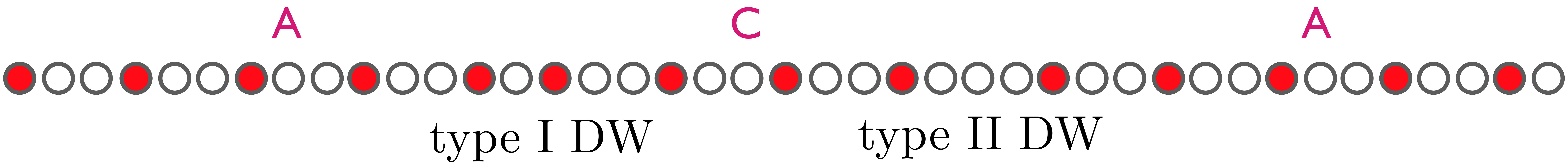
Quantum field theory for the chiral clock transition in one spatial dimension

PHYSICAL REVIEW B **98**, 205118 (2018)

S. Whitsitt, R. Samajdar, and S. Sachdev

Density wave order Φ: $\mathbb{Z}_3$ symmetry $\Phi \to e^{2\pi i/3}\Phi$





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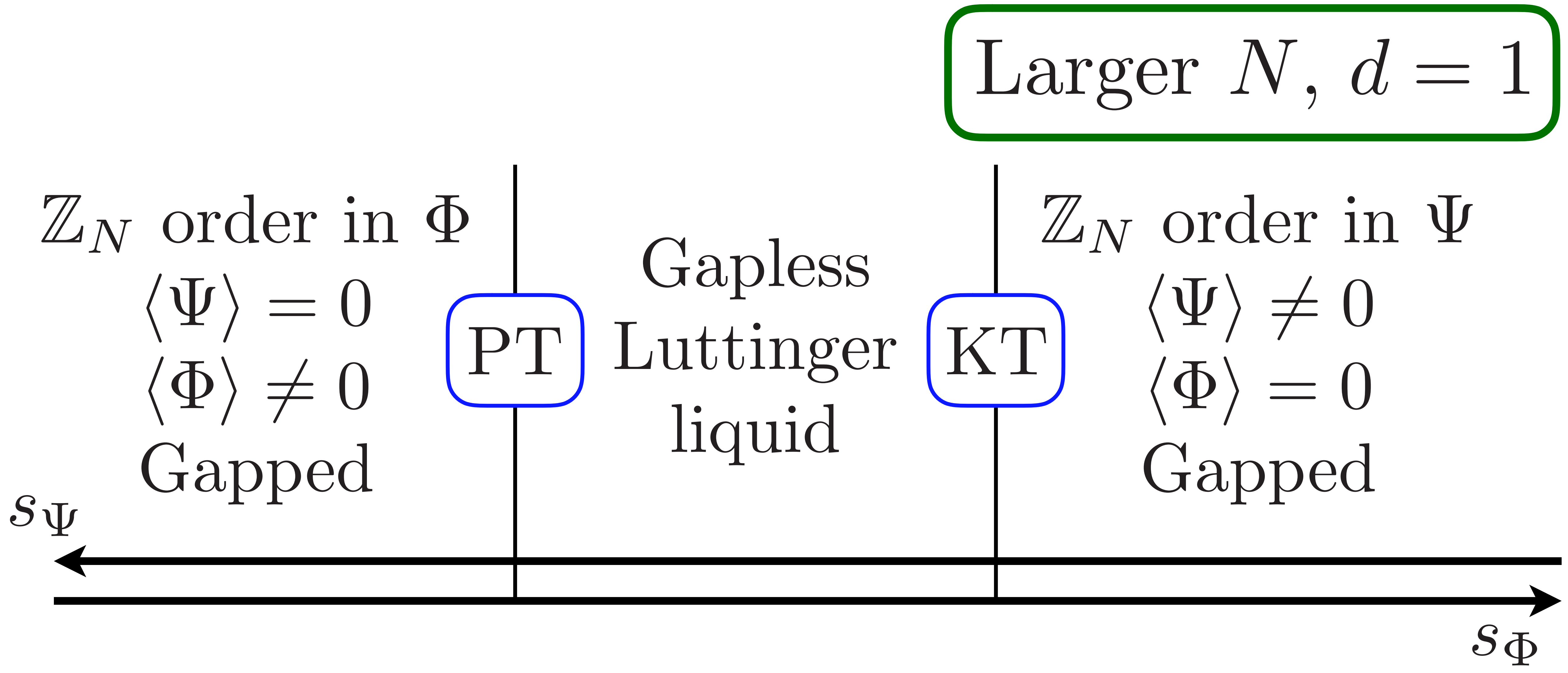
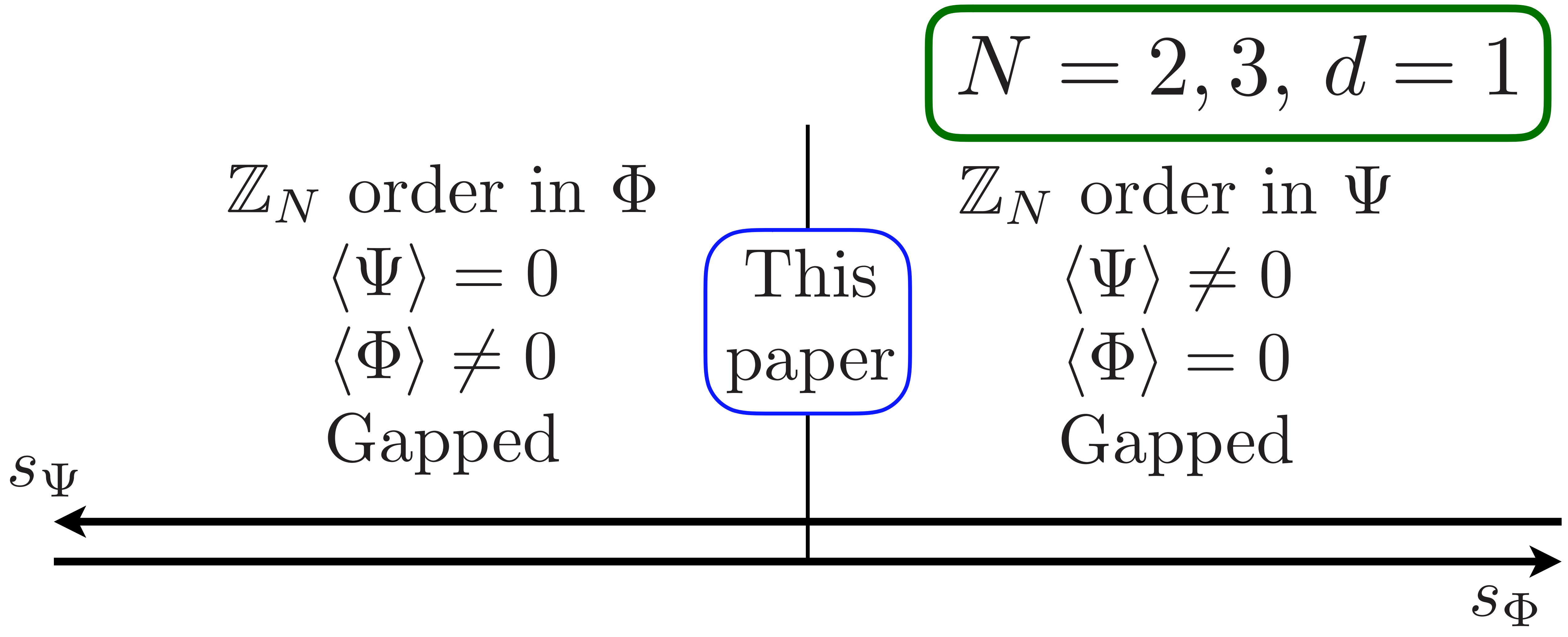
Kramers-Wannier duality: we map the $\mathbb{Z}_N$ case to the condensation of a boson Ψ in the presence of a N boson condensate

$$\mathcal{S}_\Psi = \int d\tau dx \left[\Psi^* \partial_\tau \Psi + |\partial_x \Psi|^2 + s_\Psi |\Psi|^2 + u|\Psi|^4 + \lambda(\Psi^N + \Psi^{*N})\right]$$

RG analysis leads to a strongly-coupled quantum critical point with dynamic critical exponent $z \neq 1$.





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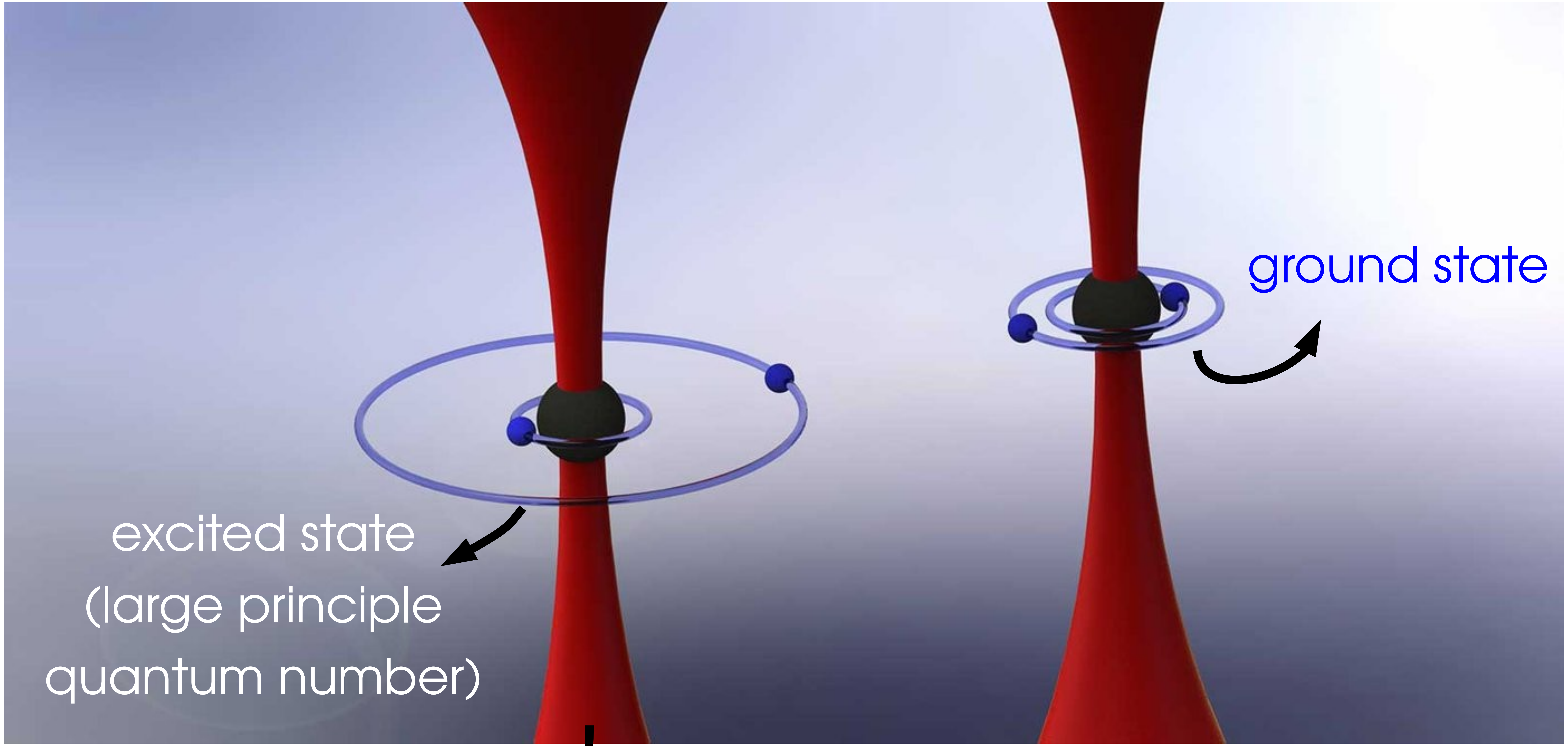

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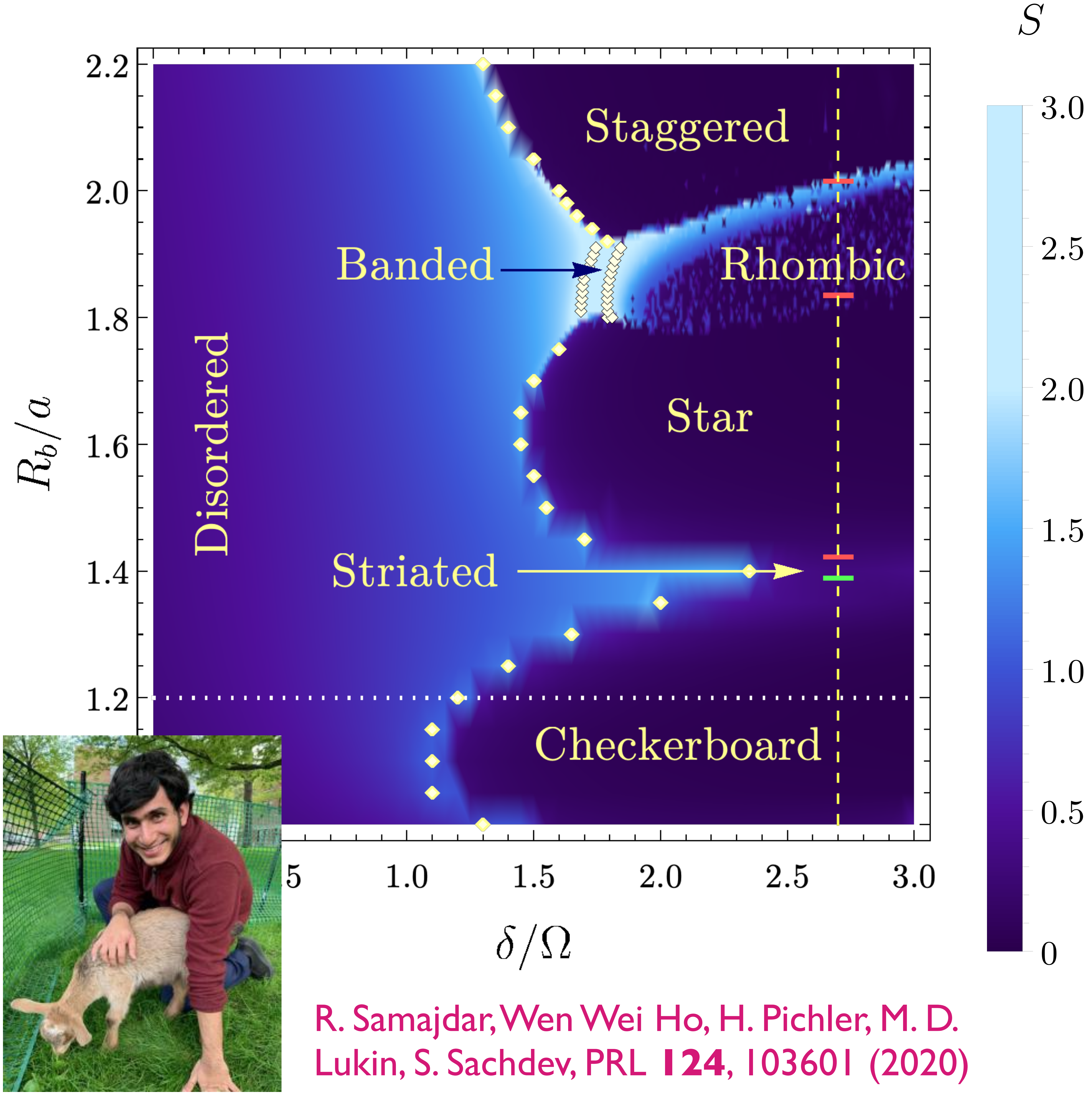
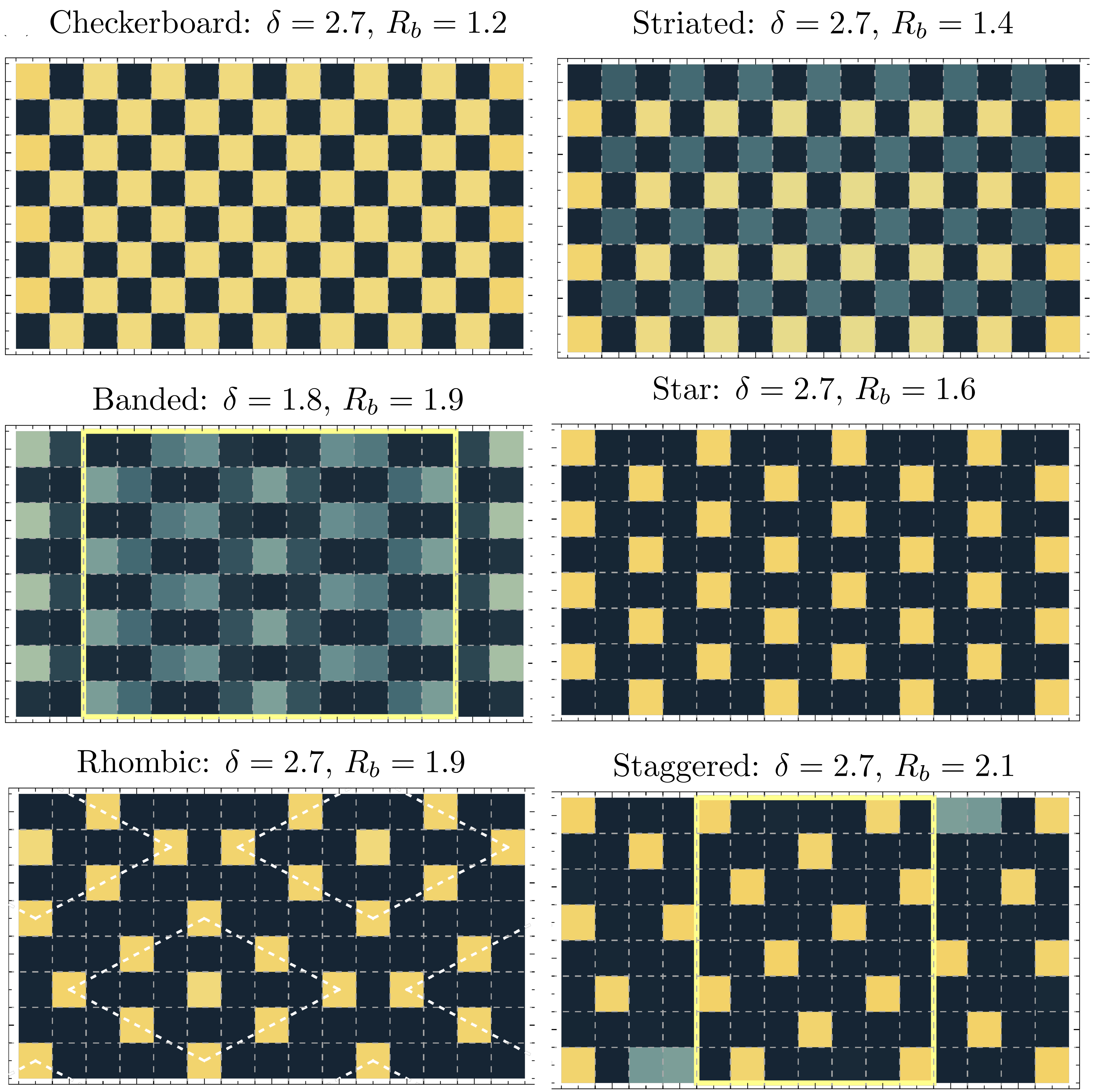
Rydberg atoms on the square lattice: theory



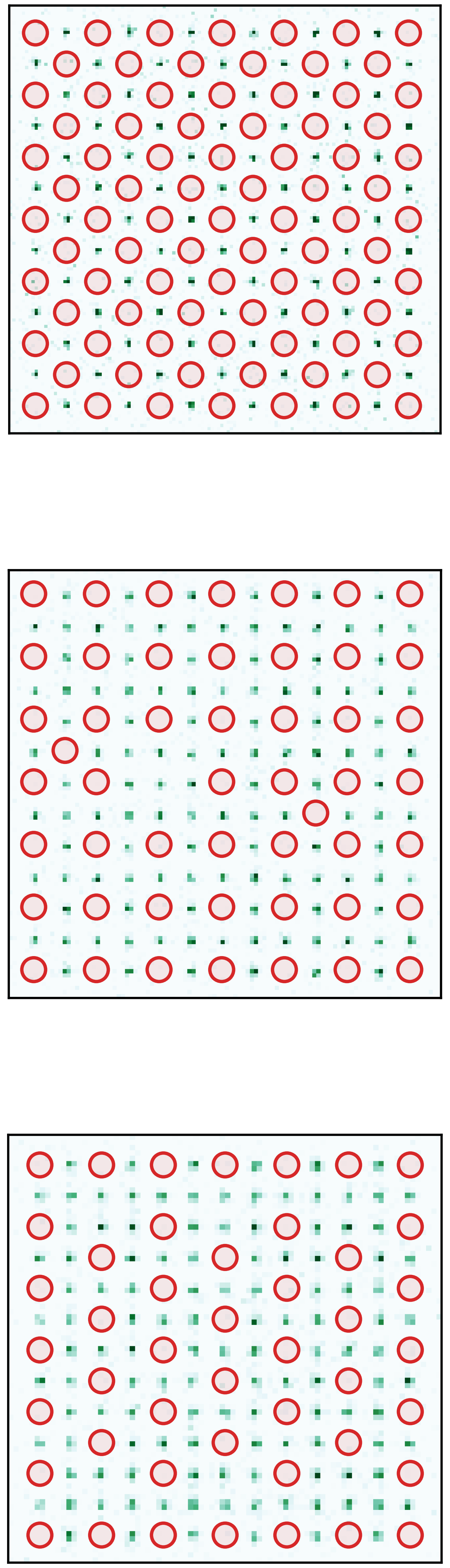


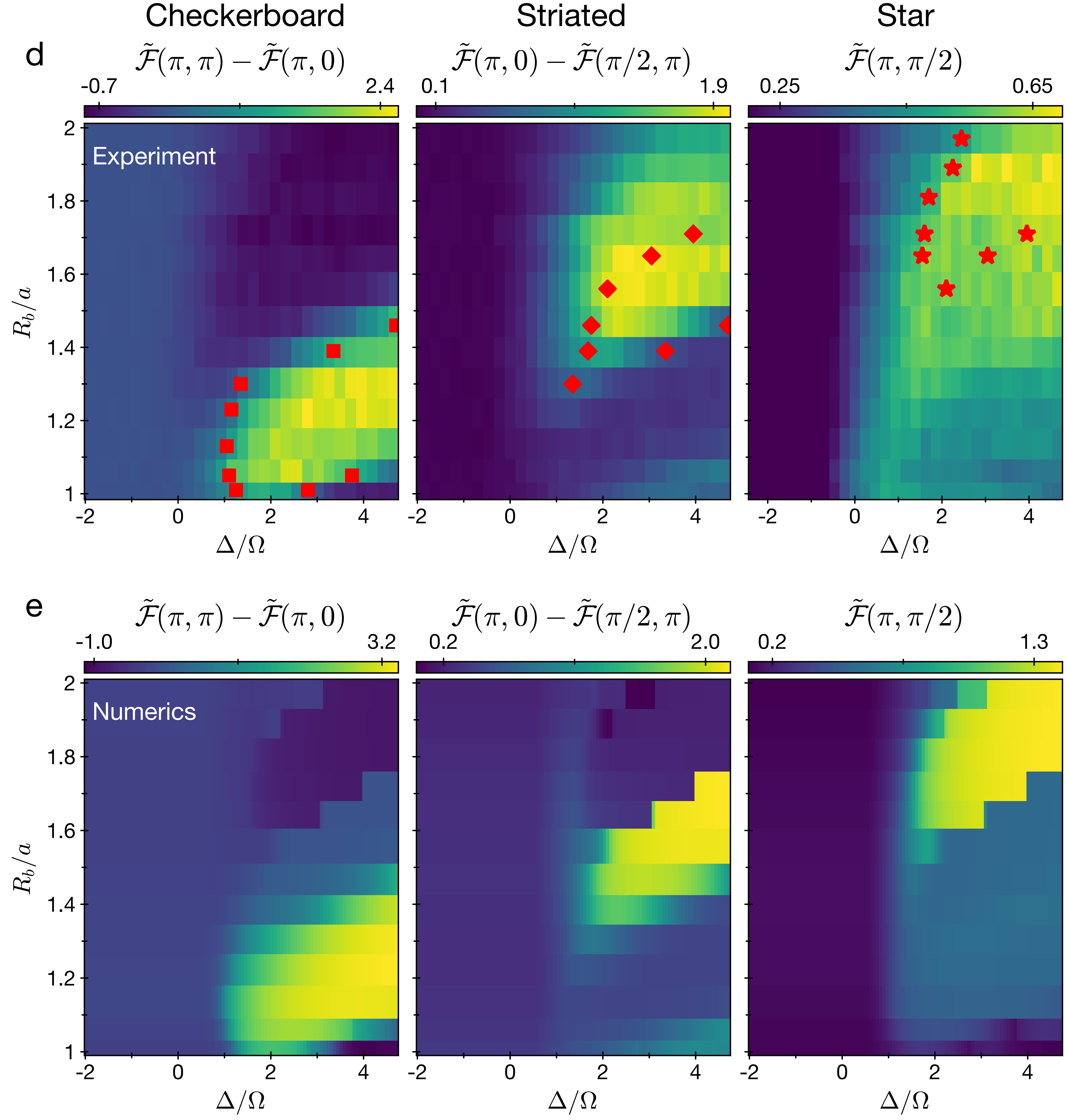
R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PRL **124**, 103601 (2020)

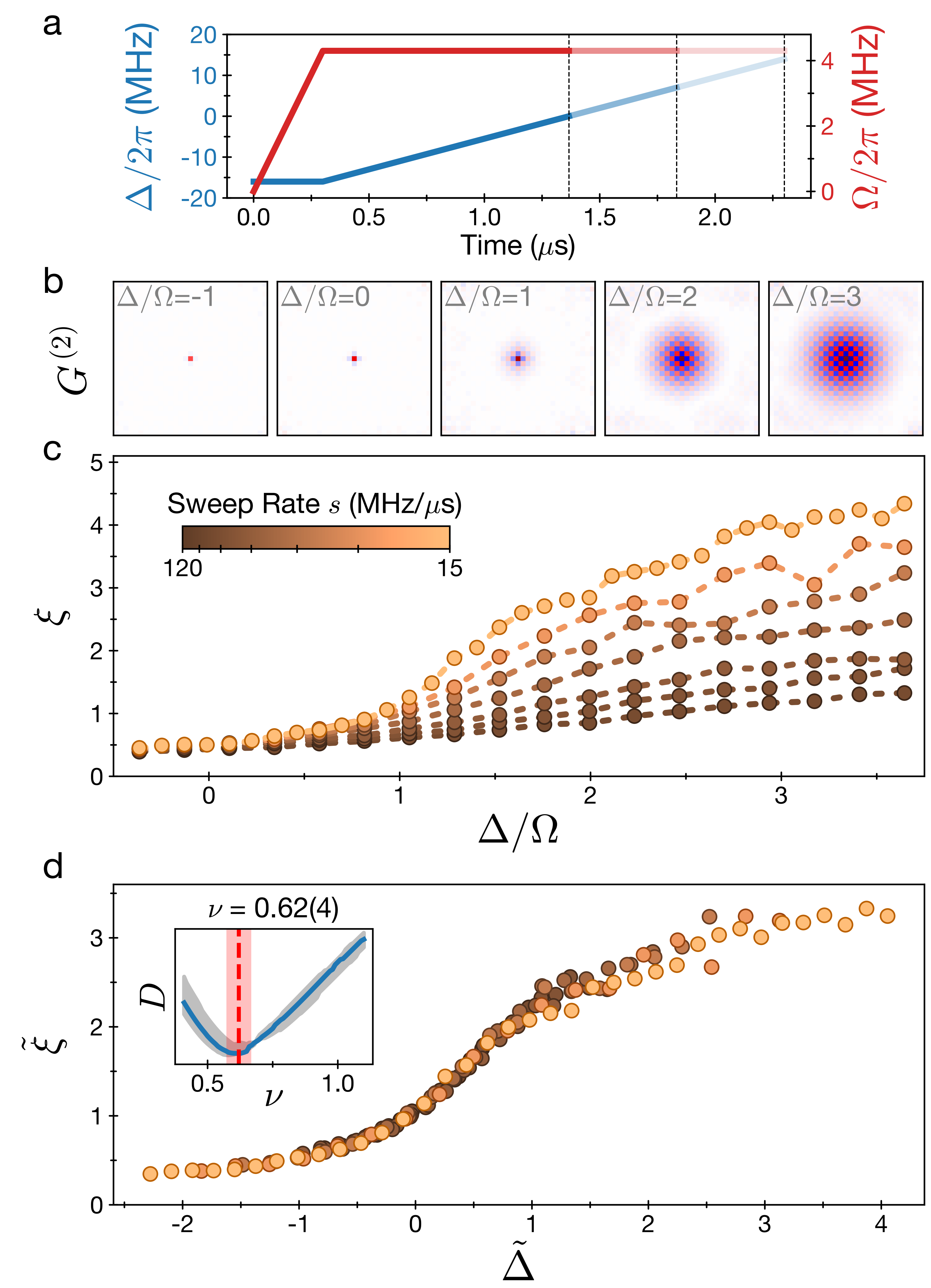
Rydberg atoms on the square lattice: experiment

$$\tilde{\xi} = \xi(s/s_0)^{\mu}$$

$$\tilde{\Delta} = (\Delta - \Delta_c)(s/s_0)^{\kappa}$$







Quantum Phases of Matter on a 256-Atom Programmable Quantum Simulator, Sepehr Ebadi, Tout T. Wang, Harry Levine, Alexander Keesling, Giulia Semeghini, Ahmed Omran, Dolev Bluvstein, Rhine Samajdar, Hannes Pichler, Wen Wei Ho, Soonwon Choi, Subir Sachdev, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, Nature **595**, 227 (2021); Pascal Scholl et al. Nature **595**, 233 (2021)

