Theories of non-Fermi liquids

Low Energy Challenges for High Energy Physicists II
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FOR THEORETICAL PHYSICS

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What is a non-Fermi liquid ?

- A compressible phase at $T = 0$: the density $\mathcal{Q}$ varies smoothly as a function of μ. Global U(1) symmetry is unbroken.

- No quasiparticle excitations

- Shortest possible local-equilibration/de-phasing/ transition-to-quantum-chaos with

$$\tau_\varphi \geq C \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions* (1999)
K. Damle and Sachdev, PRB **56**, 8714 (1997)

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

P. Kovtun, D.T. Son, and A.O. Starinets, PRL **94**, 111601 (2005)

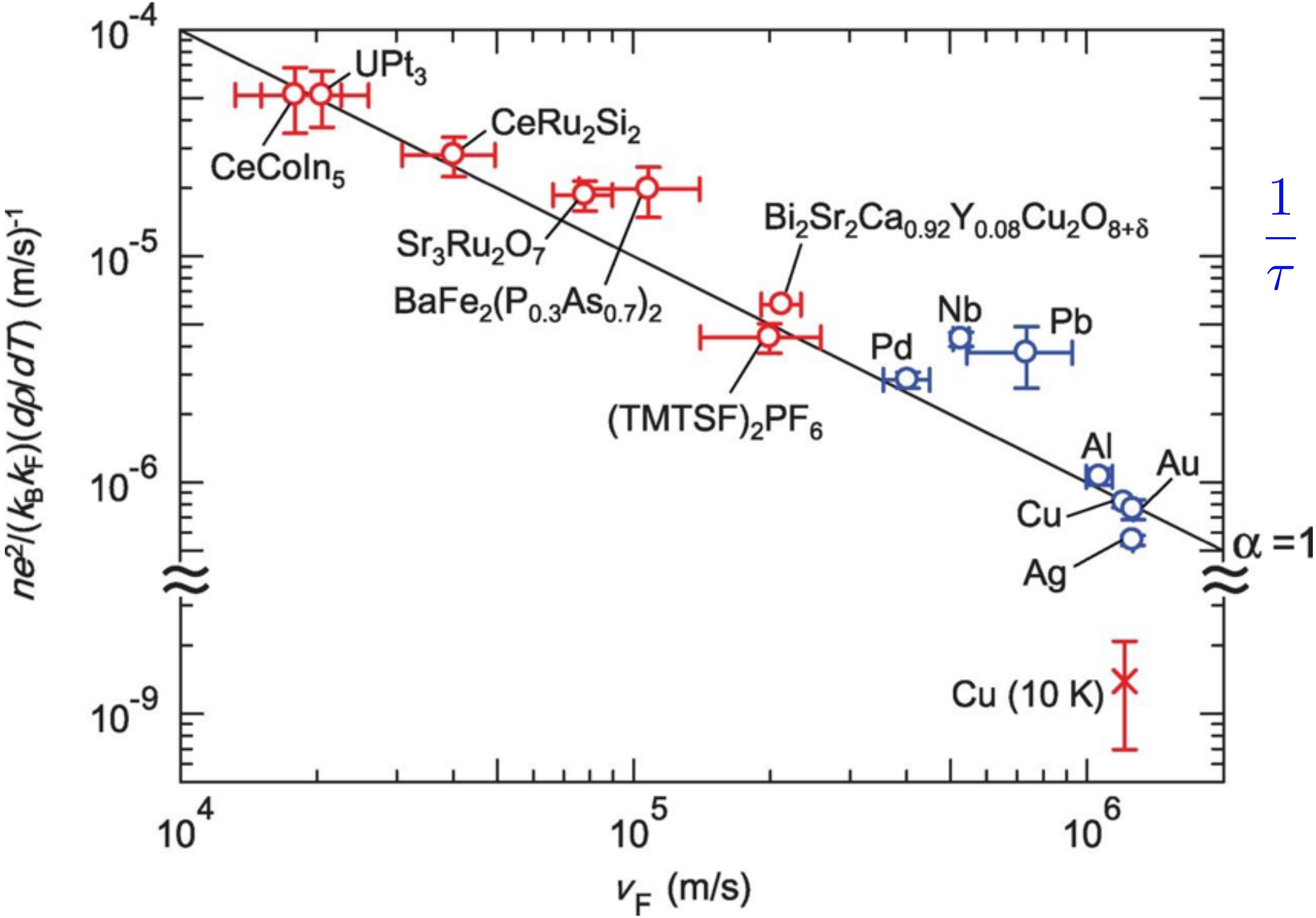
$$\frac{D}{v_b^2} \geq \tilde{C} \frac{\hbar}{k_B T}$$

Saturation requires fixed point with disorder *and* interactions
S.A. Hartnoll, Nature Physics **11**, 54 (2015)
M. Blake, PRL **117**, 091601 (2016)

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

J. Maldacena, S. H. Shenker and D. Stanford, JHEP 08 (2016)106

In Fermi liquids, $\tau \sim 1/T^2$; in gapped systems, $\tau \sim e^{\Delta/T}$.



$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie, *Science*. **339,** 804 (2013)

Theories of non-Fermi liquids

- Sachdev-Ye-Kitaev (SYK) model
- Ising-nematic criticality in $d=2$
- Higgs criticality in the cuprates

Theories of non-Fermi liquids

- Sachdev-Ye-Kitaev (SYK) model

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Infinite-range model with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

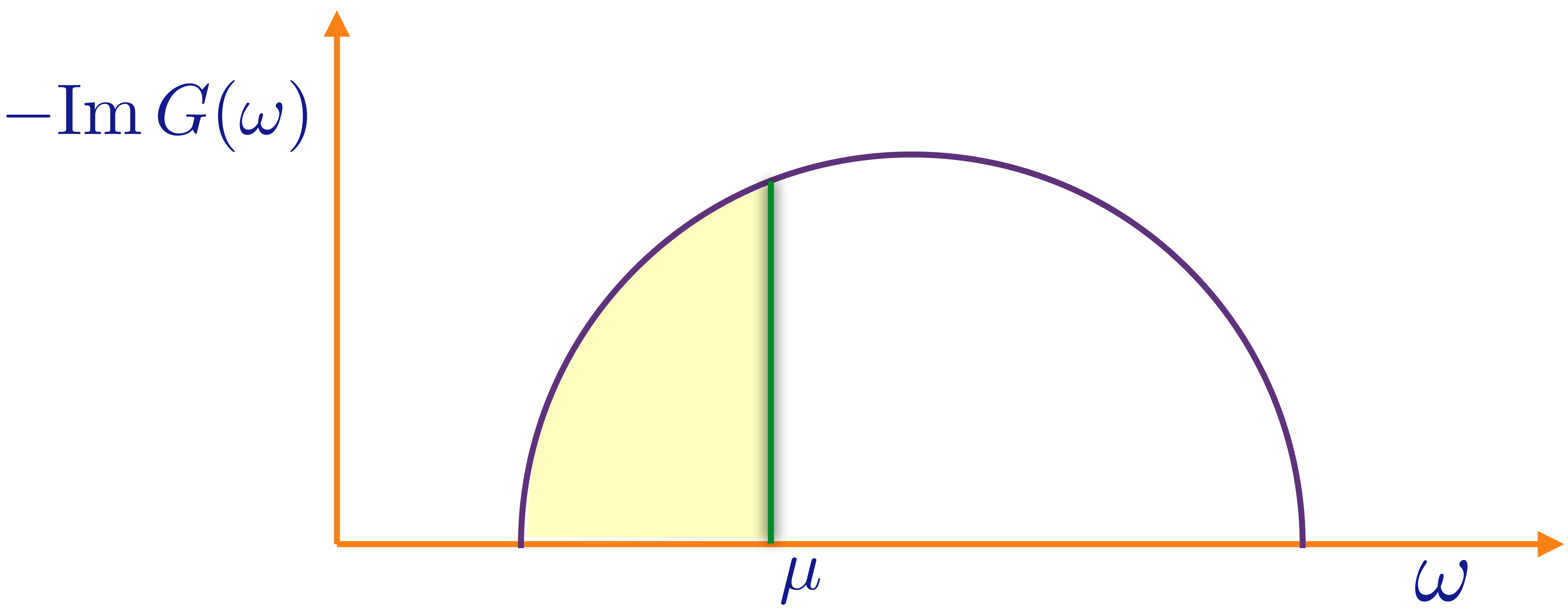
Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy E_α. By Fermi's Golden rule, for E_α at the Fermi energy

$$\begin{aligned}\frac{1}{\tau_\alpha} &= \pi J^2 \rho_0^3 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta) \\ &= \frac{\pi^3 J^2 \rho_0^3}{4} T^2\end{aligned}$$

where ρ_0 is the density of states at the Fermi energy.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

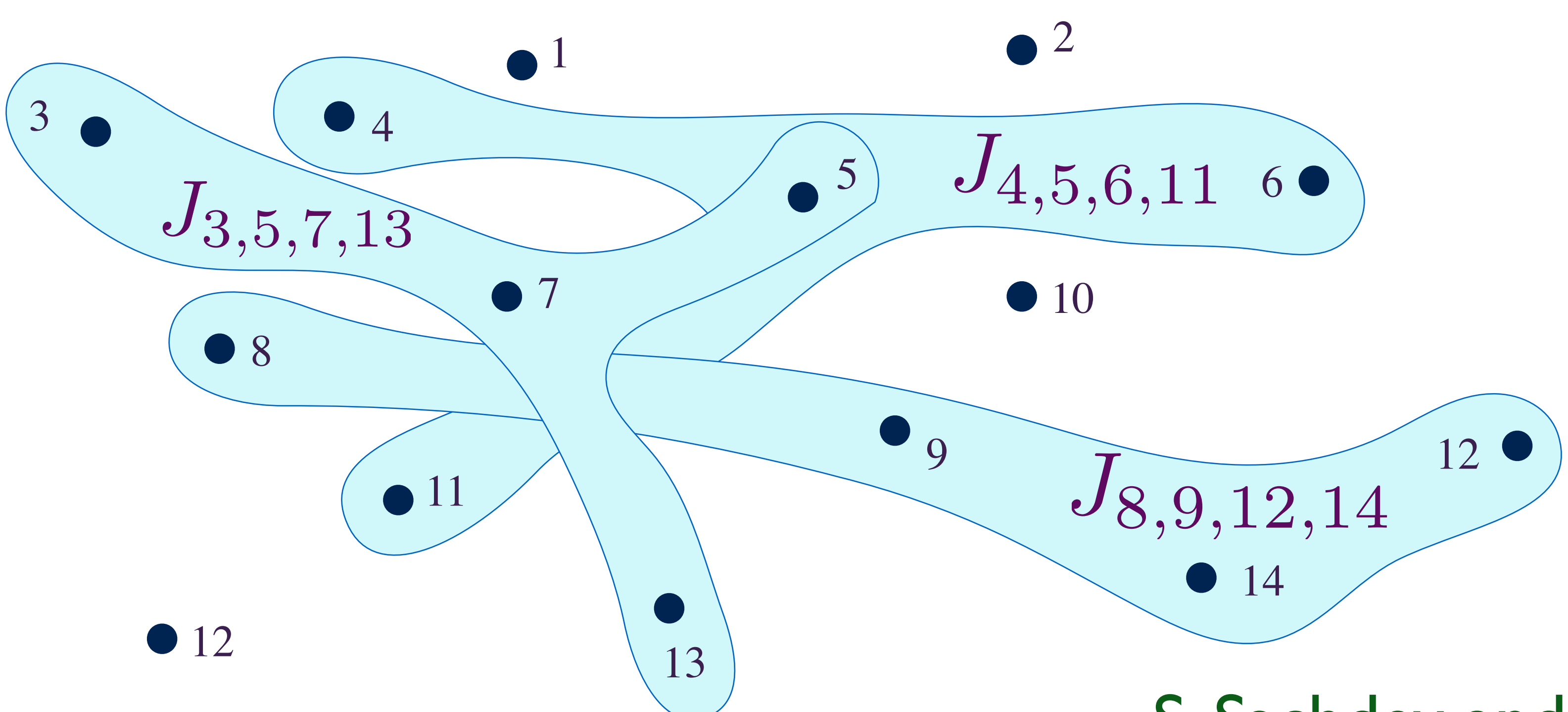
SYK model

To obtain a non-Fermi liquid, we set $t_{ij} = 0$:

$$H_{\text{SYK}} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

H_{SYK} is similar, and has identical properties, to the SY model.



A fermion can move only by entangling with another fermion: the Hamiltonian has "nothing but entanglement".

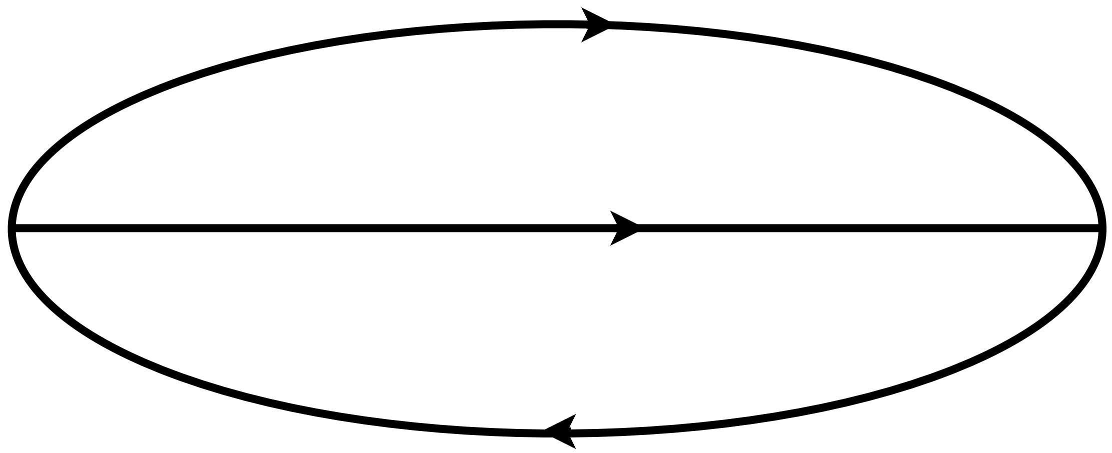
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

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$$G(\tau = 0^-) = \mathcal{Q}.$$

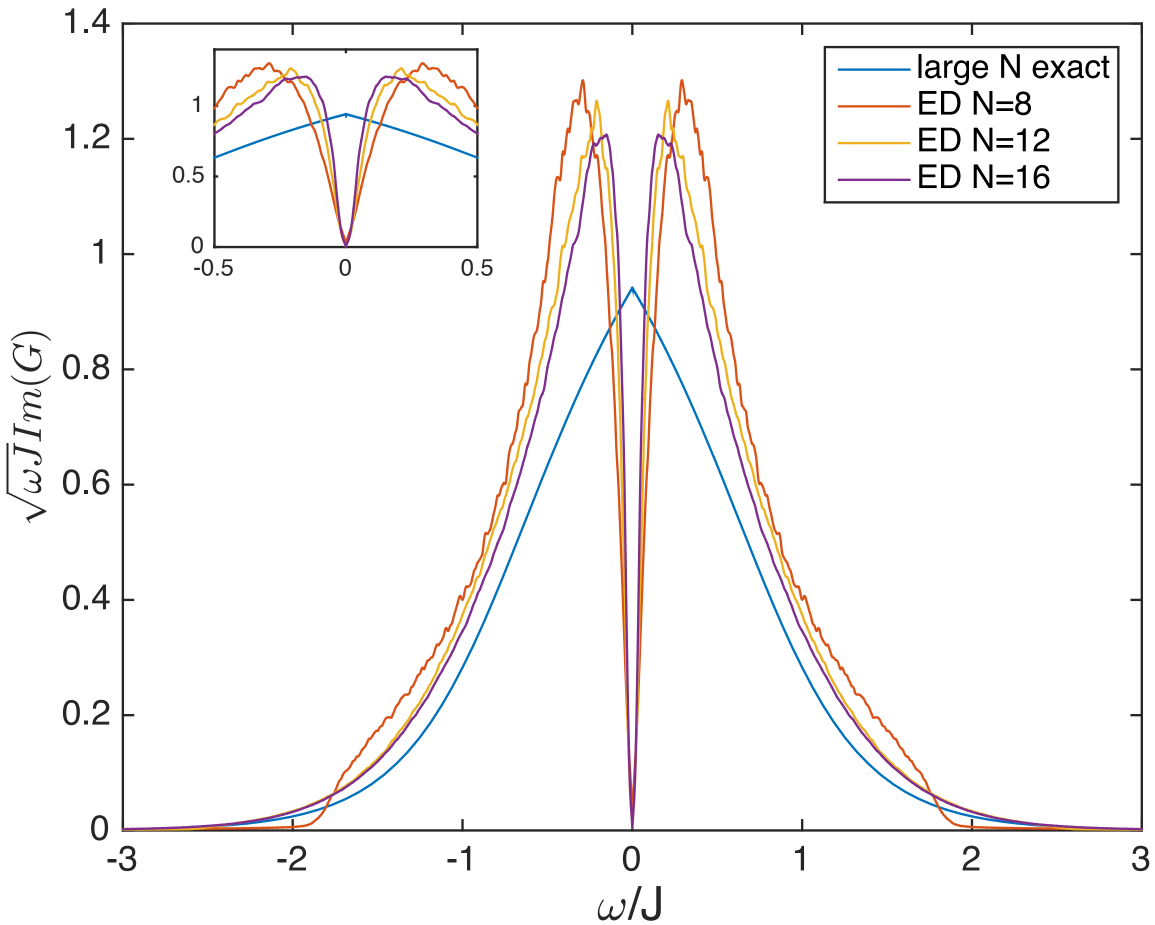
Low frequency analysis shows that the solutions must be gapless and obey

