

# A quantum dimer model for the pseudogap metal

College de France, Paris  
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PERIMETER INSTITUTE  
FOR THEORETICAL PHYSICS



JOHN TEMPLETON  
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PHYSICS



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Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)



Andrea Allais



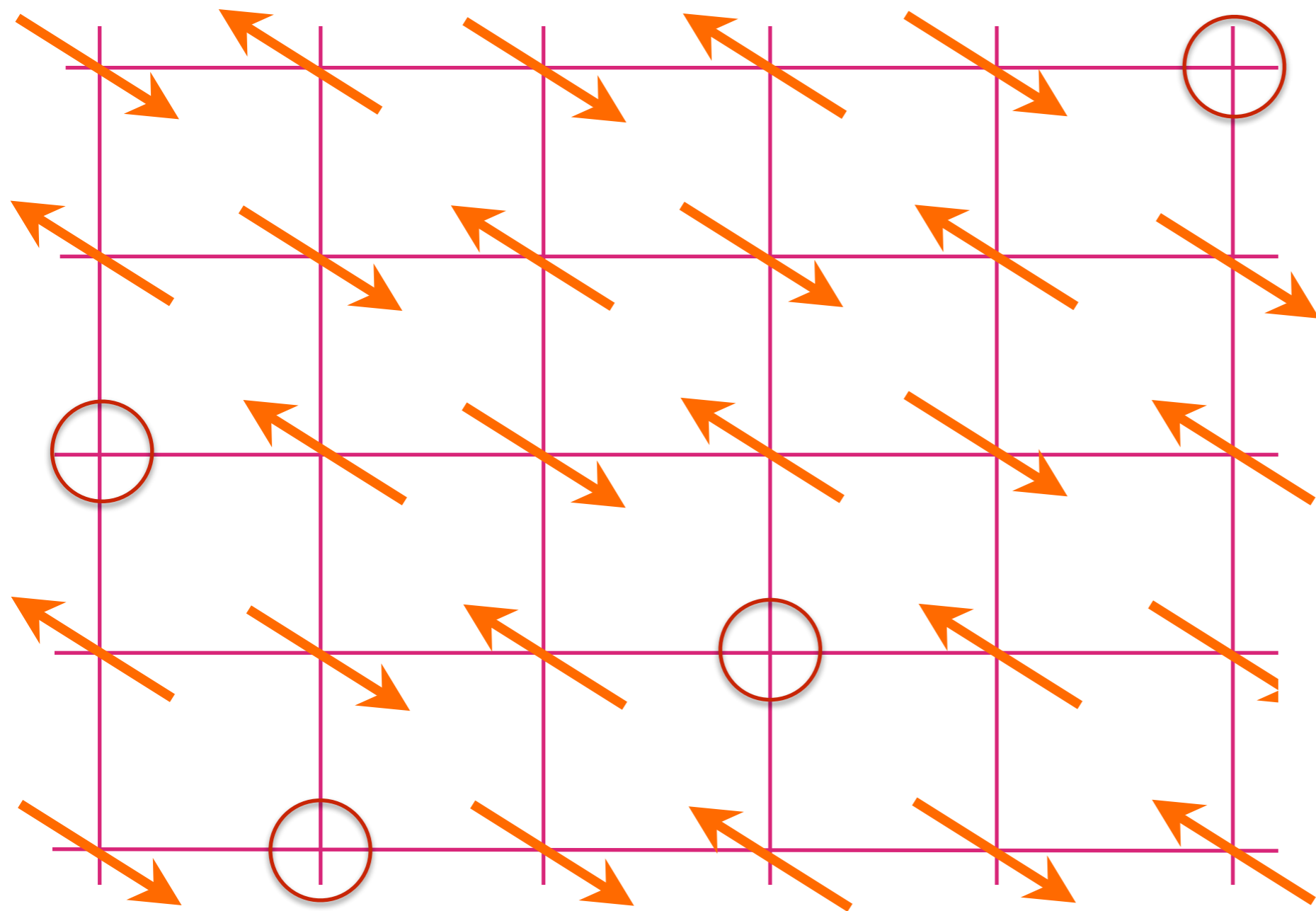
Debanjan  
Chowdhury



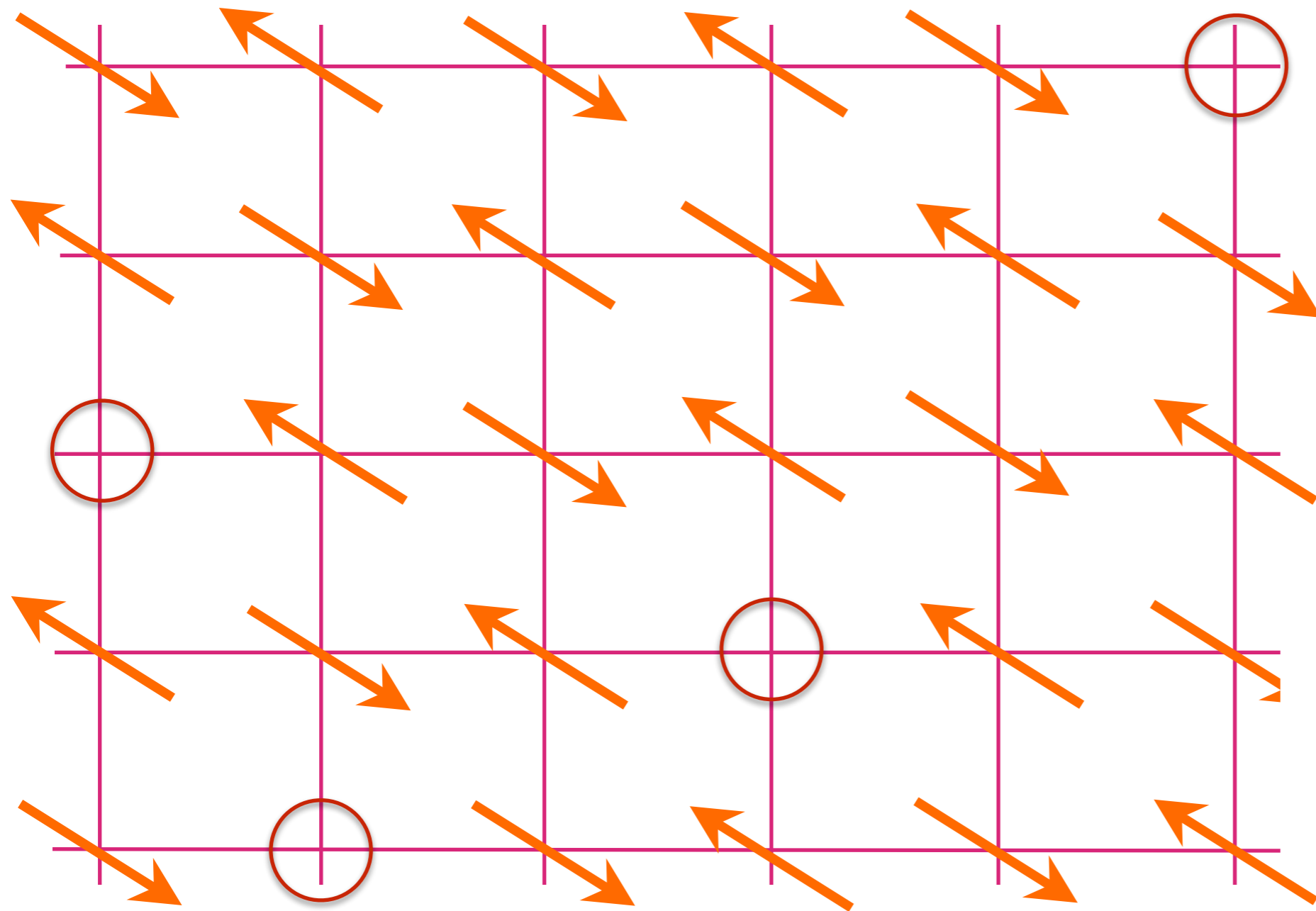
Alexandra  
Thomson



Matthias Punk  
(Innsbruck)

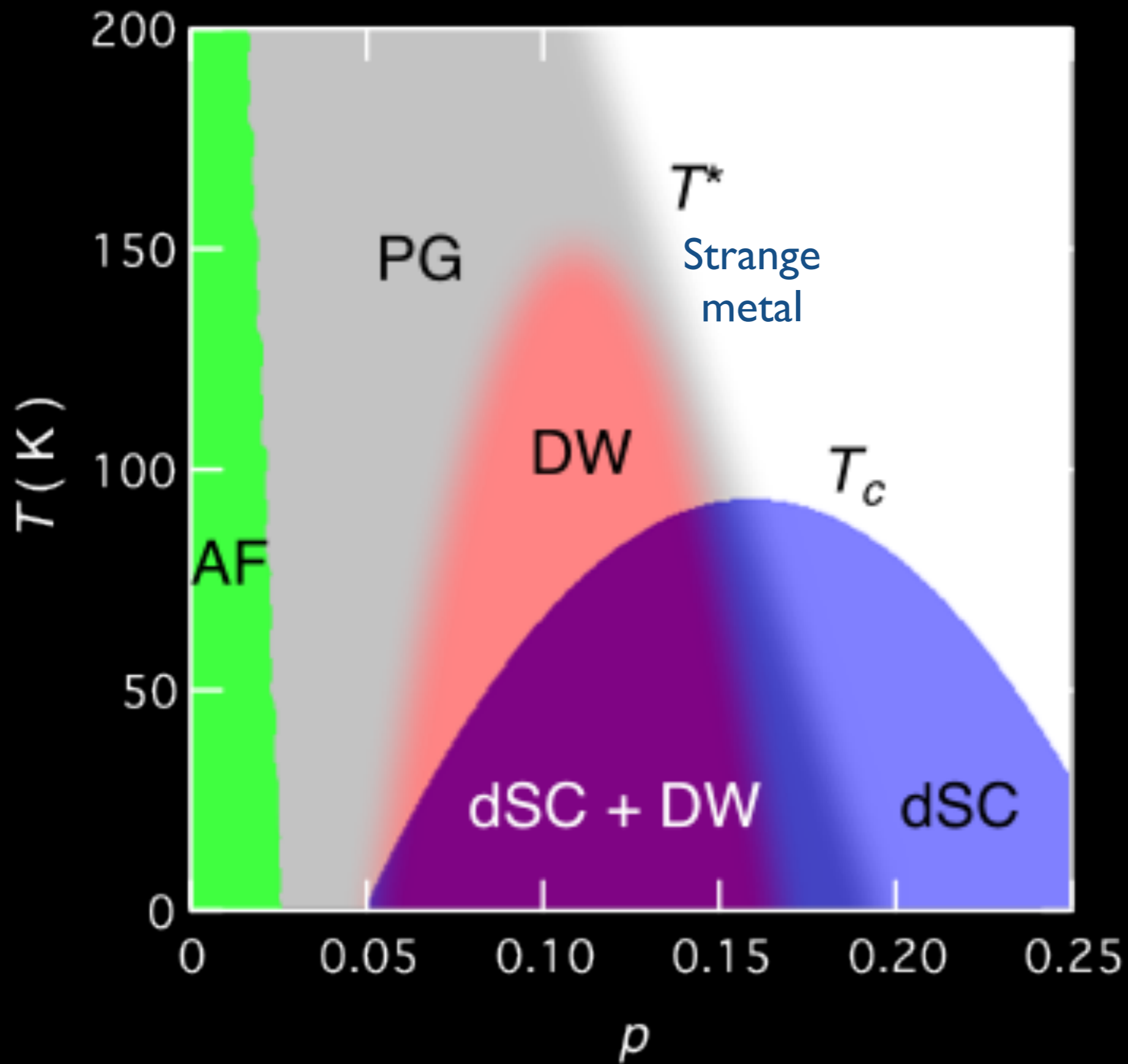


Anti-ferromagnet  
with  $p$  holes  
per square

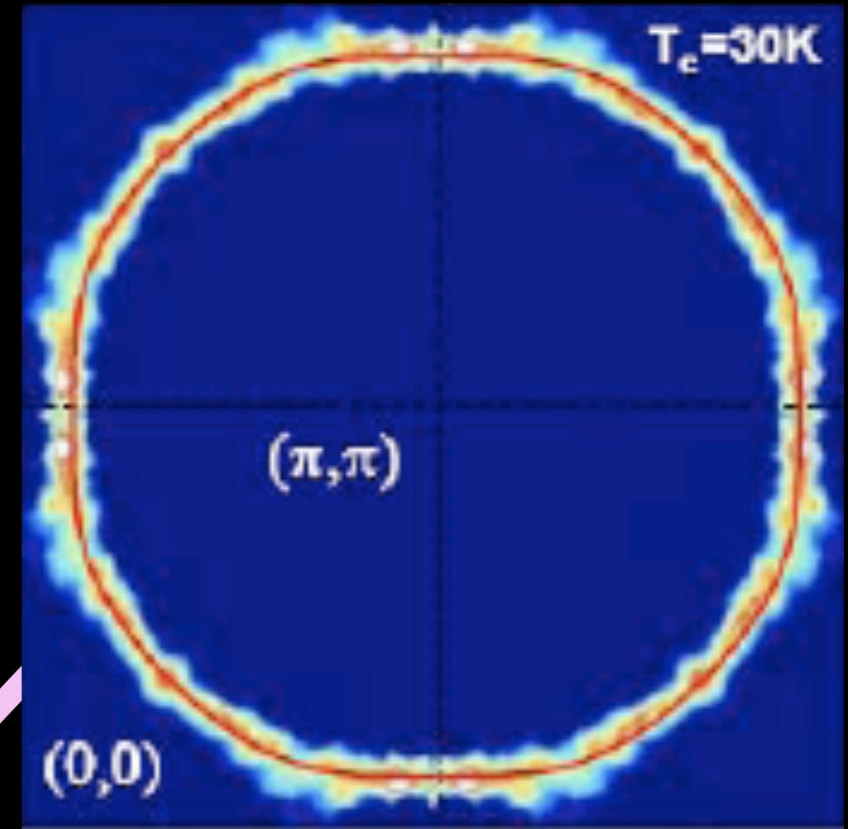
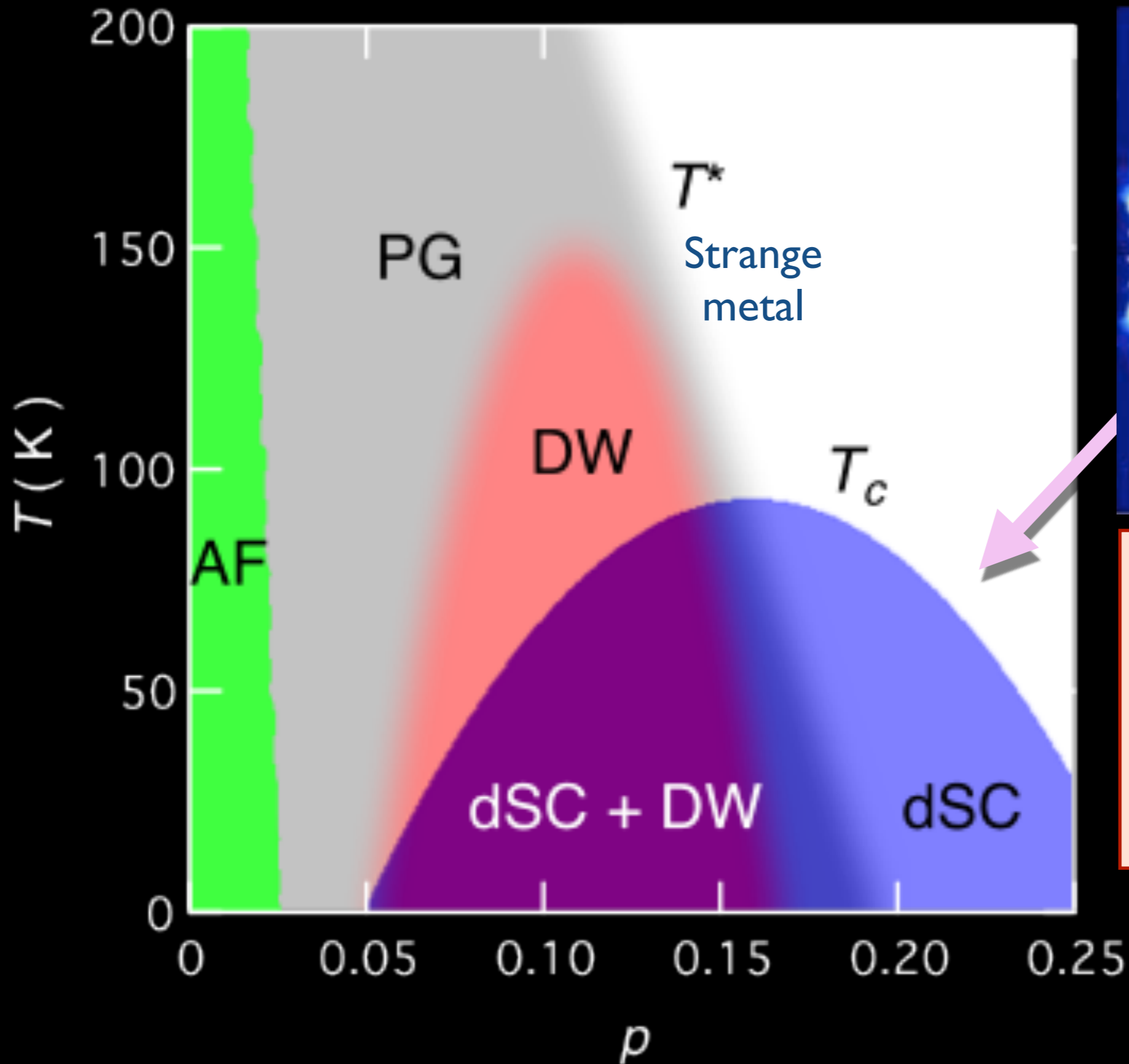


Anti-ferromagnet  
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per square

But relative  
to the band  
insulator,  
there are  
 $1 + p$  holes  
per square

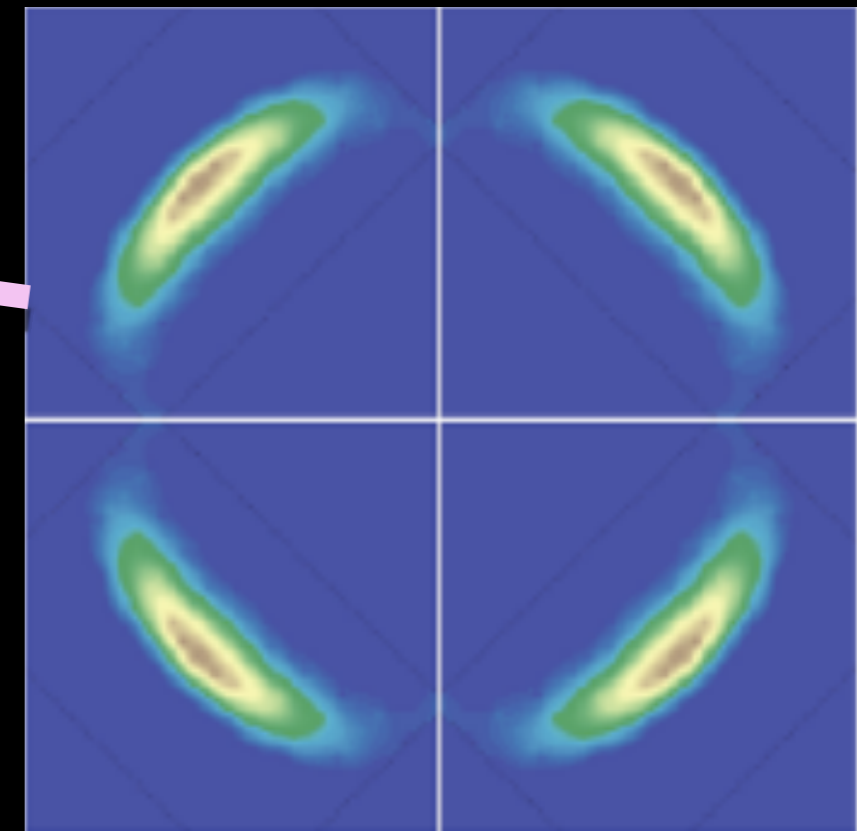
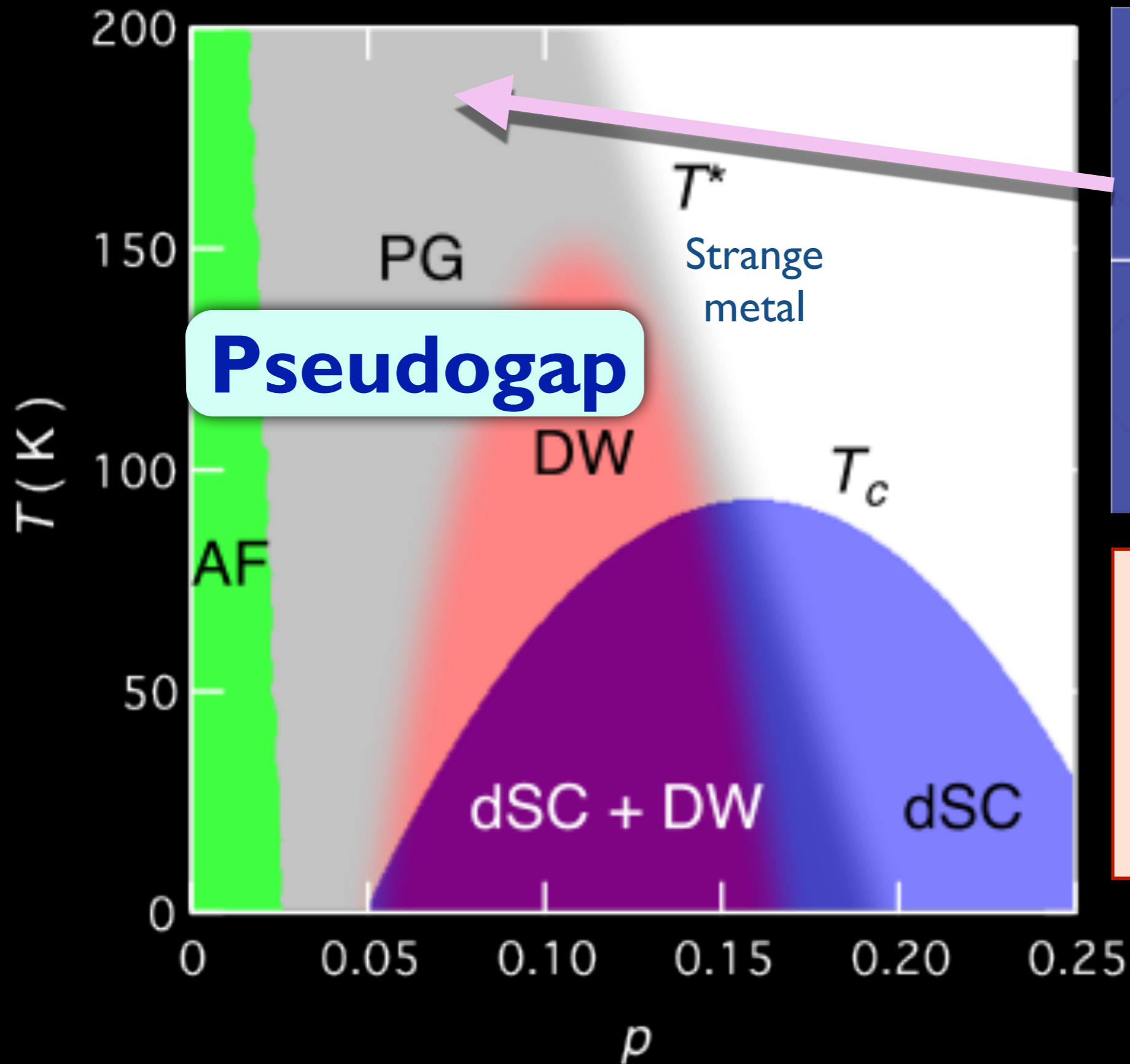


M. Platié, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

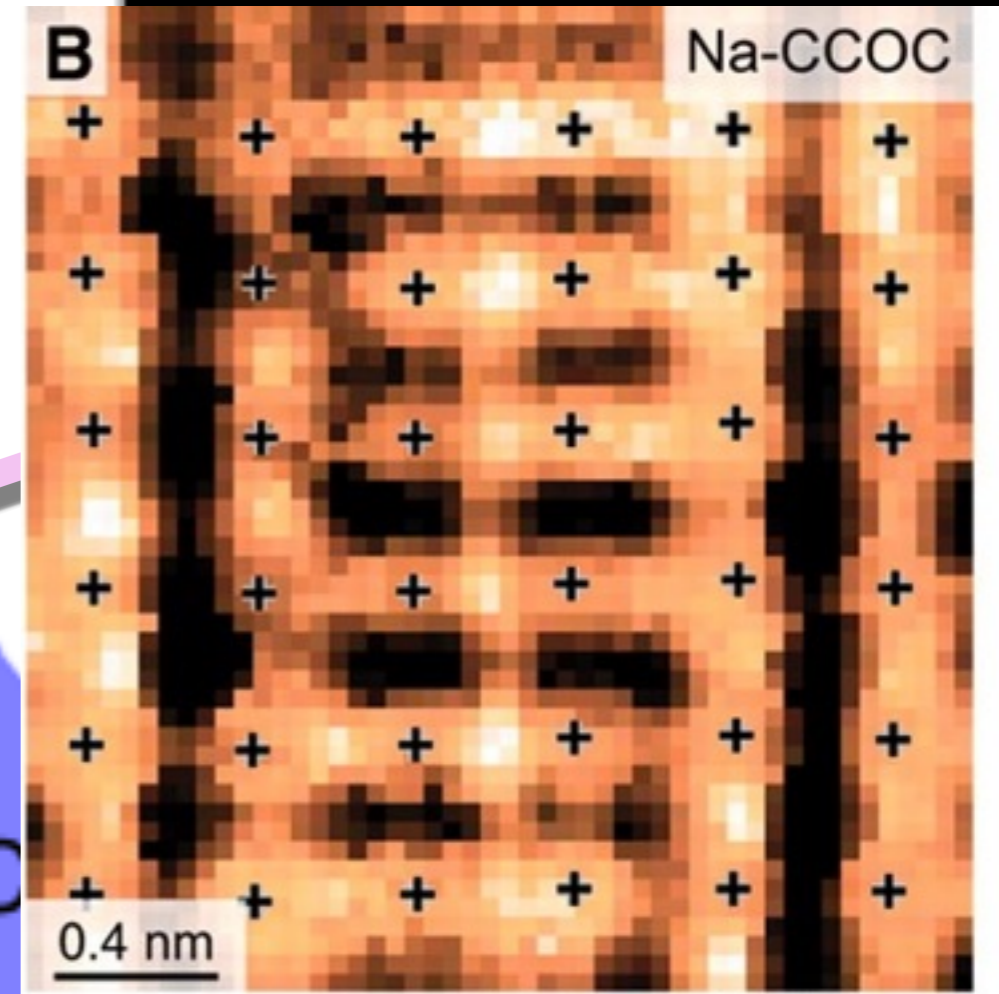
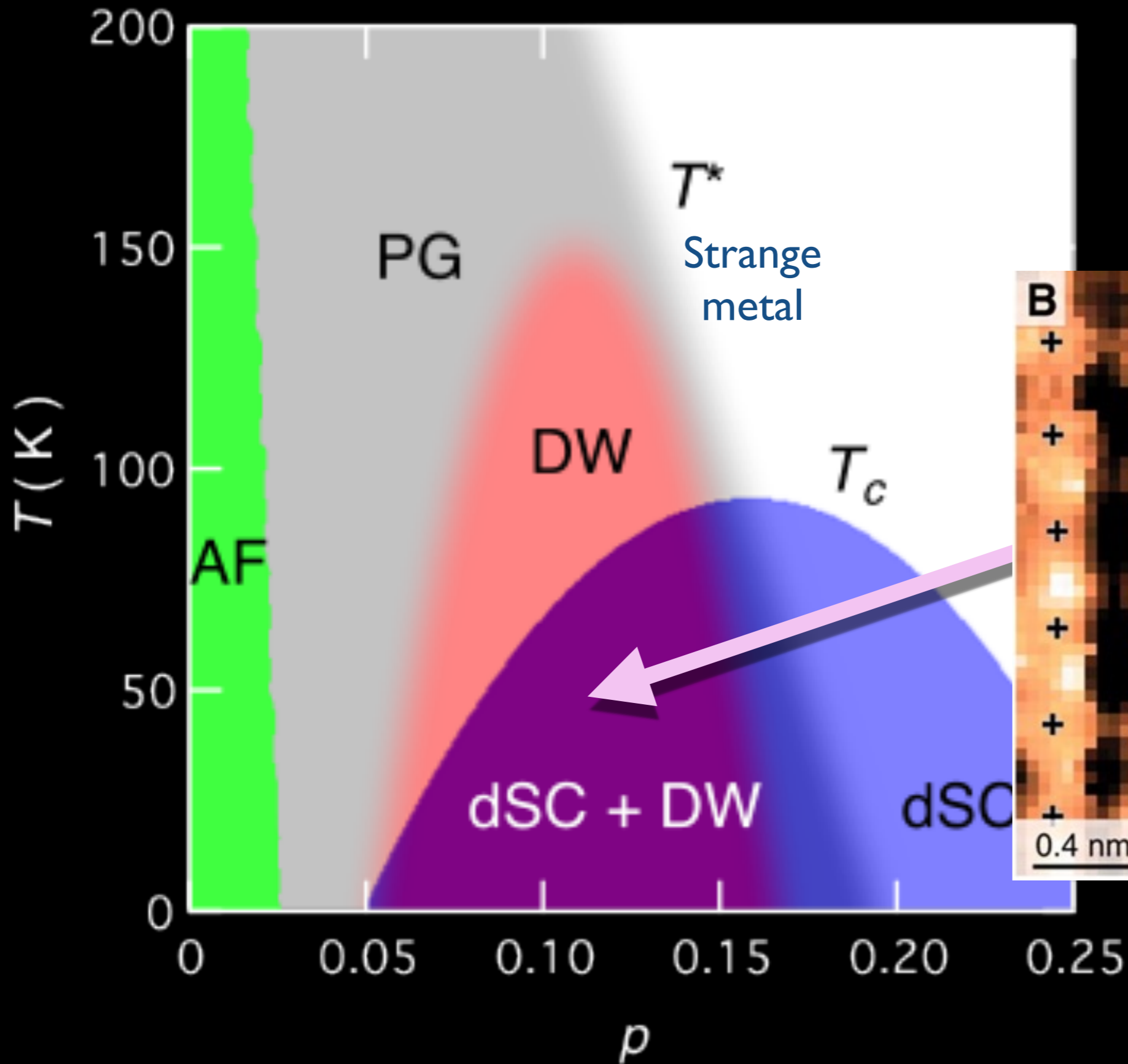


Fermi liquid:  
Area enclosed  
by Fermi surface  
 $= | + p$

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)

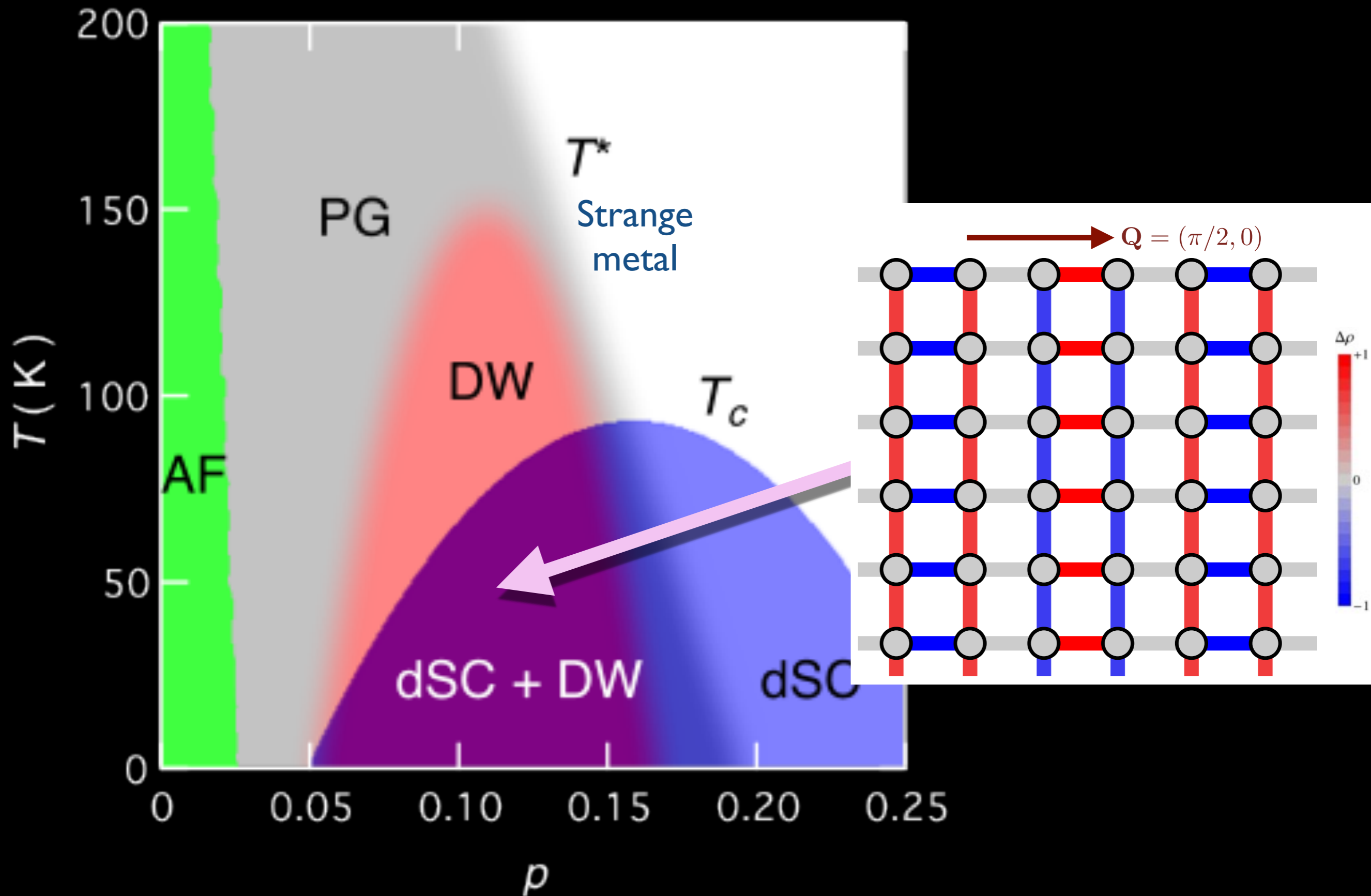


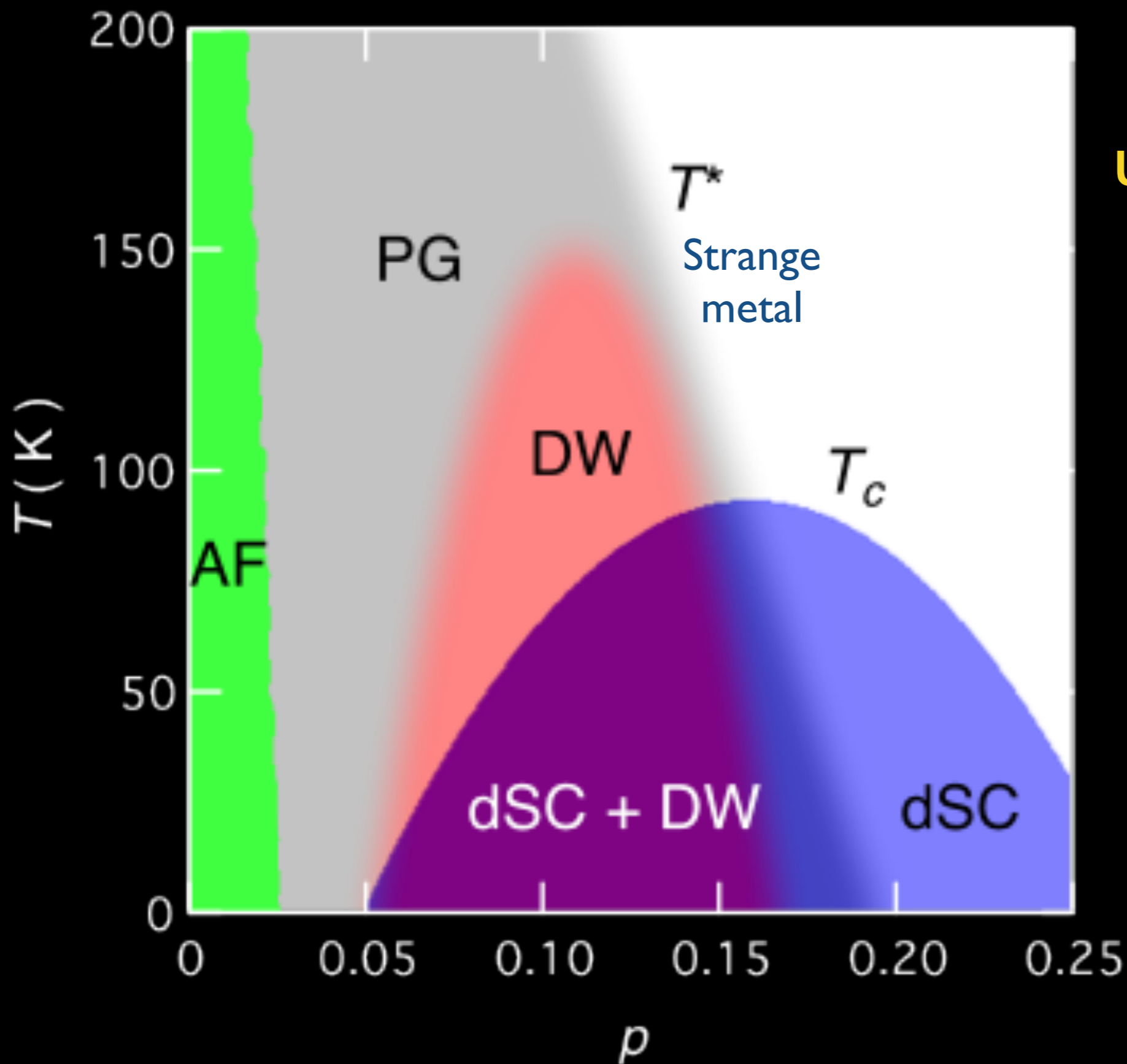
“Fermi arcs”  
at  
low  $p$



M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).

