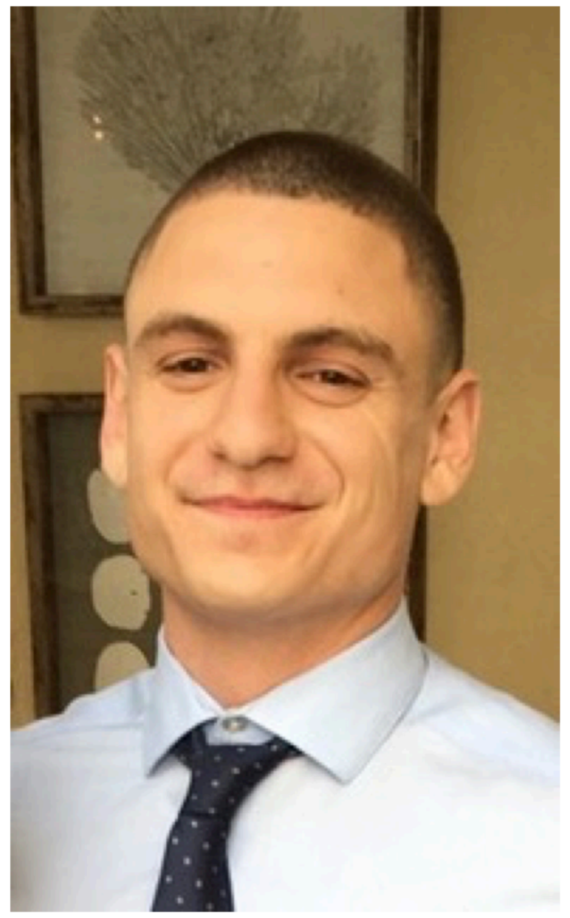
Universal theory of strange metals

Subir Sachdev

Perimeter Institute, Waterloo
March 14, 2024

PHYSICS
VE RI TAS
HARVARD



Ilya Esterlis
Wisconsin

Haoyu Guo
Cornell

Aavishkar Patel
Flatiron

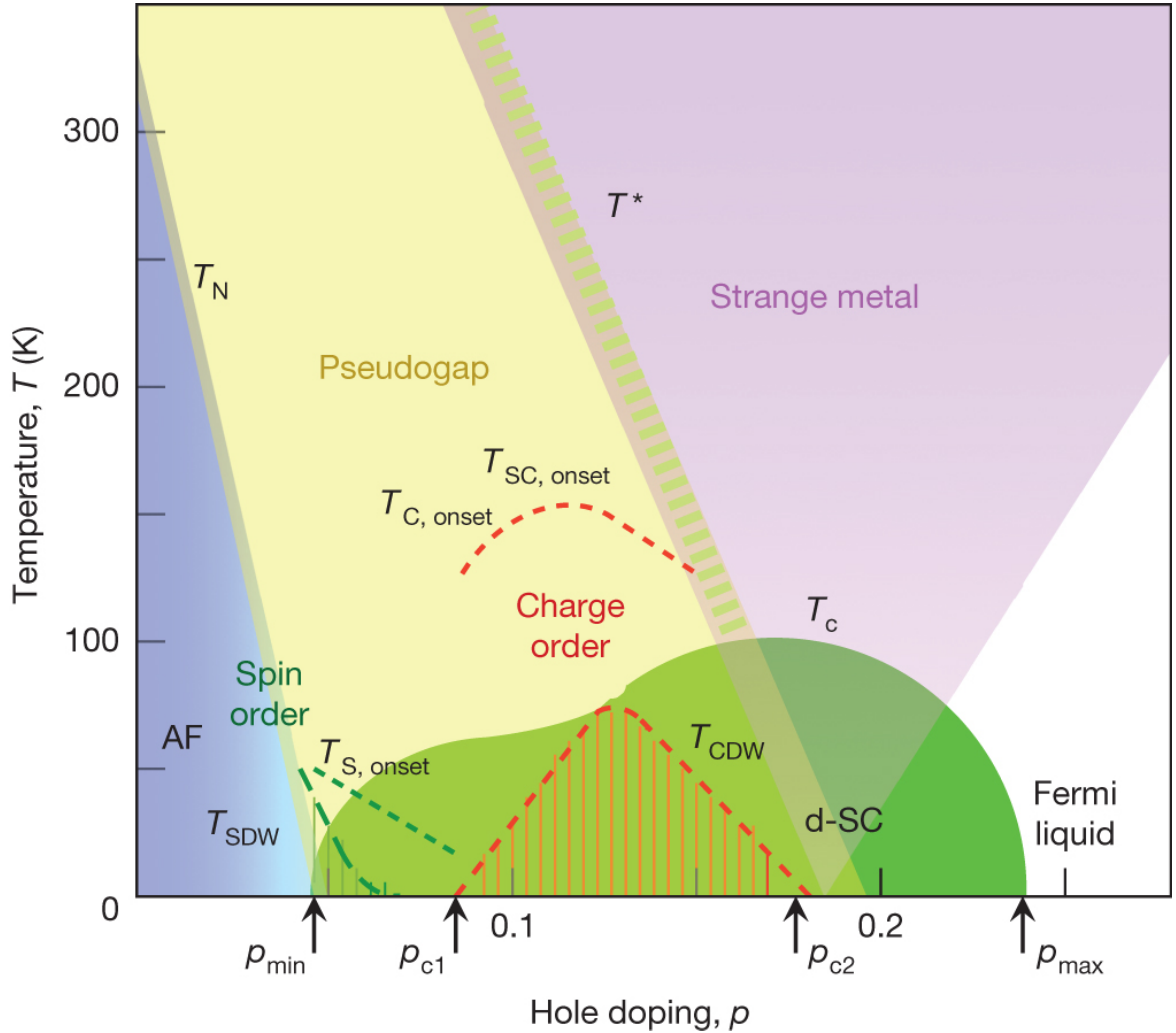
Chenyuan Li
Harvard

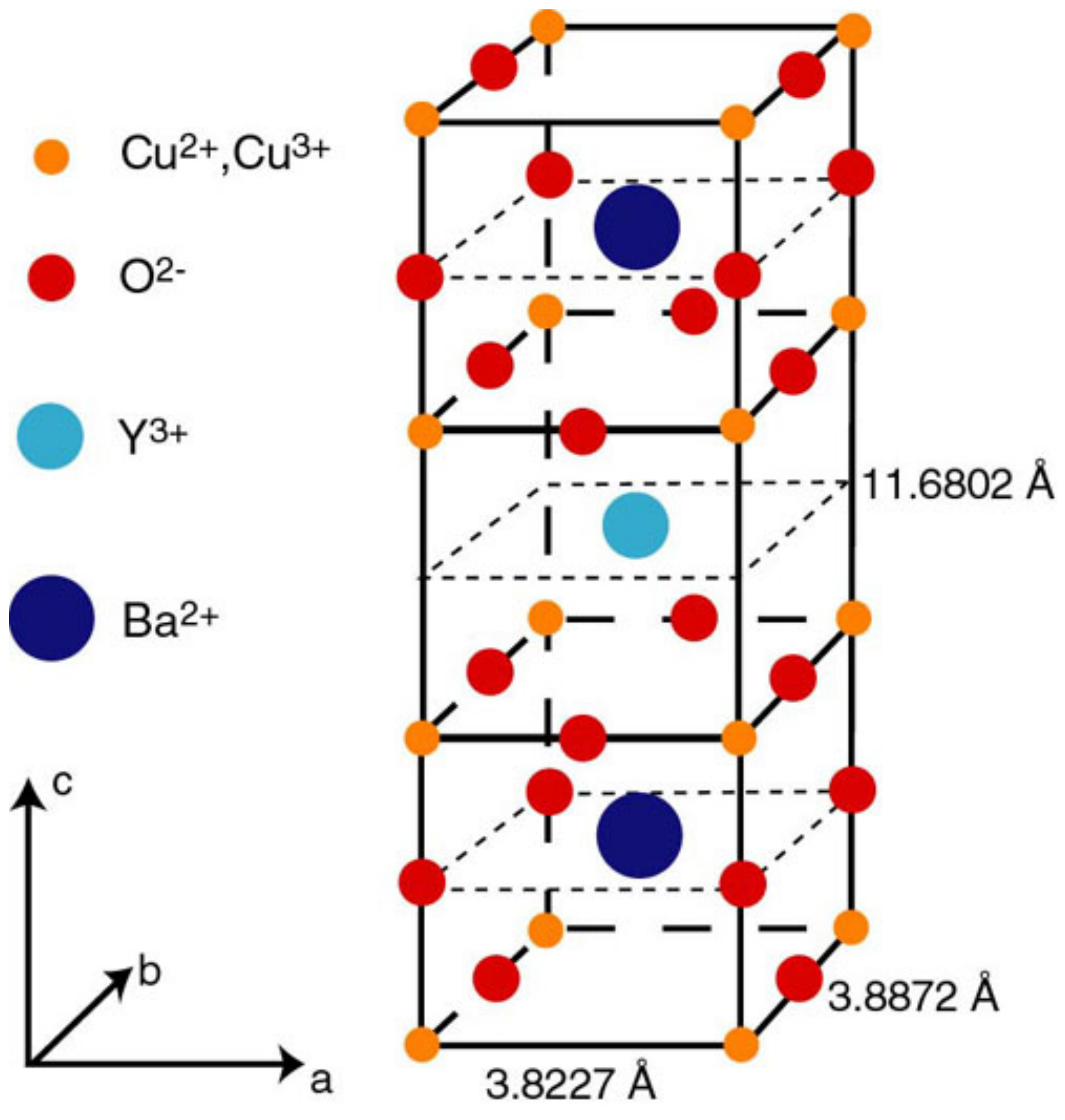
Davide Valentinis
KIT

Joerg Schmalian
KIT

Peter Lunts
Harvard

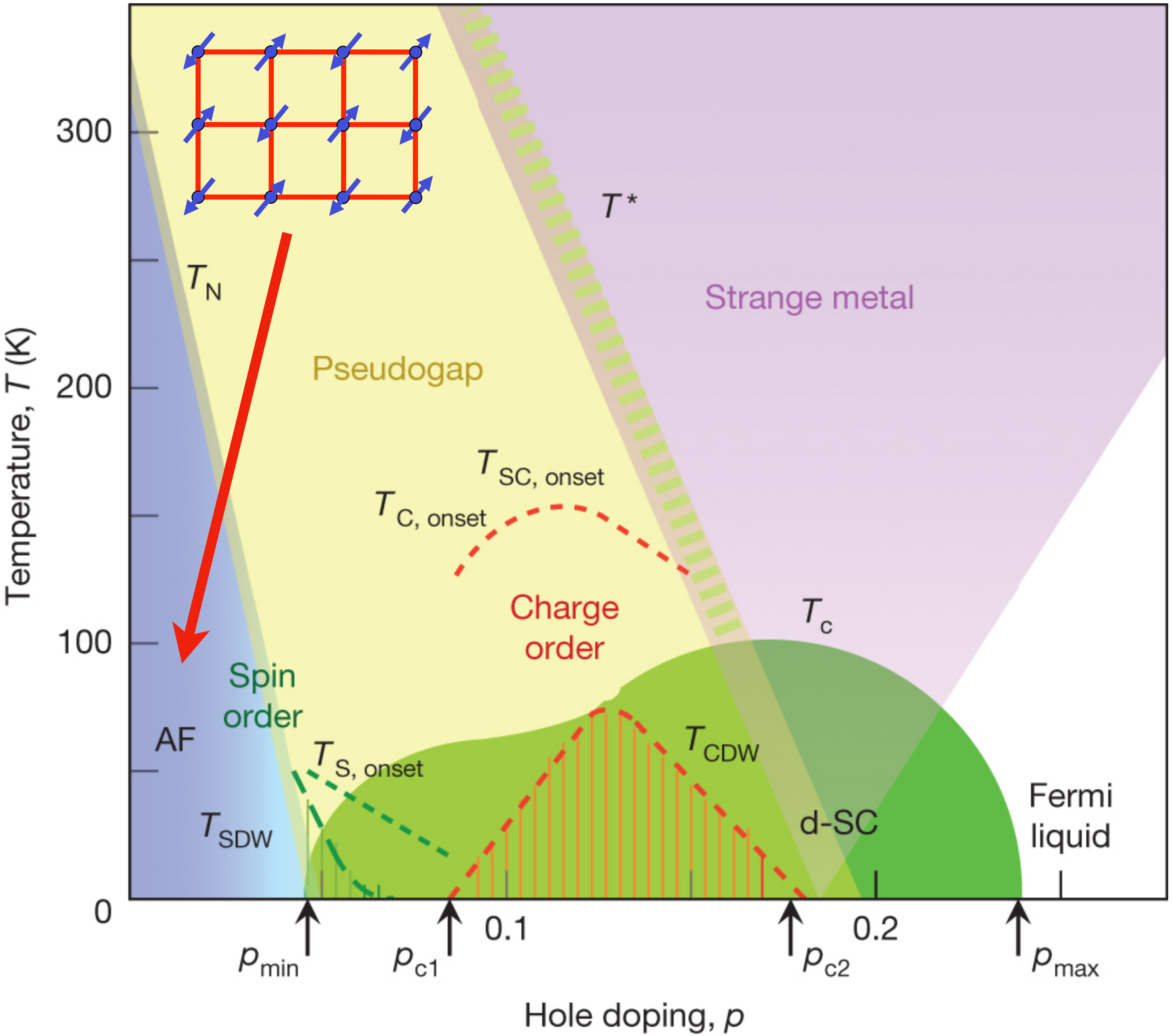
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

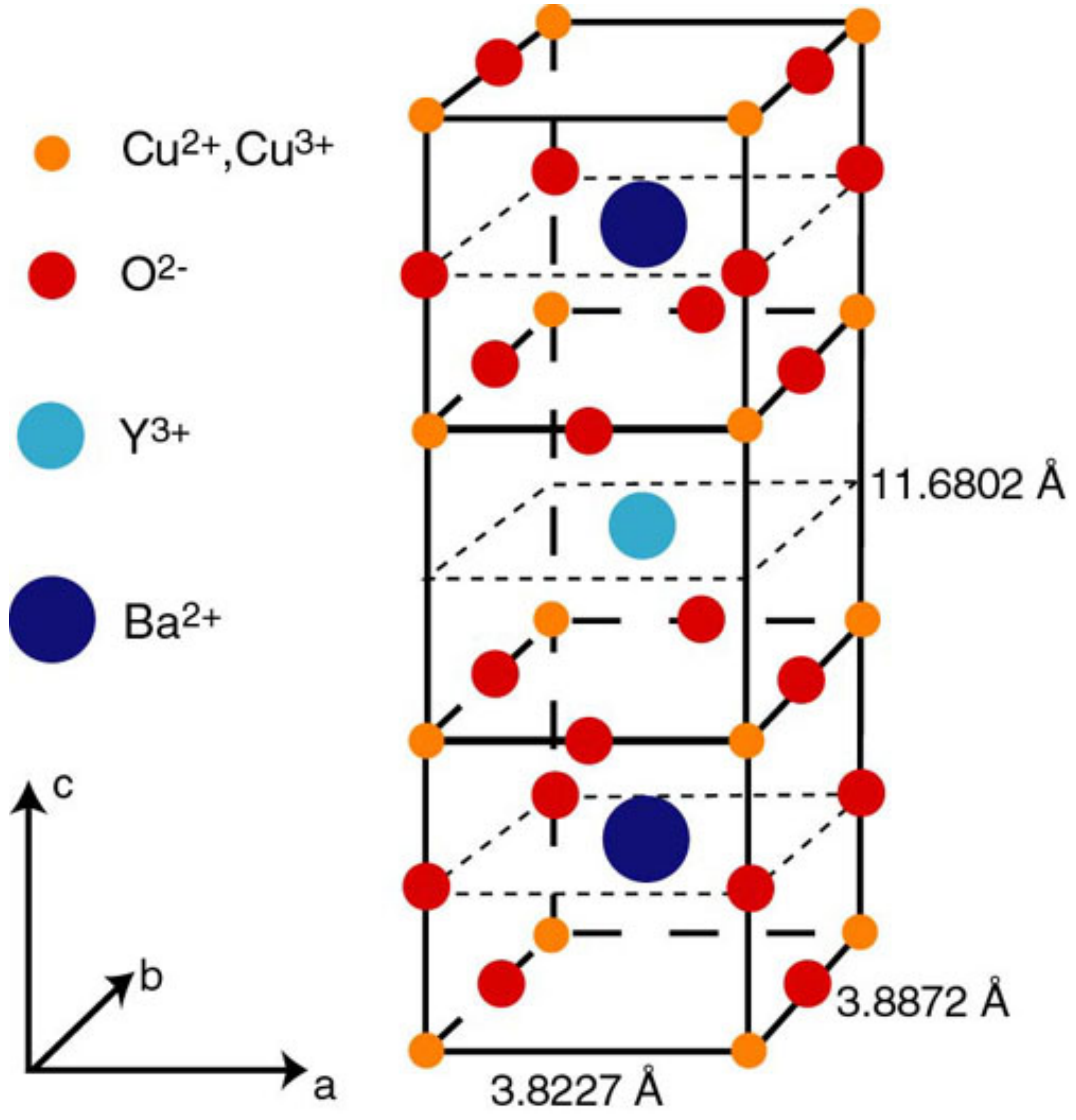




$YBa_2Cu_3O_{6+x}$

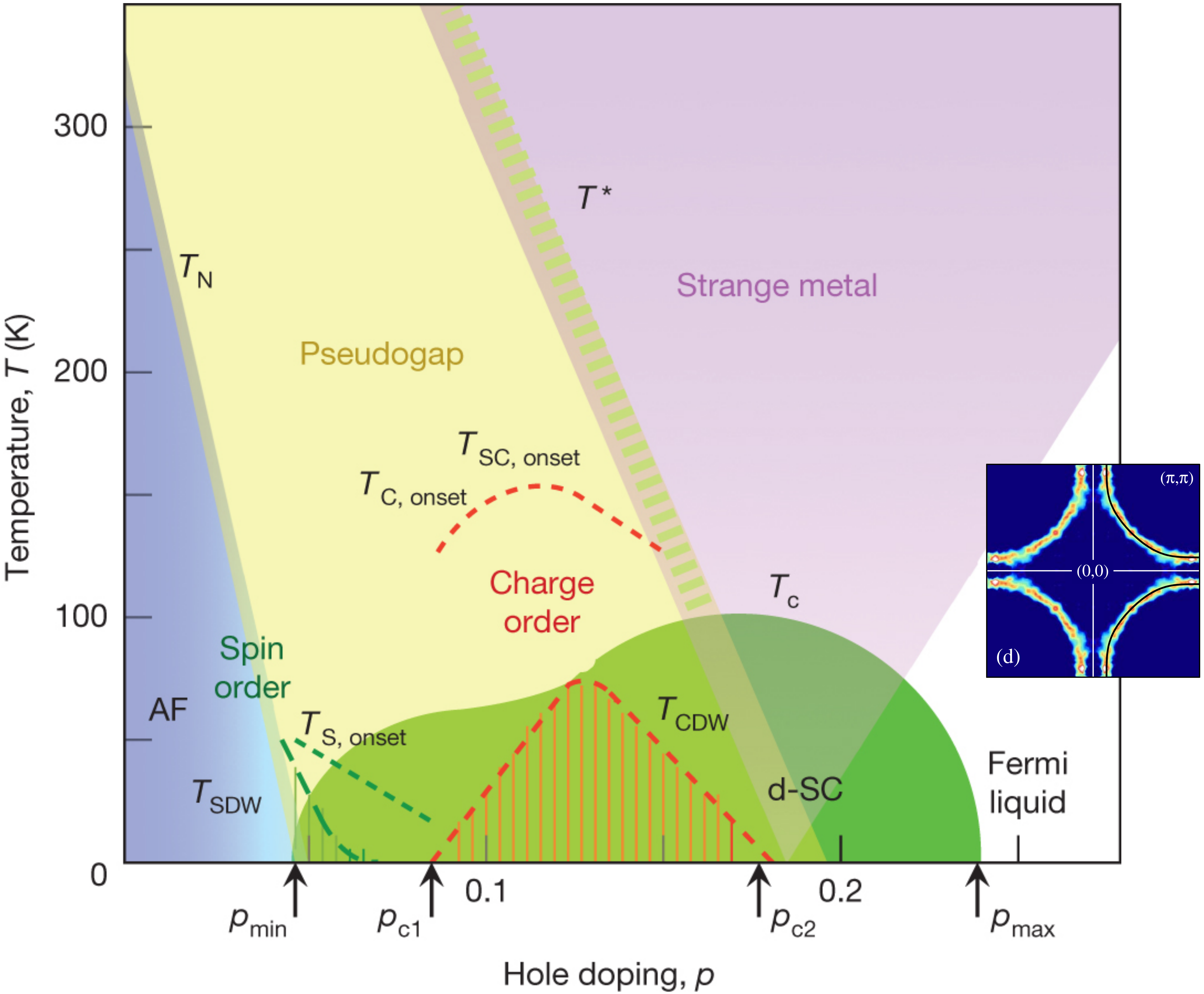
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





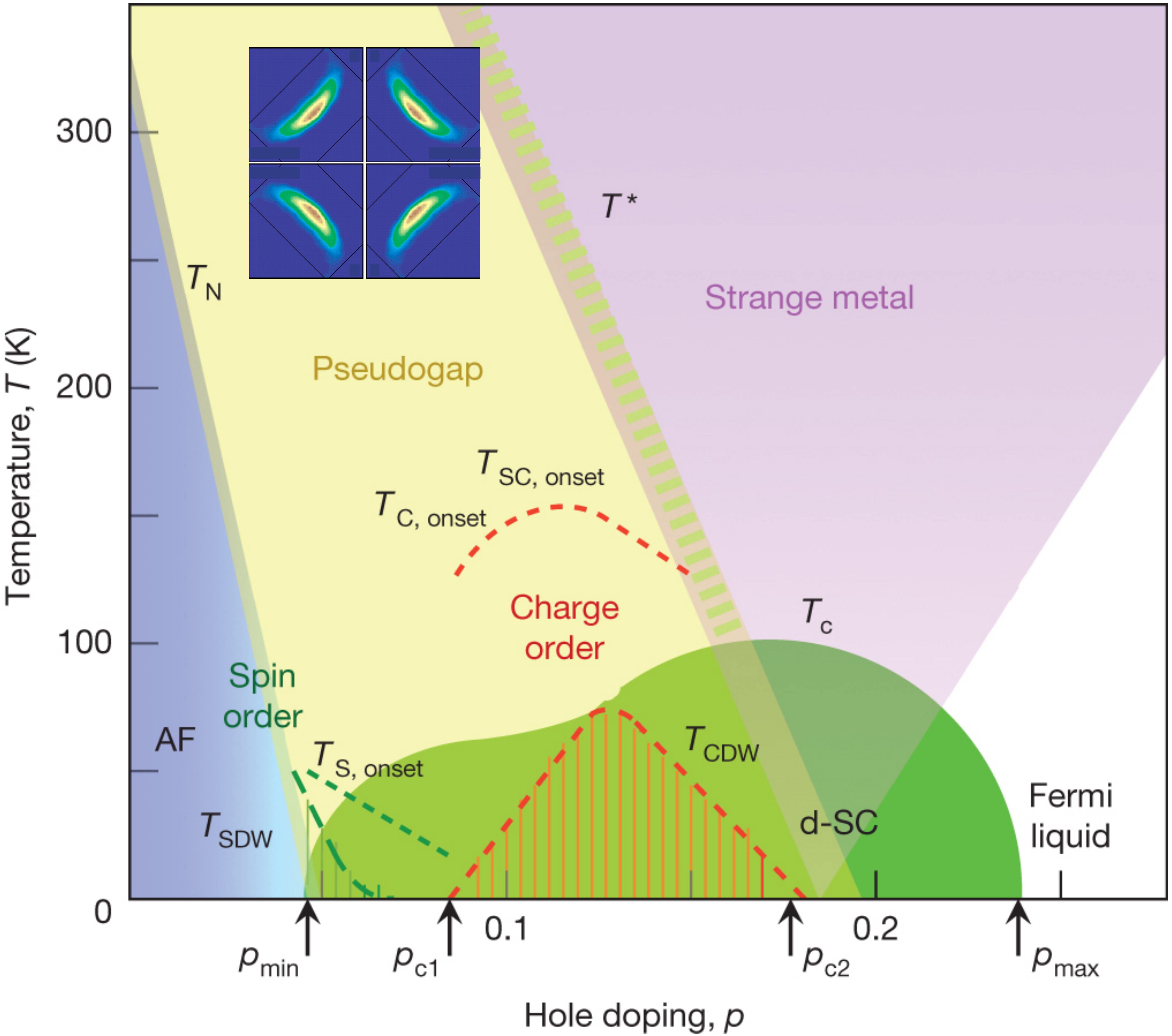
$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



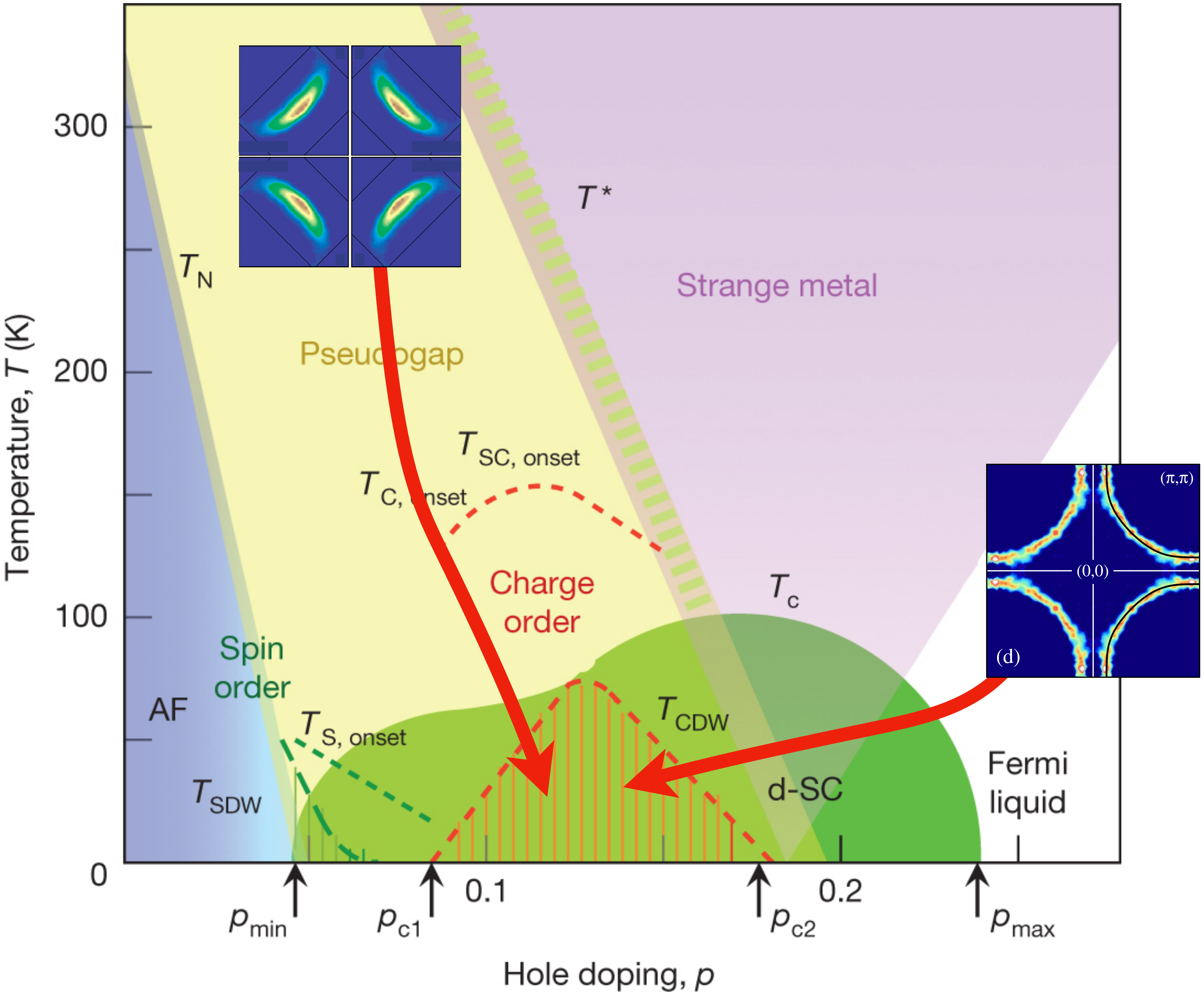
Fermi liquid in the overdoped metal

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Pseudogap metal with "Fermi arcs"

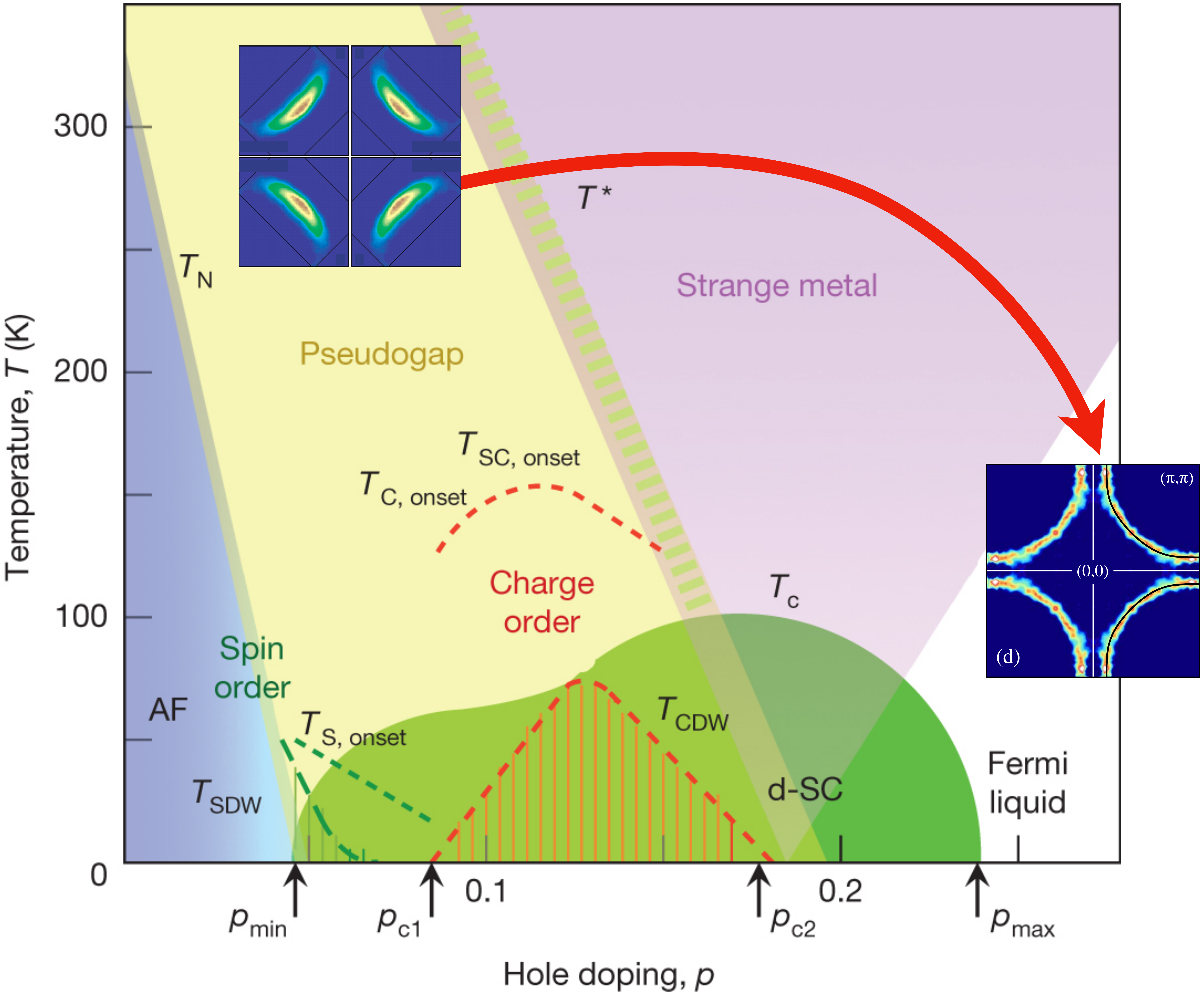
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Build a theory for the phase diagram from a theory of the pseudogap metal as a 'metastable' $T = 0$ quantum phase.

Lowest T phases obtained from pseudogap metal should connect smoothly to conventionally order phases obtained from the Fermi liquid.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Build a theory for the phase diagram from a theory of the pseudogap metal as a 'metastable' $T = 0$ quantum phase.

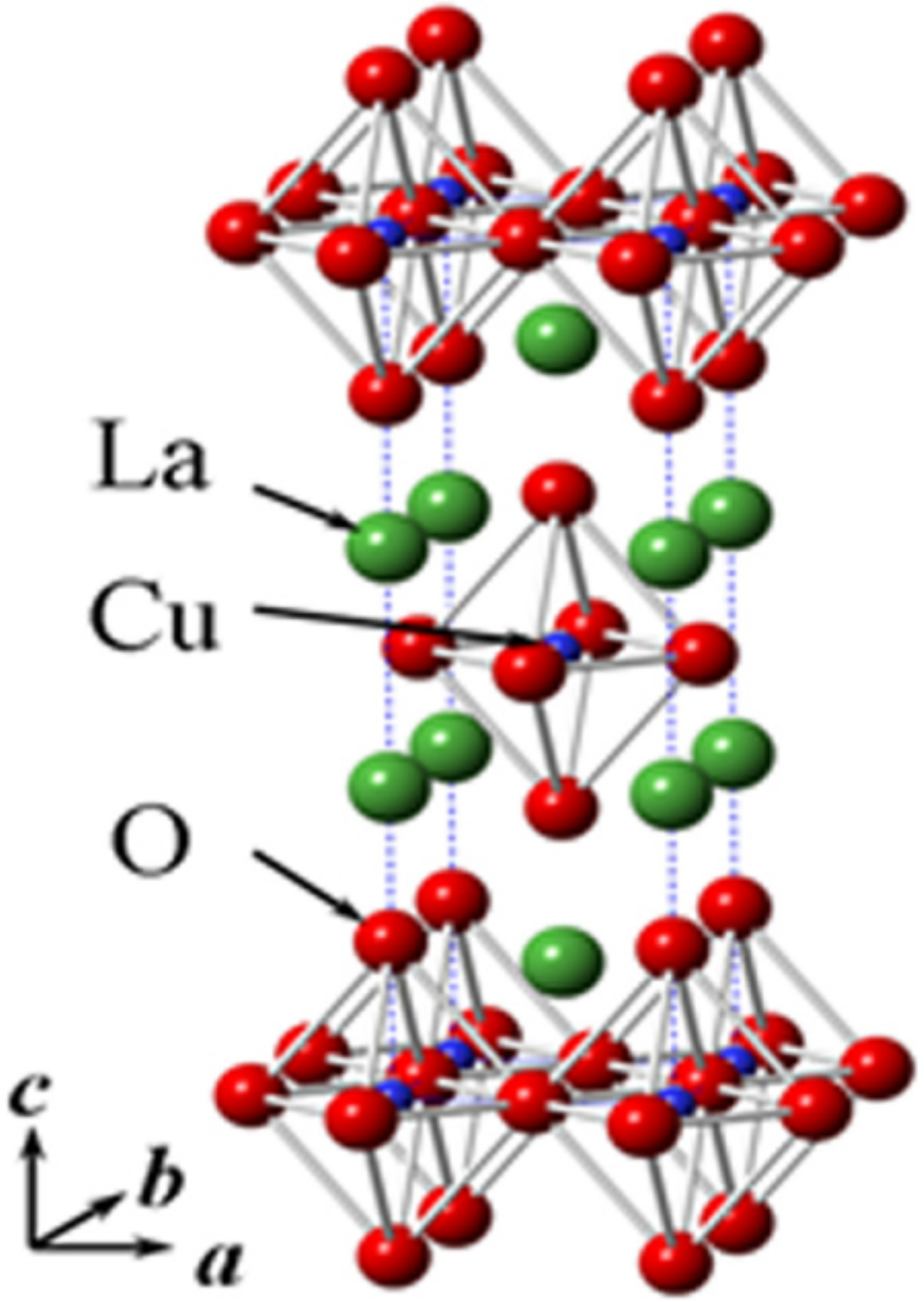
Strange metal described by quantum critical region of a $T = 0$ quantum transition between the pseudogap metal (FL*) and the Fermi liquid (FL).

