The quantum phase transitions of metals in two dimensions

Talk online: sachdev.physics.harvard.edu

PHYSICS

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Outline

1. Theory of Ising-nematic ordering in a metal

Field theories and line singularities in momentum space

2. Fermion pairing in metals with fluctuating spin density waves

Non-Fermi liquids and superconductivity in a doped CFT

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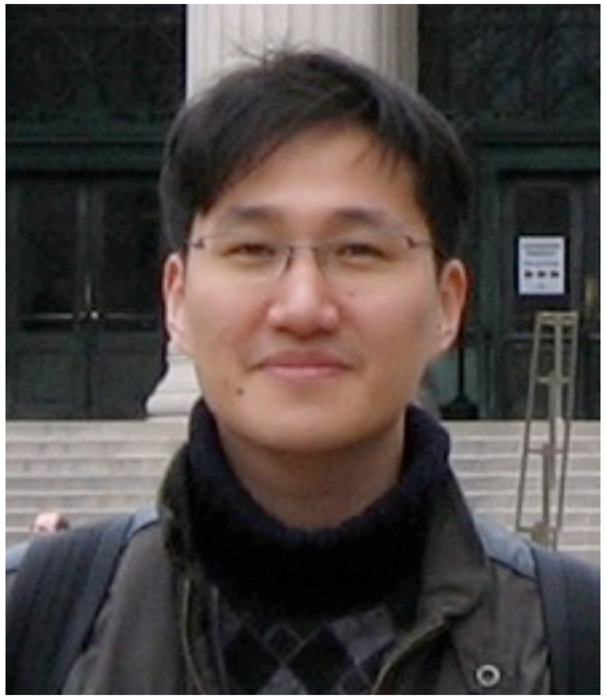
Non-Fermi liquids and superconductivity in a doped CFT



Max Metlitski, Harvard

arXiv:1001.1153

Eun Gook Moon, Harvard

Phys. Rev. B **80**, 035117 (2009)

PHYSICS

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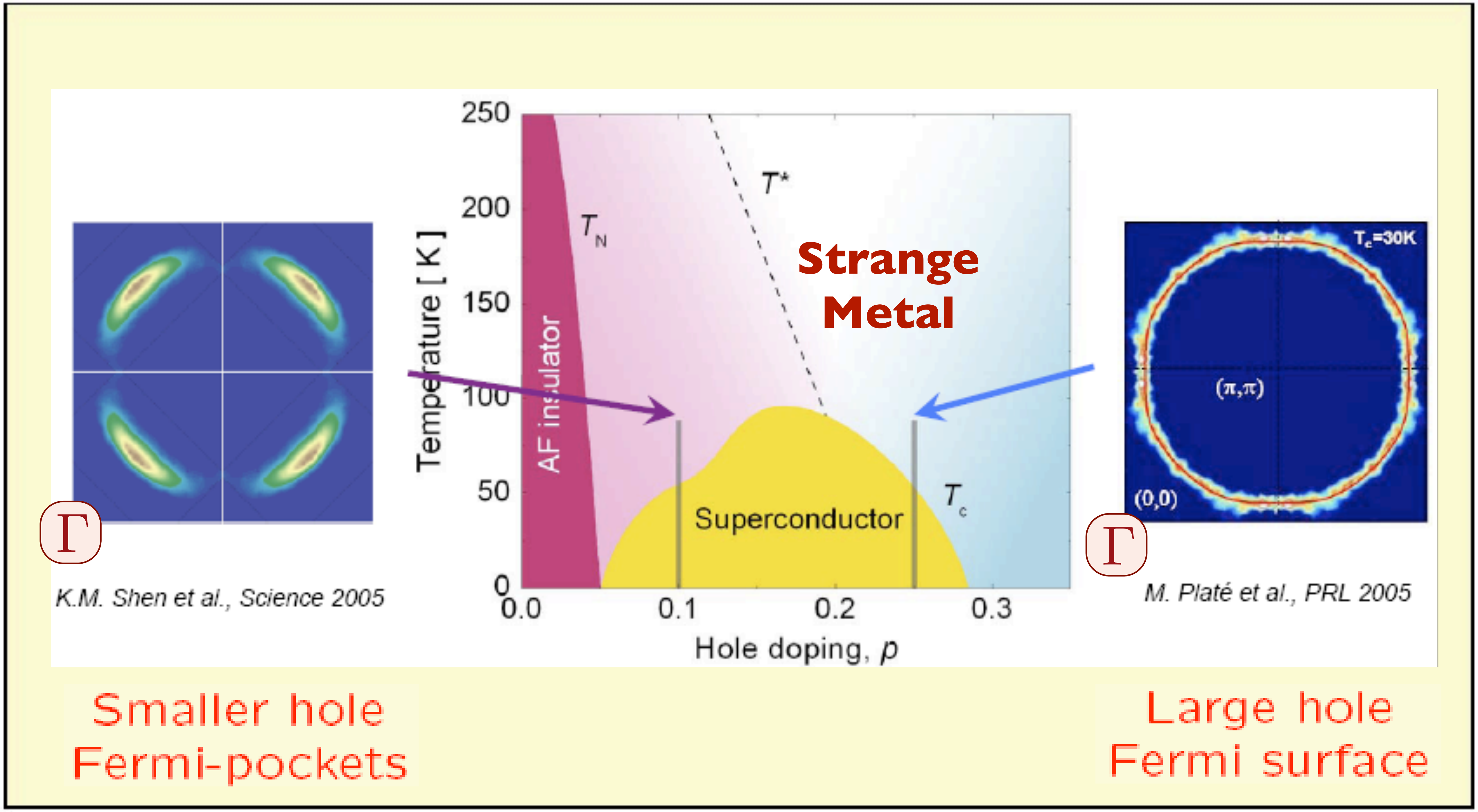
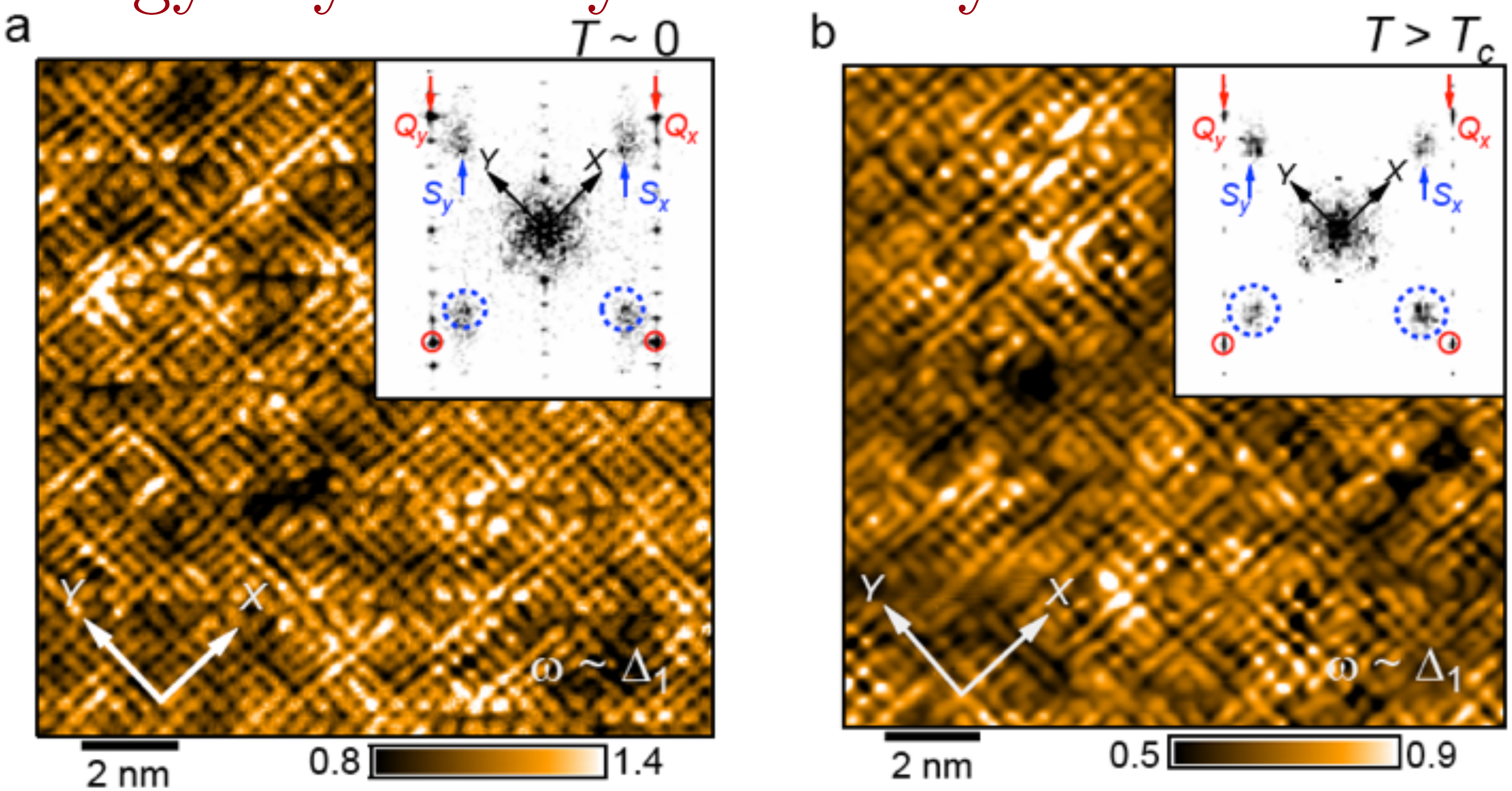
Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



STM measurements of $Z(r)$, energy asymmetry in density of states



M. J. Lawler, K. Fujita, Jhinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, preprint

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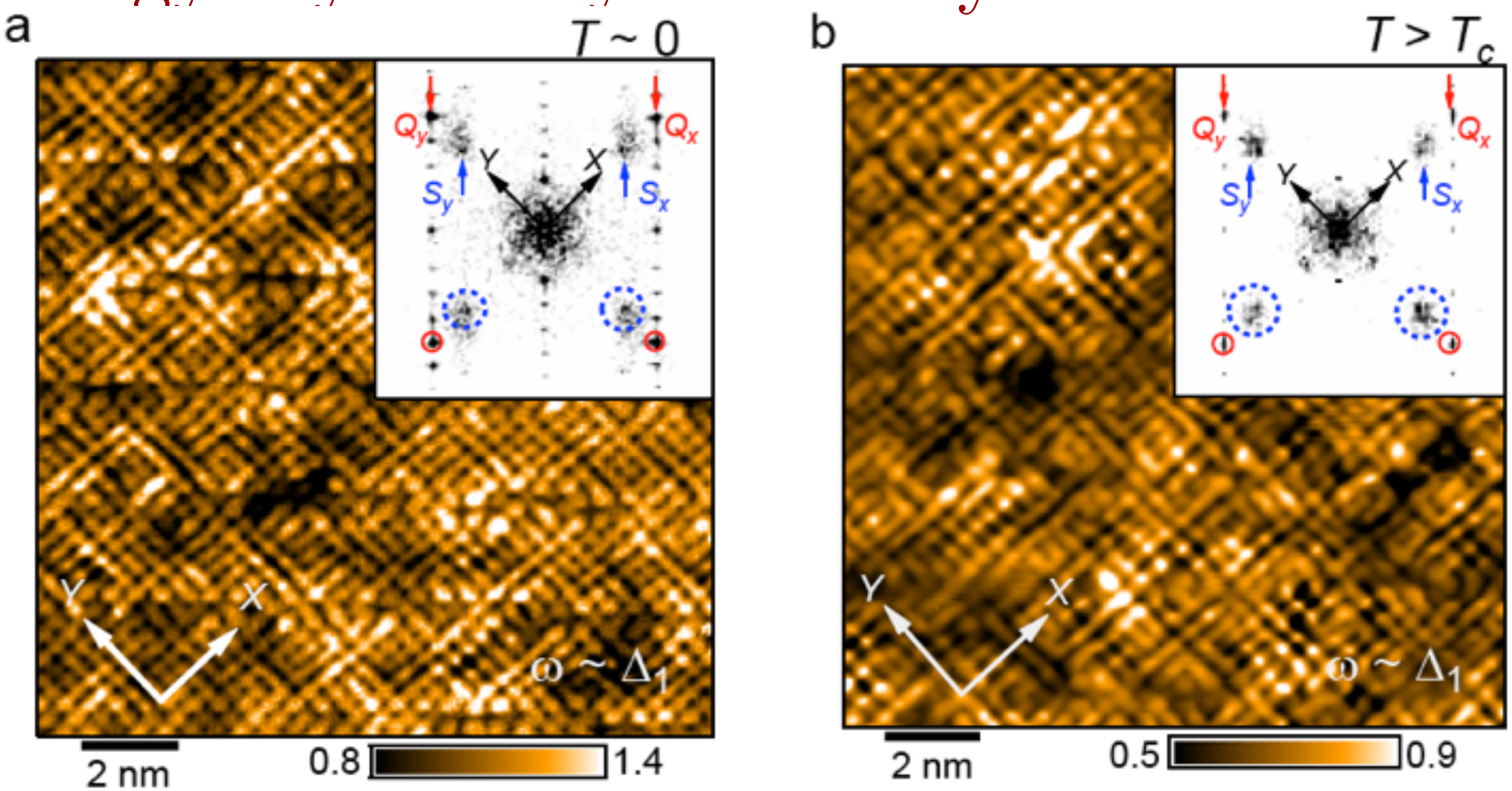

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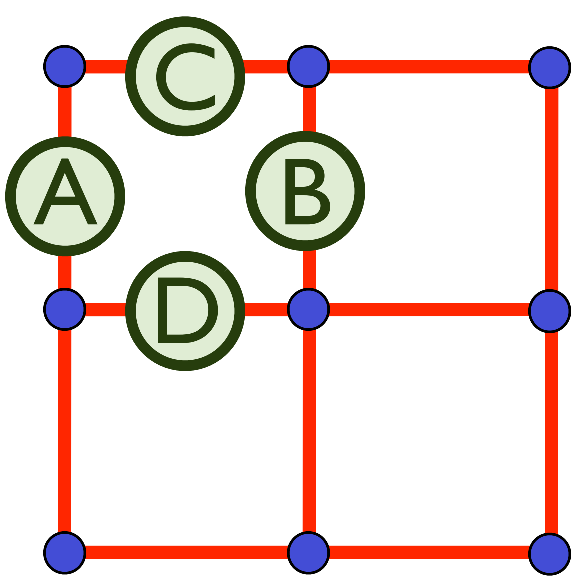
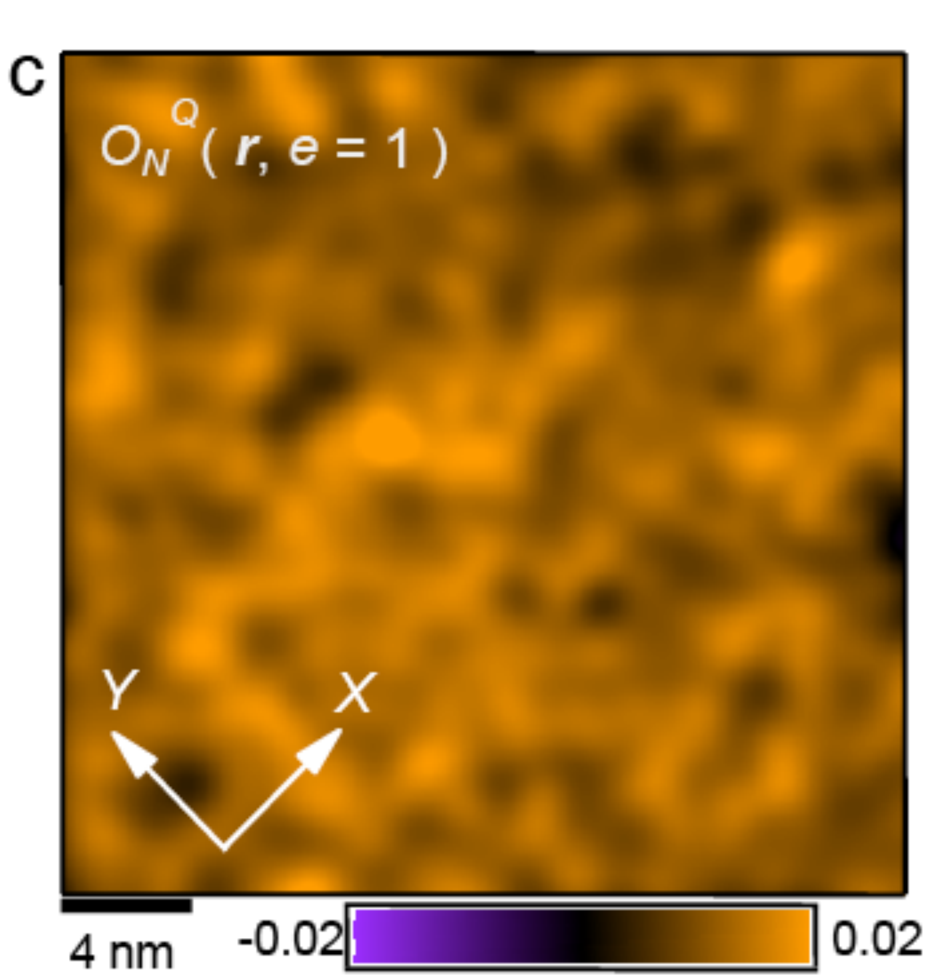
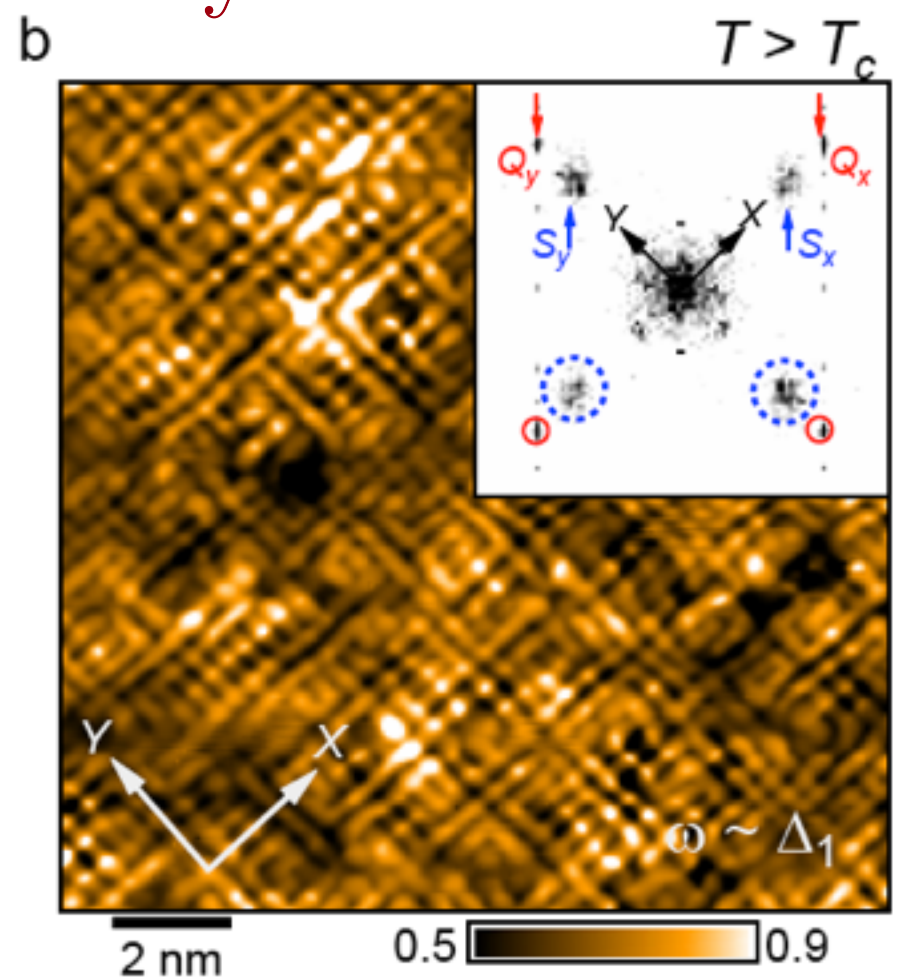


$$O_N = Z_A + Z_B - Z_C - Z_D$$

STM measurements of $Z(r)$, energy asymmetry in density of states





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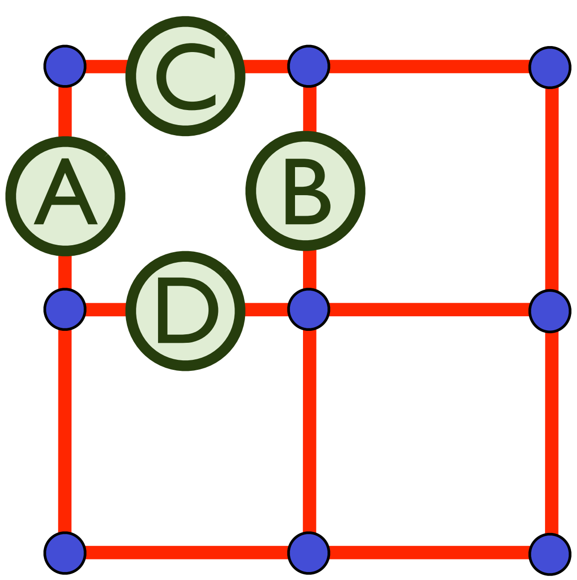
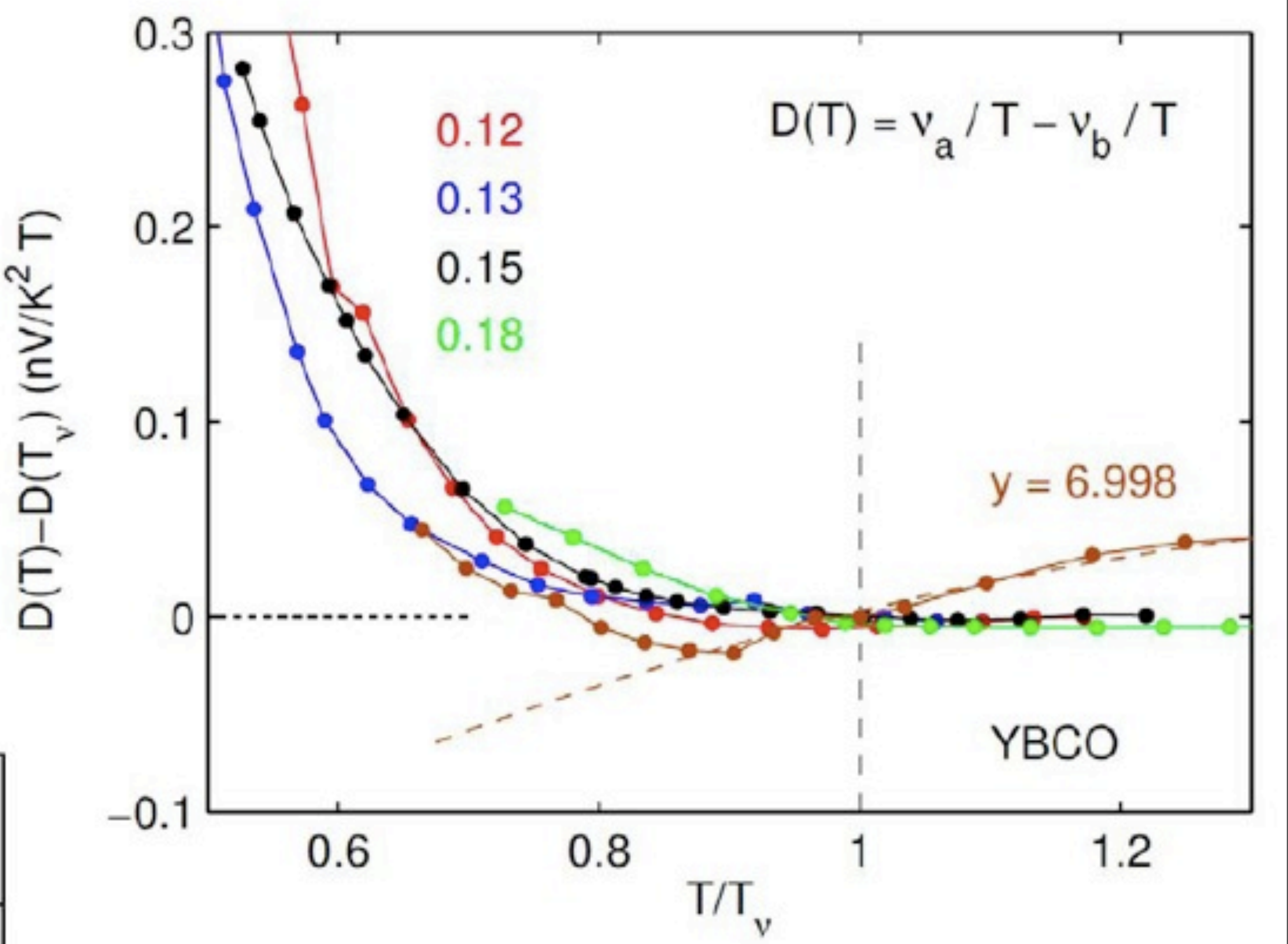


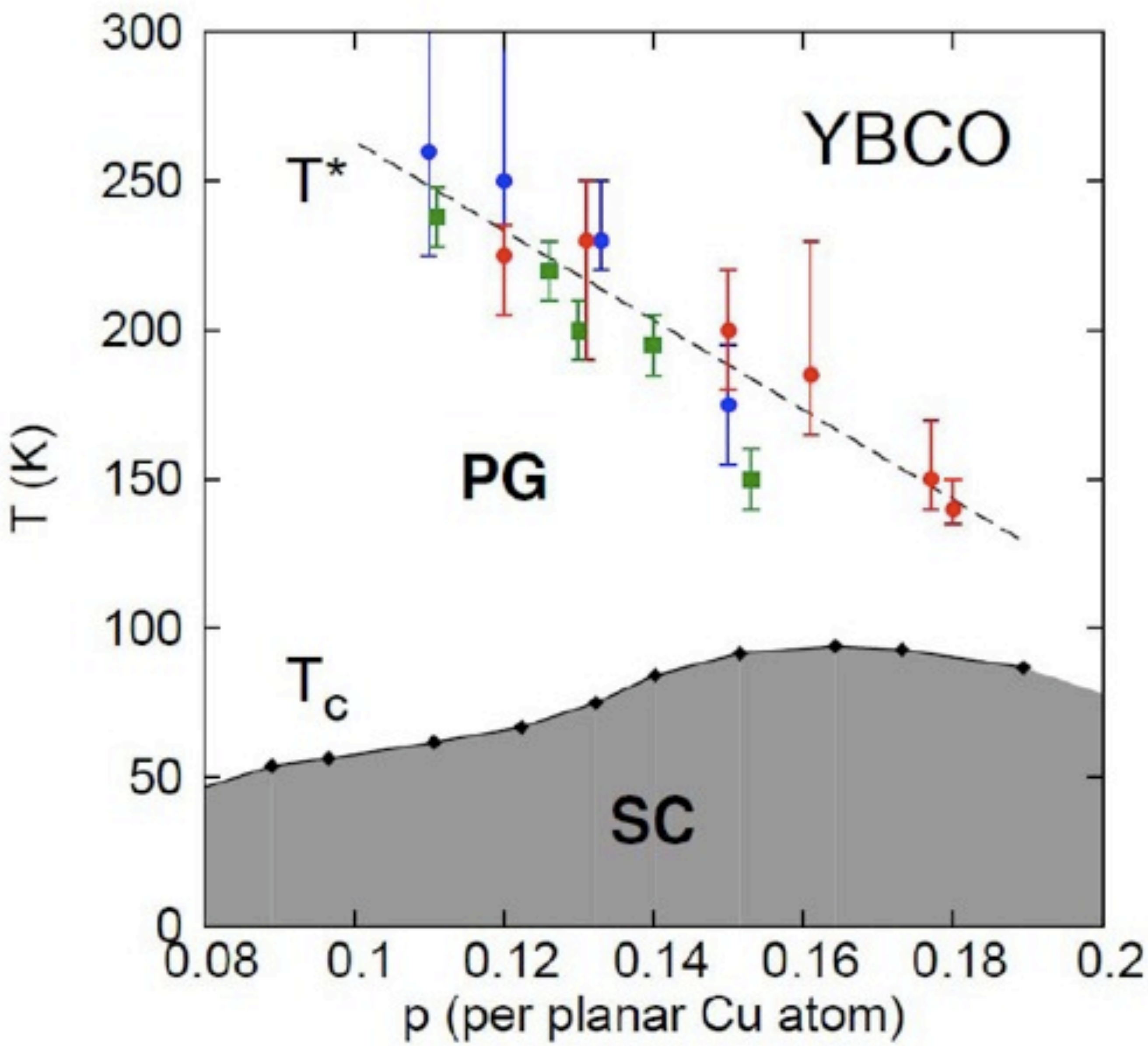
$$O_N = Z_A + Z_B - Z_C - Z_D$$

Strong anisotropy of electronic states between *x* and *y* directions: Electronic "Ising-nematic" order

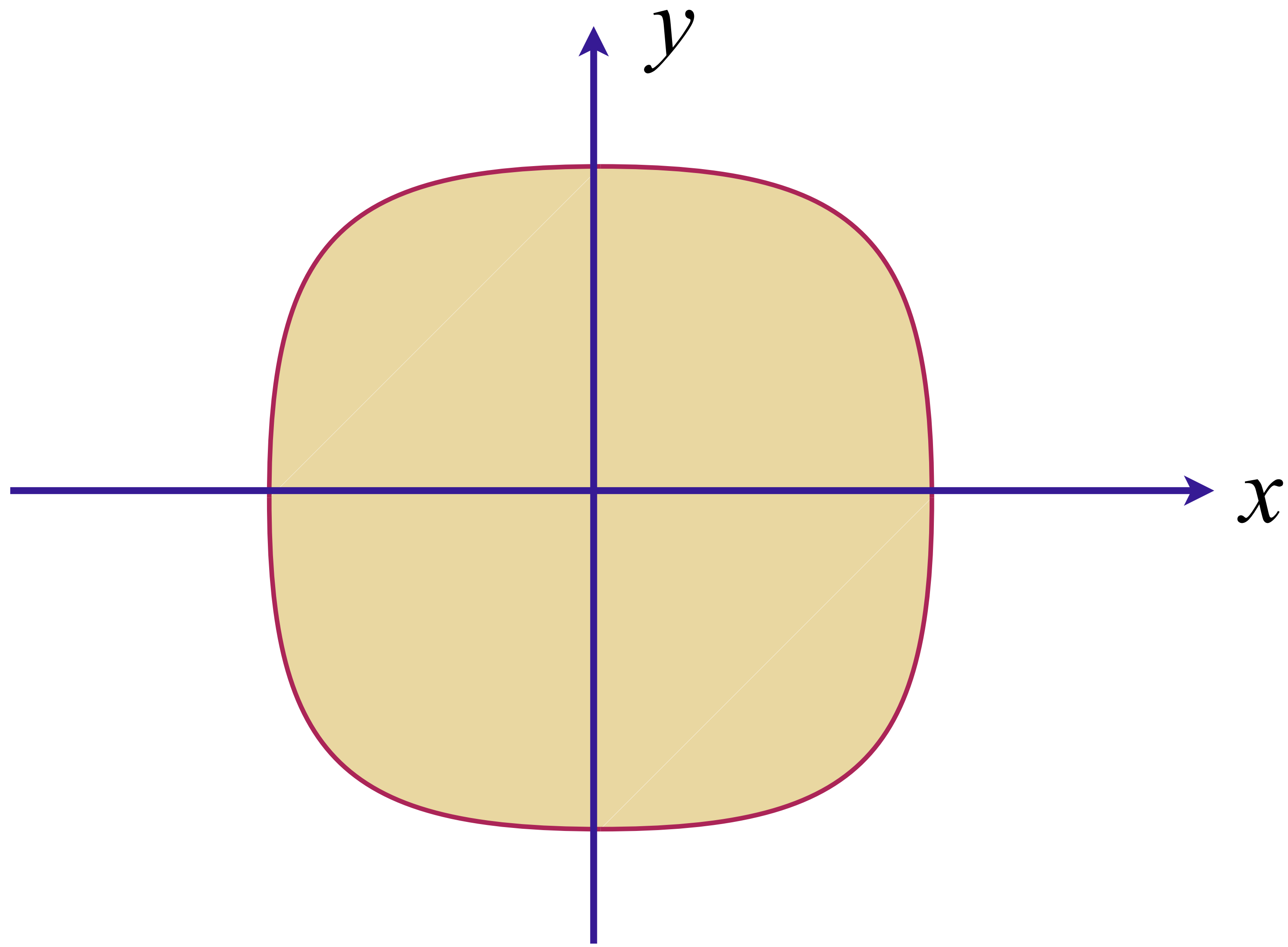
Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
Nature, **463**, 519 (2010).