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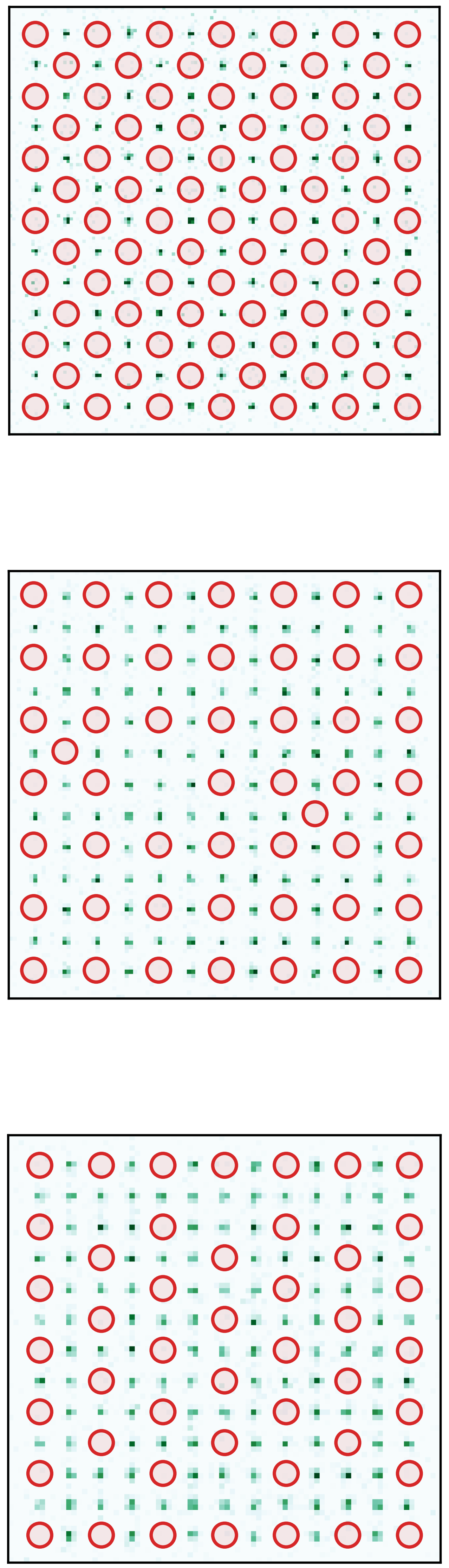

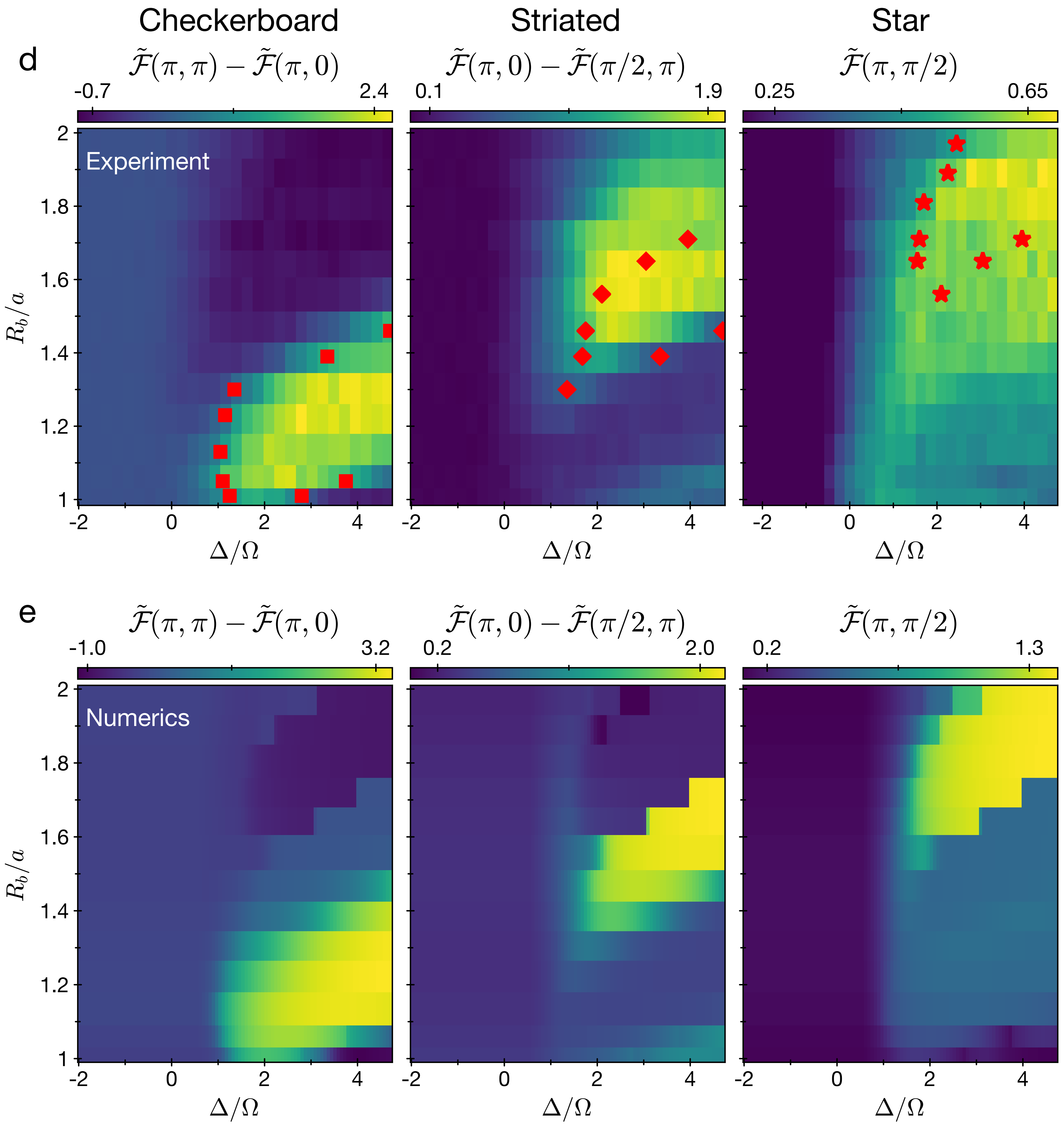

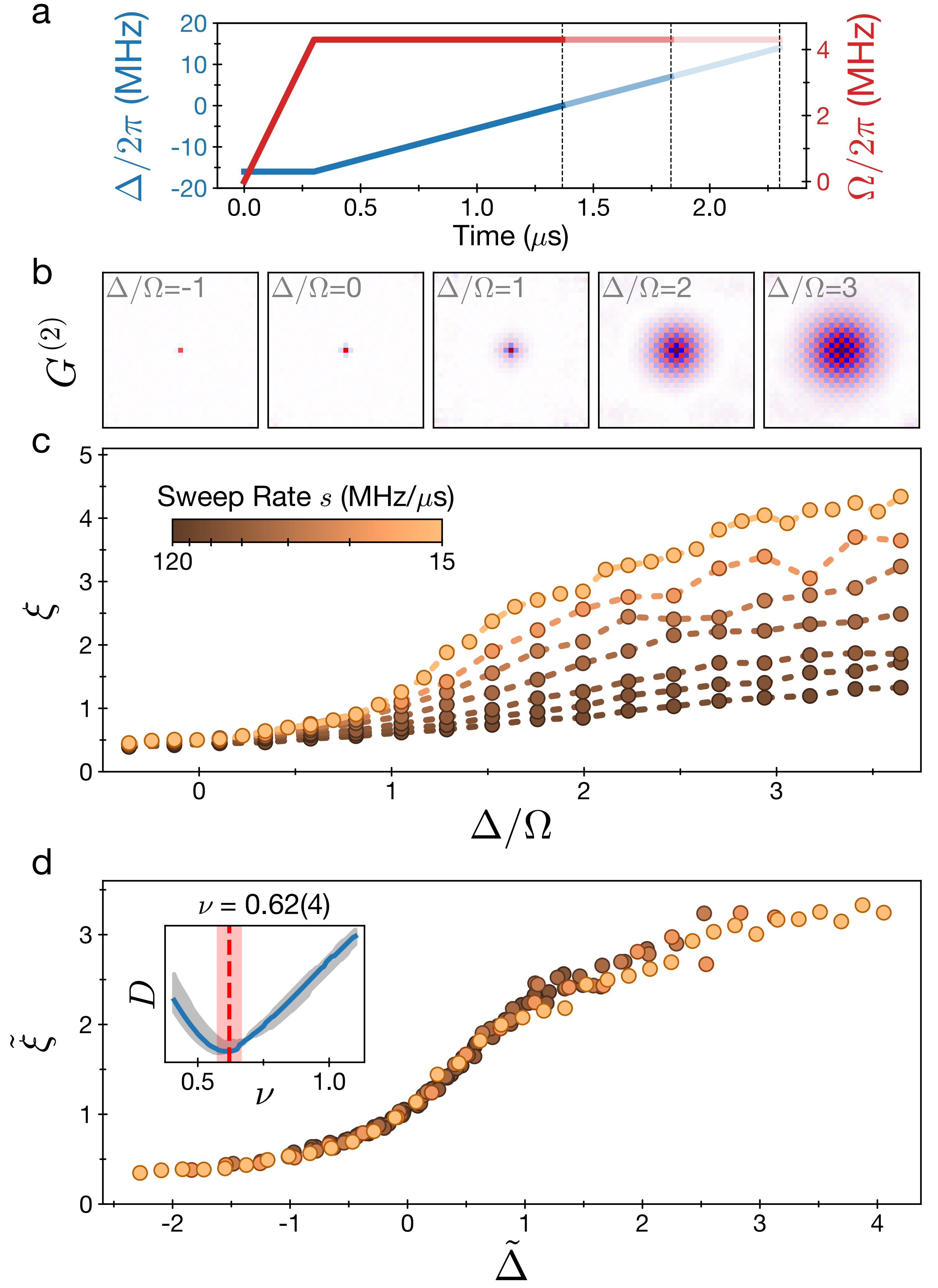

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First observation of Ising quantum phase transition in 2+1 dimensions

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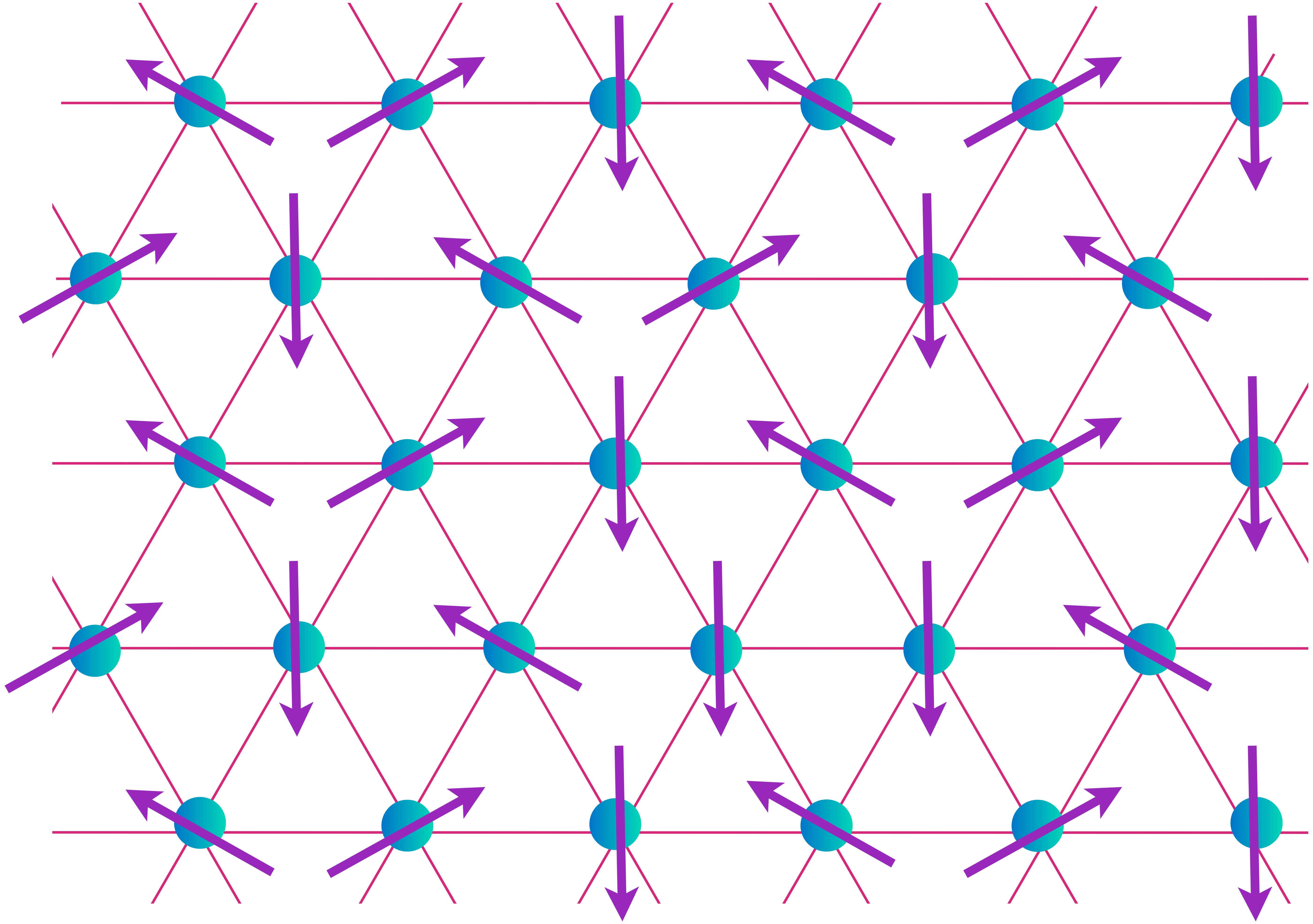
Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Mapping of bosons and spins

Boson B	Spin $\vec{S}$
$B^\dagger B \leq 1$	$\vec{S}^2 = 3/4$
States $\|0\rangle$, $B^\dagger\|0\rangle$	States $\|\uparrow\rangle$ $\|\downarrow\rangle$
Operators	Operators
$B^\dagger$ $\longleftrightarrow$	$S_+ = S_x + iS_y$
B $\longleftrightarrow$	$S_- = S_x - iS_y$
$B^\dagger B - \frac{1}{2}$ $\longleftrightarrow$	S_z

Resonating valence bonds

Is there a ground state with an energy gap to all excitations which does not break any symmetries with mean boson number $\langle B_i^\dagger B_i \rangle = 1/2$ ($i.e.$ $\langle S_{iz} \rangle = 0$) ?

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P.W. Anderson (1973):
Yes – the resonating valence bond (RVB) state.

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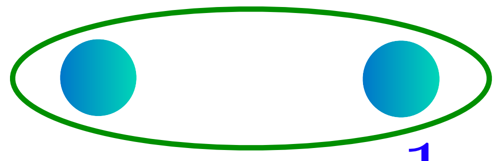
Yes – the resonating valence bond (RVB) state.

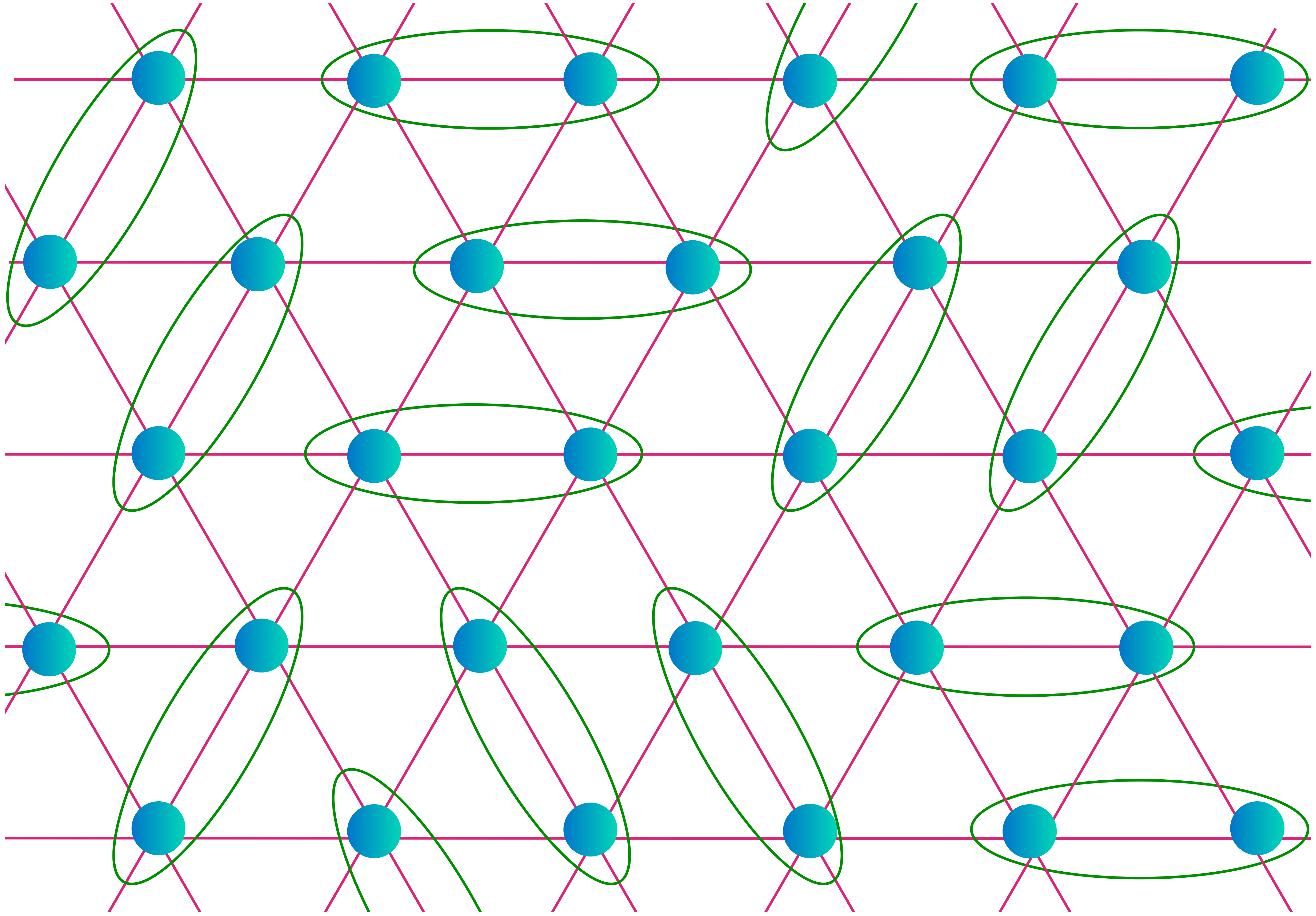
Kivelson, Rokhsar, Sethna; Baskaran, Anderson (1987):
Such a state must have excitations with fractional quantum numbers: with total boson number $\sum_i B_i^\dagger B_i = \pm 1/2$, or total spin $\sum_i S_{zi} = \pm 1/2$.

RVB

Bosons at half-filling,
or a spin model with S=1/2 per unit cell



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) = \frac{1}{\sqrt{2}} \left(B_1^\dagger - B_2^\dagger \right) |0\rangle$$



$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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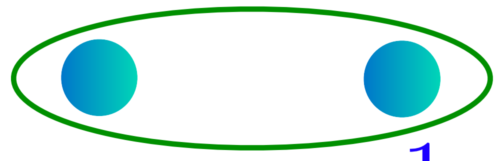

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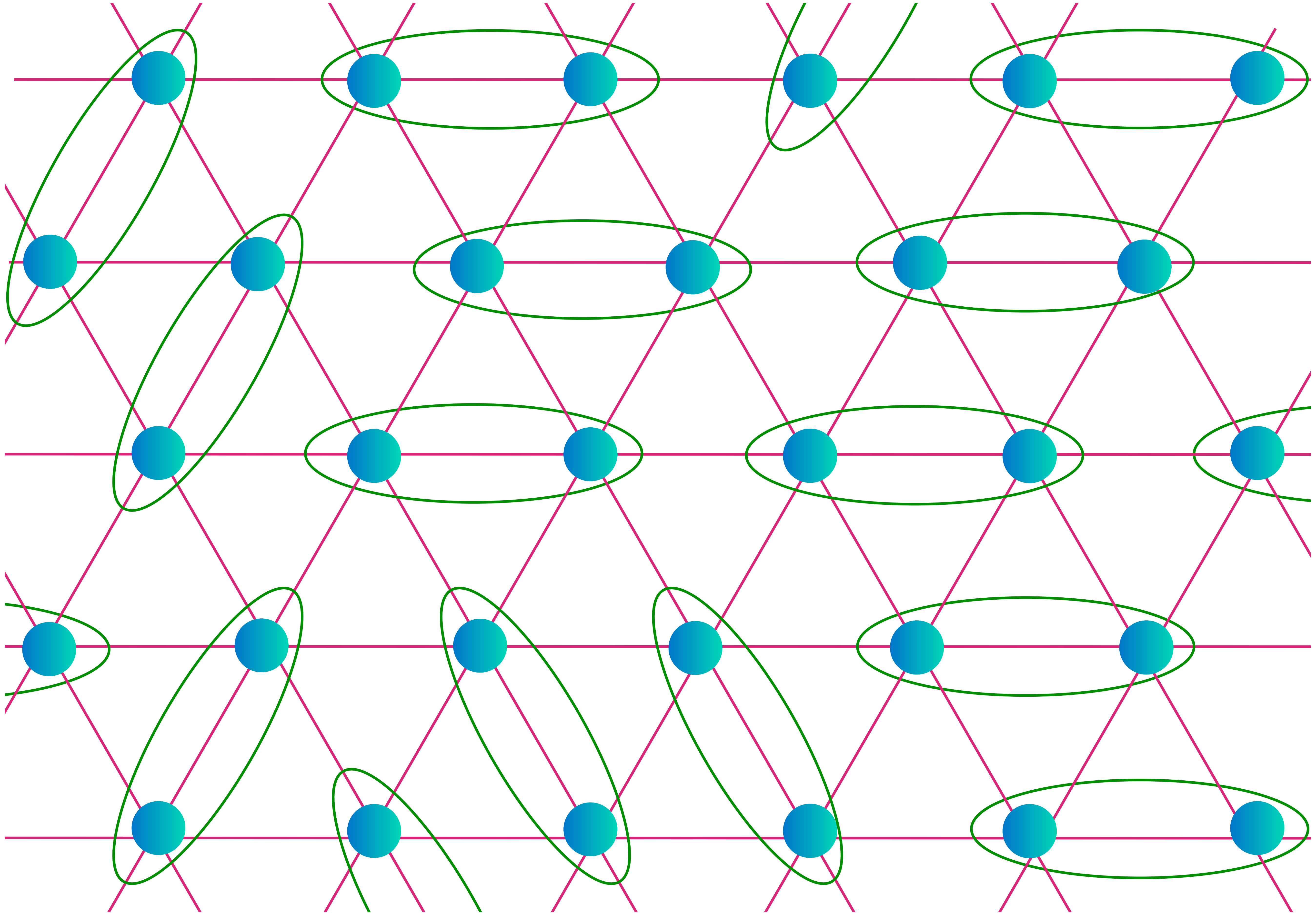

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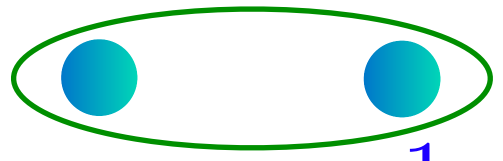

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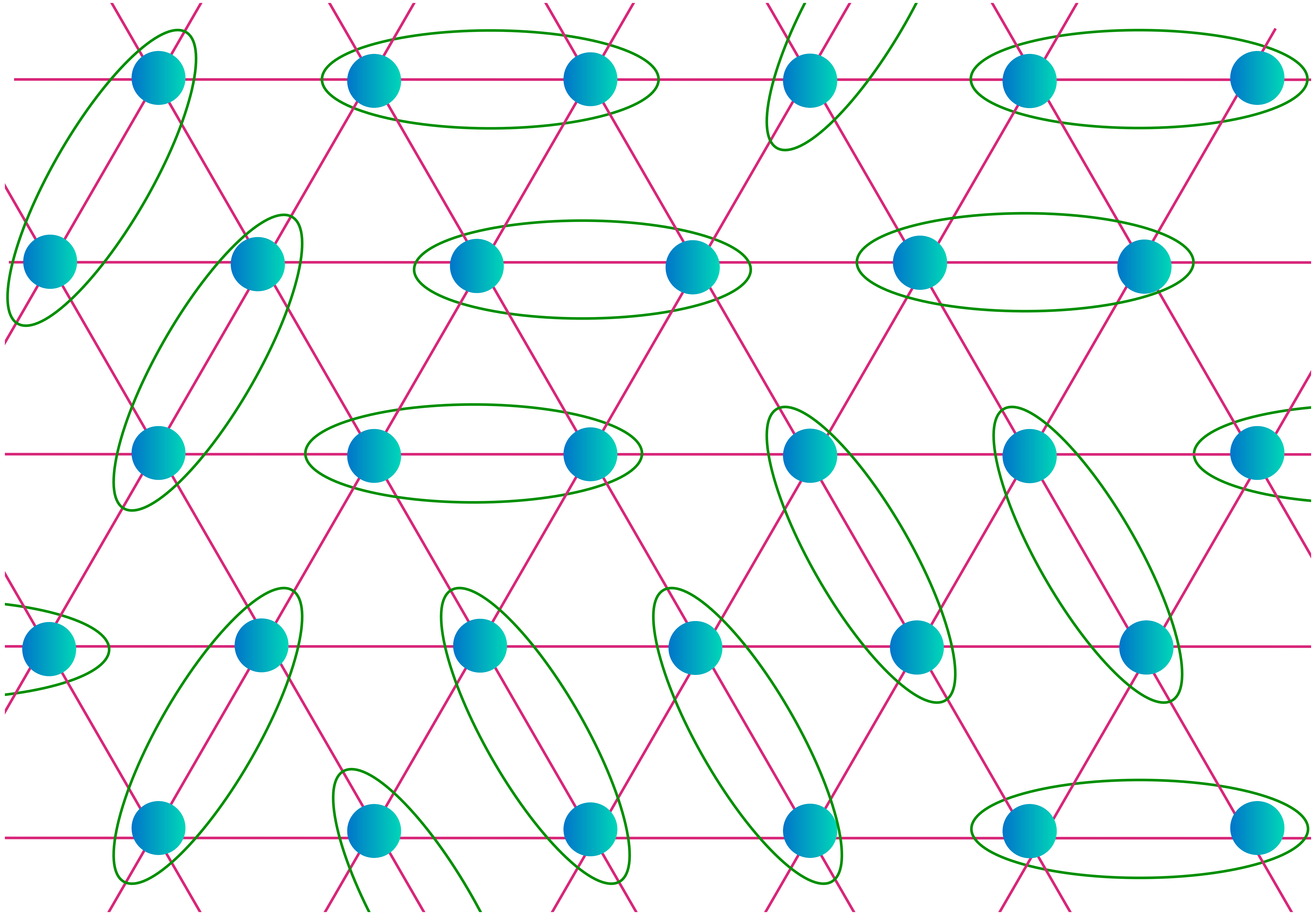