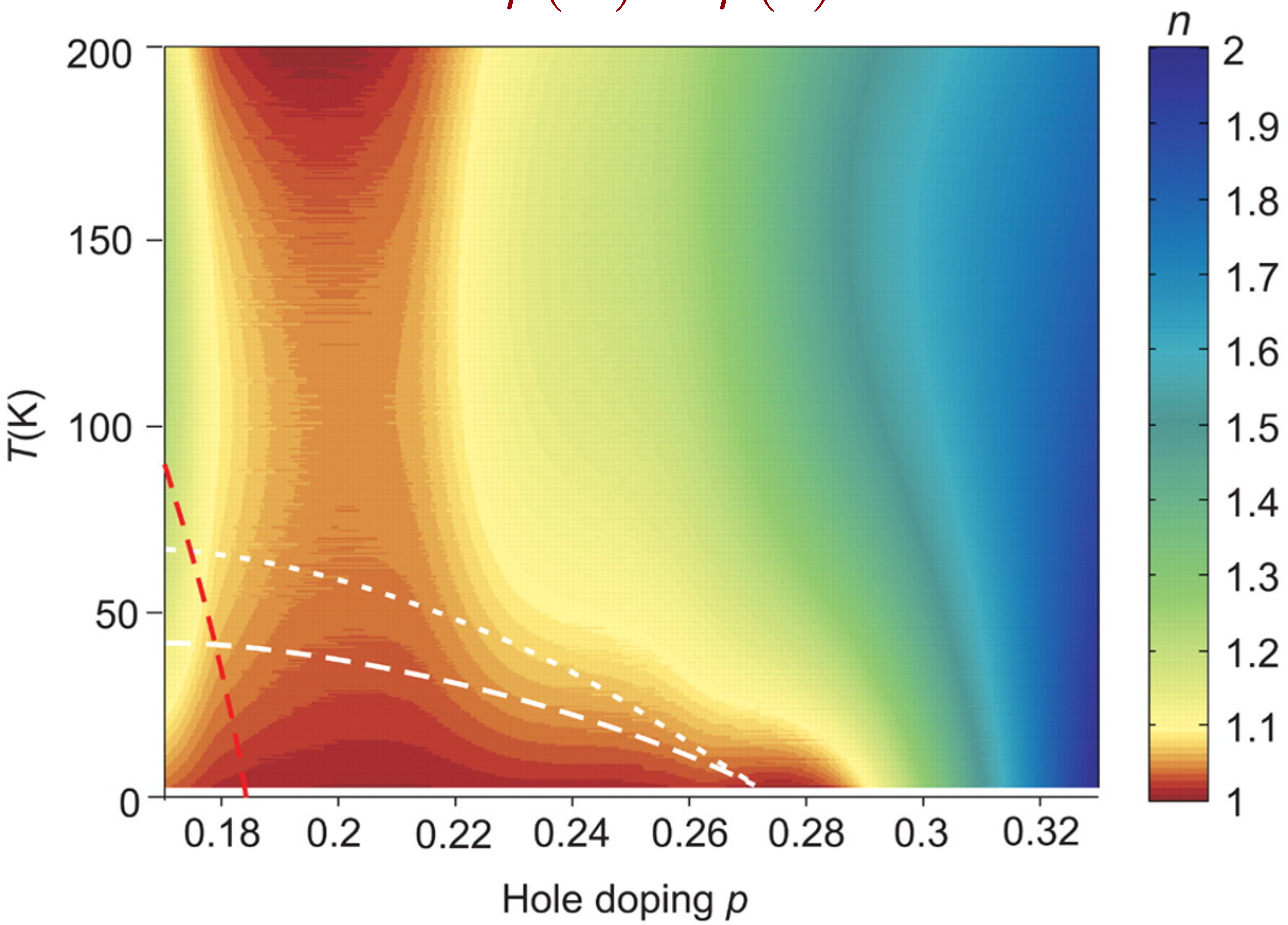
Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3]
Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

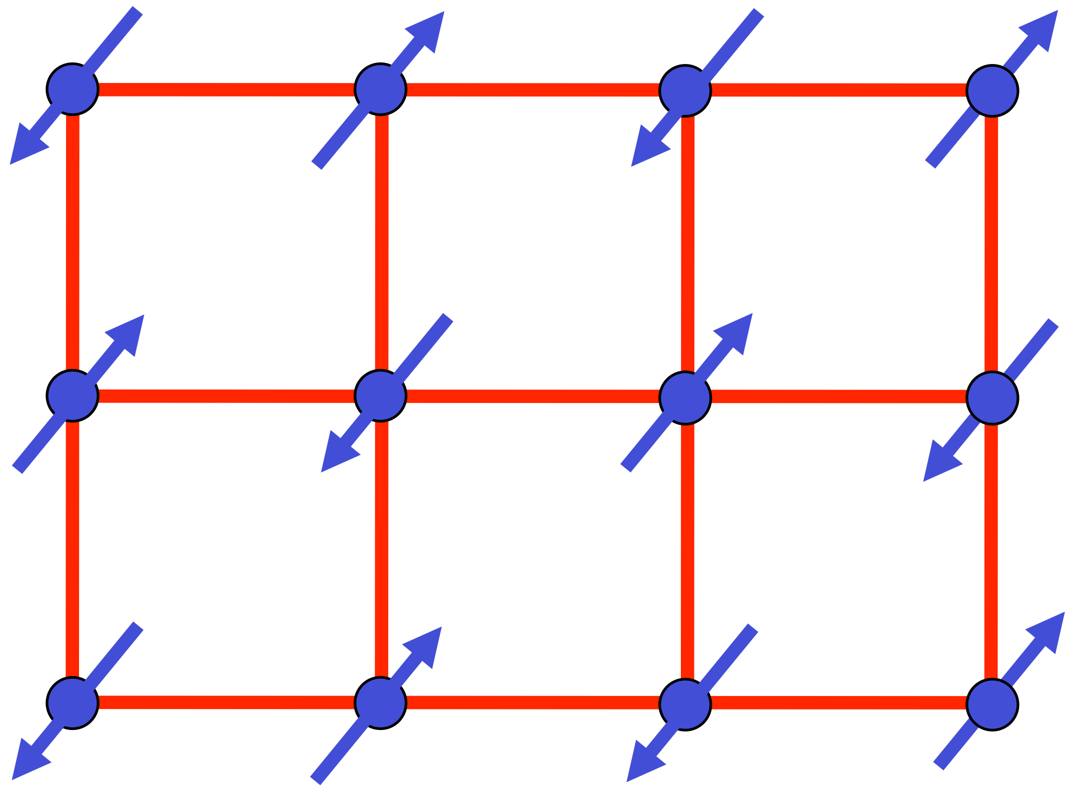
SCIENCE VOL 323 603 2009

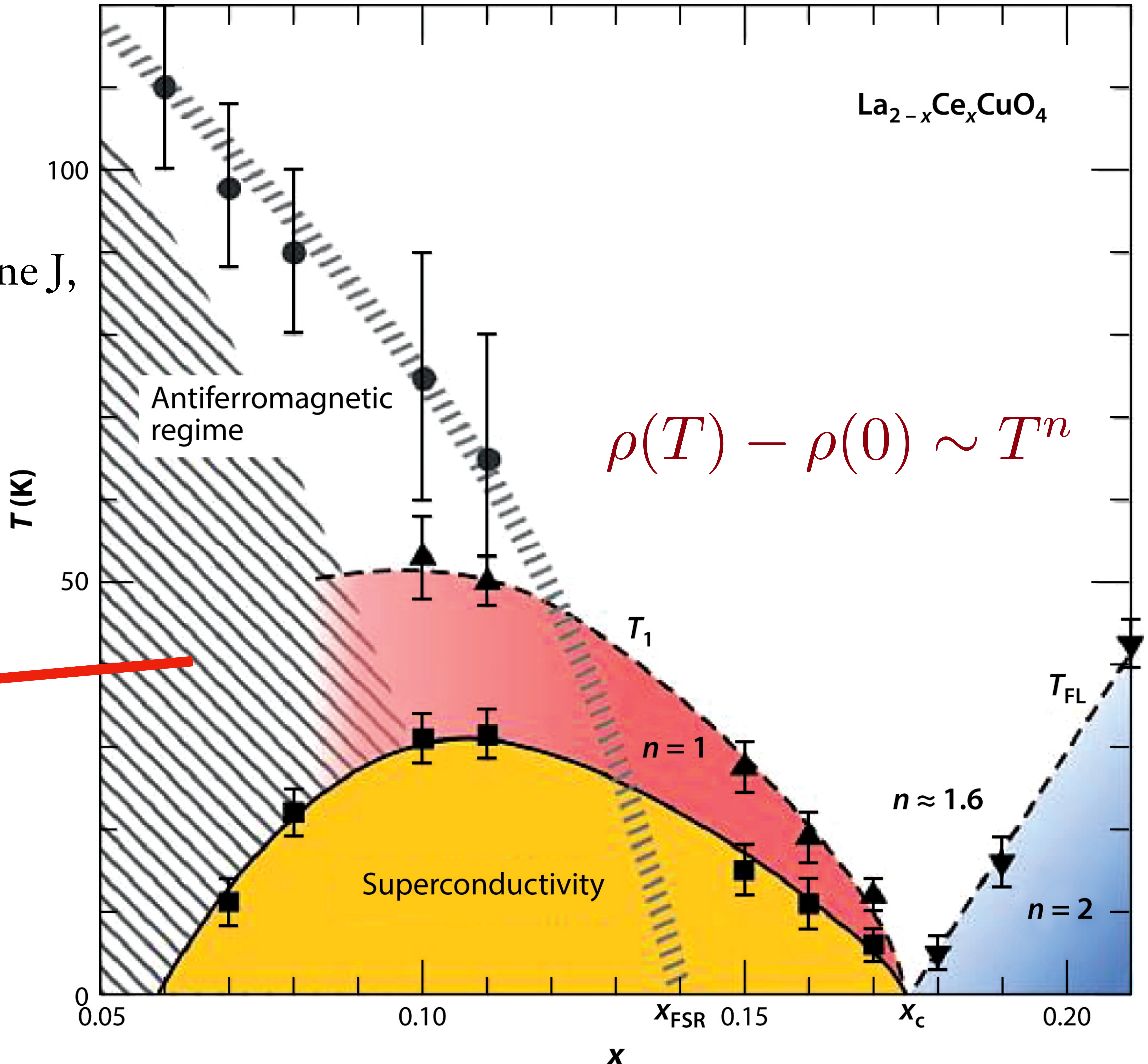
$$\rho(T) - \rho(0) \sim T^n$$





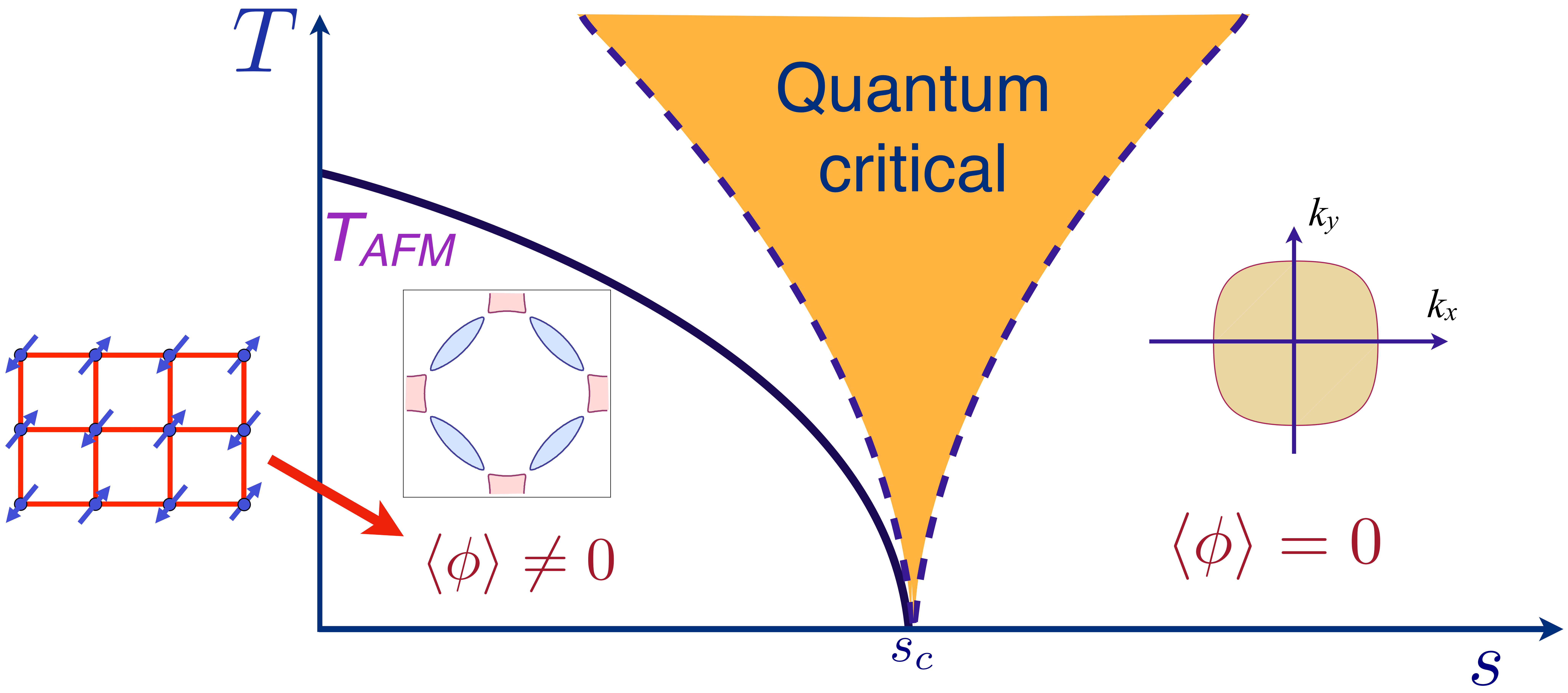
Jin K, Butch NP, Kirshenbaum K, Paglione J, Greene RL. 2011. *Nature* 476:73–75





$$\rho(T) - \rho(0) \sim T^n$$

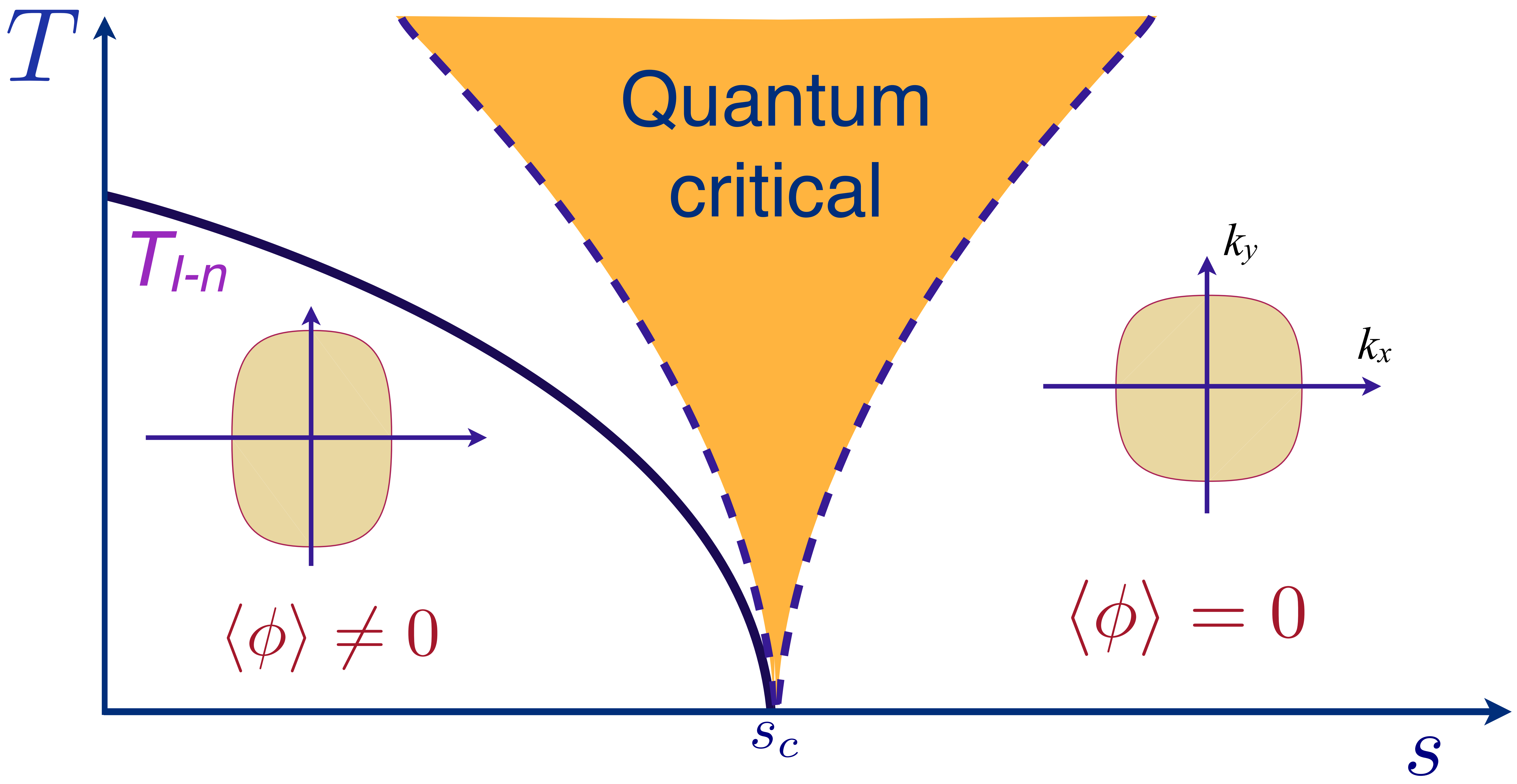
Quantum criticality of AF ordering in a metal



Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlief, Sung-Sik Lee

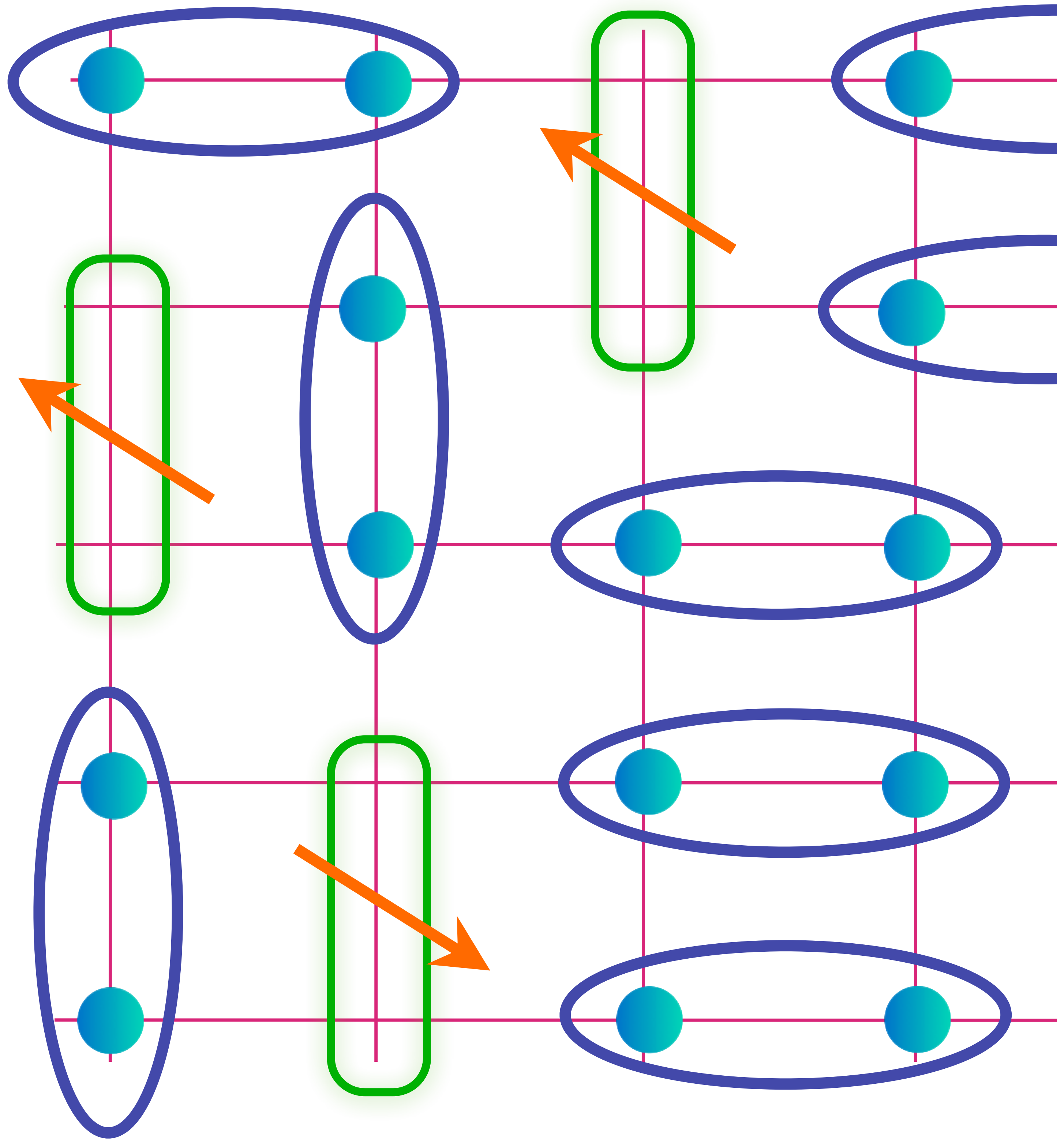
Annals of Physics 450, 169221 (2023)

Quantum criticality of Ising-nematic ordering in a metal

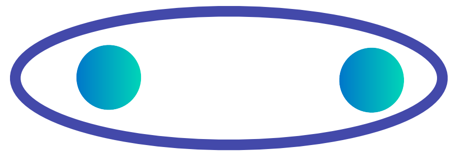


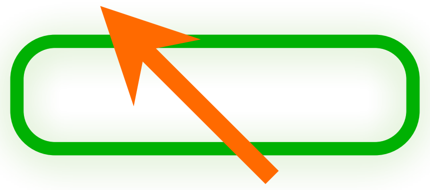
Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlief, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

Pseudogap metal to Fermi liquid in single band model



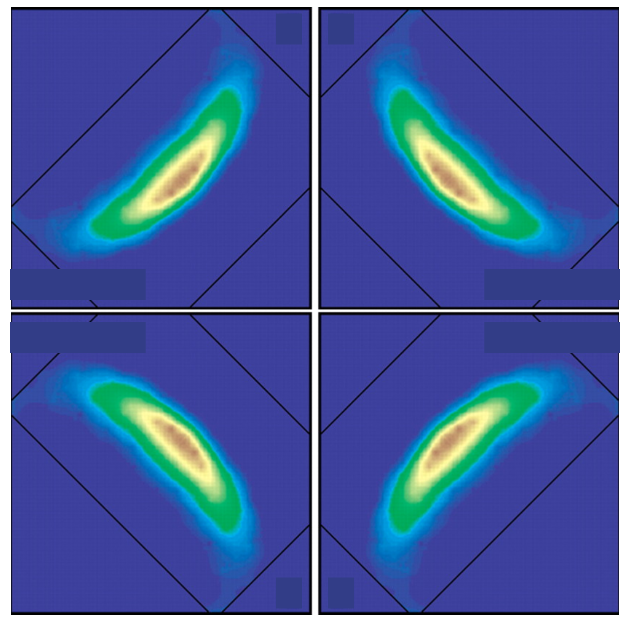
Φ Higgs boson with the fundamental gauge charge of an emergent SU(2) gauge field.

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

Small Fermi surface of size p + spin liquid.

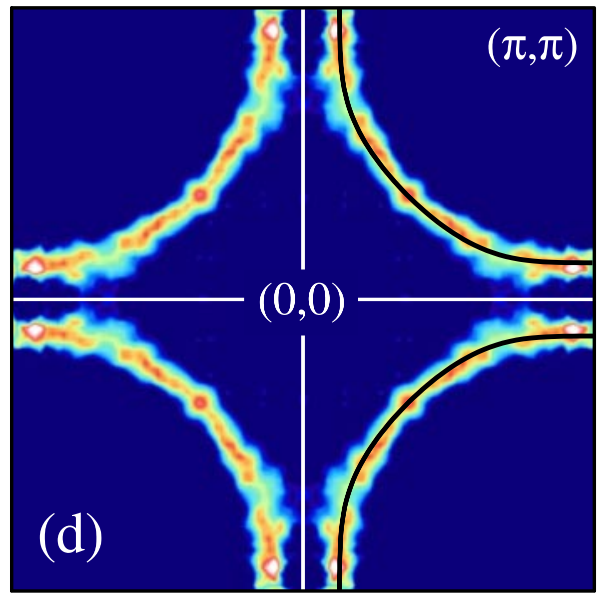
FL*



$\langle \Phi \rangle \neq 0$

Large Fermi surface of size $1 + p$

FL

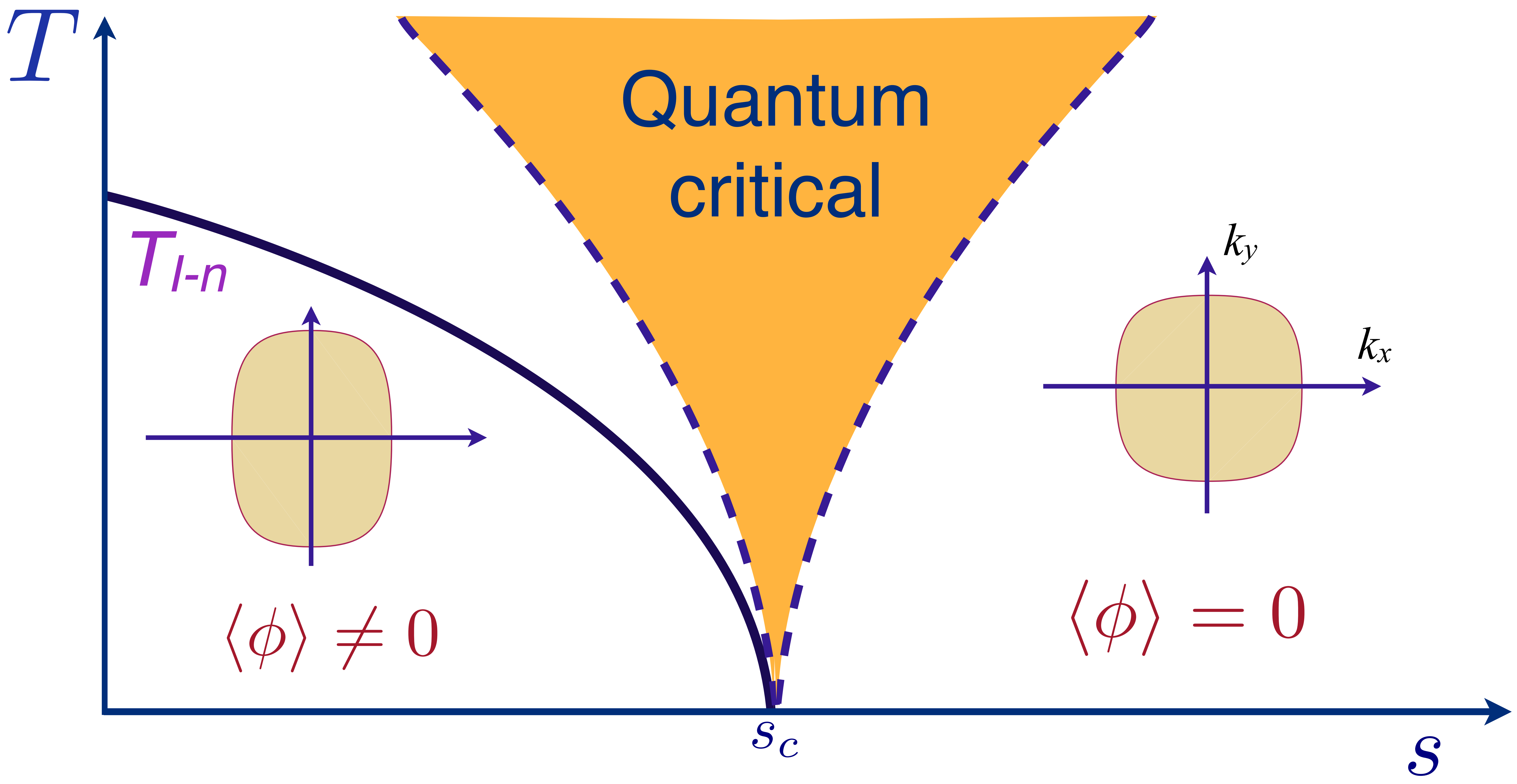


$\langle \Phi \rangle = 0$

doping p

Ya-Hui Zhang and S.S. *Phys. Rev. Research* **2**, 023172 (2020); *Phys. Rev. B* **102**, 155124 (2020)

Quantum criticality of Ising-nematic ordering in a metal

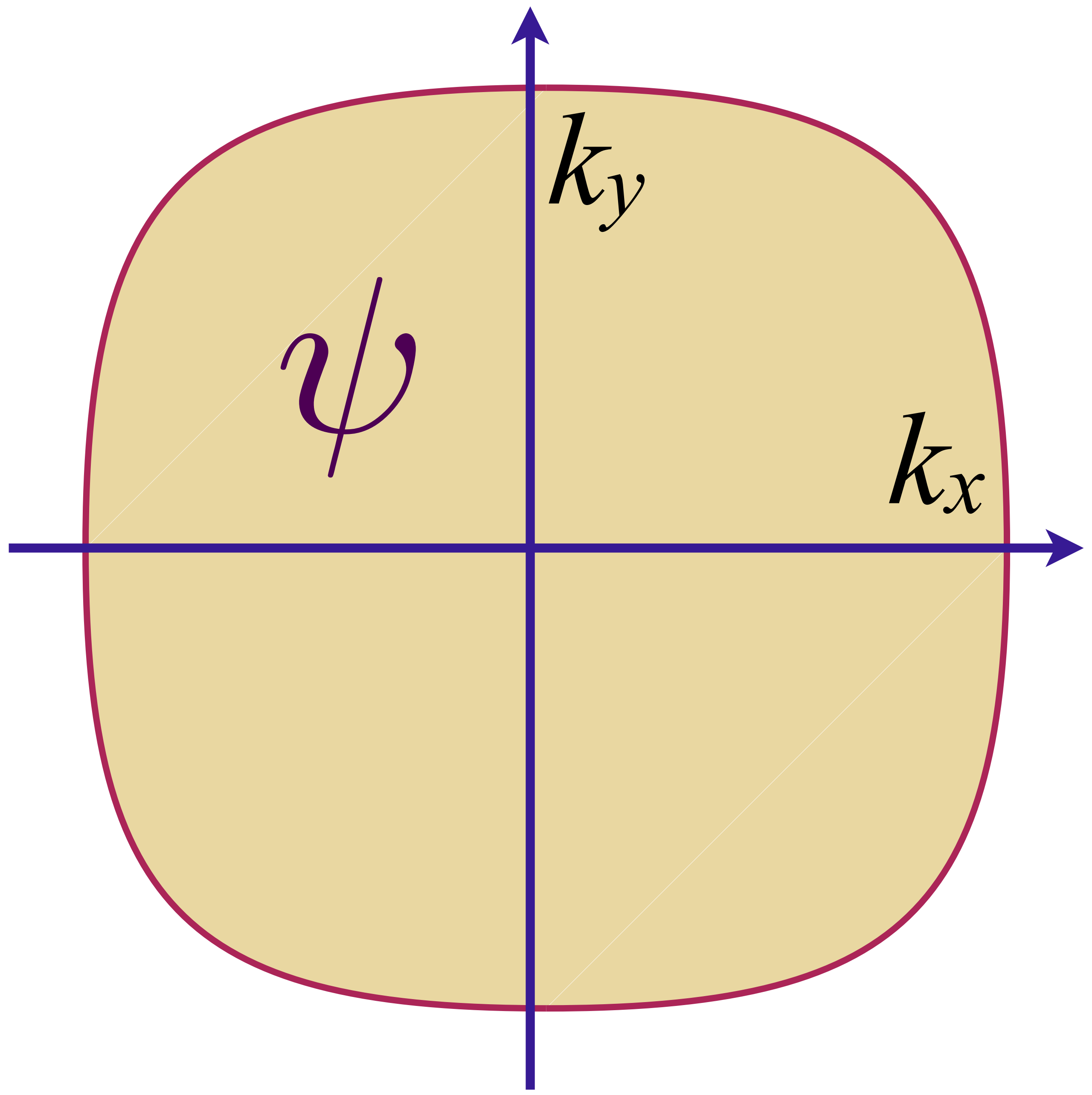


Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlief, Sung-Sik Lee

Annals of Physics 450, 169221 (2023)

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s \, [\phi(\boldsymbol{r})]^2 \qquad + g \, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}) \, \phi(\boldsymbol{r})$$

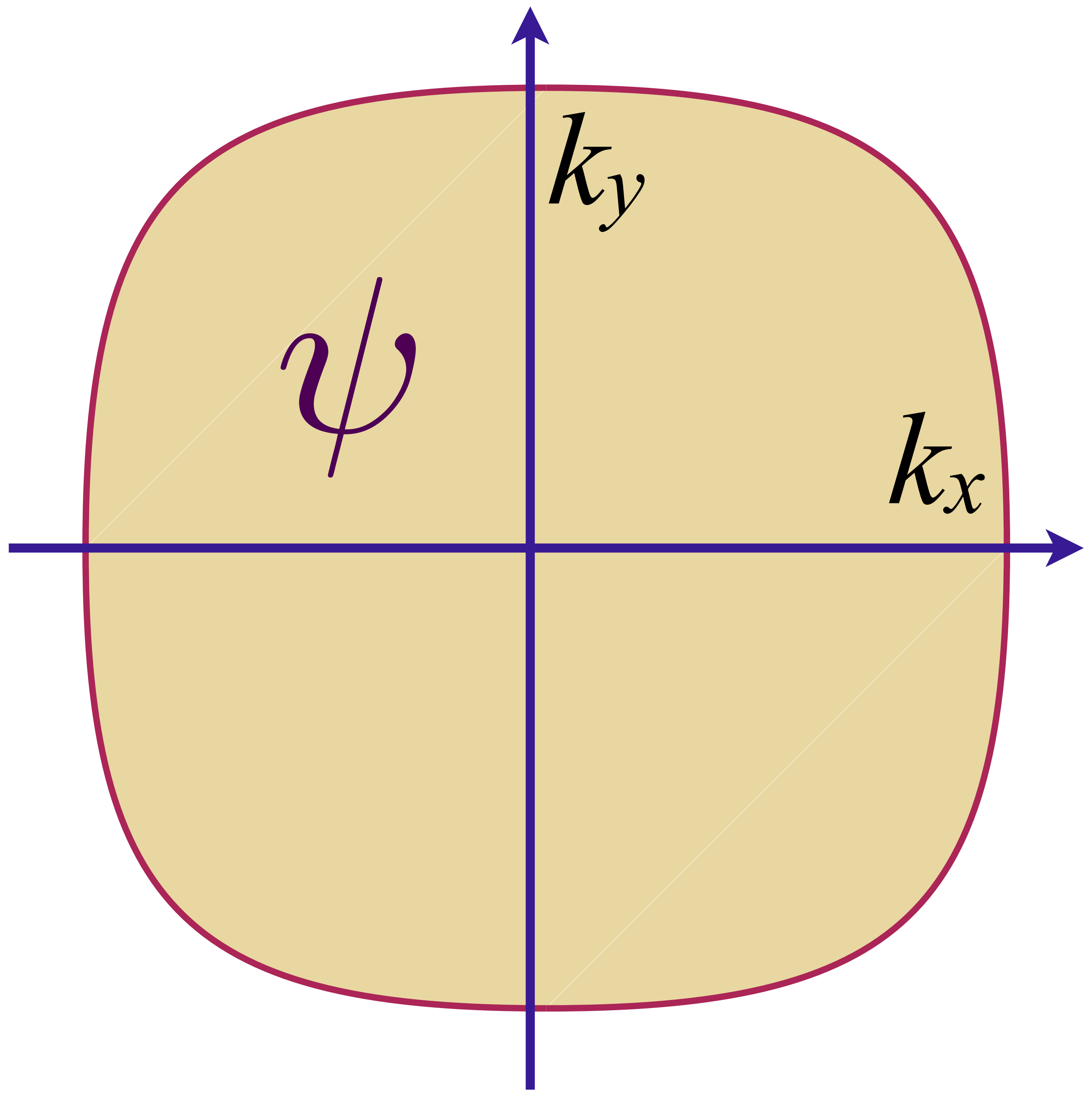
$$+K \, [\nabla_{\boldsymbol{r}} \phi(\boldsymbol{r})]^2 + u \, [\phi(\boldsymbol{r})]^4$$

Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlief, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition



$$+s\,[\phi(\boldsymbol{r})]^2 \qquad + g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

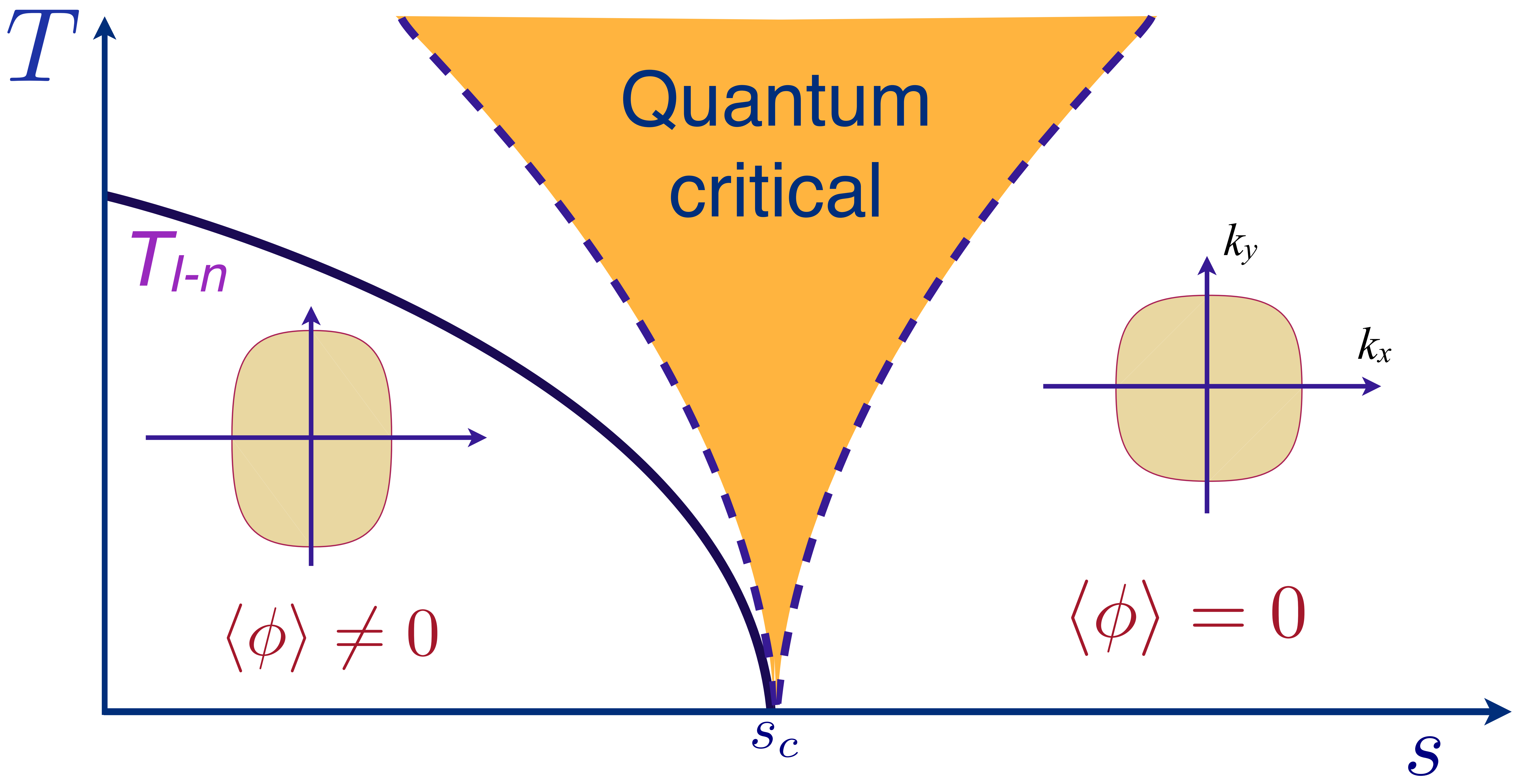
$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