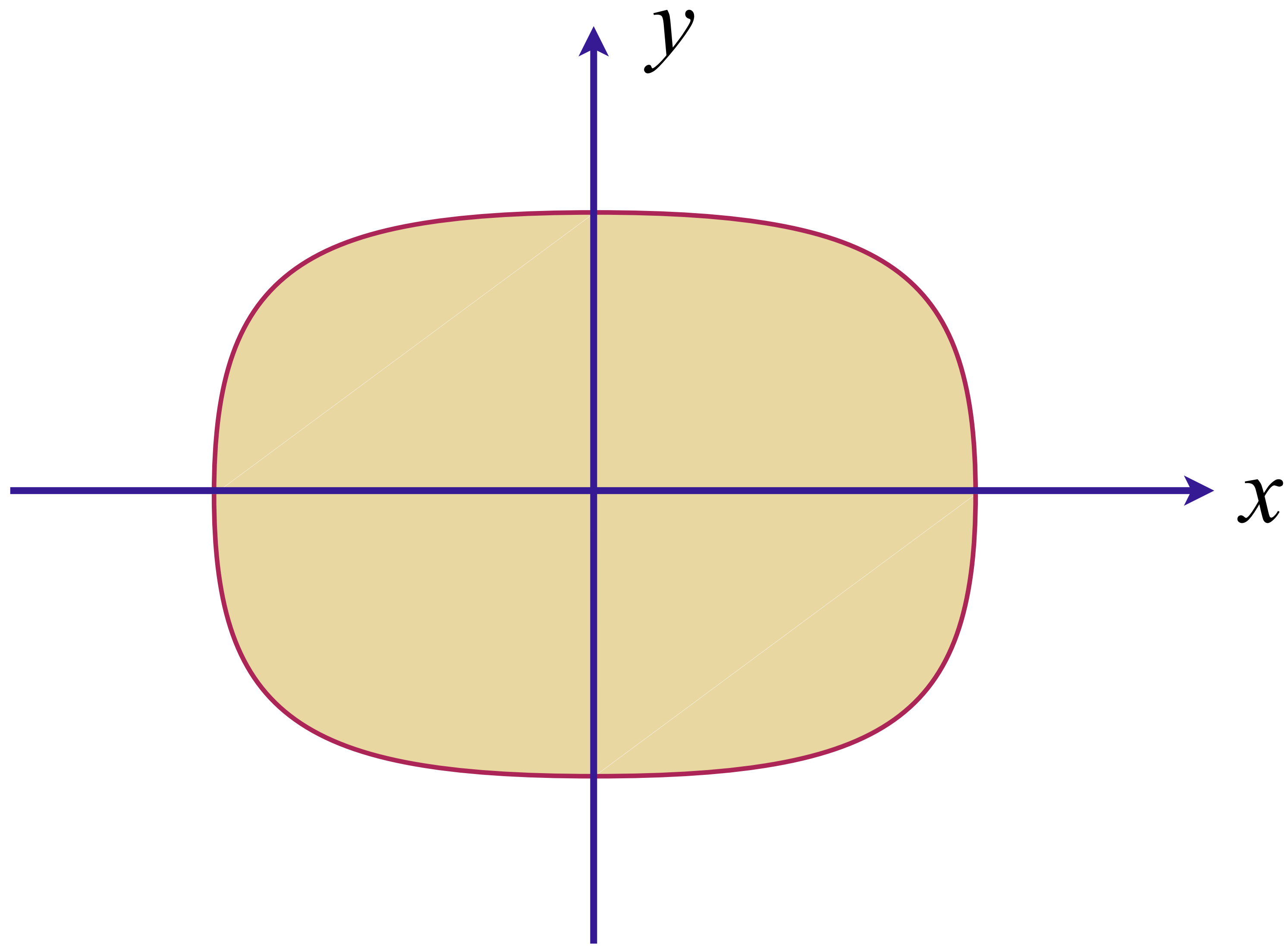


Quantum criticality of Ising-nematic ordering



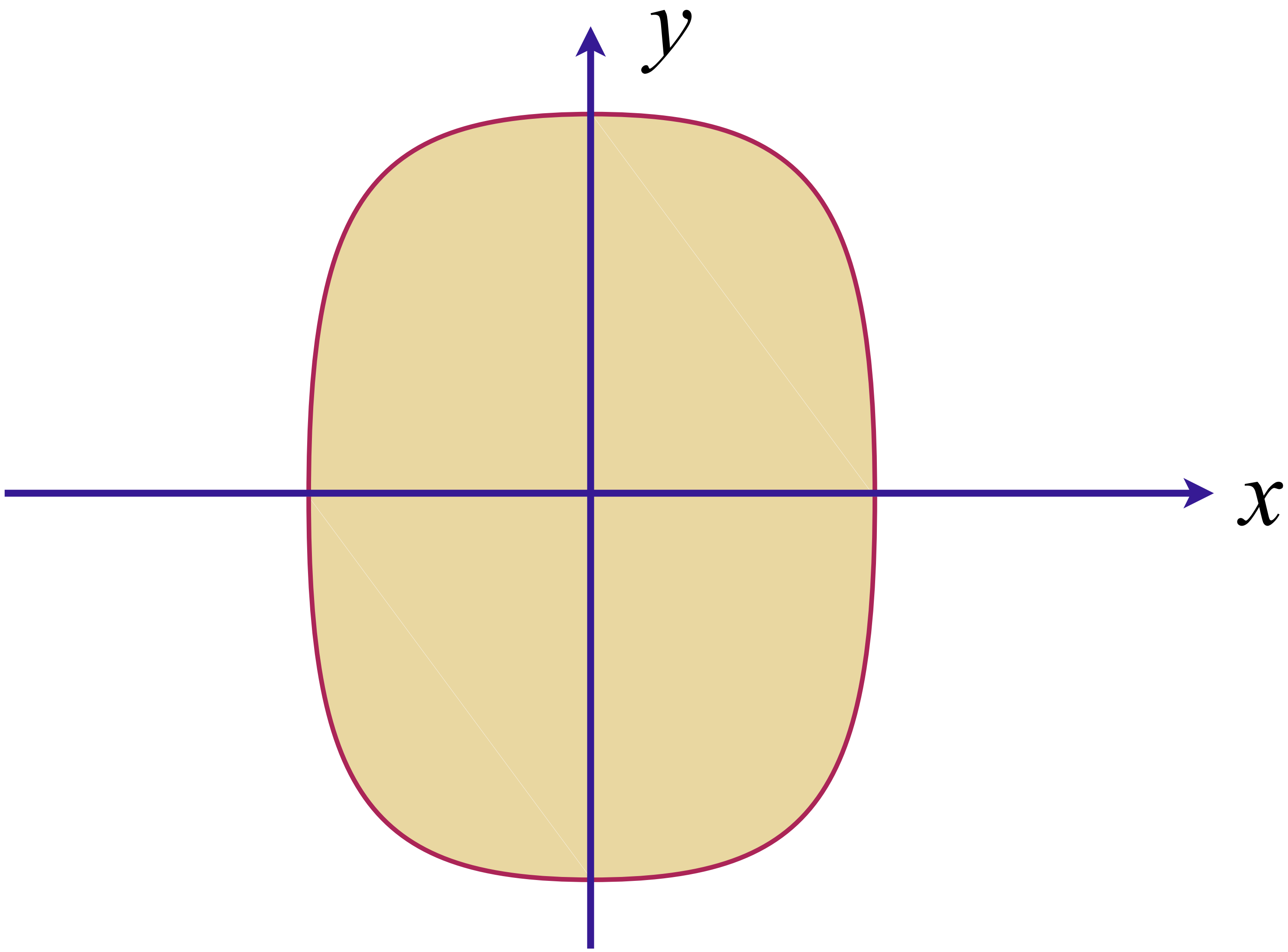
Fermi surface with full square lattice symmetry

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:

Quantum criticality of Ising-nematic ordering



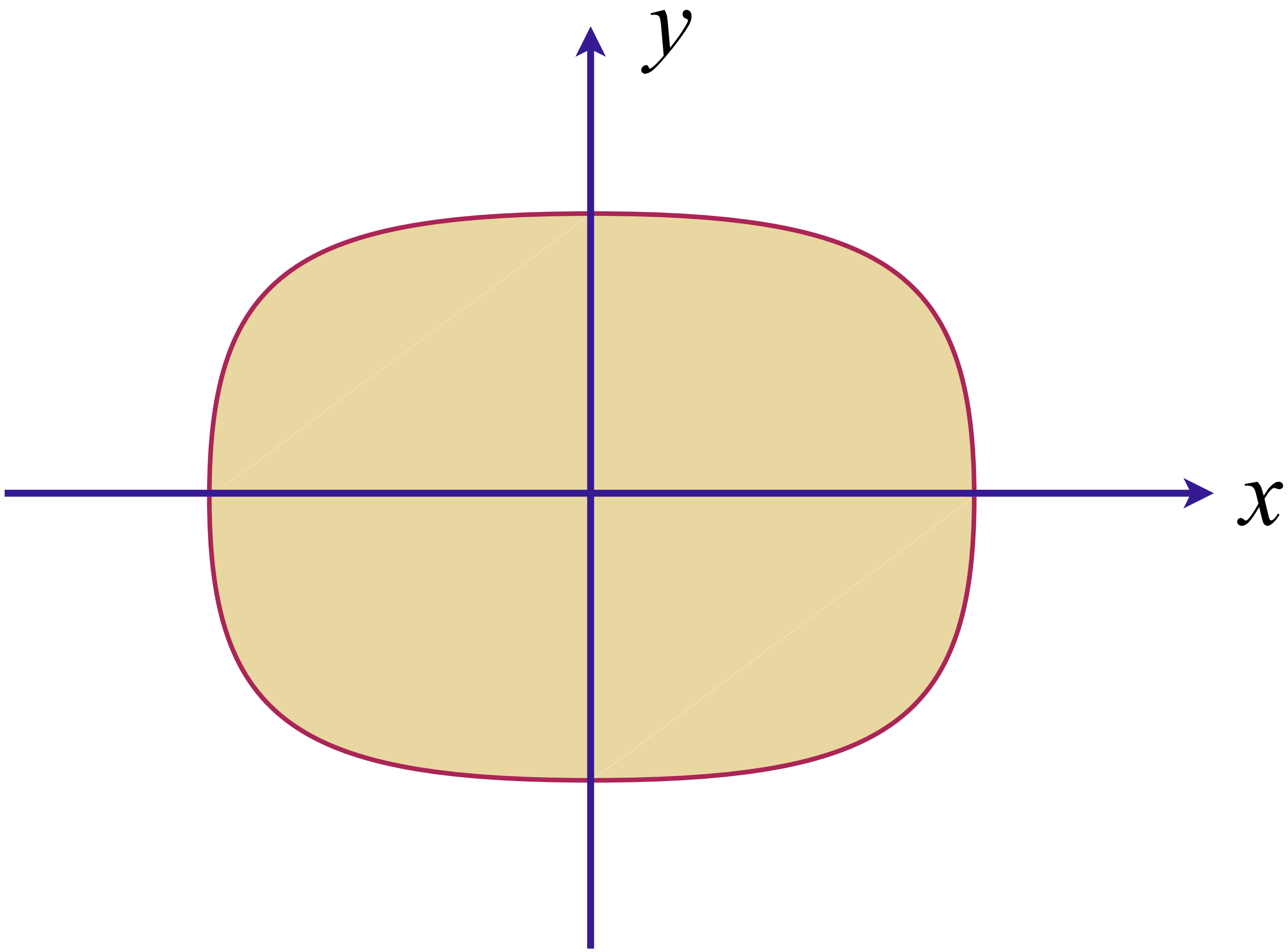
Spontaneous elongation along y direction:

Ising-nematic order parameter

$$\phi \sim \int d^2 k \left(\cos k_x - \cos k_y\right) c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}$$

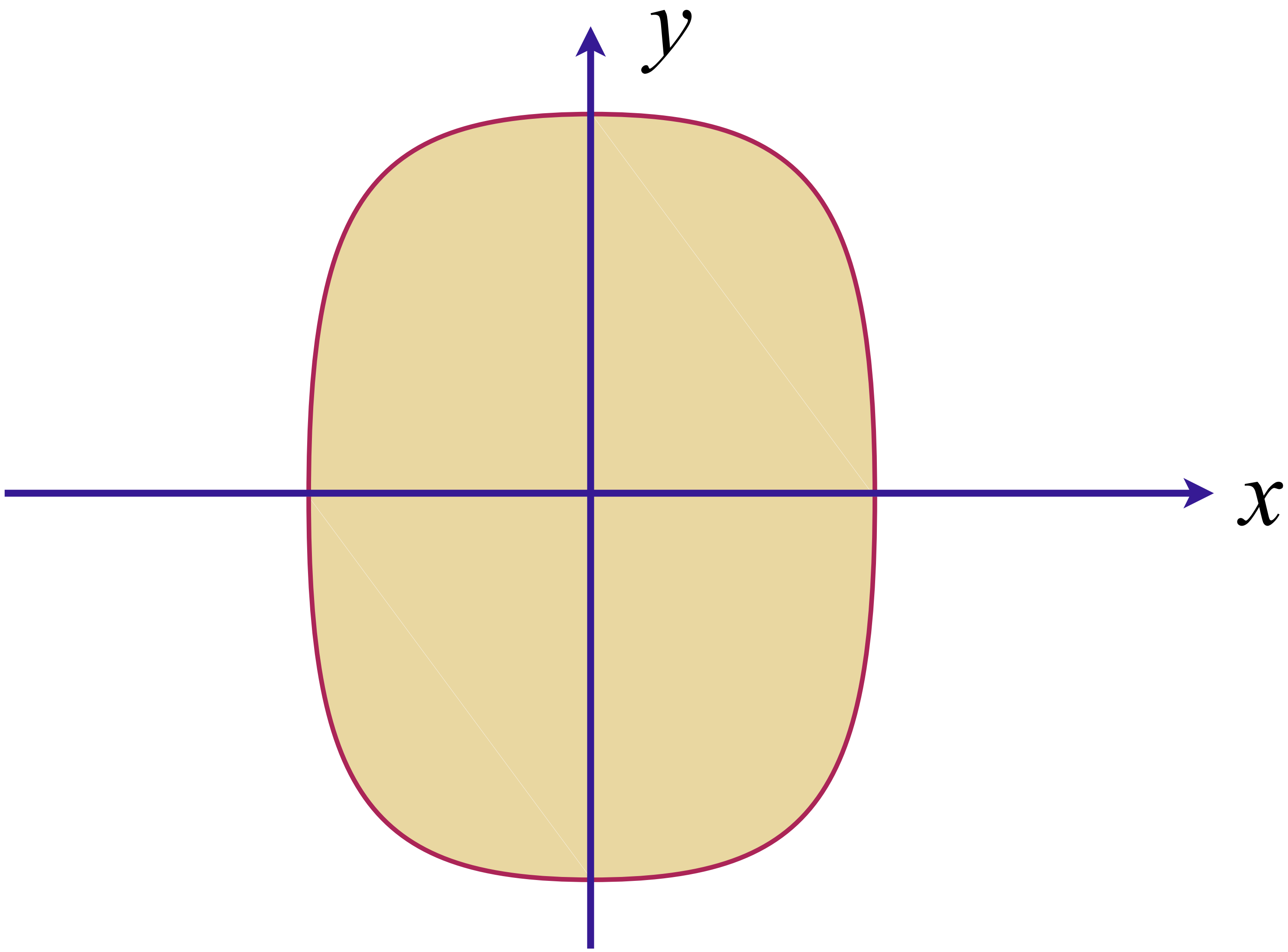
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Ising-nematic ordering



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Ising order parameter $\phi < 0$.

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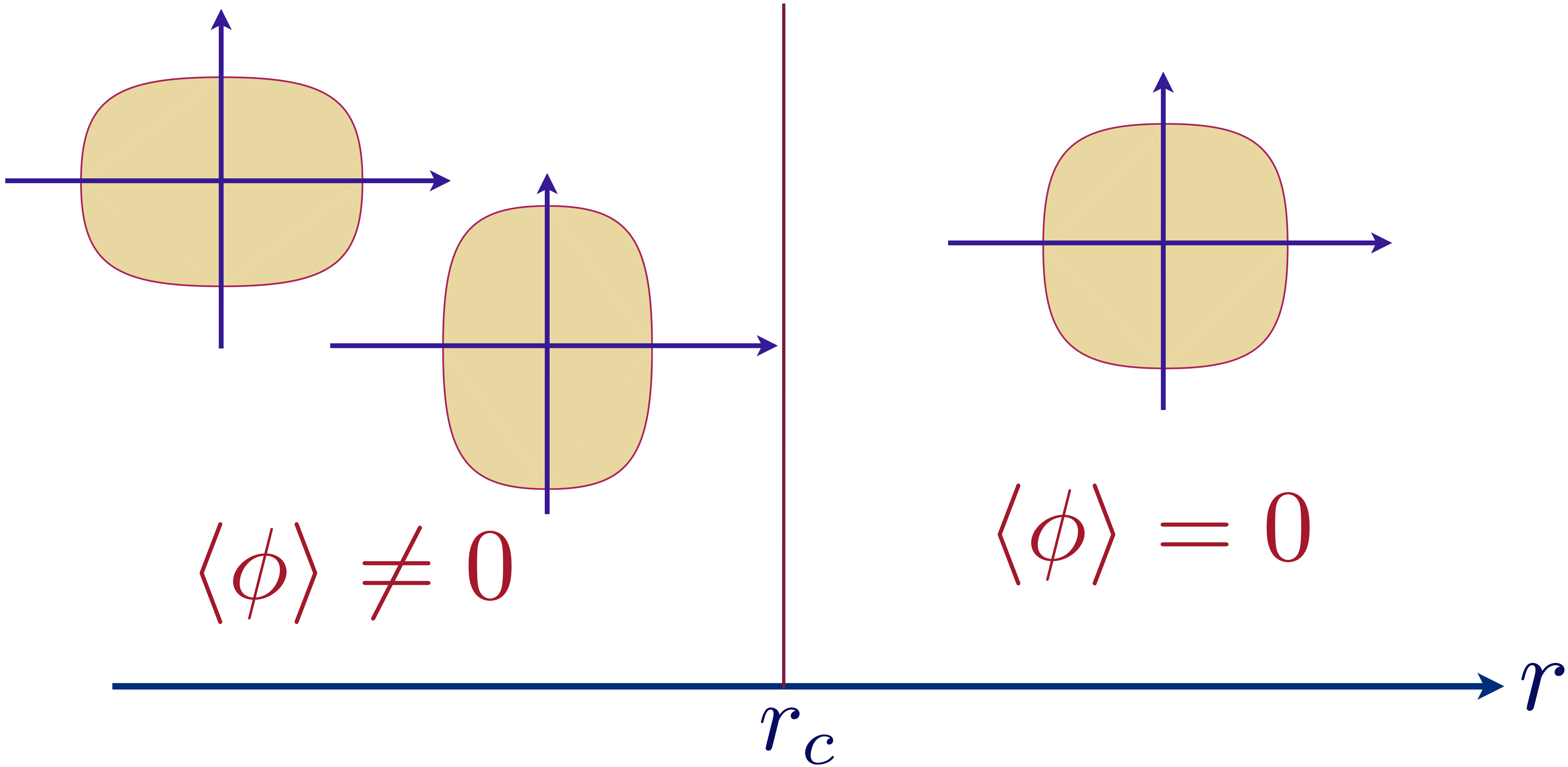
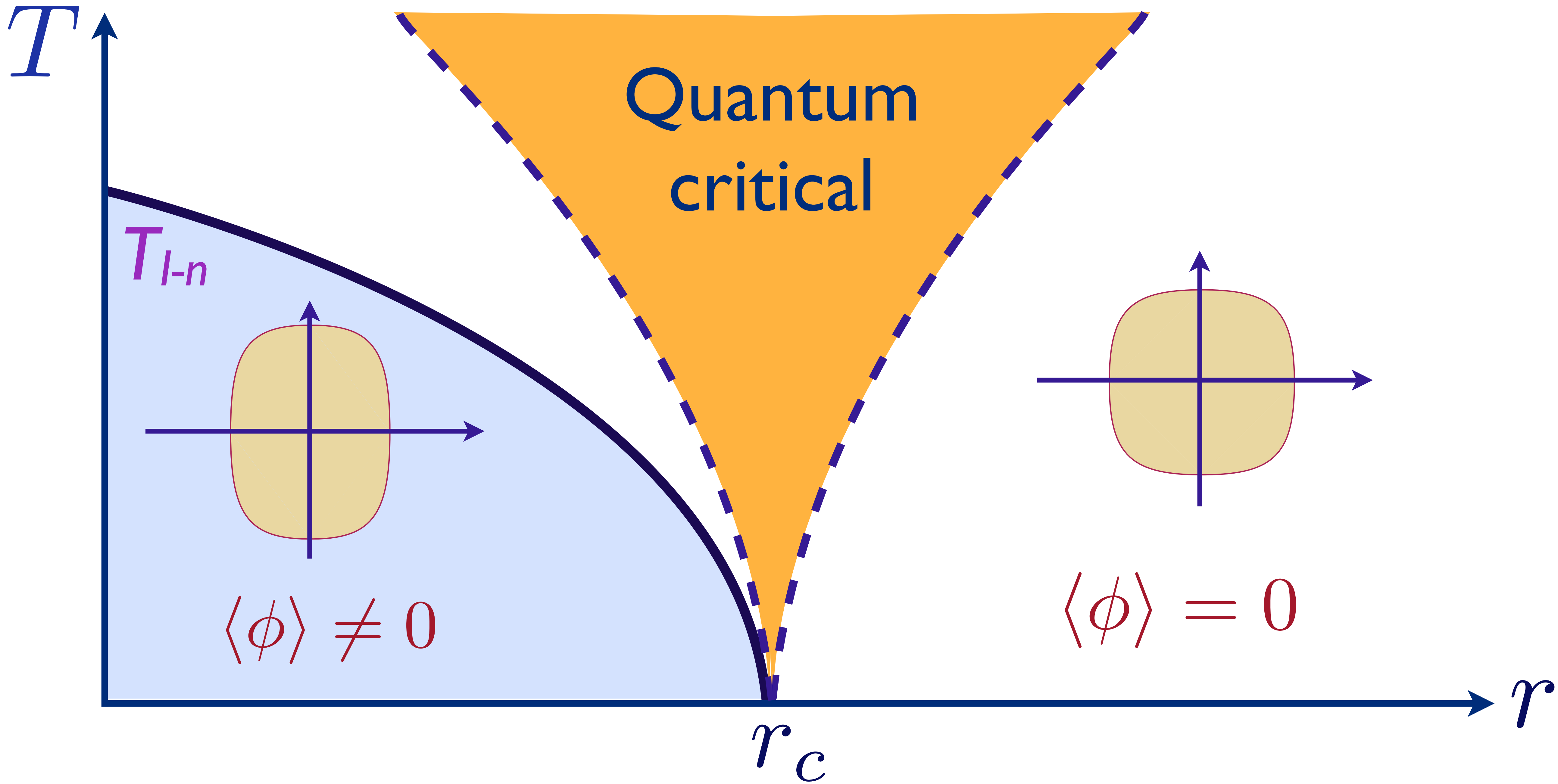


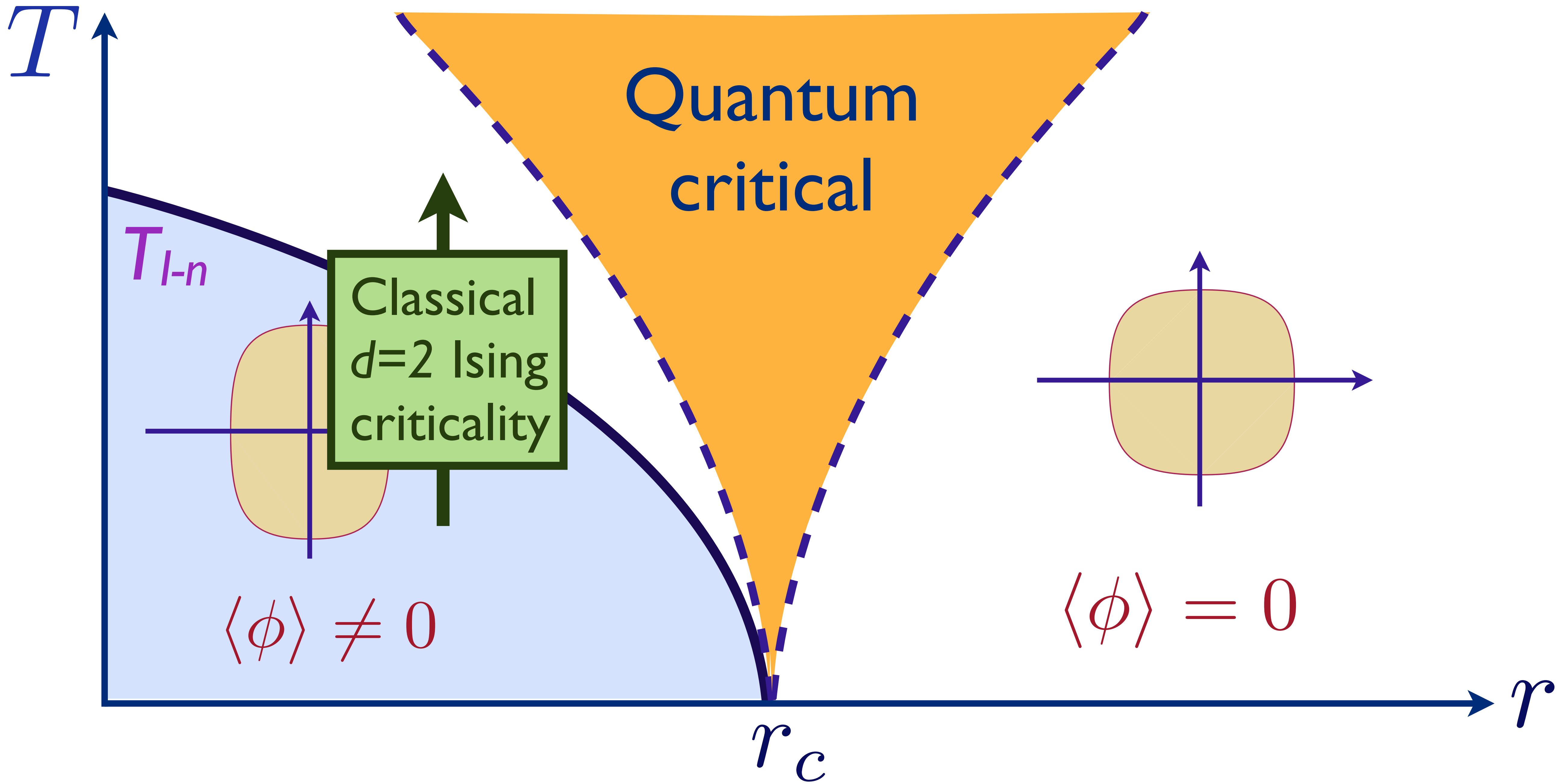
Phase diagram as a function of coupling r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



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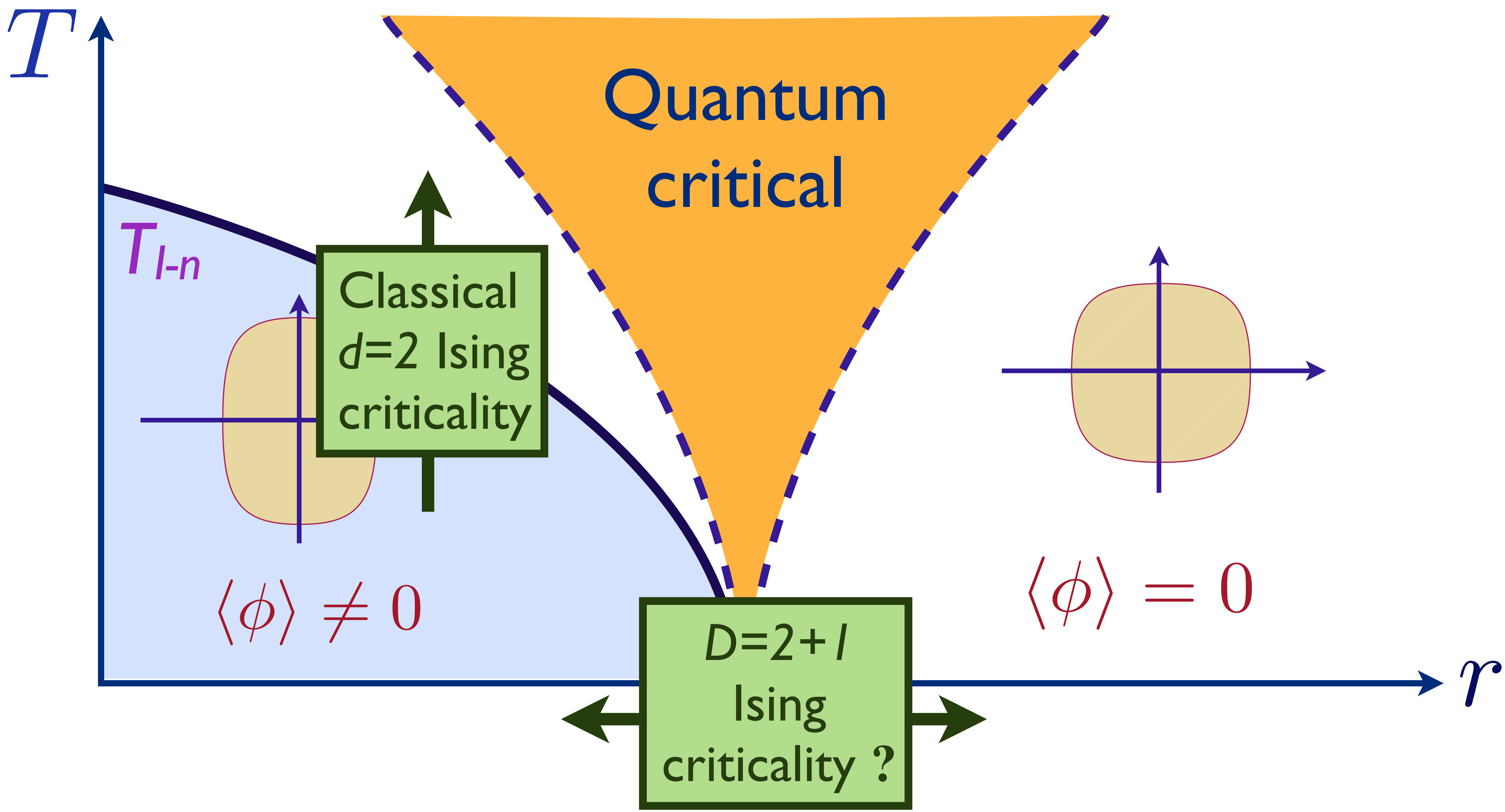