K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)





How do we understand the Fermi arc spectrum, and what is its relationship to the density wave (DW) order at lower  $T$ ?

Is the higher temperature pseudogap  
(with ``Fermi arc" spectra) described by

(A) Thermal fluctuations of the low  
temperature orders (superconductivity,  
density wave, antiferromagnetism...)

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(A) Thermal fluctuations of the low  
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OR

(B) A new type of metal, which can be stable  
(in principle) as a quantum ground state

# Reasons for working with option (B)

- Pseudogap appears already at high temperatures where there are no observed density wave correlations, and no pairing fluctuations.

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- No room for antinodal Fermi surfaces in high field, low  $T$  specific heat.
- Suppressed paramagnetic susceptibility at high fields and low  $T$

Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size  $p$  ?

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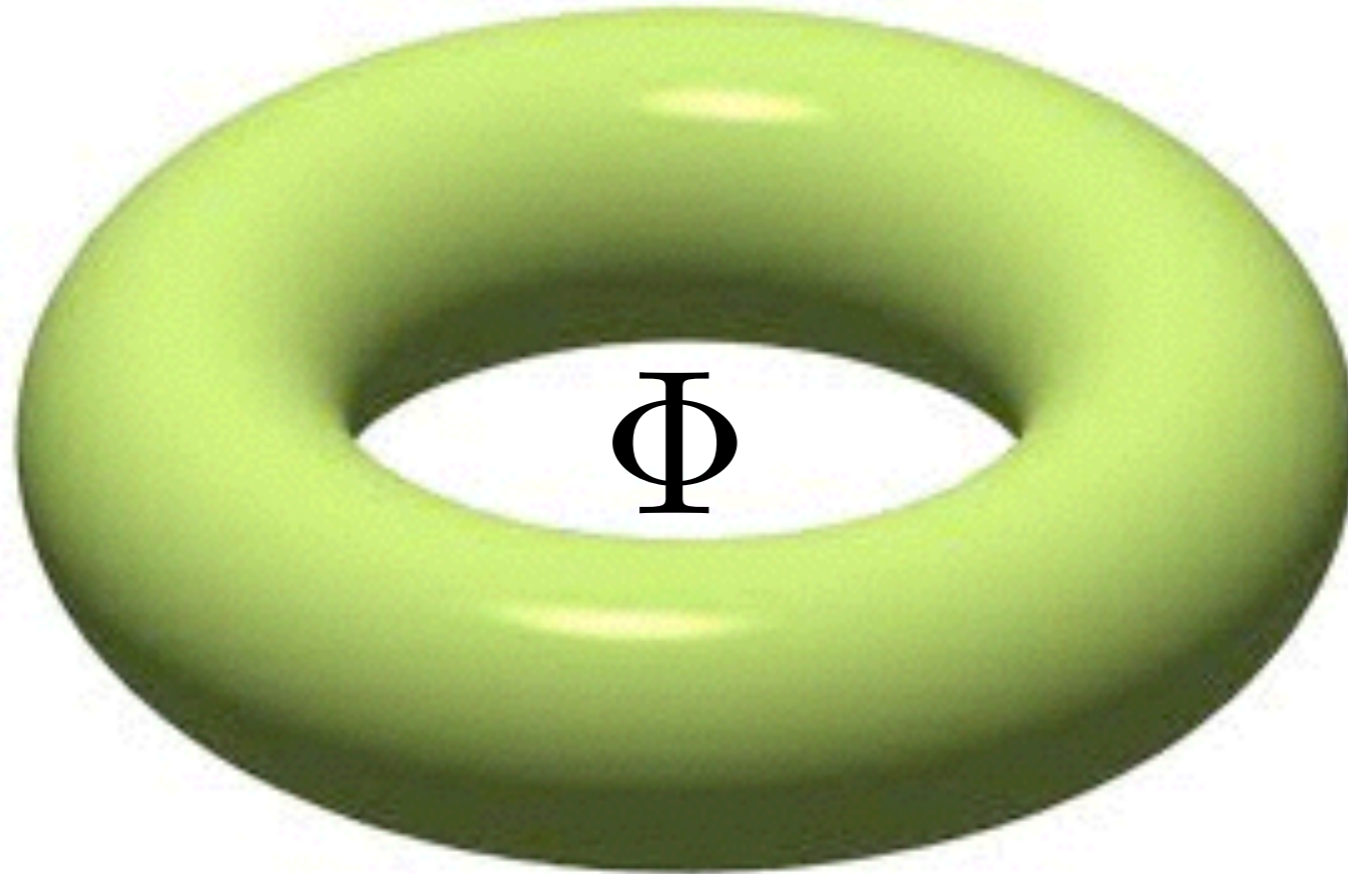
The Luttinger theorem for a Fermi liquid requires a Fermi surface of size  $1+p$ .

Answer: Yes.

There can be a Fermi surface of size  $p$ , but it must be accompanied by “topological order”.

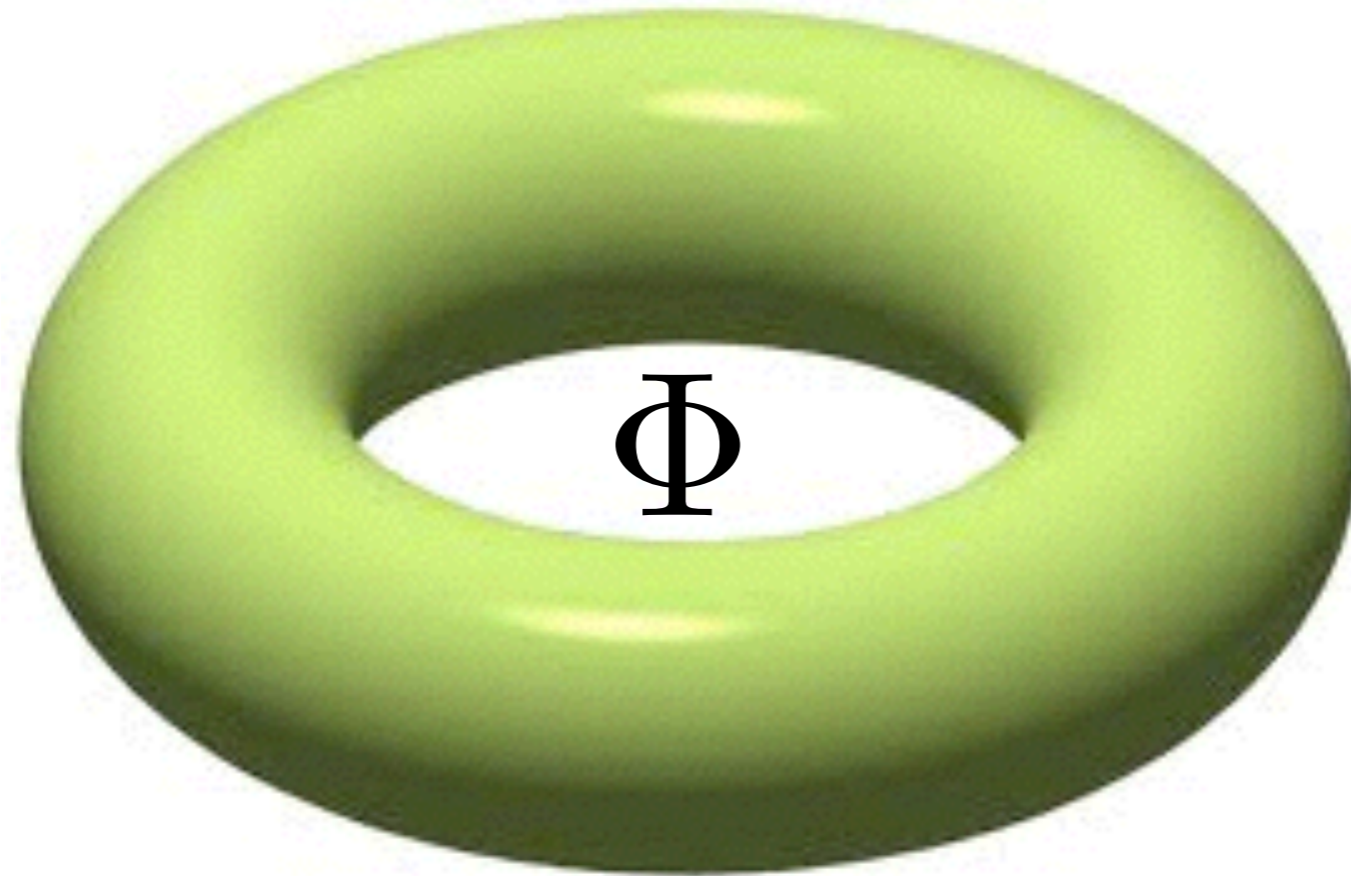
At  $T=0$ , such a metal must be separated from a Fermi liquid (with a Fermi surface of size  $1+p$ ) by a quantum phase transition

## Topological argument

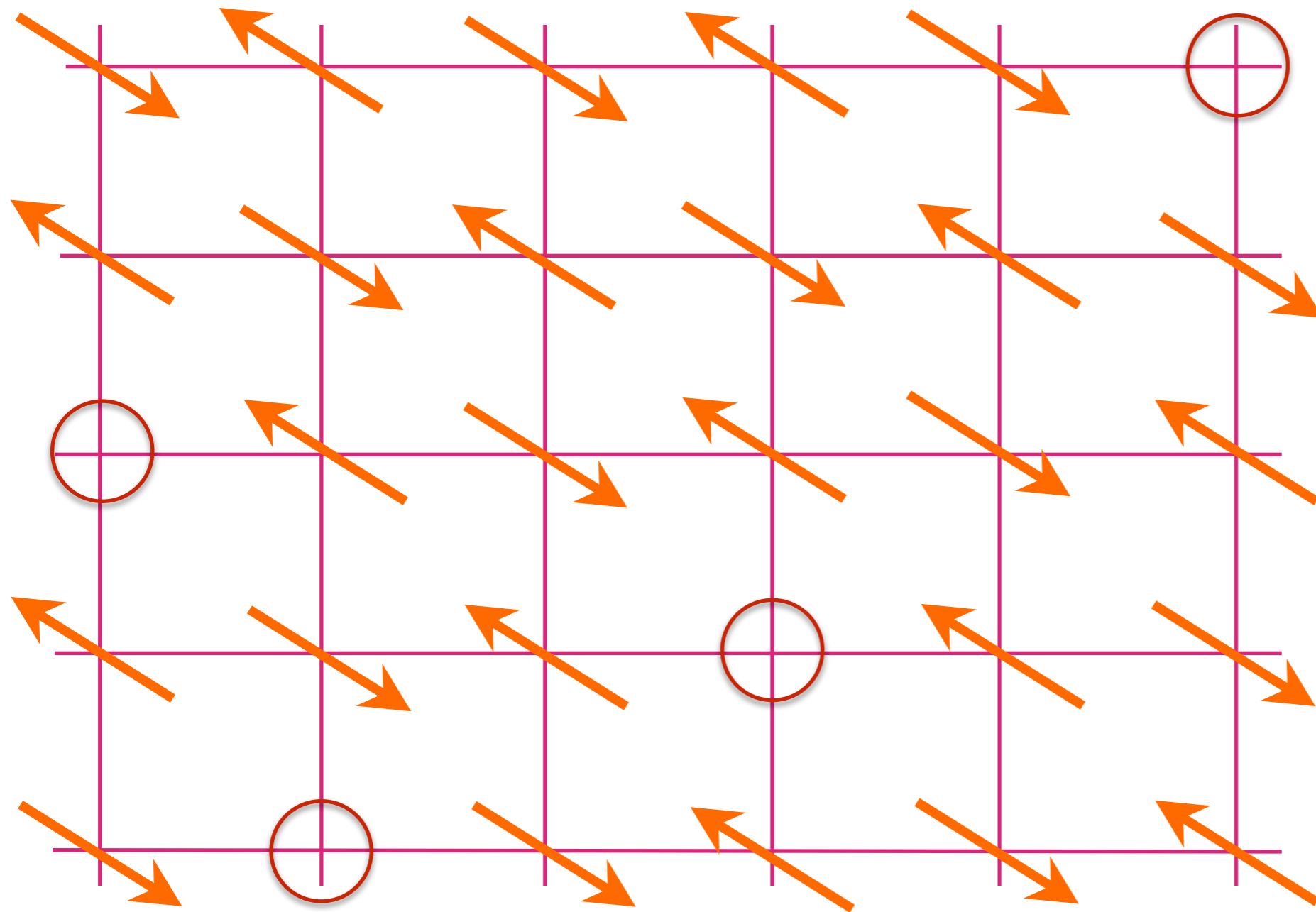


Put metal on a torus, adiabatically insert flux  $\Phi = h/e$  through hole, and measure change in momentum. In a FL, we can assume the only low energy excitations are quasiparticles near the Fermi surface, and this leads to a non-perturbative proof of the Luttinger relation on the area enclosed by the Fermi surface.

## Topological argument

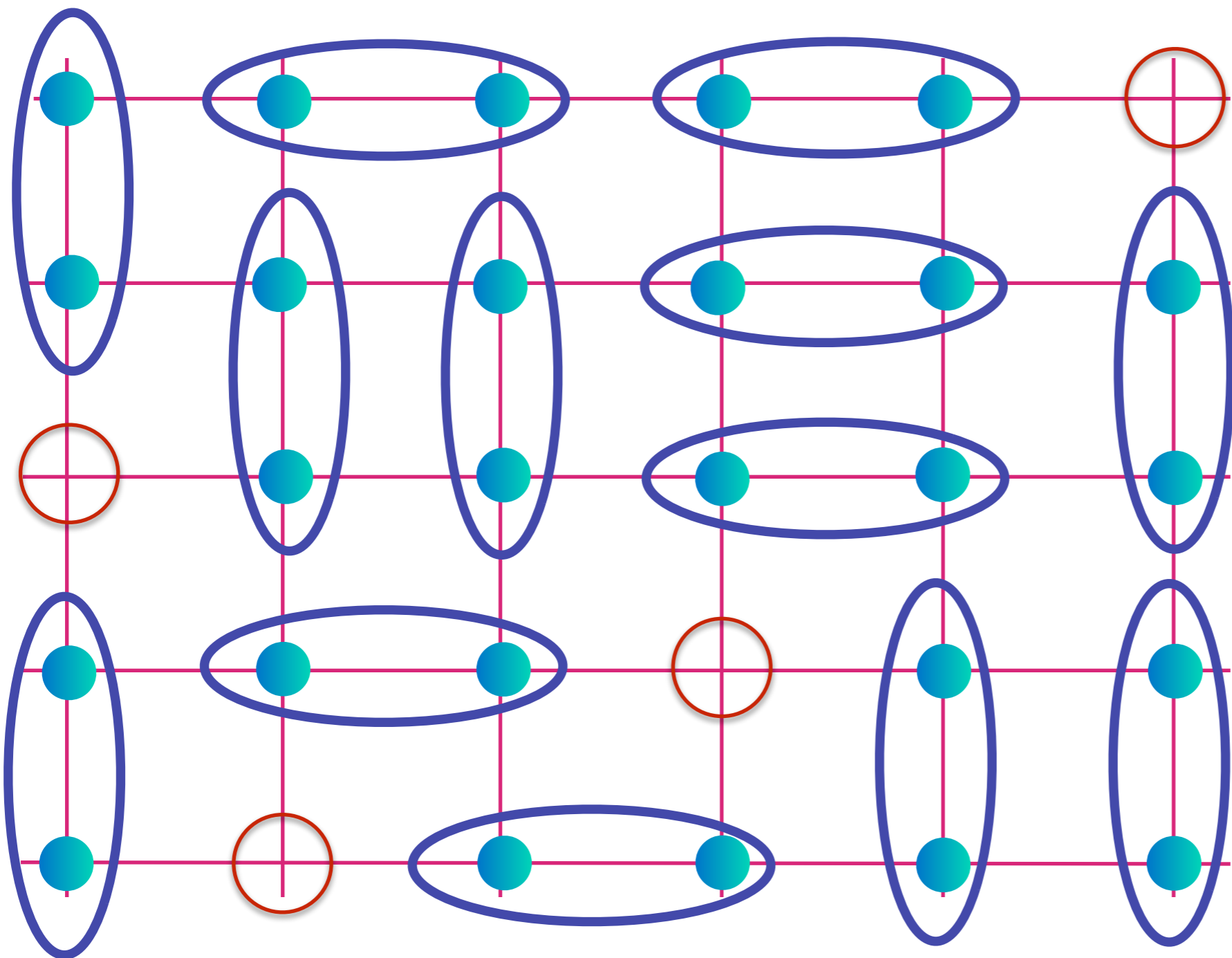


Violations of the Luttinger relation are possible in a fractionalized Fermi liquid (FL\*) because there are “topological” low energy excitations associated with a flux of the emergent gauge field in the hole of the torus.




Anti-ferromagnet with  $p$  holes per square

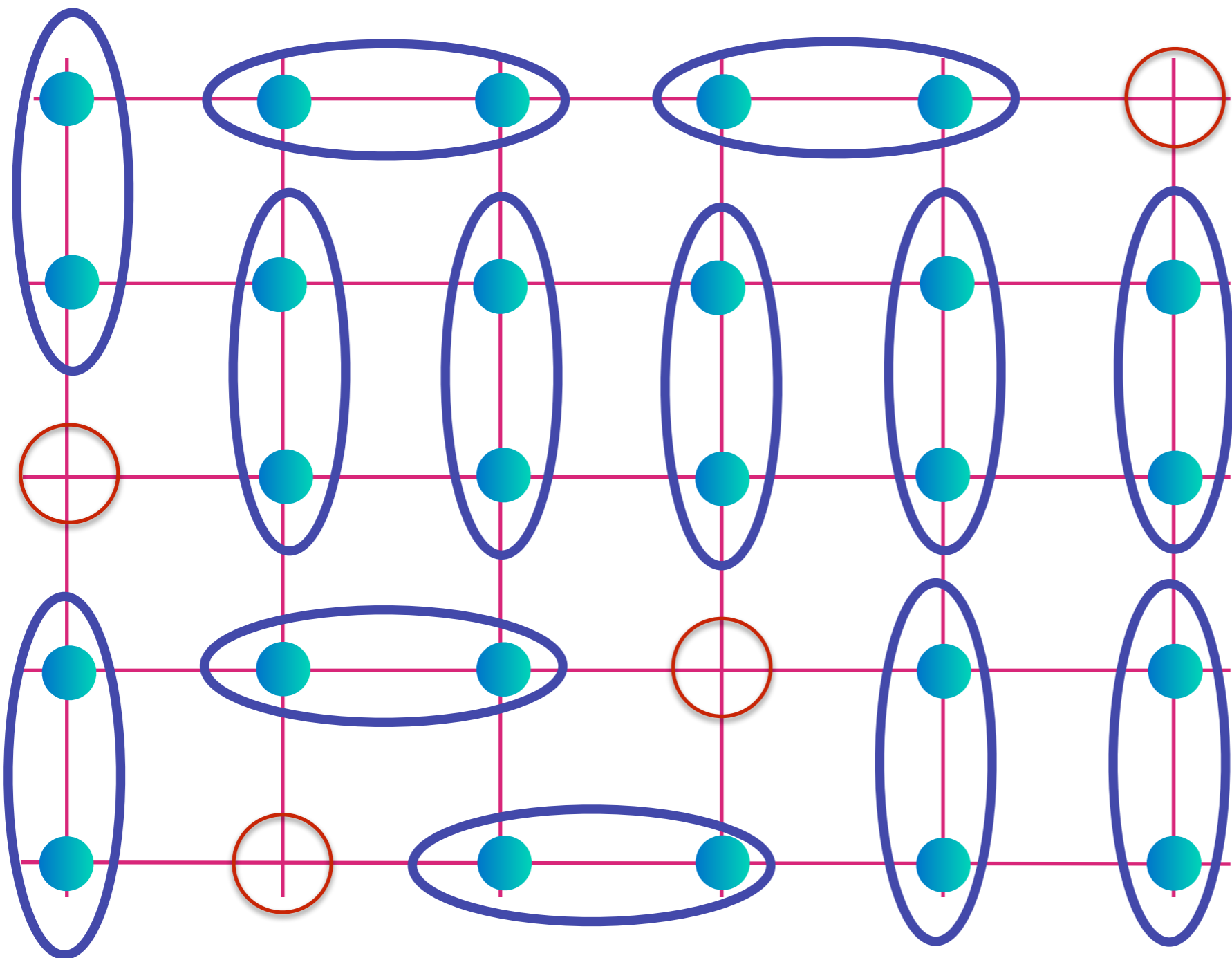
Note: relative to the fully-filled band insulator, there are  $1+p$  holes per square



Spin liquid  
with emergent  
gauge field  
*and*  
 $p$  "holons"  
(gauge-charged,  
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charge  $+e$   
quasiparticles)  
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 =  $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

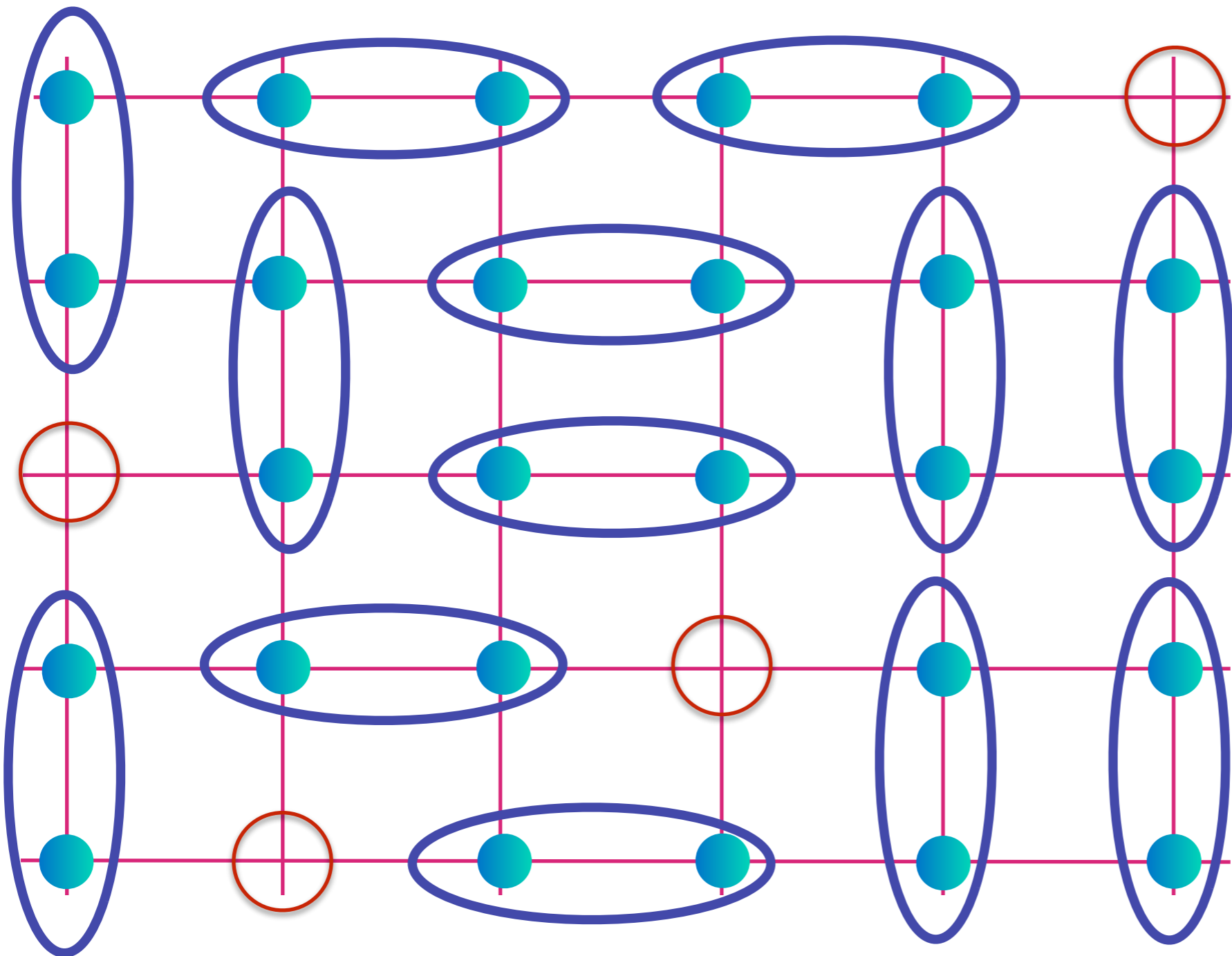
Baskaran, Zou, Anderson, Fradkin, Kivelson...



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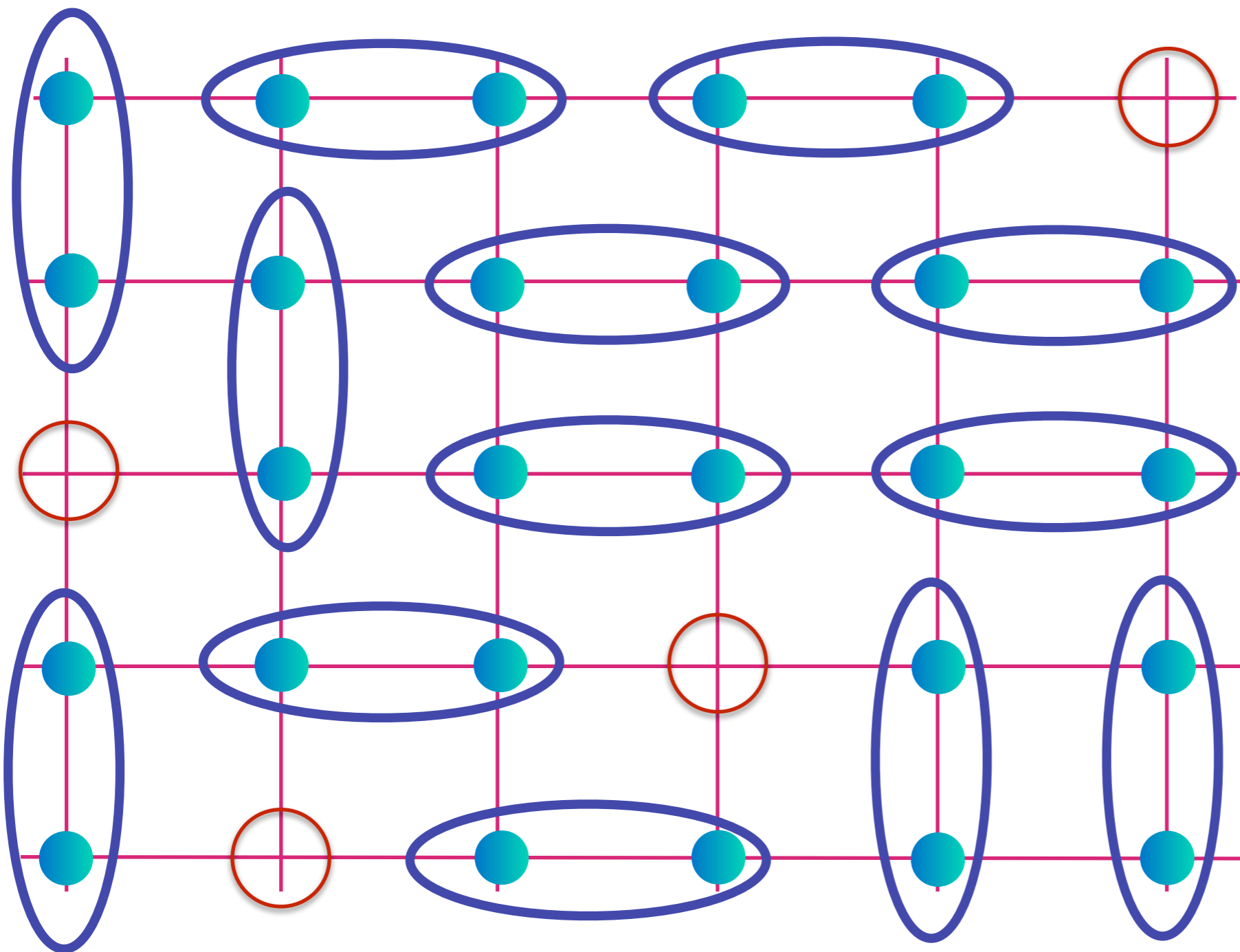
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$$\text{[Pair of sites]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

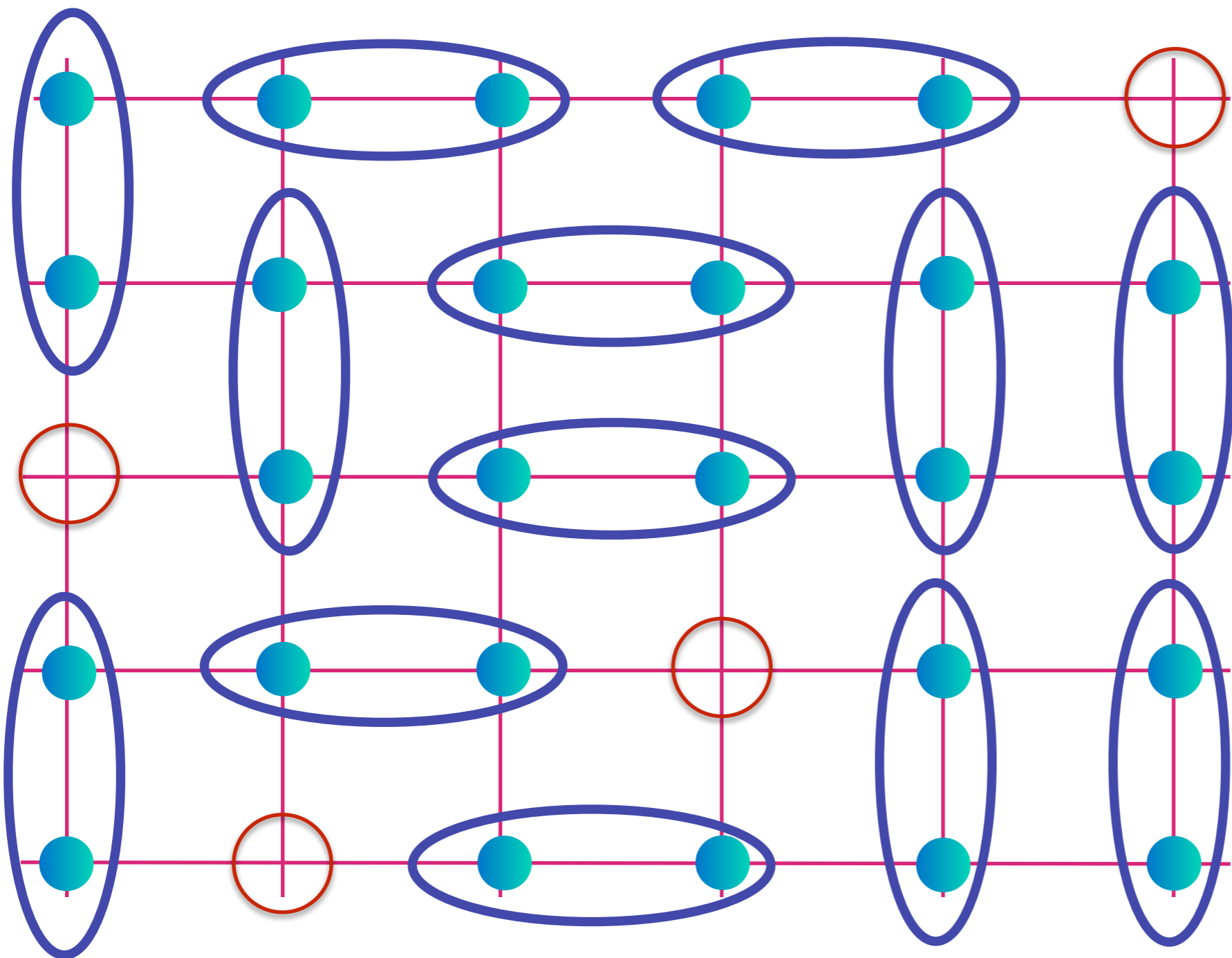
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$$\text{[blue oval with two cyan dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