A non-Fermi liquid in the electron spectral function
but a perfect metal in transport!

$$\Sigma(\omega) \sim \omega^{2/3}$$
$$\sigma(\omega) = iD/(\omega - \omega_c) + \omega^0 + \ldots$$

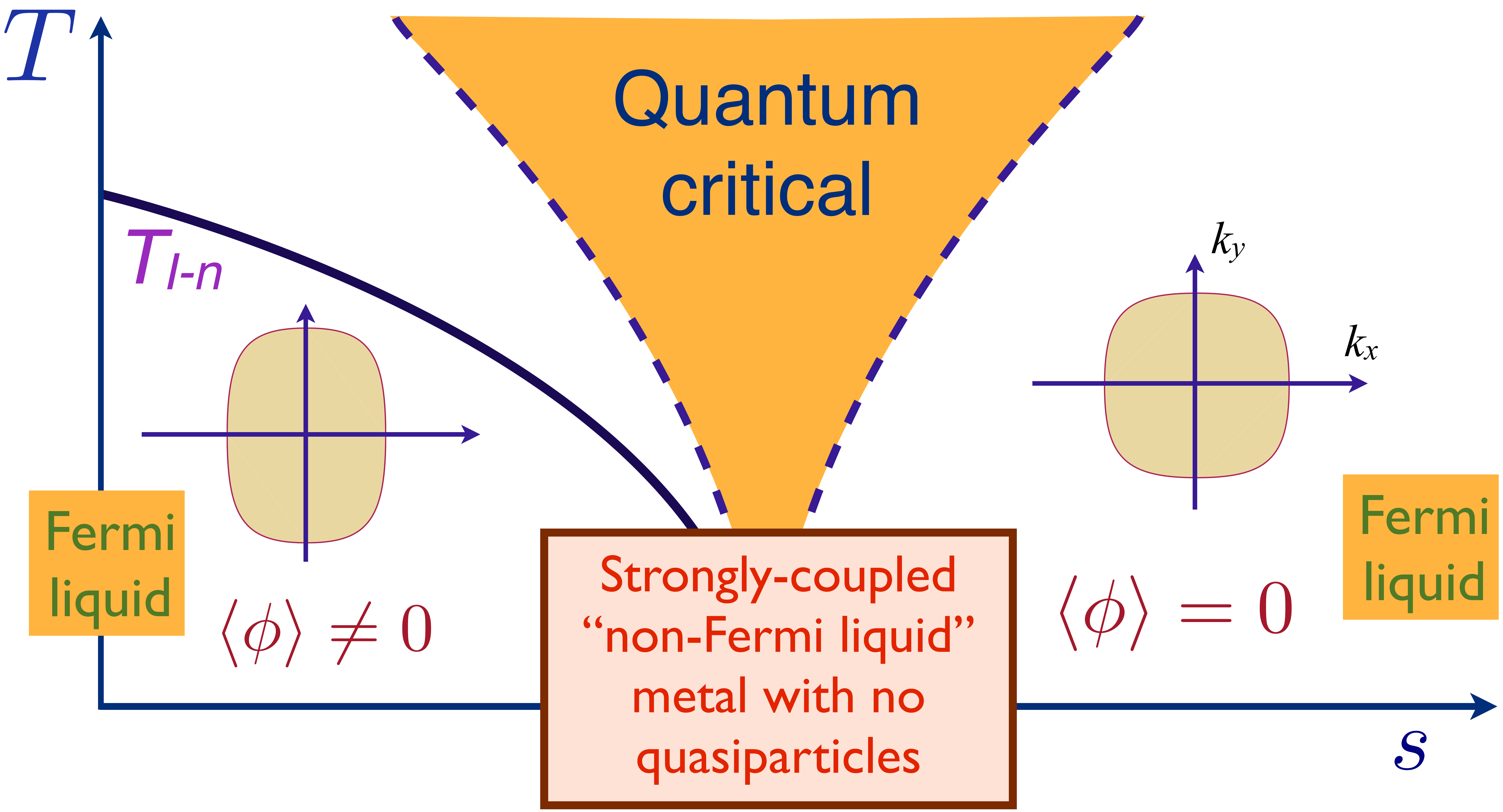
Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022)
Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024)
D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017)
Zhengyan Darius Shi, Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023)

Quantum criticality of Ising-nematic ordering in a metal



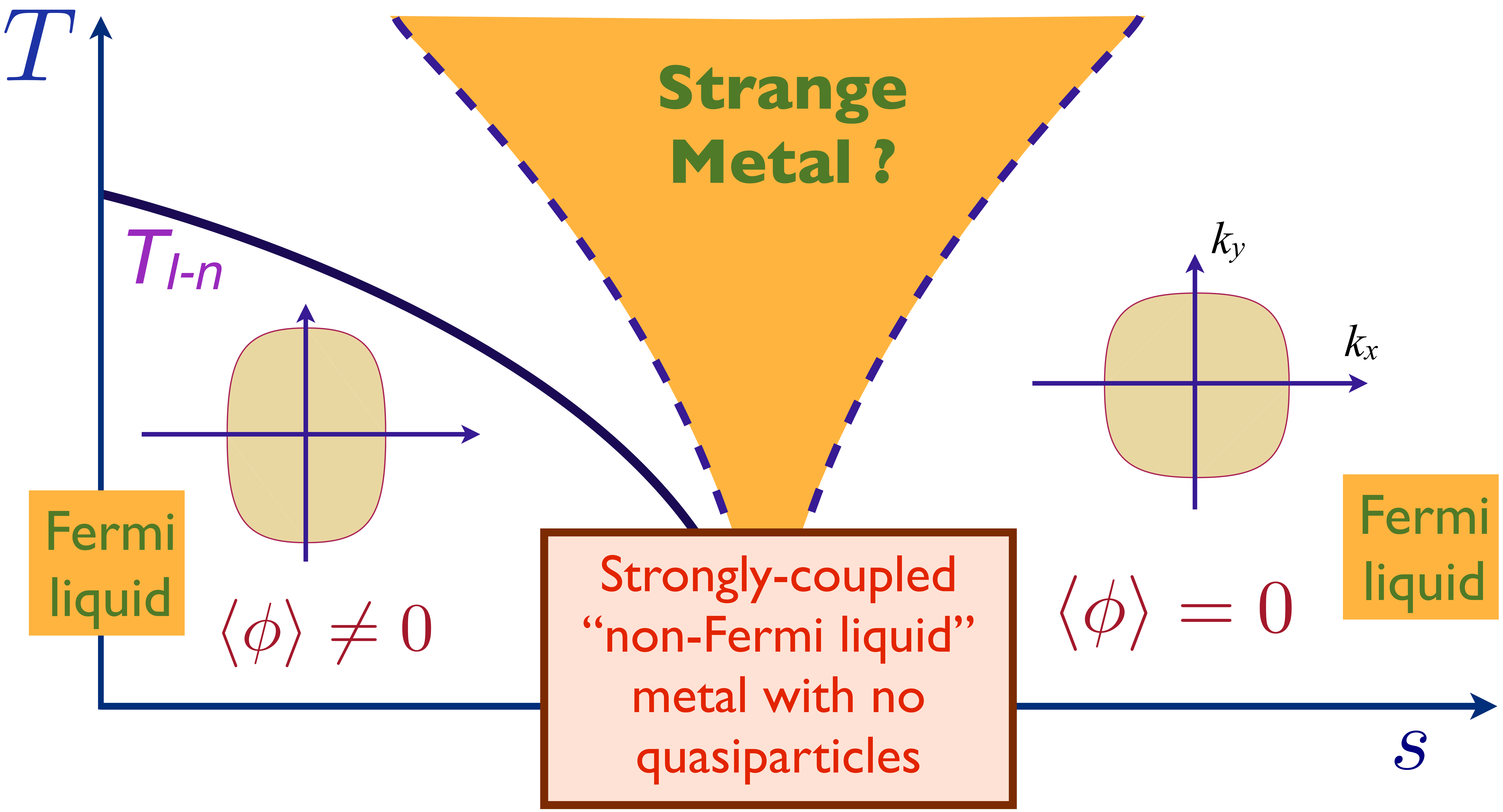
Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlief, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

Quantum criticality of Ising-nematic ordering in a metal

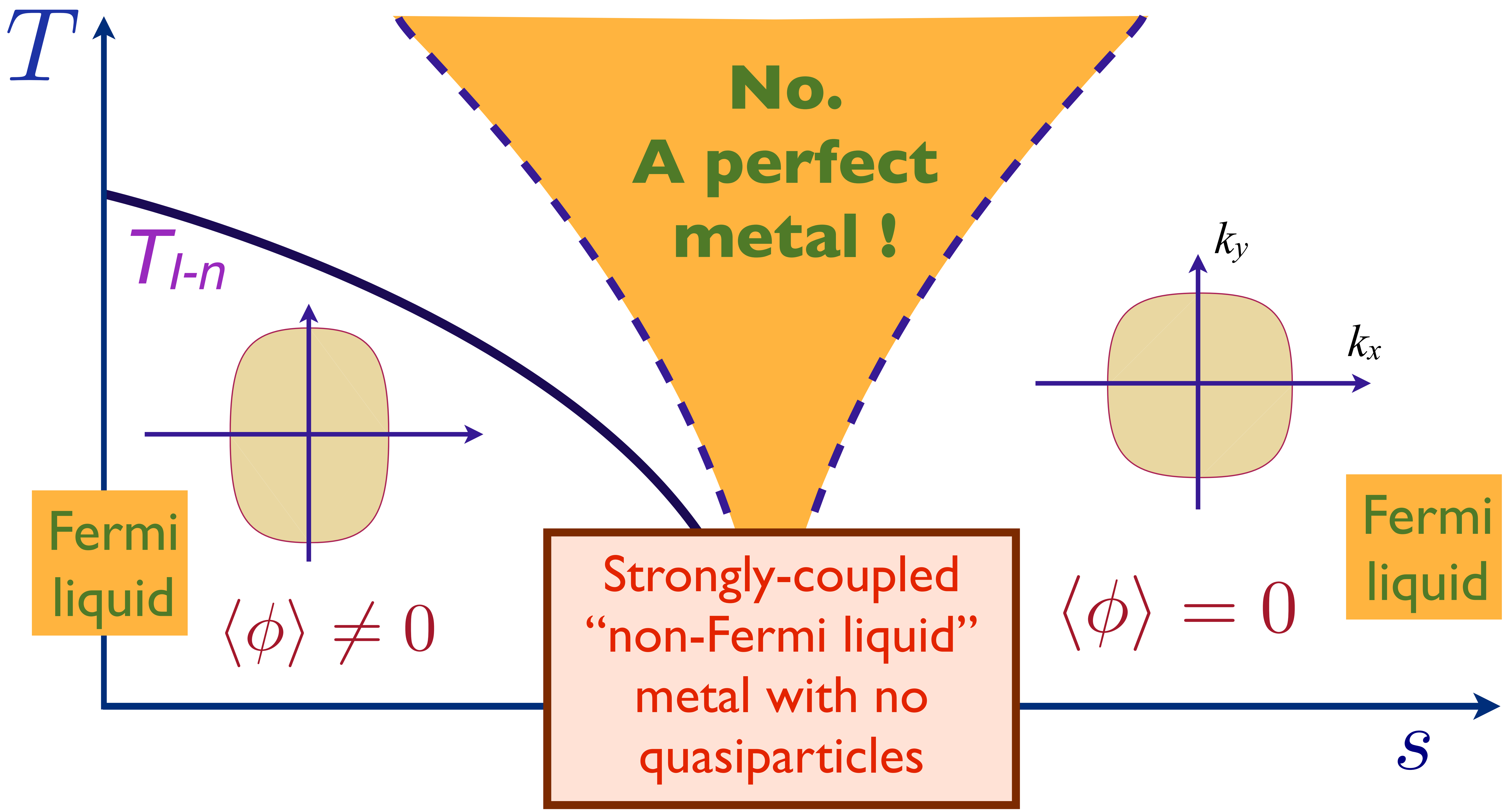


Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlief, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

Quantum criticality of Ising-nematic ordering in a metal



Quantum criticality of Ising-nematic ordering in a metal



I. Yukawa-SYK model

II. Universal Yukawa-SYK theory
in d=2 spatial dimensions

III. Random “mass” Hertz theory at low T

I. Yukawa-SYK model

II. Universal Yukawa-SYK theory in d=2 spatial dimensions

III. Random "mass" Hertz theory at low T

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

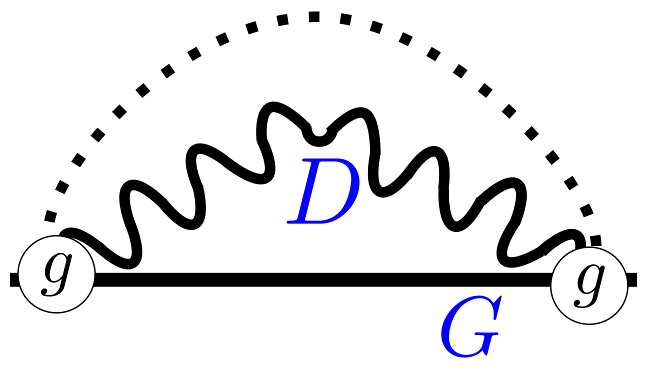
with $g_{ij\ell}$ independent random numbers with $\overline{g_{ij\ell}} = 0$, $\overline{g_{ij\ell}^2} = g^2$.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

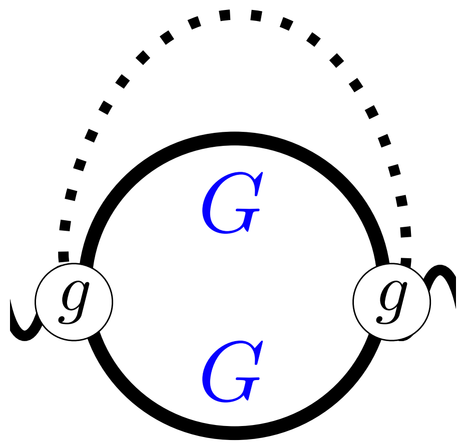
$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with $\overline{g_{ij\ell}} = 0$, $\overline{g_{ij\ell}^2} = g^2$. Large N saddle-point equations:

$\Sigma =$ 

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

$\Pi =$ 

Make the low frequency ansatz

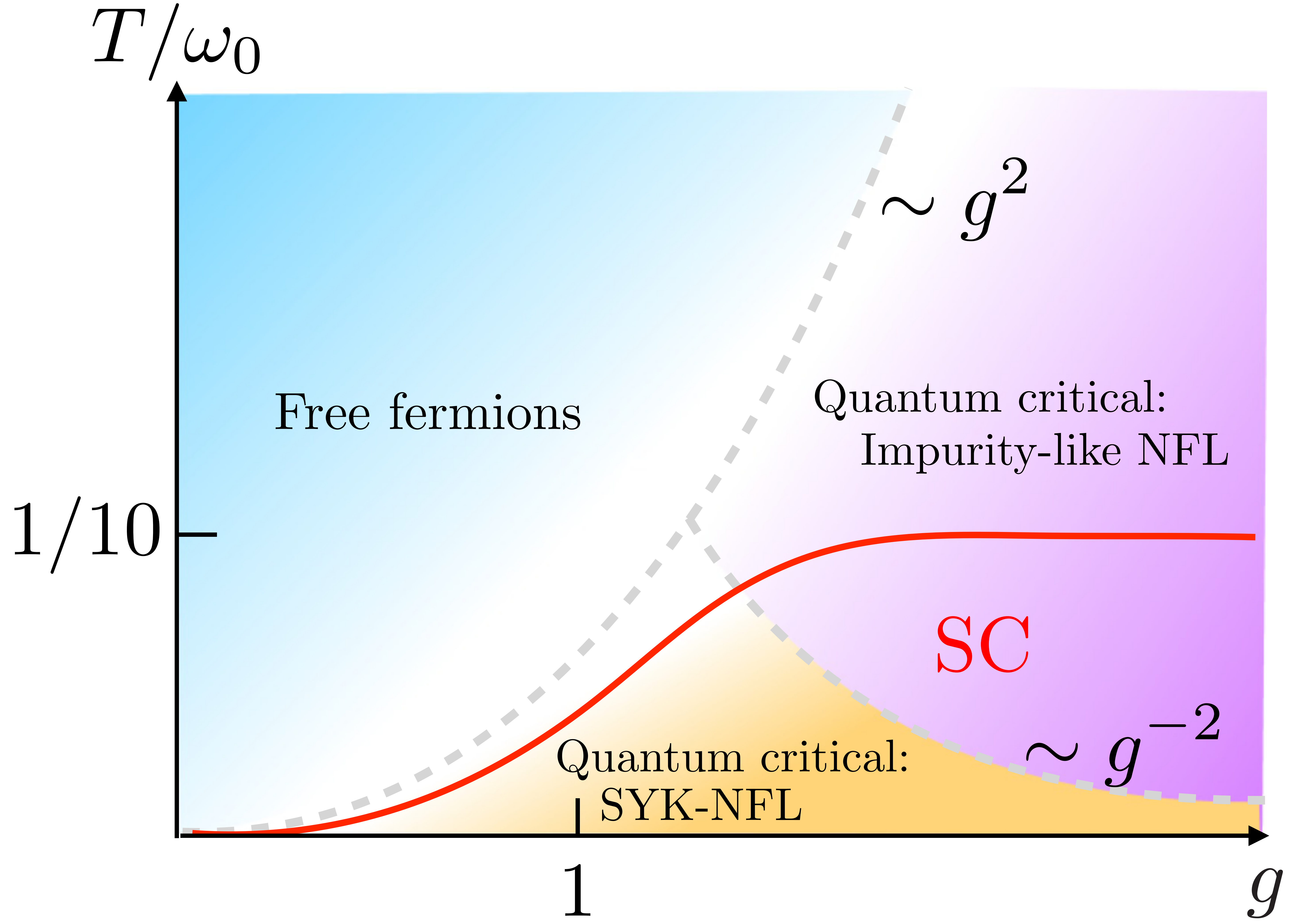
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK models



I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

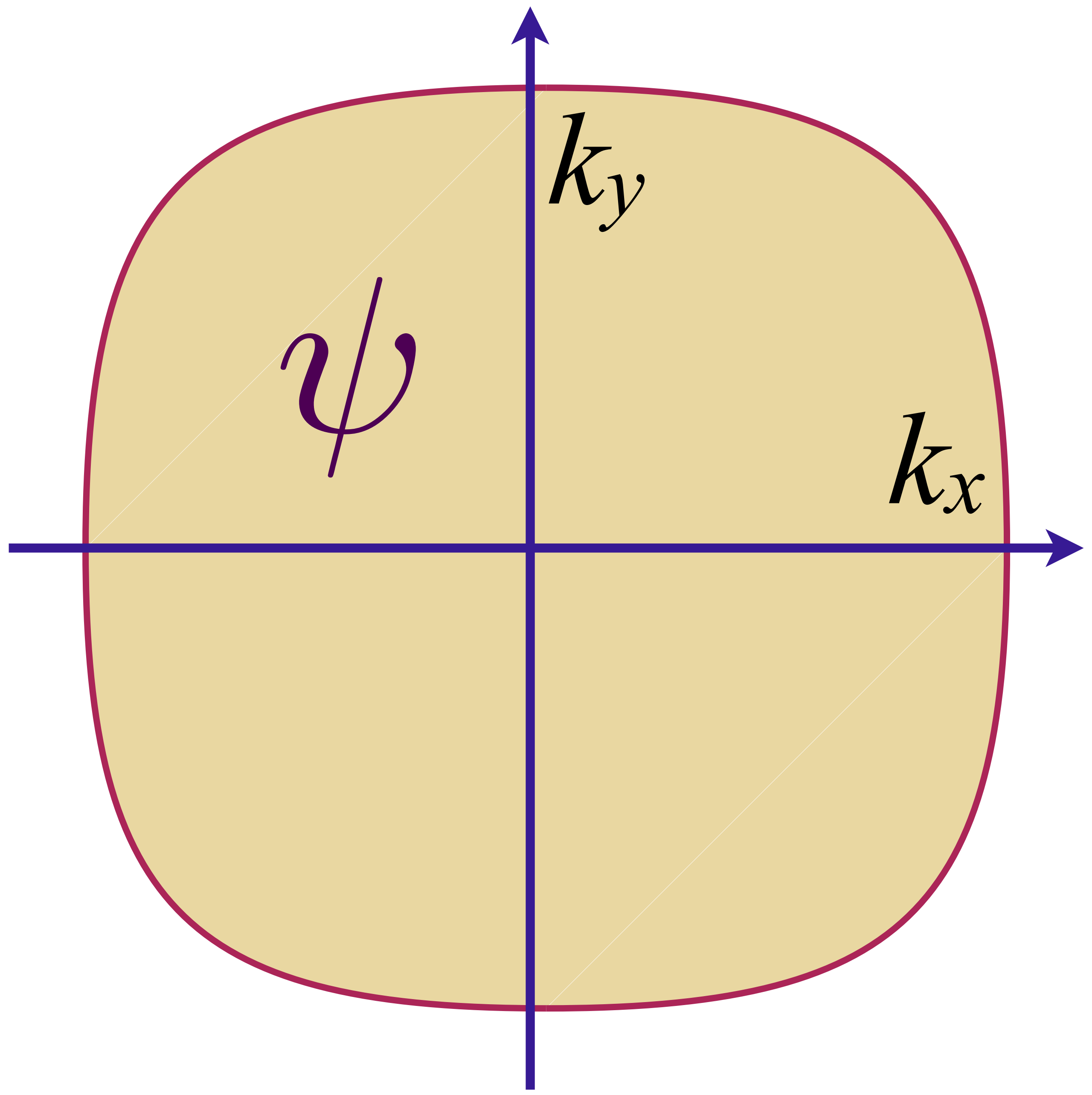
I. Yukawa-SYK model

II. Universal Yukawa-SYK theory in d=2 spatial dimensions

III. Random “mass” Hertz theory at low T

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



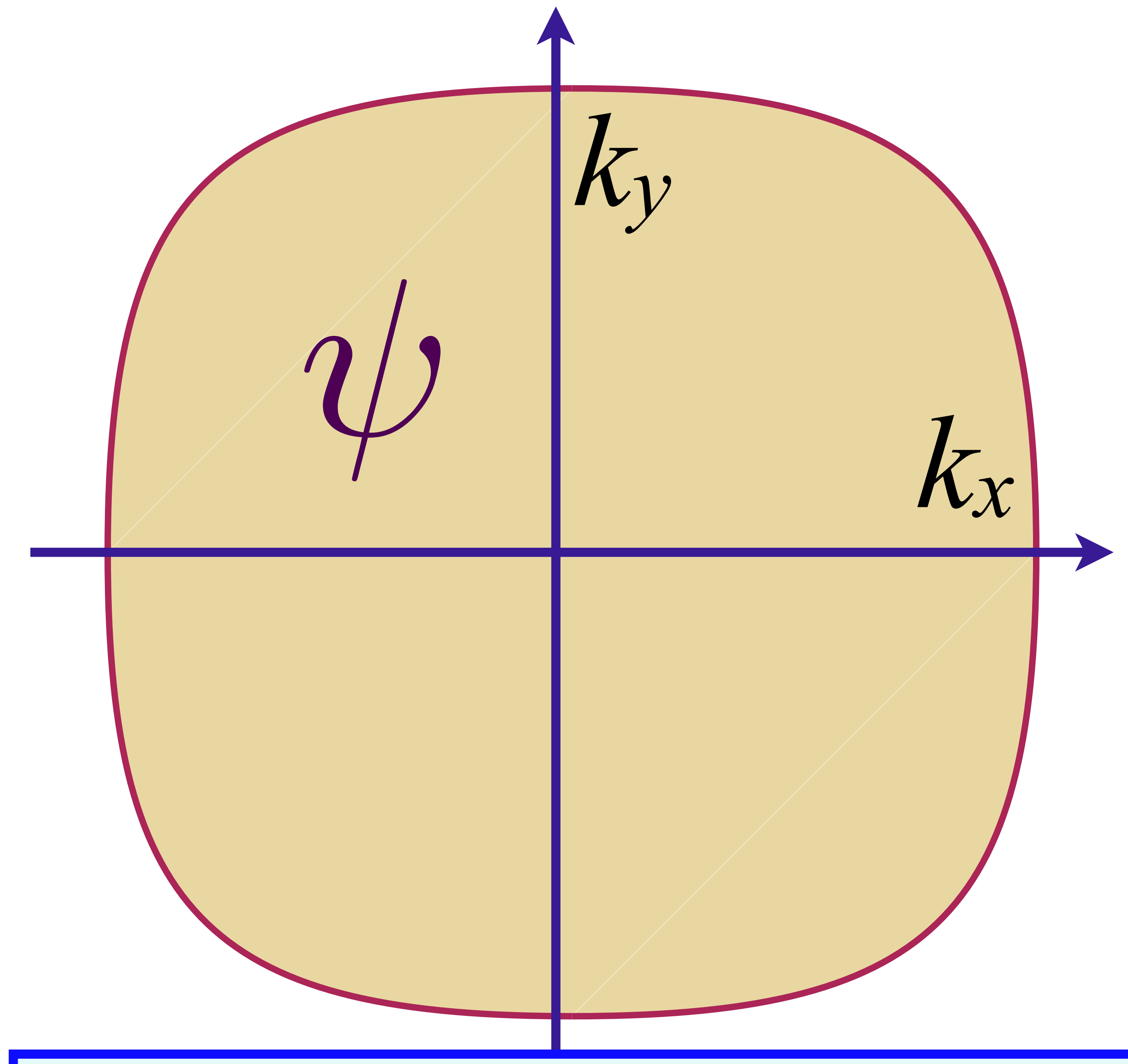
A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

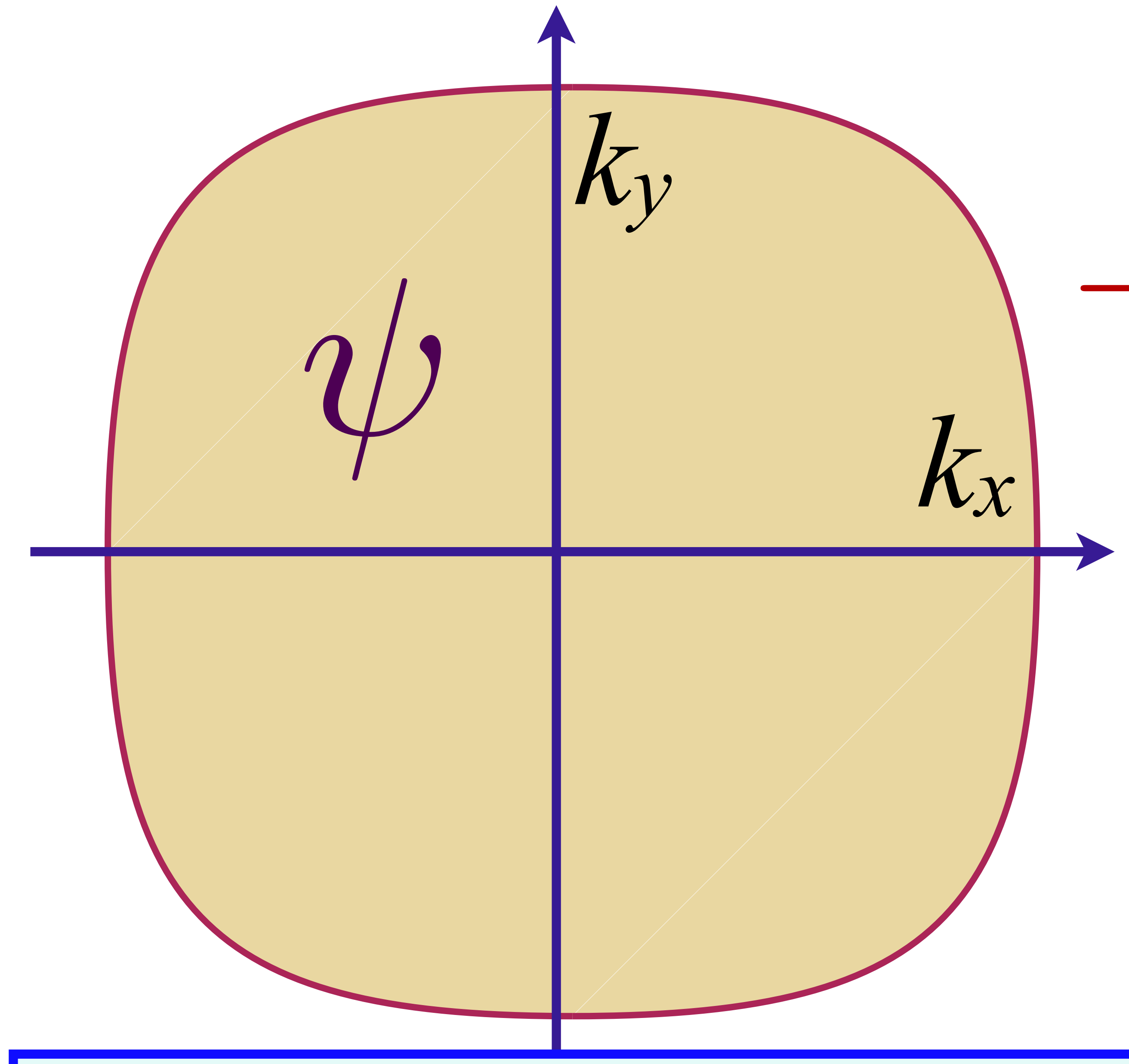
$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 \; + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition



$$+[s + \delta s(\boldsymbol{r})] \left[\phi(\boldsymbol{r})\right]^2 + \quad + g\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+K\left[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})\right]^2 + u\left[\phi(\boldsymbol{r})\right]^4 \; + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

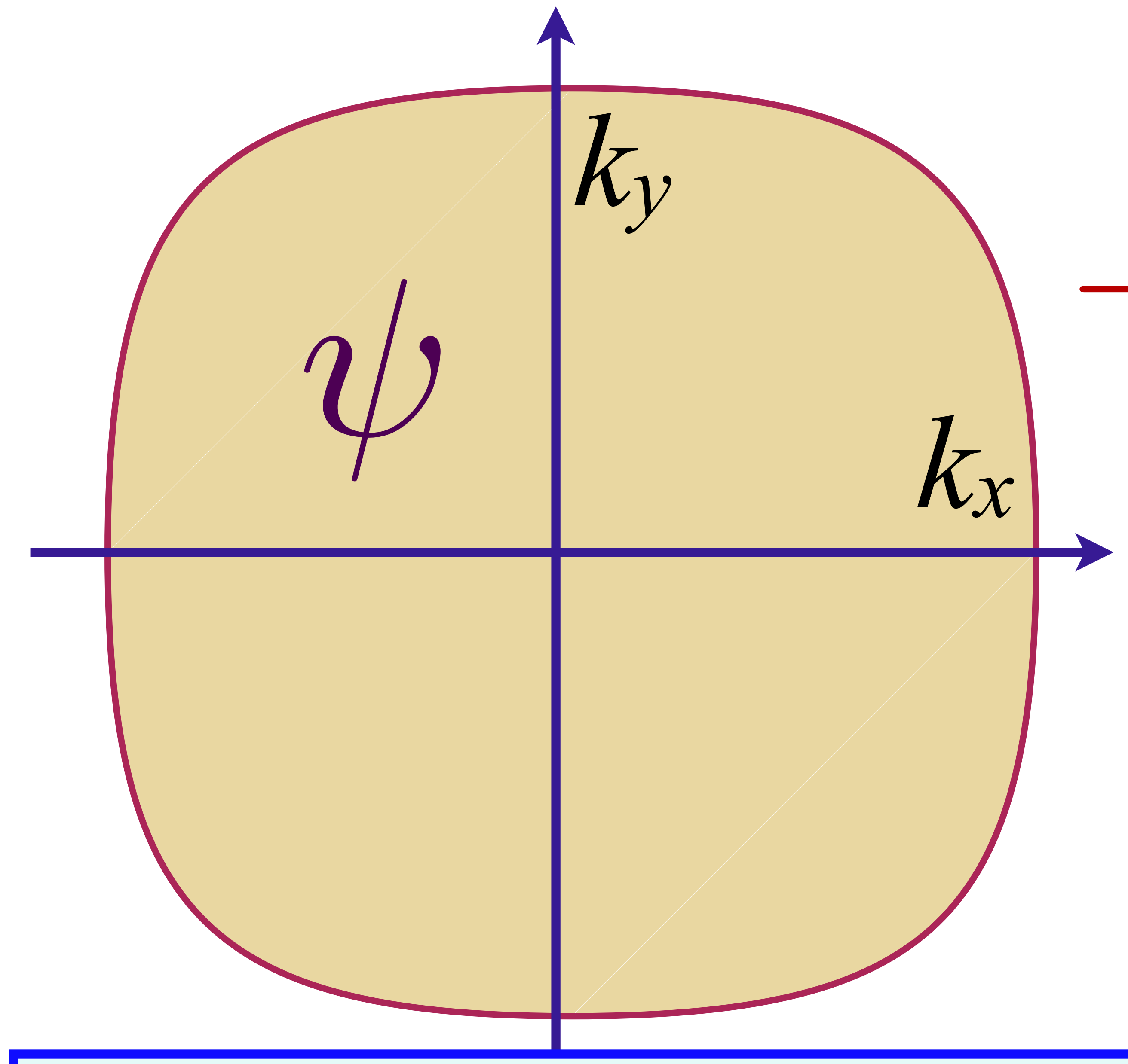
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})} = 0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')} = \delta s^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition



$$+[s + \delta s(\boldsymbol{r})] \, [\phi(\boldsymbol{r})]^2 + \qquad + g \, \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r}) \, \phi(\boldsymbol{r})$$

$$+K \, [\nabla_{\boldsymbol{r}} \phi(\boldsymbol{r})]^2 + u \, [\phi(\boldsymbol{r})]^4 \; + v(\boldsymbol{r}) \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r})$$

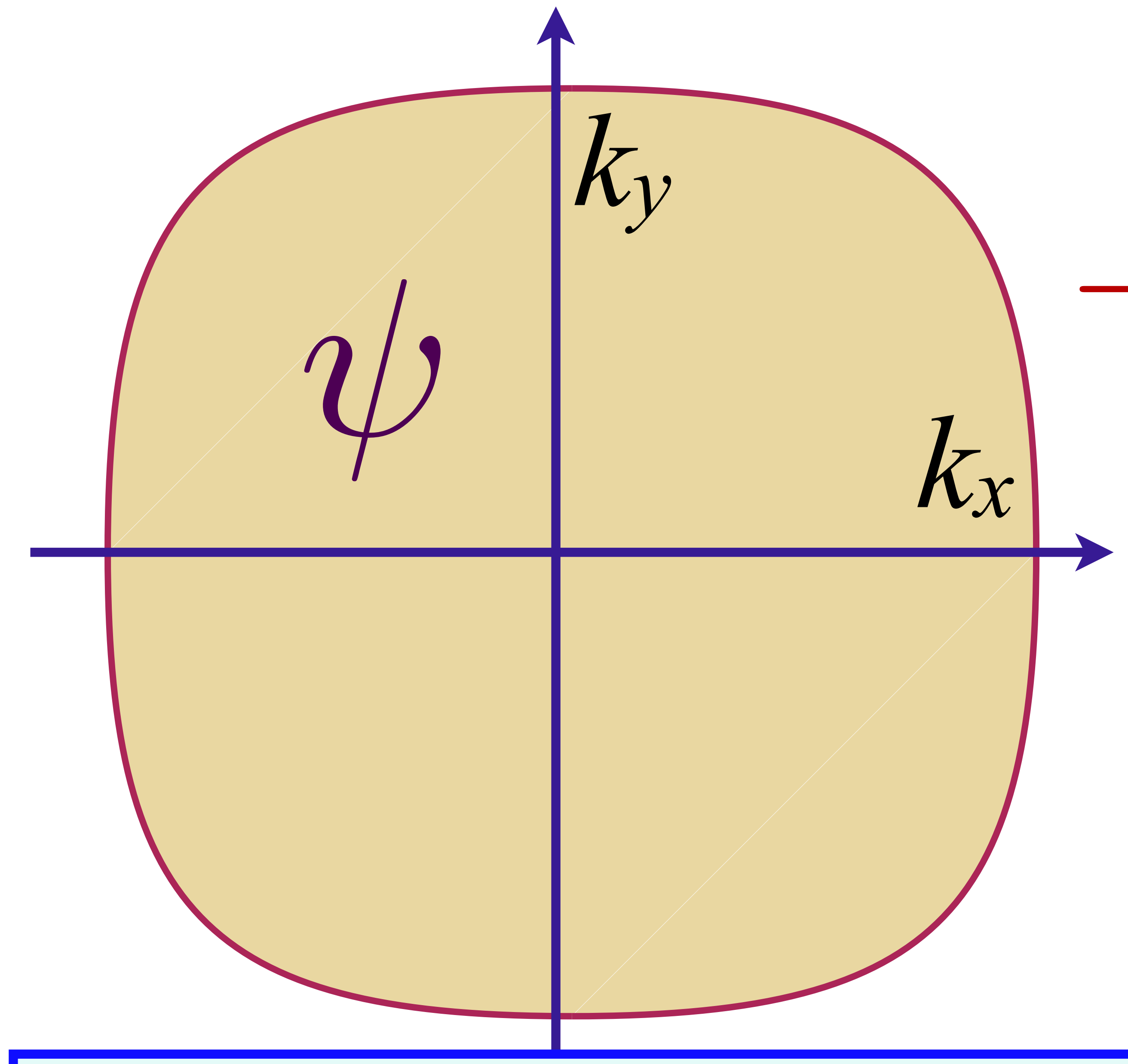
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r}) v(\boldsymbol{r}')} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})} = 0$, $\overline{\delta s(\boldsymbol{r}) \delta s(\boldsymbol{r}')} = \delta s^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

