The non-superconducting states of the cuprates

Perimeter Institute, Waterloo
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PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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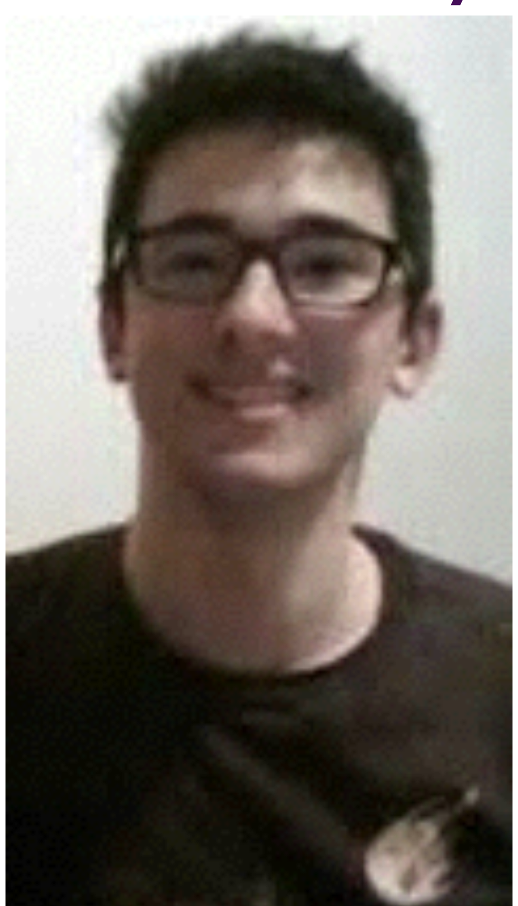
Talk online: sachdev.physics.harvard.edu

Debanjan Chowdhury

Andrea Allais

Aavishkar Patel



Andrew Lucas

Alexandra Thomson

Philipp Strack Cologne

Matthias Punk (Innsbruck)

Flavors of Quantum Matter

A. Ordinary quantum matter

Independent electrons, or pairs of electrons

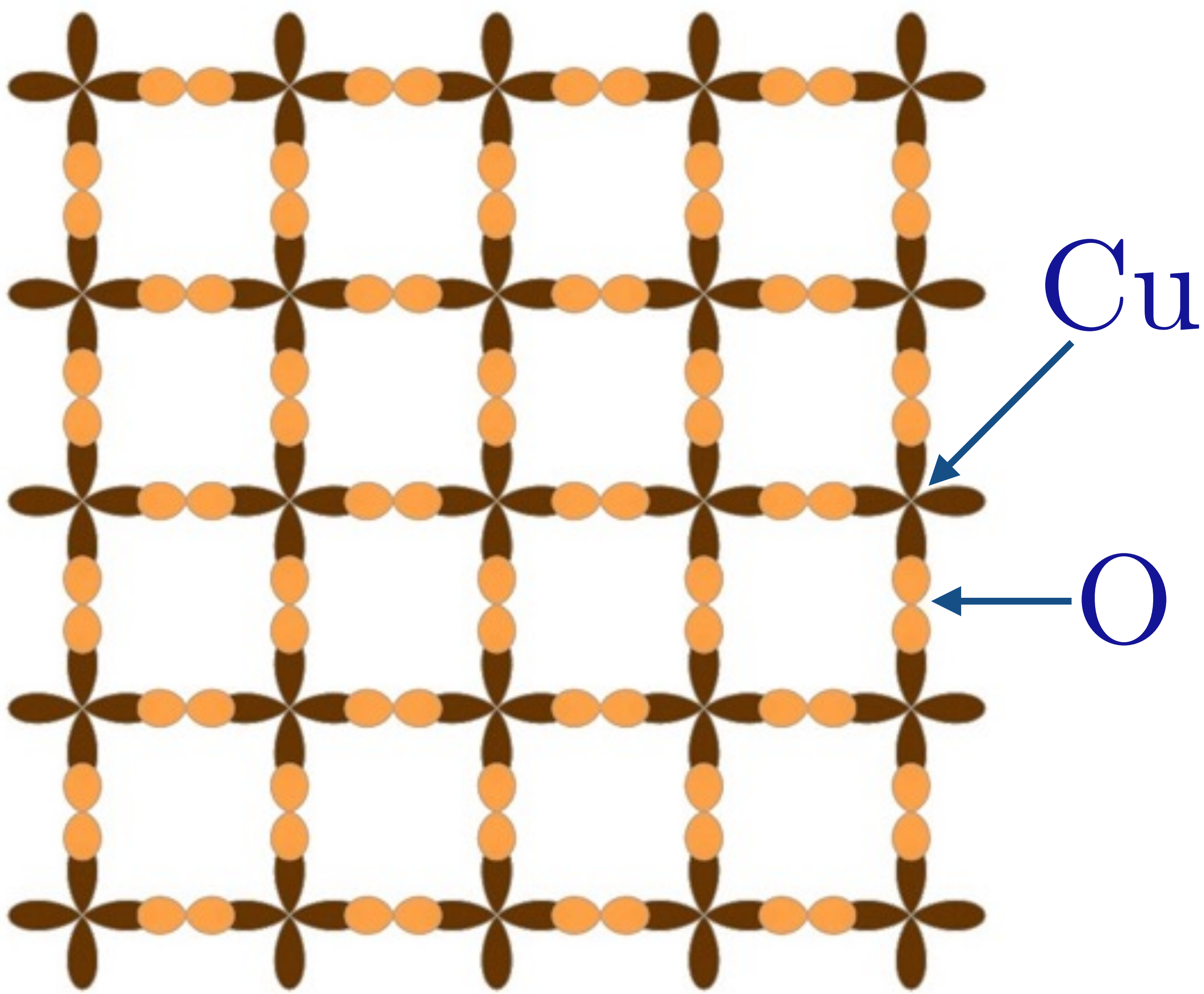
B. Topological quantum matter

Long-range quantum entanglement leads to sensitivity to spatial topology

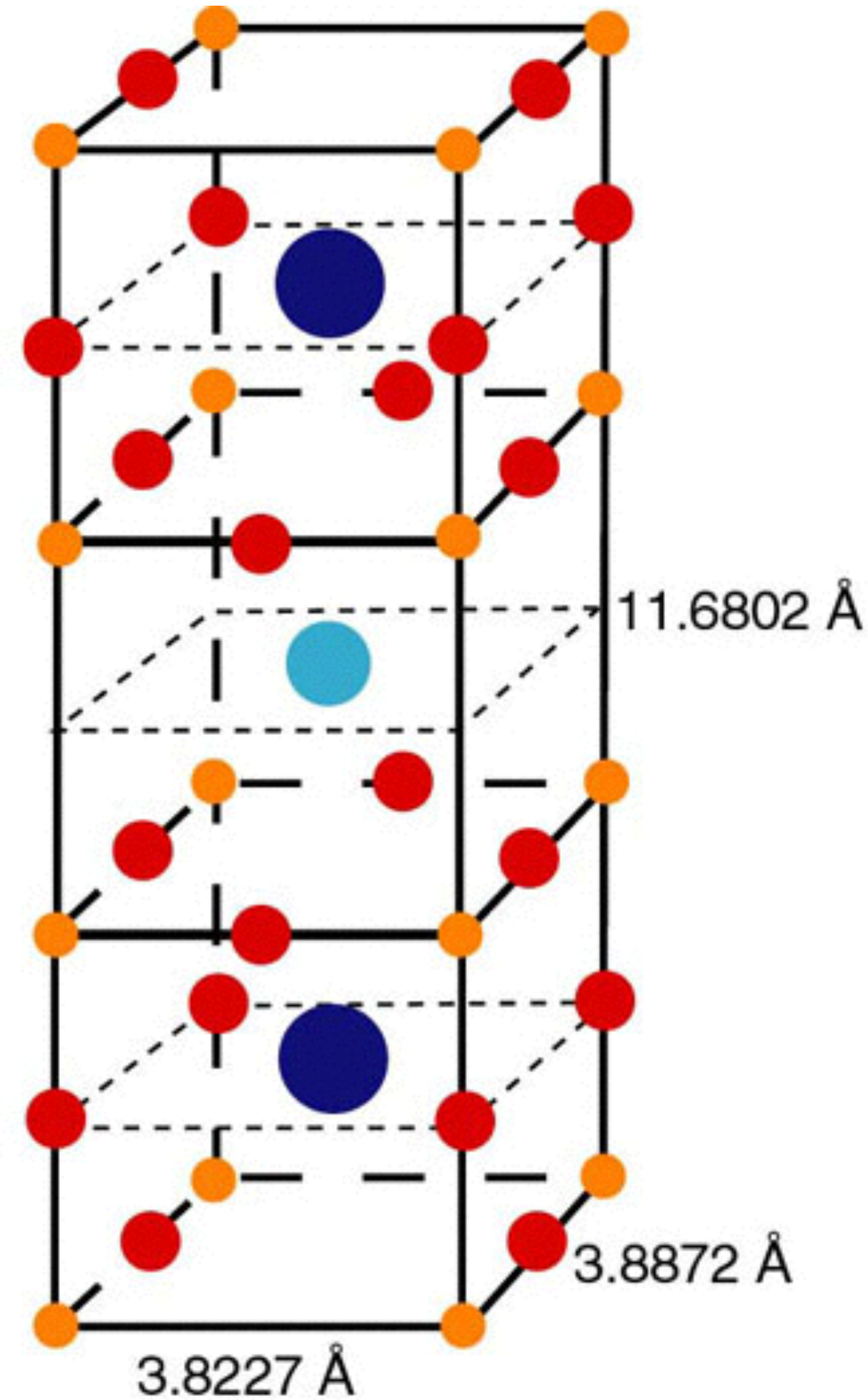
C. Quantum matter without quasiparticles

Hydrodynamics, memory functions, holography, and field theory

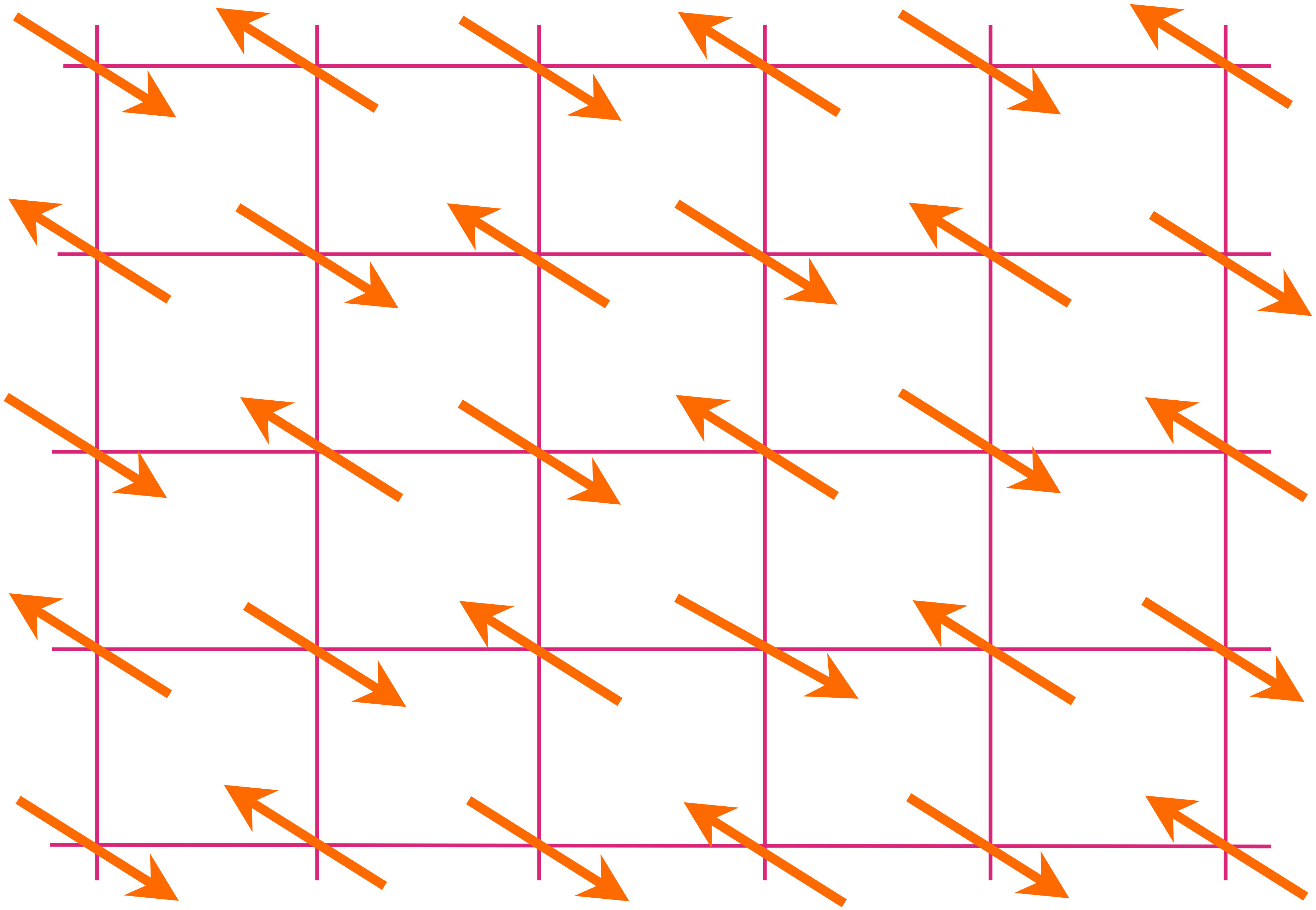
High temperature superconductors



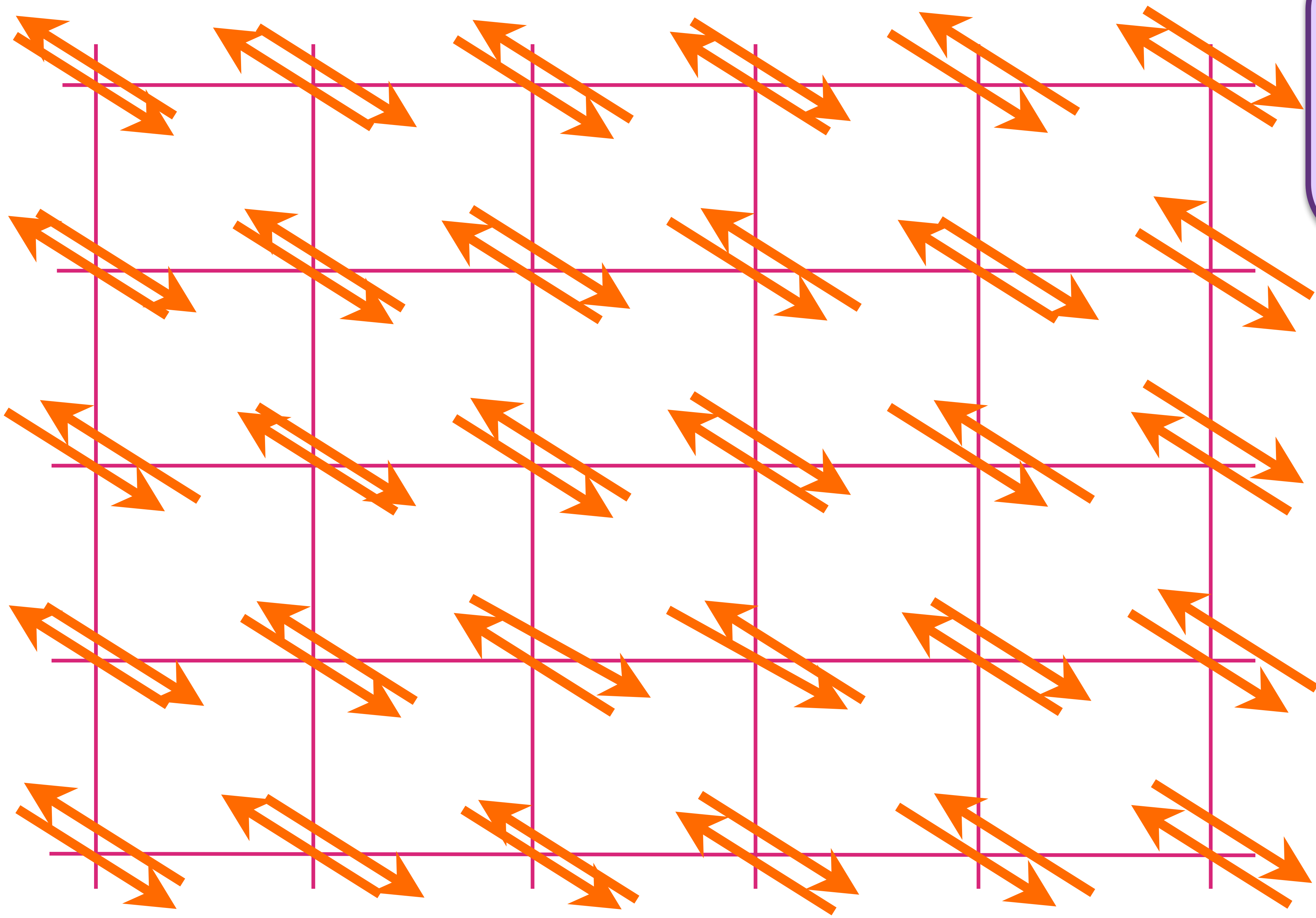
CuO_2 plane



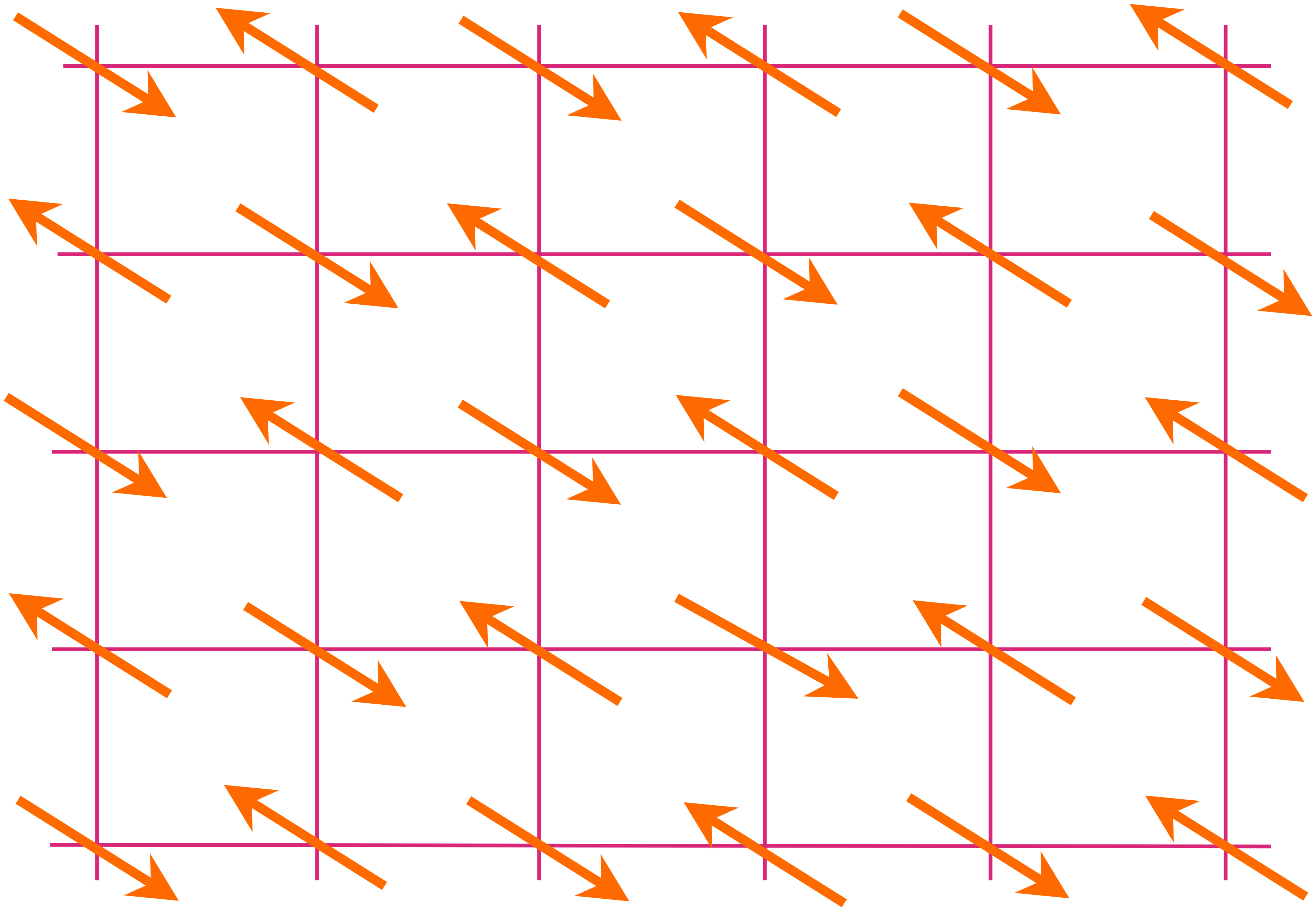
$YBa_2Cu_3O_{6+x}$



“Undoped” Anti-ferromagnet



Filled Band



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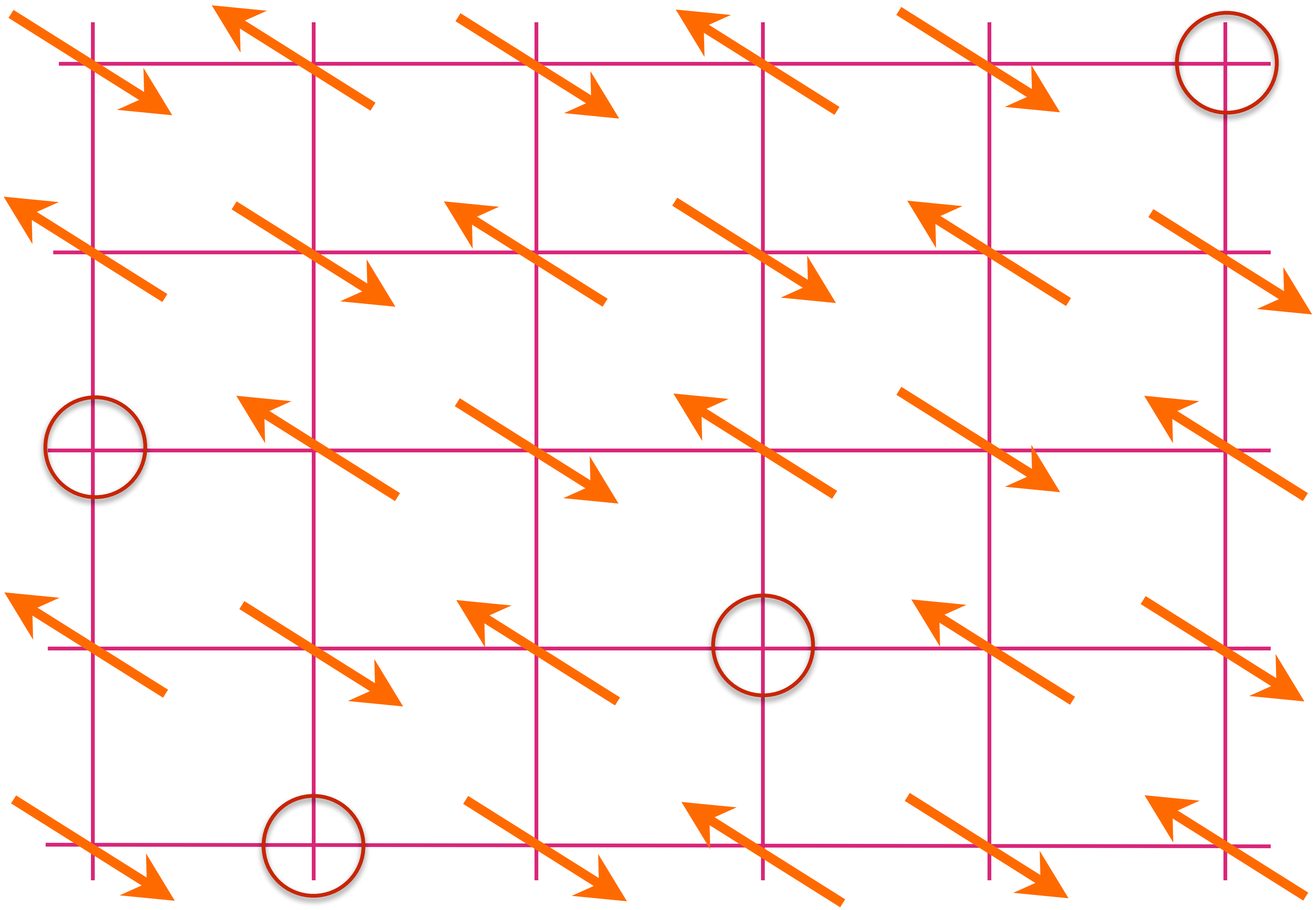
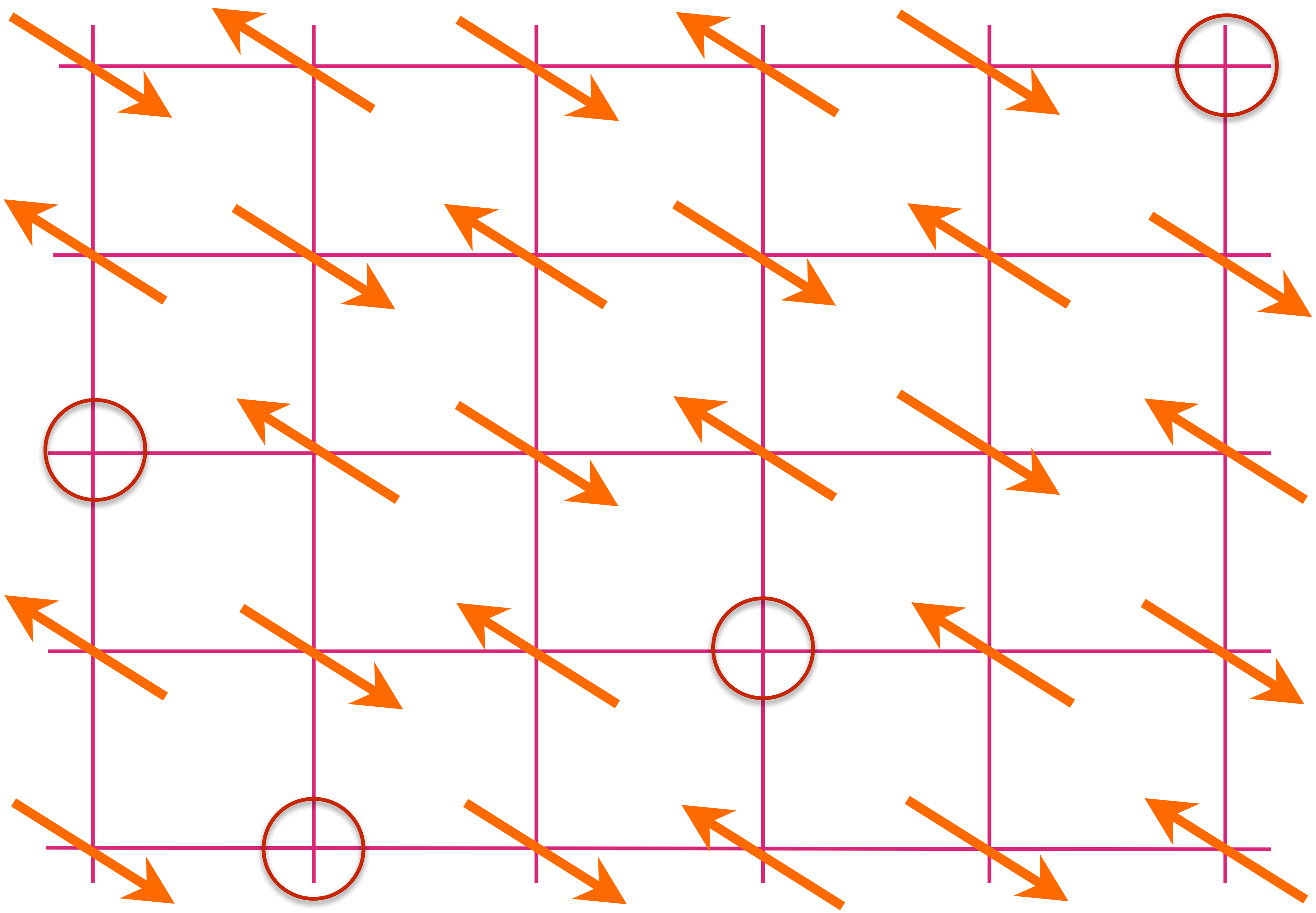


Anti-ferromagnet with p holes per square



Anti-ferromagnet with p holes per square

But relative to the band insulator, there are $1 + p$ holes per square

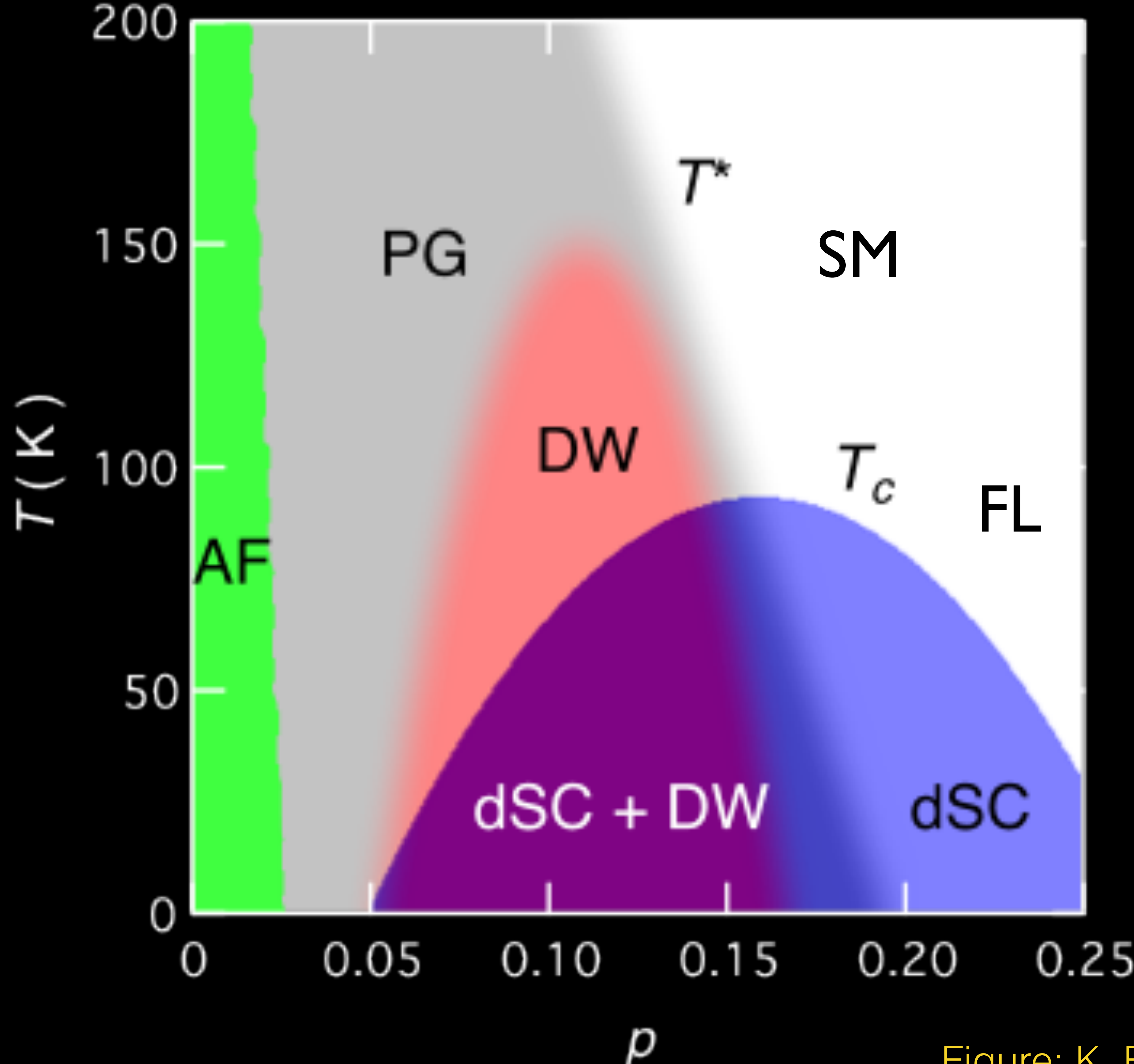


Figure: K. Fujita and J. C. Seamus Davis

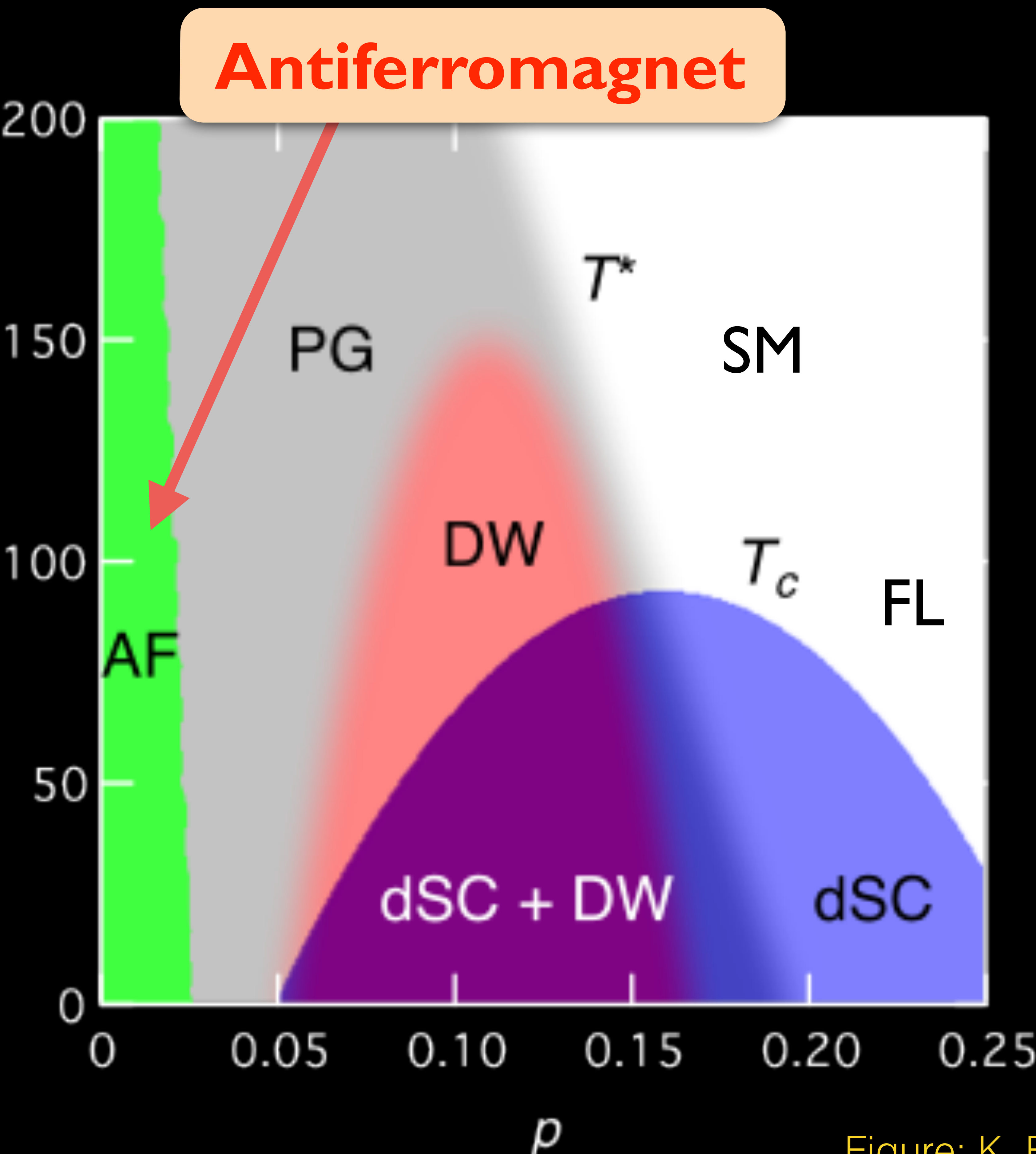


Figure: K. Fujita and J. C. Seamus Davis

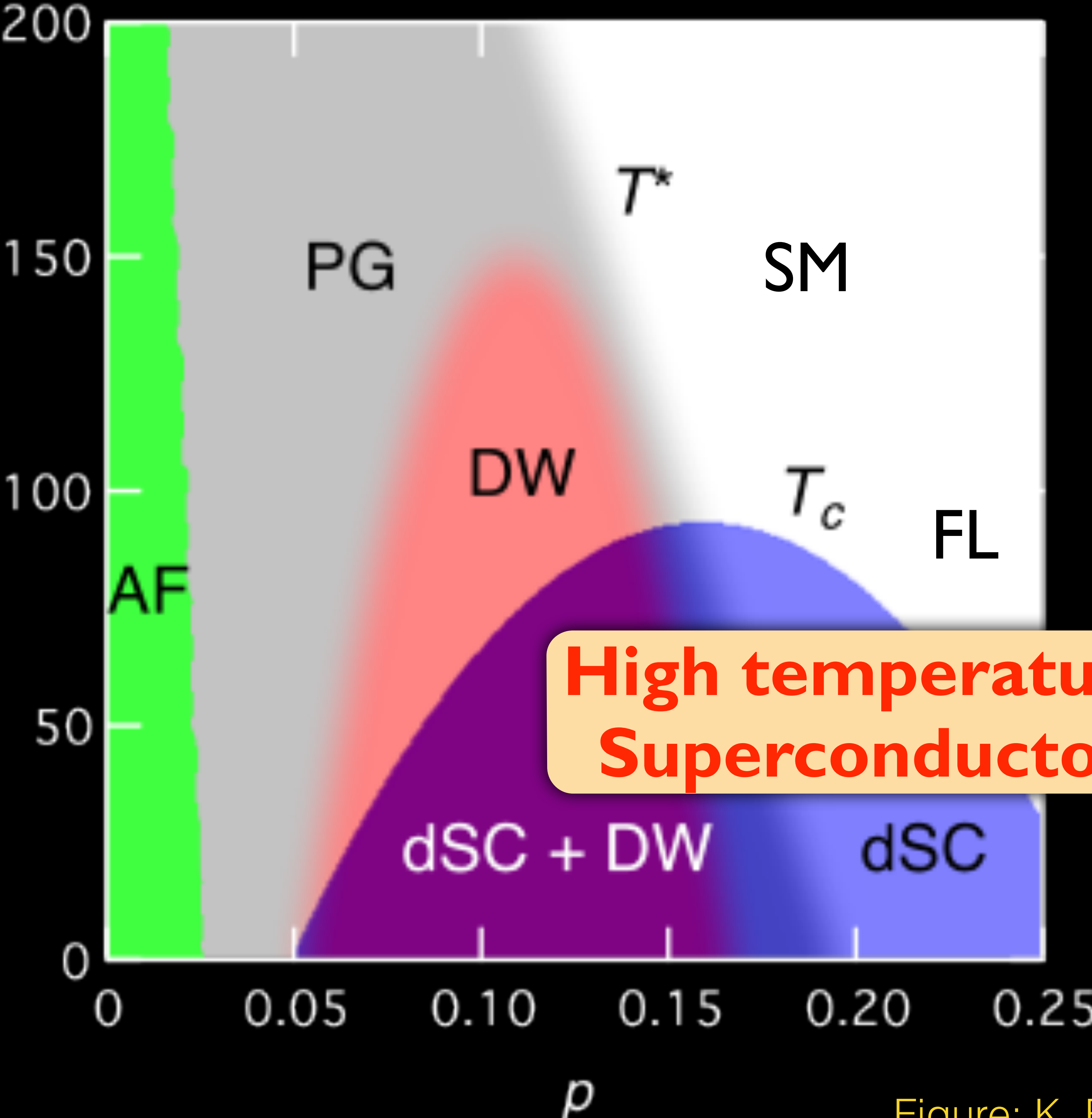
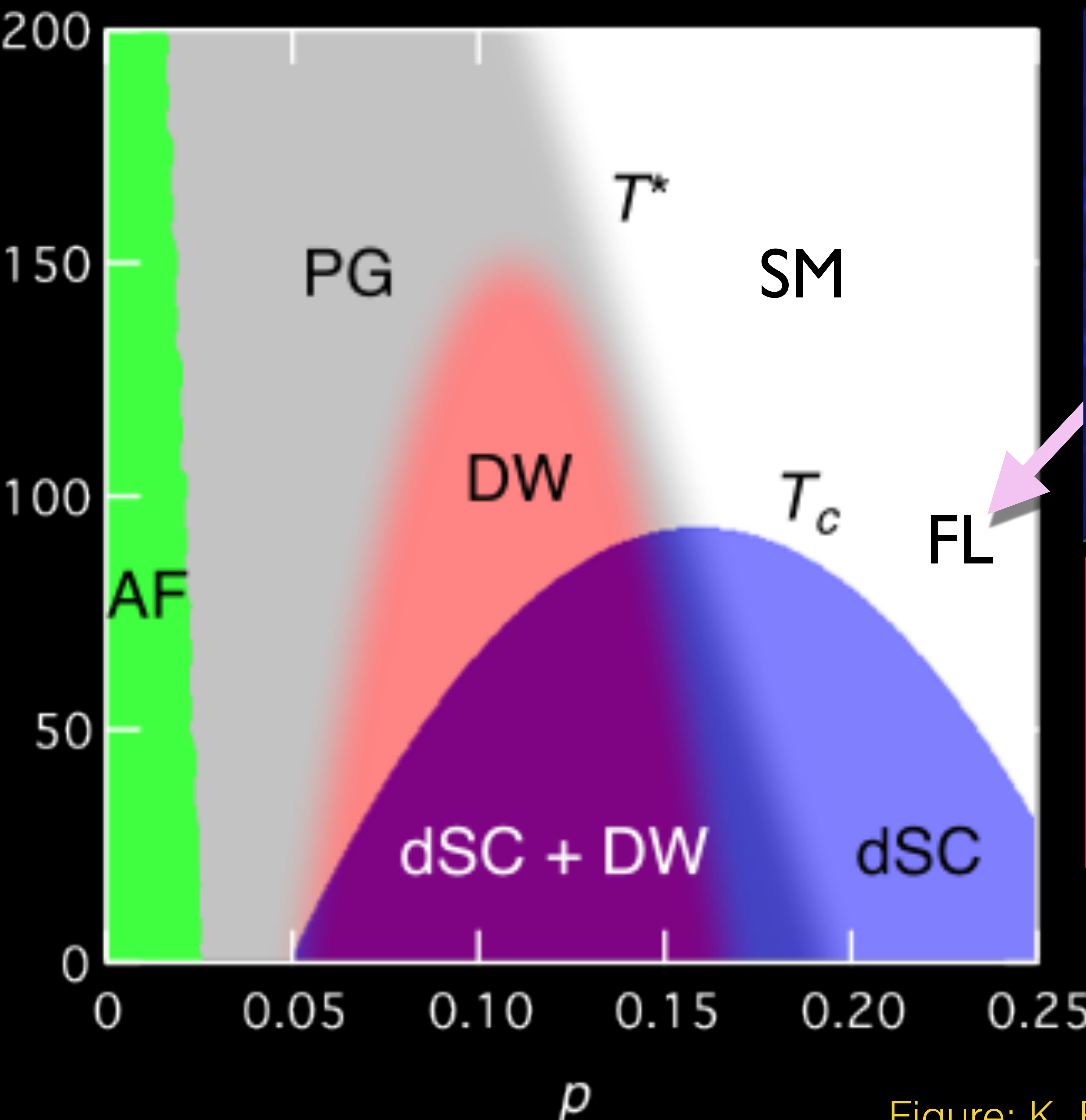
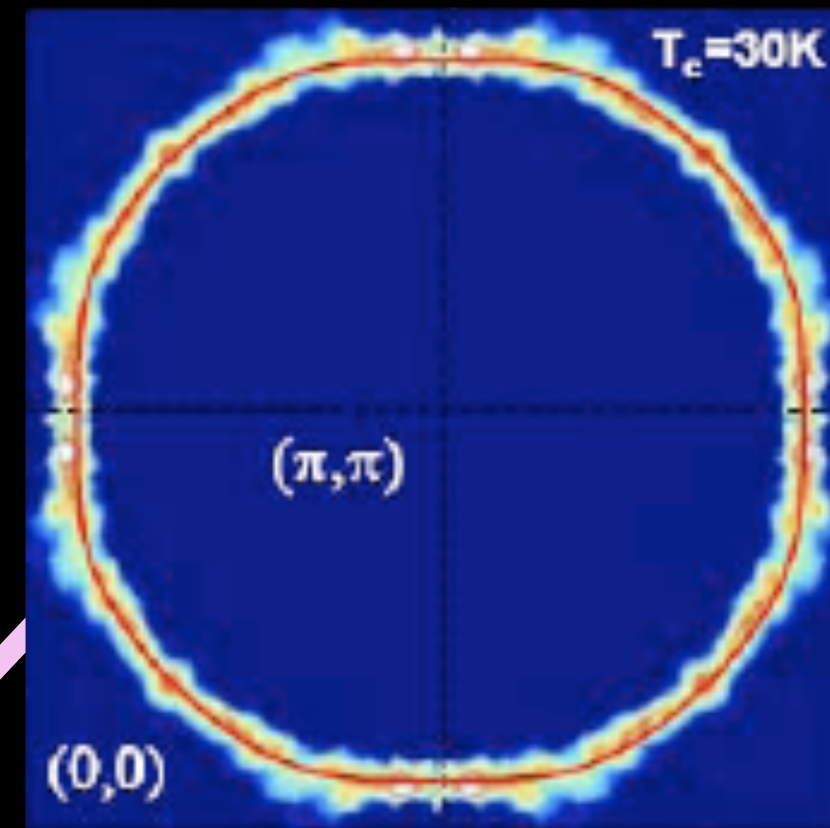


Figure: K. Fujita and J. C. Seamus Davis

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



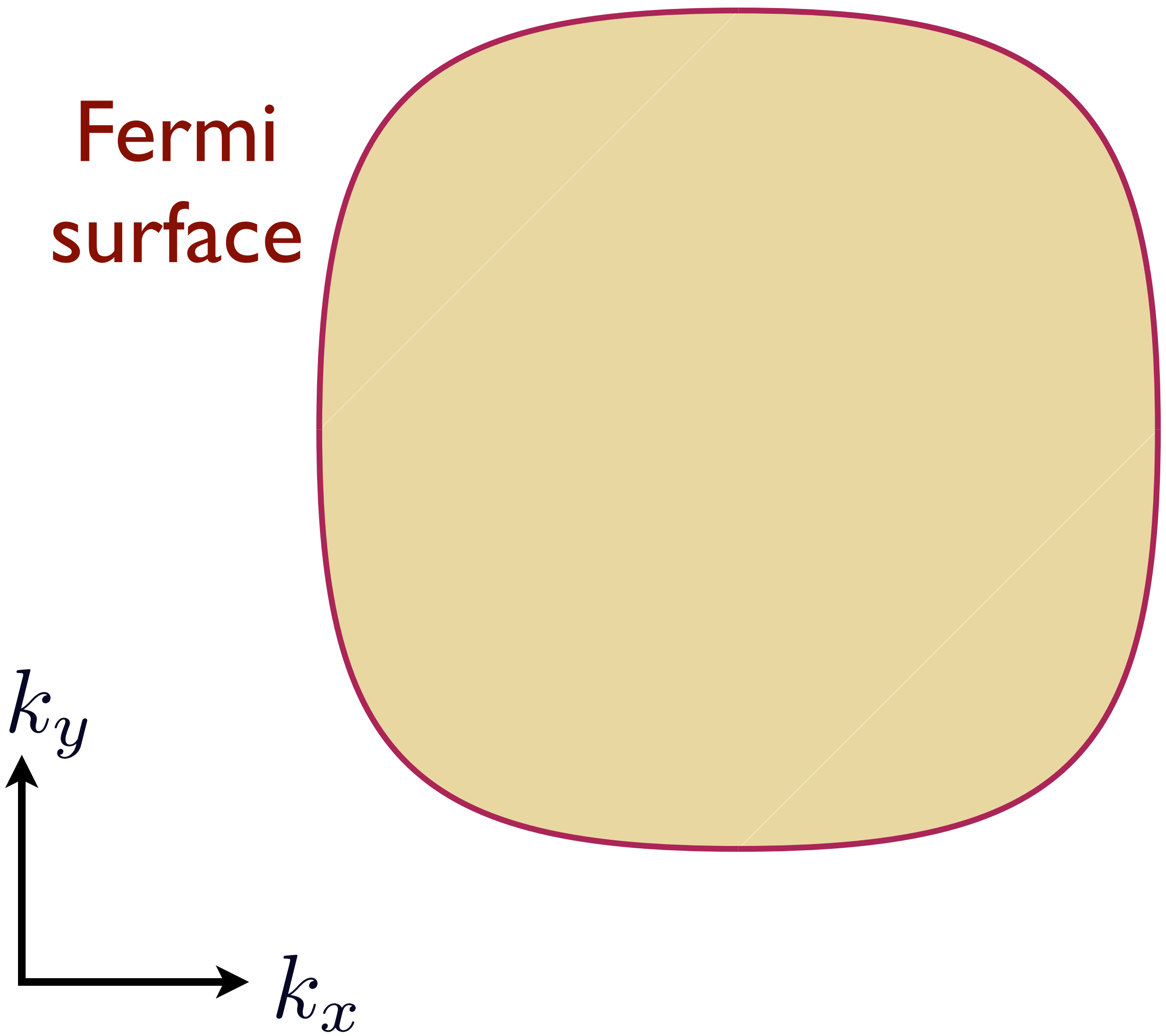


Conventional metal

Area enclosed by Fermi surface =1+p

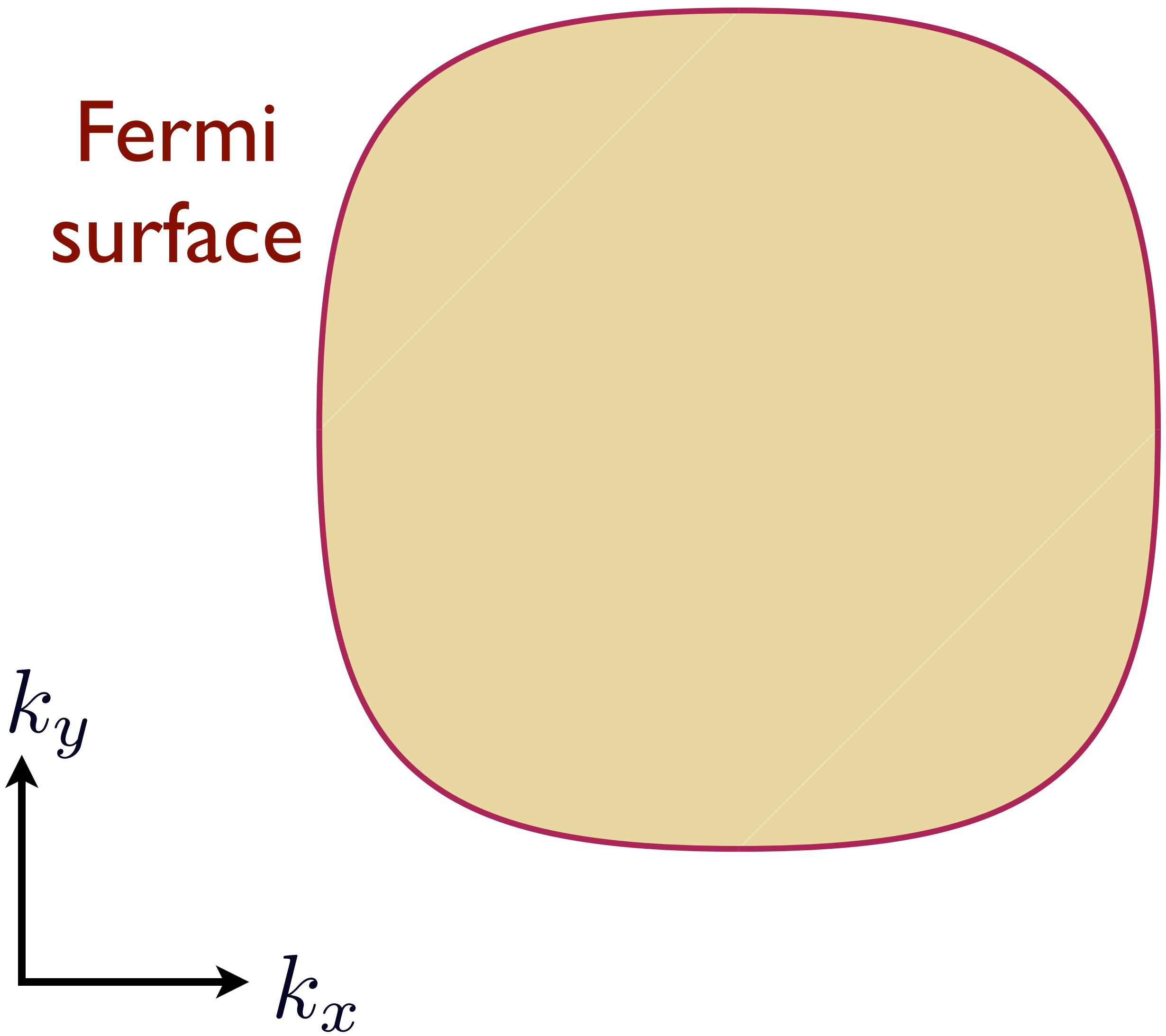
Figure: K. Fujita and J. C. Seamus Davis

Ordinary quantum matter: the Fermi liquid (FL)



- Fermi surface separates empty and occupied states in momentum space.

