New Horizons in Quantum Matter

Physics Next:
From Quantum Fields to Condensed Matter

Hyatt Place Long Island Hotel, NY
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Subir Sachdev

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PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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Quantum condensed matter physics before the 1980s:

- The ground state of metals and insulators is adiabatically connected to the free electron state
- Excitations are electron-like quasiparticles
- Pairing of electrons into Cooper pairs, and their condensation leads to superconductivity
- Breaking of symmetry describes superconductivity, ferromagnetism, antiferromagnetism, and other ordered states

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- Breaking of symmetry describes superconductivity, ferromagnetism, antiferromagnetism, and other ordered states
- Hints of physics beyond quasiparticles: the Wilson-Fisher theory of the Ising transition at finite temperature, the theory of dynamic critical phenomena, Nozieres-Blandin multi-channel Kondo critical point

The integer quantum Hall effect

VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

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Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

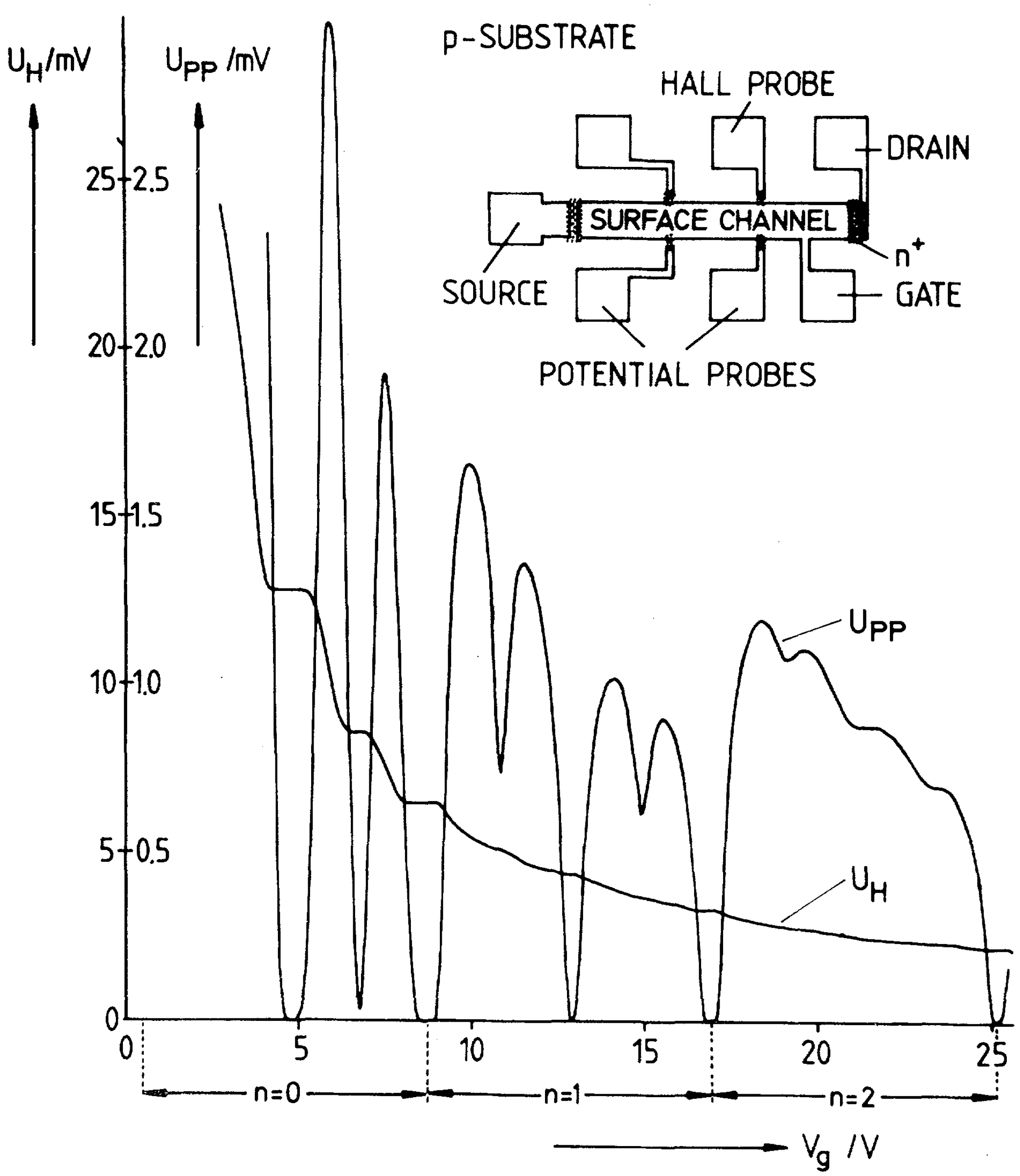
and

G. Dorda
Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

M. Pepper
Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom
(Received 30 May 1980)

Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.



$$\sigma_{xy} = \frac{ne^2}{h}$$

The fractional quantum Hall effect

VOLUME 48, NUMBER 22 PHYSICAL REVIEW LETTERS 31

Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,[a],[b] H. L. Stormer,[a] and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 5 March 1982)

A quantized Hall plateau of $\rho_{xy} = 3h/e^2$, accompanied by a minimum in ρ_{xx}, was observed at $T < 5$ K in magnetotransport of high-mobility, two-dimensional electrons, when the lowest-energy, spin-polarized Landau level is $\frac{1}{3}$ filled. The formation of a Wigner solid or charge-density-wave state with triangular symmetry is suggested as a possible explanation.

PACS numbers: 72.20.My, 71.45.-d, 73.40.Lq, 73.60.Fw

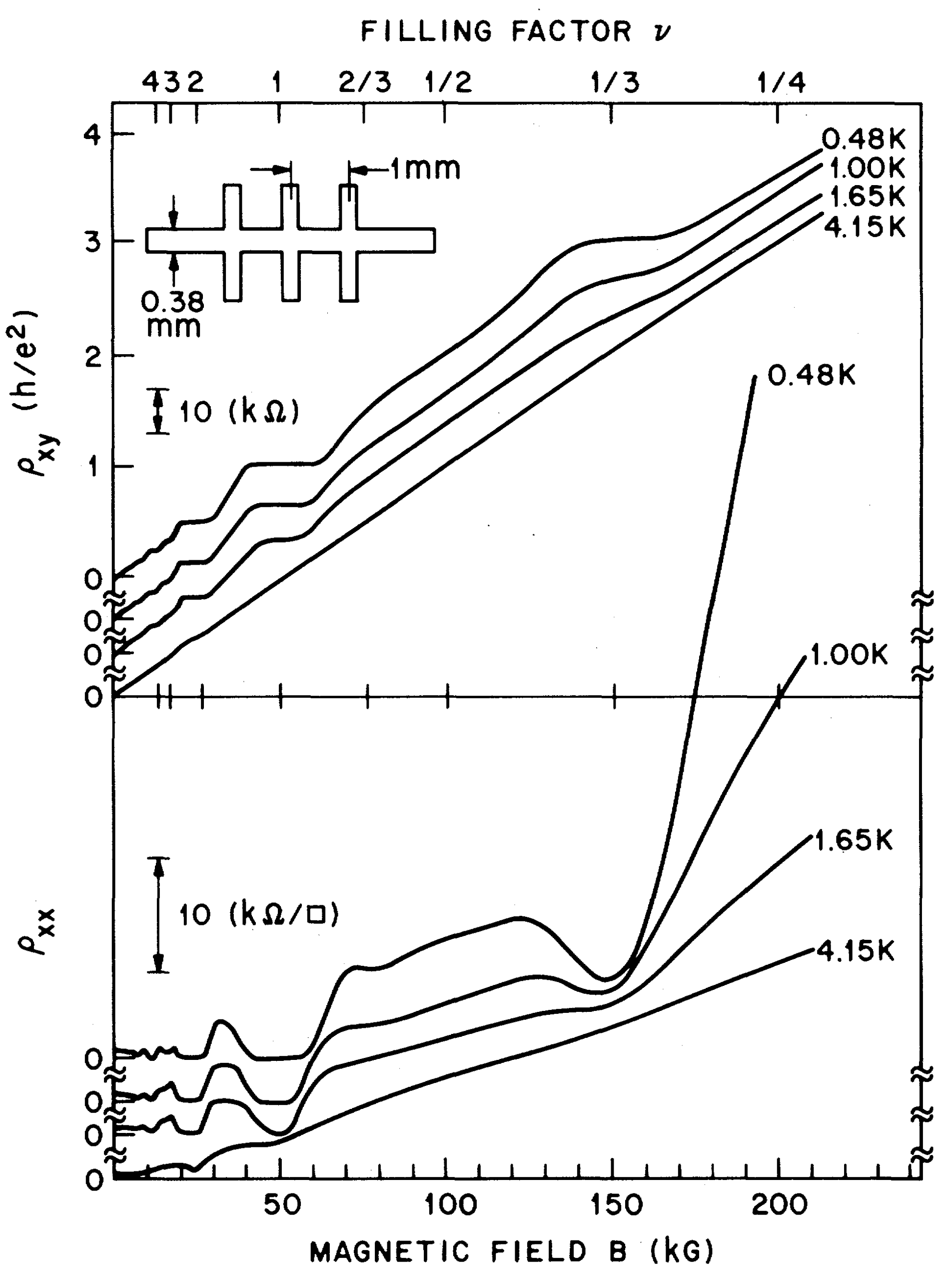


FIG. 1. ρ_{xy} and ρ_{xx} vs B, taken from a GaAs-$Al_{0.3}$-$Ga_{0.7}$As sample with $n = 1.23 \times 10^{11}/\mathrm{cm}^2$, $\mu = 90\,000$ cm²/V sec, using $I = 1\ \mu$A. The Landau level filling factor is defined by $\nu = nh/eB$.

$$\sigma_{xy} = \frac{\nu e^2}{h}$$

High temperature superconductivity

Possible High T_c Superconductivity in the Ba – La – Cu – O System

J.G. Bednorz and K.A. Müller
IBM Zürich Research Laboratory, Rüschlikon, Switzerland

Received April 17, 1986

Metallic, oxygen-deficient compounds in the Ba – La – Cu – O system, with the composition $Ba_xLa_{5-x}Cu_5O_{5(3-y)}$ have been prepared in polycrystalline form. Samples with $x=1$ and 0.75, $y>0$, annealed below 900 °C under reducing conditions, consist of three phases, one of them a perovskite-like mixed-valent copper compound. Upon cooling, the samples show a linear decrease in resistivity, then an approximately logarithmic increase, interpreted as a beginning of localization. Finally an abrupt decrease by up to three orders of magnitude occurs, reminiscent of the onset of percolative superconductivity. The highest onset temperature is observed in the 30 K range. It is markedly reduced by high current densities. Thus, it results partially from the percolative nature, bute possibly also from 2*D* superconducting fluctuations of double perovskite layers of one of the phases present.

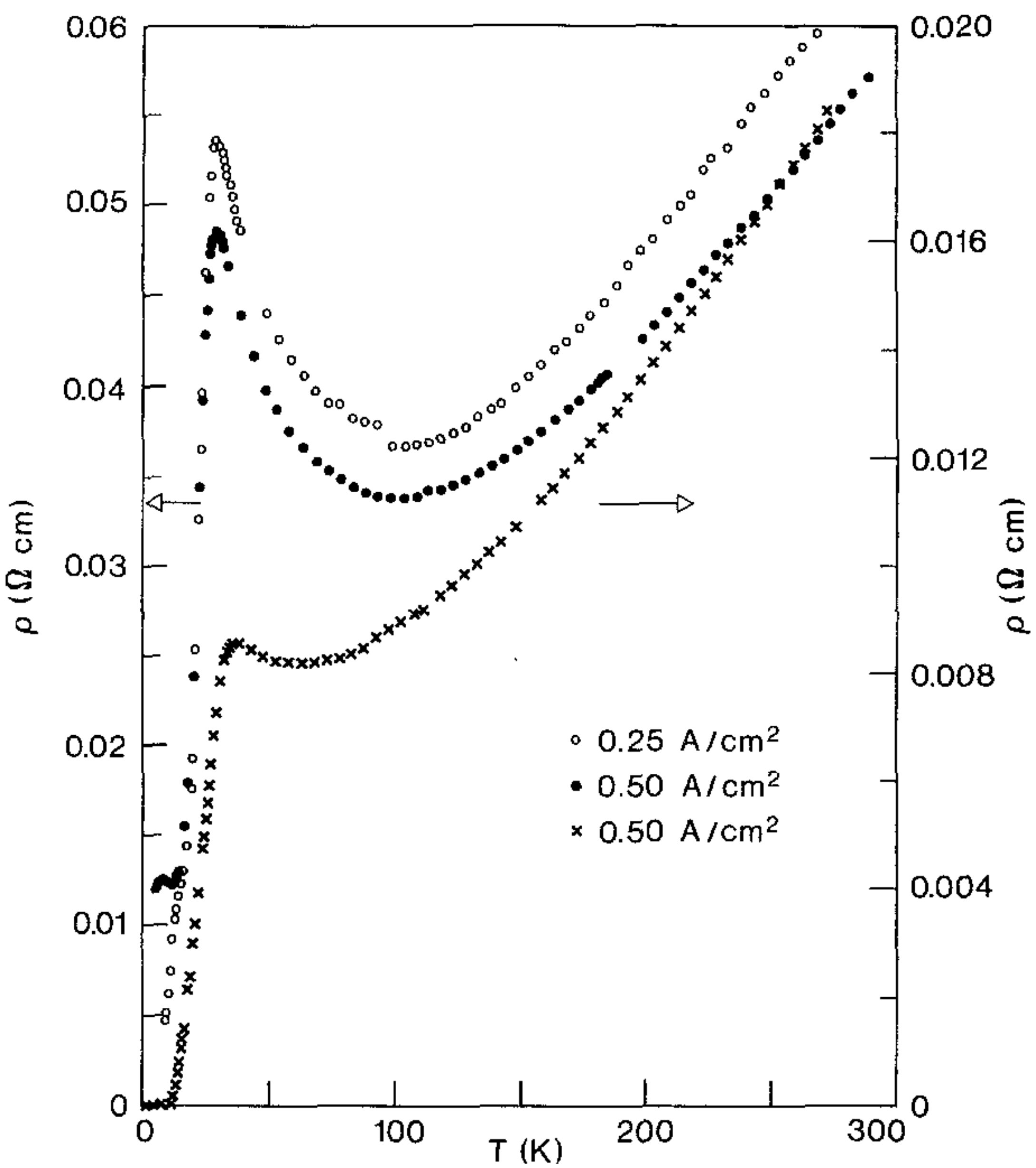


Fig. 1. Temperature dependence of resistivity in $Ba_xLa_{5-x}Cu_5O_{5(3-y)}$ for samples with $x(\text{Ba})=1$ (upper curves, left scale) and $x(\text{Ba})=0.75$ (lower curve, right scale). The first two cases also show the influence of current density

Z. Phys. B – Condensed Matter 64, 189–193 (1986)

1. Descendants of the integer quantum Hall effect

2. Descendants of the fractional quantum Hall effect

3. Quantum matter without quasiparticles: strange metals and black holes

1. Descendants of the integer quantum Hall effect

Protected gapless edge states,
while bulk excitations are “trivial”

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,[a] M. P. Nightingale, and M. den Nijs
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 30 April 1982)

The Hall conductance of a two-dimensional electron gas has been studied in a uniform magnetic field and a periodic substrate potential U. The Kubo formula is written in a form that makes apparent the quantization when the Fermi energy lies in a gap. Explicit expressions have been obtained for the Hall conductance for both large and small $U/\hbar\omega_c$.

$$\sigma_H = \frac{ie^2}{2\pi h}\sum\int d^2k\int d^2r\left(\frac{\partial u^*}{\partial k_1}\frac{\partial u}{\partial k_2} - \frac{\partial u^*}{\partial k_2}\frac{\partial u}{\partial k_1}\right)$$
$$= \frac{ie^2}{4\pi h}\sum\oint dk_j\int d^2r\left(u^*\frac{\partial u}{\partial k_j} - \frac{\partial u^*}{\partial k_j}u\right),$$

A physical observable is related to a topological invariant: the Chern number. Closely connected to the Chern-Simons invariant for the vector potential:

$$\mathcal{S}_{CS} = \frac{1}{4\pi}\int d^2x\,dt\,\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

PHYSICAL REVIEW B VOLUME 25, NUMBER 4 15 FEBRUARY 1982

Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential

B. I. Halperin
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 21 August 1981)

When a conducting layer is placed in a strong perpendicular magnetic field, there exist current-carrying electron states which are localized within approximately a cyclotron radius of the sample boundary but are extended around the perimeter of the sample. It is shown that these quasi-one-dimensional states remain extended and carry a current even in the presence of a moderate amount of disorder. The role of the edge states in the quantized Hall conductance is discussed in the context of the general explanation of Laughlin. An extension of Laughlin's analysis is also used to investigate the existence of extended states in a weakly disordered two-dimensional system, when a strong magnetic field is present.

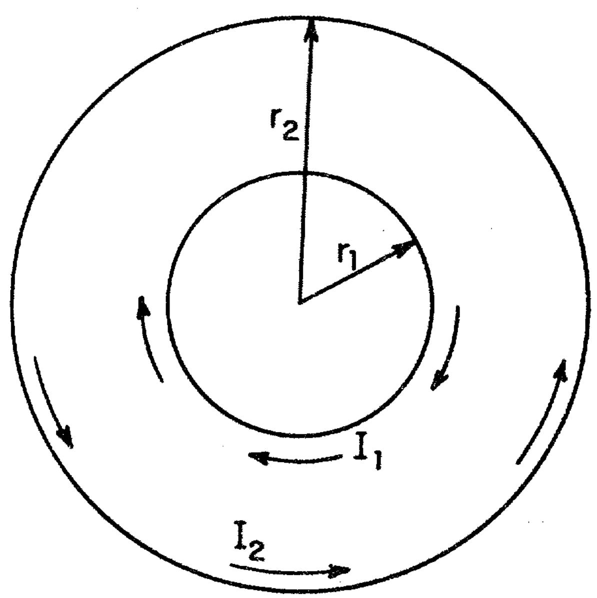


FIG. 1. Geometry of sample. Annular film, in region $r_1 < r < r_2$ is placed in uniform magnetic field B_0, pointing out of the page. Additional magnetic flux Φ is confined to region $r < r_1$. Curved arrows show direction of currents I_1 and I_2 at the boundaries of film.

Protected one-dimensional chiral edge states which cannot appear on their own in a one-dimensional system. All bulk two-dimensional excitations are "trivial". These edge states are required by 'anomaly matching' of the bulk field theory.

Integer spin antiferromagnets in one dimension

VOLUME 50, NUMBER 15 PHYSICAL REVIEW LETTERS 11 APRIL 1983

Nonlinear Field Theory of Large-Spin Heisenberg Antiferromagnets: Semiclassically Quantized Solitons of the One-Dimensional Easy-Axis Néel State

F. D. M. Haldane
Department of Physics, University of Southern California, Los Angeles, California 90089
(Received 31 January 1983)

The continuum field theory describing the low-energy dynamics of the large-spin one-dimensional Heisenberg antiferromagnet is found to be the O(3) nonlinear sigma model. When weak easy-axis anisotropy is present, soliton solutions of the equations of motion are obtained and semiclassically quantized. Integer and half-integer spin systems are distinguished.

$$\mathcal{S} = \int dx dt \left[\frac{1}{2g} (\partial_\mu \mathbf{n})^2 + i \frac{\theta}{4\pi} \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_t \mathbf{n}) \right] \quad , \quad \theta = 2\pi S$$

VOLUME 59, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1987

Rigorous Results on Valence-Bond Ground States in Antiferromagnets

Ian Affleck,[(a)] Tom Kennedy, Elliott H. Lieb, and Hal Tasaki
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
(Received 26 May 1987)

 $= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

 $= |+\rangle\langle\uparrow\uparrow| + |0\rangle \frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}} + |-\rangle\langle\downarrow\downarrow|$

Integer spin antiferromagnets in one dimension

VOLUME 65, NUMBER 25 PHYSICAL REVIEW LETTERS 17 DECEMBER 1990

Observation of $S = \frac{1}{2}$ Degrees of Freedom in an $S = 1$ Linear-Chain Heisenberg Antiferromagnet

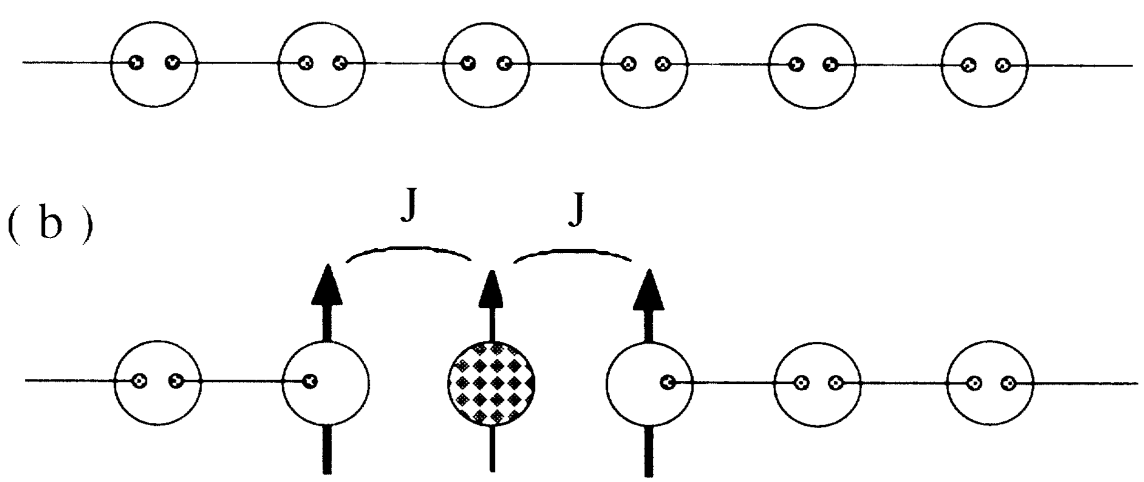
M. Hagiwara and K. Katsumata
The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-01, Japan

Ian Affleck
Canadian Institute for Advanced Research and Physics Department, University of British Columbia, Vancouver, British Columbia, Canada V6T 2A6

B. I. Halperin
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

J. P. Renard
Institut d'Electronique Fondamentale, Bâtiment 220, Université Paris-Sud, 91405 Orsay CEDEX, France
(Received 31 July 1990)

The ground state of the typical spin-1 linear-chain Heisenberg antiferromagnet $Ni(C_2H_8N_2)_2NO_2(ClO_4)$ containing a small amount of Cu^{2+} is studied by the electron-spin-resonance (ESR) technique. The ESR results are quantitatively explained by the model that the valence bonds are broken at the Cu^{2+} sites resulting in spin-$\frac{1}{2}$ states at the Ni^{2+} sites neighboring the Cu^{2+}. Thus the present study gives experimental evidence for the existence of the valence-bond-solid ground state in $S = 1$ linear-chain Heisenberg antiferromagnets.



Protected $S = 1/2$ excitations at the ends of the antiferromagnet, but only $S = 1$ excitations in the bulk

Protected chiral edge states without a magnetic field, and with time-reversal symmetry. Bulk excitations remain "trivial".

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane
Department of Physics, University of California, San Diego, La Jolla, California 92093
(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.

Z_2 Topological Order and the Quantum Spin Hall Effect

C. L. Kane and E. J. Mele
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
(Received 22 June 2005; published 28 September 2005)

The quantum spin Hall (QSH) phase is a time reversal invariant electronic state with a bulk electronic band gap that supports the transport of charge and spin in gapless edge states. We show that this phase is associated with a novel Z_2 topological invariant, which distinguishes it from an ordinary insulator. The Z_2 classification, which is defined for time reversal invariant Hamiltonians, is analogous to the Chern number classification of the quantum Hall effect. We establish the Z_2 order of the QSH phase in the two band model of graphene and propose a generalization of the formalism applicable to multiband and interacting systems.

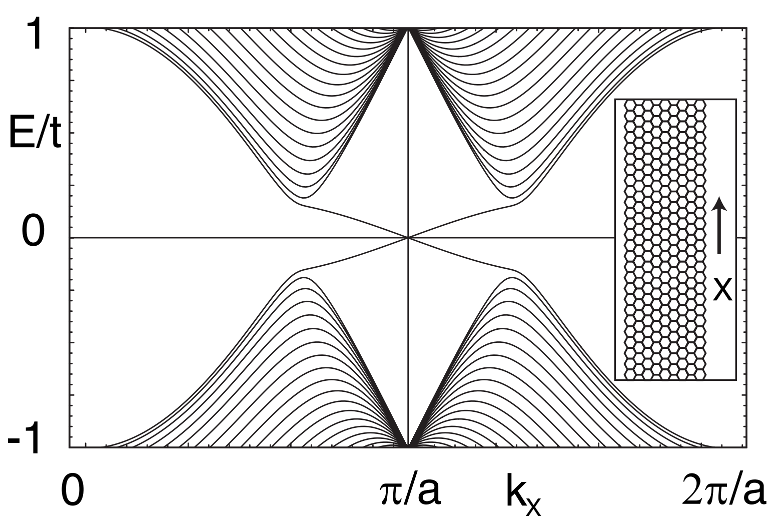
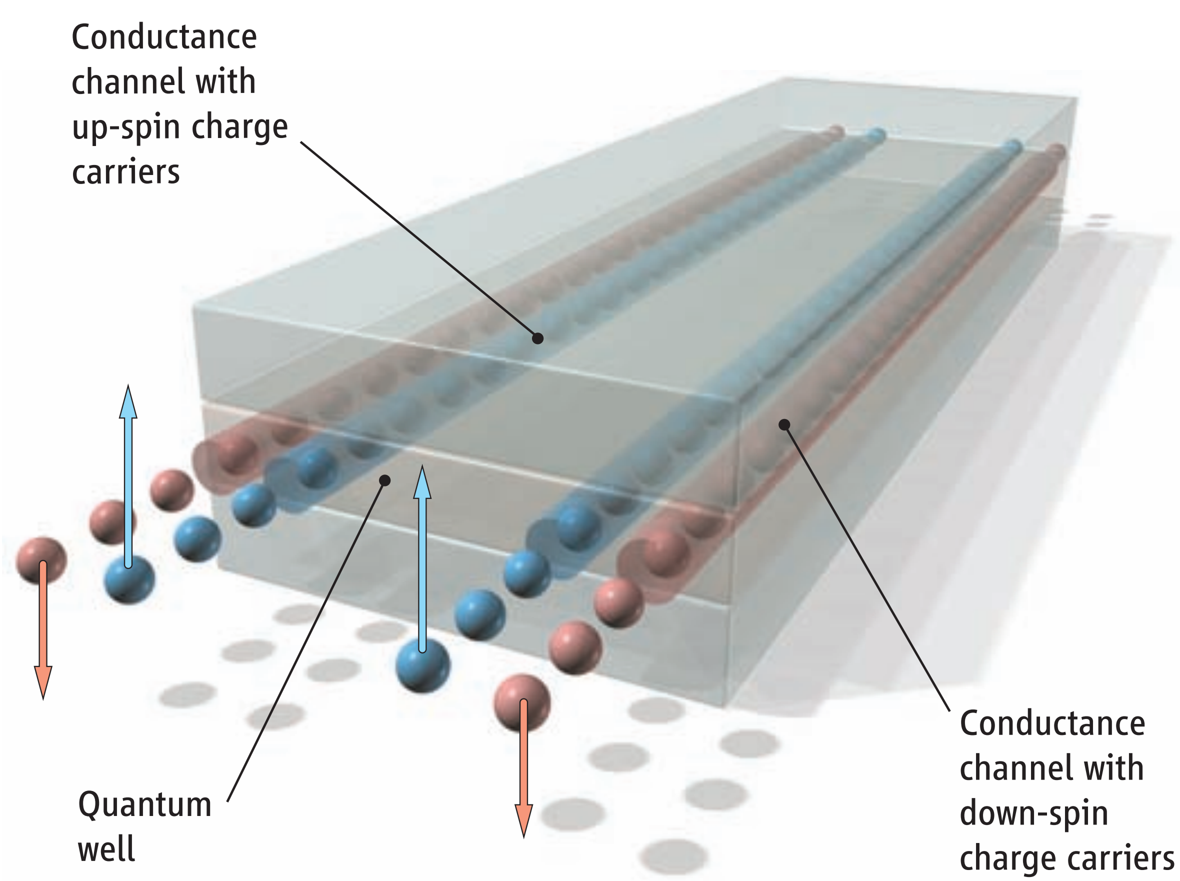


FIG. 1. (a) One-dimensional energy bands for a strip of graphene (shown in inset) modeled by (7) with $t_2/t = 0.03$. The bands crossing the gap are spin filtered edge states.