RG analysis (Harris criterion) shows that $\delta s(\boldsymbol{r})$ is most relevant disorder.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+[s + \delta s(\boldsymbol{r})]\,[\phi(\boldsymbol{r})]^2 + \qquad + g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

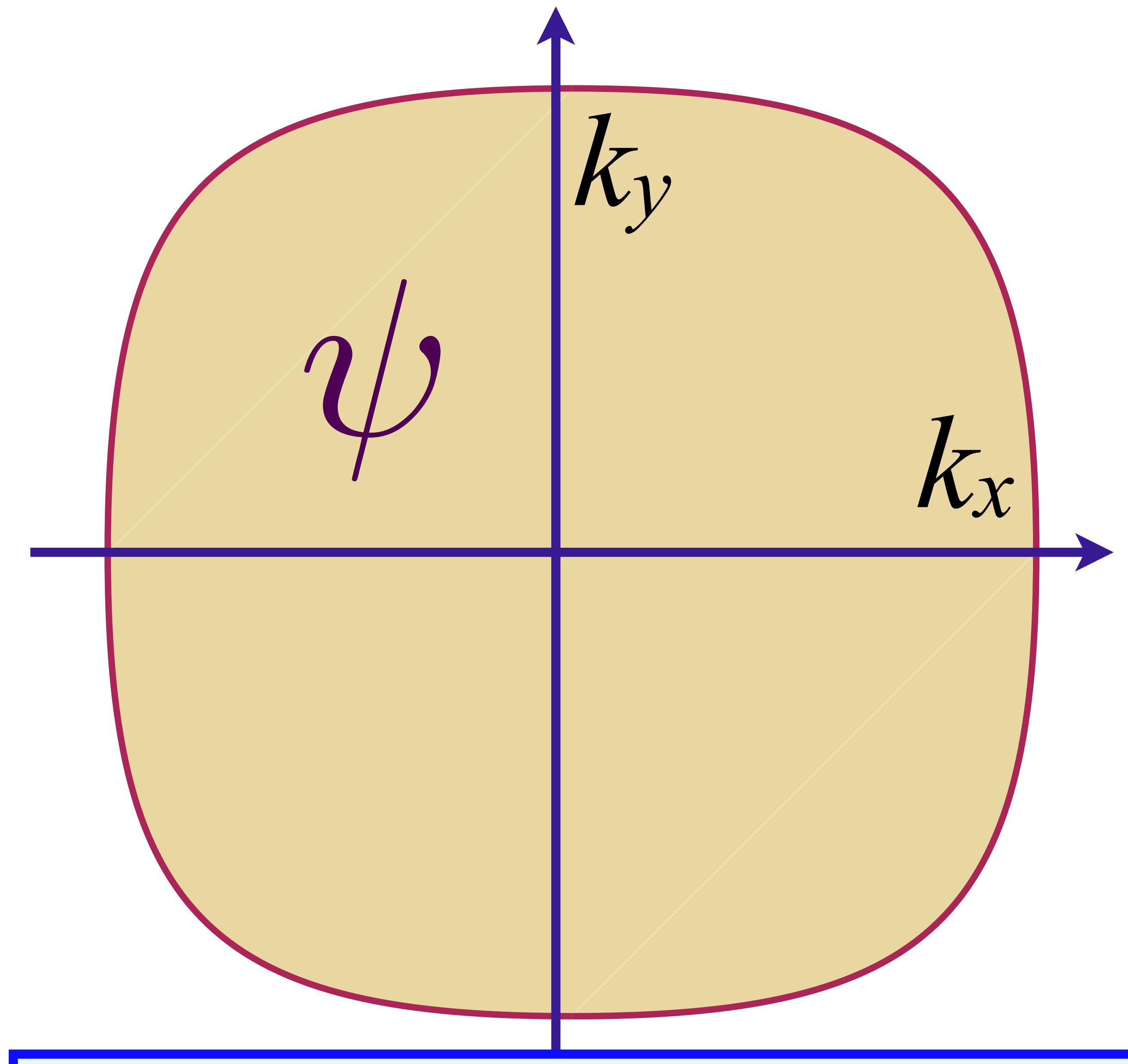
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})} = 0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')} = \delta s^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

RG analysis (Harris criterion) shows that $\delta s(\boldsymbol{r})$ is most relevant disorder.
Rescale $\phi(\boldsymbol{r})$ to obtain a theory with $\delta s(\boldsymbol{r}) = 0$.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

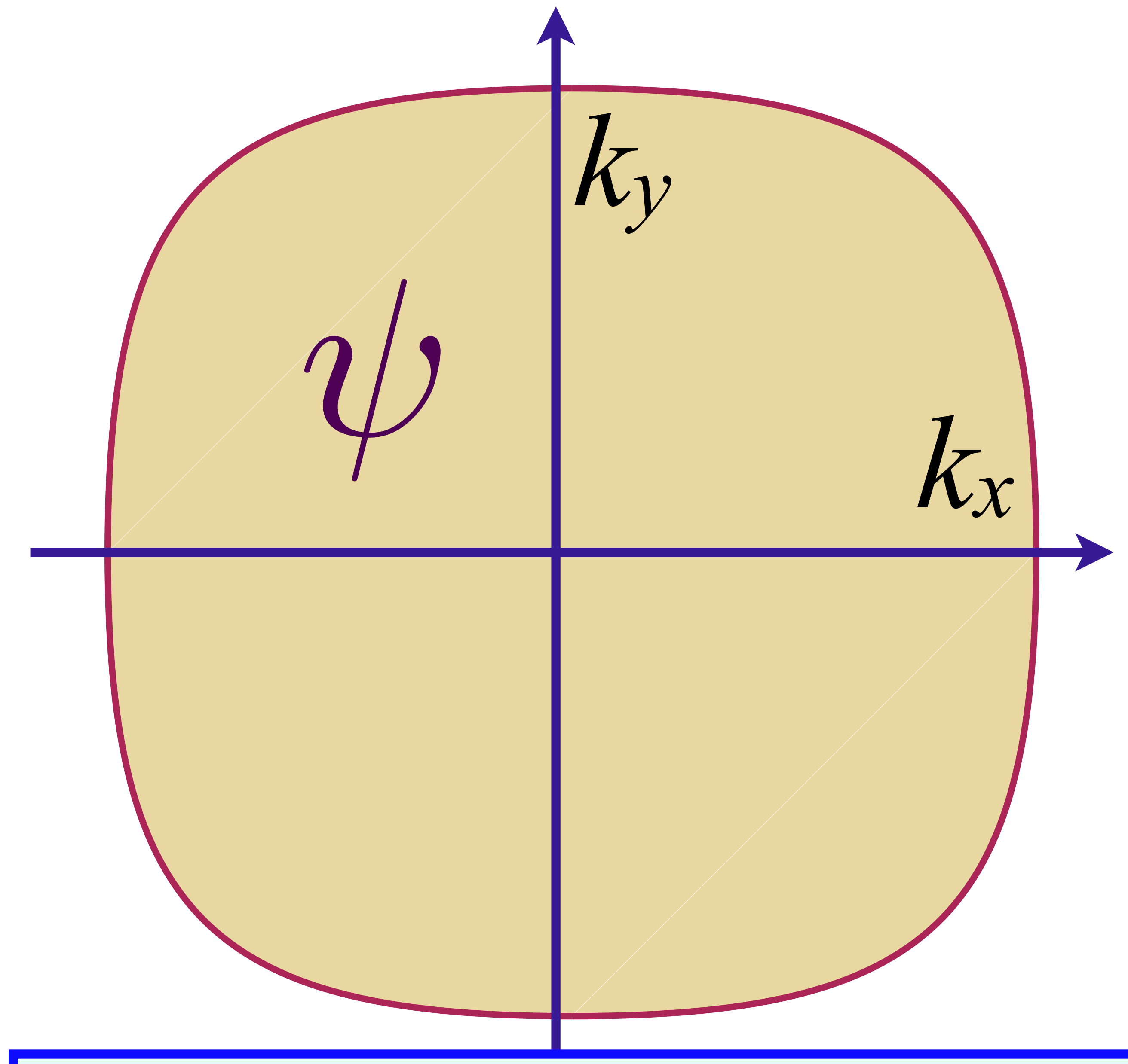
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

RG analysis (Harris criterion) shows that $\delta s(\boldsymbol{r})$ is most relevant disorder.
Rescale $\phi(\boldsymbol{r})$ to obtain a theory with $\delta s(\boldsymbol{r}) = 0$.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

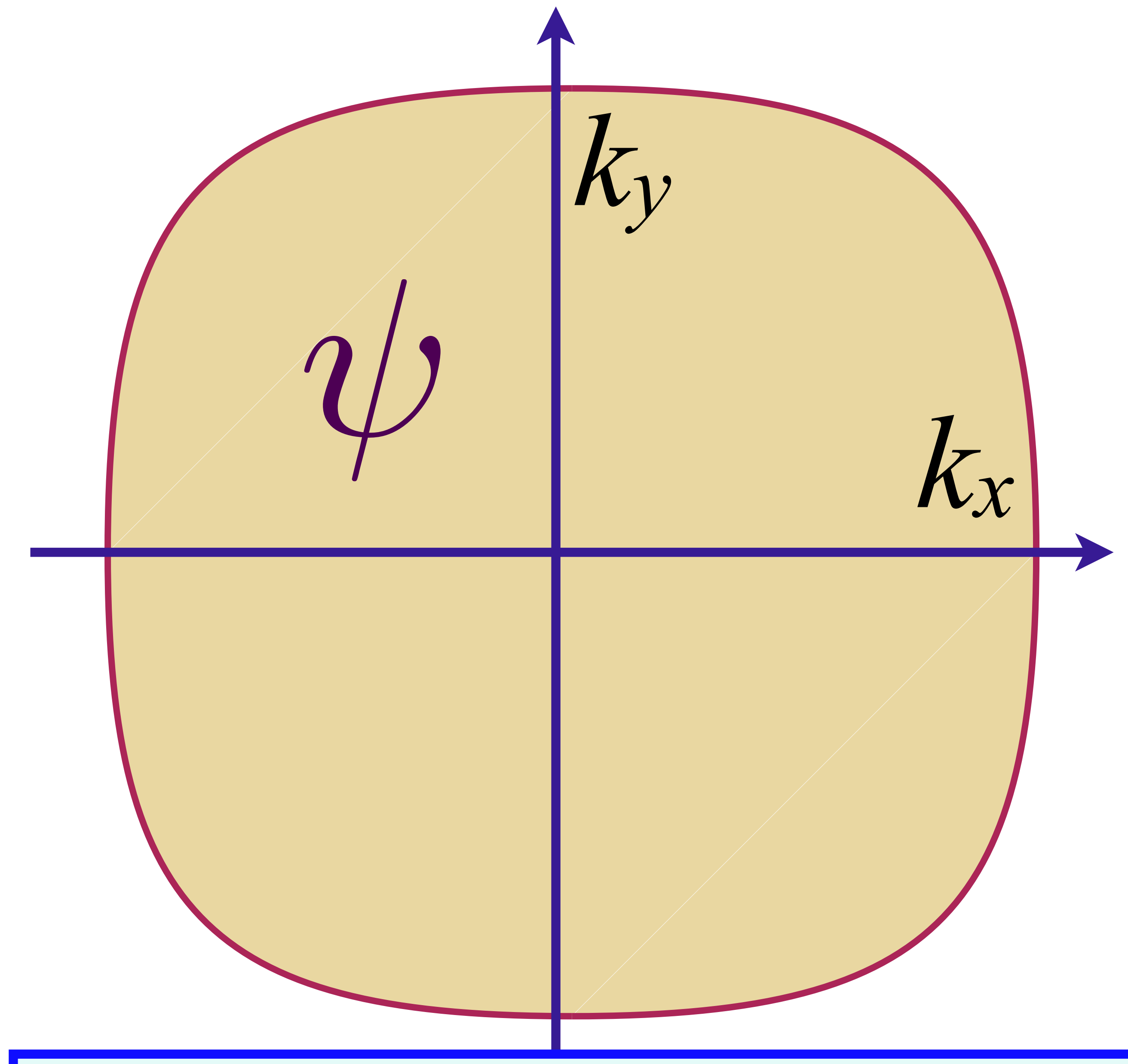
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Analyze such a theory in a self-averaging manner as in the Yukawa-SYK model.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Analyze such a theory in a self-averaging manner as in the Yukawa-SYK model.
Should be applicable as long as eigenmodes of $\phi(\boldsymbol{r})$ are extended.

Fermi surface + critical boson with potential and interaction disorder

SYK-type self-consistent equations

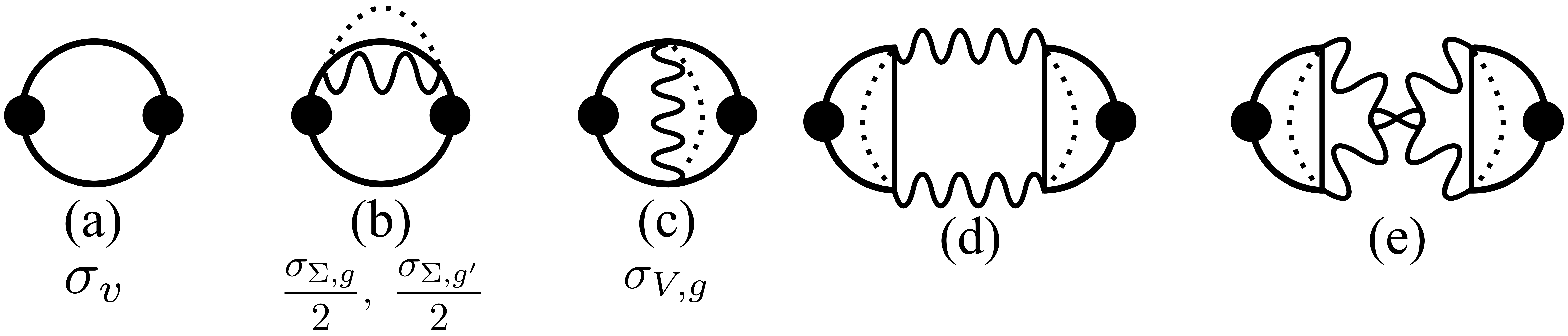
$$\Sigma(\tau,\mathbf{r}) = g^2 D(\tau,\mathbf{r}) G(\tau,\mathbf{r}) + v^2 G(\tau,\mathbf{r})\delta^2(\mathbf{r}) + {g'}^2 G(\tau,\mathbf{r}) D(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$

$$\Pi(\tau,\mathbf{r}) = -g^2 G(-\tau,-\mathbf{r}) G(\tau,\mathbf{r}) - {g'}^2 G(-\tau,\mathbf{r}) G(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$

$$G(i\omega,\mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega,\mathbf{k})},$$

$$D(i\Omega,\mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega,\mathbf{q})}.$$

Conductivity:



+ all ladders and bubbles…..

Fermi surface + critical boson with potential and interaction disorder

Electron Green's function: $$G(\omega) \sim \frac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)}\right)\mathrm{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S.S., *Science* **381**, 790 (2023)

Fermi surface + critical boson with potential and interaction disorder

Electron Green's function: $$G(\omega) \sim \frac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)}\right)\mathrm{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Conductivity: $$\sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega\dfrac{m^*_{\rm trans}(\omega)}{m}}$$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

B. Michon……A. Georges, Nat. Commun. **14**, 3033 (2023)

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.
Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2;
Transport insensitive to g;

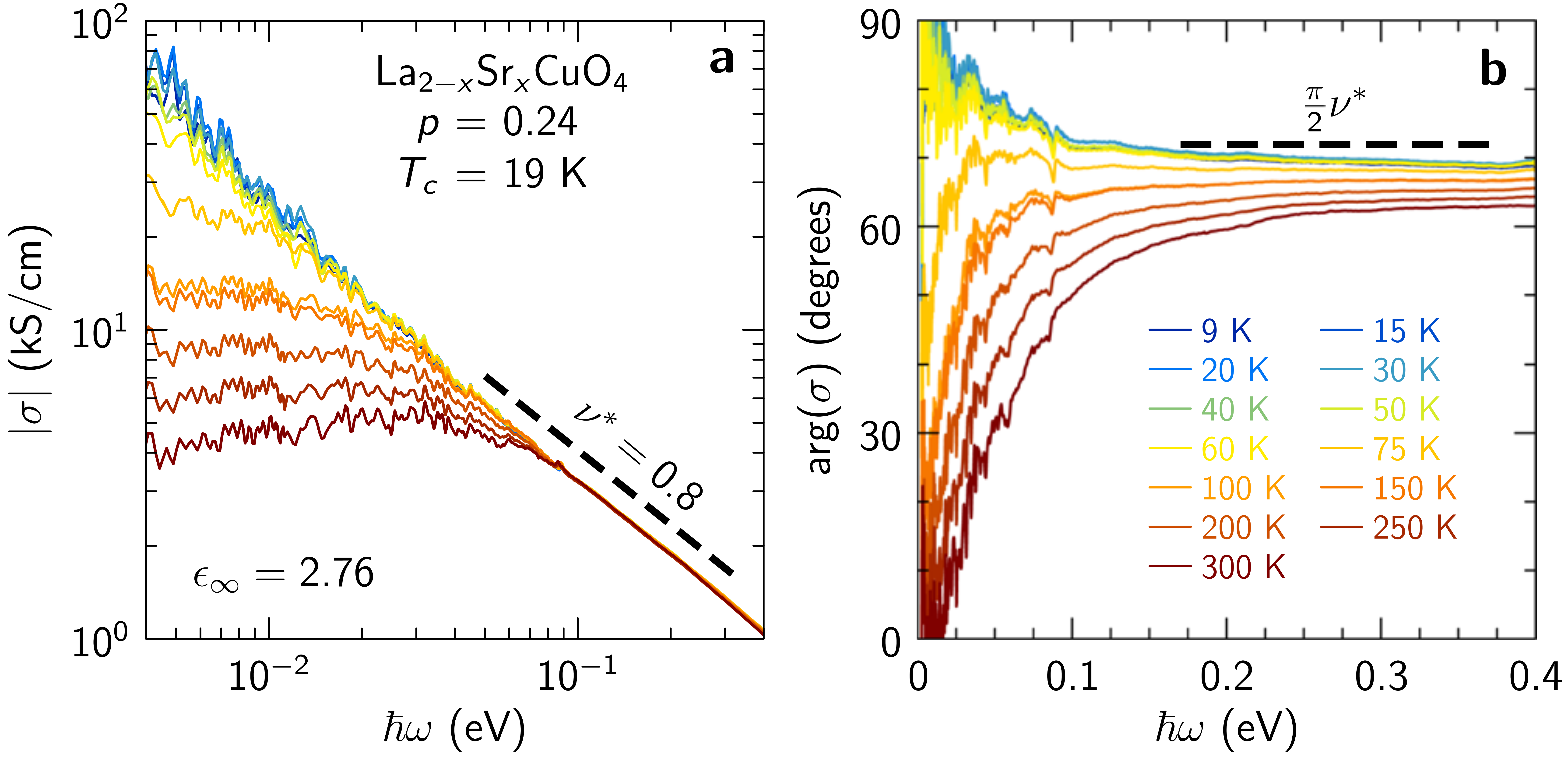
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S.S., *Science* **381**, 790 (2023)

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$

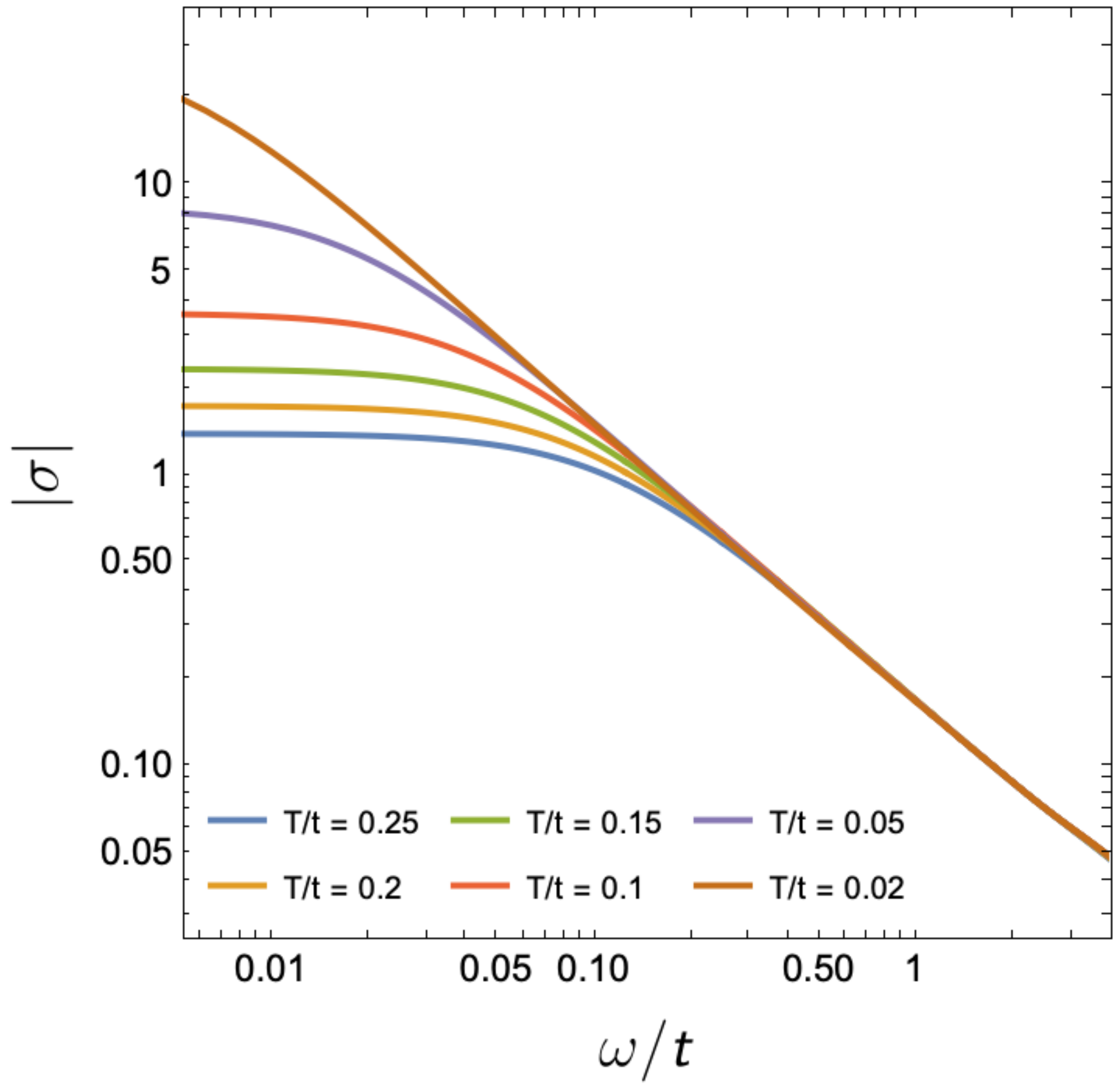


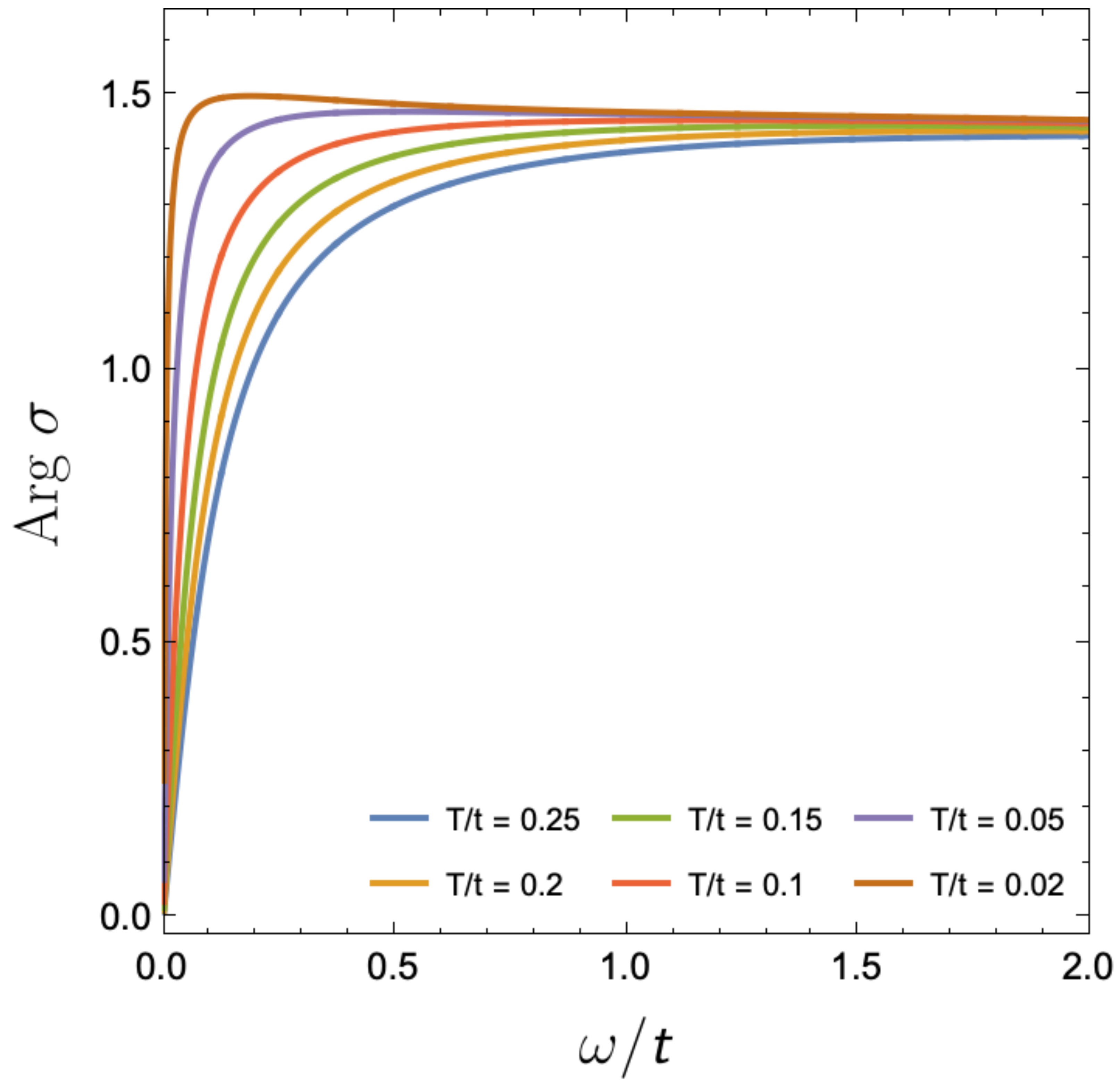
Strange metal and superconductor in the two-dimensional Yukawa-SYK model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esters, to appear

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \dfrac{m^*(\omega)}{m} + i \dfrac{\hbar}{\tau(\omega)}}$$



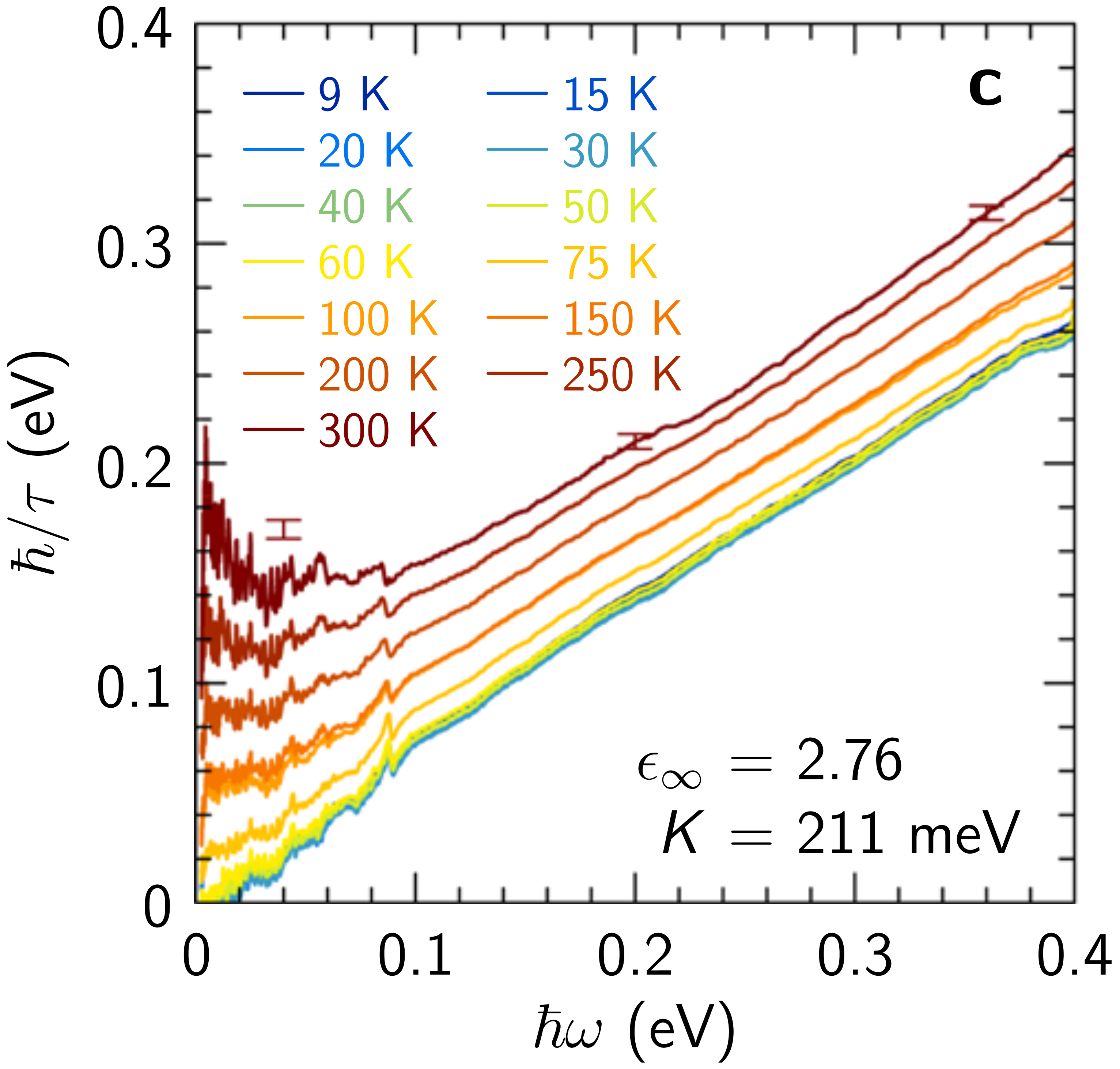


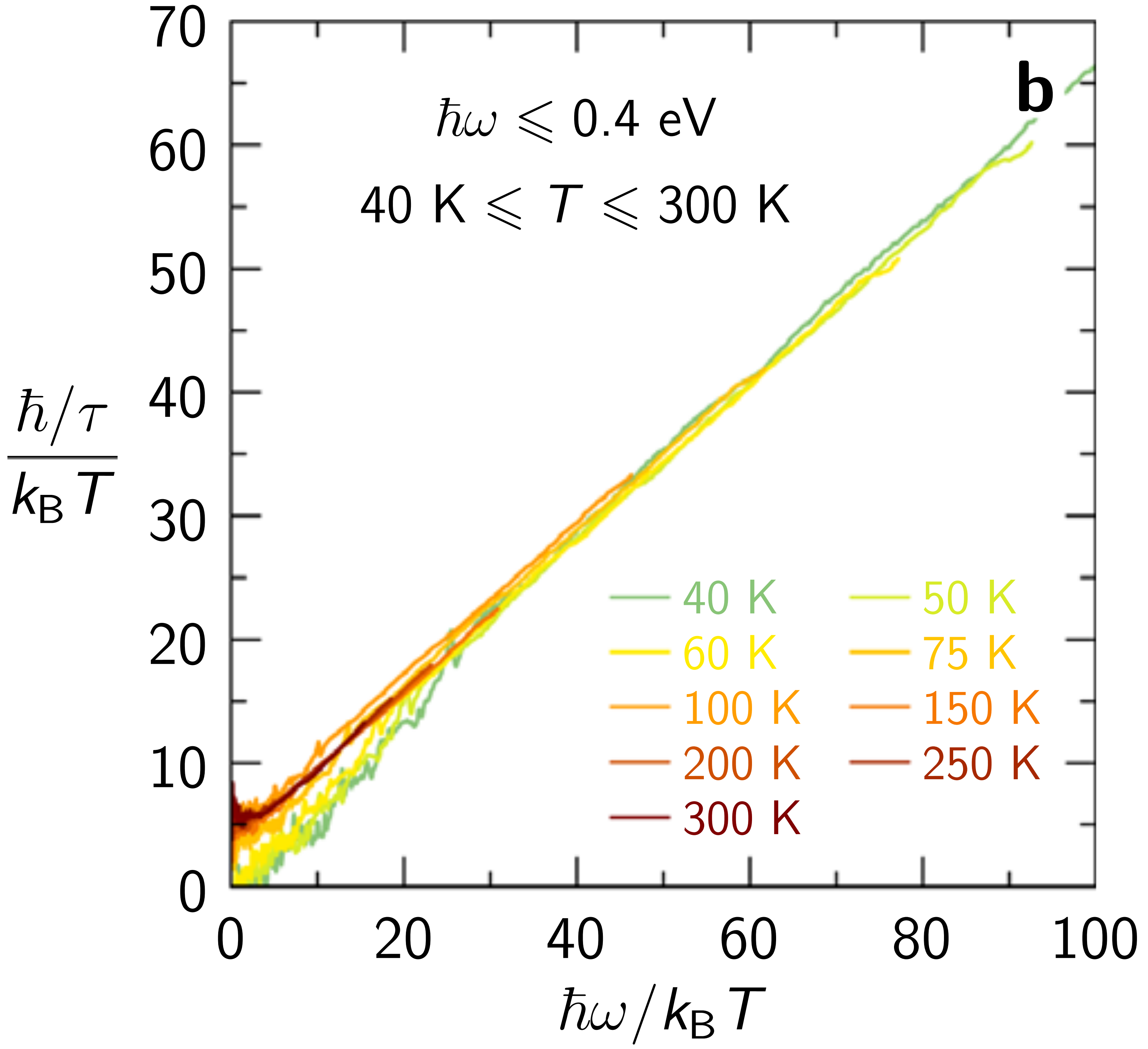
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$



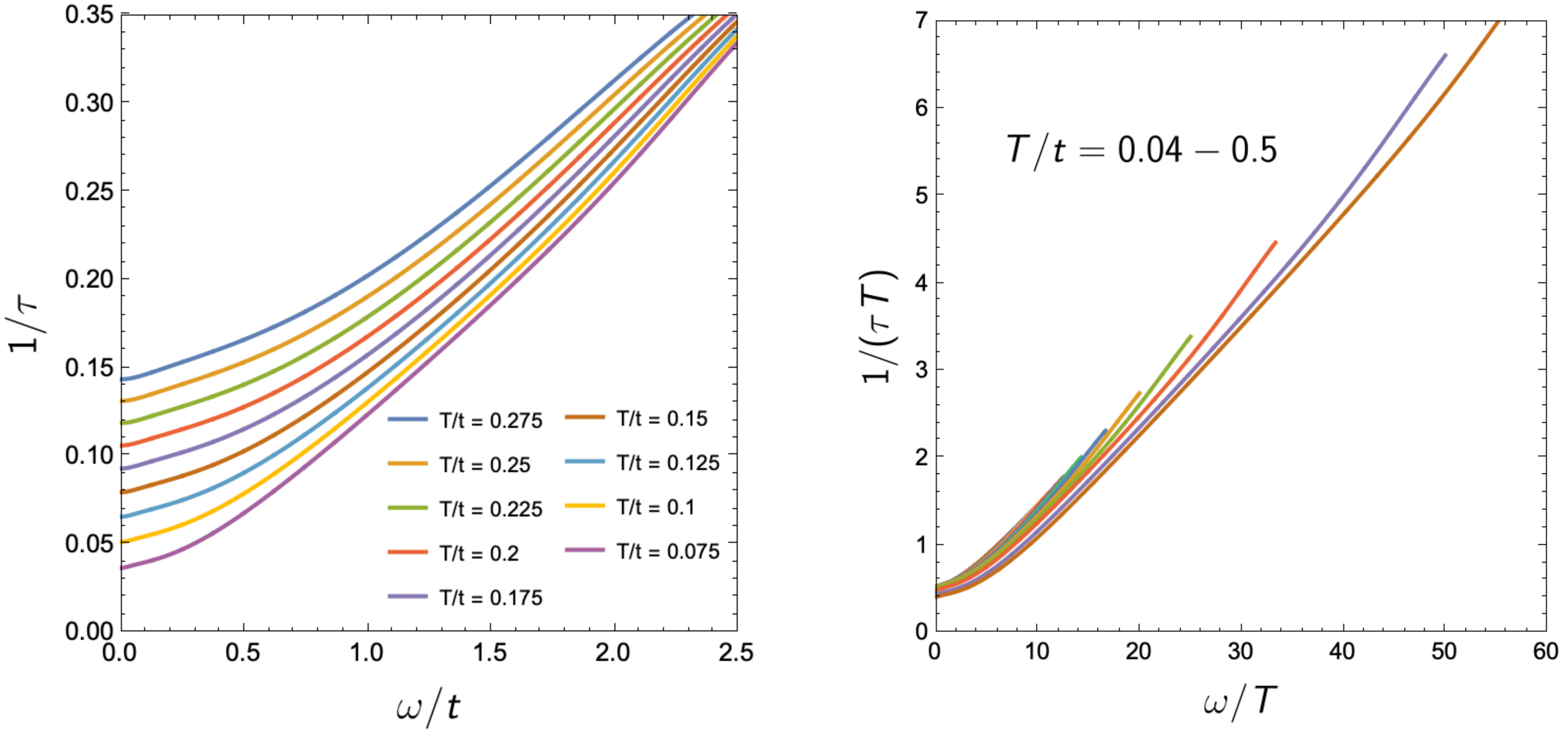


Strange metal and superconductor in the two-dimensional Yukawa-SYK model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esters, to appear

$$\sigma(\omega) = i\frac{e^2 K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

