

Stringing together the quantum phases of matter

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Theory of the Quantum World
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Talk online at sachdev.physics.harvard.edu



“Complex entangled” states of
quantum matter,
not adiabatically connected to independent particle states

Gapped quantum matter

Z_2 Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

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A. Field theory:

Honeycomb lattice

Hubbard model

B. Gauge-gravity duality

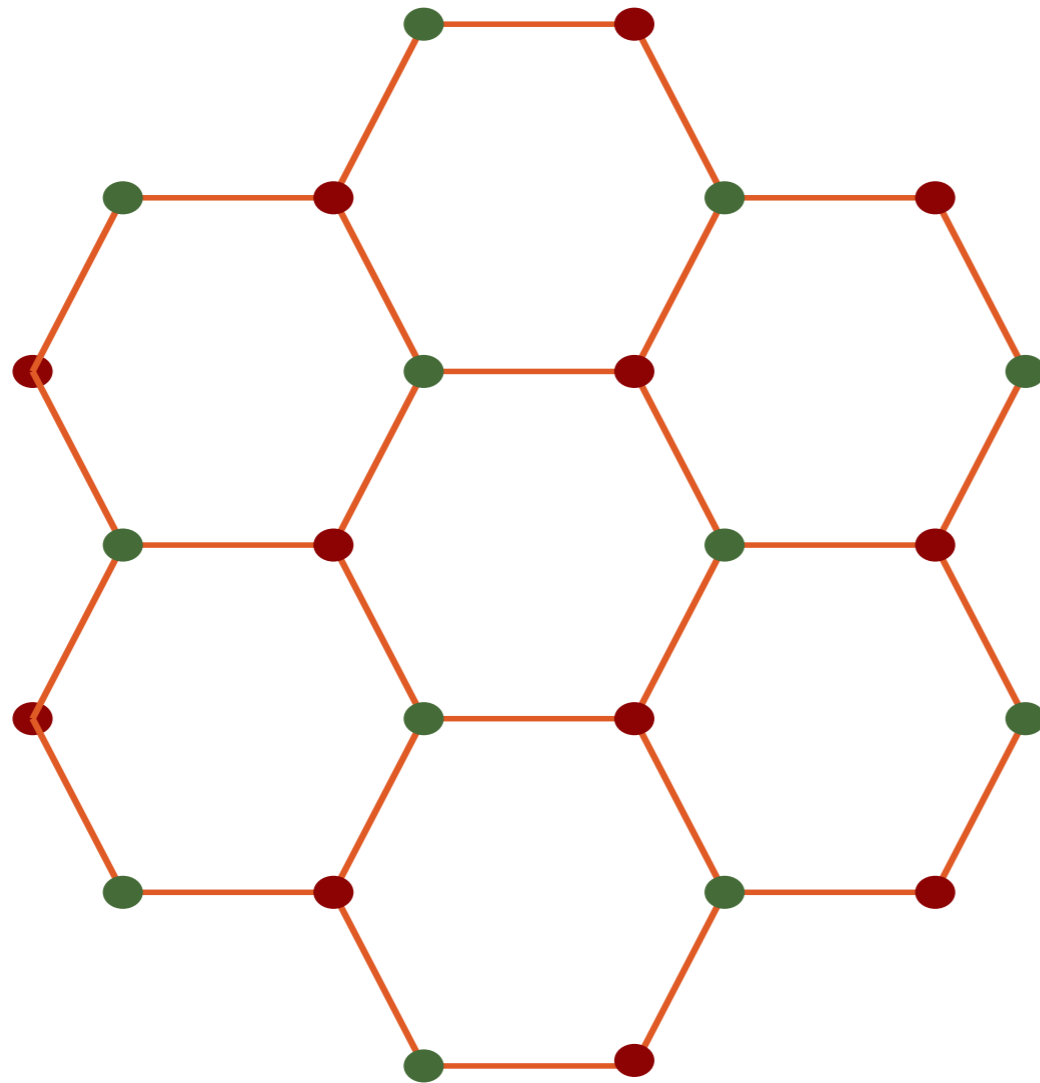
Conformal quantum matter

*A. Field theory:
Honeycomb lattice
Hubbard model*

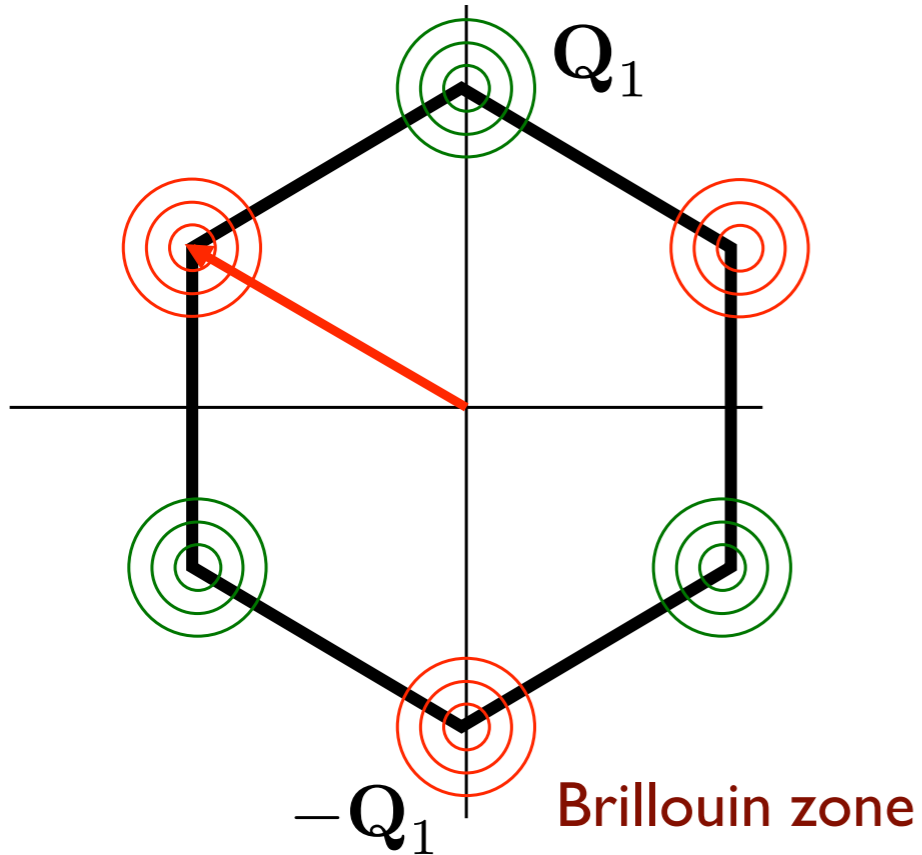
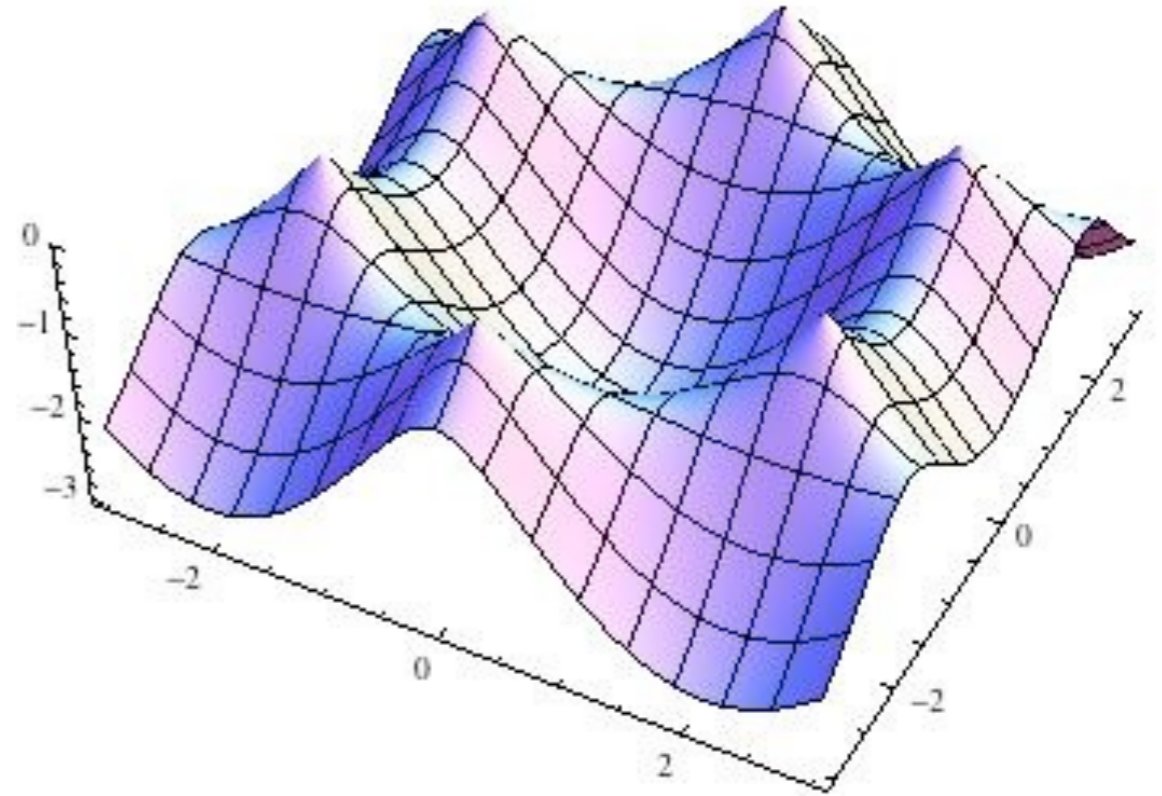
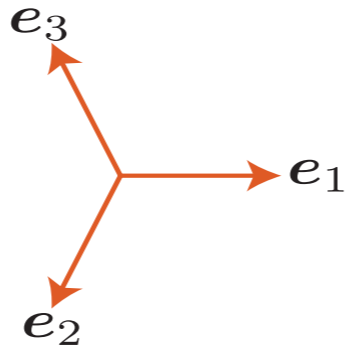
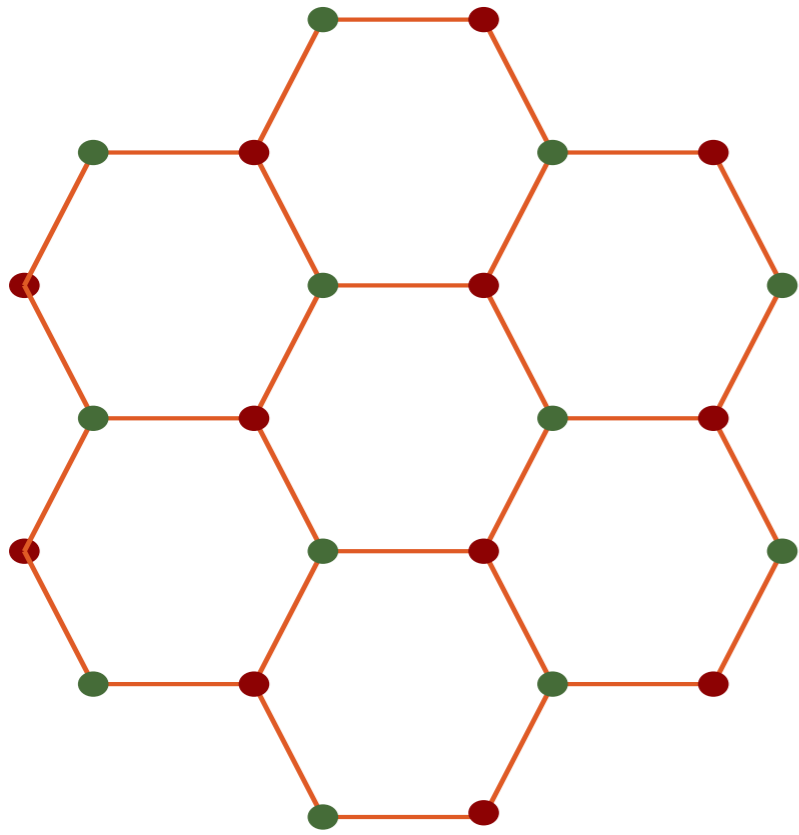
B. Gauge-gravity duality

Honeycomb lattice

(describes graphene after adding long-range Coulomb interactions)



$$H = -t \sum_{\langle ij \rangle} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right)$$



**Semi-metal with
massless Dirac fermions
at small U/t**

We define the Fourier transform of the fermions by

$$c_A(\mathbf{k}) = \sum_{\mathbf{r}} c_A(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} \quad (4)$$

and similarly for c_B . **A** and **B** are sublattice indices. The hopping Hamiltonian is

$$H_0 = -t \sum_{\langle ij \rangle} \left(c_{Ai\alpha}^\dagger c_{Bj\alpha} + c_{Bj\alpha}^\dagger c_{Ai\alpha} \right) \quad (5)$$

where α is a spin index. If we introduce Pauli matrices τ^a in sublattice space ($a = x, y, z$), this Hamiltonian can be written as

$$H_0 = \int \frac{d^2k}{4\pi^2} c^\dagger(\mathbf{k}) \left[-t \left(\cos(\mathbf{k} \cdot \mathbf{e}_1) + \cos(\mathbf{k} \cdot \mathbf{e}_2) + \cos(\mathbf{k} \cdot \mathbf{e}_3) \right) \tau^x + t \left(\sin(\mathbf{k} \cdot \mathbf{e}_1) + \sin(\mathbf{k} \cdot \mathbf{e}_2) + \sin(\mathbf{k} \cdot \mathbf{e}_3) \right) \tau^y \right] c(\mathbf{k}) \quad (6)$$

The low energy excitations of this Hamiltonian are near $\mathbf{k} \approx \pm \mathbf{Q}_1$.

In terms of the fields near \mathbf{Q}_1 and $-\mathbf{Q}_1$, we define

$$\begin{aligned}
 \Psi_{A1\alpha}(\mathbf{k}) &= c_{A\alpha}(\mathbf{Q}_1 + \mathbf{k}) \\
 \Psi_{A2\alpha}(\mathbf{k}) &= c_{A\alpha}(-\mathbf{Q}_1 + \mathbf{k}) \\
 \Psi_{B1\alpha}(\mathbf{k}) &= c_{B\alpha}(\mathbf{Q}_1 + \mathbf{k}) \\
 \Psi_{B2\alpha}(\mathbf{k}) &= c_{B\alpha}(-\mathbf{Q}_1 + \mathbf{k})
 \end{aligned} \tag{7}$$

We consider Ψ to be a 8 component vector, and introduce Pauli matrices ρ^a which act in the 1, 2 valley space. Then the Hamiltonian is

$$H_0 = \int \frac{d^2k}{4\pi^2} \Psi^\dagger(\mathbf{k}) \left(v\tau^y k_x + v\tau^x \rho^z k_y \right) \Psi(\mathbf{k}), \tag{8}$$

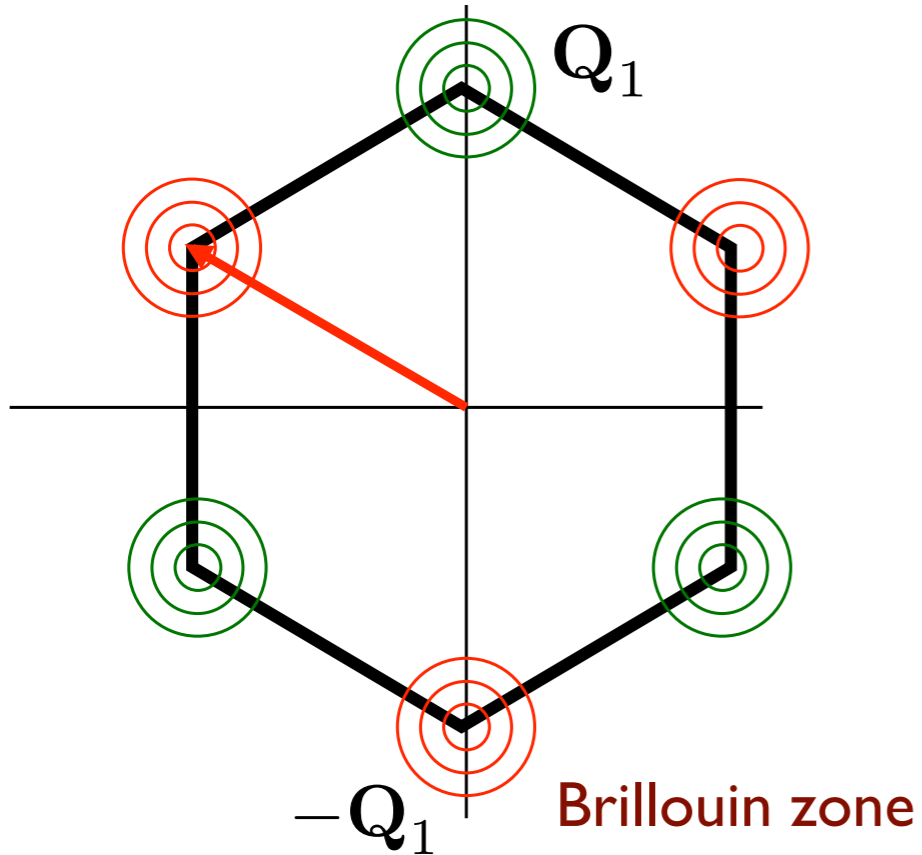
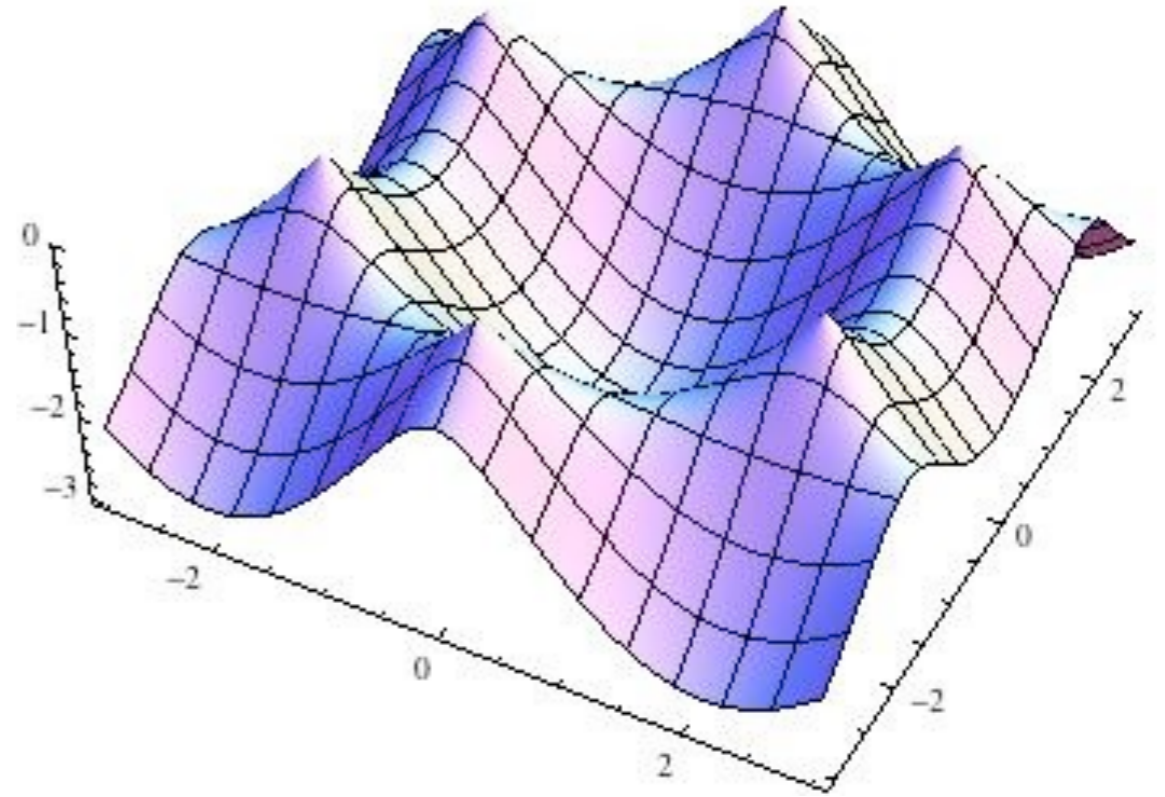
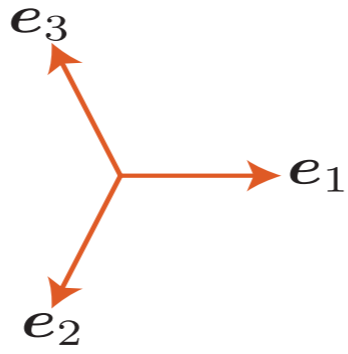
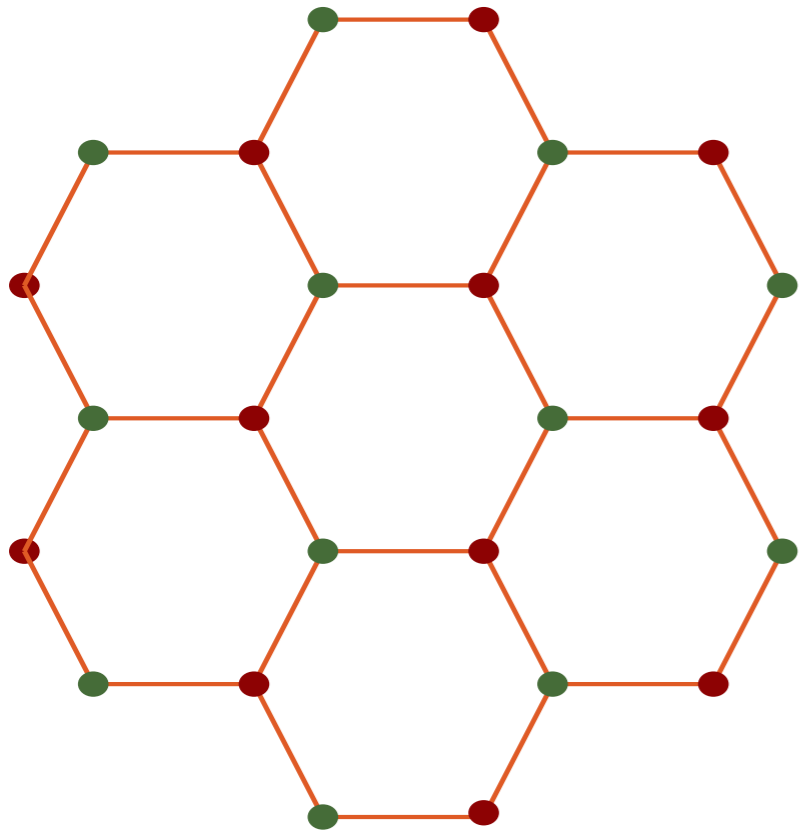
where $v = 3t/2$; below we set $v = 1$. Now define $\bar{\Psi} = \Psi^\dagger \rho^z \tau^z$. Then we can write the imaginary time Lagrangian as

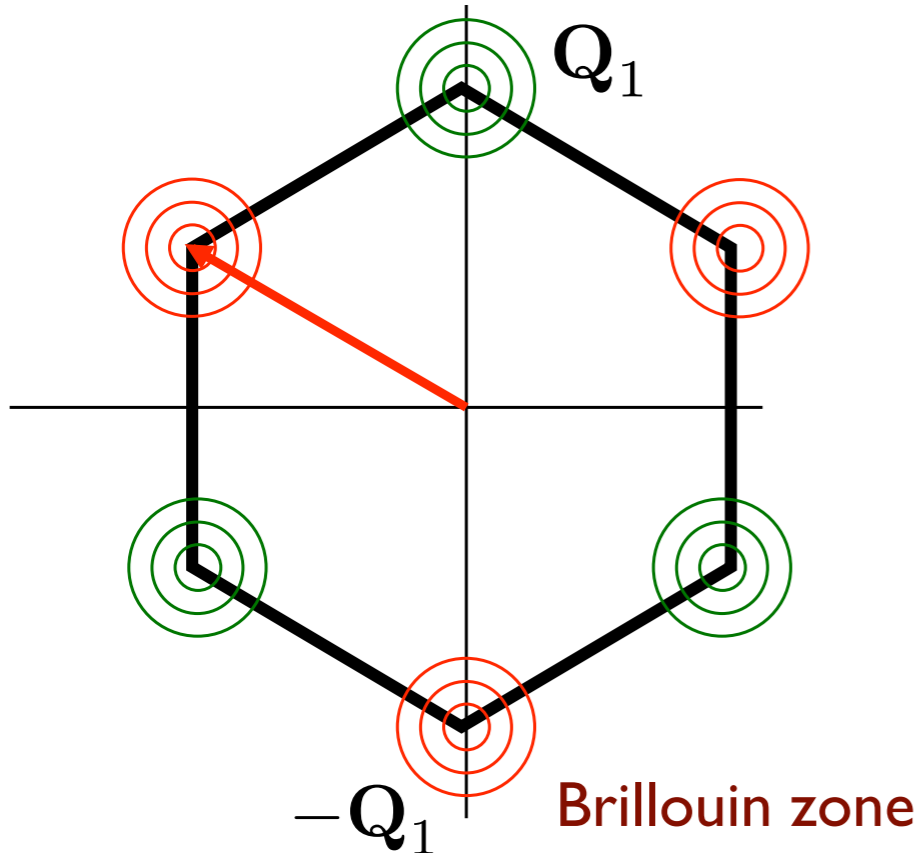
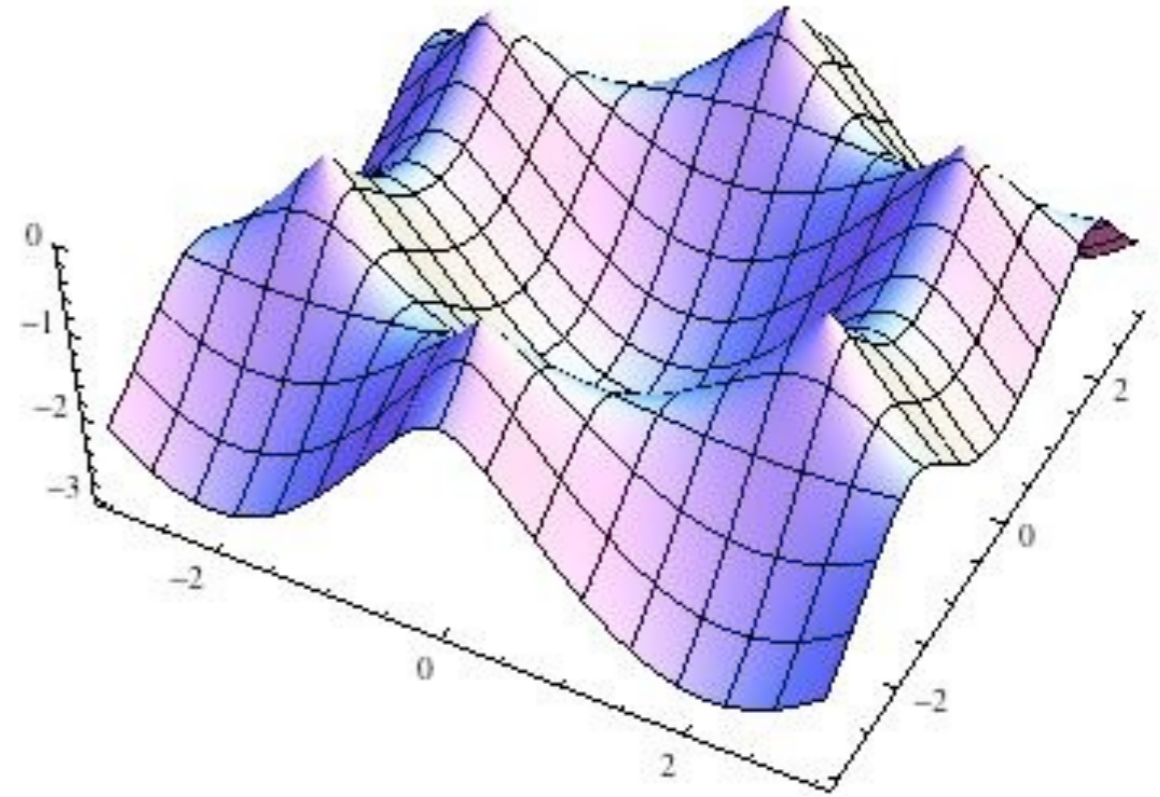
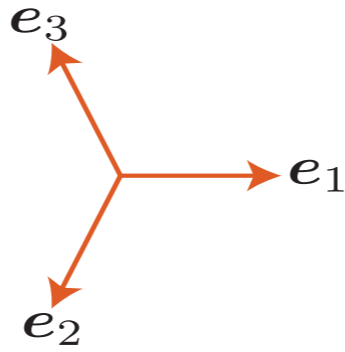
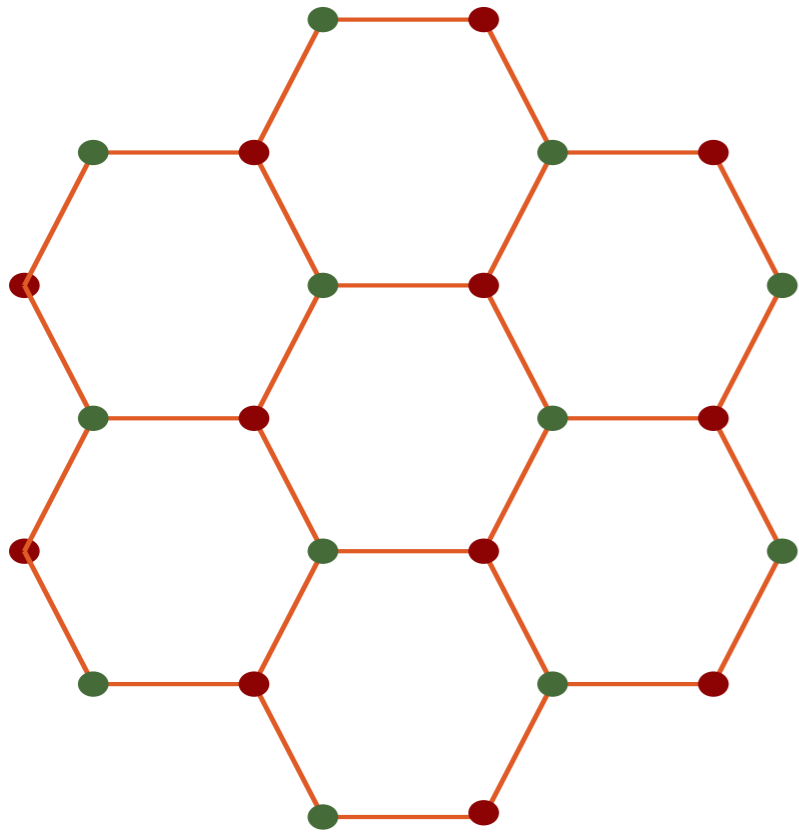
$$\mathcal{L}_0 = -i\bar{\Psi} (\omega\gamma_0 + k_x\gamma_1 + k_y\gamma_2) \Psi \tag{9}$$

where

$$\gamma_0 = -\rho^z \tau^z \quad \gamma_1 = \rho^z \tau^x \quad \gamma_2 = -\tau^y \tag{10}$$

Exercise: Observe that \mathcal{L}_0 is invariant under the scaling transformation $x' = xe^{-\ell}$ and $\tau' = \tau e^{-\ell}$. Write the Hubbard interaction U in terms of the Dirac fermions, and show that it has the tree-level scaling transformation $U' = Ue^{-\ell}$. So argue that all short-range interactions are *irrelevant* in the Dirac semi-metal phase.





The theory of free Dirac fermions is invariant under conformal transformations of spacetime. This is a realization of a simple conformal field theory in $2+1$ dimensions: a CFT3

The Hubbard Model at large U

$$H = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

In the limit of large U , and at a density of one particle per site, this maps onto the Heisenberg antiferromagnet

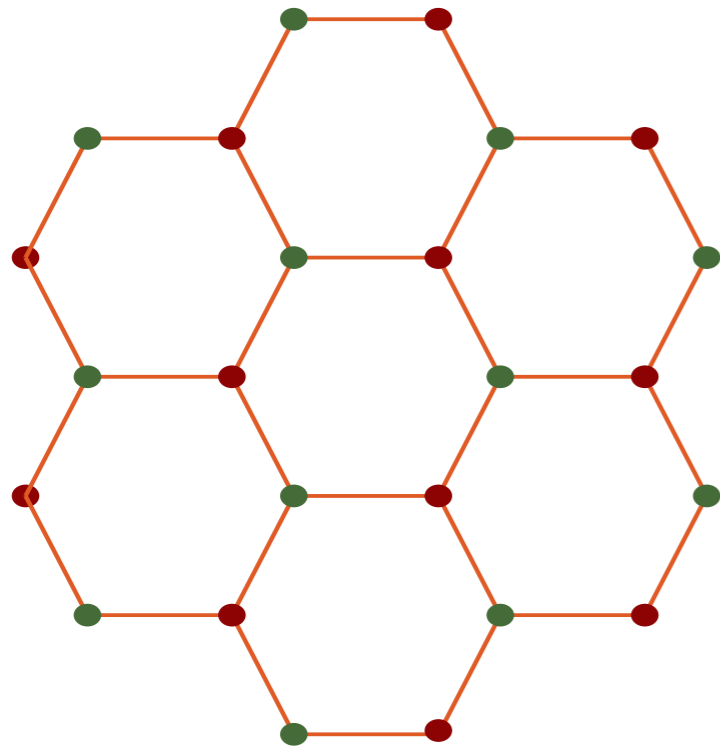
$$H_{AF} = \sum_{i < j} J_{ij} S_i^a S_j^a$$

where $a = x, y, z$,

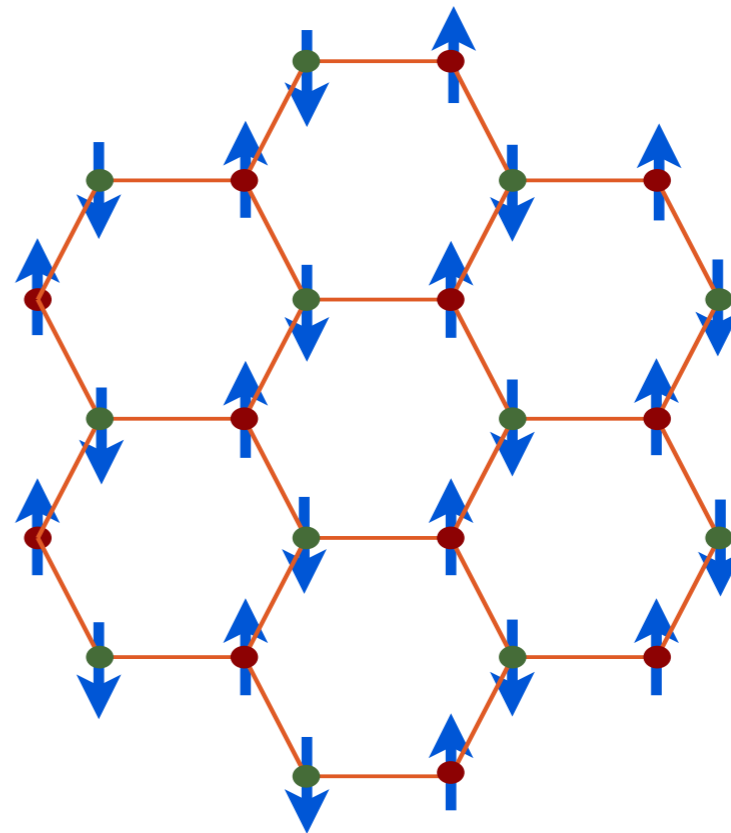
$$S_i^a = \frac{1}{2} c_{i\alpha}^{a\dagger} \sigma_{\alpha\beta}^a c_{i\beta},$$

with σ^a the Pauli matrices and

$$J_{ij} = \frac{4t_{ij}^2}{U}$$



Dirac
semi-metal



Insulating
antiferromagnet
with Neel order

U/t

Antiferromagnetism

We use the operator equation (valid on each site i):

$$U \left(n_{\uparrow} - \frac{1}{2} \right) \left(n_{\downarrow} - \frac{1}{2} \right) = -\frac{2U}{3} S_i^{a2} + \frac{U}{4} \quad (11)$$

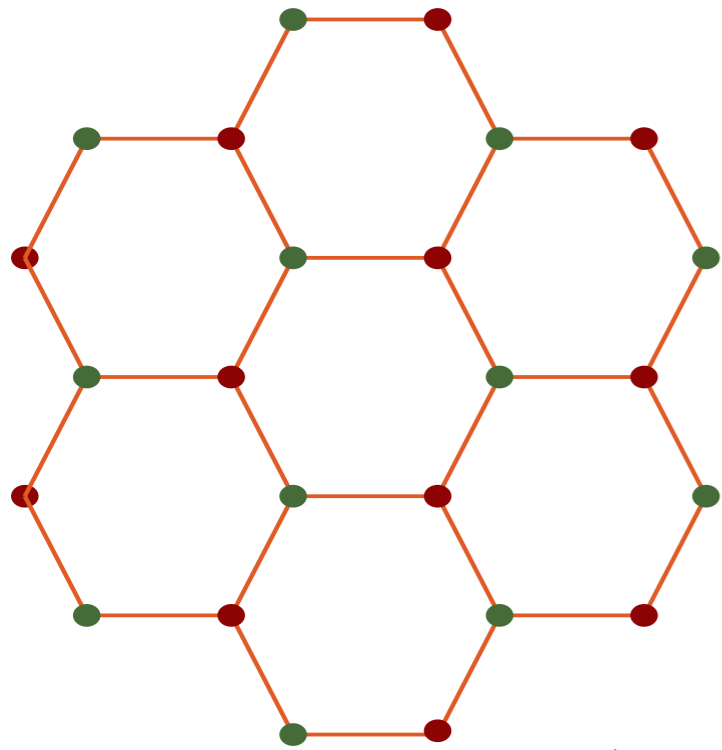
Then we decouple the interaction via

$$\exp \left(\frac{2U}{3} \sum_i \int d\tau S_i^{a2} \right) = \int \mathcal{D} J_i^a(\tau) \exp \left(- \sum_i \int d\tau \left[\frac{3}{8U} J_i^{a2} - J_i^a S_i^a \right] \right) \quad (12)$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for J_i^a .

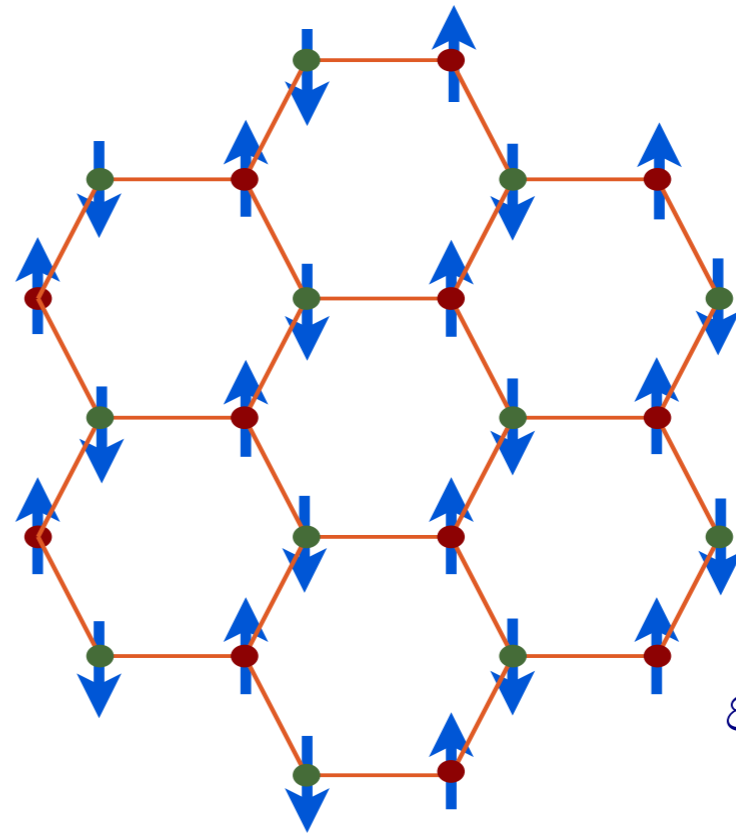
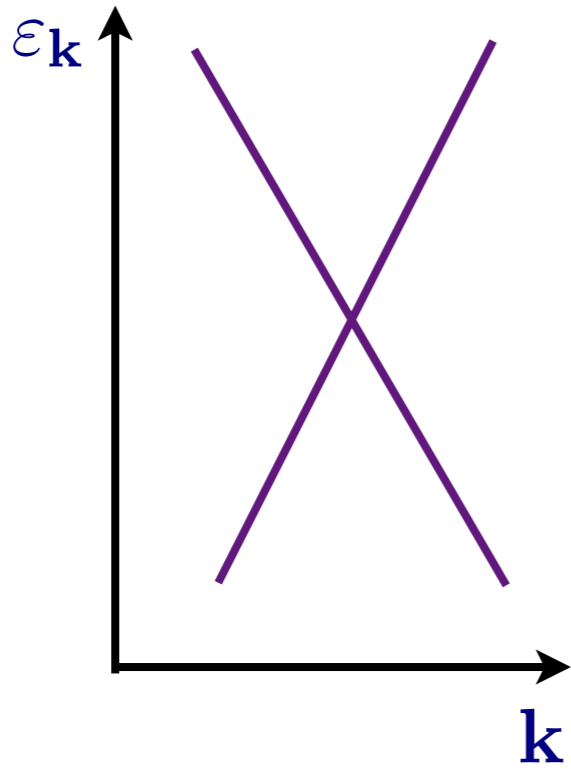
Long wavelength fluctuations about this saddle point are described by a field theory of the Néel order parameter, φ^a , coupled to the Dirac fermions in the **Gross-Neveu** model.

$$\mathcal{L} = \bar{\Psi} \gamma_{\mu} \partial_{\mu} \Psi + \frac{1}{2} \left[(\partial_{\mu} \varphi^a)^2 + s \varphi^{a2} \right] + \frac{u}{24} (\varphi^{a2})^2 - \lambda \varphi^a \bar{\Psi} \rho^z \sigma^a \Psi$$



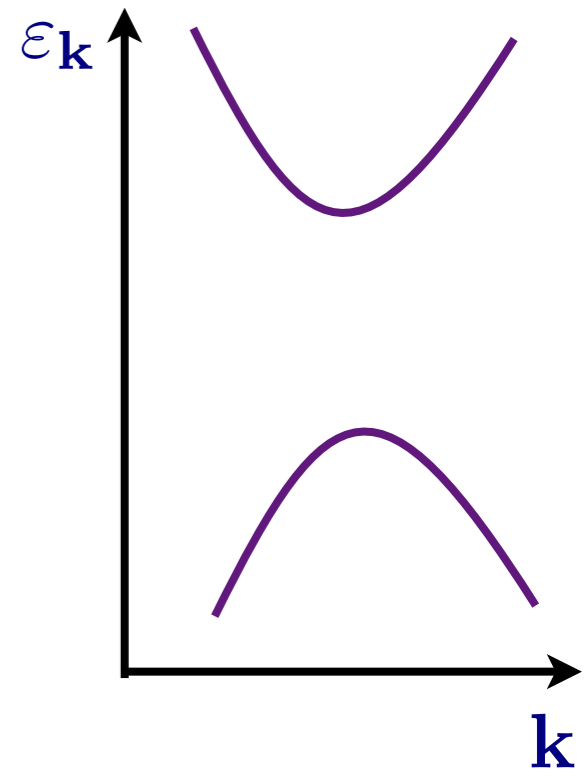
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$

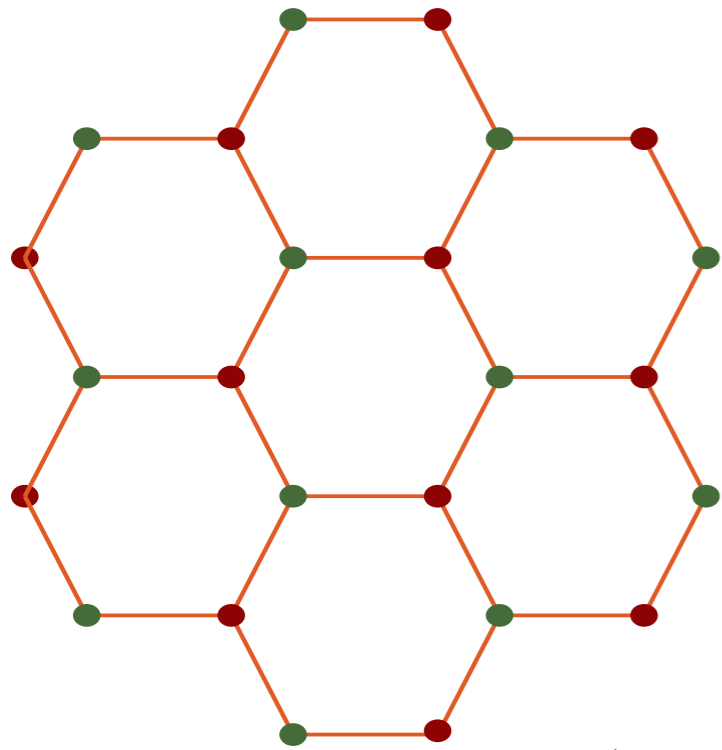


Insulating
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with Neel order

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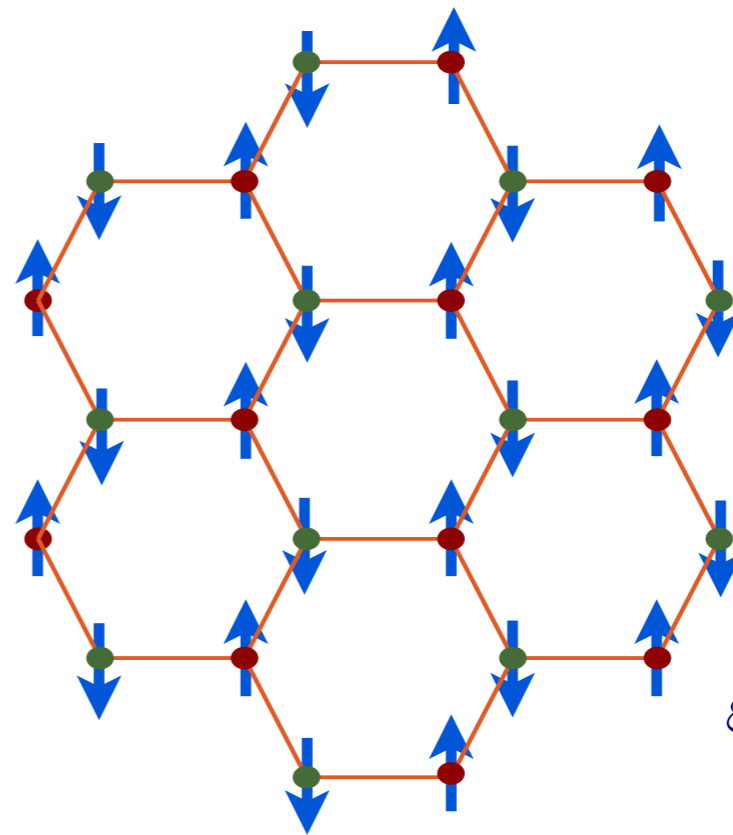
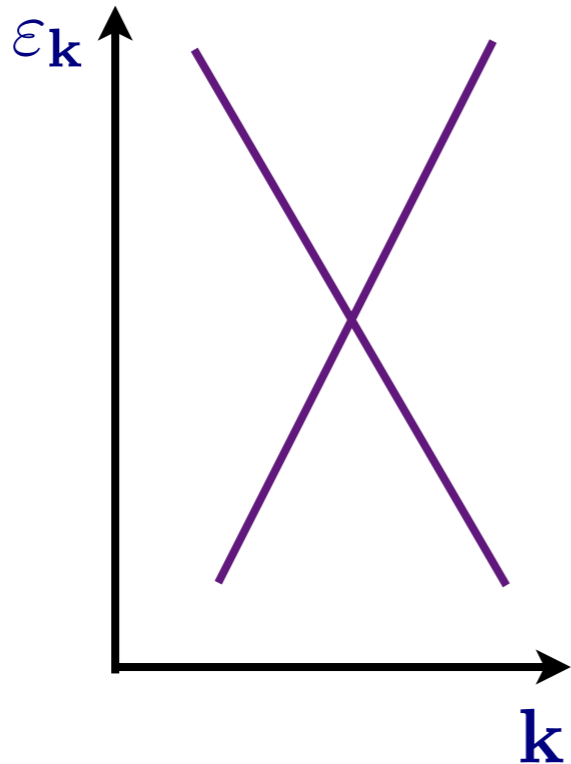


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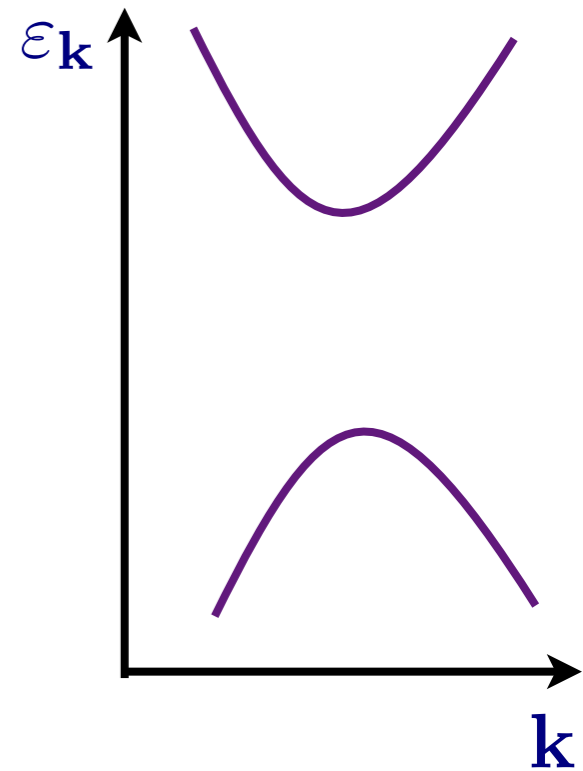
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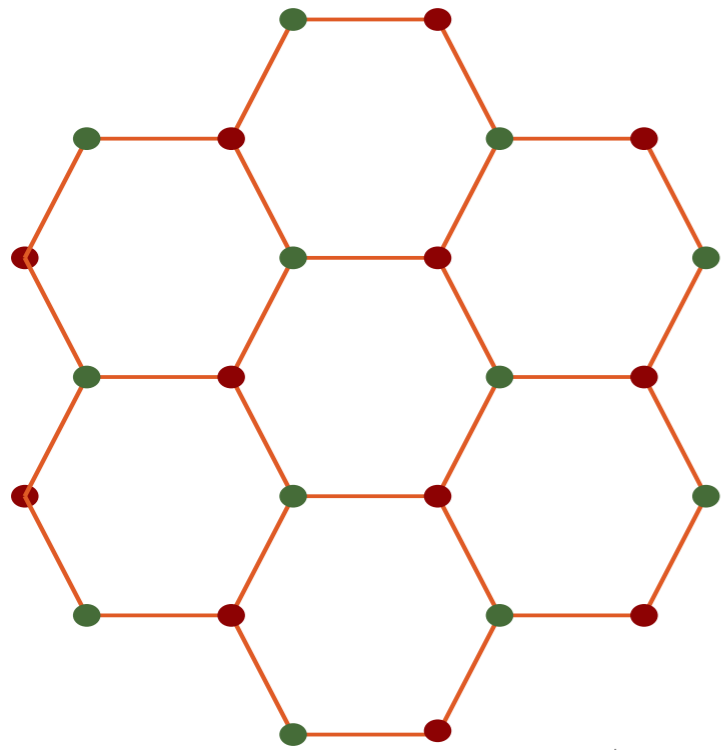
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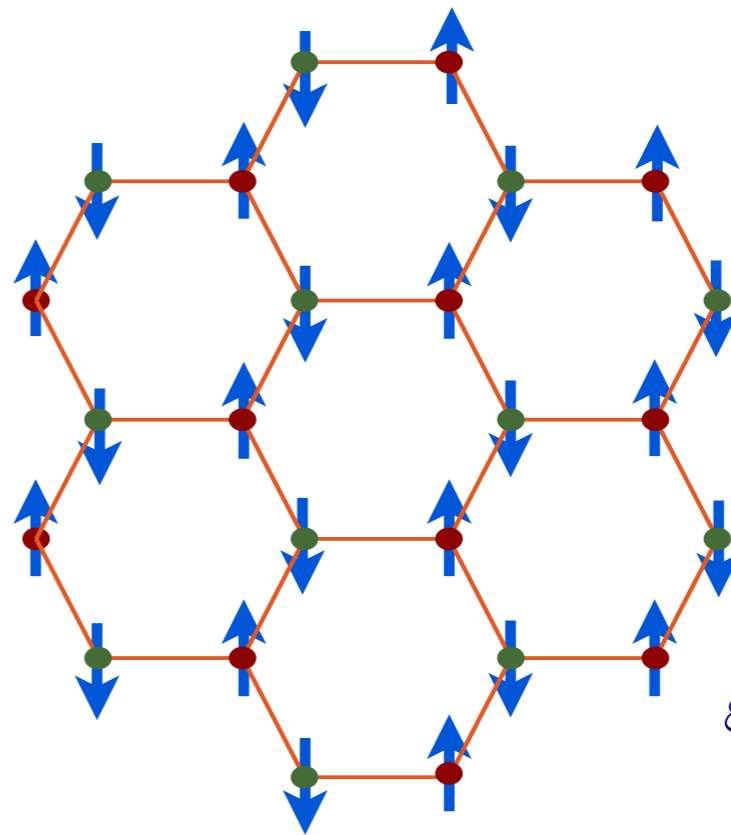
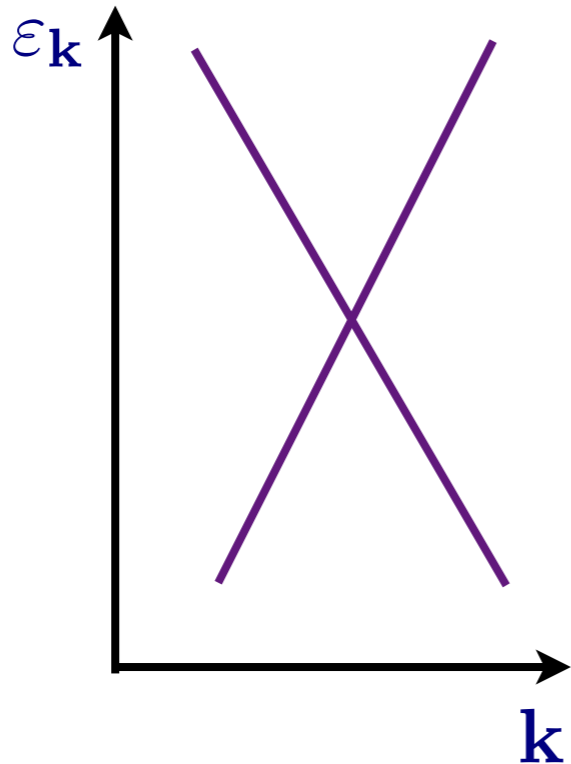
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At the quantum critical point, the non-linear couplings λ and u in the Gross-Neveu model reach non-zero fixed-point values under the renormalization group flow. The critical theory is an *interacting* CFT₃



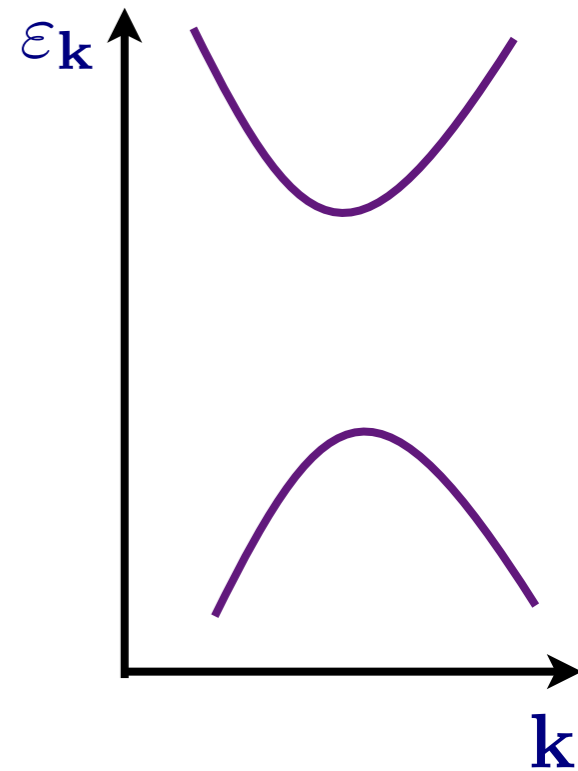
Dirac
semi-metal

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Insulating
antiferromagnet
with Neel order

$$\langle \varphi^a \rangle \neq 0$$



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Free CFT3

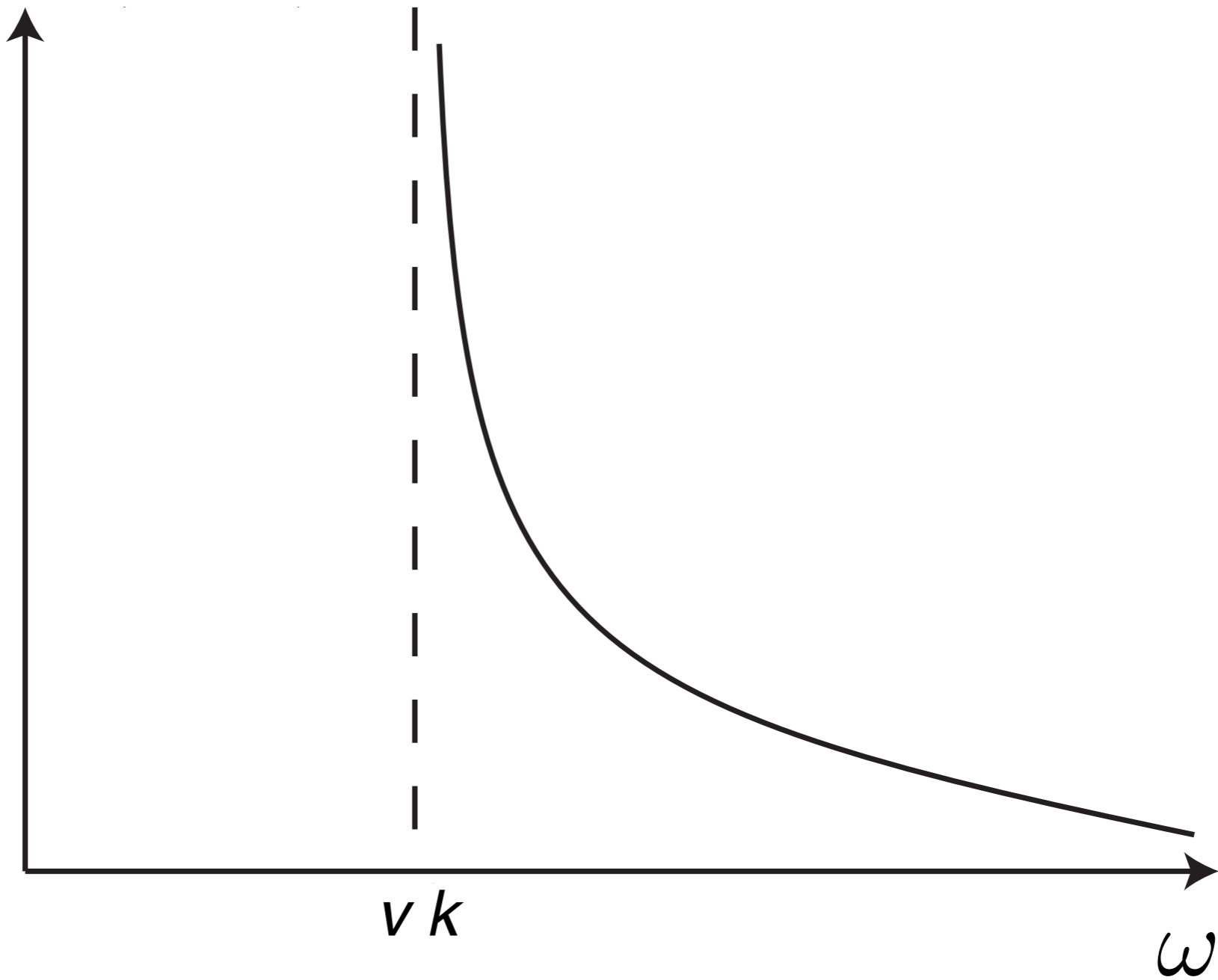
Interacting CFT3
with many-body entanglement

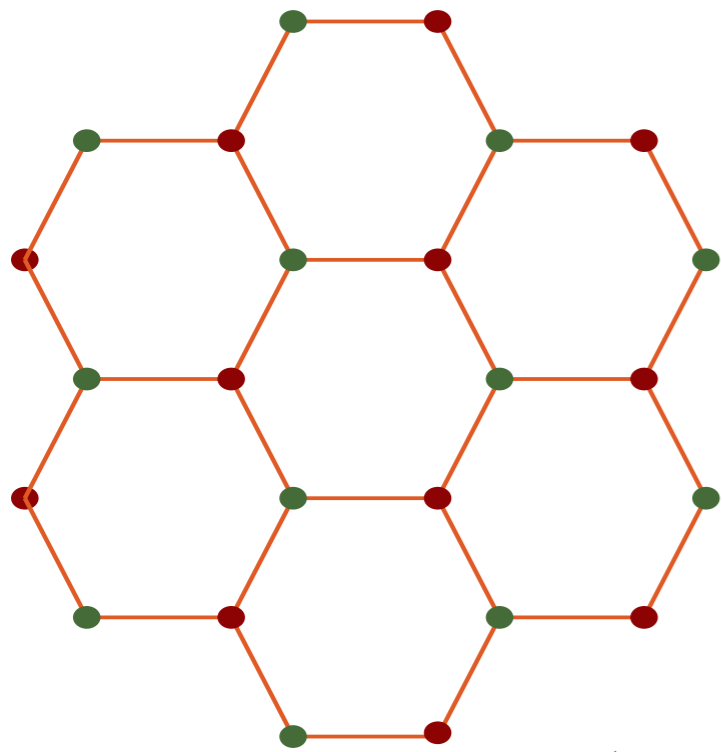
Electron Green's function for the interacting CFT3

$$G(k, \omega) = \langle \Psi(k, \omega); \Psi^\dagger(k, \omega) \rangle \sim \frac{i\omega + vk_x \tau^y + vk_y \tau^x \rho^z}{(\omega^2 + v^2 k_x^2 + v^2 k_y^2)^{1-\eta/2}}$$

where $\eta > 0$ is the *anomalous dimension* of the fermion. Note that this leads to a fermion spectral density which has no quasiparticle pole: thus the quantum critical point has no well-defined quasiparticle excitations.