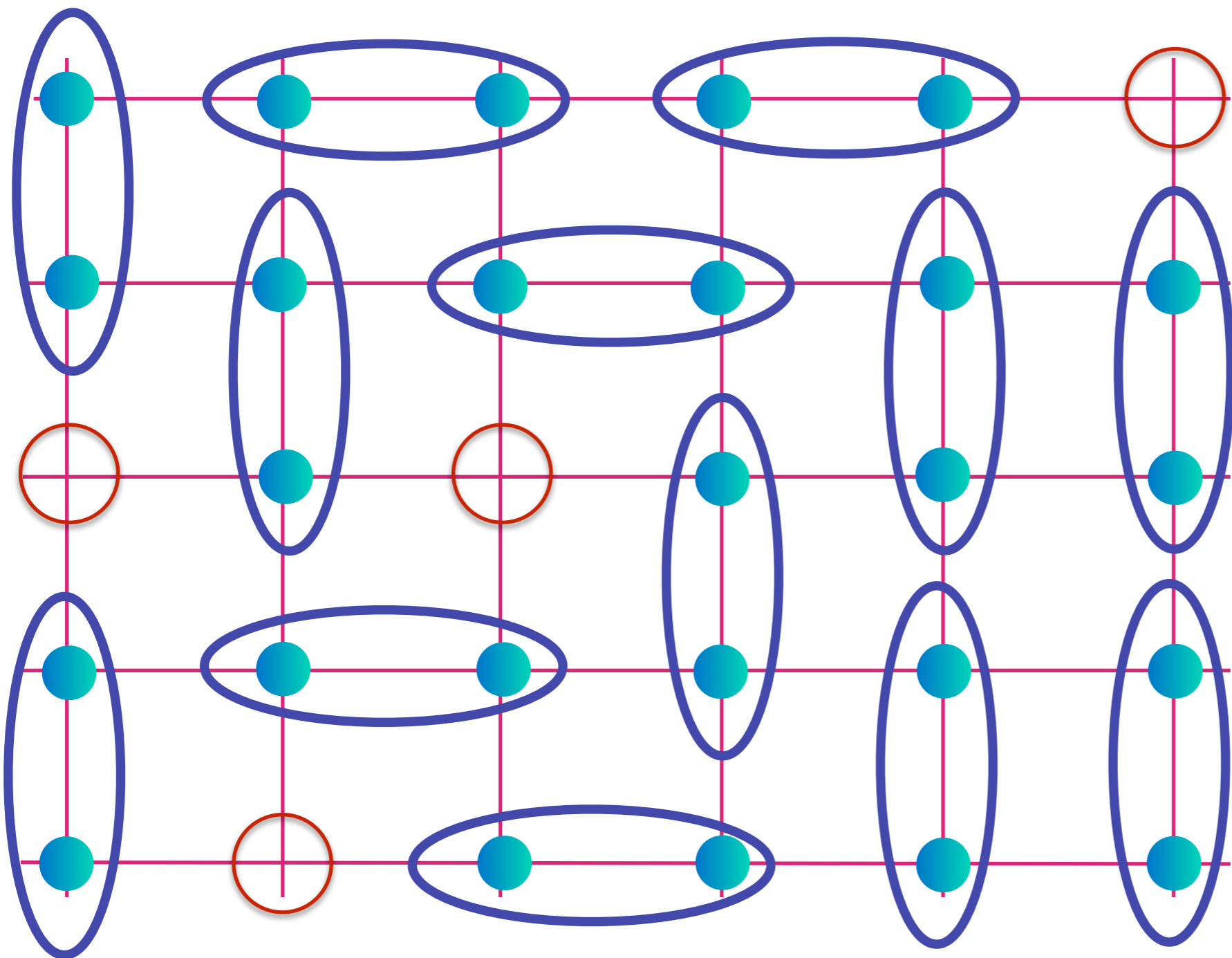
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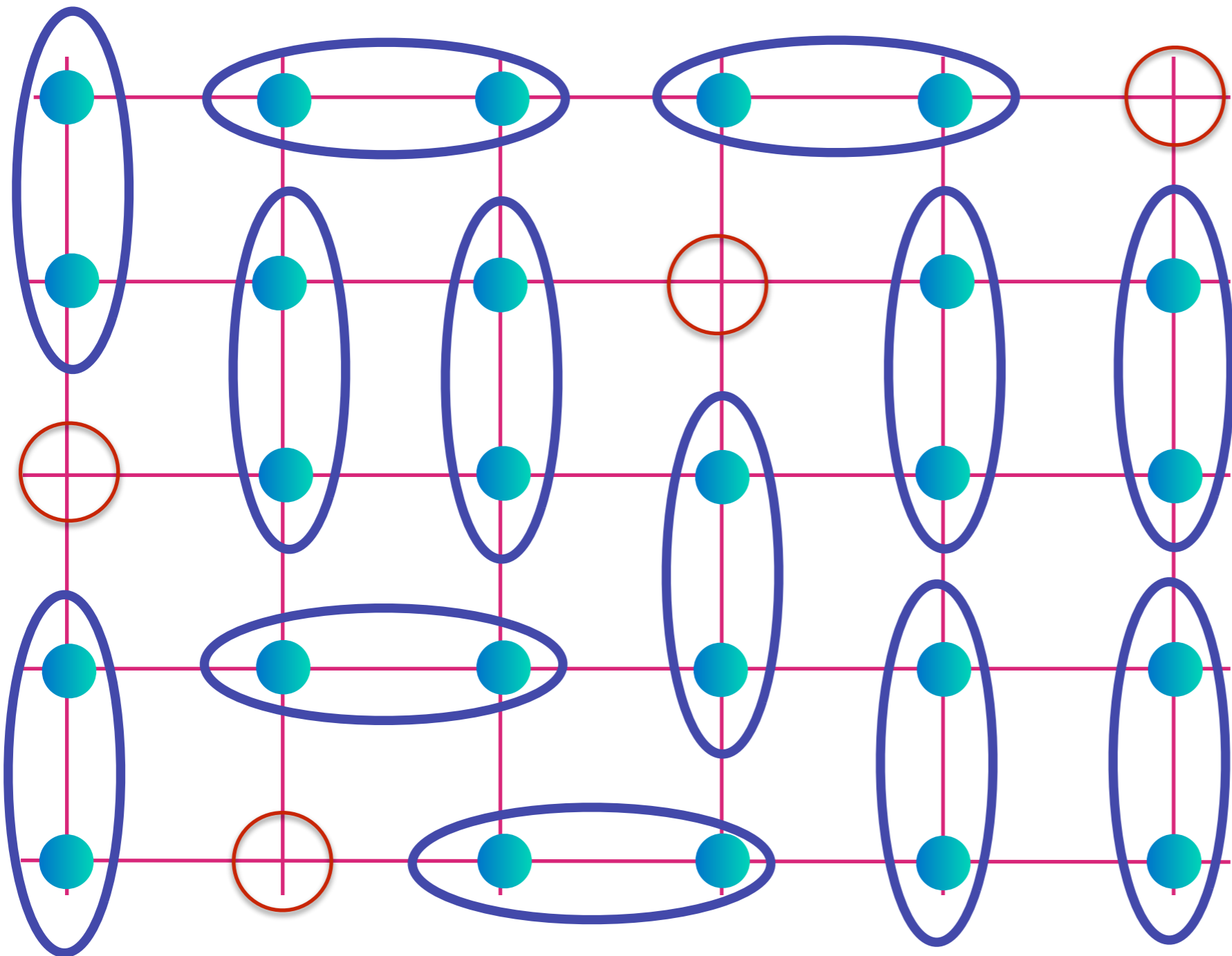
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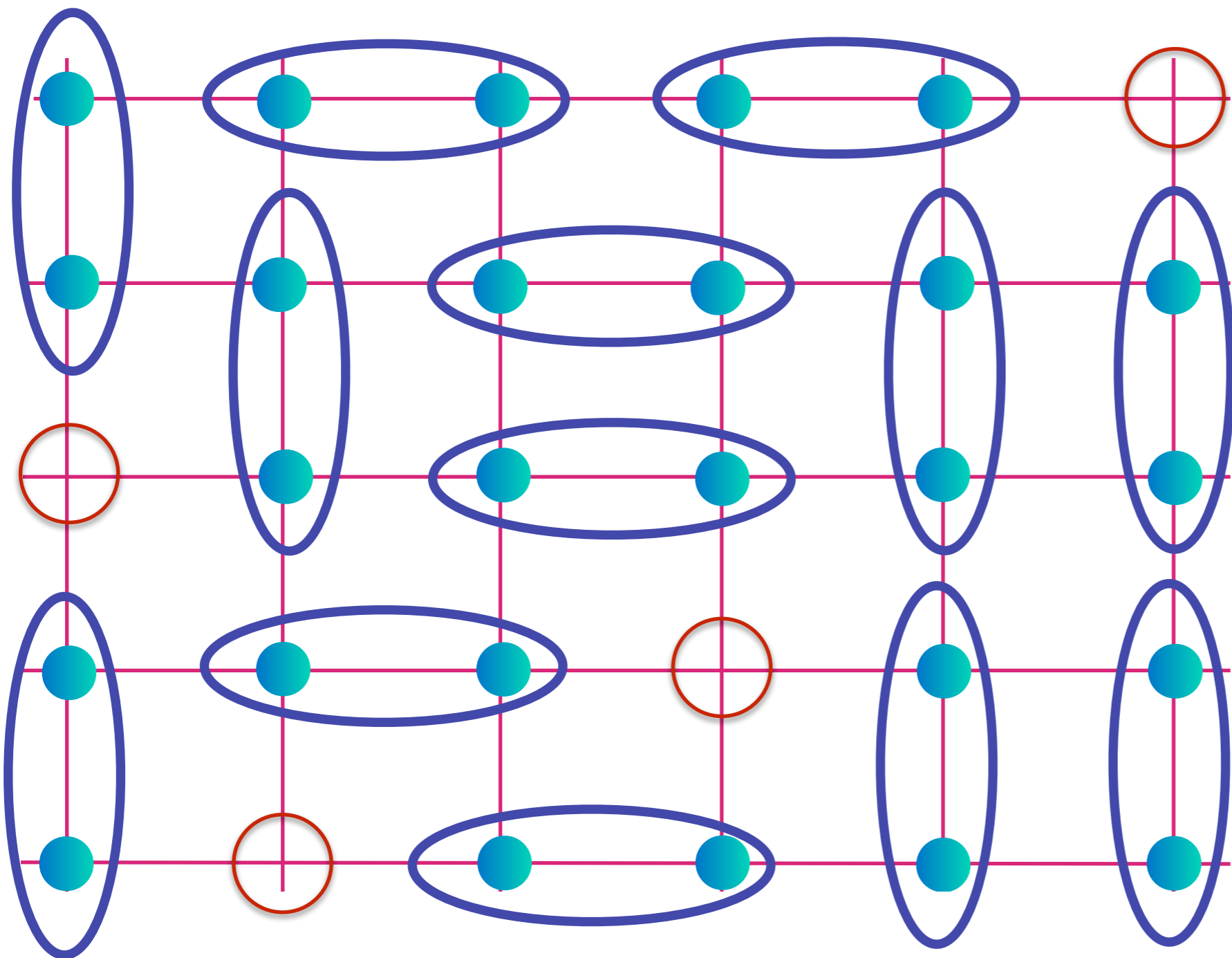
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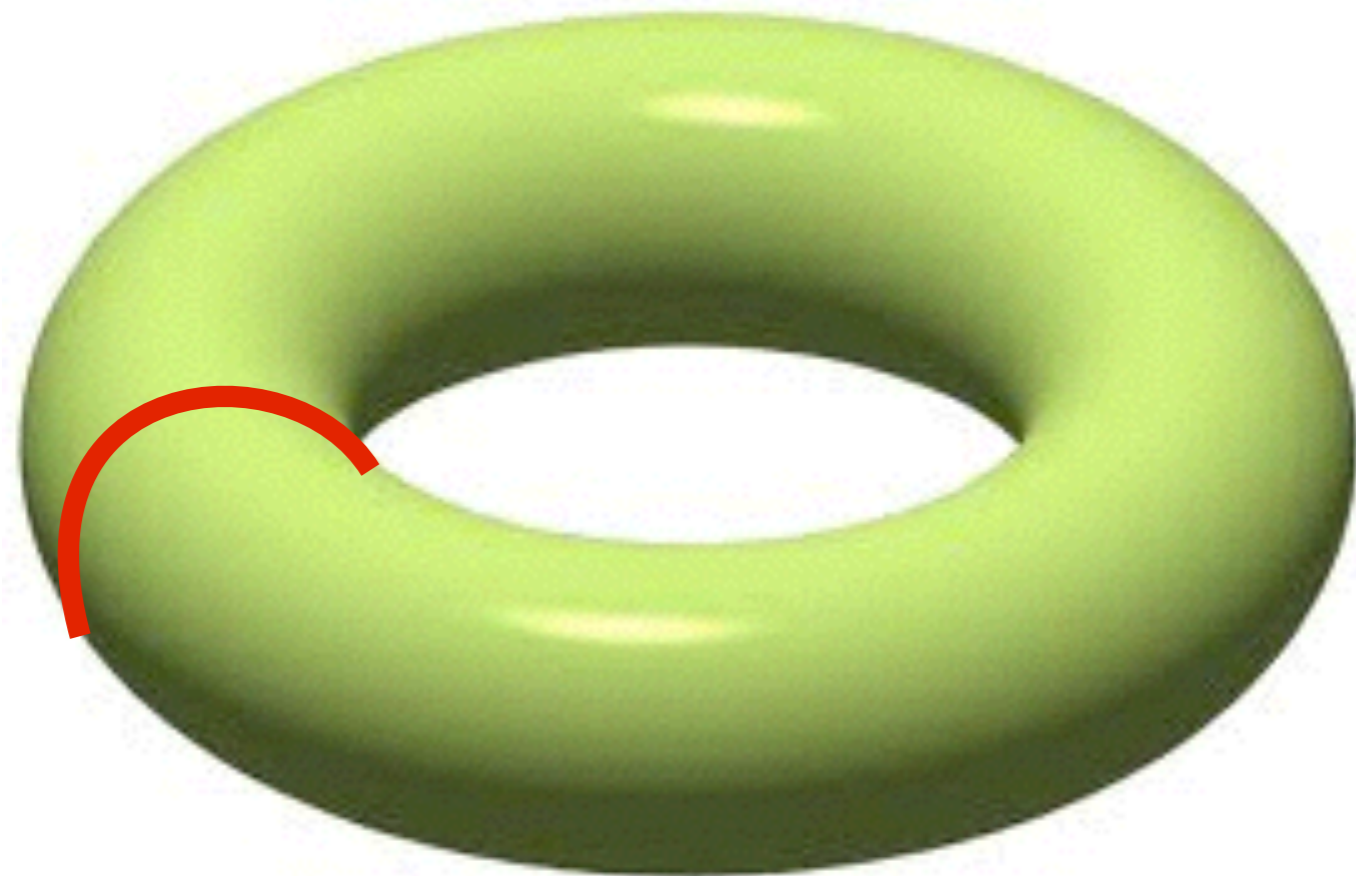
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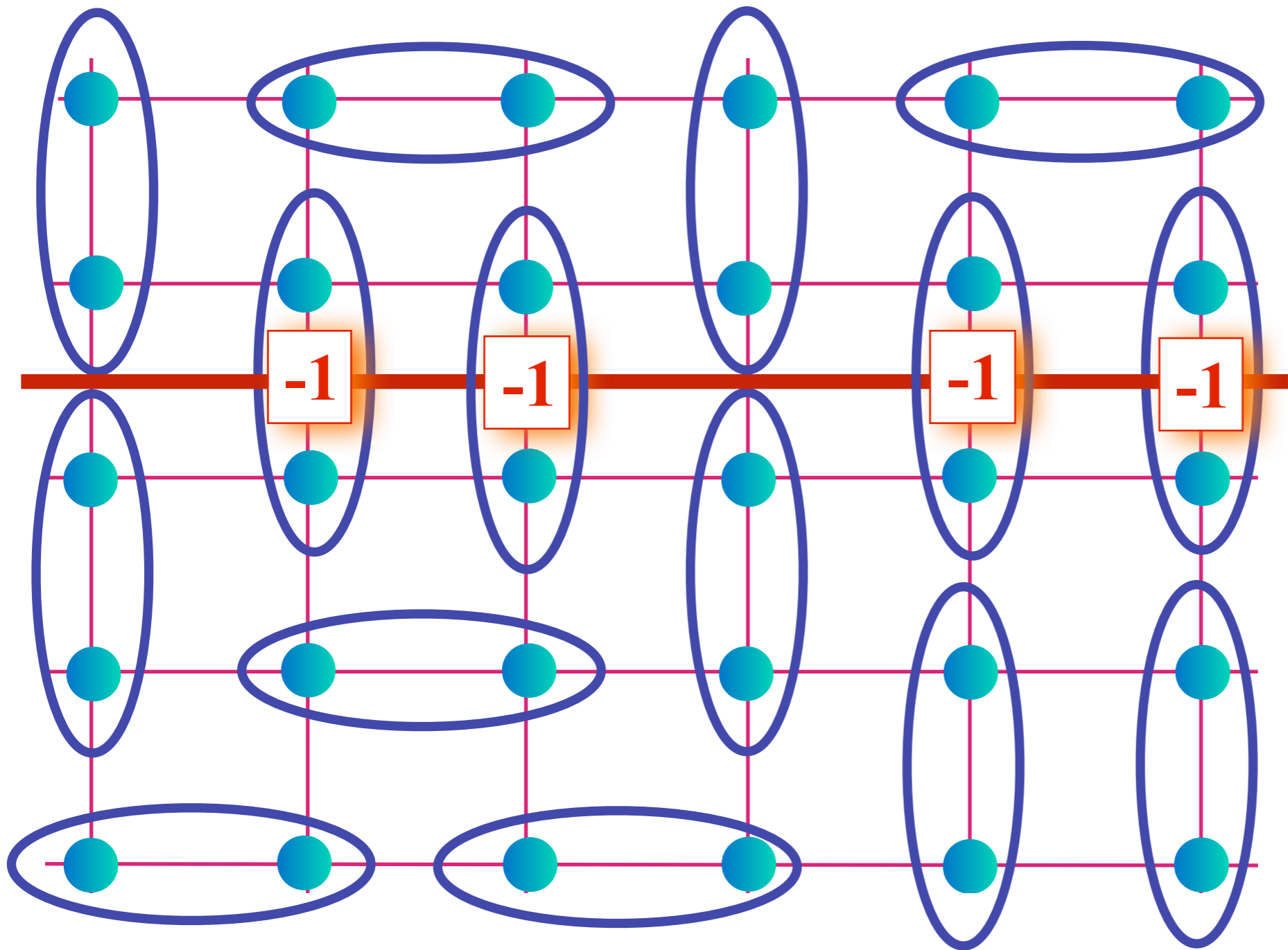


Spin liquid.  
Place on a  
torus;



## Spin liquid.

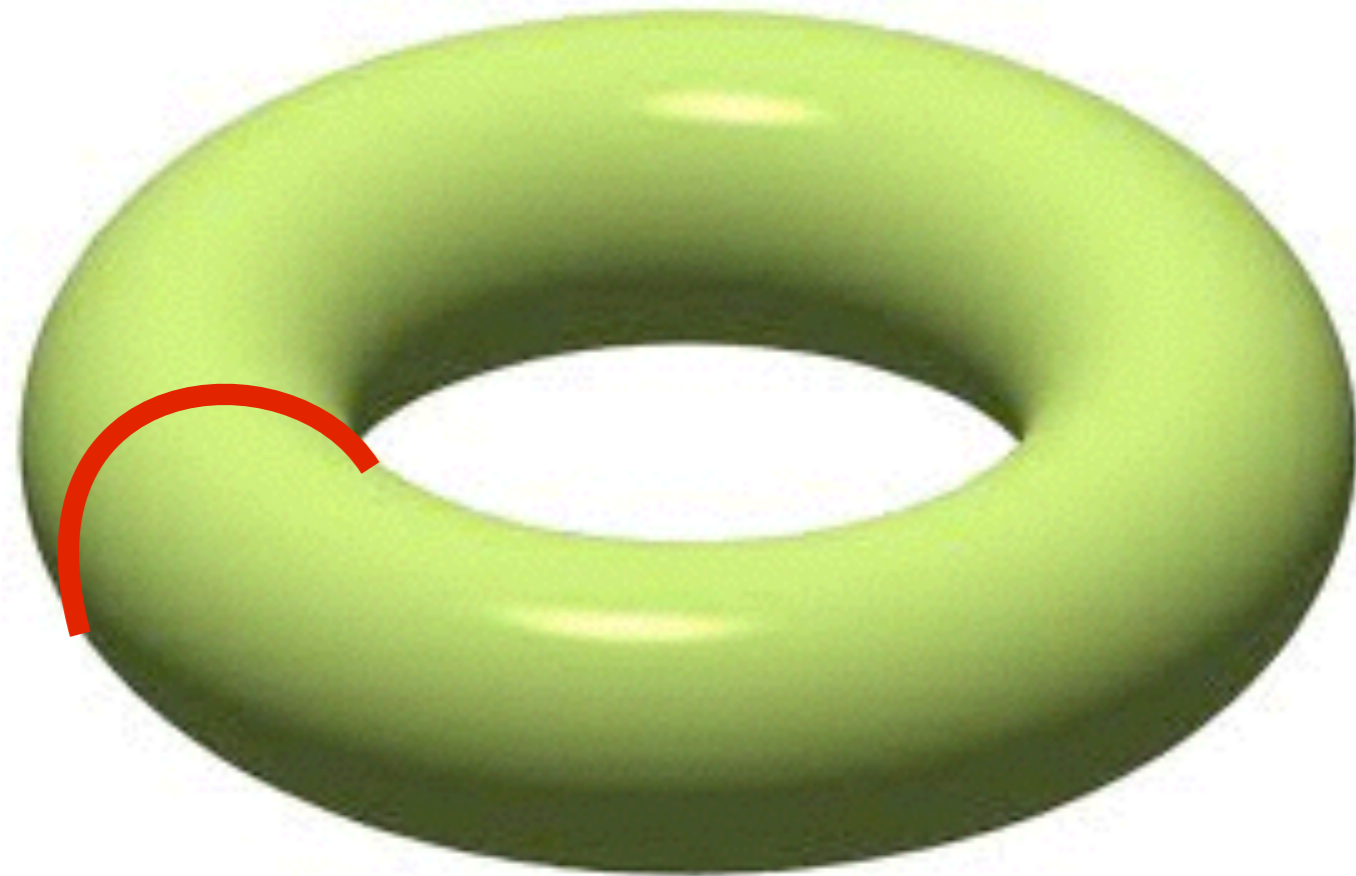
Place on a torus;  
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degenerate  
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change sign of  
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red line



## Spin liquid.

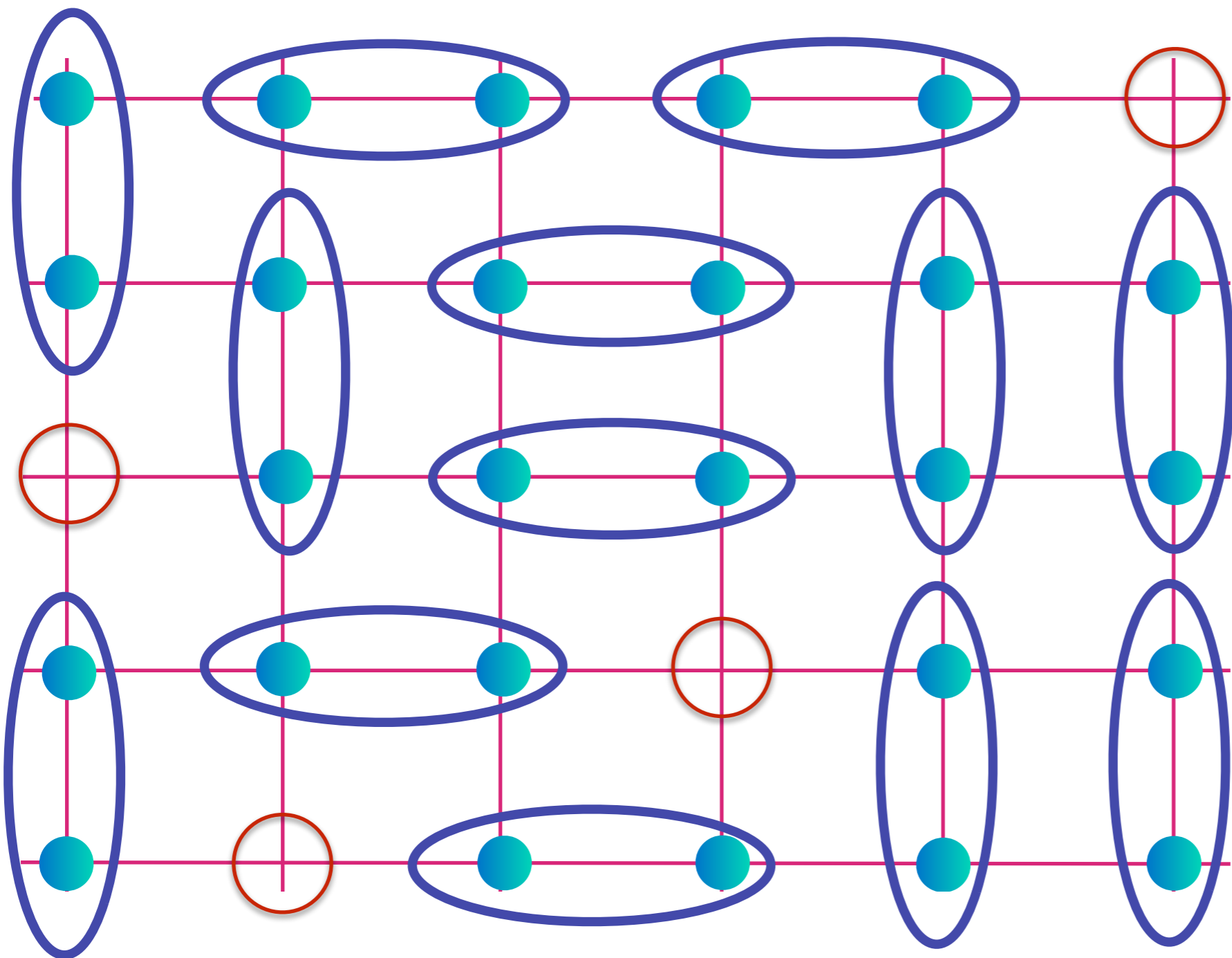
Place on a torus; to obtain “topological” states nearly degenerate with the ground state: change sign of every singlet bond across red line

$$\text{[Oval with two dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



## Spin liquid.

These  
“topological”  
states are  
needed to  
allow for Fermi  
surfaces of  
total size  $p$   
(and not  $1+p$ )

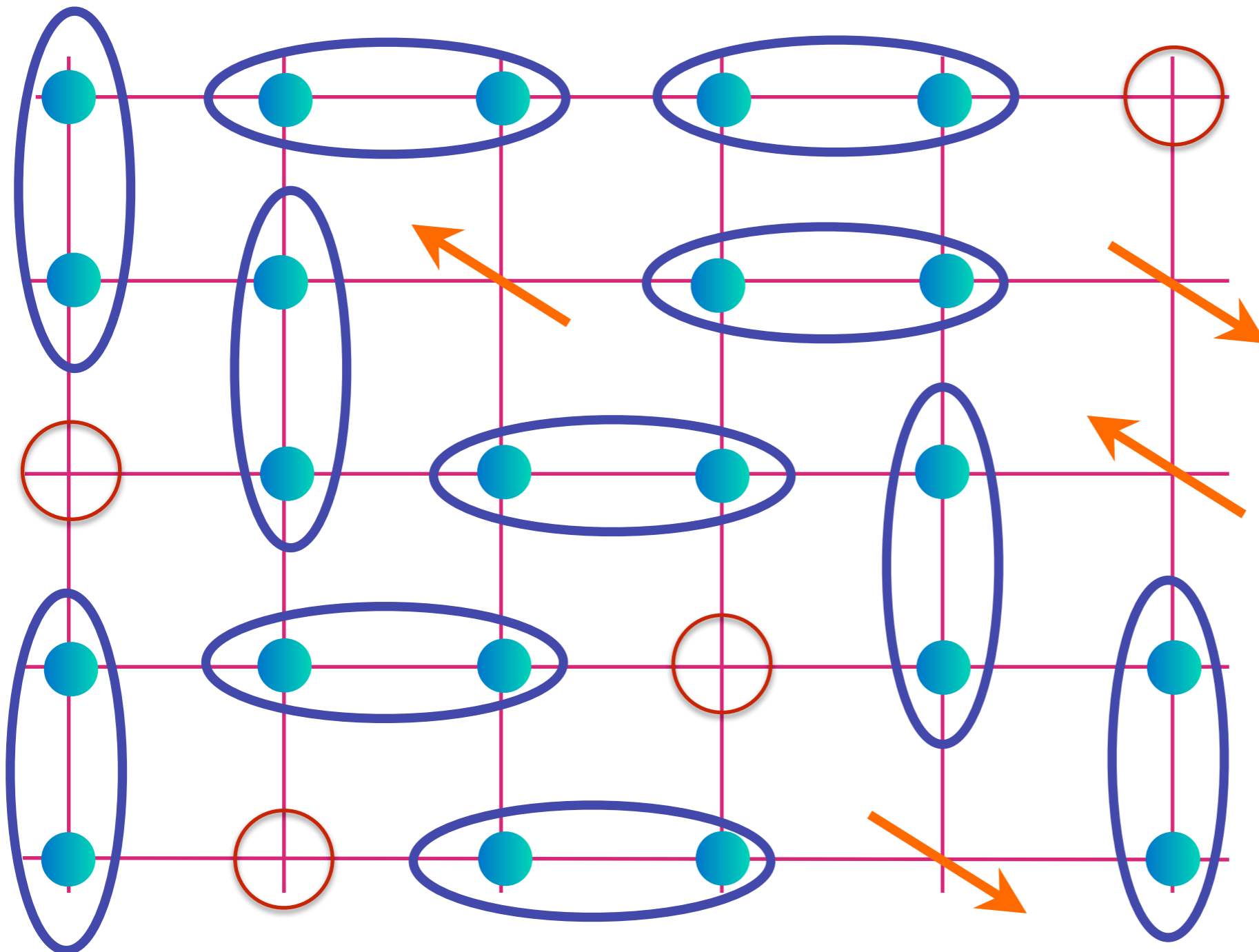


Spin liquid  
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 $p$  "holons"  
(gauge-charged,  
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per square

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Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Gauge-charged, spin $S=1/2$ , neutral “spinon” excitations

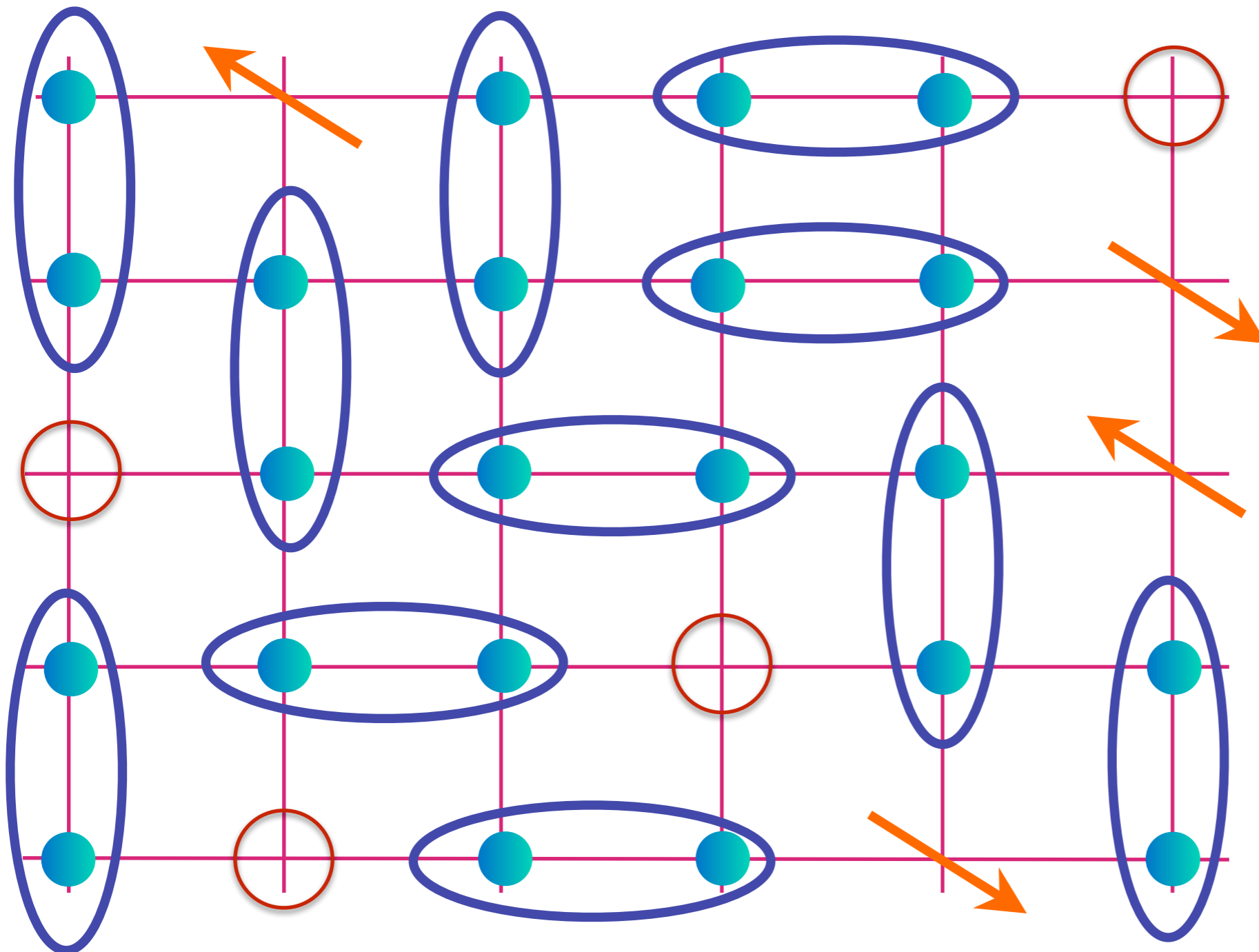


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$$\text{[Blue oval with two cyan dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

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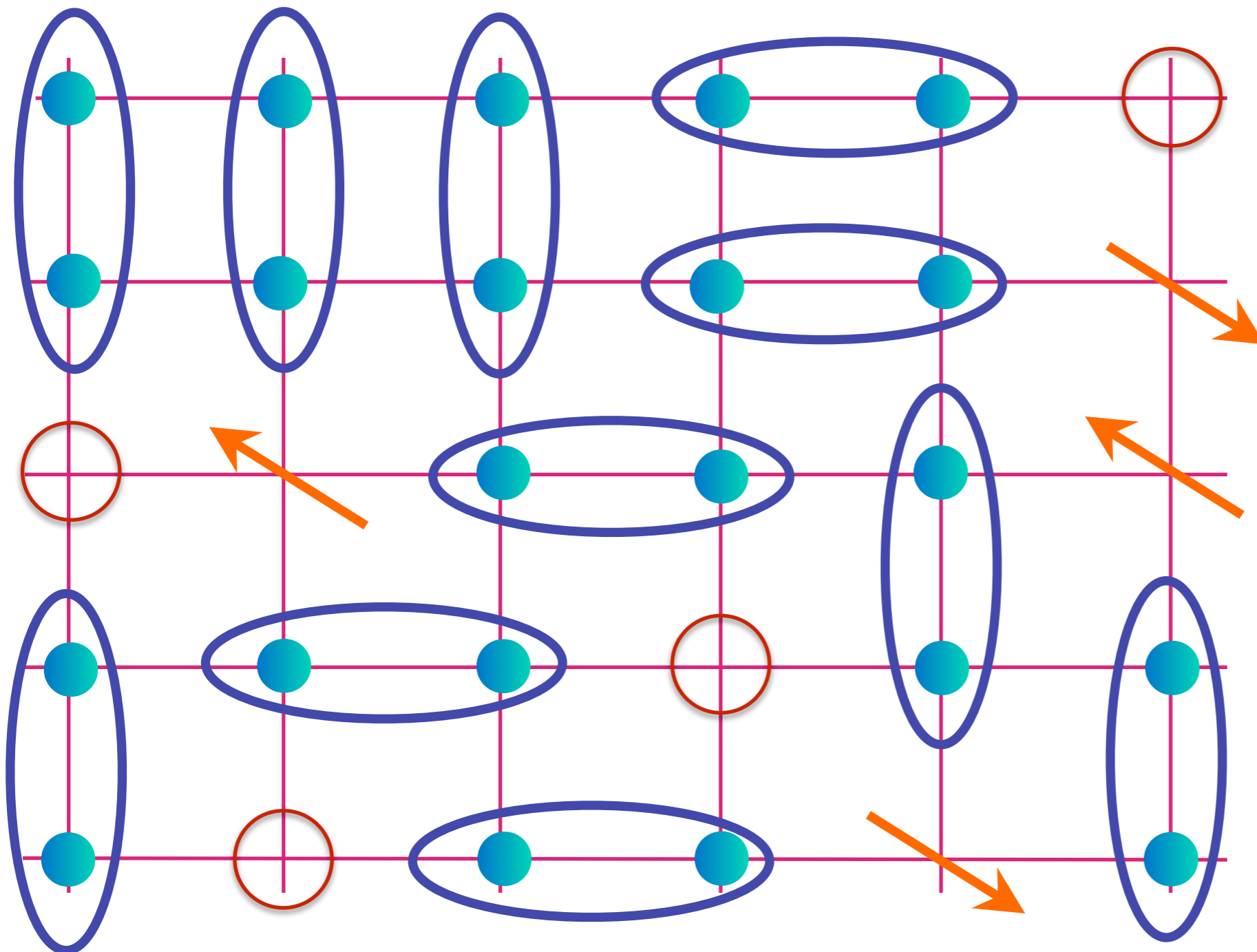