Ordinary quantum matter: the Fermi liquid (FL)



- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2) = $1+p$.

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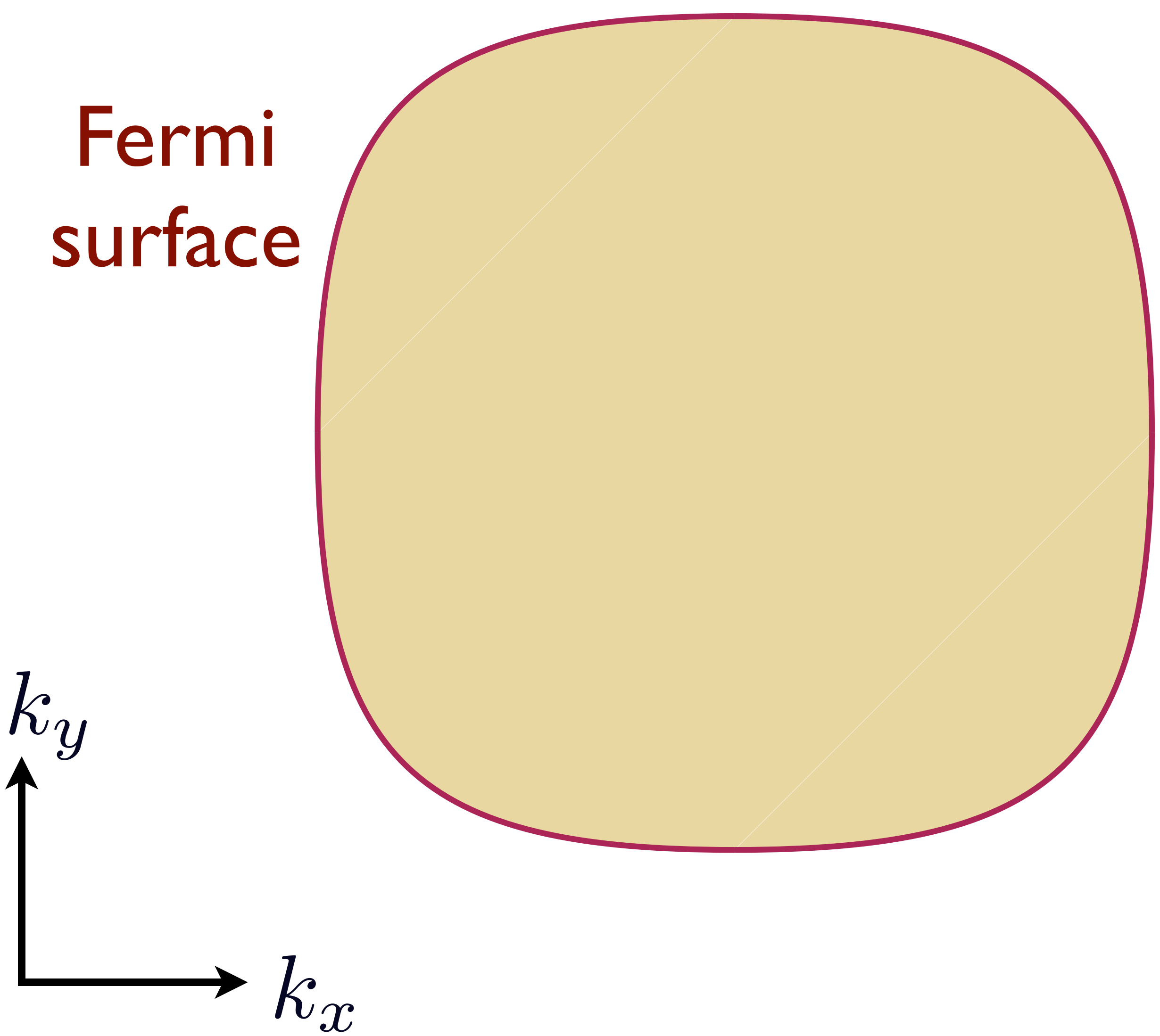

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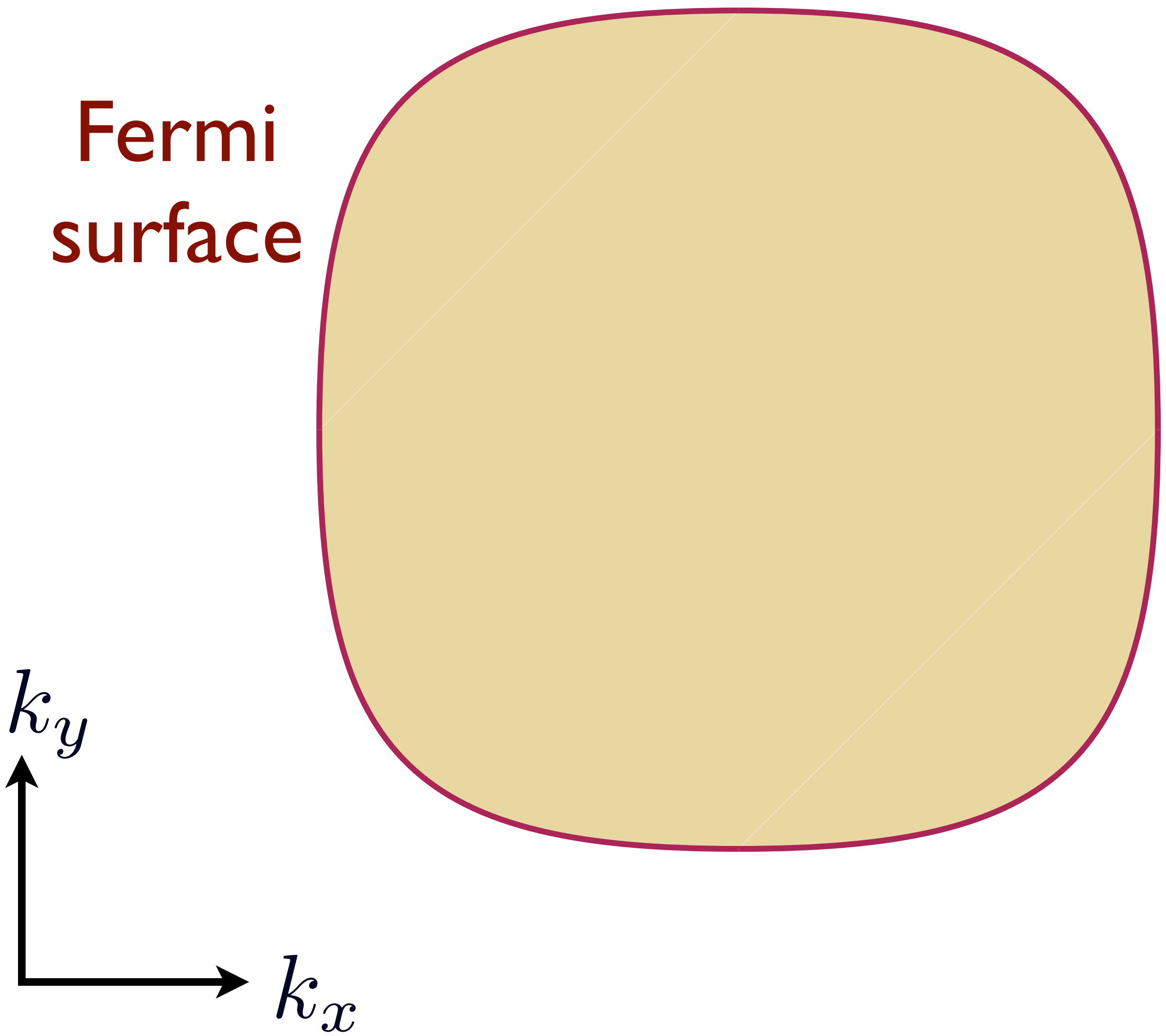
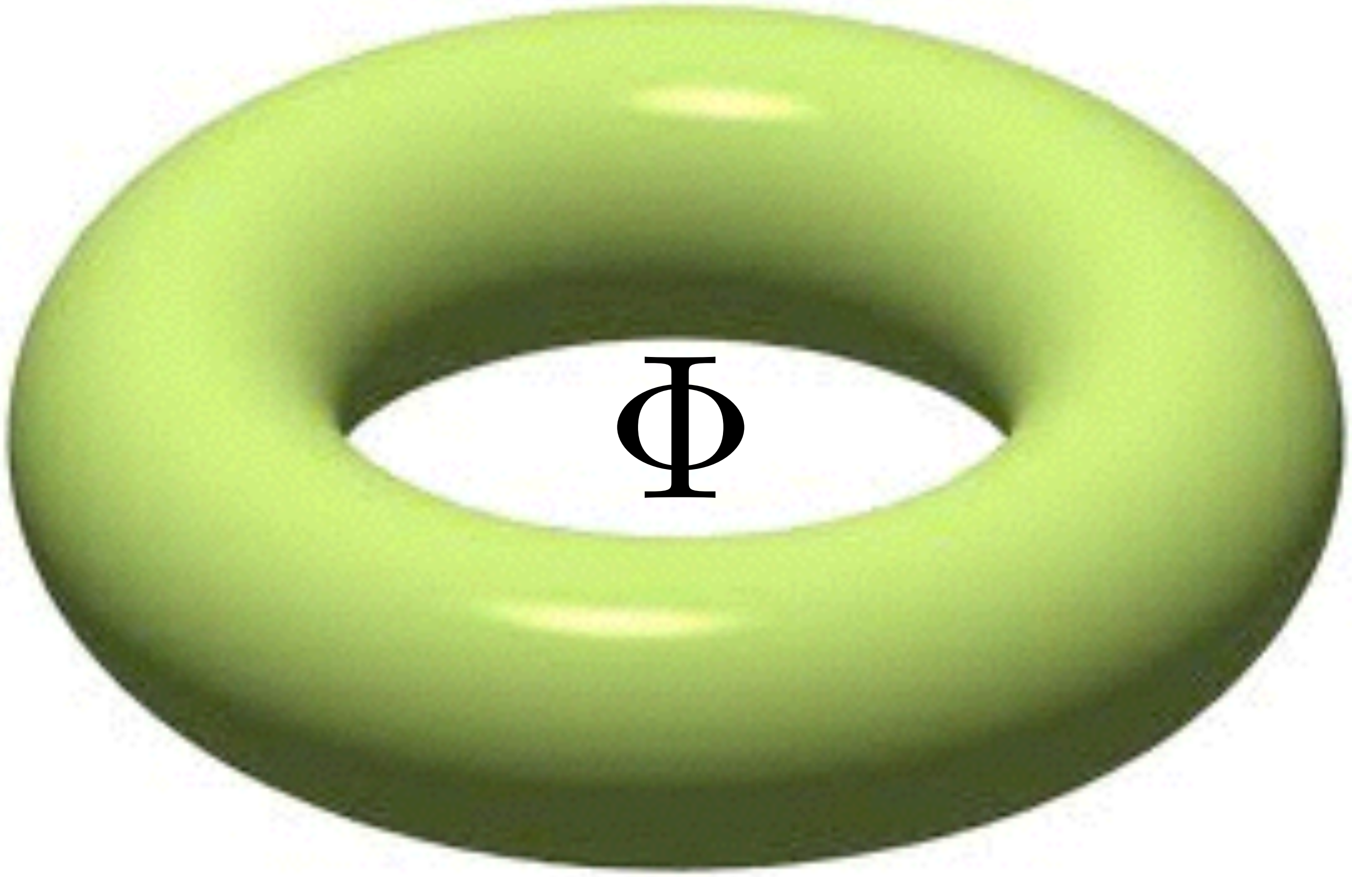


- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2) $= 1+p$.
- Density of electrons can be continuously varied at zero temperature.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

Fermi liquid (FL)

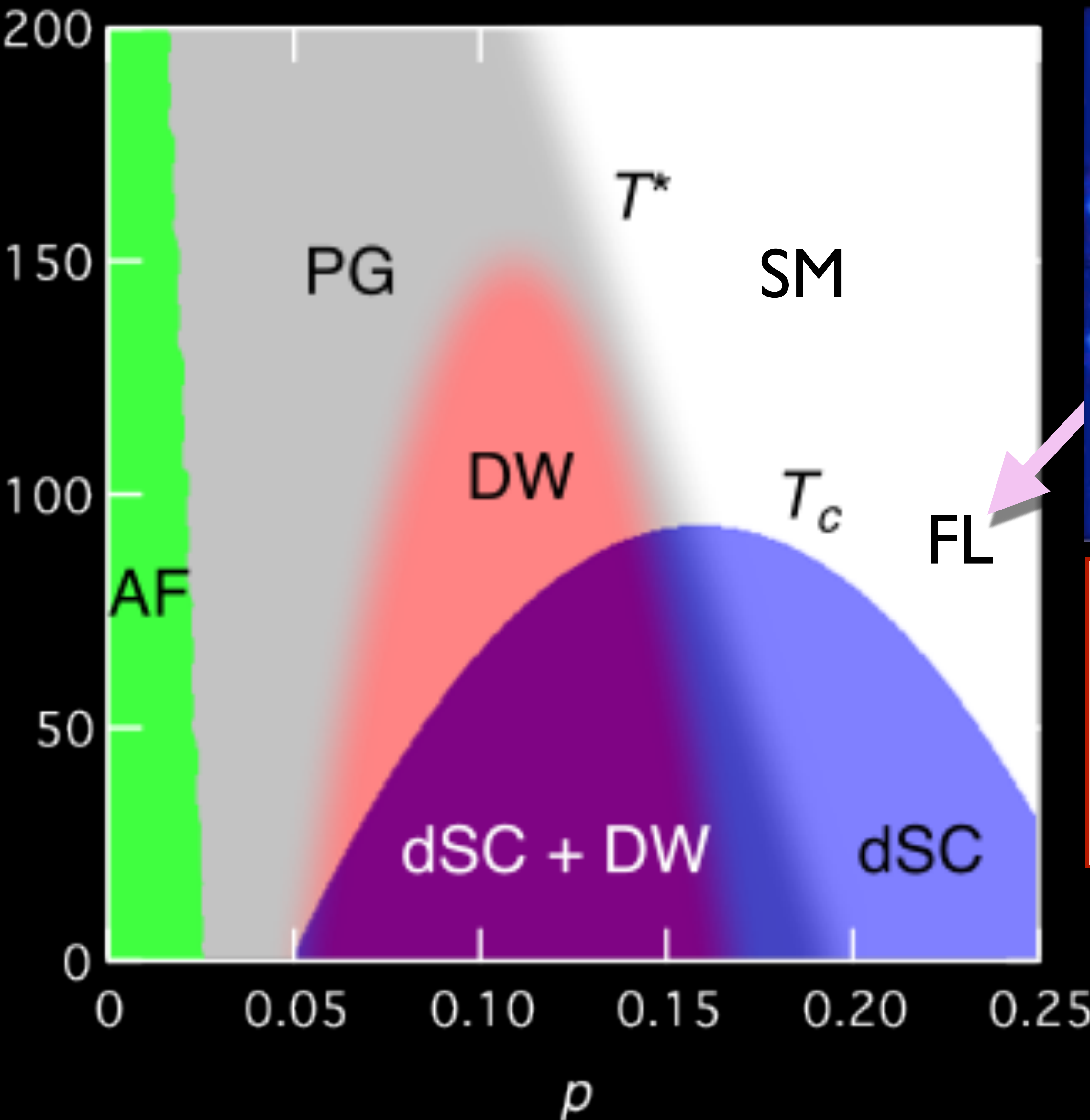
Topological argument for the area of Fermi surface

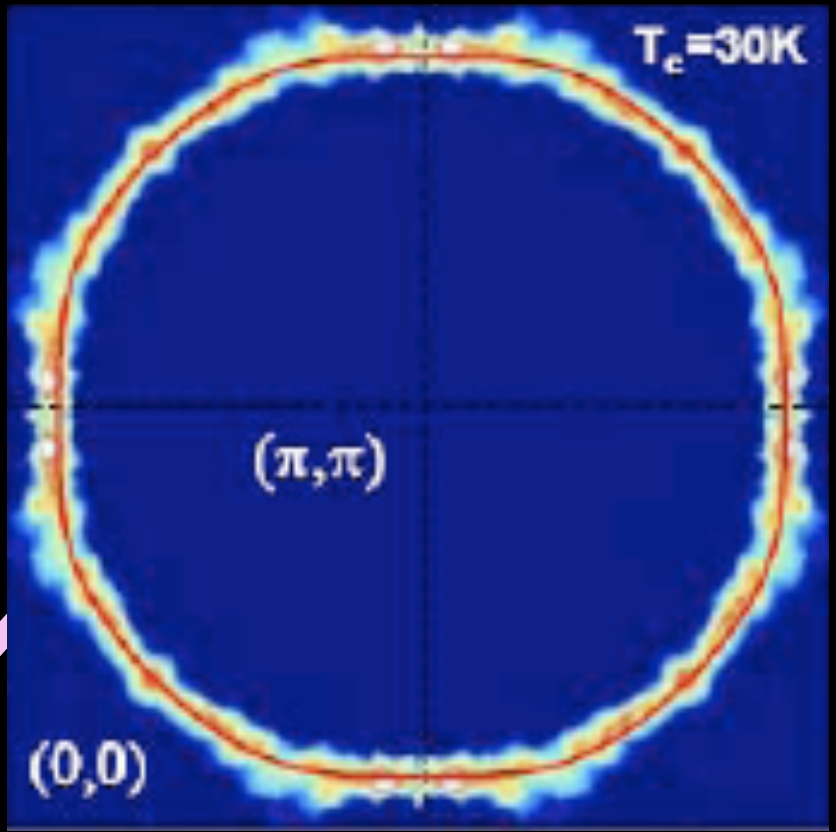


Put metal on a torus, adiabatically insert flux $\Phi = h/e$ through hole, and measure change in momentum. In a FL, we can assume the only low energy excitations are quasiparticles near the Fermi surface, and this leads to a non-perturbative proof of the Luttinger relation on the area enclosed by the Fermi surface.

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





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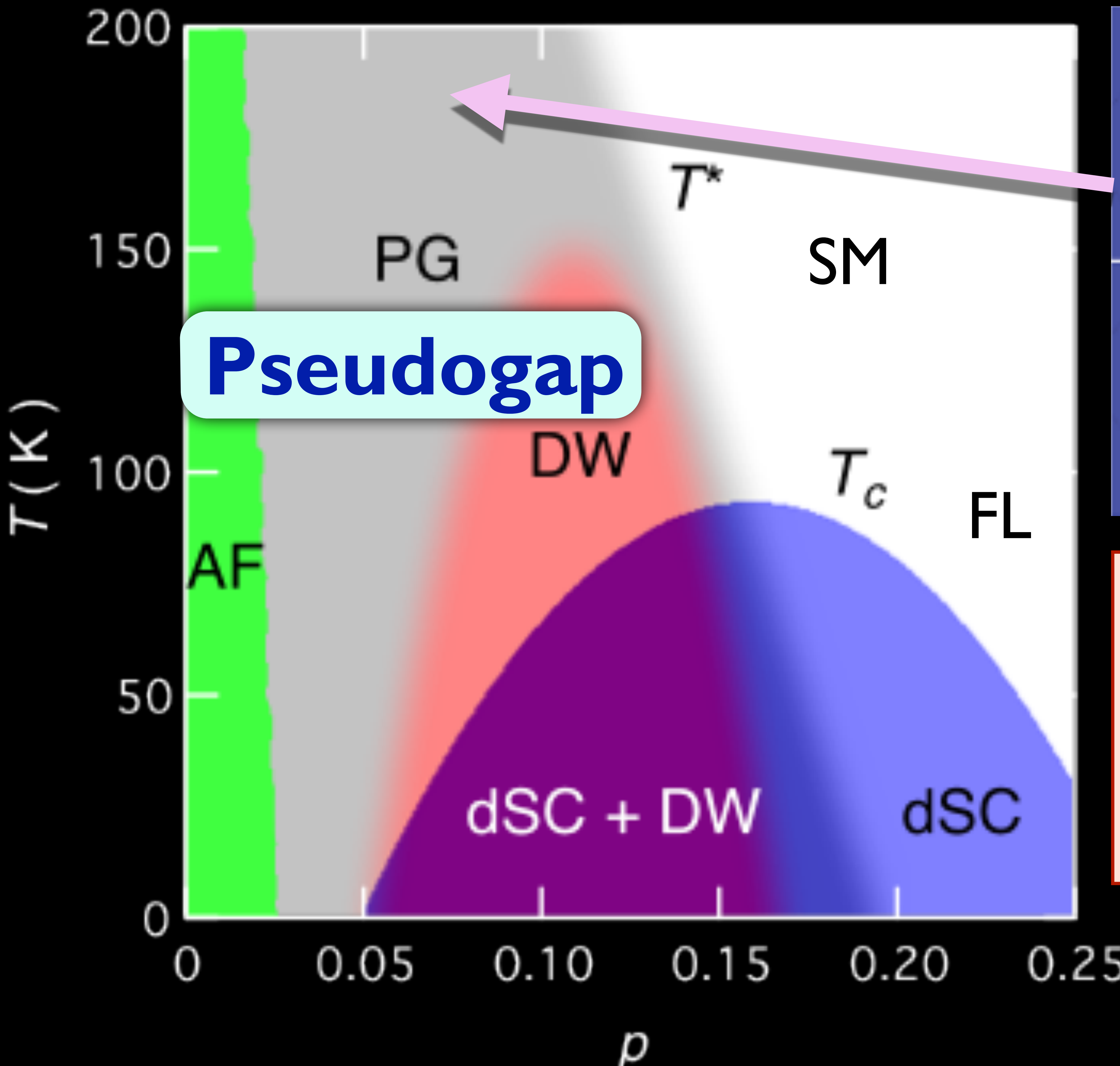
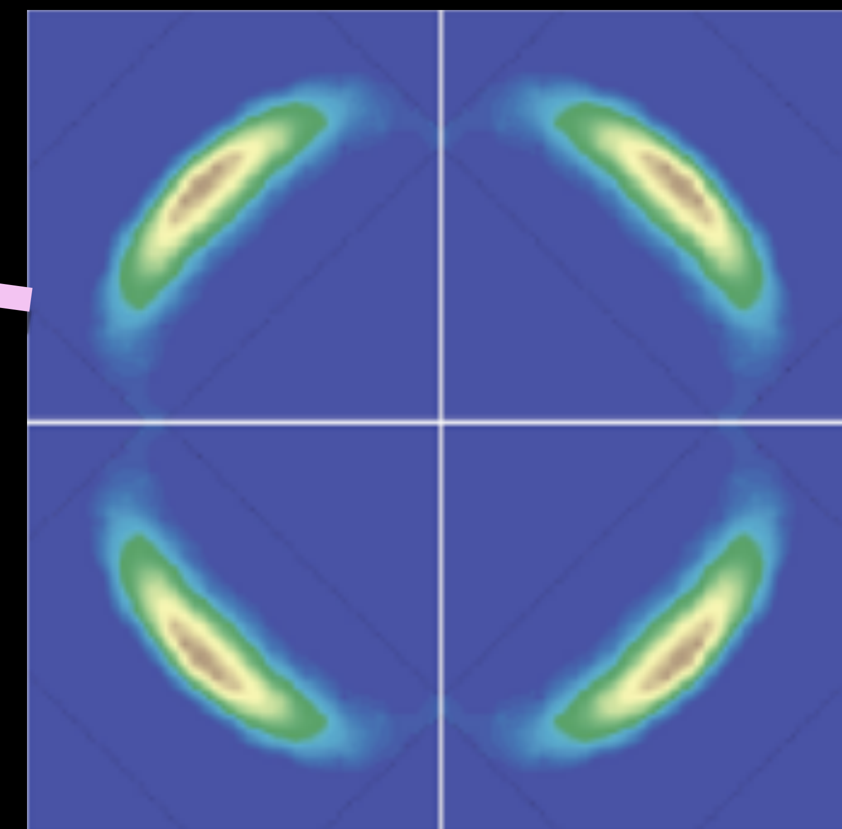
Area enclosed by Fermi surface =1+p

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)





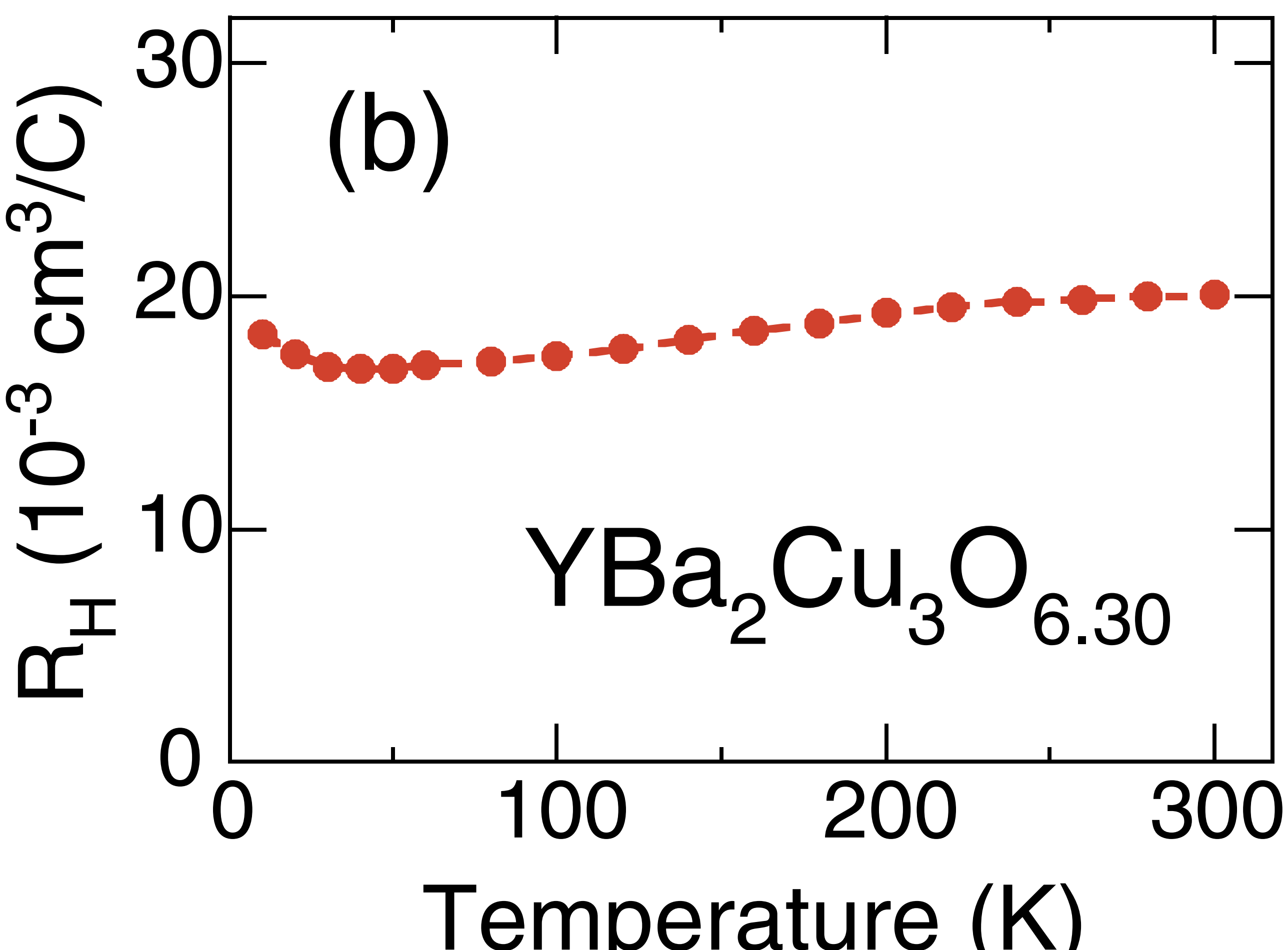
“Fermi arcs” at low p

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density *p*

Evolution of the Hall Coefficient and the Peculiar Electronic Structure of the Cuprate Superconductors

Yoichi Ando,* Y. Kurita,† Seiki Komiya, S. Ono, and Kouji Segawa

PRL **92**, 197001 (2004)



T-independent Hall effect in a magnetic field of fermions of charge +e and density *p*

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density p

Spectroscopic evidence for Fermi liquid-like energy and temperature dependence of the relaxation rate in the pseudogap phase of the cuprates

Seyed Iman Mirzaei[a], Damien Stricker[a], Jason N. Hancock[a,b], Christophe Berthod[a], Antoine Georges[a,c,d], Erik van Heumen[a,e], Mun K. Chan[f], Xudong Zhao[f,g], Yuan Li[h], Martin Greven[f], Neven Barišić[f,i,j], and Dirk van der Marel[a,1]

PNAS 110, 5774 (2013)

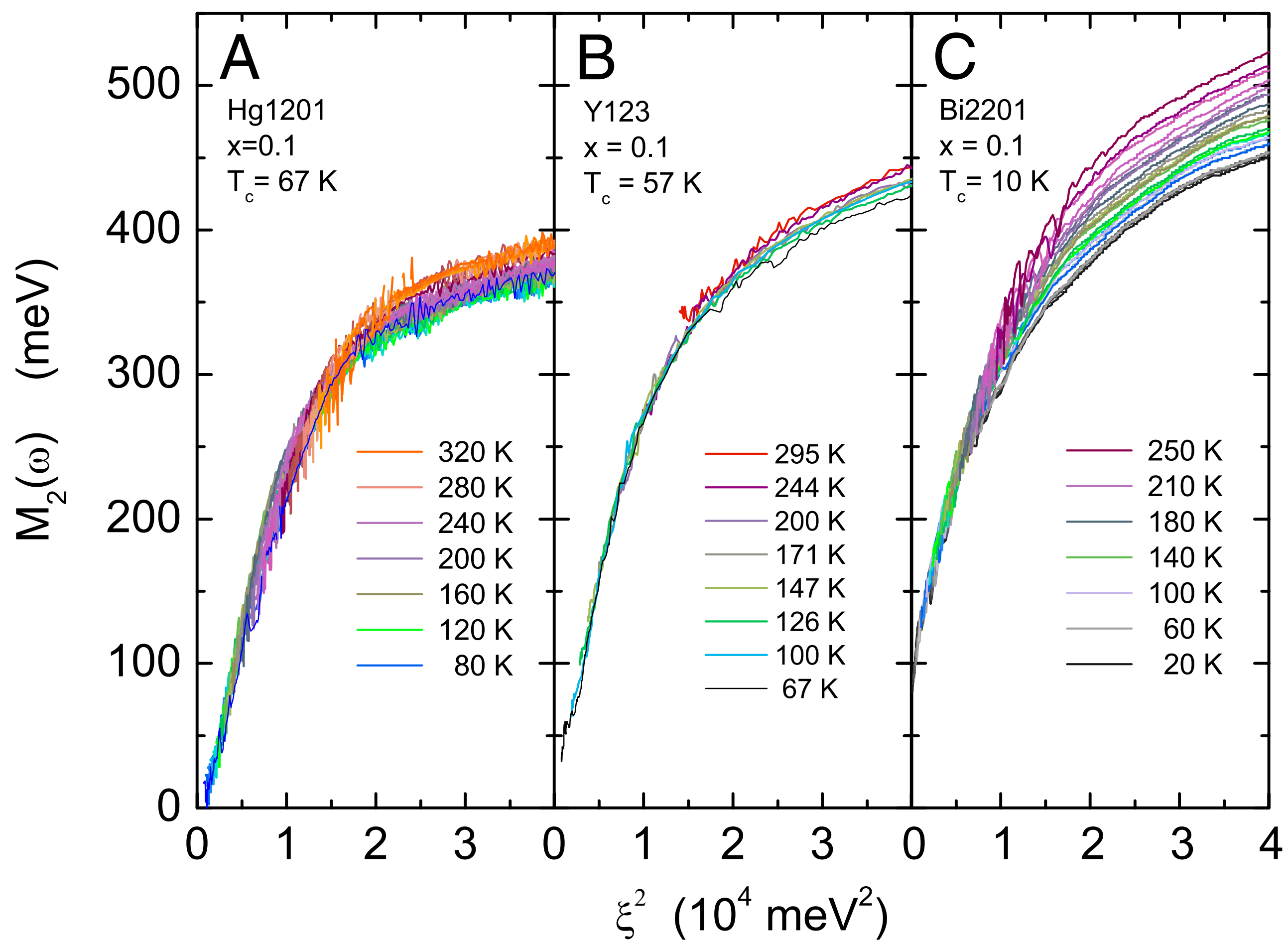


Fig. 6. Collapse of the frequency and temperature dependence of the relaxation rate of underdoped cuprate materials. Normal state $M_2(\omega, T)$ as a function of $\xi^2 \equiv (\hbar\omega)^2 + (p\pi k_B T)^2$

$$\sigma_{xx} \sim \frac{1}{(-i\omega + 1/\tau)}$$

$$\text{with } \frac{1}{\tau} \sim \omega^2 + T^2$$

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density p

In-Plane Magnetoresistance Obeys Kohler's Rule in the Pseudogap Phase of Cuprate Superconductors

M. K. Chan,[1,*] M. J. Veit,[1] C. J. Dorow,[1,†] Y. Ge,[1] Y. Li,[1] W. Tabis,[1,2] Y. Tang,[1] X. Zhao,[1,3] N. Barišić,[1,4,5,‡] and M. Greven[1,§]

PRL **113**, 177005 (2014)

We report in-plane resistivity (ρ) and transverse magnetoresistance (MR) measurements for underdoped $HgBa_2CuO_{4+\delta}$ (Hg1201). Contrary to the long-standing view that Kohler's rule is strongly violated in underdoped cuprates, we find that it is in fact satisfied in the pseudogap phase of Hg1201. The transverse MR shows a quadratic field dependence, $\delta\rho/\rho_0 = aH^2$, with $a(T) \propto T^{-4}$. In combination with the observed $\rho \propto T^2$ dependence, this is consistent with a single Fermi-liquid quasiparticle scattering rate. We show that this behavior is typically masked in cuprates with lower structural symmetry or strong disorder effects.

$$\rho_{xx} \sim \frac{1}{\tau}(1 + aH^2\tau^2 + \dots)$$

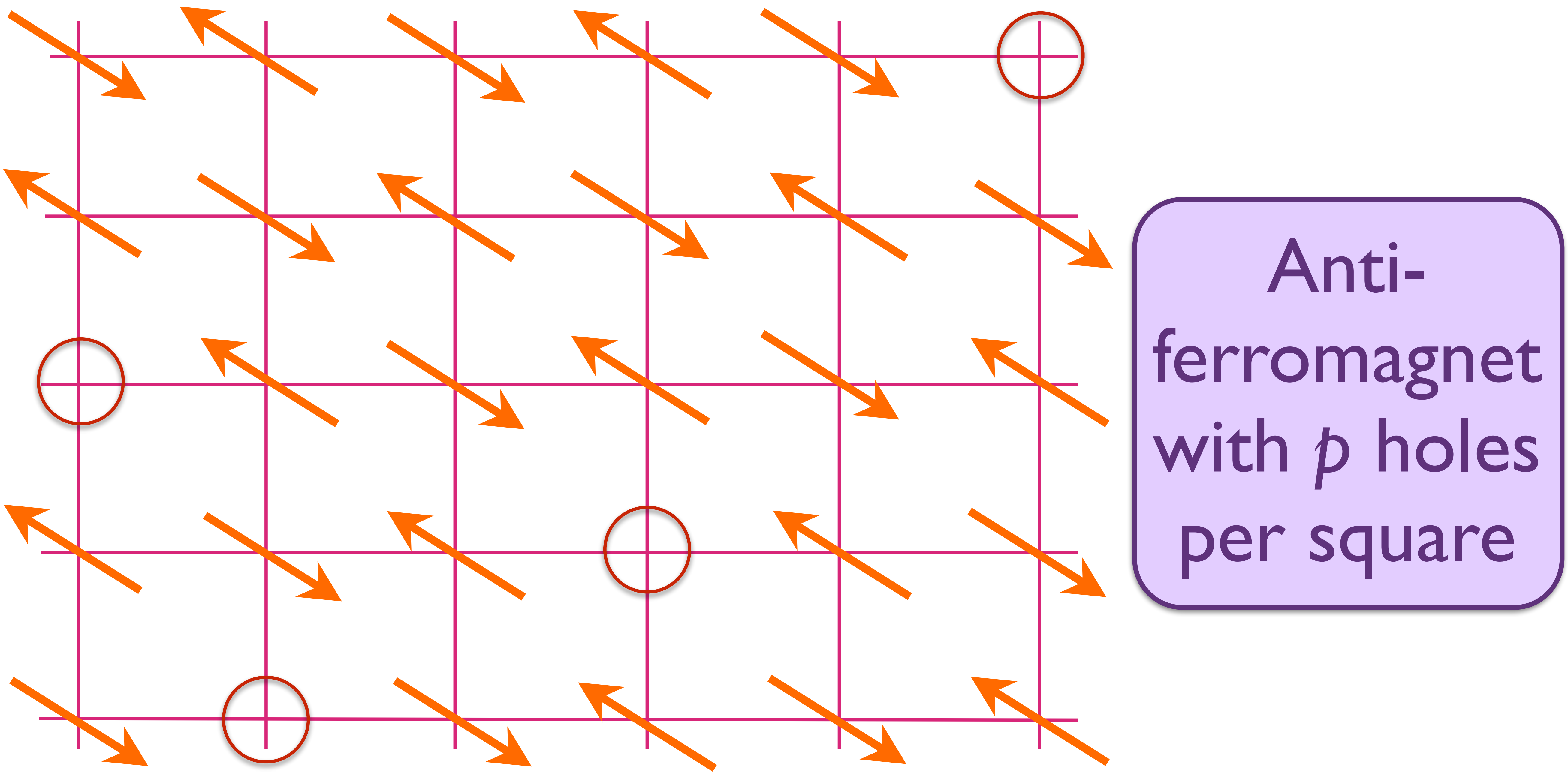
$$\text{with } \frac{1}{\tau} \sim T^2$$

Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size p ?

Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size p ?

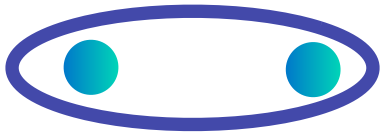
Answer: **Yes.**
There can be a Fermi surface of size p,
but it must be accompanied by
topological order, in a
"fractionalized Fermi liquid".