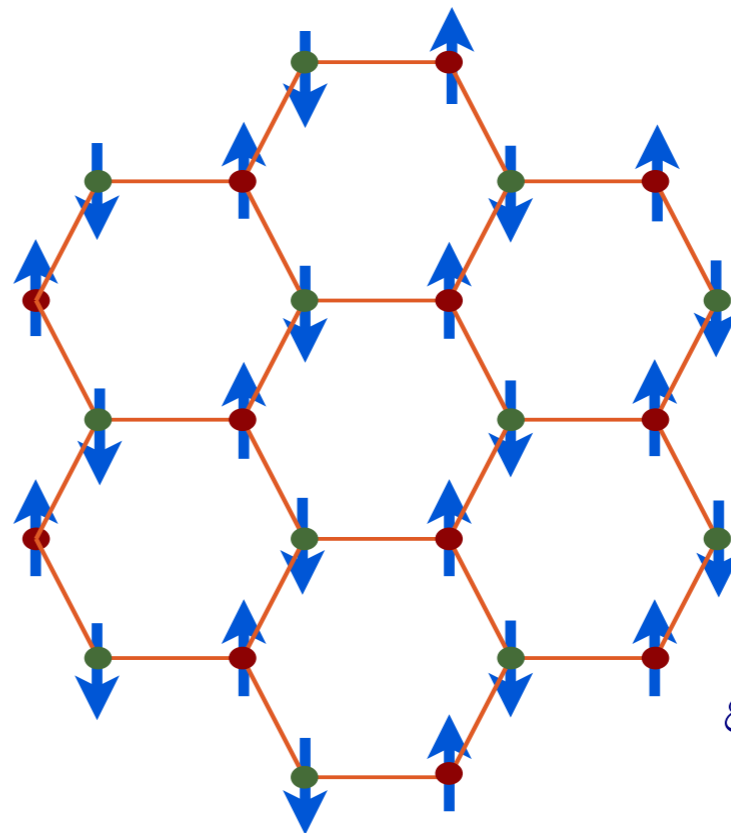
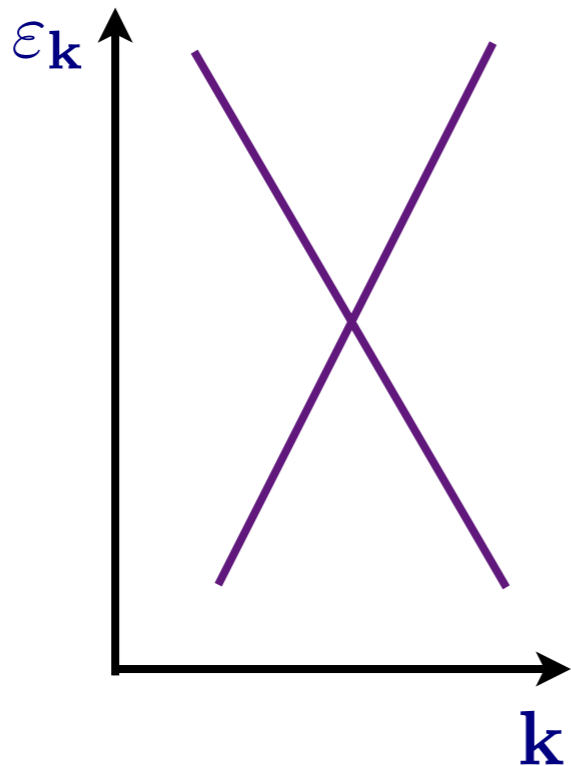
$\text{Im}G(k, \omega)$





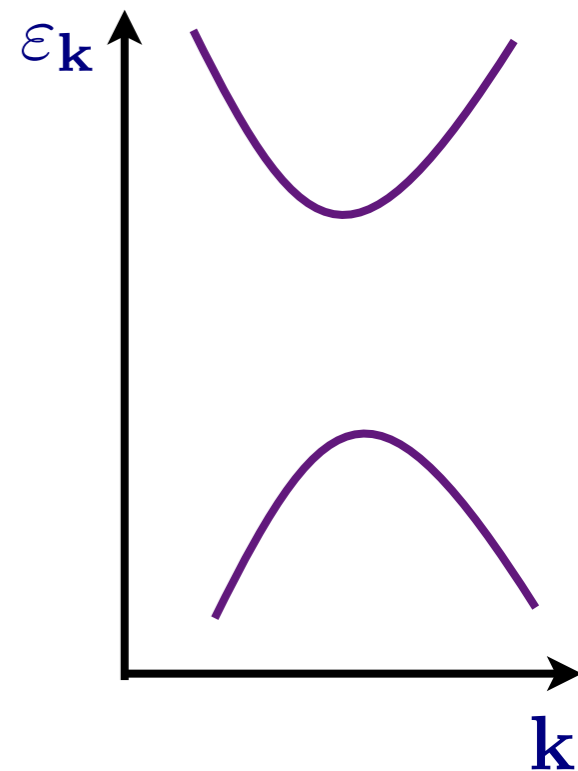
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

$$\langle \varphi^a \rangle \neq 0$$



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Quantum phase transition described by a strongly-coupled conformal field theory without well-defined quasiparticles

Electrical transport

The conserved electrical current is

$$J_\mu = -i\bar{\Psi}\gamma_\mu\Psi. \quad (1)$$

Let us compute its two-point correlator, $K_{\mu\nu}(k)$ at a spacetime momentum k_μ at $T = 0$. At leading order, this is given by a one fermion loop diagram which evaluates to

$$\begin{aligned} K_{\mu\nu}(k) &= \int \frac{d^3p}{8\pi^3} \frac{\text{Tr} [\gamma_\mu (i\gamma_\lambda p_\lambda + m\rho^z \sigma^z) \gamma_\nu (i\gamma_\delta (k_\delta + p_\delta) + m\rho^z \sigma^z)]}{(p^2 + m^2)((p+k)^2 + m^2)} \\ &= -\frac{2}{\pi} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \int_0^1 dx \frac{k^2 x(1-x)}{\sqrt{m^2 + k^2 x(1-x)}}, \end{aligned} \quad (2)$$

where the mass $m = 0$ in the semi-metal and at the quantum critical point, while $m = |\lambda N_0|$ in the insulator. Note that the current correlation is purely transverse, and this follows from the requirement of current conservation

$$k_\mu K_{\mu\nu} = 0. \quad (3)$$

Of particular interest to us is the K_{00} component, after analytic continuation to Minkowski space where the spacetime momentum k_μ is replaced by (ω, k) . The conductivity is obtained from this correlator via the Kubo formula

$$\sigma(\omega) = \lim_{k \rightarrow 0} \frac{-i\omega}{k^2} K_{00}(\omega, k). \quad (4)$$

In the insulator, where $m > 0$, analysis of the integrand in Eq. (2) shows that the spectral weight of the density correlator has a gap of $2m$ at $k = 0$, and the conductivity in Eq. (4) vanishes.

These properties are as expected in any insulator.

In the metal, and at the critical point, where $m = 0$, the fermionic spectrum is gapless, and so is that of the charge correlator. The density correlator in Eq. (2) and the conductivity in Eq. (4) evaluate to the simple universal results

$$\begin{aligned} K_{00}(\omega, k) &= \frac{1}{4} \frac{k^2}{\sqrt{k^2 - \omega^2}} \\ \sigma(\omega) &= 1/4. \end{aligned} \quad (5)$$

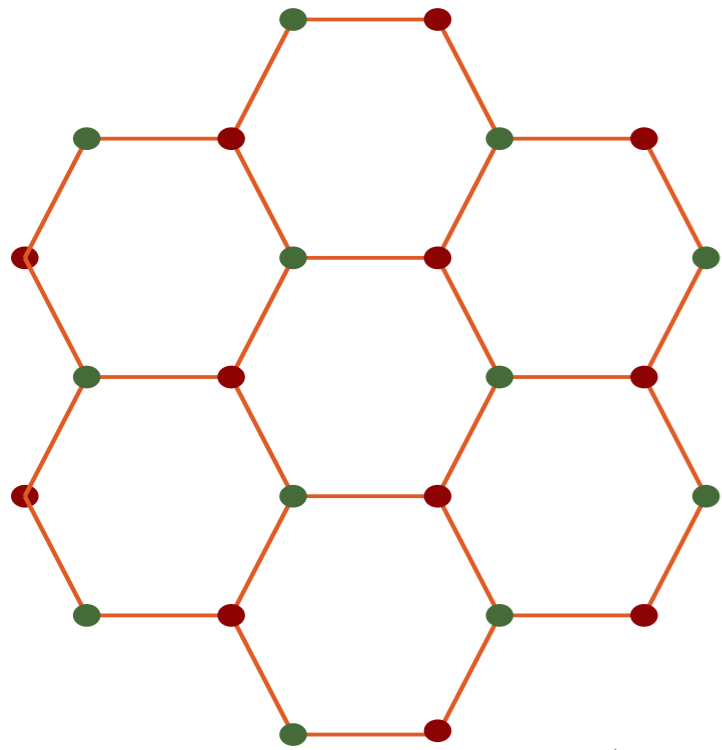
Going beyond one-loop, we find *no change* in these results in the

semi-metal to all orders in perturbation theory. At the quantum critical point, there are no anomalous dimensions for the conserved current, but the amplitude does change yielding

$$\begin{aligned} K_{00}(\omega, k) &= \mathcal{K} \frac{k^2}{\sqrt{k^2 - \omega^2}} \\ \sigma(\omega) &= \mathcal{K}, \end{aligned} \tag{6}$$

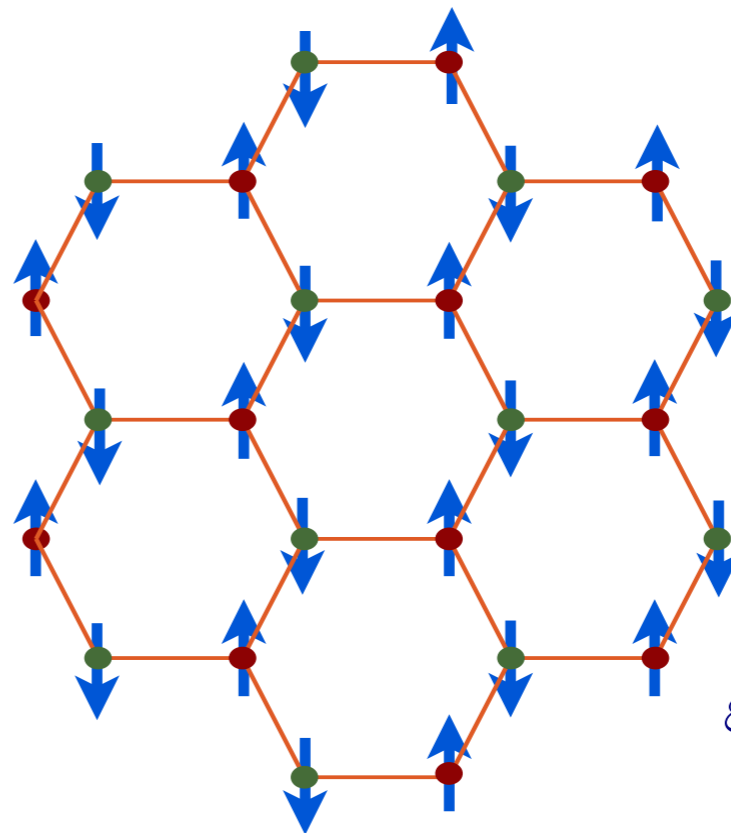
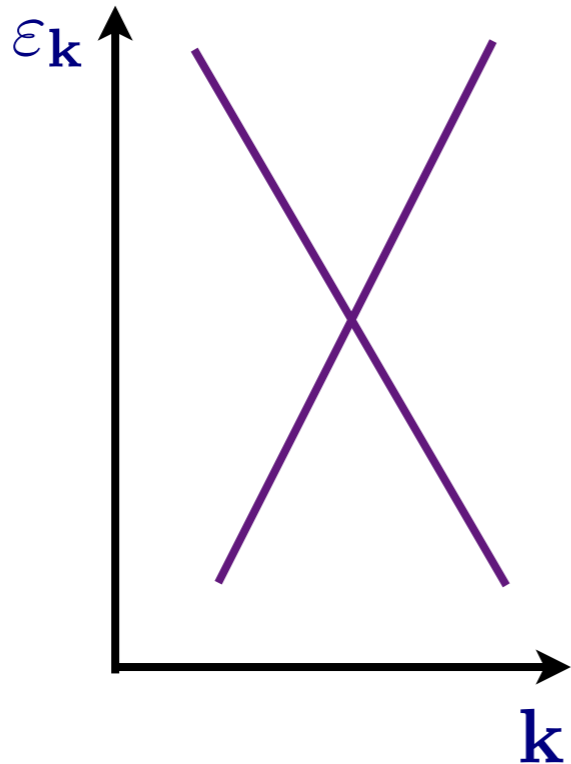
where \mathcal{K} is a universal number dependent only upon the universality class of the quantum critical point. The value of the \mathcal{K} for the Gross-Neveu model is not known exactly, but can be estimated by computations in the $(3 - d)$ or $1/N$ expansions.

$$\text{Also note } K_{\mu\nu} = \mathcal{K}|k| \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$$



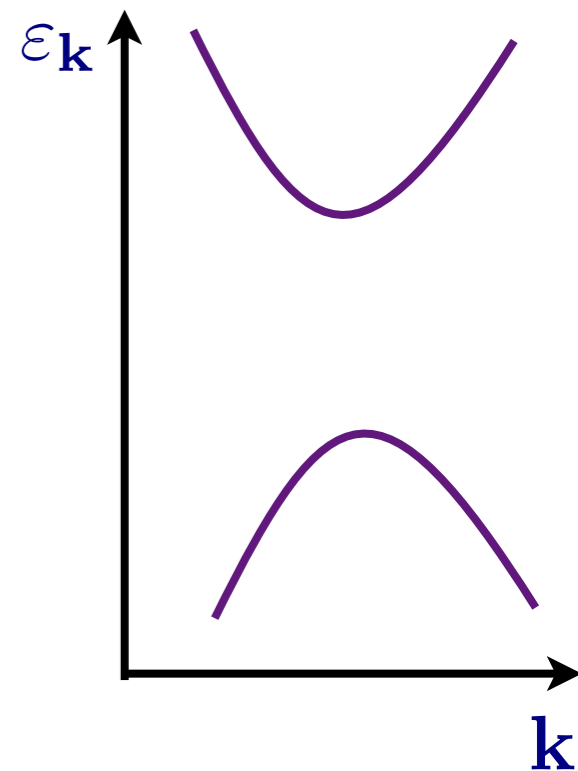
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

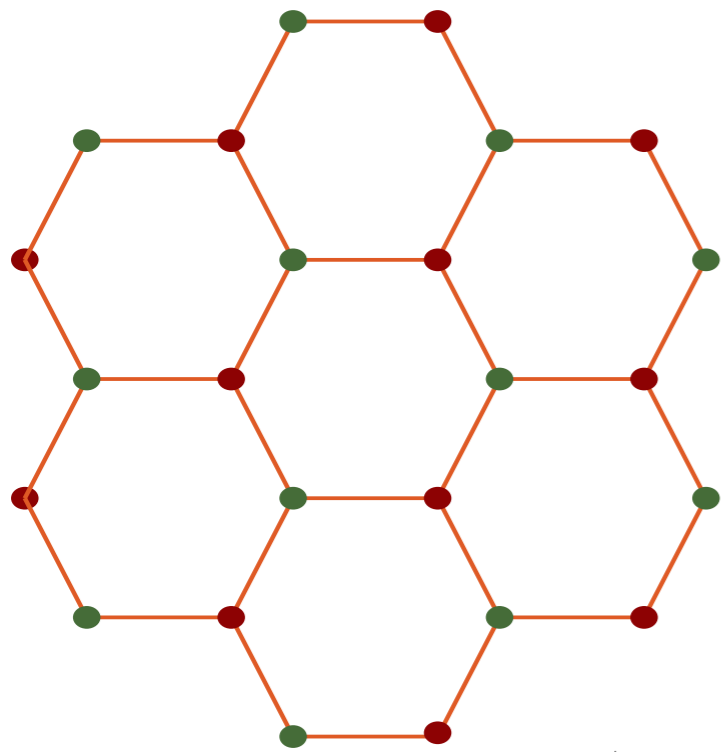
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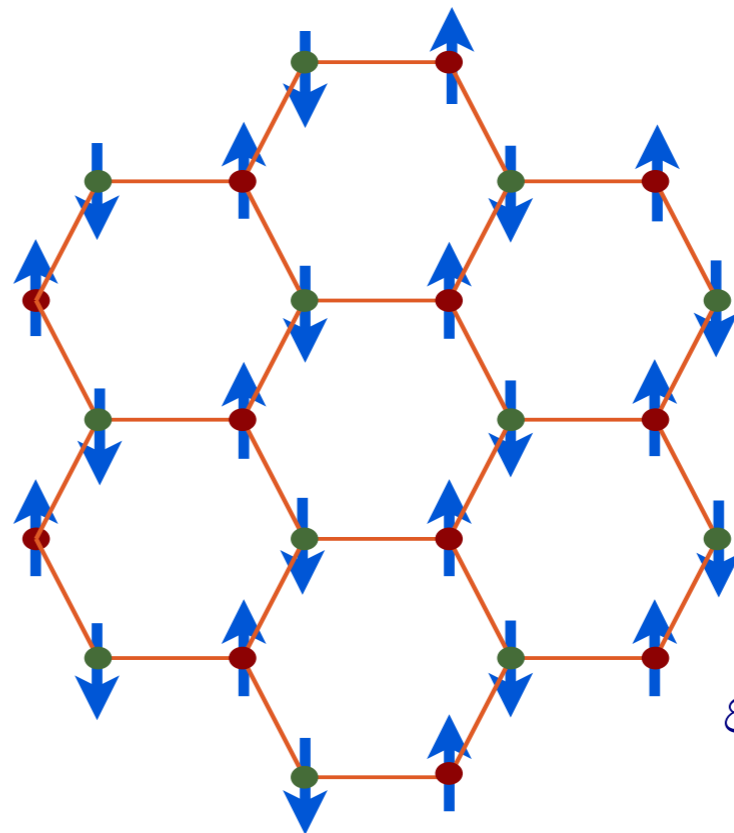
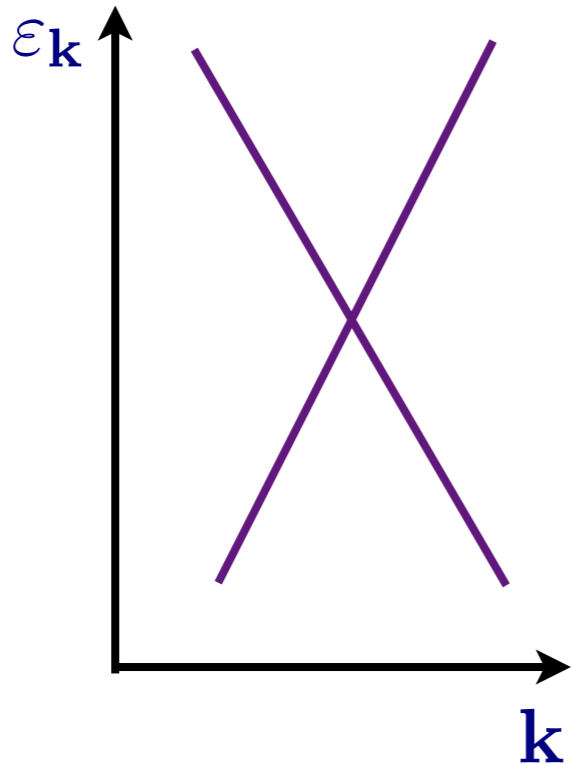
Free CFT3

Interacting CFT3
with long-range entanglement



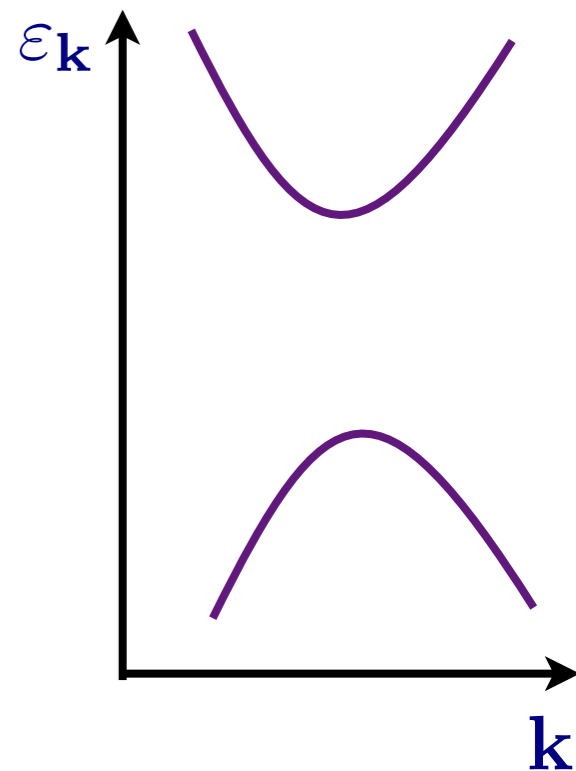
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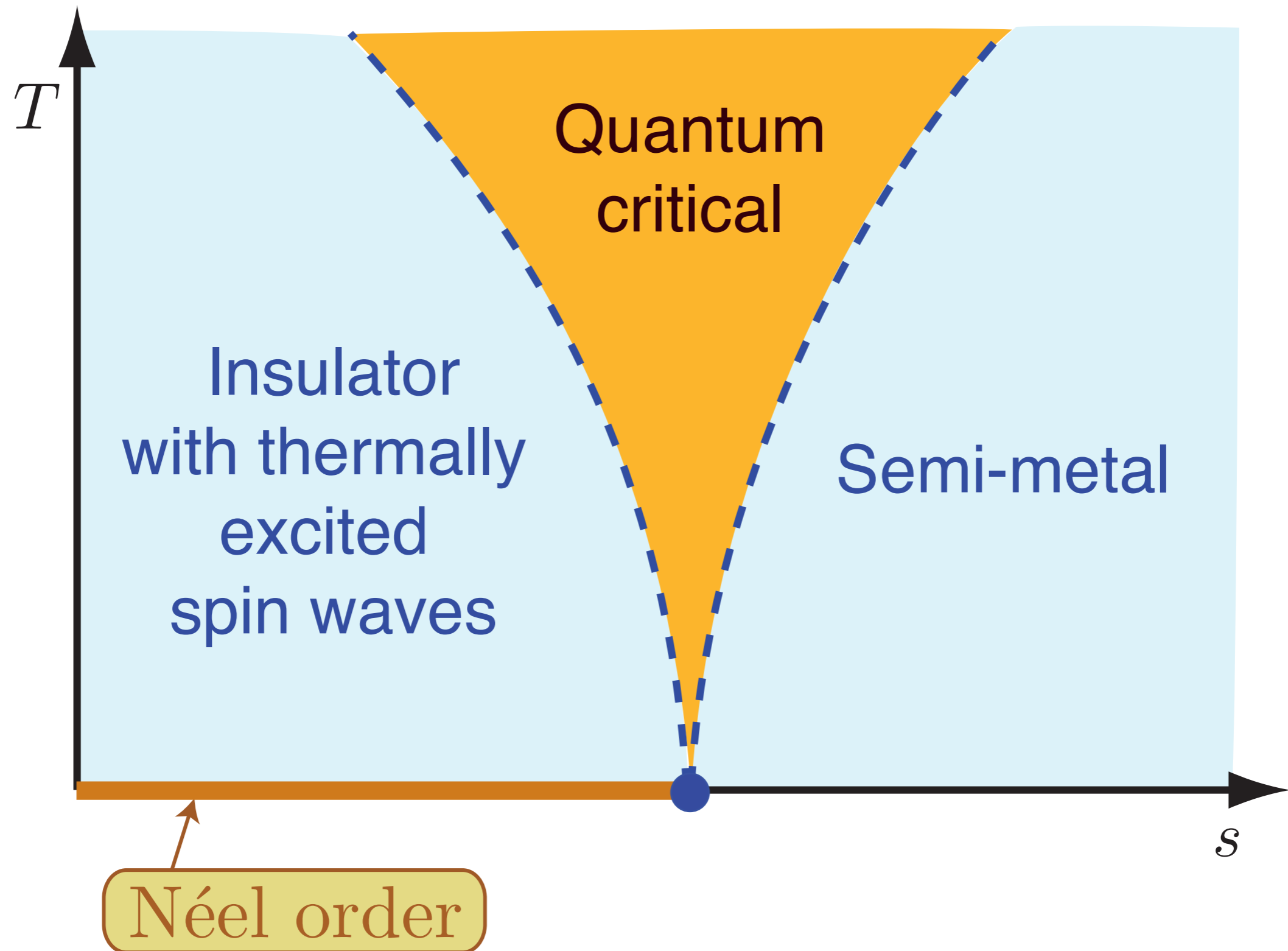


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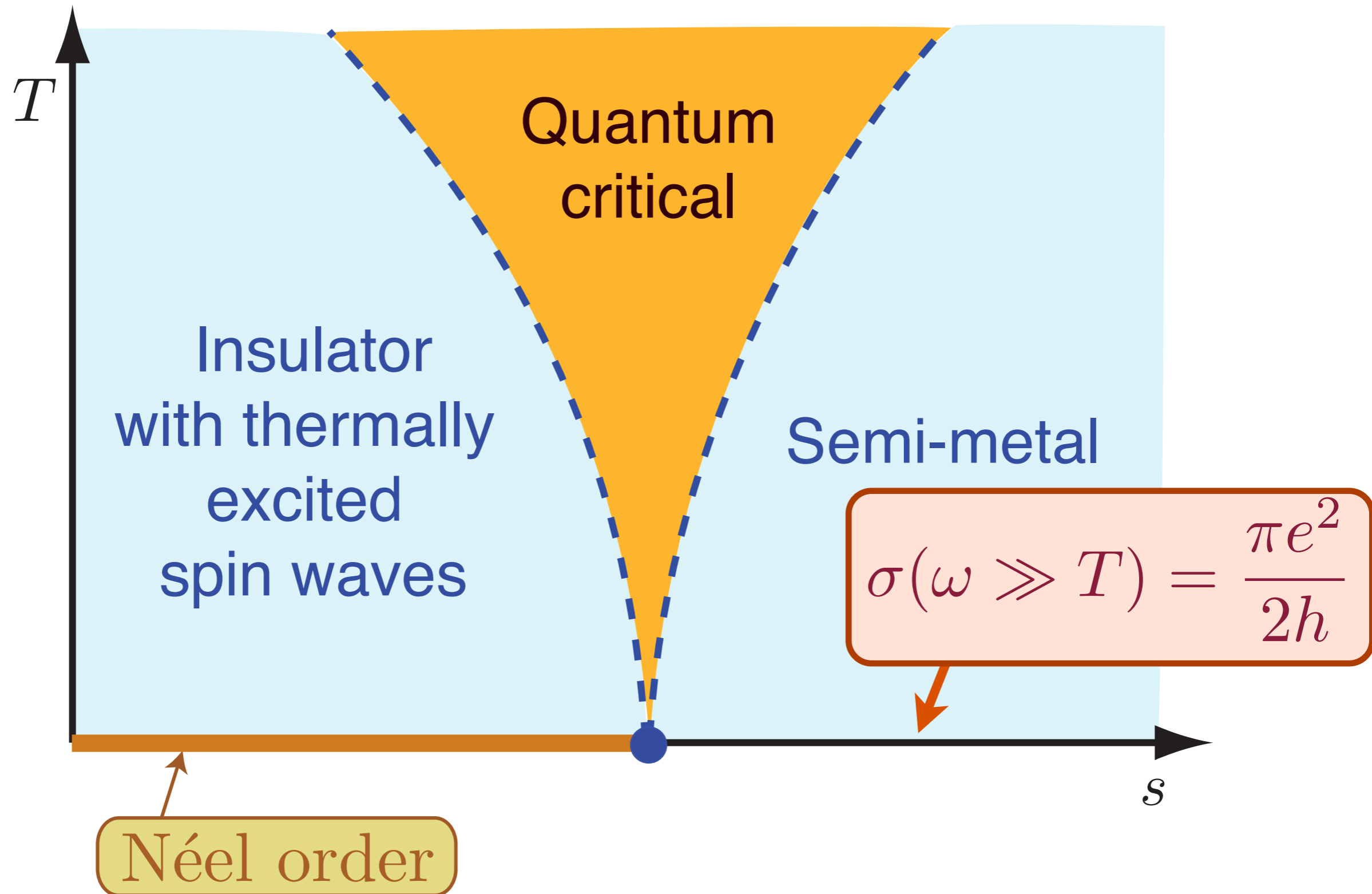
$$\sigma(\omega) = \frac{\pi e^2}{2h}$$

$$\sigma(\omega) = \frac{\mathcal{K} e^2}{h}$$

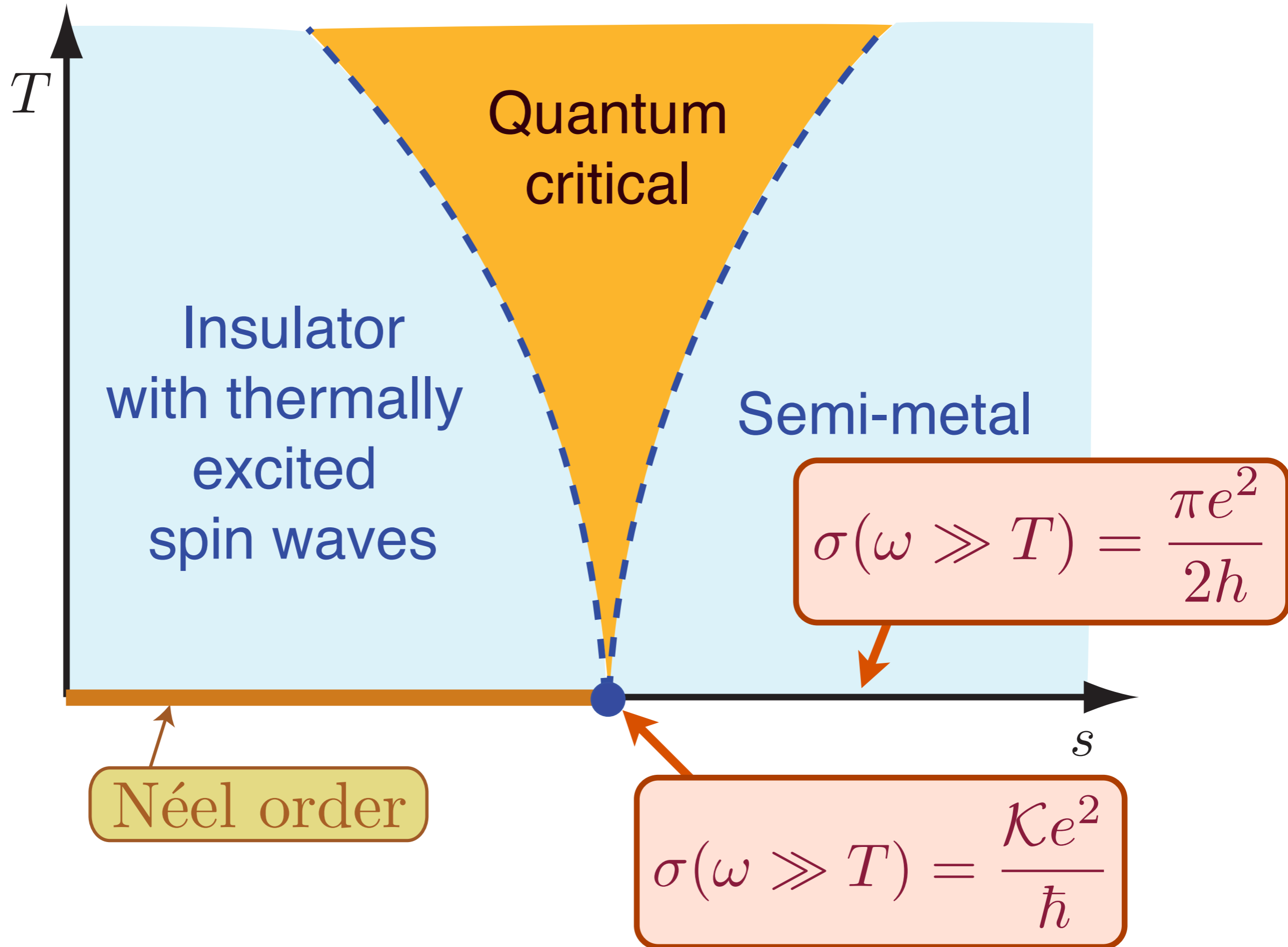
Phase diagram at non-zero temperatures



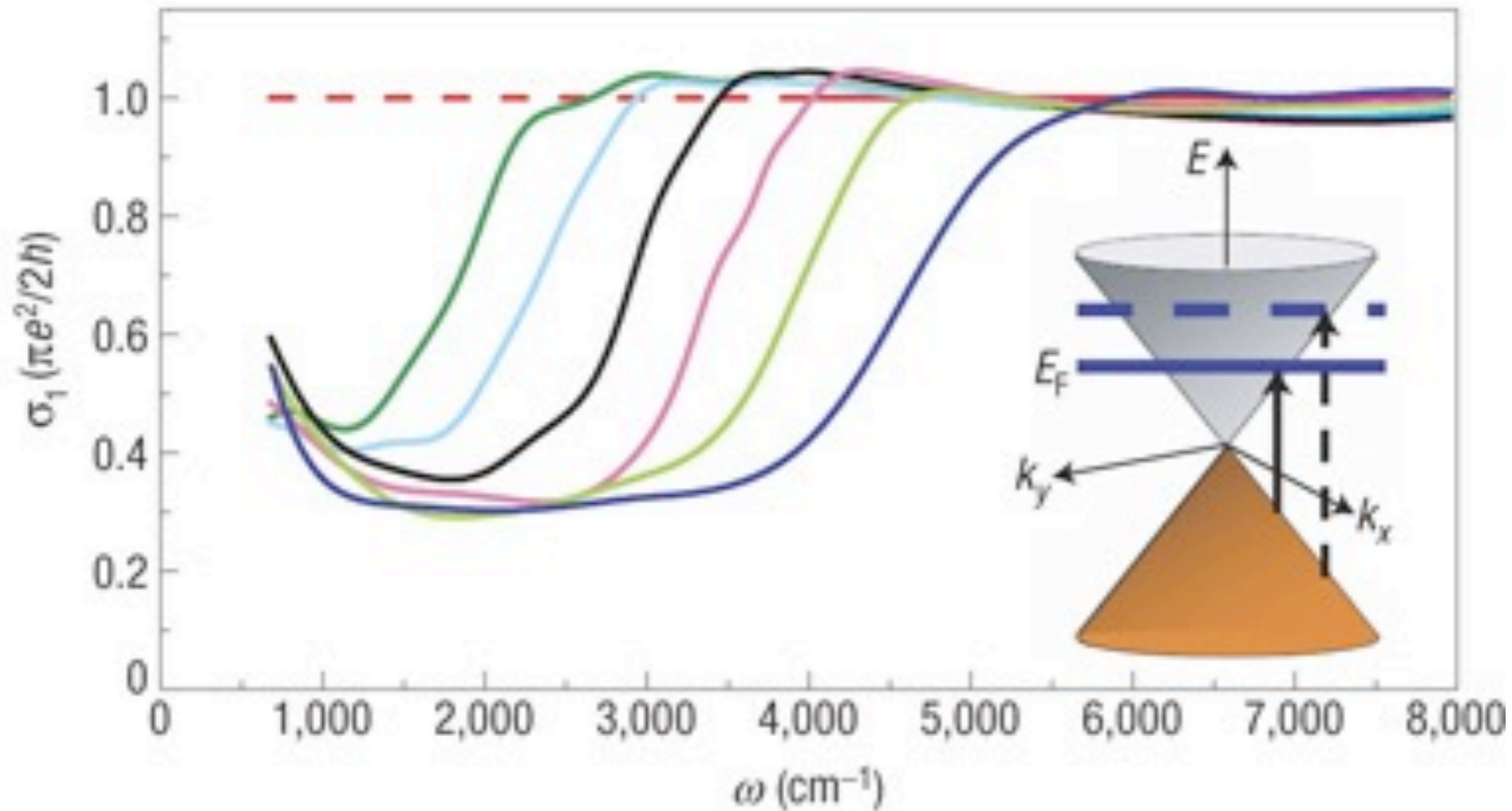
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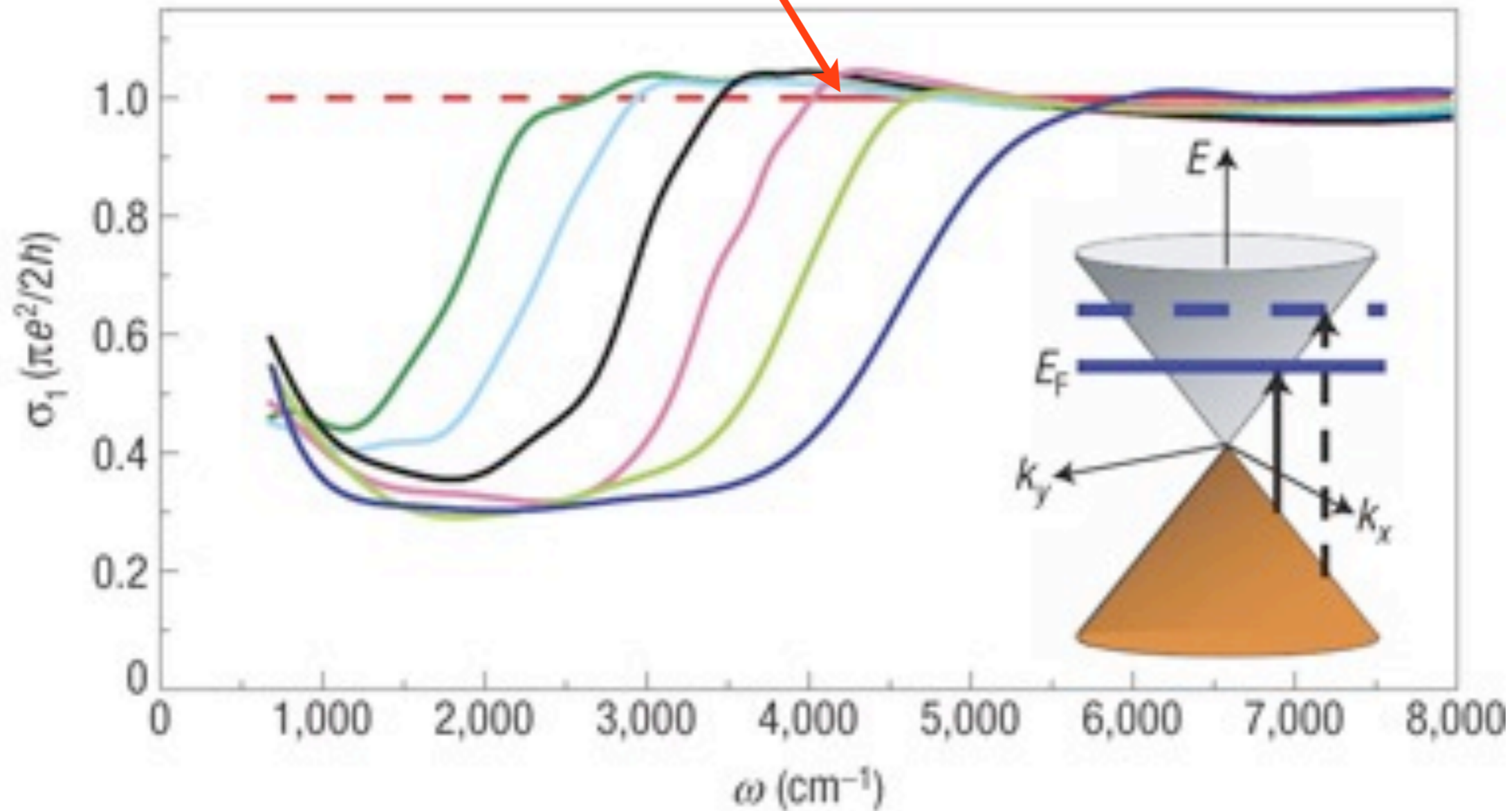
Optical conductivity of graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* 4, 532 (2008).

Optical conductivity of graphene

Undoped graphene



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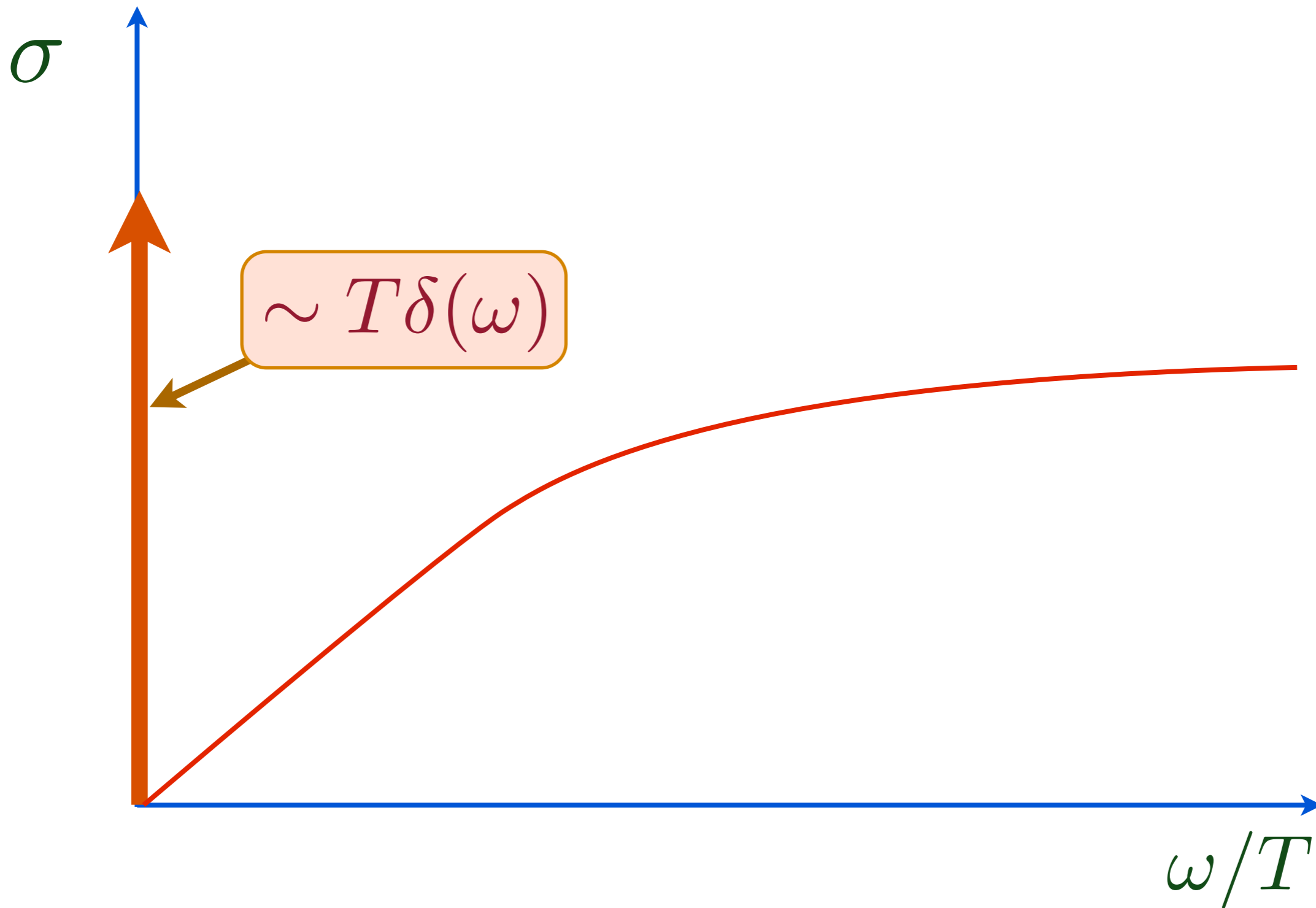
Non-zero temperatures

At the quantum-critical point at one-loop order, we can set $m = 0$, and then repeat the computation in Eq. (2) at $T > 0$. This only requires replacing the integral over the loop frequency by a summation over the Matsubara frequencies, which are quantized by odd multiples of πT . Such a computation, via Eq. (4) leads to the conductivity

$$\text{Re}[\sigma(\omega)] = (2T \ln 2) \delta(\omega) + \frac{1}{4} \tanh\left(\frac{|\omega|}{4T}\right); \quad (7)$$

the imaginary part of $\sigma(\omega)$ is the Hilbert transform of $\text{Re}[\sigma(\omega)] - 1/4$. Note that this reduces to Eq. (5) in the limit $\omega \gg T$. However, the most important new feature of Eq. (7) arises for $\omega \ll T$, where we find a delta function at zero frequency in the real part. Thus the d.c. conductivity is infinite at this order, arising from the collisionless transport of thermally excited carriers.

Electrical transport in a free CFT3 for $T > 0$



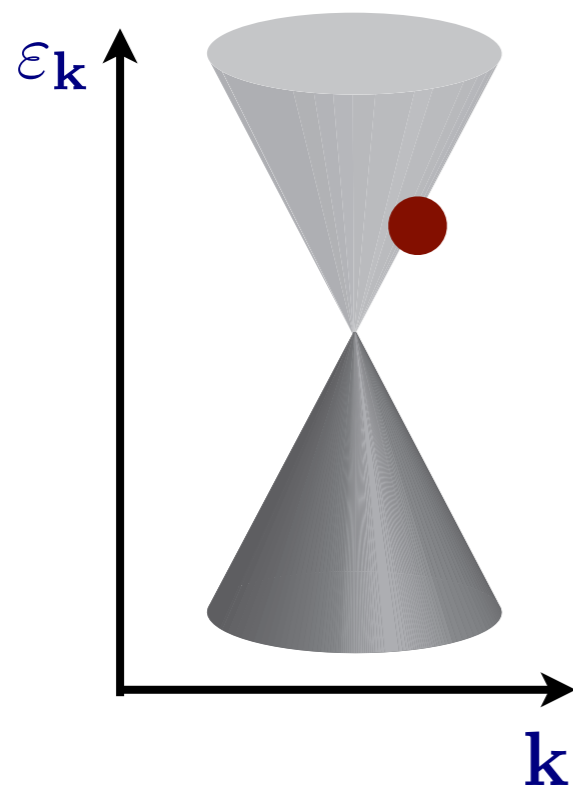
Particles



Momentum



Current



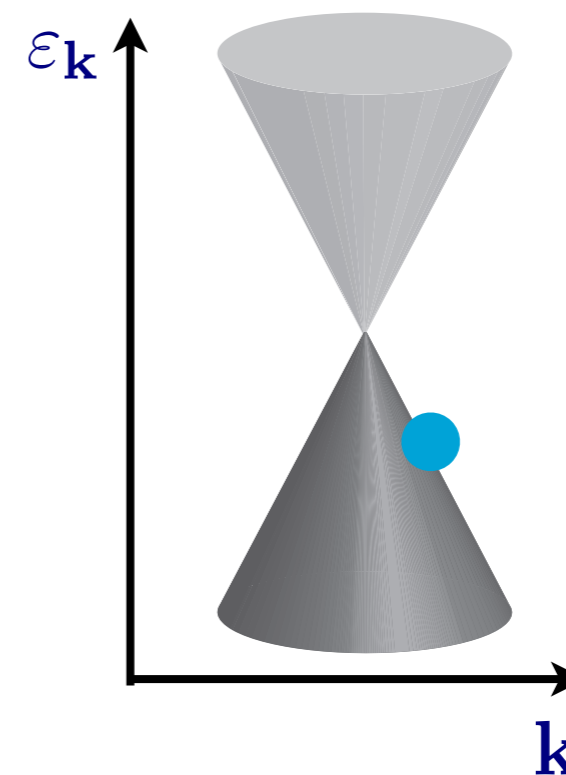
Holes



Momentum



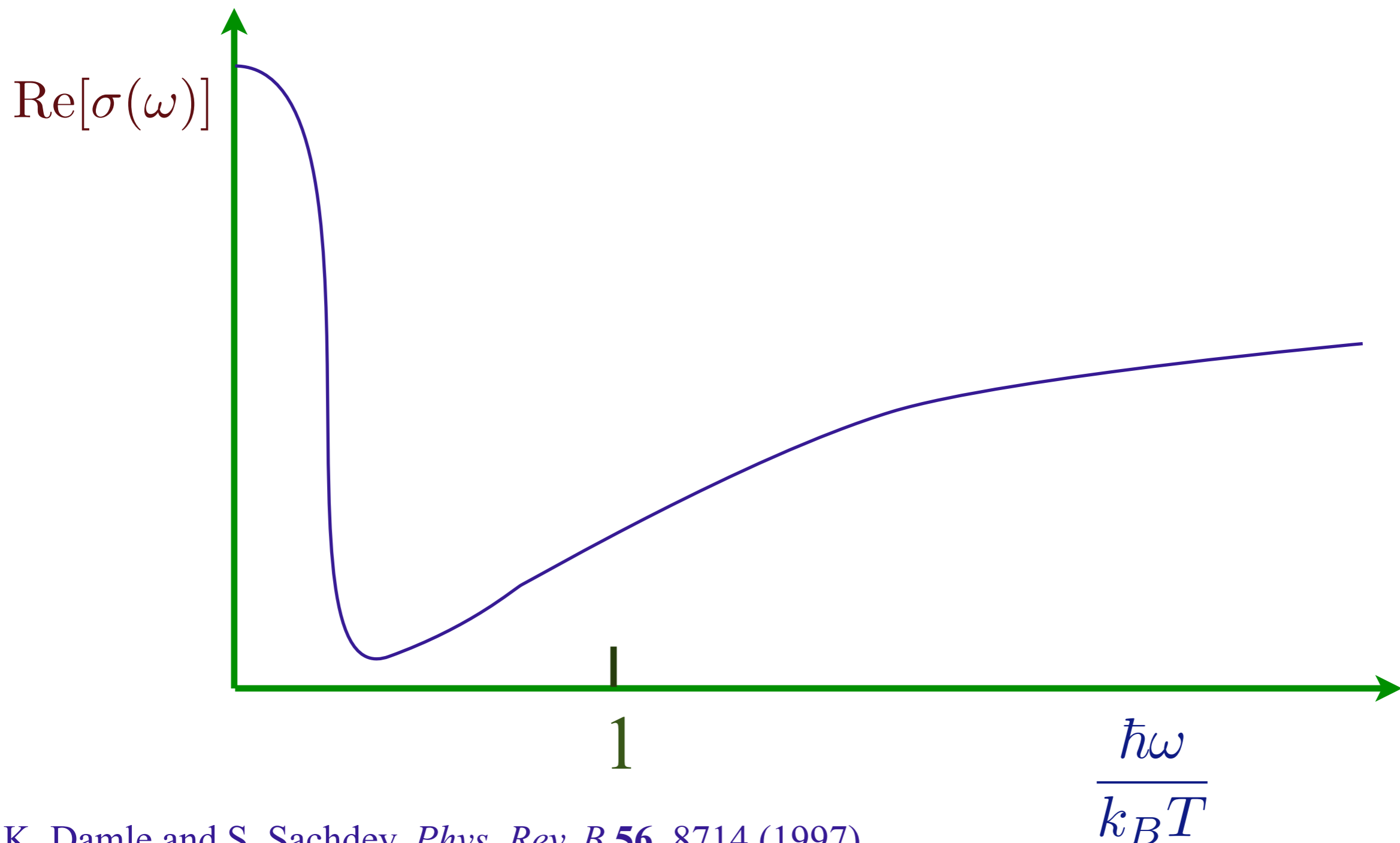
Current



Particle hole symmetry: current carrying state has zero momentum, and collisions can relax current to zero

Electrical transport for a (weakly) interacting CFT3

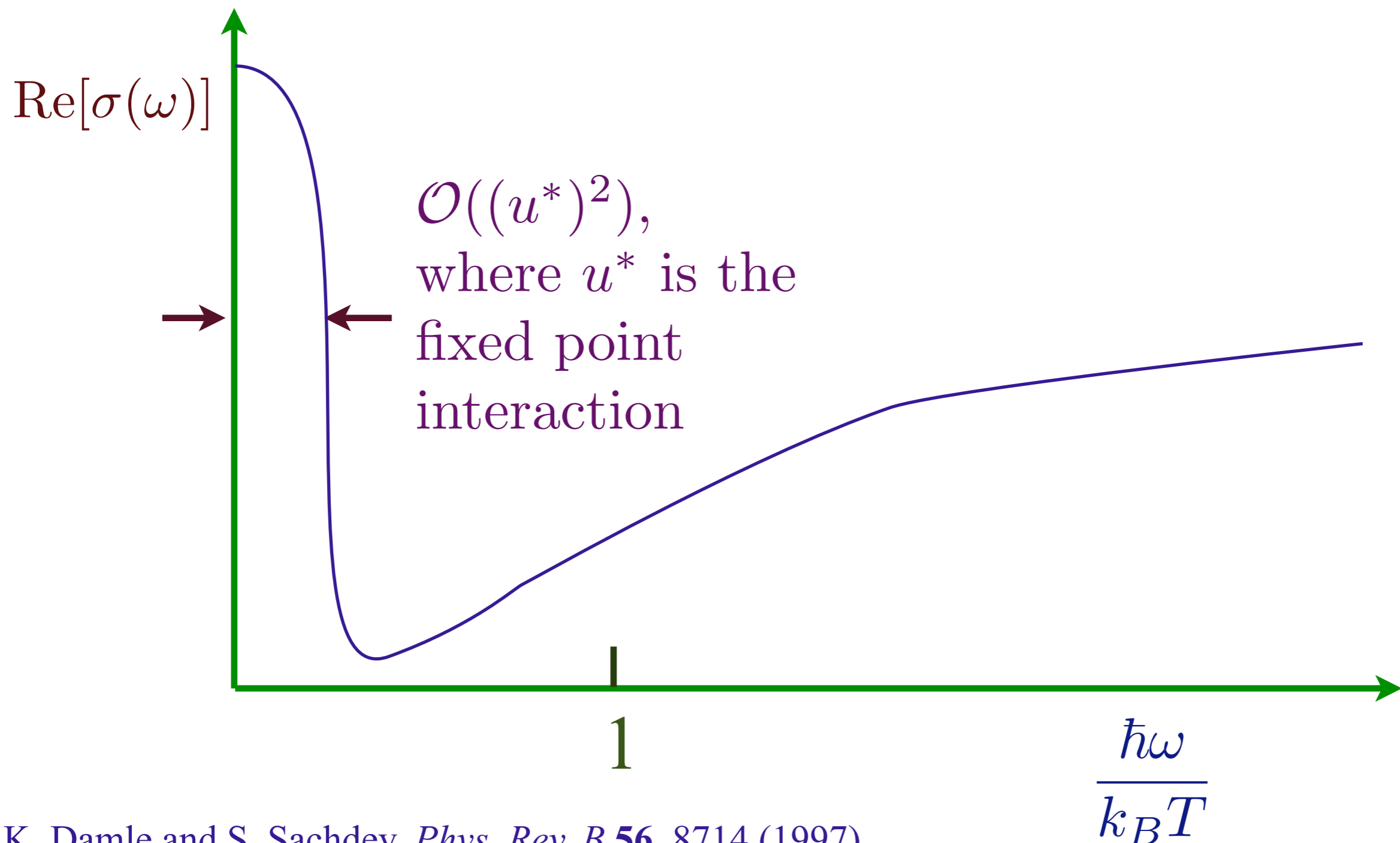
$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right) ; \quad \Sigma \rightarrow \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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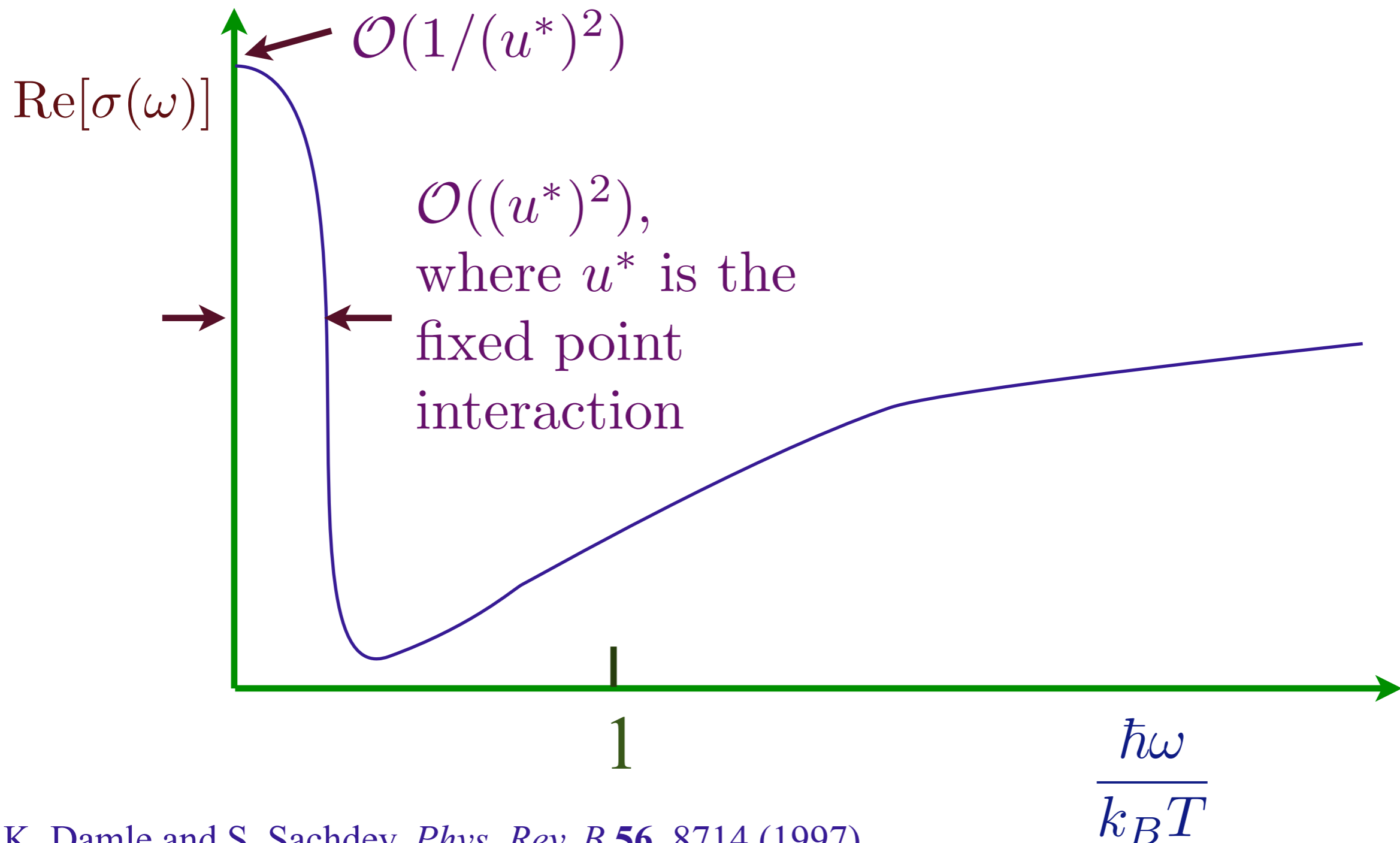
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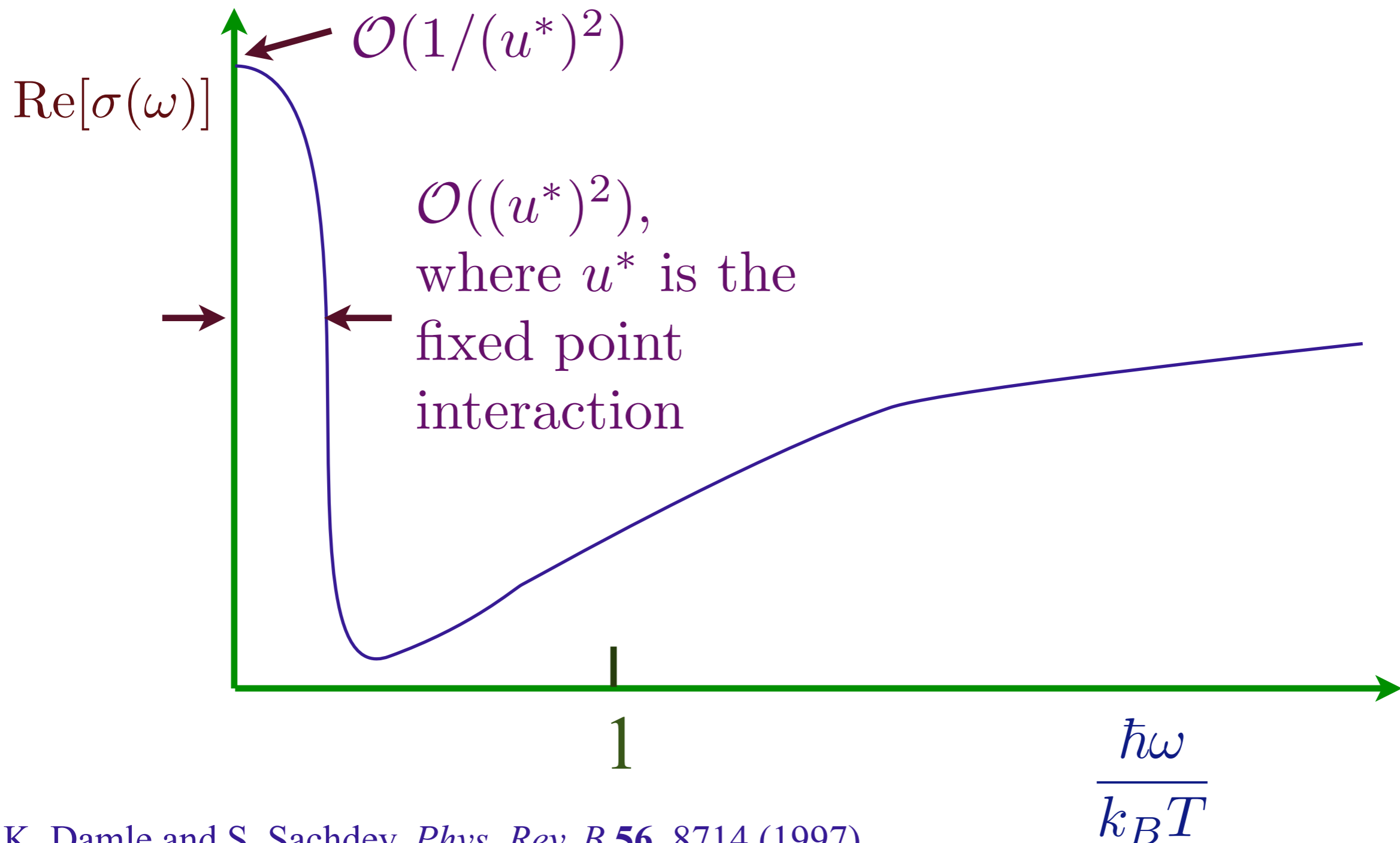
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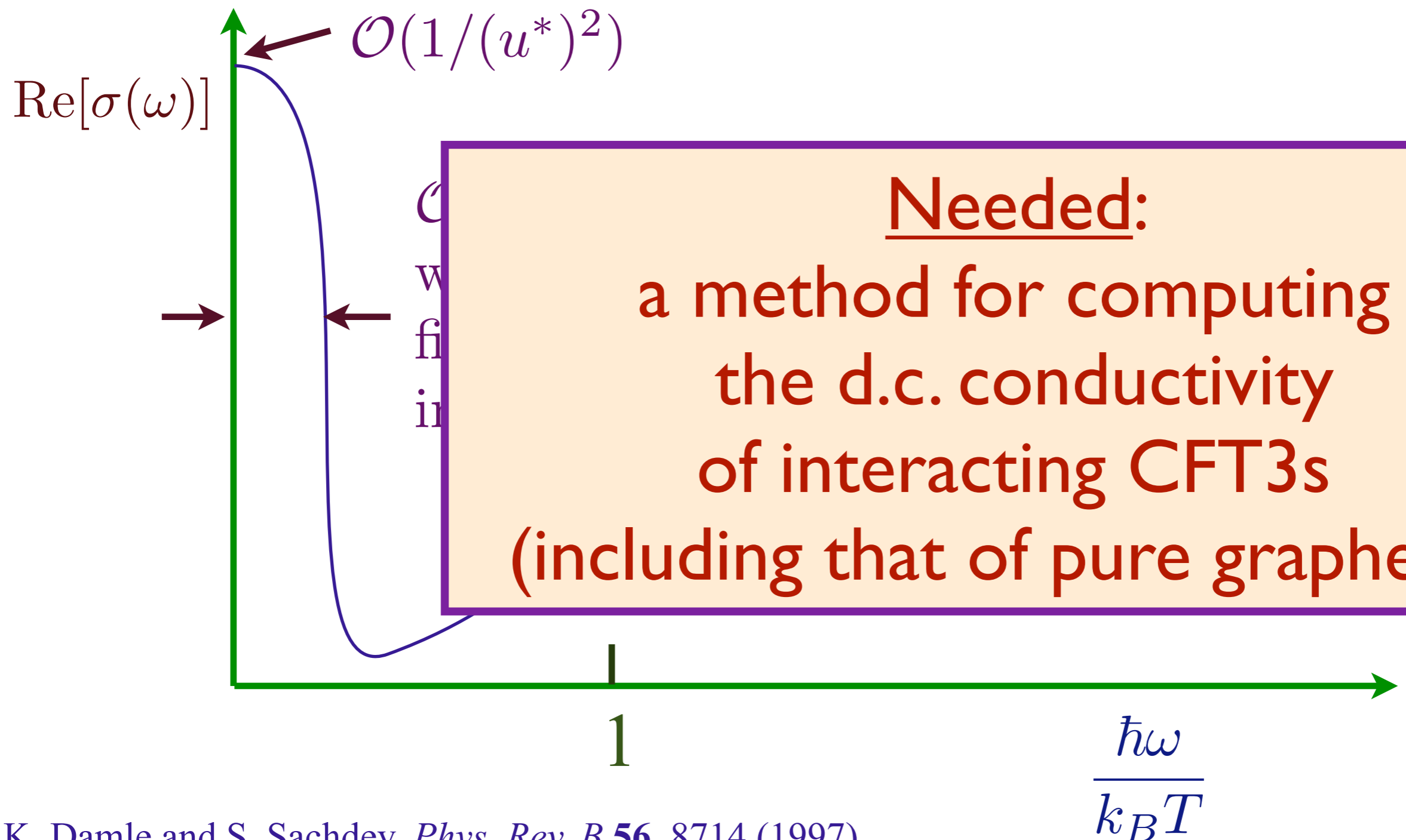
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$$\frac{\hbar\omega}{k_B T}$$

Quantum critical transport

Quantum “*nearly perfect fluid*”
with shortest possible
equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C} \frac{\hbar}{k_B T}$$

where \mathcal{C} is a *universal* constant

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Zaanen: Planckian dissipation

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999).