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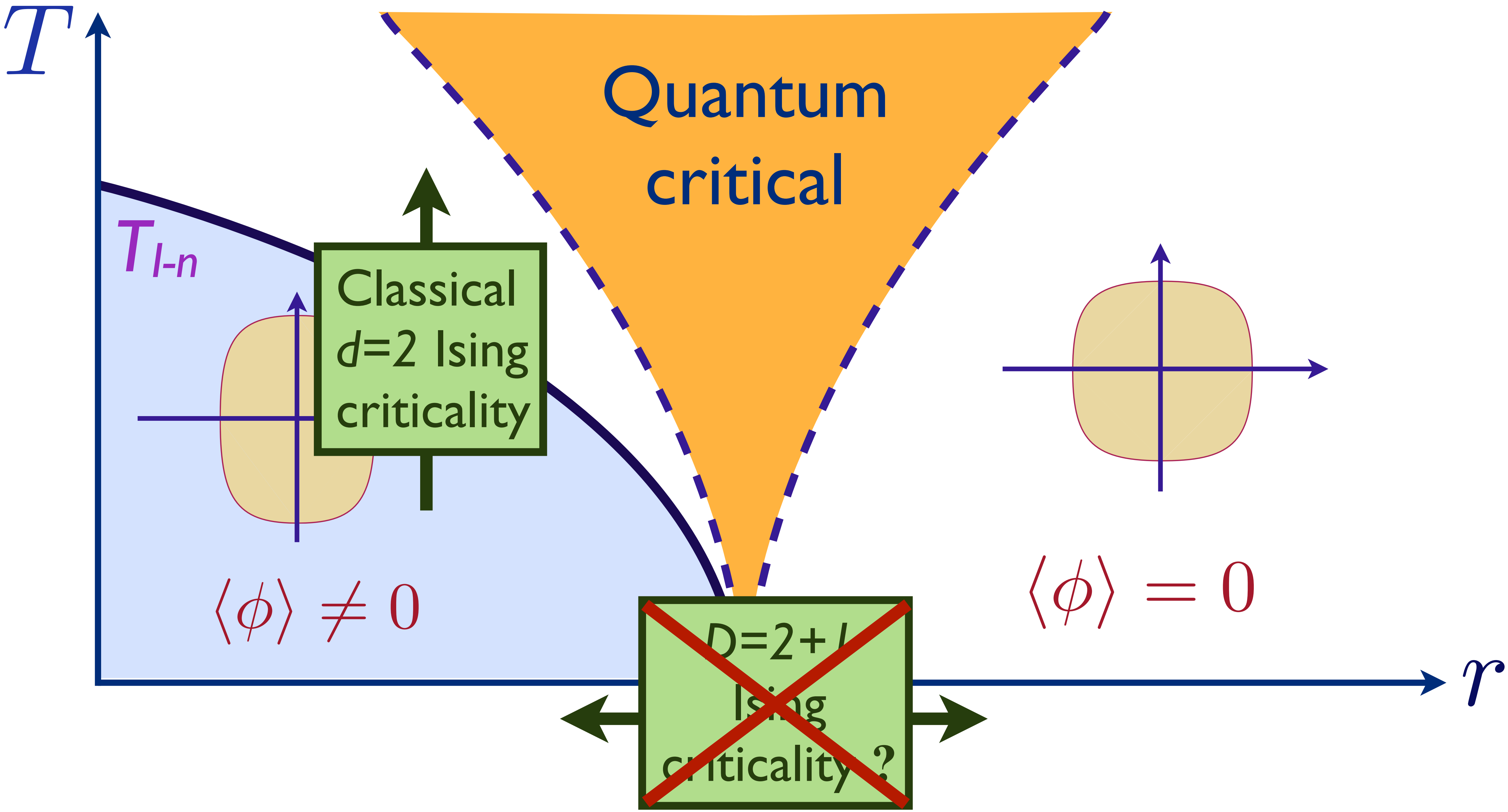

Phase diagram as a function of T and r

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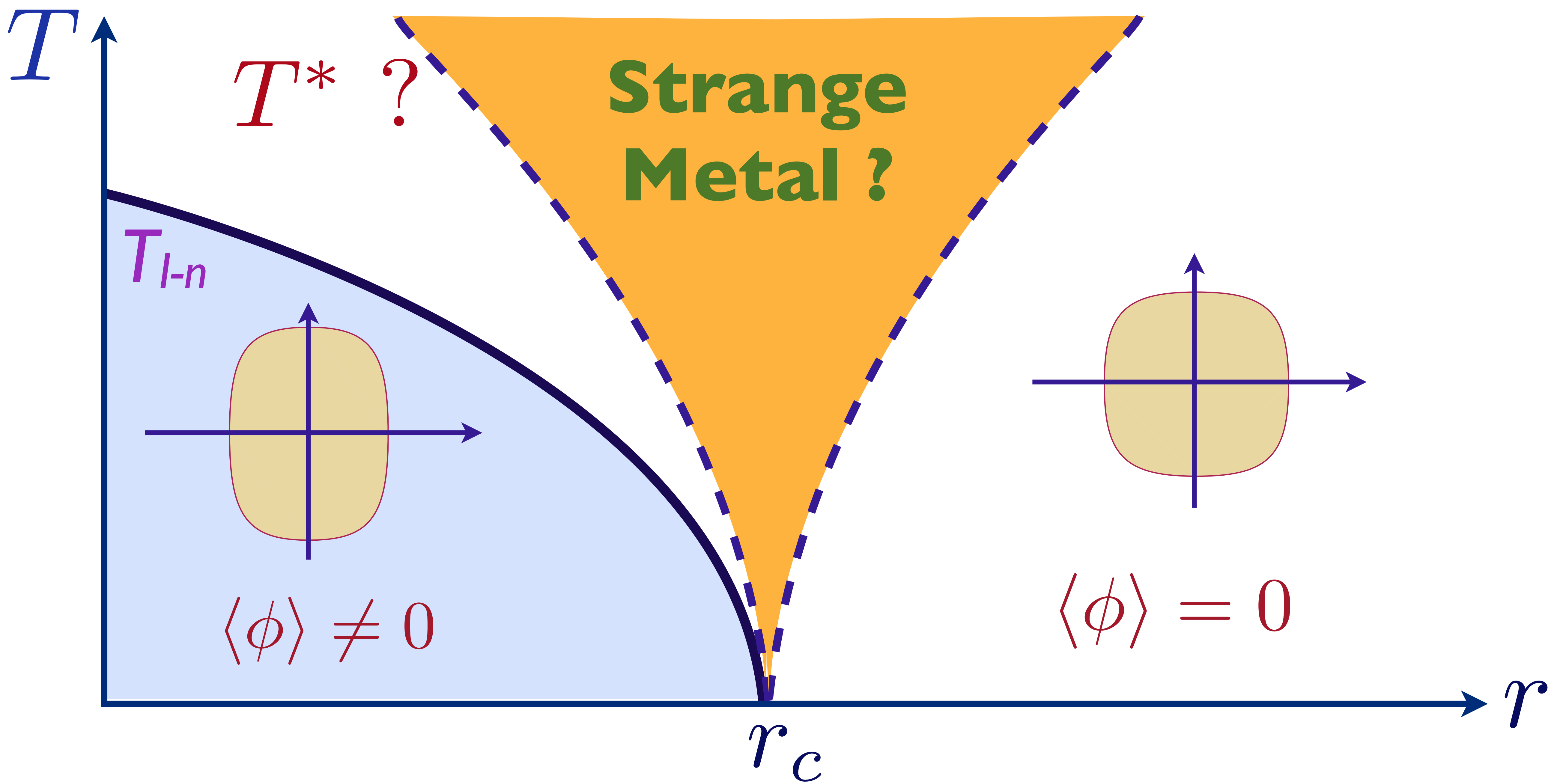

Phase diagram as a function of T and r

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2x d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (r - r_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

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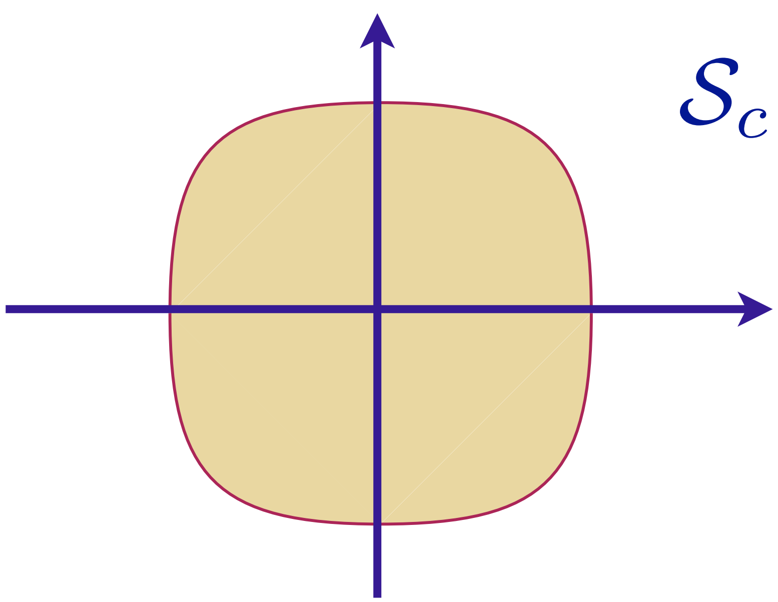
Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

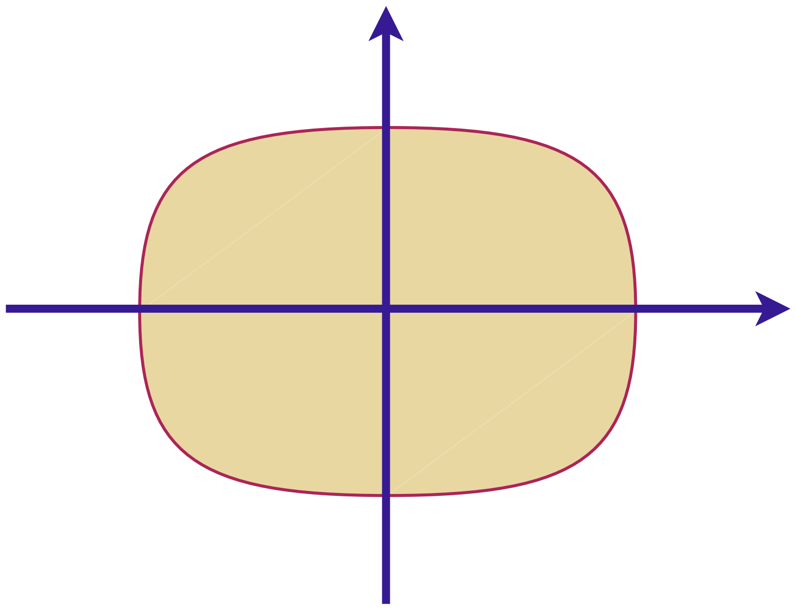
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Pomeranchuk instability

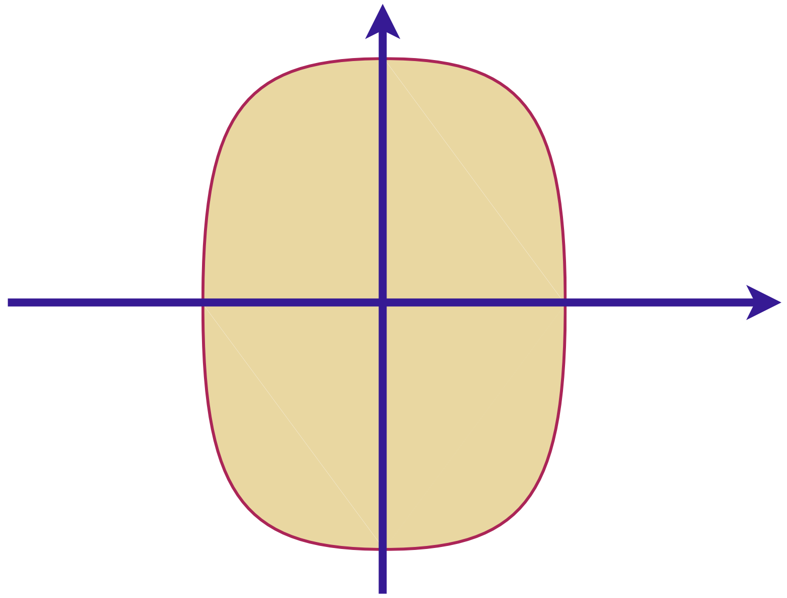
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \, \phi \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

for spatially independent ϕ



$$\langle \phi \rangle > 0$$



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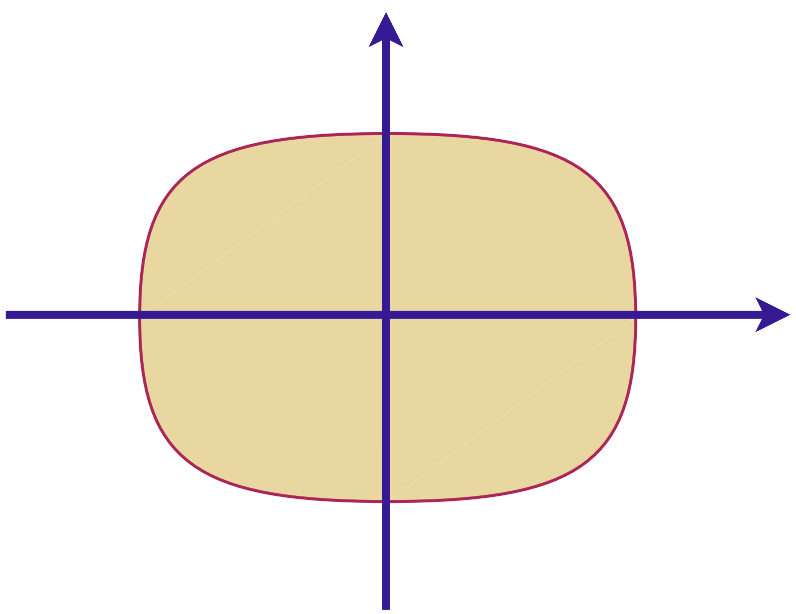
Quantum criticality of Pomeranchuk instability

Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



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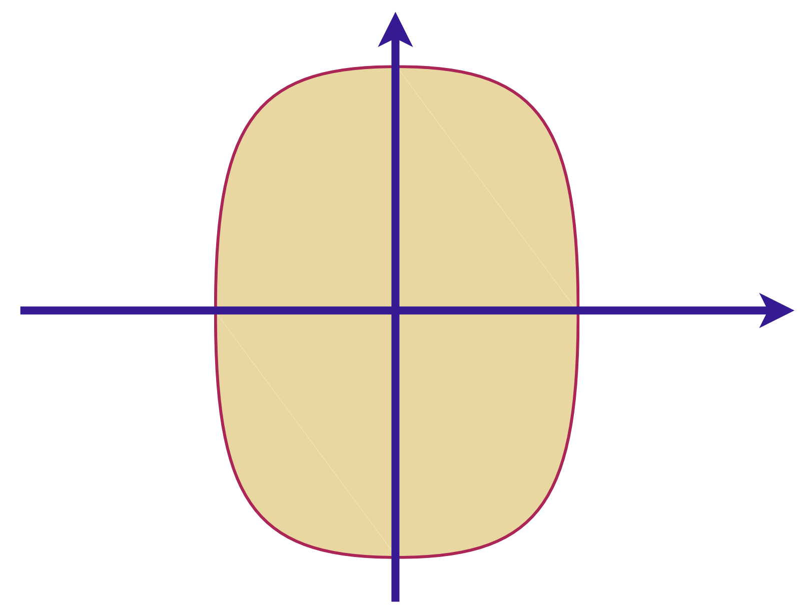

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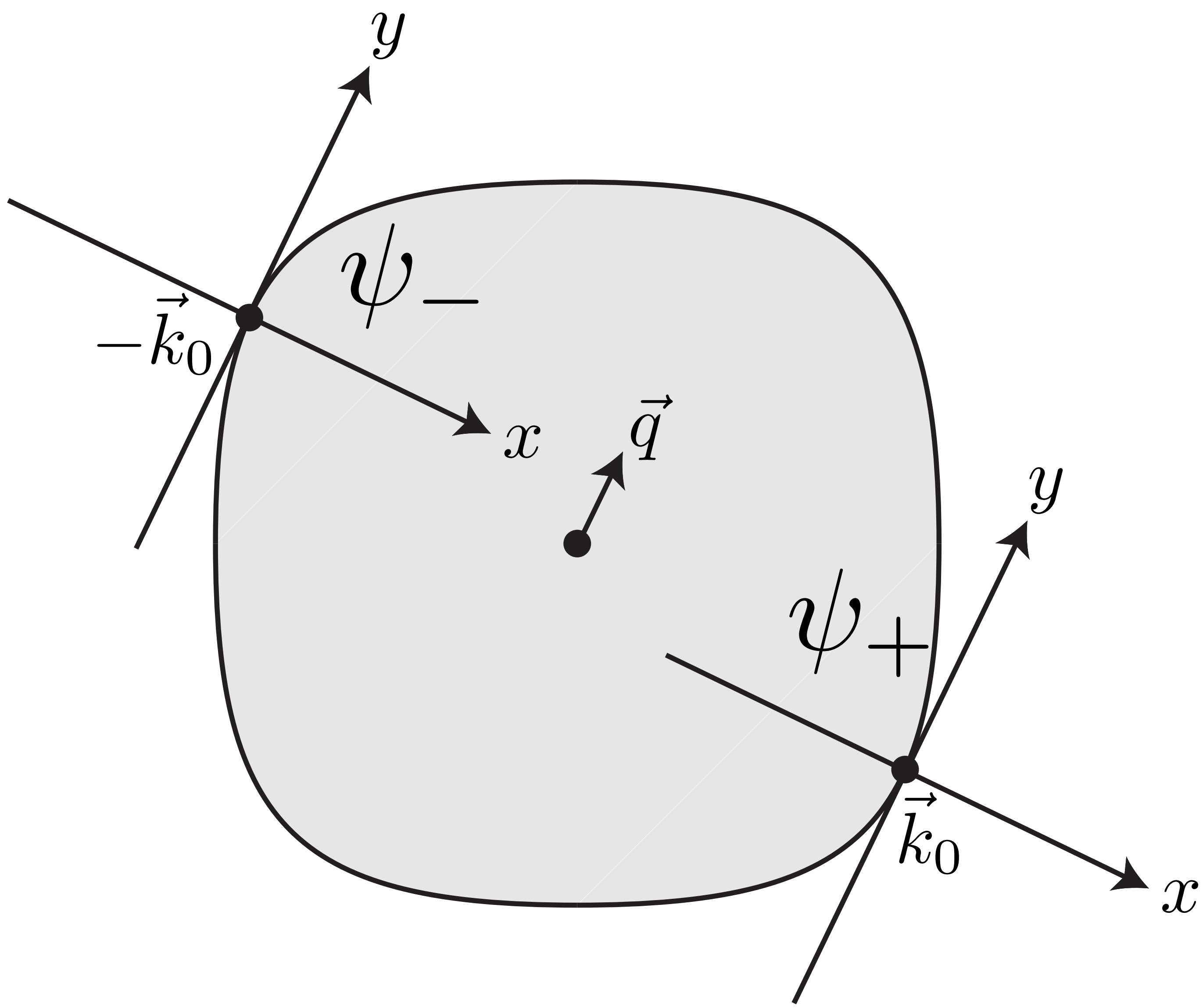
Quantum criticality of Pomeranchuk instability

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (r - r_c)\phi^2 + u\phi^4 \right]$$

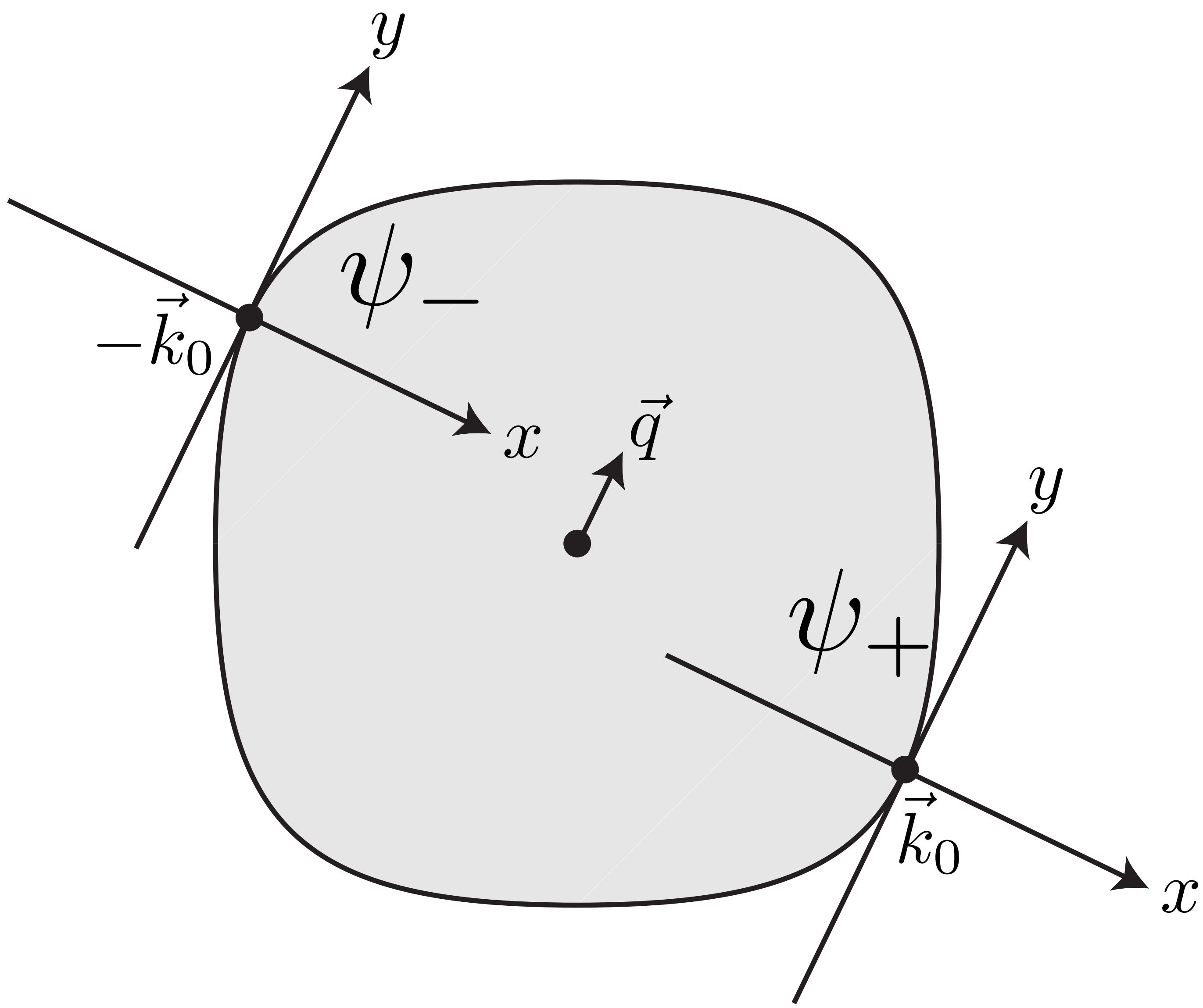
$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c^\dagger_{\mathbf{k}\alpha} \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$



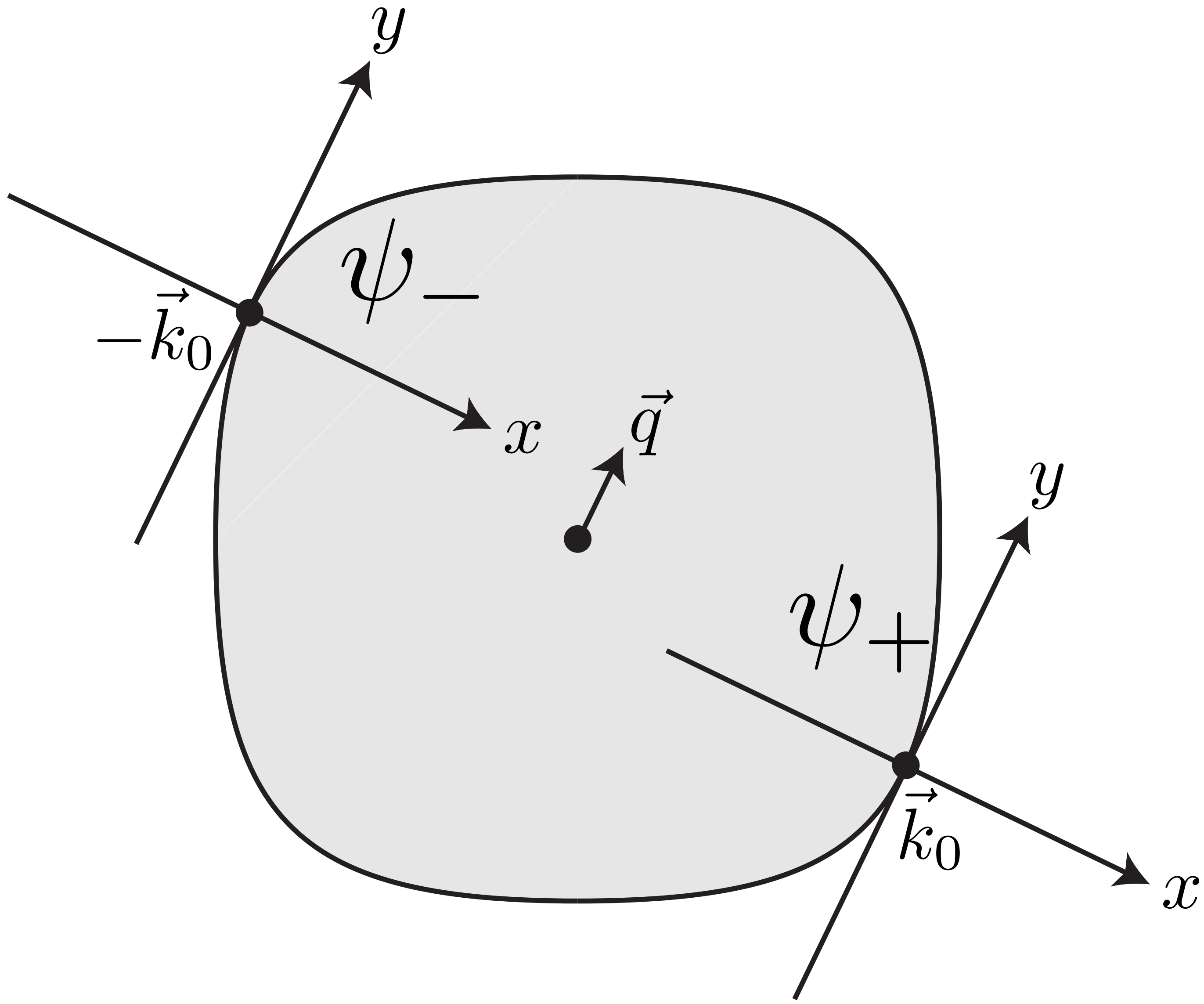
A ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Expand fermion kinetic energy at wavevectors about $\vec{k}_0$



$$\mathcal{L} = \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \lambda\phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2$$

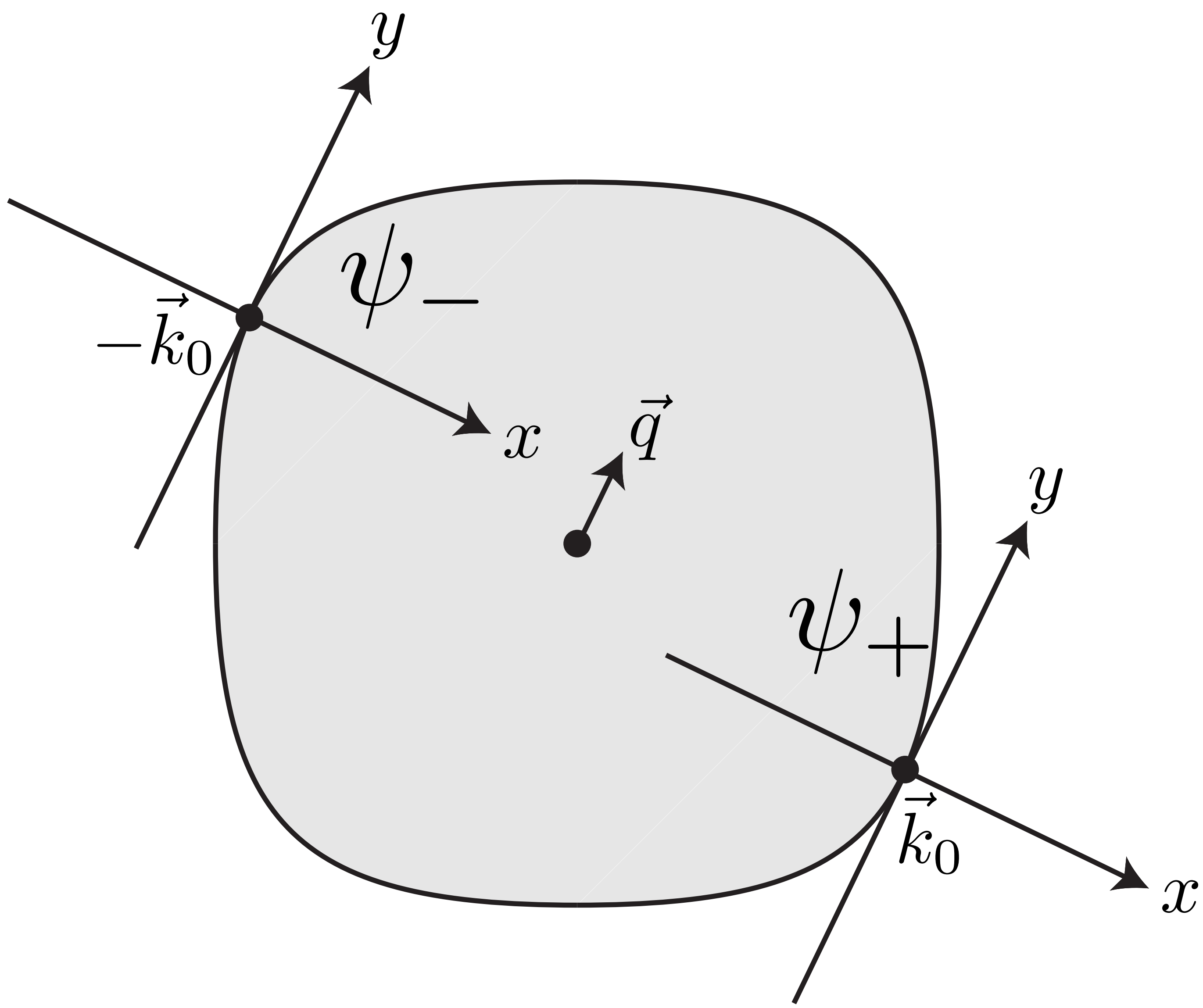
Theory of Ising-nematic transition



$$\mathcal{L} = \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$- \lambda\phi\left(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2$$

Theory of a Fermi surface minimally coupled to an Abelian or non-Abelian gauge field with $\phi \sim A_x$



