The AdS/CFT description of the quantum phases of matter

Penn State University, September 17, 2010

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Outline

1. "Zero"-density quantum citical points
   Diffusion and transport in strongly interacting "perfect fluids"

2. Quantum matter at non-zero density
   Holographic superconductors and strange metals

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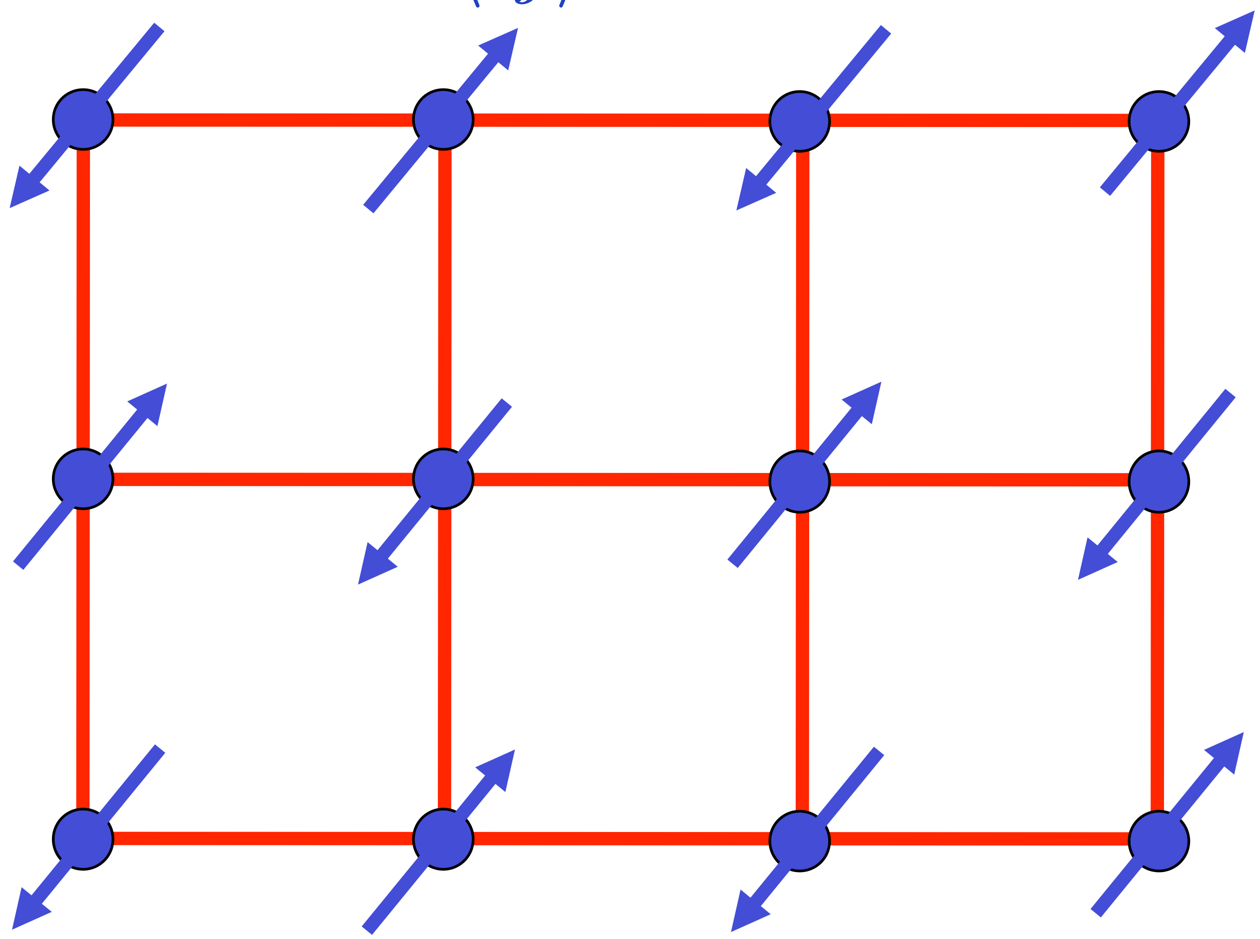
   Diffusion and transport in strongly interacting "perfect fluids"

2. Quantum matter at non-zero density
   Holographic superconductors and strange metals

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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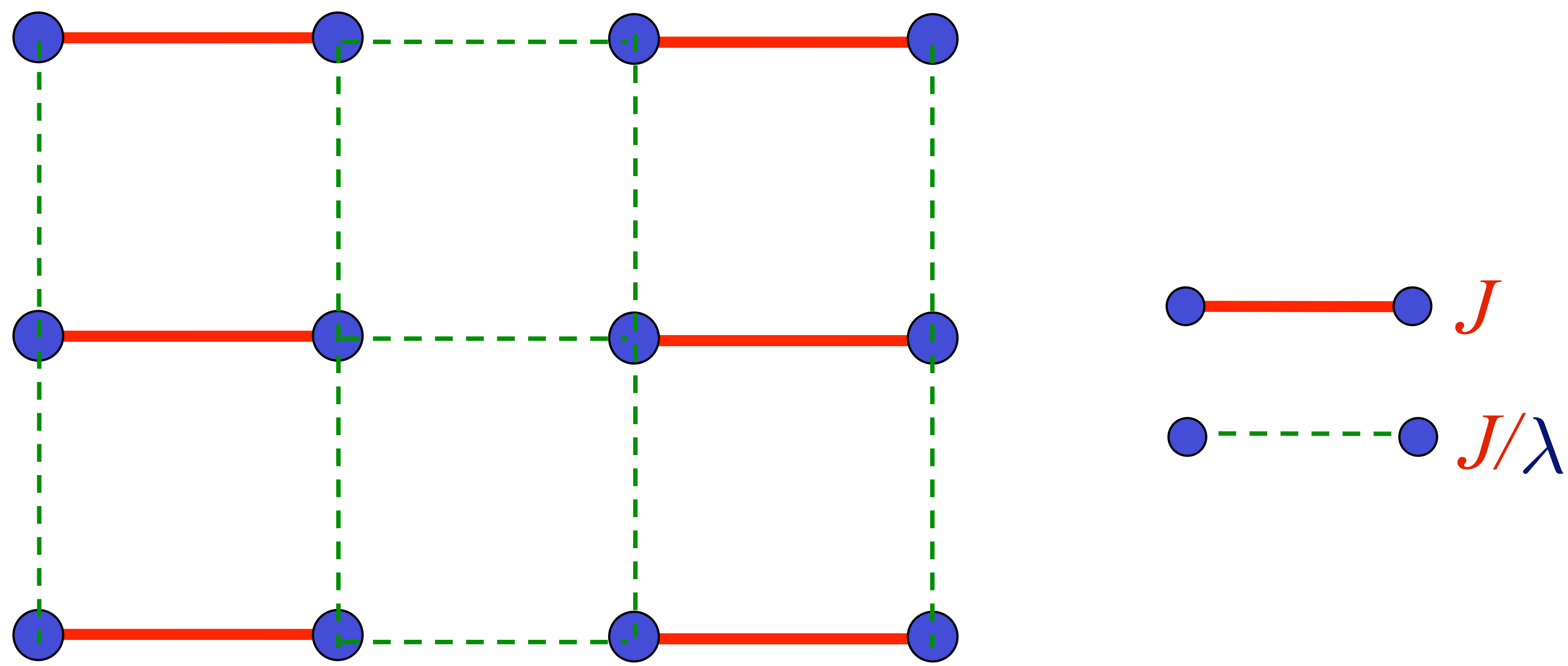
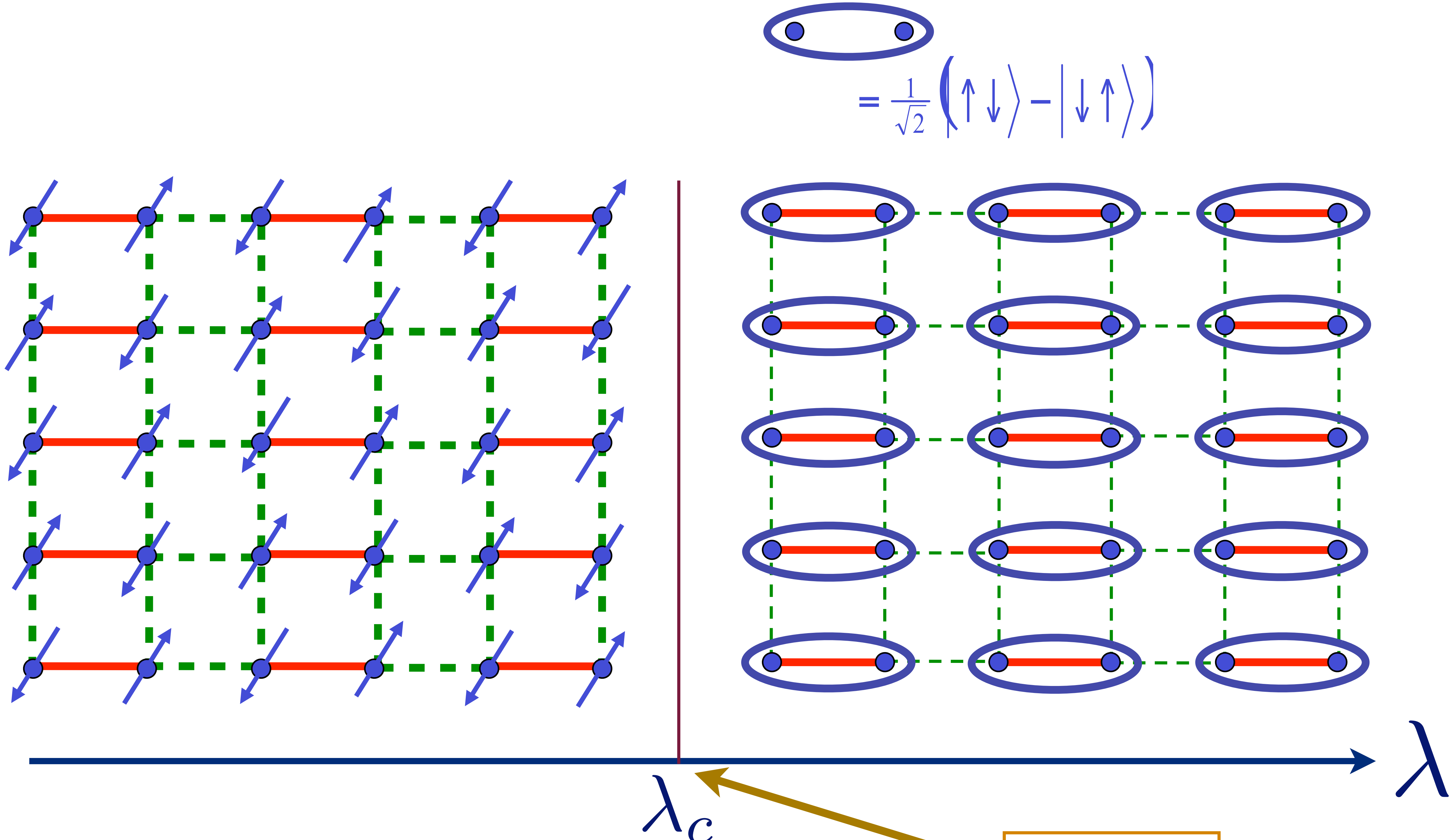
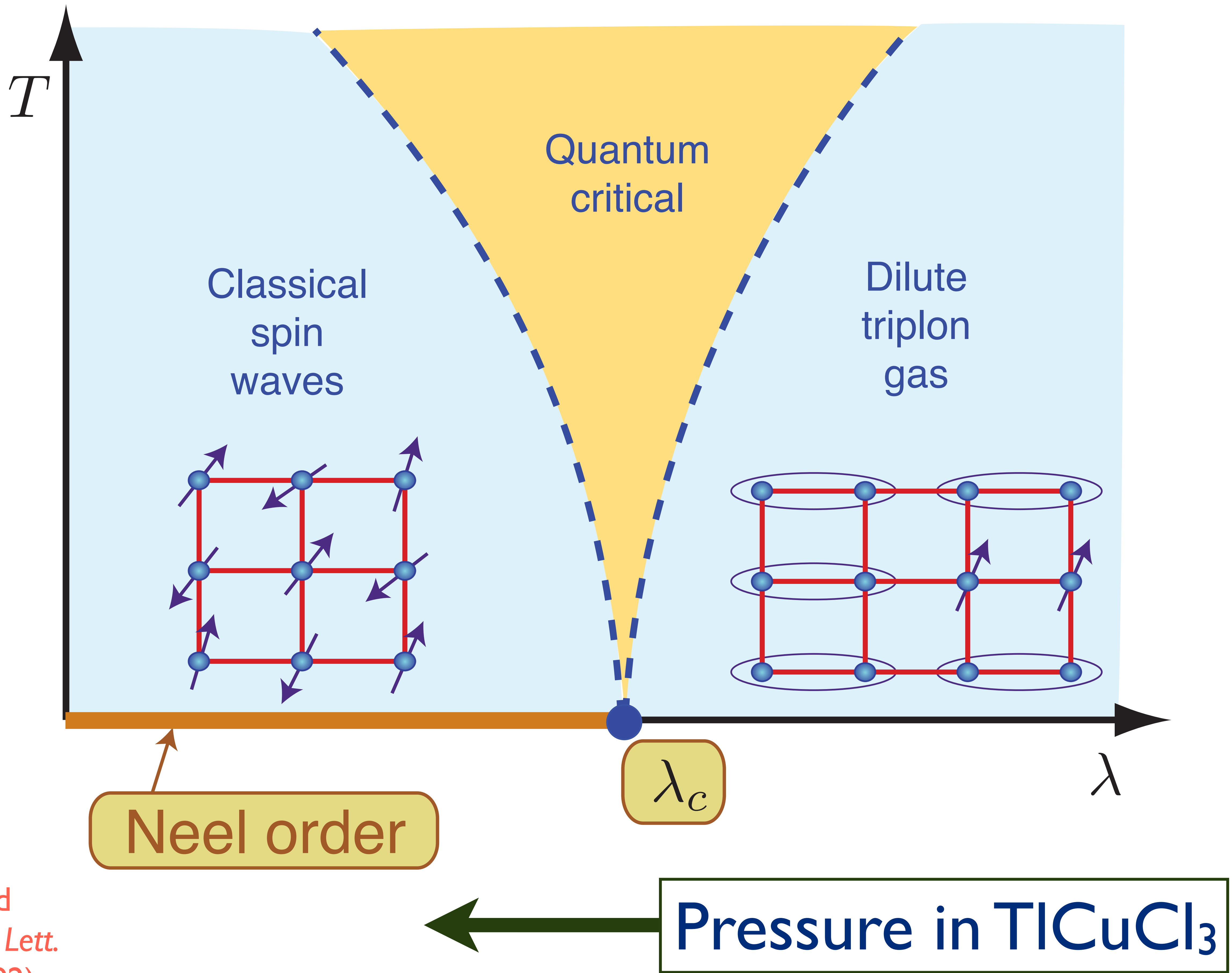


Weaken some bonds to induce spin entanglement in a new quantum phase

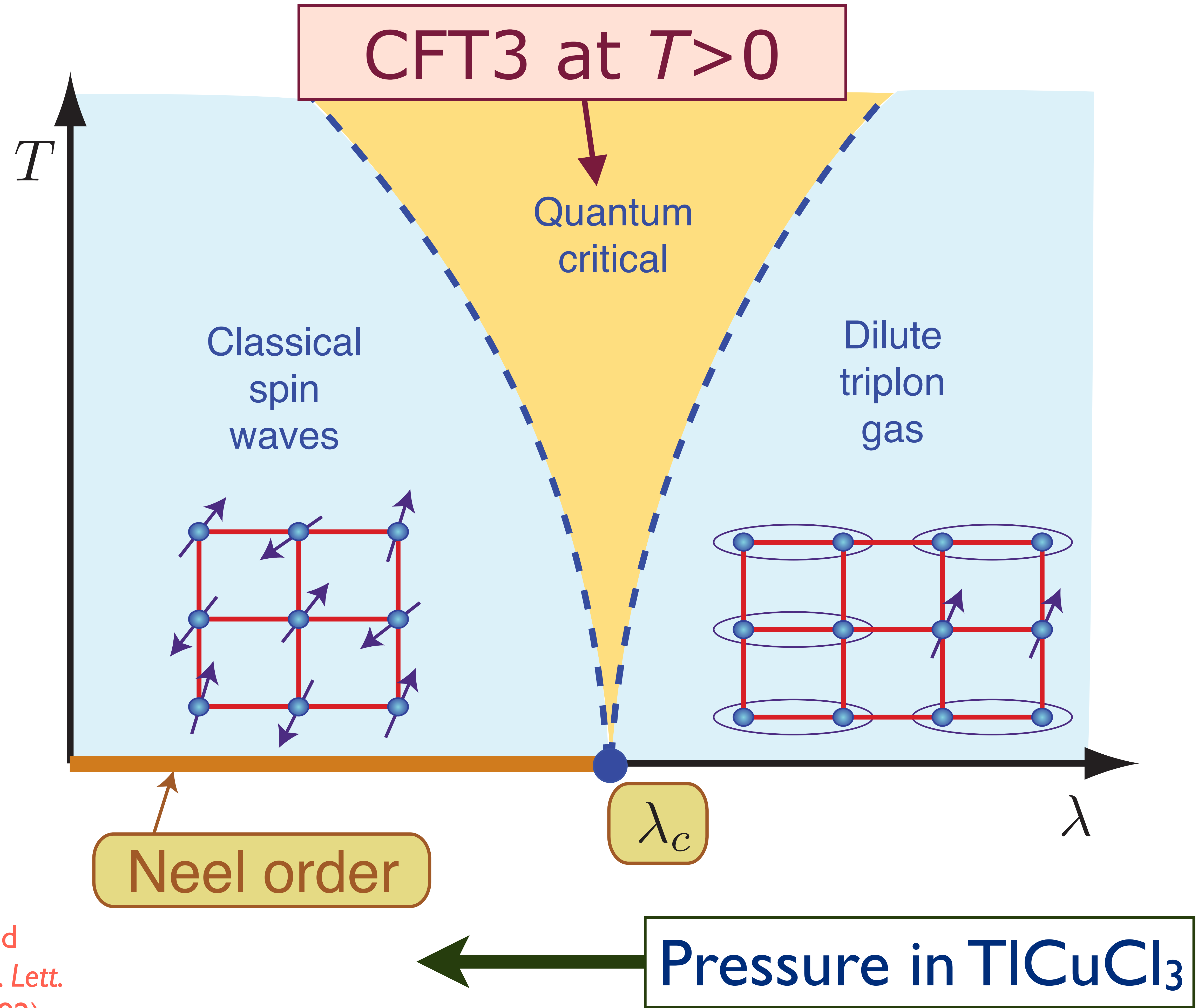


O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2 \right]$$



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).



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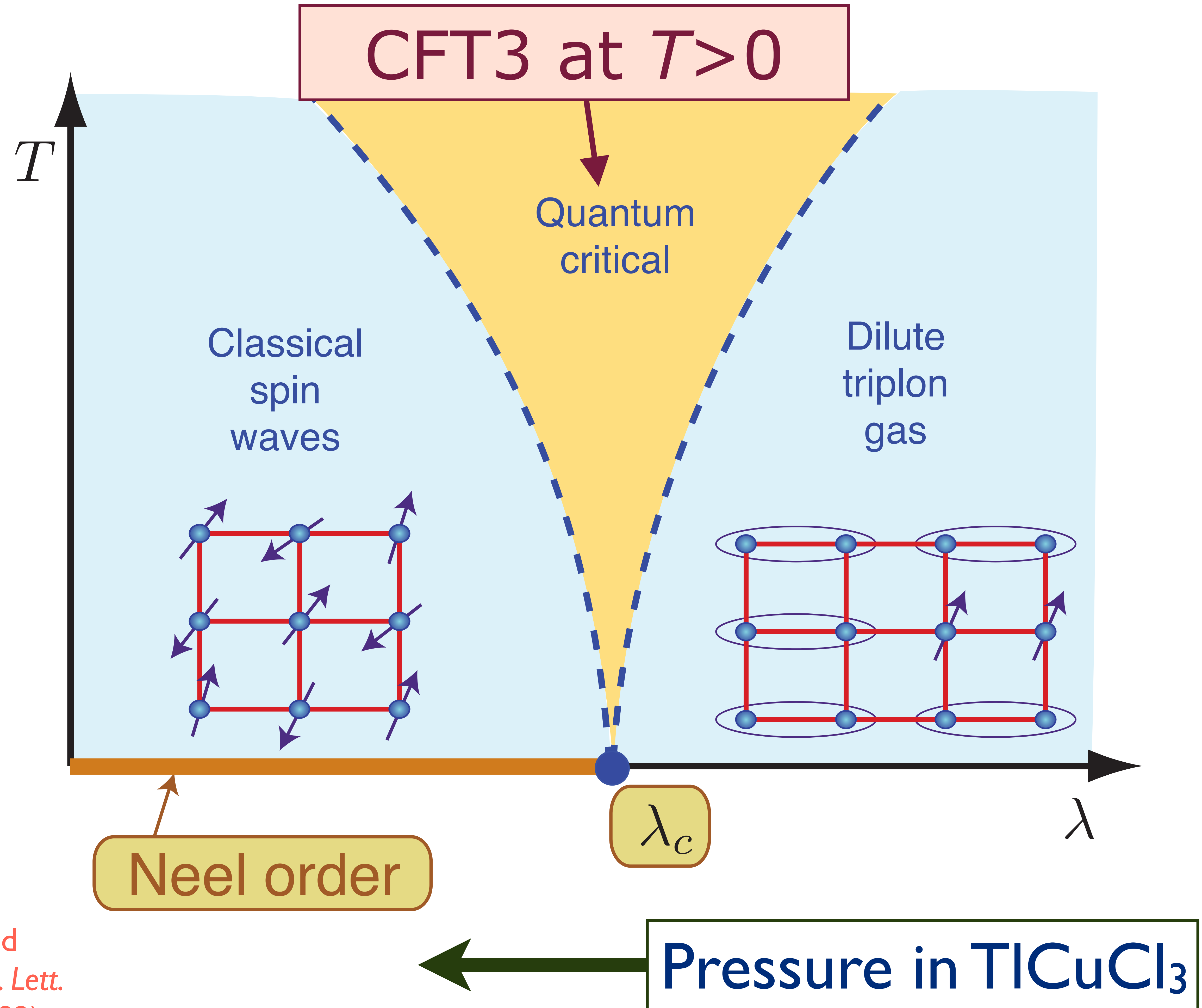