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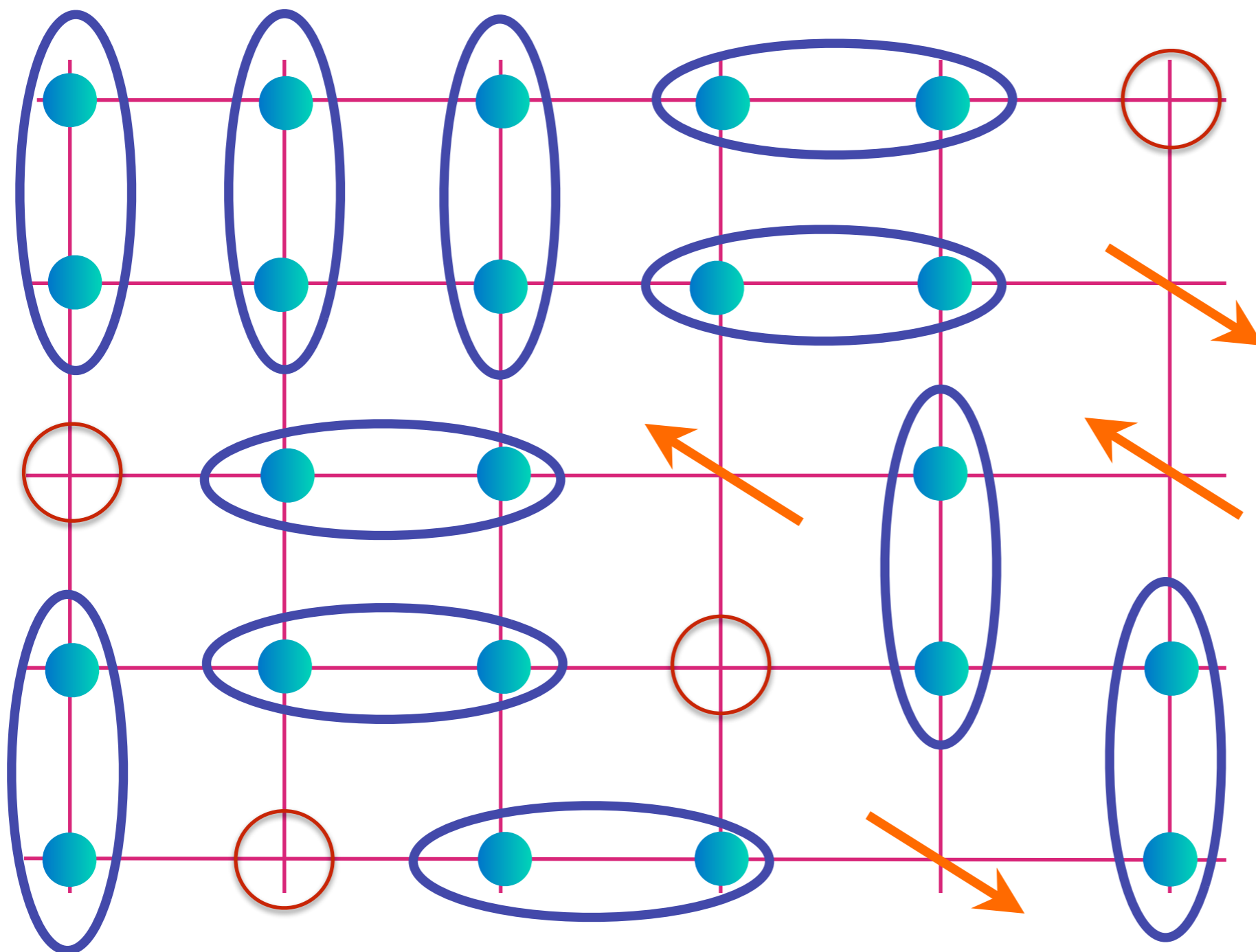


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$$\text{[Pair of teal dots in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

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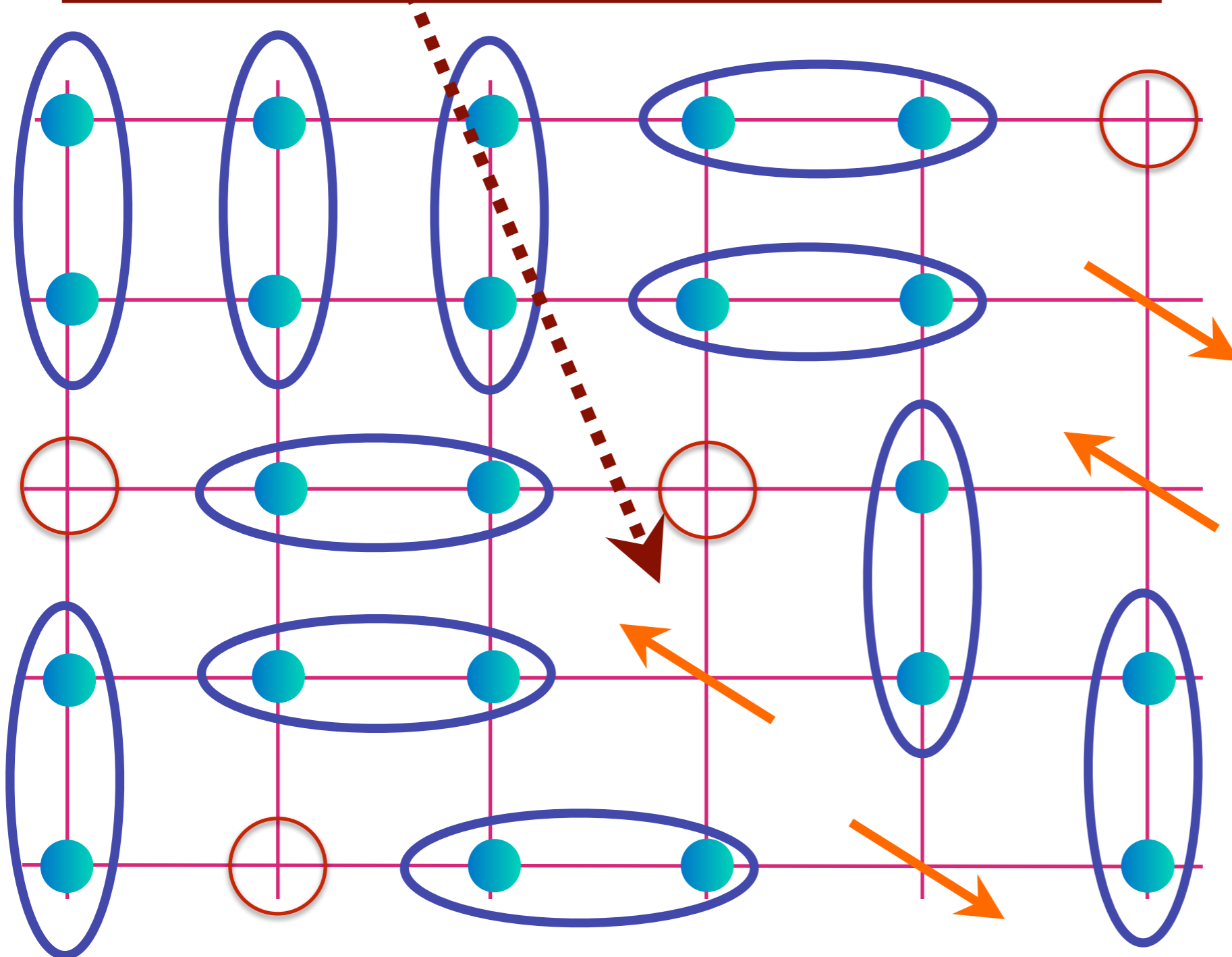


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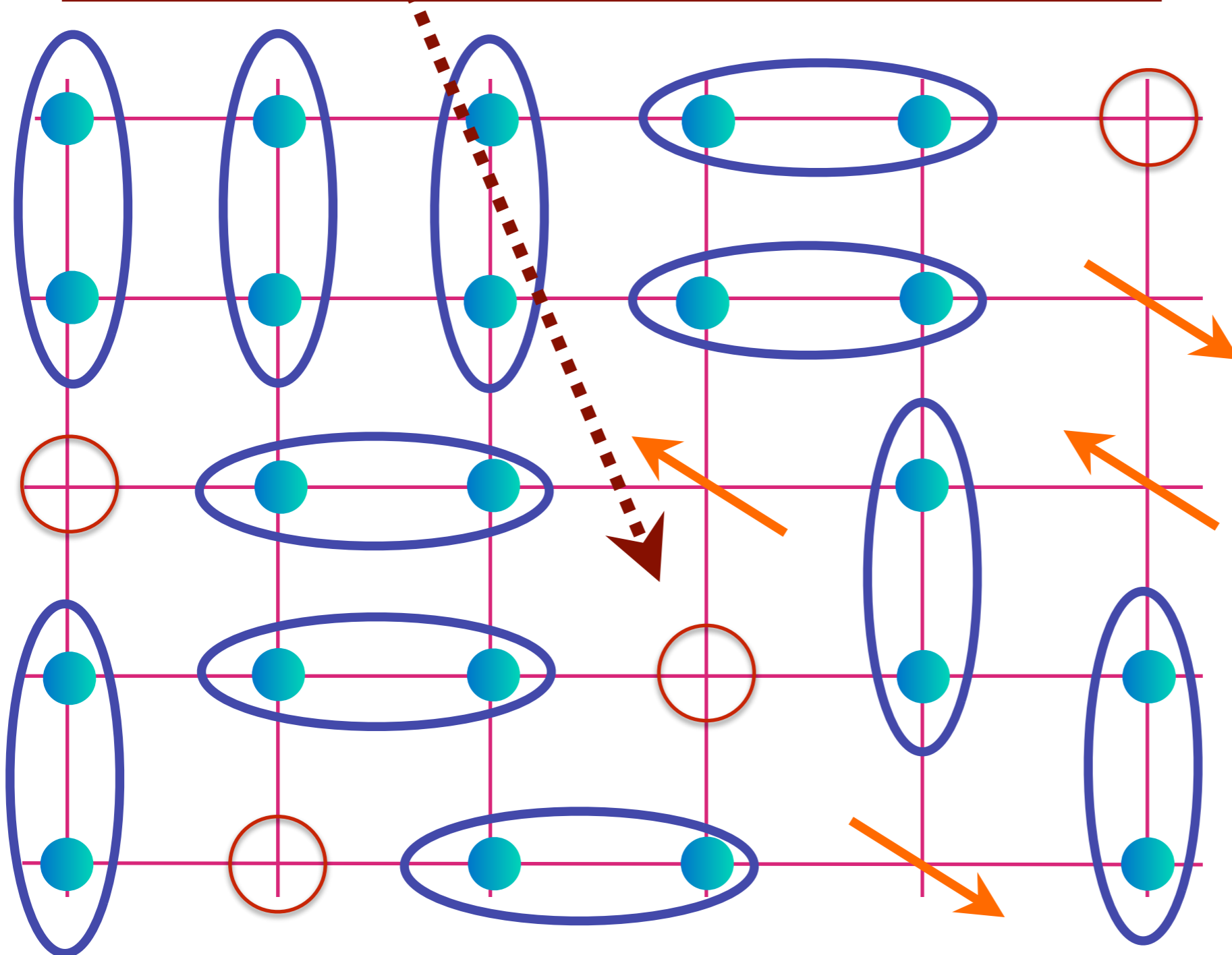
Nearest-neighbor hopping leads to attraction between holon and spinon, which can pay for the energy needed to create the spinon



Spin liquid with emergent gauge field and  $p$  "holons" (gauge-charged, spinless, charge  $+e$  quasiparticles) per square

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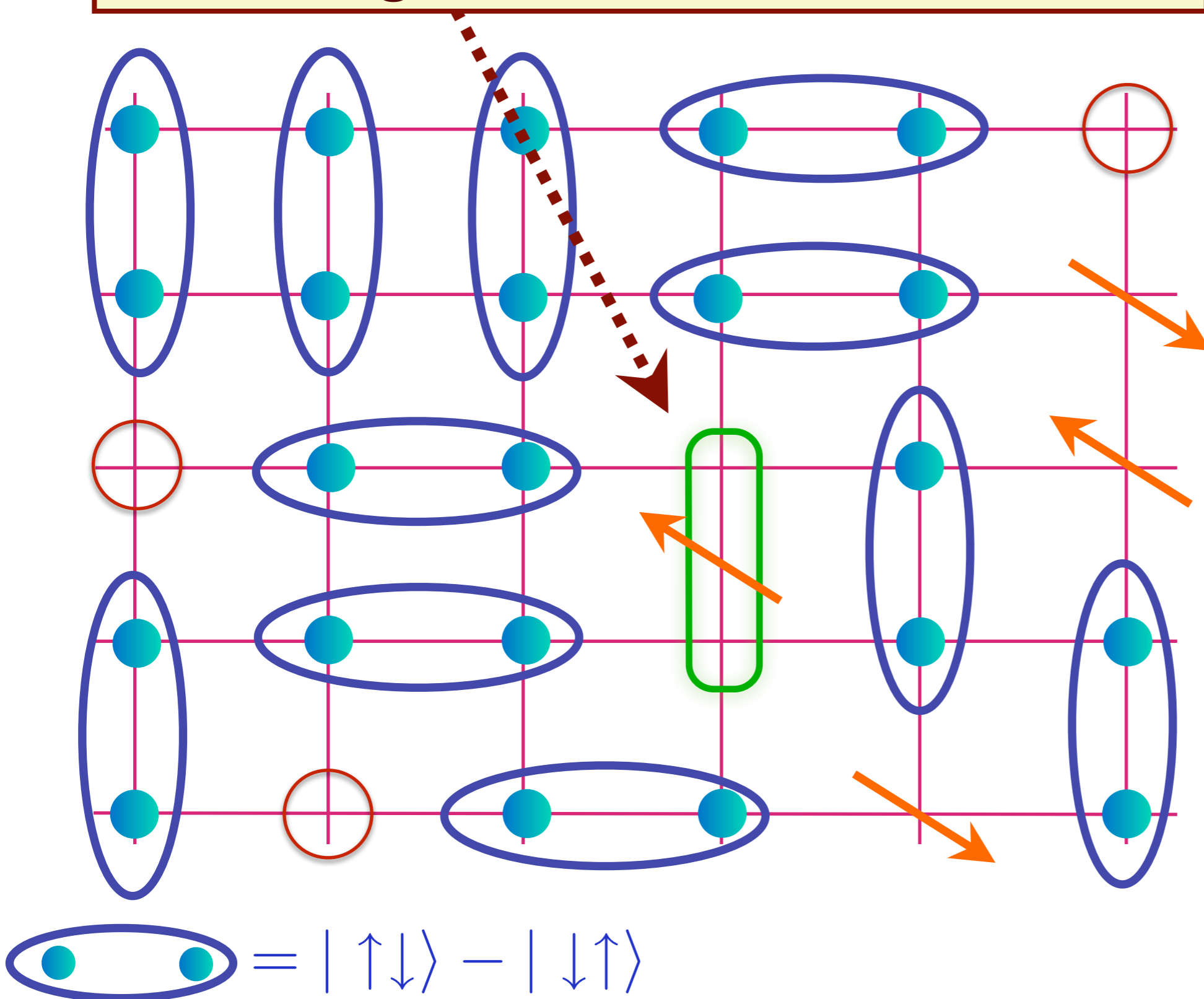
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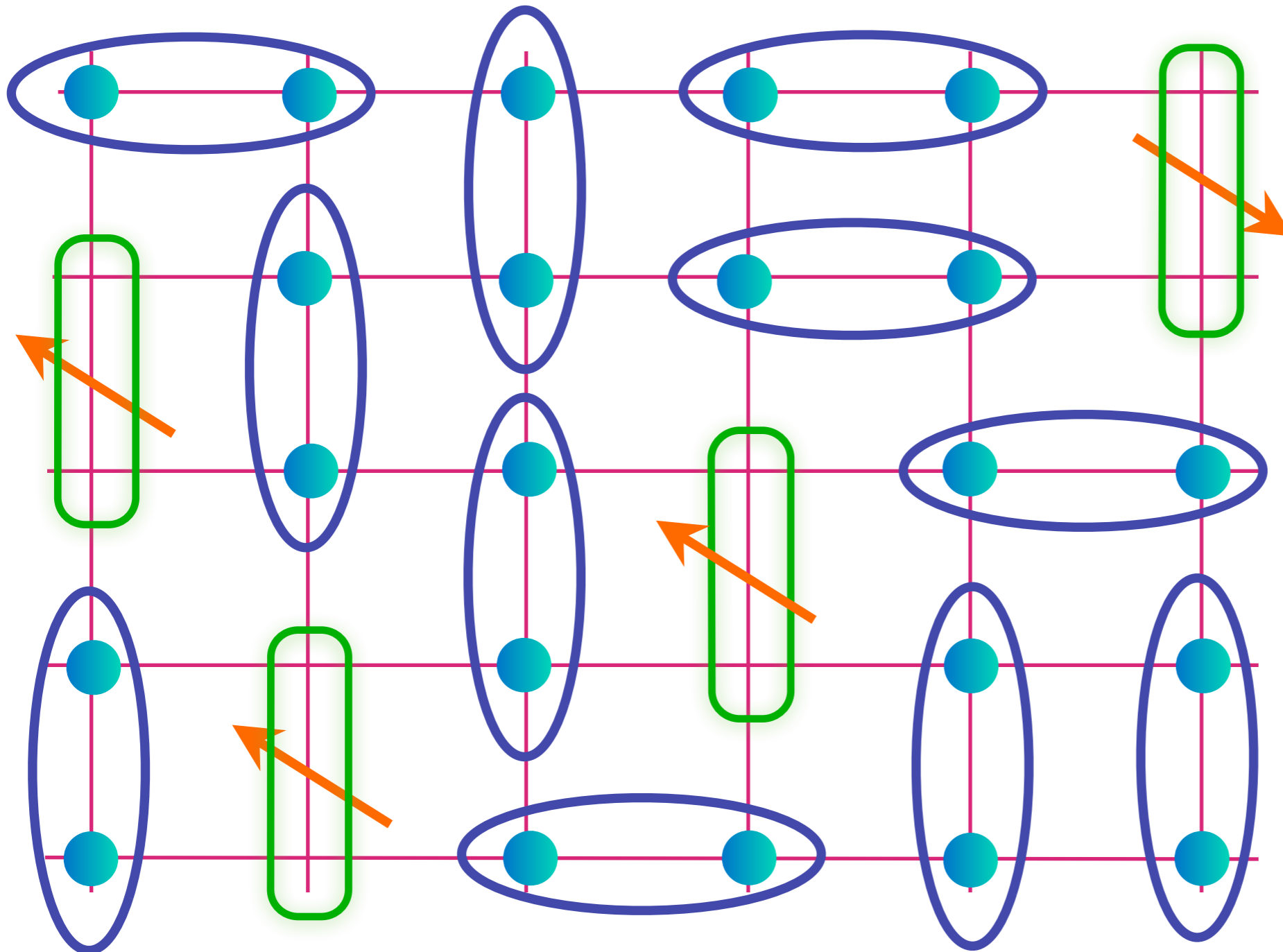
Spinon-holon bound state resides on a “bonding” orbital between two sites



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# Fractionalized Fermi liquid (FL\*)


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



Emergent gauge field  
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spin  $S=1/2$ ,  
charge  $+e$   
fermions  
of density  $p$

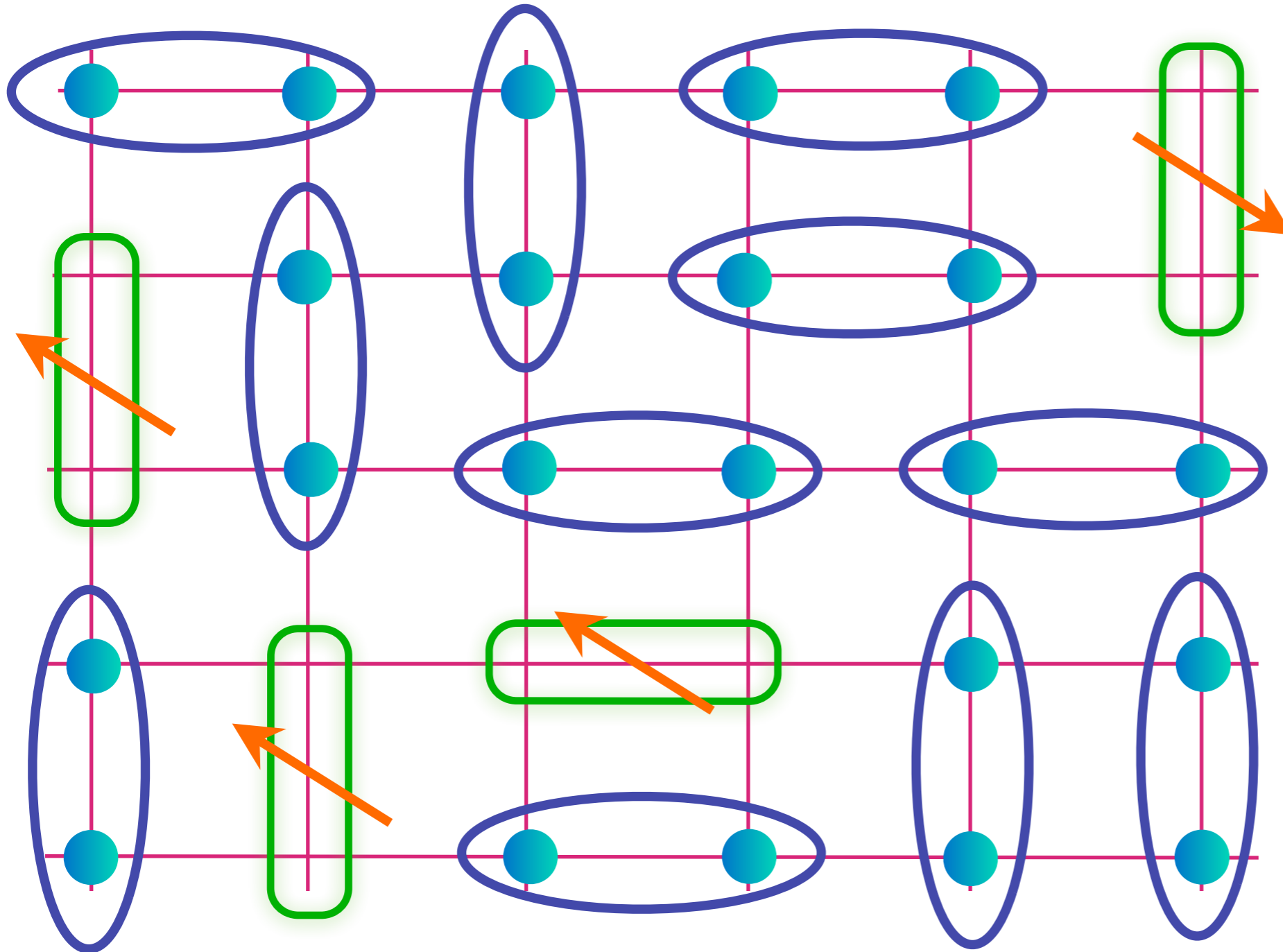
T. Senthil, S. S., M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

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$$\text{Diagram of two blue dots in a blue oval} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



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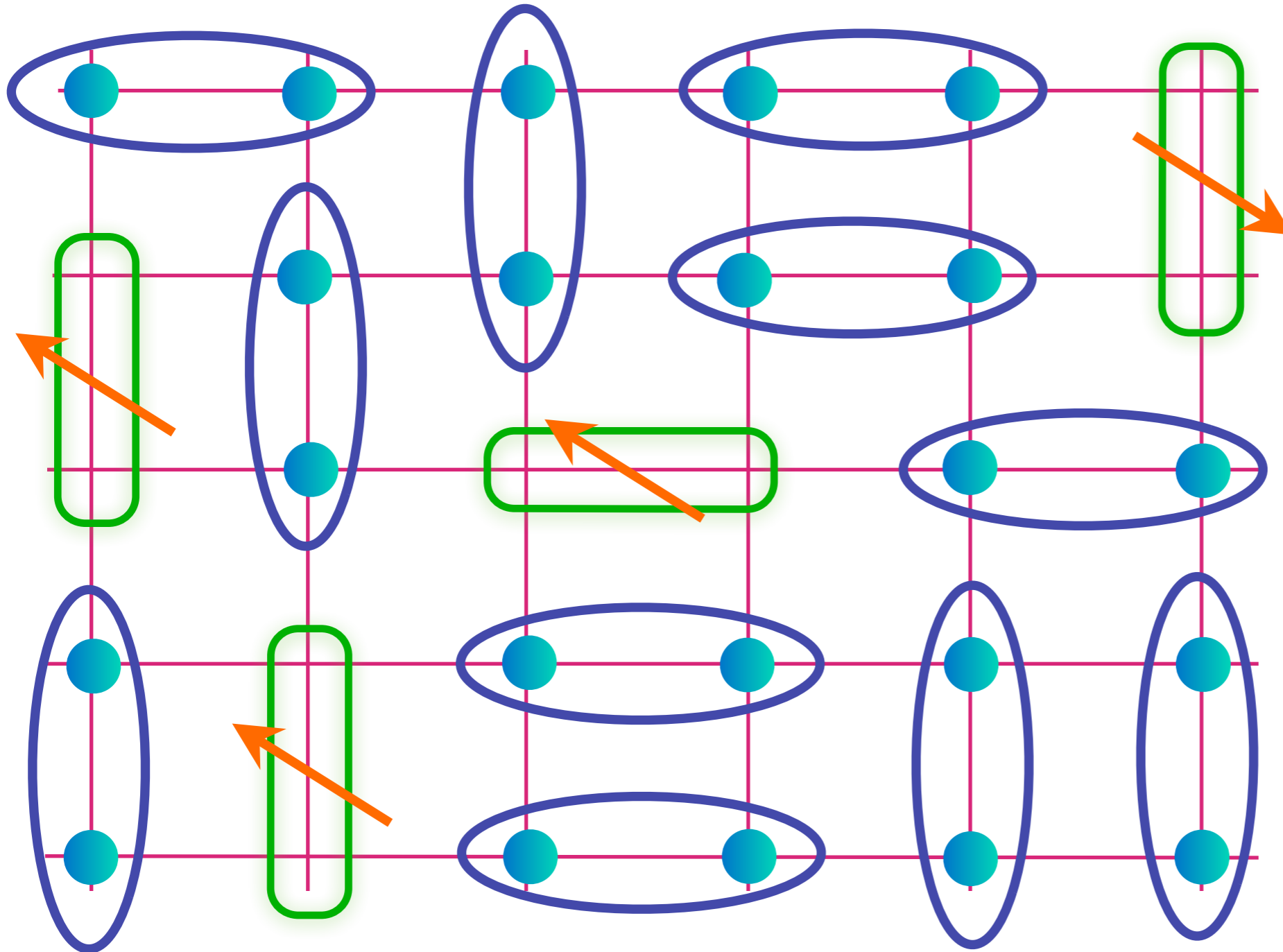
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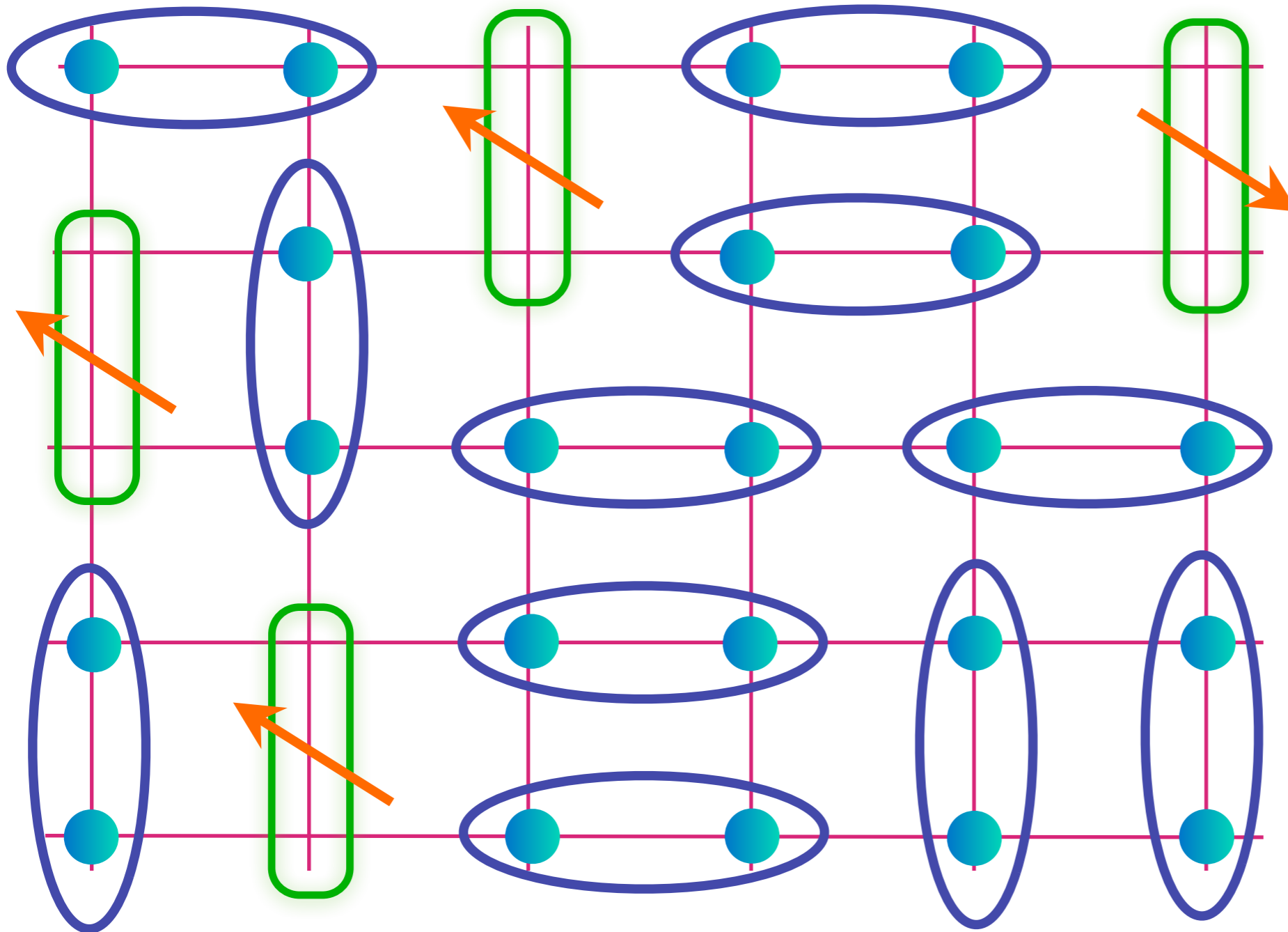
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$$\text{blue oval with 2 dots} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



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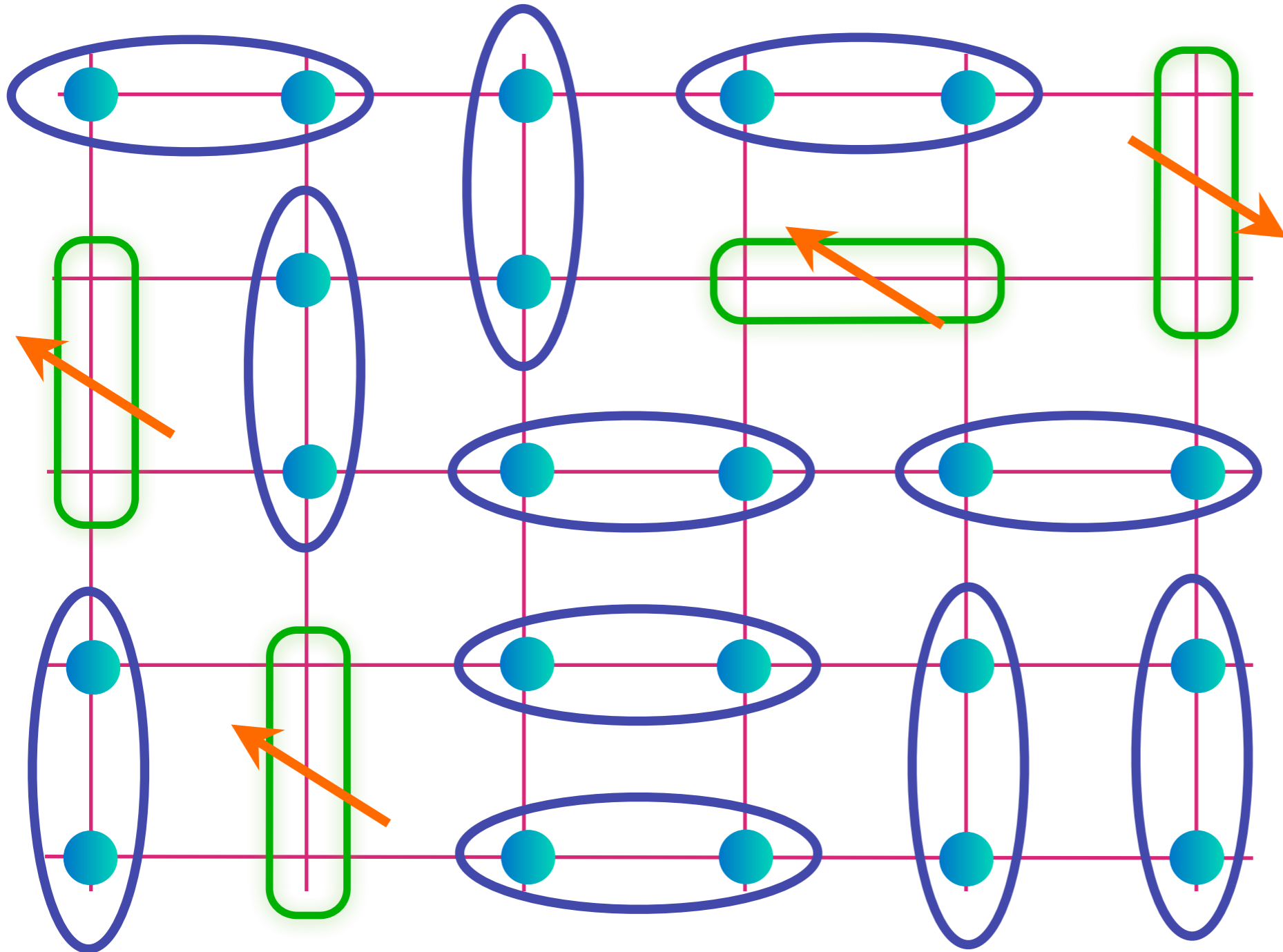
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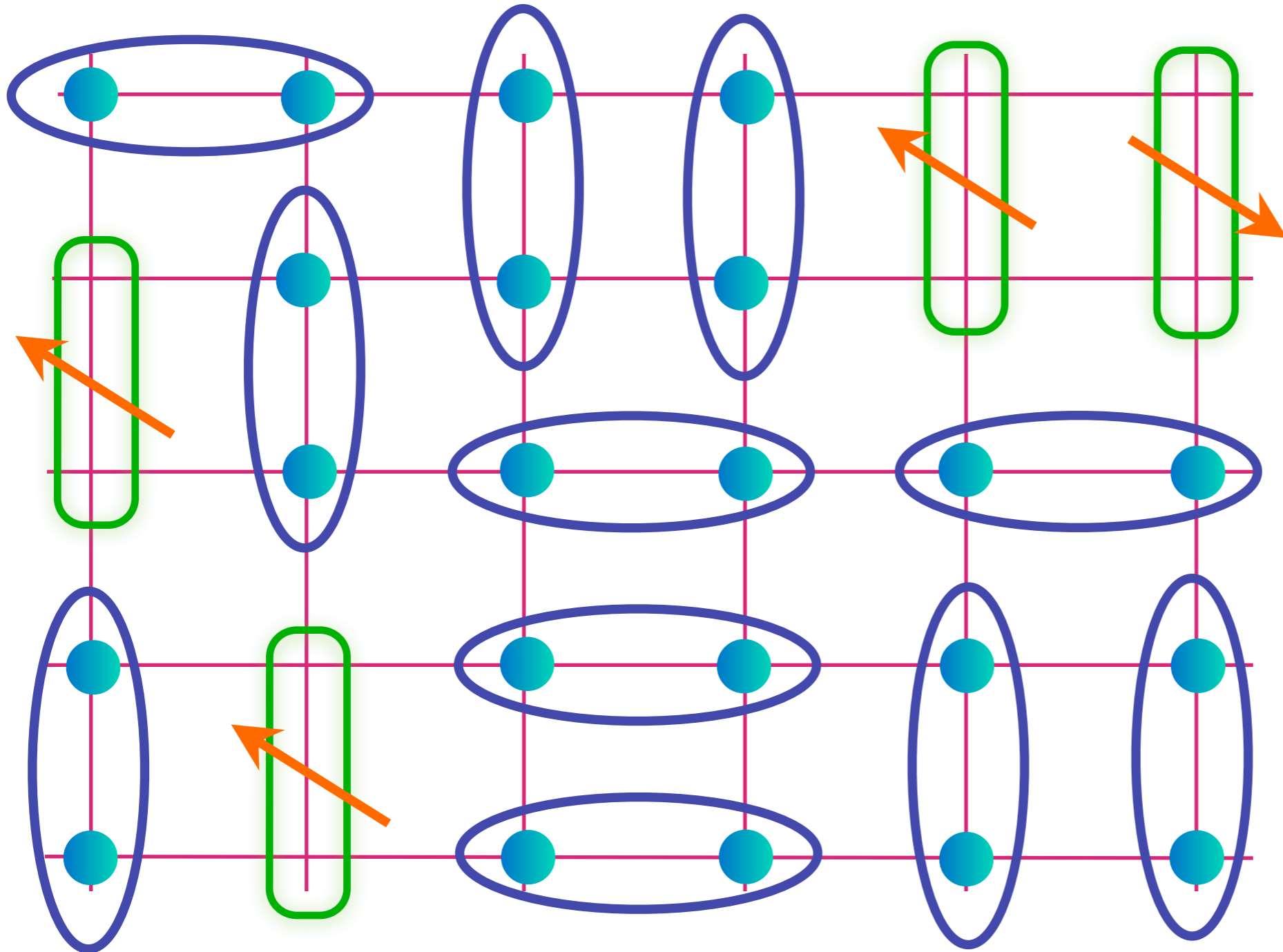
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$$\text{blue oval with 2 dots} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



