Universal, low temperature, T-linear resistivity in two-dimensional quantum-critical metals from spatially random interactions

Low-Dimensional Holography and Black Holes,
Princeton Center for Theoretical Sciences
March 30, 2022

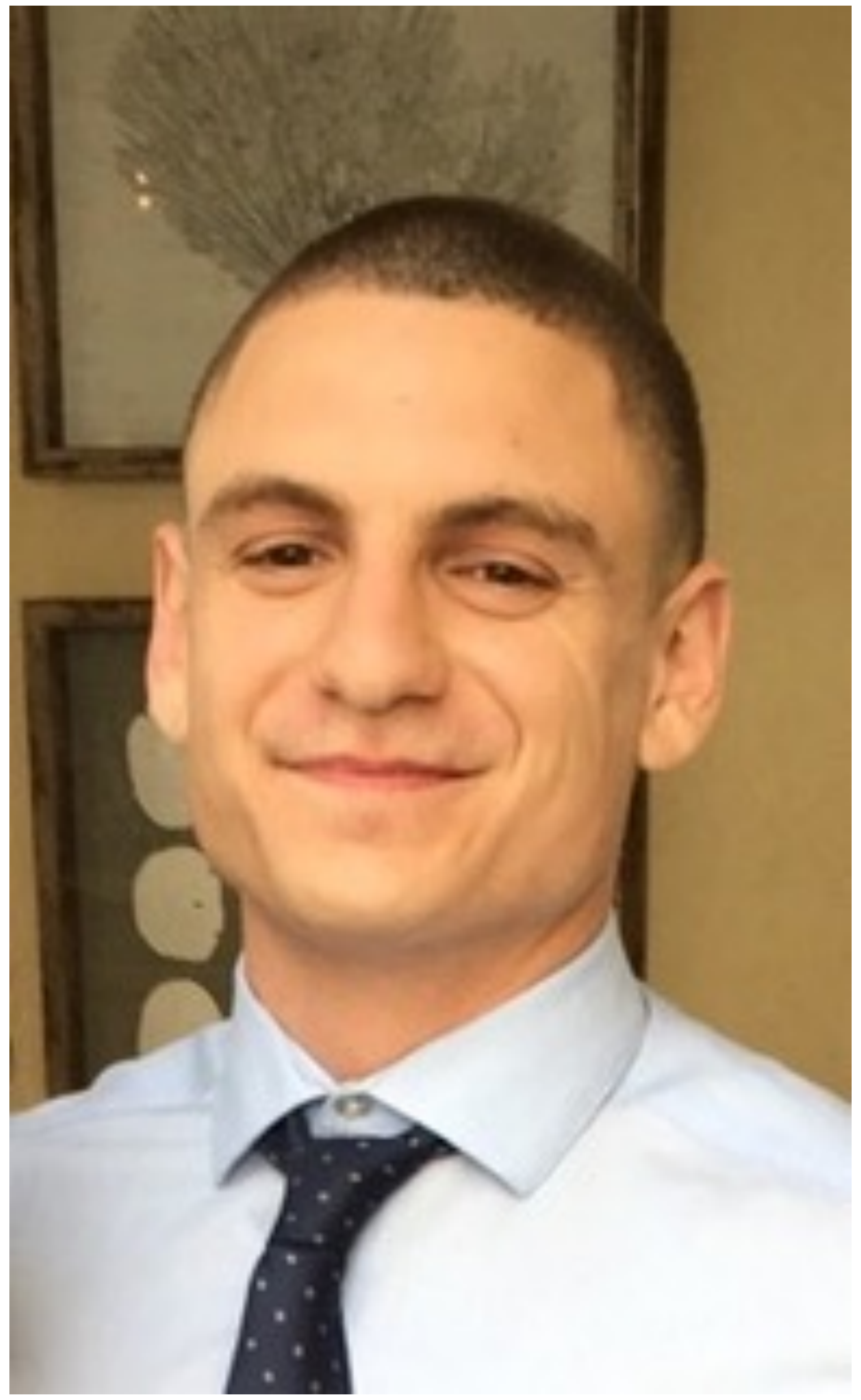
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