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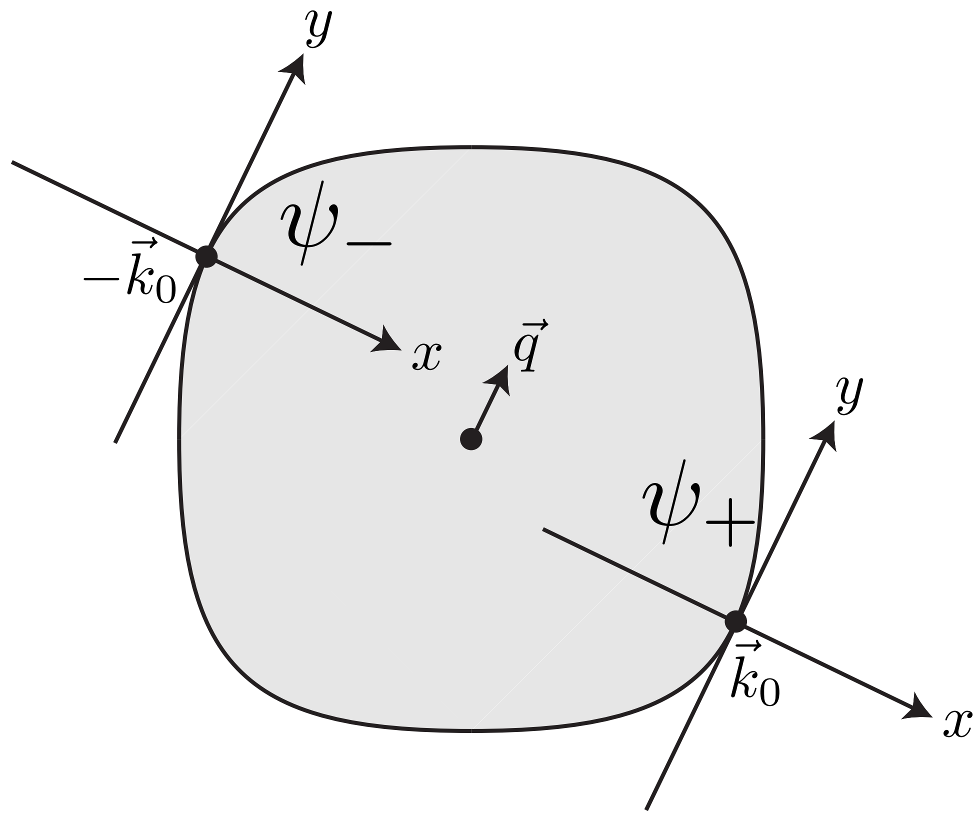
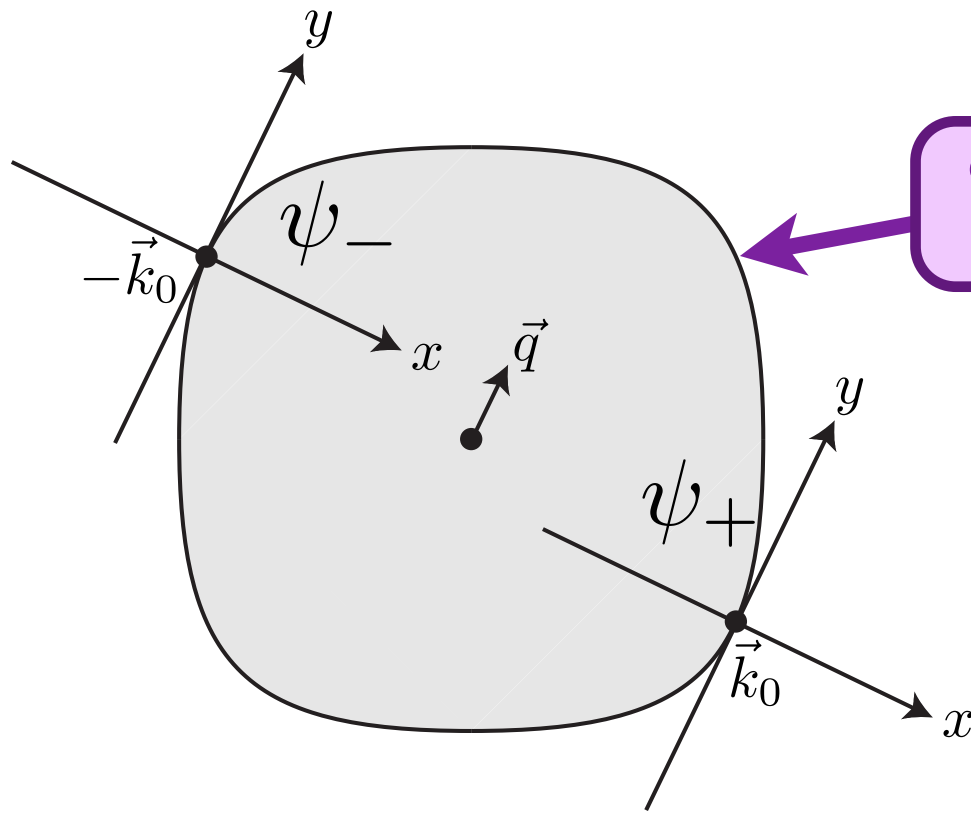


Emergent "Galilean invariance" at low energy $(s = \pm)$:

$$\phi(x,y) \to \phi(x, y+\theta x), \quad \psi_s(x,y) \to e^{-is(\frac{\theta}{2}y + \frac{\theta^2}{4}x)}\psi_s(x, y+\theta x)$$

which implies for the fermion Green's function

$$G(q_x, q_y) = G(sq_x + q_y^2).$$



"Hot" Fermi surfaces

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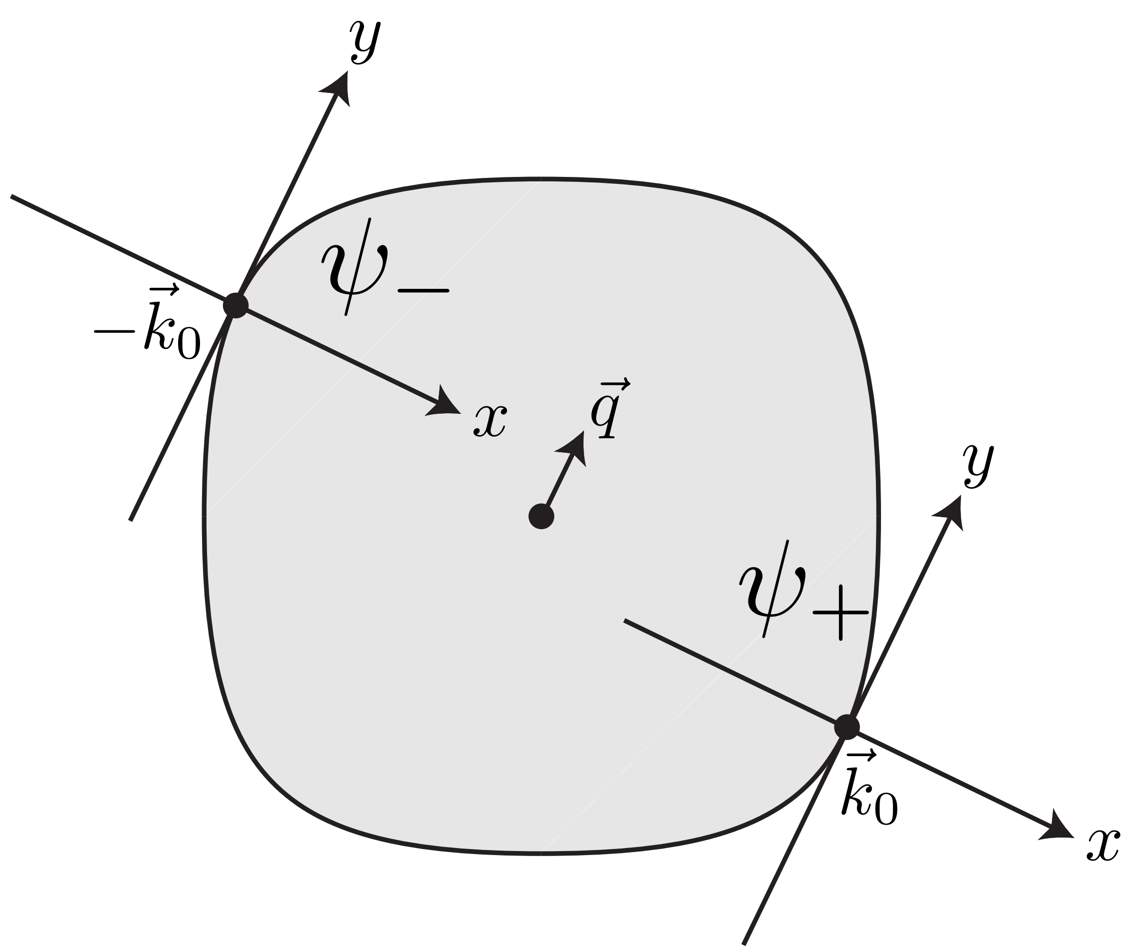
which implies for the fermion Green's function

$$G(q_x, q_y) = G(sq_x + q_y^2).$$

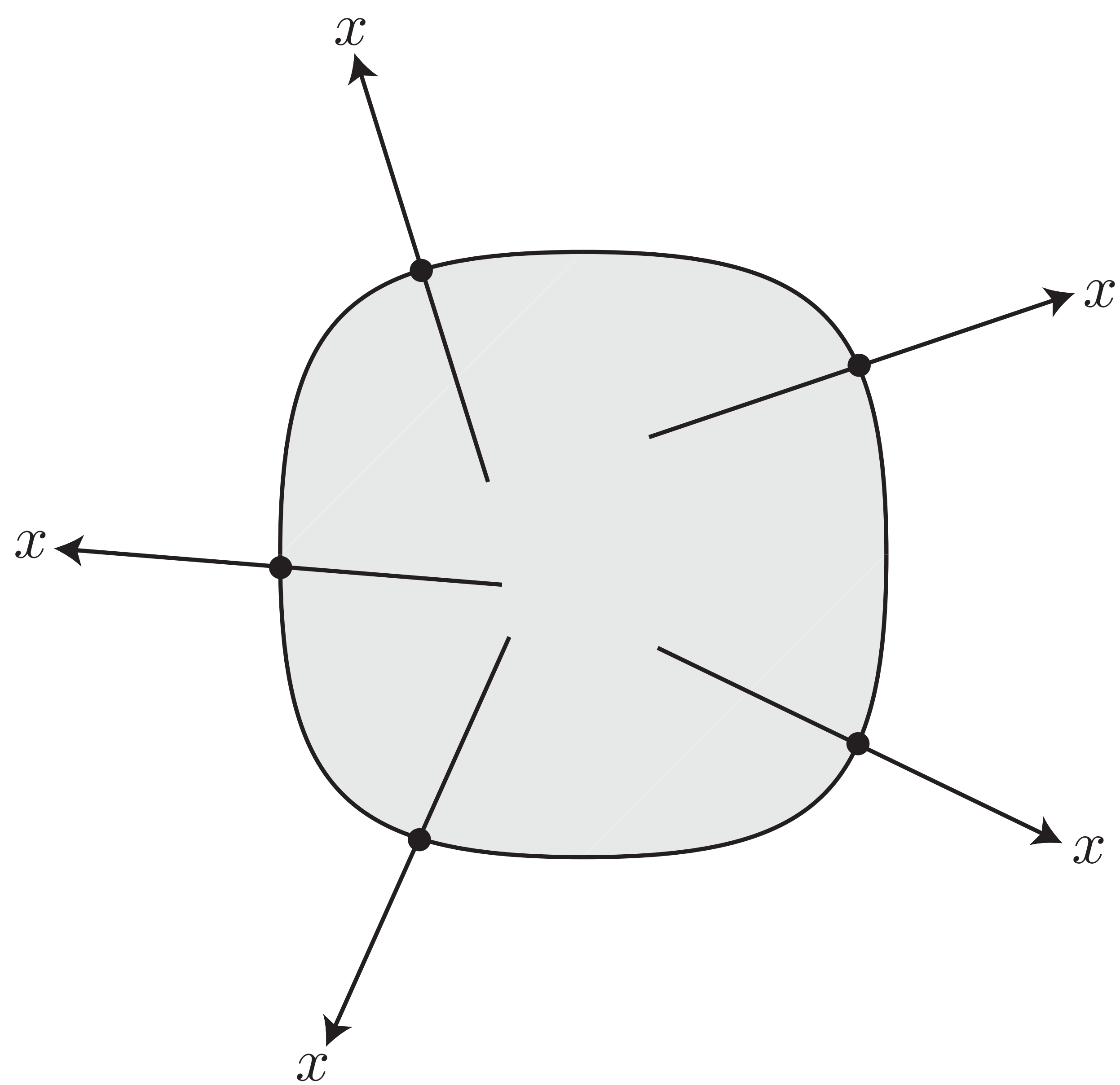
Line of singularities in momentum space on the "hot" Fermi surface $sq_x + q_y^2 = 0$.

- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction $\hat{q}$.



- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction $\hat{q}$.

- Contrast with "Fermi surface bosonization" methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$.



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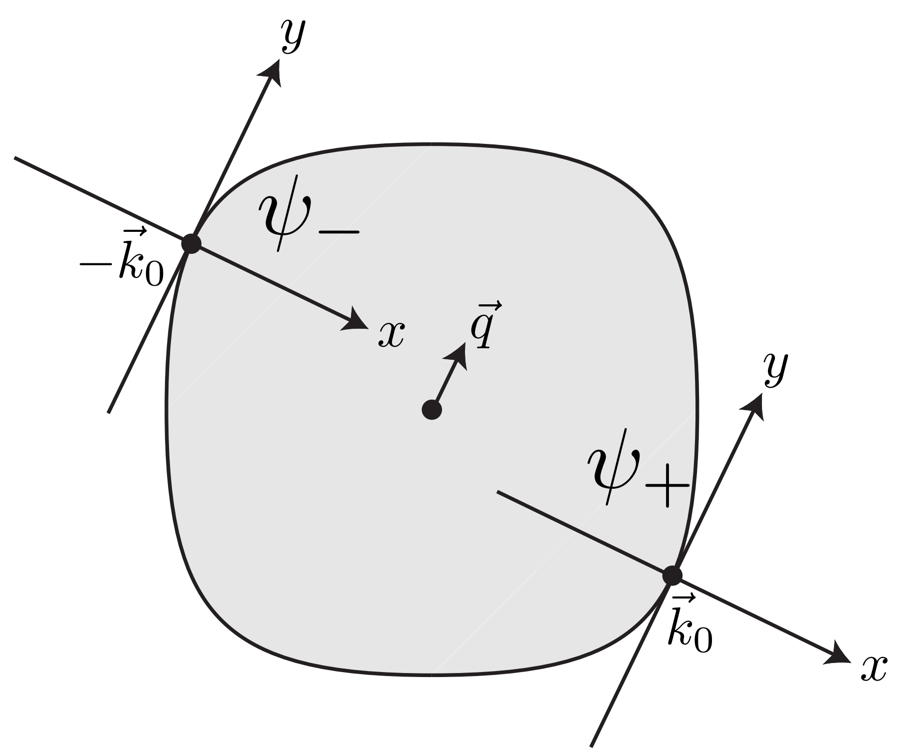
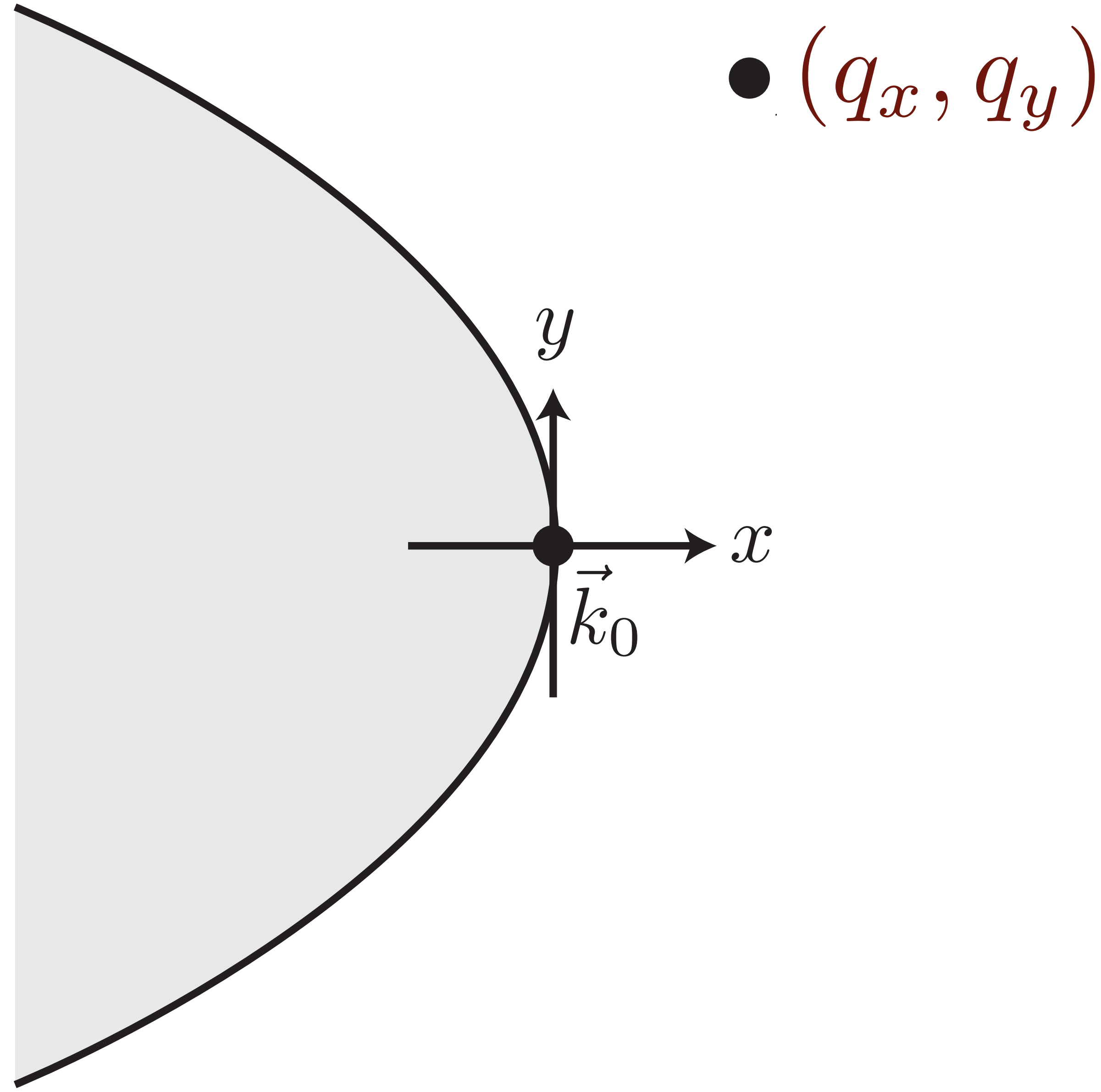
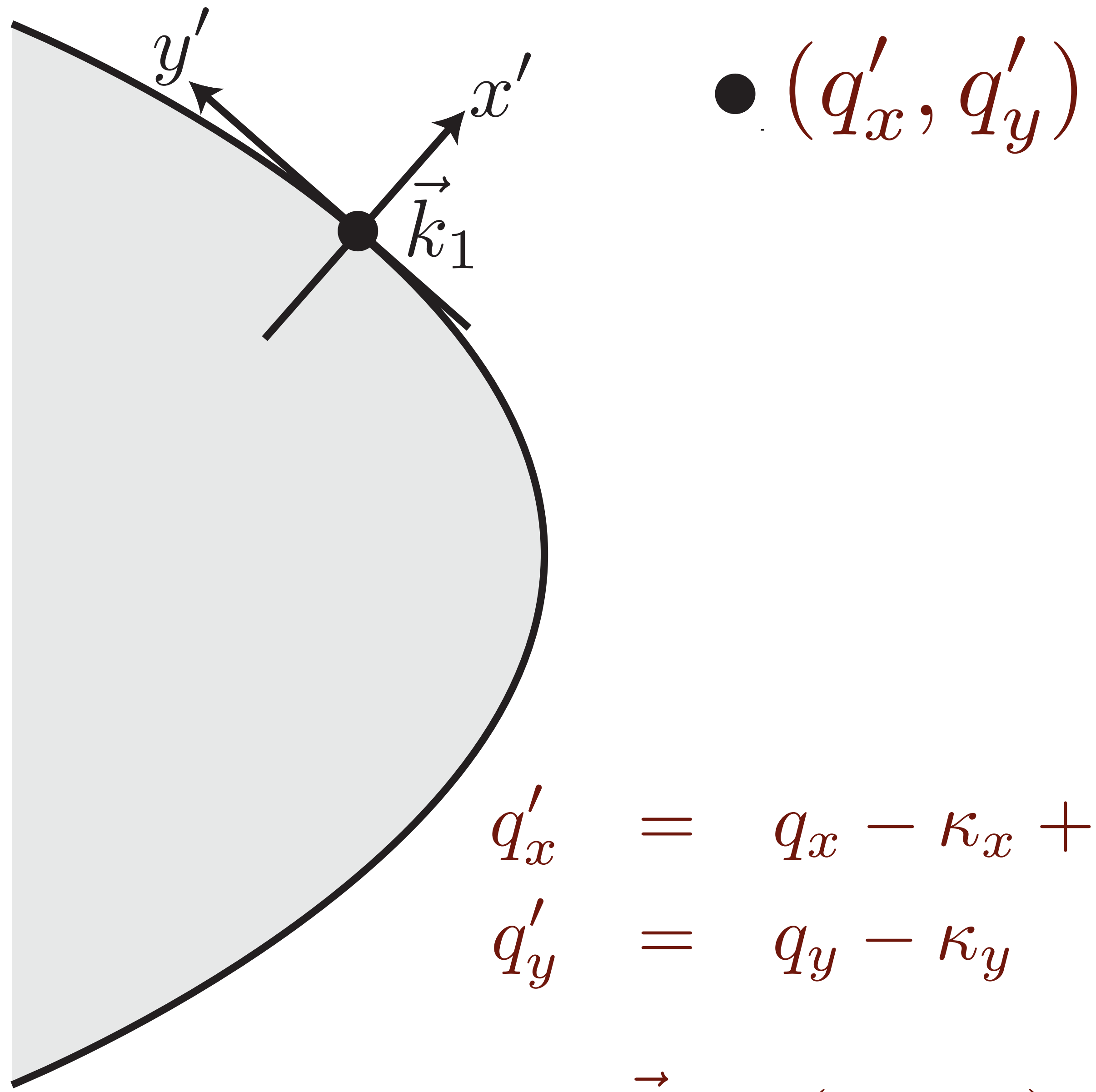
- Our approach leads to a redundant description of underlying degrees of freedom. The "Galilean symmetry" ensures consistency of redundant description.



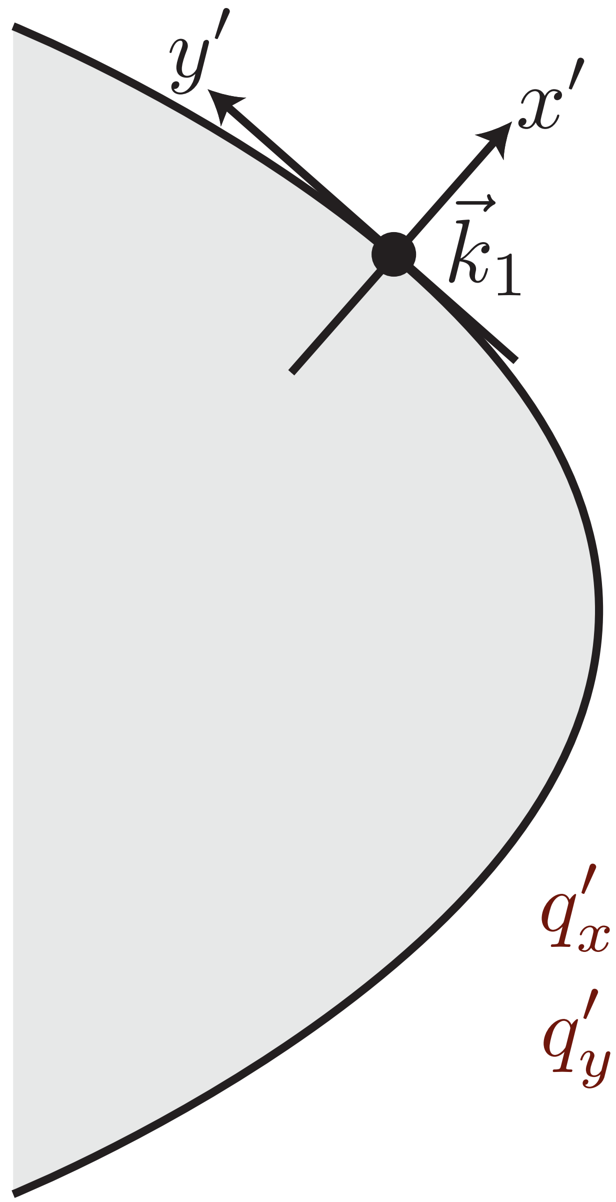


(q_x, q_y)
y
x
$\vec{k}_0$



$$
\begin{aligned}
q'_x &= q_x - \kappa_x + 2\kappa_y(q_y - \kappa_y) \\
q'_y &= q_y - \kappa_y \qquad ,
\end{aligned}
$$

where $\vec{k}_1 = (\kappa_x, \kappa_y)$ and $\kappa_x + \kappa_y^2 = 0$.



• (q'_x, q'_y)

$$\begin{aligned} q'_x &= q_x - \kappa_x + 2\kappa_y(q_y - \kappa_y) \\ q'_y &= q_y - \kappa_y \qquad , \end{aligned}$$

where $\vec{k}_1 = (\kappa_x, \kappa_y)$ and $\kappa_x + \kappa_y^2 = 0$.

Note $q'_x + q'^2_y = q_x + q_y^2$: ensures compatibility of redundant 2+1 dimensional field theories.

- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction $\hat{q}$.

- Contrast with "Fermi surface bosonization" methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$.

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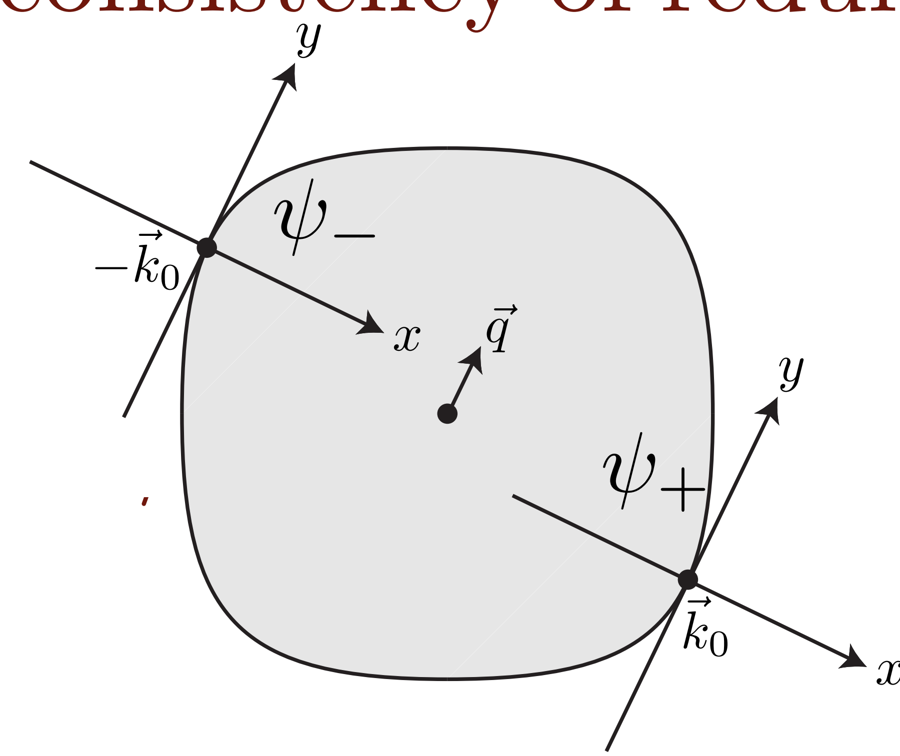

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- <u>Infinite set of 2+1 dimensional field theories</u>: Does this imply an "emergent dimension" and is there a direct connection to AdS/CFT ?

$$\mathcal{L} = \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger\left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \lambda\phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2$$

After tuning the single parameter $r \sim \lambda - \lambda_c$, and sending $\zeta \to 0$, $\mathcal{L}$ describes a critical theory with no coupling constants. There is a separate copy of this critical theory for each direction $\hat{q}$. This theory has 2 independent exponents z and η, and the correlation length and susceptibility exponents are given by

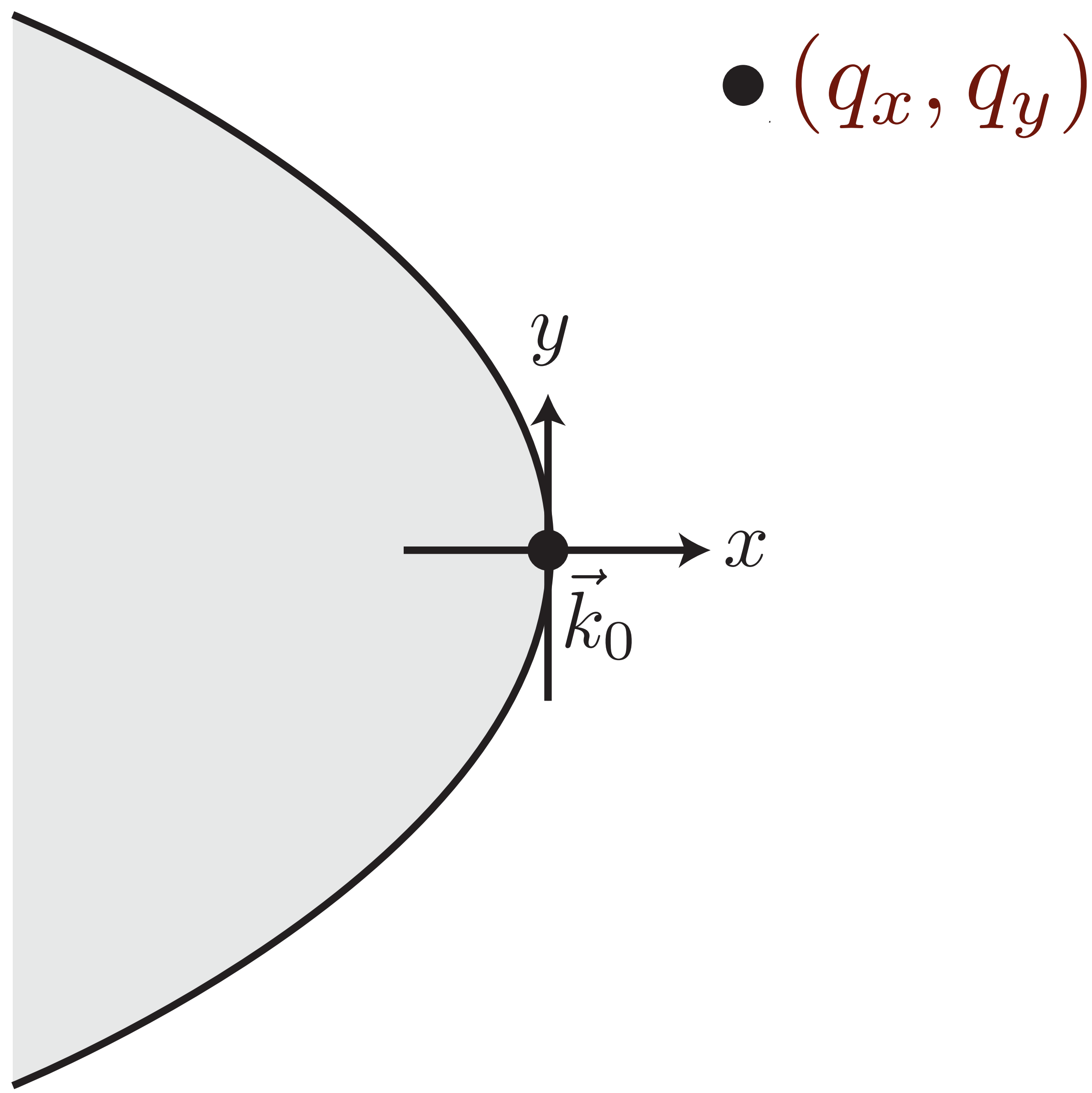
$$\nu = \frac{1}{z-1} \quad ; \quad \gamma = 1$$

The fermion and order parameter Green's functions obey the scaling forms

$$G(\vec{q},\omega) = \xi^{2-\eta}\Phi_\psi\left((q_x+q_y^2)\xi^2, \omega\xi^z\right) \; ; \; D(\vec{q},\omega) = \xi^{z-1}\Phi_\phi\left(q_y\xi, \omega\xi^z\right)$$

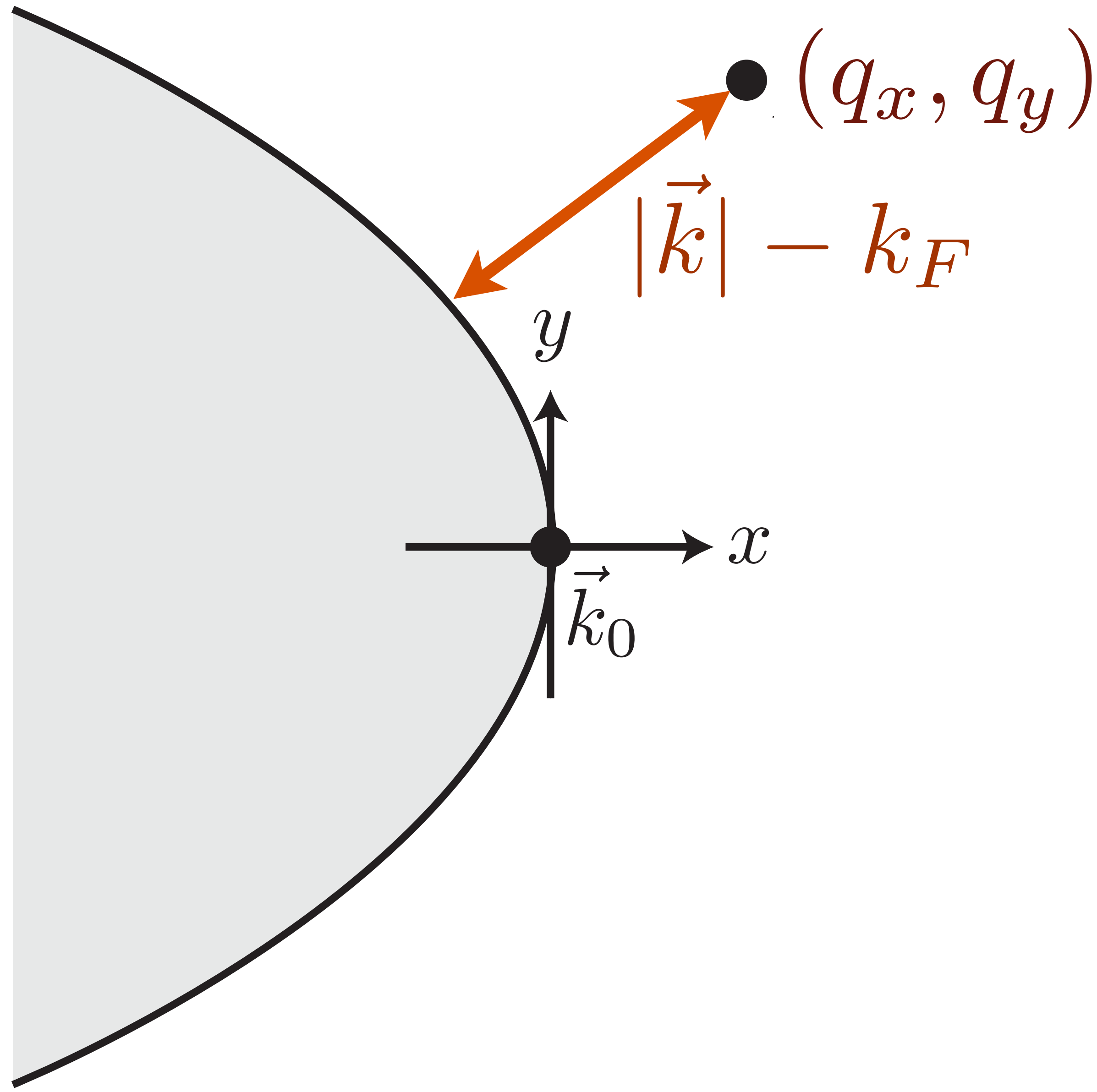
We have computed the exponents to three loops, and find $z=3$ and $\eta = 0.06824$ at this order.