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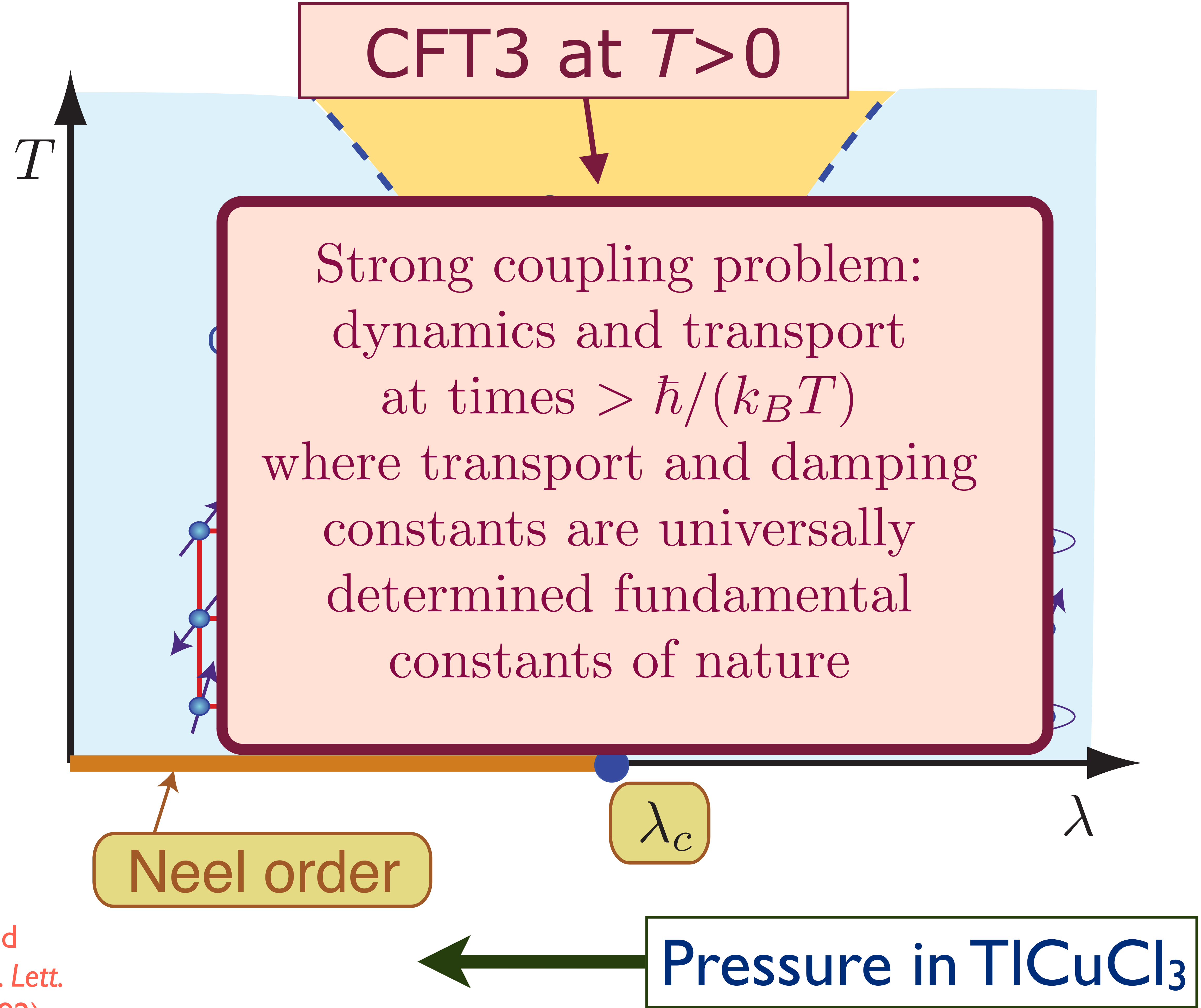

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Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

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Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $D+1$ dimensions.

At the RG fixed point, $\beta(g) = 0$, the D dimensional field theory is invariant under the scale transformation

$$x^\mu \rightarrow x^\mu / b \quad , \quad u \rightarrow b\, u$$

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This is an invariance of the *metric* of the theory in $D + 1$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS_{D+1}, and L is the AdS radius.

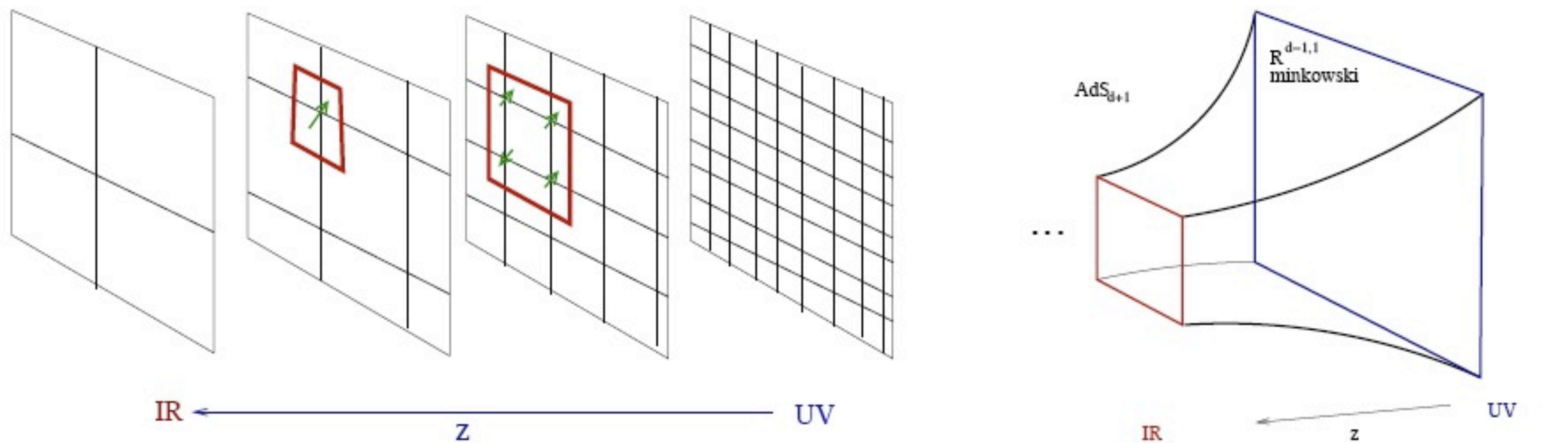
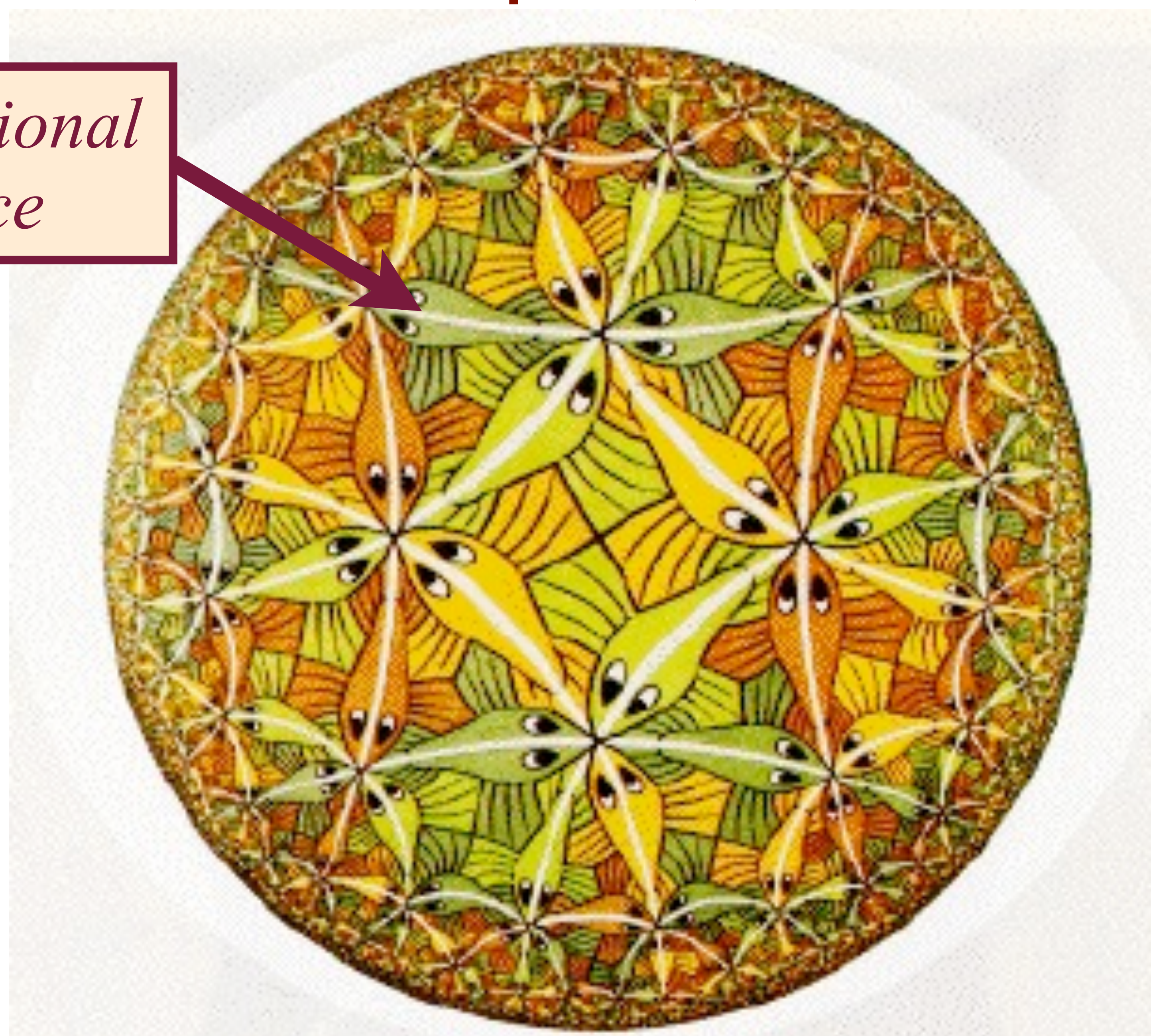


Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

J. McGreevy, arXiv0909.0518

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



Maldacena, Gubser, Klebanov, Polyakov, Witten

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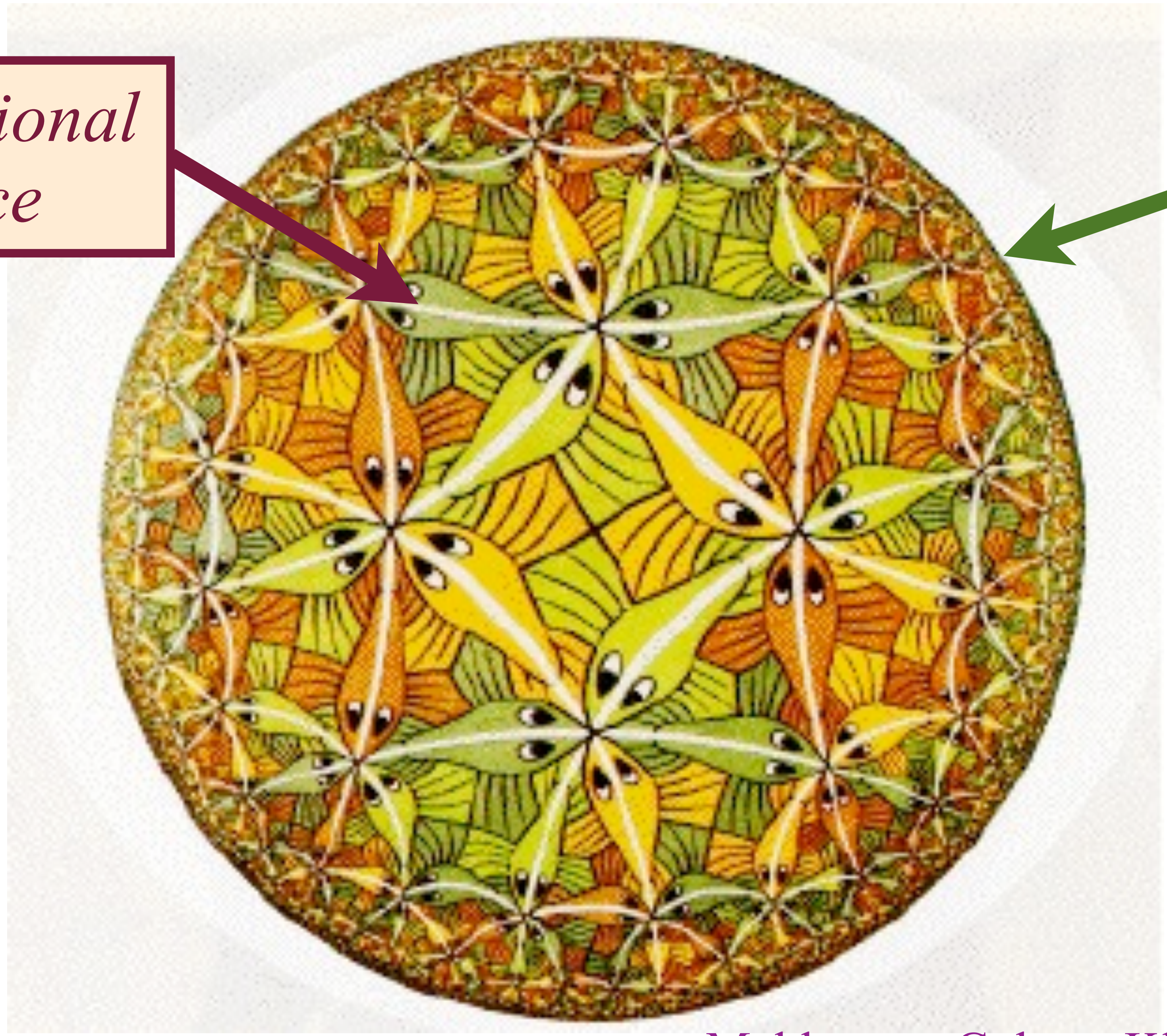
3+1 dimensional AdS space



A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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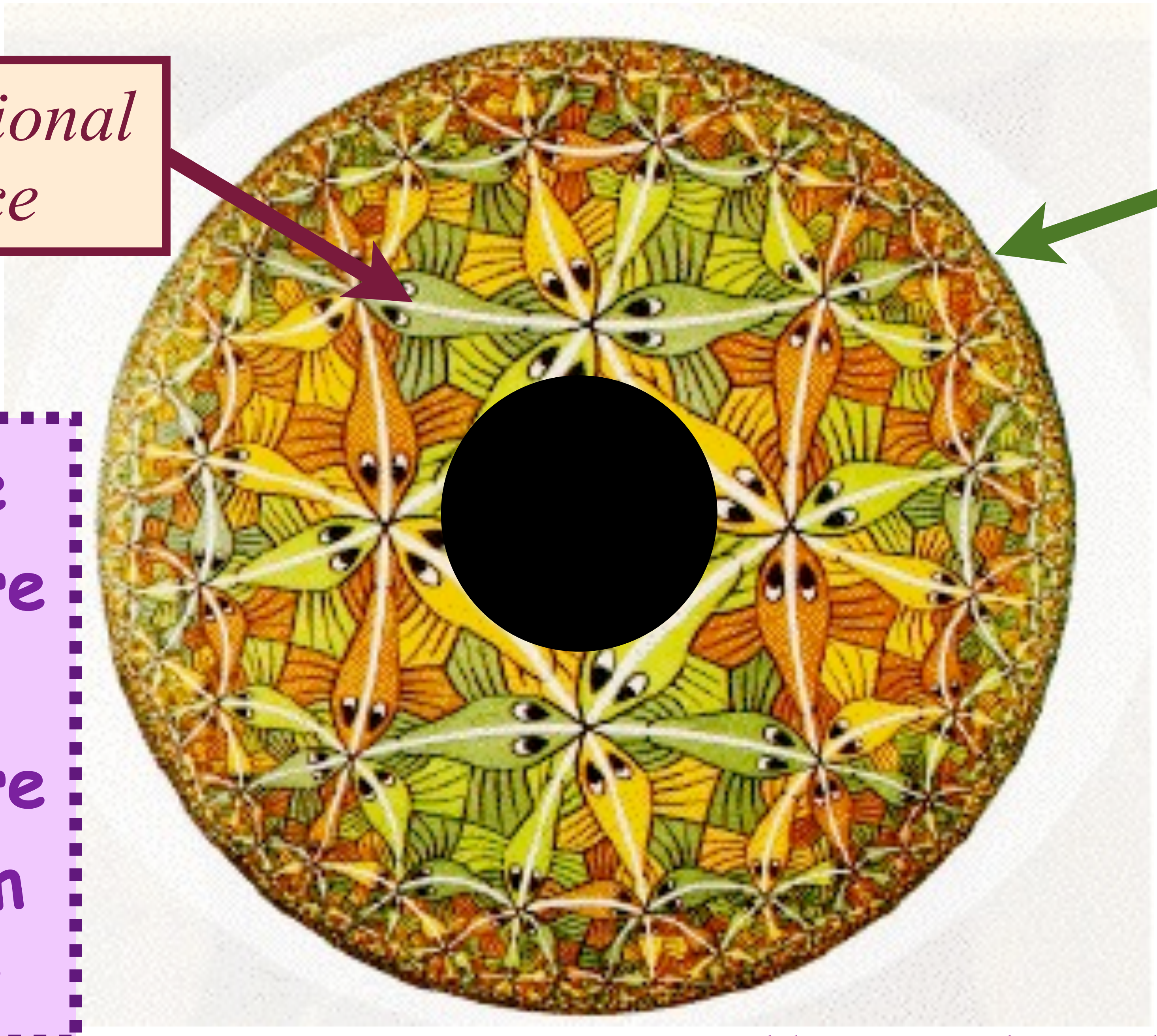

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

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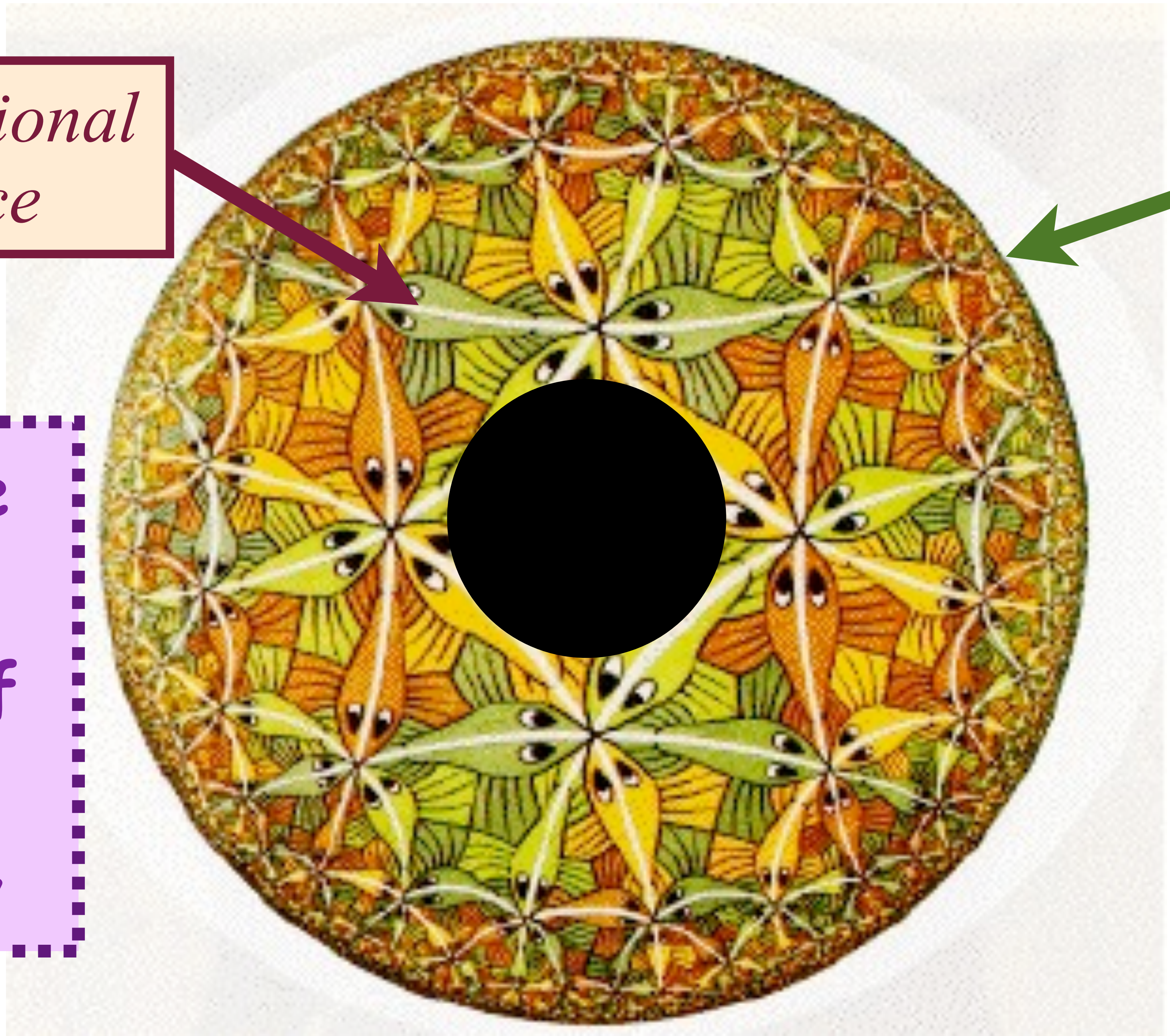

Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

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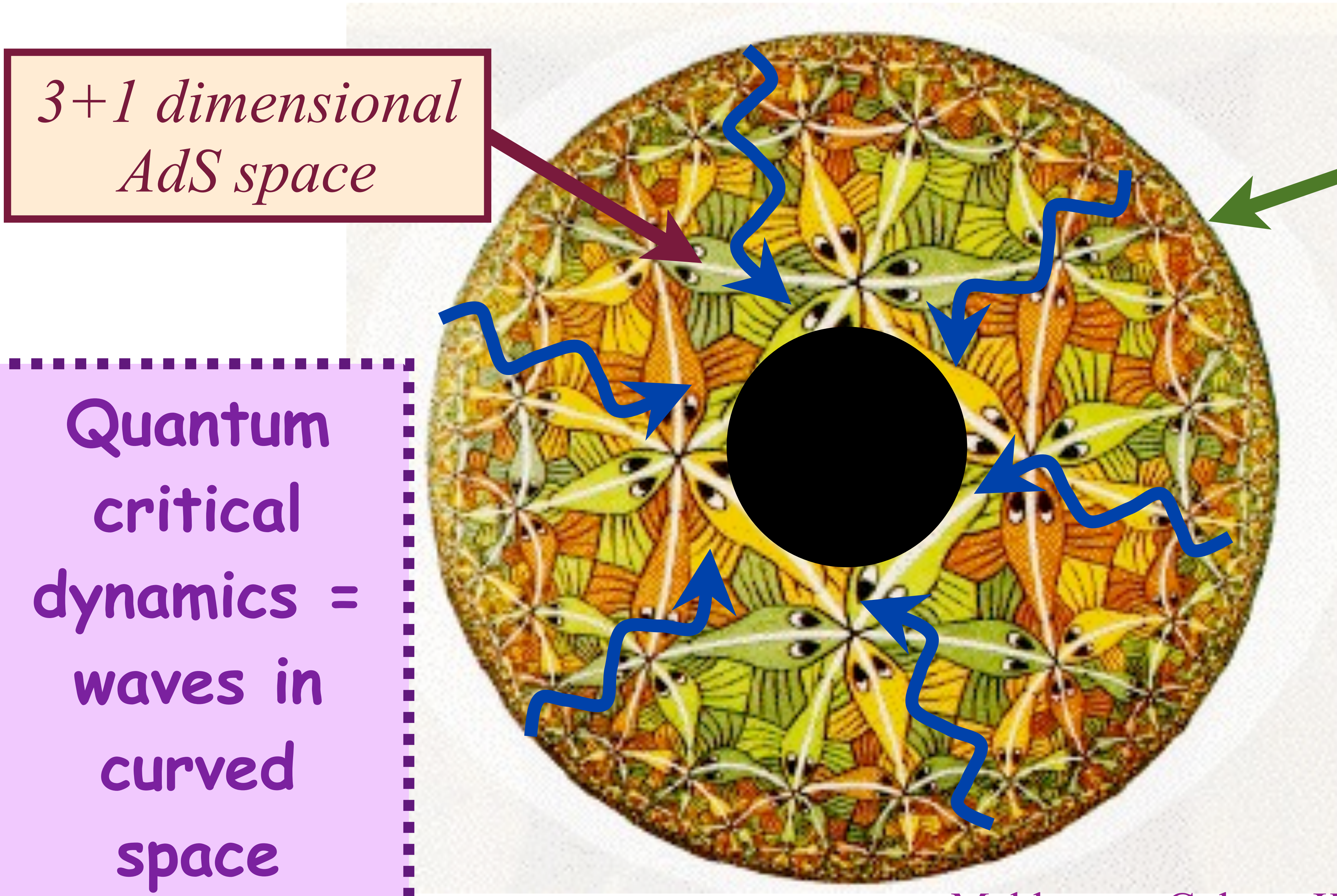

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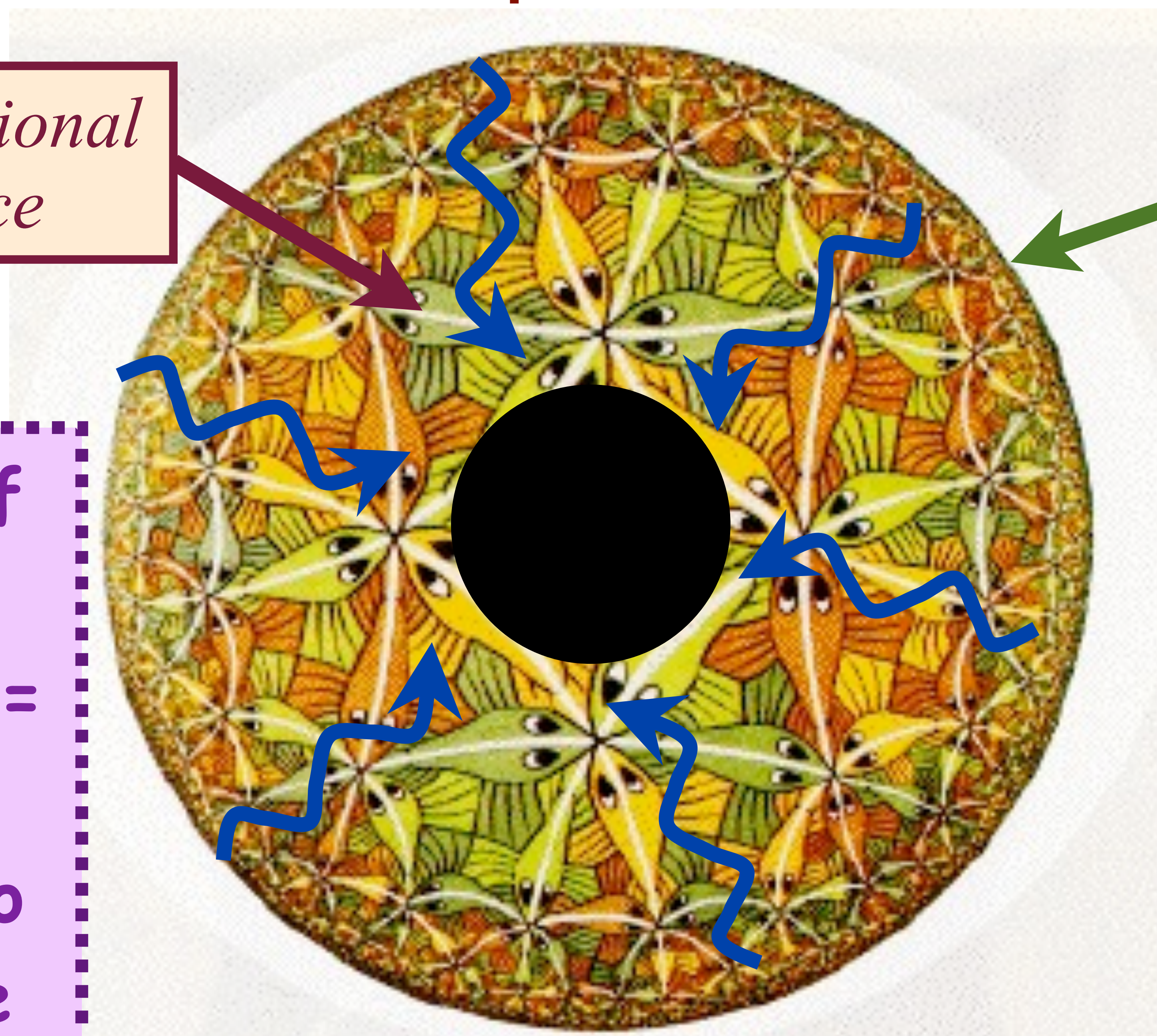

Quantum criticality in 2+1 dimensions

Friction of quantum criticality = waves falling into black hole

Kovtun, Policastro, Son

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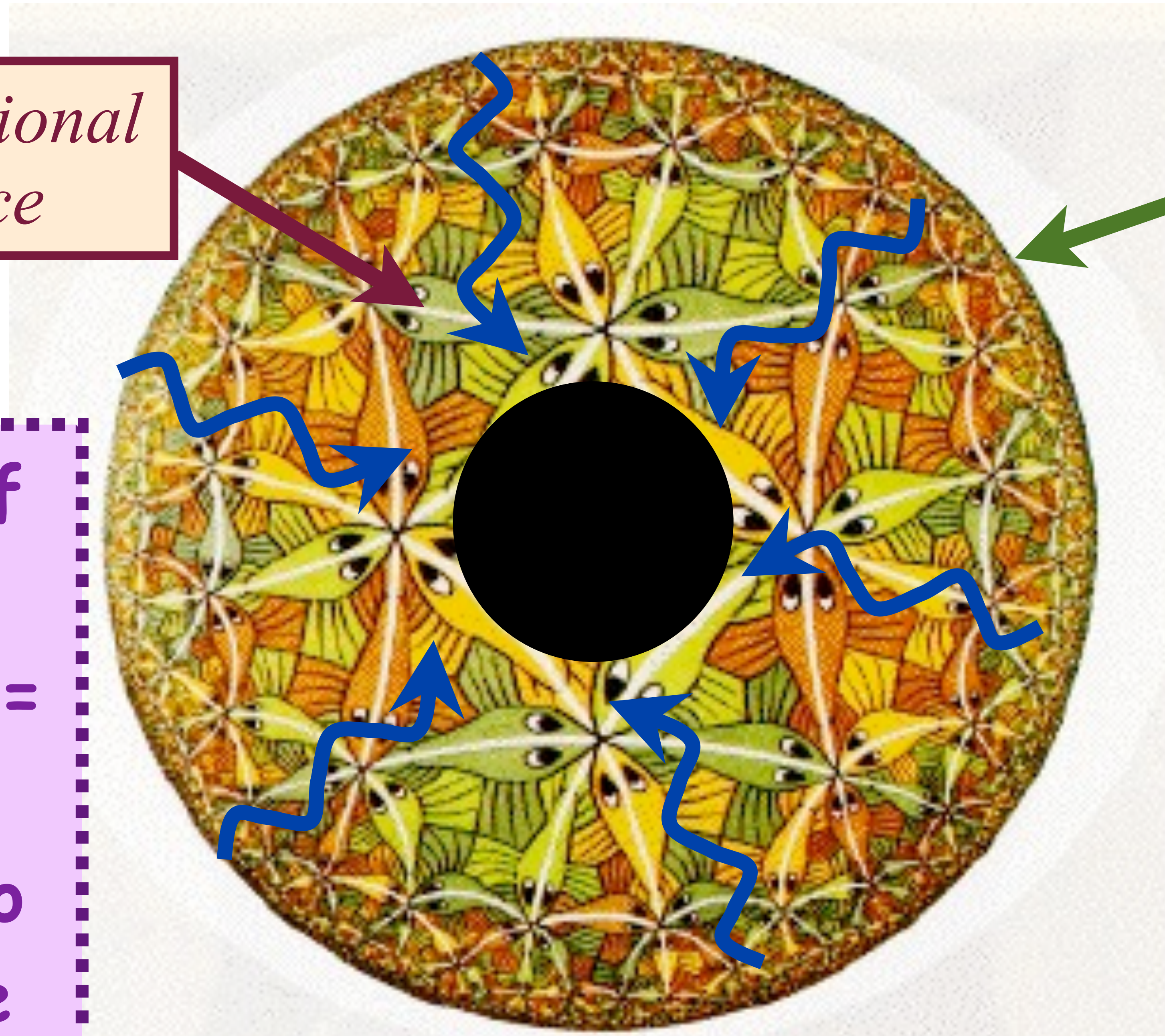

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Strong coupling problem: General solution of spin and magneto-thermo-electric transport in quantum critical region.

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).
S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76**, 144502 (2007).

Kovtun, Policastro, Son

Quantum critical transport

Quantum *"perfect fluid"* with shortest possible relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Spin/charge conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1) \ ]$$

(Q is the quantum of spin/charge)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

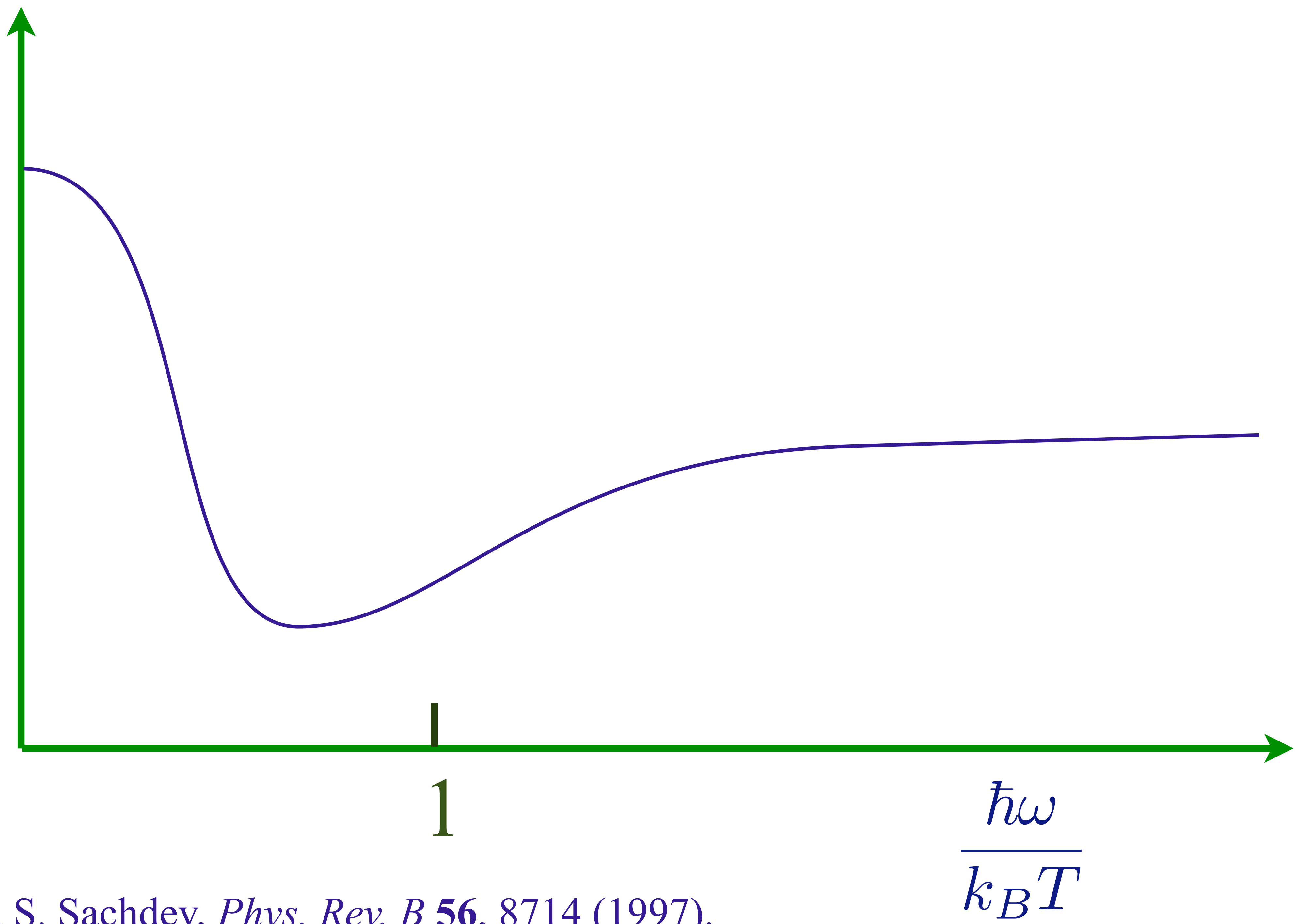
Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

Boltzmann theory of quantum critical transport

$$\sigma = \frac{4e^2}{h} \Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad ; \quad \Sigma \to \text{a universal function}$$



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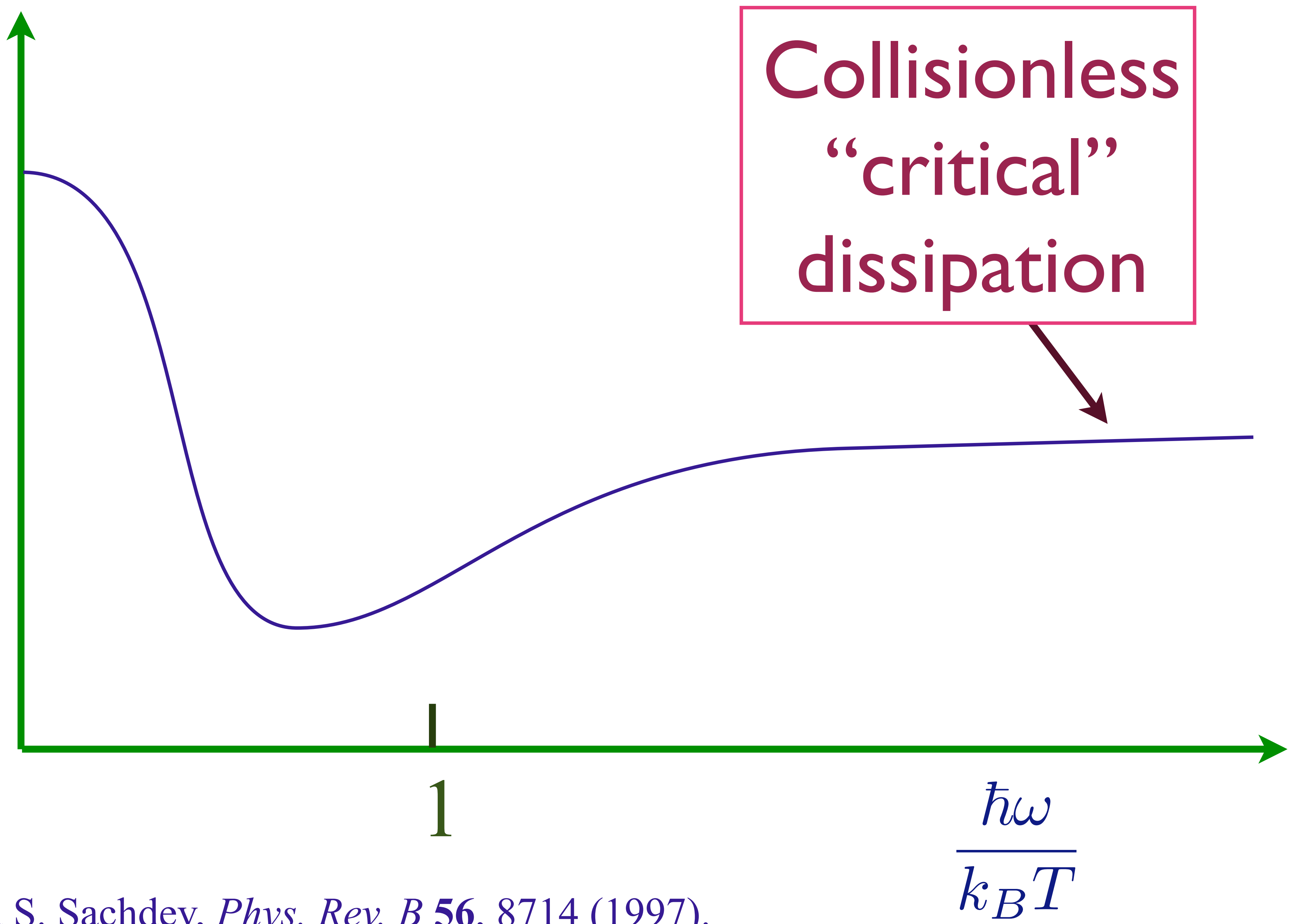

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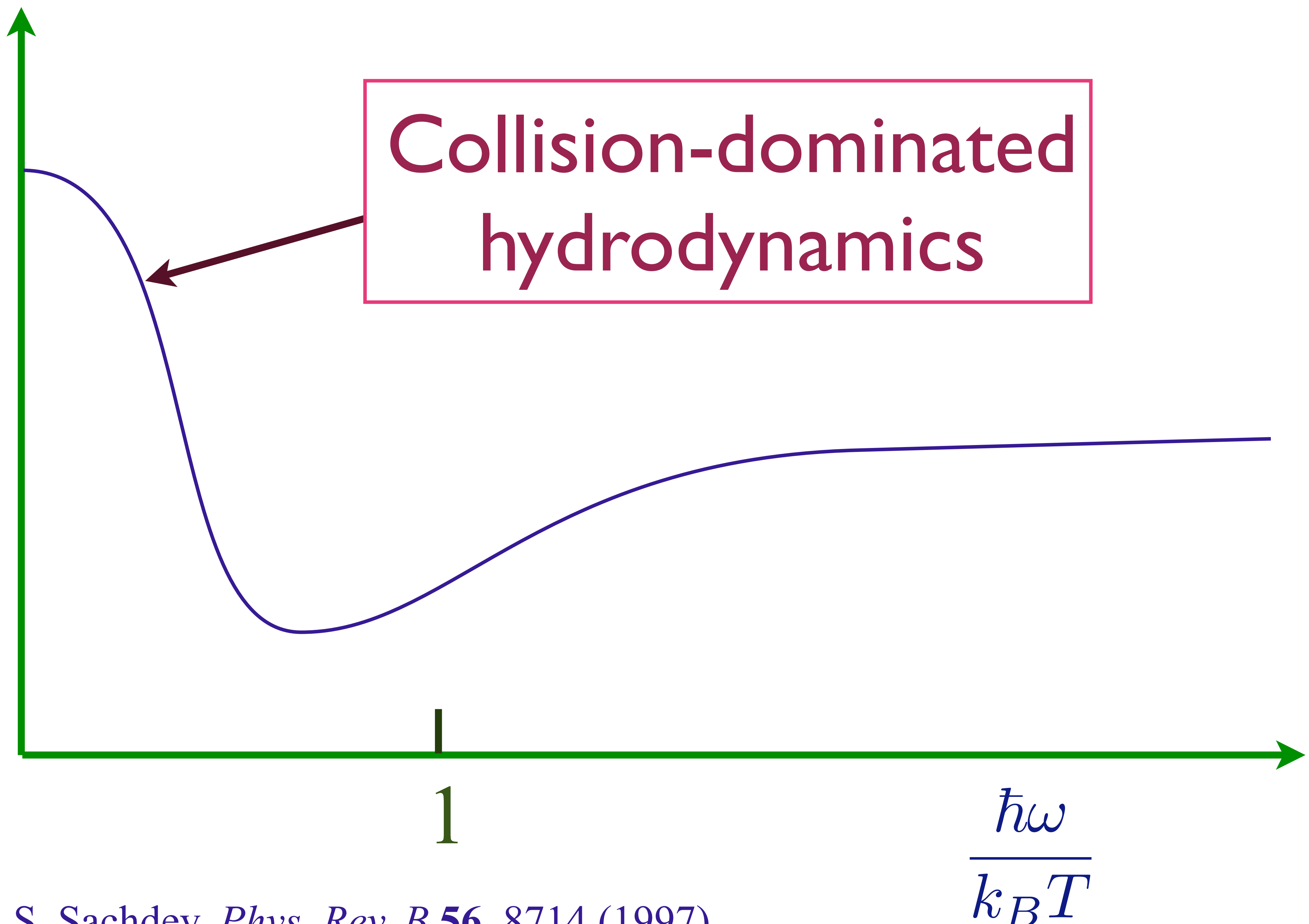

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AdS theory of strongly interacting “perfect fluids”

An infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4 with the following common properties:

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An infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4 with the following common properties:

- Exact solutions for correlators of conserved densities as a function of $\mathbf{k}$ and ω exhibit the collisionless-hydrodynamic crossover.