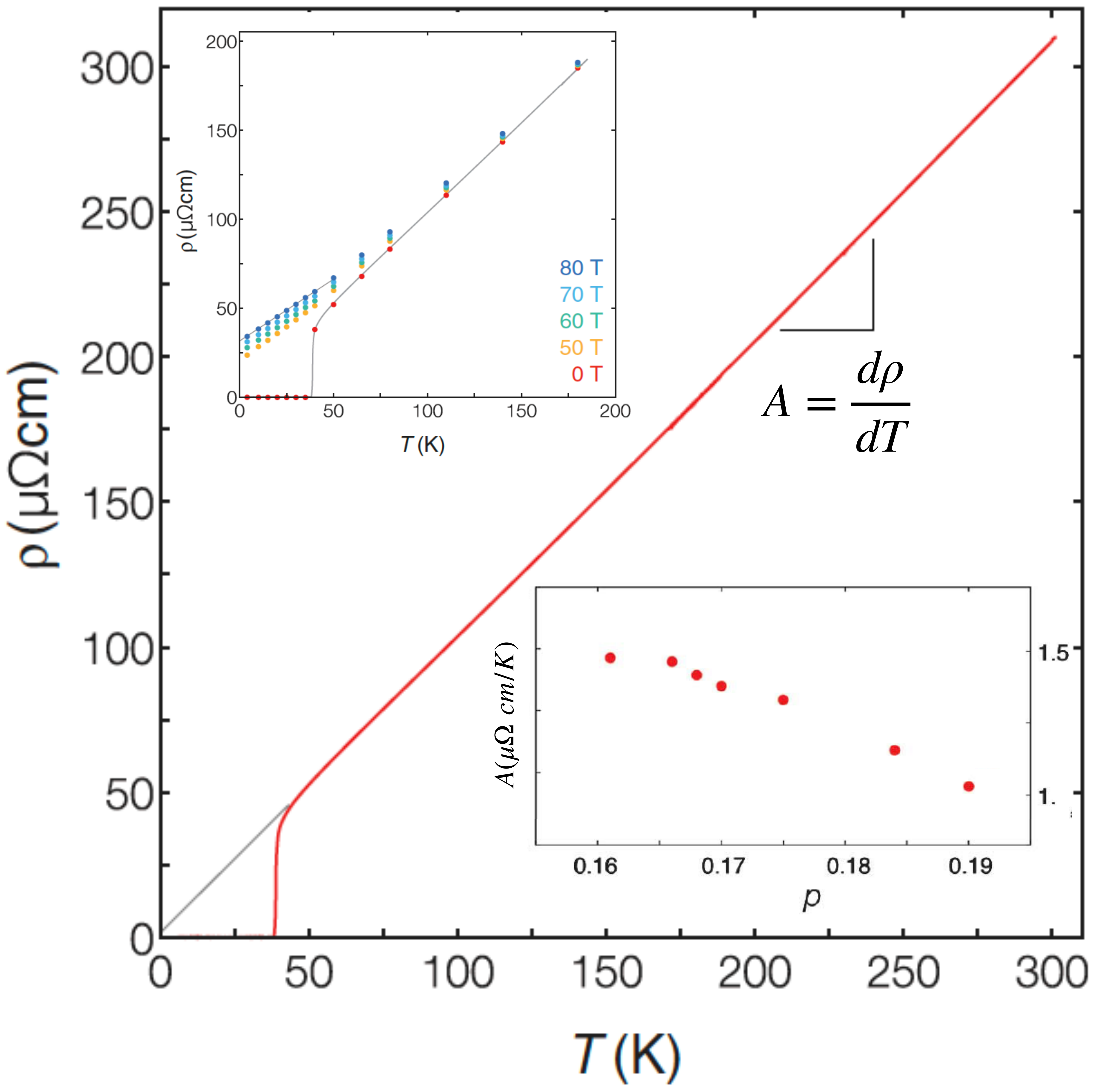
INSTITUTE FOR ADVANCED STUDY

PHYSICS

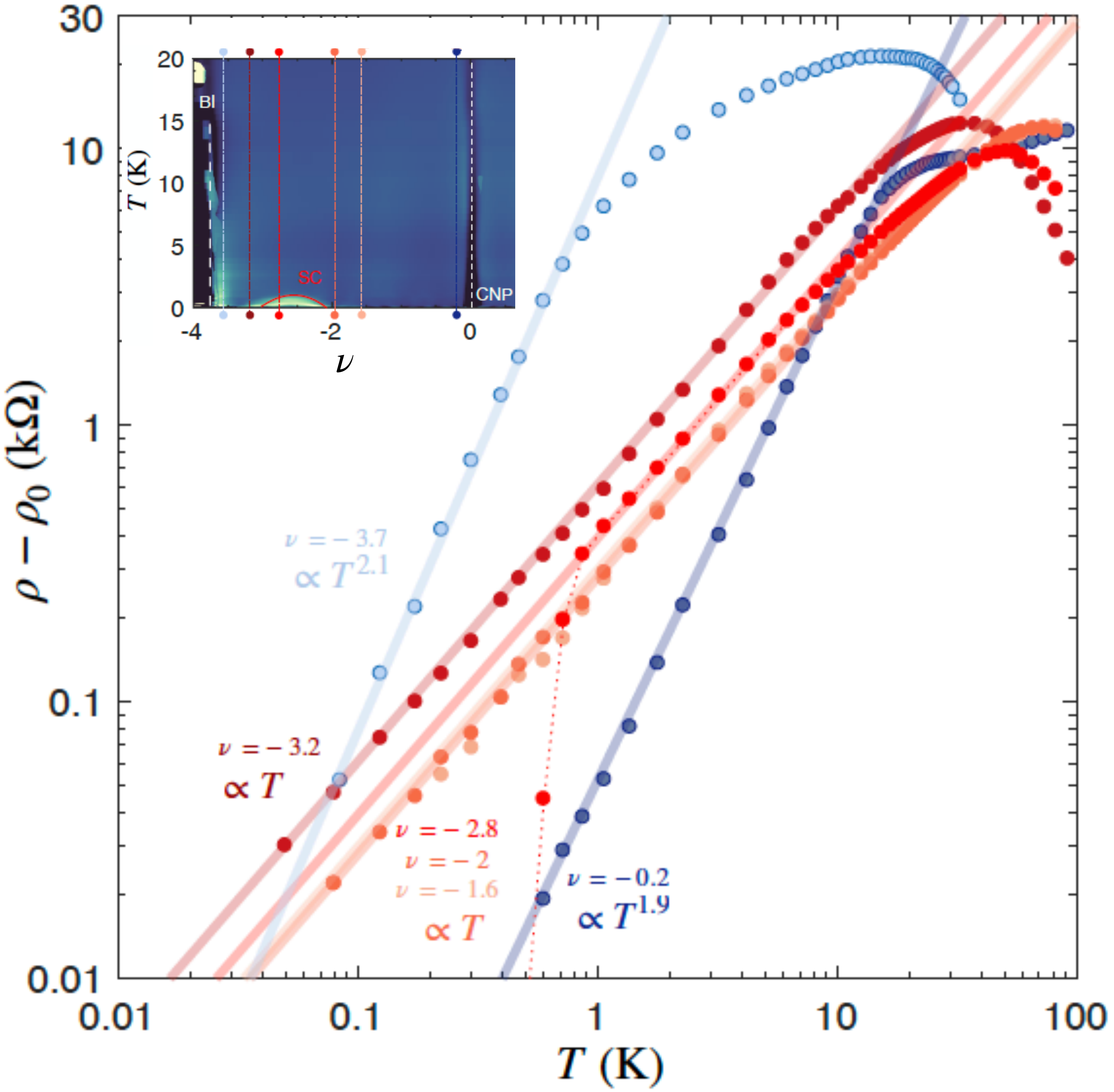
HARVARD



Aavishkar Patel, Haoyu Guo, Ilya Esterlis, and S.S. arXiv:2203.04990

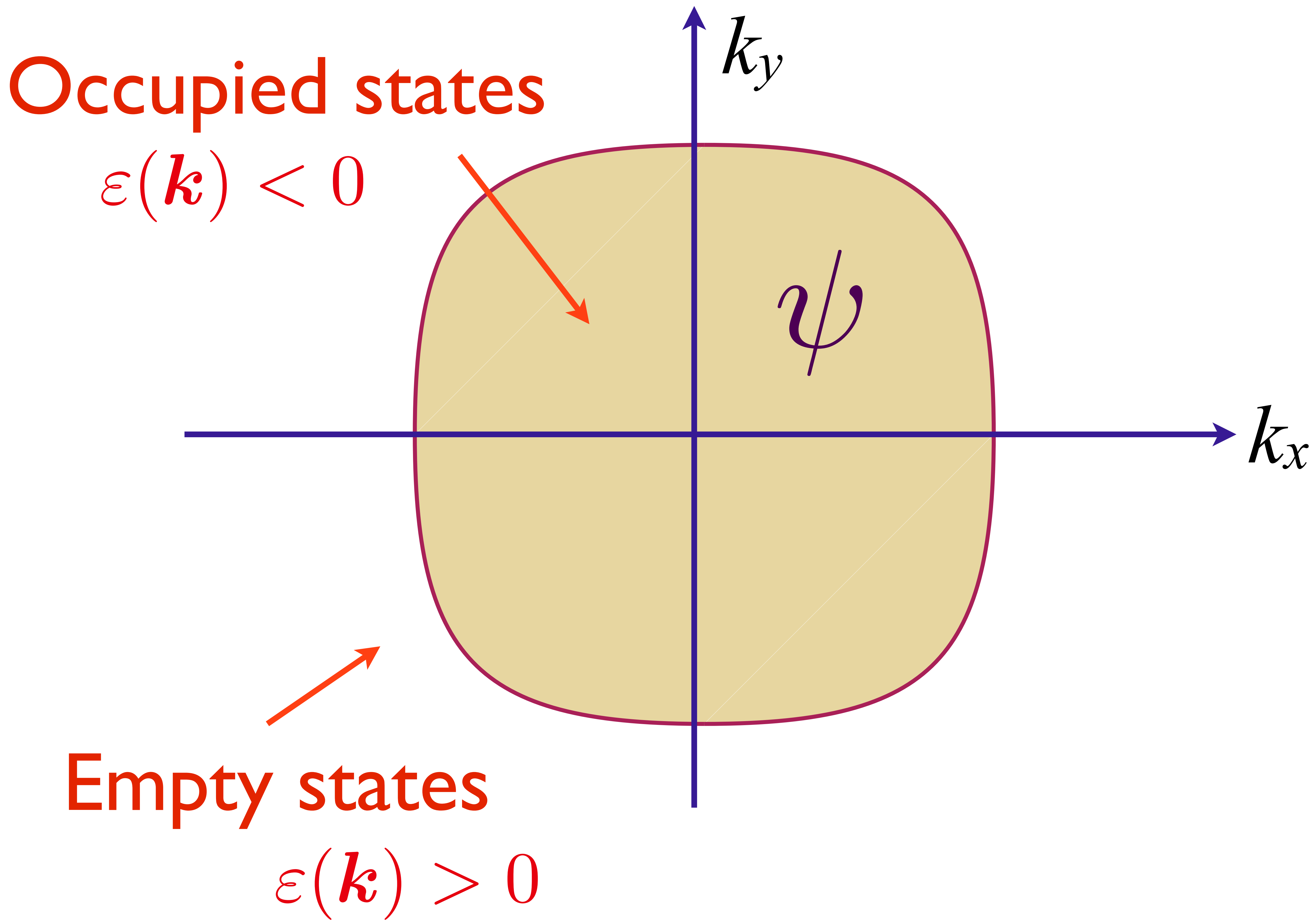


LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Fermi surface coupled to a critical boson