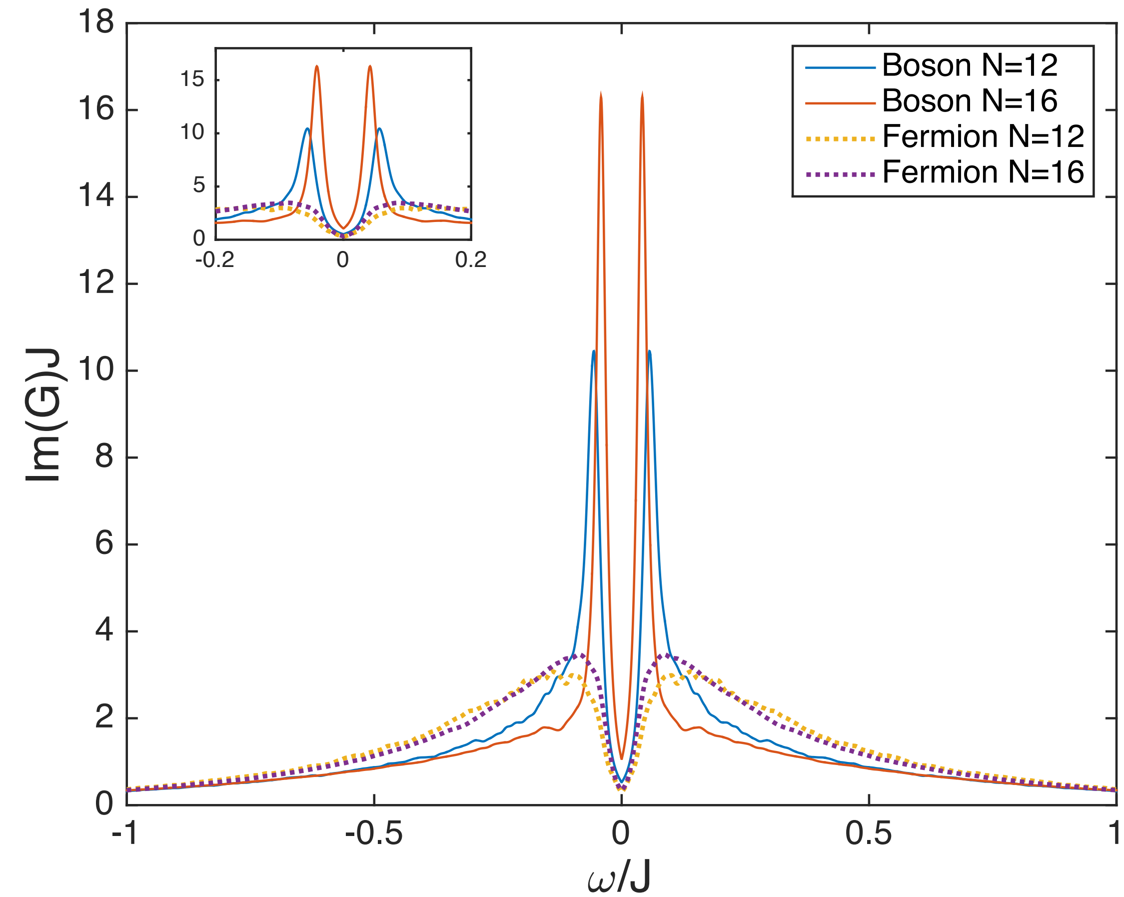
$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

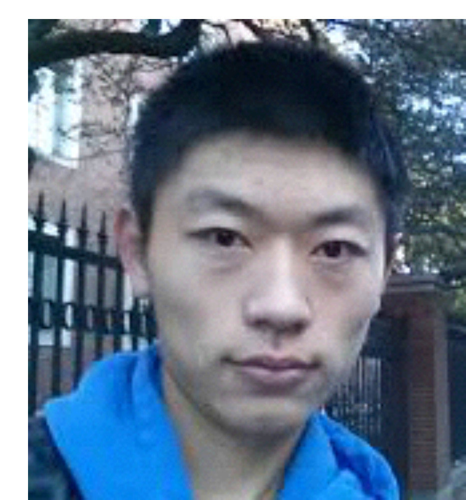
SYK model





Large N solution of equations for G and Σ agree well with exact diagonalization of the finite N Hamiltonian $\Rightarrow$ no spin-glass order

However, exact diagonalization of the same model with hard-core bosons indicates the presence of spin-glass order in the ground state.



W. Fu and S. Sachdev, arXiv: 1603.05246

It would be nice to have a solvable model of holography.

theory	bulk dual	anom. dim.	chaos	solvable in $1/N$	black hole
SYM	Einstein grav.	large	maximal	no	yes
$O(N)$	Vasiliev	$1/N$	$1/N$	yes	no
SYK	"$\ell_s \sim \ell_{AdS}$"	$O(1)$	maximal	yes	yes

column added by SS

Slide by D. Stanford at Strings 2016, Beijing

SYK model

- $T = 0$ Green's function $G \sim 1/\sqrt{\tau}$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK model

- $T = 0$ Green's function $G \sim 1/\sqrt{\tau}$
- $T > 0$ Green's function implies conformal invariance
  $G \sim 1/(\sin(\pi T \tau))^{1/2}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

SYK model

- $T = 0$ Green's function $G \sim 1/\sqrt{\tau}$
- $T > 0$ Green's function implies conformal invariance $G \sim 1/(\sin(\pi T \tau))^{1/2}$
- Non-zero entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$

A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

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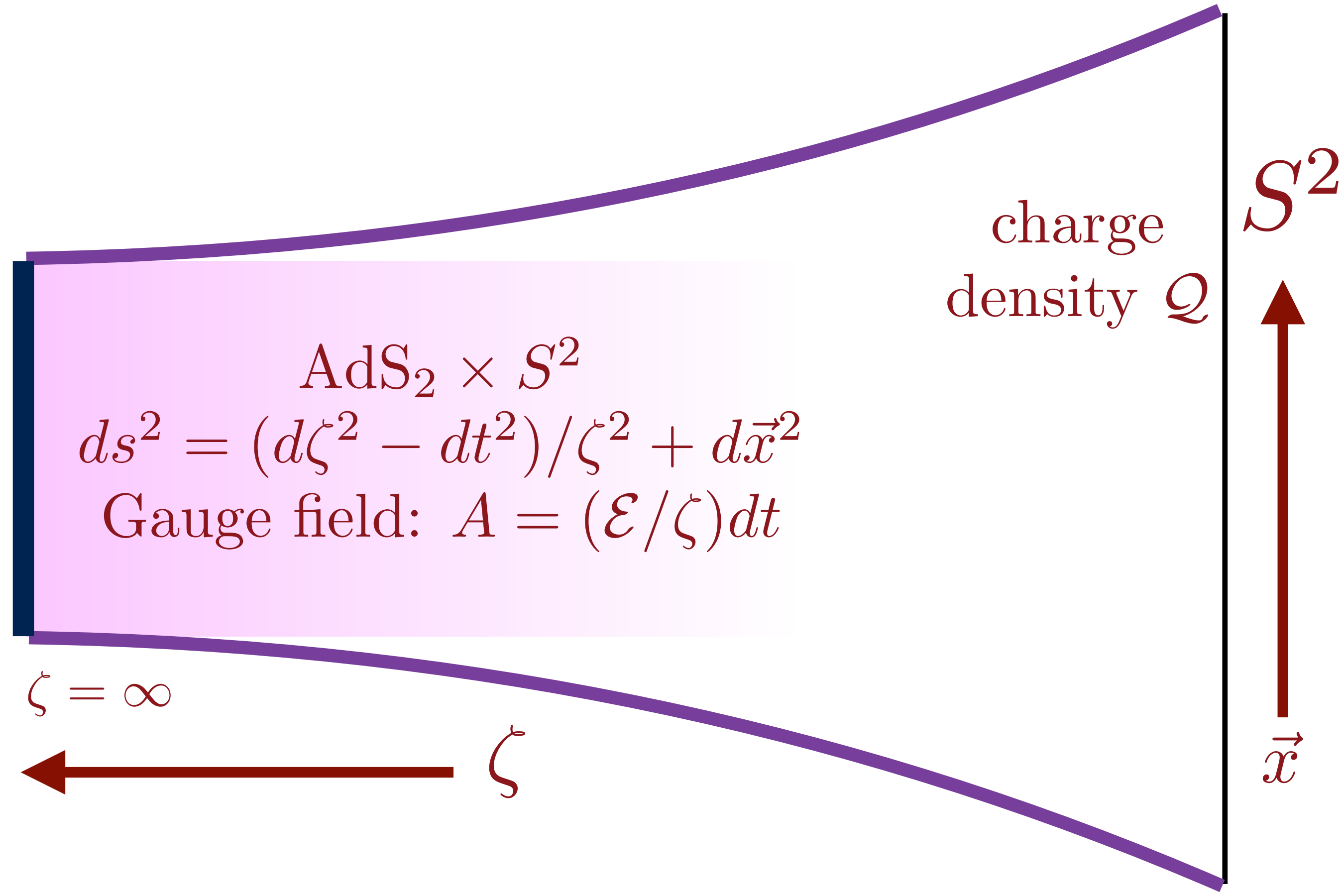
- Non-zero entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$
- These features indicate that the SYK model is dual to the low energy limit of a quantum gravity theory of black holes with AdS_2 near-horizon geometry. The Bekenstein-Hawking entropy is $N S_0$.

S. Sachdev, PRL **105**, 151602 (2010)

SYK and AdS_2



PHYSICAL REVIEW LETTERS 105, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.

SYK model

- $T = 0$ Green's function $G \sim 1/\sqrt{\tau}$
- $T > 0$ Green's function implies conformal invariance $G \sim 1/(\sin(\pi T \tau))^{1/2}$
- Non-zero entropy as $T \to 0$, $S(T \to 0) = N S_0 + \dots$
- These features indicate that the SYK model is dual to the low energy limit of a quantum gravity theory of black holes with AdS_2 near-horizon geometry. The Bekenstein-Hawking entropy is $N S_0$.
- There is a scalar zero mode associated with the breaking of reparameterization invariance down to SL(2,R). The same pattern of symmetries is present in gravity theories on AdS_2.

A. Kitaev, KITP talk, 2015

SYK model

- The dependence of S_0 on the density $\mathcal{Q}$ matches the behavior of the Wald-Bekenstein-Hawking entropy of AdS_2 horizons in a large class of gravity theories.

S. Sachdev PRX **5**, 041025 (2015)

SYK model

- The dependence of S_0 on the density $\mathcal{Q}$ matches the behavior of the Wald-Bekenstein-Hawking entropy of AdS_2 horizons in a large class of gravity theories.

- The scalar zero mode leads to a linear-in-T specific heat

$$S(T \to 0) = NS_0 + N\gamma T + \ldots.$$

An identical scalar zero model is also present in the low energy limit of theories of quantum gravity on AdS_2.

J. Maldacena and D. Stanford, arXiv:1604.07818

SYK model

- The dependence of S_0 on the density Q matches the behavior of the Wald-Bekenstein-Hawking entropy of AdS_2 horizons in a large class of gravity theories.

- The scalar zero mode leads to a linear-in-T specific heat

$$S(T \to 0) = NS_0 + N\gamma T + \ldots.$$

  An identical scalar zero model is also present in the low energy limit of theories of quantum gravity on AdS_2.

- The Lyapunov time to quantum chaos saturates the lower bound both in the SYK model and in quantum gravity.

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

A. Kitaev, KITP talk, 2015

J. Maldacena and D. Stanford, arXiv:1604.07818

SYK model

After integrating the fermions, the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

$$S = \ln \det \left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right] + \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left[G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

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$$S = \ln \det \left[\delta(\tau_1 - \tau_2)(\cancel{\partial_{\tau_1}} + \cancel{\mu}) - \Sigma(\tau_1, \tau_2)\right]$$
$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left[G(\tau_2, \tau_1) + (J^2/2) G^2(\tau_2, \tau_1) G^2(\tau_1, \tau_2)\right]$$

A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, PRX **5**, 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

SYK model

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1-\tau_2) \sim (\tau_1-\tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1-\tau_2) \sim (\tau_1-\tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau+b}{c\tau+d} \quad , \quad ad-bc=1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

Reparametrization zero mode

Expand about the saddle point by writing

$$G(\tau_1,\tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1)-f(\tau_2))$$

(and similarly for Σ) and obtain an effective action for $f(\tau)$. This action does not vanish because of the time derivative in the determinant which is not reparameterization invariant.

J. Maldacena and D. Stanford, arXiv:1604.07818

See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

SYK model

However the effective action must vanish for SL(2,R) transformations because G_s, Σ_s are invariant under it. In this manner we obtain the effective action as a Schwarzian

$$N S_{\text{eff}} = -\frac{N\gamma}{4\pi^2} \int d\tau \, \{f, \tau\} \quad , \quad \{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2} \left(\frac{f''}{f'} \right)^2 ,$$

where the specific heat, $\mathcal{C} = N\gamma T$.

The Schwarzian effective action implies that the SYK model *saturates* the lower bound on the Lyapunov time

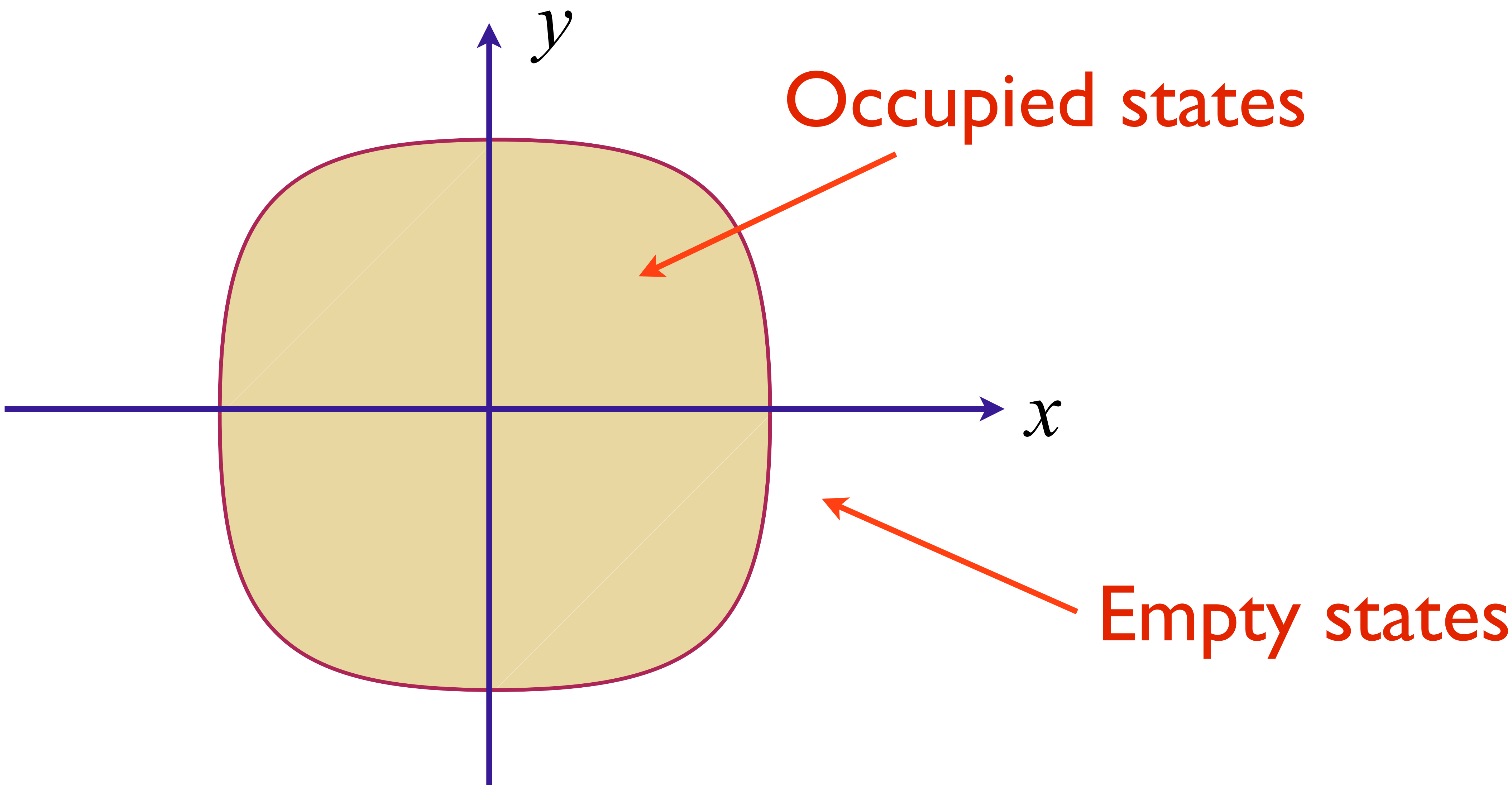
$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

Theories of non-Fermi liquids

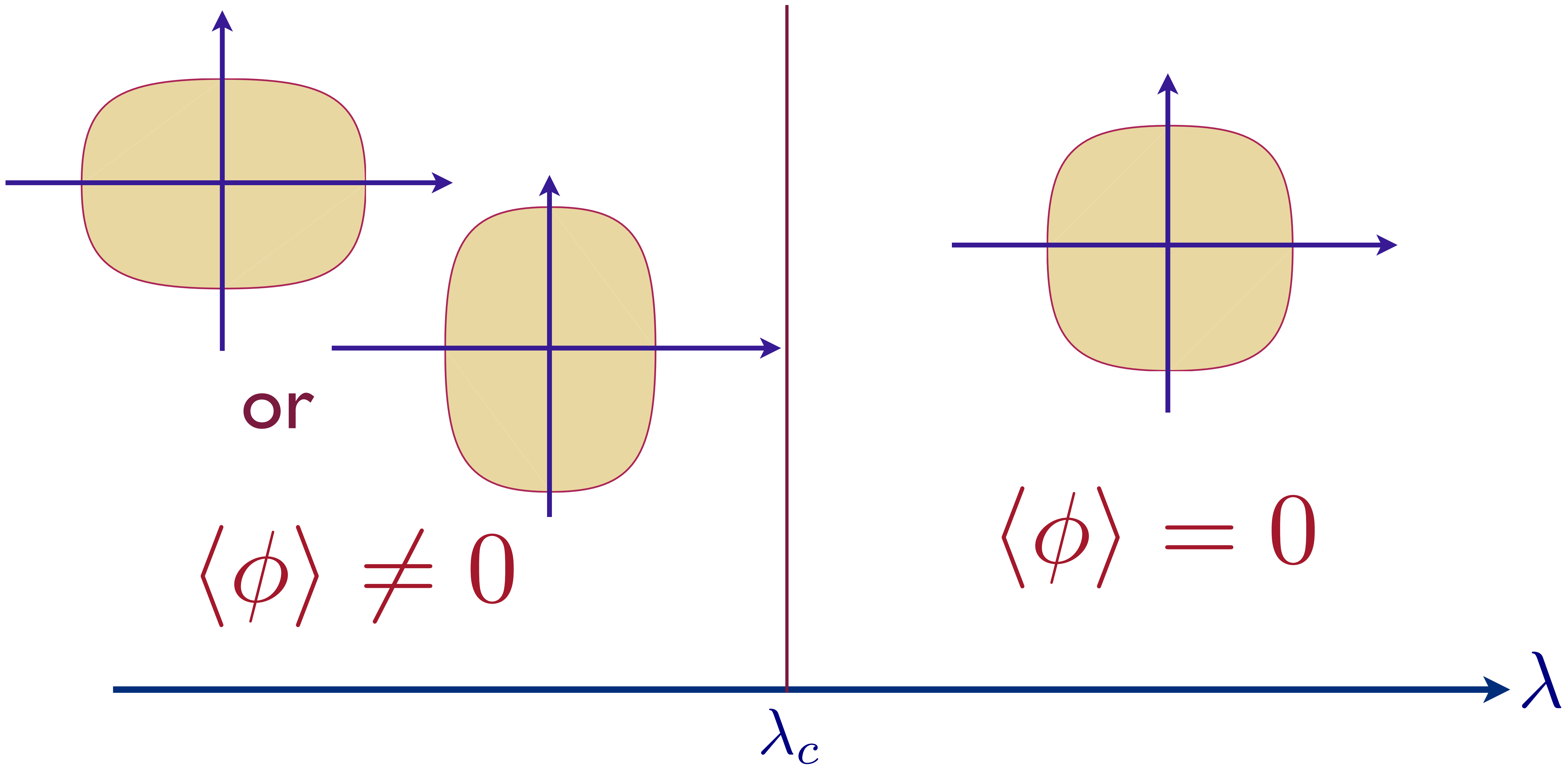
- Sachdev-Ye-Kitaev (SYK) model
- Ising-nematic criticality in d=2
- Higgs criticality in the cuprates