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS theory of strongly interacting "perfect fluids"

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- The viscosity/entropy-density ratio is

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

AdS theory of strongly interacting "perfect fluids"

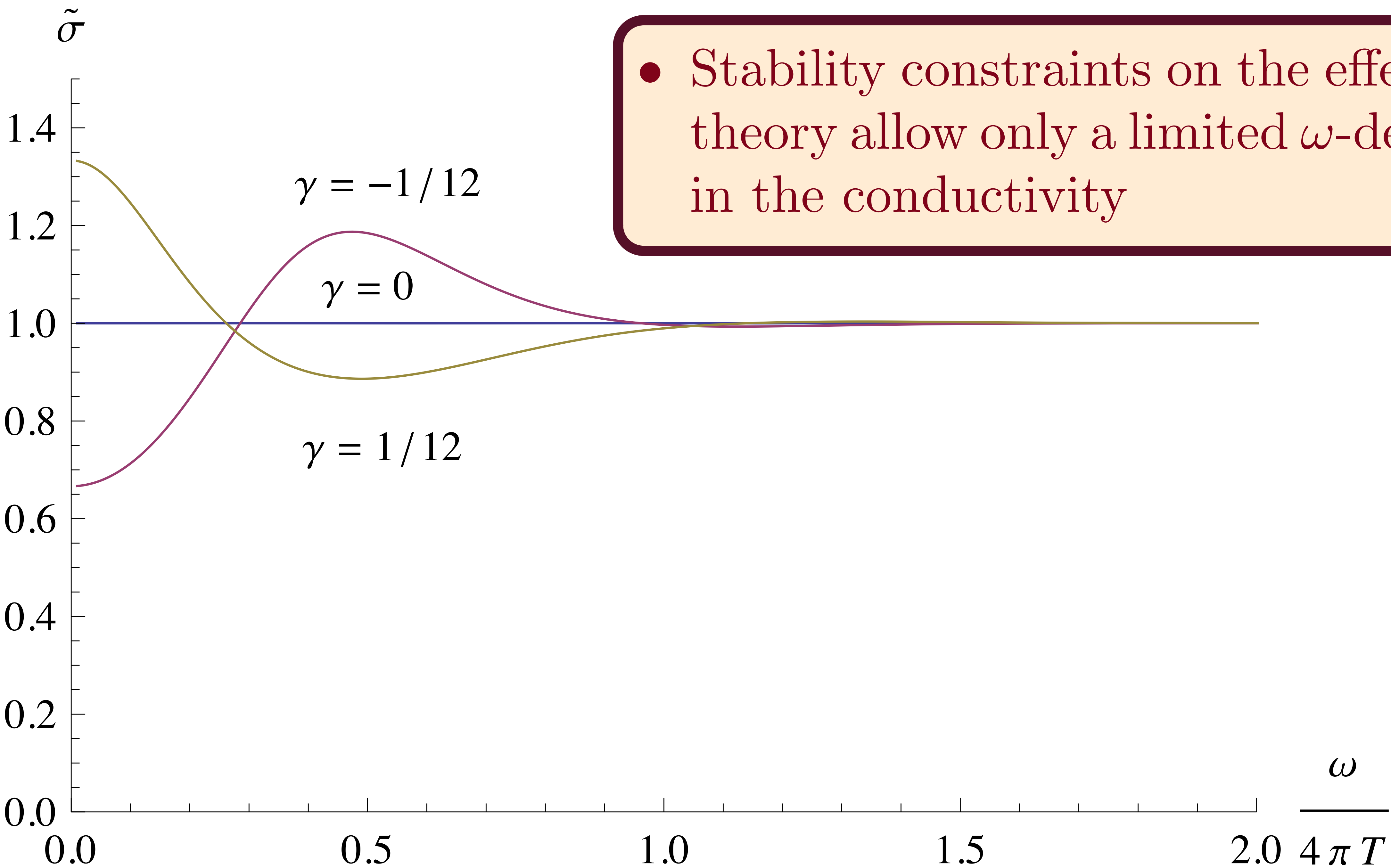
An infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4 with the following common properties:

- All continuous global symmetries are self dual.
- The conductivity σ is ω-independent and equal to the self-dual value. Frequency-dependent corrections are obtained from higher-derivative gravity theories.

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

AdS theory of strongly interacting "perfect fluids"

Examine quantum gravity effects by computing the consequences of higher-derivative corrections to the Einstein-Maxwell action.



- Stability constraints on the effective theory allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, arXiv:1009.xxxx

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$
$$\sigma_{\text{Insulator}}(T \to 0) = 0$$
$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

- Self-dual value $= 4e^2/h$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

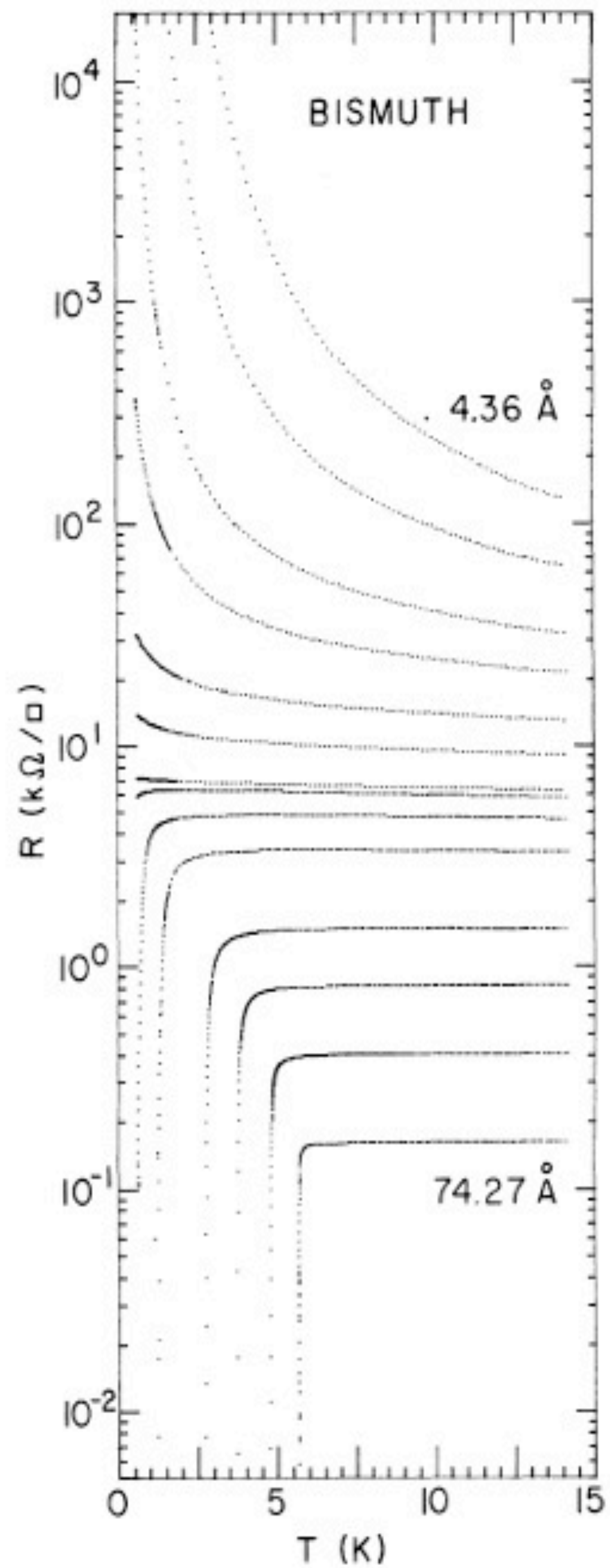


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value

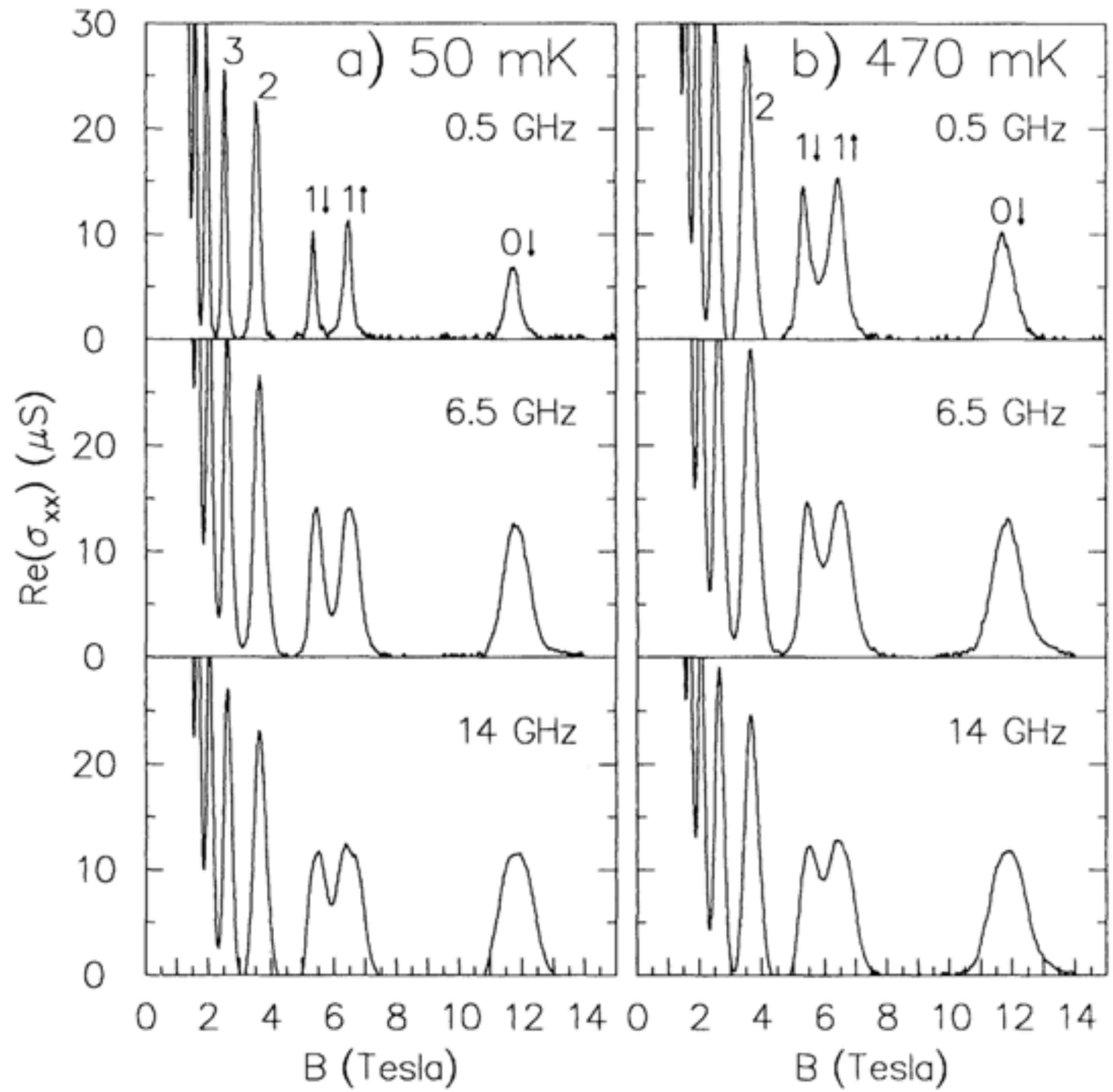


FIG. 3. Re(σ_{xx}) vs B at three frequencies and two temperatures. Peaks are marked with Landau level index N and spin.

L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui,
Physical Review Letters **71**, 2638 (1993).

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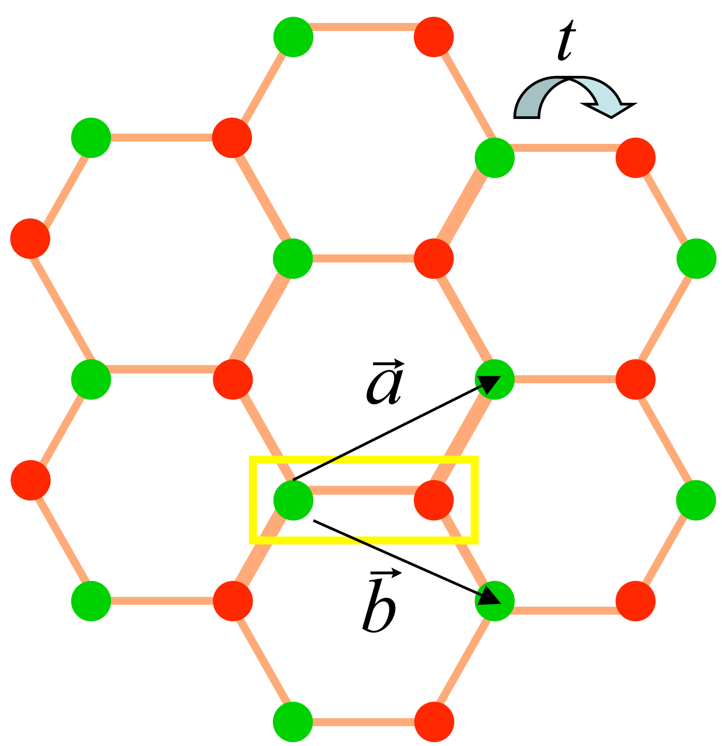
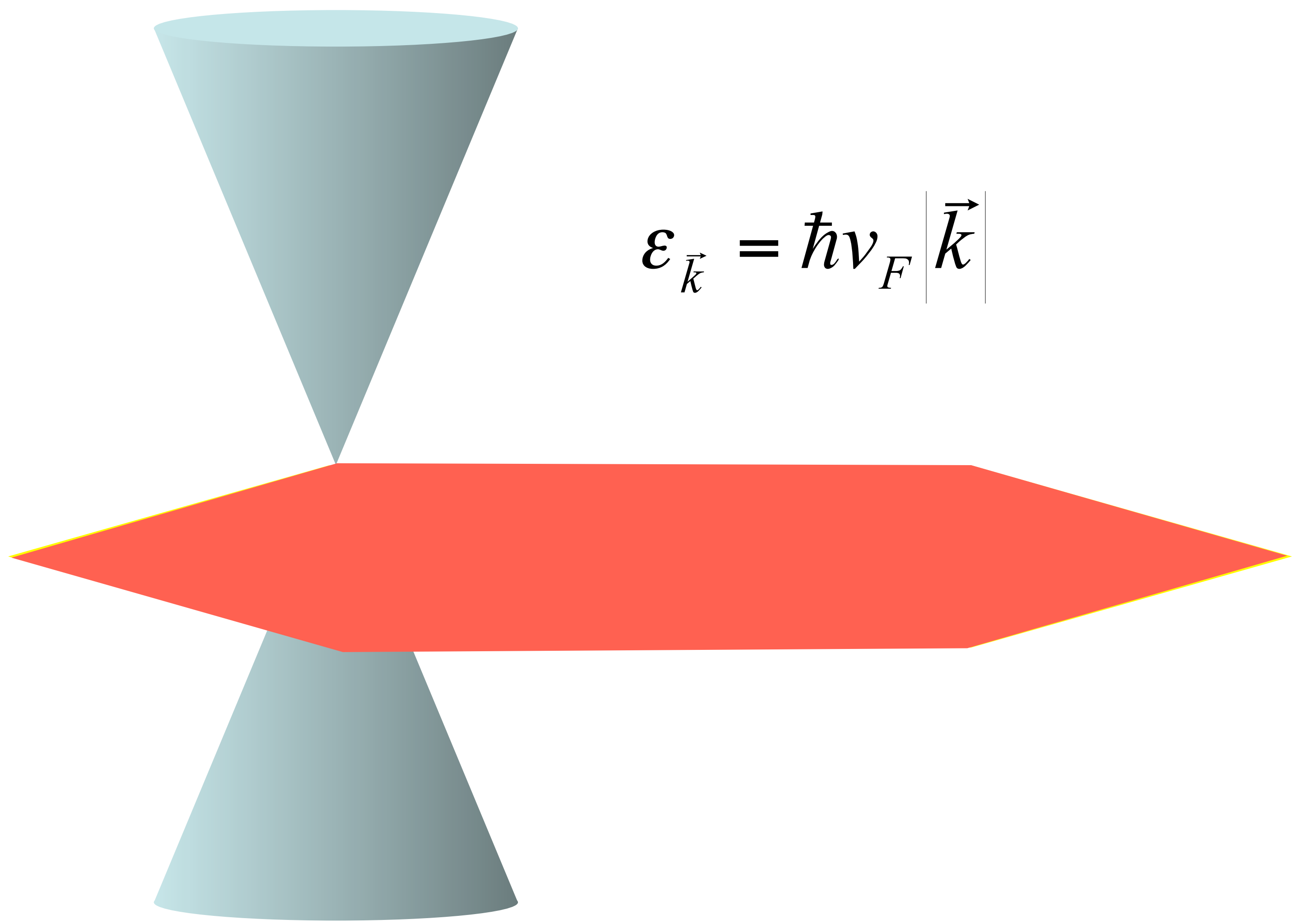
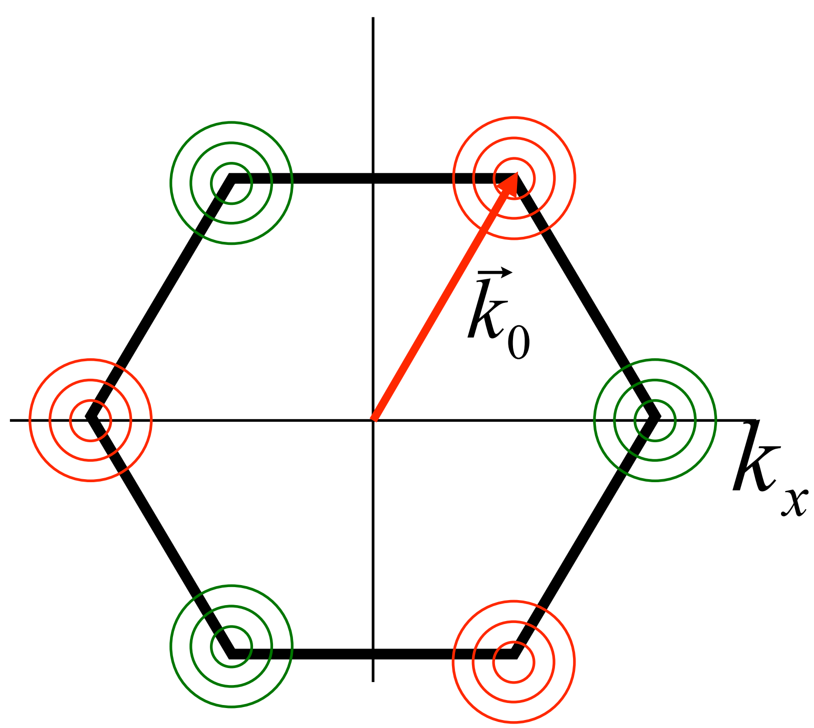
Graphene



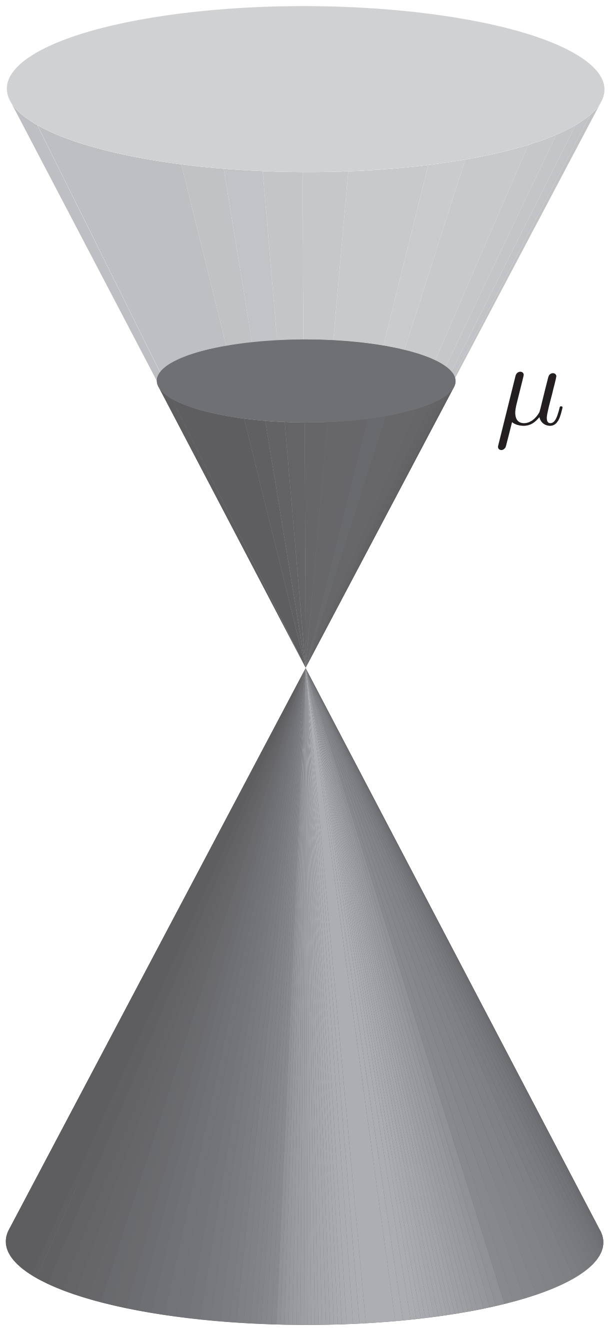


$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$



Conical Dirac dispersion

Quantum phase transition in graphene tuned by a gate voltage



Electron
Fermi surface

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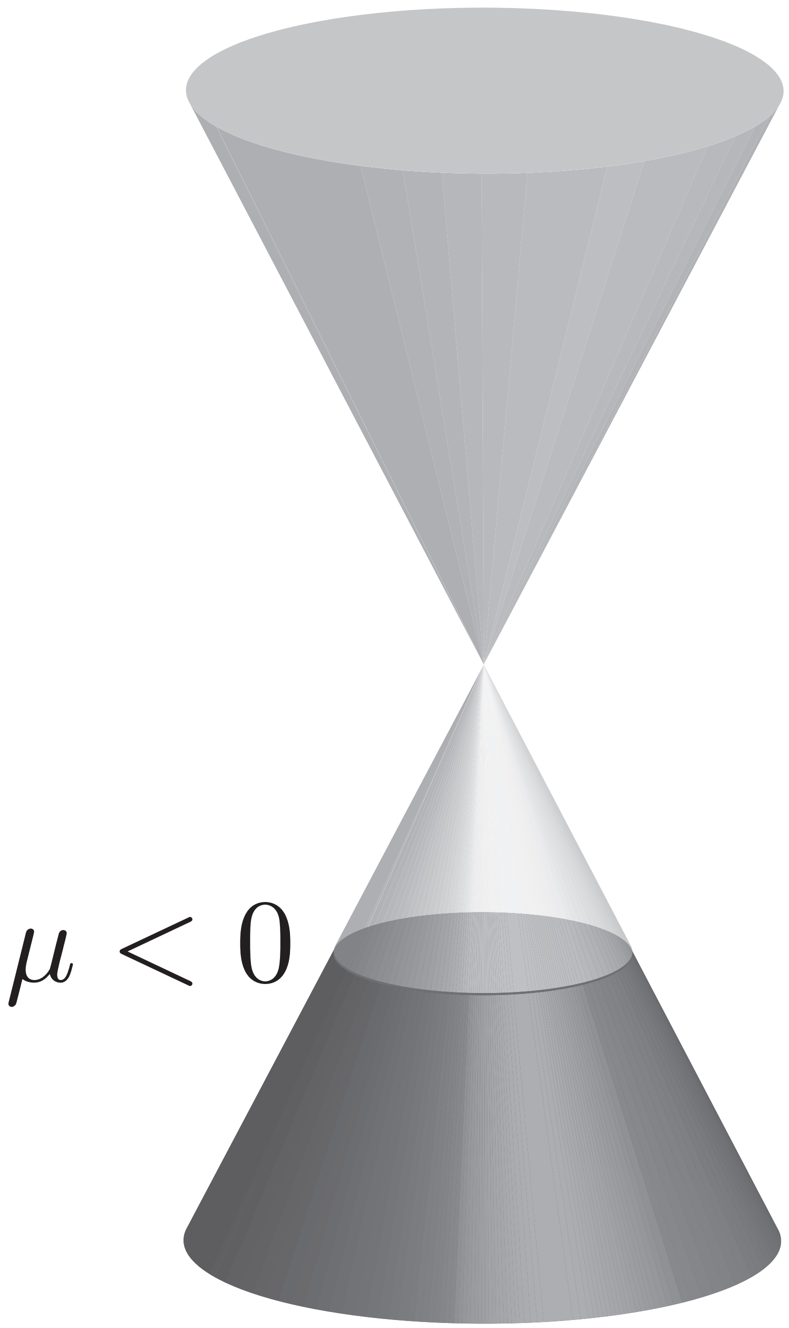