+

a critical boson

ϕ

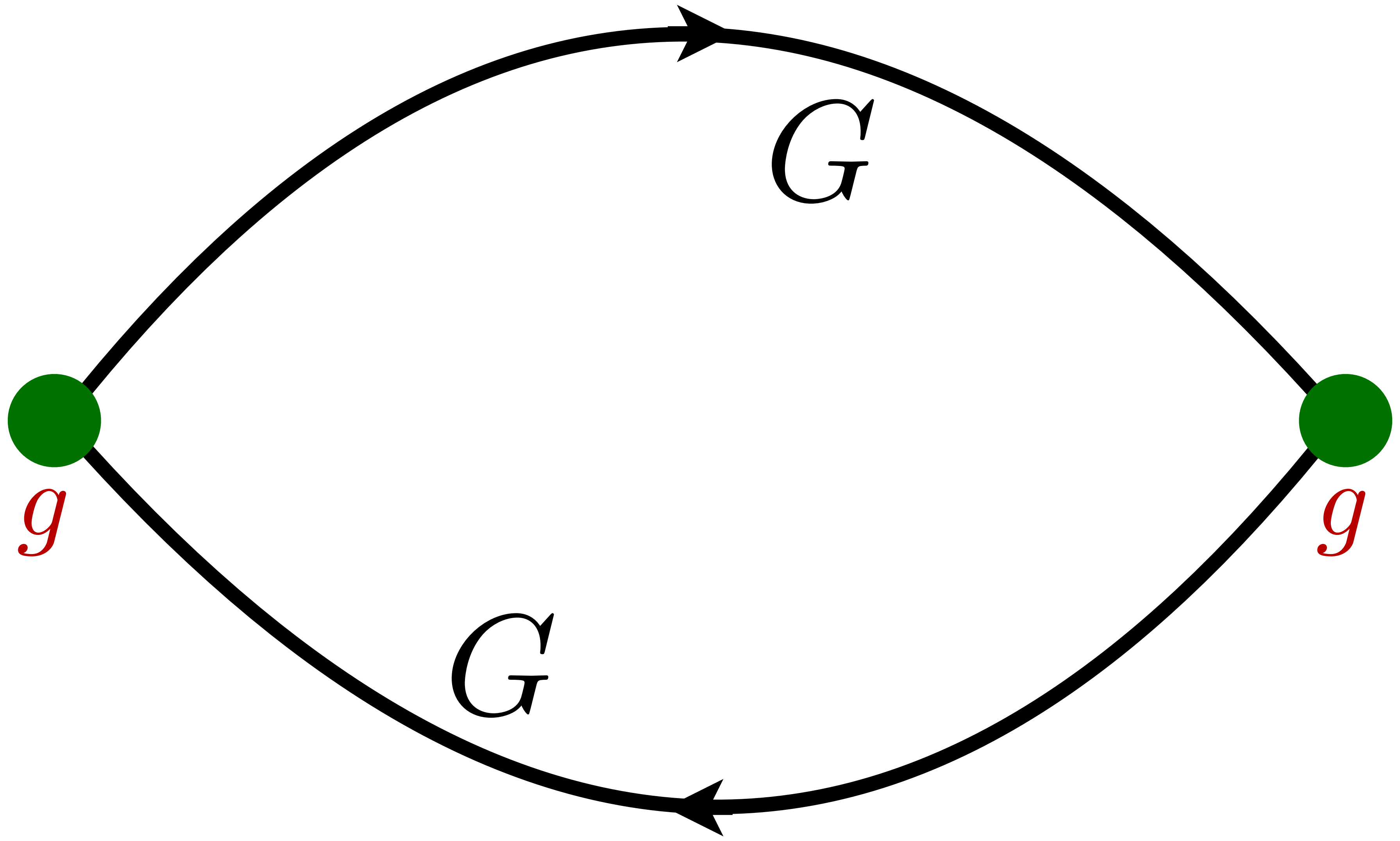
- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field

$$\mathcal{S}_\psi = \int d\tau d^2k \, \psi^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon(\boldsymbol{k}) \right) \psi$$

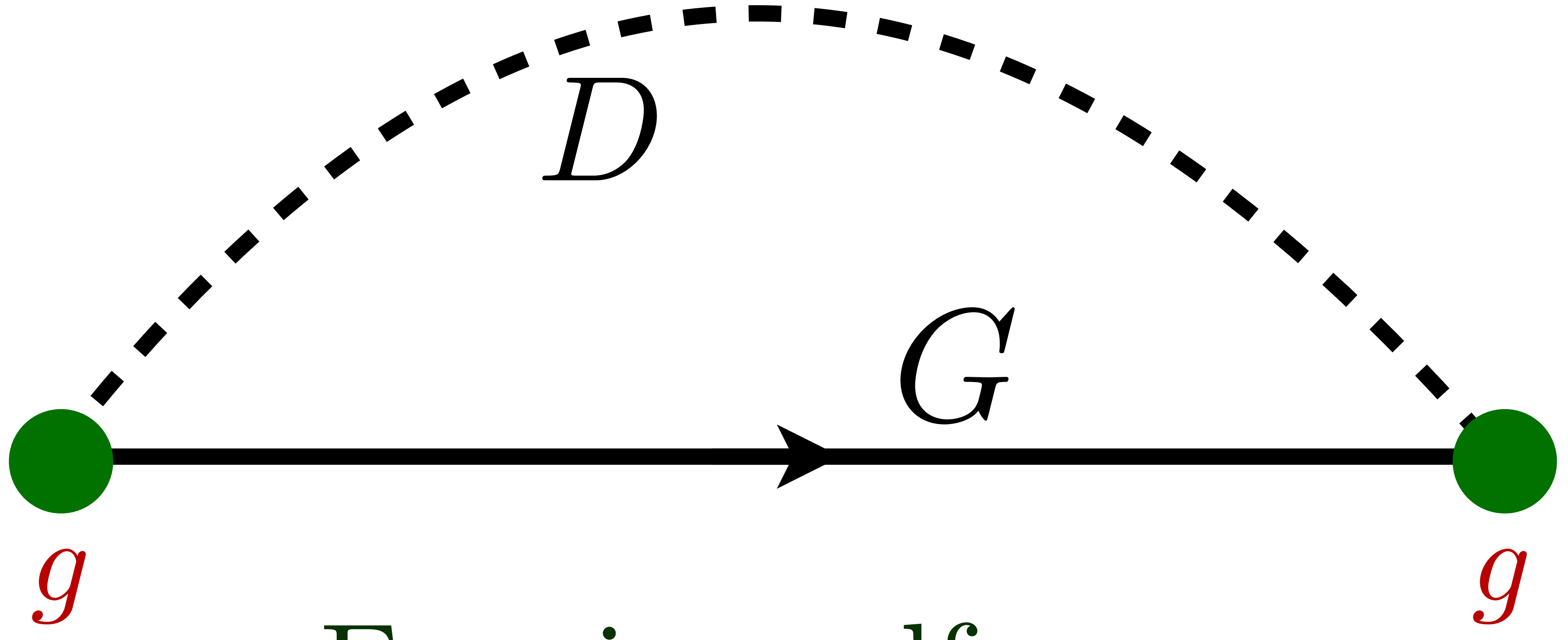
$$\mathcal{S}_\phi = \int d^3x \, (\partial_\mu \phi)^2 + \ldots$$

Fermi surface coupled to a critical boson

"Yukawa" coupling: $g\int d^2r d\tau\, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

$\Pi =$ 

Boson self energy

$\Sigma =$ 

Fermion self energy

$$G_\psi = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) + iB\,\mathrm{sgn}(\omega)|\omega|^{2/3}}$$

P. A. Lee (1989)

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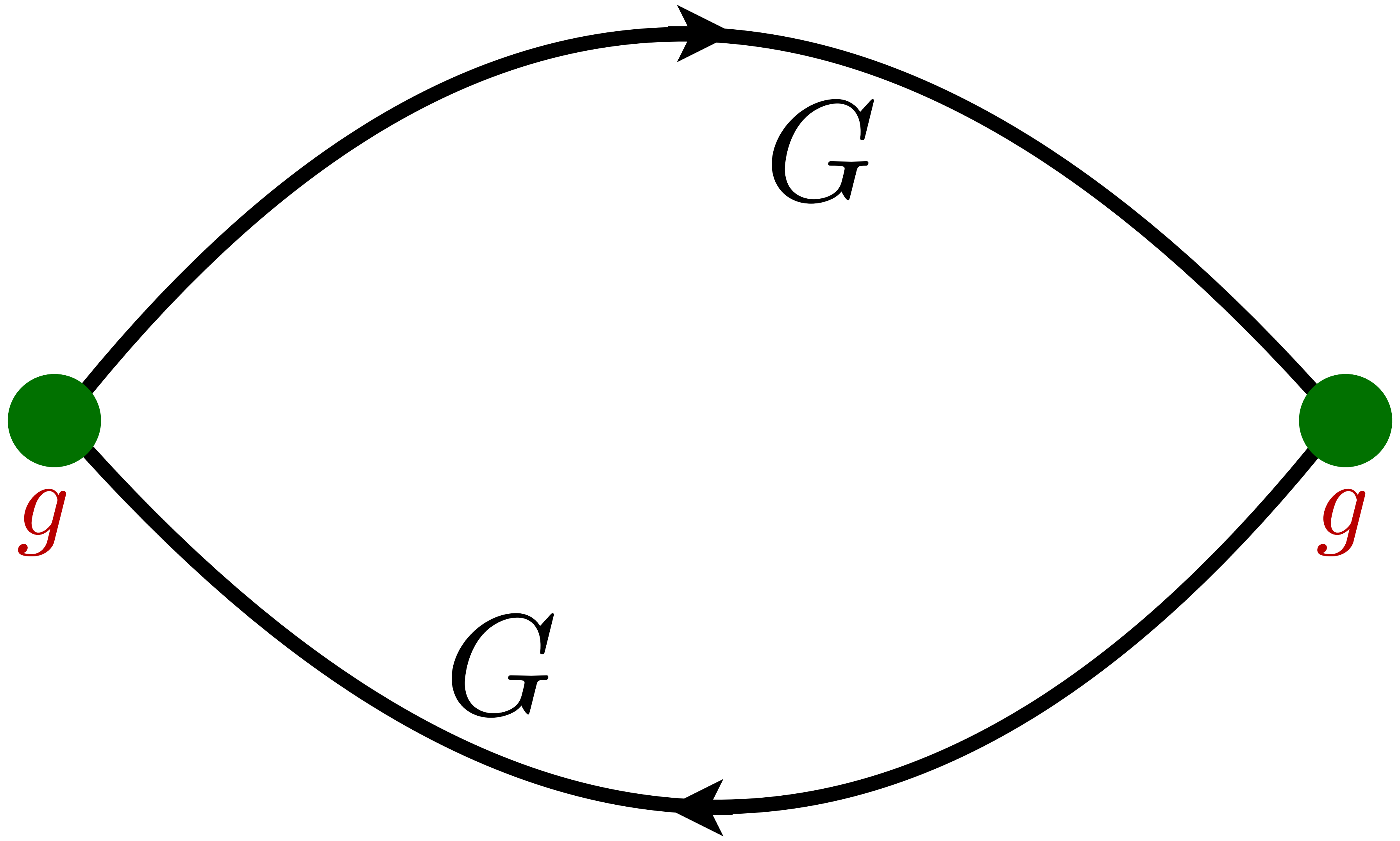

