Entanglement, holography, and the quantum phases of matter

Perimeter Institute
July 13, 2012

PHYSICS
VE RI TAS
HARVARD

sachdev.physics.harvard.edu

Liza Huijse

Max Metlitski

Brian Swingle

"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

"Complex entangled" states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

topological field theory

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Strange metals, Bose metals

“Complex entangled” states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

topological field theory

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

conformal field theory

Compressible quantum matter

Strange metals, Bose metals

“Complex entangled” states of quantum matter, *not* adiabatically connected to independent particle states

Gapped quantum matter

Spin liquids, quantum Hall states

topological field theory

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

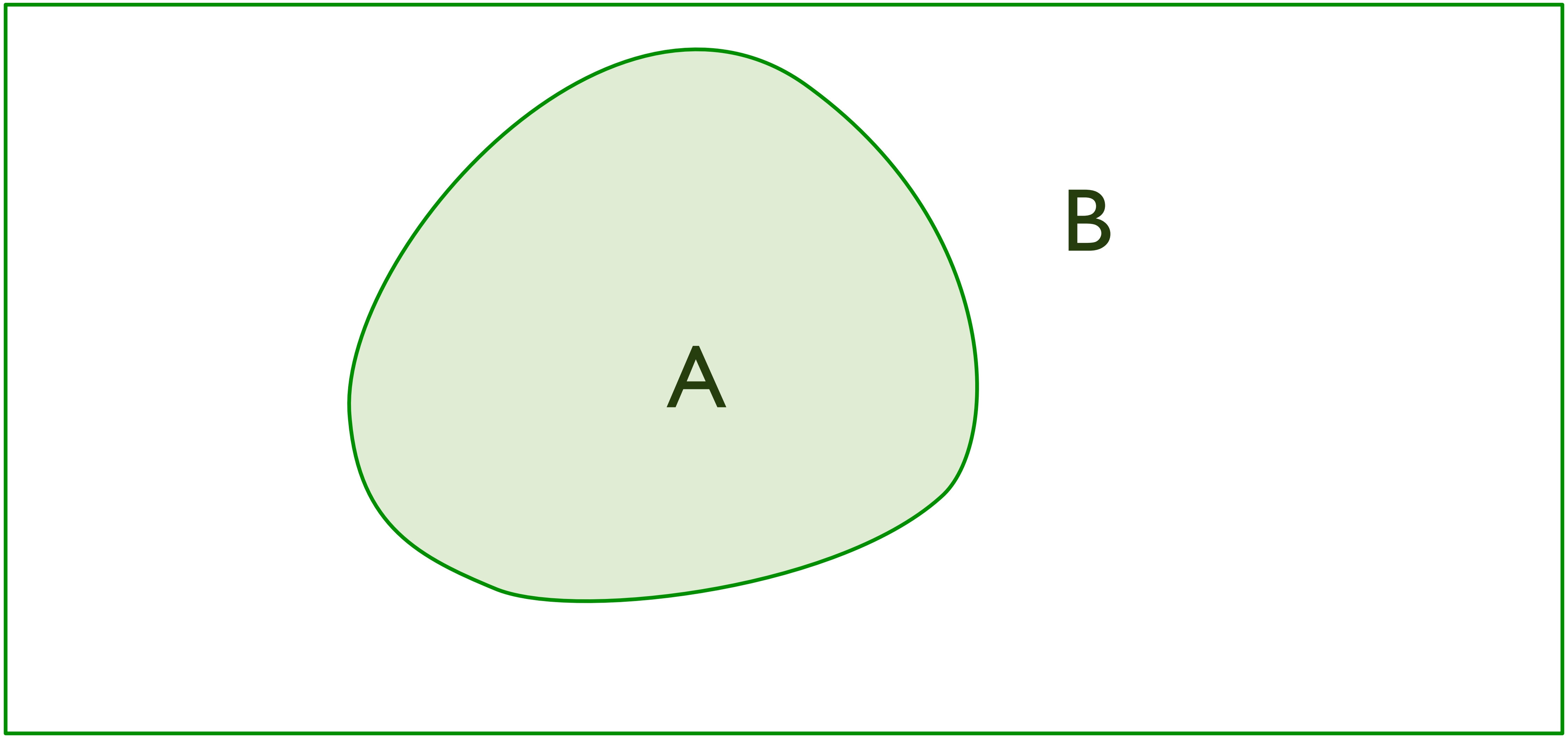
conformal field theory

Compressible quantum matter

Strange metals, Bose metals

?

Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

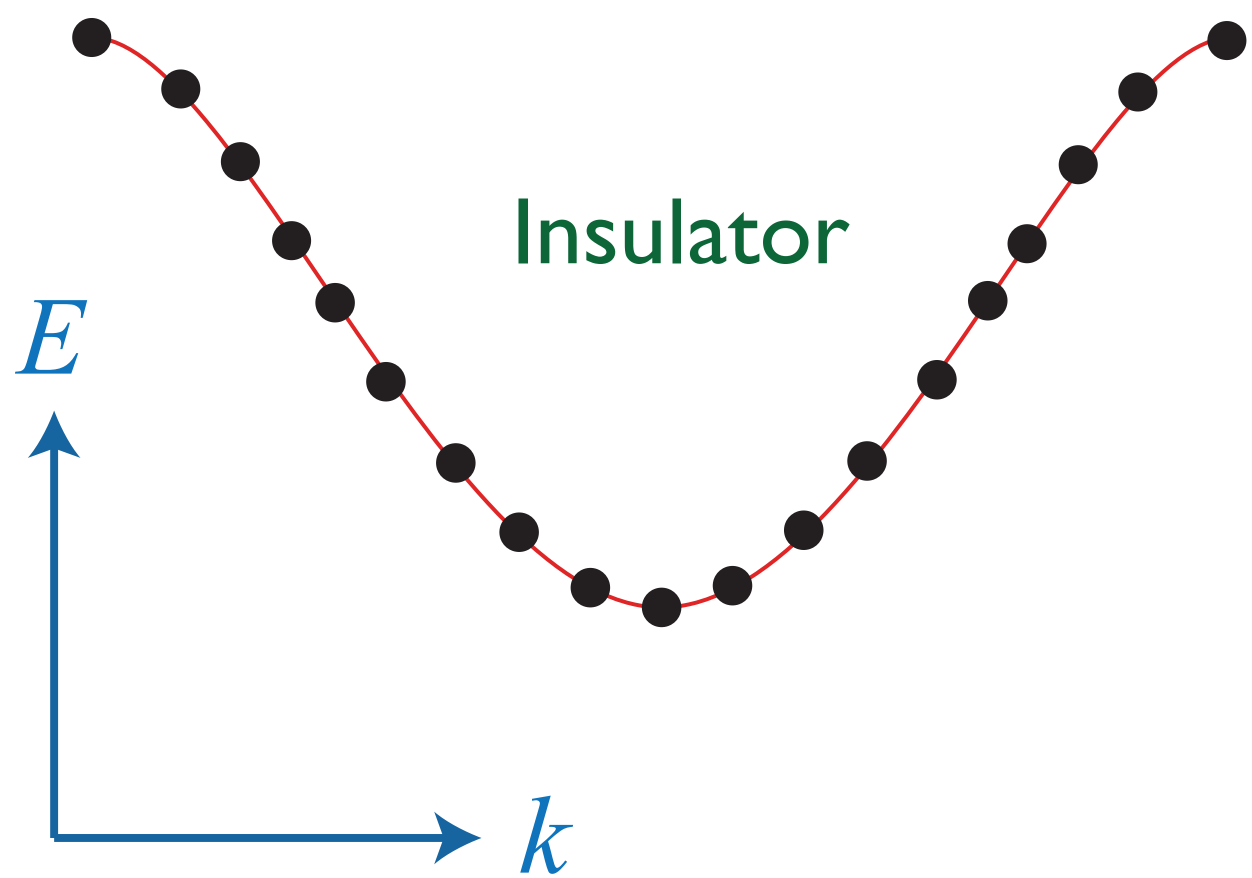
$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

Entanglement entropy $S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$

M. Srednicki, Phys. Rev. Lett. **71**, 666 (1993)

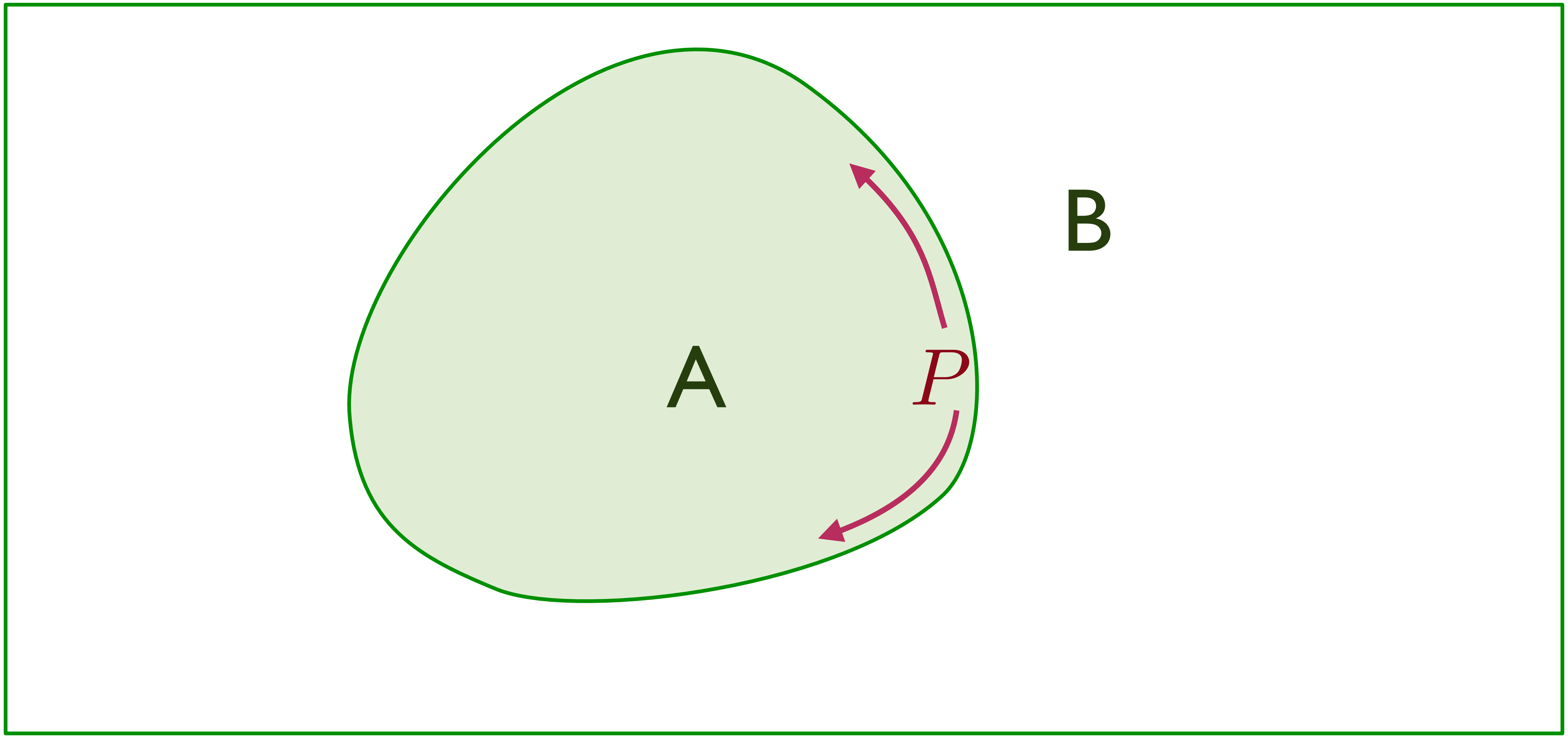
Gapped quantum matter

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator

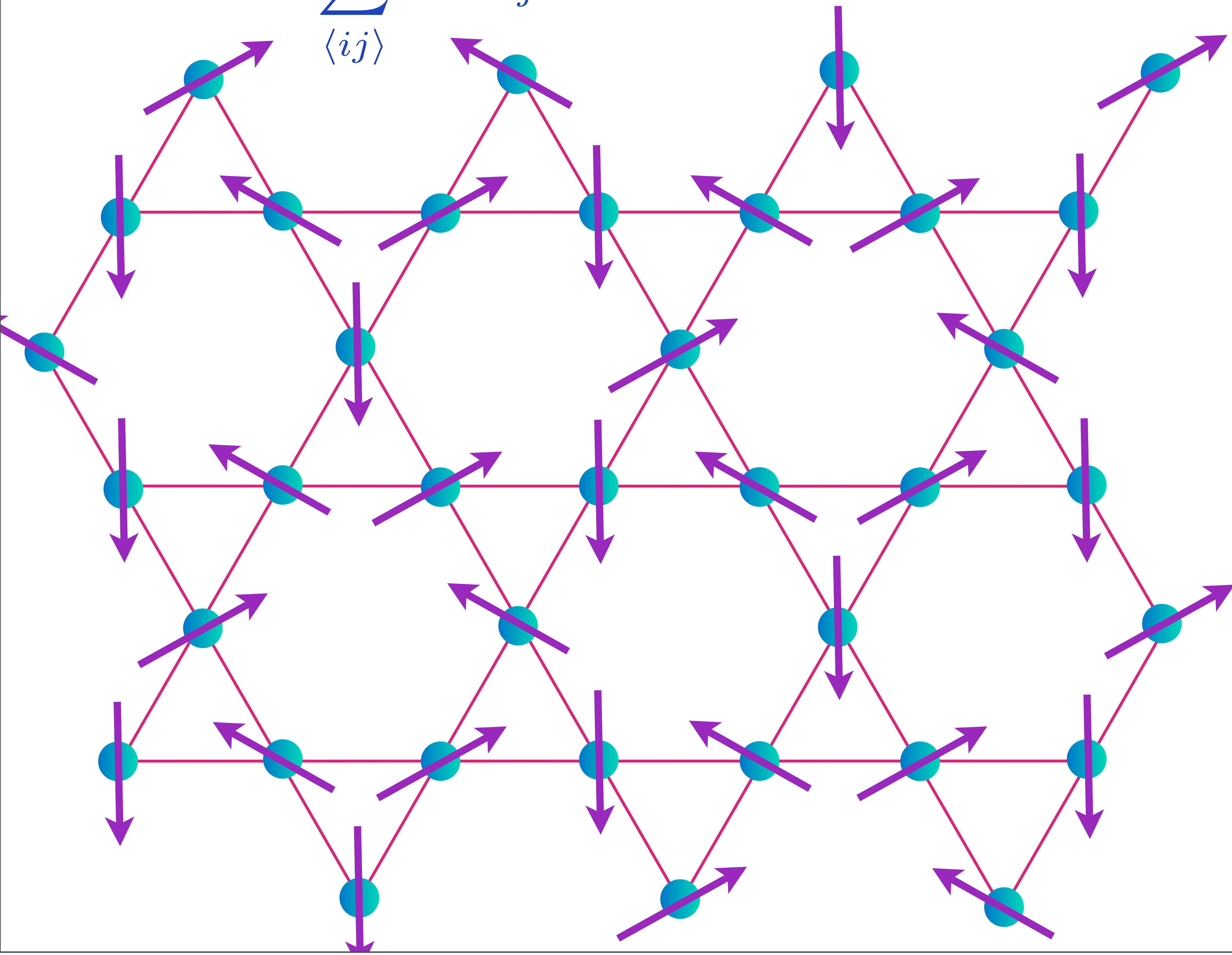


$$S_E = aP - b\exp(-cP)$$

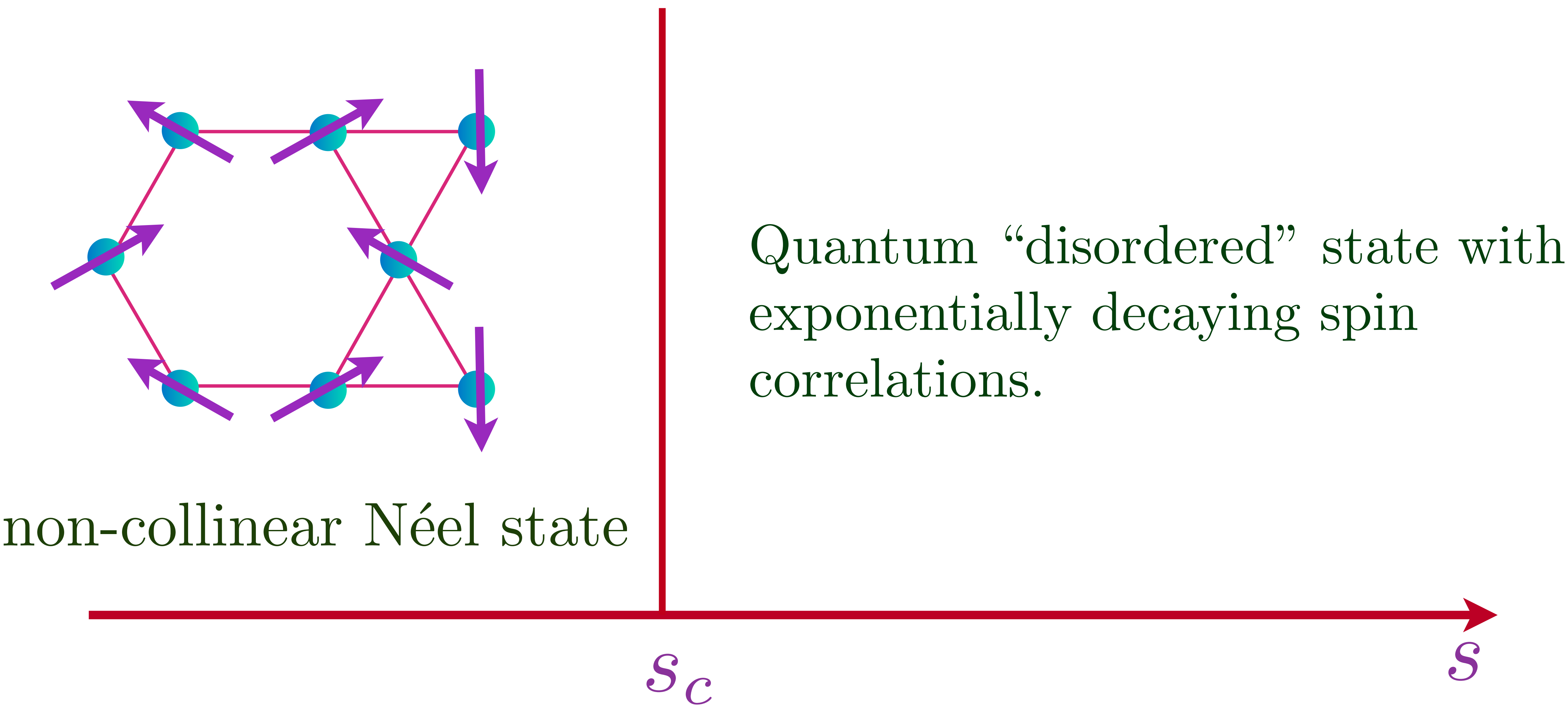
where P is the surface area (perimeter) of the boundary between A and B.

Kagome antiferromagnet

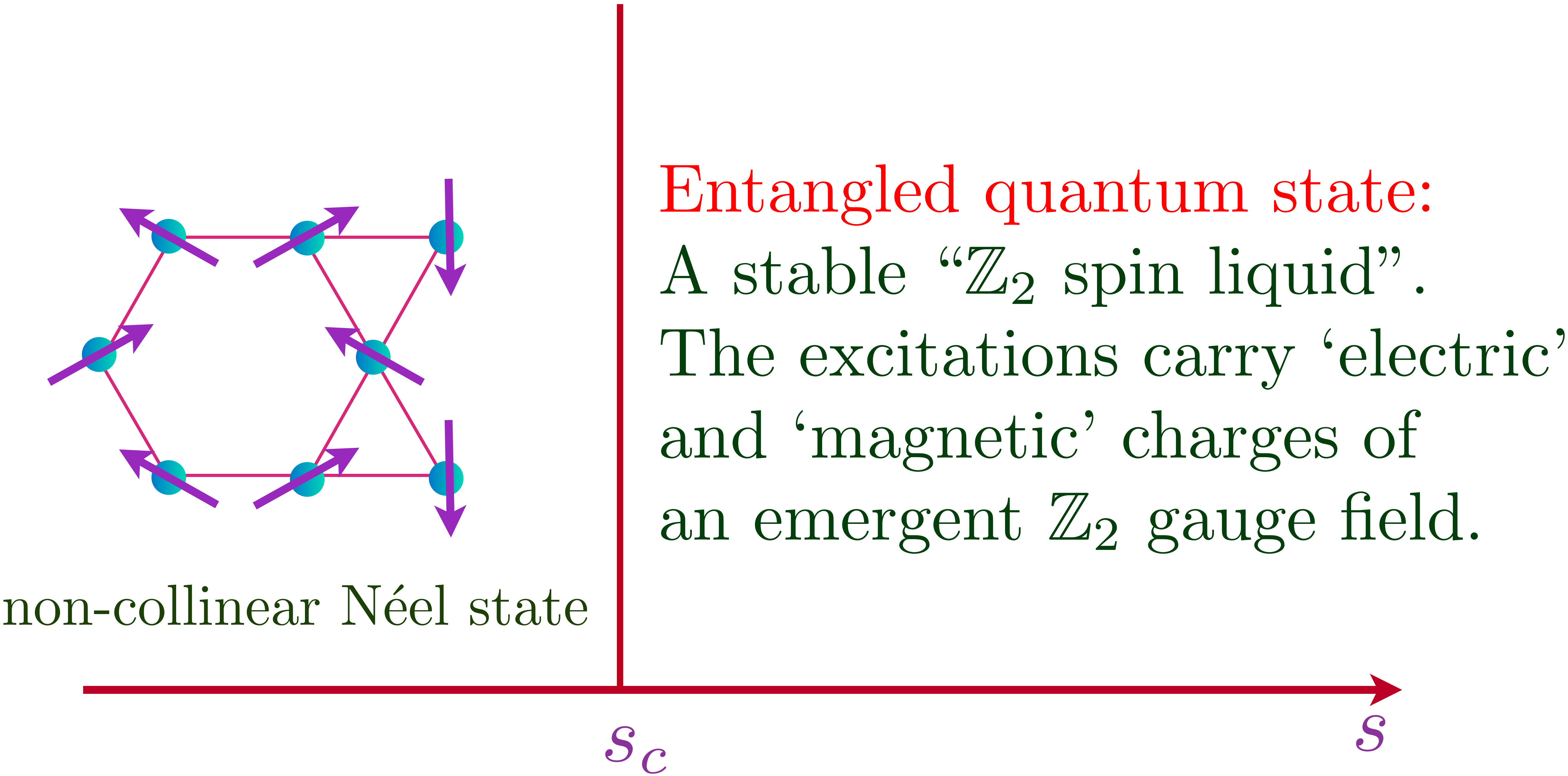
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Kagome antiferromagnet



Kagome antiferromagnet: Z_2 spin liquid

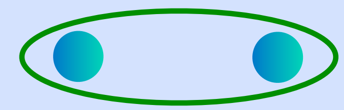


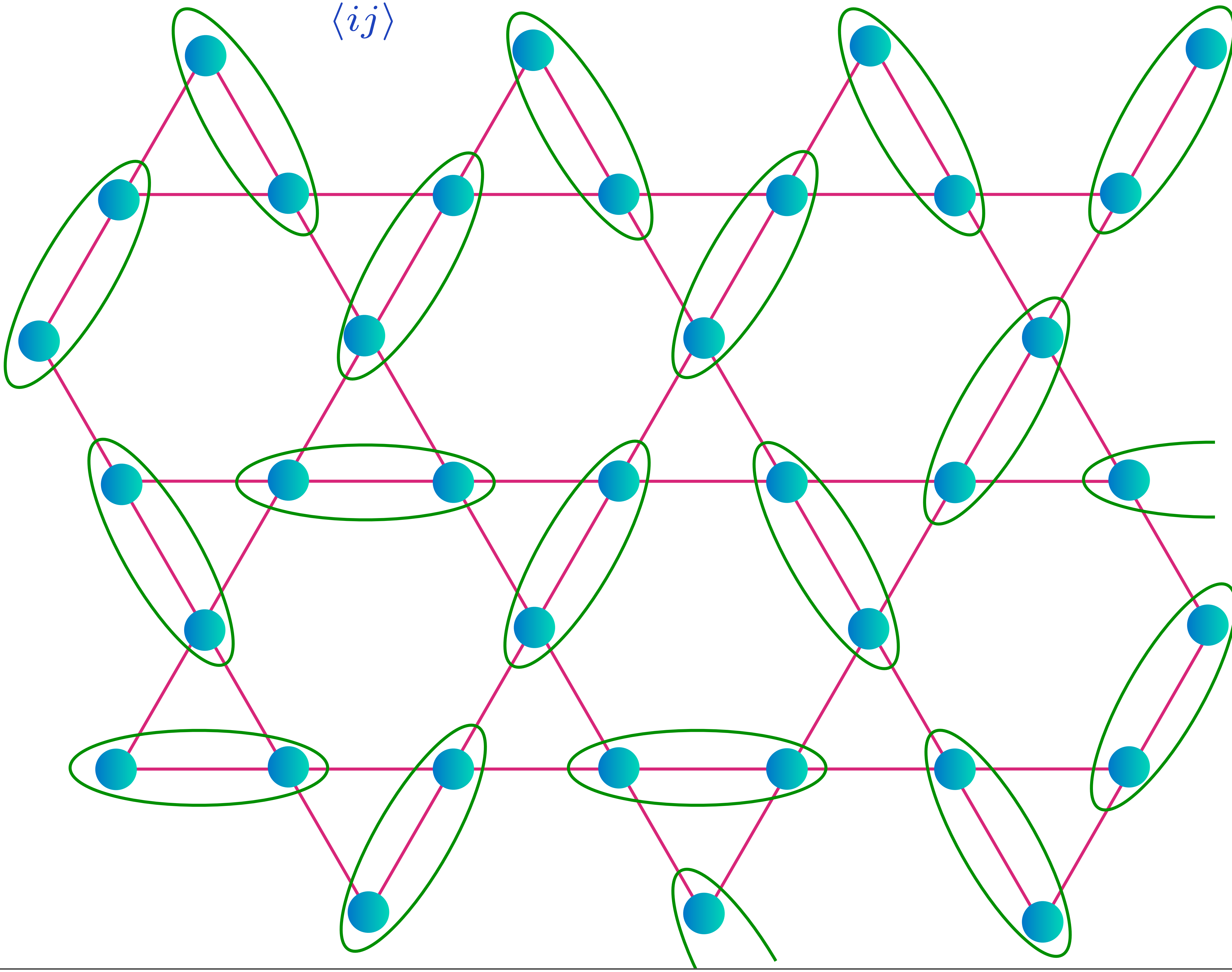
S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)
Y. Huh, M. Punk, and S. Sachdev, *Phys. Rev.* B **84**, 094419 (2011)

The Z_2 spin liquid was introduced in
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991),
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

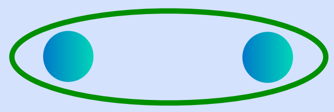
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