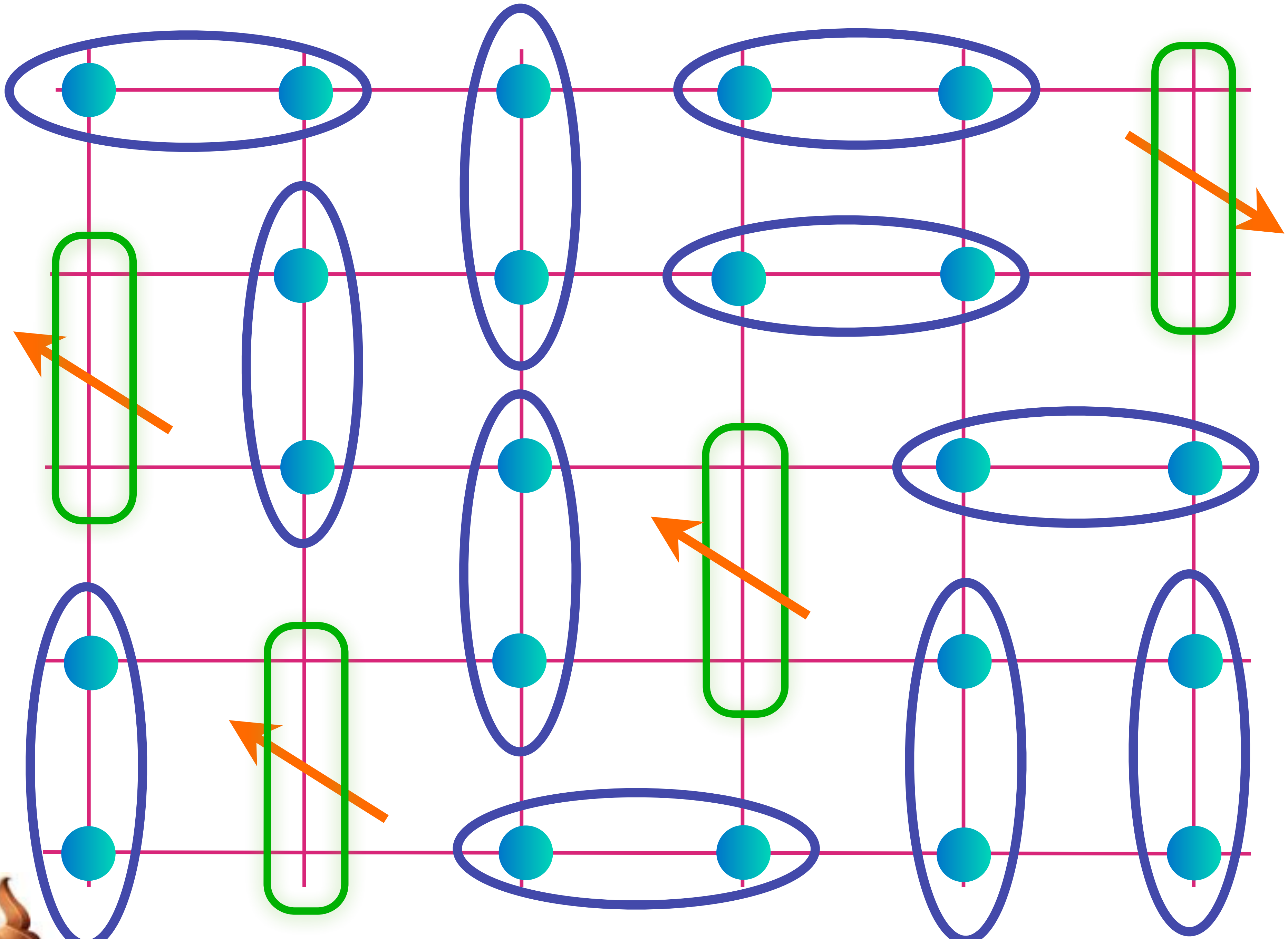
At $T=0$, such a metal must be separated from a Fermi liquid (with a Fermi surface of size $1+p$) by a quantum phase transition



Note: relative to the fully-filled band insulator,
there are $1+p$ holes per square

Fractionalized Fermi liquid (FL*)

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



Realizes a metal with a Fermi surface of area p co-existing with "topological order"

M. Punk, A. Allais, and S. S., arXiv:1501.00978.

Fractionalized Fermi liquid (FL*)

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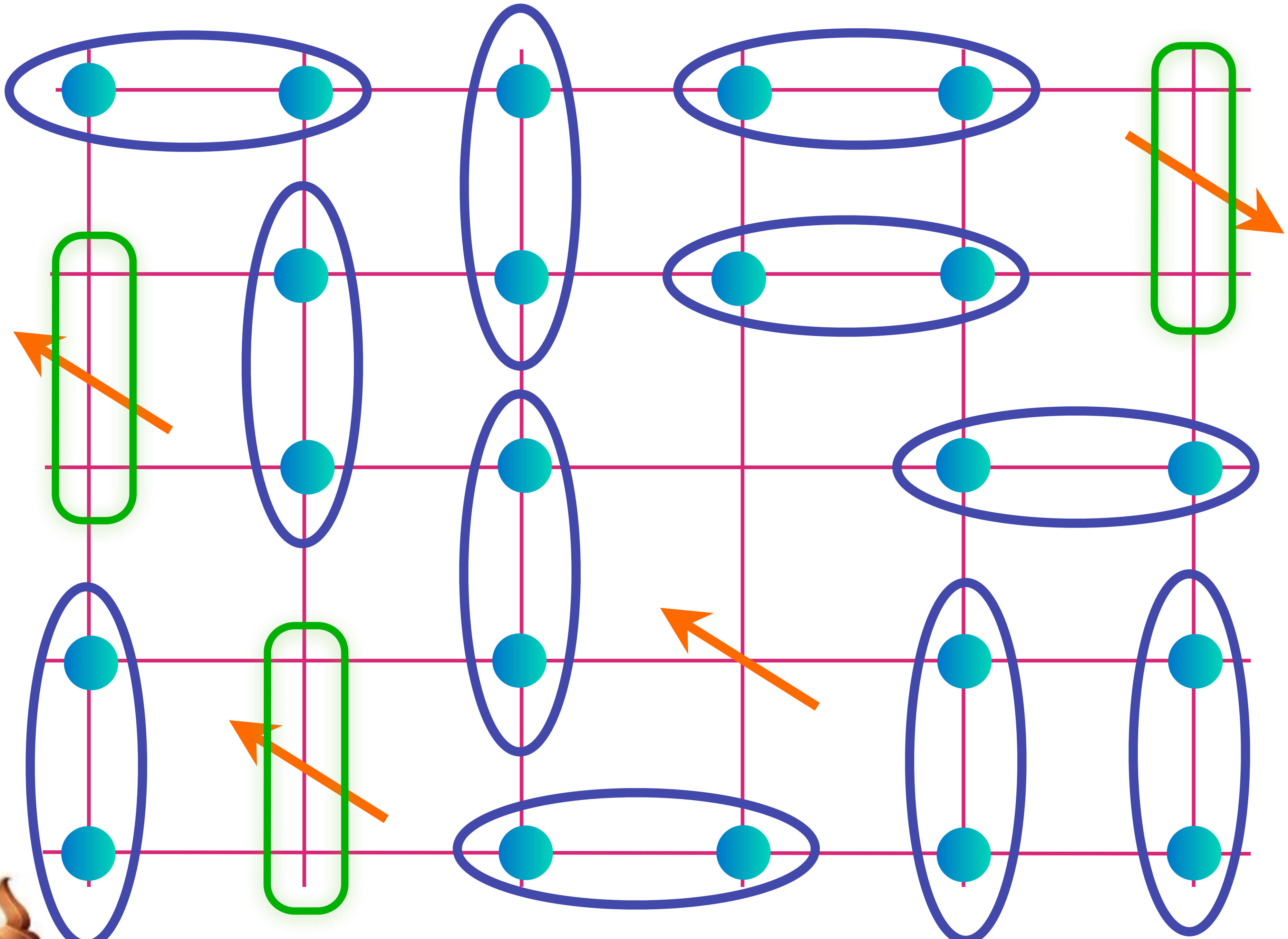

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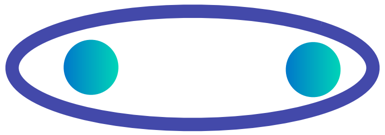
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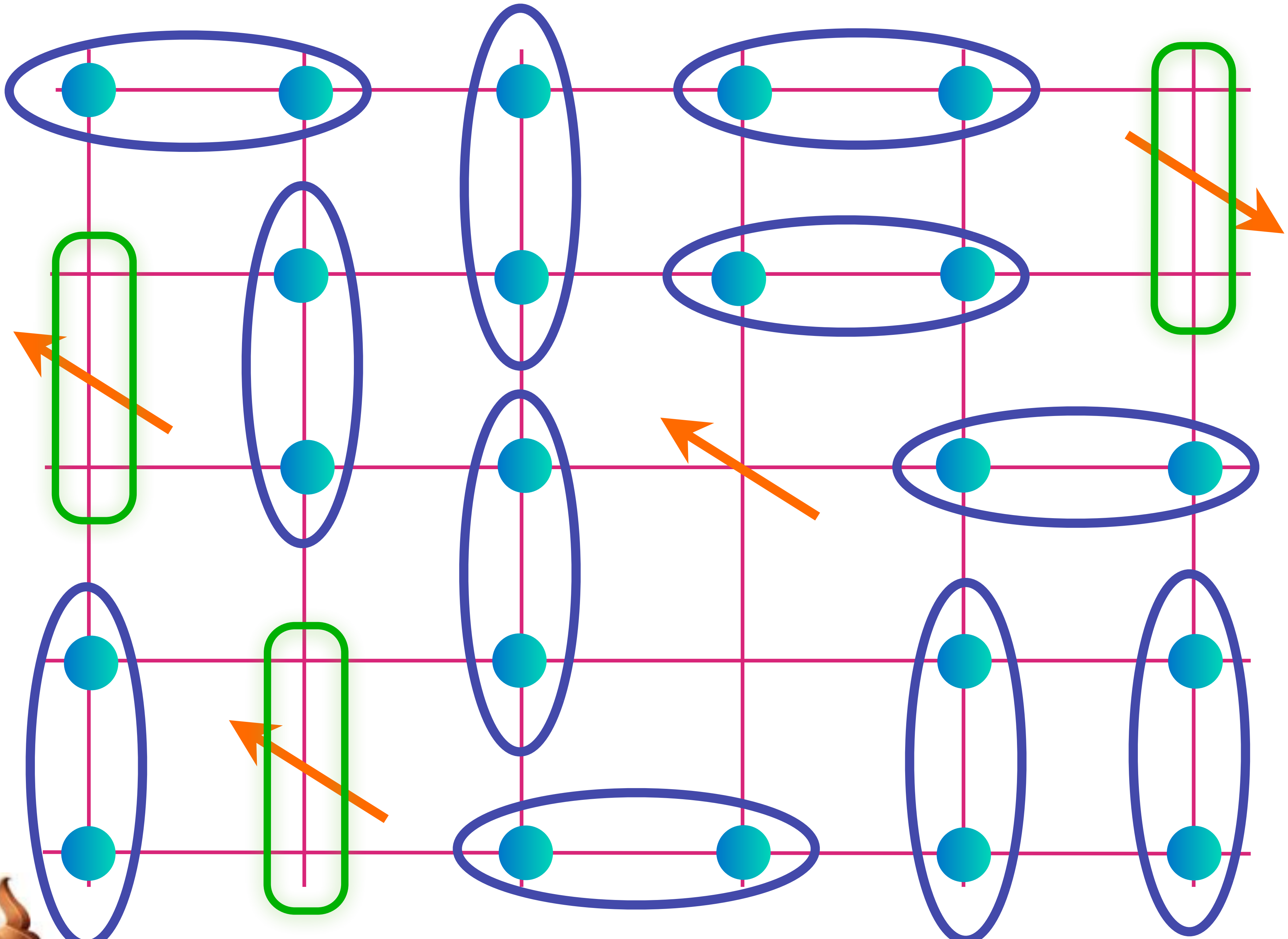

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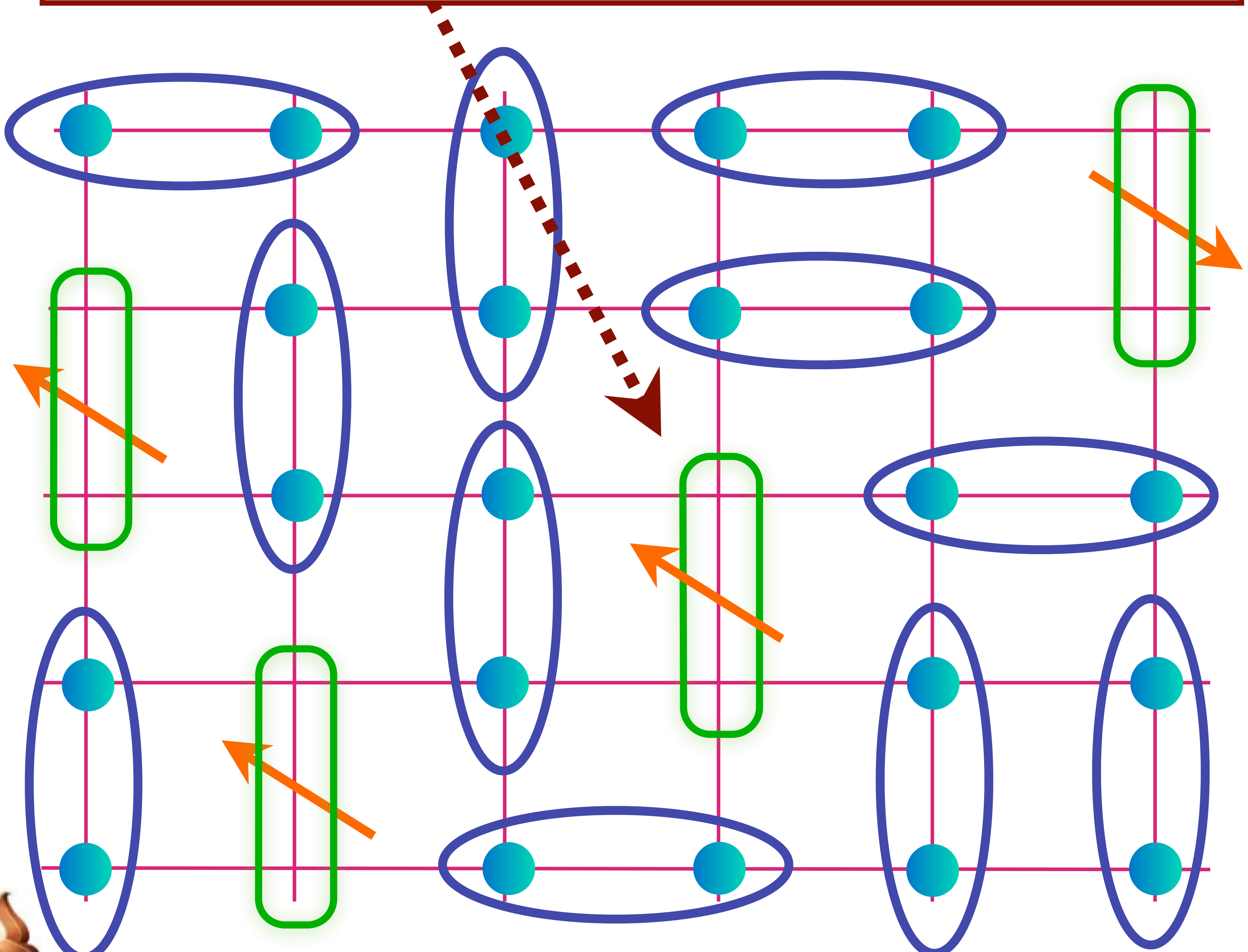
A fermionic “dimer” describing a “bonding” orbital between two sites



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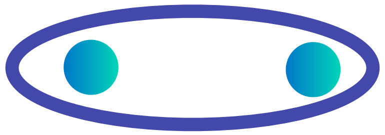
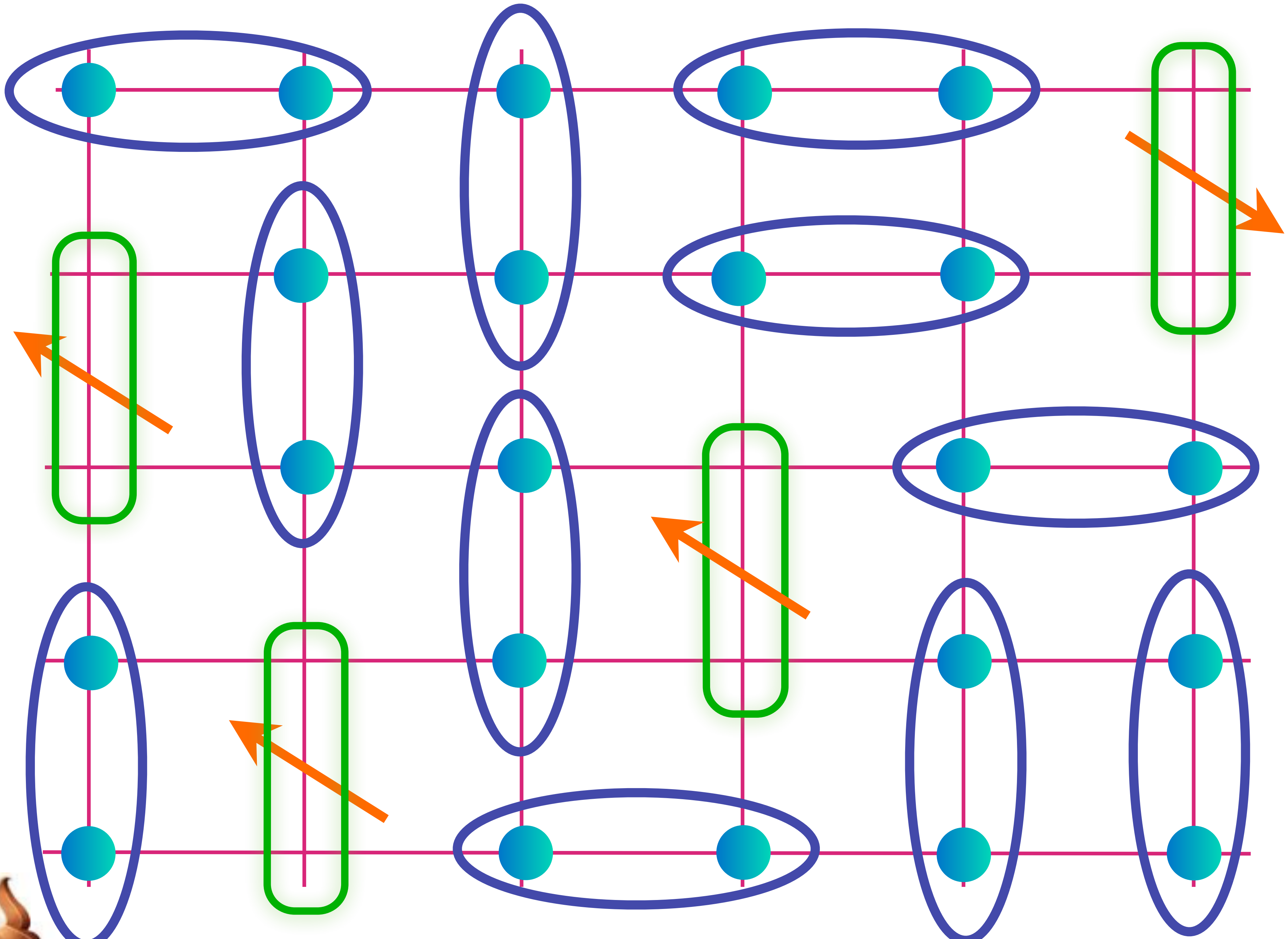
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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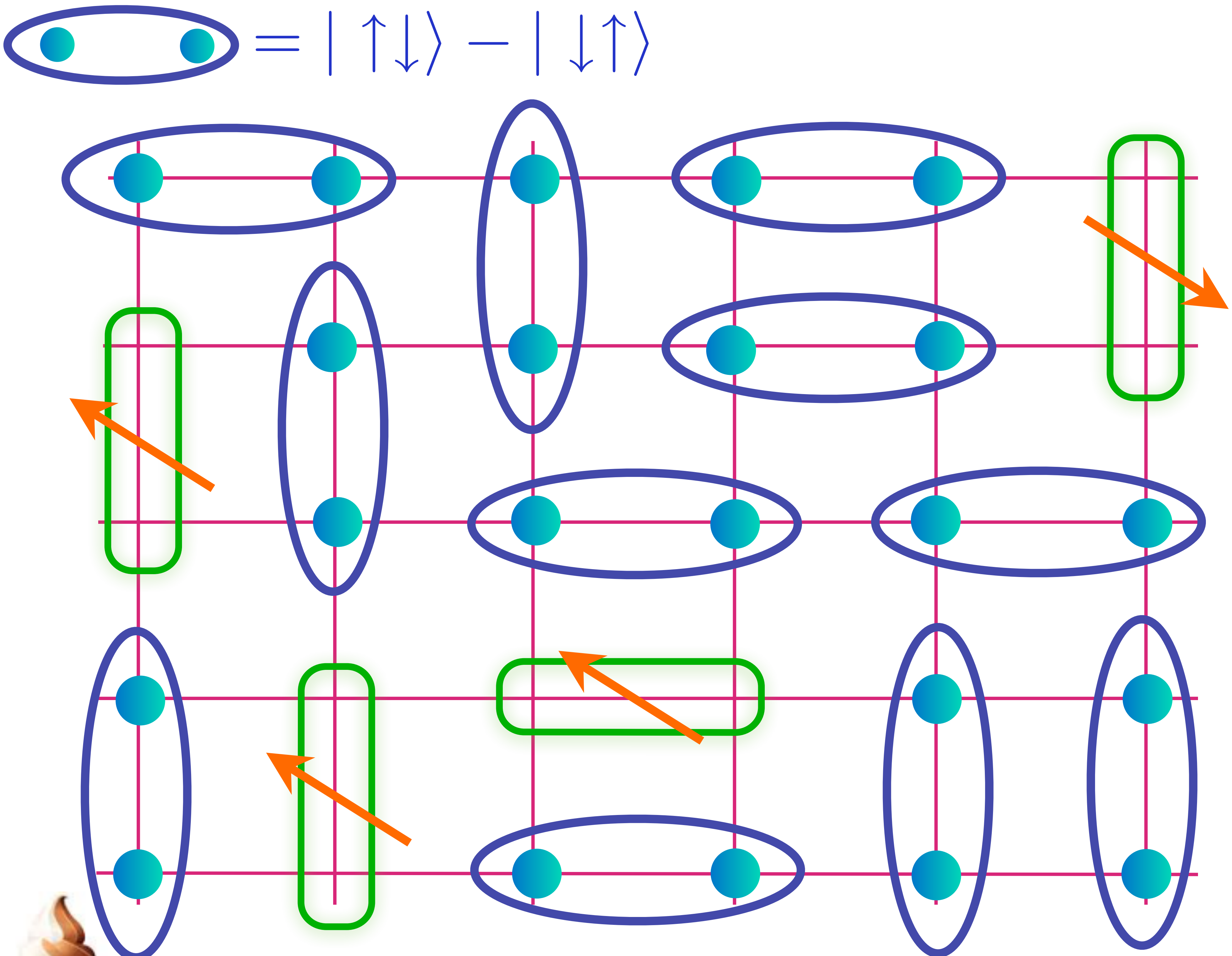

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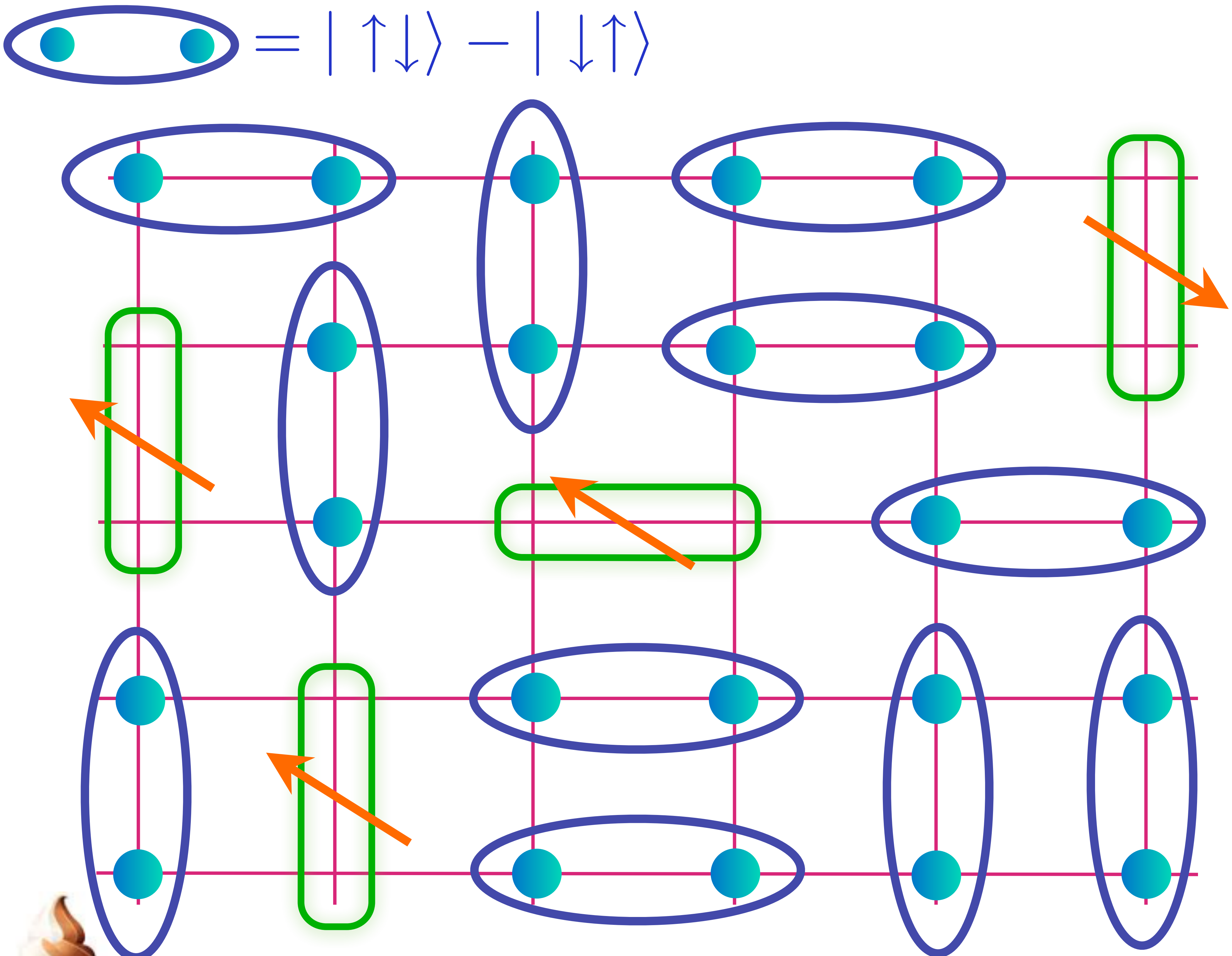

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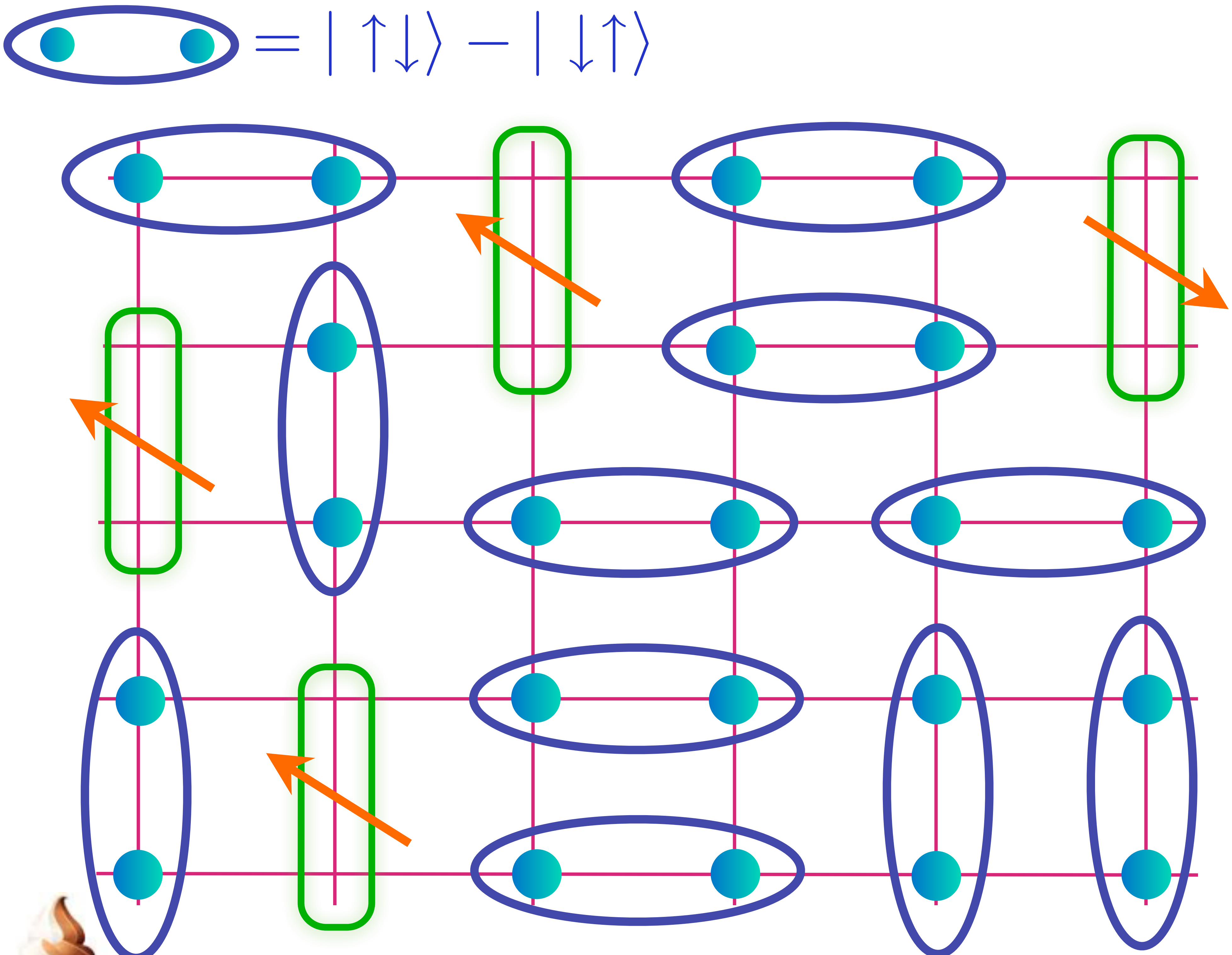

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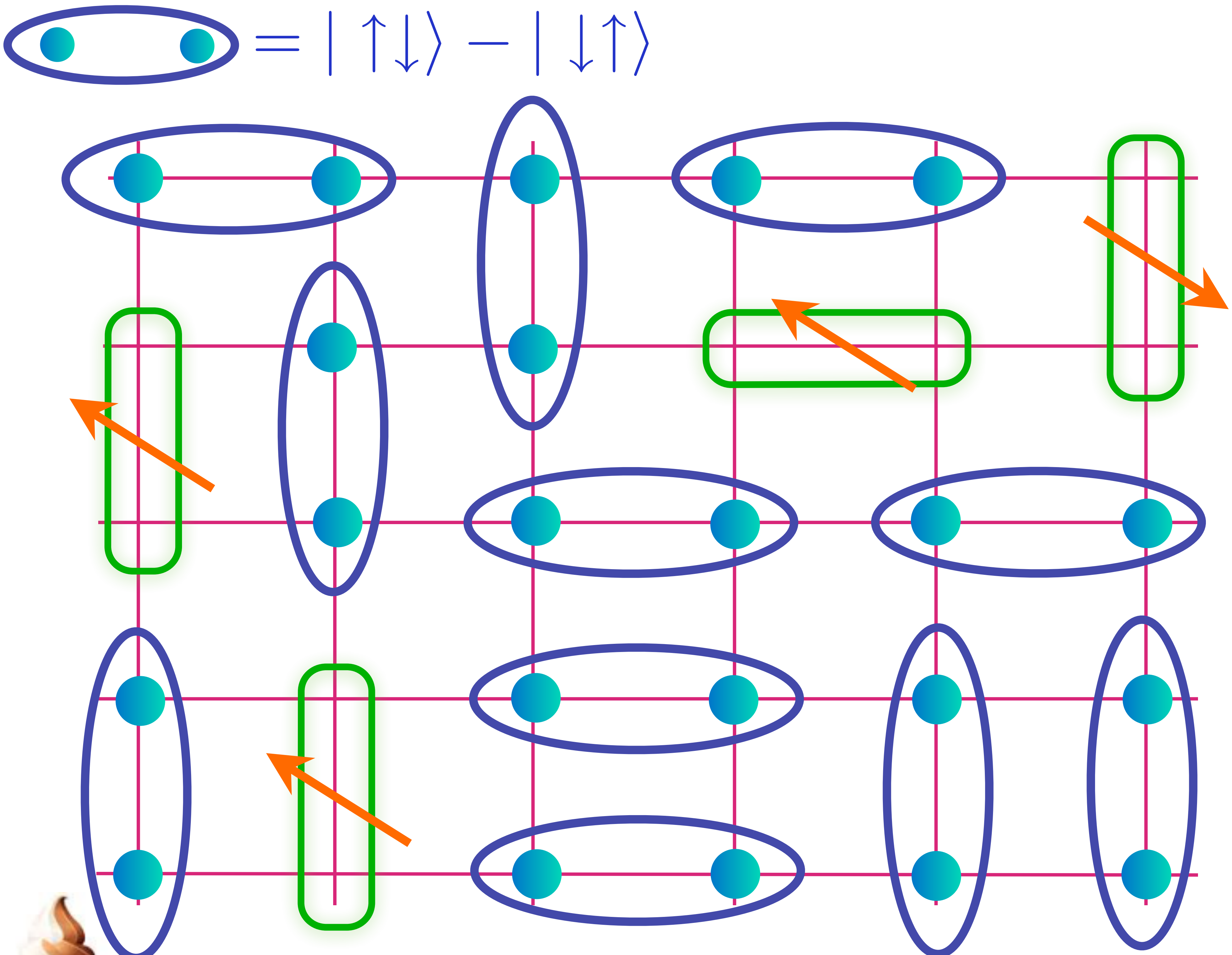

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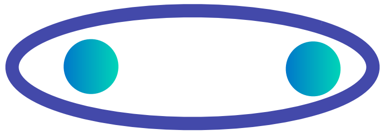
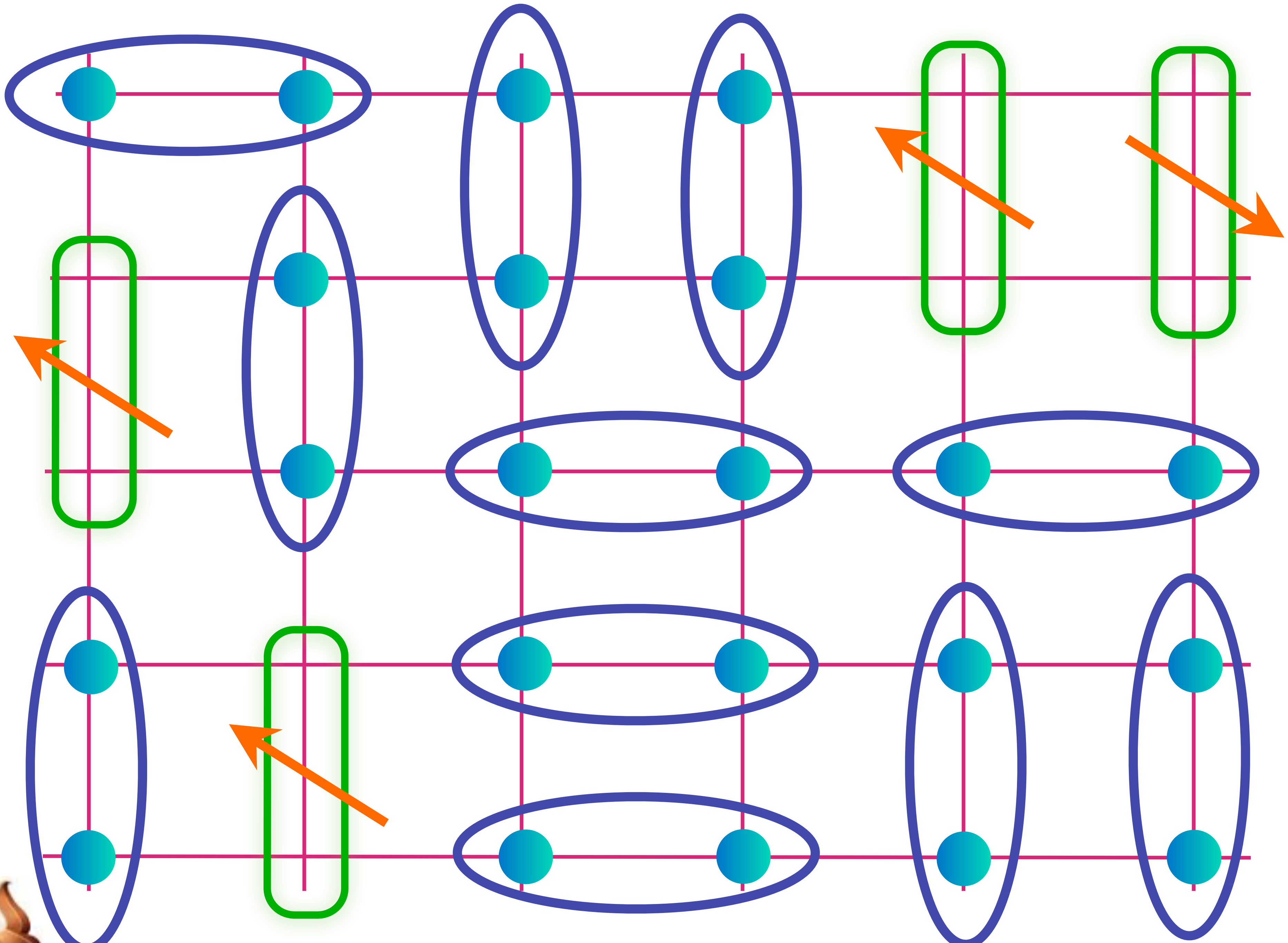
 $= | \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle$



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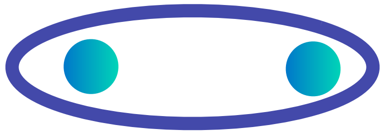
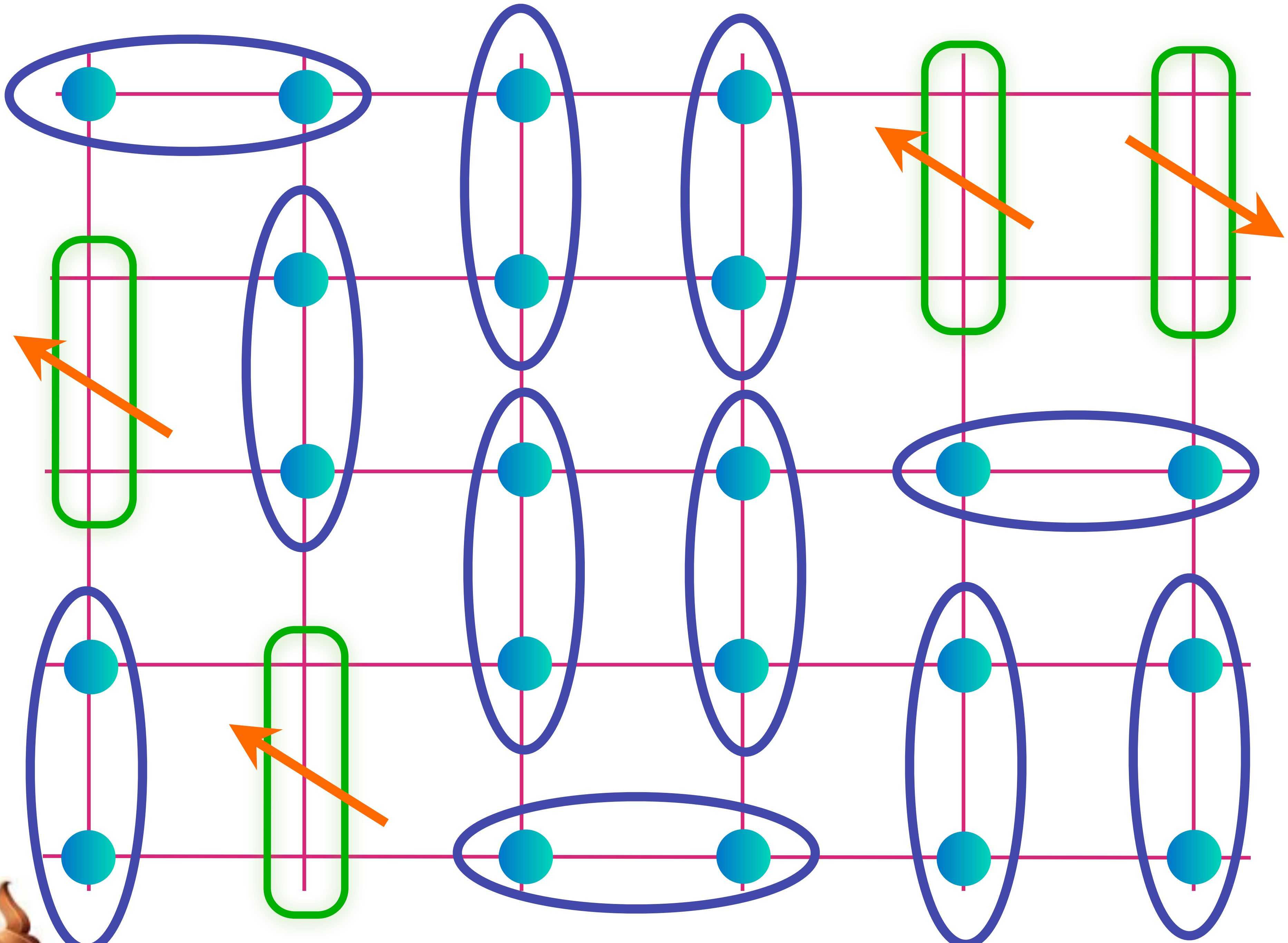
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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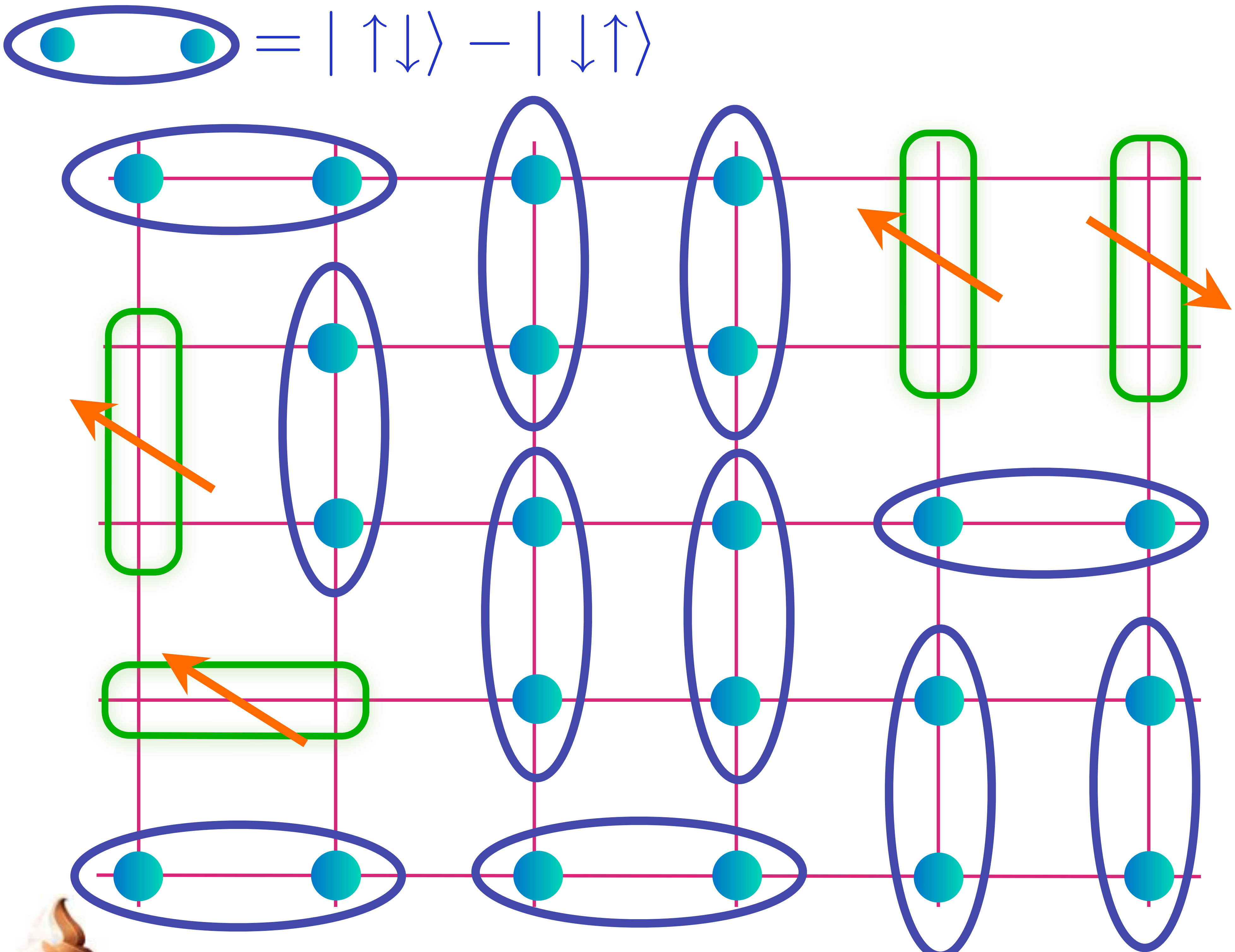

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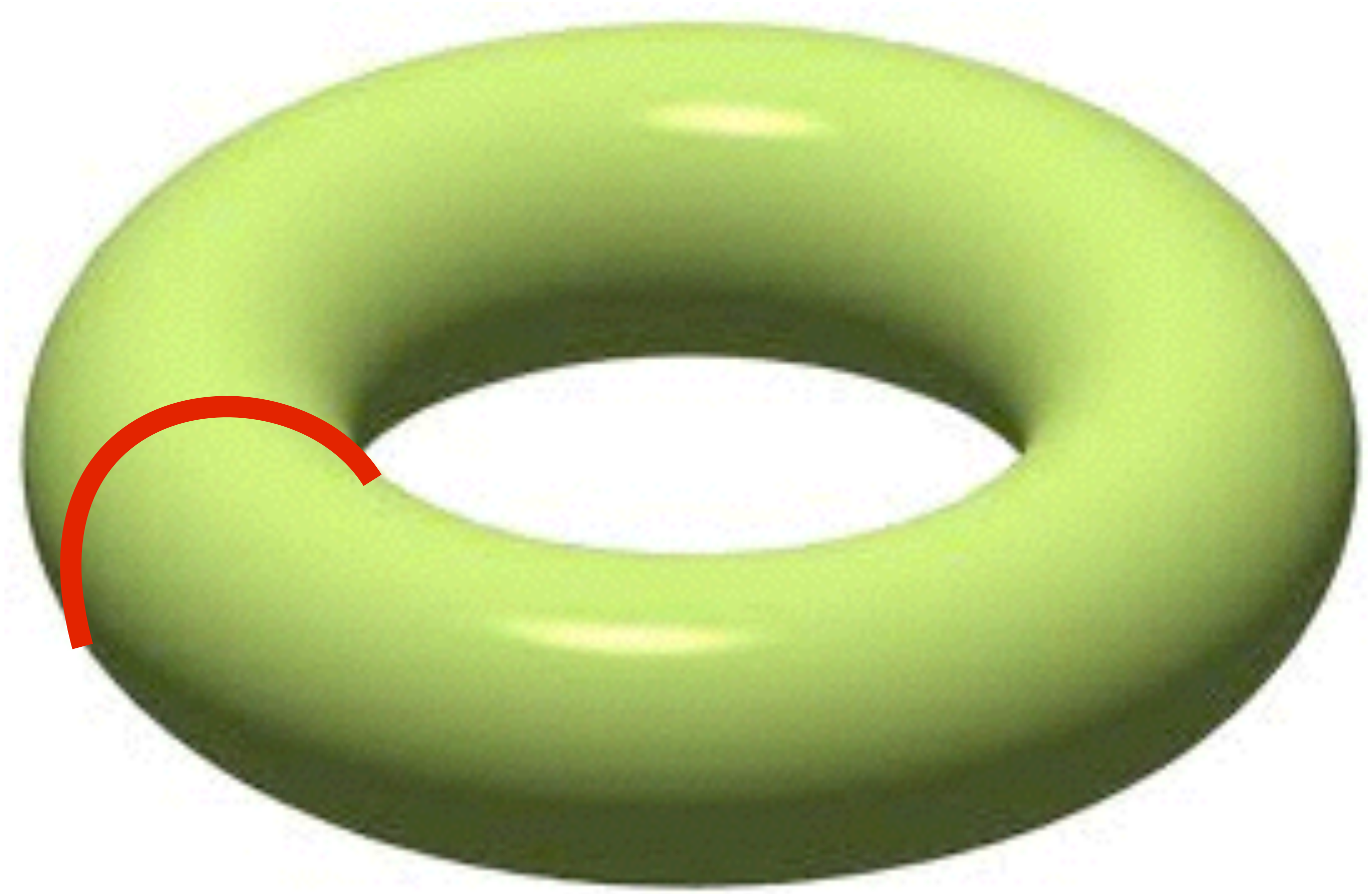
Topological order

Place pseudogap metal on a torus;

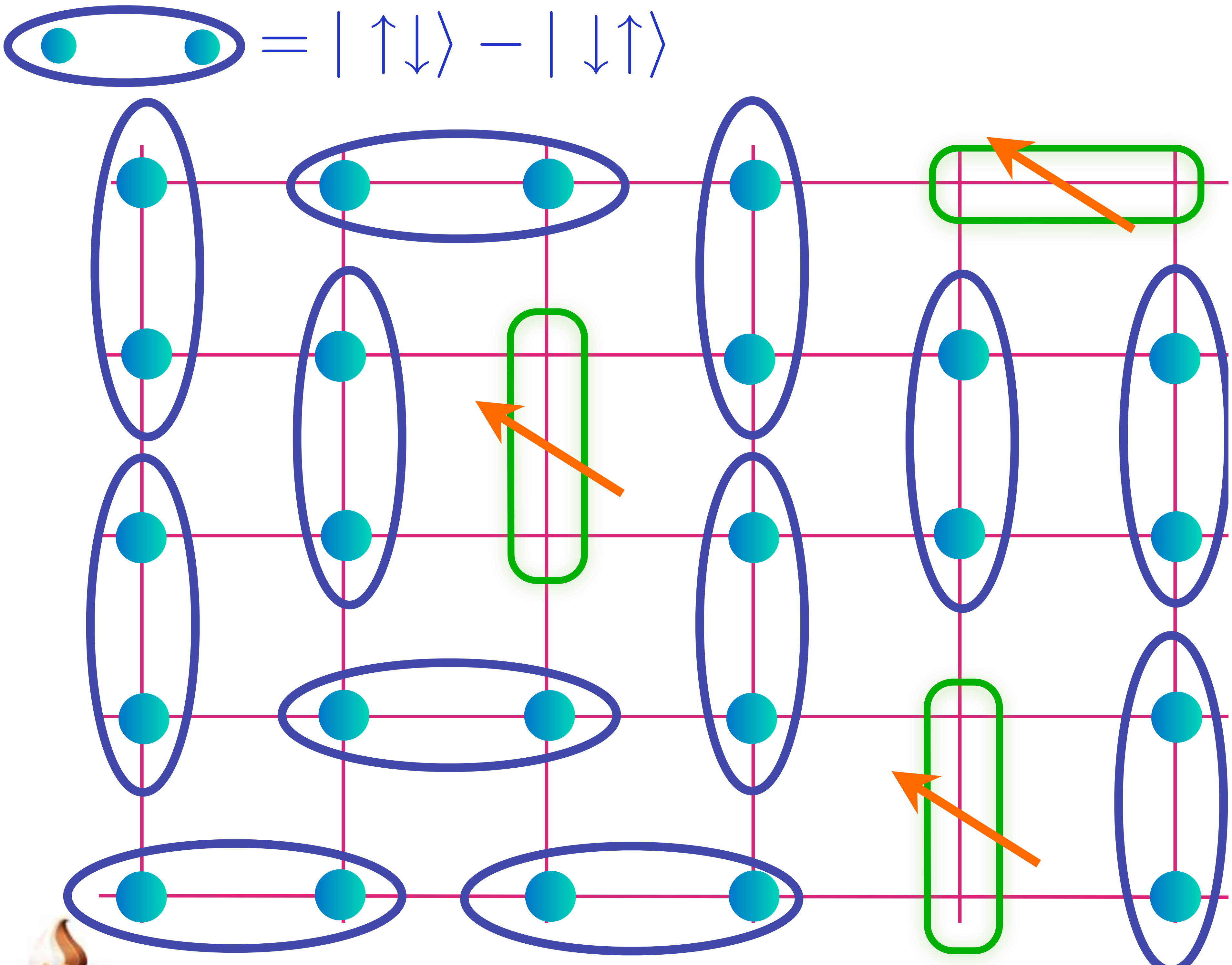
Topological order



Place pseudogap metal on a torus;

obtain "topological" states nearly degenerate with the ground state: change sign of every dimer across red line