Leading order fermion Green's function



$$G(\vec{q}, \omega) = \frac{1}{\omega - q_x - q_y^2 + \frac{ic}{N}\omega^{2/3}}$$

Leading order fermion Green's function



$$G(\vec{q}, \omega) = \frac{1}{\omega - v_F(|\vec{k}| - k_F) + \frac{ic}{N}\omega^{2/3}}$$

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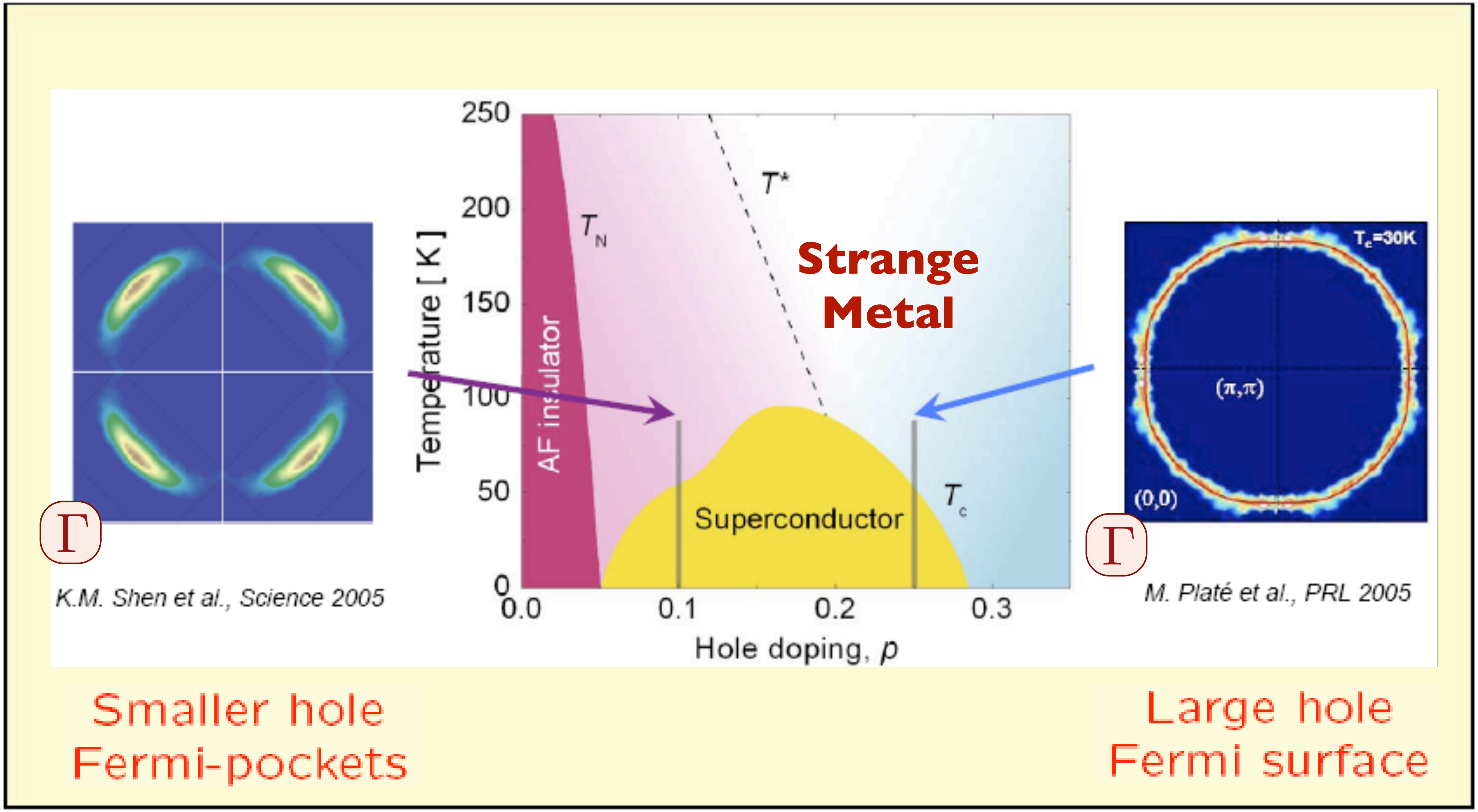
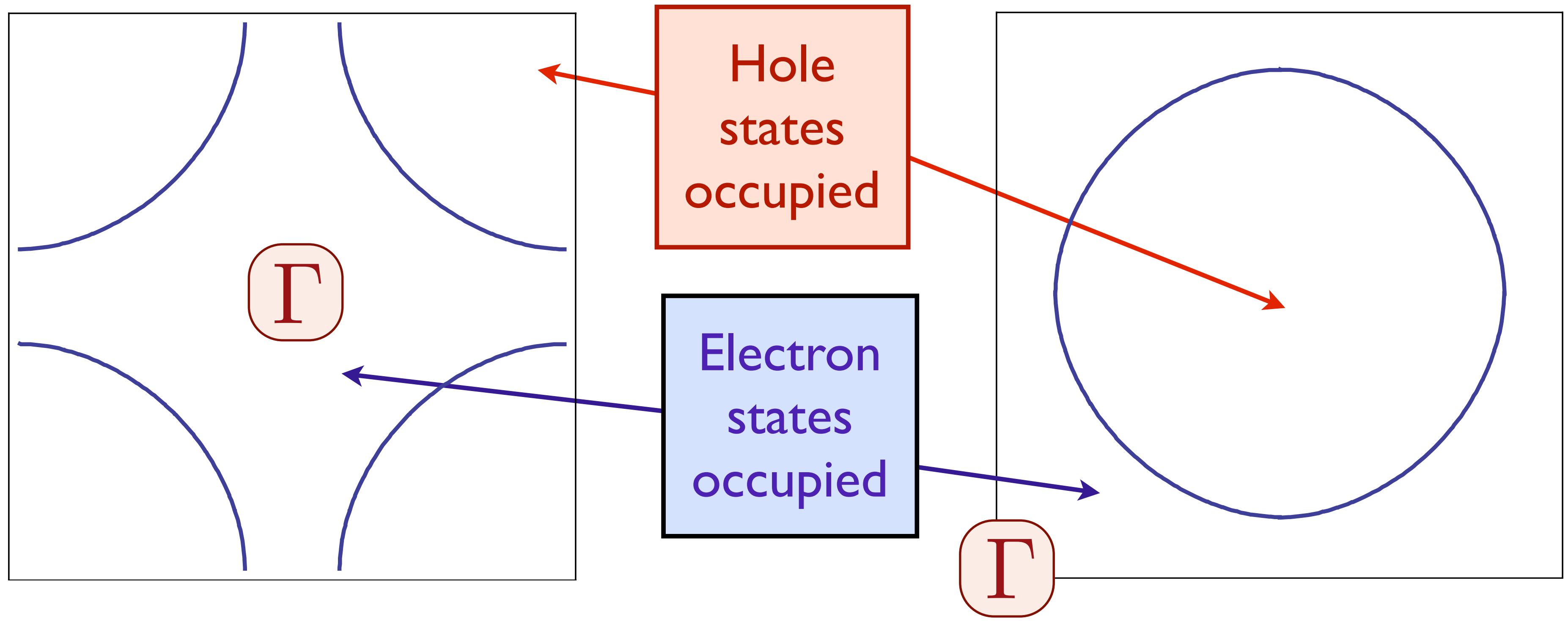
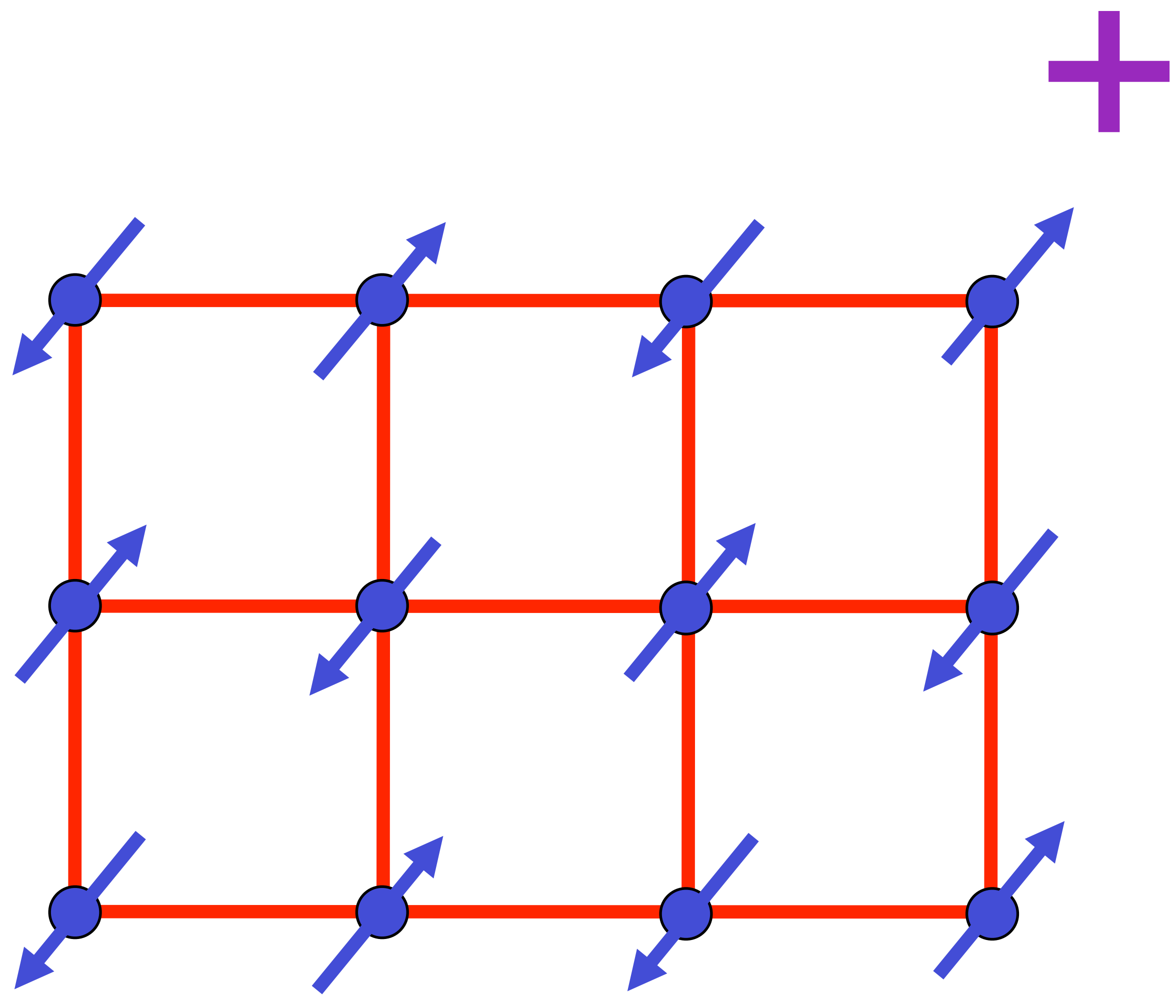
Non-Fermi liquids and superconductivity in a doped CFT

Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Fermi surface+antiferromagnetism





The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Start from the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

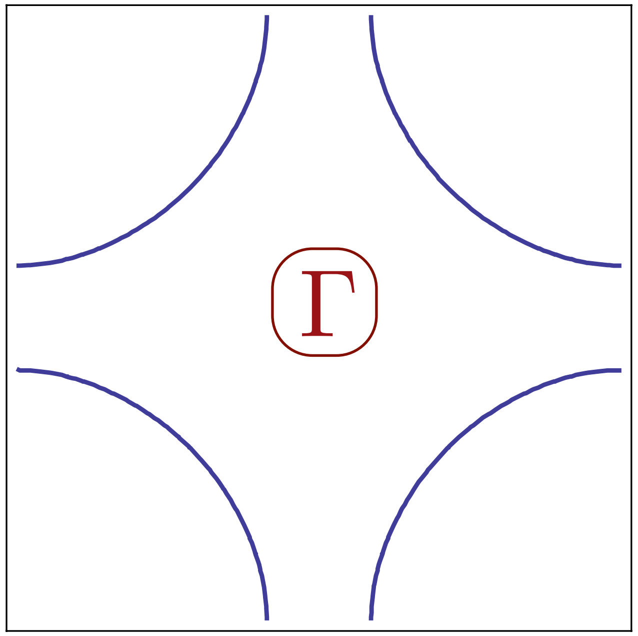
$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c_{i\alpha}^\dagger \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{\widetilde{\zeta}}{2} \left(\partial_\tau \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]$$

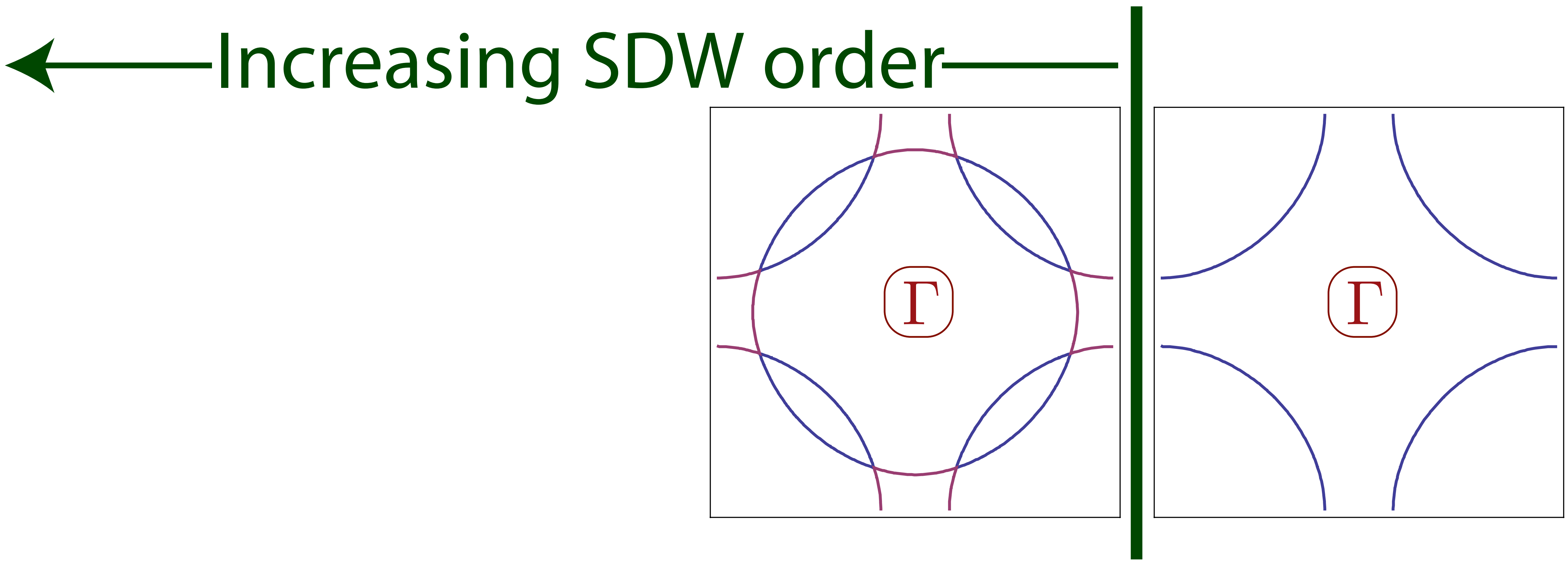
Hole-doped cuprates

Increasing SDW order



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



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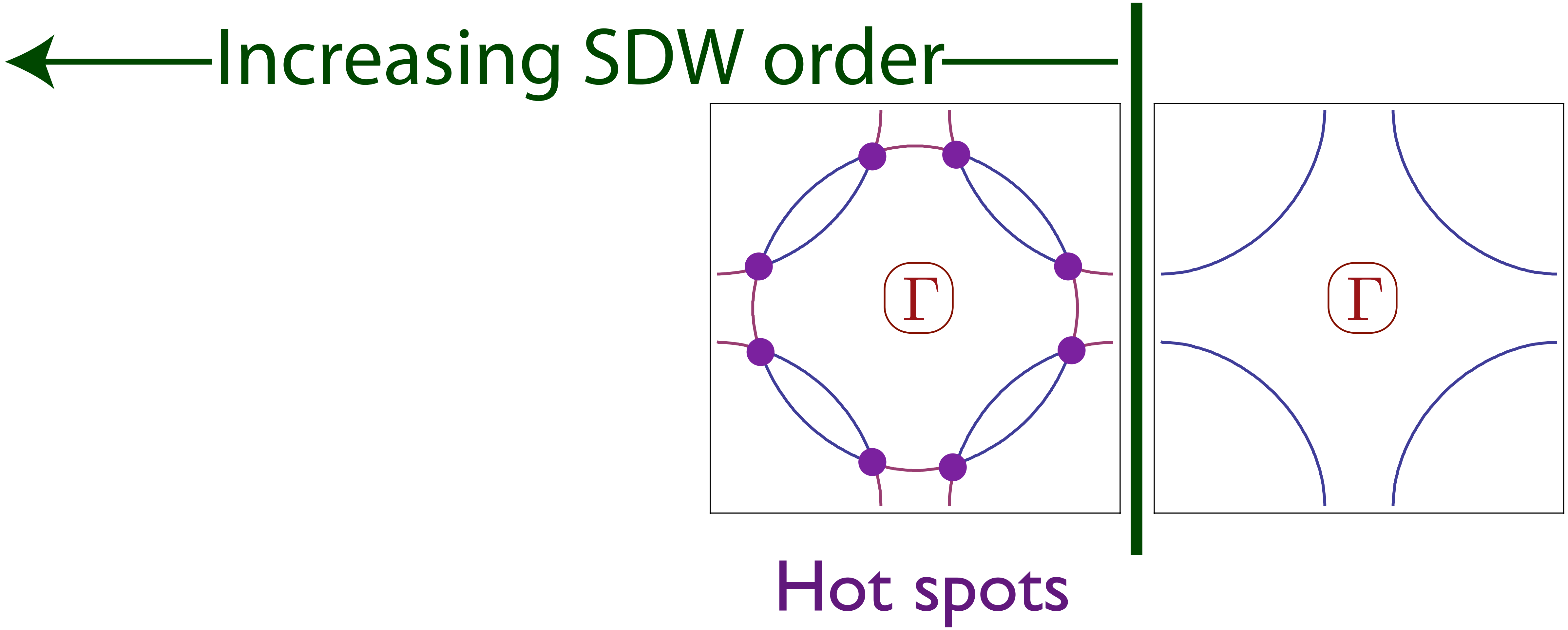

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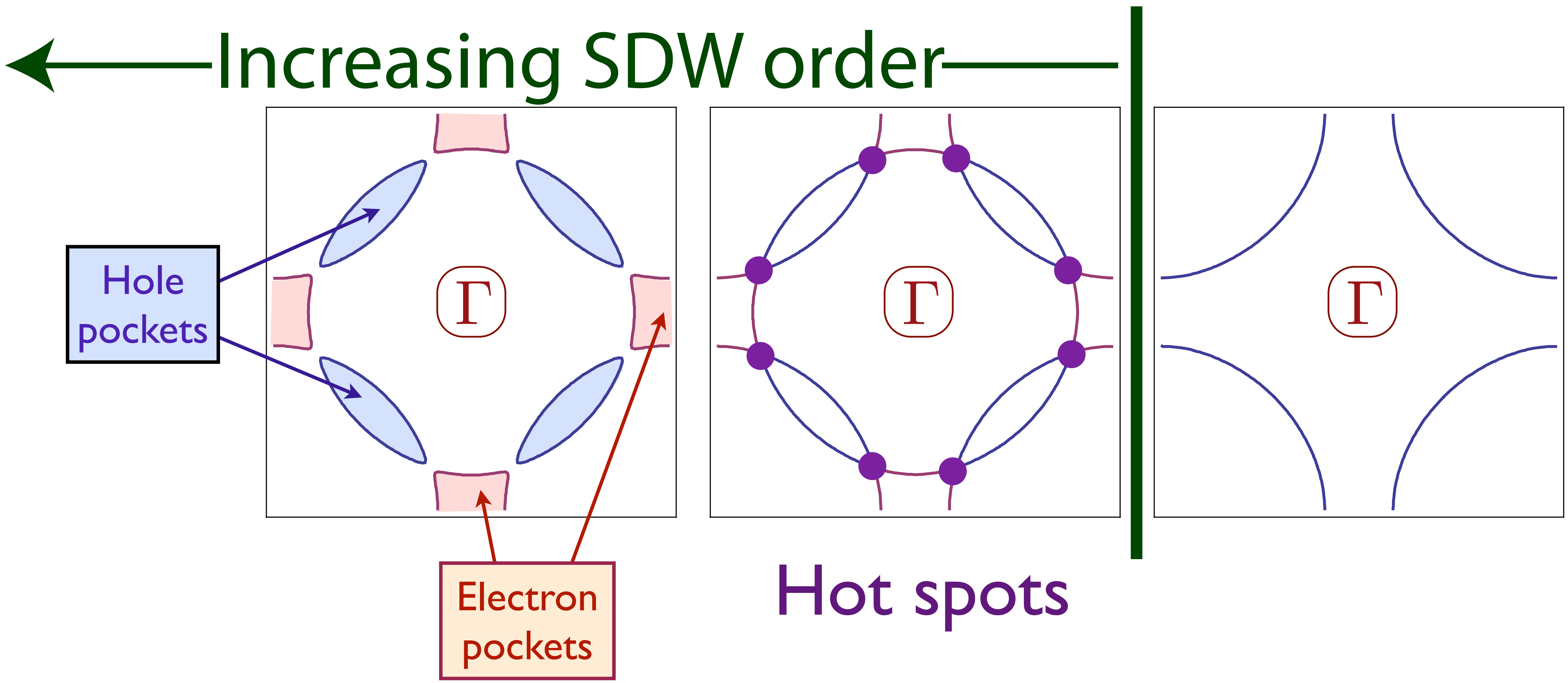

Fermi surface breaks up at hot spots into electron and hole "pockets"

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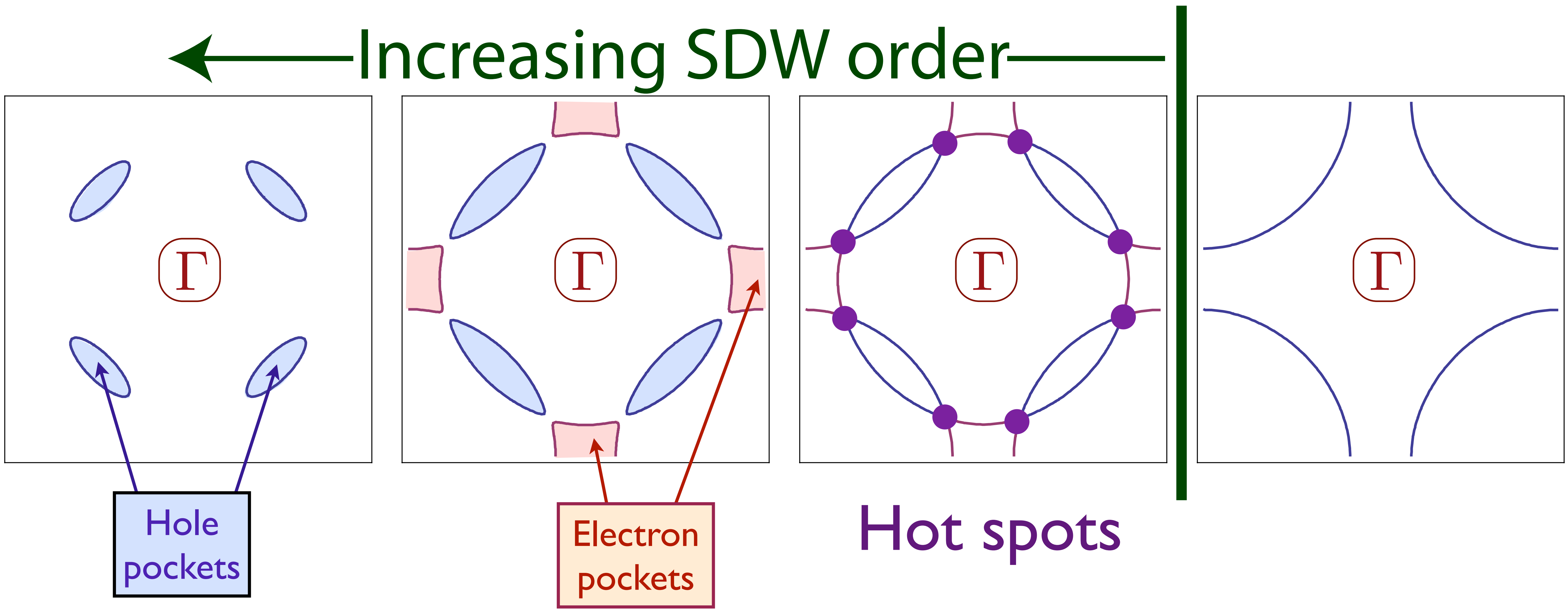

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Evidence for small Fermi pockets

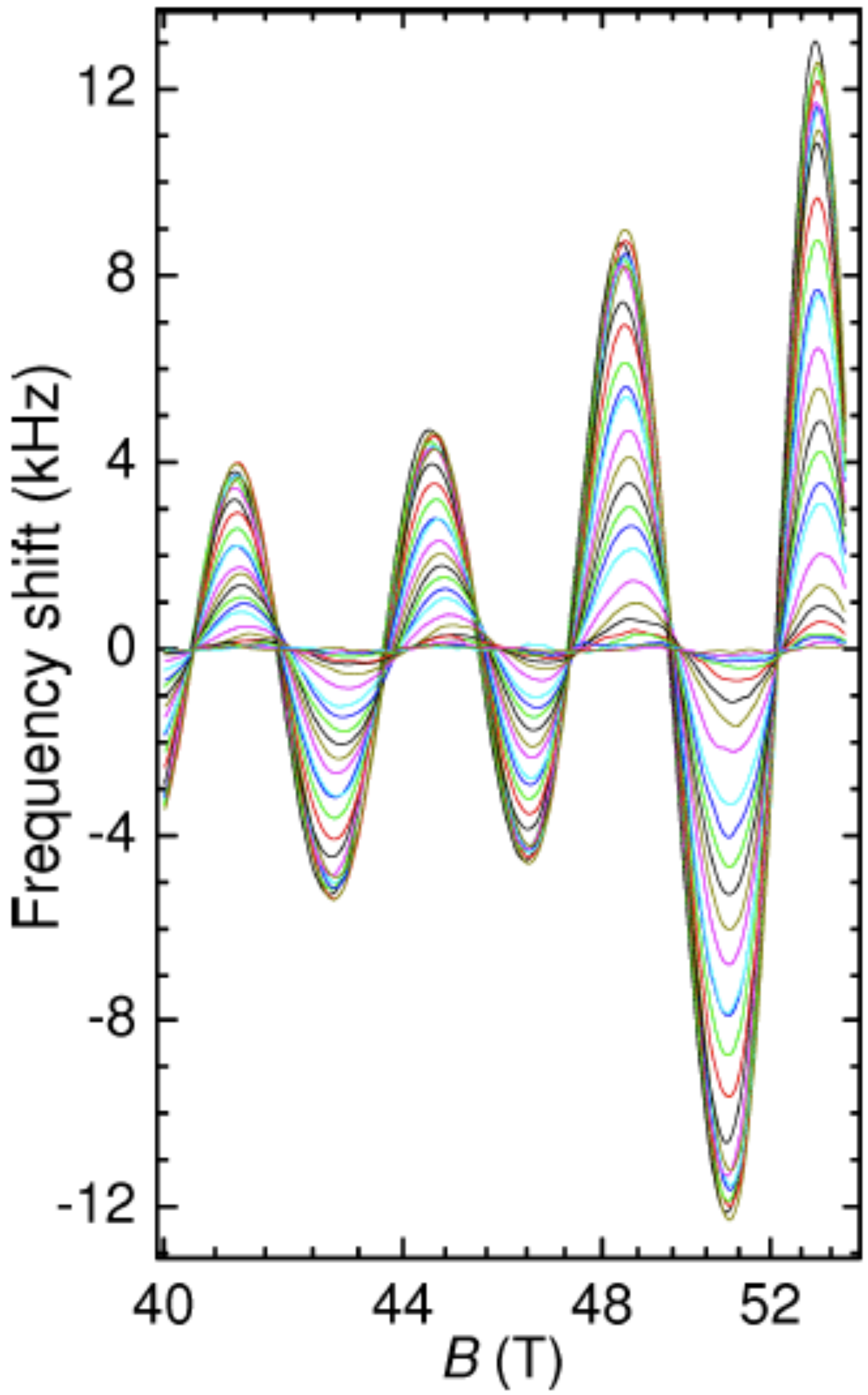


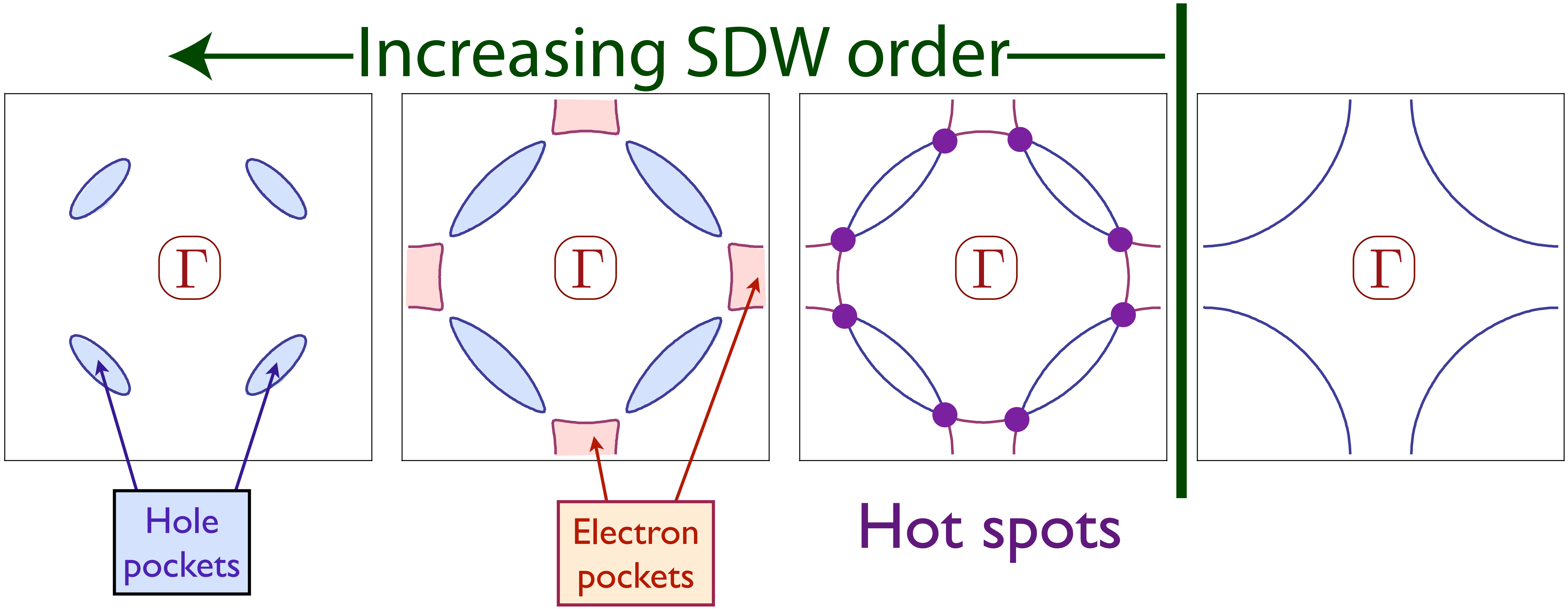
FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of $\sim$ 50 dB between T = 1 and 18 K. The actual T values are provided in Fig. 3.