Emergent gauge field and gauge-neutral, spin  $S=1/2$ , charge  $+e$  fermions of density  $p$

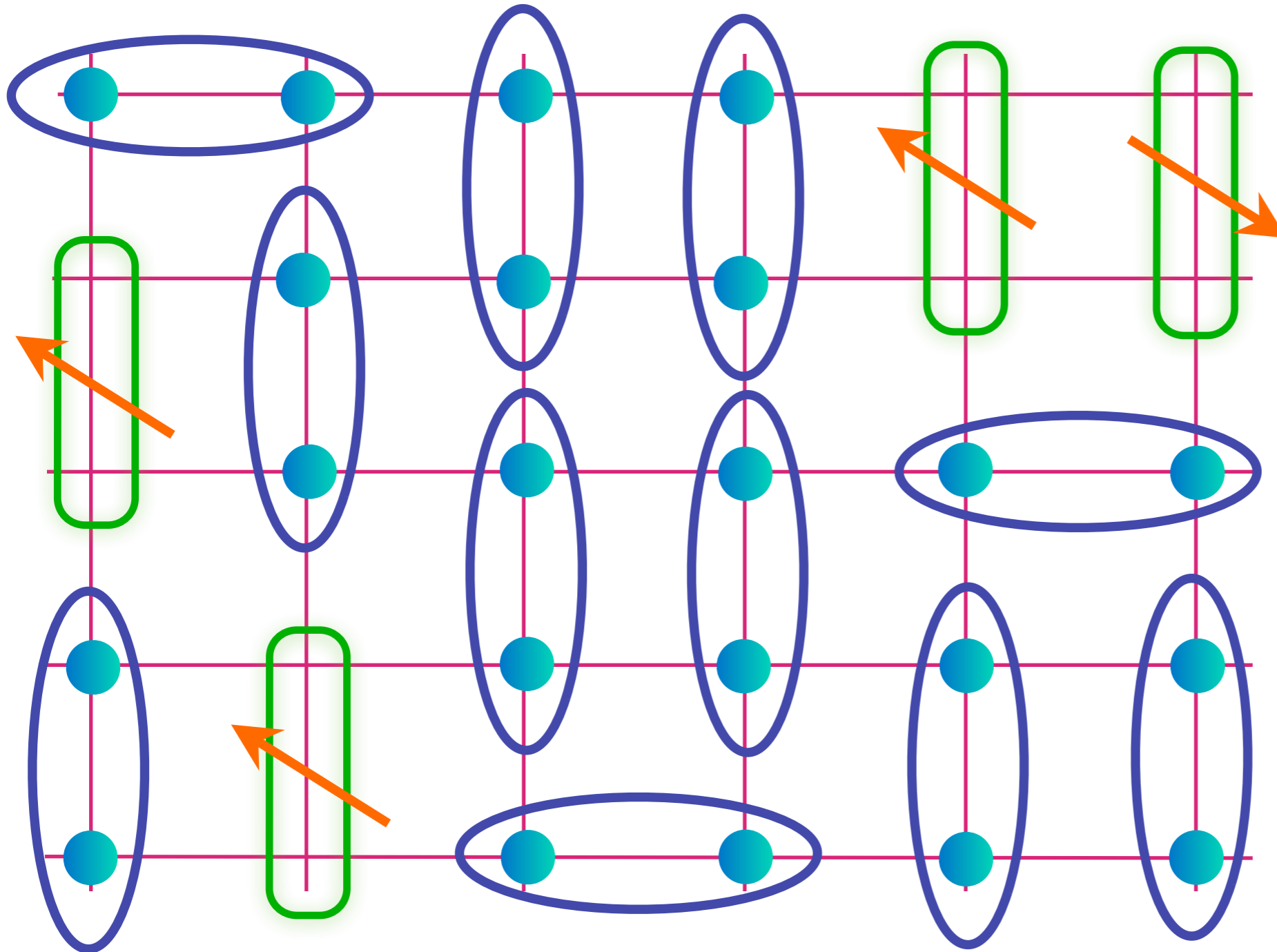
T. Senthil, S. S., M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

R. K. Kaul, A. Kolezhuk, M. Levin, S. S., and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)

E. G. Moon and S. S. *Phys. Rev. B* **83**, 224508 (2011); M. Punk, A. Allais, and S. S., arXiv:1501.00978.

# Fractionalized Fermi liquid (FL\*)


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



Emergent gauge field  
*and*  
gauge-neutral,  
spin  $S=1/2$ ,  
charge  $+e$   
fermions  
of density  $p$

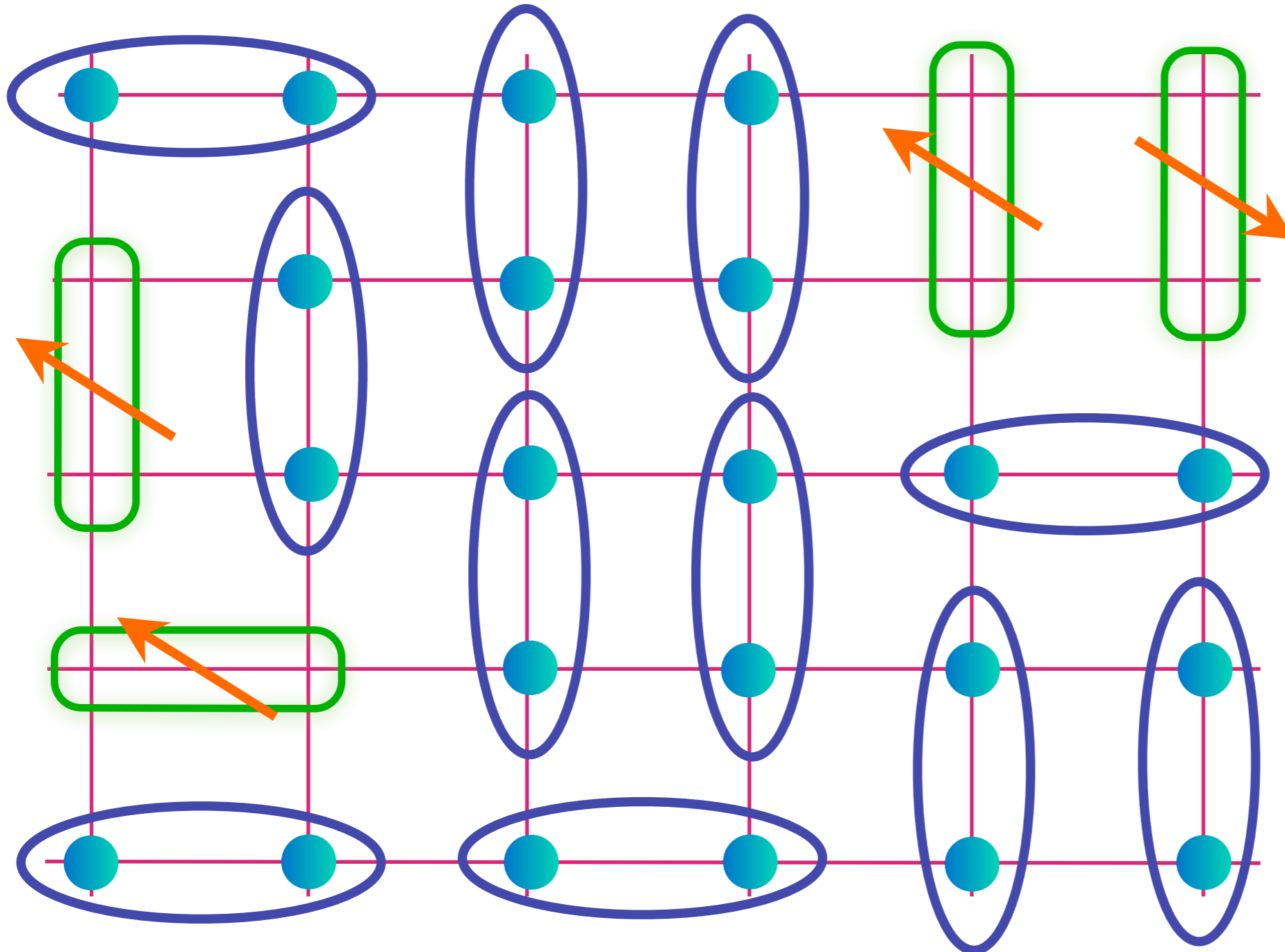
T. Senthil, S. S., M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

R. K. Kaul, A. Kolezhuk, M. Levin, S. S., and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)

E. G. Moon and S. S. *Phys. Rev. B* **83**, 224508 (2011); M. Punk, A. Allais, and S. S., arXiv:1501.00978.

# Fractionalized Fermi liquid (FL\*)


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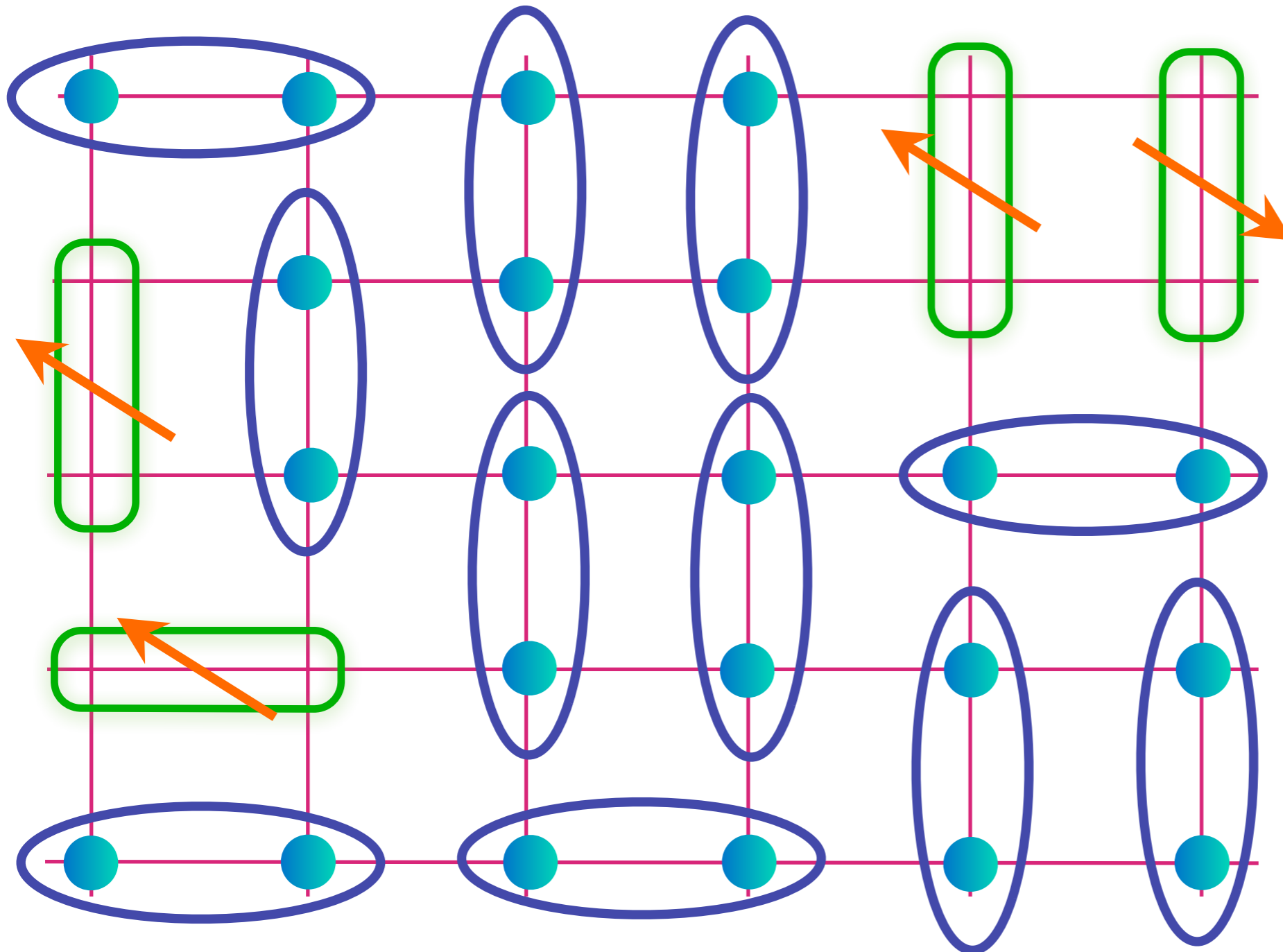
T. Senthil, S. S., M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

R. K. Kaul, A. Kolezhuk, M. Levin, S. S., and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)

E. G. Moon and S. S. *Phys. Rev. B* **83**, 224508 (2011); M. Punk, A. Allais, and S. S., arXiv:1501.00978.

# Fractionalized Fermi liquid (FL\*)

$$\text{dimer} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



Note:  
electron-like  
quasiparticle  
can only be a  
dimer because  
of spin-liquid  
background; it  
is not possible  
to have it on a  
single site

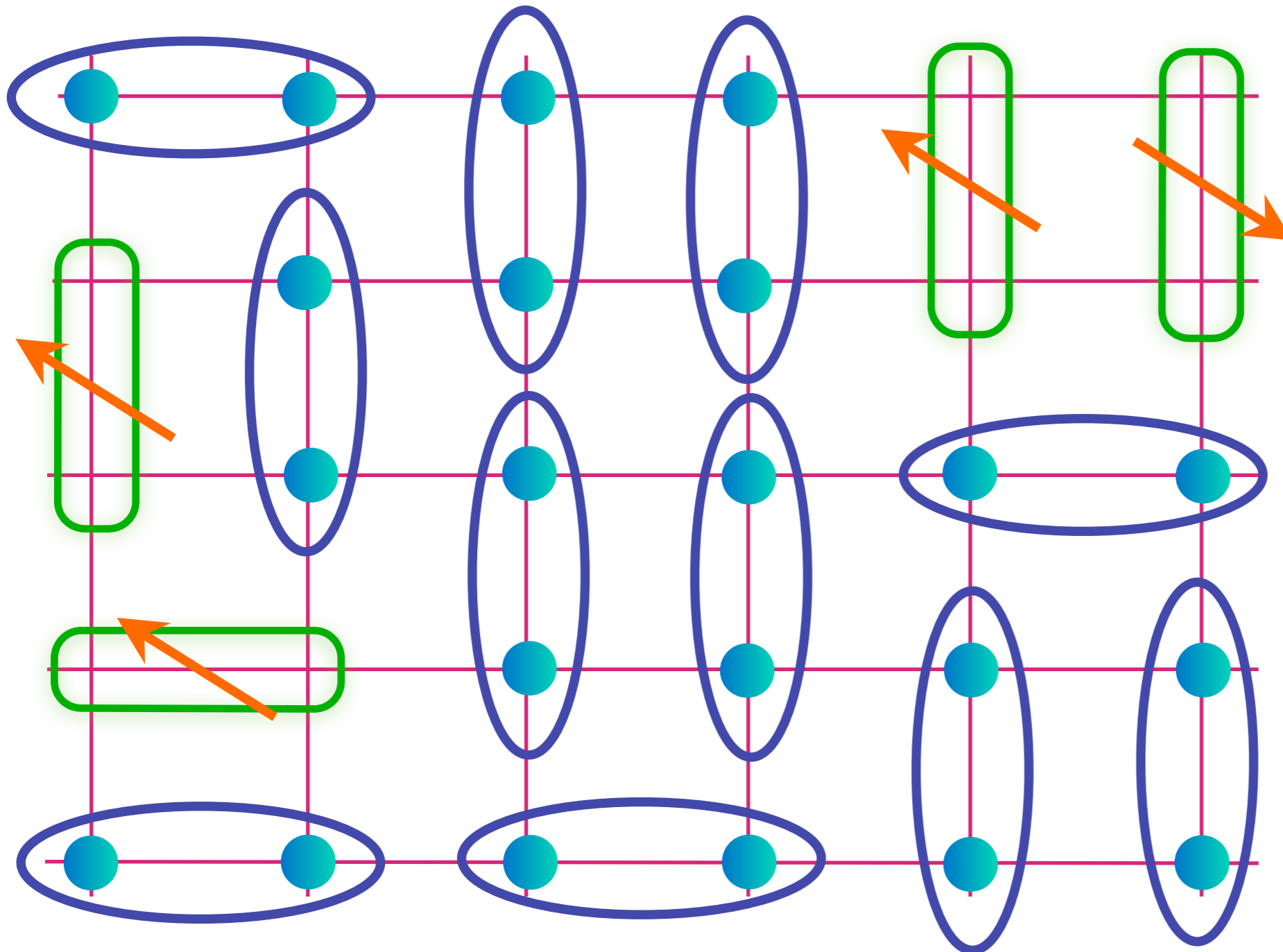
T. Senthil, S. S., M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

R. K. Kaul, A. Kolezhuk, M. Levin, S. S., and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)

E. G. Moon and S. S. *Phys. Rev. B* **83**, 224508 (2011); M. Punk, A. Allais, and S. S., arXiv:1501.00978.

# Fractionalized Fermi liquid (FL\*)

$$\text{Diagram of two particles in an oval} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



Realizes a metal with a Fermi surface of area  $p$  and “topological order”

T. Senthil, S. S., M. Vojta *Phys. Rev. Lett.* **90**, 216403 (2003)

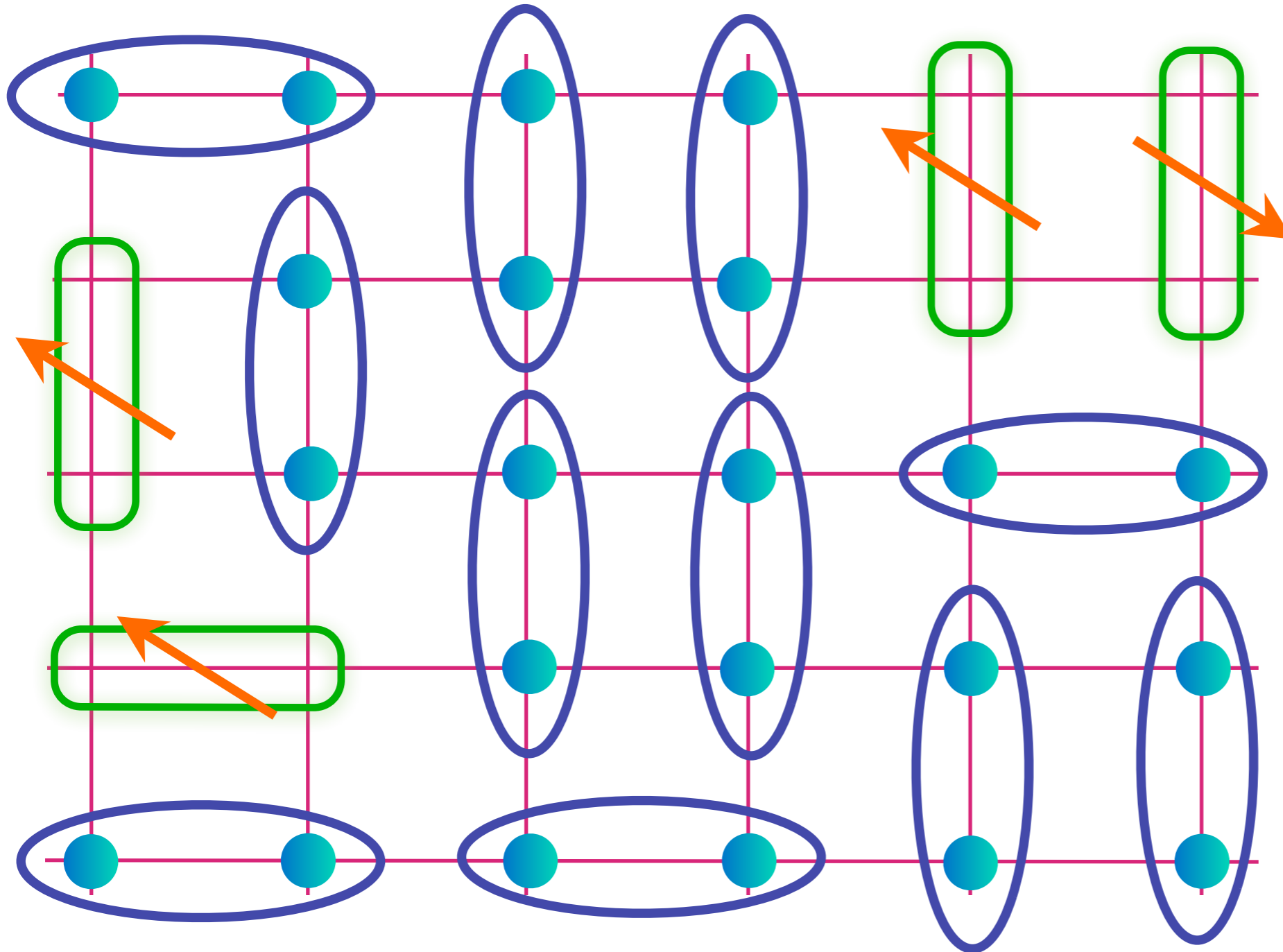
R. K. Kaul, A. Kolezhuk, M. Levin, S. S., and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)

E. G. Moon and S. S. *Phys. Rev. B* **83**, 224508 (2011); M. Punk, A. Allais, and S. S., arXiv:1501.00978.

The high  $T$  pseudogap:

*A quantum dimer model for a metal  
with topological order*

# Quantum dimer model with bosonic and fermionic dimers



# Early discussion of a fermionic hole on a dimer

PHYSICAL REVIEW B

VOLUME 38, NUMBER 16

1 DECEMBER 1988

## Quasiparticles in the copper-oxygen planes of high- $T_c$ superconductors: An exact solution for a ferromagnetic background

V. J. Emery

*Brookhaven National Laboratory, Upton, New York 11973*

G. Reiter

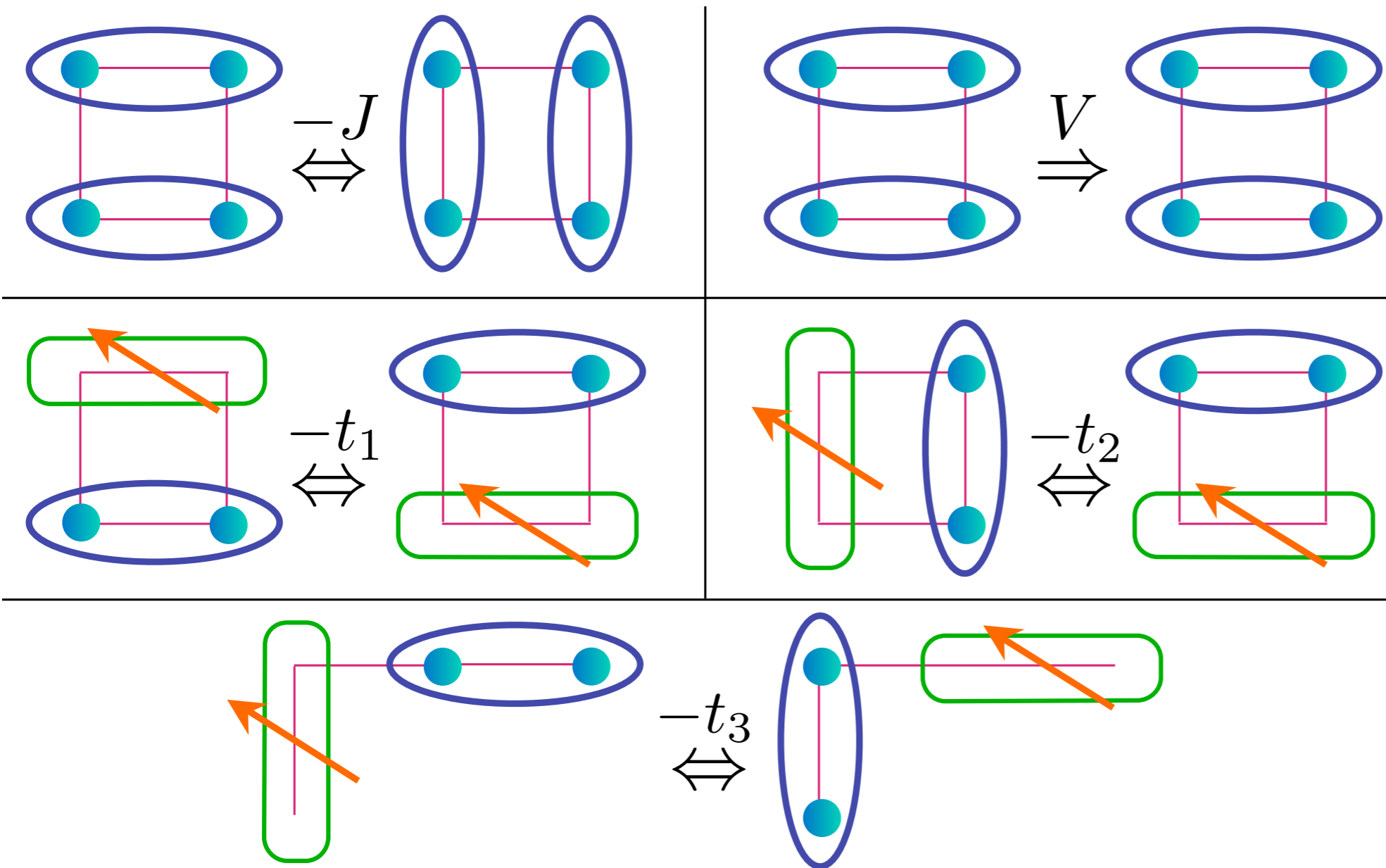
*Physics Department, University of Houston, Houston, Texas 77204-5504*

(Received 22 July 1988)

A model for a mobile hole in the copper oxide planes of high-temperature superconductors is solved exactly. The hole moves on the oxygen atoms through a lattice of spins localized on the copper atoms. In order to obtain a solvable problem, it is assumed that the copper atoms provide a ferromagnetic background. The resulting quasiparticles have both charge and spin in contrast to the Cu-O singlets occurring in proposed effective single-band Hubbard models derived from the Cu-O network. Thus these two models of high-temperature superconductors may have different low-energy physics.

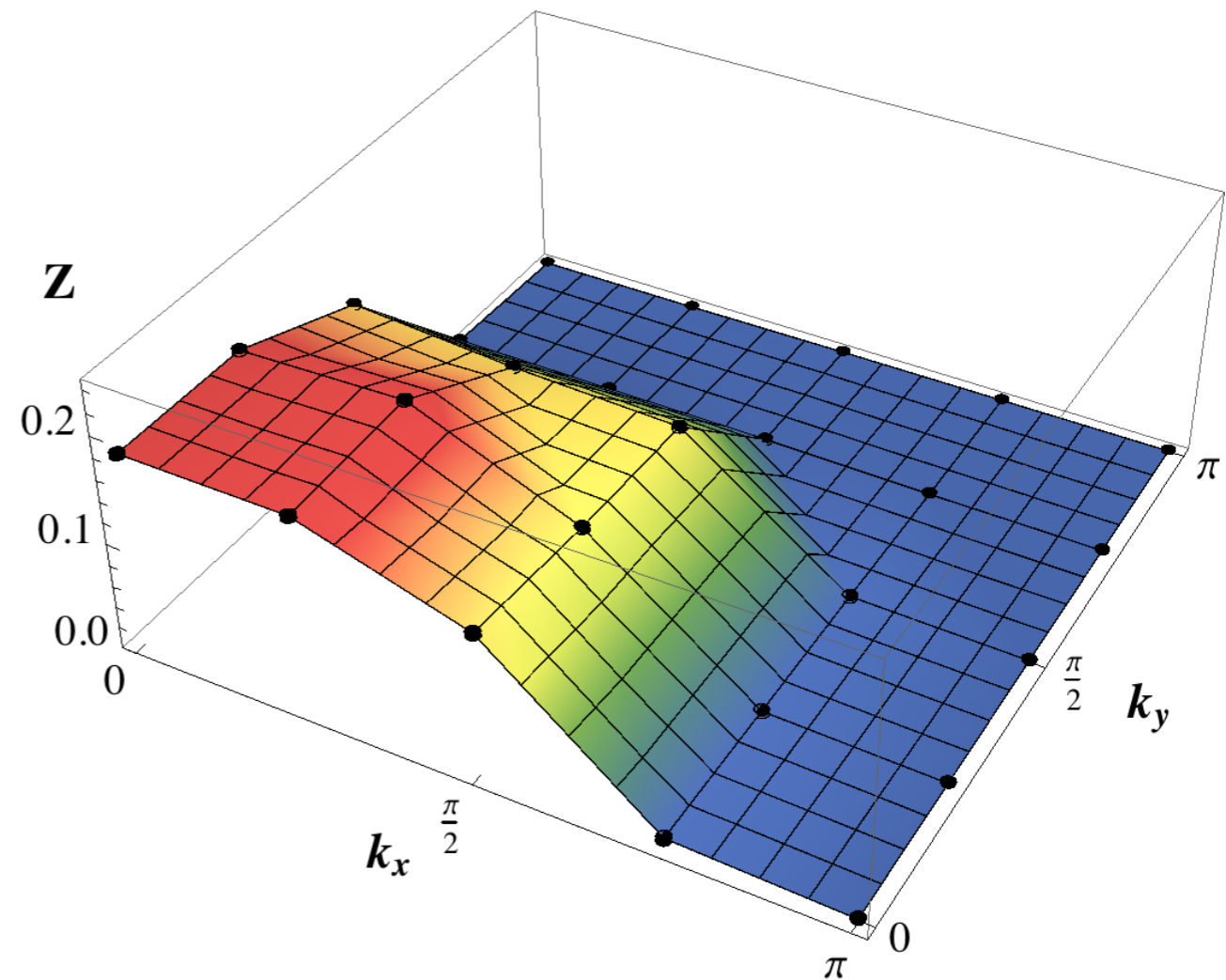
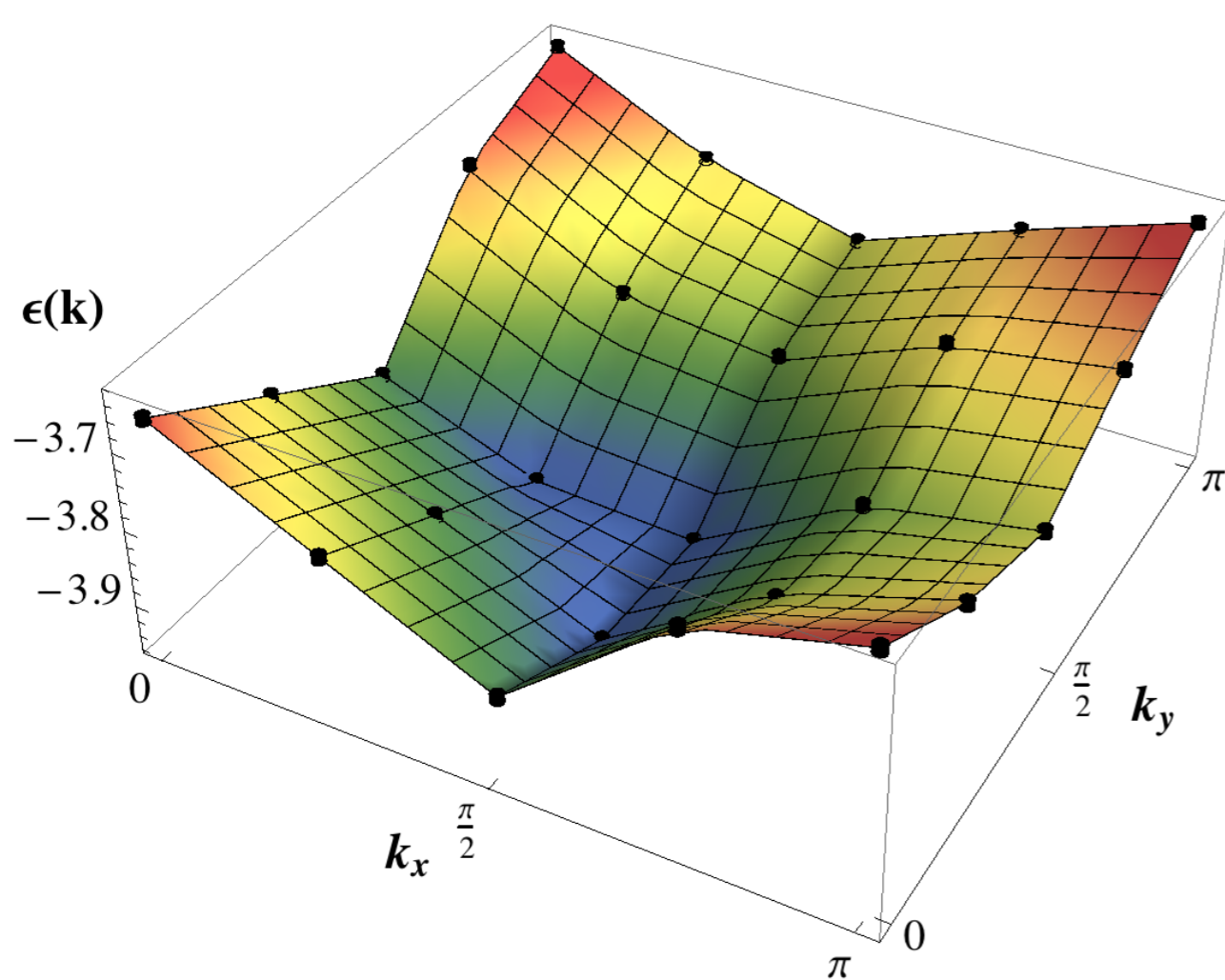
propose that the correct basis to describe the result is composed of a hole on a single O atom coupled to its two neighboring Cu atoms, i.e., basis states of the form  $6^{-1/2}[2|\downarrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle]$ .