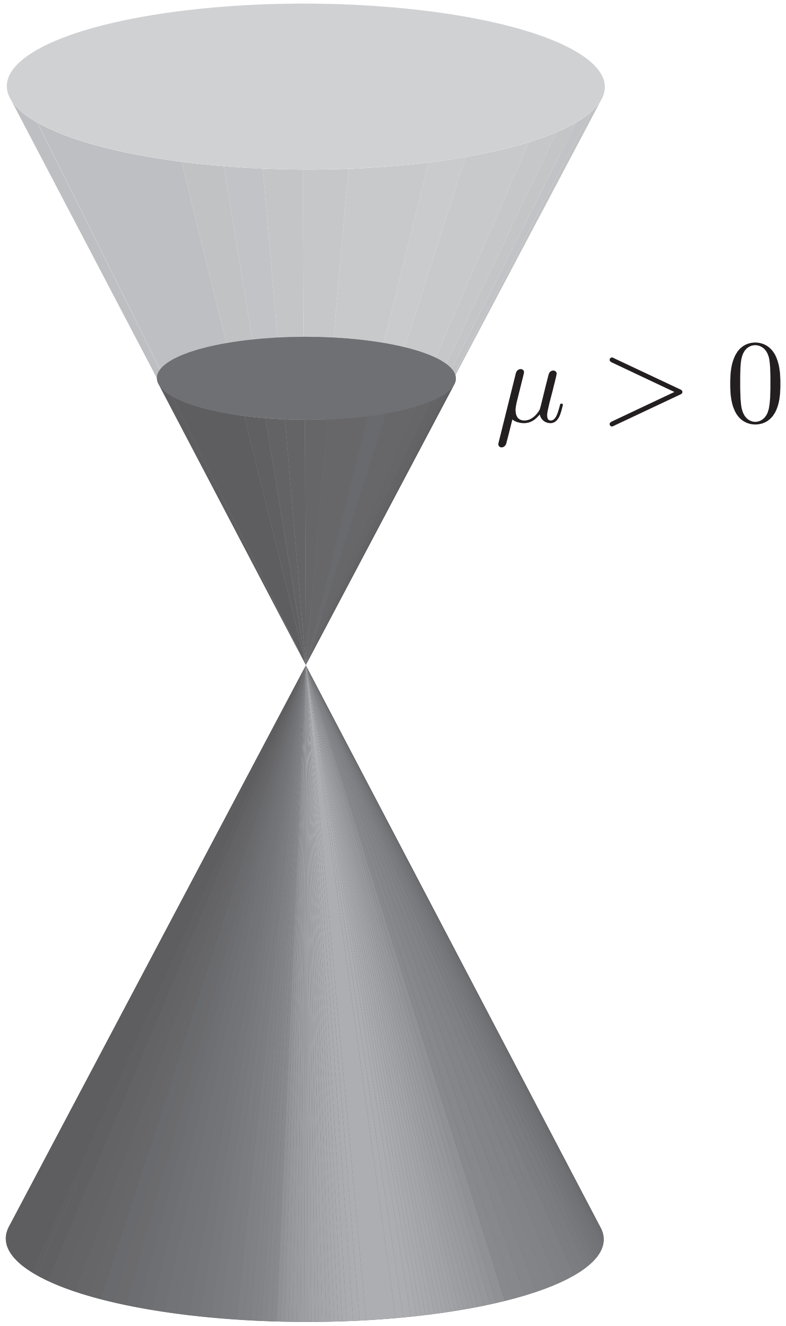

Hole
Fermi surface

Electron
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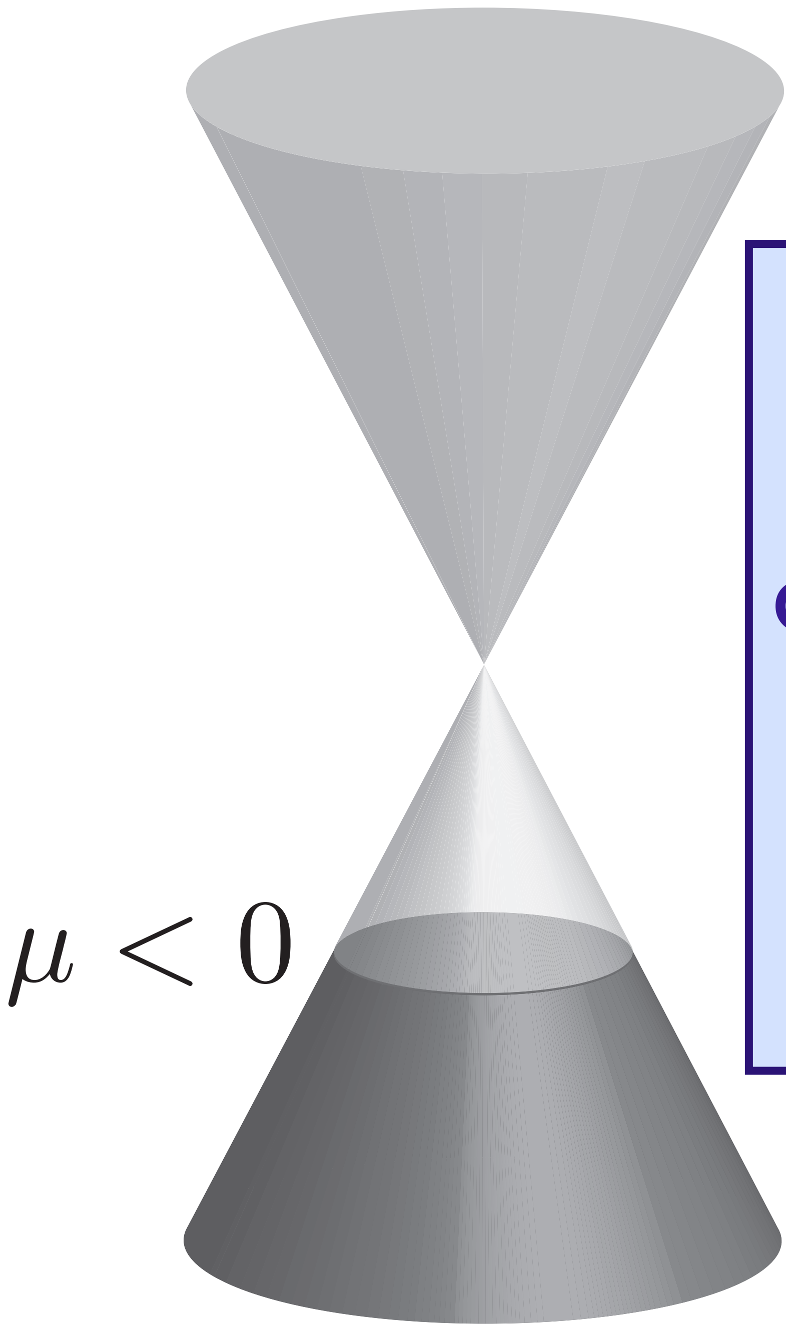
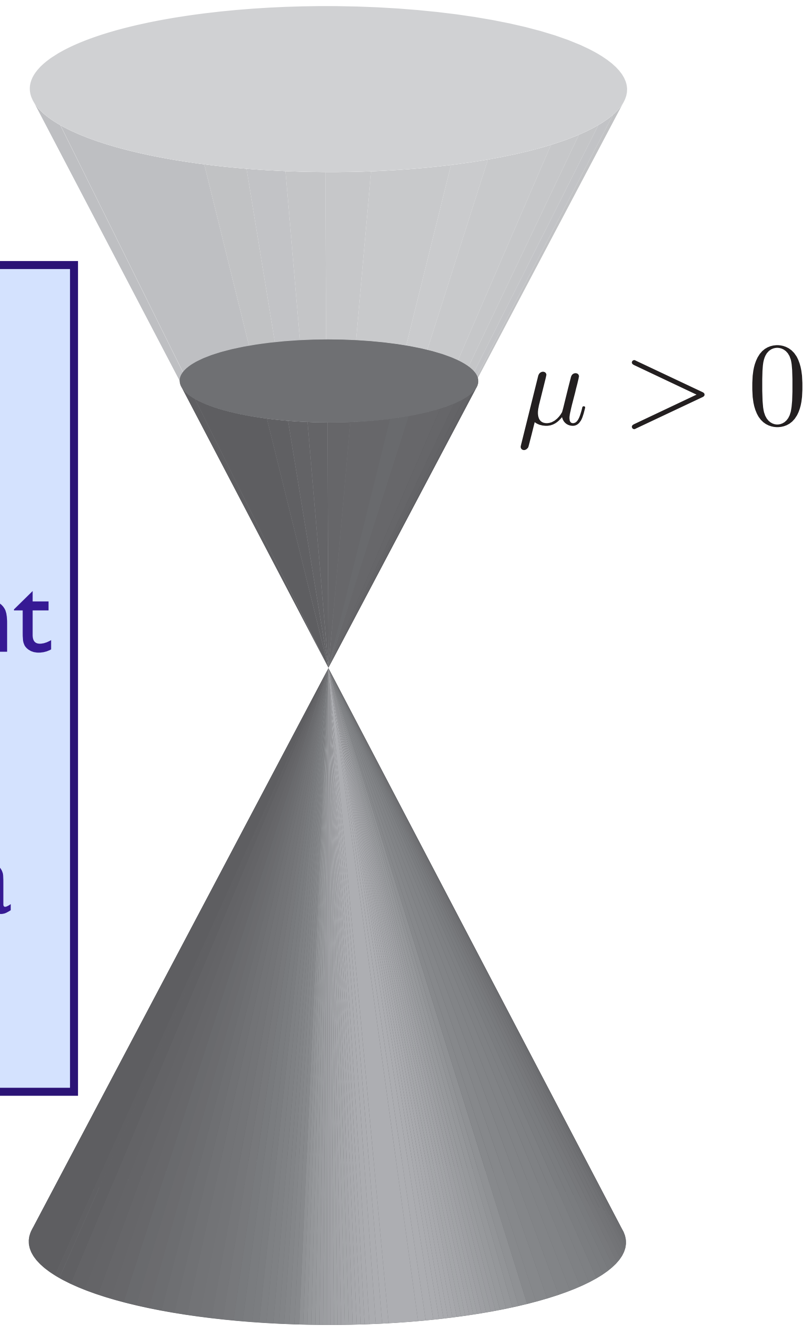


Hole
Fermi surface

There must be an intermediate quantum critical point where the Fermi surfaces reduce to a Dirac point



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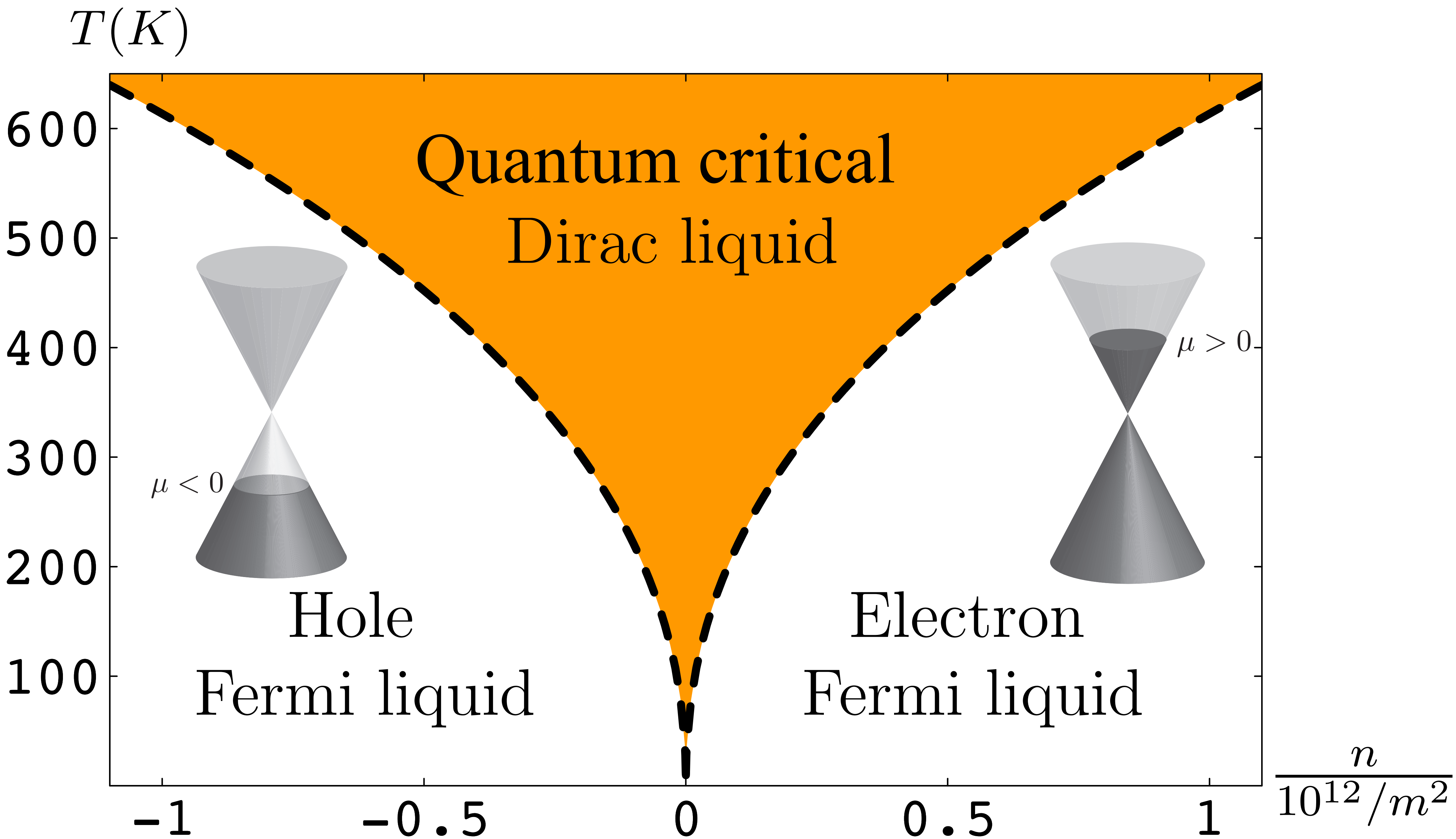

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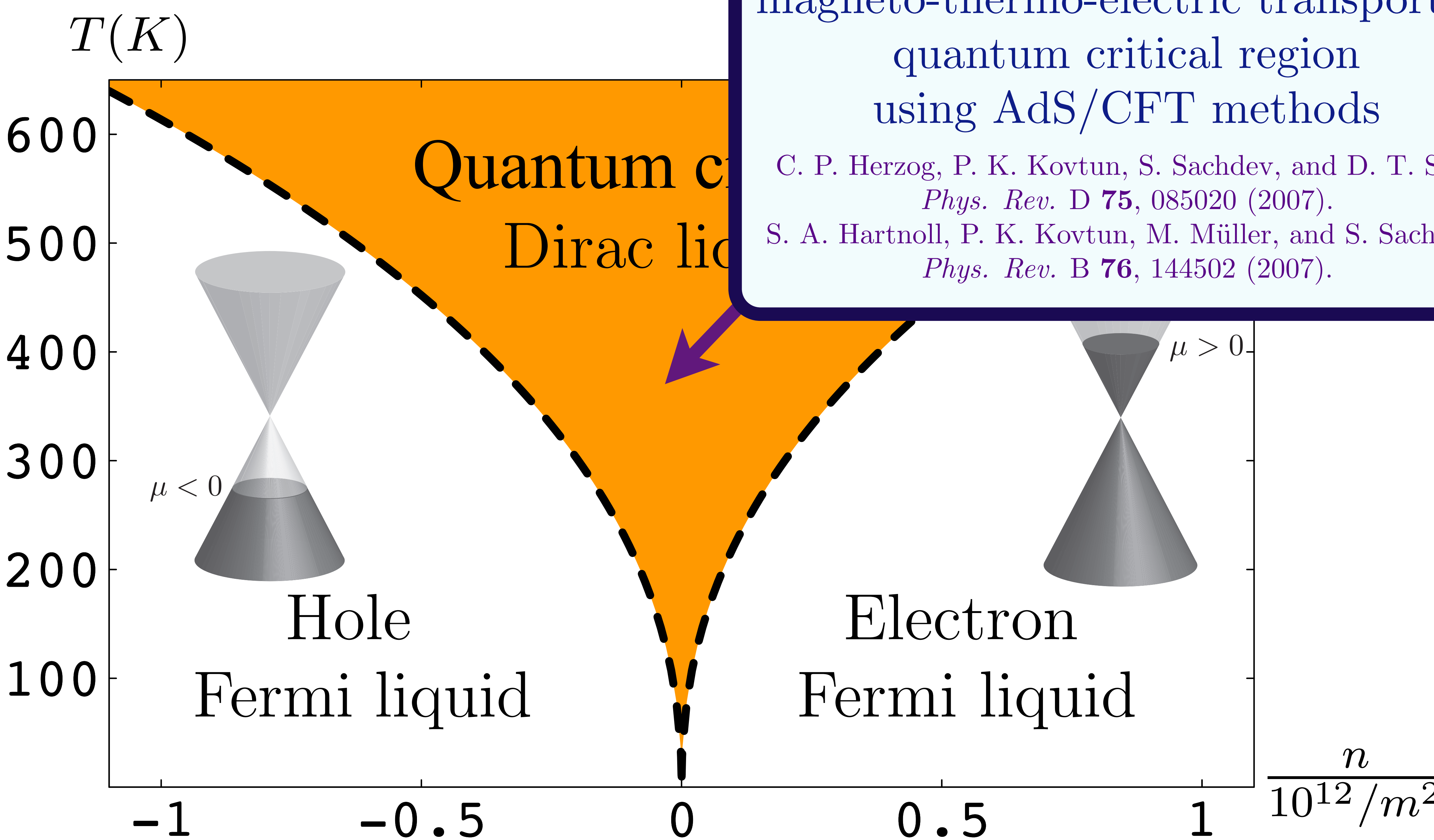

General solution of spin and magneto-thermo-electric transport in quantum critical region using AdS/CFT methods