Quantum criticality of Ising-nematic ordering in a metal



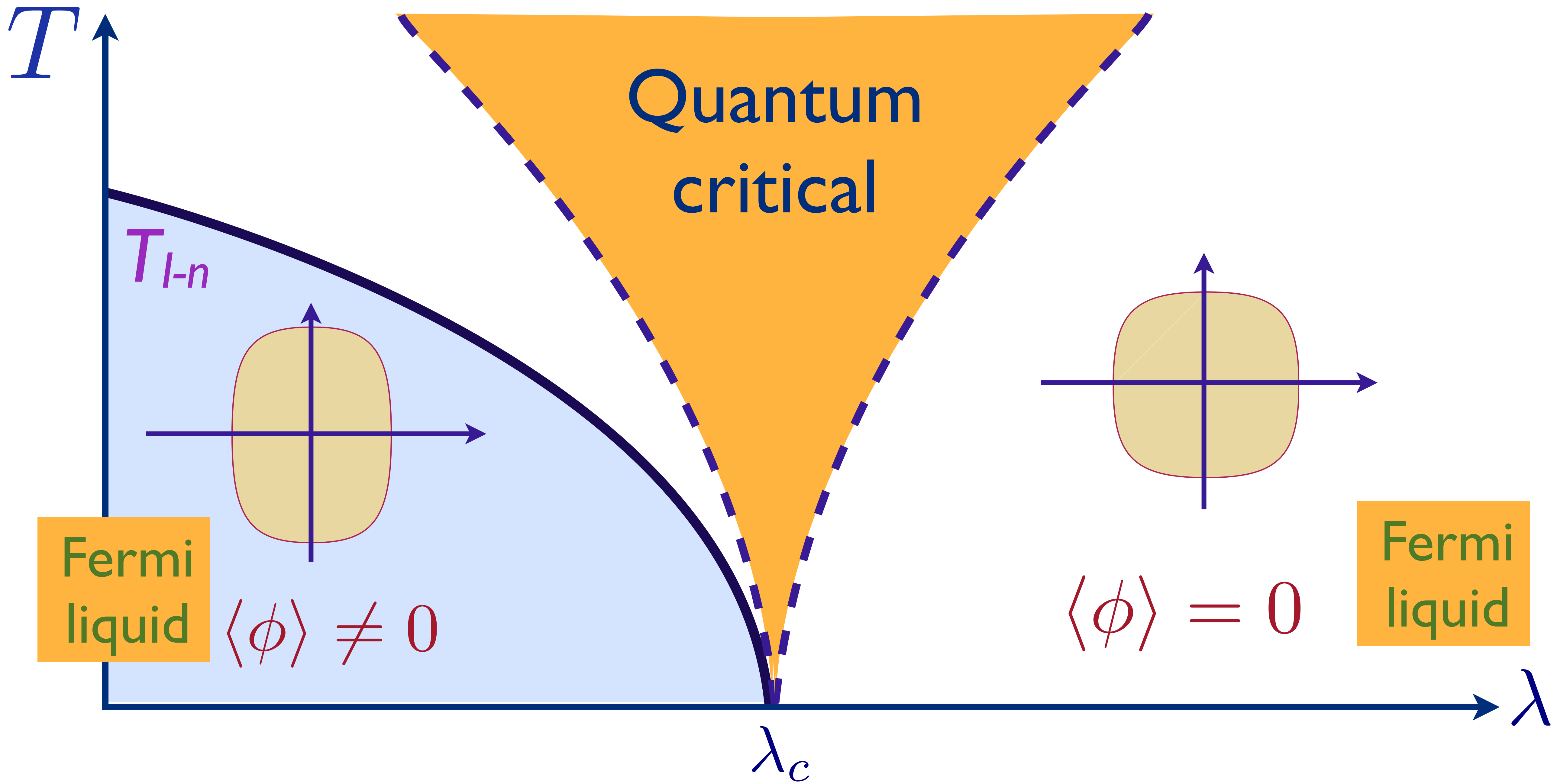
A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



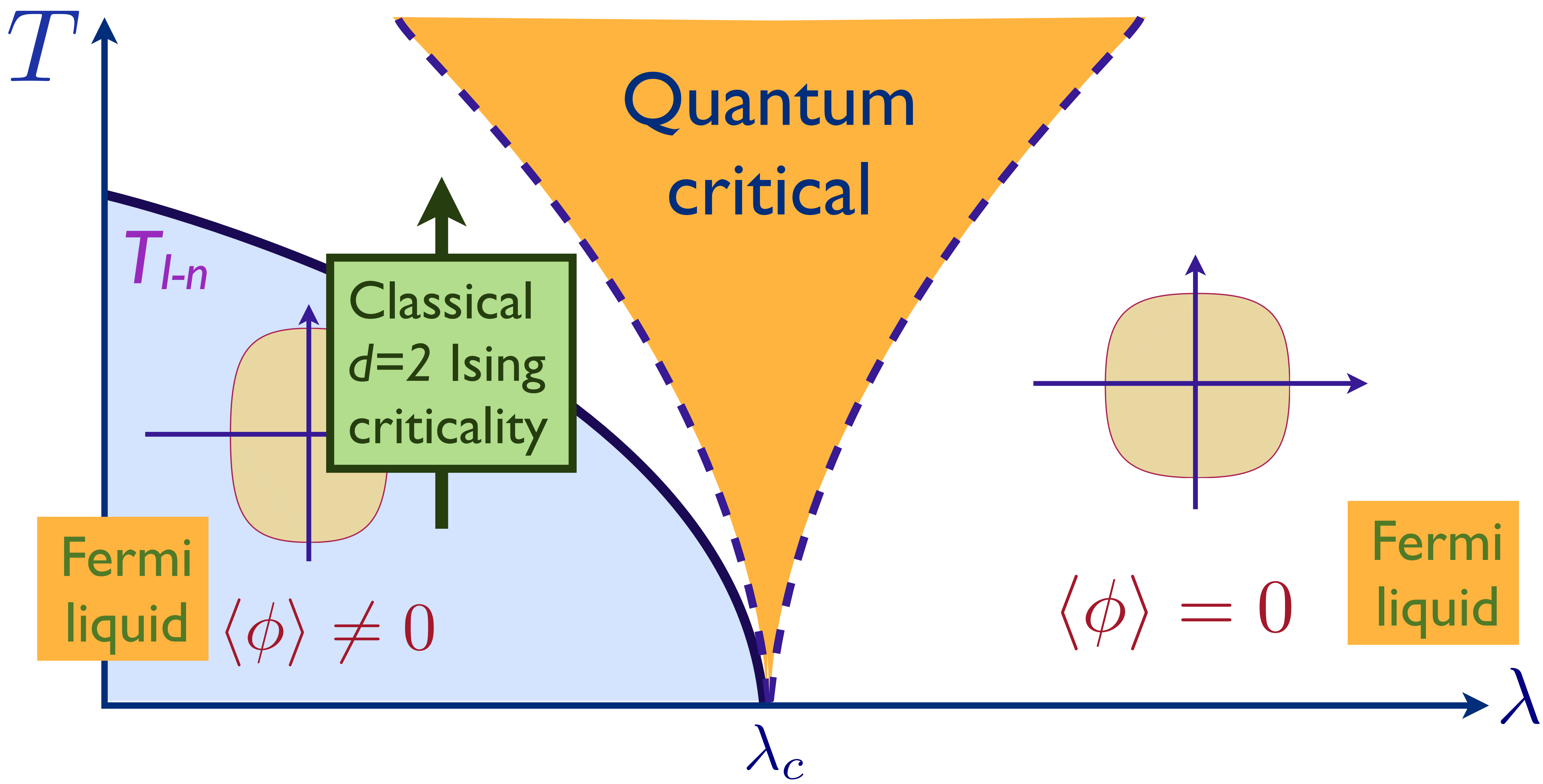
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

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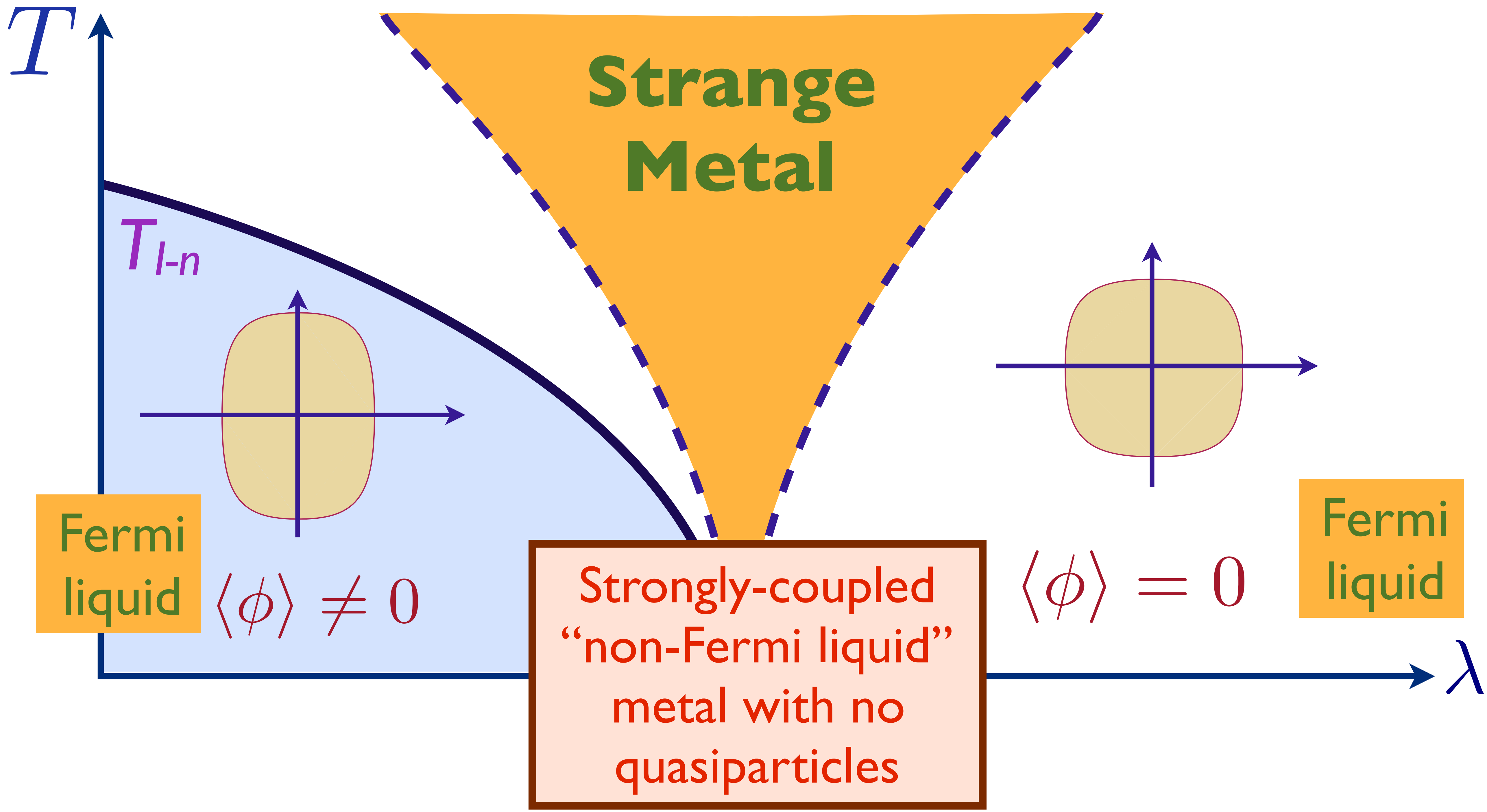

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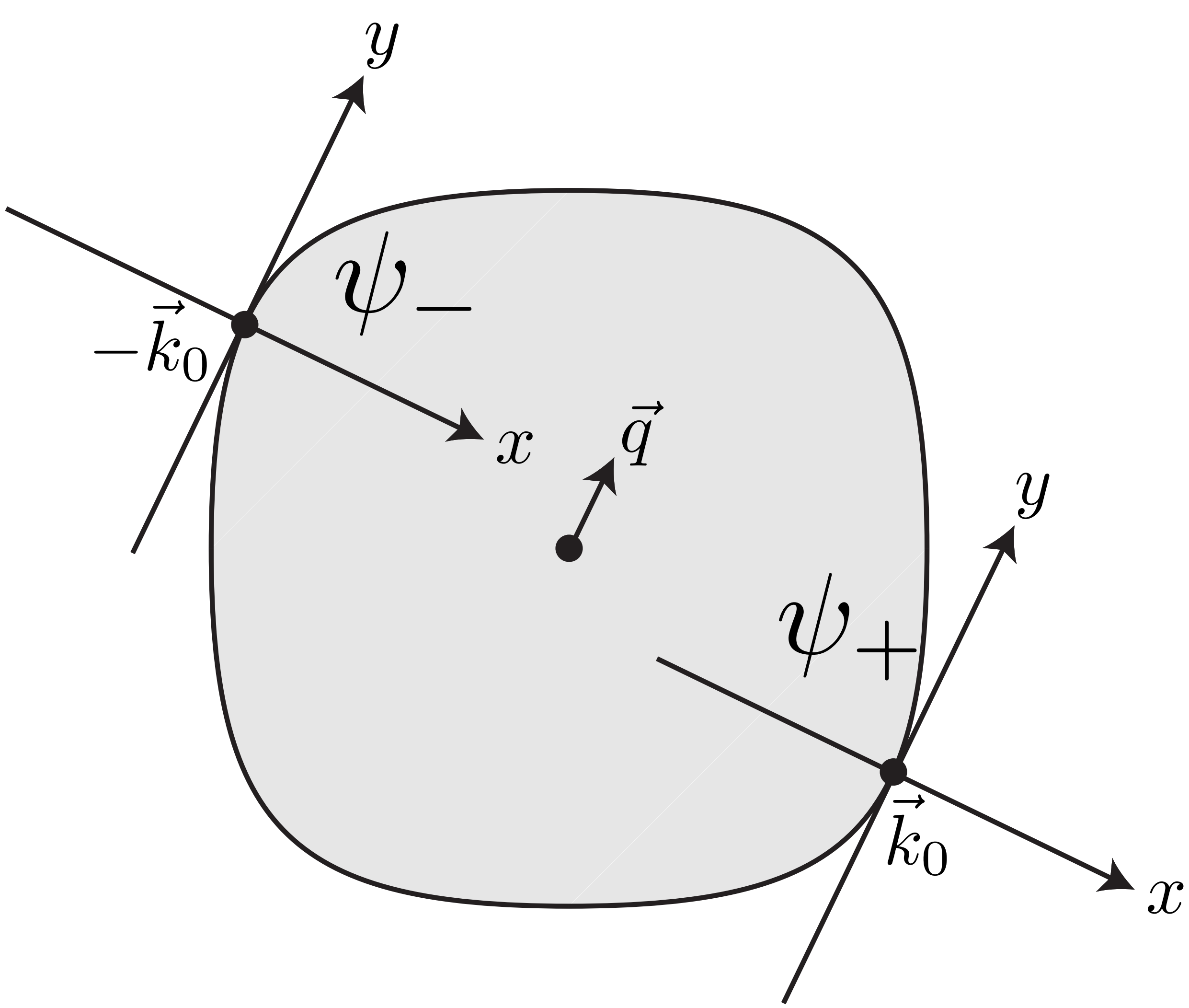
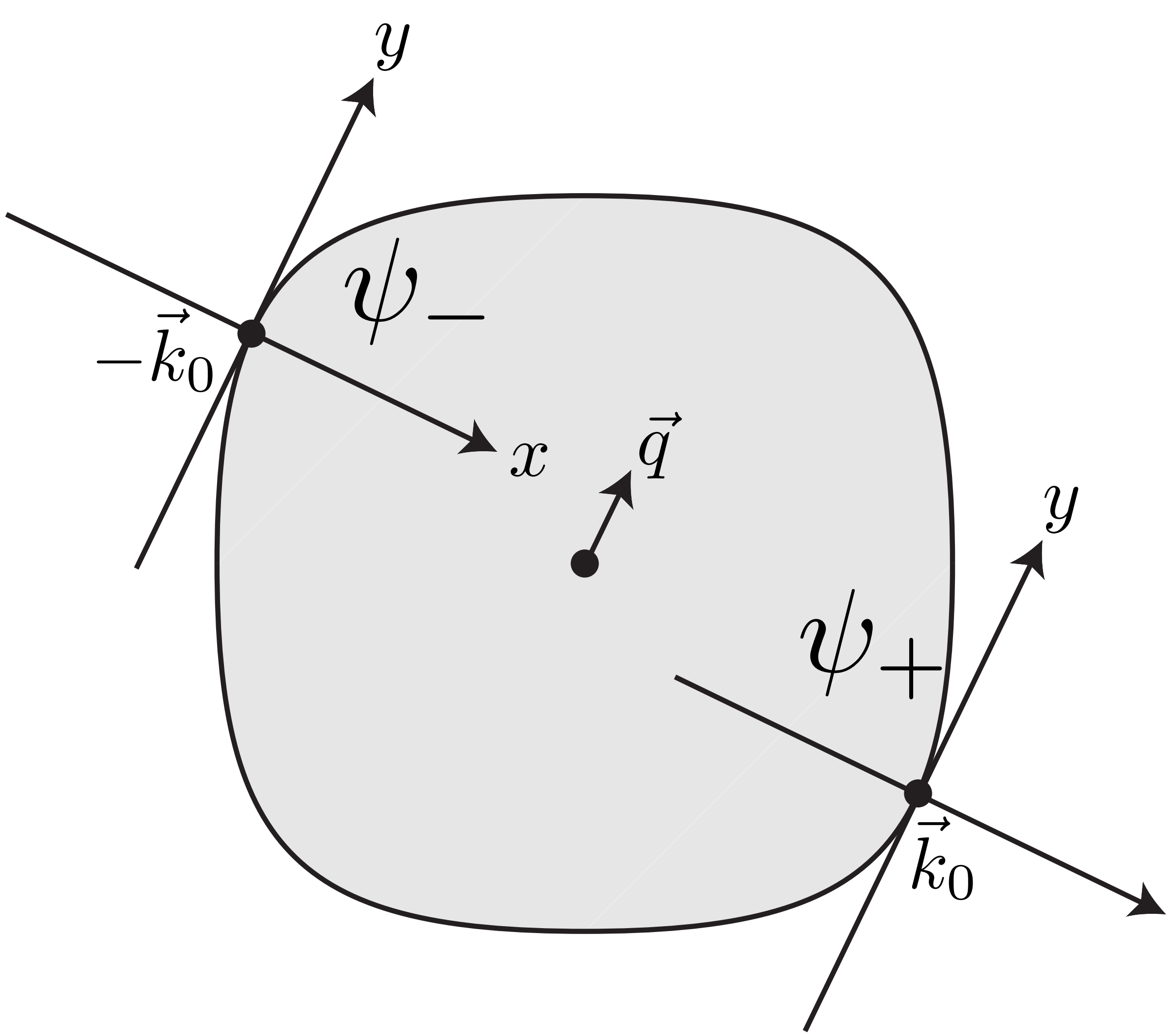


- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering in a metal



- Exact solution for some exponents for the 'one-sided' chiral model (Shouvik Sur and Sung-Sik Lee, PRB **90**, 045121 (2014)).
- Expansion in $\epsilon = 5/2 - d$ (D. Dalidovich and Sung-Sik Lee, PRB **88**, 245106 (2013)).

$$\mathcal{L}[\psi_\pm, \phi] =$$

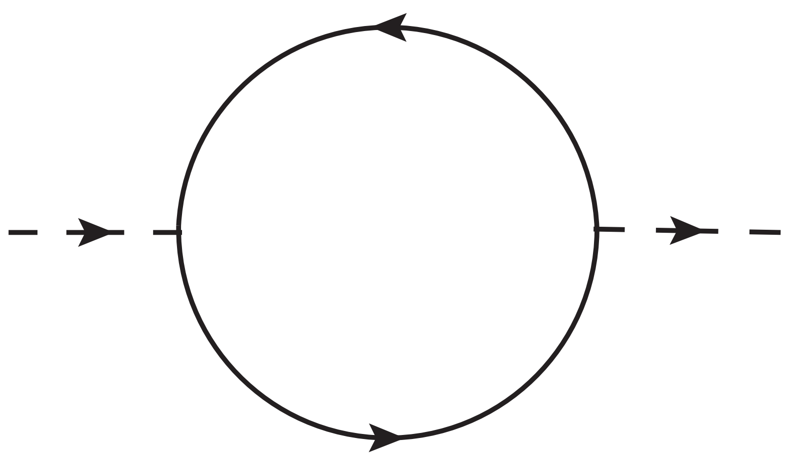
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi\left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y \phi\right)^2$$



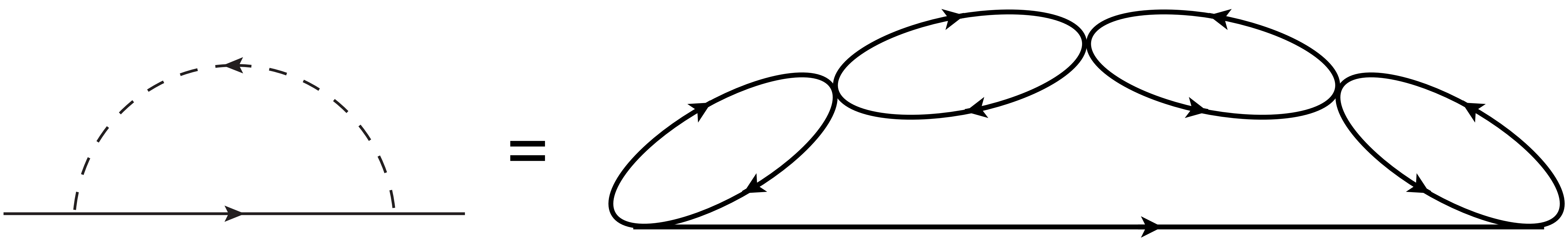
One loop ϕ self-energy with N_f fermion flavors:

$$\Sigma_\phi(\vec{q},\omega) = N_f \int \frac{d^2k}{4\pi^2}\frac{d\Omega}{2\pi} \frac{1}{\left[-i(\Omega+\omega) + k_x + q_x + (k_y+q_y)^2\right]\left[-i\Omega - k_x + k_y^2\right]} = \frac{N_f}{4\pi}\frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi\left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y \phi\right)^2$$



Diagrams similar to SYK!

Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int \frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\frac{q_y^2}{g^2}+\frac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\operatorname{sgn}(\Omega)|\Omega|^{2/3}$$

$\sim |\Omega|^{d/3}$ in dimension d.

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

Schematic form of ϕ and fermion Green's functions in d dimensions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_{\|}^2 + \dfrac{|\omega|}{|q_{\|}|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_{\|}^2 - i \text{sgn}(\omega) |\omega|^{d/3} / N_f}$$

In the boson case, $q_{\|}^2 \sim \omega^{1/z_b}$ with $z_b = 3/2$.
In the fermion case, $q_x \sim q_{\|}^2 \sim \omega^{1/z_f}$ with $z_f = 3/d$.

Note $z_f < z_b$ for $d > 2$ $\Rightarrow$ Fermions have *higher* energy than bosons, and perturbation theory in g is OK.

Quantum criticality of Ising-nematic ordering in a metal

$$
\begin{aligned}
\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- \\
&\quad - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2
\end{aligned}
$$

Schematic form of ϕ and fermion Green's functions in $d = 2$

$$
D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \dfrac{|\omega|}{|q_y|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i\,\mathrm{sgn}(\omega)|\omega|^{2/3}/N_f}
$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2 \right) \psi_-$$
$$- \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left(\partial_y \phi \right)^2$$

Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} \phi &\to \phi\, s \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

Quantum criticality of Ising-nematic ordering in a metal

The entropy density, s, obeys

$$s \sim T^{(d-\theta)/z} \sim T^{2/3}$$

Y. B. Kim, A. Furusaki, X.-G. Wen, and P.A. Lee, PRB **50**, 17917 (1994)

where $z = 3/2$ is the dynamic critical exponent for fermionic excitations dispersing normal to the Fermi surface, and $d - \theta = 1$ is the number of dimensions normal to the Fermi surface.

A RG analysis using a dimensionality expansion below $d = 5/2$ shows that the opitcal conductivity obeys

$$\sigma \sim \omega^{(d-\theta-2)/z} \sim \omega^{-2/3}$$

A. Eberlein, I. Mandal, and S. Sachdev, PRB **94**, 045133 (2016)

We also computed the shear viscosity and found

$$\eta \sim T^{(d-\theta-2)/z} \sim T^{-2/3}$$

Note that $\eta/s \sim T^{-2/z}$ does not scale to a constant: so for the viscosity we do not have a system of reduced dimensionality, a consequence of the anisotropic scaling between the directions normal and parallel to the Fermi surface.

A. A. Patel, A. Eberlein, and S. Sachdev, arXiv:1607.03894

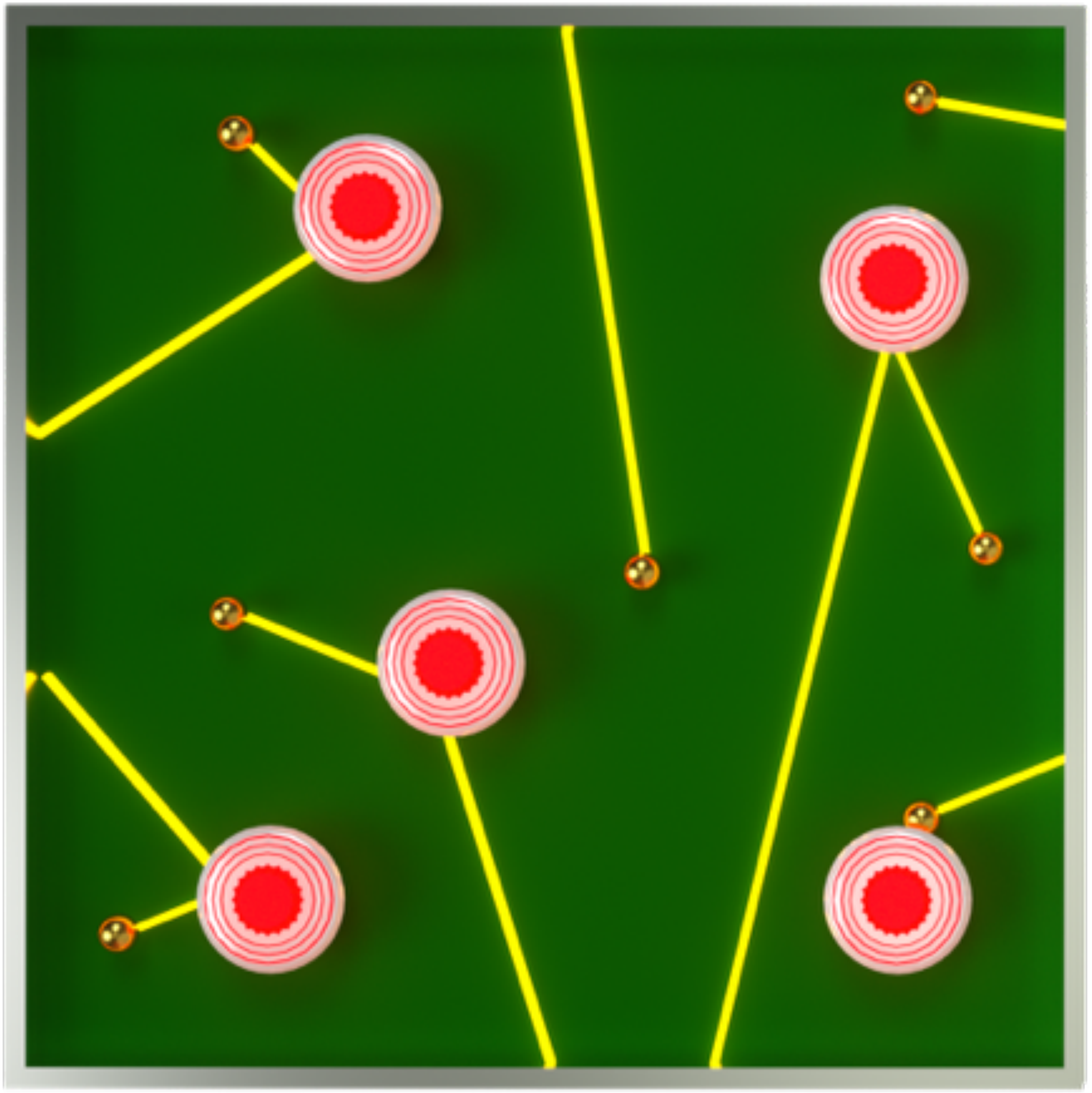
Quantum criticality of Ising-nematic ordering in a metal

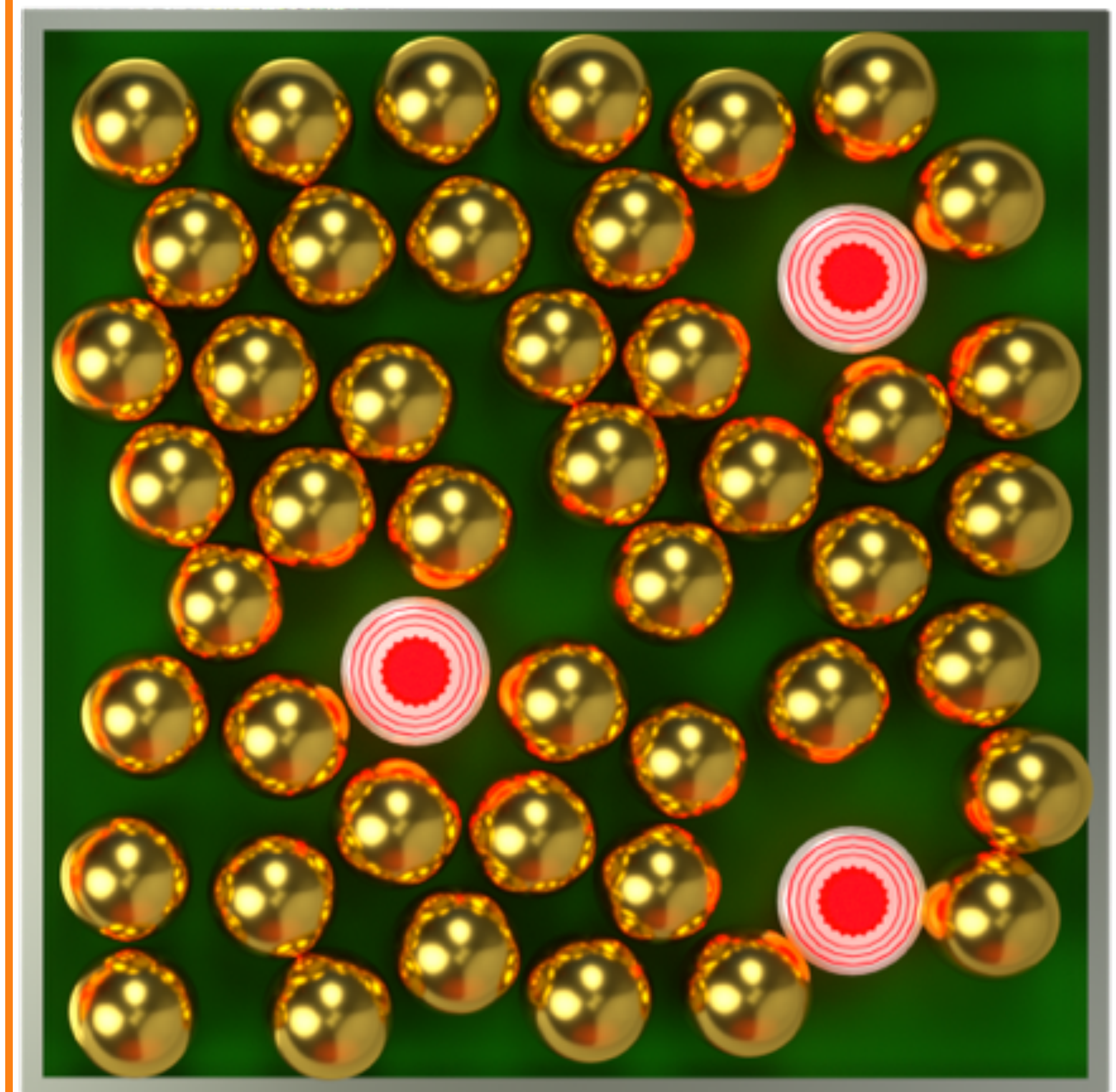
- Early computations of fermions scattering off ϕ fluctuations obtained the d.c. conductivity $\sigma \sim T^{-4/3}$.

- However, this ignores "momentum drag": the ϕ momentum is ultimately given back to the fermions, and this happens rapidly because the fermion-ϕ coupling is (infinitely) strong.

D. L. Maslov, V.I. Yudson, A.V. Chubukov, PRL **106**, 106403 (2011)

- The critical theories of the clean non-Fermi liquid can always be defined in the continuum in a manner so that there is a conserved "momentum" operator $\vec{P}$, so that $[\vec{P}, \mathcal{O}(x)] = i\nabla_x \mathcal{O}(x)$.





Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events

Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron "liquid" then "flows" around impurities

Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \pi \delta(\omega) + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \pi \delta(\omega) + \alpha_Q(\omega)$$

$$\bar{\kappa} = \frac{T\mathcal{S}^2}{\mathcal{M}} \pi \delta(\omega) + \bar{\kappa}_Q(\omega)$$

with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x, P_x}$, and $\mathcal{M} \equiv \chi_{P_x, P_x}$.

In theories which are relativistic at high energies (including graphene), $T\alpha_Q(\omega) = -\mu\sigma_Q(\omega)$, $T\bar{\kappa}_Q(\omega) = \mu^2\sigma_Q(\omega)$, $\mathcal{M} = T\mathcal{S} + \mu\mathcal{Q} = \mathcal{H}$ the enthalpy density, and $\mathcal{Q} = n$ the electron density

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Translational symmetry breaking

Momentum relaxation by an external source h coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h(x) \, \mathcal{O}(x).$$

Leads to an additional term in equations of motion:

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\text{imp}}} + \ldots$$

"Memory function" methods yield an explicit expression for τ_{imp}:

$$\frac{\mathcal{M}}{\tau_{\text{imp}}} = \lim_{\omega \to 0} \int d^d q \, |h(q)|^2 q_x^2 \frac{\text{Im}\left(G^{\text{R}}_{\mathcal{O}\mathcal{O}}(q,\omega)\right)_{H_0}}{\omega} + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Thermoelectric transport coefficients

Transport has two components: a "momentum drag" term, and a "quantum critical" term.

$$\sigma = \frac{\mathcal{Q}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \sigma_Q(\omega)$$

$$\alpha = \frac{\mathcal{S}\mathcal{Q}}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_{\rm imp})} + \alpha_Q(\omega)$$

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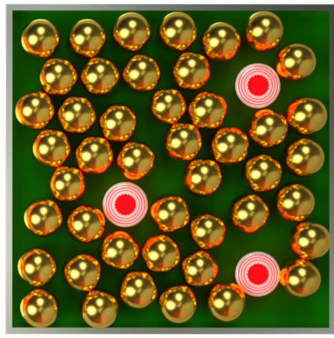


with entropy density $\mathcal{S}$, $\mathcal{Q} \equiv \chi_{J_x,P_x}$, and $\mathcal{M} \equiv \chi_{P_x,P_x}$.

Obtained in hydrodynamics, and by memory functions

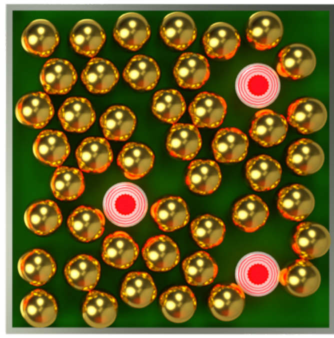
S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Quantum criticality of Ising-nematic ordering in a metal

- If we choose $\mathcal{O}$ = number or energy density, then the results are similar to obtained from solving hydrodynamic equations in the presence of impurities.