2 NOVEMBER 2007 VOL 318 **SCIENCE** www.sciencemag.org

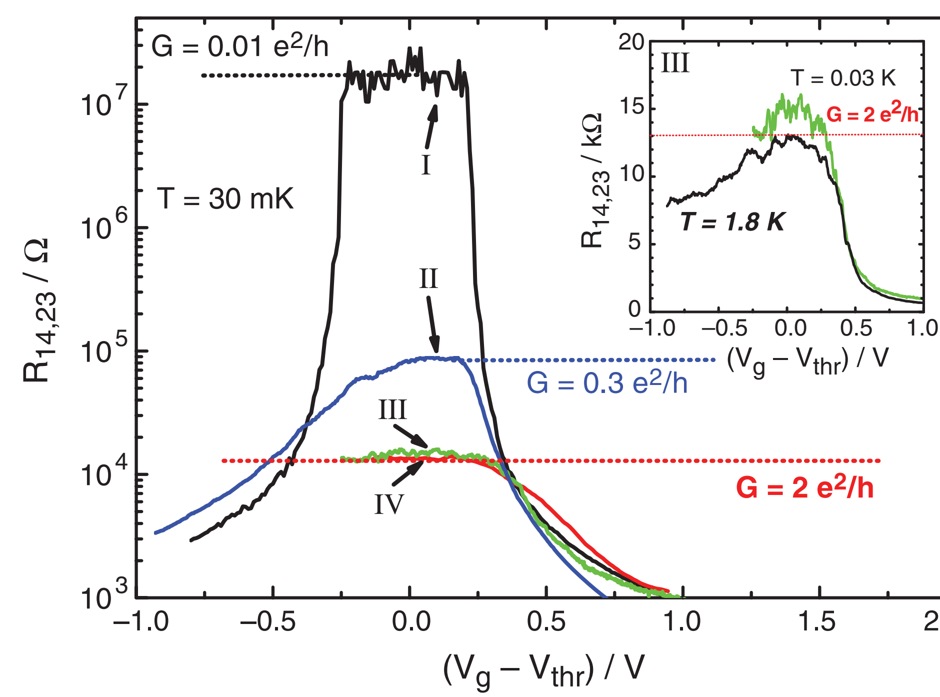
Quantum Spin Hall Insulator State in HgTe Quantum Wells

Markus König,[1] Steffen Wiedmann,[1] Christoph Brüne,[1] Andreas Roth,[1] Hartmut Buhmann,[1] Laurens W. Molenkamp,[1]* Xiao-Liang Qi,[2] Shou-Cheng Zhang[2]



Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

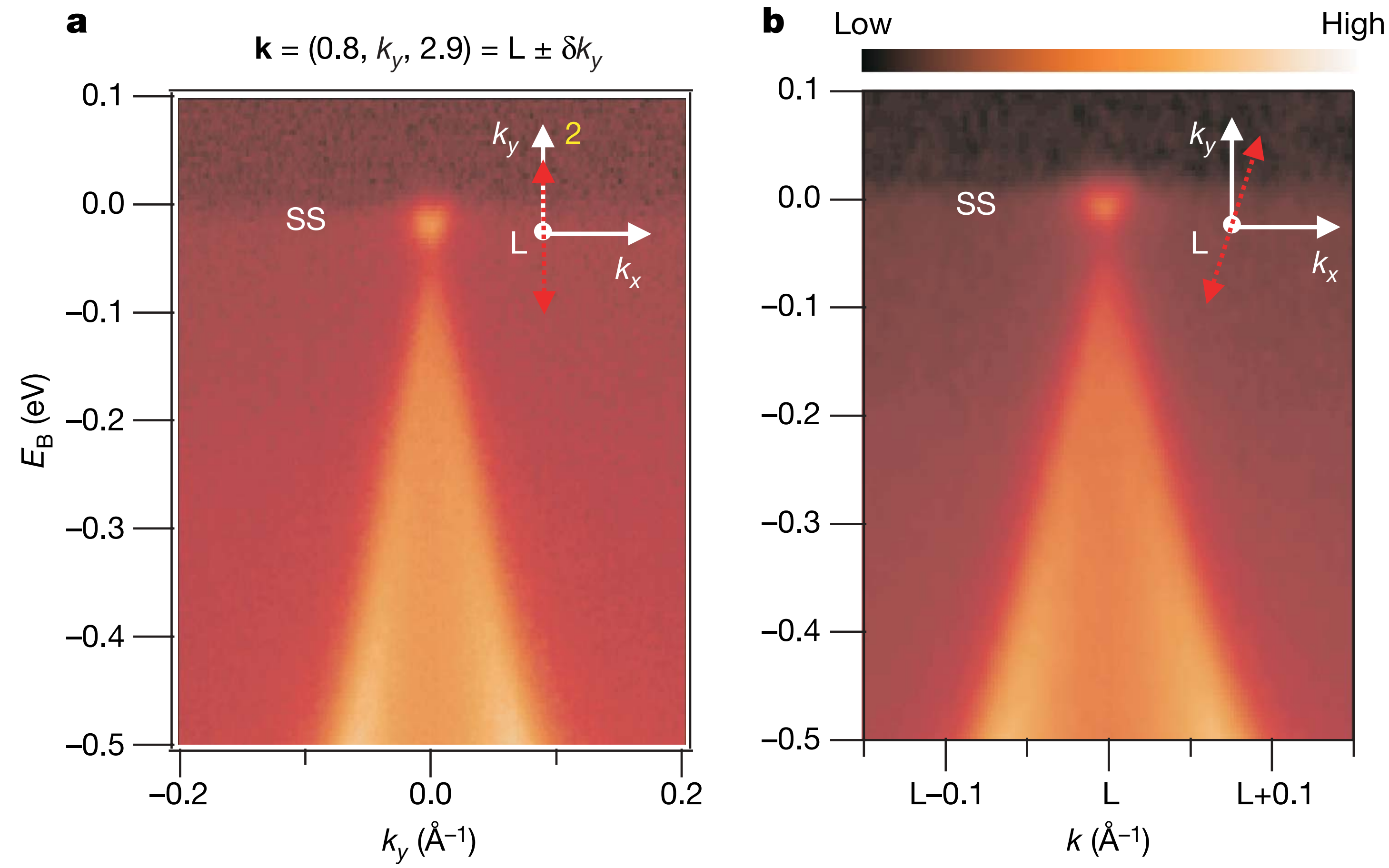
Fig. 4. The longitudinal four-terminal resistance, $R_{14,23}$, of various normal (d = 5.5 nm) (I) and inverted (d = 7.3 nm) (II, III, and IV) QW structures as a function of the gate voltage measured for B = 0 T at T = 30 mK. The device sizes are (20.0 × 13.3) μm^2 for devices I and II, (1.0 × 1.0) μm^2 for device III, and (1.0 × 0.5) μm^2 for device IV. The inset shows $R_{14,23}(V_g)$ of two samples from the same wafer, having the same device size (III) at 30 mK (green) and 1.8 K (black) on a linear scale.



A topological Dirac insulator in a quantum spin Hall phase

D. Hsieh[1], D. Qian[1], L. Wray[1], Y. Xia[1], Y. S. Hor[2], R. J. Cava[2] & M. Z. Hasan[1,3]

When electrons are subject to a large external magnetic field, the conventional charge quantum Hall effect[1,2] dictates that an electronic excitation gap is generated in the sample bulk, but metallic conduction is permitted at the boundary. Recent theoretical models suggest that certain bulk insulators with large spin–orbit interactions may also naturally support conducting topological boundary states in the quantum limit[3–5], which opens up the possibility for studying unusual quantum Hall-like phenomena in zero external magnetic fields[6]. Bulk $Bi_{1-x}Sb_x$ single crystals are predicted to be prime candidates[7,8] for one such unusual Hall phase of matter known as the topological insulator[9–11]. The hallmark of a topological insulator is the existence of metallic surface states that are higher-dimensional analogues of the edge states that characterize a quantum spin Hall insulator[3–13]. In addition to its interesting boundary states, the bulk of $Bi_{1-x}Sb_x$ is predicted to exhibit three-dimensional Dirac particles[14–17], another topic of heightened current interest following the new findings in two-dimensional graphene[18–20] and charge quantum Hall fractionalization observed in pure bismuth[21]. However, despite numerous transport and magnetic measurements on the $Bi_{1-x}Sb_x$ family since the 1960s[17], no direct evidence of either topological Hall states or bulk Dirac particles has been found. Here, using incident-photon-energy-modulated angle-resolved photoemission spectroscopy (IPEM-ARPES), we report the direct observation of massive Dirac particles in the bulk of $Bi_{0.9}Sb_{0.1}$, locate the Kramers points at the sample's boundary and provide a comprehensive mapping of the Dirac insulator's gapless surface electron bands. These findings taken together suggest that the observed surface state on the boundary of the bulk insulator is a realization of the 'topological metal'[9–11]. They also suggest that this material has potential application in developing next-generation quantum computing devices that may incorporate 'light-like' bulk carriers and spin-textured surface currents.



Nature **452**, 970 (2008)

Chernogolovka 2000: Mesoscopic and strongly correlated electron systems *Usp. Fiz. Nauk* (Suppl.) **171** (10)

Unpaired Majorana fermions in quantum wires

A Yu Kitaev

Abstract. Certain one-dimensional Fermi systems have an energy gap in the bulk spectrum while boundary states are described by one Majorana operator per boundary point. A finite system of length L possesses two ground states with an energy difference proportional to $\exp(-L/l_0)$ and different fermionic parities. Such systems can be used as qubits since they are intrinsically immune to decoherence. The property of a system to have boundary Majorana fermions is expressed as a condition on the bulk electron spectrum. The condition is satisfied in the presence of an arbitrary small energy gap induced by proximity of a three-dimensional p-wave superconductor, provided that the normal spectrum has an odd number of Fermi points in each half of the Brillouin zone (each spin component counts separately).

PRL **105,** 077001 (2010) PHYSICAL REVIEW LETTERS week e 13 AUGU

Majorana Fermions and a Topological Phase Transition in Semiconductor-Superconductor Heterostructures

Roman M. Lutchyn, Jay D. Sau, and S. Das Sarma
Joint Quantum Institute and Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA
(Received 24 February 2010; published 13 August 2010)

We propose and analyze theoretically an experimental setup for detecting the elusive Majorana particle in semiconductor-superconductor heterostructures. The experimental system consists of one-dimensional semiconductor wire with strong spin-orbit Rashba interaction embedded into a superconducting quantum interference device. We show that the energy spectra of the Andreev bound states at the junction are qualitatively different in topologically trivial (i.e., not containing any Majorana) and nontrivial phases having an even and odd number of crossings at zero energy, respectively. The measurement of the supercurrent through the junction allows one to discern topologically distinct phases and observe a topological phase transition by simply changing the in-plane magnetic field or the gate voltage. The observation of this phase transition will be a direct demonstration of the existence of Majorana particles.

PRL **105,** 177002 (2010) PHYSICAL REVIEW LETTERS week 22 OCTO

Helical Liquids and Majorana Bound States in Quantum Wires

Yuval Oreg,[1] Gil Refael,[2] and Felix von Oppen[3]
[1]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, 76100, Israel*
[2]*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*
[3]*Dahlem Center for Complex Quantum Systems and Fachbereich Physik, Freie Universität Berlin, 14195 Berlin, Ger*
(Received 16 March 2010; published 20 October 2010)

We show that the combination of spin-orbit coupling with a Zeeman field or strong interactions may lead to the formation of a helical electron liquid in single-channel quantum wires, with spin and velocity perfectly correlated. We argue that zero-energy Majorana bound states are formed in various situations when such wires are situated in proximity to a conventional *s*-wave superconductor. This occurs when the external magnetic field, the superconducting gap, or, most simply, the chemical potential vary along the wire. These Majorana states do not require the presence of a vortex in the system. Experimental consequences of the helical liquid and the Majorana states are also discussed.

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik,[1]* K. Zuo,[1]* S. M. Frolov,[1] S. R. Plissard,[2] E. P. A. M. Bakkers,[1,2] L. P. Kouwenhoven[1]†

Majorana fermions are particles identical to their own antiparticles. They have been theoretically predicted to exist in topological superconductors. Here, we report electrical measurements on indium antimonide nanowires contacted with one normal (gold) and one superconducting (niobium titanium nitride) electrode. Gate voltages vary electron density and define a tunnel barrier between normal and superconducting contacts. In the presence of magnetic fields on the order of 100 millitesla, we observe bound, midgap states at zero bias voltage. These bound states remain fixed to zero bias, even when magnetic fields and gate voltages are changed over considerable ranges. Our observations support the hypothesis of Majorana fermions in nanowires coupled to superconductors.

SCIENCE VOL 336 25 MAY 2012

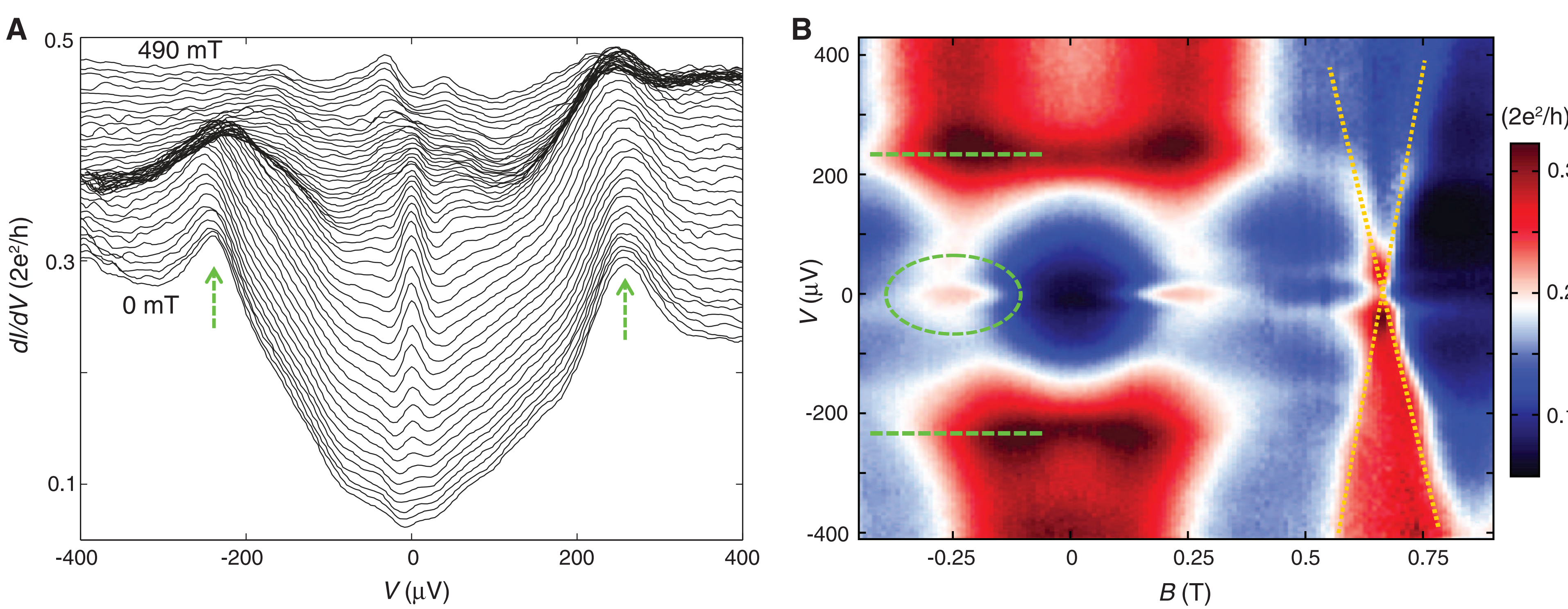


Fig. 2. Magnetic field–dependent spectroscopy. (**A**) *dI/dV* versus *V* at 70 mK taken at different *B* fields (from 0 to 490 mT in 10-mT steps; traces are offset for clarity, except for the lowest trace at $B = 0$). Data are from device 1. Arrows indicate the induced gap peaks. (**B**) Color-scale plot of *dI/dV* versus *V* and *B*. The ZBP is highlighted by a dashed oval; green dashed lines indicate the gap edges. At ~0.6 T, a non-Majorana state is crossing zero bias with a slope equal to ~3 meV/T (indicated by sloped yellow dotted lines). Traces in (A) are extracted from (B).

1. Descendants of the integer quantum Hall effect

2. Descendants of the fractional quantum Hall effect

3. Quantum matter without quasiparticles: strange metals and black holes

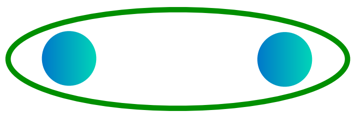
2. Descendants of the fractional quantum Hall effect

Bulk topological excitations which cannot be created from the ground state by the action of a local operator

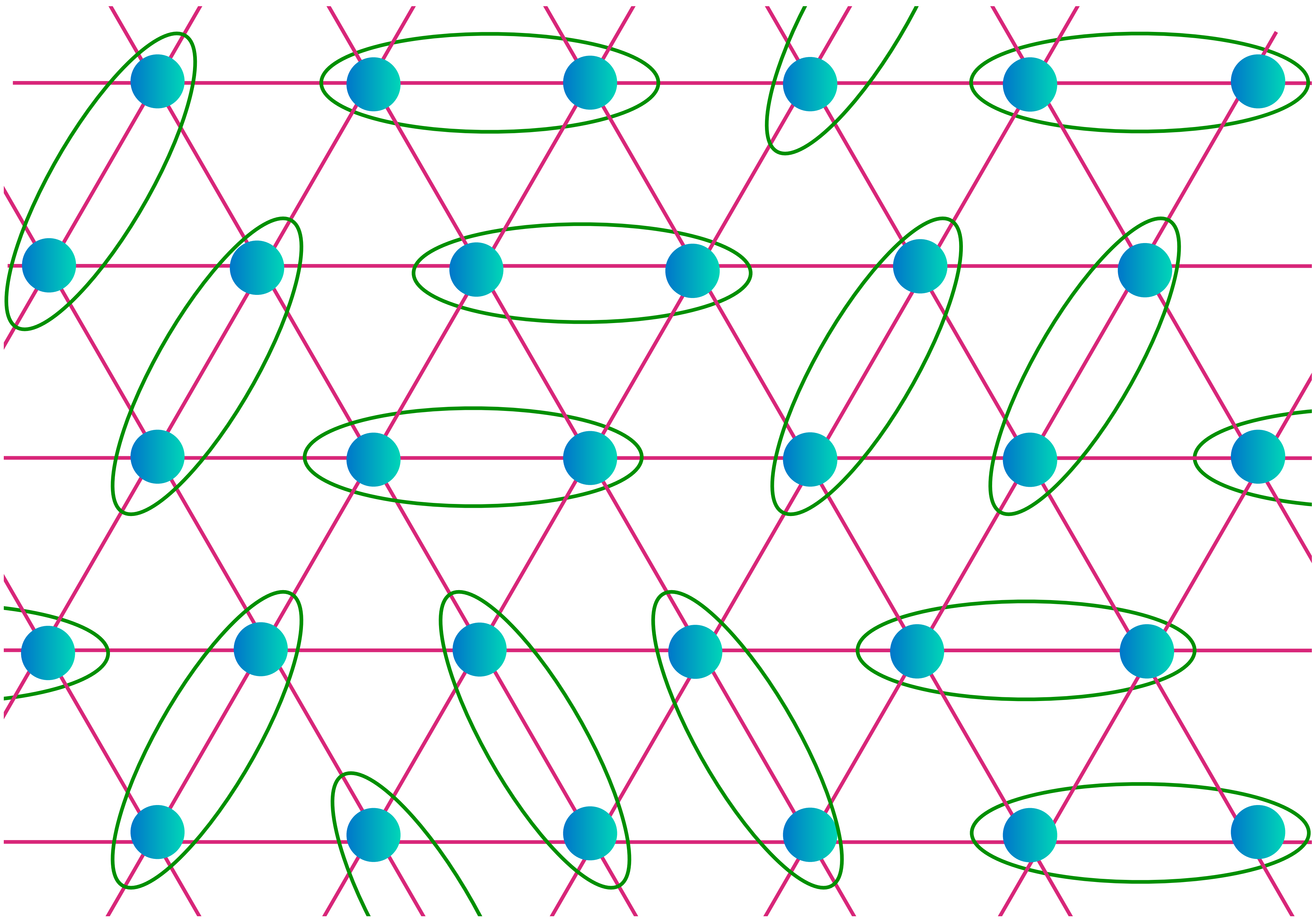
Quantum antiferromagnets on the triangular lattice

Resonating valence bond state with Z_2 topological order.

J. Wildeboer, A. Seidel, and R.G. Melko Phys. Rev. B **95**, 100402(R) (2017)



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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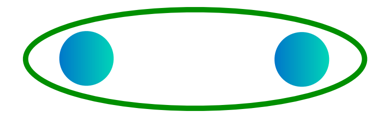

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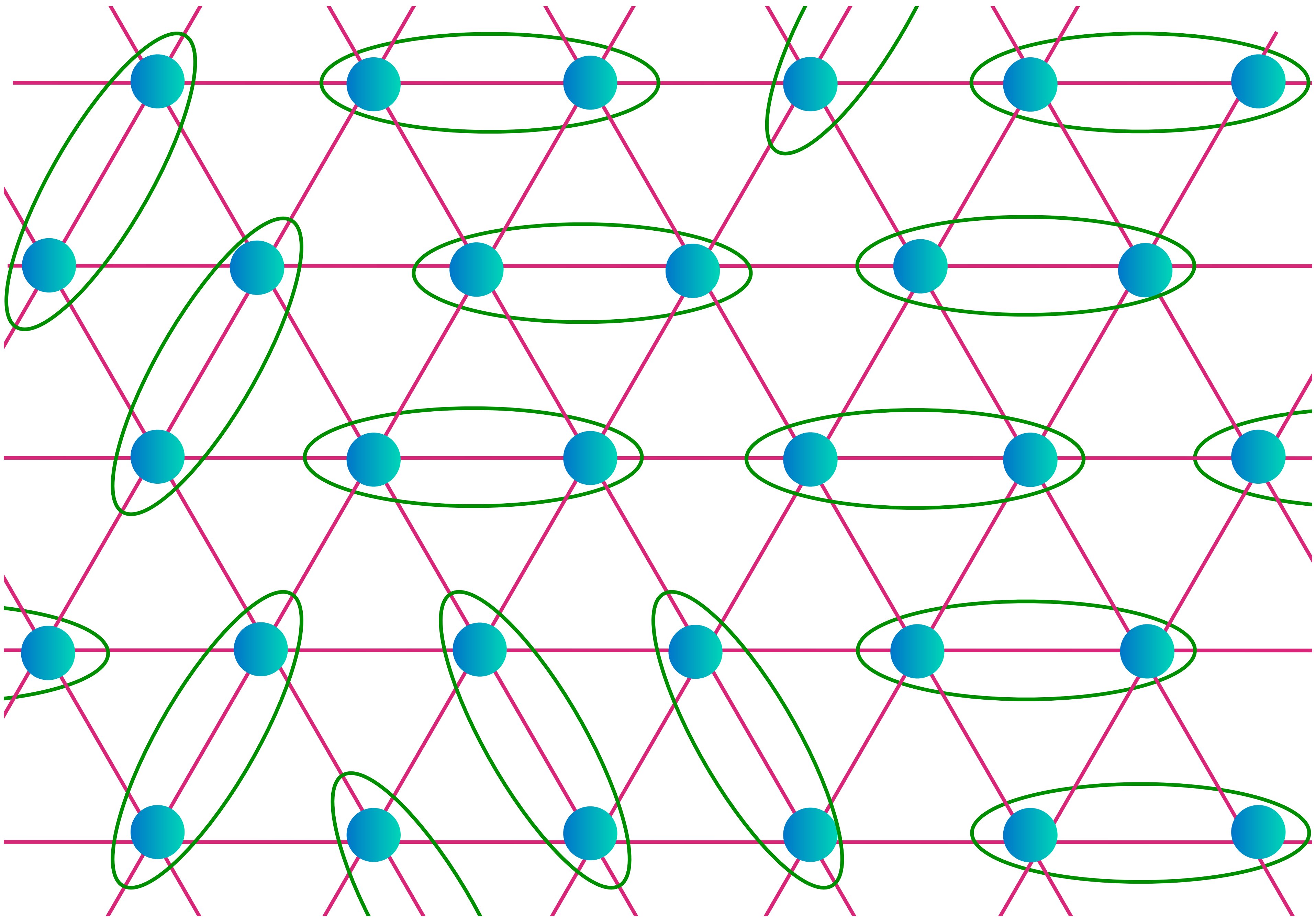

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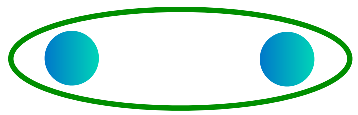

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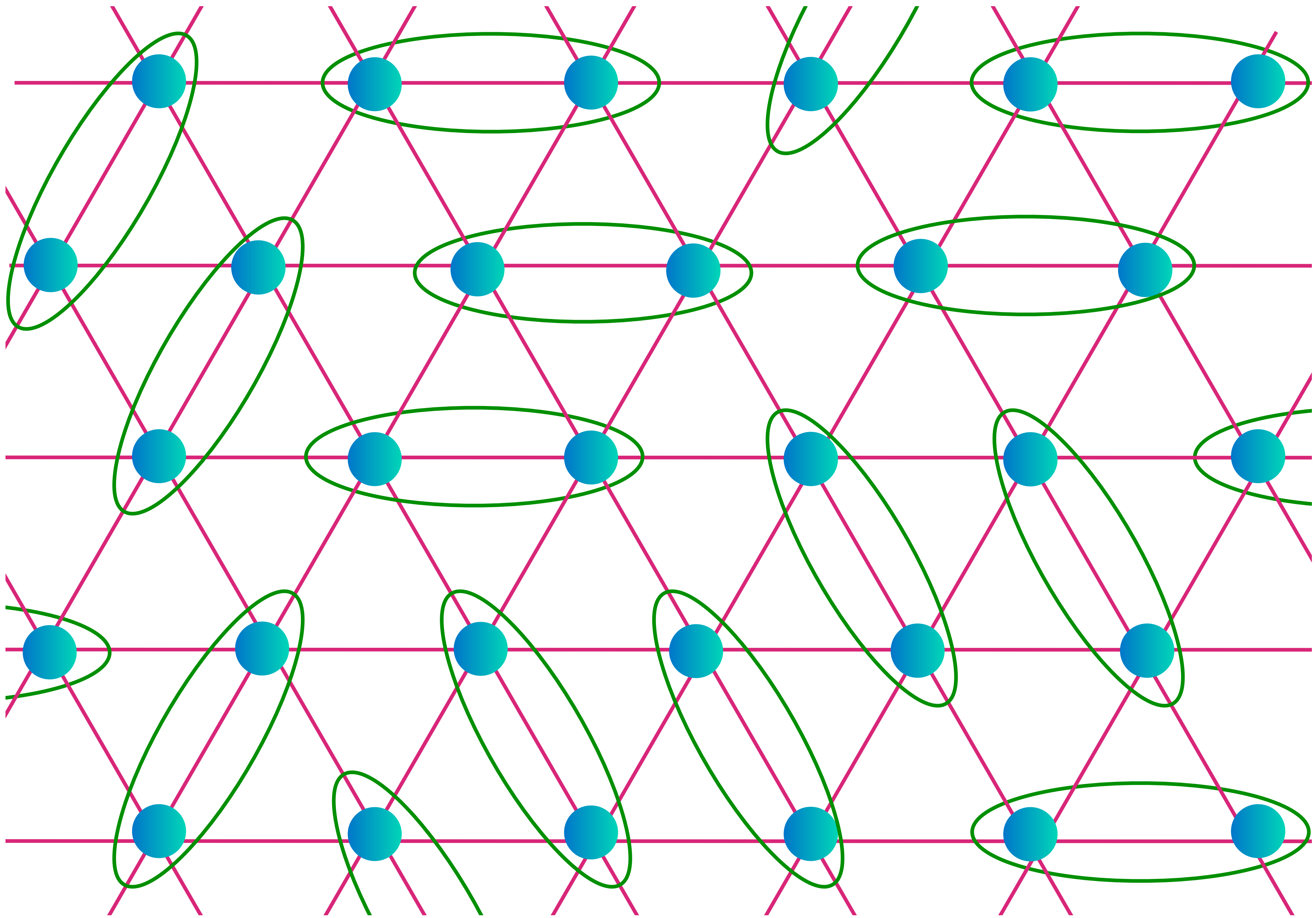