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).
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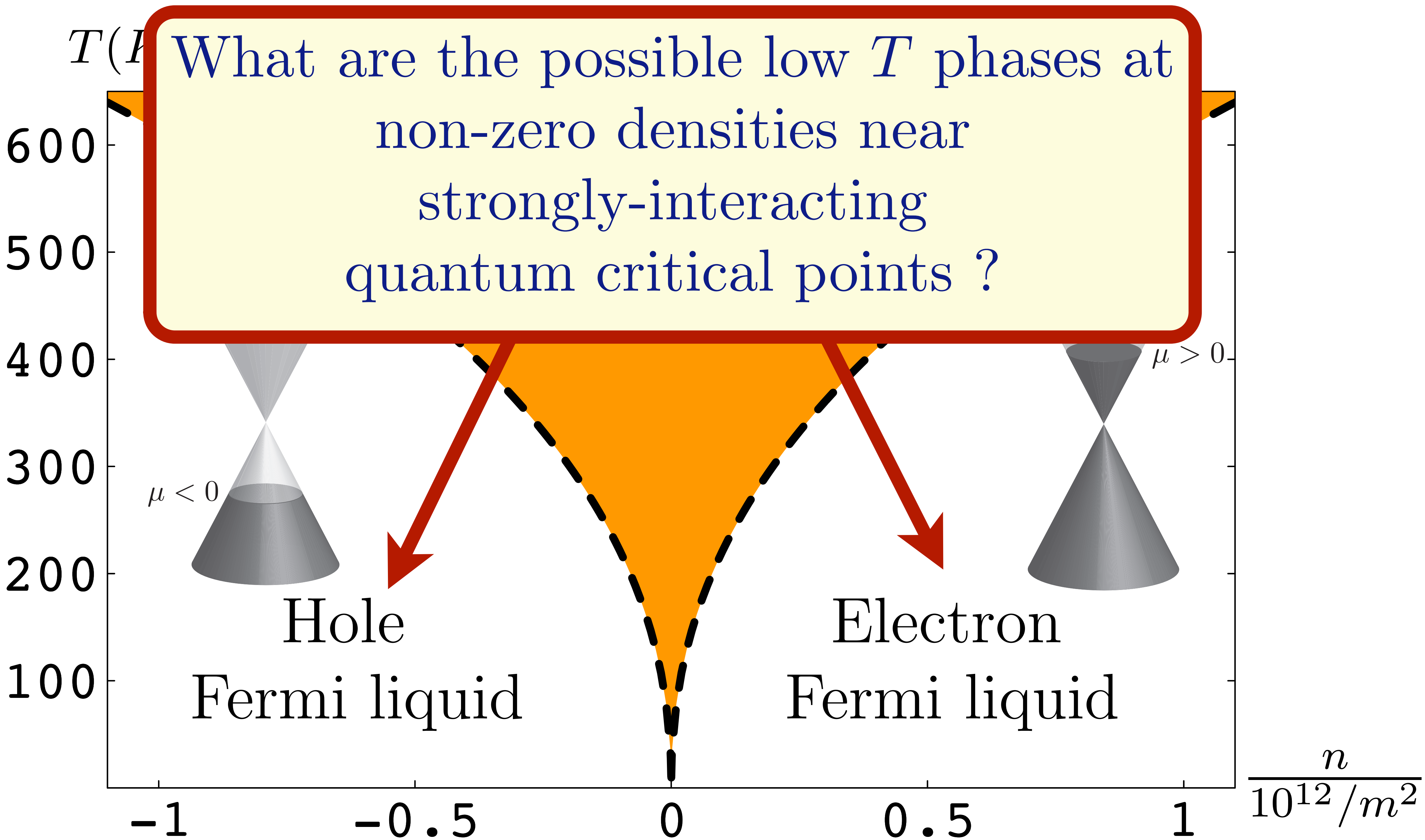
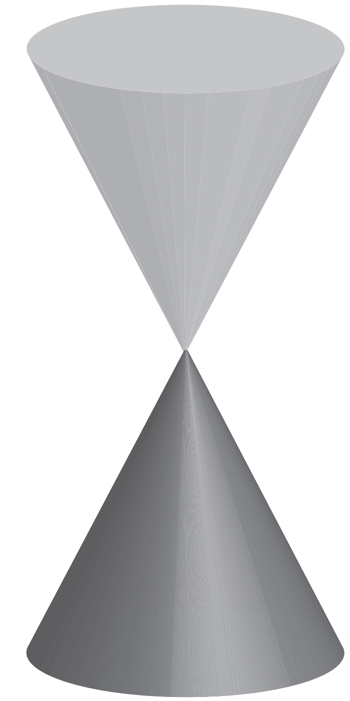
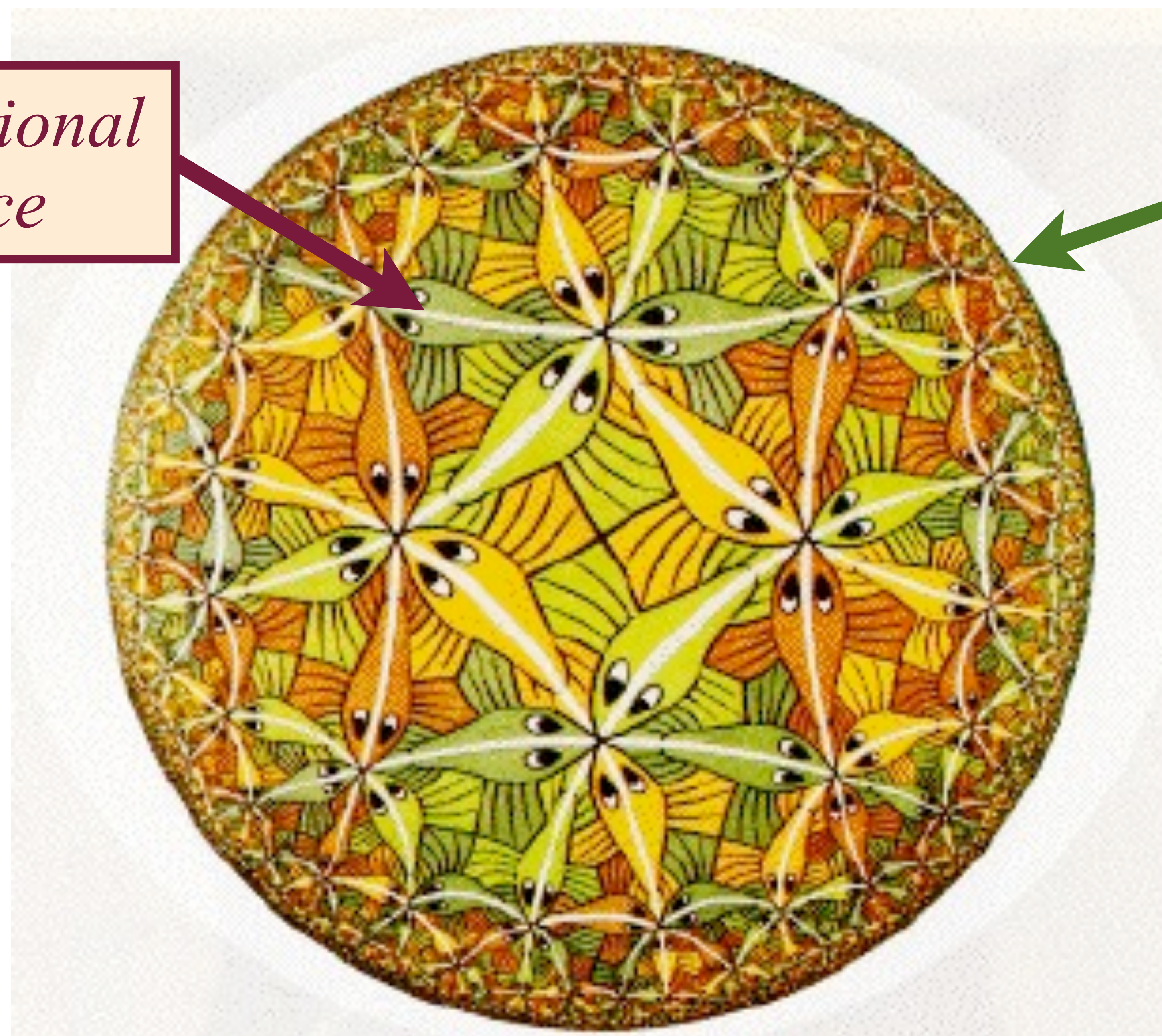


AdS/CFT correspondence

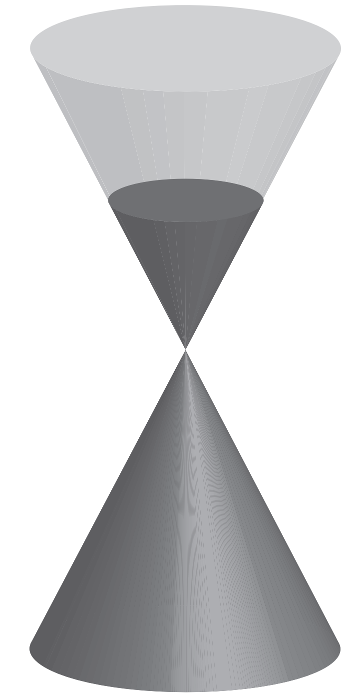


3+1 dimensional AdS space

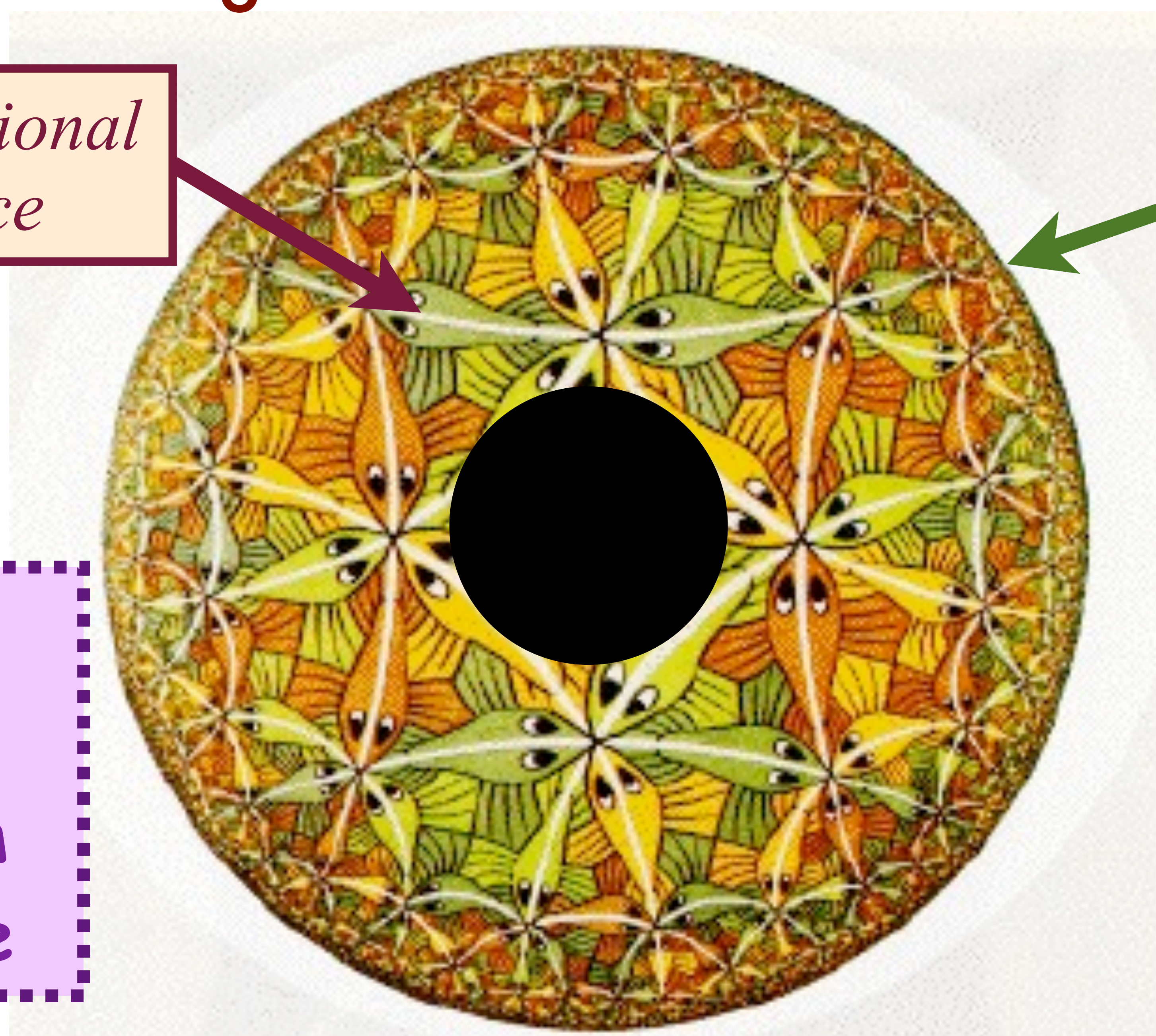


Quantum criticality in 2+1 dimensions

AdS/CFT correspondence



Move away from the quantum critical point to a system of matter at non-zero density: equivalent to adding an electrical charge to the black hole.



One of the most common phases of finite density quantum matter at zero temperature is a *superfluid* or a *superconductor*

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- This is obtained in the dual gravity theory by condensing a scalar field in the background of a charged black hole.

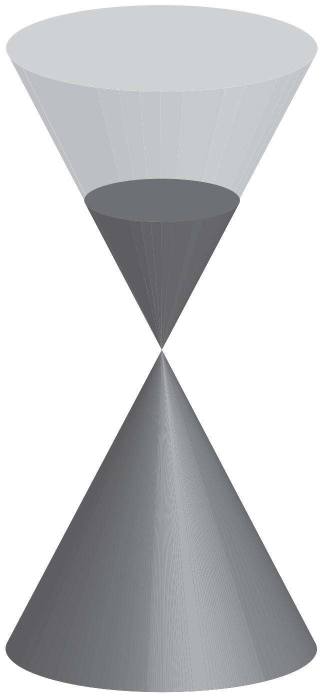
- The resulting theory has response functions indicating that the superfluid condensate is made up of pairs of fermions, just as in BCS theory.

- This yields the first mean-field-theory of a superconductor which has a value of Δ/T_c, (where Δ is the fermion energy gap) different from the BCS value.

S. S. Gubser, Phys. Rev. D **78**, 065034 (2008).
S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, Phys. Rev. Lett. **101**, 031601 (2008)

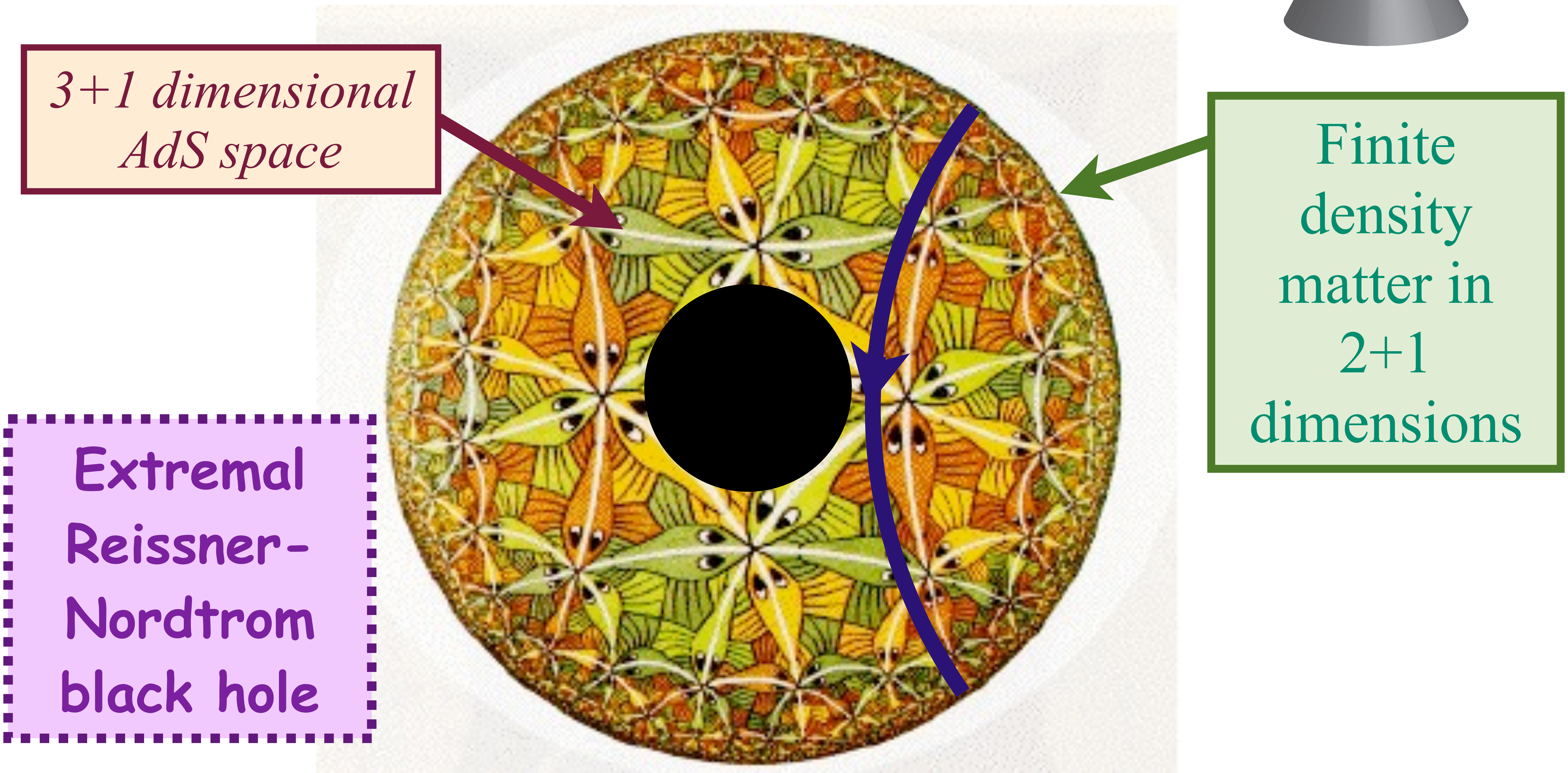
The other common phase of finite density quantum matter at zero temperature is a *metal*

AdS/CFT correspondence

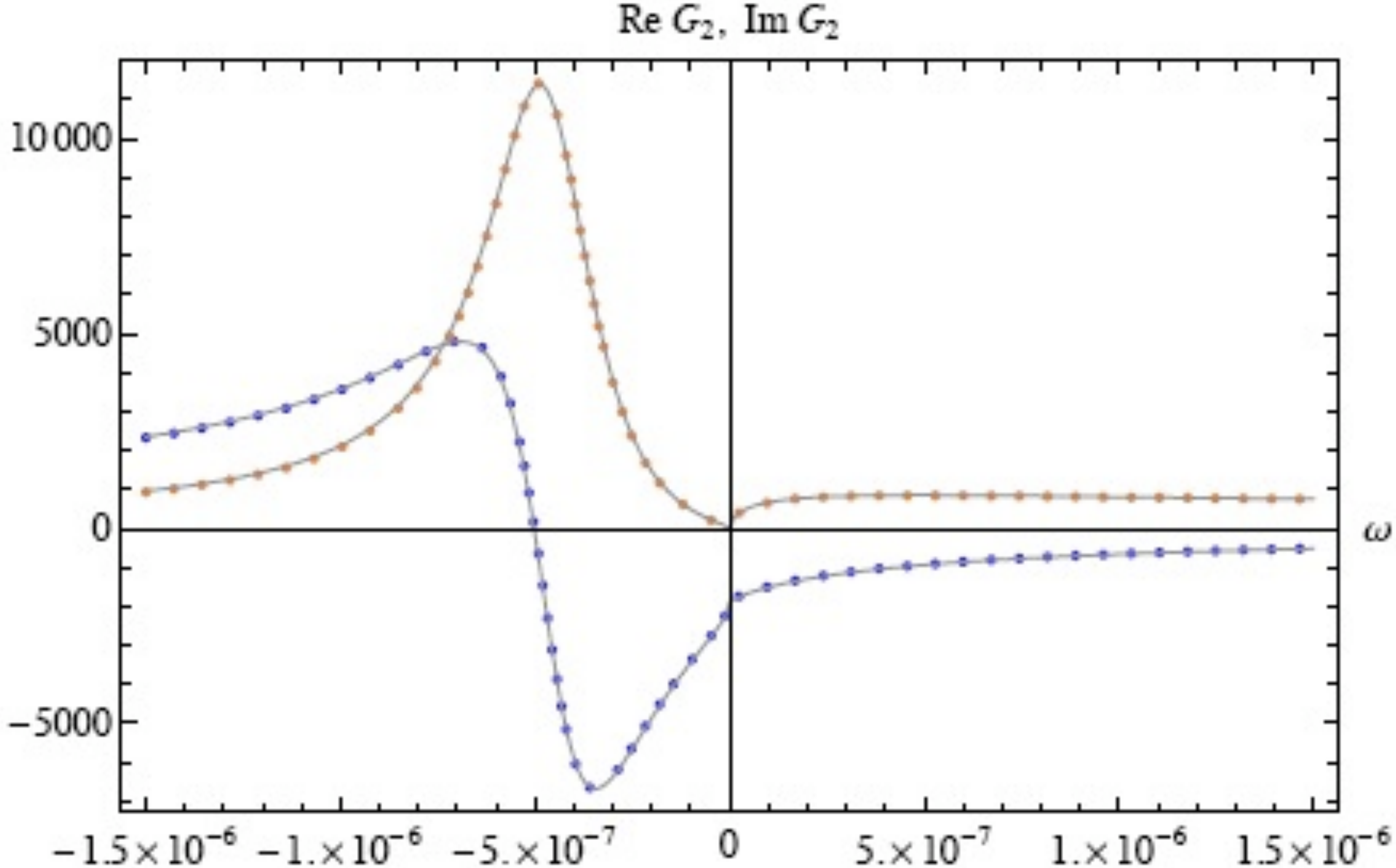


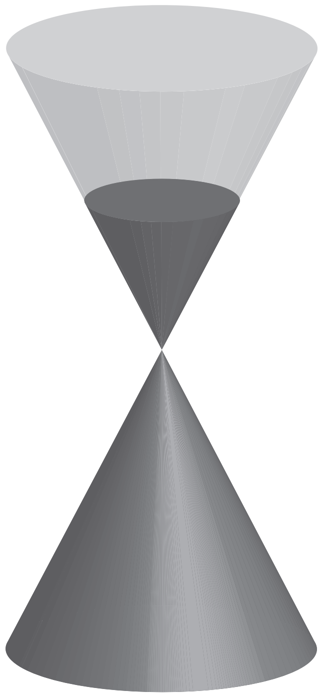
Examine the free energy and Green's function of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



