

This is the most general metric which is invariant under the scale transformation

$$\begin{aligned}x_i &\rightarrow \zeta x_i \\ t &\rightarrow \zeta^z t \\ ds &\rightarrow \zeta^{\theta/d} ds.\end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points). We will see shortly that θ is the violation of hyperscaling exponent.

We have used reparametrization invariance in r to define it so that it scales as

$$r \rightarrow \zeta^{(d-\theta)/d} r.$$