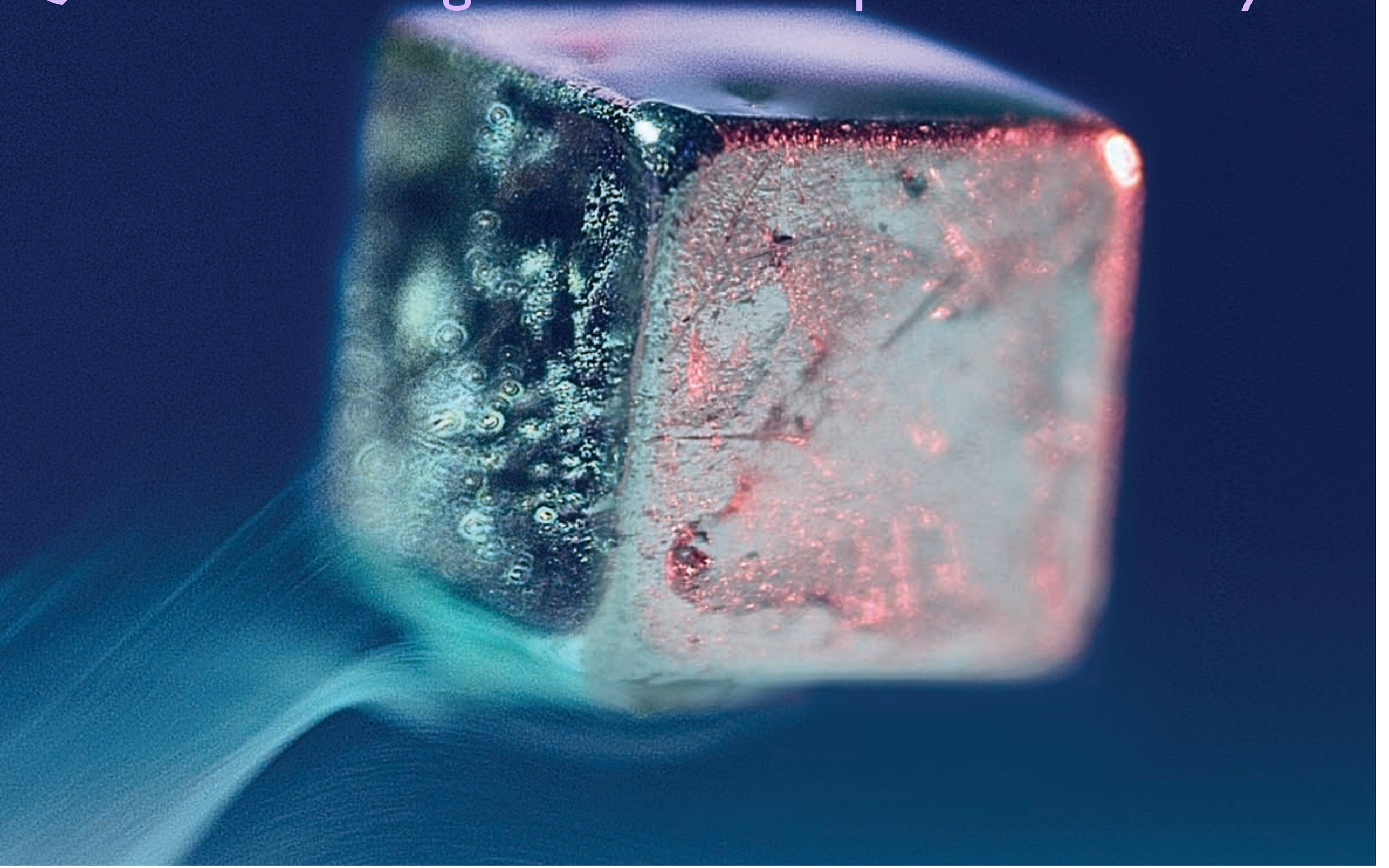
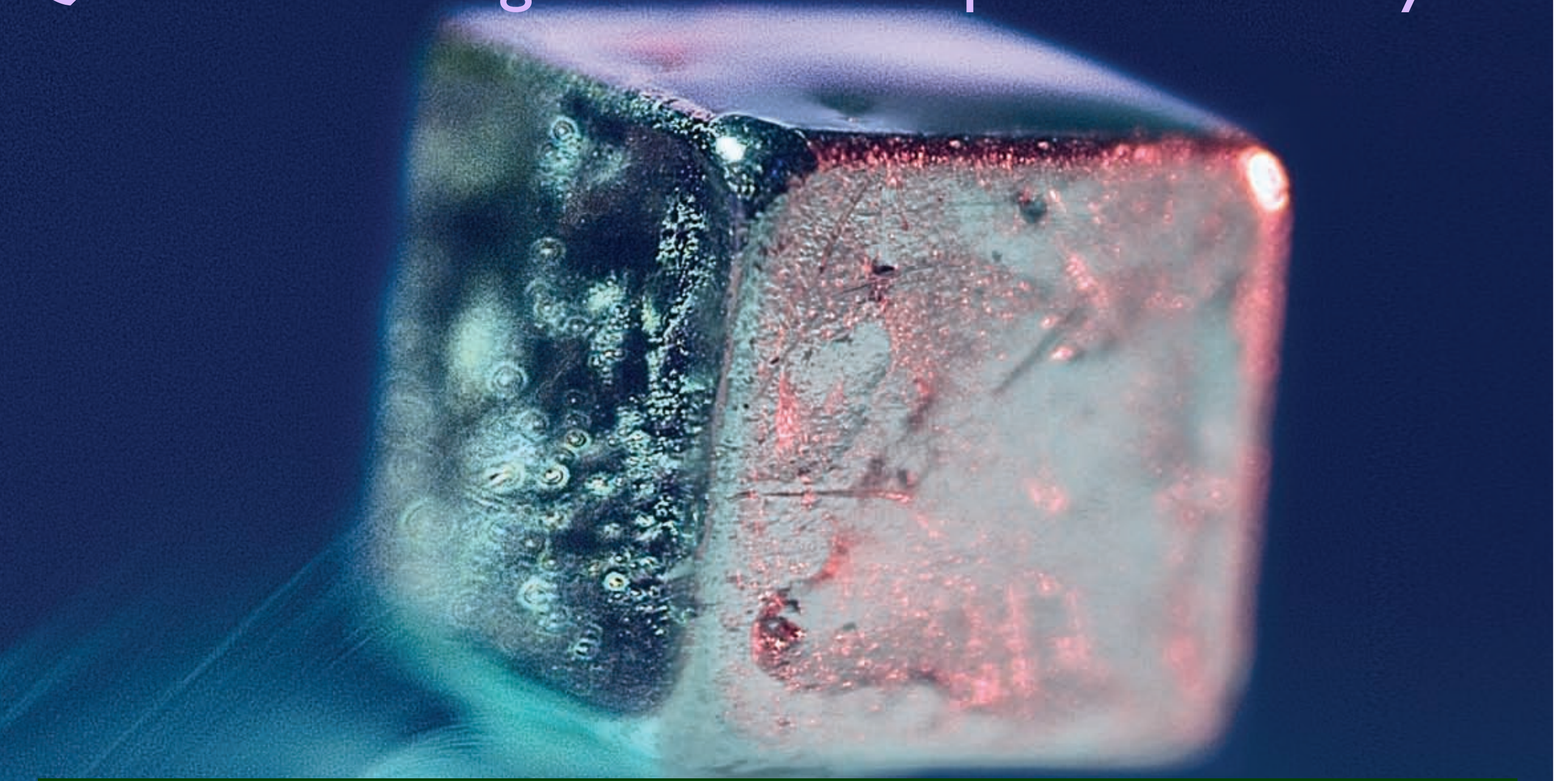
Quantum Entanglement and Superconductivity



Subir Sachdev, Perimeter Institute and Harvard University

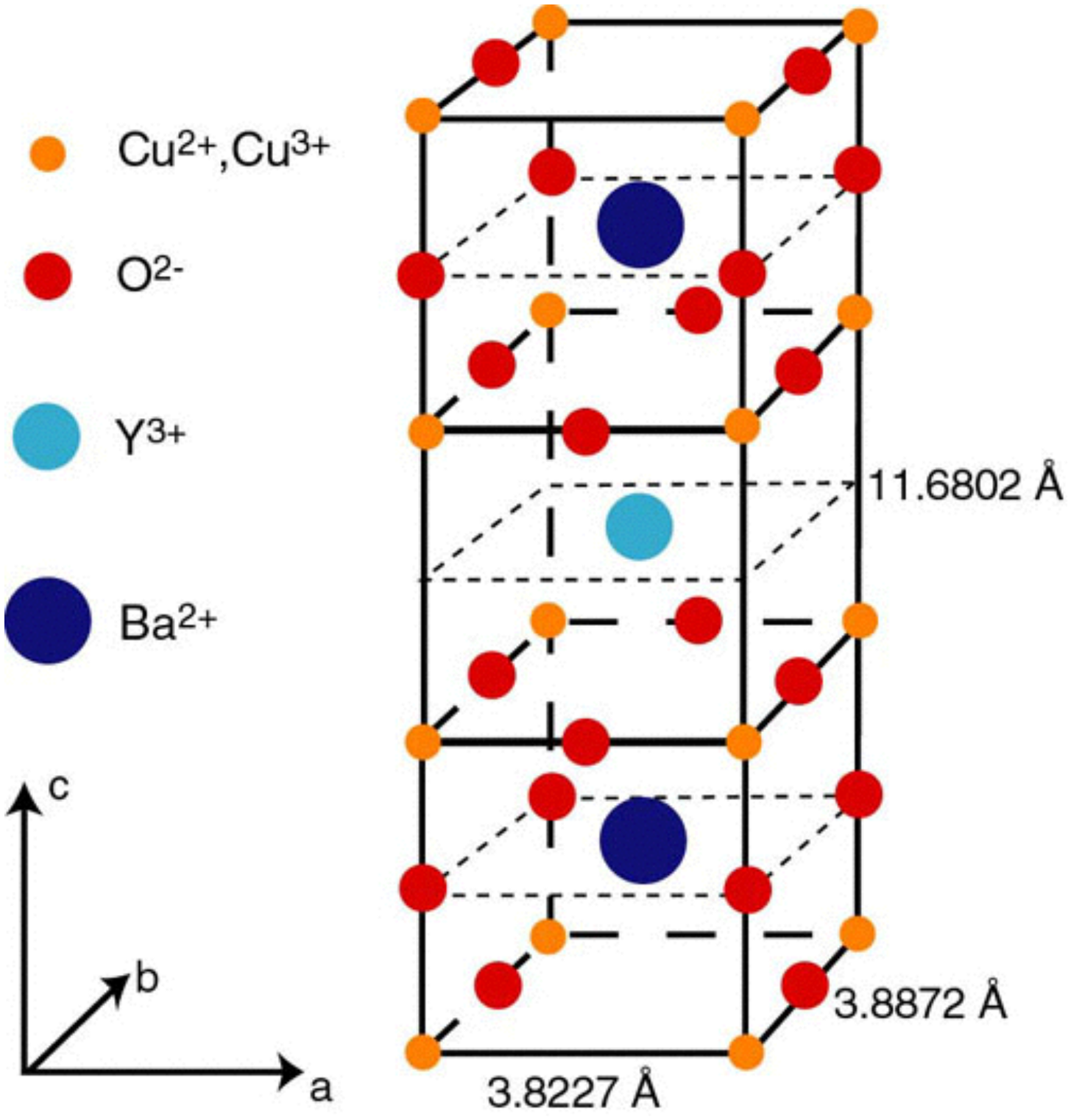
Quantum Entanglement and Superconductivity



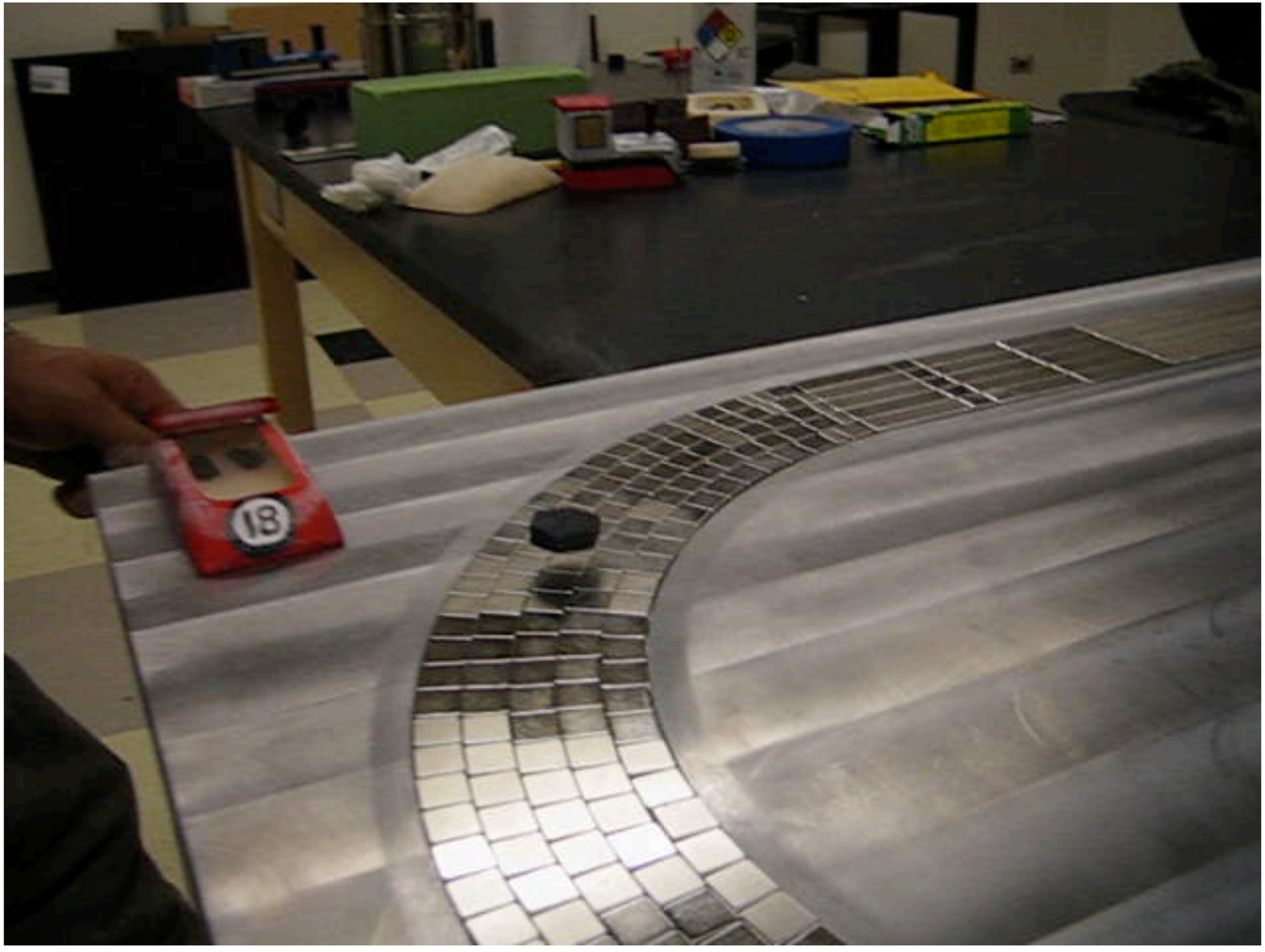
Superconductor, levitated by an unseen magnet, in which countless trillions of electrons form a vast interconnected quantum state.
Scientific American, January 2013

Subir Sachdev, Perimeter Institute and Harvard University

High temperature superconductors

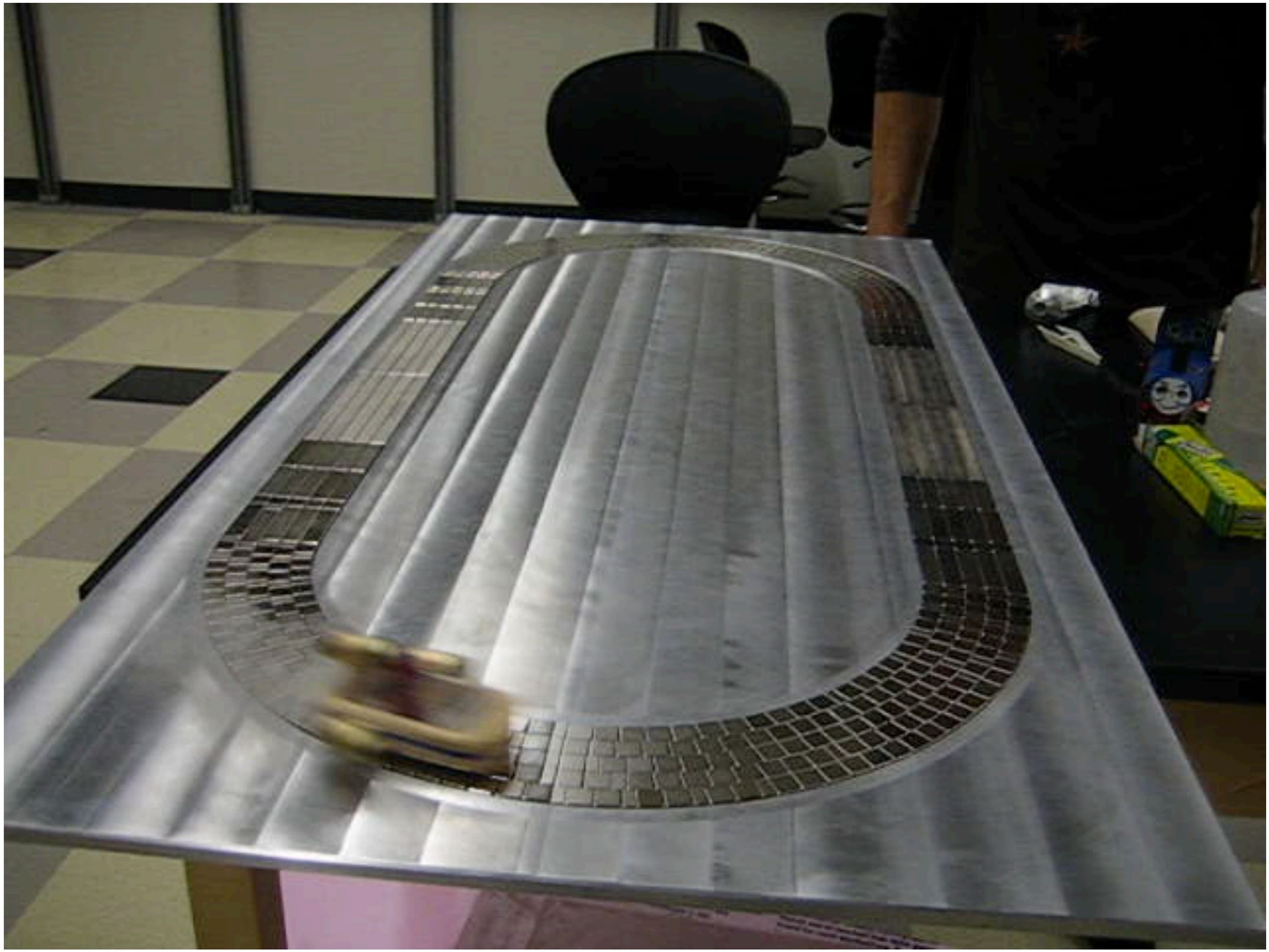


$$YBa_2Cu_3O_{6+x}$$



Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

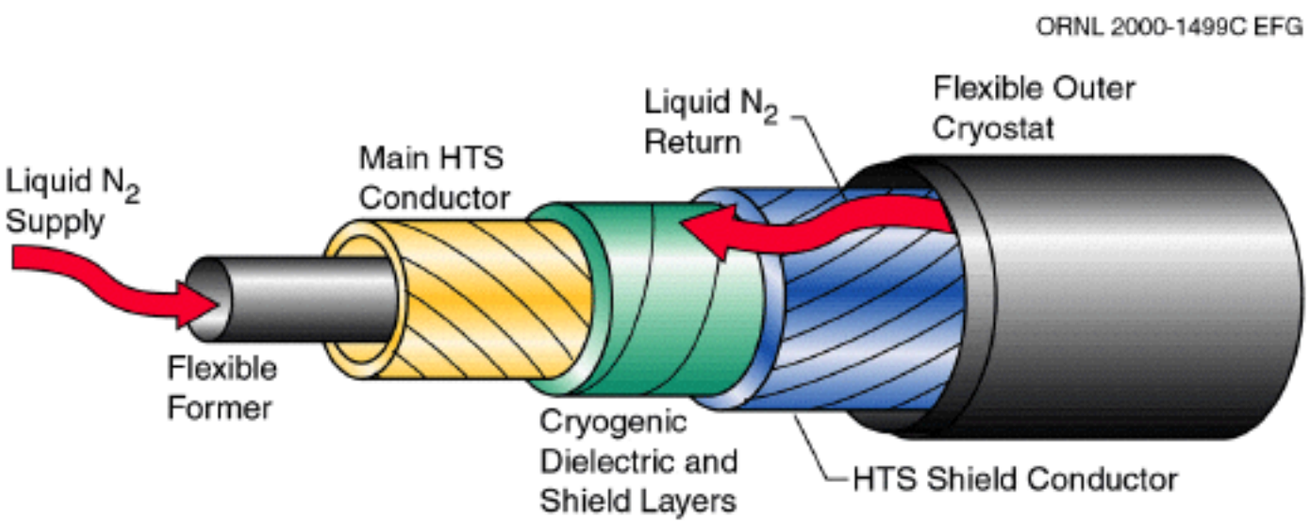


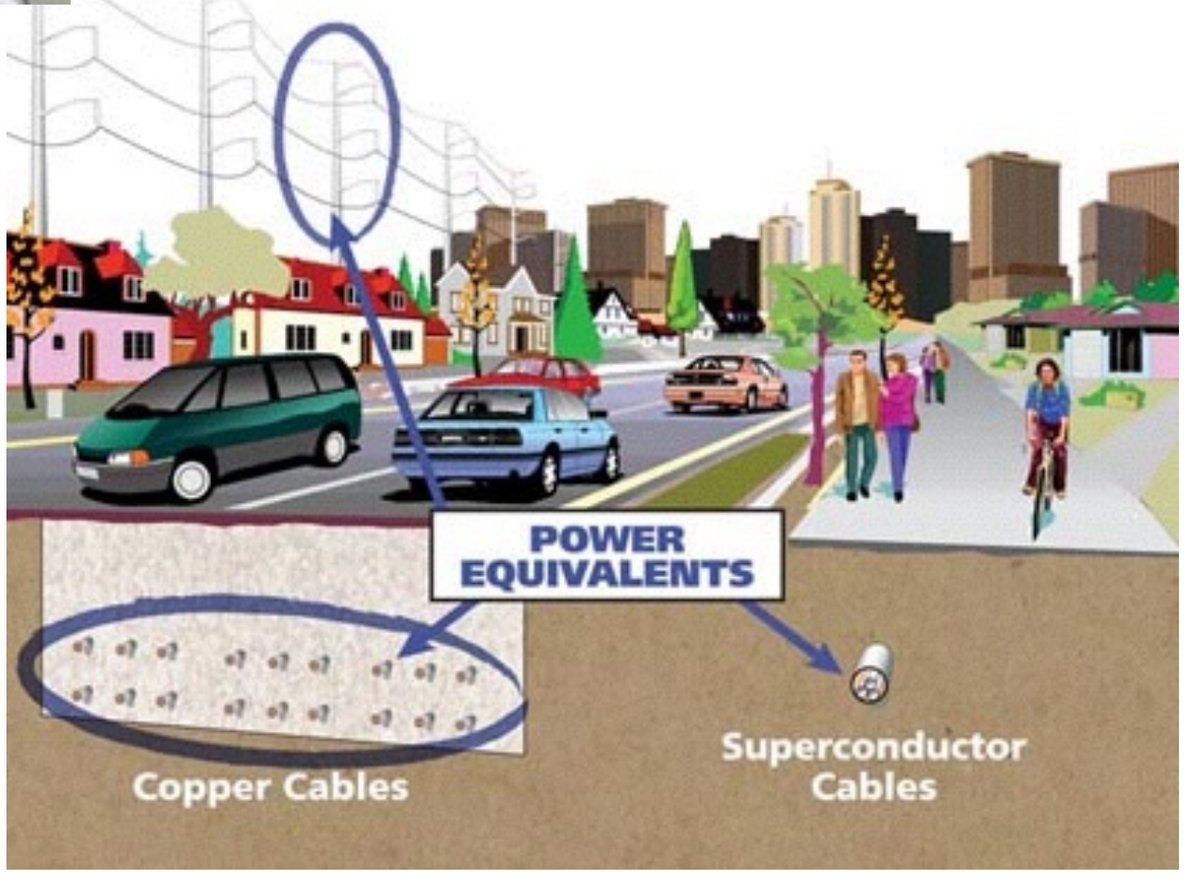
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

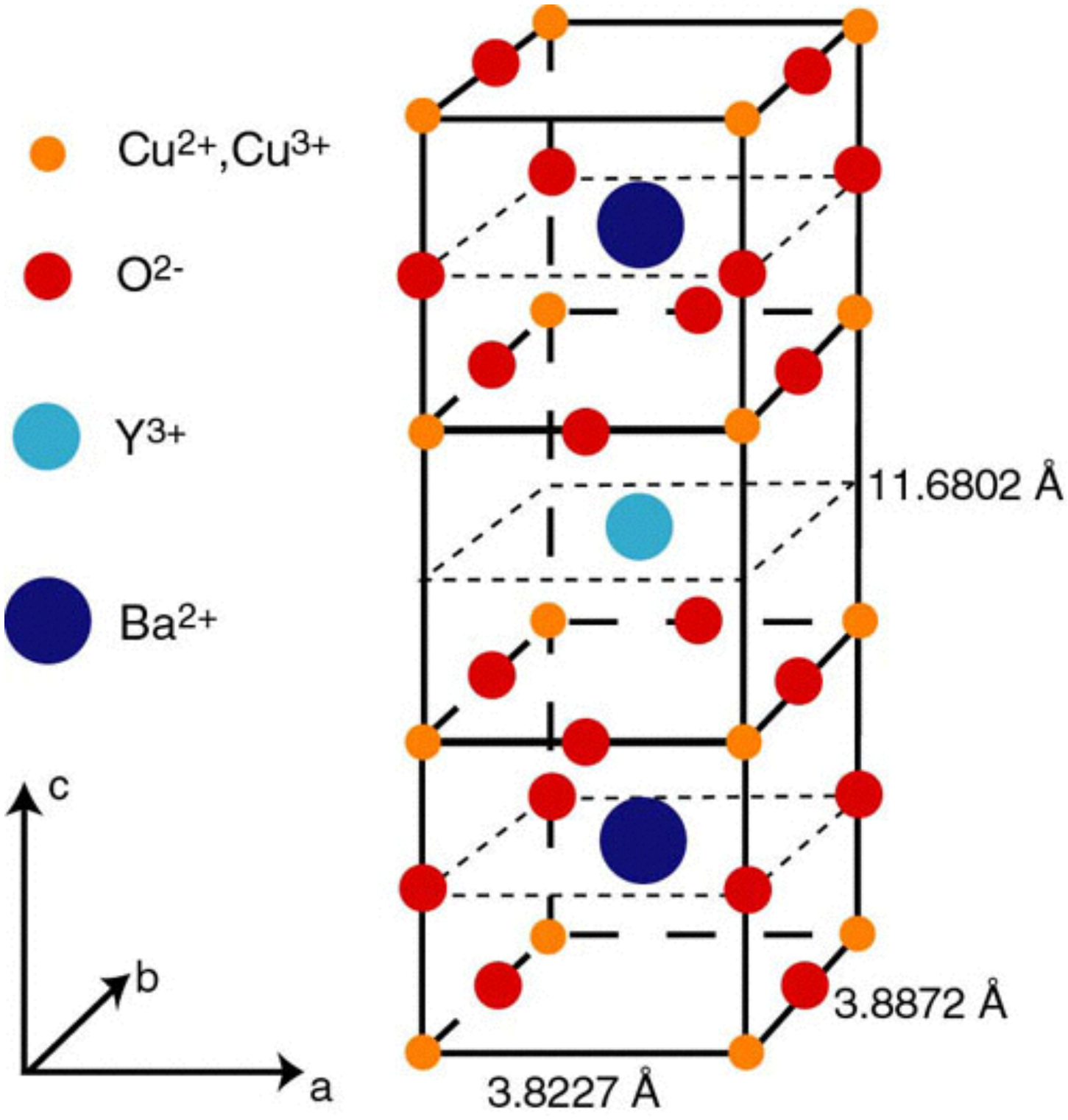
YBCO cables

American Superconductor Corporation





High temperature superconductors



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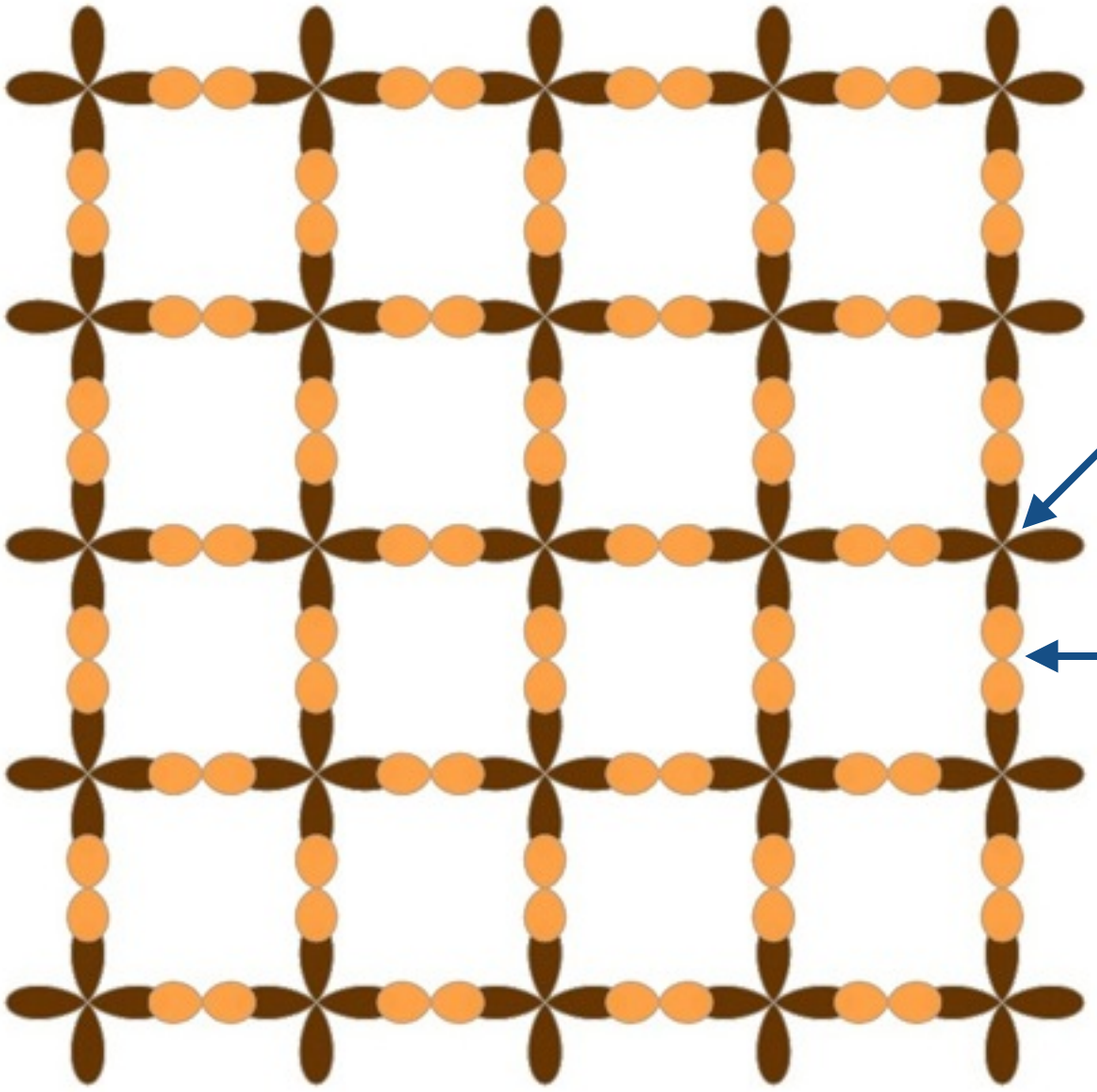
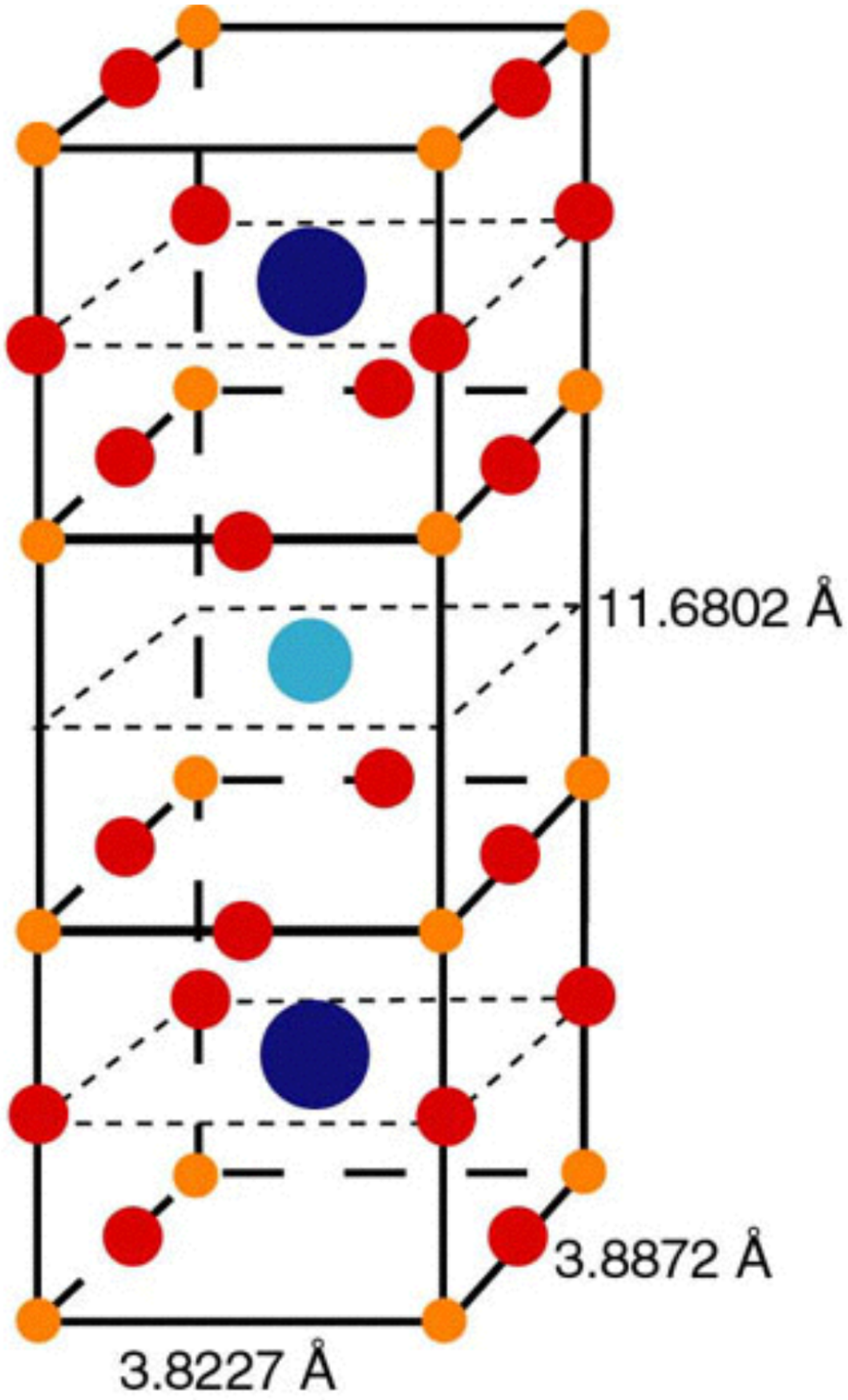
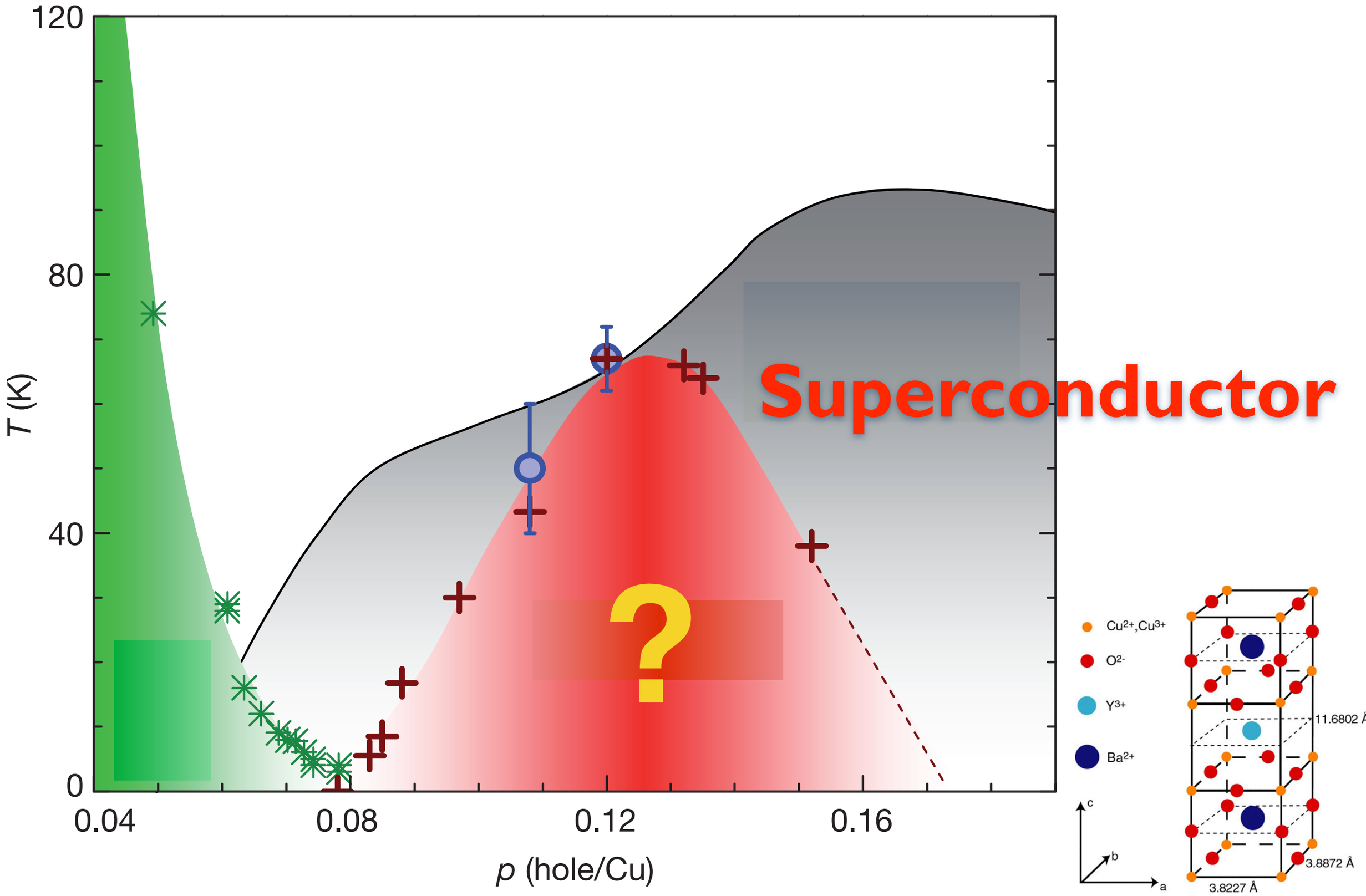
High temperature superconductors