Quantum critical transport

Transport co-efficients not determined
by collision rate, but by
universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)]$$

(Q is the “charge” of one particle)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-efficients not determined
by collision rate, but by
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Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)]$$

Conformal quantum matter

A. Field theory:

Honeycomb lattice

Hubbard model

B. Gauge-gravity duality

Conformal quantum matter

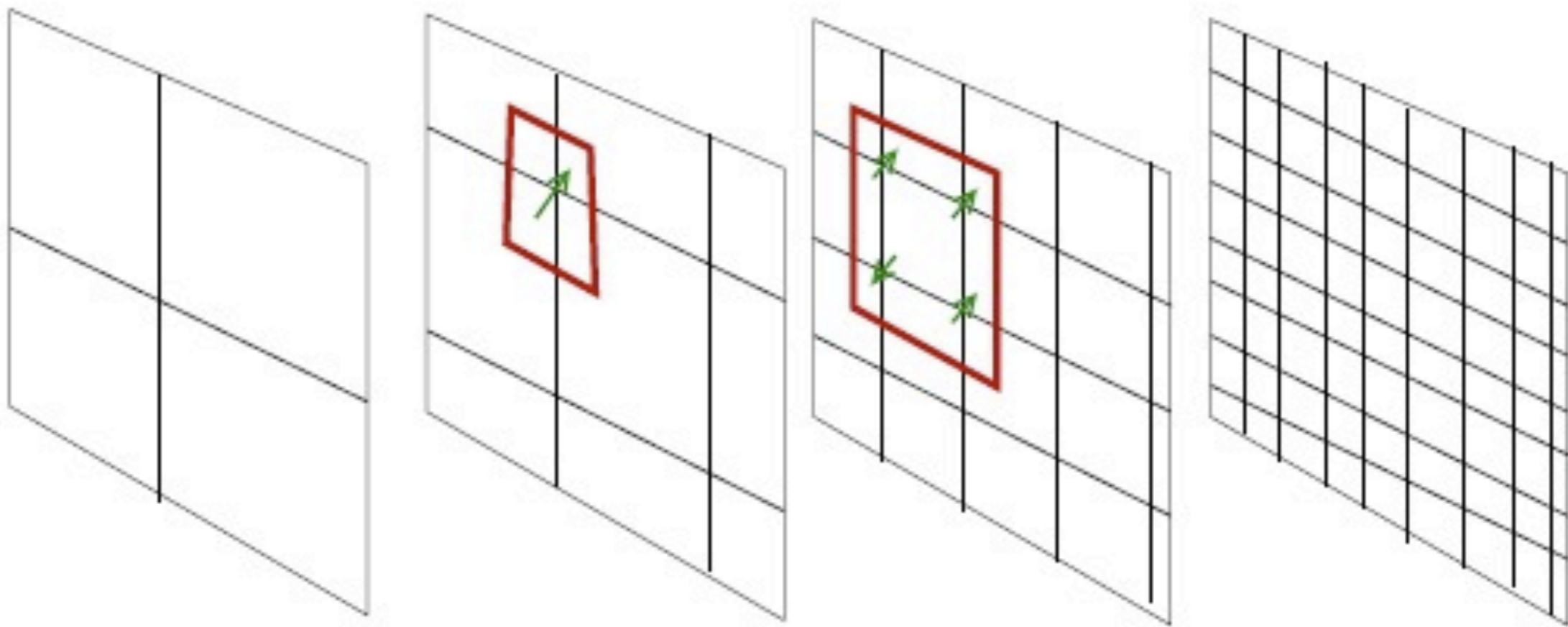
*A. Field theory:
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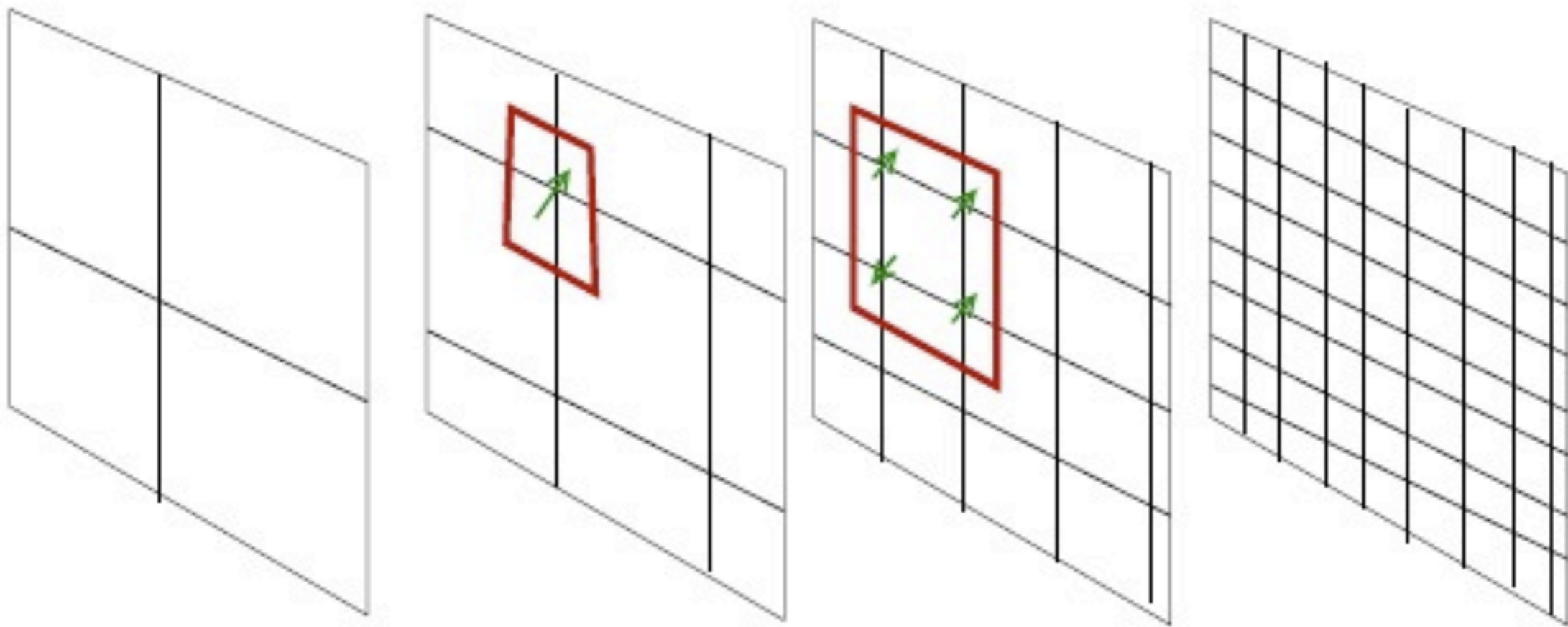
Field theories in $d + 1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

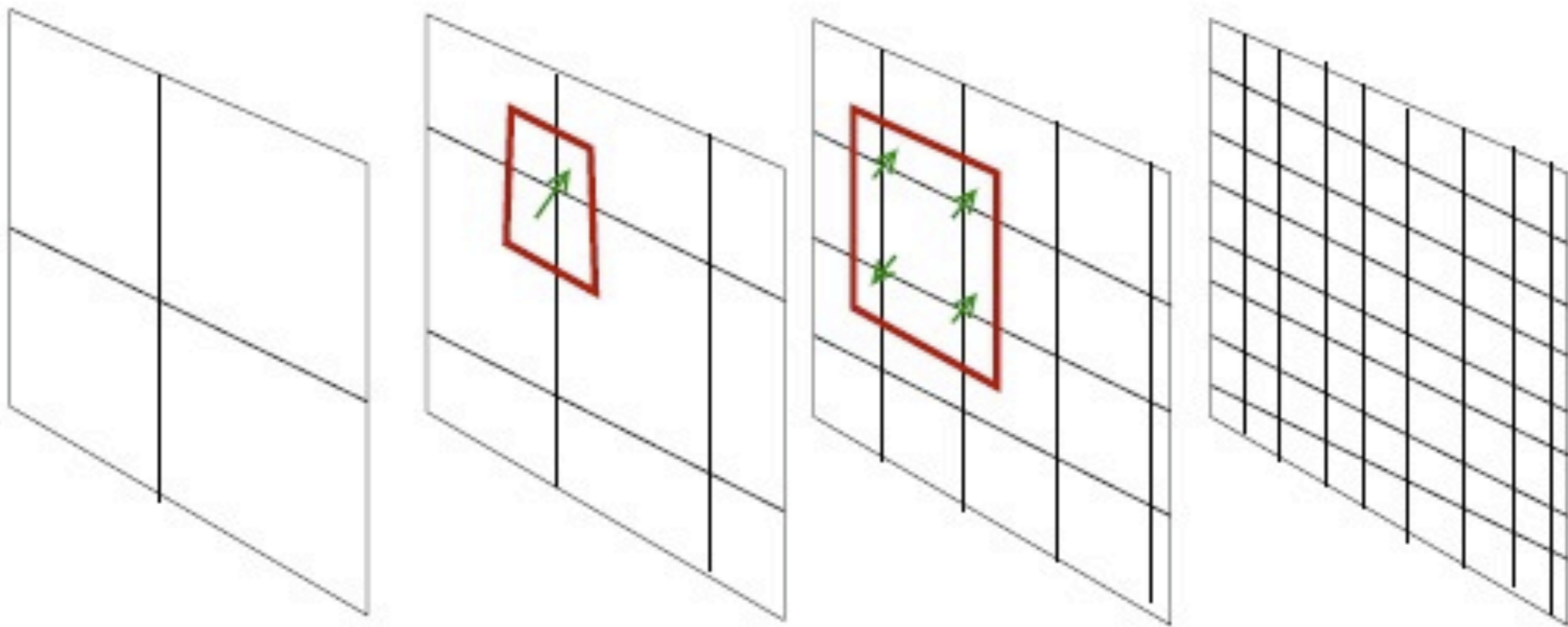
where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u .



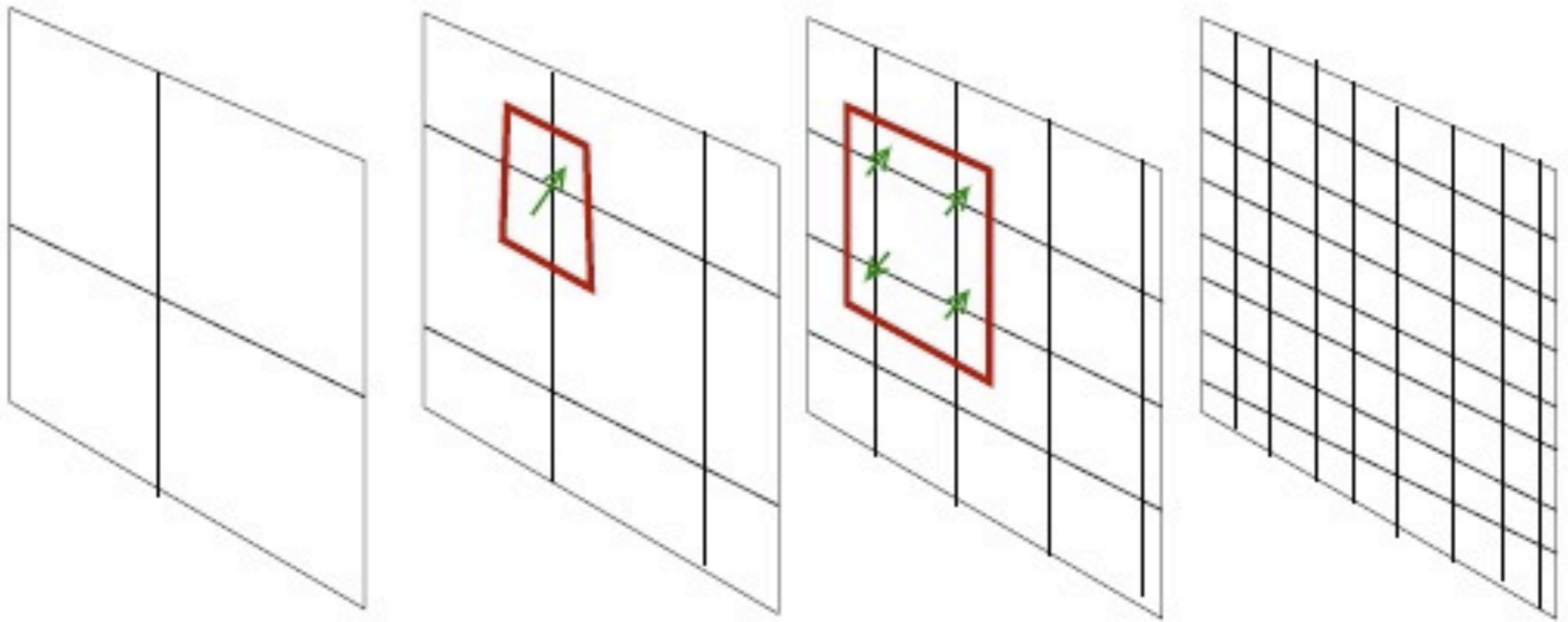
—————→ u



r ←



r ←



r ←

Key idea: \Rightarrow Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformation ($i = 1 \dots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

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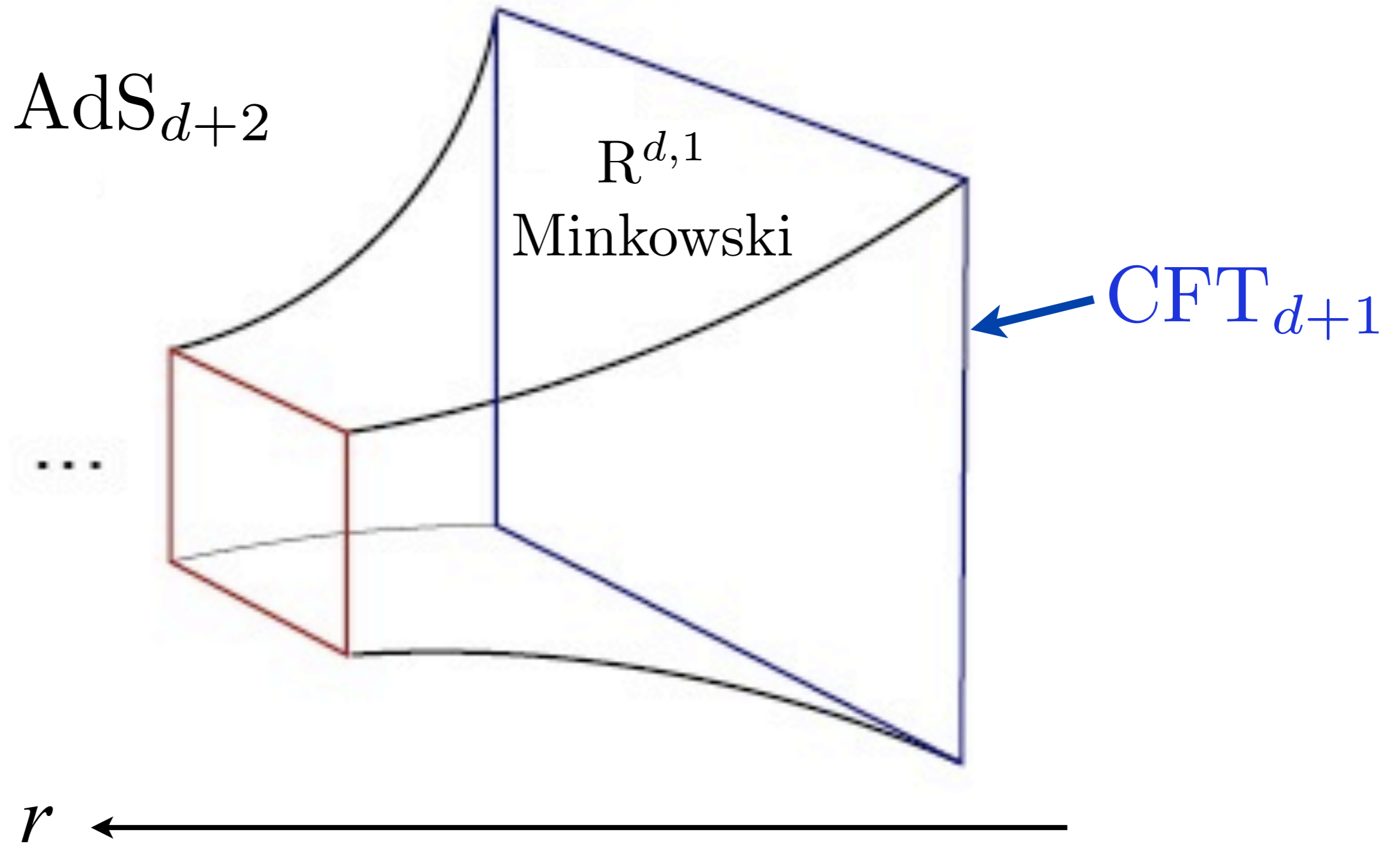
$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

This gives the unique metric

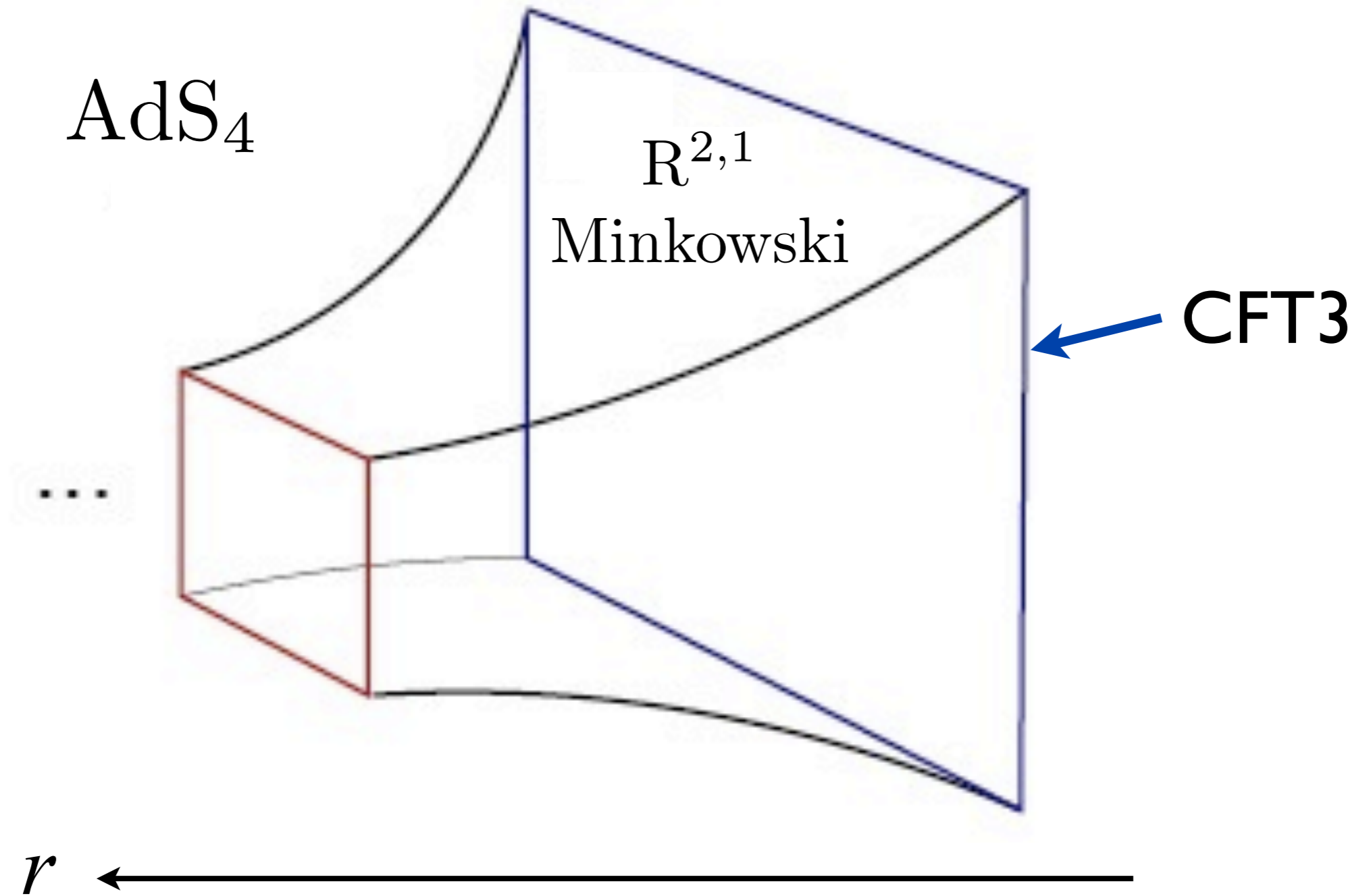
$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \rightarrow \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2} .

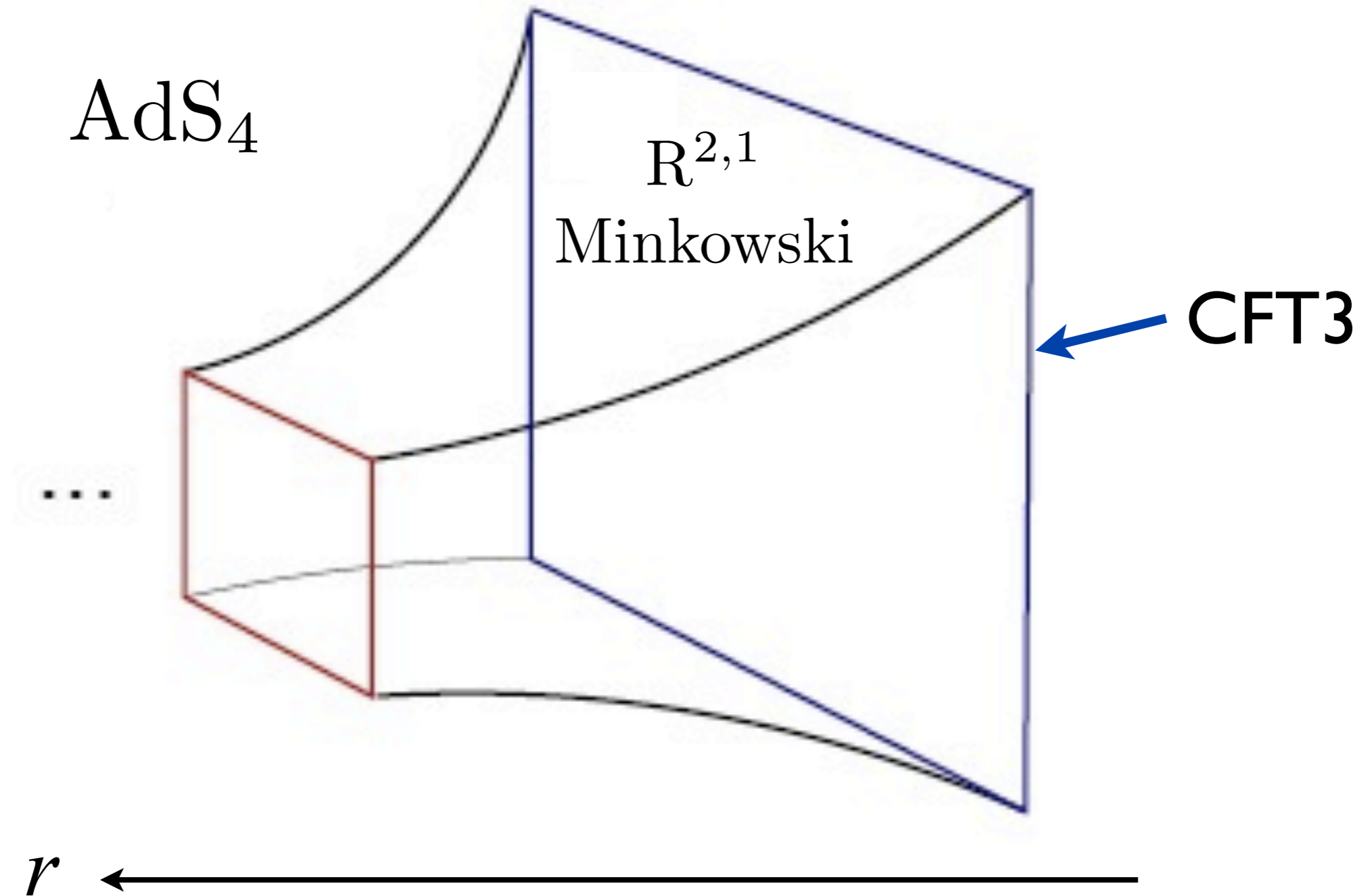
AdS/CFT correspondence



AdS/CFT correspondence



AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS/CFT correspondence at zero temperature

Suvrat Raju

Consider a CFT in D space-time dimensions with a scalar operator $O(\mathbf{x})$ with scaling dimension Δ . This is presumed to be equivalent to a dual gravity theory on AdS_{D+1} with action $\mathcal{S}_{\text{bulk}}$. The CFT and the bulk theory are related by the GKPW ansatz

$$\int \mathcal{D}\phi \exp(-\mathcal{S}_{\text{bulk}}) \Big|_{\text{bdy}} = \left\langle \exp \left(\int d^D x \phi_0(\mathbf{x}) O(\mathbf{x}) \right) \right\rangle_{\text{CFT}}$$

where the boundary condition is

$$\lim_{r \rightarrow 0} \phi(\mathbf{x}, r) = r^{D-\Delta} \phi_0(\mathbf{x}).$$

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We consider the simplest case of a single scalar field, where the bulk action is

$$\mathcal{S}_{\text{bulk}} = \frac{1}{2} \int d^{D+1}x \sqrt{g} [g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2]$$

where g_{ab} is the AdS_{D+1} metric (we are working with a Euclidean signature, and a, b extend over $D+1$ dimensions) and $g = \det(g_{ab})$. After Fourier transforming space-time co-ordinates to momenta \mathbf{k} , the saddle-point equation for $\phi(\mathbf{k}, r)$ is

$$-r^{D-1} \frac{d}{dr} \left(\frac{1}{r^{D-1}} \frac{d\phi}{dr} \right) + \left(k^2 + \frac{m^2}{r^2} \right) \phi = 0.$$

This equation has two solutions as $r \rightarrow 0$, with $\phi \sim r^\Delta$ or $\phi \sim r^{D-\Delta}$ where

$$\Delta = \frac{D}{2} \pm \sqrt{\frac{D^2}{4} + m^2}.$$

We will choose the positive sign in the manipulations below, but the final results hold for both signs. The complete solution of the saddle-point equation with the needed boundary condition can be written as

$$\phi(\mathbf{k}, r) = G_{\text{bulk-bdy}}(k, r)\phi_0(\mathbf{k})$$

where

$$G_{\text{bulk-bdy}}(k, r) = \frac{2^{1-\Delta+D/2}}{\Gamma(\Delta - D/2)} k^{\Delta-D/2} r^{D/2} K_{\Delta-D/2}(kr)$$

where $K_{\Delta-D/2}$ is a modified Bessel function.

Next, it is useful to obtain the bulk-bulk Green's function by inverting the operator in the equation of motion. A standard computation yields

$$G_{\text{bulk-bulk}}(k, r_1, r_2) = (r_1 r_2)^{D/2} I_{\Delta-D/2}(kr_{<}) K_{\Delta-D/2}(kr_{>})$$

where $r_{<}$ ($r_{>}$) is the smaller (larger) of $r_{1,2}$. This bulk-bulk Green's function is evaluated in the absence of any sources on the boundary, and so we have to impose

the boundary condition $\phi(\mathbf{k}, r) \sim r^\Delta$ as $r \rightarrow 0$ in solving the saddle-point equation. The utility of this bulk-bulk Green's function is that it now allows us to extend our results to include interactions in $\mathcal{S}_{\text{bulk}}$ by the usual Feynman graph expansion. We can account for the presence of the boundary source $\phi_0(\mathbf{k})$ in the CFT by imagining there is a corresponding bulk source field $J_0(\mathbf{k}, r)$ which is localized at very small values of r . Then this bulk source field will generate a bulk $\phi(\mathbf{k}, r)$ via the propagator $G_{\text{bulk-bulk}}$. We now note that

$$\lim_{r_2 \rightarrow 0} G_{\text{bulk-bulk}}(k, r_1, r_2) = \frac{r_2^\Delta}{(2\Delta - D)} G_{\text{bulk-bdy}}(k, r_1).$$

This is a key relation which shows us that functional derivatives of the full action w.r.t. $J_0(\mathbf{k}, r)$ (which yield bulk-bulk correlation functions) are the *same* as functional derivatives w.r.t. $\phi_0(\mathbf{k})$ (which yield correlators of the CFT). This yields the second statement of the equivalence between the bulk and boundary theories

$$\langle O(\mathbf{x}_1) \dots O(\mathbf{x}_n) \rangle_{\text{CFT}} = Z^n \lim_{r \rightarrow 0} r_1^{-\Delta} \dots r_n^{-\Delta} \langle \phi(\mathbf{x}_1, r_1) \dots \phi(\mathbf{x}_n, r_n) \rangle_{\text{bulk}}$$

where the “wave function renormalization” factor $Z = (2\Delta - D)$. Note that this relationship holds for arbitrary bulk actions, and permits full quantum fluctuations in the bulk theory. Also, both correlators are evaluated in the absence of external sources; for the bulk theory this means that we have the boundary condition

$\phi(\mathbf{k}, r) \sim r^\Delta$ as $r \rightarrow 0$. From this general relation we can evaluate the two-point correlator of the CFT for the case of a bulk Gaussian action:

$$\begin{aligned}\langle O(\mathbf{k})O(-\mathbf{k}) \rangle_{\text{CFT}} &= Z^2 \lim_{r \rightarrow 0} (r)^{-2\Delta} G_{\text{bulk-bulk}}(k, r, r) \\ &= \lim_{r \rightarrow 0} (2\Delta - D)r^{-(2\Delta - D)} - (2\Delta - D) \left(\frac{k}{2}\right)^{2\Delta - D} \frac{\Gamma(1 - \Delta + D/2)}{\Gamma(1 + \Delta - D/2)}\end{aligned}$$

The first term is divergent, but it is independent of k : so it does not contribute to the long-distance correlations of the CFT, and can be dropped. The second term has the singular dependence $\sim k^{2\Delta - D}$, which is just as expected for a field with scaling dimension Δ , for the Fourier transformation yields

$$\langle O(\mathbf{x}_1)O(\mathbf{x}_2) \rangle_{\text{CFT}} \sim |\mathbf{x}_1 - \mathbf{x}_2|^{-2\Delta}.$$

The final formulation of the bulk-boundary correspondence appears by using the above relations for arbitrary multi-point correlators in the absence of a source, to make a statement for the one-point function in the presence of a source, working to all orders in the source and all bulk interactions. As we noted earlier, the CFT source $\phi_0(\mathbf{k})$ can be simulated by a source $J_0(\mathbf{k}, r)$ which is localized near the boundary but acts on the bulk theory. Because of the identity above between the source-free correlators, we can conclude that $\langle O(\mathbf{x}) \rangle$ equals $Z \lim_{r \rightarrow 0} r^{-\Delta} \langle \phi(\mathbf{x}, r) \rangle$. However, we have to remember that the source $J_0(\mathbf{k}, r)$ is actually realized by a boundary condition on $\phi(\mathbf{x}, r)$, and so the complete statement is

$$\lim_{r \rightarrow 0} \langle \phi(\mathbf{x}, r) \rangle = r^{D-\Delta} \phi_0(\mathbf{x}) + \frac{r^\Delta}{Z} \langle O(\mathbf{x}) \rangle,$$

in the presence of the source $\phi_0(\mathbf{x})$. Note that this result is not just linear response, and holds to all orders in the source; it also allows for arbitrary bulk interactions and quantum fluctuations. It can be checked that it is indeed obeyed by the correlators above of the Gaussian theory. This relationship is frequently used in practice, because it is often straightforward to implement, especially when we are using the classical saddle-point approximation for the bulk theory. Then we simply have to extract the subleading behavior in $\phi(\mathbf{x}, r)$ as $r \rightarrow 0$ to extract the full non-linear response function to the perturbation $\phi_0(\mathbf{x})$ to the CFT.

A similar analysis can be applied to operators of the CFT with non-zero Lorentz spin. Of particular interest are correlators of a conserved current, J_μ , associated with a global ‘flavor’ symmetry, and the conserved stress energy tensor $T_{\mu\nu}$.

We couple the conserved current to a source a_μ and so are interested in evaluating

$$\mathcal{Z}(a_\mu) = \left\langle \exp \left(\int d^D x a_\mu(\mathbf{x}) J_\mu(\mathbf{x}) \right) \right\rangle_{\text{CFT}} .$$

The conservation law $\partial_\mu J_\mu = 0$ now implies that this partition function is invariant under the gauge transformation $a_\mu \rightarrow a_\mu + \partial_\mu \alpha$. So the bulk field dual to a (say) U(1) conserved current J_μ is a U(1) gauge field, which we denote $A_a(\mathbf{x}, r)$. We assume the gauge field has a Maxwell action

$$\mathcal{S}_M = \frac{1}{4g_M^2} \int d^{D+1} x \sqrt{g} F_{ab} F^{ab}$$

plus other possible gauge couplings to the bulk fields. By an analysis very similar to the scalar field, we can establish the following bulk-boundary correspondences

$$\lim_{r \rightarrow 0} \langle A_\mu(\mathbf{x}, r) \rangle = a_\mu(\mathbf{x}) + \frac{r^{D-2}}{Z g_M^{-2}} \langle J_\mu(\mathbf{x}) \rangle$$

$$\lim_{r \rightarrow 0} \langle A_r(\mathbf{x}, r) \rangle = 0$$

$$\langle J_\mu(\mathbf{x}_1) \dots J_\nu(\mathbf{x}_n) \rangle_{\text{CFT}} = (Z g_M^{-2})^n \lim_{r \rightarrow 0} r_1^{2-D} \dots r_n^{2-D} \langle A_\mu(\mathbf{x}_1, r_1) \dots A_\nu(\mathbf{x}_n, r_n) \rangle_{\text{bulk}}$$

with $Z = D - 2$. Working with only the Maxwell action these relations yield

$$\langle J_\mu(\mathbf{k}) J_\nu(\mathbf{k}) \rangle_{\text{CFT}} = \frac{(D-2) \Gamma(2-D/2)}{g_M^2 \Gamma(D/2)} \left(\frac{k}{2}\right)^{D-2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$$

This is precisely the expected form for the correlator of a conserved current in a CFT in D space-time dimensions. For the case $D = 3$ it has the expected form

$$\langle J_\mu(\mathbf{k}) J_\nu(\mathbf{k}) \rangle_{\text{CFT}} = \mathcal{K} k \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$$

with

$$\mathcal{K} = \frac{1}{g_M^2}.$$

A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the metric of the bulk theory. Now the needed partition function is

$$\mathcal{Z}(\chi_{\mu\nu}) = \left\langle \exp \left(\int d^D x \chi_{\mu\nu}(\mathbf{x}) T_{\mu\nu}(\mathbf{x}) \right) \right\rangle_{\text{CFT}}$$

and the source is related to the metric g_{ab} via

$$\begin{aligned} \lim_{r \rightarrow 0} g_{rr} &= \frac{L^2}{r^2} \\ \lim_{r \rightarrow 0} g_{r\mu} &= 0 \\ \lim_{r \rightarrow 0} g_{\mu\nu} &= \frac{L^2}{r^2} (\delta_{\mu\nu} + \chi_{\mu\nu}) \end{aligned}$$

The bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(\mathbf{x}_1) \dots T_{\rho\sigma}(\mathbf{x}_n) \rangle_{\text{CFT}} = \left(\frac{Z L^2}{\kappa^2} \right)^n \lim_{r \rightarrow 0} r_1^{-D} \dots r_n^{-D} \langle \chi_{\mu\nu}(\mathbf{x}_1, r_1) \dots \chi_{\rho\sigma}(\mathbf{x}_n, r_n) \rangle_{\text{bulk}}$$

with $Z = D$. Applying this prescription to the Einstein action, we obtain in $D = 3$

$$\langle T_{\mu\nu}(\mathbf{k})T_{\rho\sigma}(-\mathbf{k}) \rangle_{\text{CFT}} = C_T |k|^3 \left(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\nu\rho}\delta_{\mu\sigma} - \delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\nu}\frac{k_\rho k_\sigma}{k^2} + \delta_{\rho\sigma}\frac{k_\mu k_\nu}{k^2} - \delta_{\mu\rho}\frac{k_\nu k_\sigma}{k^2} - \delta_{\nu\rho}\frac{k_\mu k_\sigma}{k^2} - \delta_{\mu\sigma}\frac{k_\nu k_\rho}{k^2} - \delta_{\nu\sigma}\frac{k_\mu k_\rho}{k^2} + \frac{k_\mu k_\nu k_\rho k_\sigma}{k^4} \right)$$

This is the most-general form expected for any CFT, and the “central charge” is related to a dimensionless combination of the gravitational constant and the AdS radius

$$C_T \propto \frac{L^2}{2\kappa^2}.$$

So, to recapitulate, we have equated the correlators of the CFT3 to a bulk theory on AdS₄ with the Einstein-Hilbert action

$$\mathcal{S} = \frac{1}{4g_M^2} \int d^4x \sqrt{g} F_{ab} F^{ab} + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right].$$

This action is characterized by two dimensionless parameters: g_M and L^2/κ^2 . These parameters determine, respectively, the two-point correlators of a conserved U(1) current J_μ and the stress-energy tensor $T_{\mu\nu}$.

However, this action is non-linear, and it also implies non-zero multipoint correlators of these operators, even at tree-level in the bulk theory. For the simplest 3-point correlator, a lengthy computation from the bulk theory yields

$$\langle J_\mu(\mathbf{k}_1) J_\nu(\mathbf{k}_2) T_{\rho\sigma}(-\mathbf{k}_1 - \mathbf{k}_2) \rangle \sim \frac{k_1 k_2}{(k_1 + k_2)^5} k_{1\mu} k_{1\nu} k_{1\rho} k_{1\sigma} + 175 \text{ terms}$$

with co-efficients determined by g_M and L^2/κ^2 .

We can now compare this 3-point correlator with that obtained by direct computation on a CFT3. A general analysis of the constraints from conformal invariance (Osborn and Petkou, 1993) shows that this 3-point correlator is fully determined by the values \mathcal{K} , C_T , and *one* additional dimensionless constant which is characteristic of the CFT3.

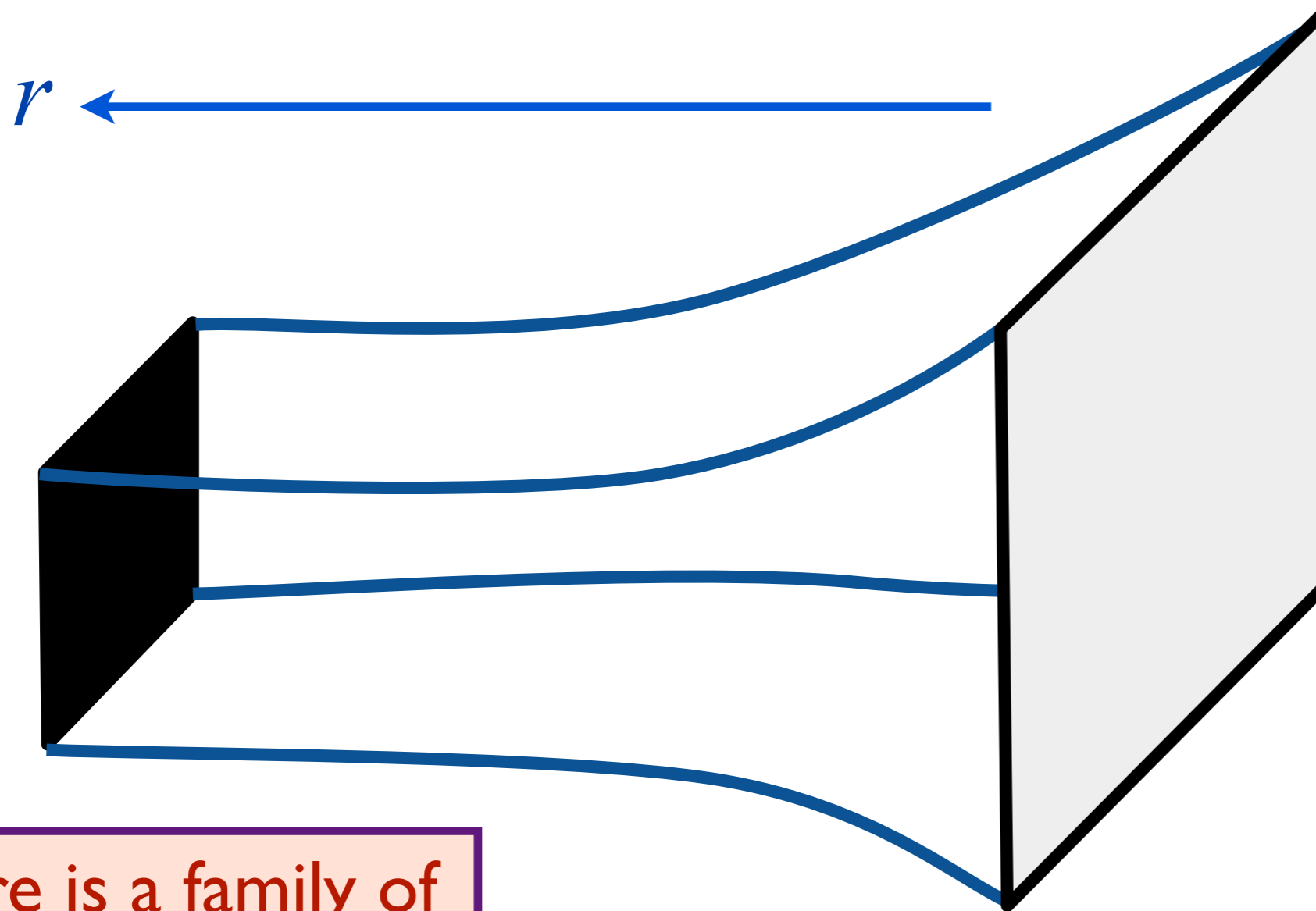
To fix this additional constant by the bulk theory, we have to go beyond the Einstein-Maxwell action. This action is the simplest action with up to 2 derivatives of the bulk fields. So, in the spirit of effectively field theory, let us now include all terms up to 4 derivatives. We want to work in linear response for the conserved current, and so we exclude terms which have more than 2 powers of F_{ab} . Then, up to some field redefinitions, there turns out to be a unique 4 derivative term, and the extended action of the bulk theory now becomes

$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

where C_{abcd} is the Weyl tensor. Now we have a new dimensionless parameter, γ ; stability constraints on this action restrict $|\gamma| < 1/12$. The Weyl tensor vanishes on the AdS metric, and consequently γ does not modify the previous results on the 2-point correlators of J_μ and $T_{\mu\nu}$. However, γ does change the structure of the 3-point correlator. We found that for a suitable choice of γ it is possible to reproduce the 3-point correlator of free conformal fields; we expect we can match the properties of any CFT3 for a suitable γ .

It is clear that similar results apply at higher orders: matching higher multipoint correlators of the CFT requires higher derivative terms in the effective bulk theory.

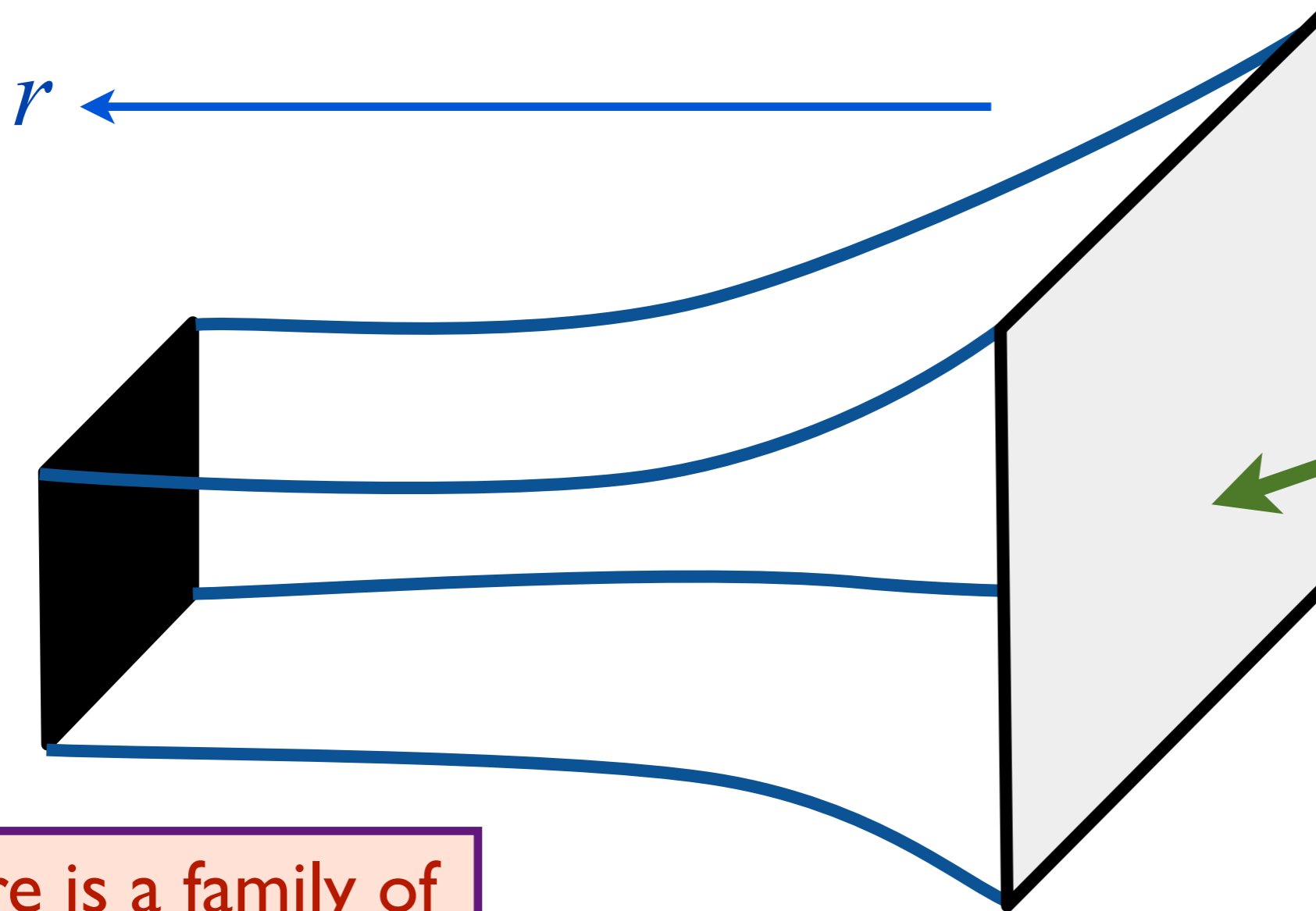
AdS₄-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

AdS₄-Schwarzschild black-brane



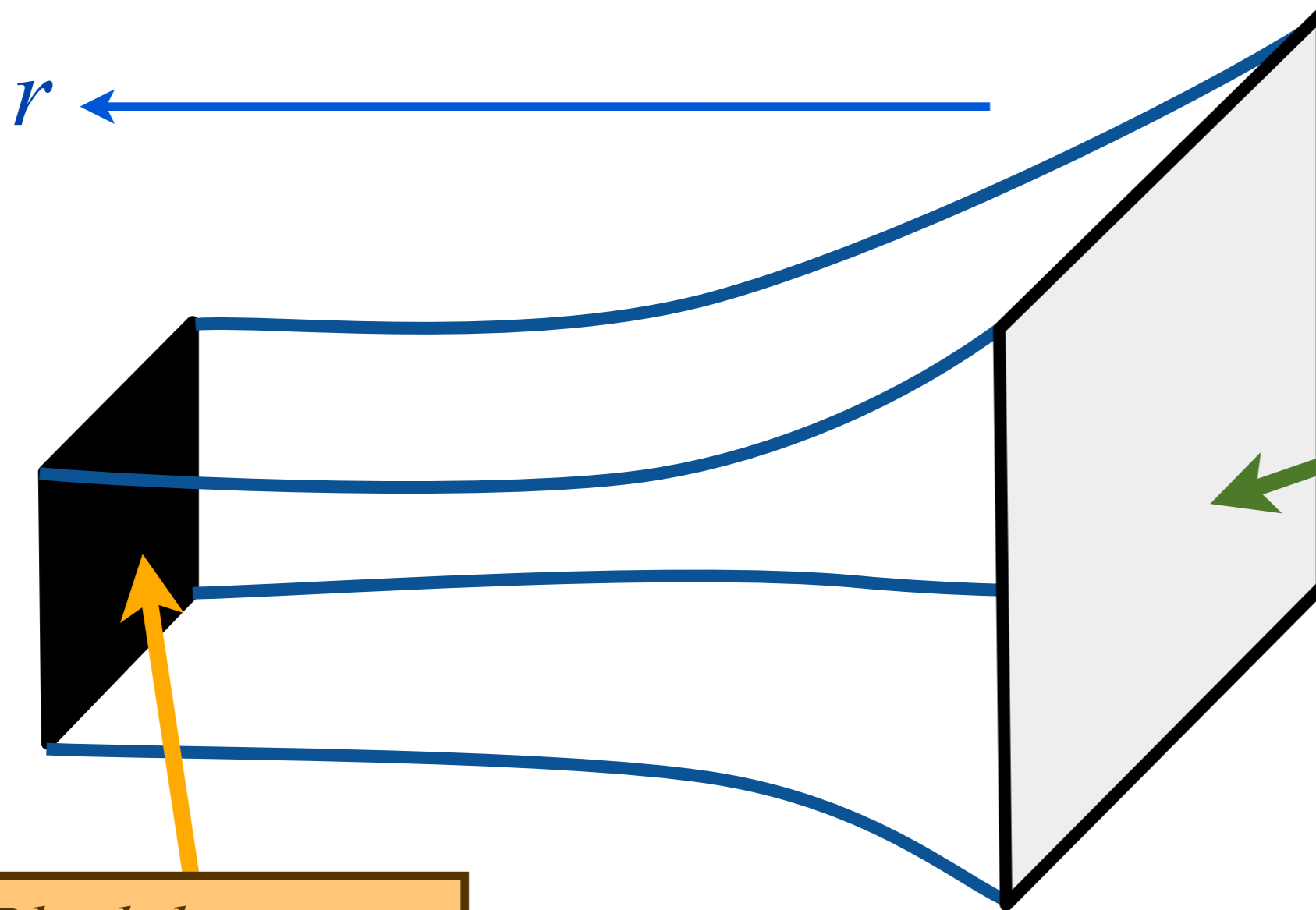
A 2+1 dimensional system at its quantum critical point:
 $k_B T = \frac{3\hbar}{4\pi R}$

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$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1 - (r/R)^3$

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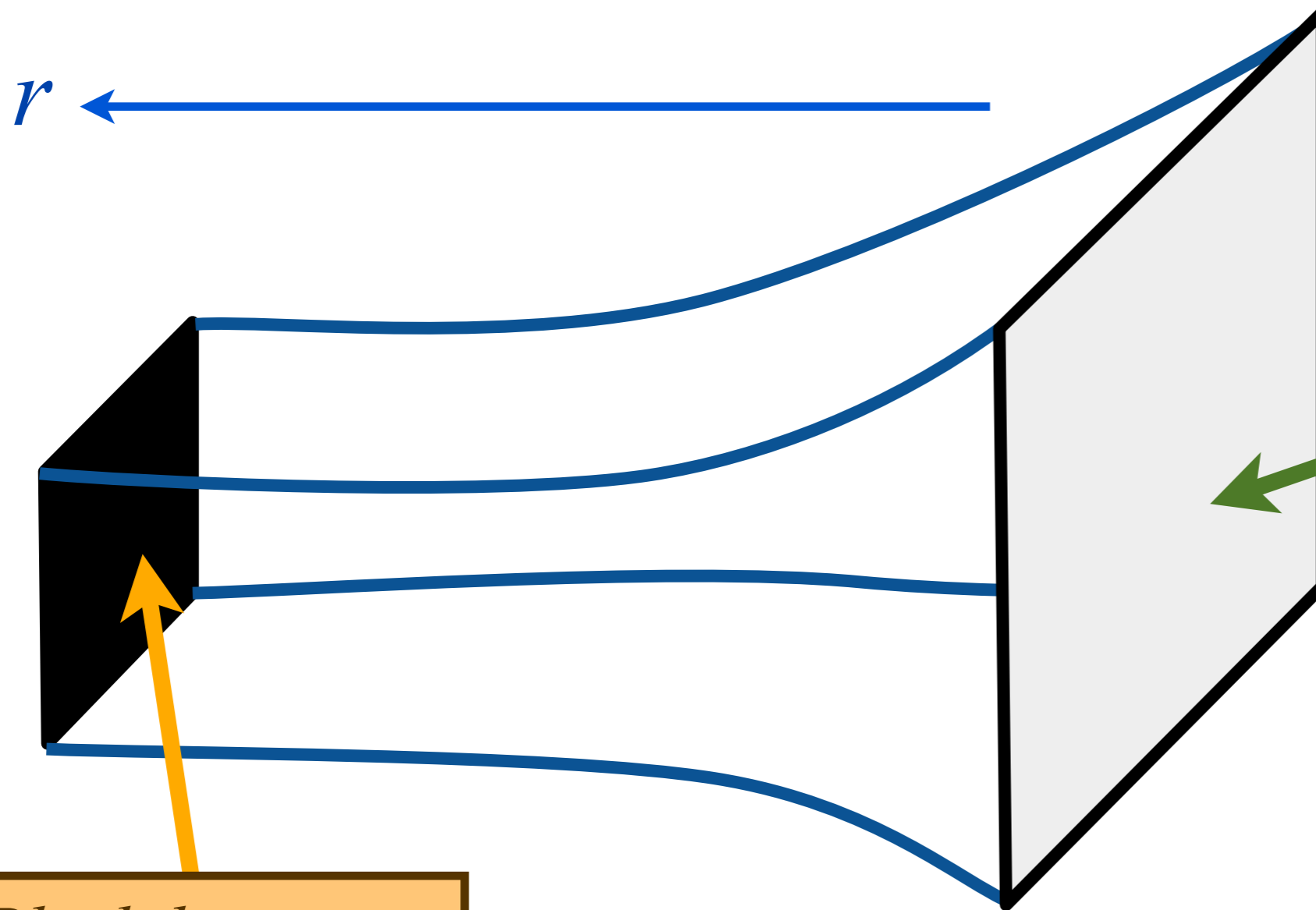
Black-brane at temperature of 2+1 dimensional quantum critical system

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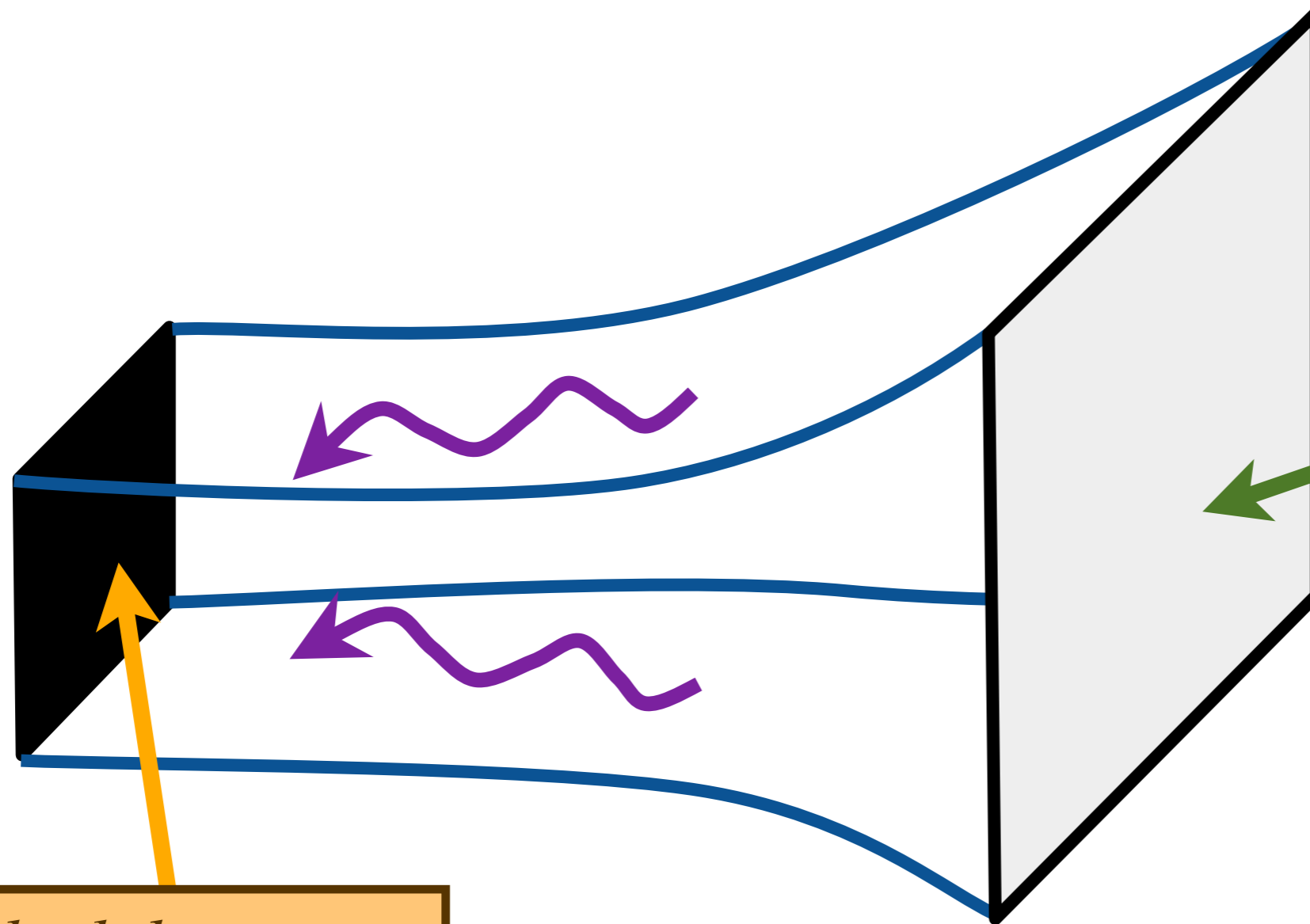


A 2+1 dimensional system at its quantum critical point:
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Black-brane at temperature of 2+1 dimensional quantum critical system

Beckenstein-Hawking entropy of black brane = entropy of CFT3

AdS₄-Schwarzschild black-brane



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Friction of quantum criticality = waves falling into black brane

AdS/CFT correspondence at non-zero temperatures

At non-zero temperatures, we consider a Euclidean metric with a horizon at $r = R$:

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} + f(r)d\tau^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1 - (r/R)^3$; note $f(R) = 0$. In the near horizon region we define $z = R - r$ and write this metric as

$$ds^2 = \left(\frac{L}{R}\right)^2 \left[\frac{dz^2}{|f'(R)|z} + |f'(R)|z d\tau^2 + dx^2 + dy^2 \right]$$

Now we introduce co-ordinates $\rho = 2\sqrt{z/|f'(R)|}$ and $\theta = 2\pi T\tau$, and then the metric is

$$ds^2 = \left(\frac{L}{R}\right)^2 \left[d\rho^2 + \left(\frac{f'(R)}{4\pi T}\right)^2 \rho^2 d\theta^2 + dx^2 + dy^2 \right]$$

Now if we choose the Hawking temperature

$$T = \frac{|f'(R)|}{4\pi}$$

then the spacetime is periodic under $\tau \rightarrow \tau + 1/T$, and there is no singularity at the horizon.