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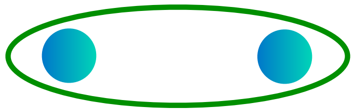

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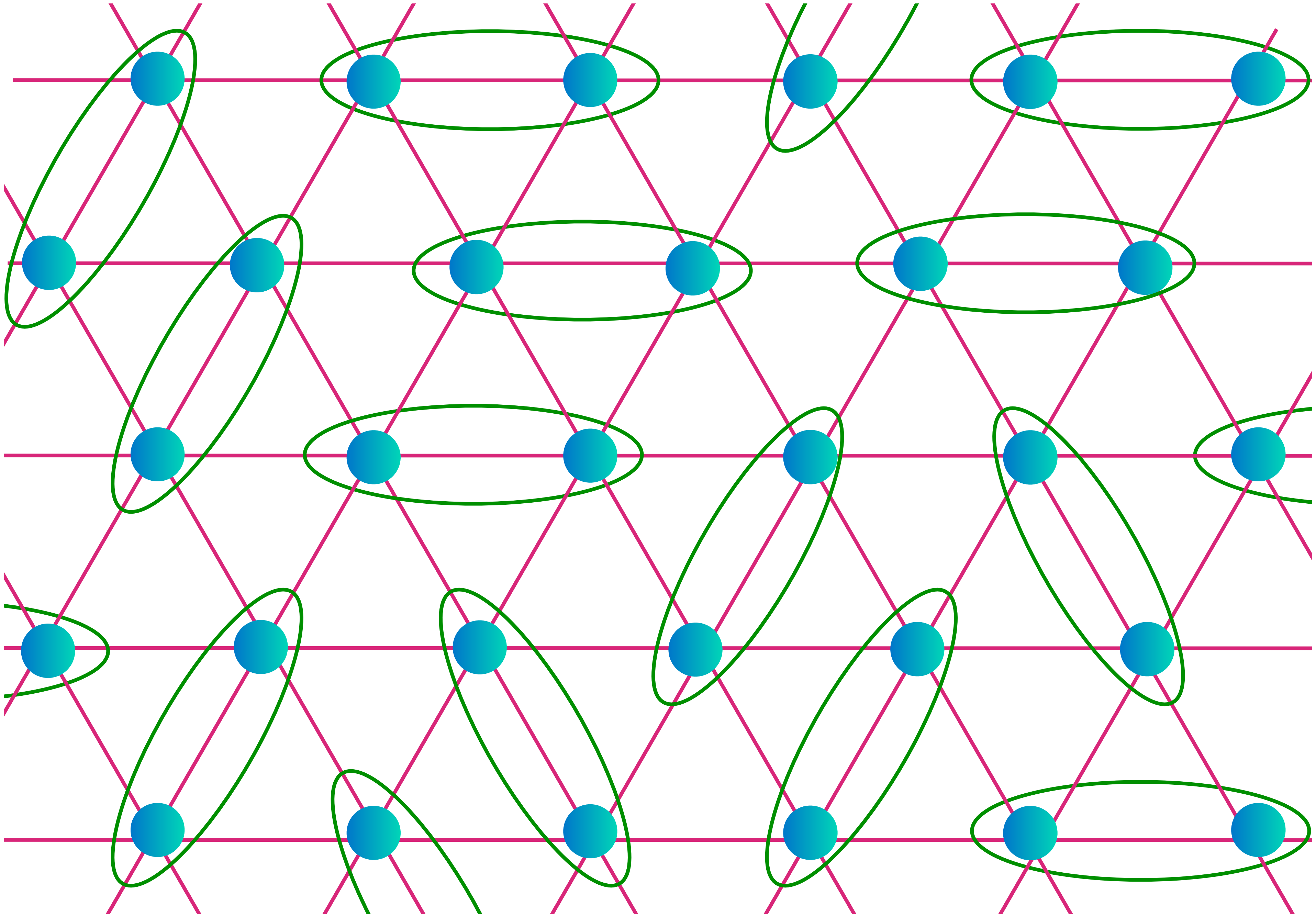

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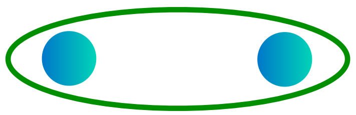

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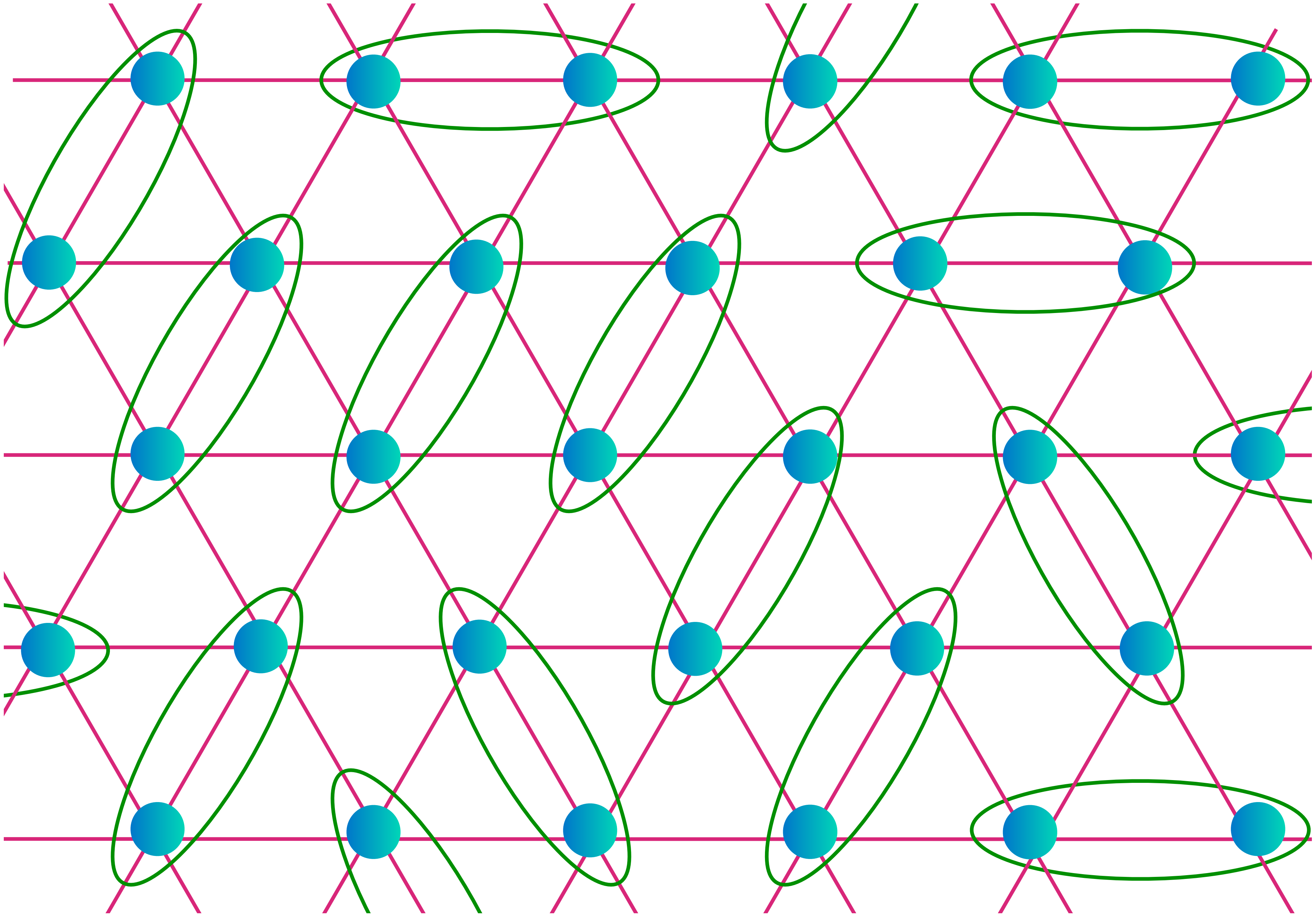

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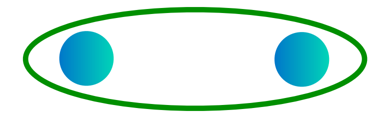
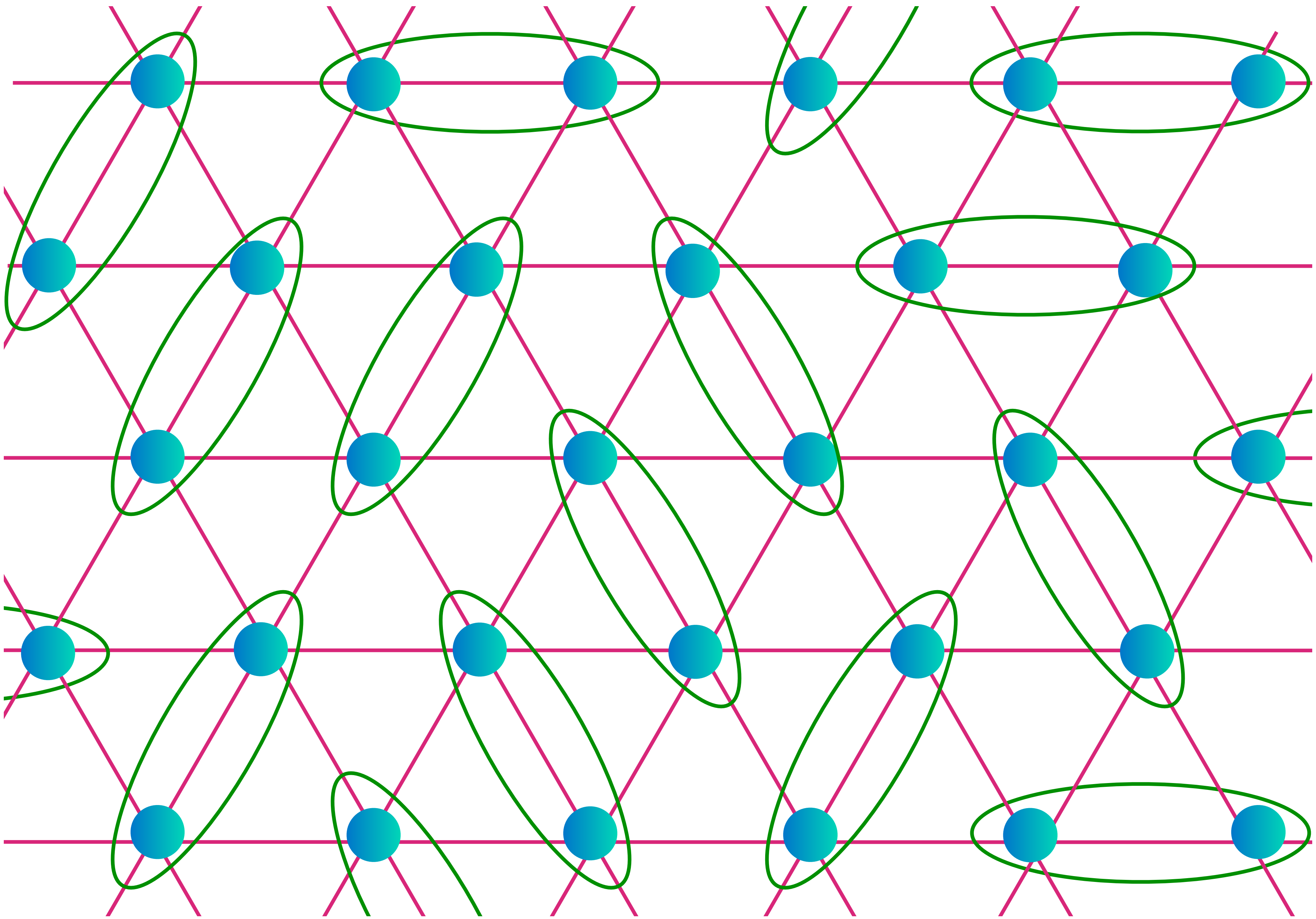
Resonating valence bond state with Z_2 topological order.

J. Wildeboer, A. Seidel, and R.G. Melko Phys. Rev. B **95**, 100402(R) (2017)



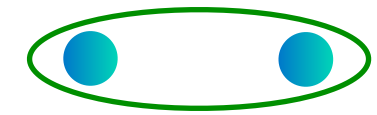
$$= \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\Big)$$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Quantum antiferromagnets on the triangular lattice

Has "topological" excitations which cannot be created individually by any local operator acting on the ground state



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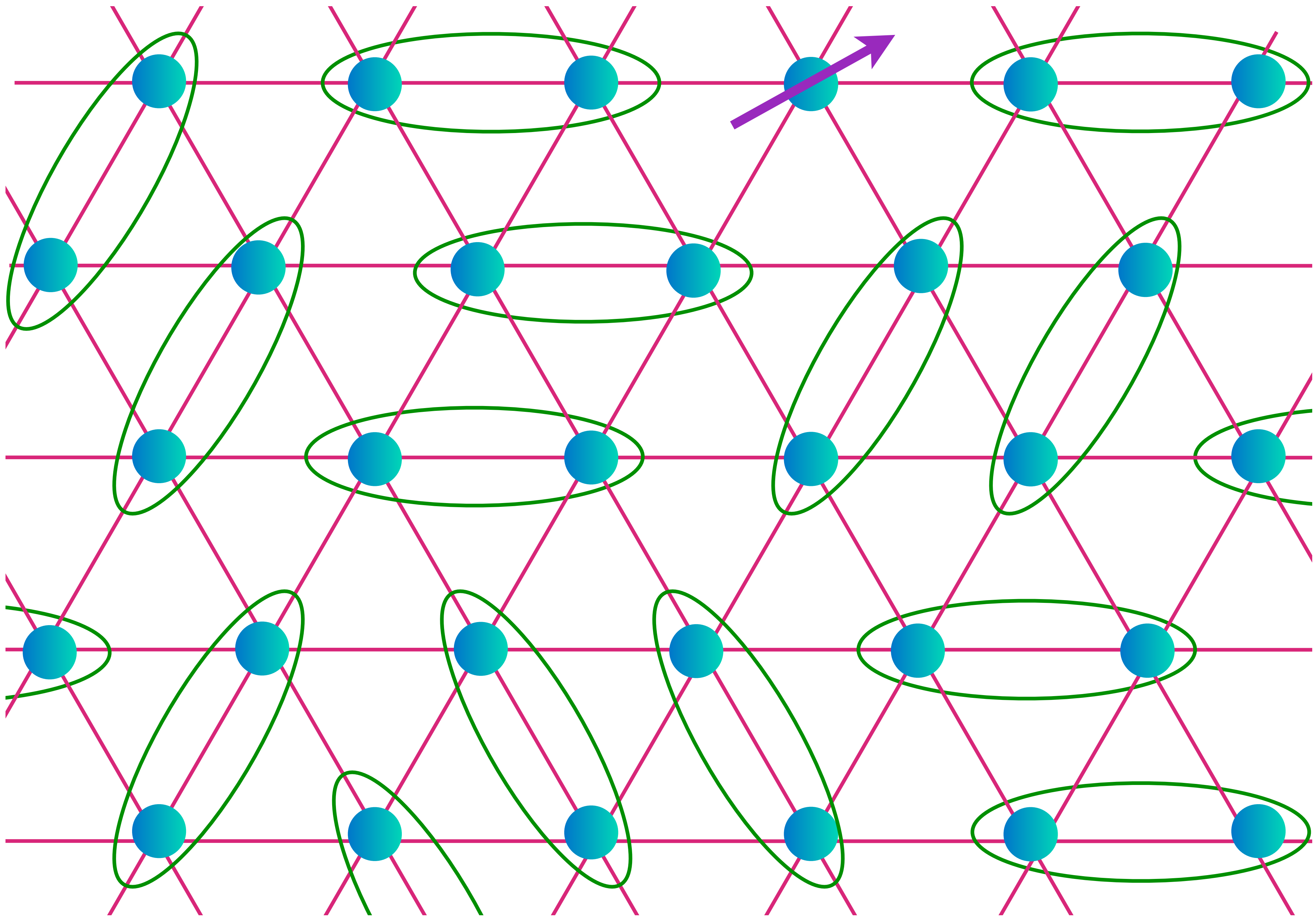

Quantum antiferromagnets on the triangular lattice

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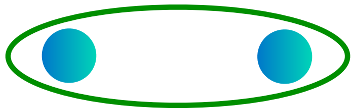

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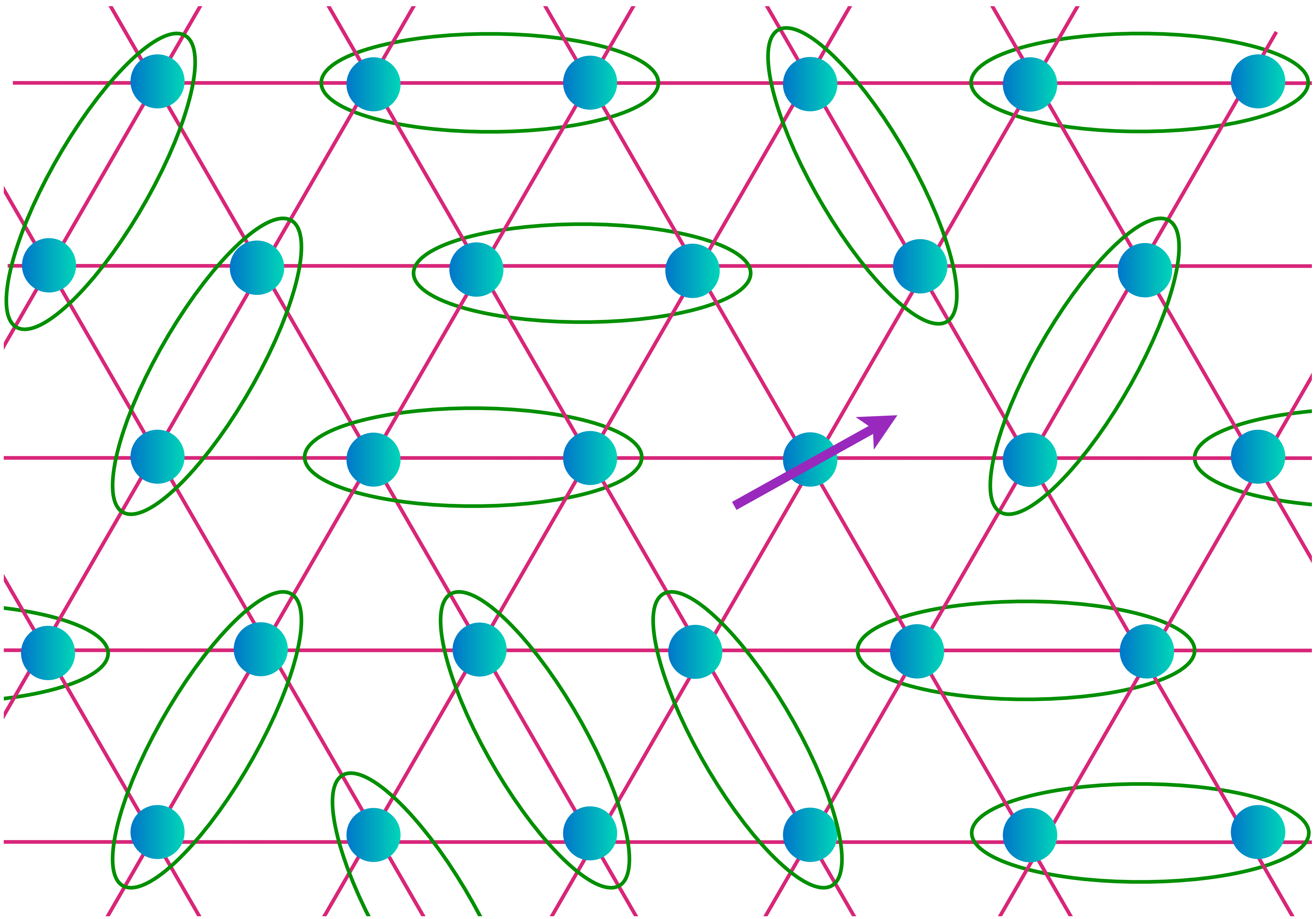

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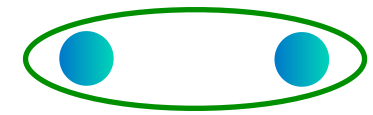

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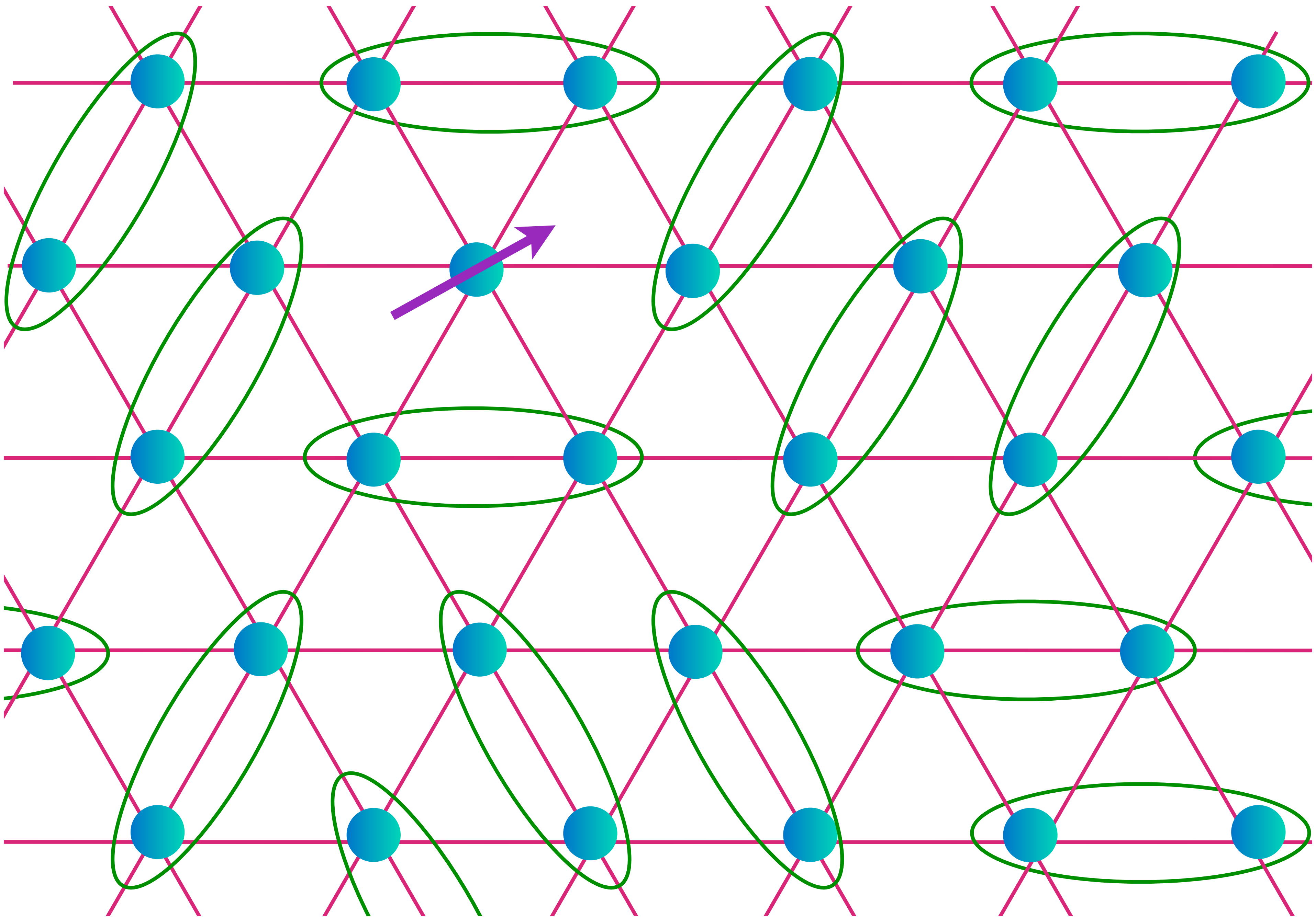

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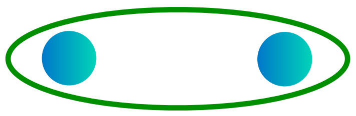

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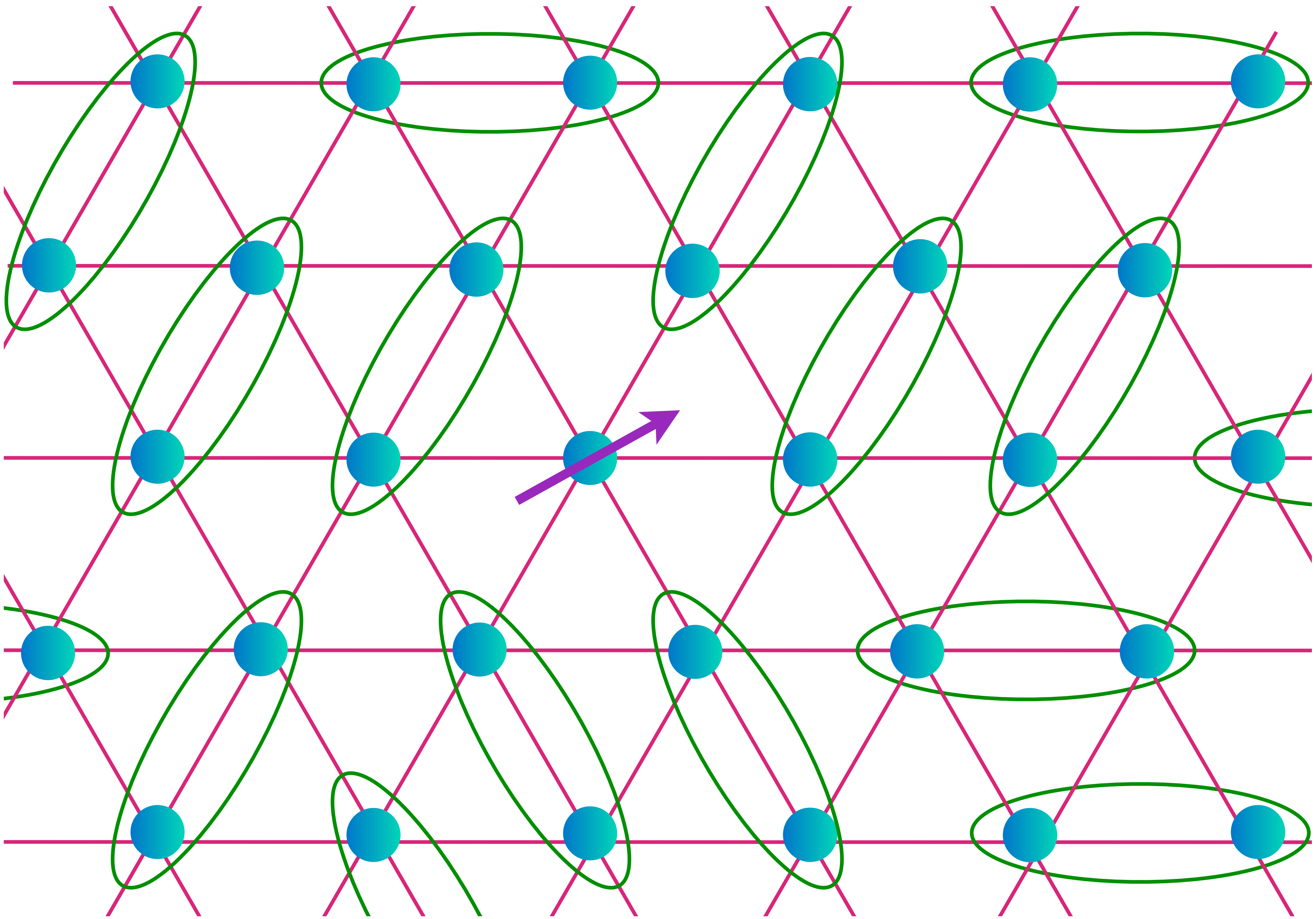