# Quantum dimer model with bosonic and fermionic dimers



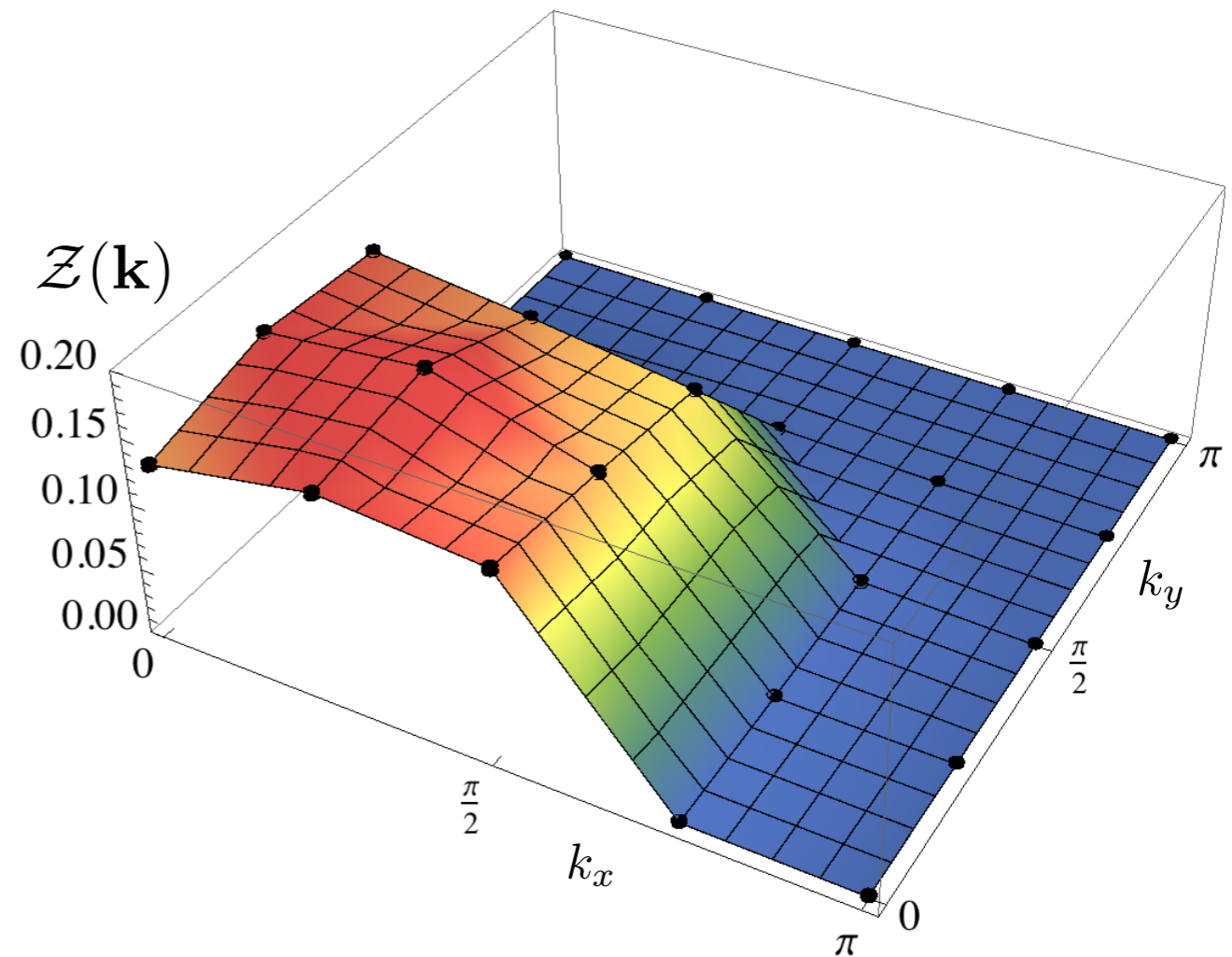
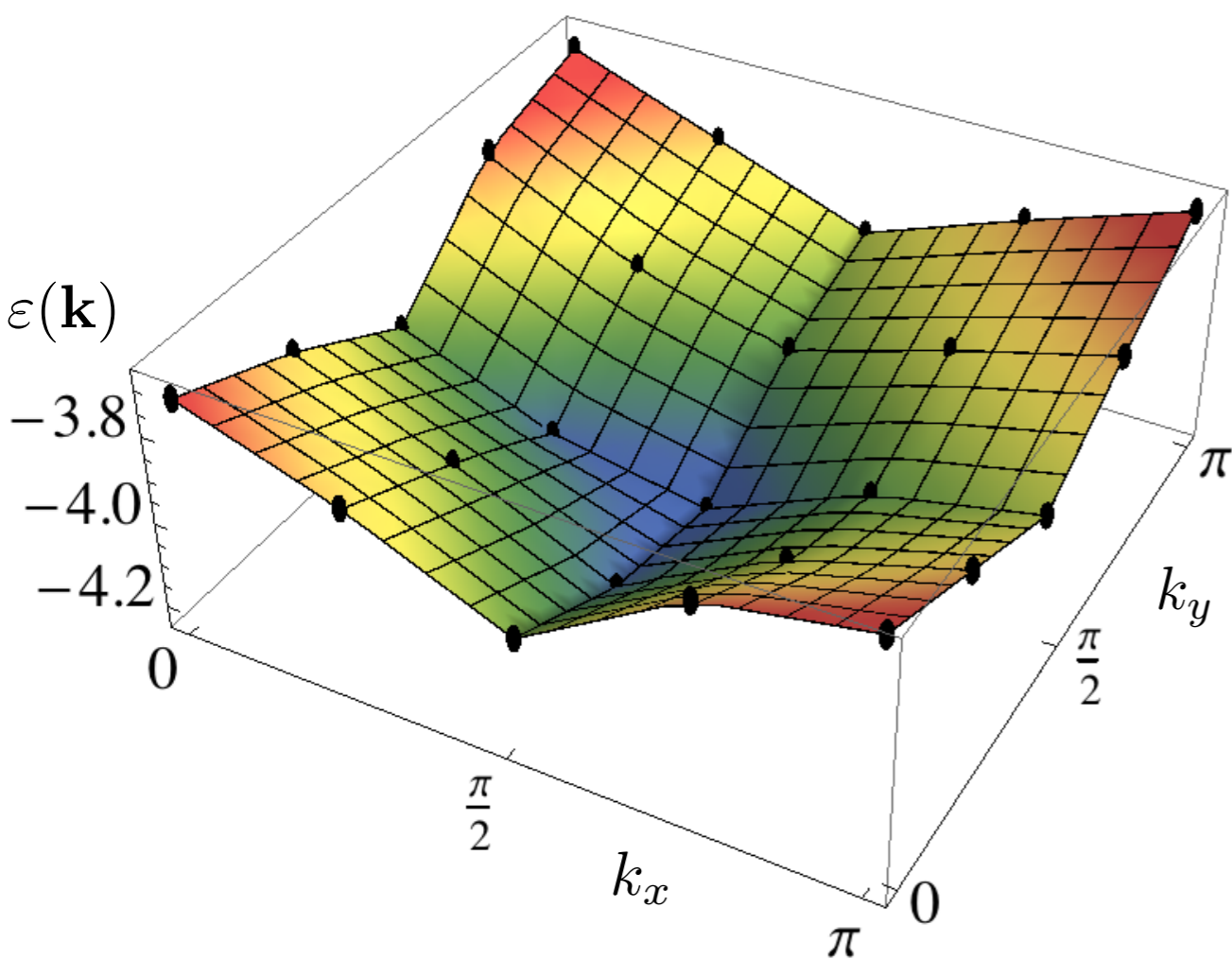
Connection to the  $t-t'-t''-J$  model:  
 $t_1 = -(t + t')/2$   
 $t_2 = (t - t')/2$   
 $t_3 = -(t + t' + t'')/4$

# Quantum dimer model with bosonic and fermionic dimers



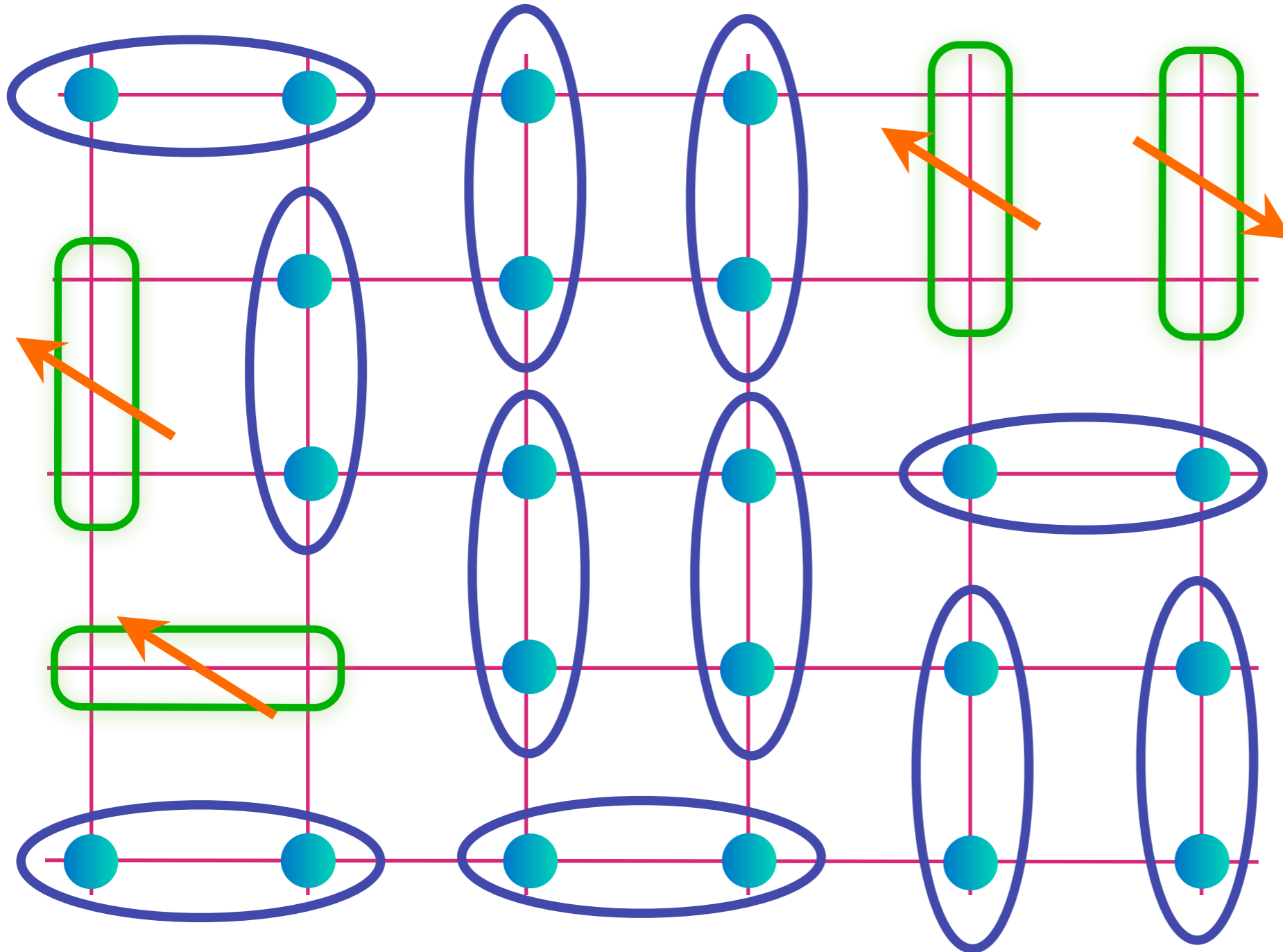
Dispersion and quasiparticle residue of a single fermionic dimer for  $J = V = 1$ , and hopping parameters obtained from the  $t$ - $J$  model for the cuprates,  $t_1 = -1.05$ ,  $t_2 = 1.95$  and  $t_3 = -0.6$ , on a  $8 \times 8$  lattice.

# Quantum dimer model with bosonic and fermionic dimers



Dispersion and quasiparticle residue for  $J = V = 1$ , and hopping parameters  $t_1 = 1$ ,  $t_2 = 1$  and  $t_3 = -1$ , on a  $8 \times 8$  lattice.

# Quantum dimer model with bosonic and fermionic dimers

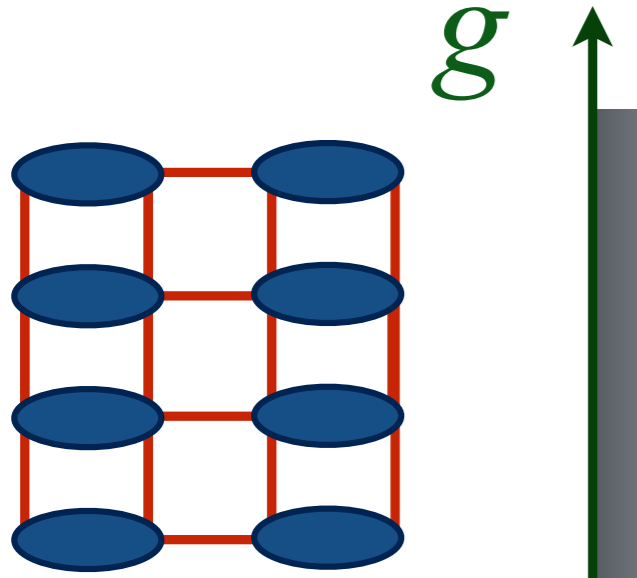


Dimer nature of charge carriers makes them “nematogens”: the metallic state is likely to have nematic order/ correlations

The high  $T$  pseudogap:

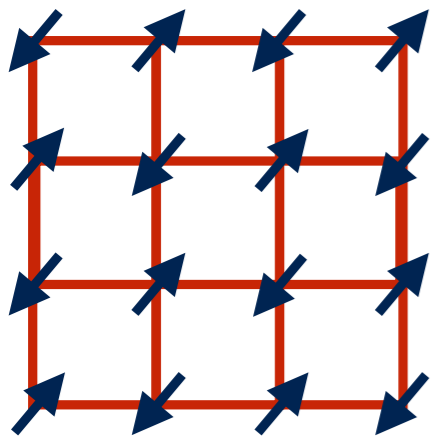
*Doping deconfined quantum critical points  
in frustrated antiferromagnets*

# Deconfined criticality



$U(1)$  SL  $\rightarrow$  VBS  
+ confinement

$g_c$

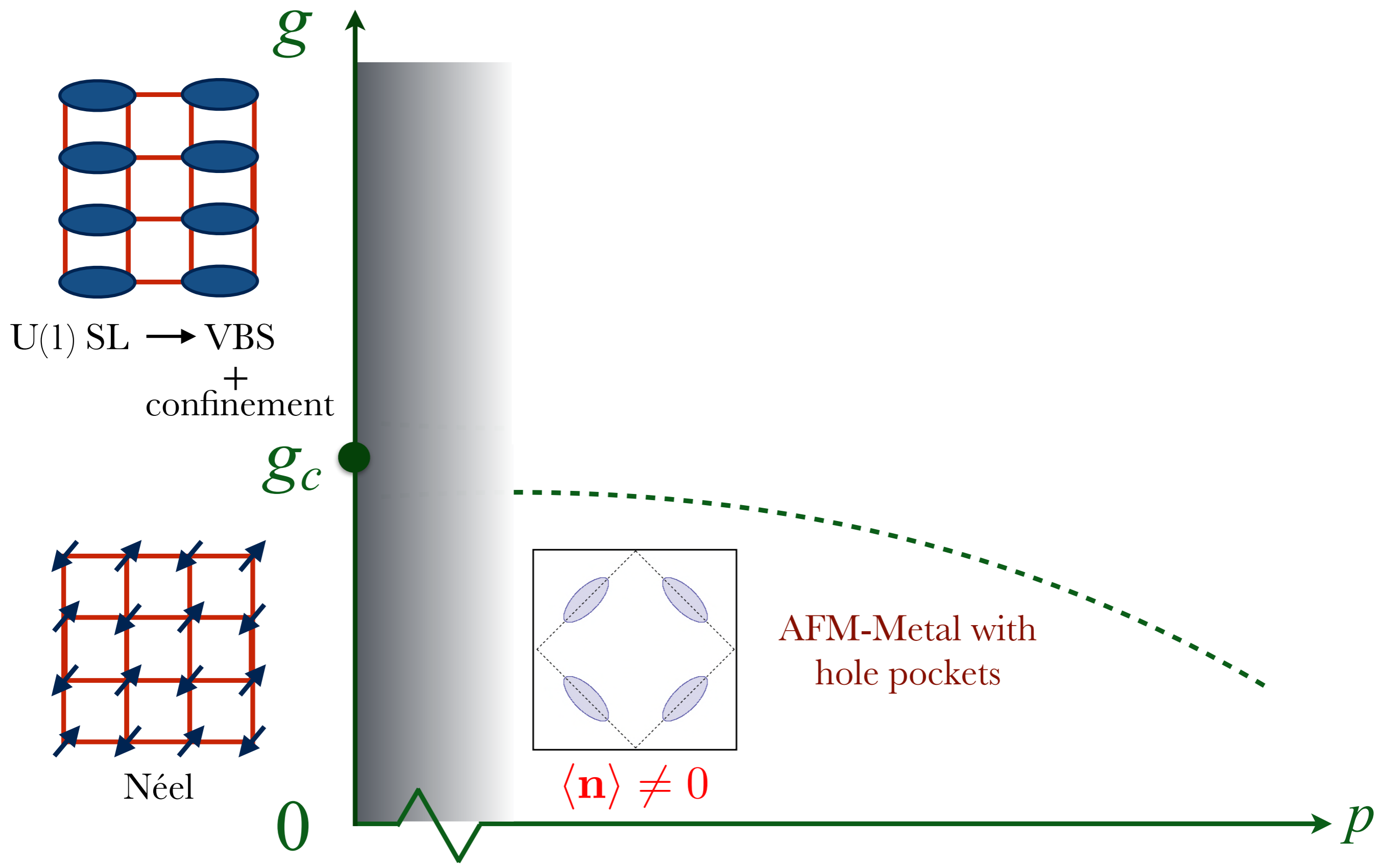


Néel

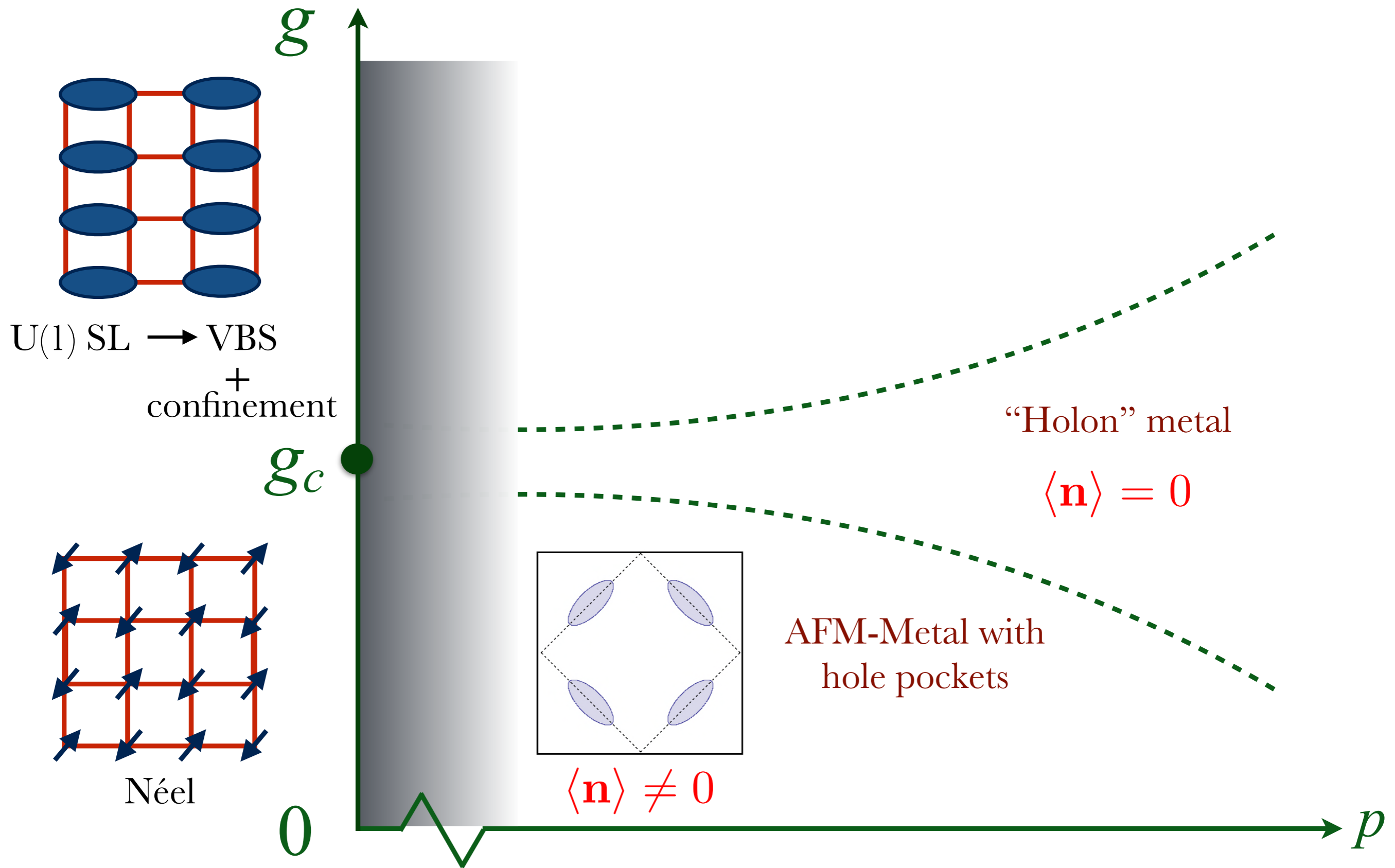
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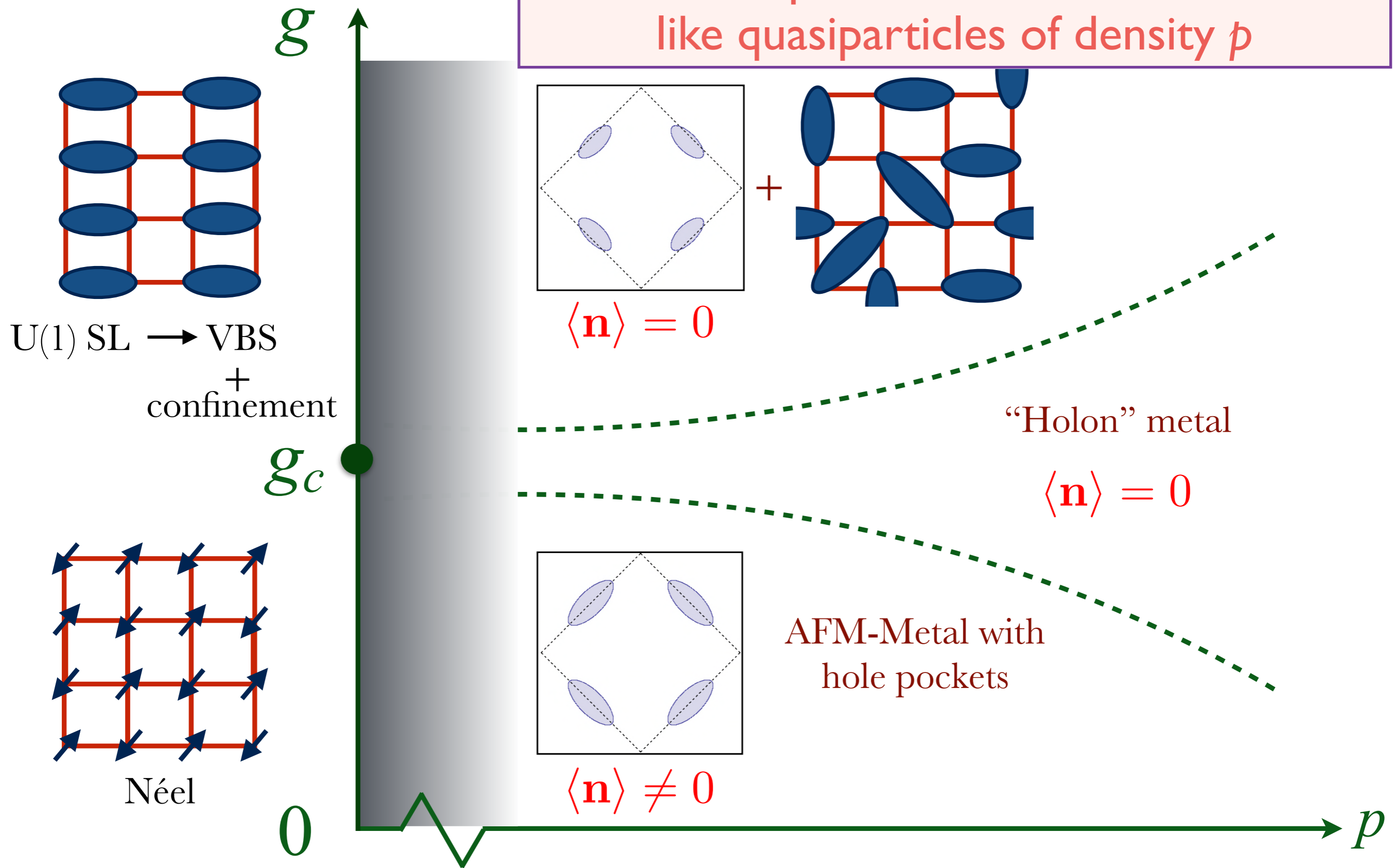
# Deconfined criticality



# Deconfined criticality



# Deconfined criticality

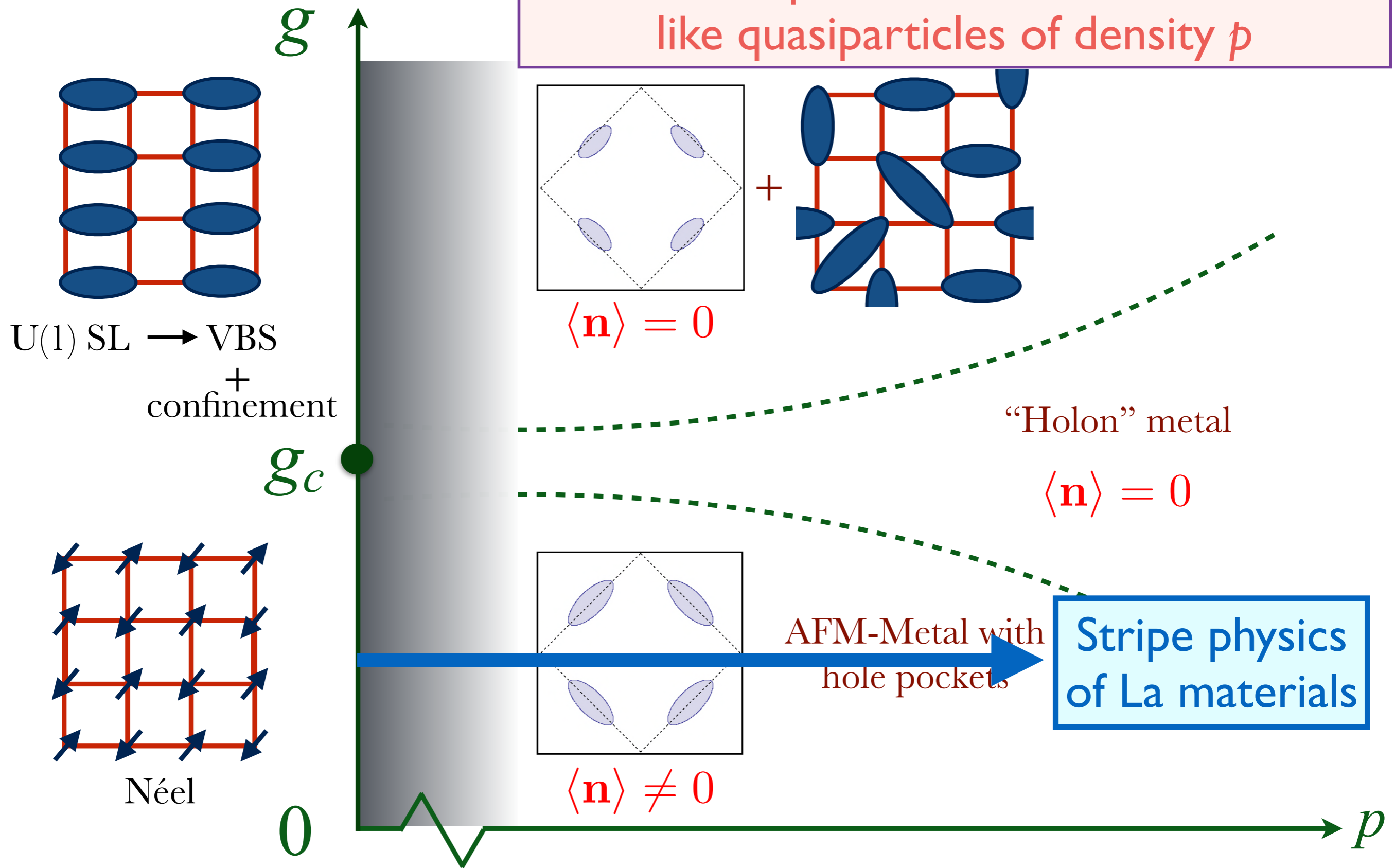


R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, Phys. Rev. B **75**, 235122 (2007)

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)

D. Chowdhury and S. Sachdev, Phys. Rev. B **90**, 245136 (2014).

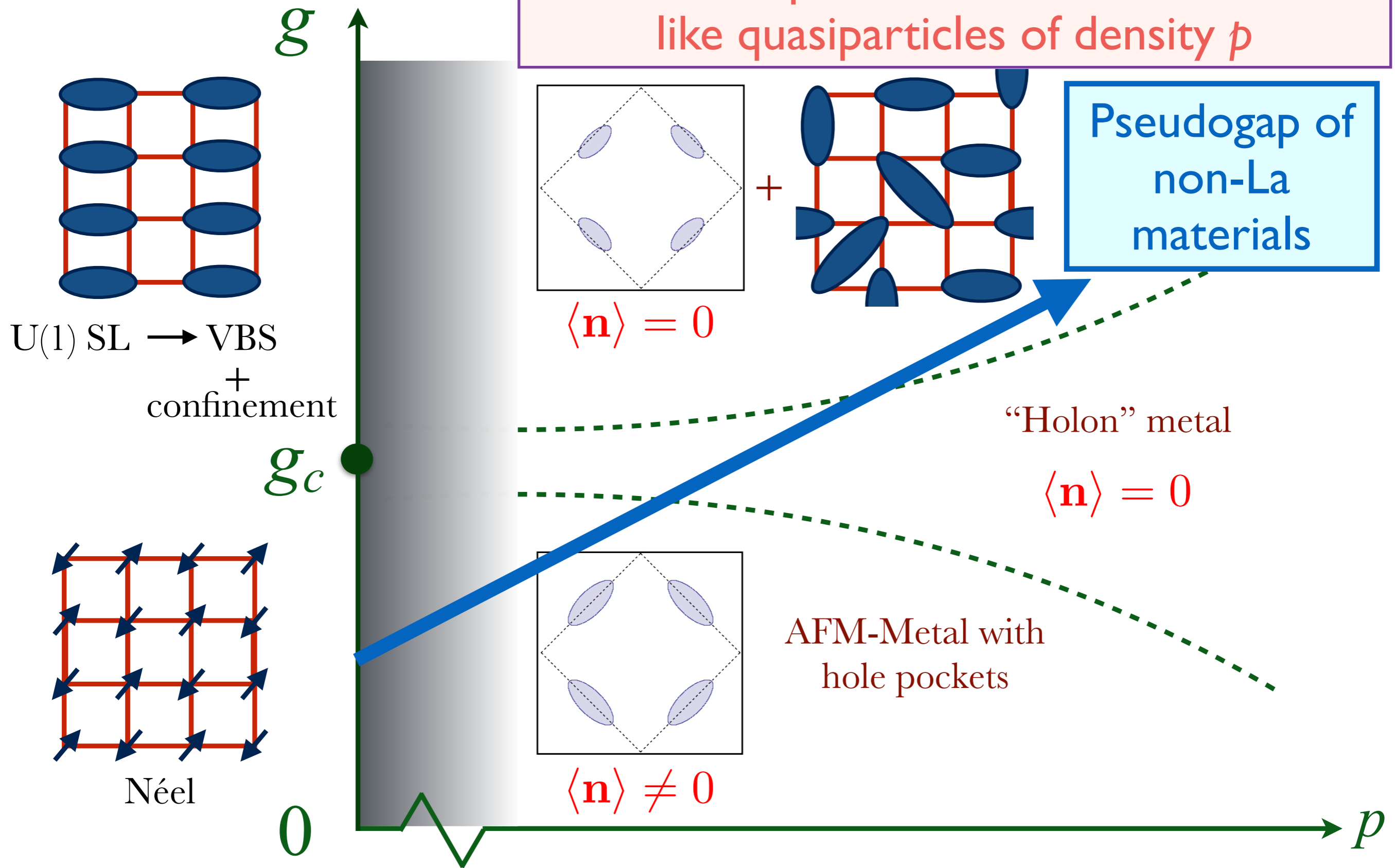
# Deconfined criticality

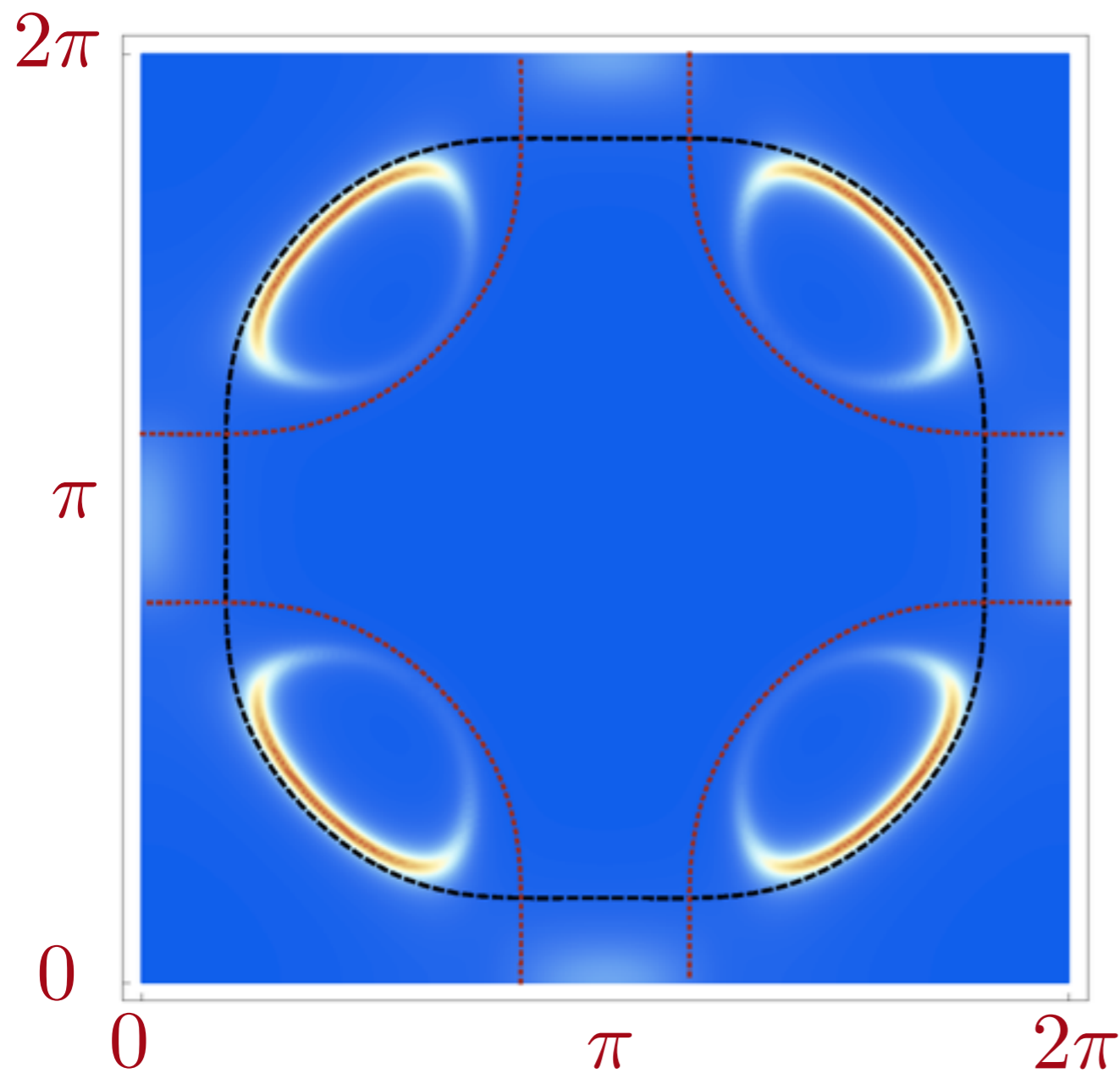


D. Chowdhury and S. Sachdev, Phys. Rev. B **90**, 245136 (2014)

A. Thomson and S. Sachdev, arXiv:1410.3483

# Deconfined criticality



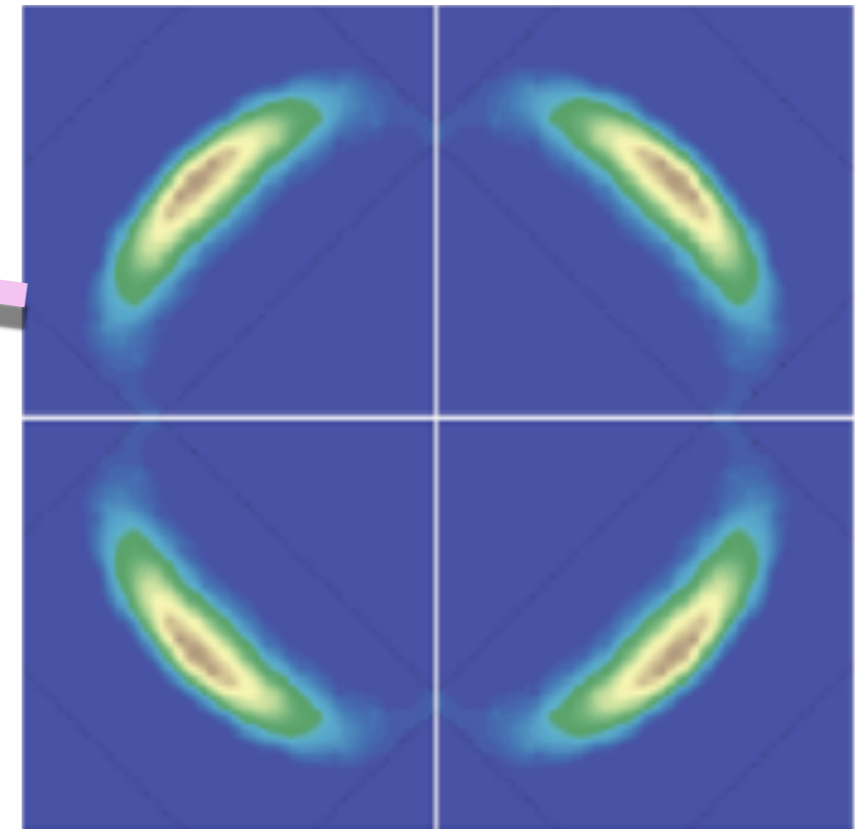
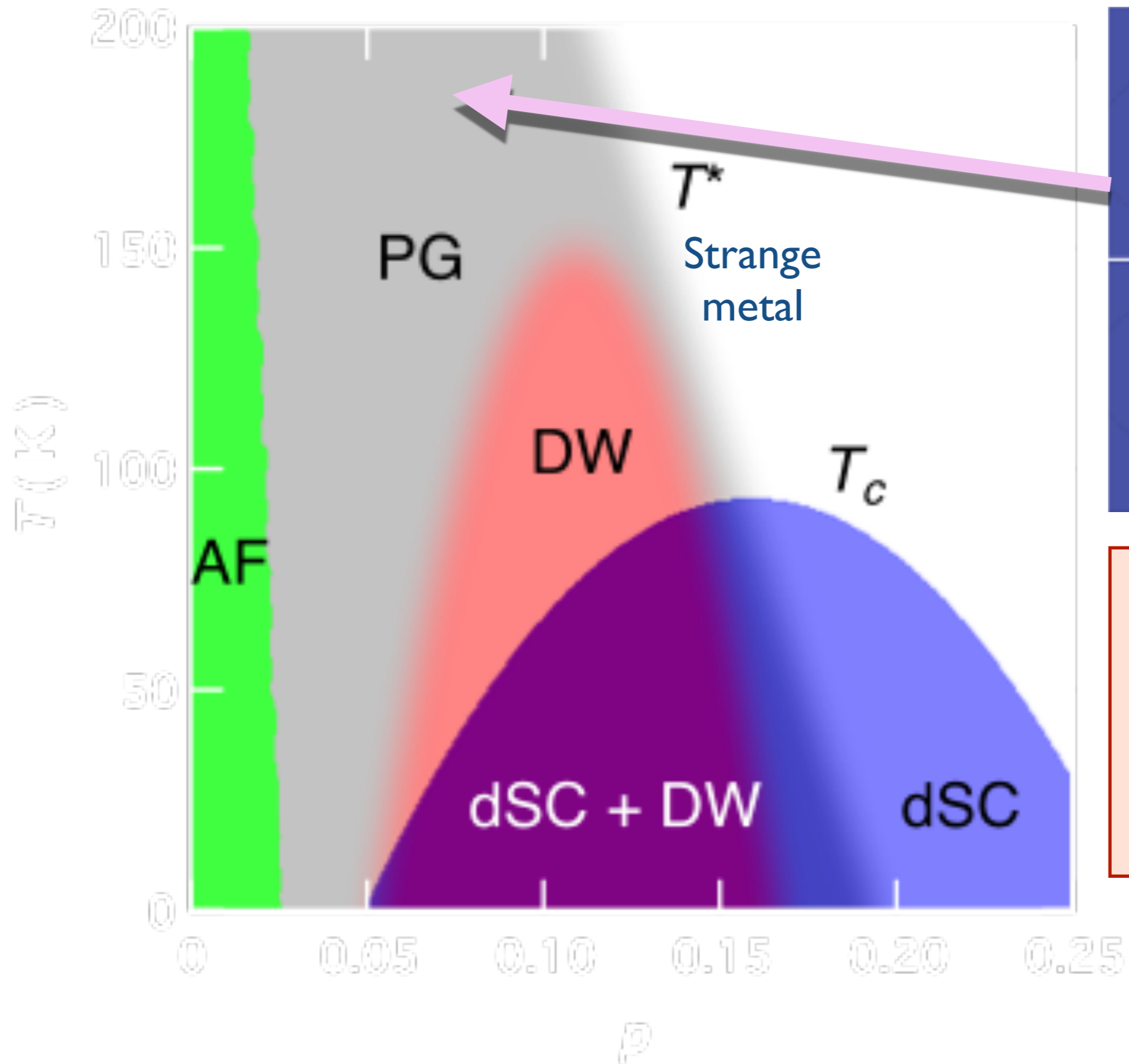