Computing conductivity at non-zero temperatures

In Euclidean signature, all the correspondences between the bulk and boundary correlators remain exactly the same as before. We need only add the additional requirement that the bulk solutions remain integrable at the horizon.

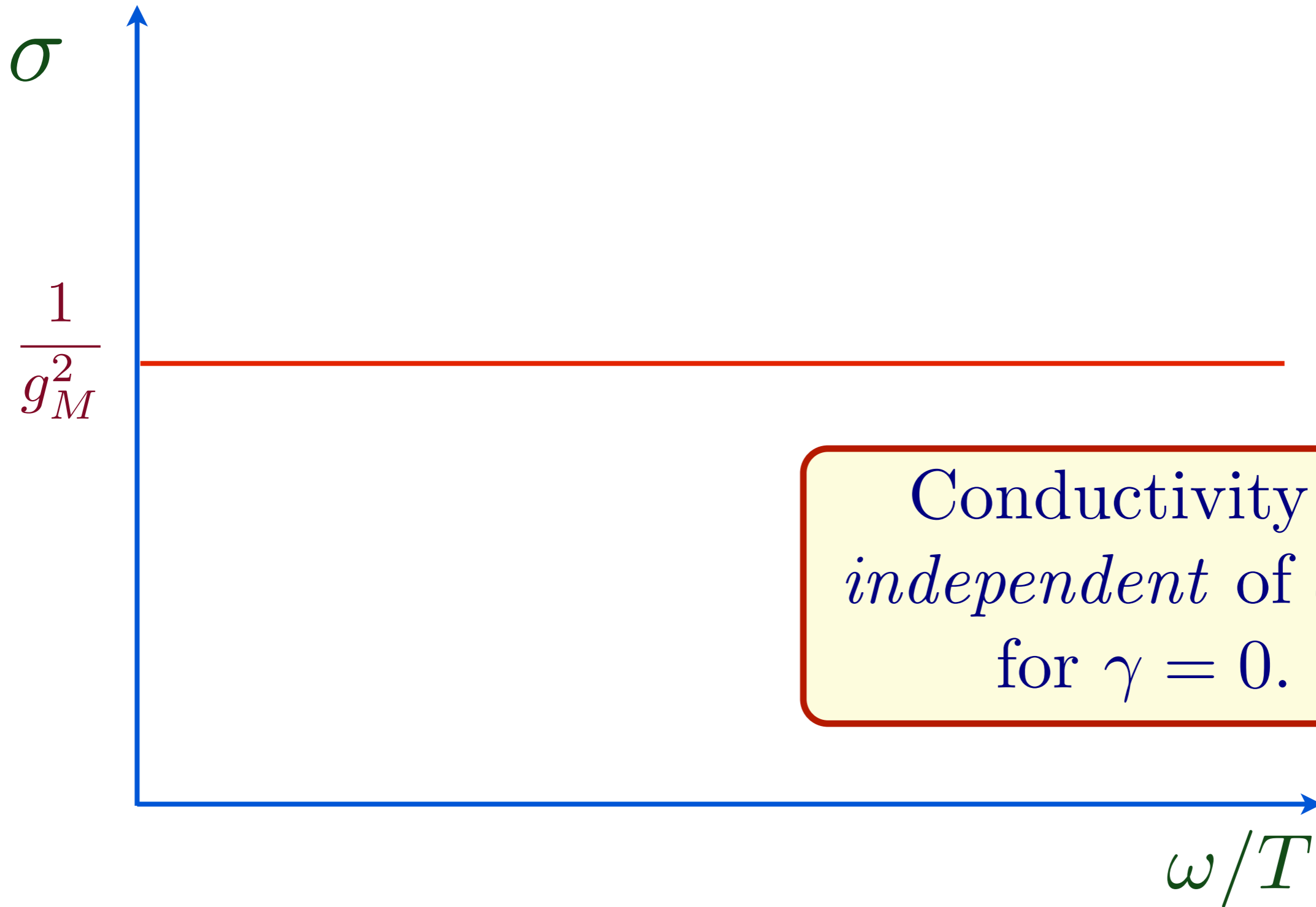
However, it is often convenient to work directly in real time and frequencies, and obtain the corresponding response functions directly, rather than by analytic continuation. It can be shown that the process of analytic continuation translates into the requirement of *in-going waves* at the horizon. The only other change in the equations is due to the change in the metric from AdS_4 to AdS_4 -Schwarzschild, via the factor $f(r)$.

In terms of the co-ordinate $u = r/R$, the equation for $A_x(u)$ in the presence of a probe oscillating at frequency ω is

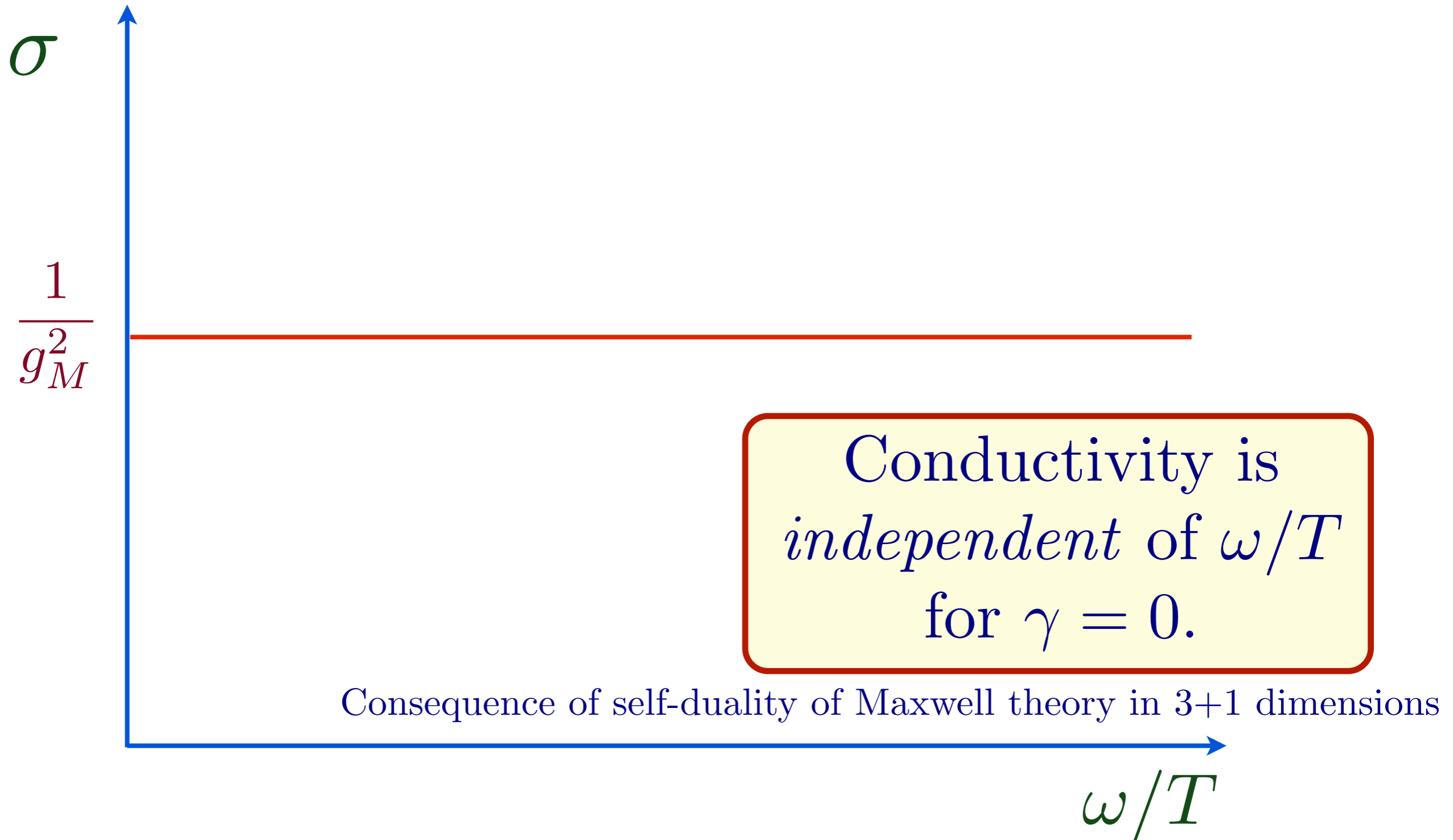
$$A_x'' + \frac{f'(3 - 2u^2\gamma f'') - 2u\gamma f(2f'' + uf''')}{f(3 - 2u^2\gamma f'')} A_x' + \frac{L^4 \omega^2}{R^2 f^2} A_x = 0,$$

where the primes are derivatives w.r.t u . Solution of this equation, subject to the boundary conditions discussed earlier yields the conductivity.

AdS4 theory of electrical transport in a strongly interacting CFT3 for $T > 0$

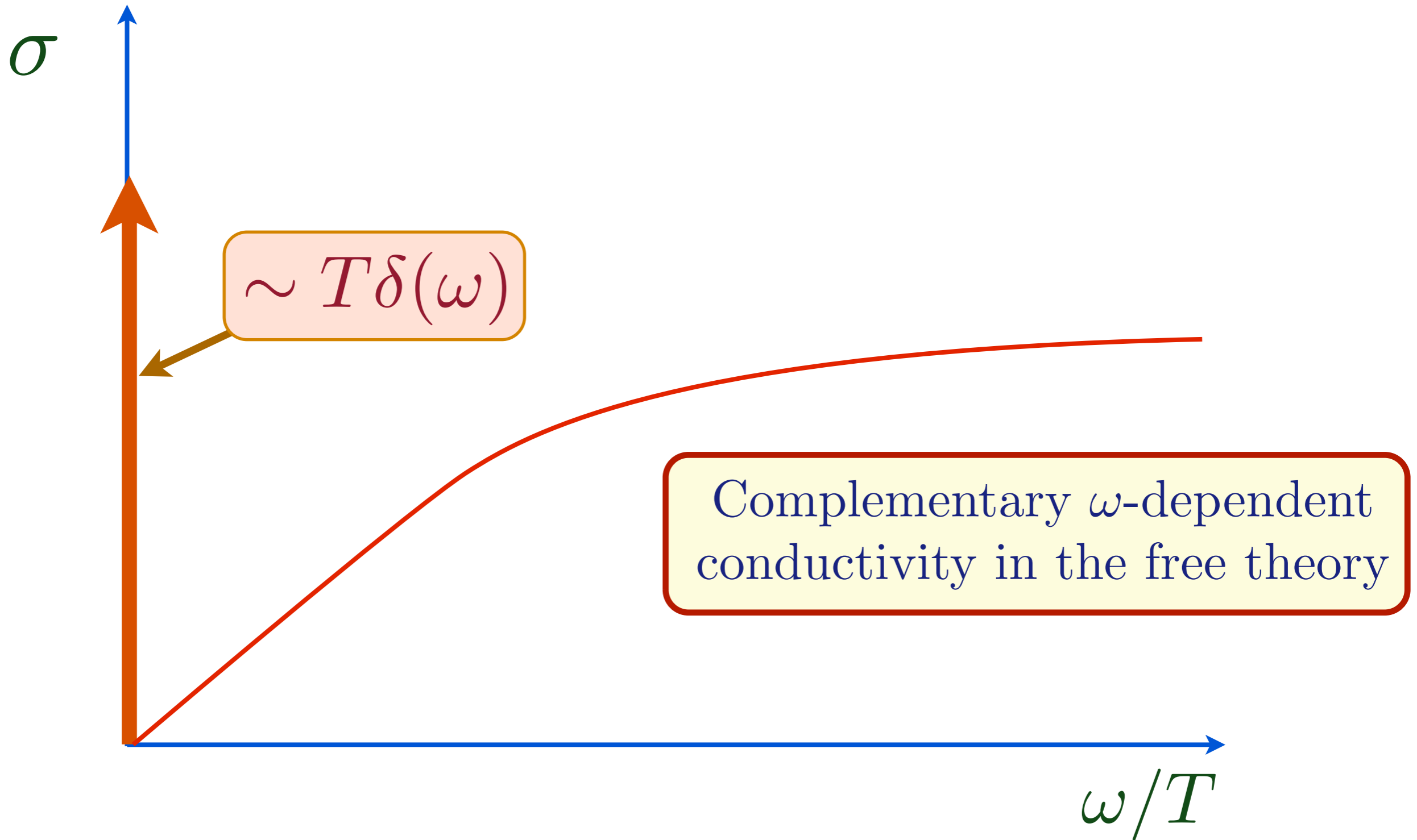


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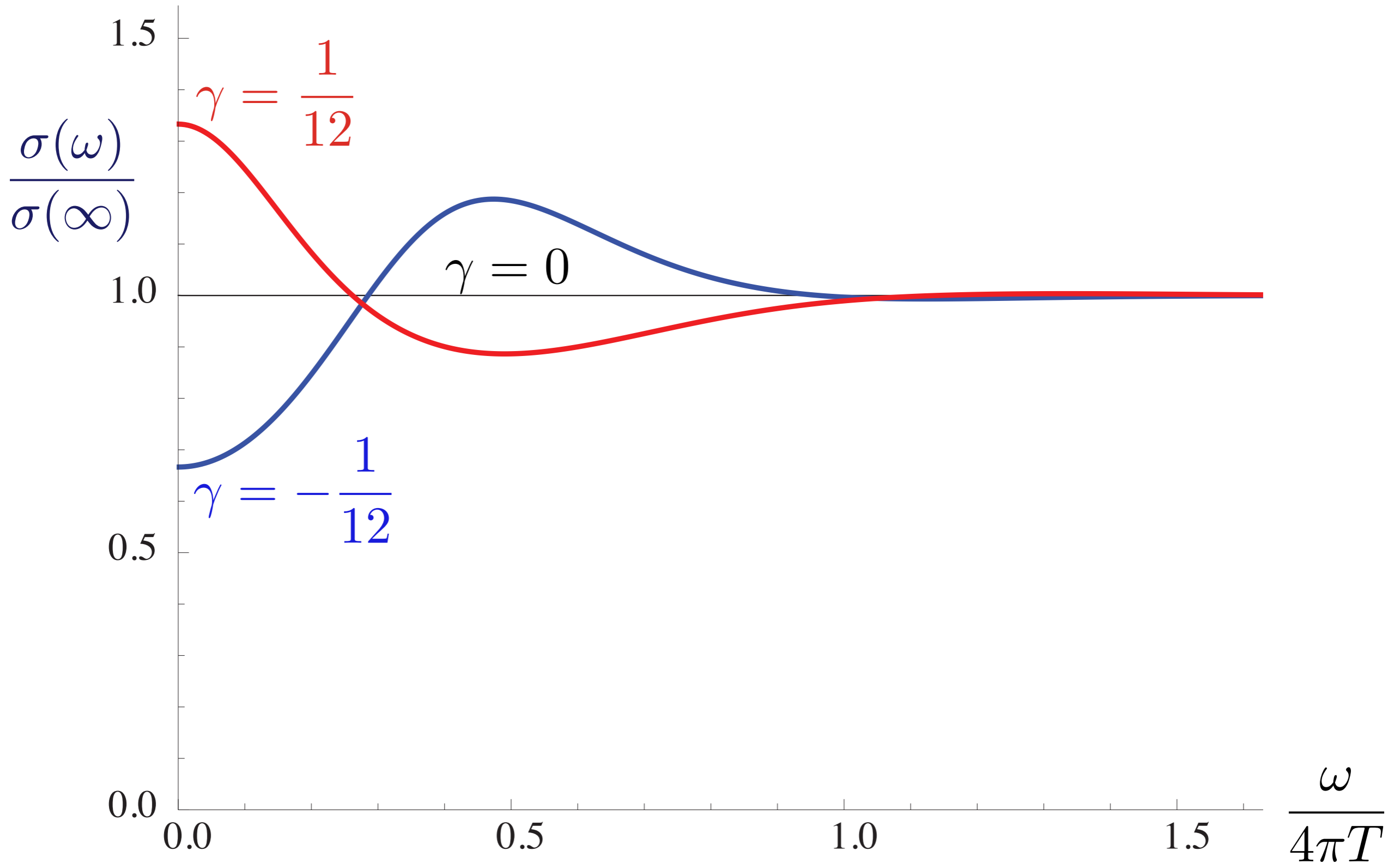


C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

Electrical transport in a free CFT3 for $T > 0$

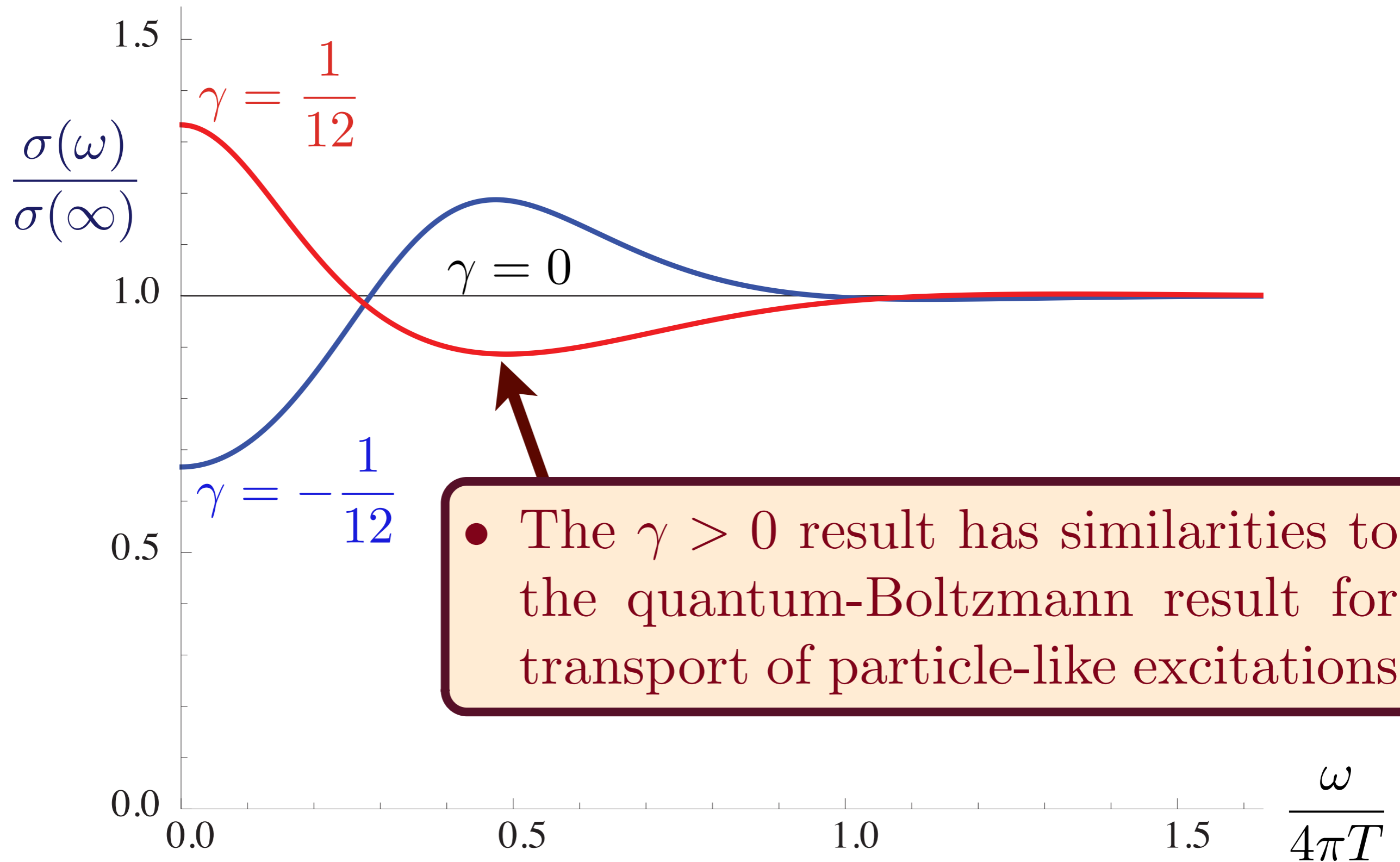


AdS₄ theory of “nearly perfect fluids”



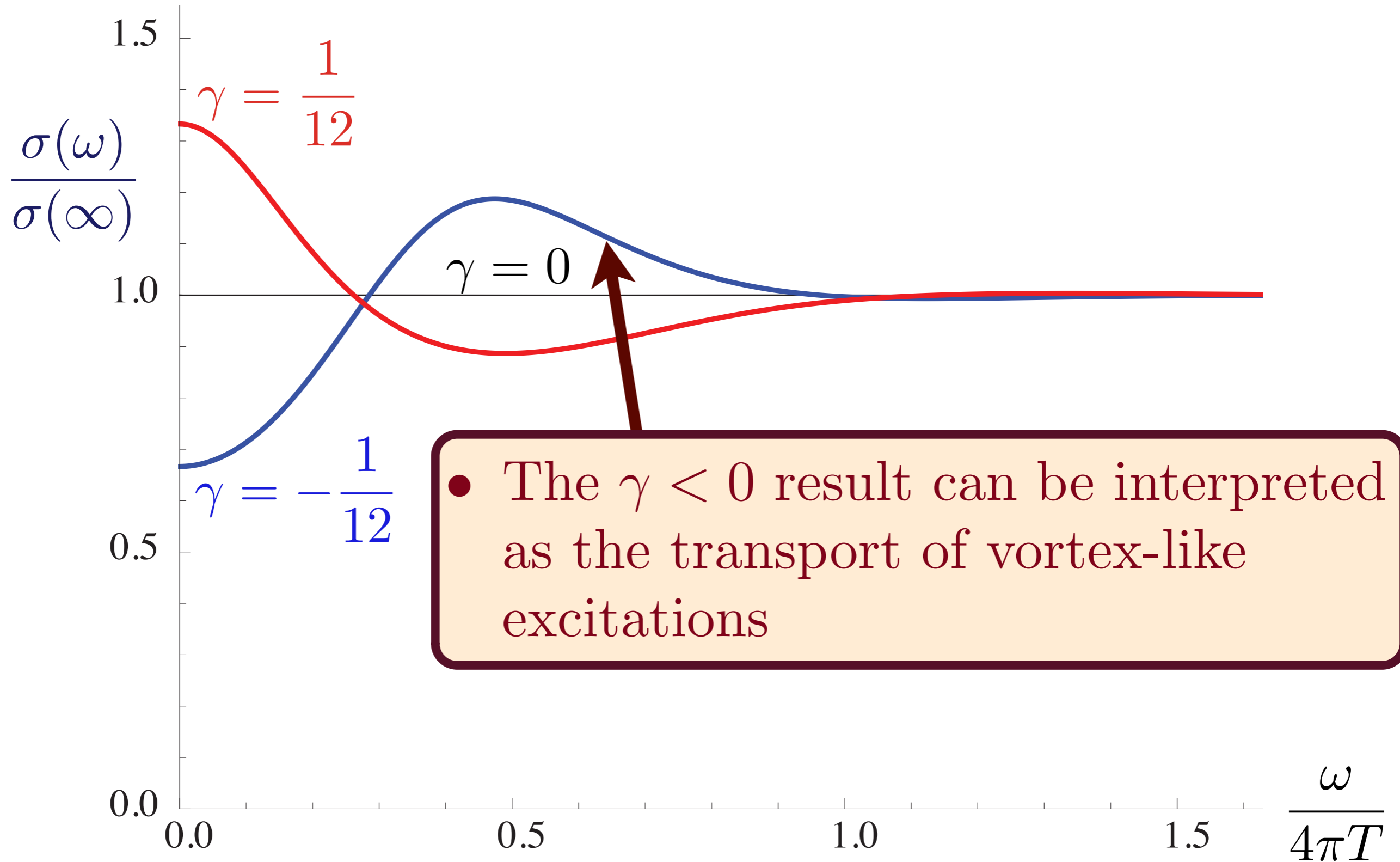
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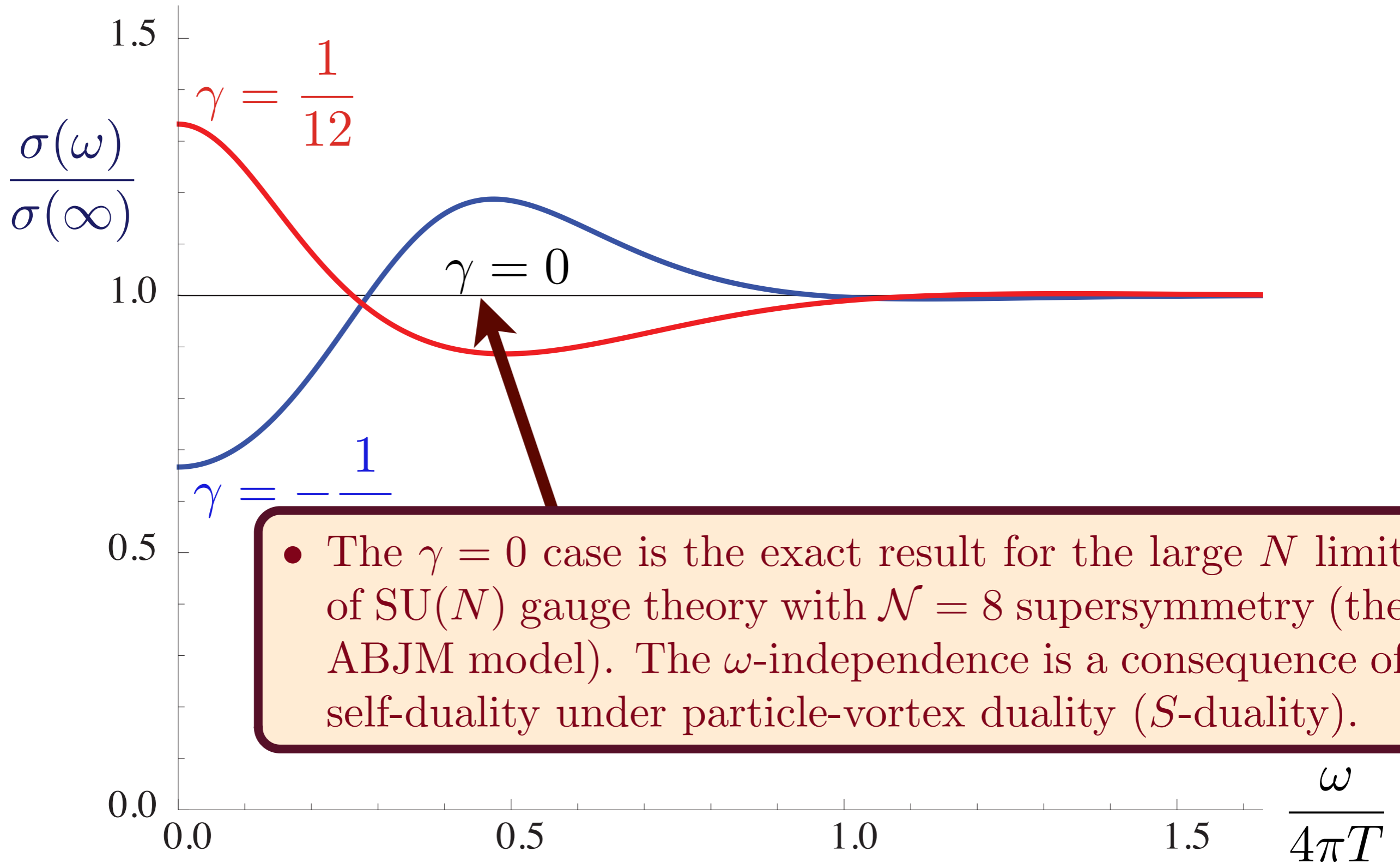
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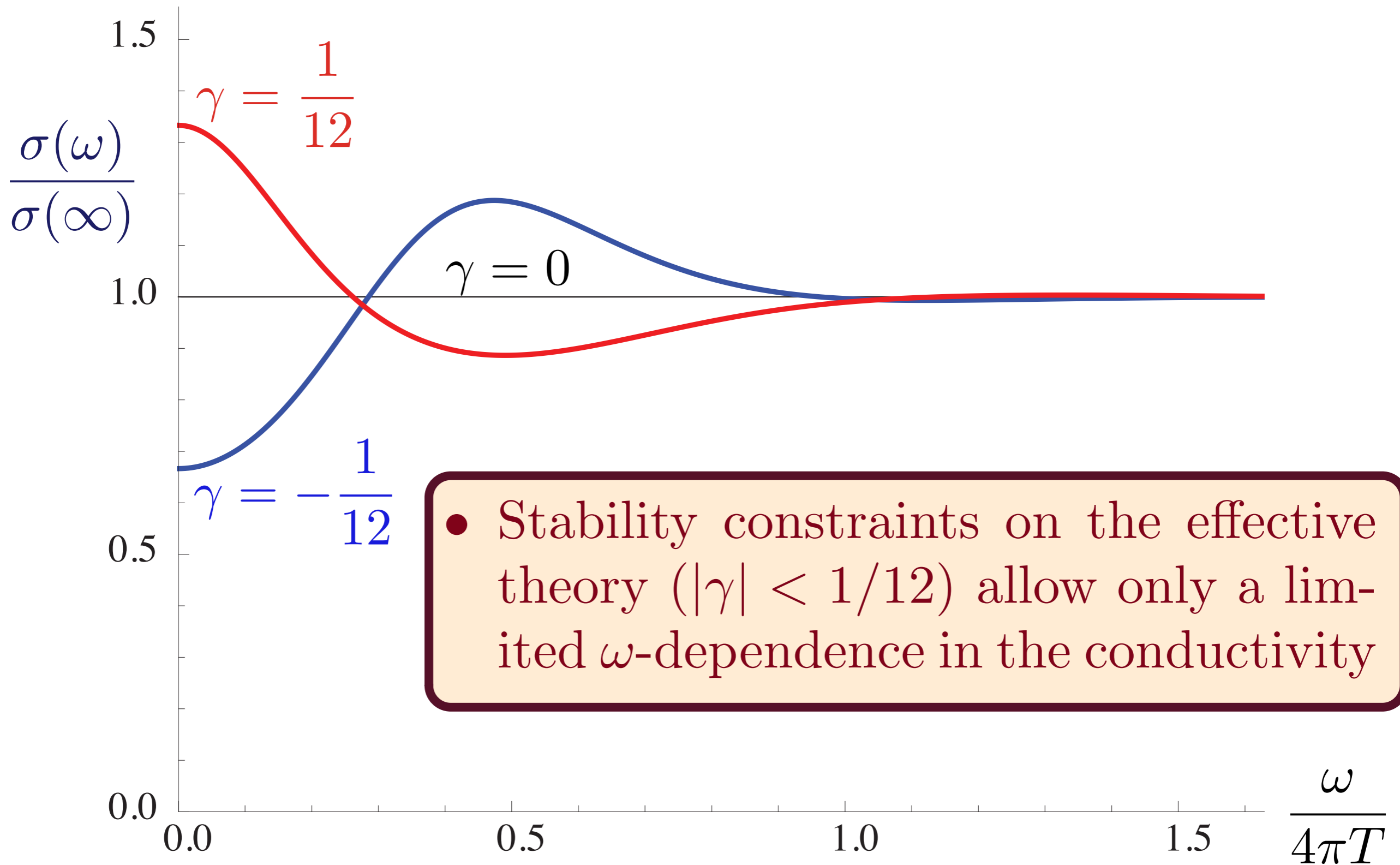
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Traditional CMT

● Identify quasiparticles and their dispersions

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- Solve Einstein-Maxwell-... equations, allowing for a horizon at non-zero temperatures.

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Conformal quantum matter

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- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points
- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.
- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Title: Holographic Superfluids and the Dynamics of Symmetry Breaking

Authors: [M. J. Bhaseen](#), [Jerome P. Gauntlett](#), [B. D. Simons](#), [Julian Sonner](#), [Toby Wiseman](#)

(Submitted on 17 Jul 2012)

Abstract: We explore the far from equilibrium response of a holographic superfluid using the AdS/CFT correspondence. We establish the dynamical phase diagram corresponding to quantum quenches of the order parameter source field. We find three distinct regimes of behaviour that are related to the spectrum of black hole quasi-normal modes. These correspond to damped oscillations of the order parameter, and over-damped approaches to the superfluid and normal states. The presence of three regimes, which includes an emergent dynamical temperature scale, is argued to be a generic feature of time-reversal invariant systems that display continuous symmetry breaking.

Comments: 6 pages, 4 figures

Subjects: High Energy Physics - Theory (hep-th); Strongly Correlated Electrons (cond-mat.str-el)

Cite as: [arXiv:1207.4194v1](#) [hep-th]

“Complex entangled” states of
quantum matter,
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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge Q (the “electron density”) in spatial dimension $d > 1$.

Compressible quantum matter

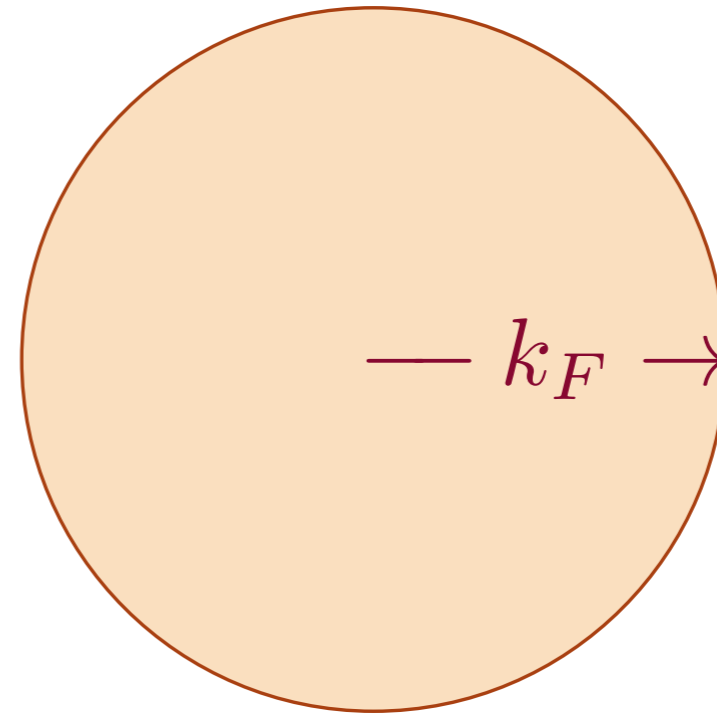
- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge Q (the “electron density”) in spatial dimension $d > 1$.
- Describe zero temperature phases where $d\langle Q \rangle / d\mu \neq 0$, where μ (the “chemical potential”) which changes the Hamiltonian, H , to $H - \mu Q$.

The only compressible phase of traditional condensed matter physics which does not break the translational or $U(1)$ symmetries is the Landau Fermi liquid

The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

+ 4 Fermi terms

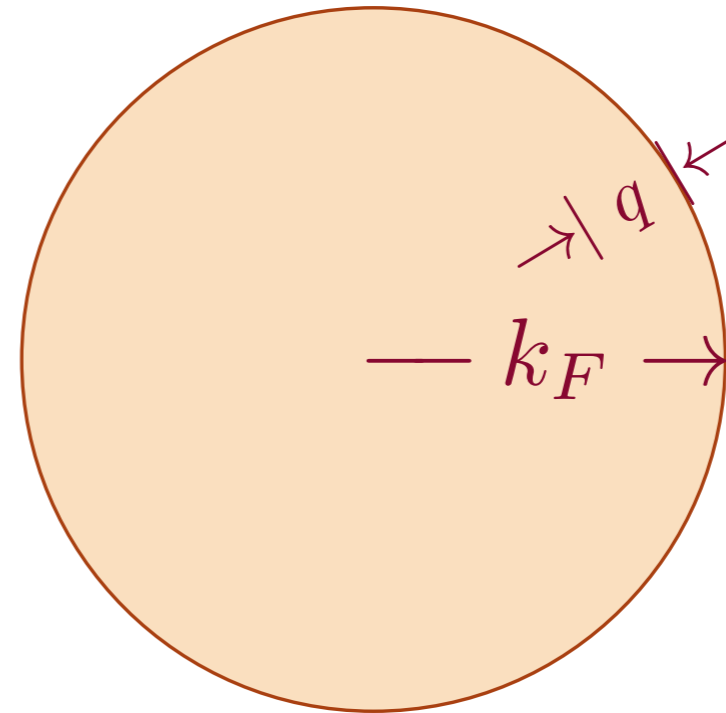


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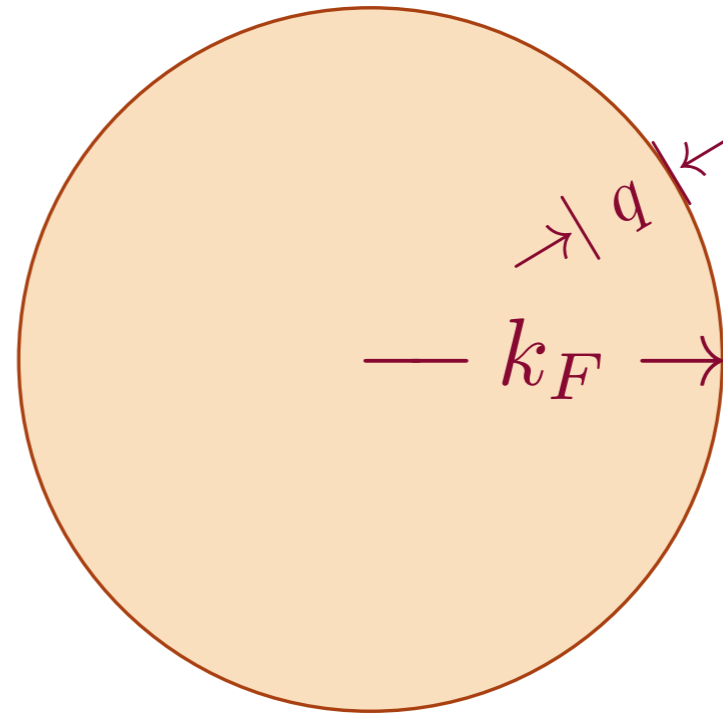


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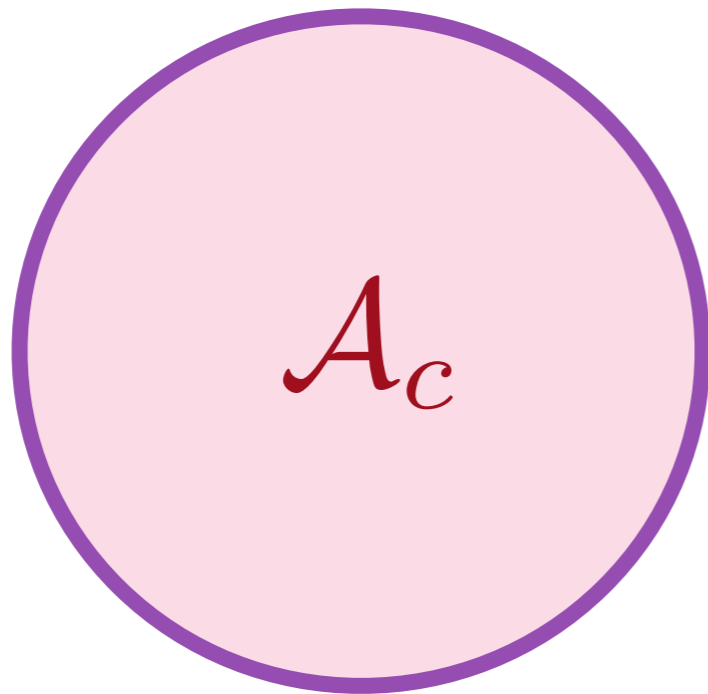


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

Consider a model of interacting bosons, b , whose density is $Q = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

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$$Q = b^\dagger b$$

$$A_c = \langle Q \rangle$$

S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review B* **72**, 024534 (2005)

P. Coleman, I. Paul, and J. Rech, *Physical Review B* **72**, 094430 (2005)

Fermi liquid

We have bosons, b , and spectator fermions, s , interacting via an attractive interaction u :

$$\mathcal{L} = b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) b + s^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_s} - \mu_s \right) s - u b^\dagger b s^\dagger s$$

This theory has two conserved U(1) charges

$$Q = b^\dagger b \quad , \quad Q_s = s^\dagger s$$

We can decouple the quartic term by a *fermonic* field, c , and write

$$\mathcal{L} = b^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) b + s^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_s} - \mu_s \right) s + \frac{1}{u} c^\dagger c - c^\dagger b s - s^\dagger b^\dagger c$$

Depending upon parameters, one or both of the f or s fermions can acquire Fermi surfaces. Naturally the b bosons cannot have Fermi surfaces, although they may condense. *Provided* the b don't condense, two Luttinger constraints, associated with the 2 U(1) charges, can be established on the areas enclosed by the Fermi surfaces:

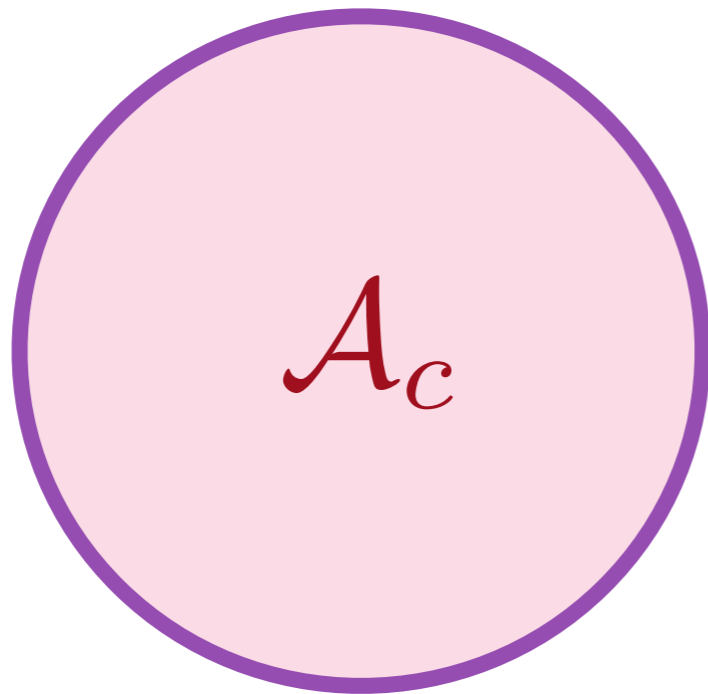
$$\langle Q \rangle = \mathcal{A}_c \quad , \quad \langle Q_s \rangle = \mathcal{A}_c + \mathcal{A}_s$$

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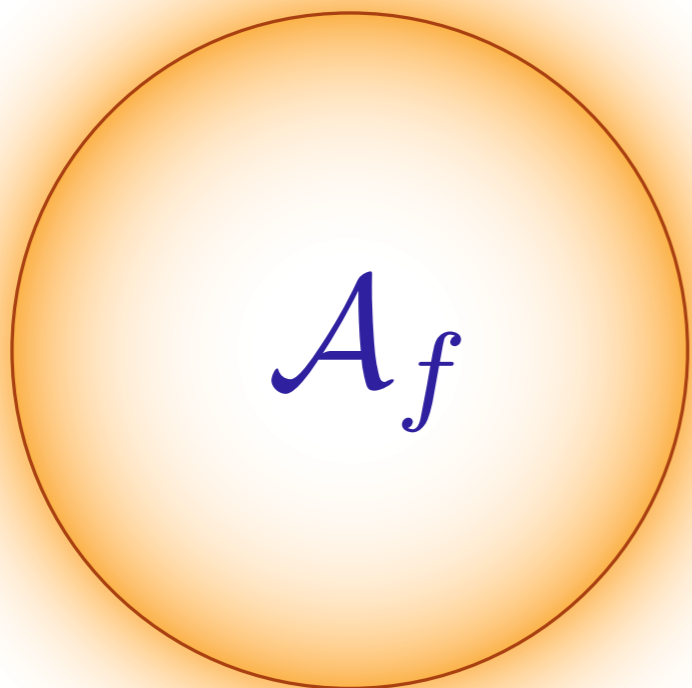
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- **NFL**, the non-Fermi liquid *Bose metal*. The boson fractionalizes into (say) 2 fermions, f_1 and f_2 , each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are “*hidden*”.



$$Q = b^\dagger b$$

$$A_f = \langle Q \rangle$$

O. I. Motrunich and M. P.A. Fisher,
Physical Review B **75**, 235116 (2007)

L. Huijse and S. Sachdev,
Physical Review D **84**, 026001 (2011)

Bose Metal

For suitable interactions, we can have the boson, b , *fractionalize* into two fermions $f_{1,2}$:

$$b \rightarrow f_1 f_2$$

This implies the effective theory for $f_{1,2}$ is invariant under the U(1) gauge transformation

$$f_1 \rightarrow f_1 e^{i\theta(\mathbf{x},\tau)} \quad , \quad f_2 \rightarrow f_2 e^{-i\theta(\mathbf{x},\tau)}$$

Consequently, the effective theory of the Bose metal has an emergent gauge field A_μ and has the structure

$$\mathcal{L} = f_1^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_1 + f_2^\dagger \left(\partial_\tau + iA_\tau - \frac{(\nabla + i\mathbf{A})^2}{2m} - \mu \right) f_2$$

The gauge-dependent $f_{1,2}$ Green's functions have Fermi surfaces obeying $\mathcal{A}_f = \langle Q \rangle$. However, these Fermi surfaces are not directly observable because it is gauge-dependent. Nevertheless, gauge-independent operators, such as b or $b^\dagger b$, will exhibit *Friedel oscillations* associated with fermions scattering across these hidden Fermi surfaces.

Bose Metal

The integrable $S = 1/2$ Heisenberg chain

$$H = J \sum_i \vec{S}_i \cdot \vec{S}_j$$

provides a nice example of a Bose metal. Spin correlations of this model decay as

$$\langle \vec{S}_i \cdot \vec{S}_j \rangle \sim \frac{(-1)^{i-j}}{|i-j|}$$

Writing $S_{+i} = b_i^\dagger$, we can also re-interpret this model as one of hard-core bosons with nearest-neighbor interactions.

However, let us write $\vec{S} = f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, where f_\uparrow and f_\downarrow are the ‘hidden’ gauge-charged fermions. The fermions obey the constraints

$$f_\alpha^\dagger f_\alpha = 1 \quad , \quad \varepsilon_{\alpha\beta} f_\alpha f_\beta = 0$$

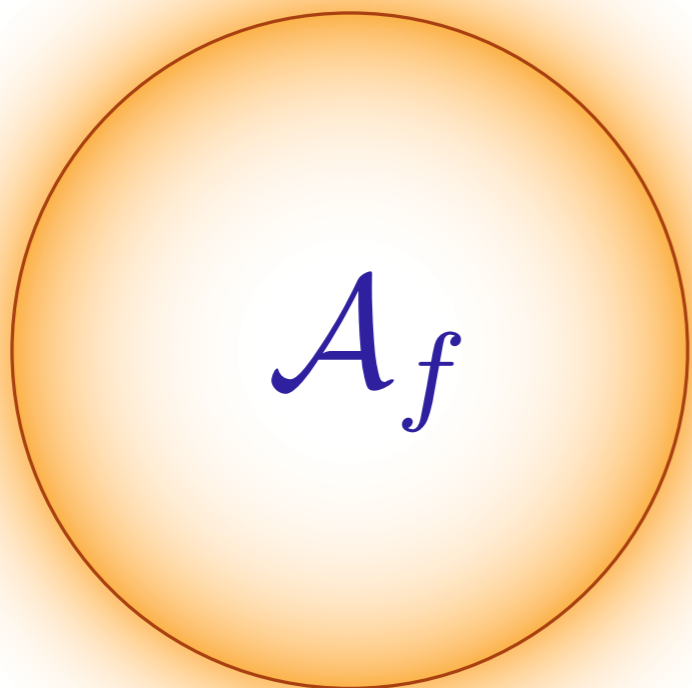
on each site. These constraints are imposed by a SU(2) gauge field.

Initially ignoring the gauge field, both fermions are at half-filling, and so have Fermi wavevectors at $k_F = \pm\pi/2$. The low-energy theory consists of excitations across these Fermi surfaces coupled to an emergent SU(2) gauge field.

In this interpretation, the oscillatory spin correlation of the antiferromagnet above is a *Friedel oscillation* of the hidden Fermi surfaces at wavevector $2k_F = \pi$.

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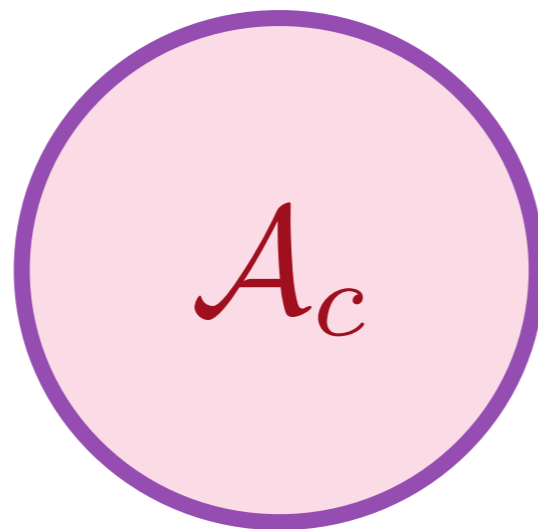
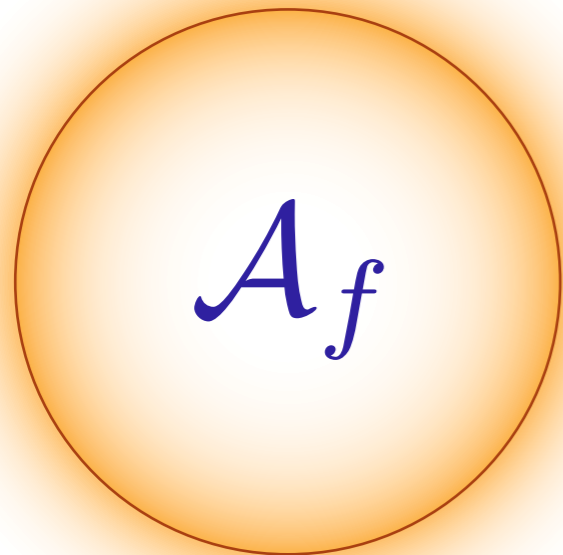
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T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)
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Fractionalized Fermi liquid

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$$\mathcal{L} = f_1^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_1 + f_2^\dagger \left(\partial_\tau + iA_\tau - \frac{(\nabla + i\mathbf{A})^2}{2m} - \mu \right) f_2 \\ + s^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m_s} - \mu_s \right) s + \frac{1}{u} c^\dagger c - c^\dagger f_1 f_2 s - s^\dagger f_2^\dagger f_1^\dagger c$$

Now the Luttinger constraints are

$$\langle Q \rangle = \mathcal{A}_c + \mathcal{A}_f \quad , \quad \langle Q_s \rangle = \mathcal{A}_c + \mathcal{A}_s$$

where recall that $Q = b^\dagger b$ and $Q_s = s^\dagger s$.

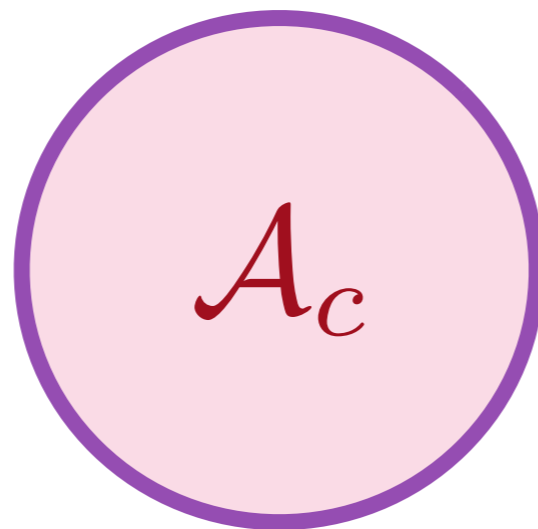
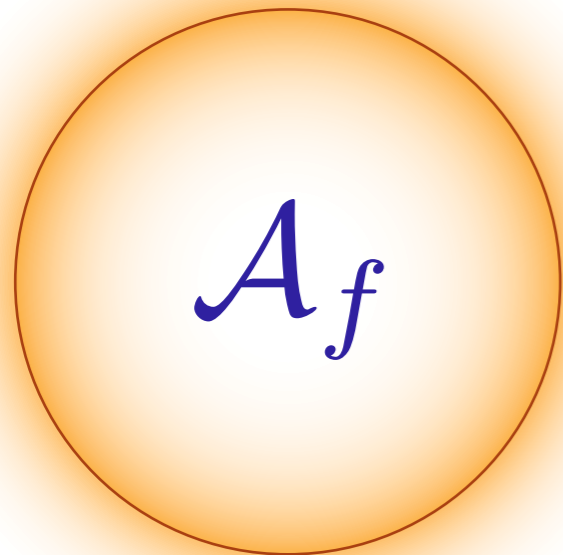
Analogous phases arise in Kondo lattice models, where c and s are conduction electron Fermi surfaces, and $f_{1,2}$ are spinon Fermi surfaces associated with the local moments.

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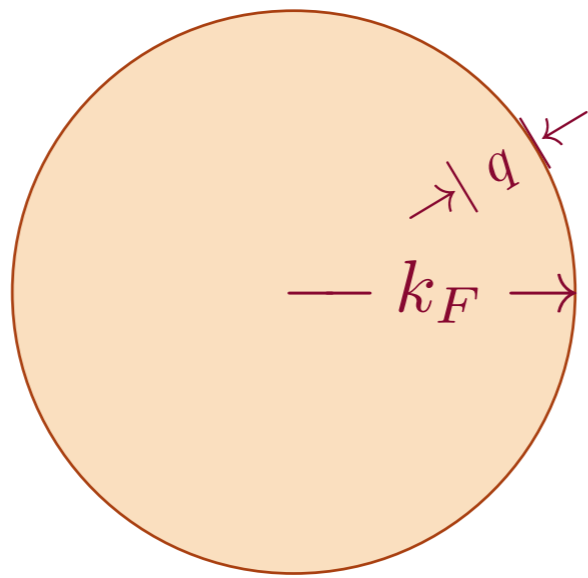


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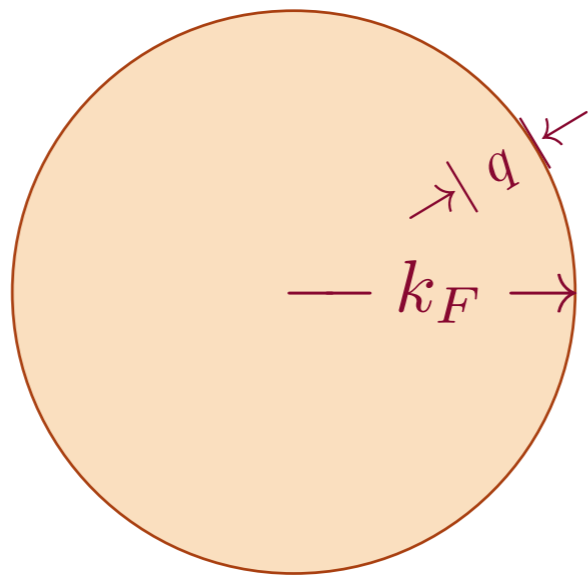
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FL Fermi liquid



- $k_F^d \sim Q$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

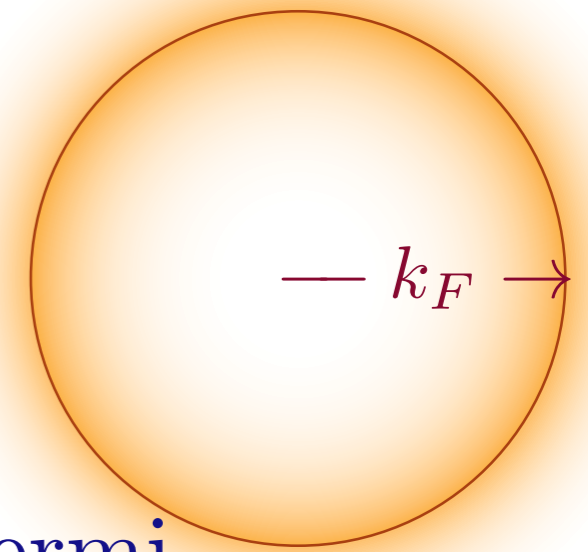
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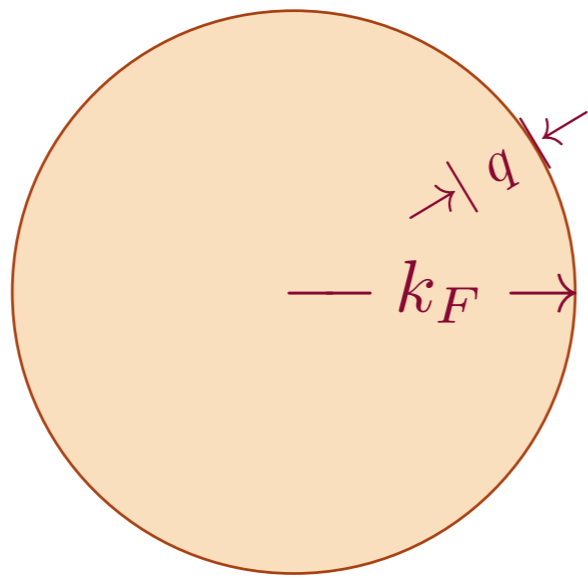
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NFL Bose metal



- Hidden Fermi surface with $k_F^d \sim Q$.