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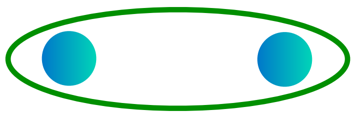

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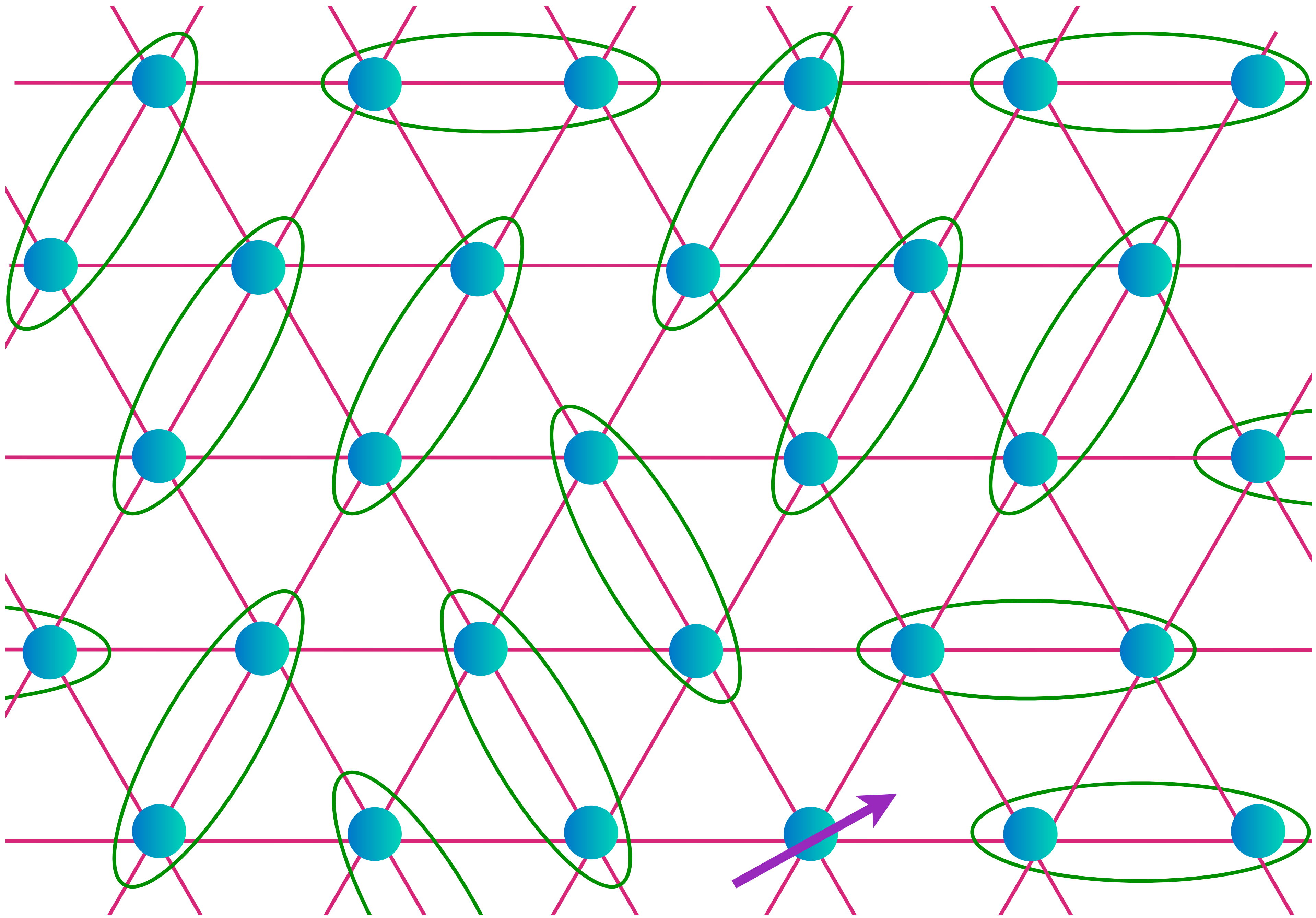


Equivalence of the Resonating-Valence-Bond and Fractional Quantum Hall States

V. Kalmeyer
Department of Physics, Stanford University, Stanford, California 94305

and

R. B. Laughlin
Department of Physics, Stanford University, Stanford, California 94305, and University of California, Lawrence Livermore National Laboratory, Livermore, California 94550
(Received 24 July 1987)

We present evidence that the ground state of the frustrated Heisenberg antiferromagnet in two dimensions is well described by a fractional quantum Hall wave function for bosons. This is compatible with the resonating-valence-bond concept of Anderson in being a liquid with neutral spin-$\frac{1}{2}$ excitations. Our results suggest strongly that the resonating-valence-bond and fractional quantum Hall states are the same thing. We also argue that the excitation spectrum has an energy gap.

Chiral spin states and superconductivity

X. G. Wen, Frank Wilczek,* and A. Zee
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 9 December 1988)

It is shown that several different order parameters can be used to characterize a type of *P*- and *T*-violating state for spin systems, that we call chiral-spin states. There is a closely related, precise notion of chiral-spin-liquid states. We construct soluble models, based on *P*- and *T*-symmetric local-spin Hamiltonians, with chiral-spin ground states. Mean-field theories leading to chiral spin liquids are proposed. Frustration is essential in stabilizing these states. The quantum numbers of quasiparticles around the chiral spin liquids are analyzed. They generally obey fractional statistics. Based on these ideas, it is speculated that superconducting states with unusual values of the flux quantum may exist.

66, NUMBER 13 PHYSICAL REVIEW LETTERS 1 APRIL 1991

Large-N Expansion for Frustrated Quantum Antiferromagnets

N. Read and Subir Sachdev
Department of Applied Physics, P.O. Box 2157, and Center for Theoretical Physics, P.O. Box 6666, Yale University, New Haven, Connecticut 06520
(Received 31 August 1990)

A large-N expansion technique based on symplectic [Sp(N)] symmetry for frustrated magnetic systems is proposed and applied to the square-lattice quantum antiferromagnet with first-, second-, and third-neighbor antiferromagnetic coupling. In addition to disordered states similar to those in unfrustrated systems, phases with incommensurate coplanar spin correlations and unconfined bosonic spinons are found. The occurrence of "order from disorder" is discussed. Neither chirally ordered nor spin-nematic states are found.

L REVIEW B VOLUME 44, NUMBER 6 1 AUGUST 1991-II

Mean-field theory of spin-liquid states with finite energy gap and topological orders

X. G. Wen
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 13 July 1990; revised manuscript received 18 March 1991)

The mean-field theory of a T- and P-symmetric spin-liquid state is developed. The quasiparticle excitations in the spin-liquid state are shown to be spin-$\frac{1}{2}$ neutral fermions (the spinons) and charge e spinless bosons (the holons). The spin-liquid state is shown to be characterized by a nontrivial topological order. Although our discussions are based on the mean-field theory, the concept of the topological order and the associated universal properties (e.g., the quantum number of the quasiparticles) are expected to be valid beyond the mean-field theory. We also discuss the dynamical stability of the mean-field theory.

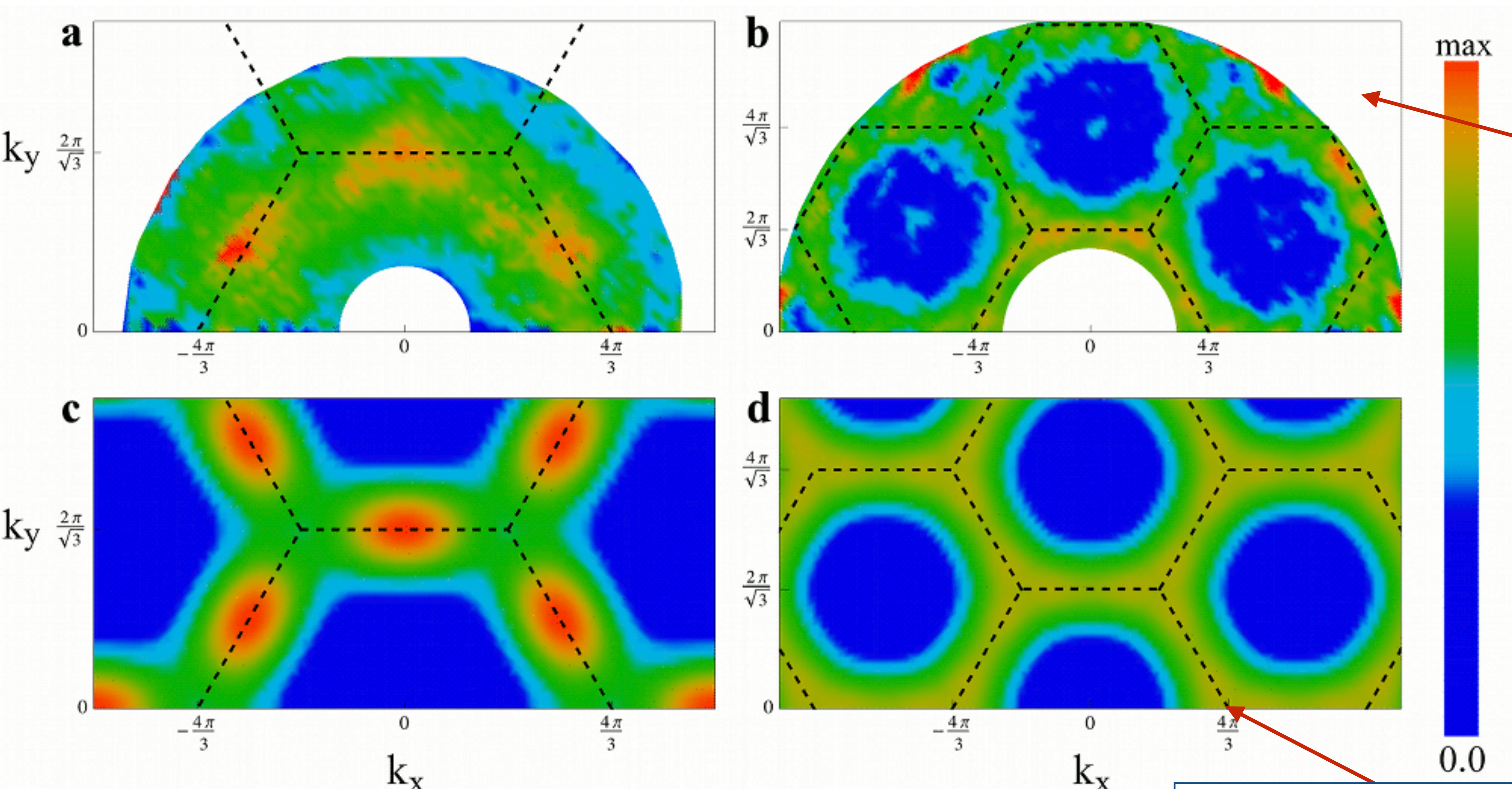
$\mathbb{Z}_2$ spin liquids: States with topological order (*i.e.* there are bulk excitations that cannot be created individually by a local operator) which can preserve time-reversal symmetry. No protected edge states in general (but some varieties do have protected edge states)

Evidence for a gapped spin-liquid ground state in a kagome Heisenberg antiferromagnet

Mingxuan Fu,[1] Takashi Imai,[1,2*] Tian-Heng Han,[3,4] Young S. Lee[5,6]

The kagome Heisenberg antiferromagnet is a leading candidate in the search for a spin system with a quantum spin-liquid ground state. The nature of its ground state remains a matter of active debate. We conducted oxygen-17 single-crystal nuclear magnetic resonance (NMR) measurements of the spin-1/2 kagome lattice in herbertsmithite [$ZnCu_3(OH)_6Cl_2$], which is known to exhibit a spinon continuum in the spin excitation spectrum. We demonstrated that the intrinsic local spin susceptibility χ_{kagome}, deduced from the oxygen-17 NMR frequency shift, asymptotes to zero below temperatures of 0.03*J*, where *J* ~ 200 kelvin is the copper-copper superexchange interaction. Combined with the magnetic field dependence of χ_{kagome} that we observed at low temperatures, these results imply that the kagome Heisenberg antiferromagnet has a spin-liquid ground state with a finite gap.

Gap to topological $S = 1/2$ spinon excitations: $\approx J/20$.



SCIENCE 6 NOVEMBER 2015 • VOL 350 ISSUE 6261 655

Neutron scattering observations

T Han, Young Lee, et al, Nature **492**, 406 (2012)

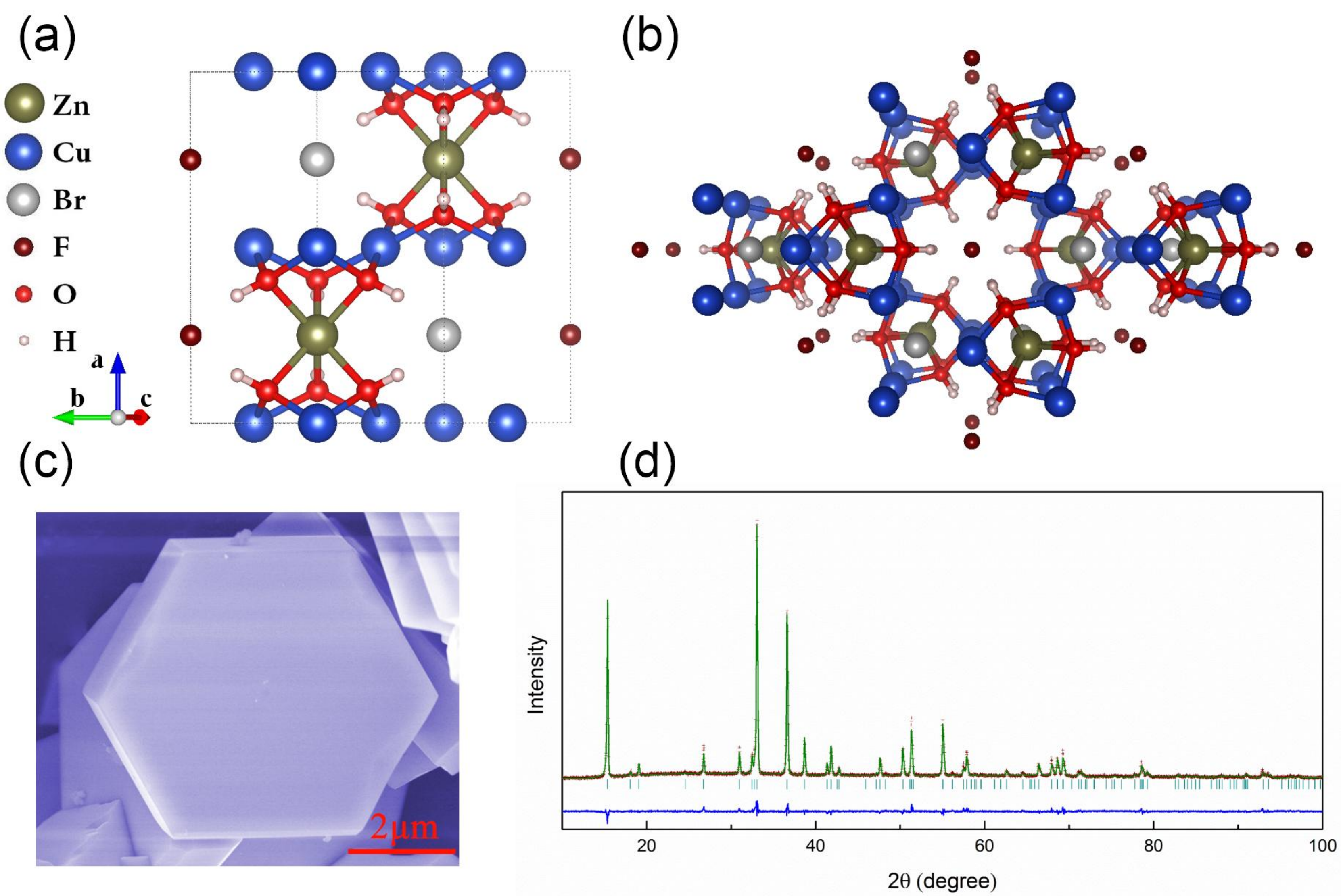
Theory by Punk, Chowdhury, Sachdev Nature Physics, 2013

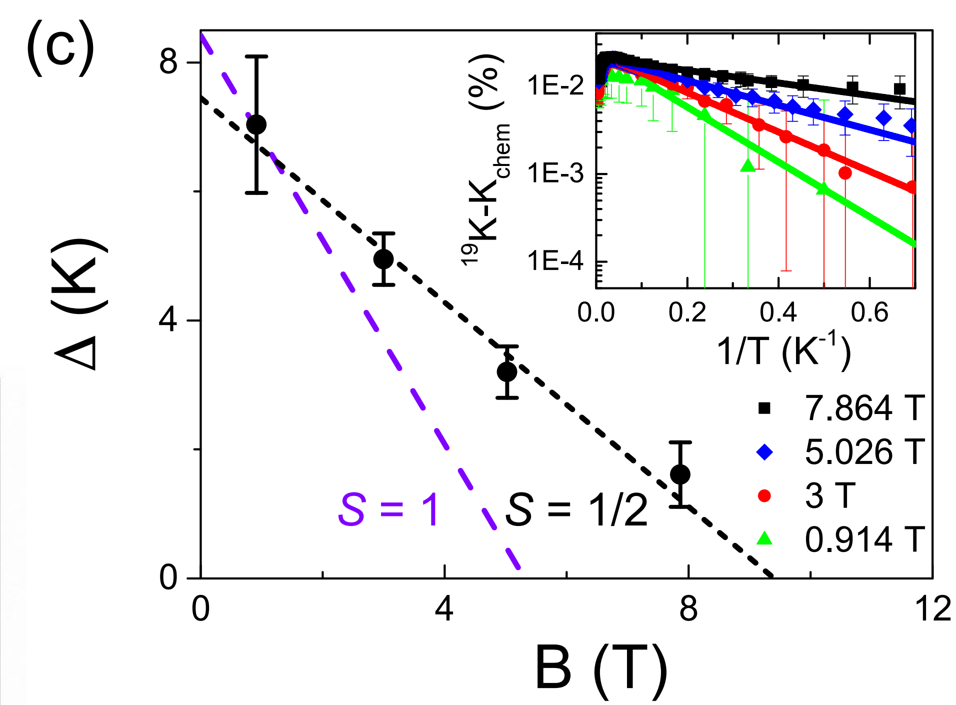
Gapped Spin-1/2 Spinon Excitations in a New Kagome Quantum Spin Liquid Compound $Cu_3Zn(OH)_6FBr$*

Zili Feng (冯子力)[1], Zheng Li (李政)[1,2], Xin Meng (孟鑫)[1], Wei Yi (衣玮)[1], Yuan Wei (魏源)[1], Jun Zhang (张骏)[3], Yan-Cheng Wang (王艳成)[1], Wei Jiang (蒋伟)[4], Zheng Liu (刘峥)[5], Shiyan Li (李世燕)[3,6]

Chinese Physics Letters, Volume 34, Number 7

Gap to topological $S = 1/2$ spinon excitations: $\approx J/20$.





States with bulk topological order are described by theories with emergent gauge fields.

$\mathbb{Z}_2$ topological order as the Higgs phase of a SU(2) gauge theory

We start with the Hubbard model

$$H_U = -\sum_{i,j} t_{ij} \hat{c}^\dagger_{i\alpha} \hat{c}_{j\alpha} - \mu \sum_i \hat{c}^\dagger_{i\alpha} \hat{c}_{i\alpha} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

The exact path integral of H_U in the "spin-fermion" form has action $S = S_c + S_{\text{int}}$.

$$S_c = \int_0^\beta d\tau \left[\sum_i c^\dagger_{i\alpha} (\partial_\tau - \mu) c_{i\alpha} - \sum_{i,j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} \right].$$

$$S_{\text{int}} = \int_0^\beta d\tau \left[\sum_i c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta} \cdot \vec{\Phi}_i + \frac{3}{2U} \vec{\Phi}_i^2 \right].$$

$\mathbb{Z}_2$ topological order as the Higgs phase of a SU(2) gauge theory

We rewrite the path integral as a SU(2) gauge theory by transforming to a rotating reference frame

$$c_i(\tau) = R_i(\tau)\psi_i(\tau)$$

Here, the unitary 2×2 matrices $R_i(\tau)$ are the bosonic spinon, and the $\psi_i(\tau)$ the 2-component fermionic chargon operators. This parameterization introduces an additional redundancy leading to an emergent local SU(2) gauge invariance,

$$R_i(\tau) \rightarrow R_i(\tau)V_i^\dagger(\tau), \quad \psi_i(\tau) \rightarrow V_i(\tau)\psi_i(\tau).$$

We introduce the Higgs field via

$$\vec{\sigma} \cdot \vec{H}_i(\tau) = R_i^\dagger(\tau)\vec{\sigma}R_i(\tau) \cdot \vec{\Phi}_i(\tau)$$

and then

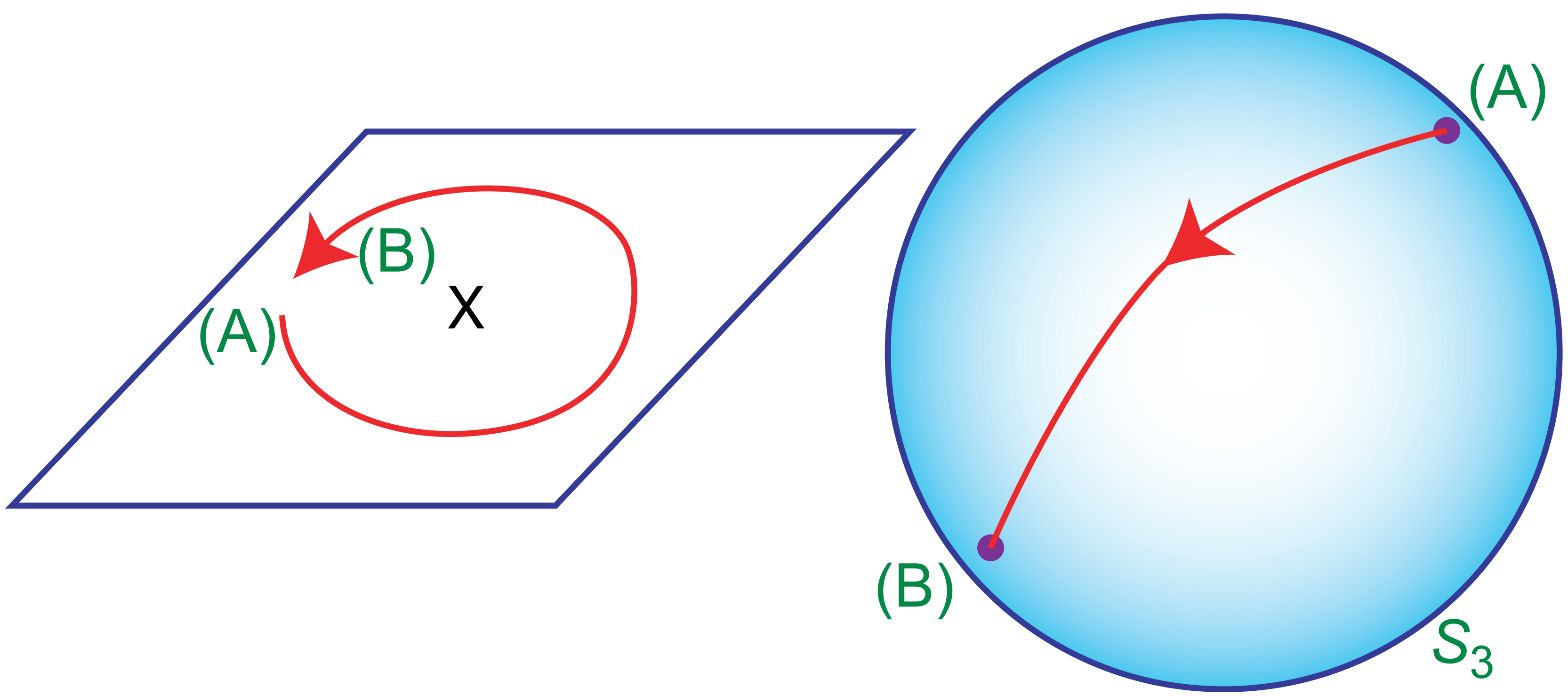
$$S_{\text{int}} = \int_0^\beta d\tau \left[\sum_i \psi_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta}\psi_{i\beta} \cdot \vec{H}_i + \frac{3}{2U}\vec{H}_i^2 \right].$$

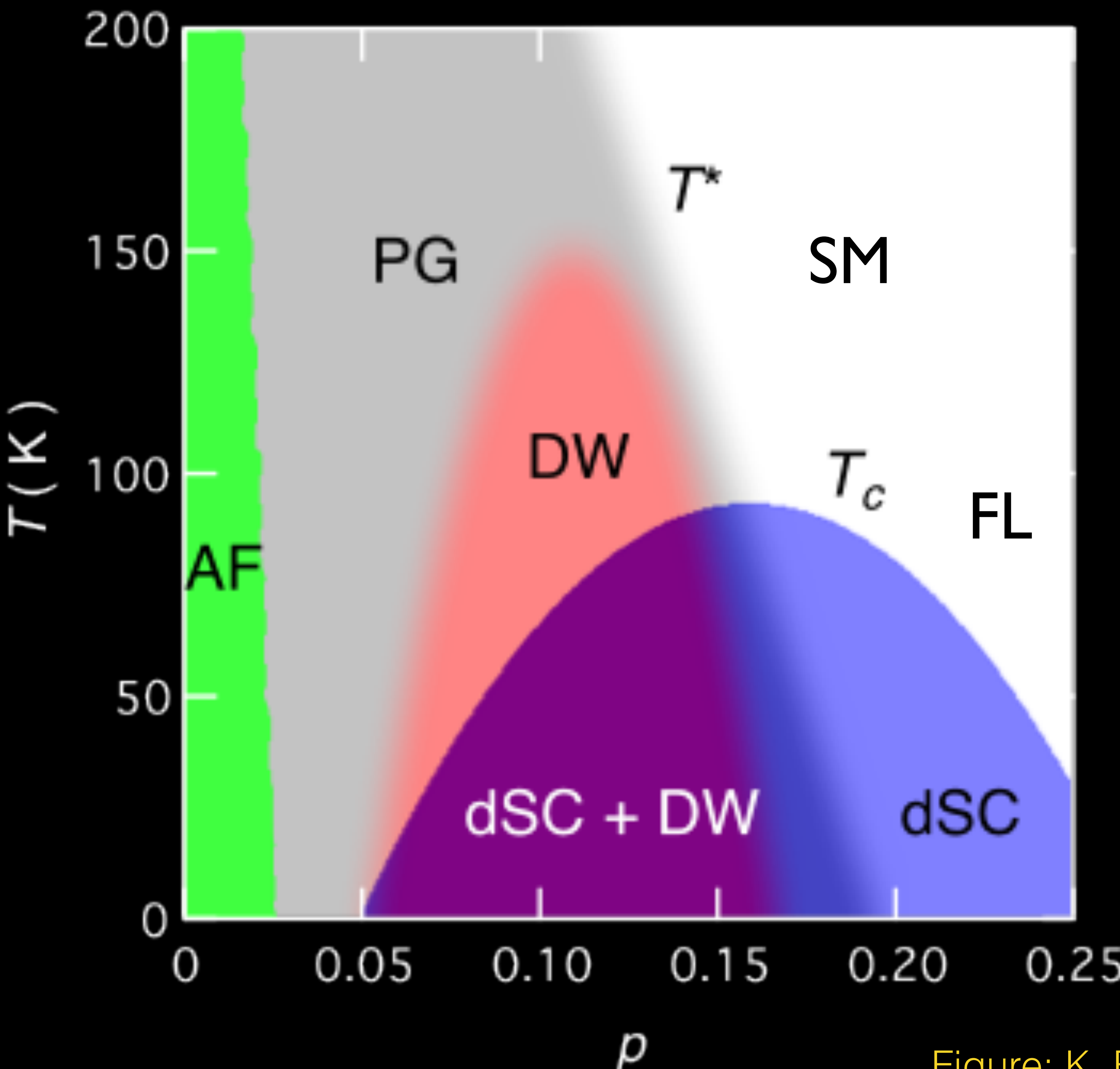
So the Higgs field is the local magnetic moment in the rotating frame of reference.