Electron spectral  
function of FL\*

Semi-phenomenological theory of a FL\* state with hole pockets of volume  $p$ , along with a background spin liquid with an emergent U(1) gauge field. Note that the quasiparticle excitations around the Fermi surface do not carry U(1) gauge charges.

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)  
M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978

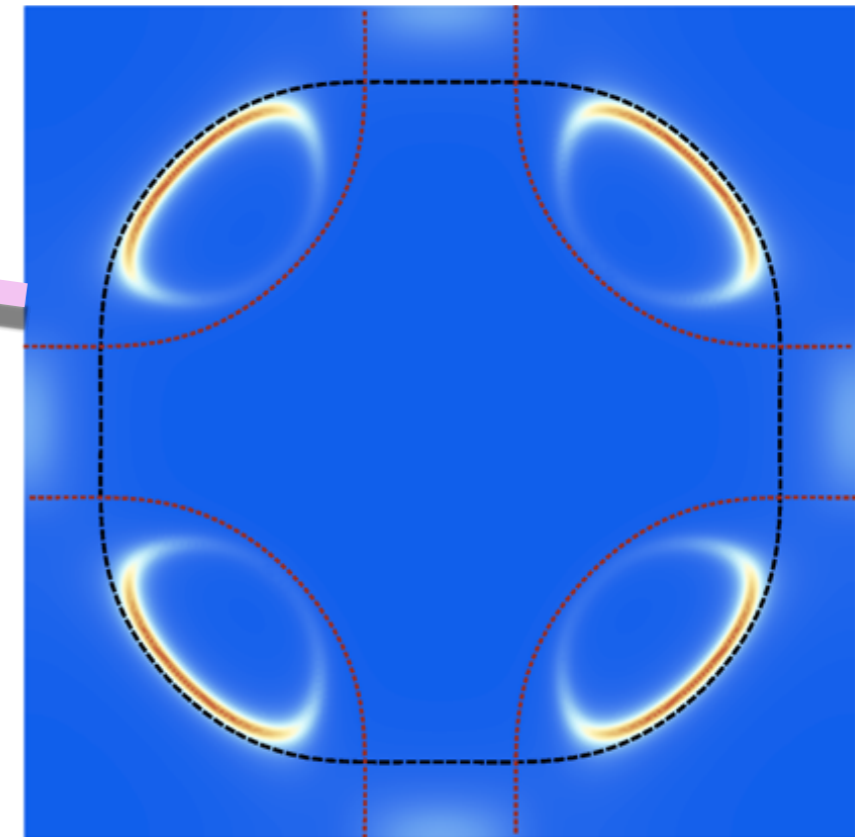
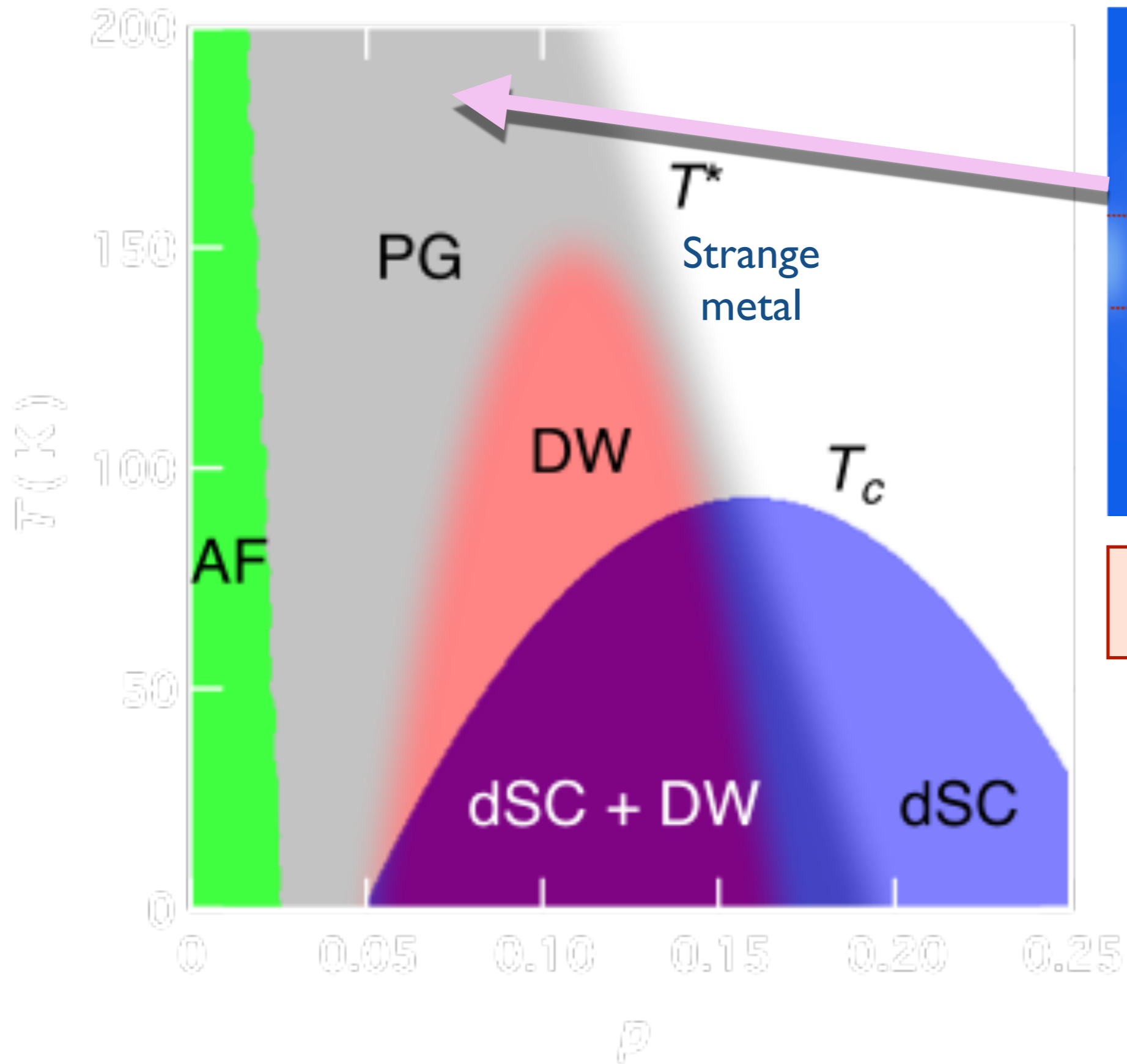
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)



“Fermi arcs”  
at  
low  $p$

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)

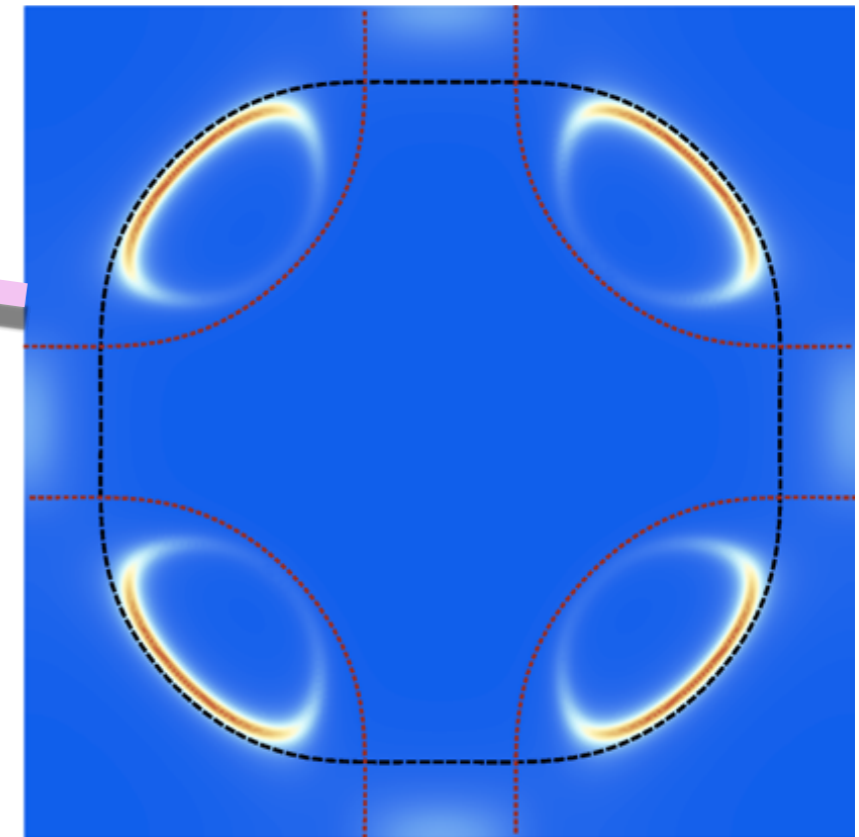
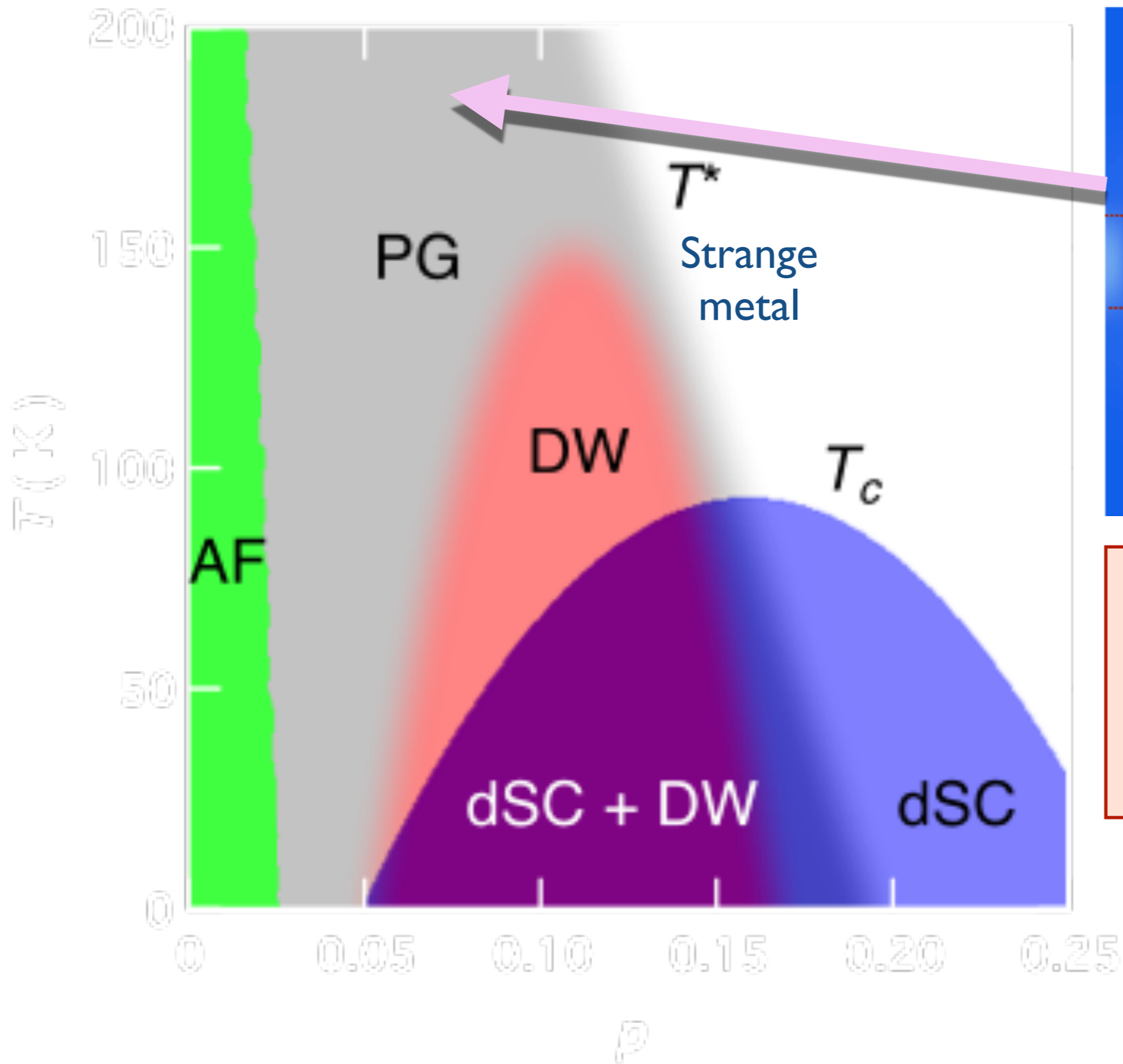
M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978



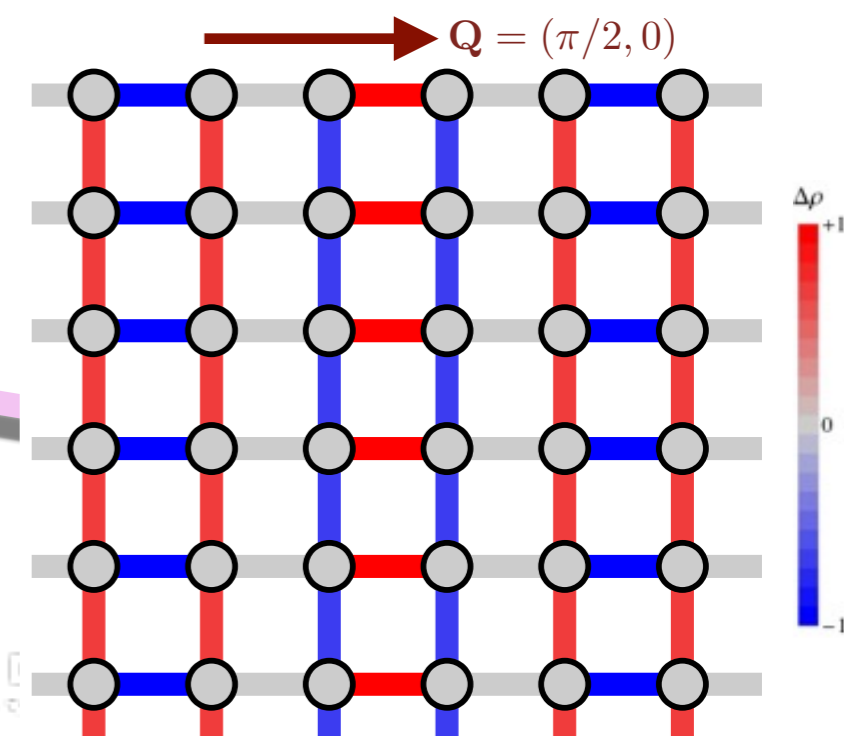
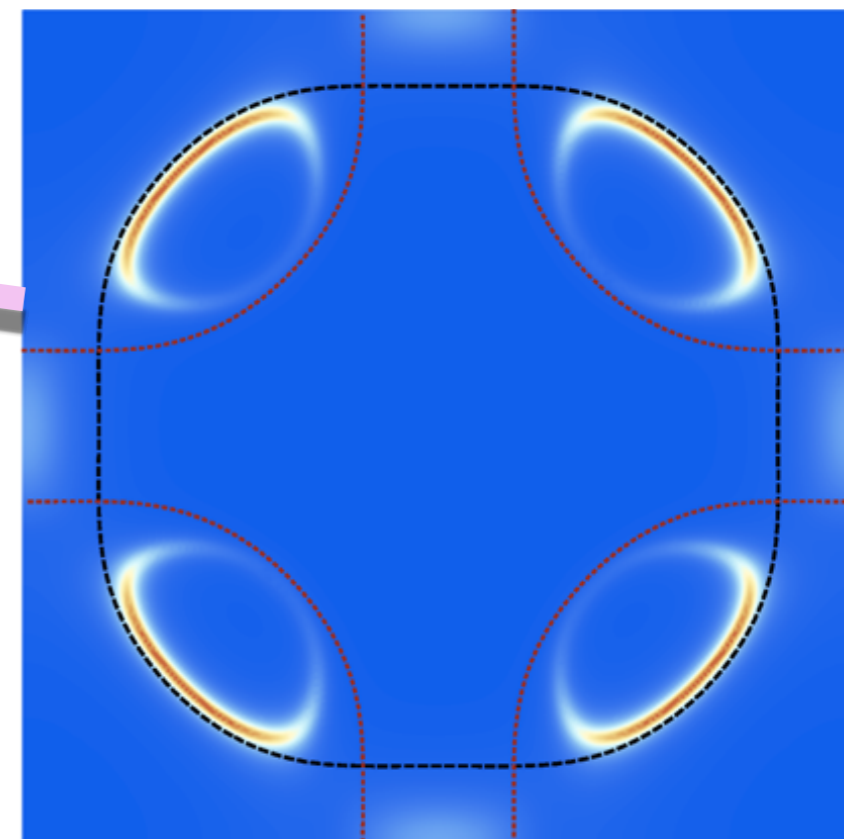
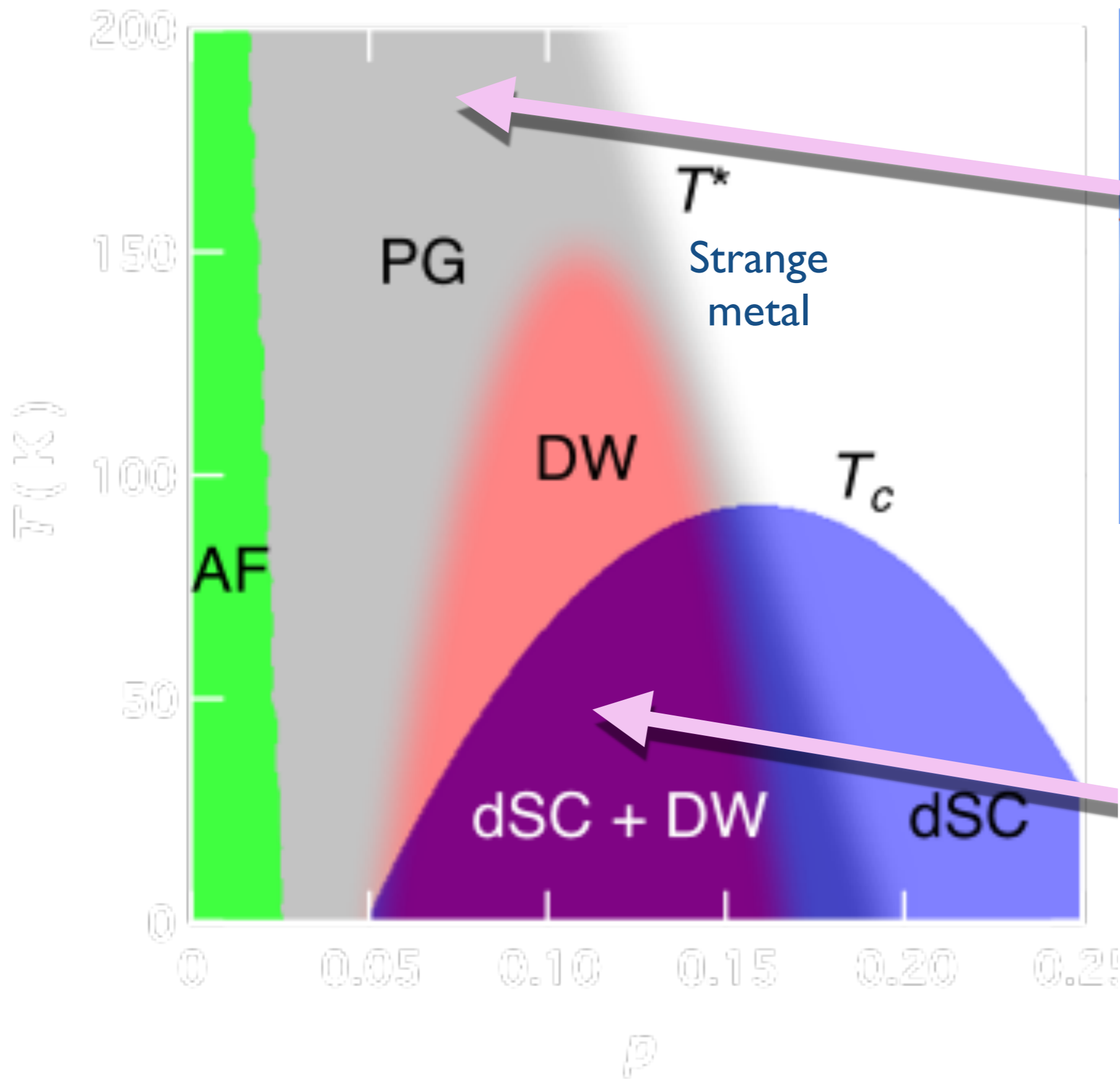
**FL\***.

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978



**FL\*.**  
Expts to detect  
hole pockets ?



Connecting high and low  $T$

*Density wave instabilities*

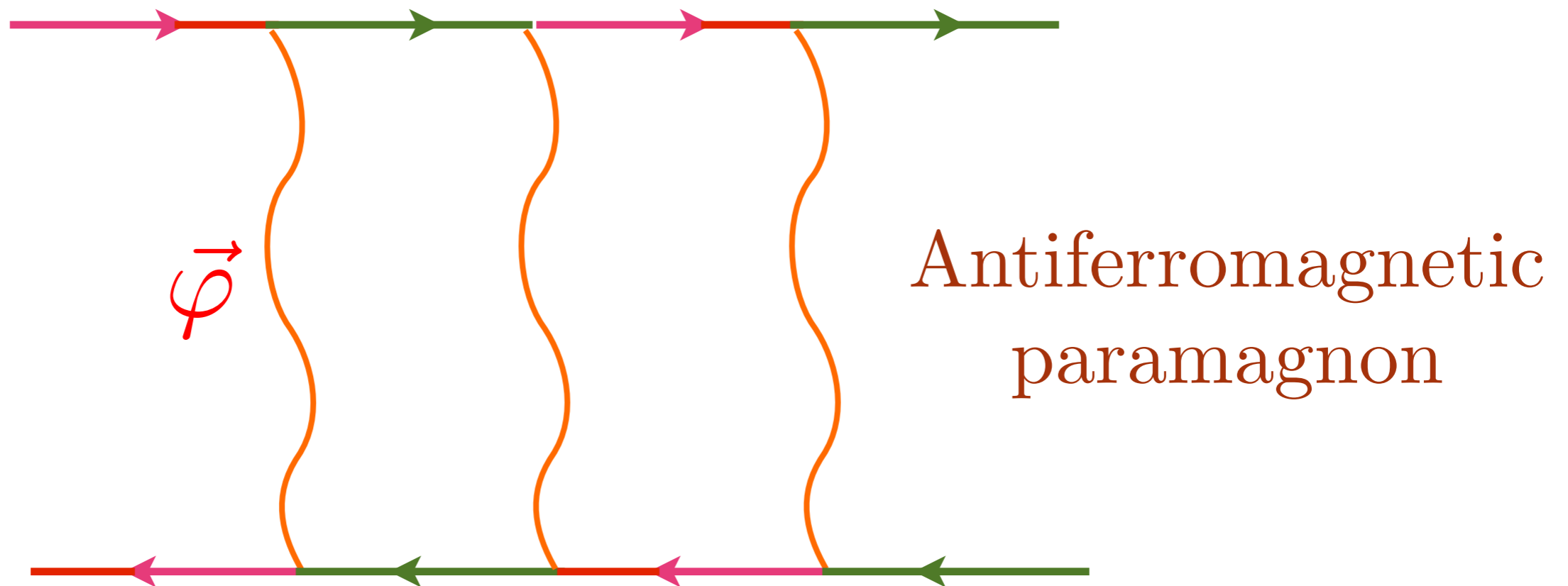
# Pairing “glue” for d-wave superconductivity from antiferromagnetic fluctuations



Leads to  $\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$

- V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)  
 D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)  
 K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)  
 P. Monthoux, A. V. Balatsky, and D. Pines, *Phys. Rev. Lett.* **67**, 3448 (1991)

# Same glue can lead to “d-wave” particle-hole pairing

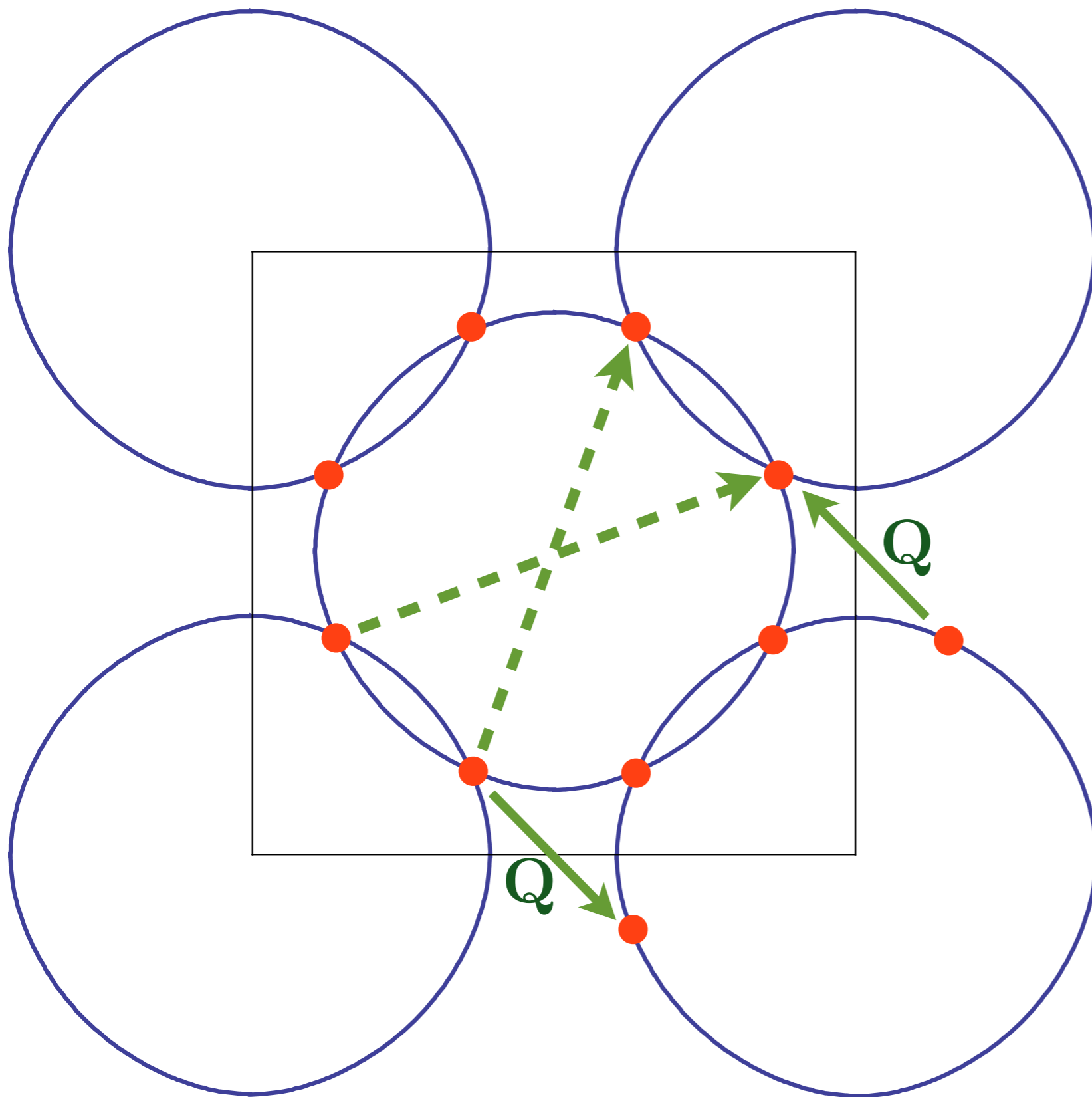


- M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)  
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)  
M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)  
S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)  
K. B. Efetov, H. Meier, and C. Pépin, Nat. Phys. **9**, 442 (2013)  
J. D. Sau and S. Sachdev, Phys. Rev. B **89**, 075129 (2014)  
Y. Wang and A. V. Chubukov, Phys. Rev. B **90**, 035149 (2014)

# Same glue can lead to “d-wave” particle-hole pairing

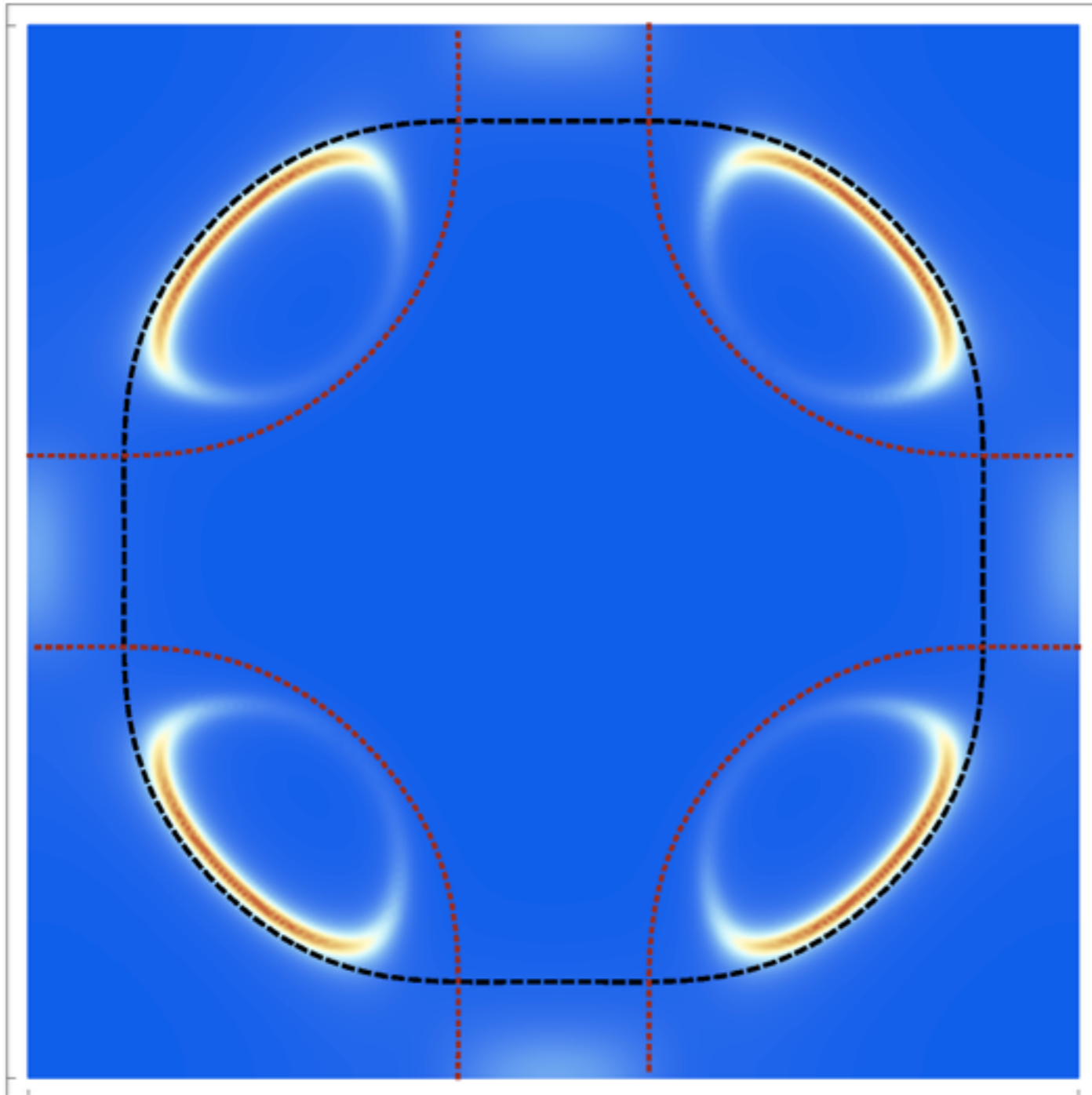


Leads to  $\left\langle c_{\mathbf{k}+\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{Q}/2,\alpha} \right\rangle =$   
 $\mathcal{P}_s + \mathcal{P}_{s'} (\cos k_x + \cos k_y) + \mathcal{P}_d (\cos k_x - \cos k_y)$   
 with  $\mathcal{P}_d$  dominant.