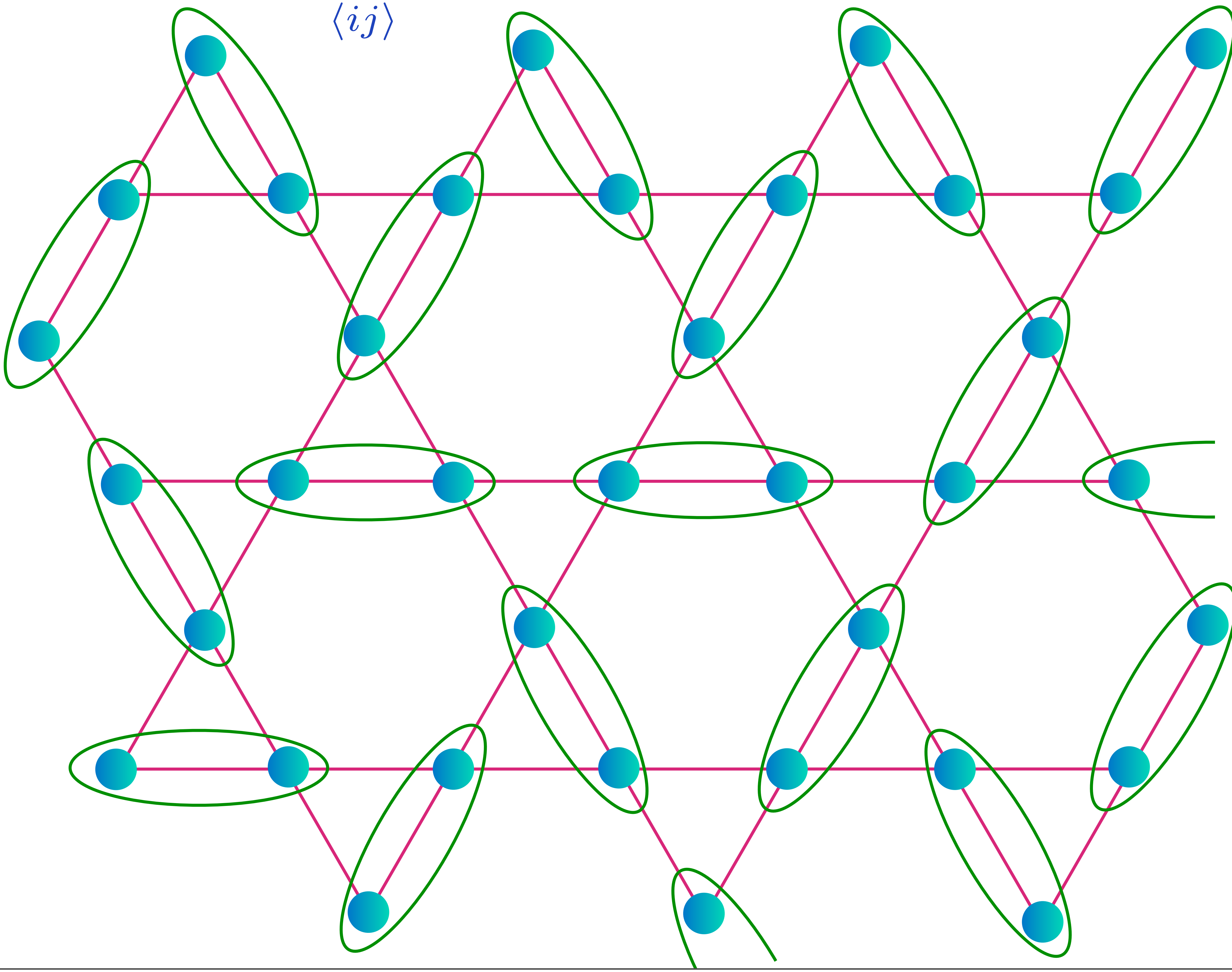
P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



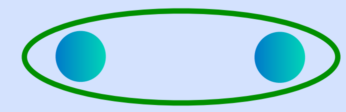
$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



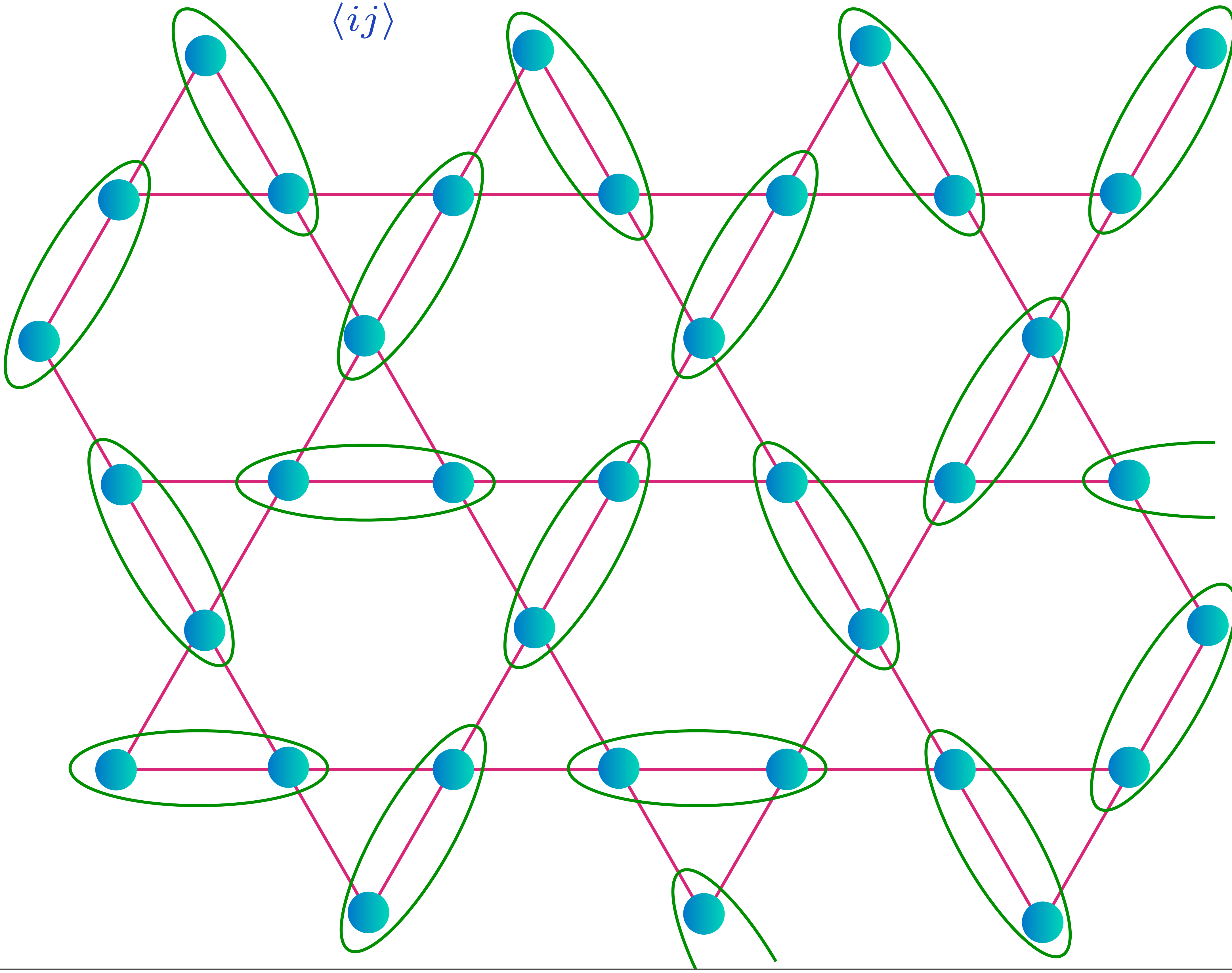
P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



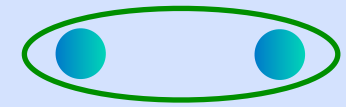
$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

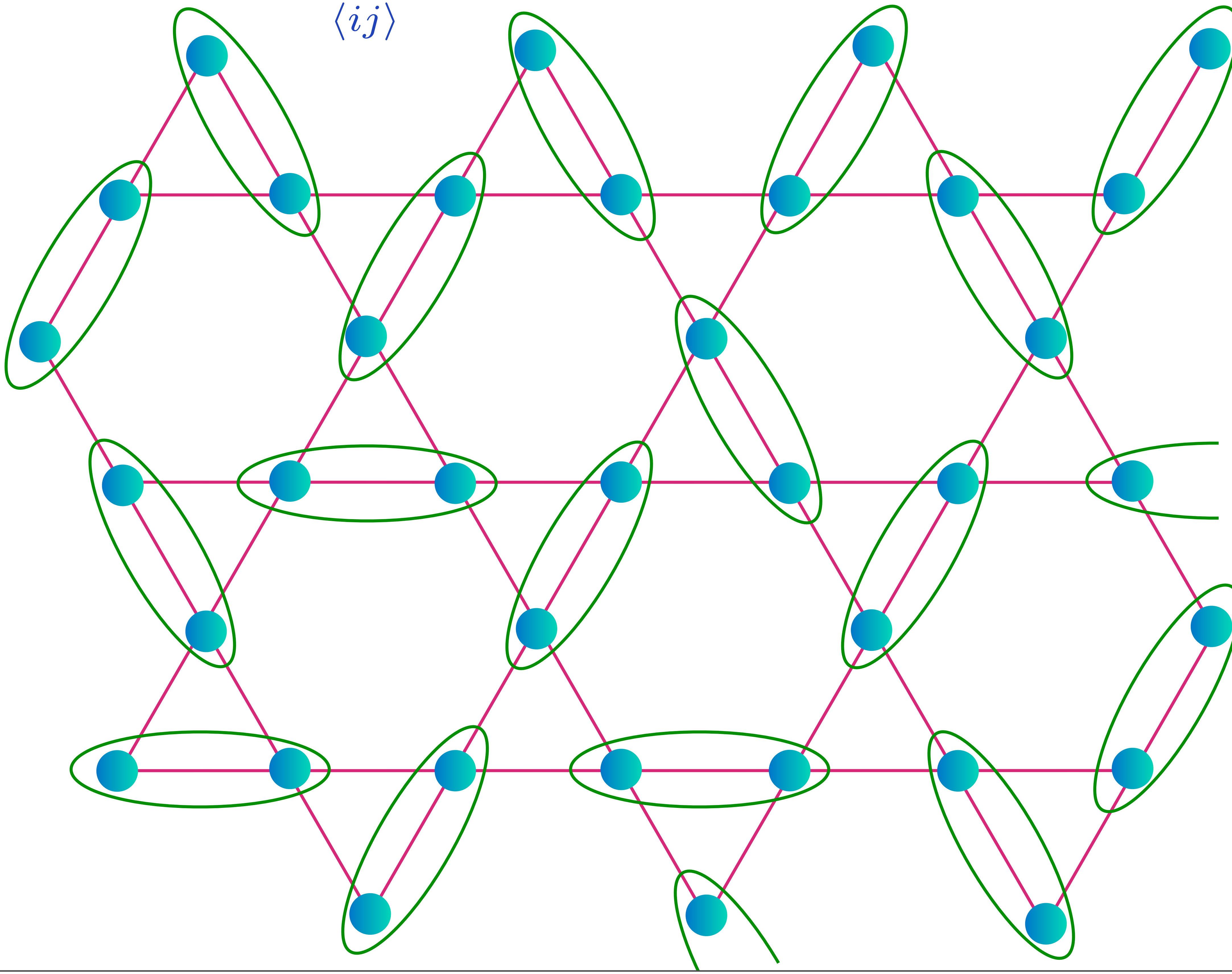


P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

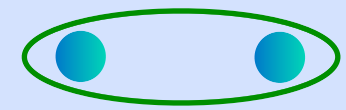
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



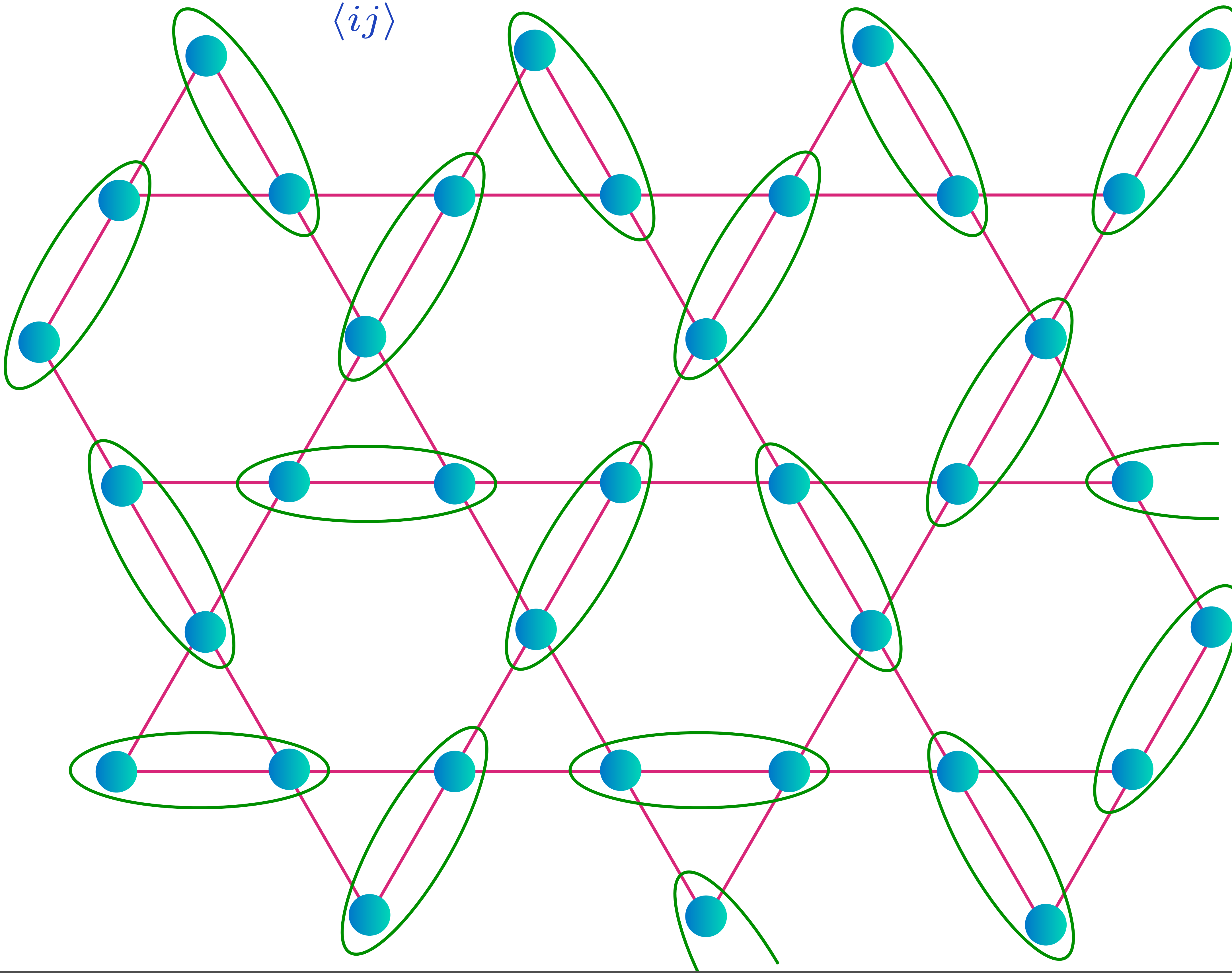
P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



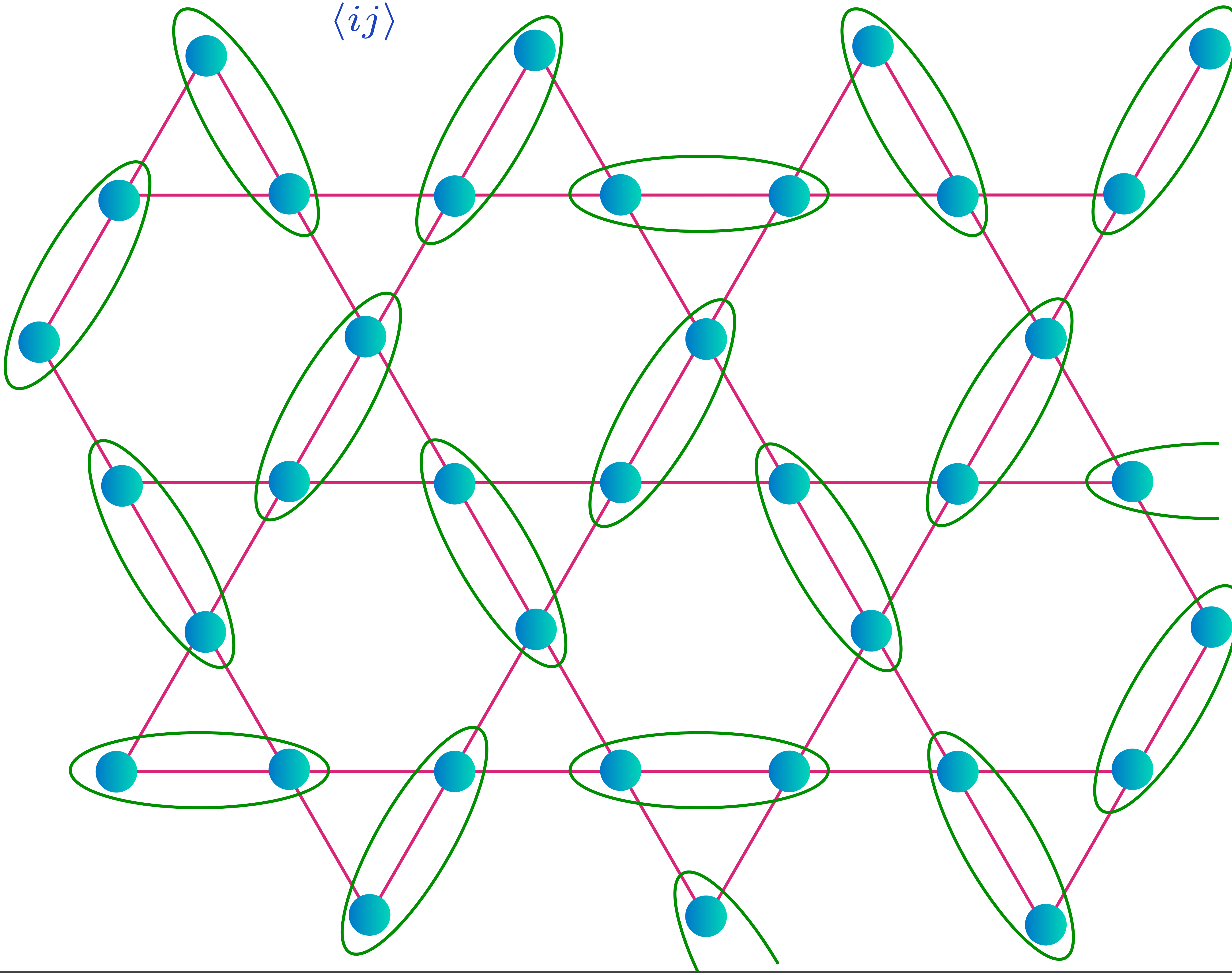
$$= \frac{1}{\sqrt{2}} \left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

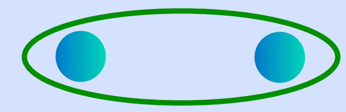


P. Fazekas and
P. W. Anderson,
Philos. Mag.
30, 23 (1974).

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

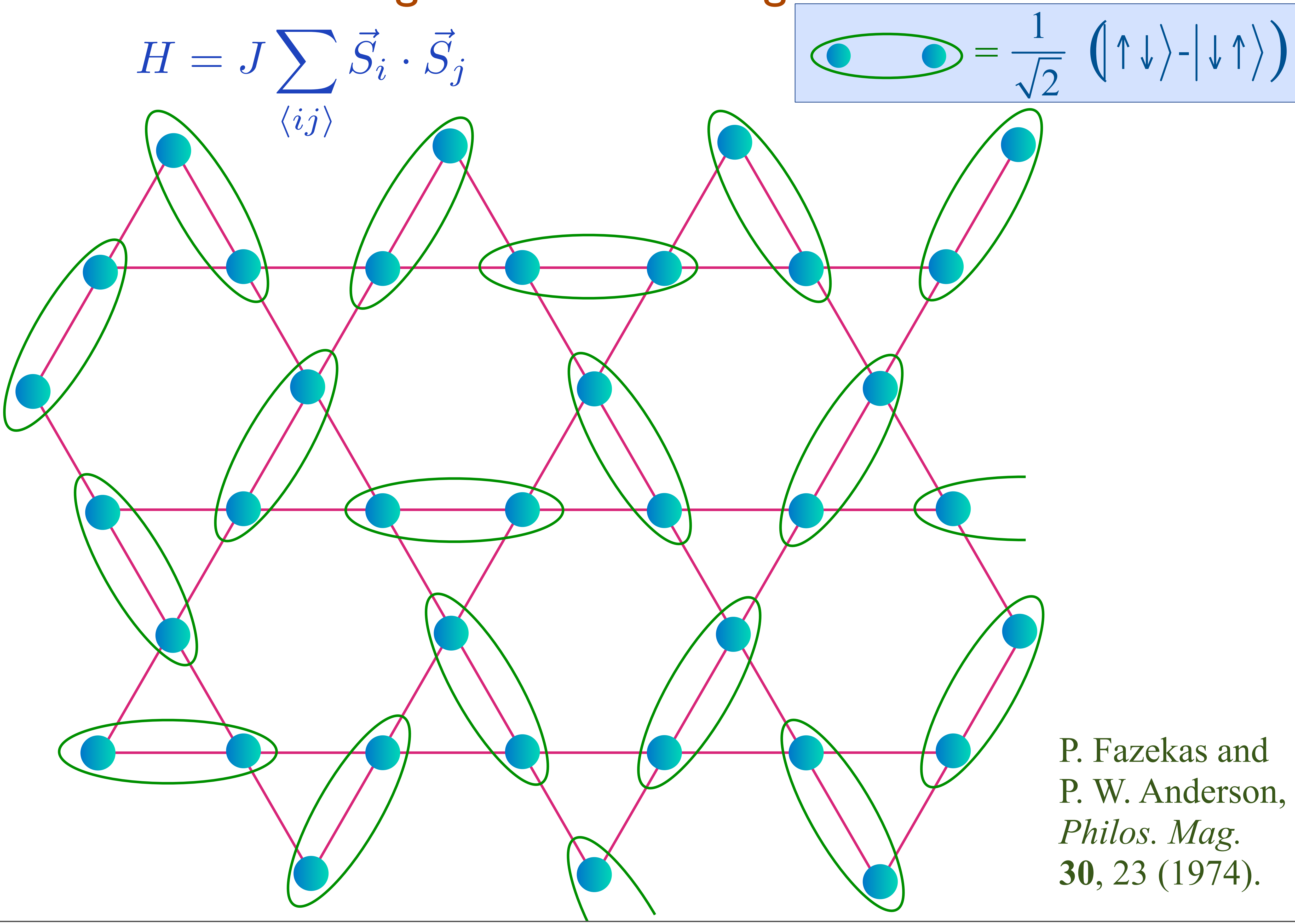




P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

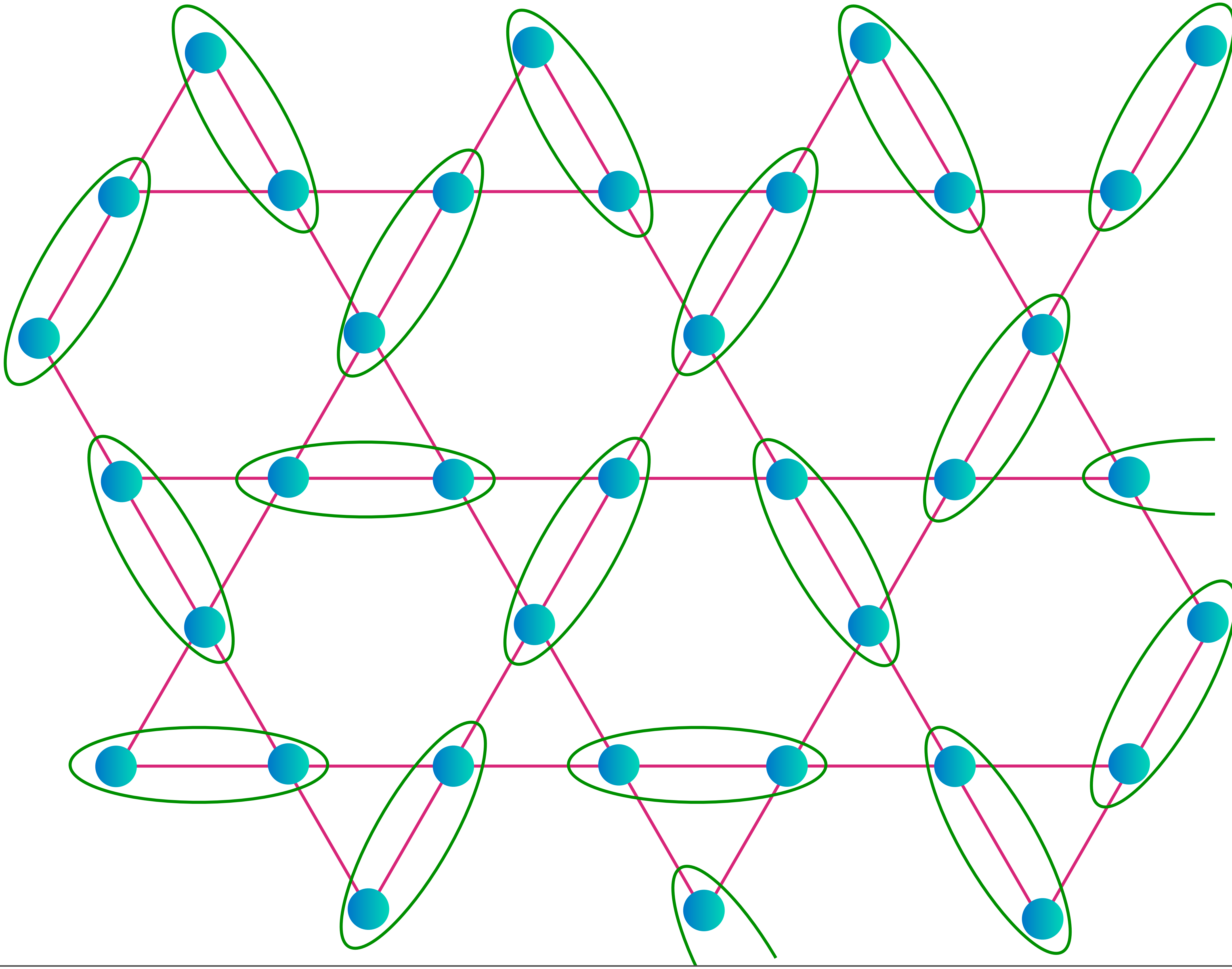


P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Mott insulator: Kagome antiferromagnet