Fermi liquid behaviour in an underdoped high Tc superconductor

Suchitra E. Sebastian, N. Harrison, M. M. Altarawneh, Ruixing Liang, D. A. Bonn, W. N. Hardy, and G. G. Lonzarich

arXiv:0912.3022

Theory of underdoped cuprates



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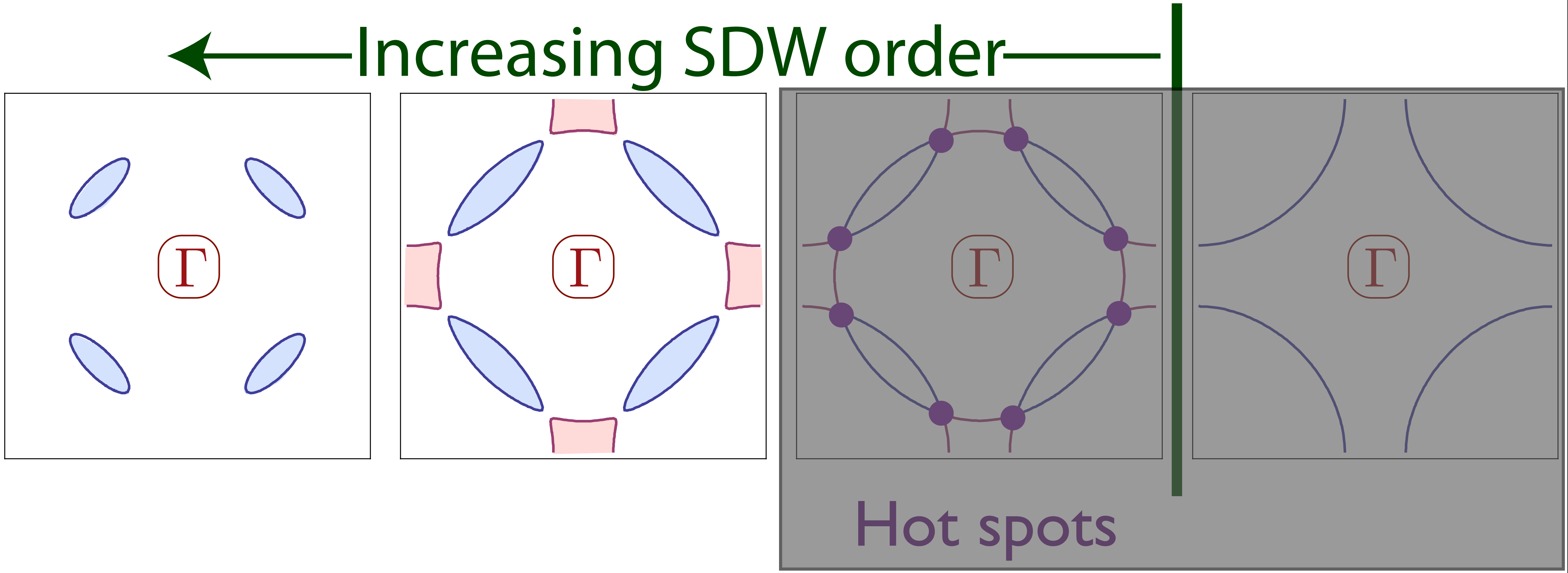


Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

H. J. Schulz, *Physical Review Letters* **65**, 2462 (1990)

B. I. Shraiman and E. D. Siggia, *Physical Review Letters* **61**, 467 (1988).

J. R. Schrieffer, *Journal of Superconductivity* **17**, 539 (2004)

Theory of underdoped cuprates

With $R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}$ or $\hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$

the theory is invariant under

$$z_\alpha \to e^{i\theta} z_\alpha \; ; \; \psi_+ \to e^{-i\theta}\psi_+ \; ; \; \psi_- \to e^{i\theta}\psi_{.-}$$

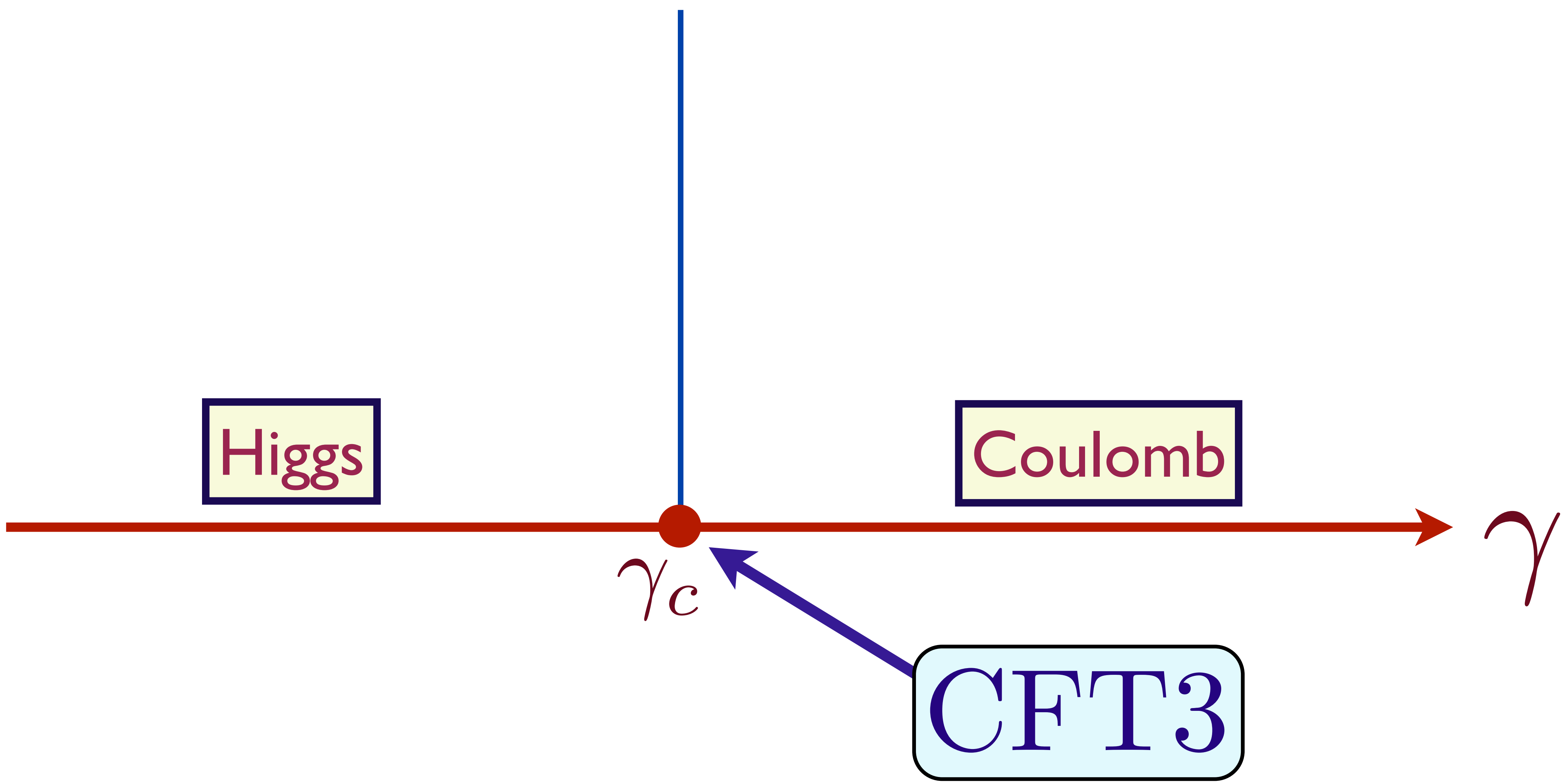
We obtain a U(1) gauge theory of

- bosonic neutral spinons z_α;
- spinless, charged fermions $\psi_\pm$ with small 'pocket' Fermi surfaces;
- an emergent U(1) gauge field A_μ.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Phys. Rev. B* **80**, 155129 (2009).

- Begin with a CFT3: the CP^1 model.

$$\mathcal{L}_z = \frac{1}{\gamma}|(\partial_\mu - iA_\mu)z_\alpha|^2 \quad ; \quad |z_\alpha|^2 = 1$$



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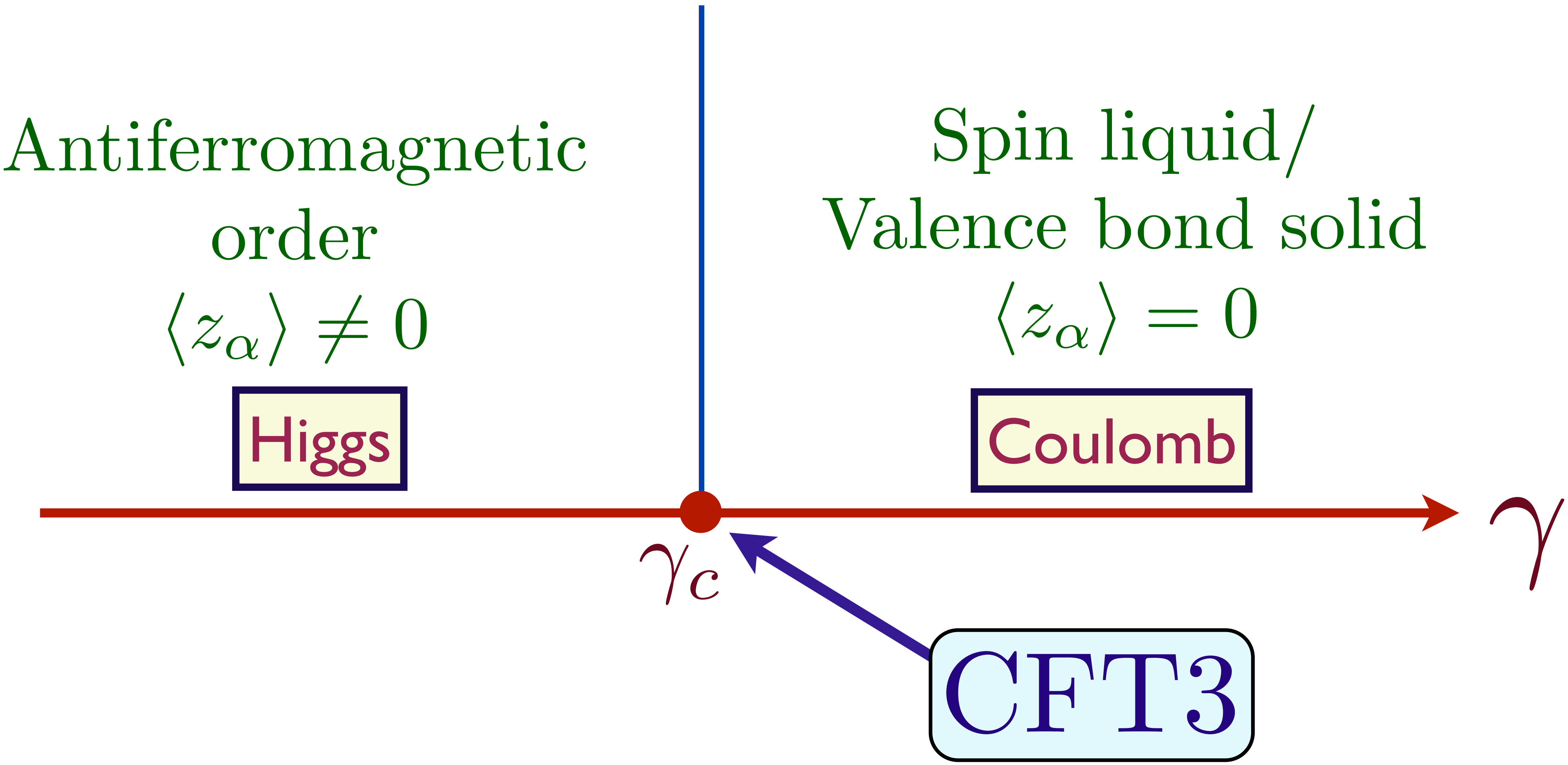

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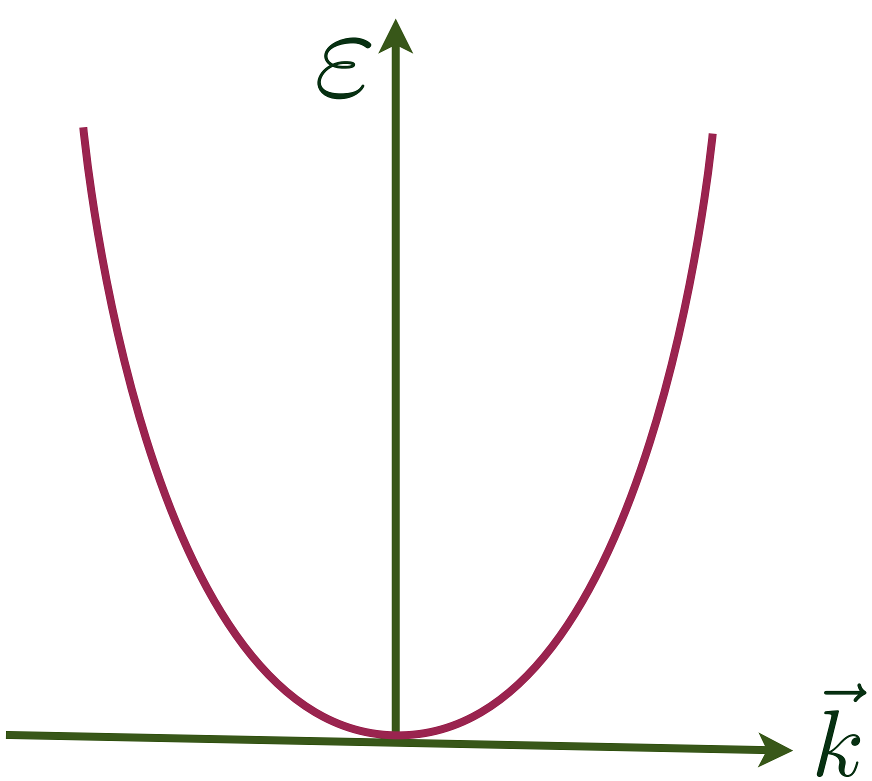
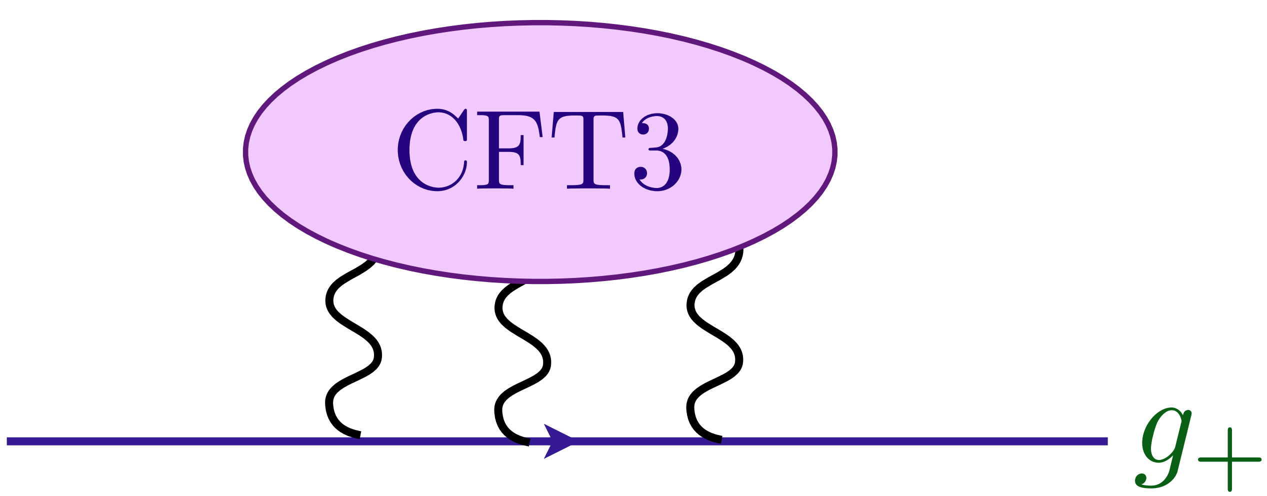
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges

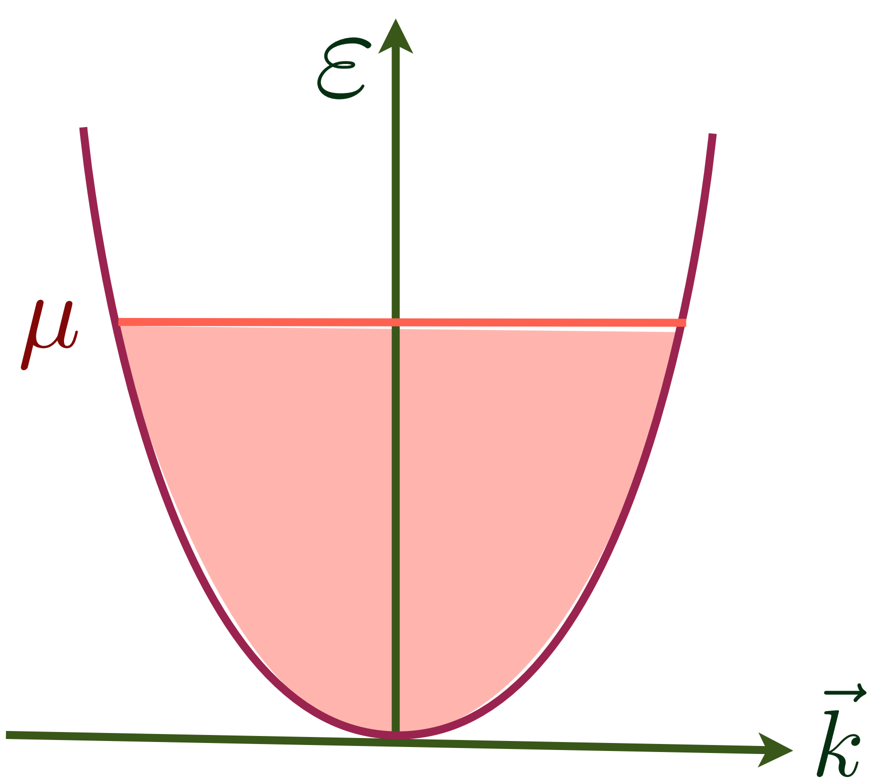
$$\mathcal{L}_f = g_+^\dagger \left(\frac{\partial}{\partial \tau} - i A_\tau - \frac{1}{2m} \left(\vec{\nabla} - i \vec{A} \right)^2 \right) g_+ + g_-^\dagger \left(\frac{\partial}{\partial \tau} + i A_\tau - \frac{1}{2m} \left(\vec{\nabla} + i \vec{A} \right)^2 \right) g_-$$

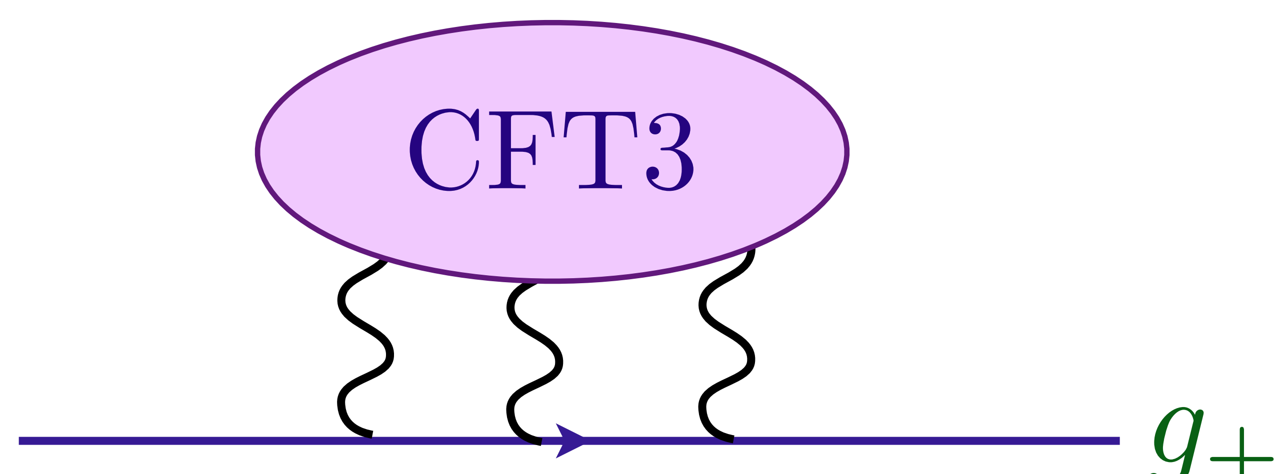




- Begin with a CFT3: the CP^1 model.
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges
- Turn on fermion chemical potential:

$$\mathcal{L}_f = g_+^\dagger \left(\frac{\partial}{\partial \tau} - iA_\tau - \mu - \frac{1}{2m}\left(\vec{\nabla} - i\vec{A}\right)^2 \right) g_+ + g_-^\dagger \left(\frac{\partial}{\partial \tau} + iA_\tau - \mu - \frac{1}{2m}\left(\vec{\nabla} + i\vec{A}\right)^2 \right) g_-$$





Complete theory

$$\mathcal{L} = \mathcal{L}_z + \mathcal{L}_f$$

$$\mathcal{L}_z = \frac{1}{\gamma} \left| (\partial_\mu - i A_\mu) z_\alpha \right|^2 \quad ; \quad |z_\alpha|^2 = 1$$

$$\begin{aligned} \mathcal{L}_f \quad = \quad & g_+^\dagger \left(\frac{\partial}{\partial \tau} - i A_\tau - \mu - \frac{1}{2m} \left(\vec{\nabla} - i \vec{A} \right)^2 \right) g_+ \\ & + g_-^\dagger \left(\frac{\partial}{\partial \tau} + i A_\tau - \mu - \frac{1}{2m} \left(\vec{\nabla} + i \vec{A} \right)^2 \right) g_- \end{aligned}$$

V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

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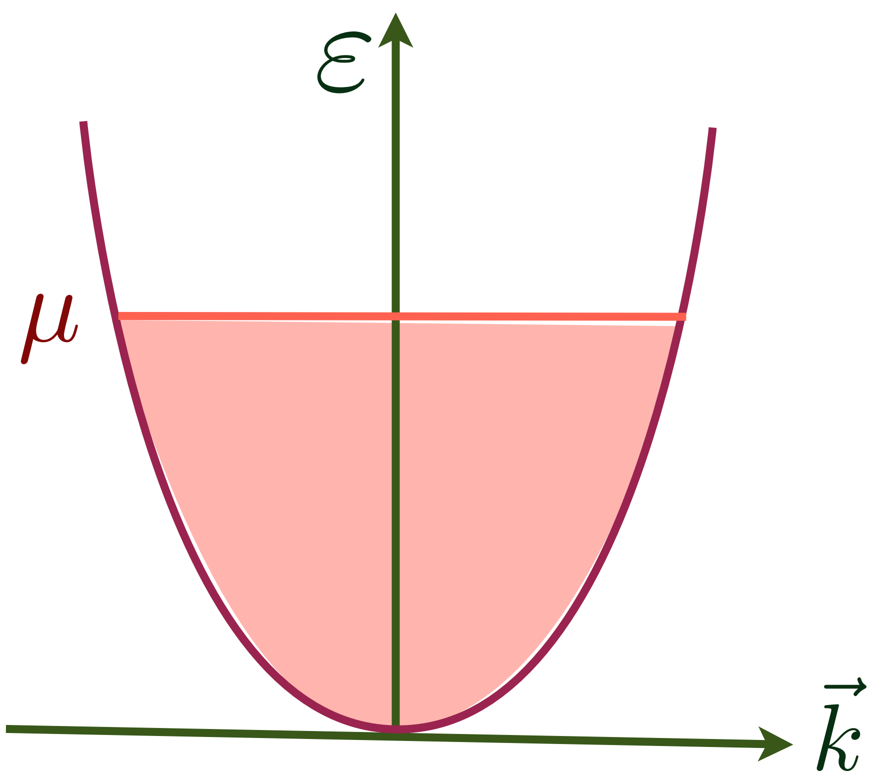
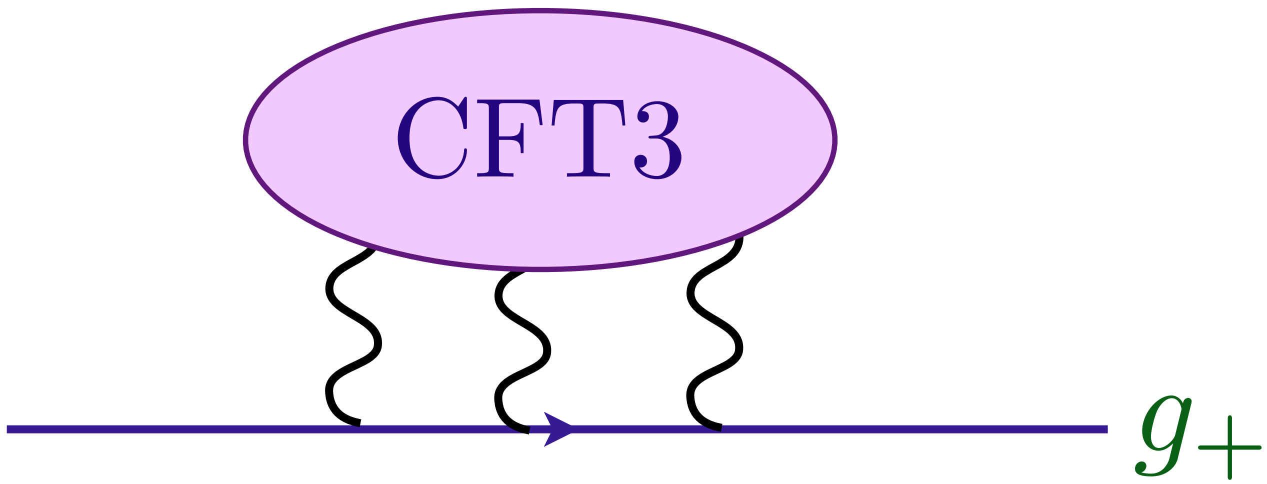
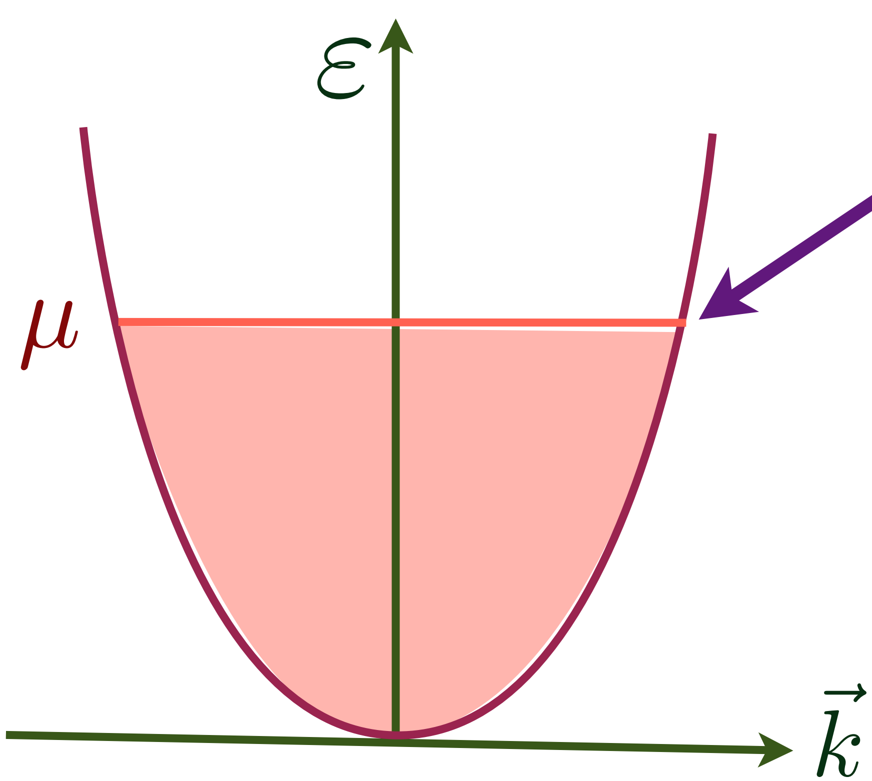
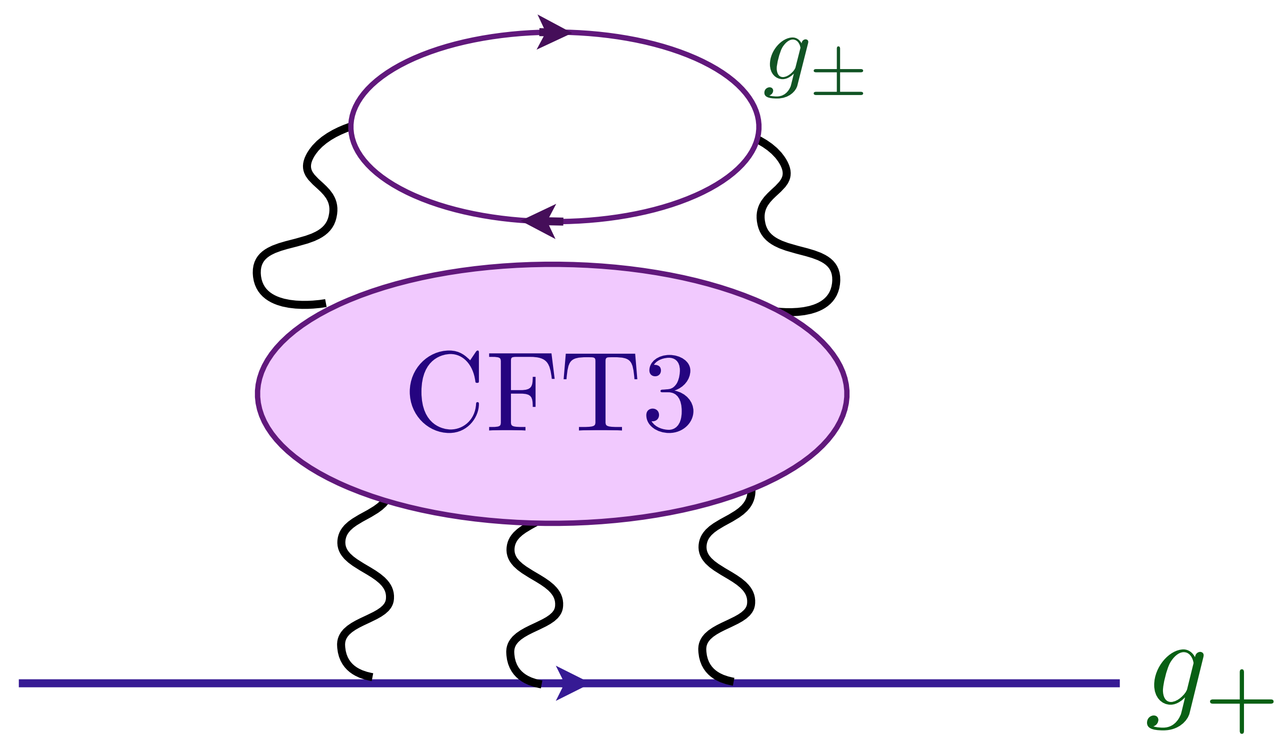