Topological order



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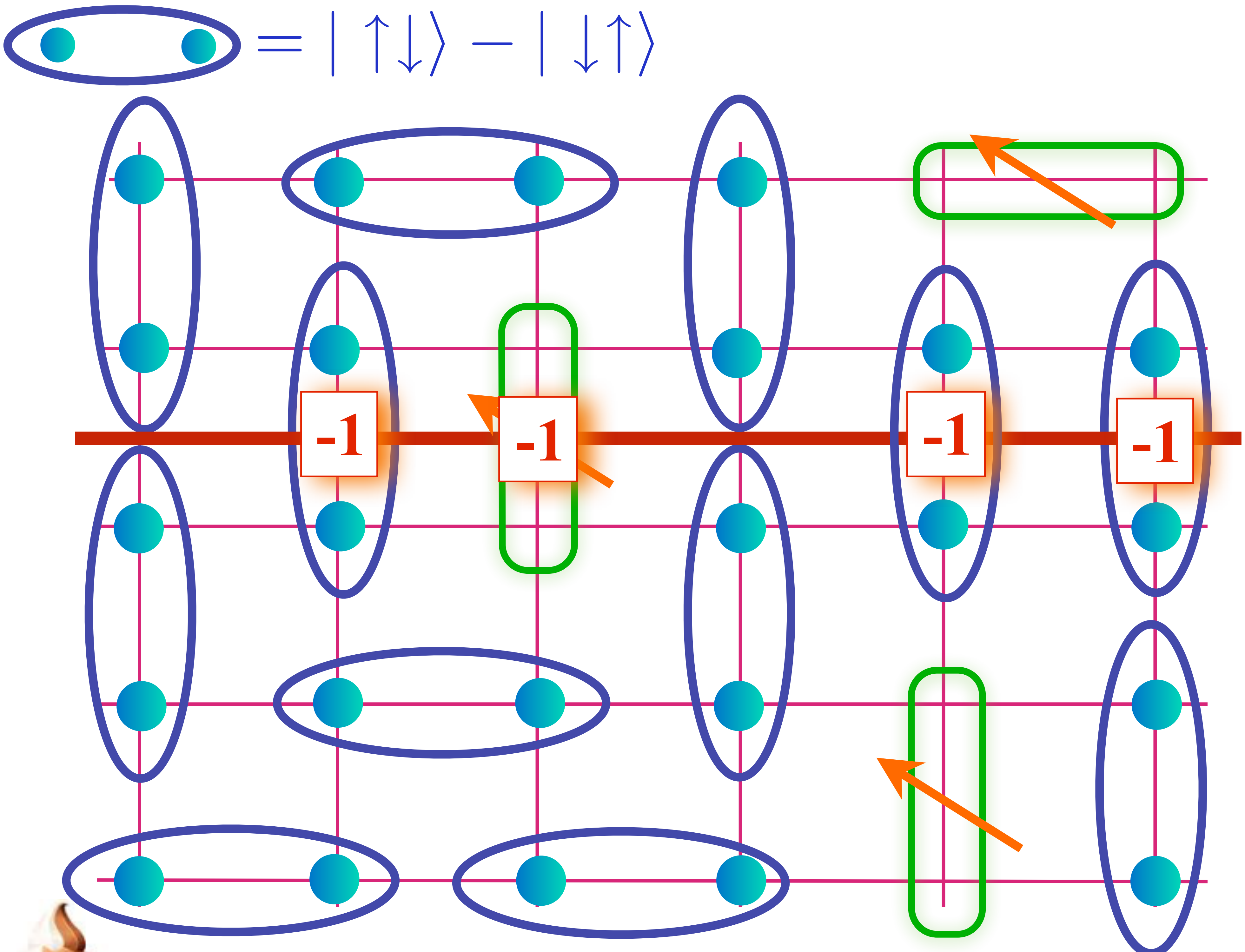

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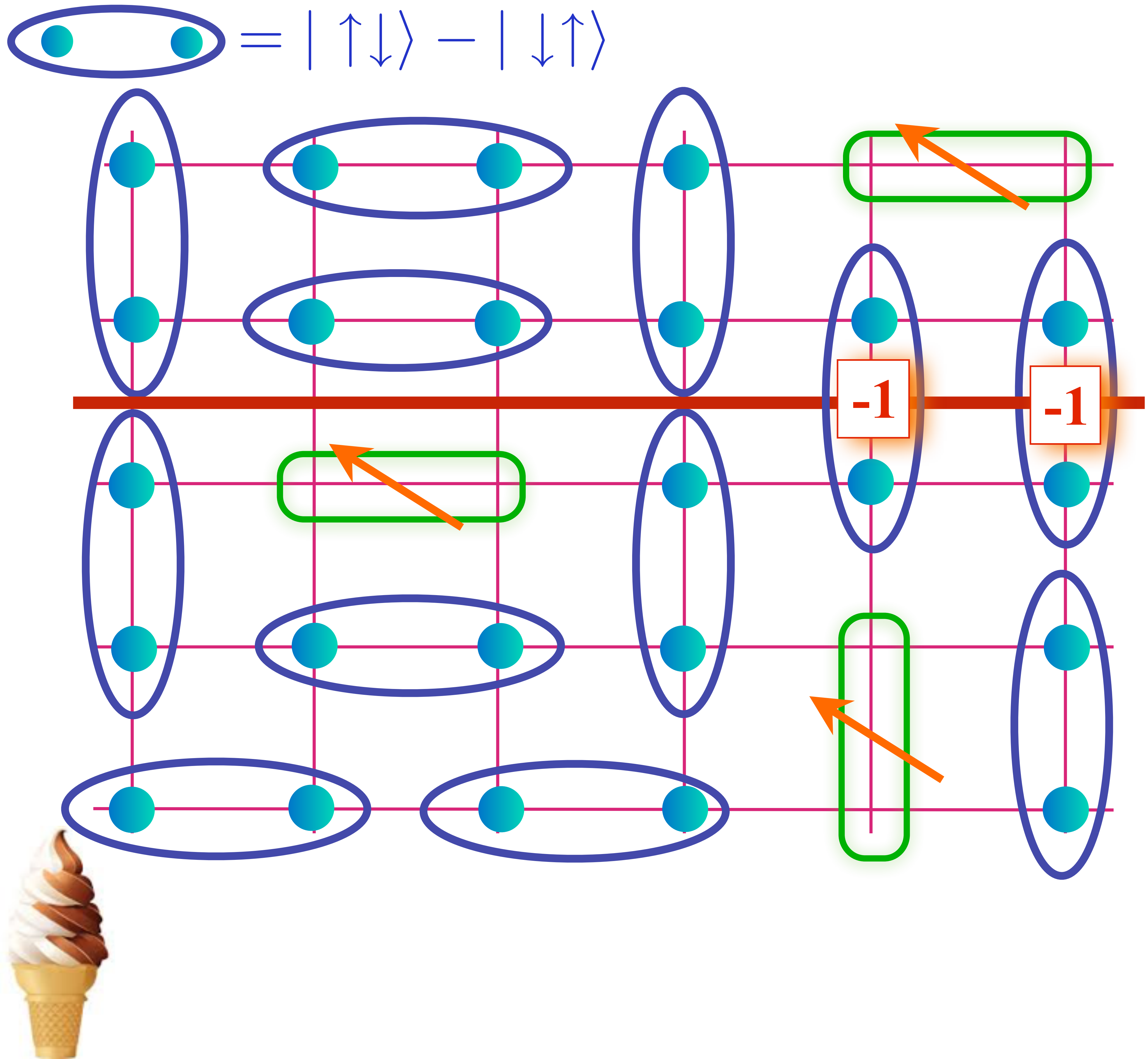

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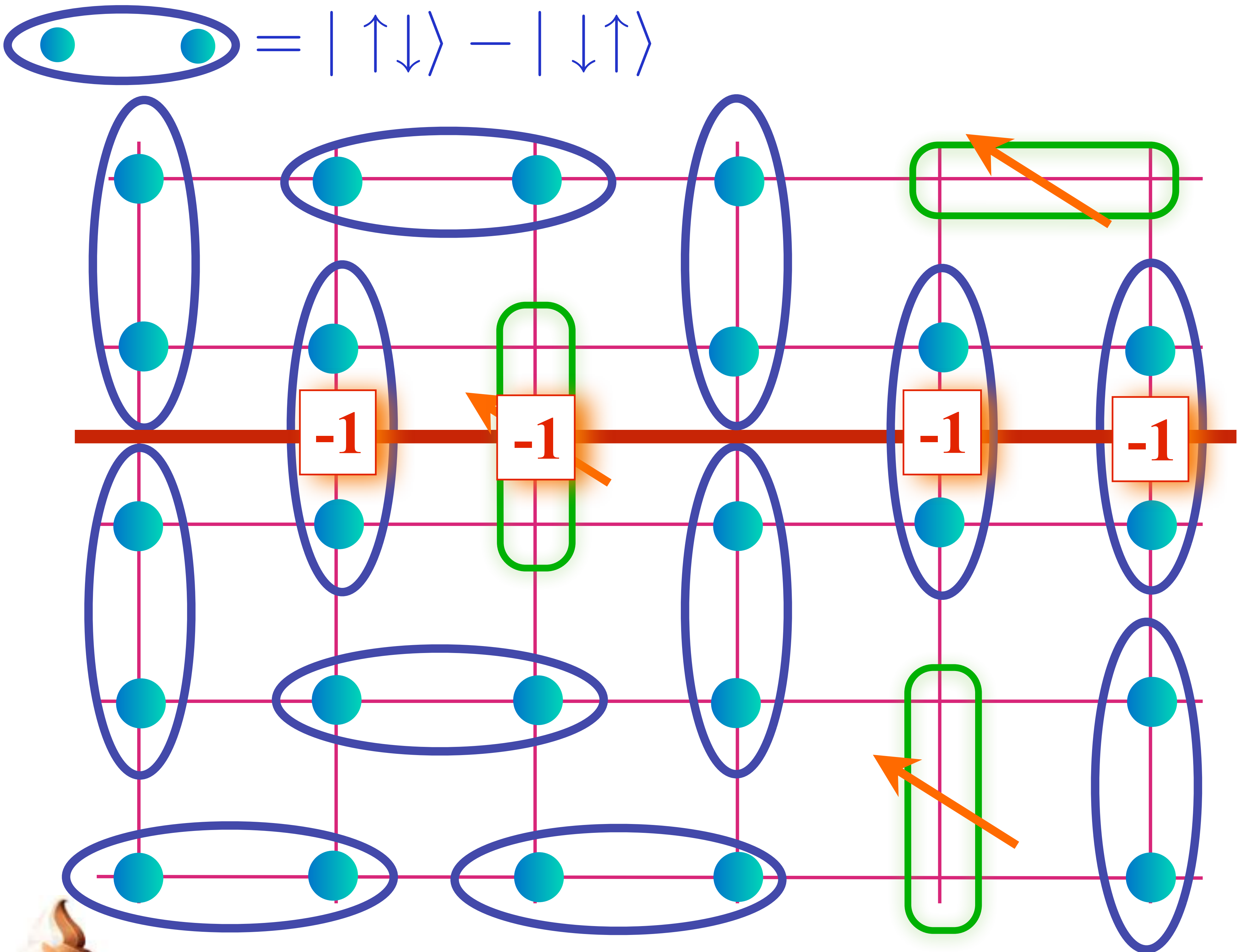

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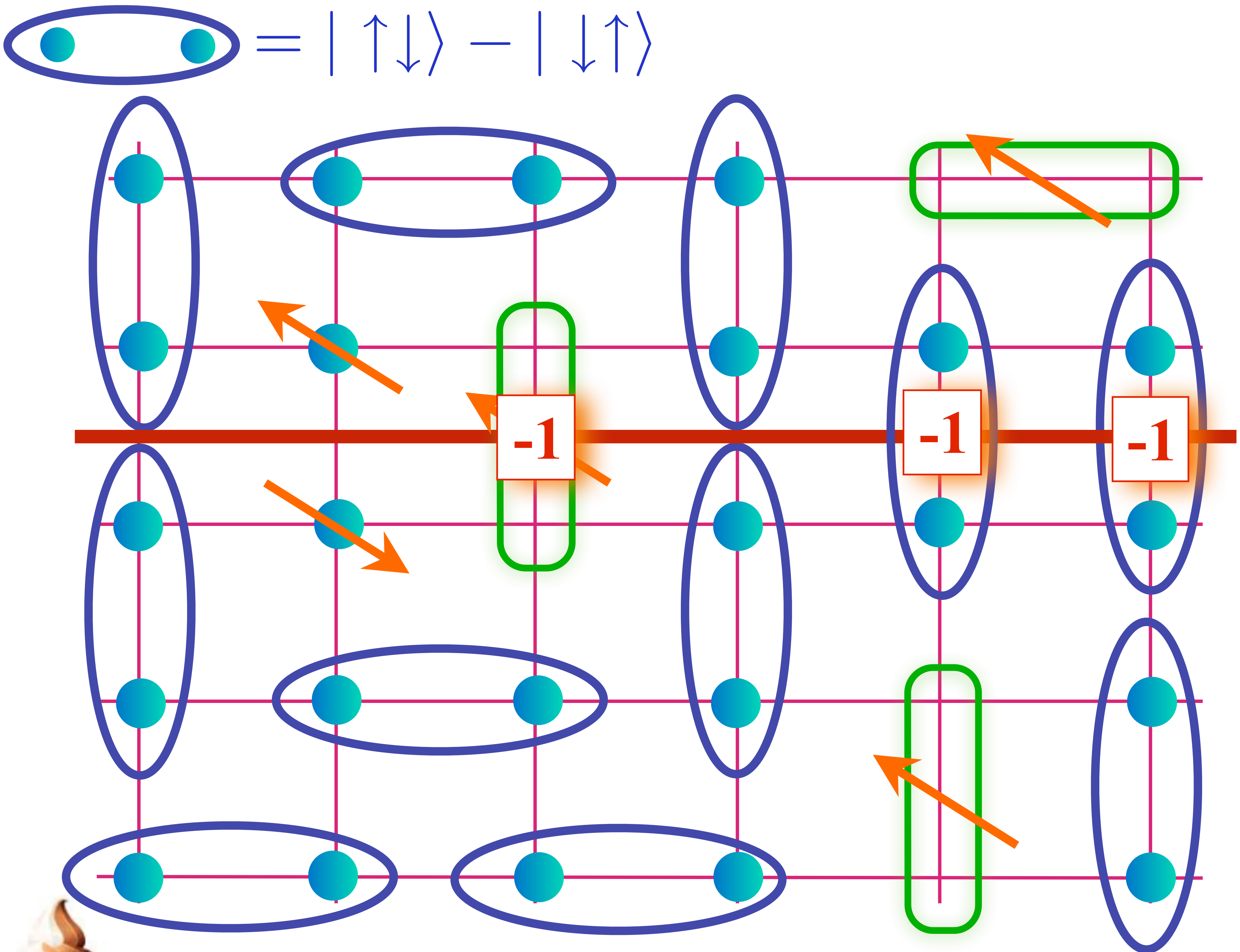
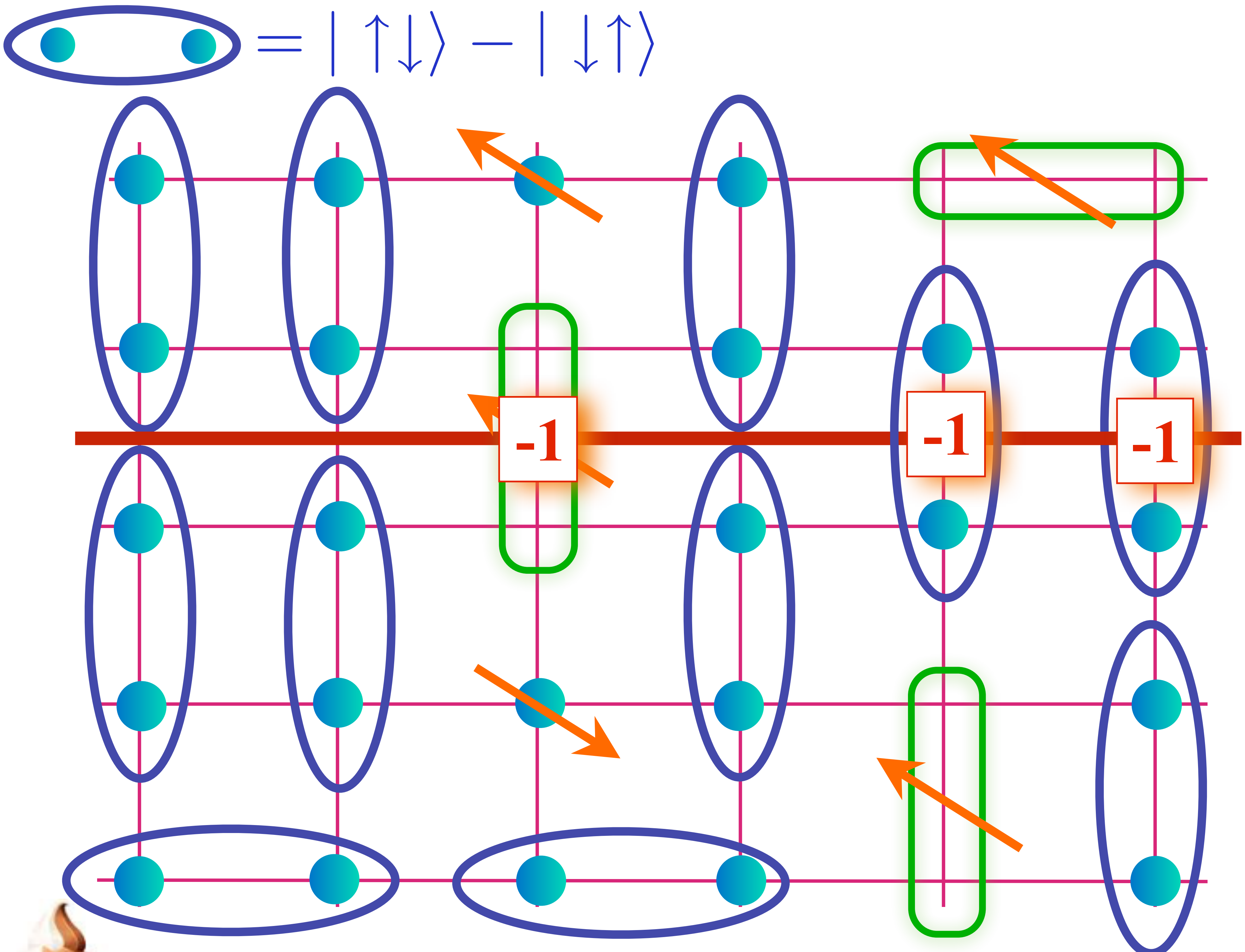


Place pseudogap metal on a torus;

to change overall sign, a pair of "spinons" have to be moved globally around a circumference of the torus

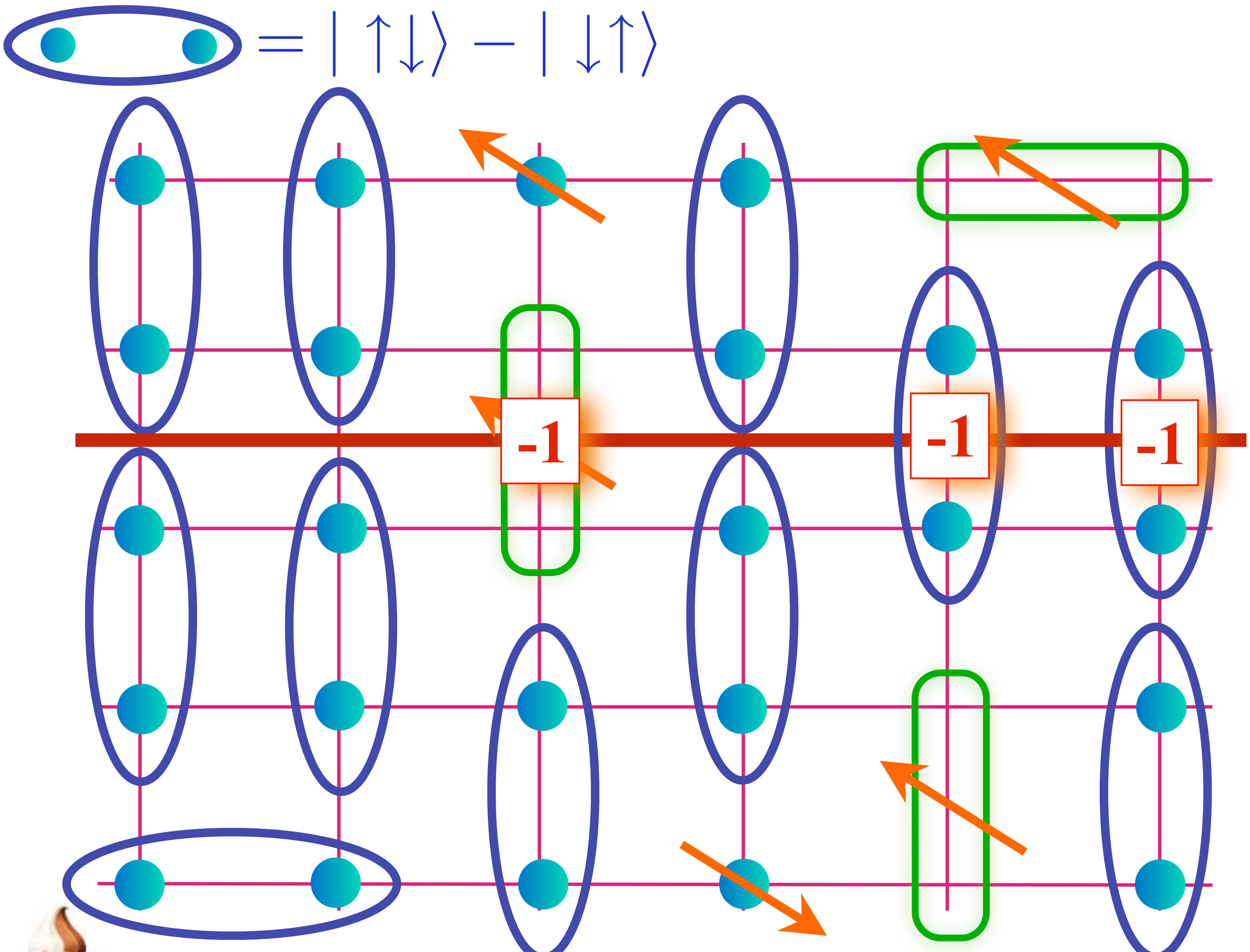
Topological order



Place pseudogap metal on a torus;

to change overall sign, a pair of "spinons" have to be moved globally around a circumference of the torus

Topological order



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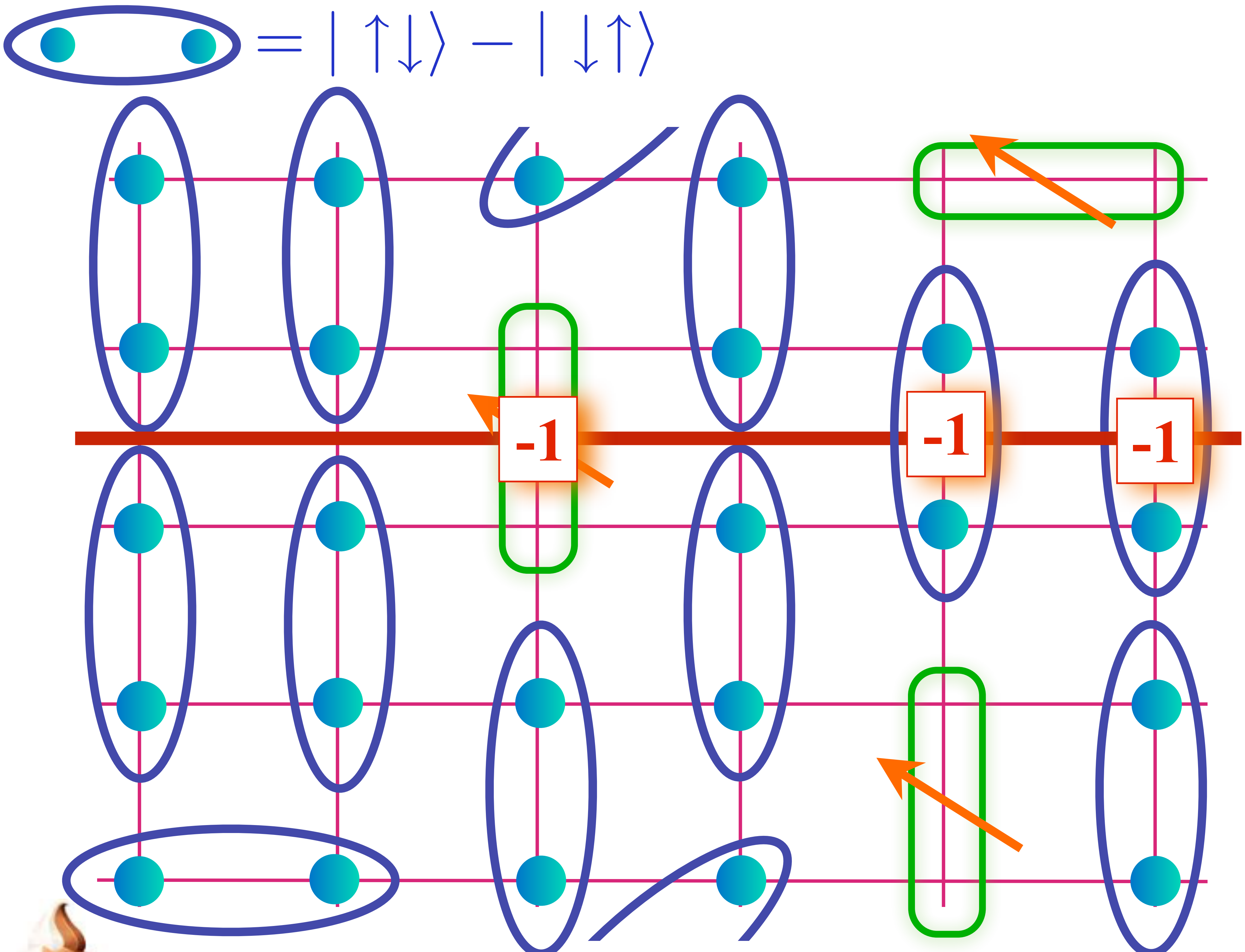
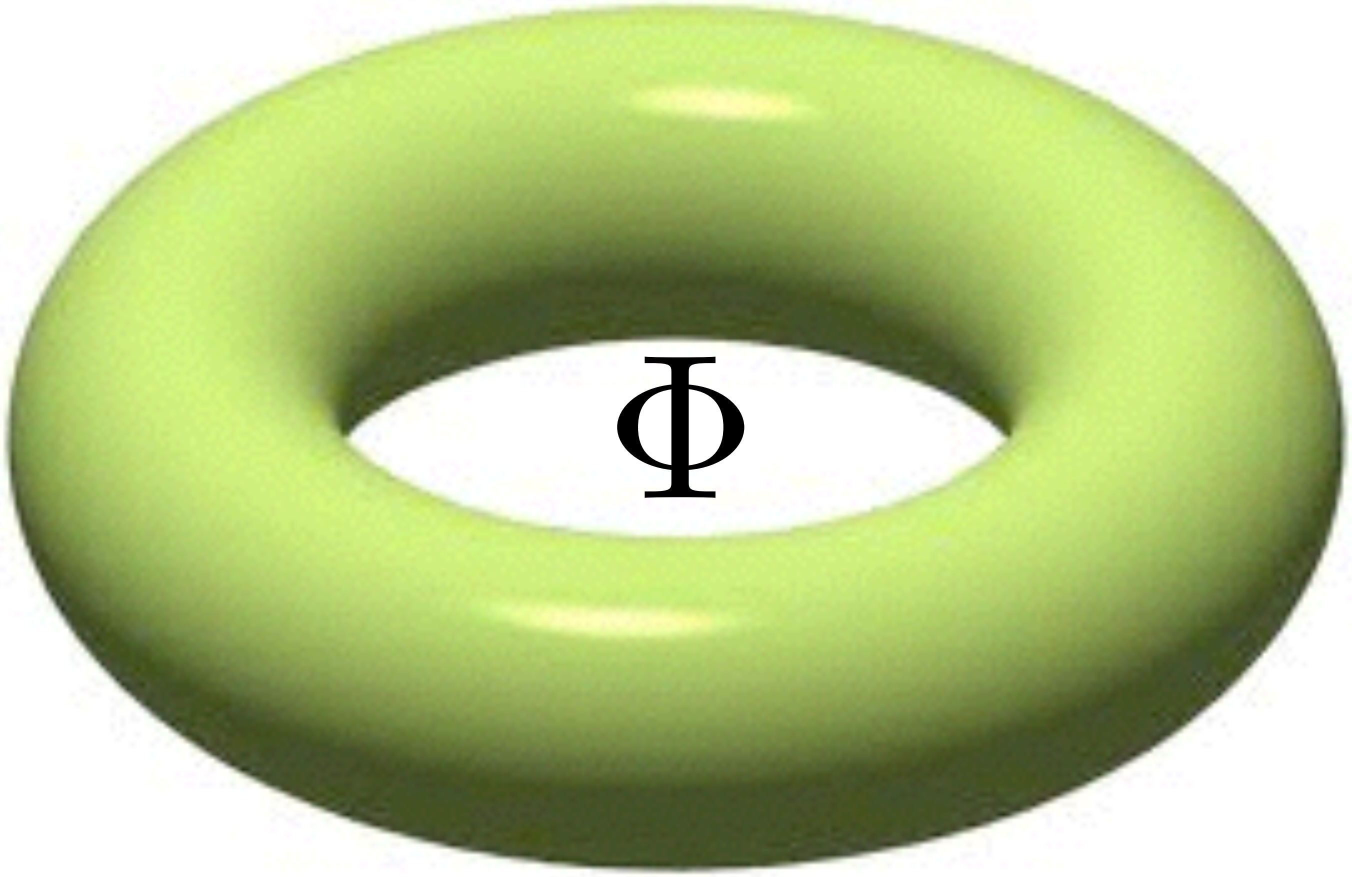


Place pseudogap metal on a torus;

to change overall sign, a pair of "spinons" have to be moved globally around a circumference of the torus

Fermi liquid (FL)

Topological argument for the area of Fermi surface

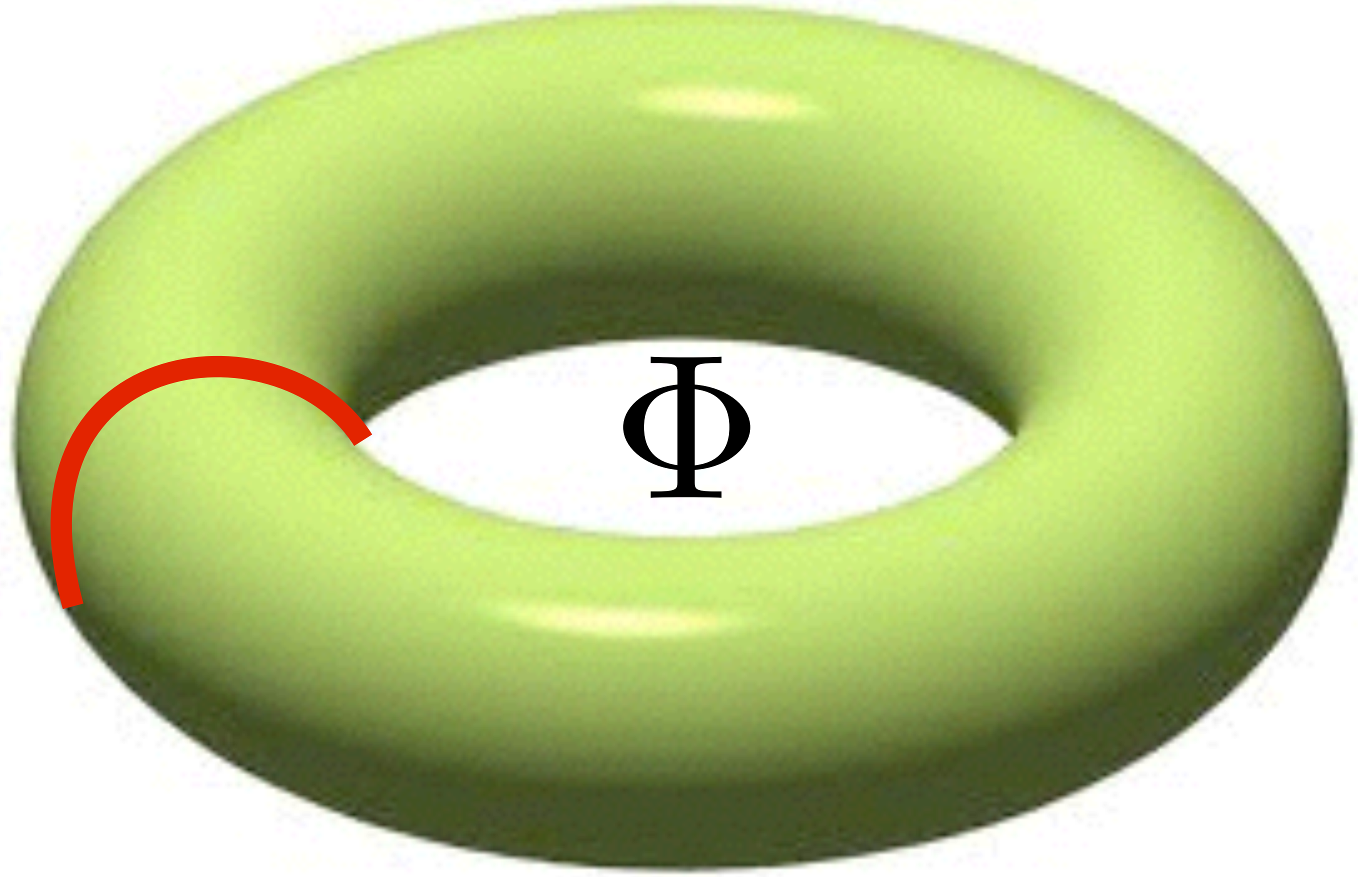


Put metal on a torus, adiabatically insert flux $\Phi = h/e$ through hole, and measure change in momentum. In a FL, we can assume the only low energy excitations are quasiparticles near the Fermi surface, and this leads to a non-perturbative proof of the Luttinger relation on the area enclosed by the Fermi surface.

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)

Fractionalized Fermi liquid (FL*)

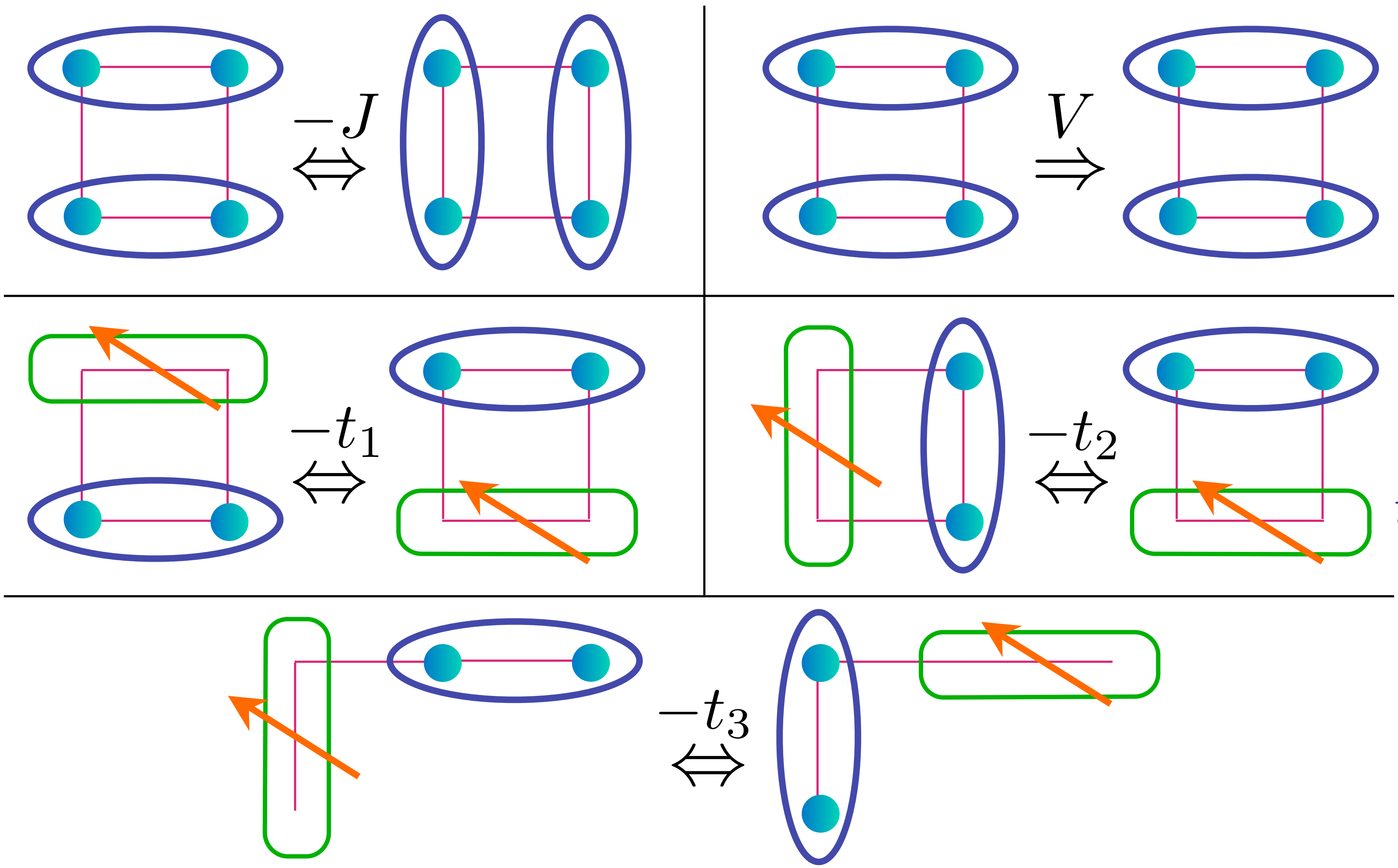
Topological argument for the area of Fermi surface



Violations of the Luttinger relation are possible in a fractionalized Fermi liquid (FL*) because there are "topological" low energy excitations associated with a flux of the emergent gauge field in the hole of the torus.

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

Quantum dimer model with bosonic and fermionic dimers

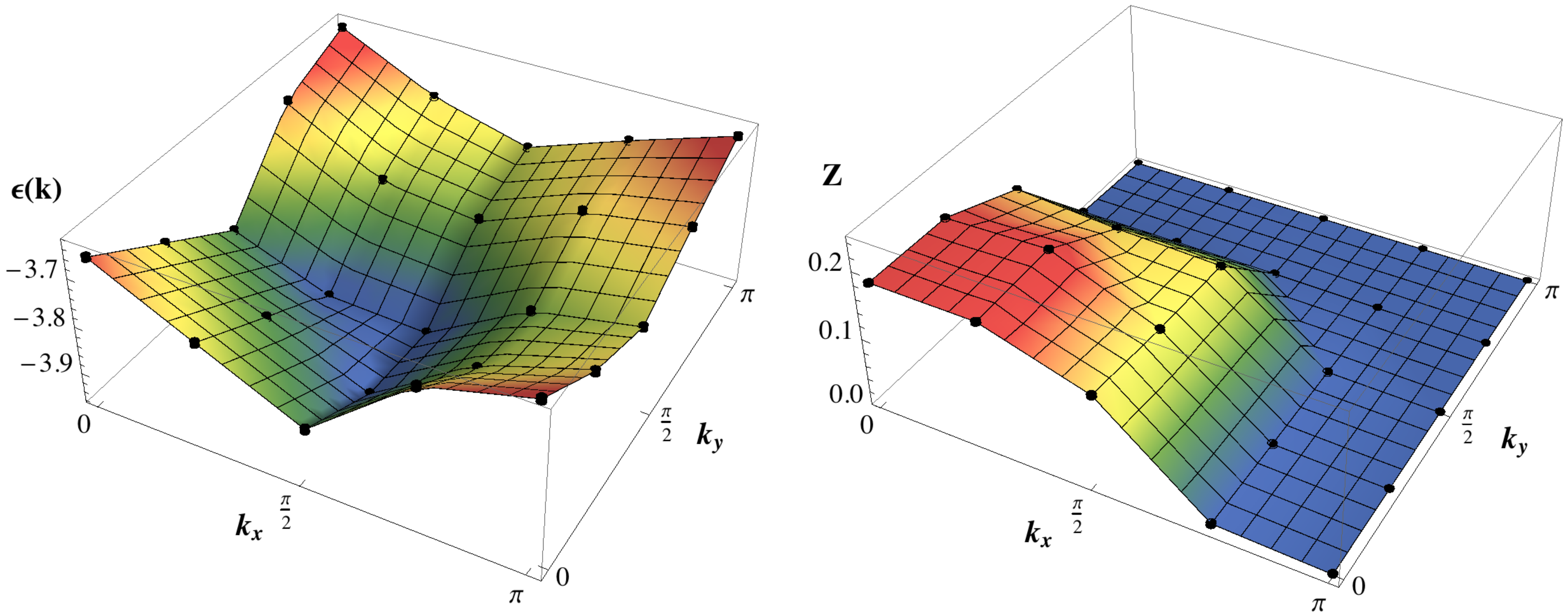


Connection to the t-t'-t''-J model:

$$t_1 = -(t + t')/2$$
$$t_2 = (t - t')/2$$
$$t_3 = -(t + t' + t'')/4$$

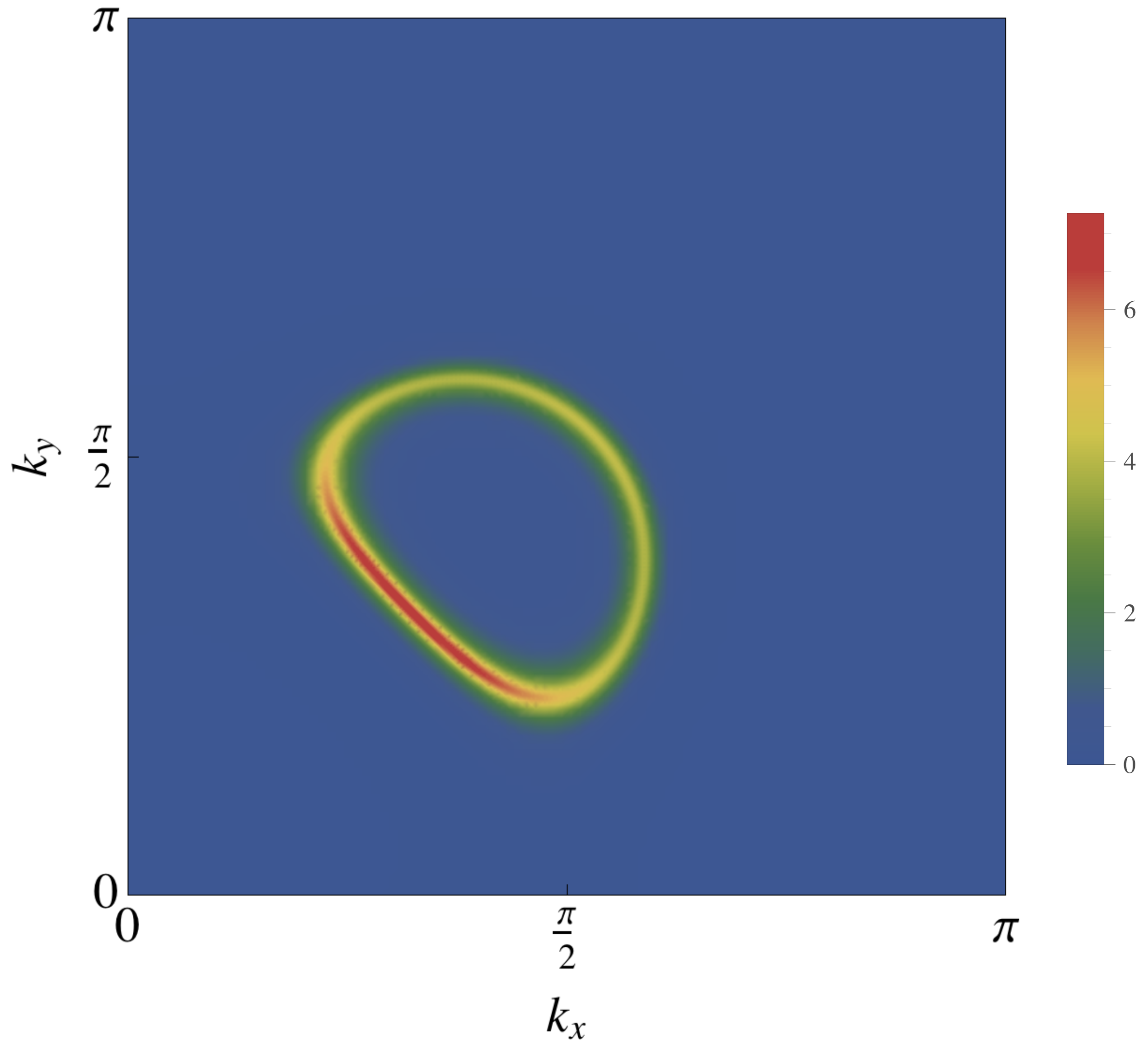
M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978

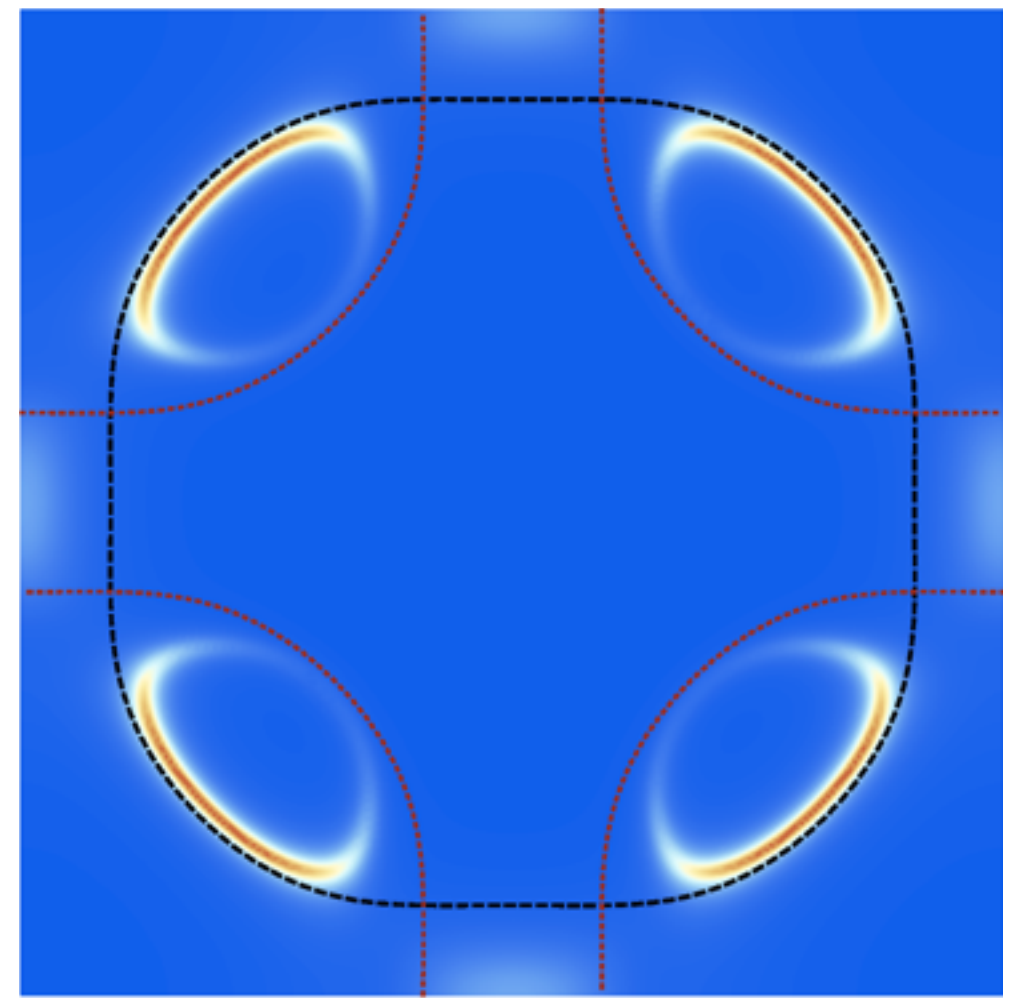
Quantum dimer model with bosonic and fermionic dimers



Dispersion and quasiparticle residue of a single fermionic dimer for $J = V = 1$, and hopping parameters obtained from the t-J model for the cuprates, $t_1 = -1.05$, $t_2 = 1.95$ and $t_3 = -0.6$, on a 8×8 lattice.