Alternative view

Pick a reference configuration

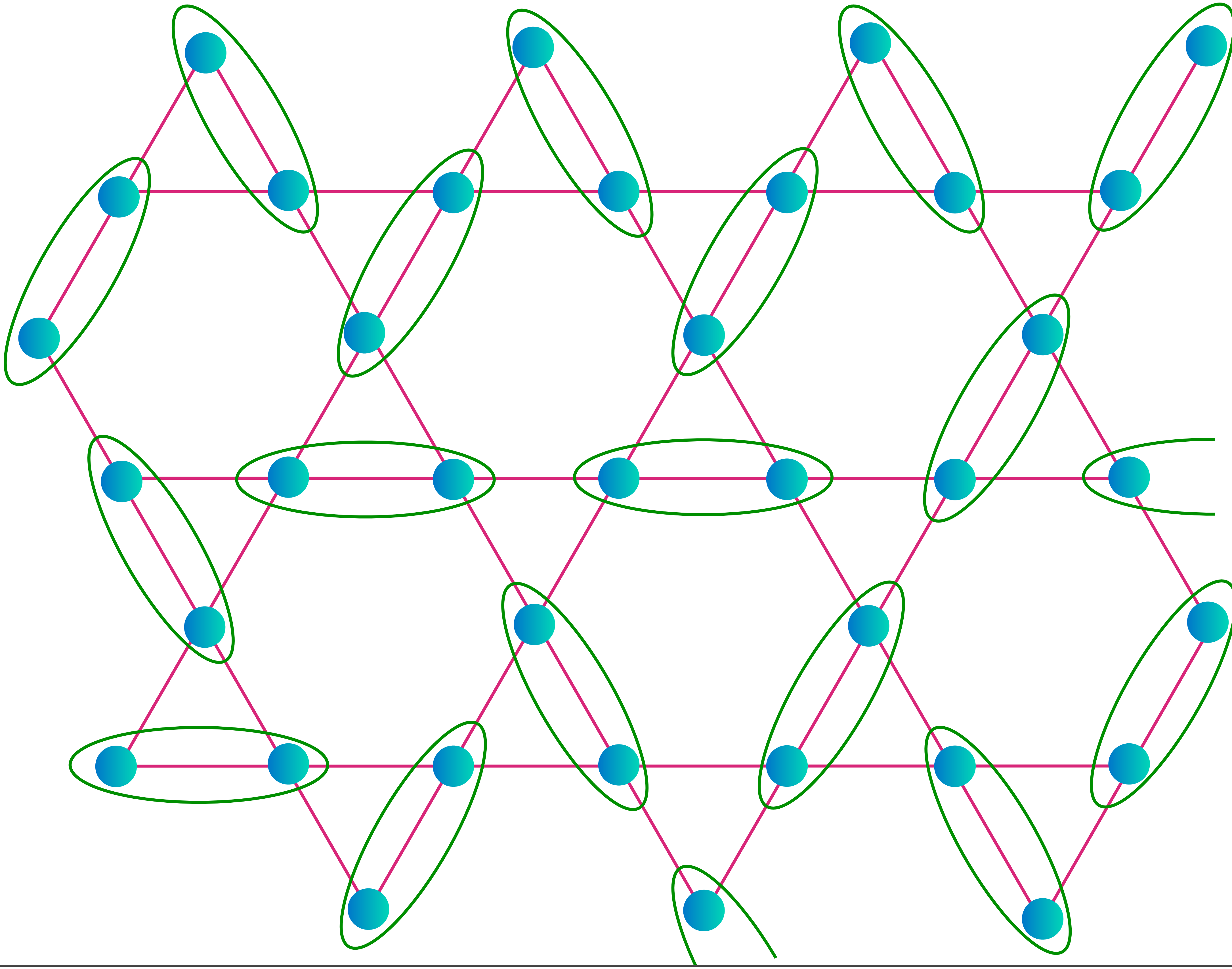


D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

A nearby configuration

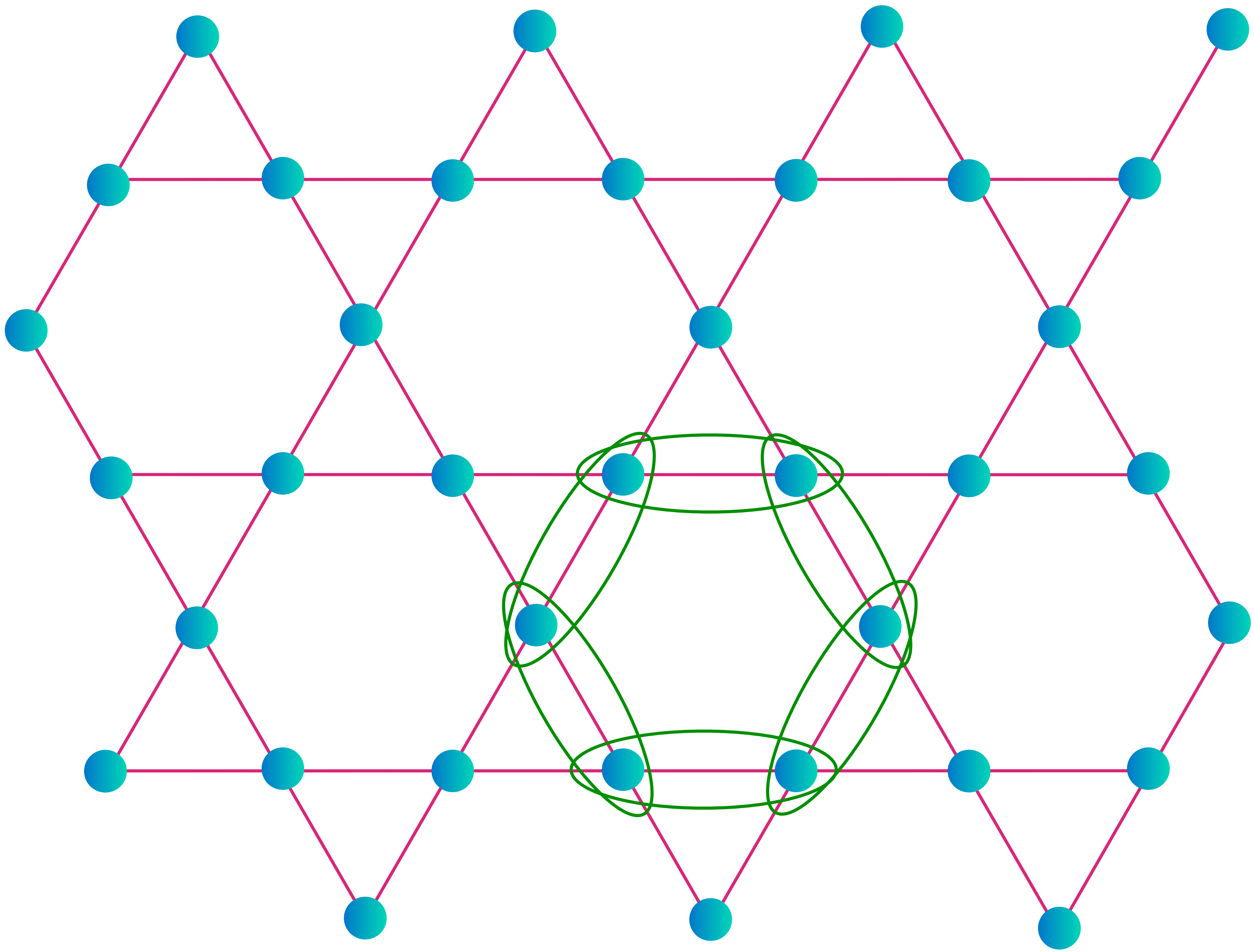


D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

Difference: a closed loop

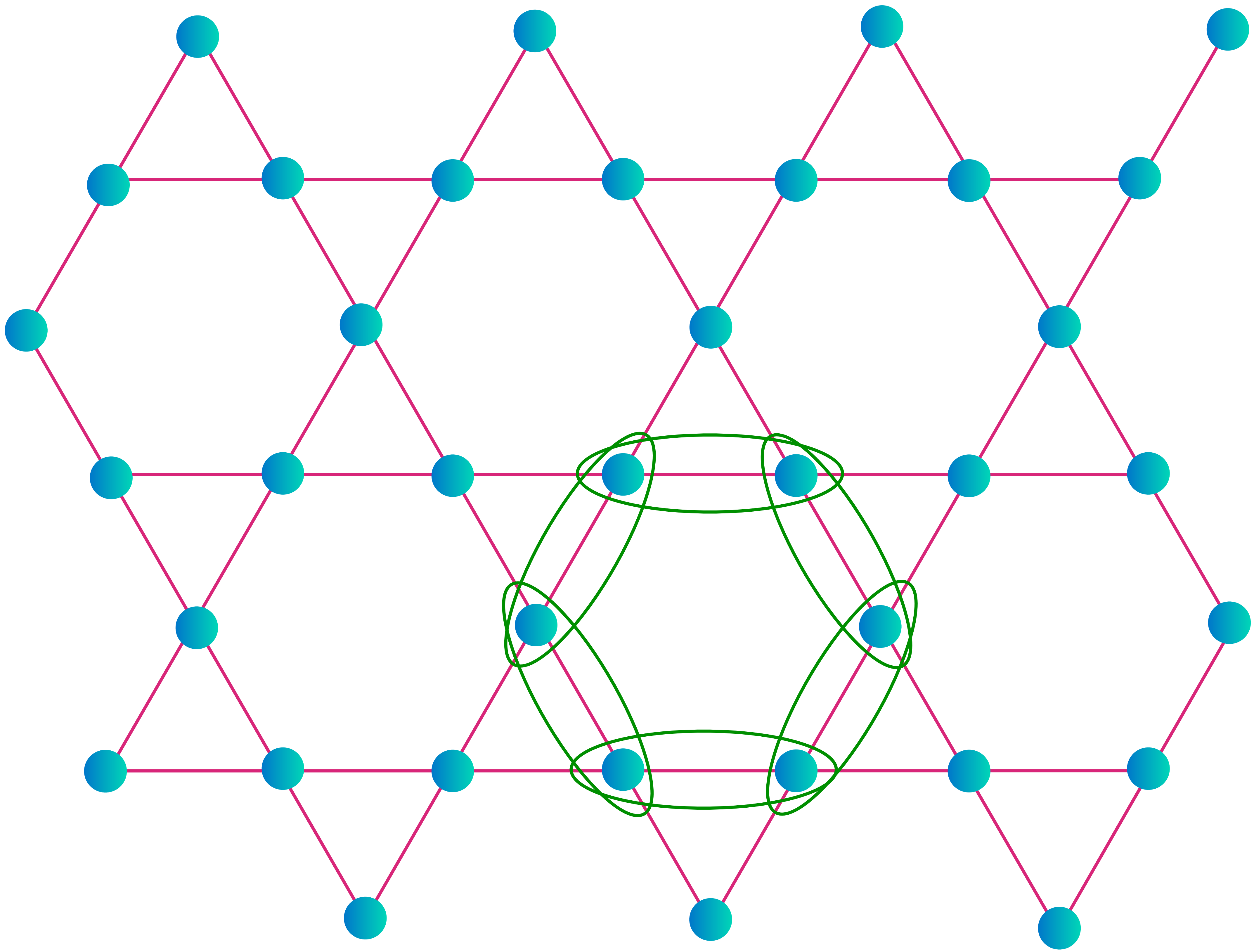


D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops

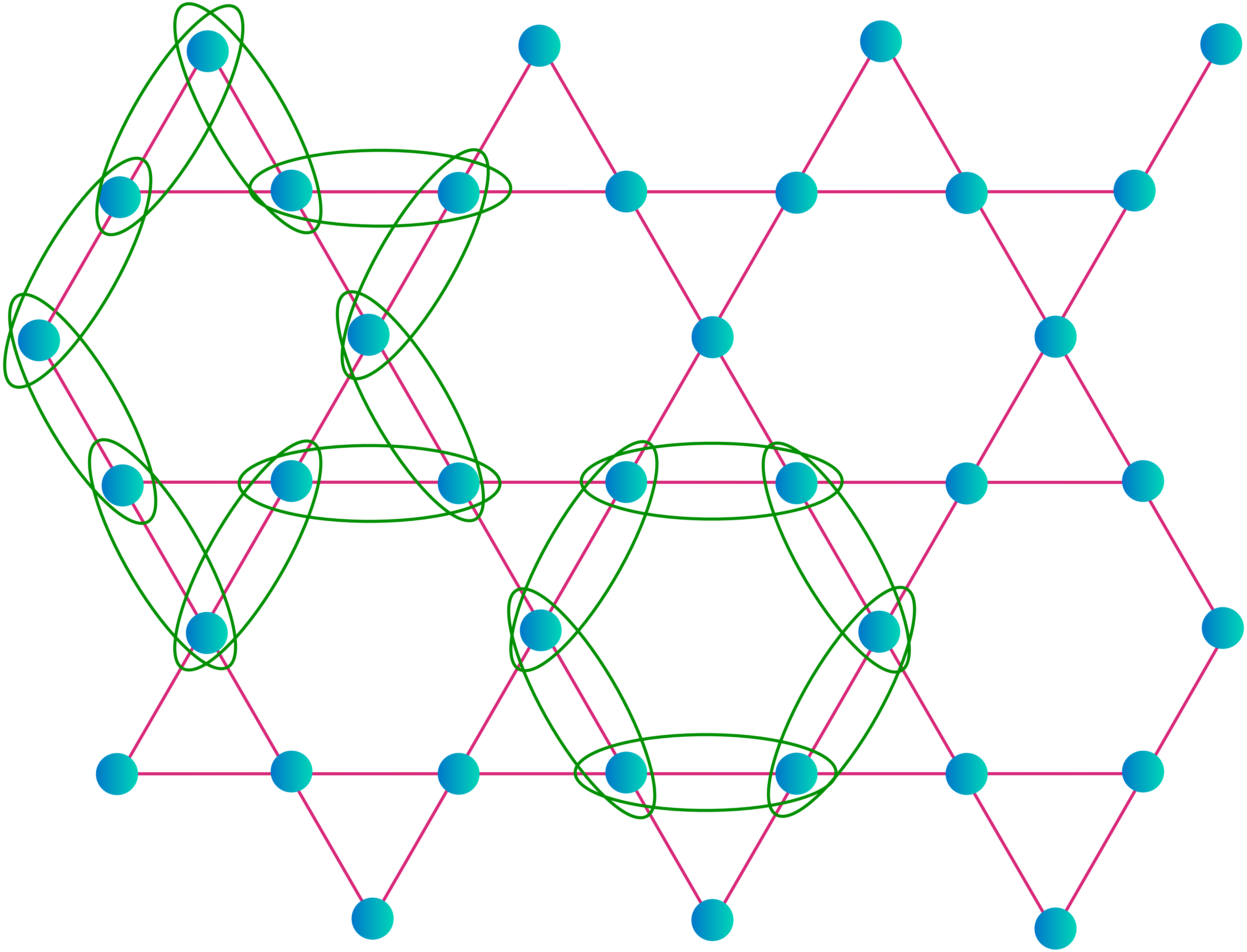


D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops

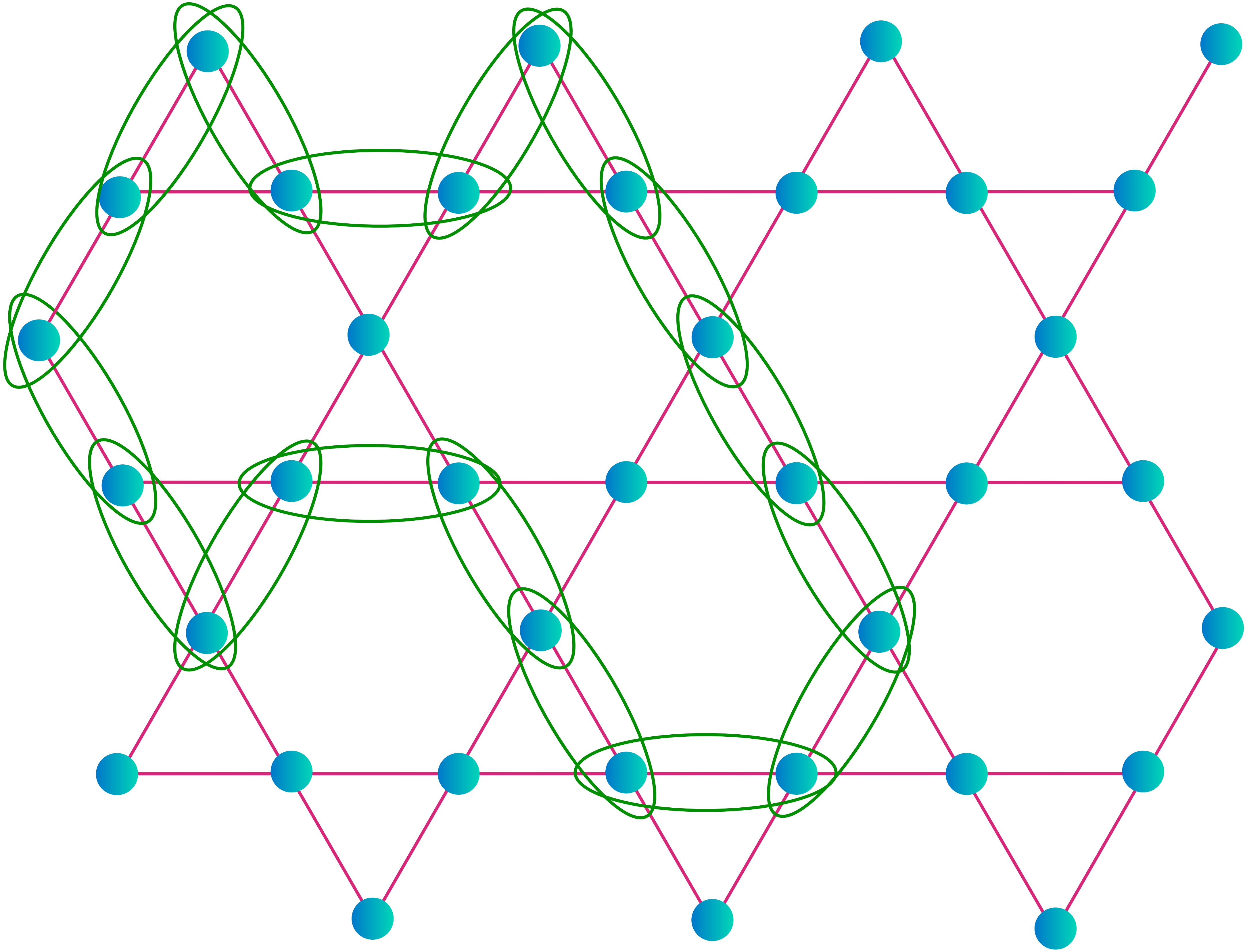


D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops

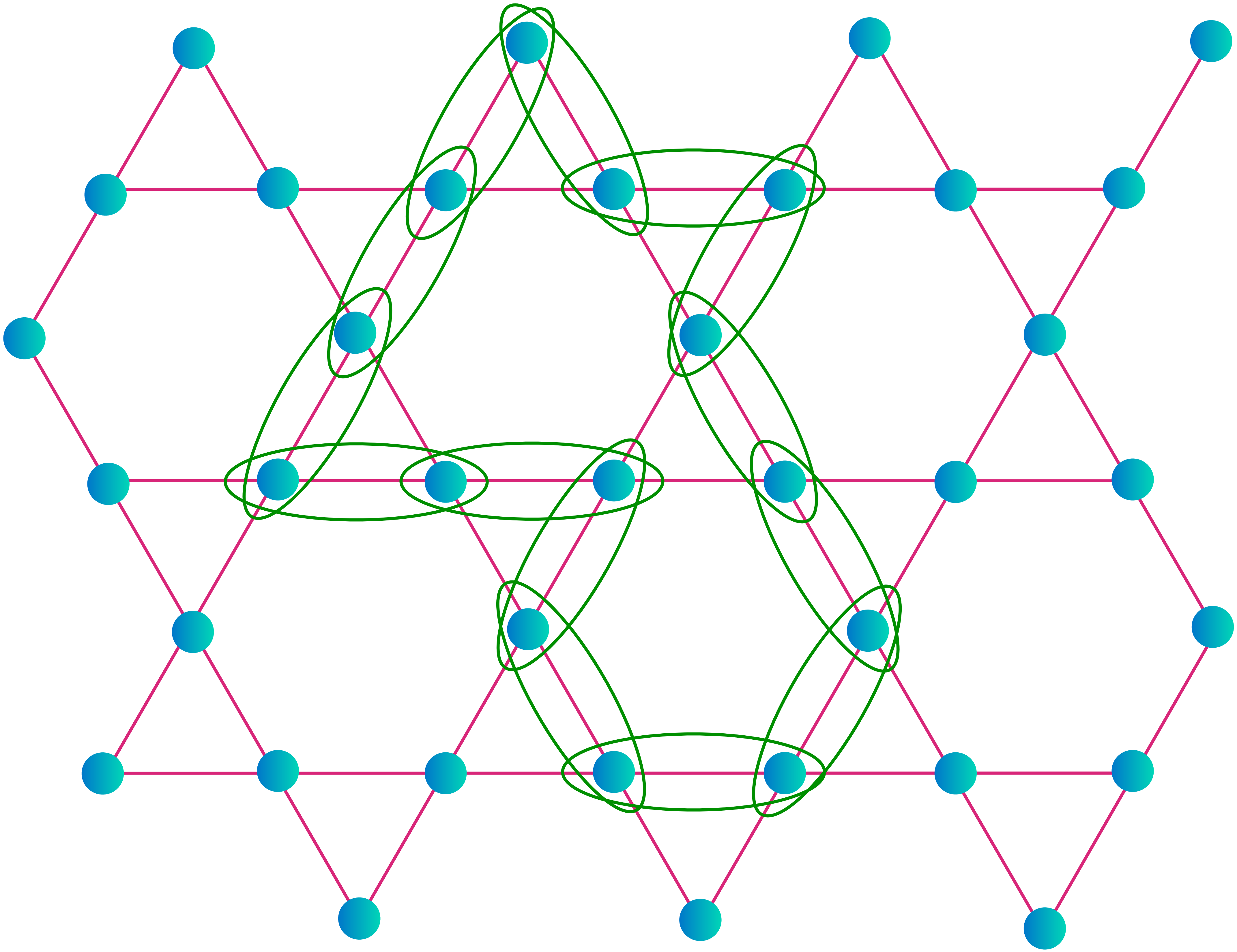


D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Mott insulator: Kagome antiferromagnet

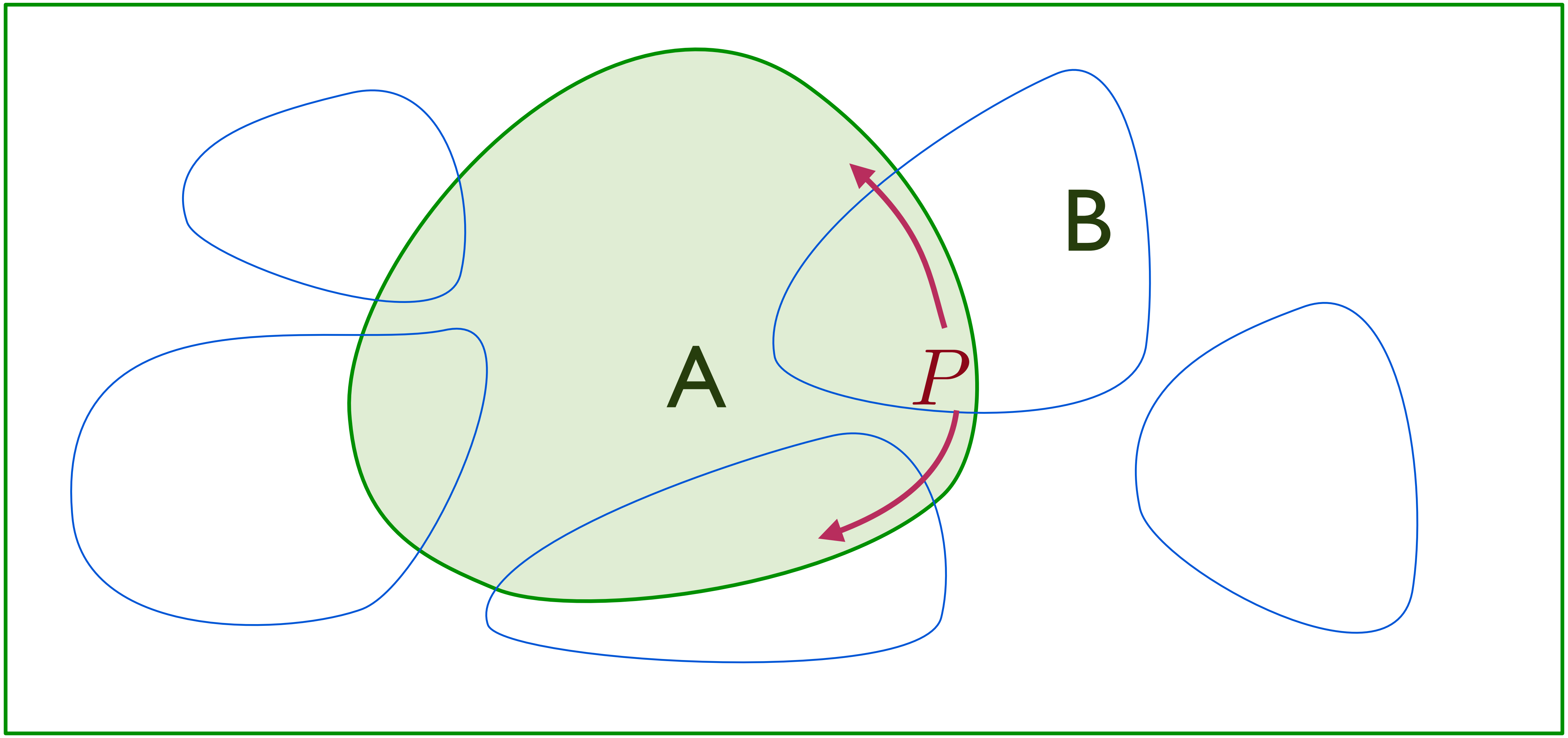
Alternative view

Ground state: sum over closed loops



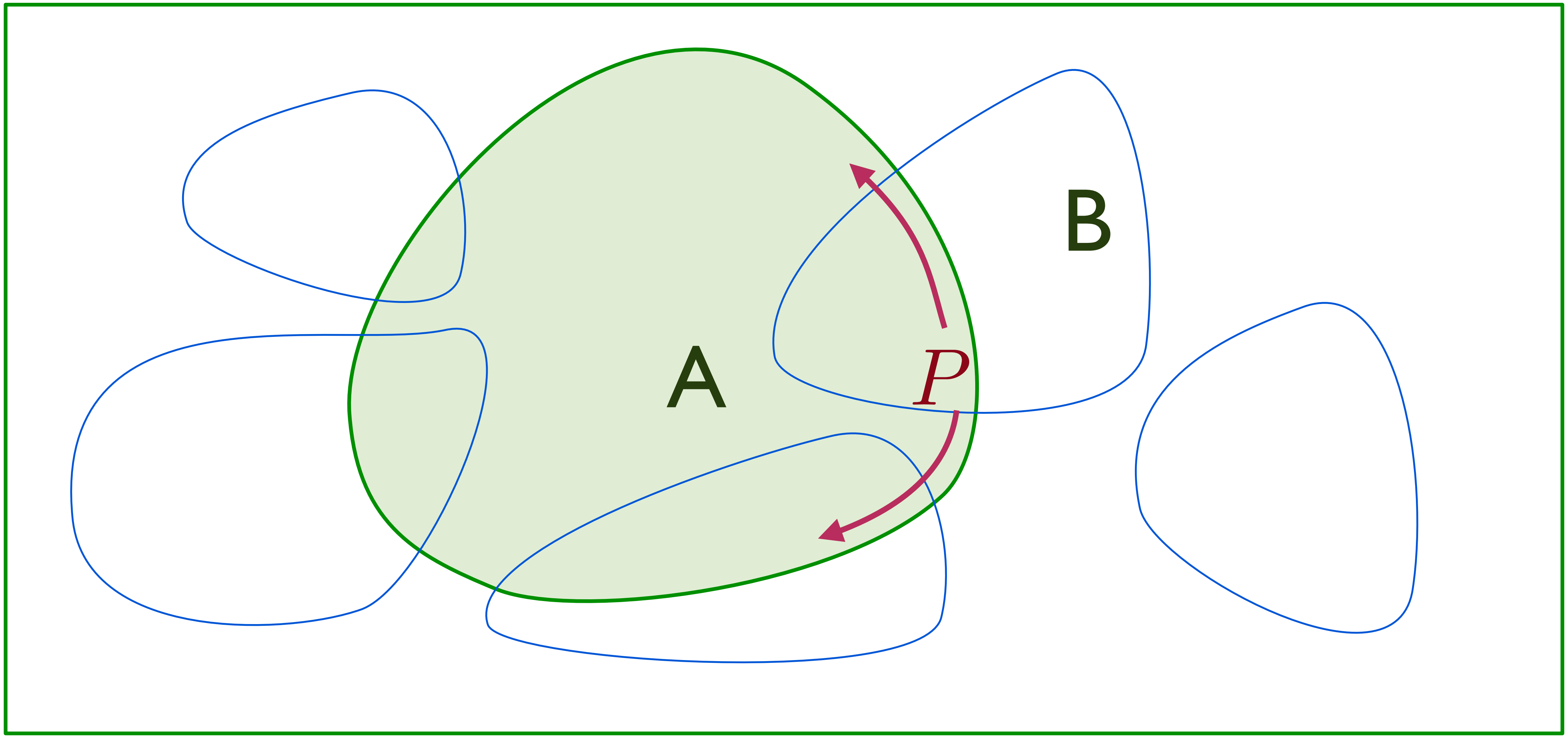
D. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988).