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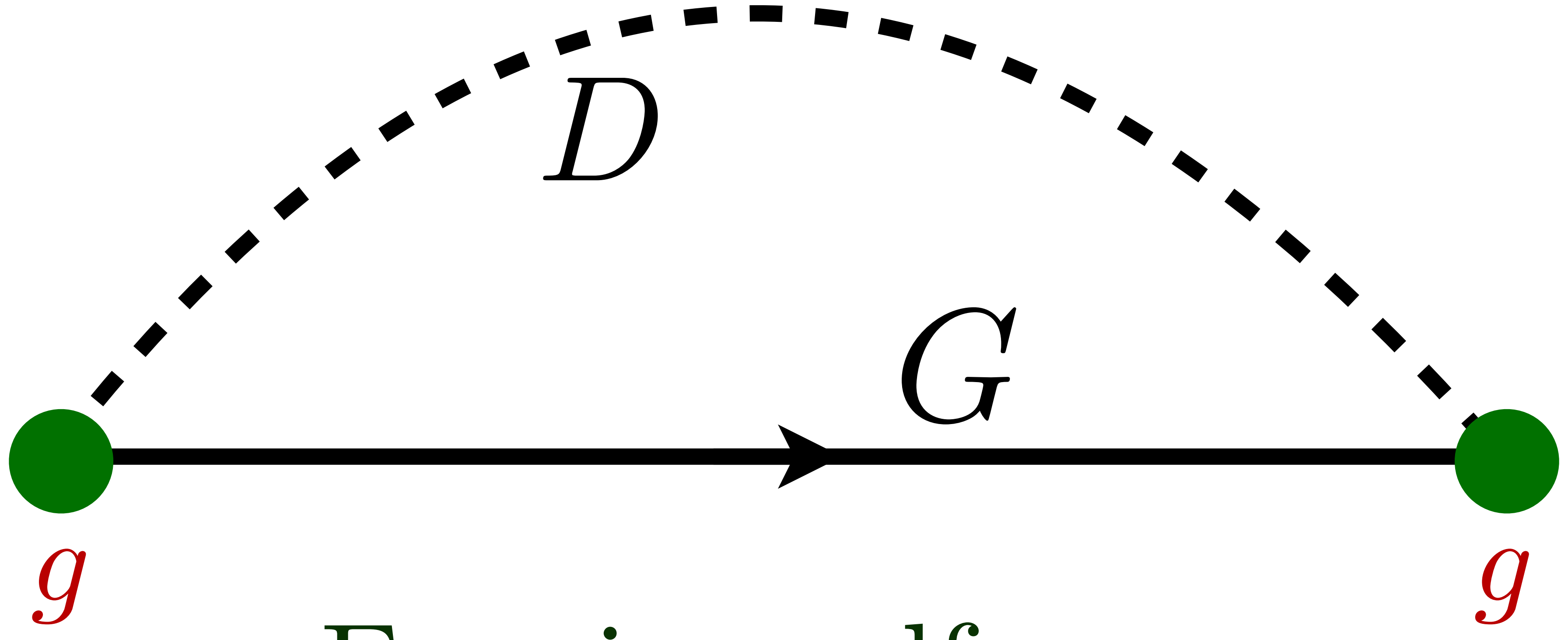

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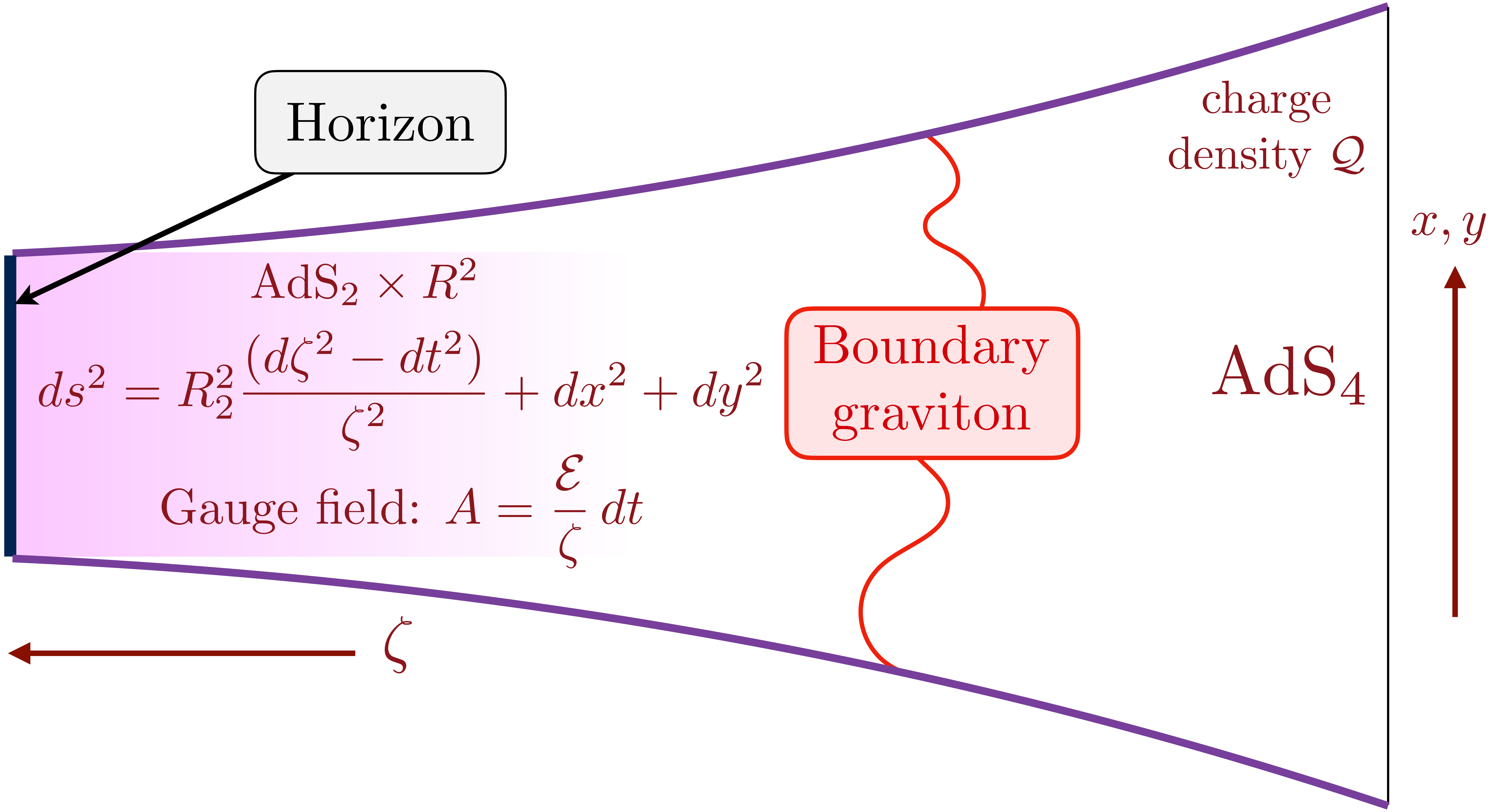
Yields a state without quasiparticle excitations, but the theory is not systematic. A theory with N fermion flavors breaks down at higher loop order because $B \sim 1/N$.

Sung-Sik Lee (2009)

Holographic non-Fermi liquid



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Probe
YM-gauge-invariant
fermion f
carrying
global U(1) charge

$$G_f = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) + iB\,\mathrm{sgn}(\omega)|\omega|^{\sigma(k)}}$$

Tom Faulkner, Hong Liu, John McGreevy, David Vegh (2009)
Mihailo Cubrovic, Jan Zaanen, Koenraad Schalm (2009)

Transport in non-Fermi liquids

- Common past claims:
  - d.c. resistivity in critical boson theory $\sim T^{4/3}$, arising from

$$\frac{1}{\tau_{\rm transport}} = \frac{1}{\tau_{\rm single-particle}} \times \left[1 - \cos(\theta_{\rm scattering\ angle})\right]$$

  - In holographic theory, $\langle \cos(\theta) \rangle = 0$ because the AdS_2 leads to angle-independent scattering.