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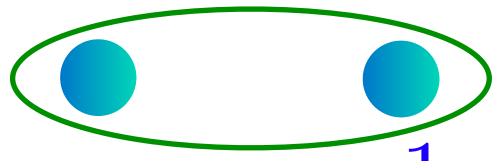

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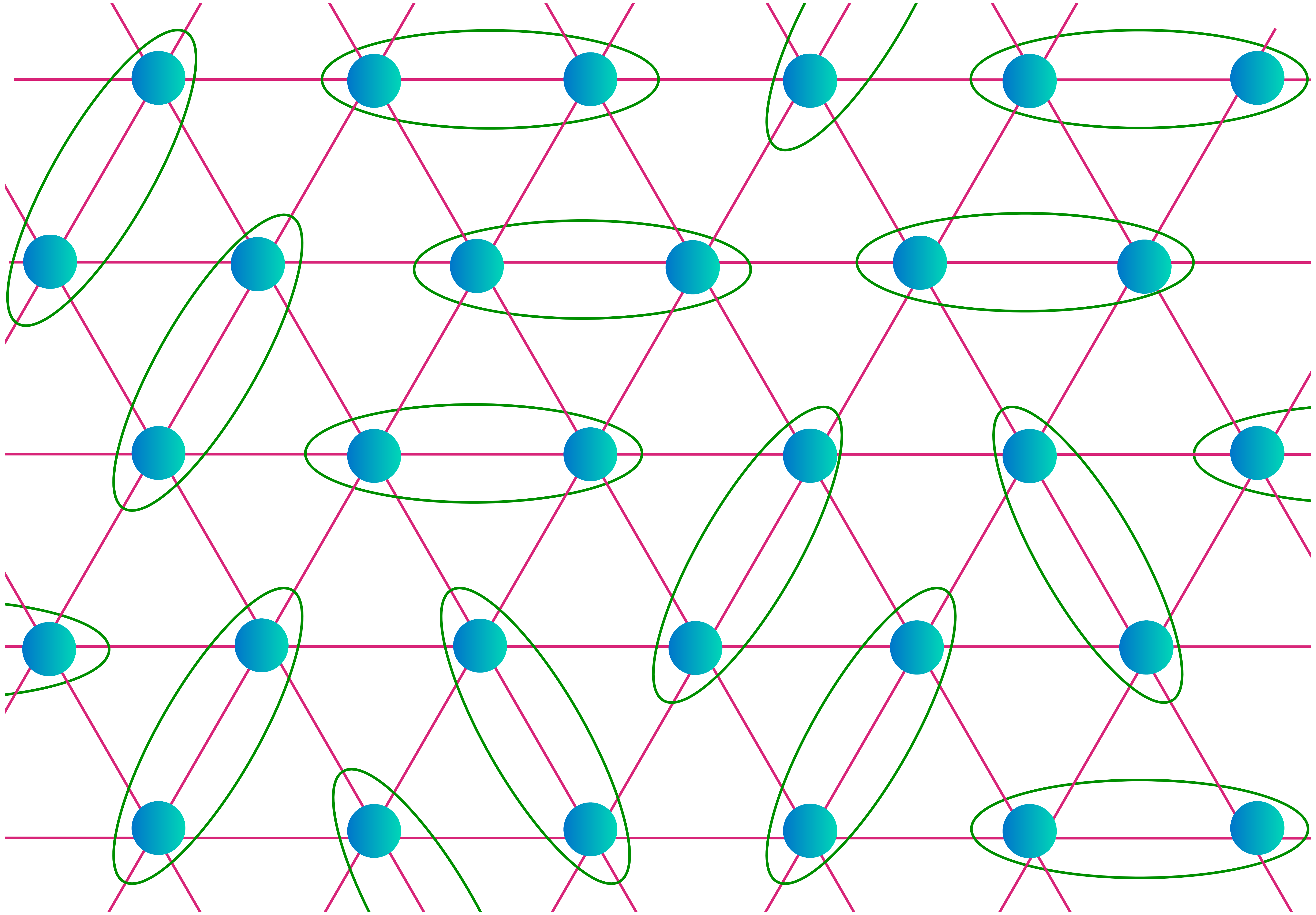

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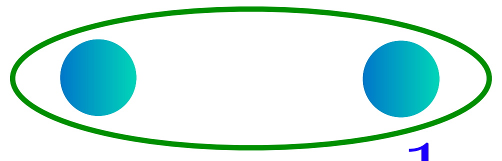

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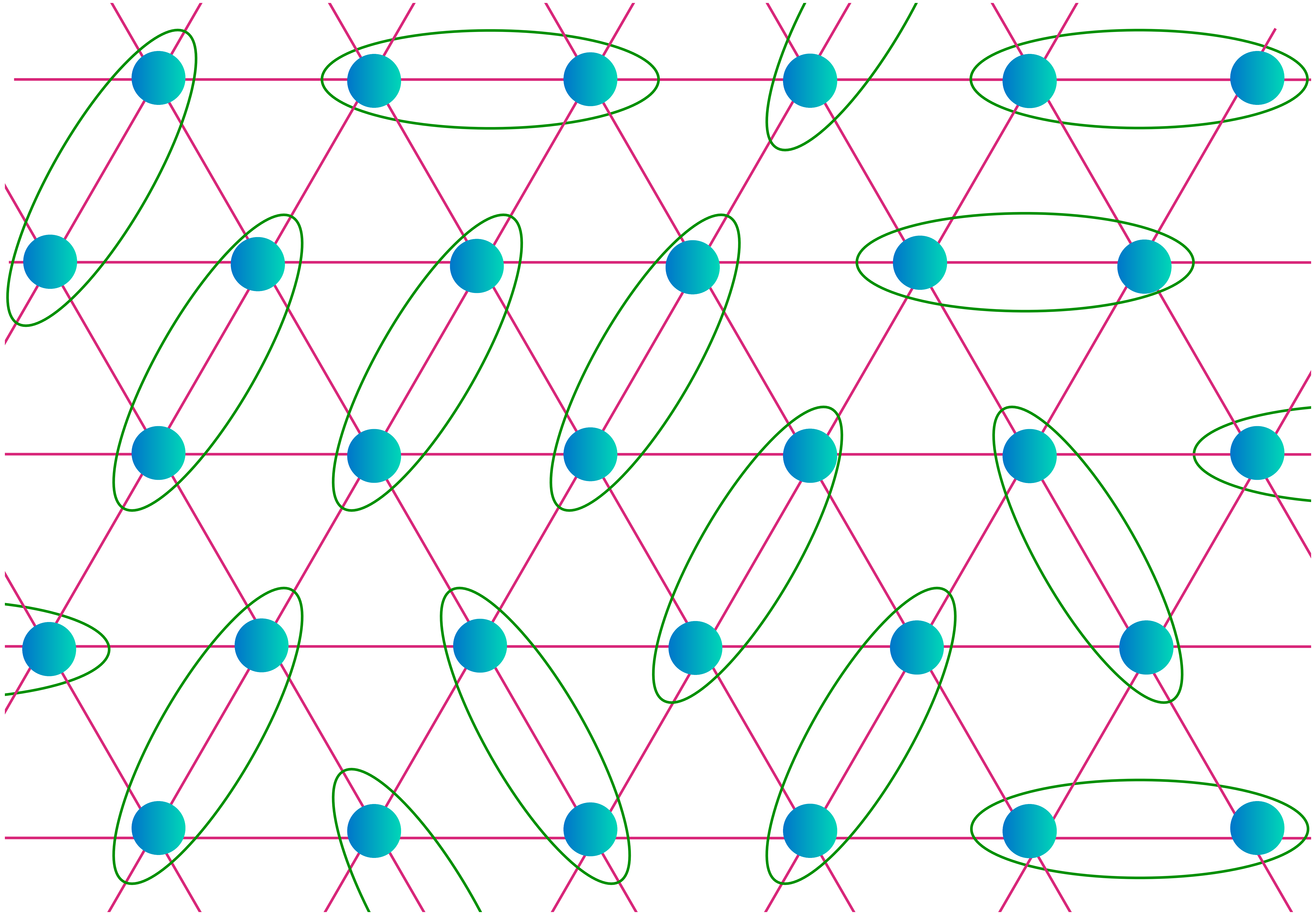

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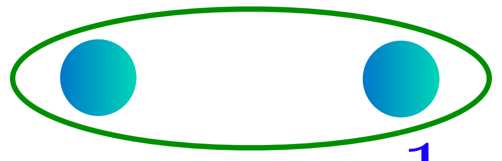

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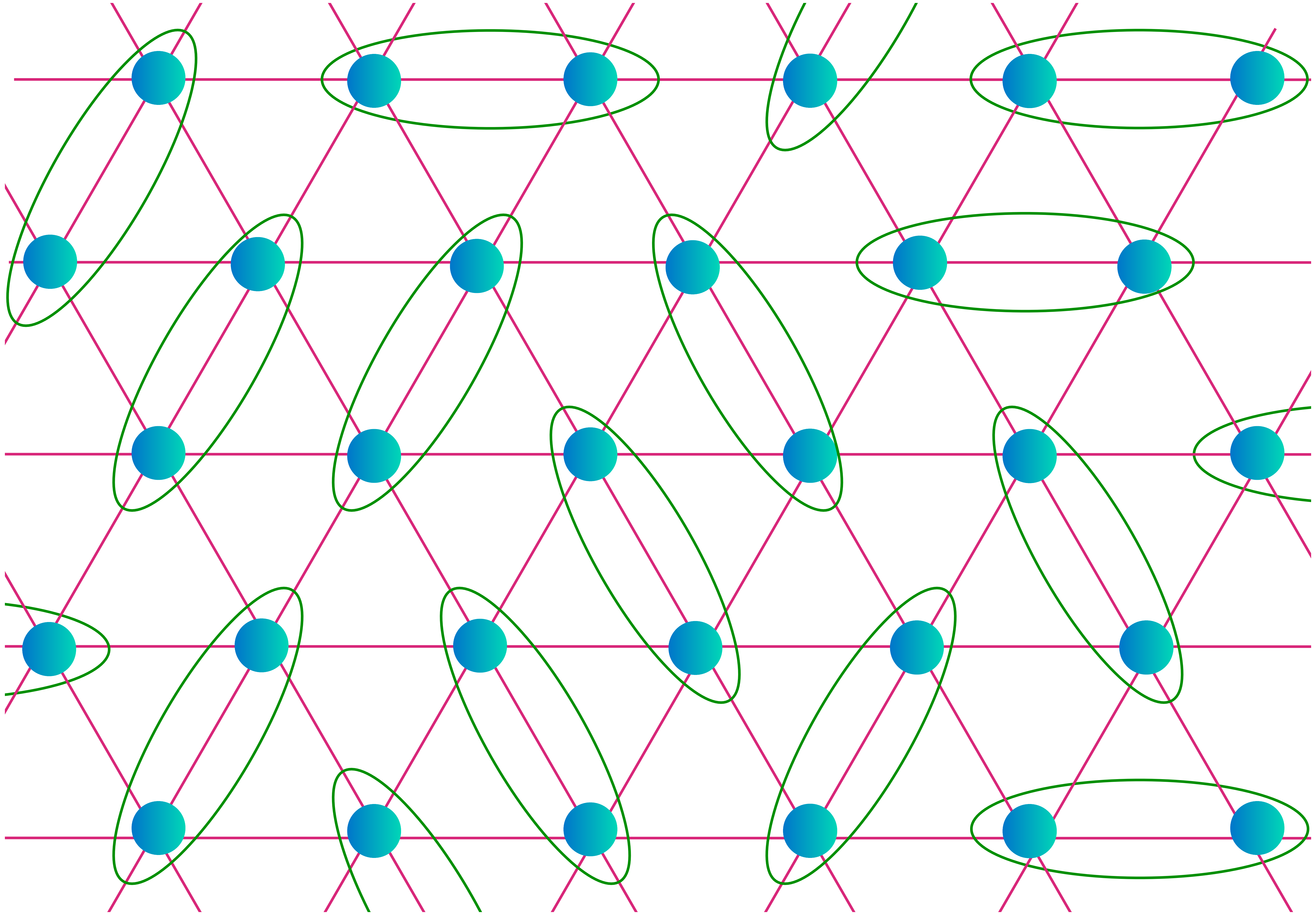

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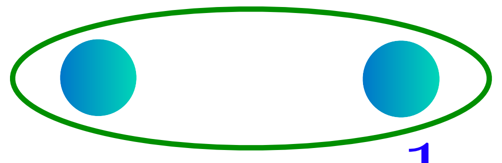
RVB

Excitations with boson number 1/2



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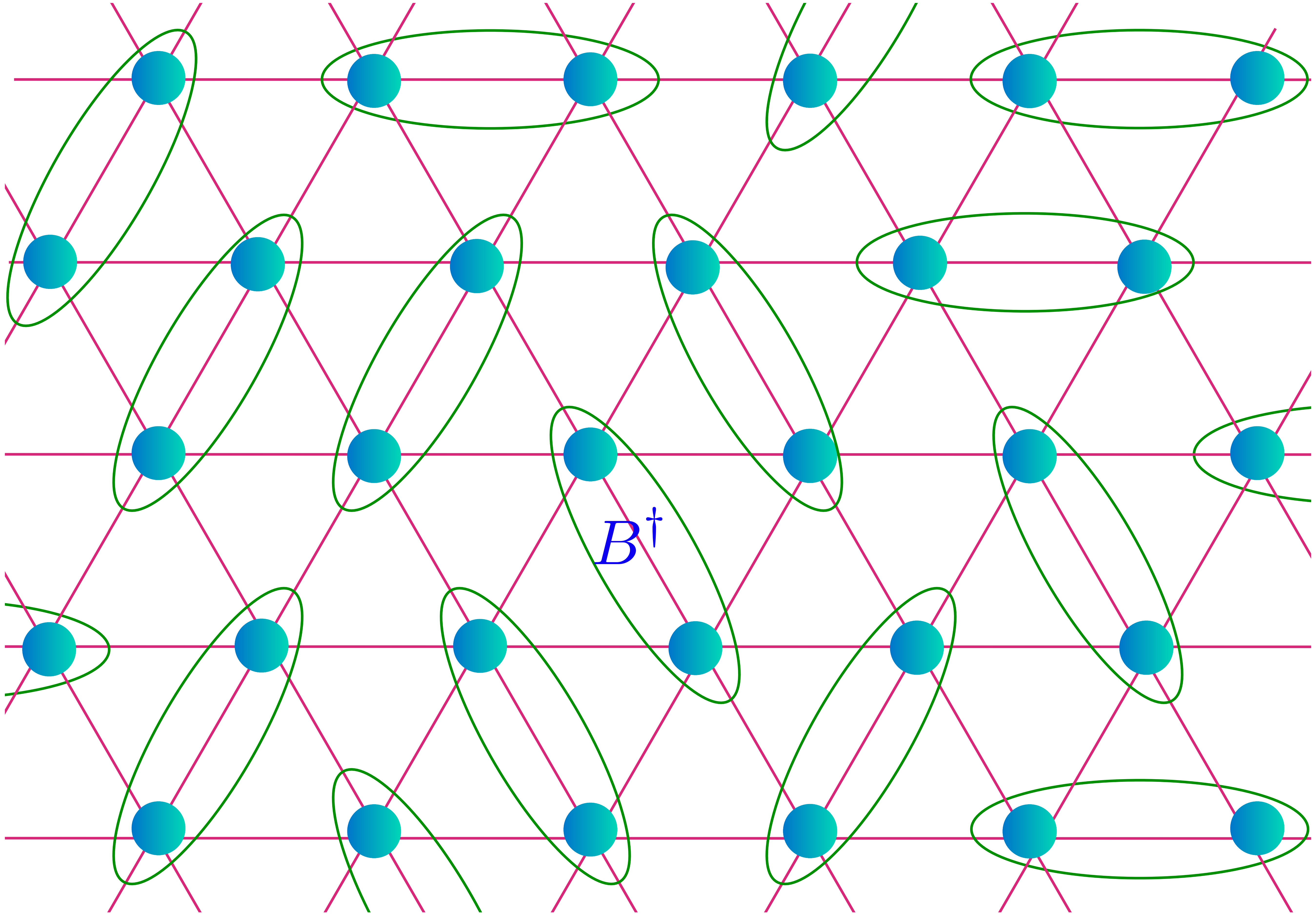
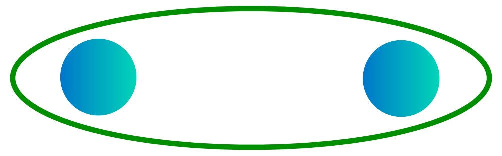


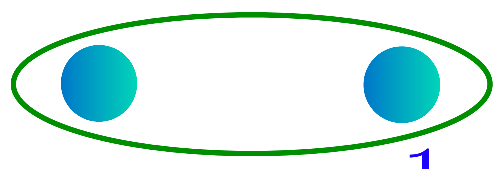
$B_2^\dagger$



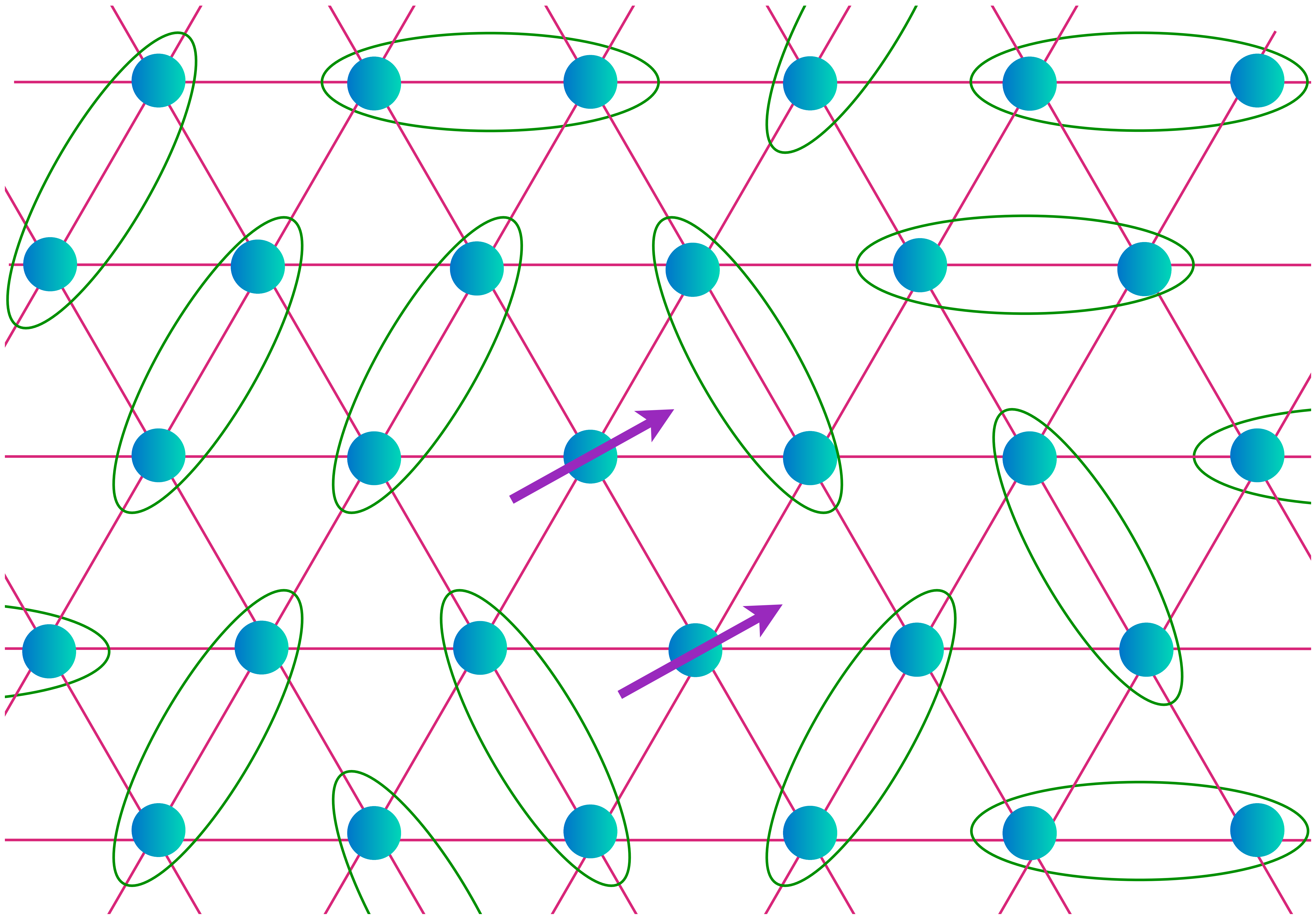
$$= \frac{1}{\sqrt{2}} B_1^\dagger B_2^\dagger |0\rangle = \frac{1}{\sqrt{2}} | \uparrow\uparrow \rangle$$

RVB

Excitations with boson number 1/2
a "spinon"



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) = \frac{1}{\sqrt{2}} \left(B_1^\dagger - B_2^\dagger \right) |0\rangle$$



- The boson creation operator $B^\dagger$ creates a *pair* of spinons.
- A single spinon carries boson number $B^\dagger B = 1/2$: fractionalization!

RVB

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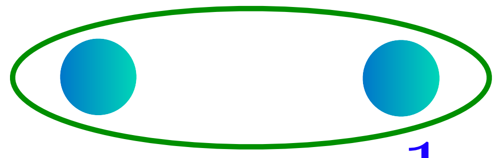

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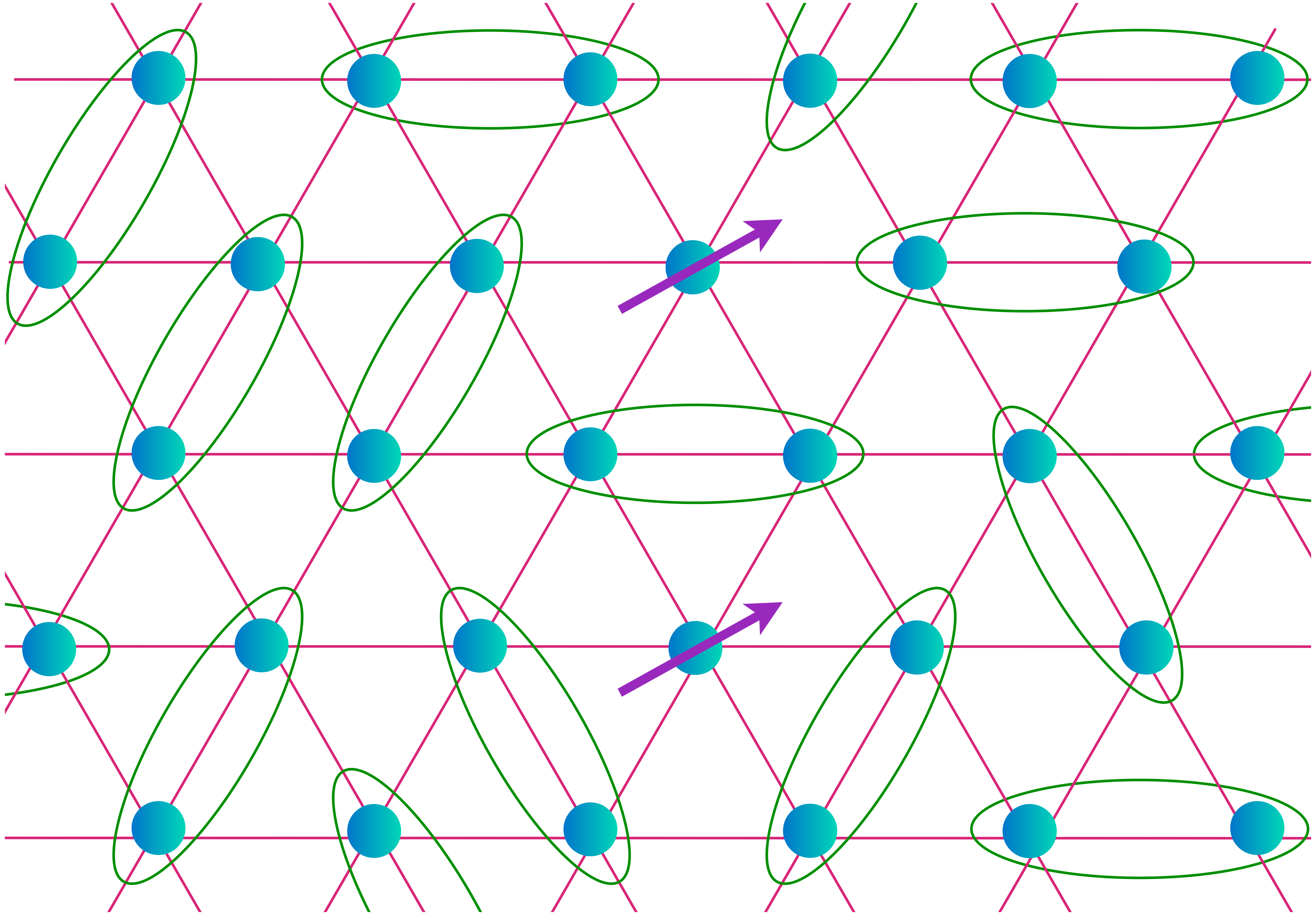

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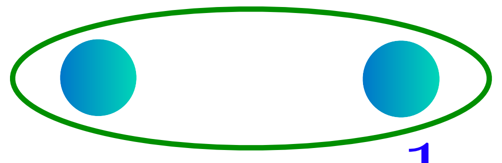

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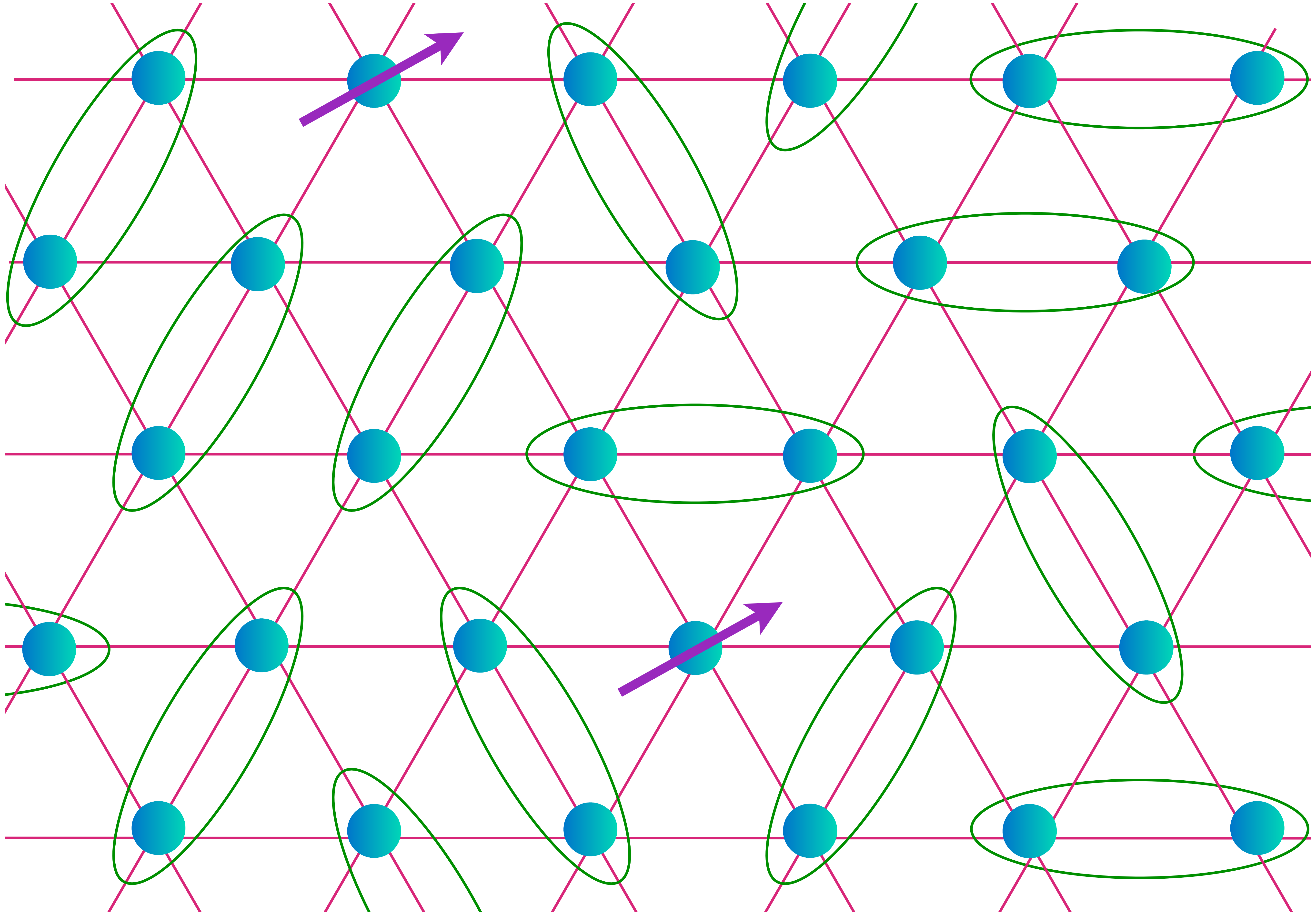