Entanglement in the Z_2 spin liquid



Sum over closed loops: only an even number of links cross the boundary between A and B

Entanglement in the Z_2 spin liquid

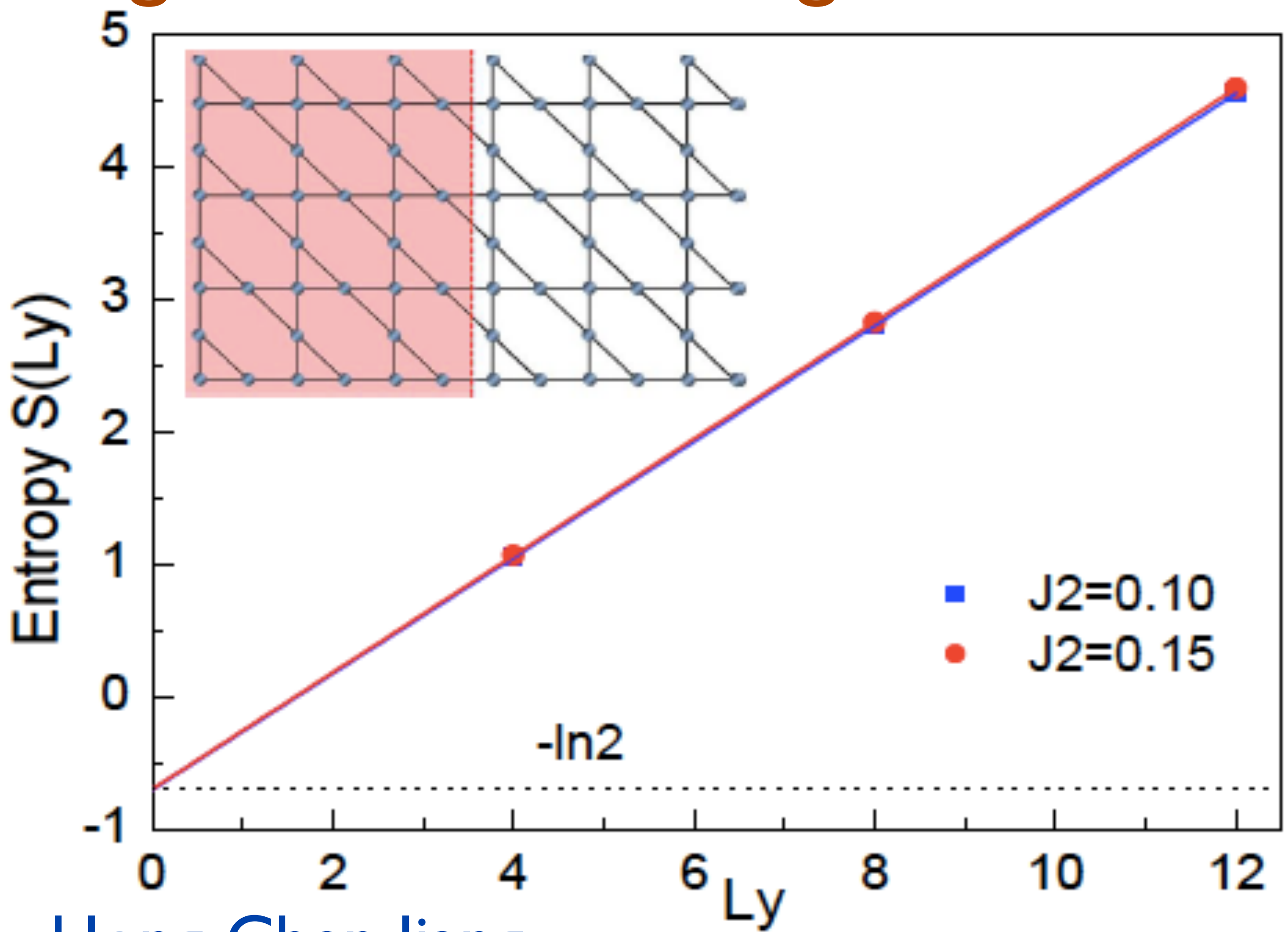


$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

A. Hamma, R. Ionicioiu, and P. Zanardi, Phys. Rev. A **71**, 022315 (2005)
M. Levin and X.-G. Wen, Phys. Rev. Lett. **96**, 110405 (2006); A. Kitaev and J. Preskill, Phys. Rev. Lett. **96**, 110404 (2006)
Y. Zhang, T. Grover, and A. Vishwanath, Phys. Rev. B **84**, 075128 (2011)

Kagome antiferromagnet

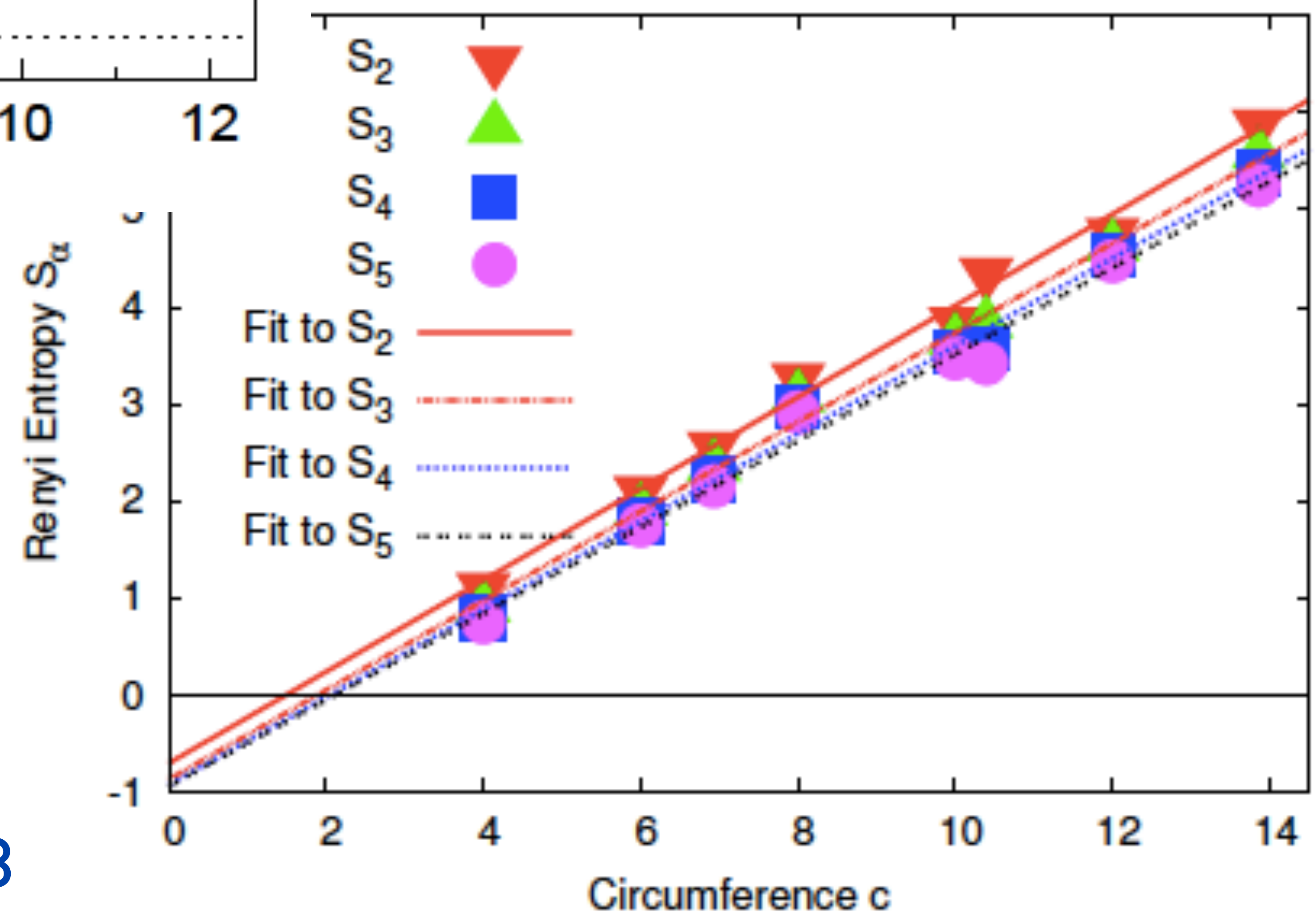


Strong numerical evidence for a Z_2 spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289

S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, arXiv:1205.4858

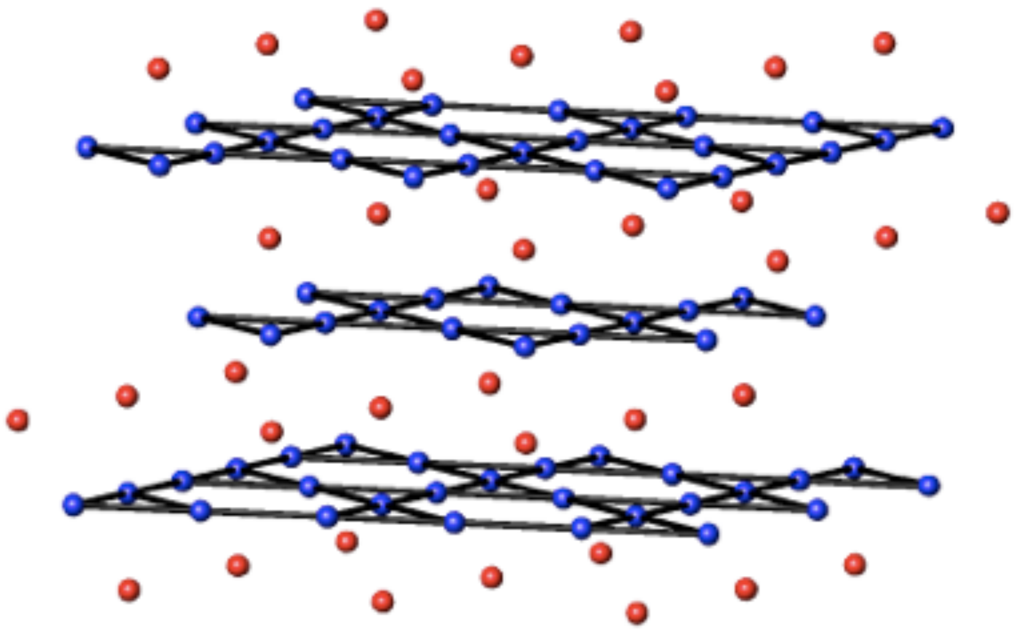


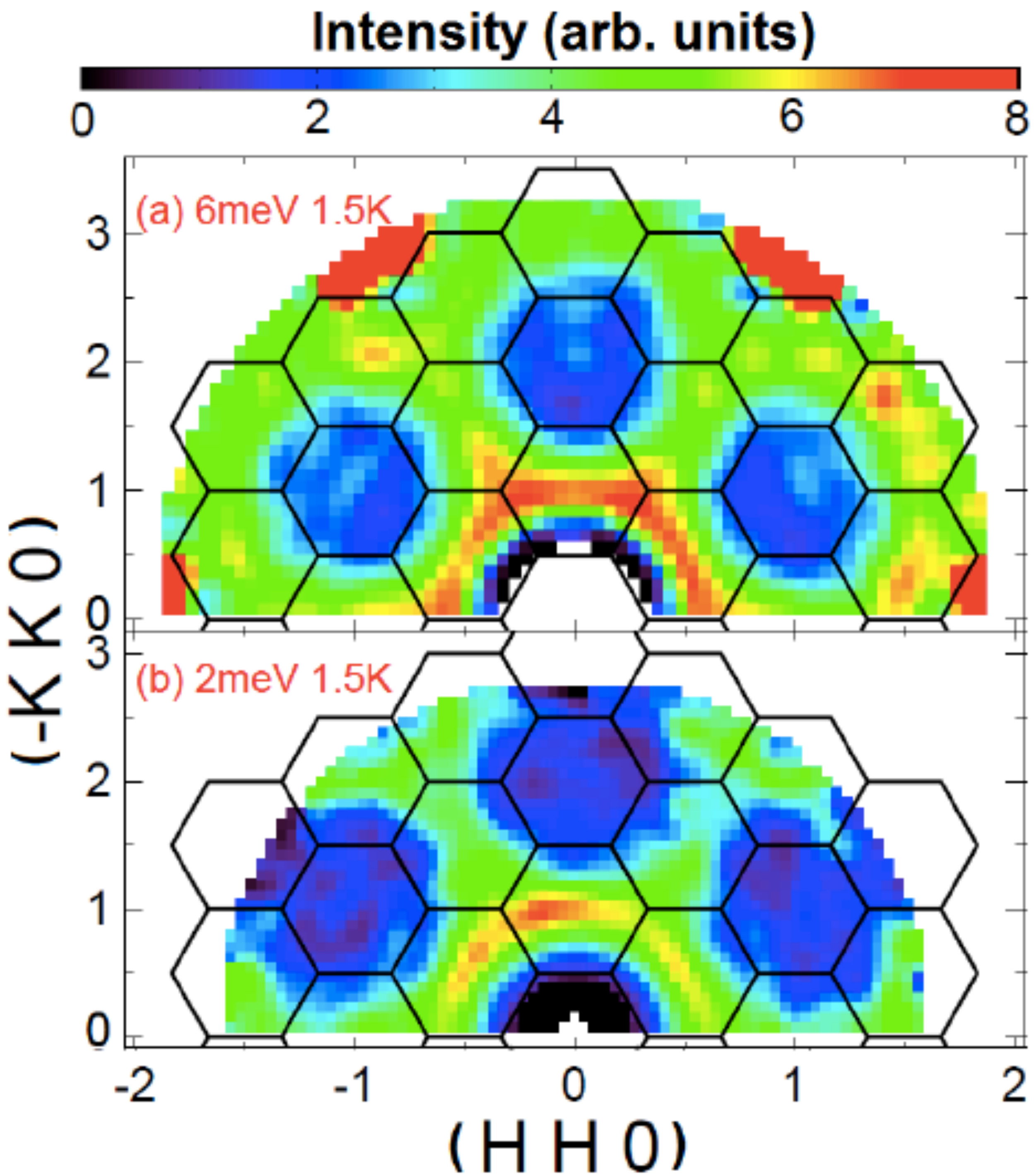
Kagome antiferromagnet

Evidence for spinons
Young Lee,
APS meeting, March 2012

http://meetings.aps.org/link/BAPS.2012.MAR.H8.5

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)

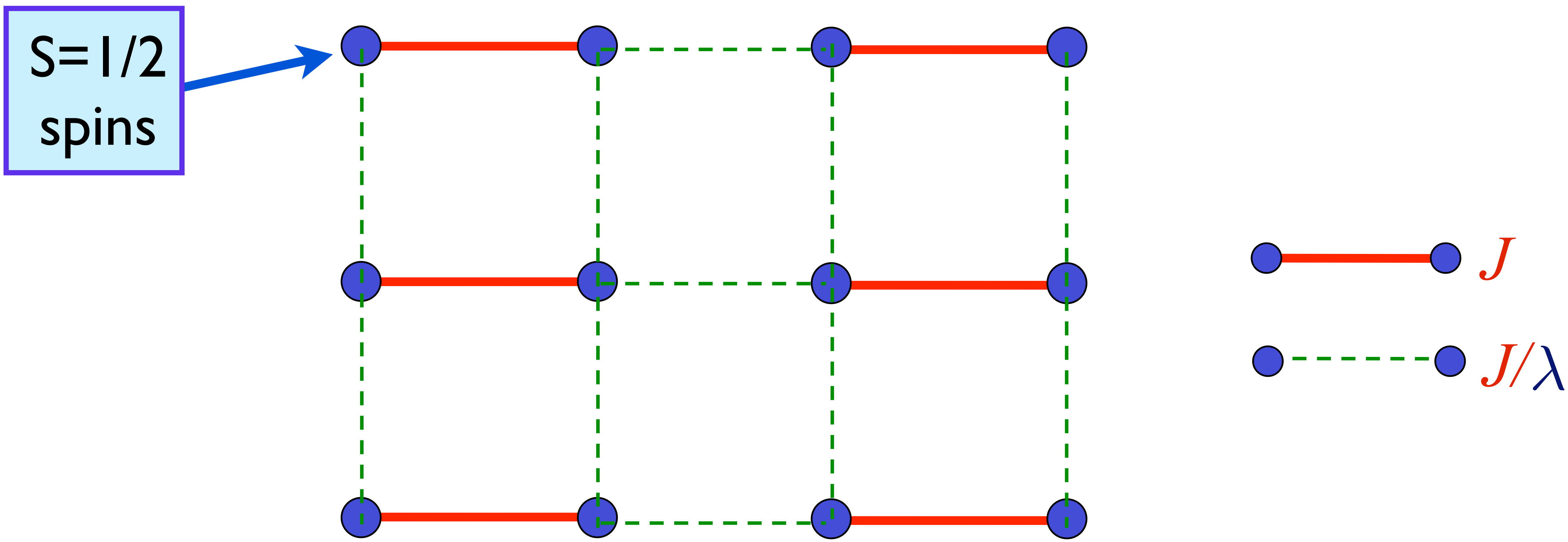




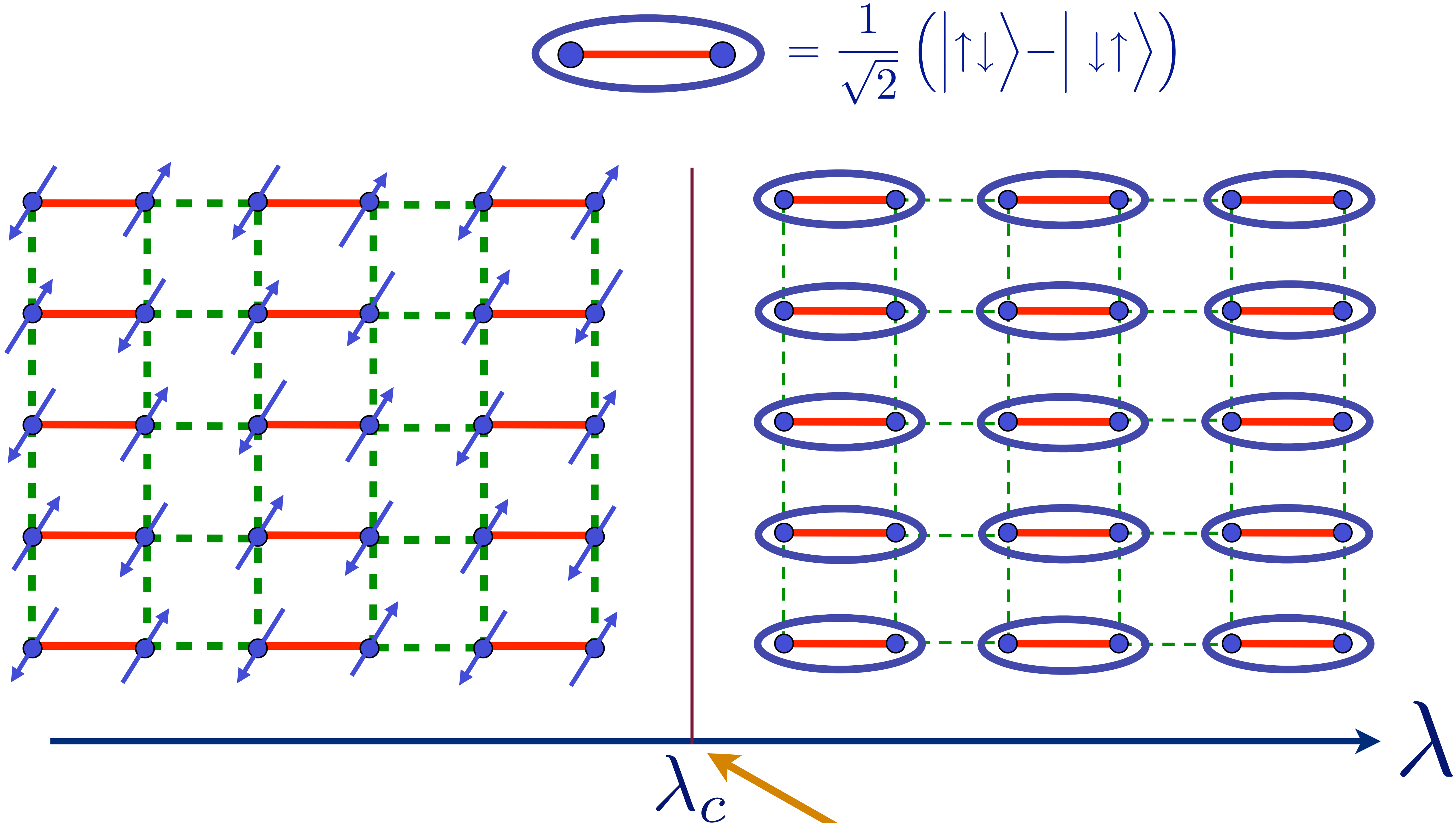
Conformal quantum matter

Coupled dimer antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

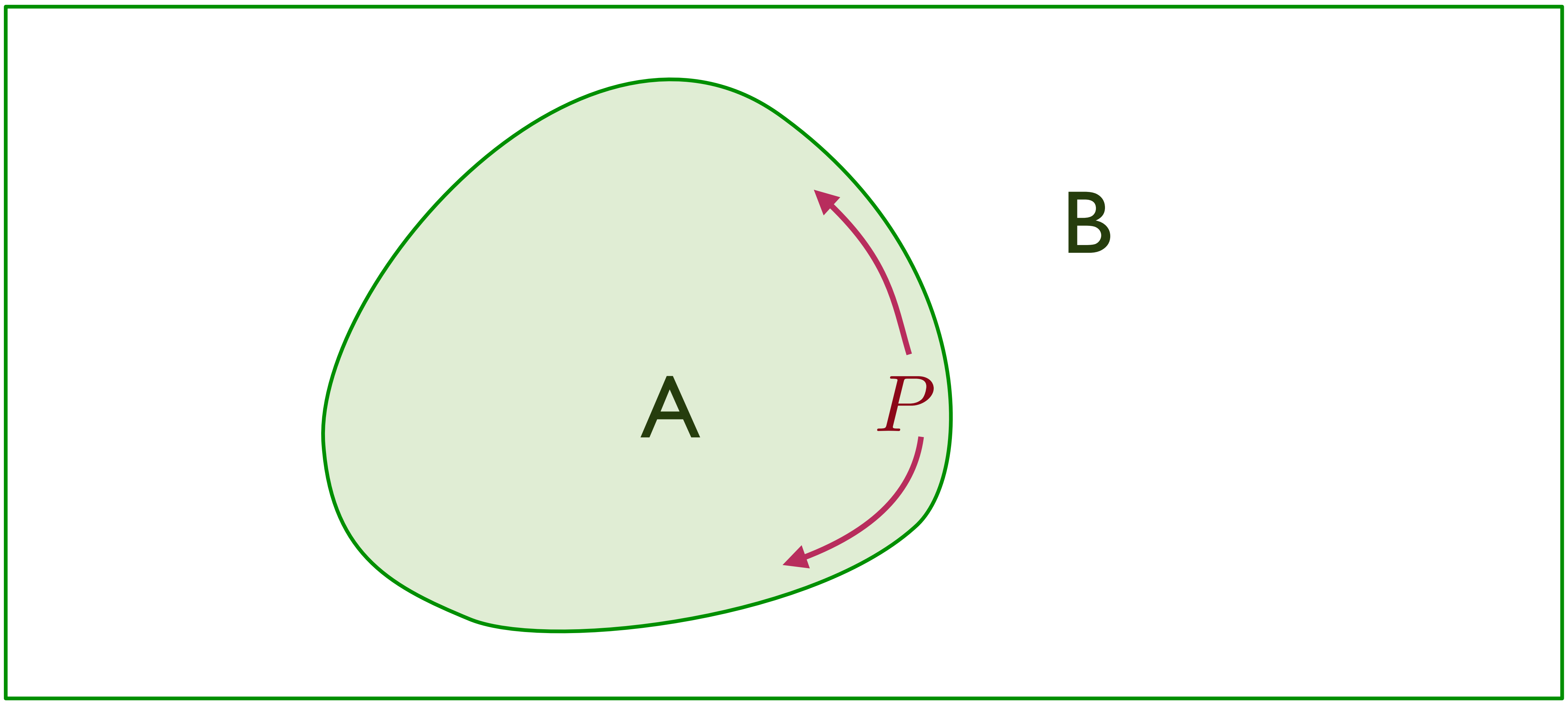


Quantum critical point described by a CFT3 (O(3) Wilson-Fisher)

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. Lett. **60**, 1057 (1988).
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).
A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).

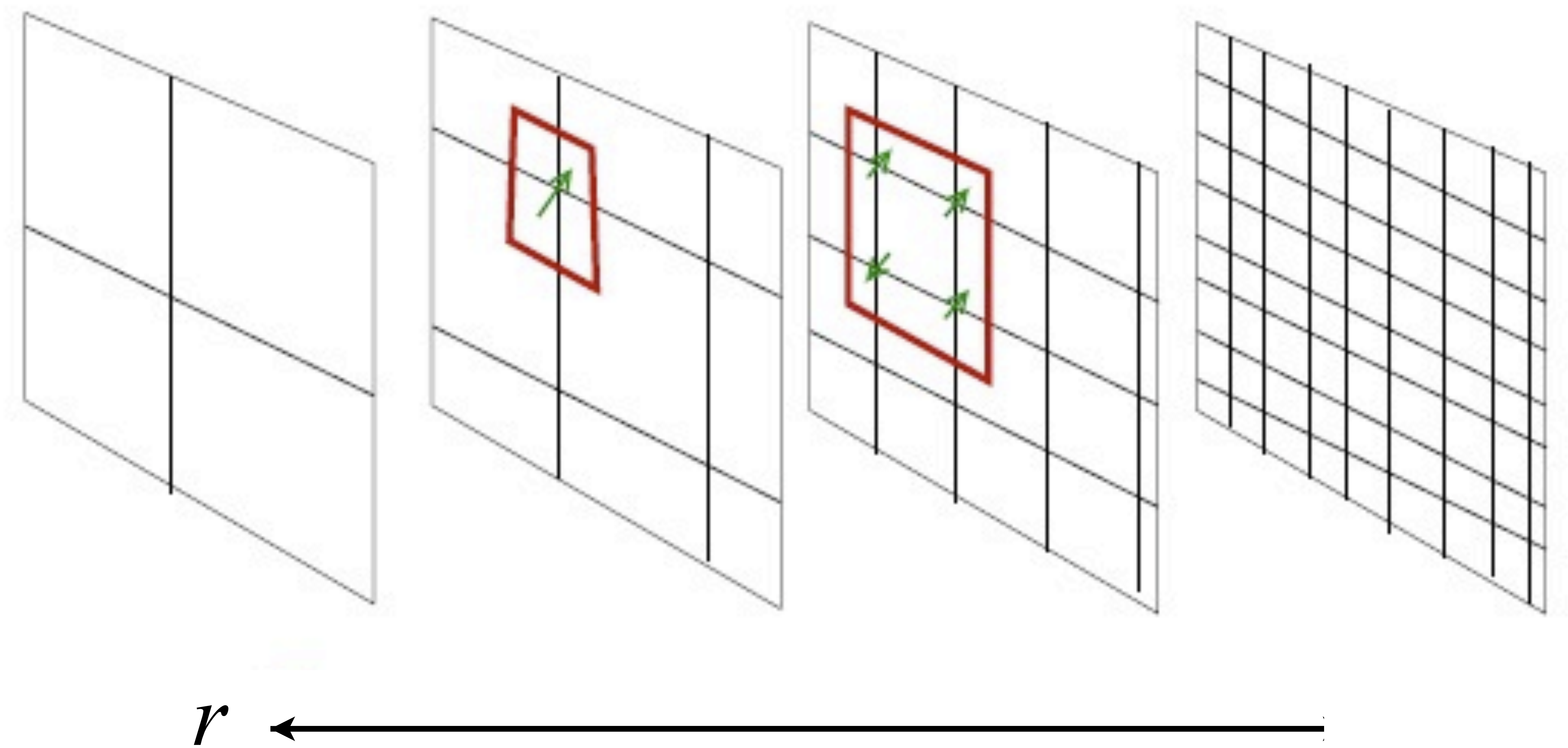
Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Phys. Rev. B 80, 115122 (2009)
B. Hsu, M. Mulligan, E. Fradkin, and Eun-Ah Kim, Phys. Rev. B 79, 115421 (2009)
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Holography



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

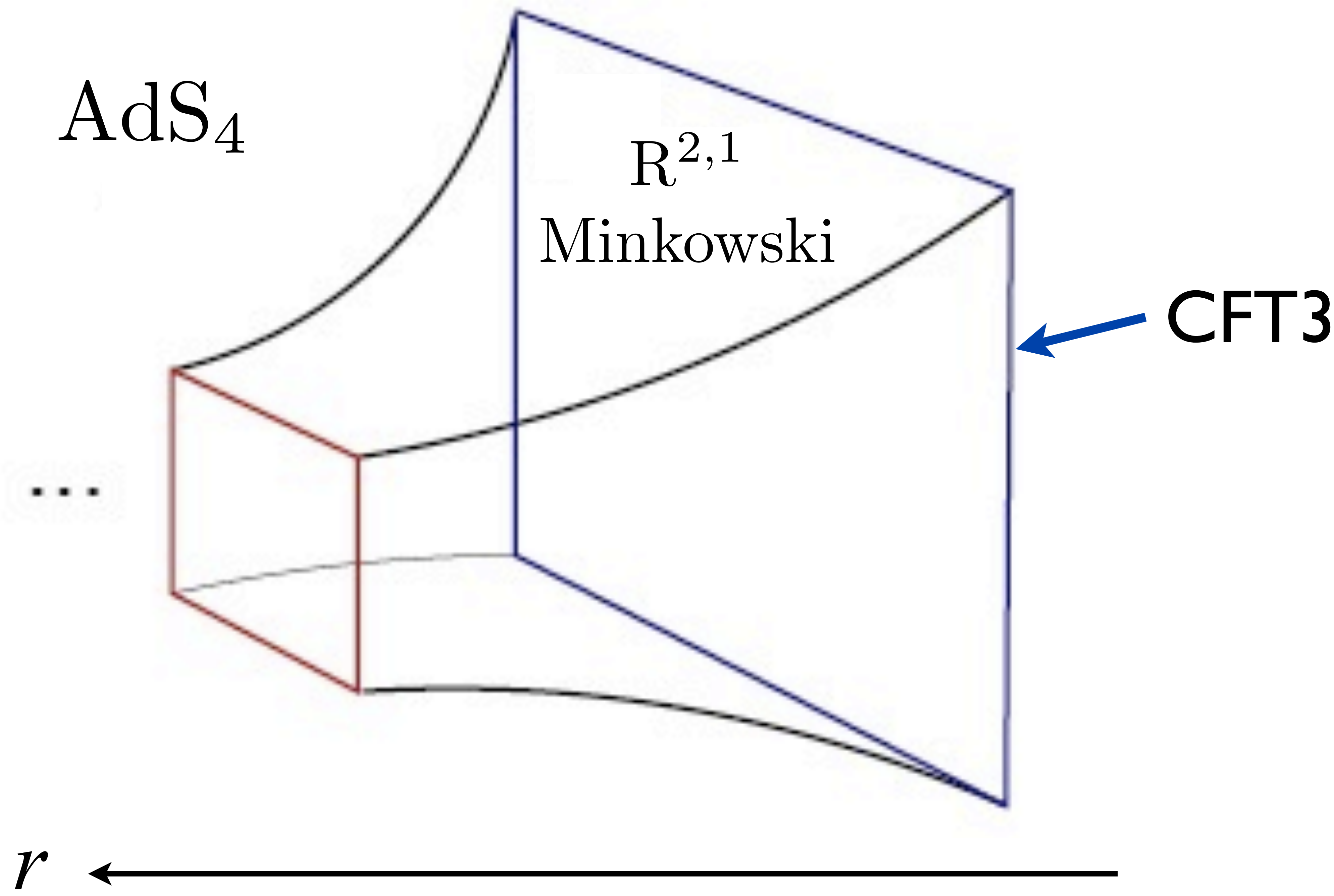
$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