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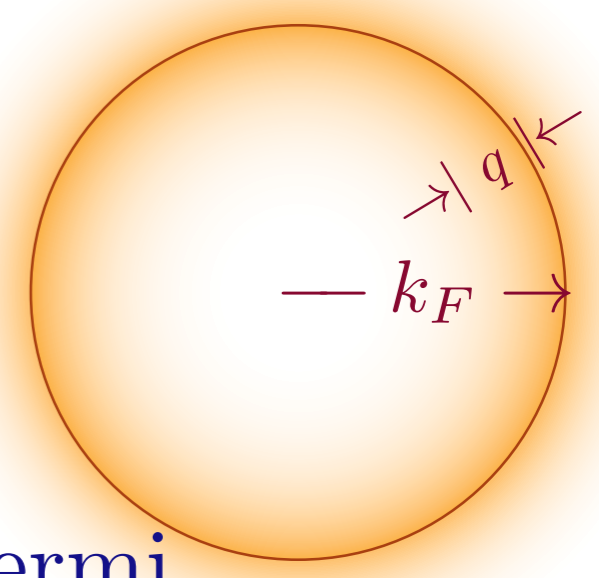


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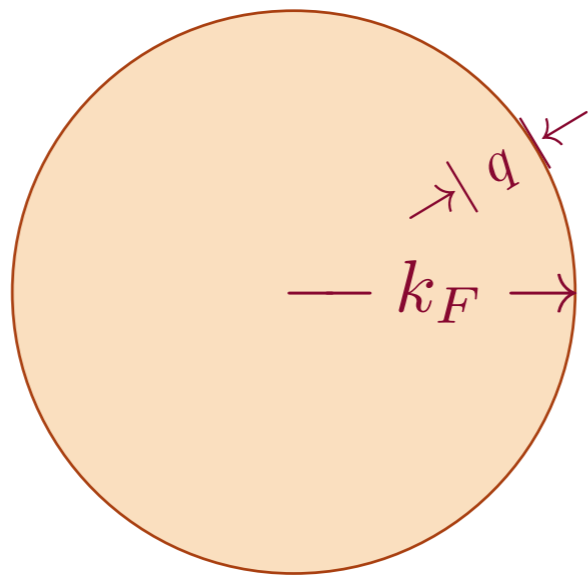


- Hidden Fermi surface with $k_F^d \sim Q$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

M. A. METLITSKI and S. SACHDEV,
Phys. Rev. B **82**, 075127 (2010)

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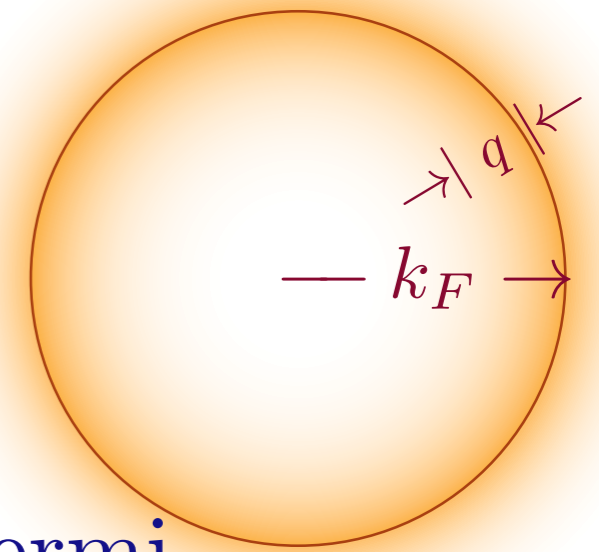


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NFL Bose metal

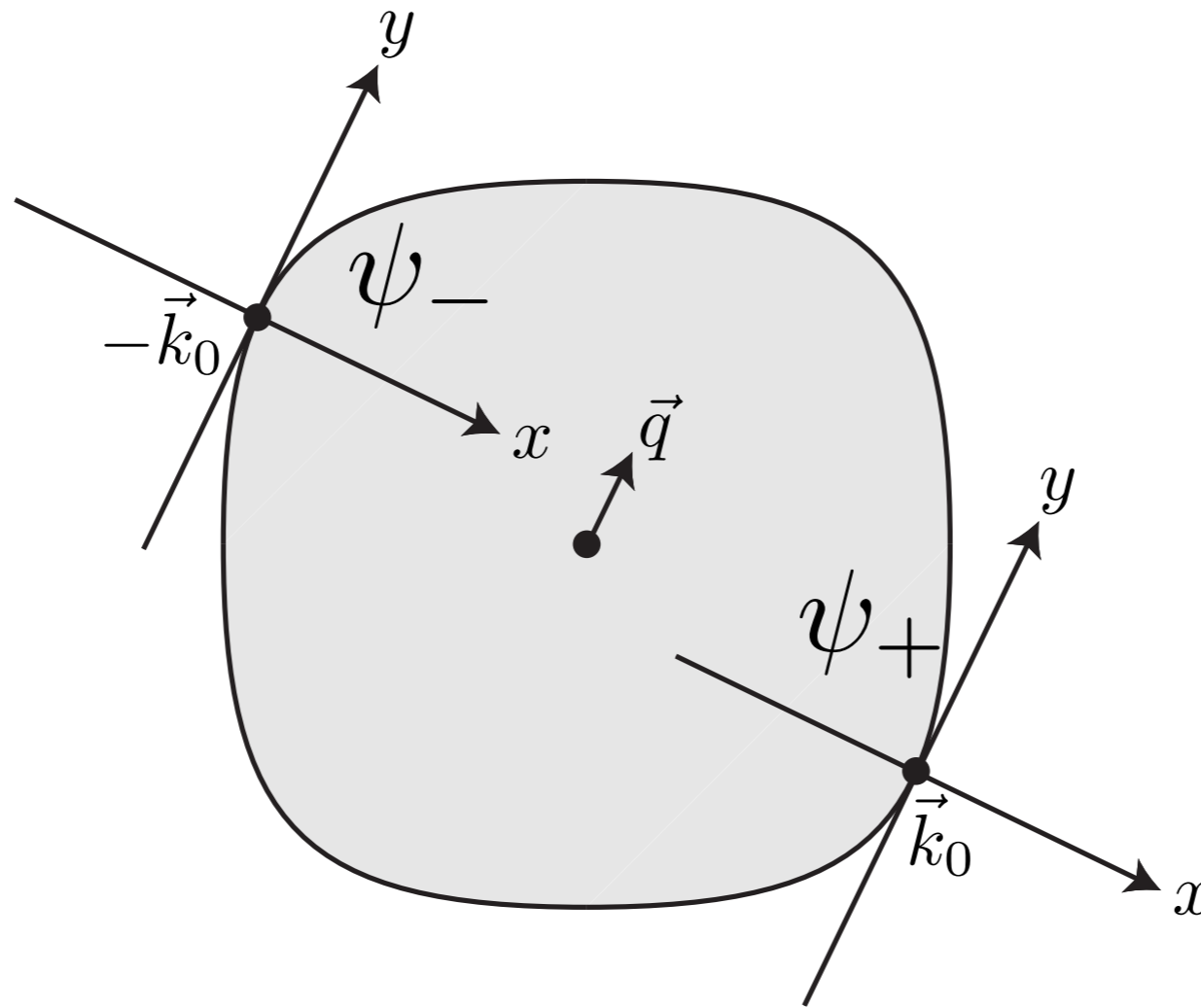


- Hidden Fermi surface with $k_F^d \sim Q$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

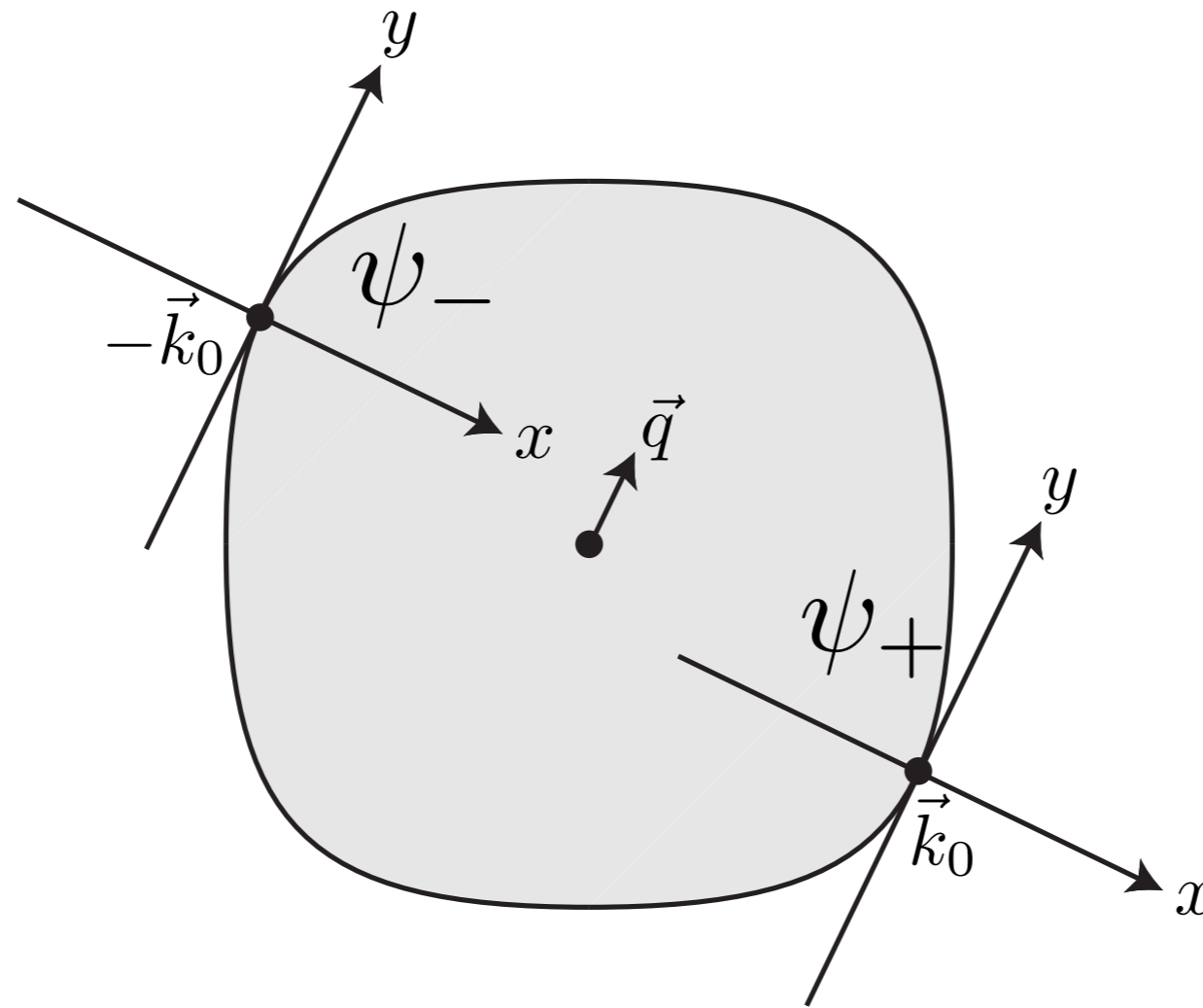
- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

Field theory of hidden Fermi surfaces



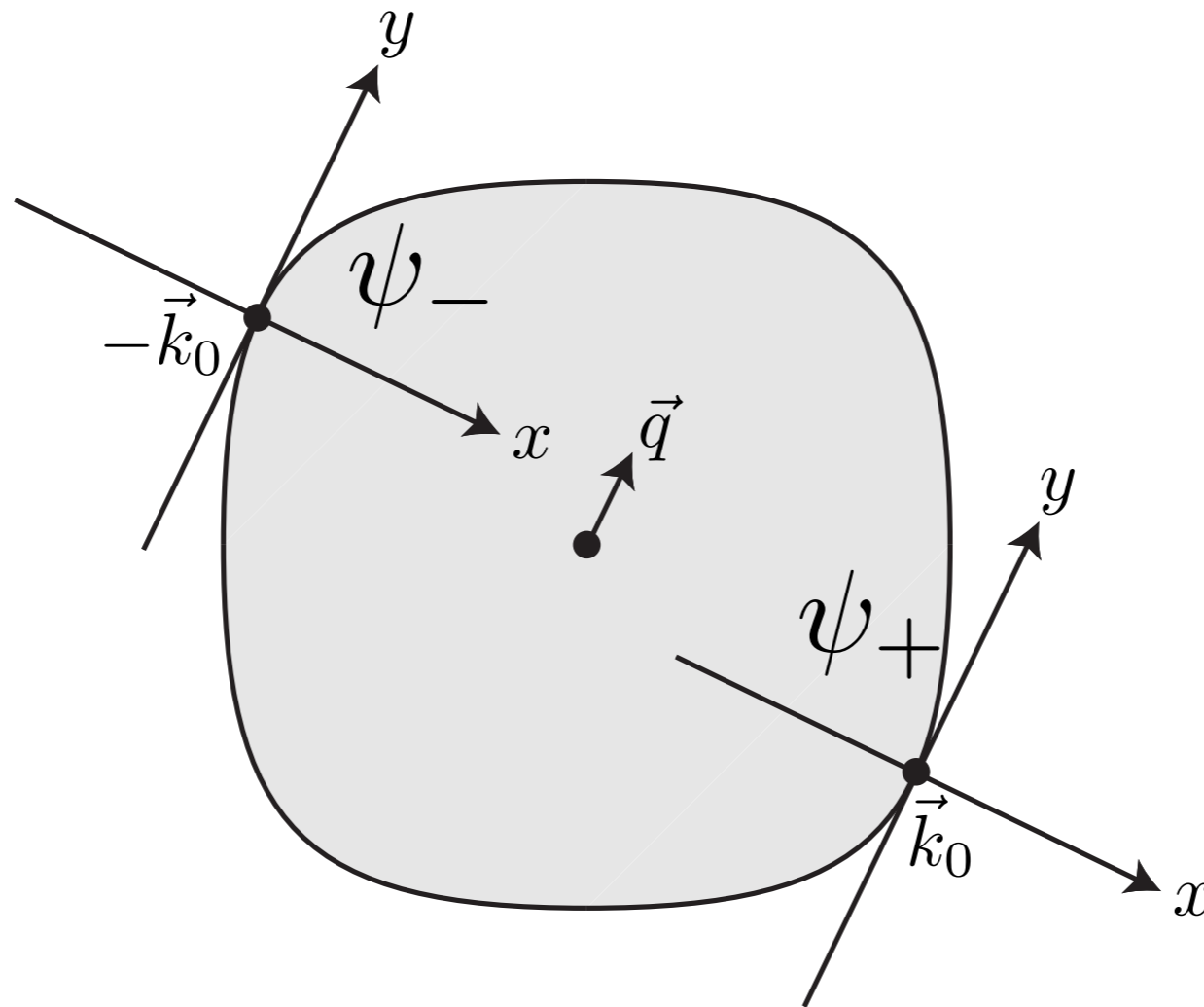
- \vec{A} fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.

Field theory of hidden Fermi surfaces



- \vec{A} fluctuation at wavevector \vec{q} couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$. In Landau gauge $\vec{A} = (a, 0)$.

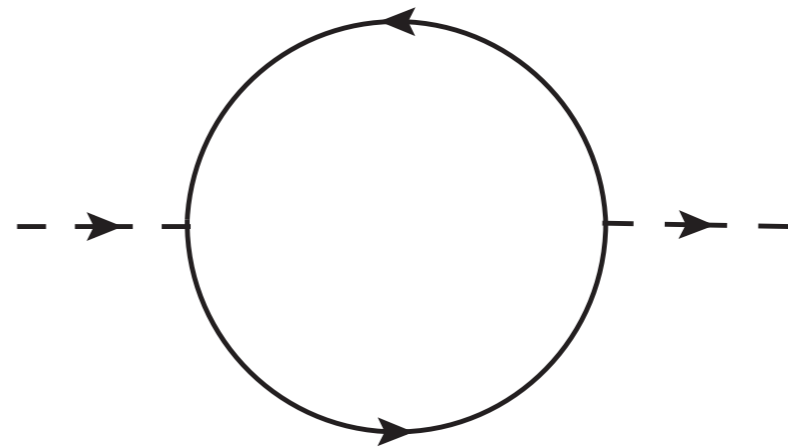
Field theory of hidden Fermi surfaces



$$\mathcal{L}[\psi_{\pm}, a] = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2$$

Field theory of hidden Fermi surfaces

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2$$



One loop a self-energy with N_f fermion flavors:

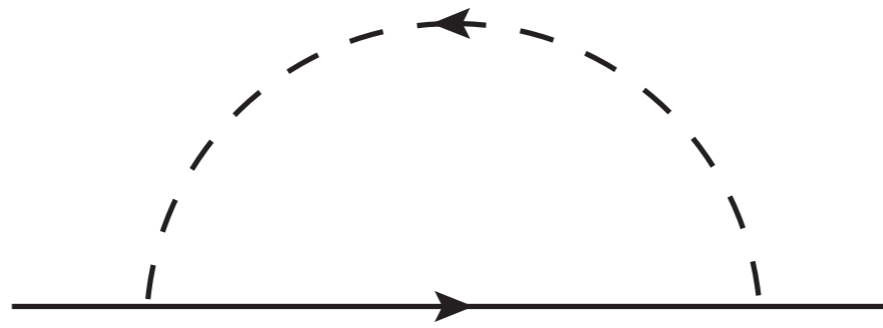
$$D(\vec{q}, \omega) = N_f \int \frac{d^2 k}{4\pi^2} \frac{d\Omega}{2\pi} \frac{1}{[-i(\Omega + \omega) + k_x + q_x + (k_y + q_y)^2] [-i\Omega - k_x + k_y^2]}$$

$$= \frac{N_f}{4\pi} \frac{|\omega|}{|q_y|}$$

Landau-damping

Field theory of hidden Fermi surfaces

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - a (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) + \frac{1}{2g^2} (\partial_y a)^2$$



Electron self-energy at order $1/N_f$:

$$\begin{aligned} \Sigma(\vec{k}, \Omega) &= -\frac{1}{N_f} \int \frac{d^2q}{4\pi^2} \frac{d\omega}{2\pi} \frac{1}{[-i(\omega + \Omega) + k_x + q_x + (k_y + q_y)^2] \left[\frac{q_y^2}{g^2} + \frac{|\omega|}{|q_y|} \right]} \\ &= -i \frac{2}{\sqrt{3}N_f} \left(\frac{g^2}{4\pi} \right)^{2/3} \text{sgn}(\Omega) |\Omega|^{2/3} \end{aligned}$$

Field theory of hidden Fermi surfaces

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2$$

Schematic form of a and fermion Green's functions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \frac{|\omega|}{|q_y|}}, \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i \text{sgn}(\omega) |\omega|^{2/3} / N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Field theory of hidden Fermi surfaces

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- \\ - a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2$$

Simple scaling argument for $z = 3/2$.

Field theory of hidden Fermi surfaces

$$\begin{aligned} \mathcal{L}_{\text{scaling}} = & \psi_+^\dagger (-i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (+i\partial_x - \partial_y^2) \psi_- \\ & - g a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + (\partial_y a)^2 \end{aligned}$$

Simple scaling argument for $z = 3/2$.

Field theory of hidden Fermi surfaces

$$\mathcal{L}_{\text{scaling}} = \psi_+^\dagger (-i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (+i\partial_x - \partial_y^2) \psi_- - g a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + (\partial_y a)^2$$

Simple scaling argument for $z = 3/2$.

Under the rescaling $x \rightarrow x/s$, $y \rightarrow y/s^{1/2}$, and $\tau \rightarrow \tau/s^z$, we find invariance provided

$$\phi \rightarrow \phi s^{(2z+1)/4}$$

$$\psi \rightarrow \psi s^{(2z+1)/4}$$

$$g \rightarrow g s^{(3-2z)/4}$$

So the action is invariant provided $z = 3/2$.

Field theory of hidden Fermi surfaces

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2$$

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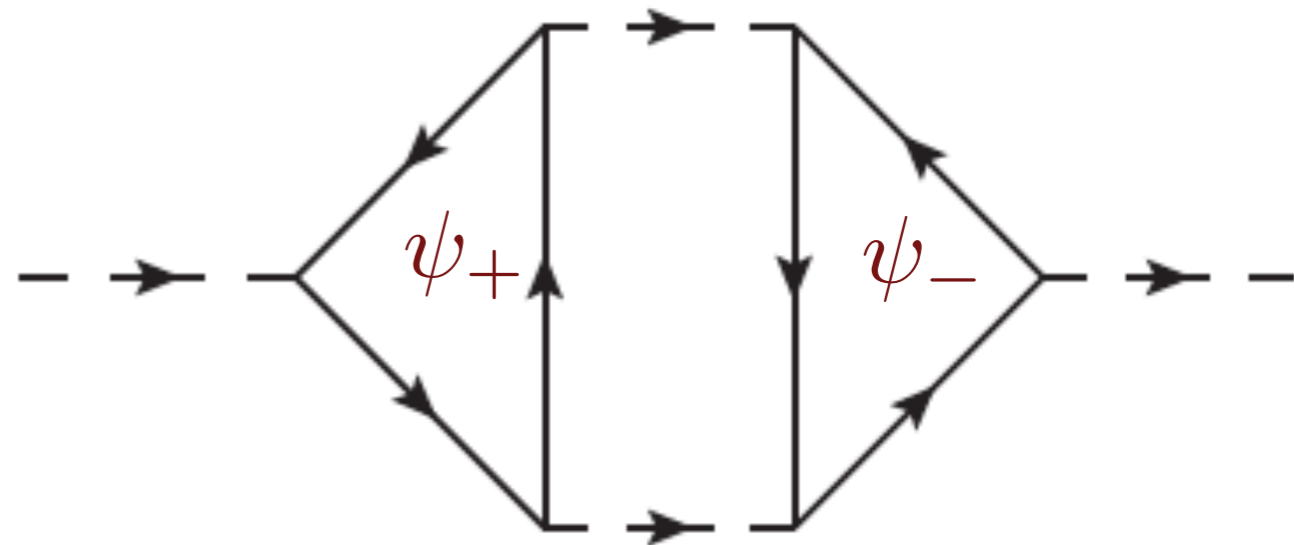
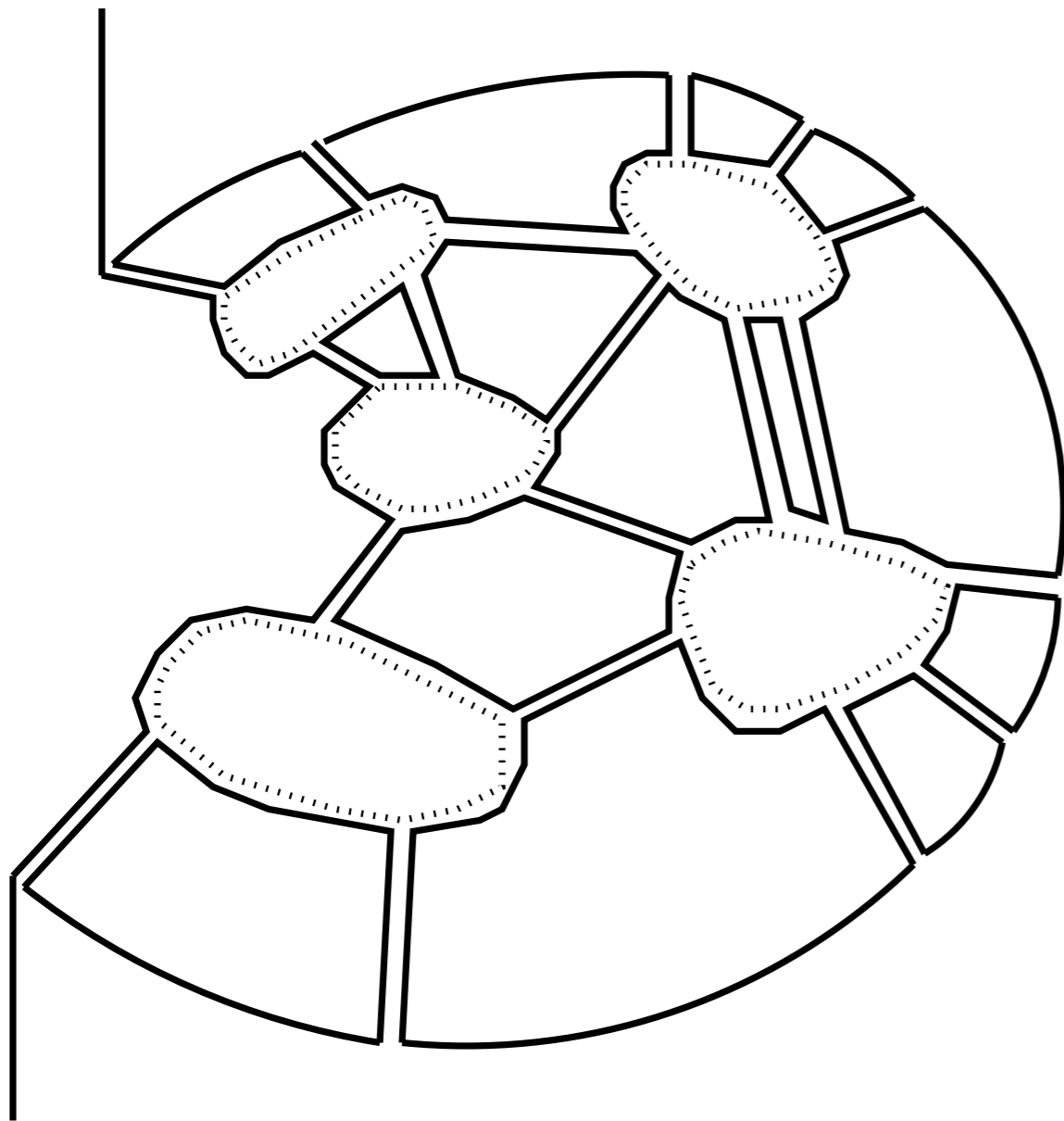
Strongly-coupled theory without quasiparticles.

Field theory of hidden Fermi surfaces

$$\mathcal{L} = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2$$

The $1/N_f$ expansion is *not* determined by counting fermion loops, because of infrared singularities created by the Fermi surface. The $|\omega|^{2/3}/N_f$ fermion self-energy leads to additional powers of N_f , and a breakdown in the loop expansion.

Computations in the $1/N$ expansion



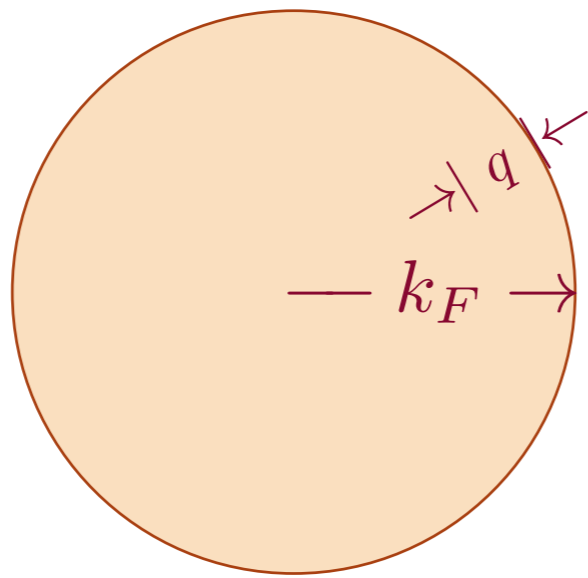
Graph mixing ψ_+ and ψ_- is $\mathcal{O}(N^{3/2})$ (instead of $\mathcal{O}(N)$), violating genus expansion

All planar graphs of ψ_+ alone are as important as the leading term

M. A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review B* **80**, 165102 (2009)

FL Fermi liquid

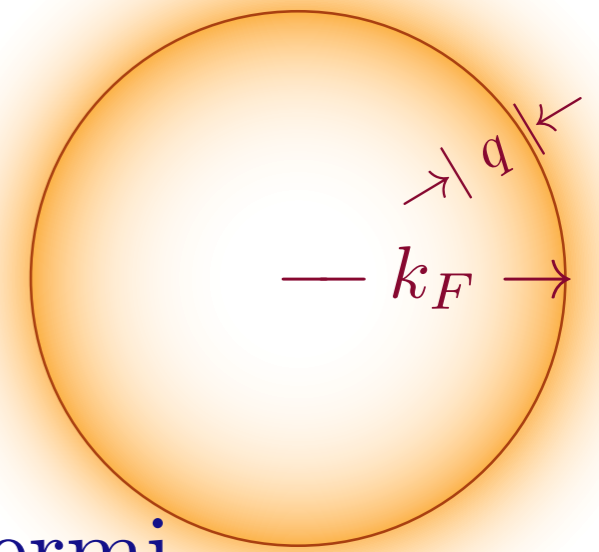


- $k_F^d \sim Q$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

NFL Bose metal



- Hidden Fermi surface with $k_F^d \sim Q$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

Compressible quantum matter *the cond-mat perspective*

- Fermi liquid (**FL**): the entire charge Q is contained within visible Fermi surfaces
- Bose metal (**NFL**): the entire charge Q is contained within hidden Fermi surfaces of gauge-charged fermions.
- Fractionalized Fermi liquid (**FL***): the charge Q is divided between visible and hidden Fermi surfaces.

Compressible quantum matter

*A. Field theory:
Condensed matter
classification*

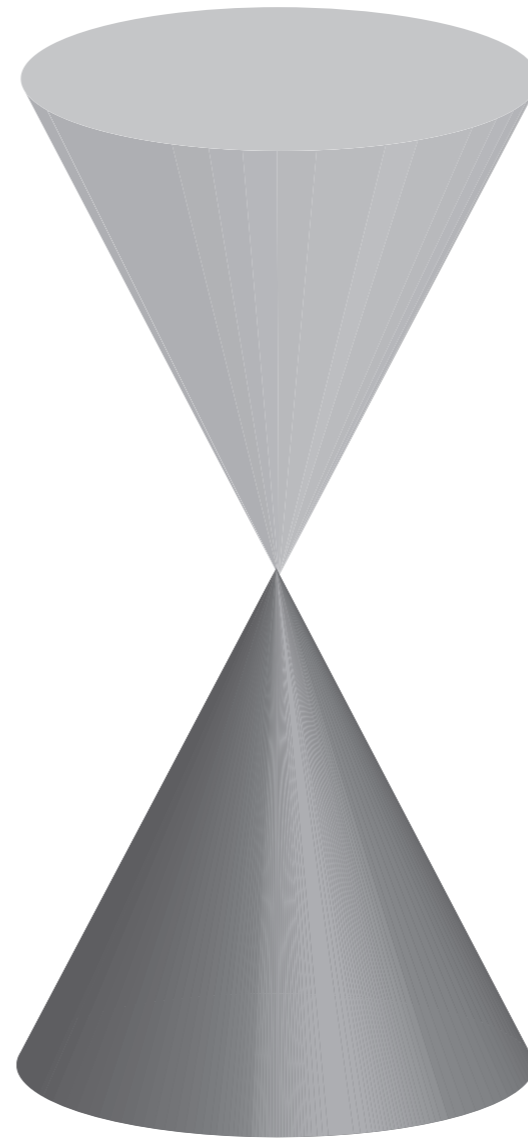
B. Gauge-gravity duality

Compressible quantum matter

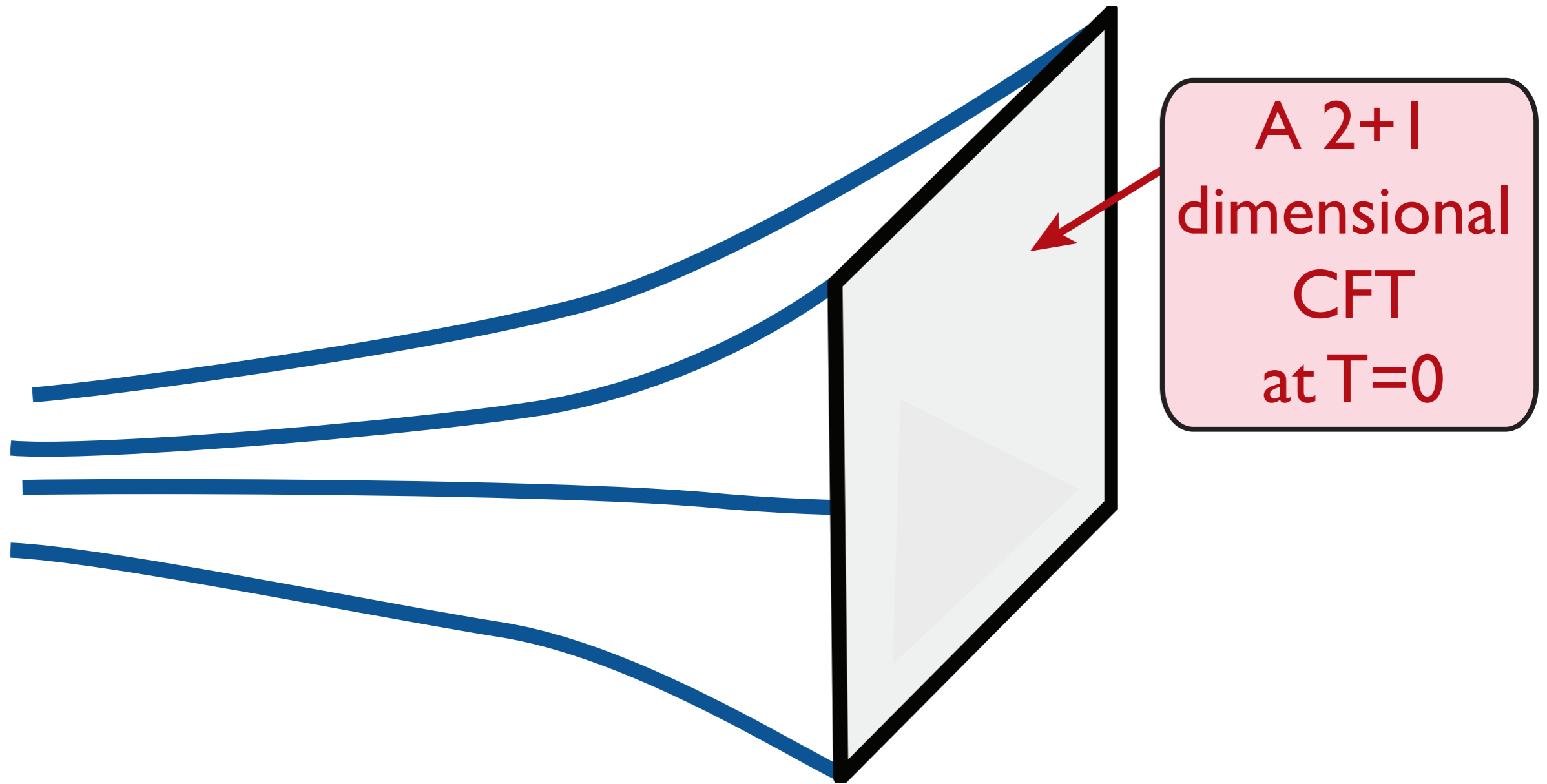
*A. Field theory:
Condensed matter
classification*

B. Gauge-gravity duality

Begin with a CFT

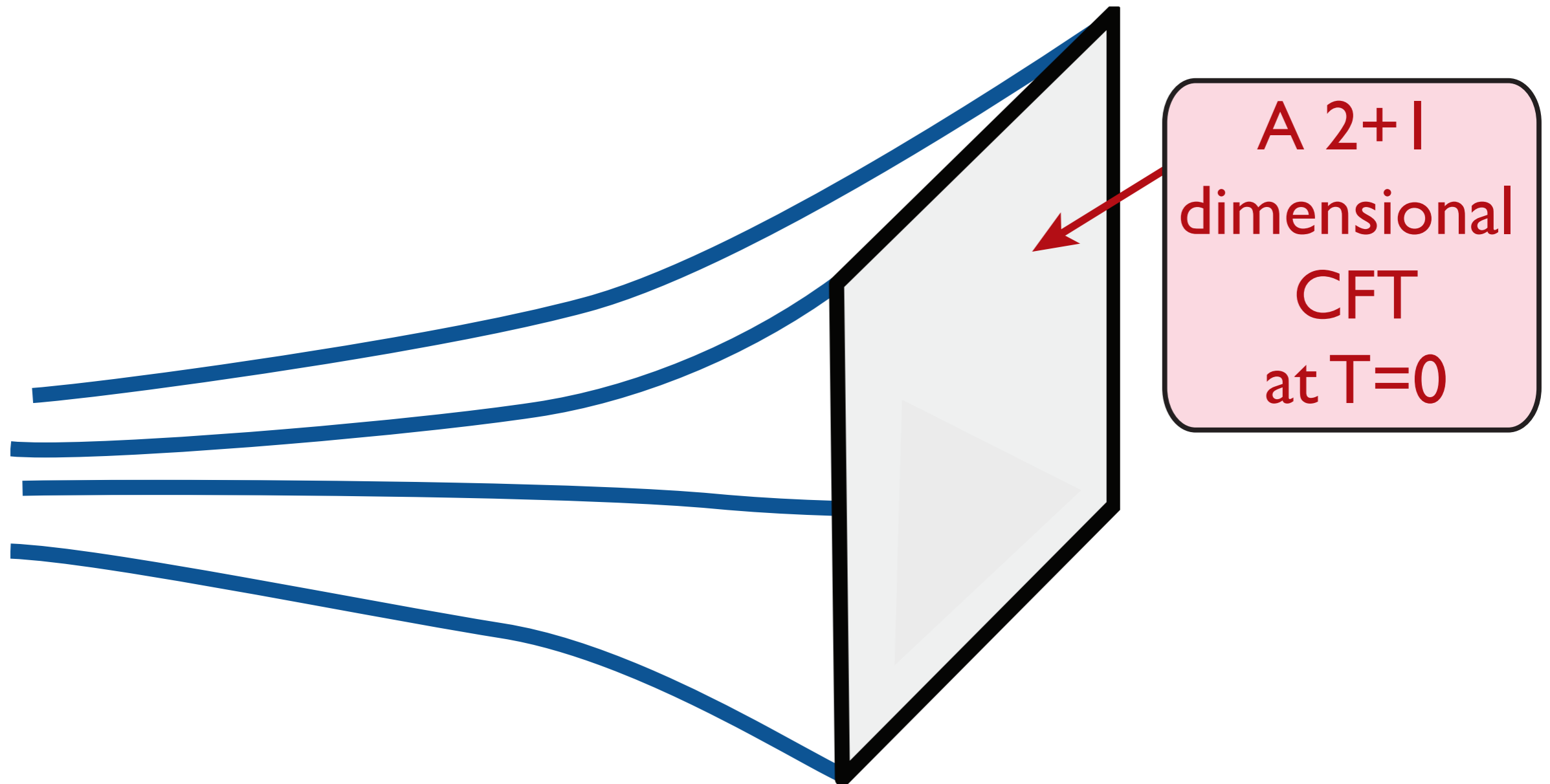


Holographic representation: AdS₄



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

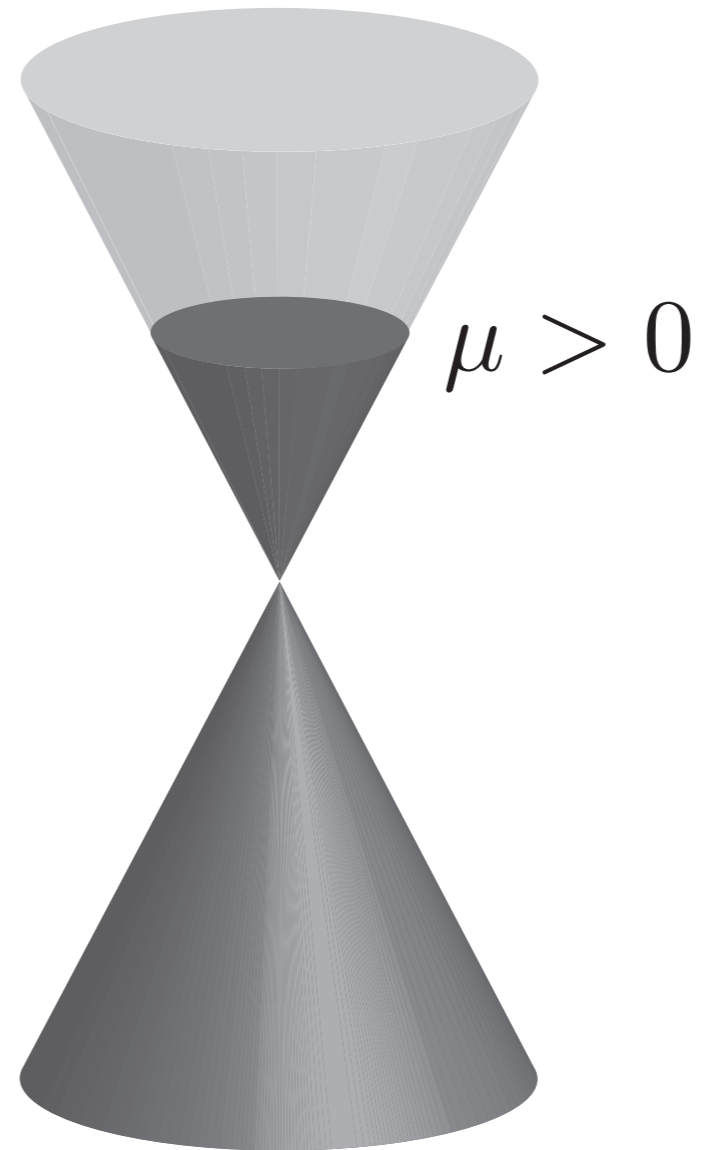
Holographic representation: AdS₄



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1$

Apply a chemical potential



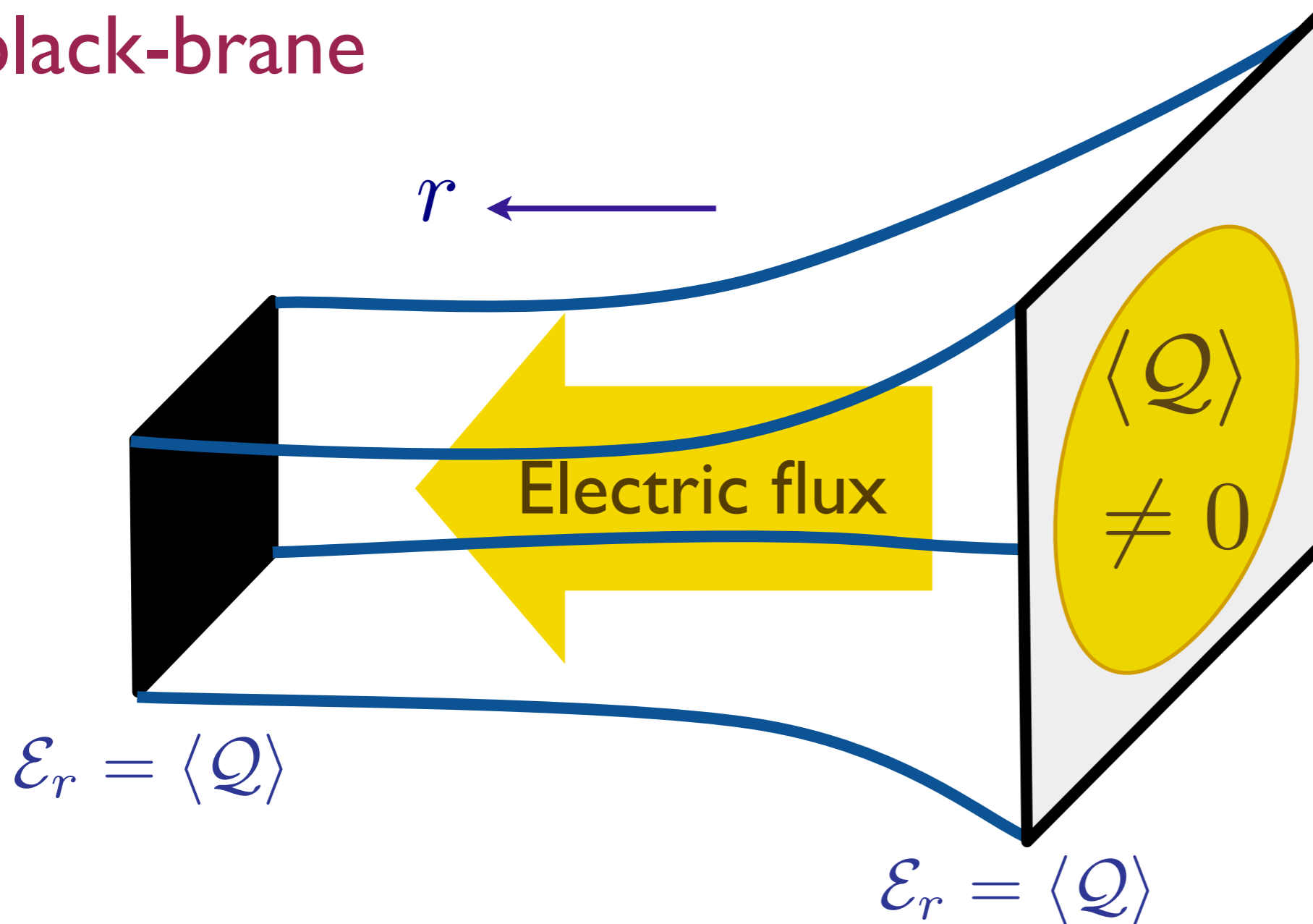
Holographic theory of a compressible state

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_M^2} F_{ab} F^{ab} \right]$$

At non-zero chemical potential
we simply require $\mathcal{A}_t(r \rightarrow 0, \vec{x}, t) = \mu$.

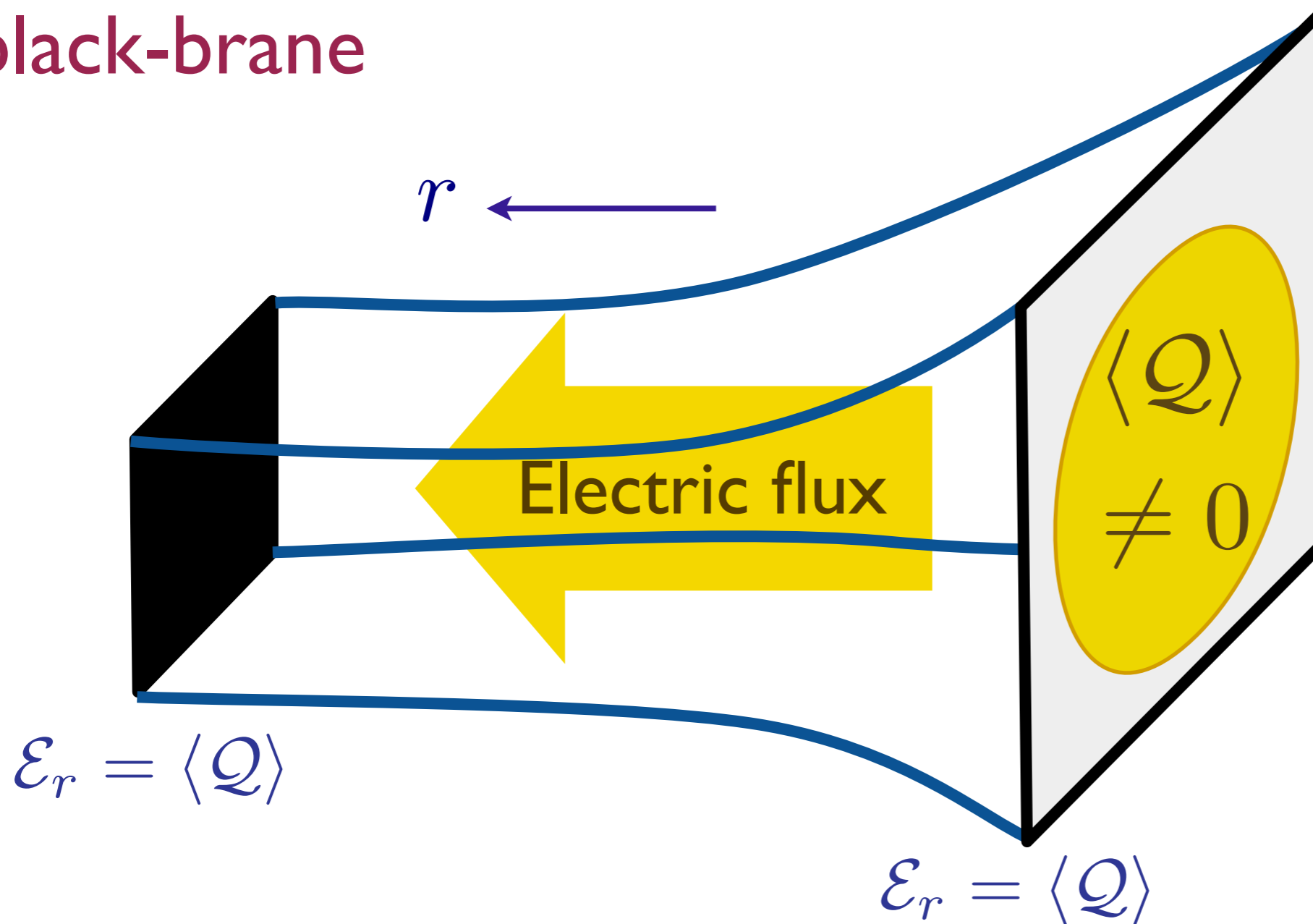
The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_M^2} F_{ab} F^{ab} \right]$$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane

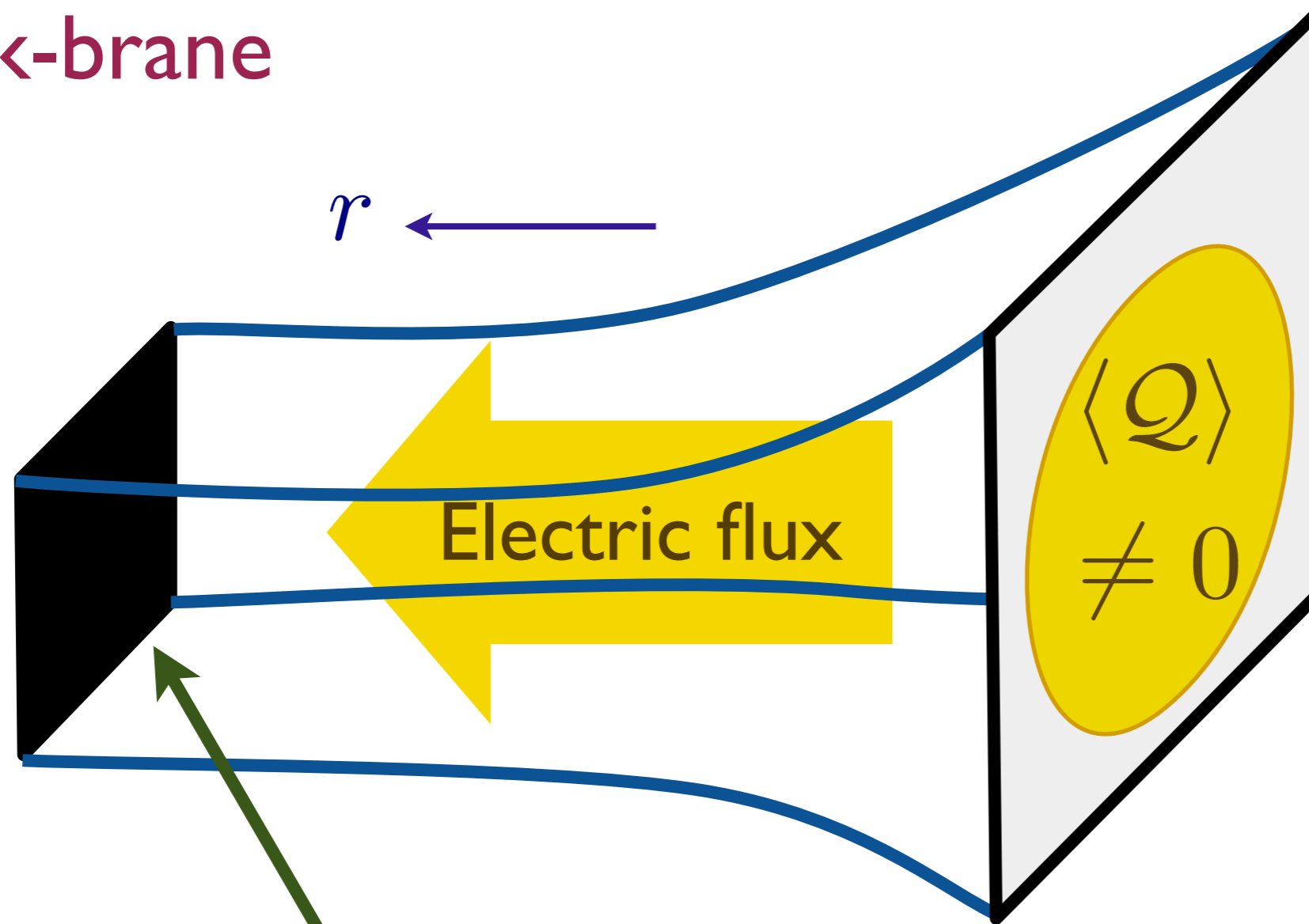


$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

$$\text{with } f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right) \text{ and } R = \frac{\sqrt{6}Lg_M}{\kappa\mu}, \text{ and } A_t = \mu \left(1 - \frac{r}{R}\right)$$

A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, Phys. Rev. D **60**, 064018 (1999)

The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4 -Reissner-Nordström black-brane



At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\text{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6} \left(\frac{-dt^2 + dr^2}{r^2} \right) + dx^2 + dy^2$$

T. Faulkner, H. Liu,
J. McGreevy,
and D. Vegh,
arXiv:0907.2694

Features of $\text{AdS}_2 \times R^2$

- Has non-zero entropy density at $T = 0$. This can be fixed by adding “dilaton” fields, which correspond to relevant operators on the boundary CFT (see later).

Features of $\text{AdS}_2 \times R^2$

- Has non-zero entropy density at $T = 0$. This can be fixed by adding “dilaton” fields, which correspond to relevant operators on the boundary CFT (see later).
- Describes a compressible phase at zero temperature. So what is the fate of the Luttinger theorem? The bulk electric flux extending to the horizon at infinity measures the density, \mathcal{Q} , contained in *hidden* Fermi surfaces of gauge-charged particles (‘quarks’).

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

L. Huijse and S. Sachdev, *Physical Review D* **84**, 026001 (2011).

Title: Friedel oscillations and horizon charge in 1D holographic liquids

Authors: [Thomas Faulkner](#), [Nabil Iqbal](#)

(Submitted on 17 Jul 2012)

Abstract: In many-body fermionic systems at finite density correlation functions of the density operator exhibit Friedel oscillations at a wavevector that is twice the Fermi momentum. We demonstrate the existence of such Friedel oscillations in a 3d gravity dual to a compressible finite-density state in a (1+1) dimensional field theory. The bulk dynamics is provided by a Maxwell U(1) gauge theory and all the charge is behind a bulk horizon. The bulk gauge theory is compact and so there exist magnetic monopole tunneling events. We compute the effect of these monopoles on holographic density-density correlation functions and demonstrate that they cause Friedel oscillations at a wavevector that directly counts the charge behind the bulk horizon. If the magnetic monopoles are taken to saturate the bulk Dirac quantization condition then the observed Fermi momentum exactly agrees with that predicted by Luttinger's theorem, suggesting some Fermi surface structure associated with the charged horizon. The mechanism is generic and will apply to any charged horizon in three dimensions. Along the way we clarify some aspects of the holographic interpretation of Maxwell electromagnetism in three bulk dimensions and show that perturbations about the charged BTZ black hole exhibit a hydrodynamic sound mode at low temperature.

Comments: 59 pages, 5 figures

Subjects: High Energy Physics - Theory (hep-th); Strongly Correlated Electrons (cond-mat.str-el)

Report number: NSF-KITP-12-122

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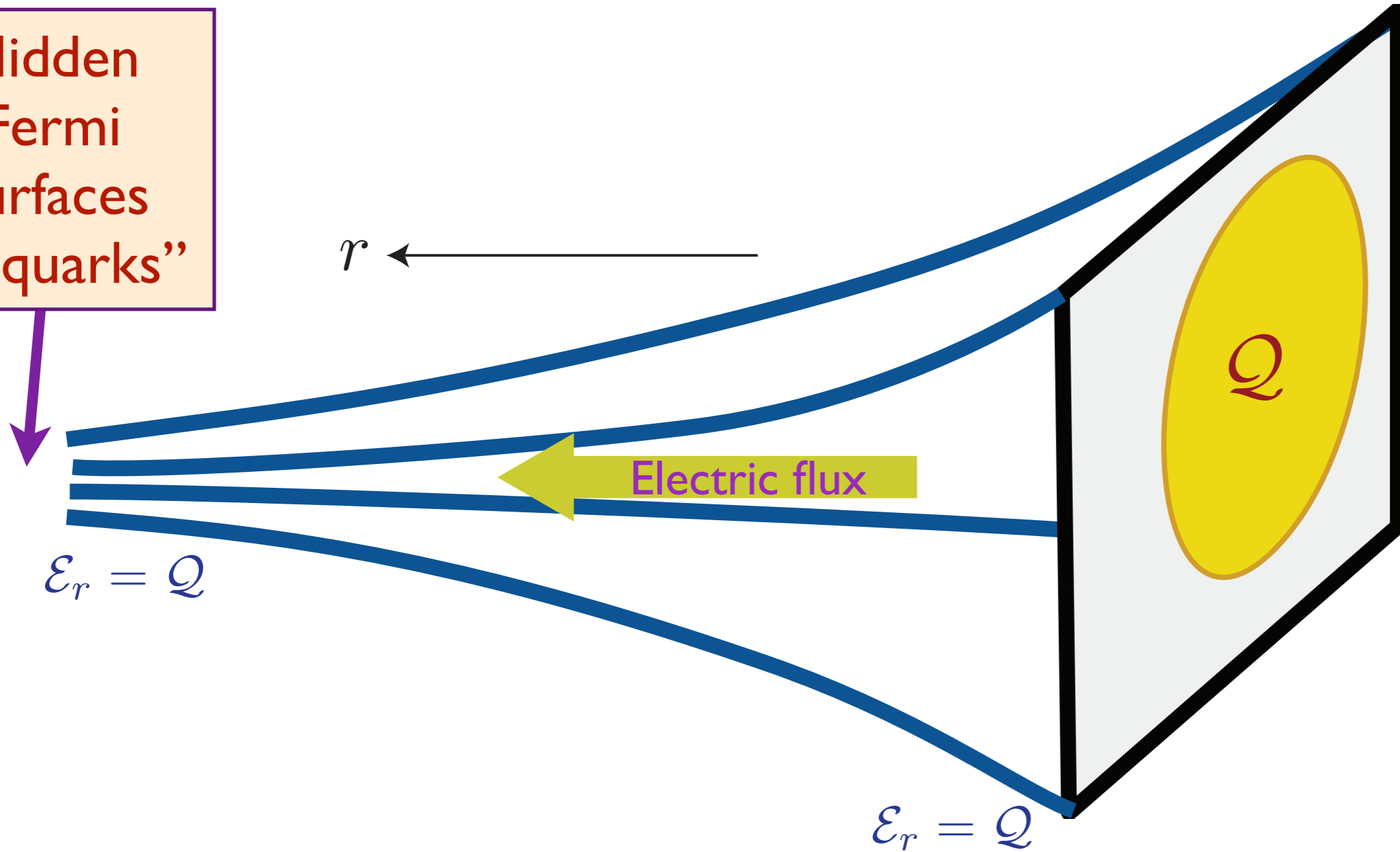
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Cite as: [arXiv:1207.4208v1](#) [hep-th]

Holographic theory of a non-Fermi liquid (NFL)

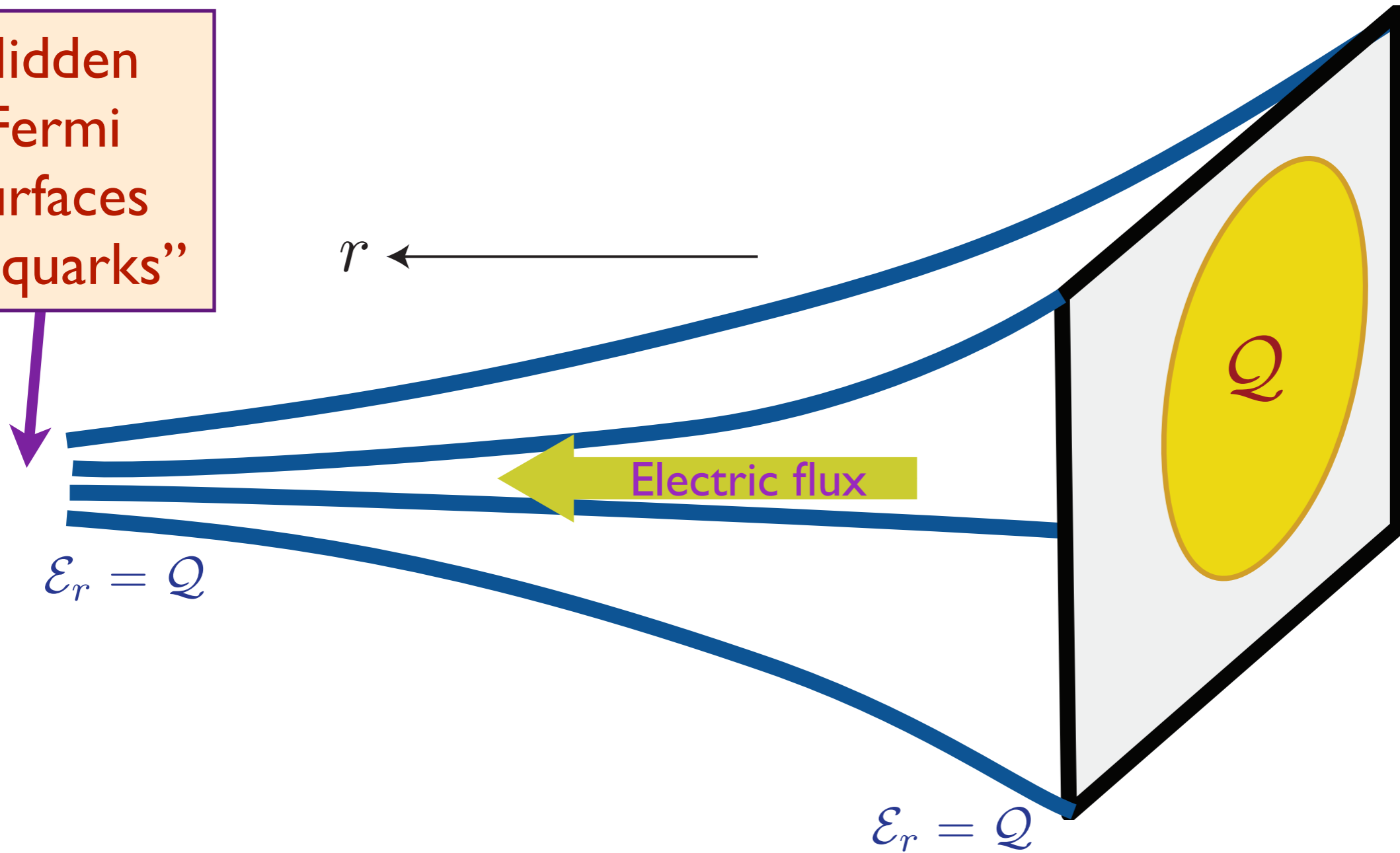
Hidden Fermi surfaces of “quarks”



This is a “bosonization” of the *hidden* Fermi surface

Holographic theory of a non-Fermi liquid (NFL)

Hidden Fermi surfaces of "quarks"



Fully fractionalized state has all the electric flux exiting to the horizon at $r = \infty$

Holographic theory of a compressible state

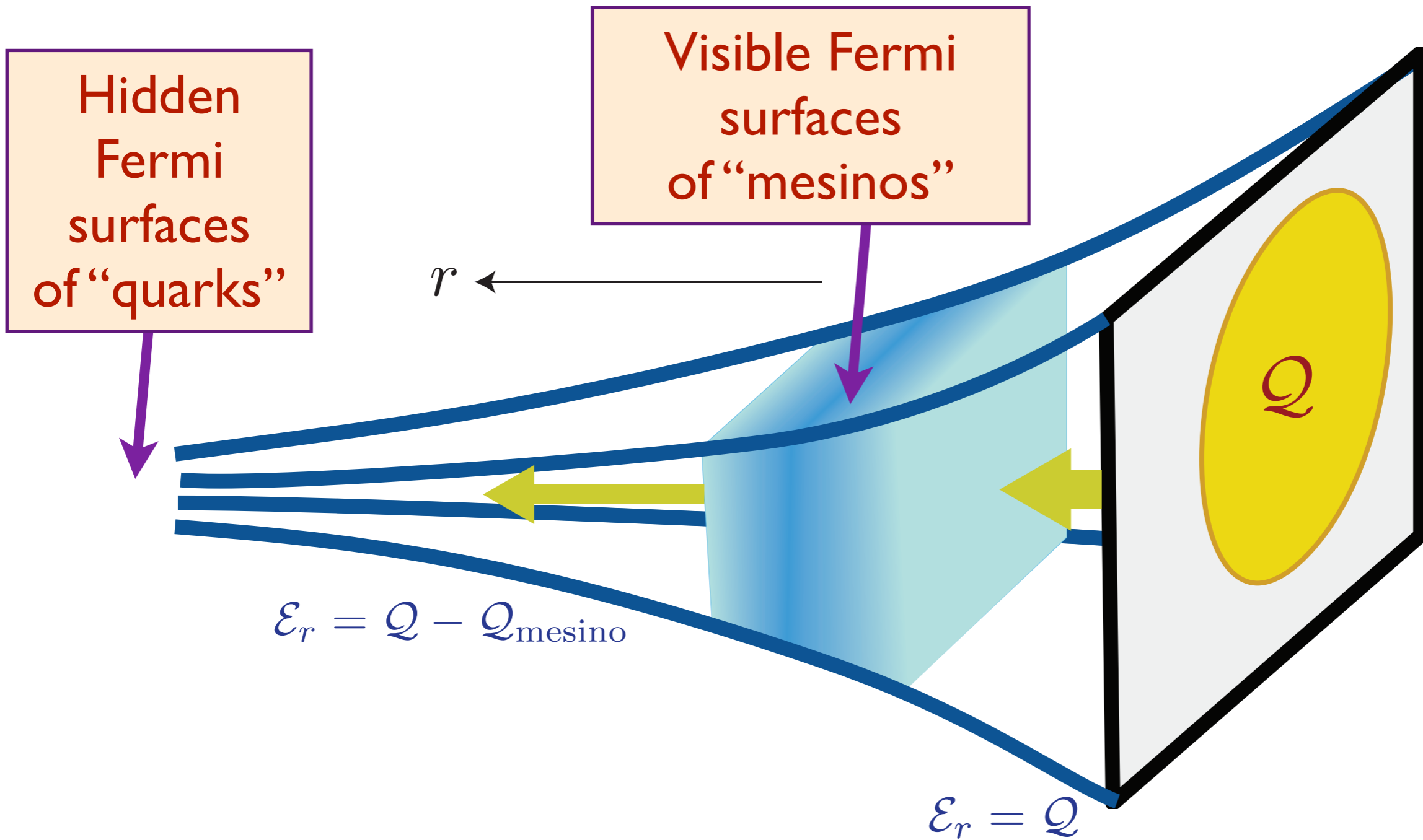
Add a fermionic field ψ to the bulk effective action, carrying the U(1) charge of the bulk gauge field: consequently, this field corresponds to a boundary fermion which carries charge Q , but is *neutral* w.r.t to any gauge fields in the boundary theory. We refer to such fermions as *mesinos*.

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_M^2} F_{ab} F^{ab} + i(\bar{\psi} \Gamma^M D_M \psi + m \bar{\psi} \psi) \right]$$

For a finite density state, we impose the boundary condition $A_t(r \rightarrow 0) = \mu$. Procedure to solve the bulk theory:

1. Assume some reasonable form for the electric potential $A_t(r)$ and the metric $g_{\mu\nu}(r)$.
2. Solve Dirac equation for fermions in this background.
3. Occupy negative energy fermions states.
4. Compute the U(1) density and $T_{\mu\nu}$ of the occupied states.
5. Use Poisson's equation and Einstein's equations to recompute $A_t(r)$ and the metric $g_{\mu\nu}(r)$.
6. Return to step 2.