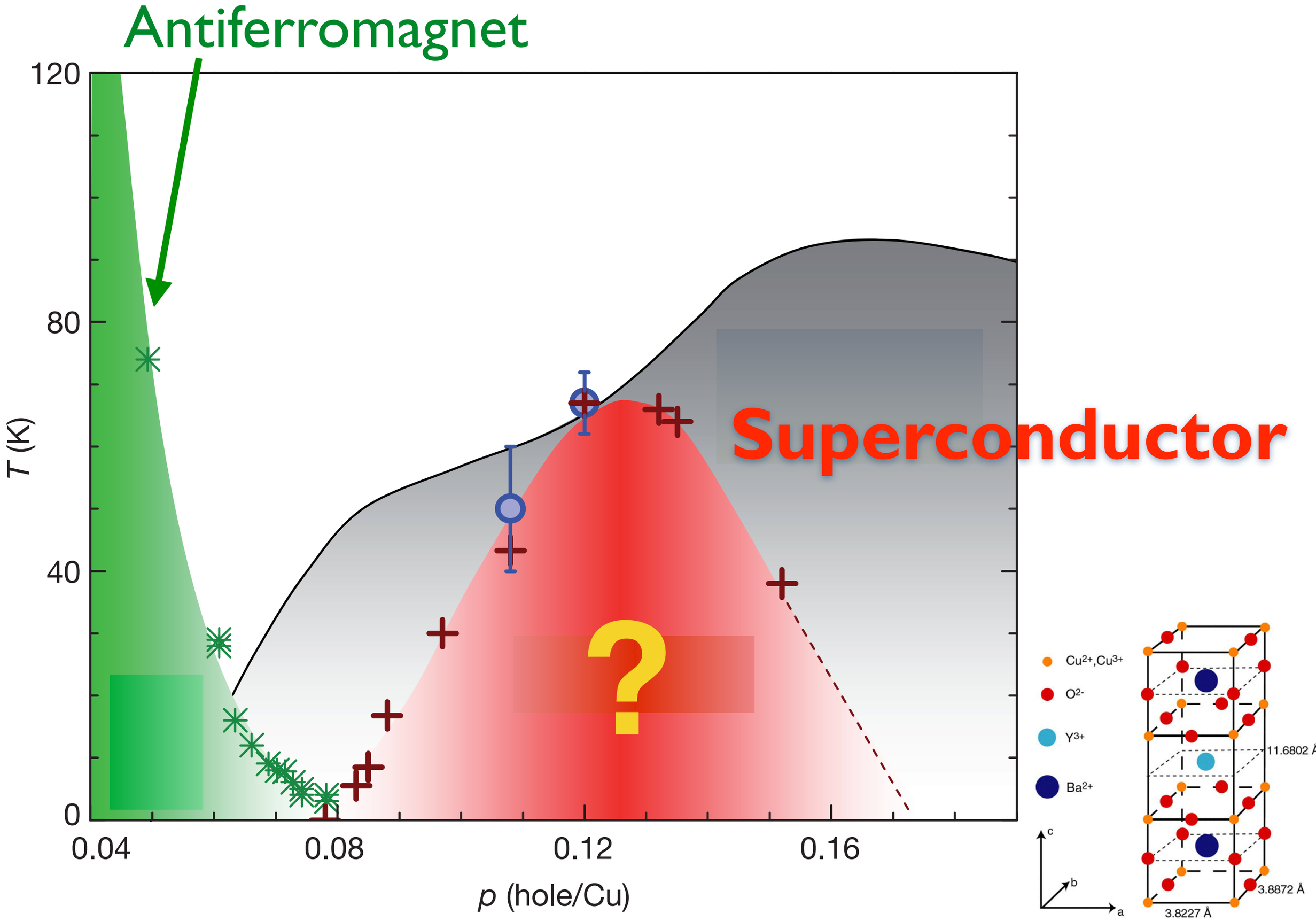
CuO_2 plane



$YBa_2Cu_3O_{6+x}$



T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, Nature **477**, 191 (2011).



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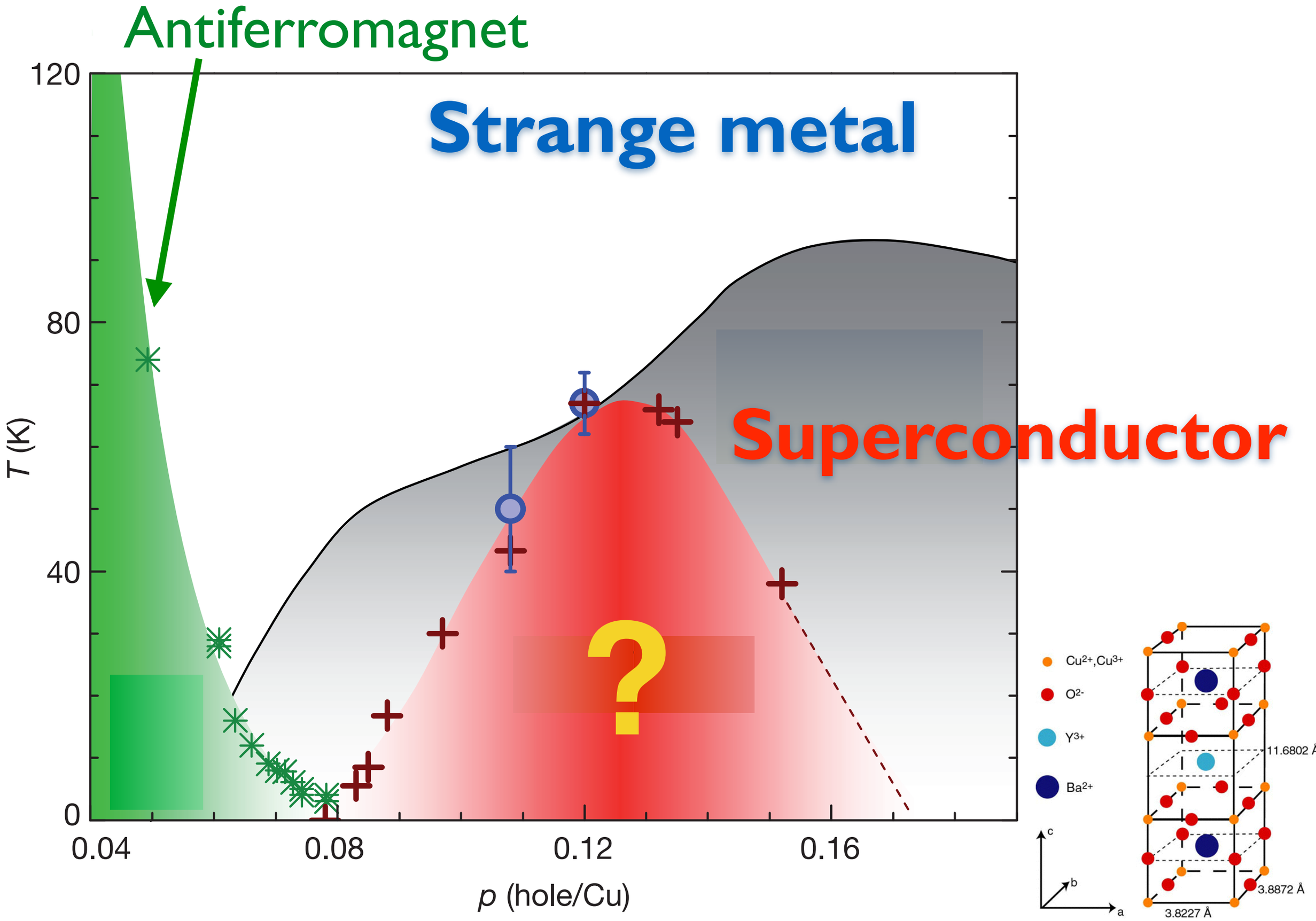

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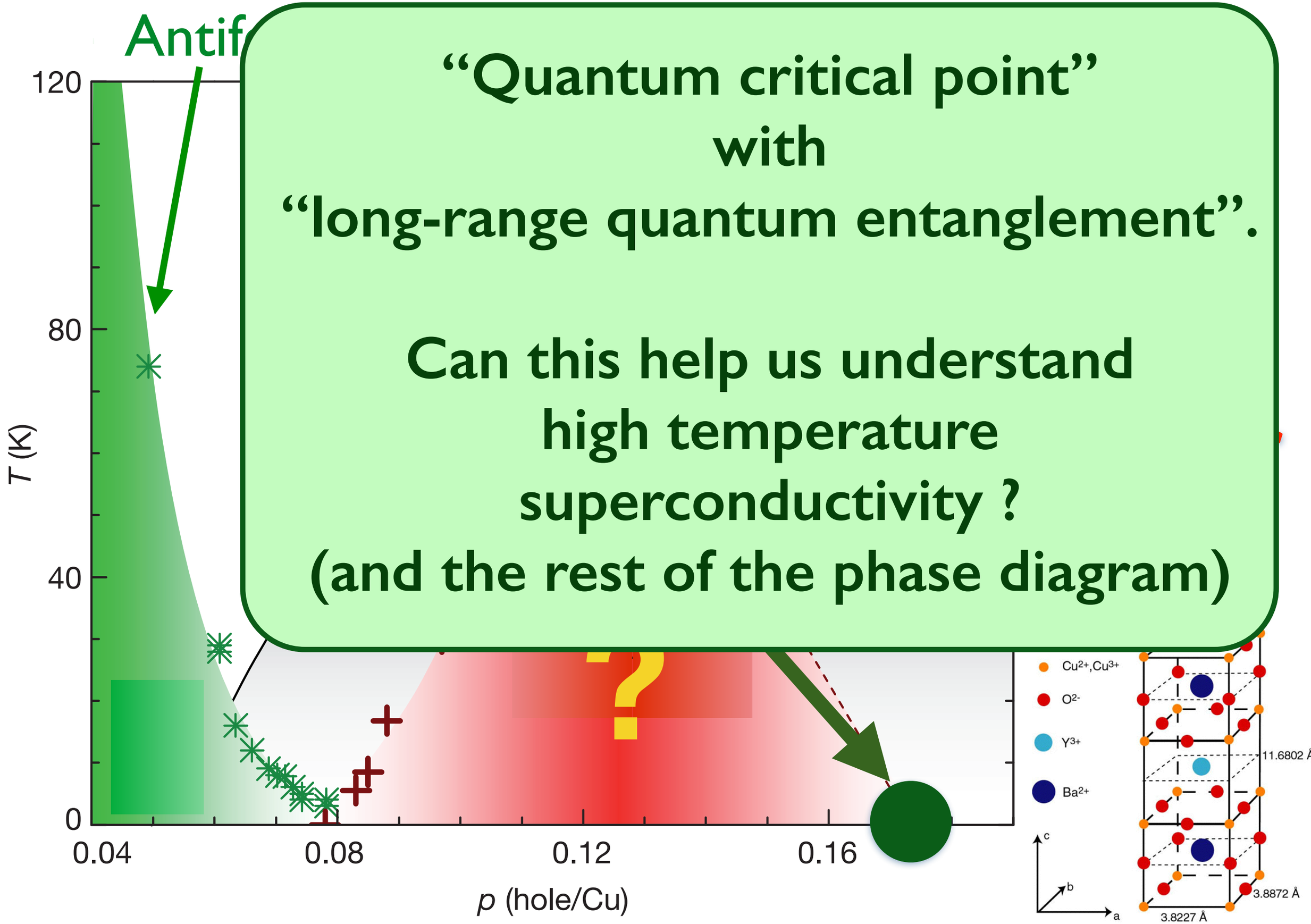


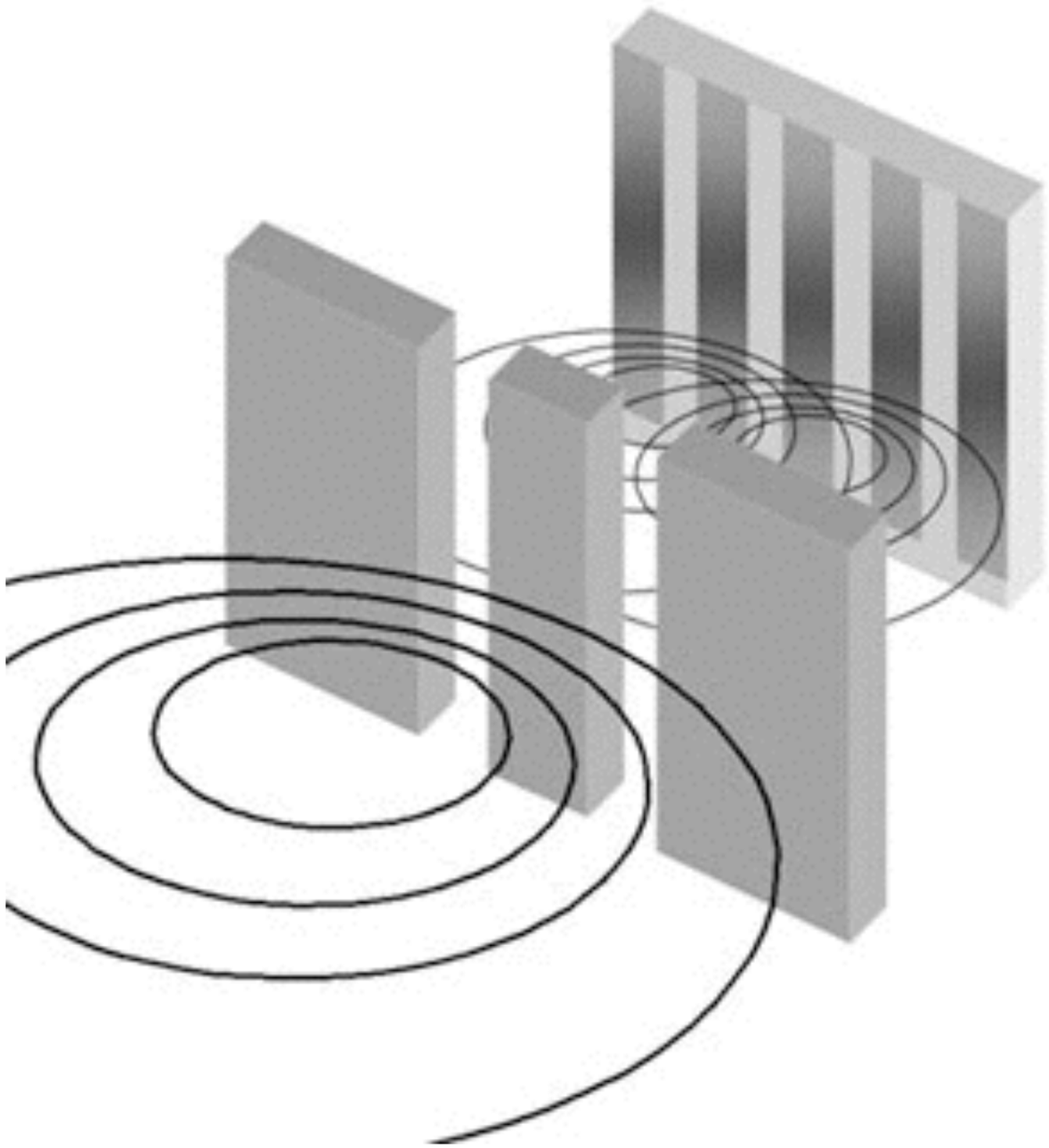
"Quantum critical point"
with
"long-range quantum entanglement".

Can this help us understand high temperature superconductivity ?
(and the rest of the phase diagram)

T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, Nature **477**, 191 (2011).

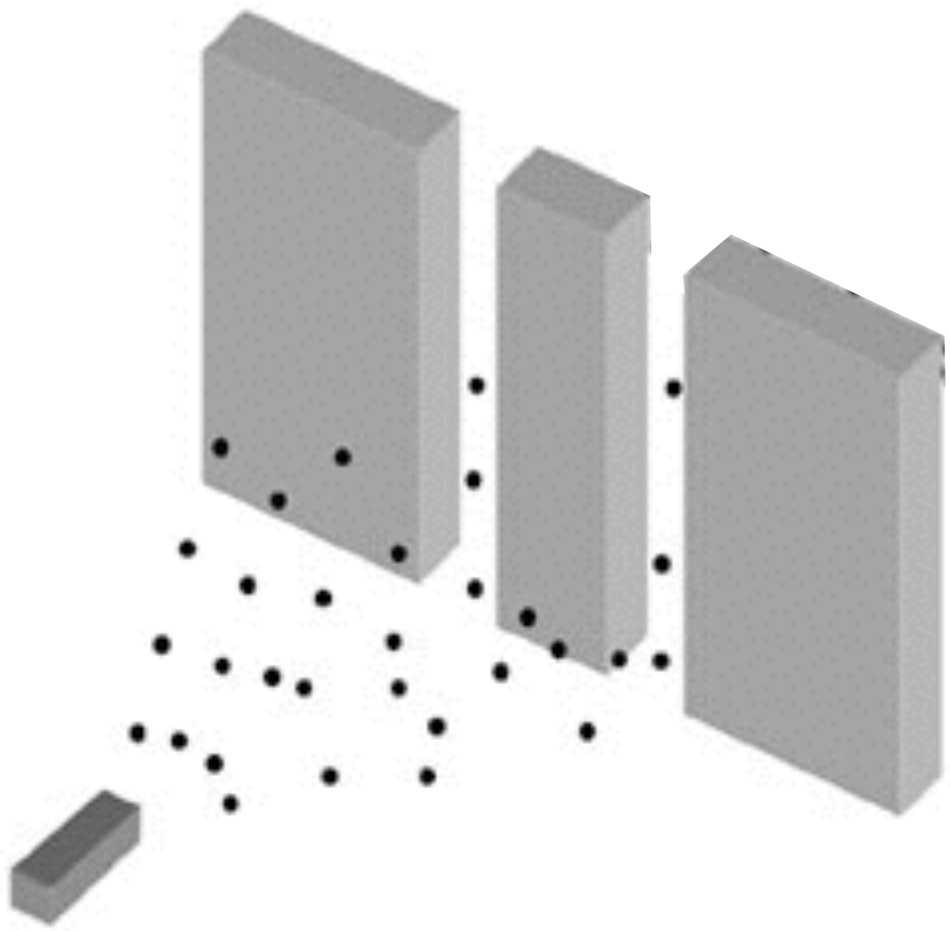
Quantum superposition and entanglement

The double slit experiment



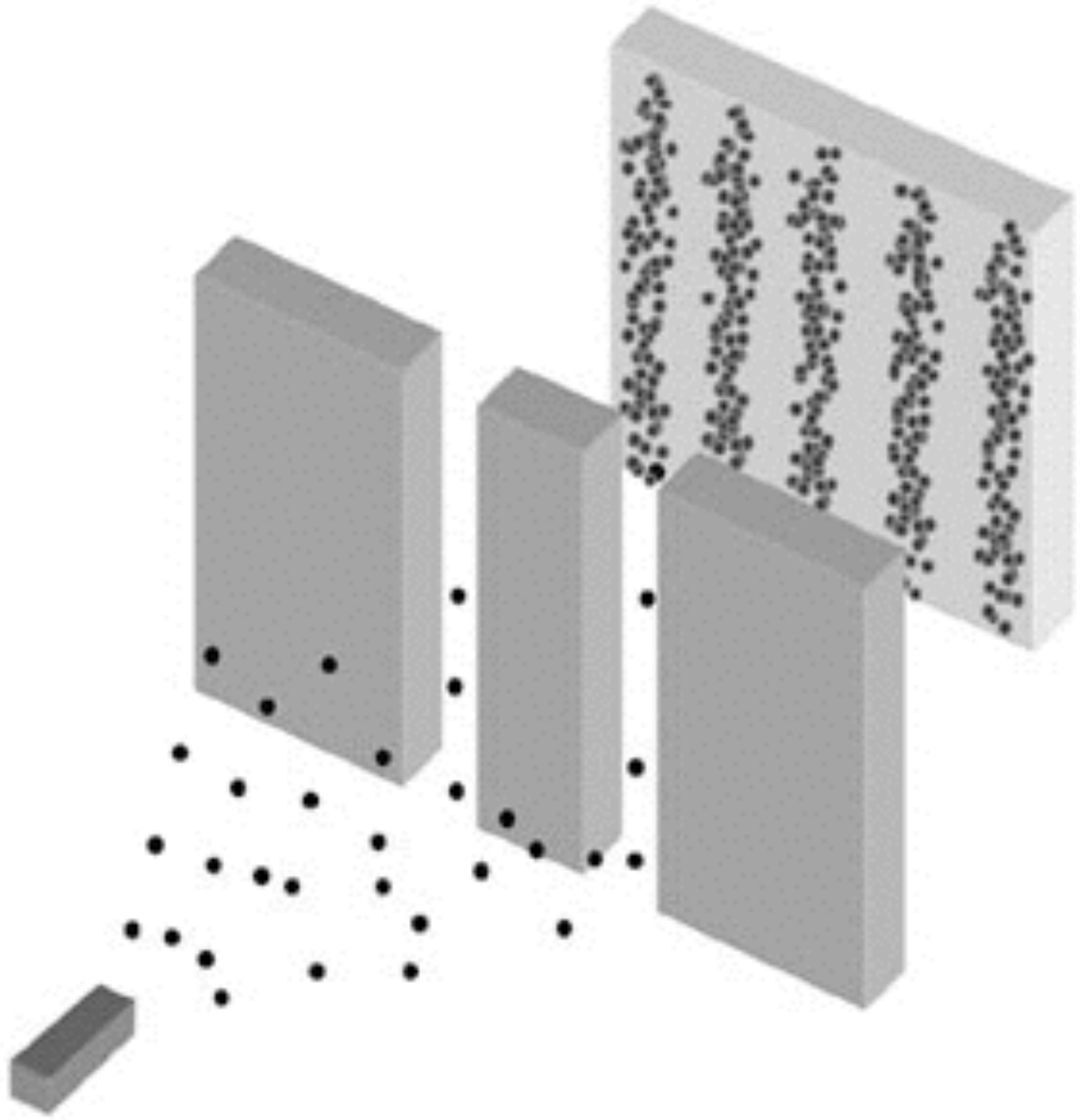
Interference of water waves

The double slit experiment



Send electrons through the slits

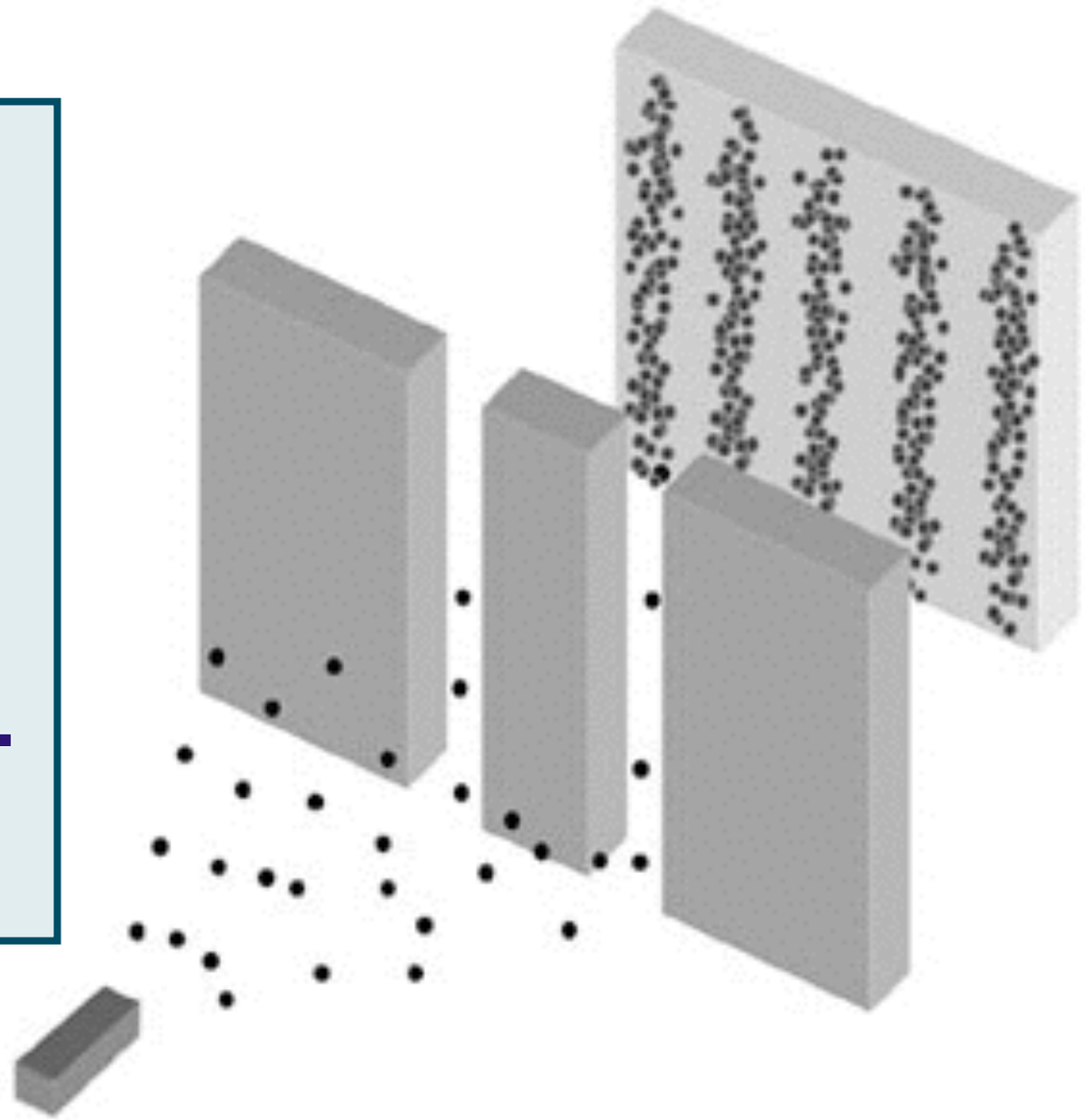
The double slit experiment



Interference of electrons

The double slit experiment

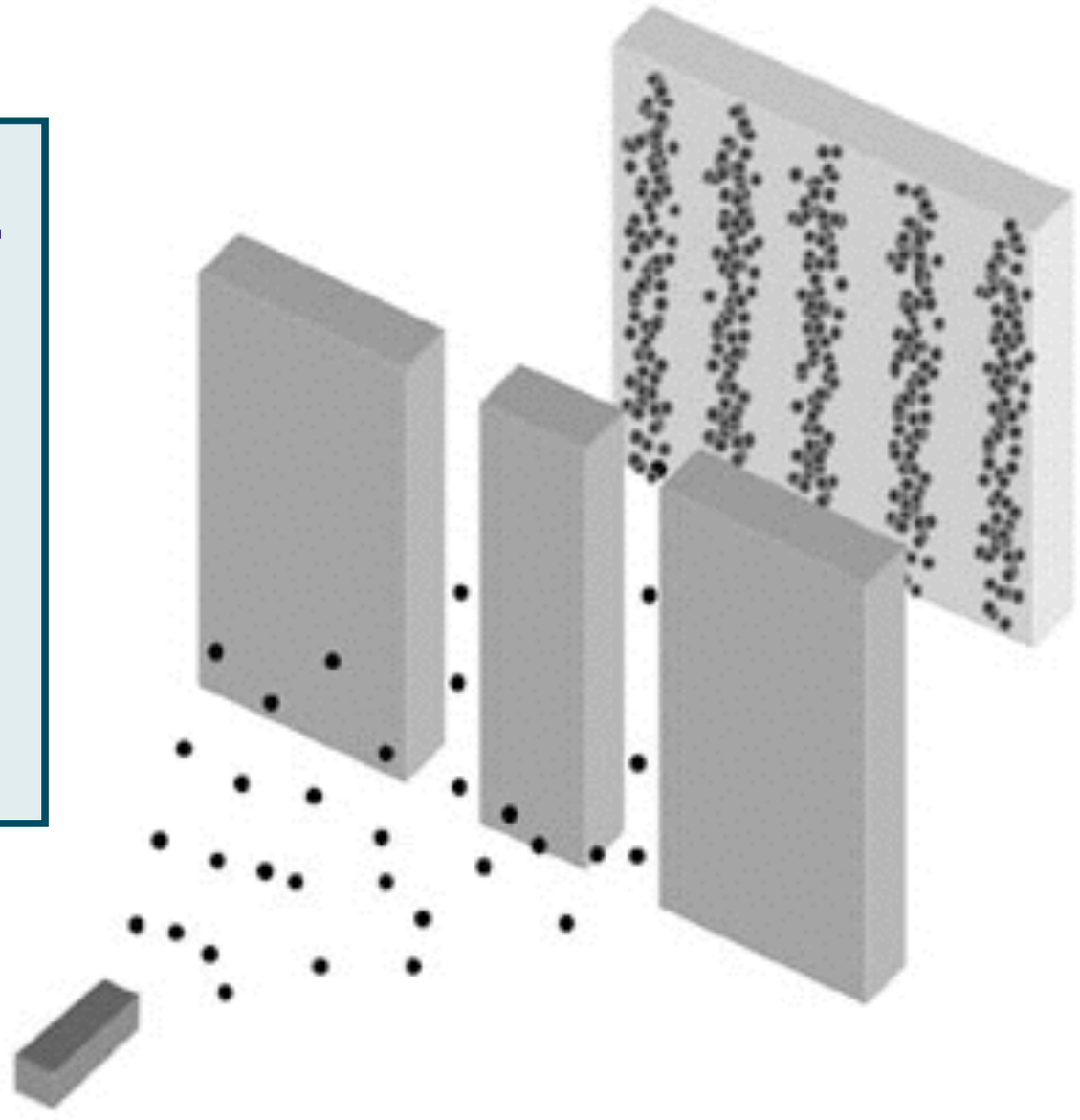
Unlike water waves, electrons arrive one-by-one



Interference of electrons

The double slit experiment

Which slit does an electron pass through ?