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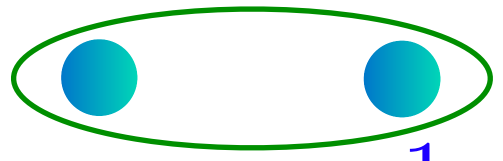

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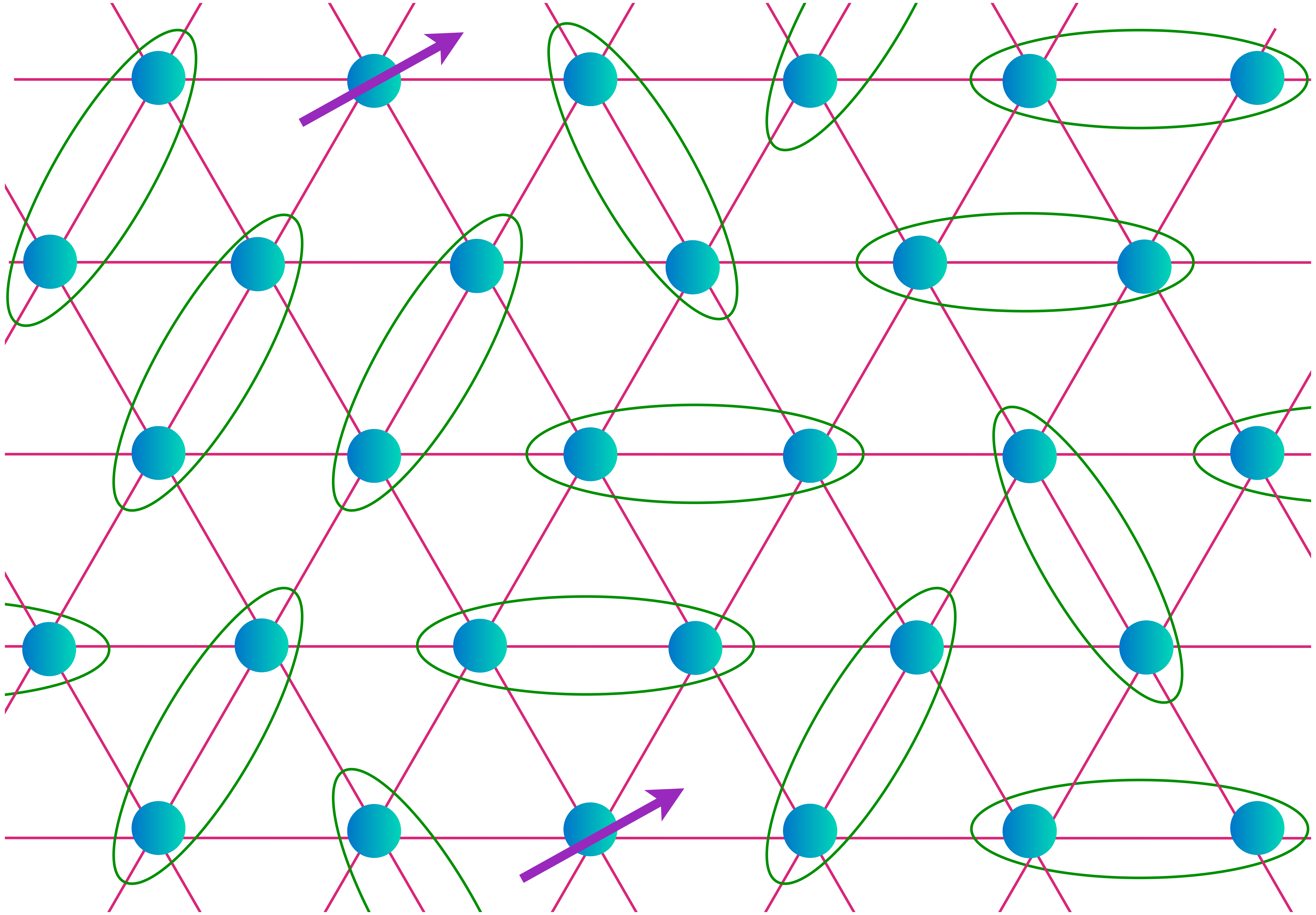

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RVB

A theory is needed to specify the $c_{\mathcal{D}}$ in the ground state wavefunction, and to show that the fractional excitations are not confined in pairs.

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Kalmeyer and Laughlin (1987):

The bosons form a fractional quantum Hall state with filling fraction $\nu = 1/2$. This has excitations with boson number $\pm 1/2$ which have fractional statistics. The excitation is an anyon: a semion, halfway between a boson and a fermion.

This state, now known as the chiral spin liquid, or a fractional Chern insulator, breaks time-reversal symmetry.

RVB: Z_2 spin liquid

A theory is needed to specify the $c_{\mathcal{D}}$ in the ground state wavefunction, and to show that the fractional excitations are not confined in pairs.

Read and Sachdev (1990); Wen (1991)

There is a state which has deconfined excitations with boson number 1/2 which need not break any symmetry, including time-reversal. This state has 3 distinct anyon excitations (superselection sectors):

Anyon	e (spinon)	ϵ (spinon)	m (vison)
Boson number	1/2	1/2	0
Self-statistics	boson	fermion	boson

Any pair of e, ϵ, m are mutual semions.

The existence of this $\mathbb{Z}_2$ spin liquid state was established by a connection to the deconfined phase of $\mathbb{Z}_2$ gauge theory.

RVB: Z_2 spin liquid

Excitations with boson number 0
a vison (*m* particle)

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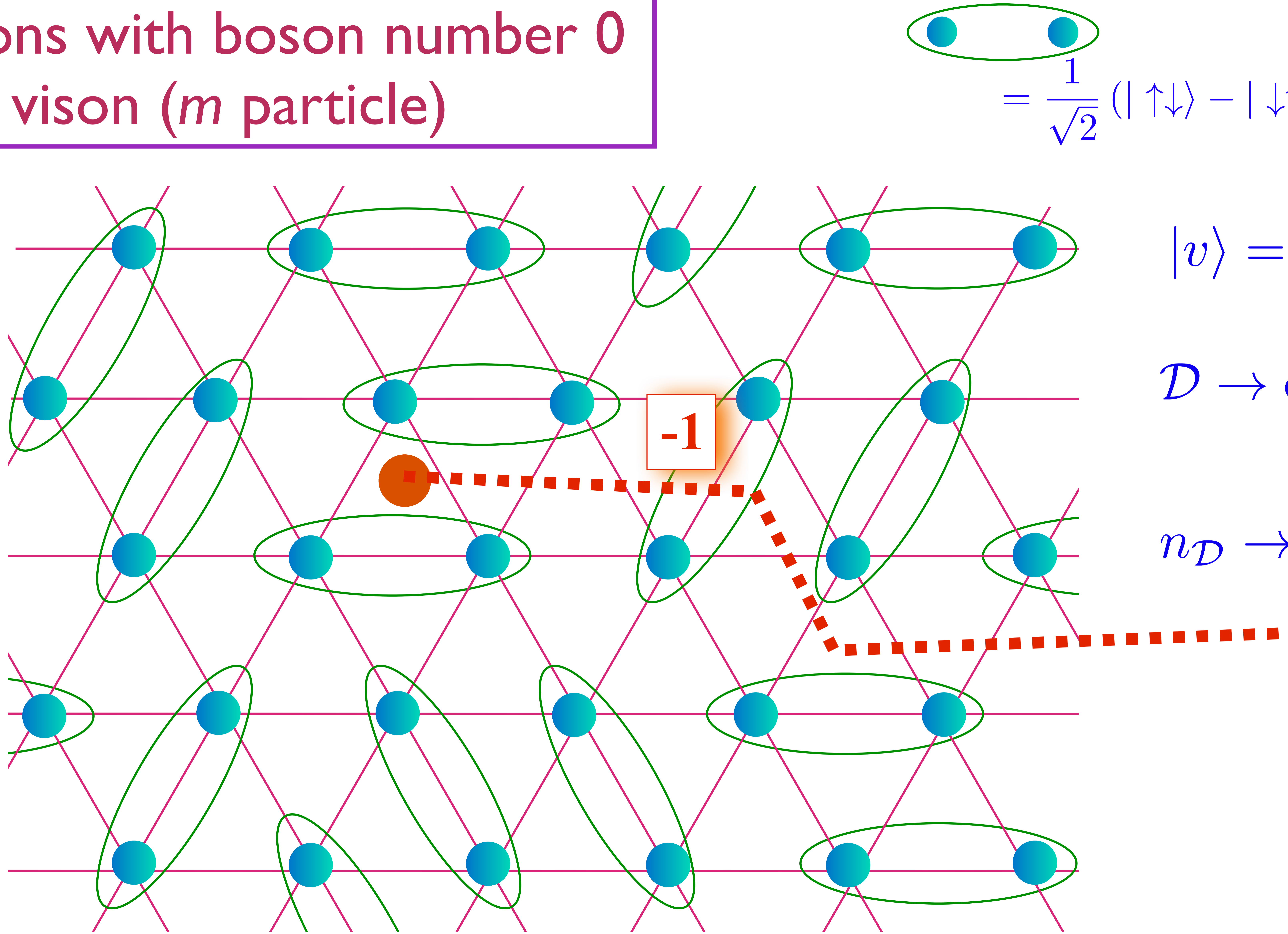

$$|v\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} (-1)^{n_{\mathcal{D}}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

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N. Read and B. Chakraborty, Phys. Rev. B 40, 7133 (1989)

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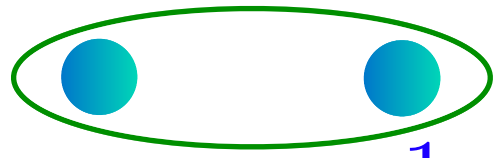

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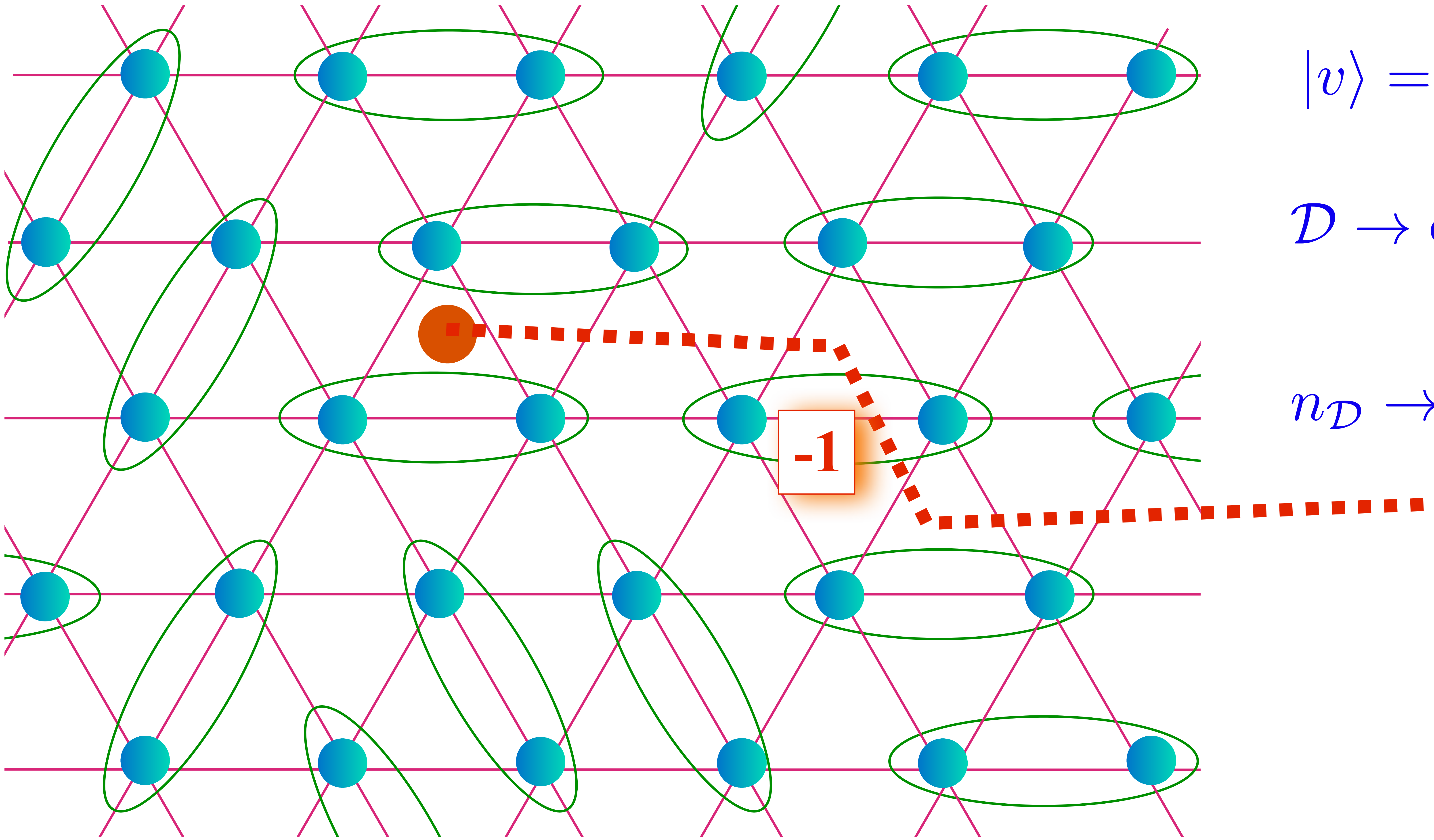

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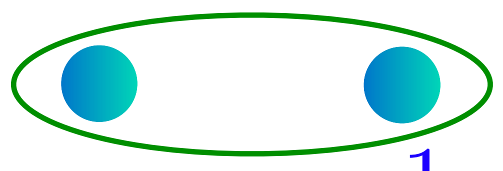

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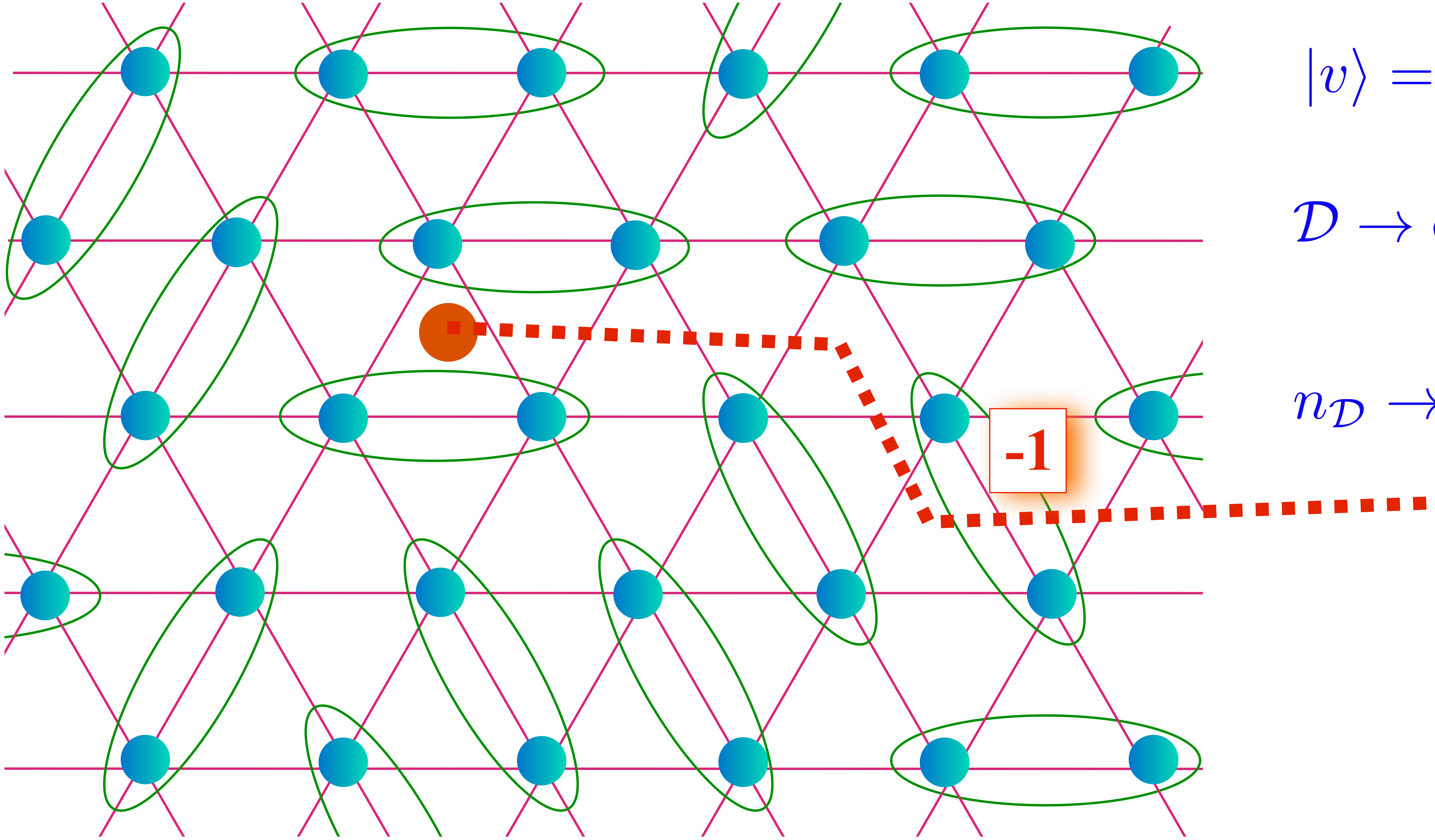

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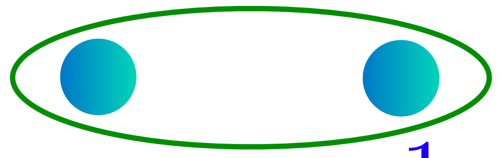

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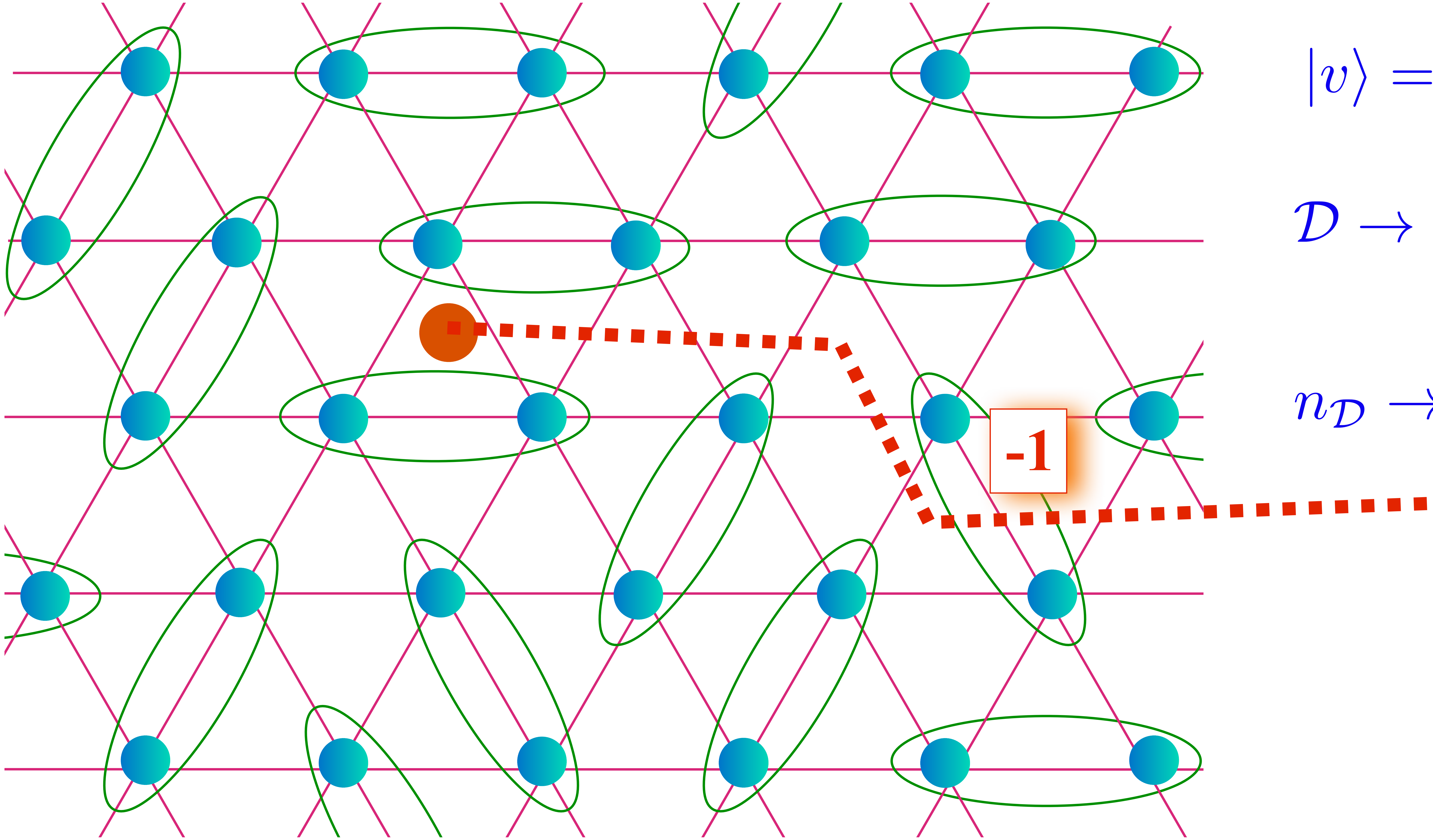

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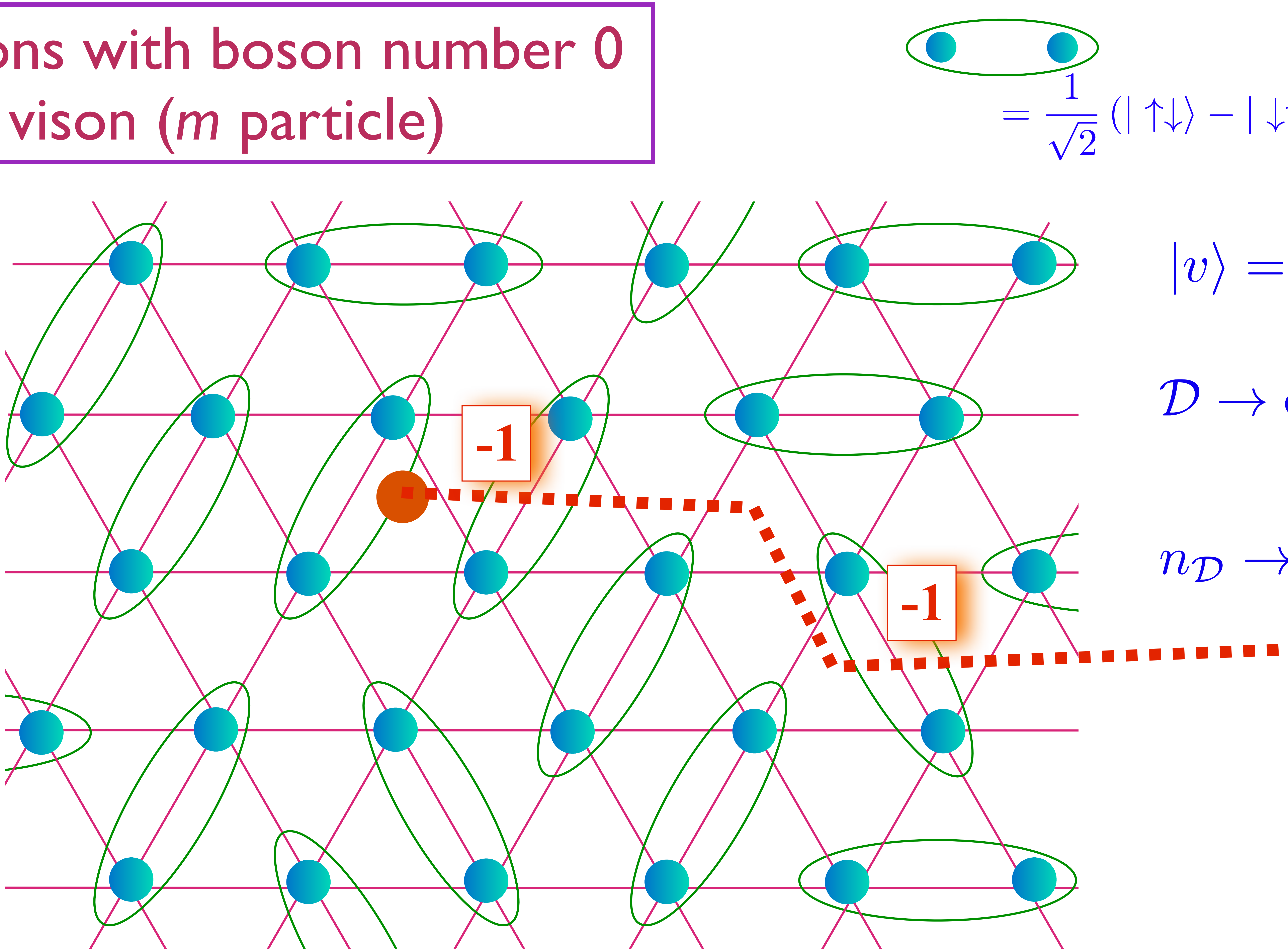

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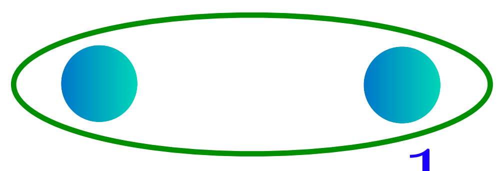