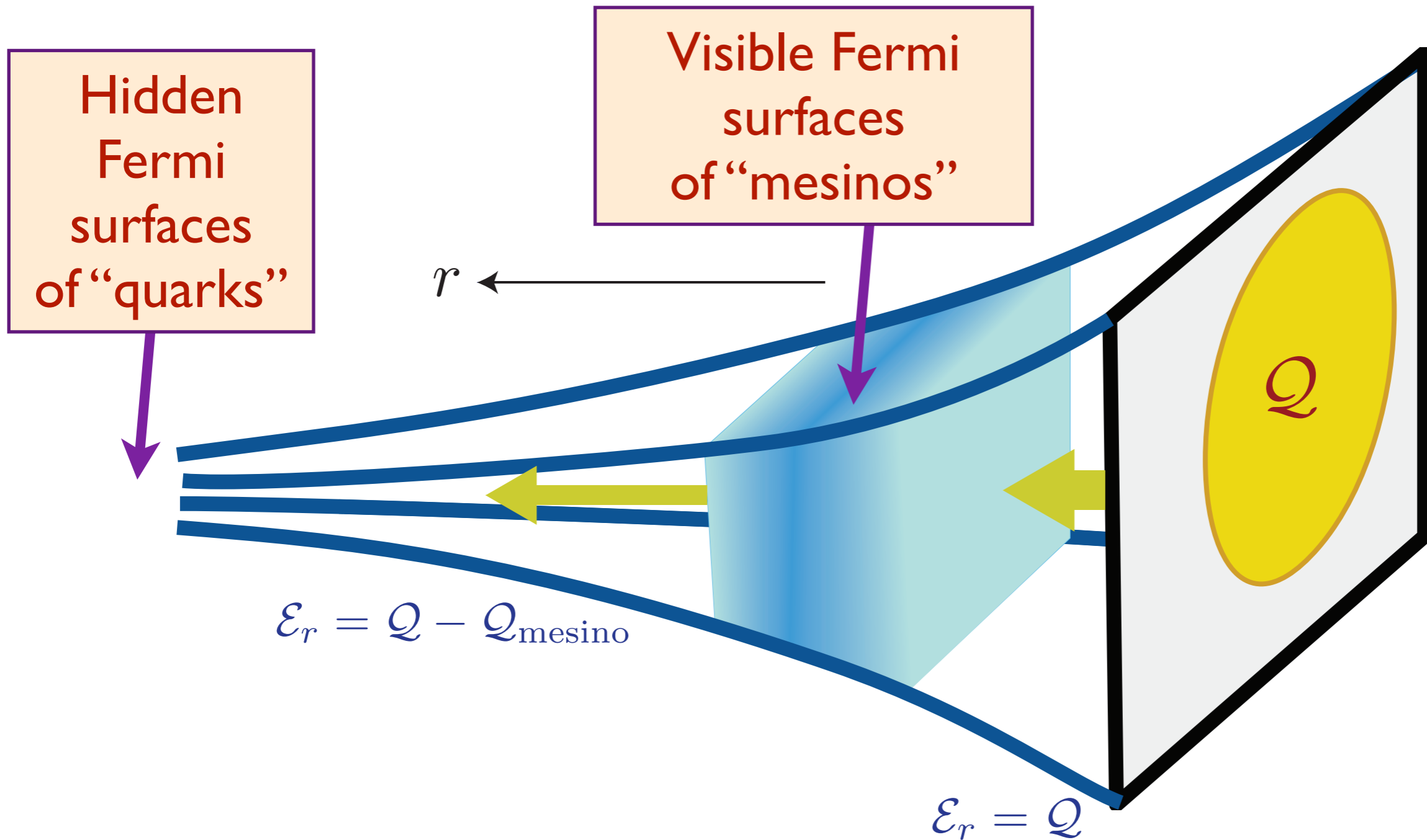
Holographic theory of a fractionalized-Fermi liquid (FL*)



A state with partial fractionalization, and partial electric flux exiting horizon

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010); S. Sachdev, *Physical Review D* **84**, 066009 (2011)

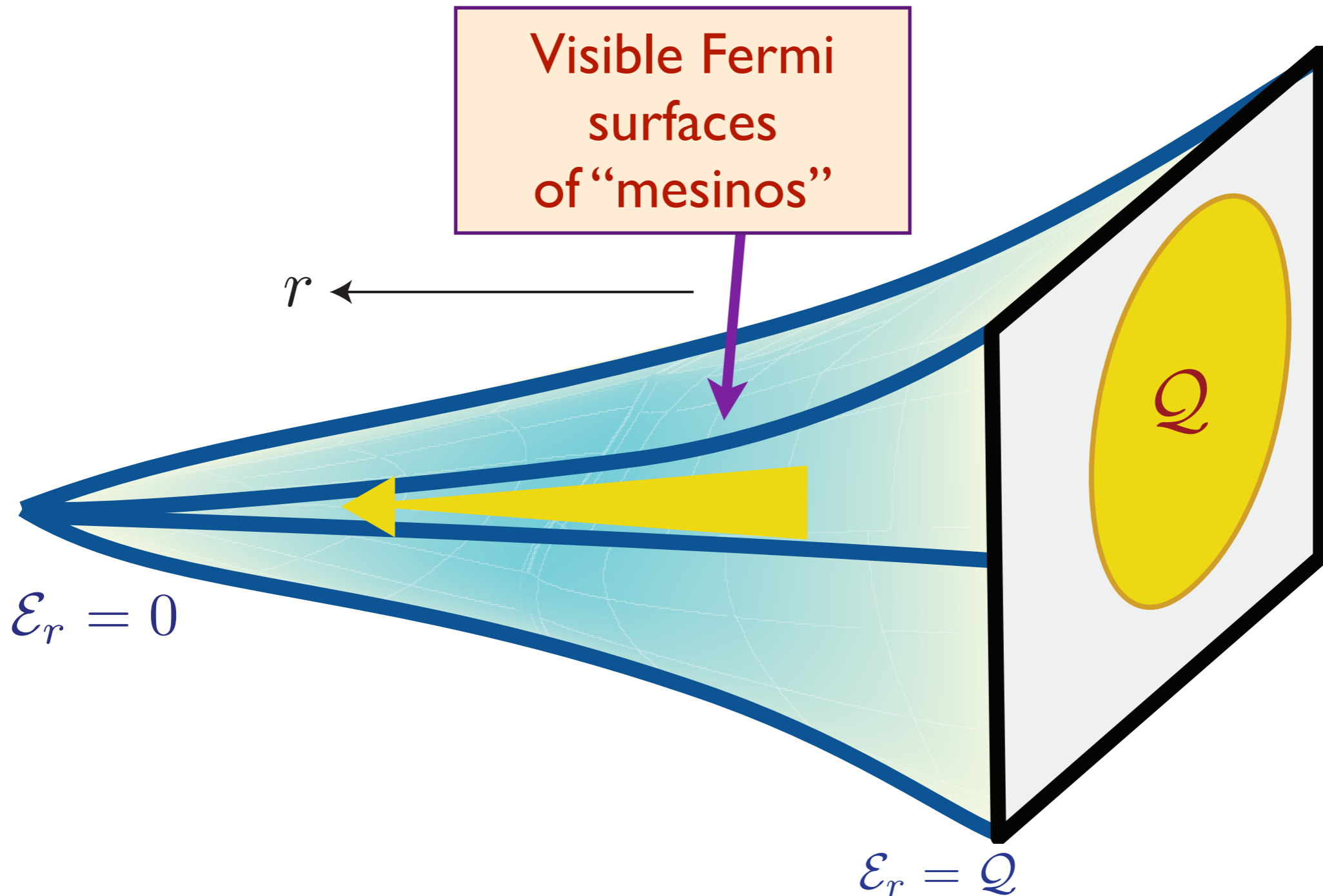
Holographic theory of a fractionalized-Fermi liquid (FL*)



The “mesinos” corresponds to the Fermi surfaces obtained in the early probe fermion computation (S.-S. Lee, Phys. Rev. D **79**, 086006 (2009); H. Liu, J. McGreevy, and D. Vegh, arXiv:0903.2477; M. Čubrović, J. Zaanen, and K. Schalm, Science **325**, 439 (2009)).

These are spectators, and are expected to have well-defined quasiparticle excitations.

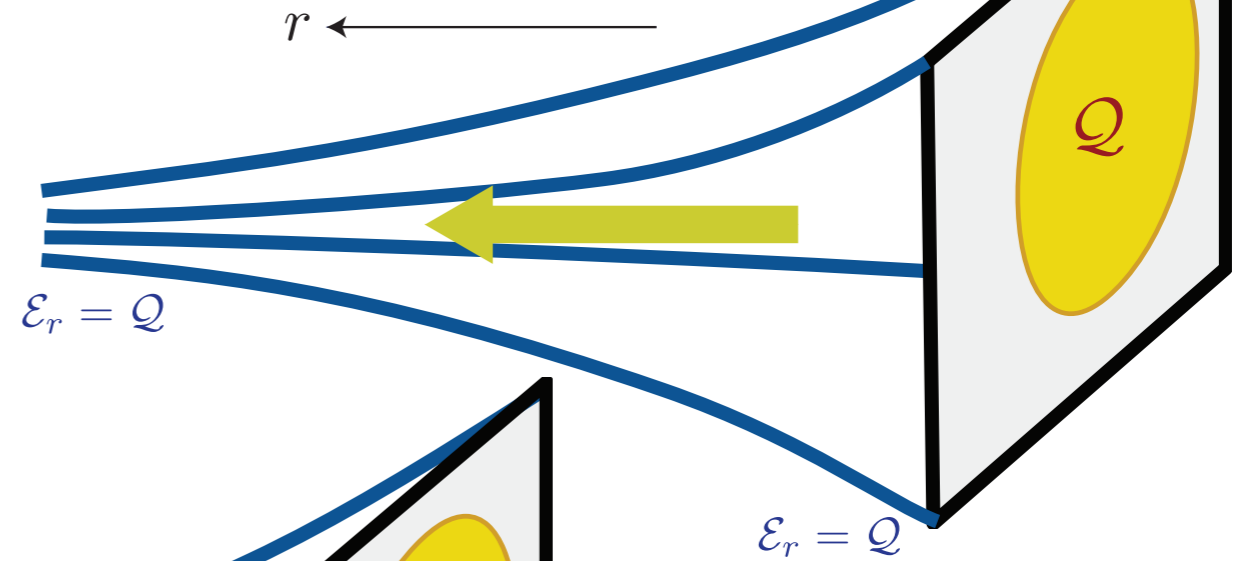
Holographic theory of a Fermi liquid (FL)



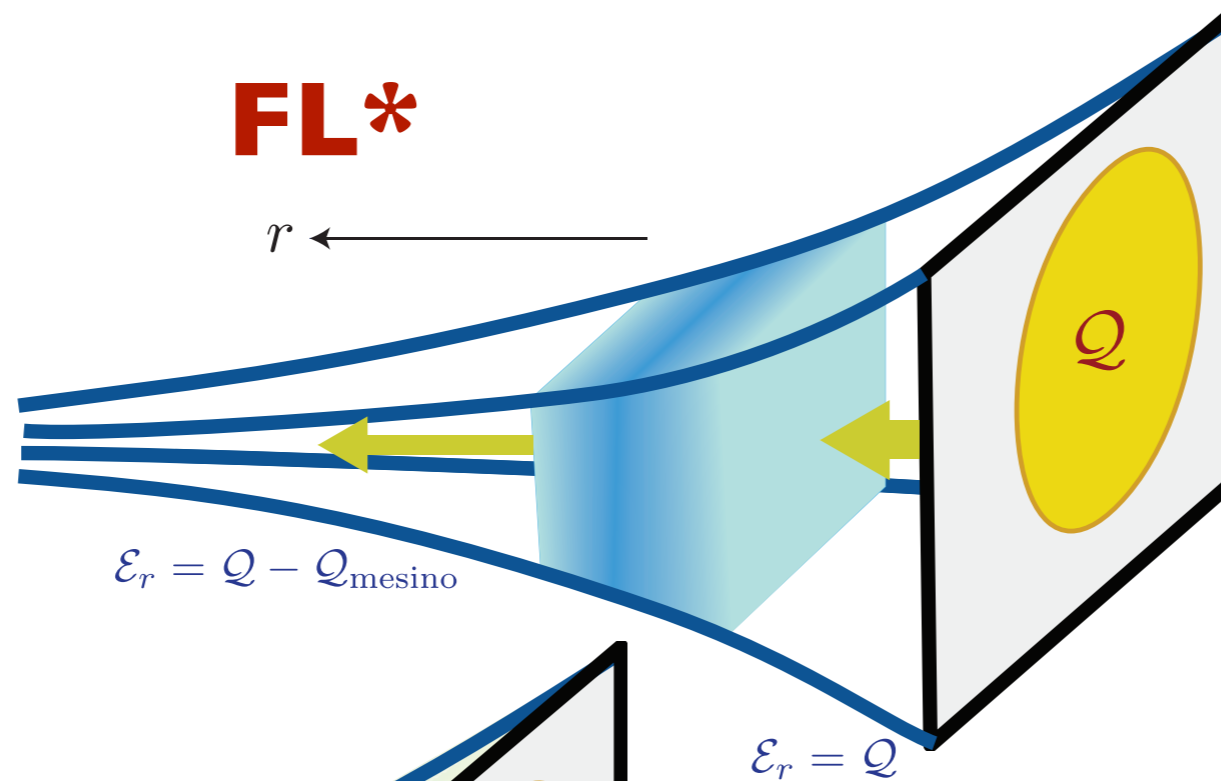
- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)

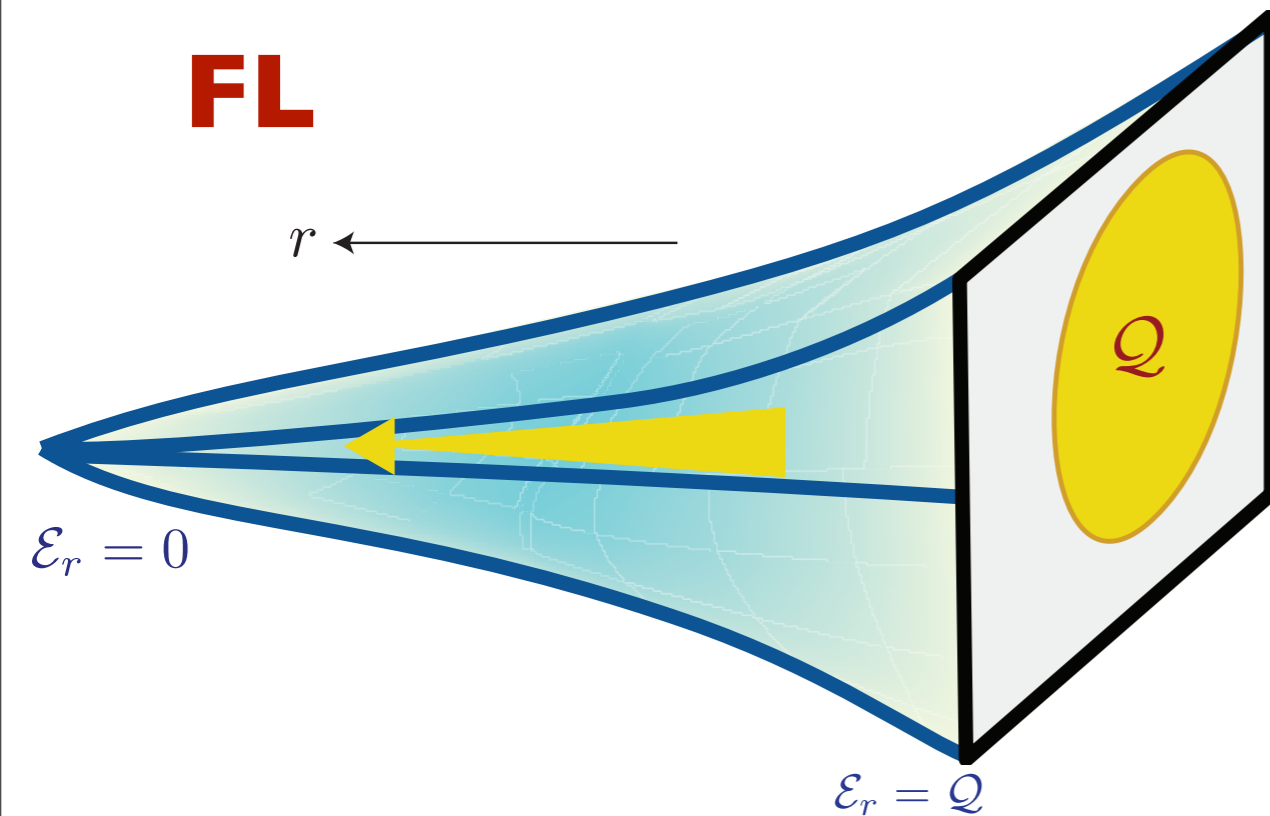
NFL



FL*



FL



Gauss Law in the bulk
 \Leftrightarrow Luttinger theorem
on the boundary

S. Sachdev, Physical Review D **84**, 066009 (2011)

Compressible quantum matter *the holographic perspective*

- Fermi liquid (**FL**): the entire charge Q is contained in the bulk, and there is no electric flux leaking to infinity.
- Bose metal (**NFL**): All the electric flux leaks to infinity, and this is linked to hidden Fermi surface of gauge-charged ‘quarks’.
- Fractionalized Fermi liquid (**FL***): Part of the electric flux leaks to infinity, and remainder is within visible Fermi surfaces in the bulk.

Compressible quantum matter *the cond-mat perspective*

- Fermi liquid (**FL**): the entire charge Q is contained within visible Fermi surfaces
- Bose metal (**NFL**): the entire charge Q is contained within hidden Fermi surfaces of gauge-charged fermions.
- Fractionalized Fermi liquid (**FL***): the charge Q is divided between visible and hidden Fermi surfaces.

Entanglement, holography, and the quantum phases of matter

Princeton University, July 26, 2012

Subir Sachdev

sachdev.physics.harvard.edu





Liza Huijse



Max Metlitski



Brian Swingle

“Complex entangled” states of
quantum matter,
not adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

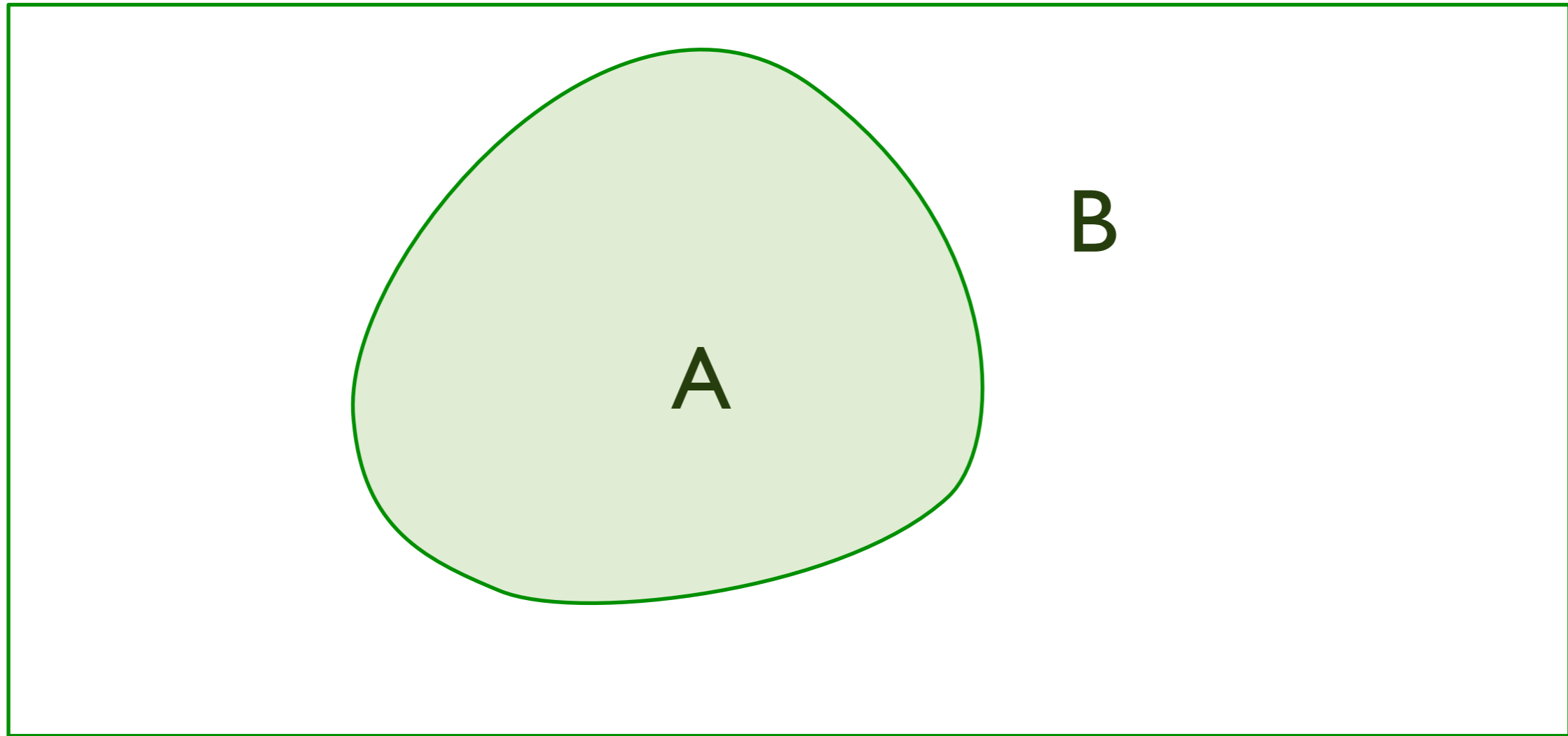
Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

Entanglement entropy



$|\Psi\rangle \Rightarrow$ Ground state of entire system,
 $\rho = |\Psi\rangle\langle\Psi|$

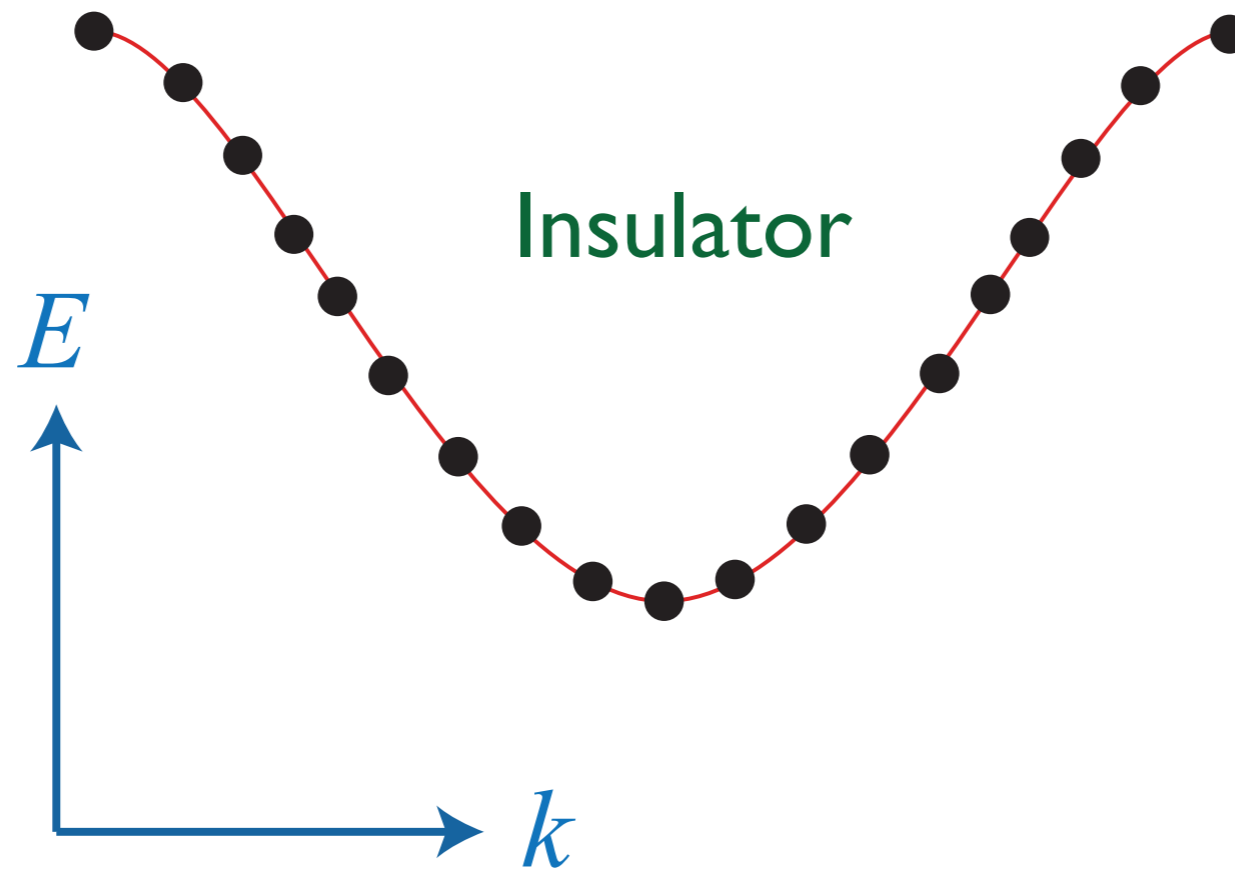
$\rho_A = \text{Tr}_B \rho =$ density matrix of region A

Entanglement entropy $S_E = -\text{Tr}(\rho_A \ln \rho_A)$

M. Srednicki, Phys. Rev. Lett. **71**, 666 (1993)

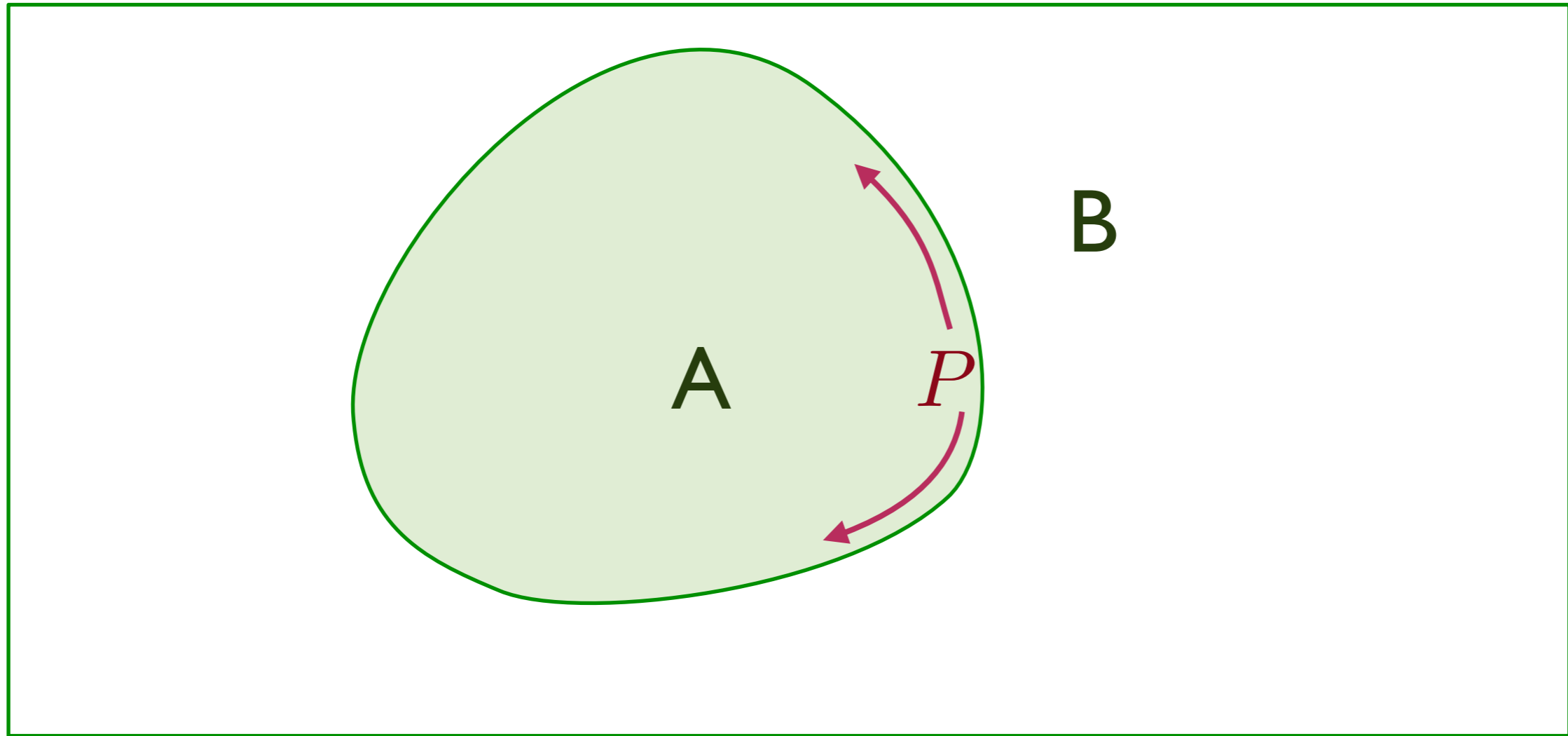
Gapped quantum matter

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator

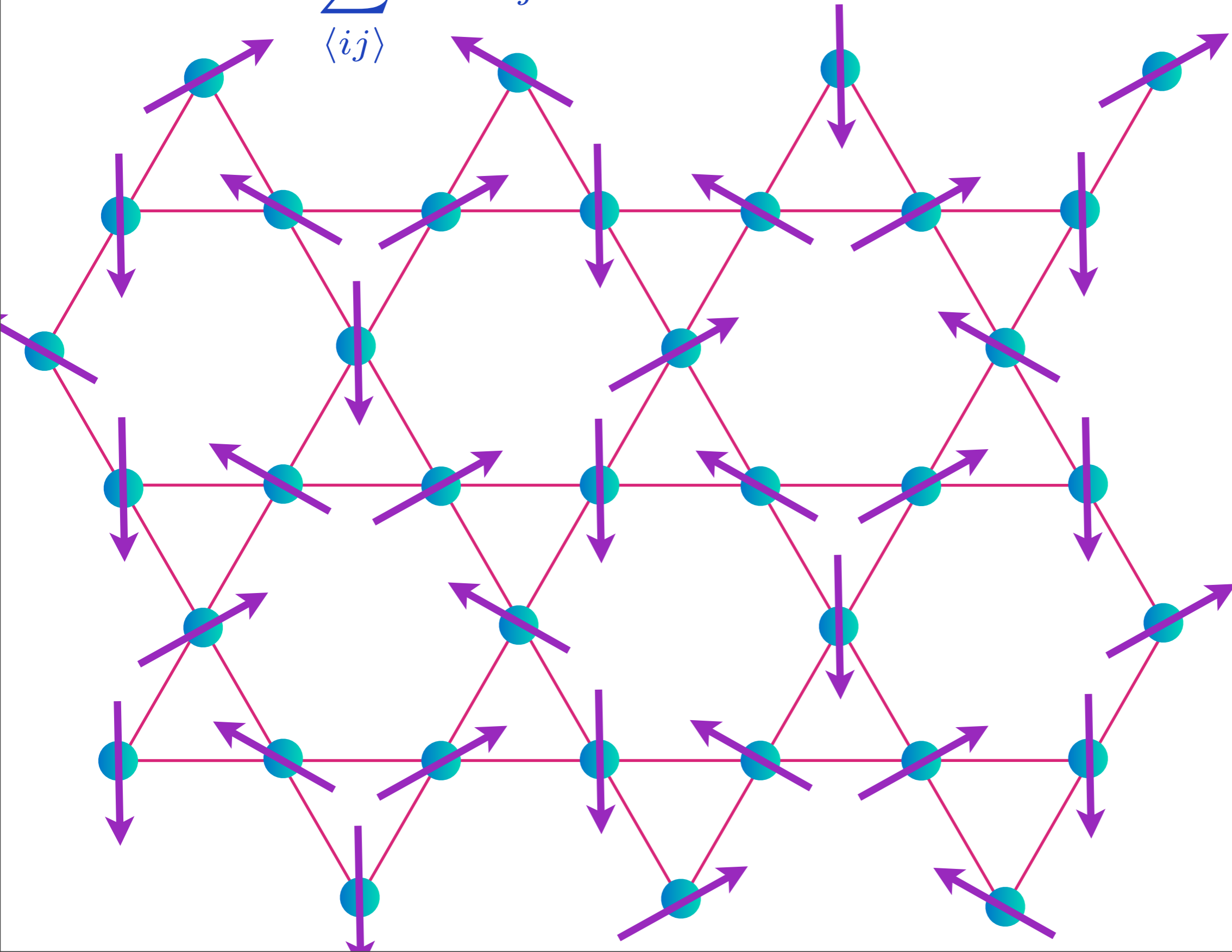


$$S_E = aP - b \exp(-cP)$$

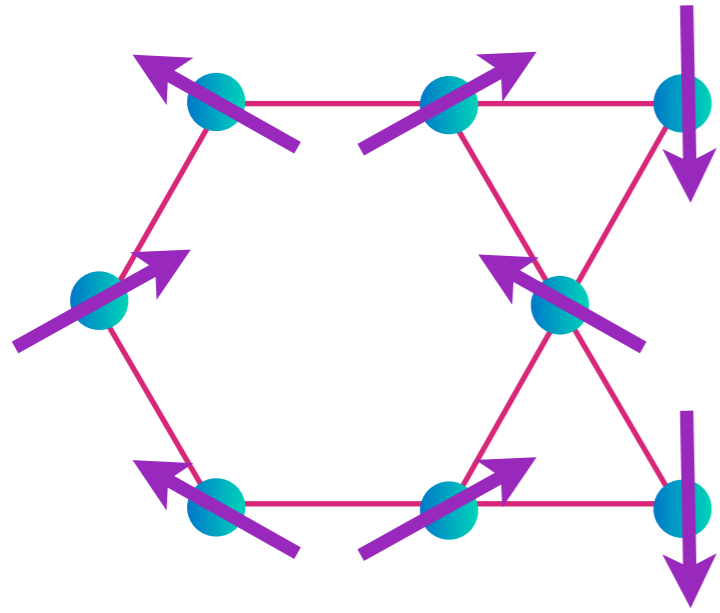
where P is the surface area (perimeter) of the boundary between A and B.

Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Kagome antiferromagnet



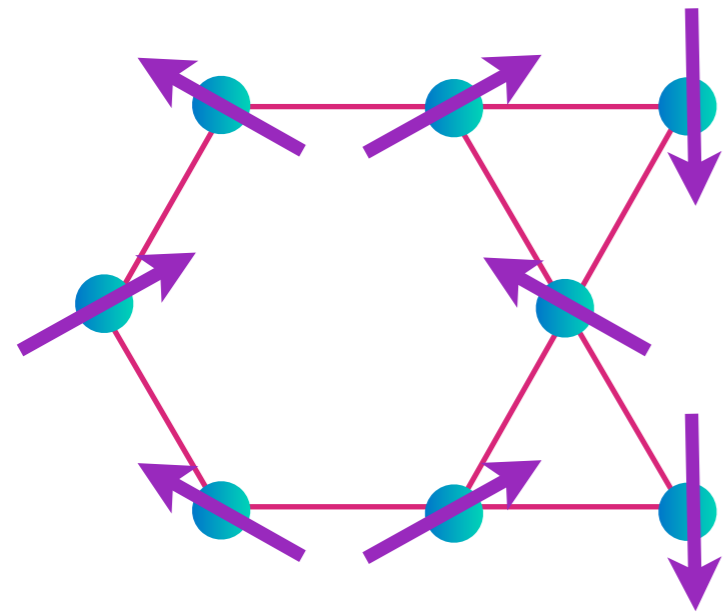
non-collinear Néel state

Quantum “disordered” state with exponentially decaying spin correlations.

S_c

S

Kagome antiferromagnet: Z_2 spin liquid



non-collinear Néel state

Entangled quantum state:
A stable “ Z_2 spin liquid”.
The excitations carry ‘electric’
and ‘magnetic’ charges of
an emergent Z_2 gauge field.

S_c

S

S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)

Y. Huh, M. Punk, and S. Sachdev, *Phys. Rev. B* **84**, 094419 (2011)

The Z_2 spin liquid was introduced in

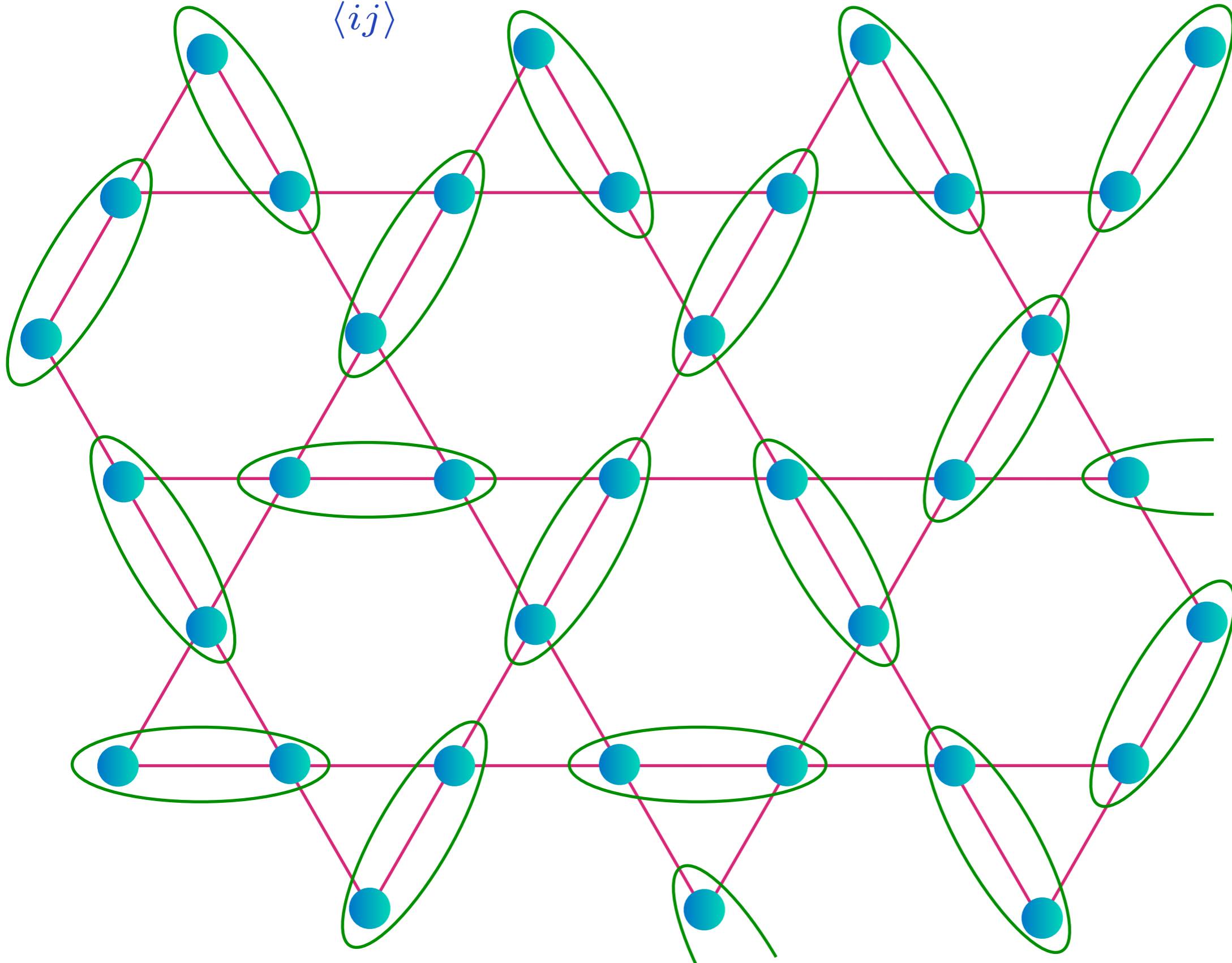
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991),

X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$$\left(\begin{array}{c} \bullet \\ \bullet \end{array} \right) = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

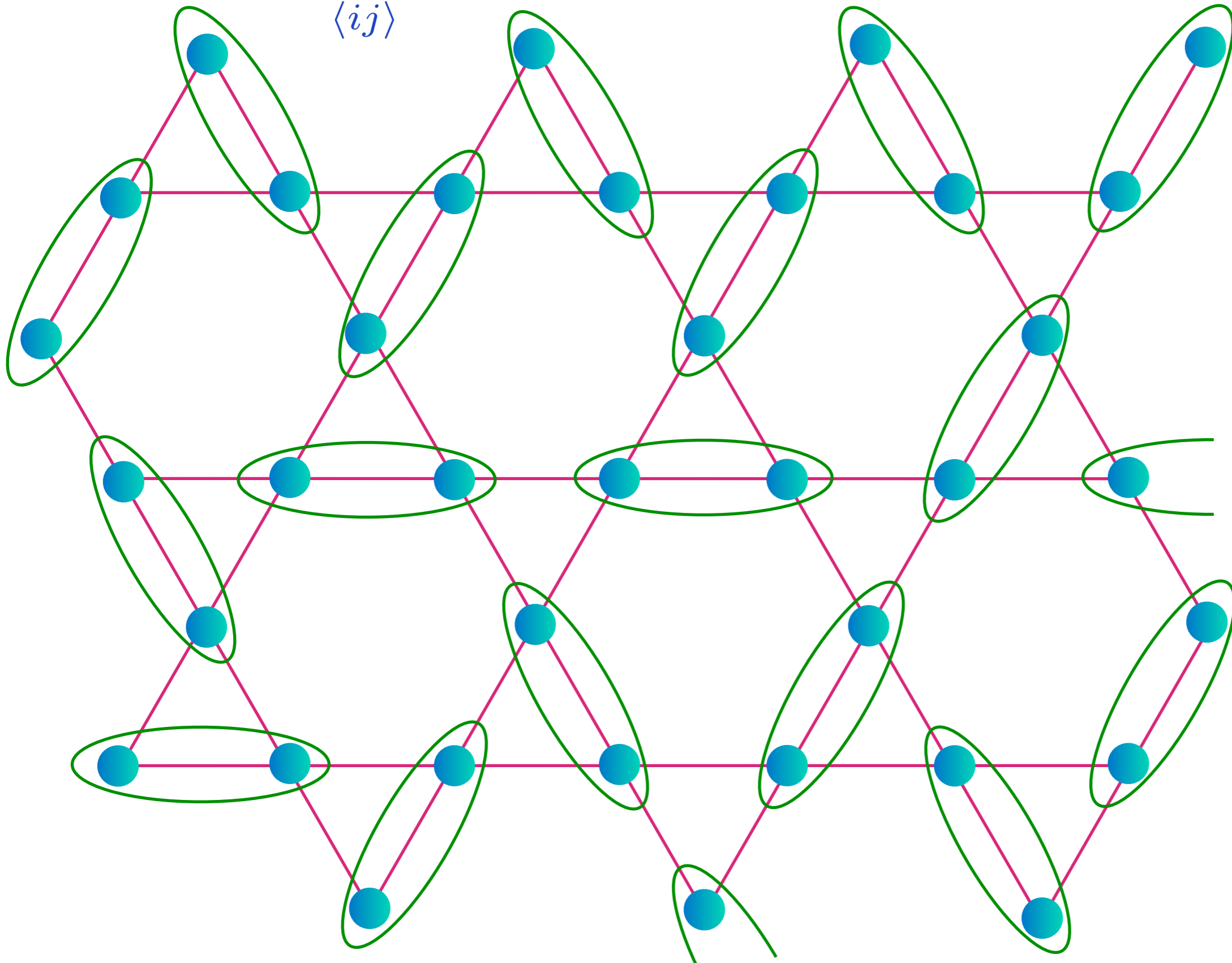


P. Fazekas and
P. W. Anderson,
Philos. Mag.
30, 23 (1974).

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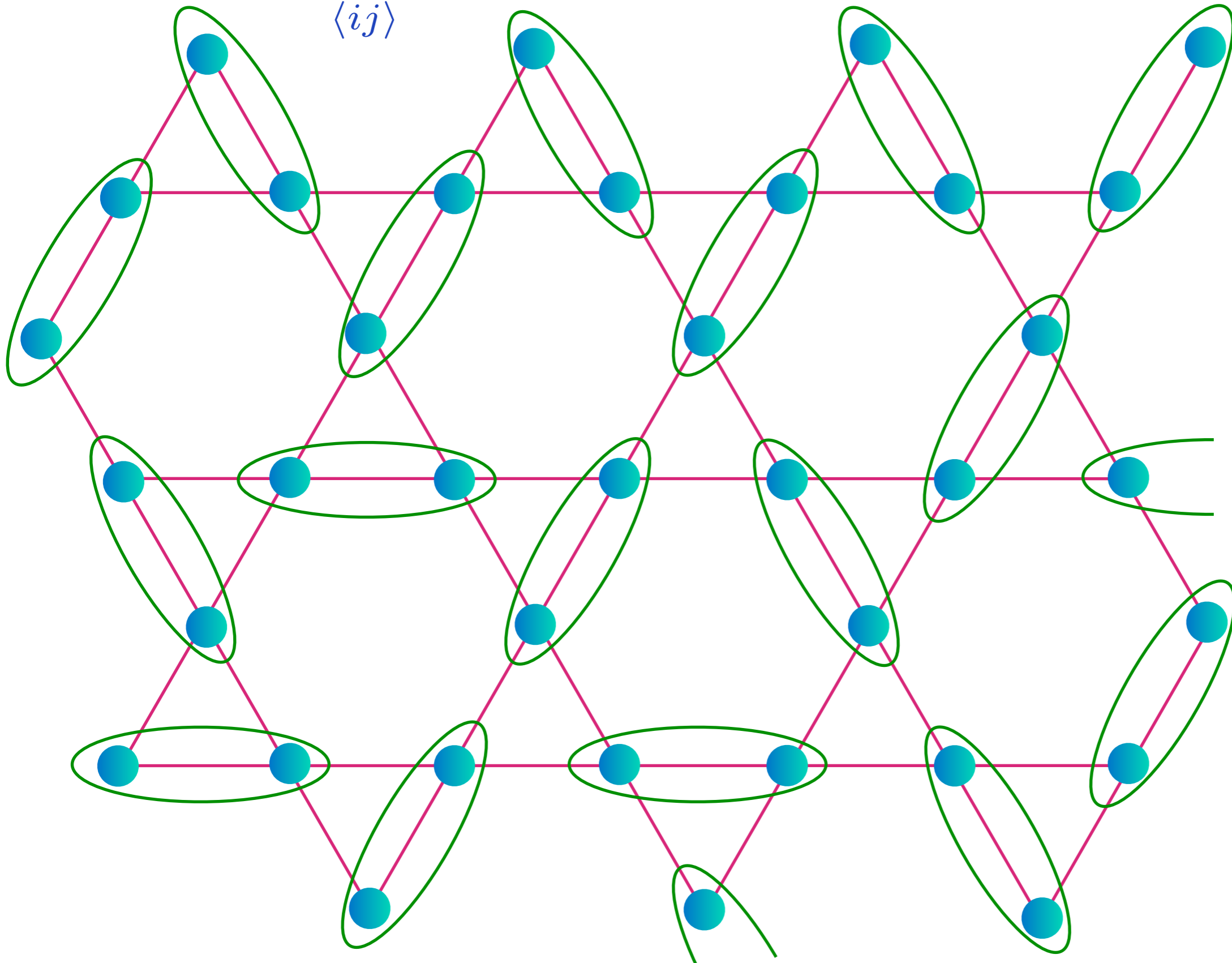


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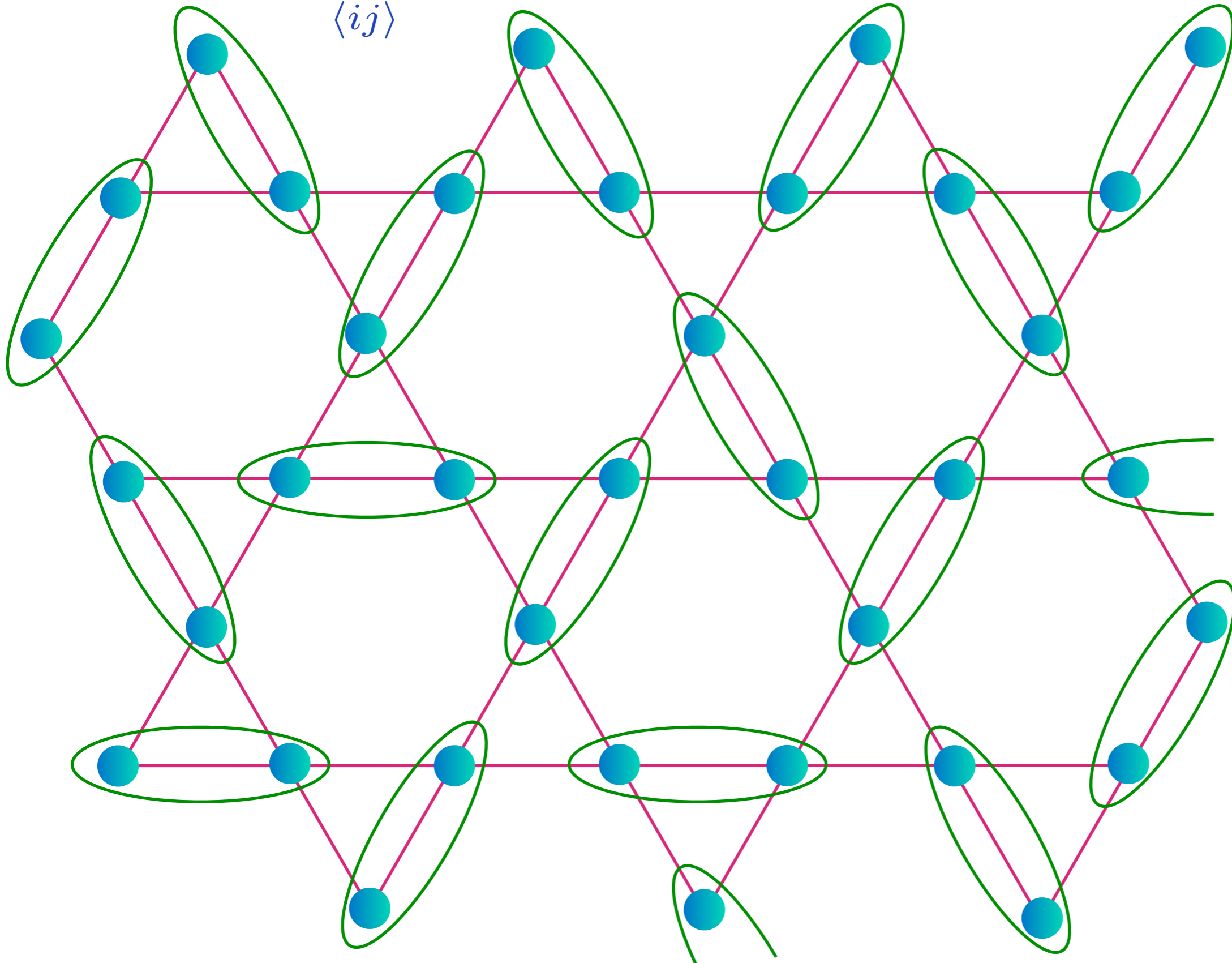


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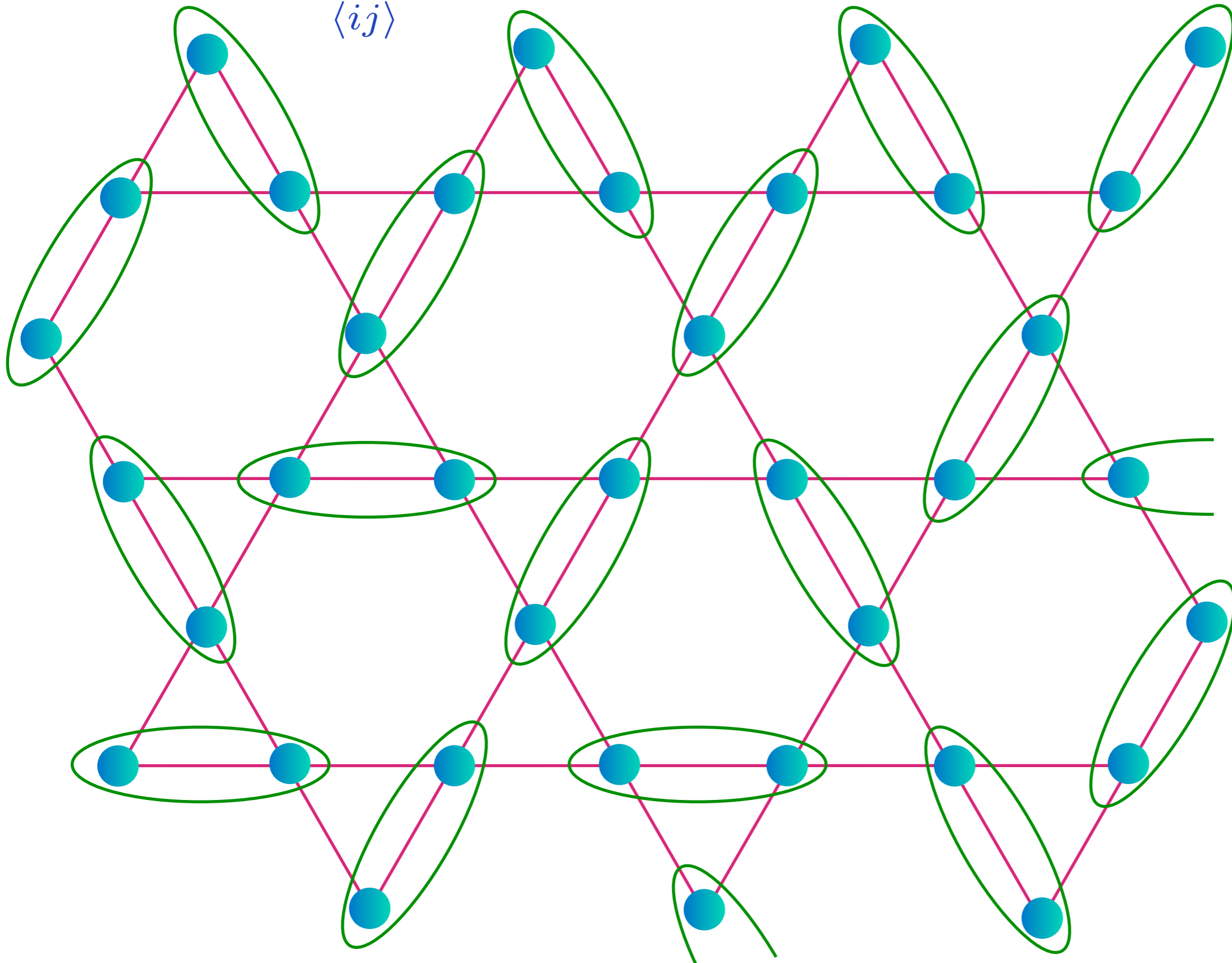


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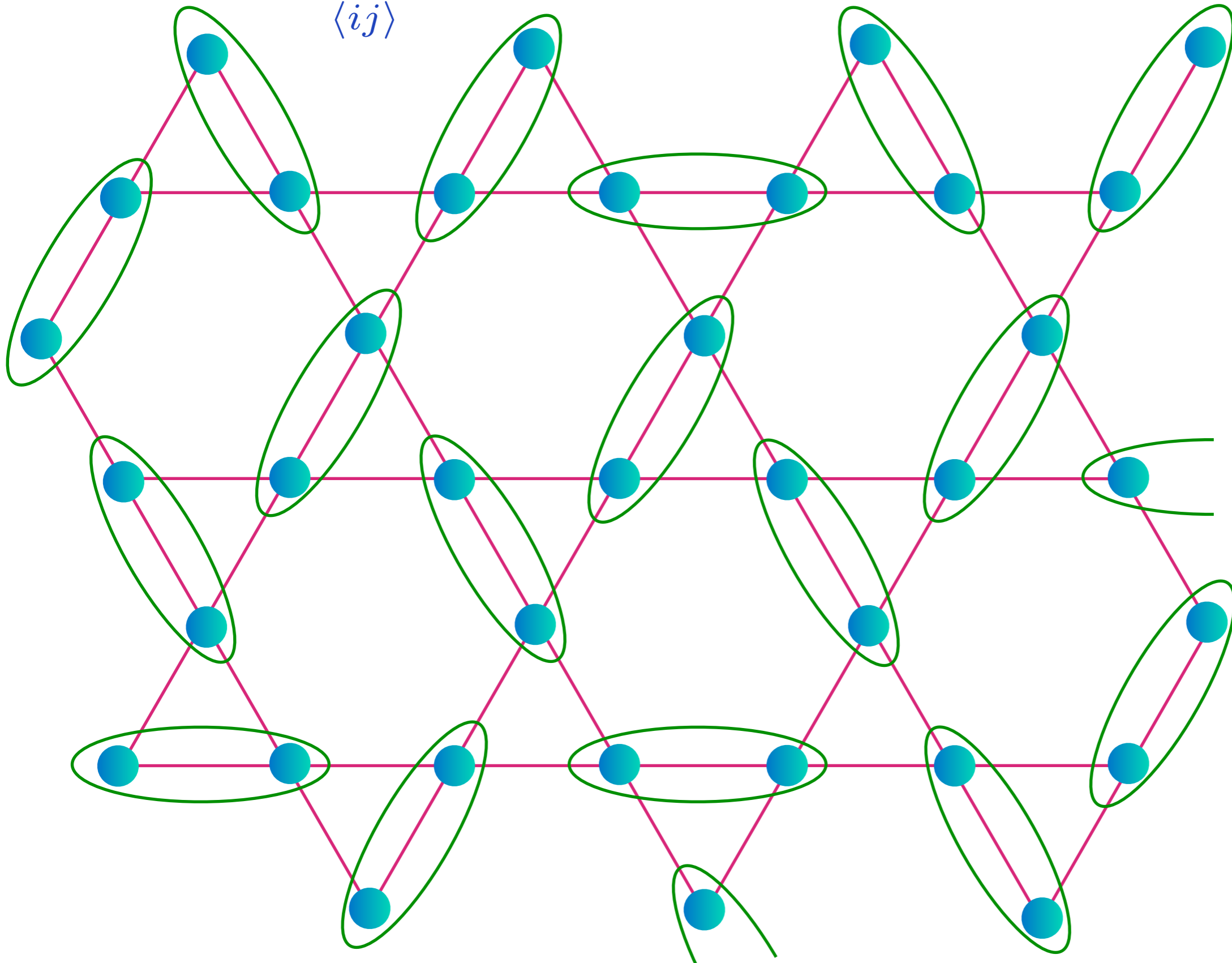


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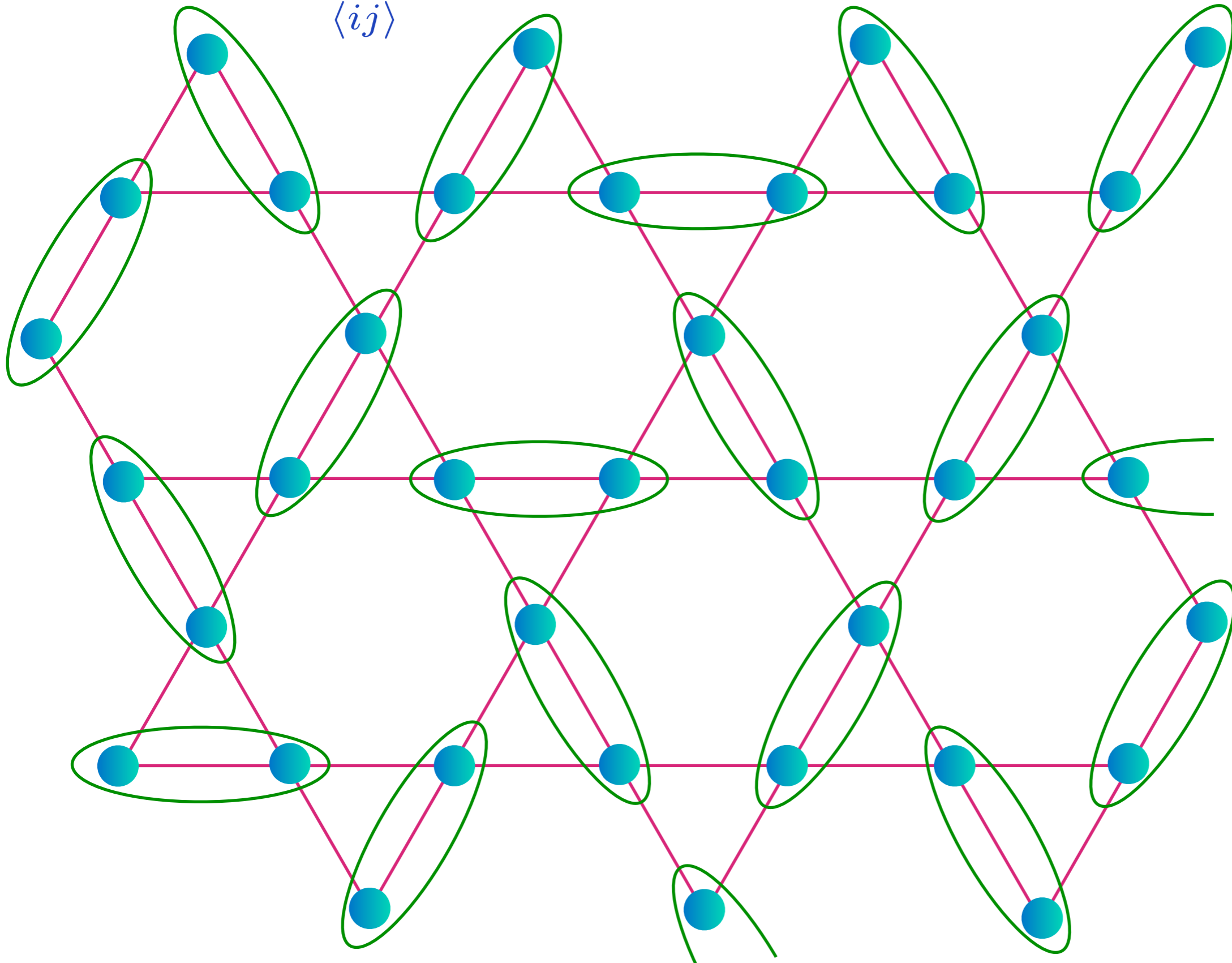