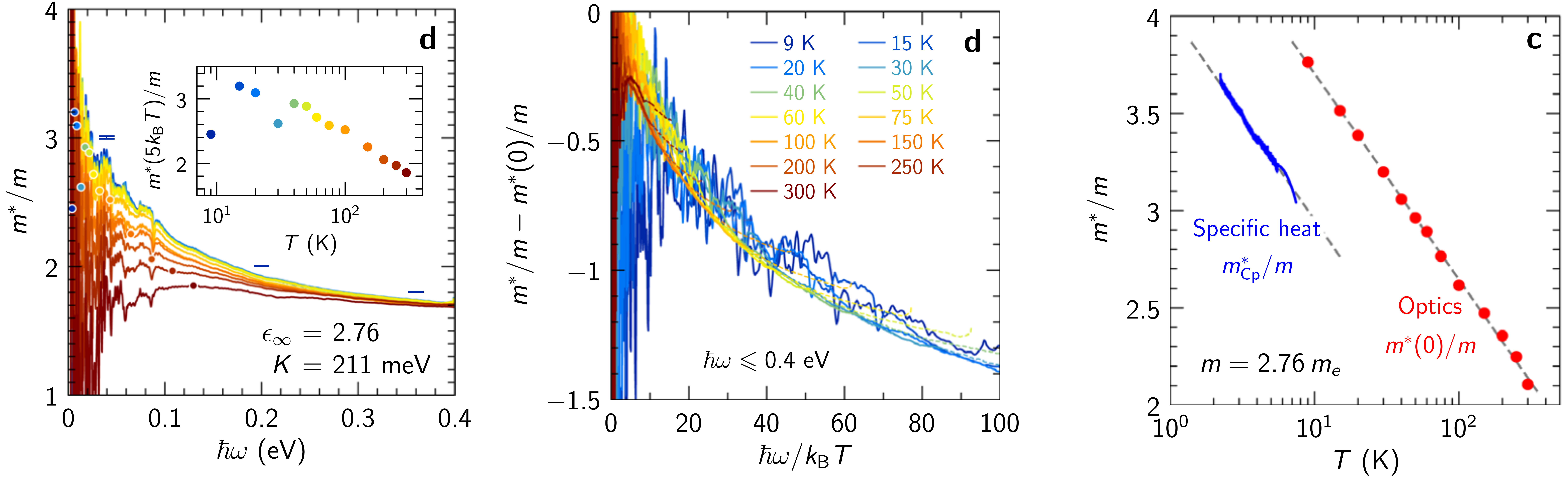


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

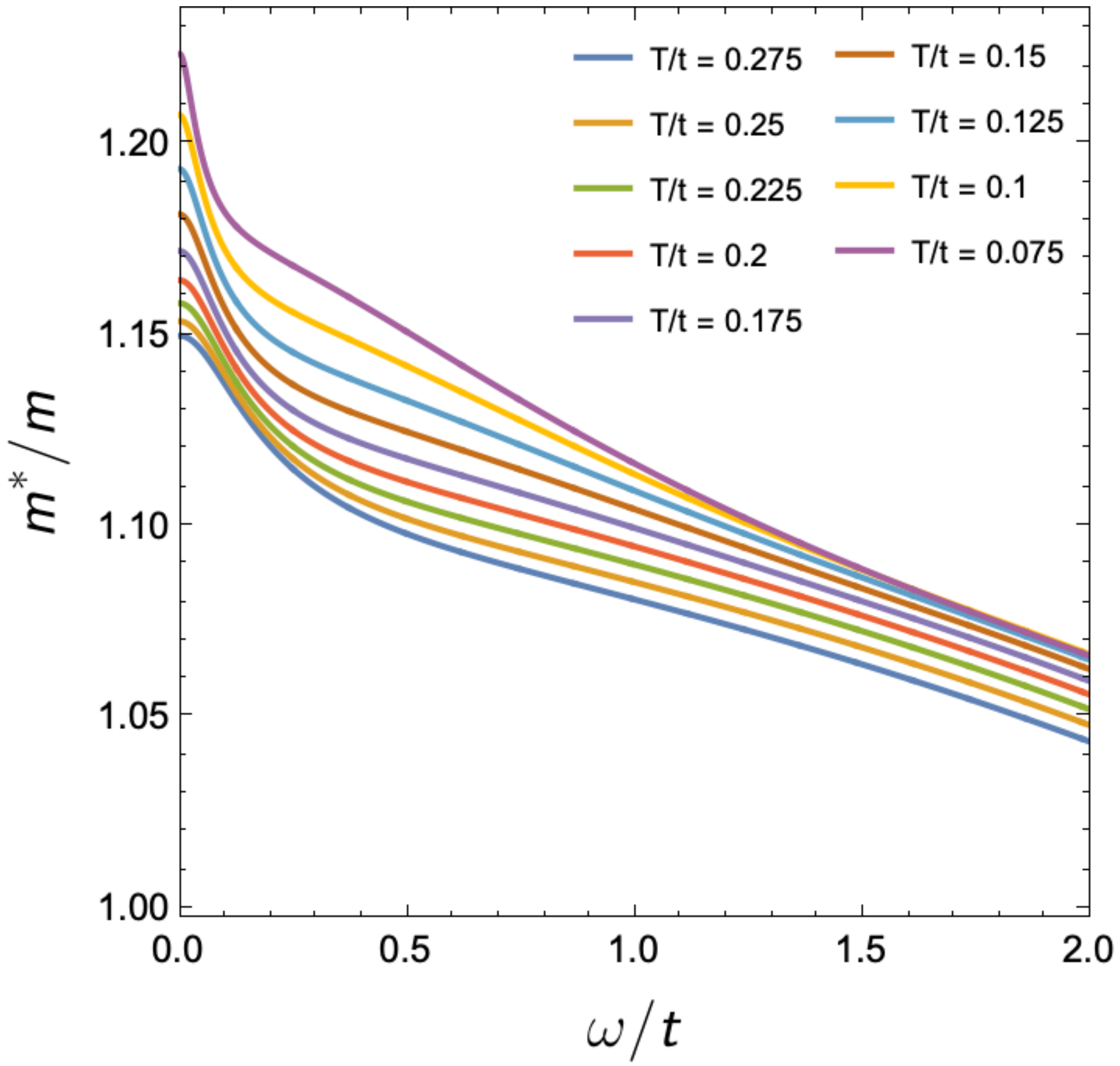


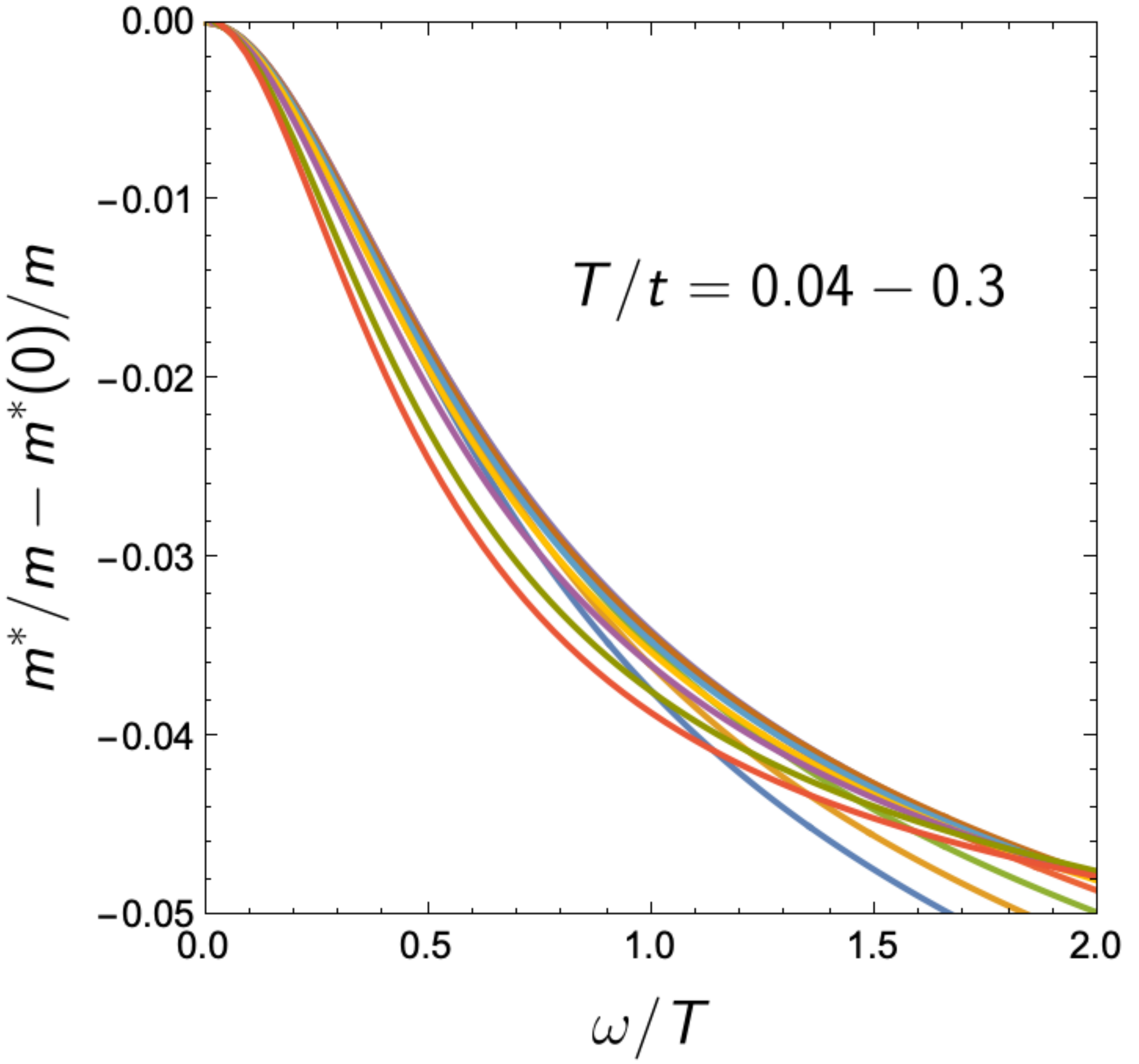
$g = 0$

Strange metal and superconductor in the two-dimensional Yukawa-SYK model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esters, to appear

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$





I. Yukawa-SYK model

II. Universal Yukawa-SYK theory in d=2 spatial dimensions

III. Random "mass" Hertz theory at low T

Quantum Interference of Hydrodynamic Modes in a Dirty Marginal Fermi Liquid

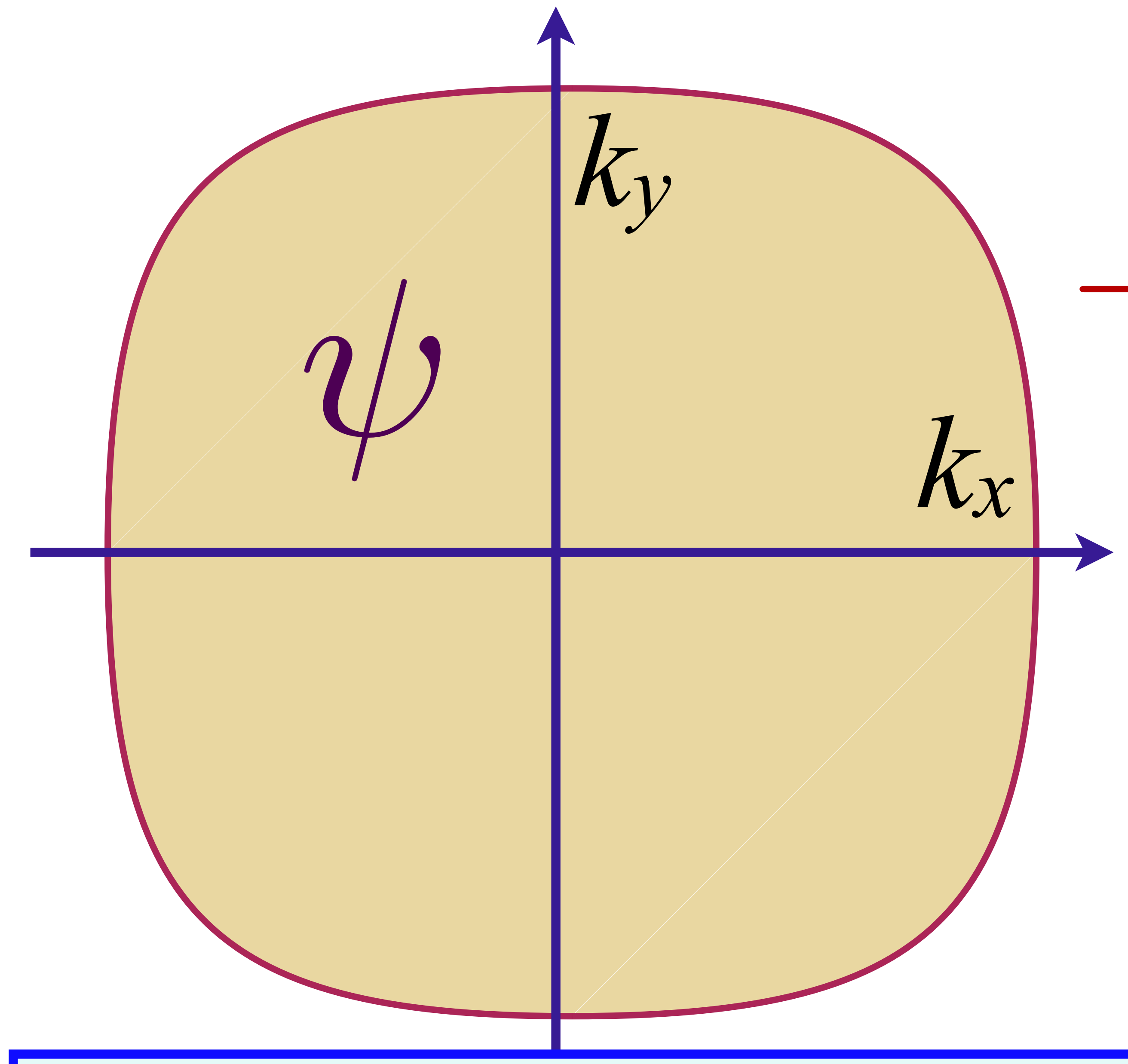
Tsz Chun Wu, Yunxiang Liao, Matthew S. Foster
Phys. Rev. B **106**, 155108 (2022)

Diffusion (Altshuler-Aronov) corrections to conductivity are singular $\sim -1/T$.

Our interpretation:
need to consider the feedback on the boson propagator from stronger disorder, where the dominant effect is the localization of overdamped bosonic modes …

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume changing transition

$$+[s + \delta s(\boldsymbol{r})]\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})} = 0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')} = \delta s^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

RG analysis (Harris criterion) shows that $\delta s(\boldsymbol{r})$ is most relevant disorder.
Mapping of $\delta s(\boldsymbol{r})$ to $g'(\boldsymbol{r})$ only works if eigenmodes of $\phi(\boldsymbol{r})$ are extended.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \Big(\frac{s + s'_j}{2} \phi_{ja}^2 + \frac{u}{4M} \left(\phi_{ja}^2\right)^2 \Big) \Big]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j \left(\gamma |\Omega| + \Omega^2/c^2 \right) |\phi_{ja}(i\Omega)|^2 \,,$$

where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \Big(\frac{s + s'_j}{2} \phi_{ja}^2 + \frac{u}{4M} \left(\phi_{ja}^2\right)^2 \Big) \Big]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_\Omega \sum_j \left(\gamma |\Omega| + \Omega^2/c^2 \right) |\phi_{ja}(i\Omega)|^2 \,,$$

where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\overline{\mathcal{S}}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\overline{s}'_j}{2} \phi_{ja}^2 \Big]$$

Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev, Phys. Rev. Lett. **101**, 035701 (2008).

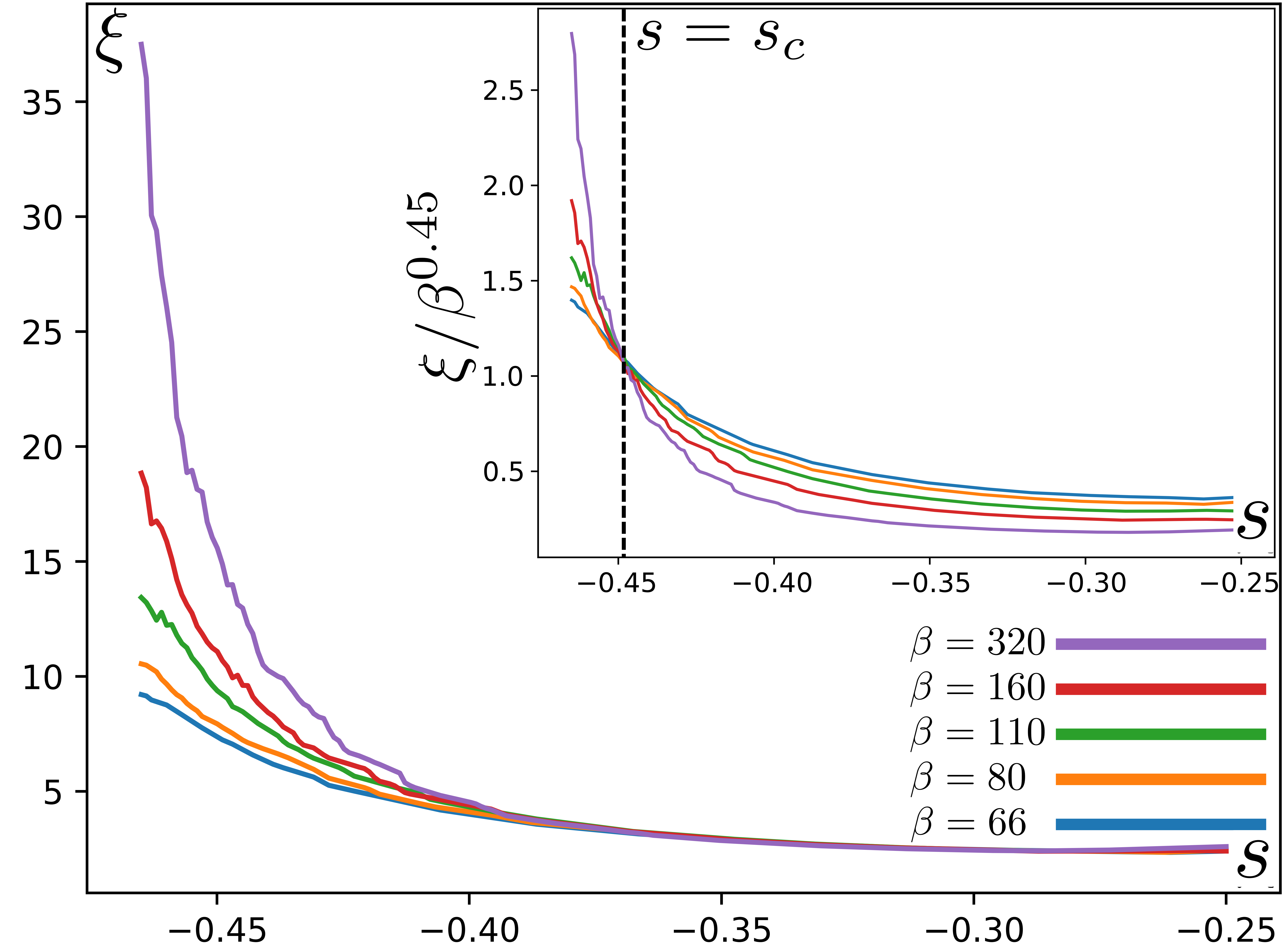
$$\overline{s}'_j = s + s'_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\overline{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + s'_j + uT \sum_\Omega \sum_\alpha \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma |\Omega| + \Omega^2/c^2 + e_\alpha}$$

where e_α and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\overline{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \ldots L^2$ for a $L \times L$ sample.

Aavishkar A. Patel, Peter Lunts, S.S., PNAS to appear, *arXiv:2312.06751*

Bosonic eigenmodes in random mass Hertz theory

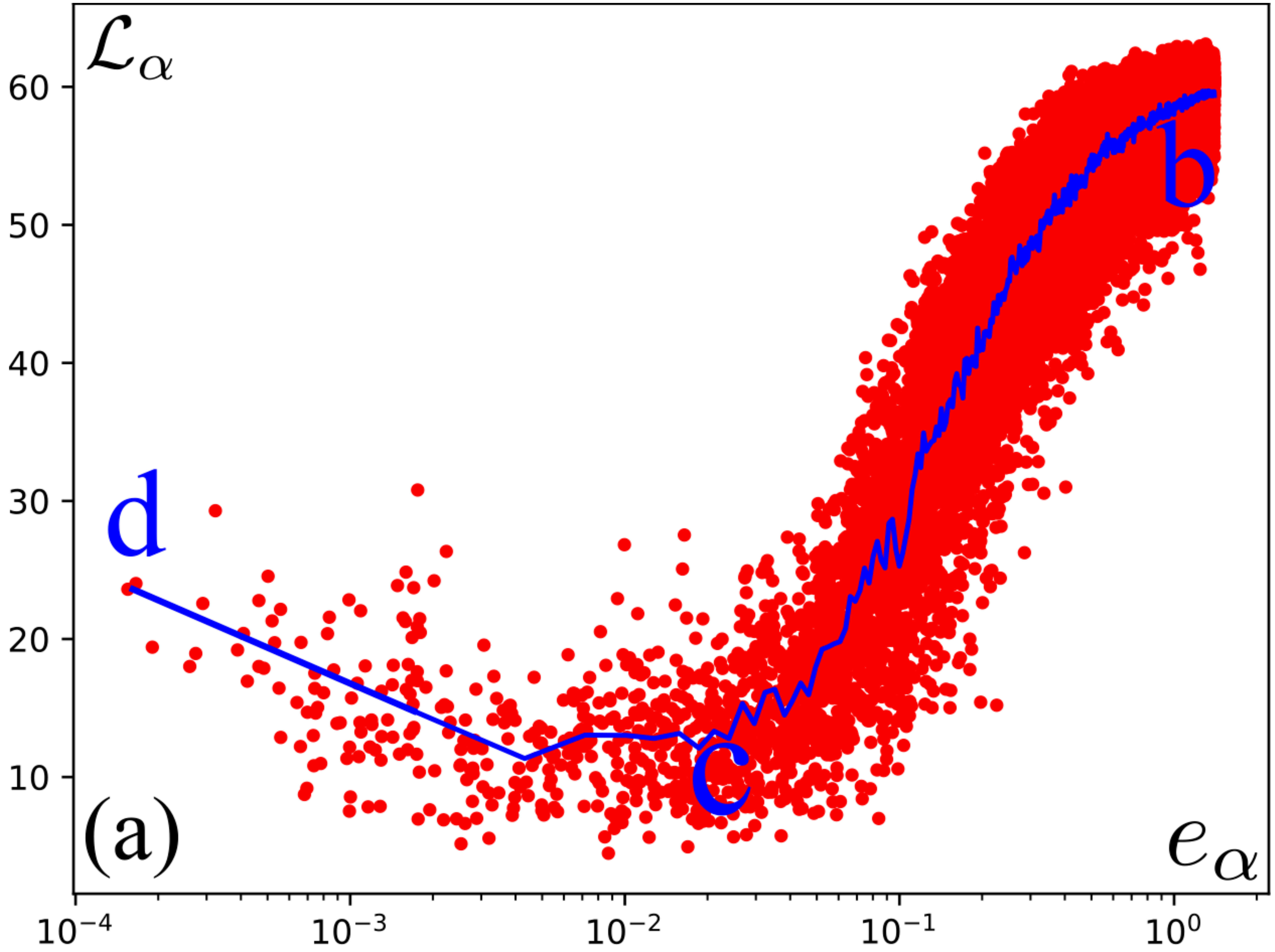
ϕ correlation length ξ



Aavishkar A. Patel, Peter Lunts, S.S., *PNAS to appear, arXiv:2312.06751*

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$

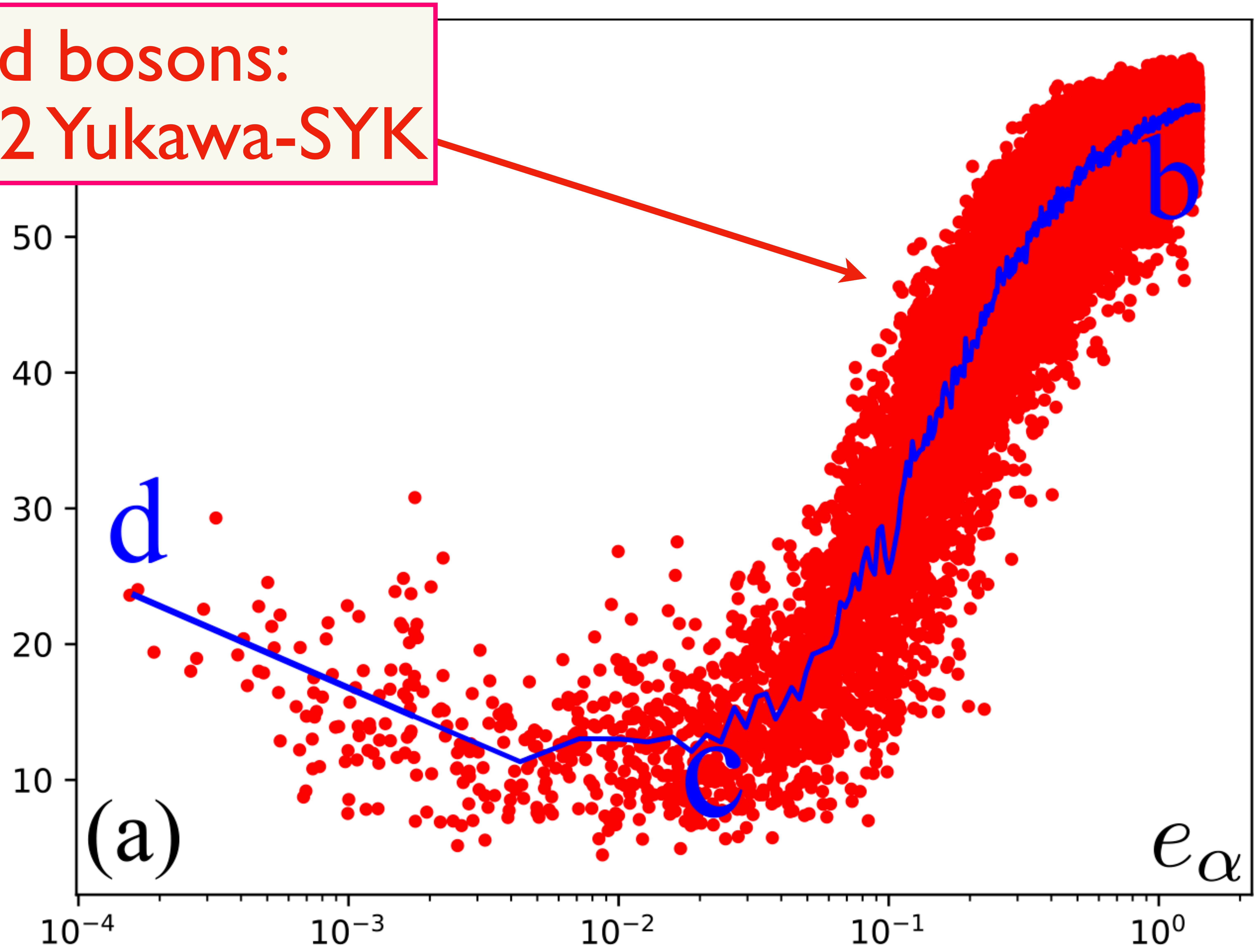


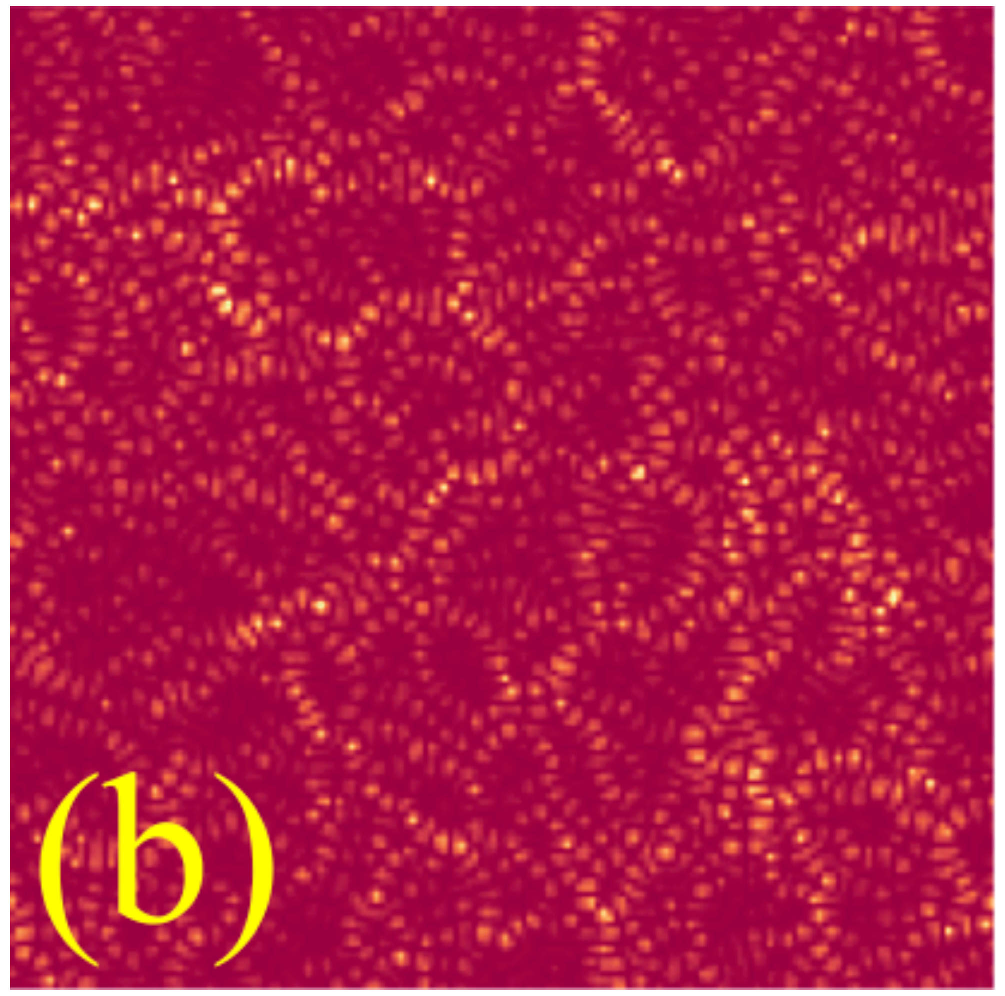
Aavishkar A. Patel, Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Extended bosons: physics of d=2 Yukawa-SYK

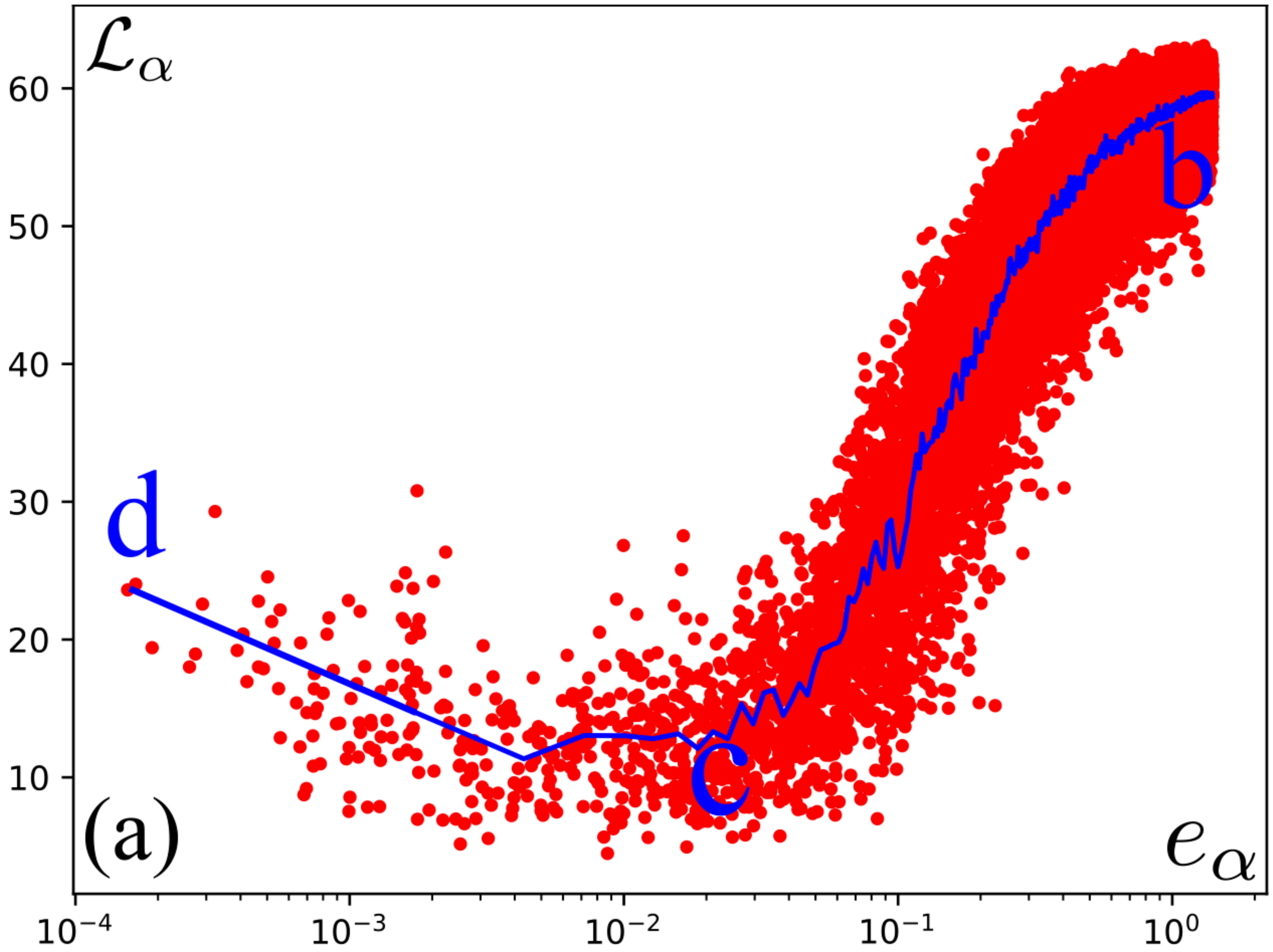


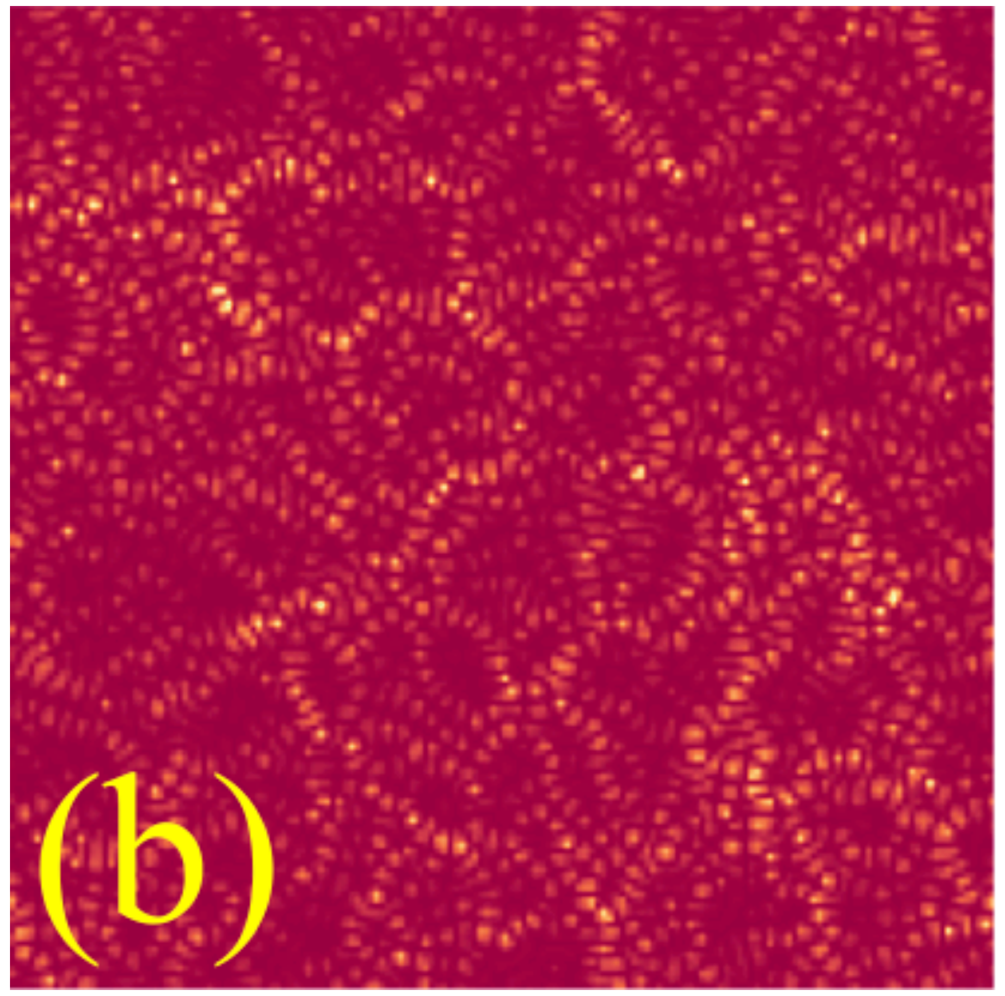


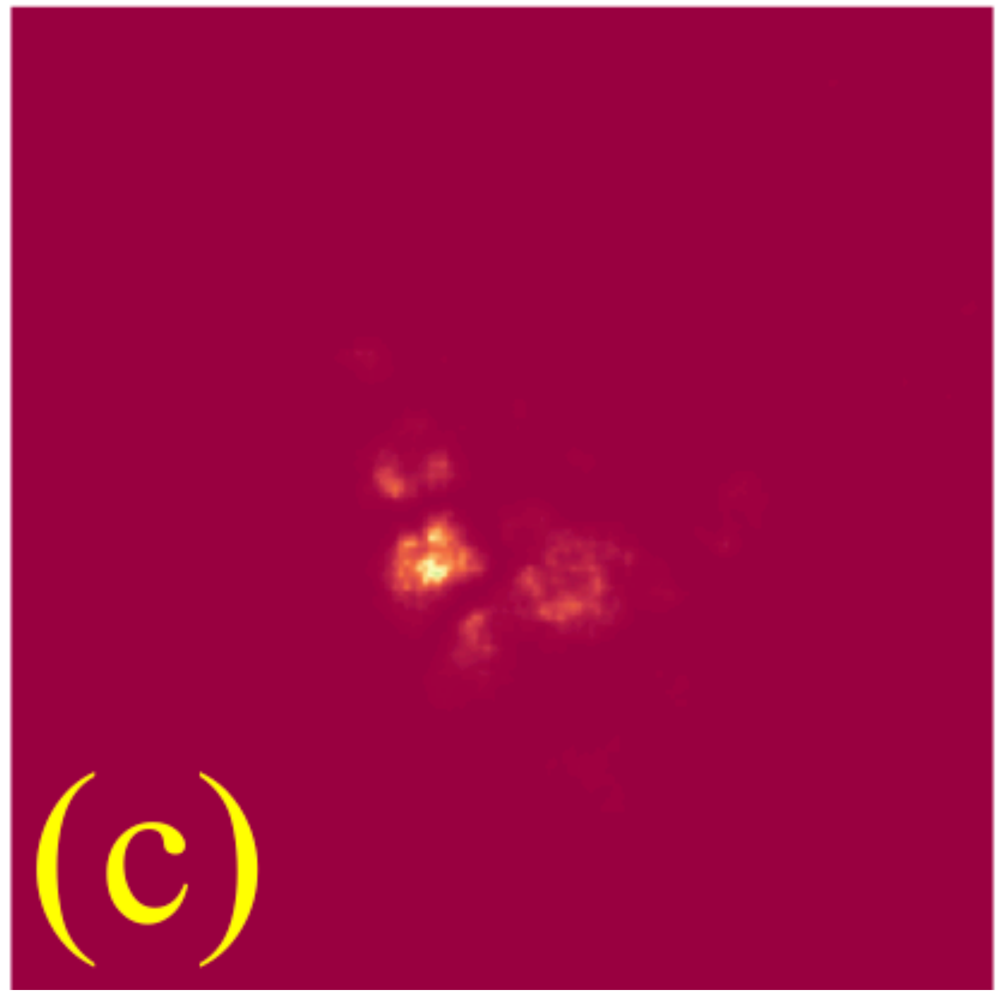
Aavishkar A. Patel, Peter Lunts, S.S., *PNAS to appear, arXiv:2312.06751*