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  - In holographic theory, $\langle \cos(\theta) \rangle = 0$ because the AdS_2 leads to angle-independent scattering.

- This is incorrect:

  The strong coupling between the Fermi surface and the boson places the system in the limit of strong 'boson drag'; this is in stark contrast to the electron-phonon system, where the weak electron-phonon coupling makes 'phonon drag' a factor only in ultrapure samples.

  Conservation of momentum implies $\mathrm{Re}\,\sigma(\omega) \sim \delta(\omega)$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. S., PRB **76**, 144502 (2007)
S.A. Hartnoll, R. Mahajan, M. Punk, and S. S., PRB **89**, 155130 (2014); Aavishkar Patel, Haoyu Guo, Ilya Esterlis, and S.S. arXiv:2203.04990

Needed:

- Systematic large N theory of Fermi surface without quasiparticles
- Theory for the resistivity in the presence of momentum relaxation

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$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

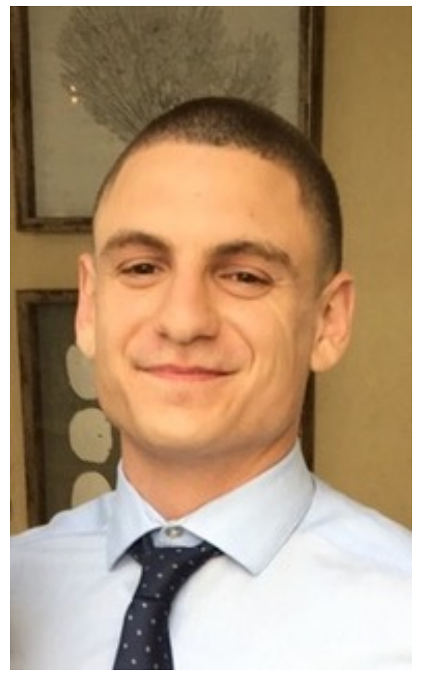
This theory still has conductivity $\mathrm{Re}\,\sigma \sim \delta(\omega)$

Main idea:

Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

W. Fu, D. Gaiotto, J. Maldacena, and S. S., PRD **95**, 026009 (2017); J. Murugan, D. Stanford, and E. Witten, JHEP **08**, 146 (2017)
E. Marcus and S. Vandoren, JHEP 166 (2019); Ilya Esterlis, J. Schmalian, PRB **100**, 115132 (2019); Yuxuan Wang and A.V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763 ; Ilya Esterlis, Haoyu Guo, Aavishkar Patel, and S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory



The theory self-averages, and the average partition function can be written exactly as a 'G-Σ' theory involving a path integral over *bilocal in spacetime*. We introduce the spacetime co-ordinate $X \equiv (\tau, x, y)$, and all Green's functions and self energies in the path integral are functions of two spacetime co-ordinates X_1 and X_2.

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(X_1, X_2)\mathcal{D}\Sigma(X_1, X_2)\mathcal{D}D(X_1, X_2)\mathcal{D}\Pi(X_1, X_2) \exp\left[-NI(G, \Sigma, D, \Pi)\right] .$$

The G-Σ-D-Π action is now

$$\begin{aligned} I(G, \Sigma, D, \Pi) &= \frac{g^2}{2}\mathrm{Tr}\left(G \cdot [GD]\right) - \mathrm{Tr}(G \cdot \Sigma) + \frac{1}{2}\mathrm{Tr}(D \cdot \Pi) \\ &- \ln\det\left[(\partial_{\tau_1} + \varepsilon(-i\boldsymbol{\nabla}_1))\,\delta(X_1 - X_2) + \Sigma(X_1, X_2)\right] \\ &\quad + \frac{1}{2}\ln\det\left[\left(-\partial_{\tau_1}^2 - \boldsymbol{\nabla}_1^2 + s\right)\delta(X_1 - X_2) - \Pi(X_1, X_2)\right] . \end{aligned}$$

where we have introduced notation

$$\mathrm{Tr}\,(f \cdot g) \equiv \int dX_1 dX_2\, f(X_2, X_1) g(X_1, X_2) .$$

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory

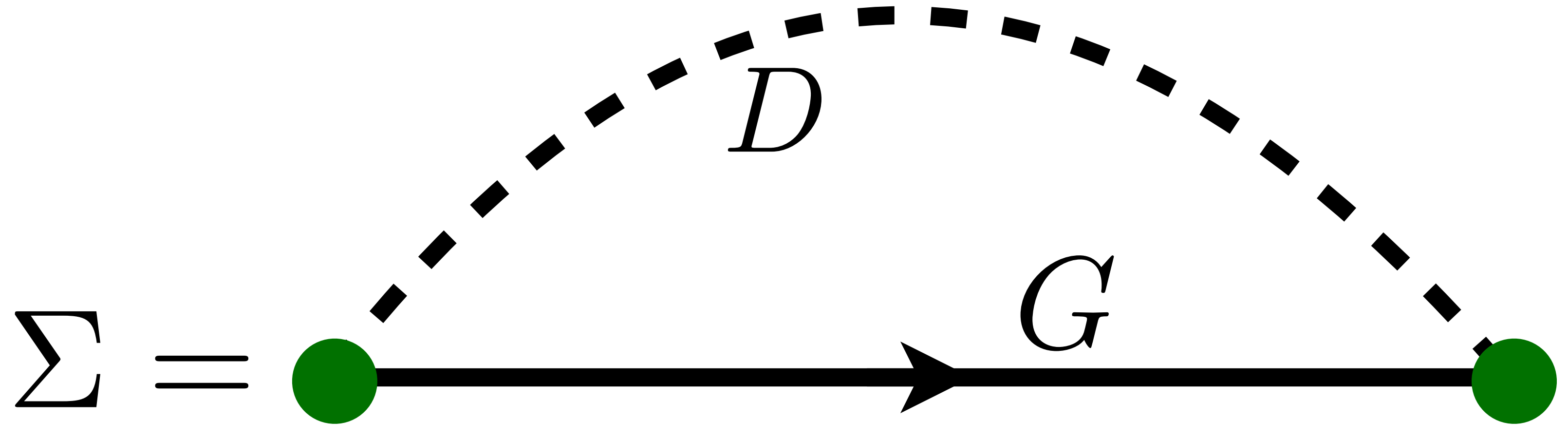
The saddle point equations are

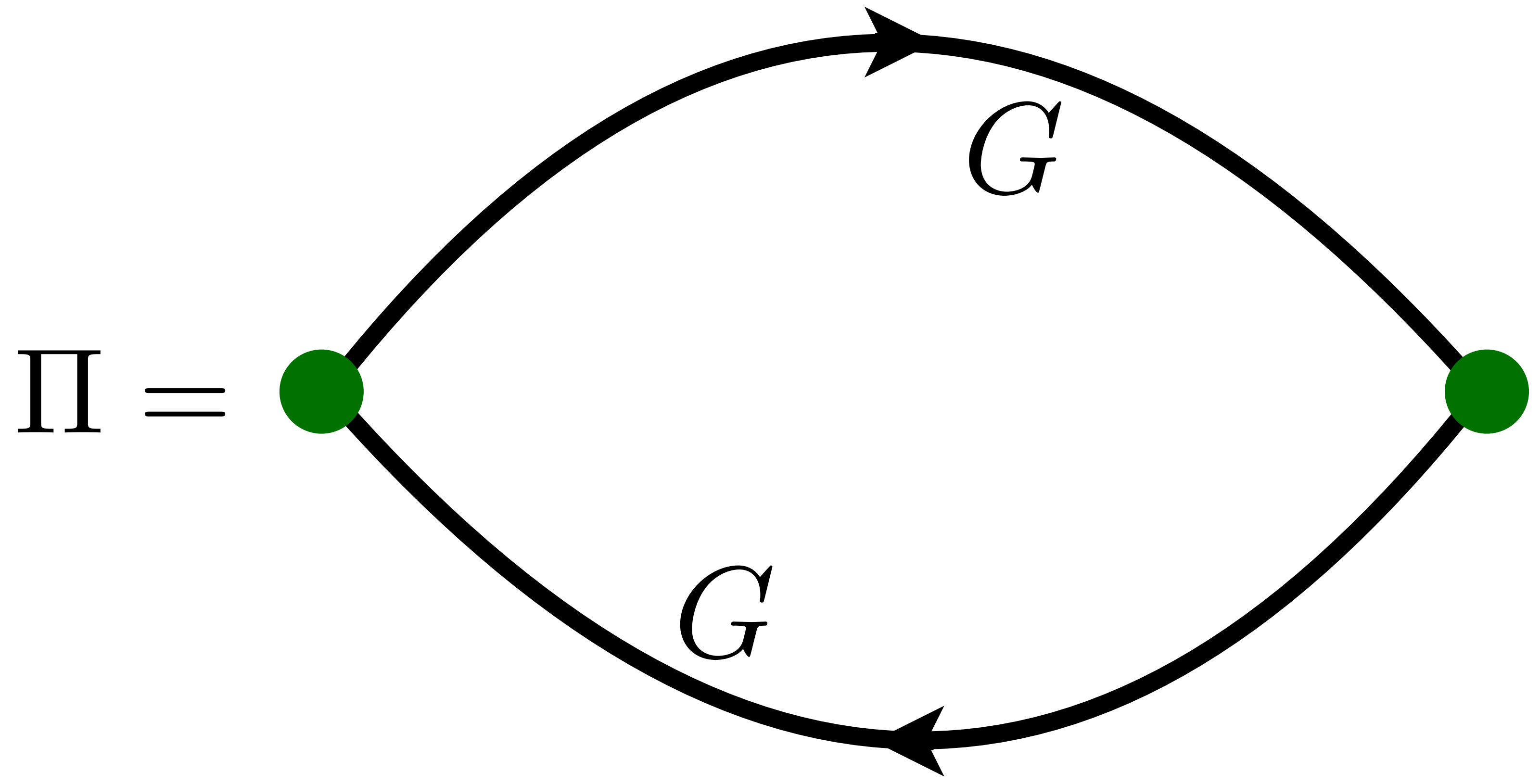
$$\Sigma(\boldsymbol{r},\tau) = g^2\lambda D(\boldsymbol{r},\tau)G(\boldsymbol{r},\tau),$$