R. A. Davison, K. Schalm, and Jan Zaanen, PRB 89, 245116 (2014)

- For the Ising-nematic critical point, the strongest scattering arises from the choice $\mathcal{O} = \phi$, with the disorder coupling to the local orientation of the nematic ordering. This leads to



$$\text{Resistivity } \rho \sim [T \ln(1/T)]^{-1/2}$$

- If Landau damping of the ϕ fluctuations can be neglected, then

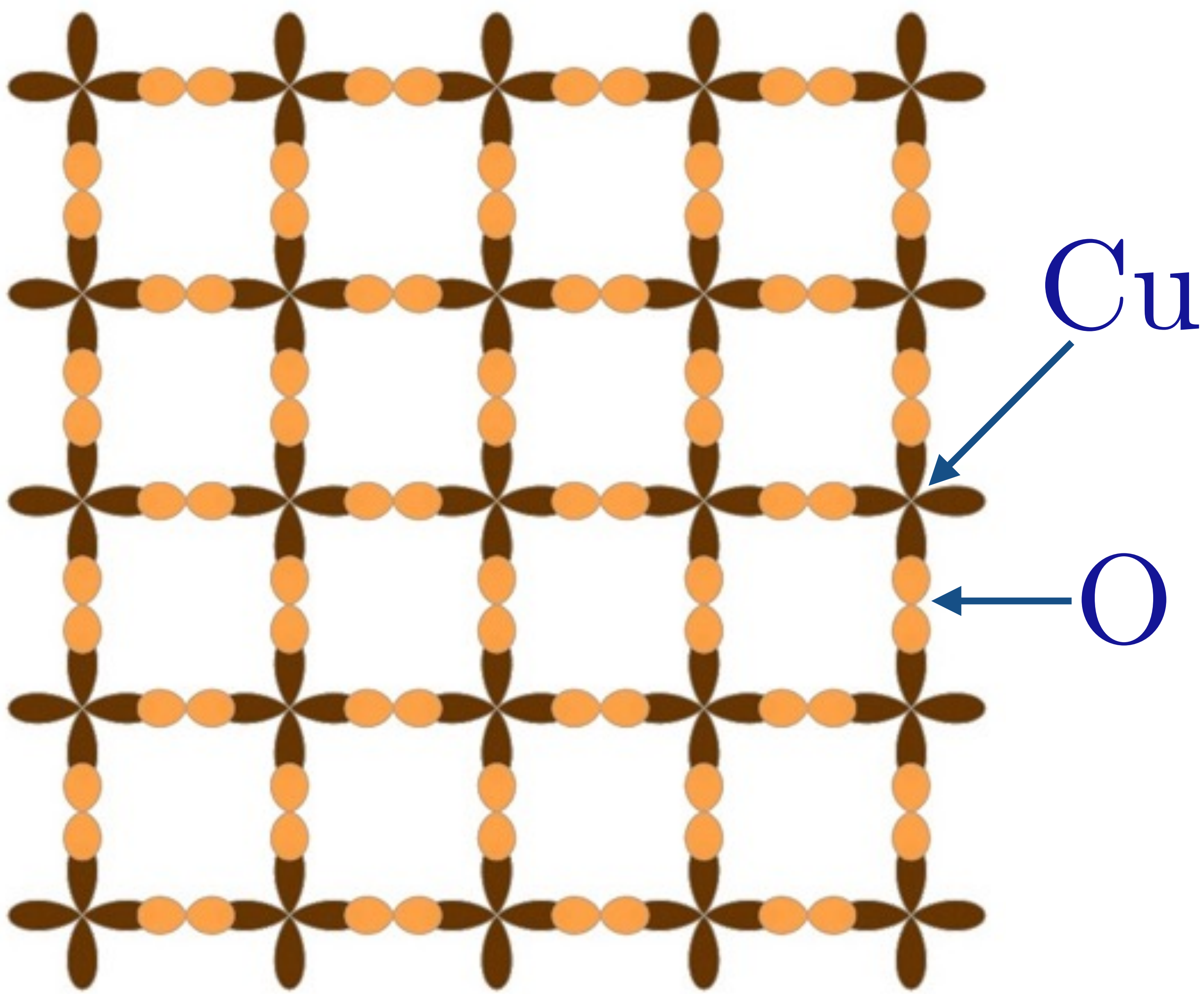
$$\text{Resistivity } \rho \sim T$$

S. A. Hartnoll, R. Mahajan, M. Punk, and S. Sachdev, PRB **89**, 155130 (2014)

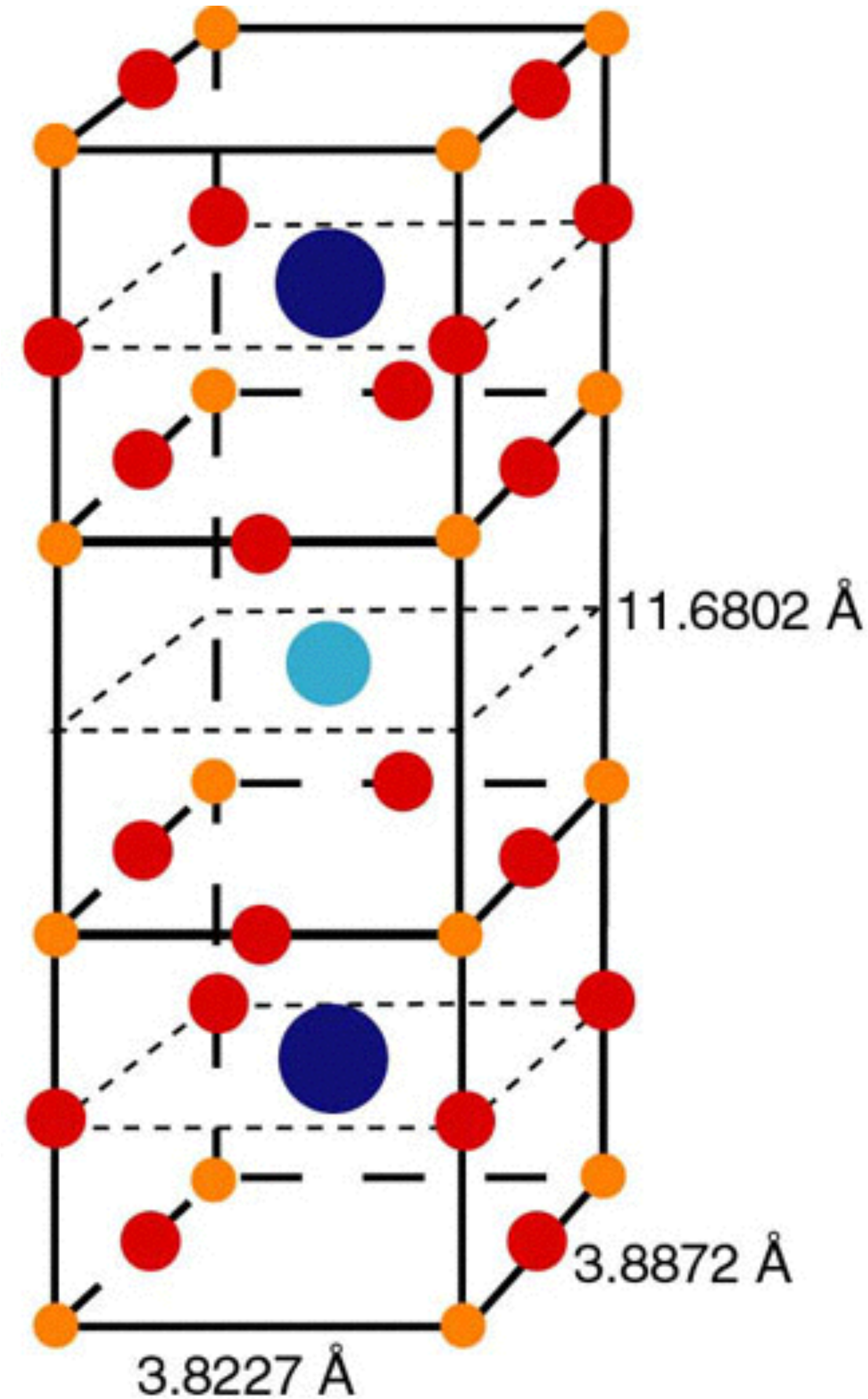
Theories of non-Fermi liquids

- Sachdev-Ye-Kitaev (SYK) model
- Ising-nematic criticality in d=2
- Higgs criticality in the cuprates

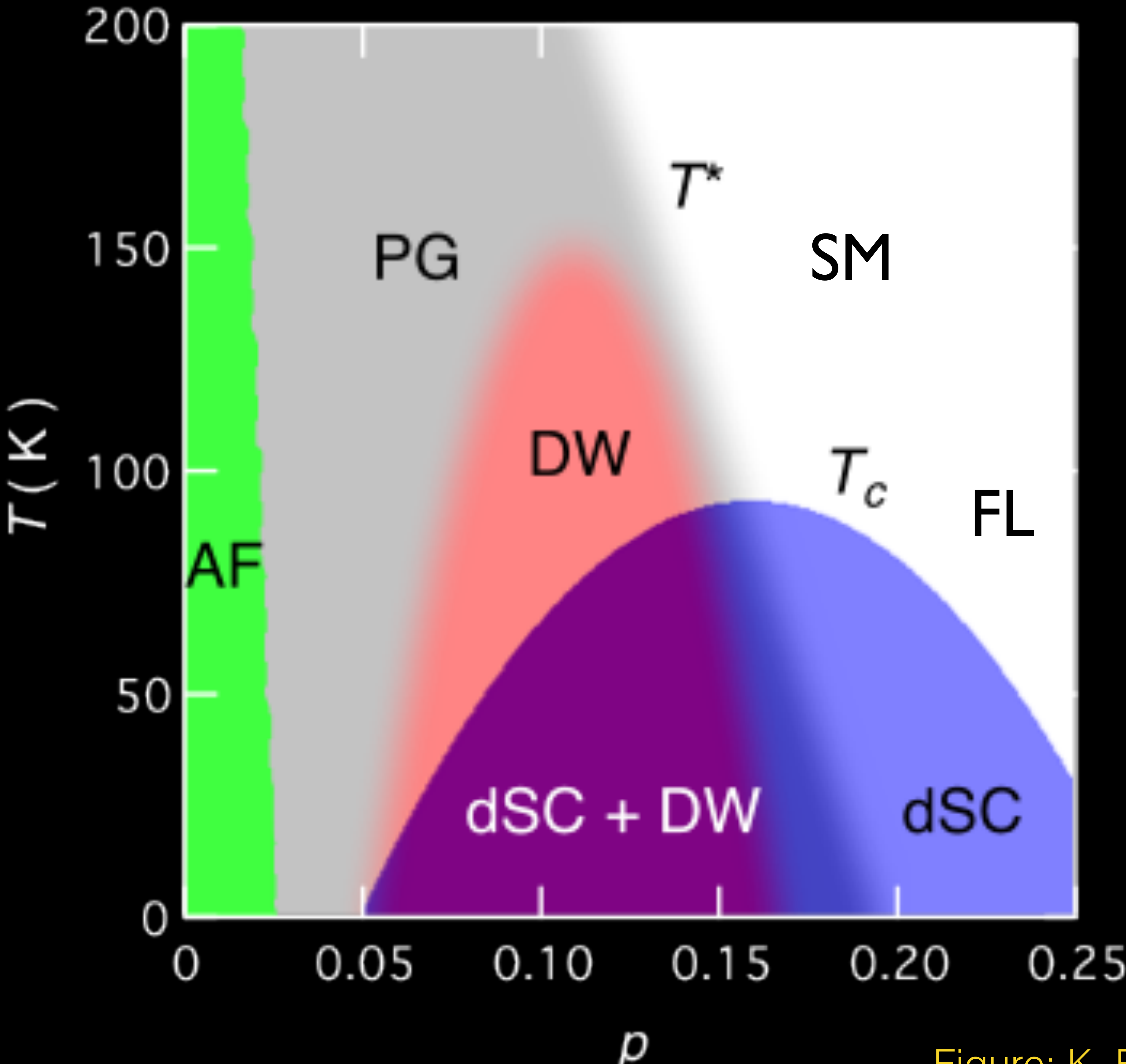
High temperature superconductors

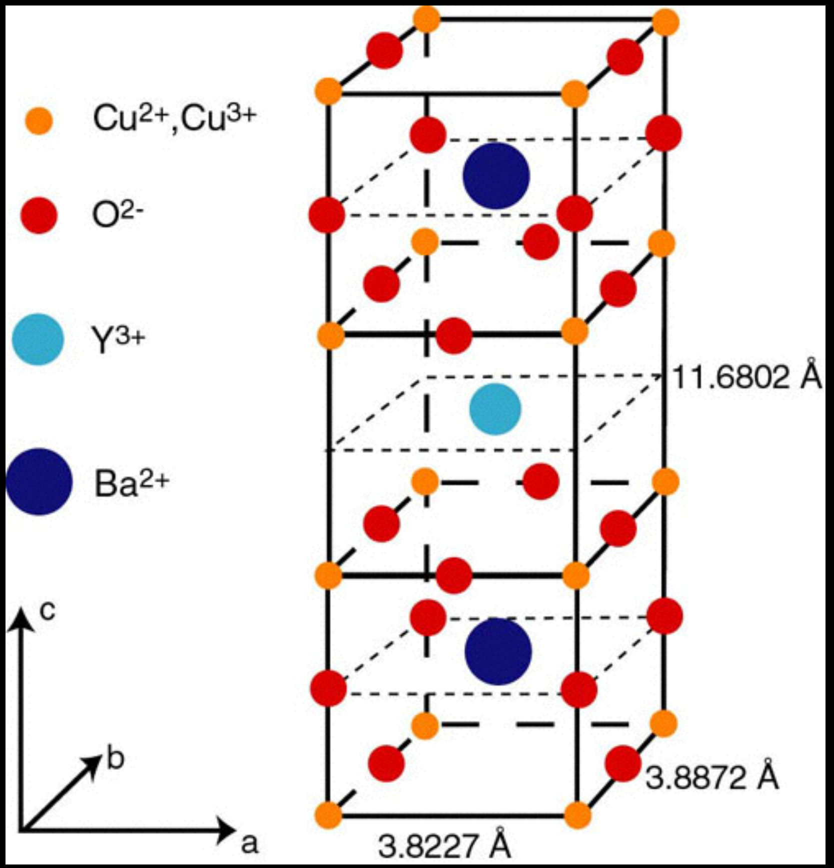


CuO_2 plane



$YBa_2Cu_3O_{6+x}$

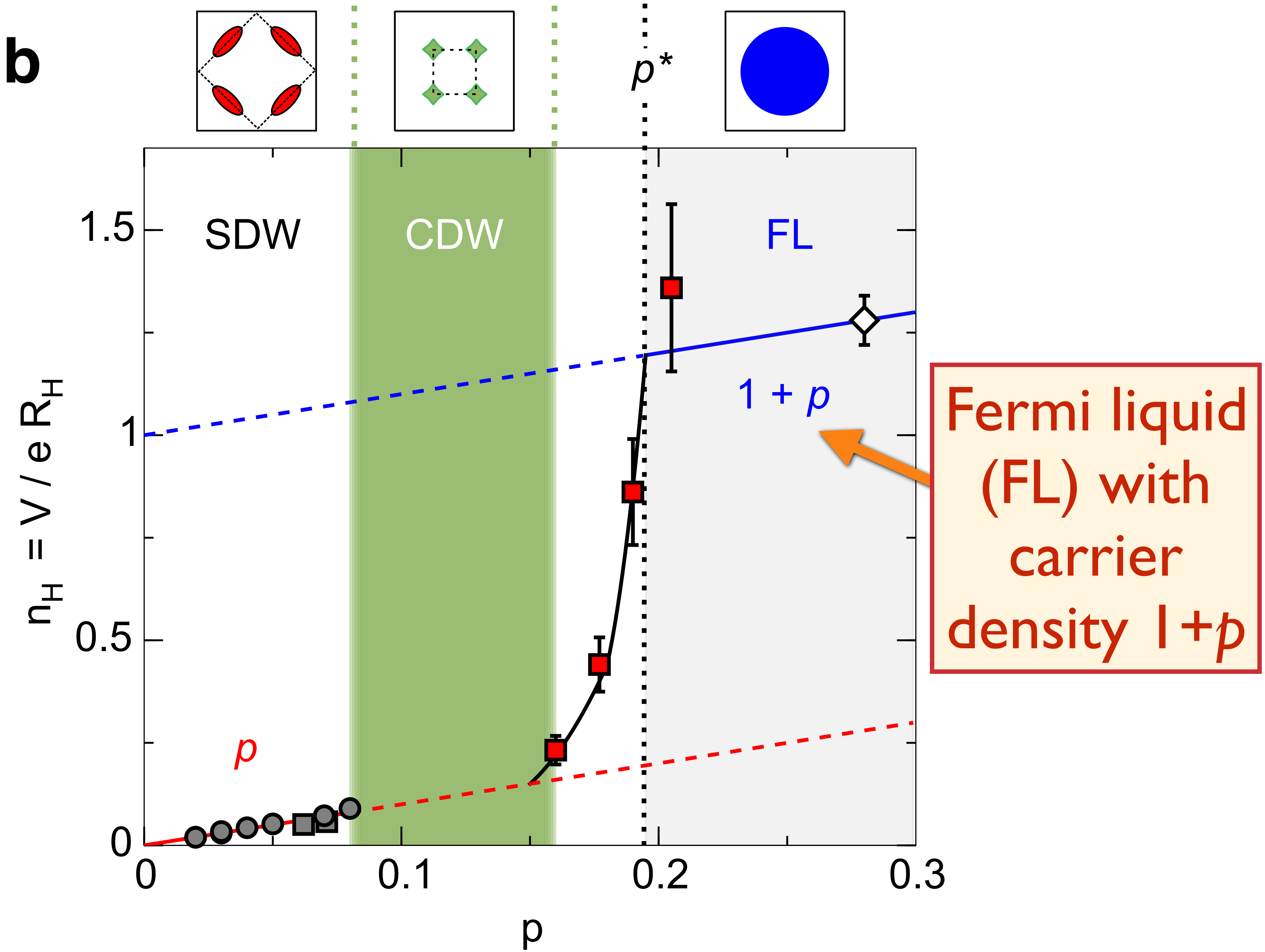




$YBa_2Cu_3O_{6+x}$

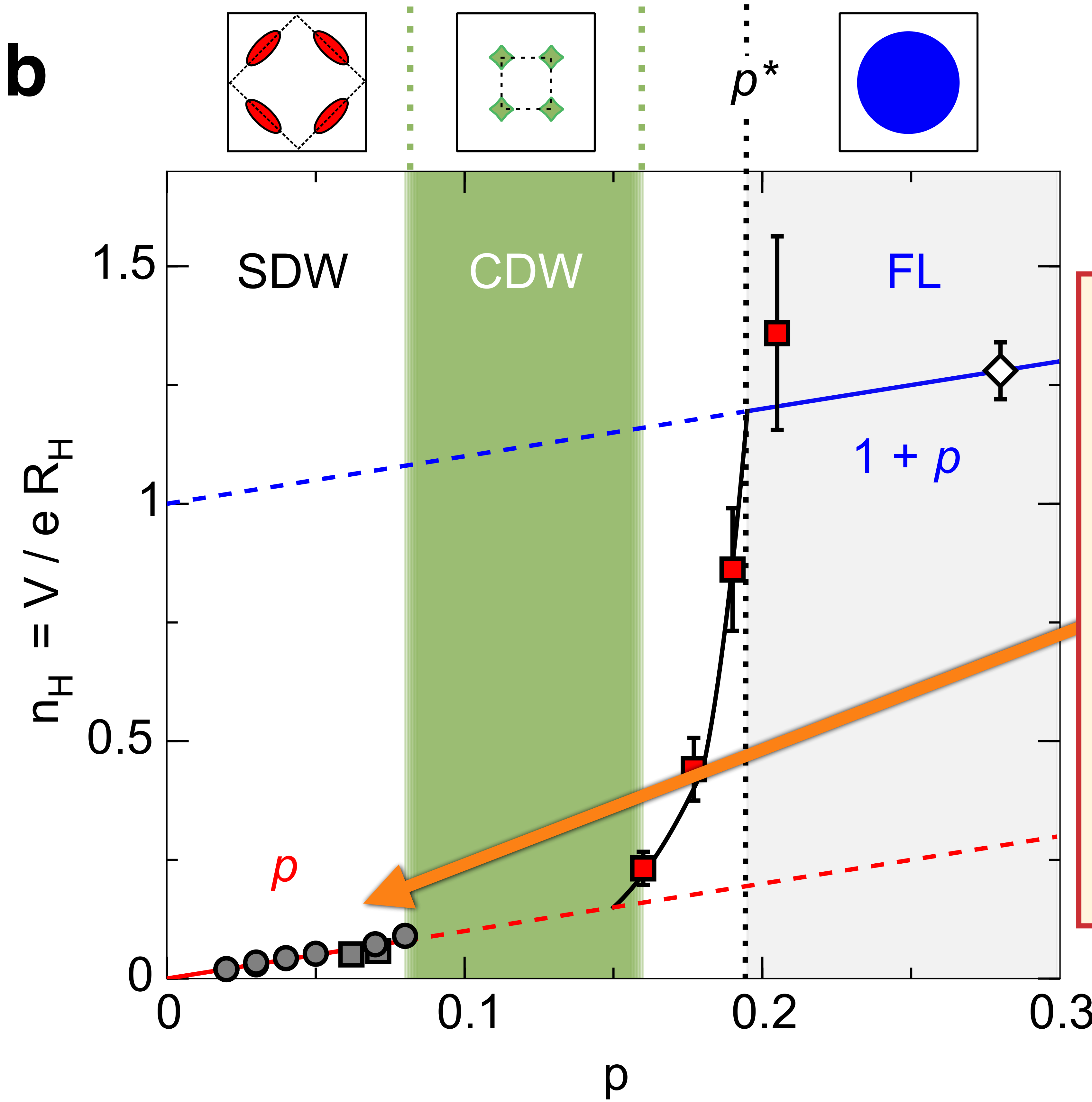
Figure: K. Fujita and J. C. Seamus Davis