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



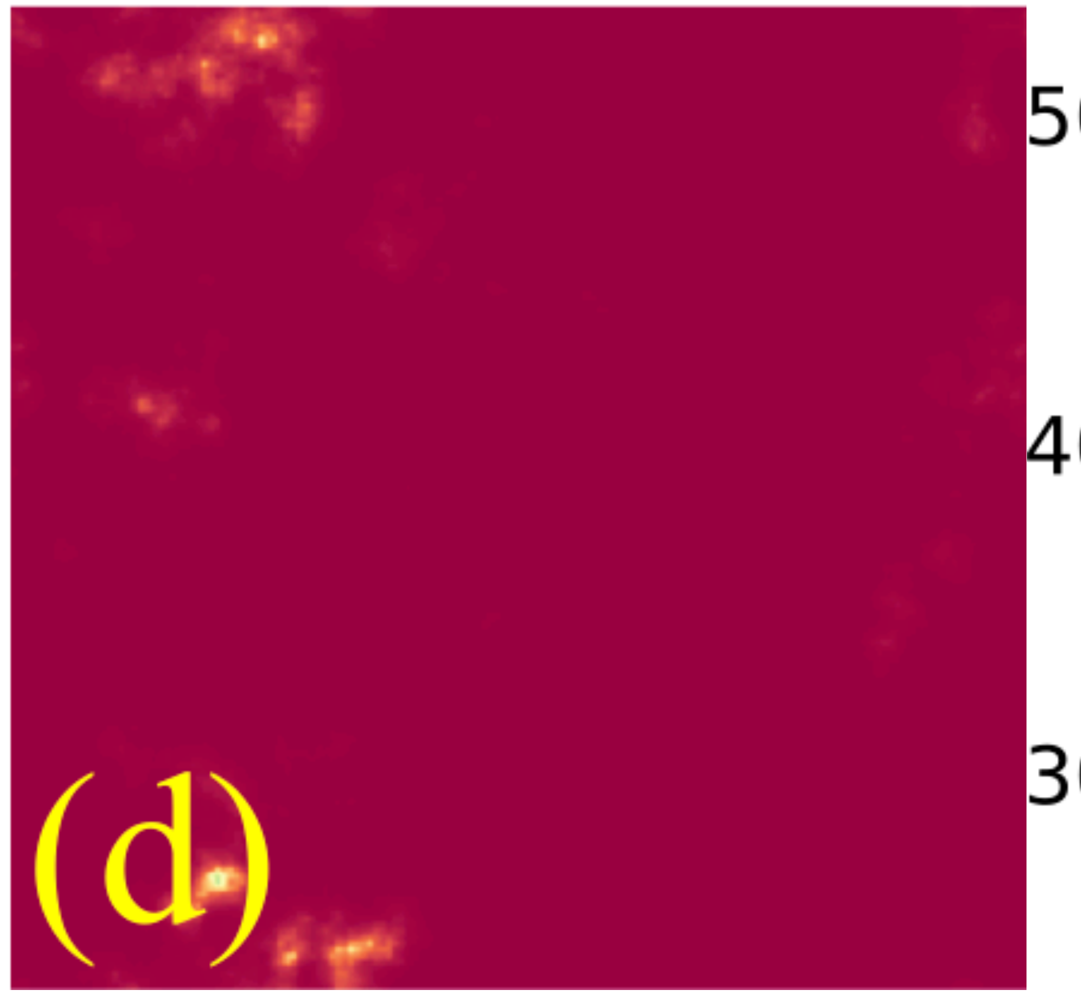


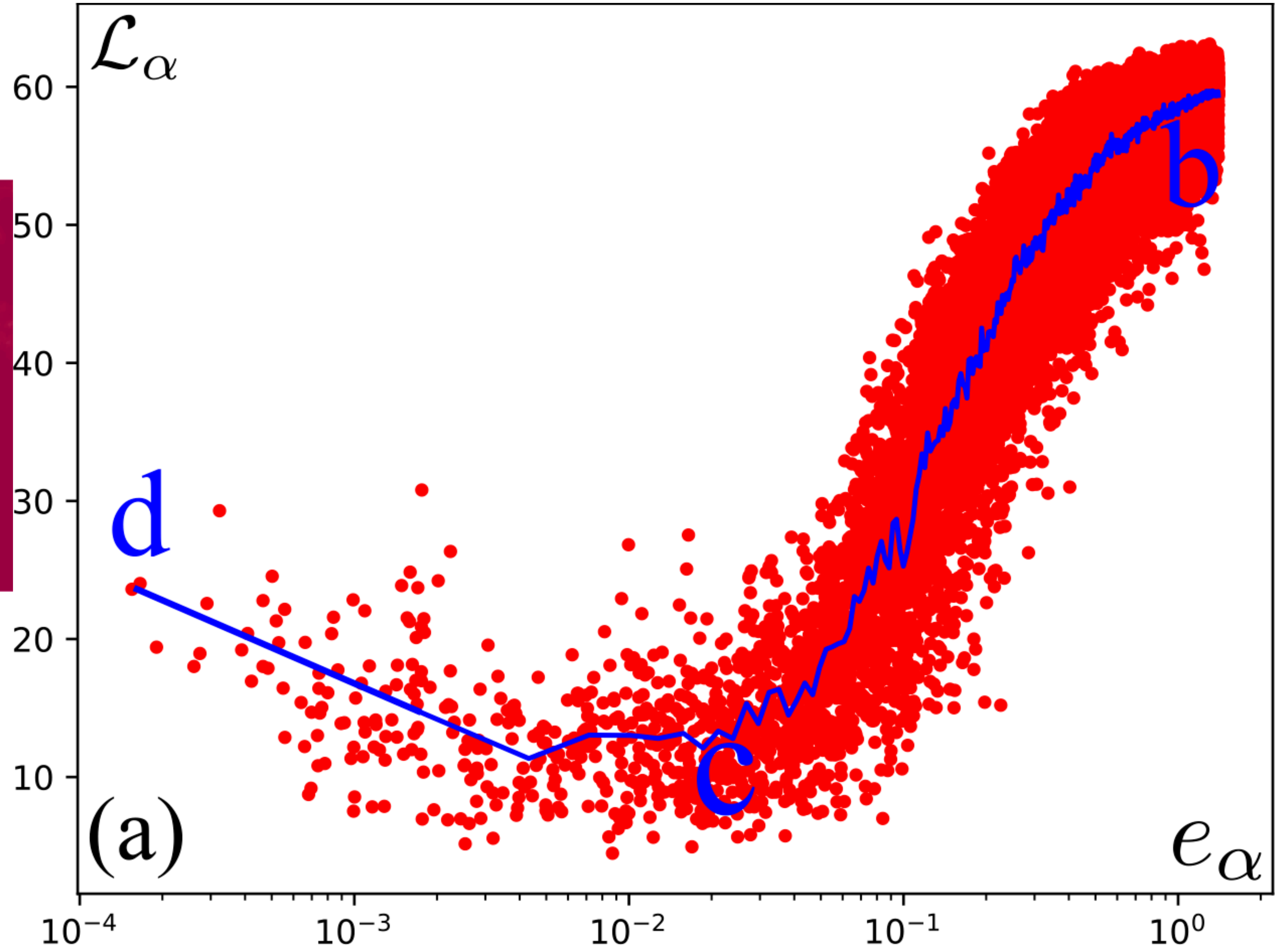


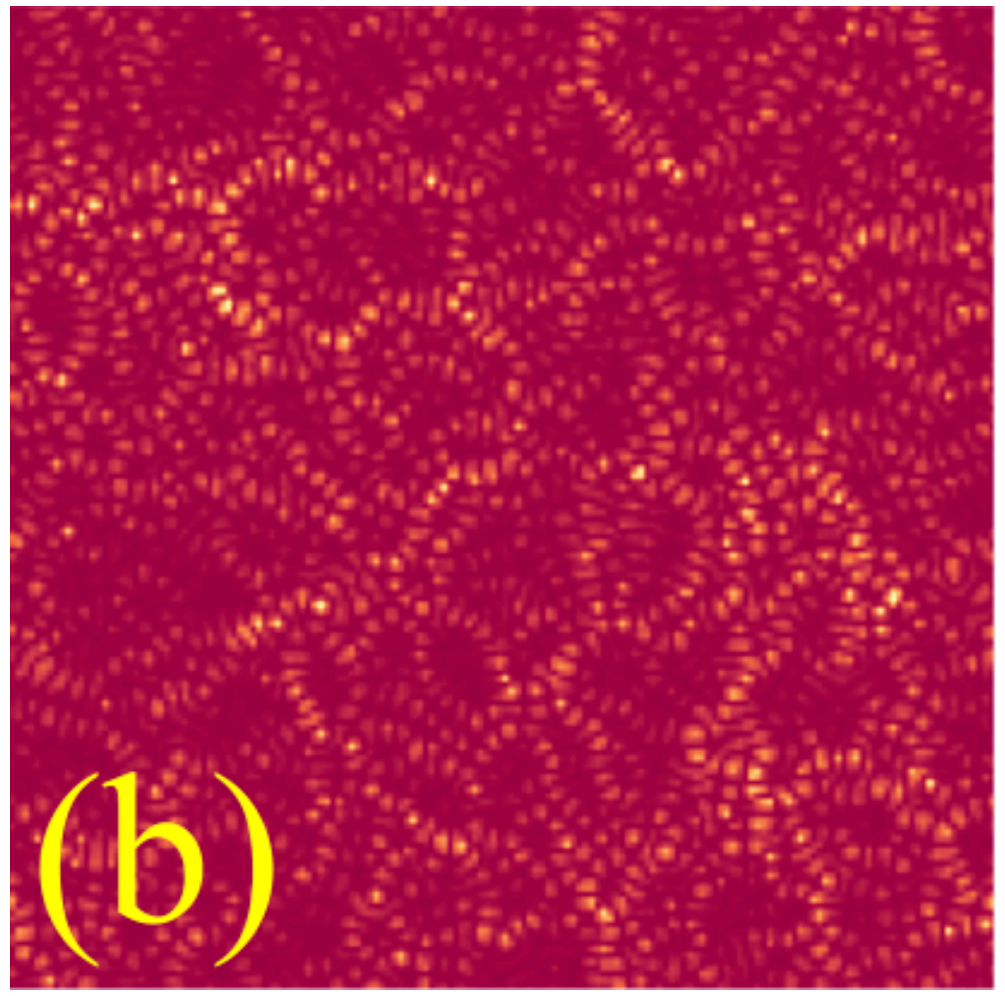
Aavishkar A. Patel, Peter Lunts, S.S., *PNAS to appear, arXiv:2312.06751*

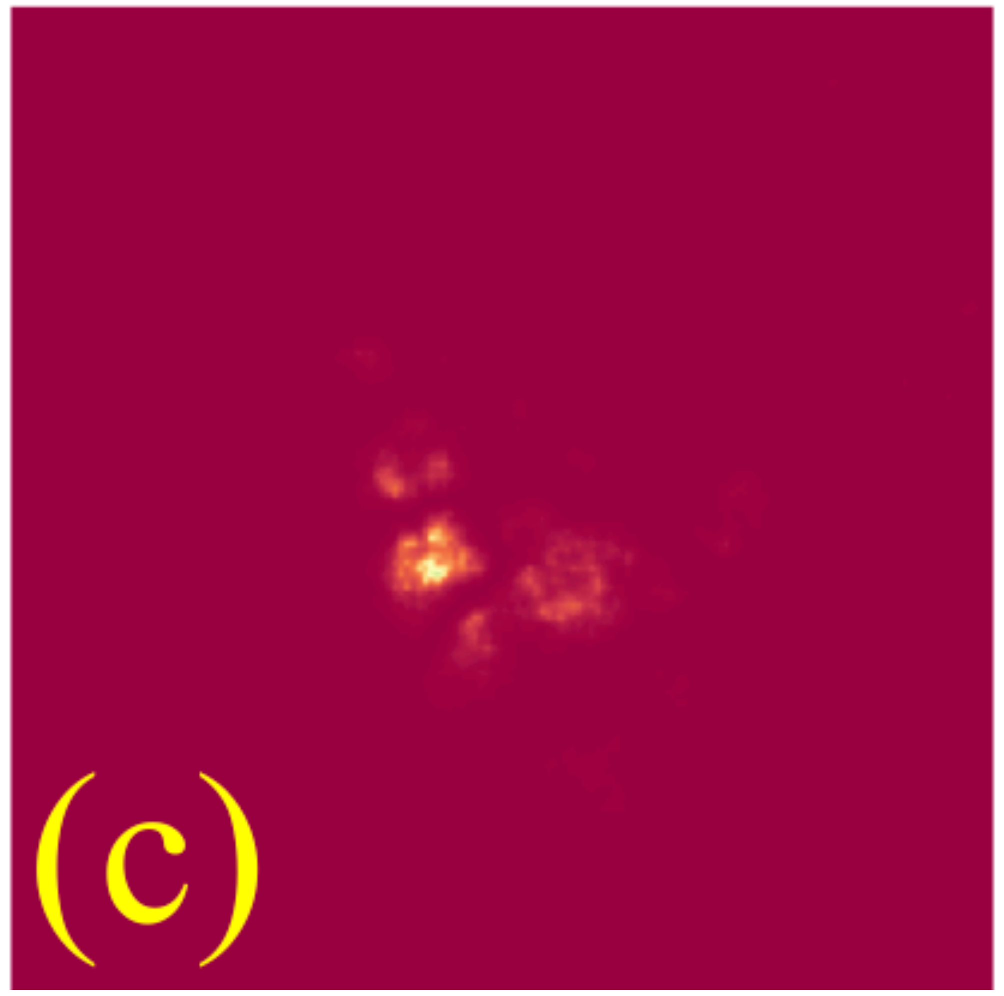
Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$









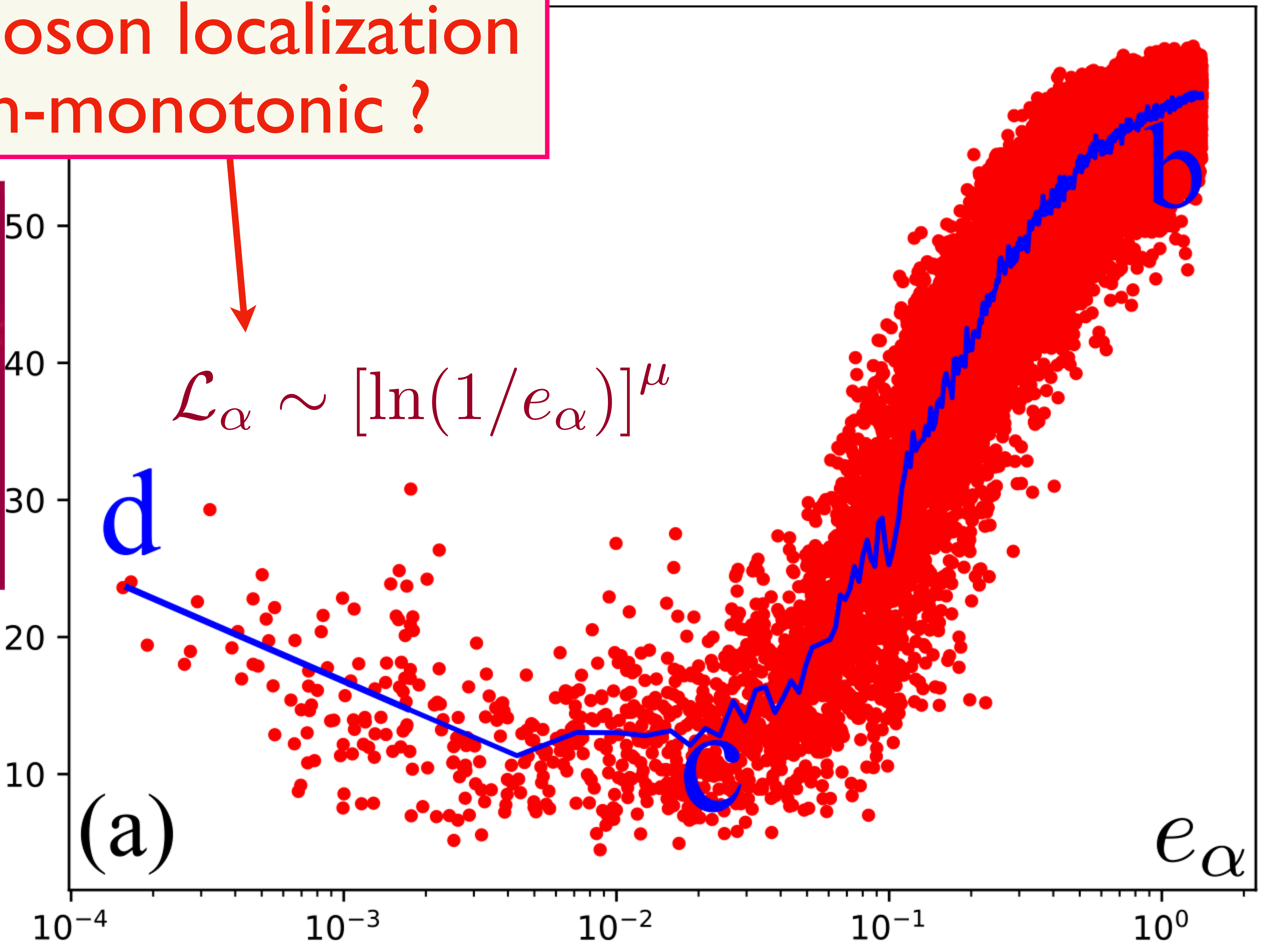
Aavishkar A. Patel,
Peter Lunts, S.S.,

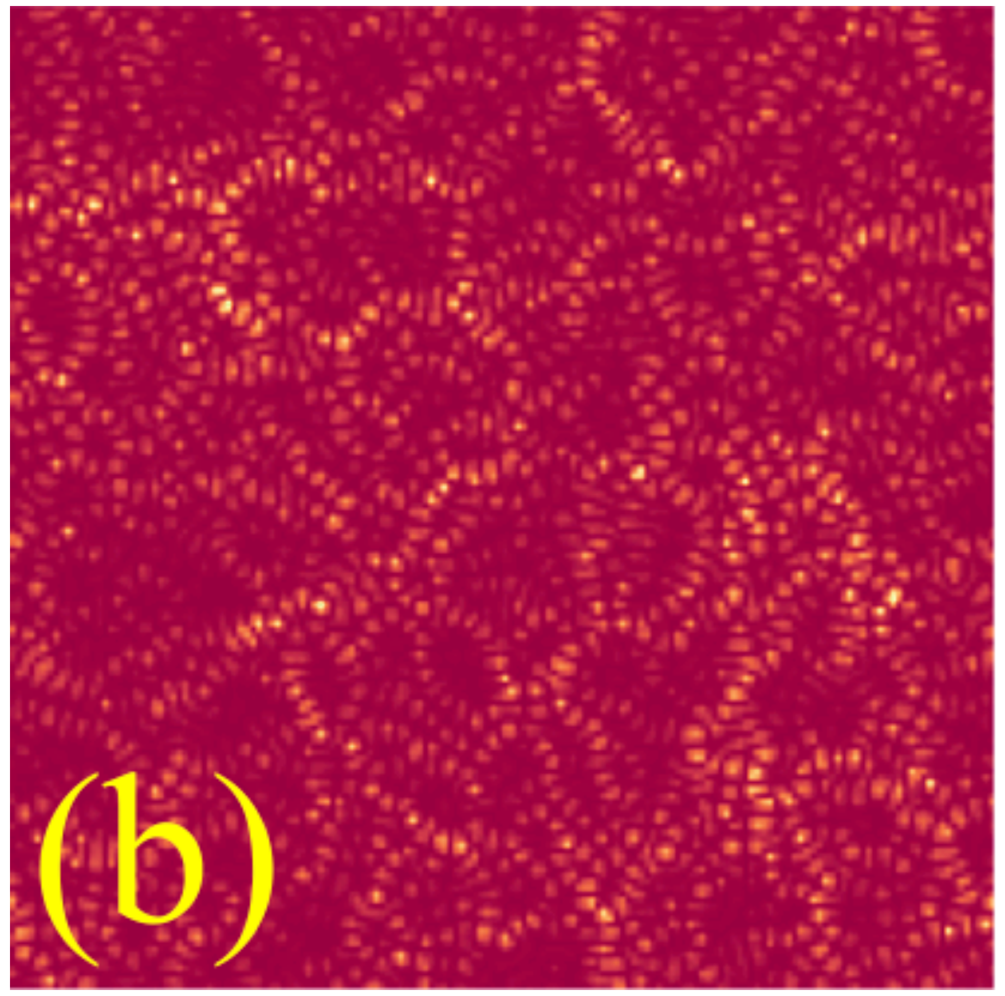
PNAS to appear,
arXiv:2312.06751

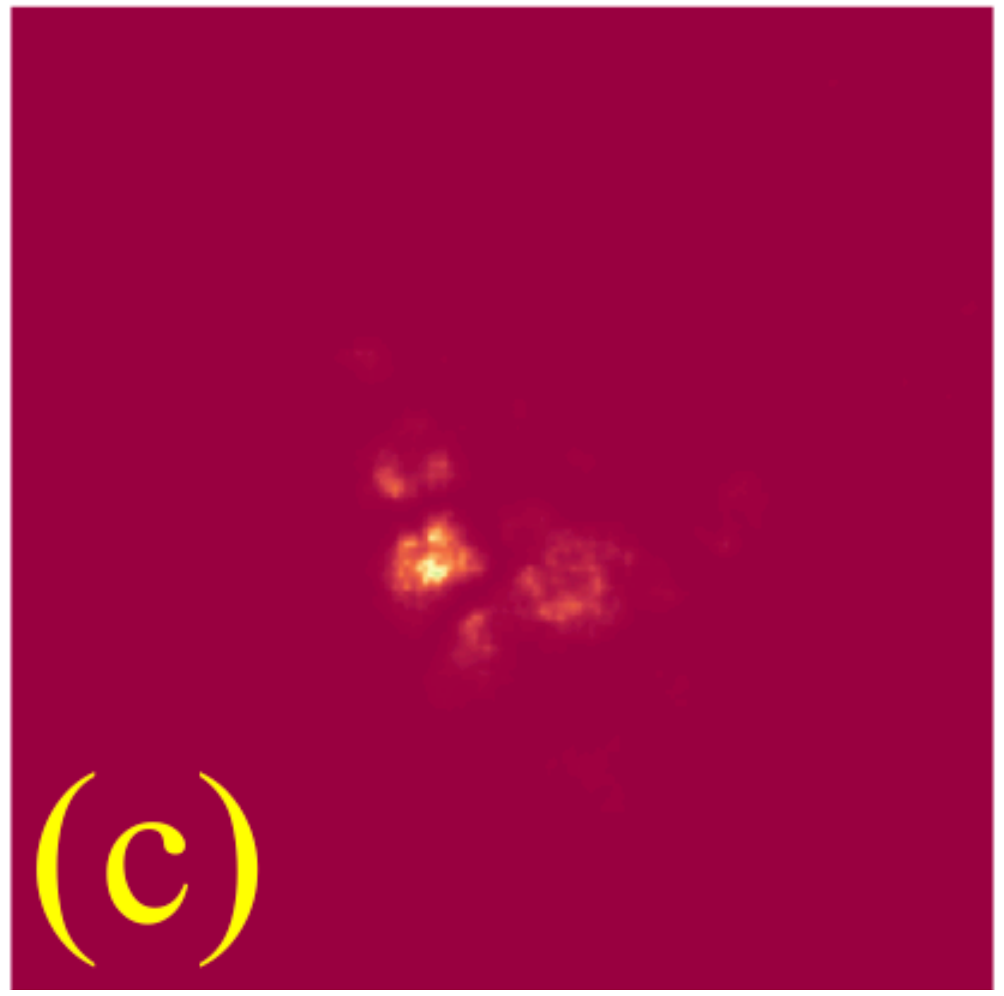
Landau-damped bosonic eigenmodes with random mass

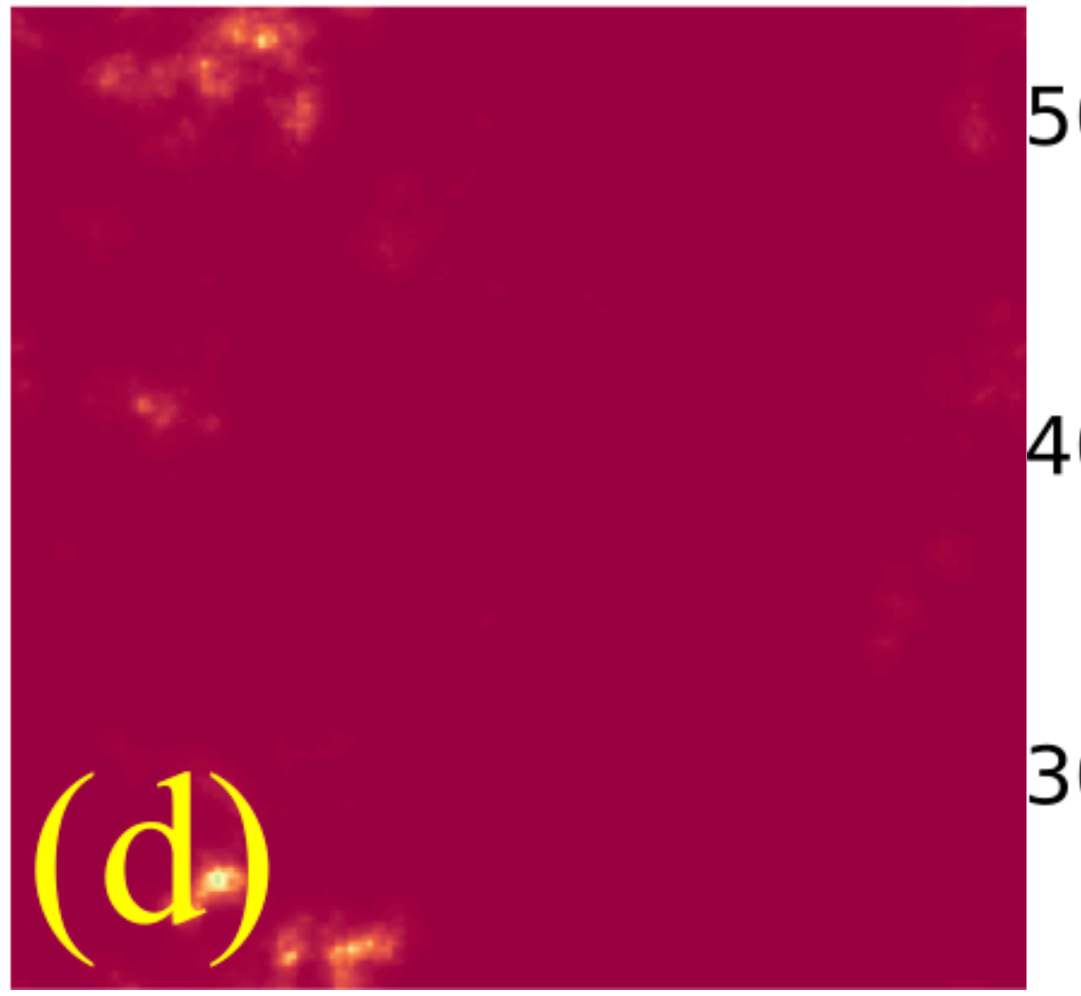
ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Why is the boson localization length non-monotonic ?









Aavishkar A. Patel,
Peter Lunts, S.S.,

PNAS to appear,
arXiv:2312.06751

T. Vojta and J. Schmalian, PRB **72**, 045438 (2005)

PRL **99,** 230601 (2007) PHYSICAL REVIEW LETTERS

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta
Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

$$\mathcal{S}_b = \int d\tau \left(-\sum_{\langle ij \rangle} J_{ij} \phi_{ia} \phi_{ja} + \sum_j \left[\frac{s_j}{2} \phi_{ja}^2 + \frac{u}{4} \left(\phi_{ja}^2 \right)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

T. Vojta and J. Schmalian, PRB **72**, 045438 (2005)

PRL **99,** 230601 (2007)   PHYSICAL REVIEW LETTERS

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

Strong disorder RG identical to that for the RTFIM

$$\widetilde{J}_{ij} = J_{ij} + \frac{J_{i2} J_{2j}}{s_2}$$

$$\widetilde{s}_2 = 2\frac{s_2 s_3}{J_{23}}$$

$$H_{\mathrm{RTFIM}} = -\sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

Numerically studied in *d*=2 by
O. Motrunich, S.-C. Mau, D.A. Huse and D.S. Fisher,
PRB **61** (2000) 1160

T. Vojta and J. Schmalian, PRB **72**, 045438 (2005)

PRL **99,** 230601 (2007) PHYSICAL REVIEW LETTERS

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta
Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

Strong disorder RG identical to that for the RTFIM

$$\widetilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\widetilde{s}_2 = 2\frac{s_2 s_3}{J_{23}}$$

$$H_{\mathrm{RTFIM}} = -\sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

Numerically studied in *d*=2 by
O. Motrunich, S.-C. Mau, D.A. Huse and D.S. Fisher,
PRB **61** (2000) 1160

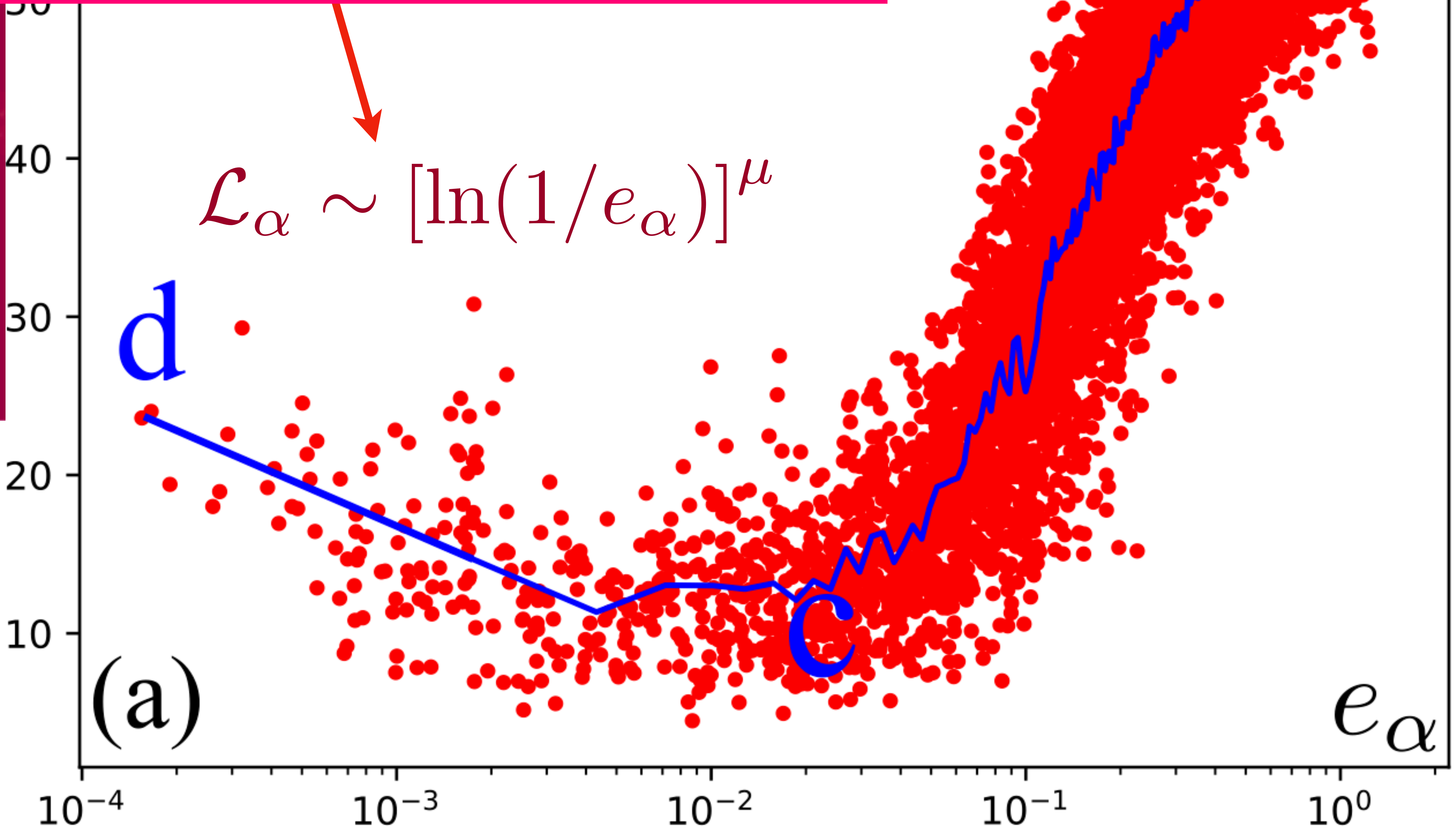
Key fact: Similarity of classical $O(N \geq 2)$ chain with $1/r^2$ interactions, and classical Ising chain with short-range interactions.