Interference of electrons

The double slit experiment

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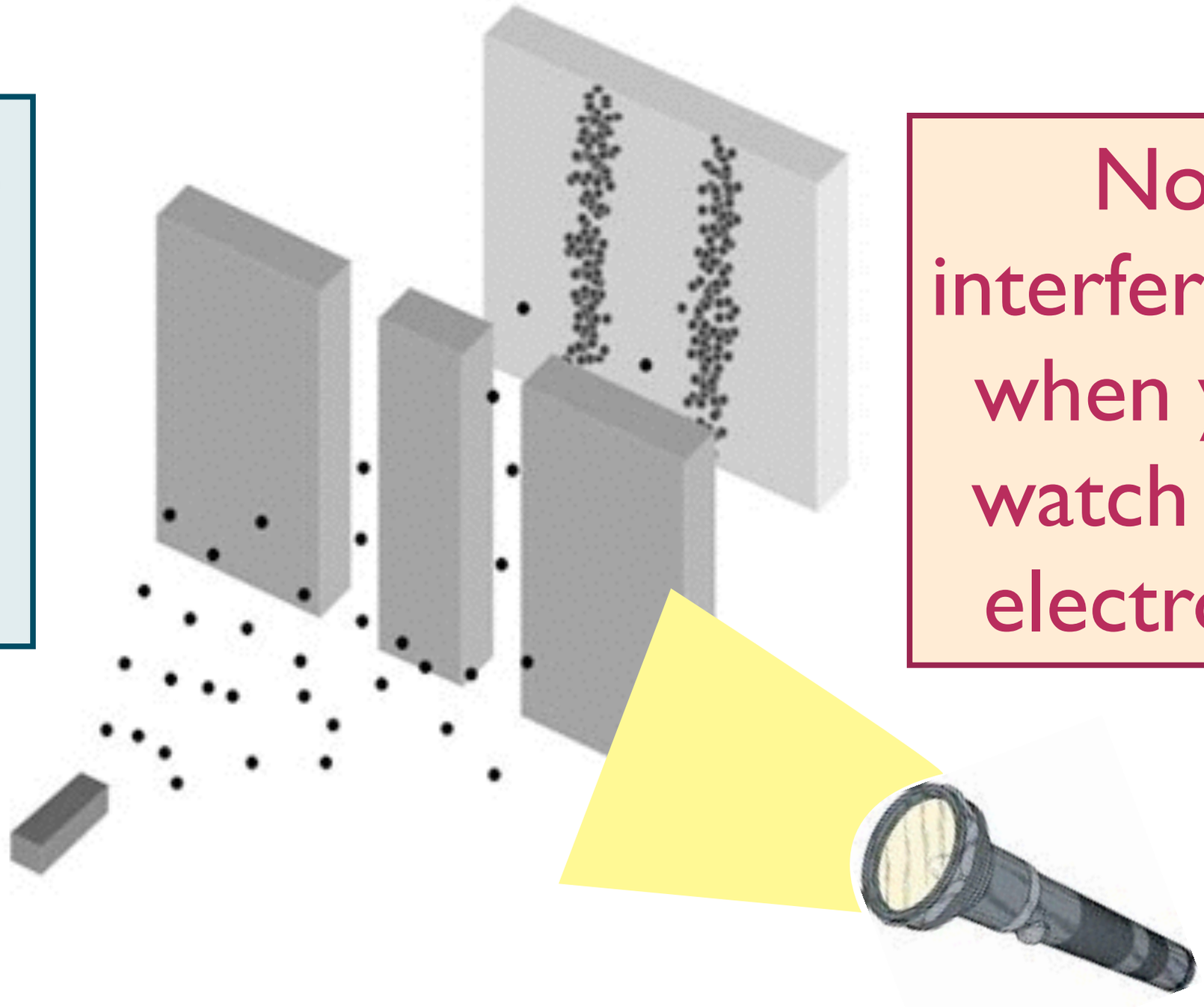


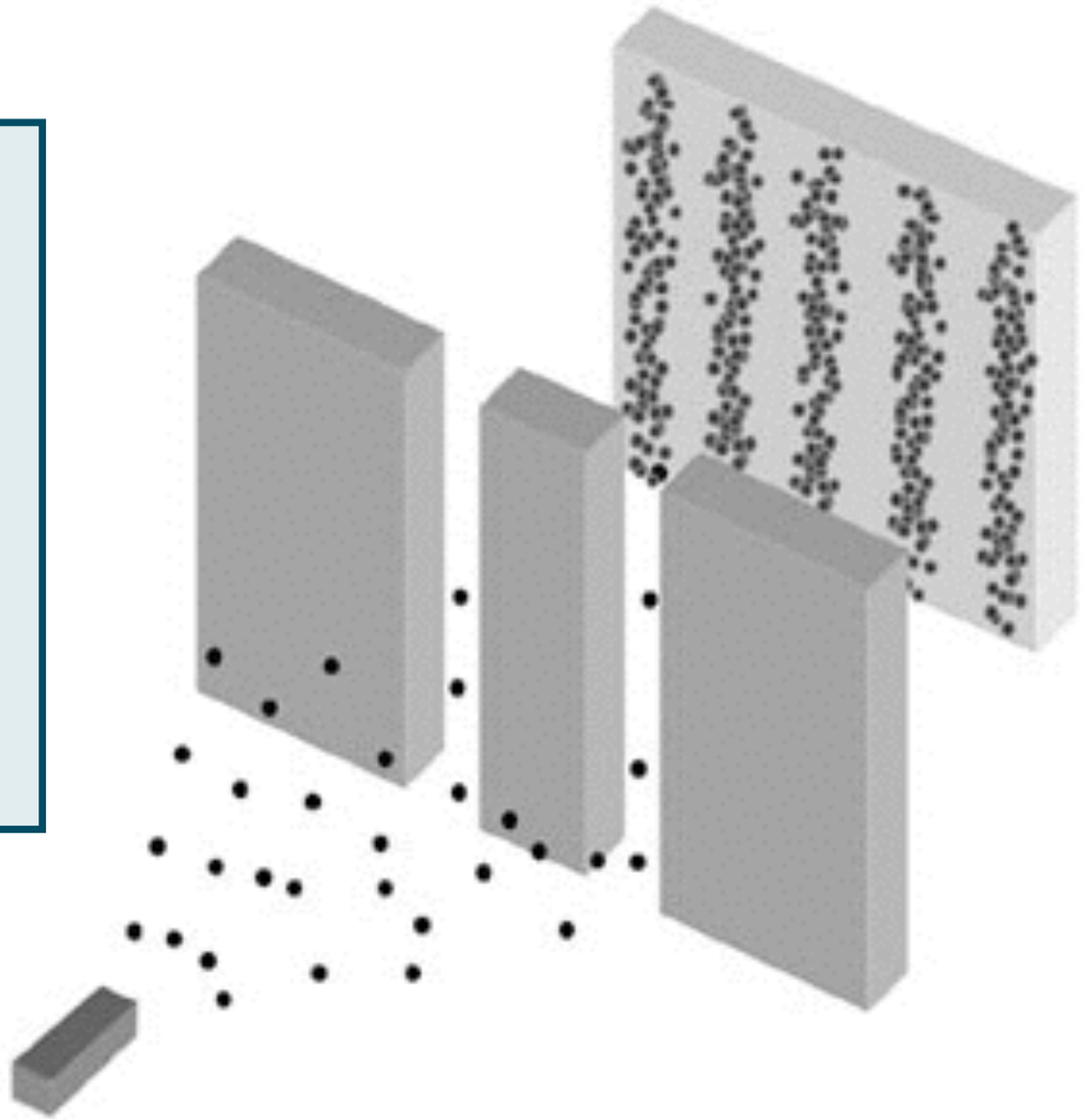
No interference when you watch the electrons

Interference of electrons

Principles of Quantum Mechanics: 1. Quantum Superposition

The double slit experiment

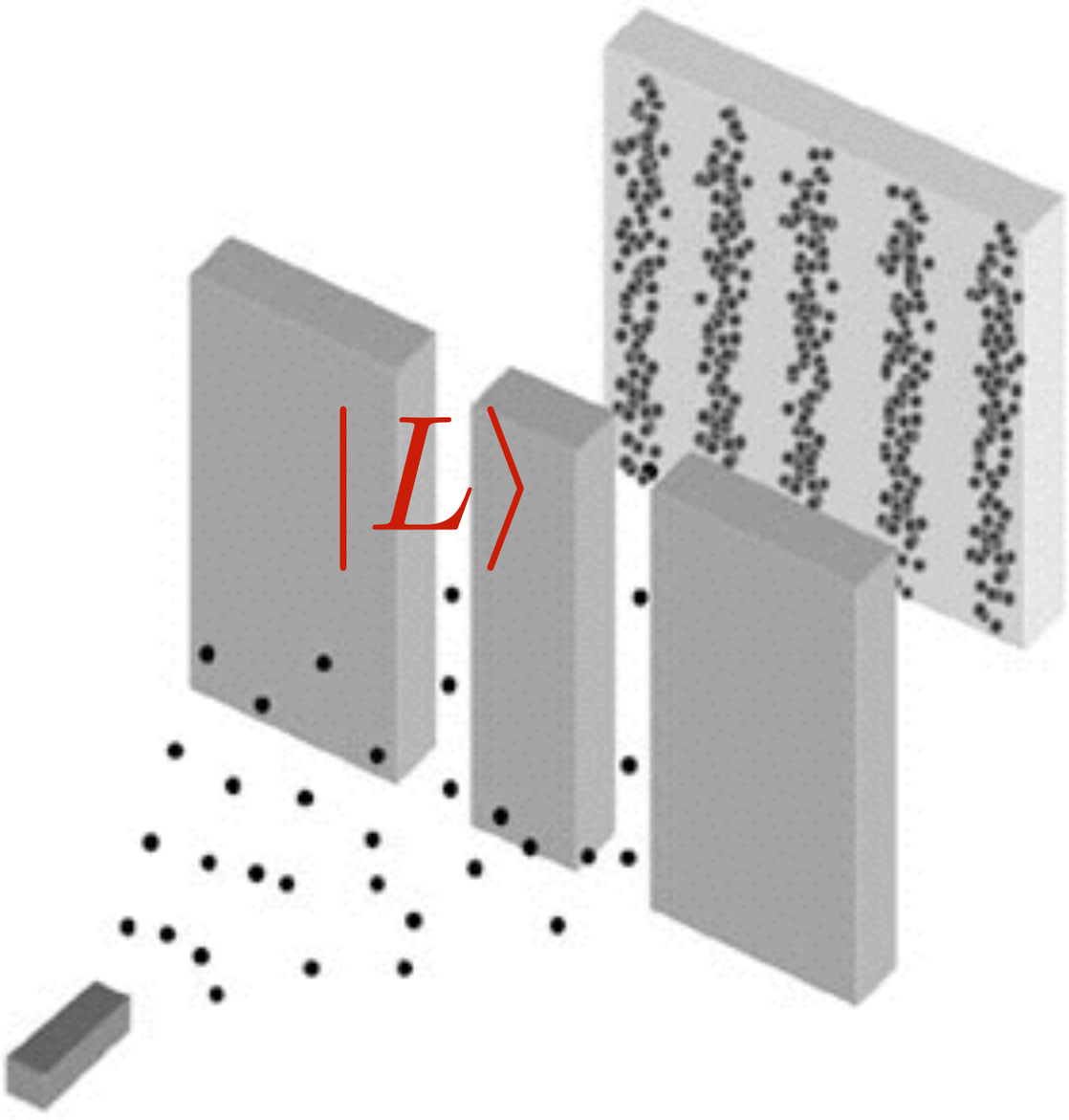
Which slit does an electron pass through ?



Each electron passes through both slits !

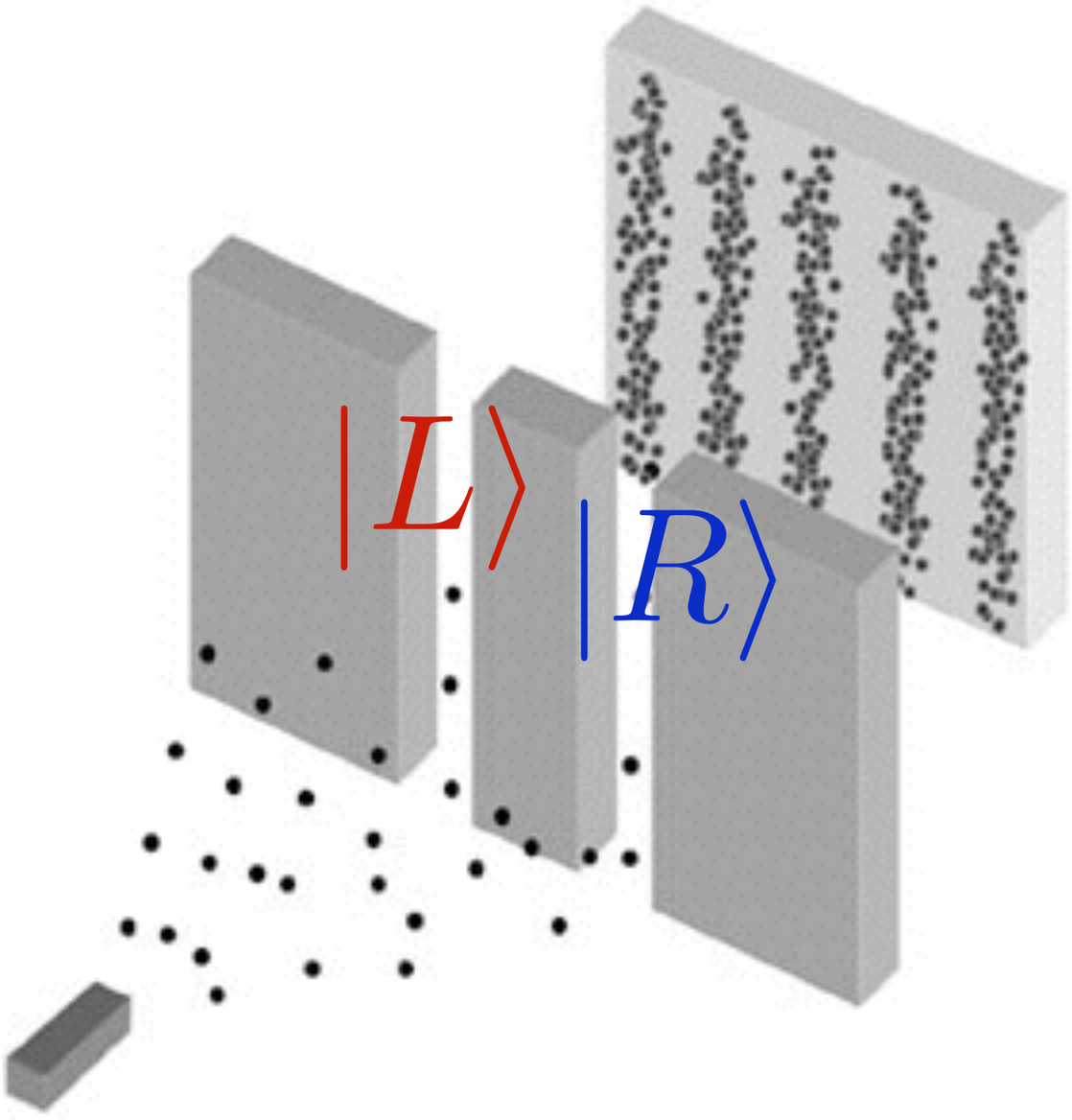
Interference of electrons

The double slit experiment



Let $|L\rangle$ represent the state
with the electron in the left slit

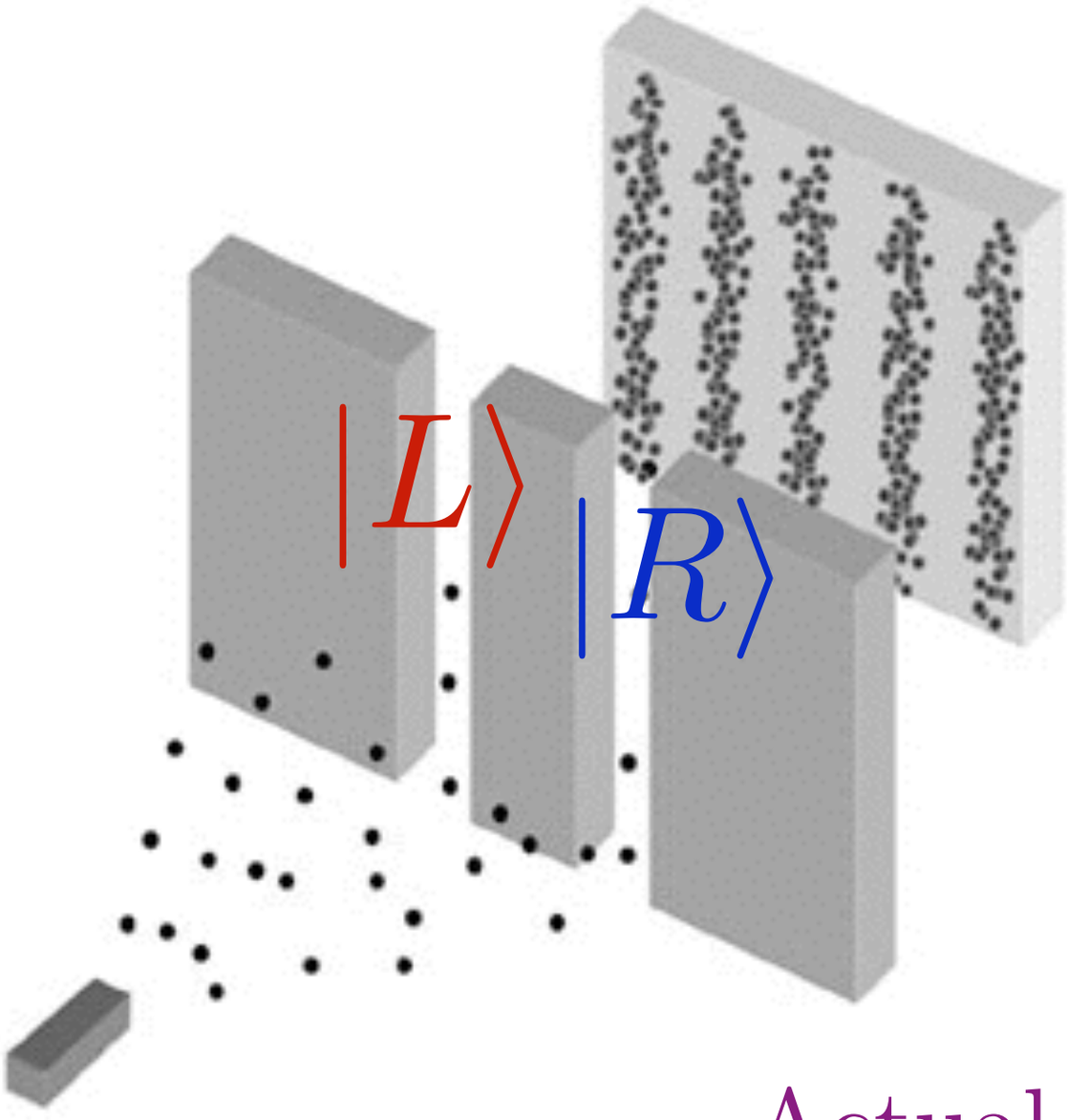
The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

The double slit experiment



Let $|L\rangle$ represent the state
with the electron in the left slit

And $|R\rangle$ represents the state
with the electron in the right slit

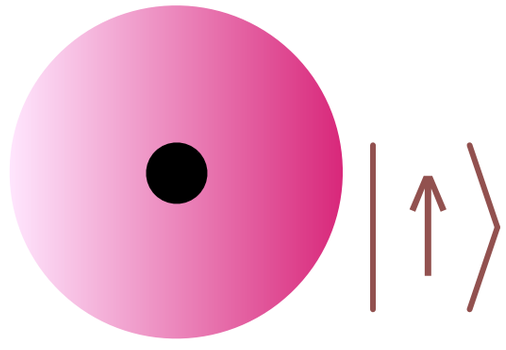
Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

Quantum Entanglement: quantum superposition with more than one particle

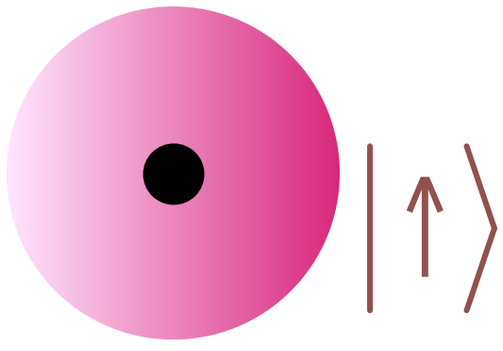
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

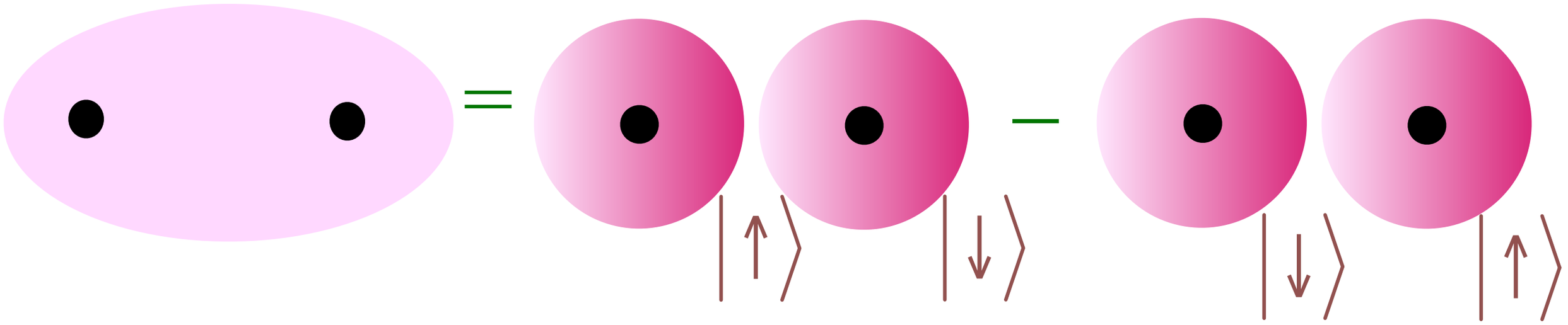


Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

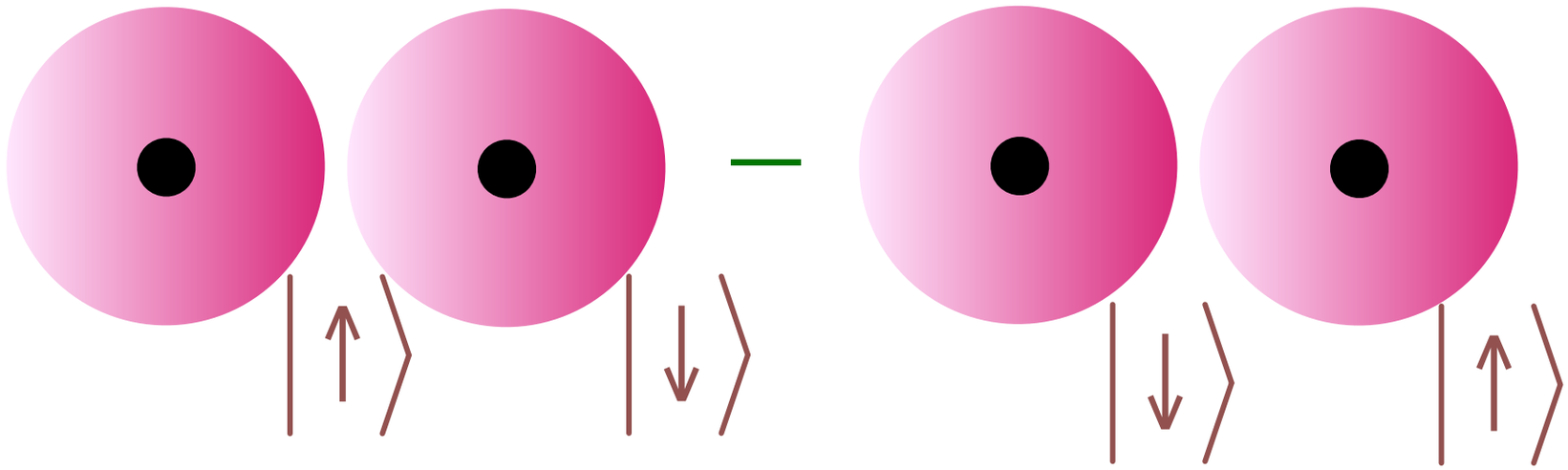


Hydrogen molecule:

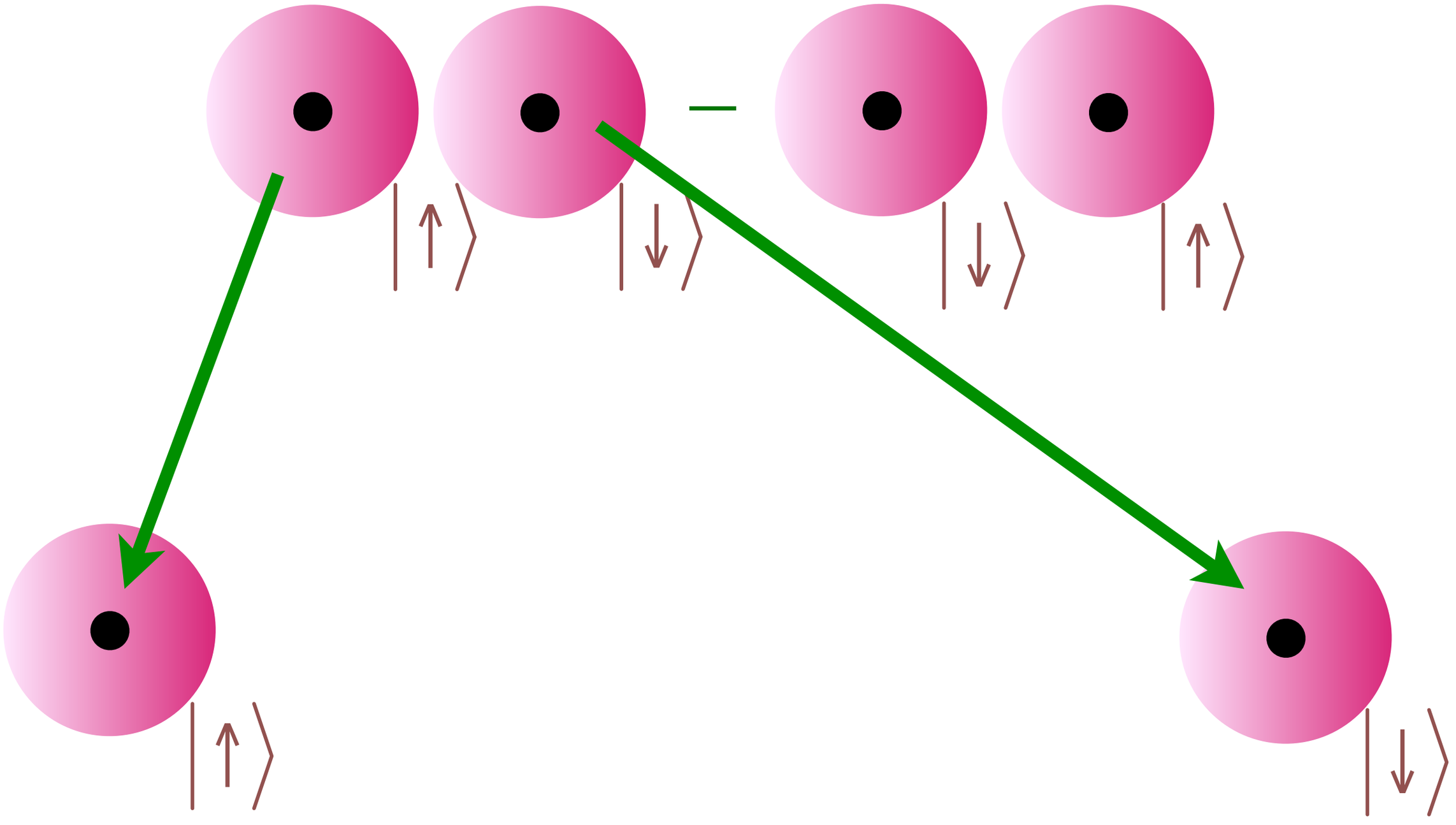


$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

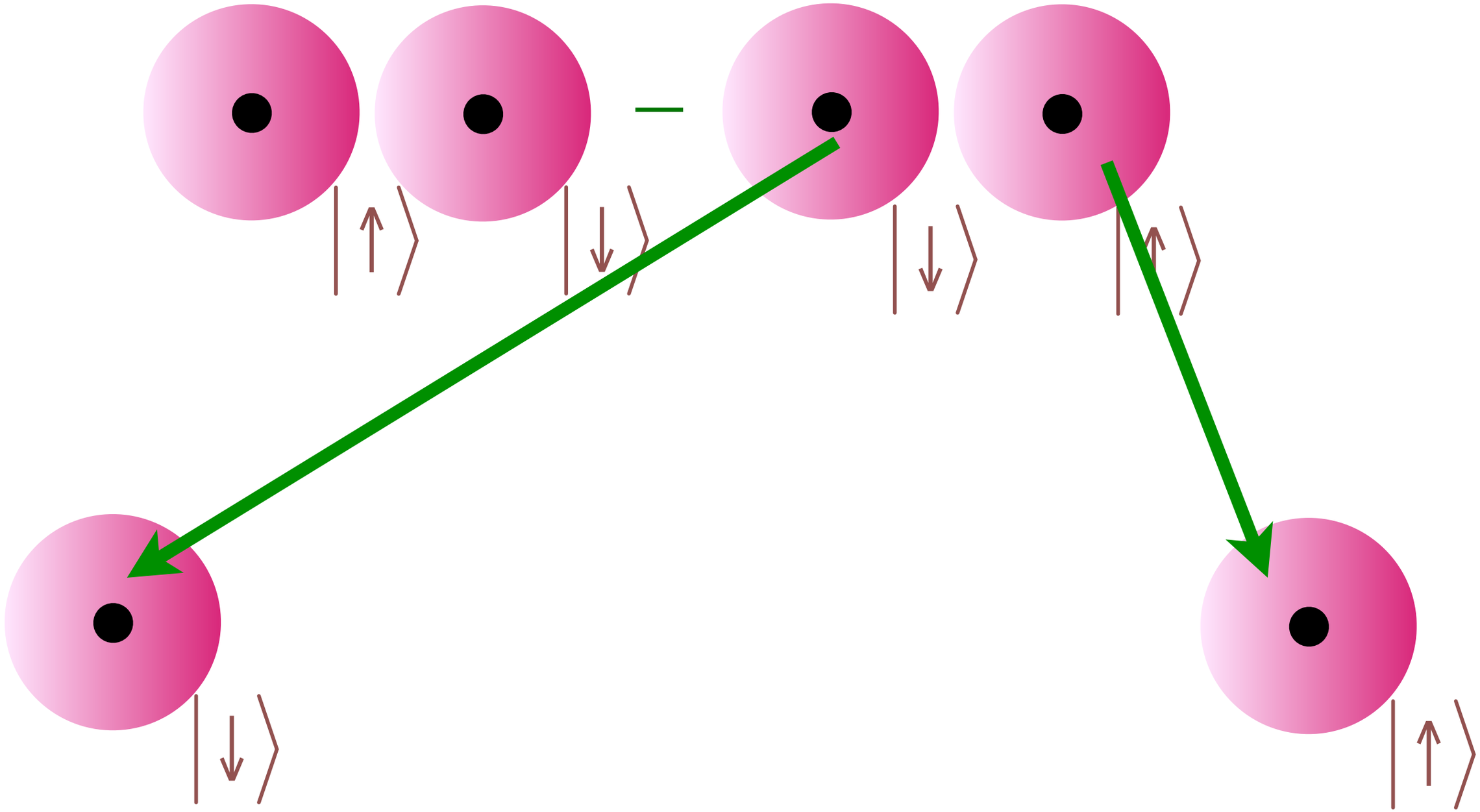
Quantum Entanglement: quantum superposition with more than one particle



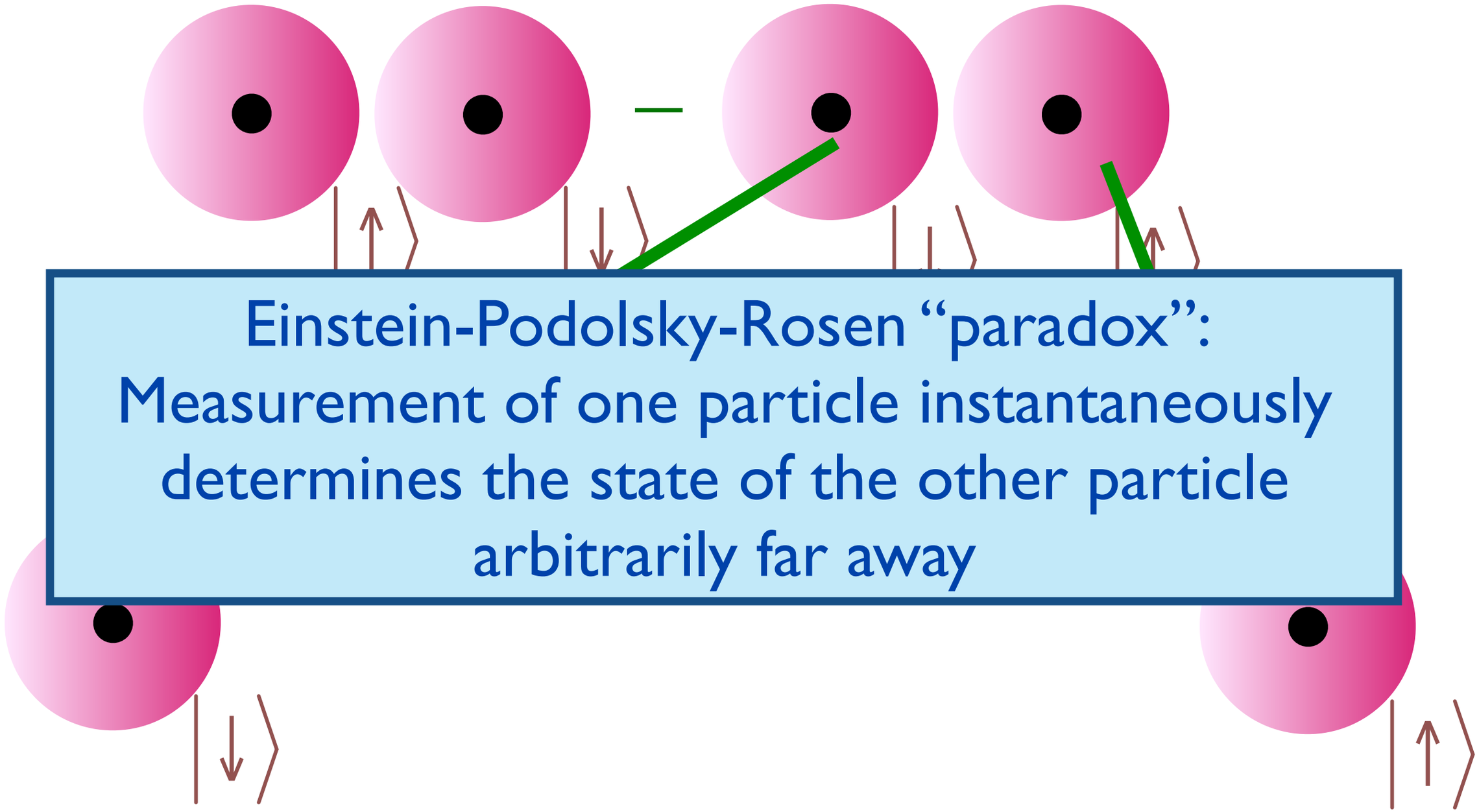
Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



Quantum superposition and entanglement

Quantum superposition and entanglement

Quantum critical points and long-range entanglement of electrons in crystals

String theory and black holes

Quantum superposition and entanglement

Quantum critical points and long-range entanglement of electrons in crystals

String theory and black holes

Antiferromagnet

Strange metal

Superconductor

?