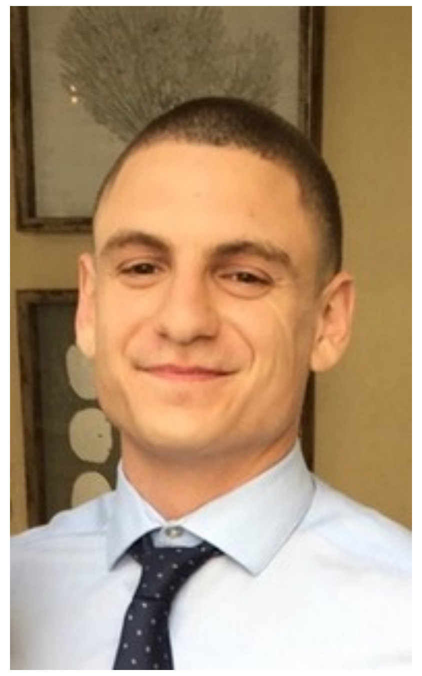
$$\Pi(\boldsymbol{r},\tau) = -g^2 G(-\boldsymbol{r},-\tau)G(\boldsymbol{r},\tau),$$

$$G(\boldsymbol{k},i\omega_n) = \frac{1}{i\omega_n - \varepsilon(\boldsymbol{k}) - \Sigma(\boldsymbol{k},i\omega_n)},$$

$$D(\boldsymbol{q},i\Omega_m) = \frac{1}{\Omega_m^2 + q^2 + s - \Pi(\boldsymbol{q},i\Omega_m)}.$$







Exact Solution at small ω:

$$\Sigma(\hat{\boldsymbol{k}},i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k},i\omega) = \frac{-1}{\varepsilon(\boldsymbol{k}) + \Sigma(\hat{\boldsymbol{k}},i\omega)}$$

where the co-efficient is known exactly in terms of the Fermi velocity and Fermi surface curvature at the Fermi surface point along the direction $\hat{\boldsymbol{k}}$.

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

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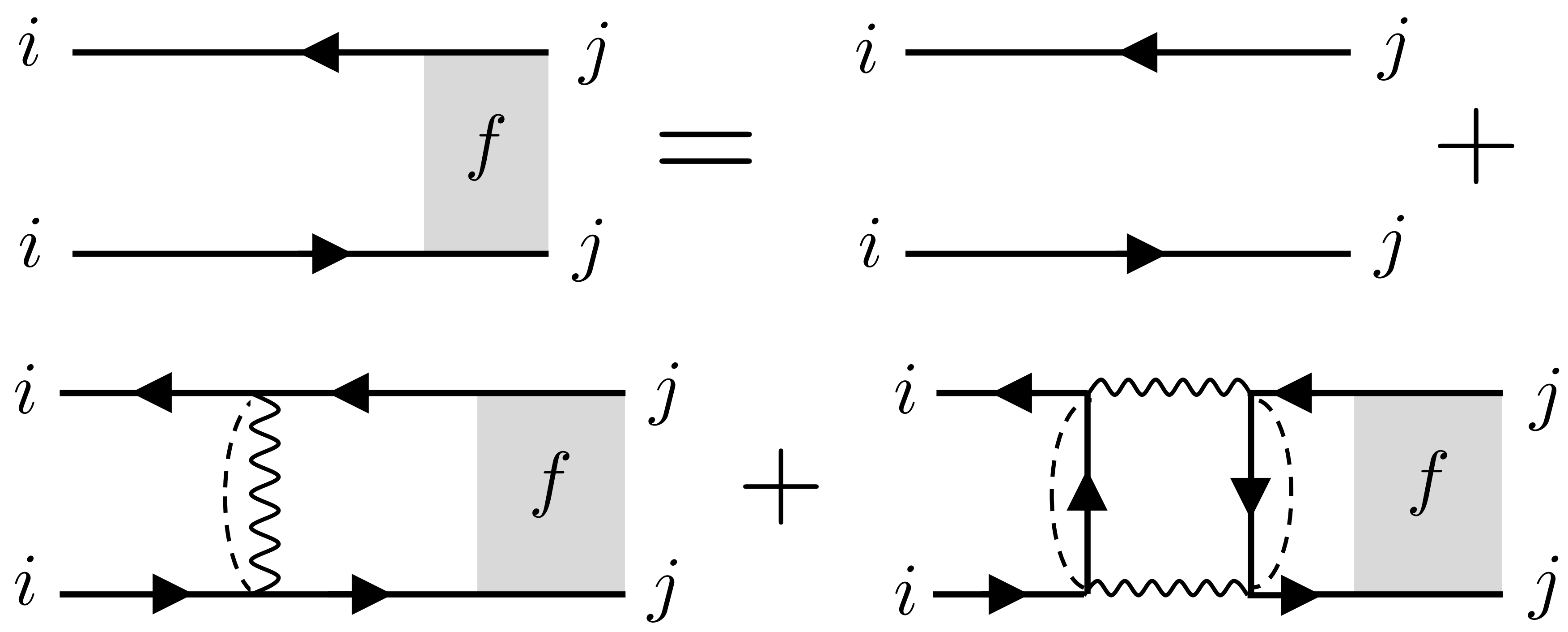

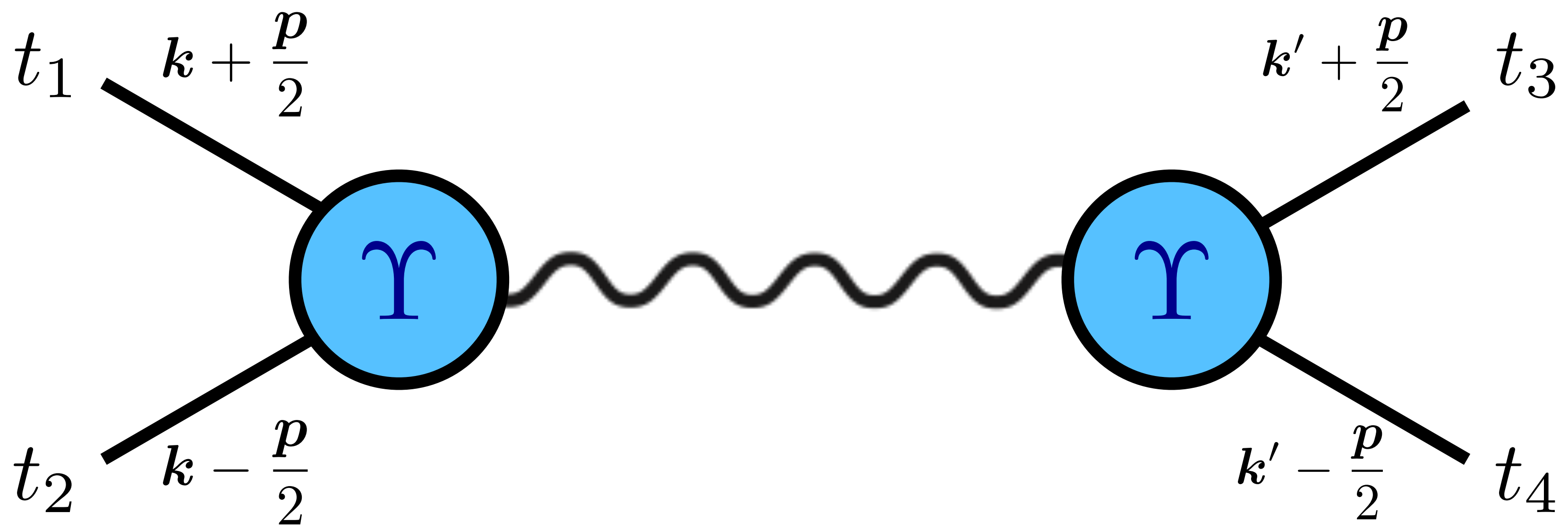

- There is many-body quantum chaos in the out-of-time-order correlator (OTOC) with maximal Lyapunov exponent $\lambda_L = 2\pi k_B T/\hbar$. This follows from the ladder identity of Gu and Kitaev, and a pole in the 'scramblon' propagator $[\cos(\lambda_L(\boldsymbol{k})/(4T)]^{-1}$.