Hall effect measurements in YBCO



Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

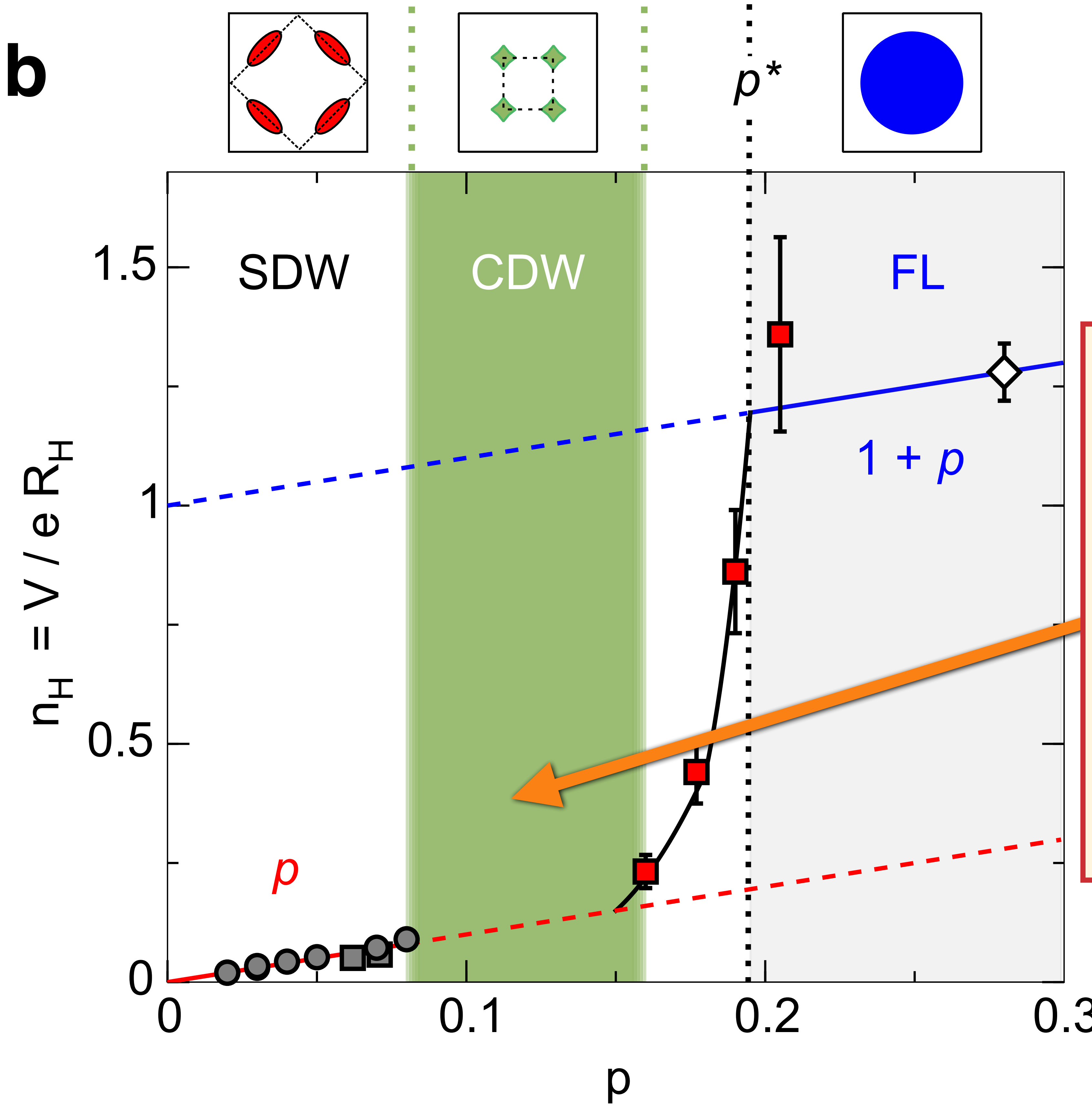
Hall effect measurements in YBCO



Spin density wave (SDW) breaks translational invariance, and the Fermi liquid then has carrier density p

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

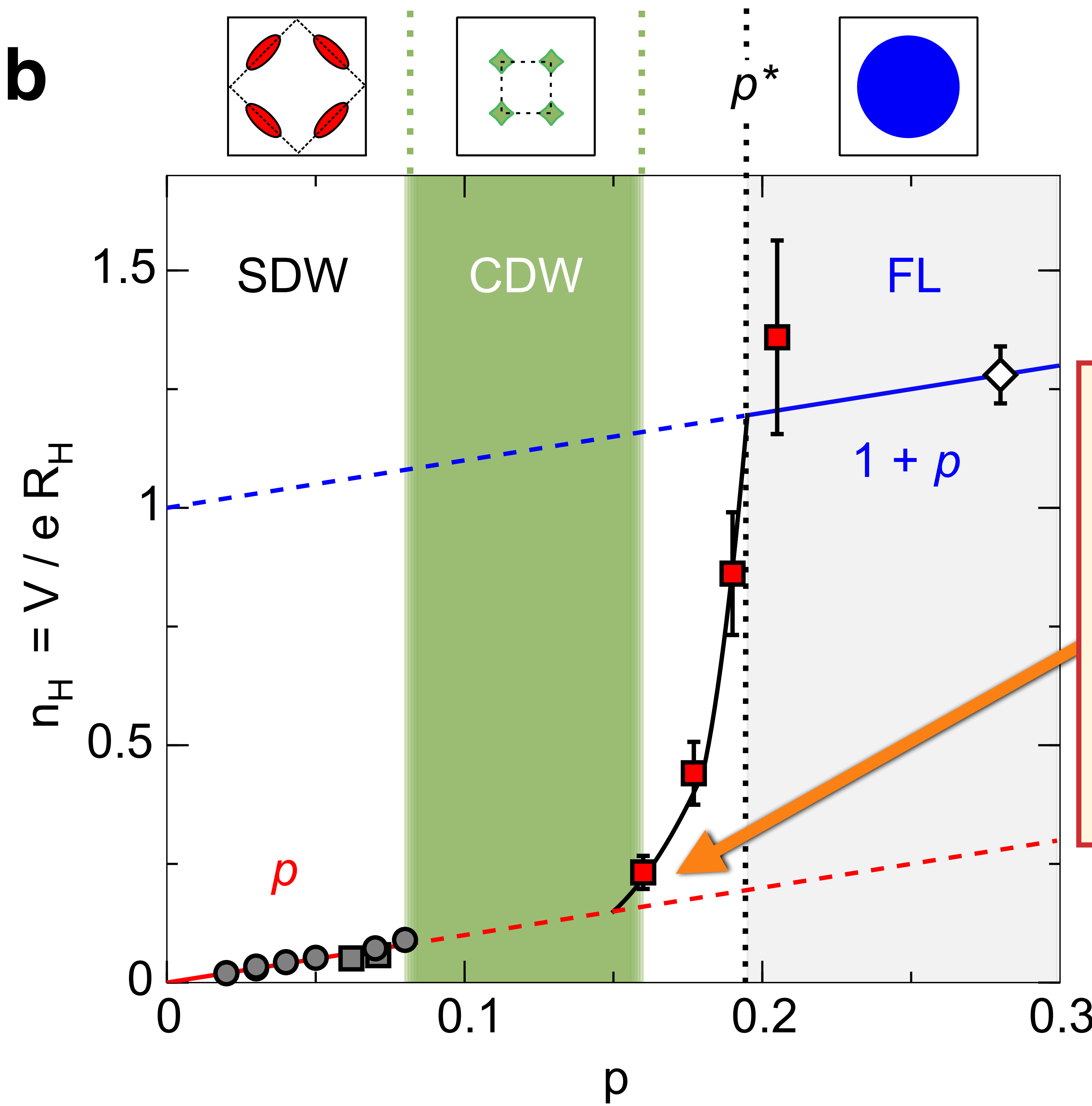
Hall effect measurements in YBCO



Charge density wave (CDW) leads to complex Fermi surface reconstruction and negative Hall resistance

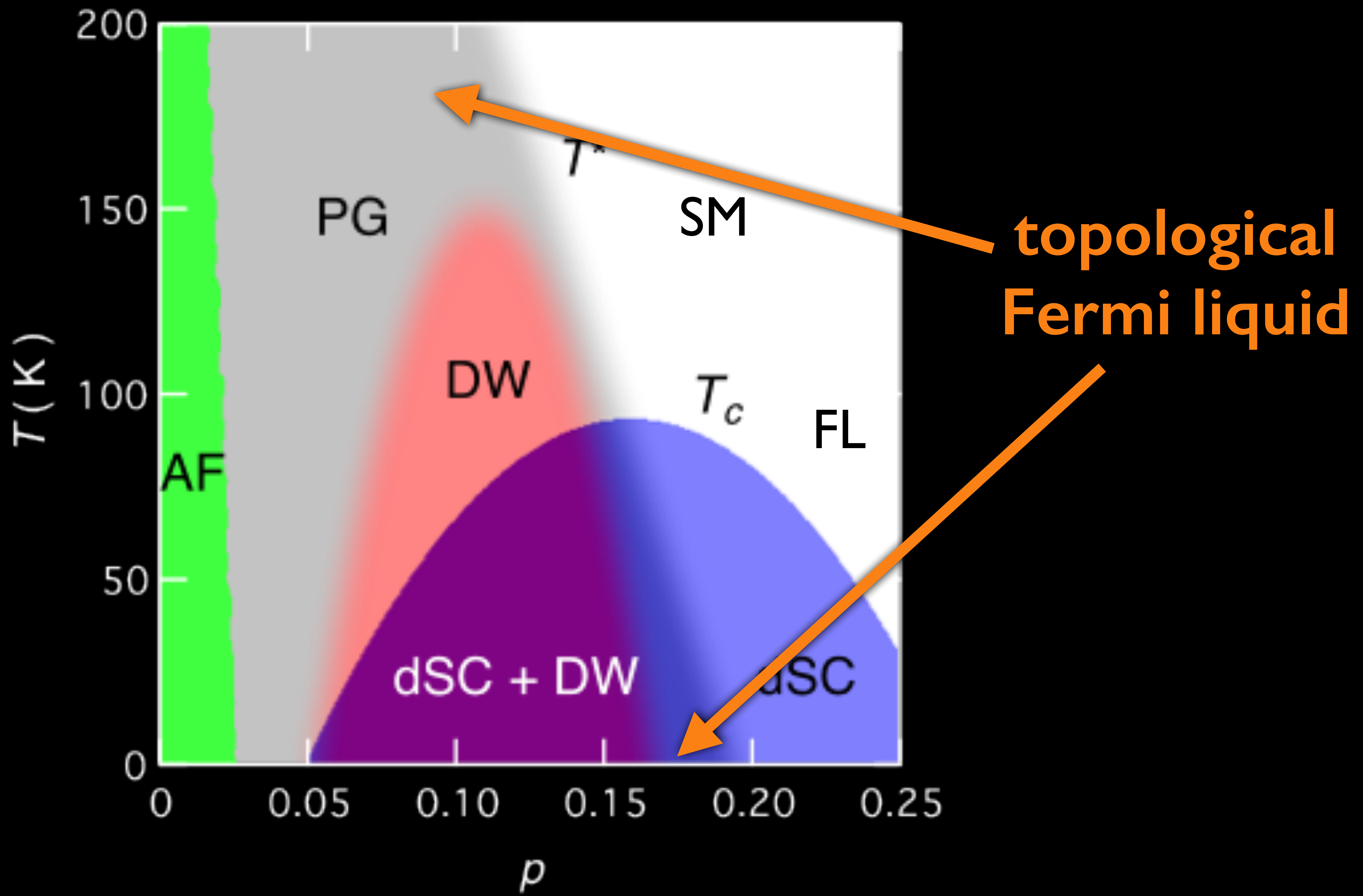
Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

Hall effect measurements in YBCO



Evidence for <u>topological</u> metal with Fermi surface of size p !

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)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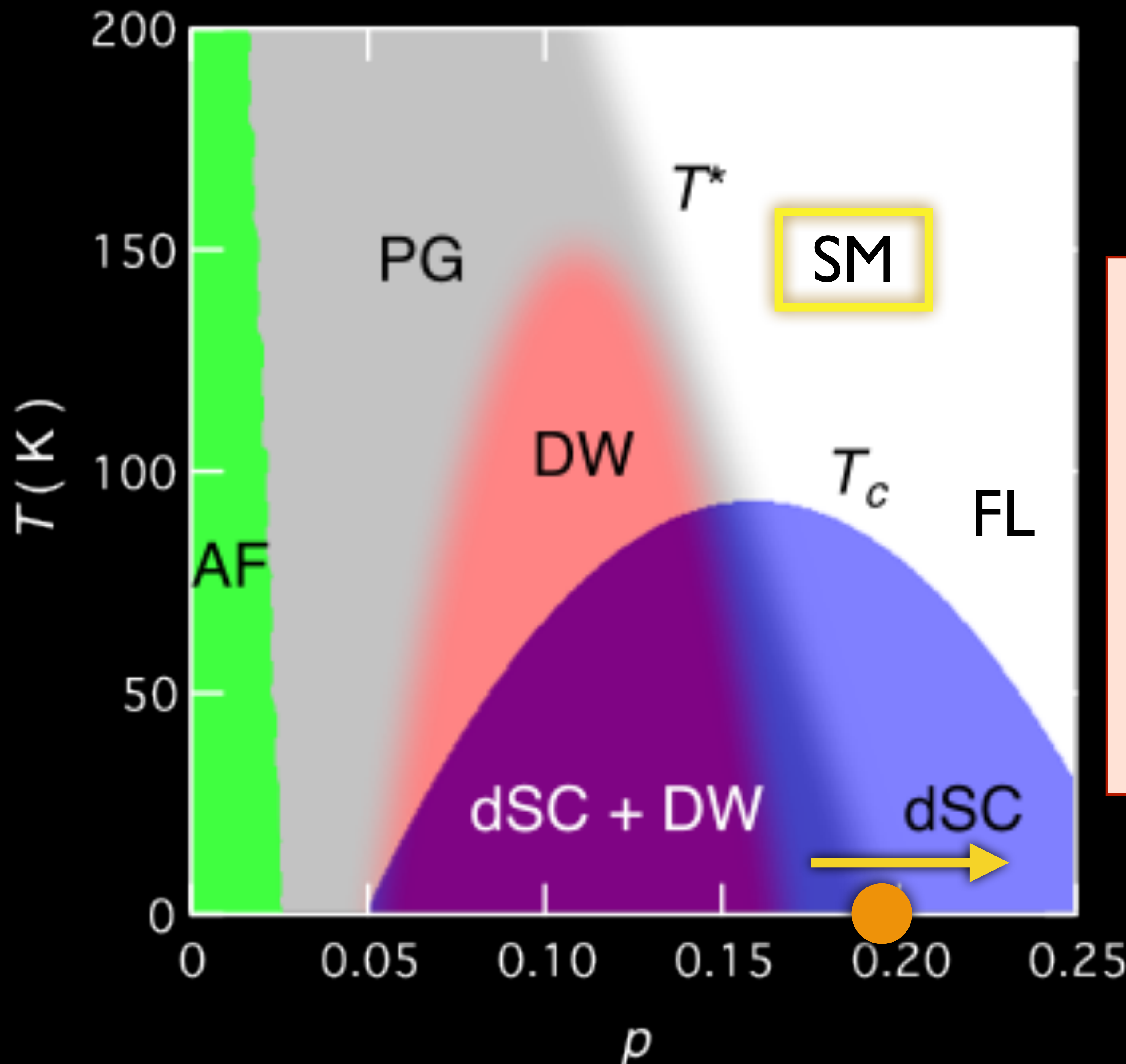
PG
T*
SM
DW
Tc
FL
dSC + DW
dSC
topological
Fermi liquid



Transition from Z_2 topological metal to FL as a theory of the strange metal (SM)

Quantum critical point at optimal doping

- Transition is primarily “topological”. Main change is in the size of the Fermi surface.

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- The main symmetry breaking which appears co-incident with the transition is Ising-nematic ordering. But this symmetry cannot change the size of the Fermi surface; similar comments apply to time-reversal symmetry.