Quantum critical point

T (K)

120

80

40

0

0.04

0.08

0.12

0.16

p (hole/Cu)

Cu^{2+},Cu^{3+}

O^{2-}

Y^{3+}

Ba^{2+}

c

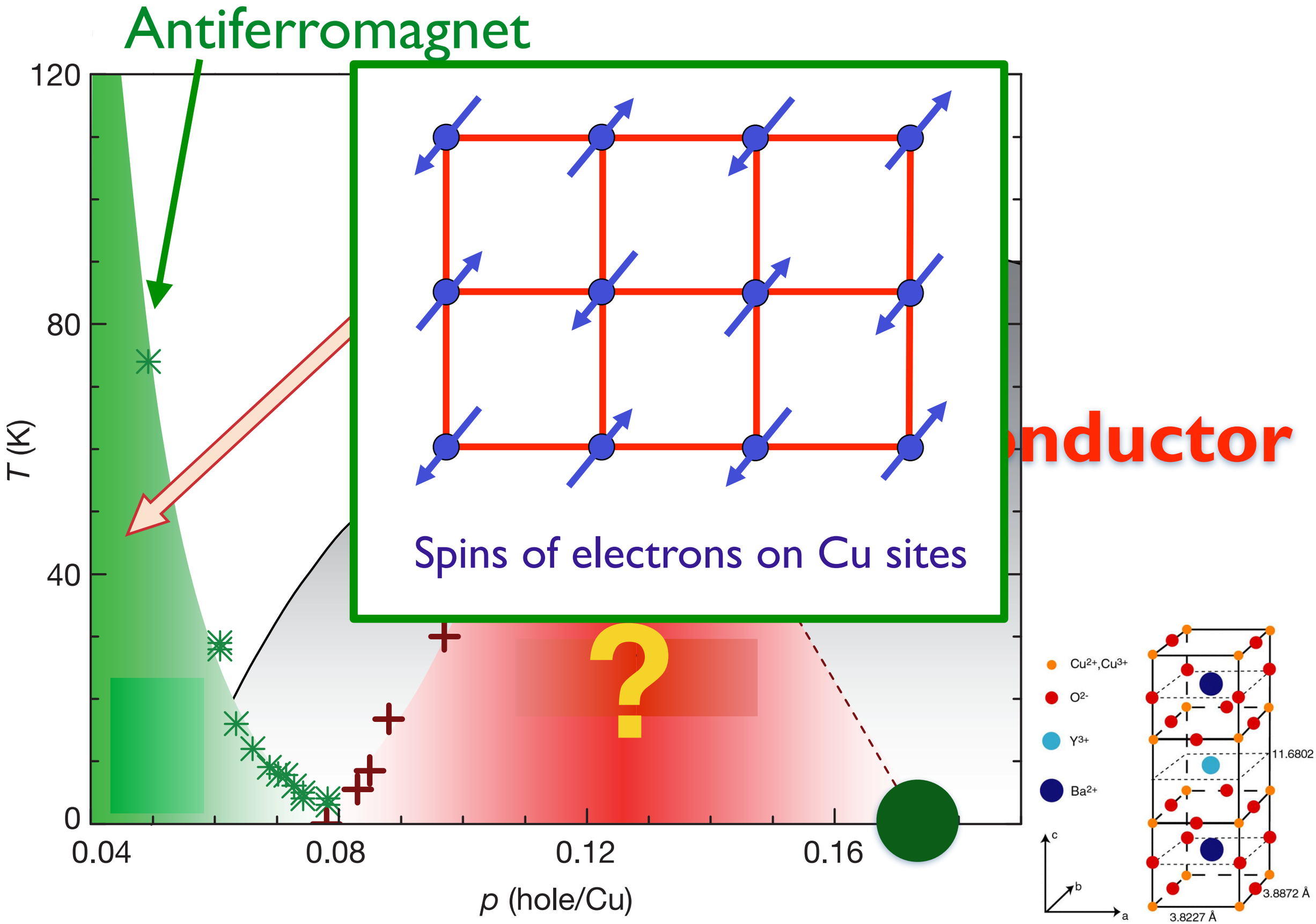
b

a

11.6802 Å

3.8872 Å

3.8227 Å



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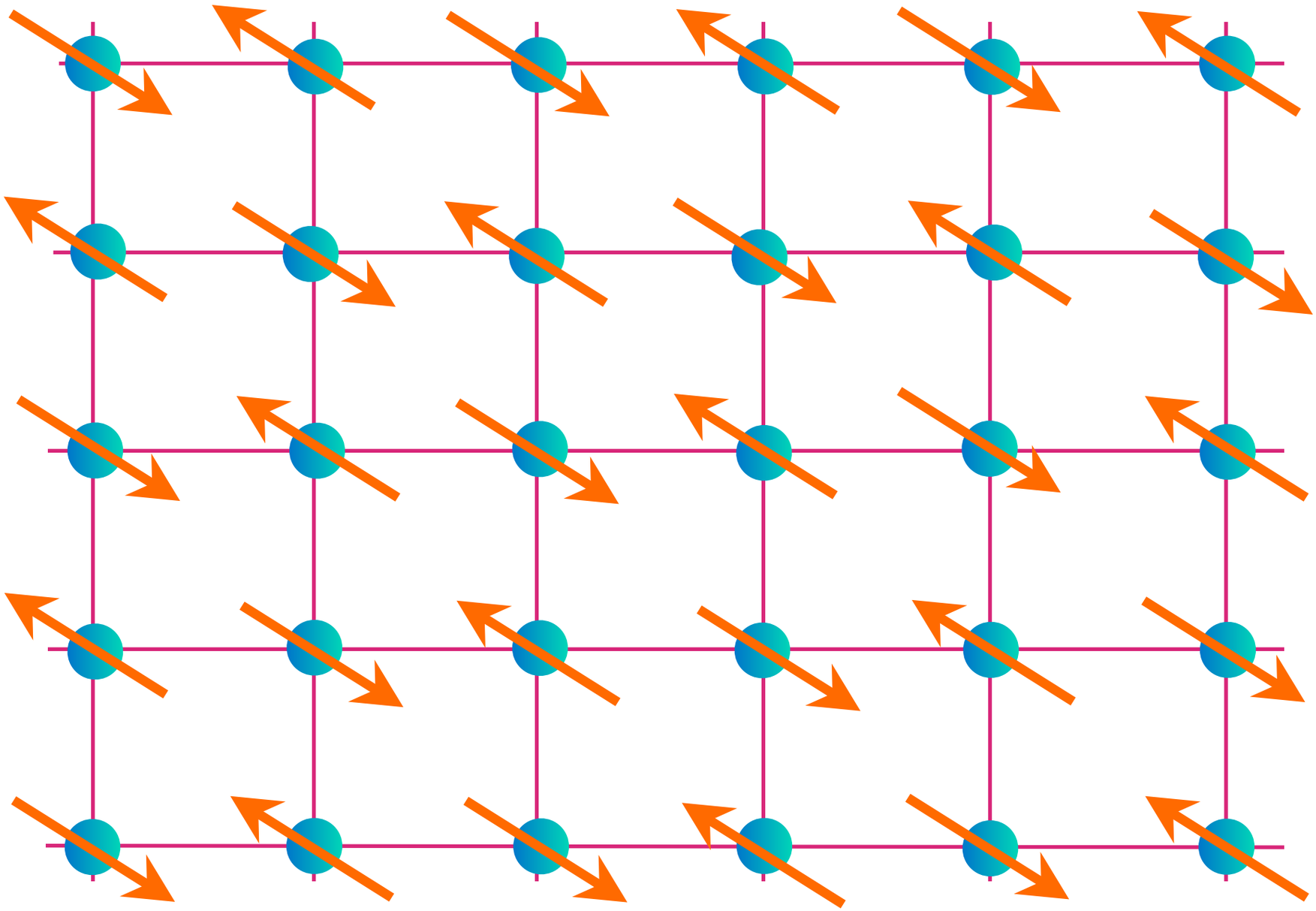
Square lattice of Cu sites



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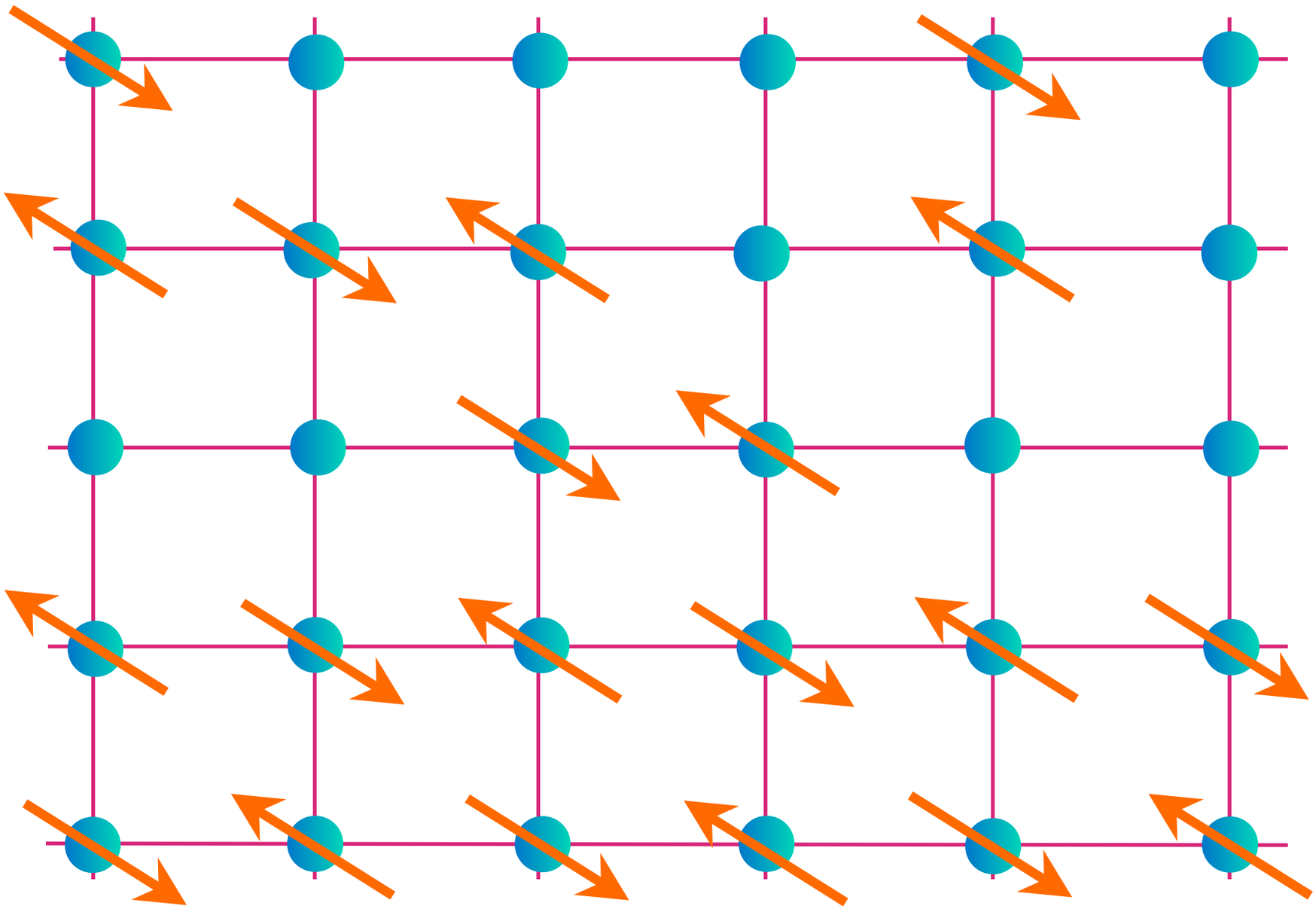
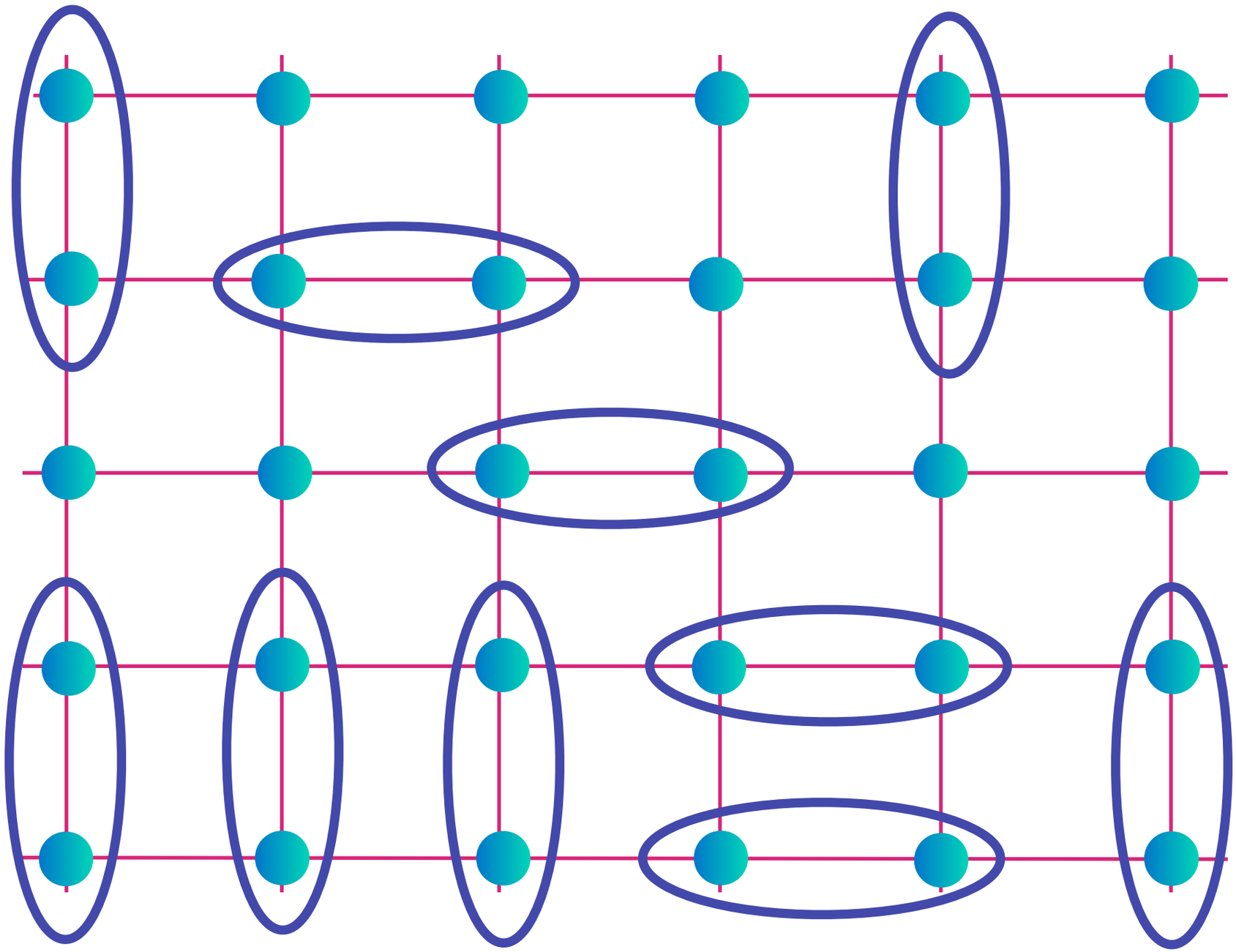
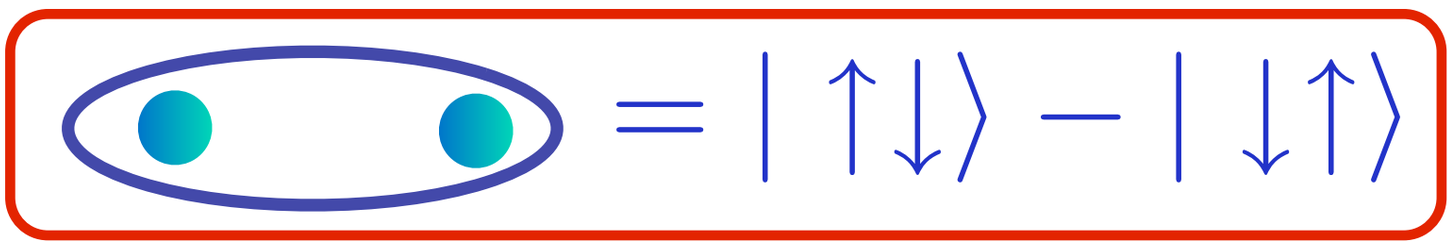


Remove some electrons

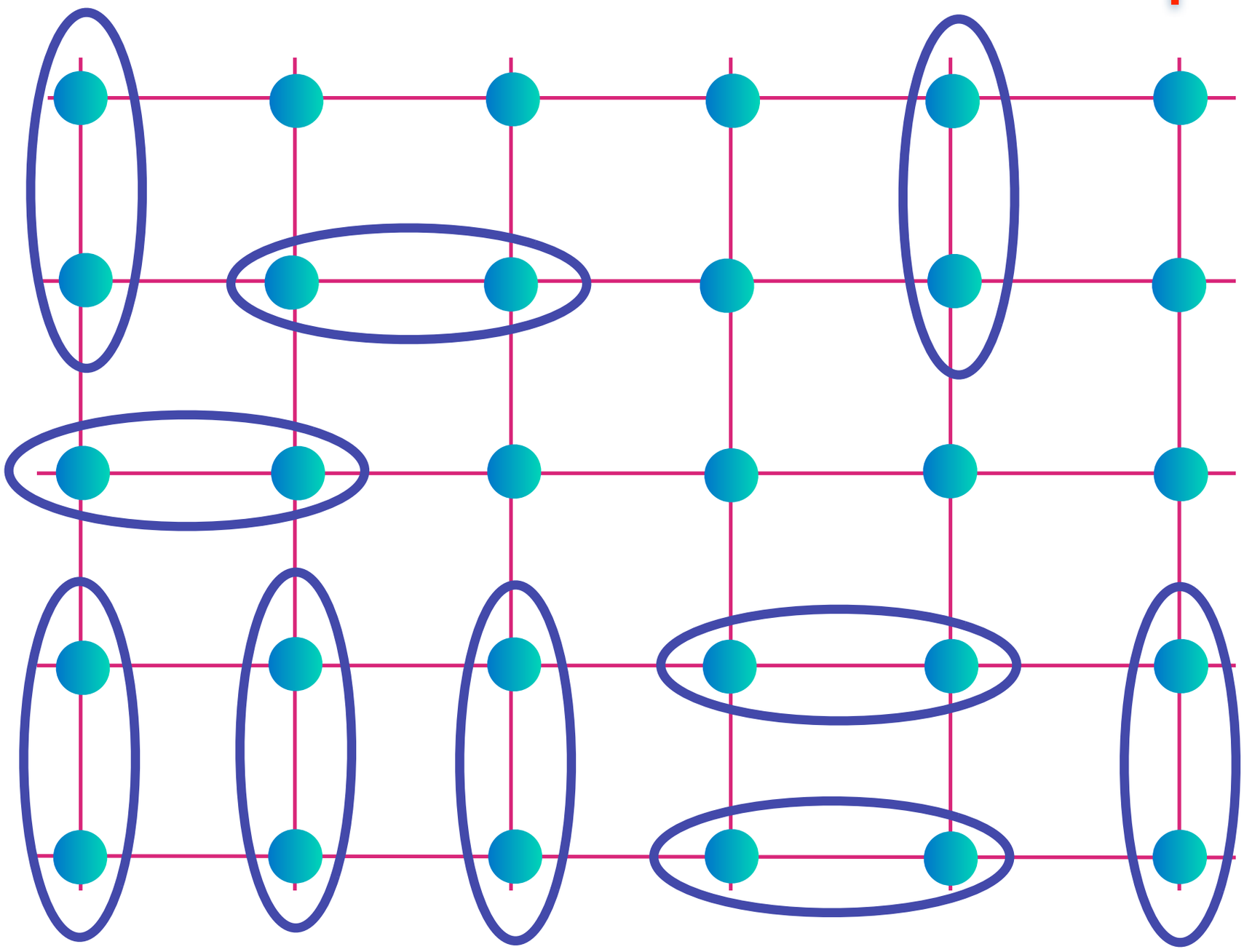
Square lattice of Cu sites



Electrons entangle in ("Cooper") pairs into chemical bonds

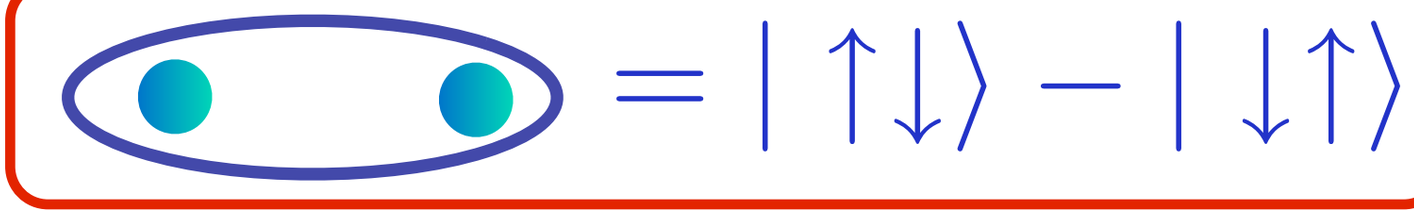


Square lattice of Cu sites



Superconductivity !

Cooper pairs form quantum superpositions at different locations: "Bose-Einstein condensation" in which all pairs are "everywhere at the same time"



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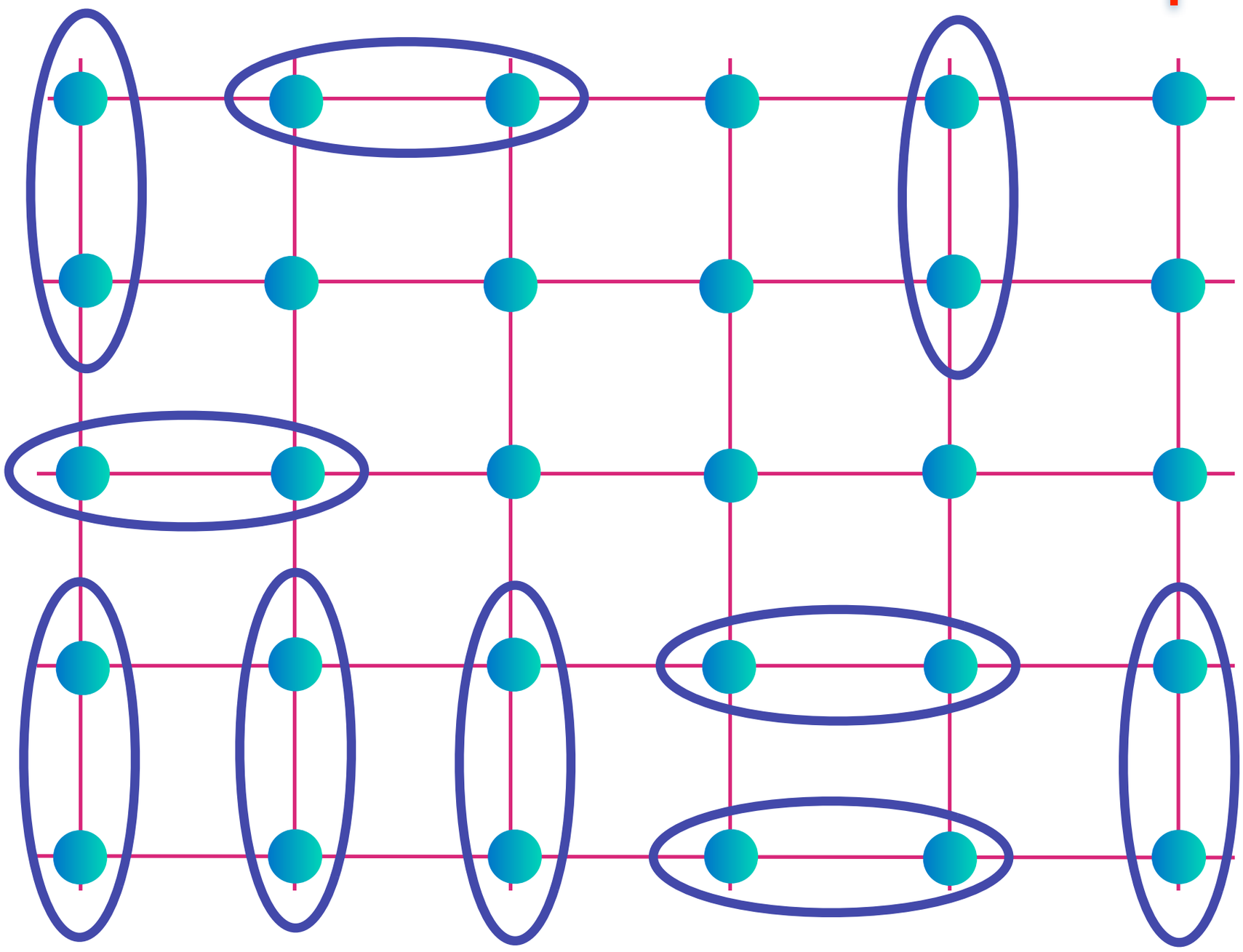

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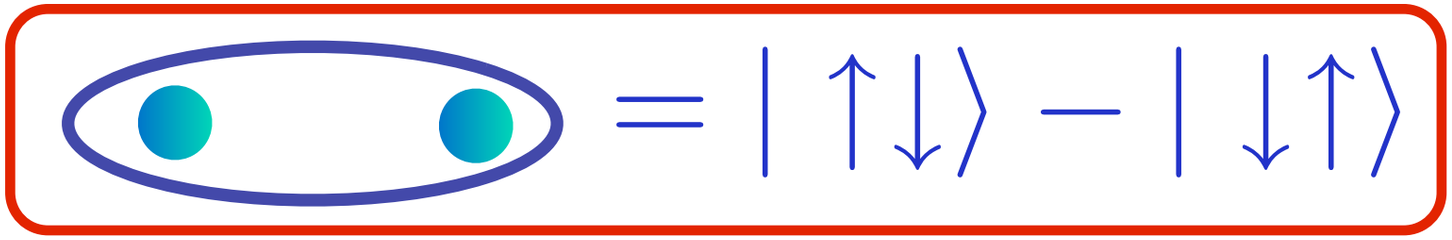

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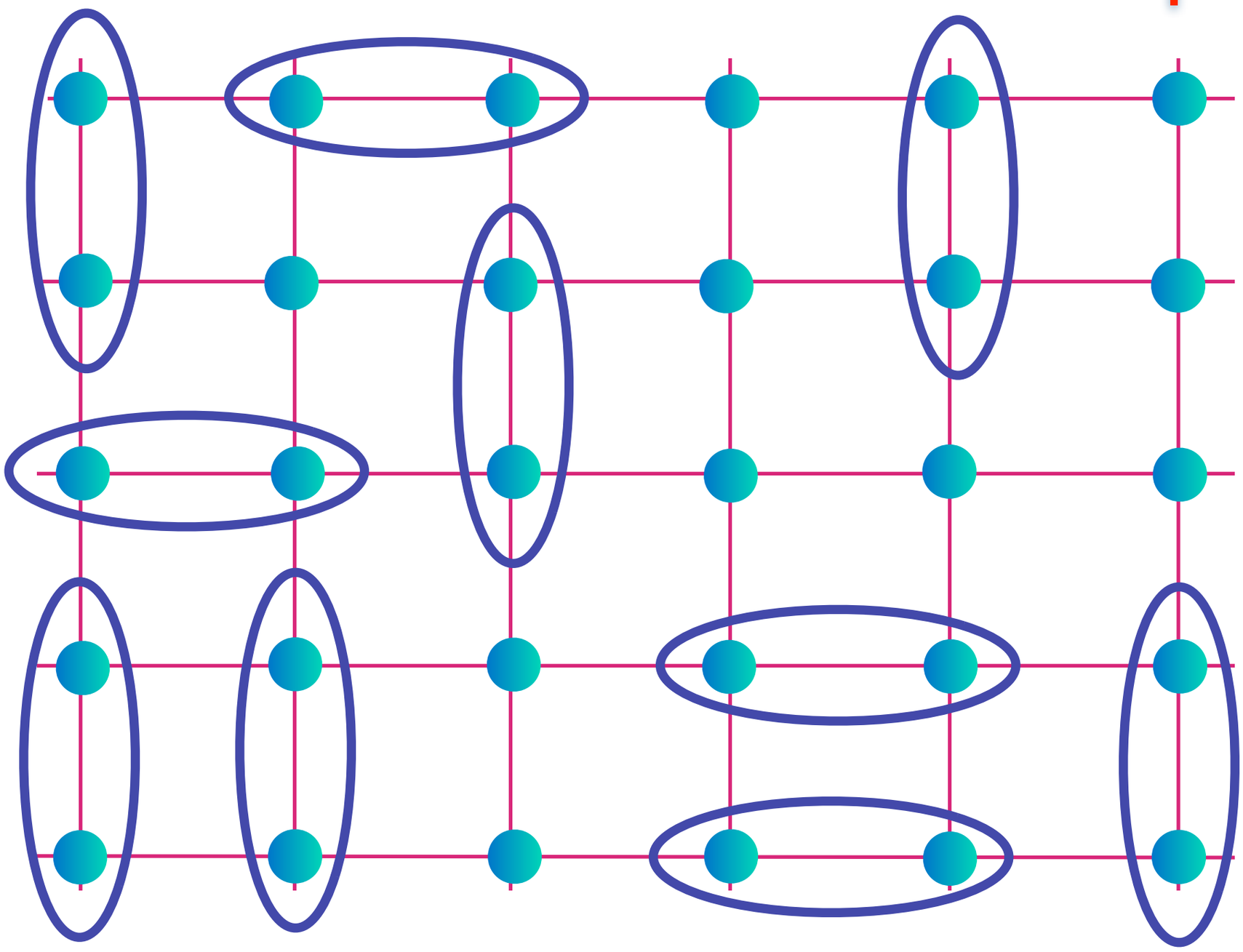

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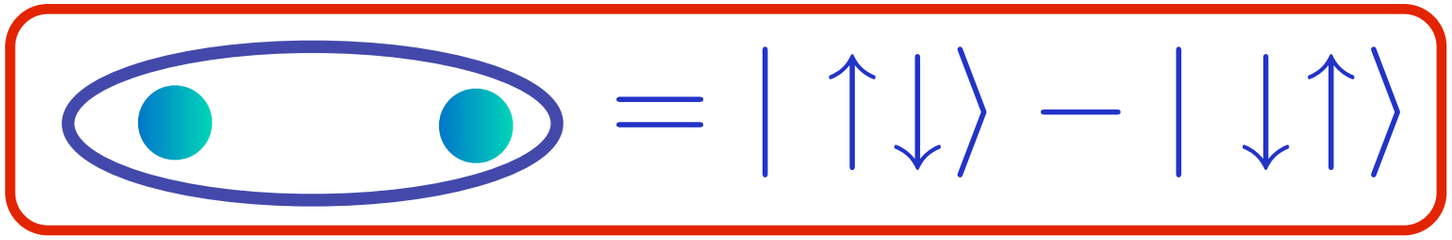

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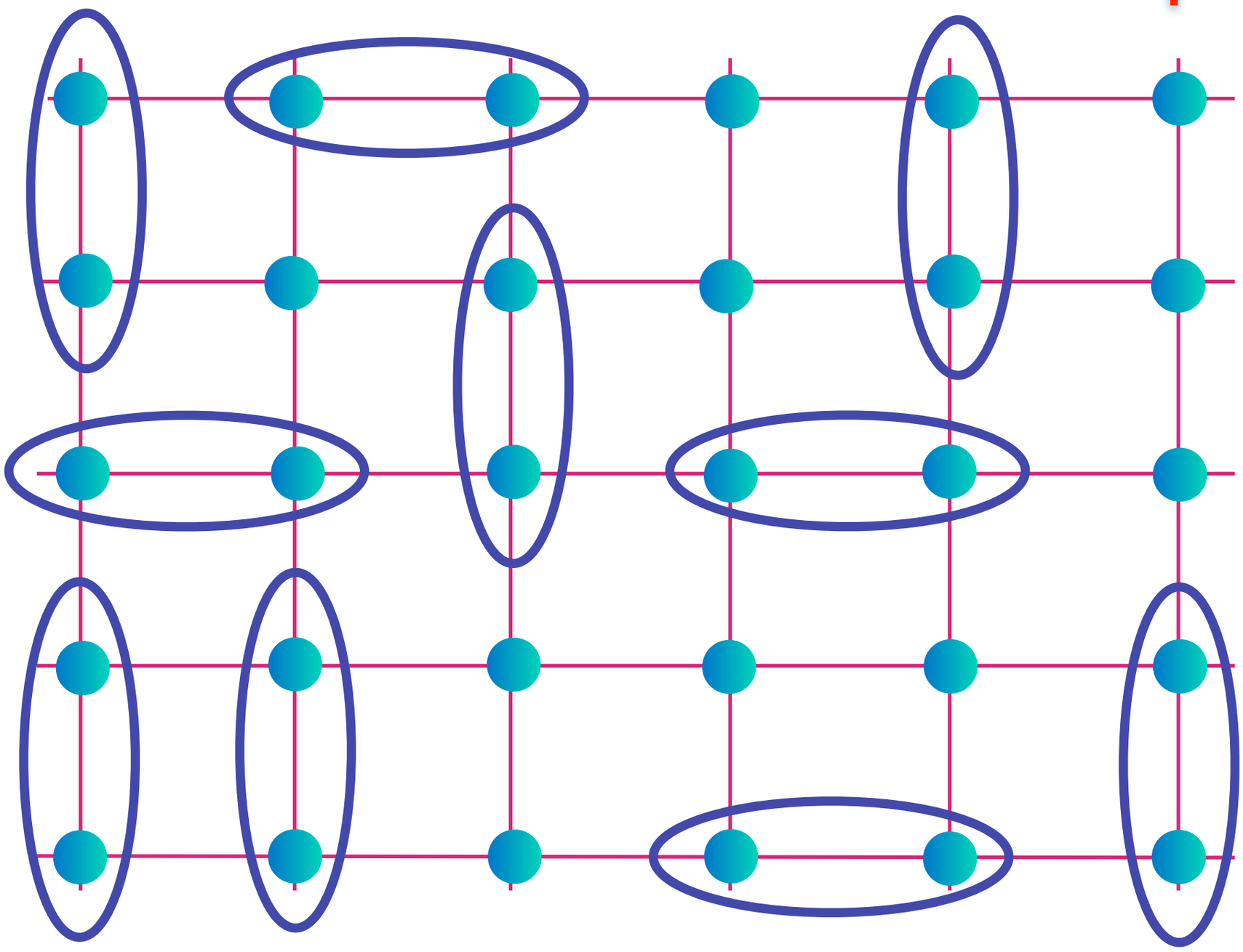

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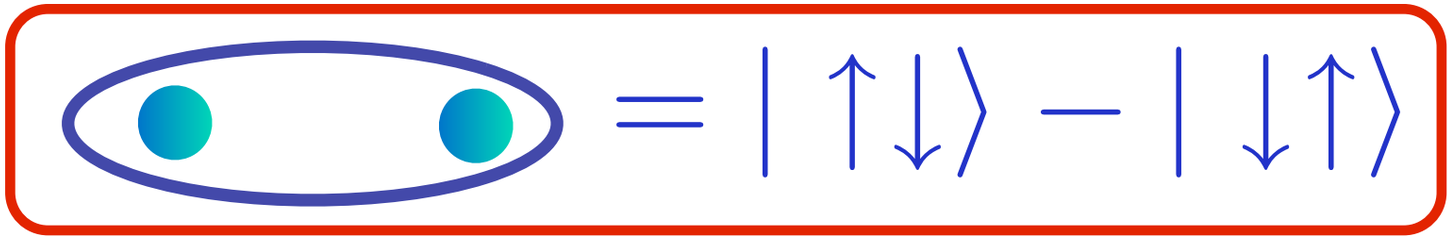