P. Fazekas and
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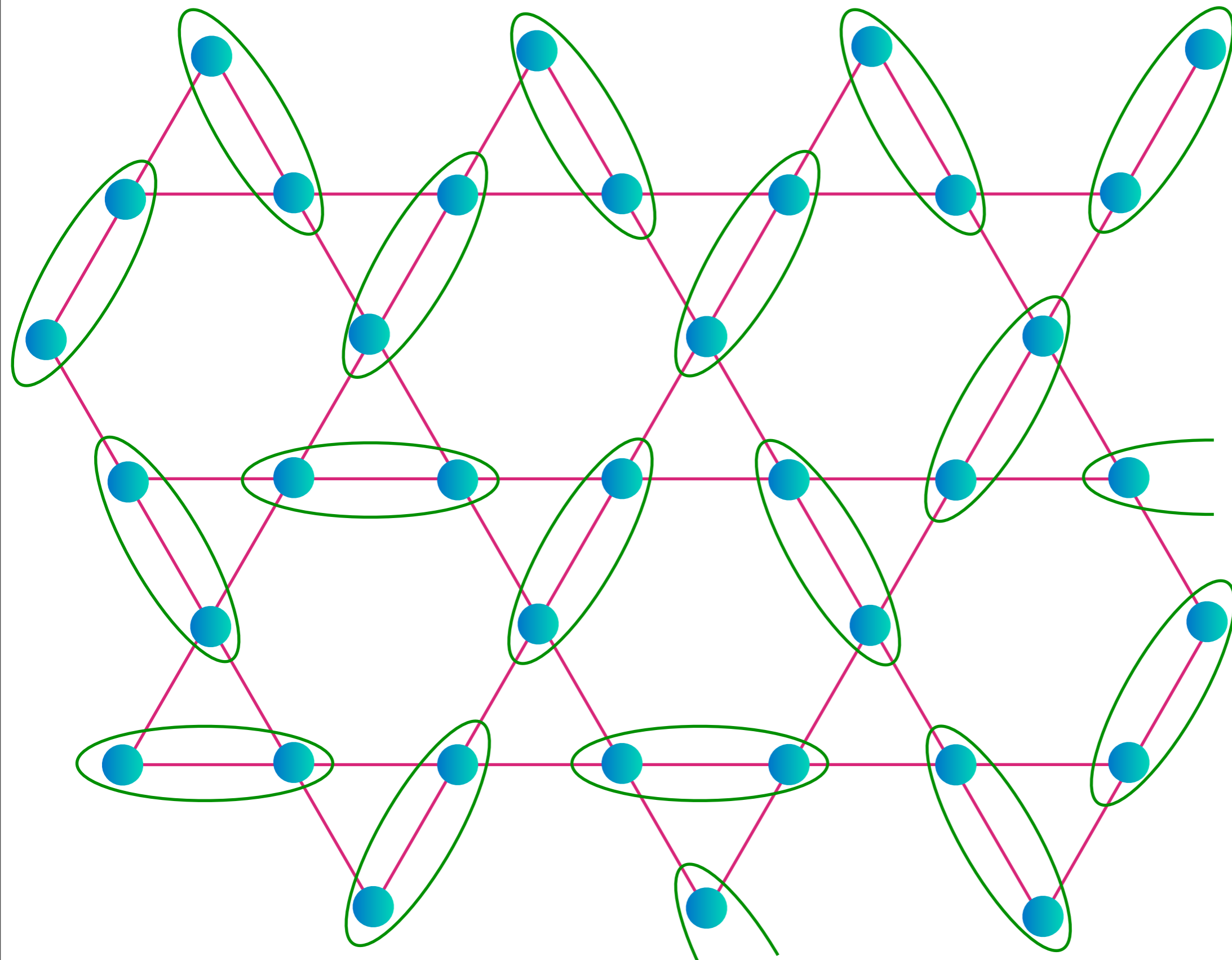


P. Fazekas and
P. W. Anderson,
Philos. Mag.
30, 23 (1974).

Mott insulator: Kagome antiferromagnet

Alternative view

Pick a reference configuration

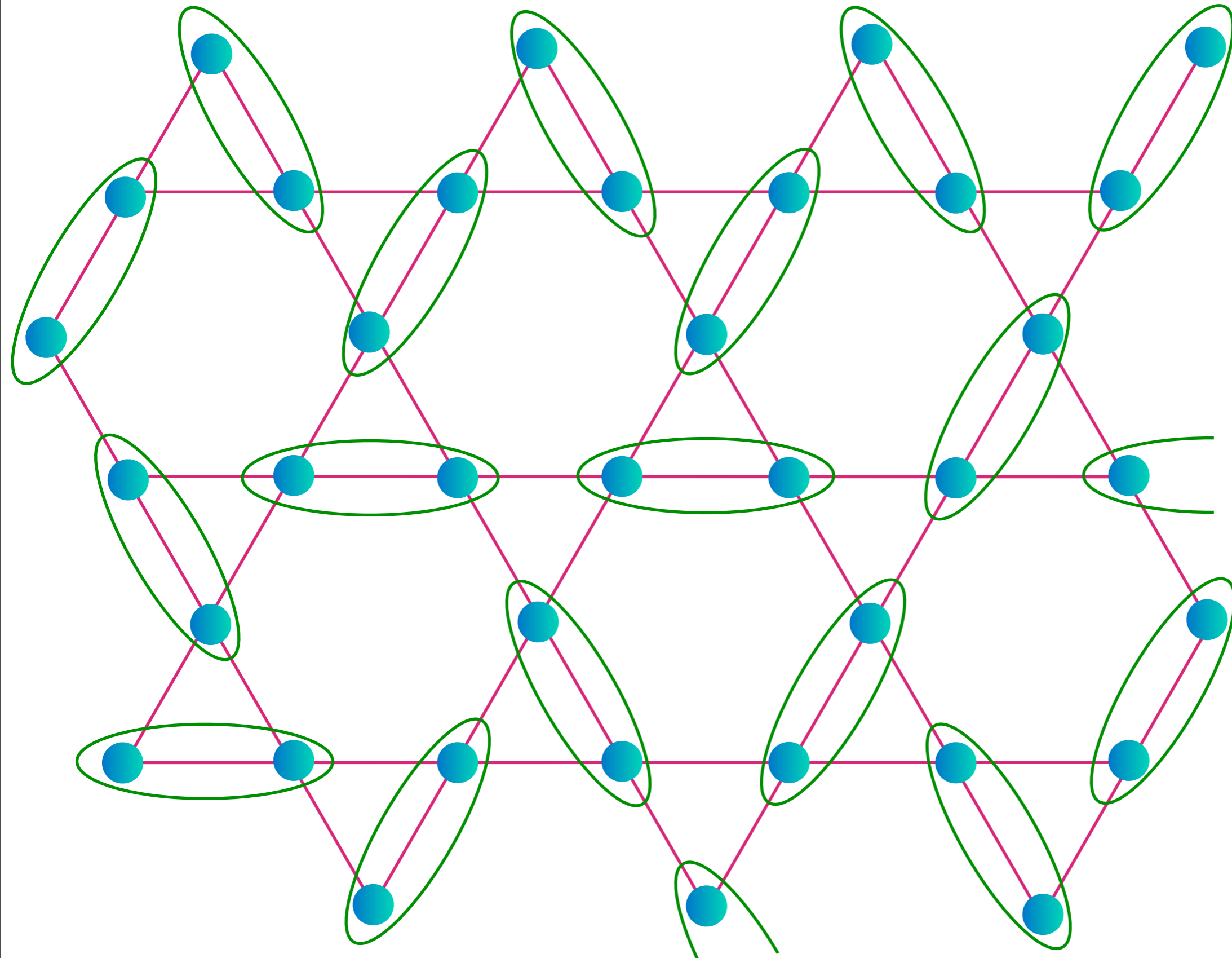


D. Rokhsar and
S. Kivelson,
Phys. Rev. Lett.
61, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

A nearby configuration

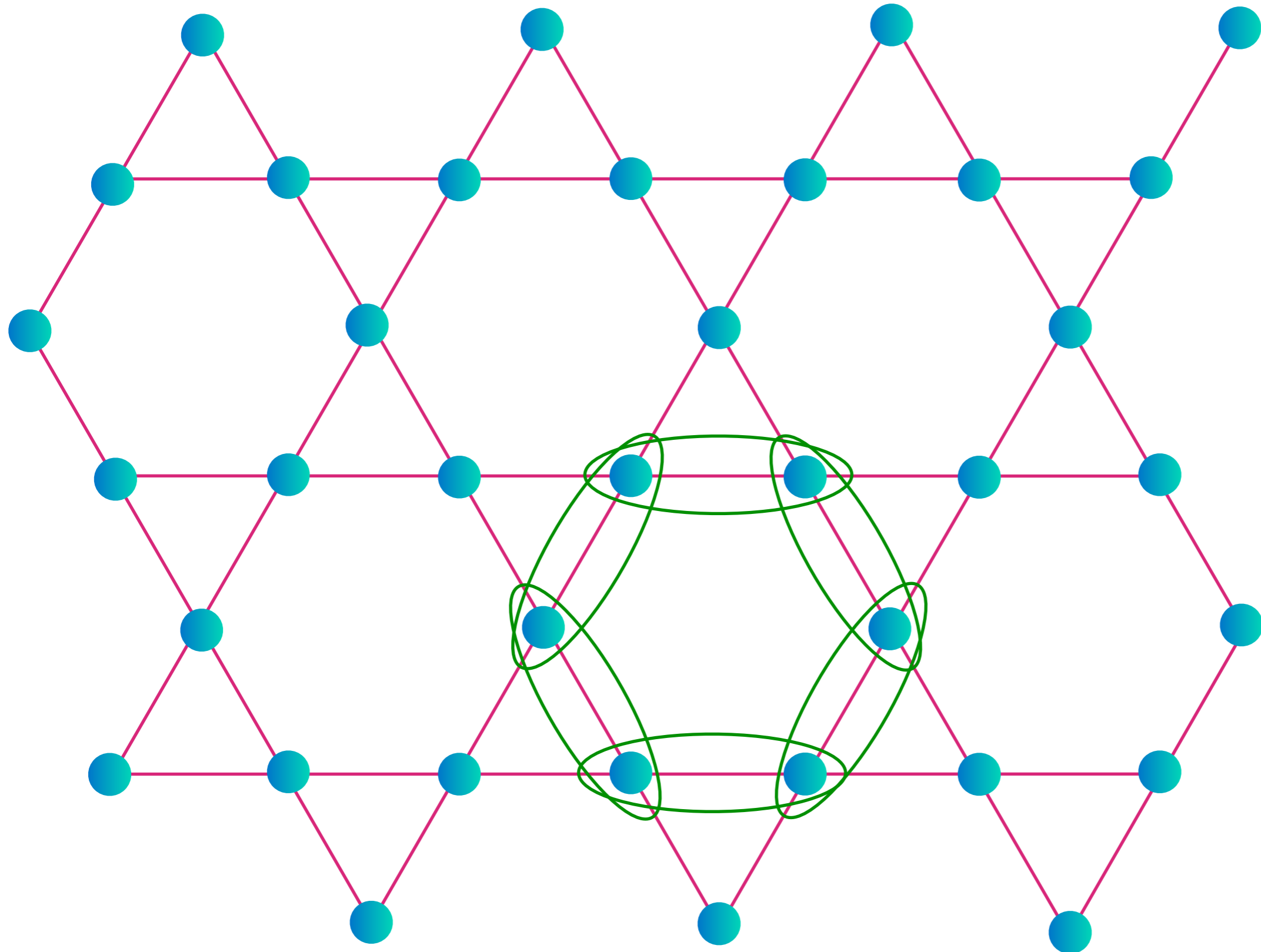


D. Rokhsar and
S. Kivelson,
Phys. Rev. Lett.
61, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

Difference: a closed loop

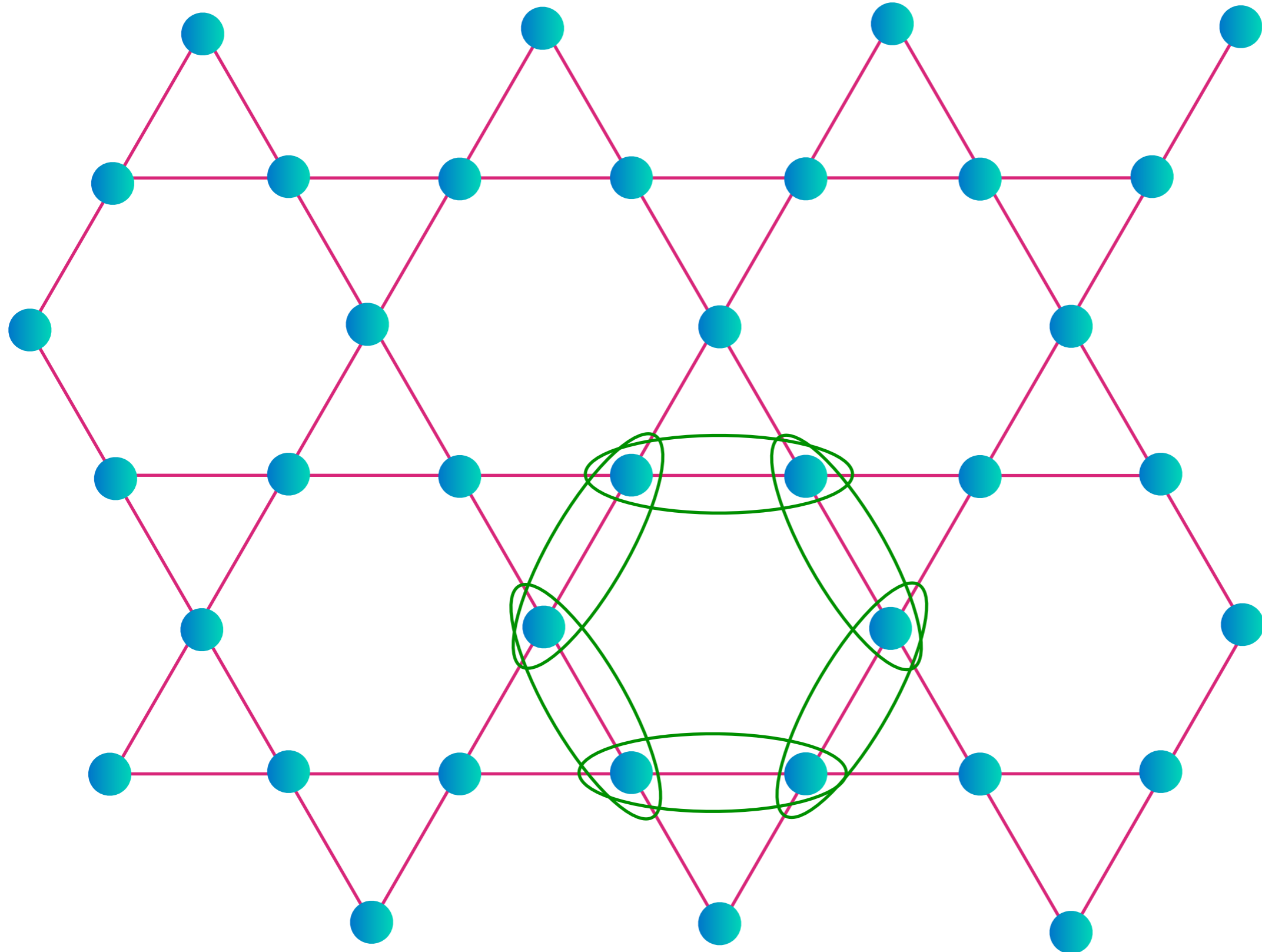


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Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops

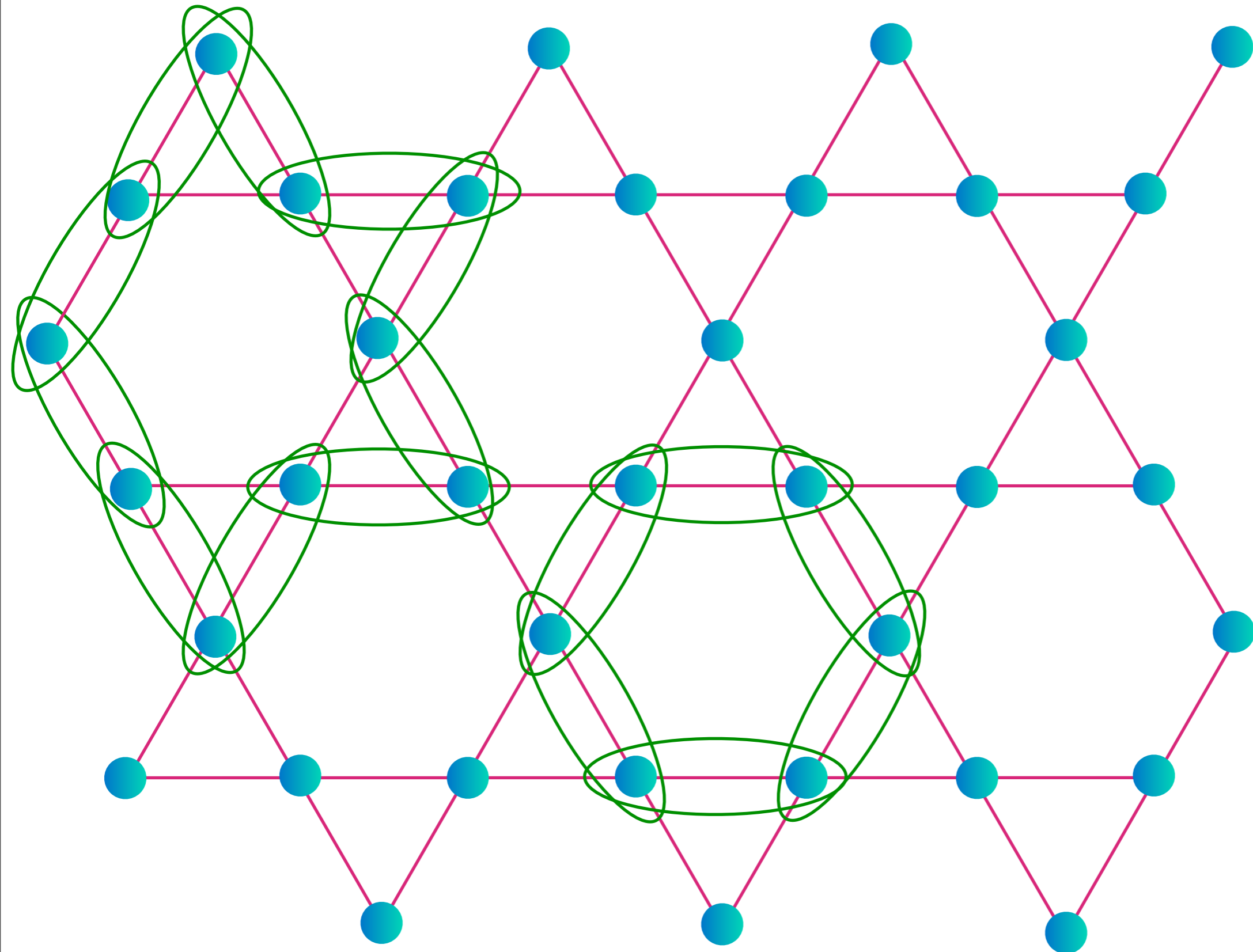


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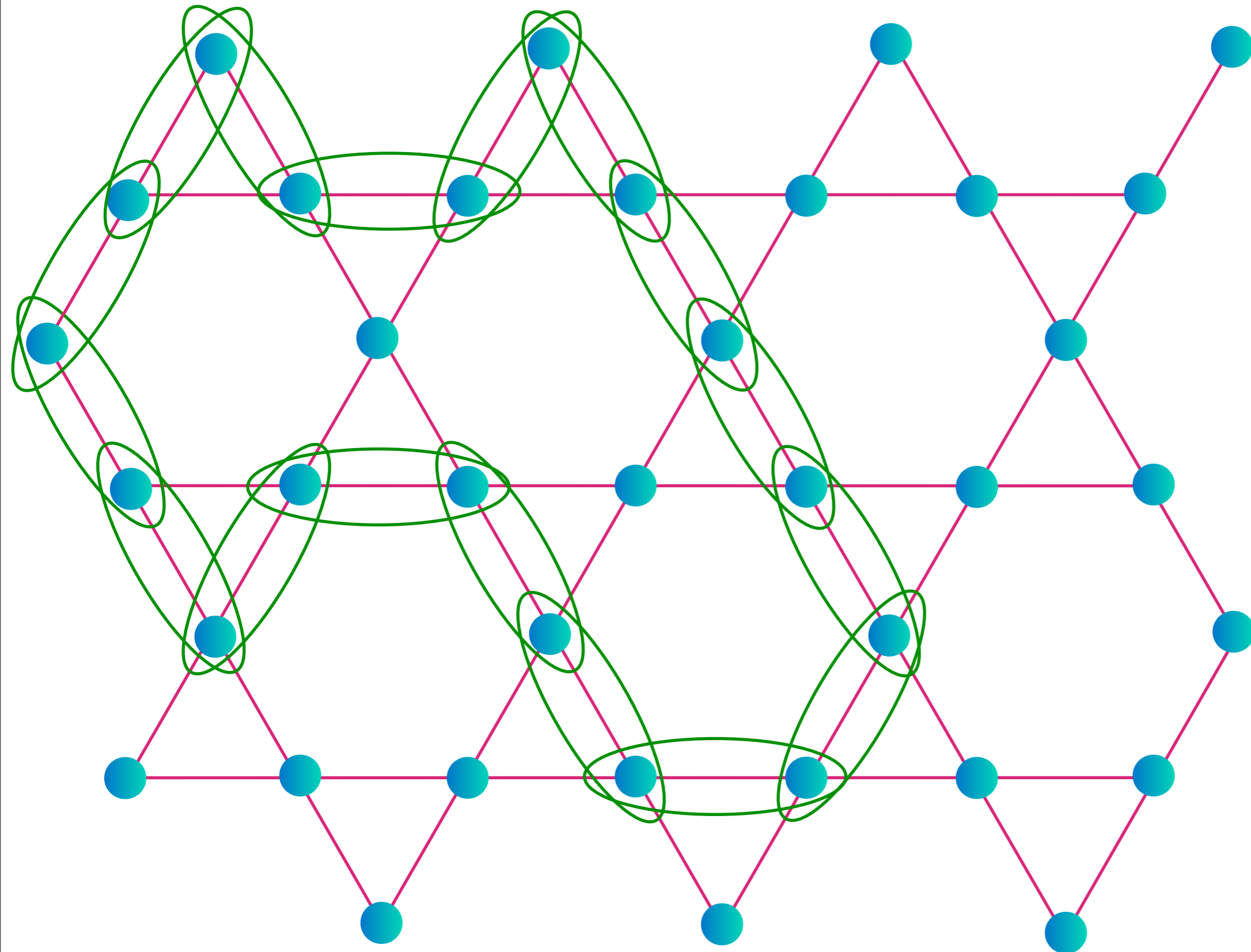


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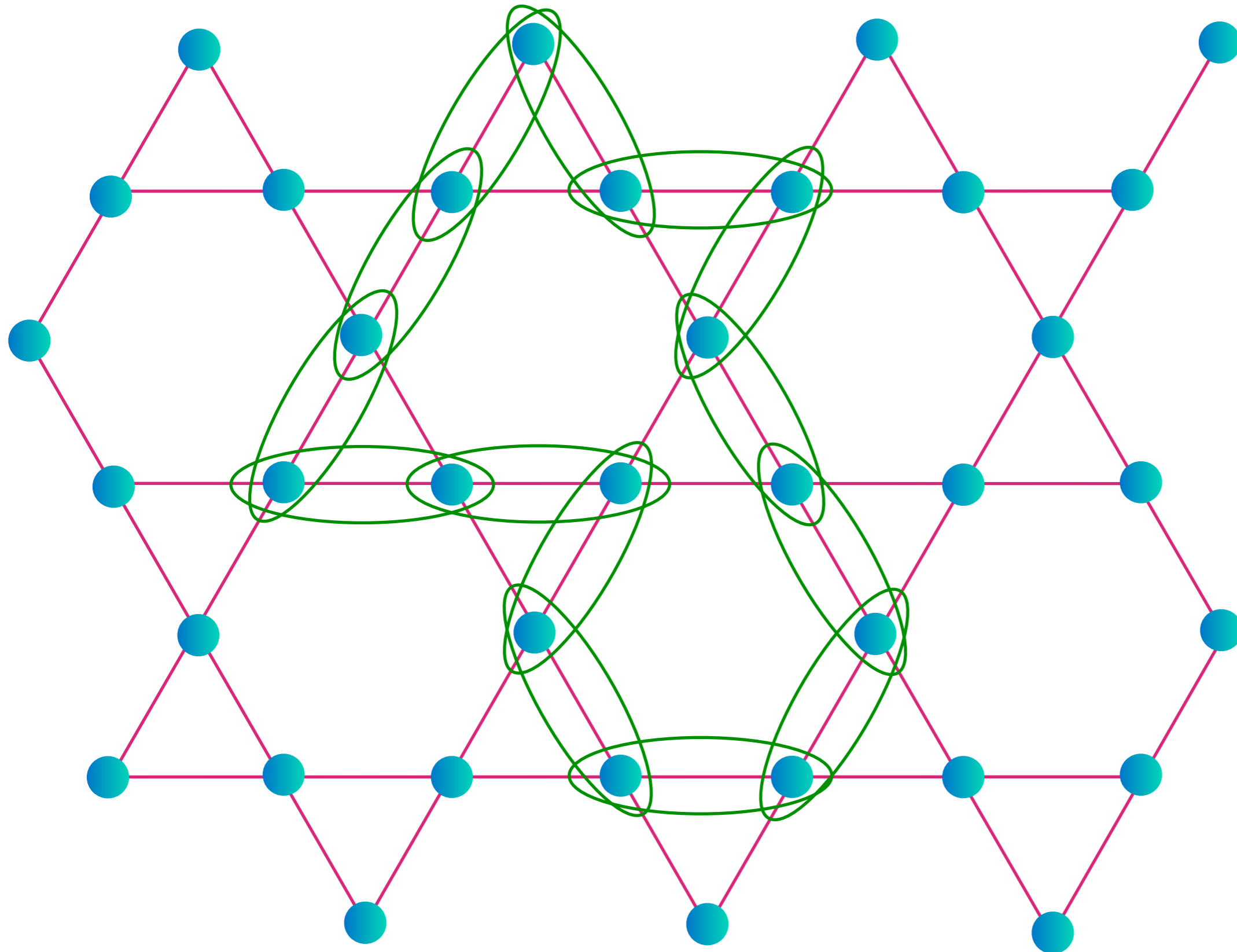


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Mott insulator: Kagome antiferromagnet

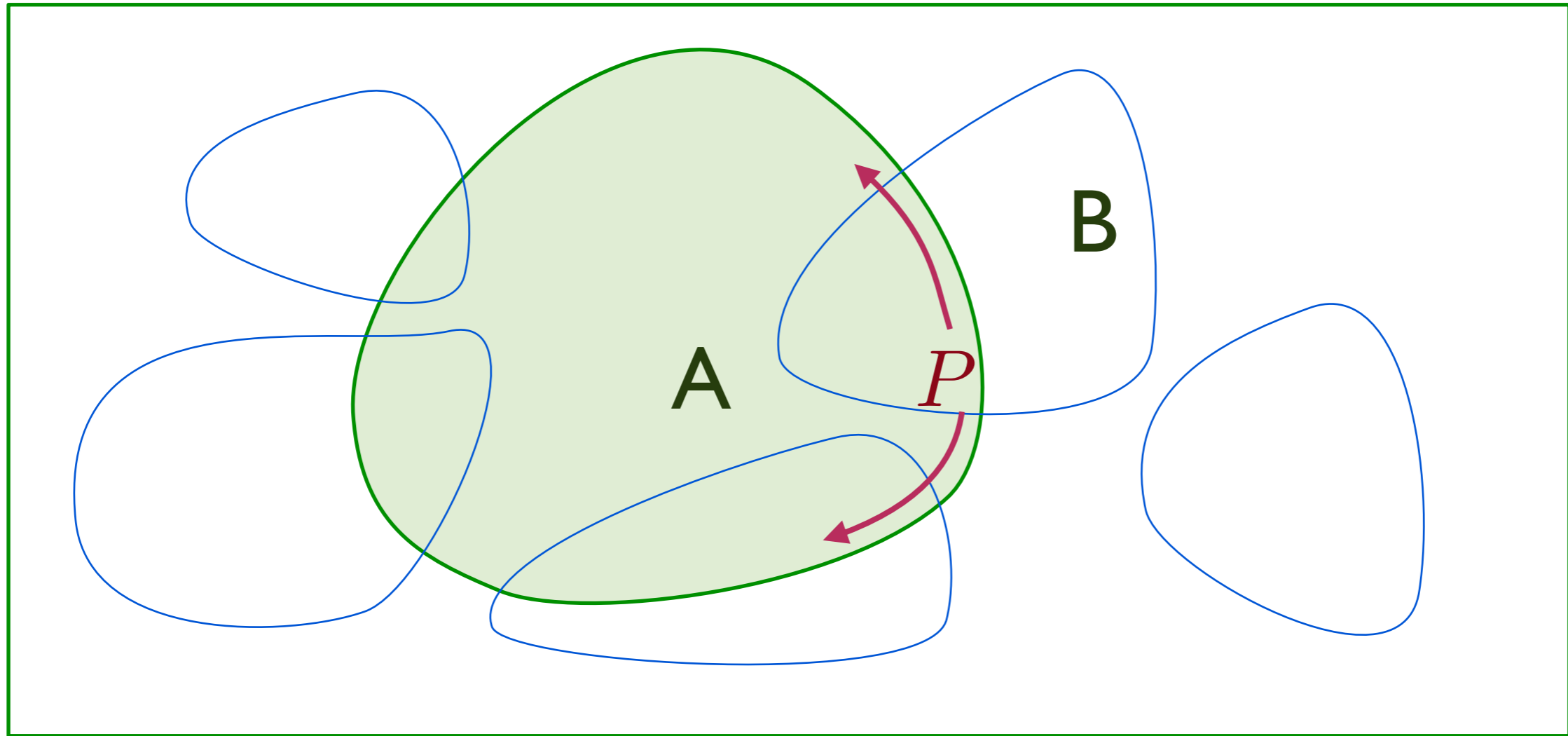
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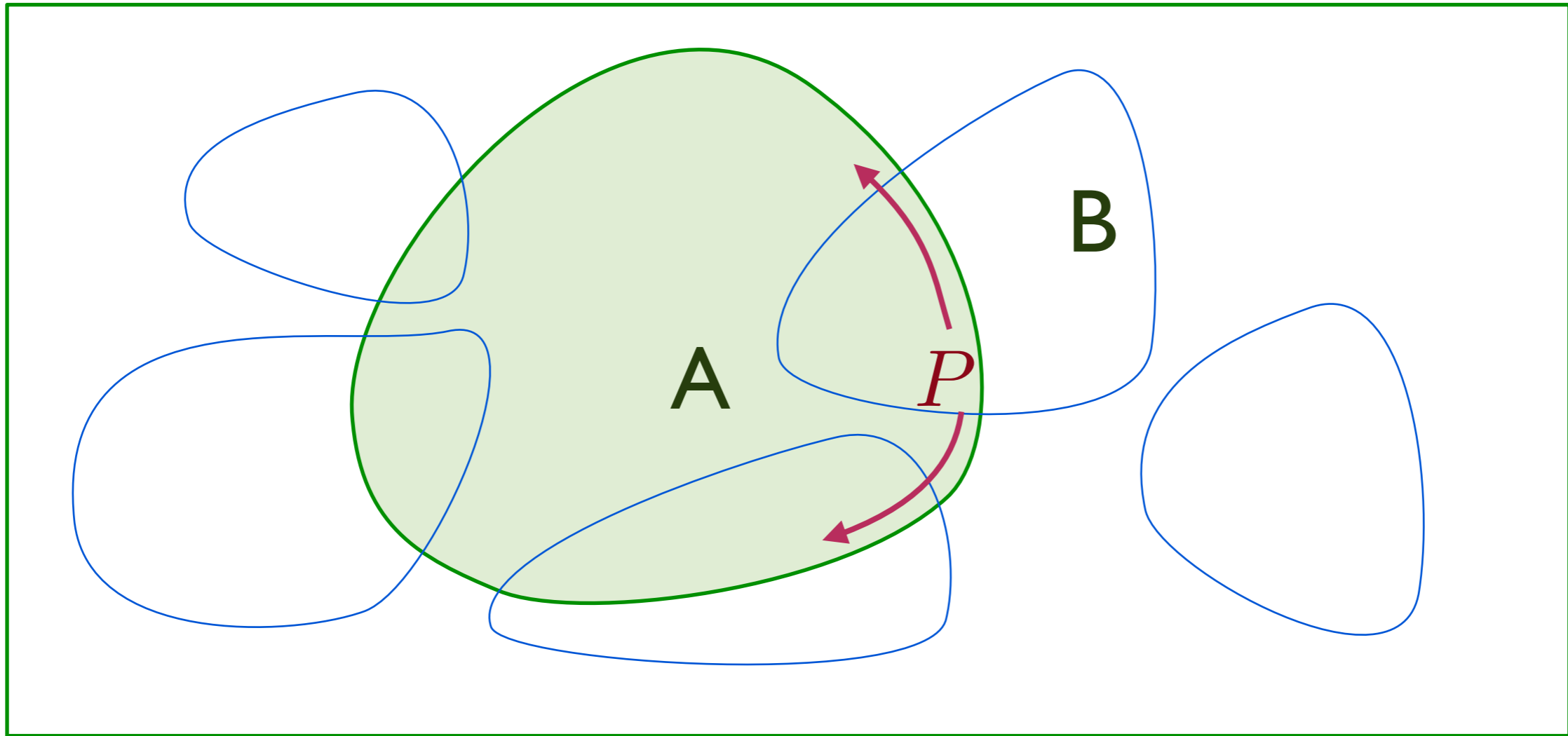
D. Rokhsar and
S. Kivelson,
Phys. Rev. Lett.
61, 2376 (1988).

Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

Entanglement in the Z_2 spin liquid

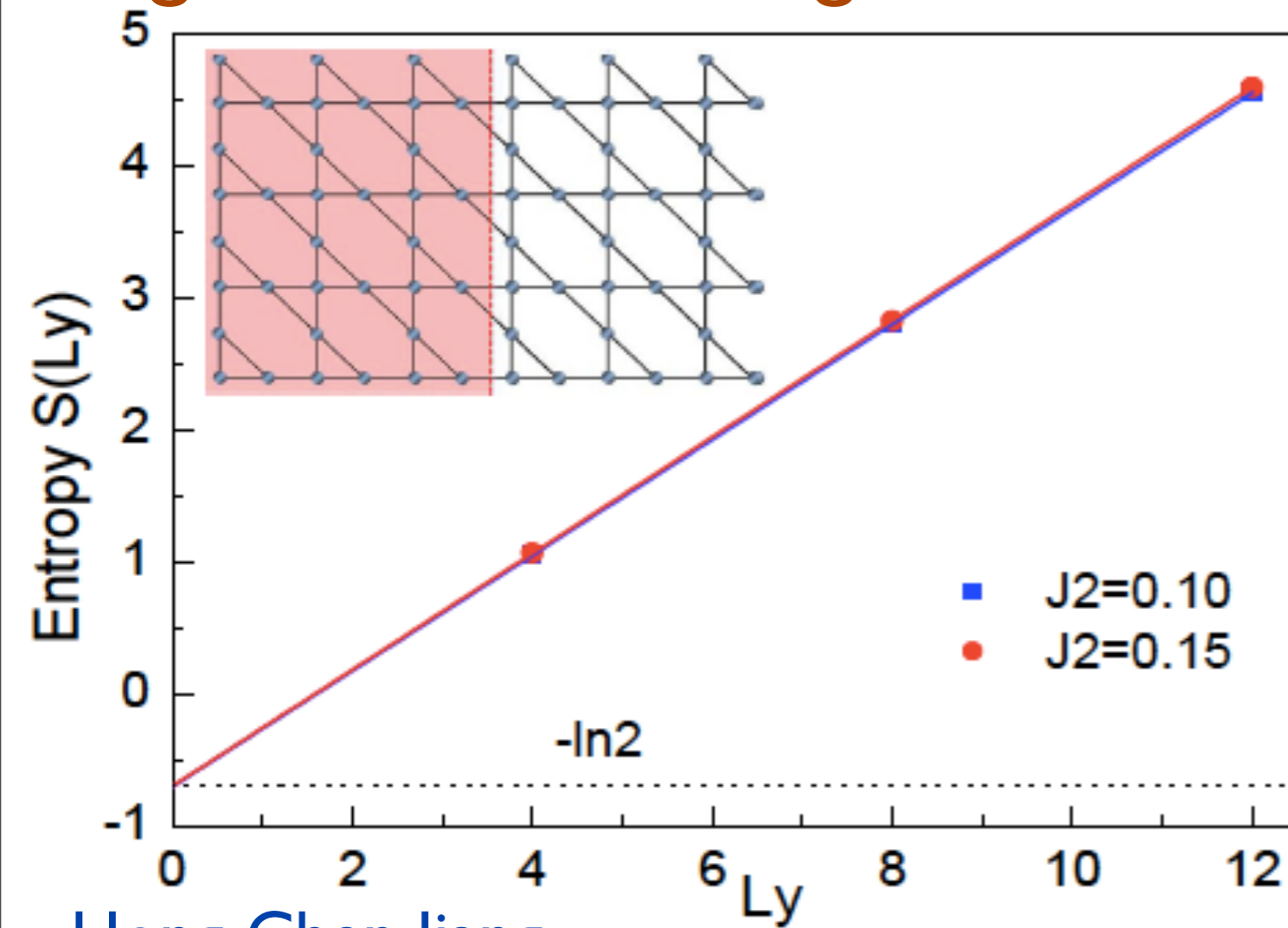


$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

A. Hamma, R. Ionicioiu, and P. Zanardi, Phys. Rev. A **71**, 022315 (2005)
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006); A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006)
Y. Zhang, T. Grover, and A. Vishwanath, Phys. Rev. B **84**, 075128 (2011)

Kagome antiferromagnet

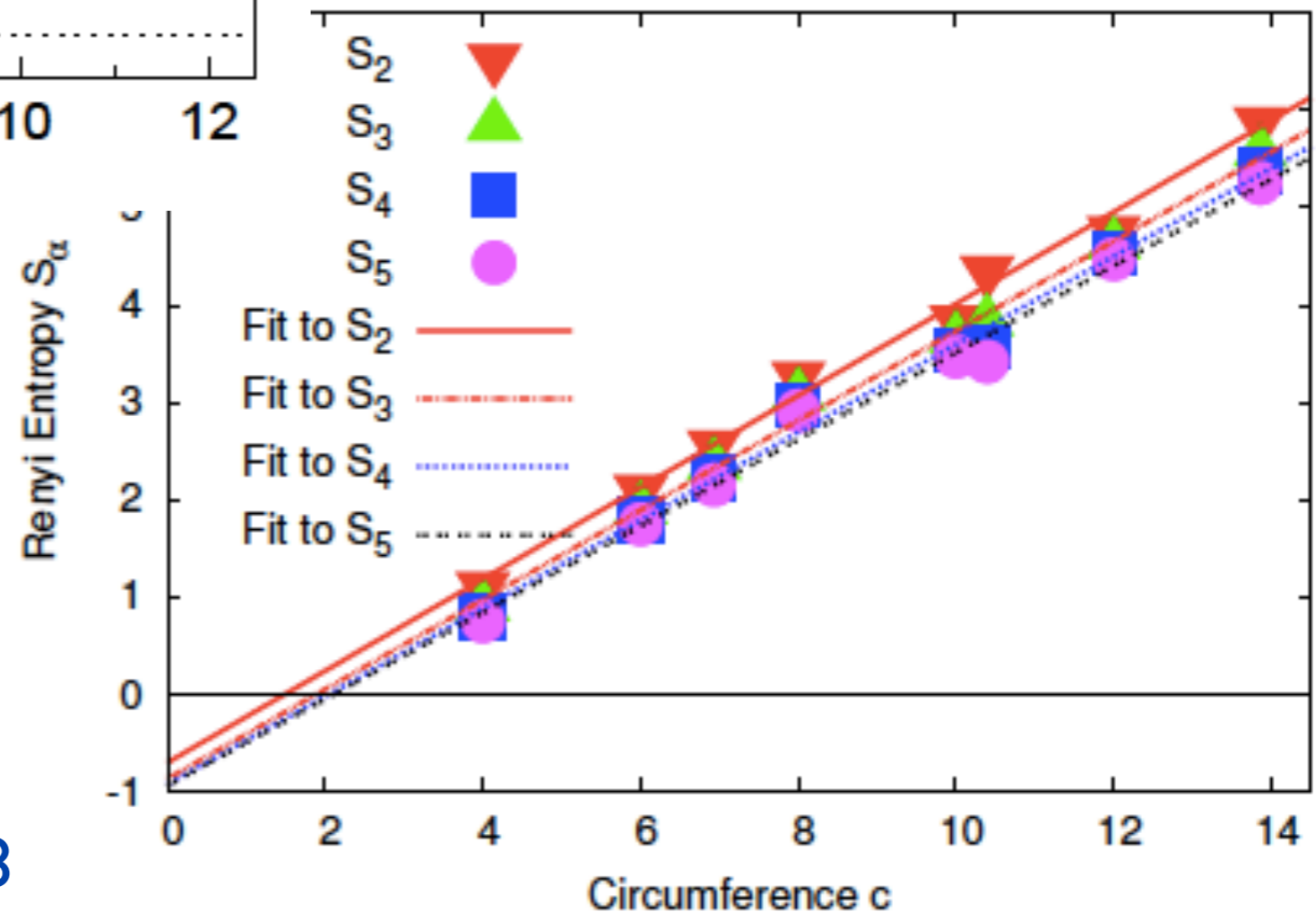


Hong-Chen Jiang,
Z. Wang,
and L. Balents,
arXiv:1205.4289

S. Depenbrock,
I. P. McCulloch,
and
U. Schollwoeck,
arXiv:1205.4858

Strong numerical evidence
for a Z_2 spin liquid

Simeng Yan, D.A. Huse,
and S. R. White,
Science **332**, 1173 (2011).



Kagome antiferromagnet

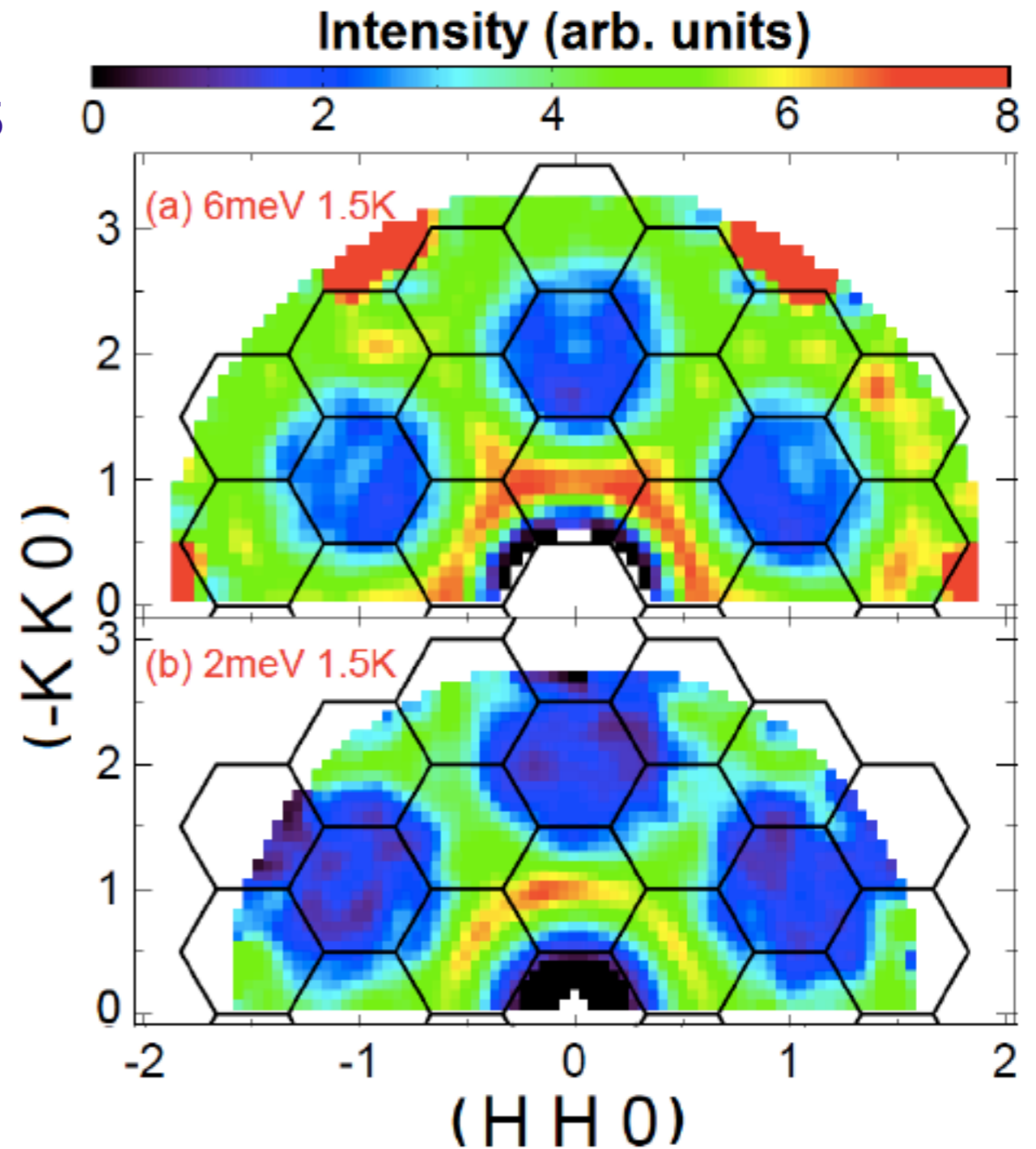
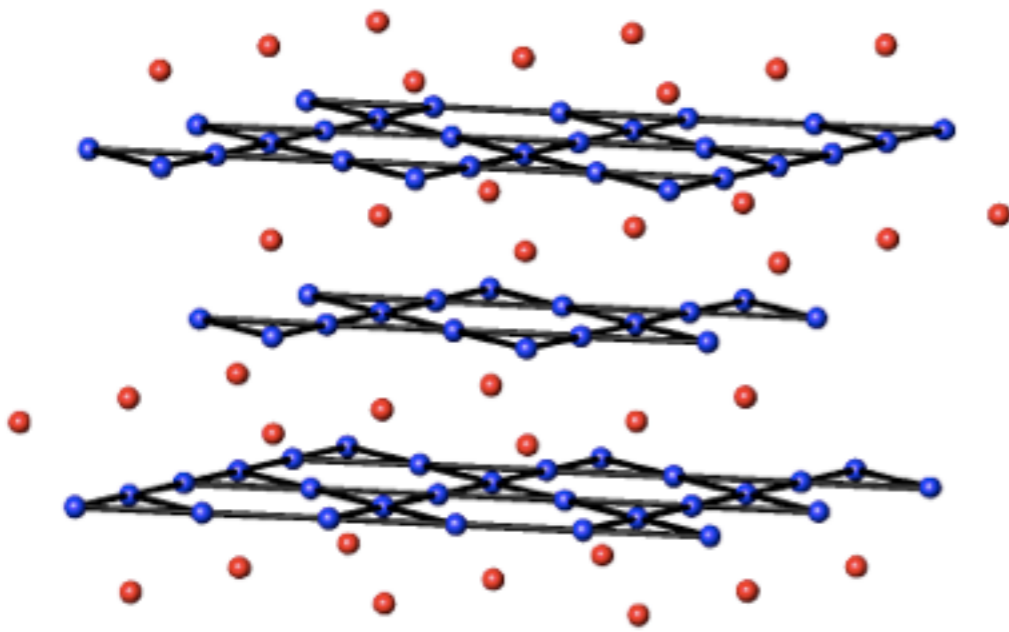
Evidence for spinons

Young Lee,

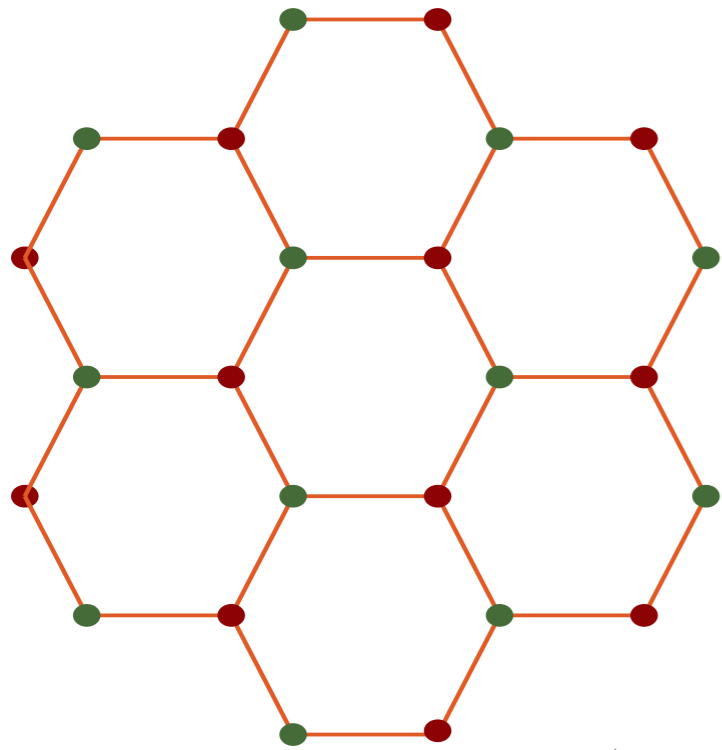
APS meeting, March 2012

<http://meetings.aps.org/link/BAPS.2012.MAR.H8.5>

$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$ (also called Herbertsmithite)

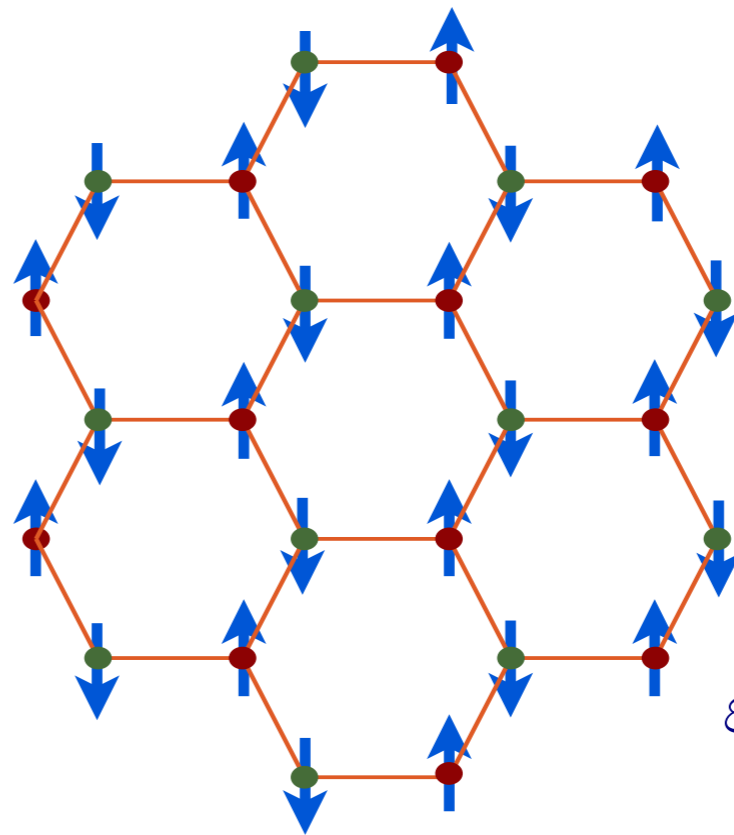
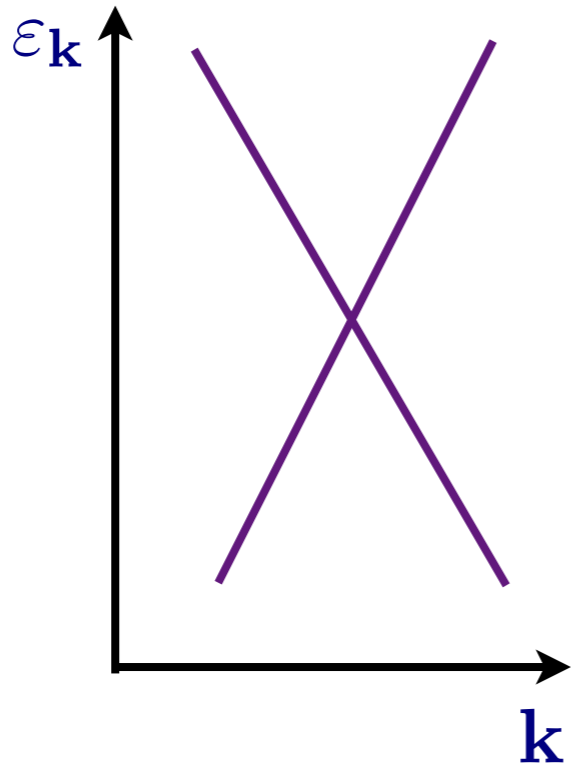


Conformal quantum matter



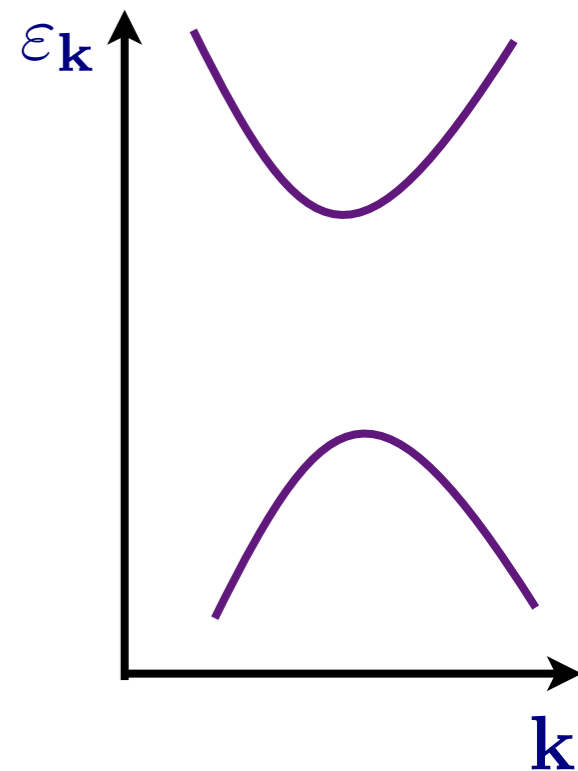
Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order

$$\langle \varphi^a \rangle \neq 0$$



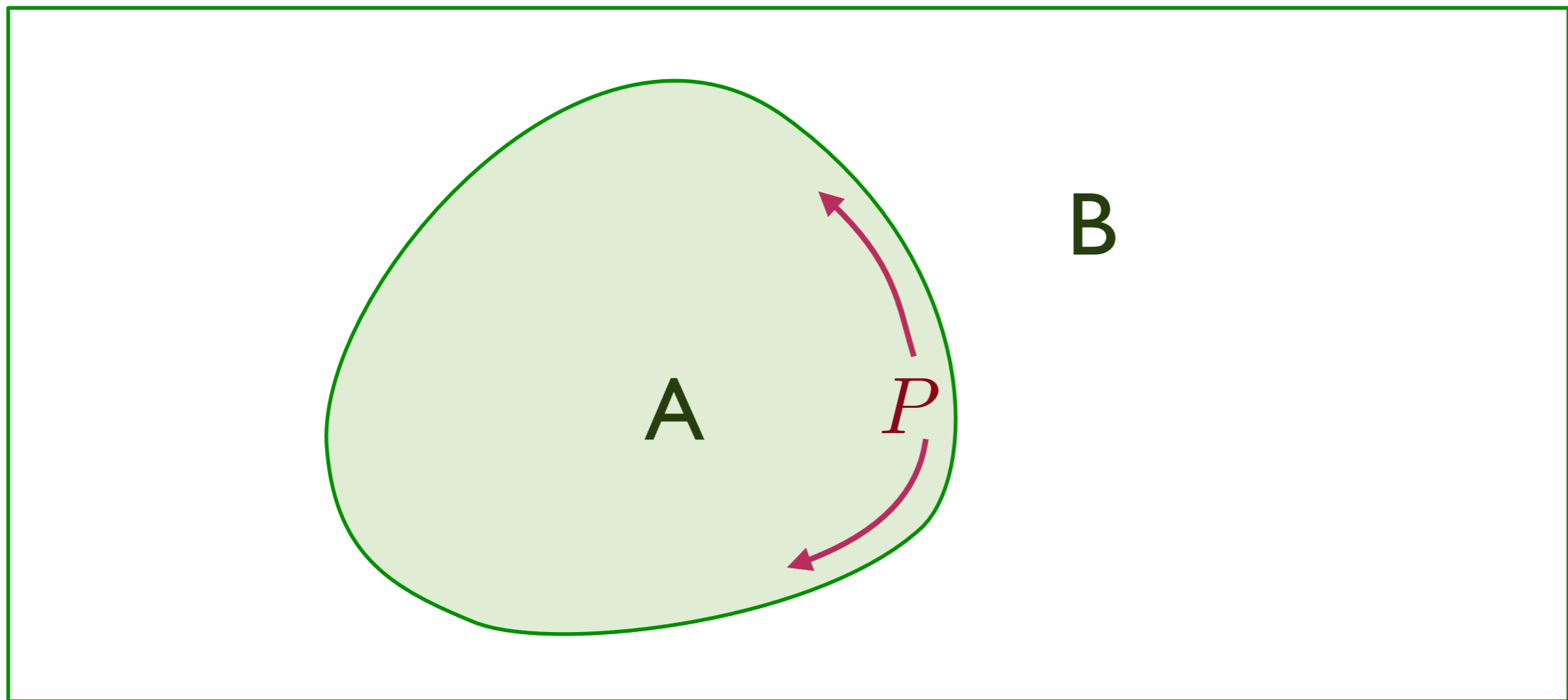
S

Free CFT3

Interacting CFT3
with long-range entanglement

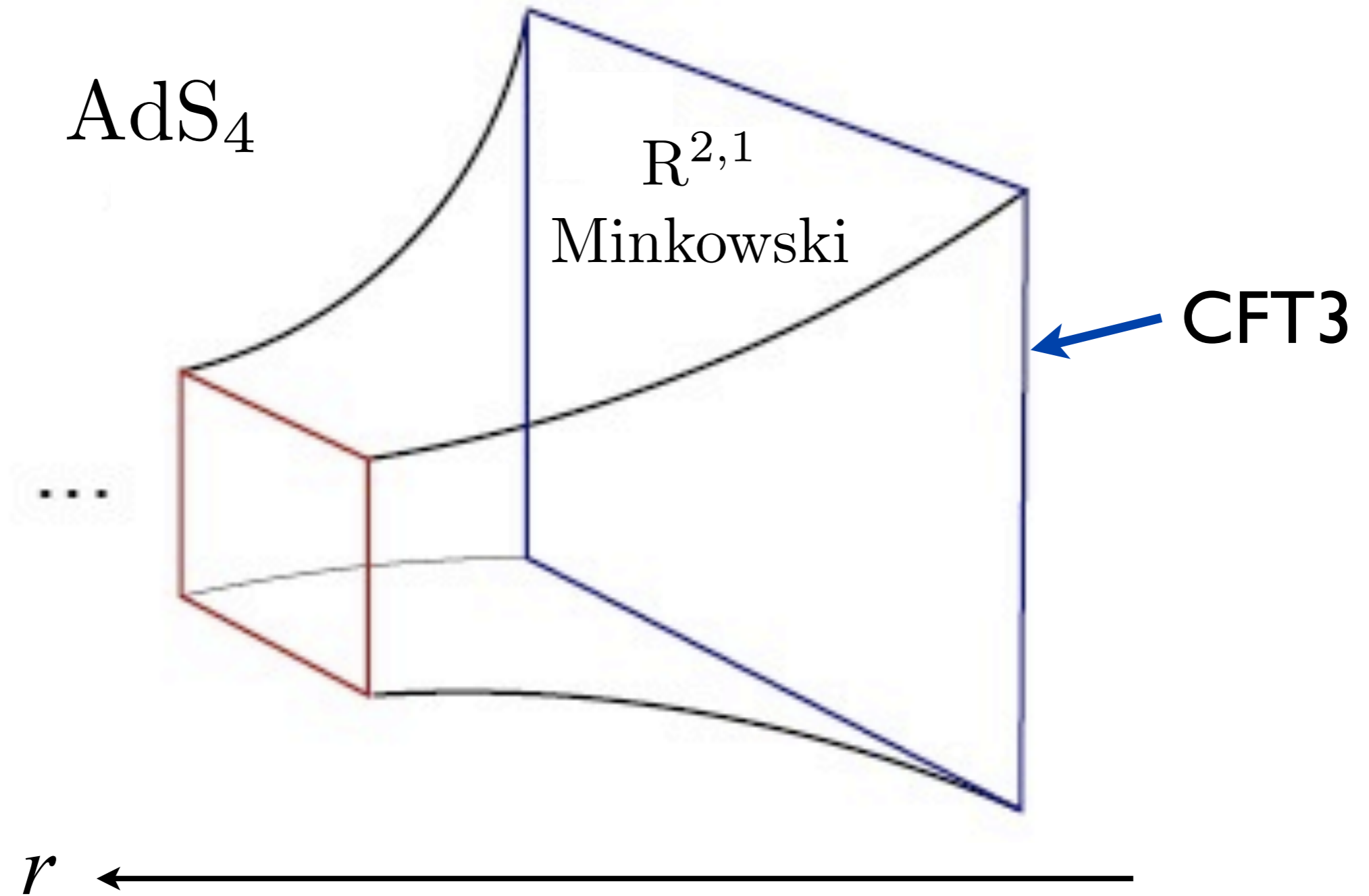
Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.