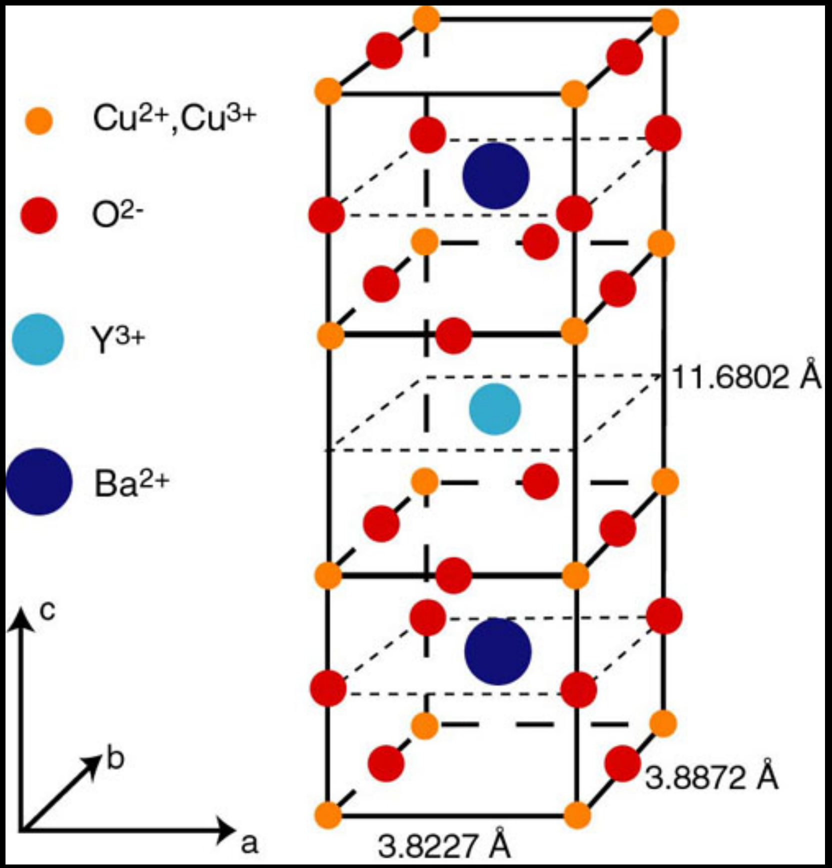
$\mathbb{Z}_2$ topological order as the Higgs phase of a SU(2) gauge theory

Phases with topological order are obtained by entering a Higgs phase where $\langle \vec{H} \rangle \neq 0$, while maintaining $\langle R \rangle = 0$. Any such phase will preserve spin-rotation invariance. The vanishing of $\langle R \rangle$ arises from large fluctuations in the local rotating reference frame, and so we are considering states with local magnetic order whose *orientation* undergoes large quantum fluctuations. However, the *magnitude* of the local magnetic order remains large, and this is captured by the Higgs field with $\langle \vec{H} \rangle \neq 0$.

A non-collinear spatial configuration of $\langle \vec{H}_i \rangle$ breaks SU(2) down to SO(3) $\equiv S_3/\mathbb{Z}_2$. This leads to a phase with $\mathbb{Z}_2$ topological order, which has stable, topological, finite energy vortex ("vison") excitations.



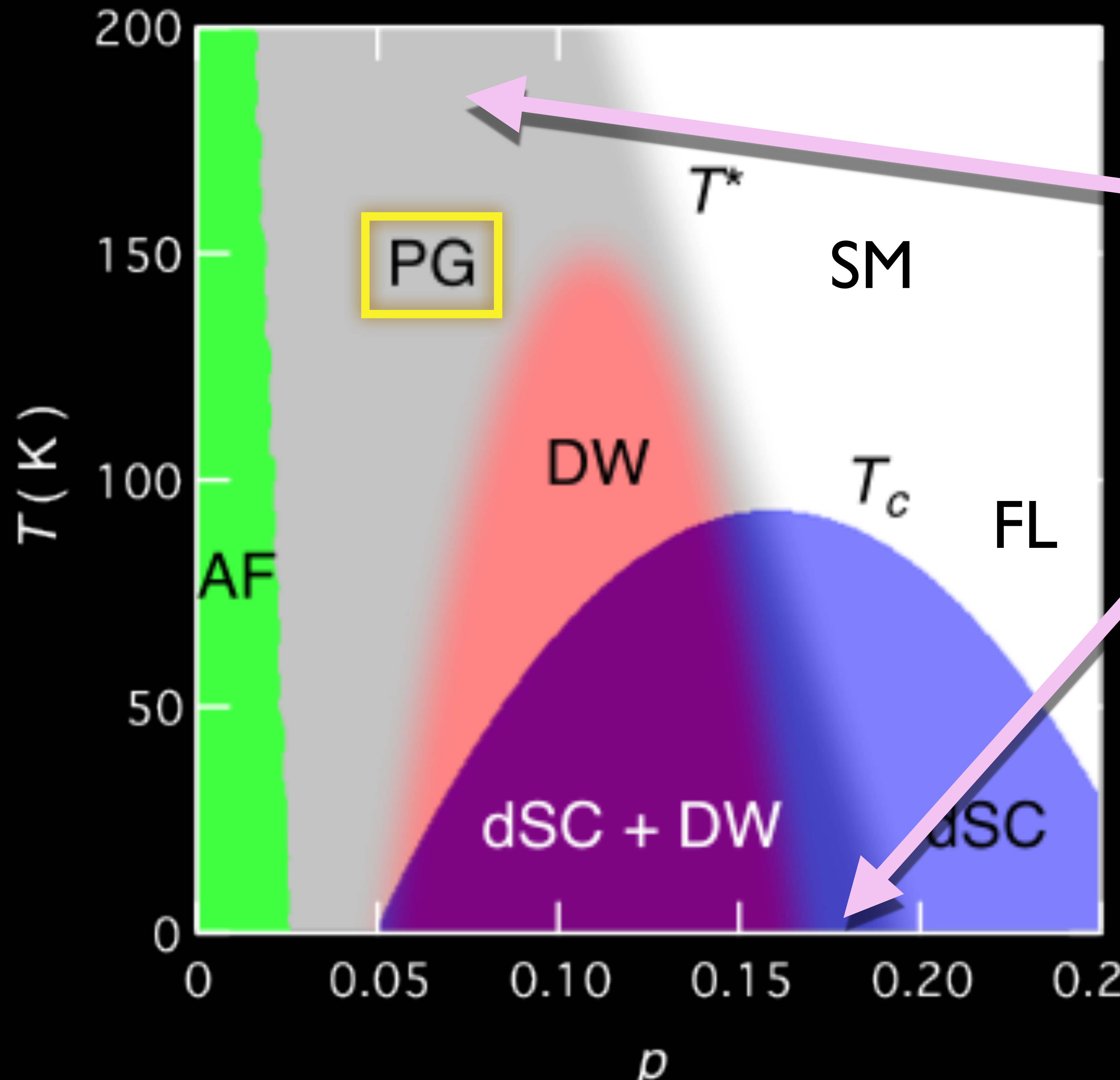




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

T. Senthil, M. Vojta and S. Sachdev, PRB **69**, 035111 (2004)



Pseudogap metal at low p

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

If present at $T=0$, a metal with a size p Fermi surface (and translational symmetry preserved) has ***bulk topological order***

1. Descendants of the integer quantum Hall effect

2. Descendants of the fractional quantum Hall effect

3. Quantum matter without quasiparticles: strange metals and black holes

High temperature superconductivity

Possible High T_c Superconductivity in the Ba – La – Cu – O System

J.G. Bednorz and K.A. Müller
IBM Zürich Research Laboratory, Rüschlikon, Switzerland

Received April 17, 1986

Metallic, oxygen-deficient compounds in the Ba – La – Cu – O system, with the composition $Ba_xLa_{5-x}Cu_5O_{5(3-y)}$ have been prepared in polycrystalline form. Samples with $x=1$ and 0.75, $y>0$, annealed below 900 °C under reducing conditions, consist of three phases, one of them a perovskite-like mixed-valent copper compound. Upon cooling, the samples show a linear decrease in resistivity, then an approximately logarithmic increase, interpreted as a beginning of localization. Finally an abrupt decrease by up to three orders of magnitude occurs, reminiscent of the onset of percolative superconductivity. The highest onset temperature is observed in the 30 K range. It is markedly reduced by high current densities. Thus, it results partially from the percolative nature, bute possibly also from $2D$ superconducting fluctuations of double perovskite layers of one of the phases present.

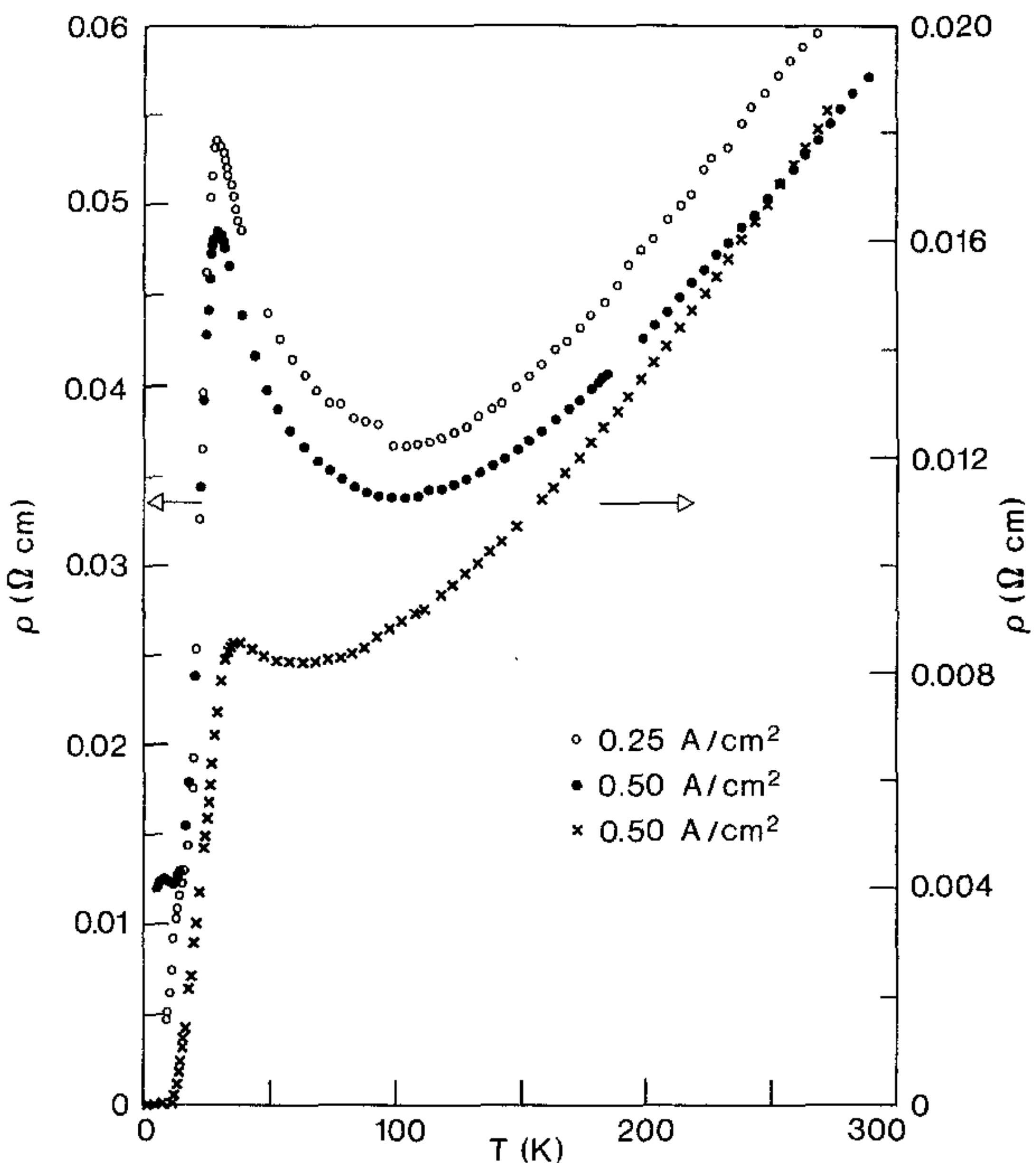


Fig. 1. Temperature dependence of resistivity in $Ba_xLa_{5-x}Cu_5O_{5(3-y)}$ for samples with x(Ba) = 1 (upper curves, left scale) and x(Ba) = 0.75 (lower curve, right scale). The first two cases also show the influence of current density

Z. Phys. B – Condensed Matter 64, 189–193 (1986)

Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

  - **Note:** The electron liquid in one dimension and the fractional quantum Hall state both have quasiparticles; however, the quasi-particles do not have the same quantum numbers as an electron.

Quantum matter with quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time is of order $\hbar E_F/(k_B T)^2$ as $T \to 0$, where E_F is the Fermi energy.

Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states
- Rapid thermalization

Local thermal equilibration or phase coherence time, τ_φ:

- There is an *lower bound* on τ_φ in all many-body quantum systems as $T \to 0$,

$$\tau_\varphi \geq C \frac{\hbar}{k_B T},$$

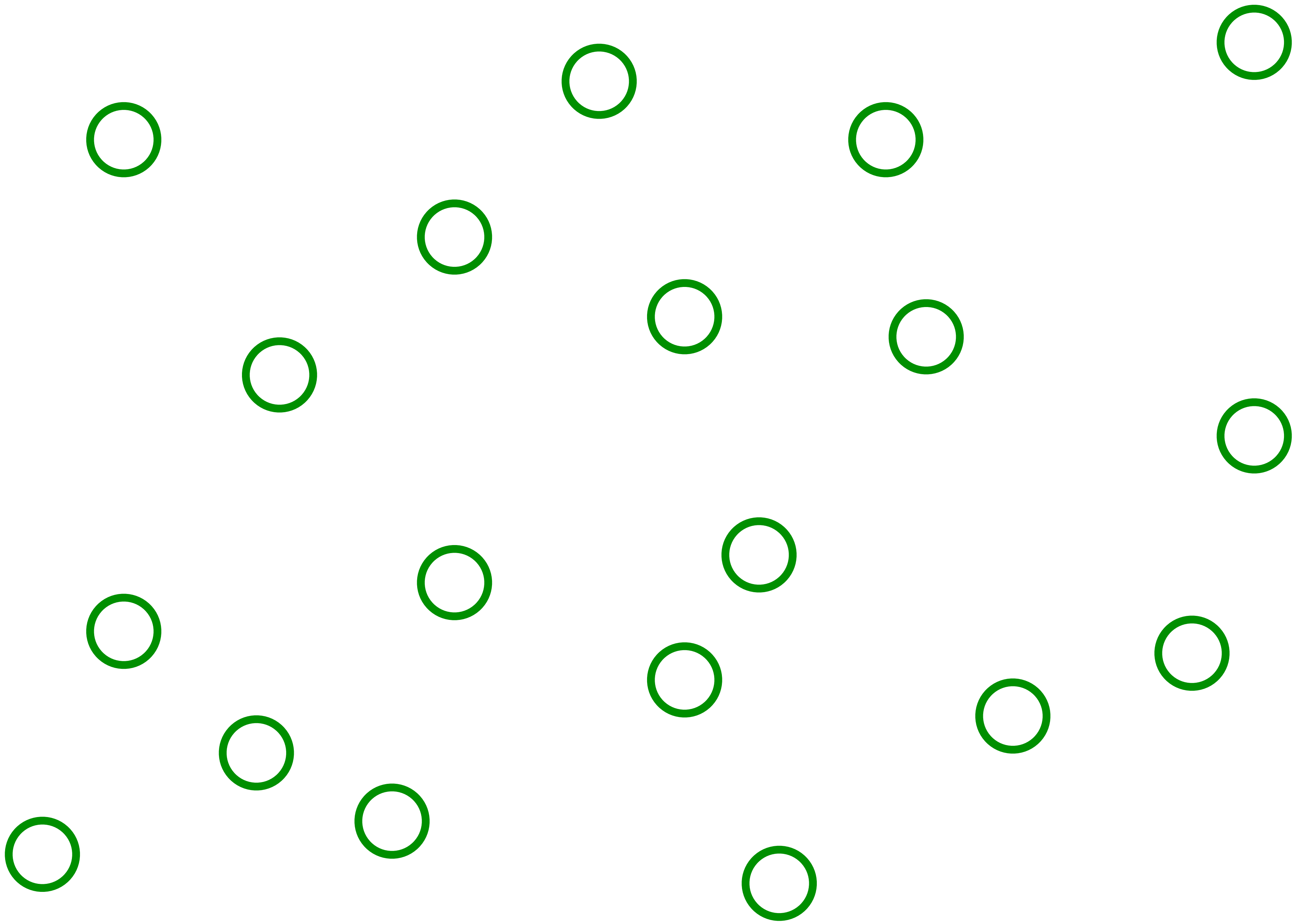
where C is a T-independent constant.

- Systems *without* quasiparticles have

$$\tau_\varphi \sim \frac{\hbar}{k_B T},$$

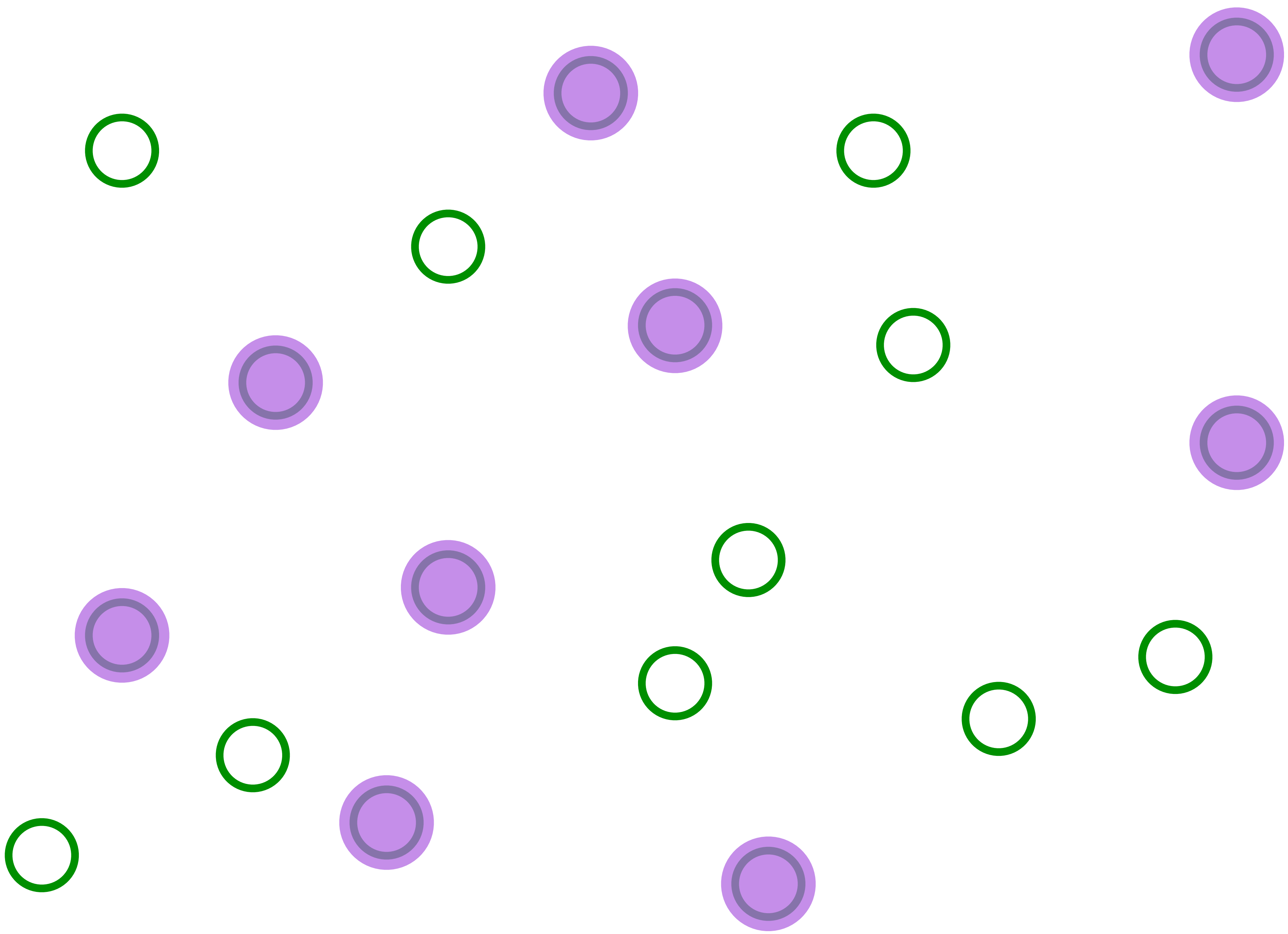
K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A simple model of a metal with quasiparticles



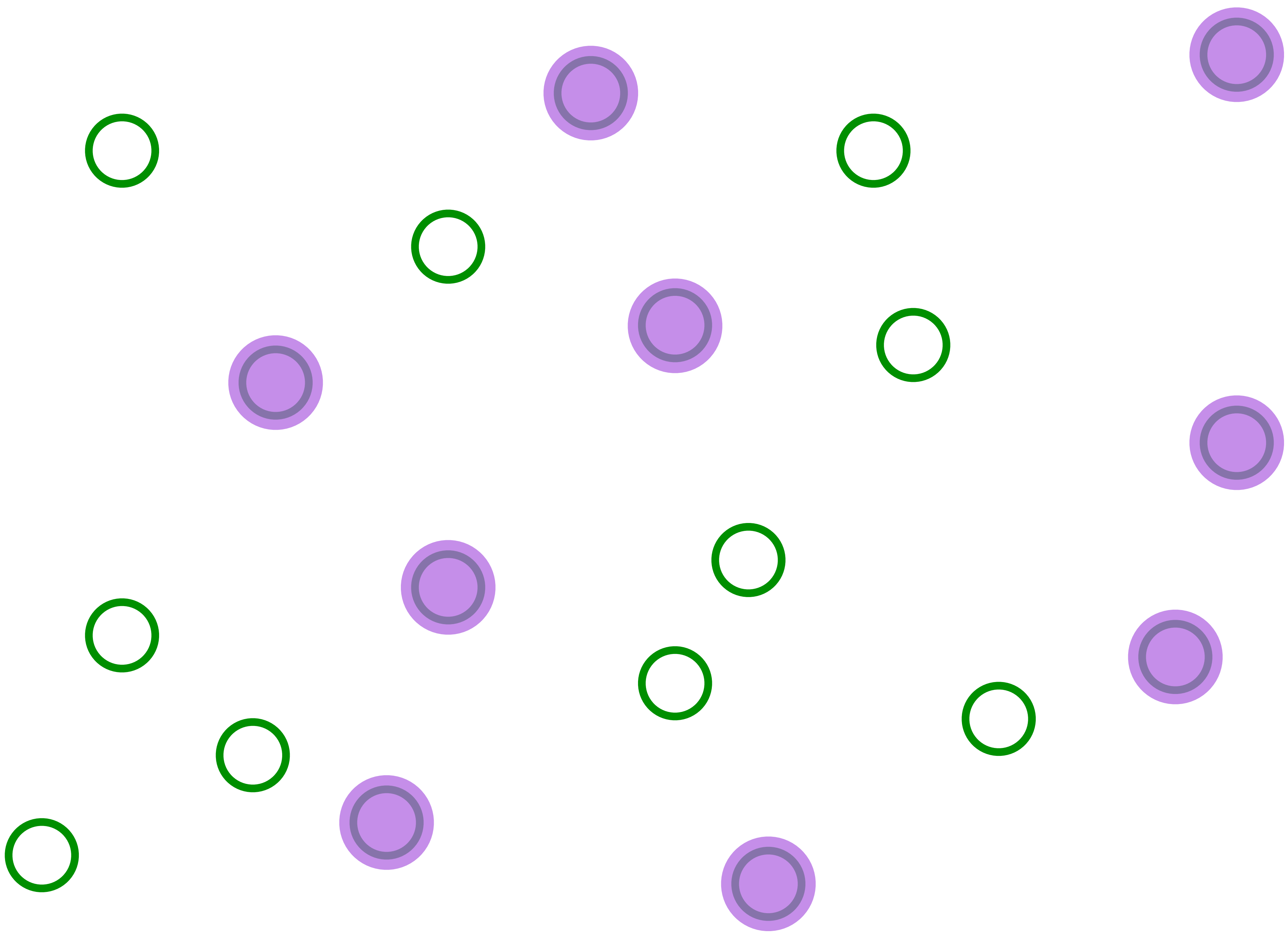
Pick a set of random positions

A simple model of a metal with quasiparticles



Place electrons randomly on some sites

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

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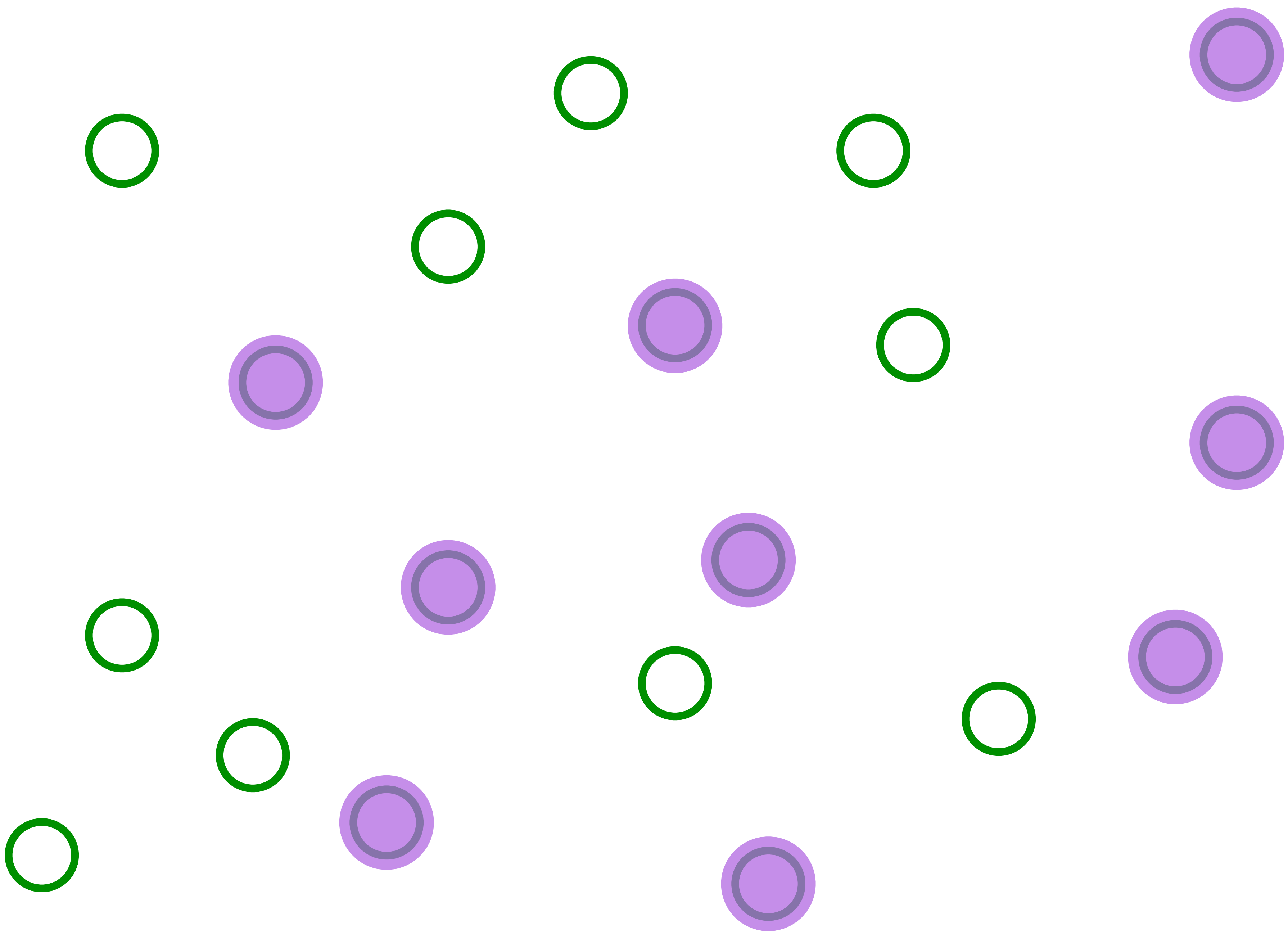