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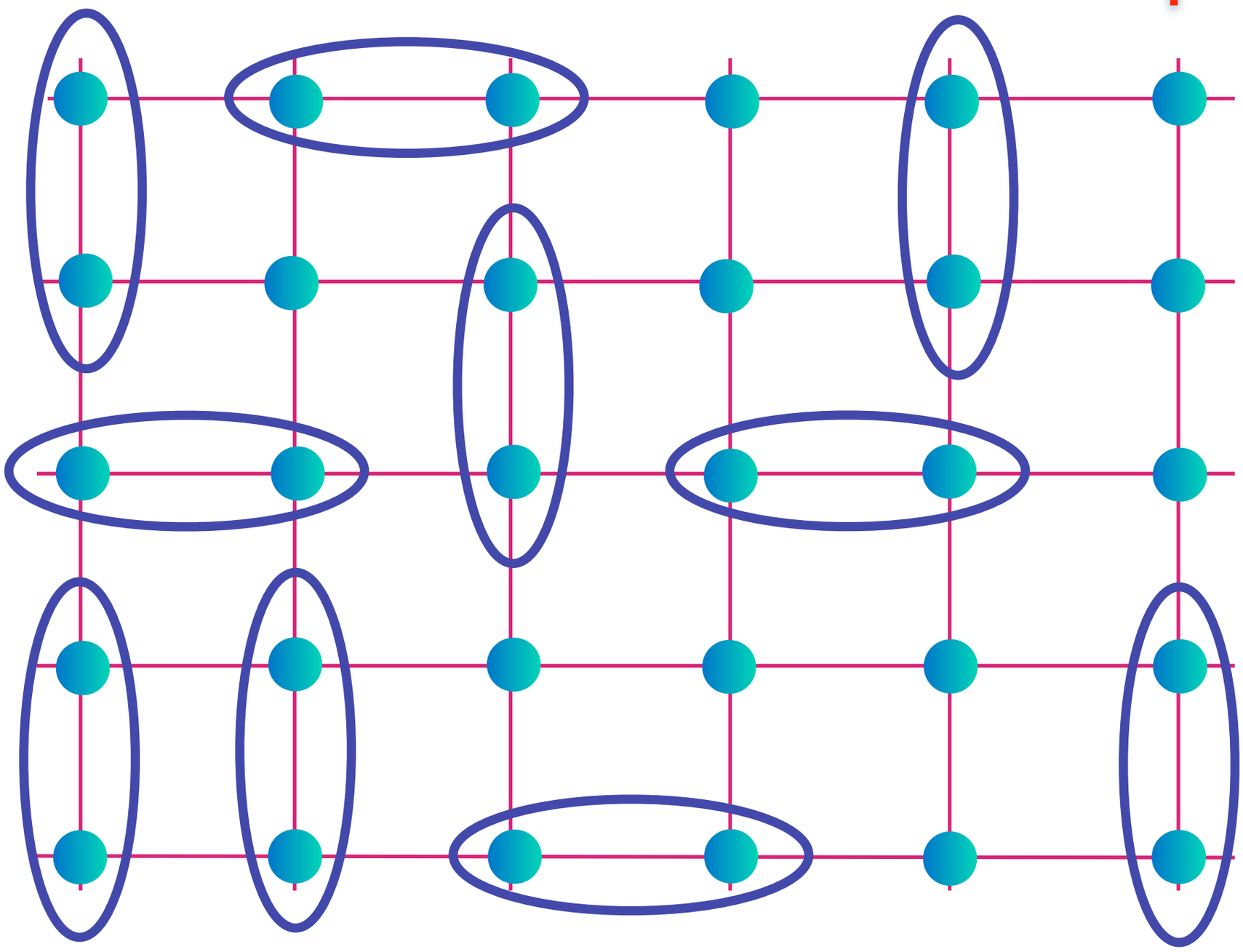

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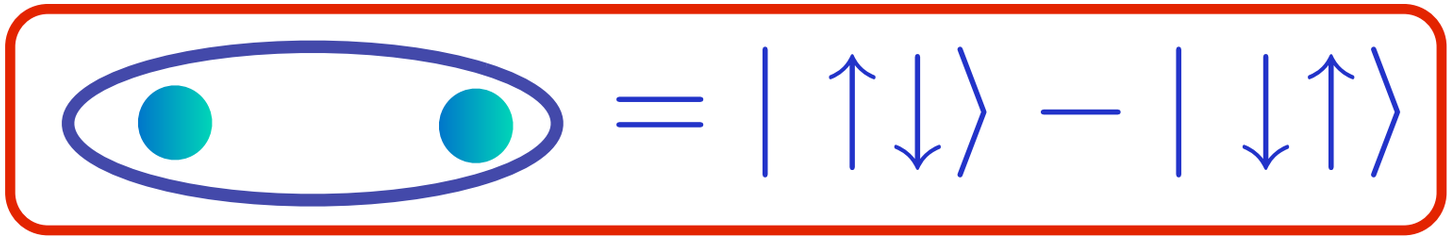

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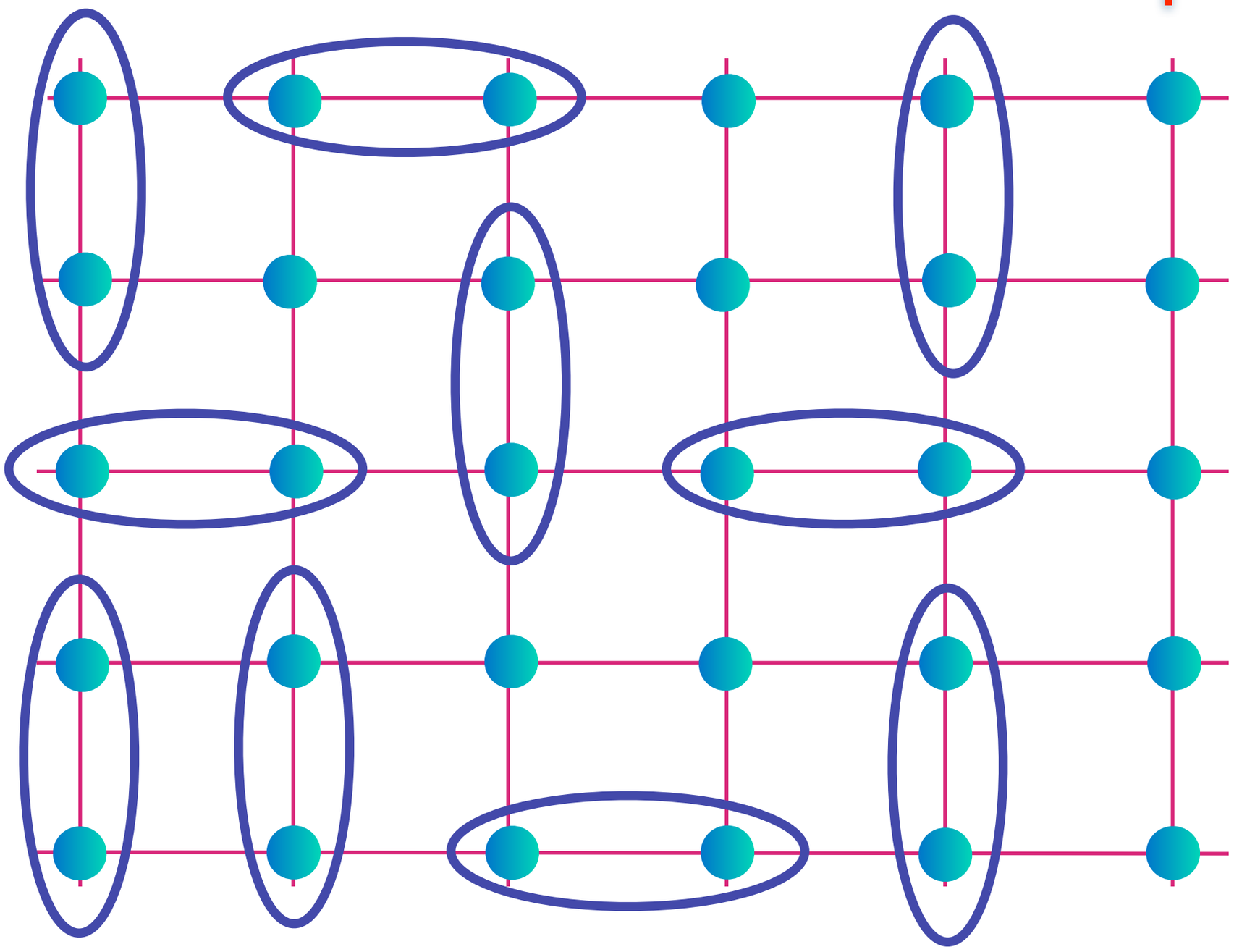

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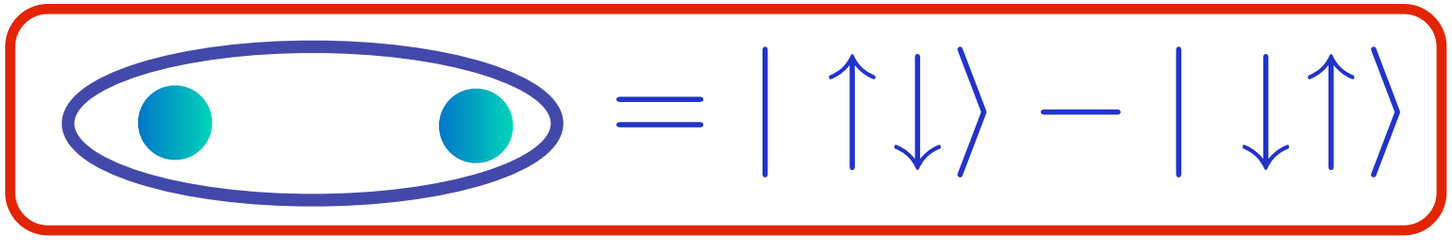

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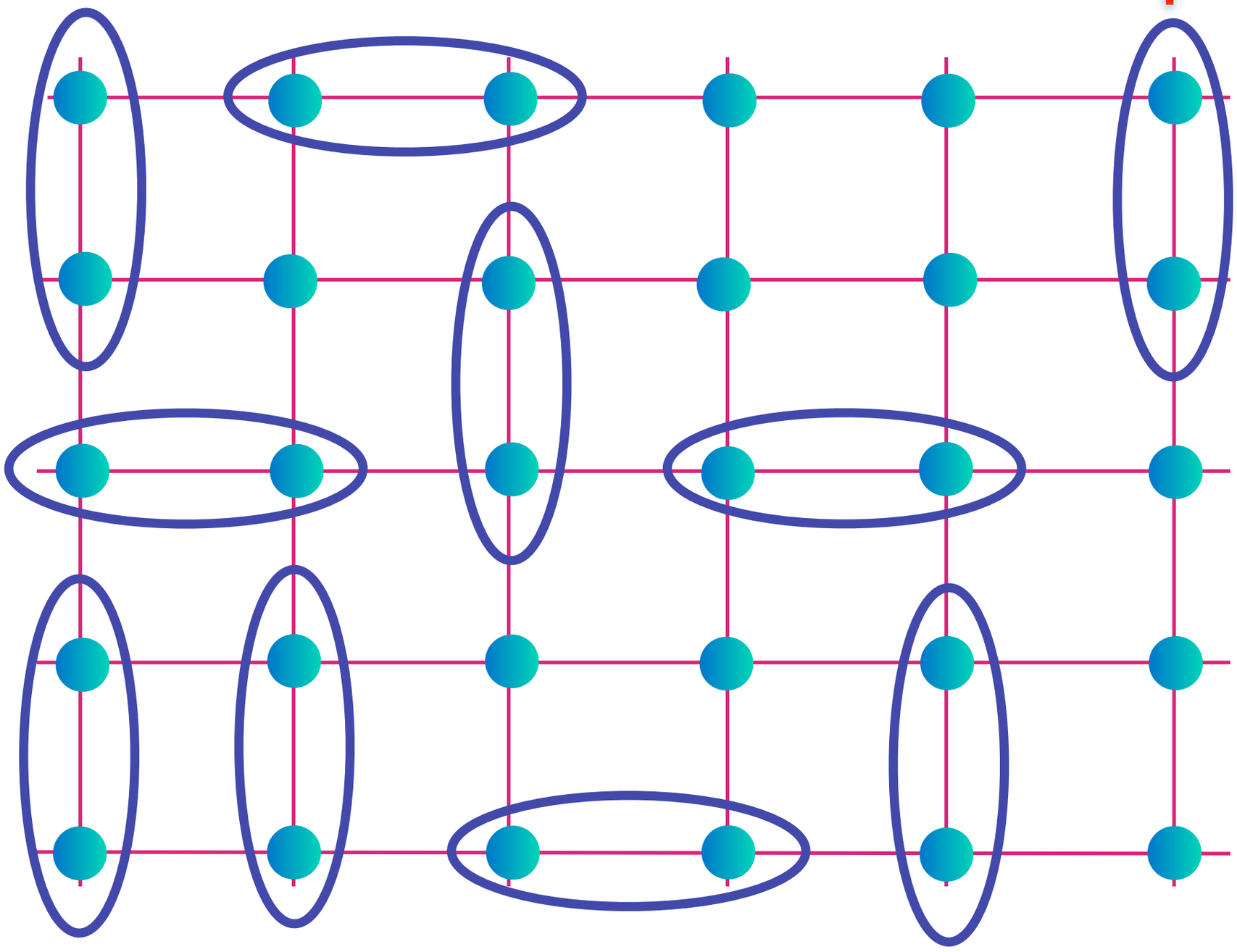
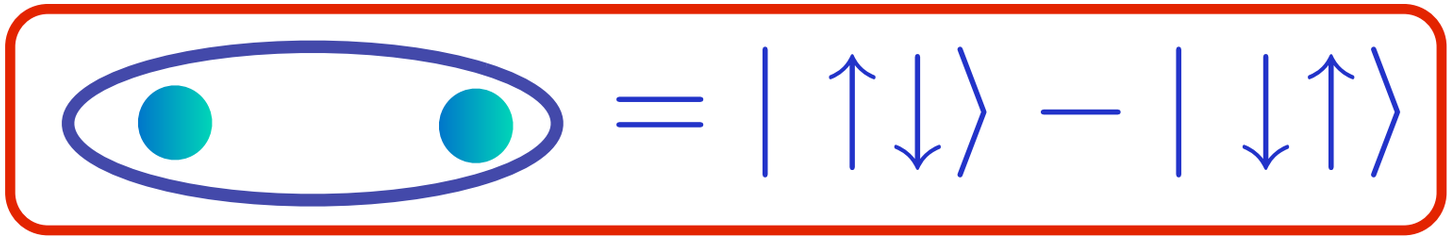
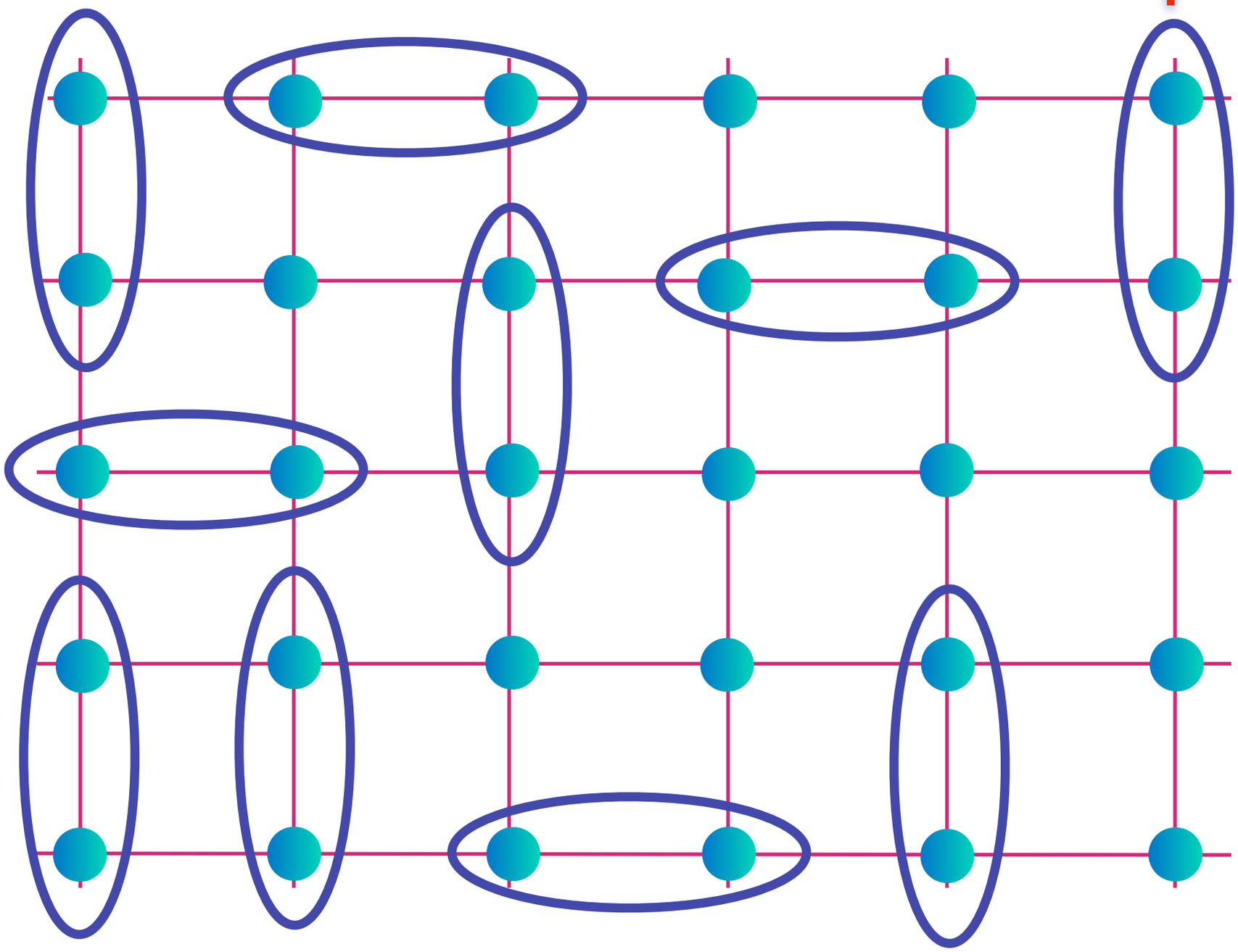


Cooper pairs form quantum superpositions at different locations: "Bose-Einstein condensation" in which all pairs are "everywhere at the same time"



Square lattice of Cu sites

Superconductivity !



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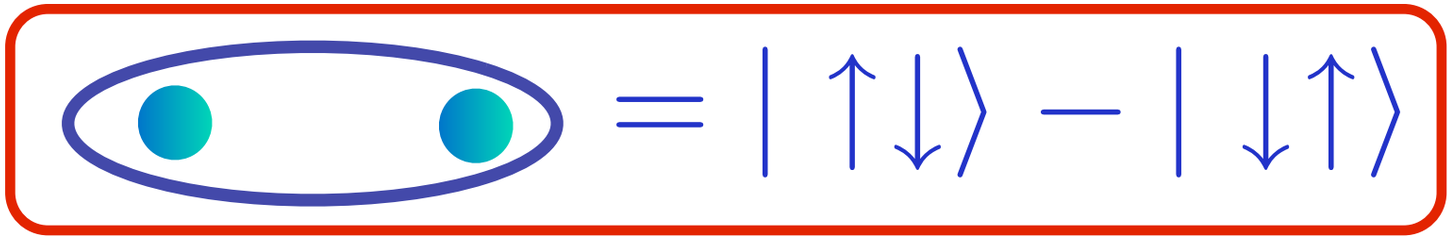

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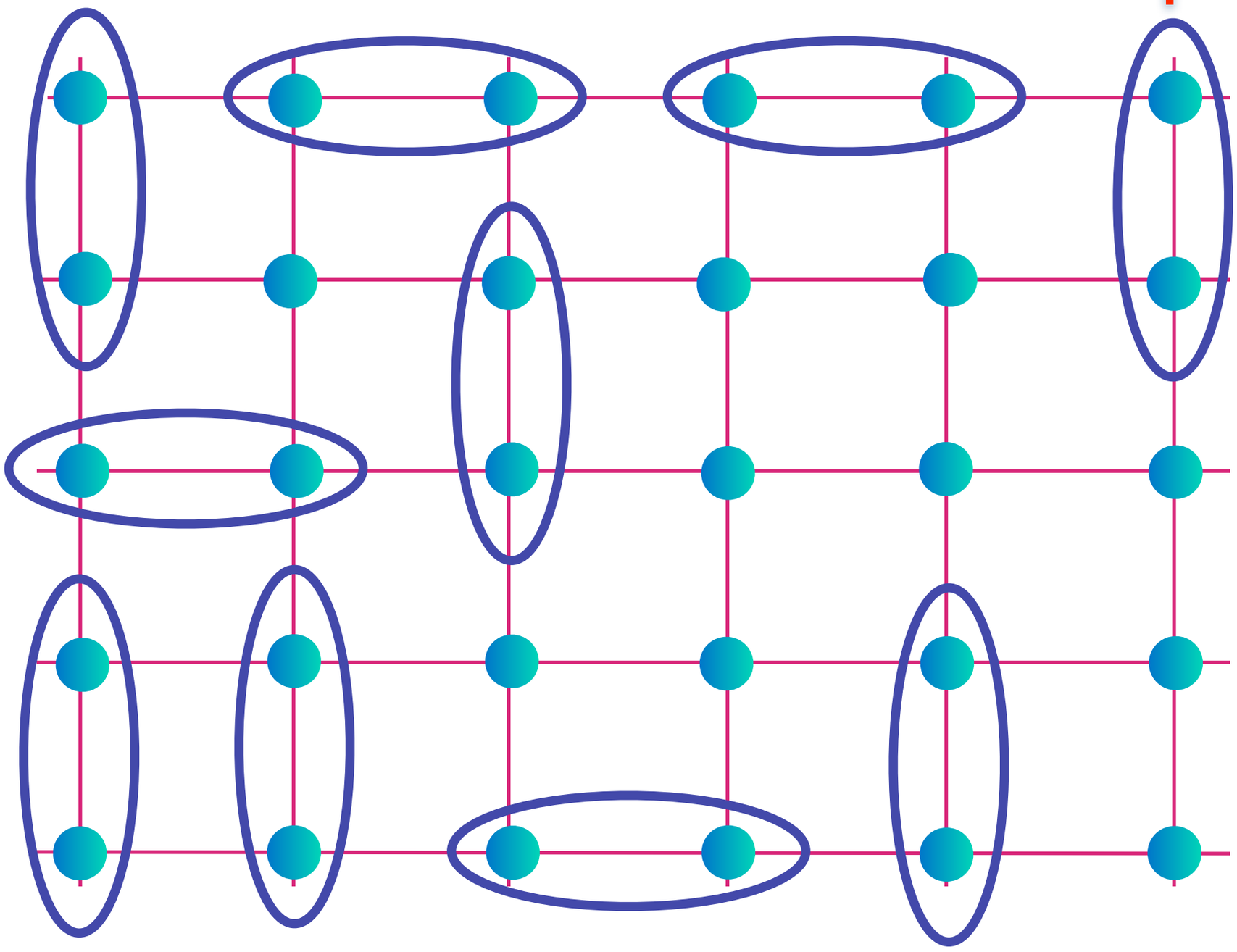

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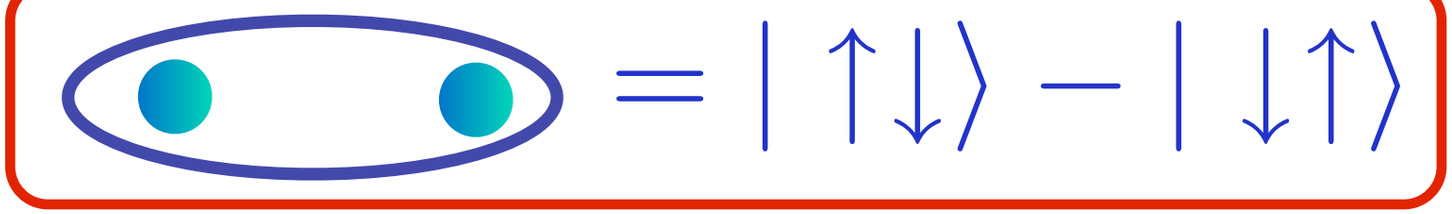

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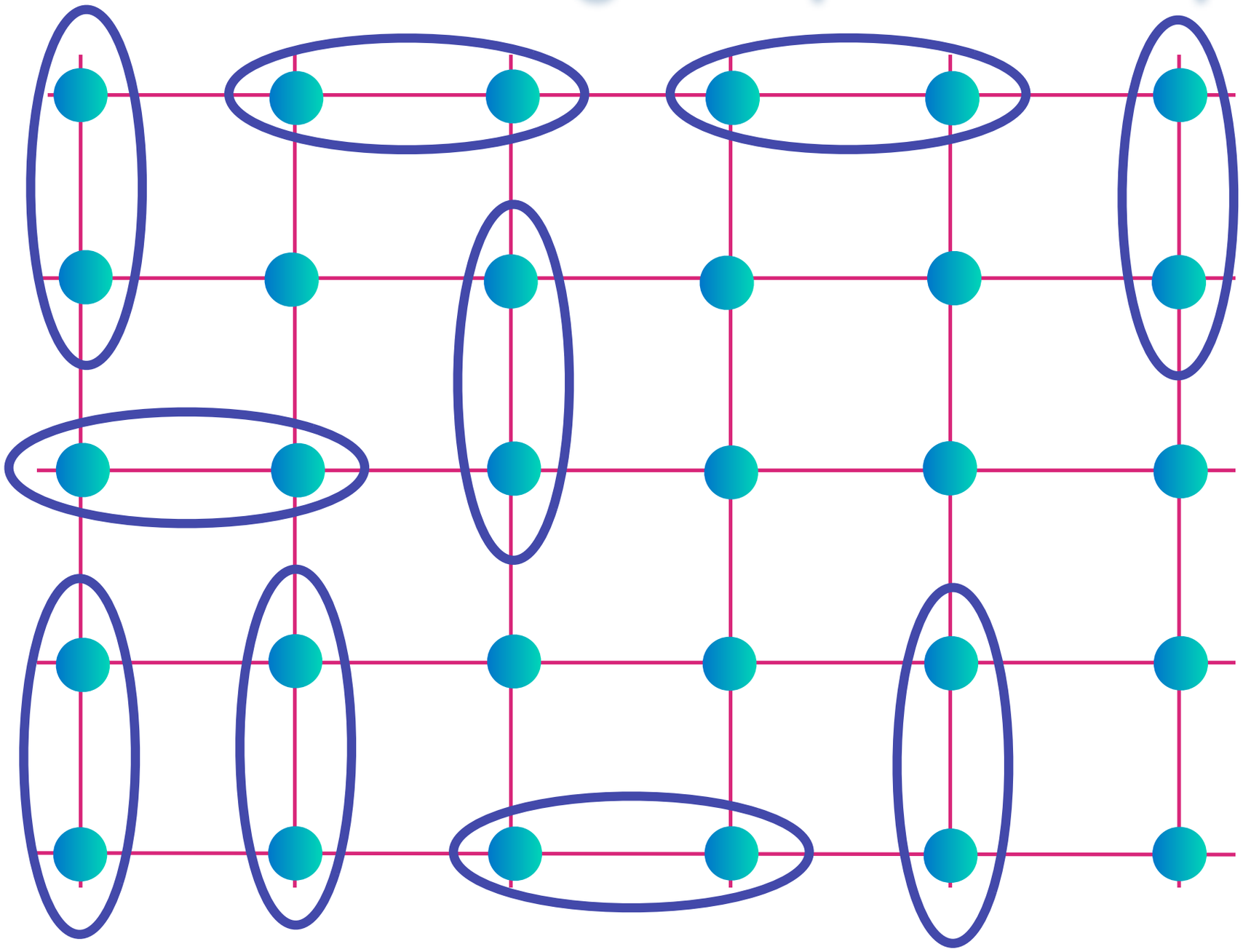
High temperature superconductivity ?



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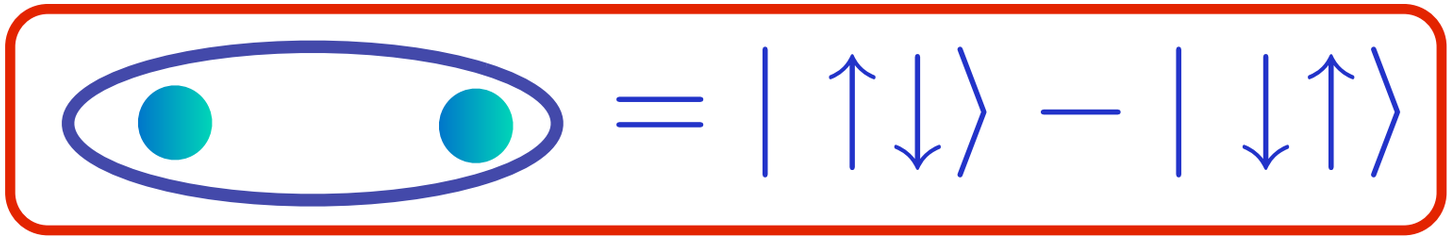

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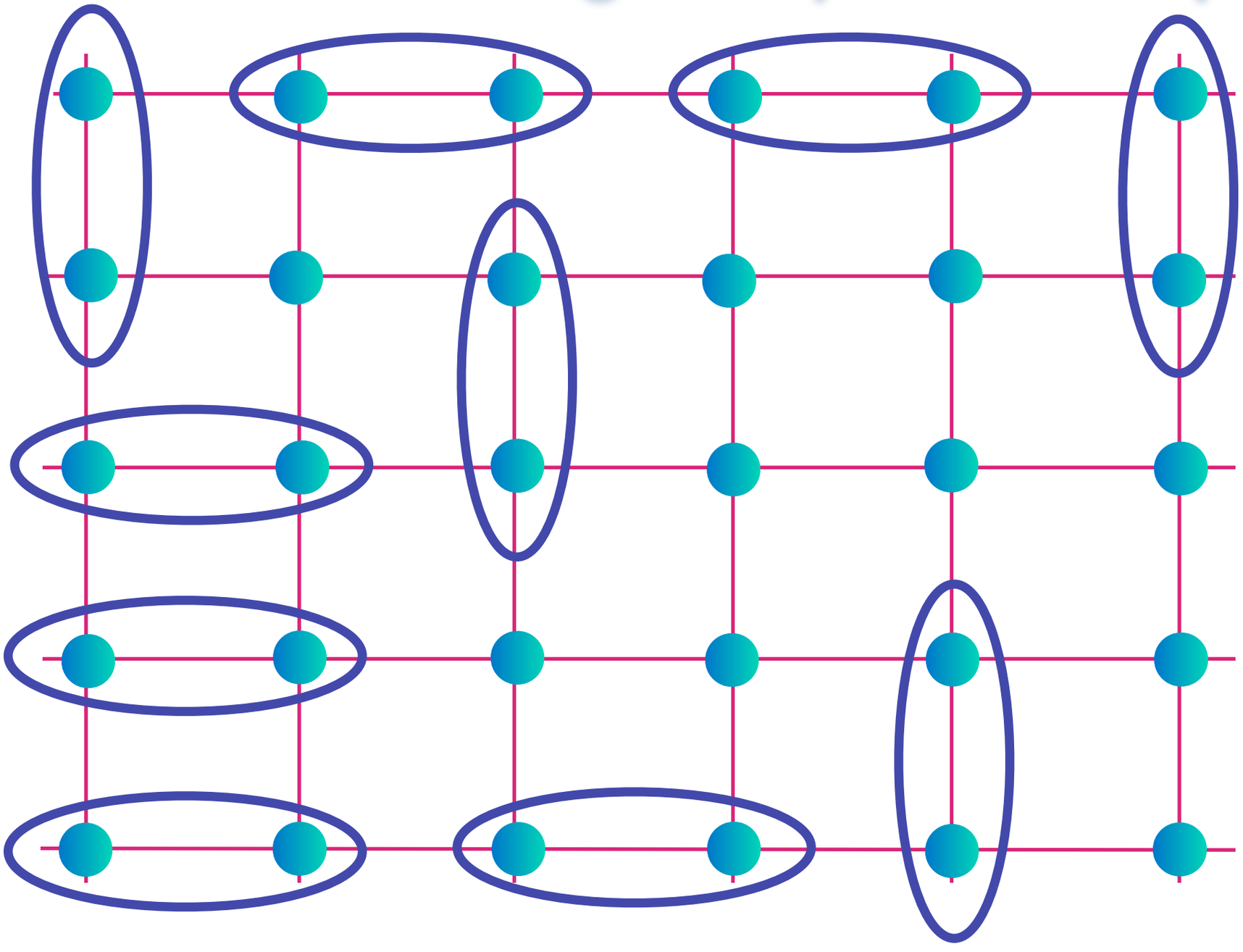
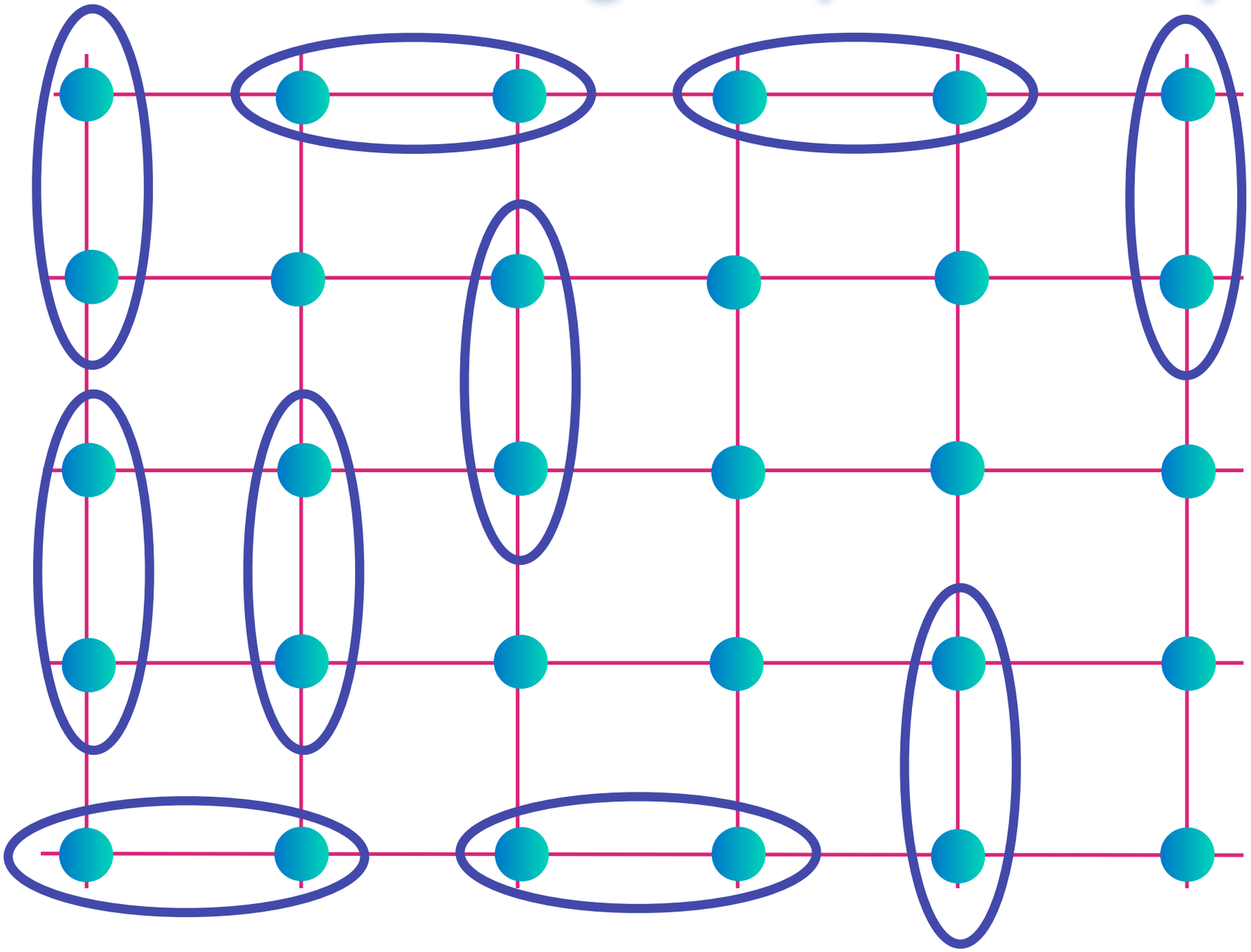


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Electrons <u>entangle</u> by exchanging partners, and there is <u>long-range quantum entanglement</u> near the quantum critical point.