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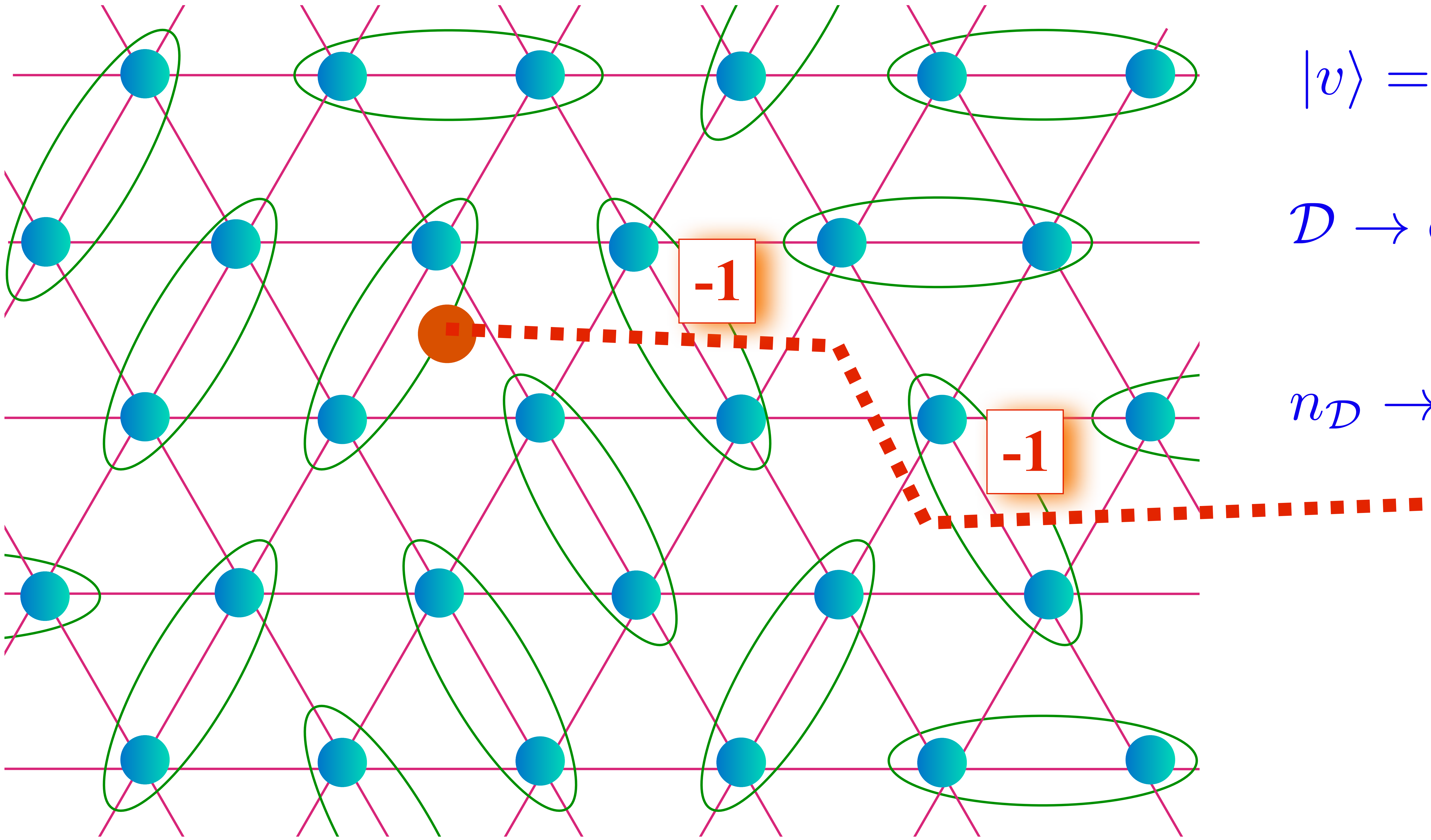

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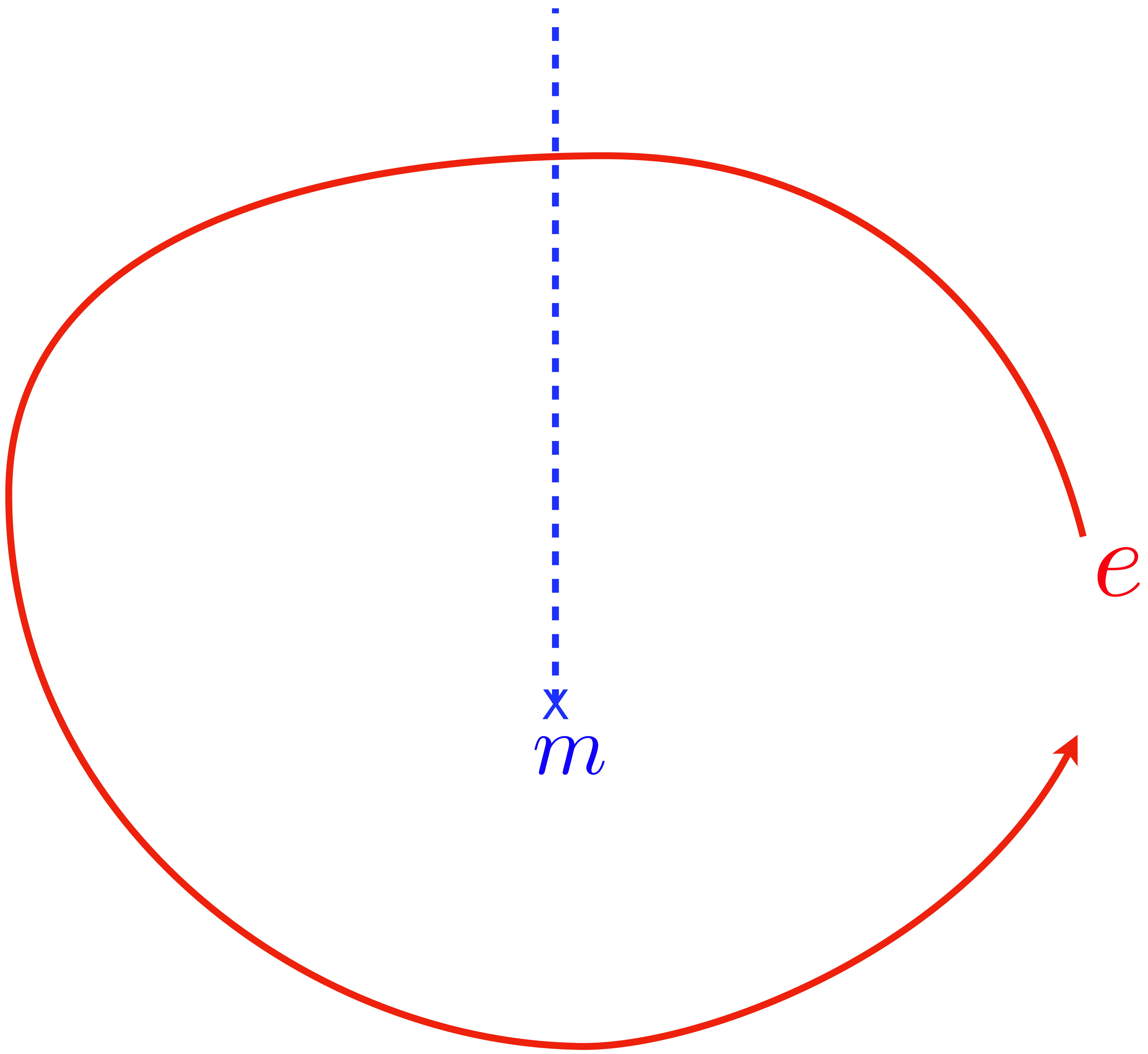

The e spinon and the m vison are *mutual* semions

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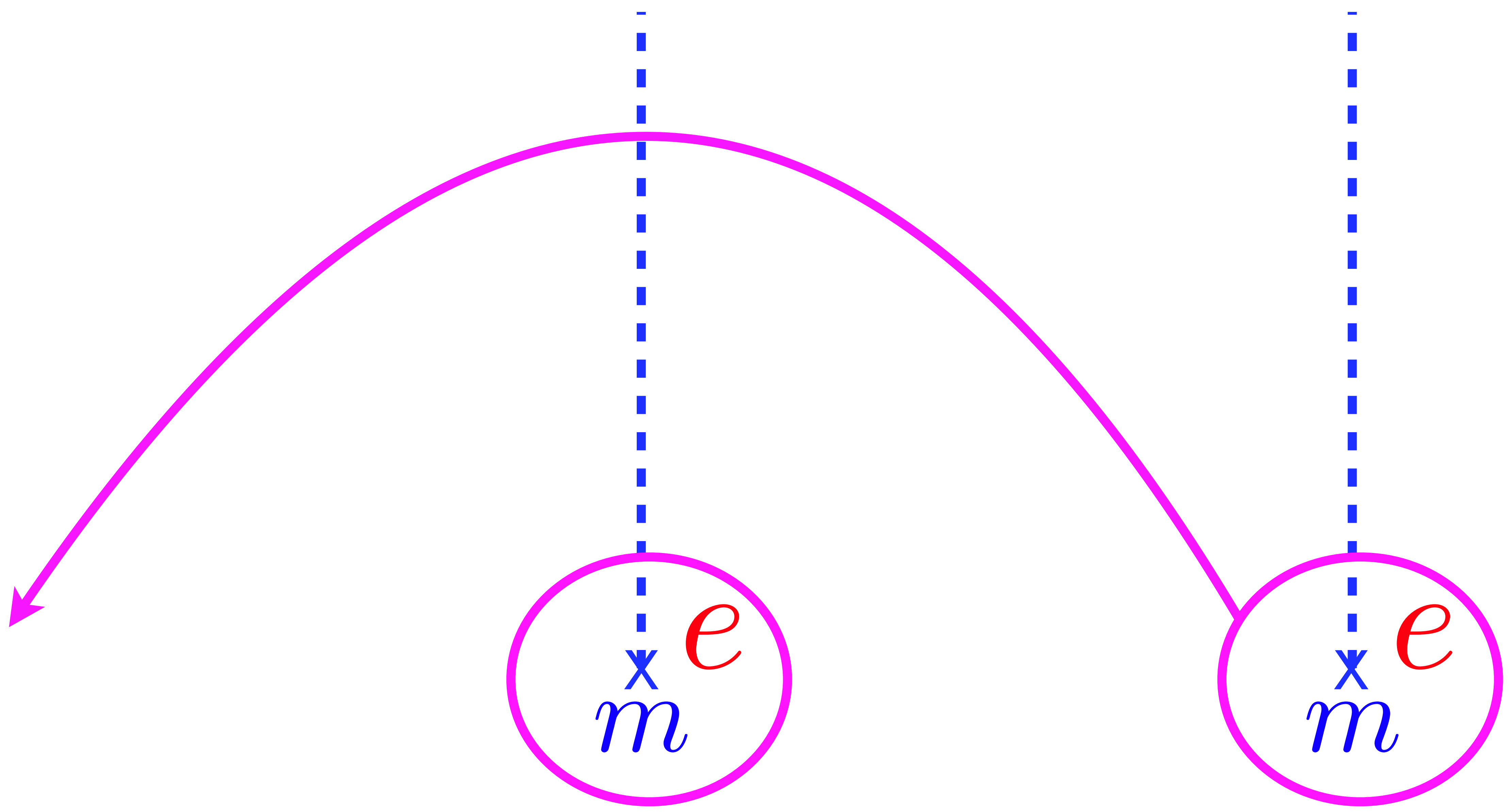

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Ground state degeneracy on the torus

Place insulator on a torus:

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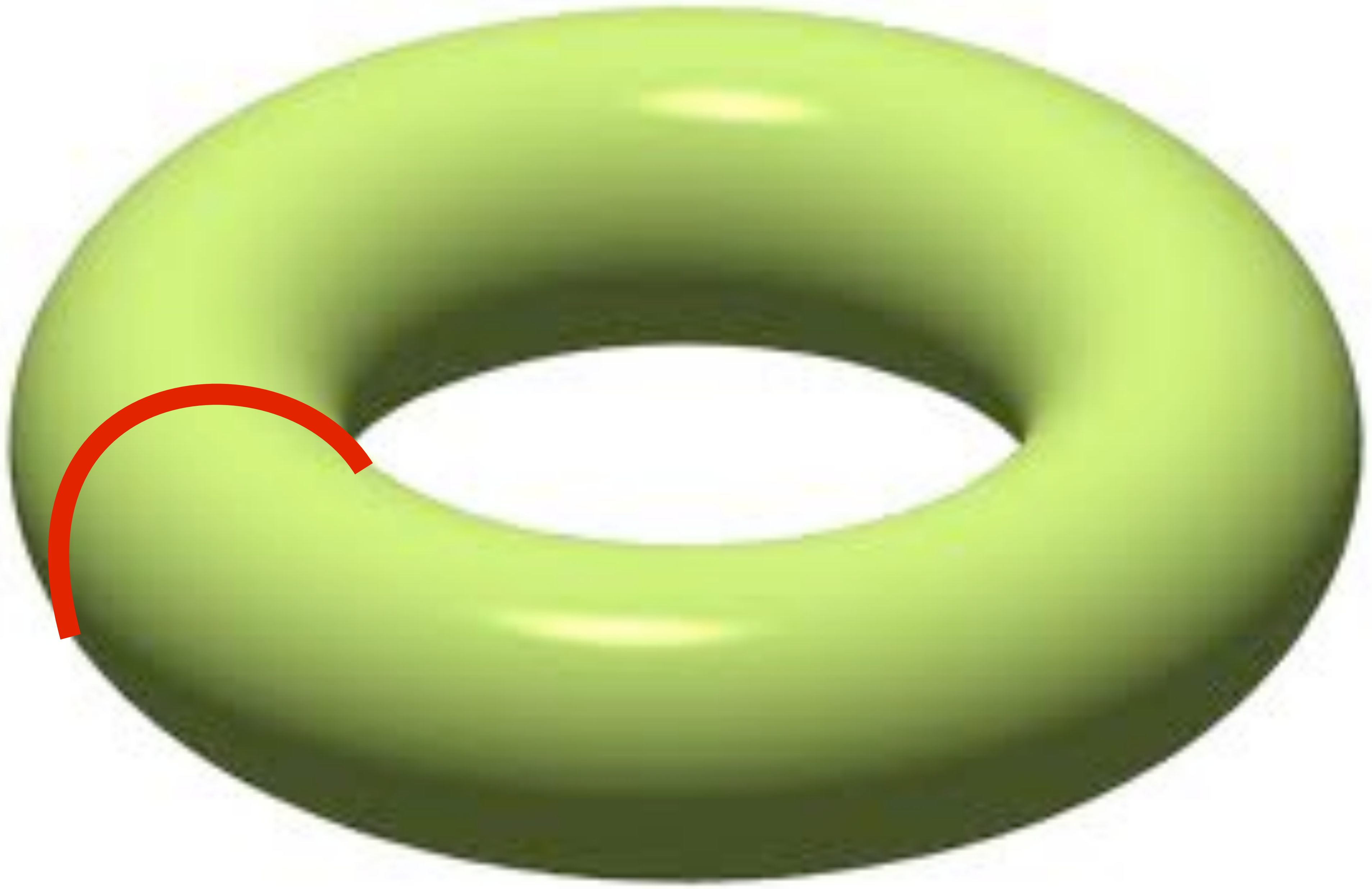
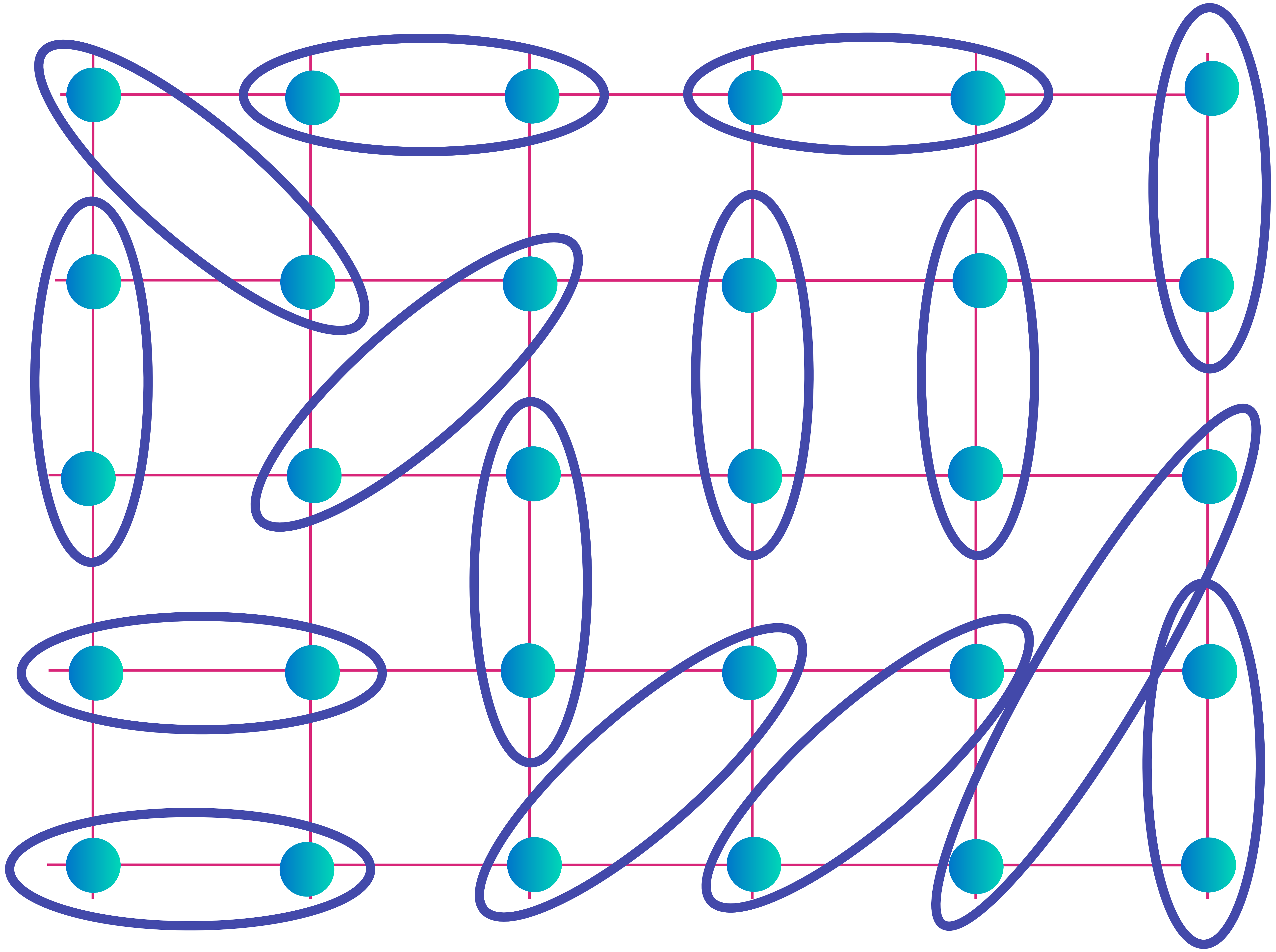


Place insulator on a torus:

Obtain a degenerate orthogonal state by modifying the wavefunction on a “branch-cut” encircling the torus.

RVB: Z_2 spin liquid

$$\text{(dimer)} = \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



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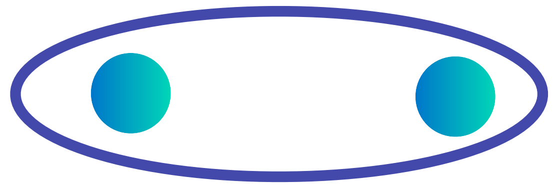

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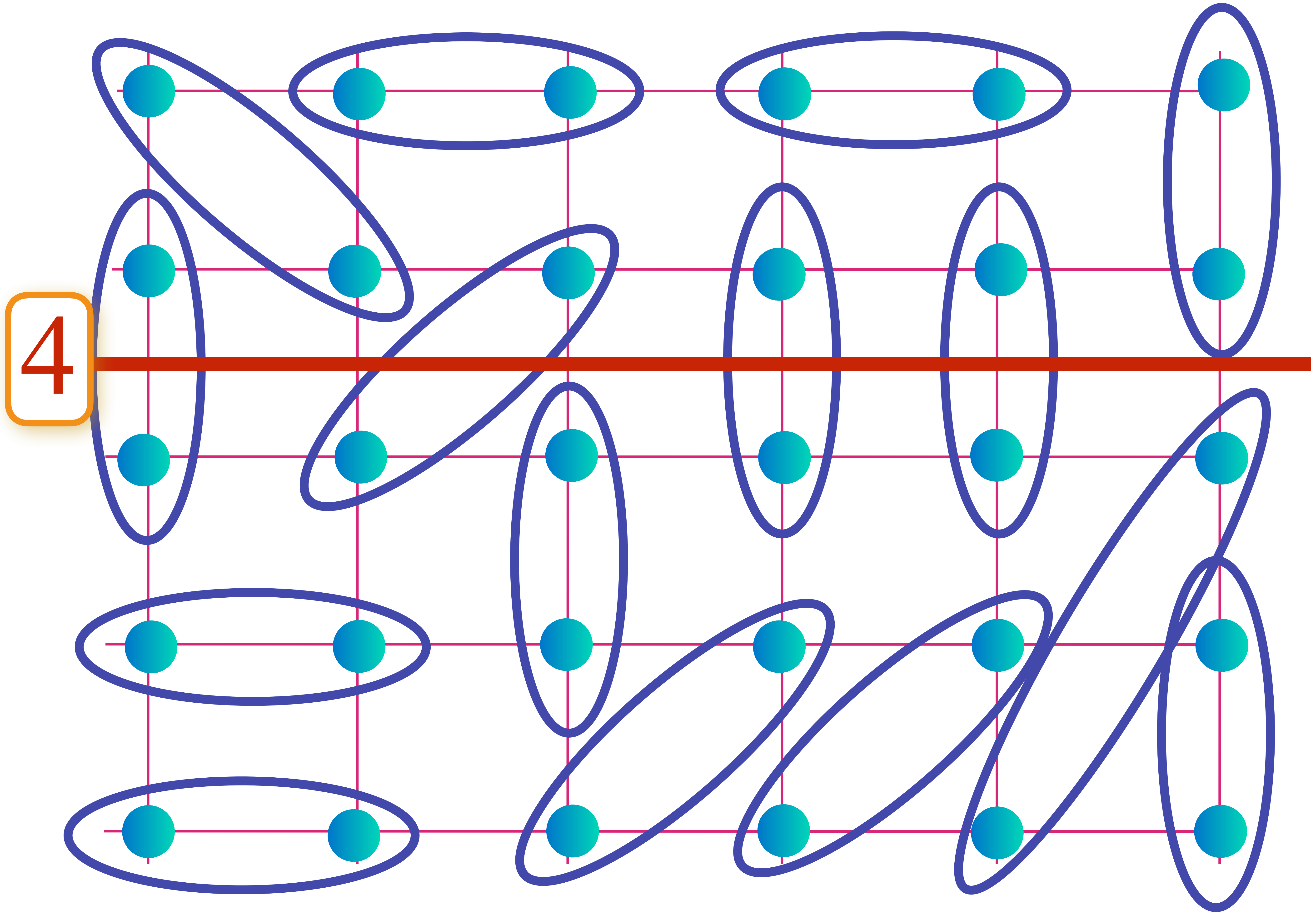

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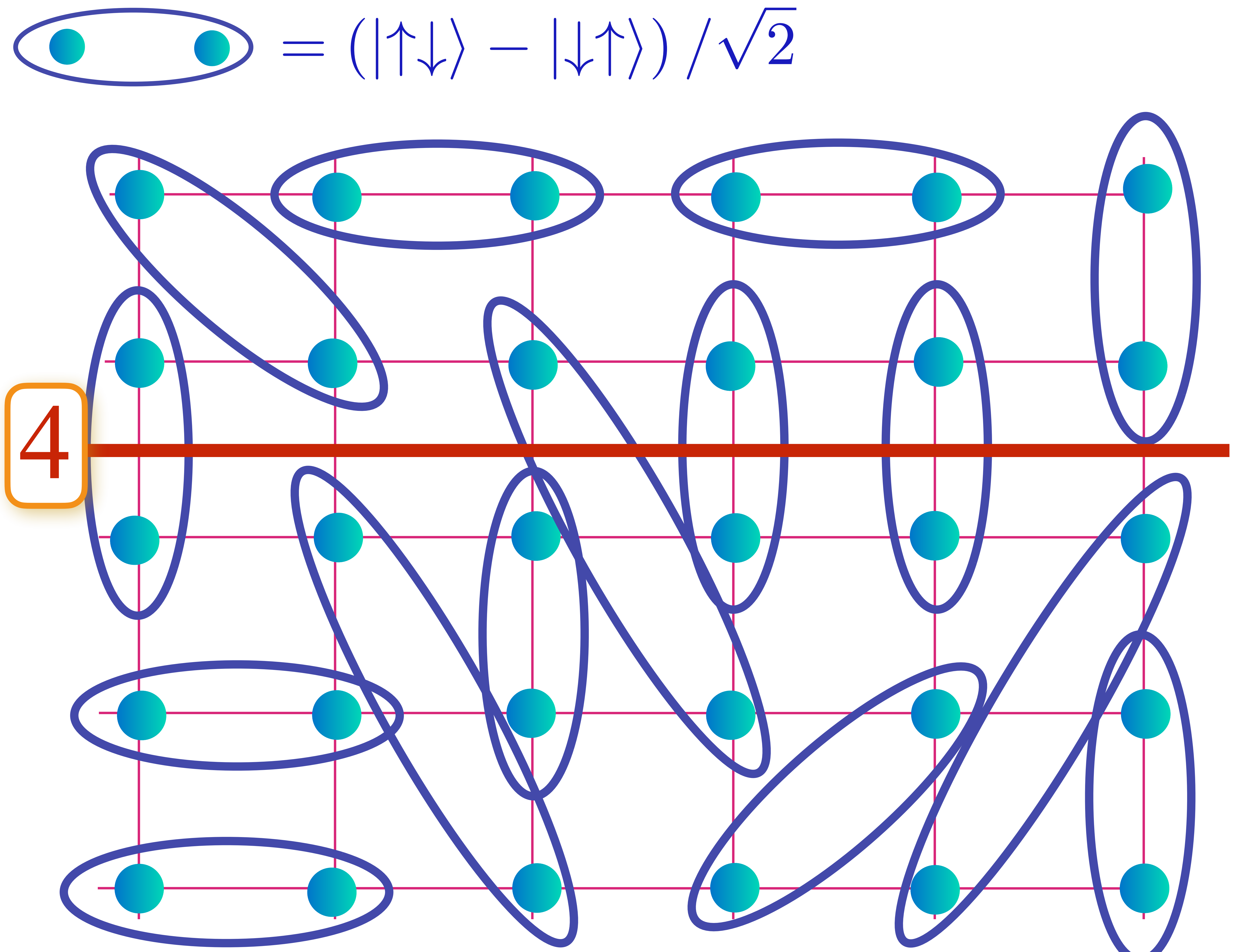

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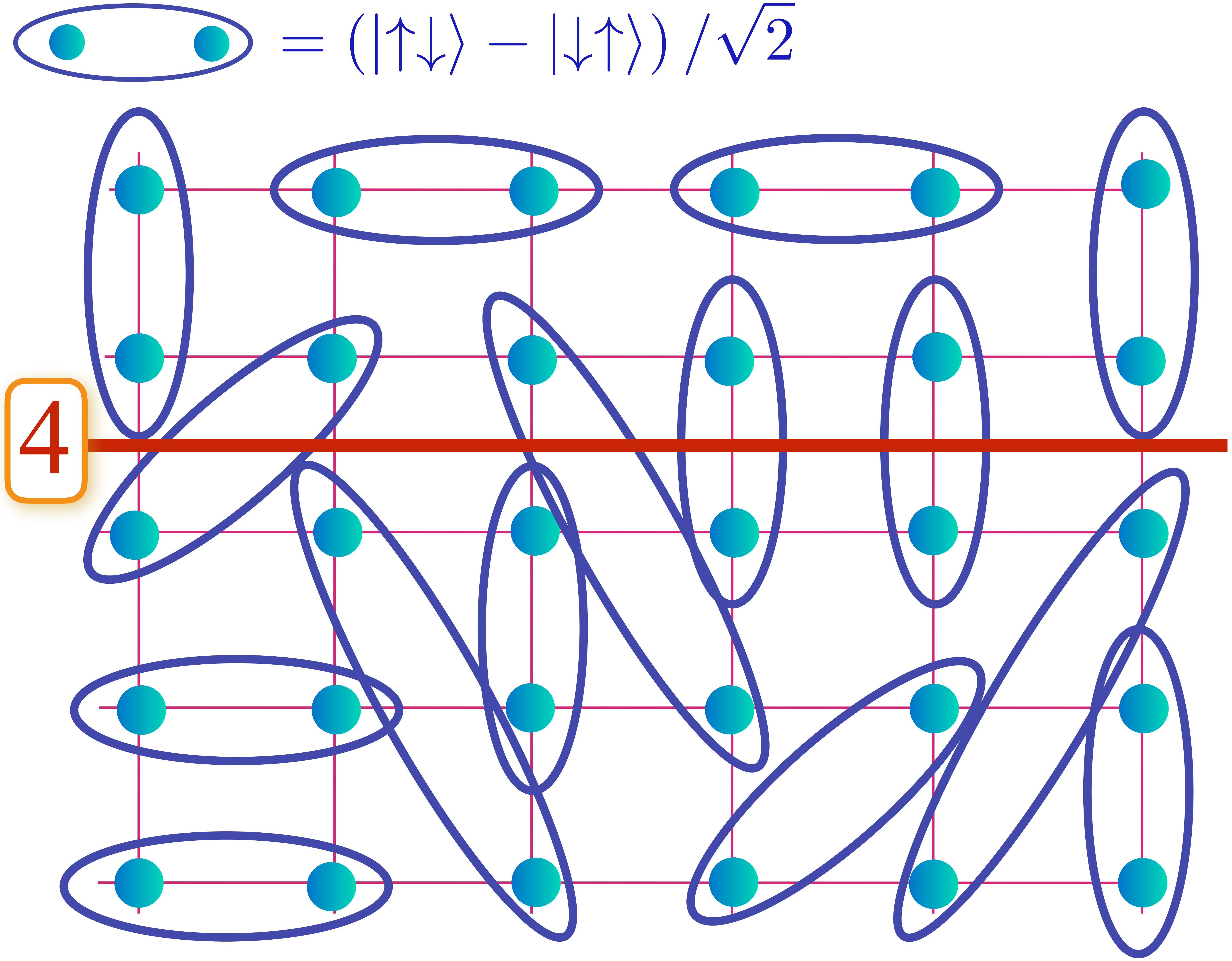