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978



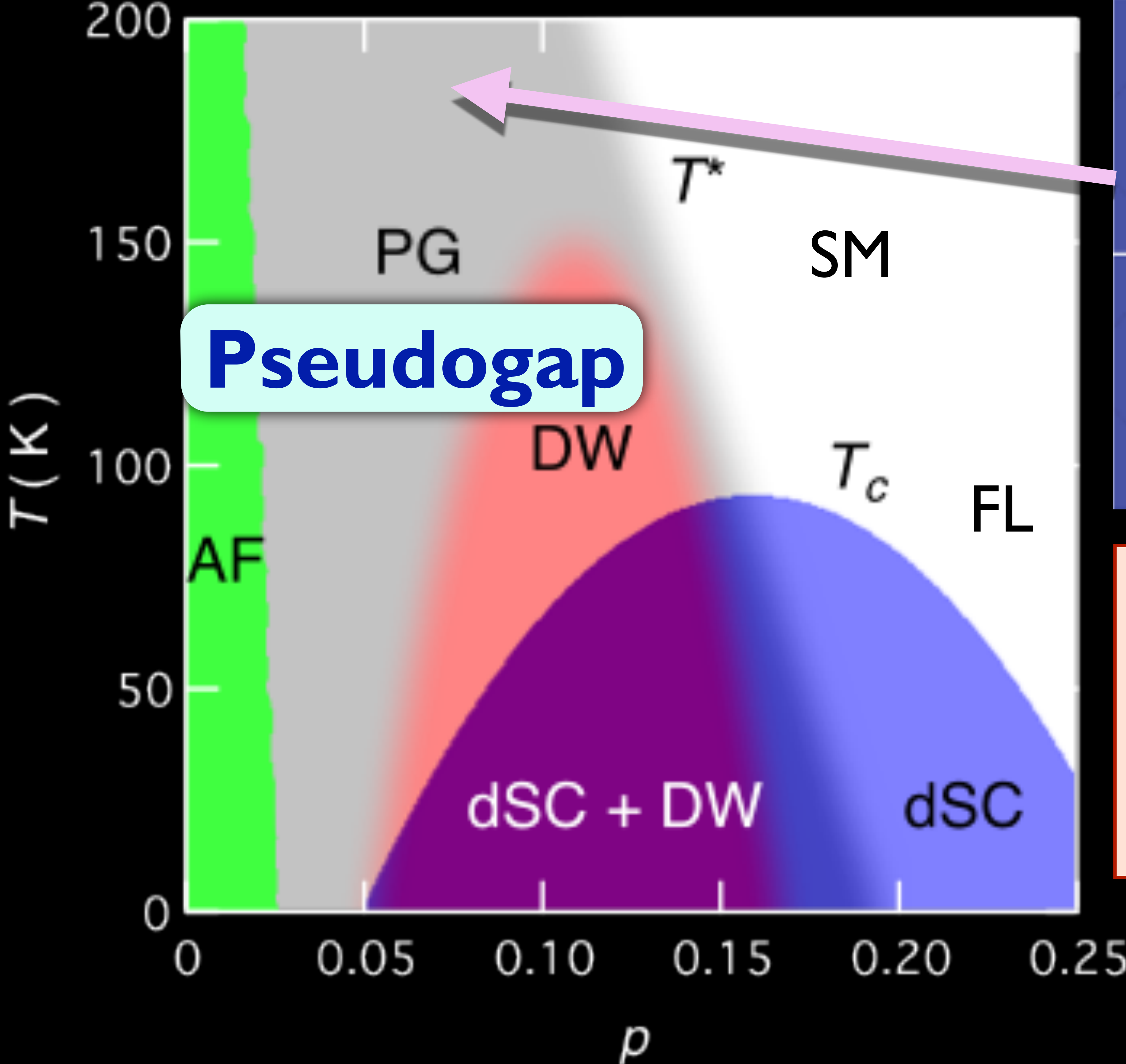


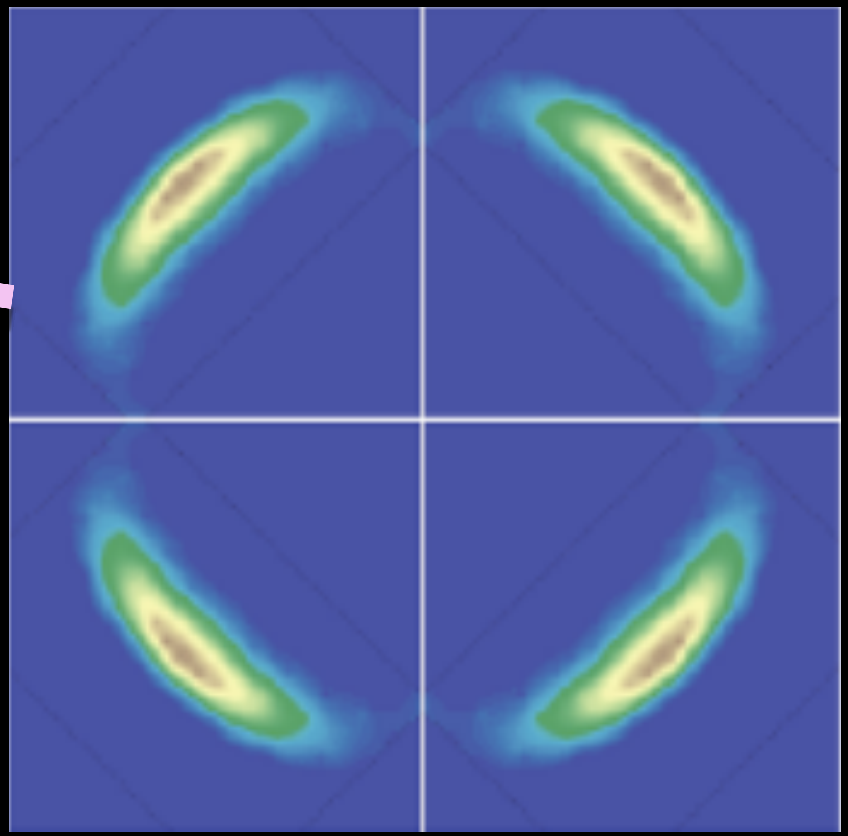
M. Punk, A. Allais, and S. Sachdev,
arXiv:1501.00978

Y. Qi and S. Sachdev,
Phys. Rev. B **81**, 115129 (2010)

"Back side" of Fermi surface is suppressed for observables which change electron number in the CuO_2 layer

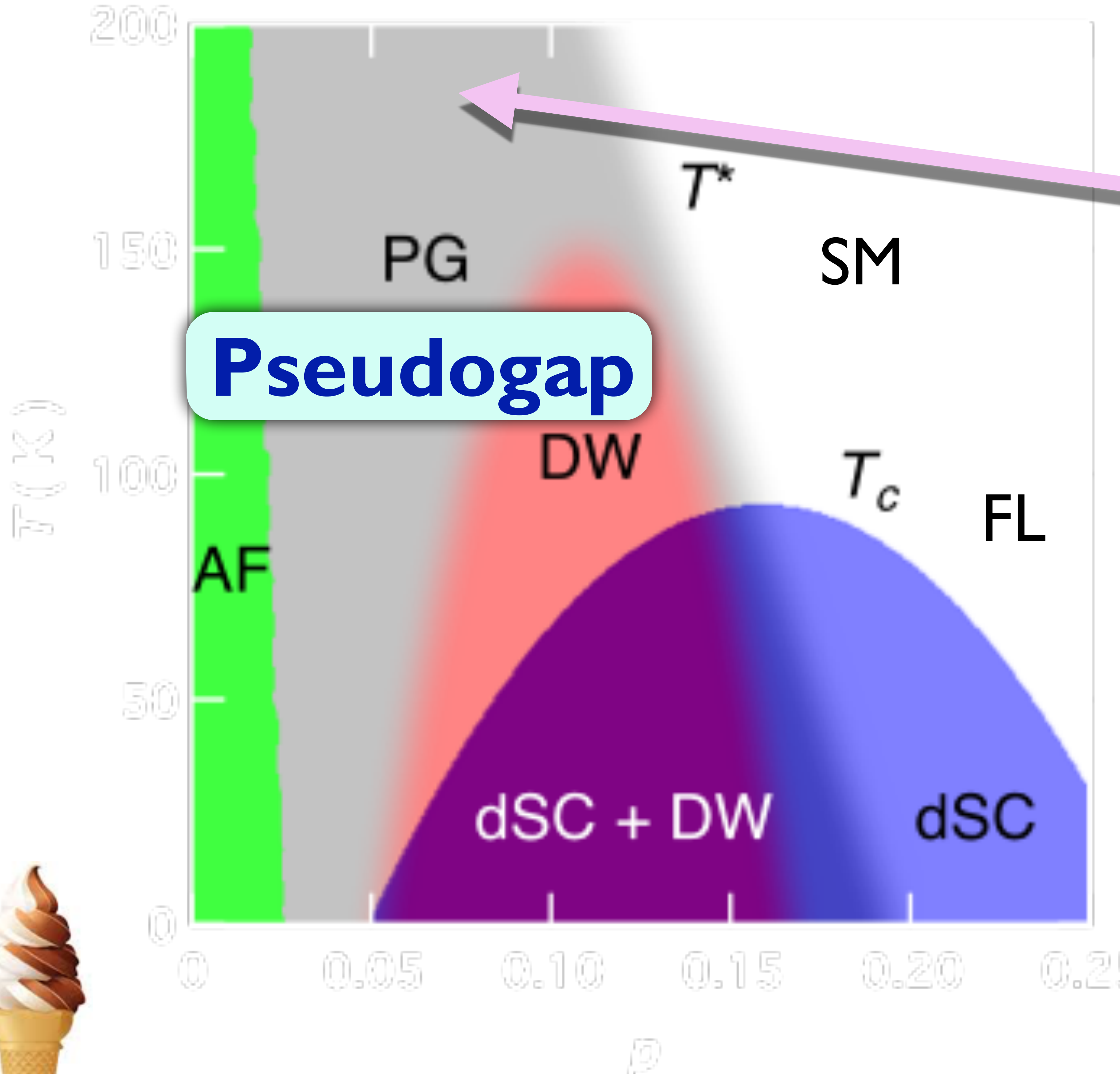
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

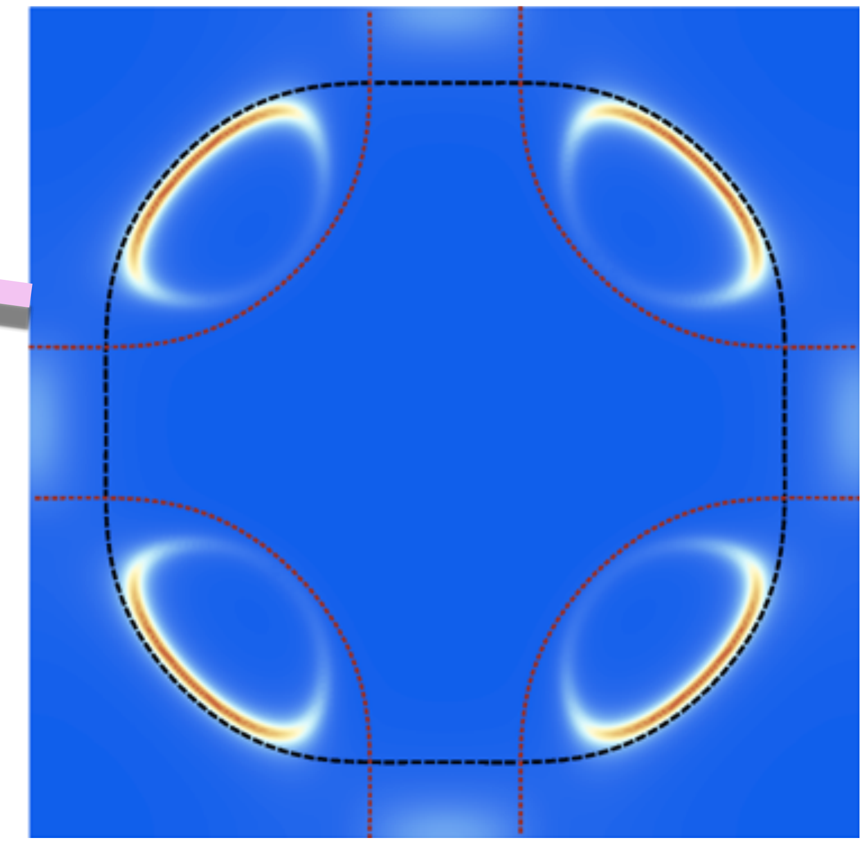




"Fermi arcs"
at
low p

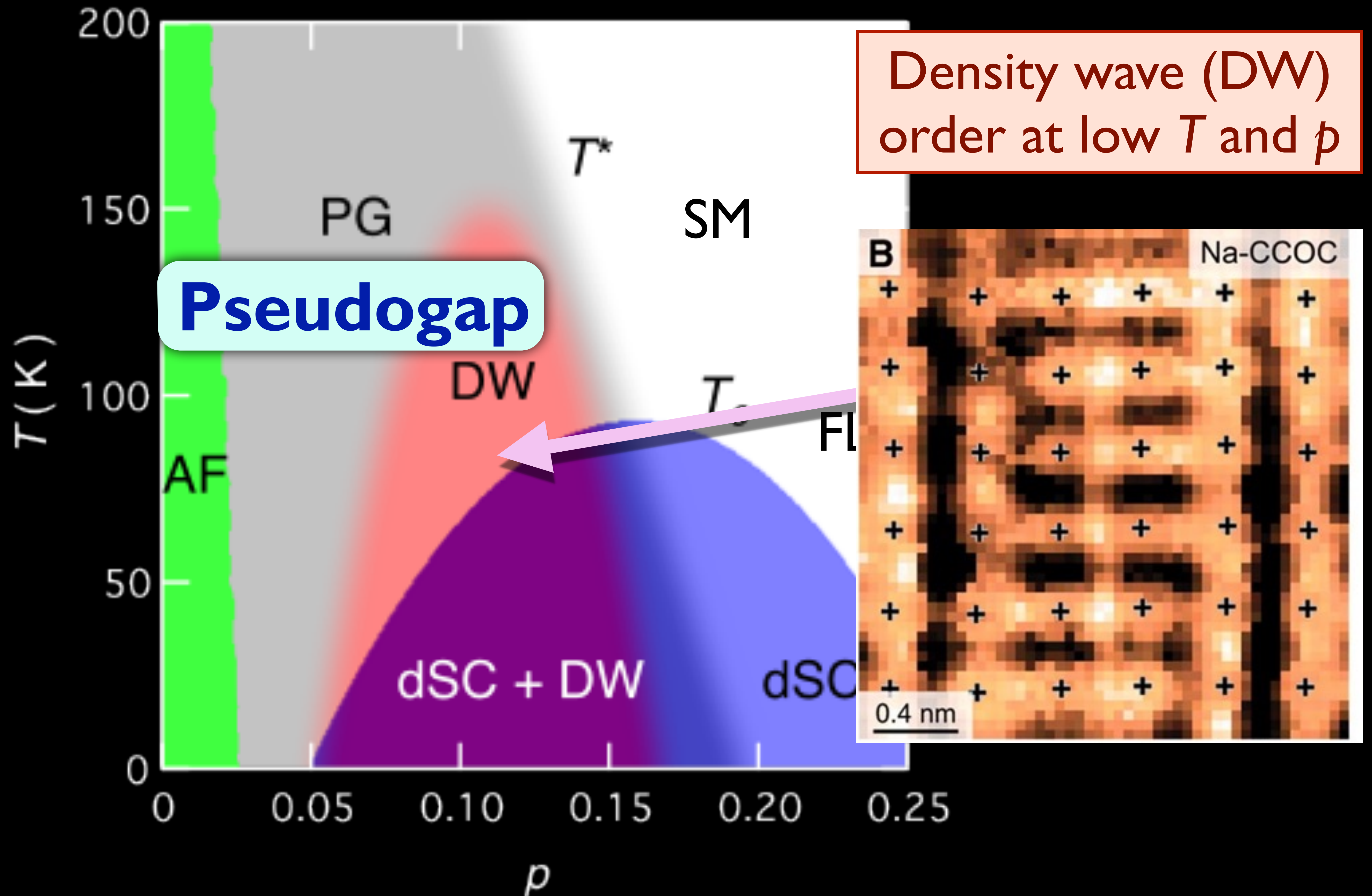
Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978





A new metal —
a fractionalized
Fermi liquid (FL*)
— with electron-
like quasiparticles
on a Fermi surface
of size p ?

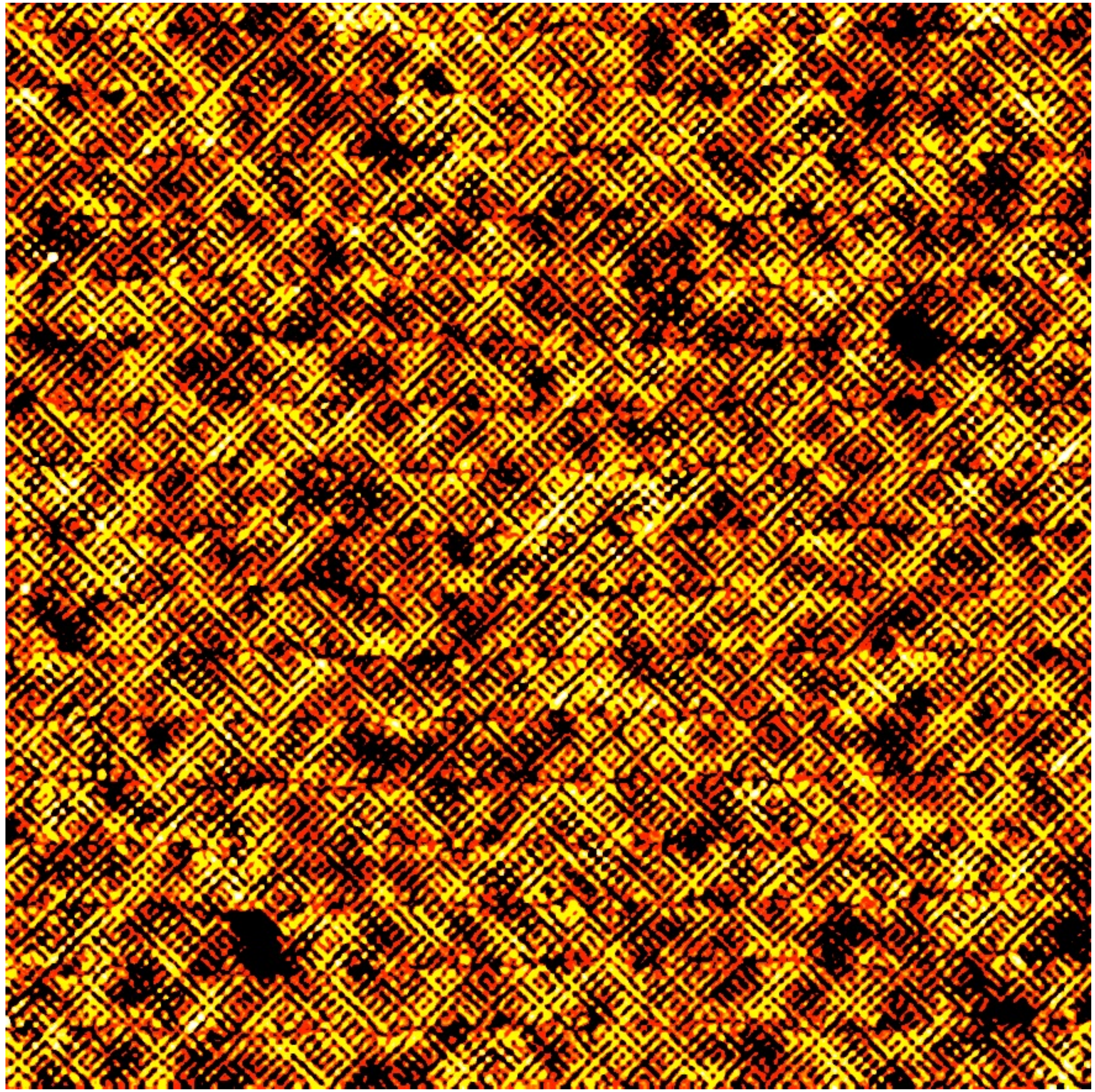
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

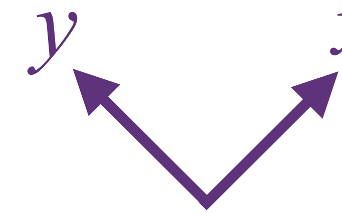


See also
C. Howald, H. Eisaki, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. B **67**, 014533 (2003);

M. Vershinin, S. Misra, S. Ono, Y. Abe, Yoichi Ando, and A. Yazdani, Science **303**, 1995 (2004).

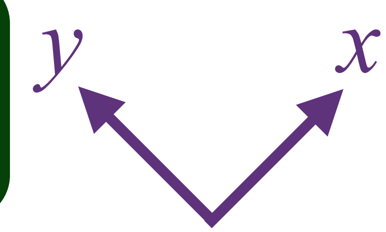
W. D. Wise, M. C. Boyer, K. Chatterjee, T. Kondo, T. Takeuchi, H. Ikuta, Y. Wang, and E. W. Hudson, Nature Phys. **4**, 696 (2008).





"R-map" of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.

Disordered uni-directional charge density waves ("stripes") with wavelength ≈ 4 lattice sites ?



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C. Howald, H. Eisaki, N. Kaneko, M. Greven, and A. Kapitulnik, Phys. Rev. B **67**, 014533 (2003);

M. Vershinin, S. Misra, S. Ono, Y. Abe, Yoichi Ando, and A. Yazdani, Science **303**, 1995 (2004).

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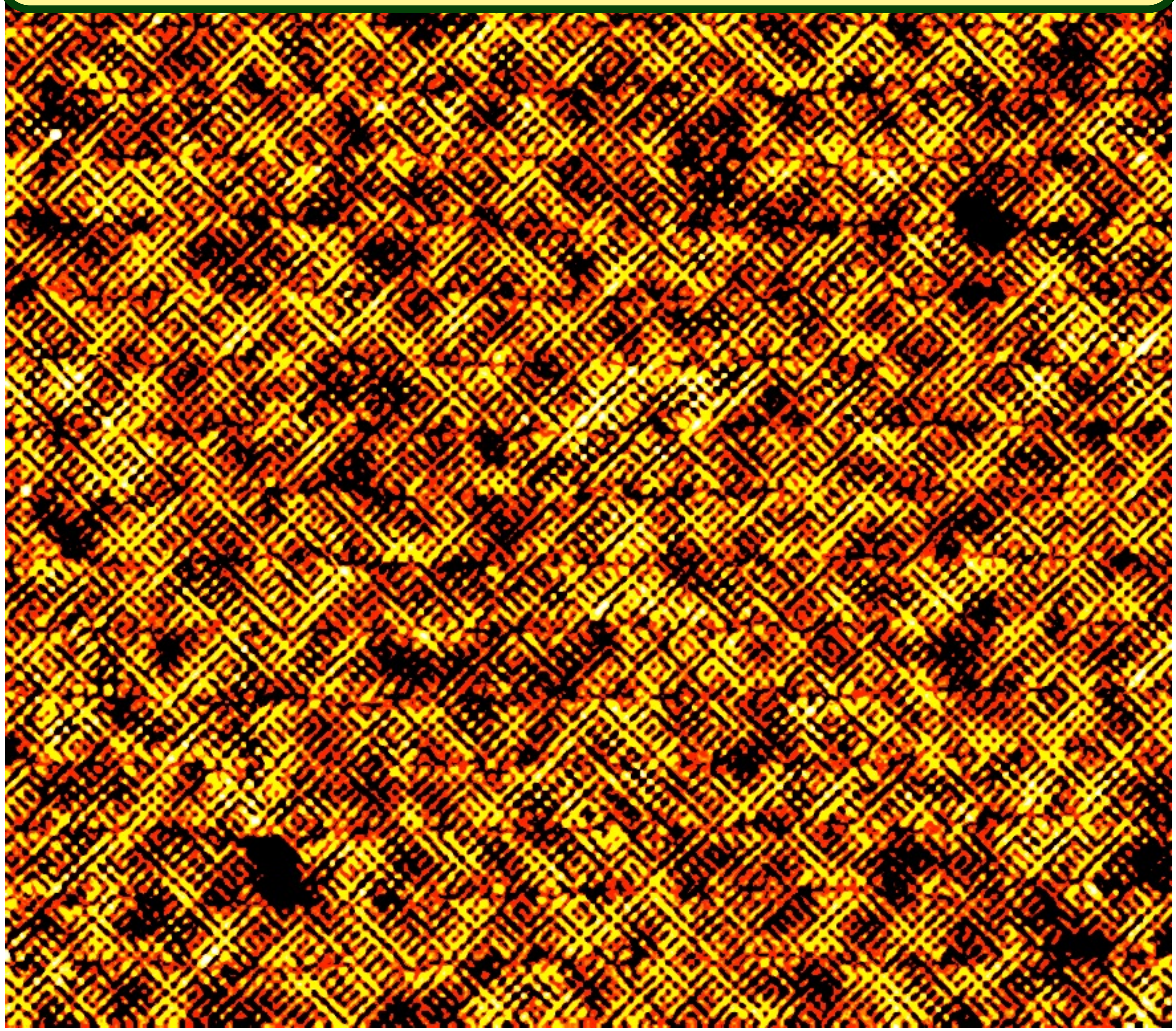

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Unconventional density wave (DW) : Bose condensation of particle-hole pairs

$$\left\langle c_\alpha^\dagger(\mathbf{r}_1) c_\alpha(\mathbf{r}_2) \right\rangle$$

$$= \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q}\cdot(\mathbf{r}_1 + \mathbf{r}_2)/2}$$

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Unconventional density wave (DW) :
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Crucial "center-of-mass" co-ordinate.
(Not used in previous work)
Simplifies action of time-reversal

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Unconventional density wave (DW) : Bose condensation of particle-hole pairs

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Density wave **form factor** (internal particle-hole pair wavefunction)

$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2 k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

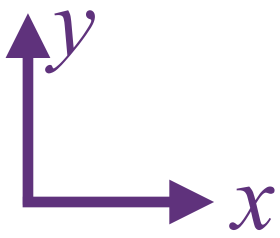
Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand (using reflection symmetry for $\mathbf{Q}$ along axes or diagonals)

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

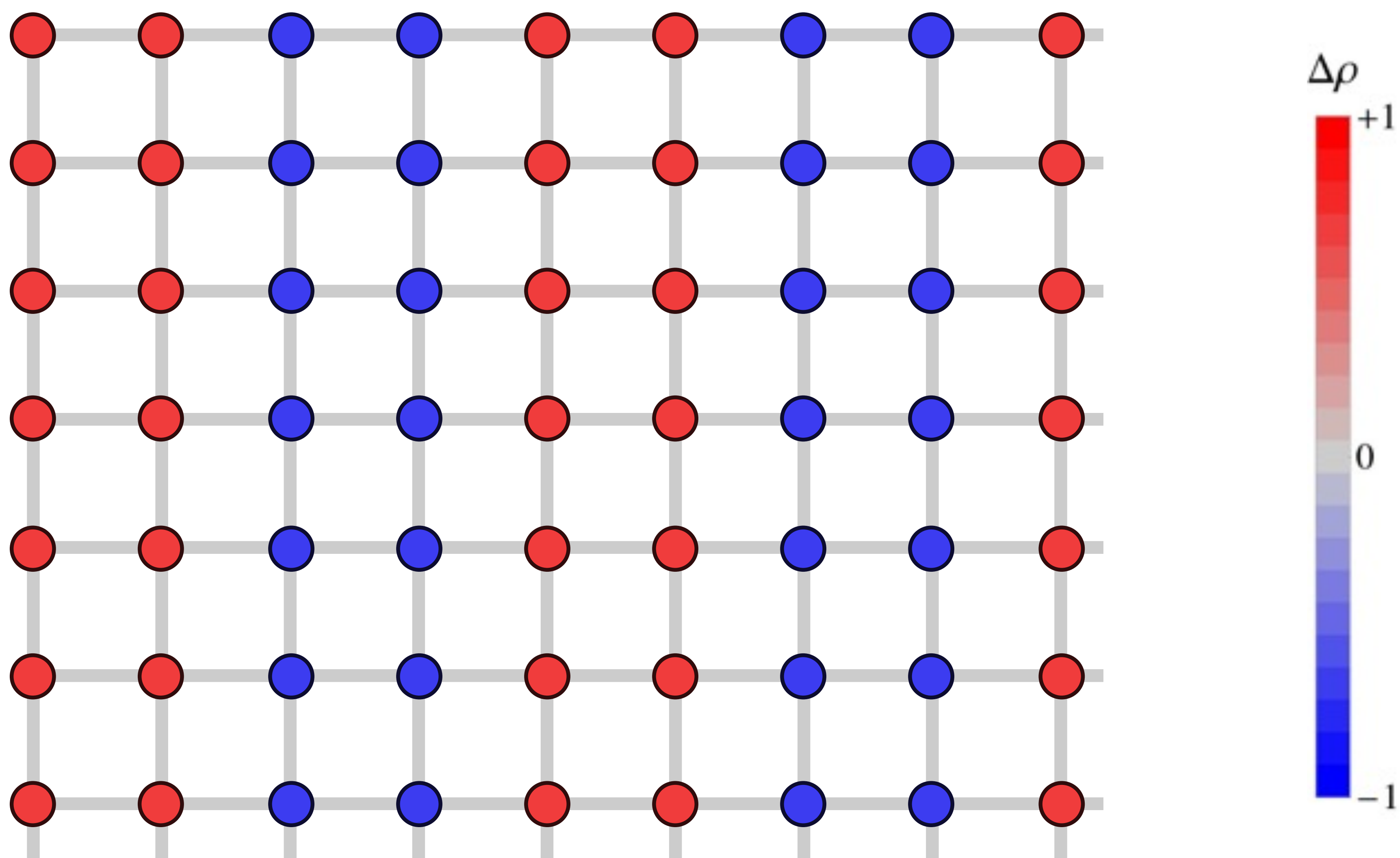
Conventional CDW order: s-form factor



Plot of $P_{ij} = \left\langle c_{i\alpha}^{\dagger} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

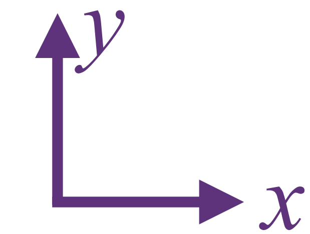
$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



$\mathbf{Q} = (\pi/2, 0)$

Unconventional DW order: s'-form factor



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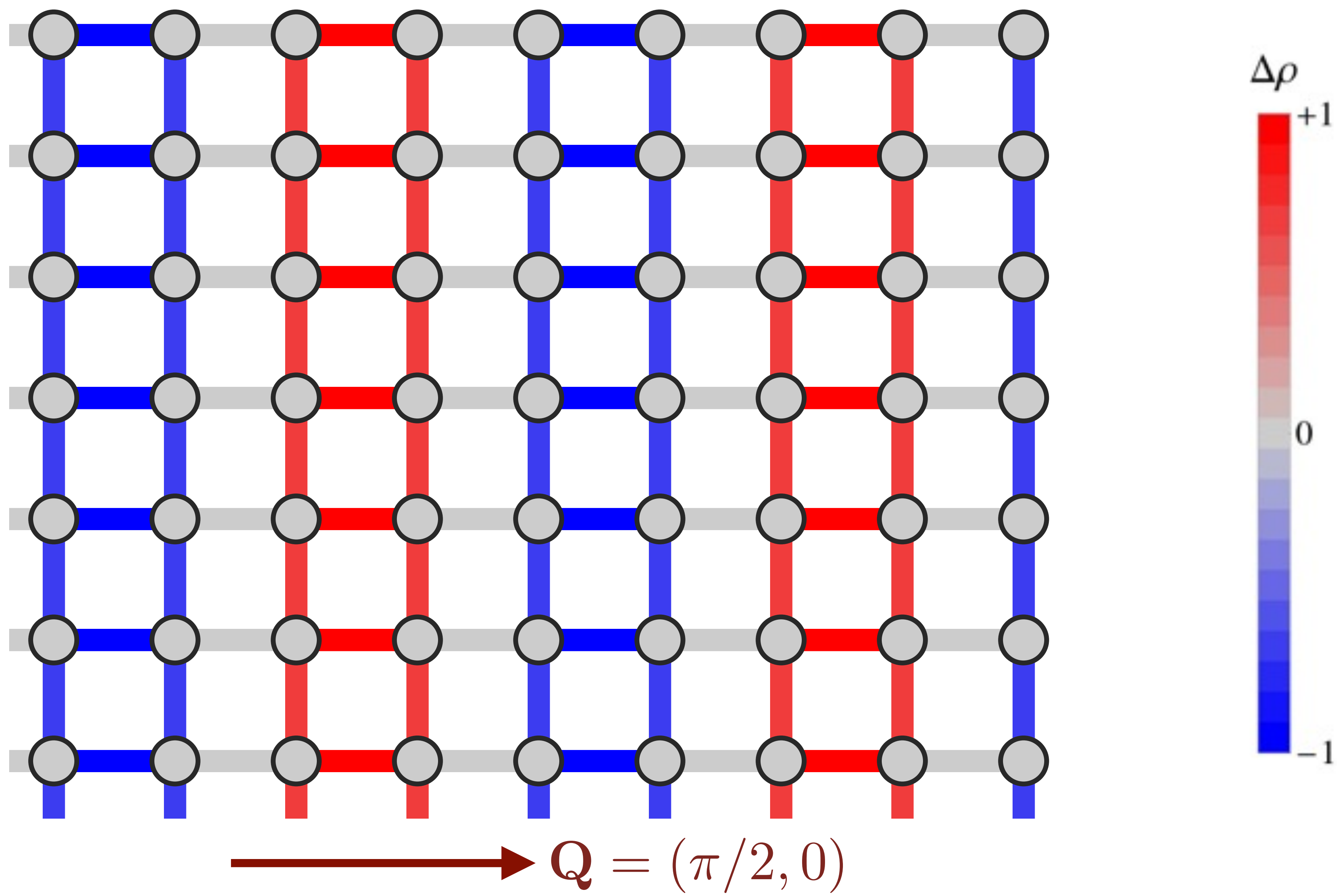
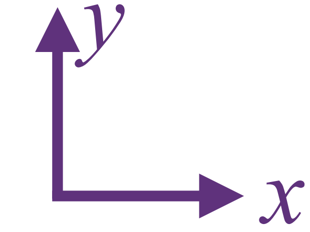
$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[\cos(k_x) + \cos(k_y)\right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



Unconventional DW order: $s + s'$-form factor



Plot of $P_{ij} = \left\langle c^\dagger_{i\alpha} c_{j\alpha} \right\rangle$ for $i = j$, and i, j nearest neighbors.

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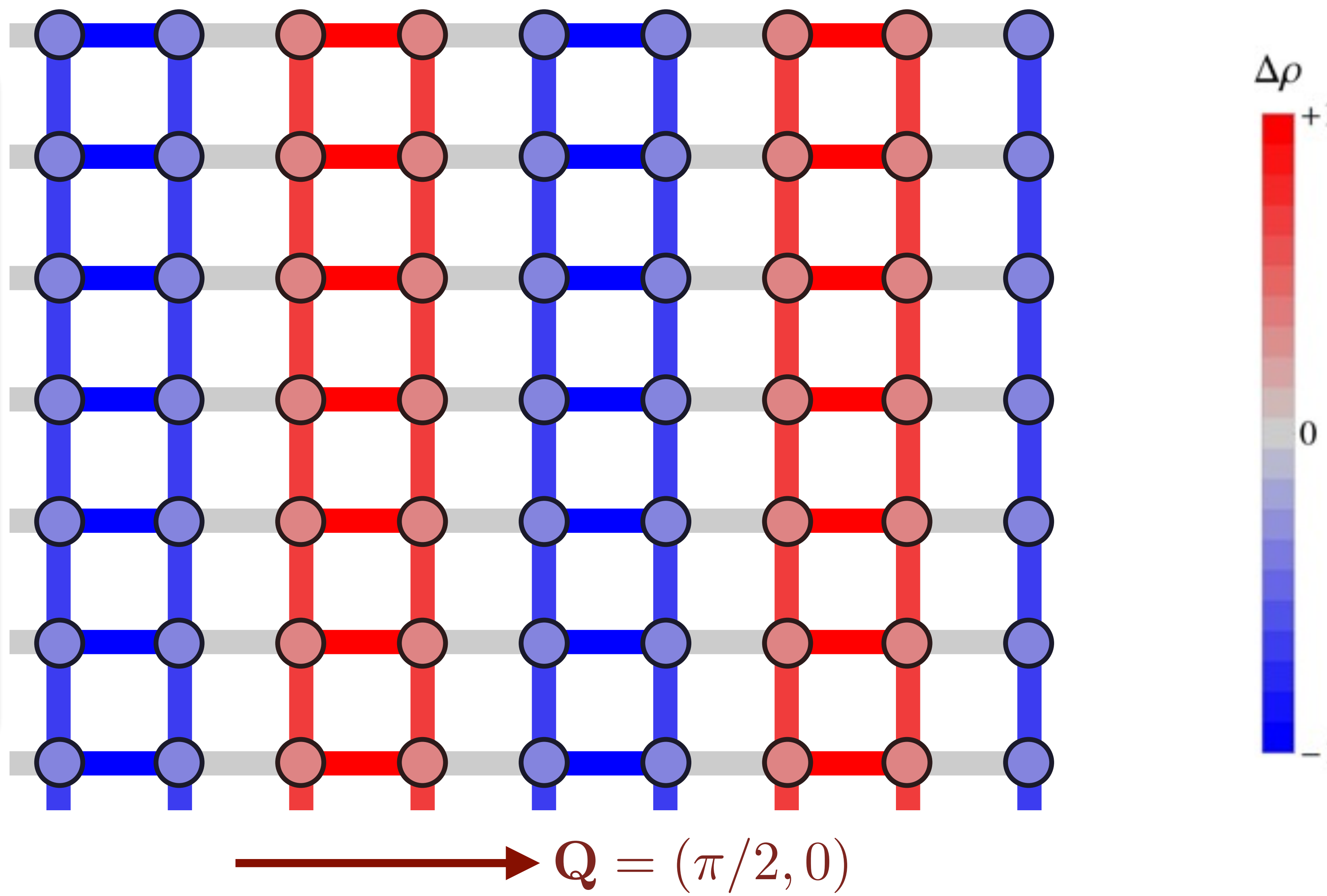
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} \left[1/2 + \cos(k_x) + \cos(k_y) \right] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

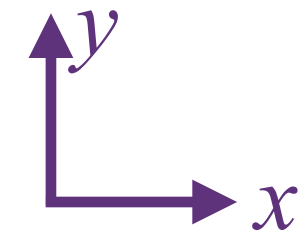
The pattern of charge densities and bond energies in the "stripe" model yields a combination of s and s' components



$\mathbf{Q} = (\pi/2, 0)$

M. H. Fischer, Si Wu, M. Lawler, A. Paramekanti, and Eun-Ah Kim, arXiv:1406.2711

Unconventional DW order: $s + s'$-form factor



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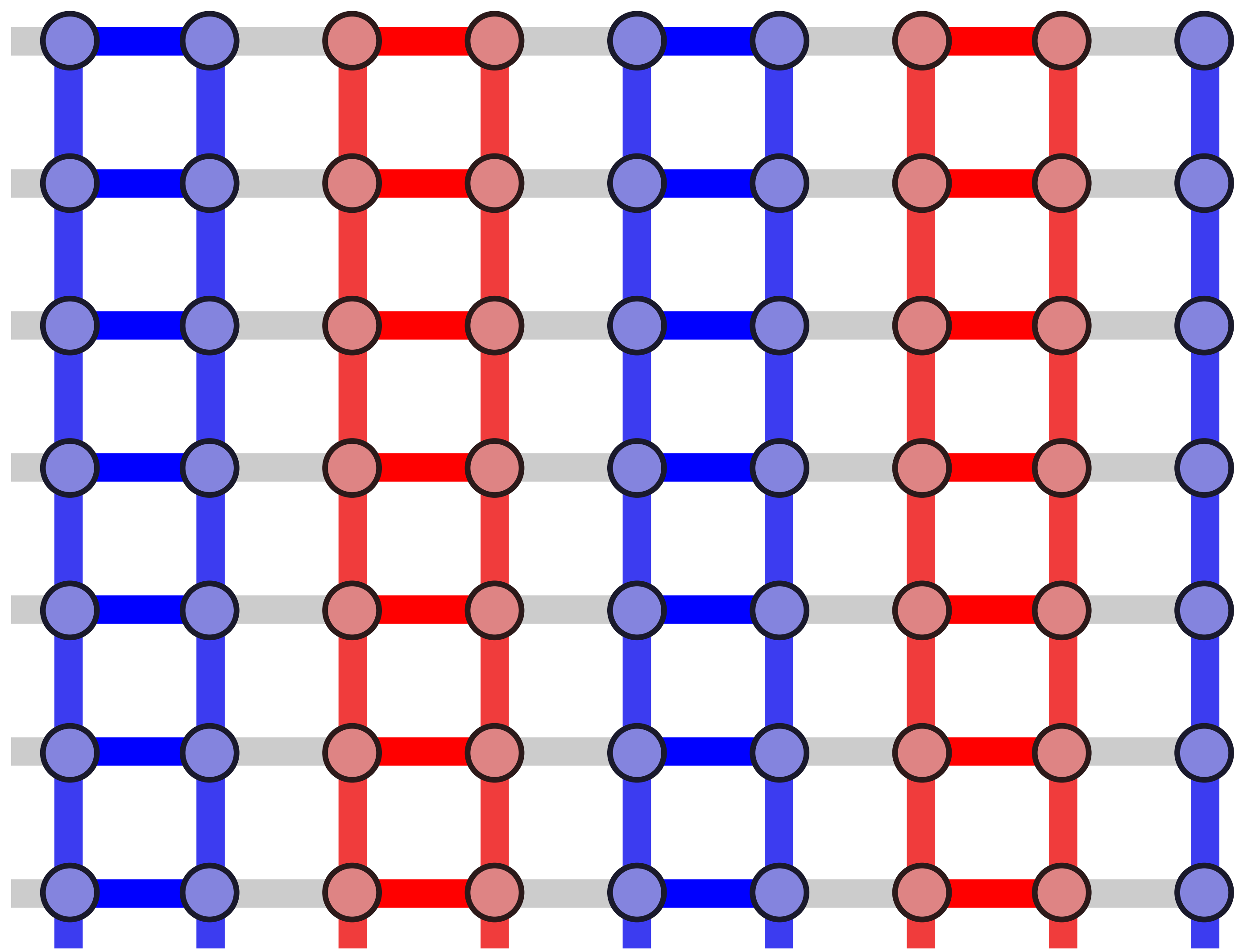
X-ray observations indicate strong s' component in LBCO

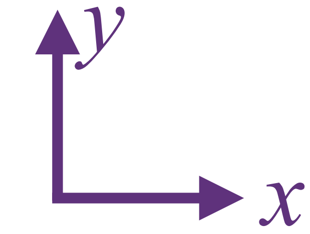


$\longrightarrow \mathbf{Q} = (\pi/2, 0)$

David Hawthorn, Waterloo

A. J. Achkar, F. He, R. Sutarto, Christopher McMahon, M. Zwiebler, M. Hucker, G. D. Gu, Ruixing Liang, D. A. Bonn, W. N. Hardy, J. Geck, and D. G. Hawthorn, arXiv:1409.6787

Unconventional DW order: d-form factor



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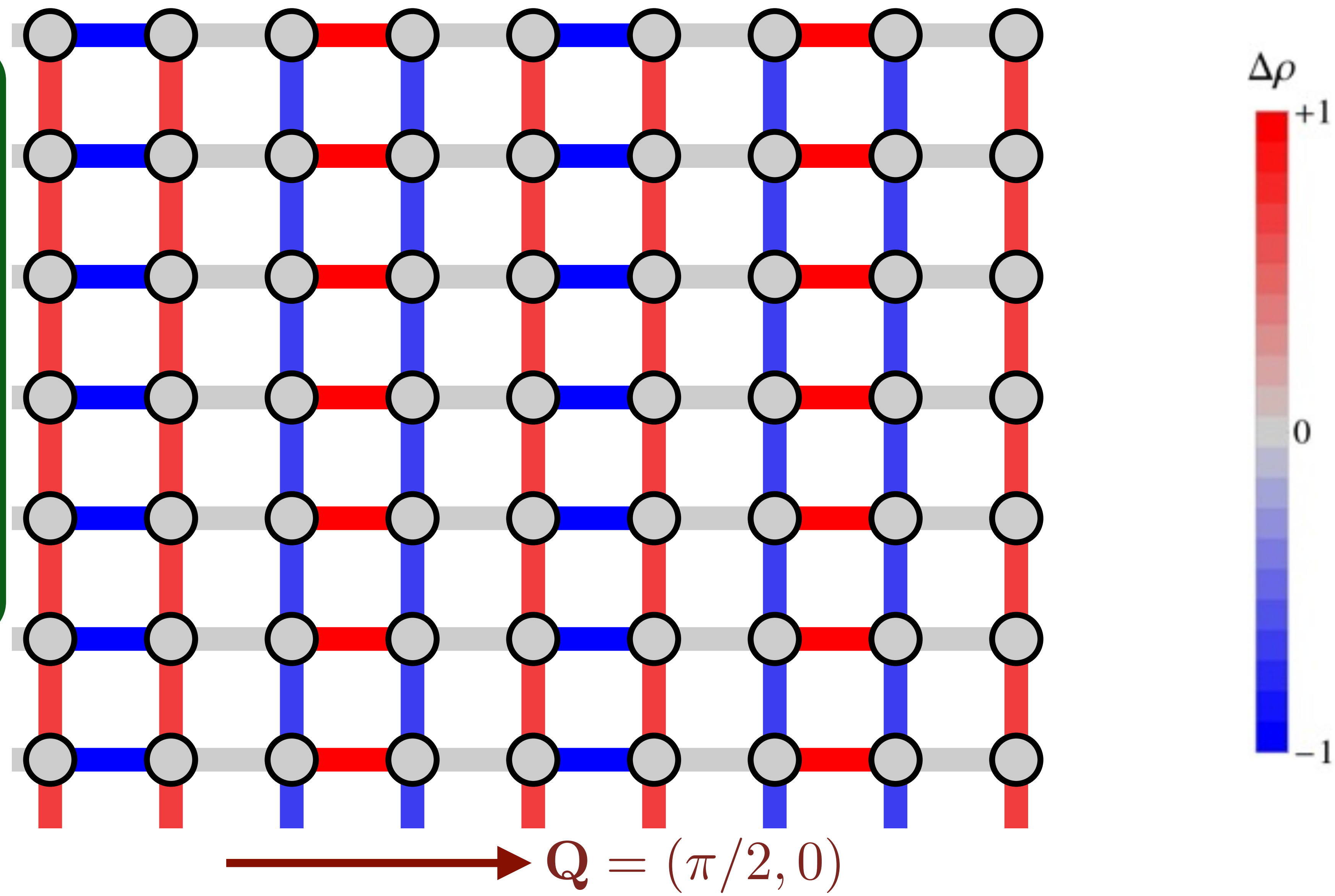
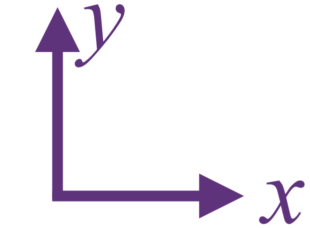
Our prediction:

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Unconventional DW order: d-form factor



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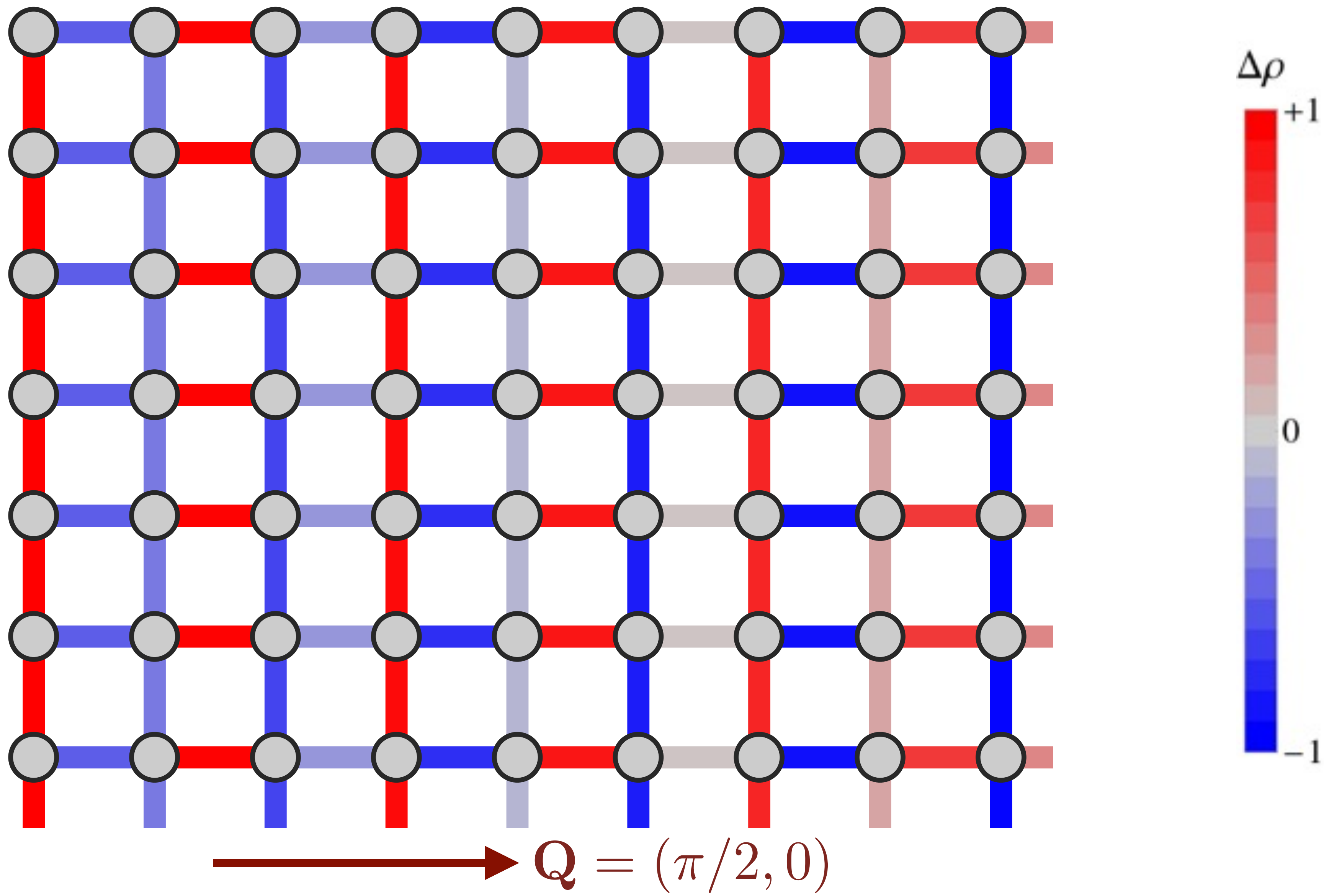

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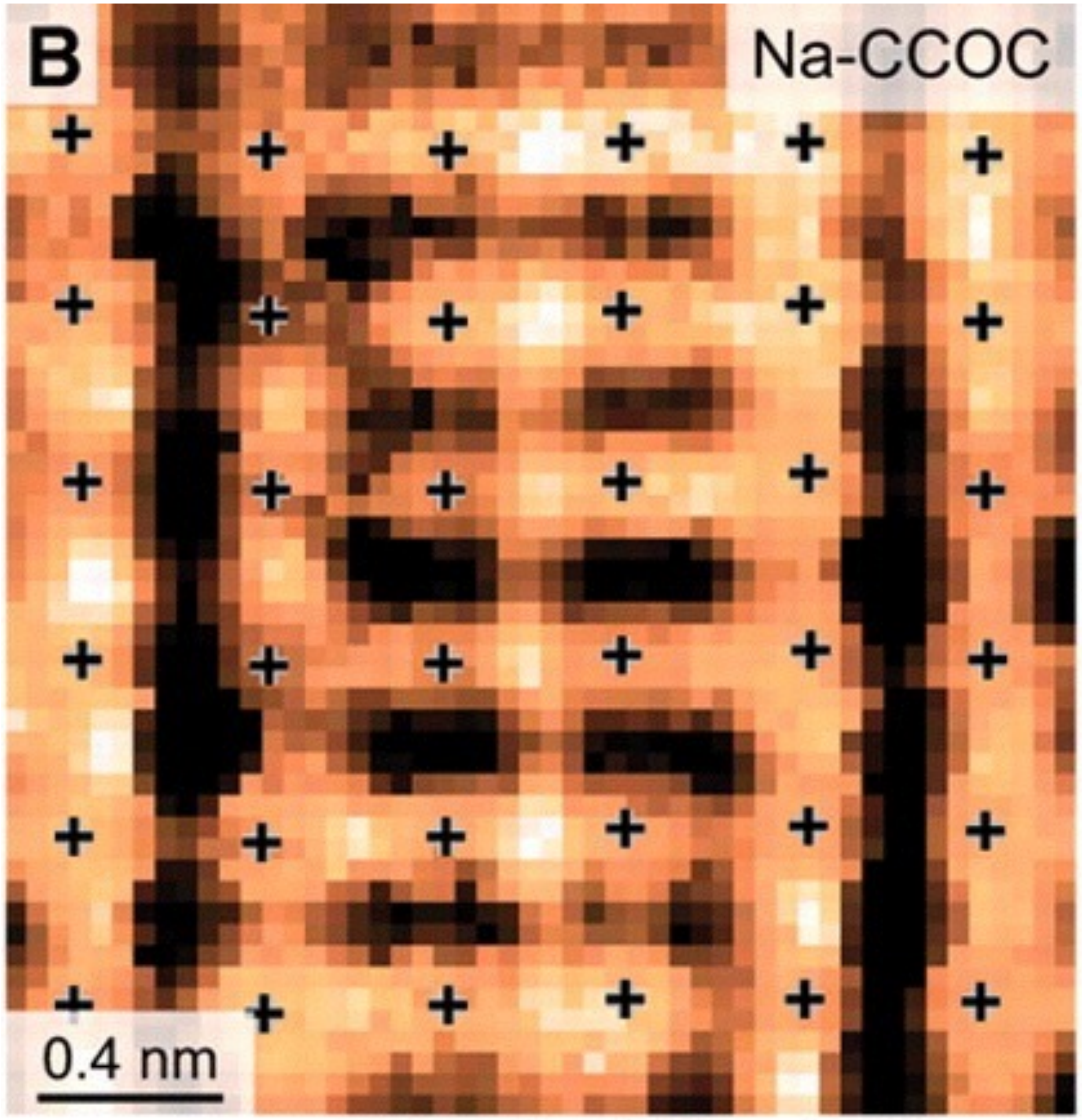
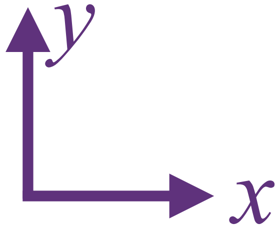
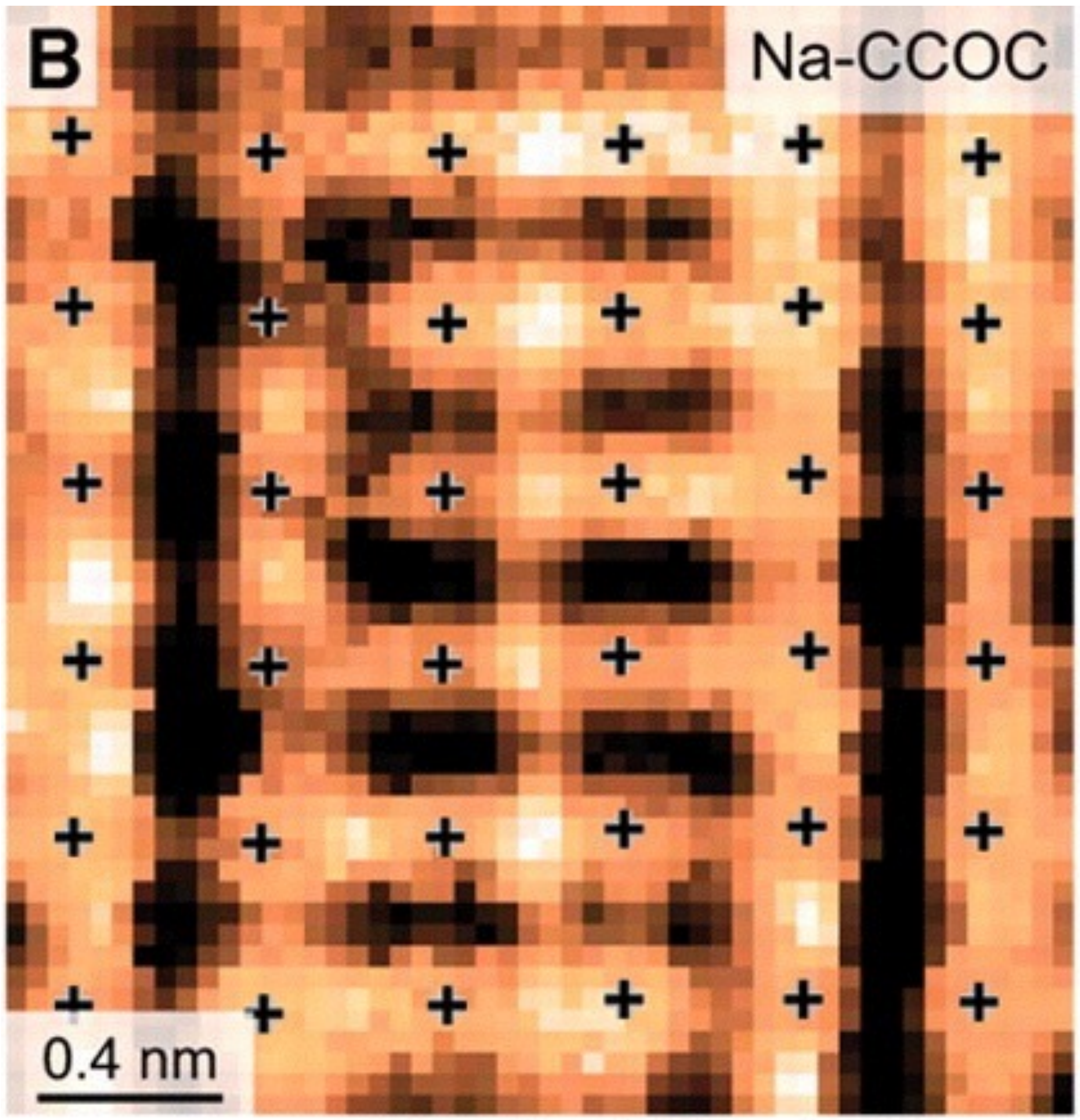


Y. Kohsaka *et al.*, Science **315**, 1380 (2007)





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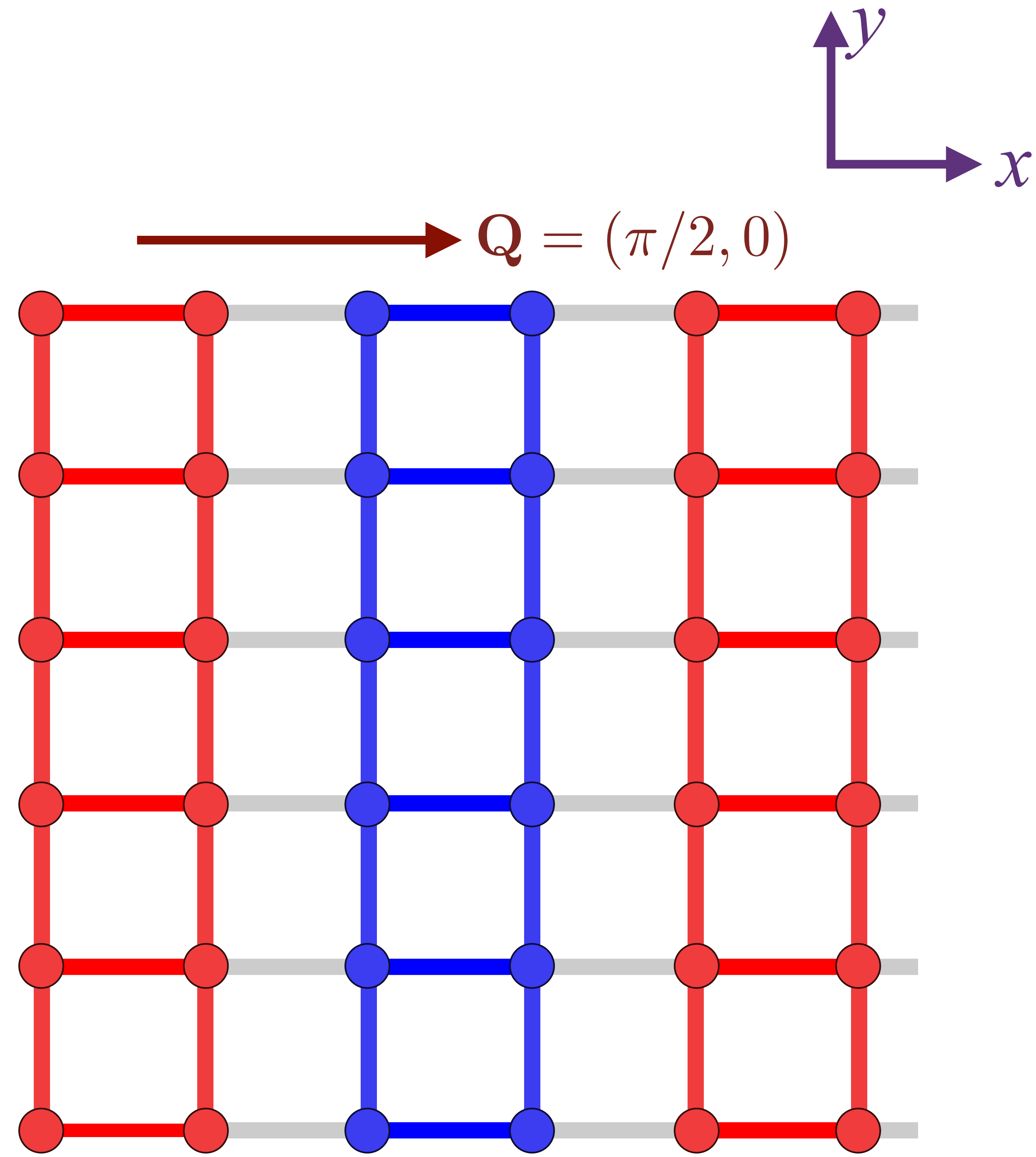
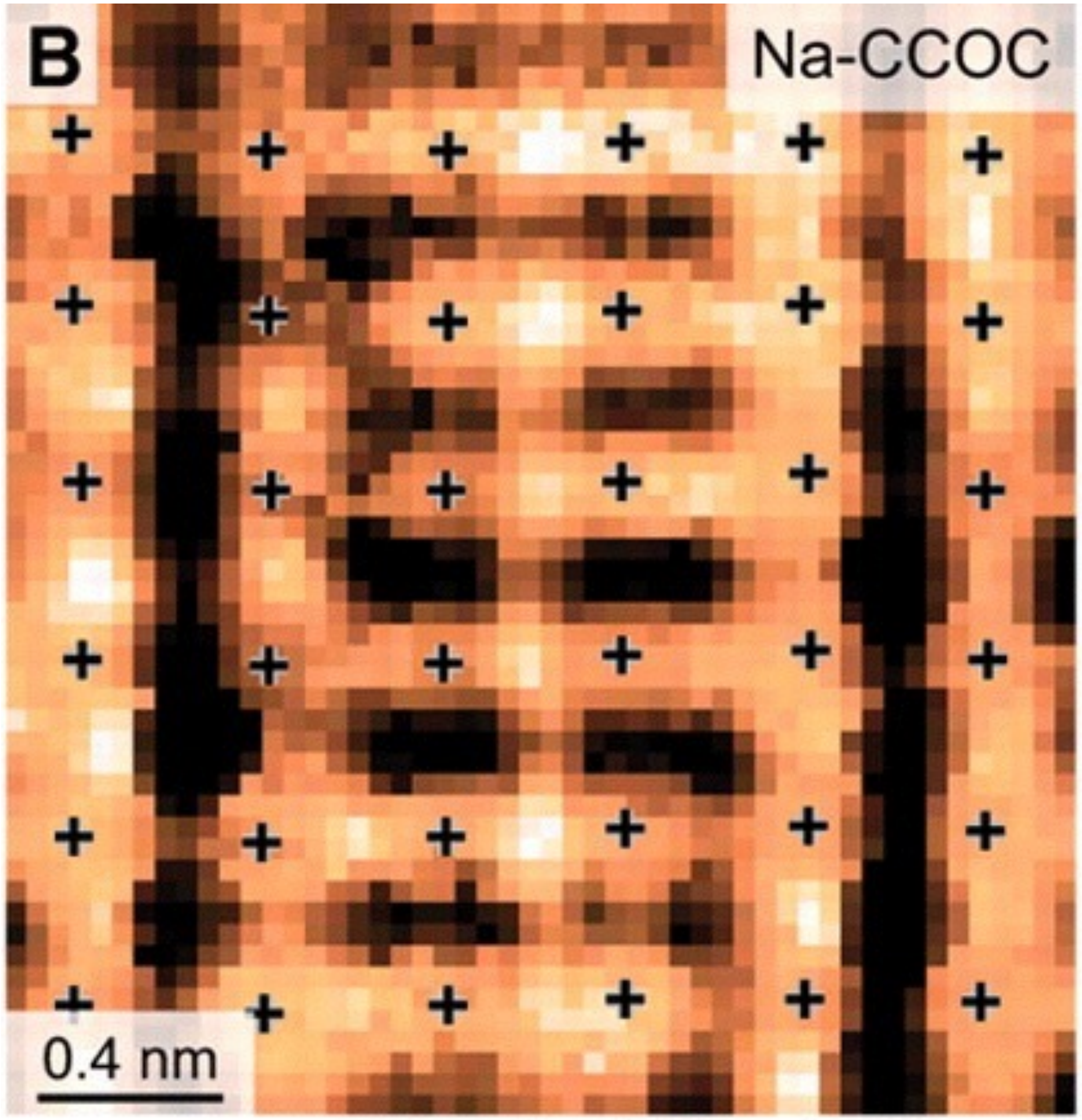
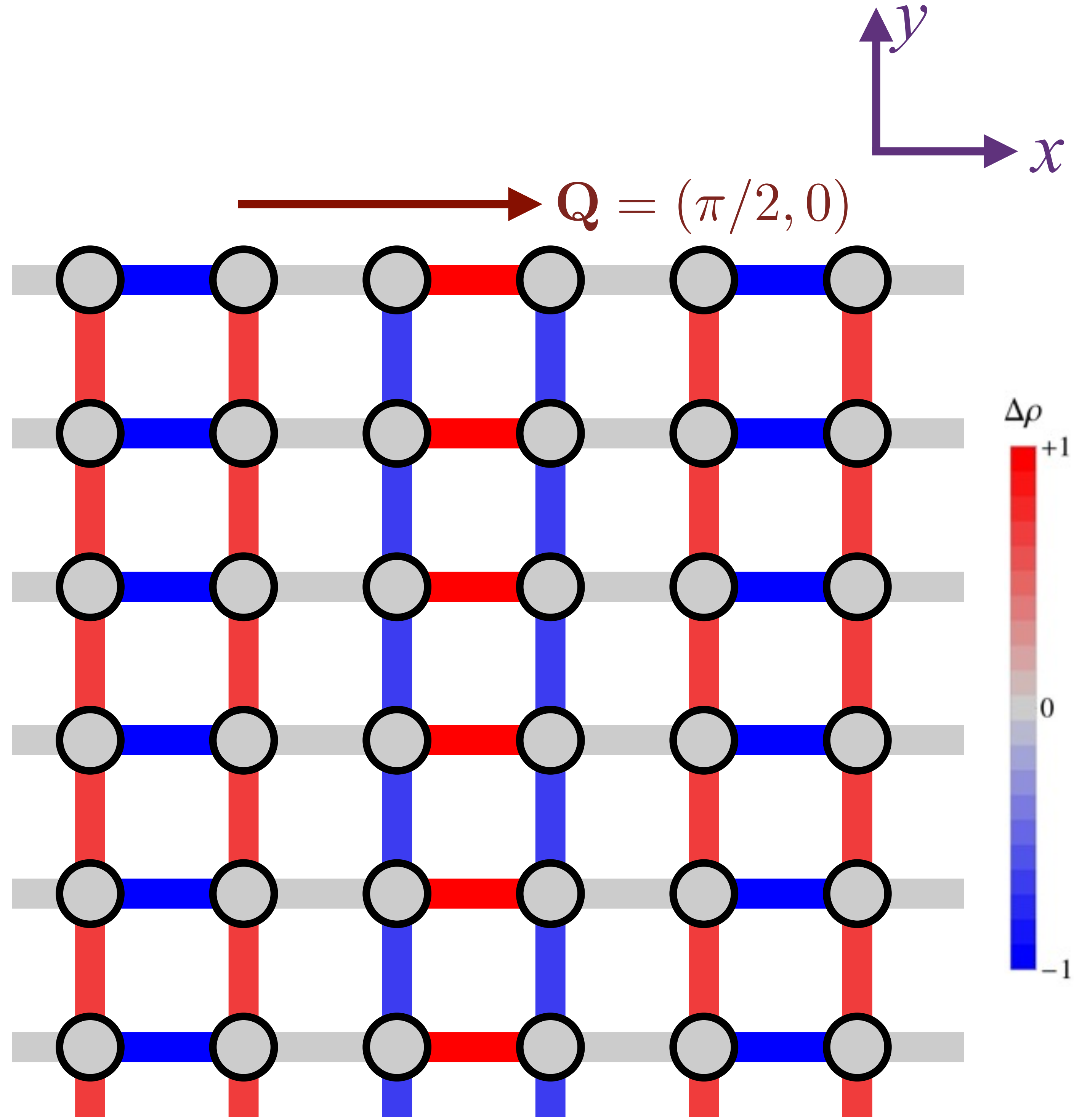


$s + s'$-form factor density wave

$s + s'$ form factor (stripe model) does **not** match STM measurements on BSCCO, Na-CCOC.



Y. Kohsaka *et al.*, Science **315**, 1380 (2007)



d-form factor density wave order

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
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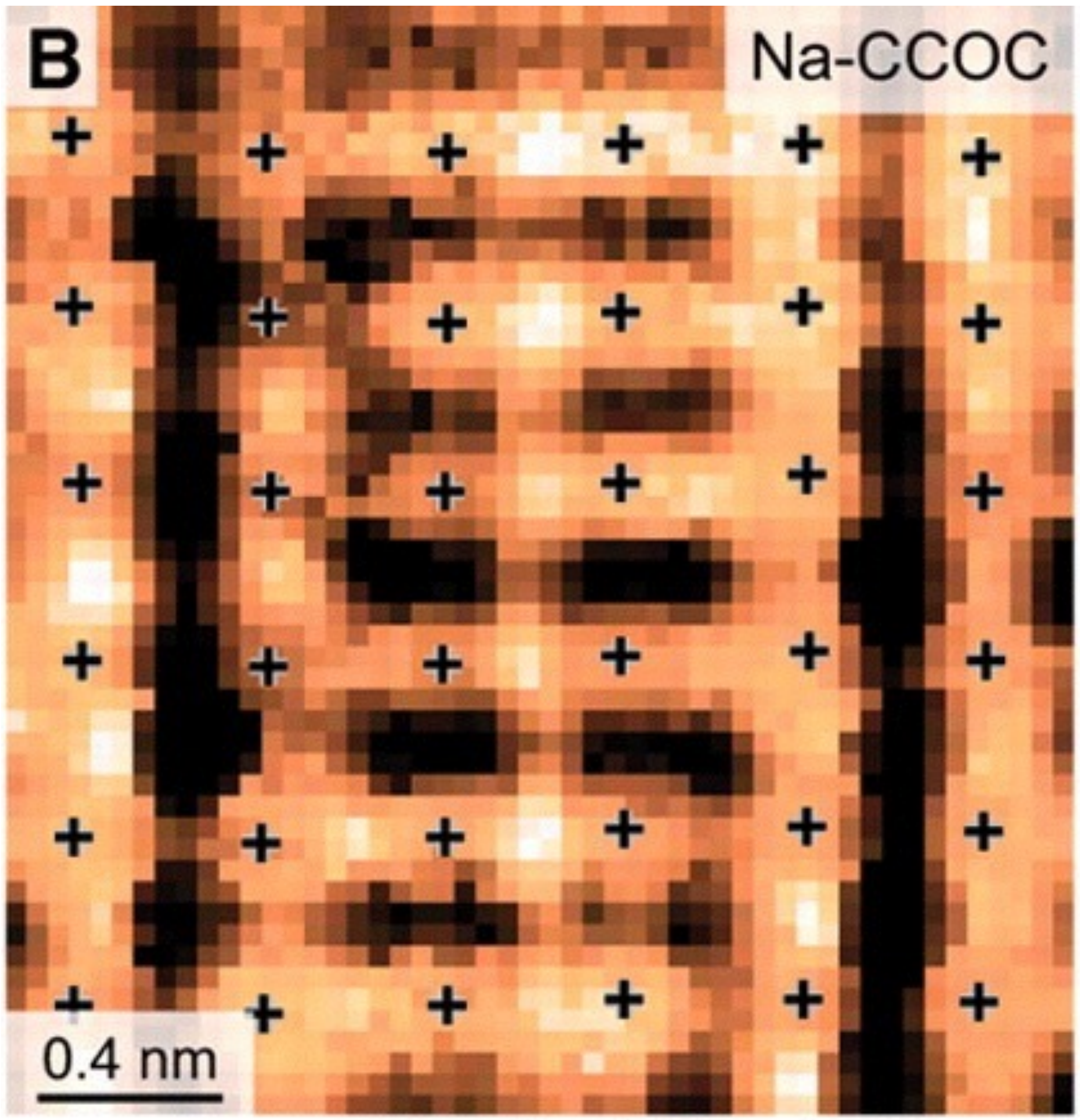

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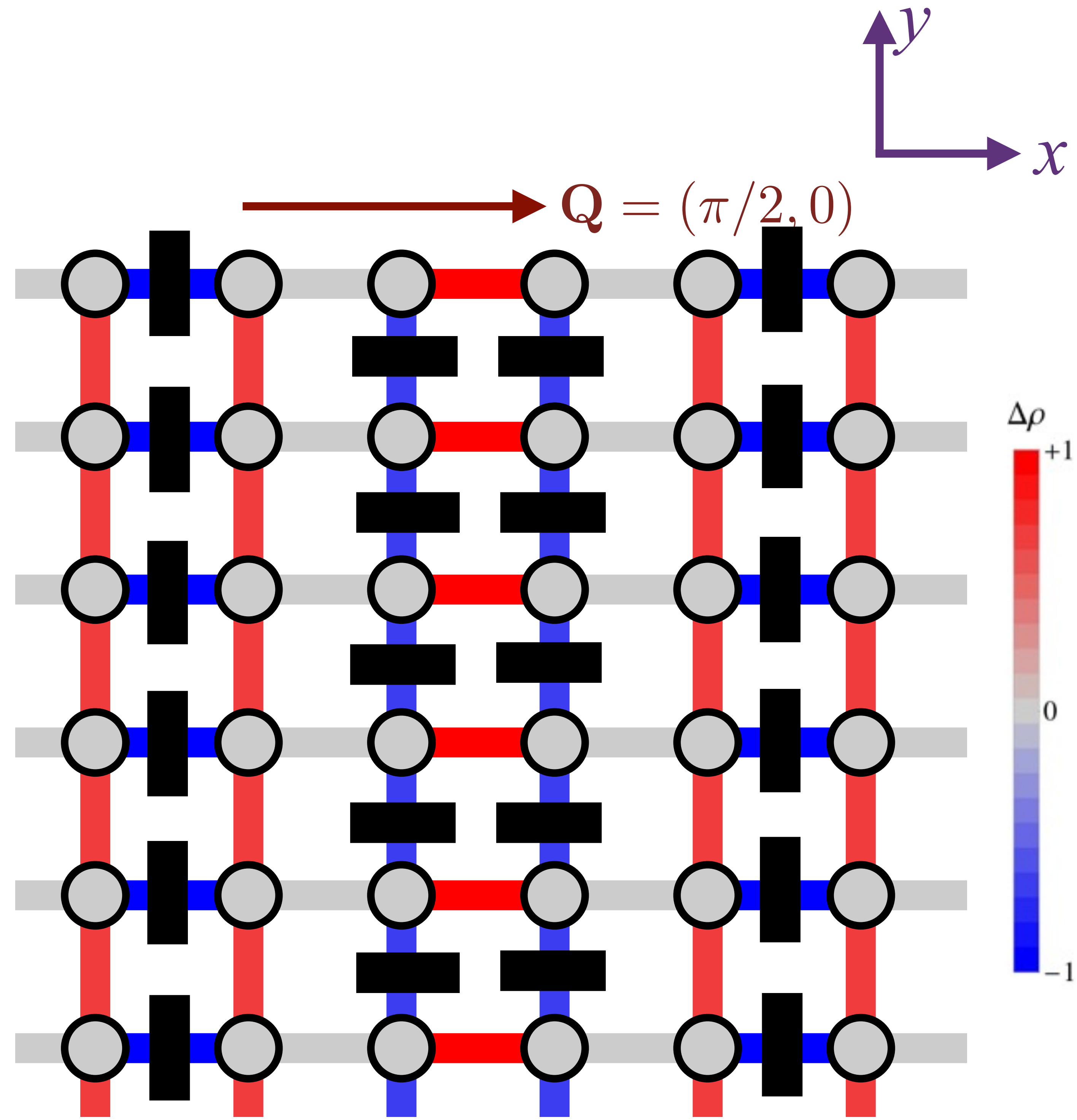
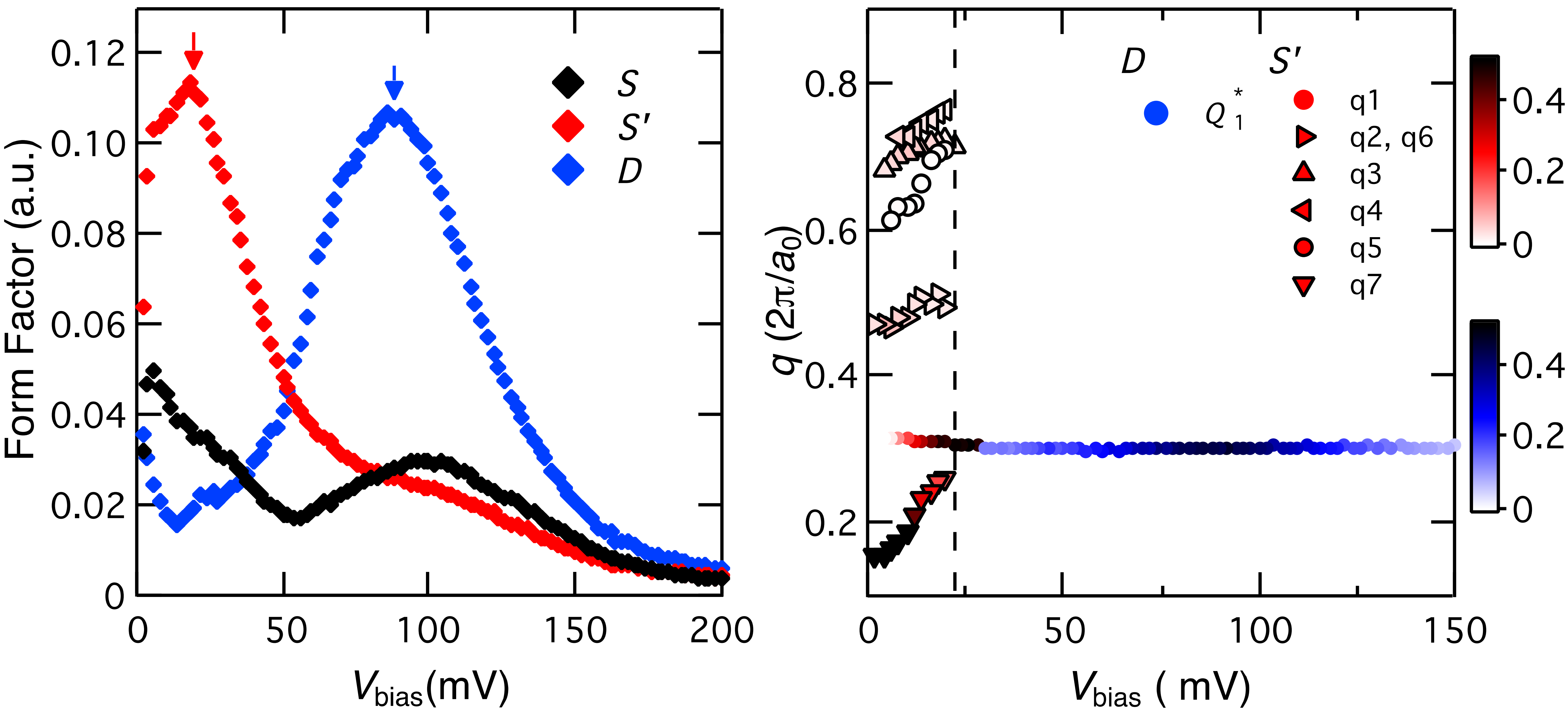


d-form factor density wave order

Predicted *d* form factor observed
in STM measurements on BSCCO, Na-CCOC !

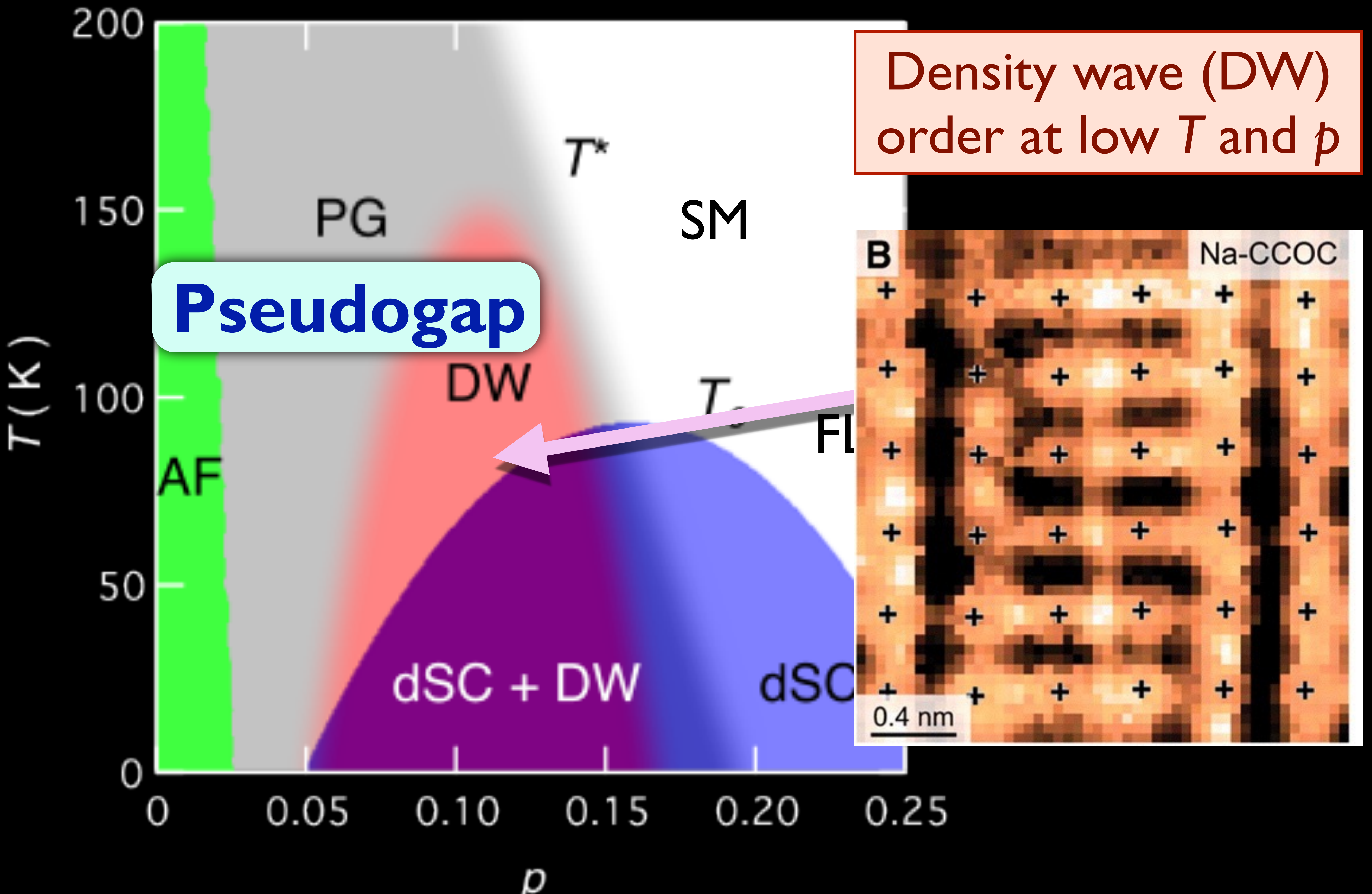
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

d-form factor is peaked at the pseudogap energy, and does not disperse as a function of wavevector

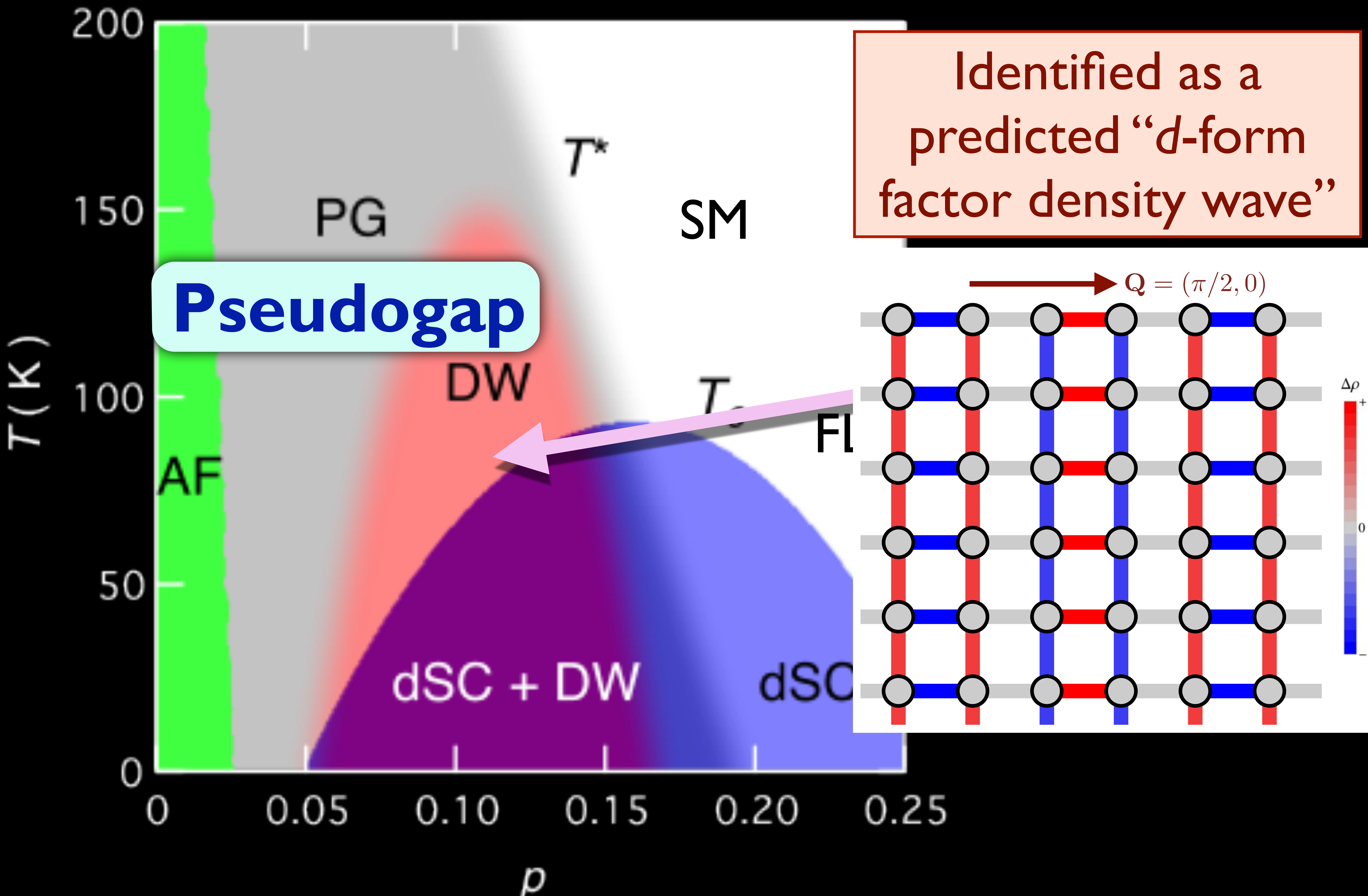


K. Fujita, M. H. Hamidian, S. D. Edkins, Chung Koo Kim, A. P. MacKenzie, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, to appear

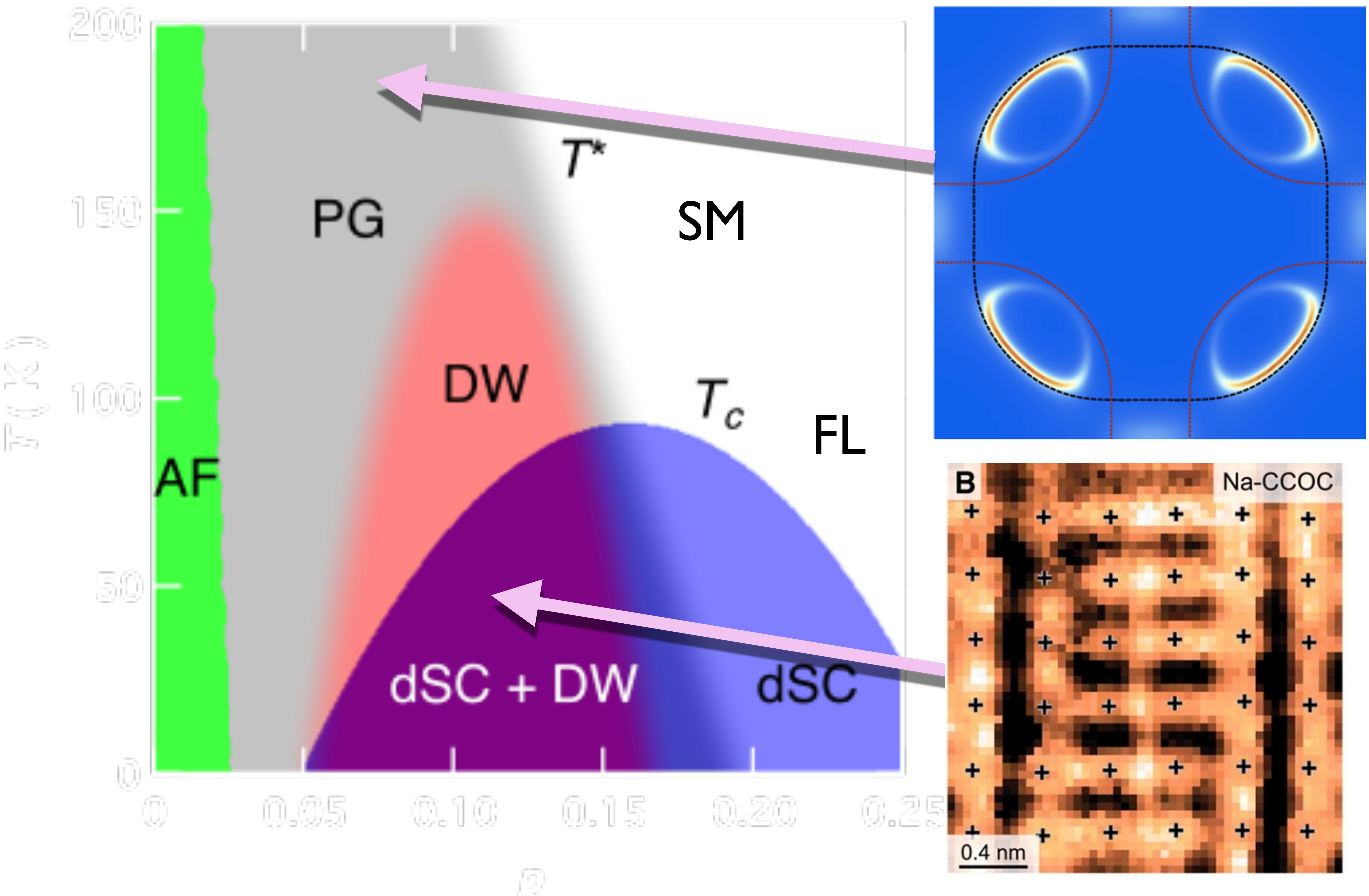
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



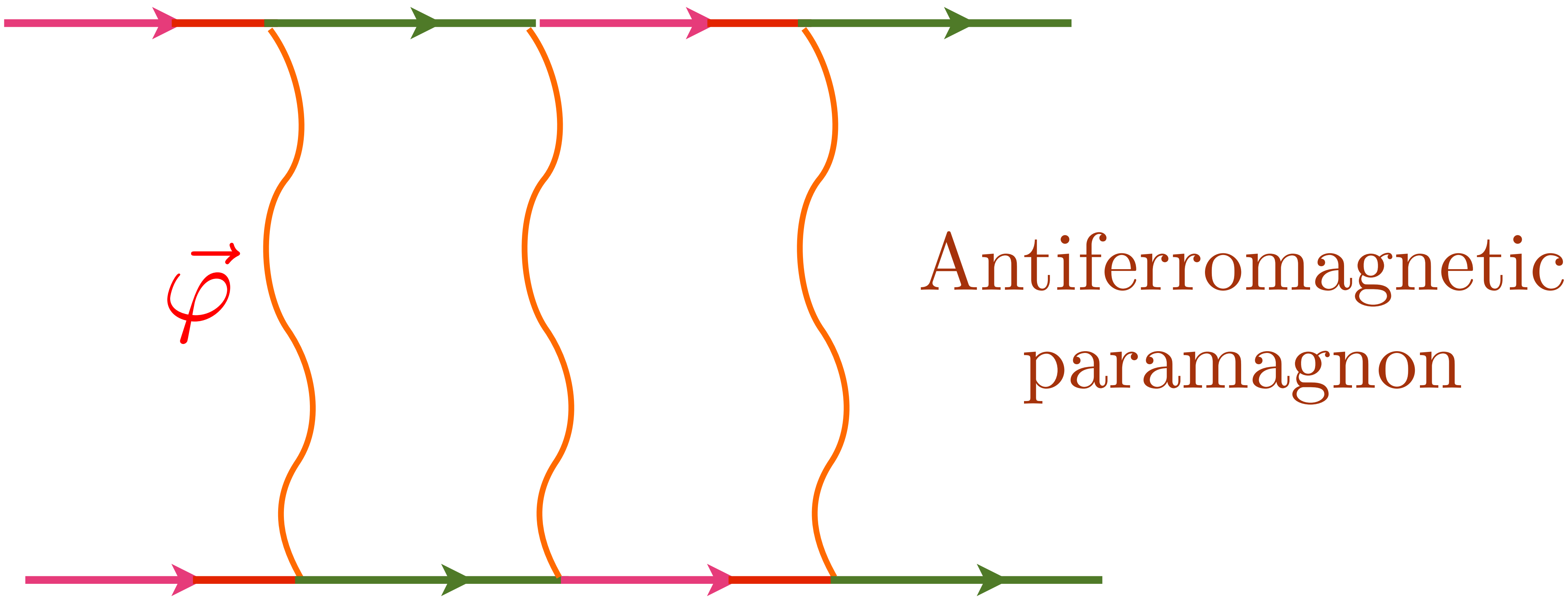
M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013). K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)



What is the relationship between the FL* at high T and the "d-form factor density wave" at low T ?