Square lattice of Cu sites

High temperature superconductivity ?



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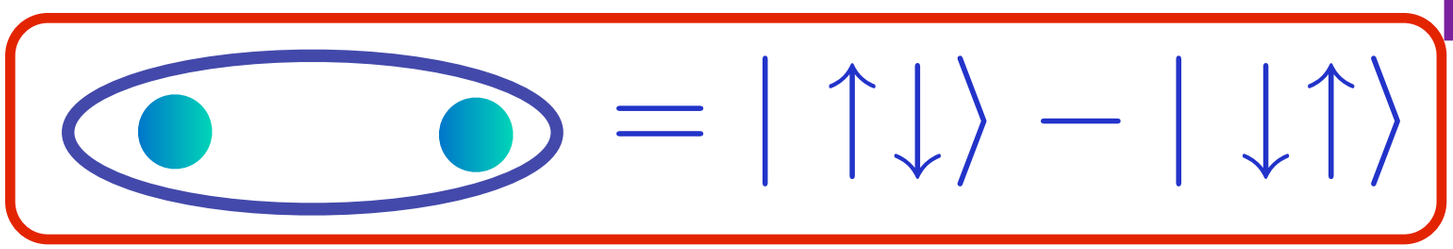

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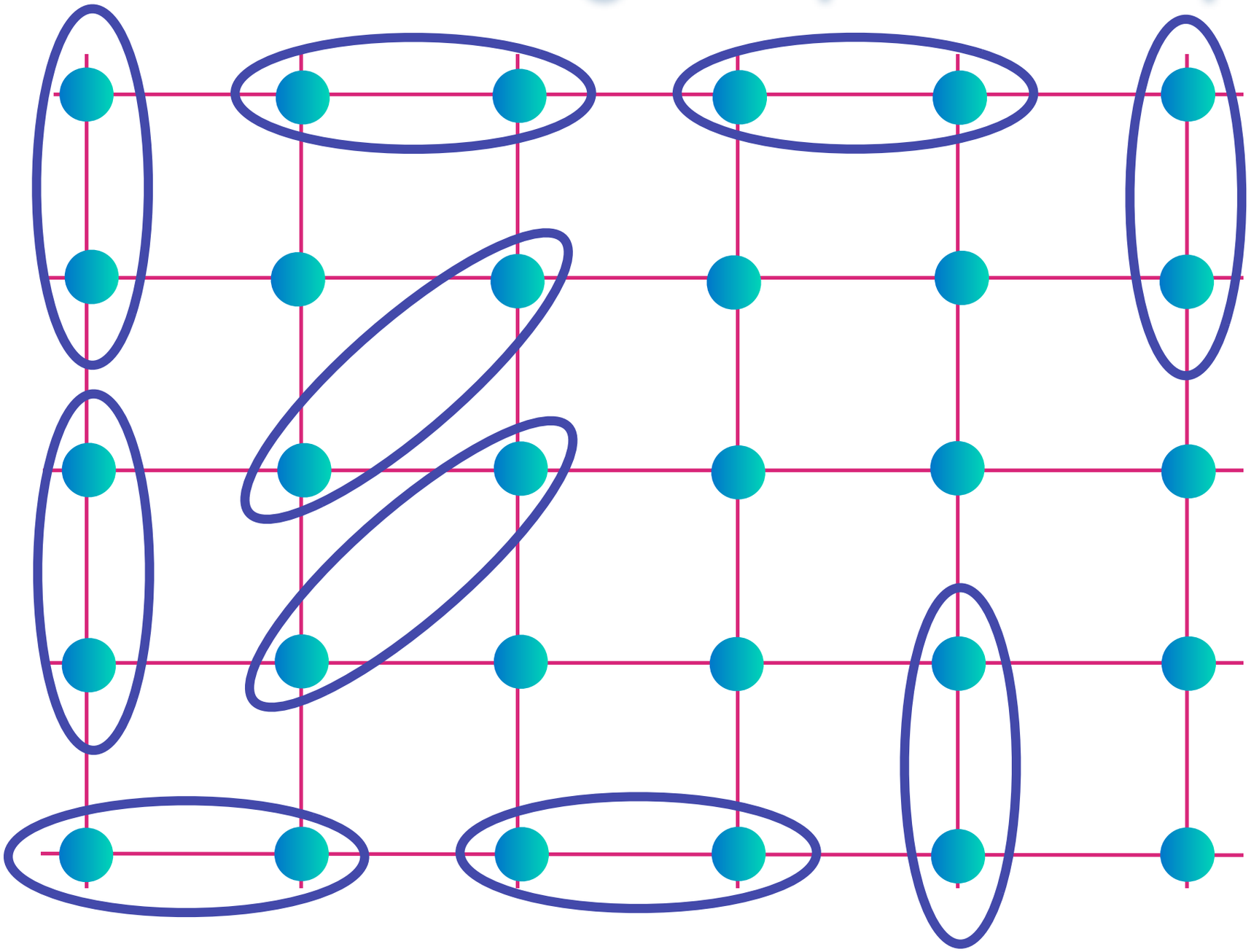

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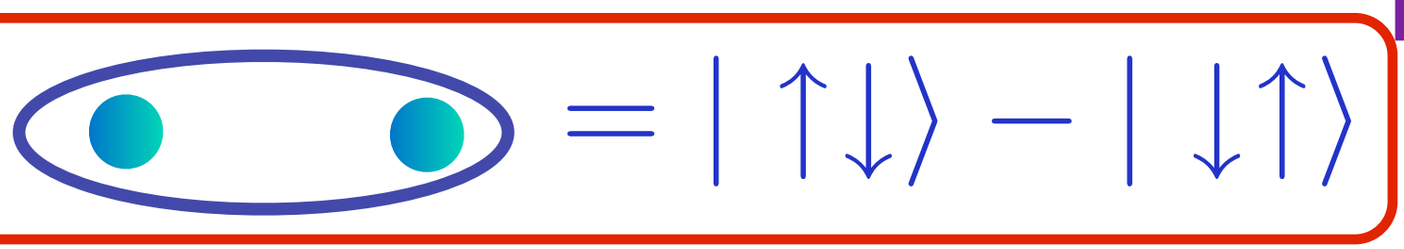

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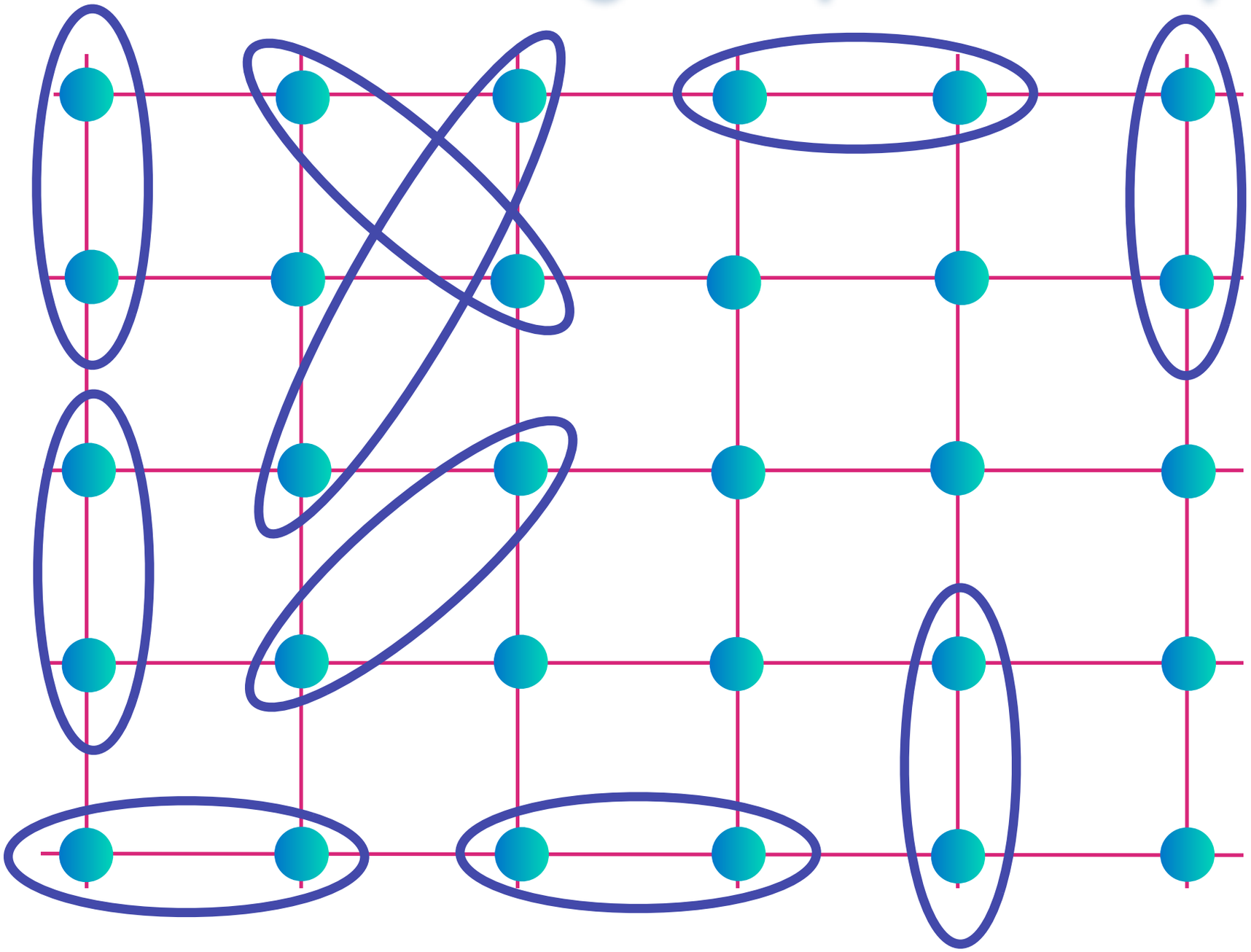

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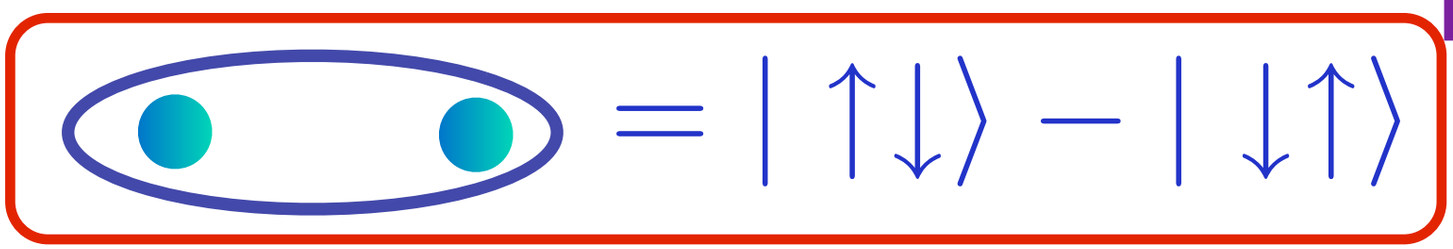

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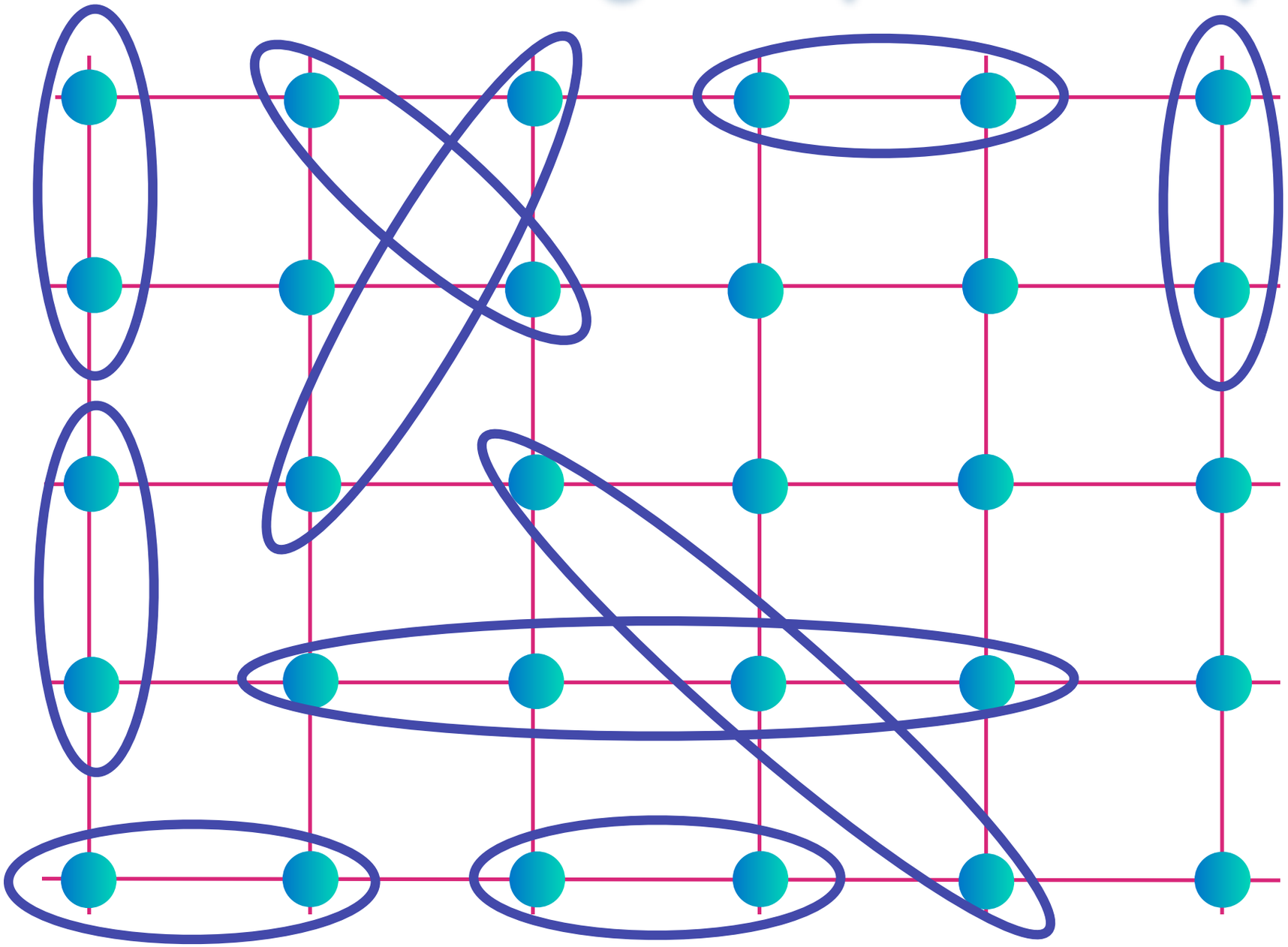

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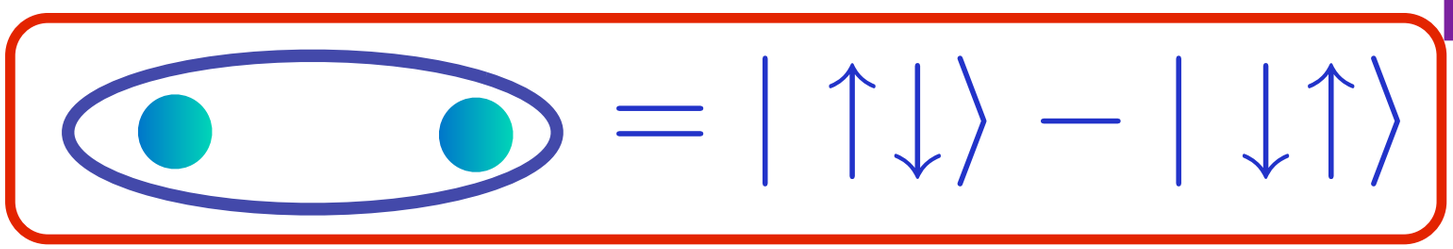

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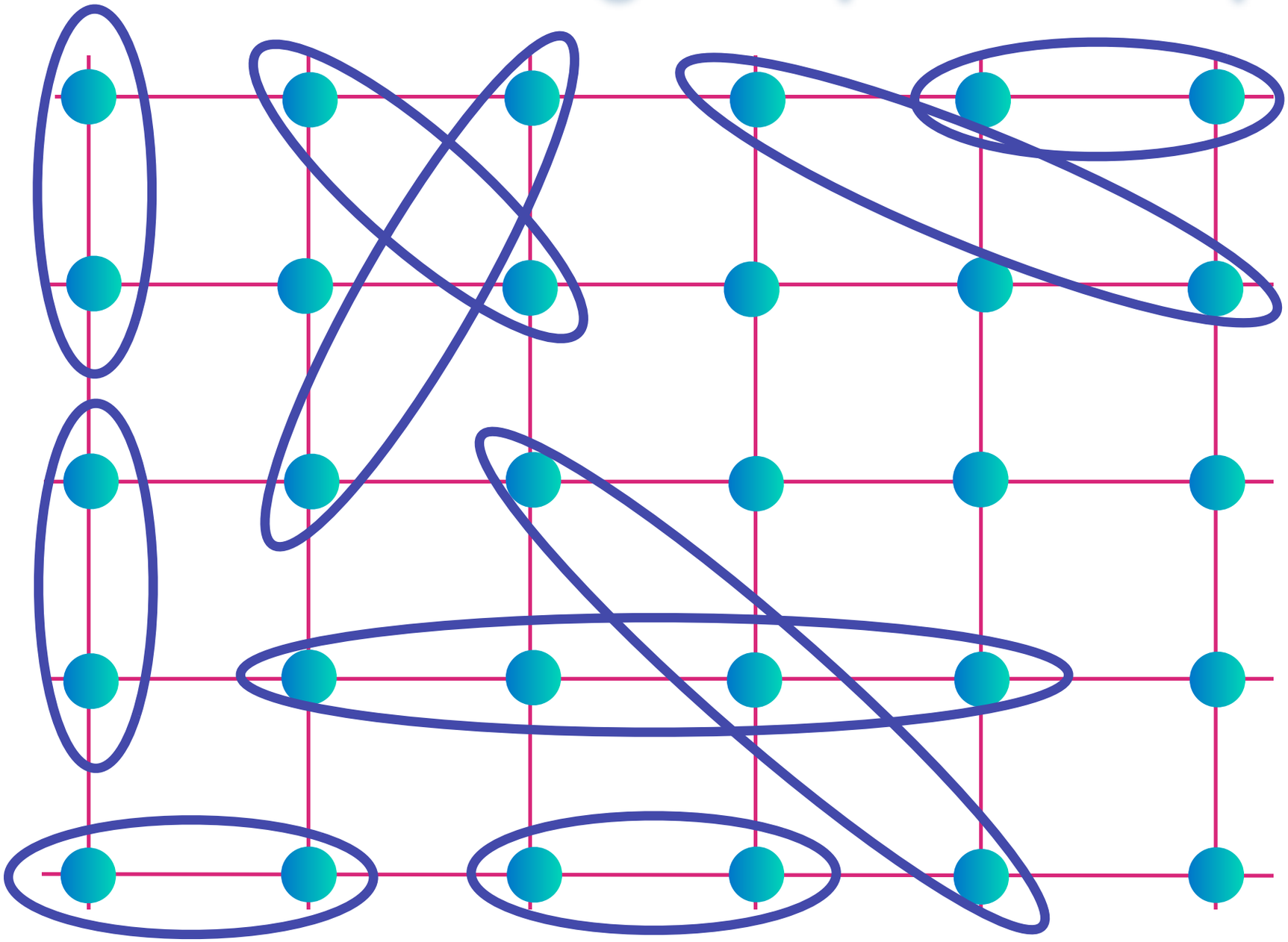


Electrons entangle by exchanging partners, and there is long-range quantum entanglement near the quantum critical point.

$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Antiferromagnet

Strange metal

Superconductor

?

Quantum critical point

T (K)

120

80

40

0

0.04

0.08

0.12

0.16

p (hole/Cu)

Cu^{2+},Cu^{3+}

O^{2-}

Y^{3+}

Ba^{2+}

c

b

a

11.6802 Å

3.8872 Å

3.8227 Å

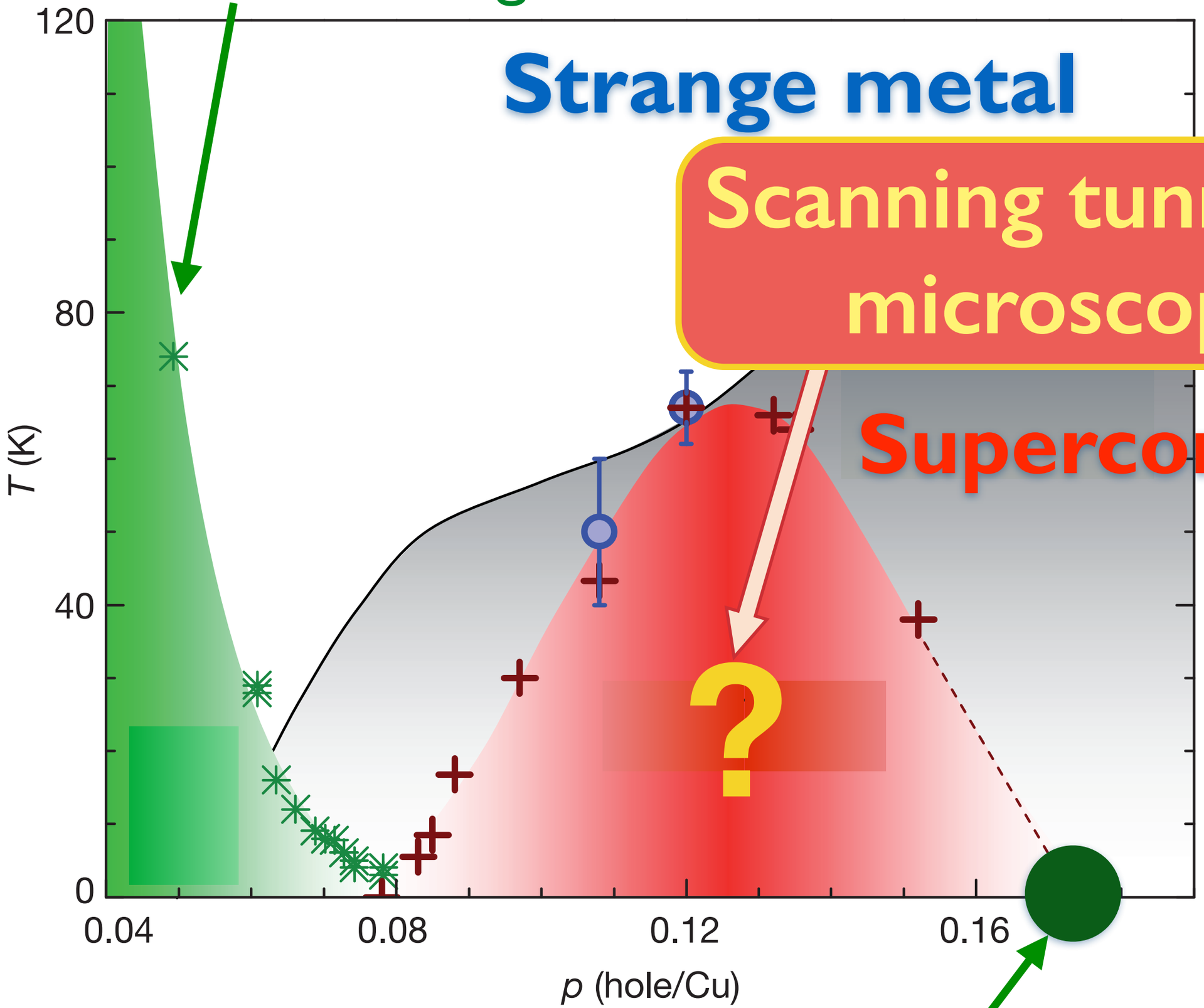
Antiferromagnet

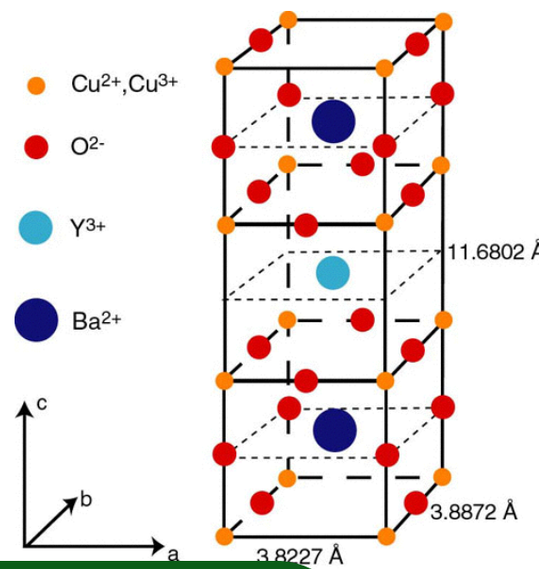
Strange metal

Scanning tunneling microscopy

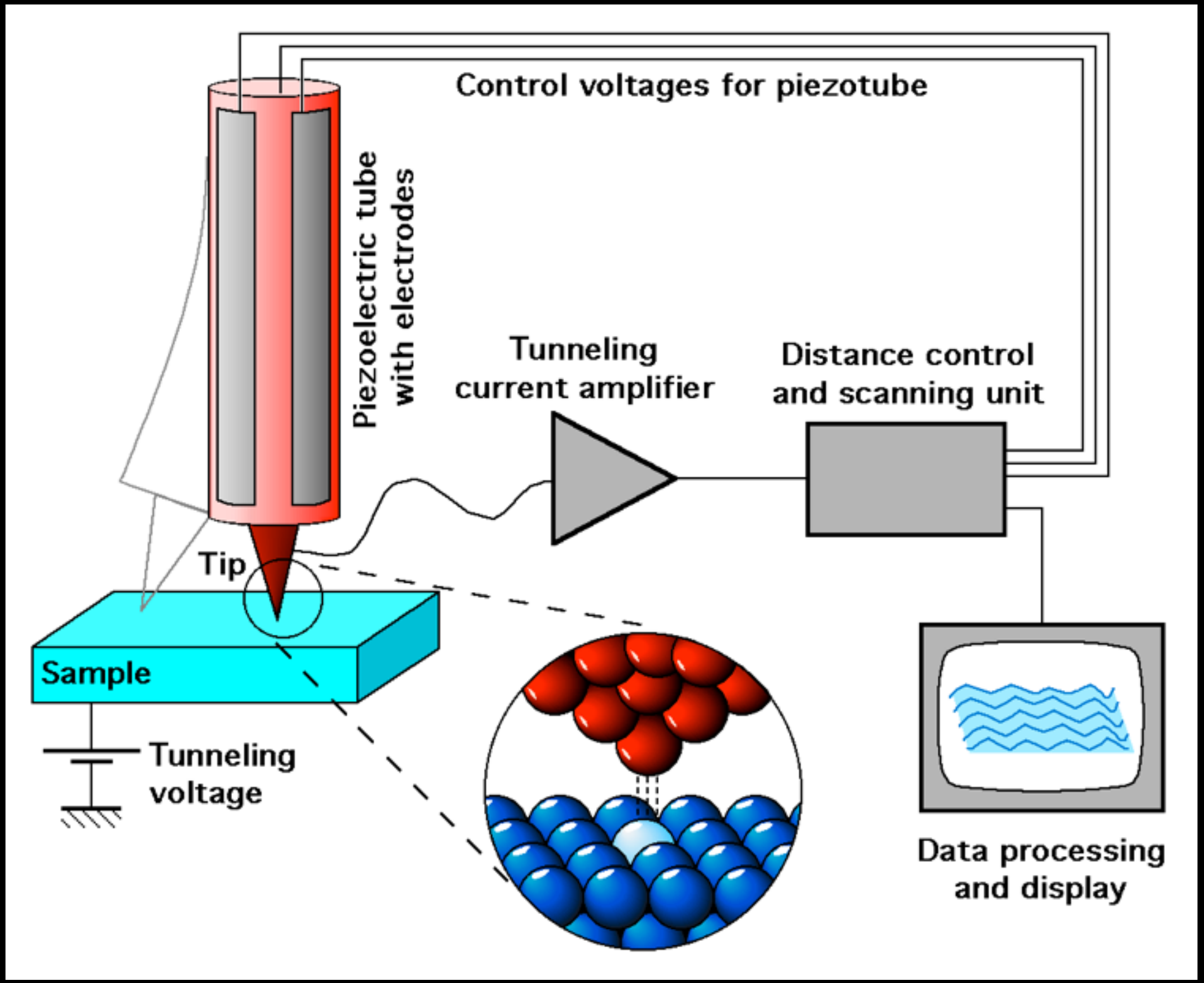
Superconductor

Quantum critical point



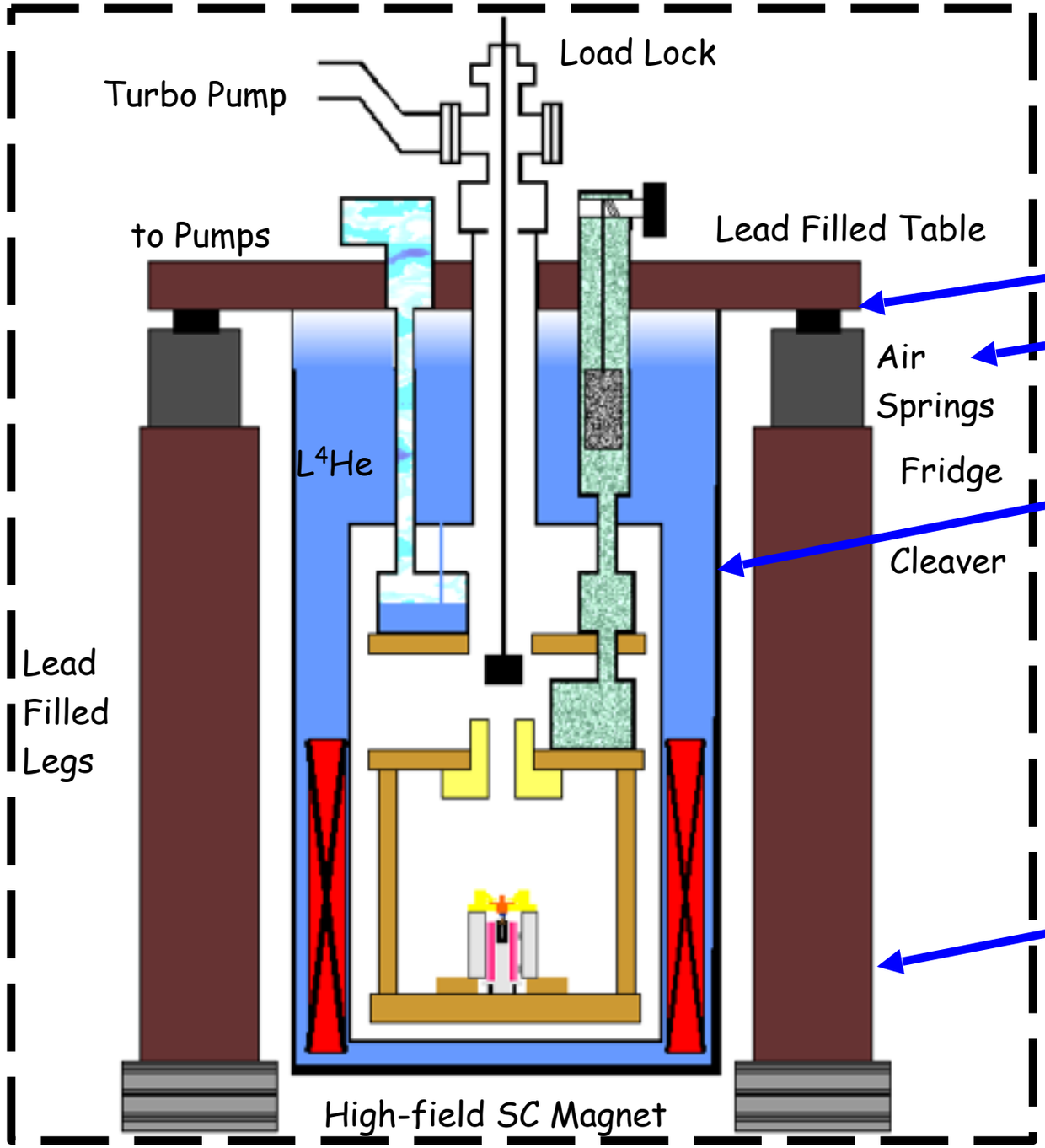


Scanning Tunneling Microscopy



SI-STM System

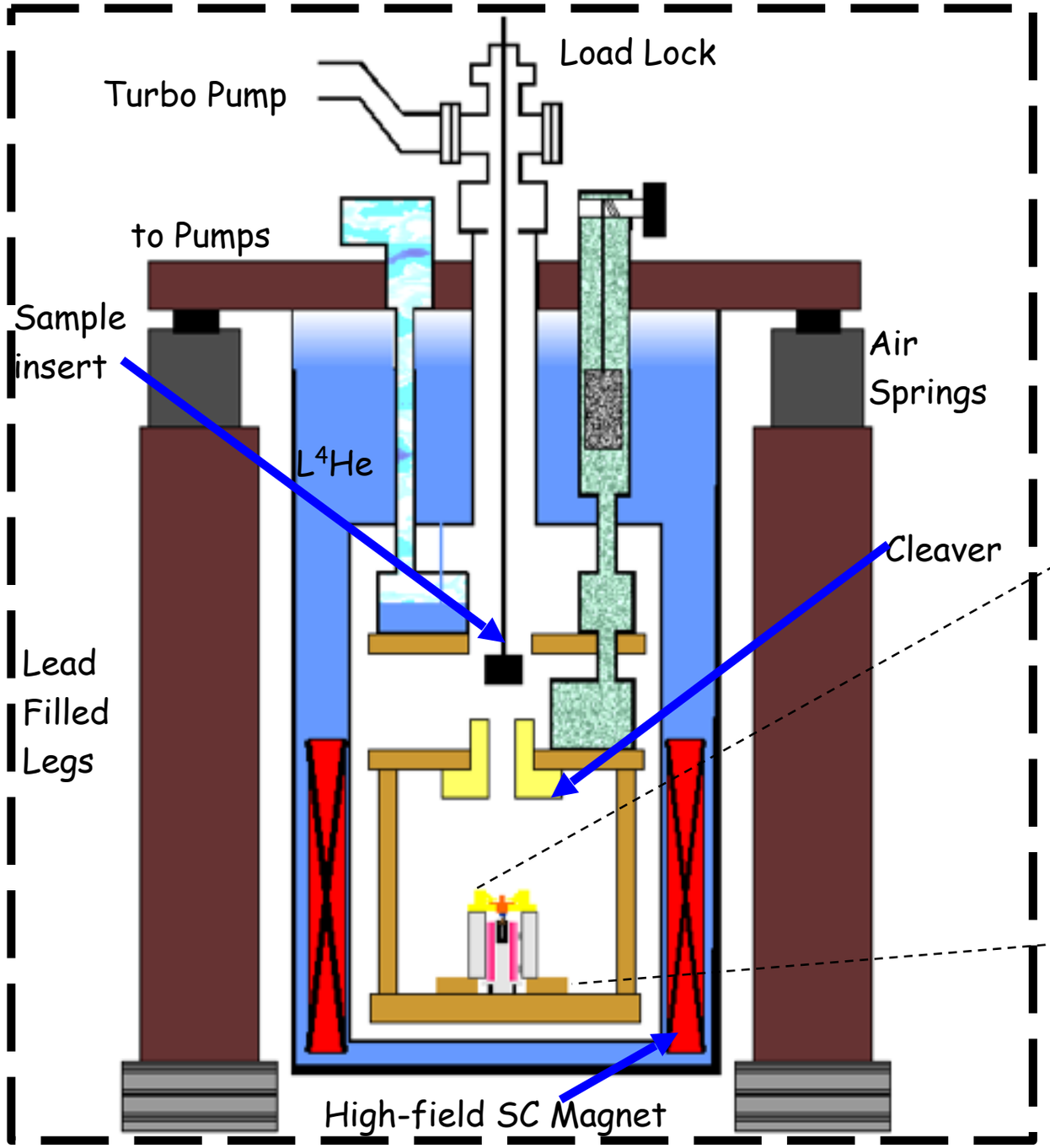
J. C. Davis group, Rev. Sci. Inst. 70, 1459 (1999).