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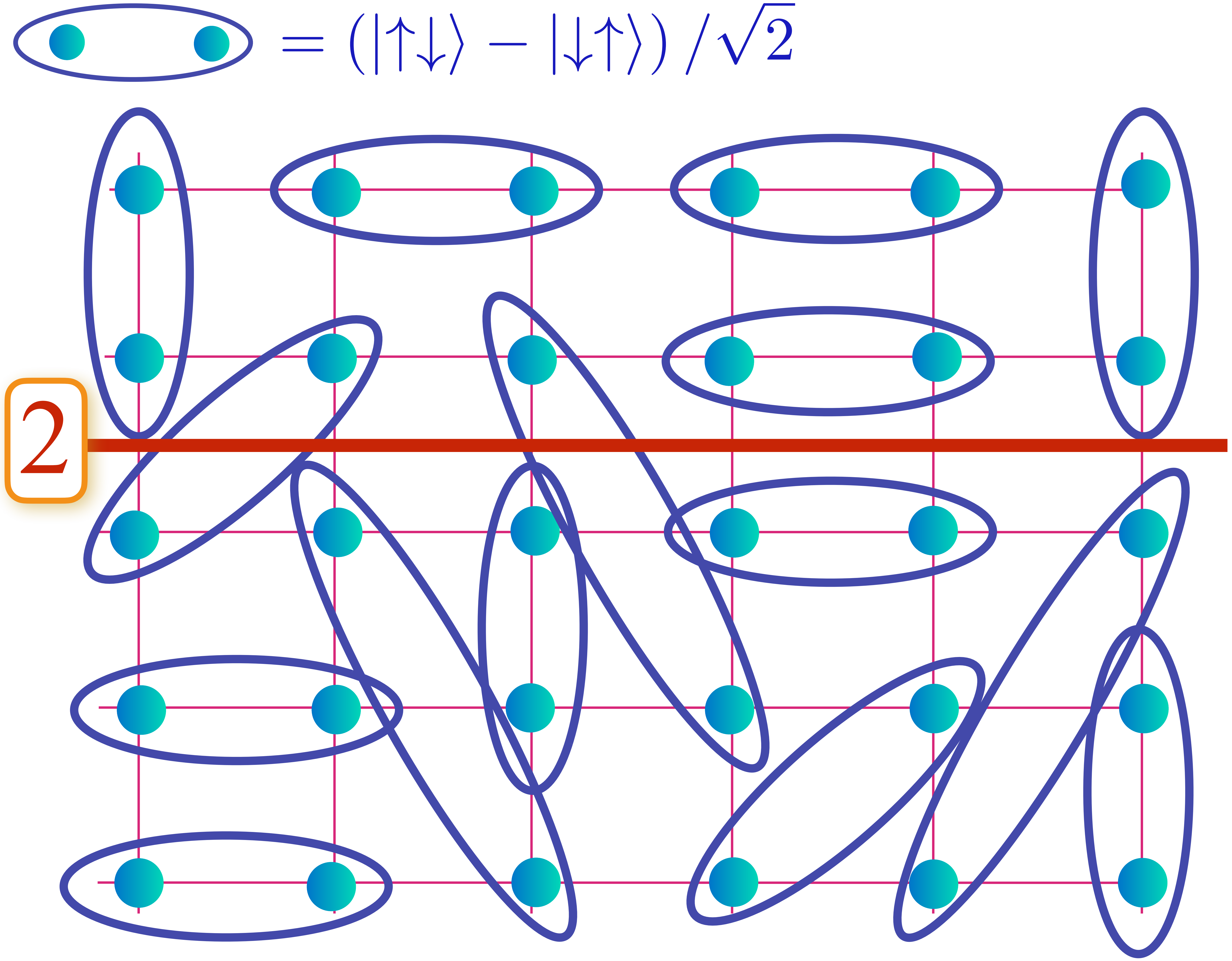

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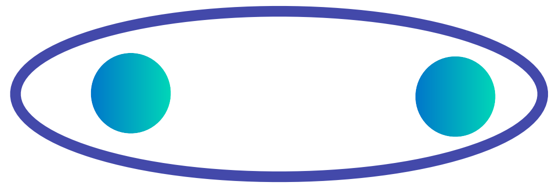

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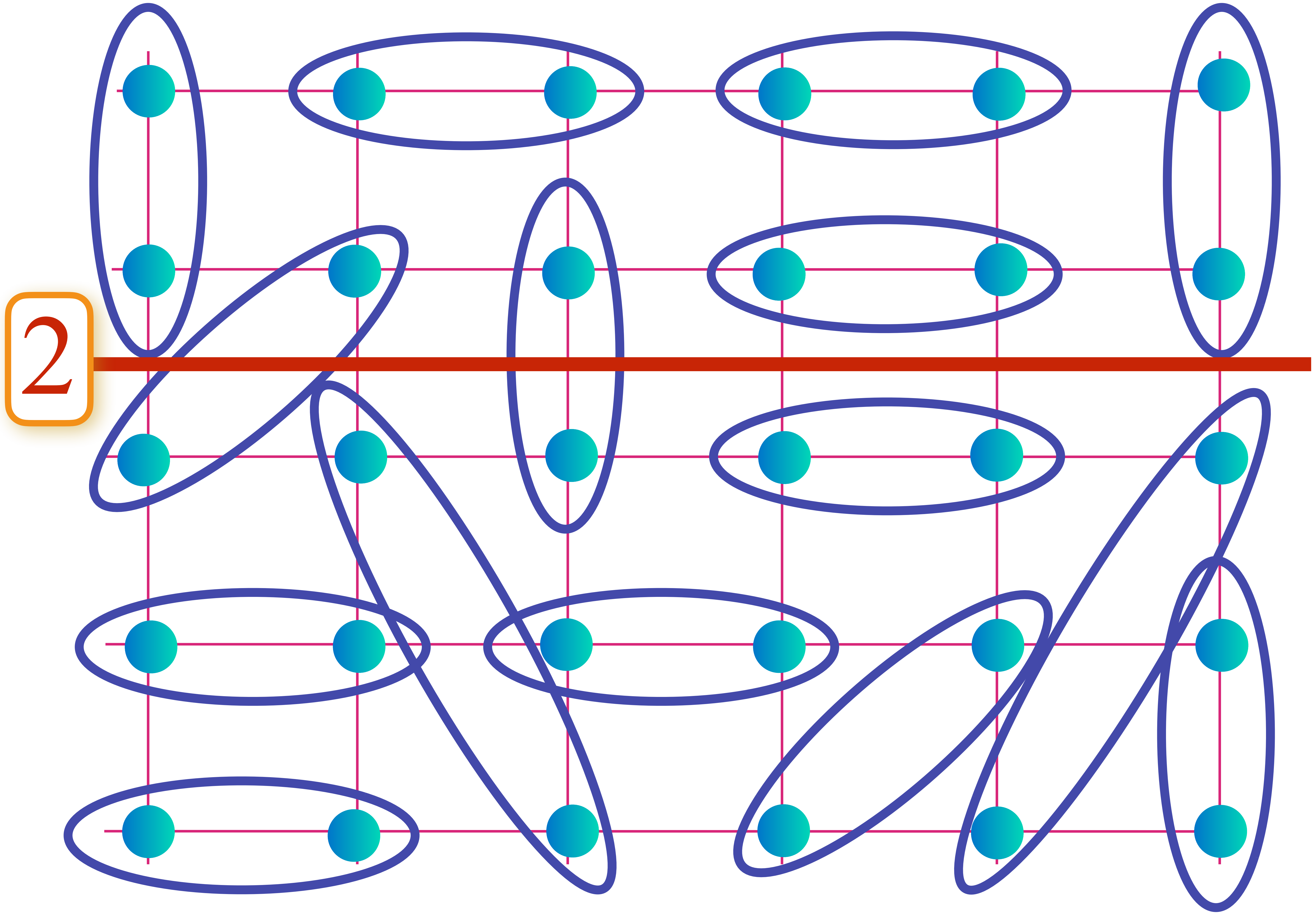

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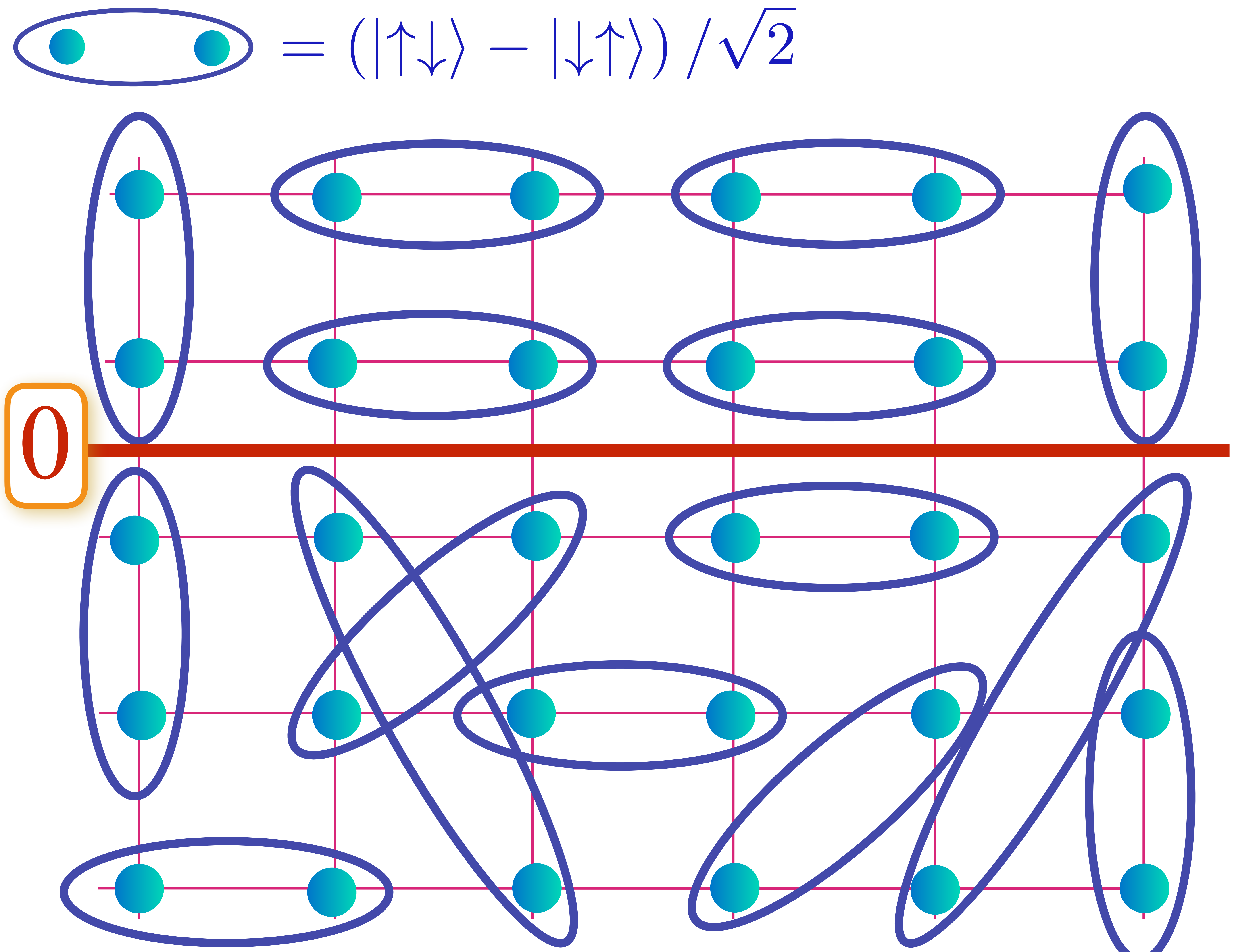

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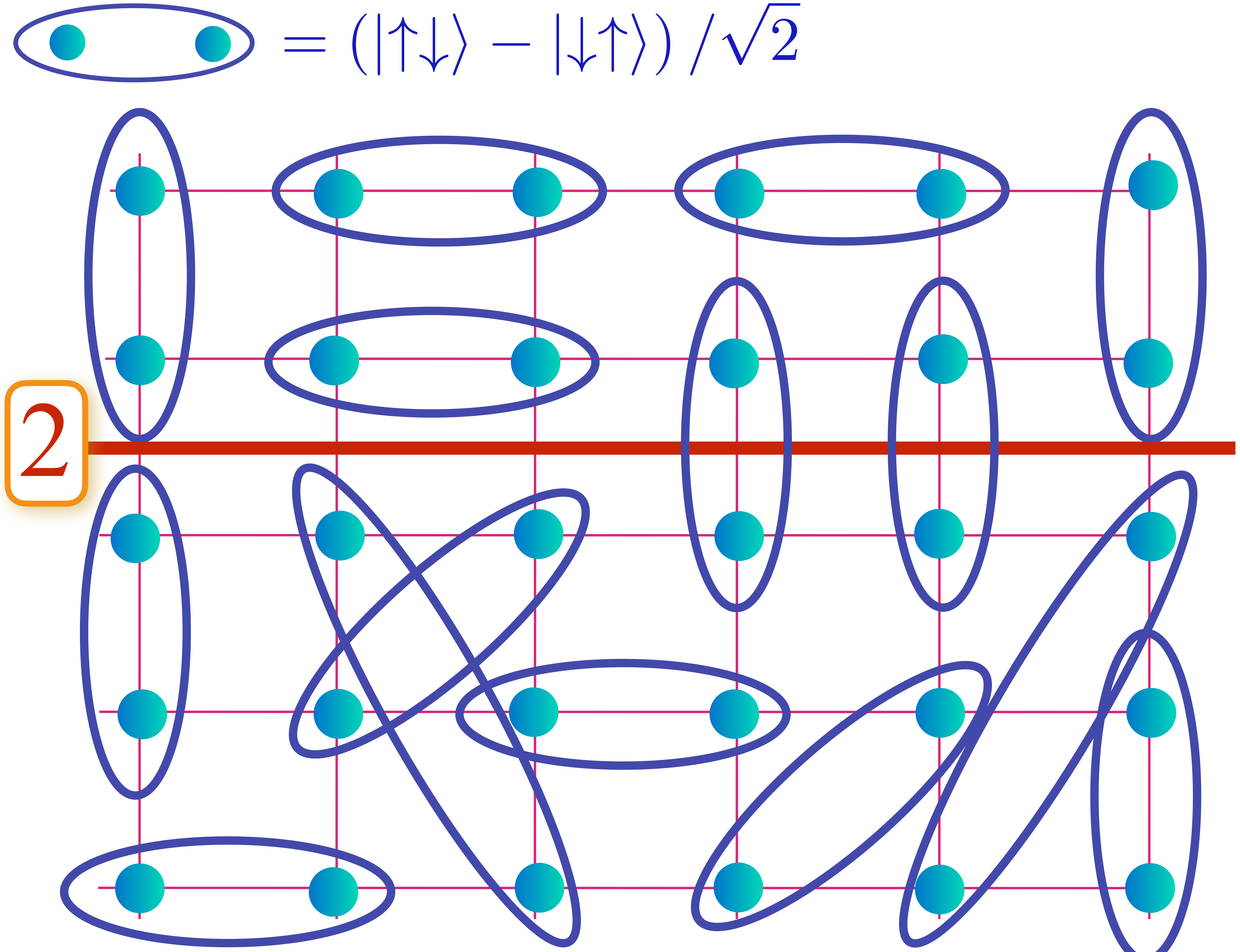

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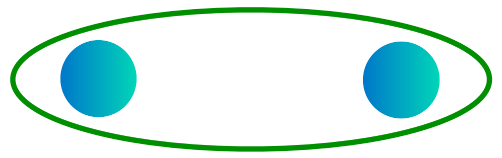
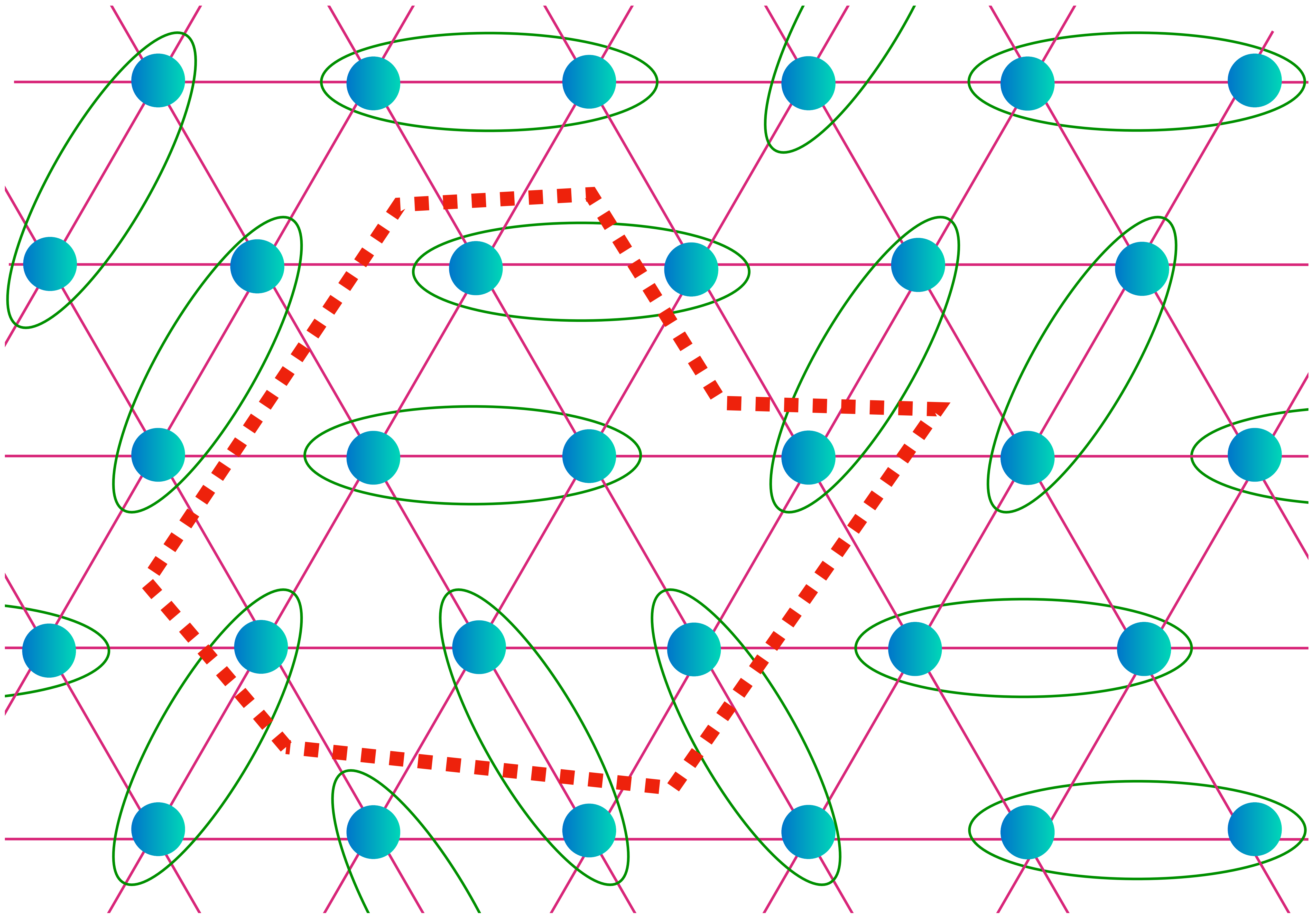
Topological Z operator of the Z_2 Spin liquid



$$= \frac{1}{\sqrt{2}} \Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \Big)$$

5



$Z = (-1)^{n_{\mathcal{D}}}$

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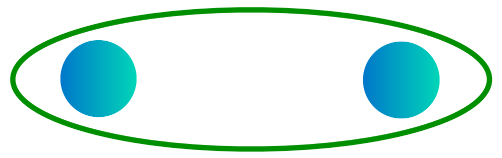

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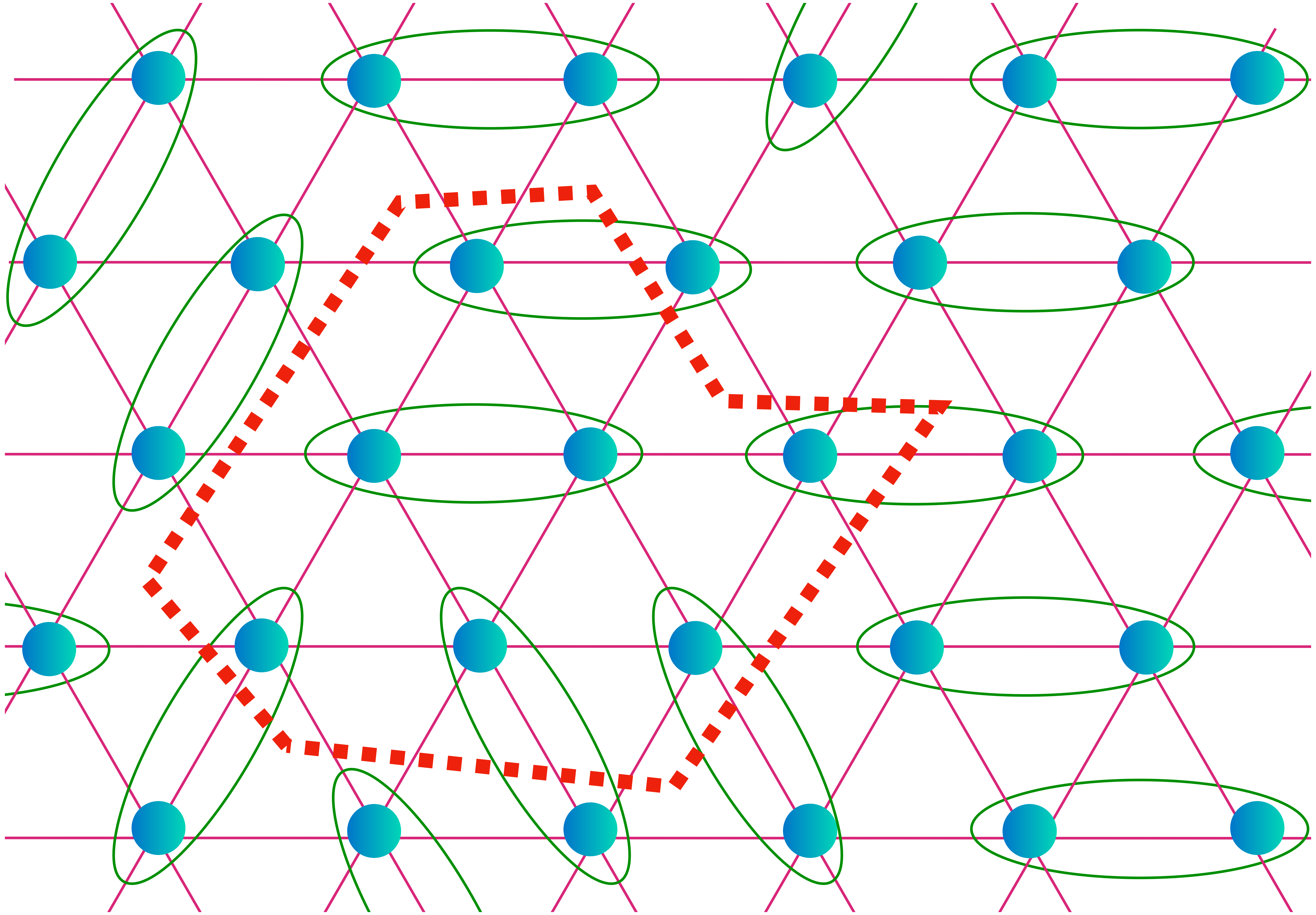

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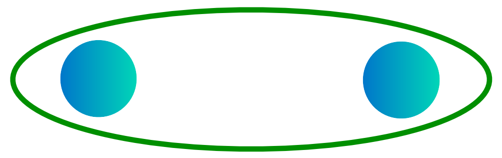

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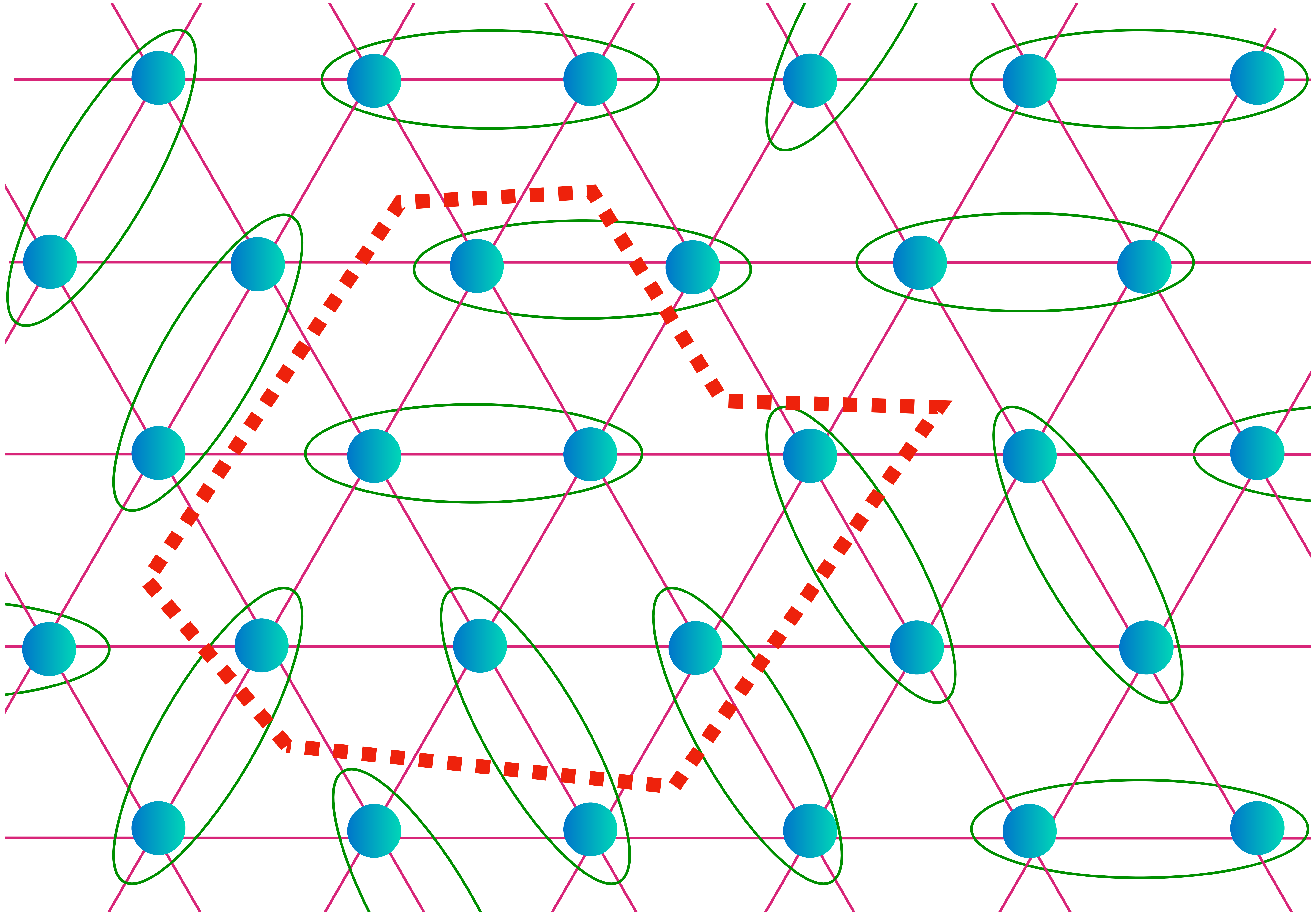