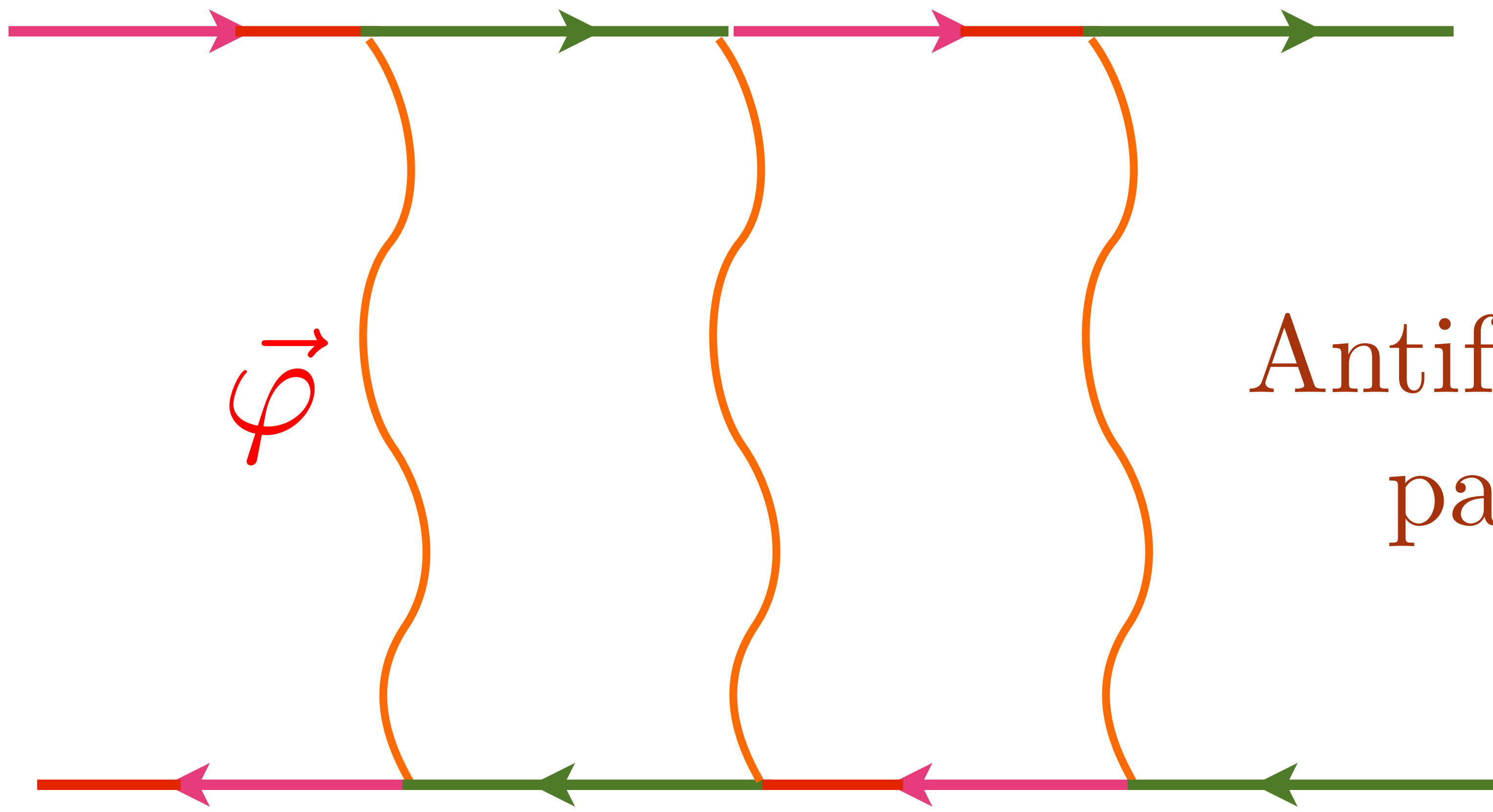
Pairing "glue" for d-wave superconductivity from antiferromagnetic fluctuations



Leads to $\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta}\Delta(\cos k_x - \cos k_y)$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
P. Monthoux, A.V. Balatsky, and D. Pines, *Phys. Rev. Lett.* **67**, 3448 (1991)

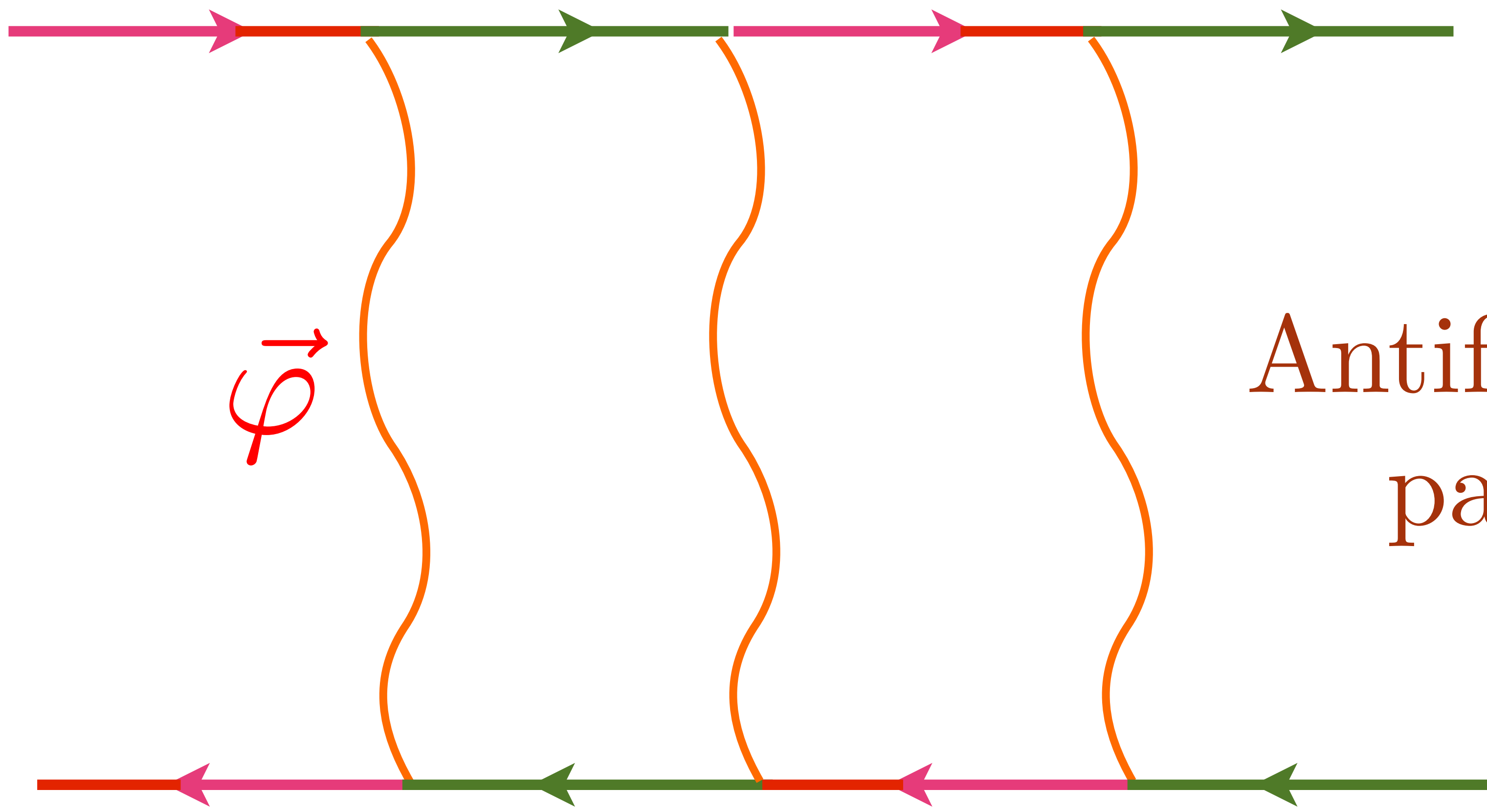
Same glue can lead to "d-wave" particle-hole pairing



Antiferromagnetic paramagnon

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)
M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)
K. B. Efetov, H. Meier, and C. Pépin, Nat. Phys. **9**, 442 (2013)
J. D. Sau and S. Sachdev, Phys. Rev. B **89**, 075129 (2014)
Y. Wang and A. V. Chubukov, Phys. Rev. B **90**, 035149 (2014)

Same glue can lead to "d-wave" particle-hole pairing



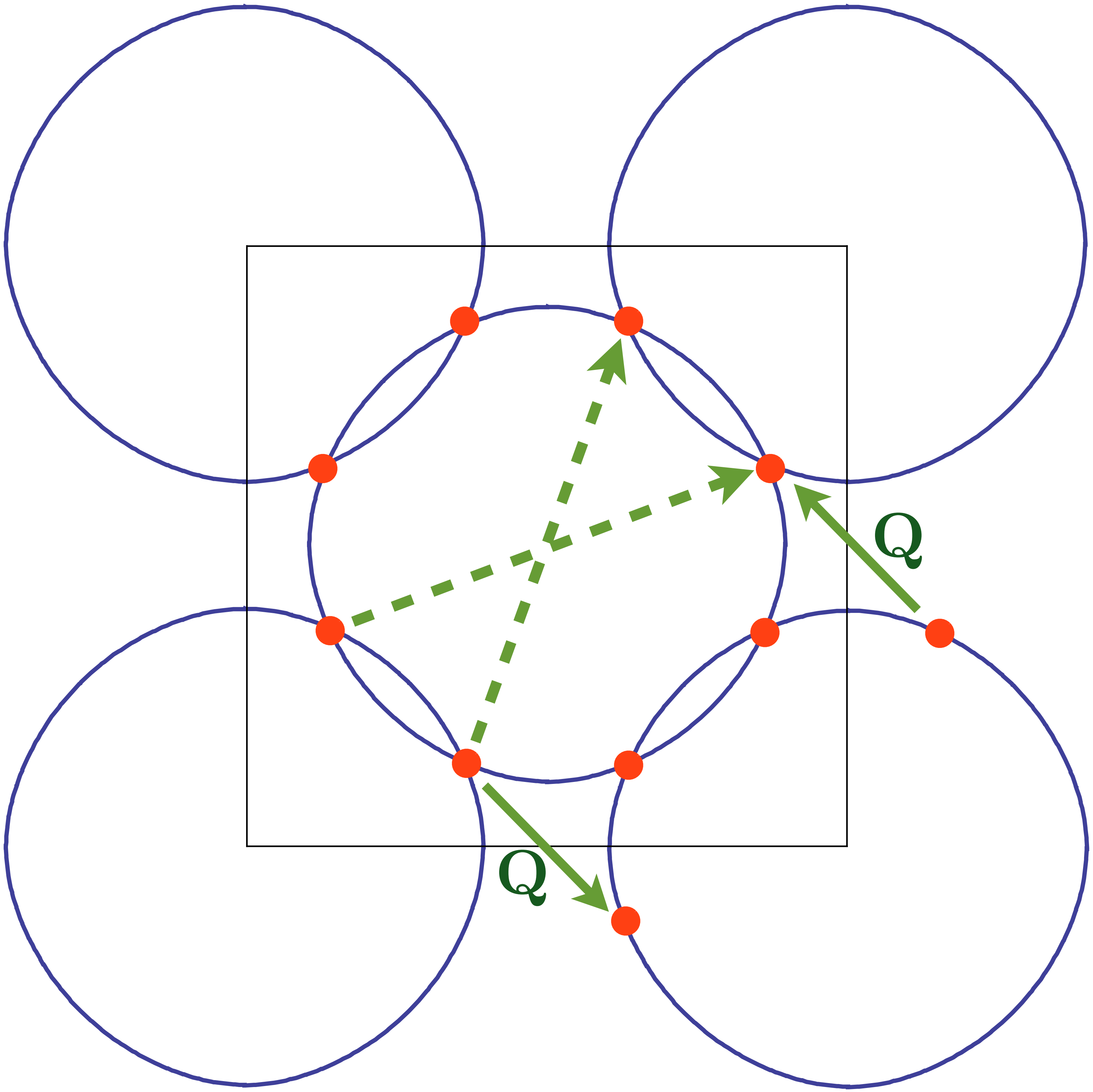
Antiferromagnetic paramagnon

Leads to $\left\langle c^\dagger_{\mathbf{k}+\mathbf{Q}/2,\alpha} c_{\mathbf{k}-\mathbf{Q}/2,\alpha} \right\rangle =$

$$\mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

with $\mathcal{P}_d$ dominant.

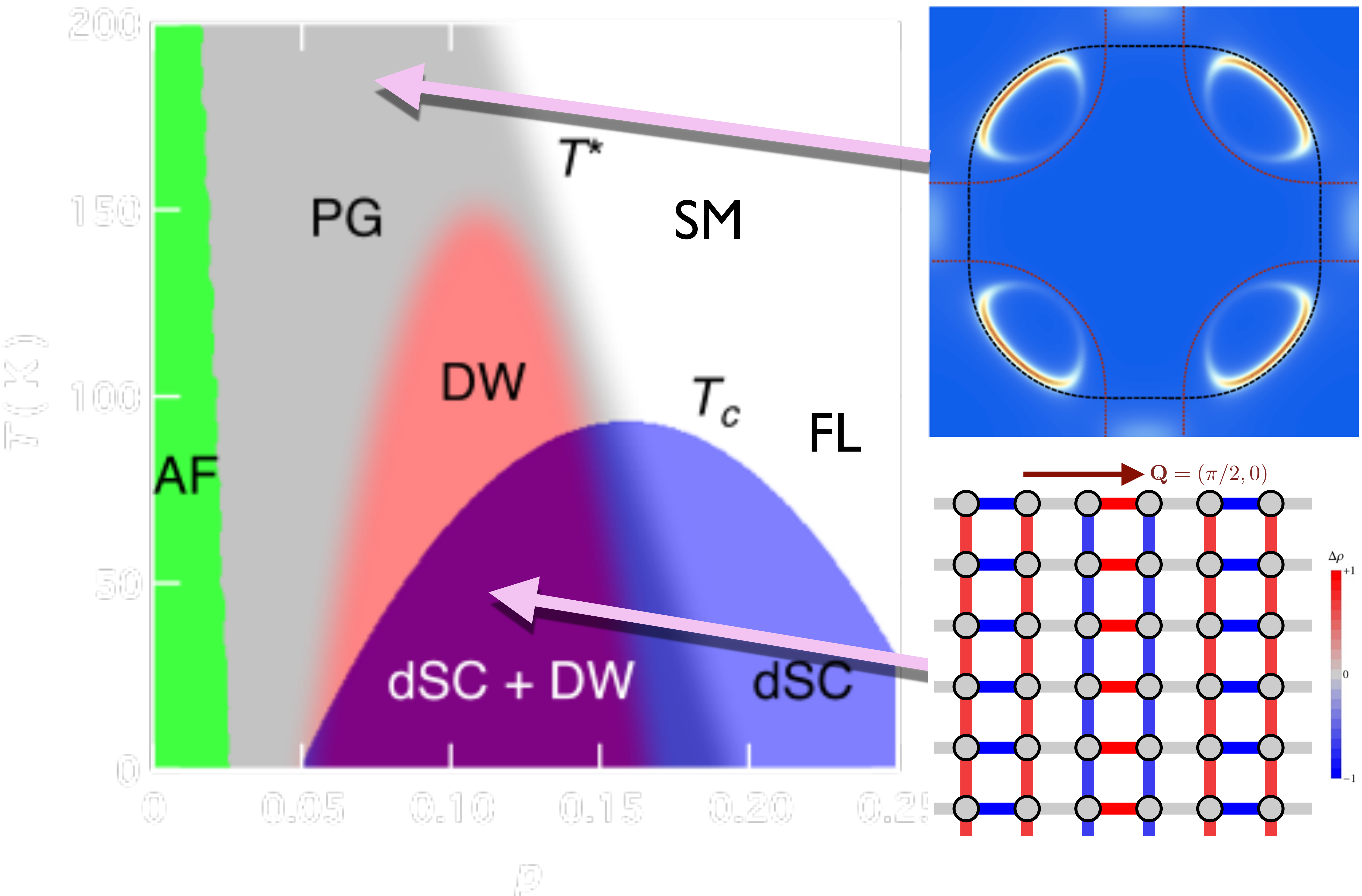
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)



Density wave instability of large Fermi surface (FL) leads to an incorrect "diagonal" wavevector

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \mathcal{P}_d (\cos k_x - \cos k_y)$$

M. A. Metlitski and S. Sachdev, PRB **85**, 075127 (2010); A. Thomson and S. Sachdev, PRB **91**, 115142 (2015)
W. A. Atkinson, A. P. Kampf, and S. Bulut, New Journal of Physics **17**, 013025 (2015)



200
150
100
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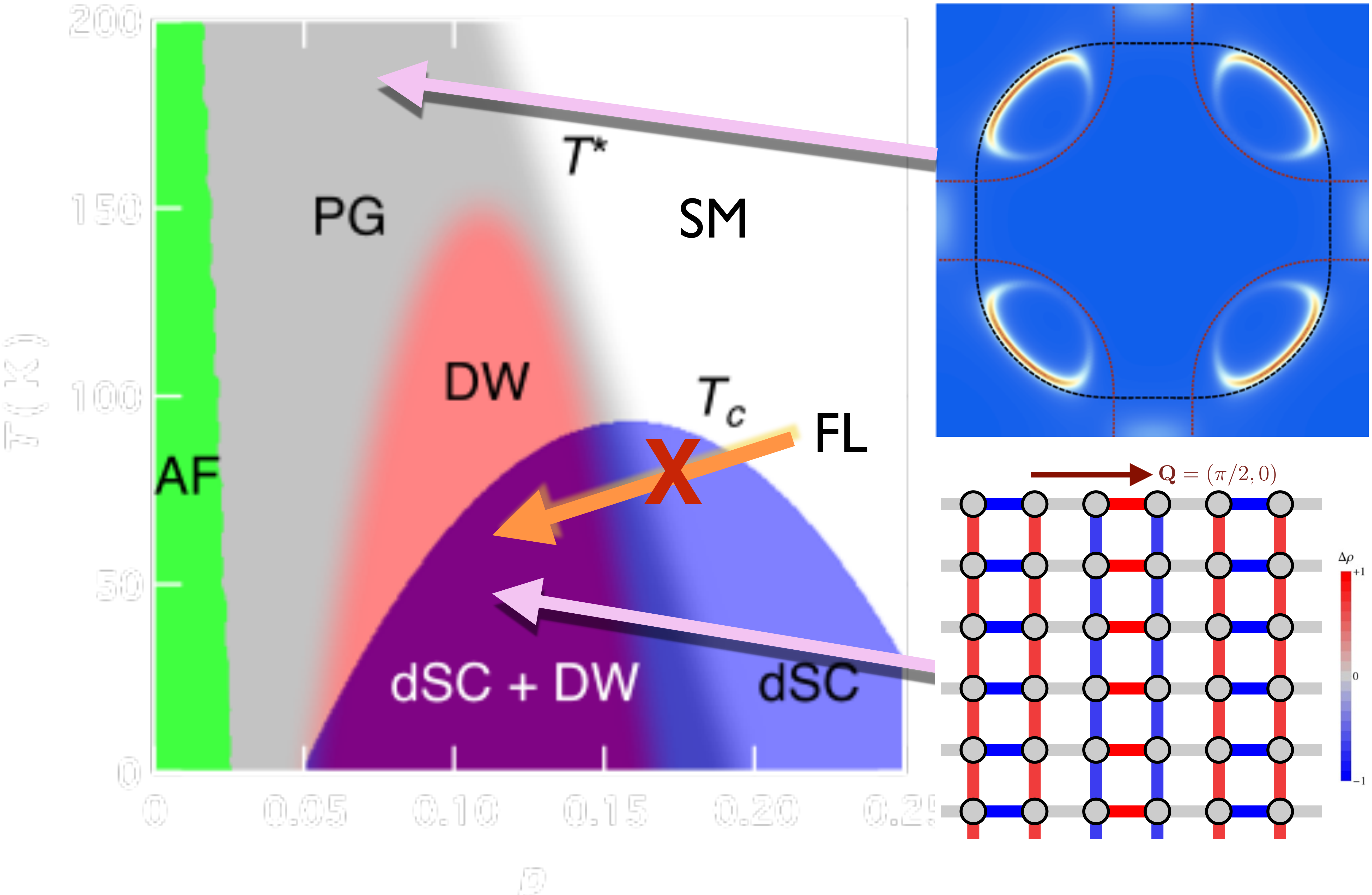
0
T (K)
0
0.05
0.10
0.15
0.20
p
AF
PG
T*
SM
DW
Tc
FL
dSC + DW
dSC
Q = (π/2, 0)
Δρ
+1
0
−1



200
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T (K)
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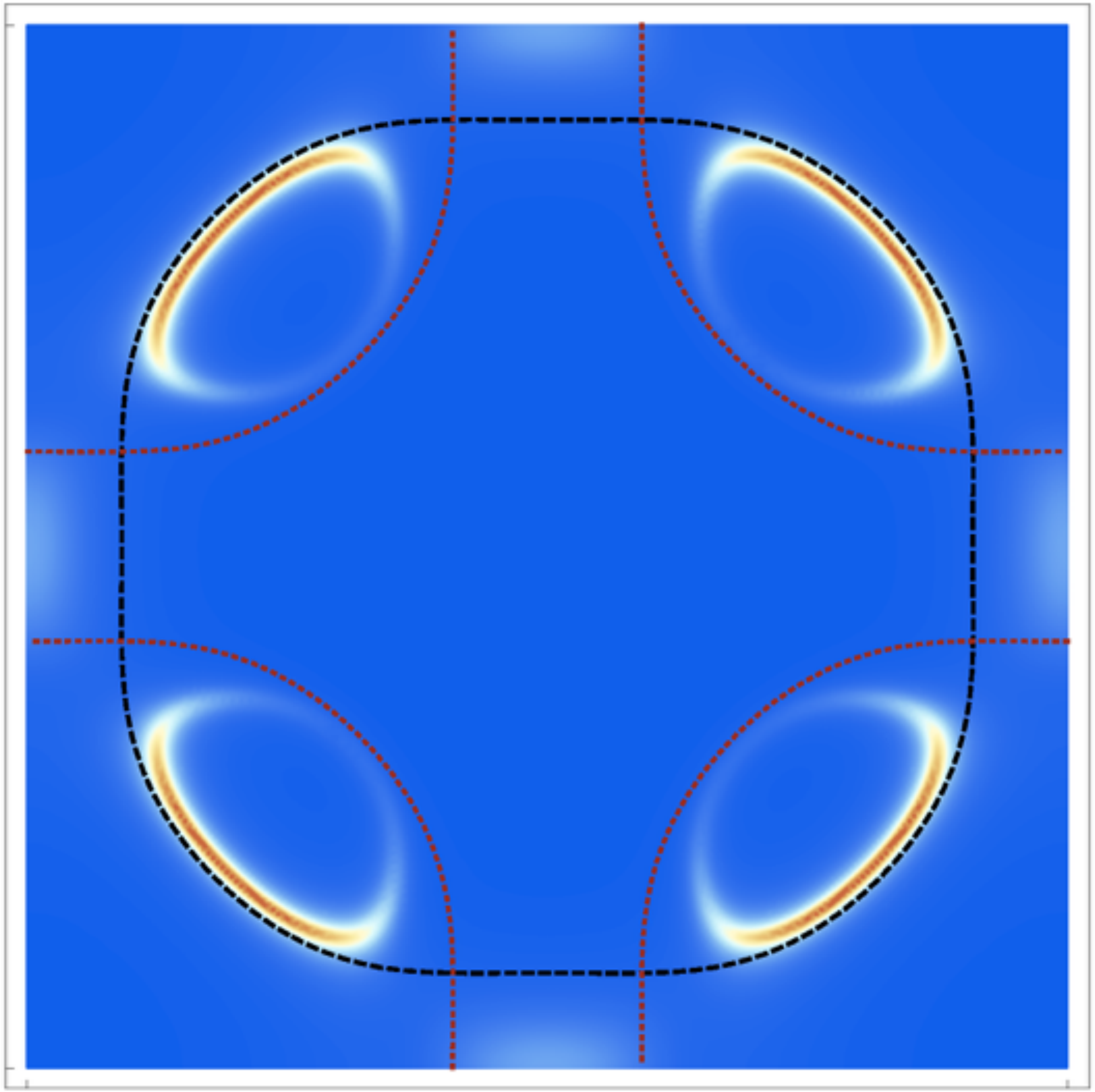
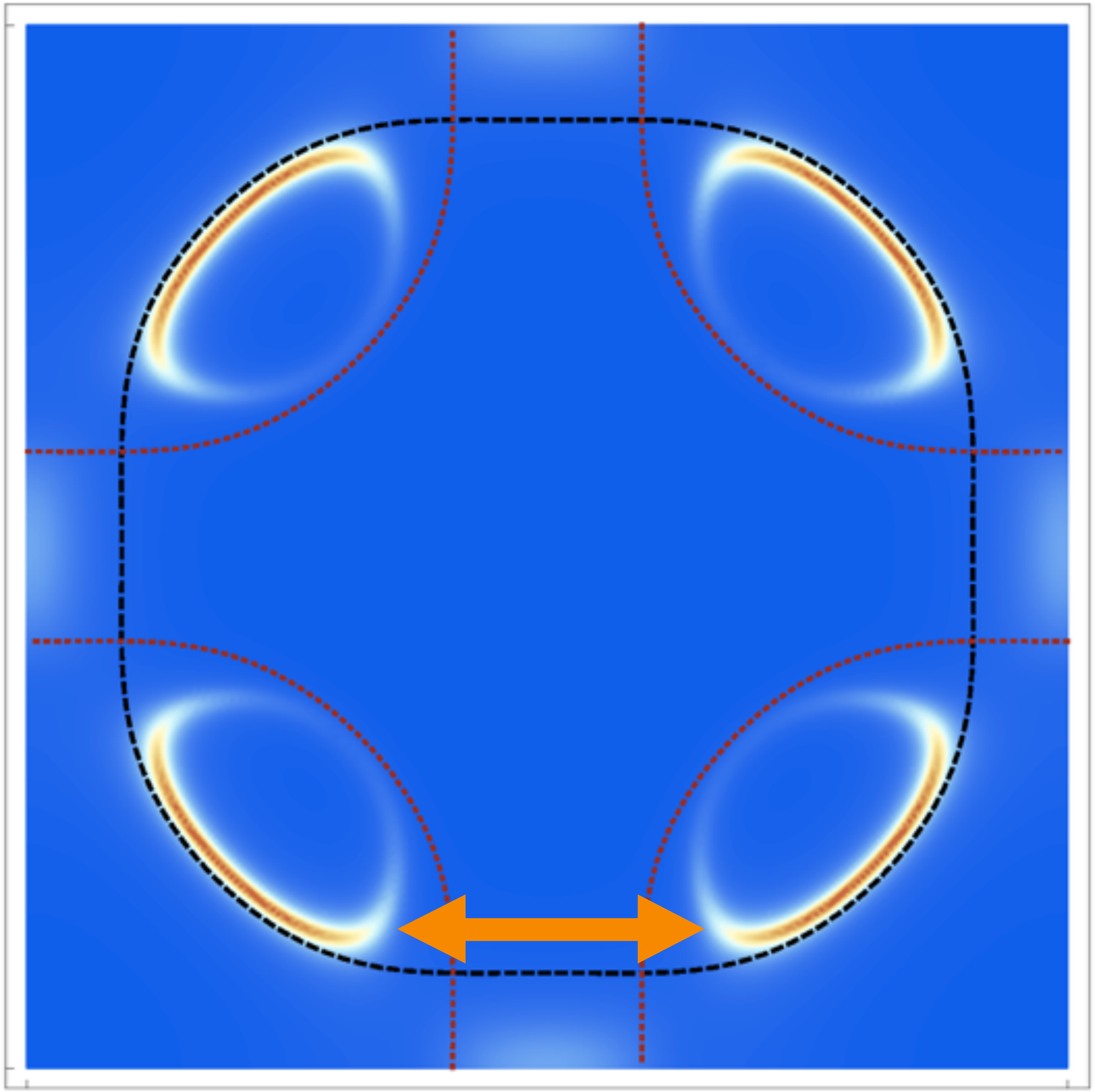
AF
dSC + DW
dSC
Q = (π/2, 0)
Δρ
+1
0
−1



Fermi surface of a fractionalized Fermi liquid (FL*)

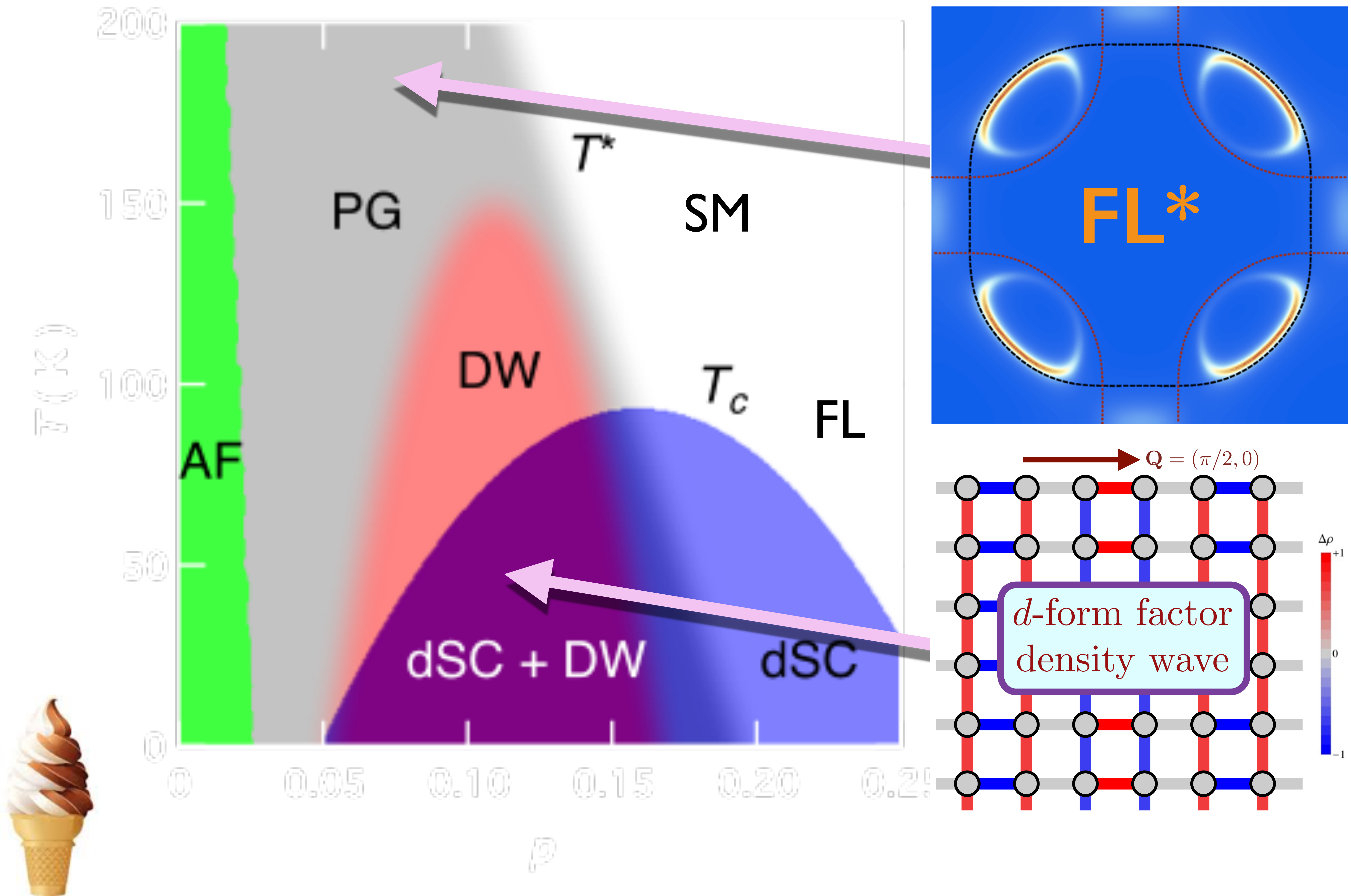
Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)



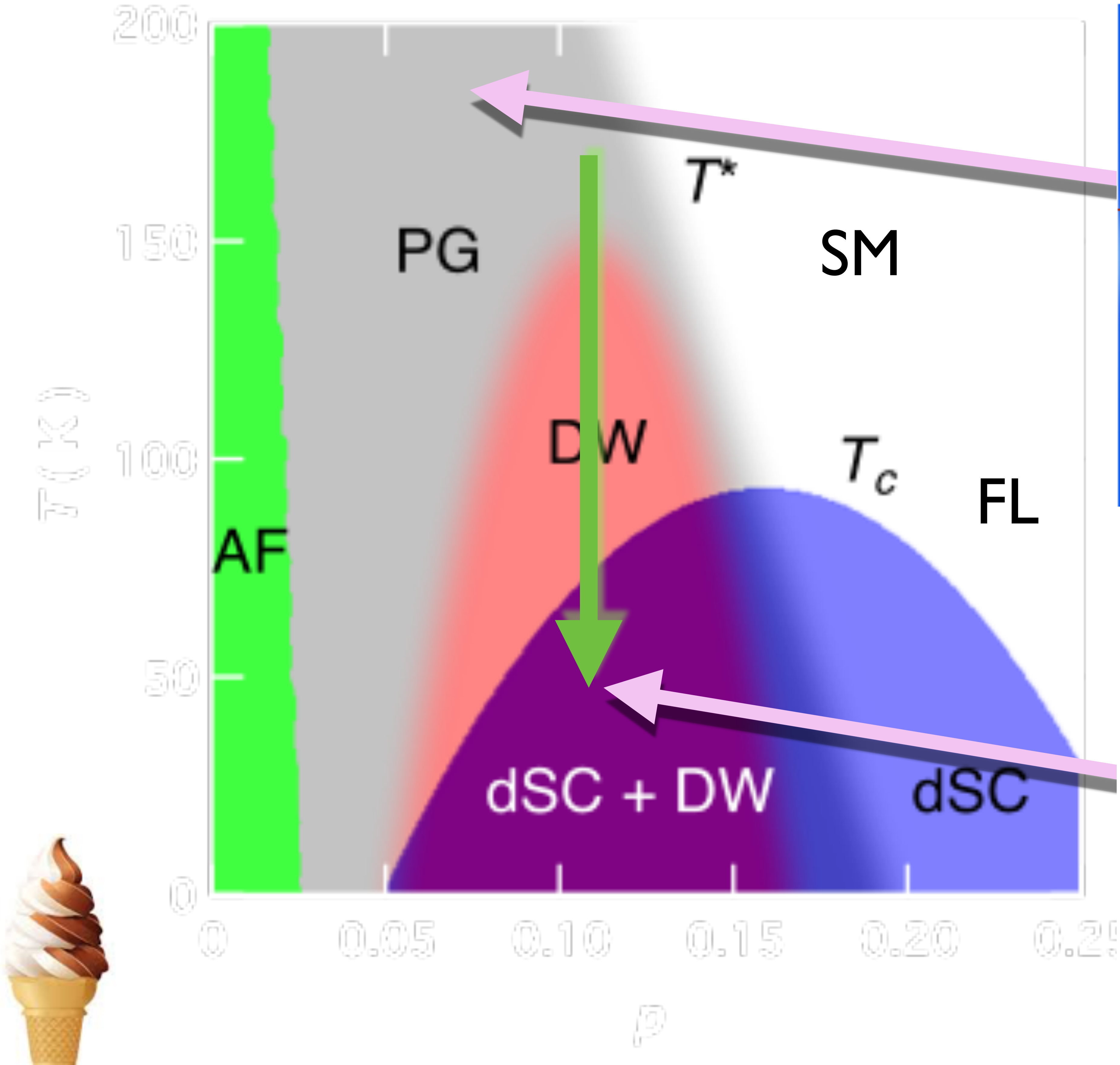
Density wave instability of FL* leads to the observed wavevector and form-factor

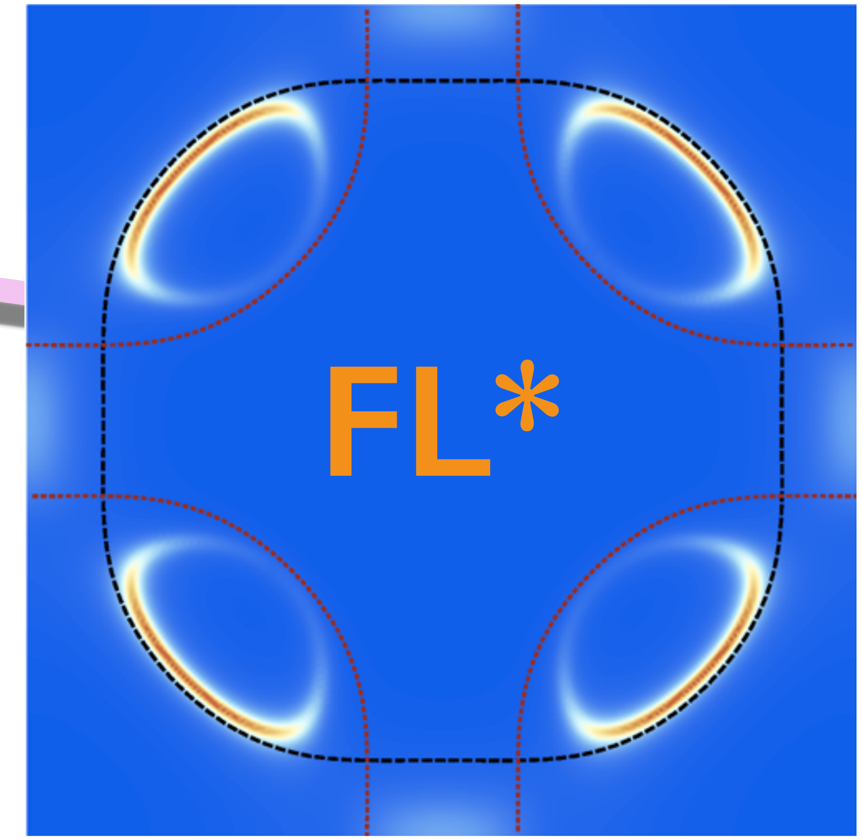
D. Chowdhury and S. Sachdev, Physical Review B **90**, 245136 (2014).