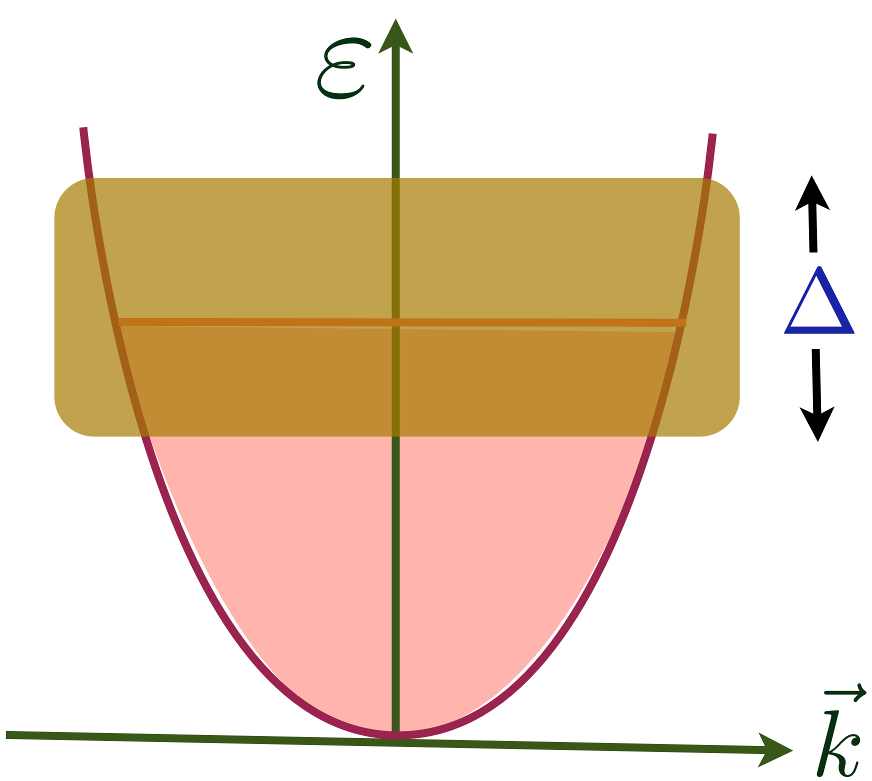
- Begin with a CFT3: the CP^1 model.
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges
- Turn on fermion chemical potential: leads to a marginal Fermi liquid of $g_\pm$ (not electrons)

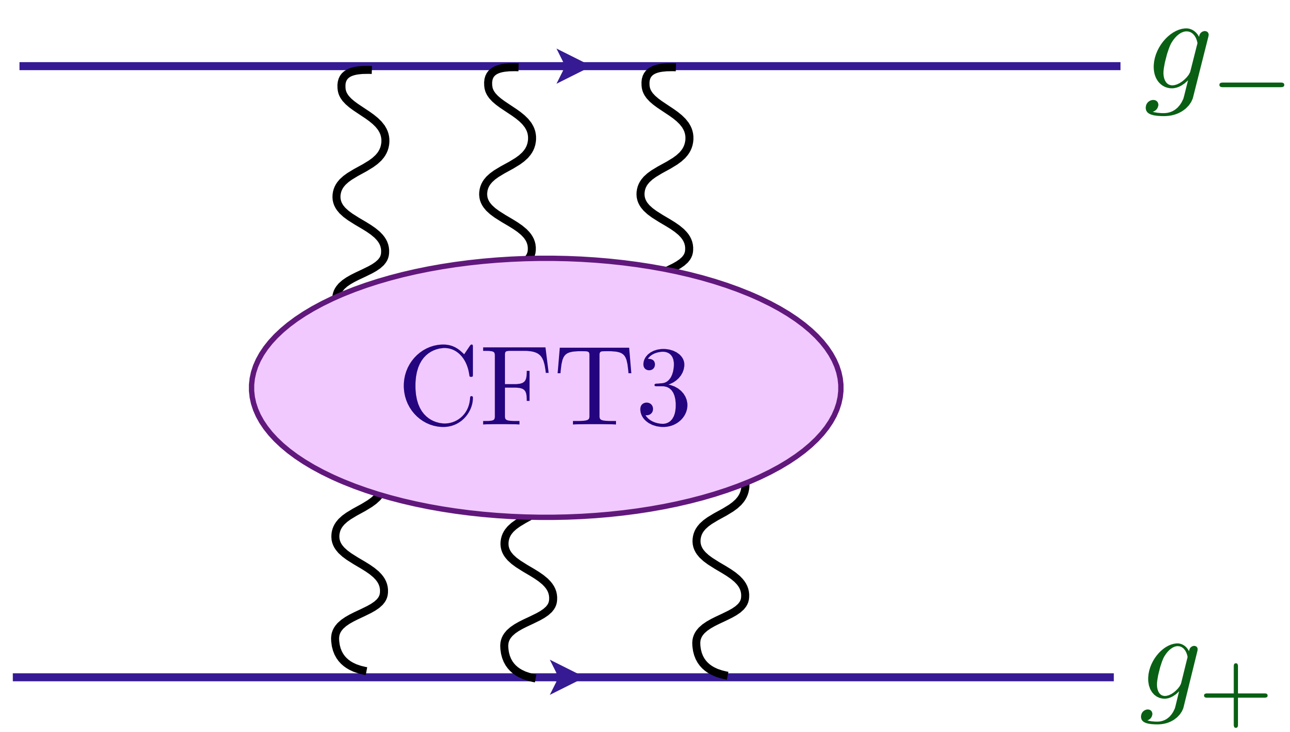
$$G(\vec{k},\omega) = \frac{1}{\omega - v_F(|\vec{k}| - k_F) + c\,\omega[\ln(|\omega|) + i\pi\mathrm{sgn}(\omega)]}$$



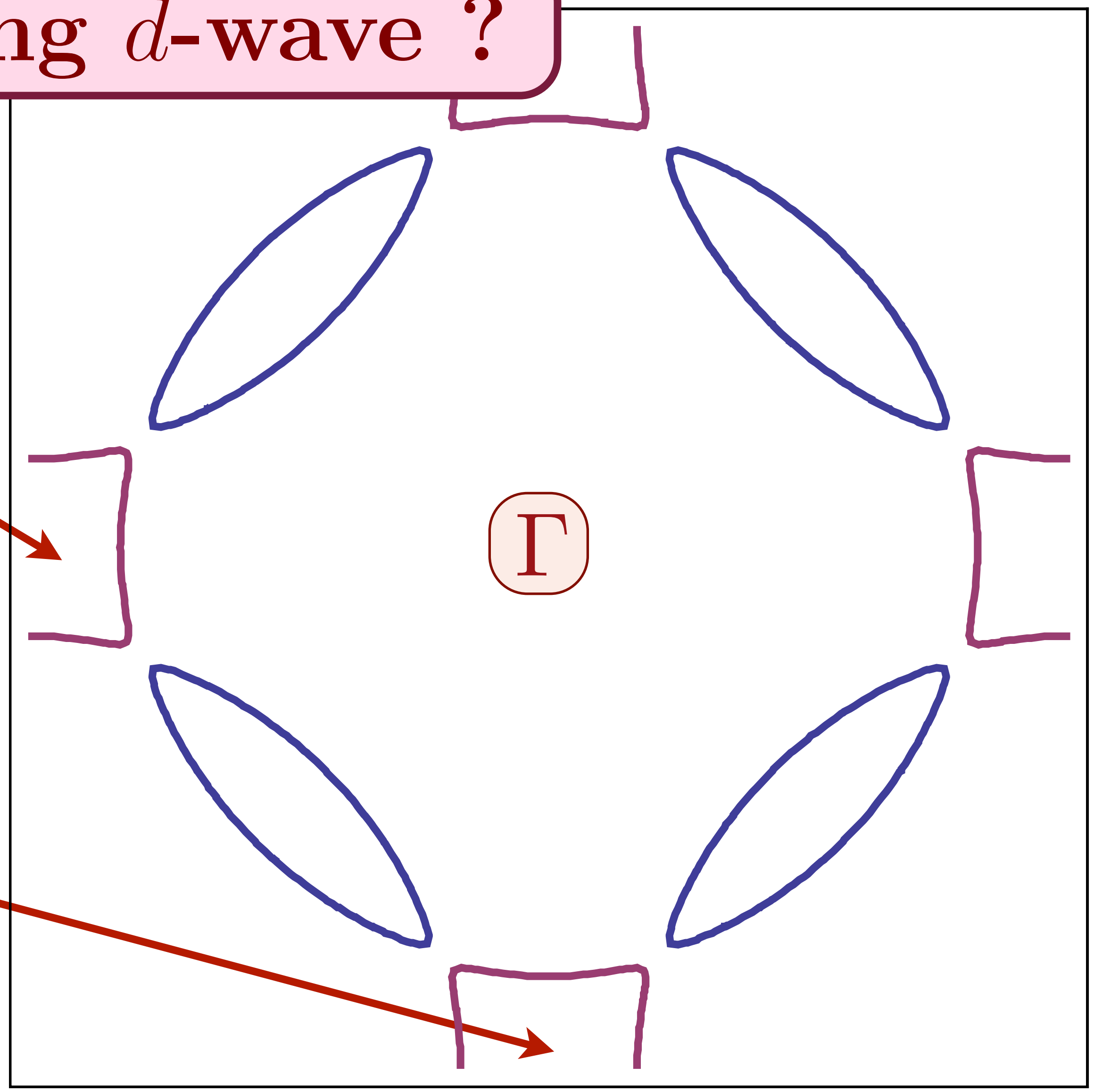


- Begin with a CFT3: the CP^1 model.
- Add "probe" non-relativistic fermions, g_+ and g_-, with opposite gauge charges
- Turn on fermion chemical potential: leads to a marginal Fermi liquid of $g_\pm$ (not electrons)
- Low T state is a superconductor with $\langle g_+ g_- \rangle = \Delta \neq 0$





Why is the pairing d-wave ?



Focus on pairing near $(\pi, 0)$, $(0, \pi)$, where $\psi_\pm \equiv g_\pm$,
and the electron operators are

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix}$$

$$\mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

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Electron $c_{2\alpha}$,
spinless fermion $g_\pm$

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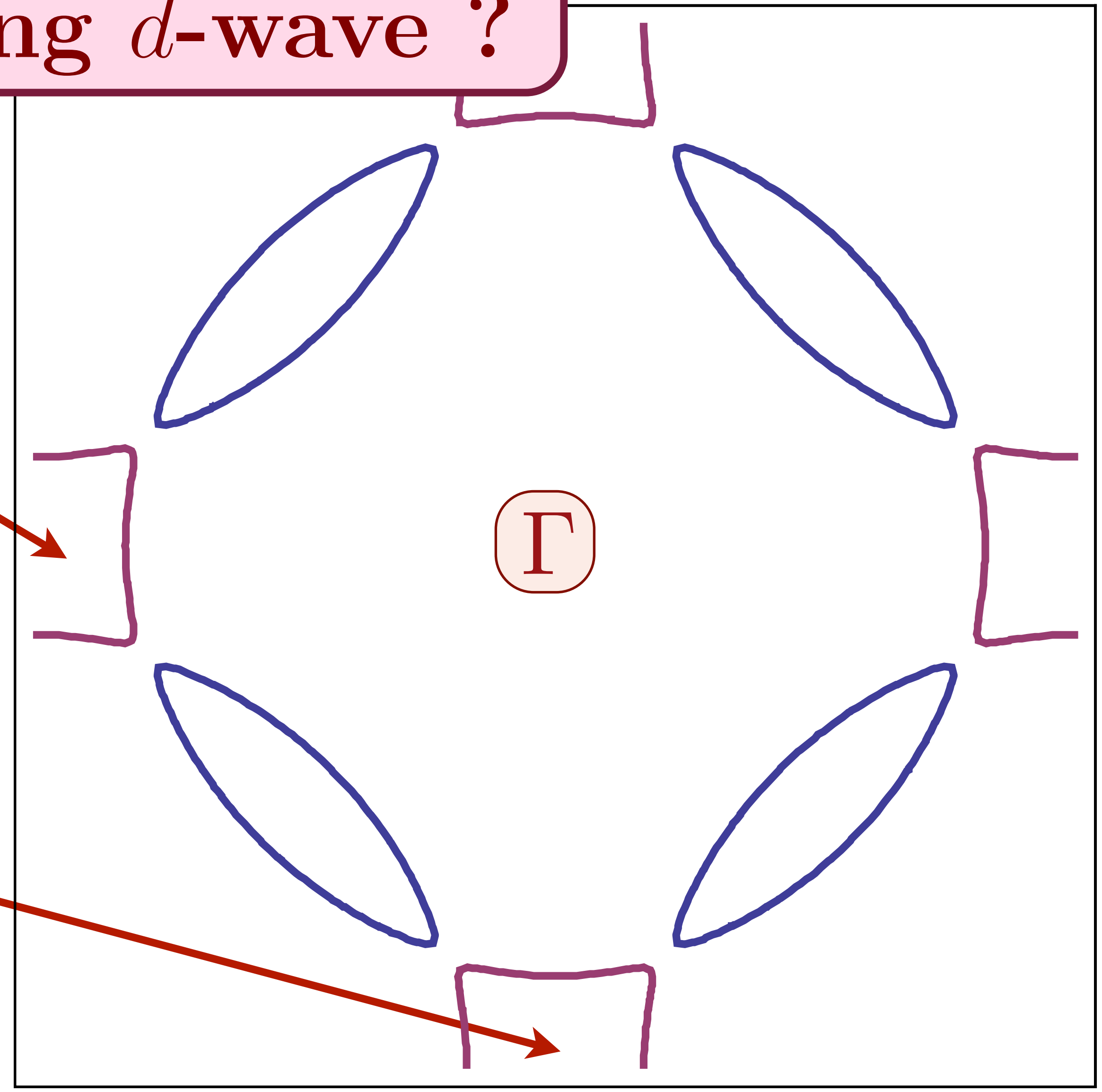

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Why is the pairing d-wave ?

Fluctuating pocket theory for electrons near $(0, \pi)$ and $(\pi, 0)$

Attractive gauge forces lead to simple s-wave pairing of the $g_\pm$

$$\langle g_+ g_- \rangle = \Delta$$

For the physical electron operators, this pairing implies

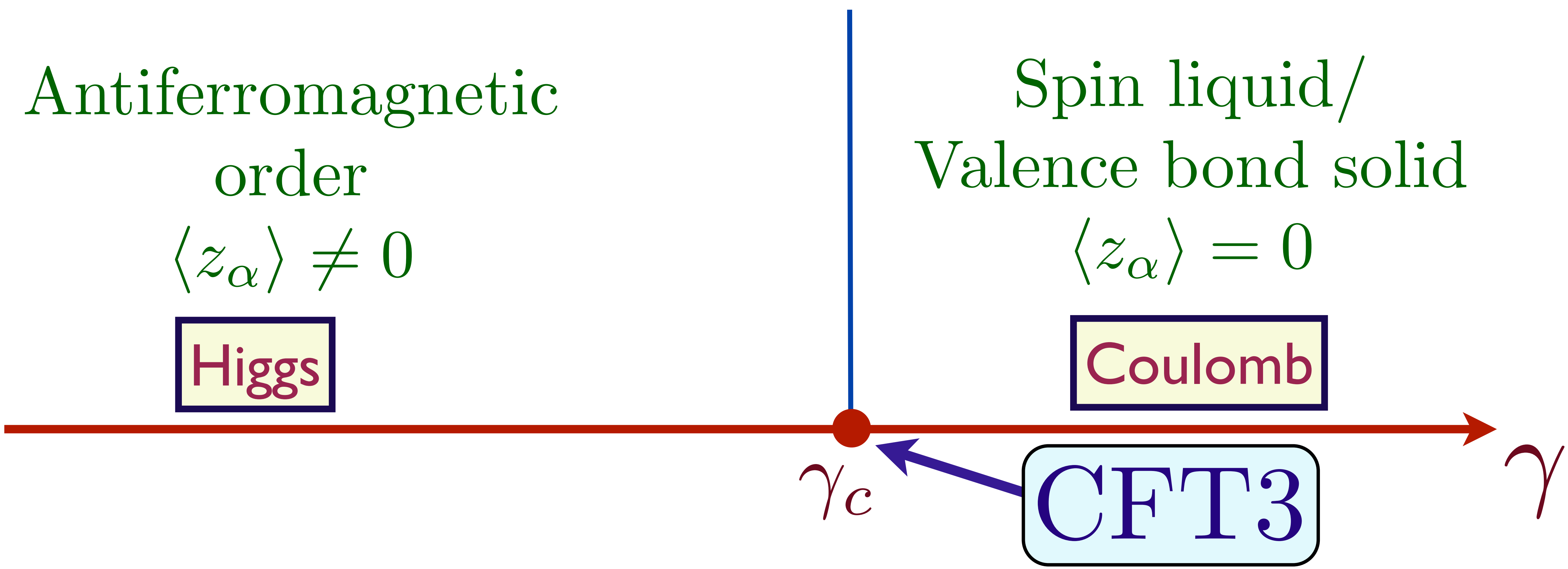
$$\begin{aligned} \langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \left\langle |z_\alpha|^2 \right\rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \left\langle |z_\alpha|^2 \right\rangle \end{aligned}$$

$i.e.$ d-wave pairing !

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

T=0 Phase diagram

$$\mathcal{L}_z = \frac{1}{\gamma}|(\partial_\mu - iA_\mu)z_\alpha|^2 \quad ; \quad |z_\alpha|^2 = 1$$



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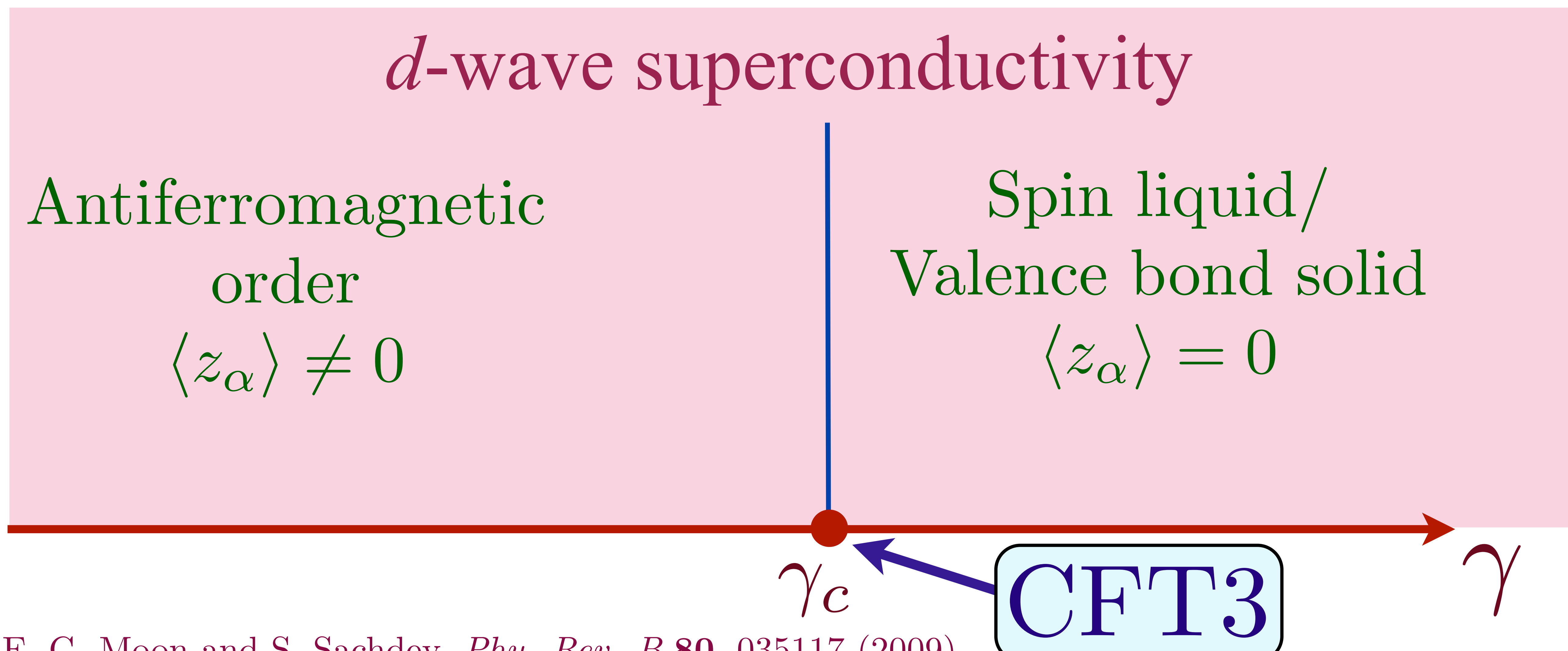

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

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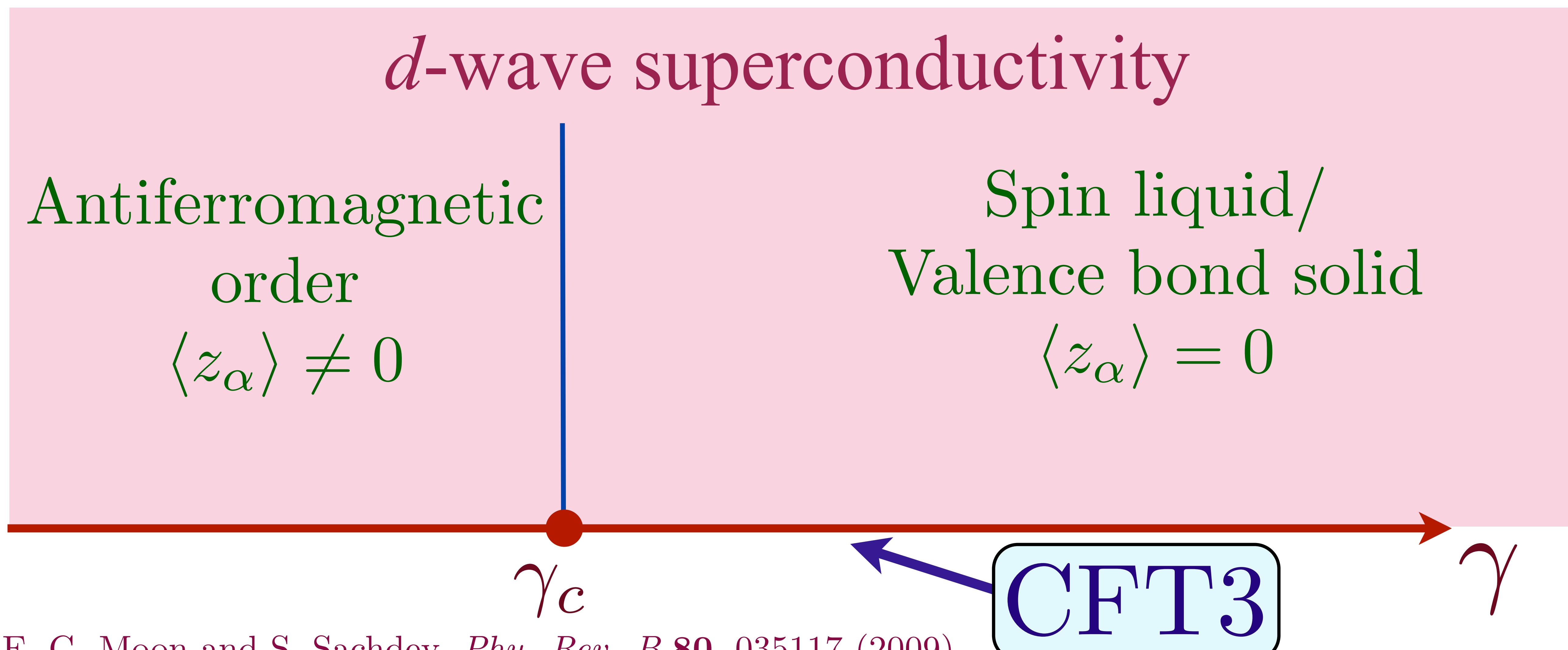

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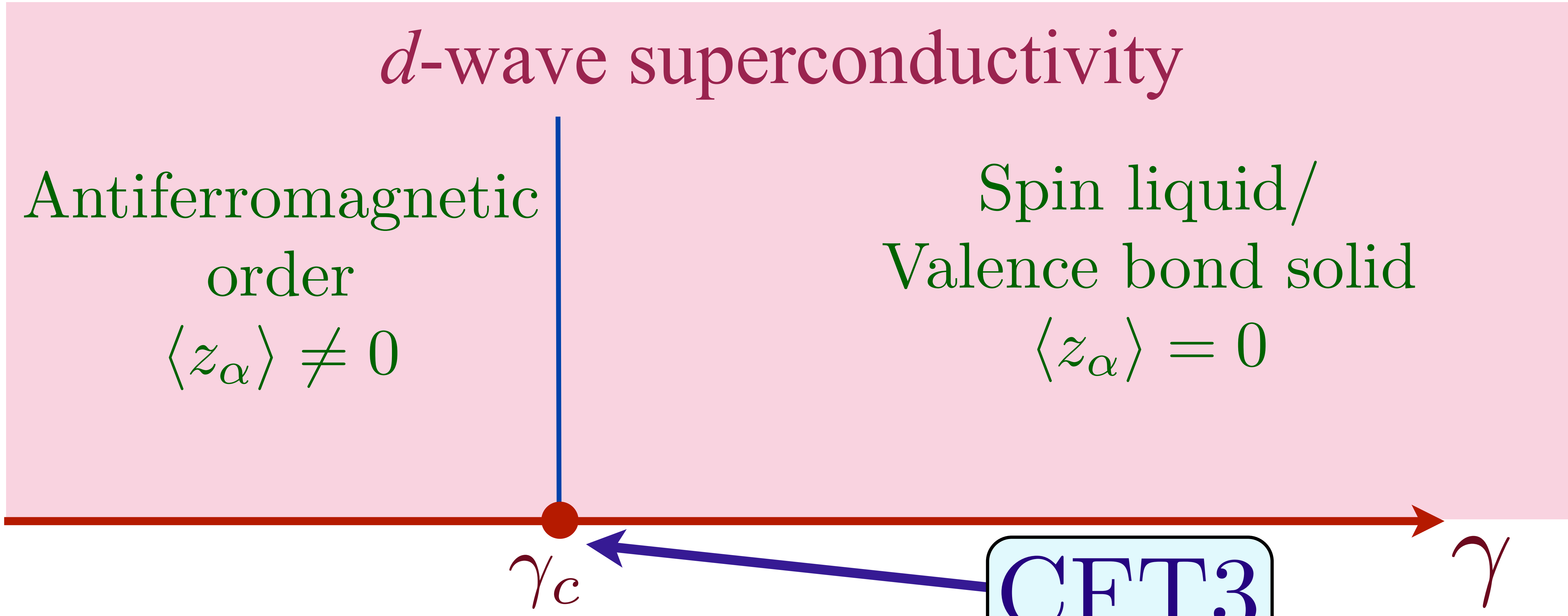
T=0 Phase diagram

Competition between antiferromagnetism and superconductivity shrinks region of antiferromagnetic order: feedback of "probe fermions" on CFT is important



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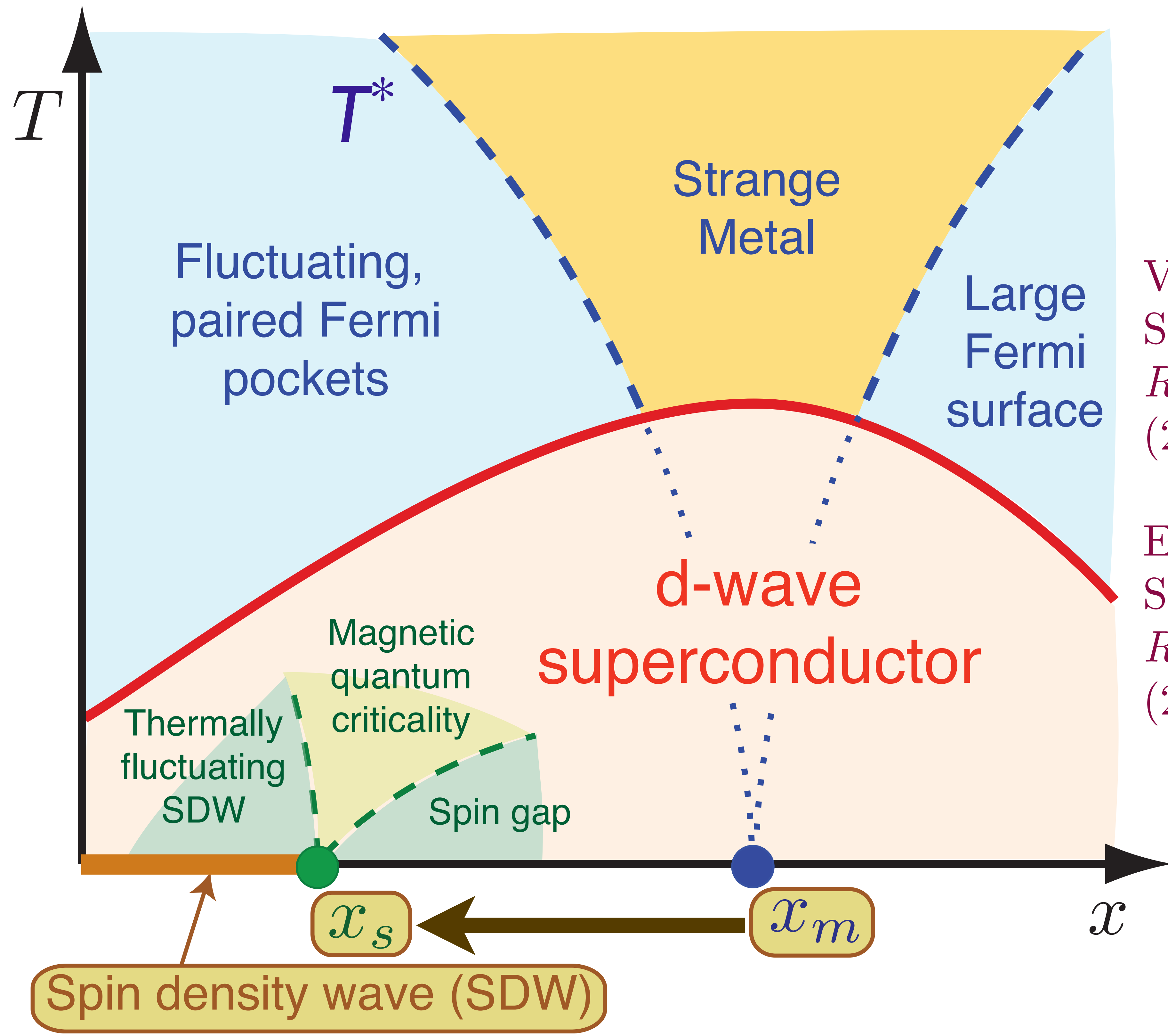
Theory of quantum criticality in the cuprates