- Need a gauge theory for transition from "topological" to "confined" state.

SU(2) gauge theory by transformation to a rotating reference frame

Write the electron operator c_α ($\alpha = \uparrow, \downarrow$ are spin indices) as

$$\left(\begin{array}{c} c_\uparrow \\ c_\downarrow \end{array} \right) = R \left(\begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) \quad , \quad R = \left(\begin{array}{cc} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{array} \right)$$

where R is a SU(2) matrix which determines the orientation of the local antiferromagnetic order, and $\psi_\pm$ are spinless fermions which carry the global electron U(1) charge.

This parameterization is invariant under a SU(2) *gauge* transformation

$$\left(\begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) \to U \left(\begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) \quad ; \quad R \to R U^\dagger$$

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009); D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579

SU(2) gauge theory by transformation to a rotating reference frame

- Spinless fermion ψ (the fermionic chargon) transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a "large" Fermi surface, and carries electromagnetic charge

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009); D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579

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- A SU(2) fundamental scalar z_α (the bosonic spinon), carrying electron-spin and electromagnetically neutral.

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Let $\Phi_{x\ell}$ and $\Phi_{y\ell}$ ($\ell = x, y, z$) be the complex spin density wave order parameters at wave vectors $\mathbf{K}_x$ and $\mathbf{K}_y$. Then in the rotating reference frame, these become the complex Higgs fields, H_x^a and H_y^a.

$$H_x^a = \frac{1}{2}\Phi_{x\ell}\,\mathrm{Tr}\left[\sigma^\ell R \sigma^a R^\dagger\right] \quad , \quad H_y^a = \frac{1}{2}\Phi_{y\ell}\,\mathrm{Tr}\left[\sigma^\ell R \sigma^a R^\dagger\right]$$

howdhury and
S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579

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- A SU(2) fundamental scalar z_α (the bosonic spinon), carrying electron-spin and electromagnetically neutral.

- A SU(2) gauge boson.

- Two complex Higgs fields, H_x^a and H_y^a, transforming as gauge SU(2) adjoints, and carrying non-zero lattice momentum.

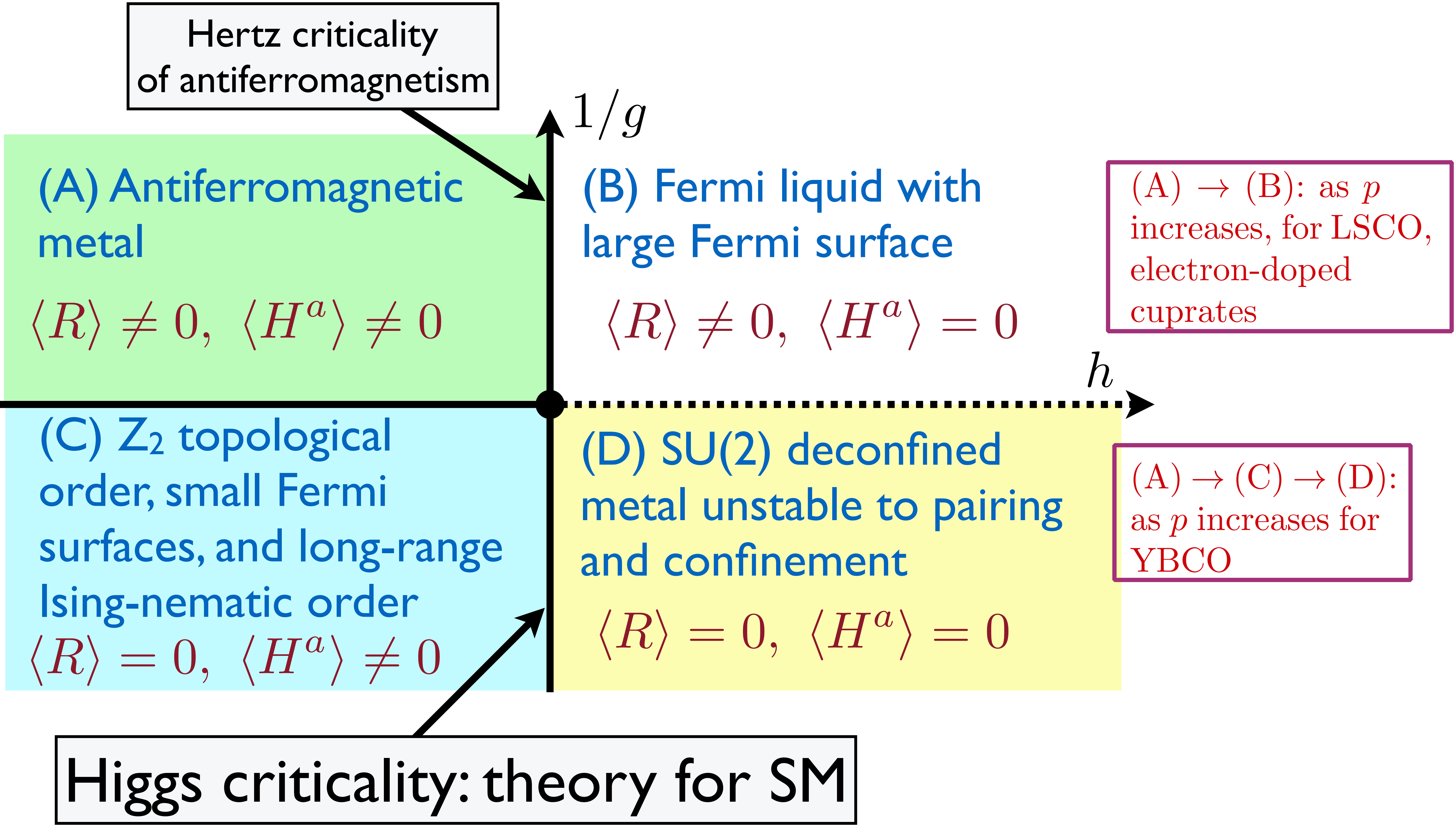
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009); D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579

SU(2) gauge theory by transformation to a rotating reference frame

Field	Symbol	Statistics	$SU(2)_{\text{gauge}}$	$SU(2)_{\text{spin}}$	$U(1)_{\text{e.m.charge}}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009); D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015); S. Sachdev and D. Chowdhury, arXiv:1605.03579

SU(2) gauge theory by transformation to a rotating reference frame



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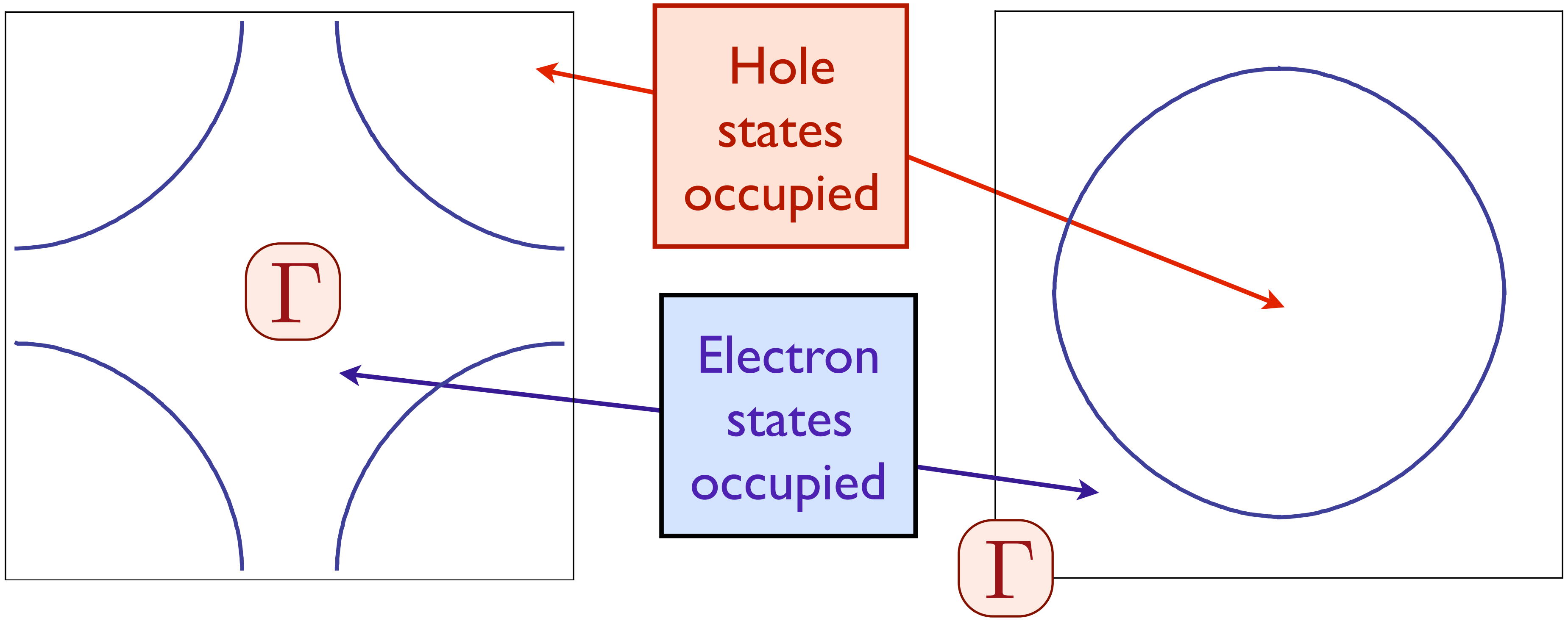
Fermi surface+antiferromagnetism



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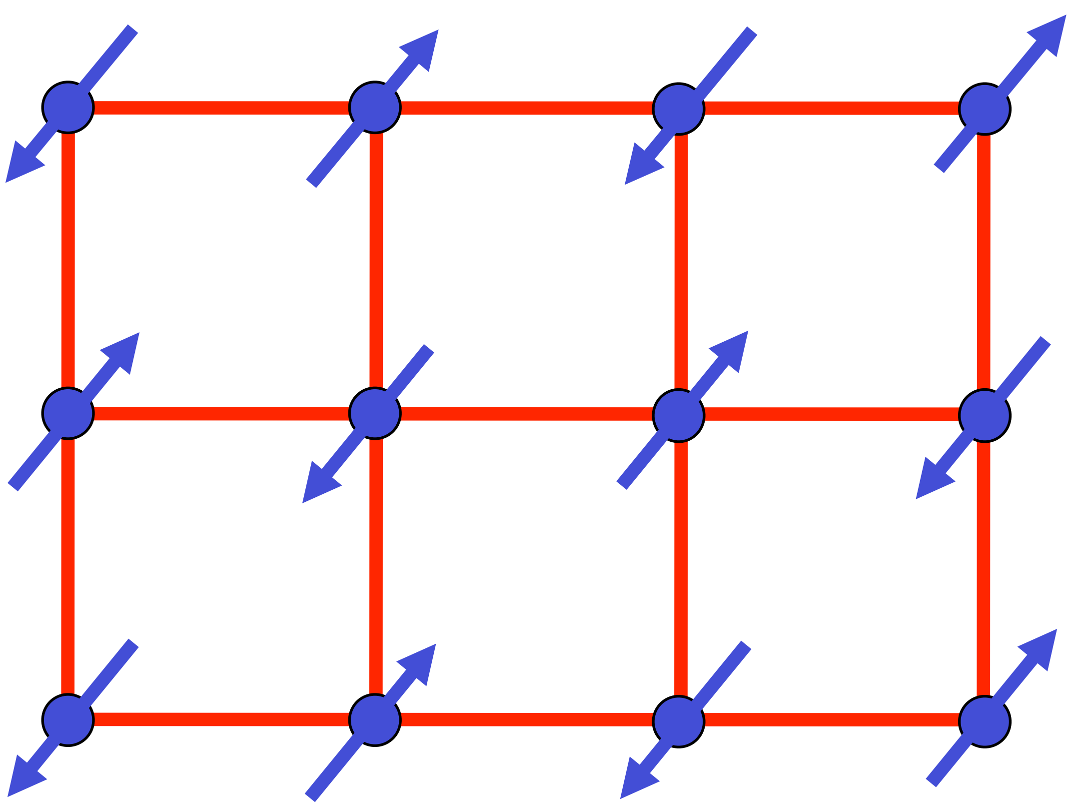


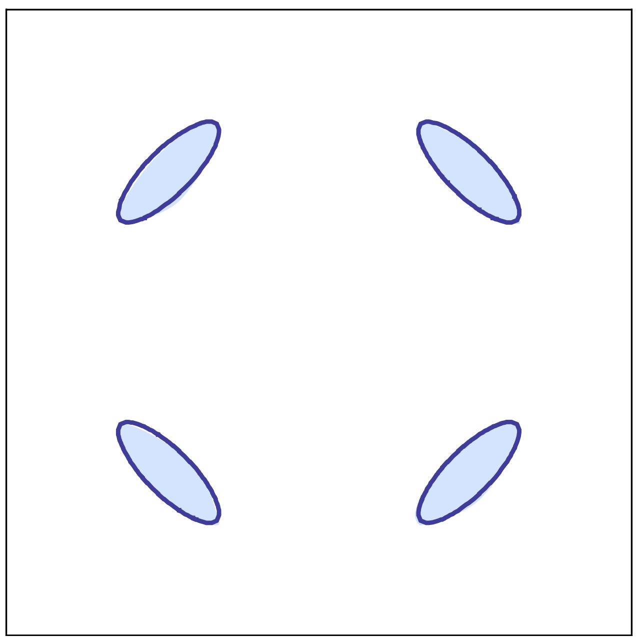
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where **K** is the ordering wavevector.

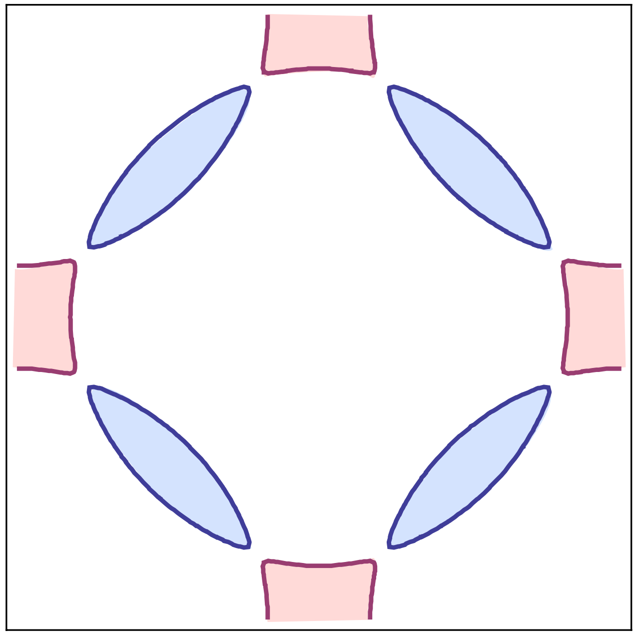
Fermi surface+antiferromagnetism

$\langle \vec{\varphi} \rangle \neq 0$
and large



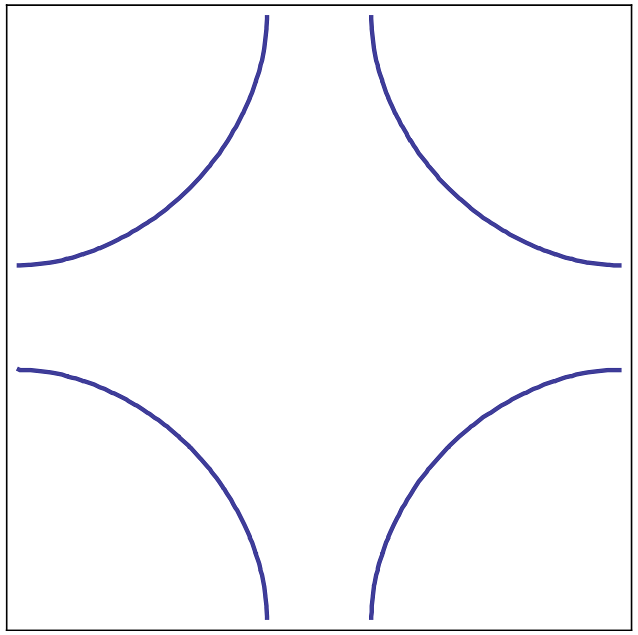
Metal with
hole pockets

$\langle \vec{\varphi} \rangle \neq 0$
and small



Metal with
electron and
hole pockets

$\langle \vec{\varphi} \rangle = 0$

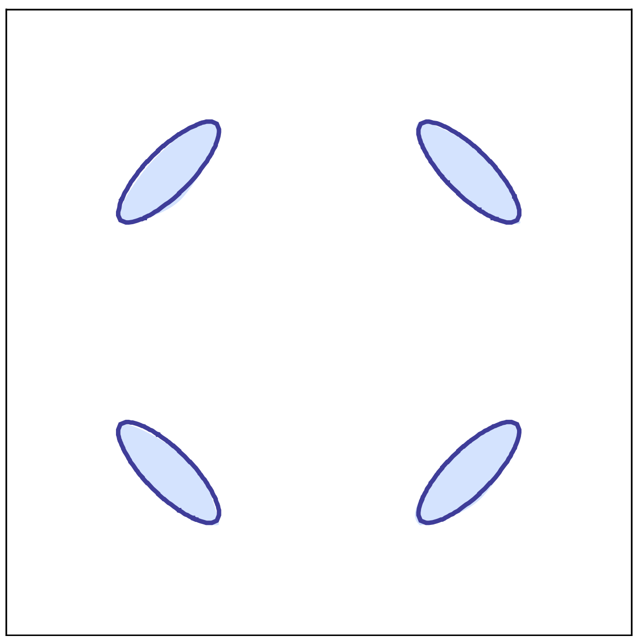


Metal with
“large” Fermi
surface

s

Fermi surface+antiferromagnetism

$\langle \vec{\varphi} \rangle \neq 0$ and large



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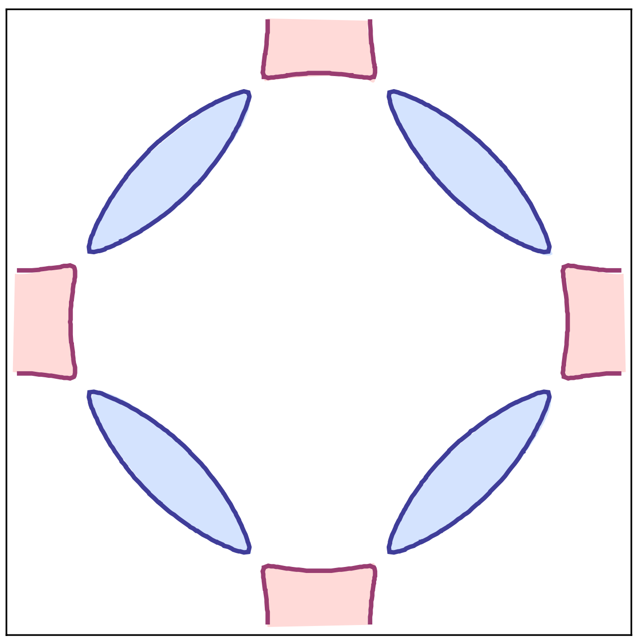
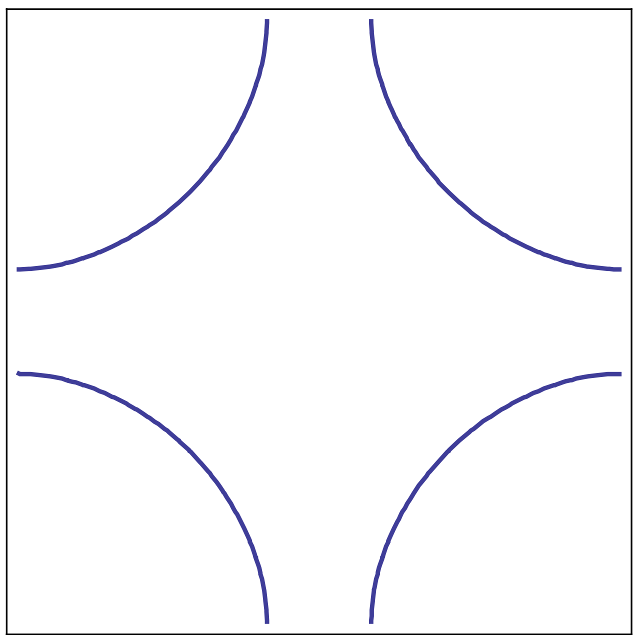


$\langle \vec{\varphi} \rangle = 0$



The same Fermi surface transformations apply to the chargons, ψ, in the presence of the Higgs condensate. The large Fermi surfaces appear when the Higgs field is uncondensed, and the SU(2)-gauge is unbroken, and the small Fermi surfaces appear when the Higgs field is condensed, and there is $\mathbb{Z}_2$ topological order.