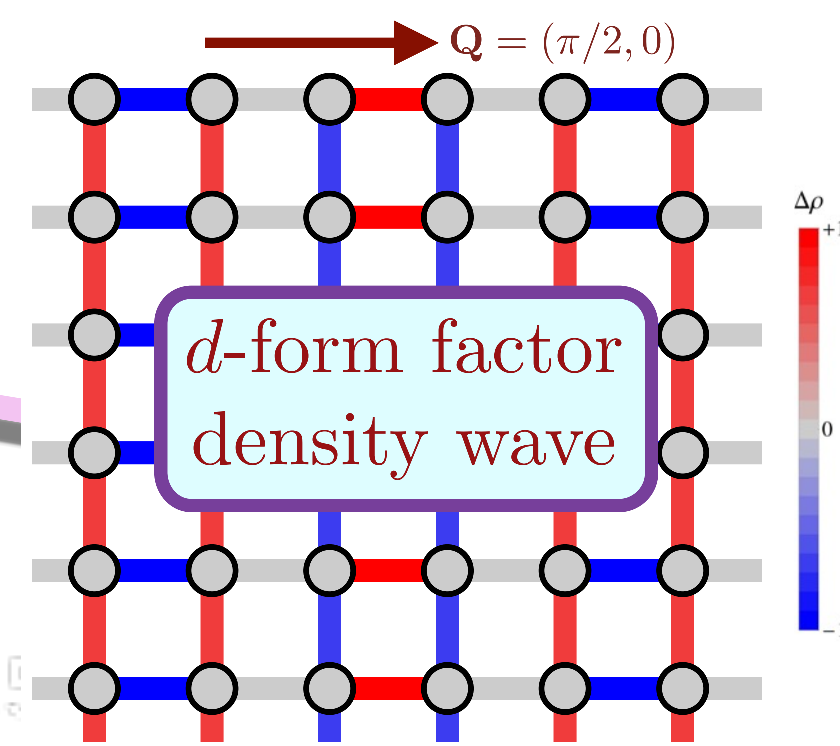
The high T FL* can help explain the "d-form factor density wave" observed at low T

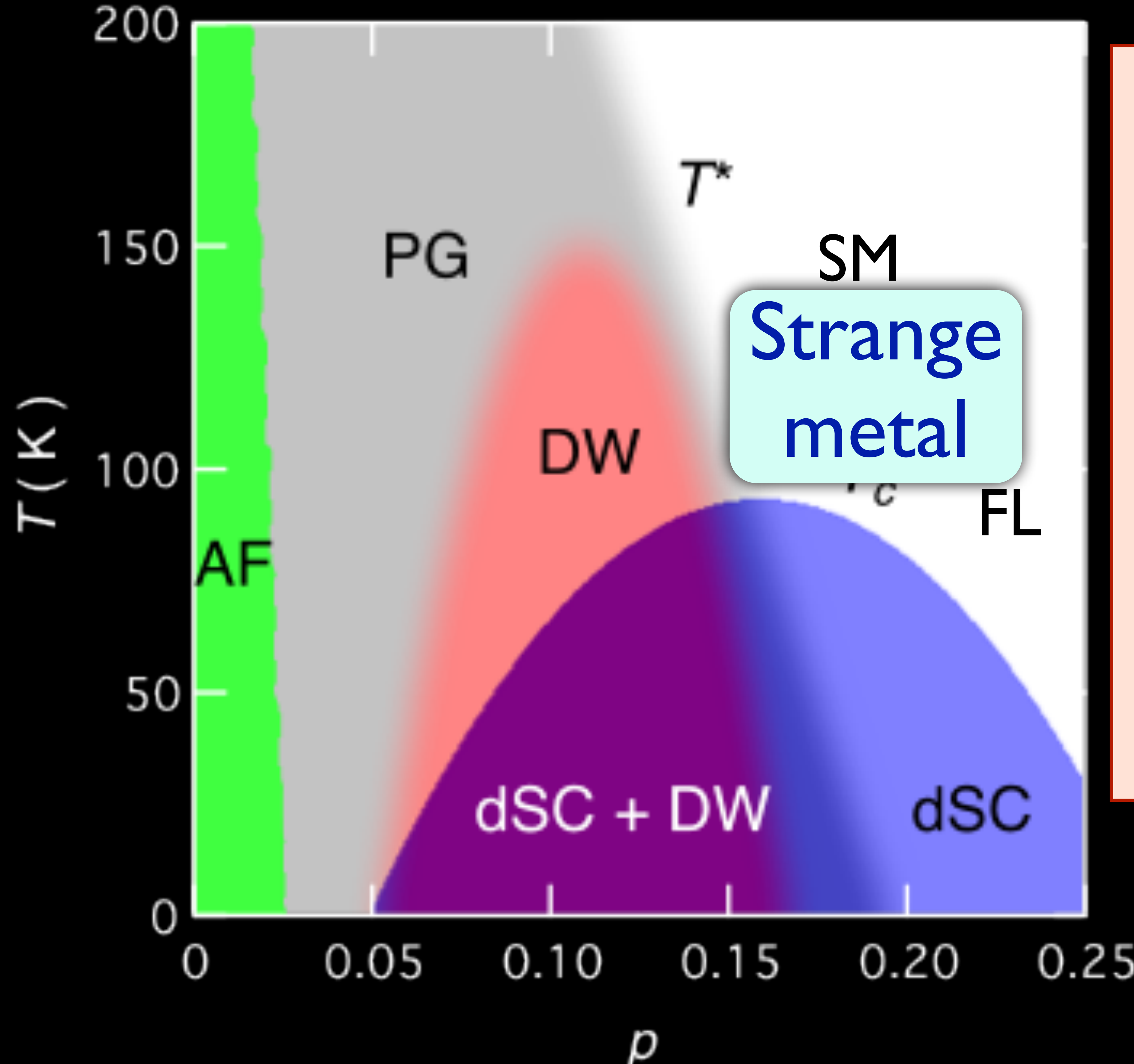


The high T FL* can help explain the "d-form factor density wave" observed at low T

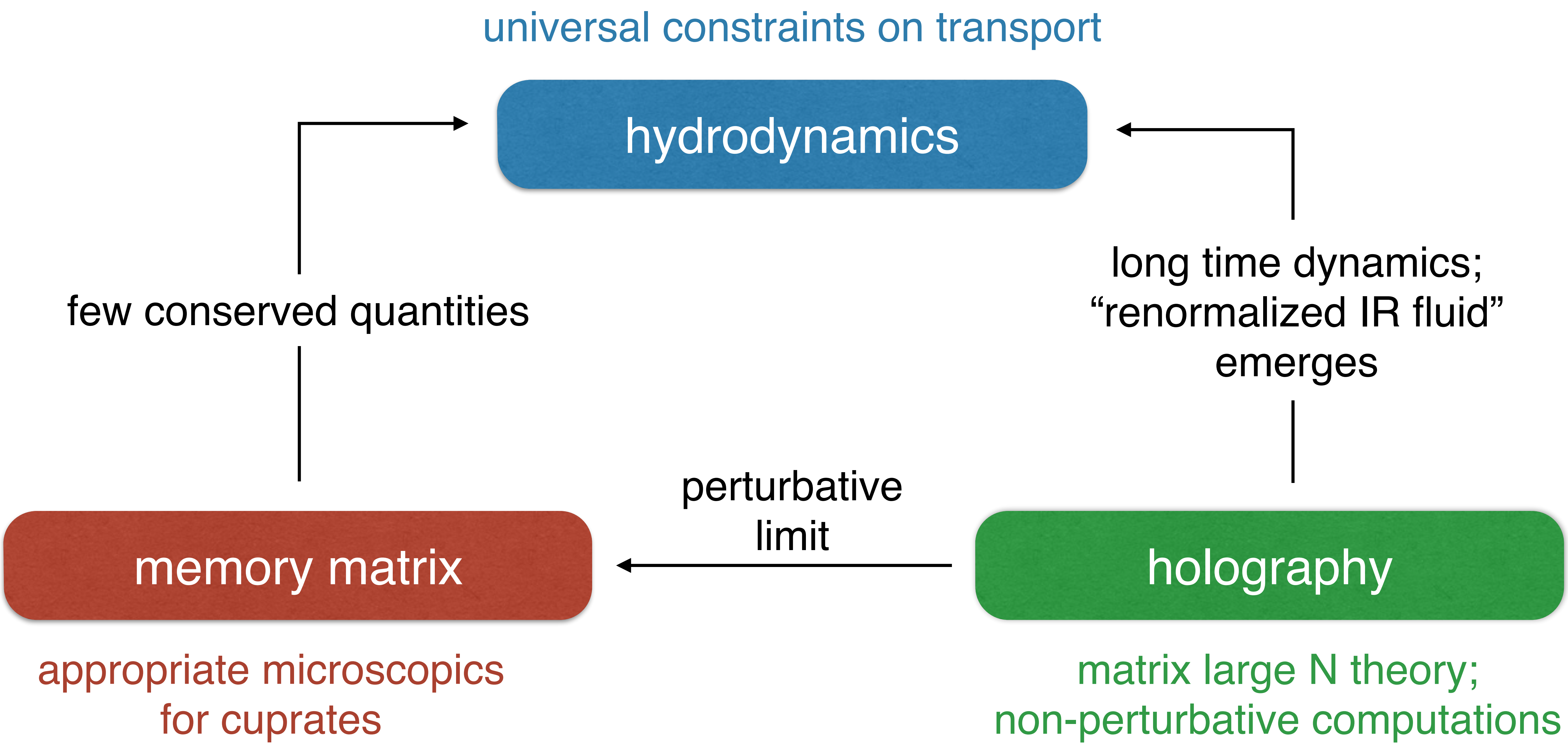








Metal (gapless, compressible state) without quasi-particles



A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Electrical transport at a strongly-coupled critical theory *with* particle-hole symmetry, obeying hyperscaling, in d spatial dimensions with dynamic critical exponent z

$$\sigma = \sigma_Q \sim T^{(d-2)/z}$$

Follows from gauge invariance

$(\sigma = 1/\rho = \text{conductivity})$

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)

Electrical transport at a strongly-coupled critical theory *without* particle-hole symmetry, *with* a conserved momentum P

$$\sigma = \sigma_Q + \frac{\mathcal{Q}^2}{\mathcal{M}} \pi \delta(\omega)$$

with $\mathcal{Q} \equiv \chi_{J_x, P_x}$ and $\mathcal{M} \equiv \chi_{P_x, P_x}$ thermodynamic response functions

Obtained in hydrodynamics, holography, and by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Electrical transport at a strongly-coupled critical theory *without* particle-hole symmetry, with an *almost* conserved momentum P

$$\sigma = \sigma_Q + \frac{\mathcal{Q}^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_L)}$$

with $\mathcal{Q} \equiv \chi_{J_x,P_x}$ and $\mathcal{M} \equiv \chi_{P_x,P_x}$ thermodynamic response functions

Obtained in hydrodynamics, holography, and by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Electrical transport at a strongly-coupled critical theory *without* particle-hole symmetry, with an *almost* conserved momentum P

$$\sigma = \sigma_Q + \frac{Q^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_L)}$$

Momentum relaxation by an external source h_L coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h_L(x) \, \mathcal{O}(x).$$

$$\frac{\mathcal{M}}{\tau_L} = \lim_{\omega \to 0} \int d^d q \, |h_L(q)|^2 q_x^2 \frac{\mathrm{Im}\left(G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(q,\omega)\right)_{H_0}}{\omega} + \text{higher orders in } h_L$$

Obtained by memory functions

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
S. A. Hartnoll and D. M. Hofman, PRL **108**, 241601 (2012)
A. Lucas, S. Sachdev and K. Schalm, PRD **89**, 066018 (2014)

Electrical transport at a strongly-coupled critical theory *without* particle-hole symmetry, with an *almost* conserved momentum P, and an applied magnetic field B

$$\sigma_{xx} = \frac{(\tau_L^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau_L^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}\mathcal{M}\left(\frac{1}{\tau_L} - i\omega\right),$$

$$\sigma_{xy} = \frac{2(\tau_L^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau_L^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}B\mathcal{Q}.$$

Obtained in hydrodynamics, holography, and by memory functions

.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Blake and A. Donos, PRL **114**, 021601 (2015)
A. Lucas and S. Sachdev, arXiv:1502.04704

Electrical transport at a strongly-coupled critical theory *without* particle-hole symmetry, with an *almost* conserved momentum P, and an applied magnetic field B

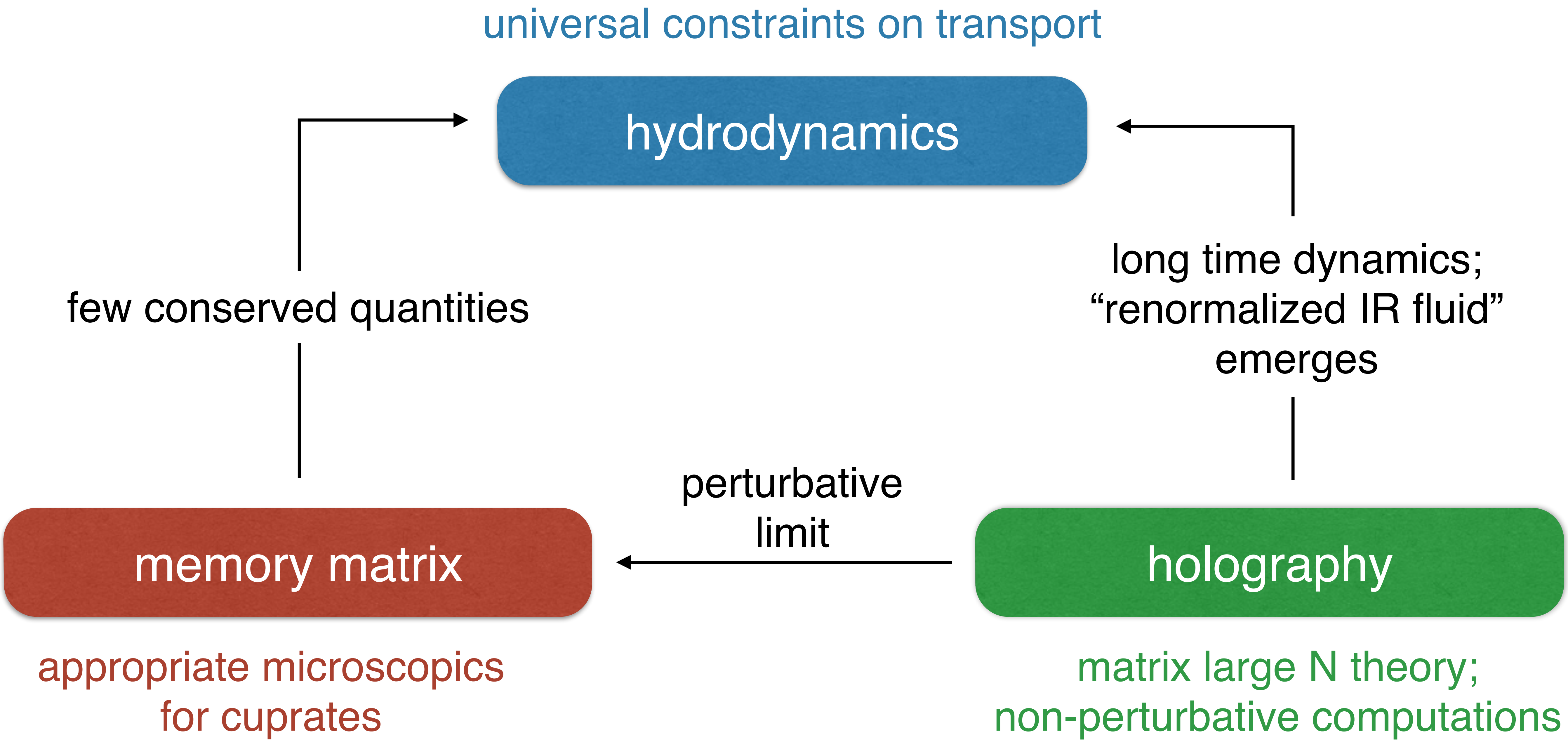
$$\sigma_{xx} = \frac{(\tau_L^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau_L^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}\mathcal{M}\left(\frac{1}{\tau_L} - i\omega\right),$$

$$\sigma_{xy} = \frac{2(\tau_L^{-1} - i\omega)\mathcal{M}\sigma_Q + \mathcal{Q}^2 + B^2\sigma_Q^2}{\mathcal{Q}^2 B^2 + ((\tau_L^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}B\mathcal{Q}.$$

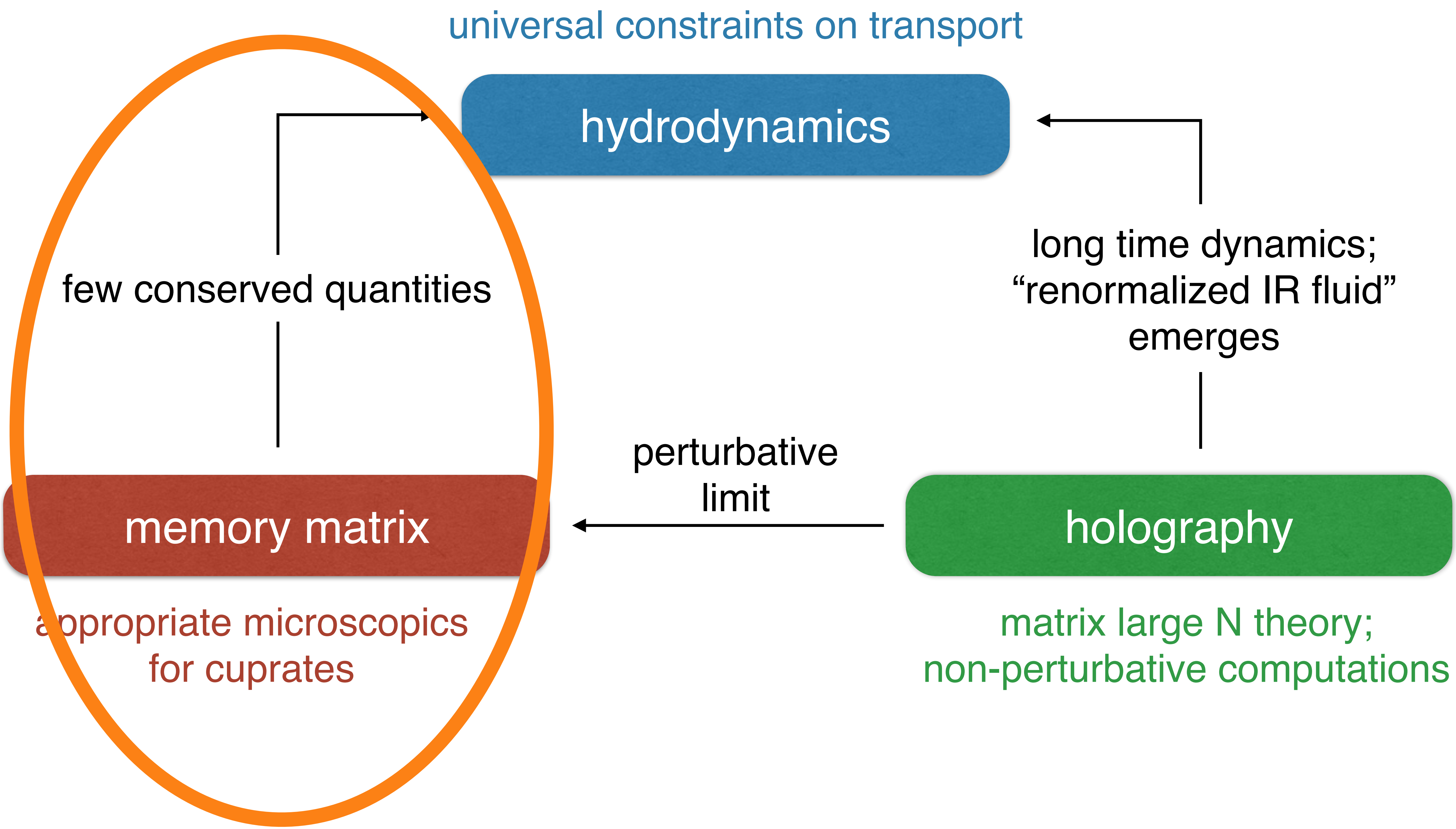
Blake and Donos: With $\sigma_Q \sim 1/T$ and $\tau_L \sim 1/T^2$, we obtain $\sigma_{xx} \sim 1/T$ and $\tan(\theta_H) = \sigma_{xy}/\sigma_{xx} \sim 1/T^2$, in agreement with strange metal data on cuprates (such data cannot be explained in a quasiparticle model).

Obtained in hydrodynamics, holography, and by memory functions

.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Blake and A. Donos, PRL **114**, 021601 (2015)
A. Lucas and S. Sachdev, arXiv:1502.04704

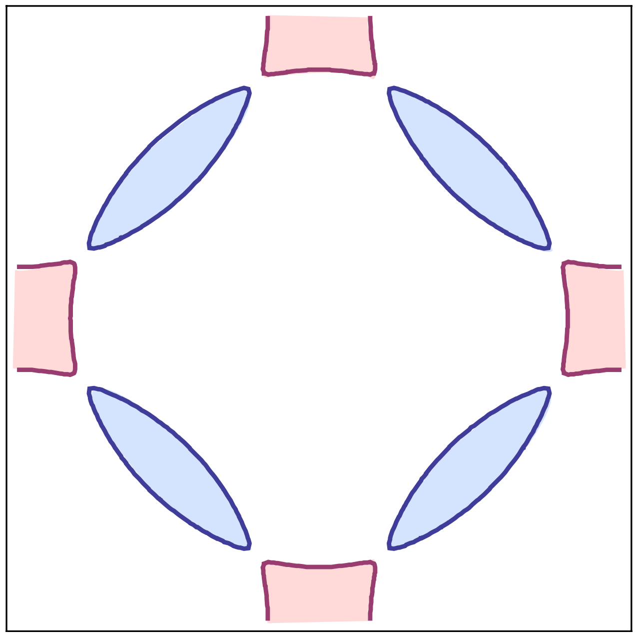


A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)



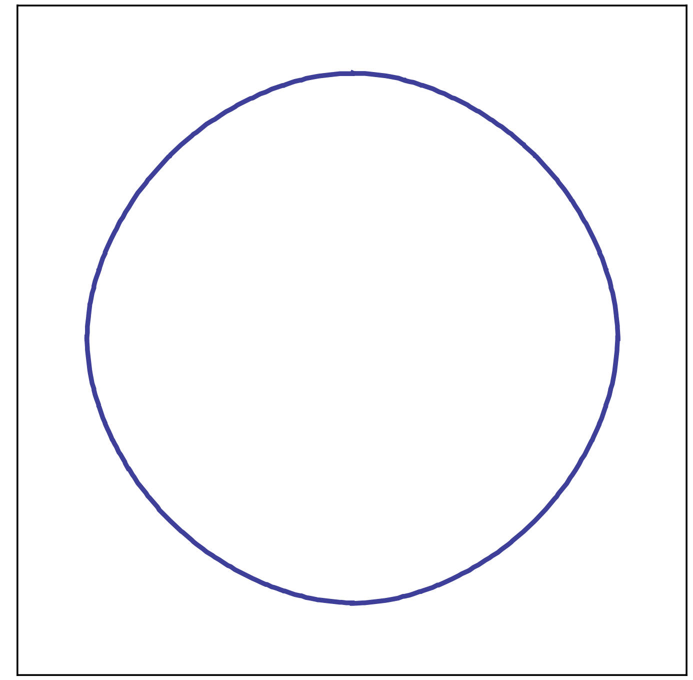
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Fermi surface+antiferromagnetism



$\langle \vec{\varphi} \rangle \neq 0$

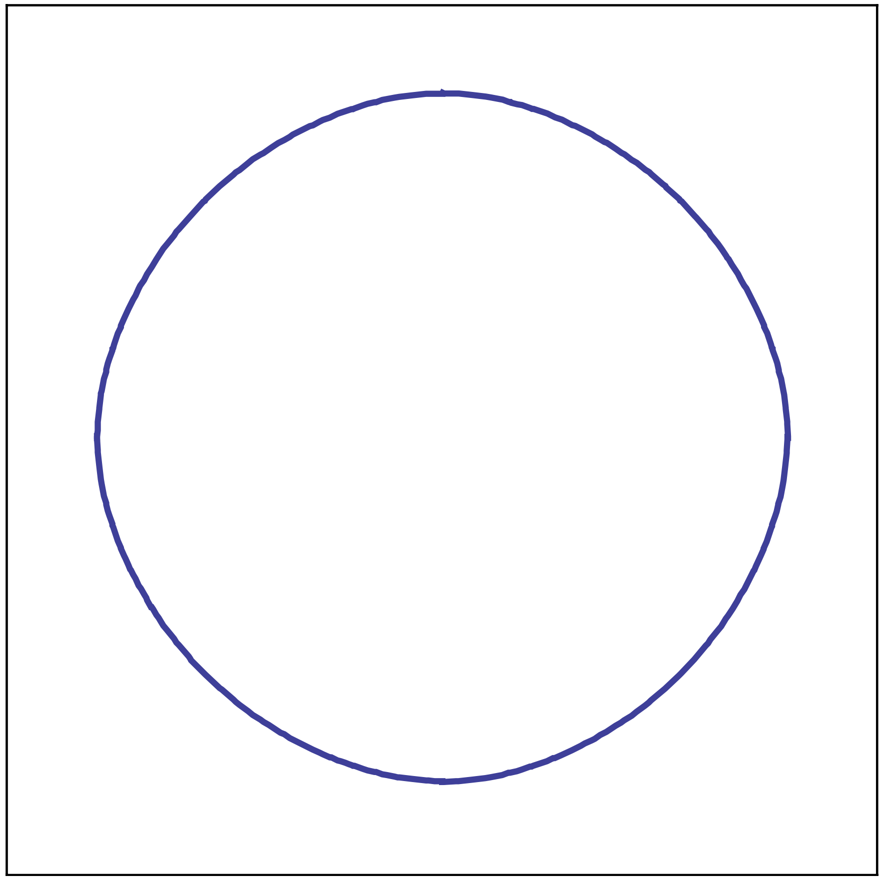
Metal with electron and hole pockets



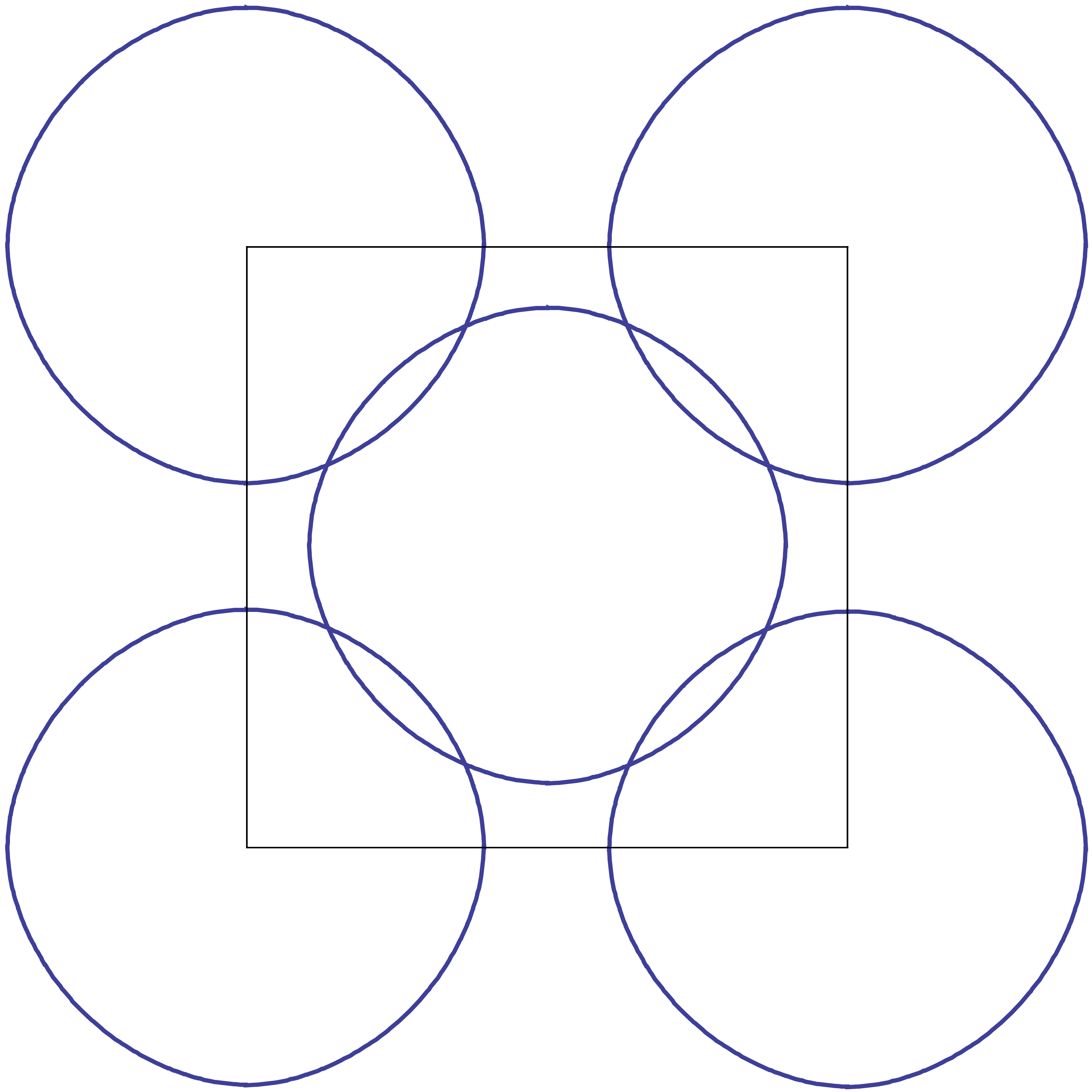
$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

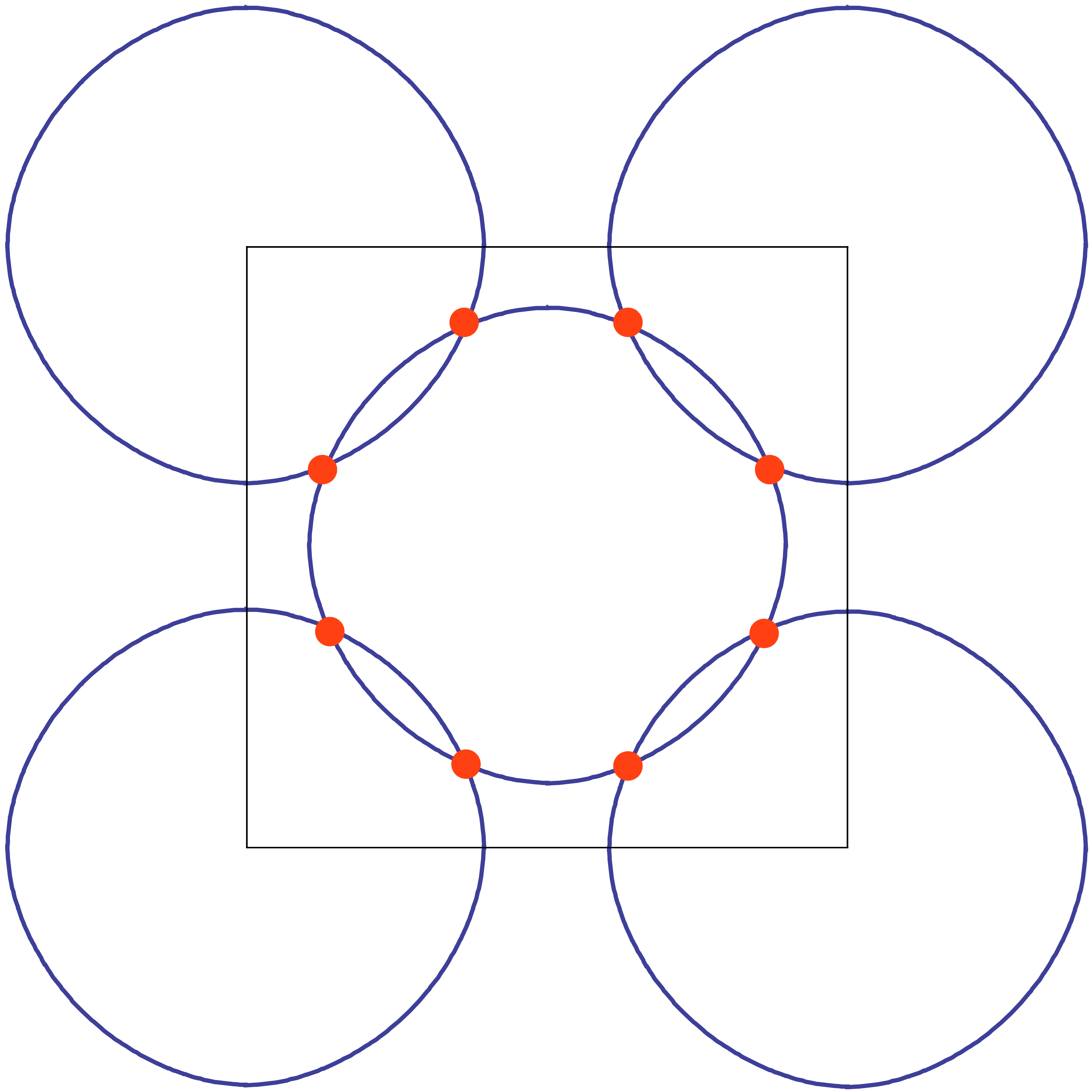
Increasing interaction



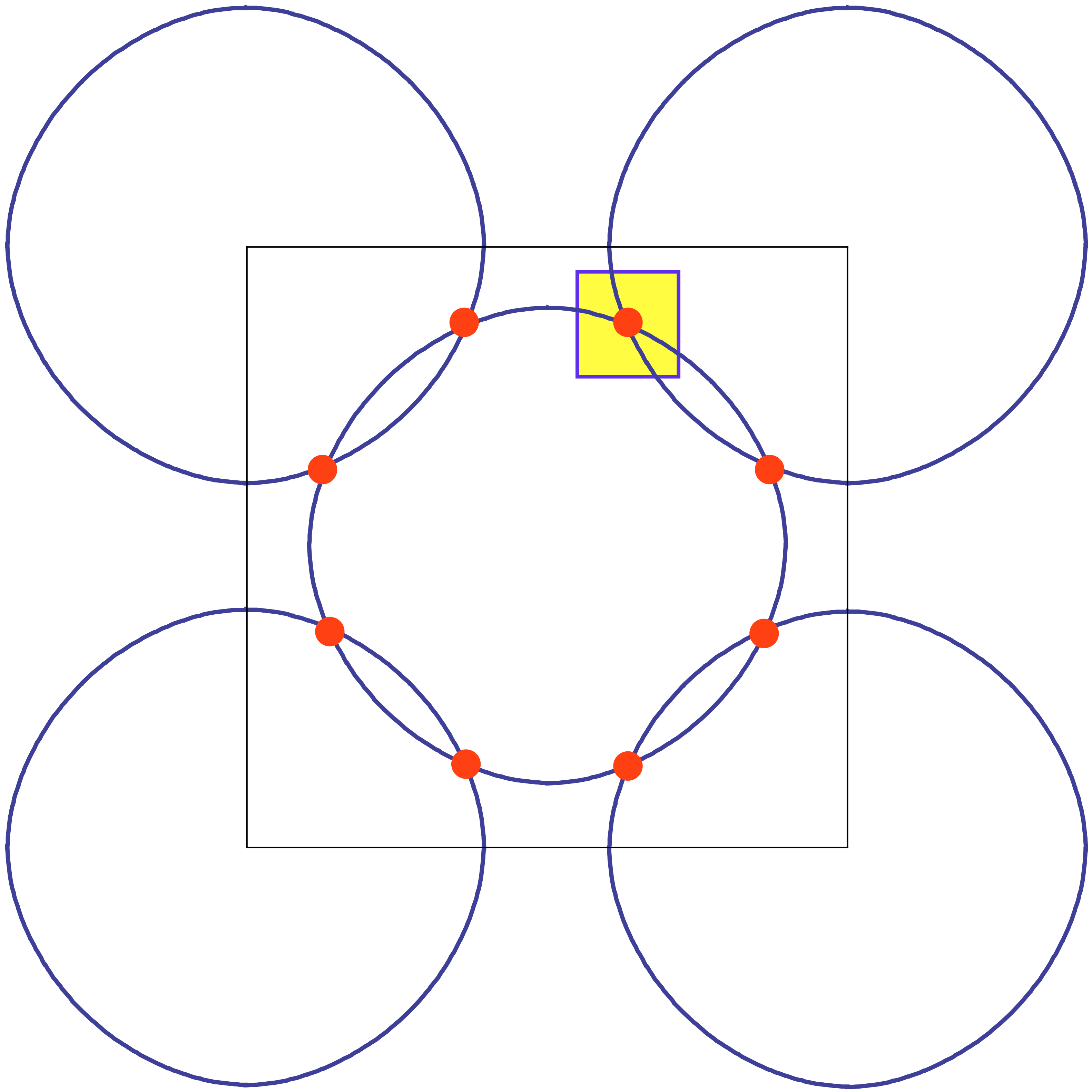
Metal with “large” Fermi surface



Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

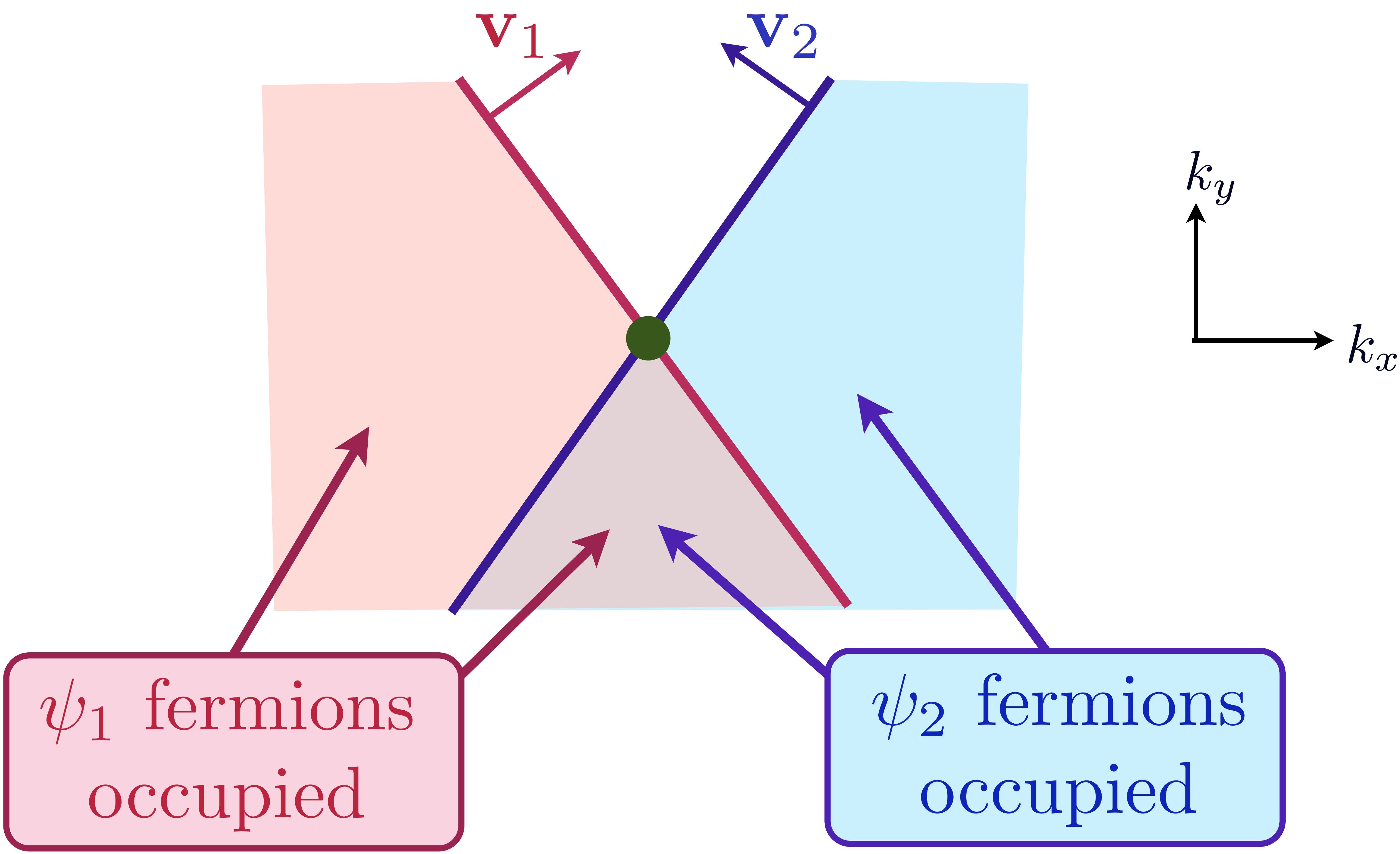


“Hot” spots



Low energy theory for critical point near hot spots

Hot-spot theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, and a "Yukawa" coupling λ. This theory is particle-hole symmetric.



$$\sigma = \sigma_Q + \frac{\mathcal{Q}^2}{\mathcal{M}} \pi \delta(\omega)$$

There is a natural separation into the two contributions to transport:

- Particle-hole symmetric hot-spot theory yields the value of σ_Q.
- Remaining "cold" regions of the Fermi surface yield the contribution of the (nearly) conserved momentum mode.

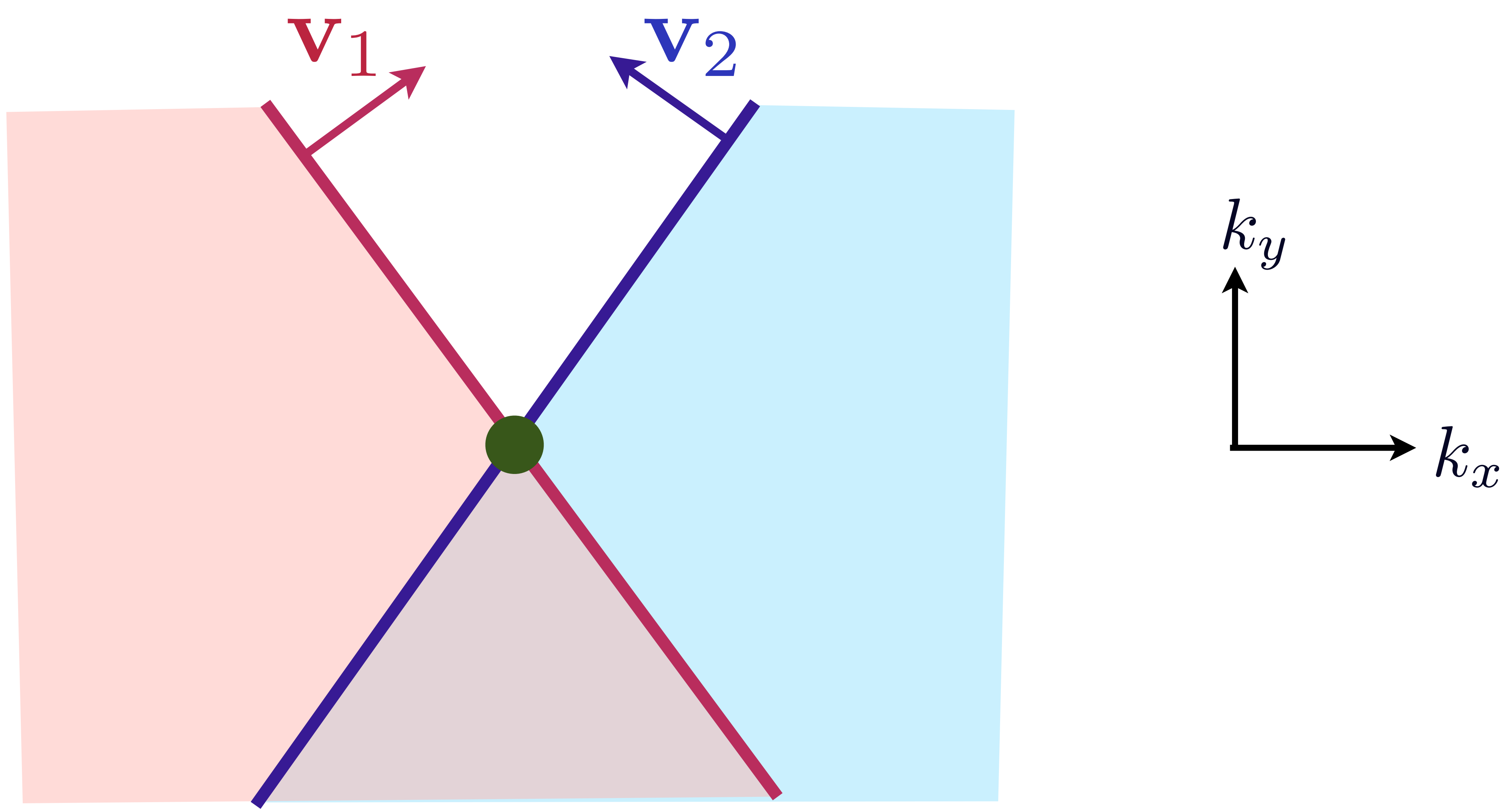
$$\sigma = \sigma_Q + \frac{\mathcal{Q}^2}{\mathcal{M}} \pi \delta(\omega)$$

There is a natural separation into the two contributions to transport:

- Particle-hole symmetric hot-spot theory yields the value of σ_Q.
- Remaining "cold" regions of the Fermi surface yield the contribution of the (nearly) conserved momentum mode.

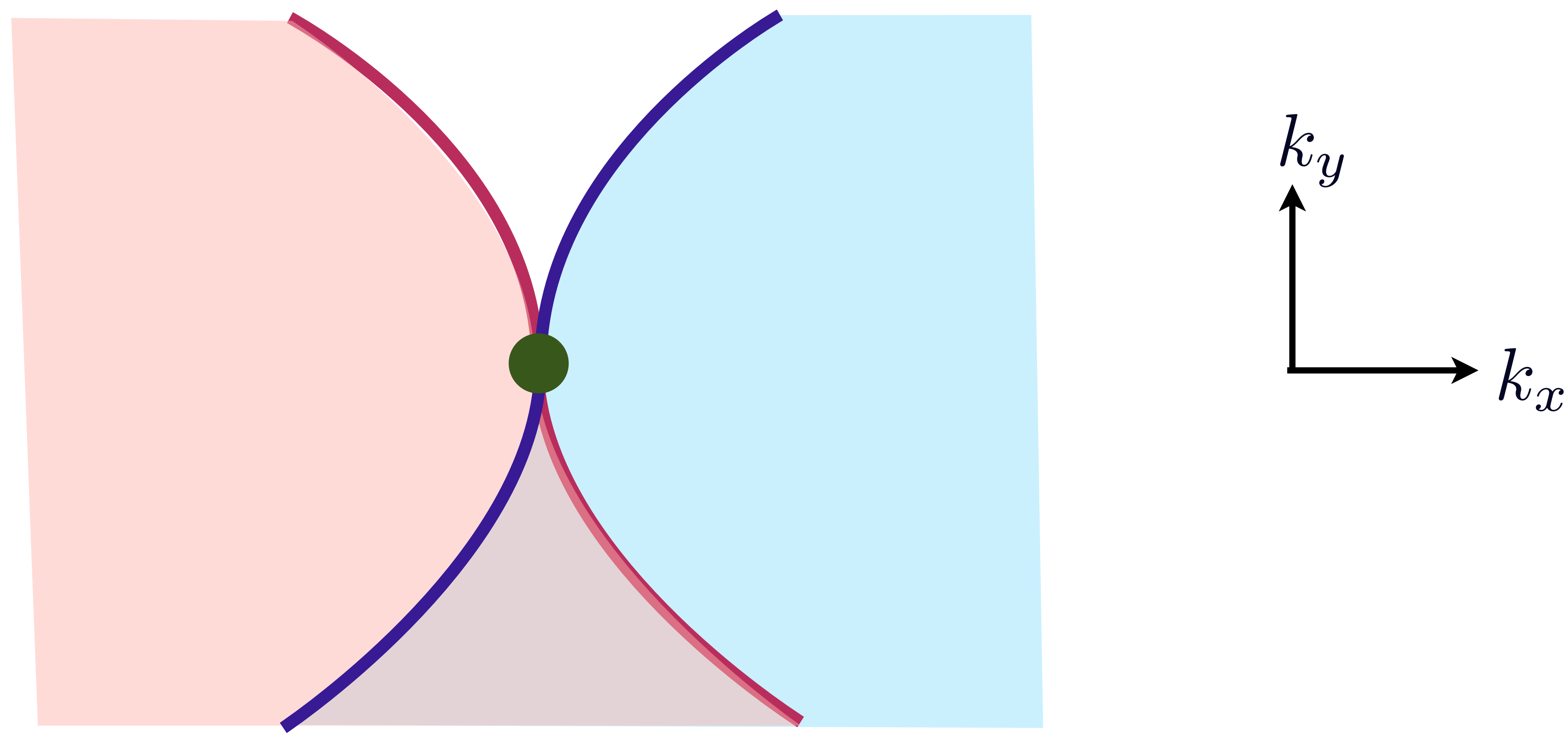
<u>But</u>, all known particle-hole symmetric, strongly coupled critical theories obey hyperscaling, and so have $\sigma_Q \sim T^0$ in $d = 2$.

Hot-spot theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, and a “Yukawa” coupling λ. This theory is particle-hole symmetric.



Hot-spot theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, and a "Yukawa" coupling λ. This theory is particle-hole symmetric.

Renormalized Fermi surface has $k_y \sim k_x \ln(1/k_x)$



M.A. Metlitski and S. Sachdev, PRB **85**, 075127 (2010)

A RG fixed point for the spin density wave critical point has recently been found by Shouvik Sur and Sung-Sik Lee (PRB **91**, 125136 (2015)) using a novel ϵ-expansion.

- We find that the presence of gapless lines of zero energy excitations at this fixed point leads to hyper-scaling violation only in the limit of vanishing boson velocity.

- Upto logarithmic corrections, the hyperscaling violation leads to the entropy density $S \sim T^{(2-\theta)/z}$, and $\sigma_Q \sim T^{-\theta/z}$ with $\theta = 1$, and $z = 1 + \mathcal{O}(\epsilon)$.

A. A. Patel, S. Sachdev, and P. Strack, to appear universal constraints on transport

hydrodynamics

few conserved quantities

long time dynamics;
“renormalized IR fluid”
emerges

perturbative
limit

memory matrix

holography

appropriate microscopics
for cuprates

matrix large N theory;
non-perturbative computations

A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

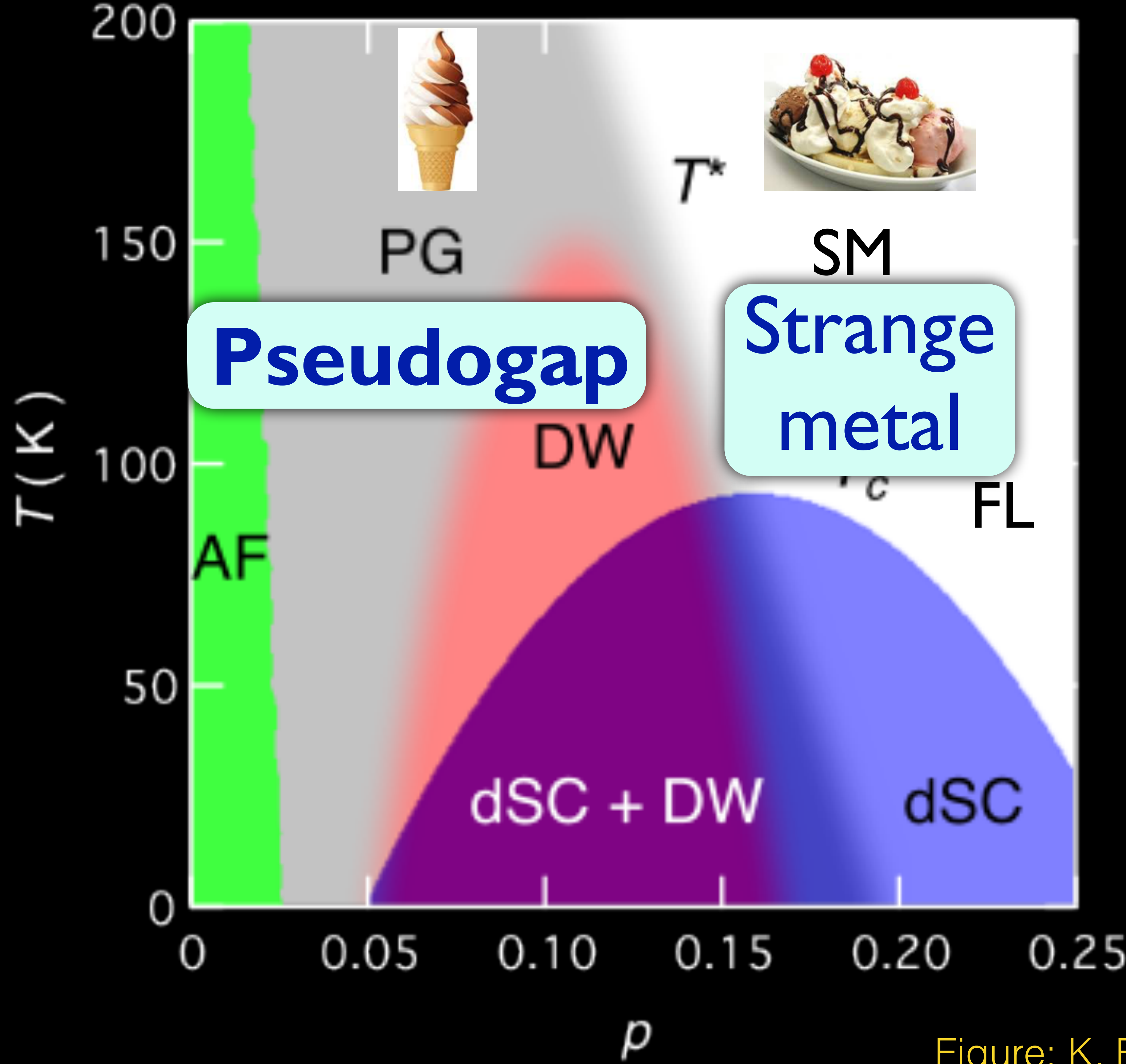


Figure: K. Fujita and J. C. Seamus Davis

Conclusions

1. Predicted *d*-form factor density wave order observed in the non-La hole-doped cuprate superconductors.

2. Proposed the pseudogap metal is a fractionalized Fermi liquid (FL*): a Fermi liquid co-existing with topological order.

3. Can we experimentally detect possible "topological order" in the pseudogap metal ? (topological order is directly linked to Fermi surface size)

4. Hydrodynamic, memory-function, holographic, and field-theoretic approaches to transport without quasiparticles