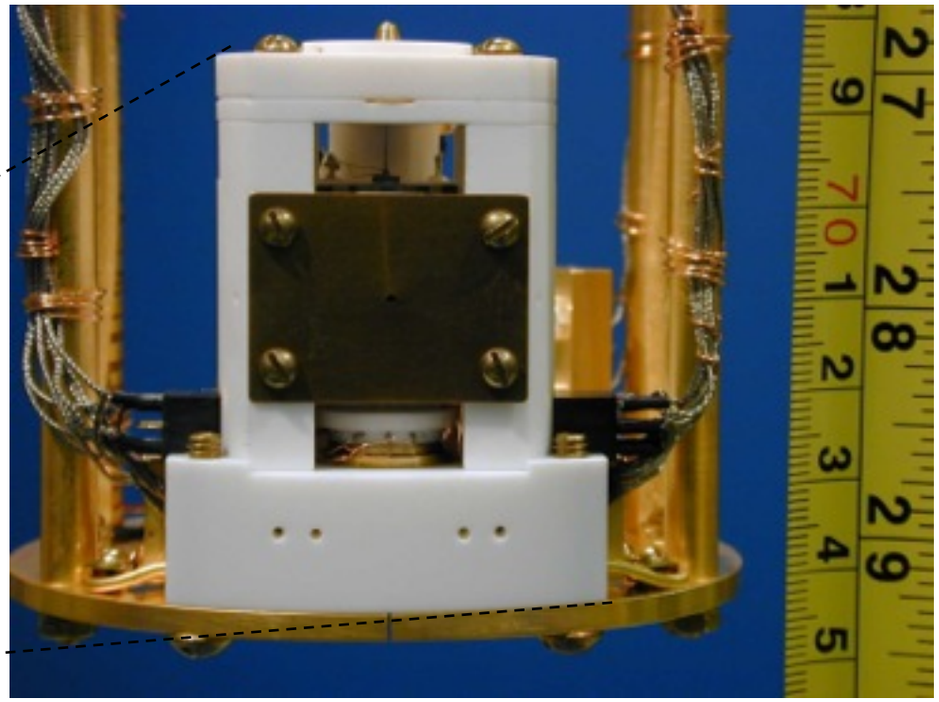
Ultra low vibration cryostat.

SI-STM System

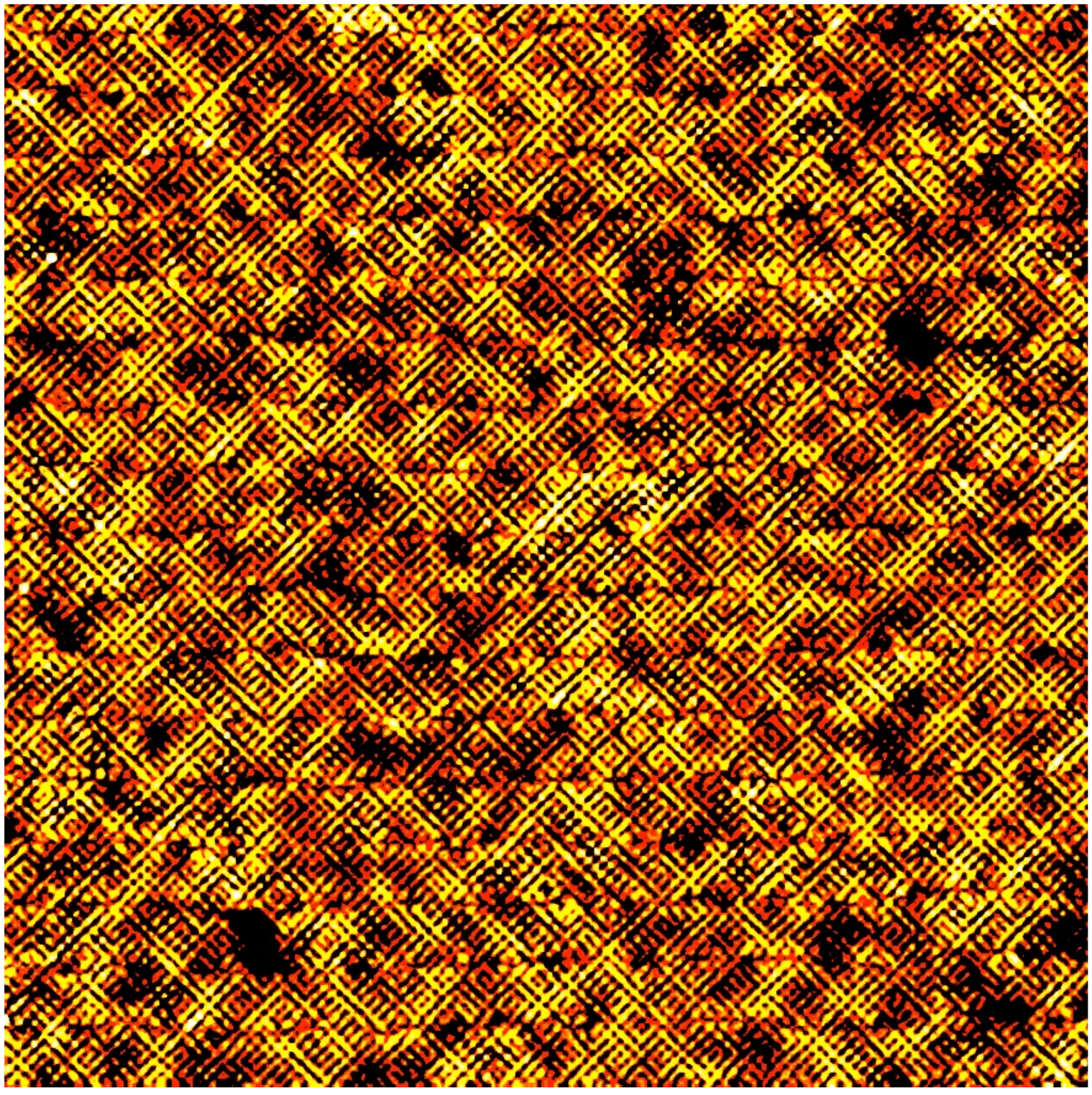
J. C. Davis group, Rev. Sci. Inst. 70, 1459 (1999).



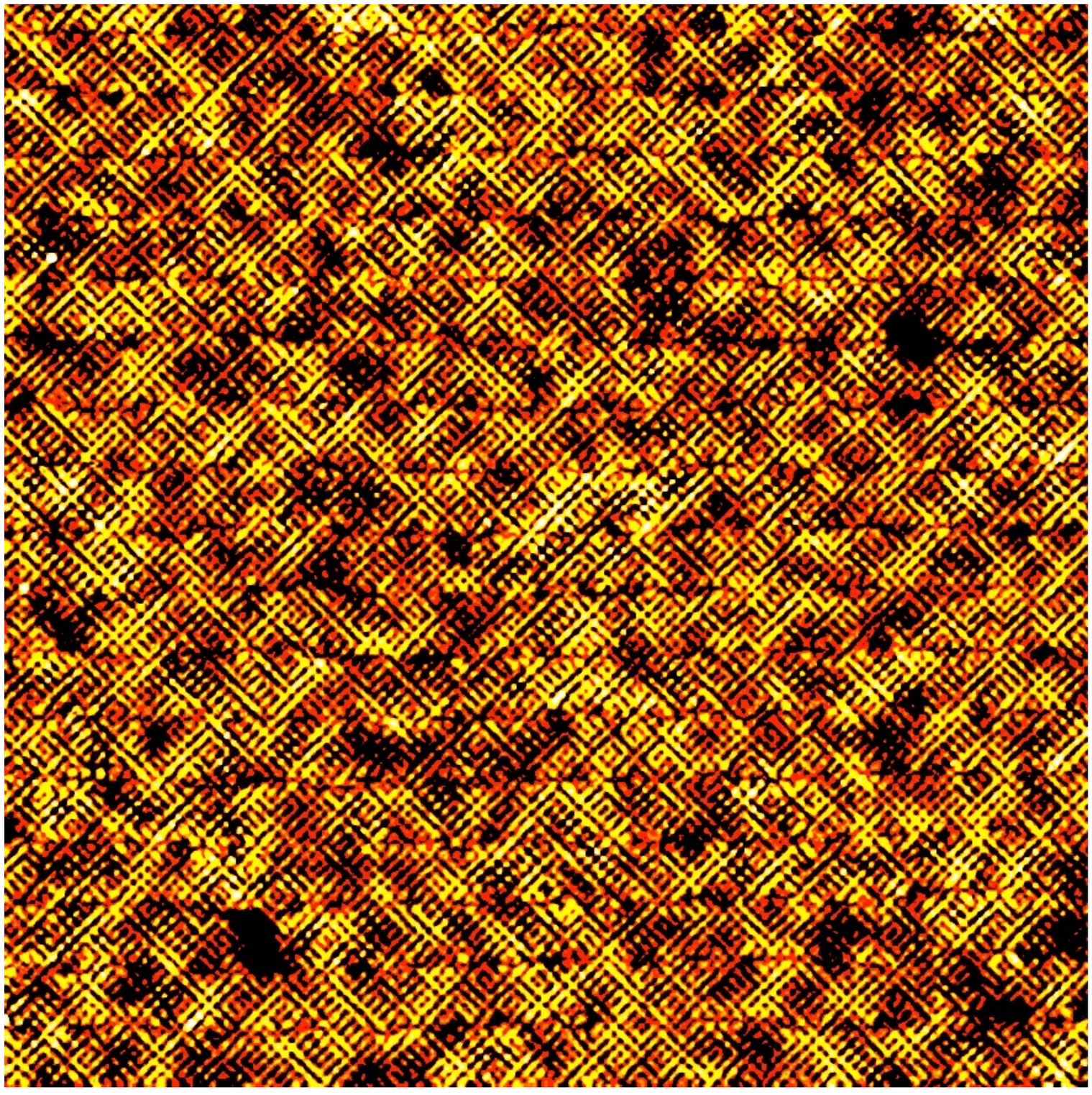
- Massive ULV Cryostat
- Subkelvin Fridge
- STM + High Field Magnet
- Sample Exchange from RT
- Cryogenic UHV Cleave



STM Head



Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007).



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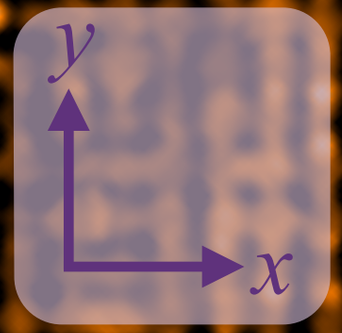
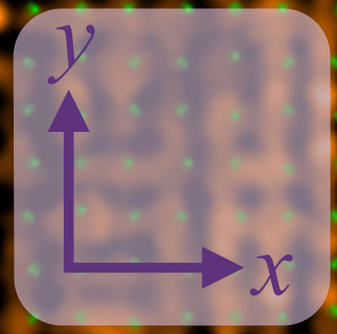
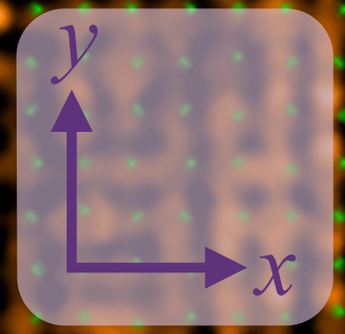

y
x

Cu sites



Intricrate pattern of tunneling currents contains signatures of long-range quantum entanglement !!(?)



Quantum superposition and entanglement

Quantum critical points and long-range entanglement of electrons in crystals

String theory and black holes

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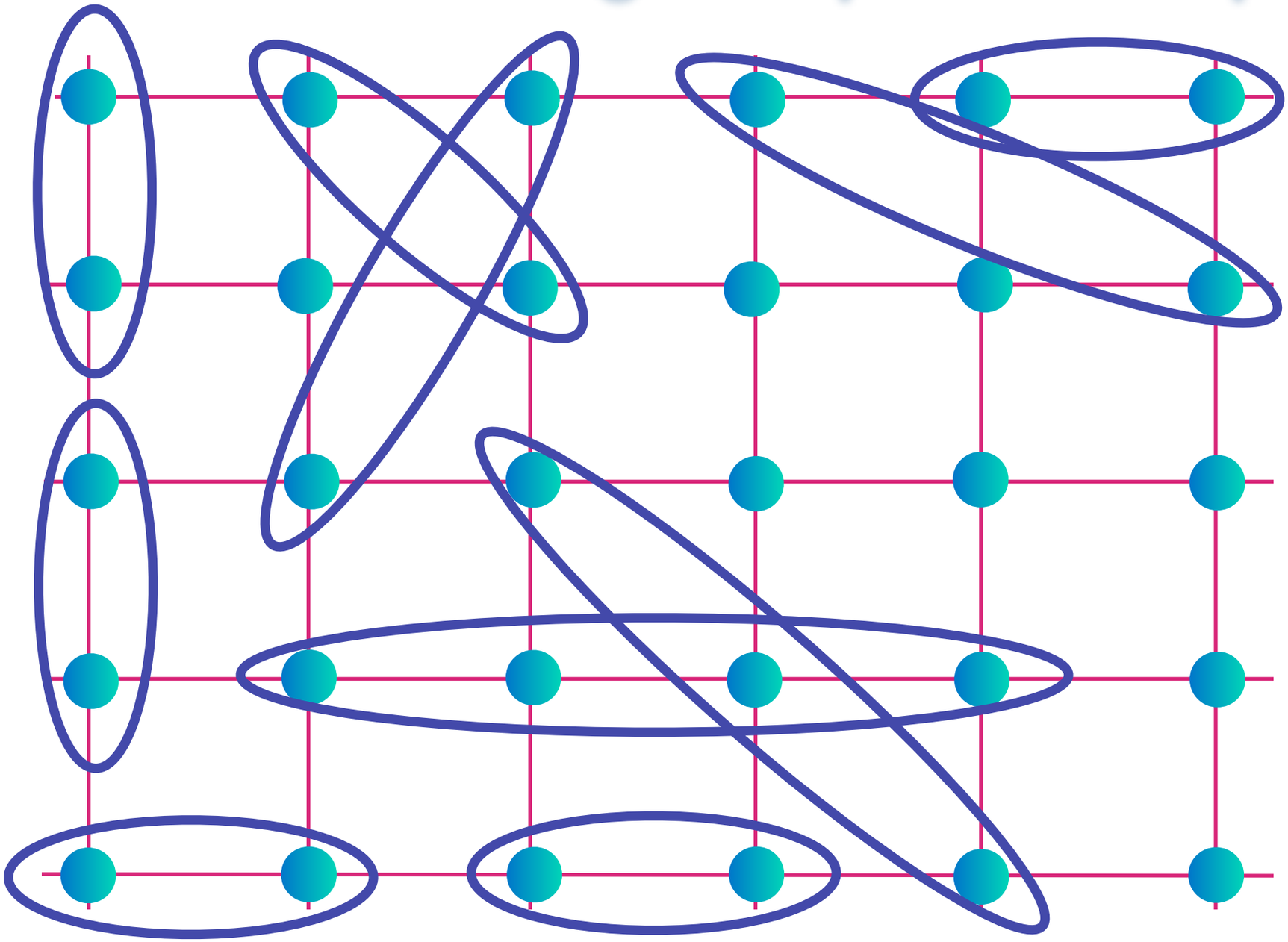
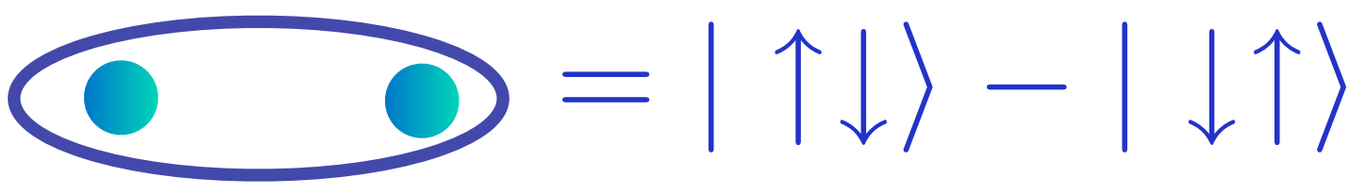
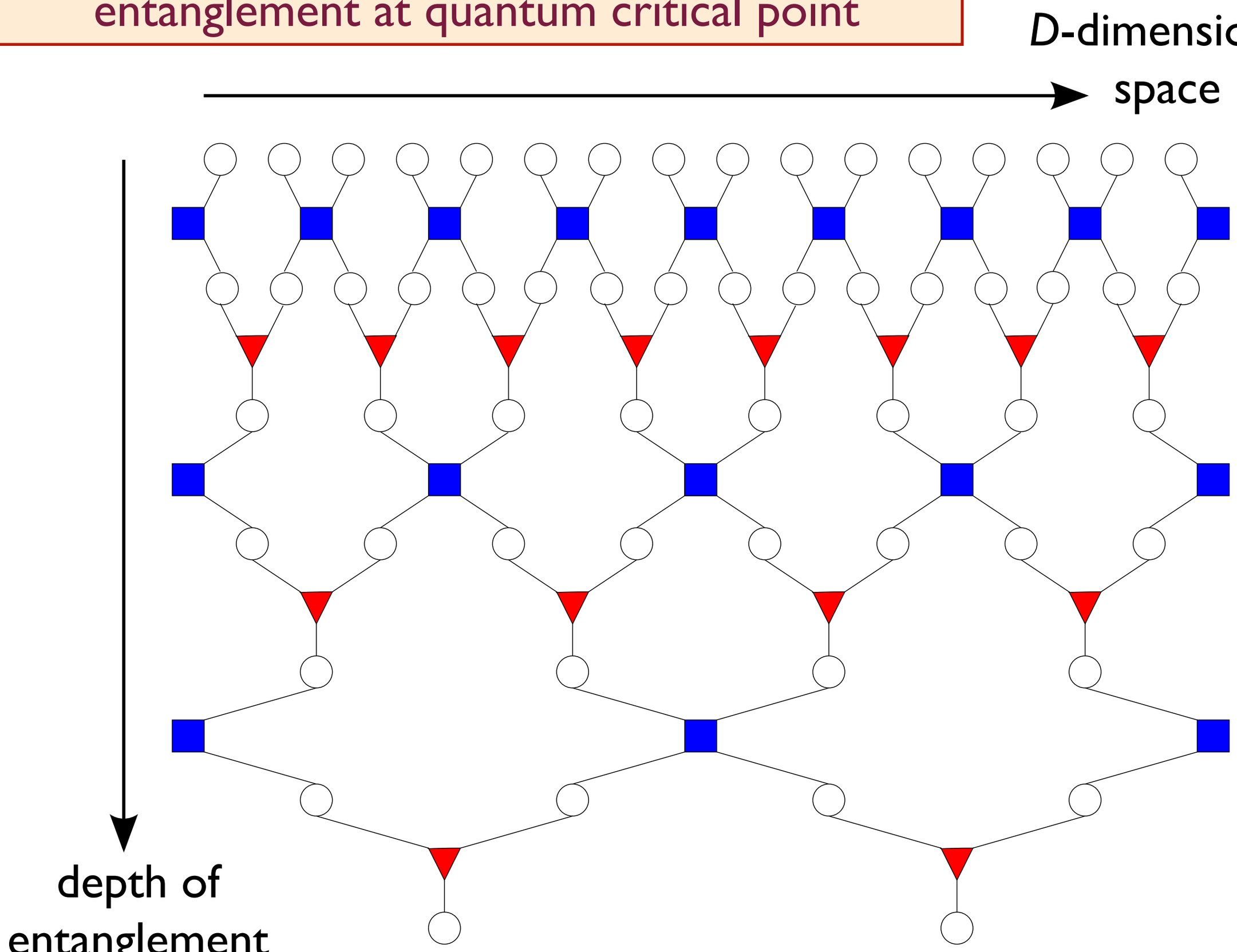




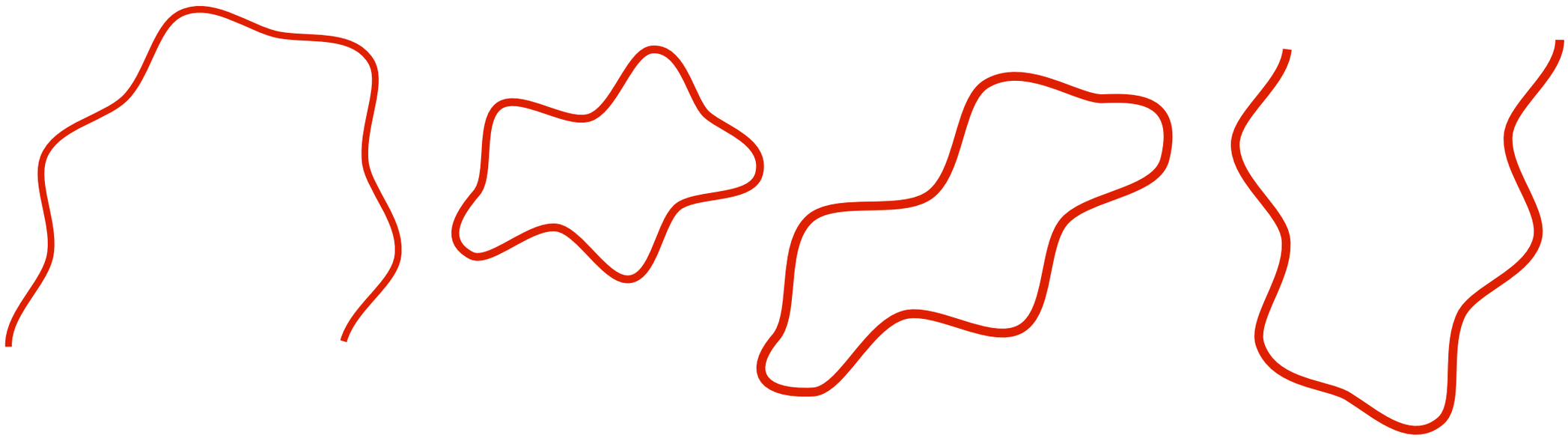
Long-range entanglement has a heierarchical structure: electrons entangle in pairs, pairs entangle with pairs, and so on……

Tensor network representation of hierarchical entanglement at quantum critical point

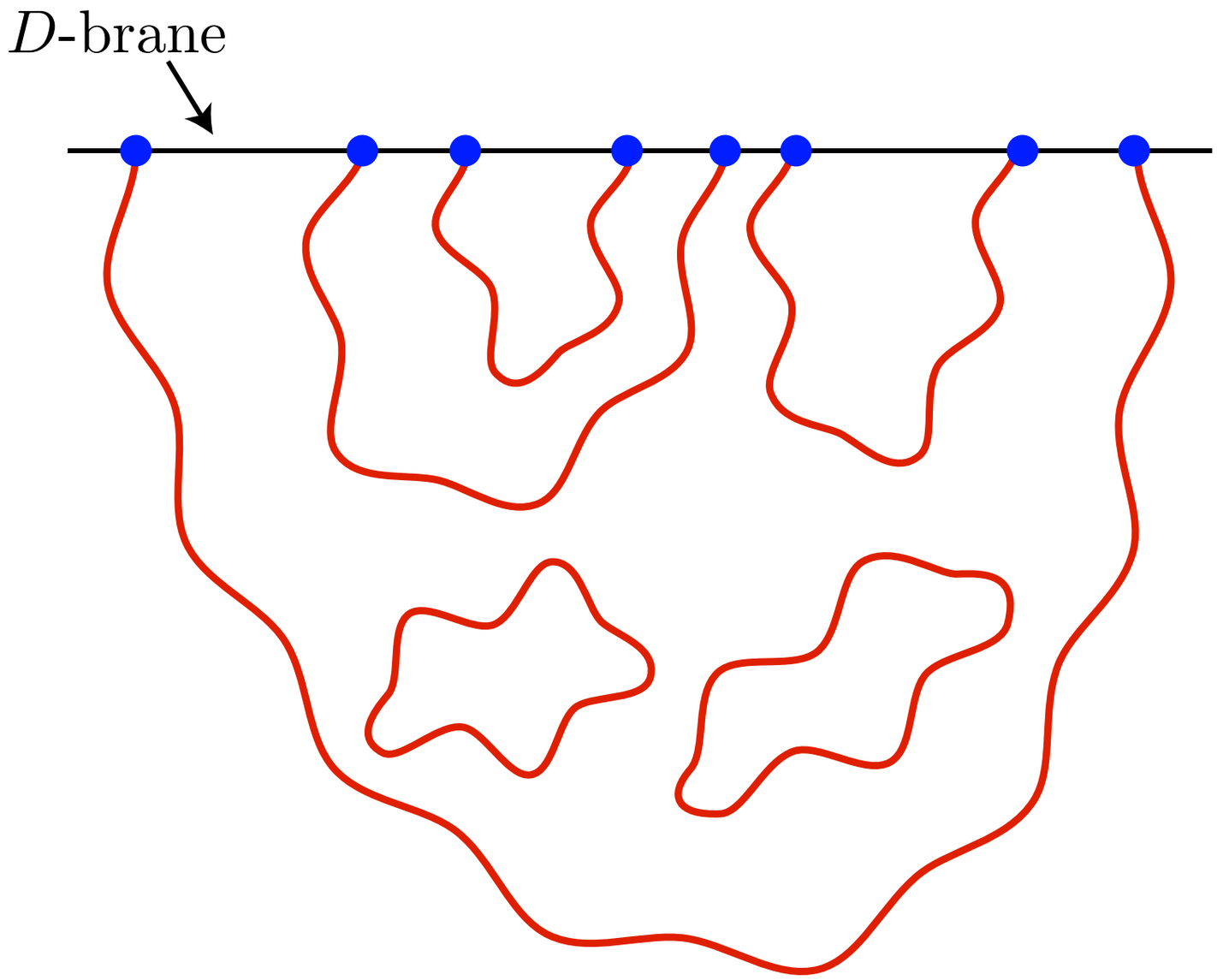


G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)

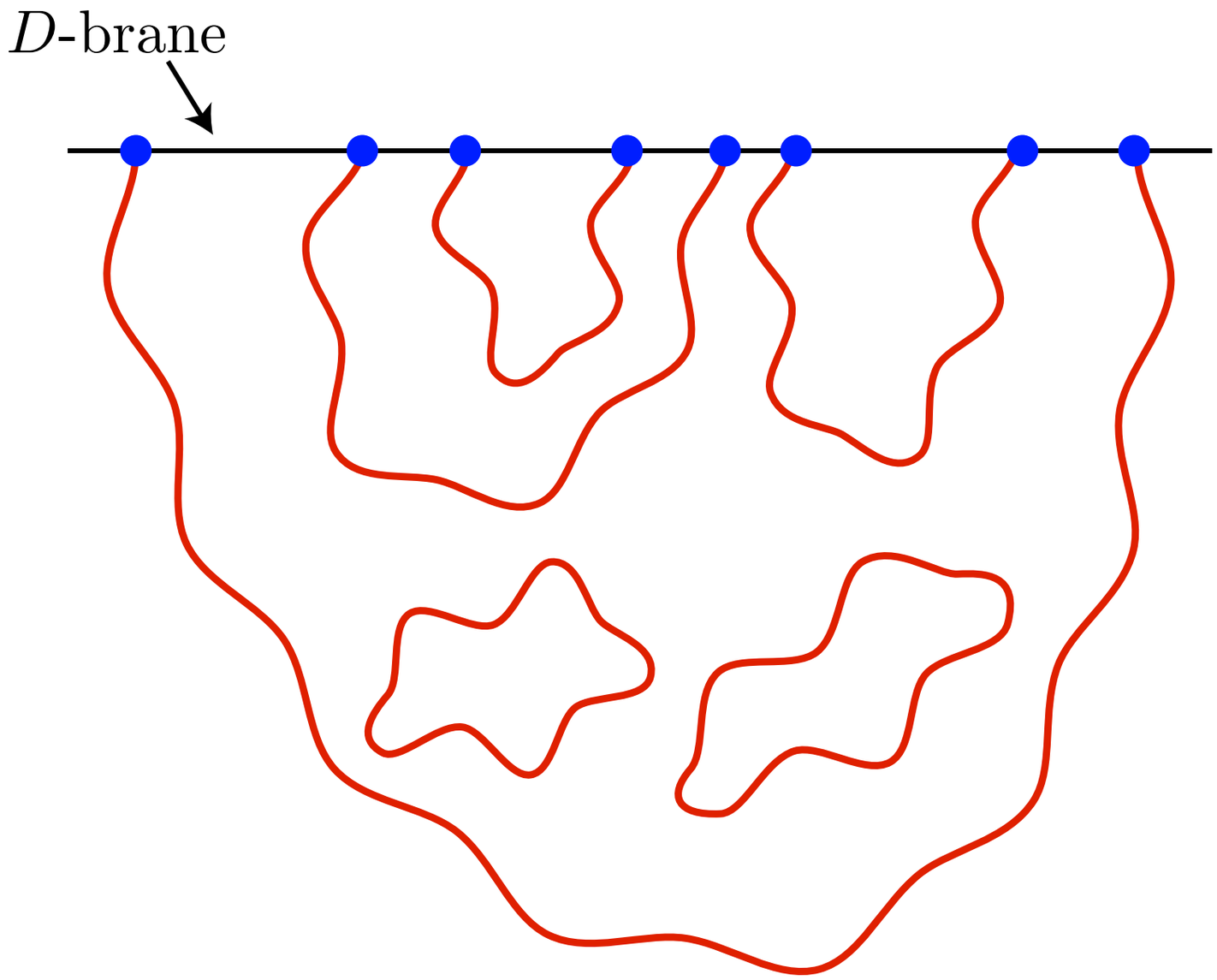
String theory



- Allows unification of the standard model of particle physics with Einstein's theory of gravitation (general relativity).
- Vibrations of a string (its "musical notes") correspond to quarks, gravitons, the Higgs boson, photons, gluons .....



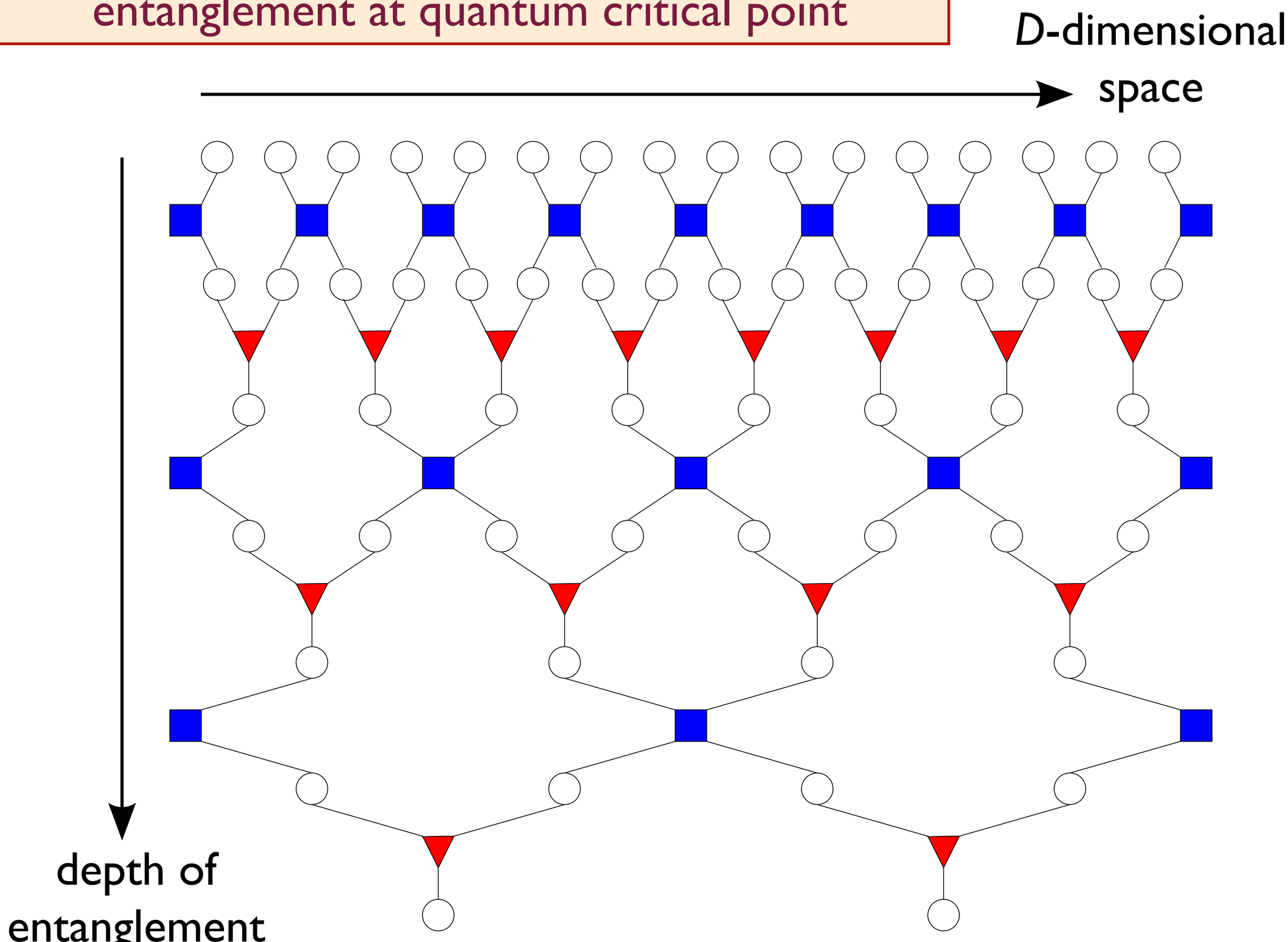
- A D-brane is a D-dimensional surface on which strings can end.