Green's function of a fermion





T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
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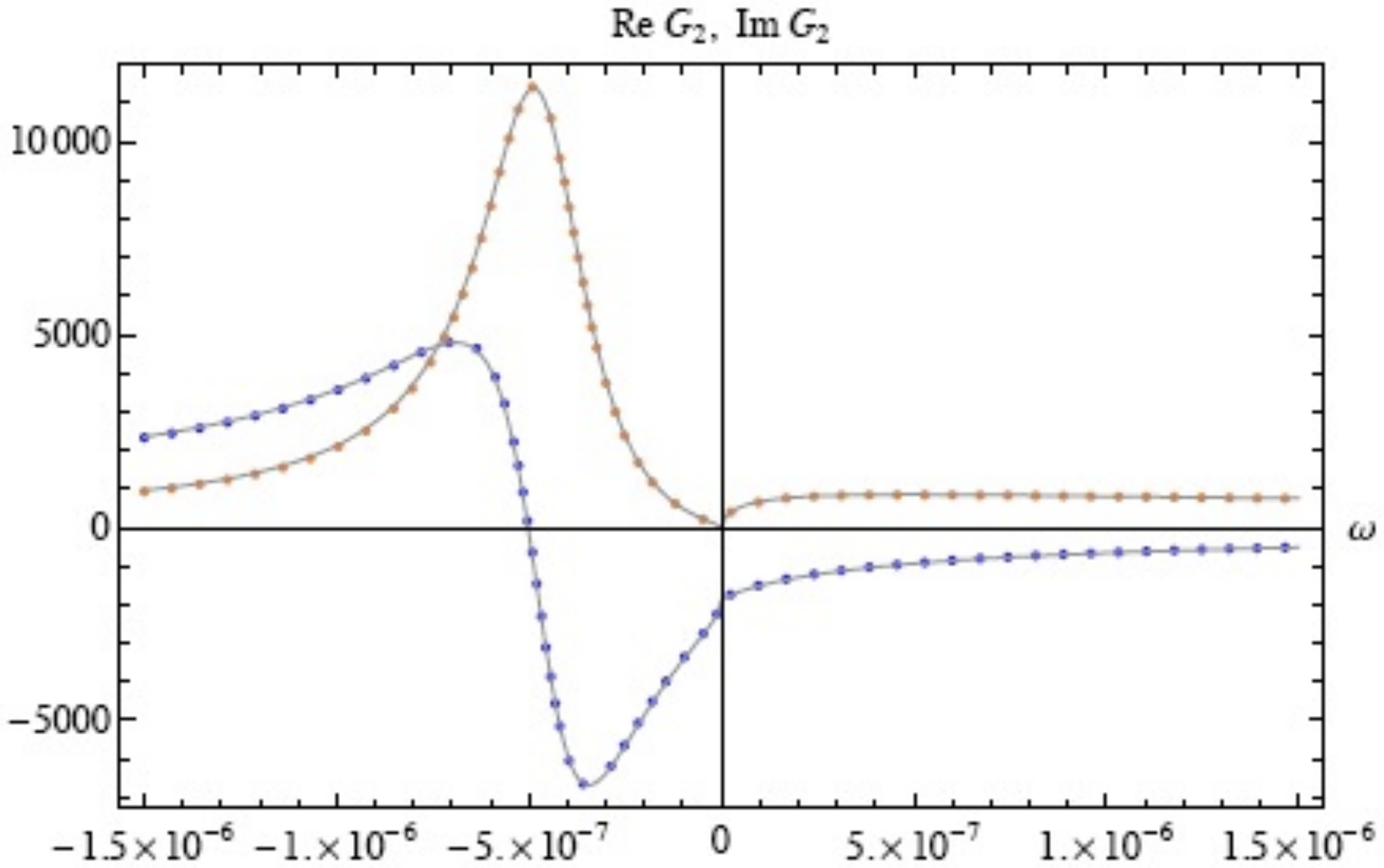
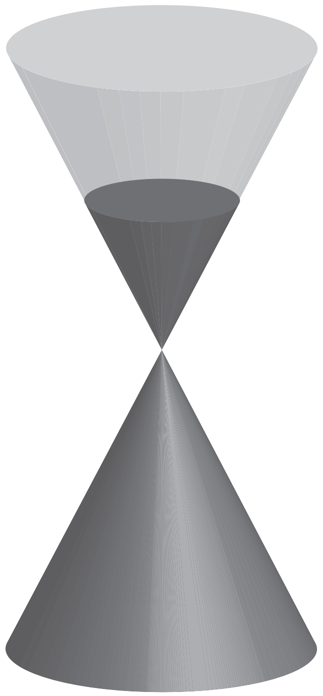
Green's function of a fermion





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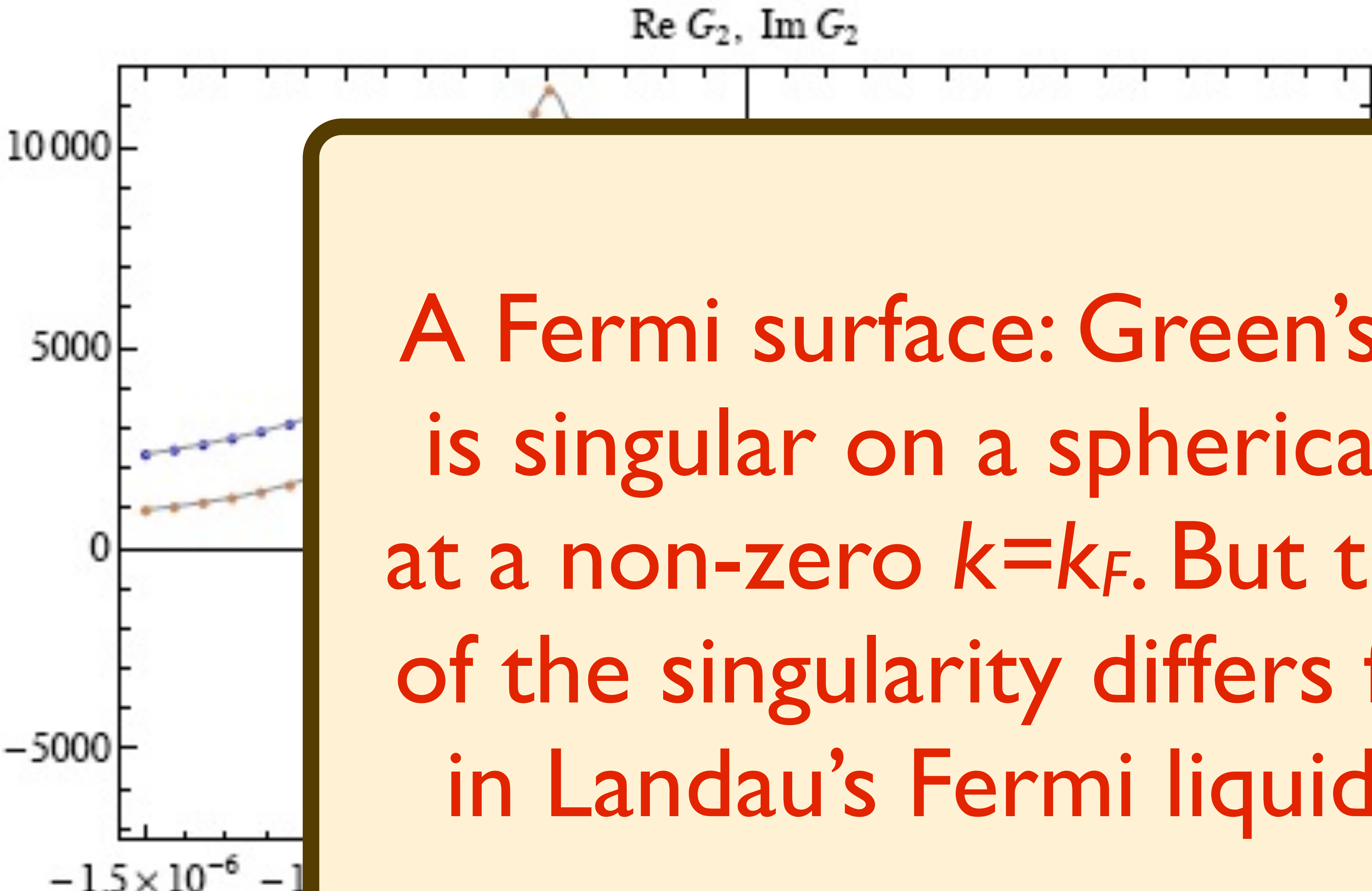

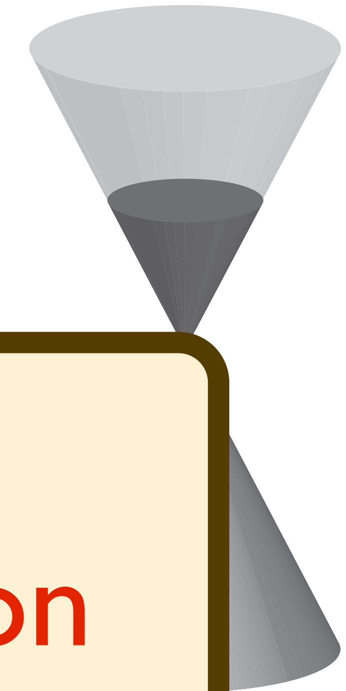

A Fermi surface: Green's function is singular on a spherical surface at a non-zero $k = k_F$. But the nature of the singularity differs from that in Landau's Fermi liquid theory.

H. Liu,
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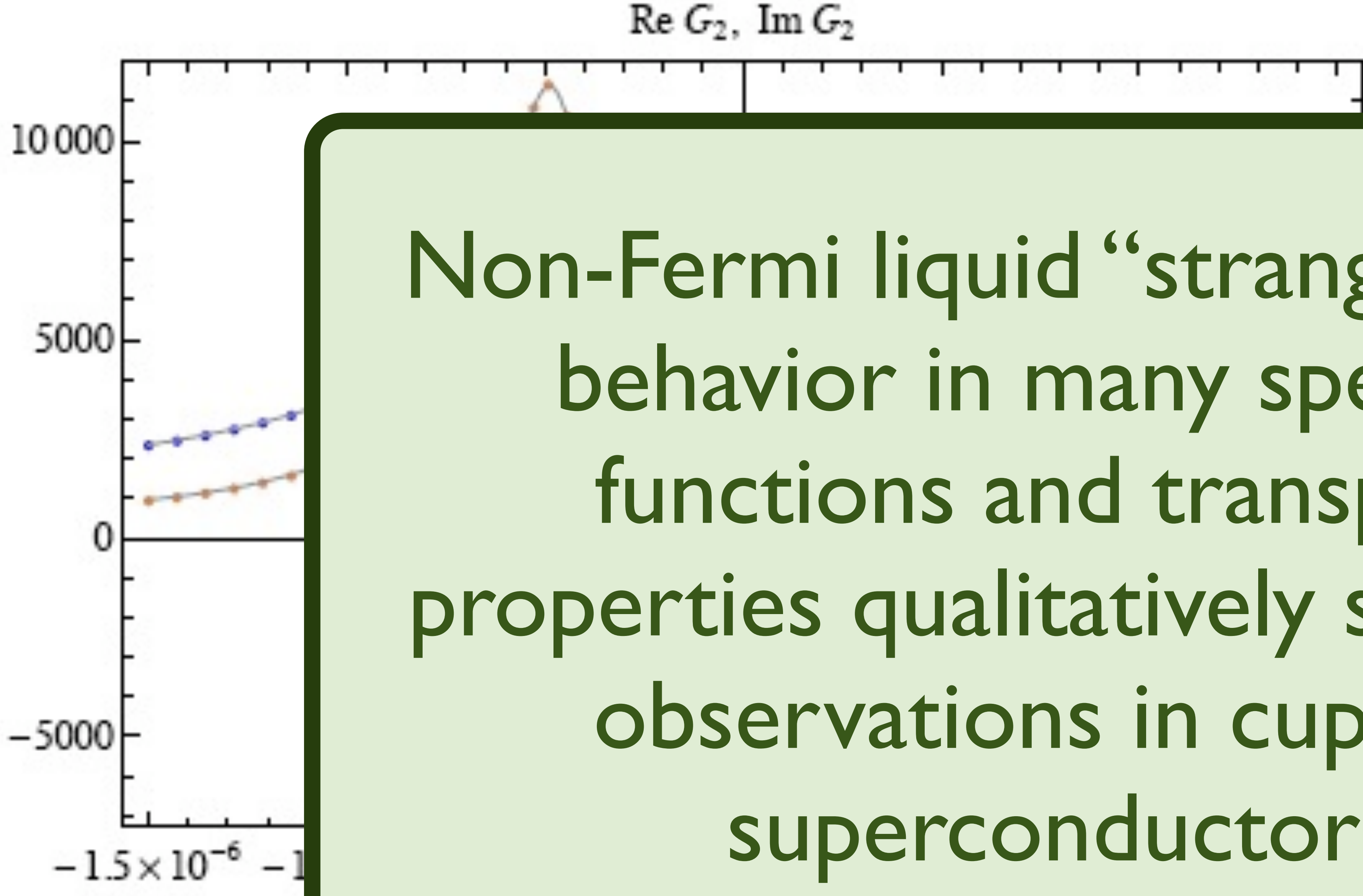

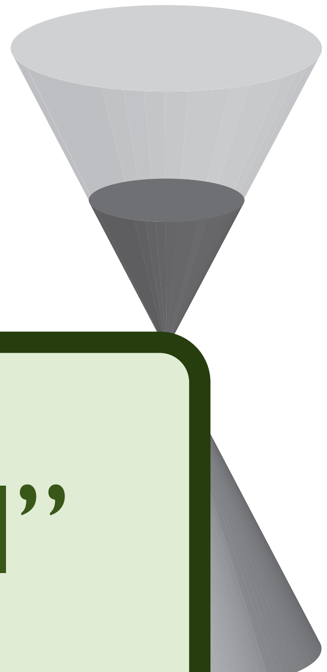

Non-Fermi liquid "strange metal" behavior in many spectral functions and transport properties qualitatively similar to observations in cuprate superconductors

H. Liu,
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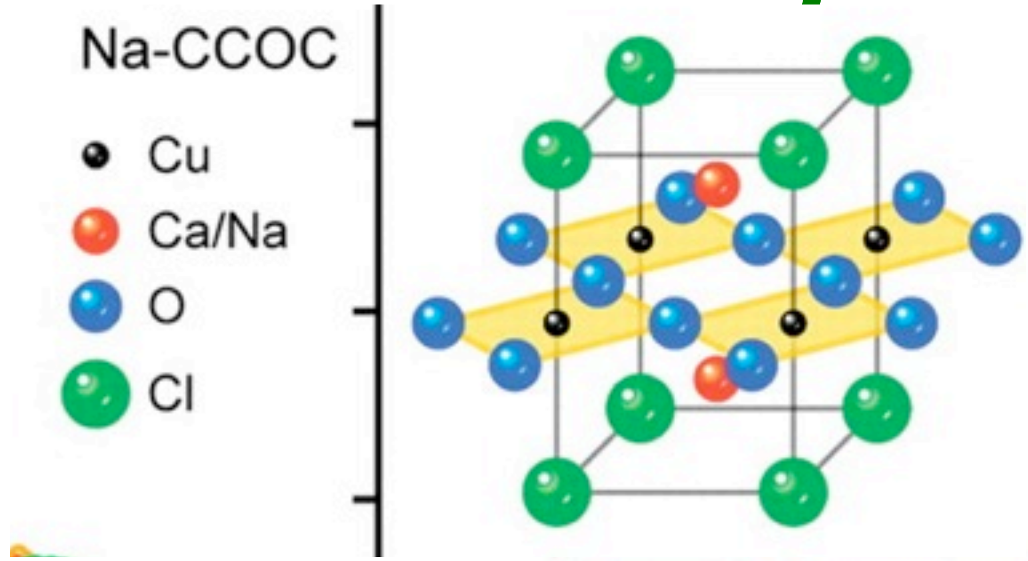
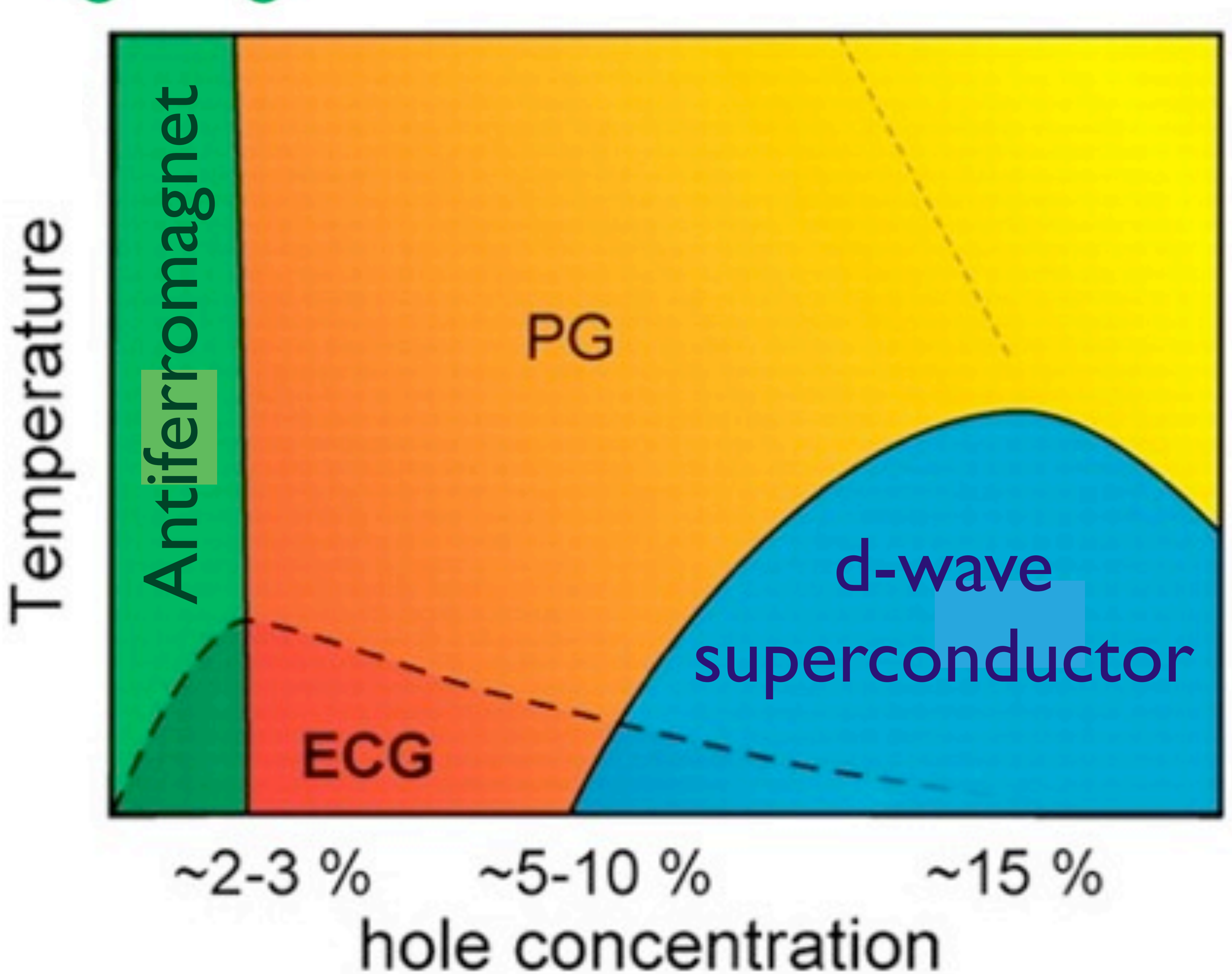
The cuprate superconductors





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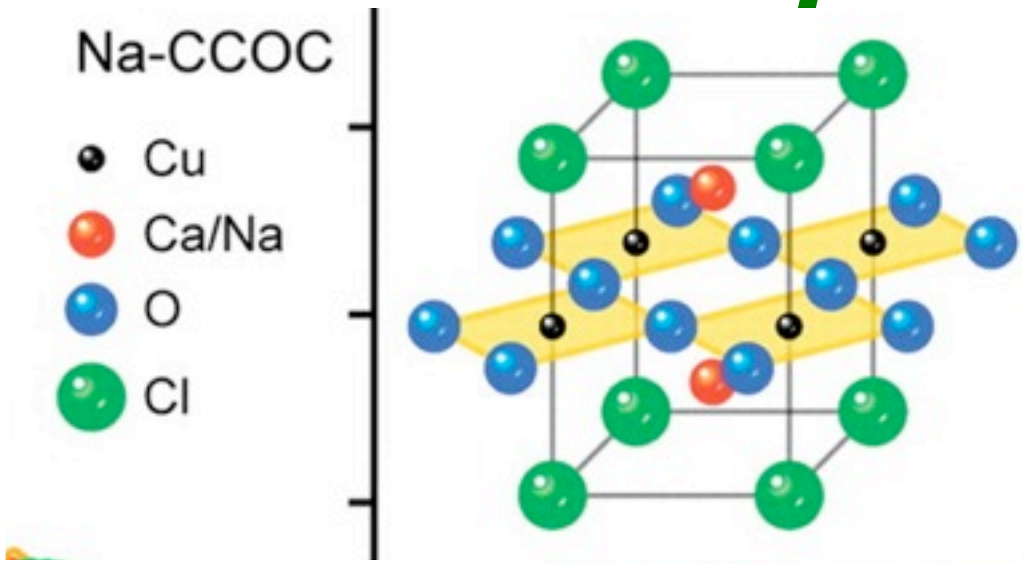