M. Tikhanovskaya, Aavishkar Patel, S.S. arXiv:2202.01845

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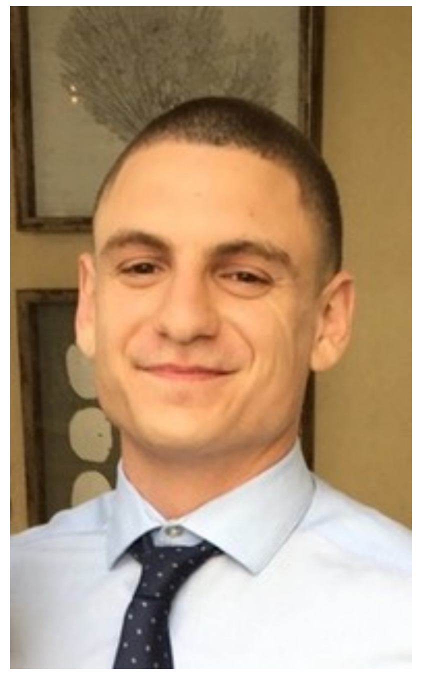
Fermi surface coupled to a critical boson with spatial disorder



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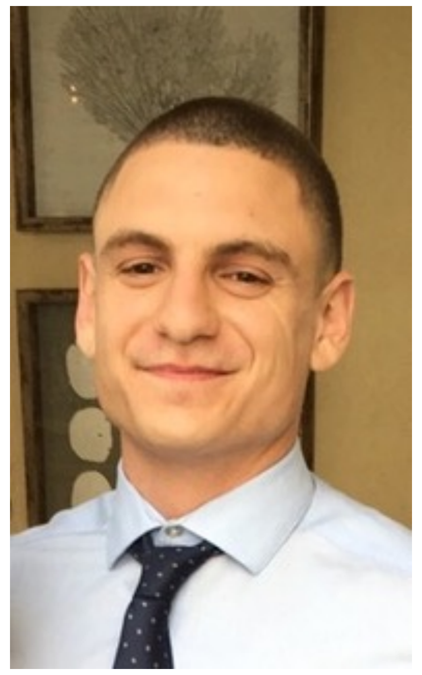
Solve saddle-point of G-Σ-D-Π theory with additional term $\propto v^2$

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, and S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder





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Marginal Fermi liquid self energy and $T \log T$ specific heat

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, and S.S. arXiv:2203.04990

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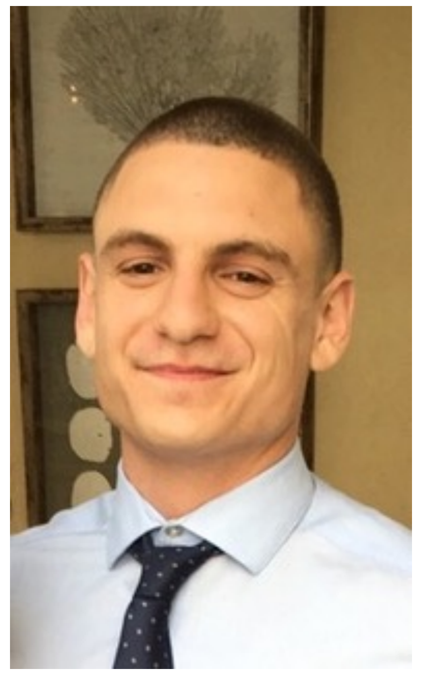
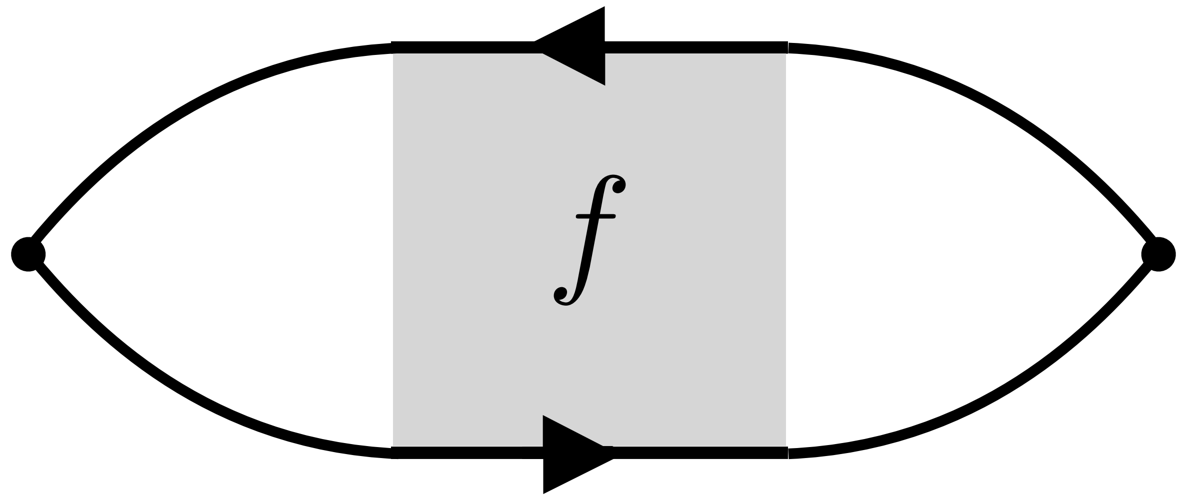
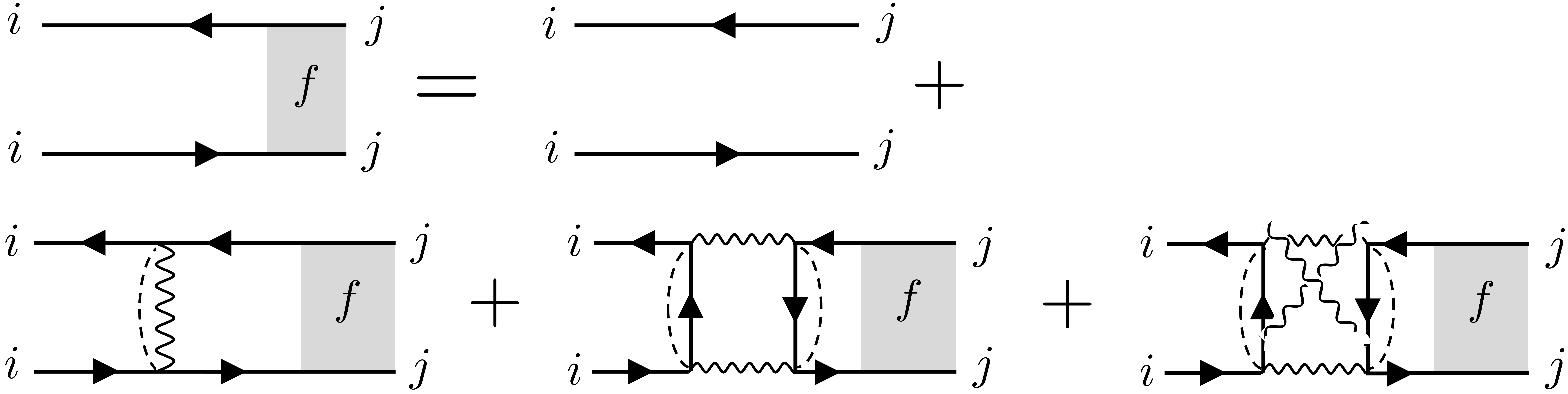


$\sigma =$ 



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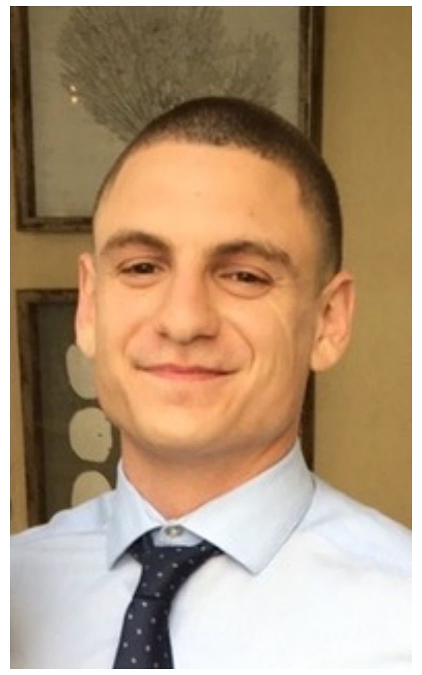