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Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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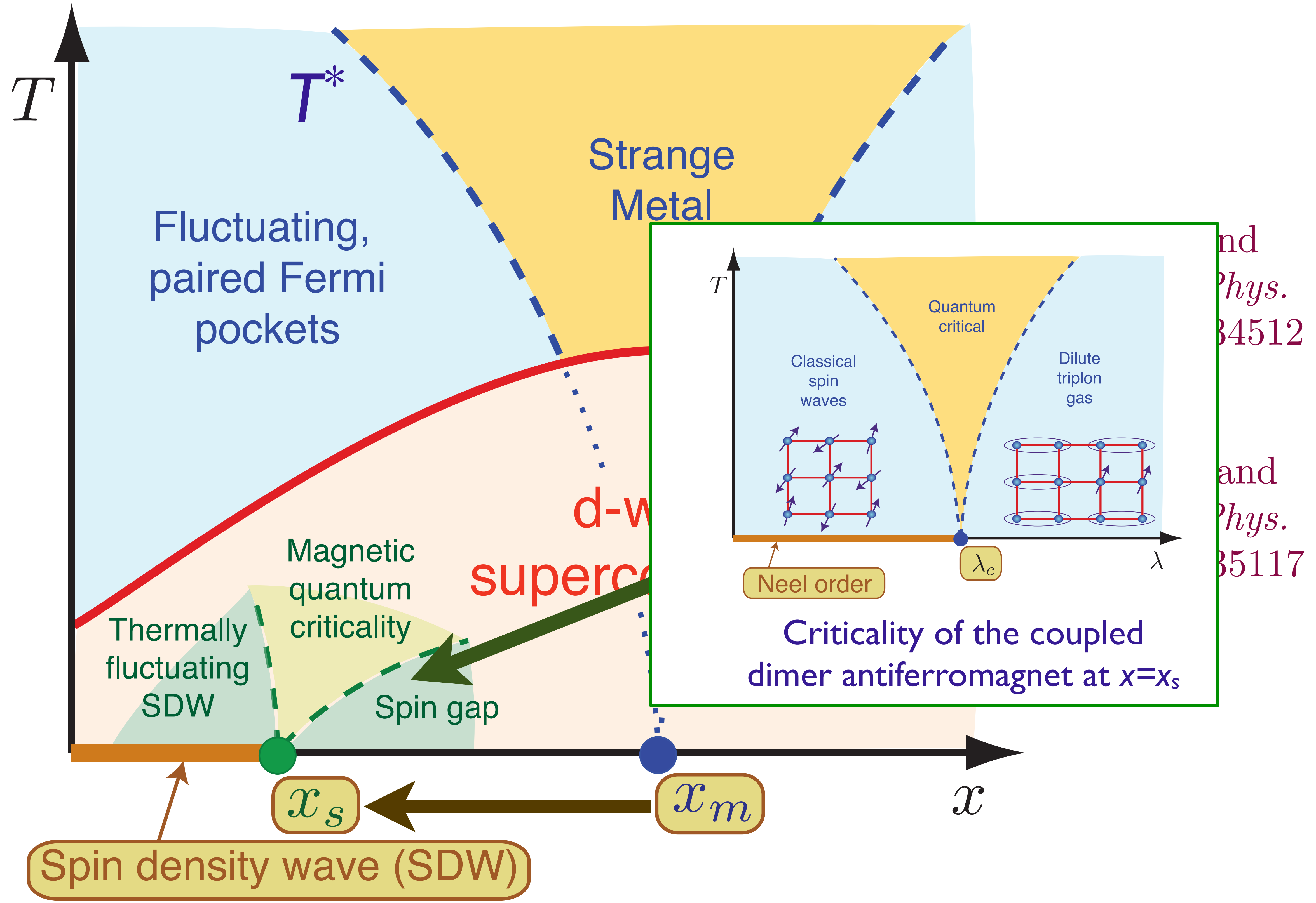

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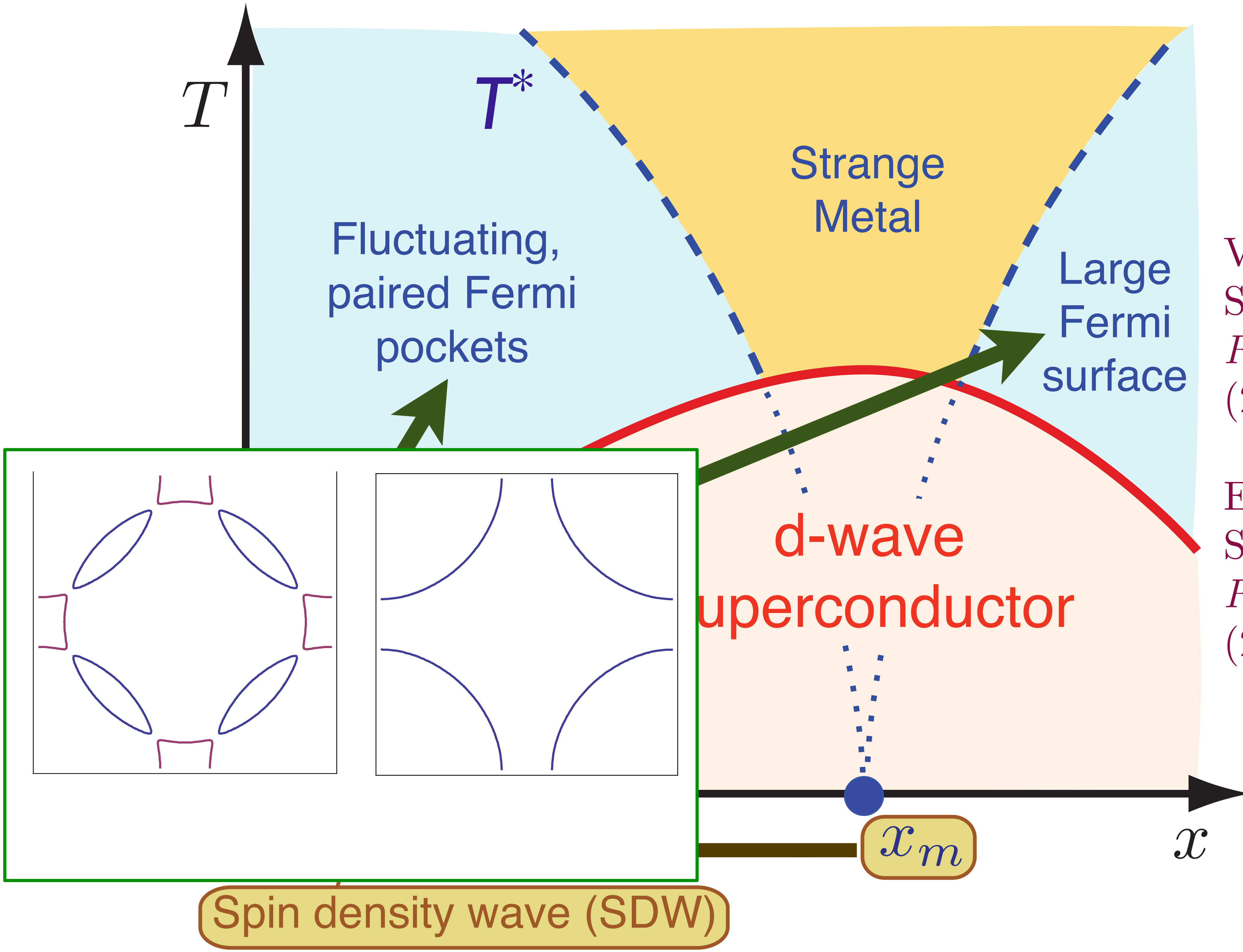

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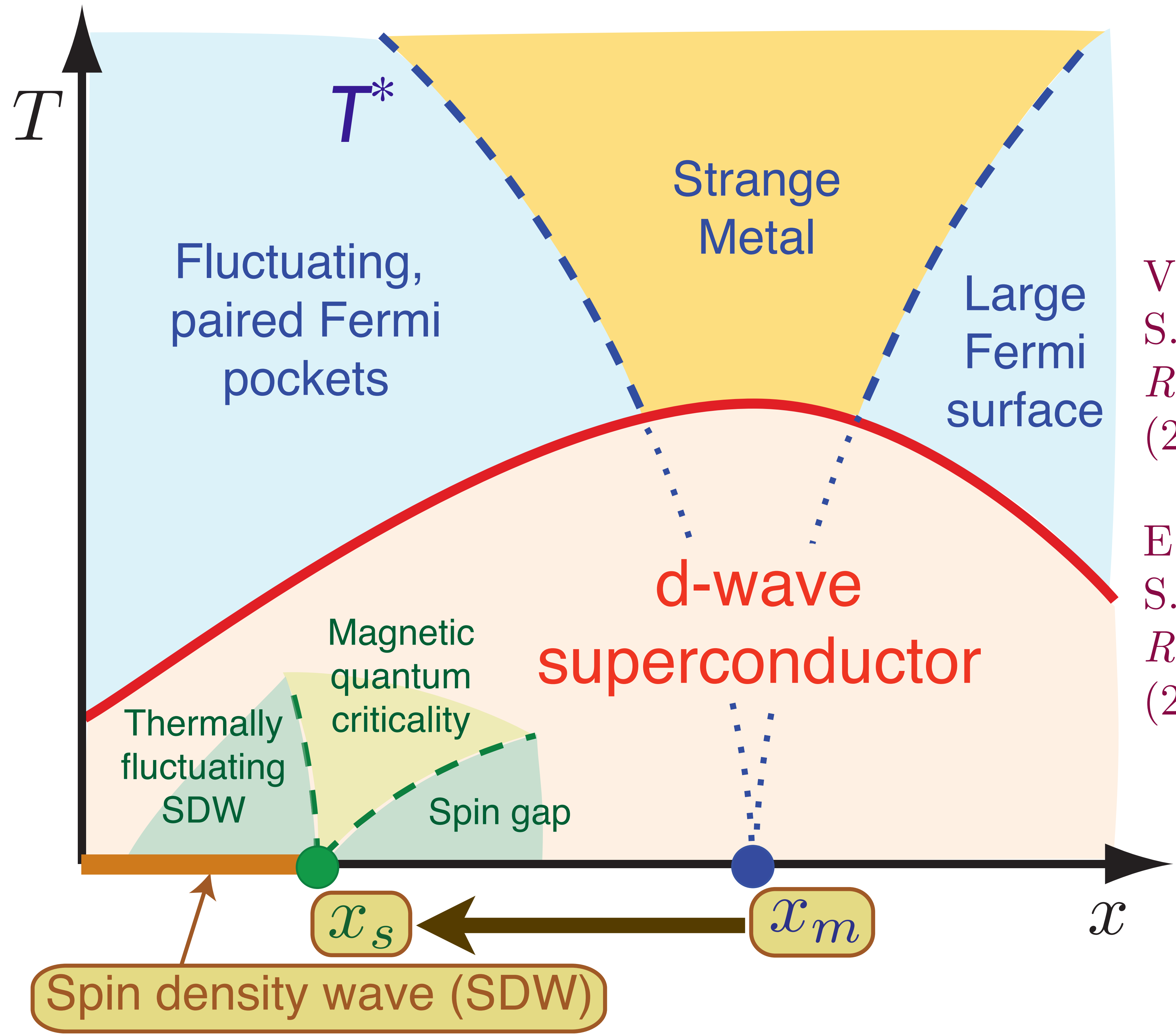
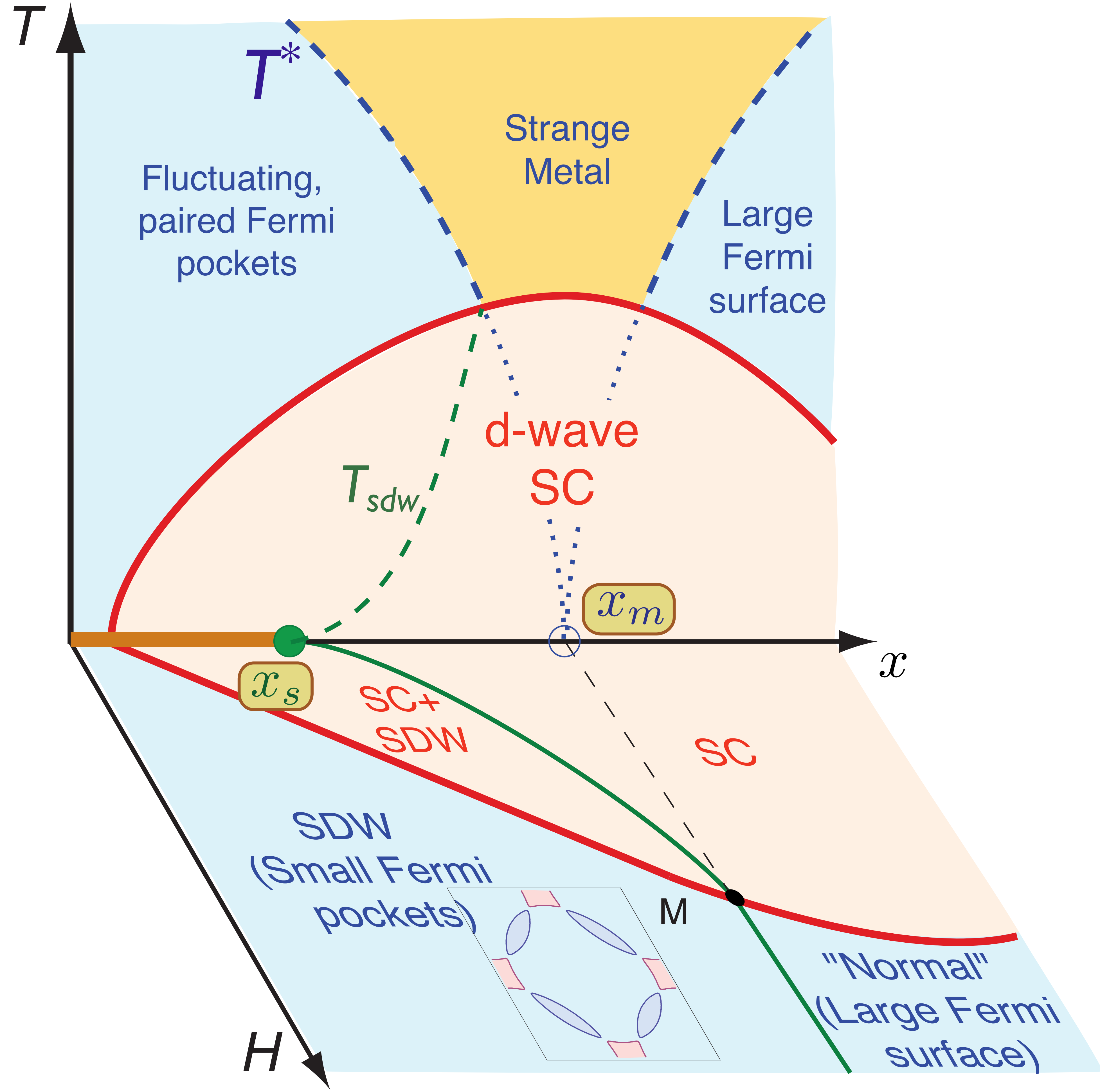


V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

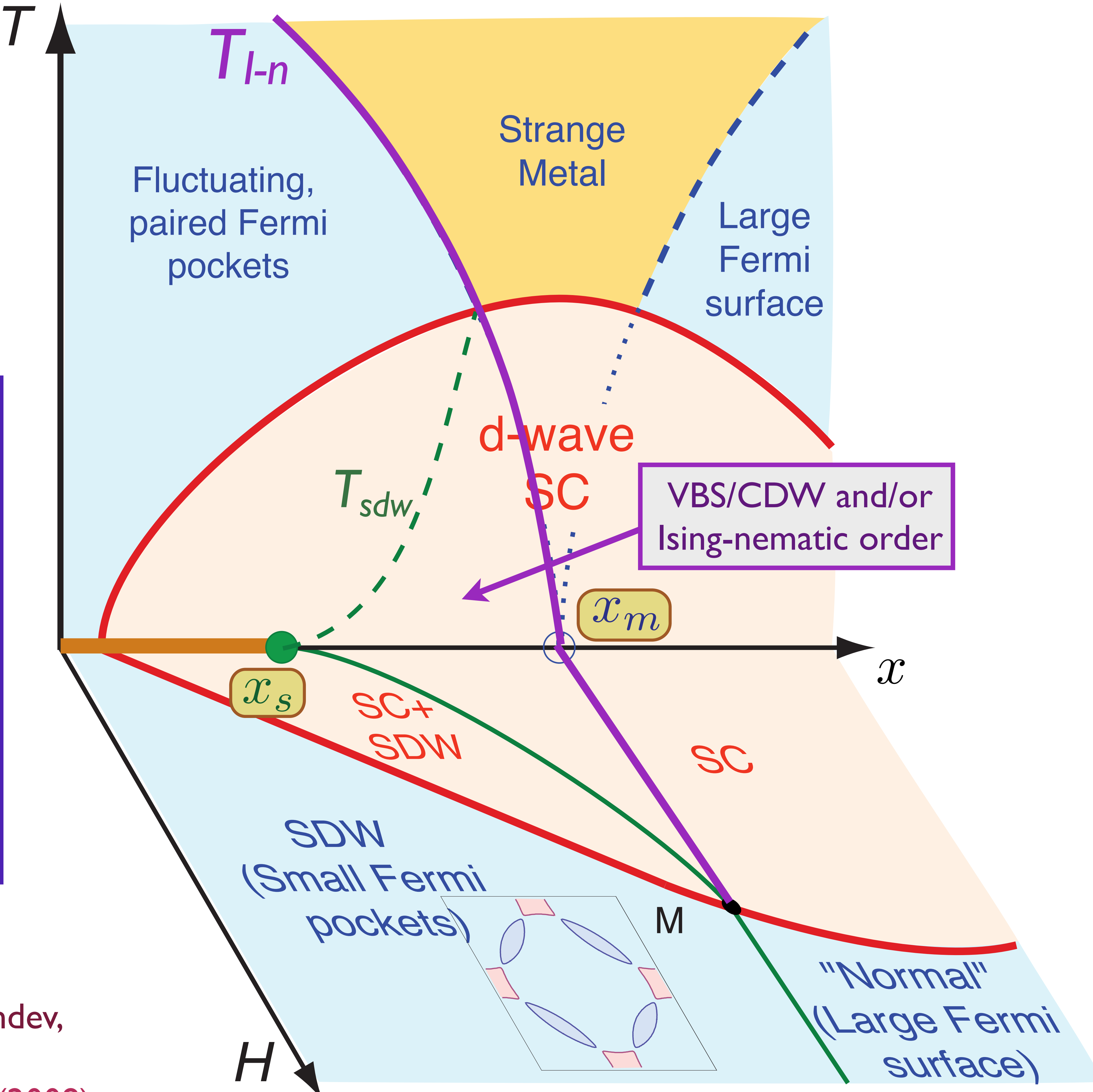
E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Physics of competition: *d*-wave SC and SDW "eat up" same pieces of the large Fermi surface.



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

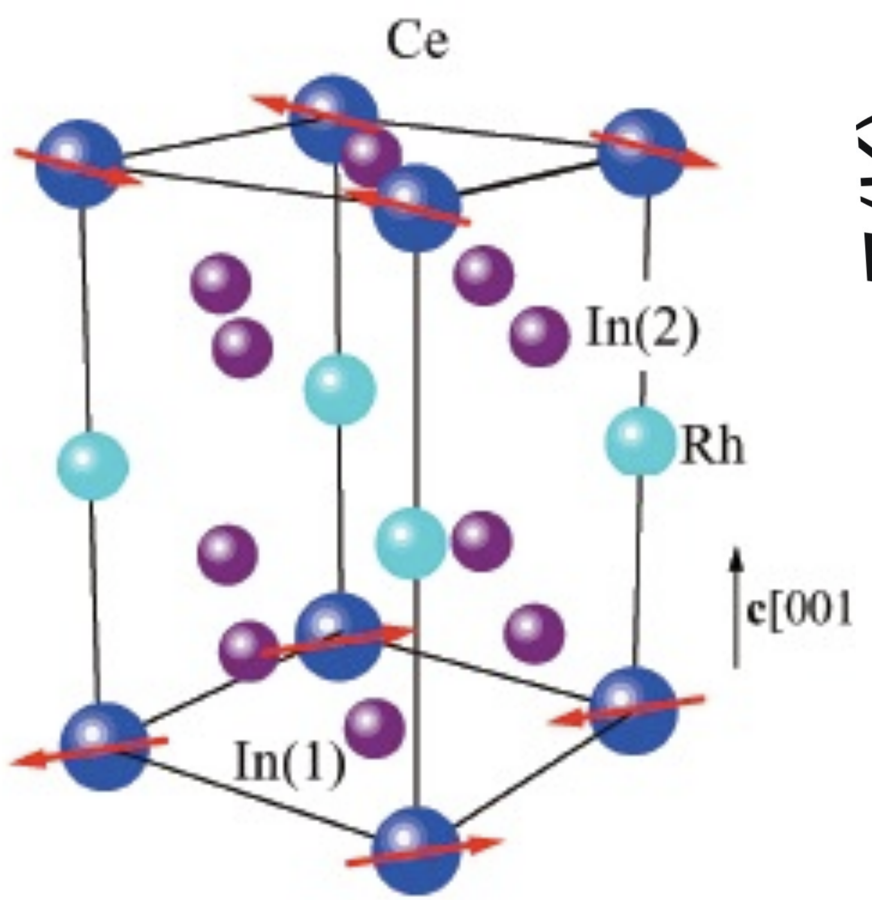
E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

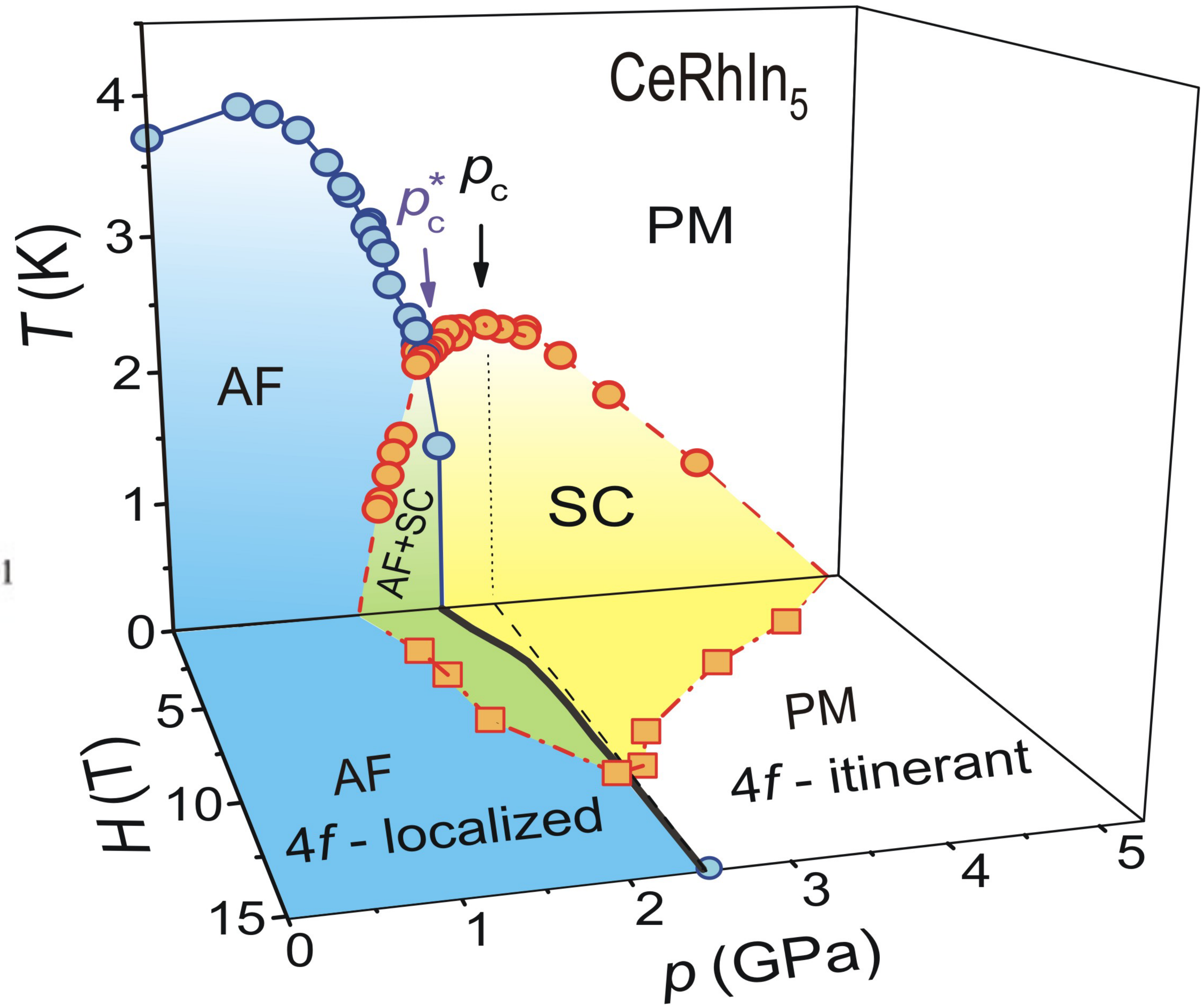


Onset of superconductivity disrupts SDW order, but VBS/CDW/ Ising-nematic ordering can survive

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

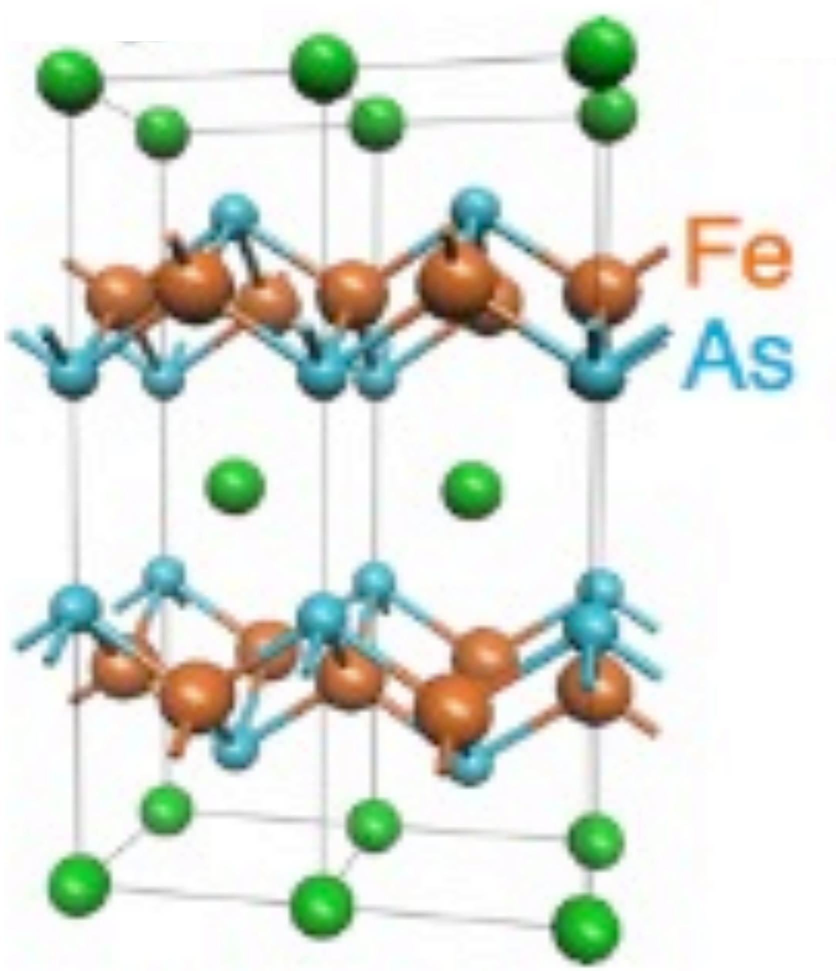
Similar phase diagram for $CeRhIn_5$



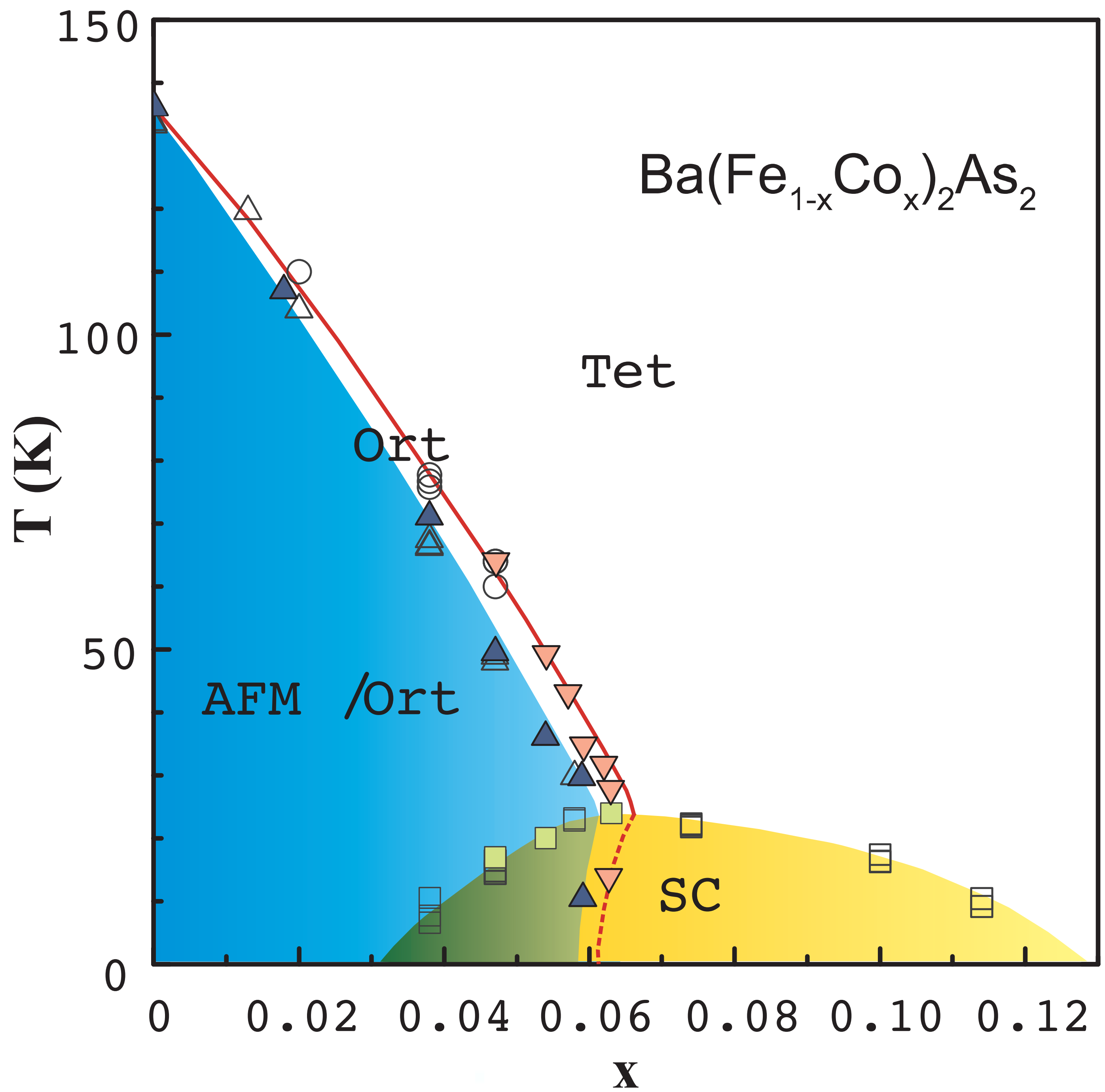


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Similar phase diagram for the pnictides



$BaFe_2As_2$
Unit cell



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Conclusions

Theory of Ising-nematic ordering in a two-dimensional metal:
line singularities in momentum space, and an emergent dimension: 2+1 dimensional field theories labeled by points on the Fermi surface

Conclusions

Gauge theory for pairing of Fermi pockets in a metal with fluctuating spin density wave order: Many qualitative similarities to holographic strange metals and superconductors