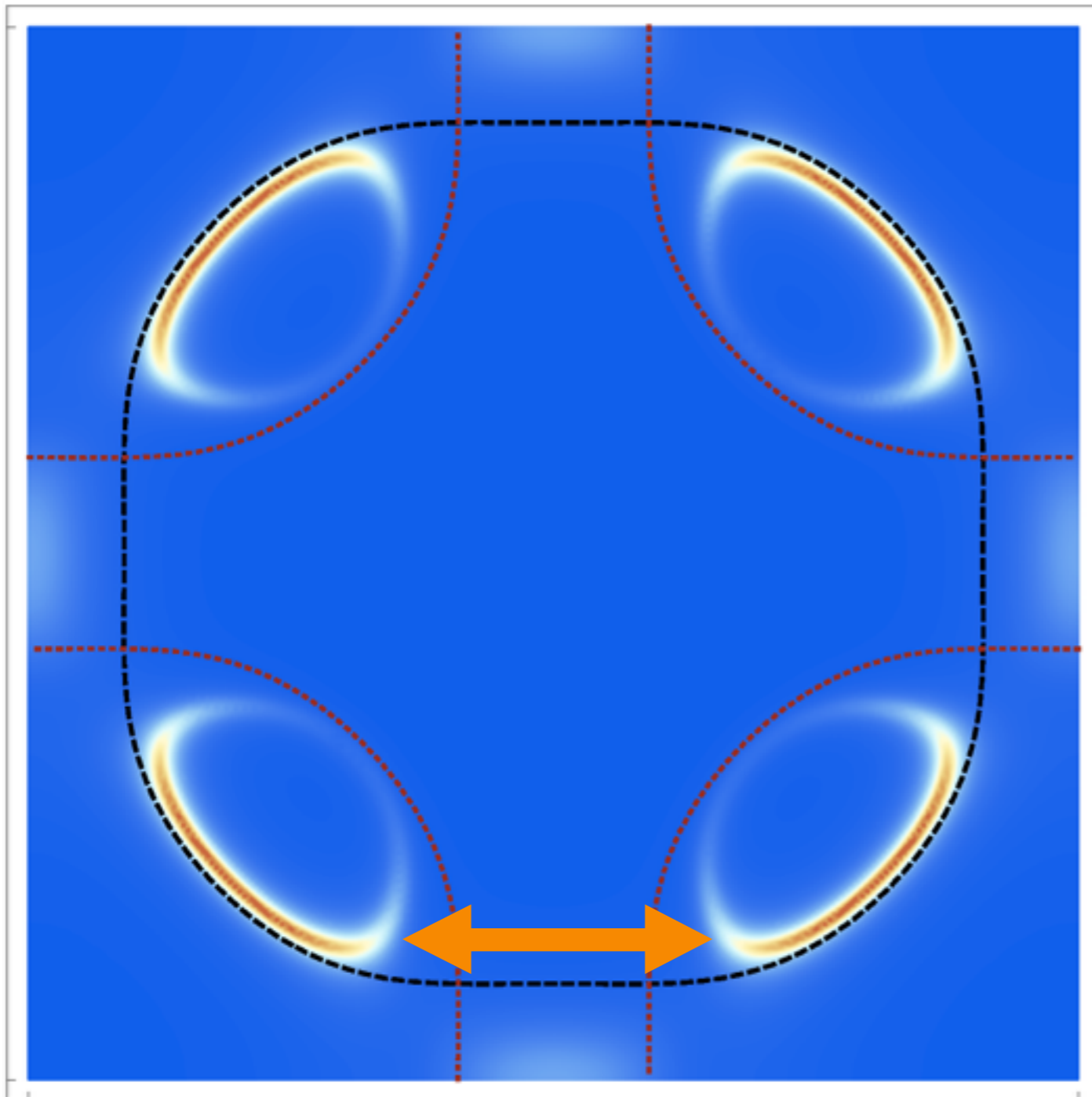


Density wave instability of large Fermi surface leads to an incorrect “diagonal” wavevector

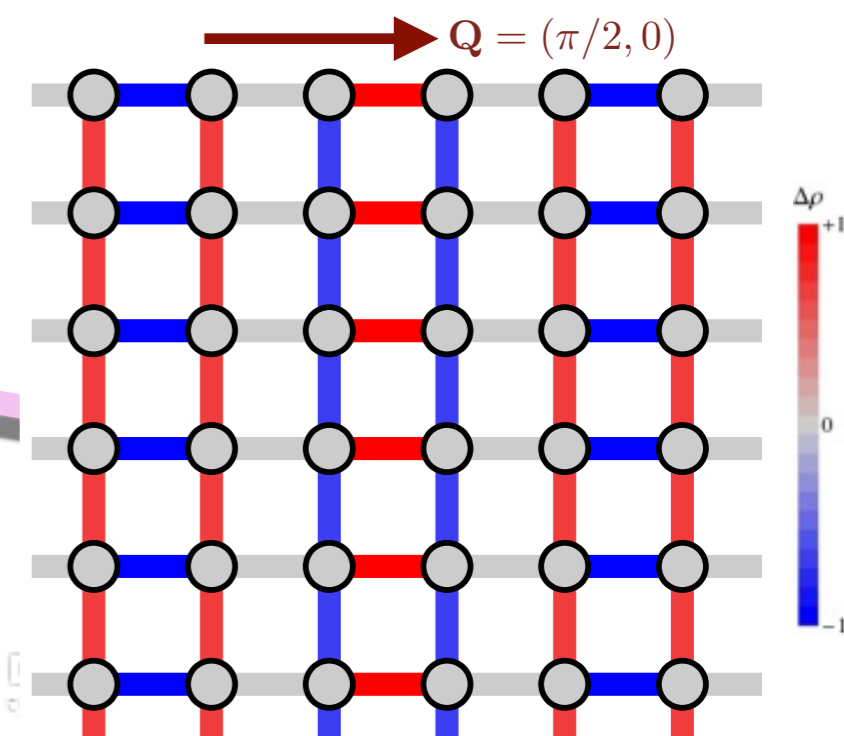
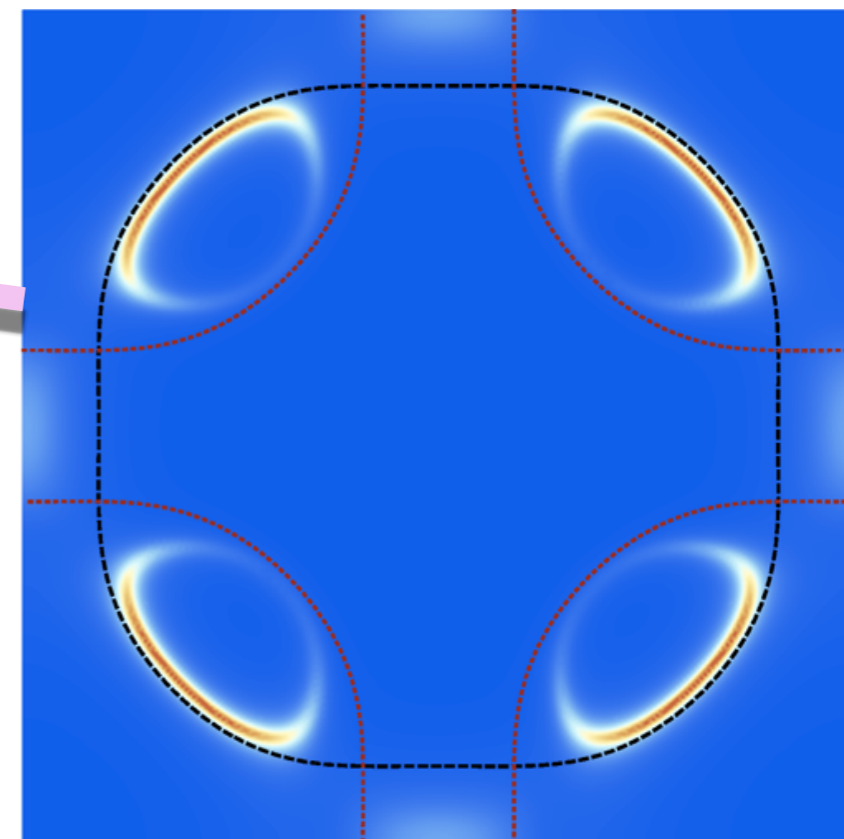
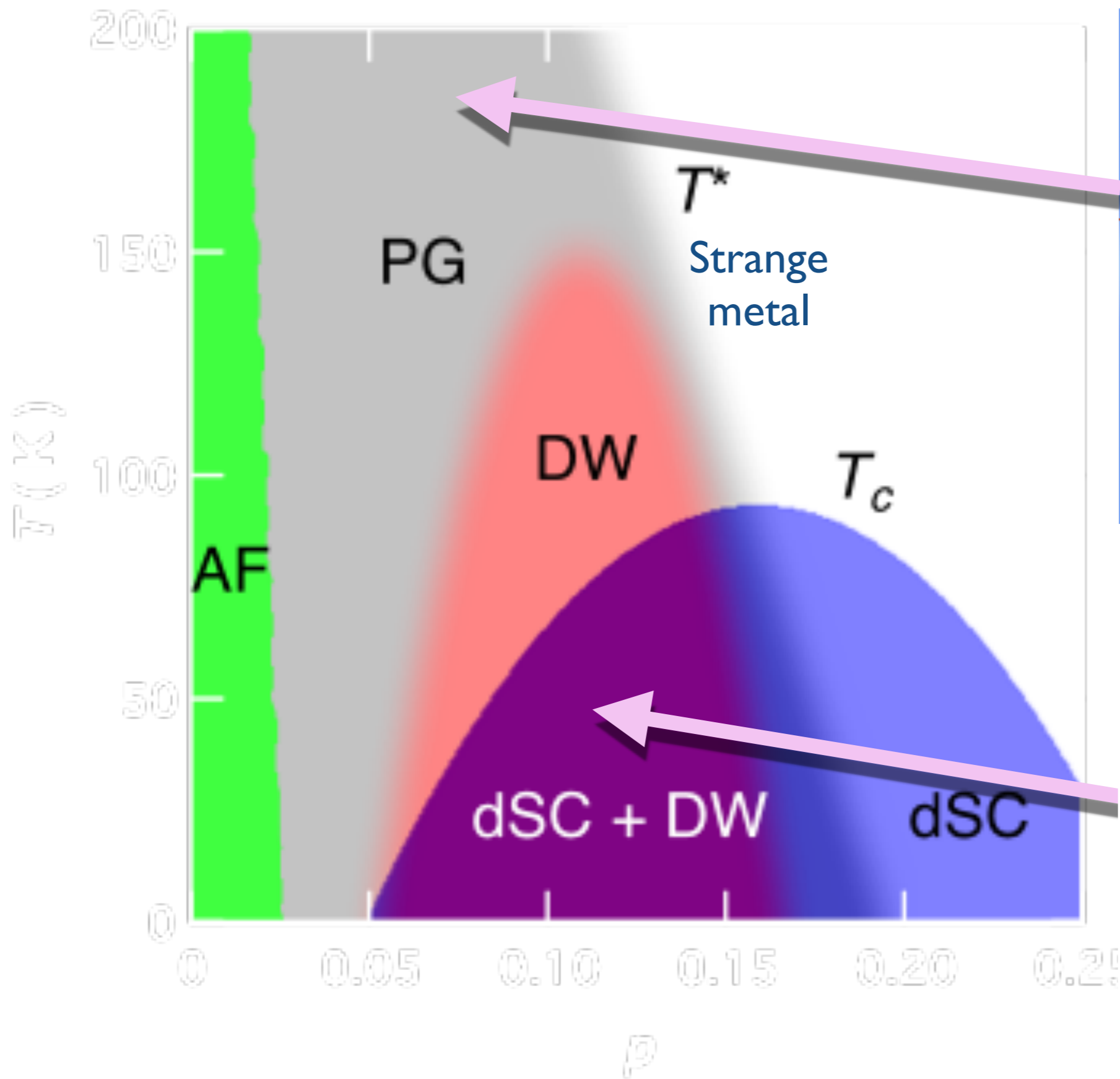
$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \mathcal{P}_d(\cos k_x - \cos k_y)$$

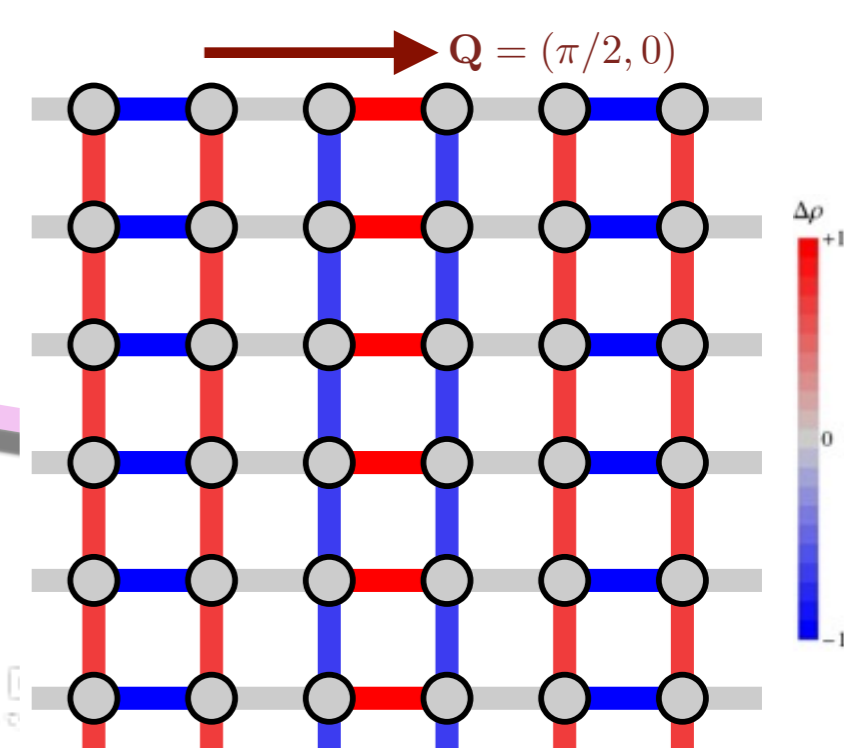
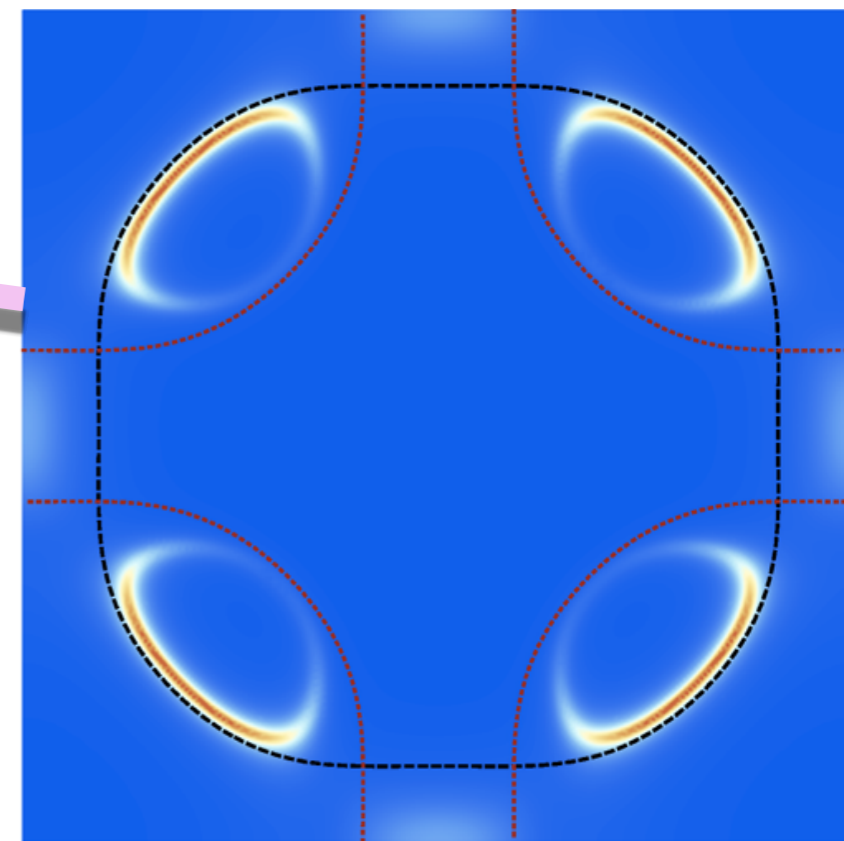
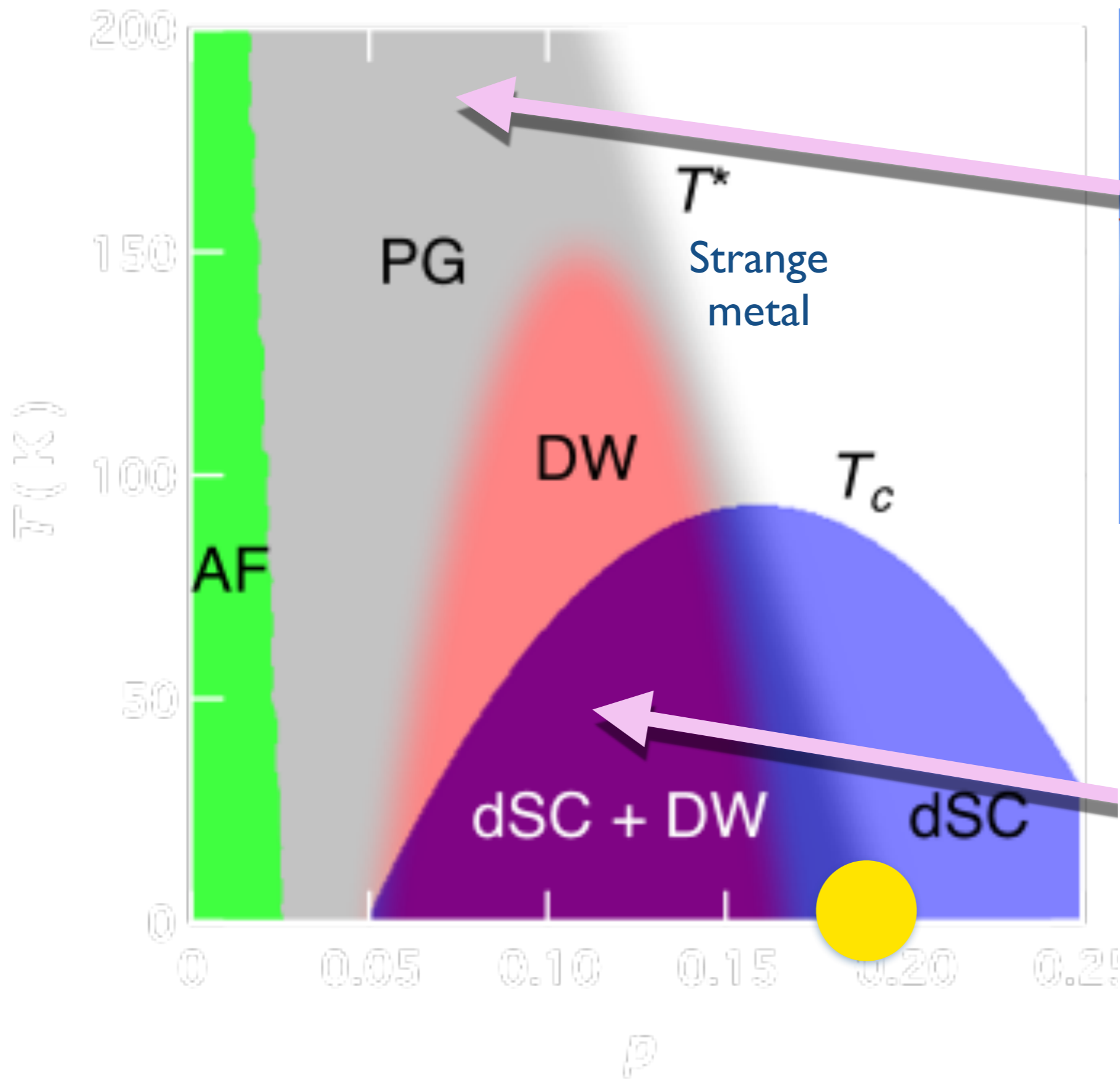


Fermi  
surface  
of FL\*



Density wave  
instability of  
 $FL^*$  leads to the  
observed  
wavevector  
and form-factor





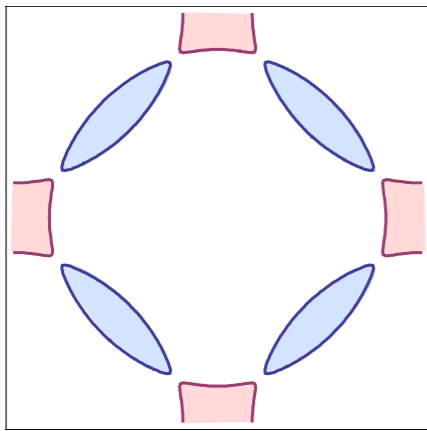
Quantum critical point near optimal  $p$

*Higgs transition in a metal,  
not directly involving any broken symmetry*

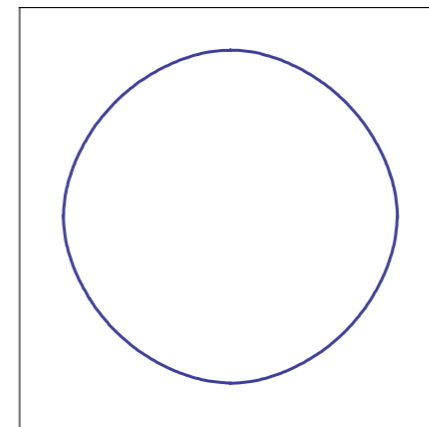
Hertz-Millis  
criticality  
of AFM order

Conventional  
Fermi liquids

(A) AFM order with  
small Fermi pockets



(B) Fermi liquid with  
large Fermi surface



$1/g$



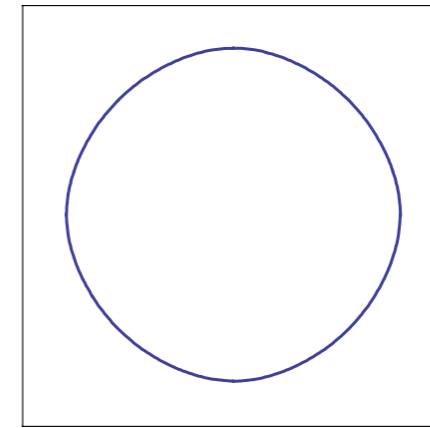
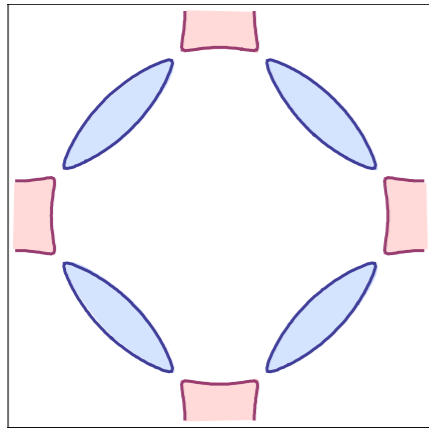
$s$

Hertz-Millis  
criticality  
of AFM order

Conventional  
Fermi liquids

(A) AFM order with  
small Fermi pockets

(B) Fermi liquid with  
large Fermi surface

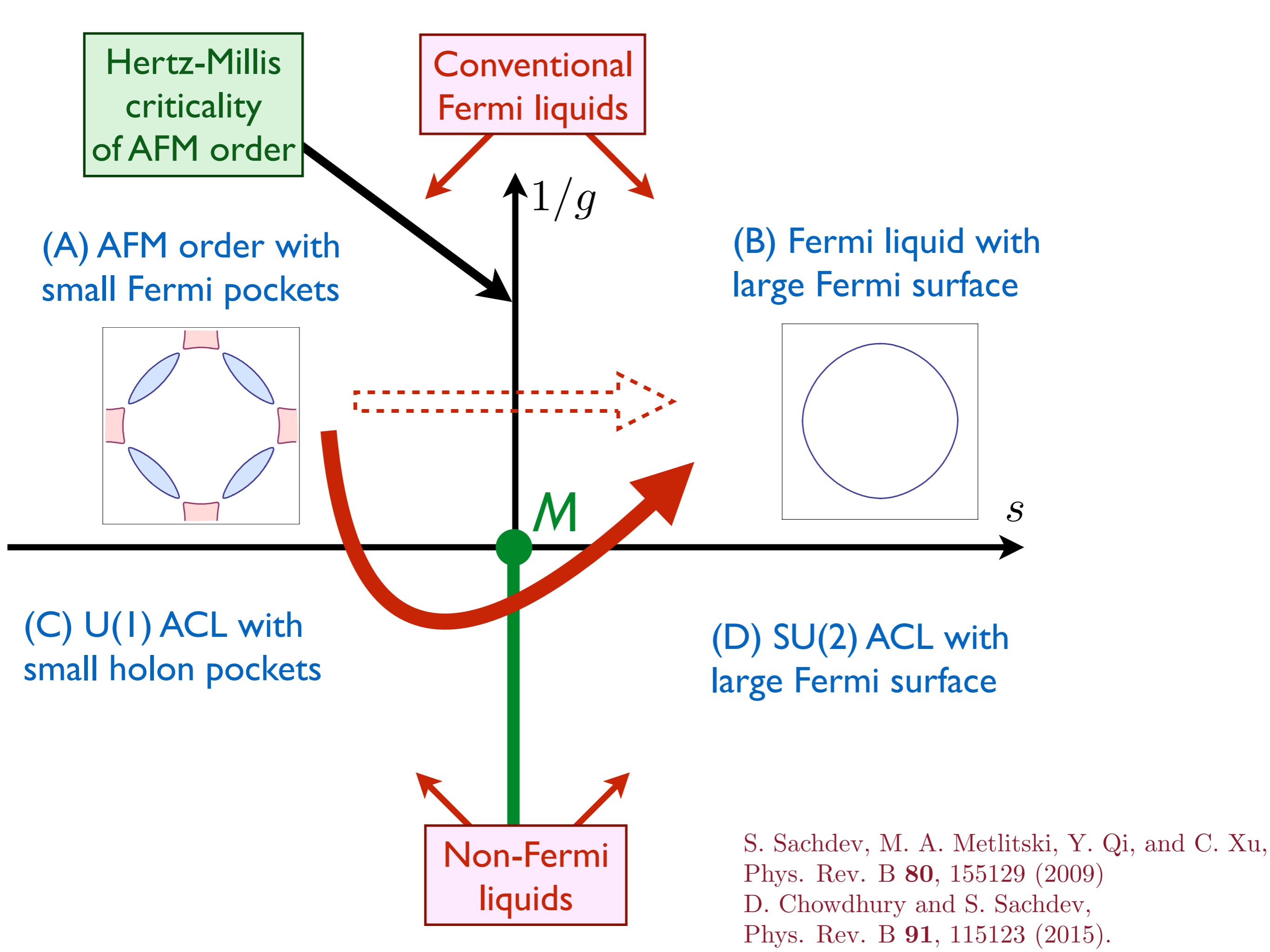


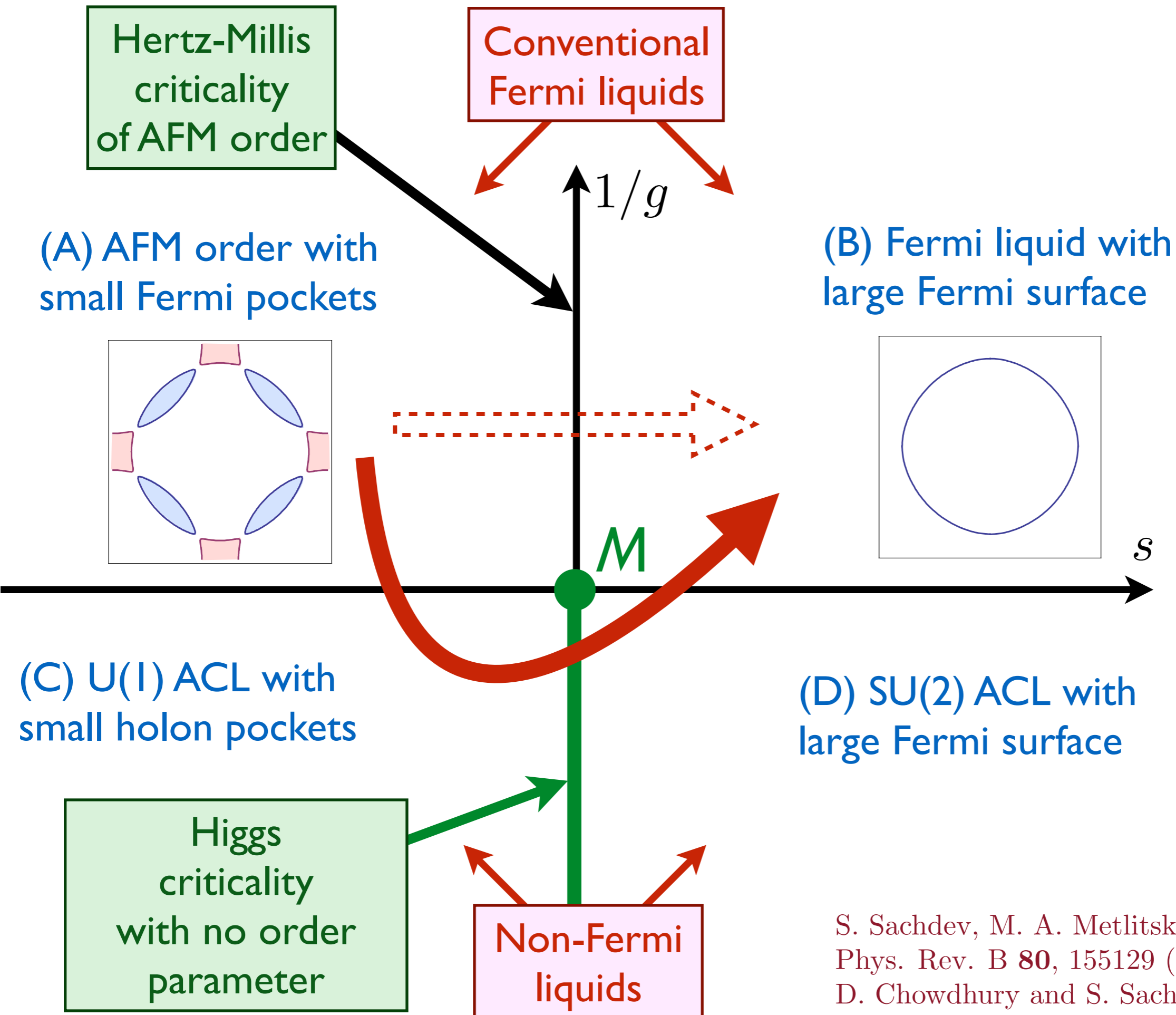
(C) U(1) ACL with  
small holon pockets

(D) SU(2) ACL with  
large Fermi surface

Non-Fermi  
liquids

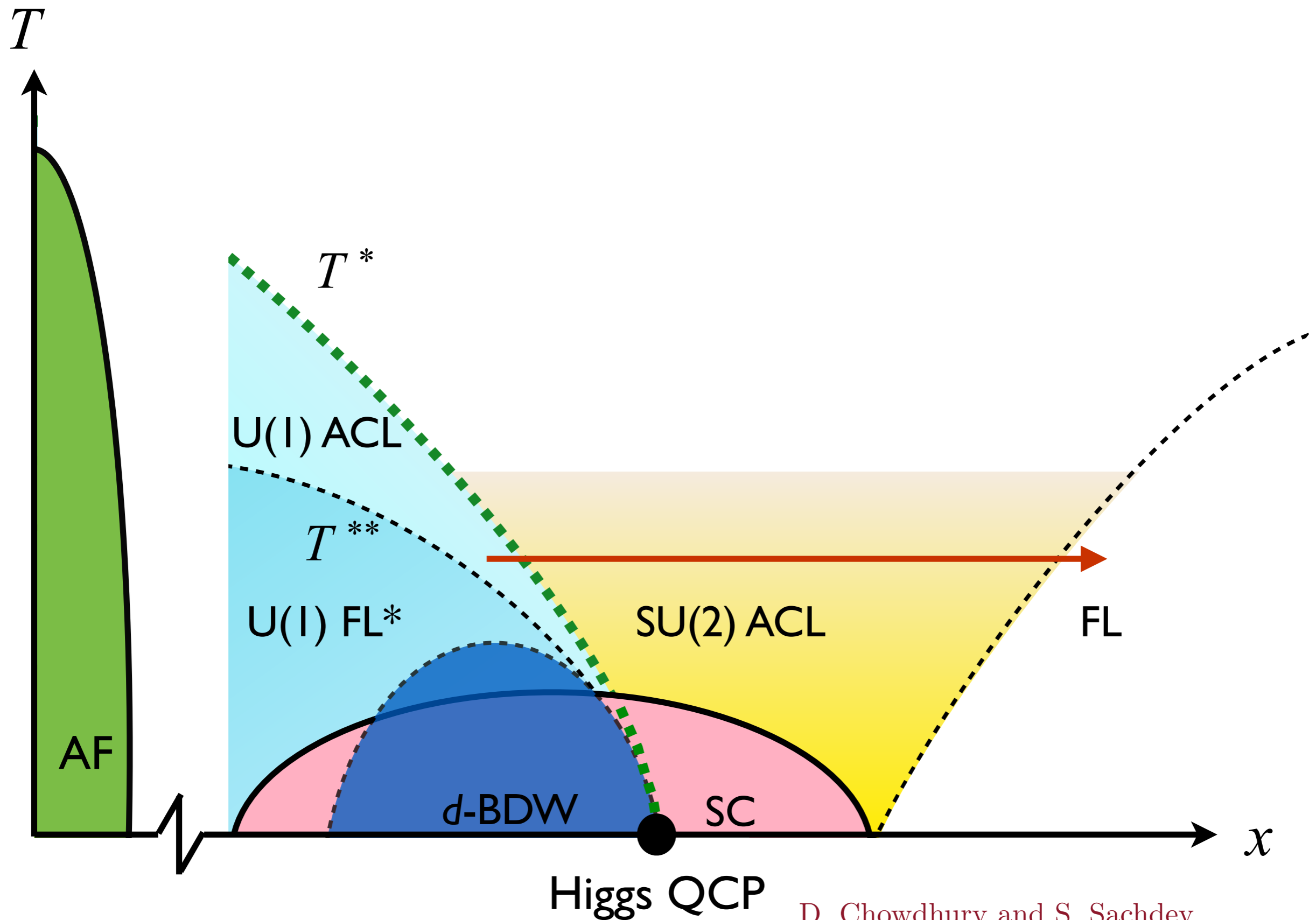
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu,  
Phys. Rev. B **80**, 155129 (2009)  
D. Chowdhury and S. Sachdev,  
Phys. Rev. B **91**, 115123 (2015).





S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu,  
 Phys. Rev. B **80**, 155129 (2009)  
 D. Chowdhury and S. Sachdev,  
 Phys. Rev. B **91**, 115123 (2015).

# SU(2) gauge theory for underlying quantum critical point



# Conclusions

1. Predicted  $d$ -form factor density wave order observed in the non-La hole-doped cuprate superconductors.
2. The “electron becomes a dimer” in the pseudogap metal: proposed a quantum dimer model.
3. Can we experimentally detect possible “topological order” in the pseudogap metal ?  
(topological order is directly linked to Fermi surface size)