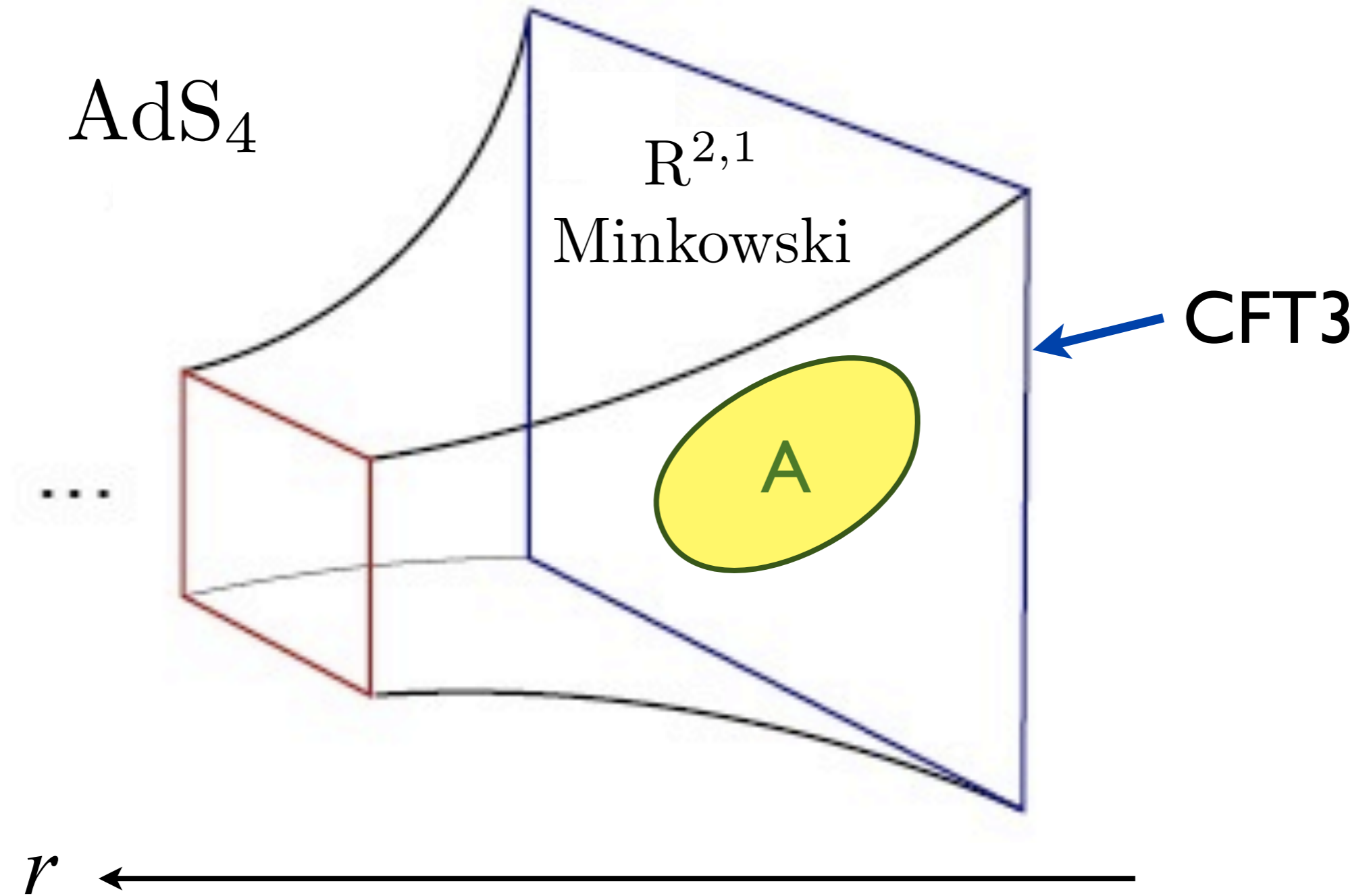


M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Phys. Rev. B 80, 115122 (2009)
B. Hsu, M. Mulligan, E. Fradkin, and Eun-Ah Kim, Phys. Rev. B 79, 115421 (2009)
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

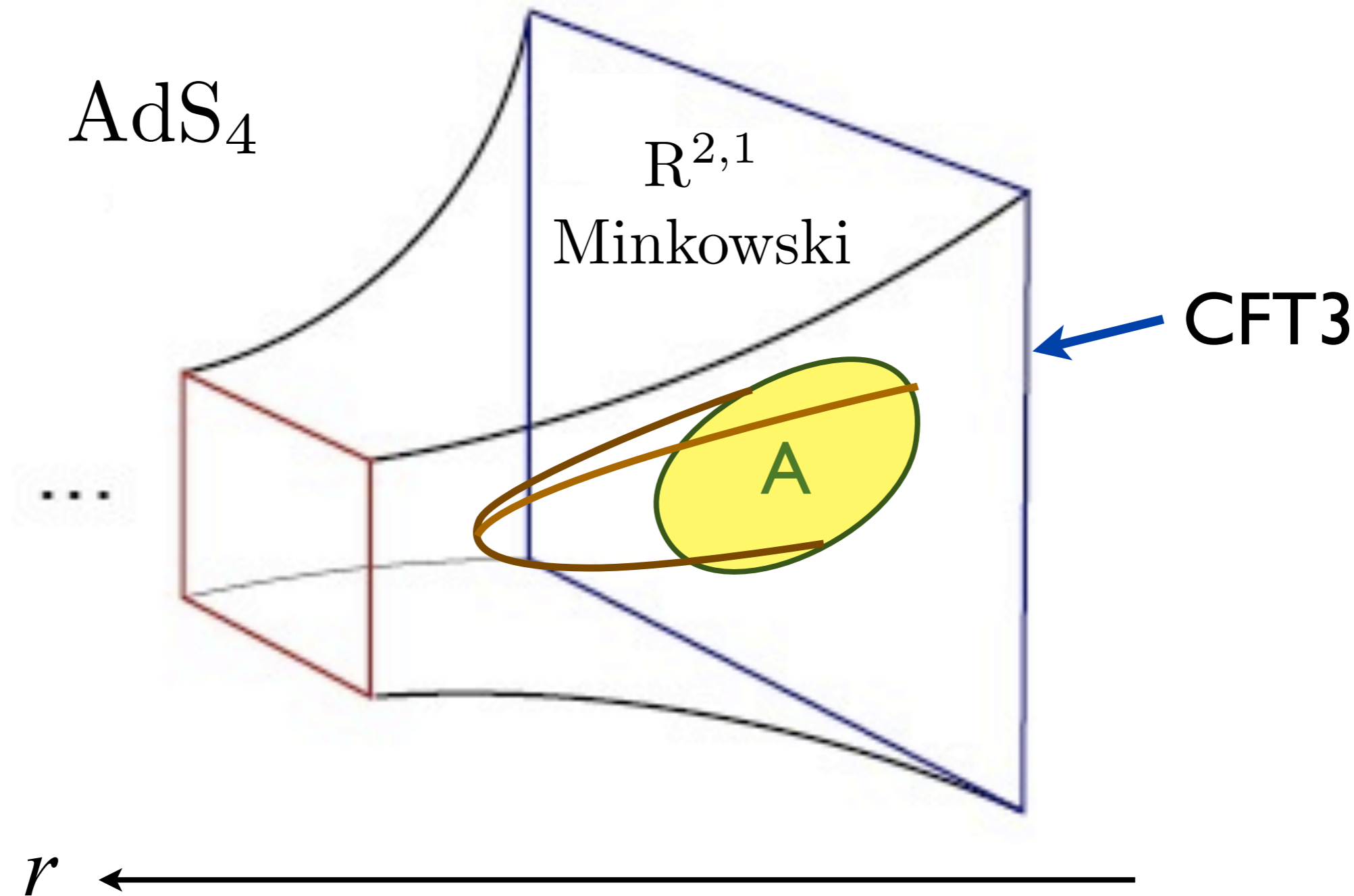
AdS/CFT correspondence



AdS/CFT correspondence



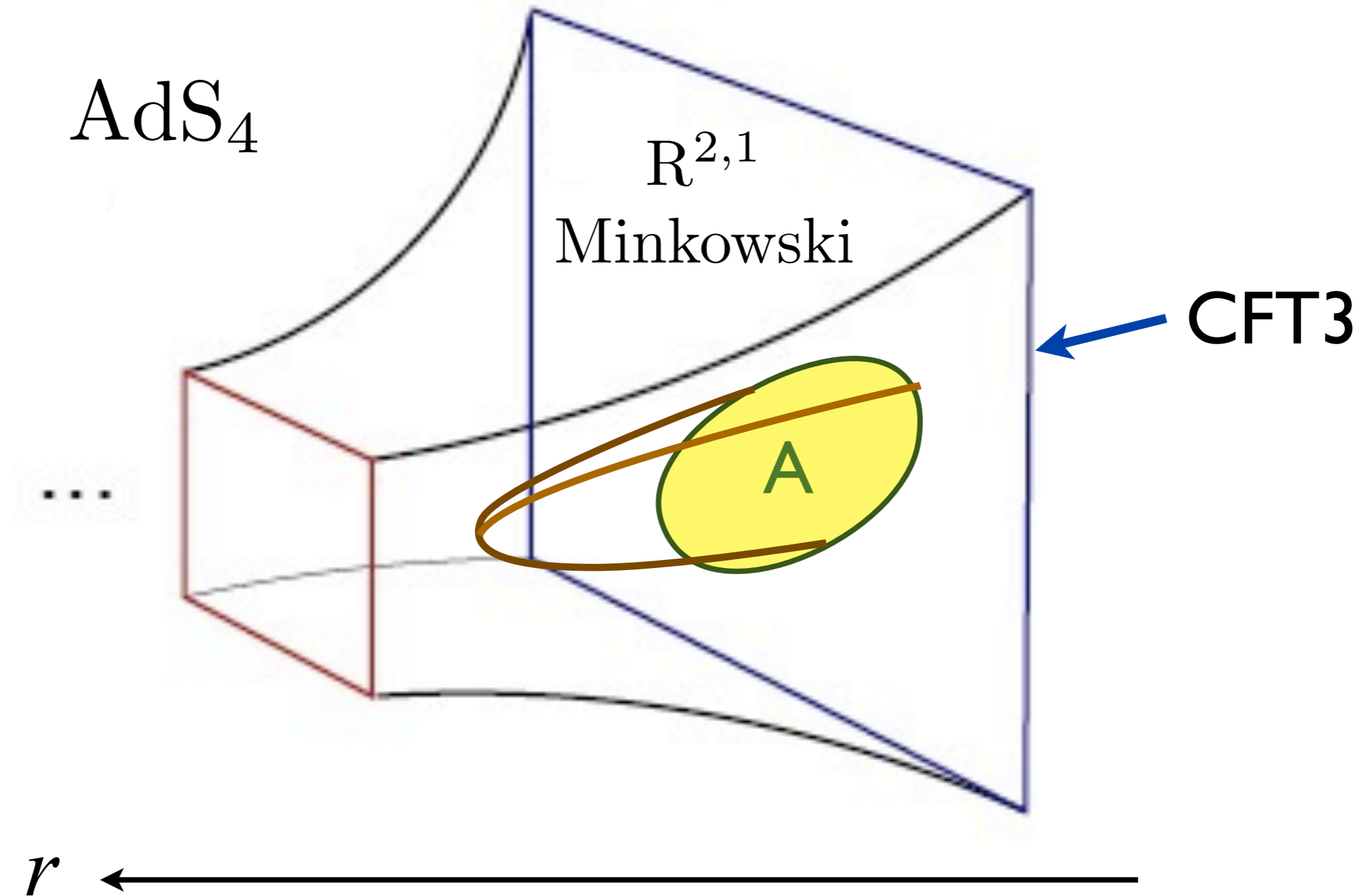
AdS/CFT correspondence



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe “outside” : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

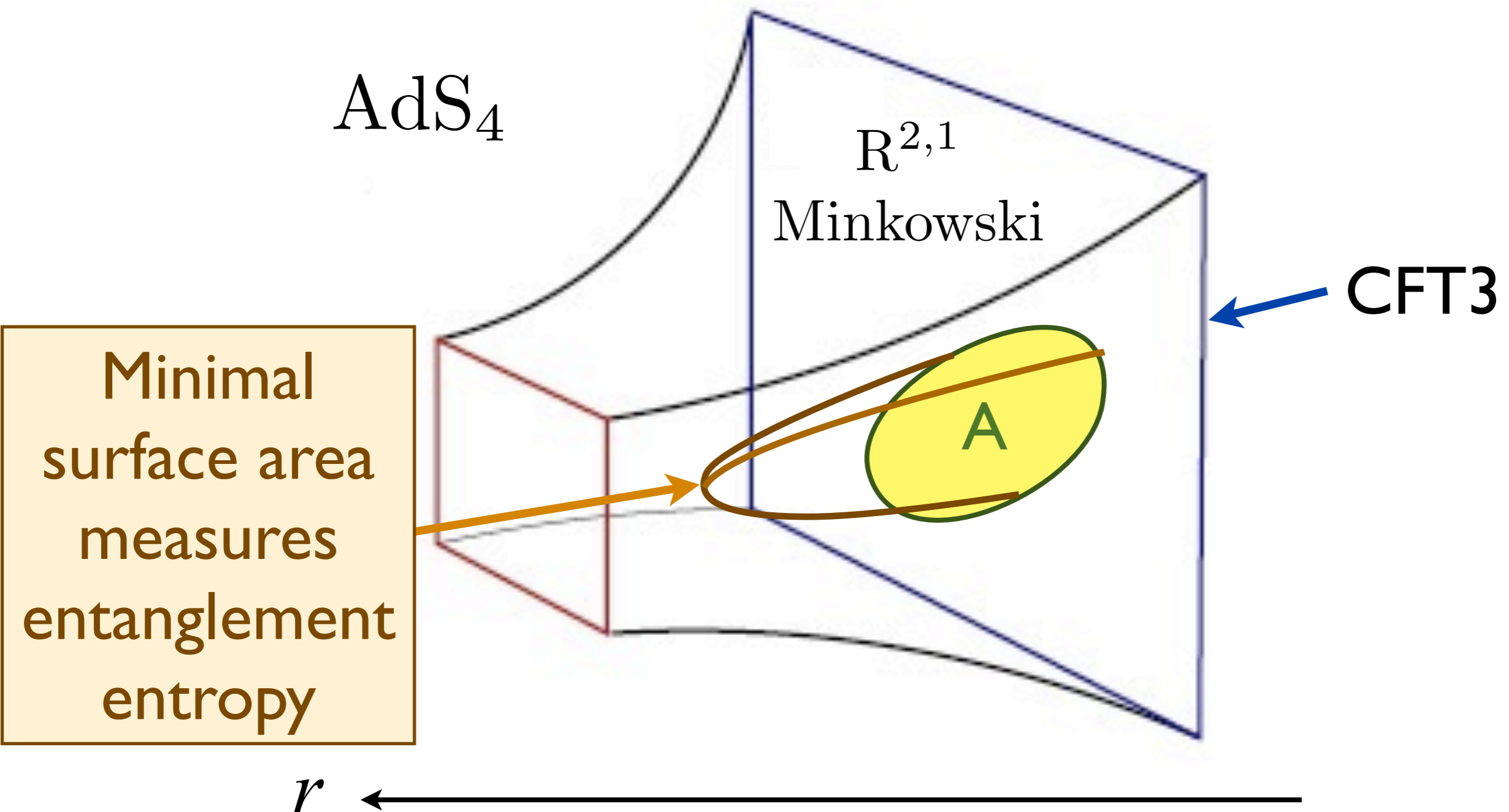
AdS/CFT correspondence



The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

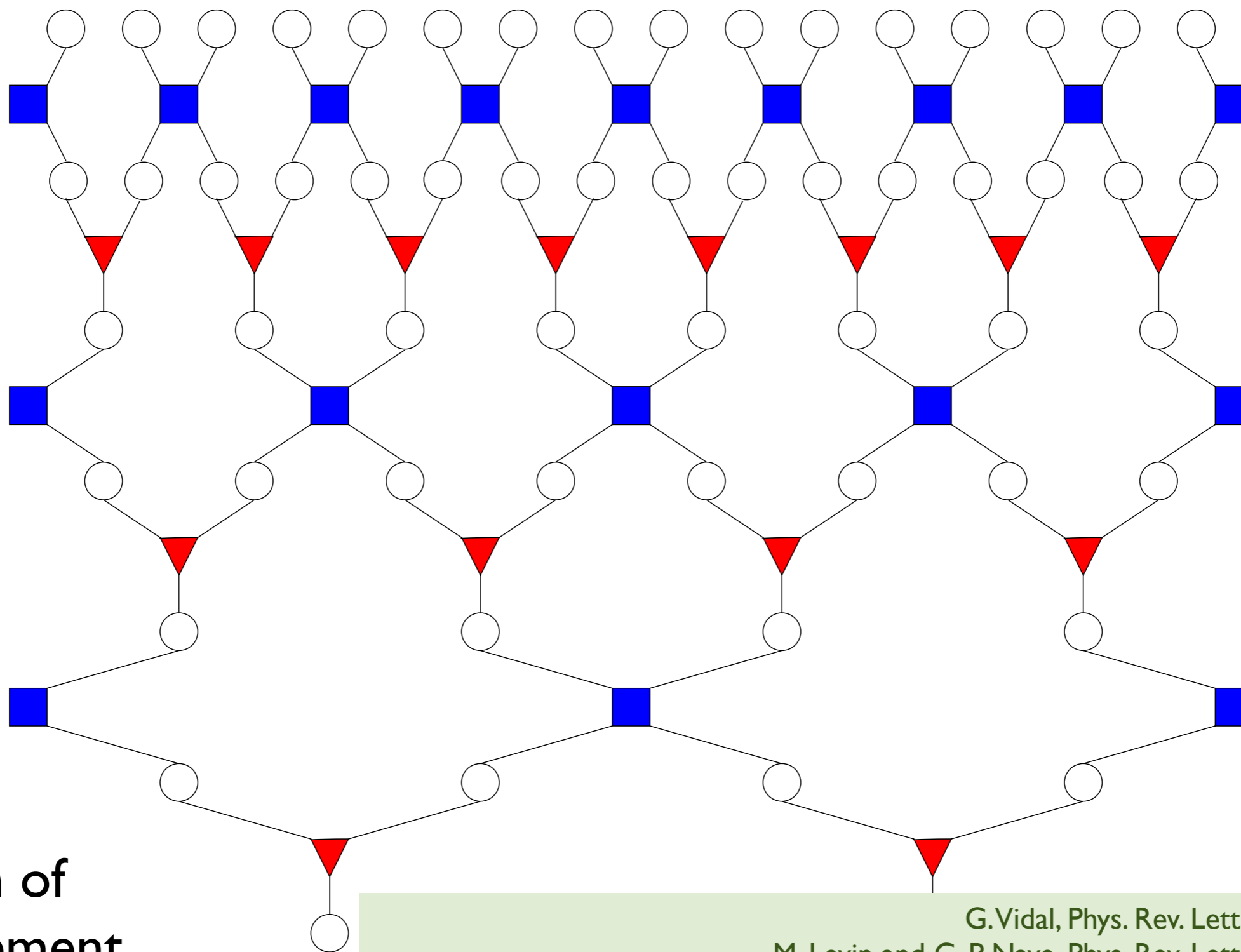
AdS/CFT correspondence



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Tensor network representation of entanglement at quantum critical point

d -dimensional
space

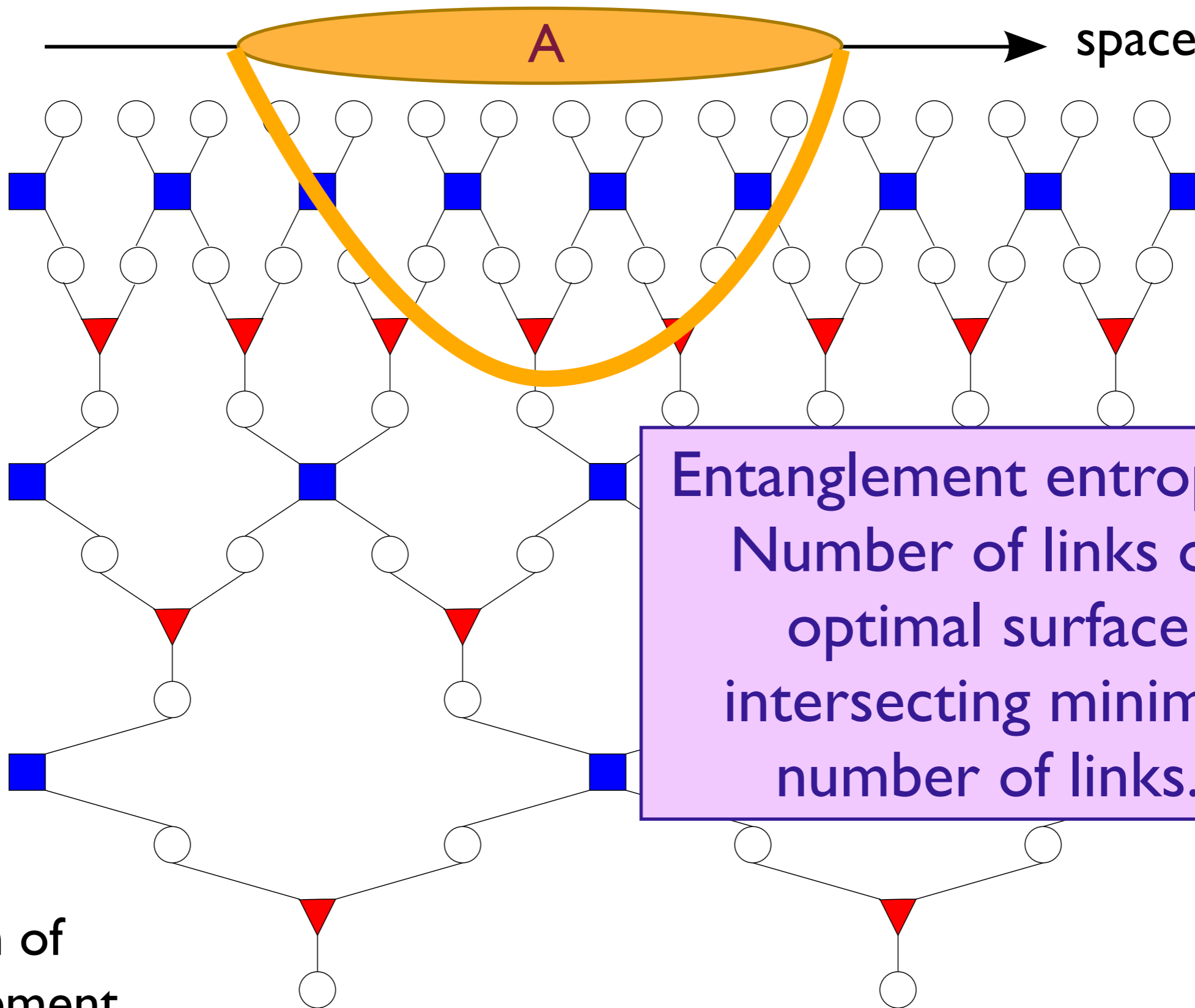


depth of
entanglement

G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

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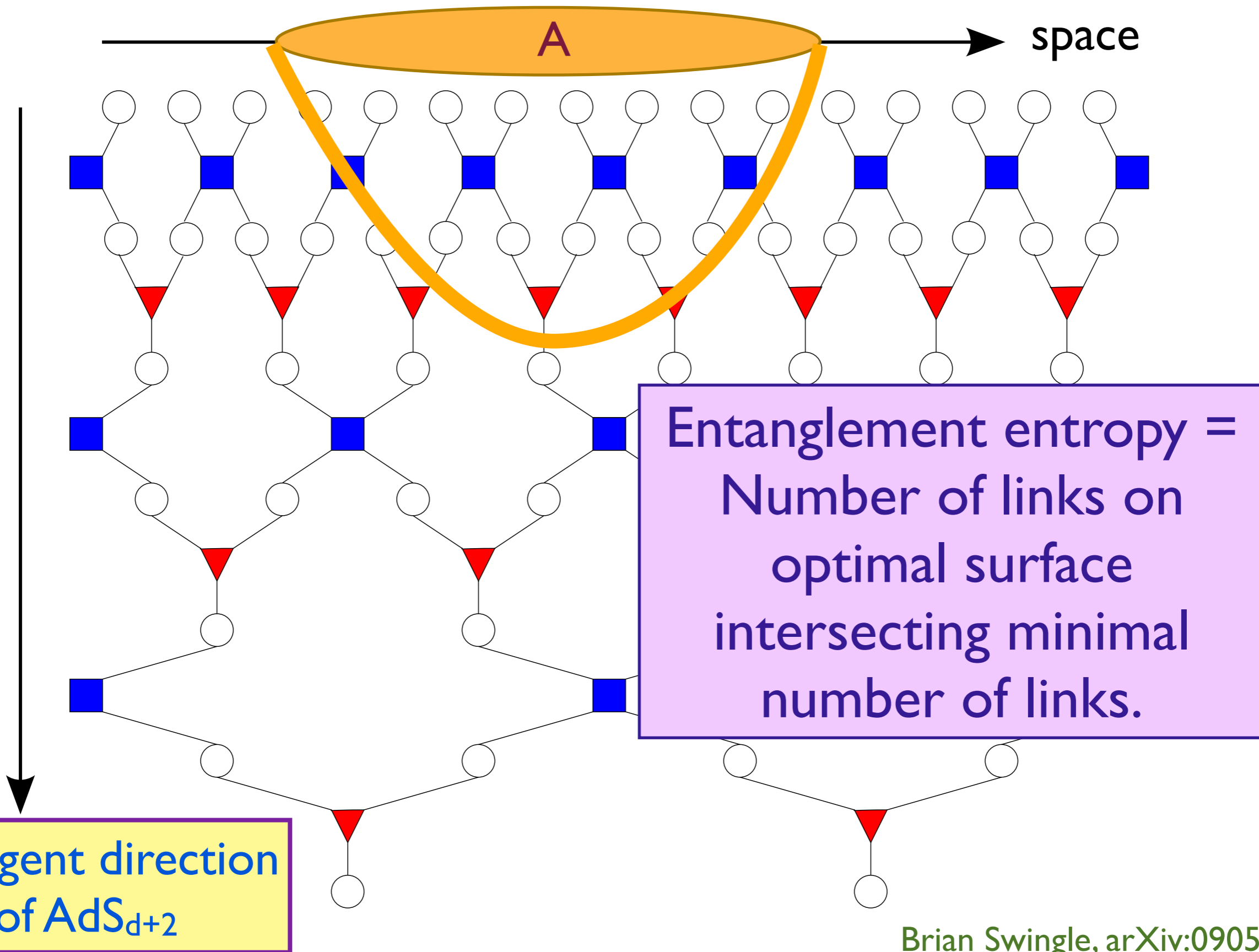


Entanglement entropy =
Number of links on
optimal surface
intersecting minimal
number of links.

depth of entanglement

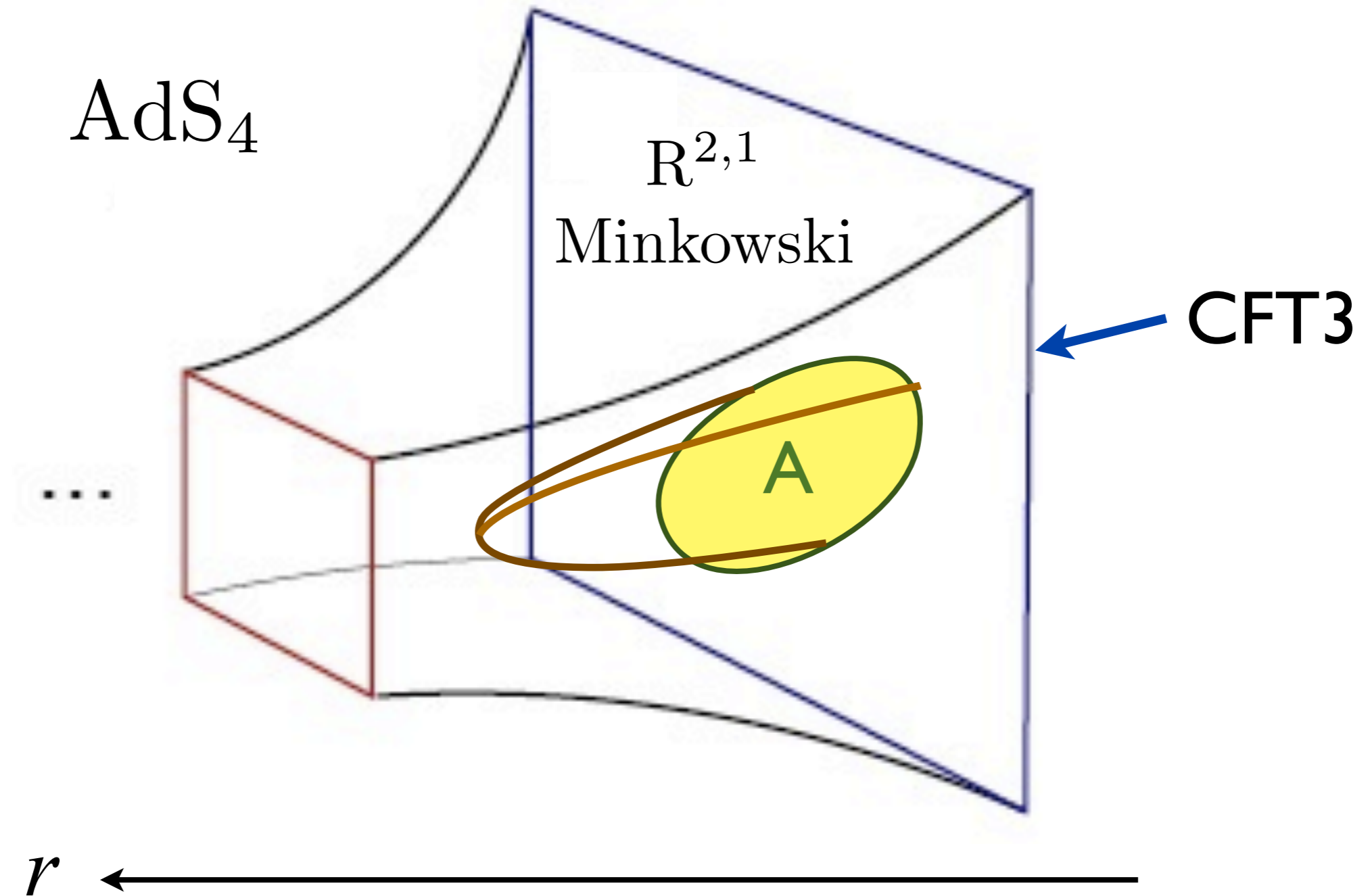
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Brian Swingle, arXiv:0905.1317

AdS/CFT correspondence



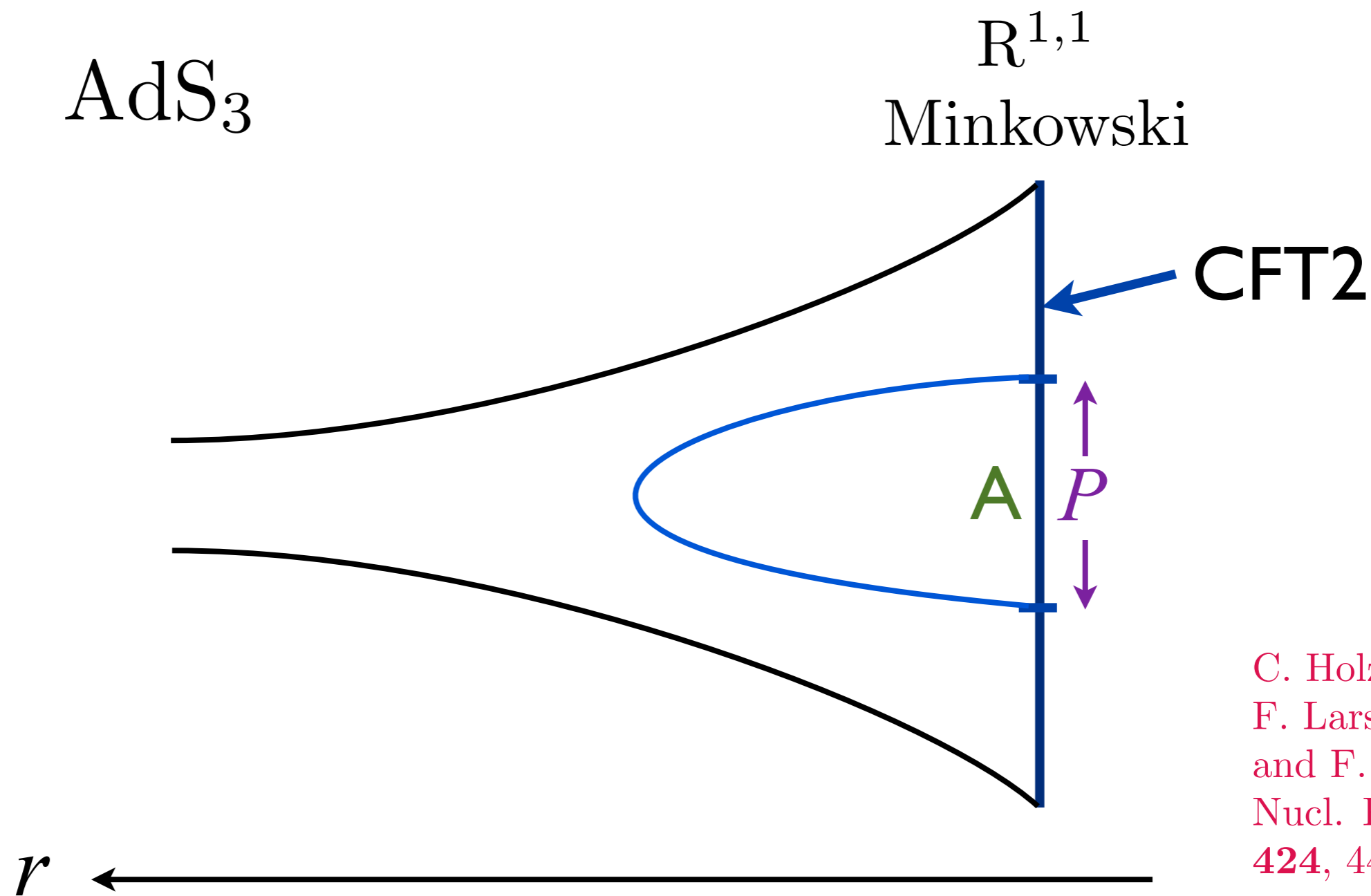
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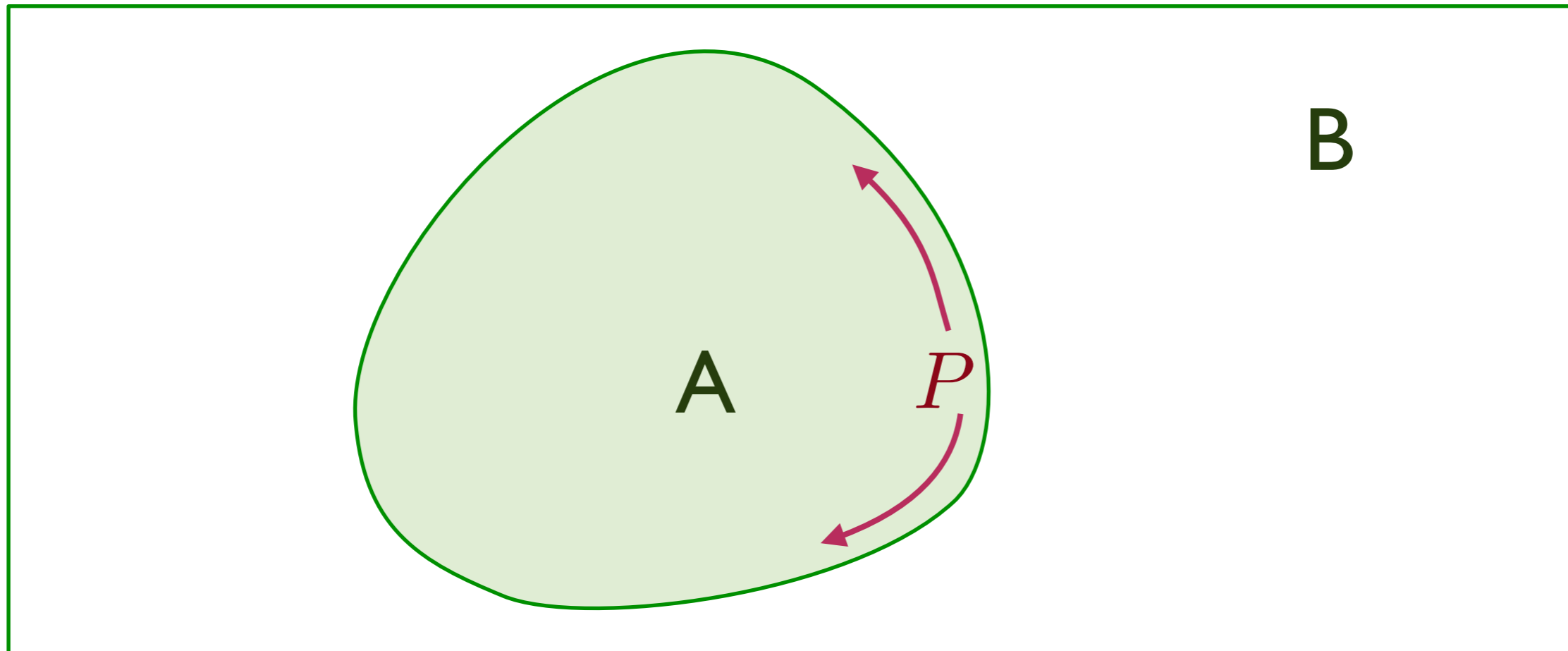
C. Holzhey,
F. Larsen
and F. Wilczek,
Nucl. Phys. B
424, 443 (1994).

- Computation of minimal surface area, or direct computation in CFT2, yield $S_E = (c/6) \ln P$, where c is the central charge.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Compressible quantum matter

Entanglement entropy of Fermi surfaces



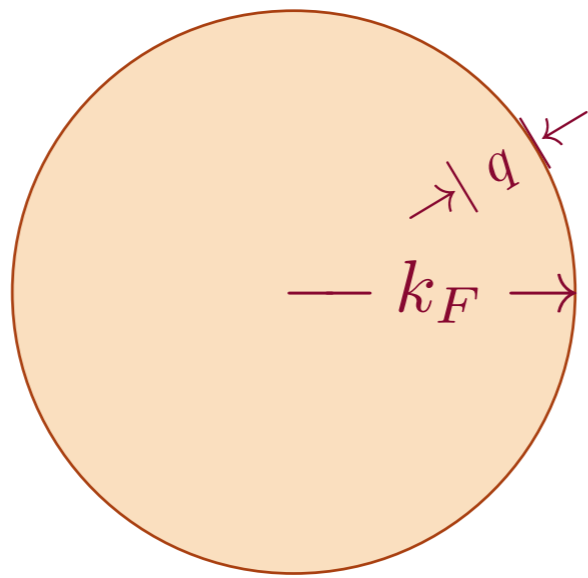
Logarithmic violation of “area law”: $S_E = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F , where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

FL Fermi liquid

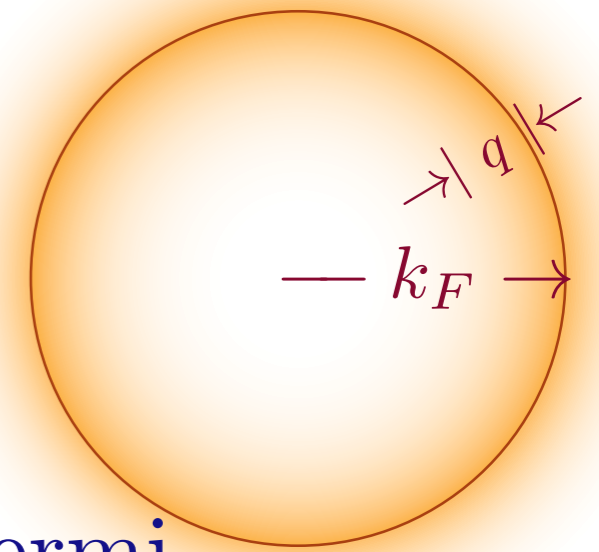


- $k_F^d \sim Q$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

NFL Bose metal

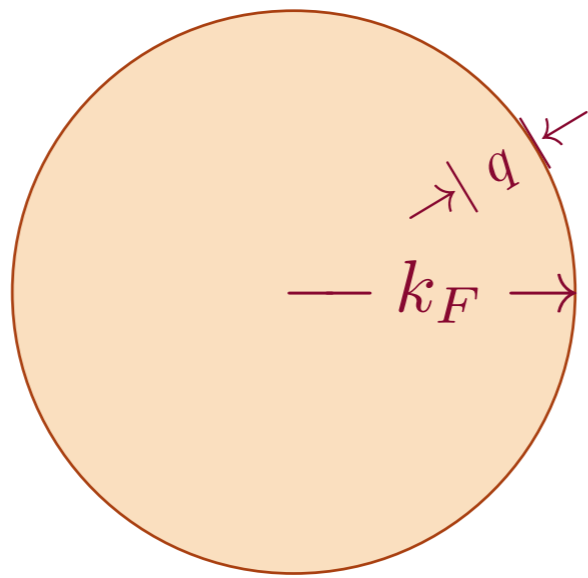


- Hidden Fermi surface with $k_F^d \sim Q$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

FL Fermi liquid



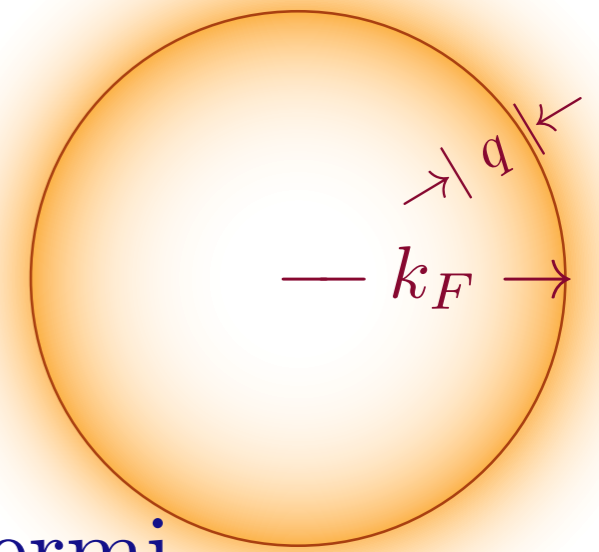
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NFL Bose metal

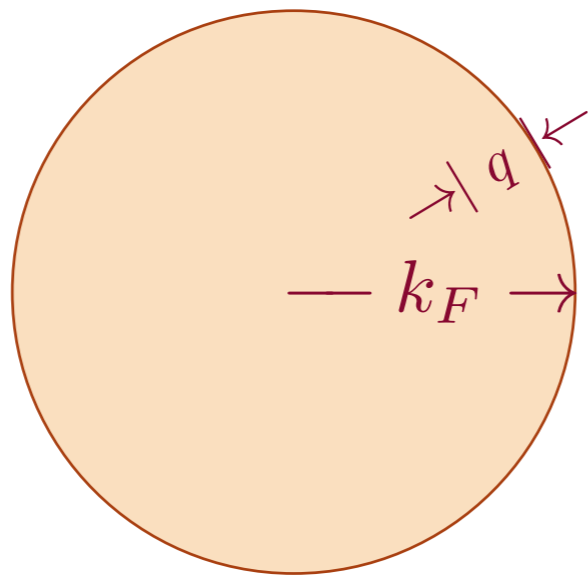


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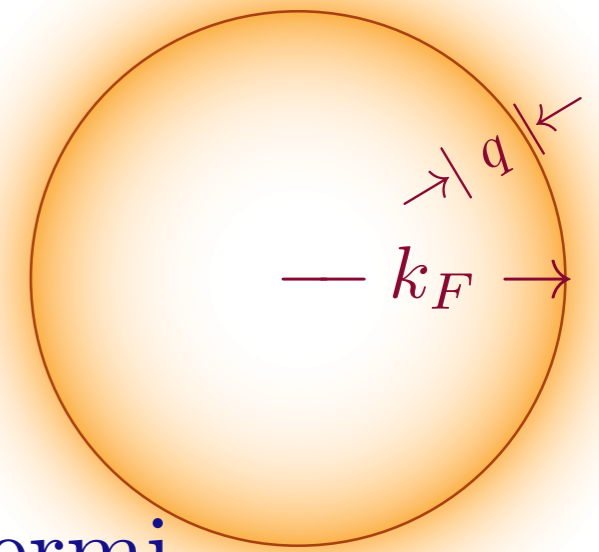
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NFL Bose metal



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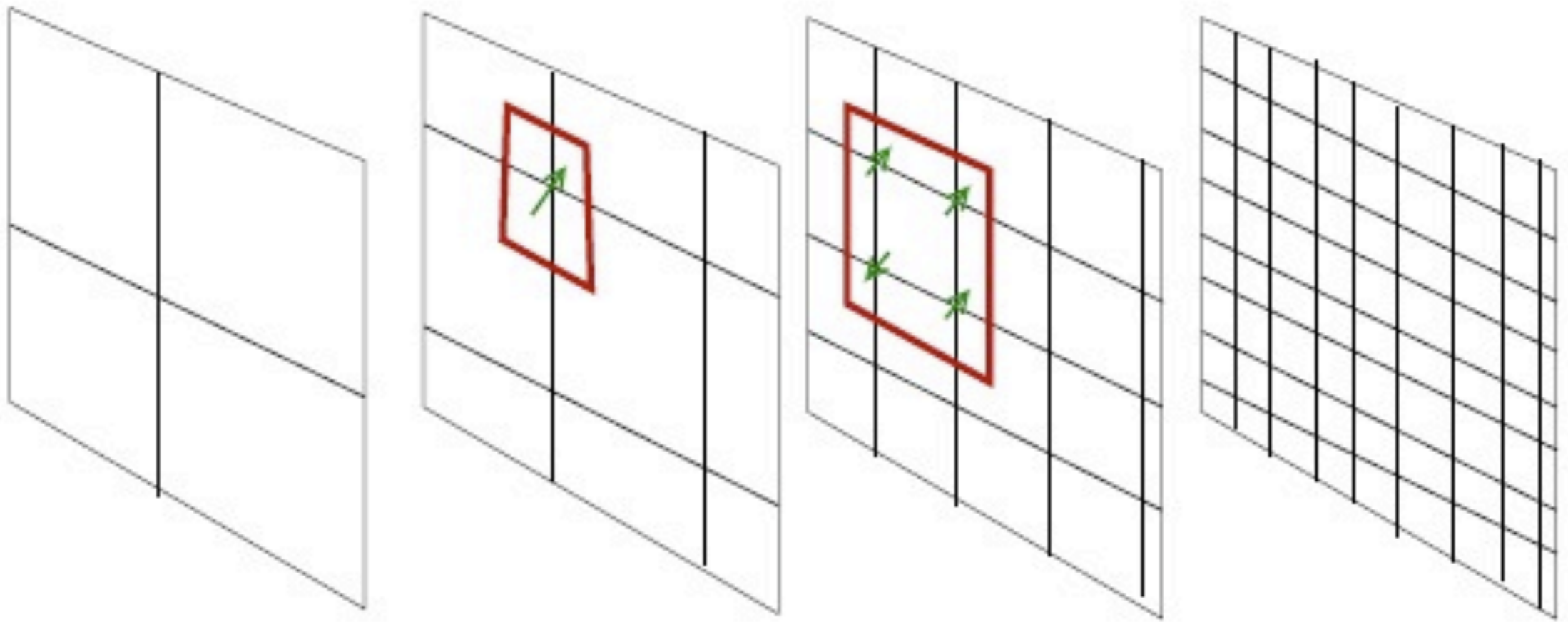
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- $S_E \sim k_F^{d-1} P \ln P$.

Y. Zhang, T. Grover, and A. Vishwanath,
Phys. Rev. Lett. **107**, 067202 (2011)

Holography



r ←

Consider the metric which transforms under rescaling as

$$\begin{aligned}x_i &\rightarrow \zeta x_i \\t &\rightarrow \zeta^z t \\ds &\rightarrow \zeta^{\theta/d} ds.\end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points).

θ is the violation of hyperscaling exponent.

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θ is the violation of hyperscaling exponent.

The most general choice of such a metric is

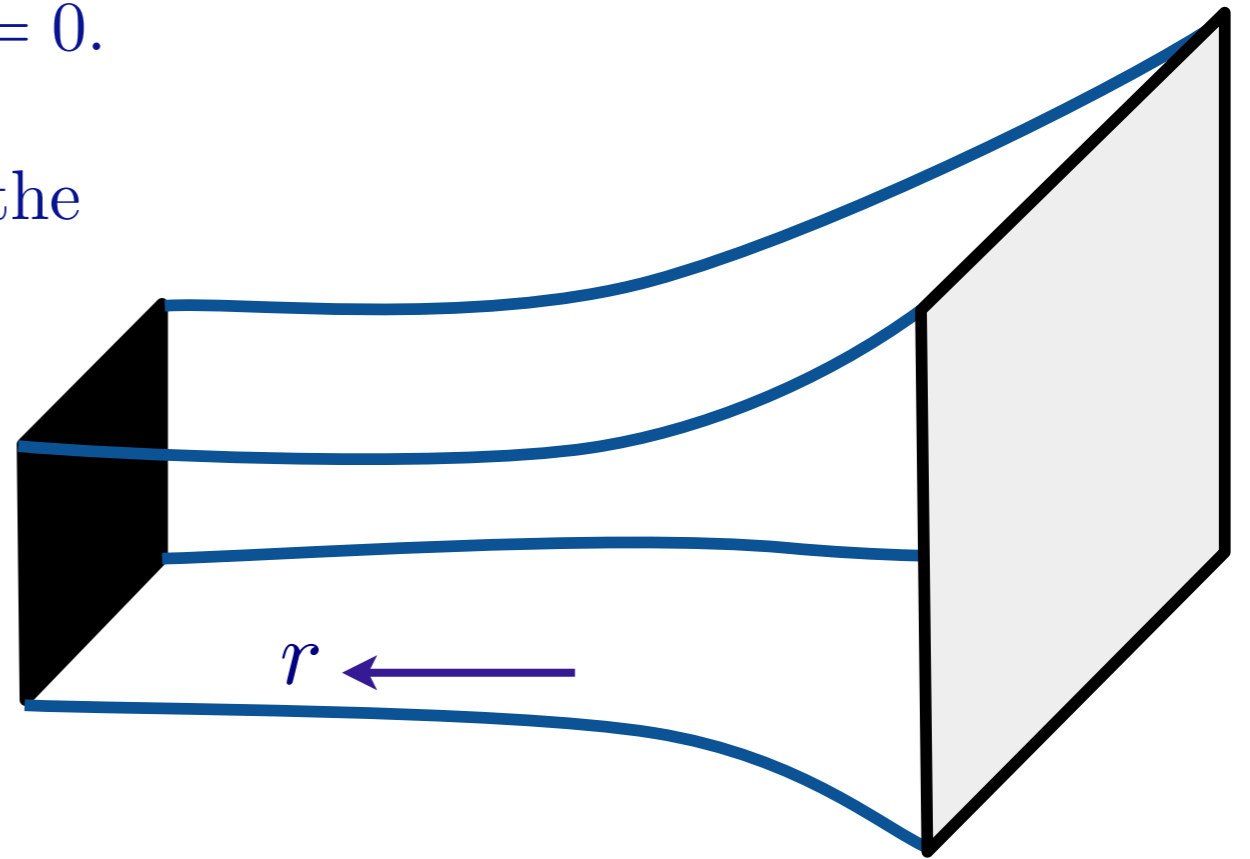
$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

At $T > 0$, there is a “black-brane” at $r = r_h$.

The Bekenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

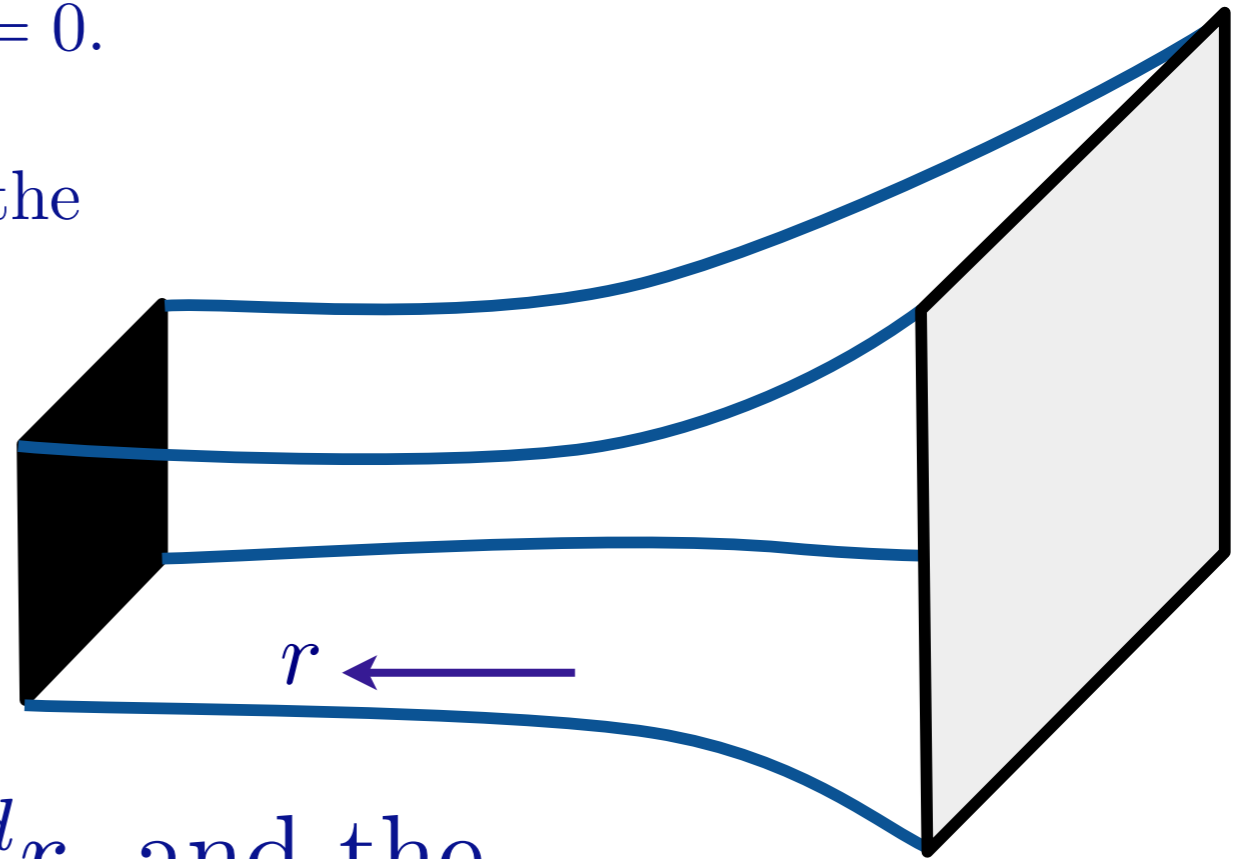
The entropy density, S , is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$



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Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$ measures “dimension deficit” in the phase space of low energy degrees of a freedom.

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

At $T > 0$, there is a *horizon*, and computation of its Bekenstein-Hawking entropy shows

$$S \sim T^{(d-\theta)/z}.$$

So θ is indeed the violation of hyperscaling exponent as claimed. For a compressible quantum state we should therefore *choose* $\theta = d - 1$.

No additional choices will be made, and all subsequent results are consequences of the assumption of the existence of a holographic dual.

Holography of strange metals

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

In $d = 2$, this implies $z \geq 3/2$. So the lower bound is precisely the value obtained from the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$\theta = d - 1$$

Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

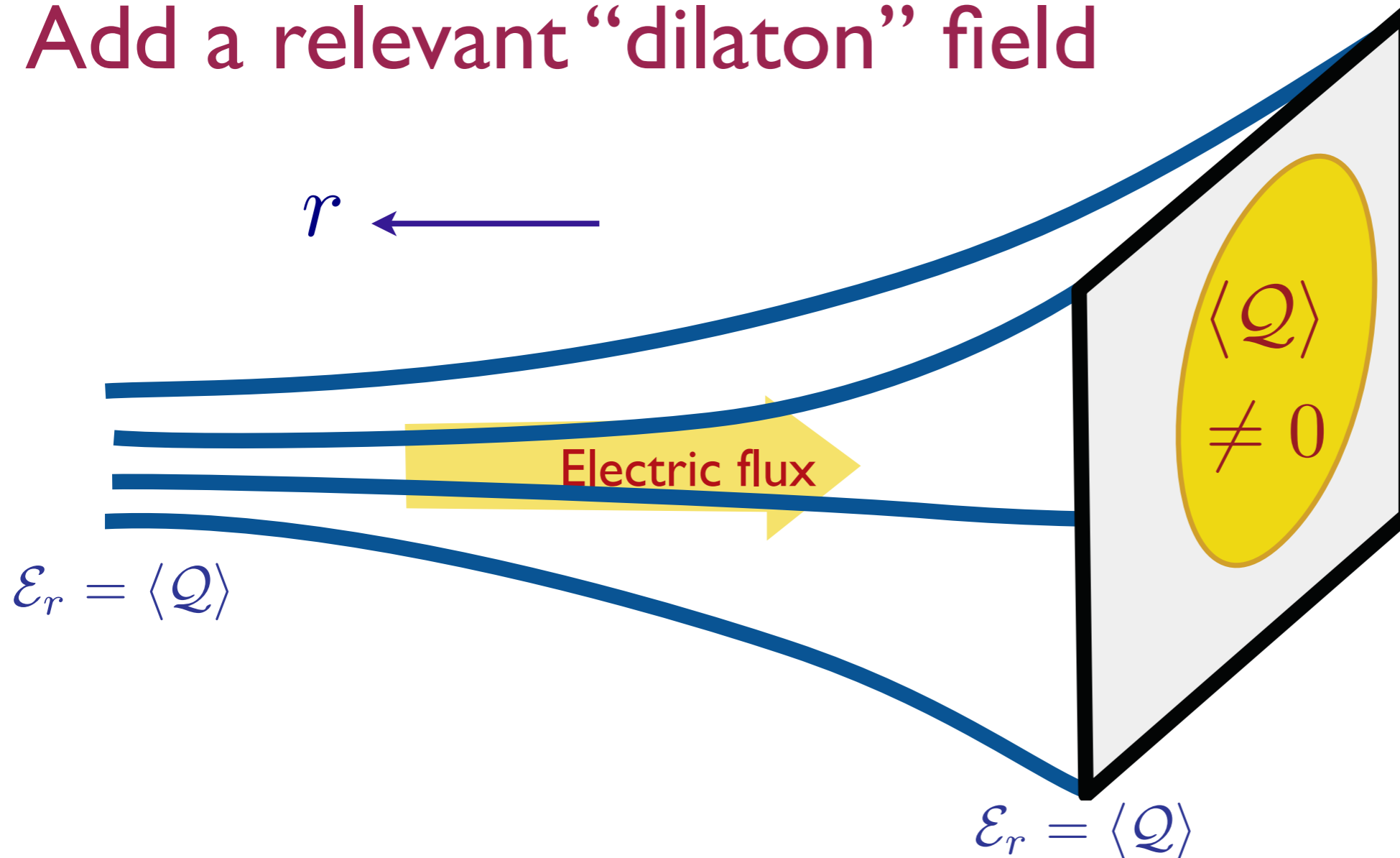
$$S_E \sim Q^{(d-1)/d} P \ln P$$

with a co-efficient *independent* of UV details and of the shape of the entangling region. These properties are just as expected for a circular Fermi surface with $Q \sim k_F^d$.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



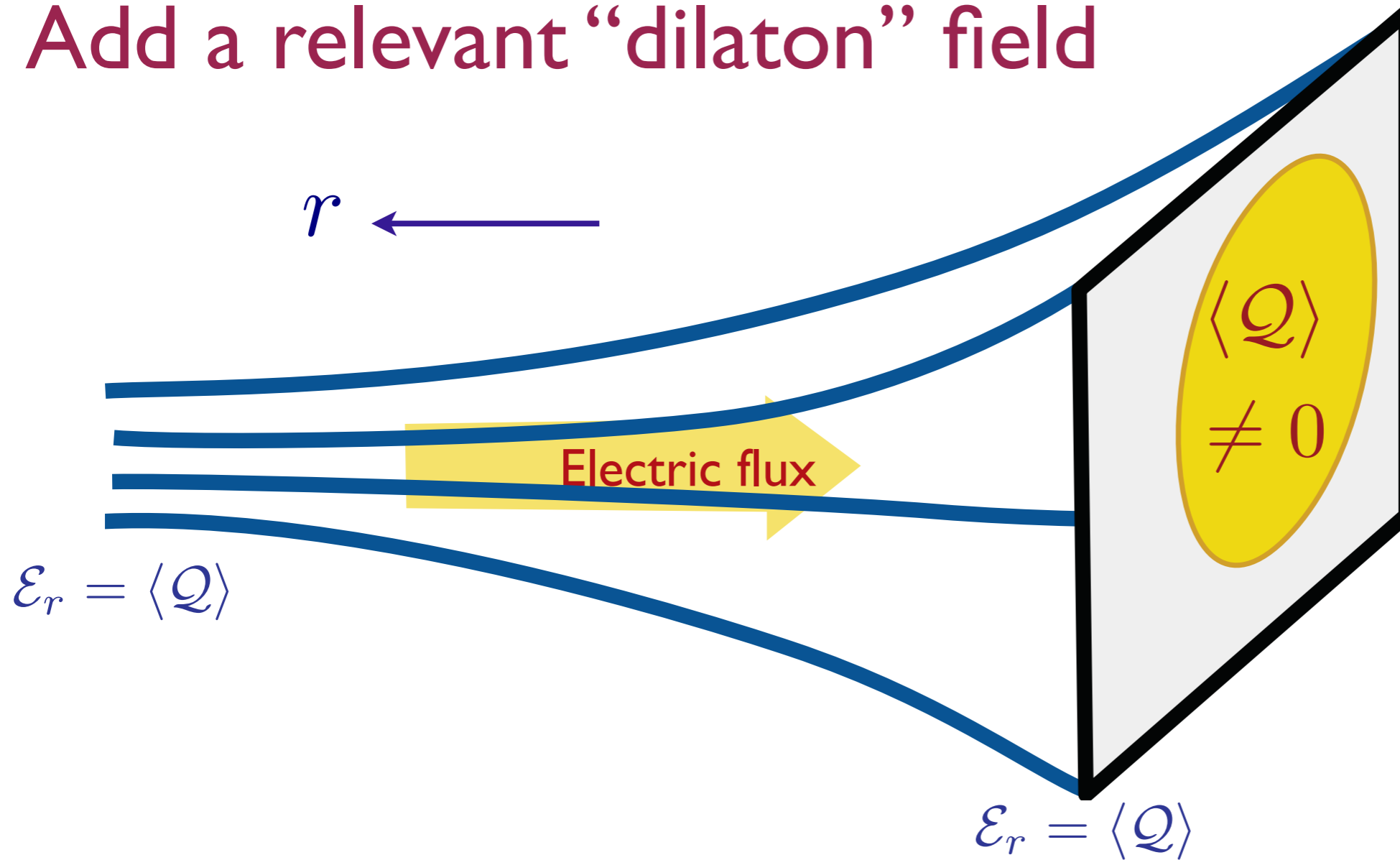
$$\mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \rightarrow \infty$.

- C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field



Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$
with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^\delta$, $\Phi(r) \sim \ln(r)$ as $r \rightarrow \infty$.

- C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

Holographic theory of a non-Fermi liquid (NFL)

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

The $r \rightarrow \infty$ metric has the above form with

$$\theta = \frac{d^2 \beta}{\alpha + (d-1)\beta}$$
$$z = 1 + \frac{\theta}{d} + \frac{8(d(d-\theta) + \theta)^2}{d^2(d-\theta)\alpha^2}.$$

Note $z \geq 1 + \theta/d$.

In the present theory, we have to choose α or β so that $\theta = d - 1$.

Needed: a dynamical quantum analysis which automatically selects this value of θ .


Conclusions

Gapped quantum matter

- Numerical and experimental observation of a spin liquid on the kagome lattice. Likely a Z_2 spin liquid.

Conclusions

Conformal quantum matter

 Numerical and experimental observation in coupled-dimer antiferromagnets, and at the superfluid-insulator transition of bosons in optical lattices.

Conclusions

Compressible quantum matter

- Holographic theory yields models of non-Fermi liquids (NFL), fractionalized Fermi liquids (FL*), and Fermi liquids (FL), in close correspondence with the phases expected from field theory

Thank you !