Hall effect with spiral magnetic order or with fluctuating magnetism and Z_2 topological order

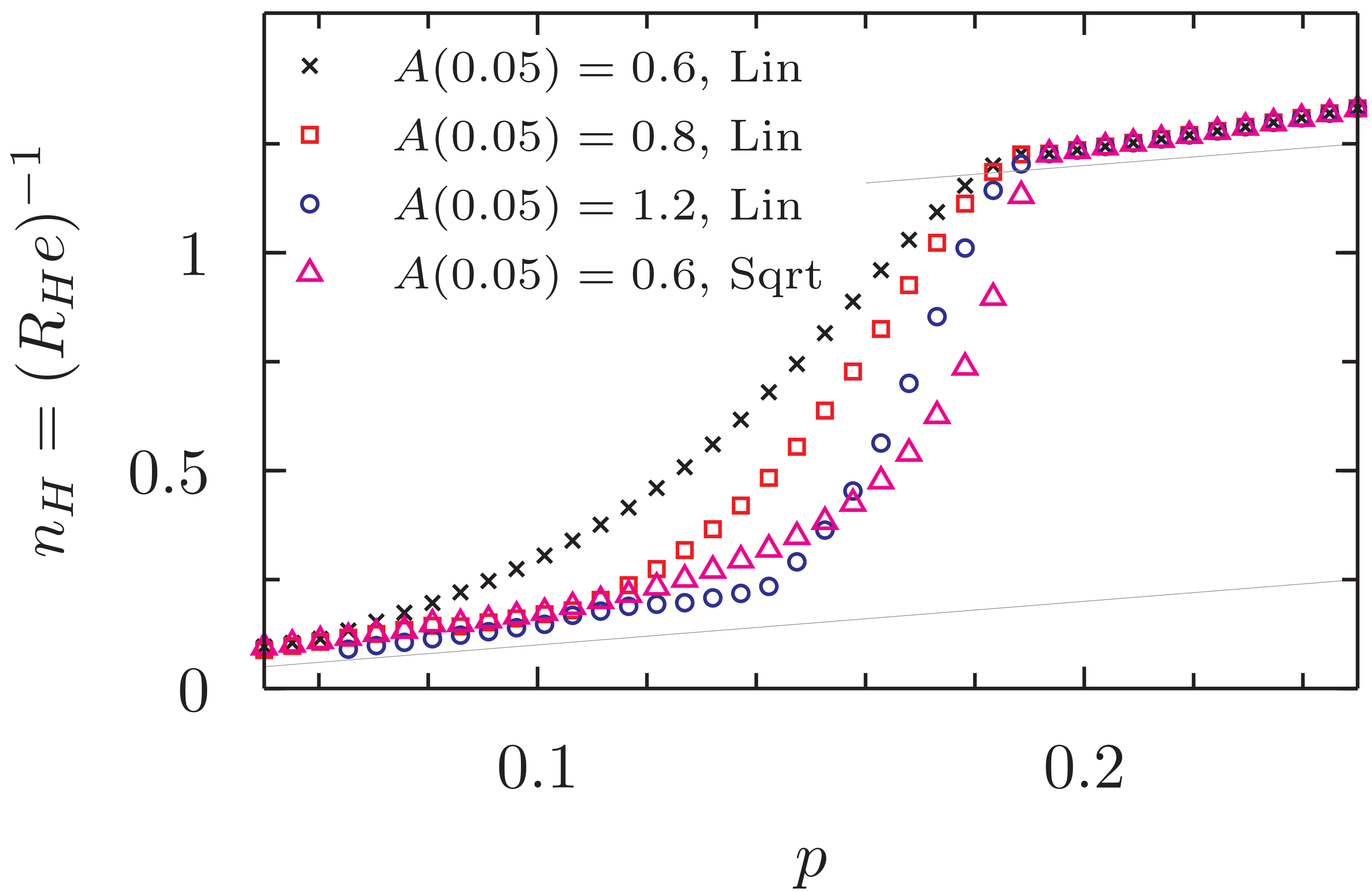
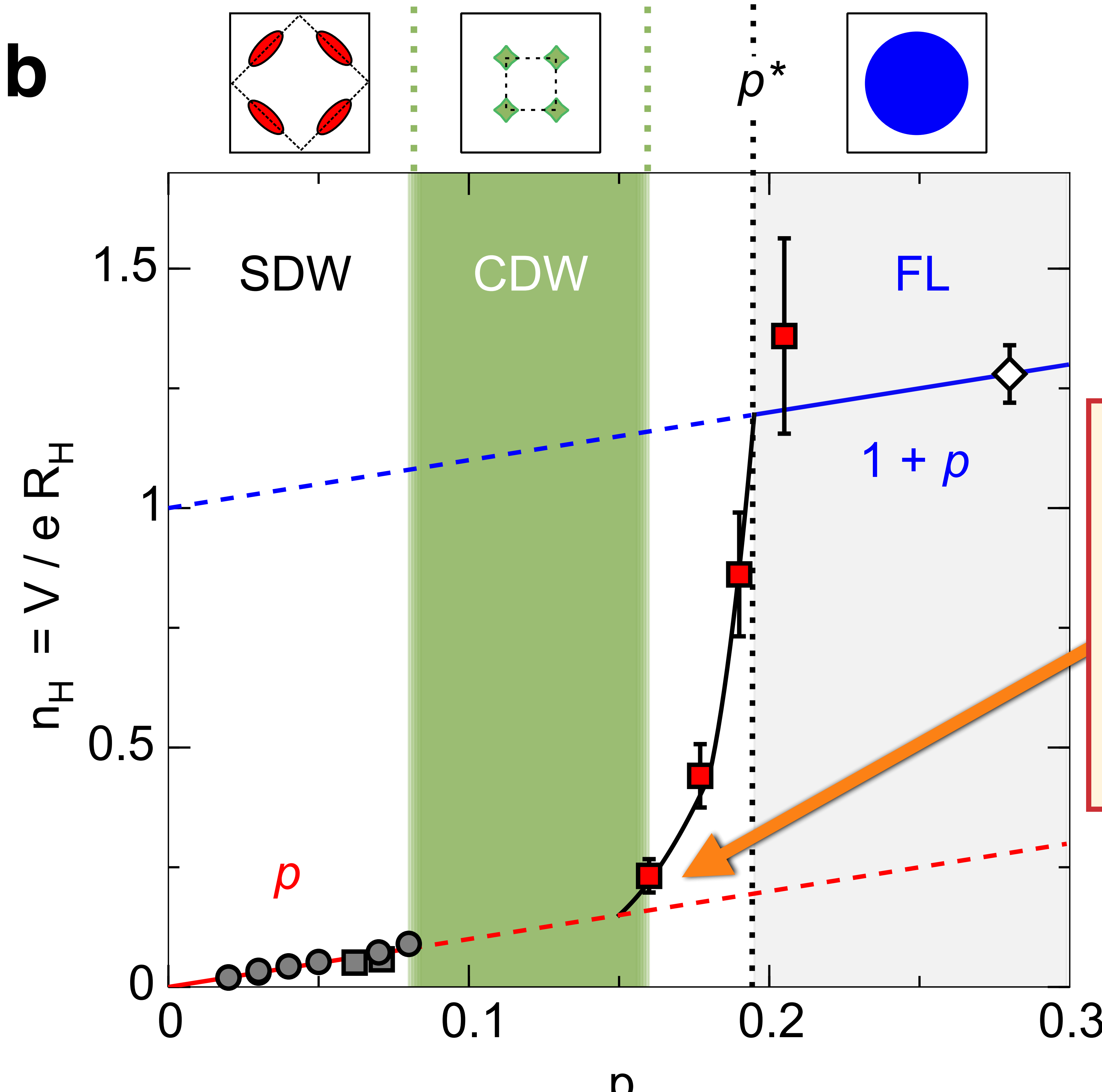


Figure 2: Hall number n_H as a function of doping for $t' = -0.35$. Results for a linear dependence, $A(p) \sim (p^* - p)$, and a square root dependence, $A(p) \sim \sqrt{p^* - p}$, where $p^* = 0.19$ in both cases, are labeled as "Lin" and "Sqrt", respectively.

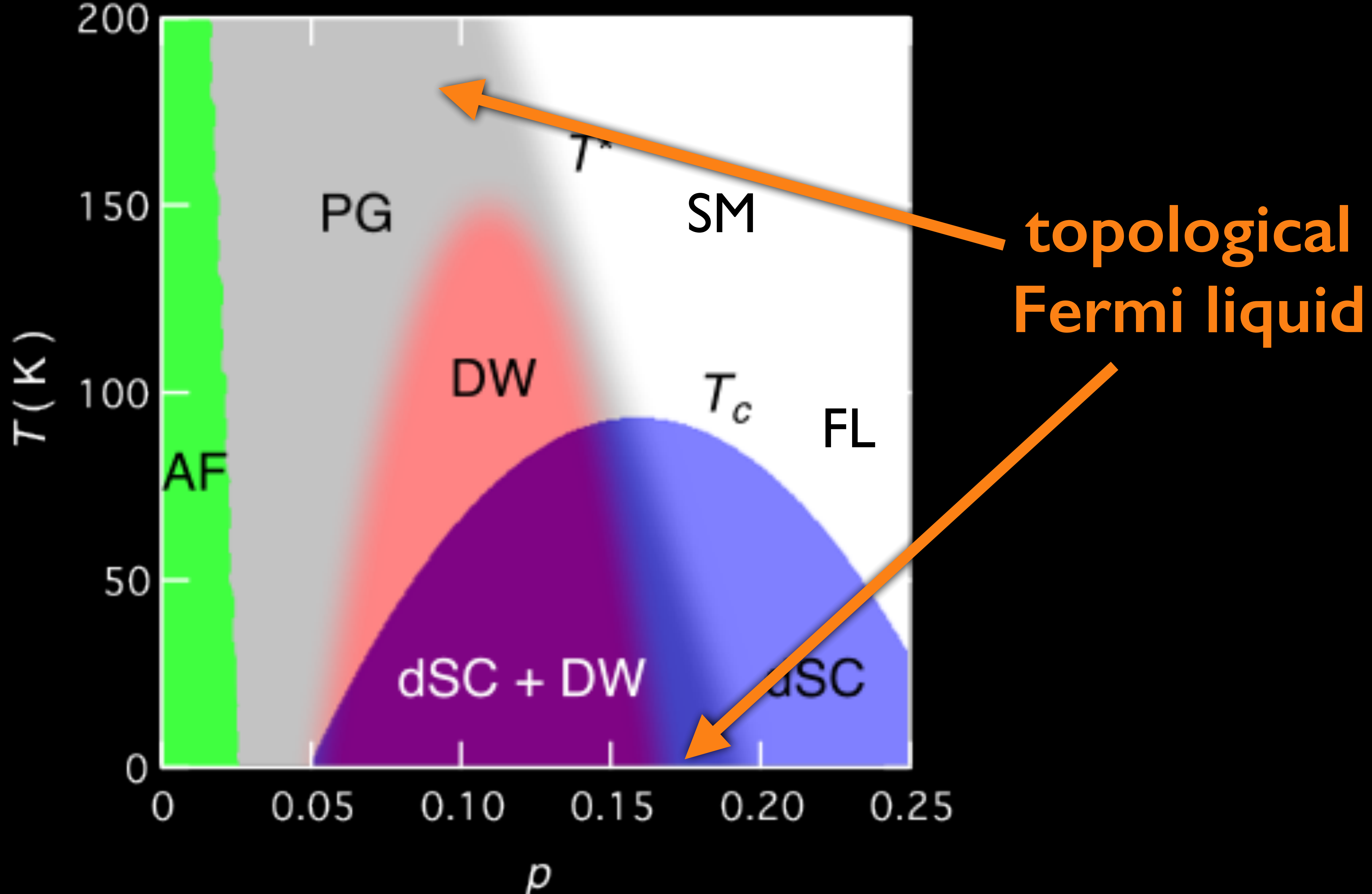
A. Eberlein, W. Metzner, S. Sachdev, and H. Yamase, arXiv:1607.06087

Hall effect measurements in YBCO



Topological metal with Fermi surface of size *p* ?

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)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DW
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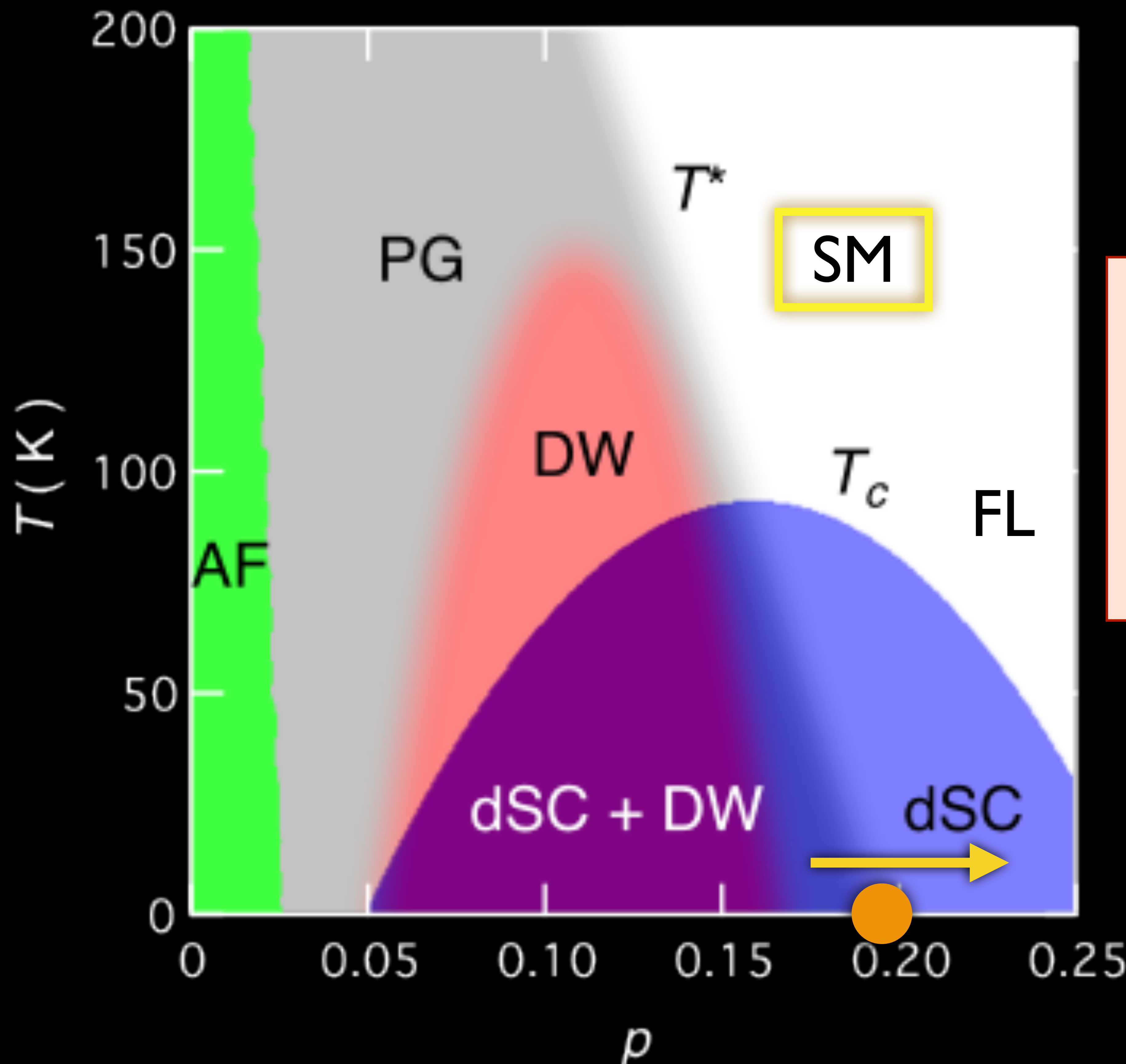
SM
Tc
FL
dSC + DW
dSC
topological Fermi liquid



Higgs criticality: theory of the strange metal (SM)

Non-Fermi liquids

- Shortest possible "phase coherence" time, fastest possible local equilibration time, or fastest possible Lyapunov time towards quantum chaos, all of order
$$\frac{\hbar}{k_B T}$$

- Realization in solvable SYK model, which saturates the lower bound on the Lyapunov time. Its properties have some similarities to non-rational, large central charge CFT2s.

- Remarkable match between SYK and quantum gravity of black holes with AdS_2 horizons, including a SL(2,R)-invariant Schwarzian effective action for thermal energy fluctuations.

- Transport of non-Fermi liquids described by collective flow of fluid around impurities. Non-Fermi liquids with a critical Fermi surface have a divergent η/s as $T \to 0$. This is a consequence of the spatial anisotropy in the vicinity of a Fermi surface point, and unlike existing holographic models.

- Higgs criticality is an attractive model for the strange metal in the cuprates, consistent with recent measurements of the Hall effect as a function of doping.