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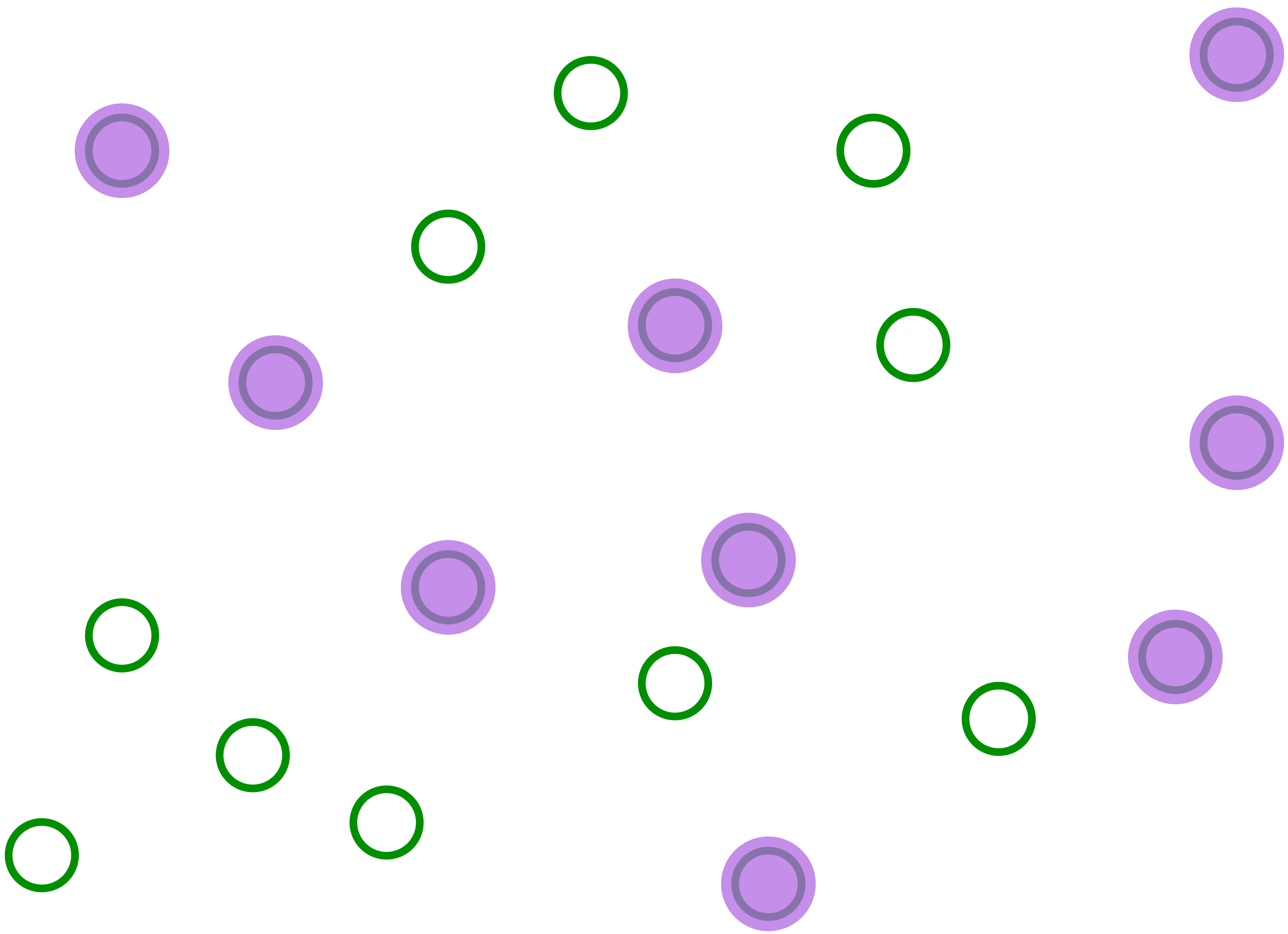

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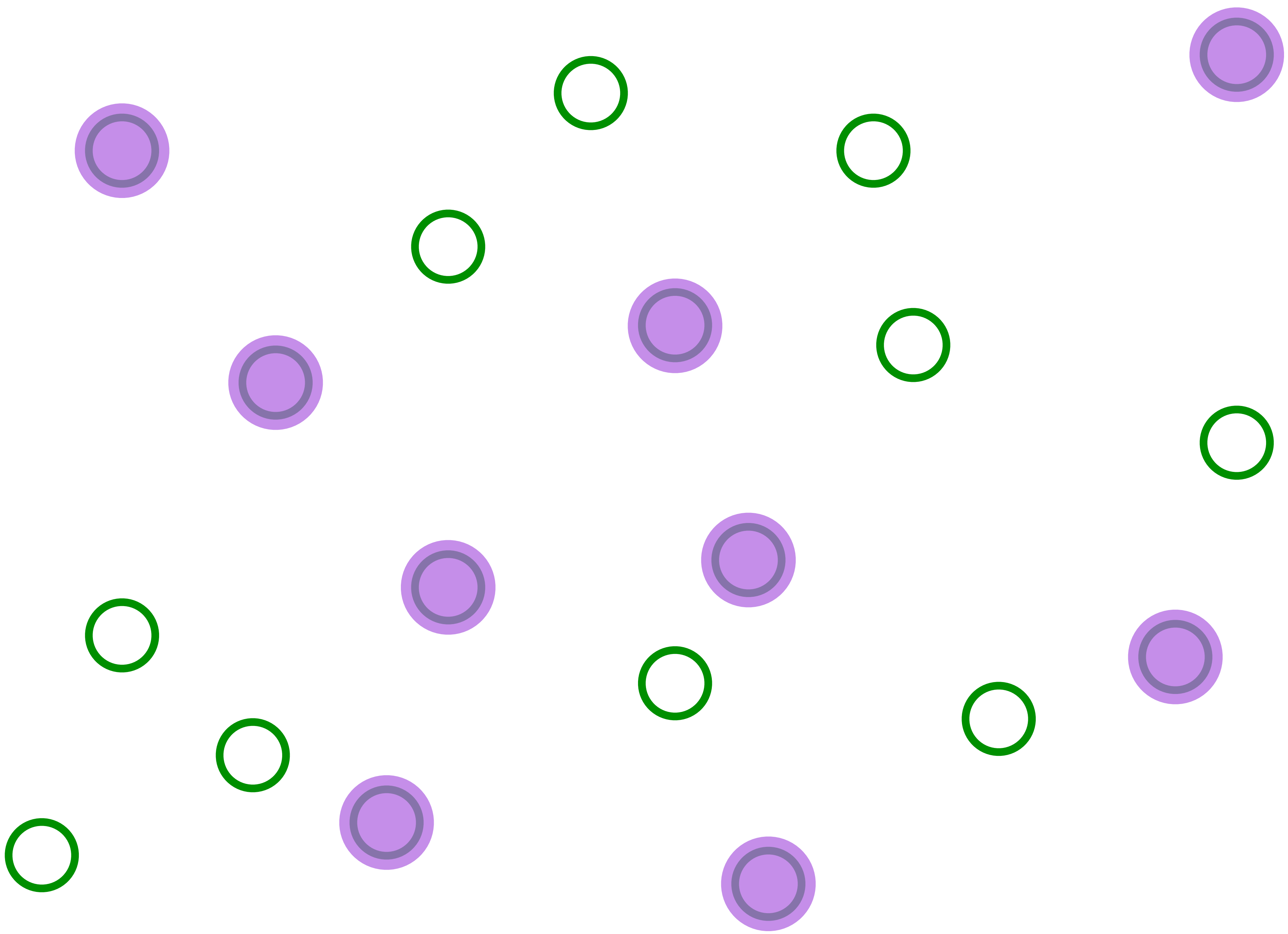


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

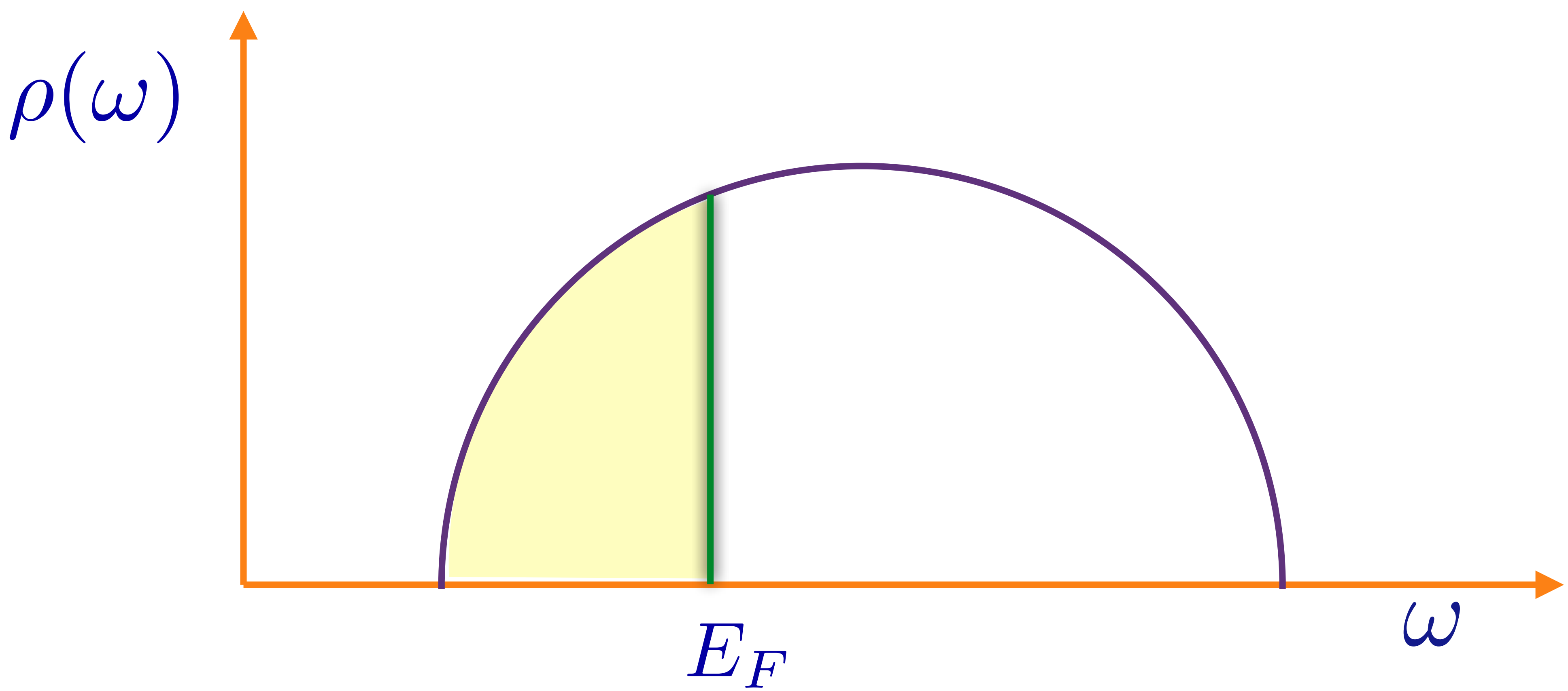
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



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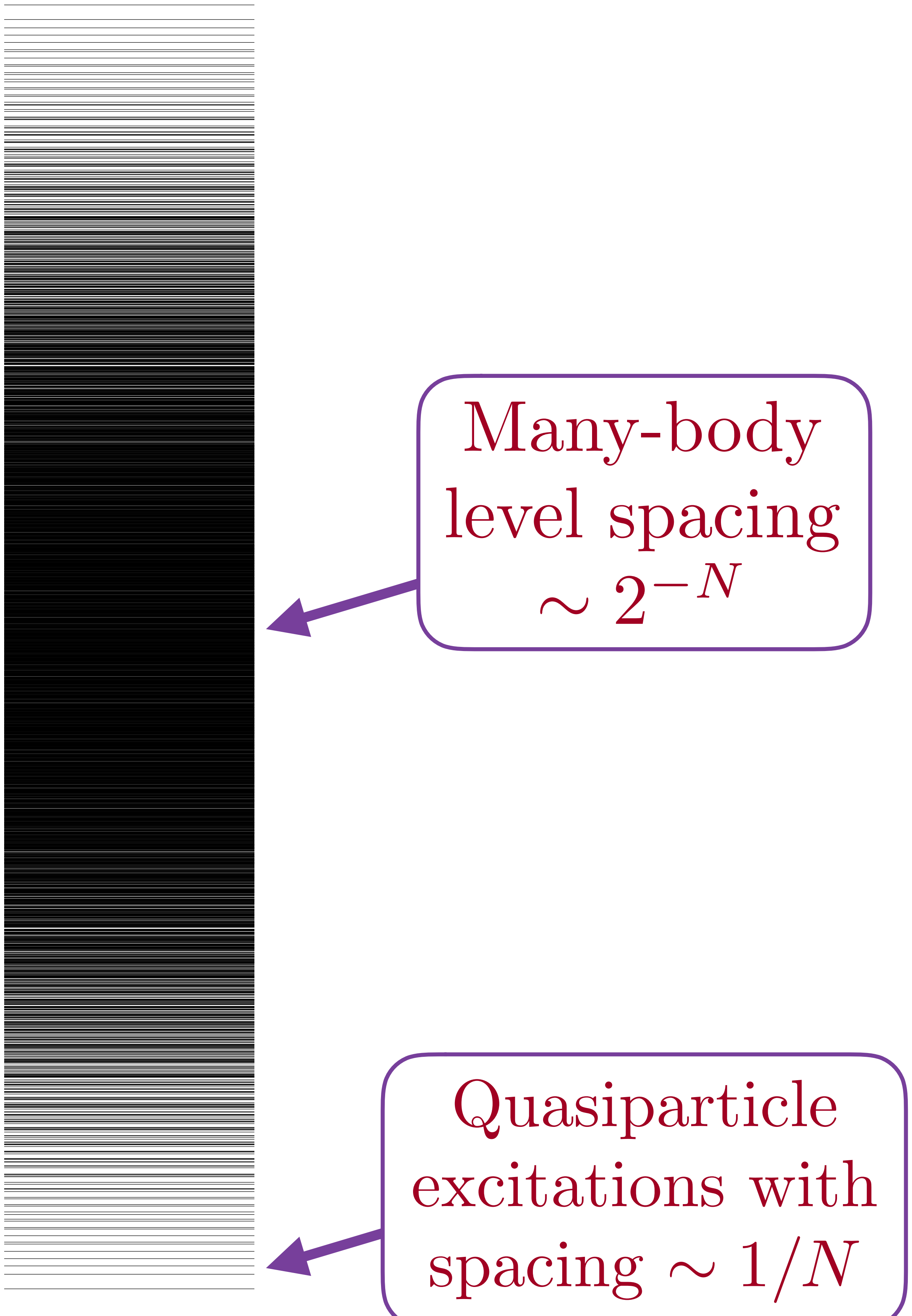
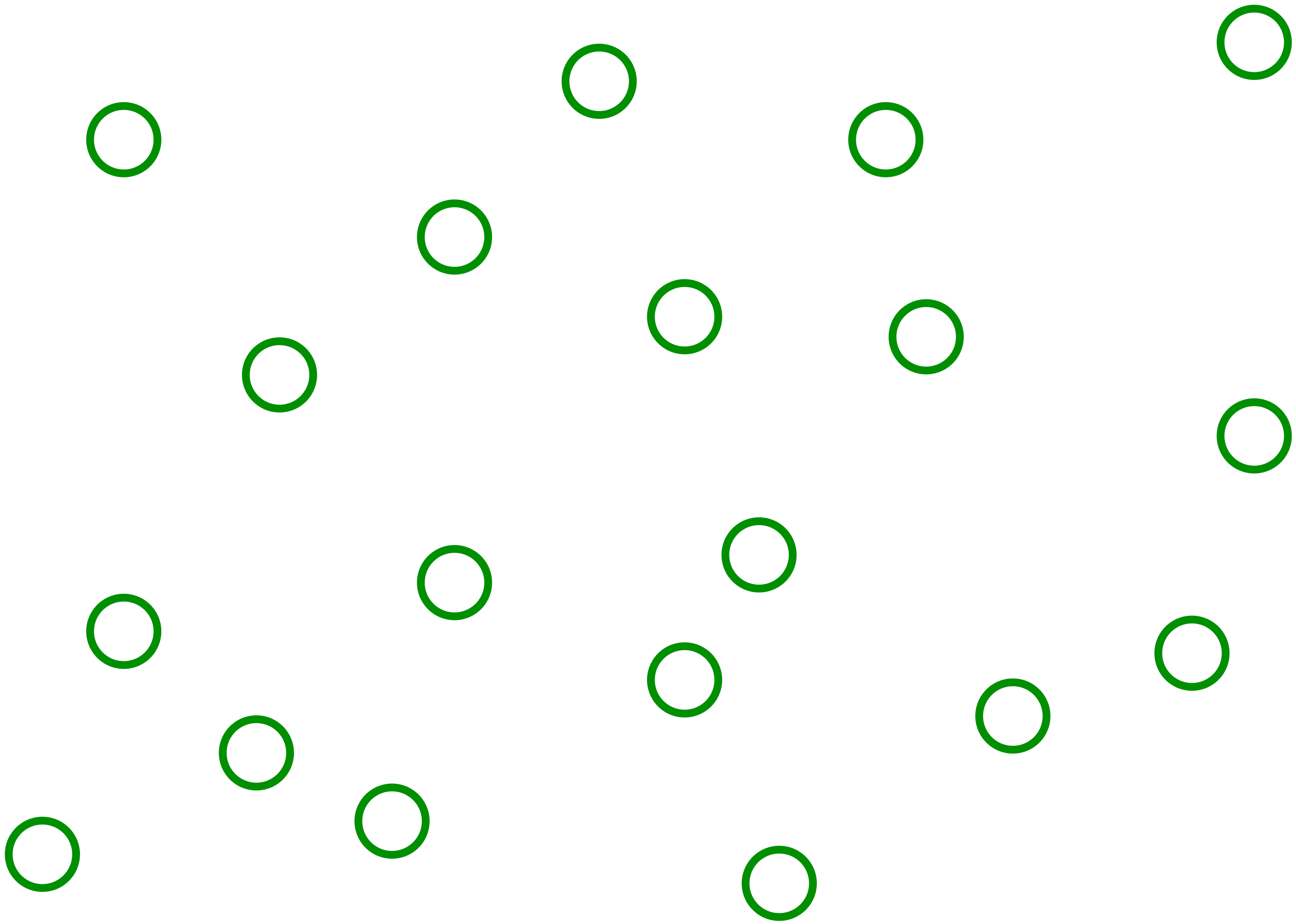


There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha ,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

The Sachdev-Ye-Kitaev (SYK) model



Pick a set of random positions

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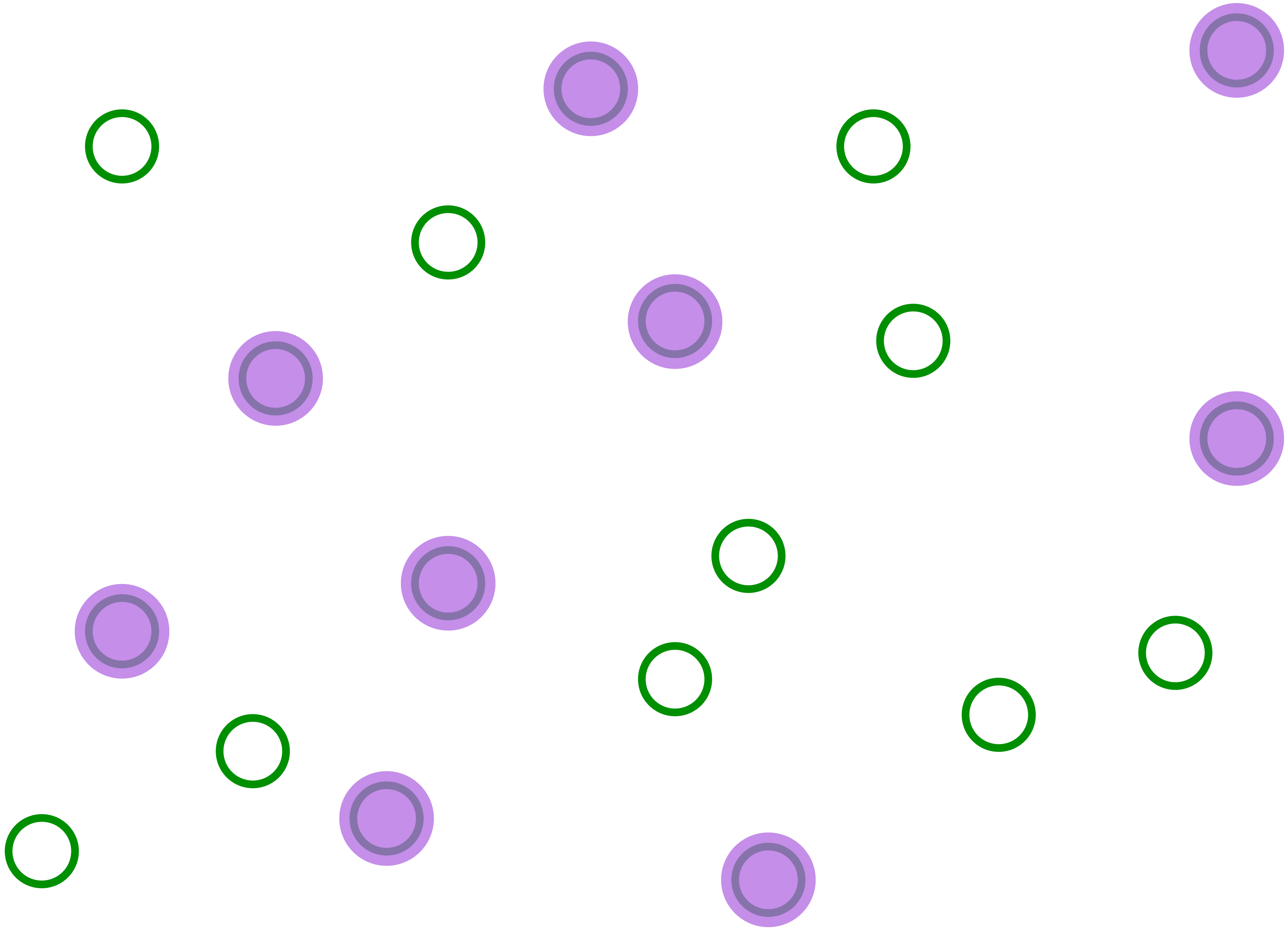
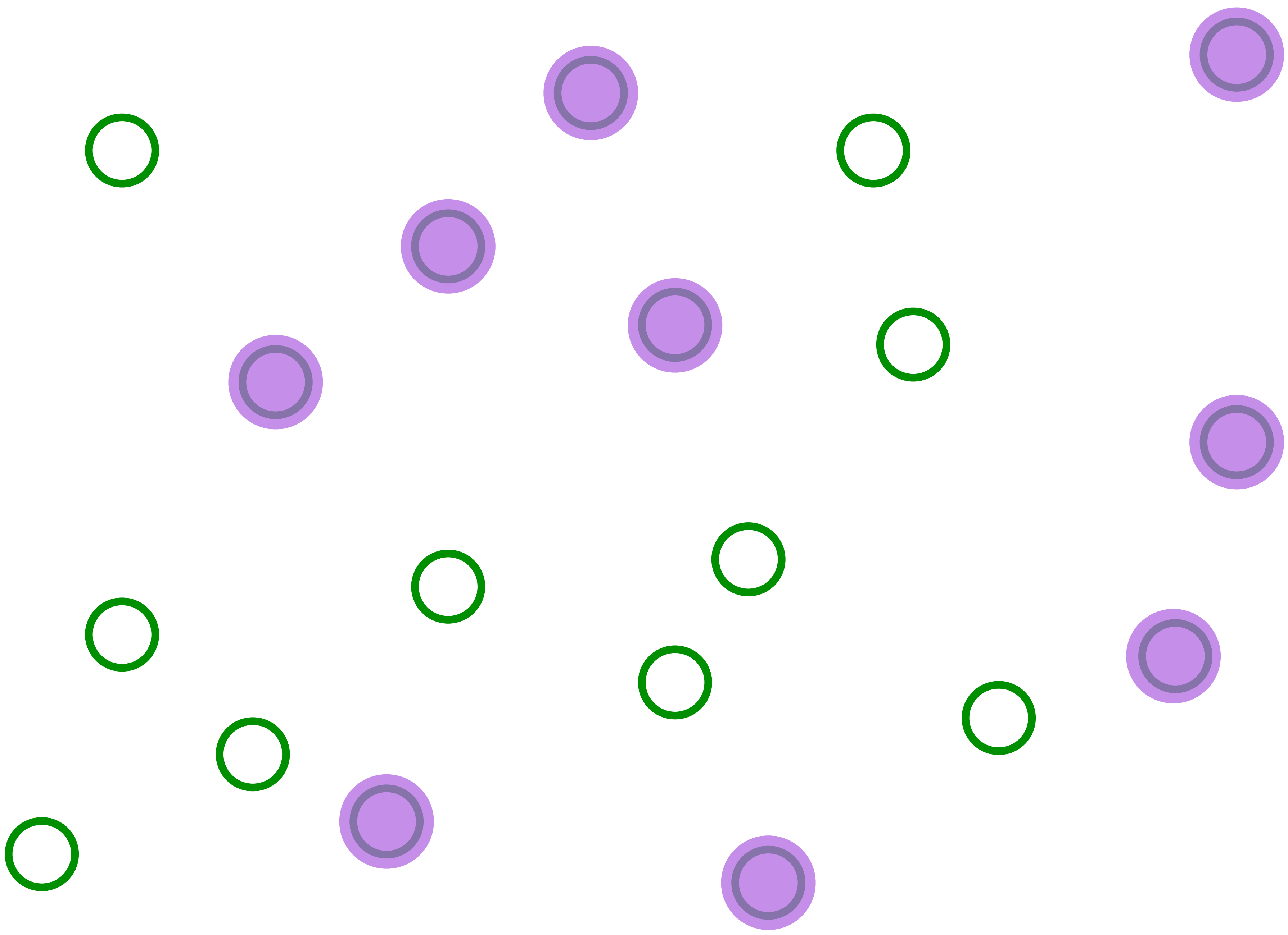