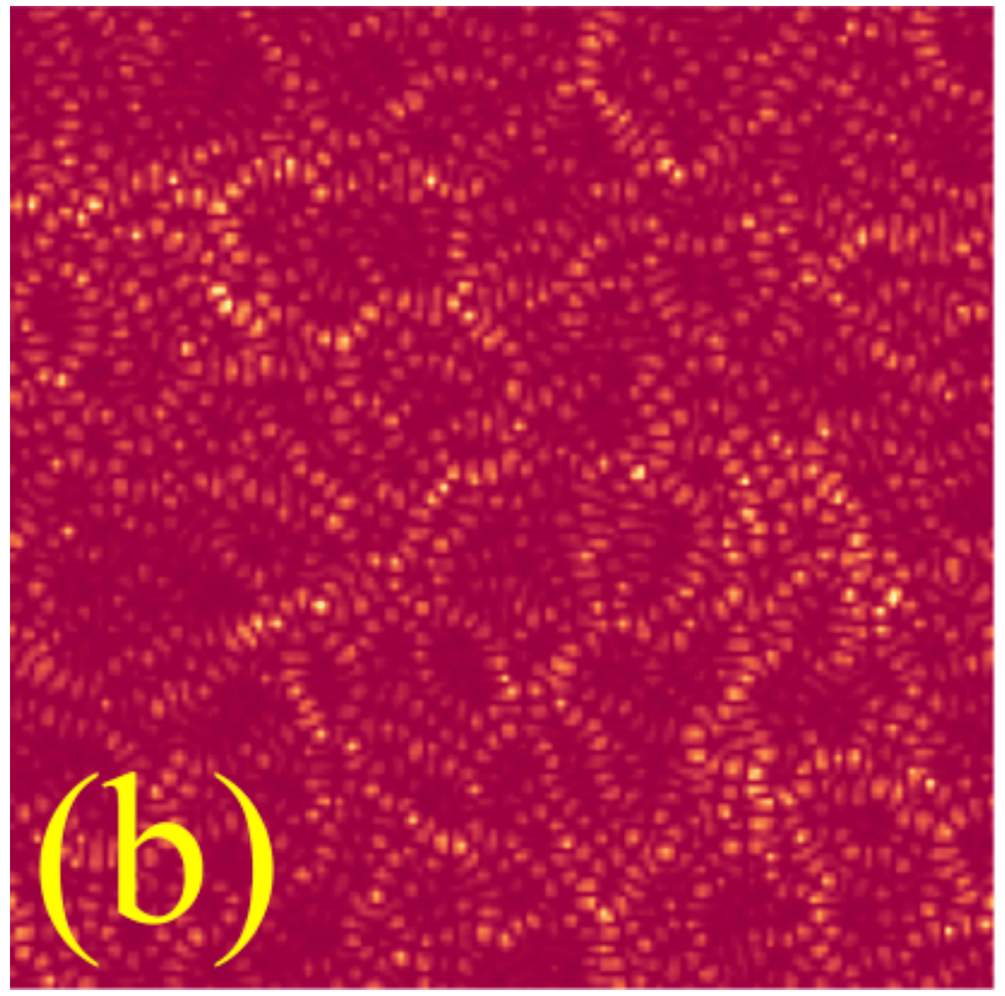
Bosonic eigenmodes in random mass Hertz theory

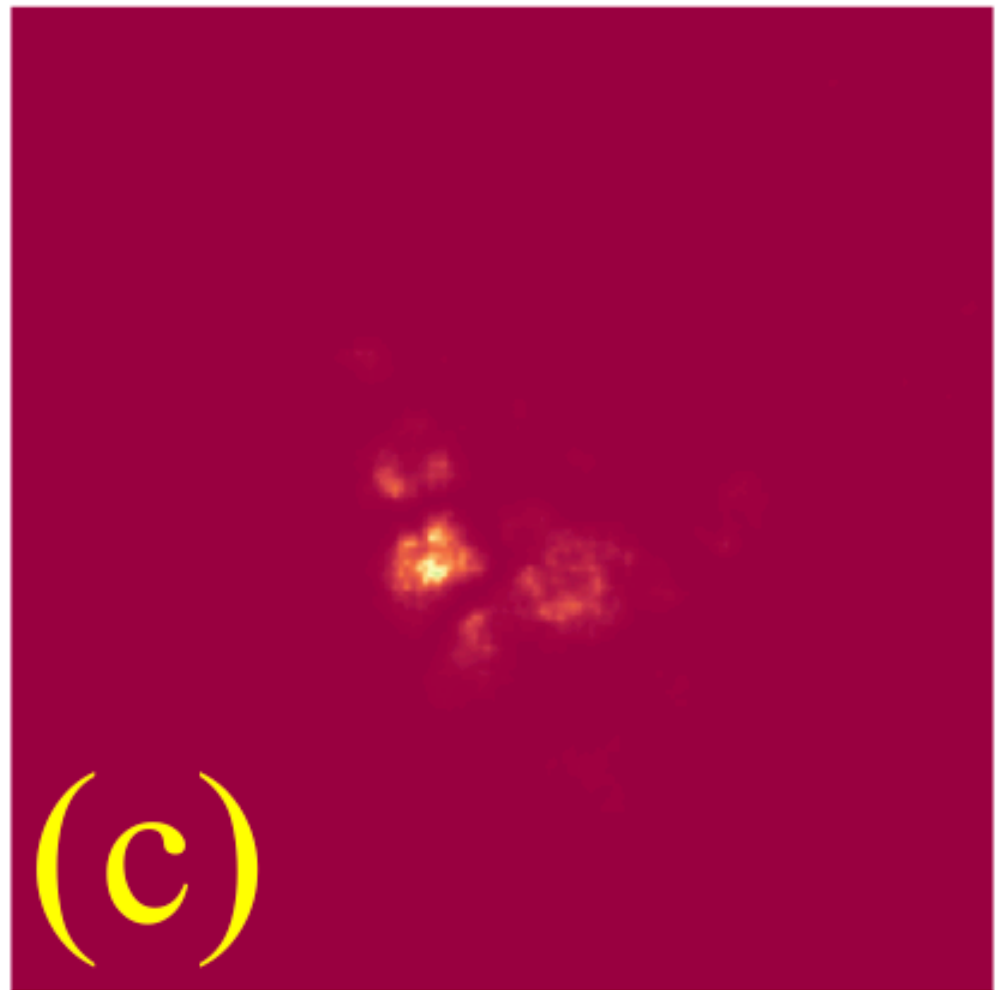
ϕ eigenmodes localization length $\mathcal{L}_\alpha$

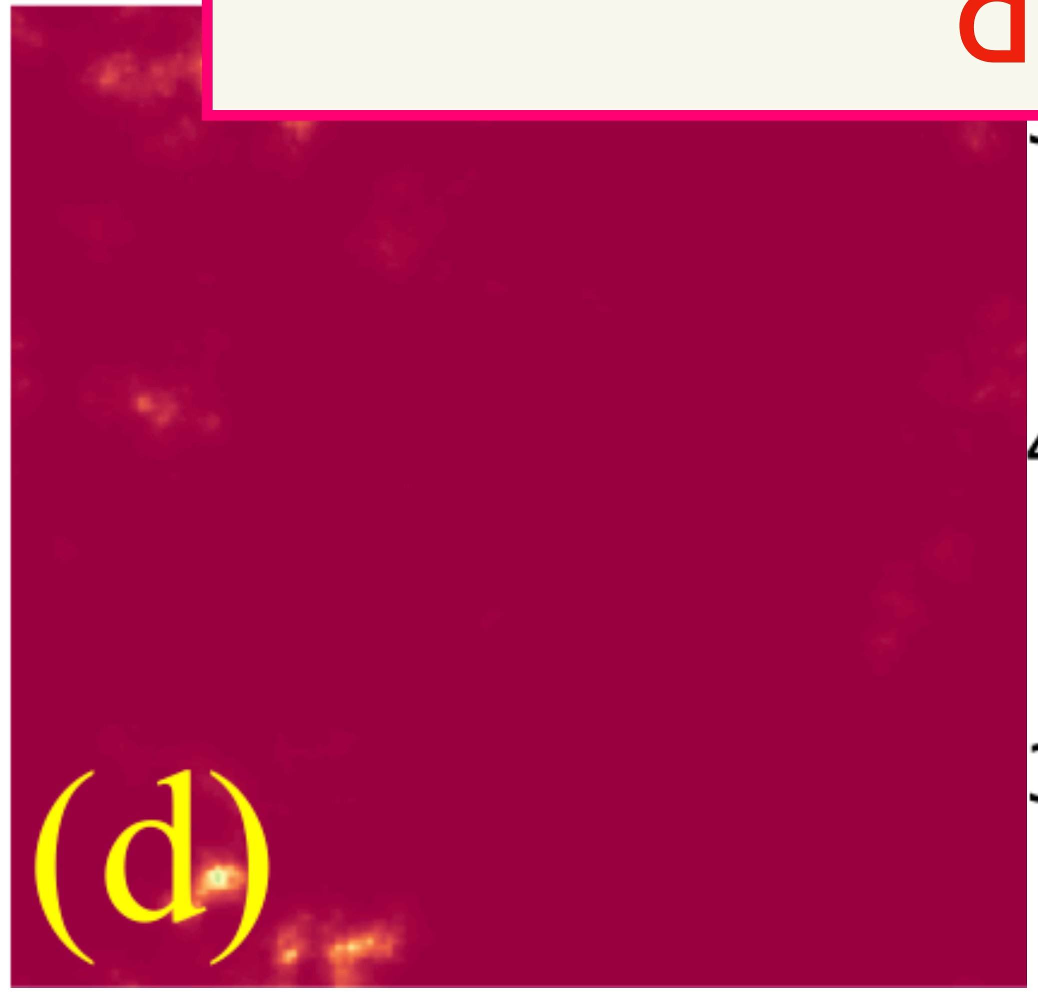
Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy

$$\mathcal{L}_\alpha \sim [\ln(1/e_\alpha)]^\mu$$









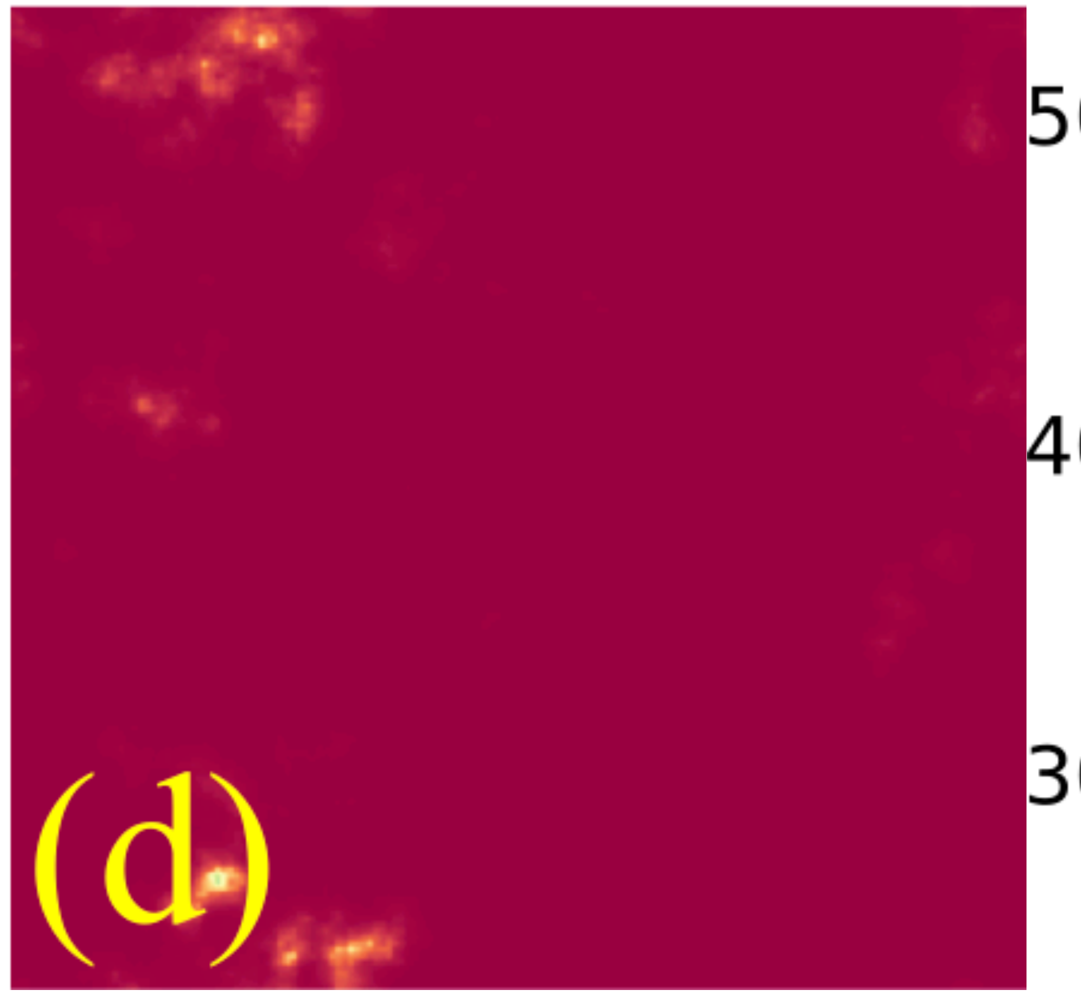
Aavishkar A. Patel,
Peter Lunts, S.S.,

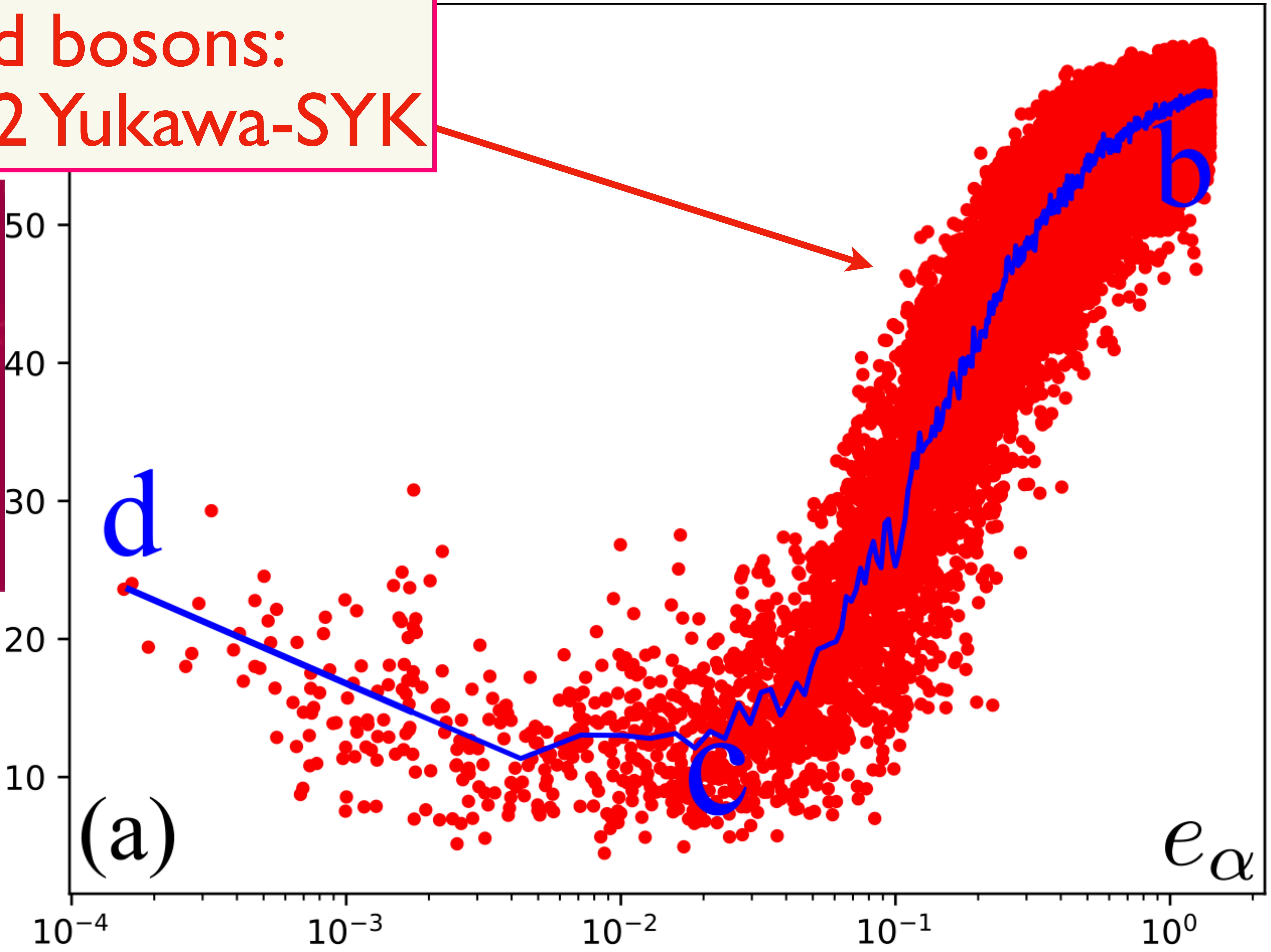
PNAS to appear,
arXiv:2312.06751

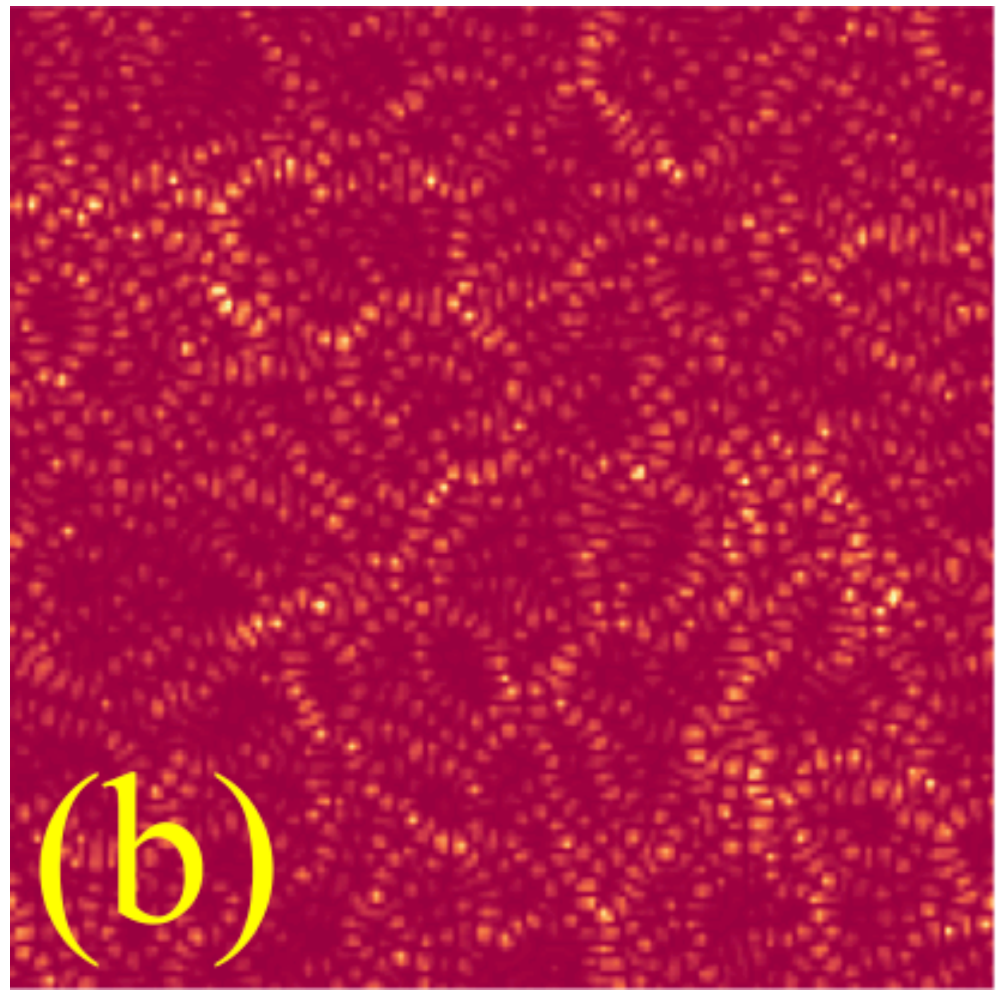
Bosonic eigenmodes in random mass Hertz theory

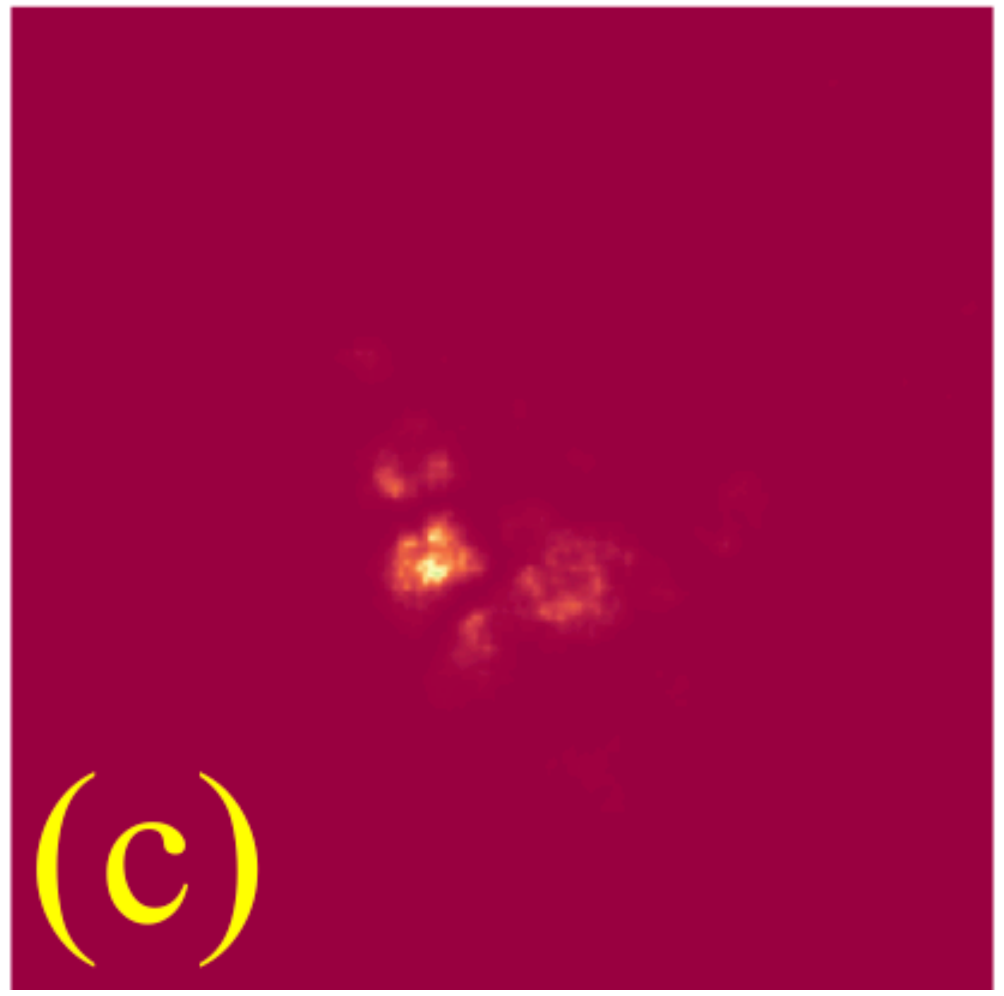
ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Extended bosons: physics of *d*=2 Yukawa-SYK









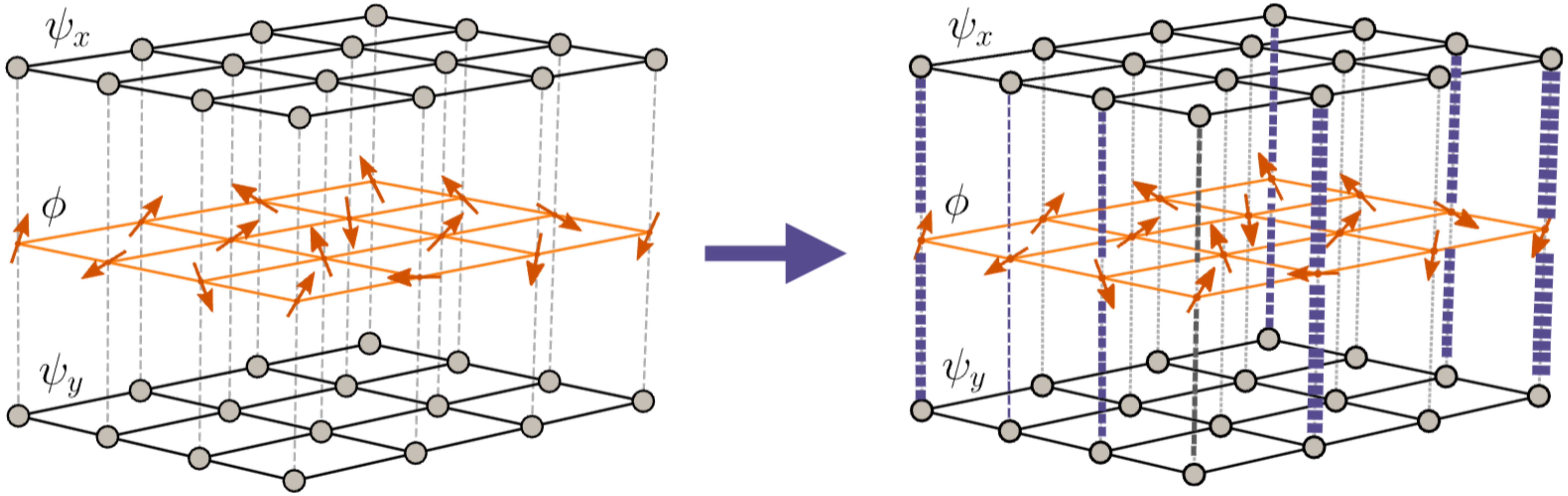
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751

Model for sign-free QMC

$$\mathcal{S} = \int d\tau \sum_{\sigma=\uparrow,\downarrow} \sum_{\alpha=0,1} \sum_{j=1}^{2} \sum_{\vec{r},\vec{r}'} \psi^{\dagger}_{\alpha,\sigma,j,\vec{r}} \left[\partial_\tau - (-1)^{\alpha}\mu - \delta_{\vec{r},\vec{r}'} - t_{\alpha,\vec{r},\vec{r}'}\right] \psi_{\alpha,\sigma,j,\vec{r}'}$$

$$+ \int d\tau \sum_{\vec{r}} \left[\frac{1}{c^2}(\partial_\tau \vec{\phi}_{\vec{r}})^2 + \frac{1}{2}(\nabla \vec{\phi}_{\vec{r}})^2 + \frac{r}{2}(\vec{\phi}_{\vec{r}})^2 + \frac{u}{4}(\vec{\phi}_{\vec{r}})^4\right] + \sum_{\sigma,\sigma'=\uparrow,\downarrow} \sum_{j=1}^{2} \int d\tau \sum_{\vec{r}} \boxed{g'(\vec{r}) e^{i\vec{Q}_{\mathrm{AF}}\cdot\vec{r}}}\, \vec{\phi}_{\vec{r}} \cdot \left[\psi^{\dagger}_{0,\sigma,\vec{r}}\, \vec{\tau}_{\sigma,\sigma'}\, \psi_{1,\sigma',\vec{r}} + \mathrm{H.c.}\right].$$

$g = 0$



coupling g = const

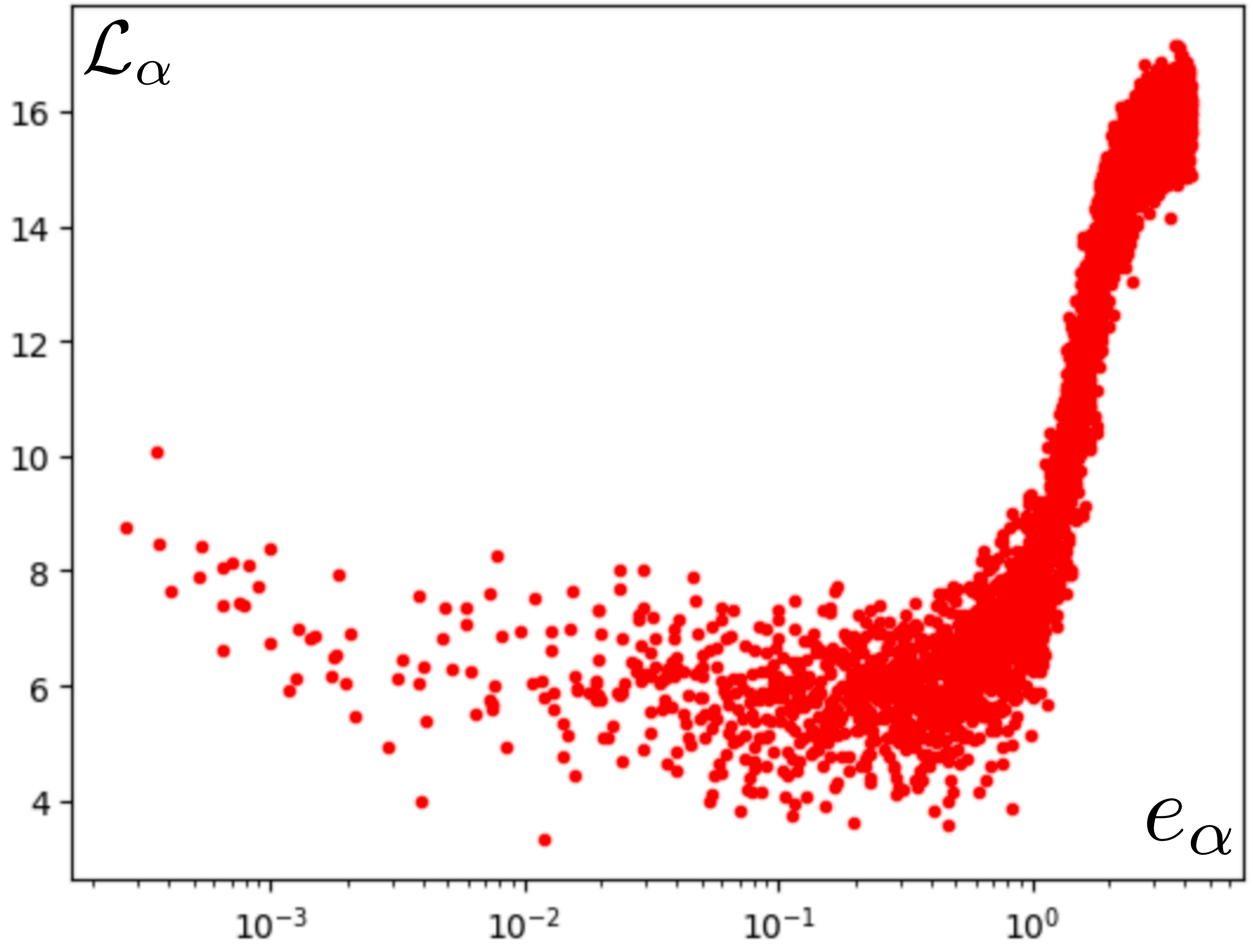
coupling g = spatially random

Two-band structure: Berg, Metlitsk
Sachdev, Science **338** 1606-1609 (2012).

Aavishkar A. Patel, Peter Lunts, M. Albergo **(to appear)**

Boson eigenmodes at $\Omega = 0$ from QMC

- Localization length vs eigenvalue



10 disorder samples, $L = 40,\ \beta t = 66,\ {g'}^2 = t/2,\ \lambda \approx \lambda_c$

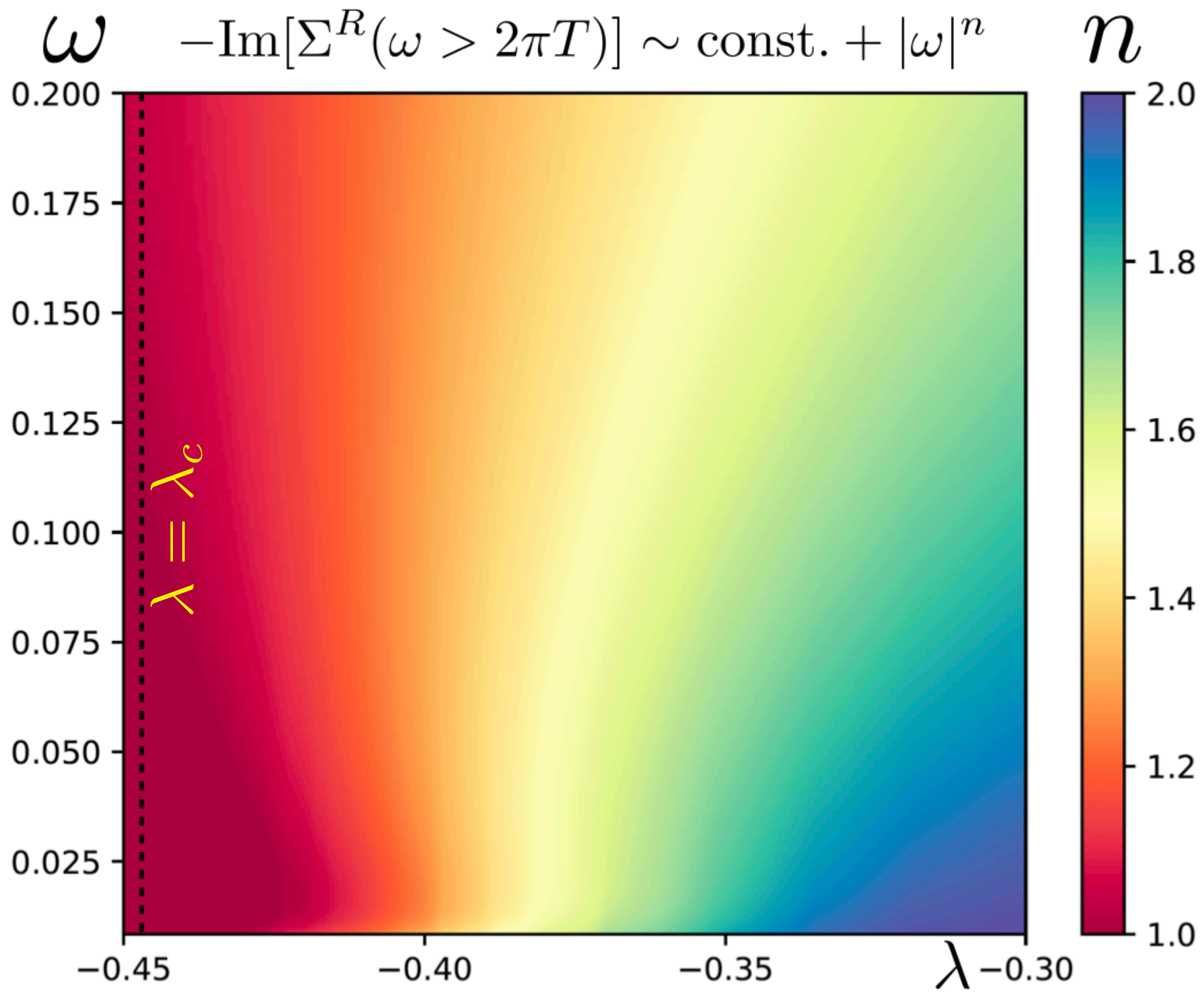
- Same qualitative features as large-M: localization followed by slow delocalization as energy is reduced.