- A D-brane is a D-dimensional surface on which strings can end.

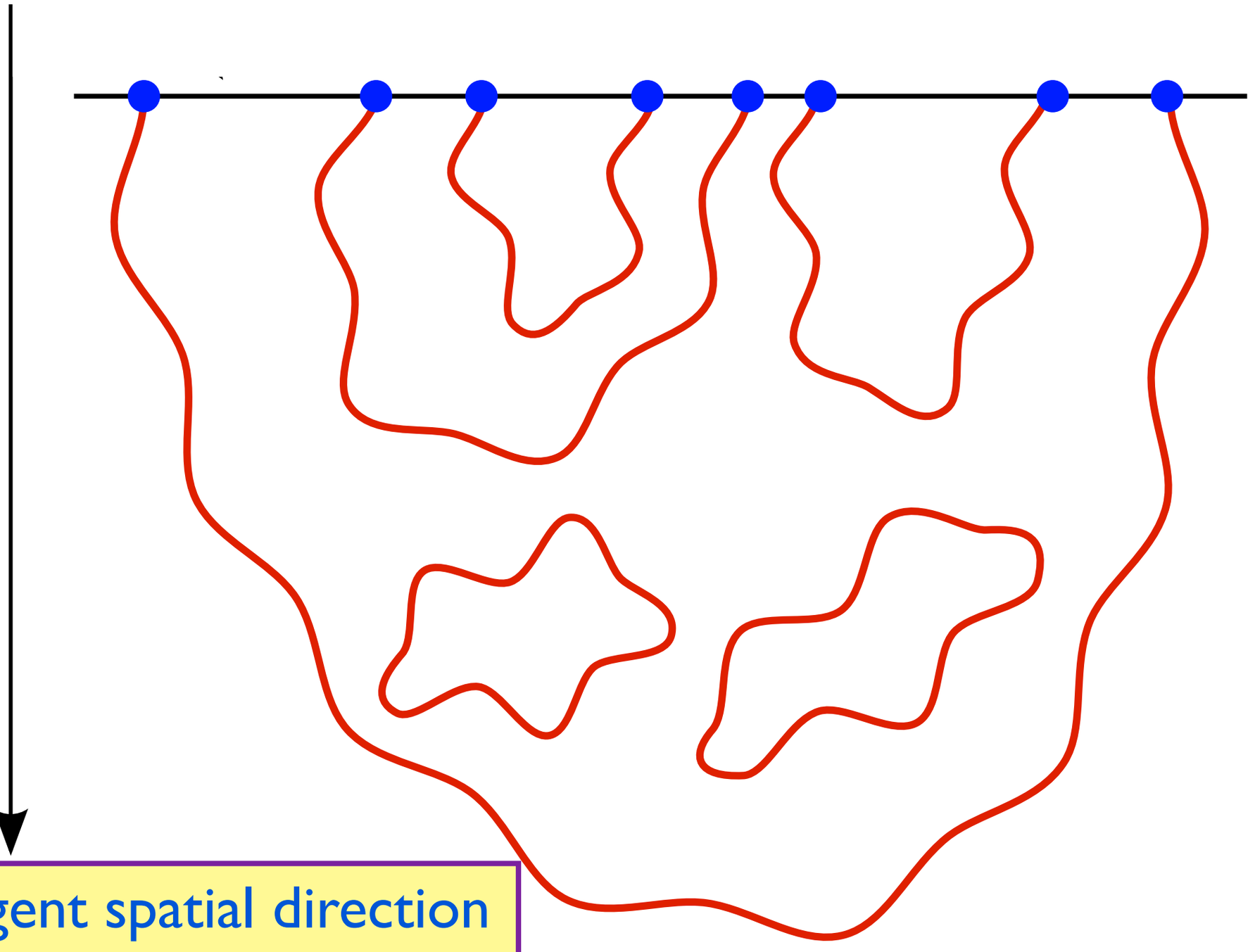
- If we focused only on the blue points on the D-dimensional surface, they would appear to us to have long-range quantum entanglement !

Tensor network representation of hierarchical entanglement at quantum critical point



String theory near
a D-brane

D-dimensional
space

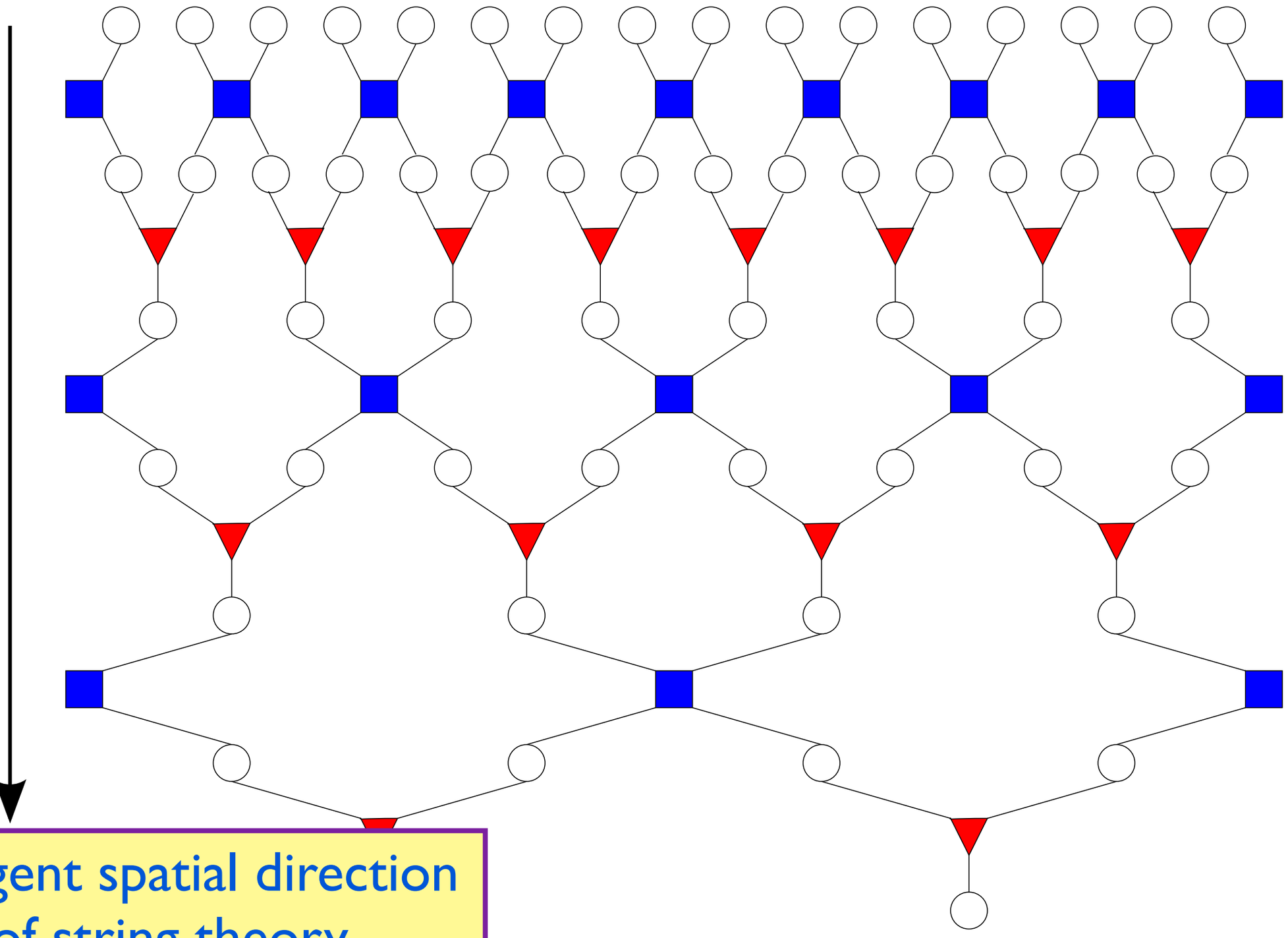


Emergent spatial direction
of string theory

Brian Swingle, arXiv:0905.1317

Tensor network representation of hierarchical entanglement at quantum critical point

D-dimensional space



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States of matter with
long-range quantum entanglement
in D dimensions

String theory and
Einstein's General Relativity
in D+1 dimensions

States of matter with
long-range quantum entanglement
in D dimensions

Are there solutions of Einstein's General Relativity in D+1 dimensions which correspond to superconductors and "strange metals" ?

String theory and
Einstein's General Relativity
in D+1 dimensions

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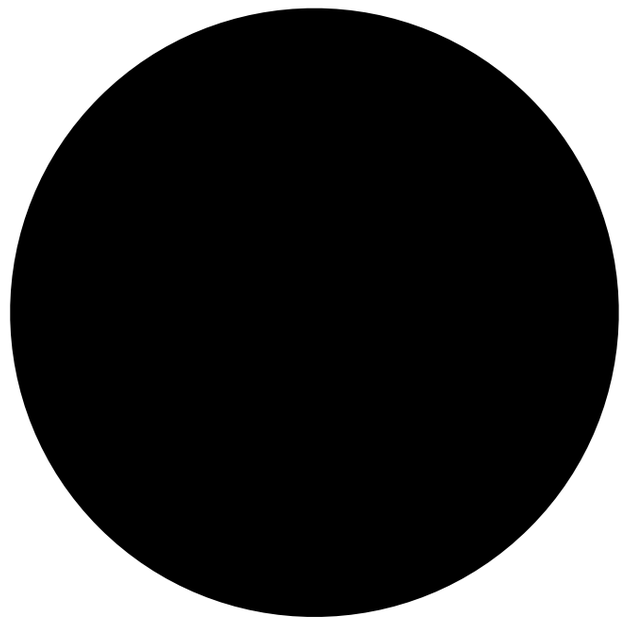
Black Holes

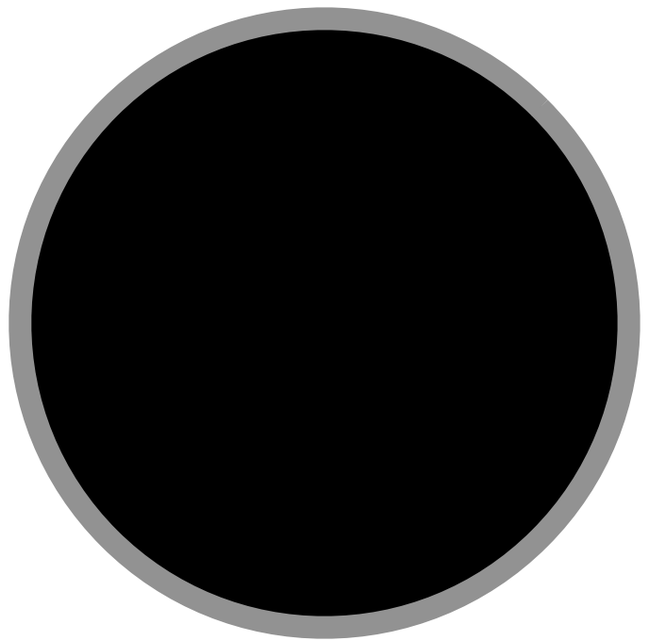
Objects so massive that light is gravitationally bound to them.



Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.

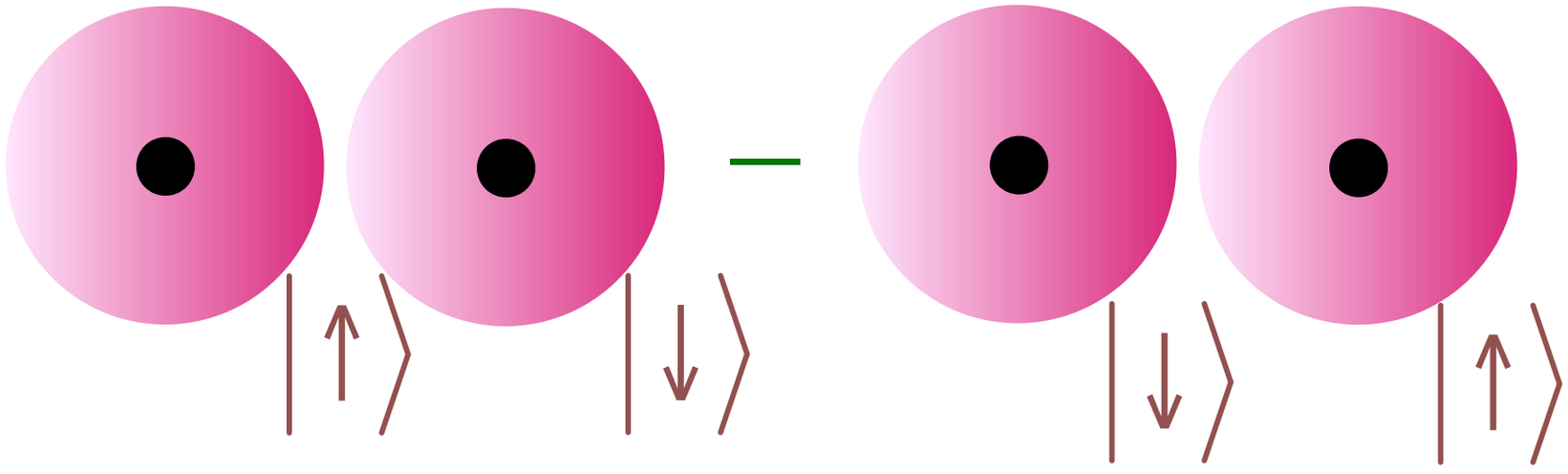


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

Quantum Entanglement across a black hole horizon



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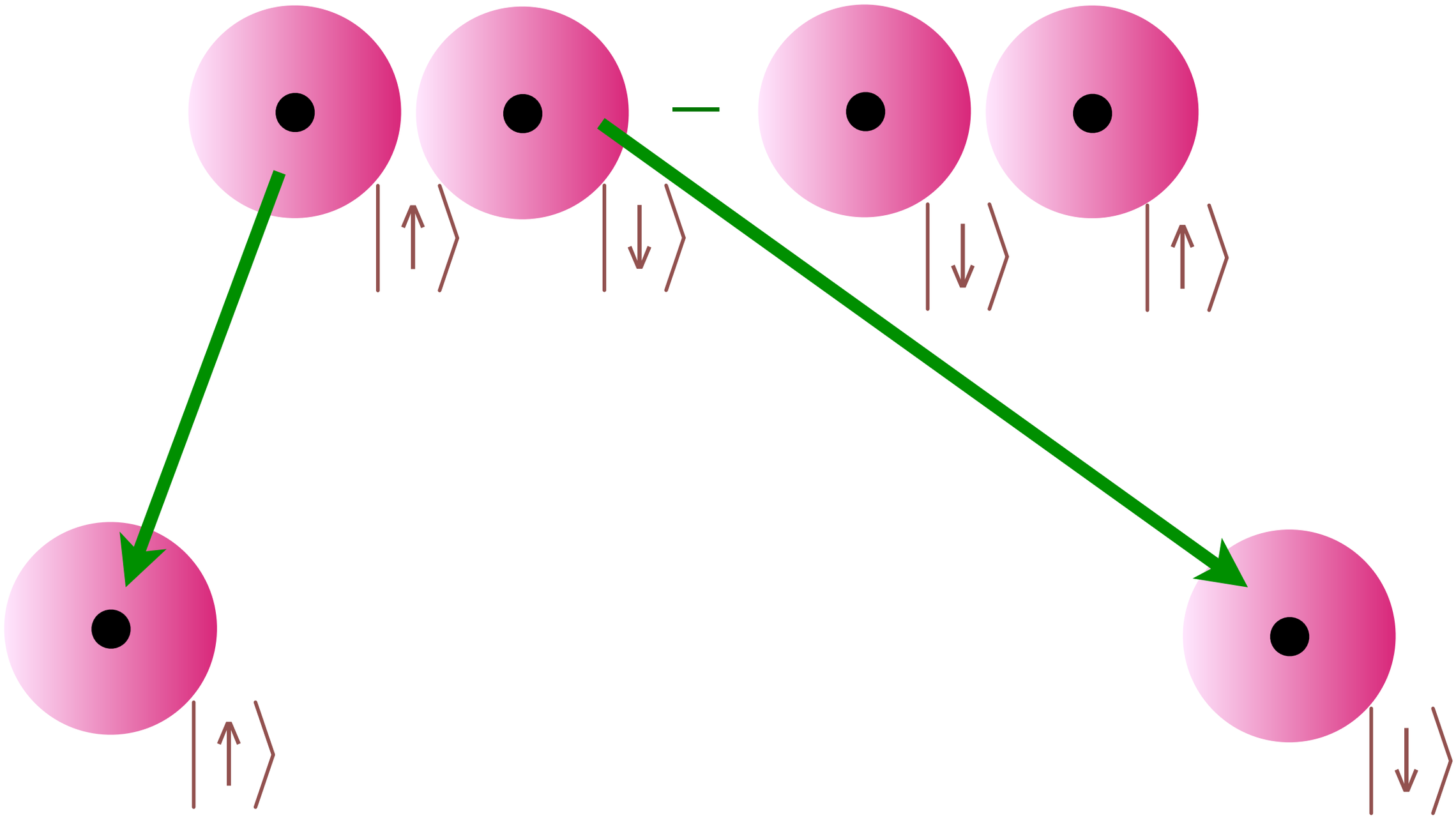
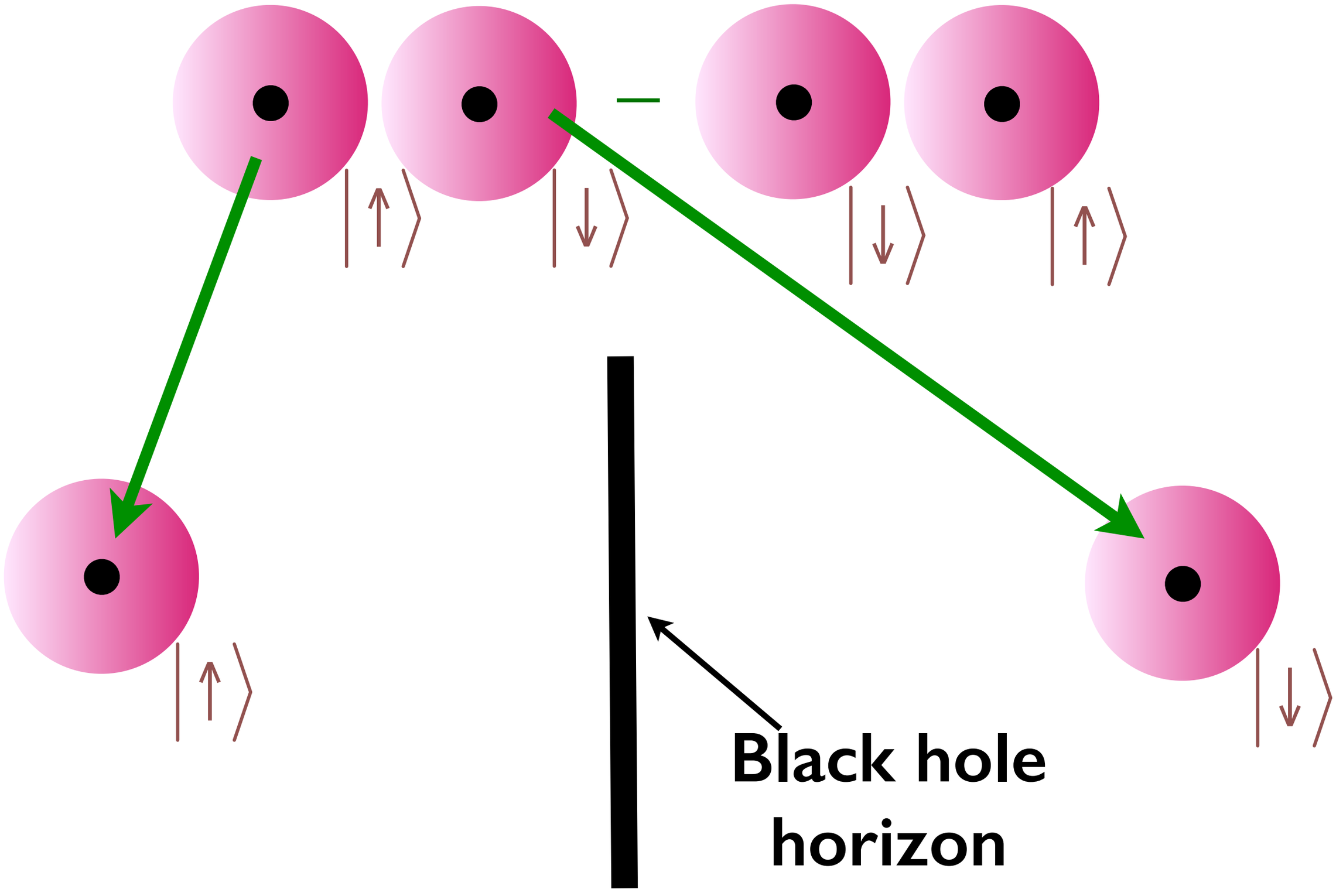
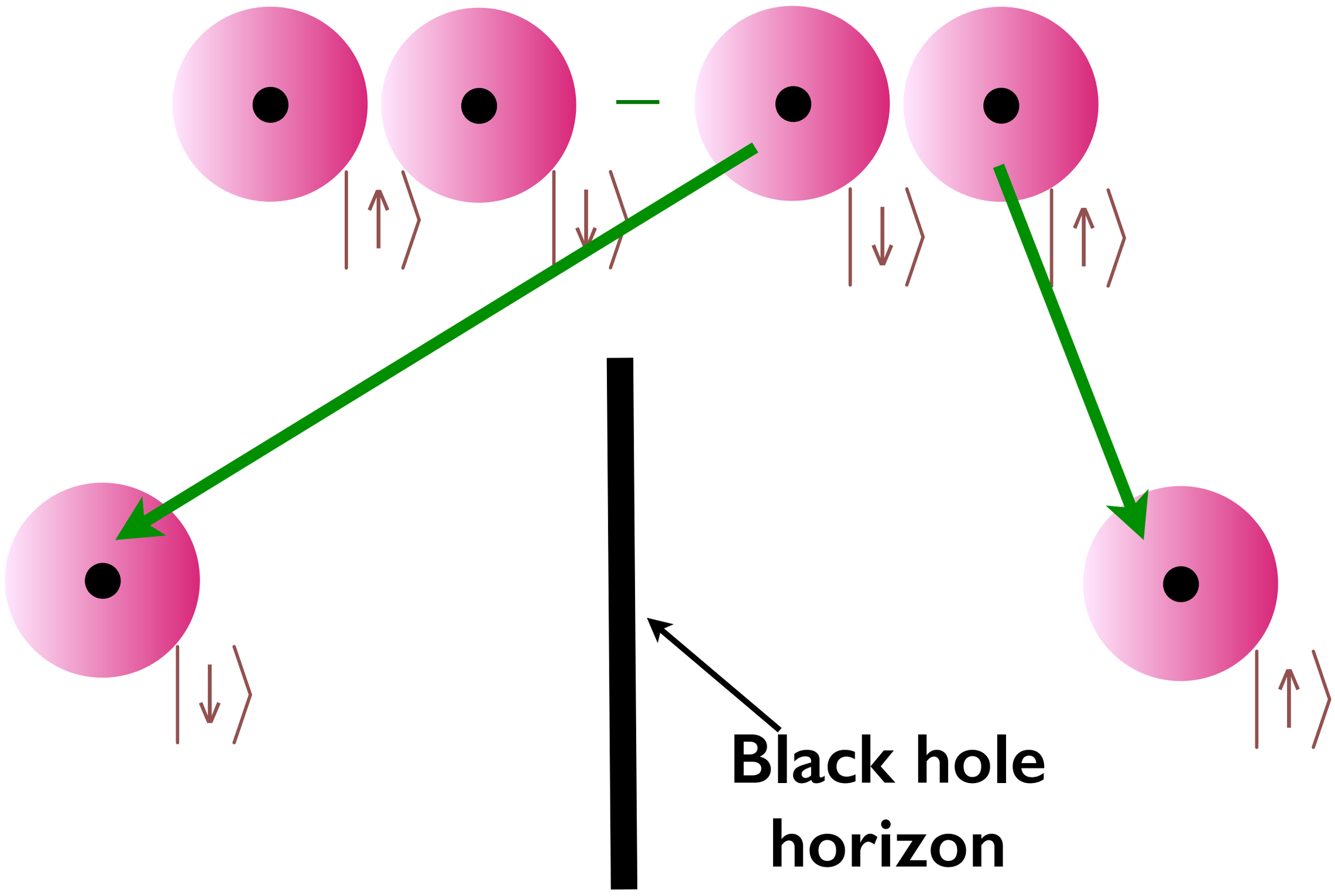


Quantum Entanglement across a black hole horizon

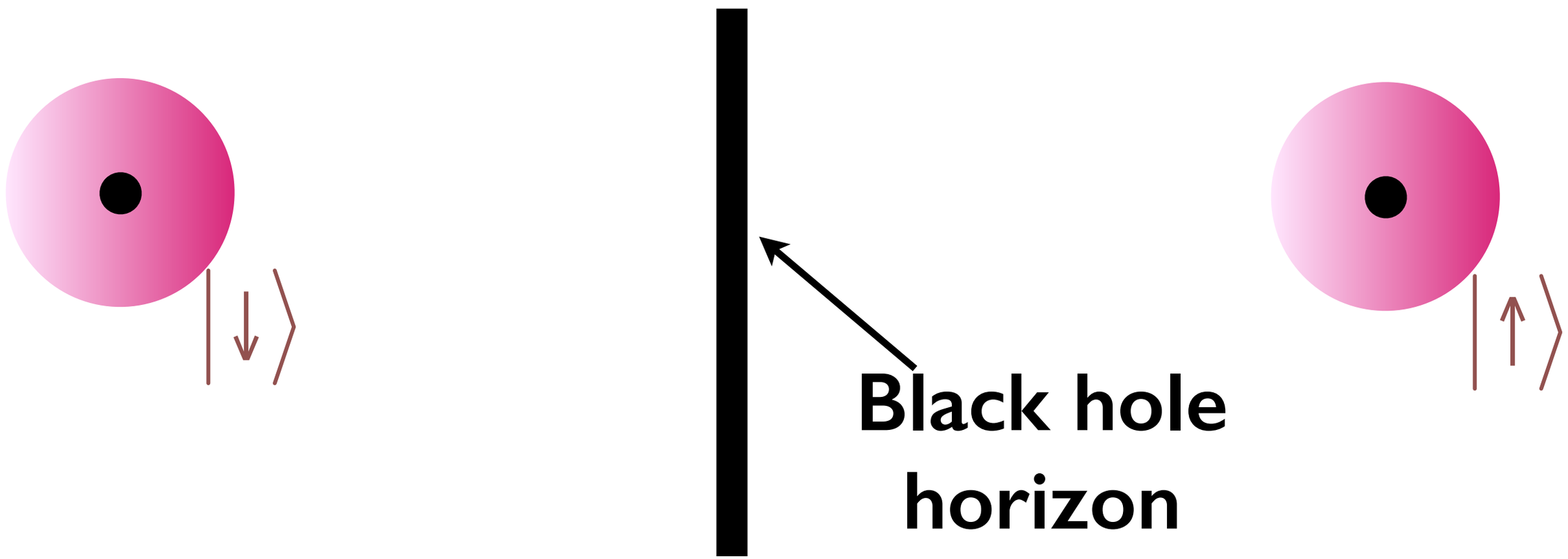


Quantum Entanglement across a black hole horizon



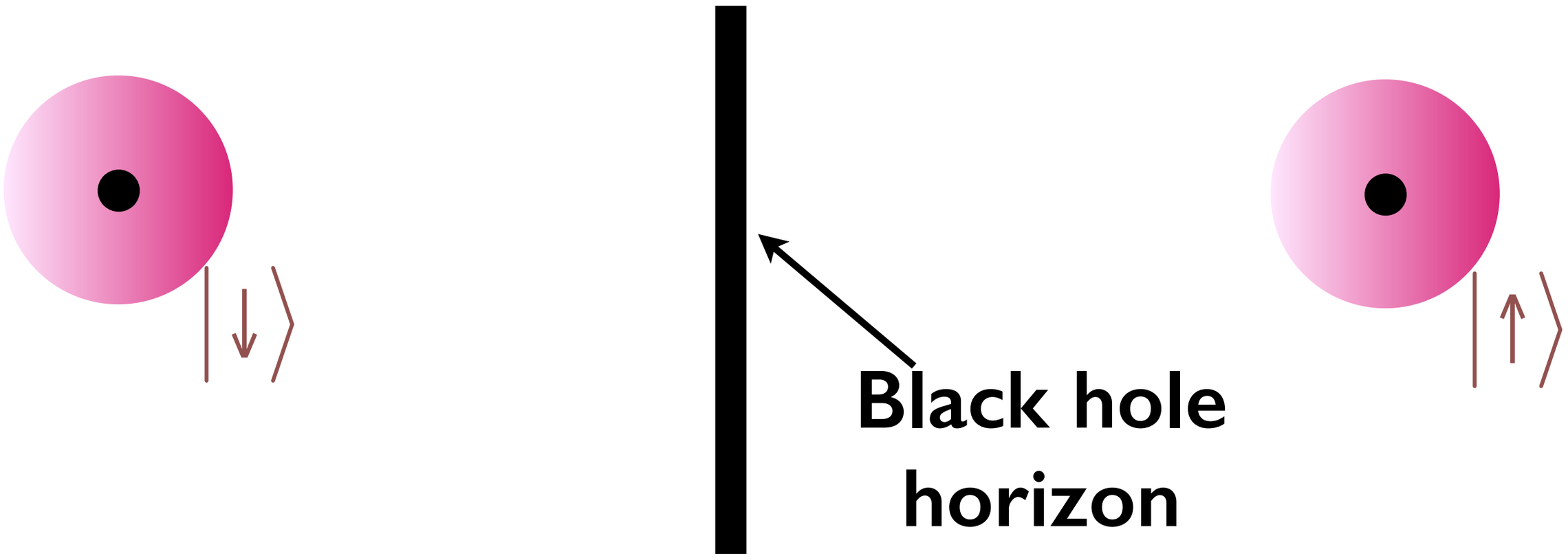
Quantum Entanglement across a black hole horizon

There is long-range quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

There are special black hole solutions which mimic strange metals and superconductors !



Quantum Entanglement across a black hole horizon

There are special black hole solutions which mimic strange metals and superconductors !

These black hole states of General Relativity in D+1 dimensions correspond to (and allow us to compute the properties of) superconductors and strange metals in D dimensions

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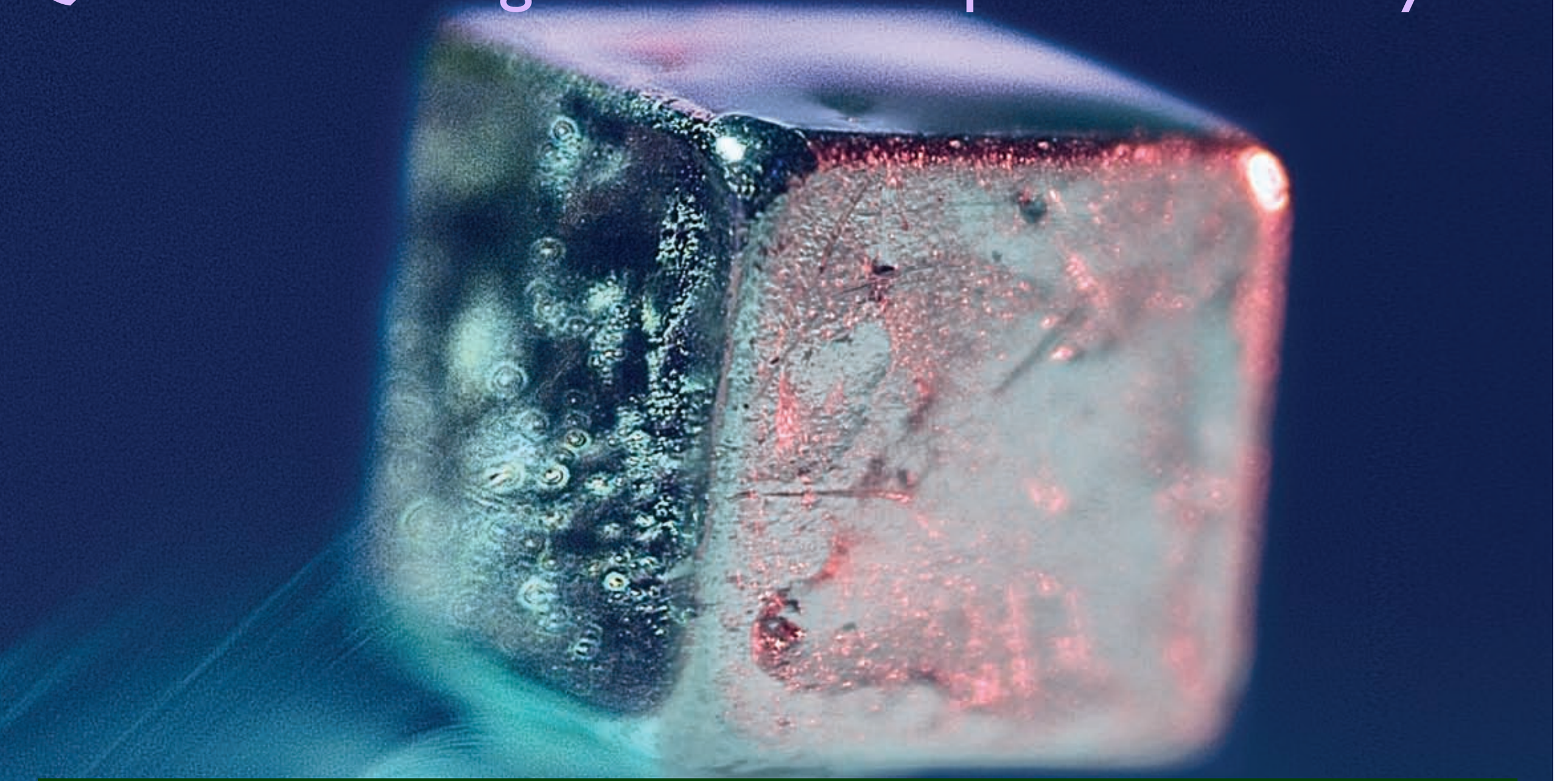
Quantum Entanglement and Superconductivity



Superconductor, levitated by an unseen magnet, in which countless trillions of electrons form a vast interconnected quantum state.
Scientific American, January 2013

Subir Sachdev, Perimeter Institute and Harvard University