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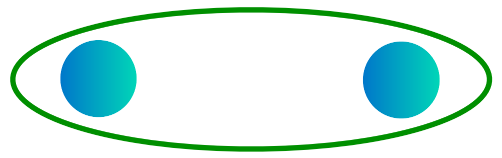

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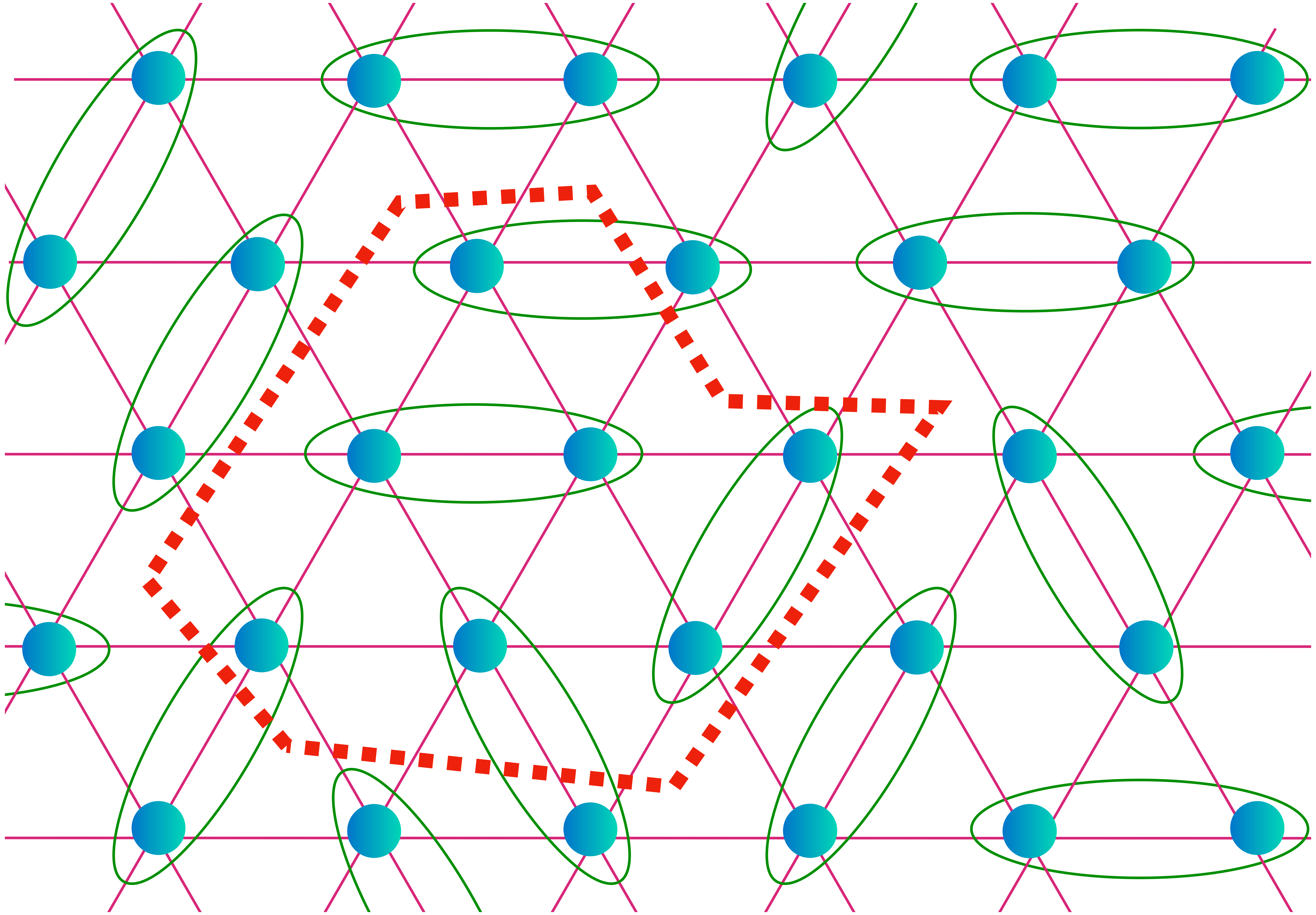

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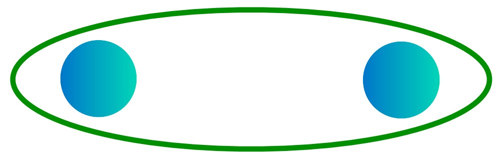

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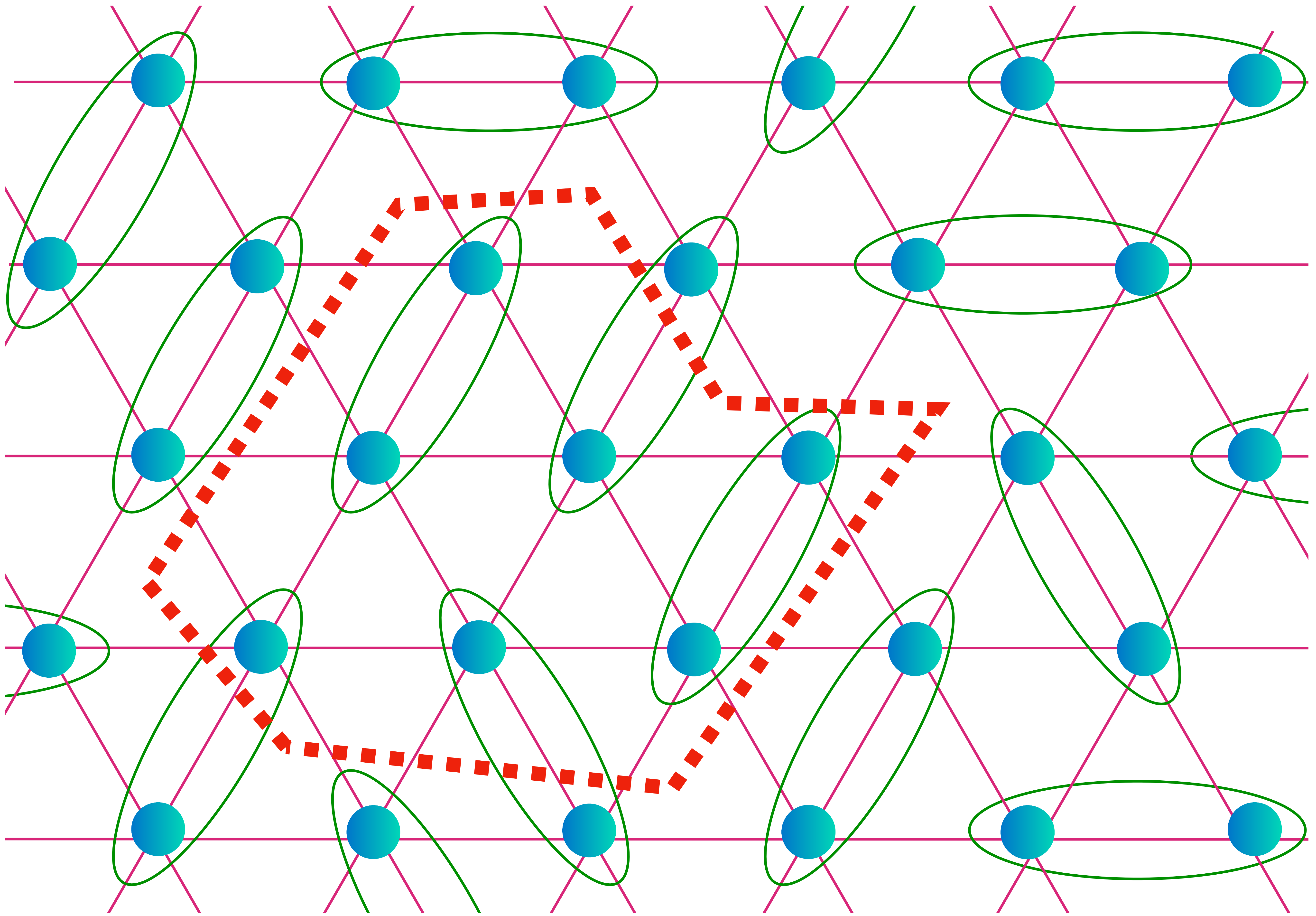

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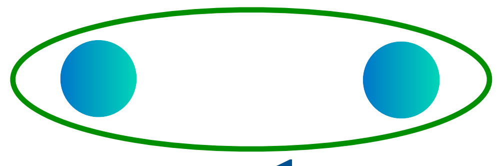
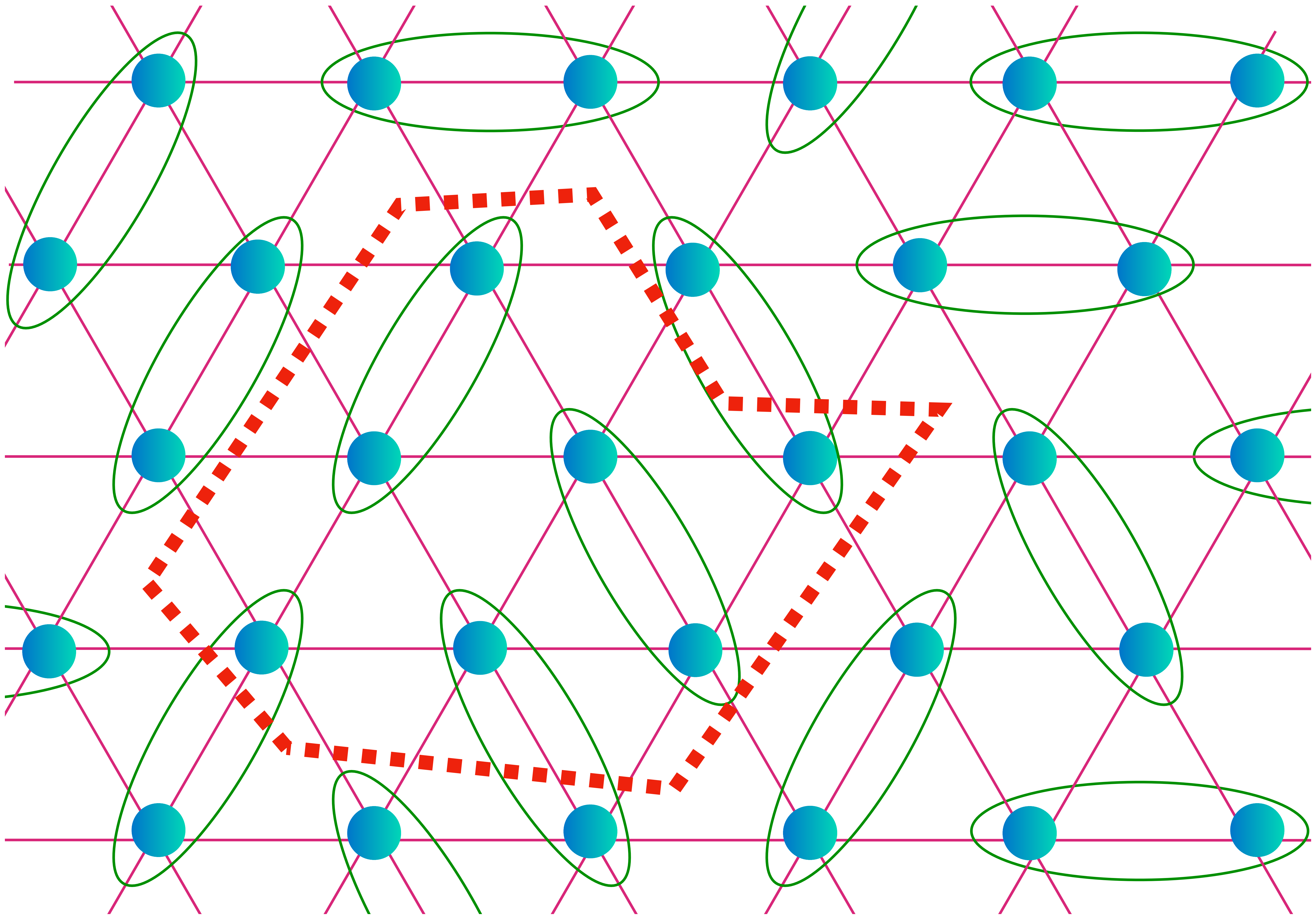
RVB: Z_2 spin liquid

Topological Z operator of the Z_2 Spin liquid



$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

3



$Z = (-1)^{n_{\mathcal{D}}}$

$=$ parity of sites enclosed by red line

$n_{\mathcal{D}} \rightarrow$ number of dimers crossing red line

1. Rydberg chains

    The Z_3 chiral clock transition

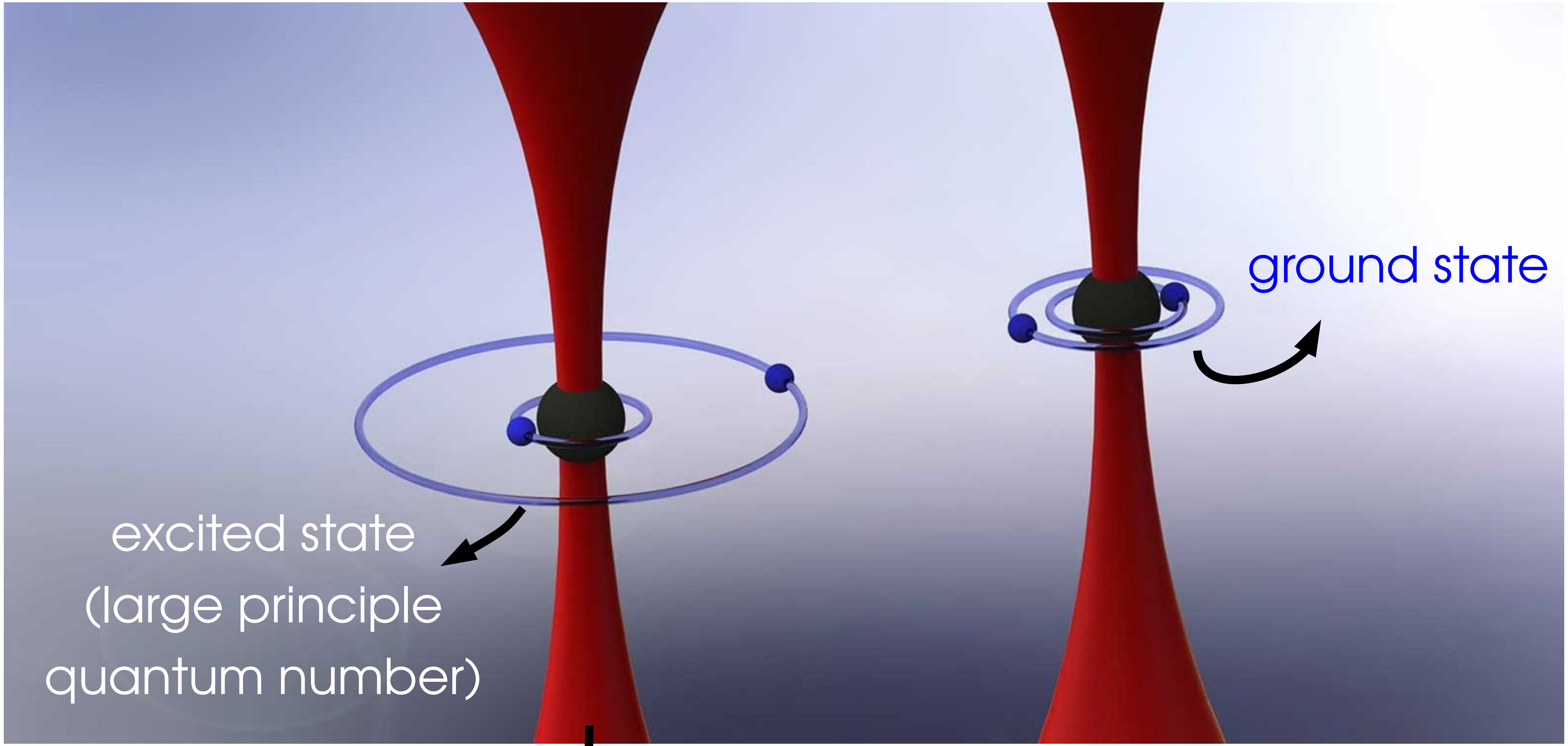
2. Square lattice

    Quantum Ising criticality in 2+1 dimensions

3. Resonating valence bonds and Z_2 spin liquids

4. Kagome symmetry lattices

    Probing topological spin liquids



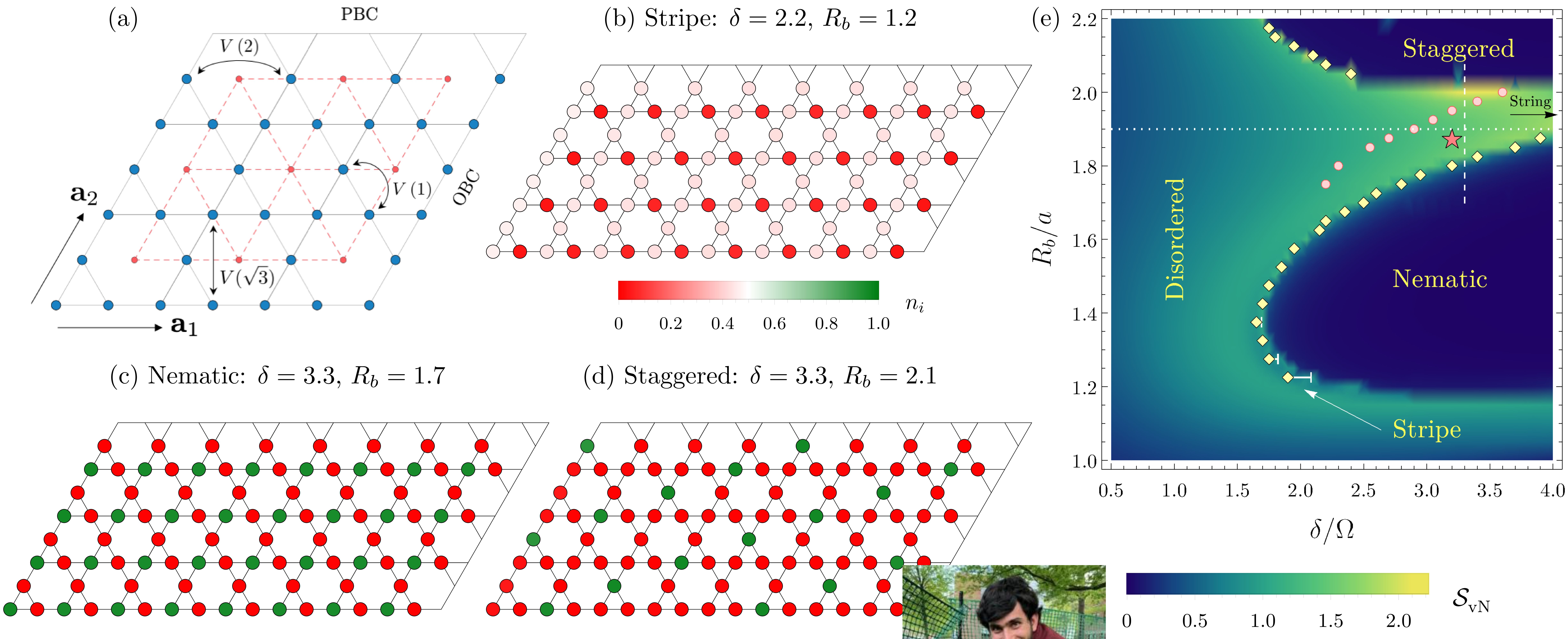
$$V_{|i-j|} \sim |i-j|^{-6}$$

optical tweezer (traps atom)

Fig: https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms

$$\mathcal{H} = \sum_j \left[\frac{\Omega}{2} \left(b_j + b_j^\dagger \right) - \Delta n_j \right] + \sum_{i<j} V_{|i-j|} n_i n_j \,, \quad n_j \equiv b_j^\dagger b_j = 0, 1.$$

Rydberg atoms on site-kagome lattice: theory



R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

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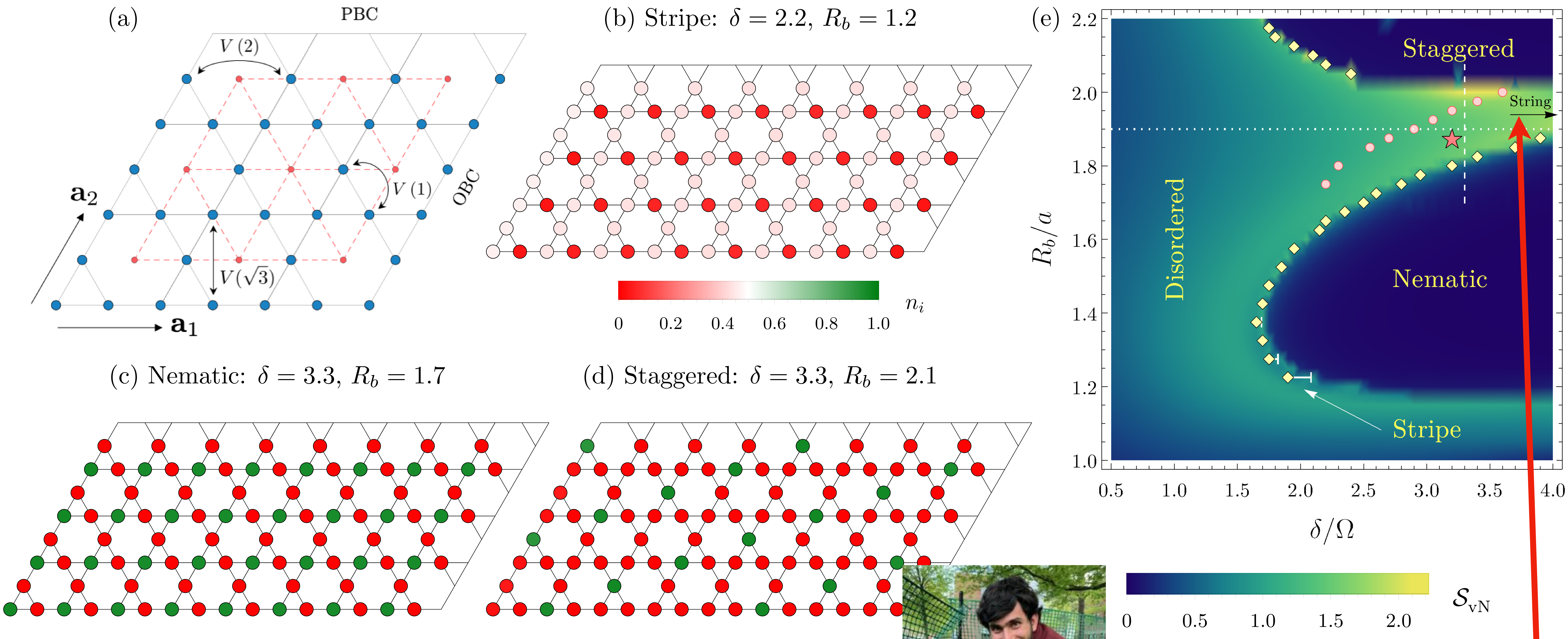

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$\mathbb{Z}_2$ spin liquid with b_j annhilating a dimer on the triangular lattice?

Rydberg atoms on link-kagome lattice: theory

$$\mathcal{H} = \sum_j \left[\frac{\Omega}{2} \left(b_j + b_j^\dagger \right) - \Delta n_j \right] + \sum_{i<j} V_{|i-j|} n_i n_j \,, \quad n_j \equiv b_j^\dagger b_j = 0, 1.$$

The sites j are on the links of the kagome lattice.

Examine the PXP model, $V_{6 \text{ nearest neighbors}} = \infty$, other $V_k = 0$.

R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)

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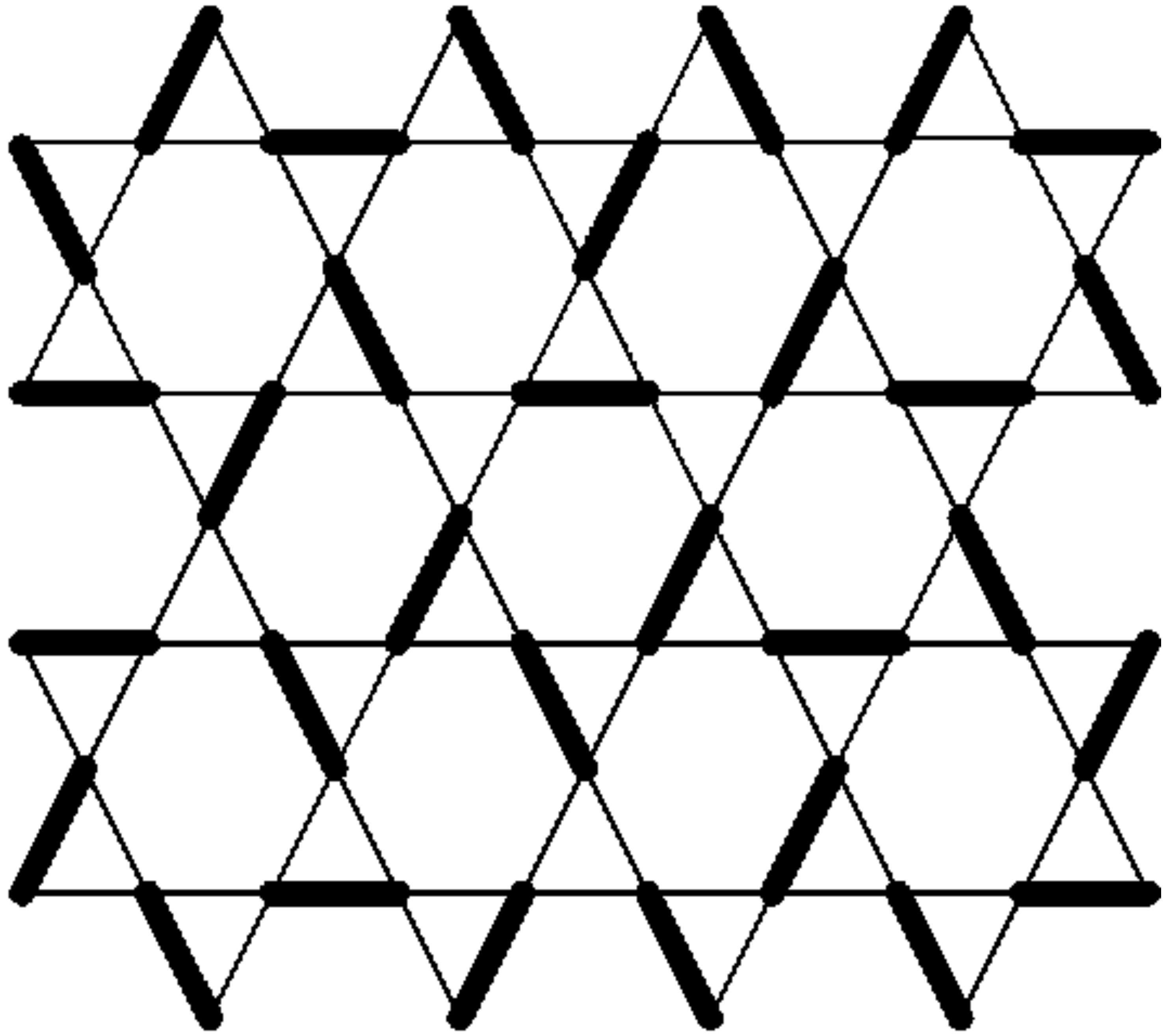

The 'dimers' represent the sites with $n_j = 1$.