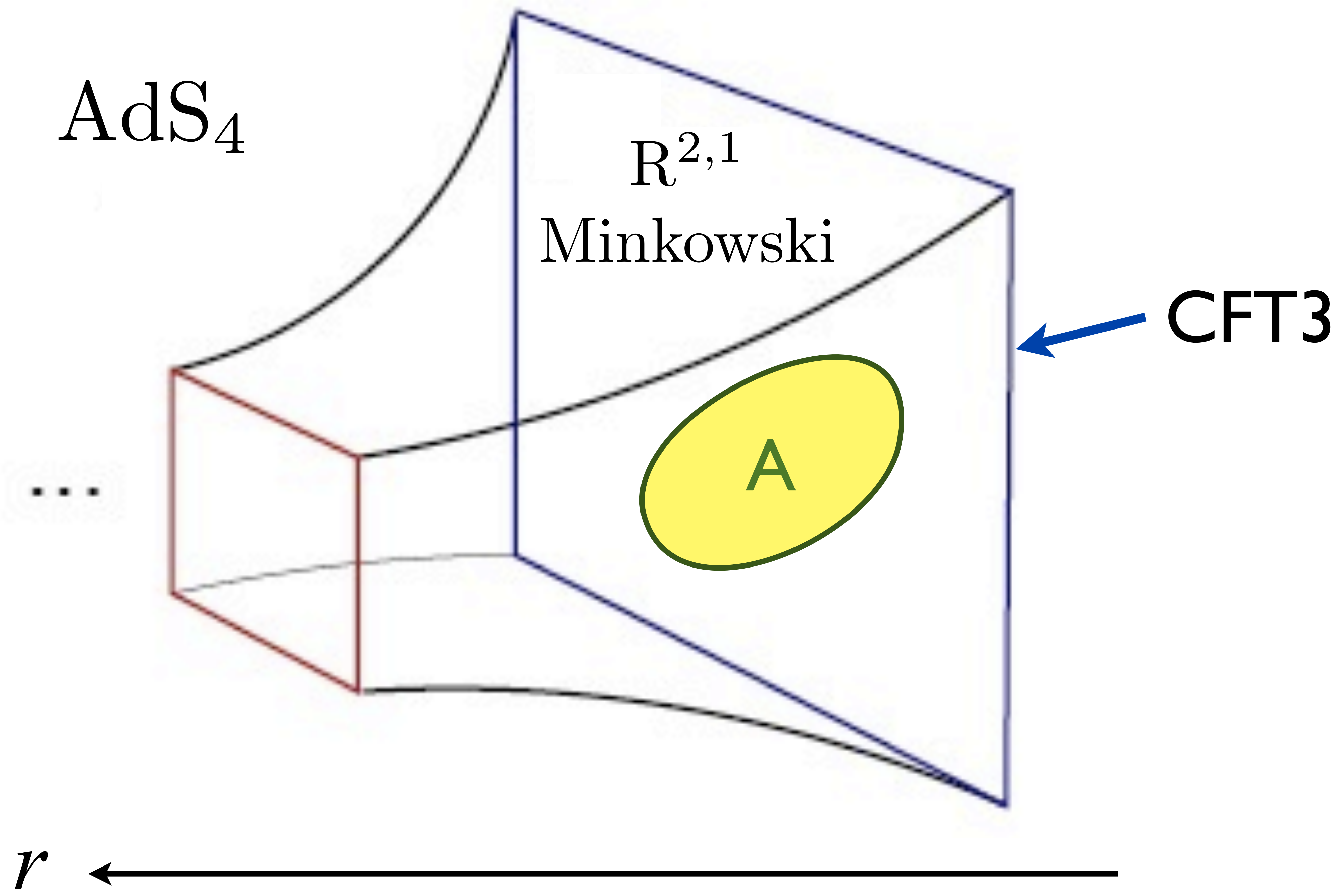
$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

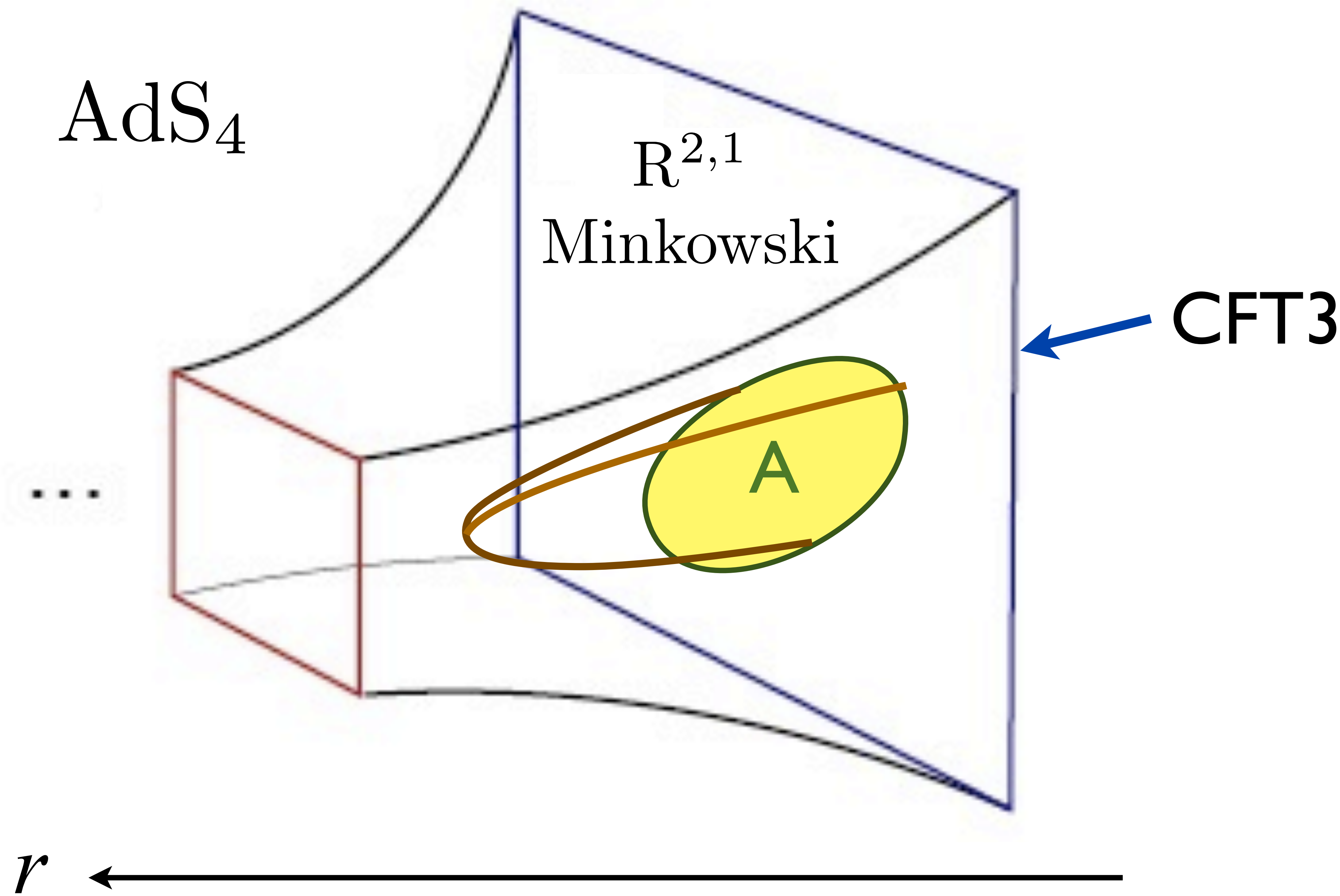
AdS/CFT correspondence



AdS/CFT correspondence



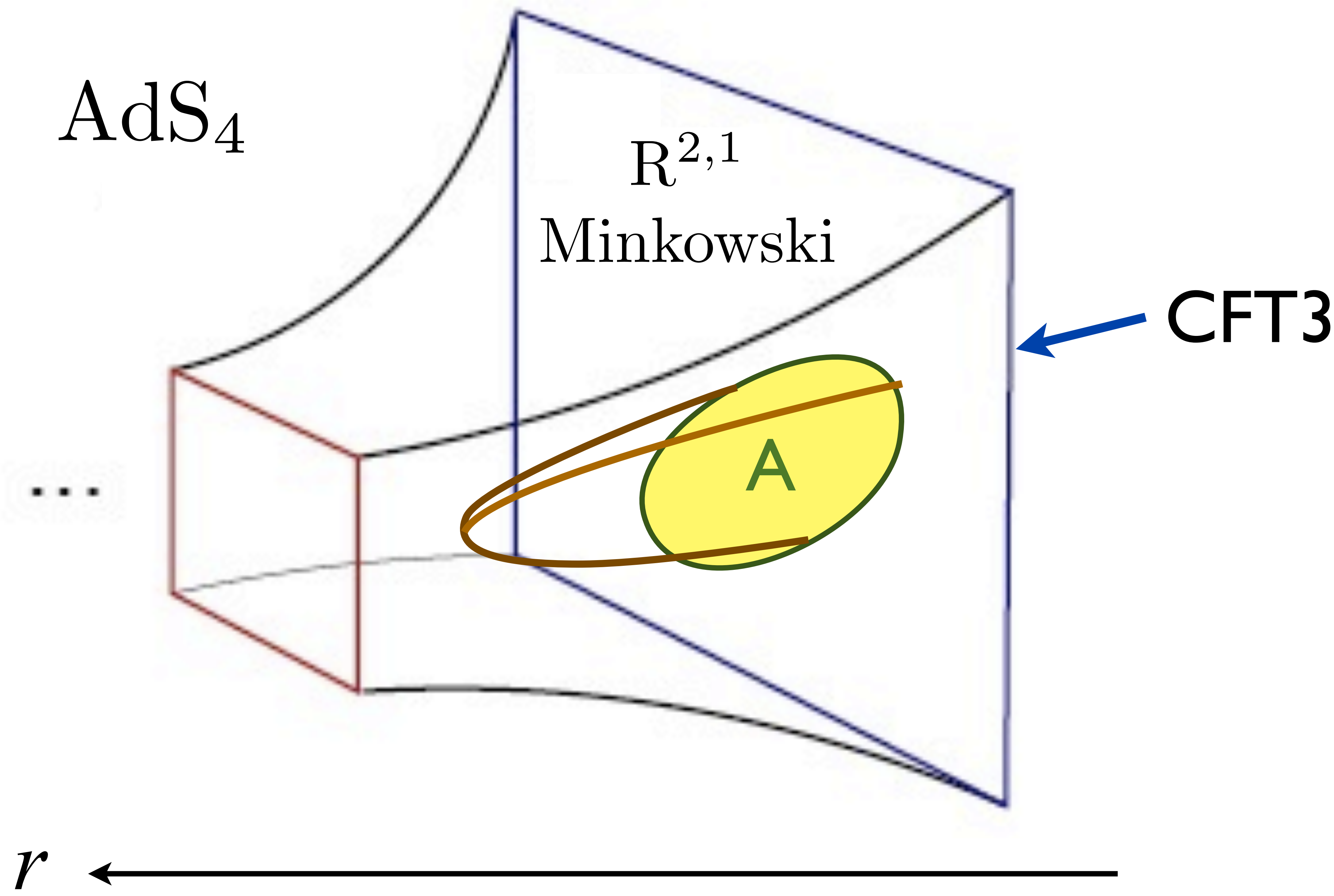
AdS/CFT correspondence



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

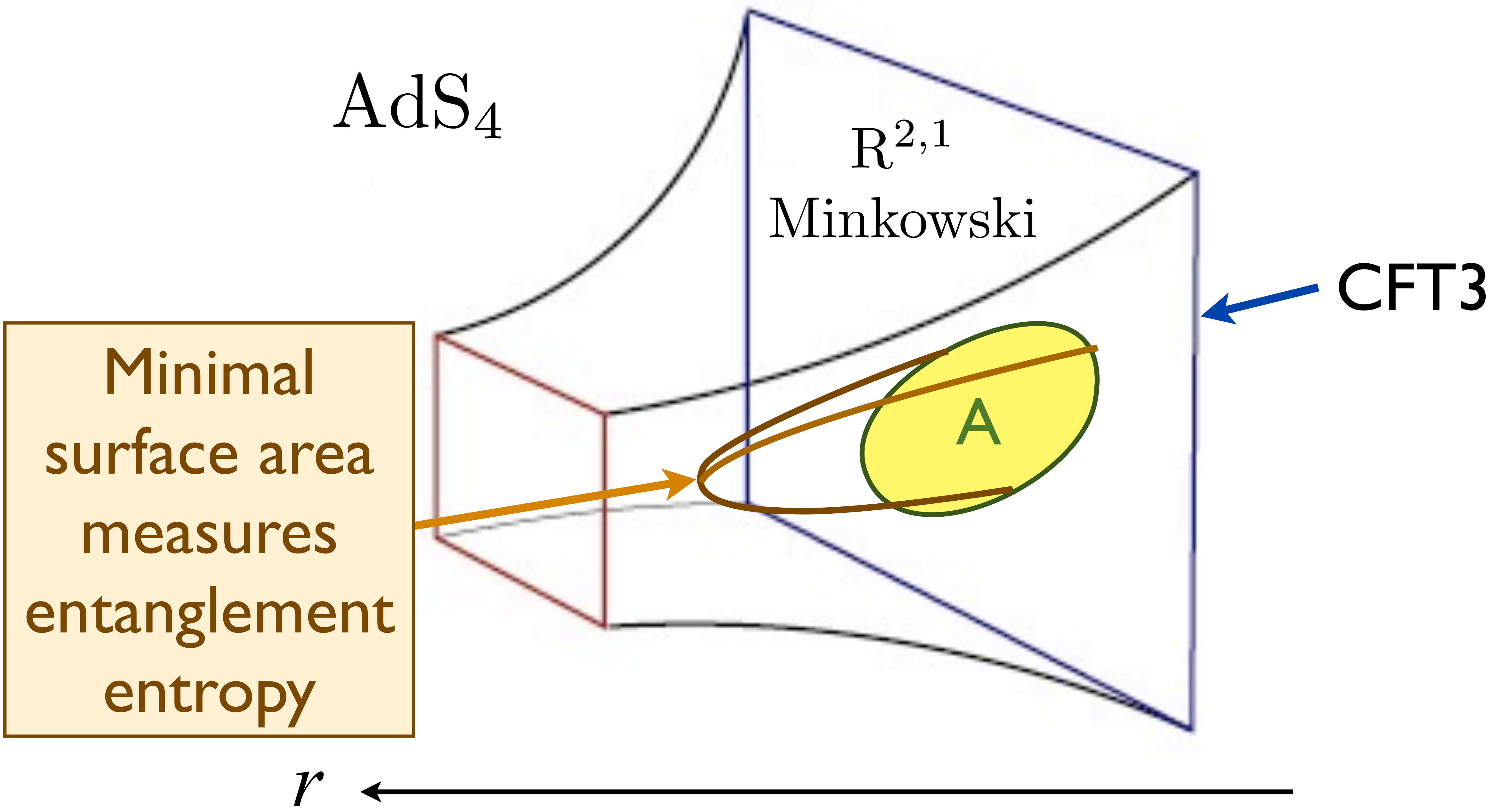
AdS/CFT correspondence



The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

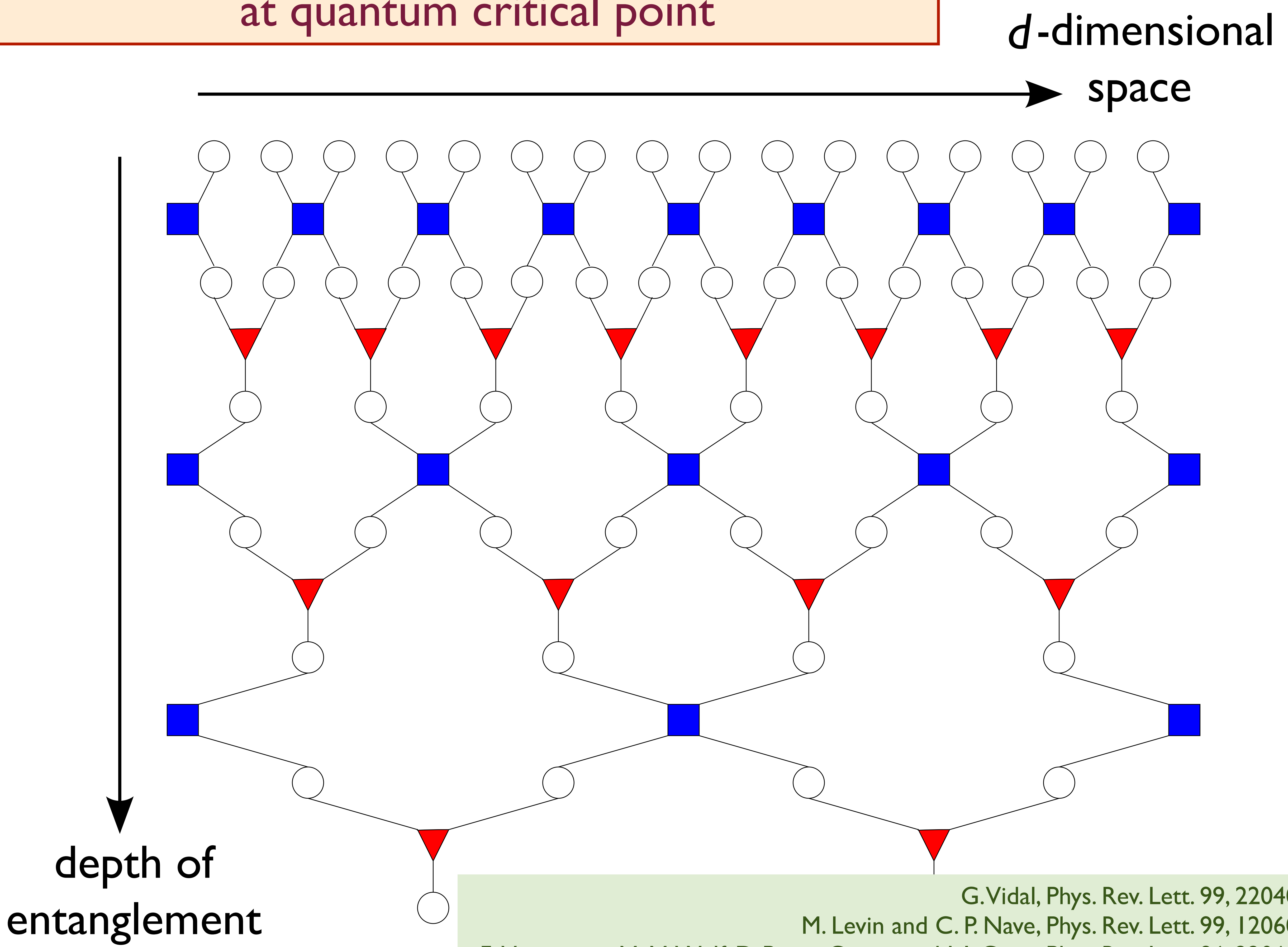
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence



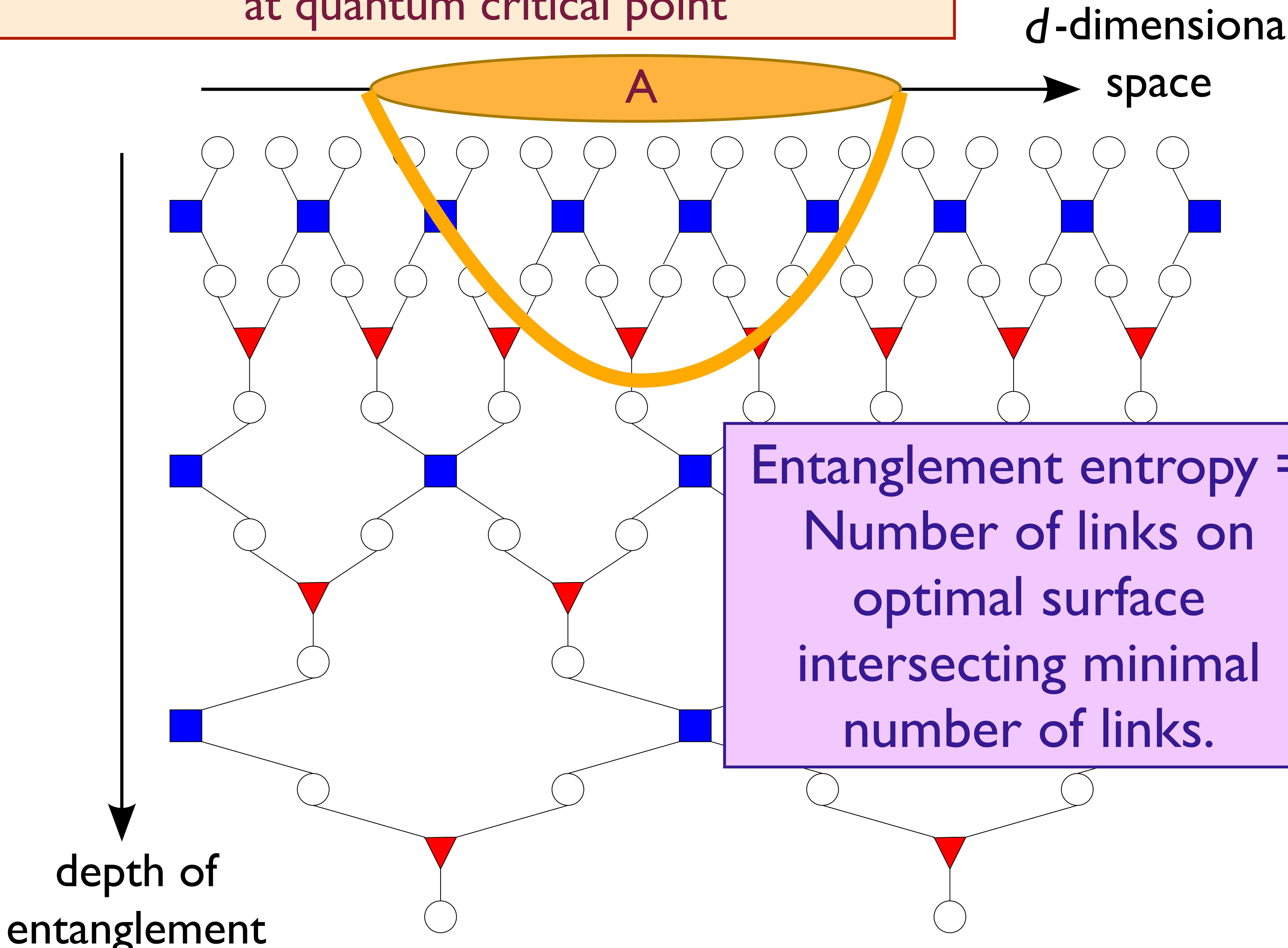
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Tensor network representation of entanglement at quantum critical point



G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

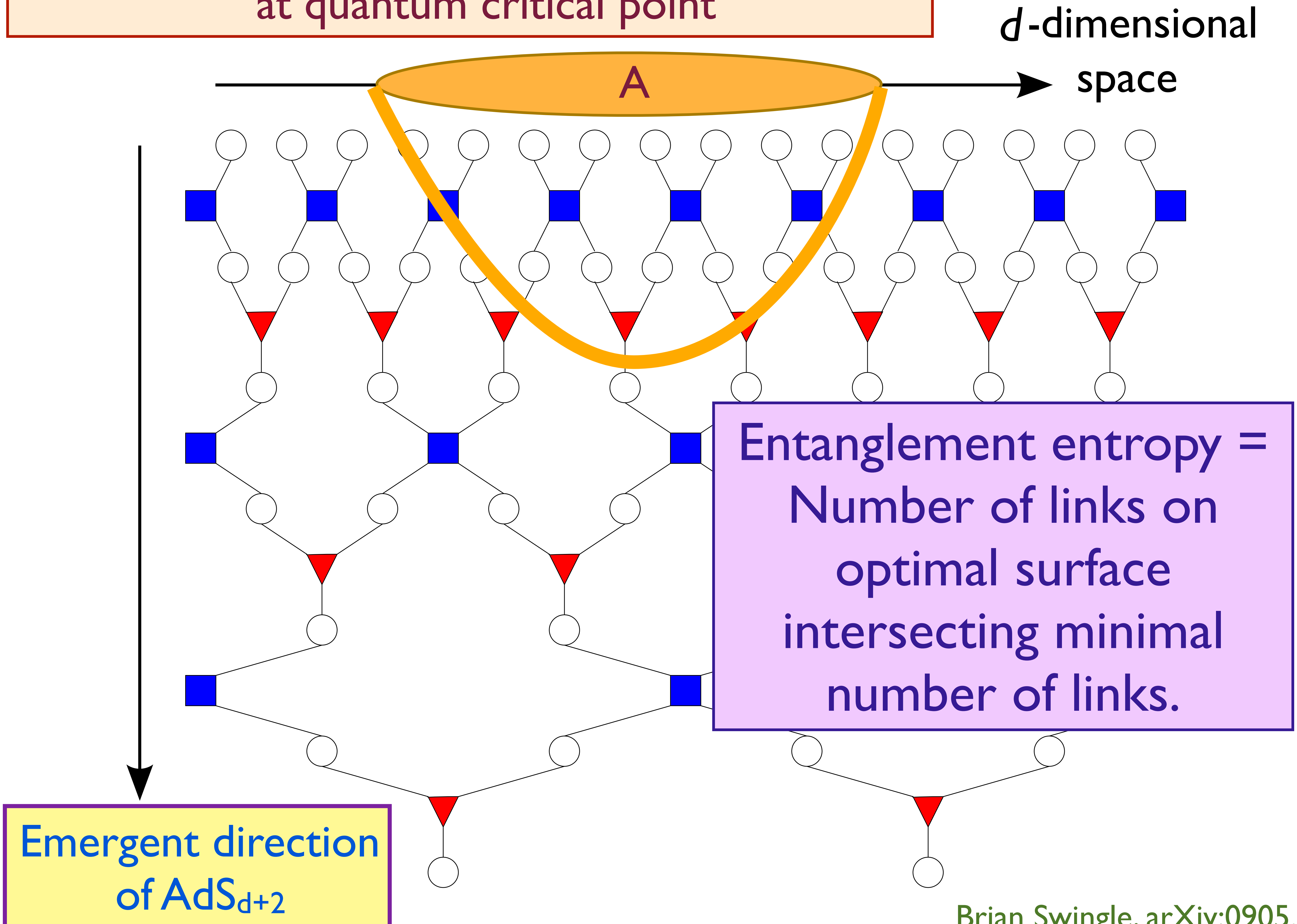
Tensor network representation of entanglement at quantum critical point



Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

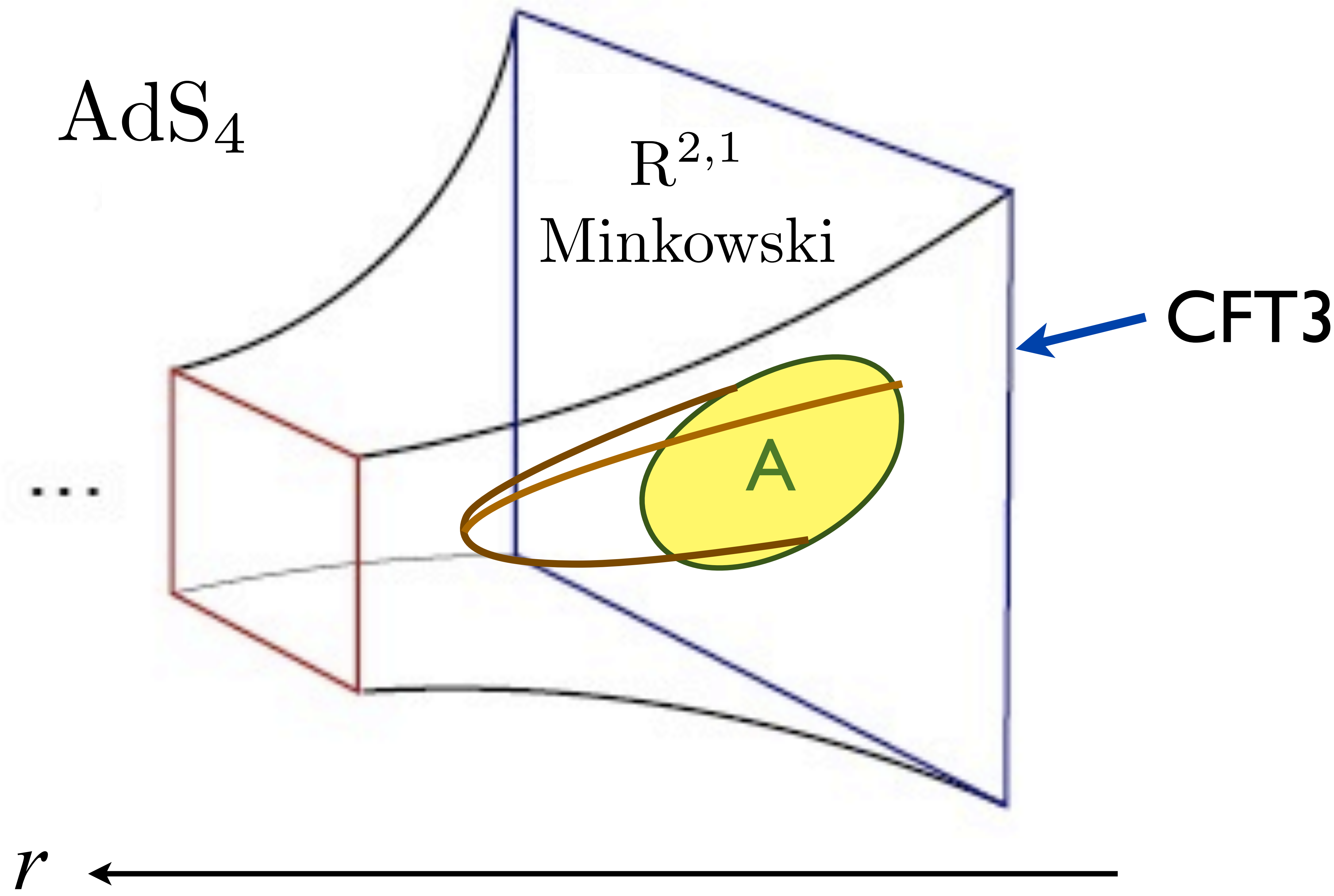
Brian Swingle, arXiv:0905.1317

Tensor network representation of entanglement at quantum critical point



Brian Swingle, arXiv:0905.1317

AdS/CFT correspondence



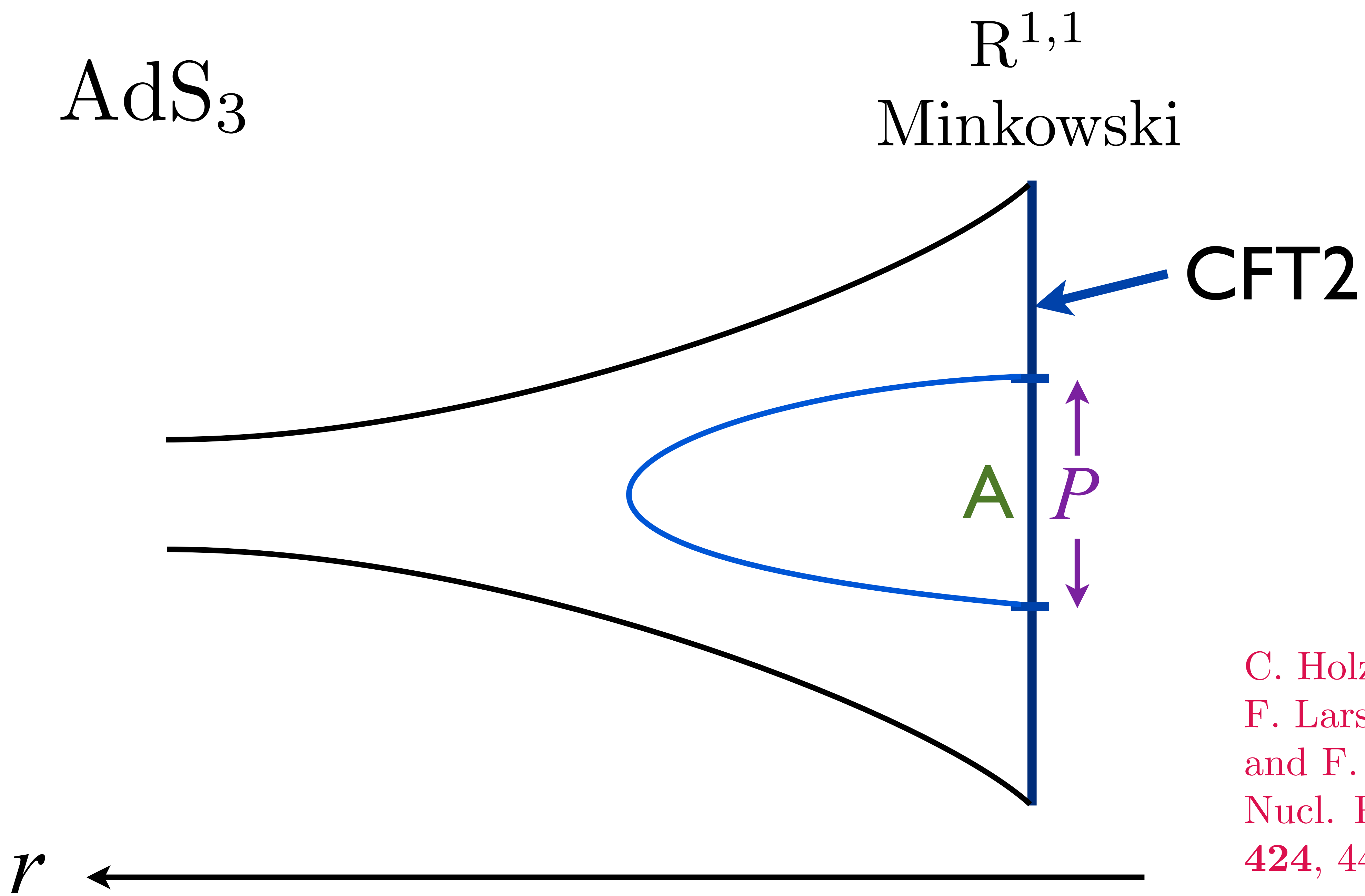
- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

AdS/CFT correspondence

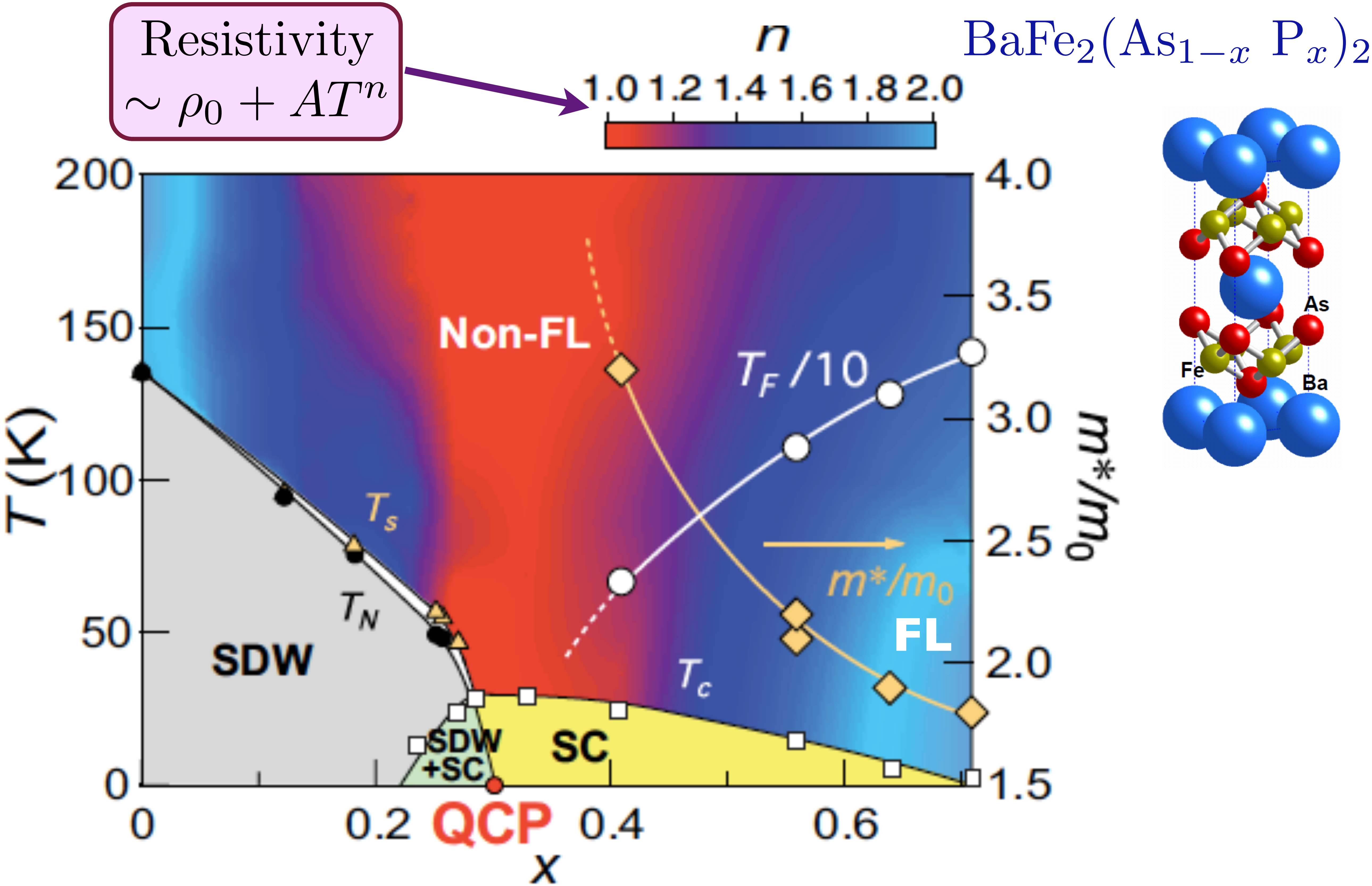


C. Holzhey, F. Larsen and F. Wilczek, Nucl. Phys. B **424**, 443 (1994).

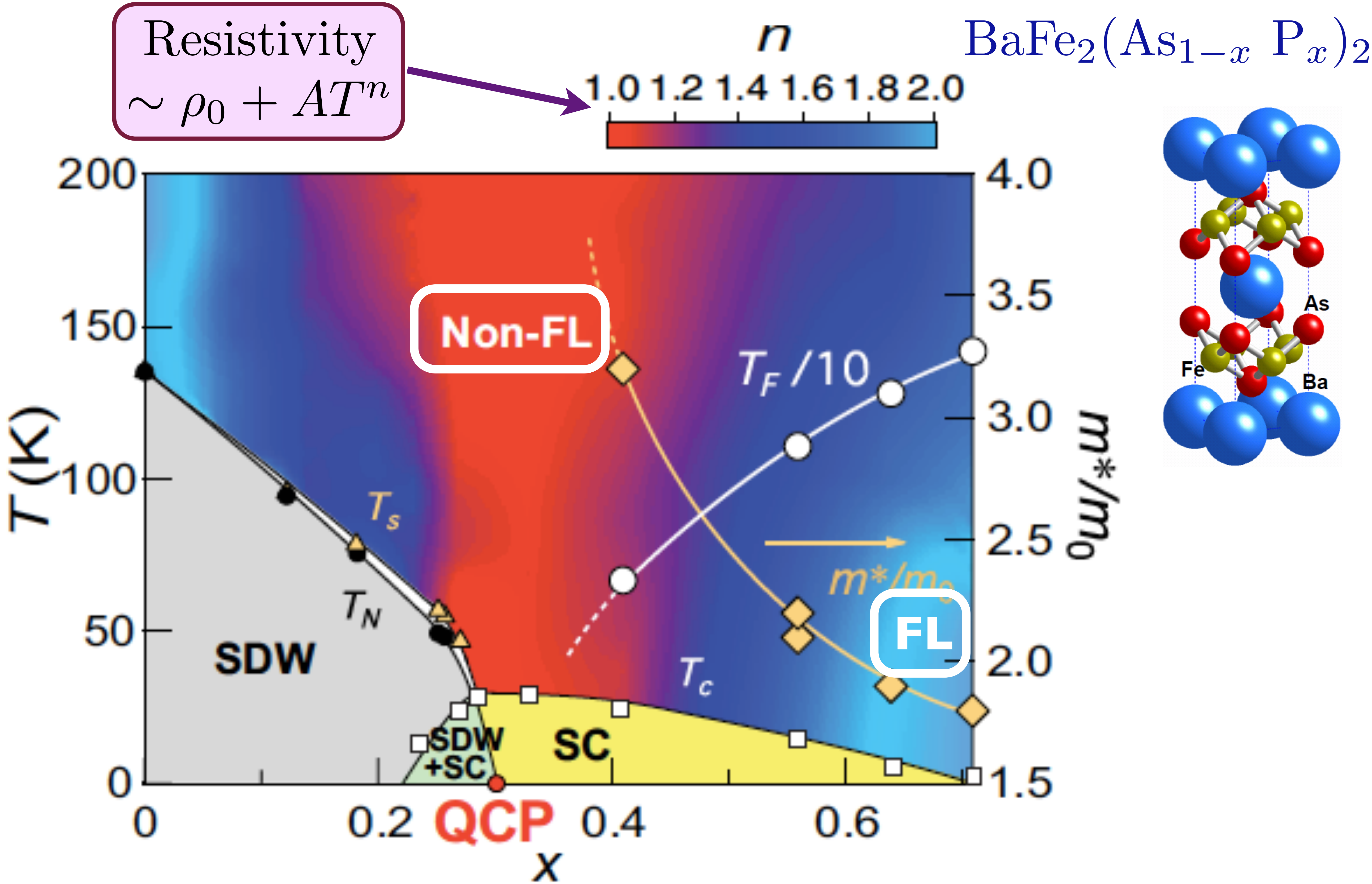
- Computation of minimal surface area, or direct computation in CFT2, yield $S_E = (c/6) \ln P$, where c is the central charge.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Compressible quantum matter

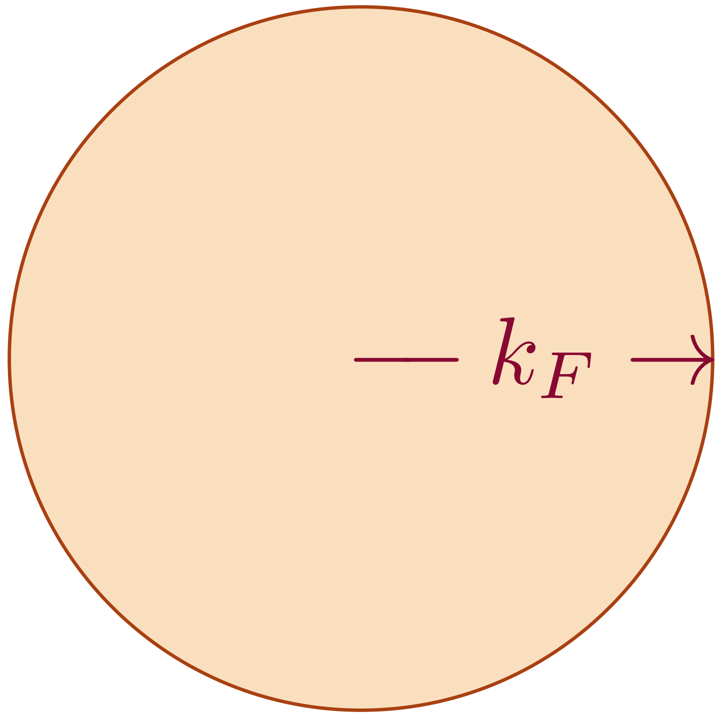


K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).



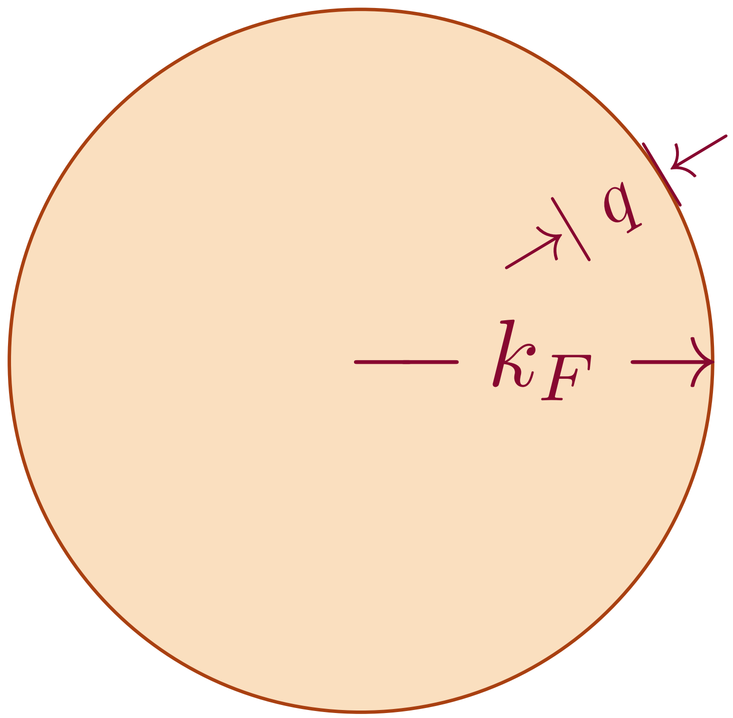
K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).

The Fermi liquid



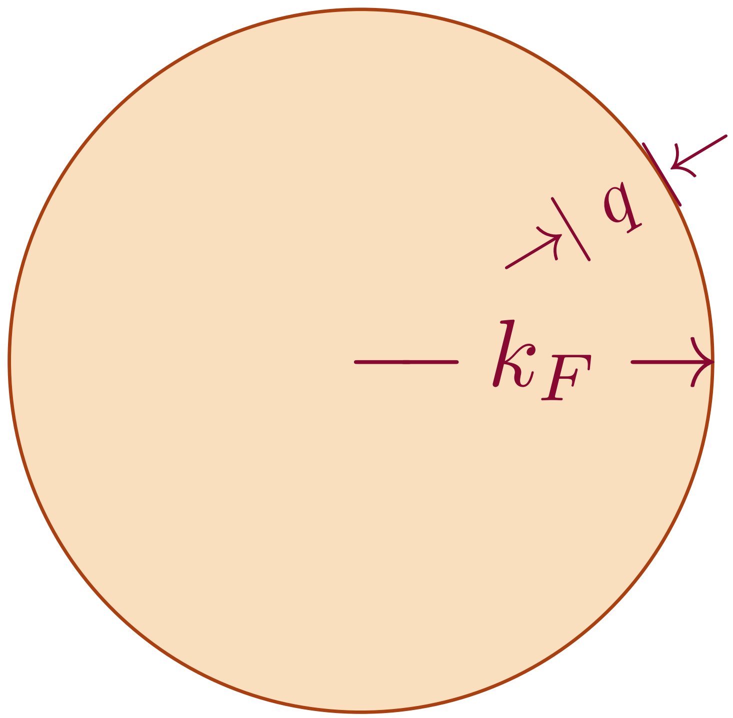
- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density

The Fermi liquid



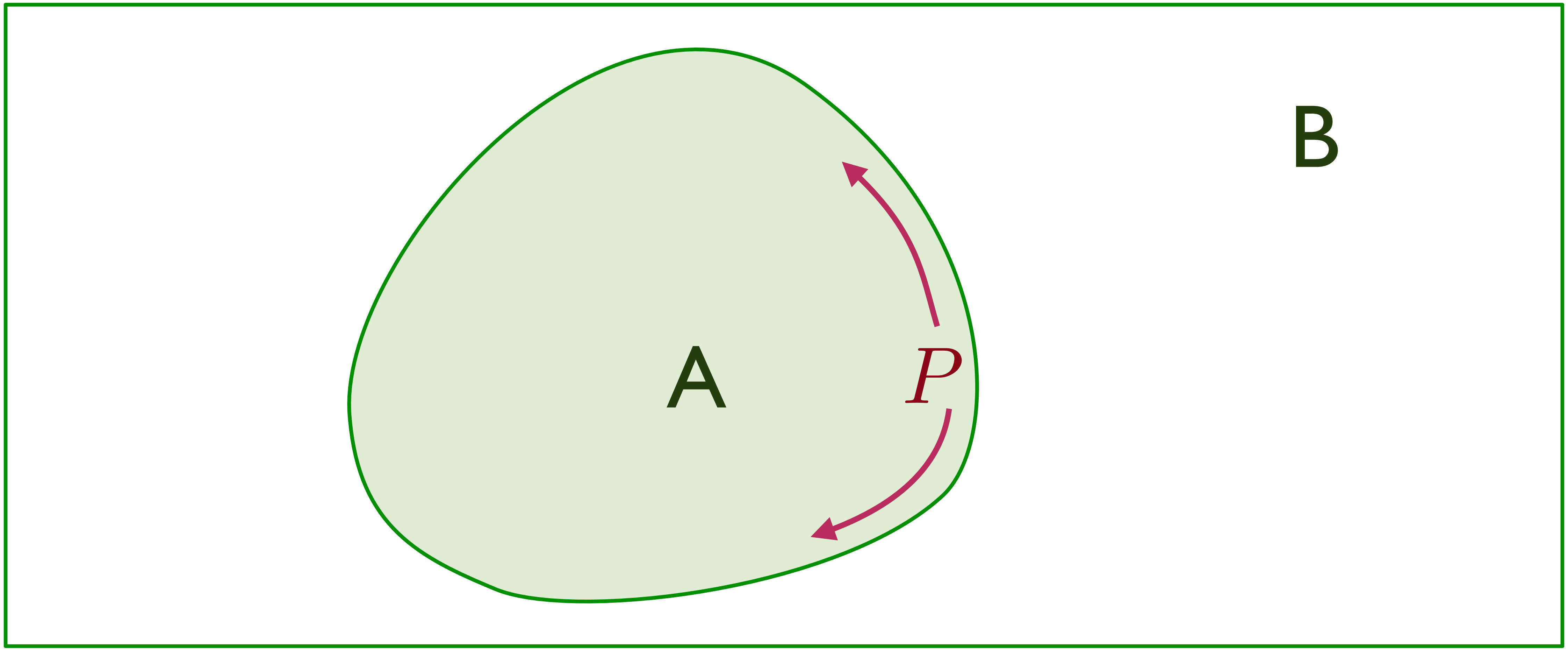
- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.

The Fermi liquid



- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$, with dynamic exponent $z = 1$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyper-scaling exponent $\theta = d - 1$.

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

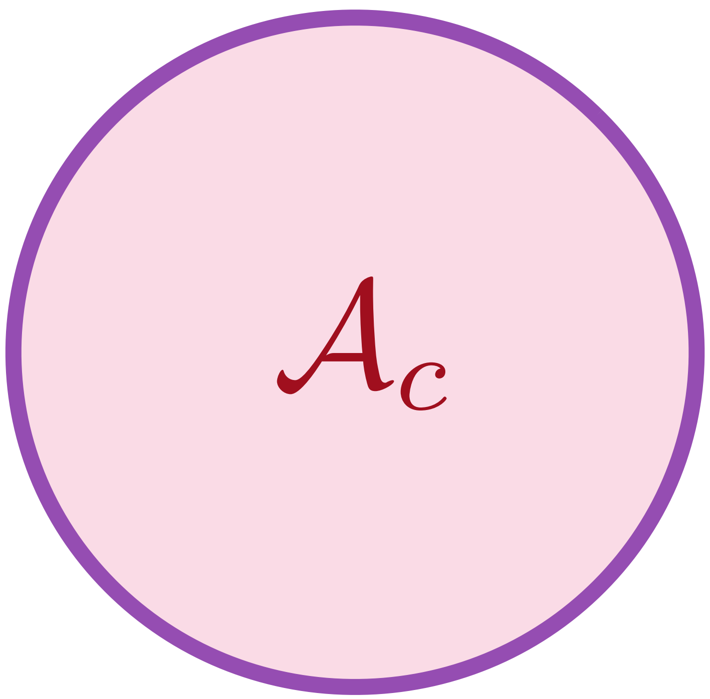
for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Consider a model of interacting bosons, b, whose density is $\mathcal{Q} = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

Consider a model of interacting bosons, b, whose density is $\mathcal{Q} = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

- **FL:** The bosons forms a bound state, c, with some spectator fermions, and there is a Fermi liquid of the fermionic c molecules.



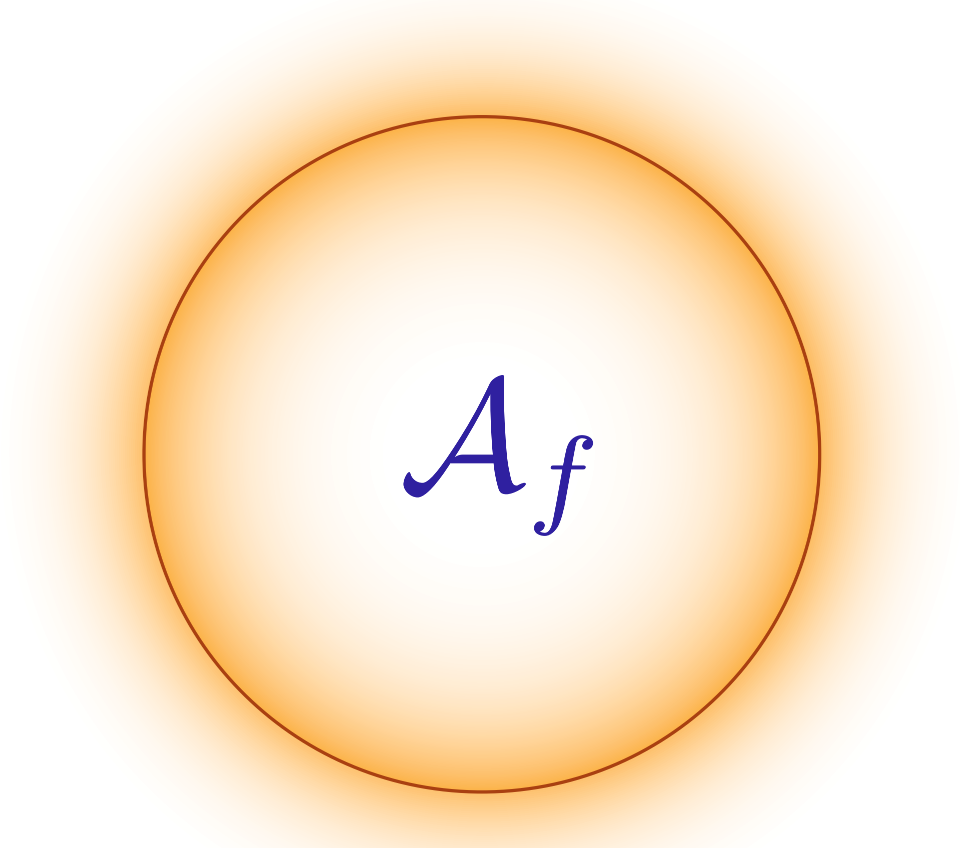
$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

S. Powell, S. Sachdev, and H. P. Büchler, *Physical Review* B **72**, 024534 (2005)
P. Coleman, I. Paul, and J. Rech, *Physical Review* B **72**, 094430 (2005)

Consider a model of interacting bosons, b, whose density is $\mathcal{Q} = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

- **NFL**, the non-Fermi liquid *Bose metal*. The boson fractionalizes into (say) 2 fermions, f_1 and f_2, each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are "*hidden*".