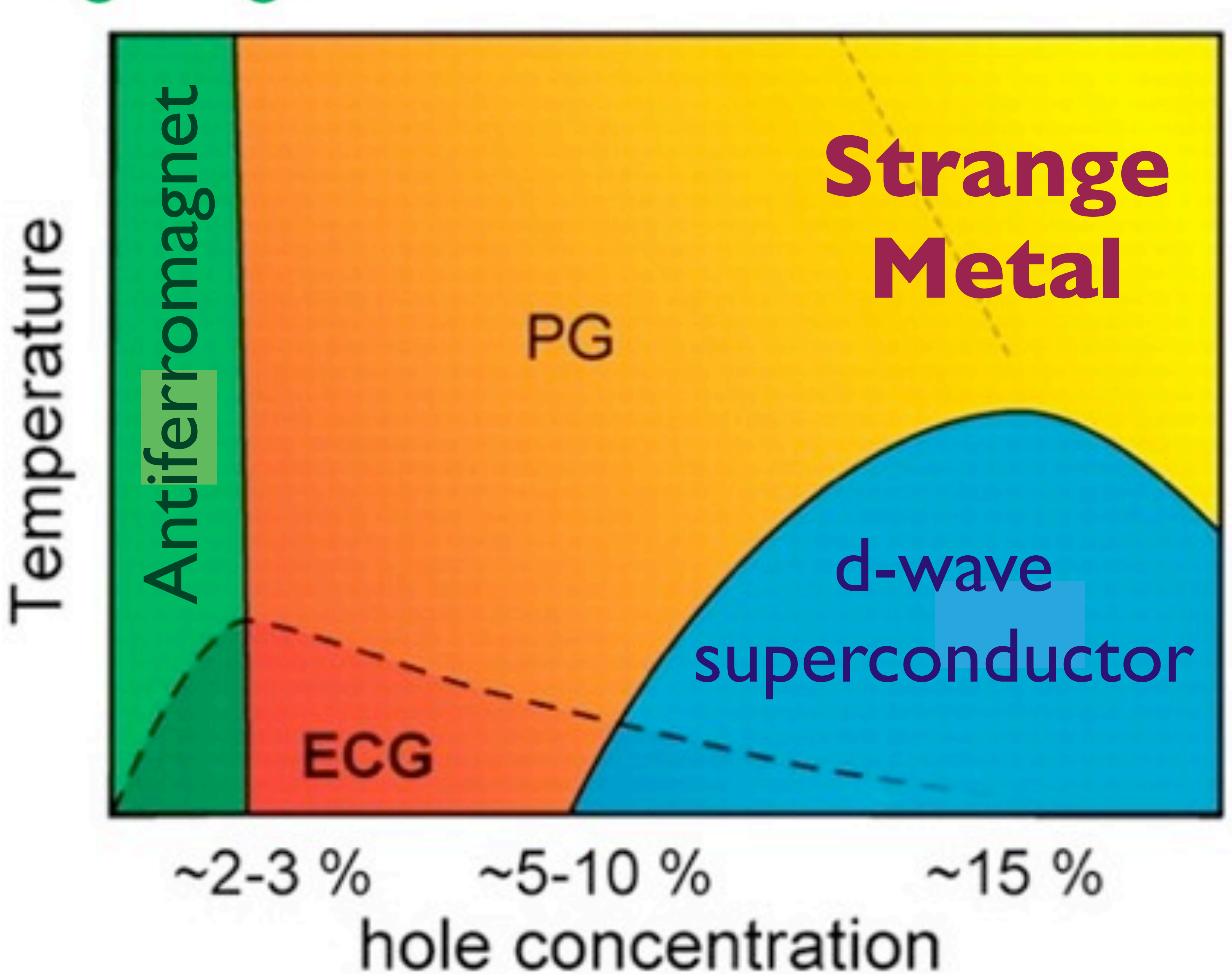

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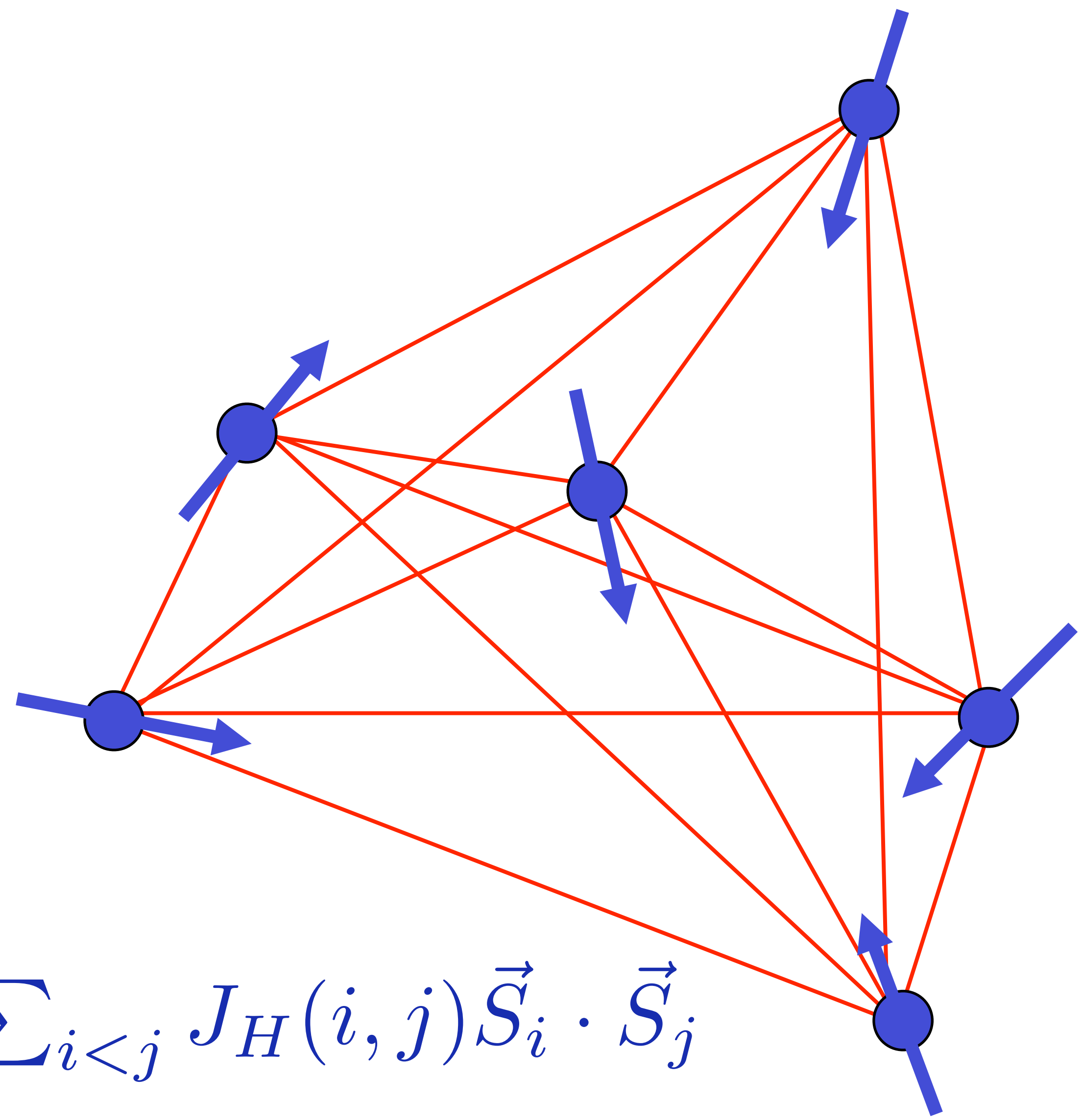
- Single particle self energies are momentum independent, and their singular behavior is the same on and off the Fermi surface.
- Low energy singularities are described by "conformal quantum mechanics": a 0+1 dimensional defect in a 2+1 dimensional CFT. This is linked to the factorization of the near-horizon metric to $AdS_2 \times R^2$,

A Kondo lattice model for the AdS_2 x R^d region of an extremal Reissner-Nordstrom black hole



$$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$$

$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick model of SU(N) spins.

S. Sachdev, arXiv:1006.3794

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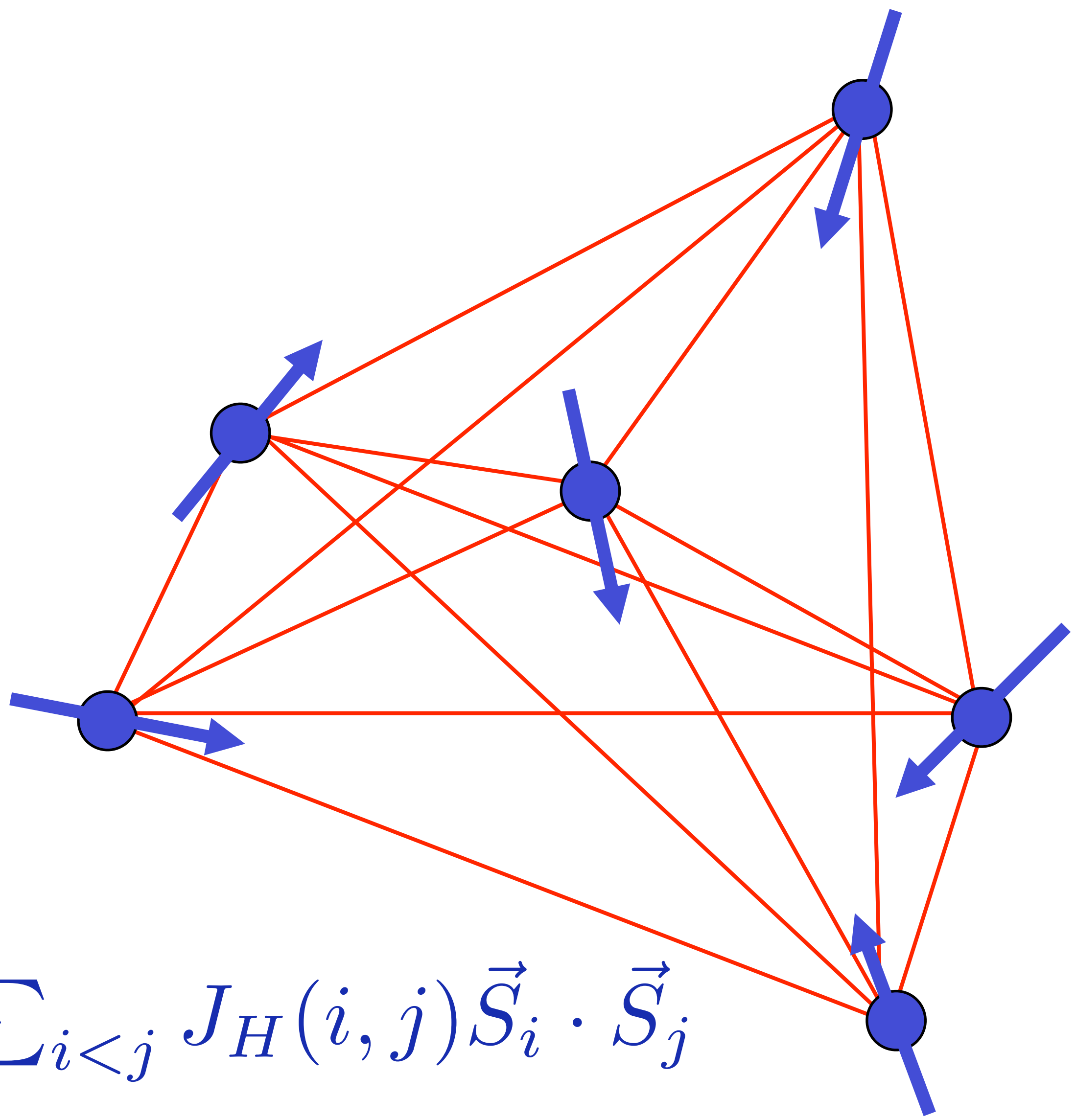

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Described by the conformal quantum mechanics of a quantum spin fluctuating in a self-consistent time-dependent magnetic field: a realization the finite entropy density $\mathrm{AdS}_2 \times R^d$ state

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S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
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AdS$_2$ realization in the quantum SK model

Focus on a single $\vec{S}$ spin, and represent its imaginary time fluctuations by a unit vector $\vec{S} = \vec{n}(\tau)/2$ which is controlled by the partition function

$$\mathcal{Z} = \int \mathcal{D}\vec{n}(\tau)\, \delta(\vec{n}^2(\tau) - 1) \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \frac{i}{2}\int_0^1 du \int_0^{1/T} d\tau\, \vec{n} \cdot \left(\frac{\partial \vec{n}}{\partial u} \times \frac{\partial \vec{n}}{\partial \tau}\right) - \int_0^{1/T} d\tau\, \vec{h}(\tau) \cdot \vec{n}(\tau)$$

The first term is a Wess-Zumino term, with the "extra dimension" u defined so that $\vec{n}(\tau, u = 1) \equiv \vec{n}(\tau)$ and $\vec{n}(\tau, u = 0) = (0, 0, 1)$.

The field $\vec{h}(\tau)$ represents the "environment", which is determined as a Gaussian field obeying the self-consistency condition

$$\left\langle \vec{h}(\tau) \cdot \vec{h}(0) \right\rangle \propto \langle \vec{n}(\tau) \cdot \vec{n}(0) \rangle$$

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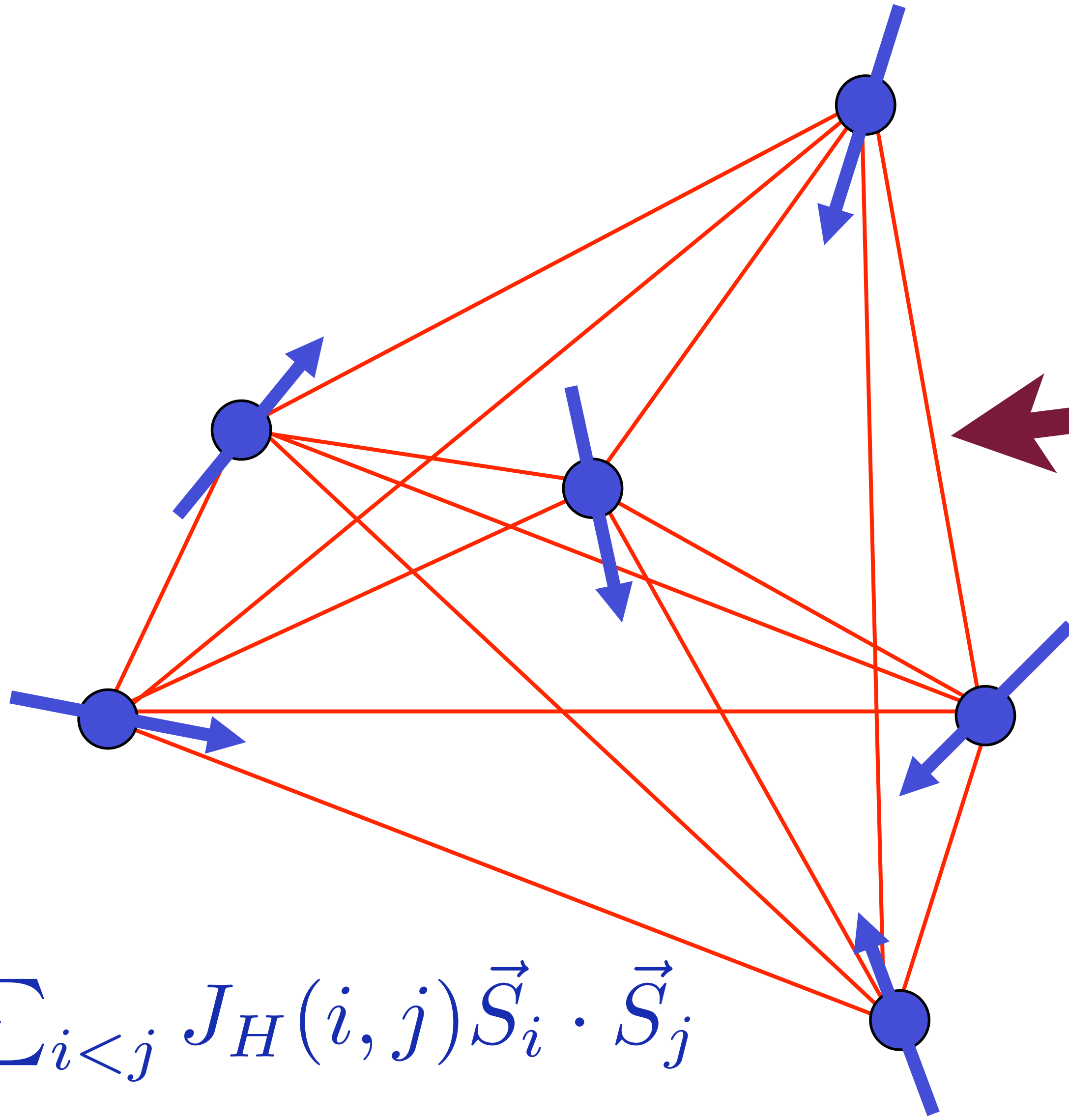

Kondo exchange J_K

A small Fermi surface of probe fermions

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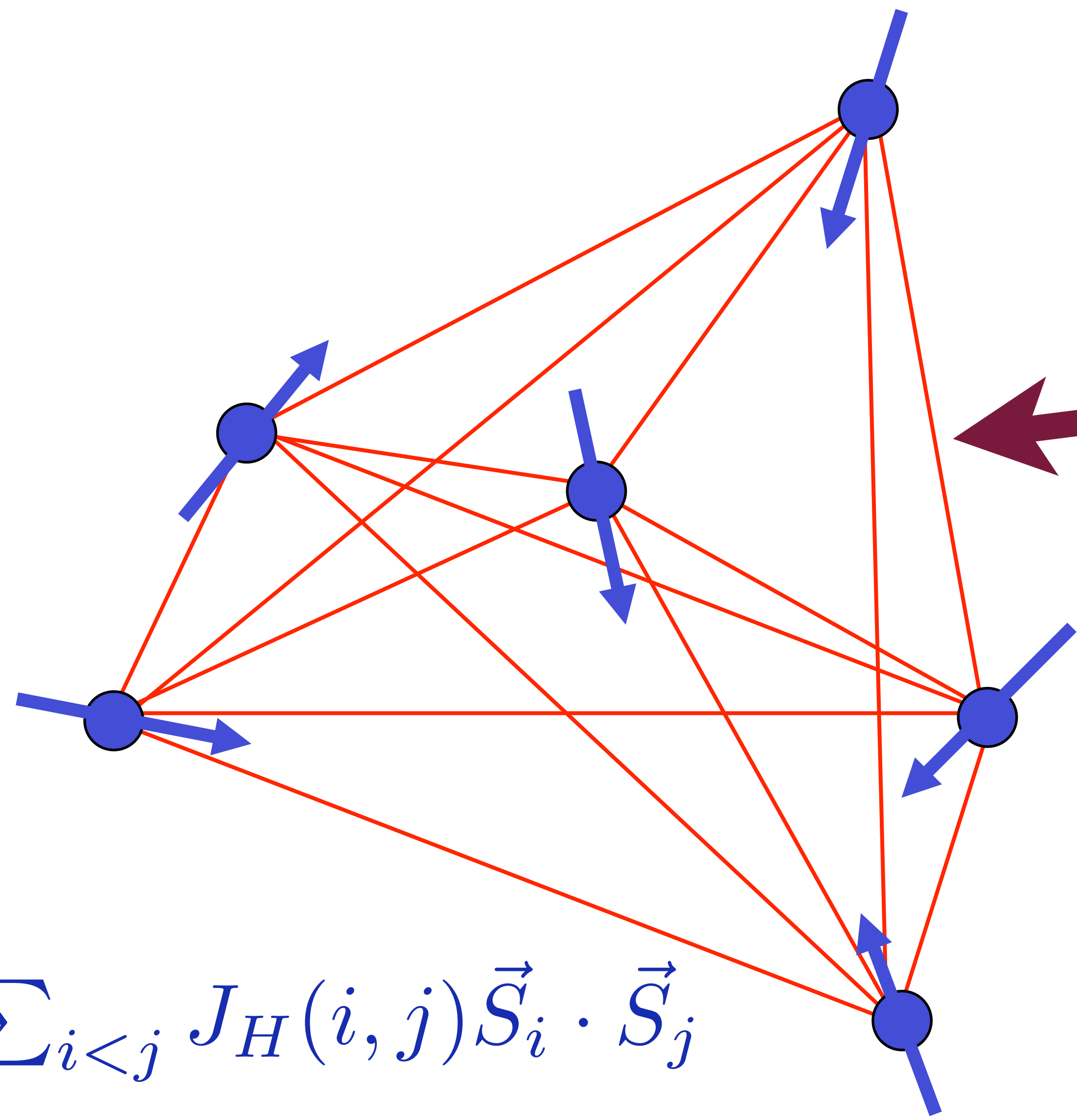

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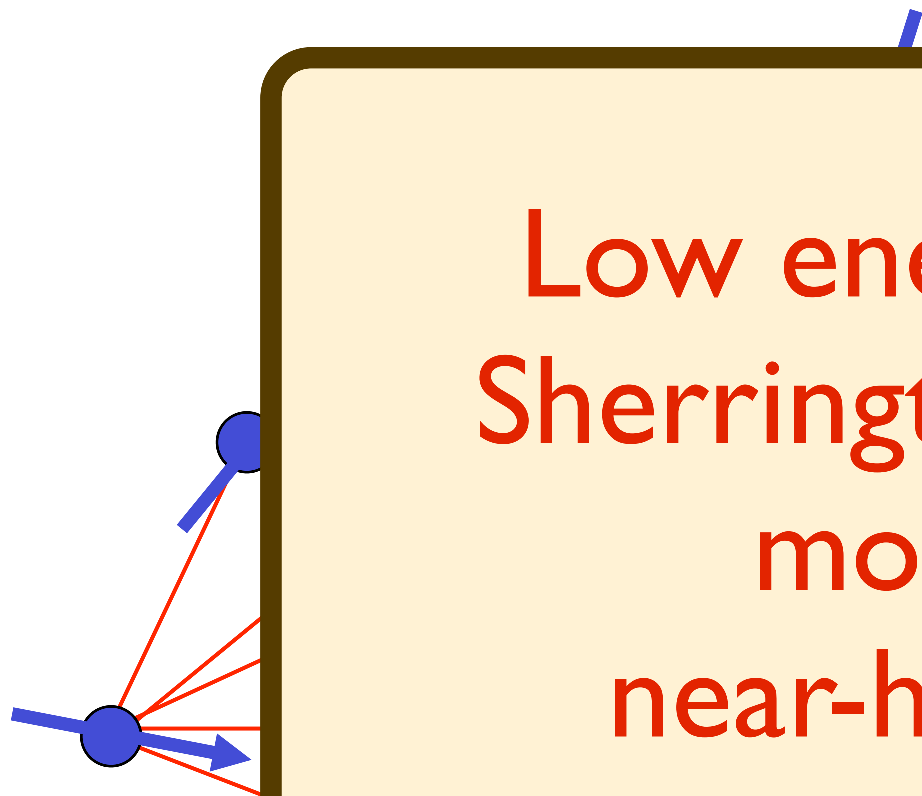

Low energy properties of the Sherrington-Kirkpatrick-Kondo model map onto the near-horizon physics of an extremal Reissner-Nordstrom black hole

surface

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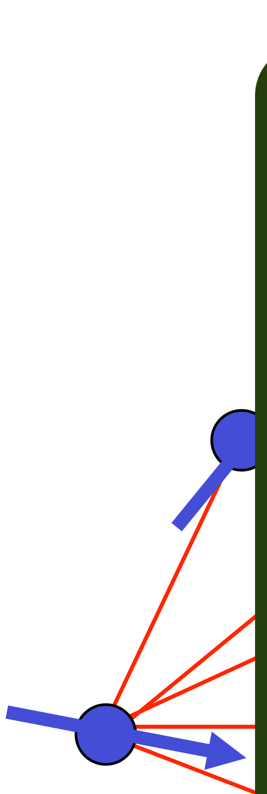

surface

nions

Much work remains in extending
these solvable models
(the AdS theories or
the large-connectivity limits of
Kondo lattice models)
to a realistic theory of the
cuprate superconductors

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Conclusions

New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle picture.

Conclusions

The AdS/CFT correspondence offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density