Aavishkar A. Patel, Peter Lunts, M. Albergo **(to appear)**

Bosonic eigenmodes in random mass Hertz theory

Transport scattering rate

$$\Sigma(i\omega) = -i\pi g'^2 \mathcal{N}_0 \frac{T}{L^2} \sum_{\alpha,\Omega} \frac{\mathrm{sgn}(\omega+\Omega)}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha}\,.$$



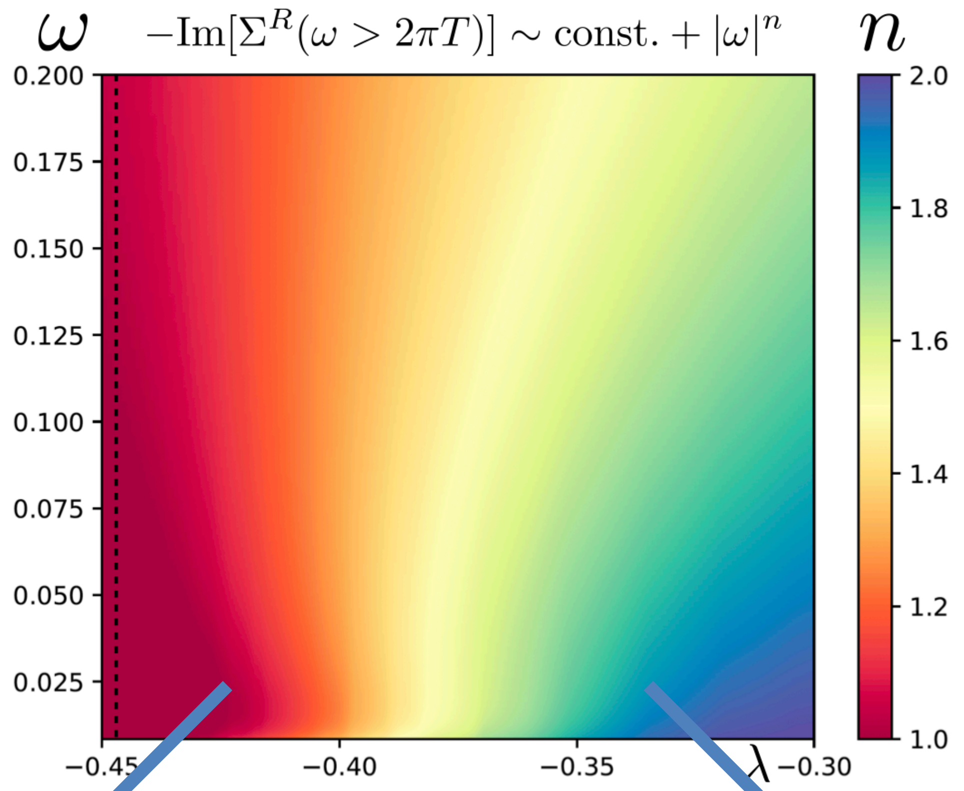
$L = 160, \beta = 800$, 10 disorder samples

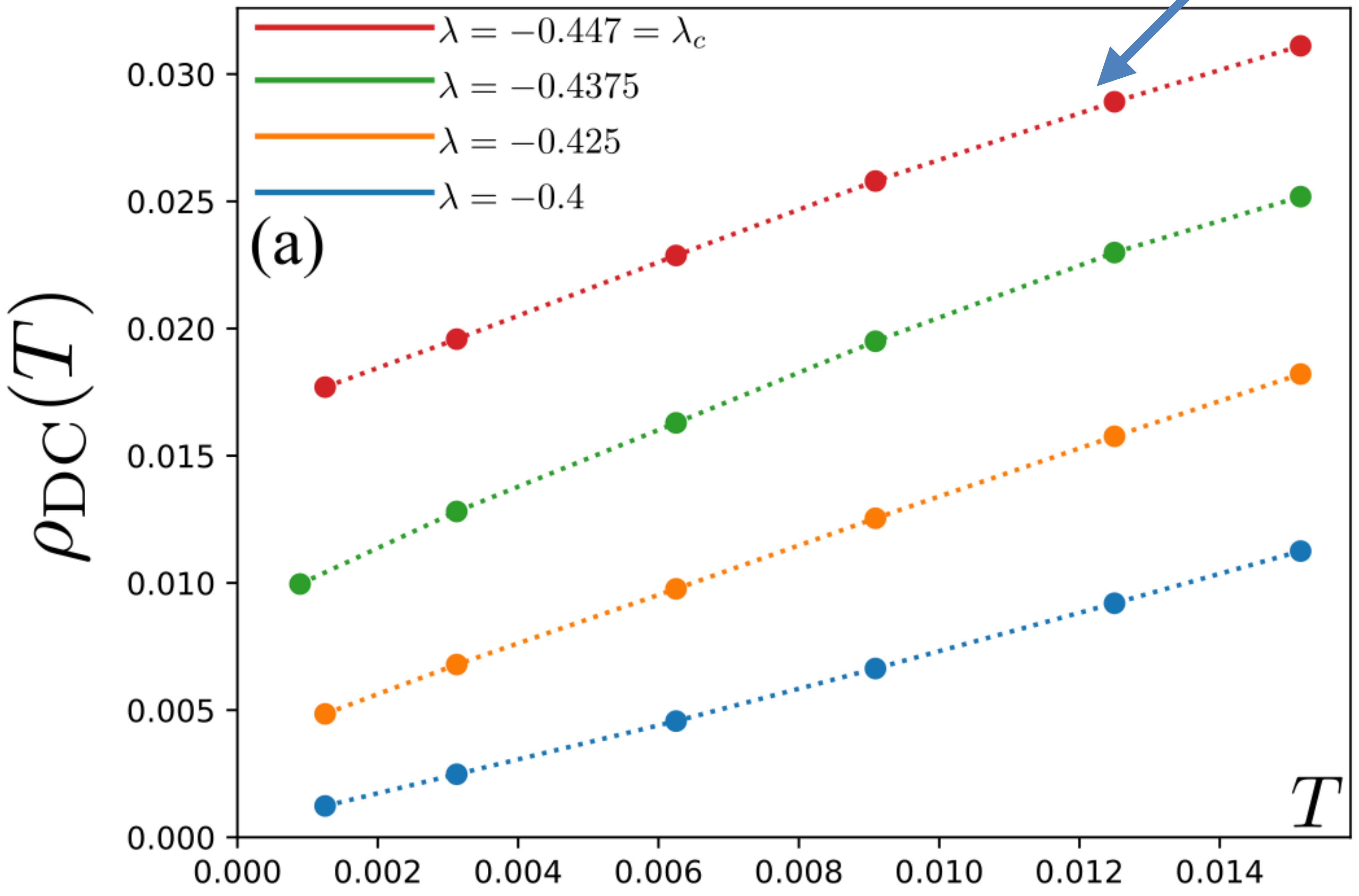
Extended region in λ with $n \approx 1$ - a strange metal *phase*

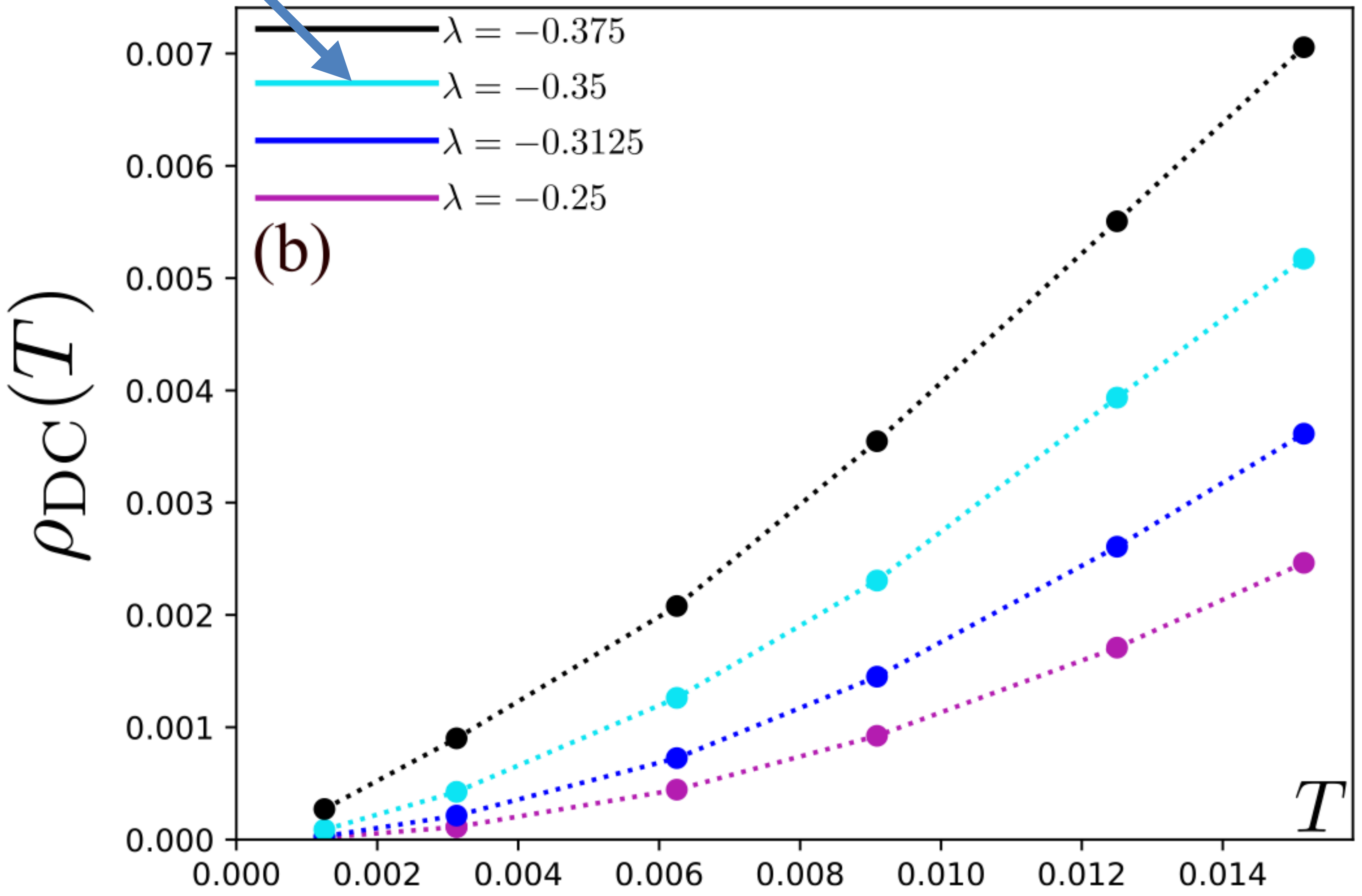
Aavishkar A. Patel, Peter Lunts, S.S., *PNAS to appear, arXiv:2312.06751*

Bosonic eigenmodes in random mass Hertz theory

DC resistivity



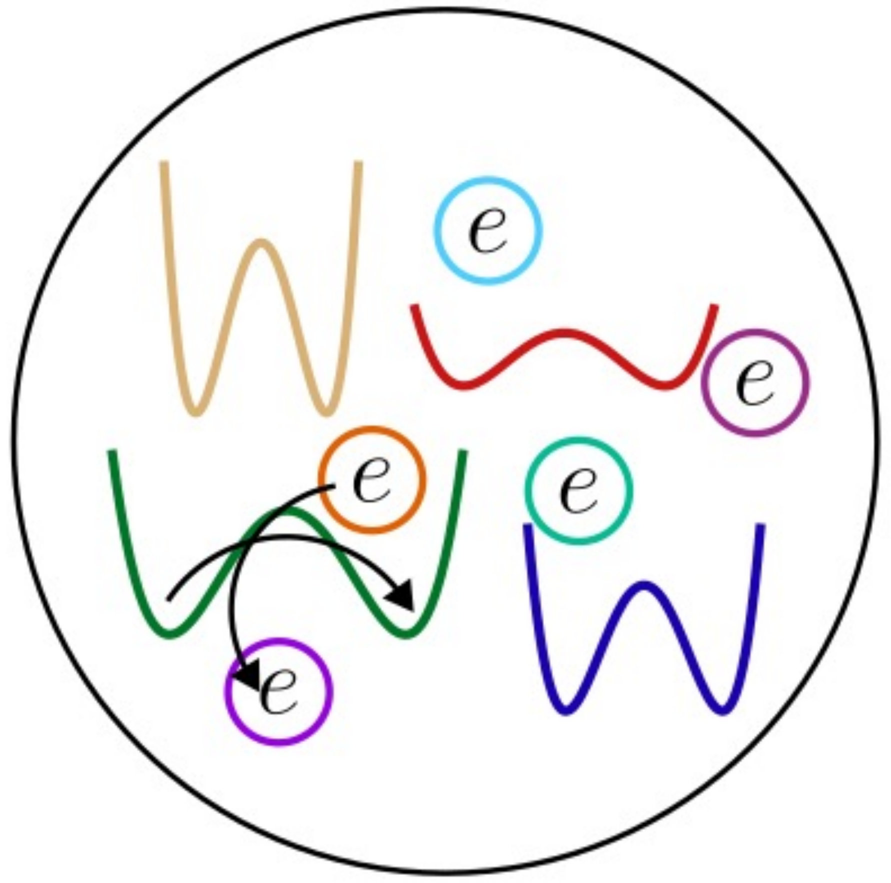


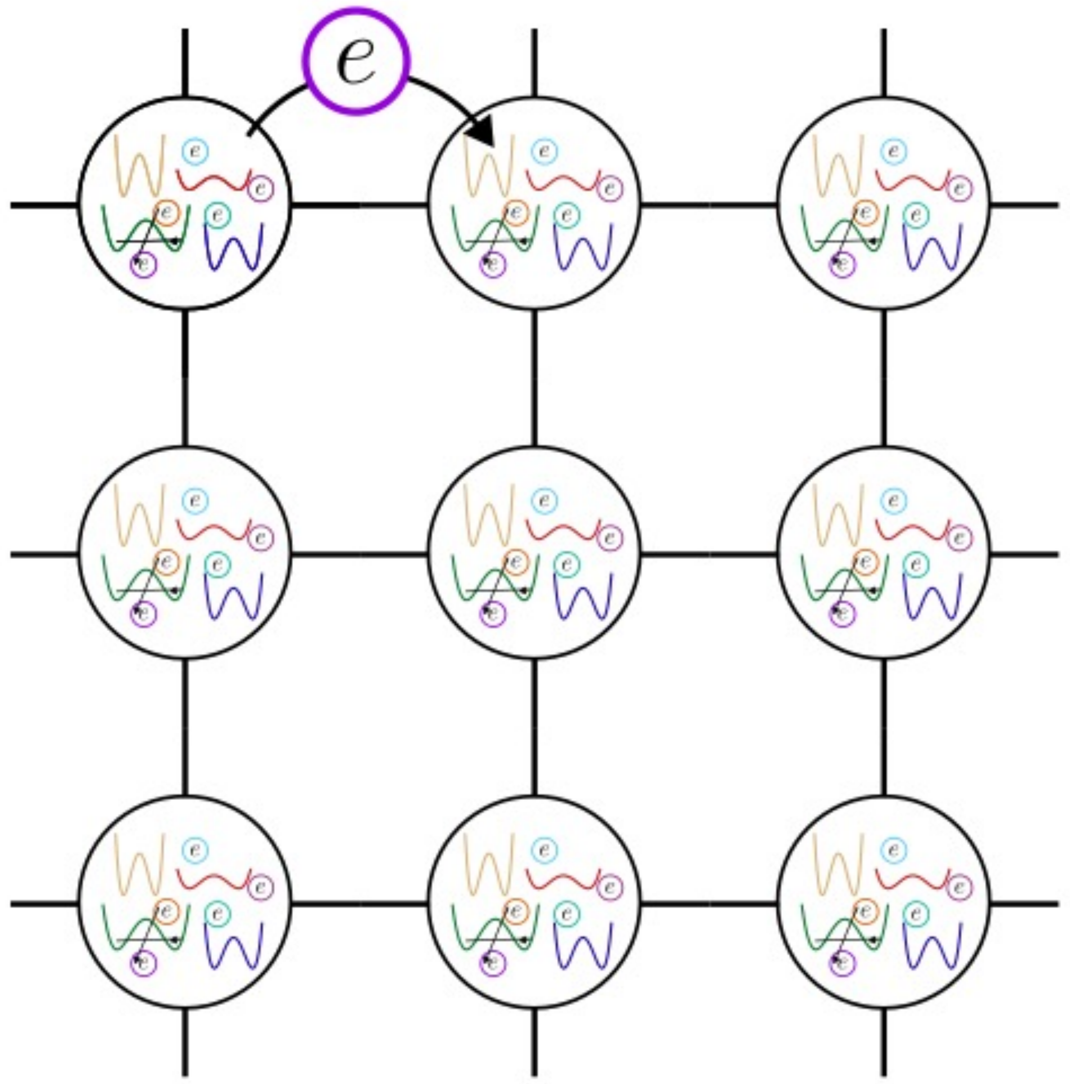


Extended region in λ with T-linear resistivity - a strange metal *phase*

Aavishkar A. Patel, Peter Lunts, S.S., PNAS to appear, *arXiv:2312.06751*

Tuneable non-Fermi liquid phase in electronic glasses



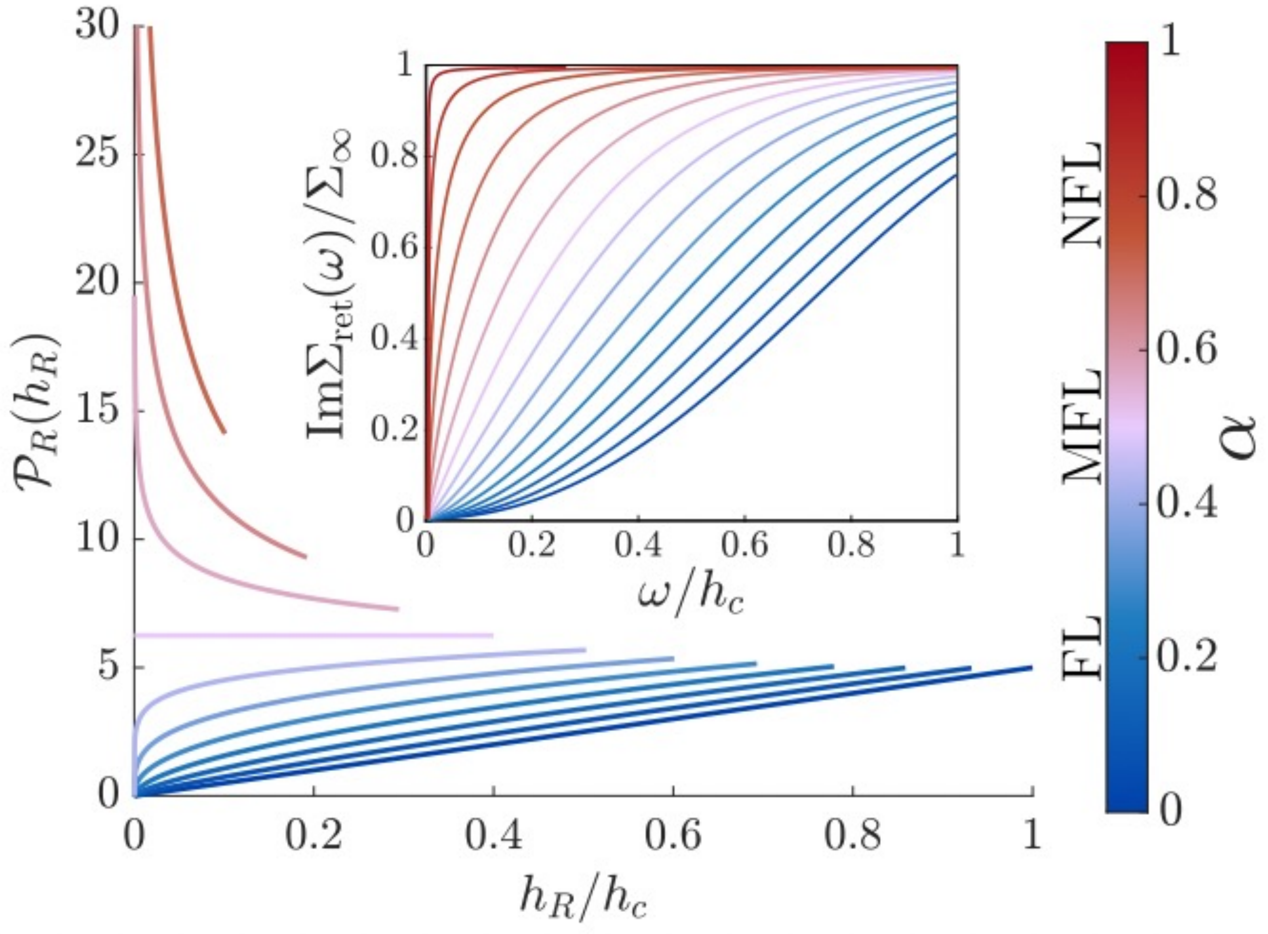


generalization of the Sachdev-Ye-Kitaev approach to two-level systems in glasses

$\Downarrow$

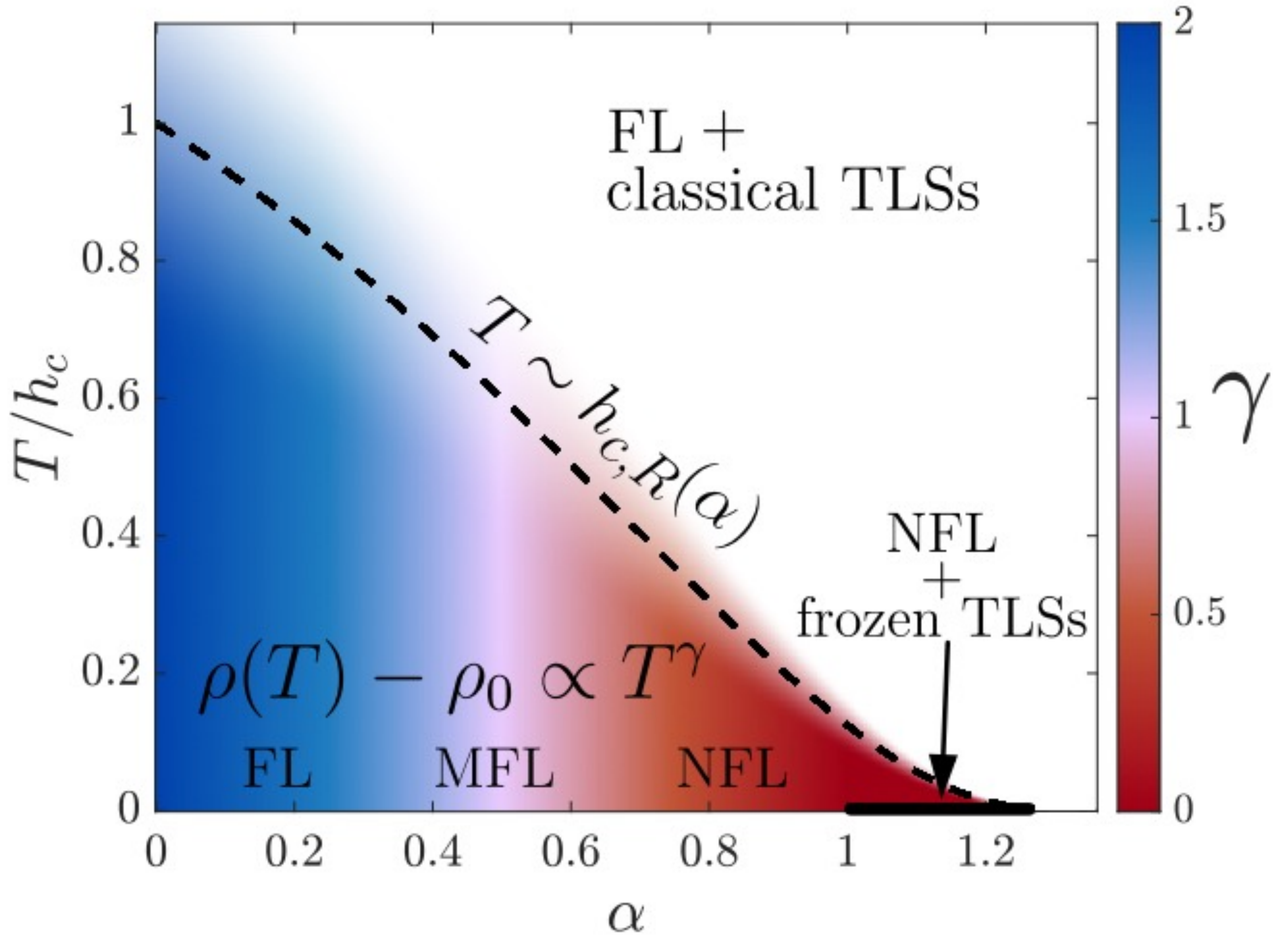
controlled approach to obtain a tunable Non-Fermi liquid state

strong renormalizations of the TLS distribution function



non-Fermi liquid physics in electronic glasses

Is this relevant to the physics of correlated quantum materials?

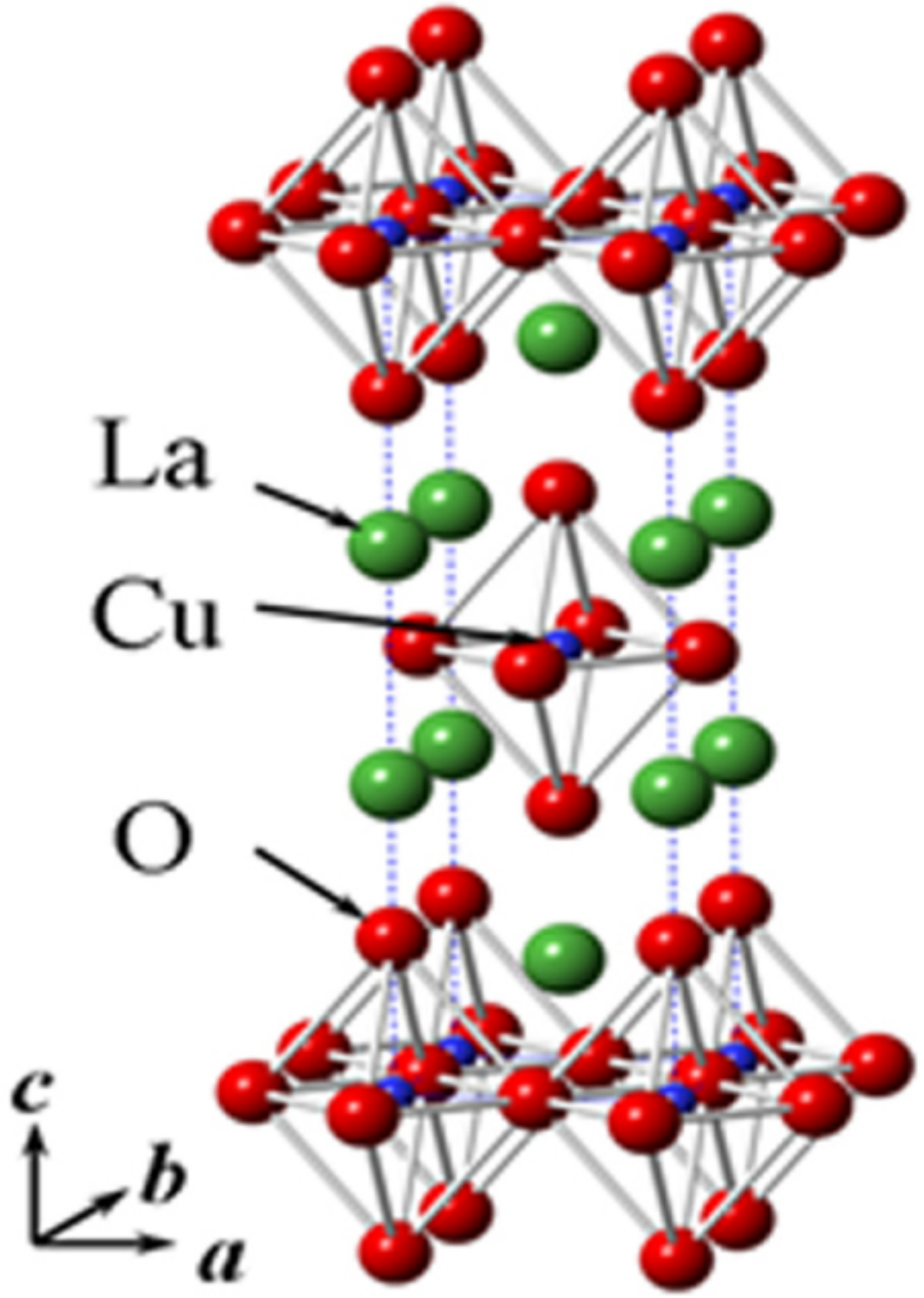


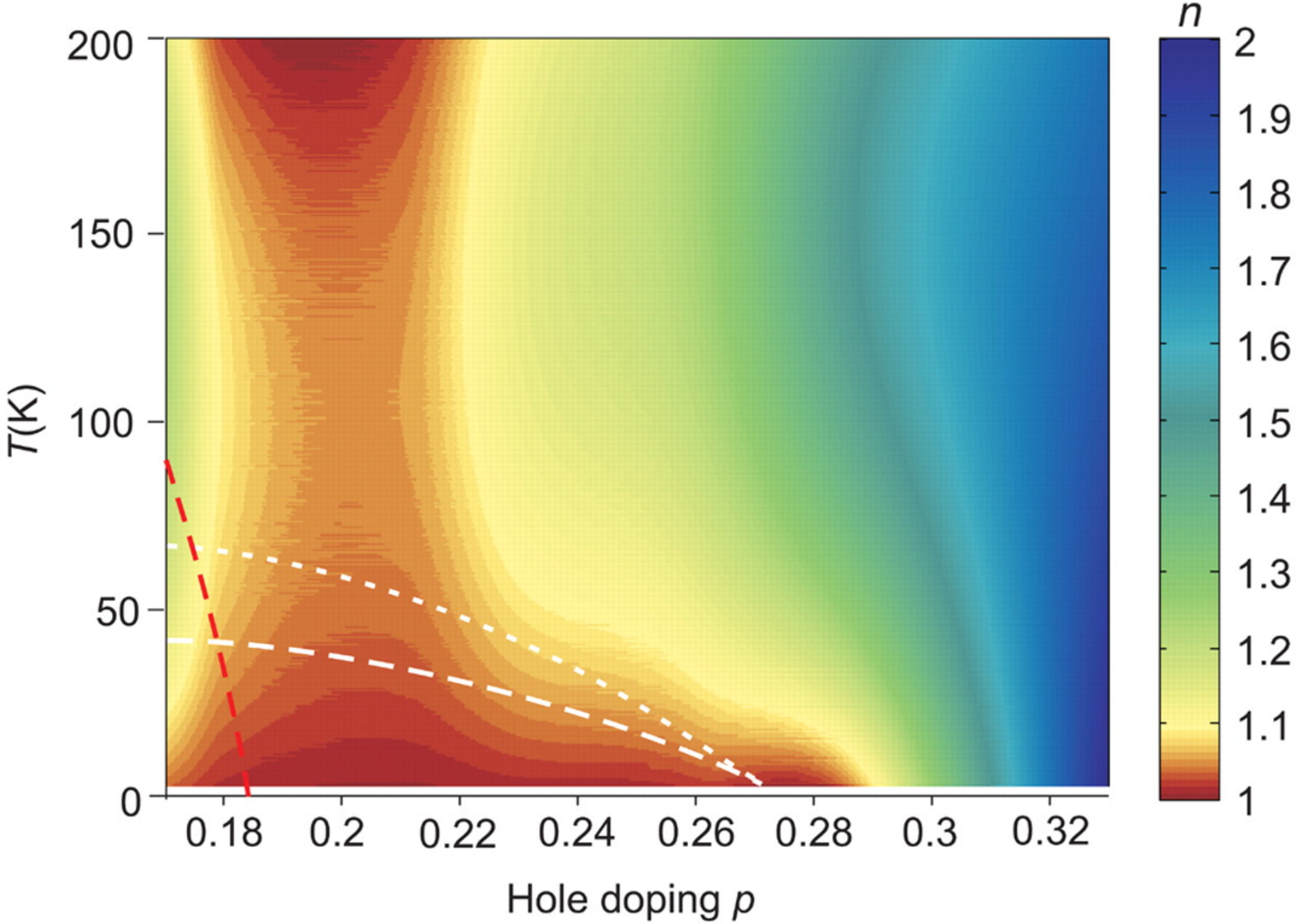
Noga Bashan, Evyatar Tulipman, Jörg Schmalian, Erez Berg, arXiv:2310.07768

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 **603** 2009



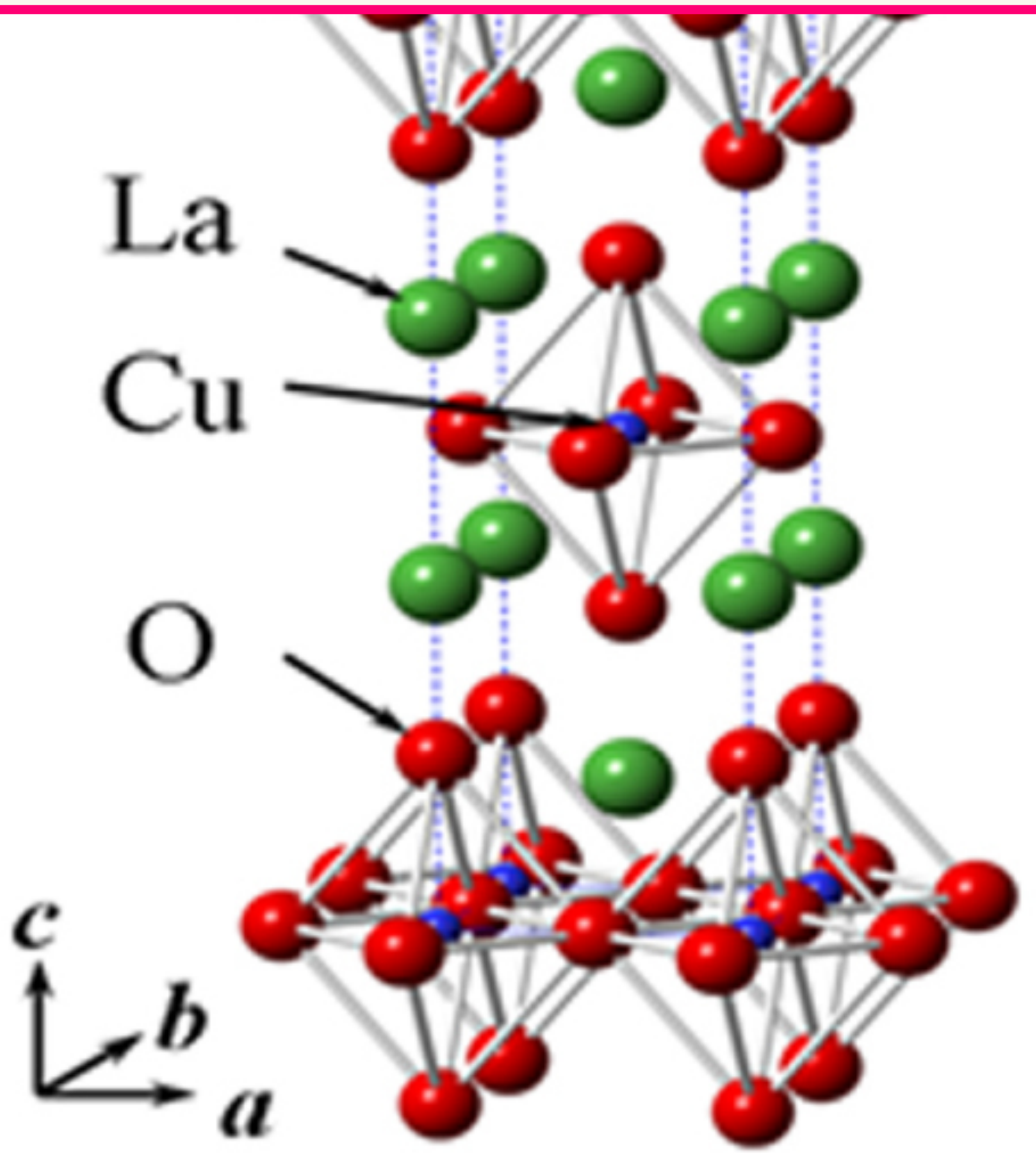


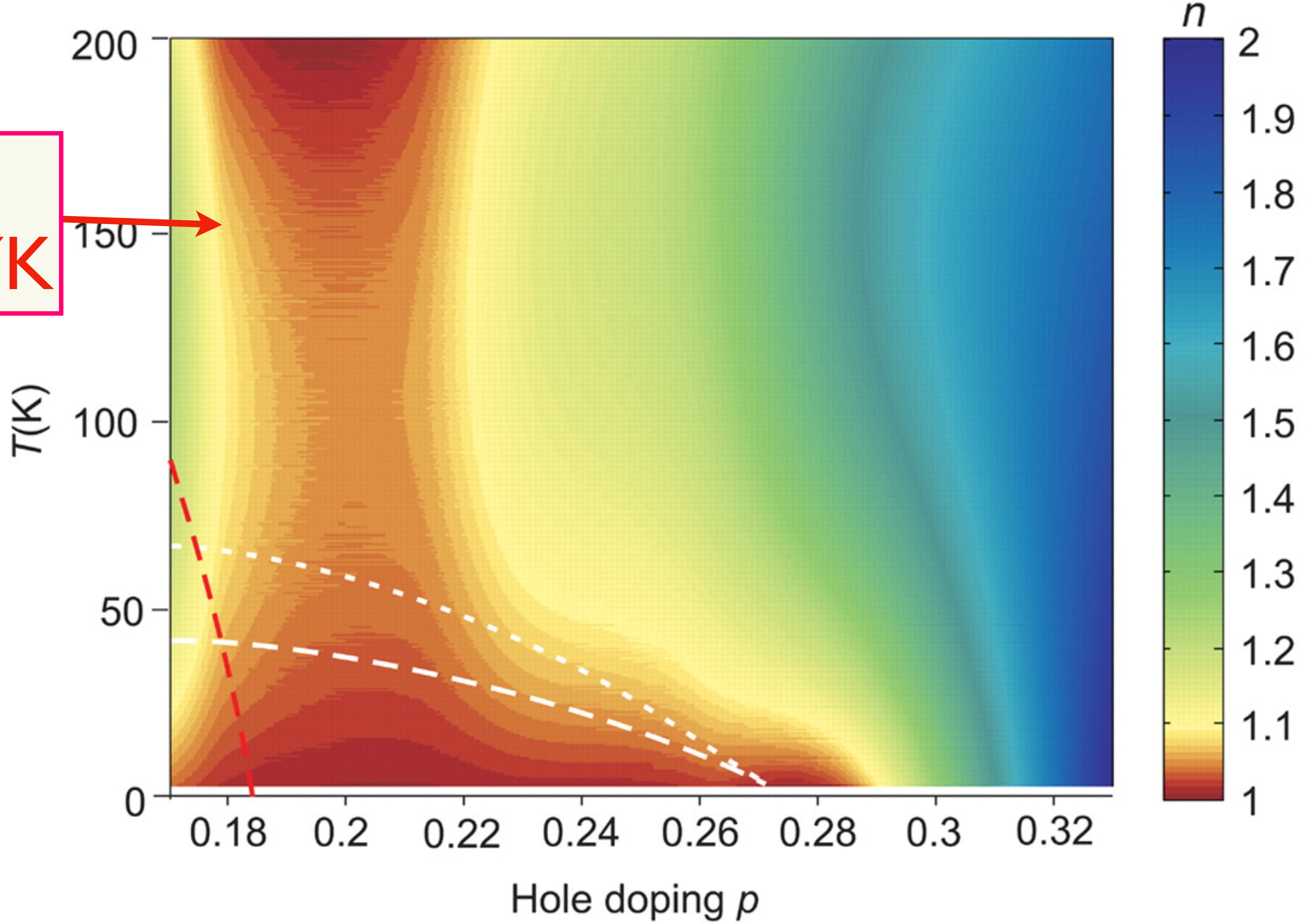
Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 **603** 2009

Extended bosons:
physics of *d*=2 Yukawa-SYK





Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 **603** 2009

Extended bosons:
physics of *d*=2 Yukawa-SYK

Localized
overdamped
bosons,
but extended
fermions