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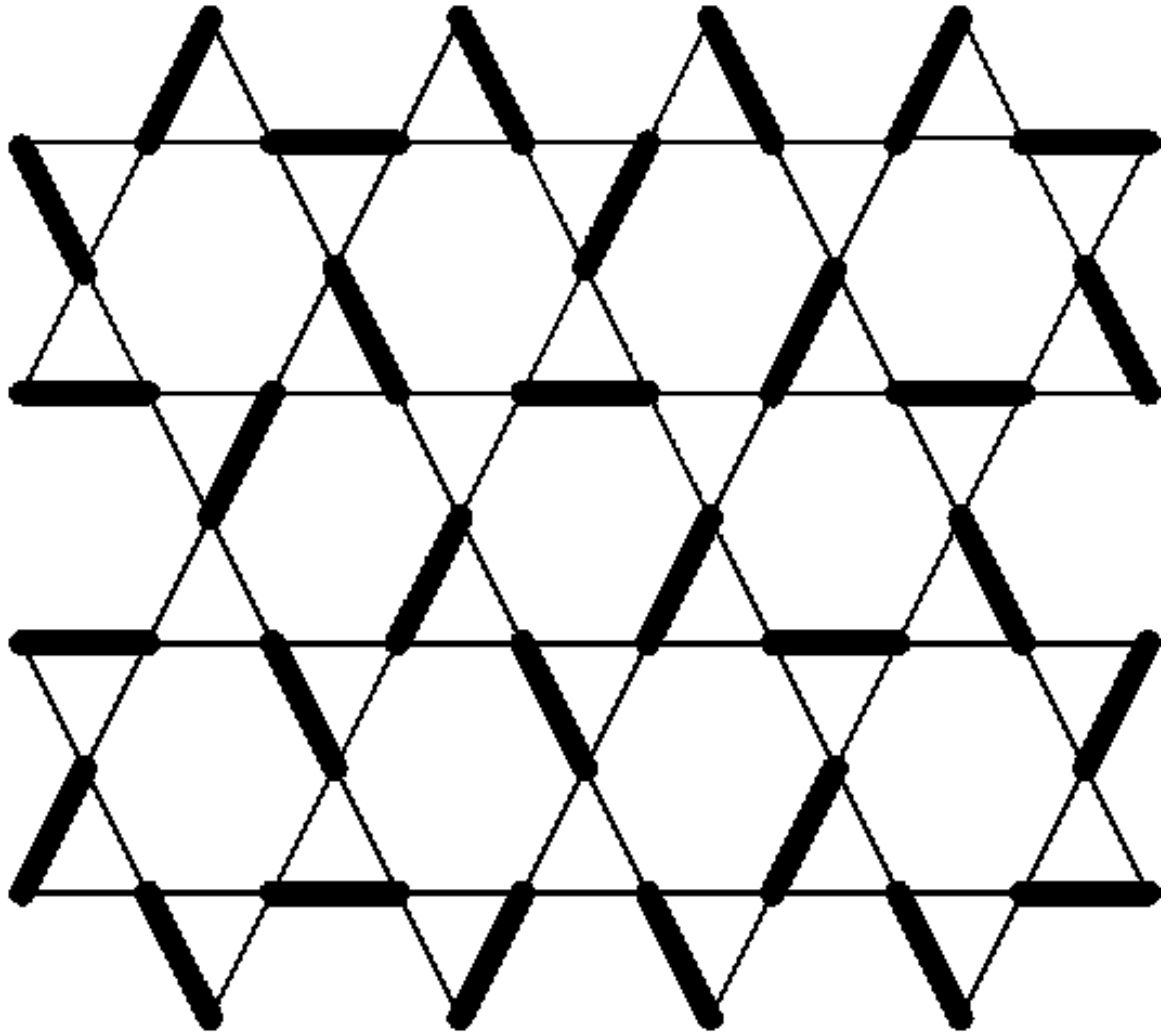

The 'dimers' represent the sites with $n_j = 1$.

For $\Omega \neq 0$, there is a $\mathbb{Z}_2$ spin liquid ground state, of the type studied in S. Sachdev, PRB **45**, 12377 (1992). The $\mathbb{Z}_2$ 'topological order' survives even though there is no conserved boson number $(\sum_i S_{zi})$ in the Rydberg case.

The b_j operators annihilate dimers on the kagome lattice.

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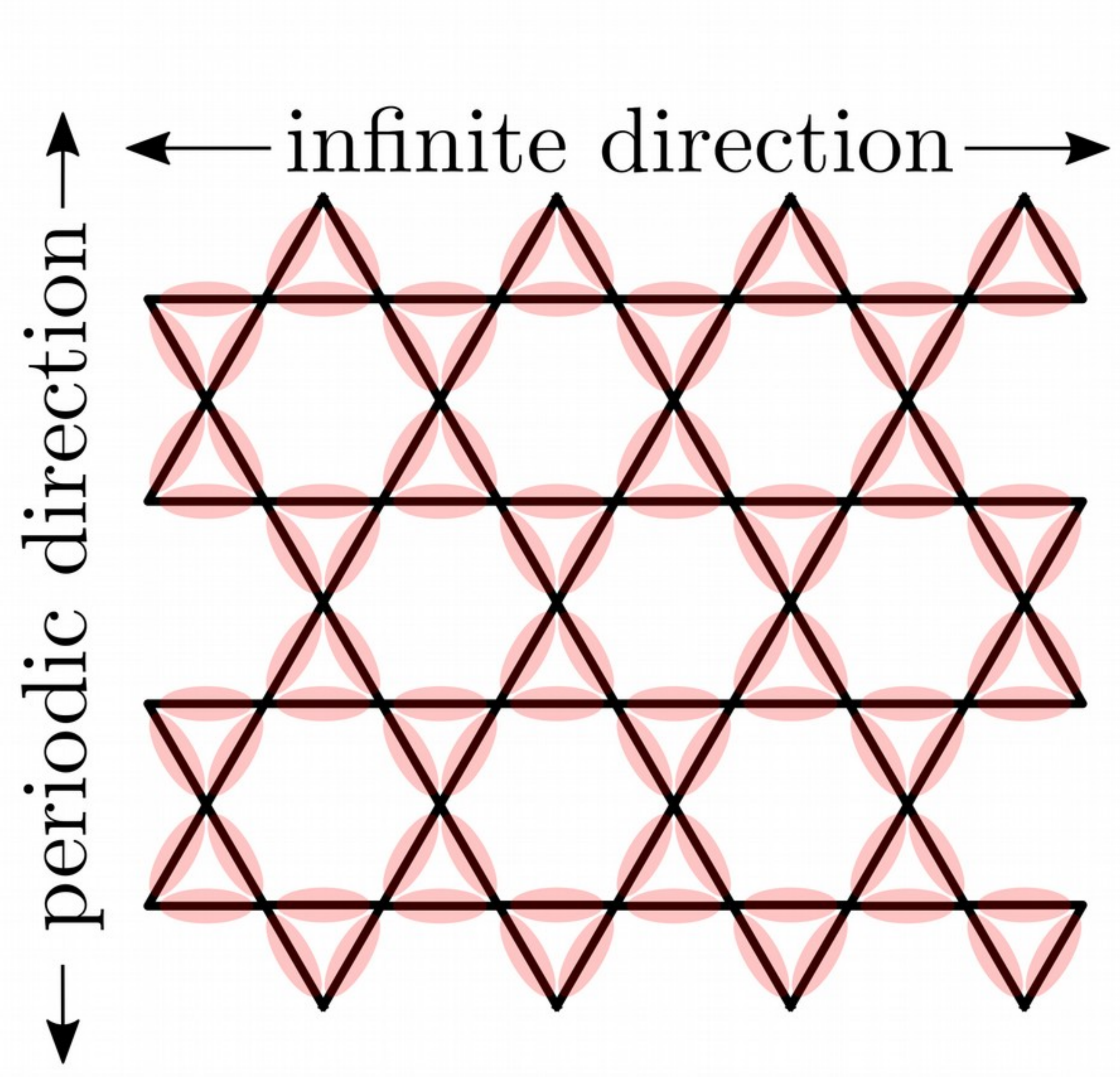
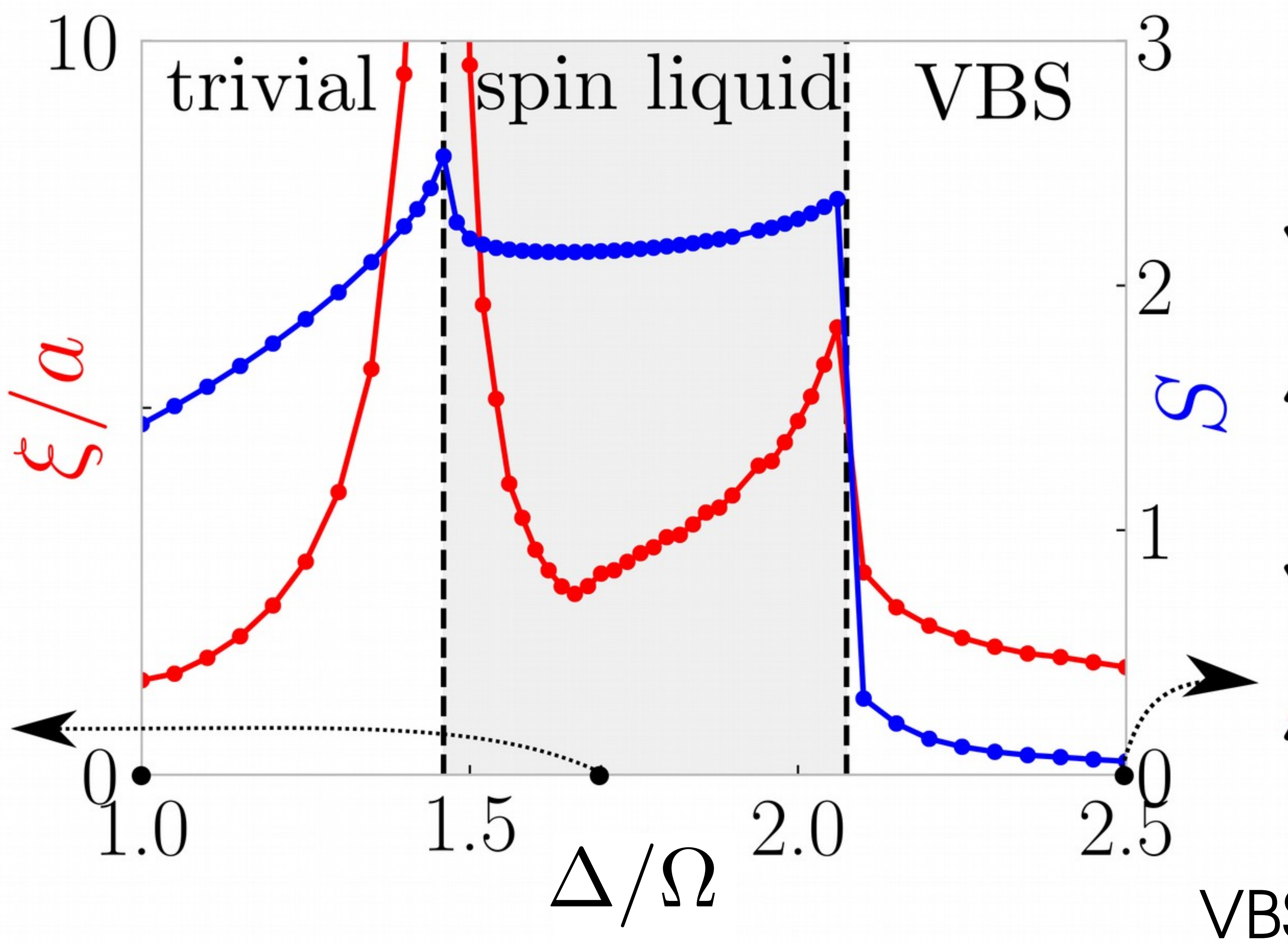
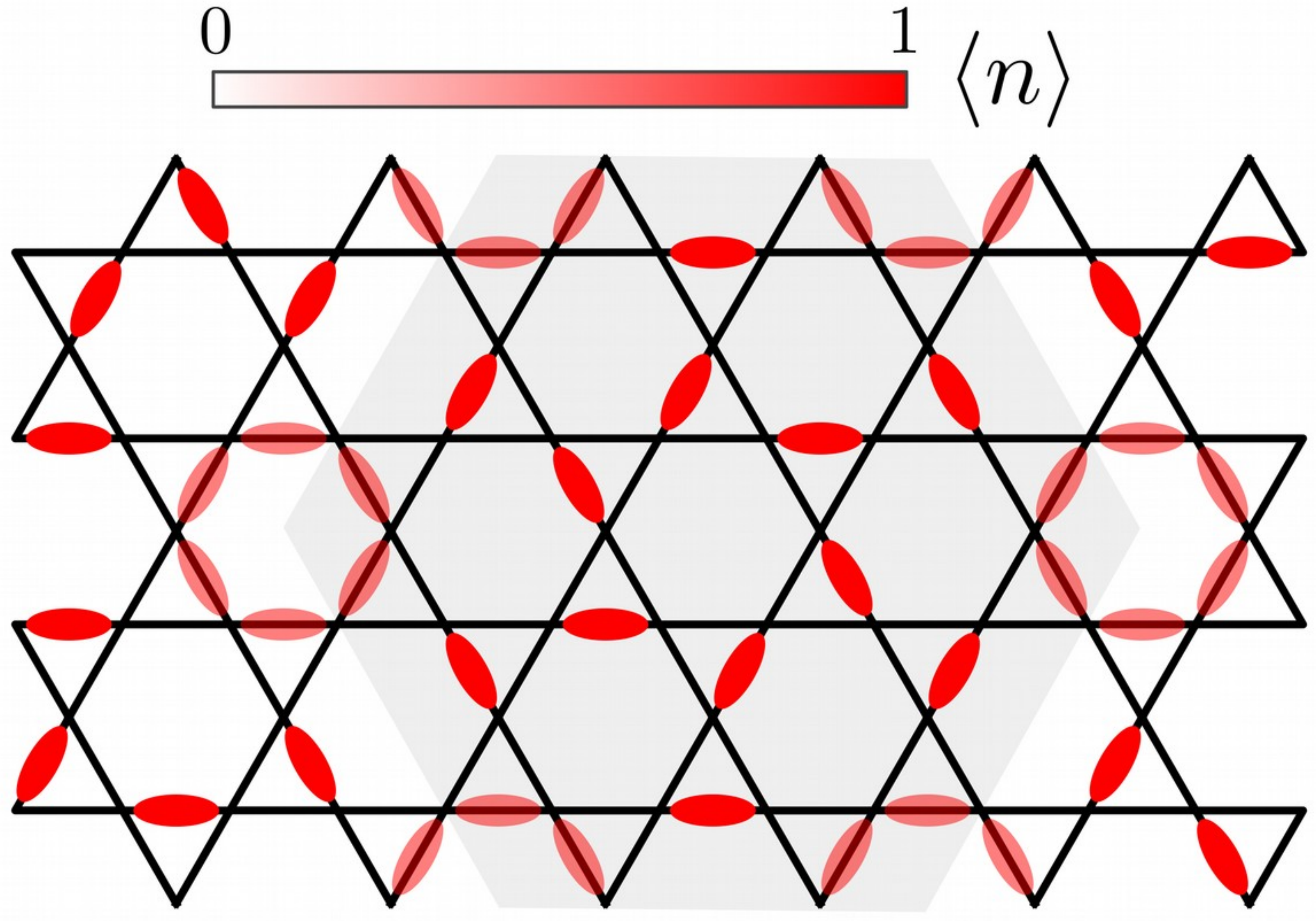


intermediate featureless phase



DMRG



VBS pattern studied before in context of Heisenberg model on kagome lattice (Nikolic and Senthil, Singh and Huse, ...)

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Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, arXiv:2104.04119; Science, to appear.

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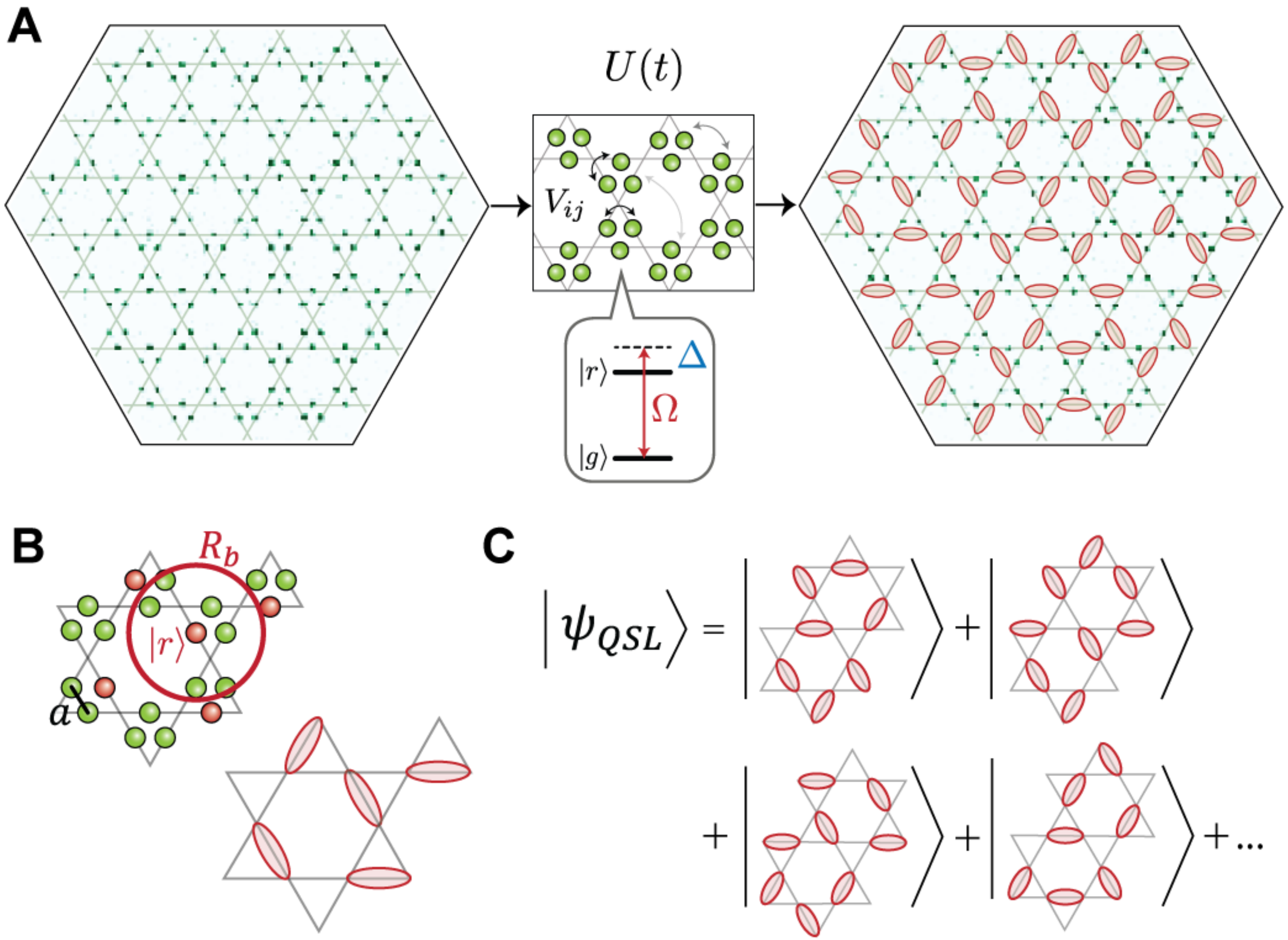
experiment



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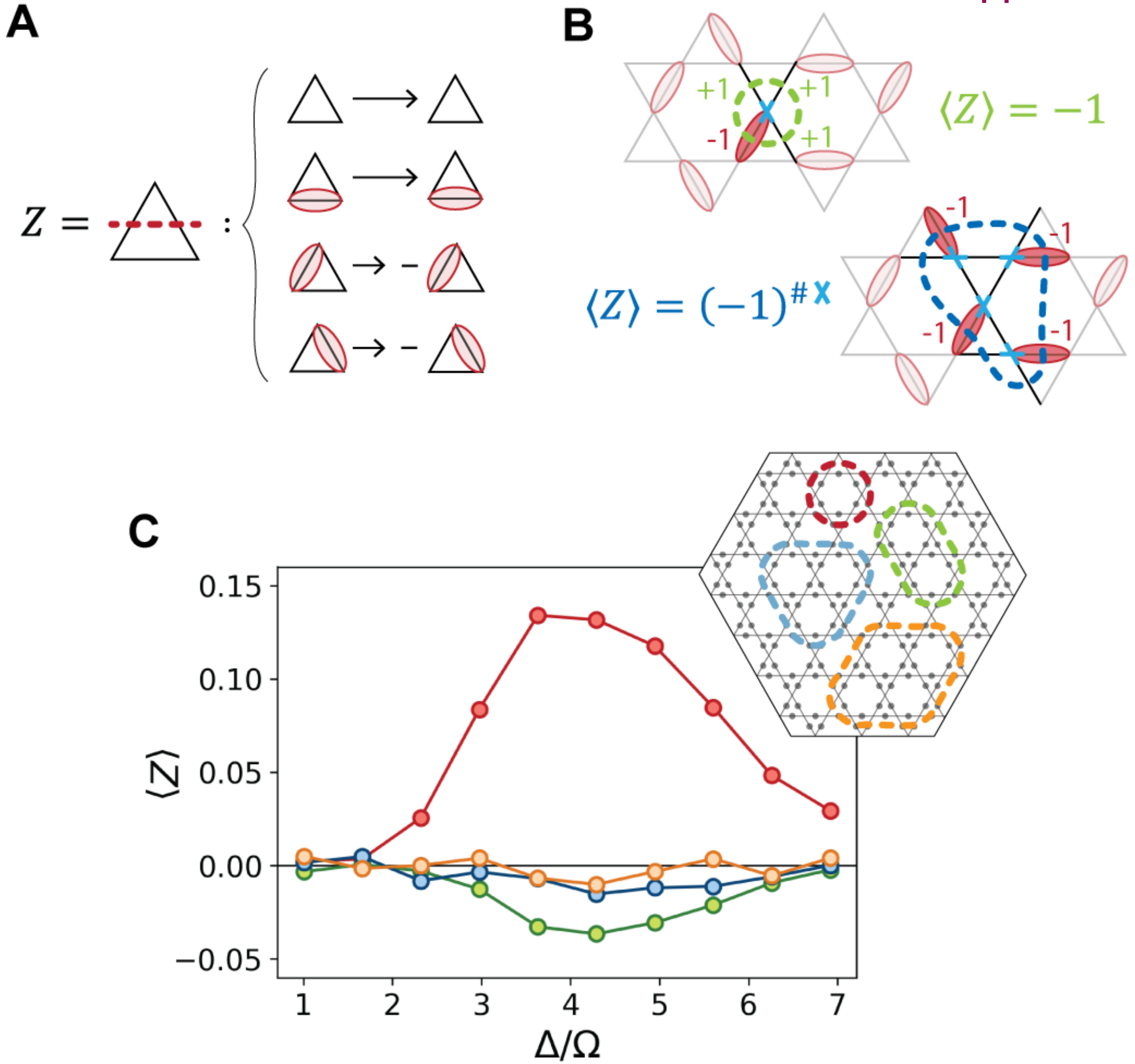

Measurement of the topological Z operator

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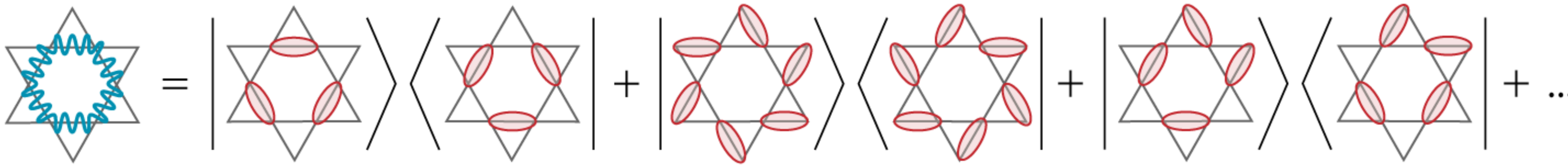
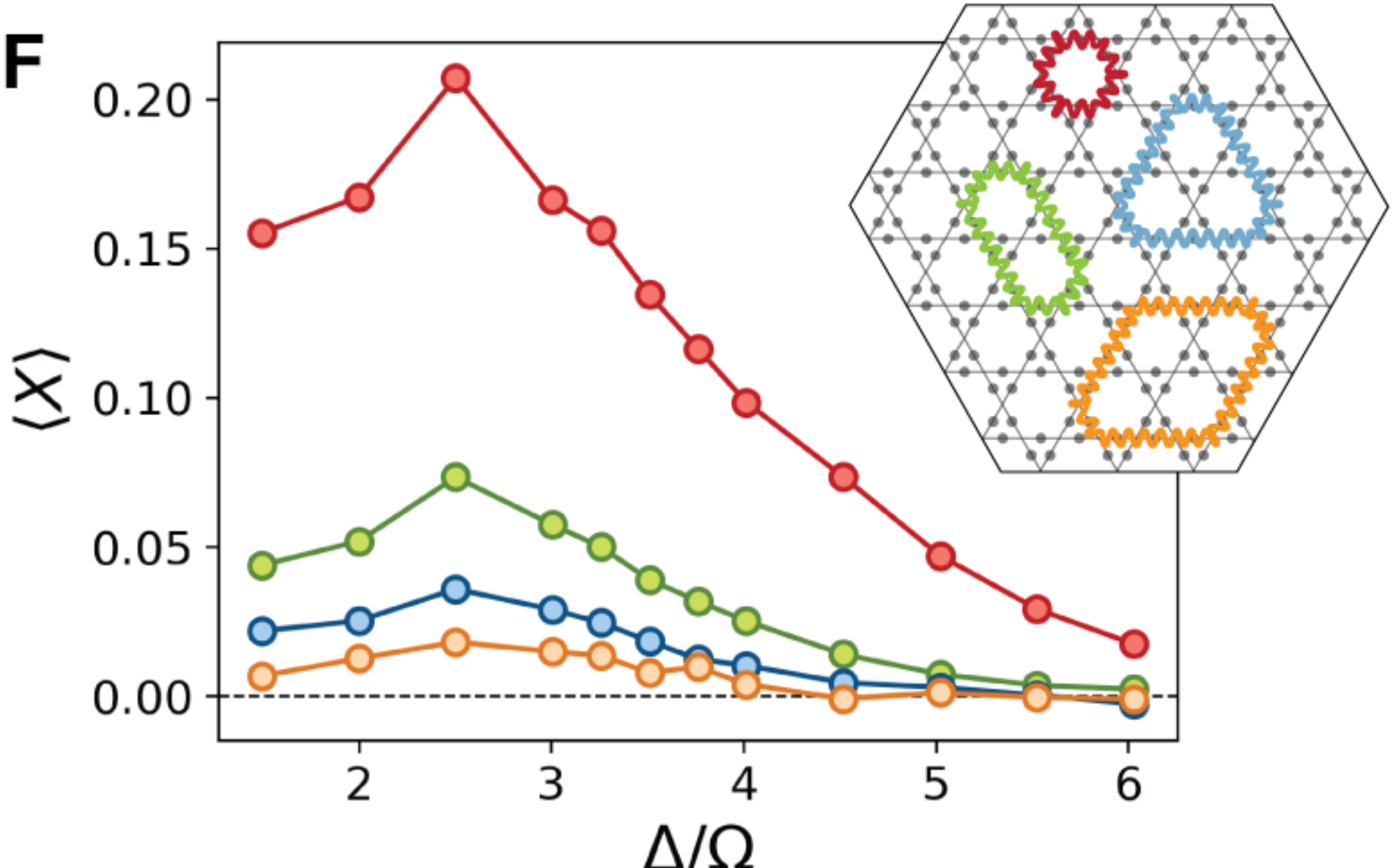


Can also define a topological X operator which resonates between different dimer configurations of the kagome dimer model.

$$XZ = (-1)^{\text{number of intersections}}\, ZX$$

R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)



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