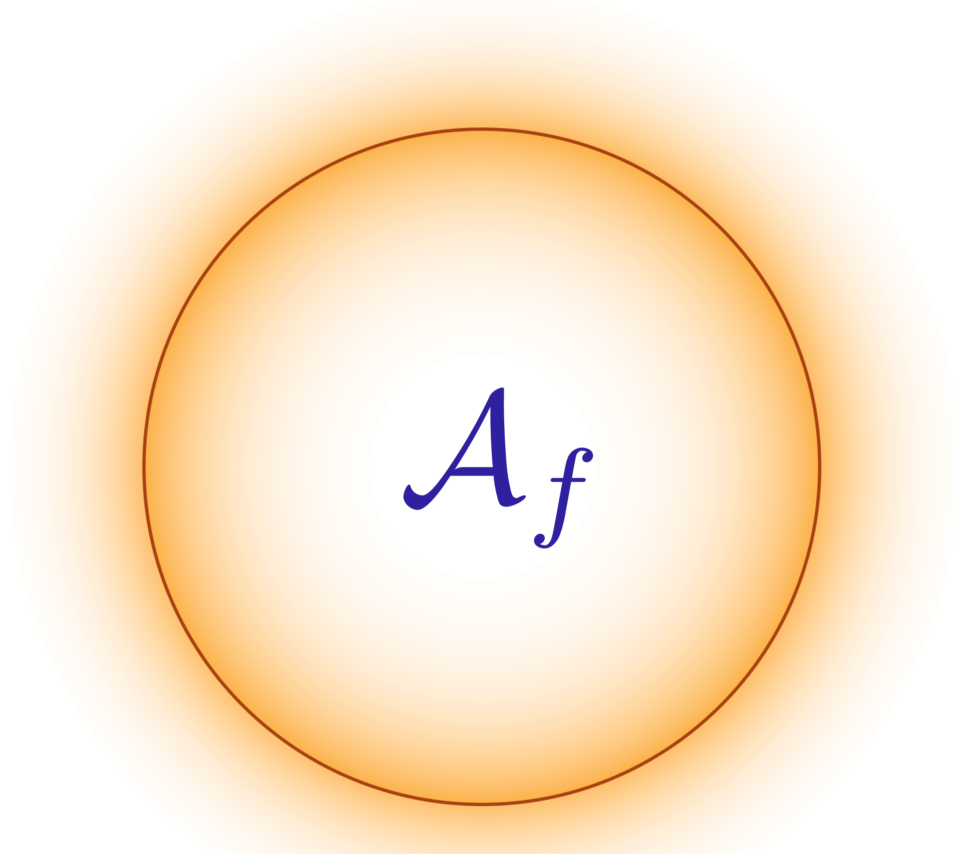
$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher,
Physical Review B **75**, 235116 (2007)
L. Huijse and S. Sachdev,
Physical Review D **84**, 026001 (2011)

Consider a model of interacting bosons, b, whose density is $\mathcal{Q} = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

- **NFL**, the non-Fermi liquid *Bose metal*. The boson fractionalizes into (say) 2 fermions, f_1 and f_2, each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are "*hidden*".



$$b \to f_1 f_2$$

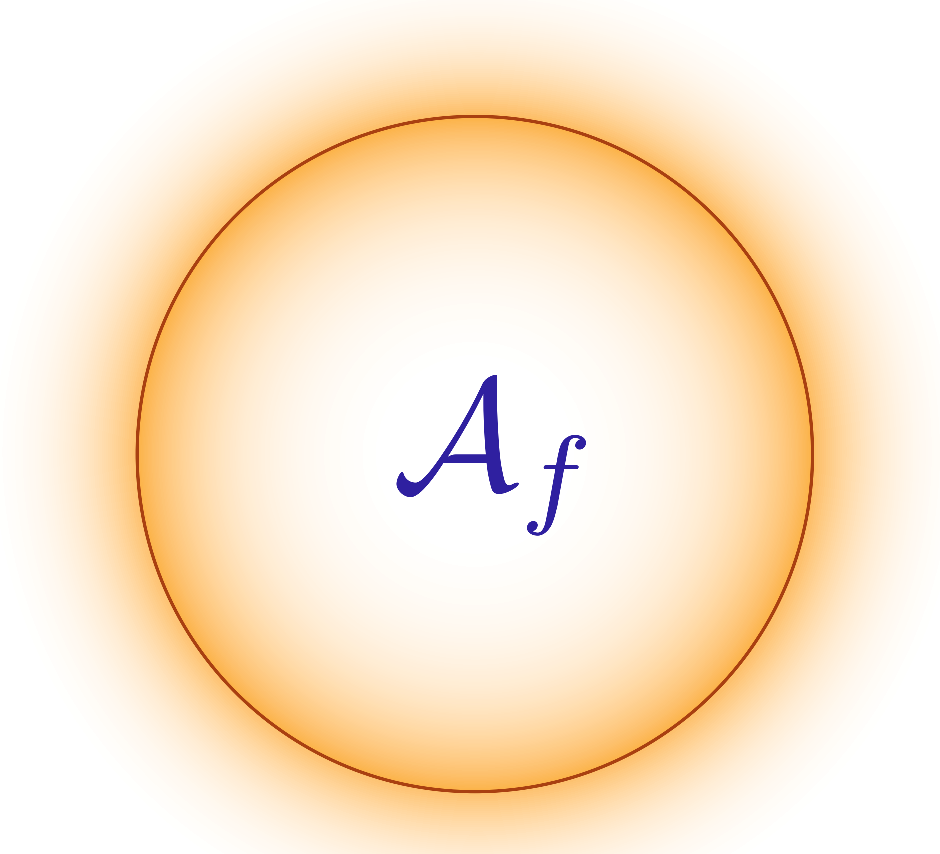
Gauge invariance:

$$f_1(x) \to f_1(x) e^{i\theta(x)},$$
$$f_2(x) \to f_2(x) e^{-i\theta(x)}$$

Fisher,
2007)
chdev,
Physical Review D **84**, 026001 (2011)

Consider a model of interacting bosons, b, whose density is $\mathcal{Q} = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

- **NFL**, the non-Fermi liquid *Bose metal*. The boson fractionalizes into (say) 2 fermions, f_1 and f_2, each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are "*hidden*".



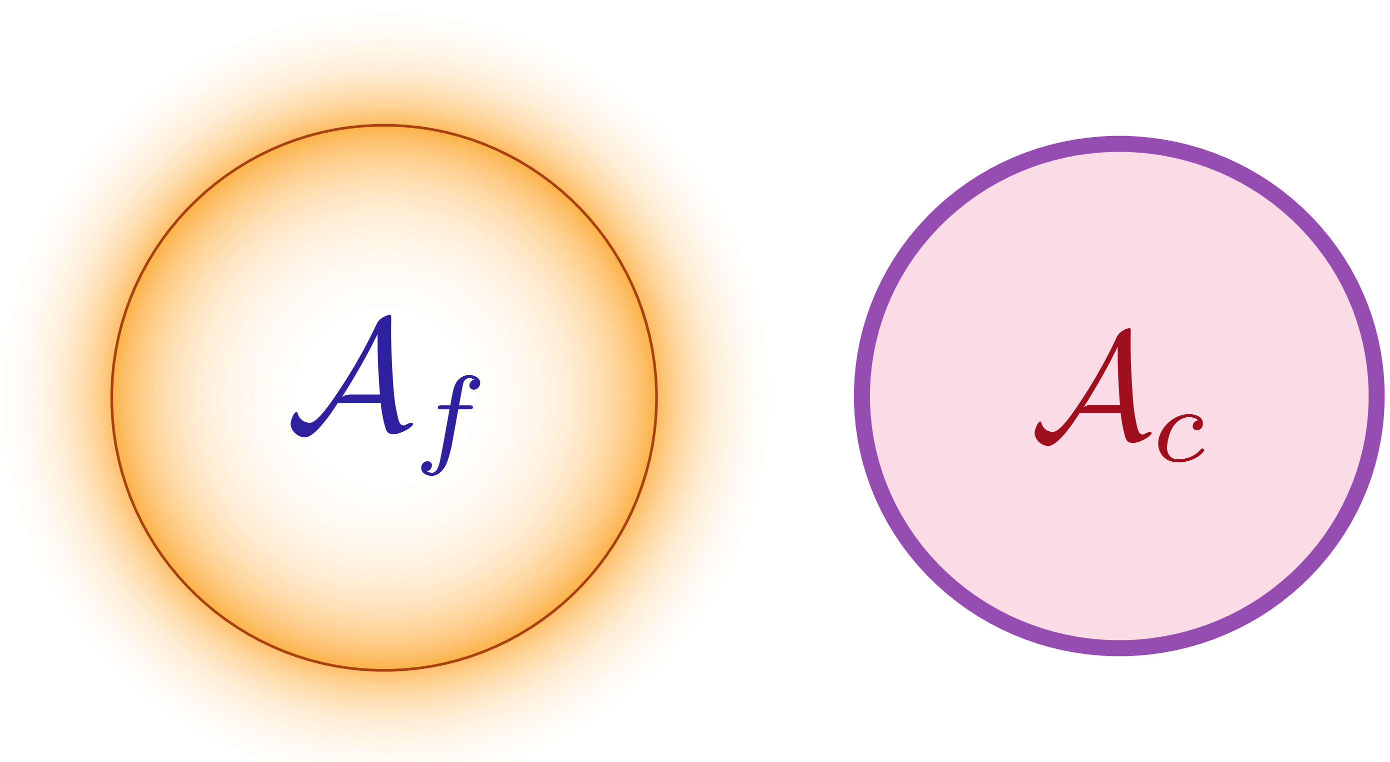
$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher, *Physical Review* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011)

Consider a model of interacting bosons, b, whose density is $\mathcal{Q} = b^\dagger b$ is conserved. We want a ground state which does not break any symmetries (and so solids and superfluids are excluded). The only known possibilities are:

- **FL*** Partially fractionalized state, with co-existence of visible and hidden Fermi surfaces.

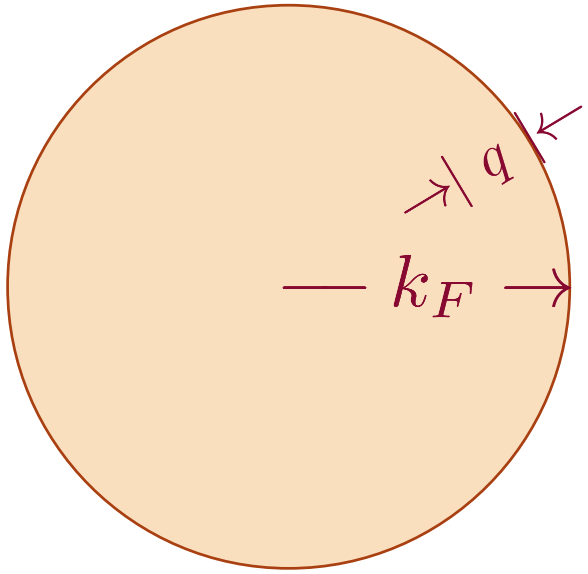


$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_c + \mathcal{A}_f = \langle \mathcal{Q} \rangle$$

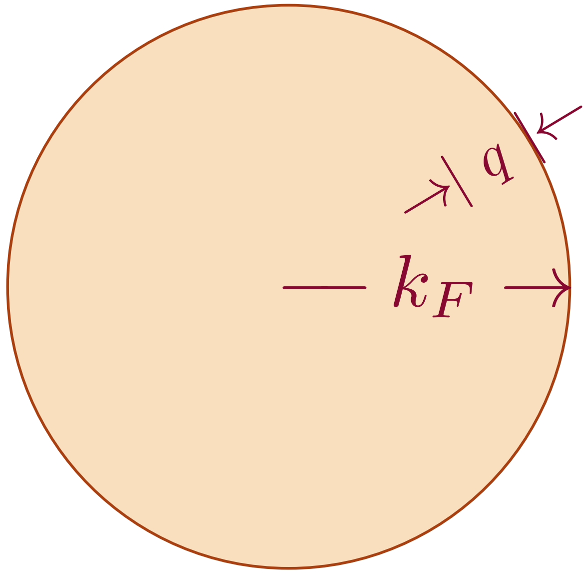
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)
L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011).

FL Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

FL
Fermi
liquid



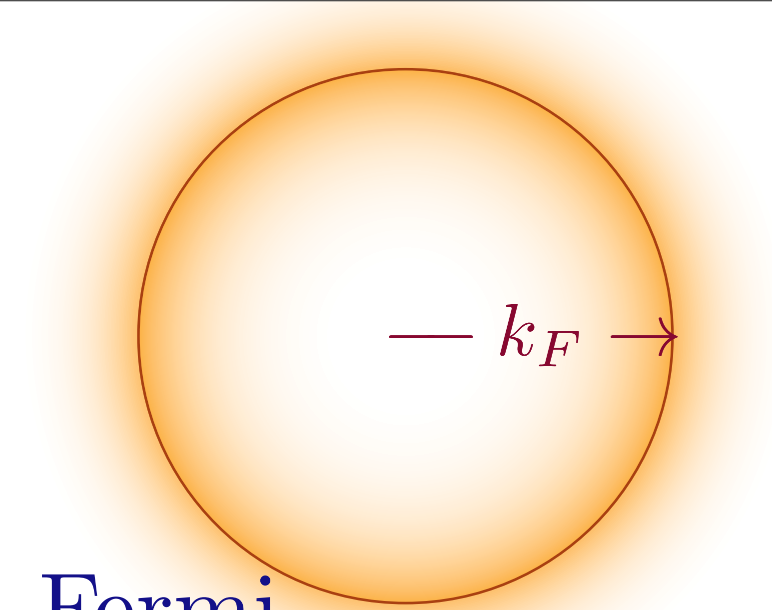
- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

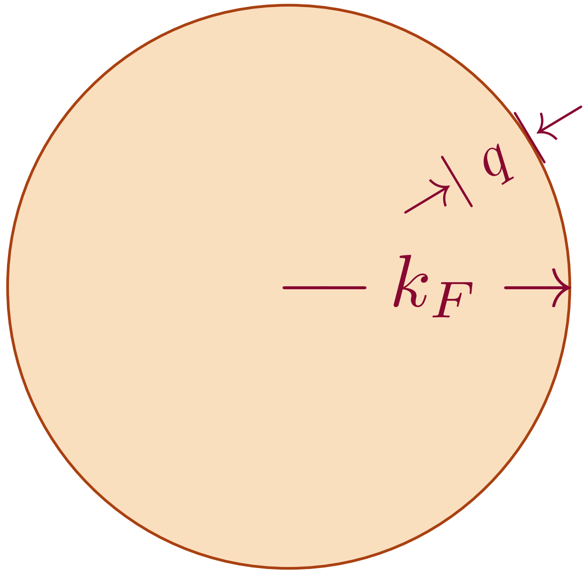
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

NFL
Bose
metal



- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.

FL
Fermi
liquid



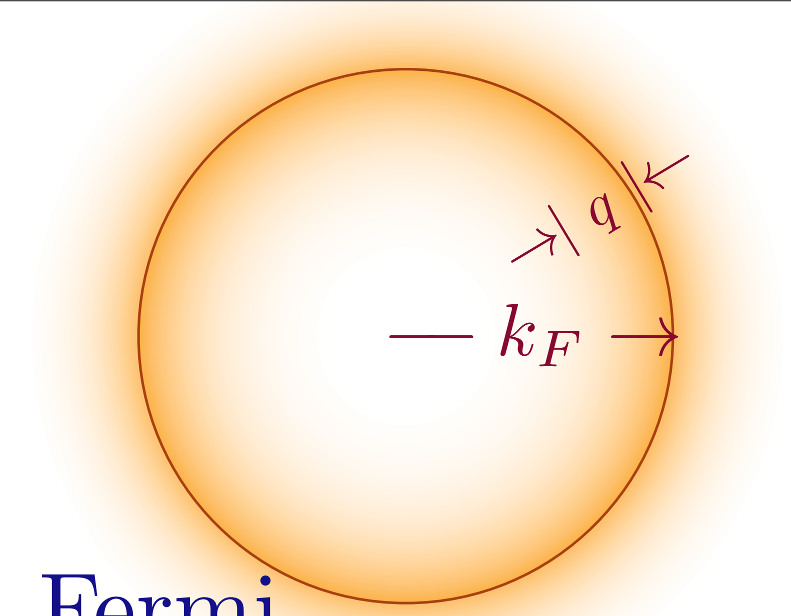
- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

NFL
Bose
metal

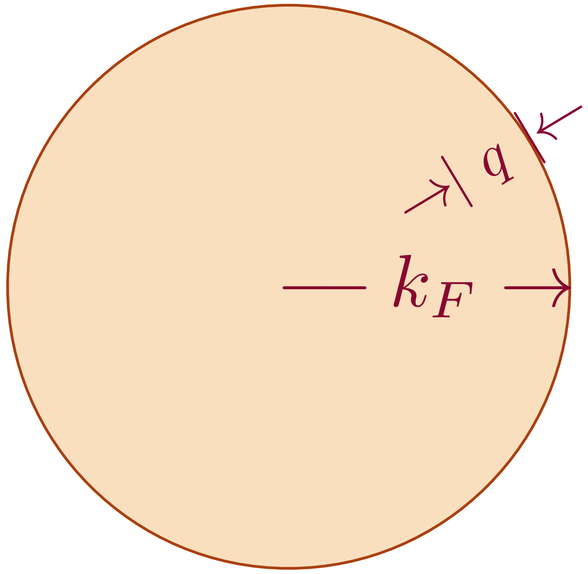


- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.

- Diffuse fermionic excitations with $z = 3/2$ to three loops.

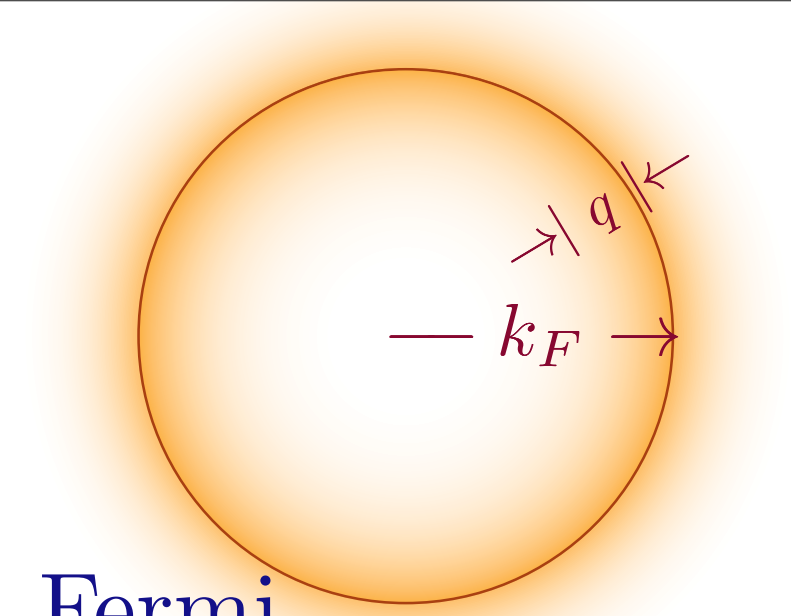
M. A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075127 (2010)

FL
Fermi liquid

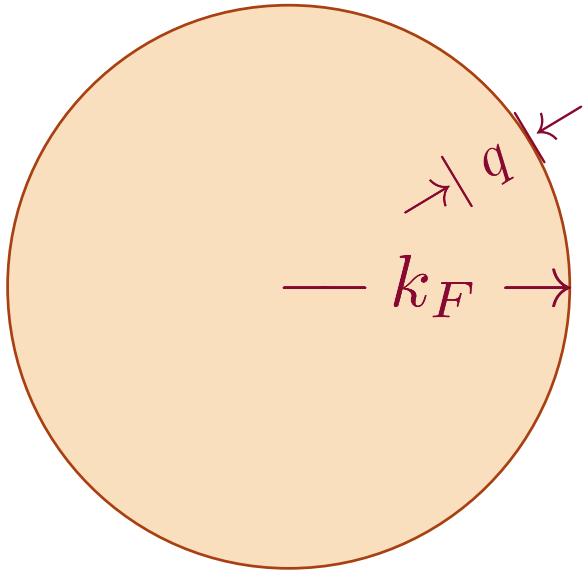


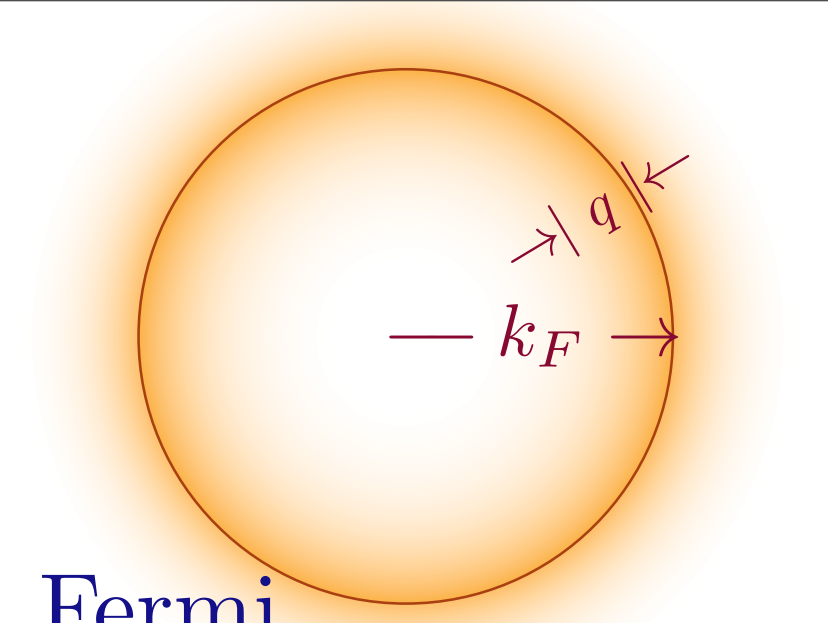
- $k_F^d \sim \mathcal{Q}$, the fermion density
- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.
- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.
- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

NFL
Bose metal



- Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.
- $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.

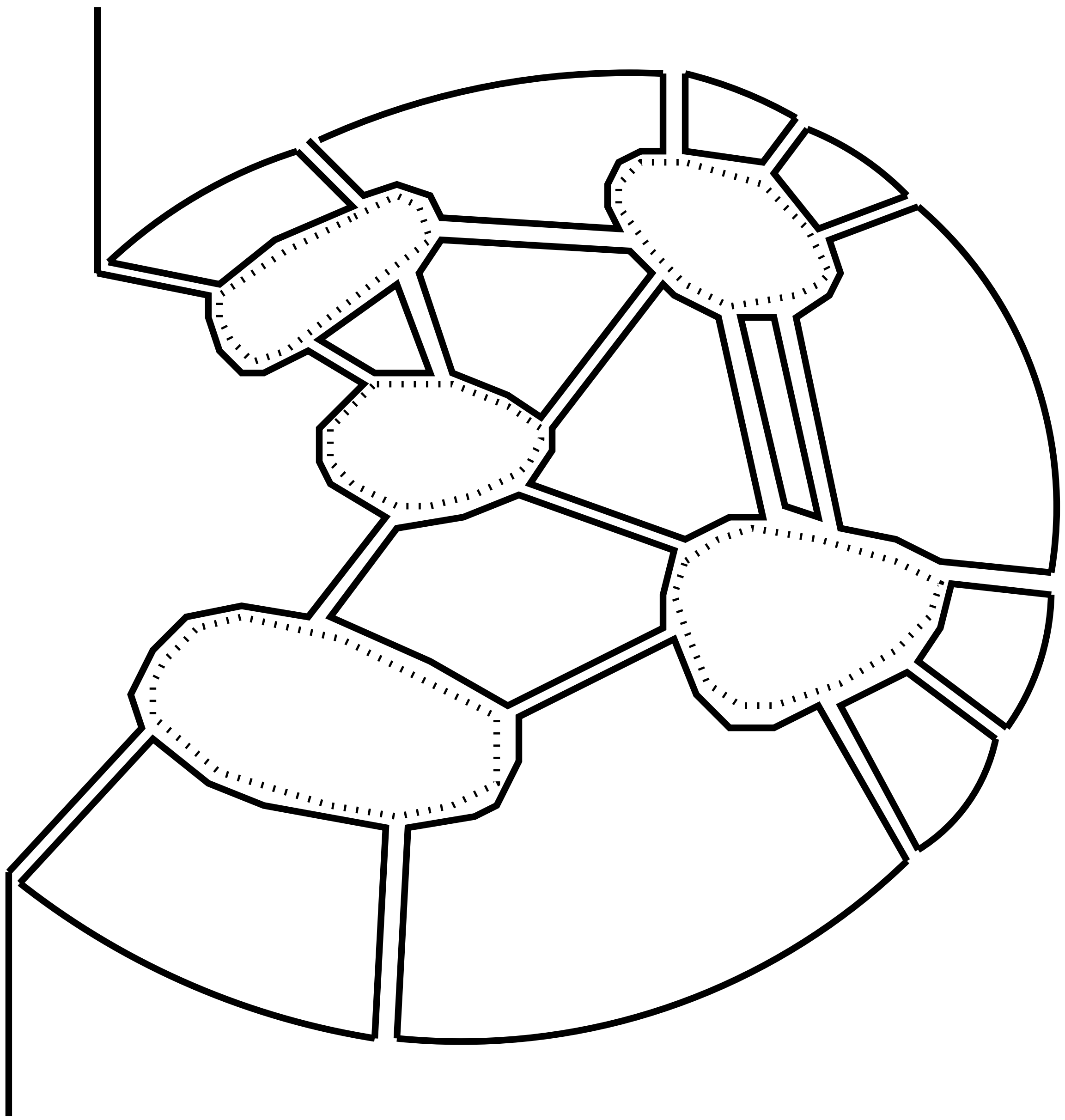


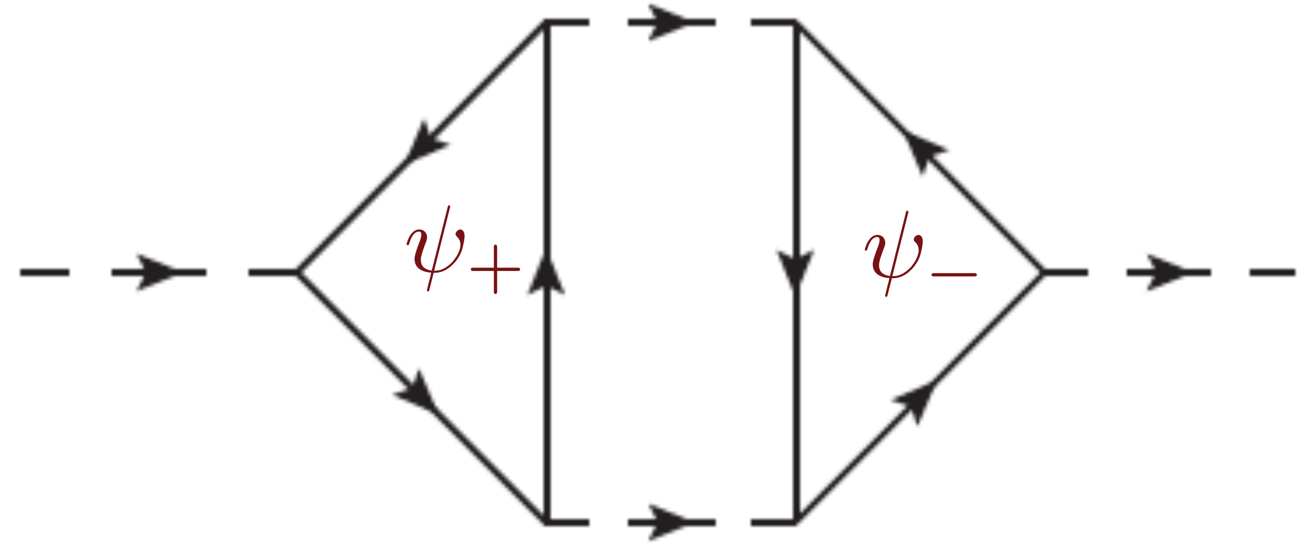


FL Fermi liquid	NFL Bose metal
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Hidden Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \|q\|^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	• $S \sim T^{(d-\theta)/z}$ with $\theta = d - 1$.
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	• $S_E \sim k_F^{d-1} P \ln P$.

Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. Lett.* **107**, 067202 (2011)

Computations in the $1/N$ expansion





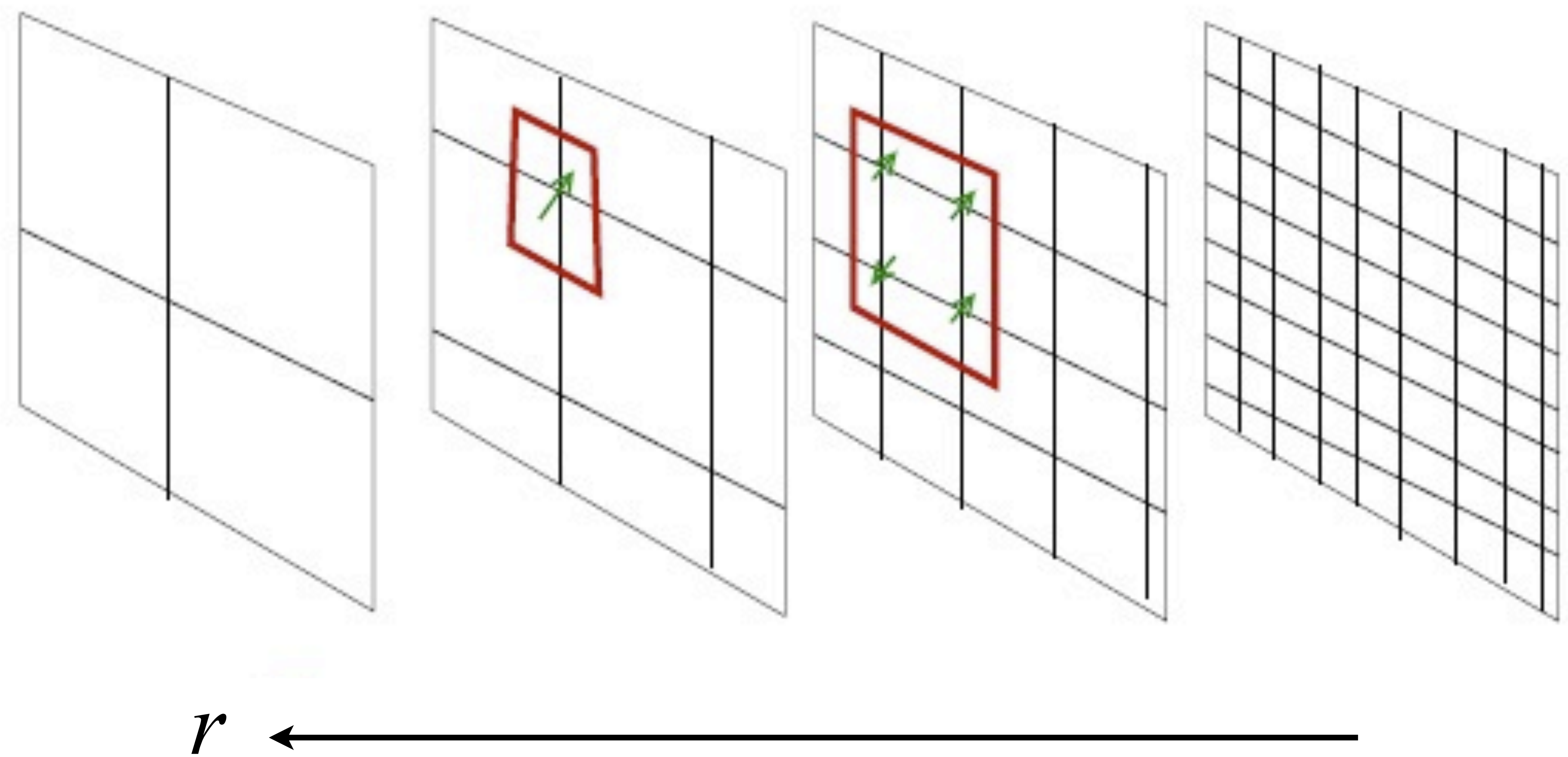
Graph mixing antipodal arcs is $\mathcal{O}\left(N^{3/2}\right)$ (instead of $\mathcal{O}\left(N\right)$), violating genus expansion

All planar graphs of fermions on an arc of the Fermi surface are as important as the leading term

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Holography



Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Consider the metric which transforms under rescaling as

$$
\begin{aligned}
x_i &\rightarrow \zeta\, x_i \\
t &\rightarrow \zeta^z\, t \\
ds &\rightarrow \zeta^{\theta/d}\, ds.
\end{aligned}
$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$
ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)
$$

We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

At $T > 0$, there is a *horizon*, and computation of its Bekenstein-Hawking entropy shows

$$S \sim T^{(d-\theta)/z}.$$

So θ is indeed the violation of hyperscaling exponent as claimed. For a compressible quantum state we should therefore *choose* $\theta = d - 1$.
No additional choices will be made, and all subsequent results are consequences of the assumption of the existence of a holographic dual.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

In $d = 2$, this implies $z \geq 3/2$. So the lower bound is precisely the value obtained from the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of strange metals

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim \mathcal{Q}^{(d-1)/d} P \ln P$$

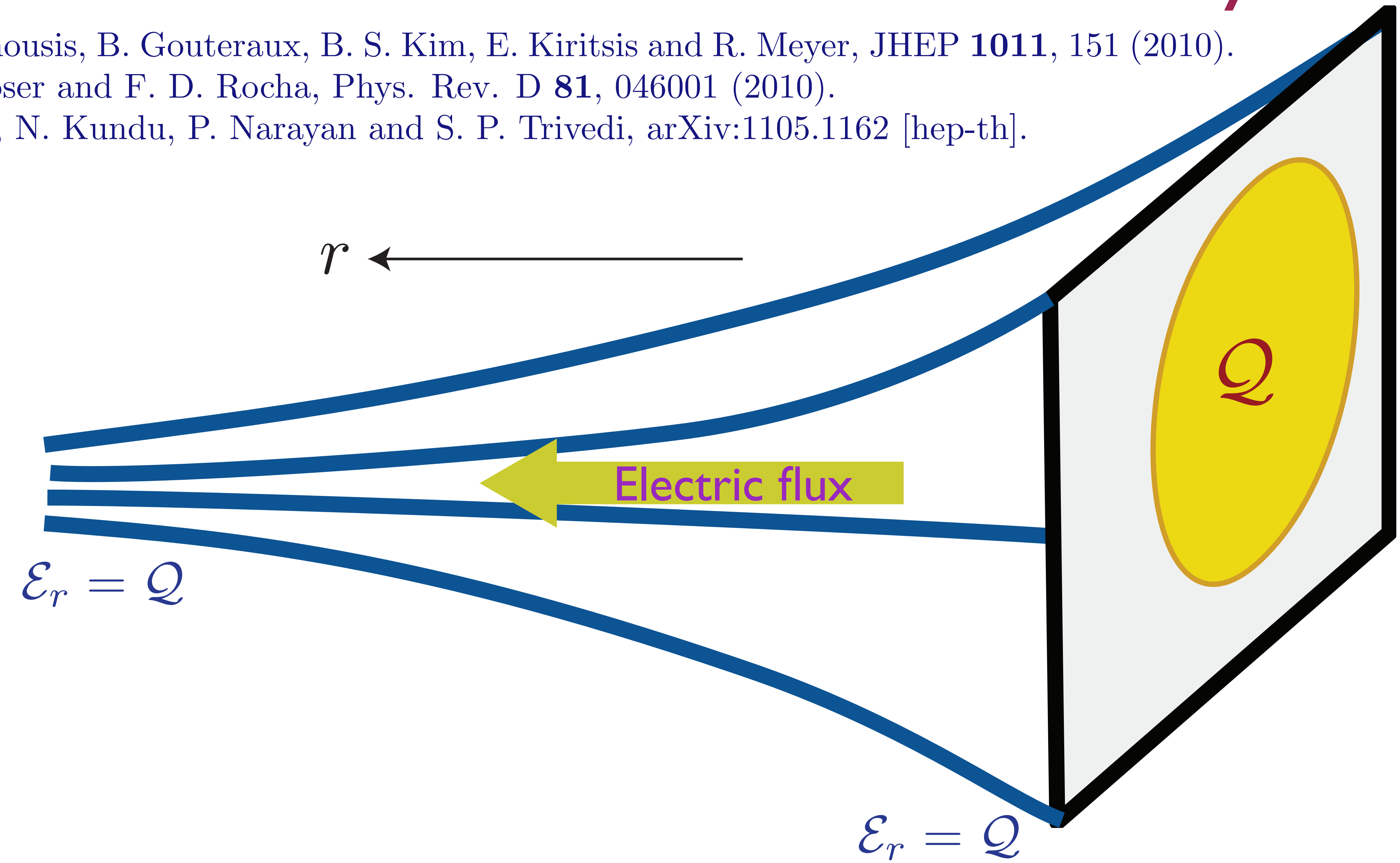
with a co-efficient *independent* of UV details and of the shape of the entangling region. These properties are just as expected for a circular Fermi surface with $\mathcal{Q} \sim k_F^d$.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holographic theory of a non-Fermi liquid (NFL)

Einstein-Maxwell-dilaton theory

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

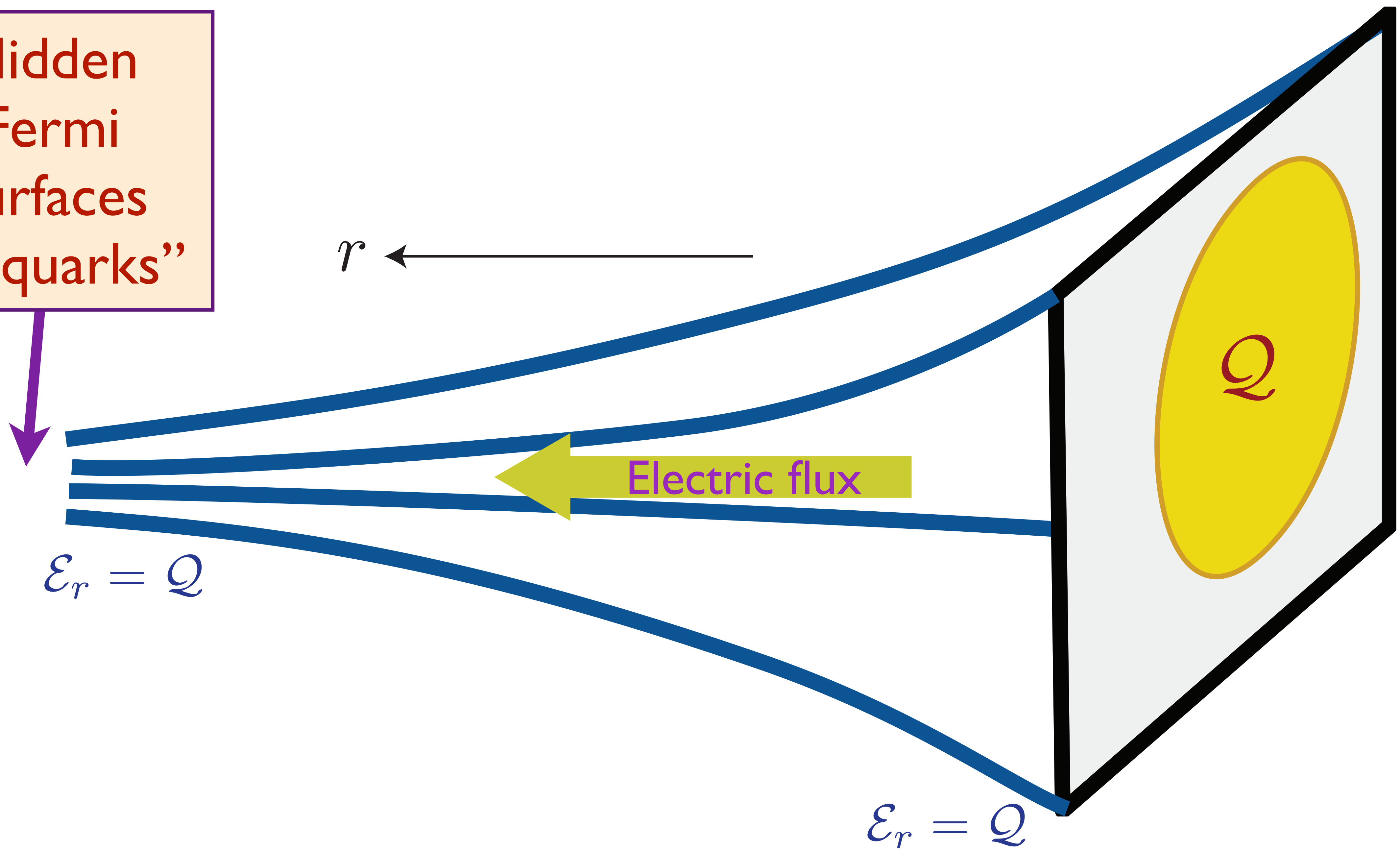


$$\mathcal{S} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2}\right) - \frac{Z(\Phi)}{4e^2}F_{ab}F^{ab}\right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

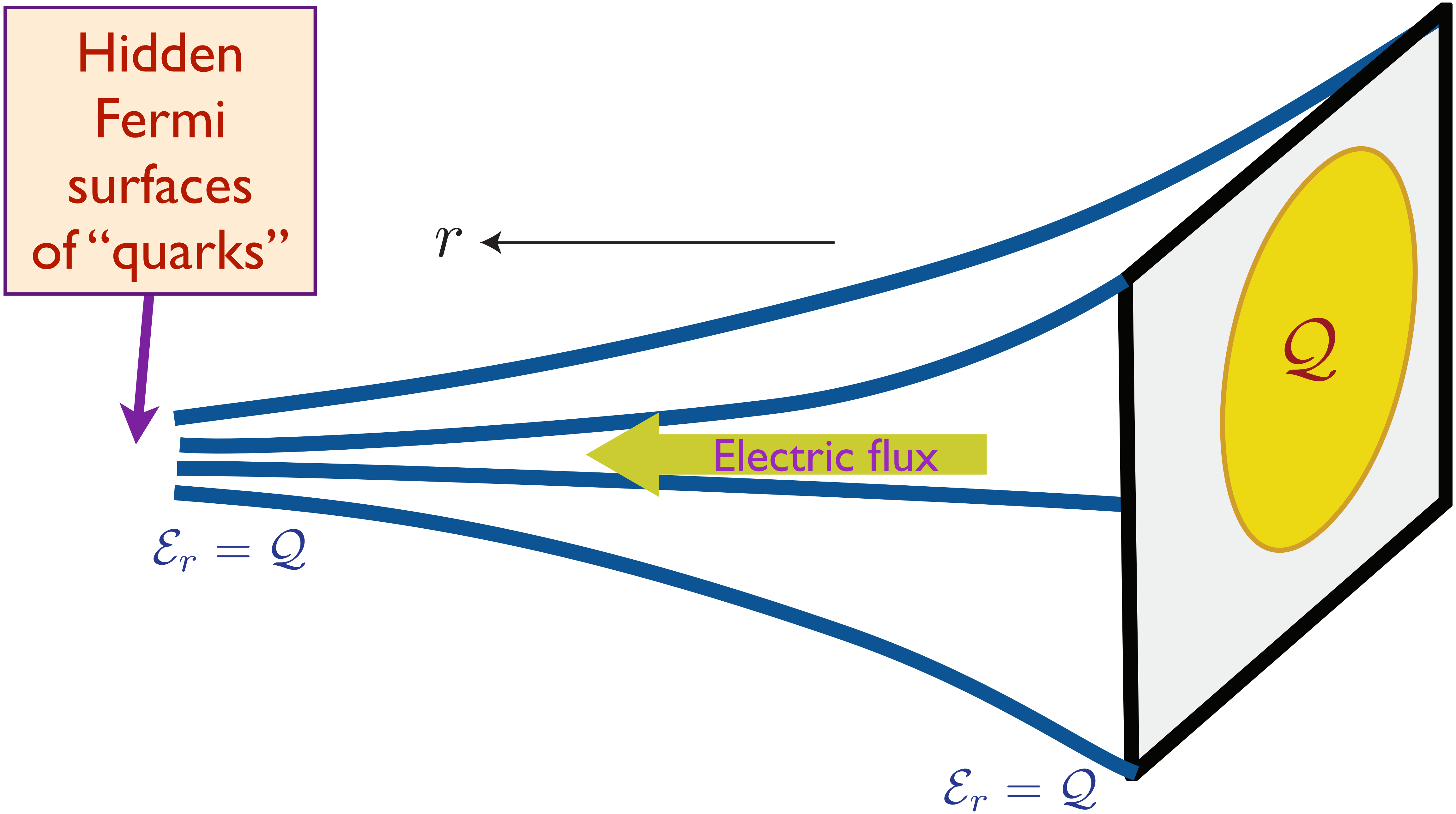
Holographic theory of a non-Fermi liquid (NFL)

Hidden Fermi surfaces of “quarks”



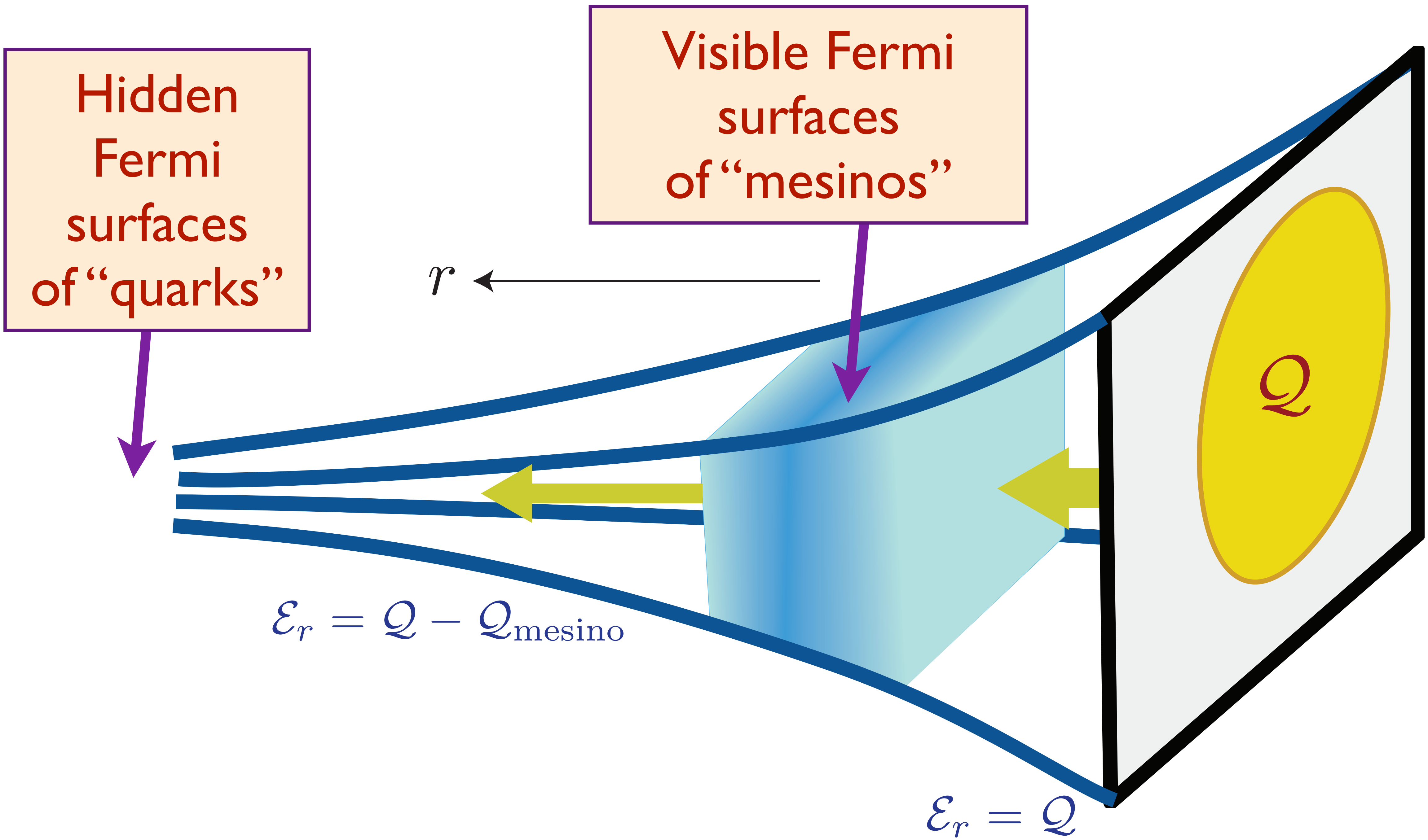
This is a “bosonization” of the *hidden* Fermi surface

Holographic theory of a non-Fermi liquid (NFL)



Fully fractionalized state has all the electric flux exiting to the horizon at $r = \infty$

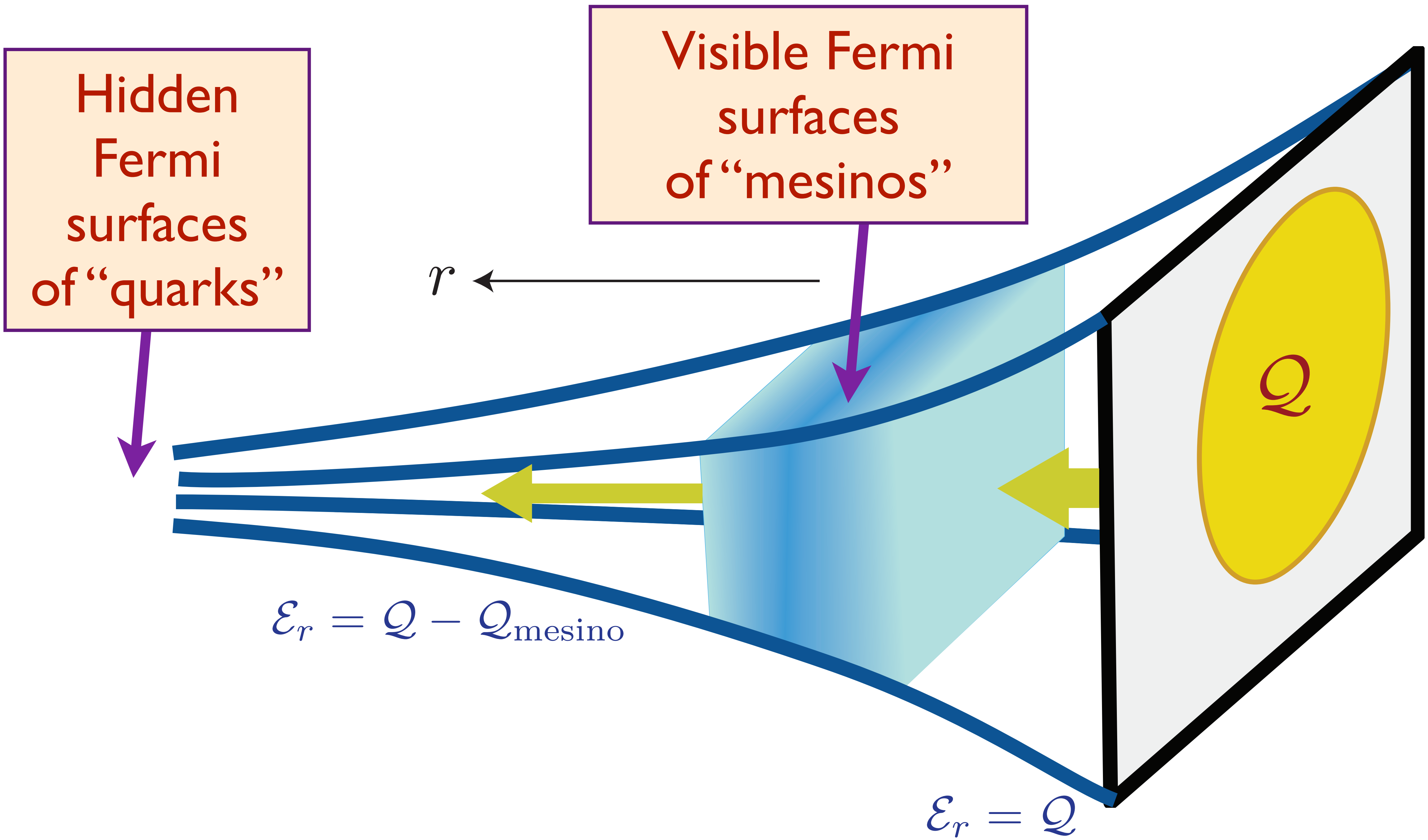
Holographic theory of a fractionalized-Fermi liquid (FL*)



A state with partial fractionalization, and partial electric flux exiting horizon

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010); S. Sachdev, *Physical Review D* **84**, 066009 (2011)

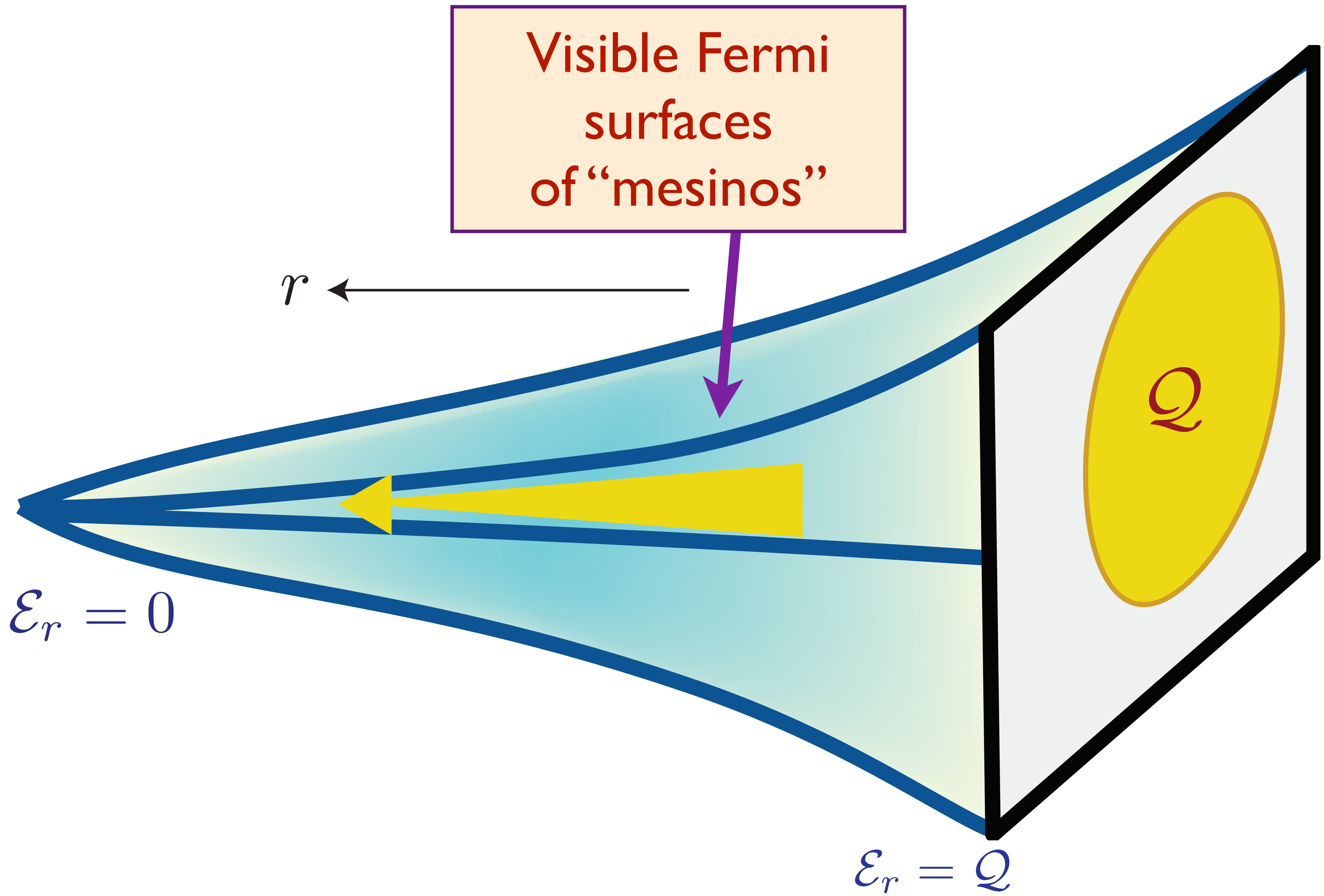
Holographic theory of a fractionalized-Fermi liquid (FL*)



The “mesinos” corresponds to the Fermi surfaces obtained in the early probe fermion computation (S.-S. Lee, Phys. Rev. D **79**, 086006 (2009); H. Liu, J. McGreevy, and D. Vegh, arXiv:0903.2477; M. Čubrović, J. Zaanen, and K. Schalm, Science **325**, 439 (2009)).

These are spectators, and are expected to have well-defined quasiparticle excitations.

Holographic theory of a Fermi liquid (FL)



- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)

Holography, fractionalization, and hidden Fermi surfaces

- Electric flux exiting the horizon corresponds to fractionalized component of the conserved density $\mathcal{Q}$, which is proposed to be associated with "hidden" Fermi surfaces of gauge-charged particles.

- Gauss Law and the "attractor" mechanism in the bulk
$\Leftrightarrow$ Luttinger theorem on the boundary theory.

Conclusions

Gapped quantum matter

- Numerical and experimental observation of a spin liquid on the kagome lattice. Likely a Z_2 spin liquid.

Conclusions

Conformal quantum matter

- Numerical and experimental observation in coupled-dimer antiferromagnets, and at the superfluid-insulator transition of bosons in optical lattices.

Conclusions

Compressible quantum matter

- Holographic theory yields models of non-Fermi liquids (NFL), fractionalized Fermi liquids (FL*), and Fermi liquids (FL) , in close correspondence with the phases expected from field theory