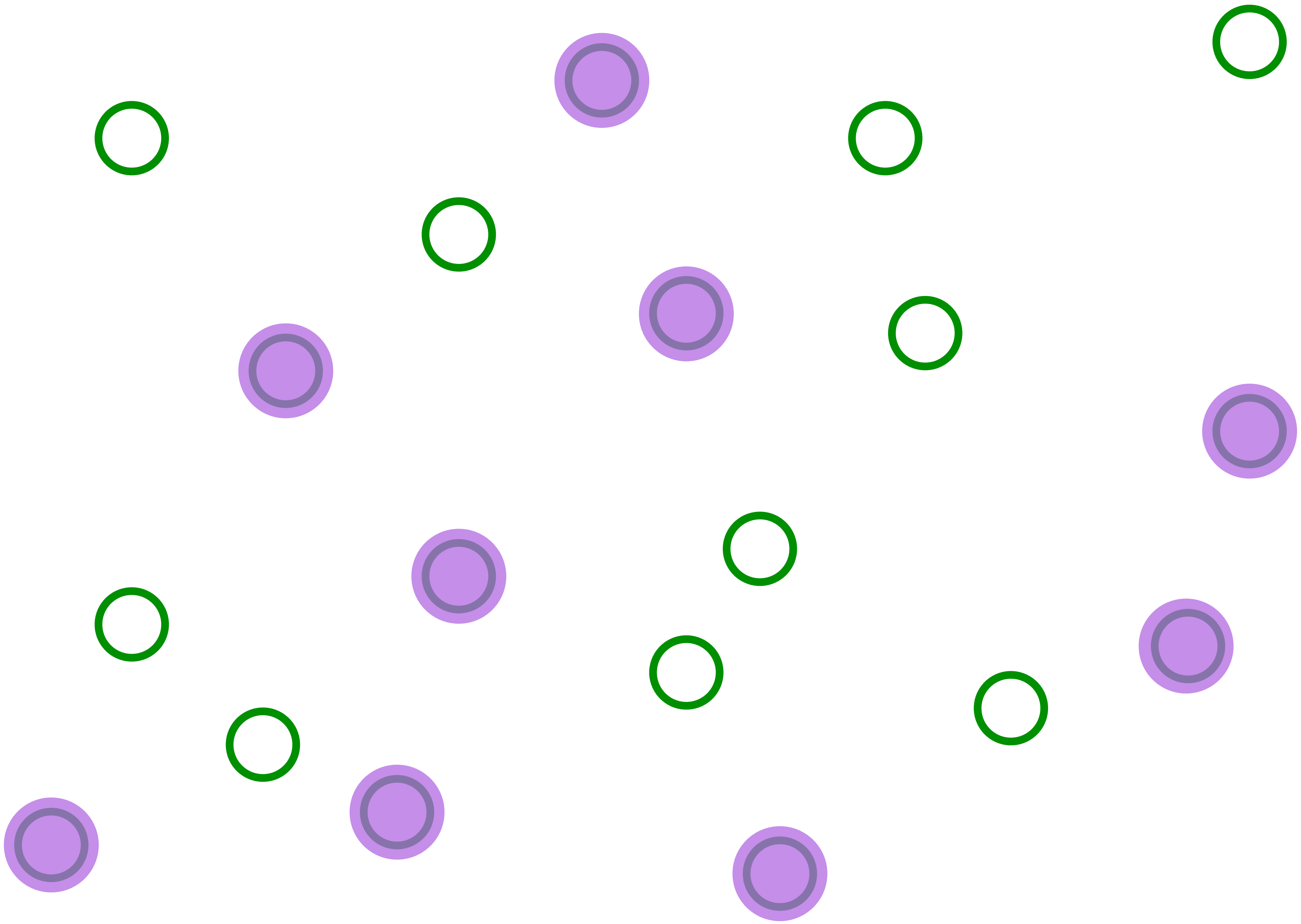
Place electrons randomly on some sites

The Sachdev-Ye-Kitaev (SYK) model



Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



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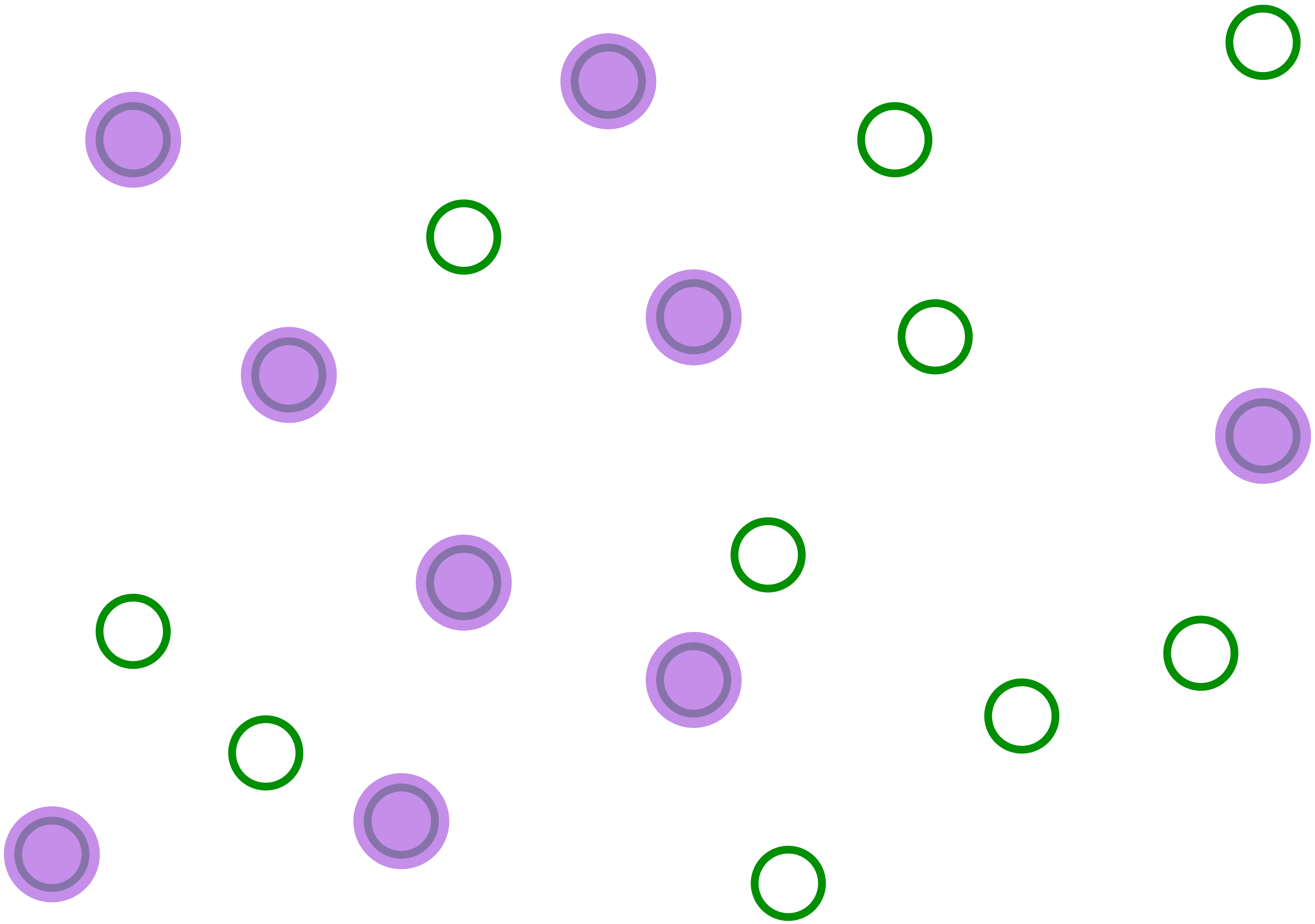

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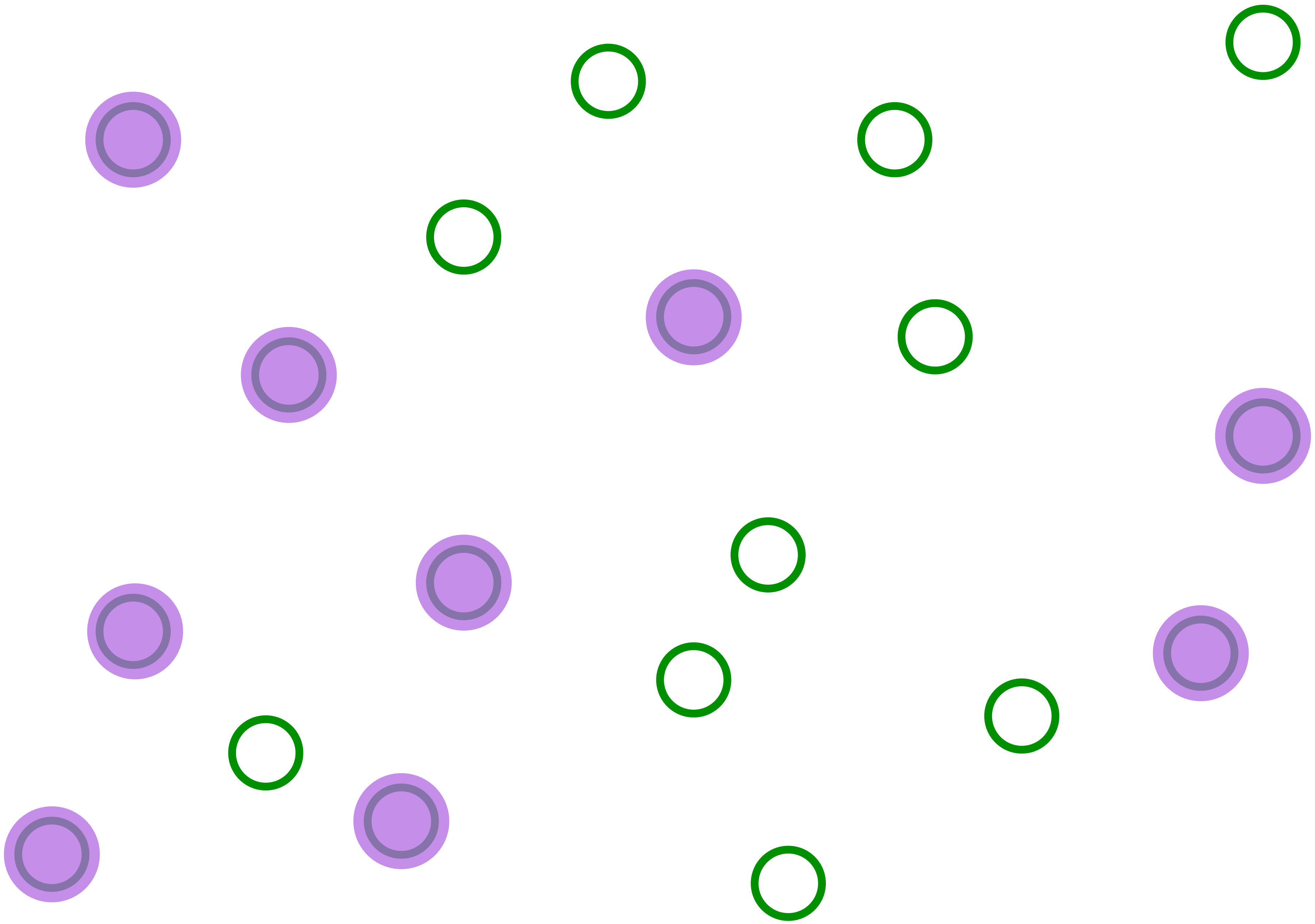

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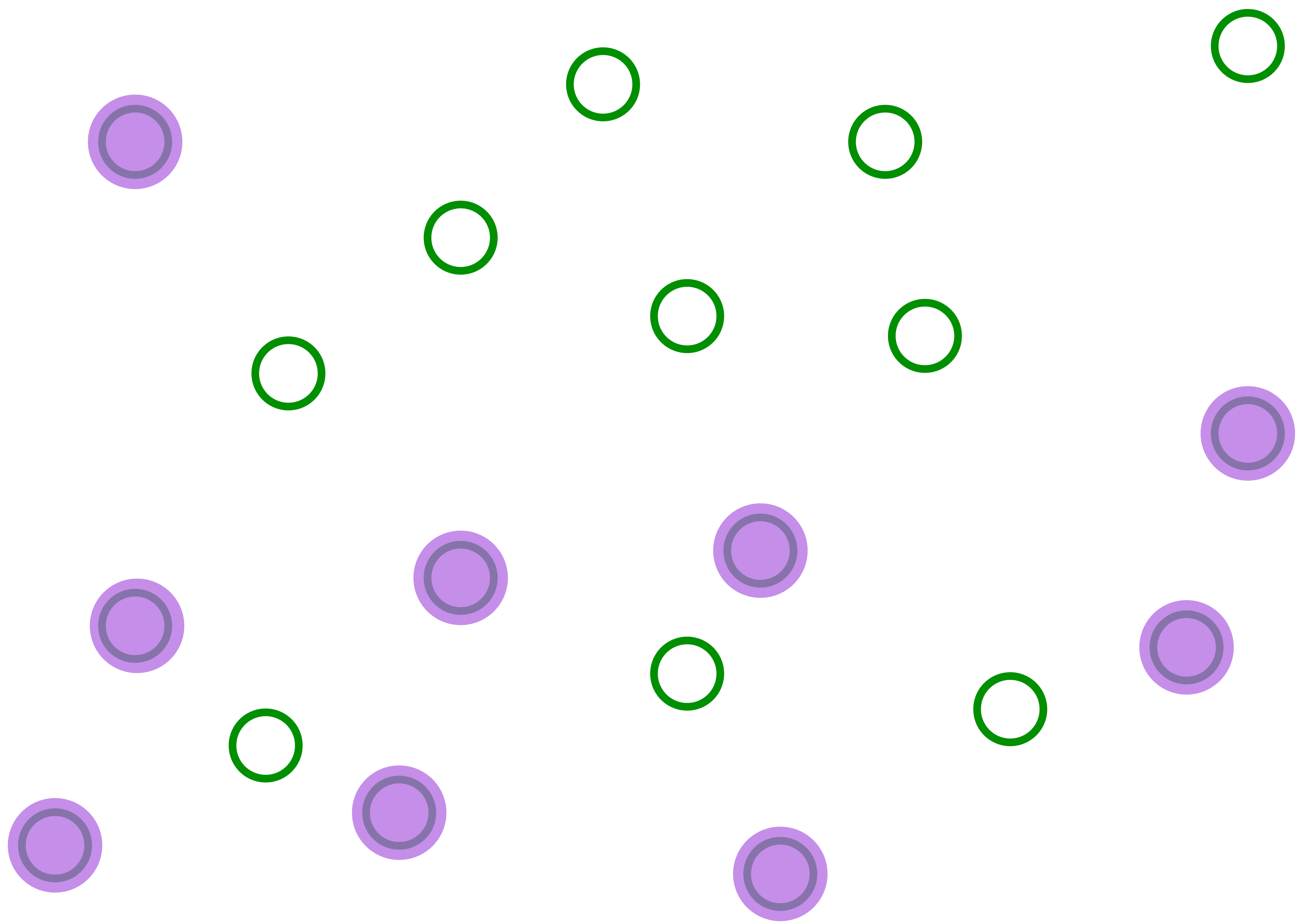

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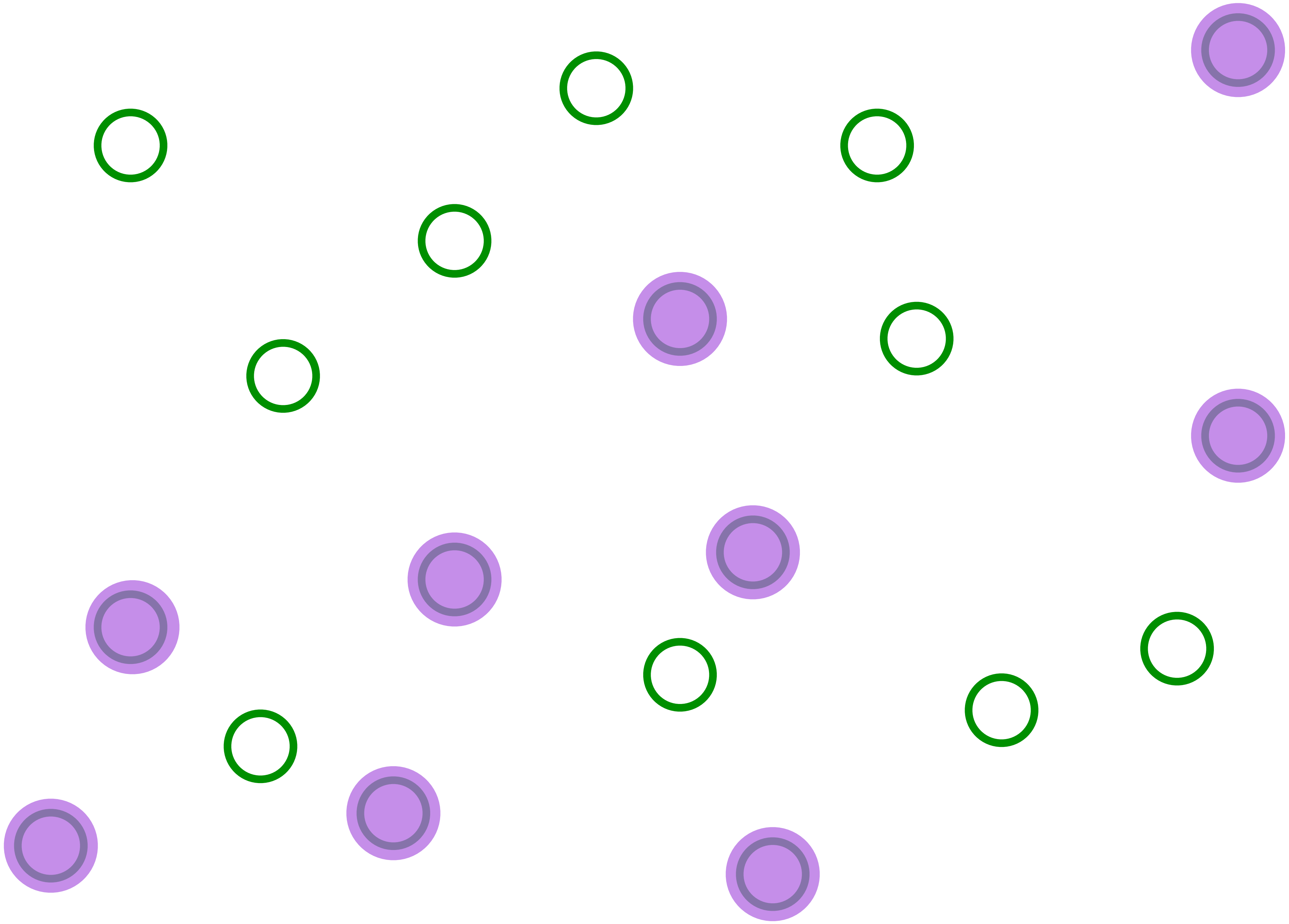
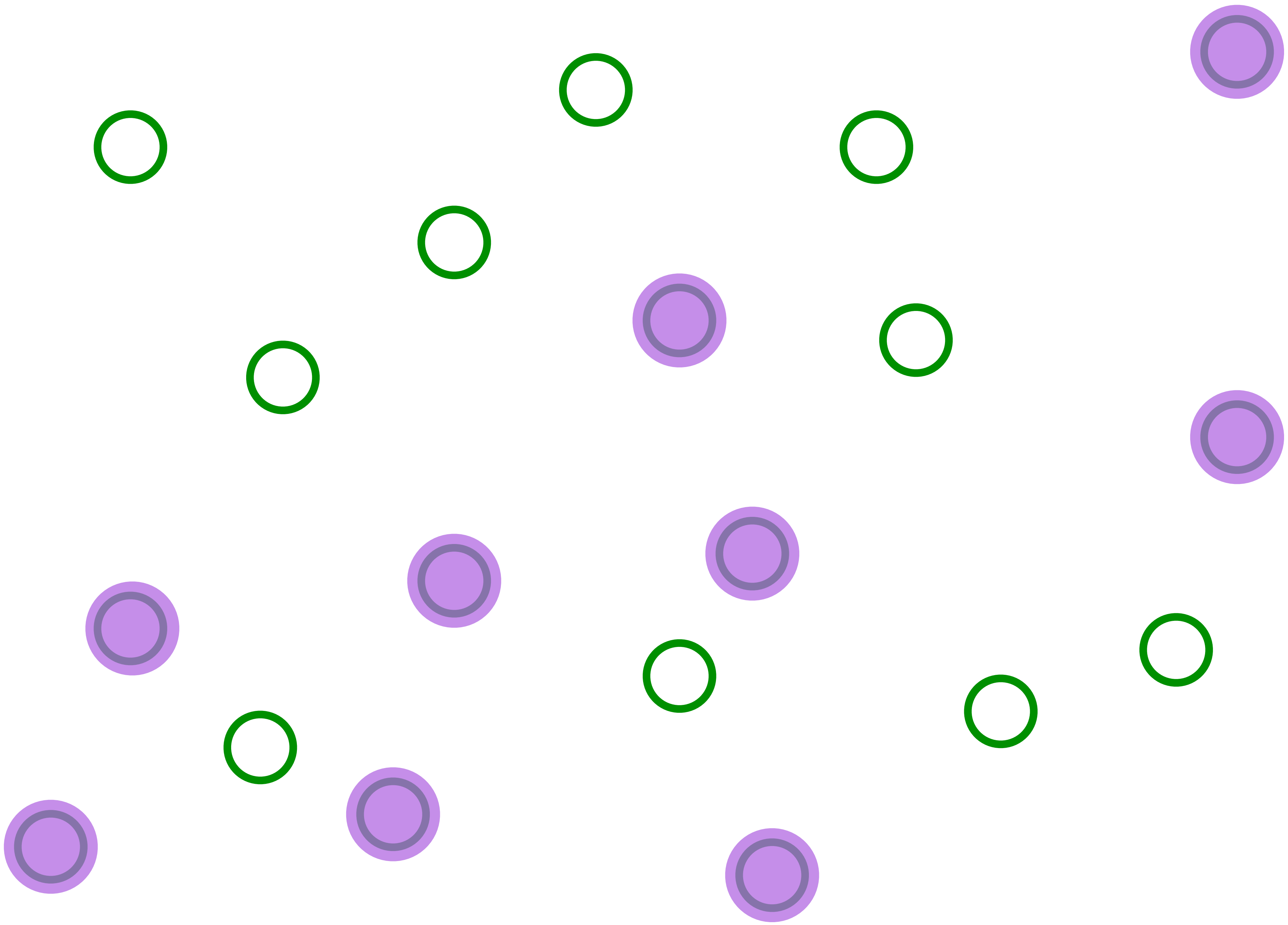


Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



This describes both a strange metal and a black hole!