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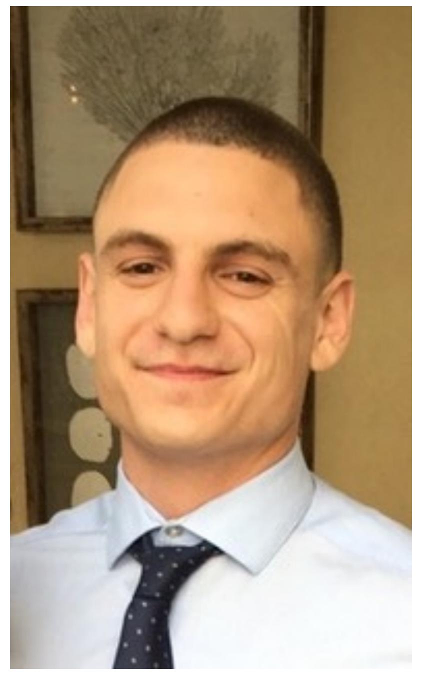
The g^2 log term does not contribute to transport

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, and S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder





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With g and v non-zero, we obtain a non-zero residual resistivity and Fermi liquid like corrections

$$\rho(T) = \rho(0) + AT^2 + \ldots.$$

with $1/\rho(0) \sim 1/\tau_{\text{trans}} \sim v^2$.

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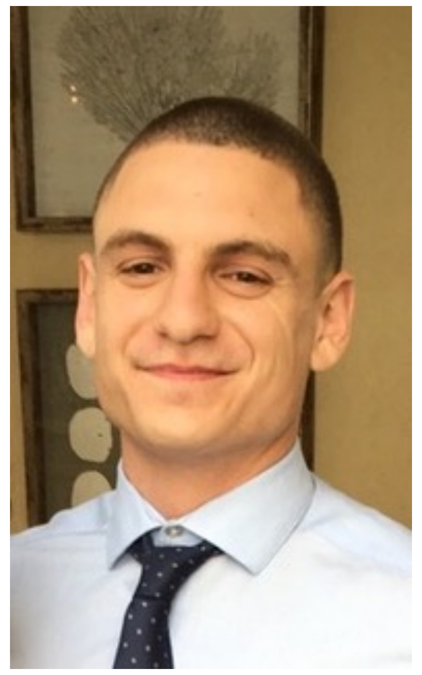

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$$\text{Random interactions:} \quad + \frac{1}{N} \int d^2r d\tau \, g'_{ijl}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

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Solve saddle-point of G-Σ-D-Π theory with additional terms $\propto v^2$ and ${g'}^2$

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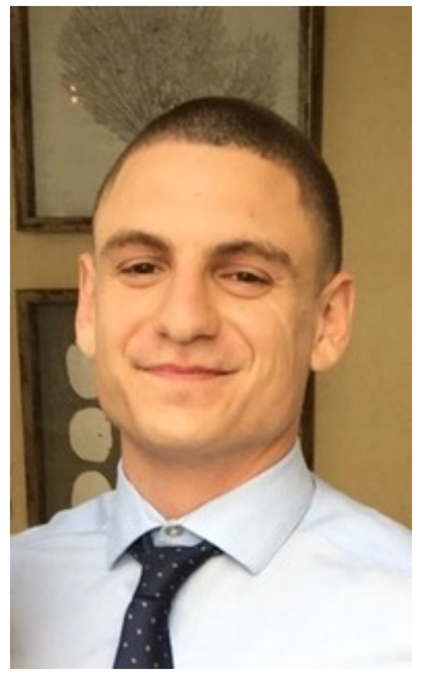

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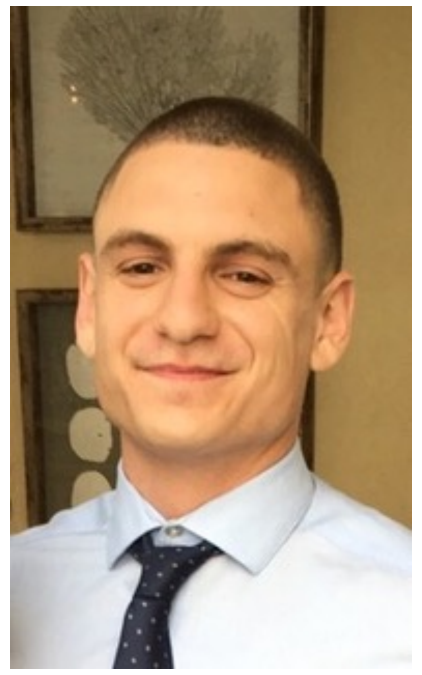

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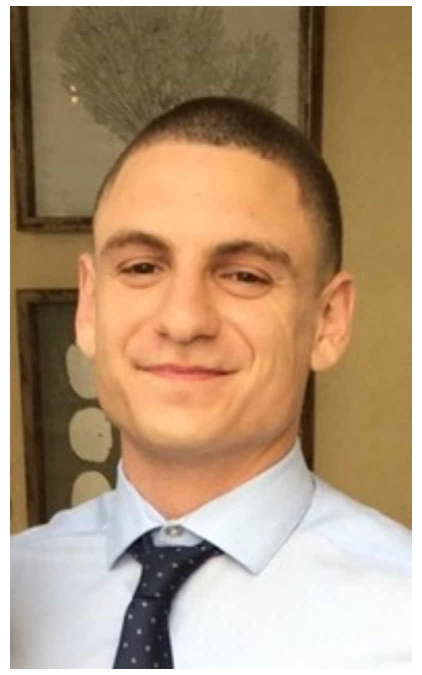

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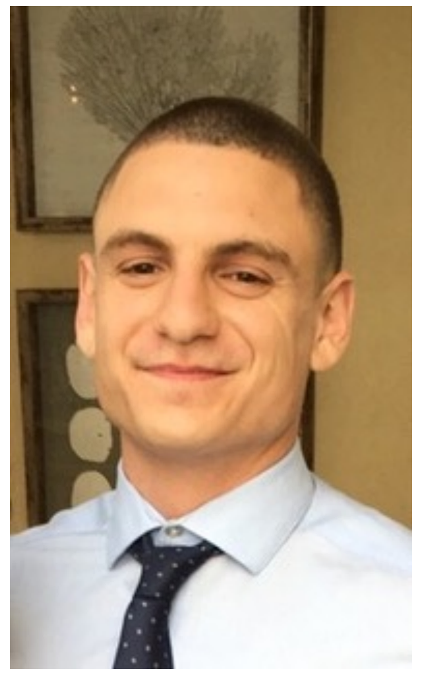
The g^2 log term does not contribute to transport but the g'^2 log term does!

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, and S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder





$$\text{“Yukawa” coupling:} \quad \frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\text{Random potential:} \quad + \frac{1}{\sqrt{N}} \int d^2 r d\tau \, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$$

$$\text{Random interactions:} \quad + \frac{1}{N} \int d^2 r d\tau \, g'_{ijl}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\Sigma_v(i\omega) \sim -iv^2 \text{sgn}(\omega), \quad \Sigma_g(i\omega) \sim -i\frac{g^2}{v^2}\omega \ln(1/|\omega|), \quad \Sigma_{g'}(i\omega) \sim -ig'^2 \omega \ln(1/|\omega|)$$

$$\text{Conductivity:} \ \sigma(\omega) \sim \tau_{\text{trans}}(\omega)$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega|$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2.

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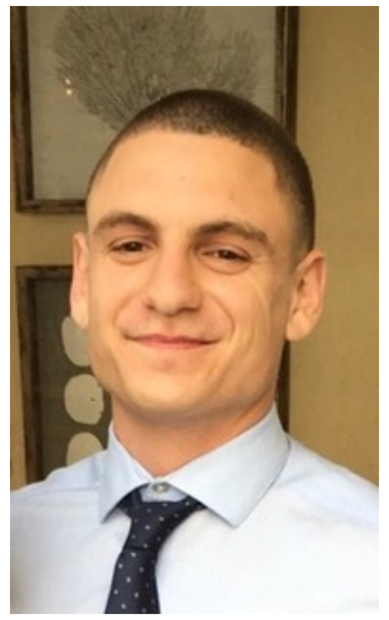
Summary

- Two-dimensional Fermi surface coupled to a critical boson has no quasiparticle excitations, and exhibits Planckian time dynamics and maximal chaos with Lyapunov exponent $2\pi k_B T/\hbar$.

- Linear-T resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.





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