The Sachdev-Ye-Kitaev (SYK) model

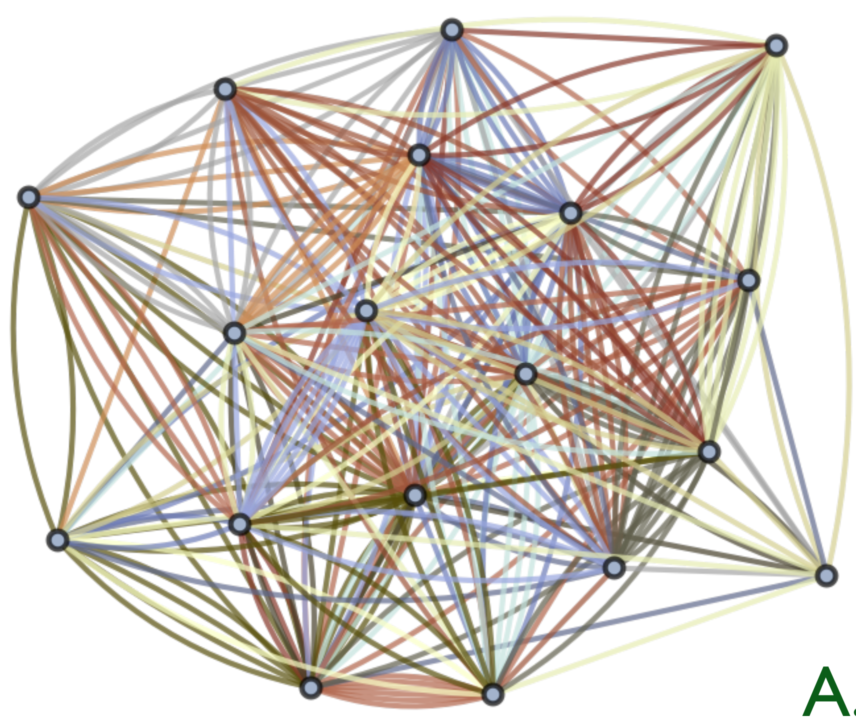
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

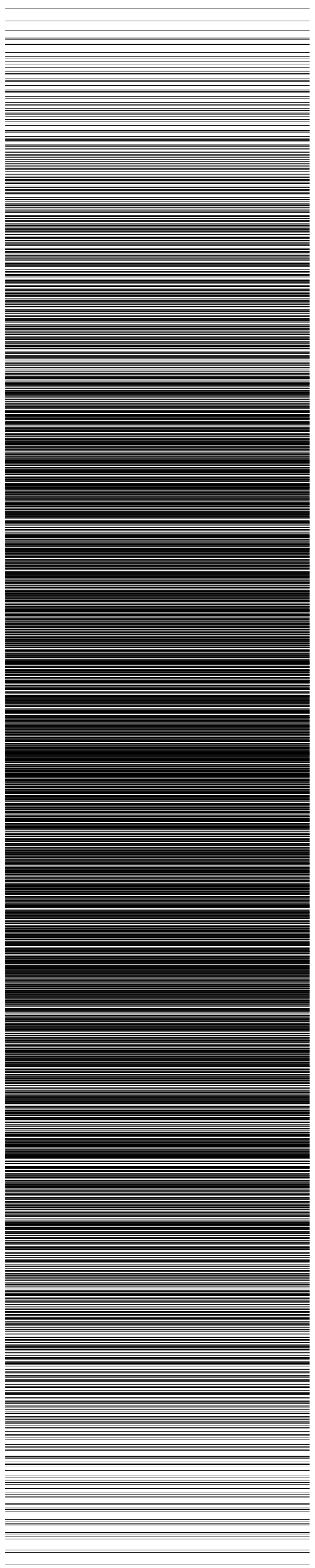
$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The Sachdev-Ye-Kitaev (SYK) model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-S_{GPS}}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy S_{GPS} with

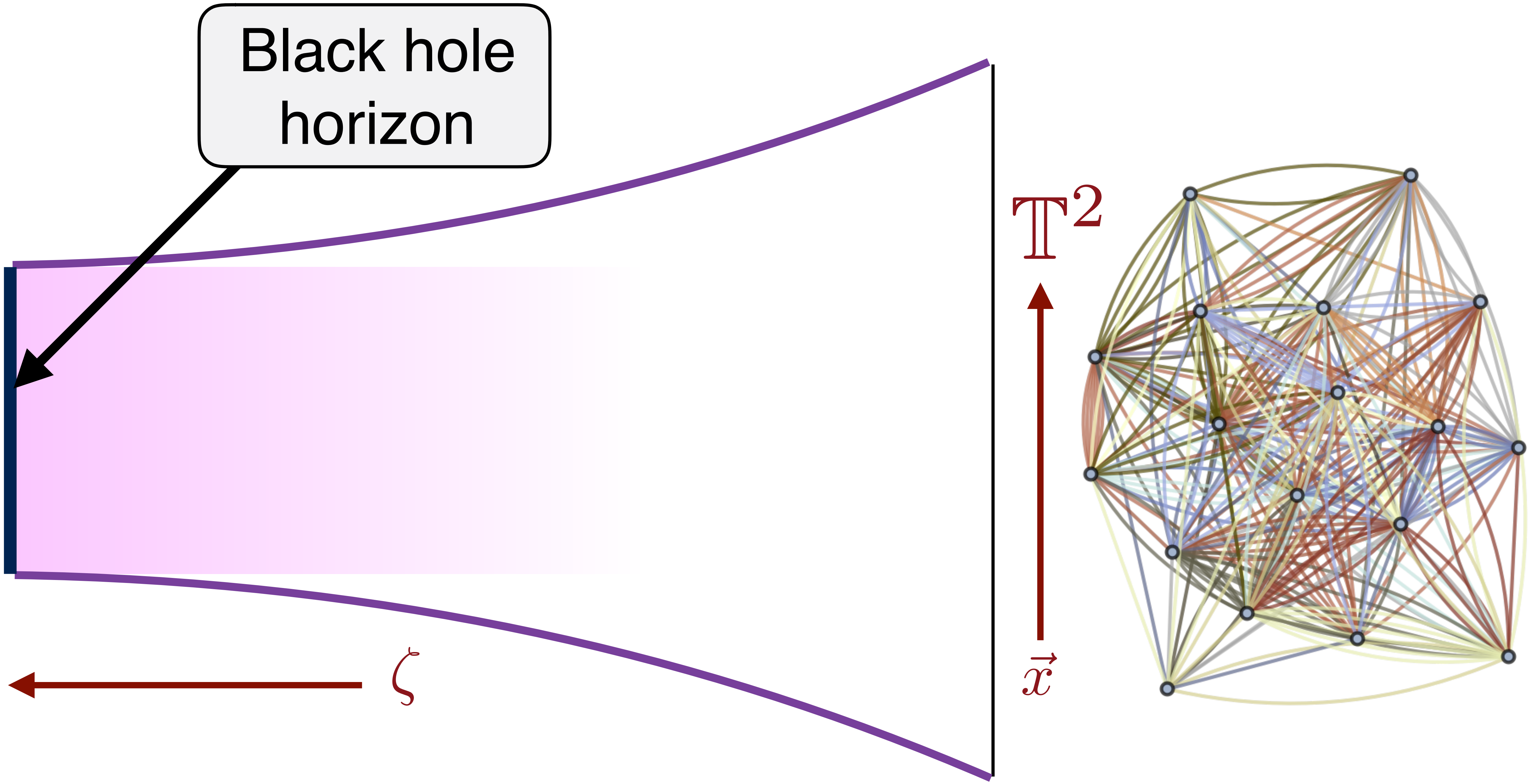
$$\frac{S_{GPS}}{N} = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$
$$< \ln 2$$

where G is Catalan's constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

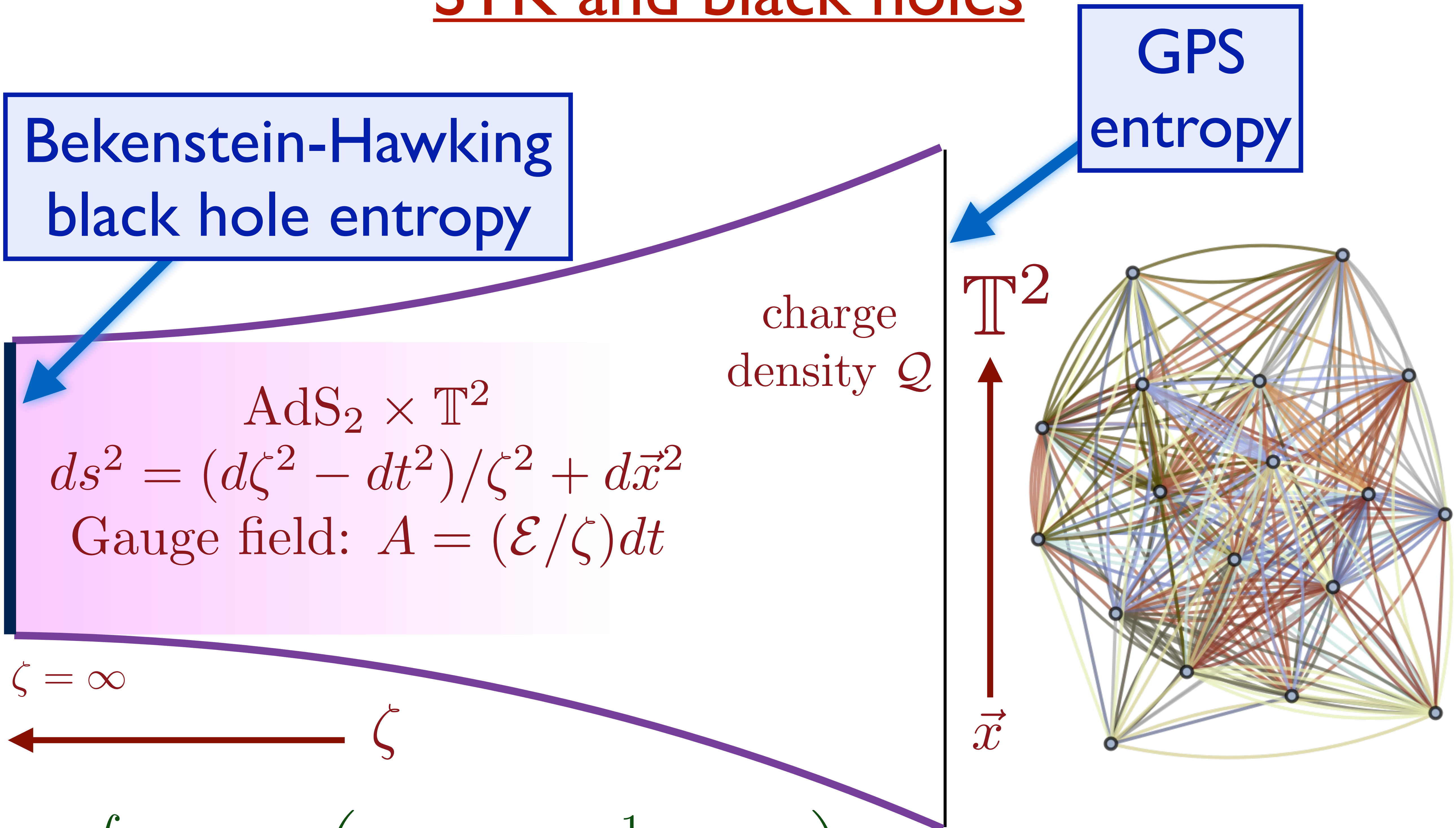
W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

SYK and black holes



The SYK model has "dual" description
in which an extra spatial dimension, ζ, emerges.
The curvature of this "emergent" spacetime is described
by Einstein's theory of general relativity

SYK and black holes

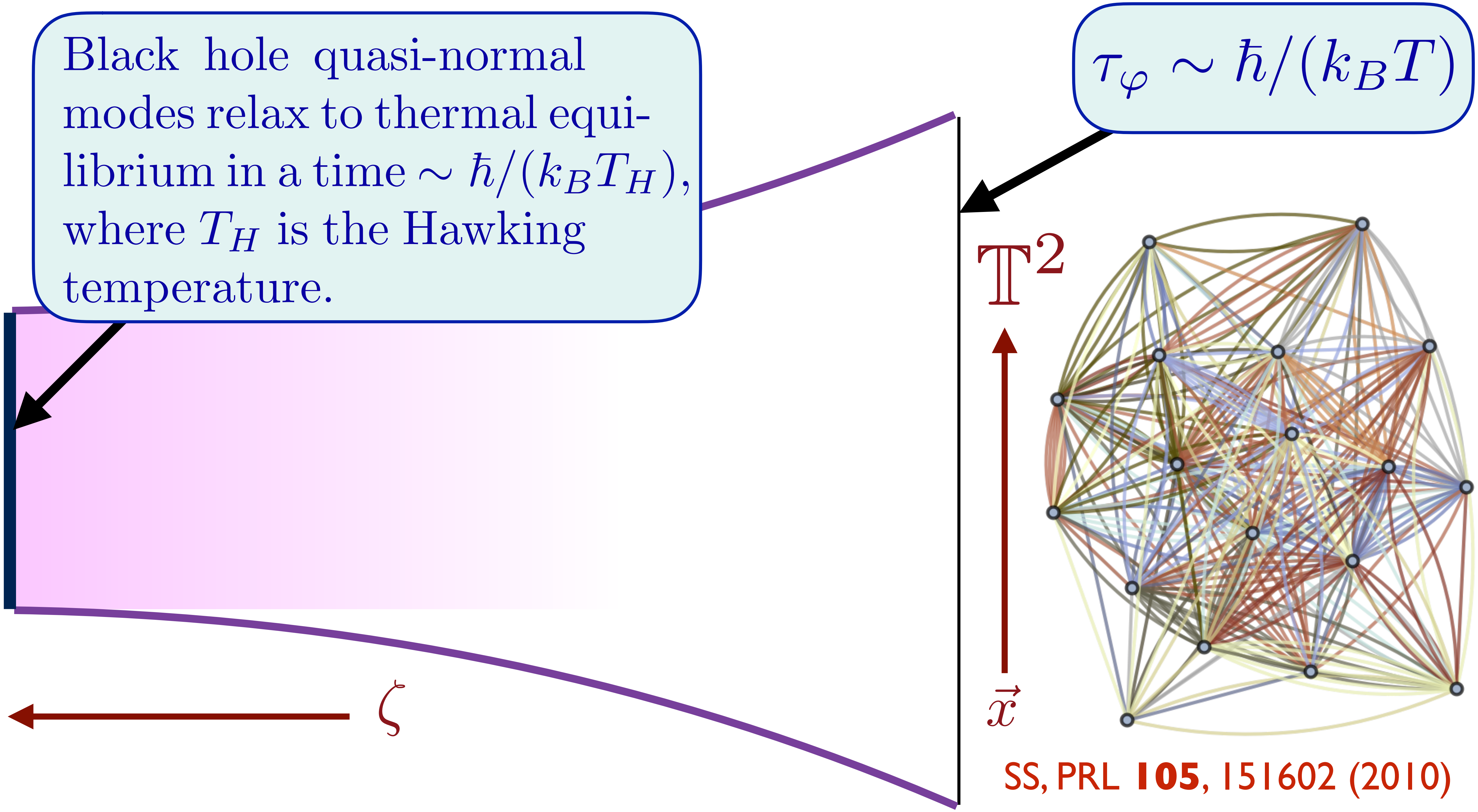


$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

SS, PRL **105**, 151602 (2010)

The BH entropy is proportional to the size of $\mathbb{T}^2$, and hence the surface area of the black hole. Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$.

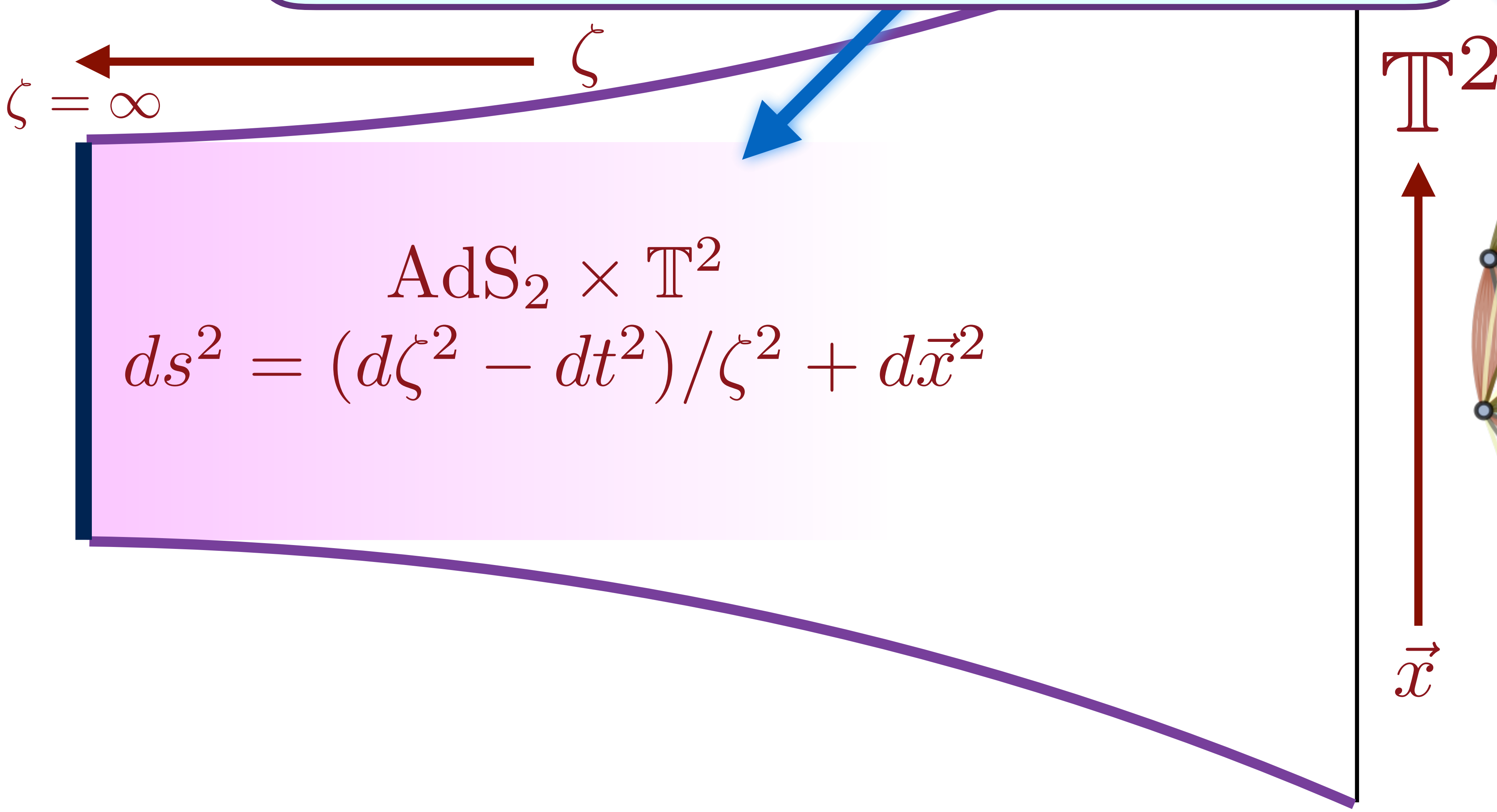
SYK and black holes

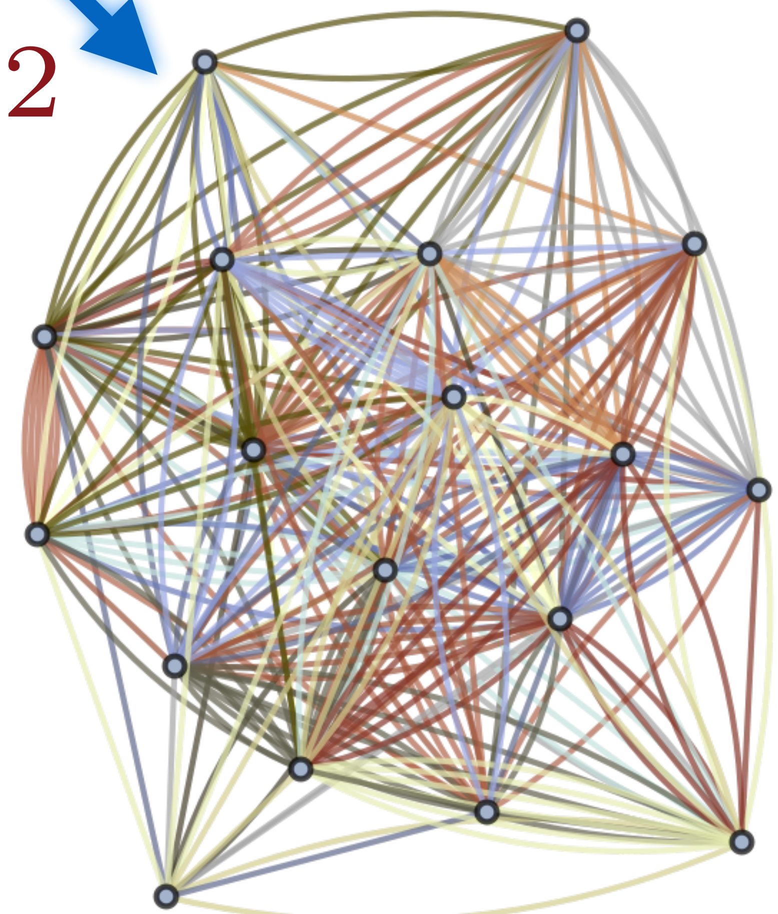


The SYK model has "dual" description in which an extra spatial dimension, ζ, emerges. The curvature of this "emergent" spacetime is described by Einstein's theory of general relativity

SYK and AdS_2

Equilibrium and non-equilibrium* dynamics described by a theory with SL(2,R) invariance, and effective Schwarzian action, $S[h(\tau)]$, of a time reparameterization $\tau \to h(\tau)$.





A. Kitaev, unpublished; J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438
*A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

SYK and AdS$_2$

Equilibrium and non-equilibrium* dynamics described by a theory with SL(2,R) invariance, and effective Schwarzian action, $S[h(\tau)]$, of a time reparameterization $\tau \rightarrow h(\tau)$.

ζ

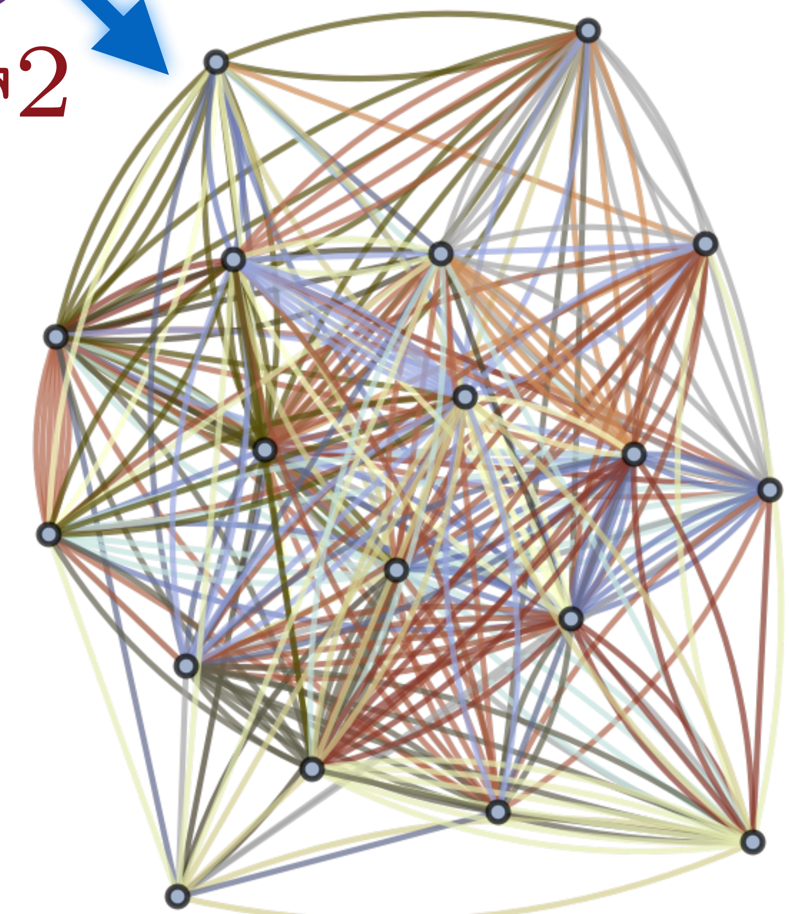
$\zeta = \infty$

$\mathrm{AdS}_2 \times \mathbb{T}^2$

$$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$$

$\mathbb{T}^2$

$\vec{x}$



SL(2,R) is the isometry group of AdS$_2$: $ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}$$

with $ad - bc = 1$.

A. Kitaev, unpublished; J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438
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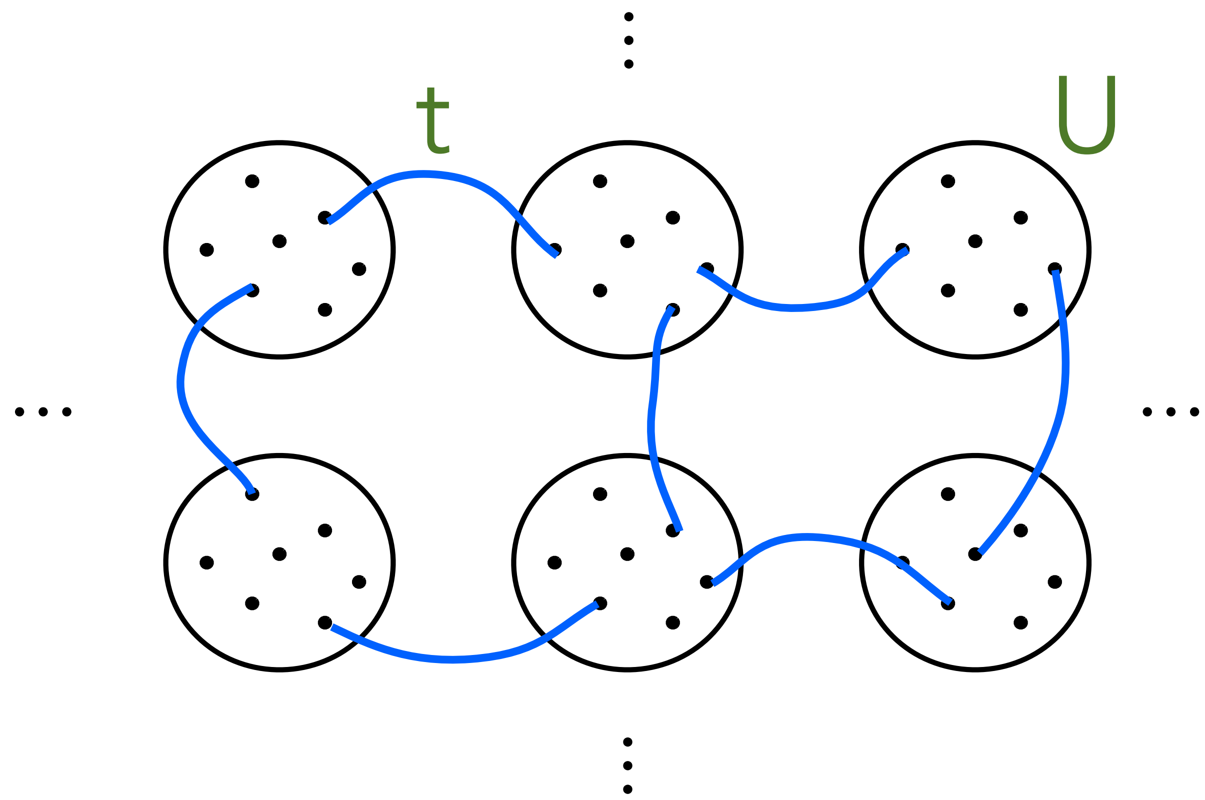
arXiv:1705.00117

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

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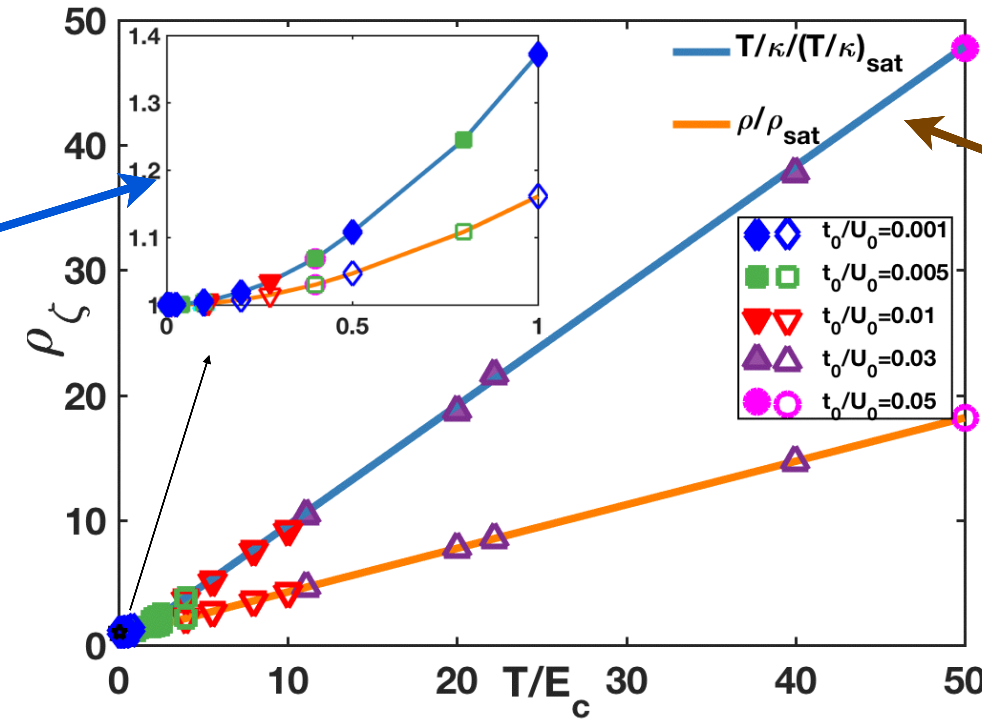
Fermi liquid
$R=R_0+AT^2$
for $T \ll E_c$



Linear in T for
$E_c \ll T \ll U$

Crossover from heavy FL to strange metal

- Small coherence scale $E_c=t^2/U$
- Heavy mass $\gamma \sim m^*/m \sim U/t$
- Small QP weight $Z \sim t/U$
- Kadowaki-Woods A/γ^2 = constant
- Linear in T resistivity and T/κ
- Lorenz ratio crosses over from FL to NFL value

1. Descendants of the integer quantum Hall effect

Protected gapless edge states,
while bulk excitations are "trivial"

2. Descendants of the fractional quantum Hall effect

Bulk topological excitations which cannot be created from the ground state by the action of a local operator

3. Quantum matter without quasiparticles: strange metals and black holes

- Is there a connection between strange metals and black holes?
  Yes, the SYK model leads to an explicit duality mapping.

- Why do they have the same local equilibration time $\sim \hbar/(k_B T)$?
  Strange metals don't have quasiparticles and thermalize rapidly;
  General relativity leads to black hole quasi-normal modes, whose decay time $\sim \hbar/(k_B T_H)$, where T_H is the Hawking temperature.

- Theoretical predictions for strange metal transport in graphene agree well with experiments